

This item was submitted to Loughborough's Institutional Repository by the author and is made available under the following Creative Commons Licence conditions.



For the full text of this licence, please go to:
<http://creativecommons.org/licenses/by-nc-nd/2.5/>

The Minimum Capacity of HVAC Secondary Systems (with Capacity Reduction by Interzonal Airflow) (RP-1049)

Jonathan Wright, PhD, CEng
Member ASHRAE

Yi Zhang, PhD
Member ASHRAE

Received October 3, 2006; accepted December 12, 2007

This paper is based on findings resulting from ASHRAE Research Project RP-1049.

To date, the performance of alternative HVAC secondary systems has been compared using either the systems' energy use directly or by a life-cycle cost analysis. This paper introduces the concept of rating a system's performance by comparing its capacity at a particular operating point to the thermodynamic minimum capacity. A simple ratio, termed the "system effectiveness," is also introduced to indicate the extent to which the system operates with the minimum possible capacity.

This paper describes the calculation of the minimum system capacity as a nonlinear, single-criterion, constrained optimization problem. In particular, it describes the case for the minimization of the system capacity by use of interzonal airflow (the interzonal airflow rates and zone thermal conditions being variables of the optimization). This optimization problem is multi-modal in that the same system capacity can result from more than one interzonal airflow configuration. The optimization problem has been solved here using a genetic algorithm (GA) search method.

This paper illustrates the use of the minimum system capacity as a benchmark for the performance of a typical multizone heating, ventilating, and air-conditioning (HVAC) system. The example also illustrates the potential benefit of using interzonal airflow to reduce the required capacity of a system. It can be concluded from the example that the arrangement of the system components is a significant factor in determining the thermal effectiveness of HVAC systems.

INTRODUCTION

The energy use of HVAC secondary systems¹ is often much higher than is necessary to condition the outdoor air and meet the zone sensible and latent loads. The extra energy use can be a result of the characteristics of HVAC processes, the performance limits of HVAC components, the choice of system configuration and the restrictions in system operation that this imposes, and the optimality of the HVAC system supervisory control strategy (Zhang et al. 2006). The potential for poor system performance increases with the number of zones served by the system and the degree to which temperature, humidity, and indoor air quality must be controlled. Perhaps the

1. Within the context of this paper, *HVAC secondary systems* refers to that part of the building's environmental conditioning system that performs the final conditioning of the zone air. This includes conventional air-based systems, such as VAV systems; water-based systems, such as chilled ceilings; and refrigerant-based heat-pump systems that are used to transfer energy between zones in the building.

Jonathan Wright is professor of building optimization in the Department of Civil and Building Engineering, Loughborough University, Leicestershire, UK. **Yi Zhang** is a senior research fellow at the Institute of Energy and Sustainable Development, Queens Building, De Montfort University, Leicester, UK.

best known example of a system configuration that results in higher-than-necessary energy use is a variable air volume (VAV) system with central cooling and zonal reheat; such systems have an energy overhead due to the simultaneous cooling and reheating of the supply air.

The global need to reduce building energy use and associated carbon emissions has increased interest in the optimum design and operation of HVAC secondary systems (Wright et al. 2004; Zhang 2005). However, to date, there is no means of judging the optimality of a particular design solution. The optimality of a system and its operating strategy is commonly judged by comparing the system performance with that of similar systems (or by measuring the improvement in system performance resulting from a change in the design or operation of a system). However, this approach is unsatisfactory, since it is possible that all systems in the comparison have poor performance and, as such, all that is identified is the best among a set of poorly performing systems. Further, when the system performance is measured in terms of energy use, there is a loss of transparency, as energy use can only be used to indicate the average performance of the system over the operating period (since energy use is the time-integral of the rate of system energy transfer at each operating condition).

This paper describes an approach to calculating the minimum HVAC system capacity² required at a particular operating point; the minimum capacity forms a benchmark against which the system performance may be judged. By basing the analysis on the system capacity at a particular operating condition, the cause of poor performance can be determined in most cases as a choice of system components and topology and/or the system operating strategy. The effect of system configuration and operating strategy on system performance would be less transparent if the analysis were based on the system energy input, as the system configuration and operation effects may be masked by the inefficiency in energy transfer between input and output (although most HVAC secondary-system components are passive devices for which the useful output energy equals the input energy).

The minimum capacity of an HVAC secondary system at a particular operating condition is conventionally assumed equal to the outdoor air and zone thermal loads (the sensible and latent loads are treated separately). However, this assumption is only valid for a single-zone system, as the potential reduction in required system capacity due to the system transferring energy between zones must be considered when calculating the minimum capacity of a multi-zone system. The calculation of the reduction in system capacity achievable through the use of interzonal energy transfer is a complicated task for any system serving more than a few zones. This problem is addressed for the case in which the energy transfer between zones is facilitated by interzonal airflow. This paper also describes an extension to the approach in which the minimum system capacity is used to evaluate the thermal effectiveness of the system. Finally, an example minimum capacity calculation is given for a multizone system operating with zone conditions that have been optimized to promote the energy transfer by interzonal airflow. The concepts presented herein have also been applied in a related study of the performance analysis of both novel and conventional HVAC systems (Wright and Zhang 2008).

Minimum System Capacity

The minimum system capacity at a given operating point is a function of the ambient and zone boundary conditions together with the potential for energy transfer between zones. The zone

2. Note that the term “capacity” is used here to denote the instantaneous rate of energy output at a given operating point; therefore, it is not restricted to the “peak” duty of the system, but applies equally to all load conditions. Note also that the capacity of a given HVAC secondary system is equal to the sum of the rate of energy output of the individual system components.

boundary conditions include the zone temperature, humidity, and the minimum outdoor airflow rate required to maintain indoor air quality. Ultimately, the zone boundary conditions would include the radiant temperature and be expressed as temperature, humidity, and air pollutant distributions (fields). Specification of the zone boundary conditions in terms of a distribution within the room are necessary in order to evaluate the performance of displacement ventilation systems and radiant cooling and heating systems. However, for the purposes of this paper, the zone boundary conditions are considered to have point values and, as such, relate to fully mixed conditions in each zone.

The potential for energy transfer between zones exists when there is a difference in the air temperature and/or humidity ratio between one or more zones served by the system. When these conditions exist, it may be possible to reduce the HVAC system capacity by transferring energy between zones. An HVAC system can facilitate the energy transfer in two ways: (1) by promoting airflow between zones and/or (2) by use of a heat pump. To some extent, the two approaches are complimentary, as each can provide energy transfer under some conditions when the other can not. For instance, a heat pump is ineffective in transferring energy between two or more zones when the zones have the same thermal load; however, provided there is a difference in the zone boundary conditions, energy transfer using interzonal airflow could be effective in reducing the required system capacity.³ For instance, when all zones require heating, but one or more zones is at a higher temperature than the others, the sensible load on the HVAC system can be reduced by exhausting some air from the high temperature zone(s) through the low temperature zone(s). Further, interzonal airflow has the ability to transfer both latent and sensible energy, whereas this generally is not possible using a heat pump. In this paper, the potential for energy transfer between zones is restricted to that achievable by interzonal airflow only, but includes the transfer of both sensible and latent heat energy.

The calculation of the minimum system capacity can be considered in two parts: (1) the calculation of the minimum capacity associated with moving air through the system (the *fan* capacity) and (2) the calculation of the minimum capacity associated with heating, cooling, dehumidifying, and the humidification processes. Calculation of the minimum fan capacity requires the minimum possible flow resistance for the given airflow rate to be defined. Since there is no logical thermodynamic basis for determining the minimum possible resistance, it is defined as zero, with the result that, in concept, the minimum possible fan capacity is also zero. Defining the minimum fan capacity as zero also allows the analysis to be applied to naturally ventilated (and mixed-mode) buildings, which have a zero fan capacity.

However, unlike the fan capacity, the minimum capacity associated with heating, cooling, dehumidifying, and the humidification processes can be calculated as a thermodynamic function of the energy transfer between the ambient environment and the zones, together with potential to transfer energy between zones.

Calculation of Minimum System Capacity with Outdoor and Interzonal Airflow

The calculation of the minimum system capacity with outdoor and interzonal airflow is described here as an optimization problem. The problem is formulated for the optimization of the outdoor and interzonal airflow rates, together with the zone air temperatures and humidity ratios. By including the air temperature and humidity ratios in the set of optimization problem variables, the approach can be used in the study of zone setpoint supervisory control strategies.

3. The extent to which interzonal airflow can be used to transfer energy between zones may be limited for applications where the transfer of "pollutants" between rooms is strictly controlled. Note also, that in the concepts presented here, indoor air quality is controlled by ensuring that the outdoor airflow rate to each zone is greater than a specified minimum.

Note that, in this respect, it is perceived that an HVAC secondary system operating with the minimum system capacity would not only have a system configuration that facilitated interzonal airflow, but would also operate with an optimum supervisory control strategy.

Formulation of the Minimum System Capacity Optimization Problem

Figure 1 illustrates the energy transfer for a given zone that is associated with the outdoor and interzone airflow. In Figure 1, \dot{Q} is the net heat gain or loss to the zone due to heat transfer through the building fabric, internal sources of heat gain, and heat gain or loss due to infiltration.⁴ Note that in order to simplify the notation used here, \dot{Q} has not been represented as a function of the zone and ambient conditions, although, in fact, it is in the analysis. \dot{E} is the energy transfer due to an air mass flow into or out of the zone (excluding mass flows due to infiltration). Both \dot{Q} and \dot{E} are given as a rate of energy transfer. Superscripts s and l indicate sensible heat and latent heat, respectively. Subscripts o and e signify the outdoor air and the extract air, respectively; note that the extract air could be exhausted directly to outdoors and/or another zone. Subscript i indicates the current operating condition. Subscripts j and k are zone indices where j is the zone under consideration and k is a different zone (note that $j \neq k$); nz is the number of zones. The subscript $k \rightarrow j$ signifies an airflow from zone k to zone j ; similarly, the subscript $o \rightarrow j$ signifies the flow of outdoor air into the zone, and $j \rightarrow e$ signifies the

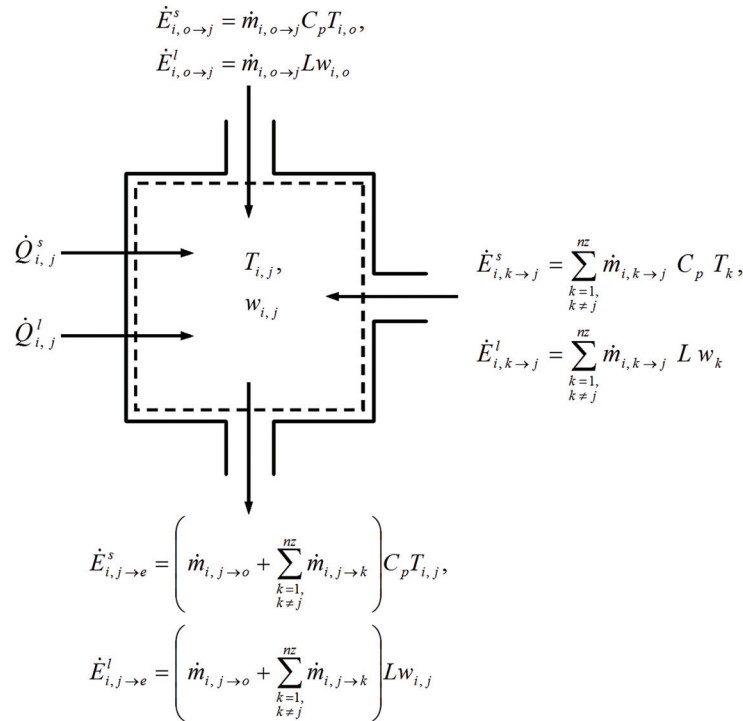


Figure 1. Zone energy transfer.

4. Strictly, *heat* is defined to be the energy transfer due to a temperature difference. However, although infiltration is the energy transfer due to a mass flow, it is included in \dot{Q} to follow the industry convention and simplify the problem formulation. In effect, \dot{Q} is the conventional *zone load*.

extract airflow, which is a function of the airflow to other zones ($j \rightarrow k$) and to outdoors ($j \rightarrow o$). \dot{m} is air mass flow rate, T air temperature, and w air humidity ratio. C_p is sensible heat capacity at constant pressure, and L is the latent heat of evaporation; note that in order to simplify the representation, the specific and latent heats are shown as constants but are in fact a function of the air temperature. Finally, in formulating the equations shown in Figure 1, it is assumed that in an energy balance on the zone, the differences in kinetic or potential energy associated with the energy transfer due to mass flow (\dot{E}) will balance as zero and, therefore, these energy terms may be ignored.

For a given operating condition, i , performing an energy balance on the zone energy transfers illustrated in Figure 1 will result in an energy residual; the sum of the residuals for all zones are equal to the required capacity of the HVAC system (\dot{Q}_i^{req}):

$$\dot{Q}_i^{req} = \sum_{j=1}^{nz} (|\dot{Q}_{i,j}^s + \dot{E}_{i,o \rightarrow j}^s + \dot{E}_{i,k \rightarrow j}^s - \dot{E}_{i,j \rightarrow e}^s| + |\dot{Q}_i^l + \dot{E}_{i,o \rightarrow j}^l + \dot{E}_{i,k \rightarrow j}^l - \dot{E}_{i,j \rightarrow e}^l|) \quad (1)$$

or

$$\dot{Q}_i^{req} = \sum_{j=1}^{nz} \left[\left| \dot{Q}_{i,j}^s + \dot{m}_{i,o \rightarrow j} C_p T_{i,o} + \sum_{\substack{k=1 \\ k \neq j}}^{nz} \dot{m}_{i,k \rightarrow j} C_p T_k - \left(\dot{m}_{i,j \rightarrow o} + \sum_{\substack{k=1 \\ k \neq j}}^{nz} \dot{m}_{i,j \rightarrow k} \right) C_p T_{i,j} \right| \right. \\ \left. + \left| \dot{Q}_{i,j}^l + \dot{m}_{i,o \rightarrow j} L w_{i,o} + \sum_{\substack{k=1 \\ k \neq j}}^{nz} \dot{m}_{i,k \rightarrow j} L w_k - \left(\dot{m}_{i,j \rightarrow o} + \sum_{\substack{k=1 \\ k \neq j}}^{nz} \dot{m}_{i,j \rightarrow k} \right) L w_{i,j} \right| \right] \quad (2)$$

In order to find the minimum system capacity at a given operating condition, i , the air mass flow rates, temperatures, and humidity ratios for all zones must be optimized simultaneously such that \dot{Q}_i^{req} is minimized:

$$\dot{Q}_i^{min} = \min \dot{Q}_i^{req} (\vec{M}_i, \vec{T}_i, \vec{W}_i) \quad (3)$$

subject to

$$\dot{m}_{i,j \rightarrow o} \geq 0.0, \quad \forall j \in \{1, \dots, nz\} \quad (4)$$

$$l_j^{RH} \leq g_{i,j}^{RH}(T_{i,j}, w_{i,j}) \leq u_j^{RH}, \quad \forall j \in \{1, \dots, nz\} \quad (5)$$

where, \dot{Q}_i^{min} is the minimum thermal capacity for operating condition, i . \vec{M}_i , \vec{T}_i , and \vec{W}_i are vectors of the optimization problem variables for the air mass flow rates, zone temperatures, and zone humidity ratios, respectively. The vector of air mass flow rates, \vec{M}_i , is given by the following:

$$\hat{\vec{M}}_i = \begin{cases} \dot{m}_{i,o \rightarrow 1}, \dot{m}_{i,2 \rightarrow 1}, \dots, \dot{m}_{i,nz \rightarrow 1} \\ \dot{m}_{i,1 \rightarrow 2}, \dot{m}_{i,o \rightarrow 2}, \dots, \dot{m}_{i,nz \rightarrow 2} \\ \vdots \\ \dot{m}_{i,1 \rightarrow nz}, \dot{m}_{i,2 \rightarrow nz}, \dots, \dot{m}_{i,o \rightarrow nz} \end{cases} \in \Re^{nz^2} \quad (6)$$

subject to

$$\begin{aligned}
 l_q &\leq \dot{m}_{i,q} \leq u_q, \forall q \in \{1, \dots, nz^2\}, \\
 \dot{m}_{i,q} &\in \Re^{nz^2} \\
 \dot{m}_{i,q}, l_q, u_q &\in \Re^{nz}, \\
 l_q &\leq u_q, \forall q \in \{1, \dots, nz\}
 \end{aligned} \tag{7}$$

where the notation $\hat{M}_i \Rightarrow \dot{M}_i$ denotes that the vector of mass flow rates, \dot{M}_i , is formulated by concatenating the rows of the matrix \hat{M}_i . The vector of mass flow rates, \dot{M}_i , is of length nz^2 , with each mass flow rate being subject to lower (l) and upper (u) bounds on their values. Note that, in general, the lower bounds on the mass flow rates will be set to zero except in the case of the outdoor airflow rates, where the lower bound is set to maintain the minimum standard of indoor air quality.

The mass flow rates are given here in matrix form, as each row of the matrix conveniently represents the airflow rates entering a particular zone. For instance, the second row contains the air mass flow rates entering zone 2, with the first entry in the second row, $\dot{m}_{i,1 \rightarrow 2}$, being the airflow rate from zone 1 to 2. The diagonals of the matrix (such as positions having subscripts equivalent to, $1 \rightarrow 1$ or $2 \rightarrow 2$) contain the outdoor airflow rate entering the zone that has the same index number as the row number. Finally, excluding the diagonal elements, a column in the matrix contains the air mass flow rates extracted from a particular zone and subsequently supplied to other zones. Note that, for a single zone, the optimization problem reduces to a single optimization variable—the outdoor air mass flow rate entering the zone ($\dot{m}_{i,o \rightarrow 1}$)—but that the number of variables increases with the number of zones as $O(nz^2)$.

Note that the matrix of mass flow rates, \hat{M}_i , does not include the flow rates for the air being exhausted to outdoors. This can be calculated from a mass balance on other airflows in the matrix, since the sum of the airflow rates entering a zone, less the airflow rates being extracted and supplied to other zones, is equal to the airflow rate being exhausted to outdoors:

$$\dot{m}_{i,j \rightarrow o} = \left(\dot{m}_{i,o \rightarrow j} + \sum_{\substack{k=1 \\ k \neq j}}^{nz} \dot{m}_{i,k \rightarrow j} \right) - \sum_{\substack{k=1 \\ k \neq j}}^{nz} \dot{m}_{i,j \rightarrow k} \tag{8}$$

Excluding the mass flow rates to outdoors from the optimization variables has two advantages. First, the number of optimization variables is reduced by an amount equal to the number of zones (nz); second, the mass balance in Equation 8 can be used to guarantee conservation of mass through the use of an inequality constraint on the optimization (that is, the flow to outdoors from a given zone must be greater than or equal to zero [see Equation 4]).

The vectors of zone temperature and humidity ratio optimization variables are given by the following:

$$\begin{aligned}
 \vec{T}_i &= (T_{i,1}, T_{i,2}, \dots, T_{i,nz}) \in \Re^{nz}, \\
 \vec{W}_i &= (w_{i,1}, w_{i,2}, \dots, w_{i,nz}) \in \Re^{nz}
 \end{aligned} \tag{9}$$

subject to

$$\begin{aligned}
 l_r &\leq T_{i,r} \leq u_r, \forall r \in \{1, \dots, nz\}, \\
 l_s &\leq w_{i,s} \leq u_s, \forall s \in \{1, \dots, nz\}, \\
 T_{i,r}, w_{i,s}, l_r, u_r, l_s, u_s &\in \mathfrak{R}^{nz}, \\
 l_r &\leq u_r, \forall r \in \{1, \dots, nz\}, \\
 l_s &\leq u_s, \forall s \in \{1, \dots, nz\}
 \end{aligned} \tag{10}$$

where each vector is of length nz , and l and u are the lower and upper bounds on the optimization variables.

Finally, the constraints given by Equation 5 define the feasible range of zone relative humidity (RH), where the zone RH, $g_{i,j}^{RH}(T_{i,j}, w_{i,j})$, is a function of the zone temperature and humidity ratio optimization variables. This constraint has been included since the RH is commonly used to specify the required zone air condition (together with the zone air temperature). In a more advanced problem formulation, the zone RH constraint could be replaced by a constraint on zone thermal comfort.

The minimum system capacity optimization problem described by Equations 3–10 is nonlinear in the objective and constraint functions and is multimodal in that there can be several alternative sets of air mass flow rates that result in the same solution. Further, the number of optimization variables can be high, with the number increasing in $O(nz^2 + 2nz)$. The number of constraints (Equations 4 and 5) also increases in $O(3nz)$; note that Equation 5, in effect, defines two constraint functions—one relating to the lower bound and a second relating to the upper bound.

Solution of the Minimum Capacity Optimization Problem

The solution of the minimum capacity optimization problem requires the use of an optimization algorithm that is able to solve nonlinear, constrained multimodal optimization problems. Algorithms in the class of probabilistic evolutionary algorithms (EA) are particularly suitable for solving multimodal, constrained optimization problems. The particular form of EA used here to solve the minimum capacity optimization problem is a real-coded GA.

The principle operators of a GA are fitness formulation, selection, recombination, mutation, and replacement (Bäck 1996; De Jong 2006). The fitness formulation used here is the stochastic ranking method for constrained optimization problems, which proves to be effective in solving a range of constrained optimization problems (Runarsson and Yao 2000). A binary tournament is used for the selection of solutions (Bäck 1996). The simulated binary crossover operator and parameter-based mutation operators are used for recombination and mutation (Deb 2000). Finally, the replacement strategy includes the preservation of a single elite solution.

System Effectiveness

Given the minimum possible capacity of an HVAC secondary system, the extent to which the system operates can be expressed by a simple *system effectiveness* ratio; the system effectiveness, ε_i , at operating condition, i , is defined by the following:

$$\varepsilon_i = \begin{cases} \frac{\dot{Q}_i^{min}}{\dot{Q}_i^{system}}, & \dot{Q}_i^{system} \neq 0 \\ 1.0, & \dot{Q}_i^{system} = 0 \end{cases} \quad (11)$$

where \dot{Q}_i^{min} is the minimum possible system capacity found by the optimization algorithm, and \dot{Q}_i^{system} is the capacity in use of the system being evaluated. The system capacity is given by the following:

$$\dot{Q}_i^{system} = \sum_{j=1}^{nc} |\dot{Q}_{i,j}^s| + \sum_{j=1}^{nc} |\dot{Q}_{i,j}^l| \quad (12)$$

where $\dot{Q}_{i,j}^s$ is the sensible capacity of component j ; $\dot{Q}_{i,j}^l$ is the latent capacity of component j ; and nc is the number of HVAC components in the system. Note that all components are included in the system capacity calculation, including the fans and heat recovery equipment.

It is evident from Equation 11 that the system effectiveness is linear and in the range 0.0 to 1.0 and is discontinuous for cases where the minimum capacity is zero ($\dot{Q}_i^{min} = 0$). In this case, the effectiveness is 0.0 for all system capacities (\dot{Q}_i^{system}), except when the system capacity equals 0.0, in which case the effectiveness is equal to 1.0. The effectiveness is defined to equal 1.0 when the system capacity is zero, since the system capacity can not be lower than zero and, therefore, a zero capacity is equivalent to the maximum effectiveness. Note that, since the minimum possible fan capacity has been defined as zero, a system having one or more fans is likely to have an effectiveness of less than 1.0 (particularly during the free-cooling and active-cooling periods, where the fan energy constitutes an energy penalty).

Example System Analysis

The application of the minimum system capacity in evaluating the performance of a multi-zone HVAC system is demonstrated here for a system serving five zones of a midlevel office building. The example demonstrates the following:

- The optimization of the zone temperature and humidity setpoints to promote energy transfer by interzonal airflow—hence, the minimization of the required system capacity
- The use of the minimum capacity as a benchmark for system performance

In addition to comparing the system capacity to the minimum capacity (and, therefore, the effectiveness of the system), the system capacity is also compared to the capacity required to offset the outdoor and zone loads absent interzonal airflow (referred to here as the *reference capacity*). Comparing the system capacity to the reference capacity gives some indication of the system performance from a conventional perspective (most systems are designed to simply offset the outdoor air and zone thermal loads). Finally, a comparison of the minimum and reference capacities indicates the extent to which the system capacity can be reduced by interzonal airflow.

Example Building

The example system analysis is performed for an HVAC system serving five midlevel zones of a small commercial office building. The five zones are divided among four perimeter zones and one core (internal) zone. The perimeter zones are orientated to true north, east, south, and

west. The five zones have a total floor area of 280 m² and an occupancy of 28 people. The perimeter zones have highly insulated external walls, and windows occupying 25% of the external wall area. For simplicity, the heat transfer through the construction between the core and perimeter zones is considered to be zero. The perimeter zones have an infiltration rate of 0.5 ach and the core zone 0.05 ach. Table 1 gives the internal heat gains due to occupants and equipment, solar transmission, and the total heat conductance due to heat transfer across the building fabric and infiltration (note that the conductances are approximate, as the sensible and latent heat capacities of the zone air are temperature dependent; the zone temperature is a variable in this example). The ambient boundary conditions correspond to a winter morning with an ambient temperature of -10.0°C and an ambient humidity ratio of 0.0014 kg_{vapor}/kg_{dry_air} (approximately 87% RH). The minimum outdoor-air ventilation rate for the perimeter zones is set at 0.048 kg/s and the core zone at 0.077 kg/s (equating to 8.0 L/s/person, set in order to maintain the indoor air quality).

The Minimum System Capacity and Zone Setpoints

The procedure for finding the minimum system capacity requires that the feasible range of optimization variables are defined (Equations 7 and 10). The optimization variables are the air mass flow rates entering and leaving each zone (Equation 6) and the temperature and humidity ratio in each zone (Equation 9). The RH in each zone is also constrained (Equation 5) together with the air mass flow rate being exhausted to outdoors (Equation 4). The bounds on the optimization variables and the zone RH constraints are given in Table 2.

The zone thermal loads due to heat transfer through the building construction were calculated using a simple steady-state heat transfer relationship. No thermal storage effects have been modelled, although the heat loads due to a constant rate of infiltration of outdoor air into each zone have been modeled.

Optimum Solution

The zone conditions and interzonal airflow rates relating to the minimum capacity are illustrated in Figure 2. The optimized outdoor airflow rates to each zone are at the specified minimum values of 0.048 kg/s for the perimeter zones and 0.077 kg/s for the core zone. The zone temperatures have been optimized such that they tend to or are equal to the minimum specified temperature of 18.0°C. Similarly, the zone humidity ratios tend to the minimum value but are prevented from reaching this by the constraint on zone RH (all zones have RHs equal to the specified minimum of 40%).

Figure 2 illustrates that the optimization of zone temperature and humidity promoted interzonal airflow from the east to the south zone, from the north and south zones to the core zone,

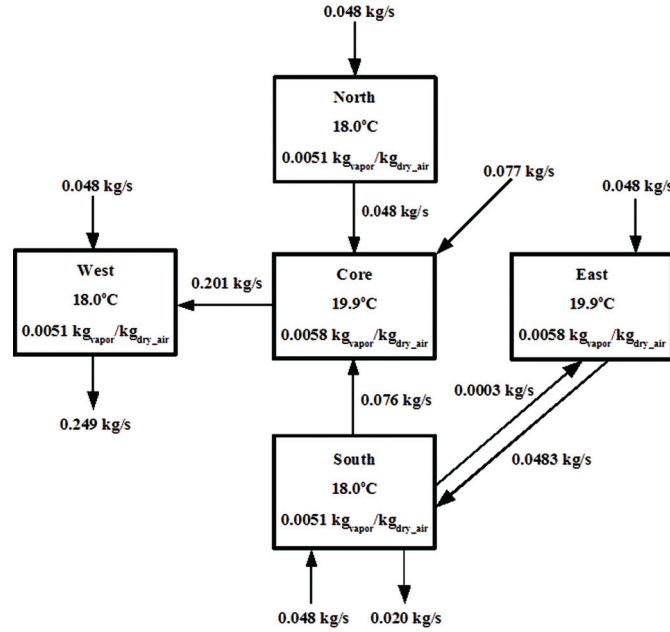
Table 1. Example Building Heat Flux Parameters

Zone	Internal Gain		Solar Gain, W	Envelope Conductance	
	Sensible, W	Latent, W		Sensible, W/k	Latent, W/kg _{vapor} kg _{dry_air} ⁻¹
North	1860	125	0	36	51,215
East	1860	125	675	36	51,215
South	1860	125	225	36	51,215
West	1860	125	0	36	51,215
Core	2640	200	0	2	6070

Table 2. Optimization Variable and Constraint Bounds

Optimization Parameter	Lower Bounds, l	Upper Bounds, u	Units
Interzone air mass flow rate, $(\forall (\dot{m}_{i,k \rightarrow j}, \dot{m}_{i,j \rightarrow k}) \in \dot{M}_i, \neg \dot{m}_{i,o \rightarrow j})$	0.0	0.5	kg/s
Outdoor air mass flow rate to the perimeter zones, $(\dot{m}_{i,o \rightarrow j}, \forall j \in \{1, \dots, 4\})$	0.048	0.5	kg/s
Outdoor air mass flow rate to the core zone, $\dot{m}_{i,o \rightarrow 5}$	0.077	0.5	kg/s
Zone air temperature, $(T_{i,j}, \forall j \in \{1, \dots, 5\})$	18.0	26.0	$^{\circ}\text{C}^1$
Zone air humidity ratio, $(w_{i,j}, \forall j \in \{1, \dots, 5\})$	0.0	0.03	$\text{kg}_{\text{vapor}}/\text{kg}_{\text{dry_air}}$
Zone air RH constraint, $(g_{i,j}^{RH}(T_{i,j}, w_{i,j}), \forall j \in \{1, \dots, 5\})$	40.0	70.0	%

1. Strictly, according to the problem formulation illustrated in Figure 1, the temperatures should be expressed in Kelvin. However, since the energy balance (Equation 1) results in a difference in the energy transfer by mass flow, $^{\circ}\text{C}$ will result in the same numerical values as Kelvin. It is also common working practice to express the air temperatures in $^{\circ}\text{C}$ rather than Kelvin.

**Figure 2. Optimized zone conditions and interzonal airflow rates.**

and from the core zone to the west zone. There is also a small flow rate from the south zone to the east zone. Note that, since the sensible and latent loads and the zone temperatures and RHs of the north and west zones are identical, the interzonal airflow paths to and from these zones are interchangeable; this is indicative of the multimodal nature of the optimization problem (in which more than one possible solution gives the same result).

Table 3 gives the zone loads for the optimized zone temperatures and humidity ratios. All zones require sensible cooling, which is a function of the high internal loads and the highly insulated external

Table 3. Zone Loads, Outdoor Air Loads, and Reference Capacity at the Optimized Zone Temperature and Humidity Ratios¹

Zone	Sensible Energy				Latent Energy			
	Zone Load, W	Outdoor Air Load, W		Reference Capacity, W	Zone Load, W	Outdoor Air Load, W		Reference Capacity, W
North	841	-1352	→	511	-66	-457	→	523
East	1446	-1445	→	-1	-100	-539	→	639
South	1066	-1352	→	286	-66	-457	→	523
West	841	-1352	→	511	-66	-457	→	523
Core	2568	-2322	→	-246	173	-867	→	694
Total				1555				2902

1. Positive load indicates heat gain; positive capacity indicates heating/humidification.

walls; all zones except the core zone require humidification. Table 3 also gives the outdoor air loads and the reference capacities. The reference capacities are taken as the sum of the zone and outdoor air loads, with, for example, a negative heat load (loss) offset by a positive (heating/humidification) capacity. Since the boundary conditions are for a winter day, the outdoor airflow rate results in both a sensible and latent heat loss. The reference capacities indicate that the north, south, and west zones are associated with a net heating capacity, but the east and core zones are associated with a net cooling capacity. All zones are associated with a net humidification capacity.

Table 4 gives the capacity available through interzonal airflow, together with the resulting minimum capacity; the capacity associated with the interzonal airflow is given as the capacity required to offset or remove the effect of the interzonal airflow rate (for instance, the interzonal airflow to the east zone provides 1 W of cooling, represented as 1 W of heating which would be required to offset the benefit of the interzonal airflow). This approach allows the minimum capacity to be calculated from the sum of the reference and interzonal offset capacities. A comparison of the total reference and total minimum capacities indicates that the use of interzonal airflow can reduce the required system capacity from 4457 to 3502 W, a 24% reduction in capacity (the total capacities being the sum of the sensible and latent capacities). The reduction in capacity is achieved by the west and south zones being heated and humidified by air from the core and east zones, respectively, and the core zone being cooled by air from the north and south zones (Figure 2). Unfortunately, the interzonal airflow to the core zone results in an increase in the humidification capacity associated with the core zone, although, the increase is less than the reduction in sensible cooling capacity (an increase in humidification capacity of 217 W compared to a reduction in sensible cooling capacity of 246 W). The 1 W cooling capacity associated with the east zone has been removed by a small interzonal airflow rate from the south zone (this airflow resulted in a negligible change in latent capacity). Note, also, that interzonal airflow has eliminated the need for any cooling capacity at this load condition, where all zones are associated with either a heating or zero sensible capacity.

The results given in Table 4 also illustrate that, at first sight, the use of interzonal airflow can seem to be illogical. For instance, it may appear illogical that the south zone is heated by air from the east zone when the south zone loads (see Table 3) indicate that it requires cooling. However, Table 3 also indicates that the south zone outdoor air load (heat loss) is higher than the zone load (cooling load) and, as a result, there is a 286 W net heating capacity *associated* with the south zone. Conventionally, the outdoor air load would be offset by heating the air at the

Table 4. Reference Capacity, Rate of Interzonal Energy Transfer, and Minimum System Capacity at the Optimized Zone Temperature and Humidity Ratios

Zone	Sensible Energy				Latent Energy			
	Reference Capacity, W	Interzonal Energy Transfer Offset Capacity, W		Minimum Capacity, W	Reference Capacity, W	Interzonal Energy Transfer Offset Capacity, W		Minimum Capacity, W
North	511	0	→	511	523	0	→	523
East	−1	1	→	0	639	0	→	639
South	286	−94	→	192	523	−82	→	442
West	511	−400	→	111	523	−351	→	173
Core	−246	246	→	0	694	216	→	911
Total				814				2688

central air-handling unit. However, in the case of interzonal energy transfer, 94 of the 286 W net heating capacity is offset by heating the south zone with air from the east zone. This increases the heat gain to the south zone; the extra heat is absorbed by the free-cooling effect of the outdoor airflow (the interzonal energy transfer, in effect, reduces the outdoor air load associated with the south zone).

The Example HVAC System and its Performance

The example multizone HVAC system is illustrated in Figure 3. The system consists of an air-mixing section, a cooling coil, a heating coil, a steam humidifier, and supply and extract fans. Air supplied to each zone may be further conditioned using separate reheater coils positioned in the zone supply air stream. Since the system configuration does not have a direct air path between zones, it does not have the ability to facilitate interzonal airflow and, as such, is expected to require a capacity that is higher than the optimized minimum.

The performance of the heating, cooling, and humidification components was modeled using conventional psychrometric relationships and, as such, these components are assumed to have an idealized linear characteristic. The supply and extract fan capacities were modeled as a cubic function of the supply air mass flow rate.⁵ For simplicity, the fan efficiency was assumed to be constant and the fan motors were assumed to be located in the airstream. It was also assumed that the extract airflow rate is equal to the supply airflow rate.

HVAC System Control Optimization

HVAC systems operate to maintain two groups of setpoints—those associated with the zone temperature and humidity and those that define the condition of the air at various locations within the system. In this example, it is assumed that the supervisor has optimized the zone setpoints and that they are equal to the values obtained from the minimum capacity optimization (Figure 2). The control of the system in meeting these setpoints is then achieved here by optimizing the controlled

5. $\dot{Q}_{fan} = \frac{k \cdot m_{supply}^3}{\eta}$, where \dot{Q}_{fan} is the fan capacity, m_{supply} is the supply airflow rate, and k is equal to

600.0 W/(kg/s)³ for the supply fan and 500.0 W/(kg/s)³ for the extract fan. The fan efficiency, η , was taken as 0.65 for both fans.

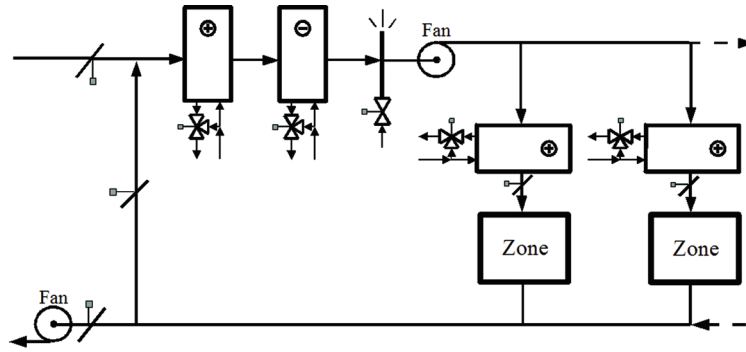


Figure 3. Example HVAC system.

variables directly (the optimization is performed using the same real parameter GA used to minimize the system capacity). Optimizing the system operation makes the effects of the system configuration on poor performance more transparent (as these may otherwise be masked by a poor control strategy). The objective of the optimization is to minimize the sum of the HVAC system capacity (at load condition i):

$$\min \dot{Q}_i^{\text{system}}(\dot{M}_i^{\text{supply}}, \dot{m}_{i,o \rightarrow j}^{\text{total}}, T_{i,s}),$$

$$\dot{Q}_i^{\text{system}}(.) = \dot{Q}_i^{\text{heat}}(.) + |\dot{Q}_i^{\text{cool}}(.)| + \sum_{i=1}^5 \dot{Q}_{i,j}^{\text{reheat}}(.) + \dot{Q}_i^{\text{humidifier}}(.) + \dot{Q}_i^{\text{supply_fan}}(.) + \dot{Q}_i^{\text{extract_fan}}(.)$$
(13)

where \dot{Q}_i is the capacity at load condition i , and superscripts *heat* and *cool* relate to the centrally located heating and cooling coils; *reheat* relates to the zone reheating coils; *humidifier* relates to the centrally located humidifier; and *supply_fan* and *extract_fan* relate to the supply and extract fans.

$\dot{M}_i^{\text{supply}}$, $\dot{m}_{i,o \rightarrow j}^{\text{total}}$, and $T_{i,s}$ are the control and optimization variables. $\dot{M}_i^{\text{supply}}$ is a vector of the zone supply air mass flow rates ($\dot{M}_i^{\text{supply}} = \{\dot{m}_{i,s \rightarrow 1}, \dot{m}_{i,s \rightarrow 2}, \dots, \dot{m}_{i,s \rightarrow 5}\}$), the optimization of the supply air mass flow rates representing the action of the VAV terminal units. $\dot{m}_{i,o \rightarrow j}^{\text{total}}$ is the total outdoor air mass flow rate entering the system⁶ and represents the action of the mixing box in providing free cooling. $T_{i,s}$ is the supply air temperature (the temperature of the air leaving the supply fan). The bounds on the optimization variables are given in Table 5. The lower bound on the outdoor air mass flow rate is equal to the sum of the minimum outdoor airflow rates that must be supplied to each zone (see Table 2). The lower bound on the supply air temperature is set to ensure that the maximum difference in zone supply air and zone air temperature is limited to a practicable value (in this case, the largest difference would be 7.9°C between a supply air temperature of 12.0°C and the core or east zone temperatures of 19.9°C). The variable upper bounds are to some extent arbitrary (although having a supply air temperature greater than 20.0°C would result in all zones being heated, when they all require cooling at this load condition).

6. $\dot{m}_{i,o \rightarrow j}^{\text{total}} = \sum_{j=1}^5 \dot{m}_{i,o \rightarrow j}$

Table 5. System Control Optimization Variable Bounds

Control Optimization Variables	Lower Bound	Upper Bound	Units
Zone supply air mass flow rate, $\dot{m}_{i,s \rightarrow j}, \forall j \in \{1, \dots, 5\}$	0.0	0.5	kg/s
Total outdoor air mass flow rate, $\sum_{j=1}^5 \dot{m}_{i,o \rightarrow j}$	0.269	1.0	kg/s
Supply air temperature, $T_{i,s}$	12.0	20.0	°C

Note that the supply air humidity ratio is not included in the optimization variables since the supply air humidity ratio must be equal to the maximum supply air humidity ratio required to offset any of the zone latent loads and meet the zone RH setpoint in order to ensure that the RH is no less than the optimized setpoint value of 40% (this value is also equal to the lower bound on RH in Table 2). This results in one or more zones being controlled to the specified RH, while the RH of other zones may be higher than desired (the inability to maintain the humidity in all zones being a consequence of the centrally located humidifier).

The constraints on the optimization (see Table 6) are that the outdoor air mass flow rate entering each zone must be equal to or be greater than the minimum required to maintain the indoor air quality. The zone reheater capacities must be positive (constraints on the reheater capacity are necessary as the capacity is calculated to be equal to that required to maintain the zone air temperature and, therefore, it is possible that the optimization may result in a cooling capacity rather than heating capacity).

System Performance

Tables 7, 8, and 9 give the optimized performance of the HVAC system. Table 7 indicates that the supply air temperature was optimized to be equal to the lower bound value of 12.0°C. This appears to be a logical solution, as the lower the supply air temperature is, the lower the supply air mass flow rate must be to offset the zone cooling load; the lower the mass flow rate, the lower the capacity of the supply and extract fans.

Table 8 indicates that the north and west zones require reheat because the supply air mass flow rate to these zones is set in order to meet the constraint on the outdoor air mass flow rate (the need for reheat indicates that the supply air mass flow rate is higher than required to offset the zone cooling load). This is evident by the fact that the outdoor air mass flow rates to the north and west zones lie on the constraint bound of 0.048 kg/s (see Table 6). Table 8 also indicates that the total outdoor air mass flow rate of 0.289 kg/s is higher than the specified minimum of 0.269 kg/s (see Table 5). The increase ensures that the heat energy from the supply fan⁷ and the recirculated air results in the optimized supply air temperature of 12.0°C. However, since increasing the outdoor air mass flow rate provides free cooling, it appears that with a 2 W central heating coil capacity, the outdoor air mass flow rate is slightly high and suboptimum (it is expected that the outdoor air mass flow rate is optimized to give a zero coil capacity). Although it is not considered to be the main factor governing the increase in outdoor air mass flow rate,

7. The need to remove part of the supply fan heat energy by free-cooling from the outdoor air, indicates that recovering the heat energy from the extract fan is unlikely to improve the performance of the system. In fact, positioning the extract fan before, rather than after, the re-circulation duct results in only a 2% decrease in the total system capacity (repositioning the extract fan energy resulted in an increase in the outdoor air mass flow rate, which in turn led to a decrease in the supply air mass flow rate and re-heat capacity of the north and west zones; however, the decrease in re-heat capacity was offset by an increase in humidification capacity resulting from the increased outdoor air mass flow rate).

Table 6. System Control Optimization Constraint Bound

Control Optimization Constraints	Constraint Bound	Units
Outdoor air mass flow rate to the perimeter zones, $\dot{m}_{i,o \rightarrow j}, \forall j \in \{1, \dots, 4\}$	0.048	kg/s
Outdoor air mass flow rate to the core zone, $\dot{m}_{i,o \rightarrow 5}$	0.077	kg/s
Zone reheat capacity, $\dot{Q}_{i,j}^{reheat}, \forall j \in \{1, \dots, 5\}$	0.0	W

Table 7. Air Conditions

Condition	Temperature, °C	Humidity Ratio, kg _{vapor} /kg _{dry_air}
Supply Air	12.0	0.0060
Mixed Air	11.0	0.0047
Recirculated Air	19.0	0.0059
Outdoor Air	-10.0	0.0014

Table 8. System Operation Results

	Zone					
	North	East	South	West	Core	Total
Supply Airflow Rate, kg/s	0.1725	0.1812	0.1766	0.1752	0.3198	1.03
Outdoor Airflow Rate, kg/s	0.0480	0.0496	0.0484	0.0480	0.0876	0.282
Reheat Capacity, W	217	0	0	217	0	434
RH, %	45	40	45	45	43	—

Table 9. AHU Component Capacities

Central Heating Coil Capacity, W	2
Central Cooling Coil Capacity, W	0
Central Humidifier Capacity, W	3481
Supply Fan Capacity, W	1003
Extract Fan Capacity, W	836

the higher total outdoor air mass flow rate makes it easier to satisfy the constraints on the minimum outdoor air mass flow rate entering each zone.

The supply air humidity ratio (see Table 7) was set to meet the east zone latent loads, which resulted in the humidity in all other zones being higher than the desired value of 40% (see Table 8). This is an inevitable consequence of having a single, centrally located humidifier unavoidably dominated, in this case, by the east zone (the humidifier capacity is 3481 W; see Table 9).

Table 10 indicates that the system capacity is higher than the reference system capacity. However, since the extract fan heat energy is exhausted directly to outdoors, it does not contribute to the sensible energy supplied to the conditioned zones. A comparison of the sum of

the supply-side component capacities and the reference capacity indicates that the capacity associated directly with maintaining the zone air temperatures is lower than the reference capacity (the sum of the supply fan, central heating coil, and re-heating coil capacities is 1439 W, whereas the sensible reference capacity is 1555 W [see Tables 8 and 9]).

Since the reference capacity is equivalent to the sum of the outdoor air and zone thermal loads, it might be expected that, given the absence of interzonal airflow paths, the supply-side sensible capacity of the system would be equal to the sensible reference capacity. However, Table 11 indicates that the recirculated air is able to provide some energy transfer between zones and, in particular, reduces the heating capacity associated with the north, south, and west zones by 123 W each. Similarly, the interzonal energy transfer reduces the cooling capacity associated with the core zone by 239 W but results in the cooling capacity associated with the east zone becoming a heating capacity of higher magnitude.

Finally, Table 11 indicates that the increase in outdoor air mass flow rate increases the heating capacity associated with the south zone and overcompensates for the cooling loads associated with the east and core zones (this is particularly the case when the outdoor air offset capacity⁸ is combined with the recirculation airflow offset capacity). The net result of the increased outdoor airflow and interzonal energy transfer by recirculation is that the system sensible capacity is 116 W lower than the reference sensible capacity. All zones are now also associated with a heating capacity, which is provided by the heat energy from the supply fan and by the re-heat coils.

Table 12 indicates that, in addition to the increase in outdoor air mass flow rate and the recirculation airflow, the required system latent capacity was effected by the increase in RH of the north, south, west, and core zones.⁹ Although the interzonal airflow due to the recirculated air reduced the humidification capacity associated with the north, east, south and west zones, it increased the humidification capacity associated with the core zone. The humidification capacity associated with all zones was increased by the increase in outdoor air mass flow rate. The capacity associated with the north, south, west, and core zones was also increased by the increase in the RH in these zones. The net effect of the recirculated airflow and the increase in outdoor air mass flow rate and zone RH is that the system latent capacity was increased by 579 W, compared to the reference latent capacity (see Table 12).

System Effectiveness

Table 13 gives the minimum system capacity, the capacity overhead, and the system effectiveness. The capacity overhead is the difference between the actual system capacity and the minimum capacity. In terms of the magnitude of the overhead, and as a percentage of the minimum capacity, the overhead is dominated by the 197.5% overhead in the sensible capacity (compared to the 29.5% overhead in the latent capacity). The overhead in sensible capacity is due to the extract fan energy being exhausted directly to outdoors, to the increase in outdoor air mass flow rate necessary to absorb part of the supply fan heat energy, and to the inability of the system to capitalize on the reduction in system capacity achievable by the use of interzonal airflow (although some reduction in the sensible capacity was achieved through the recirculation of the extract air). The overhead in latent capacity is due to the inability of the system to capitalize on the reduction in system capacity achievable by interzonal airflow, to the use of a single centrally located humidifier (and associated overhumidification of some zones), and to the increase

8. Note that in Tables 11 and 12, the concept of *offset capacity* was used to represent the impact of systematic effects on system performance; the offset capacity is the capacity required to offset or cancel the impact of the systematic effect. This enables the required system capacity to be calculated from the sum of the reference and offset capacities.

9. Note that the magnitude of the offset capacity for the outdoor airflow rate and the recirculation airflow includes the effect the increase in zone RH has on these capacities.

Table 10. System and Reference Capacity

	Sensible	Latent	Total
HVAC System Capacity, W	2275	3481	5756
Reference Capacity, W	1555	2902	4457

Table 11. Comparison between System and Reference Sensible Capacities

Zone	Reference Capacity, W	Offset Capacity			Required System Capacity, W	Capacity Difference, W
		Increased Outdoor Air Mass-Flow Rate, W	Recirculated Air Mass-Flow Rate, W			
North	511	0	-123	→	388	-123
East	-1	49	129	→	177	176
South	286	10	-123	→	173	-113
West	511	0	-123	→	388	-123
Core	-246	320	239	→	313	67
Total	1555				1439	-116

Table 12. Comparison between System and Reference Latent Capacities

Zone	Reference Capacity, W	Offset Capacity				Required System Capacity, W	Capacity Difference, W
		Increased Outdoor Air Flow Rate, W	Recirculated Airflow, W	Increased Zone RH, W			
North	523	80	-44	34	→	593	70
East	639	18	-43	0	→	614	-25
South	523	85	-44	34	→	598	75
West	523	80	-44	34	→	593	70
Core	694	210	176	3	→	1083	389
Total	2902					3481	579

Table 13. System Capacity, Minimum Capacity, and Effectiveness

	Sensible	Latent	Total
HVAC System Capacity, W	2275	3481	5756
Minimum Capacity, W	814	2688	3502
Capacity Overhead, W	1461	793	—
Capacity Overhead, %	179.5	29.5	—
Effectiveness	—	—	0.61

in outdoor air mass flow. The total overhead in system capacity is 2254 W, with the result that the system has an effectiveness of only 0.61.

A system configuration capable of achieving a maximum effectiveness of 1.0 is described by Zhang et al. (2006) and Zhang and Wright (2008). Zhang and Wright (2008) have also used the concepts of minimum system capacity and system effectiveness in a performance comparison of

both novel and contemporary HVAC system configurations. However, further research is required to investigate the annual impact of interzonal airflow on the reduction of HVAC system capacity together with the development of a practicable approach to facilitating the interzonal airflow.

CONCLUSIONS

To date, the performance of alternative HVAC secondary systems has, in general, been compared using either the systems energy use directly or, in some instances, using a life-cycle cost analysis. However, no metric exists that indicates the extent to which a system operates with the minimum possible energy use (or capacity). This paper addresses these with the introduction of a procedure for determining the minimum possible capacity of a system at a given operating condition; the minimum capacity then provides a thermodynamic benchmark for system performance. For multizone systems, the minimum system capacity is a function of the extent to which the zone loads are minimized by the transfer of energy between zones. Energy transfer between zones can be facilitated in two ways: (1) by interzonal airflow and (2) by the use of a heat pump. This paper addresses the case for energy transfer by interzonal airflow.

Given that it is impossible to define a thermodynamic minimum for flow resistance, we concluded that the minimum should be taken as zero, with the consequence that the minimum capacity associated with moving air through the system is also defined as zero (which enables the concept to be applied to naturally ventilated buildings). Conversely, the thermodynamic minimum heating, cooling, humidification, and dehumidification capacities at a given operation point can be defined in terms of an energy balance on each zone. In this paper, an energy balance on each zone is used to define an optimization problem for the minimization of the system capacity as a function of the interzonal and outdoor airflow rates, as well as the air temperature and humidity ratio in each zone. The optimization problem has a single criterion (system capacity), is nonlinear, constrained, and multimodal (in that more than one solution can give the same result). The optimization problem was solved here using a GA search method. The approach can be extended in future research to include interzonal energy transfer by a heat pump, and the inclusion of thermal comfort constraints for each zone (thermal comfort is only loosely constrained in the example given here).

The example analysis illustrates the potential benefit of optimizing the zone conditions and interzonal airflow rates in order to minimize the zone loads and associated system capacity. The example also illustrates the application of the minimum system capacity as a benchmark for system performance. It could be concluded from the particular example that the ineffectiveness of the example system was due to factors related to its configuration. In particular, the system was only partially able to capitalize on the potential for capacity reduction through the use of interzonal airflow (in this case, through the central recirculation of the extract air). The use of a single centrally located humidifier led to the overhumidification of some zones, with a consequent overhead in humidification capacity. Finally, since the extract fan energy was exhausted directly to outdoors, it was a significant factor in reducing the effectiveness of the system.

The impact of the system configuration on the thermal effectiveness of the system is the subject of a related study (Zhang et al. 2006; Wright and Zhang 2008). The study also identified a system configuration that is able to facilitate interzonal air and operate with maximum thermal effectiveness (although the practicable implementation of the concept requires further research).

ACKNOWLEDGEMENT

This research was funded in part by the American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc., under research project 1049-RP.

REFERENCES

- Bäck, T. 1996. *Evolutionary Algorithms in Theory and Practice: Evolution Strategies, Evolutionary Programming, Genetic Algorithms*. London: Oxford University Press.
- Deb, K. 2000. An efficient constraint handling method for genetic algorithms. *Computer Methods in Applied Mechanics and Engineering* 186:311–38.
- De Jong, K.A. 2006. *Evolutionary Computation: A Unified Approach*. Cambridge, MA: MIT Press.
- Runarsson, T.P. and X. Yao. 2000. Stochastic ranking for constrained evolutionary optimisation. *IEEE Transactions on Evolutionary Computation* 4(3):284–94.
- Wright, J.A., Y. Zhang, P.P. Angelov, R. Buswell, and V.I. Hanby. 2004. Building system design synthesis and optimization. Final Report, ASHRAE RP-1049, the American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc., Atlanta.
- Wright, J.A., Y. Zhang, P.P. Angelov, R.A. Buswell, and V.I. Hanby. 2008. Evolutionary synthesis of HVAC system configurations: Algorithm development. *HVAC&R Research* 14(1):33–56.
- Wright, J.A., and Y. Zhang. 2008. Evolutionary synthesis of HVAC system configurations: Example synthesis of a two-zone system. *HVAC&R Research* 14(1):57–72.
- Zhang, Y. 2005. Synthesis of optimum HVAC system configurations by evolutionary algorithm. Doctoral thesis, Department of Civil and Building Engineering, Loughborough University, Loughborough, Leicestershire, UK.
- Zhang, Y., J.A. Wright, and V.I. Hanby. 2006. Energy aspects of HVAC system configurations: Problem definition and test cases. *HVAC&R Research*. 12(3c):871–88.