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# Quantum measurement and the quantum to classical transition in a non-linear quantum oscillator

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**Abstract.** We study a non-linear quantum mechanical oscillator, acting as a measurement device. Candidate systems for realising such apparatus range from superconducting devices through to nano-mechanical resonators. The measurement device comprises an oscillator circuit where the dynamics of expectation values, in its correspondence limit, are either chaotic-like or periodic depending on the measured state of the quantum object – in this case a qubit. In a previous work we showed how the classical like trajectories of such a quantum system can act as a model of a projective measurement process. Here we investigate the quantum to classical transition of the measurement device and postulate criteria for realisation of an effective implementation of such a device.

## 1. Introduction

The measurement problem has existed in quantum mechanics since Born's paper in 1926 [1] until the present day (see, for example, [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]). Even now discussions, that can become quite lively, frequently occur on the metaphysical aspects of measurement. With the emergence of quantum technologies and specific applications such as feedback and control (see, for example, [16]) a more pragmatic approach is perhaps timely. As a detailed exploration of what might be achieved through modelling correspondence limit measurement devices simply in terms of Schrödinger evolution has yet to be undertaken, many of these metaphysical discussions could be considered somewhat pre-emptive. In other words, it is not yet clear if the process of measurement could be fully represented on a subset of the current axioms of quantum mechanics - i.e. without postulates relating directly to measurement such as the projection hypothesis.

Within the foundations theme of the DICE conference series, it is worth reflecting on the view that any attempt to identify a canonical set of axioms for quantum theory from a top down approach is confused by the very many different ways quantum mechanics can be motivated - from Schrödinger wave functions to Feynman path integrals. The method followed in this paper, as in [17], could be considered as tentative first steps taking the opposite, bottom up approach.

In other words, we seek to understand if it is possible to reproduce the effects associated with a subset of the standard axioms of quantum mechanics (in this case those relating to measurement) from other axioms. If this turns out to be possible, then we may reclassify once considered axioms as corollaries and better understand their origin and applicability in terms of a reduced and therefore simplified framework. In this way it may be possible to reduce the number of axioms in quantum theory and therefore remove some restrictions on those, such as the quantum gravity community, seeking to extend the theory.

In this work we illustrate how a classical record of a quantum measurement can arise and consider a mechanism for the realisation of effective projective measurements simply from such a perspective. The system dynamics are modified to allow for, and indeed operationally rely on, the effects of environmental degrees of freedom. Specifically, our model comprises an open quantum mechanical measurement device, which is arranged to operate in its classical-correspondence limit, coupled to a quantum system – in this case a qubit – whose state it measures through the natural emergence of a classical dynamical record.

In [17] we demonstrated that our proposed system could be considered to fully quantum mechanically model a projective measurement process. We studied both the ensemble and individual trajectories of the system using the master equation and quantum state diffusion [18] respectively. We showed that the oscillator, operating in its correspondence limit, develops dynamics in terms of its expectation values that are either chaotic-like or periodic depending on the measured value and projected state of a qubit. In our model no preferred basis is assumed to exist *a priori* – this simply emerges from the form of the coupling mechanism. Furthermore, our results were consistent with the Born rule and the Zeno effect. However, we did not demonstrate that our model necessarily requires the oscillators classical limit - an omission that we address in this paper.

## 2. The Model

In our model the (quantum) measurement device comprises a quantum oscillator that has a well defined correspondence limit and whose classical like dynamics is manifestly different depending on the quantum state of the other component of the system – in this case a qubit. The Hamiltonian takes the form [17]:

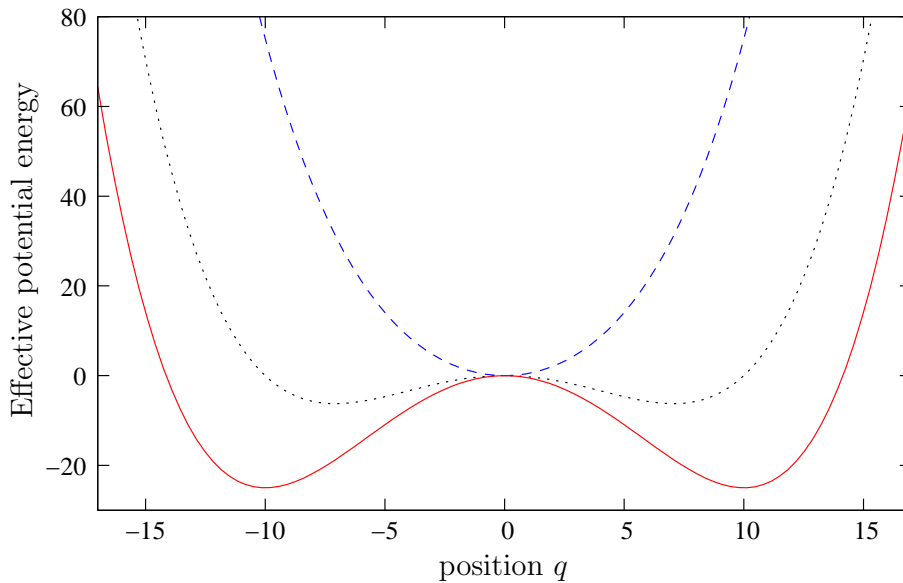
$$H = \frac{3}{4}p^2 + \frac{\beta^2}{4}q^4 - \frac{1}{4}q^2 + \frac{g}{\beta}\cos(t)q - H_{\text{int}} . \quad (1)$$

Here the interaction term

$$H_{\text{int}} = \frac{1}{4}(p^2 + q^2)\sigma_z = \frac{1}{2}\left(a^\dagger a + \frac{1}{2}\right)\sigma_z \quad (2)$$

represents a dispersive Jaymes Cummings coupling to a degenerate qubit. Here  $q$  and  $p$  are dimensionless position and conjugate momentum for the oscillator mode (i.e. defined such that the annihilation operator  $a = (q + ip)/\sqrt{2}$ ) and  $\sigma_z$  is the usual Pauli operator for the qubit. The dimensionless parameter  $\beta$  represents a scaling of the classical action of the oscillator with respect to a Planck cell where, subject to appropriate decoherence effects,  $\beta = 1.0$  is the quantum limit and the classical limit is achieved as  $\beta$  becomes vanishingly small (see [18, 19] for a comprehensive explanation of this quantum to classical transition). Finally, we note  $g = 0.3$  is the strength of the applied driving term. We further note that the Hamiltonian is normalised by a characteristic oscillator energy  $\hbar\omega$ . Furthermore, time  $t$  is similarly rendered dimensionless with respect to the same characteristic frequency  $\omega$ .

In order to gain an intuitive idea of the *modus operandi* of our model measurement device we present in Fig. 1 the (non-linear) potential of Eq. (1) as a dotted line. Invoking a hand-waving



**Figure 1.** (colour online) Effective potential for the Hamiltonian of Eq. (1) where  $\beta = 0.1$  and (a)  $\langle \sigma_z \rangle = 1$  solid (red) line (b)  $\langle \sigma_z \rangle = -1$  dashed (blue) line and (c)  $\langle \sigma_z \rangle = 0$  dotted line (Figure and caption reproduced from [17]).

Born-Oppenheimer type approximation one may associate with the dashed and solid lines of Fig. 1 an effective oscillator potential associated with the ground state  $|g\rangle$  ( $\langle \sigma_z \rangle = -1$ ) or the excited state  $|e\rangle$  ( $\langle \sigma_z \rangle = 1$ ) respectively. That is, with the ground state of the qubit we associate a quartic potential in the oscillator and with the excited state we associate the well known potential of the Duffing or anharmonic oscillator. Here we have chosen  $\beta = 0.1$  (see Eq. (1)) which, for the double well potential, has been shown to produce dynamics in sufficiently good agreement the system's classical counterpart [18, 19].

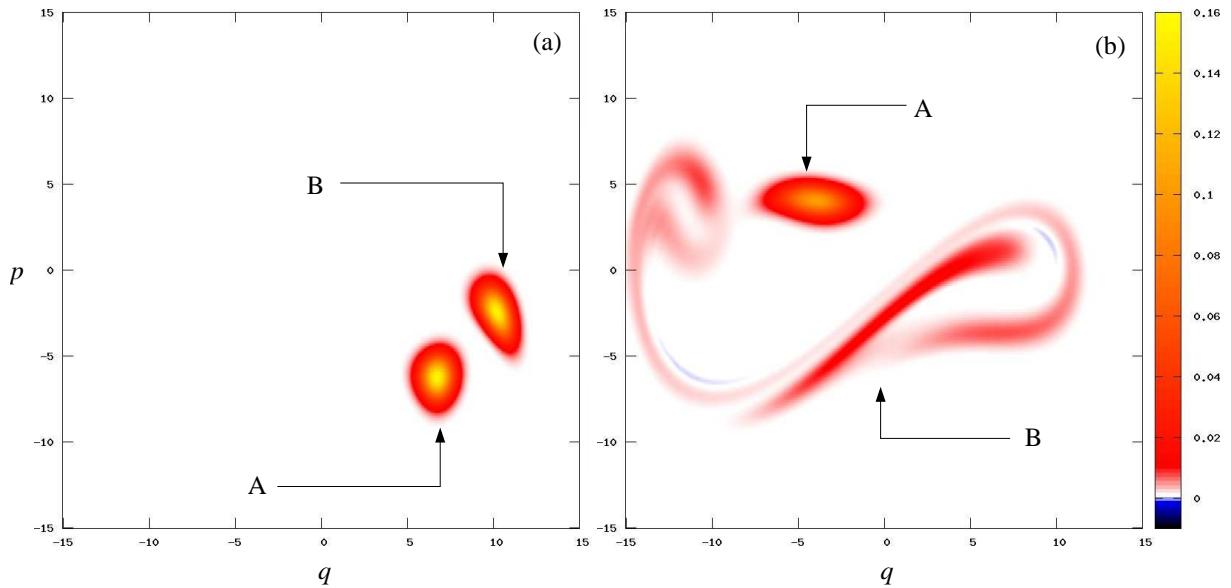
In [17] we considered both the evolution of the ensemble average as well as individual quantum trajectories. For the purpose of this work the former will be sufficient. Hence, we model the effects associated with coupling the measurement device to an environment using a Lindblad [20] type master equation in the Markovian limit at zero temperature. The dynamics of this (reduced) density operator is given by:

$$\dot{\rho} = -i[H, \rho] + \sum_m \left[ L_m \rho L_m^\dagger - \frac{1}{2} L_m^\dagger L_m \rho - \frac{1}{2} \rho L_m^\dagger L_m \right]. \quad (3)$$

In this work we assume only Ohmic type damping of the measurement device at zero temperature and hence have only one Lindblad operator  $L = \sqrt{2\Gamma}a$  where  $a$  is the oscillator annihilation operator and  $\Gamma = 0.125$ .

In order to understand the dynamical evolution of the measurement device we will make use of its Wigner function representation. For a detailed discussion of this and other phase space methods in quantum systems see, for example, [21]. The Wigner function takes the form of a pseudo probability density function in the  $(q, p)$  phase space and can be defined, for the field, by:

$$W(q, p) = \frac{1}{2\pi} \int d\zeta \left\langle q + \frac{\zeta}{2} \left| \text{Tr}_{\text{qubit}} [\rho(t)] \right| q - \frac{\zeta}{2} \right\rangle \exp(-i\zeta p).$$

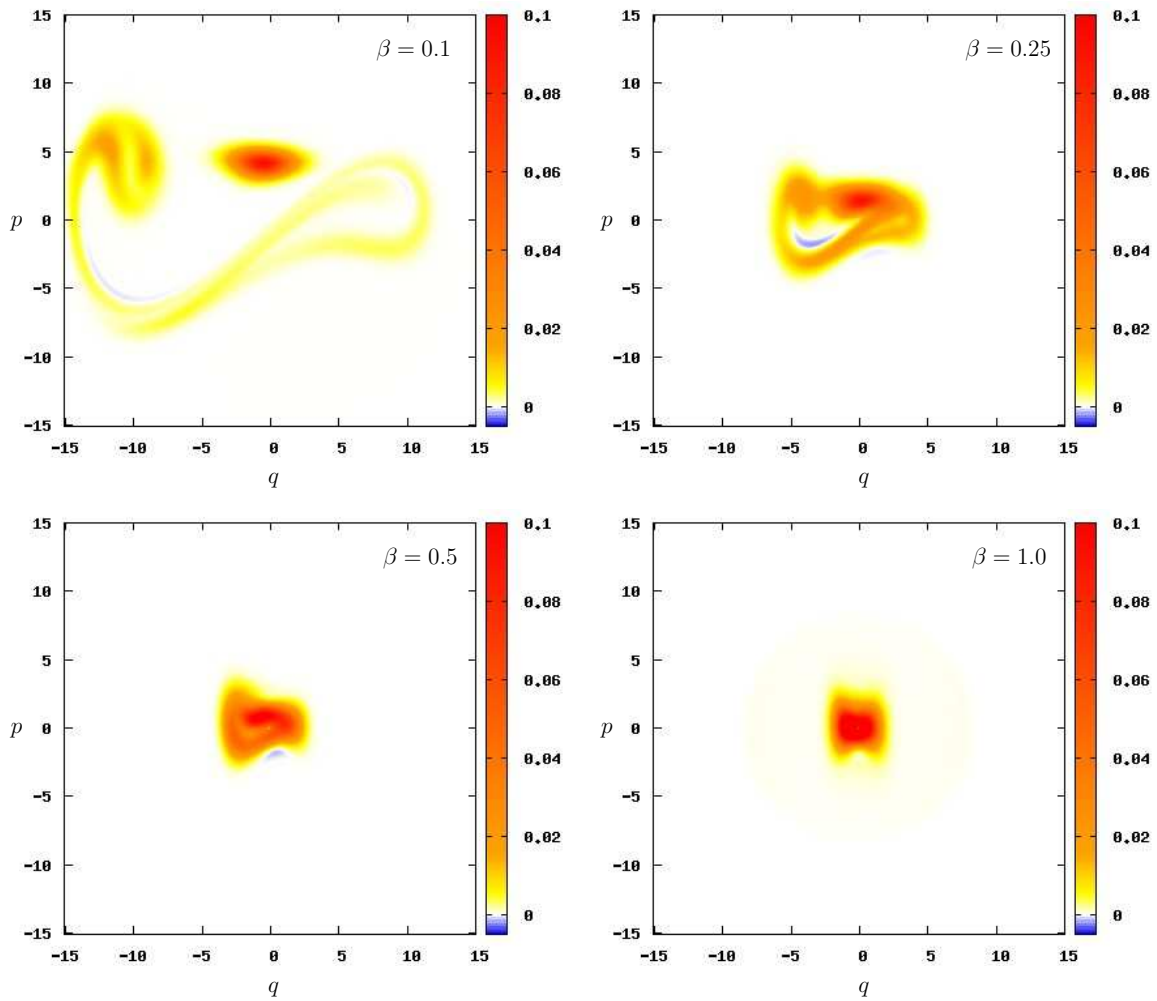


**Figure 2.** (colour online) Wigner function of the field for solutions to the master equation where the initial density operator corresponds to the pure state  $\frac{1}{\sqrt{2}}(|g\rangle + |e\rangle) \otimes |\alpha \approx 6.8\rangle$ . (a) at  $t/2\pi = 0.1$  and (b) at  $t/2\pi = 2.25$ . Here region A and B corresponds to measuring the qubit in state  $|g\rangle$  and  $|e\rangle$  respectively (Figure and caption reproduced from [17]).

Where  $\text{Tr}_{\text{qubit}}\rho(t)$  indicates a partial trace over the qubit space yielding a reduced density operator for the oscillator only. In Fig. 2 we show two snapshots of this Wigner function where, as stated in [17], “(a) is taken very early on in the system evolution at  $t/2\pi = 0.1$ . Here we see that the effect of the qubit on the measurement device is to split the coherent state up into two coherent state-like lumps. This is not surprising, as a very similar effect is seen in collapse and revival phenomena of the Jaynes-Cummings model of a qubit interacting with a harmonic oscillator. However, as the system evolves we see in Fig. 2(b) ( $t/2\pi = 2.25$ ) the distribution becomes somewhat more interesting. One lump is still quite small and is associated with the qubit’s ground state. The second is much larger, arising from the chaotic-like behaviour of the Duffing oscillator and is associated with the qubit’s excited state.”. In [17] we went on, amongst other things, to consider the qubits dynamics and demonstrate through an analysis of various entropic quantities that each of these lumps does indeed correspond to a measurement of one or the other state of the qubit.

### 3. Results

The quantum to classical transition of the Duffing oscillator has been studied in a great deal of detail. This transition was first clearly presented by Brun, Percival and Schack in [19]. In this work the authors demonstrated, using a quantum state diffusion approach, that the oscillator’s Poincaré section begins to be clearly resolved for values of  $\beta = 0.1$  (and becoming excellent by  $\beta = 0.01$ ). It is for this reason, together with computational constraints, that we chose  $\beta = 0.1$  in [17]. We now demonstrate that our model fails to achieve a measurement in the oscillator’s quantum limit – i.e. its dynamics are no longer useful in clearly distinguishing if the qubit is in  $|g\rangle$  or  $|e\rangle$ . Our model is therefore valid in, and only in, the device’s classical limit. We will show this, as in Fig. 2, through examining the oscillator’s Wigner function for different values of  $\beta$ . In Fig. 3 we show Wigner functions for the reduced density operator of the oscillator for  $\beta = 0.1, 0.25, 0.5$  and  $1.0$ . We note that, unlike in Fig. 2, in Fig. 3 we have allowed the dynamics



**Figure 3.** Wigner function of the field for solutions to the master equation where the initial density operator corresponds to the pure state  $\frac{1}{\sqrt{2}}(|g\rangle + |e\rangle) \otimes |\alpha \approx 6.8\rangle$  at  $t/2\pi = 3.4$  for four different values of the scaling parameter  $\beta$ . Here we see that the regions corresponding to measuring the qubit in state  $|g\rangle$  and  $|e\rangle$  merge as  $\beta \rightarrow 1$ .

to continue until  $t/2\pi = 3.4$  so as to enable the effects of decoherence to become more clearly manifest. We note that we have plotted each of these Wigner functions on the same scale to make clear the effect of  $\beta$  on the action.

In Fig. 3 for  $\beta = 0.1$  we see that the Wigner function has evolved into two disjoint lumps. This would imply that, except possibly on a set of measure zero, it should be possible even in individual experiments to uniquely determine the state of the qubit. Indeed, such behaviour was observed in [17]. For  $\beta = 0.25$  we see that, while these lumps are still identifiable, they have begun to merge. In terms of individual measurements, taken by examining the oscillator's phase space at a single point in time for  $\beta = 0.25$ , occasional errors would be therefore made in identifying the qubit's state. Nevertheless, it would still be possible to distinguish periodic from chaotic motion over an interval of time and therefore build up a measurement record. For  $\beta = 0.5$  it appears that not even this would be possible as the structure becomes increasingly obscured until at  $\beta = 1.0$  no discrimination whatsoever can be made.

Our above observations lead us to postulate criteria for the realisation of an instantaneous

measurement for a device of this kind. That is, in order to perform a reliable measurement at a specific time the Wigner function representing the ensemble average of the measurement device must have separated into two disjoint and clearly identifiable regions in phase space. We note that due to environmental decoherence the state associated with individual trajectories will be localised to approximately one Planck cell. Hence, our condition could be made more strict by also requiring that the space between the disjoint lumps should be at least of the order of a Planck cell or more.

In constructing the above criteria we have based our observations on a somewhat idealised setup. In general it will not always be possible to realise dynamics of this kind. For example, if we consider a non-degenerate qubit being measured in an incompatible basis, this will lead to more complicated dynamics - which will be reflected in the device's Wigner function and our above criteria is unlikely to be effective. Nevertheless, while it may not be possible to produce an instantaneous measurement under such conditions, it may be possible to build up a measurement record by examining the oscillator's dynamics over an interval of time. In [17] we showed that our model measurement apparatus in its correspondence limit could indeed track the state of a qubit under such circumstances. However, a detailed investigation of the quantum to classical transition in this more complex situation is beyond the scope of this paper.

#### 4. Conclusion

In [17] we demonstrated (i) a fully quantum mechanical model of a projective measurement process where the dynamics of expectation values, in the correspondence limit, are either chaotic-like or periodic depending on the measured value and projected state of a qubit. (ii) The preferred basis emerged from the coupling between the measurement device and the quantum object and was not assumed to exist *a priori*. (iii) Individual classical-like trajectories of an open quantum system can act as a record of the measurement of an individual qubit. (iv) Both the Born rule and Zeno effect arise as natural consequences of the systems dynamics.

In this paper we have further extended our analysis and have proposed, based on the Wigner function phase space representation of the measurement device, criteria for the effective realisation of pointer states for instantaneous measurement.

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