Transverse vibration of slender sandwich beams with viscoelastic inner layer via a Galerkin-type state-space approach

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Keywords: Sandwich beams, Euler-Bernoulli beams, State-space, Viscoelastic damping, Standard Linear Solid (SLS) model.

ABSTRACT

Nowadays composite structural elements are widely used in state-of-art engineering systems, such as in civil, automotive, aeronautical and aerospace engineering, providing beneficial stiffness/mass ratios and enhanced performance for a full range of different applications. Indeed, they have been responsible for important progresses in terms of improved safety, increased durability and reduced costs during the through-life manufacturing, operation and maintenance of structural systems. The valuable performance of laminate composites in both static and dynamic applications has motivated the continuous developments on new materials and laminate configurations, improved and cost-effective manufacturing processes and, specially, on more efficient modelling and design methods (e.g. Refs. [1, 2]). An interesting laminate configuration, usually referred to as sandwich beam, combines the high stiffness of the external layers with an internal core designed to provide significant damping capabilities. Such configuration enables an efficient use of a highly resistant material applied as skin and a core material able to dissipate large amounts of energy, thus reducing the amplitude of vibration and providing an effective method of passive control for the dynamics of these elements.

In this work, a novel state-space model for studying free and forced transverse vibrations of sandwich beams, made of two outer elastic beams of the same length, continuously joined by an inner shear-type viscoelastic layer, is presented and numerically validated. The proposed technique enables one to consider: i) inhomogeneous systems, i.e. abscissa-dependent mass, stiffness and damping; ii) any boundary conditions, which in general vary from layer to layer; and iii) rate-dependent constitutive laws for the inner layer, which can be represented either through Generalised Maxwell's (GM) model or Laguerre's Polynomial Approximation (LPA) [3, 4].

For the viscoelastic model of the inner layer, without loss of generality, the dynamic behaviour is described by the Standard Linear Solid (SLS) model, which is made of a primary elastic spring in parallel with a Maxwell's element, given in turn by a secondary elastic spring in series with a viscous dashpot. Importantly, it can be proved that the SLS model coincides with the simplest case for both GM and LPA representations.

The kinematics of the outer beams is developed by means of Galerkin-type approximations for the fields of both axial and transverse displacements in the outer beams, and imposing the pertinent compatibility conditions at interface. In the proposed formulation, the assumed modes are selected as the first modes of axial vibration and of lateral buckling for each layer with homogenised mechanical properties and their own boundary conditions. As such, layers' assumed modes are known in closed form and involve simple harmonic functions.

In a first stage, the Lagrange's equations of motion are derived for undamped sandwich beams, and then arranged in a compact state-space form, in which mass and stiffness matrices can be easily obtained through simple numerical integrations (see e.g. [5]).

In a second stage, two different sources of damping are introduced, namely a viscous damping for outer beams and a viscoelastic damping for the core, therefore addressing the very general case of non-viscous non-proportional damping. To do so, a set of additional internal variables are appended to the classical state variables (i.e. Lagrangian displacements and velocities), which take into account the rate-dependent rheology of the inner layer .

Numerical examples using a novel direct integration method for calculating the response of the dynamic system demonstrate the accuracy and versatility of the proposed formulation, in both frequency- and time-domain analyses.

Experimental and computational validation of the proposed approach in presence of different nano-enhanced viscoelastic layers will be pursued throughout the EPSRC-funded project "TREVIS: Tailoring nano-Reinforced Elastomers to Vibrating Structures", started in October 2011. As part of this project, a benchmark study will be presented during the ECCOMAS2012 Conference.

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