The scattering properties of a system of structures in water waves

THE SCATTERING PROPERTIES OF A SYSTEM OF STRUCTURES IN WATER WAVES

by M. MCIVER

(*Department of Mathematical Sciences, Loughborough University, Loughborough, Leicestershire, LE11 3TU, UK*)

[Received 2014 Revise 2014]

Summary

A fixed, rigid structure is said to be cloaked in small amplitude, time-harmonic water waves if it is possible to surround it with other fixed or moving structures or variations in the sea-bed topography in such a way that there are no scattered waves in the far field. The resulting system of structures is said to be transparent to the incident wave field. By finding inequalities which the kinetic and potential energy in the total wave field for such a system must satisfy, it is shown that transparency is impossible for a system of structures which has non-zero volume and satisfies the condition that the free surface is connected and a vertical line drawn from every point on the free surface does not intersect another structure but intersects the sea bed at a point at which the bed is horizontal.

1. Introduction

In recent years much attention has been given to the possibility of constructing a cloak for a structure, so that when it is irradiated by an electromagnetic wave, the wave is bent around the structure and the structure rendered invisible to an observer in the far field (**1**), (**2**), (**3**). The concept of cloaking has also been explored in acoustics (**4**), surface waves (**5**) and elasticity (**6**), (**7**) and a special issue of Wave Motion is devoted to work which discusses cloaking in a variety of physical situations (**8**). In linear water-wave theory Porter and Newman (**9**), (**10**), (**11**), (**12**) use the term cloaking to describe the elimination of the far-field scattered waves that arise when a monochromatic plane wave is incident upon a fixed, rigid structure and this is the definition that will be used in this work. In particular, they investigated whether is is possible to cloak a vertical circular cylinder in water waves, by either varying the sea-bed topography in the neighbourhood of the cylinder or by surrounding it with other structures. Numerical calculations showed that it is possible to significantly reduce the amplitude of the scattered wave field at certain frequencies by a suitable construction of the surrounding system, but whether perfect cloaking is possible at one or more frequencies, is an open question. In related work, an experimental study by Dupont et al (**13**) investigated the possibility of eliminating the sloshing modes produced by a structure situated at the end of a wave guide by putting vertical posts in front of it.

In order to be able to cloak a structure by other structures in water waves, it is necessary that the total system made up from the structure and its cloak does not produce any scattered waves in the far field when a monochromatic plane wave is incident upon it. Such a system is said to be transparent to the incident wave field. In two dimensions, transparency occurs if the reflection coefficient associated with the system is zero and the transmission coefficient has the same phase as the incident wave. The first condition is shown to be impossible in §2, for a two-dimensional system of structures which satisfies the so-called 'John condition' (defined below) and for which there is no isolated portion of the free surface. Brief details of the argument for a fluid of infinite depth were presented in (**14**) and an expanded version of the proof for a fluid of finite depth is given below. In §3 the proof is extended to show that a system of three-dimensional structures of non-zero volume which satisfies the John condition and for which the free surface is connected, cannot be transparent to the incident wave field. The implications of this result for the cloaking of a single structure are discussed.

2. The reflection coefficient for a two-dimensional system of structures

A small amplitude plane wave with angular frequency ω is incident from the left on a fixed system of rigid, two-dimensional structures in water of finite depth, as illustrated in Fig 1. There are a finite number of piecewise smooth structures in the system and their total cross-sectional area is assumed to be non-zero. One of the structures intersects the mean free surface at $x = \pm a$ and is contained within these lines. The free surface is connected on either side of this structure and any remaining structures, or variations in the depth of the fluid, are contained in the fluid region below the surface-piercing structure. This means that a vertical line, drawn from each point on the mean free surface does not intersect any structure but intersects the sea bed at $z = -h$, where h is a constant. This last condition is referred to as the 'John condition' because John (**15**) established that the solution to the linear scattering problem is unique for such a system. An extension of his method will now be used to show that it is impossible for the reflection coefficient R to be zero when a monochromatic plane wave is incident on such a system of structures. The result will be established by first assuming that $R = 0$ at a particular frequency and then showing that this leads to a contradiction.

The scattering potential is given by $Re[\phi(x,z)\,\mathrm{e}^{-i\omega t}]$ where ϕ has continuous second partial derivatives in the fluid and is continuous onto the boundary. The function ϕ satisfies

$$\nabla^2\phi = 0 \quad \text{in the fluid} \tag{2.1}$$

and the boundary conditions

$$\frac{\partial\phi}{\partial n} = 0 \quad \text{on the structures and sea bed,} \tag{2.2}$$

where $\partial/\partial n$ is the derivative in the outward normal direction to the fluid. The linearised free surface condition is given by

$$K\phi - \frac{\partial\phi}{\partial z} = 0 \quad \text{on } z = 0,\ |x| > a, \tag{2.3}$$

where

$$K = \frac{\omega^2}{g}. \tag{2.4}$$

There is assumed to be no reflection of the incident wave and so

$$\phi \sim \begin{cases} \mathrm{e}^{ikx}\,\frac{\cosh k(z+h)}{\cosh kh}, & x \to -\infty, \\ T\,\mathrm{e}^{ikx}\,\frac{\cosh k(z+h)}{\cosh kh}, & x \to \infty, \end{cases} \tag{2.5}$$

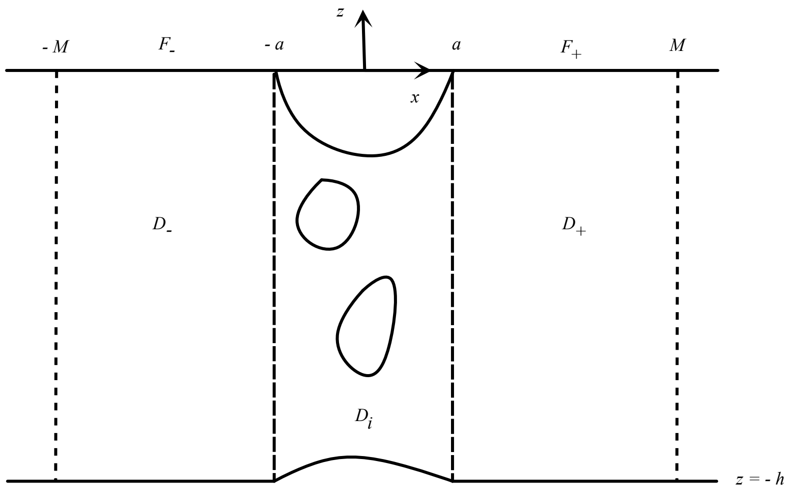

Fig. 1 A two-dimensional system of structures which satisfies the John condition

where energy conservation gives $|T| = 1$ and the wave number k satisfies the dispersion relation

$$K = k \tanh kh. \tag{2.6}$$

The portion of the mean free surface in $-M < x < -a$ is denoted by F_- and that in $a < x < M$ by F_+. The fluid regions below $F_\pm$ are denoted by $D_\pm$. The remaining fluid region surrounding the bodies and contained in the region $-a < x < a$ is denoted by D_i and is assumed to have a non-zero, cross-sectional area. As ϕ satisfies (2.1), integration of $\nabla.[\phi \nabla \overline{\phi}]$ over the total fluid region contained in $-M < x < M$ and an application of the divergence theorem gives

$$\int_{D_- \cup D_+ \cup D_i} |\nabla \phi|^2 \, \mathrm{d}V = \int_{\partial(D_- \cup D_+ \cup D_i)} \phi \frac{\partial \overline{\phi}}{\partial n} \, \mathrm{d}S, \tag{2.7}$$

where the overbar denotes complex conjugate and $\partial(D_- \cup D_+ \cup D_i)$ is the total boundary of the region $D_- \cup D_+ \cup D_i$. The boundary condition on the structures and the sea bed (2.2) means that there is no contribution to the integral on the right-hand side of (2.7) from these surfaces. Furthermore, by energy conservation and the fact that $R = 0$, the contributions from the lines $x = \pm M$ cancel in the limit as $M \to \infty$. Thus (2.7) yields

$$\int_{D_i} |\nabla \phi|^2 \, \mathrm{d}V + \lim_{M \to \infty} \left[\int_{D_- \cup D_+} |\nabla \phi|^2 \, \mathrm{d}V - K \int_{F_- \cup F_+} |\phi|^2 \, \mathrm{d}x \right] = 0, \tag{2.8}$$

where (2.3) has been used to rewrite the integral over $F_- \cup F_+$ and it is convenient to separate out the integral over the finite fluid region D_i from the integral over $D_- \cup D_+$. It is not possible to separate the integrals over $D_- \cup D_+$ from the integrals over $F_- \cup F_+$ before the limit as $M \to \infty$ is taken, because ϕ doesn't decay as $x \to \pm\infty$ and so the limits of these individual integrals in (2.8) do not exist.

If ϕ were equal to a constant in the non-zero region D_i, by analytic continuation ϕ would have to equal a constant everywhere in the fluid and so would not satisfy the far-field condition (2.5). Thus ϕ is not equal to a constant in D_i, and so the integral of $|\nabla\phi|^2$ over this region is strictly positive and (2.8) gives

$$\lim_{M\to\infty}\left[\int_{D_-\cup D_+} |\nabla\phi|^2\,\mathrm{d}V - K\int_{F_-\cup F_+} |\phi|^2\,\mathrm{d}x\right] < 0. \tag{2.9}$$

Green's theorem is applied to ϕ and $\mathrm{e}^{ik(x-b)}\cosh k(z+h)/\cosh kh$ in the region $x > b \geq a$. Both functions represent outward-going waves as $x \to \infty$ and so the only contribution comes from the line $x = b$ and gives

$$\int_{-h}^{0}\left[\frac{\partial\phi}{\partial x} - ik\phi\right]_{x=b}\frac{\cosh k(z+h)}{\cosh kh}\,\mathrm{d}z = 0. \tag{2.10}$$

The second term in (2.10) is integrated by parts to give

$$\phi(b,0) = \frac{1}{\sinh kh}\int_{-h}^{0}\left[\frac{\partial\phi}{\partial z}\sinh k(z+h) - i\frac{\partial\phi}{\partial x}\cosh k(z+h)\right]_{x=b}\mathrm{d}z. \tag{2.11}$$

It is convenient to define

$$\sin\alpha = \frac{\sinh k(z+h)}{\left[\sinh^2 k(z+h) + \cosh^2 k(z+h)\right]^{1/2}} = \frac{\sinh k(z+h)}{\left[\cosh 2k(z+h)\right]^{1/2}}, \tag{2.12}$$

where α is real but not constant, then

$$\cos\alpha = \frac{\cosh k(z+h)}{\left[\sinh^2 k(z+h) + \cosh^2 k(z+h)\right]^{1/2}} = \frac{\cosh k(z+h)}{\left[\cosh 2k(z+h)\right]^{1/2}} \tag{2.13}$$

and (2.11) becomes

$$\begin{aligned}\phi(b,0) &= \frac{1}{\sinh kh}\int_{-h}^{0}\left[\frac{\partial\phi}{\partial z}\sin\alpha - i\frac{\partial\phi}{\partial x}\cos\alpha\right]_{x=b}\left[\cosh 2k(z+h)\right]^{1/2}\,\mathrm{d}z \\ &= -\frac{i}{2\sinh kh}\int_{-h}^{0}\left[\mathrm{e}^{i\alpha}\left(\frac{\partial\phi}{\partial z} + \frac{\partial\phi}{\partial x}\right) + \mathrm{e}^{-i\alpha}\left(\frac{\partial\phi}{\partial x} - \frac{\partial\phi}{\partial z}\right)\right]_{x=b}\left[\cosh 2k(z+h)\right]^{1/2}\,\mathrm{d}z.\end{aligned} \tag{2.14}$$

For general complex A and B, $|A+B|^2 = 2(|A|^2+|B|^2) - |A-B|^2 \leq 2(|A|^2+|B|^2)$, so an application of the Cauchy-Schwarz inequality to (2.14) gives

$$\begin{aligned}|\phi(b,0)|^2 &\leq \frac{1}{4\sinh^2 kh}\int_{-h}^{0}\left|-\mathrm{e}^{i\alpha}\left(\frac{\partial\phi}{\partial z} + \frac{\partial\phi}{\partial x}\right) + \mathrm{e}^{-i\alpha}\left(\frac{\partial\phi}{\partial z} - \frac{\partial\phi}{\partial x}\right)\right|_{x=b}^2\mathrm{d}z\int_{-h}^{0}\cosh 2k(z+h)\,\mathrm{d}z \\ &\leq \frac{1}{4k\tanh kh}\int_{-h}^{0}2\left(\left|\frac{\partial\phi}{\partial z} + \frac{\partial\phi}{\partial x}\right|^2 + \left|\frac{\partial\phi}{\partial z} - \frac{\partial\phi}{\partial x}\right|^2\right)_{x=b}\mathrm{d}z = \frac{1}{K}\int_{-h}^{0}|\nabla\phi|^2_{x=b}\,\mathrm{d}z.\end{aligned} \tag{2.15}$$

Integration of (2.15) from $b = a$ to $b = M$ gives, after some rearrangement,

$$\int_{D_+} |\nabla\phi|^2 \, \mathrm{d}V - K \int_{F_+} |\phi|^2 \mathrm{d}x \geq 0. \tag{2.16}$$

As R is assumed to be equal to zero, an application of Green's theorem to ϕ and $\mathrm{e}^{ik(x+b)} \cosh k(z+h)/\cosh kh$ in the region $x < -b \leq -a$, leads to the similar result

$$\int_{D_-} |\nabla\phi|^2 \, \mathrm{d}V - K \int_{F_-} |\phi|^2 \mathrm{d}x \geq 0. \tag{2.17}$$

Inequalities (2.16) and (2.17) are valid for arbitrary M, so they may be added together and the limit as $M \to \infty$ taken, to give

$$\lim_{M\to\infty} \left[\int_{D_- \cup D_+} |\nabla\phi|^2 \, \mathrm{d}V - K \int_{F_- \cup F_+} |\phi|^2 \mathrm{d}x \right] \geq 0. \tag{2.18}$$

This last inequality is incompatible with the one in (2.9) and so the original assumption that the reflection coefficient is zero, is impossible.

3. A system of three-dimensional structures

In this section, the scattering properties of a three-dimensional system of piecewise smooth structures which satisfies the John condition and for which the free surface is connected, are investigated. An illustration of such a system where the water depth is constant everywhere, is given in Fig 2. It will be shown that when a monochromatic plane wave is incident on a system which satisfies these conditions, scattered waves are produced in the far field unless the system consists of vertical plates which are aligned with the wave direction. The result is established in a similar way to that for the two-dimensional case, by first making the assumption that there are no scattered waves in the far field and then showing that this leads to a contradiction.

Rectangular Cartesian coordinates (x, y, z) are defined so that the origin is at the level of the mean free surface F, the z-axis points vertically upwards and the incident wave propagates in the positive x direction. The parameter C is chosen so that the structures are contained within the region $r \leq C$, where cylindrical polar coordinates (r, θ, z) are defined by

$$x = r\cos\theta, \quad y = r\sin\theta. \tag{3.1}$$

The combined region of fluid directly below all the structures is denoted by D_i and the remaining fluid region in $r \leq C$ is denoted by D_o. The fluid depth in D_o is assumed to be constant and equal to h but the topography of the seabed may vary below the structures in the region D_i. Unless the structures extend throughout the depth or are thin plates, D_i is a non-zero region. The scattering potential $Re[\phi(x, y, z)\,\mathrm{e}^{-i\omega t}]$ is harmonic in the fluid, continuous onto the boundary and satisfies the boundary conditions given in (2.2) and the linearised free surface condition (2.3) on F. As the incident wave propagates in the positive x direction, the radiation condition gives

$$\phi = \left[\mathrm{e}^{ikx} + \left(\frac{2}{\pi kr}\right)^{1/2} A(\theta)\,\mathrm{e}^{ikr - i\pi/4} + O\left(\frac{1}{r^{3/2}}\right) \right] \frac{\cosh k(z+h)}{\cosh kh}, \quad \text{as} \quad r \to \infty, \tag{3.2}$$

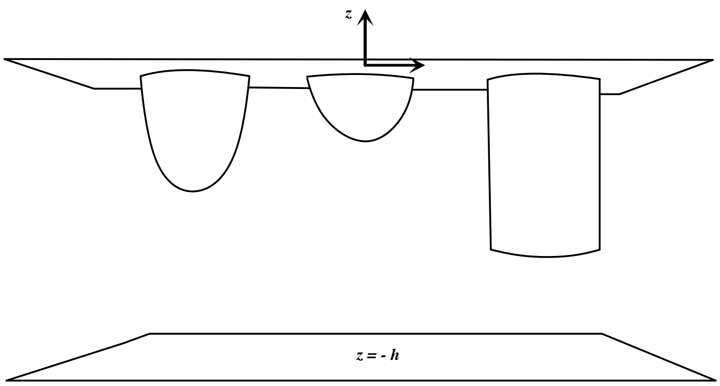

Fig. 2 A system of three-dimensional structures which satisfies the John condition

where $A(\theta)$ measures the amplitude of the scattered waves in the far field. For the system of structures to be transparent to the incident wave $A(\theta) = 0$ for all θ and (3.2) becomes

$$\phi = \left[\mathrm{e}^{ikx} + O\left(\frac{1}{r^{3/2}}\right)\right] \frac{\cosh k(z+h)}{\cosh kh}, \quad \text{as} \quad r \to \infty. \tag{3.3}$$

If each of the structures extends throughout the depth with a uniform horizontal cross-section and the fluid depth h is constant everywhere, then the $z-$dependence may be subtracted out from the potential and ϕ written as

$$\phi = u(x,y)\frac{\cosh k(z+h)}{\cosh kh}, \tag{3.4}$$

where $u(x,y)$ satisfies the two-dimensional Helmholtz equation on F,

$$\nabla^2 u + k^2 u = 0. \tag{3.5}$$

From (3.3) u may be written as

$$u = \mathrm{e}^{ikx} + u_1(x,y), \tag{3.6}$$

where

$$u_1 = O\left(\frac{1}{r^{3/2}}\right), \quad \text{as} \quad r \to \infty. \tag{3.7}$$

In general u_1 may be expanded in terms of Bessel functions in any annulus in $r > C$ but in order to satisfy (3.7) all the coefficients in the expansion must be zero. So $u_1 = 0$ in any annulus in $r > C$ and by analytic continuation $u_1 = 0$ everywhere on F. Thus the only possibility is that the diffraction potential is given by the incident wave everywhere in the fluid, ie

$$\phi = \mathrm{e}^{ikx}\,\frac{\cosh k(z+h)}{\cosh kh}. \tag{3.8}$$

This potential only satisfies the boundary condition (2.2) on each of the structures if they are all vertical plates, aligned with the direction of propagation of the incident wave. So all other structures, which extend throughout the depth with a uniform horizontal cross-section, generate a non-zero scattered wave field.

John's uniqueness theorem (**15**) is now extended to show that transparency cannot occur if the combined fluid region below all the structures is non-zero and the free surface is connected. As in the two-dimensional case, an application of the divergence theorem gives

$$\int_{D_i \cup D_o} |\nabla\phi|^2 \,\mathrm{d}V = \int_{\partial(D_i \cup D_o)} \phi \frac{\partial\overline{\phi}}{\partial n}\,\mathrm{d}S, \tag{3.9}$$

where $\partial(D_i \cup D_o)$ is the boundary of the region. As ϕ satisfies (2.2), there is no contribution to the integral on the right-hand side of (3.9) from the structure surfaces or the seabed. If there are no scattered waves in the far field, the radiation condition holds in the form (3.3) and the integral over the surface $r = C$ vanishes in the limit as $C \to \infty$. So the only contribution to the boundary integral in (3.9) comes from the mean free surface F. As in the previous section, it is convenient to separate out the integral over the region D_i on the left-hand side of (3.9) and then take the limit as $C \to \infty$ to give

$$\int_{D_i} |\nabla\phi|^2 \,\mathrm{d}V + \lim_{C\to\infty}\left[\int_{D_o} |\nabla\phi|^2 \,\mathrm{d}V - K\int_F |\phi|^2 \,\mathrm{d}S\right] = 0, \tag{3.10}$$

where (2.3) has been used to rewrite the integral over F. In order that the analytic continuation of ϕ should satisfy the far-field condition (3.3), ϕ cannot be equal to a constant in any non-zero region between a structure and the seabed. So

$$\int_{D_i} |\nabla\phi|^2 \,\mathrm{d}V > 0 \tag{3.11}$$

and (3.10) becomes

$$\lim_{C\to\infty}\left[\int_{D_o} |\nabla\phi|^2 \,\mathrm{d}V - K\int_F |\phi|^2 \,\mathrm{d}S\right] < 0. \tag{3.12}$$

As the system of structures satisfies the John condition, it is convenient to follow (**16**) and define

$$w(x,y) = \int_{-h}^{0} \phi(x,y,z)\frac{\cosh k(z+h)}{\cosh kh}\,\mathrm{d}z, \tag{3.13}$$

at all points (x, y) on F. The two-dimensional Laplacian of w is taken and after some manipulation with the use of the governing equation and boundary conditions for ϕ, it may be shown that w satisfies the two-dimensional Helmholtz equation (3.5). From (3.3) and (3.13)

$$w = \int_{z=-h}^{0} \left[\mathrm{e}^{ikx} + O\left(\frac{1}{r^{3/2}}\right)\right] \frac{\cosh^2 k(z+h)}{\cosh^2 kh}\,\mathrm{d}z = N\,\mathrm{e}^{ikx} + O\left(\frac{1}{r^{3/2}}\right), \quad \text{as} \quad r \to \infty, \tag{3.14}$$

where integration gives

$$N = \frac{[kh + \sinh kh \cosh kh]}{2k\cosh^2 kh}. \tag{3.15}$$

The same argument to that used previously for a system of structures which extends throughout the depth shows that $w = N\,\mathrm{e}^{ikx}$ in every annulus for which $r > C$. As the free surface is connected, analytic continuation of w to the whole of F means that

$$w = \int_{-h}^{0} \phi(x,y,z)\frac{\cosh k(z+h)}{\cosh kh}\,\mathrm{d}z = N\,\mathrm{e}^{ikx} \tag{3.16}$$

everywhere on F. Integration of (3.16) by parts gives

$$N\,\mathrm{e}^{ikx} = \phi(x,y,0)\frac{\sinh kh}{k\cosh kh} - \int_{-h}^{0} \frac{\partial\phi}{\partial z}\frac{\sinh k(z+h)}{k\cosh kh}\,\mathrm{d}z, \tag{3.17}$$

whereas differentiation of (3.16) with respect to x shows that

$$ikN\,\mathrm{e}^{ikx} = \int_{-h}^{0} \frac{\partial\phi}{\partial x}\frac{\cosh k(z+h)}{\cosh kh}\,\mathrm{d}z. \tag{3.18}$$

The combination ik(3.17)-(3.18) is formed and then rearranged to give

$$\phi(x,y,0) = \frac{1}{\sinh kh}\int_{-h}^{0}\left[\frac{\partial\phi}{\partial z}\sinh k(z+h) - i\frac{\partial\phi}{\partial x}\cosh k(z+h)\right]\mathrm{d}z, \tag{3.19}$$

at all points on F. This is the same representation for ϕ at a point on the free surface that was obtained in the two-dimensional problem in (2.11). So a similar argument to that used to obtain (2.15) gives

$$|\phi(x,y,0)|^2 \le \frac{1}{K}\int_{-h}^{0} |\nabla\phi|^2\,\mathrm{d}z. \tag{3.20}$$

Both sides of (3.20) may be integrated over F, the resulting equation rearranged and the limit as $C \to \infty$ taken to give

$$\lim_{C\to\infty}\left[\int_{D_o} |\nabla\phi|^2\,\mathrm{d}V - K\int_F |\phi|^2\,\mathrm{d}S\right] \ge 0. \tag{3.21}$$

A comparison of (3.21) and (3.12) shows that they are incompatible. So the original assumption that there are no scattered waves in the far field is impossible, and a system of structures of non-zero volume which satisfies the John condition and for which the free surface is connected, cannot be transparent to an incident wave.

4. Discussion

The results of the previous section show that it is impossible to cloak a single structure with non-zero volume which satisfies the John condition, by surrounding it with other structures which also satisfy the John condition, in such a way that the free surface remains connected. However, this does not rule out the possibility of forming a perfect cloak for such a structure with a torus and Newman (**12**) showed that by optimizing the cross-section of the torus, significant reductions in the scattered wave amplitude from a vertical circular cylinder may be obtained numerically at certain frequencies. In addition, the argument presented in this work only applies to the situation in which any variations in the sea bed are directly below a structure. This is consistent with the results in (**9**), (**10**) and (**11**), where near-cloaking of a vertical circular cylinder is found to be possible numerically at certain frequencies, when the bathymetry in the neighbourhood of the cylinder has a particular form. However, the question of whether any structures exist which may be cloaked perfectly at one or more frequencies, remains open.

Further work will use the extensions of John's uniqueness proof described in (**16**), to investigate whether transparency over certain frequency ranges may be ruled out for systems of structures in two- and three- dimensions, which satisfy the John condition but for which the free surface is not connected.

References

1. U. Leonardt. Optical conformal mapping. *Science*, 312:1777–1780, 2006.

2. J. B. Pendry, D. Schurig, and D. R. Smith. Controlling electromagnetic fields. *Science*, 312:1780–1782, 2006.

3. D. Schurig, J. J. Mock, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith. Metamaterial electromagnetic cloak at microwave frequencies. *Science*, 314:977–980, 2006.

4. A. N. Norris. Acoustic cloaking theory. *Proc. Roy. Soc. A*, 464:2411–2434, 2008.

5. M. Farhat, S. Enoch, S. Guenneau, and A. B. Movchan. Broadband cylindrical acoustic cloak for linear surface waves in a fluid. *Phys. Rev. Letters*, 101:134501, 2008.

6. G. W. Milton, M. Briane, and J. R. Willis. On cloaking for elasticity and physical equations with a transformation invariant form. *New. J. Phys*, 8:248–267, 2006.

7. M. Farhat, S. Guenneau, S Enoch, and A. B. Movchan. Cloaking bending waves propagating in thin elastic plates. *Physical Review B*, 79:033102, 2009.

8. A. N. Norris. (ed): Special issue on cloaking of wave motion. *Wave Motion*, 48(6), 2011.

9. R. Porter. Cloaking of a cylinder in waves. *Proceedings 26th International Workshop on Water Waves and Floating Bodies, Athens*, 2011.

10. J. N. Newman. Scattering by a cylinder with variable bathymetry. *Proceedings 27th International Workshop on Water Waves and Floating Bodies, Copenhagen*, 2012.

11. R. Porter and J. N. Newman. Cloaking of a vertical cylinder in waves using variable bathymetry. *J. Fluid Mech.*, 750:124–143, 2014.

12. J. N. Newman. Cloaking a circular cylinder in water waves. *Eur. J. Mech. B/Fluids*, 47:145–150, 2014.

13. G. Dupont, O. Kimmoun, B. Molin, S. Guenneau, and S. Enoch. An experimental study of near-cloaking. *Proceedings 28th International Workshop on Water Waves and Floating Bodies, Marseille*, 2013.

14. D.V. Evans, M. McIver, and R. Porter. Transparency of structures in water waves. *Proceedings 29th International Workshop on Water Waves and Floating Bodies, Osaka*, 2014.

15. F. John. On the motion of floating bodies, II. Simple harmonic motions. *Communications Pure Applied Mathematics*, 3:45–101, 1950.

16. N. Kuznetsov, V. Maz'ja, and B. Vainberg. *Linear Water Waves - A Mathematical Approach*. Cambridge University Press, The Edinburgh Building, Cambridge, CB2 2RU, UK, 2002.