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REPRESENTATION OF BOND IN FINITE ELEMENT  
ANALYSES OF REINFORCED CONCRETE STRUCTURES

by

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A Doctoral Thesis submitted in partial fulfilment of the  
requirements for the Award of the Doctor of Philosophy of  
the Loughborough University of Technology.

May 1984

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## SYNOPSIS

A non-linear finite element model has been developed to analyse reinforced concrete structures taking into account :

- (1) non-linear concrete behaviour under biaxial stress,
  - (2) progressive cracking of the concrete,
- and (3) interaction between the reinforcement and the concrete matrix commonly known as bond.

Three dimensional reinforced concrete components are analysed by an approximate two dimensional plane stress model. Bond is considered to be a concentric layer surrounding the reinforcement modelled by a 6 noded rectangular 'shearing' element. The concrete is represented by 8 noded isoparametric membrane elements and the reinforcement by 3 noded isoparametric bar elements. The finite element model uses, an incremental iterative solution technique known as the 'Initial stress method' and a special solution technique to allow for cracking of the concrete. Stiffnesses within elements are evaluated by numerical integration using Gaussian Quadrature, with elastic moduli stored at the sampling positions.

The bond model is based upon an assumed non-linear relationship between bond stress and slip in which the localised ultimate bond stress is a function of both the lateral pressures exerted by the concrete on the reinforcement and the radial contraction of the bar due to Poisson's effect. Allowance is also made for the deterioration of bond when the slip exceeds a tolerance value. The concrete model is a non-linear elastic fracture model based upon the 'Equivalent uniaxial strain approach' as developed by Darwin and Pecknold (1974). Cracking of the concrete is assumed to be 'smeared' within the concrete element.

Reinforced concrete components which have been analysed include; the ordinary pullout test, double ended pullout test, a transfer test, and a beam-column intersection.

A small experimental programme was conducted to obtain reliable data as to the nature of the bond stress and reinforcement strain distributions in the double-ended pullout test, the transfer test and the beam-column intersection. To determine the reinforcement strain distributions, plain round bars or ribbed reinforcement bars in the case of the beam-column, were embedded in the concrete specimens with electrical strain gauges attached .

The author's computer programs are explained and listed in the appendices.

#### ACKNOWLEDGEMENTS

The author would like to thank his Director of Research and Supervisor Dr. R.J.Allwood for his friendly guidance and encouragement throughout the duration of this work. My sincere thanks also to Dr. P.J.Robins, co-supervisor for his guidance and constructive comments.

The author would also like to thank the technical staff in the laboratory of the Civil Engineering Department, Loughborough University of Technology for their help and assistance during the experimental work.

Finally my thanks extend to Miss J. Gregory who has helped in proofreading this manuscript .

The financial assistance of the Science and Engineering Research Council is gratefully acknowledged.

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## CHAPTER 1    INTRODUCTION

Present day design methods of analysis for reinforced concrete structures are generally based on simplified assumptions of the behaviour of the constituent materials or are based on an empirical approach using the results of a large amount of experimental data. Such methods have in the past and will continue to be adequate for the analysis of simple structures. The finite element method, however offers a powerful and general analytical tool to analyse complex structures such as pressure containment vessels and unusual details in more common structures. Material models incorporated into this method can take account of the complex deformation behaviour of the constituent parts, the concrete under biaxial stresses, the reinforcement and the bonding at the interface between these two materials.

It is important to model the deformation behaviour of the interface properly since it has a direct effect on the transfer of forces between the steel and concrete and the internal distribution of forces within the materials. Ngo and Scordelis (1967) and later Lutz (1970) in their finite element analyses of reinforced concrete used special discrete bond links to connect the steel to the concrete and assumed linear bond stress-slip behaviour. Nilson (1968) developed this method of analysis and introduced a non-linear bond stress-slip relationship derived from bond experiments. In more recent analyses of reinforced concrete a distributed or embedded representation of the steel has been used which assumes perfect bonding. However the earlier researchers have shown that perfect bonding inadequately models the bonding of steel to concrete.

The work described herein has formed part of an on-going investigation within the Department of Civil Engineering at Loughborough

University of Technology into many aspects of the bond between steel and concrete. In particular investigations have been conducted into: the effects of lateral pressure on bond in pullout tests with plain or deformed bars, bond in the anchorage zones of deep beams, measurement of bar stress distributions in a number of bond tests and beam-column inter-sections, and the finite element analysis of these structural components.

This study is an attempt to establish a bond model for plain bars that makes use of existing experimental data and bond tests conducted by the author. The bond model to be described (Chapter 3) recognises that the local ultimate bond capacity of a plain bar is substantially dependent on the radial confining pressures exerted by the concrete on the bar. The radial pressures existing at the interface between the two materials are initially generated by concrete shrinkage and then modified by lateral pressure generated by external loading and by radial contraction of the steel bar. To the author's knowledge no other bond model exists which has considered all of these effects and their influence on the local bond stress-slip relationships and the local ultimate bond capacity.

It is also equally important to properly model the complex deformation behaviour of the concrete. The author has chosen the concrete model of Darwin and Pecknold (1974) and this model has been shown to very accurately reproduce the non-linear stress-strain behaviour of the concrete under all biaxial stress states. The model used also incorporates the biaxial failure envelope of Kupfer et al. (1969). Cracked concrete is treated using the smeared crack approach which involves the reduction of the appropriate elastic moduli when cracking is deemed to occur.

After reviewing the literature on bond, bond models used in finite element analyses of reinforced concrete and deformation models of concrete

behaviour, the proposed bond model is described (Chapter 3). Applications of the model to bond tests with embedded plain bars and comparisons with the respective tests, some conducted by the author and by others are described. Only cases of monotonic increasing loads are considered.

Although the fundamental mechanisms of bond for deformed bars are different from plain bars, the best estimate the author could make was to apply the present bond model to deformed bars and make estimates for the parameters involved from the existing experimental data. The model has been applied to the analysis of pullout tests and a beam-column intersection. The differences in the analytical and observed behaviour of these structural components are noted and the inadequacies of the present bond model applied to deformed bars are discussed. Proposals are outlined to model the bond of deformed bars more realistically in finite element analyses of reinforced concrete.

## CHAPTER 2    LITERATURE REVIEW

### 2.1       REVIEW OF RESEARCH ON BOND

#### 2.1.1    Introduction. .

Bond in reinforced concrete structures is the term used to describe the interaction between the reinforcement and the surrounding concrete matrix. The force in the bar is transmitted to the concrete by bond and vice-versa, and the shear force per unit area acting parallel to the bar on the interface is known as the bond stress. The bond stress is a function of the rate of change of steel stress in the reinforcement and there can be no bond stress unless the bar stress changes, and conversely there can be no bar stress without bond stress.

It is conventional to divide bond into two types, namely: local bond and anchorage bond. Local bond is required at each section along the length of the bar to make sure that the concrete and reinforcement act together and it is related to the shear at a section. Anchorage bond is required to ensure that the ends of bars are firmly embedded in the concrete and is related to the transfer of axial force over the length of reinforcement. The structural functions of these two types of bond are different, but the mechanisms by which the reinforcement and concrete interact are the same.

Unless the strains of the concrete and reinforcement are the same and constant over the region, the reinforcement attempts to move or slip in relation to the surrounding concrete. The differential movement of bar and concrete at a section is the local bond slip.

This thesis is concerned with the mechanisms by which a transference of force between the reinforcement and the concrete occurs, with particular reference to the localised relationship between bond

stress and slip and the parameters which influence this relationship. This section outlines the history of research on bond and in particular, work on bond stress distributions, bond stress-slip relationships and the mechanisms of shear failure at the reinforcement-concrete interface. A comprehensive review of the literature on bond can be found in the recent publication of Hungspreung (1981) and in a state-of-the-art report on bond in the Proceedings of the International Conference on Bond in Concrete (1982).

### 2.1.2 Bond stress and steel stress distributions.

Research work on bond began with Abrams (1913) who studied the bond resistance of round and deformed bars using pullout and beam tests. Abrams found that deformed bars were more effective in mobilising bond resistance than round bars. Abrams pointed out that the bond stress developed in any length of bar represented the change in tensile stress over that length.

Glanville (1930) published a theory on bond, and his test results showed the distribution of bond stresses in plain round bars. He used a special tube extensometer of the optical lever type to measure experimental load distributions in the bar.

The test results of Clark's (1946,1949) work on 204 specimens dealing with the comparative bond efficiency of 17 different designs of deformed bar, led to the production of the geometrical specifications of ribs in the American Code of Practice ASTM-A305 (1947). All of this work was, however, predominately concerned with the global efficiency of steel to concrete bond and it was only later that attention was focused on bond mechanisms and bond stress distributions.

It was not until the 1950's that intensive research begun into

the actual bond stress and steel stress distributions along bars embedded in the concrete, and the fundamental mechanisms of bond. Plowman (1957) measured the movement of studs welded to the reinforcement while Wilkins (1951), Mains (1951), Peattie and Pope (1956) and later Perry and Thompson (1966) and Nilson (1972) attached electrical strain gauges to bars embedded in concrete. Using a system of wedges Wilkins placed the strain gauges on the inside of steel tubes which had different outside surface deformations. These tubes were then embedded along the axis of cylinders of concrete to form pullout specimens. The test results indicated that the most important influence on bond was the character of the tubes outside surface and that bond was caused by adhesion, friction and mechanical wedging.

Mains (1951) was the first to develop a technique for measuring the localised steel stress of the reinforcement embedded in concrete, without affecting the properties of the bar. The bar was sawn longitudinally and in one half of the bar a groove milled to accommodate the electrical strain gauges, Figure 2.1, and the two halves of the bar were then tack welded back together again. Both plain bars and different types of deformed bar with either, hooked or straight ends were embedded in either pullout or beam specimens. The results of his tests indicated that cracks in beams affect the magnitude and distribution of steel stress in the reinforcement and bond stress. Very high local bond stresses always occurred near a crack in a beam and these bond stresses depended on the amount of slip. Mains also showed experimentally that the longitudinal distribution of bond stress in a pullout test was not uniform and that the localised maximum bond stress often exceeded the calculated average bond stress by a factor of two or more at loads well below pullout.

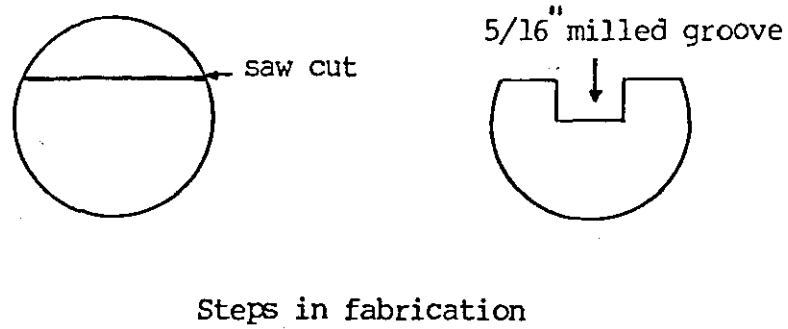
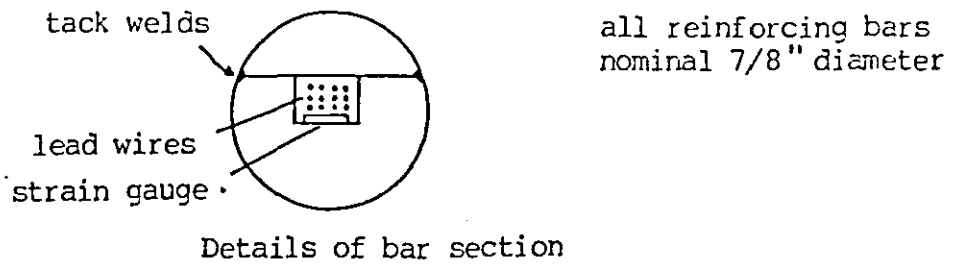


FIGURE 2.1 MAINS (1951) METHOD OF PREPARING BARS

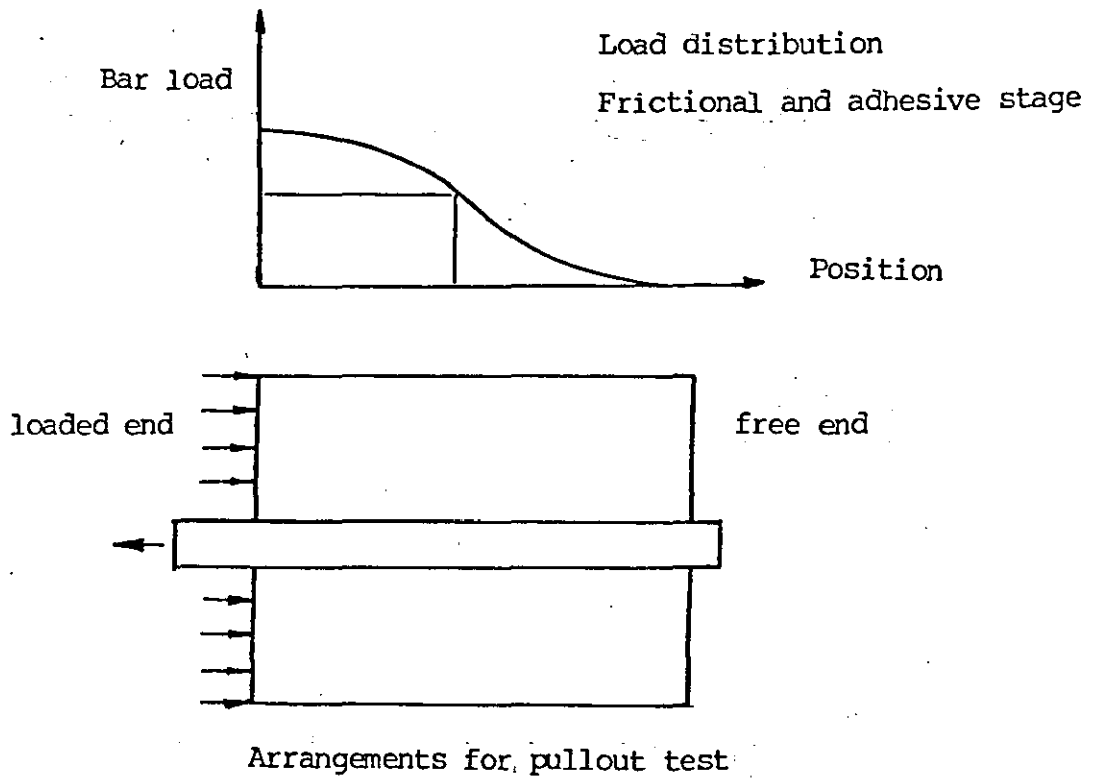


FIGURE 2.2 ARRANGEMENTS FOR THE PULLOUT TEST AND THEORETICAL LOAD DISTRIBUTION After Peattie and Pope (1956)

Peattie and Pope (1956) investigated the distribution of longitudinal steel stress in both pullout and torsion tests using plain round bars. Longitudinal slots were milled in the bar for electrical strain gauges to be mounted in, with the slot closed by a rectangular section and the composite bar turned down. From their test results Peattie and Pope developed a theoretical analysis of the pullout test which was based on three stages of bond namely: adhesion, friction and bearing. For the pullout test, Figure 2.2 , they considered that during the adhesion stage, the steel stress at any point along the bar would be in proportion to the applied load and that the distribution of load would be exponential. Further, it was considered that this proportionality would continue until a critical strain developed in the concrete and rupture of adhesion would occur at the loaded end. The point of rupture denoted the position of change from adhesion to a frictional stage and with increasing load moved towards the unloaded end of the bar.

Parland (1957) used a magneto-strictive measuring method to determine the distribution of steel stresses. This method was based on the principle that the impedance of the reinforcing bar whilst conducting an alternating current was changed by the state of axial stress in the bar. The average steel stress over a given length was determined by measuring the voltage potential at various sections along the bar. The method was simple and required no complicated instruments for measurement, however the method had the disadvantage that each specimen required special measurement of its size and quality and the embedded bars needed to be specially insulated from other metallic objects, particularly the testing machine.

Perry and Thompson (1966) used a technique that was a modification of the method used by Mains, Figure 2.3 . The reinforcement was sawn

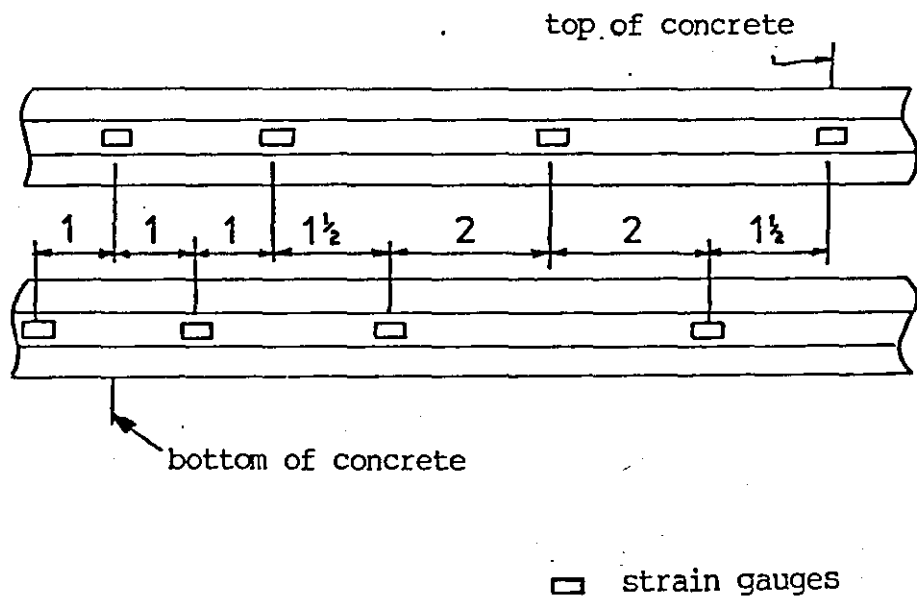
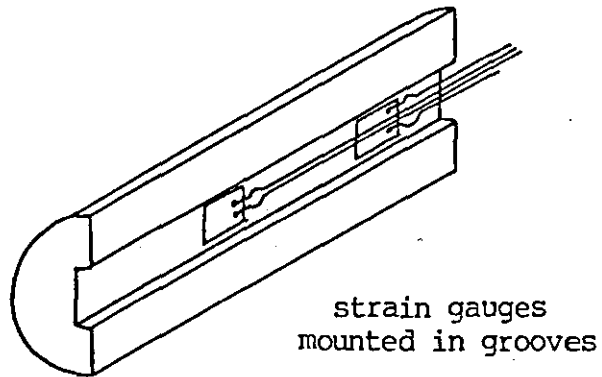


FIGURE 2.3 STEPS IN BAR PREPARATION AND GAUGE LOCATION  
After Perry and Thompson (1966)

directly in half and grooves milled in each half of the bar to accomodate the electrical strain gauges. The major objective of this investigation was to study the correlation between stress distributions in eccentric pullout specimens and those in reinforced concrete beams at a crack in constant moment regions. Very little similarity in bond stress distribution was found, but approximately the same maximum bond stress was developed for equal steel stresses at the loaded end of the specimen and at a crack in the beam.

More recent investigations have been made by Tanner (1971) and Nilson (1972), Dorr (1978), Viwathanatepa et al.(1979), Allwood (1980), Standish (1982) and Spencer, Panda and Mindess (1982). Nilson and Tanner devised a method of measuring the internal strains of both concrete and steel by using Mains method in conjunction with concrete gauges tied to the surface of the steel. The displacements for both the concrete and steel were obtained by the integration of the strain distributions. The local slip at the gauged position was the difference between these two displacements and therefore bond stress-slip relationships were deduced for any point along the reinforcing bar. Nilson observed that the bond stress-slip relationships were not unique and depended upon the strength of the concrete, and upon the distance of the gauge position from the loaded face of the specimen. Tassios and Yannopoulos (1981) have also observed that there is no unique relationship between local bond stress and local slip, but like Nilson have not offered a rational explanation of why the bond stress-slip relationship should change with position.

Dorr (1978) used a different version of Mains technique, namely that grooves were milled on the outer surface of the reinforcement to accommodate the electrical strain gauges, Figure 2.4 . Using concentric

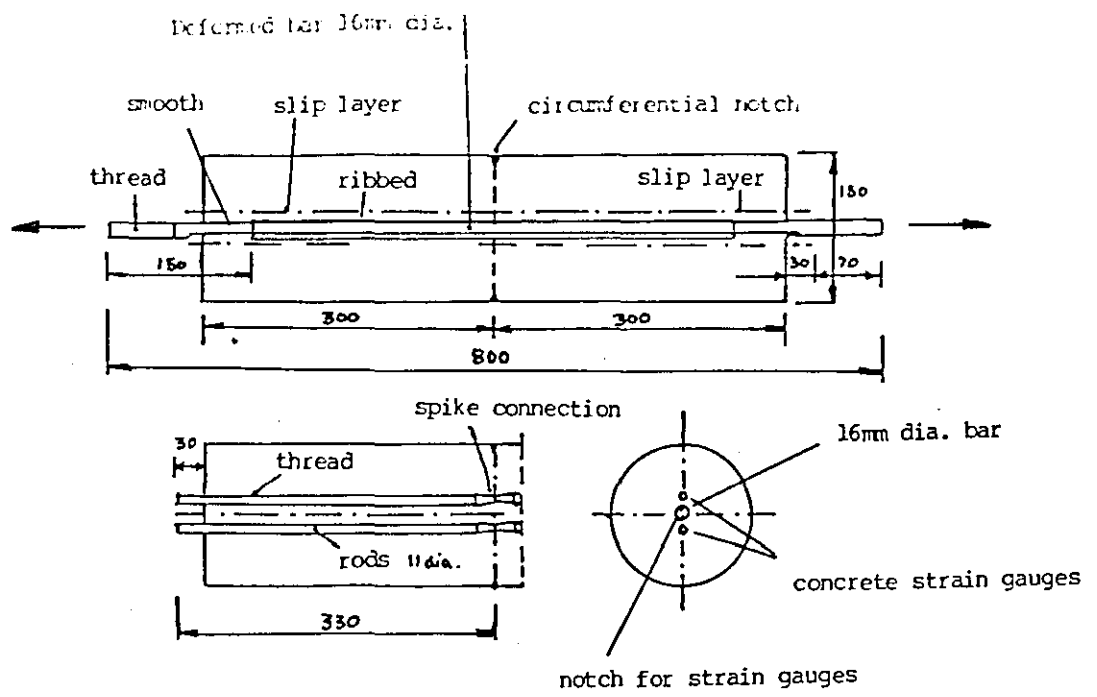


FIGURE 2.4 DETAILS OF CONCENTRIC TENSILE TEST SPECIMEN TESTED BY DORR (1978)

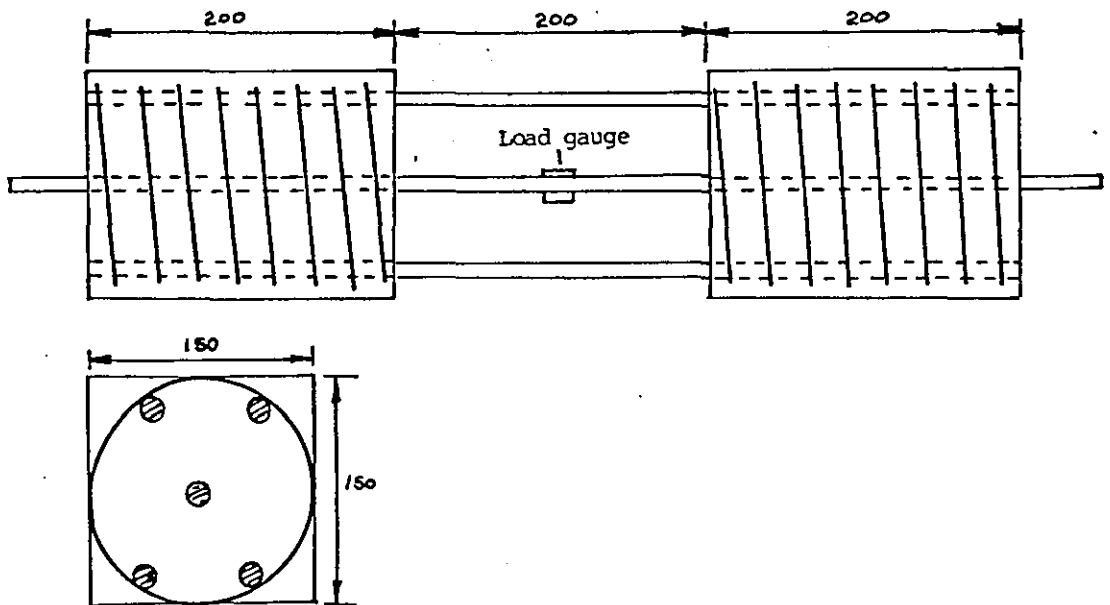


FIGURE 2.5 TRANSFER TEST SPECIMEN AS USED BY SNOWDON (1970)

tensile specimens, Dorr investigated the localised bond stress-slip relationships of 16 mm diameter reinforcement and the influence of hydrostatic pressure. The test results indicated that the local maximum bond stress increased with the hydrostatic pressure and that the slip at maximum bond increased from about 0.06mm at zero pressure to 0.15 mm at a transverse pressure of  $15 \text{ N/mm}^2$  .

Allwood (1980) used a technique of adhering the electrical strain gauges on to the polished outer surface of a reinforcing bar and heavily protected the gauges with waterproofing material and aluminium foil. Allwood investigated the bar stress distribution of the top reinforcement in a beam-column intersection and found the distribution to be markedly different from that which is normally assumed by design engineers.

Spencer, Panda and Mindess (1982) studied the bond of deformed bars in plain and fibre reinforced concrete under reversed cyclic loading. They adopted a technique, very similiar to Dorr's, in that grooves were machined on opposite sides of the deformed reinforcing bar for the electrical strain gauges to be mounted in, and then filled the grooves with epoxy resin to protect the gauges and rebuild the bar profile. The test results indicated that the fibres had no significant effect on the strain distribution in the reinforcing bar, bond stress distribution or bar displacements when compared with plain concrete tests.

Standish (1982) within his work on anchorage bond and the influence of lateral pressure conducted a small number of tests using straingauged reinforcing bars embedded in pullout and semi-beam specimens. He used a technique of gauging very similiar to Perry and Thompson (1966) and investigated the influence of lateral pressure on the bar stress distributions. Due to the limited number of tests,

conclusions based on statistical analyses of the results were not possible.

Viwathanetepa (1979) studied the performance of bars embedded in a well confined column stub. From his studies a mathematical model for predicting the force-displacement relationship for the bar embedded in the column stub was proposed.

### 2.1.3 Average bond stress-slip relationships.

Average bond stress-slip relationships have been obtained from a wide range of bond tests. In pullout type or beam type tests, average bond stress values can be obtained by simply taking the pullout force and dividing by the embedment length and the perimeter of the bar. Many researchers, notably Mathey and Watstein (1961), Ferguson et al. (1965), Losberg and Olsson (1979) and Kemp and Wilhelm (1979) have presented results from their tests as average bond stress against loaded end slip or free end slip, or average bond stress against average slip as with Edwards and Yannopoulos (1979).

Local bond stress-slip relationships have been derived with the need to model bond at the steel-concrete interface in finite element analyses of reinforced concrete structures. These relationships are generally based on measuring steel and concrete strain distributions in bond test specimens. The displacements for both concrete and steel are obtained by integration of the respective strains and the slip at each section as the difference between the steel and concrete displacements. Bond stress is related to the change in steel stress and it is therefore possible to obtain local bond stress-slip relations. Experimental curves have been used by Ngo and Scordelis (1967), Lutz

(1970), Nilson (1968), Shipman and Gerstle (1979) and Allwood (1980). This important area is discussed in more detail in Section 2.2 .

#### 2.1.4 The pullout test.

Many researchers, notably Leonhardt (1957) and Plowman (1957) doubted whether the simple ordinary pullout test was a reliable method for obtaining absolute values of anchorage bond stress. In the ordinary pullout test the concrete near the loaded face is in compression and the free-end slips are not representative of what would occur in actual beams. A variety of different bond test methods was developed in an attempt to reproduce a similiar structural action to that which occurs in actual beams. A comprehensive review and results from a large range of different bond test methods is given by Snowdon (1970). The ordinary pullout has often been modified so that the concrete is in a state of tension or in a state of shear. Abrams (1913) used a double pullout specimen and Snowdon (1970) used a transfer test specimen, Figure 2.5 .

#### 2.1.5 Mechanisms of bond.

Research work on bond has also turned to looking at the fundamental aspects of bond and its mechanisms. Abrams (1913) suggested the use of a bar of very short embedment length to obtain a true bond stress-slip relationship and avoid the embedment length effect. Rehm (1957,1961) performed a pullout test with a single annular rib and a short bond length of the order of one bar diameter. He proposed a 'basic law of bond' of the form :

$$X_R = K_1 \Delta^\alpha + K_2 \quad (2.1)$$

where  $X_R$  = bearing stress / concrete strength

$\Delta$  = slip

$\alpha, K_1, K_2$  are constants  $\alpha < 1$

Rehm also developed an analytical bond stress-slip relation which incorporated the experimentally obtained 'basic law of bond' to predict the bond slip behaviour at any point along the bar.

Several theoretical models have been proposed to account for bond failure. Pinchin (1977) studied the pullout of steel wires from concrete samples and found the stress transfer to be a frictional process. Bartos (1977) in an investigation of bond characteristics of fibrous composites in brittle matrices considered bond resistance to be in two phases. Elastic shear bond allows transfer of stresses across the interface whilst the displacements of the matrix and reinforcement are compatible and frictional shear bond allows transfer of stresses when there is slip at the interface. This failure theory is only applicable to plain round bars whereas deformed bars normally fail by splitting of the concrete. Tepfers (1979) and Cairns (1979) have proposed models that relate the ultimate bond resistance to the cracking resistance of the concrete cover and confining reinforcement.

The bond resistance of both plain bars and deformed bars can be described by the three main mechanisms which are; adhesion, friction and mechanical interlock. Bond of plain bars depends mainly on adhesion and friction, although there is some mechanical interlock. However deformed bars depend mainly on mechanical interlocking for their superior bond with chemical adhesion and friction of secondary importance.

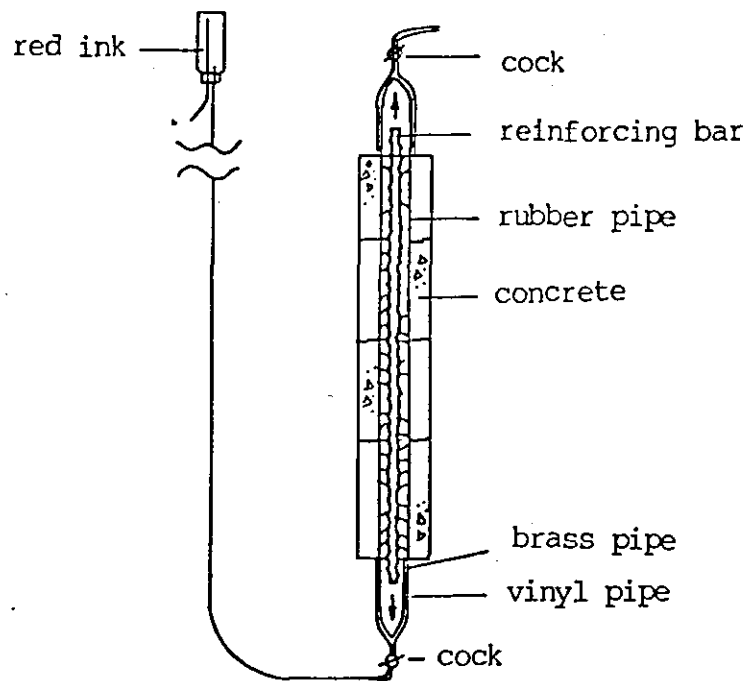
Shrinkage of the concrete matrix surrounding the reinforcement generates the normal force required for a frictional mechanism. Alexander (1969) measured the shear bond and frictional bond between a 1/2 inch cube of steel clamped between two 1/2 inch cubes of cement. The clamping pressure was varied and the load required to initiate

sliding between the steel and concrete measured. Alexander's results showed that the coefficient of friction remains constant until, on continued sliding, the surface of the cement becomes polished and the frictional bond decreases. The coefficient of friction for a cement paste of water-cement ratio of 0.35 on stainless steel decreases from 0.9 to 0.45 at a normal pressure of  $24.8 \text{ N/mm}^2$  (3600 psi). Extensive 'welding' of steel and cement asperities occurs at the higher water-cement ratios such that continued slip occurs entirely within the body of the cement.

Glanville (1930) assumed that the bond stress at any point along an embedded reinforcing bar was dependent on the strains in the concrete due to radial shrinkage and to the stress in the steel. The Poisson's ratio effect of stress in the steel would tend to reduce the interfacial stress. The same principles have been applied by Takaku and Arridge (1973) and Pinchin (1977) to steel fibres embedded in resin and concrete matrices and by Standish (1982) in developing a theory relating ultimate pullout load with lateral pressure for ordinary pullout specimens.

For deformed bars, after adhesion is destroyed and slip occurs, the ribs of the reinforcement bear against the concrete between the ribs and restrain movement. Rehm (1957) and Lutz and Gergely (1967) concluded that failure of bond can result in two ways: (1) the ribs can push the concrete away from the bar by wedging action and (2) the ribs can crush the concrete in front of the lugs. By injecting resin and dyes into the interface of tensile bond specimens Broms (1965) and Goto (1971) have shown that as the ultimate bond strength is reached, transverse cracking occurs and that the concrete moves away from the bar as illustrated in Figure 2.6 .

Goto's instrumentation to detect location and extent of internal cracks



Longitudinal section of axially loaded specimen

Cross section

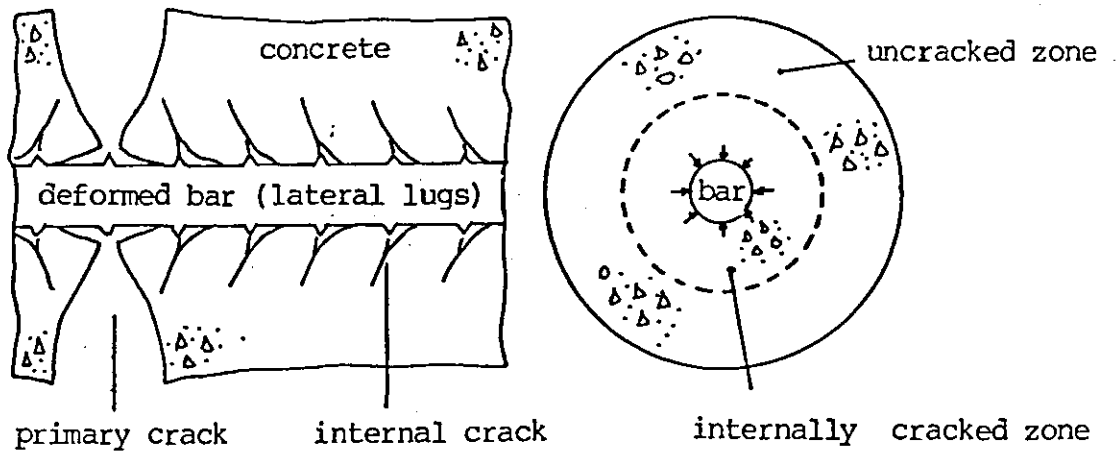


FIGURE 2.6 DEFORMATION OF CONCRETE AROUND REINFORCING BAR  
after Goto (1971)

Several researchers have studied the fundamental aspects of bond at the microscopic level by investigating the chemistry of the interface. Pogany (1940) concluded that adhesion resistance results from the micro-mechanical interlock by the in-growth of gel and crystalline mass into the bar and this has been confirmed by the observations of Plowman (1957) and Brown (1966). Alexander (1969) and Brown (1966) and Schnittgrund and Scott (1976) have shown that the failure interface for frictional sliding can vary between the steel surface and the concrete matrix, and failure for plain bars often occurs within the body of the surrounding concrete. Khalaf and Page (1979) using a scanning electron microscope have shown that an inter-facial zone forms between mild steel and Portland cement paste where failure probably occurs. This interfacial region consists of a discontinuous layer of polycrystalline portlandite which varies in thickness and contains inclusions of calcium silicate hydrate (CSH) gel.

#### 2.1.6 The effect of confining pressures on bond.

Few investigations have been made into the effects of external confining pressure on bond. Untrauer and Henry (1965) investigated the effect on the pullout load of a deformed bar of a uniaxial compressive lateral pressure across the pullout specimen. They found that bond strength increased with the square root of the compressive stress applied to the specimen. The bond strength at failure was expressed in the form of an empirical equation that:

$$f_b = ( A + B \sqrt{\sigma_n} ) \sqrt{f_c} \quad (2.2)$$

where  $f_b$  = bond strength

$\sigma_n$  = applied normal stress

$f_c$  = cylinder compressive strength

A,B = empirical constants obtained from the tests

Robins and Standish (1982) investigated the effect of lateral pressure on both plain round bars and deformed bars in ordinary pullout and semi-beam specimens using both lightweight aggregate (Lytag) and normal weight concrete. A total of 72 cube pullout tests and 79 semi-beam pullout tests were carried out using 8 mm and 12 mm diameter bars with lateral pressures ranging from zero to  $28 \text{ N/mm}^2$ . From the bond tests they developed a theoretical model to predict the pullout load for plain round bars subjected to lateral pressure and found that the pullout load increased linearly with compressive lateral pressure. The results for deformed bars indicated two distinct modes of failure. For lateral pressures from zero to  $10 \text{ N/mm}^2$  the primary cause of failure was splitting or bursting of the concrete surrounding the bar, but for higher lateral pressures failure occurred by shearing of the concrete across the tops of the ribs, leaving a smoothed surface in the concrete. For deformed bars, Standish and Robins indicated that the pullout mechanism was more complicated and could not be predicted using the frictional elasticity approach. Further they noted that any quantitative description would have to reflect the two distinct stages of behaviour.

## 2.2 REVIEW OF FINITE ELEMENT ANALYSES OF REINFORCED CONCRETE INCORPORATING BOND.

### 2.2.1 Introduction.

The finite element method of structural analysis applied to reinforced concrete has been very adequately described in a recent state-of-the-art report by the American Society of Civil Engineers (1982). The following review is limited to finite element analyses of reinforced concrete which incorporate some form of connection between the steel and concrete and which do not assume complete compatibility between the two materials. There has been in general a trend to assume perfect bond between steel and concrete in the analyses of reinforced concrete. There are three alternative representations of the reinforcement which may be used and these are: (1) distributed, (2) embedded, and (3) discrete. Distributed and embedded representations both presume perfect bond. A discrete representation of the reinforcement allows one-dimensional reinforcement elements to be superimposed on a two-dimensional mesh of concrete elements and it allows bond elements or links to be used. The importance of a realistic representation of the transfer of forces by bond has been illustrated by Khouzam (1977) in the analysis of a tensile bond specimen and by Allwood (1980) in the analysis of a beam-column intersection. Without a representation of this nature, incorrect structural behaviour may be predicted by the finite element method.

### 2.2.2 Review.

The earliest application of the finite element method to reinforced concrete structures was by Ngo and Scordelis (1967). They analysed simple beams in which the concrete and reinforcement were represented by two-dimensional triangular elements and they used special bond links to connect the steel to the concrete as shown in Figure 2.7 .

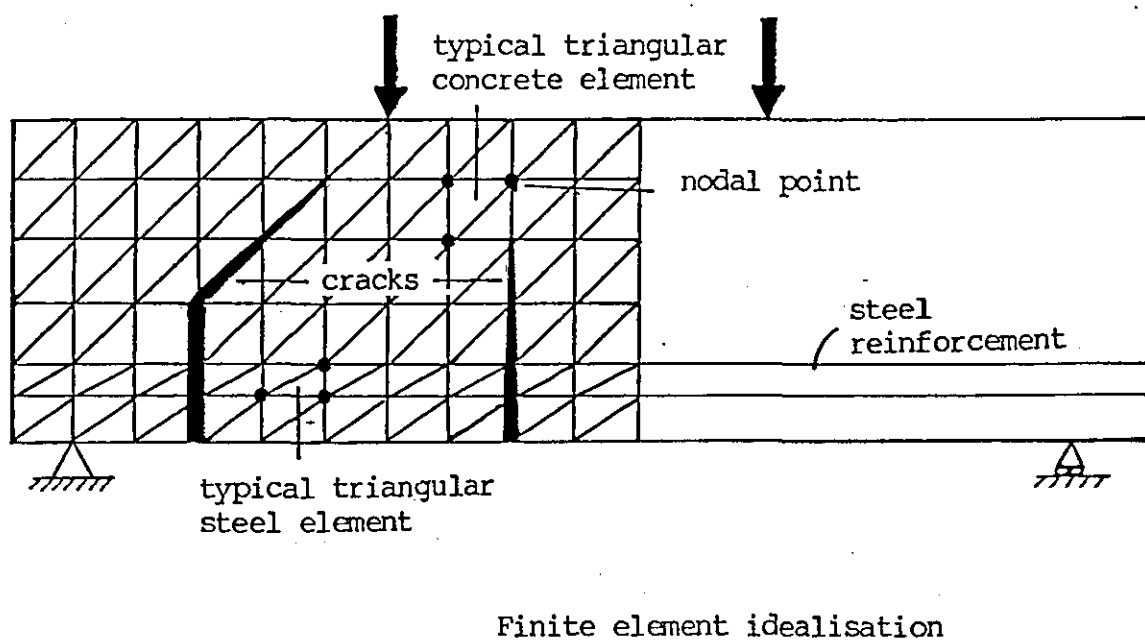
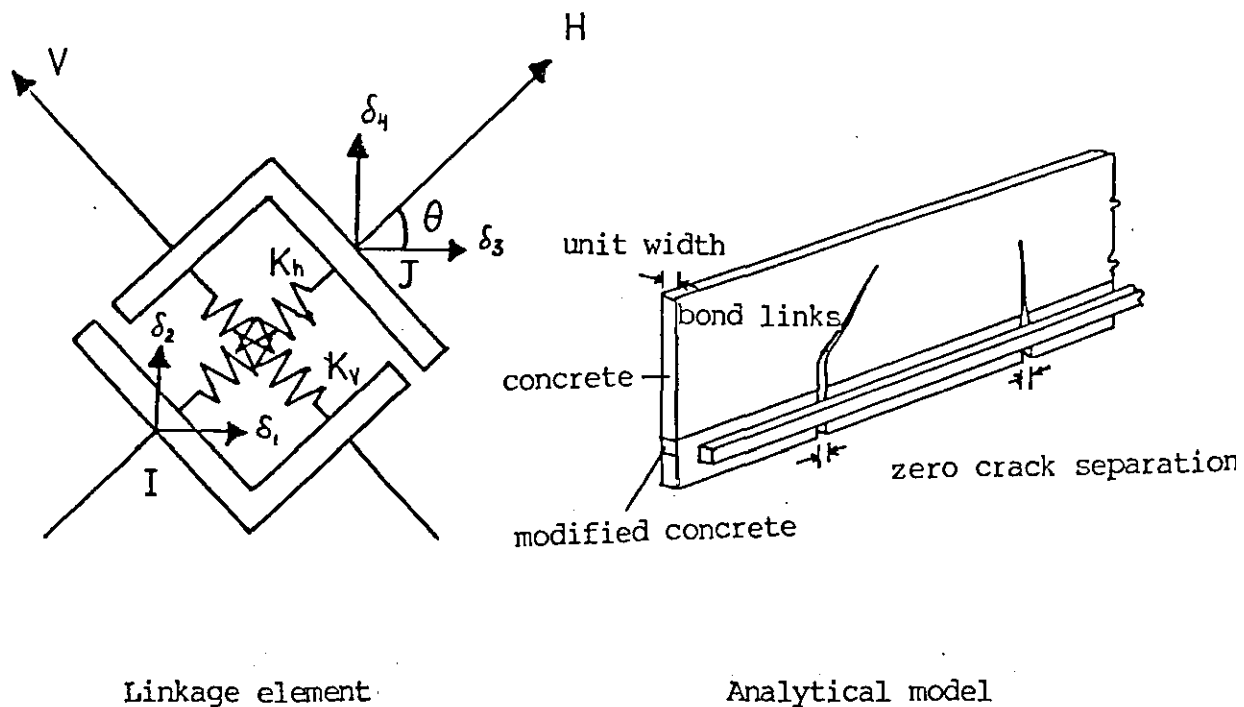


FIGURE 2.7 LINKAGE ELEMENT AND ANALYTICAL MODEL  
after Ngo and Scordelis (1967)

The linkage element which was used can be conceptually thought of as comprising of two linear springs orthogonal to each other but with no physical dimension at all. Arbitrary stiffness values were assigned to the springs, namely  $2.2 \times 10^6$  lb/in at 6 inch intervals parallel to the reinforcement and a very high stiffness to the springs orthogonal to the bar. Ngo and Scordelis performed linear elastic analyses on beams with different pre-defined crack patterns and illustrated the effect of cracking on the distribution of steel stresses and bond stresses. Nilson (1968) developed this method of analysis by introducing non-linear material properties for the concrete and a non-linear bond stress-slip relationship. The bond stress-slip equation was derived indirectly from experiments reported by Bresler and Bertero (1966), who studied the distribution of steel strain in a concentric tensile specimen. The local bond stress to local bond slip relationship derived from these tests was as follows:

$$u = 3606 \times 10^3 d - 5356 \times 10^6 d^2 + 1986 \times 10^9 d^3 \quad (2.3)$$

where  $u$  = local bond stress (psi)

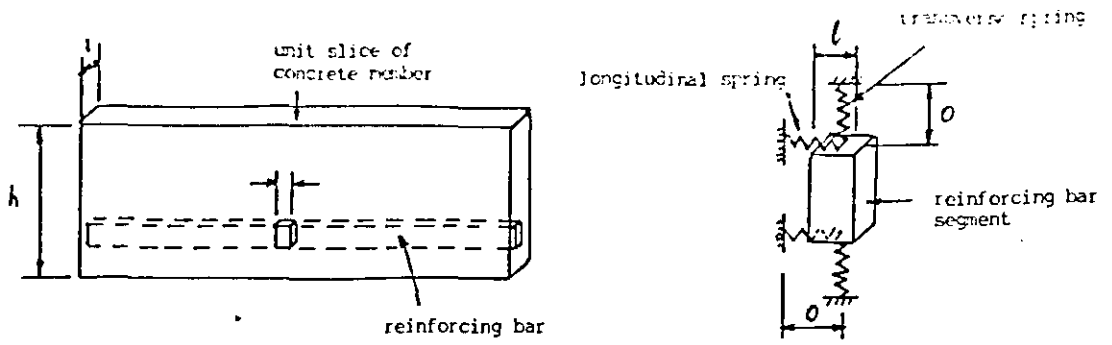
$d$  = local bond slip (in)

Nilson recognised a basic difference in bond behaviour of an elemental area deep within a concrete block (interior linkage) and that of an element near a crack face (exterior linkage). For interior linkages the local bond stress was assumed to remain constant at its maximum value for slips in excess of a certain tolerance; however for exterior linkages the bond stress at slips in excess of the tolerance was assumed to be reduced to zero. A non-linear biaxial stress-strain relationship for the concrete was adopted which assumed the concrete to be an orthotropic material and used the uniaxial compression equation proposed by Saenz (1964). The progressive cracking of the

concrete was accounted for by stopping the solution when an element indicated a tensile failure and redefining the topography of the structure. A crack was established between two elements along their common edge and the mesh redefined by disconnecting the elements at their common corners , Figure 2.8 . Both concentric and eccentric tensile pullout specimens were analysed and the results checked against the experimental results of Broms (1965).

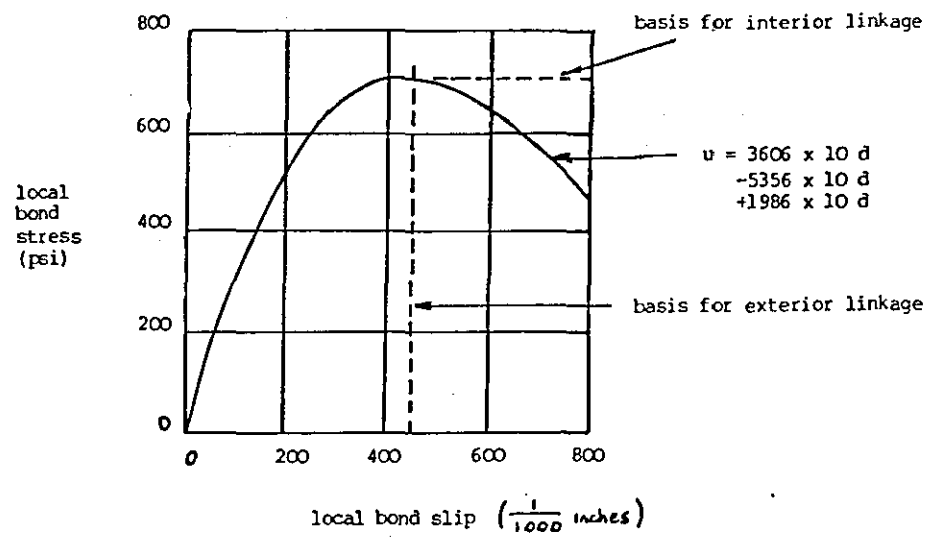
Lutz and Gergely (1967) performed an elastic finite element analysis on a concentric tensile bond specimen like Nilson but assumed perfect bond. However they found that at low working stresses in the reinforcement, the interfacial tensile stress would exceed the tensile adhesion strength even when allowing for compression due to shrinkage and concluded that separation of the bar and concrete would occur. In the case of plain bars this separation would mean complete loss of bond whereas for deformed bars bond would still occur by bearing of the concrete against transversely orientated ribs. Later Lutz (1970) performed axi-symmetric finite element analyses on the same problem but with special provision for slip and separation between the steel and the concrete. Several analyses were performed making different assumptions about the amount of slip, radial separation and the length of the reinforcement over which slip and separation were allowed to occur. Lutz and Gergely surmised that both a bond stress-slip relationship and an allowance for radial separation were required in the analyses for the best approximation to the experimental results.

Franklin (1970) enhanced the analytical method by developing a non-linear analysis which used incremental loading with iterations within each increment and automatically allowed for the non-linear properties of the materials, the cracking of the concrete elements



Model of reinforced concrete member

Detail of bond linkage



Bond stress-slip curves used in analysis

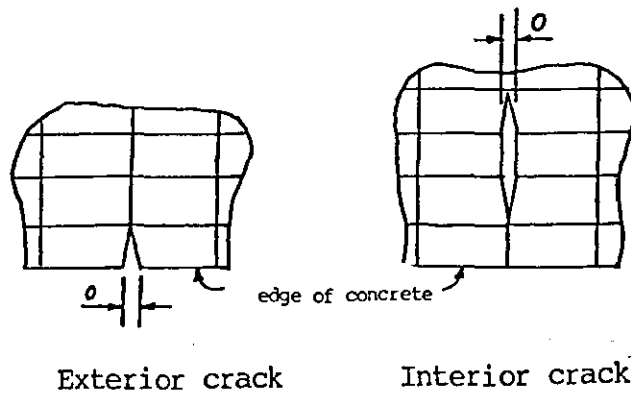


FIGURE 2.8 REPRESENTATION OF BOND FORCE AND CRACKING  
IN THE CONCRETE (Nilson, 1967)

and redistributed forces within the system. Franklin was primarily concerned with studying reinforced concrete frames with and without concrete shear walls but used two dimensional bond links within his analyses.

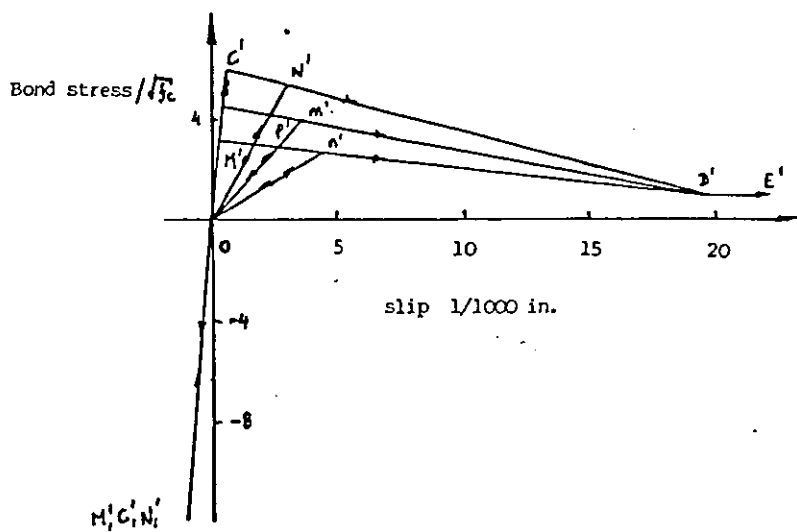
Robins (1971) used the finite element method to simulate the post-cracking behaviour of reinforced concrete deep beams. Two dimensional triangular finite elements represented the steel and the concrete, and the spring linkage elements were included to simulate bond. Loads were applied incrementally and the non-linear behaviour and cracking of the concrete automatically allowed for. Cracks were allowed to occur within the elements themselves by modifying the element moduli. Comparison of the theoretical and experimental deflections and crack pattern showed that the model gave fairly reliable information on the overall behaviour of a deep beam under test.

Houde (1973) derived an empirical bond stress-slip formula from a series of tests on 62 axially reinforced tensile specimens. This relationship, plus the use of empirical constitutive relationships for the non-linear effects of dowel action and aggregate interlock were incorporated into a finite element model. The developed analytical method used an incremental load approach and a numerical procedure of successive approximations to take into account the non-linear behaviour of the concrete and its cracking. Pullout specimens and beams with varying steel percentages and shear spans were analysed through all load stages. The results compared favourably with the available experimental data.

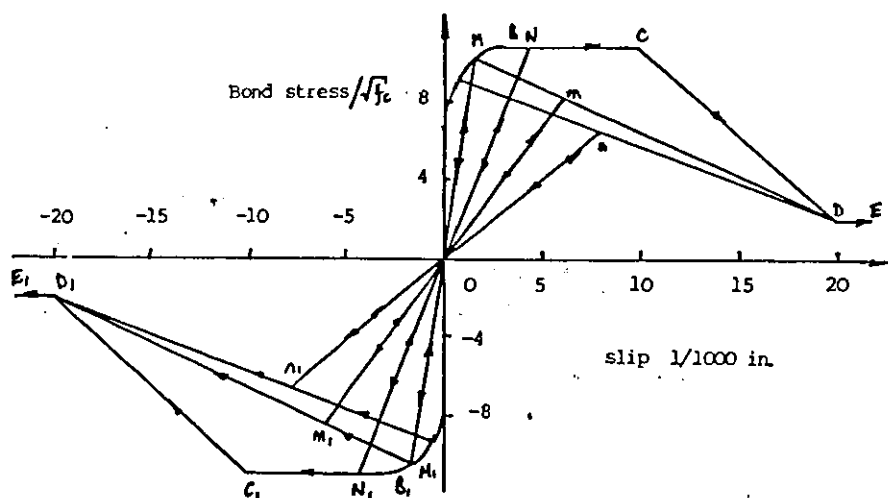
Labib (1976) and Labib and Edwards (1978) used a non-linear finite element model in an investigation of cracking of concentric

and eccentric reinforced concrete tension members. Simple four noded elements were used to represent the concrete, bar elements for the steel and longitudinal and transverse spring linkage elements were used to simulate bond. The analytical solution used an iterative solution technique known as the 'initial stress method' within an incremental loading process. The global stiffness matrix was updated at the end of each increment and during the iteration process if the number of iterations exceeded a prescribed number. A careful analysis of the cracking of the concrete was made by allowing only one concrete element to crack in any one iteration. Idealised and unsubstantiated bond stress-slip and transverse force-slip relationships were adopted as shown in Figure 2.9 . These bond curves are modified by both transverse and longitudinal cracking with a gradual deterioration in bond. For slips in excess of  $20/1000^{\text{th}}$  inch in either the longitudinal or transverse direction, a nominal retention of 20 percent of the maximum bond stress was assumed. For the analyses of both concentric and eccentric tensile members as tested by Broms (1965), there was favourable agreement with the experimental steel stress and bond stress distributions and the cracking patterns.

Khouzam (1977) used a finite element method similiar to Nilson (1968) and Houde (1973). The computer program automatically modified the structure stiffness to account for cracking of the concrete and included the non-linear effects of bond, dowel action and aggregate interlock. Khouzam analysed the concentric tensile specimen used in Broms (1965) experimental work, as did Nilson, Houde and Labib. The specimen T-RC3-1 consisted of a concrete block, 33 in. x 8.1 in. x 3.5 in. , reinforced axially with a #8 steel bar with the forces applied to the protruding ends. All four researchers have shown reasonable agreement of their finite element analyses with the



Bond perpendicular to the bar



Theoretical bond-slip curve

FIGURE 2.9 BOND STRESS-SLIP RELATIONSHIP PARALLEL AND ORTHOGONAL TO THE BAR (Labib and Edwards, 1978)

experimental results, but Nilson and Houde noted that the computed response was stiffer than the observed one. To decrease the stiffness of the model, Khouzam modified the interface behaviour by several different methods, namely: inclining the spring linkages at various angles and connecting the steel at different levels to the concrete. In addition, she modified the shear retention factor of the concrete and analysed the specimen with concrete elements which transferred no shear when cracked. However Khouzam surmised that the computed response with some shear transfer showed more favourable agreement with the experimental results than by allowing no shear transfer in the cracked concrete.

Shipman and Gerstle (1979) noted that serious discrepancies had been observed between the predicted and observed response of reinforced concrete panels subjected to load cycles. They hypothesized that these differences were due to the neglect of bond slip and incorporated the bond stress-slip relationship derived by Nilson into their finite element model. The results of their analytical studies showed that by accounting for bond slip, a closer approximation to the observed response was obtained. However bond slip did not account for all of the difference in response and they concluded that by including concrete deterioration the remaining difference might be accounted for.

Viathanatepa, Popov, Bertero (1979) conducted a detailed study into the push-pull, push only loading on single bars embedded in well confined column stubs under both monotonic and cyclic loading. The results from experimental tests provided the empirical data for the bond stress-slip relationships. A non-linear axi-symmetric finite element analysis which used incremental loading with a Newton-Raphson

iteration technique was performed. The concrete and reinforcement were represented by four-noded linear strain axi-symmetric isoparametric quadrilateral elements and bond by a soft layer of concrete surrounding the bar. The behaviour of the soft layer was described by a shear stress-shear strain relation obtained from the test data. The average shear stress and deformation at the centre of the element were used to represent the overall behavior of the element and by adopting this method the element was mathematically equivalent to a spring linkage.

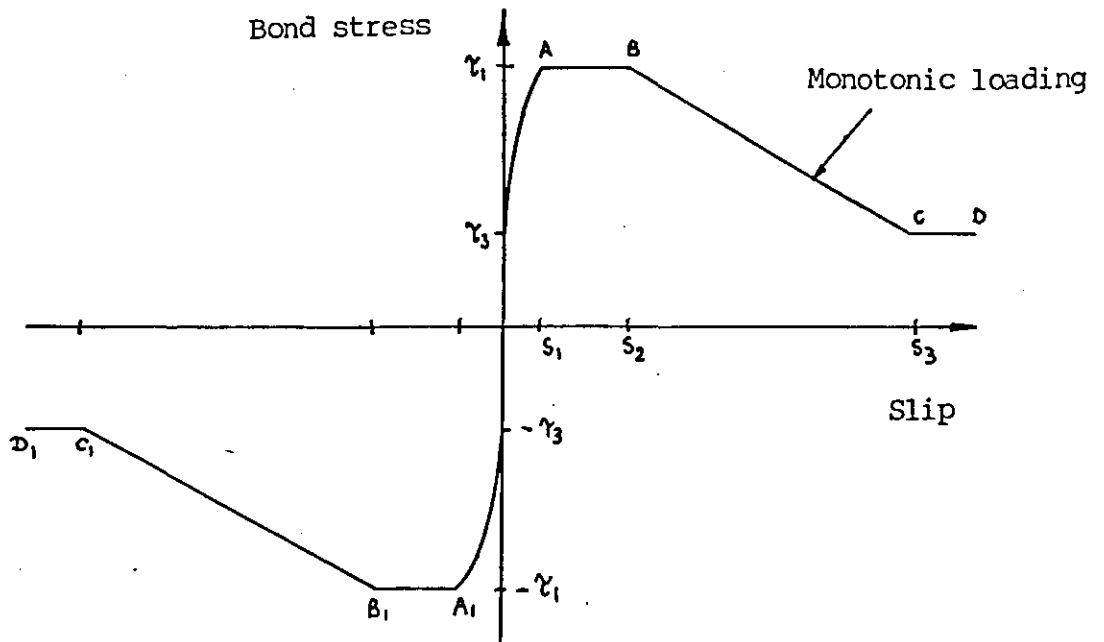
Ciampi, Eligenhausen et al. (1981) simplified the original force-displacement relations of Viathanatepa and the curves are shown in Figure 2.10 . The monotonic loading curve is non-linear to a maximum bond stress, and then the stress decreases linearly to the value of ultimate frictional bond resistance. Instead of using the finite element method to solve the anchorage problem they used a step-by-step method to solve the non-linear differential equation of bond.

Allwood (1980) analysed a beam-column intersection in stages using a simple linear elastic finite element analysis with 8-noded isoparametric elements representing the concrete, bar elements for the steel and spring connectors to simulate bond. Three separate analyses were made: (1) assuming perfect bond, (2) using a bond stress-slip relationship based on the work of Edwards and Yannopoulos (1978) and (3) a constitutive relation derived from his own test results. The analytical studies illustrated the need to model bond rather than to assume perfect compatibility between steel and concrete if the predicted behaviour is to agree reasonably with the observed behaviour.

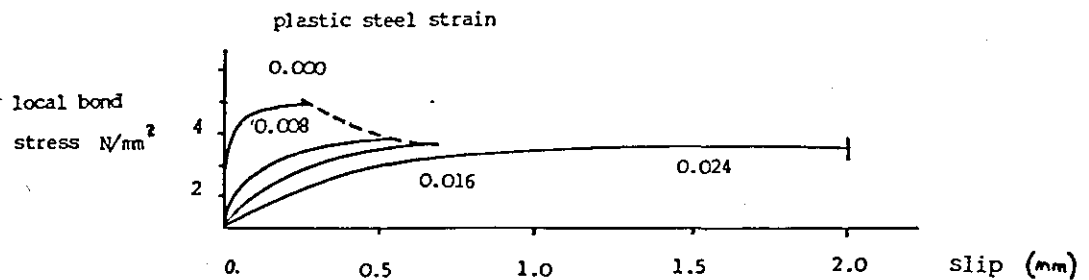
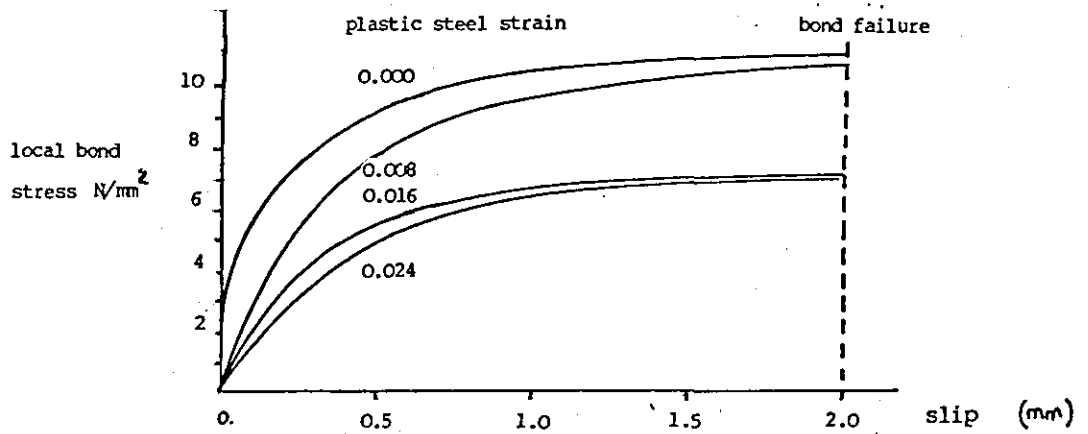
Hungspreung (1981) studied the fundamental behaviour of localised bond under high level cyclic loading, by the use of a simple

FIGURE 2.10 PROPOSED ANALYTICAL MODEL FOR BOND STRESS

After Ciampi, Eligehausen et al. (1981)



Bond stress-slip relations for bond in uncracked regions or in some distance from cracks



Bond stress-slip relations for bond near cracks

FIGURE 2.11 BOND STRESS-SLIP RELATIONSHIPS IN UNCRACKED AND CRACKED CONCRETE REGIONS (Plauk and Hees, 1981)

pullout test. The finite element studies made use of the experimentally obtained bond stress-slip relation. The idealised bond stress-slip relation used was a part non-linear, multi-linear curve and for slips less than 0.0014 inches the relationship was :

$$u = 1.77 \times 10^6 d - 1.28 \times 10^9 d^2 + 0.45 \times 10^{12} d^3 \quad (2.4)$$

where  $u$  = local bond stress (psi)

$d$  = local bond slip (in)

The ultimate load and maximum slip were expressed in terms of the confining pressure, namely :

$$u = 0.27 p + 0.1 \quad (2.5)$$

$$\text{and } P_u = 20. p + 18.75 \quad (2.6)$$

where  $u$  = slip (in)

$p$  = confining pressure (ksi)

$P_u$  = pullout load (kips)

Plauk and Hees (1981) used a two-dimensional non-linear finite element model to analyse reinforced concrete beams. For both concrete and the reinforcement, quadrilateral plane stress elements were used and spring linkages to simulate bond. The bond modelling was based on an experimental study of bond by Eifler (1974). The local bond stress-slip relations are illustrated in Figure 2.11 as a function of the plastic steel strain and for bond near cracks. Plauk and Hees found that the analytical results agreed very well with the experimentally observed behaviour and concluded that different bond stress-slip relations were required for regions near and away from cracks.

## 2.3 BRIEF REVIEW OF CONCRETE MODELS FOR FINITE ELEMENT ANALYSES OF REINFORCED CONCRETE

### 2.3.1 Introduction.

A finite element analysis of reinforced concrete must represent the multi-dimensional stress-strain relationships and failure behaviour of the concrete. There are a number of ways of defining the complicated stress-strain behaviour of concrete under various stress states and in general these fall into three groups; (1) elasticity based models including variable moduli and equivalent uniaxial approaches, (2) perfect and work-hardening plasticity theory and (3) endochronic theory. The variable moduli and equivalent uniaxial strain approaches are both very popular methods of describing the biaxial stress-strain behaviour of concrete and many researchers have derived empirical stress-strain relations in terms of principal stresses and strains based on fitting curves to biaxial test data. Triaxial analyses have also been made by assuming the concrete behaviour to be incrementally elastic with variable moduli. However the three-dimensional stress-strain behaviour under generalised loading cannot be adequately described by a variable moduli approach, and current research is heading towards the development of triaxial relations based on plasticity and endochronic theory.

The following section briefly reviews the observed behaviour of concrete under multi-axial stress and the various methods of describing the stress-strain behaviour. A comprehensive state-of-the-art report on constitutive relations and failure theories for concrete can be found in the American Society of Civil Engineers publication, 'Finite Element Analyses of Reinforced Concrete' (1982) and in the literature of Chen (1982).

### 2.3.2 Concrete behaviour under loading.

At low levels of loading up to about 30 percent of the uniaxial compressive strength ( $f'_c$ ), concrete behaves as a linear elastic material, but at higher levels of loading it exhibits a very pronounced non-linear behaviour. According to Liu et al. (1972), this behaviour is due to the onset of extensive microcracking which starts at the aggregate-mortar interfaces, and then extends between the aggregates. Typical stress-strain curves for concrete subjected to uniaxial compressive stress are shown in Figure 2.12. In tension, concrete behaves as a linear elastic material up to failure and the tensile strength ( $f'_t$ ) is an order of magnitude smaller than the compressive strength ( $f'_c$ ) and often taken to  $0.1 f'_c$  (e.g.) Buyukozturk (1977). The stress-strain behaviour under various combinations of biaxial stress is very different from that under uniaxial loading. A typical biaxial strength envelope for concrete from the test data of Kupfer et al. (1969) is illustrated in Figure 2.13. Kupfer et al. (1969) and Liu et al. (1972), both found that the compressive strength of concrete under biaxial compression increased by up to about  $1.25 f'_c$ . Under biaxial tension, concrete exhibits a constant tensile strength but under a combination of tension and compression, it exhibits a greatly reduced strength. Typical stress-strain relationships under biaxial compression, combined tension and compression, and biaxial tension are illustrated in Figures 2.14, 2.15 and 2.16.

The biaxial failure envelopes of Kupfer et al. (1969) have been widely accepted as a basis for the development of biaxial models and the failure regions can be expressed in terms of the uniaxial compressive strength ( $f'_c$ ) and the uniaxial strength ( $f_t$ ) as follows:

$$\text{Biaxial tension} \quad \sigma_{1t} = \sigma_{2t} = f_t \quad (2.7)$$

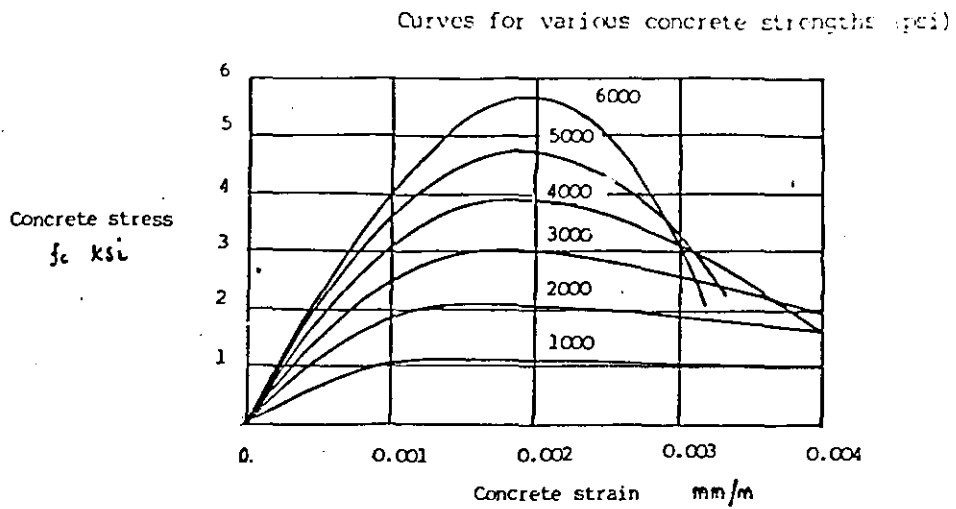


FIGURE 2.12 TYPICAL CONCRETE STRESS-STRAIN CURVE IN COMPRESSION  
After Winter and Nilson (1979)

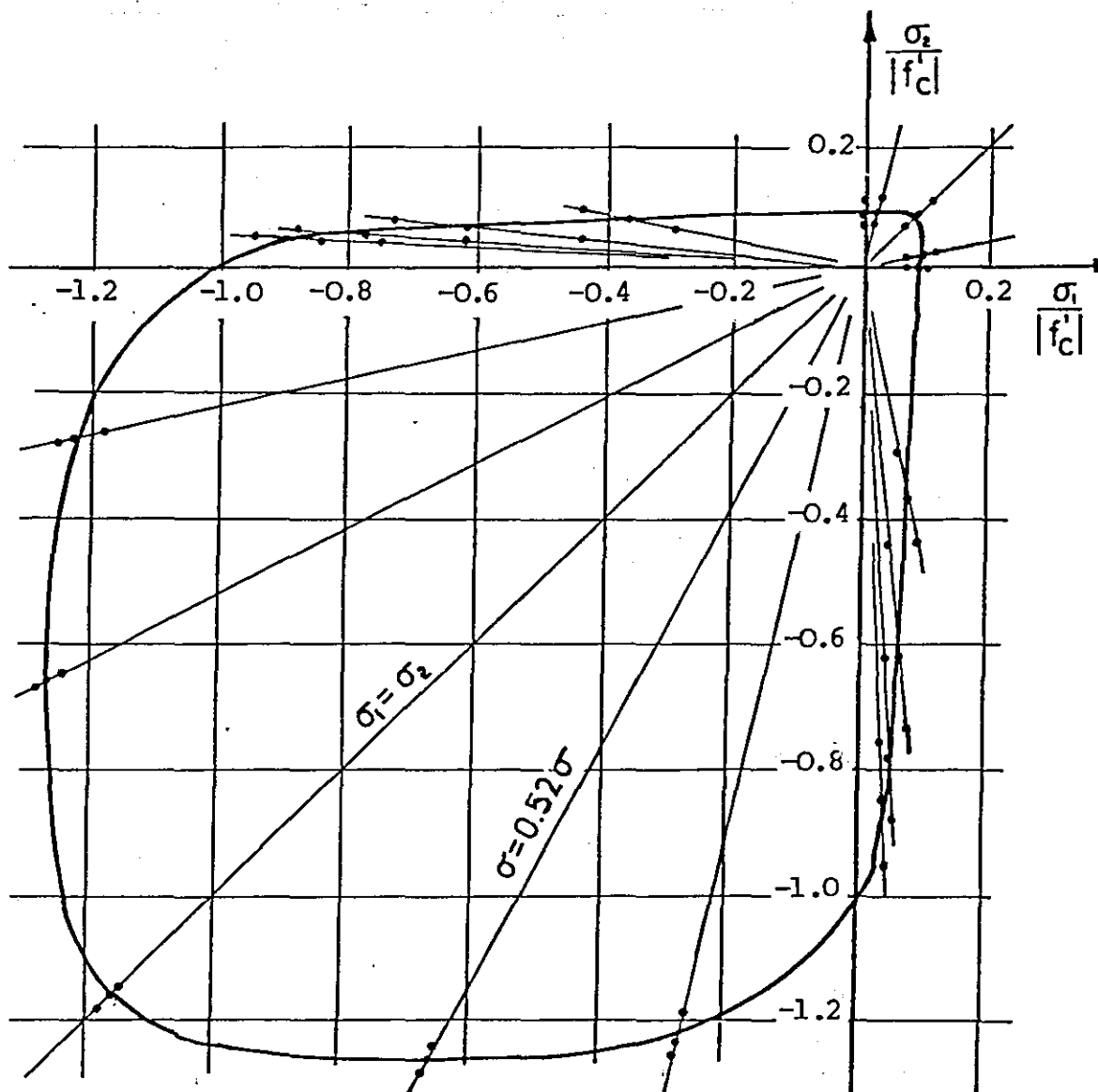


FIGURE 2.13 BIAXIAL FAILURE ENVELOPE (Kupfer et al., 1969)

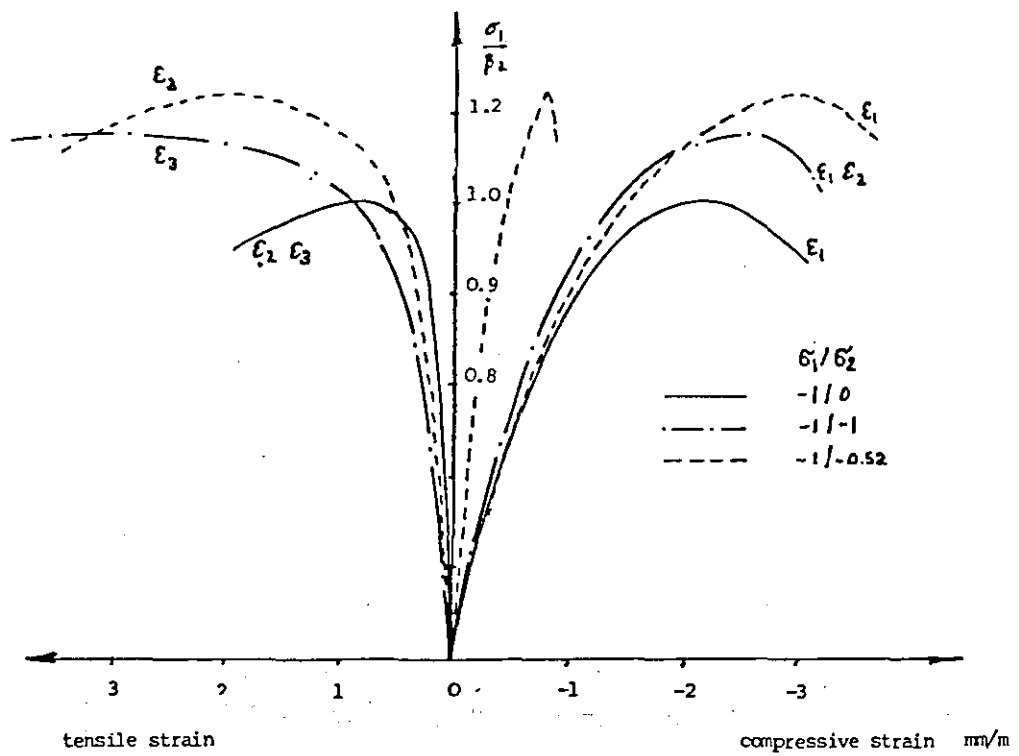


FIGURE 2.14 STRESS-STRAIN RELATIONSHIPS OF CONCRETE  
UNDER BIAXIAL COMPRESSION (Kupfer et al., 1969)

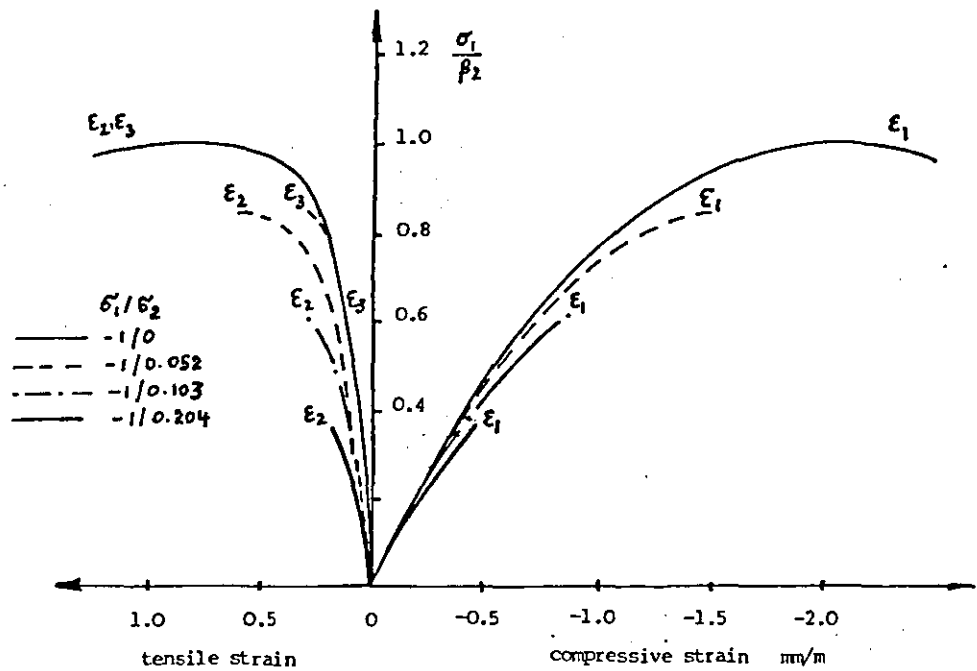


FIGURE 2.15 STRESS-STRAIN RELATIONSHIPS OF CONCRETE UNDER  
COMBINED TENSION AND COMPRESSION (Kupfer et al., 1969)

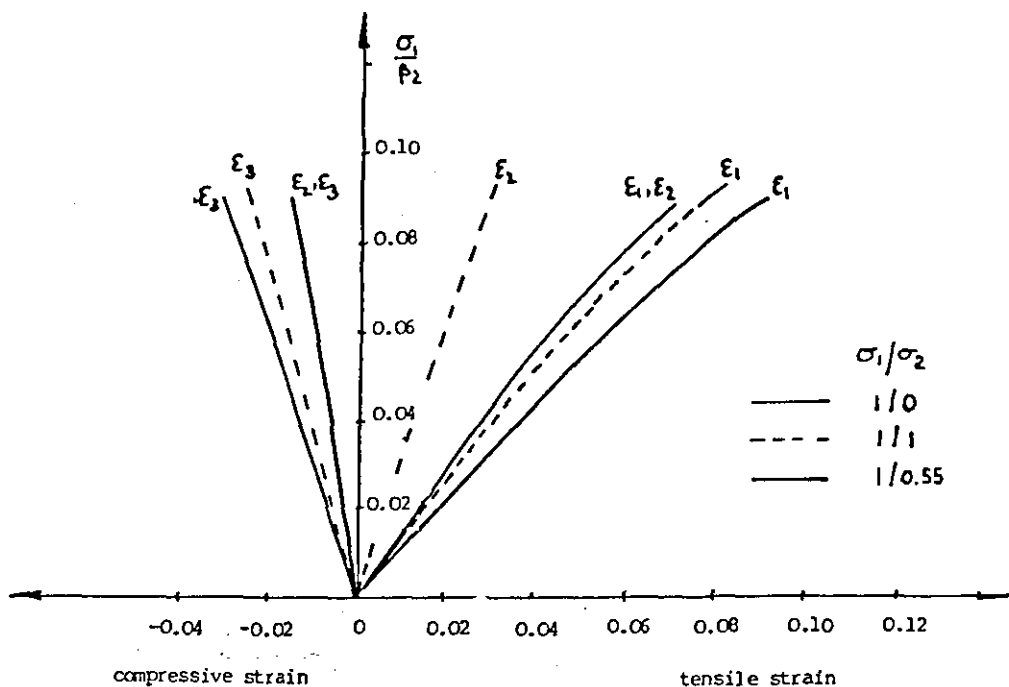


FIGURE 2.16 STRESS-STRAIN RELATIONSHIPS OF CONCRETE UNDER  
BIAXIAL TENSION (Kupfer et al., 1969)

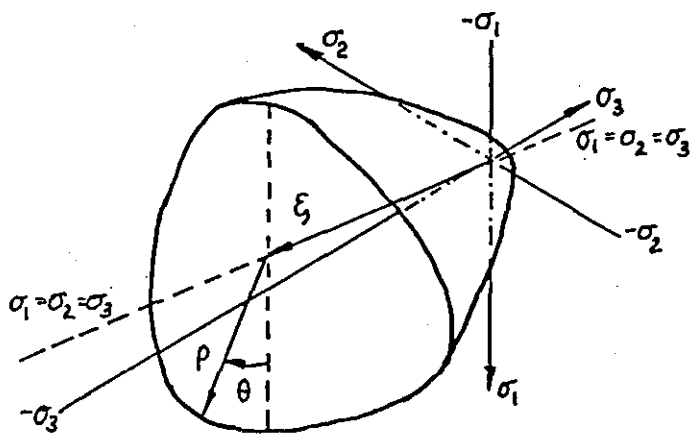


FIGURE 2.17 TRIAXIAL STRENGTH ENVELOPE IN PRINCIPAL  
STRESS SPACE

$$\text{Tension - compression} \quad \sigma_{1t} = (1 - 0.8 \frac{\sigma_2}{f'_c}) f_t \quad (2.8)$$

$$\text{Biaxial compression} \quad \sigma_{2c} = - \left( \frac{1 + 3.65 \alpha}{(1 + \alpha)^2} \right) f'_c \quad (2.9)$$

$$\text{where } \sigma_{1t} = \sigma_{2c}$$

$$\alpha = \sigma_1 / \sigma_2$$

$$\sigma_{1t}, \sigma_{1c} = \text{principal stresses in tension and compression}$$

$$\text{and where } \sigma_1 > \sigma_2$$

Concrete when subjected to triaxial stress states generates a fairly consistent failure envelope as shown in Figure 2.17 , which is expressed in terms of principal stresses. Several empirical relationships have been developed to model the concrete failure surface in three dimensions, examples are Kotsovos and Newman (1978), Ottosen (1977), William and Warnke (1974) and Ahmad and Shah (1982). Studies of the stress-strain behaviour and strength of concrete under multi-axial stress states have shown a large scatter of results and a major co-operative study is currently being undertaken by Gerstle et al. (1980) in an attempt to provide a unified formulation of triaxial stress-strain data.

### 2.3.3 Variable moduli models.

The most popular models for concrete for use in finite element analyses have made use of the fact that, general structural problems can be reasonably modeled as a two-dimensional plane stress problem. Several representations have used biaxial type models in which either isotropic total stress-strain relations or incremental isotropic or incremental orthotropic stress-strain relations were assumed. The earliest representations of concrete in finite element analyses by Ngo and Scordelis (1967) assumed concrete to be a linear isotropic

material. Franklin (1970) and Nilson (1968) advanced the concrete model by using non-linear material properties under compressive loading, however ignored the effective increase in strength under biaxial compressive stresses.

Kupfer and Gerstle (1973) from Kupfer et al.'s (1969) experimental data devised an isotropic total stress-strain model. By presenting their data in terms of octahedral stress and strain invariants, Figures 2.18, 2.19 , they were able to obtain a unique relationship for all compressive stress-states, however for uniaxial compression and compressive-tension states the uniqueness disappeared. Their expressions were presented in terms of the secant bulk ( $K_s$ ) and shear moduli ( $G_s$ ) as shown in Figures 2.20 and 2.21 . The tangential bulk and shear moduli are related to the modulus of elasticity and Poisson's ratio and the stress-strain relationships were re-expressed in terms of the tangential bulk and shear moduli. Murray et al.(1979) have shown and Kupfer and Gerstle (1973) admitted, that to replace the tangential moduli by secant moduli was not rigorously defensible. Kupfer and Gerstle also indicated that the formulation excludes load histories for which stress-induced anisotropy is important, which occurs for concrete subject to high levels of biaxial loading. By their own admission Kupfer and Gerstle obtained a poor match with the experimental data at high levels of stress. Despite the weakness of their approach, it has proved to be quite useful and several other researchers have adopted and extended the approach for use in presenting triaxial data, examples being: Kotsovos and Newman (1978), and Montague and Kormi (1982). Phillips and Zienkiewicz (1976) used a simplified version of the secant shear and bulk moduli expressions in the analysis of a prestressed concrete pressure vessel.

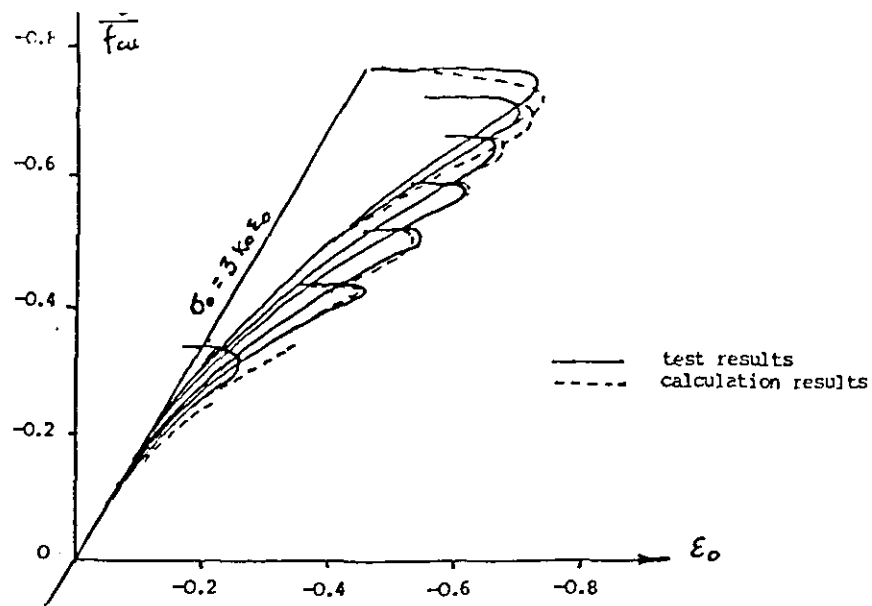


FIGURE 2.18 OCTAHEDRAL NORMAL STRESS-STRAIN RELATIONSHIP  
WITH PARAMETER  $\sigma_1/\sigma_2$  (Kupfer and Gerstle, 1973)

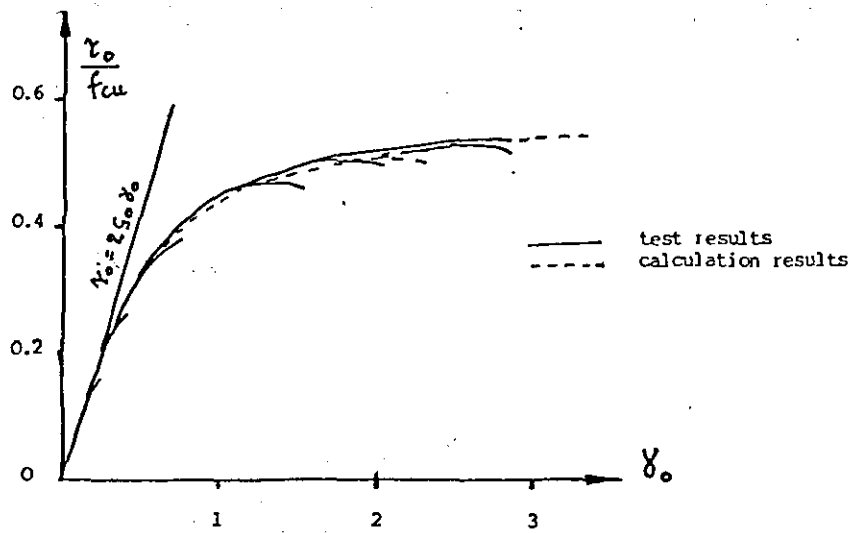


FIGURE 2.19 OCTAHEDRAL SHEAR STRESS-STRAIN RELATIONSHIP  
WITH PARAMETER  $\sigma_1/\sigma_2$  (Kupfer and Gerstle, 1973)

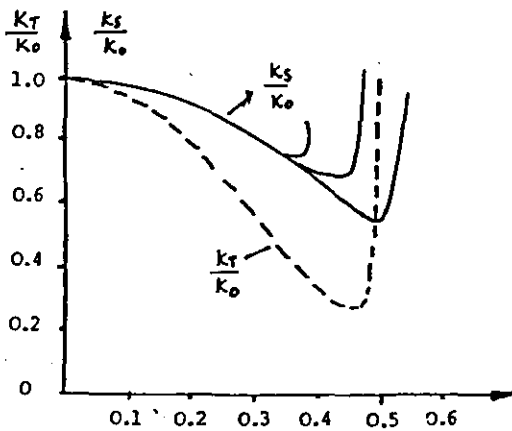


FIGURE 2.20 BULK MODULUS K - OCTAHEDRAL SHEAR STRESS  
RELATION Kupfer and Gerstle (1973)

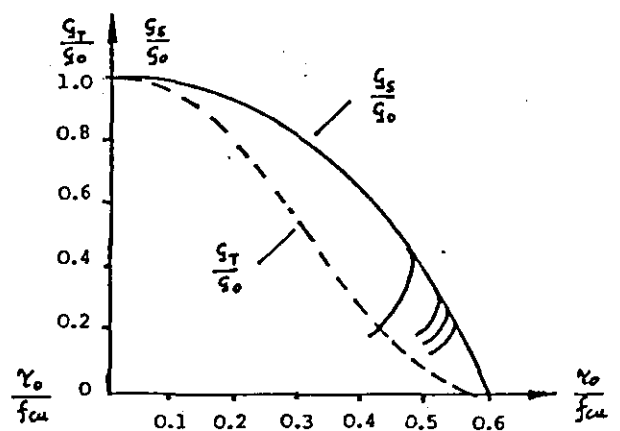


FIGURE 2.21 SHEAR MODULUS G - OCTAHEDRAL SHEAR STRESS  
RELATION Kupfer and Gerstle (1973)

#### 2.3.4 Equivalent uniaxial models.

Based on the experimental data of Kupfer et al.(1969) a number of models were developed using the observed stress-induced anisotropy under biaxial stress. Two particular models were developed which considered concrete to be a biaxial orthotropic material and in which the moduli of elasticity along the principal stress axes were a function of stress and strain.

Liu et al.(1972) devised a model based on total strains, in which the principal stresses  $\sigma_1$  and  $\sigma_2$  were functions of the principal strains  $\epsilon_1$  and  $\epsilon_2$ , the ratio of principal stresses  $\alpha = \sigma_1/\sigma_2$  and Poisson's ratio. The complete stress-strain curves were expressed independently in each principal stress direction, but in order to obtain a biaxial tangential constitutive matrix, the expressions for the principal stresses were modified since they already contained the Poisson's ratio effect. The method however, required that the principal stress axes coincided with the principal strain axes, which does not necessarily happen for concrete under general loading. A further restriction is that for a stress ratio of 0.2 , the slope of the stress-strain curve in the minor compressive direction becomes infinite. Labib (1976) using the finite element model to analyse reinforced concrete components, modified the expressions of Liu et al. to overcome the latter problem.

Darwin and Pecknold (1974) proposed an incremental orthotropic model based on the concept of 'equivalent uniaxial strain' , whereby the coupling effect due to Poisson's ratio was eliminated and the behaviour was represented by equivalent uniaxial stress-strain curves for each of the principal stress axes. The incremental equivalent uniaxial strain in the  $i^{\text{th}}$  principal direction was given by :

$$d \varepsilon_{iu} = d\sigma_i / E_i \quad (2.10)$$

$d \varepsilon_{iu}$  = incremental change in equivalent uniaxial strain

$d\sigma_i$  = incremental change in principal stress

$E_i$  = modulus of elasticity

The total equivalent uniaxial strains in the principal stress directions were given by the integration of the incremental uniaxial strains over the loading path,

$$\varepsilon_{iu} = \int \frac{d\sigma_i}{E_i} \quad (2.11)$$

The equivalent uniaxial strains were not real strains and their significance was to be regarded only as a measure of the deformation history of the concrete.

The incremental stress-strain relations for the orthotropic concrete took the form :

$$\begin{bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\sigma_{12} \end{bmatrix} = \frac{1}{(1 - \nu_1 \nu_2)} \begin{bmatrix} E_1 & \nu_2 E_1 & 0 \\ \nu_1 E_2 & E_2 & 0 \\ 0 & 0 & (1 - \nu_1 \nu_2)G \end{bmatrix} \begin{bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\varepsilon_{12} \end{bmatrix} \quad (2.12)$$

where  $\nu_1 E_2 = \nu_2 E_1$

An equivalent Poisson's ratio was defined as  $\nu^2 = \nu_1 \nu_2$  and the shear modulus  $G$  was assumed to be independent of the axes of orientation such that ,  $(1-\nu^2)G = 1/4 (E_1+E_2 - 2\nu\sqrt{E_1 E_2})$ . The modulus of elasticity in each of the two principal directions were given by the slope of the equivalent uniaxial stress-strain curves. The curve suggested by Saenz (1964):

$$\sigma_i = \varepsilon_{iu} E_0 / [1 + \left[ \frac{E_0}{E_s} - 2 \right] \left( \frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right) + \left( \frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right)^2] \quad (2.13)$$

for uniaxial compression was used for the biaxial case, where;

$E_0$  = tangent modulus of elasticity at zero stress

$\sigma_{ic}$  = maximum compressive stress

$\varepsilon_{ic}$  = equivalent uniaxial strain at maximum compressive stress

and  $E_s = \sigma_{ic} / \varepsilon_{ic}$

The values of  $E_1$  and  $E_2$  for a particular stress ratio were found from the slopes of the  $\sigma_1 - \varepsilon_{1u}$  and  $\sigma_2 - \varepsilon_{2u}$  curves at the current values of accumulated equivalent uniaxial strain  $\varepsilon_{1u}$  and  $\varepsilon_{2u}$ . Darwin and Pecknold found that their model agreed more closely with the experimental data under a wider range of conditions than the models of Kupfer and Gerstle (1973) and Liu et al. (1972), as illustrated in Figures 2.22 and 2.23. The main advantage of this model was that it was simple and the required data was readily obtainable from uniaxial tests. Many researchers have adopted this model and applied it to a wide variety of practical finite element problems, for example; Bashur and Darwin (1978), Rajagopal (1976) and Noguchi (1981).

### 2.3.5 Plasticity and endochronic models.

Several researchers, notably Chen, W.F. and Suzuki (1980), and Chen, A.C.T. and Chen, W.F. (1975) have applied the theory of plasticity with either perfect plasticity or with a work hardening function to concrete based on the pseudo-plastic behaviour of concrete. The application must be viewed in terms of the overall behaviour, since the theory is not applicable in terms of the micro-behaviour of the concrete. The cement gel does not plastically deform and the apparent plastic behaviour arises from the occurrence of internal microcracking. Accepting this idiosyncrasy this type of model has been widely used and found to

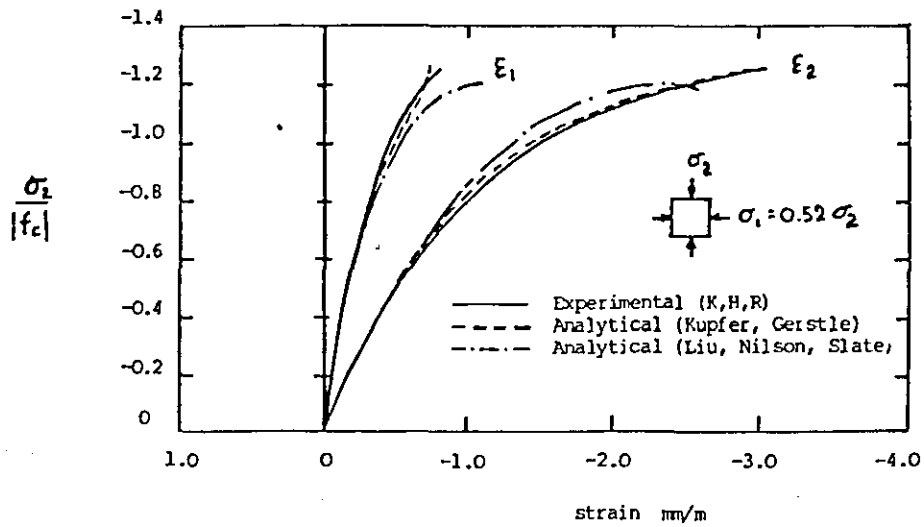


FIGURE 2.22 COMPARISON OF ANALYTICAL MODELS OF KUPFER AND GERSTLE (1973) AND LIU, NILSON AND SLATE (1972) WITH BIAXIAL COMPRESSION TESTS OF KUPFER ET AL. (1969)

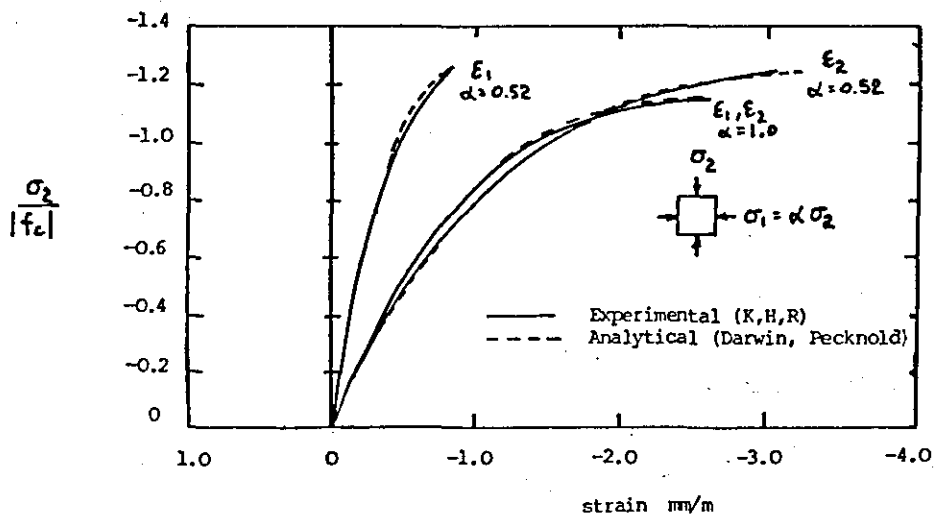


FIGURE 2.23 COMPARISON OF ANALYTICAL MODEL OF DARWIN AND PECKNOLD (1977) WITH BIAXIAL COMPRESSION TESTS OF KUPFER ET AL. (1969)

simulate reasonably well the macro-behaviour of concrete in complex situations.

A development from the flow theory of plasticity was the endochronic theory, originally proposed by Valanis (1971) and adapted to concrete behaviour by Bazant (1976). The endochronic theory of plasticity is based on the concept of intrinsic time (or endochronic time) which can be defined in terms of strain or stress used to measure the extent of damage of the material subject to deformation. This model appears to have remarkable potential for practical application to represent non-linear effects such as creep behaviour and behaviour under cyclic loading. Several researchers have used this model, for example Bazant (1980) and Arnesen et al. (1980), however the development of this type of model is in its infancy and a number of fundamental questions of the theory need further study. Its complexity is totally unnecessary in the requirements for a simple model in two-dimensional stress states.

## 2.4 SUMMARY.

It has long been recognised that there are three main mechanisms responsible for the bond between reinforcing bars and concrete namely; adhesion, friction and mechanical wedging, and that the frictional process is fundamental in explaining the bond behaviour of plain reinforcing bars. Several theoretical bond models have been developed which include the effects of concrete shrinkage, Poisson's effect of the radial contraction of the reinforcing bar under axial load and external confining pressure. Only a few researchers have realised the importance of lateral pressure on bond, however an empirical relationship for the ultimate load of a plain round bar in the

ordinary pullout test subjected to uniaxial lateral pressure has been established.

Numerous investigators have measured the steel strain distributions of bars embedded in a pullout test or transfer type test and this data forms the basis for comparisons with analytical studies. In general the steel strain distributions have been measured by a technique similiar to that first developed by Mains (1951). Bond stress-slip relationships derived from bond tests are not unique and vary with position along the reinforcement, but no rational argument has been offered to explain this phenomenon.

A limited number of studies have been conducted which have analysed reinforced concrete structures with an allowance for bond slip. The bond stress-slip relationships have in general been either linear or polynomial expressions and the effect of lateral pressure has been ignored. In general, spring linkage elements have been used to simulate bond and this has had the effect of 'lumping' bond stiffness at the connecting steel and concrete nodal positions.

Biaxial concrete models are predominately based on the experimental data of Kupfer et al. (1969) and generally one of three concrete models has been adopted either: an isotropic variable moduli model, an equivalent uniaxial model or a plasticity model.

3.1 INTRODUCTION.

The importance of making a realistic attempt to model the bond stress-slip relationship in finite element analyses of reinforced concrete has long been recognised. Allwood (1980) has shown that perfect bonding, using an infinitely stiff bond element can produce unrealistic stress transfers and the wrong deformation behaviour of the analysed structure.

Numerous experimental bond stress-slip relationships have been obtained either by measuring free end slips and the pulling force in simple pullout tests. The need to use a pullout specimen of a very short embedment length in order to obtain the local bond stress-slip relationship has also been recognised. Local bond stress-slip relationships have been calculated from the measurement of strains along the embedded bar and the strains in the concrete close to the bar in pullout and transfer tests. In general the monotonic bond stress-slip relationships which have been used in finite element analyses have been a single unique relationship invariant throughout the analysis. However Tassios and Yannopoulos (1981) and Nilson (1972) have experimentally observed that the local bond stress-slip relationship changes with position along the bar in the bond test.

The importance of the state of stress in the concrete surrounding the bar on bond strength has also been recognised. Abrams (1913) was the first to realise that concrete shrinkage plays a considerable role in developing the bond strength of plain bars. Gilkey, Chamberlain and Beal (1940) considered the bond of plain bars to be mainly a manifestation of friction resistance. The normal forces at the bar-concrete interface have been considered by Glanville (1930) and Peattie and Pope (1956),

who both developed frictional bond models for the bond of plain bars. They considered the radial contraction effect of the bar due to Poisson's ratio effect, which tends to relieve the interfacial pressures.

Untrauer and Henry (1965) and later Robins and Standish (1982) have shown that external confining pressures considerably increase the bond strengths of both plain and deformed bars in the pullout test.

The model to be described recognises that the local ultimate bond capacity of a plain bar is substantially dependent on the radial confining pressures exerted by the concrete on the bar. The initial confining pressure is generated by the shrinkage of the setting concrete and further modifications to the radial pressure at the bar-concrete interface are due to lateral pressures generated by external loading and by changes in bar diameter caused by Poisson's ratio effect. A non-linear relationship between local bond stress and bond slip up to the local ultimate bond capacity will be assumed.

Before describing the idealised bond model the fundamental mechanisms of bond for plain and deformed bars are discussed.

### 3.2 MECHANISMS OF BOND.

#### 3.2.1 Preliminary.

The purpose of this section is to explain some of the basic aspects of the bond between steel reinforcement and concrete. Bond is generally considered to be made up of three components, namely adhesion, friction and mechanical interlock. The bond of plain bars depends mainly on adhesion and friction, but it will be shown that adhesion consists of two parts, chemical adhesion and apparent adhesion caused by the roughness of the bar which actually is the mechanical interlock of the

bar asperities. Deformed bars however depend mainly on the mechanical interlock of the ribs, with friction and adhesion of secondary importance.

### 3.2.2 Adhesion

Adhesion is the term used for the bond resistance before there is slip between the reinforcement and surrounding concrete. The absolute strength of adhesion is difficult to quantify and will depend on how it is defined and on factors such as the concrete mix, bar surface and age of testing. Abrams (1913) defined the adhesion resistance as the bond stress developed before movement of a polished round bar with respect to the adjacent concrete in a pullout test. However by measuring adhesion in shear not only is chemical adhesion being measured but also the keying effect of the bar asperities with the mortar surrounding the bar. Lutz and Gergely (1967) have also recognised this difference and suggested that chemical adhesion can be best measured by loading the adhered surfaces in tension. Plowman (1963) measured the adhesion of mortar cast against flat steel plates and found that the value of adhesion to be very low and in the range 0.85 to 4.0 psi ( 0.006 to 0.03 N/mm<sup>2</sup>).

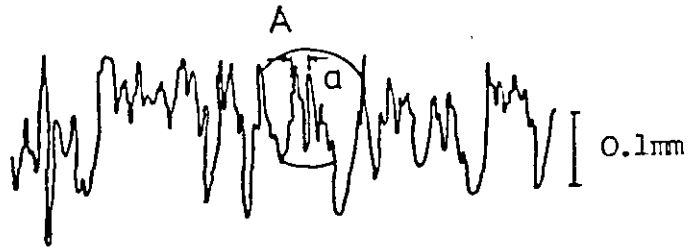
Wurzner (1937) surmised that adhesion could be attributed to cohesion arising from the suction occurring as a result of extraction or evaporation of water from capillaries in the concrete matrix during curing. Microscopic investigations by Pogany (1940) and later Plowman (1957) and Brown (1966) have shown that there is some ingrowth of the gel and crystalline mass into the steel which provides micro-mechanical interlock. Plowman (1963) considered the resistance to shearing of the small portions of mortar which project from the concrete mass into the steel depressions to be more important than chemical adhesion.

### 3.2.3 Keying.

The surface roughness of the bar will have a considerable effect on its bond resistance with the surrounding concrete matrix. Keying of the concrete and steel is the resistance to shearing of the small portions of mortar which project from the mass of concrete into the small depressions of the steel surface. Typical longitudinal roughness profiles of various steel bars, determined with a special tracer device are shown in Figure 3.1, taken from Rehm (1961).

When the bar is loaded the steel is free to move relative to the mortar either when the mortar shears or when it is forced upwards by the slopes of the depressions. As with the large deformations of a ribbed bar there will be bearing of the peaks of the plain steel bar on to the surrounding mortar. The mortar in front of the peaks will be in compression, bending of the concrete projecting into the steel will occur and some micro-cracking will take place. The different moduli of elasticity and the difference in Poisson's ratio of the two materials will cause some different amounts of longitudinal stretching and radial movement between the two materials, even at low loading. Locally this will lead to some differential displacement of the main body of the concrete and the steel bar and effectively there will be some slip between the steel and concrete before shearing of the main body of concrete takes place.

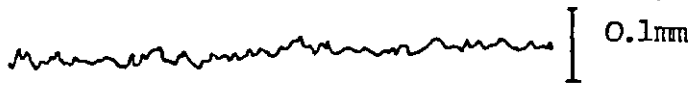
Plowman (1963) has estimated the bond strength available from this keying effect. By measuring the depressions in the surface of the bar, he estimated the depth, width and number per unit area of the depressions. Together with the shear strength of the mortar he estimated the bond stress available from keying as  $9.4$  to  $11.4 \text{ N/mm}^2$ . The calculations assume that all the depressions are filled with compacted mortar but in practice this will not be so, according to Pinchin (1977) the



plain round bar : heavily pitted      Height:Width 36:1



plain round bar : lightly rusted      Height:Width 36:1



plain round bar : as rolled      Height:Width 36:1



drawn wire      Height:Width 36:1



drawn wire      Height:Width 85:1

DETAIL A



plain round bar : heavily pitted      Height:Width 1:1

FIGURE 3.1 SURFACE CONDITION OF VARIOUS PLAIN ROUND BARS

Recorded with the aid of a special tracer device and greatly magnified. Taken from Rehm (1961).

interfacial region is of a lower hardness and so the actual bond strength available will be much lower.

#### 3.2.4 Friction.

A major contributor to bond is the friction component which is offered by the surface irregularity of the bar and the normal pressure acting at the bar-concrete interface. This frictional resistance is an important source of bond for plain bars and plays an important role until failure. The case for frictional forces being the main cause of bond development in round bars was first supported by the work of Abrams (1913), in which an external pressure was applied to the specimens during curing. The bond strengths of the samples which had set under pressures of 0.04 and 0.7 N/mm<sup>2</sup> were found to have increased by 9% and 91% respectively when compared to the corresponding values of concrete setting under normal conditions.

Shrinkage of the concrete matrix surrounding the reinforcement provides the initial radial or normal pressures around the bar for a frictional mechanism. Plowman (1963) and Abrams (1913) have both highlighted the importance of considering the effect of shrinkage in the final results of bond tests.

Both Glanville (1930) and Gilkey et al. (1940) considered the elastic drawing down of the bar due to Poisson's effect as a significant factor. Glanville in his theory assumed that the bond stress at any point along an embedded bar was dependent on the strains in the concrete due to radial shrinkage and the Poisson's ratio effect which would tend to cause radial contraction of the bar and reduce the bond capacity.

In a frictional mechanism the relationship between shear stress and normal stress for the steel-concrete interface is of considerable

importance. Alexander (1969) reports coefficients of friction for steel on cement as 0.74 and cement on cement as 1.02 and Plowman (1963) reports values of the coefficient of friction for steel sliding on paste or mortar as 0.66 to 0.73. Pinchin (1977) found little difference in the coefficients of friction for steel on concrete and concrete on concrete and gives values in the range 0.47 to 0.72. The results of Pinchin and Plowman agree reasonably well but differ markedly from Alexander. The trend towards lower values of friction than predicted by Alexander is substantiated by Cowan (1956) with results from the Cement and Concrete Association Research Laboratory who suggest a value of 0.5 for pre-stressing steel on concrete.

### 3.2.5 Mechanical interlock for deformed bars.

For a deformed bar, Lutz and Gergely (1967) have surmised that the major bond mechanism is by the bearing action of the ribs of the bar on to the surrounding concrete. Friction which would occur after slip in plain bars does not occur in deformed bars because of the presence of the ribs. Slip of a deformed bar can occur in two ways either :

(1) the ribs push the concrete away from the bar, i.e. a wedging action or (2) the ribs crush a small portion of confined concrete in front of the rib. The bearing action involves the bending of the concrete between the ribs, called the concrete key, and aggregate interlock when micro-cracks form. How these actions contribute to the bearing resistance is not yet fully understood. The non-linearity of the concrete due to crushing, the bending action of the concrete key, the cracking mechanism, the non-homogeneity of the concrete and the very localised nature of the problem makes bond resistance by bearing action a very complicated phenomenon. Tepfers (1979) has explained how the radial components of the bond forces transmitted from the ribs are balanced against rings

of tensile stress in the concrete. Eventually the concrete ring is ruptured and longitudinal cracking starts at the bar surface. If the concrete cover is not excessive then with increased loading the ultimate load capacity of the concrete cover will be reached and bursting failure occurs.

### 3.2.6 Modes of failure.

It is important to distinguish between the two types of failure generally associated with plain and deformed bars since it involves different bond and failure mechanisms. With regard to the ordinary pullout test the two types of failure are :

- (1) Pullout of the bar by shearing, leaving a smooth surface inside the concrete block.

and (2) Splitting of the concrete cover

The parameters which govern the mode of failure are:

- i) clear concrete cover,
- ii) rib geometry,
- iii) amount of confinement including lateral pressures,
- iv) bar spacing,
- and v) bar diameter.

Pulling out of the bar from concrete is very common for plain bars since no wedging action is involved in this bonding mechanism. Deformed bars generally fail by splitting of the concrete cover, unless the cover or the confinement is sufficient to restrain splitting failure and then the bars fail by pulling out.

### 3.3 EFFECT OF LATERAL PRESSURE ON BOND.

#### 3.3.1 Preliminary.

A further factor which can have considerable influence on bond strength is the stress state in the concrete surrounding the bar due to external loading. Lateral pressures due to lateral loading can significantly increase the bond strength of plain and deformed bars as has been found by the investigations of Untrauer and Henry (1965), Dorr (1978) and Robins and Standish (1982).

#### 3.3.2 Plain bars.

Robins and Standish explained the pullout of plain bars as a frictional mechanism. In the absence of any externally applied lateral pressure the resistance to pullout arises from the radial compressive stress acting across the bar-concrete interface due to concrete shrinkage, but reduced by the radial contraction of the bar due to Poisson's effect. The bar pulls out of the specimen when the change in bar force at every point along the bar exceeds the frictional force due to concrete shrinkage and radial contraction. If the effect of concrete shrinkage on the radial deformation of the bar is also included, then following Pinchin (1977) the stress in the bar at pullout is given by :

$$\sigma_f = K \frac{E_f \epsilon_c}{v_f} \quad (3.1)$$

$$\text{where } K = 1 - \exp \left( \frac{-2 \mu v_f \times \epsilon_c}{E_f \left[ \left( \frac{1 + v_m}{E_m} \right) + \left( \frac{1 - v_f}{E_f} \right) \right]} \right) \quad (3.2)$$

where  $\sigma_f$  = stress in the bar at pullout

$E_m, E_f$  = moduli of elasticity for the concrete and bar

$v_m, v_f$  = Poisson's ratio for concrete and bar

$\xi_o$  = initial concrete shrinkage

$\mu$  = coefficient of friction

$x$  = embedment length

Robins and Standish (1982) surmised that lateral loading effect-  
-ively increased the required shearing or bond stress to overcome friction.  
They estimated the average interfacial radial pressure to be 0.5 times the  
lateral pressure and then following Timoshenko (1956) the increase in  
radial strain ( $\xi'$ ) between the bar and concrete due to a uniaxial lateral  
pressure, as shown in Figure 3.2, is given by :

$$\xi' = \sigma_{av} \frac{1}{E_m} \left[ \frac{r_f^2 + (r_f + c)^2}{(r_f + c)^2 - r_f^2} + \nu_m \right]$$

where  $\sigma_{av}$  = average interfacial radial pressure

$r_f, c$  = dimensions as given in Figure 3.2

The increase in strain ( $\xi'$ ) resulting from the lateral pressure  
is now used in Equation (3.1) and thus the enhanced stress of  $\sigma'_f$  in the  
bar at pullout is given by:

$$\sigma'_f = \frac{K E_f}{\nu_f} (\xi_o + \xi') \quad (3.4)$$

A comparison of the predicted and experimental values for plain  
bars obtained by Robins and Standish (1982) is shown in Figure 3.3 . The  
lower and upper bound lines depend on the amount of initial concrete  
shrinkage, taken here to be 300 and 1000 micro-strains.

### 3.3.3 Deformed bars.

For deformed bars Robins and Standish found the pullout load-  
-lateral pressure relationship, as shown in Figure 3.4, to be much more

$\sigma'_{yy}$  = lateral pressure

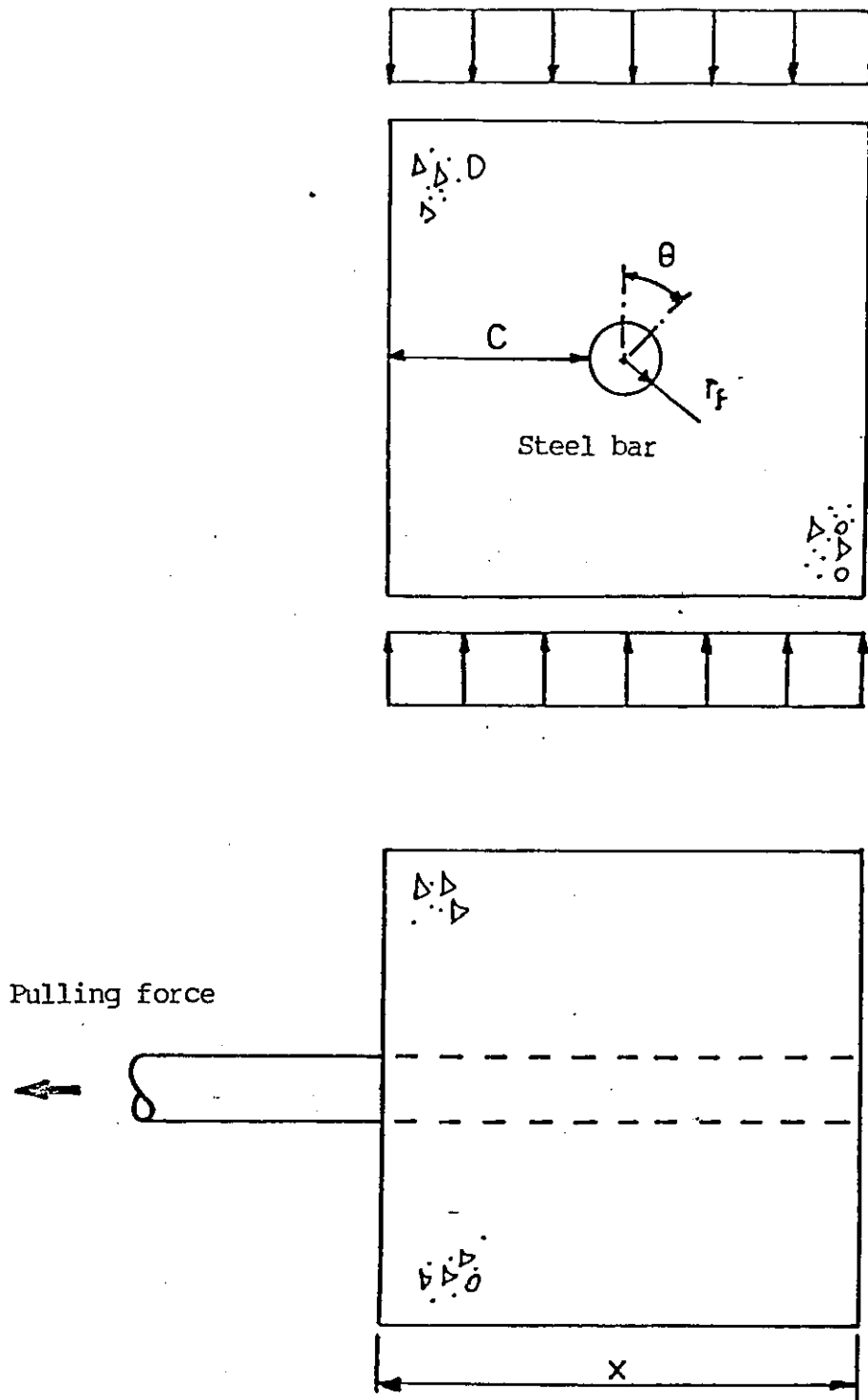


FIGURE 3.2 DIMENSIONS IN THE ORDINARY PULLOUT TEST

Pull-out load kN

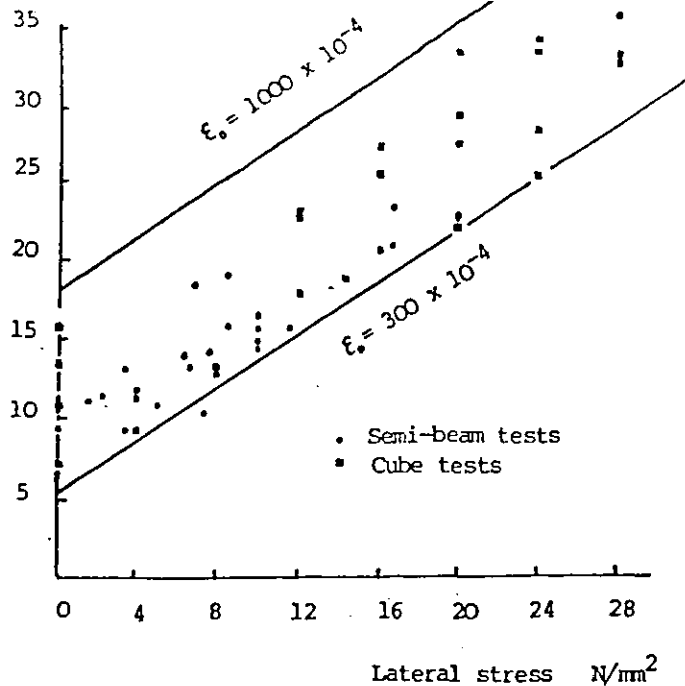


FIGURE 3.3 EFFECT OF LATERAL PRESSURE ON PULLOUT LOAD FOR 12 mm ROUND BARS. (Standish, 1982)

Pull-out load kN

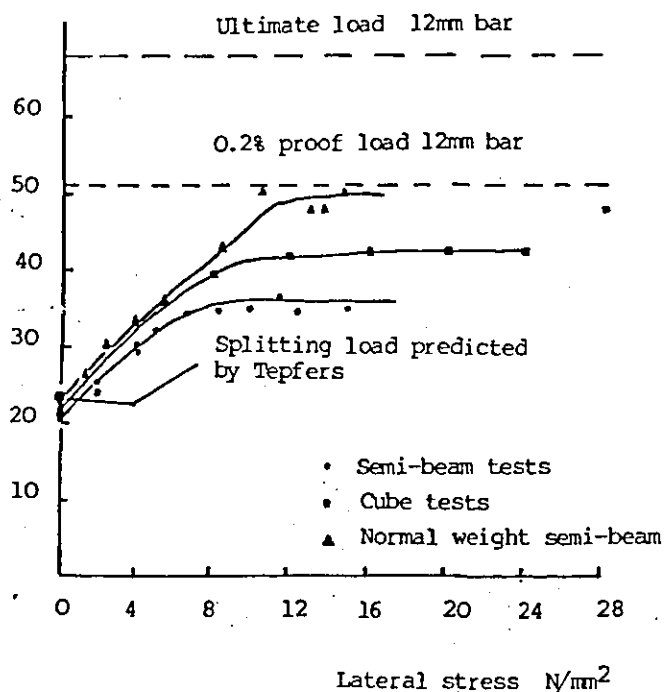


FIGURE 3.4 EFFECT OF LATERAL PRESSURE ON PULLOUT LOAD FOR 12 mm DEFORMED BARS. (Standish, 1982)

complicated and one which could not be explained by a frictional bond mechanism. There are two distinct modes of failure namely: (1) below lateral pressures of about  $10 \text{ N/mm}^2$  splitting failure occurs and (2) at larger lateral pressures a shearing type of failure occurs. The plateau effect in Figure 3.4 is due to a limiting shear failure and is seemingly independent of lateral pressure.

Untrauer and Henry (1965) in their pullout tests on deformed bars only investigated lateral pressures between zero and  $16.5 \text{ N/mm}^2$  (2370 psi) and found the ultimate bond strength to be proportional to the square root of the lateral pressure, Equation (2.2), (Chapter 2). Dorr (1978) using strain gauged deformed bars in tensile bond specimens found the local ultimate bond stress to increase with lateral pressure. The average local bond stress-slip relationships obtained by Dorr are illustrated in Figure 3.5 and compared with the results of Untrauer and Henry.

### 3.4 BOND STRESS-SLIP RELATIONSHIP.

Average bond stress-slip relationships have been obtained from a wide range of bond tests. Strictly such data should be presented as a pulling force-slip (either free or loaded end) relation, as such measurements are the overall behaviour of the specimen with the bar being pulled. Conversion to an average bond stress-slip relationship does not necessarily represent the local bond stress-slip relationship along the bar. Many researchers have attempted to reduce the effective length of the specimen to a minimum in order to produce a closer approximation to the local bond stress-slip relationship. Rehm (1961) used a short bond length of about 16 mm in his pullout tests and typical results for plain and deformed bars are shown in Figures 3.7 and 3.8 .

A superior method is to measure the bar strains along the length

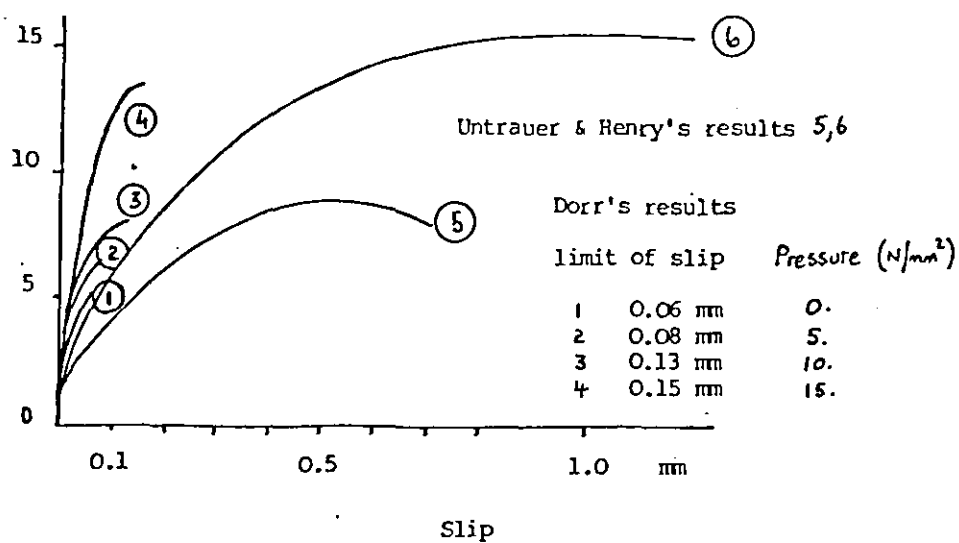
$N/mm^2$ 

FIGURE 3.5 EFFECT OF LATERAL STRESS ON LOCAL BOND STRESS SLIP RELATION. (Dorr, 1978)

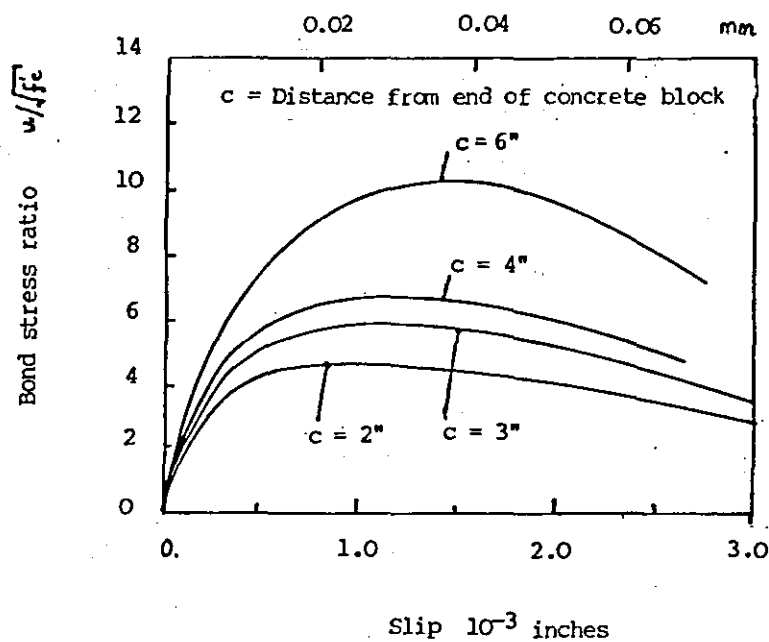


FIGURE 3.6 LOCAL BOND STRESS RATIO-SLIP RELATIONSHIPS IN A TENSILE BOND SPECIMEN (Nilson, 1972)

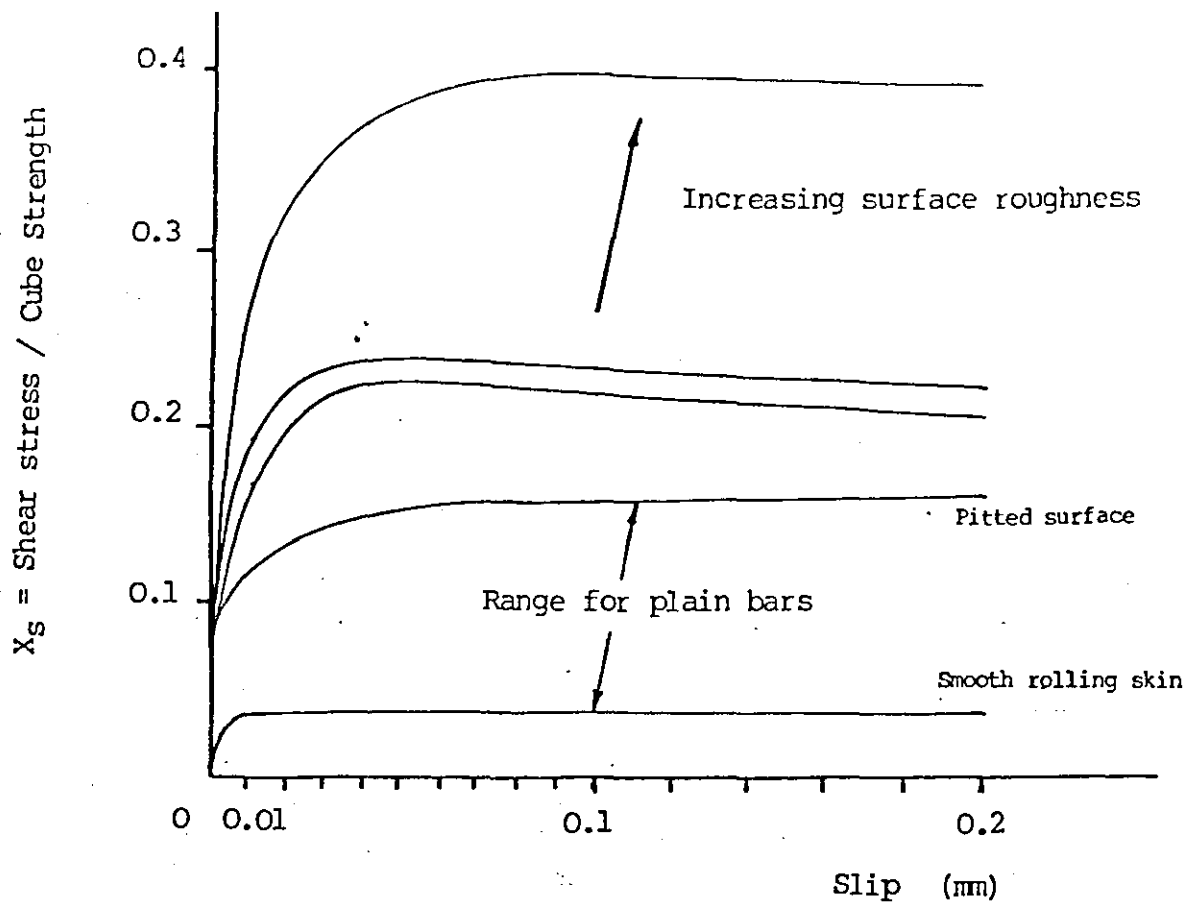


FIGURE 3.7 FUNDAMENTAL RELATIONSHIPS FOR PLAIN BARS (Rehm, 1961)

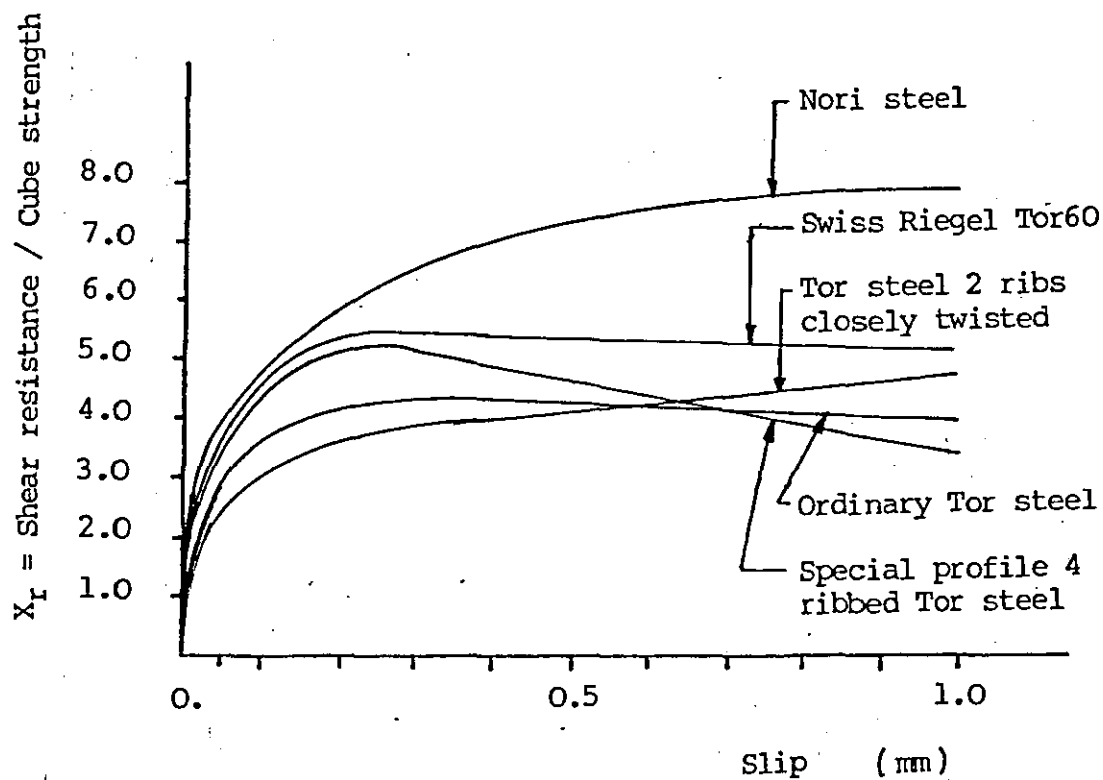


FIGURE 3.8 FUNDAMENTAL RELATIONSHIPS FOR DEFORMED BARS (Rehm, 1961)

of the bar and the concrete strains near the bar and then from this data derive the local bond stress-slip relationships. Nilson (1972) has calculated the local bond stress-slip relationship for a deformed bar in a tensile bond specimen and this is shown in Figure 3.6 . The bond stress-slip relationship varies with position along the bar. The local bond stress-slip relationships obtained by Dorr (1978) using a similar specimen are illustrated in Figure 3.5 . In both experimental tests it is very unlikely that the concrete strain gauges, although tied close to the bar (about 10 to 15 millimetres away) were able to give reliable information about the concrete strains very close to the bar, as in this region the chemistry and microstructure of the mortar is very different from that in the main body of the concrete.

### 3.5 IDEALISED BOND MODEL FOR PLAIN BARS.

#### 3.5.1 Introduction.

The objective in devising this new bond model is to form a local bond stress versus bond slip relationship in terms suitable for finite element analyses and which incorporates all relevant past research on the behaviour of plain bars embedded in concrete. In particular the model should reflect the different local conditions occurring along the reinforcement bar embedded in the concrete, as monotonically increasing loads are applied to the structure.

As the preceeding section has shown, not enough is known about the phenomenon to establish this model completely and therefore a number of assumptions have been made. The bond model for plain bars considers the bond behaviour in three stages, namely :

- (1) The ultimate local bond stress is assumed to be a function of adhesion and radial pressure between bar and concrete and that

excessive slip will occur when bond stresses reach this value.

- (2) For bond stresses below the value given by (1) there is assumed to be slip given by a non-linear relationship with bond stress.
- (3) When local bond stresses have reached (1) there may be a reduction in bond stress if further slip occurs.

From the assumptions (1), (2) and (3), the new bond model consists essentially of two constitutive relationships: (i) the ultimate bond stress - radial pressure relation and (ii) the local bond stress-slip relation, and these governing relationships are illustrated in Figure 3.9.

### 3.5.2 Ultimate bond stress-radial pressure relationship.

The experimental results of Alexander (1969), Pinchin (1977) and Plowman (1963) all show that the coefficient of friction for steel sliding on cement mortar or concrete is constant, i.e. the relationship between shear stress and normal pressure is linear.

The ultimate bond stress - radial pressure relationship to be used in the bond model may be considered to be similar to the Mohr-Coulomb friction law. A linear relationship between ultimate bond stress ( $q_u$ ) and radial pressure ( $\sigma_r$ ) is assumed of the form :

$$q_u = C + \mu \sigma_r \quad (3.5)$$

At zero radial pressure, i.e. before shrinkage takes place the only force resisting movement is adhesion (C). Concrete shrinkage generates radial pressures or normal forces at the bar-concrete interface and the ultimate bond stress due to shrinkage is developed as illustrated in Figure 3.9 . Further changes in radial pressure due to the radial contraction of the bar as it carries axial load and due to

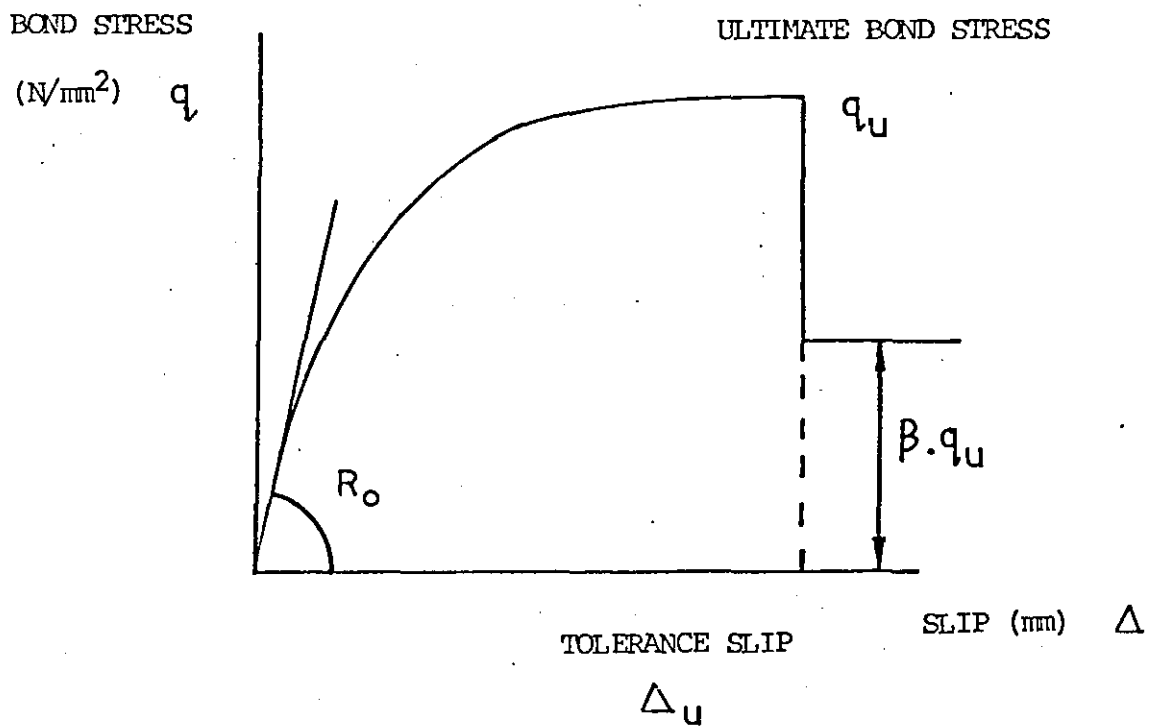
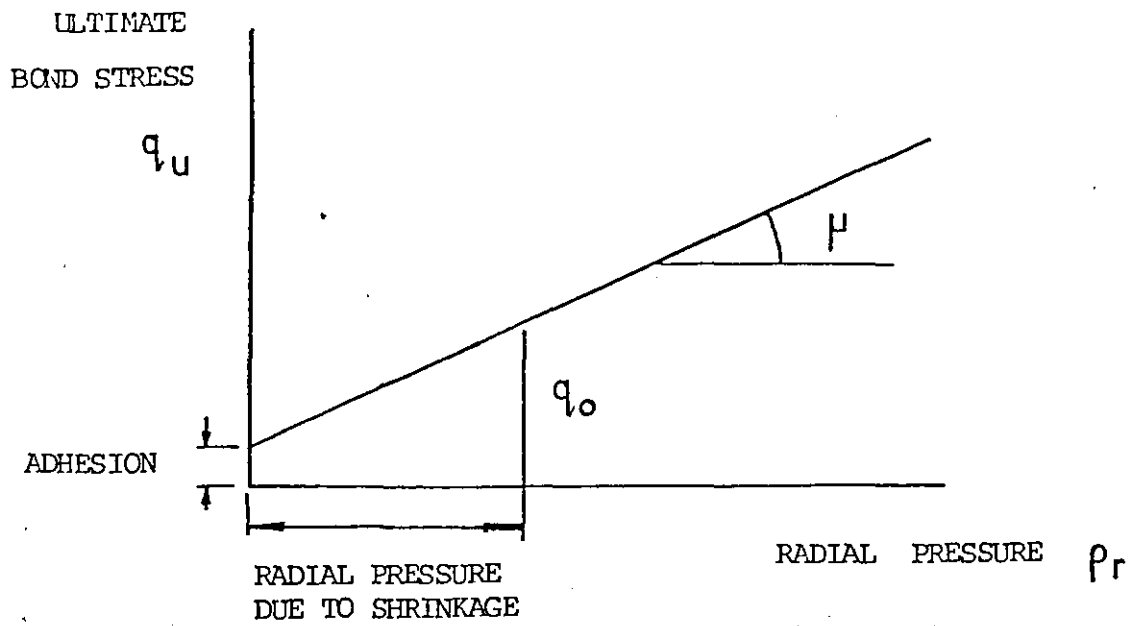


FIGURE 3.9 THE ASSUMED ULTIMATE BOND STRESS VERSUS RADIAL PRESSURE RELATIONSHIP AND THE ASSUMED BOND STRESS-SLIP RELATIONSHIP

concrete lateral pressures exerted against the bar are assumed to modify the local ultimate bond stress. For convenience, a false zero is used in the ultimate bond stress - radial pressure relationship corresponding to the initial concrete shrinkage as shown in Figure 3.10 . The governing ultimate bond stress - radial pressure relationship is then given by :

$$q_u = q_o + \mu ( \sigma_{rconc} - \sigma_{rbar} ) \quad (3.6)$$

where  $q_u$  = local ultimate bond stress

$q_o$  = ultimate bond stress due to adhesion and shrinkage

$\mu$  = 'coefficient of friction'

$\sigma_{rconc}$  = compressive radial pressure exerted by the concrete

$\sigma_{rbar}$  = tensile interfacial radial pressure due to bar contraction

#### (1) Concrete shrinkage effect.

Abrams (1913), Glanville (1930), Gilkey et al. (1940) and Robins and Standish (1982) have all surmised that concrete shrinkage generates the initial radial pressures at the bar-concrete interface for a frictional bond mechanism to occur.

Within the model, it is assumed that the pressures produced by concrete shrinkage can be calculated using Timoshenko's (1956) thick-walled cylinder theory. The pressure (p) produced between two cylinders as shown in Figure 3.11, where the external radius of the inner cylinder is larger than the internal radius of the outer cylinder by  $\Delta$  and the inner cylinder is solid is given by :

$$p = \frac{\Delta / r_s}{\frac{1}{E_c} \left( \frac{r_s^2 + r_c^2}{r_c^2 - r_s^2} + v_c \right) + \left( \frac{1 - v_s}{E_s} \right)} \quad (3.7)$$

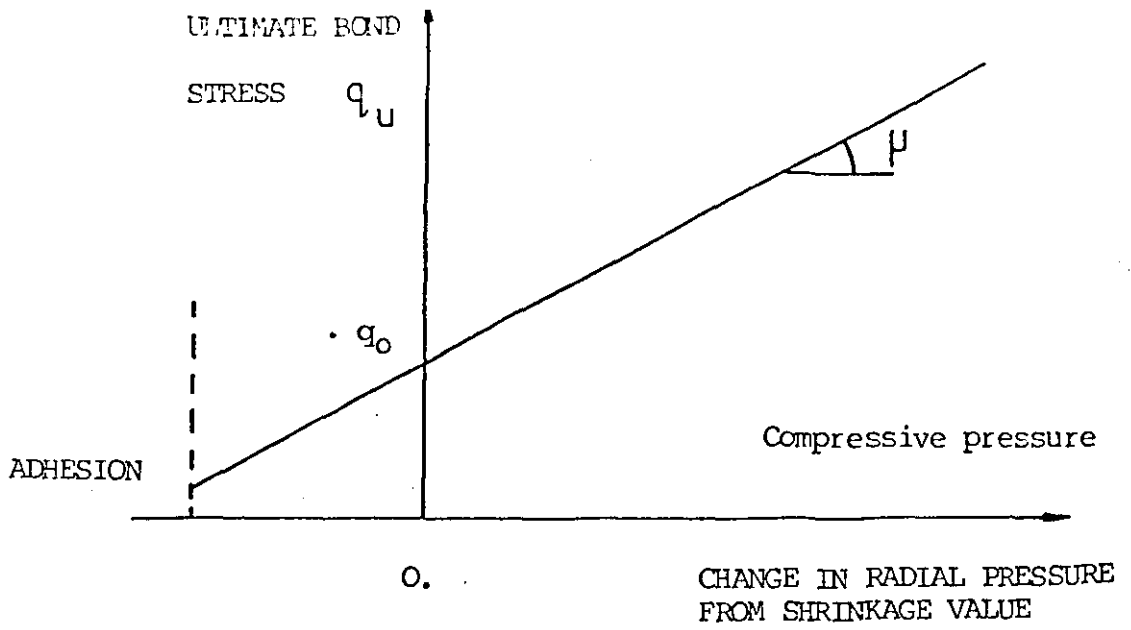
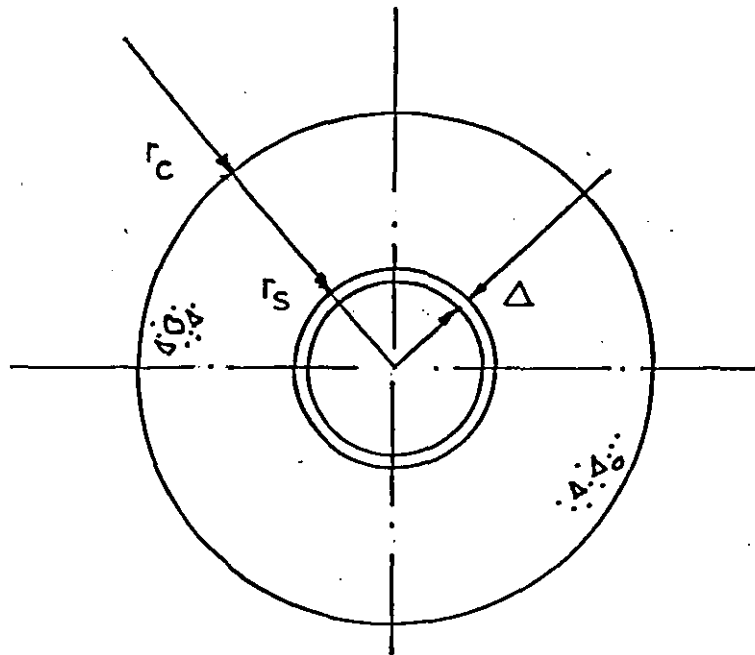


FIGURE 3.10 ULTIMATE BOND STRESS VERSUS RADIAL PRESSURE RELATIONSHIP AS USED IN THE PROPOSED MODEL.



$$r_c = 50 \text{ mm}$$

$$r_s = 6 \text{ mm}$$

$$E_c = 28 \text{ 000 N/mm}^2$$

$$E_s = 200 \text{ 000 N/mm}^2$$

$$\nu_c = 0.2$$

$$\nu_s = 0.3$$

FIGURE 3.11 INTERFACIAL PRESSURE BETWEEN TWO CYLINDERS DUE TO MISFIT

For a cylindrical pullout test of 100 mm diameter with a 12 mm diameter steel bar and typical values of elastic moduli as shown in Figure 3.11, then Equation (3.7) becomes :

$$p = \frac{\Delta/6}{4.74 \times 10^{-5}} \text{ N/mm}^2 \quad (3.8)$$

An upper value of shrinkage of 1000 micro-strains is that obtained from curing concrete samples under laboratory conditions and a lower value of 300 micro-strains is the irreversible part of shrinkage (0.3 x dry shrinkage value) that Neville (1973) suggests is the value to use where the specimens are cured under water. Using Equation (3.8) the upper and lower values of radial pressure generated by shrinkage at the bar-concrete interface are 6.3 and 21.1 N/mm<sup>2</sup> respectively. If the latter compressive stress were to exist at the interface then the associated tensile tangential stress ( $v_c \times$  radial stress) would be about 4.2 N/mm<sup>2</sup>, sufficient to cause radial cracking of the concrete at the interface. If radial cracking developed, this would tend to decrease the radial compressive stress and reduce the available bond strength.

An estimate of the ultimate bond stress due to concrete shrinkage may be obtained from the pullout load - lateral pressure relationships of Standish (1982) shown in Figure 3.3 . This experimental data may be transformed into an average bond stress - radial pressure relationship but this new relation only indicates the overall behaviour of the pullout test and is not necessarily the local ultimate bond stress - radial relation. As an initial estimate for the  $q_0$  parameter, the average bond stress at zero lateral pressure may be obtained from this transformed data and the slope of the average bond stress - radial pressure curve gives an estimate for the  $\mu$  coefficient.

(ii) Bar radial contraction effect.

The radial contraction due to Poisson's effect as the bar is axially loaded in tension tends to relieve the compressive interfacial stress due to concrete shrinkage. The radial contraction effect has been considered by Glanville (1930), Peattie and Pope (1956) and by Robins and Standish (1982) in the development of their own respective frictional bond models for plain bars in concrete.

It is assumed that the bar radial contraction and the associated interfacial radial pressure between the bar and the concrete can be estimated using Timoshenko's (1956) thick walled cylinder theory. If the longitudinal strain in the bar is  $\epsilon$ , then for a bar of radius  $r_s$  the interfacial radial pressure is given by using  $\Delta = \epsilon r_s$  in Equation (3.8). For example a 12 mm diameter plain round steel bar carrying a tensile axial load of 20 kN, the radial strain is 265.3 micro-strains and the associated interfacial radial pressure is 5.6 N/mm<sup>2</sup>.

(iii) Lateral pressure effect.

The effect of lateral pressure on the ultimate bond strength of reinforcement bars has been investigated by Dorr (1978) and by Robins and Standish (1982) and the results indicate that the local ultimate bond stress at each point along the bar and the overall pullout strength of the bar are increased when compressive lateral pressure is applied.

Consider the effect of a lateral pressure ( $\sigma_{yy}$ ) in Figure 3.2. The radial stress at the bar-concrete interface resulting from this uniaxial lateral pressure is not uniform. The problem may be considered as an infinite plate with a circular hole, into which an elastic circular disc has been inserted, and the plate subjected to a uniaxial stress field. This particular problem has been analysed by Muskhelishvili

(1956) and the analysis pertaining to a circular steel elastic bar within an assumed elastic concrete body is given in Appendix A. Following Muskhelishvili the average interfacial radial pressure ( $\sigma_{av}$ ) is 0.7704 times the uniaxial lateral pressure ( $\sigma_{yy}$ )

Within the bond model it is assumed that the concrete stresses ( $\sigma_{yy}$ ) in the y-y direction given by the plane stress analysis, can be converted into an average interfacial radial pressure ( $\sigma_{av}$ ) by the relation :

$$\sigma_{av} = 0.7704 \sigma_{yy} \quad (3.9)$$

### 3.5.3 Local bond stress-slip relationships.

The local bond stress-slip relationships obtained by Dorr (1978) and Nilson (1972), as shown in Figures 3.5 and 3.6 indicate that this relationship is non-linear up to a local maximum bond stress value.

Bowden and Tabor (1964) have shown that in experiments measuring the friction between two metal surfaces that before gross sliding of the two surfaces takes place that some amount of micro-slipping occurs. Experiments conducted by Courtney-Pratt and Eisner (1957) on the friction between two steel surfaces pressed together by a normal force show that as the tangential force is increased from zero there is a steady monotonic increase in micro-displacement. The relationship between tangential force and micro-displacement is non-linear and similar in shape to the local bond stress-slip relationship obtained by Dorr (1978). The bond stress-slip between concrete and a steel bar may be considered similar to the micro-displacement phenomenon in the friction between two metal surfaces.

Within the proposed bond model the local bond stress-slip relationship is assumed to be a non-linear curve based on the Saenz (1964)

equation, originally used to describe the uniaxial stress-strain curve for concrete. Use of the Saenz curve is an arbitrary choice since there is no evidence to suggest that the local bond stress-slip relationship is in general of a particular shape. Nilson (1972) used a third order polynomial but in this bond model the Saenz curve is used for convenience, since the initial bond stress-slip modulus, the slip at maximum bond and the maximum bond stress, are all independent variables. The bond stress-slip relationship is assumed to be :

$$q = R_o \Delta / \left[ 1 + \left( \frac{R_o^2}{R_{sec}} - 2 \right) \left( \frac{\Delta}{\Delta_u} \right) + \left( \frac{\Delta}{\Delta_u} \right)^2 \right] \quad (3.10)$$

given  $R_o > 2q_u / \Delta_u$

where  $R_{sec} = q_u / \Delta_u$

$q$  = bond stress at a slip of  $\Delta$

$q_u$  = local maximum bond stress

$\Delta_u$  = slip at maximum bond stress

The peak bond stress is governed by the ultimate bond stress-radial pressure relationship (Equation 3.6).

(i) Initial bond stress-slip modulus ( $R_o$ ).

The range of values for the initial bond stress-slip modulus as seen from the experimental results of Nilson (1972) and Dorr (1978), (Figures 3.5 and 3.6) is quite considerable and will depend on many factors, but predominately the bar roughness and the concrete mix.

By considering experimentally measured bar strain distributions which are directly dependent on the bond modulus, an initial estimate to the initial bond modulus may be obtained. For plain round bars an estimate for the initial bond modulus obtained from double-ended pullout tests (Chapter 6) is  $200 \text{ N/mm}^2$ .

(ii) Slip at maximum bond stress ( $\Delta_u$ ).

An initial estimate for the tolerance slip at which the local maximum bond stress occurs can be obtained from the experimental pullout load - free end slip data of Robins and Standish (1982). The value of tolerance slip ( $\Delta_u$ ) is within the range 0.06 to 0.1 mm .

(iii) Ultimate bond stress at slips in excess of  $\Delta_u$ .

Once excessive slip occurs, the sliding frictional resistance for plain bars is likely to decrease. Alexander (1969) found the shear bond strength between aggregate and cement, which is similar to the steel-cement interface, to decrease by approximately 50 % with increased slip, as shown in Figure 3.12 .

Within the bond model, for slips in excess of the slip tolerance the maximum bond stress is assumed to be a fixed proportion of the ultimate bond stress as shown in Figure 3.9 . For plain round bars a typical value for the  $\beta$  parameter, ratio of maximum bond stress to ultimate bond stress, is 0.5 .

3.5.4 Relative lateral displacements between bar and concrete.

The bond in this direction must represent the forces and slip between the bar and concrete which is at right angles to the bar axis and involves dowel action. The problems of dowel action are extremely involved and the author considers this to be beyond the scope of the present work. An arbitrary high bond modulus ( $R_n$ ) is assigned to the bond in this direction and equal to  $10^5 \text{ N/mm}^2/\text{mm}$ . By assuming a relatively high value, the displacements of the steel bar in the direction normal to the bar axis are the same as the displacements of the concrete in this direction.

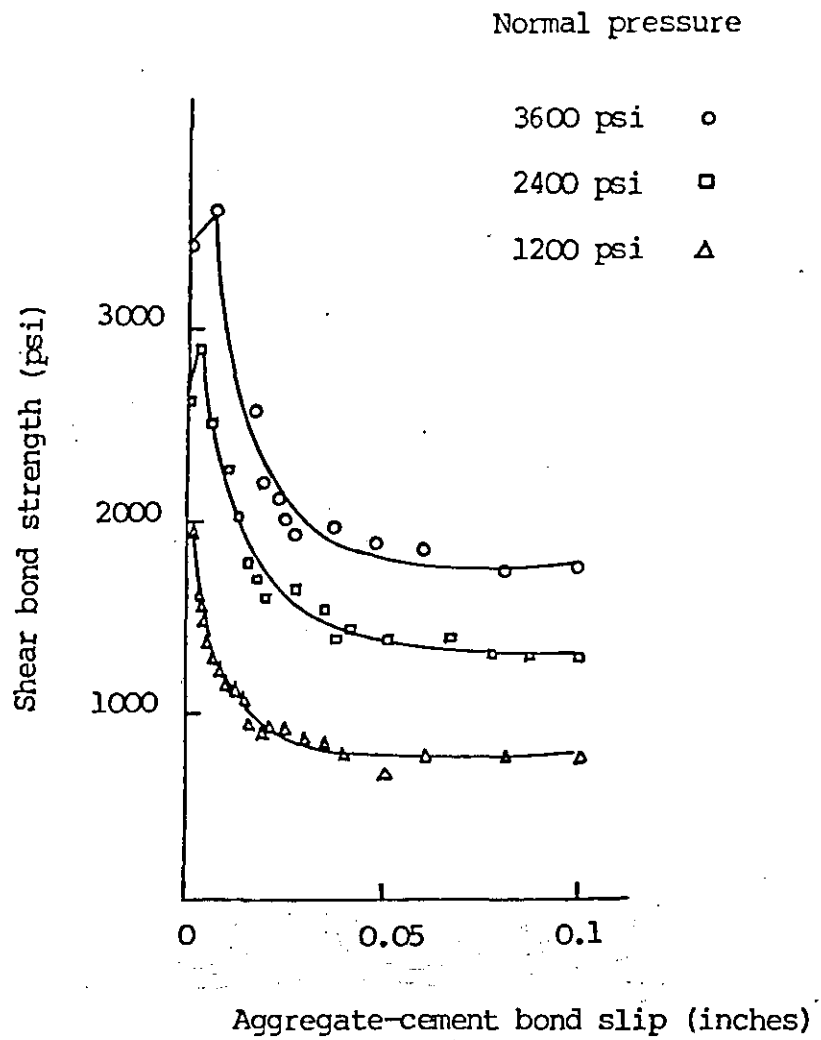


FIGURE 3.12 DEPENDENCE OF AGGREGATE-CEMENT SHEAR BOND STRENGTH ON THE MAGNITUDE OF SLIP AT VARIOUS LEVELS OF NORMAL PRESSURE (Alexander, 1969)

### 3.6 APPLICATION OF THE BOND MODEL TO DEFORMED BARS.

The bond of deformed bars relies mainly on the mechanical interlock of the ribs with the surrounding concrete and the mode of failure normally associated with this type of bar is splitting of the concrete cover. It is therefore unlikely that the frictional bond model developed for plain bars can be applied to the bond of deformed bars without major changes.

The adaptation of the bond model for plain bars for use with deformed bars and its application to selected problems is discussed in greater detail in Chapter 8 .

### 3.7 SUMMARY.

The developed bond model is applicable to plain round bars embedded in concrete where friction is the main bond mechanism and where bond would fail by shearing between the bar and the surrounding concrete. Within the developed bond model the local ultimate bond stress is a function of the initial concrete shrinkage, the radial contraction of the bar due to Poisson's effect and the concrete lateral pressures exerted against the bar. These interfacial effects between a round bar and surrounding concrete matrix are combined by converting each effect in to an equivalent radial pressure. The local ultimate bond stress is assumed to be a linear function of the combined radial pressures. A non-linear bond stress-slip relationship based on the Saenz (1964) curve for concrete is fitted up to the ultimate bond stress and a tolerance slip value. For slips in excess of the tolerance slip the local bond stress required to maintain sliding is a fixed proportion of the ultimate bond stress.

#### 4.1    Introduction.

Within the present model reinforced concrete structures are analysed by an approximate two dimensional plane stress model and plain concrete is considered to be in a state of biaxial stress. Under biaxial loading plain concrete exhibits different stress-strain behaviour and varying strength characteristics depending on the ratio of the biaxial stresses. It is a major requirement that the constitutive model for plain concrete should accurately reproduce the highly non-linear stress-strain relationships and varying strength characteristics under all combinations of biaxial stress. Further, the model should accurately predict the behaviour of fractured concrete, caused either by cracking or by crushing.

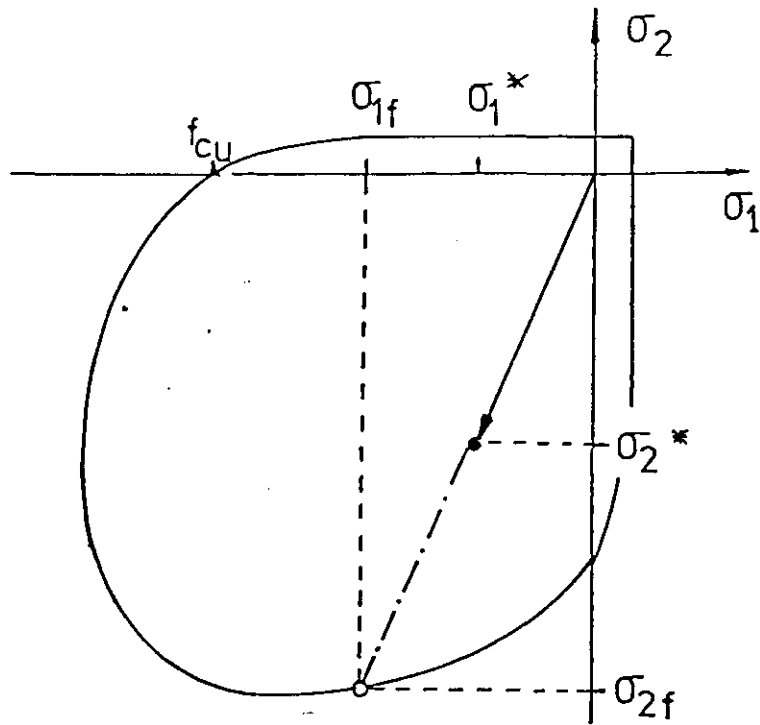
Concrete modelling is not a major part of this investigation, the main requirement being the choice of a constitutive model which has been previously well tested on a variety of reinforced concrete problems and shown to reproduce the characteristics of plain concrete reasonably well. The requirements for a suitable model are :

- (1) Empirical stress-strain relationships and definitions of parameters readily available.
- (2) Stress-strain relationships and failure criteria defined by readily obtained uniaxial test data.
- (3) Easy implementation in the finite element model.
- (4) The model previously shown to provide a good match with stress behaviour and failure characteristics of plain concrete subject to biaxial loading.

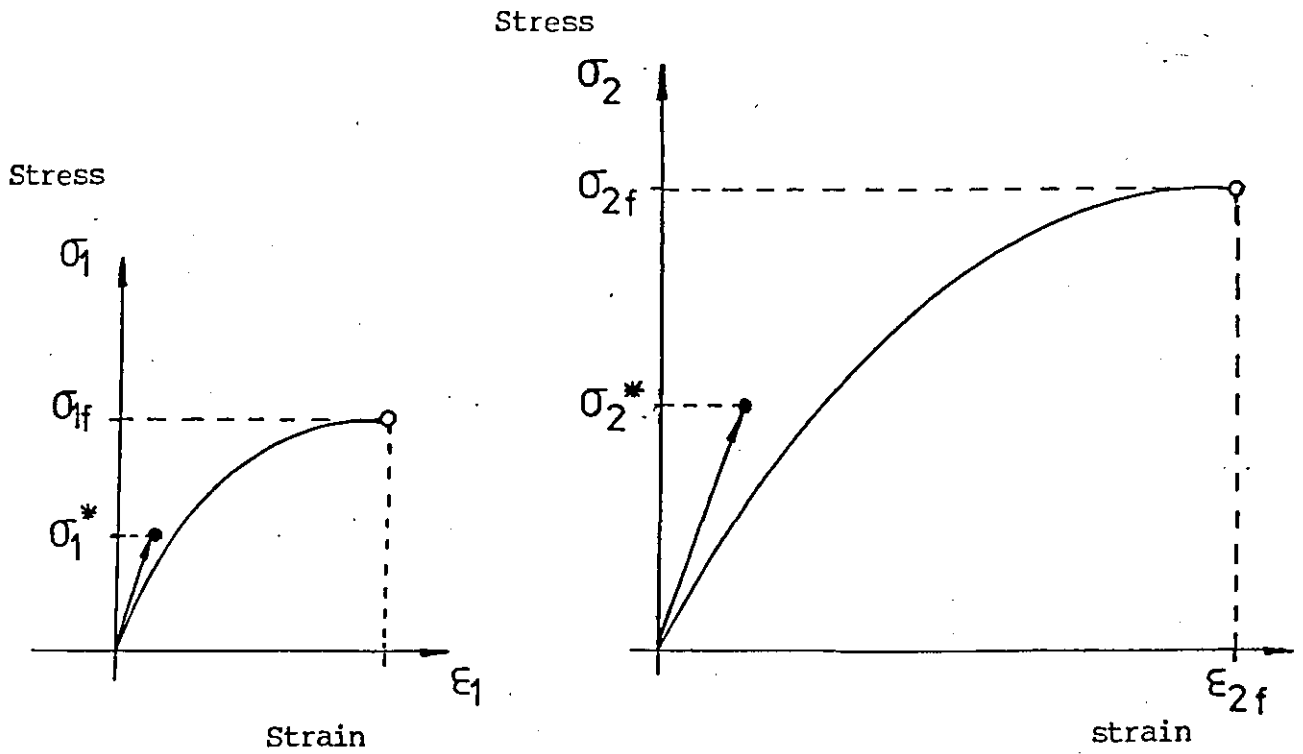
Various concrete models have already been reviewed, namely :

variable moduli models, eg. Kupfer and Gerstle (1973), equivalent uniaxial models, eg. Liu et al.(1972) and Darwin and Pecknold (1974) and plasticity and endochronic models, eg. Chen W.F. and Suzuki (1980) and Bazant (1976). Kupfer and Gerstle proposed an isotropic material model and presented a series of closed form expressions for the secant shear modulus and secant bulk modulus, based on curve fitting to their own experimental data. By their own admission they did not obtain good results for uniaxial compression, tension-compression stress states and at high levels of biaxial compression.

The model devised by Liu et al.(1972) considers concrete to be an orthotropic material and their model is expressed in terms of total stresses and strains as a variation of Saenz's (1964) equation. The model is strictly only applicable to biaxial compression. Further, the model causes the minor stress to become indefinite when the ratio of principal stresses, known as the stress ratio, equals 0.2 . Plasticity models with work hardening function, eg. Chen, W.F. (1975), have been shown to adequately reproduce the biaxial behaviour of plain concrete. However work hardening plasticity models are based on closed form expressions for the effective stress-strain curves. The strain hardening rule which is required in such a model is related to the slope of the effective stress-strain curve. Although there is considerable graphical representation of the effective stress-strain relationships, eg. Chen,W.F. (1975) and Chen, A.C.T.(1973), no empirical relationships for the effective stress-strain curves and their inter-relationship with uniaxial test parameters can be found in the literature. The recently developed endochronic models appear to have considerable potential for practical applications, however certain theoretical aspects of the model still require refinement, particularly stability in small amplitude stress and strain cycling. Further the number of functions and material



KUPFER FAILURE ENVELOPE



SAENZ CURVE FIT

FIGURE 4.1 USE OF THE KUPFER FAILURE ENVELOPE AND SAENZ CURVE FIT  
TO PRODUCE STRESS-STRAIN RELATIONSHIPS FOR THE CONCRETE

constants required makes it difficult to apply if only uniaxial test parameters are known.

The model considered most suitable is the equivalent uniaxial strain approach of Darwin and Pecknold (1974). They have adequately compared their model with the experimental data of Kupfer, Hilsdorf and Rusch (1969) and Nelissen (1972) and the model can accurately reproduce the varying stress-strain curves and varying strength characteristics under all combinations of biaxial stress.

In the present model the current stresses  $\sigma_{xx}$ ,  $\sigma_{yy}$  and  $\sigma_{xy}$  at any point within the body of plain concrete are transformed into principal stresses  $\sigma_1$  and  $\sigma_2$  and from these principal stresses the failure stresses and strains at this point may then be predicted. Within the model it is assumed that at failure the stress ratio would remain the same and therefore the principal stresses at failure may be predicted by taking the current stress state and extrapolating across the stress space to the Kupfer failure envelope as illustrated in Figure 4.1 .

For reasons given later, strains within the present model are accumulated using the Darwin and Pecknold concept of equivalent uniaxial strains. The Darwin and Pecknold model provides empirical equations for the failure strain in each principal stress direction, which are based on the experimental data of Kupfer et al. (1969). The stress-strain relationship in each principal stress direction may then be defined. For a tensile stress the stress-strain relationship is linear and for a compressive principal stress it is based on the curve proposed by Saenz (1964) as illustrated in Figure 4.1 . The parameters defining this curve are; the initial tangent slope and the failure stress and strain condition.

The concrete model as implemented is now described in detail.

## 4.2 CONCRETE MODEL.

### 4.2.1 Constitutive relationships.

Plain concrete has been idealised as an isotropic material by Kupfer and Gerstle (1973) and as an orthotropic material by Liu et al. (1972) and by Darwin and Pecknold (1974). In the present model plain concrete is considered to be an incrementally linear elastic orthotropic material. The stress-strain curves for plain concrete under biaxial stress, shown in Figure 2.14, strongly indicate stress induced orthotropic behaviour.

The equations relating change in strain to change in stress for an incrementally linear orthotropic material in the principal stress axes, but not considering shear deformation for the moment, are :

$$d \epsilon_1 = \frac{d\sigma_1}{E_1} - \nu_2 \frac{d\sigma_2}{E_2} \quad (4.1)$$

$$d \epsilon_2 = -\nu_1 \frac{d\sigma_1}{E_1} + \frac{d\sigma_2}{E_2} \quad (4.2)$$

$E_1, E_2, \nu_1, \nu_2$  are stress-dependent material properties.

Solving these equations for a change in stress and rewriting in matrix form :

$$\begin{bmatrix} d\sigma_1 \\ d\sigma_2 \end{bmatrix} = \frac{1}{1 - \nu_1\nu_2} \begin{bmatrix} E_1 & \nu_2 E_1 \\ \nu_1 E_2 & E_2 \end{bmatrix} \begin{bmatrix} d\epsilon_1 \\ d\epsilon_2 \end{bmatrix} \quad (4.3)$$

From energy considerations, it can be shown that :

$$\nu_1 E_2 = \nu_2 E_1 \quad (4.4)$$

For each load increment  $E_1, E_2, \nu_1, \nu_2$  must be known. To simplify their use and to ensure that no particular direction in the material is preferred, the relationship is modified such that :

$$v^2 = v_1 v_2 \quad (4.5)$$

where  $v$  is an 'equivalent' Poisson's ratio. The incremental stress-strain relationship becomes :

$$\begin{bmatrix} d\sigma_1 \\ d\sigma_2 \end{bmatrix} = \frac{1}{1 - v^2} \begin{bmatrix} E_1 & \sqrt{E_1 E_2} \\ \sqrt{E_1 E_2} & E_2 \end{bmatrix} \begin{bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \end{bmatrix} \quad (4.6)$$

Now introducing the shear term, the following relationship is obtained :

$$\begin{bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\sigma_{12} \end{bmatrix} = \frac{1}{1 - v^2} \begin{bmatrix} E_1 & \sqrt{E_1 E_2} & 0 \\ \text{sym} & E_2 & 0 \\ & & (1-v^2)G \end{bmatrix} \begin{bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\varepsilon_{12} \end{bmatrix} \quad (4.7)$$

As with Poisson's ratio, it is desirable that no particular direction in the material is to be favoured with respect to the shear modulus term. Darwin and Pecknold have shown that the shear modulus ( $G'$ ) is independent of the axes of orientation if  $G' = 1/4 (E_1 + E_2 - 2v\sqrt{E_1 E_2})$ . The constitutive relationship therefore becomes :

$$\begin{bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\sigma_{12} \end{bmatrix} = \frac{1}{1 - v^2} \begin{bmatrix} E_1 & \sqrt{E_1 E_2} & 0 \\ \text{sym} & E_2 & 0 \\ & & 1/4(E_1 + E_2 - 2v\sqrt{E_1 E_2}) \end{bmatrix} \begin{bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\varepsilon_{12} \end{bmatrix} \quad (4.8)$$

The elasticity moduli  $E_1$  and  $E_2$  in the principal stress directions are determined from the slopes of stress-strain curves similar to the uniaxial stress-strain curve for plain concrete. The following section defines and interprets these curves in the biaxial case.

#### 4.2.2 Equivalent uniaxial strain approach.

The concept of equivalent uniaxial strain was developed by Darwin and Pecknold in order to allow biaxial stress-strain curves to be

duplicated from uniaxial curves and to separate the Poisson's ratio effect from total stress-strain curves. For a material subject to biaxial loading, the strain in one direction is a function not only of the stress in that direction but also of the stress in a direction orthogonal to the first, due to Poisson's effect. The modulus of elasticity in each principal stress direction cannot be obtained directly as the slope of the total stress-strain curve in each direction, since the stress-strain relationship contains Poisson's effect. The equivalent uniaxial strain approach is a method of separating the Poisson effect from the cumulative strains and of obtaining equivalent uniaxial stress-strain curves from which the modulus of elasticity in each principal stress direction may be obtained directly.

The technique can be described as follows. Consider the general constitutive relationships for an orthotropic material in two dimensions :

$$\begin{bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\sigma_{12} \end{bmatrix} = \begin{bmatrix} E_1 C_{11} & E_1 C_{12} & 0 \\ E_2 C_{21} & E_2 C_{22} & 0 \\ 0 & 0 & G \end{bmatrix} \begin{bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\varepsilon_{12} \end{bmatrix} \quad (4.9)$$

and carrying out the multiplication yields

$$\begin{aligned} d\sigma_1 &= E_1 ( C_{11} d\varepsilon_1 + C_{12} d\varepsilon_2 ) \\ d\sigma_2 &= E_2 ( C_{21} d\varepsilon_1 + C_{22} d\varepsilon_2 ) \\ d\sigma_{12} &= G d\varepsilon_{12} \end{aligned} \quad (4.10)$$

which can be rewritten in matrix form as :

$$\begin{bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\sigma_{12} \end{bmatrix} = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & G \end{bmatrix} \begin{bmatrix} d\varepsilon_{1u} \\ d\varepsilon_{2u} \\ d\varepsilon_{12} \end{bmatrix} \quad (4.11)$$

The vector on the right-hand side can be defined as the vector of equivalent incremental uniaxial strains whose components are defined in terms of actual incremental strains by identifying with the appropriate terms, ie. as  $d\epsilon_{iu} = C_{i1} d\epsilon_1 + C_{i2} d\epsilon_2$  where  $i=1,2$ . For the orthotropic constitutive relationships developed in the preceeding section, Equation (4.6), these relationships written in full are :

$$d\epsilon_{1u} = \frac{1}{1-\nu^2} d\epsilon_1 + \nu \sqrt{\frac{E_2}{E_1}} d\epsilon_2 \quad (4.12)$$

$$d\epsilon_{2u} = \frac{1}{1-\nu^2} \nu \sqrt{\frac{E_1}{E_2}} d\epsilon_1 + d\epsilon_2 \quad (4.13)$$

The incremental equivalent uniaxial strains can be evaluated in the simple form as :

$$d\epsilon_{iu} = \frac{d\sigma_i}{E_i} \quad i = 1,2 \quad (4.14)$$

These relations have the same form as the uniaxial stress condition and hence the name 'equivalent uniaxial strain' is given for  $d\epsilon_{iu}$ . The total equivalent uniaxial strain can be determined by integration over the load path as :

$$\epsilon_{iu} = \int \frac{d\sigma}{E_i} \quad (4.15)$$

or its incremental equivalent

$$\epsilon_{iu} = \sum_{\substack{\text{load} \\ \text{increments}}} \frac{\sigma_i}{E_i} \quad (4.16)$$

The incremental and accumulated equivalent uniaxial strains do not transform in the same manner as stress. Both are fictitious (except in the uniaxial case) and their significance is only as a measure on which to base the deformation history of the material. Since the equivalent

uniaxial strains are not transformable, they are assumed to be defined only in the current principal stress directions. Bazant (1983) has criticised the application of orthotropic models to concrete where the principal stress directions rotate during loading. The non-linearity of concrete is due to the formation of microcracks produced within the microstructure by previous deformations. The rotation of the axes of orthotropy implies that these defects are rotated against the material. Further the defects are assumed to be caused solely by the current state of stress and not by previous deformations and this implies that the non-linearity is path independent. However Bazant also remarks that the orthotropic models are not the only models which may be criticised and that no perfect model for concrete exists free from criticism.

Isotropic variable moduli models such as <sup>that of</sup> Kupfer and Gerstle (1973) are inadequate in reproducing the stress-strain behaviour of concrete under all combinations of biaxial loading. Work-hardening plasticity models require knowledge of the effective stress-strain relationships to derive the strain hardening rule, but no empirical equations for the stress-strain relationships exist in the literature. Serious criticisms of the endochronic models have been raised by Chen, W.F. (1982), particularly with small amplitude stress and strain cycling. Further the extensive number of functions required to fit the experimental data makes it undesirable in this particular application.

The model proposed by Darwin and Pecknold has been shown to reproduce very accurately the non-linear behaviour of plain concrete subject to biaxial loading and Bazant's academic objections have not affected its practical usefulness. Many other researchers have successfully used orthotropic models and in particular the Darwin and Pecknold model, eg. Rajagopal (1976), Bashur and Darwin (1978) and Noguchi (1981).

In using the current model it is not expected that in general, at any point within the plain concrete, major rotations of the axes of orthotropy will occur during loading and even when cracking occurs the axes remain fixed at that point. There may however be locations within the plain concrete parts of a reinforced concrete structure where within the current model the axes of orthotropy might rotate considerably during loading and in such instances the model is inaccurate.

#### 4.2.3 Failure criteria.

The test data of Kupfer, Hilsdorf and Rusch (1969) has been widely accepted as a basis for the modelling of concrete behaviour under biaxial loading. Many other researchers have investigated the strength of concrete subject to biaxial loading and two other notable investigations were conducted by Nelissen (1972) and by Liu, Nilson and Slate (1972). The maximum strength criteria obtained was quite consistent between the separate investigations and is shown in Figure 2.13 .

From their tests, Kupfer et al. found that the strength envelope for biaxial compression was closely approximated by the equation :

$$\left[ \left( \frac{\sigma_1}{f'_c} \right) + \left( \frac{\sigma_2}{f'_c} \right) \right]^2 - \left( \frac{\sigma_2}{f'_c} \right) - 3.65 \left( \frac{\sigma_1}{f'_c} \right) = 0. \quad (4.17)$$

where  $\sigma_1, \sigma_2$  = principal stresses

$f'_c$  = uniaxial compressive strength

If the stress ratio  $\alpha = \sigma_1/\sigma_2$ , then this relationship may be rewritten as:

$$\sigma_{2c} = \frac{(1 + \alpha 3.65)}{(1 + \alpha)^2} f'_c \quad (4.18)$$

The peak stress in the minor compressive stress direction is then given by :

$$\sigma_{1c} = \alpha \sigma_{2c} = \frac{(1 + \alpha 3.65) \alpha f'_c}{(1 + \alpha)^2} \quad (4.19)$$

The values of  $\sigma_{1c}$  and  $\sigma_{2c}$  are used to define the shape of the equivalent uniaxial stress-strain curves for a particular value of the stress ratio ( $\alpha$ ). As the ratio  $\sigma_1$  to  $\sigma_2$  changes, so the shape of the uniaxial stress-strain curve changes as well.

For the tension-compression region, Kupfer suggested a straight line reduction in the tensile strength with increased compressive stress namely :

$$\sigma_1 = 1 - 0.8 \frac{\sigma_2}{f'_c} f_t \quad (4.20)$$

For the tension-compression region , Darwin and Pecknold used a simpler criterion of a constant tensile strength and then the compressive stress  $\sigma_{2c}$  is given by :

$$\sigma_{2c} = \frac{1 + 3.28 \alpha f'_c}{(1 + \alpha)^2} \quad (4.21)$$

The author has found that the Equation (4.21) is only applicable for part of the tension-compression region, given by:

$$\frac{f_t}{0.65 f'_c} < \alpha < 0. \quad (4.22)$$

ie. the region bounded by uniaxial compression ( $\alpha = 0$ .) and a stress ratio of  $\alpha$  approximately equal to -0.15 . For the remaining part of the tension-compression region the constant tensile strength criteria is used. Similiar types of failure envelopes for the tension-compression

have been used by Rajagopal (1976) and by Buyukozturk (1977).

For the tension-tension region, Kupfer et al. and Darwin and Pecknold recommend a constant tensile strength equal to the uniaxial tensile strength and this is used in the present model.

According to Nelissen (1972) the maximum strength envelope under biaxial loading appears to be largely independent of the loading path, but Taylor et al. (1972) indicate that non-proportional loading produces a lower strength than proportional loading for lightweight aggregate concrete. Within the present model it is assumed that the maximum strength envelope is independent of the loading path. The failure envelope as implemented in the finite element model is illustrated in Figure 4.2 .

#### 4.2.4 Loading curves.

Darwin and Pecknold used the concept of equivalent uniaxial strain to define equivalent uniaxial stress-strain curves for plain concrete subject to biaxial loading. A family of uniaxial stress-strain curves for each principal stress direction was developed with equivalent uniaxial strains as the abscissae. These curves may vary during non-proportional loading as a function of the principal stress ratio.

For compression loading the curves selected are based on the equation suggested by Saenz (1964) which is :

$$\sigma'_i = E_o \varepsilon_{iu} / \left[ 1 + \left( \frac{E_o}{E_s} - 2 \right) \left( \frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right) + \left( \frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right)^2 \right] \quad (4.23)$$

where  $E_o$  = Initial modulus of elasticity in compression

$E_s$  = ratio of maximum stress to maximum strain

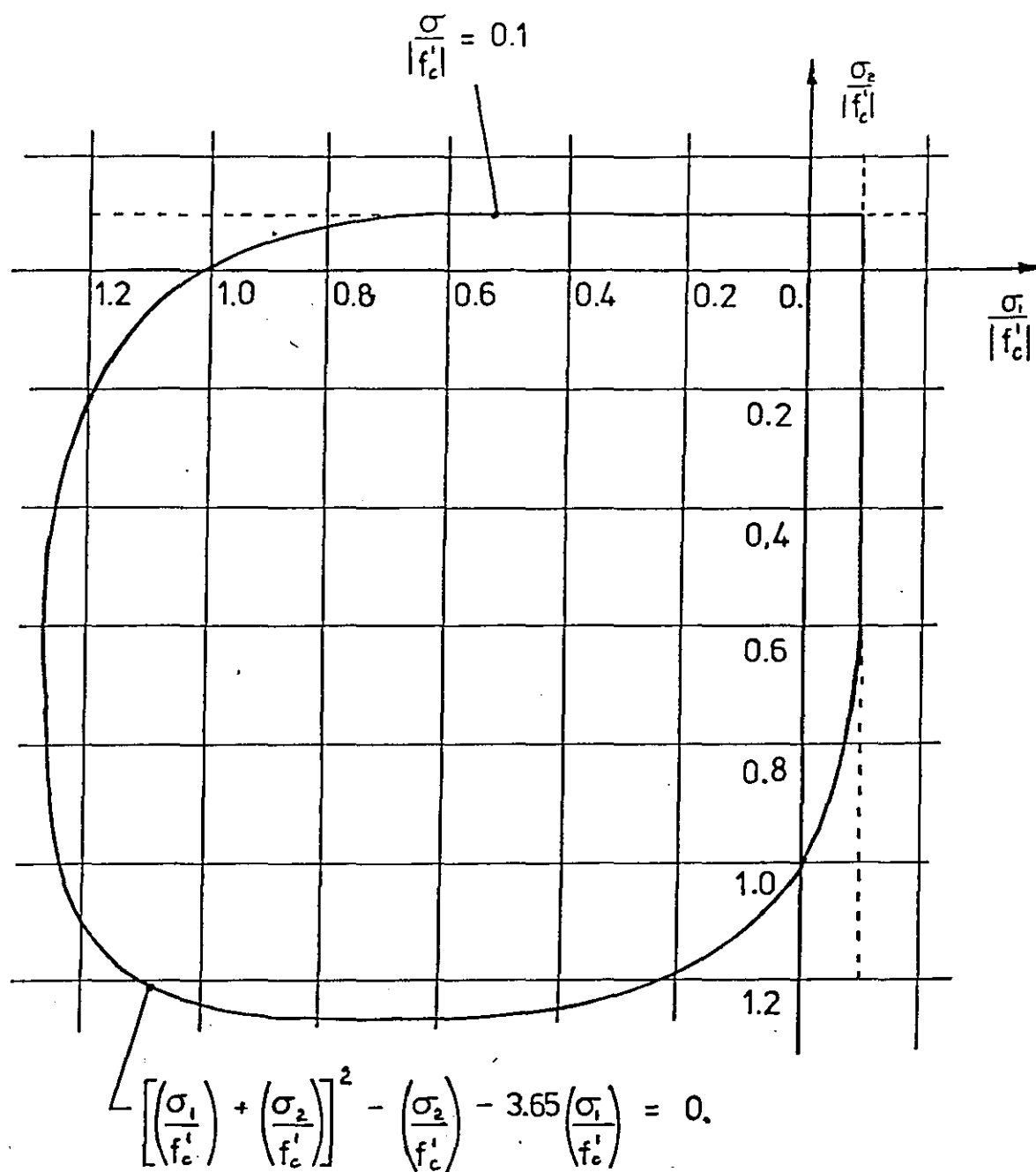


FIGURE 4.2 IDEALISED BIAXIAL FAILURE ENVELOPE FOR THE CONCRETE

$\epsilon_{iu}$  = current accumulated strain

$\epsilon_{ic}$  = failure value of strain

This equation is particularly useful since the initial modulus of elasticity and the value of peak stress and corresponding strain are independent variables.  $E_o$ ,  $\sigma_{ic}$  and  $\epsilon_{ic}$  define the compressive equivalent uniaxial stress-strain curve and  $\sigma_{ic}$  and  $\epsilon_{ic}$  will depend on the current principal stress ratio.

In determining the shape of the equivalent uniaxial curves the equivalent uniaxial strain at which the maximum compressive stress occurs is required. For values of strength greater than the uniaxial compressive strength a large increase in real strain occurs despite the Poisson's effect and to include this stress induced orthotropic behaviour, Darwin and Pecknold used the following equation that :

$$\epsilon_{ic} = \epsilon_{cu} \left[ \frac{\sigma_{ic} R - (R - 1)}{f'_c} \right] \quad (4.24)$$

where  $\epsilon_{cu}$  = strain at maximum stress in uniaxial compression

$f'_c$  = failure strength in uniaxial compression

$\epsilon_{ic}$  = equivalent uniaxial strain at maximum stress of  $\sigma_{ic}$

and

$$R = \frac{\frac{\epsilon_{ic} (\alpha=1)}{\epsilon_{cu}} - 1}{\frac{\sigma_{ic} (\alpha=1)}{f'_c}} \quad (4.25)$$

From the available experimental data Darwin and Pecknold indicate that R is approximately 3.0 . The author has accordingly used a constant value of R equal to 3.0 .

The maximum equivalent uniaxial strain at peak stress, Equation (4.24) does not give very good results for values of  $\sigma'_{ic}$  less than the uniaxial compressive strength, and Darwin and Pecknold suggest the use of a further equation where  $\epsilon_{ic}$  varies with  $\sigma'_{ic}$ , namely :

$$\epsilon_{ic} = \epsilon_{cu} \left[ -1.6 \left( \frac{\sigma'_{ic}}{f'_c} \right)^3 + 2.25 \left( \frac{\sigma'_{ic}}{f'_c} \right)^2 + 0.35 \left( \frac{\sigma'_{ic}}{f'_c} \right) \right] \quad (4.26)$$

The values of  $\epsilon_{ic}$  are constrained so that the ratio of  $E_o/E'_s$  is always greater than or equal to two and this prevents the stress-strain curve from becoming concave upwards.

For tensile loading, a linear-stress strain curve is used from zero stress and strain to a failure stress equal to the uniaxial tensile strength ( $f_t$ ) at the maximum uniaxial strain ( $\epsilon_{tu}$ ).

Darwin and Pecknold assumed Poisson's ratio to remain constant at 0.2 for biaxial tension and biaxial compression stress states, but to vary in tension-compression. In the present model a constant value of 0.2 for Poisson's ratio is used for all stress states.

#### 4.2.5 Cracking.

An important criteria in the modelling of concrete is the tensile failure condition. The progressive development of cracking in a loaded reinforced concrete structure is very important, as when cracking occurs the tensile forces that were being transferred across the crack can no longer be maintained and the internal forces need to be redistributed. The consequences of cracking and a large redistribution of internal forces can have a major influence on the overall behaviour of a reinforced concrete structure.

Cracking indicates a partial collapse of the concrete across the plane of cracking under tensile stress states and a crack is assumed to form in the plane perpendicular to the maximum principal tensile stress direction when that stress exceeds the failure strength. For biaxial tension and tension-compression stress states, the biaxial failure envelope as shown in Figure 4.1 has been adopted as the criteria for tensile failure.

An infinite number of parallel cracks are assumed to occur in the direction perpendicular to the offending principal stress. The tensile stress across the crack drops abruptly to zero and the resistance of the concrete against further deformation normal to the crack is reduced to zero. At the instant of the crack formation only the normal stress perpendicular to the cracked plane is released and the other stresses are assumed to remain unchanged. It follows that the state of stress in the cracked concrete is reduced to a uniaxial stress state parallel to the crack direction. A second crack may develop perpendicular to the first crack and the possibility of crushing is not excluded.

Cracking is modelled by reducing the modulus of elasticity along the major principal stress direction to zero. When a single crack occurs the constitutive equation becomes :

$$\begin{bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\sigma_{12} \end{bmatrix} = \frac{1}{1 - \nu^2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_2/4 \end{bmatrix} \begin{bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\varepsilon_{12} \end{bmatrix} \quad (4.27)$$

This assumes that there is some amount of shear retention with an open crack and shear may be transferred along the crack representing the friction and aggregate interlock that occurs in cracked concrete. Numerous researchers have allowed concrete to retain a small shear

stiffness after cracking. Hand, Pecknold and Schnobrich (1973) have shown that the magnitude of the shear stiffness was not as important as the fact that some stiffness was retained. The same conclusion was made by Khouzam (1977) in the analysis of a concentric tensile bond specimen. She noted that a closer approximation to the experimental bar stresses and concrete cracking behaviour was obtained by allowing some amount of shear retention rather than no shear stiffness. Phillips and Zienkiewicz (1976) in the analysis of a pre-stressed concrete pressure vessel used a shear factor of 'aG' where 'a' was equal to 0.5 .

In the present model when a single crack forms, the shear retention as given in Equation (4.8), is  $E_2/4(1-\nu^2)$ . Since  $G = E/2(1+\nu)$ , for uncracked concrete, this implies that 'a' is less than or equal to  $1/2(1-\nu)$ , ie.  $a \leq 0.625$  in the present model.

#### 4.2.6 Crushing.

Crushing is deemed to occur for concrete loaded in biaxial compression when the major compressive stress  $\sigma'_{2u}$  is :

$$\sigma'_{2u} > \frac{1 + 3.65 \alpha}{(1 + \alpha)^2} f'_c \quad (4.28)$$

where  $\alpha$  = principal stress ratio  $\sigma_1/\sigma_2$

$f'_c$  = uniaxial compressive strength

Crushing indicates the complete rupture and disintegration of the material under compressive stress states. After crushing the current stresses drop abruptly to zero and the concrete is assumed to lose its resistance completely against further deformation. The modulus of elasticity in both principal stress directions are reduced to zero and the concrete is assumed to be unable to carry any further stresses. The

stress-strain matrix [ D ] is set equal to zero.

The special numerical details to allow for both cracking and crushing in the finite element model are discussed in greater detail in Chapter 5.

#### 4.3 STEEL MODEL.

For this study a simplified bi-linear model for the stress-strain behaviour of steel is used. The model is such that steel is assumed to be an elastic-perfectly plastic material and the stress-strain curve for the reinforcement as shown in Figure 4.3 has been adopted.

In material co-ordinates the constitutive [ D ] matrix for the steel is given by :

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{bmatrix} E_{\text{steel}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{bmatrix} \quad (4.29)$$

It is expected that for the reinforced concrete structures to be analysed in Chapters 7 and 8, that in all cases the reinforcement stresses will be much less than the yield stress, even at maximum loading.

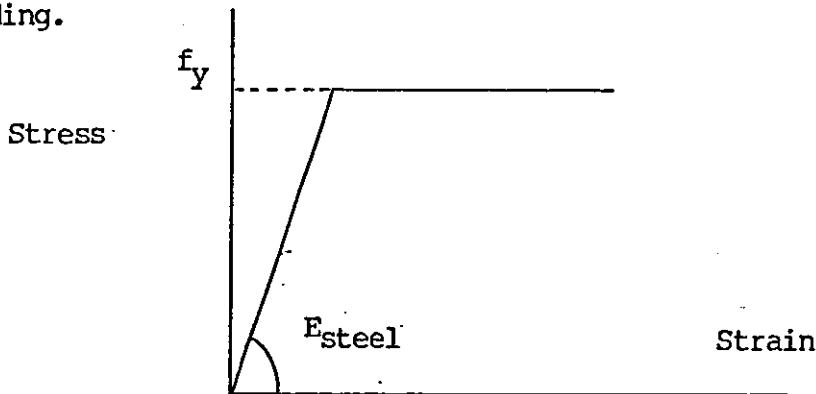


FIGURE 4.3 SIMPLIFIED STRESS-STRAIN CURVE FOR THE REINFORCEMENT

#### 4.4 SUMMARY.

Concrete under biaxial loading exhibits different stress-strain behaviour and varying strength characteristics depending on the ratio of biaxial stresses. Several concrete models were considered, the variable models, plasticity and endochronic models, and the equivalent uniaxial models. The concrete model chosen for use in this study was originally developed by Darwin and Pecknold (1974) and assumes concrete to be an orthotropic material in the two principal stress directions. Darwin developed the concept of 'equivalent uniaxial strain' whereby the coupling effect in the strains due to Poisson's ratio is eliminated and the behaviour is represented by equivalent uniaxial stress-strain curves for each of the principal stress axes. The 'equivalent uniaxial' stress-strain curves for compressive loading are based on the Saenz (1964) curve and a linear relationship is assumed for tensile loading. The values of maximum stress are obtained from a modified biaxial strength envelope based on the results of Kupfer et al.(1969). Cracking is modelled by a reduction in the modulus of elasticity perpendicular to the direction of the crack and there is some shear retention, related to the elastic modulus parallel to the crack direction. Crushed concrete is assumed to have no strength capacity.

The reinforcement is assumed to be a linear elastic material and perfectly plastic on reaching the yield stress.

## CHAPTER 5    FINITE ELEMENT METHOD OF ANALYSIS.

### 5.1        INTRODUCTION.

There is a considerable amount of literature on the theoretical and programming requirements of the finite element method, e.g. Zienkiewicz (1971), Cook (1981), Rockey et al. (1975) and its application to the structural analysis of reinforced concrete, e.g. A.S.C.E.(1982) State-of-the-art-report. Therefore within the following section, only a brief description of the finite element method is given, but where the author considers it necessary to clarify particular points in the discussion, further details will be given. Discussion is restricted to the displacement type of finite element method, the use of particular elements to represent the three phases: concrete, bond and steel, numerical integration and how the bond model of Chapter 3 and the concrete and steel models of Chapter 4 can be implemented. Special attention is given to the numerical techniques required to model the non-linear material behaviour, how to allow the concrete to crack (or crush) and to the implementation of the bond model.

### 5.2        BRIEF DESCRIPTION OF THE FINITE ELEMENT METHOD.

The finite element method in structural analysis is based on sub-dividing a structure into a number of discrete elements, connected to each other at individual joints or nodes. Within each element simple functions are chosen to approximate the variation of the displacements in terms of the nodal values. Using the principle of virtual work, a set of equations is obtained for each element which relates the displacements and applied forces at each node i.e. the elemental stiffness matrices. The global stiffness matrix is assembled from the elemental contributions and the global set of equations is modified to take into account the particular boundary conditions. The stiffness matrix contains the material

properties which in elastic analyses are represented by linear relations and therefore the elastic moduli contributing to the stiffness matrices are constants. The solution of this set of equations is a straight forward problem of solving a set of linear simultaneous equations, to obtain the nodal displacements. Stresses and strains within the elements may be calculated by further calculation from the nodal displacements.

Non-linear material behaviour, as with reinforced concrete is often modelled using incremental and/or iterative solution strategies, which are essentially combinations of piece-wise linear elastic solutions. Further consideration of the non-linear solution strategy to be adopted is given in Section 5.4 .

### 5.3 DISCRETIZATION OF THE REINFORCED CONCRETE STRUCTURE.

The model chosen by the author approximates three-dimensional reinforced concrete structures by a two-dimensional plane stress analysis. As reviewed (Chapter 2) there are various methods of representing the steel in reinforced concrete : embedded, distributed and discrete, each of which presumes the manner in which the concrete is bonded to the steel. The discrete method allows the possibility of bond slip and a more refined approach to the relationship between bond stress and bond slip and such an approach has been adopted by the author.

The finite elements which are used to represent the three phases of bond, concrete and steel within the model are now discussed in detail.

#### 5.3.1 A finite element for bond slip.

##### (i) The link element.

As reviewed (Chapter 2) the finite element most commonly used to model bond slip has been the link element as shown in Figure 5.1, e.g.

Ngo and Scordelis (1967), Lutz (1970), Labib (1976), Robins (1971), Allwood (1980) in the analyses of reinforced concrete structures and by Goodman et al. (1968) in the analysis of jointed rock. The element consists of two orthogonal springs which connect and transmit shear and normal forces between concrete node i and steel node j (Figure 5.1). The constitutive relationship for the linkage element relates these forces to the nodal displacements by :

$$\begin{bmatrix} F_r \\ F_s \end{bmatrix} = \begin{bmatrix} K_r & 0 \\ 0 & K_s \end{bmatrix} \begin{bmatrix} d_r \\ d_s \end{bmatrix} \quad (5.1)$$

where  $F_r, F_s$  = spring forces parallel and orthogonal to the direction of the reinforcement.

$K_r, K_s$  = spring stiffness in the two orthogonal directions.

$d_r, d_s$  = relative displacement of the nodes i and j in the two orthogonal directions.

The value of  $K_r$  relates to the force transfer by dowel action. The value of  $K_s$  is the interface shear stiffness and can be derived from measured bond stress-slip relationships such as those presented in Chapter 3. For a beam cross-section which contains (n) number of bars of diameter (d) and modelled with spring linkage elements at a spacing of (l), the linkage shear stiffness is given by :

$$K_s = \frac{u n d l}{a} \quad (5.2)$$

where  $u$  = bond modulus, i.e. the derivative of the bond stress-slip relationship.

If a single linkage element is used to connect the steel and concrete elements e.g. Allwood (1980) then the coefficient of 'a' is unity, but if the linkage elements are placed at the top and bottom of the bar

elements e.g. Ngo and Scordelis (1967) then  $\alpha$  is equal to two.

The linkage element however is an artificially discrete element and effectively lumps at the nodes connecting the steel and concrete elements the bond behaviour which occurs all along the interface. If planar isoparametric elements are used to represent the concrete and 3 noded bar elements (assumed quadratic displacement functions) represent the steel, then the use of the simple linkage element to inter-connect the elements is wrong, since the bond stiffnesses will be incorrectly attributed to the connecting nodes.

Consider the simple elastic analysis of a beam as shown in Figure 5.2a where the previously mentioned three types of elements have been used. The solution leads to an oscillating bond stress distribution as illustrated and a redistribution of the bond stiffness in the ratio 1:4:1 across each of the 3 noded bar elements results in an improved solution (Figure 5.2b). A bond element which avoids the discreteness of the linkage element is required and such an element is the 6 noded inter-face element. Using this element, which is described in detail in the following section, the solution to the above problem is improved still further (Figure 5.2c).

(ii) The 6 noded interface element.

The 6 noded bond interface element as shown in Figure 5.2<sup>3</sup> with an assumed quadratic displacement function and constant material properties (i.e. constant bond modulus throughout its length) has been used by Ngo (1975) and Saouma (1981). The element constitutive relationship is formulated in terms of the relative displacements of its top and bottom surfaces according to :

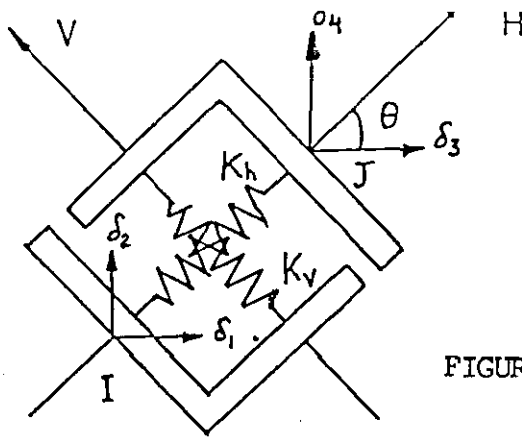


FIGURE 5.1 THE SPRING LINKAGE ELEMENT

Figure 5.2(a)

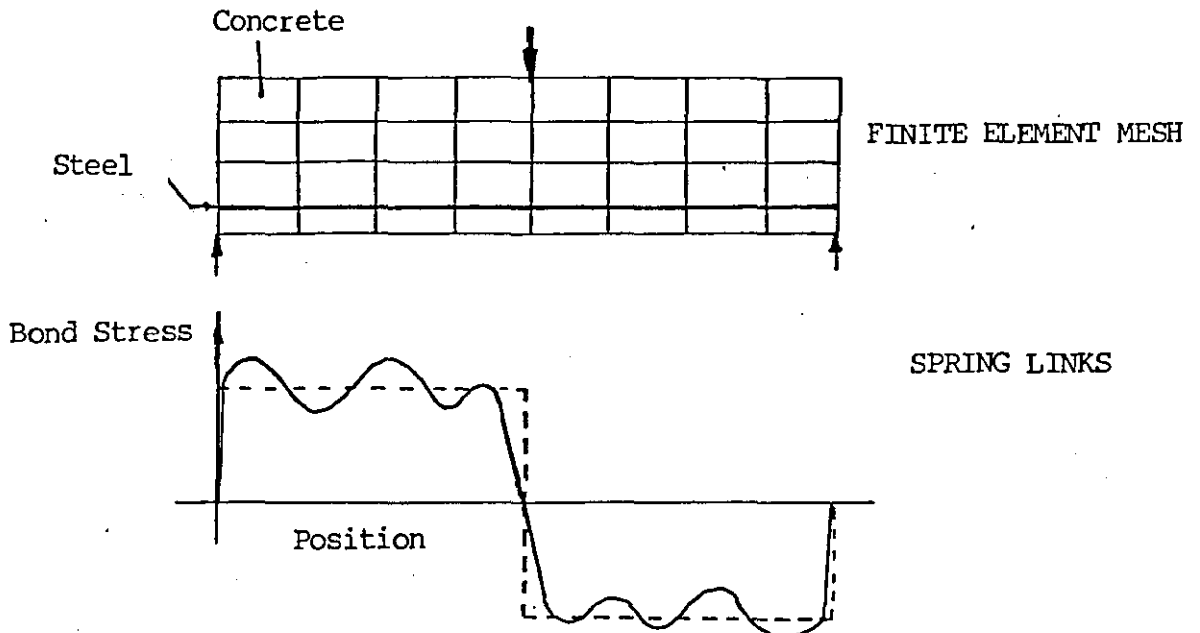


Figure 5.2(b)

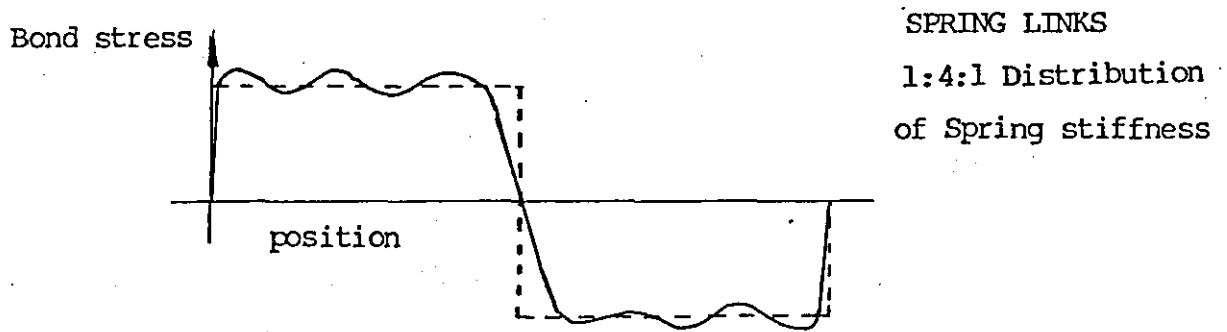


Figure 5.2(c)

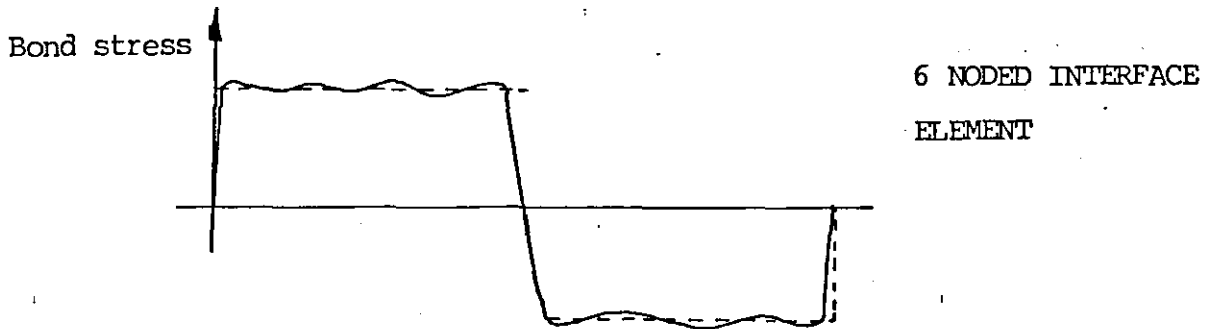


FIGURE 5.2 SIMPLE ELASTIC ANALYSIS OF A RC BEAM ILLUSTRATING THE USE OF DIFFERENT ELEMENTS TO REPRESENT BOND

$$P_r (r) = K_r W_r (r) \quad (5.3)$$

$$P_s (r) = K_s W_s (r) \quad (5.4)$$

where  $W_r (r)$  = slip of top surface relative to bottom.

$W_s (r)$  = normal displacement of the top surface relative to the bottom surface.

$K_r, K_s$  = shear and normal Stiffnesses

The stiffness matrix for such an element is relatively straightforward and can be found in the text of Ngo (1975). However it is unlikely that in practice, the bond properties remain constant throughout the length which the bond element represents. The author has therefore assumed a quadratic variation in bond properties throughout the length of an interface element. For the 6 noded element shown in Figure 5.3 with the mid node exactly halfway, then if the 'material' of the interface is considered to be a distributed spring system then :

$$q_x = K_x (u_b + u_t) \quad (5.5)$$

where  $K_x = \sum N_i K_i$

and  $q_x$  = bond stress at local co-ordinate  $x$

$K_x$  = bond modulus at  $x$

$N_i$  = shape function evaluated at  $i^{th}$  node

$$\text{However } u_b + u_t = U_i N_i \quad (5.6)$$

$$\text{and therefore } q_x = \sum K_i N_i \sum U_i N_i \quad (5.7)$$

By the Virtual Work principal the terms of the stiffness matrix are given by :

$$K_{ij} = t \int_{-a}^a q U_b dx + t \int_{-a}^a q U_t dx \quad (5.8)$$

where  $U_b$  and  $U_t$  = virtual displacements  
 $t$  = thickness of the element

which gives :

$$K_{ij} = t \int_{-a}^a \sum N_i K_i \sum N_j U_j dx$$

The definite integrals can be solved directly, however the calculations are greatly eased by the use of numerical integration to obtain the stiffnesses.

A commonly adopted quadrature rule is Gaussian integration which evaluates an integral by evaluating the function to be integrated at specified sampling points (known as Gauss points) and multiplying them by specified weighting factors. A Gauss point rule with (n) sampling points can integrate a polynomial up to degree  $2n-1$  exactly. For the bond interface element to be used in this model, where the displacement and the bond properties are assumed to be both quadratic in variation, the stiffness integrals are fourth order polynomials and a 3-point rule is used and is capable of exactly integrating the functions.

The bond stress at any point in the element is given by :

$$\sigma_x = \sum K_i N_i \sum U_j N_j \quad (5.10)$$

where  $U_j$  = relative displacement of the top and bottom  
points of the element

Within the model, for each bond element, values of bond stress are calculated at the three Gauss points and then extrapolated in a quadratic fashion to the nodes. The nodes are convenient positions to have the bond stresses output for examination, however for a node which

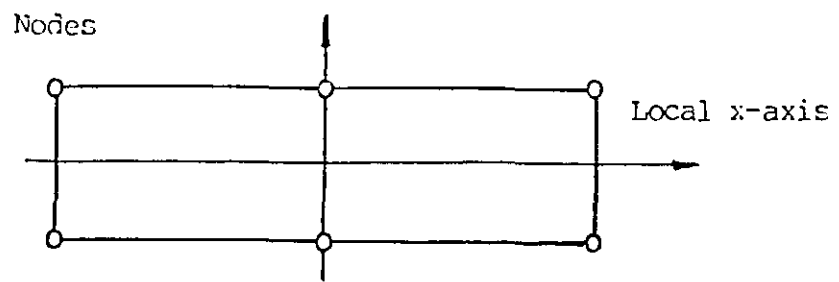


FIGURE 5.3 6 NODED BOND ELEMENT

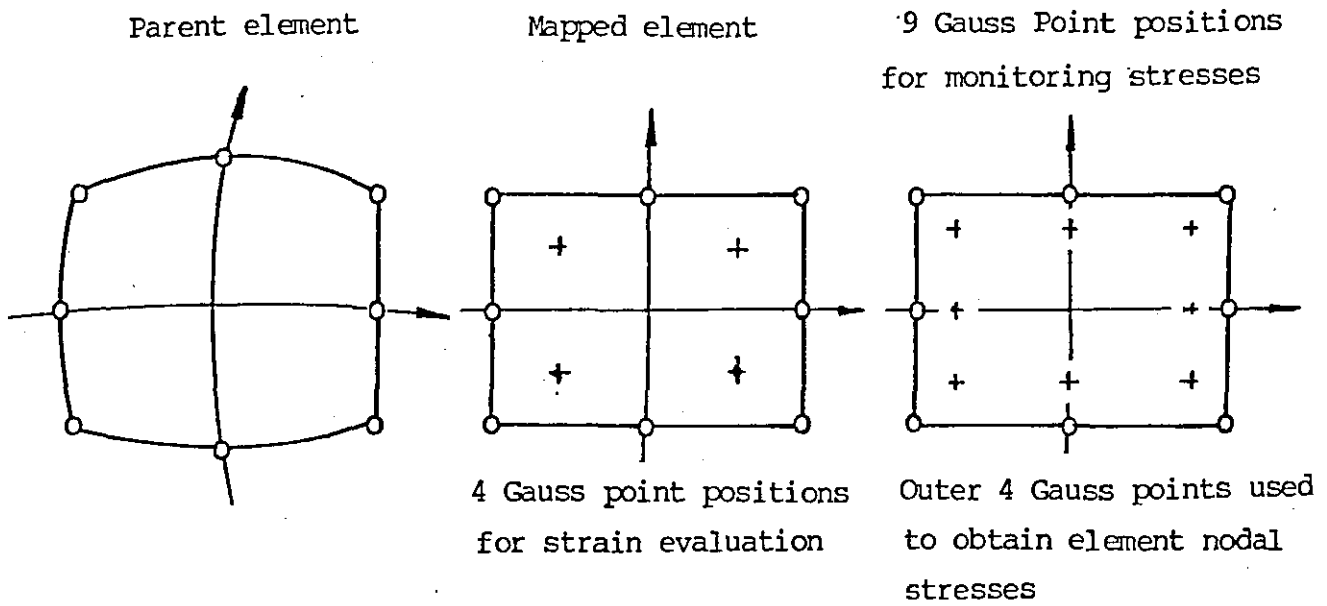


FIGURE 5.4 8 NODED ISOPARAMETRIC MEMBRANE ELEMENT

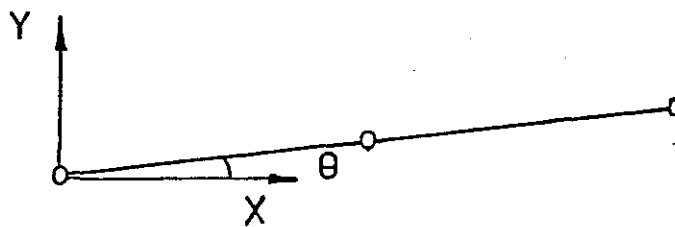


FIGURE 5.5 3 NODED BAR ELEMENT

is shared by more than one element the value of stress obtained from each element may not coincide. There is a discontinuity in the stress field at the nodes joining the bond elements. A stress-smoothing technique is employed where the values of stress obtained at the nodes of an element are averaged with adjacent element contributions where they exist. These nodal average values are used for visual inspection purposes only and the relative displacements and bond stresses at the 3 Gauss points are used to monitor and control non-linear behaviour. Details of how the bond model developed in Chapter 3 is implemented using this 6 noded interface element is described in Section 5.5.

A restriction placed upon the interface element in the present model is that the local x-axis of the element coincides with the global x-axis.

### 5.3.2 A finite element to represent the concrete.

Within the model concrete is assumed to be represented by 8-noded isoparametric membrane elements (Figure 5.4) and both a quadratic variation in the displacements and the material properties is assumed in both directions. The stiffness integrals of such an element are polynomials of sixth order and the use of numerical integration to solve these integrals is essential. A 3x3 Gaussian quadrature rule is adopted to evaluate the stiffness integrals and such a rule is capable of exactly integrating a fifth order polynomial. In this case the 'reduced' order of integration is sufficient and tends to soften the element countering overly stiff behaviour. A further consideration with respect to the appropriate order of quadrature for the stiffness integrals is the possibility of cracking within the concrete elements.

Cracking is modelled (Chapter 4) by a reduction to zero in the

appropriate elastic moduli monitored at the Gauss points. If cracking is localised to only one or two Gauss points within an element then there will be a considerable variation in the elastic moduli across the element and 3x3 Gauss integration is needed rather than a lower order rule. By monitoring the elastic moduli at the 3x3 Gauss points, if cracking occurs within an element, the crack is effectively smeared over a localised area around the affected Gauss point. This type of approach is commonly known as the 'smeared crack approach'.

For the 8-noded isoparametric element the optimal positions for evaluating strains corresponds to the 2x2 Gauss points (Barlow,1976). This result is independent of the variation of elastic moduli and strain field and depends only on the derivation of the Jacobian, which is a function of the element geometry only. Therefore within the model the 2x2 Gauss points are used to calculate the most accurate strains within the element. These strains are then extrapolated to the 3x3 Gauss points using a bi-linear expansion method similar to that used by Hinton and Campbell (1974) to obtain nodal values from the 2x2 Gauss points. Concrete stresses at the 3x3 Gauss points are then calculated directly using the extrapolated strains and the elastic moduli monitored at these positions. Nodal values of stress for an element are obtained by a bi-linear expansion on the four outer 3x3 Gauss points as shown in Figure 5.4 and the nodal averaging technique is employed to smooth nodal stresses between elements.

### 5.3.3 A finite element for the reinforcement.

The steel reinforcement is assumed to be represented by 3 noded axial force bar elements as shown in Figure 5.5. The midside node is fixed exactly halfway between the outer nodes and a quadratic variation in the displacement field is assumed. With constant material properties assumed across the element, the stiffness matrix for the 3 noded bar

element is relatively straight forward to derive. In local co-ordinates the element stiffness matrix is given by :

$$K^{(e)} = \frac{EA}{6l} \begin{bmatrix} 14 & -16 & 2 \\ -16 & 32 & -16 \\ 2 & -16 & 14 \end{bmatrix} \quad (5.11)$$

where  $K^{(e)}$  = element stiffness matrix  
 $E$  = Young's modulus  
 $l$  = overall length of the element

Bar stresses are evaluated at points corresponding to a 2 point Gauss rule. Nodal values of stress for an element are obtained by a simple linear expansion from the 2 Gauss points. As with the concrete and bond elements a nodal averaging technique is employed to smooth stresses between bar elements.

#### 5.4 NON-LINEAR ANALYSIS TECHNIQUE.

Non-linearity arises from the assumed non-linear stress-strain behaviour of the concrete and the assumed non-linear bond stress-slip between the steel and concrete. Geometric non-linearity is not considered. There are three main methods of solving non-linear material behaviour, namely : (1) incremental procedures, (2) iterative procedures and (3) mixed procedures, a combination of (1) and (2).

##### 5.4.1 Incremental load procedures.

The basis of this procedure is to subdivide the total load into small increments of load and for each increment the materials are considered to have linear elastic properties. The method is illustrated in Figure 5.6. At the end of each load increment a new stiffness matrix

is calculated using the current tangential value from the respective stress-strain or bond stress-slip relationship. The method can be improved by using a predictor-corrector method such as the Runge-Kutta scheme. However, whichever method is adopted the numerical solution tends to drift away from the exact solution. The respective non-linear material laws may be followed more closely by using smaller load increments, however greater accuracy is offset by the increased computational time spent recalculating the stiffness matrix for each load increment.

#### 5.4.2 Iterative procedures.

The basis of these procedures is to resolve the problem until some convergence criteria is satisfied and one direct method is the secant modulus method as shown in Figure 5.7. In recent years there has been a general trend amongst researchers in reinforced concrete to adopt the alternative iteration technique known as the 'equivalent load method', first advanced by Zienkiewicz, Valliappan and King (1968).

The basis of the equivalent or residual force method is to apply a set of artificial loads to bring the elastic solution closer to the non-linear solution, and there are two methods of achieving this namely: the 'initial stress' and 'initial strain' methods. The initial stress method is illustrated in Figure 5.8. Point A represents the first elastic solution. The true solution for the current strain is point B and the difference between the true stress corresponding to the elastic solution is the initial or residual stress. This difference represents the out of balance stress which is to be redistributed elastically to restore equilibrium. The whole process was originally named by Zienkiewicz et al.(1968) as one of 'stress transfer'. These residual stresses are converted to a set of nodal residual loads, via a virtual work integration and contributions from each element are

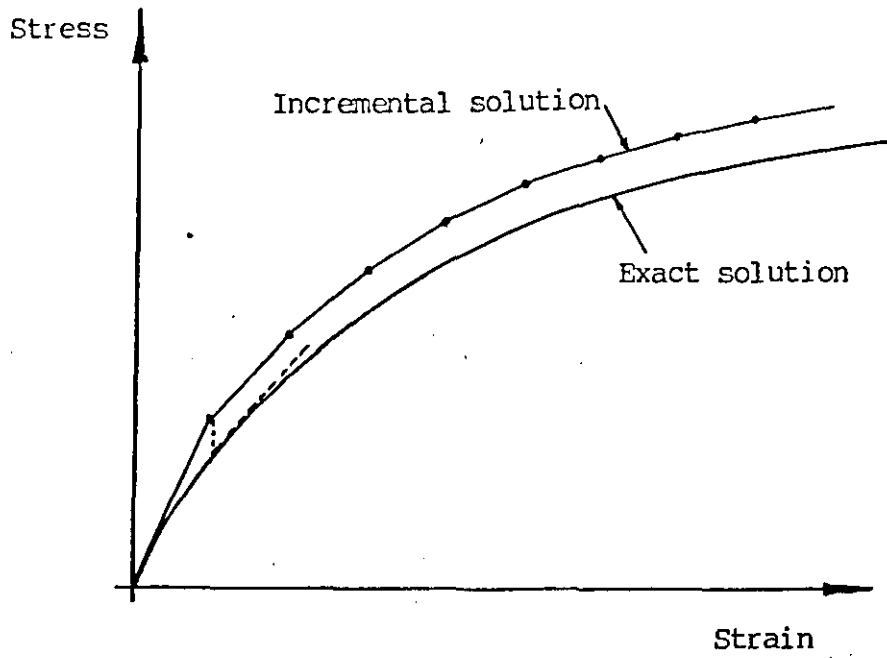


FIGURE 5.6 INCREMENTAL LOAD PROCEDURE

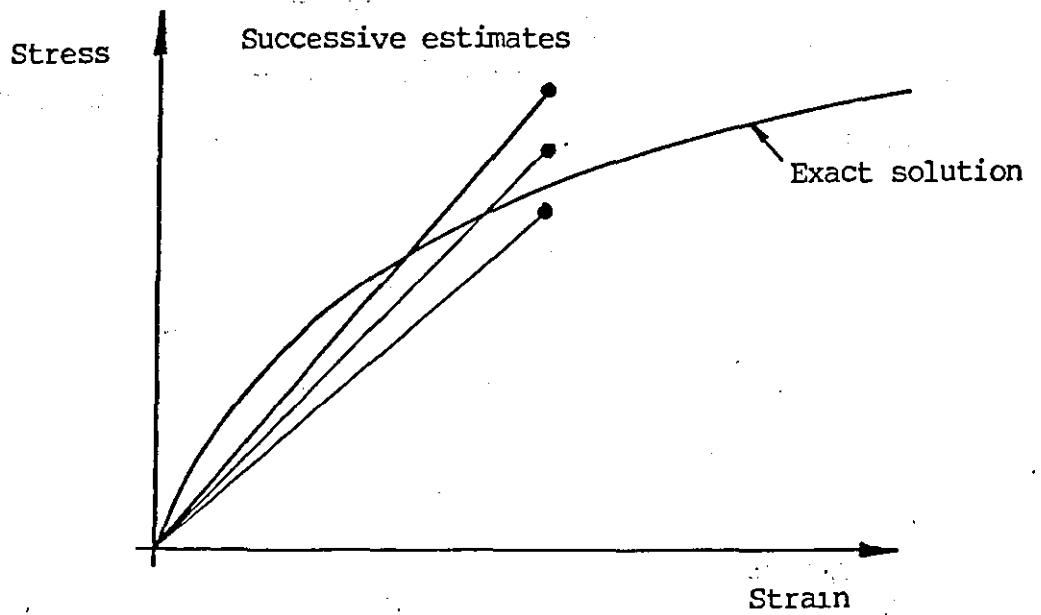


FIGURE 5.7 DIRECT ITERATION METHOD

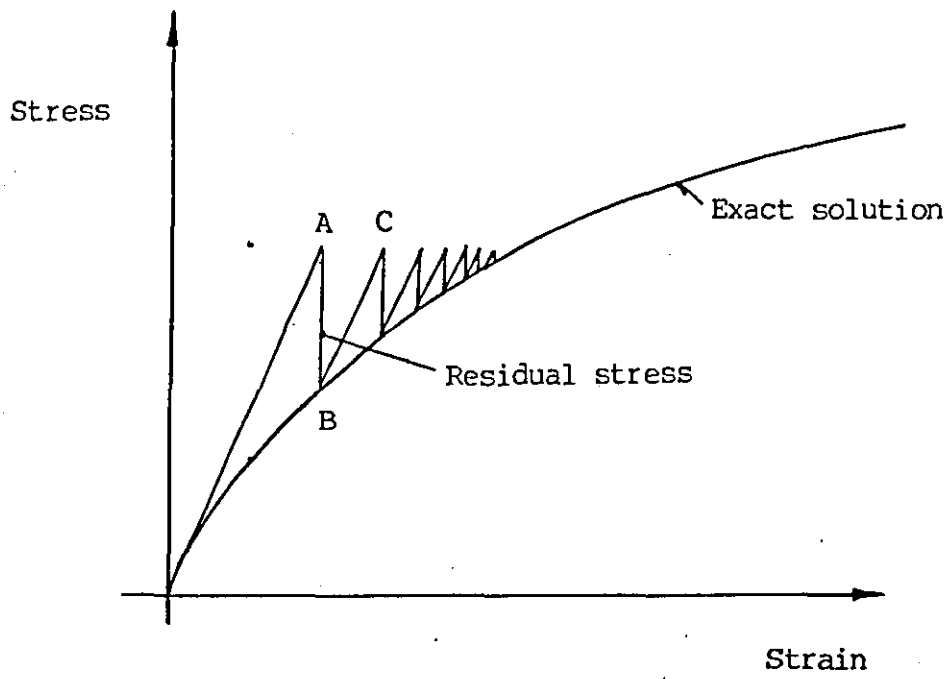


FIGURE 5.8 INITIAL STRESS METHOD

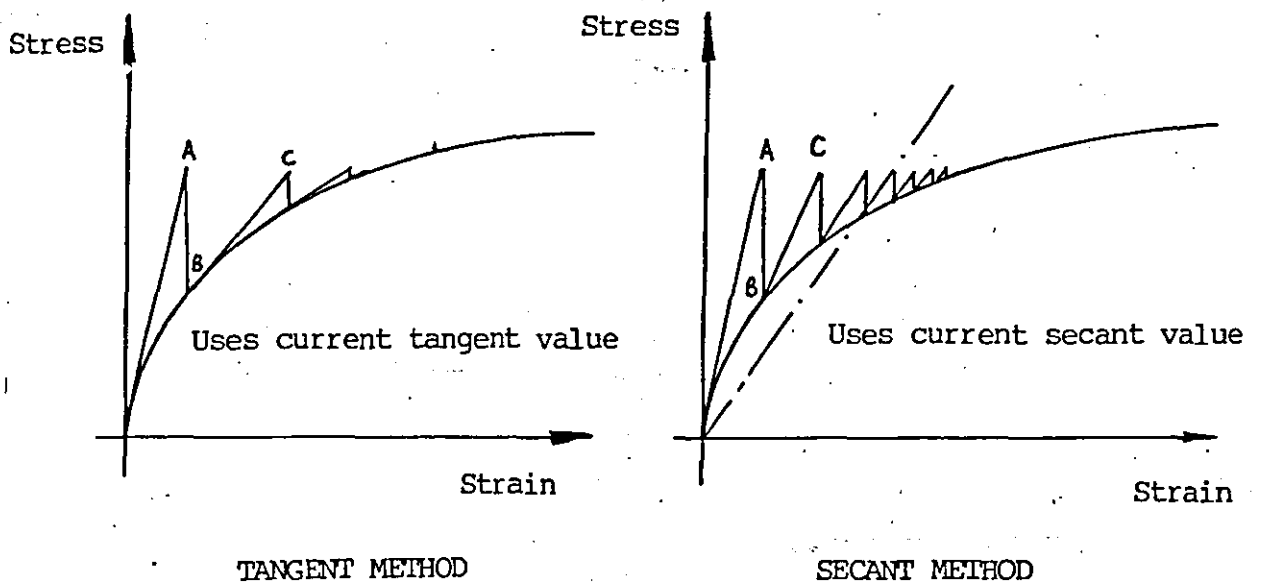


FIGURE 5.9 TANGENT AND SECANT STIFFNESS METHODS

accumulated. The set of residual loads that corresponds to the residual stresses is then applied to the structure to obtain the next solution C. The process continues until some convergence criterion or criteria are satisfied. With this method during the iteration process the stiffness matrix calculated at the start of the load increment is used throughout. Refinements can be made to the method to increase the convergence rate. Two such methods are the 'secant stiffness' and 'tangent stiffness' approaches, whereby during the iteration process the stiffness matrix is recalculated based on the accumulated level of strain (or slip) and either secant or tangent values of moduli are utilised. While all three methods initial, tangent and secant methods, as shown in Figure 5.9, satisfy in the final state, the necessary equilibrium, compatibility and constitutive laws conditions, it is evident that convergence is fastest by the tangent stiffness method and slowest by the initial stiffness method. However, the initial stiffness method has the distinct advantage that the stiffness matrix need only be calculated once.

#### 5.4.3 Mixed procedures.

It is usual to use the more desirable features of both the incremental and iterative solution techniques. The total is subdivided into load increments, which are not necessarily of equal size and within each load increment an iterative solution technique is employed.

#### 5.4.4 The basic analytical procedure adopted.

The basic input for the analysis procedure consists of a description of the topology and initial material properties of the structure. The loads are imposed as nodal forces and the material properties for the bond, concrete and steel are specified for each element. A detailed description of the input for an analysis is present-

-ed in Appendix B.

The first stage in the analysis consists of calculating the global stiffness matrix, which is based on the initial material properties. The structure is then analysed under several increments of load and for each increment the solution is carried through several iterations until a specified convergence criterion is satisfied. The force correction procedure adopted is the 'initial stress' method and the structural stiffness matrix is only recalculated at the end of the load increment before the application of the next load increment.

The non-linear behaviour of each material is followed at the respective Gauss points within each element and the calculated residual stresses at these points are used to obtain the residual nodal loads. The process of force correction continues until the solution for that load increment converges. A diverging solution is generally an indication of a failure condition.

The overall method is illustrated in the flowchart of Figure 5.10.

#### 5.4.5 Cracking of the concrete elements.

A further complication in the numerical technique applied to reinforced concrete structures is the possibility of cracking within the concrete elements. Cracking in this instance is modelled by a reduction in the appropriate elastic modulus (Chapter 4) and the releasing of the offending tensile stress at the respective Gauss point. The tensile stress-strain relationship which has been assumed is shown in Figure 5.11. If the theoretical curve was followed exactly during the iterations of a load increment then when the tensile strain exceeds the failure strain there would be a sudden increase in the residual stress, approximately the size of the tensile failure stress. The author considers

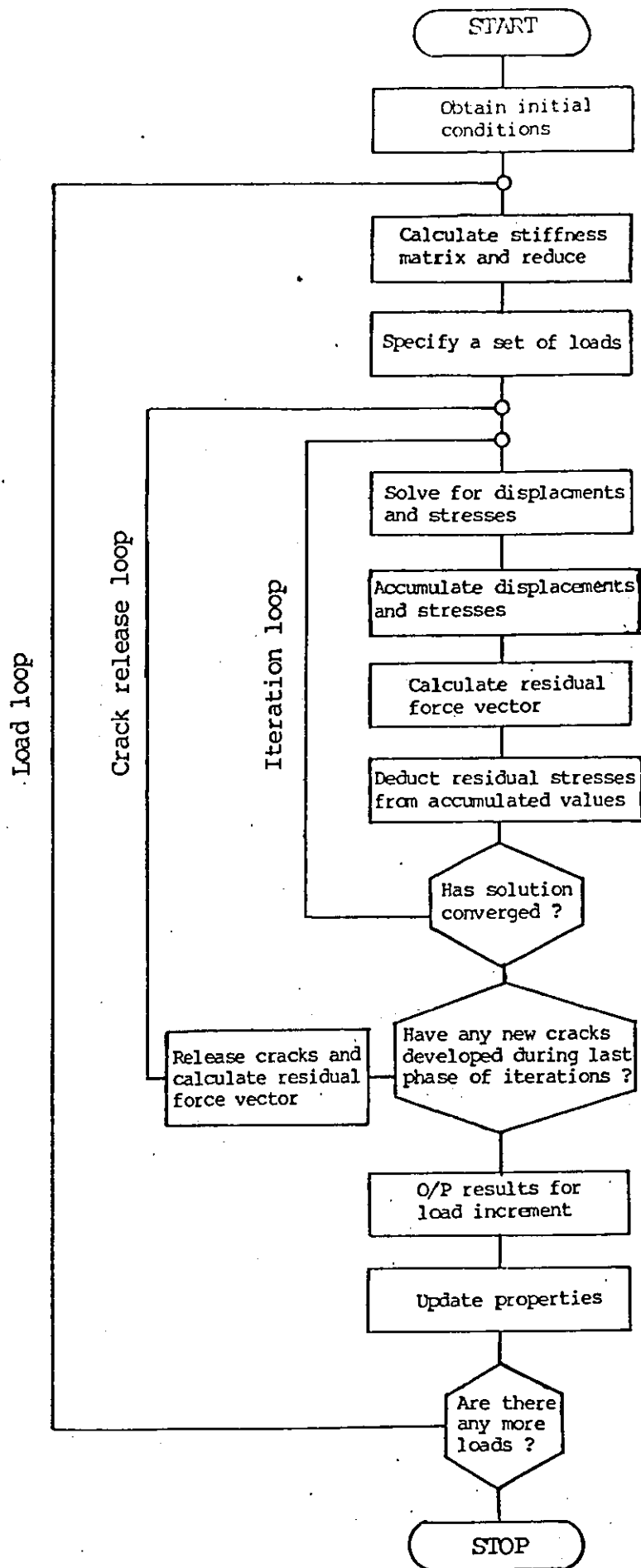


FIGURE 5.10 FLOWCHART FOR THE BASIC SOLUTION PROCEDURE BY FINITE ELEMENTS

this phenomenon undesirable for two reasons :

- (1) The residual forces during the iteration process are pseudo forces and the occurrence of a high tensile stress sufficient to cause cracking may be temporary state and may be reduced during further iterations. In such instances if cracking was allowed to occur then a different load path from the correct load path would occur. i.e further incorrect cracking would probably be initiated.
- (2) The release of a large tensile stress during the iteration process may cause considerable problems with the convergence of the solution.

To overcome these problems a particular solution technique has been adopted by the author and is now described.

During the load increment, if at a particular Gauss point within a concrete element it is deemed to have failed by cracking then the assumed stress-strain relationship is as shown in Figure 5.12. i.e the tensile stress for strains in excess of the tensile strain is assumed to be the maximum tensile value. The iteration process continues until the convergence criterion is satisfied. The tensile stresses at the Gauss points which are deemed to have cracked are then released as residual stresses. The nodal loads associated with this stress release may be considered to be similar to a further load increment. The iteration process is then restarted, continuing to use the same structural stiffness matrix until a convergence criterion is satisfied. The author terms this iteration process 'concrete failure phase 1'. If additional cracking occurs at other concrete Gauss points during this phase, the tensile stresses at these sampling points are held and the stress-strain curve of Figure 5.12 is followed. At the end of the iteration process of

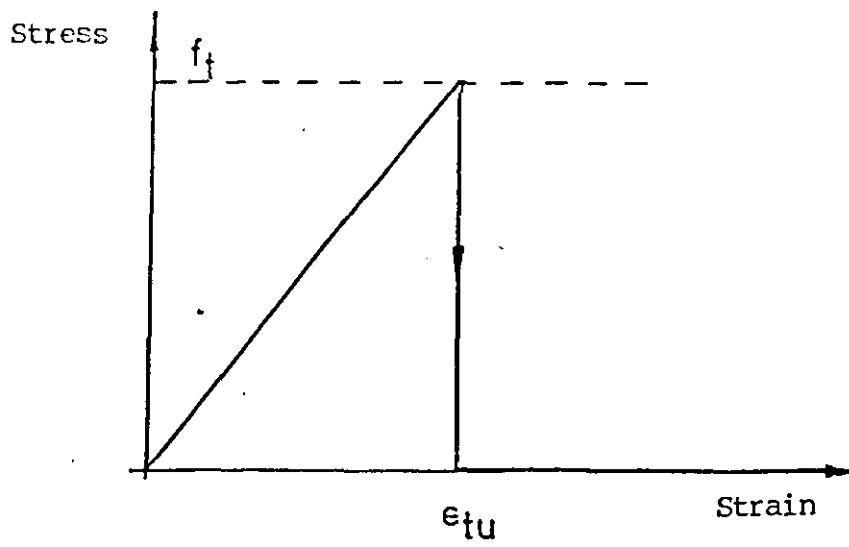


FIGURE 5.11 THEORETICAL STRESS-STRAIN CURVE FOR CONCRETE IN TENSION

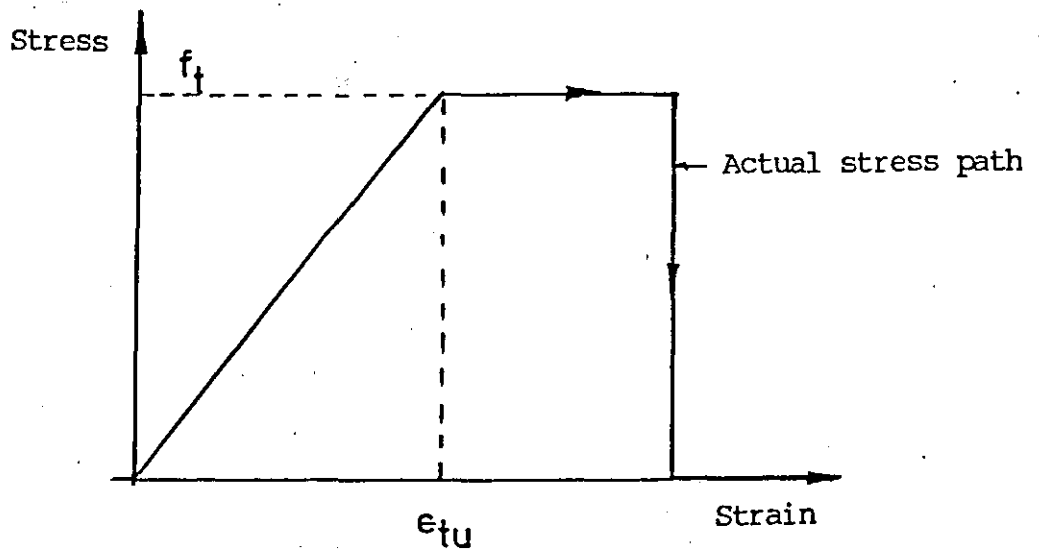


FIGURE 5.12 ADOPTED STRESS-STRAIN CURVE FOR THE CONCRETE

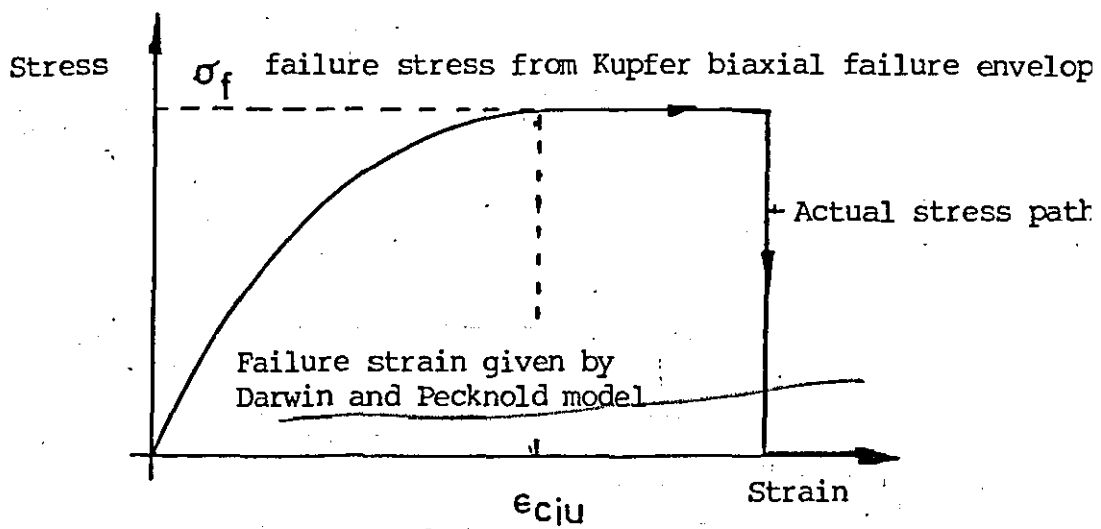


FIGURE 5.13 ADOPTED STRESS-STRAIN CURVE FOR CONCRETE IN COMPRESSION

'concrete failure phase 1', the tensile stresses associated with further cracks are released and another concrete cracking phase of iterations commences. The whole process continues until no further cracking of the concrete occurs and only then may a further imposed load increment be applied.

#### 5.4.6 Crushing of the concrete elements.

Similar to the cracking phenomenon, the possibility of crushing within the concrete elements could give rise to undesirable numerical features, since crushing is modelled by a reduction to zero stress for strains in excess of the compressive failure strain (Chapter 4). Therefore exactly the same procedure is adopted for crushing as with cracking. If during any of the iteration processes, crushing of the concrete is deemed to occur at any Gauss point then the compressive stress-strain curve adopted is that shown in Figure 5.13. At the end of the iteration process, at Gauss points where the concrete is deemed to have failed by crushing, the associated compressive crushing stresses are released and treated as residual stresses. A further phase of iterations is then commenced to redistribute the out of balance stresses.

#### 5.4.7 Convergence and updating the materials properties.

Within the model during an increment of load there will always be a phase of iterations corresponding to material non-linearity. However if concrete cracking (or crushing) occurs then there will be a subsequent phase or phases of concrete failure iterations, corresponding to concrete Gauss points cracking or crushing. A single criterion is used to establish convergence during a phase of iterations, however the tolerance value depends on whether the concrete is failing or not. The criterion is based on the magnitude of the Euclidean norm of the residual nodal load vector, which is the square root of the sum of the squares of the

components of the vector.

During a phase of iterations the Euclidean norm of the first residual load vector is calculated and the criterion to be satisfied is that the Euclidean norm of a subsequent residual load vector is less than a prescribed percentage of the norm of the first residual load vector. Duncan and Johnnary (1979) surmised that if strict static equilibrium is demanded at each load level, then the convergence is slow and often uncertain and carries with it the prospects of predicting the wrong behaviour. Further, they commented that the results of the initial stress method fit the true results well if coarse rather than close convergence tolerances are allowed. Within the model, with the onset of failure within concrete elements the convergence process becomes very slow (15 or more iterations are often required). Therefore two tolerance values are used which depend on whether the iteration phase is the material non-linearity or a phase of concrete failure iterations. It is assumed that for material non-linearity a tolerance of one percent is used and for a concrete failure iteration phase a value of five percent.

## 5.5 INCORPORATING THE BOND MODEL.

To incorporate the bond model developed in Chapter 3 and in particular to find the theoretical bond stress given the bond slip, the following quantities need to be known at each Gauss point within a bond element during the analysis :

- (1) concrete lateral pressures ( $\sigma_{yy}$ )
- (2) bar radial strains or equivalent interfacial radial pressures
- (3) bond parameters  $q_0, \mu, \Delta_u, \beta$  (Chapter 3)

The element bond parameters (3) are constants for a given problem

and are assigned during the first stages of the analysis. The concrete lateral pressures and bar radial pressures along a bond element will vary during loading. During the loading process, stresses for all elements of each element type are accumulated at their respective Gauss points. To find the theoretical bond stresses at any stage during the analysis the following procedure is adopted.

Nodal average values of stress for each element type are calculated from the current accumulated values of stress monitored at the respective Gauss points of each element. Since the bond elements are restricted to lie parallel with the global x-axis, the concrete lateral pressures need only be calculated from the  $\sigma'_{yy}$  stresses. Consider part of a reinforced concrete structure illustrated in Figure 5.14, which shows a steel element surrounded by two concrete elements, inter-connected by a single bond element. From the nodal average values of concrete stresses ( $\sigma'_{yy}$ ), the value of stress on the boundary corresponding to the positions of the three Gauss points within the bond element are calculated by linear least squares interpolation. Similarly the bar axial stresses at the three Gauss points are calculated by least squares interpolation of the nodal average bar stresses. The values of concrete lateral stress ( $\sigma'_{yy}$ ) and bar axial stresses are converted to equivalent radial pressures using Equations (3.8) and (3.9) (Chapter 3). From these two values of equivalent radial pressure and given the bond slip the theoretical bond stress may be calculated at each Gauss point of a bond element, as outlined in Chapter 3.

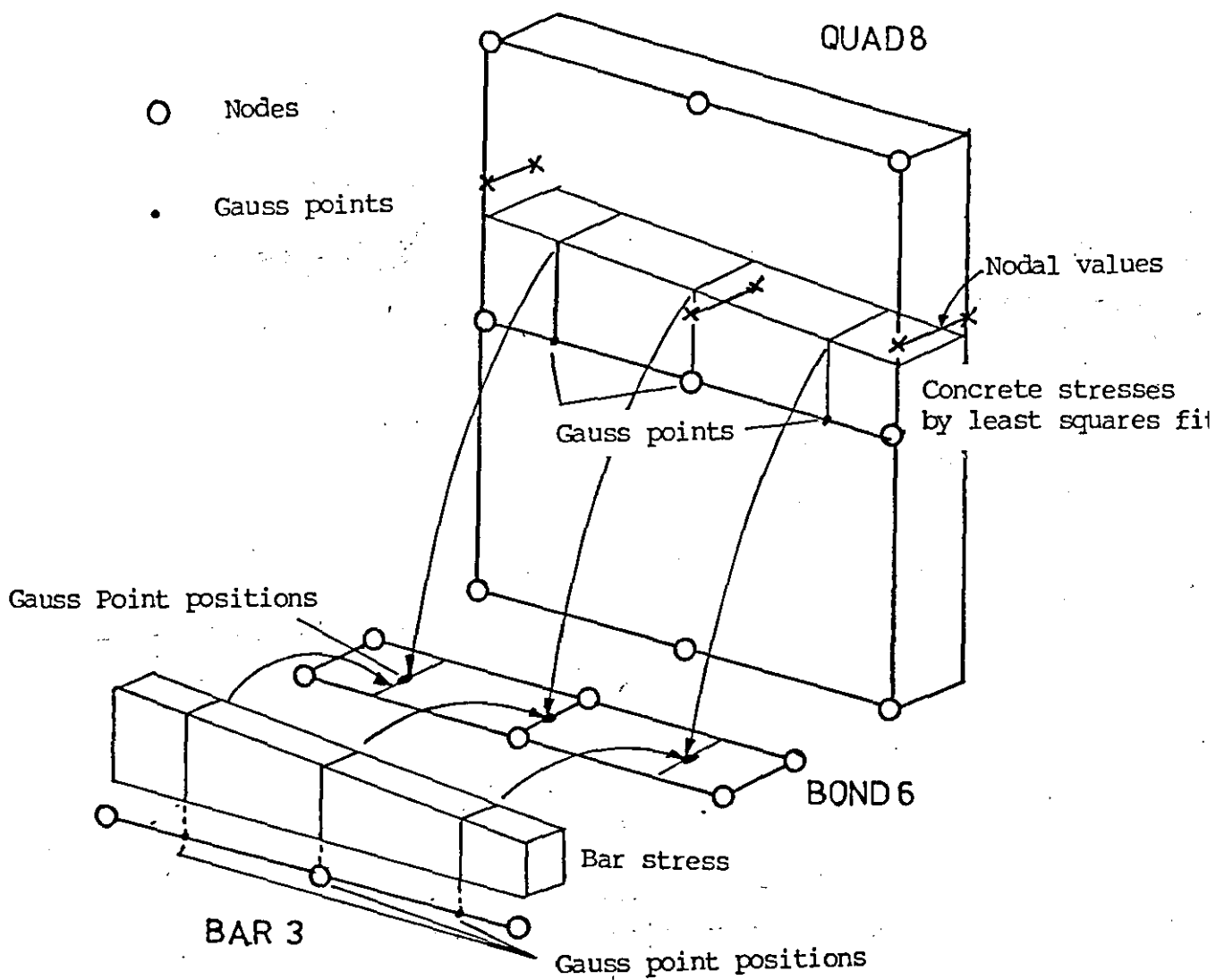


FIGURE 5.14 SCHEMATIC DIAGRAM OF HOW THE APPROPRIATE CONCRETE LATERAL PRESSURES AND BAR STRESSES ARE TRANSFERRED TO A BOND ELEMENT

## 5.6 SUMMARY

A displacement finite element approach incorporating incremental loading and the initial stress method of force correction during each load increment has been adopted. Three dimensional reinforced concrete structures are analysed by an approximate two dimensional plane stress model. Six noded rectangular interface elements with assumed quadratic displacement and material properties variation within an element, represent the bonding between steel and concrete. Three noded axial force bar elements represent the steel and 8 noded isoparametric membrane elements represent the concrete. A 3 point Gaussian quadrature rule is used for a bond element and a 3x3 Gauss rule for a concrete element to evaluate the respective element stiffness integrals. Material properties for the bond and concrete elements are monitored at the respective Gauss points, and an overall value for the steel elements is used. Cracking of the concrete elements is achieved by a smeared crack approach ,i.e. by a reduction in the appropriate elastic moduli at the affected 3x3 concrete Gauss points. A special numerical solution technique has been adopted to cope with cracking or crushing within the concrete elements. Essentially the method involves maintaining the failure value of stress during the current phase of iterations and then releasing the offending stress and redistributing the new out of balance stresses by further iterations.

## CHAPTER 6    EXPERIMENTAL WORK

### 6.1    AIMS AND OBJECTIVES.

A limited number of tests have<sup>been</sup> conducted by various researchers (Chapter 2) to find the load distributions along plain round bars embedded in concrete which form pullout specimens, i.e. Mains (1951), Wilkins (1951), Peattie and Pope (1956) and Parland (1957). Slightly larger amounts of data are available from the literature regarding the load distributions along deformed bars embedded in concrete of both pullout specimens, i.e. Mains (1951), Wilkins (1951), Peattie and Pope (1956), Parland (1957), Perry and Thompson (1966), Nilson (1972) and Standish (1982) and other reinforced concrete specimens, e.g. Mains (1951) and Allwood (1980).

A small series of experiments was therefore designed to confirm the nature of the bar distributions along plain bars embedded in pullout specimens obtained by previous researchers. Additional information regarding the load distributions along plain round bars embedded in other types of bond tests and to the nature of bonding between plain bars and the surrounding concrete was also gained from the experiments. The choice of bond tests is discussed in Section 6.2 and the adopted methods of strain gauging the bars to measure the load distributions in Section 6.3. Some additional pullout tests begun by the author and continued by Allwood (1984) within the Department of Civil Engineering, Loughborough University of Technology are also reported herein.

The information obtained from this limited range of bond tests together with the data from the previous investigations then forms a larger resource of available data on the bonding of plain bars to concrete with which to compare the finite element analyses being performed (Chapter 7).

Further experimental tests were conducted by the author on two beam-column intersections with deformed bar reinforcement to confirm the nature of the load distribution along the bar within the column width. Only one single model of this particular reinforced concrete problem had been previously tested by Allwood (1980).

The details of all the experimental tests and the strain gauging techniques employed are reported within this chapter but the test results for the beam-column intersections are reported in Chapter 8 where the results are used as a basis for comparison with the finite element analyses.

The entire experimental programme however must be viewed as only a minor part of this project.

## 6.2 CHOICE OF TESTS.

The most obvious bond test to choose is the simple and easily reproduced simple pullout test. This test however has been questioned by a number of researchers (Ferguson 1965, Leonhardt 1957, Kemp and Wilhelm 1979) since the state of stress in the concrete specimen at the loaded face is in compression and the specimen is not in simple shear. An alternative test was sought by the author to provide a closer approximation to shear bonding, and such a test is the transfer test of Snowdon (1970) as illustrated in Figure 6.1. This may be modified to produce a further test specimen which the author describes as a 'double ended pullout test' (Figure 6.2). This test is effectively two pullout specimens in one test but without the compressive restraint on the concrete prisms which is obtained in the ordinary pullout test.

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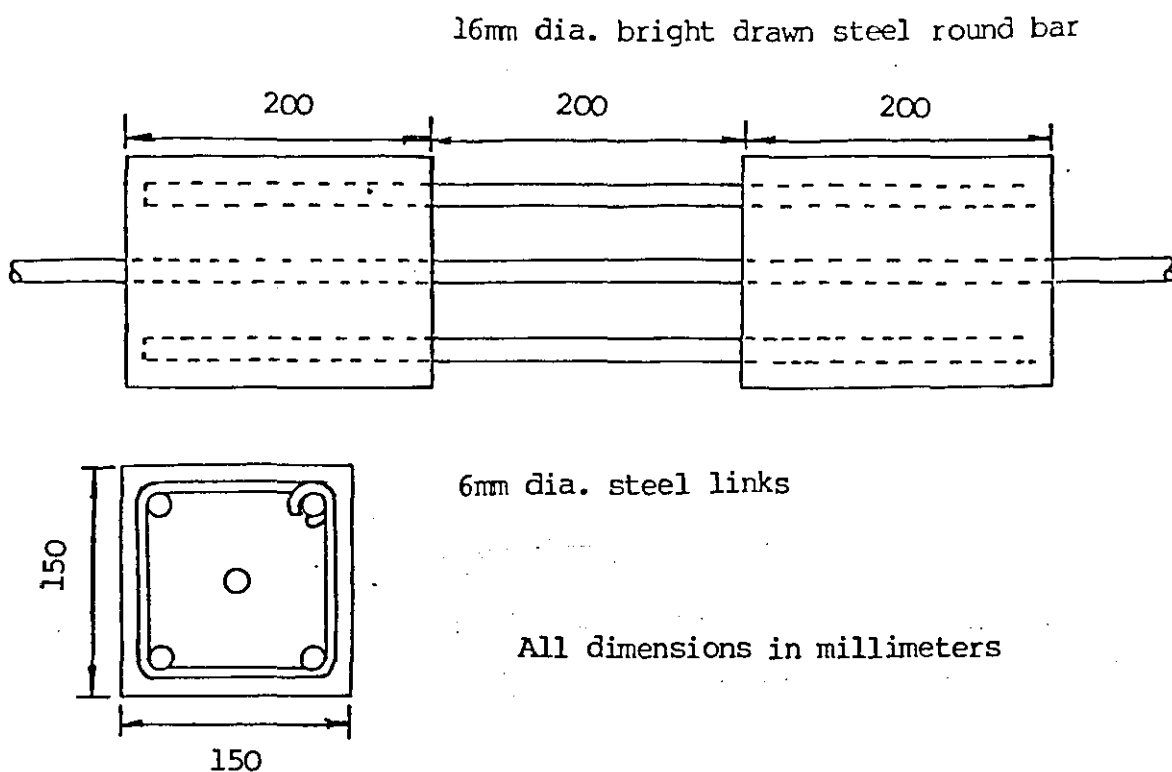


FIGURE 6.1 TRANSFER TEST

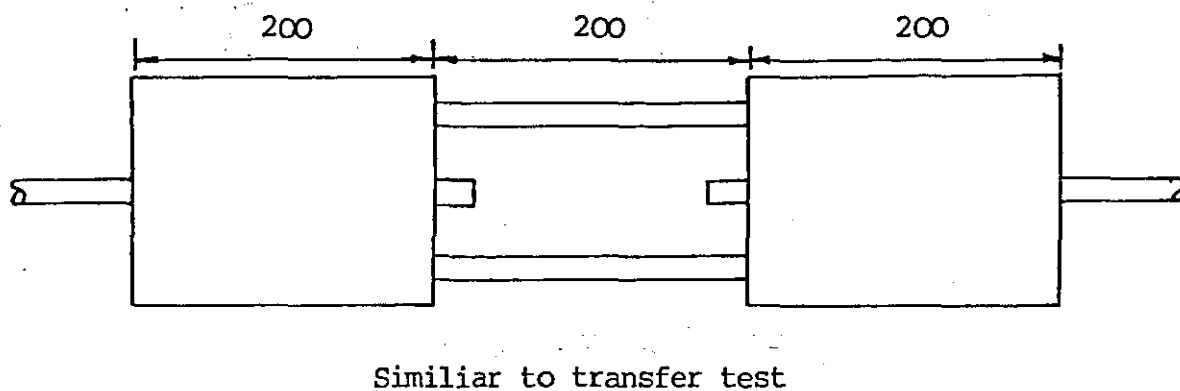


FIGURE 6.2 DOUBLE ENDED PULLOUT TEST

### 6.2.1 Simple pullout test.

The original standard pullout test BS 114:1957 was modified such that the test bar was pulled from an unreinforced cube of concrete. A  $150 \times 150 \times 150 \text{ mm}^3$  cube size was chosen and used throughout all the tests.

### 6.2.2 Transfer test.

This test was originally devised by Snowdon (1970) and consisted of two short prisms of helically reinforced concrete cast at a distance round a continuous test bar. The two prisms were reinforced longitudinally with common bars. In the original test by Snowdon, load was applied to the two free ends of the test bar and only load was measured at its centre. The reduction in load is obviously related directly to the load transferred by bond. The only modification to the original test was the use of shear links instead of helical reinforcement and the strain gauging of the test bar. Details of the dimensions of the transfer test used are illustrated in Figure 6.1 .

### 6.2.3 Double ended pullout test.

This test was a modification of the previously described transfer test. If the test bar in the transfer test is severed in the centre then effectively a 'double ended pullout' test is produced. There is a direct transference of the whole load from the centre bar to the outer bars through the concrete prisms. Instead of the concrete prisms bearing against a loading table, the outer bars of each concrete prism effectively restrain the block but without producing the undesirable compressive forces near the test bar. Details of the double ended pullout specimen used are shown in Figure 6.2 .

#### 6.2.4 Beam-Column intersection.

Within the Department of Civil Engineering, Loughborough University of Technology one beam-column intersection had already been tested by Allwood (1980). This test provides high shear stresses within the beam arms and therefore high bond stresses in a practical reinforced concrete structure. The beam-column was approximately a 1/3 scale model of part of a multi-storey car park design with continuous spans between columns (Smalley, 1976). In the model the point loads were applied to the beam arms at positions corresponding to the points of contraflexure of a beam with a span approximately ten times the column width. Details of the dimensions of the beam-column model are shown in Figure 6.3 .

A summary of all the experiments conducted and the number of specimens in each test is given in Table 6.1.

#### 6.3 STRAIN GAUGING THE TEST BARS.

The methods reviewed (Chapter 2), mainly splitting the test bar and strain gauging internally cannot be used when the length of the test bar becomes greater than about one metre, as in the transfer test, double ended pullout test and beam-column. Bars in excess of this length are too long for machining. It is also not easily possible to join short lengths of bars without a large connecting joint since the lead wires from the gauges protrude out of the ends of the bar. The author therefore sought alternative methods of strain gauging the bars and a number of different methods were developed for the positioning and protecting the gauges adhered to the bar.

The first method adopted was the 'recess and cap' method and a development from this was the quicker and easier variation known as the 'recess and resin fill' method. The various methods used: 'recess and cap'

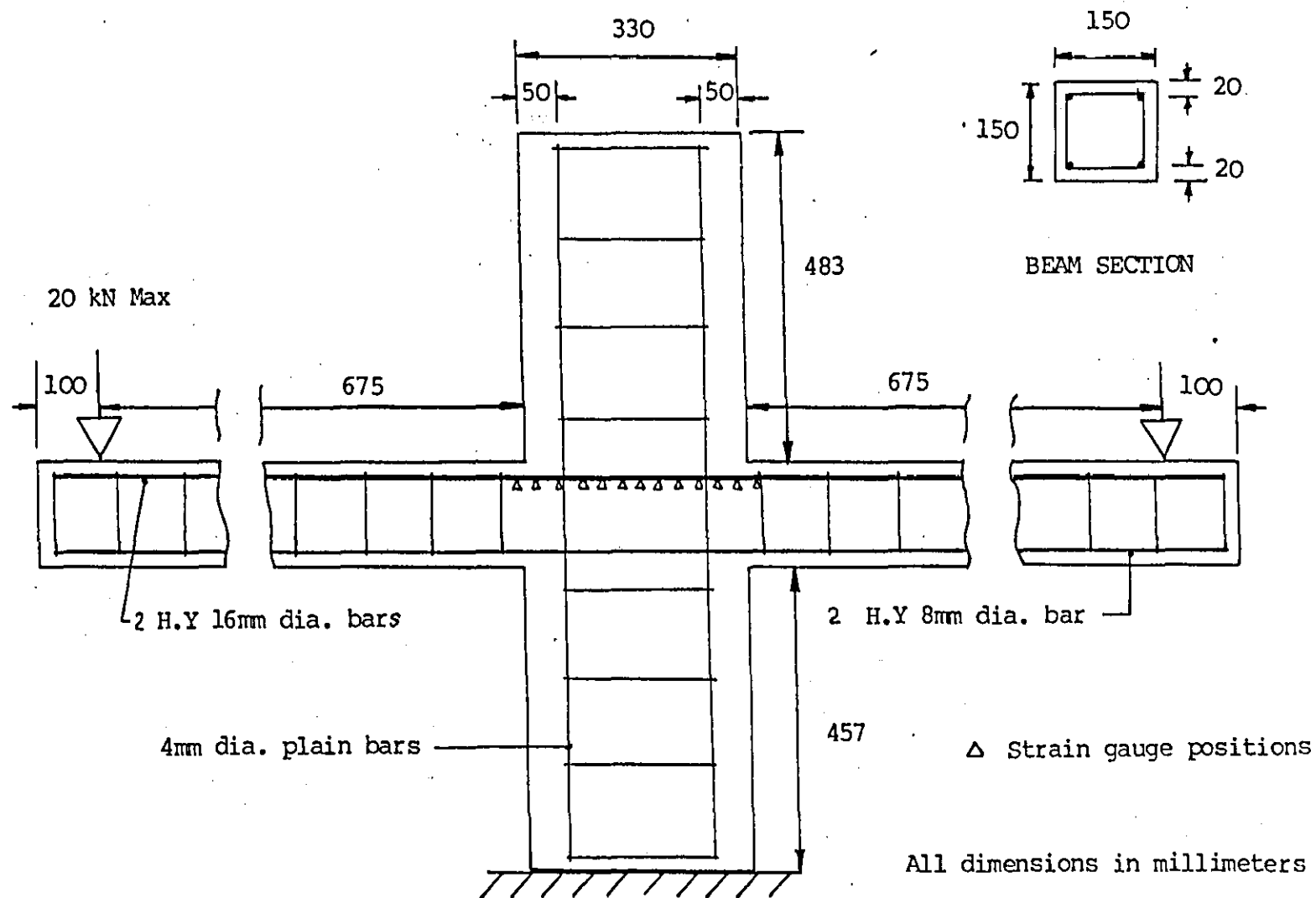


FIGURE 6.3 BEAM-COLUMN MODEL

TABLE 6.1      SUMMARY OF EXPERIMENTS

| TYPE OF TEST                                                                          | BAR TYPE                    | METHOD OF GAUGING        | BAR SIZE        | NUMBER OF<br>SPECIMENS TESTED |
|---------------------------------------------------------------------------------------|-----------------------------|--------------------------|-----------------|-------------------------------|
| BEAM-COLUMN                                                                           | GKN60<br>DEFORMED           | RECESS AND CAP           | 16mm<br>nominal | 2                             |
| PULLOUT                                                                               | BRIGHT DRAWN<br>PLAIN ROUND | RECESS AND<br>RESIN FILL | 16mm            | 3                             |
| TRANSFER                                                                              | BRIGHT DRAWN<br>PLAIN ROUND | RECESS AND<br>RESIN FILL | 16mm            | 3                             |
| DOUBLE ENDED<br>PULLOUT                                                               | BRIGHT DRAWN<br>PLAIN ROUND | RECESS AND<br>RESIN FILL | 16mm            | 3                             |
| PULLOUT *                                                                             | BRIGHT DRAWN<br>PLAIN ROUND | SPLIT BAR                | 16mm            | 3<br>reported                 |
| <p>NOTE</p> <p>*      Tests started by the author and continued by Allwood (1984)</p> |                             |                          |                 |                               |

and 'recess and resin fill' are now described.

#### 6.3.1 Recess and cap method.

In this method the strain gauges were situated in shallow recesses cut in the bar as shown in Figure 6.4. The strain gauges used were Durofix bond gauges drawn W8/120/G/K/2 manufactured by Tinsley Telcon Ltd., London, and bonded to the steel with an epoxy type adhesive. The strain gauges were protected by means of a steel cap which was screwed down on to the bar and had been machined such that the original profile of the bar was restored. The strain gauges were further protected under the caps by covering them with plastic tape and a water proofing gasket compound. Manufacture of the steel caps was however very difficult and time consuming and a quicker method was sought.

#### 6.3.2 Recess and resin fill method.

This method was similiar to the 'recess and cap' method except that instead of the steel caps the strain gauges were protected by filling the recess with an elastic epoxy resin material, 'Elastic Plastic Padding' manufactured by Plastic Padding Ltd, Goteborg, Sweden. The original bar profile was redeveloped very accurately by cutting and filing the resin material before it had completely hardened. In addition wwaterproofing strain gauges TML-WFLA-3 manufactured by Tokyo Sokki Kenkyuio Co. Ltd., Japan were used to give additional protection. These gauges comprised of an ordinary foil strain gauge with one metre vinyl lead wires pre-soldered and a pre-assembled flexible epoxy coat of about one millimetre thickness. The gauge and junctions of the lead wires were fully encapsulated with the epoxy coat and were originally designed for measuring strains under high humidity or underwater conditions. The gauges were bonded to the steel using an epoxy type resin.

All dimensions in millimeters

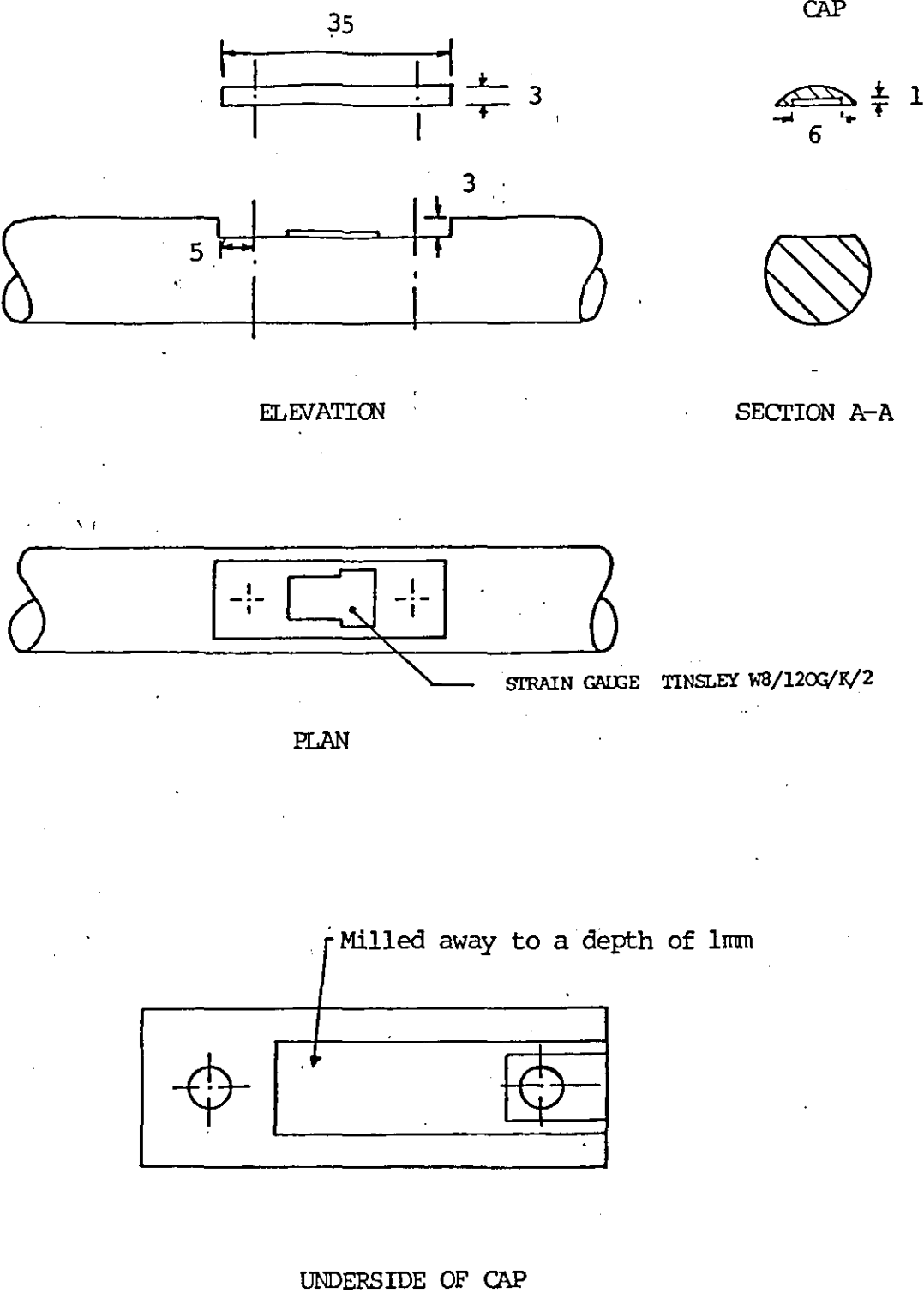


FIGURE 6.4 RECESS AND CAP METHOD OF STRAIN GAUGING THE STEEL BARS

### 6.3.3 Split bar method.

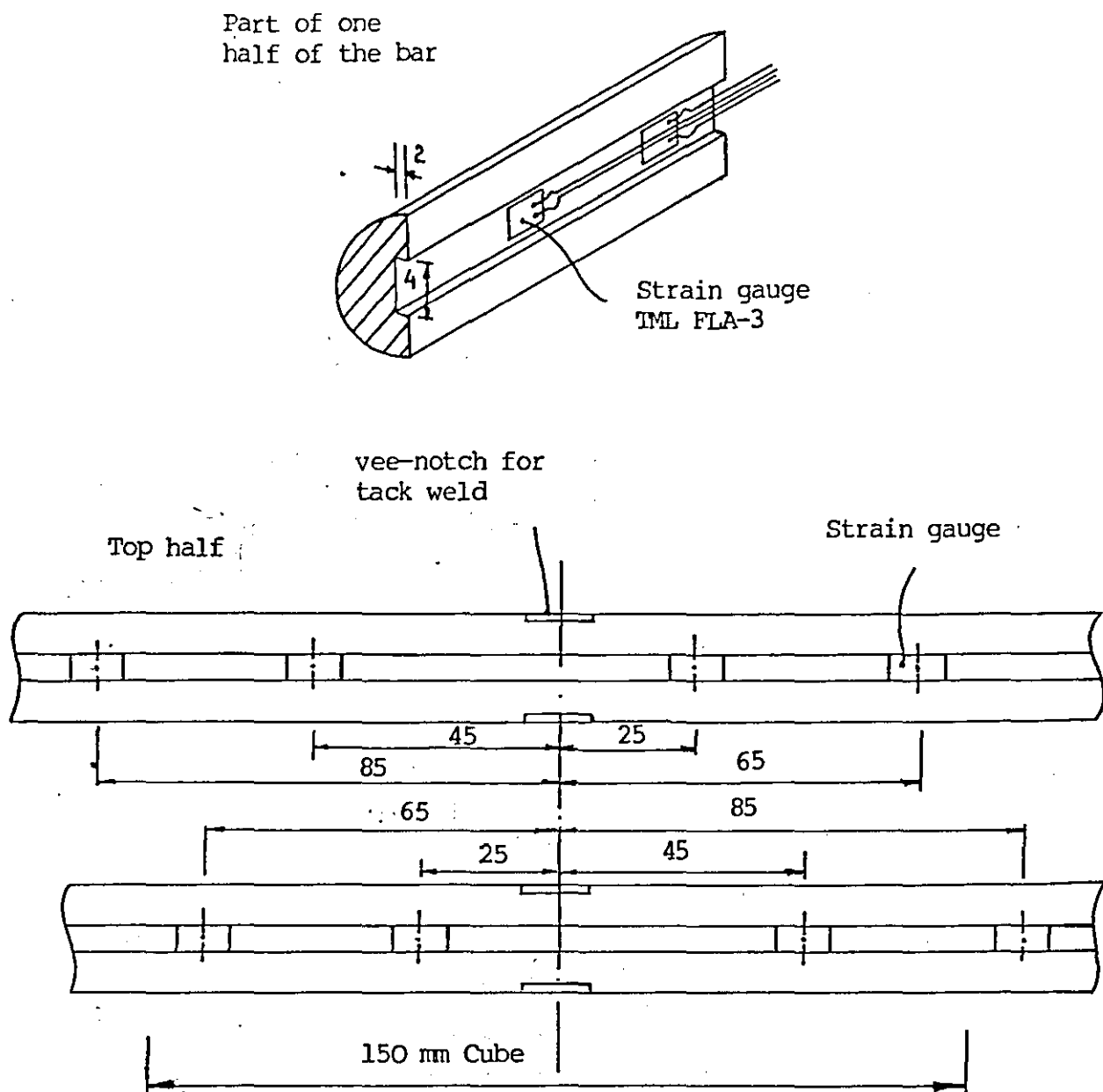
As reviewed (Chapter 2) there are a number of techniques of splitting the test bar and internally locating the strain gauges. The method adopted was similar to that used by Perry and Thompson (1966) and is shown in Figure 6.5. Foil strain gauges TML FLA-3 manufactured by Tokyo Sokki Kenyuio Co. Ltd, Japan with a gauge length of 3 mm and pre-attached vinyl coated lead wires were used. The size of the backing sheet to this gauge was 9 x 3.5 mm and fitted snugly into the 4 mm wide recess of the bar. Each individual gauge was waterproofed with a coat of epoxy resin but the internal space of the bar was mainly filled with the lead wires of the gauges, so additional waterproofing was not necessary. The two halves of the bar were tack welded back together at points at least 25 mm from the nearest strain gauge and the tack welds were filed down until the original profile of the bar was retained. Threads were cut into each end of the bar so that gripping sleeves could be attached which protected the ends of the bar where the individual lead wires protrude and were solder connected to a multi-flex cable.

For the magnitude of loads used in the pullout tests, the stresses in the bars were quite low (less than  $100 \text{ N/mm}^2$ ) and therefore after bond failure in the pullout test the bars were carefully removed, re-polished, re-calibrated and were used again.

### 6.3.4 Gauging method adopted for each test.

#### (i) Pullout

In the first tests conducted by the author the 'recess and resin fill' method was used. In the additional test started by the author and continued by Allwood (1984) the 'split bar method' was used.



All dimensions in millimeters

FIGURE 6.5 SPLIT BAR METHOD OF STRAIN GAUGING THE STEEL BARS

(ii) Transfer test and double ended pullout

In all of these test the 'recess and resin fill' method was used.

(iii) Beam-column intersection

In the two tests the 'recess and cap' method was used.

#### 6.4 MATERIALS USED.

##### 6.4.1 Concrete mix.

The concrete mix used throughout the tests was designed using the Department of the Environment method as given in 'Design of Normal Concrete mixes' (1975). A concrete target mean strength at 28 days of  $30 \text{ N/mm}^2$  was assumed. Some of the test specimens were tested at 14 days and this concrete mix is known (Parsons, 1980) to have a cube strength of about  $22 \text{ N/mm}^2$  at this date. A specified slump of 30-60 mm and the use of ordinary Portland cement were assumed.

The coarse aggregates used were 'uncrushed' river gravel type grades 20-10 mm and 10-5 mm and the fine aggregate was Zone 2 graded river sand. The gross apparent specific gravity of the aggregates was assumed to be 2.6 . The mix design was obtained using these parameters and is summarised in Table 6.2 .

##### 6.4.2 Steel bars.

The material properties for the steel bars used in the tests were obtained in accordance with BS 18:1962. The plain round bars were bright drawn steel which does not have a well defined yield so the 0.2 percent proof stress was determined. The deformed bars were GKN 60 hot rolled high yield bars with naturally hard ribs. The deformed bars have a well defined yield and so the value was determined in accordance

TABLE 6.2 CONCRETE MIX DESIGN

|                                            |                                    |                        |
|--------------------------------------------|------------------------------------|------------------------|
| TARGET MEAN STRENGTH                       |                                    | 30 N/mm <sup>2</sup>   |
| O.P. CEMENT                                | COARSE AGGREGATE - 20mm Uncrushed  |                        |
|                                            | 10mm River Gravel                  |                        |
|                                            | FINE AGGREGATE - ZONE 2 River sand |                        |
| FREEWATER/CEMENT RATIO (NO MAX. SPECIFIED) |                                    | 0.6                    |
| SPECIFIED SLUMP                            |                                    | 30 -60 mm              |
| FREE WATER CONTENT                         |                                    | 180 Kg/m <sup>3</sup>  |
| CEMENT CONTENT (NO MAX OR MIN. SPECIFIED)  |                                    | 300 Kg/m <sup>3</sup>  |
| RELATIVE DENSITY OF AGGREGATES             |                                    | 2.6                    |
| CONCRETE DENSITY                           |                                    | 2375 Kg/m <sup>3</sup> |
| TOTAL AGGREGATE CONTENT                    |                                    | 1895 Kg/m <sup>3</sup> |
| FINE AGGREGATE GRADING                     |                                    | ZONE 2                 |
| PROPORTION OF FINE AGGREGATE               |                                    | 38 %                   |
| FINE AGGREGATE CONTENT                     |                                    | 720 Kg/m <sup>3</sup>  |
| COARSE AGGREGATE CONTENT                   |                                    | 1175 Kg/m <sup>3</sup> |
| QUANTITIES PER CUBIC<br>METRE (NEAREST Kg) | O.P CEMENT                         | 300 Kg                 |
|                                            | WATER                              | 180 Kg                 |
|                                            | SAND                               | 720 Kg                 |
|                                            | 20-10 mm                           | 783 Kg                 |
|                                            | 10-5 mm                            | 392 Kg                 |

with BS 4449:1976. The values of Young's modulus and yield stress as determined were for the bright drawn steel  $200 \text{ kN/mm}^2$  and  $550 \text{ N/mm}^2$  respectively and for the GKN 60 deformed bar  $200 \text{ kN/mm}^2$  and  $460 \text{ N/mm}^2$ .

#### 6.5 FABRICATION OF THE SPECIMENS.

The pullout, transfer test and double ended pullout specimens were cast in timber moulds. The beam-column intersections were cast in forms built using steel shuttering panels and timber end pieces to obtain the required dimensions. The transfer test and double ended pullout specimens were fabricated in batches of three in one partitioned mould with the bars supported horizontally on the axis of the cube. In all cases the bars were carefully degreased and cleaned with acetone and the reinforcement placed in the mould and securely wired down.

The steel cage (double ended pullout, transfer and beam-column) provided to prevent premature shear failure of the specimens, was placed in the mould and the concrete poured in.

The control specimens consisted of six  $100 \times 100 \times 100 \text{ mm}$  cubes and  $150 \text{ mm}$  diameter cylinder. Vibration of the specimens was carried out on a small vibrating table in the laboratory. After curing under laboratory conditions for 24 hours all specimens including the control specimens were demoulded and painted with Ritecure, a curing compound, covered with plastic sheeting and left in the laboratory until testing.

#### 6.6 METHOD OF TESTING.

##### 6.6.1 Pullout tests.

The pullout tests were carried out using an Amsler 400 kN hydraulic testing machine. The cubes were placed on a loading table on the moving cross-head of the machine and adjusted so that an axial load was transmitted through the bar to the cube. M.G.A. pads (Hughes et al.,

1965) were placed between the loaded face of the cube and the loading surface to reduce the end restraint of the specimen. In the later tests the pullout specimens were bedded down on Kaffir plaster to eliminate the effect of the uneven surface and to aid the alignment of the bar axis with the direction of the pull of the grips.

#### 6.6.2 Transfer and double ended pullout.

For both types of specimen the rig illustrated in Figure 6.6 was used. The specimens were supported from the laboratory floor by means of ball bearing roller joints. The pullout load was applied using a 100 kN hydraulic jack. The load was measured using a suitable load cell with the reinforcement passing through it and positioned between the end of the jack and the CCL wedge anchor .

#### 6.6.3 Beam-column intersection.

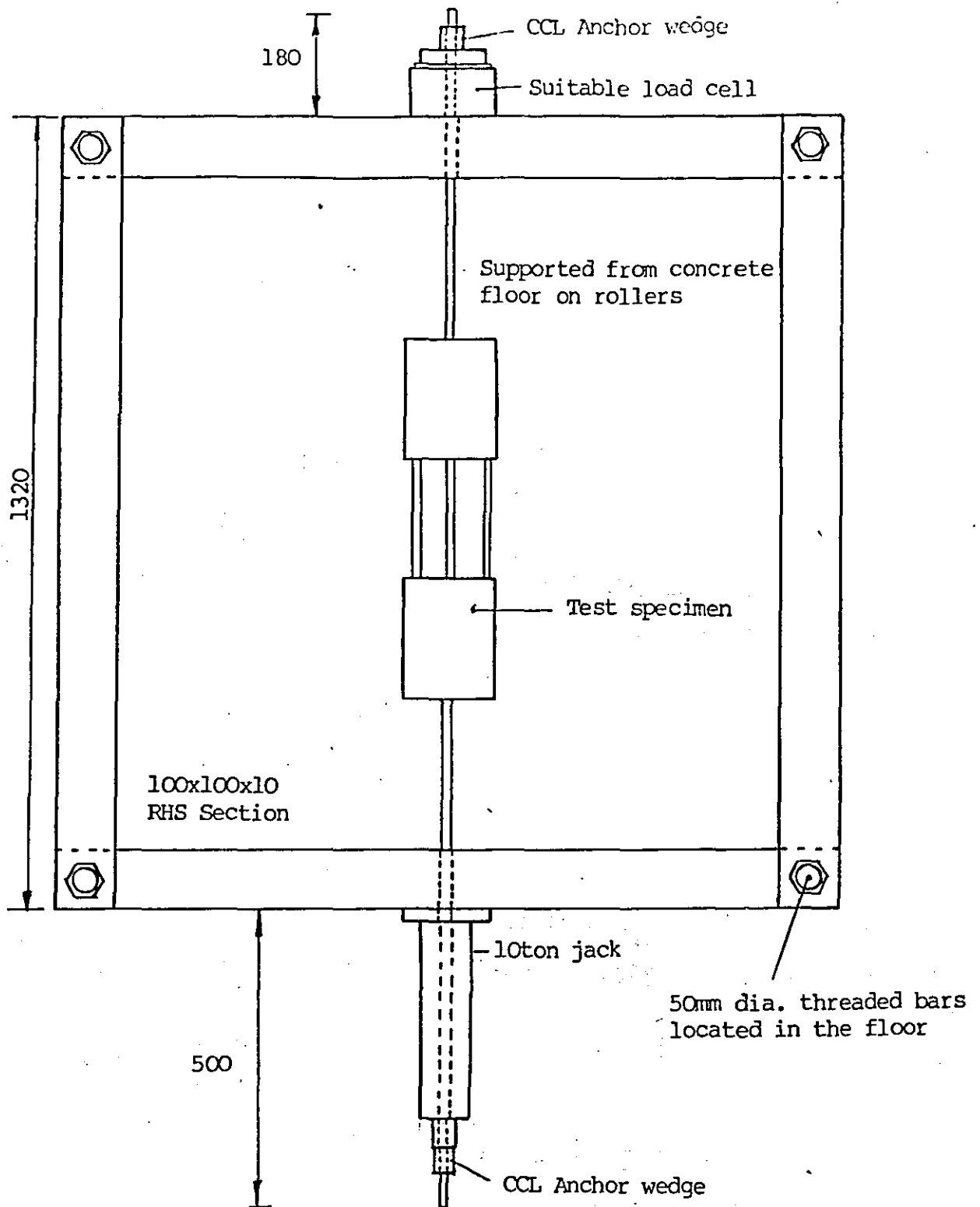
The specimens were tested with the plane of the column and beam arms lying horizontal and supported from the concrete floor on roller bearings. Loading was achieved by using two pulling jacks connected to the same hydraulic pump and load cells were positioned between the loading arms and the beam arms as illustrated in Figure 6.3.

#### 6.6.4 Testing period.

The beam-column models were tested at 28 days. All other specimens were tested at 14 days to increase the turn around of production of the specimens.

#### 6.6.5 Control specimens.

The cubes and cylinders provided the crushing and splitting strengths of the concrete at the time of testing. The control specimens were tested in a 120 Ton Denison crushing machine.



All dimensions in millimeters

FIGURE 6.6 STRAINING RIG USED TO TEST THE TRANSFER  
AND DOUBLE ENDED PULLOUT SPECIMENS

(i) Cube strengths

For the bond tests the average cube crushing strength of twelve cubes at 14 days was  $20.4 \text{ N/mm}^2$  with a standard deviation of  $1.5 \text{ N/mm}^2$  and the average cylinder strength was  $1.94 \text{ N/mm}^2$ .

For the two beam-column models the average cube crushing strength of twelve cubes at 28 days was  $35.2 \text{ N/mm}^2$  with a standard deviation of  $1.8 \text{ N/mm}^2$  and the average cylinder strength of two specimens was  $2.9 \text{ N/mm}^2$ .

(ii) Estimation Young's modulus and failure strains

The secant modulus of elasticity ( $E_{\text{sec}}$ ) can be estimated from the data provided by Neville (1973). For the concrete mix used here (1 : 2.4 : 3.9) the estimated value of  $E_{\text{sec}}$  at 14 days is  $15.5 \text{ kN/mm}^2$  and at 28 days the value is  $16.5 \text{ kN/mm}^2$ .

From the cube strengths and the cylinder strengths the estimated strains at failure are given by :

$$\epsilon_{\text{cu}} = \frac{f_{\text{cu}}}{E_{\text{sec}}} \quad \text{and} \quad \epsilon_{\text{tu}} = \frac{f_t}{E_o}$$

where  $E_o = 2 \times E_{\text{sec}}$  (Saenz, 1964)

The estimated failure strains for the 28 day concrete are  $\epsilon_{\text{cu}} = 2.13$  percent and  $\epsilon_{\text{tu}} = 0.088$  percent and for the 14 day concrete  $\epsilon_{\text{cu}} = 1.32$  percent and  $\epsilon_{\text{tu}} = 0.063$  percent. The estimated values of the initial Young's modulus are for the 14 day concrete  $31.0 \text{ kN/mm}^2$  and for the 28 day concrete  $33.0 \text{ kN/mm}^2$ .

6.6.6 Calibration of the strain gauges.

The strain gauged bars for use in the pullout specimens were

calibrated in a 400 kN Amsler hydraulic testing machine and the other longer bars were calibrated using the rig shown in Figure 6.6. For each gauge the relationship between load and voltage change per bridge volt in a wheatstone bridge network was assumed to be linear.

#### 6.6.7 Strain gauges and data logging equipment.

The lead wires from the strain gauges were connected to the wheatstone bridge network via screw-down terminal connectors. The bridge voltage was set at 1.00 volt at all times and the current was switched on at least one hour before testing to allow the gauges to stabilize. Data logging was provided by a Commodore Pet 32K Computer connected to a Solatron 7060 digital voltmeter which in turn was connected to a Solartron Minate Analog Scanner (Figure 6.7). The wheatstone bridge circuits incorporating the strain gauges were connected directly to the input terminals of the scanner.

The scanner and digital voltmeter were controlled by the micro-computer and scanned voltage readings from each gauge at the rate of about 2 channels per second. The voltage changes were converted to load measurements using the previously obtained calibration factors and the loads were continuously output on a visual display unit and when required output on an on-line printer.

### 6.7 PRESENTATION AND DISCUSSION OF THE BOND TEST RESULTS.

The results of the bond tests using the plain bars are reported here i.e. pullout, double ended pullout and transfer tests. For the ease of comparison, the beam-column experimental results are reported in Chapter 8 together with the analytical results of the finite element analyses.

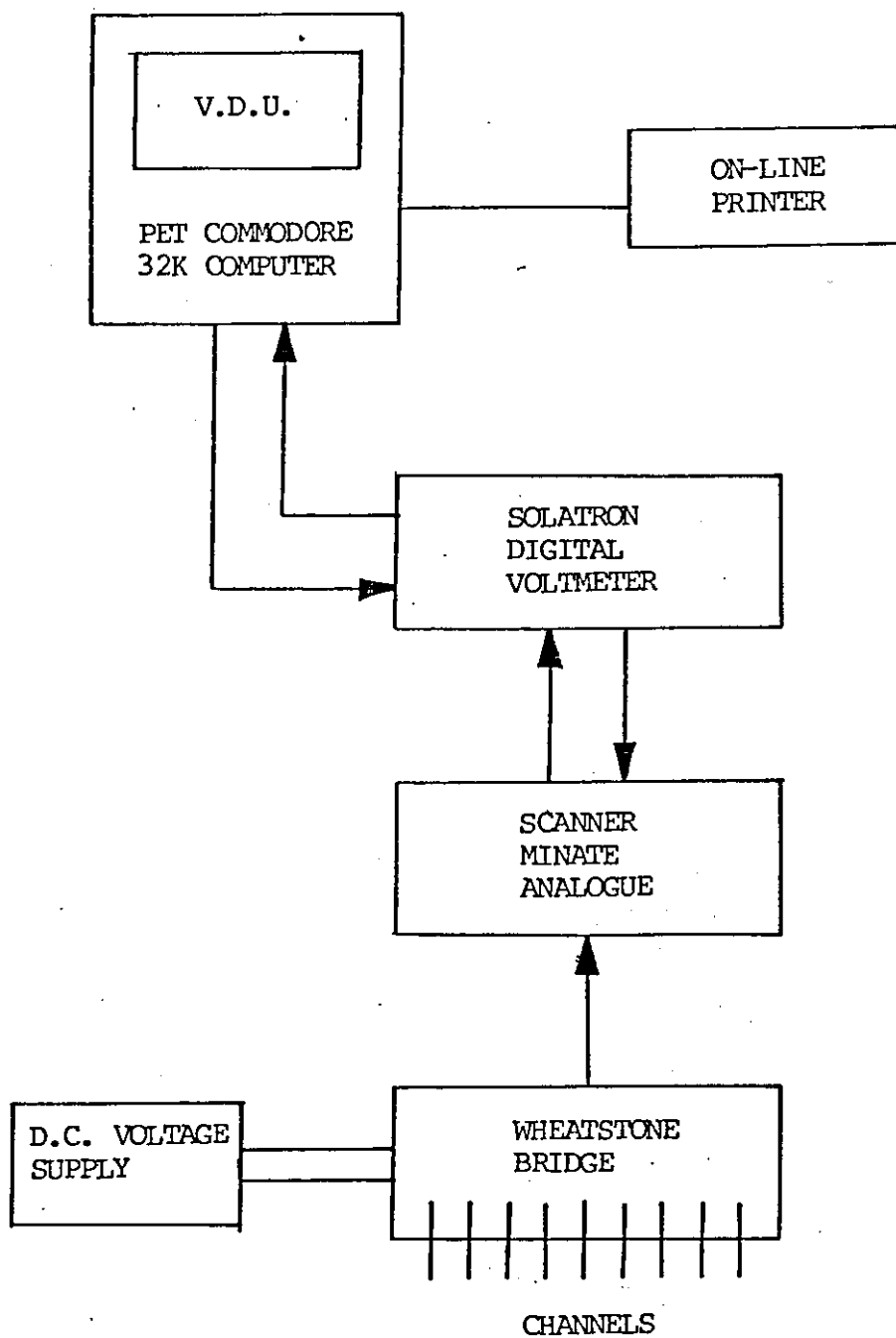


FIGURE 6.7 DATA LOGGING EQUIPMENT

#### 6.7.1 Pullout tests.

The distribution of bar stress with position is illustrated in Figure 6.8 for one of the tests using the 'recess and resin fill' method of strain gauging. The distribution of bar stress for two of the tests using the 'split bar method' of strain gauging are illustrated in Figures 6.9 and 6.10.

There is some variation in the size of the pullout load for each specimen as they were not all cast at the same time and differing amounts of shrinkage during curing will have had an effect on the size of the pullout load. For the results illustrated (Figure 6.8) the strain gauges on the test bar failed to function at pulling loads in excess of about 50 percent of the failure load due to the shearing of the lead wires which trailed through the concrete specimen.

The load distributions obtained for the plain round bars illustrate the changing nature of the bar stress distribution along the bar as the pulling load was increased. At low loads the bar stress rapidly decreases towards zero in an exponential manner away from the pulling end. With increasing pulling load the distribution tends to change towards a linear reduction along the bar. Close to the failure load, bond between the steel bar and surrounding concrete near the pulling end was decreasing and the shape of the bar stress distribution tends towards a S -shape. The changing nature of the bar stress distribution from an exponential type of distribution to a gradual S -shape is very similar to the results obtained by Mains (1951), Peattie and Pope (1956) and Parland (1957) in their tests with plain round bars in pullout tests.

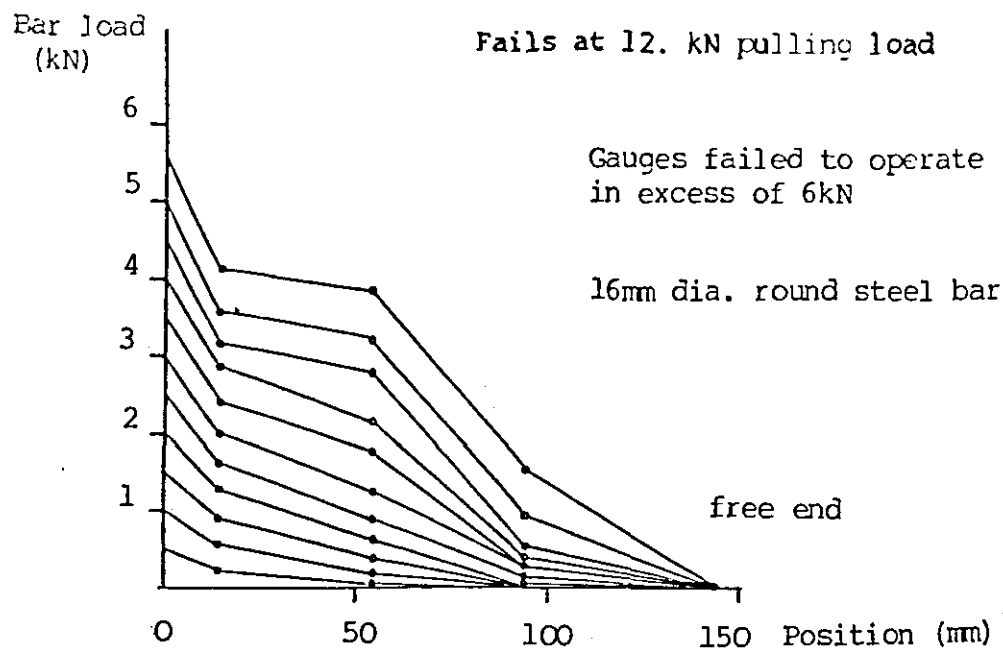


FIGURE 6.8 EXPERIMENTAL LOAD DISTRIBUTIONS IN THE PULLOUT TEST  
(RECESS AND RESIN FILL METHOD)

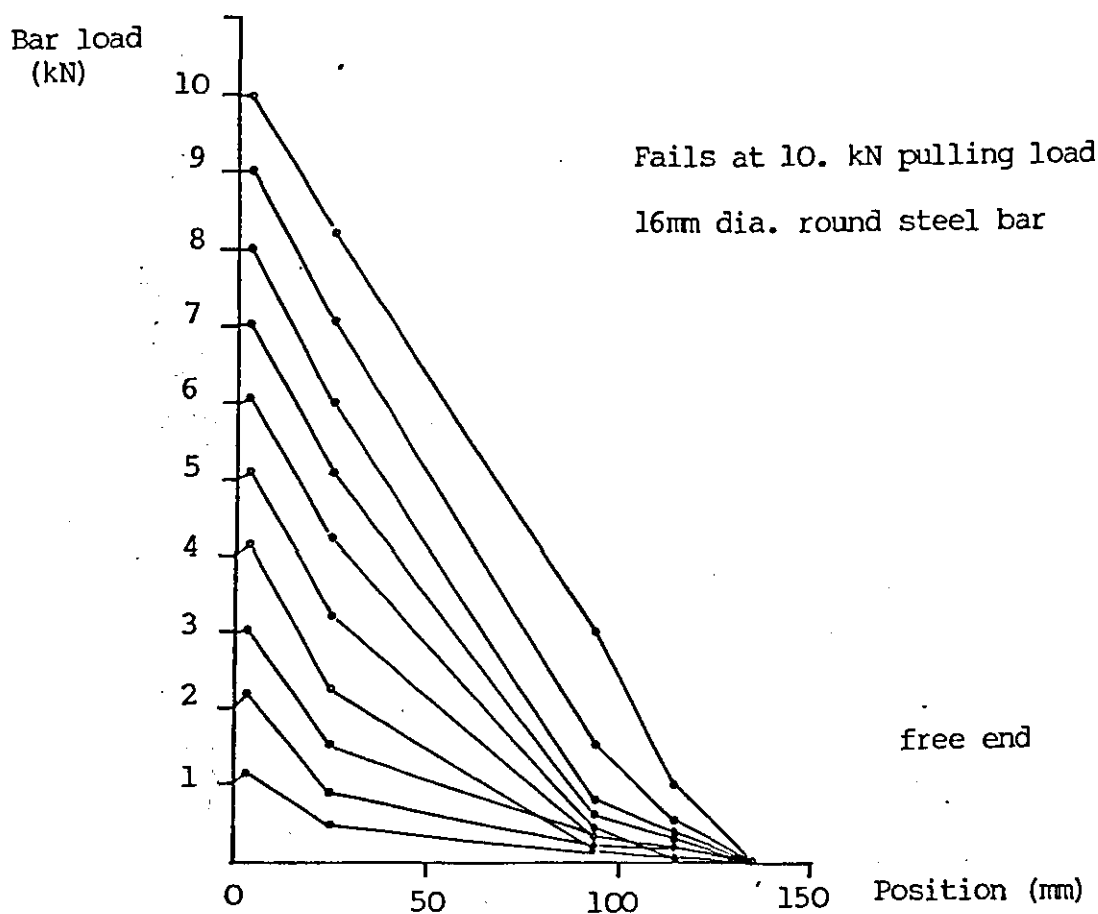


FIGURE 6.9 EXPERIMENTAL LOAD DISTRIBUTIONS IN THE PULLOUT TEST  
(SPLIT BAR METHOD)

Bar load  
(kN)

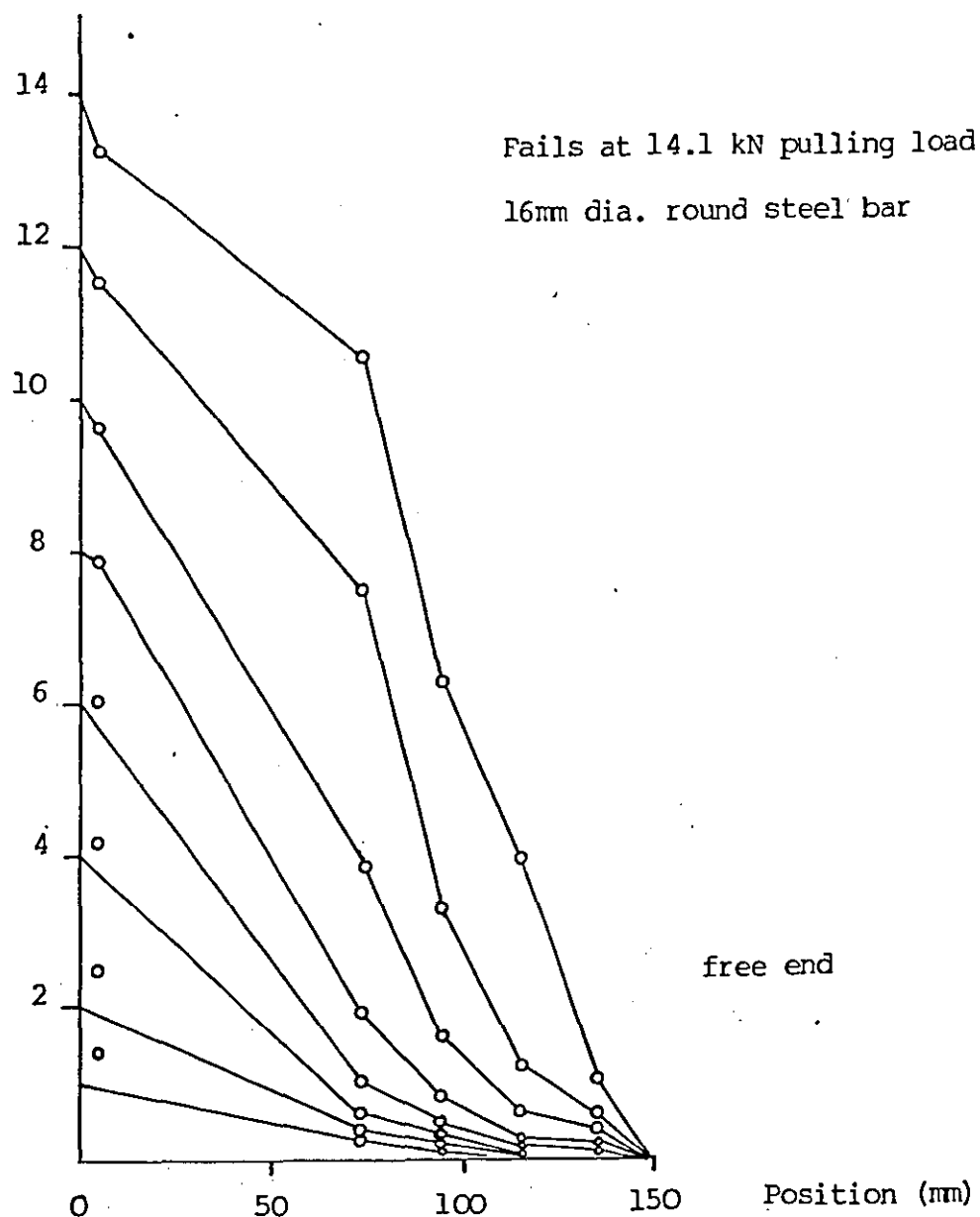


FIGURE 6.10 EXPERIMENTAL LOAD DISTRIBUTIONS IN THE PULLOUT TEST  
(SPLIT BAR METHOD)

#### 6.7.2 Double ended pullout tests.

The values of the bar stress calculated from the strains at each individual strain gauge position for all the tests performed are illustrated in Figures 6.11 and 6.12 for pulling loads of 2.5 kN, 5.0 kN, 7.5 kN and 10 kN. The individual results show a considerable amount of scatter at higher loads and interpretation is difficult. At lower loads the bar stress tends to decrease away from the pulling end in an 'exponential' type of manner, similar to the ordinary pullout results. At higher loads this type of distribution appears to be maintained and there is no indication of the distribution changing to an S-shape.

#### 6.7.3 Transfer tests.

The values of bar stress calculated from the strains measured in the test bar at each gauge position are shown for pulling loads of 10 kN, 20 kN and 60 kN in Figure 6.13. There is about the same amount of scatter in the individual values of bar stress at each gauge position in the transfer tests as compared with the double ended pullout test results. At all pulling loads the bar stress tends to decay in a gradual exponential manner with position away from the pulling end. Most of the bar force which is to be transferred from the test bar to the concrete occurs within the first 50 mm of the specimen. There is little transfer of load between the bar and the concrete within the rest of the specimen.

#### 6.7.4 Comparison between the various strain gauging techniques employed.

##### (i) Recess and cap method

This method was reasonably successful and the gauges worked reliably, however the steel caps were very difficult and time consuming to fabricate. Great care had to be taken in waterproofing the gauges once adhered to the bar. Lead wires had to be manually soldered to the gauges and considerable problems were encountered insulating and waterproofing

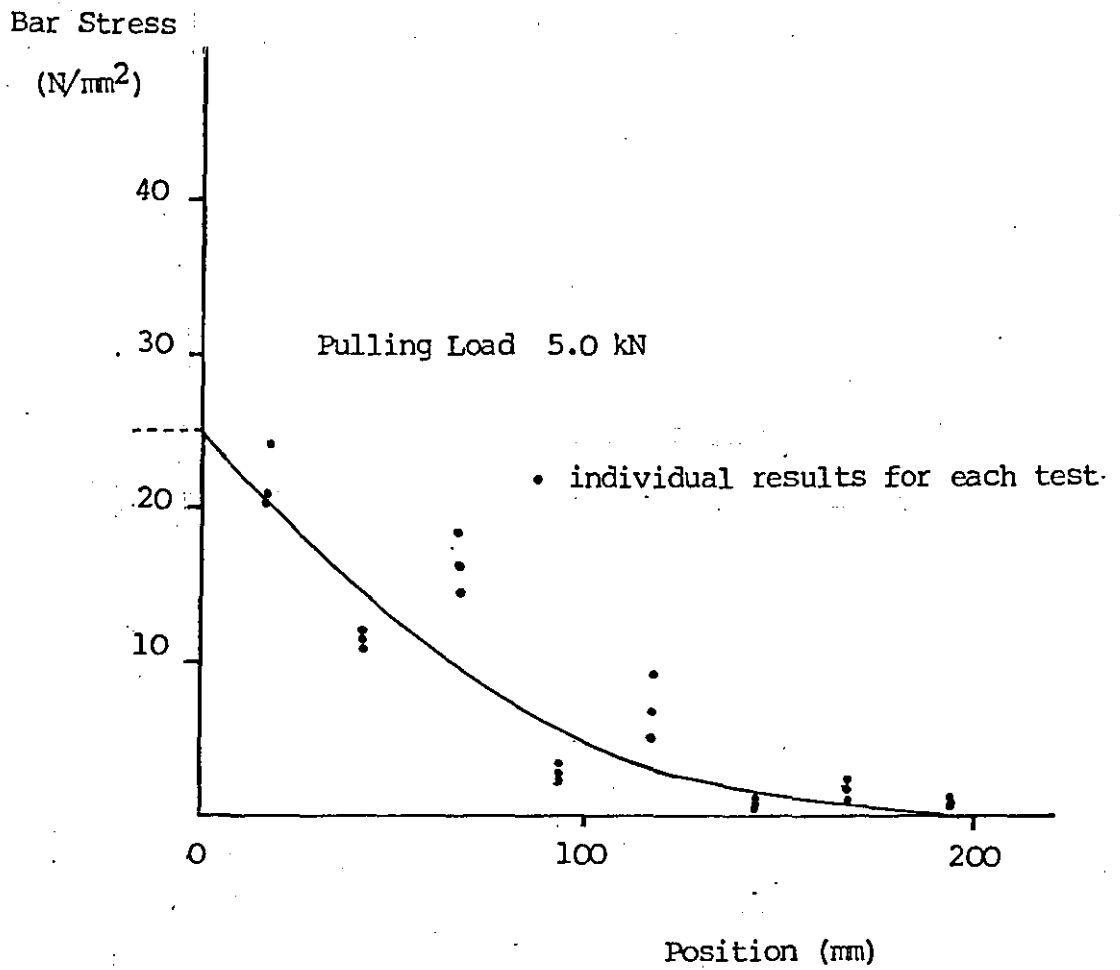
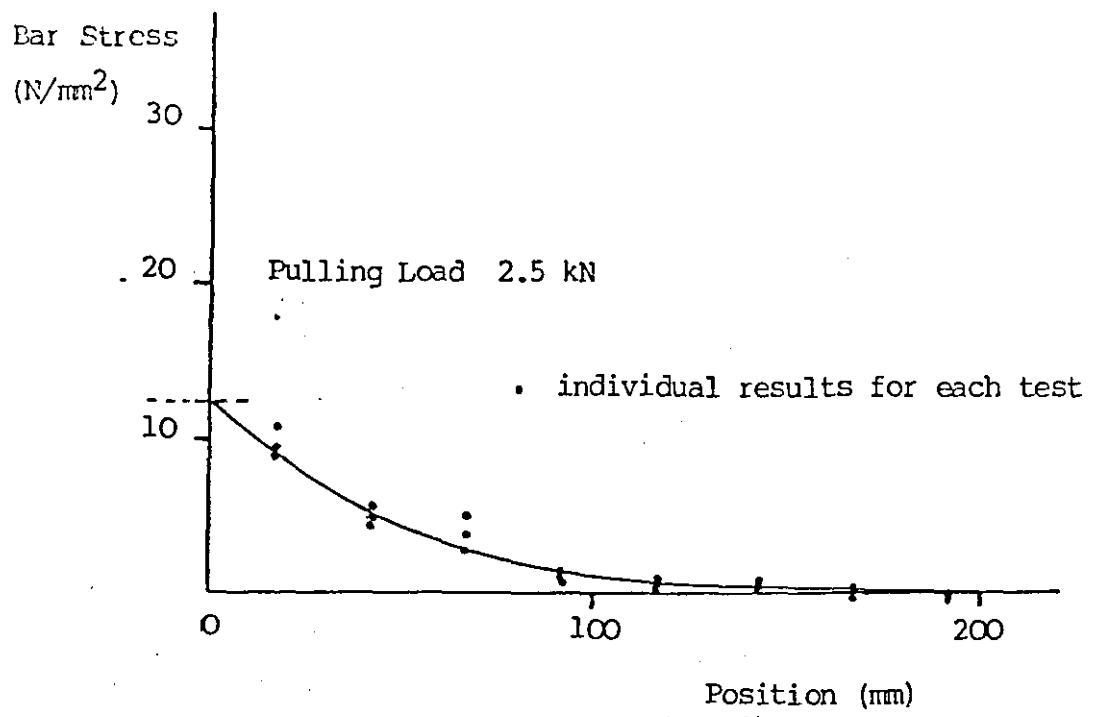


FIGURE 6.11 EXPERIMENTAL BAR STRESS DISTRIBUTIONS FOR THE DOUBLE-ENDED PULLOUT TEST

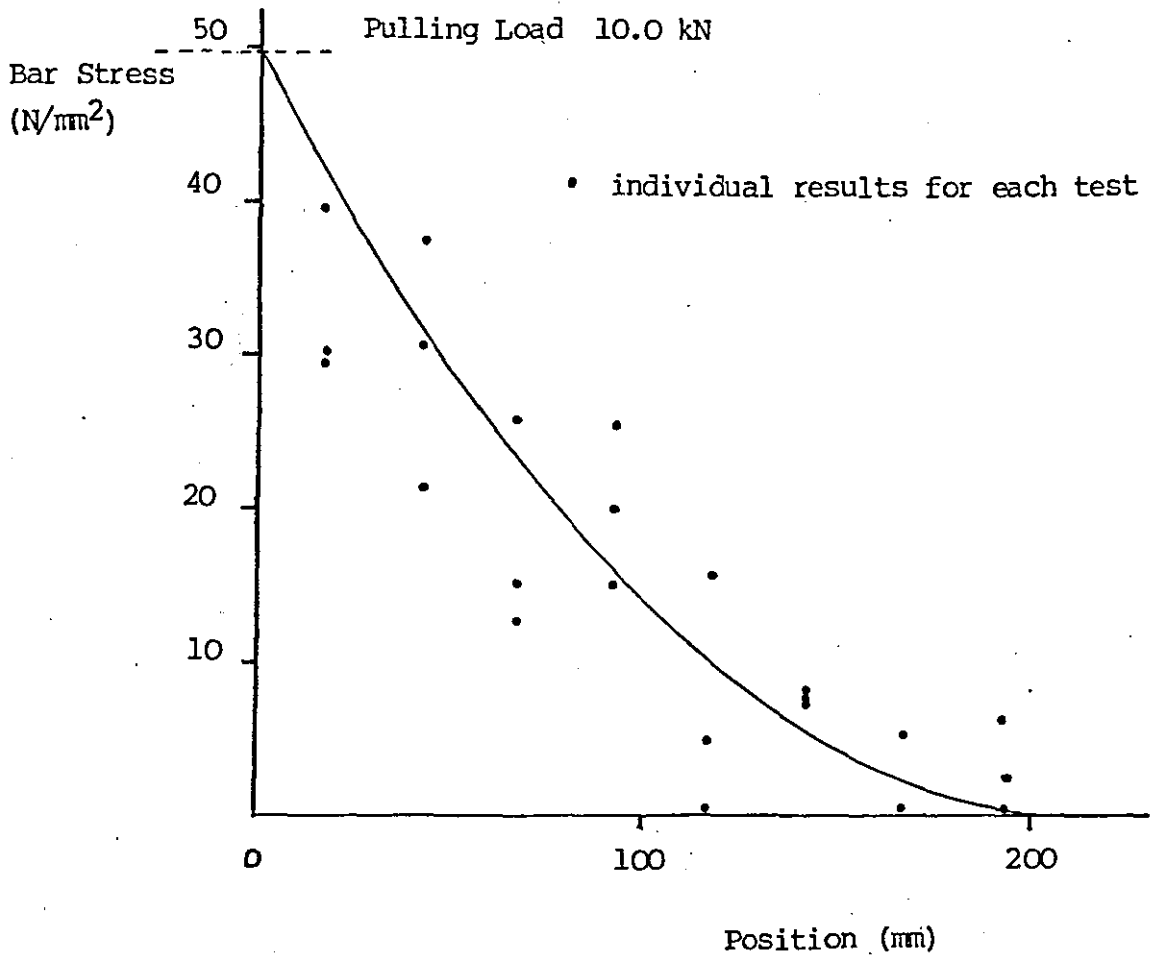
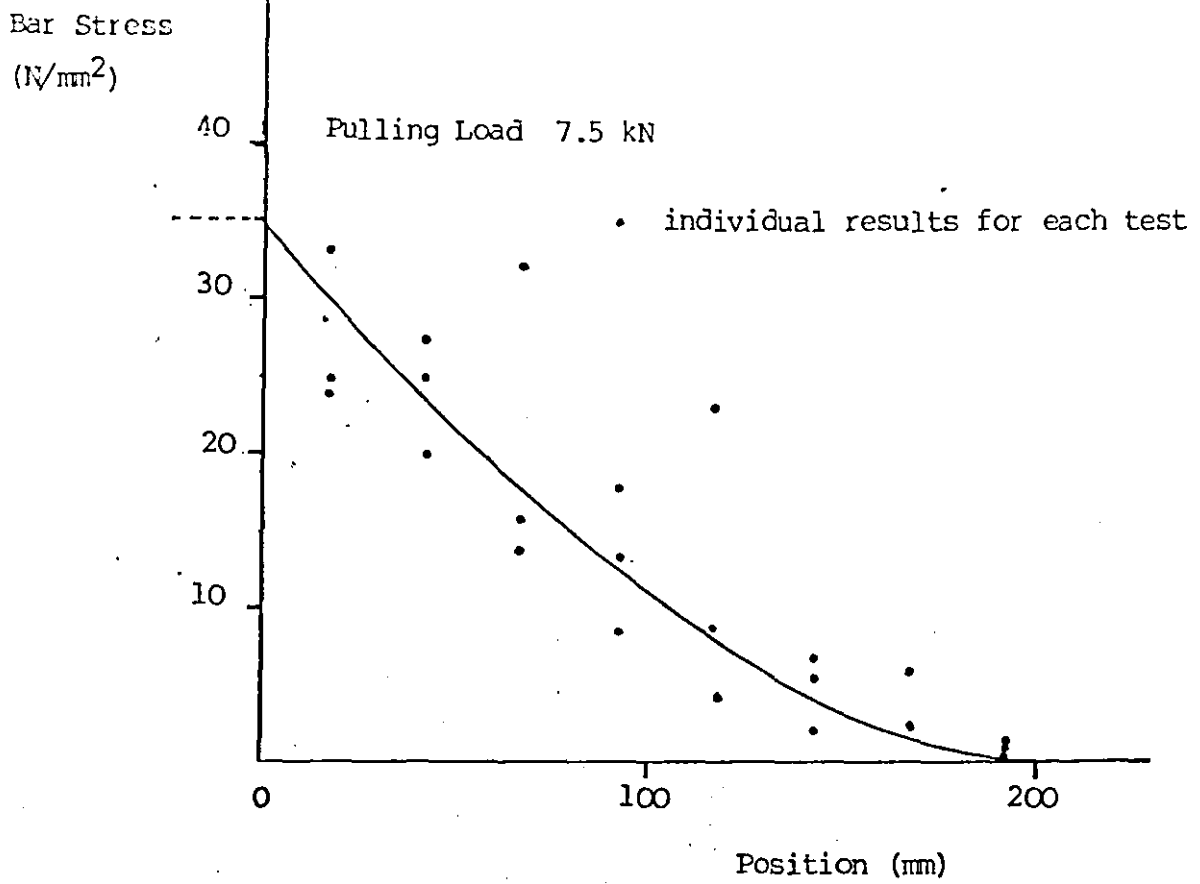


FIGURE 6.12 EXPERIMENTAL BAR STRESS DISTRIBUTIONS FOR THE  
DOUBLE-ENDED PULLOUT TEST

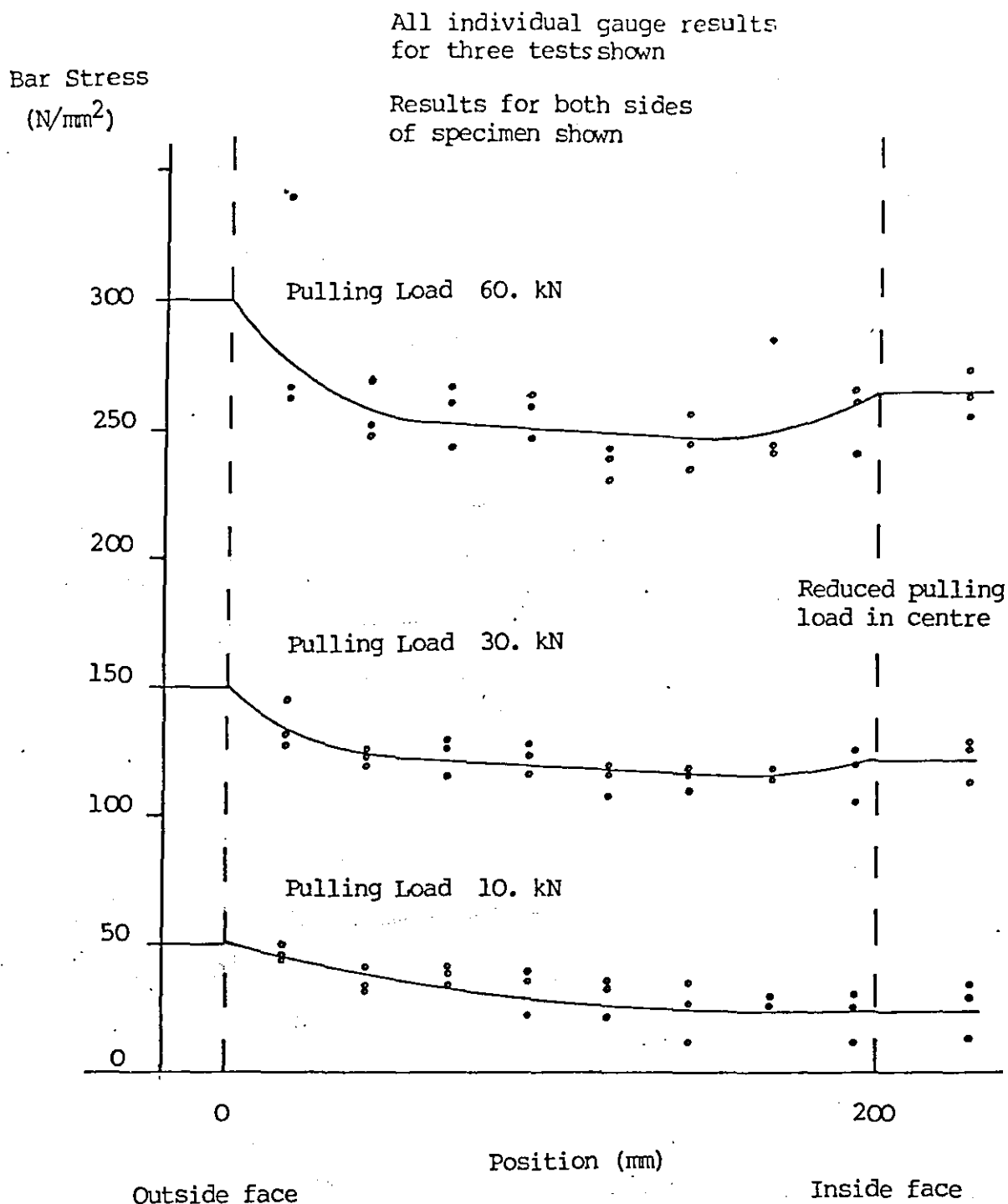


FIGURE 6.13 EXPERIMENTAL BAR STRESS DISTRIBUTIONS  
FOR THE TRANSFER TEST

the connections. This method of gauging was used in the beam-column models and after the models had been loaded to failure, the test bars were removed for inspection. It was found that water from the concrete mix had penetrated under the steel caps and had started attacking the gauges.

(ii) Recess and resin fill method

This method was quicker than (i) and relatively straight forward and simple to perform. The use of the waterproofing strain gauges with pre-attached lead wires greatly reduced the problems associated with water penetration. However there were a number of other problems, mainly due to the padding material being softer than the steel and were namely :

- (1) the resin fill material was disturbed if large amounts of slip occurred, being forced up by aggregate cutting into the surface,
- (2) local bearing pressure was found to affect the strain gauge performance,
- (3) the bonding properties of the resin with the concrete were probably different from that of steel with concrete.

In the ordinary pullout test and double ended pullout test some of the test results show an oscillating nature in the load distributions. With the strain gauges located in the recesses away from the central axis of the bar and the recesses alternating from side to side along the length of the bar, this suggests bending of the bar occurred during testing as the eccentricity of the gauges would tend to produce this effect. In the later series of pullout tests where Kaffir plaster was used to bed the concrete cubes down there is evidence to suggest that in the earlier pullout tests the test bar may have been bending during

loading due to the cube not being aligned perfectly.

For both the recess methods the lead wires trail through the concrete specimens and sheared when large movements of the bar relative to the concrete occurred.

(iii) Split bar method

This method was very successful and the gauges worked reliably. The bars could be re-used when carefully removed from the specimens and re-calibrated with only a small degree of error from the original calibration. The only limitation with this method is the length of bar that can be prepared and machined accurately as it would be very difficult to make the bars longer than about one metre. The joining of the shorter lengths of bar would require a joint connector larger than the diameter of the bar and the lack of homogeneity in a length of bar would be undesirable.

6.8 SUMMARY.

The following bond tests were performed using strain gauged plain round bars, ordinary pullout, transfer and double ended pullout. In addition two beam-column models with strain gauged deformed bars were tested. Three different techniques were used to strain gauge and protect the gauges adhered to the bars. Splitting the bar and gauging internally was the best method, however it can only be used on short lengths of bar as in the ordinary pullout test. The other methods tried consisted essentially of locating the gauges in recesses cut into the outer surface of the bar and rebuilding the profile of the bar either with a steel cap or with an epoxy resin filler material.

The load distributions obtained in the pullout tests confirmed

the general nature of the distributions previously obtained by other researchers e.g. Mains (1951) and Parland (1957), which changes from 'exponential' through to a gradual S - shape near failure pullout load.

In the double ended pullout test the bar stress distributions remain of an 'exponential' type with increasing load and show no tendency to the S- shape. However at the same pulling loads near to the failure load the load distributions from the separate tests show a considerable amount of scatter and interpretation is difficult.

In the transfer test most of the load which is to be transferred from the centre bar to the outer bars via the concrete does so within a short distance into the specimen.

The results from the ordinary pullout tests and the transfer tests with embedded plain bars are suitable as the basis for comparison with finite element analyses of these particular structures. The results from the double ended pullout at low loads are also suitable as a basis for comparison with the analytical results but the experimental results at pulling loads near to the failure load will not be as useful due to the amount scatter.

## CHAPTER 7    NUMERICAL EXAMPLES OF THE BOND MODEL APPLIED TO PLAIN BARS IN BOND TESTS

### 7.1    INTRODUCTION.

This chapter is concerned with the application of the bond model developed in Chapter 3 to reinforced concrete structures with embedded plain bars. The developed bond model is based on assumed frictional mechanisms of bond and as surmised in Chapter 3 should be applicable mainly to plain round bars.

The aims of performing the analyses are primarily to demonstrate the applicability of the proposed bond model to plain bar problems and the usefulness of the bond model. Comparison with available experimental evidence part of which is provided from the author's experimental programme (Chapter 6) will be carried out so that the applicability of the bond model can be evaluated. The objectives in applying the bond model to several selected bond test problems with plain bars are namely:

- (1) Evaluate applicability of the proposed bond model to plain bar problems and the usefulness of the model to investigate these tests.
- (2) To observe if by suitable adjustment of the parameters used in the bond model within reasonable bounds, whether the various types of observed behaviour in the bond tests can be predicted. In particular the varying degrees of S-shape bar stress distribution and the changing nature of the bar stress distributions in the ordinary pullout test.
- (3) To observe whether in the pullout test the model predicts increased pullout load with lateral pressure exerted on the concrete cube.



TABLE 7.1 BOND PARAMETERS FOR PLAIN BARS USED IN ANALYSES

| PARAMETER                                                                              | SYMBOL     | VALUE                                             |
|----------------------------------------------------------------------------------------|------------|---------------------------------------------------|
| INITIAL BOND STRESS-SLIP MODULUS                                                       | $R_0$      | $200 \text{ N/mm}^3$                              |
| SLOPE OF LOCAL ULTIMATE BOND STRESS-RADIAL PRESSURE LINE                               | $\mu$      | 0.4                                               |
| TOLERANCE SLIP                                                                         | $\Delta_u$ | 0.1 mm                                            |
| RATIO OF MAXIMUM BOND STRESS TO ULTIMATE BOND STRESS FOR SLIPS GREATER THAN $\Delta_u$ | $\beta$    | 0.5                                               |
| BOND MODULUS ORTHOGONAL TO THE BAR                                                     | $R_n$      | $10^5 \text{ N/mm}^3$                             |
| ULTIMATE LOCAL BOND STRESS DUE TO SHRINKAGE                                            | $q_0$      | $3 \text{ N/mm}^2$<br>(Robins and Standish, 1982) |
|                                                                                        |            | $2 \text{ N/mm}^2$<br>(Author's tests)            |

TABLE 7.2 CONCRETE PARAMETERS USED IN ANALYSES

| PARAMETER                                           | SYMBOL          | VALUE                   |
|-----------------------------------------------------|-----------------|-------------------------|
| INITIAL TANGENT MODULUS OF ELASTICITY               | $E_0$           | $32958 \text{ N/mm}^2$  |
| COMPRESSIVE STRENGTH                                | $f_{cu}$        | $32.062 \text{ N/mm}^2$ |
| TENSILE STRENGTH                                    | $f_t$           | $2.96 \text{ N/mm}^2$   |
| UNIAXIAL COMPRESSIVE FAILURE STRAIN                 | $\epsilon_{cu}$ | 2.16 millistrains       |
| UNIAXIAL TENSILE FAILURE STRAIN                     | $\epsilon_{tu}$ | 0.0909 millistrains     |
| POISSON'S RATIO                                     | $\nu$           | 0.2                     |
| Values taken from Kupfer, Hilsdorf and Rusch (1969) |                 |                         |

(ii)  $q_0$  parameter (ultimate local bond stress due to shrinkage)

An initial estimate for this parameter may be obtained from the value of the pullout force in the ordinary pullout test where there is no lateral loading, by converting the pullout force to an average bond stress.

One of two values has been used for the problems investigated in this chapter. From the pullout tests performed by Robins and Standish (1982) an estimate for the  $q_0$  parameter is  $3 \text{ N/mm}^2$  and from the author's own pullout tests  $q_0$  is estimated as  $2 \text{ N/mm}^2$ . The  $q_0$  parameter is an indication of the strength of bonding and will in practice be influenced by the bar type, the bar's surface characteristics and the amount of concrete shrinkage. The value of  $q_0$  estimated from the results of Robins and Standish indicates enhanced bonding at zero lateral pressure when compared to the author's tests.

(iii)  $\mu$  parameter (slope of local bond stress-radial pressure line)

This value has been estimated from the test results of Robins and Standish by converting their pullout load-lateral stress relationship to an average bond stress-radial pressure relationship. The  $\mu$  parameter has been estimated from the slope of the linear relationship fitted to the data points as 0.4 and this value is used herein.

(iv)  $\Delta_u$  parameter (tolerance slip)

The  $\Delta_u$  parameter is the slip at which the maximum local bond stress occurs and is very difficult to estimate. The best estimate is to take the value of free-end slip in the pullout test at which the bond stress first reaches or is close to its maximum value. An estimate from the free-end slip curves of Robins and Standish (1982) is that  $\Delta_u$  is about 0.1 mm and this value is used herein.

(v)  $\beta$  parameter

The  $\beta$  parameter is the ratio of maximum bond stress to ultimate bond stress for slips in excess of the tolerance slip ( $\Delta_u$ ). This parameter is very difficult to estimate but Alexander (1969) found the bond strength of aggregate and cement (similar to steel-cement) with excessive slip to decrease by about 50 percent. The assumed value of  $\beta$  used here is 0.5 .

(vi)  $R_n$  (Bond modulus orthogonal to the bar)

An arbitrary high bond modulus has been assumed to the bond in this direction and equal to  $10^5 \text{ N/mm}^3$ . Accordingly the displacements of the steel bar in the direction normal to the bar axis will be the same as the concrete in this direction.

### 7.2.2 Concrete parameters.

For all the analyses reported in this chapter the same set of concrete parameters has been used throughout. For the Mains (1951) pull-out test there is very little information about the concrete parameters. In the author's tests some of the parameters have been estimated and therefore the author considers it more consistent to use the same concrete parameters for all the problems. The concrete parameters which have been used are the original set from the work of Kupfer, Hilsdorf and Rusch (1969) and are tabulated in Table 7.2 .

### 7.3 ORDINARY PULLOUT TEST.

The finite element analyses will be compared against the experimental results reported in Chapter 6 and the test results of Standish (1982). In both sets of experimental tests there was no additional reinforcement in the cubes, and the concrete cubes are assumed to be  $150 \times 150 \times 150 \text{ mm}^3$  for the author's test and  $100 \times 100$

$\times 100 \text{ mm}^3$  for the test of Standish (1982).

Advantage of the symmetry of the problem has been taken and only one half of the specimen modelled. The finite element mesh used to analyse this problem is shown in Figure 7.1. The concrete is represented by eight QUAD8SM elements, the steel bar by five BAR3 elements and bond by four BOND6 elements.

### 7.3.1 Author's tests.

For these analyses the bond parameters are as given in Table 7.1 with the  $q_0$  parameter equal to  $2 \text{ N/mm}^2$ . The diameter of the steel bar in this test was 16mm. The load was applied in increments of 2.0 kN up to 10 kN and then in smaller increments up to the failure load of 11.5 kN. Failure occurred by the bond slip exceeding 0.1 mm for all positions along the embedded bar length.

The load distributions obtained in the finite element analysis are compared with two sets of experimental results in Figures 7.2 and 7.3. The failure loads in the experimental tests were 12 kN and 14 kN respectively. For the pullout test results shown in Figure 7.2 experimental load distributions only up to 6 kN were available and comparing the analytical and experimental results over this load range a reasonable match is obtained. At a pulling load of 2.0 kN the predicted bar stress distribution does not reduce in magnitude rapidly enough, indicating that the initial bond stress-slip modulus of  $200 \text{ N/mm}^3$  is too low. At a pulling load of 4 kN the analytical curve lies approximately in the middle of the scatter of the experiments points. There is very good agreement between the predicted failure load and the experimental pullout load.

For the second set of experimental results (Figure 7.3) the pullout load at failure (14kN) is about 16 percent higher than the

|                               |                                              |
|-------------------------------|----------------------------------------------|
| 5 Bar elements                | 48 nodes                                     |
| 4 Bond elements               |                                              |
| 8 Concrete elements           | $x = 150\text{mm}$ Author's tests            |
| All dimensions in millimeters | $x = 100\text{mm}$ Robins and Standish tests |
|                               | Thickness = 100 or 150 mm                    |

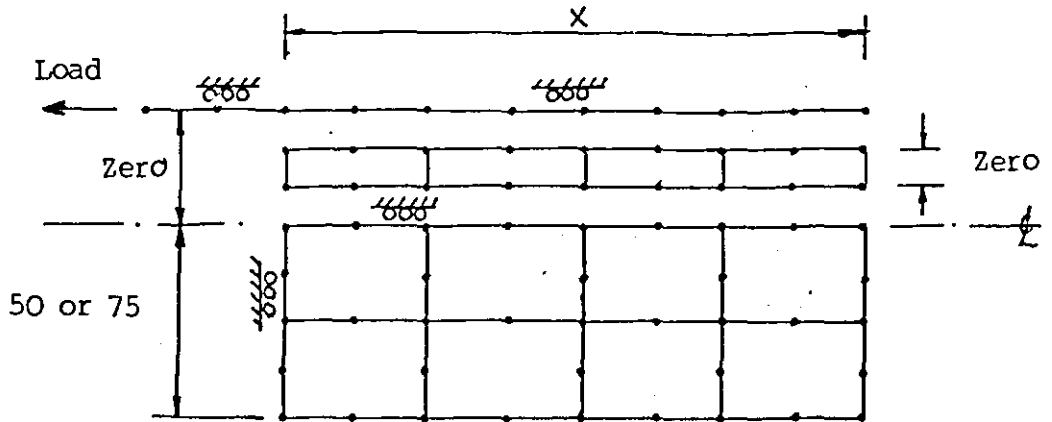


FIGURE 7.1 FINITE ELEMENT MESH AND BOUNDARY CONDITIONS TO MODEL THE ORDINARY PULLOUT TEST

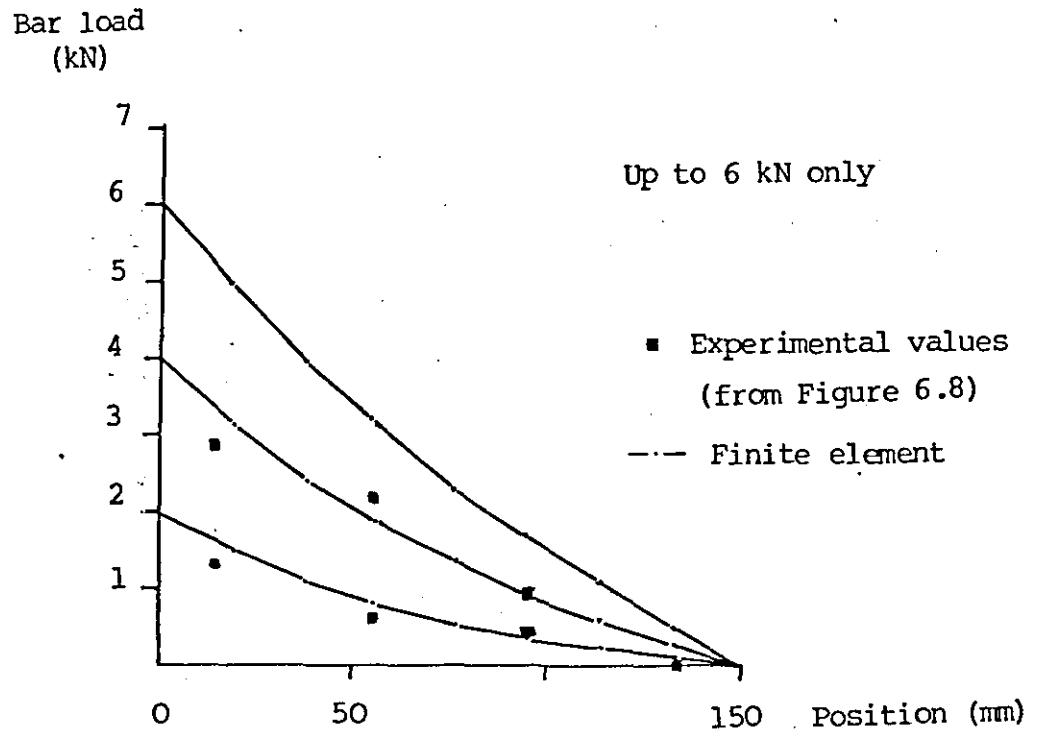


FIGURE 7.2 ANALYTICAL BAR LOAD DISTRIBUTIONS COMPARED WITH EXPERIMENTAL RESULTS FOR THE PULLOUT TEST

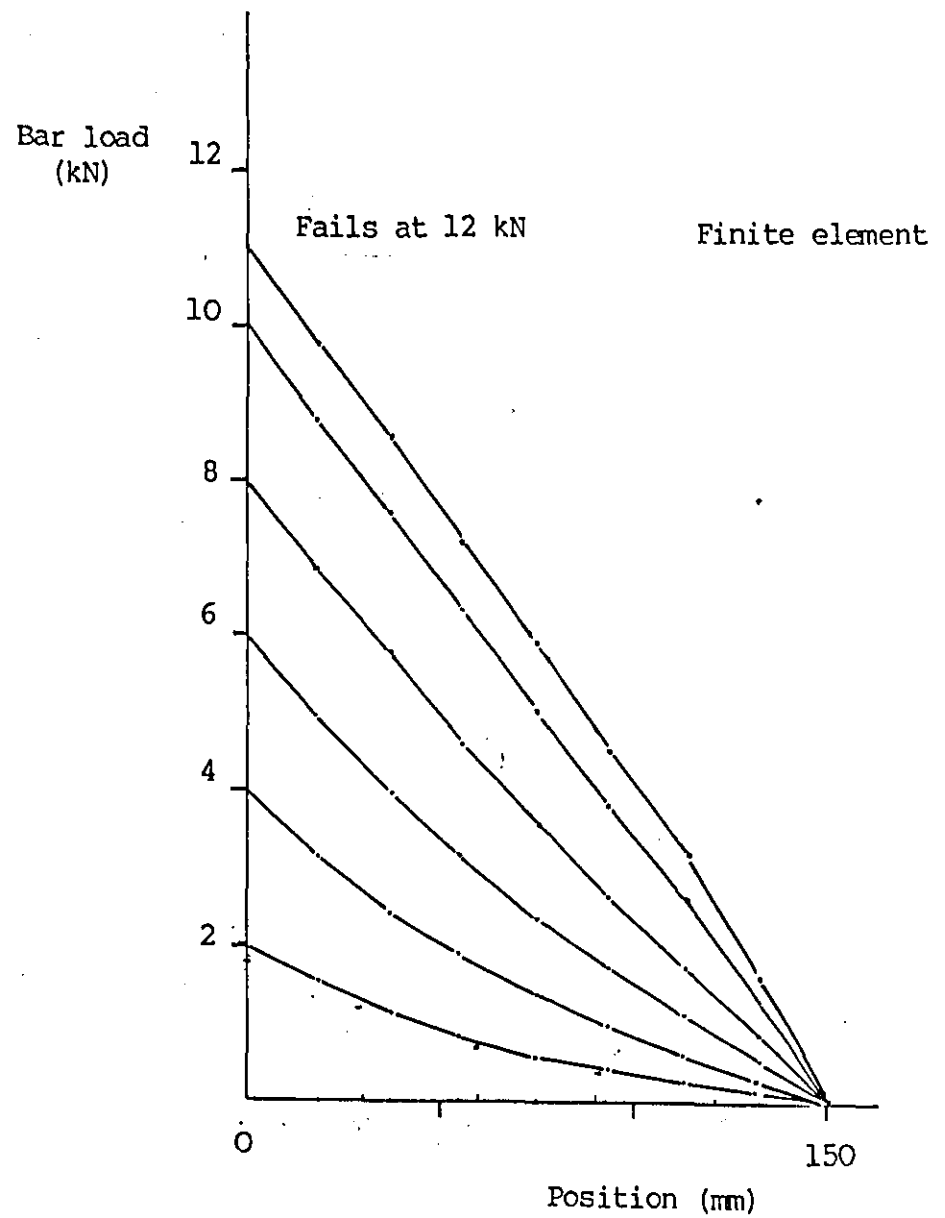
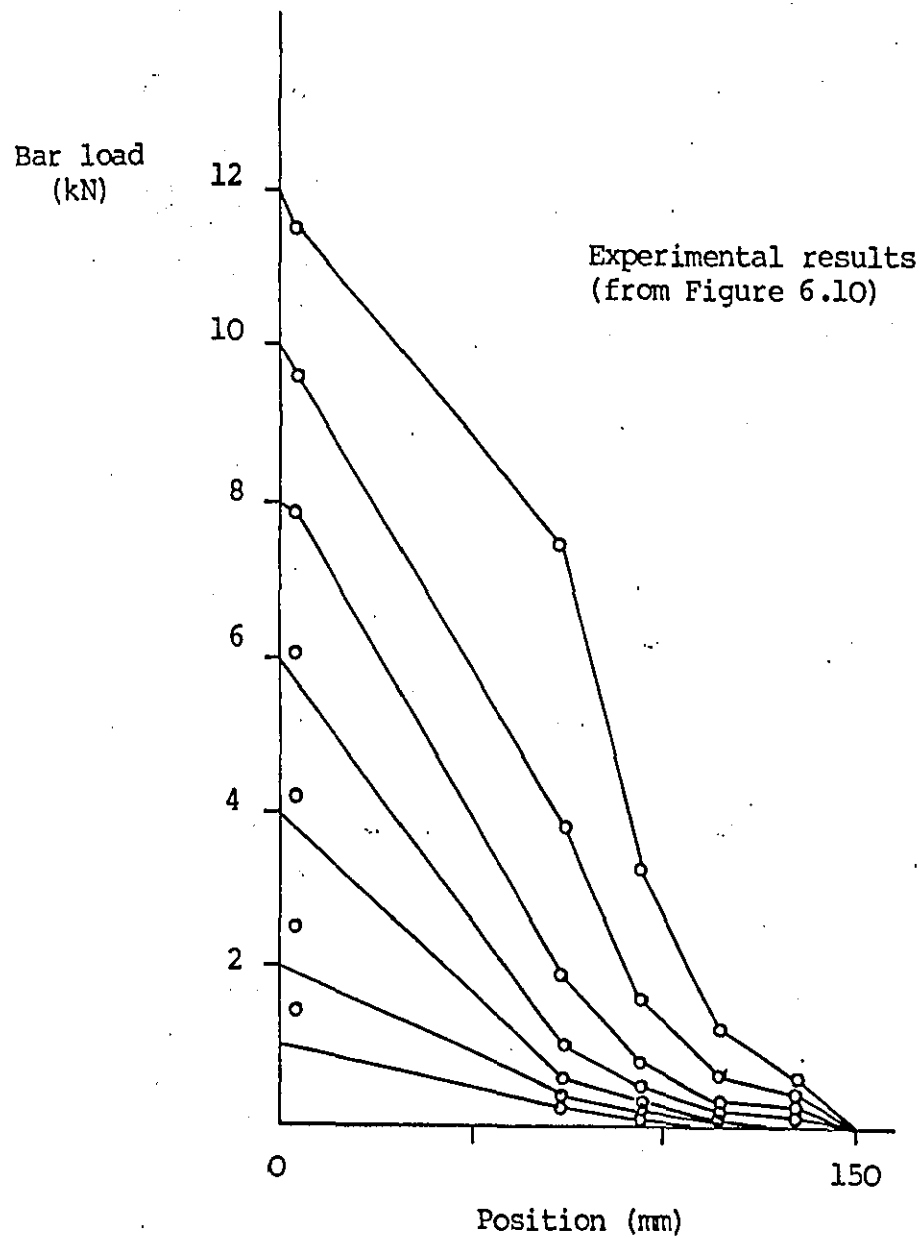


FIGURE 7.3 ANALYTICAL BAR LOAD DISTRIBUTIONS COMPARED WITH  
EXPERIMENTAL RESULTS FOR THE PULLOUT TEST

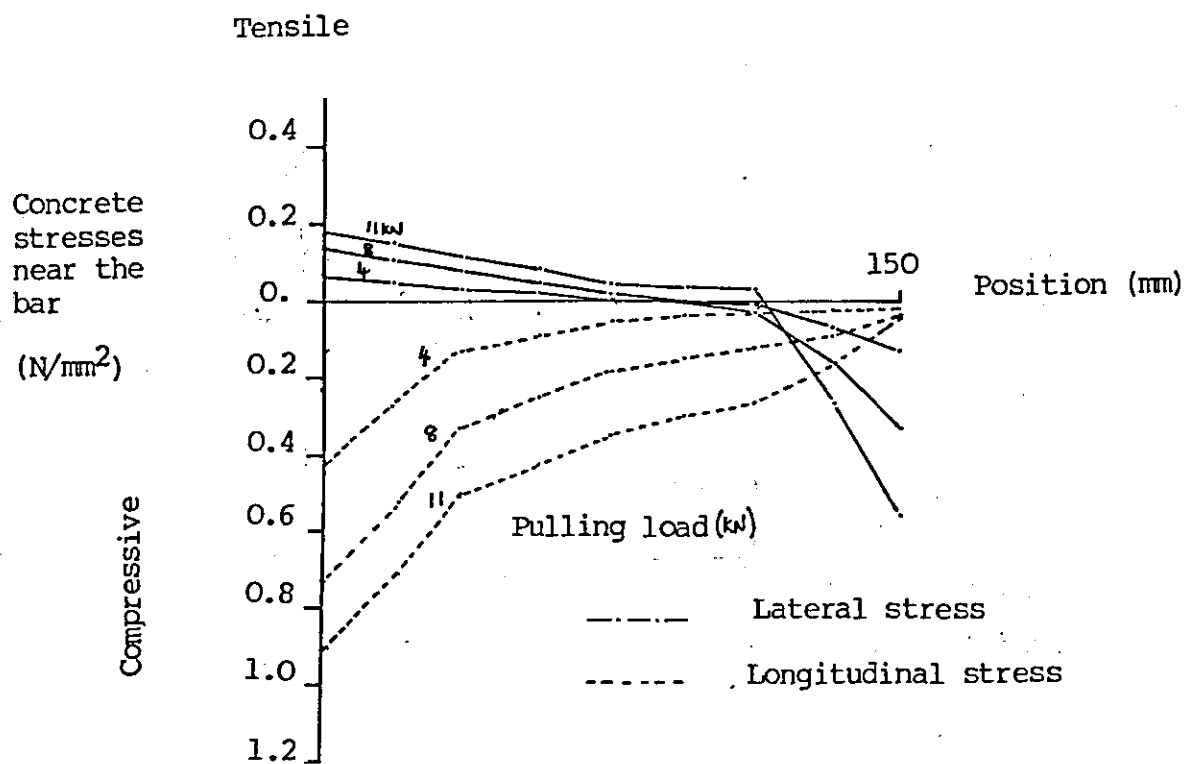
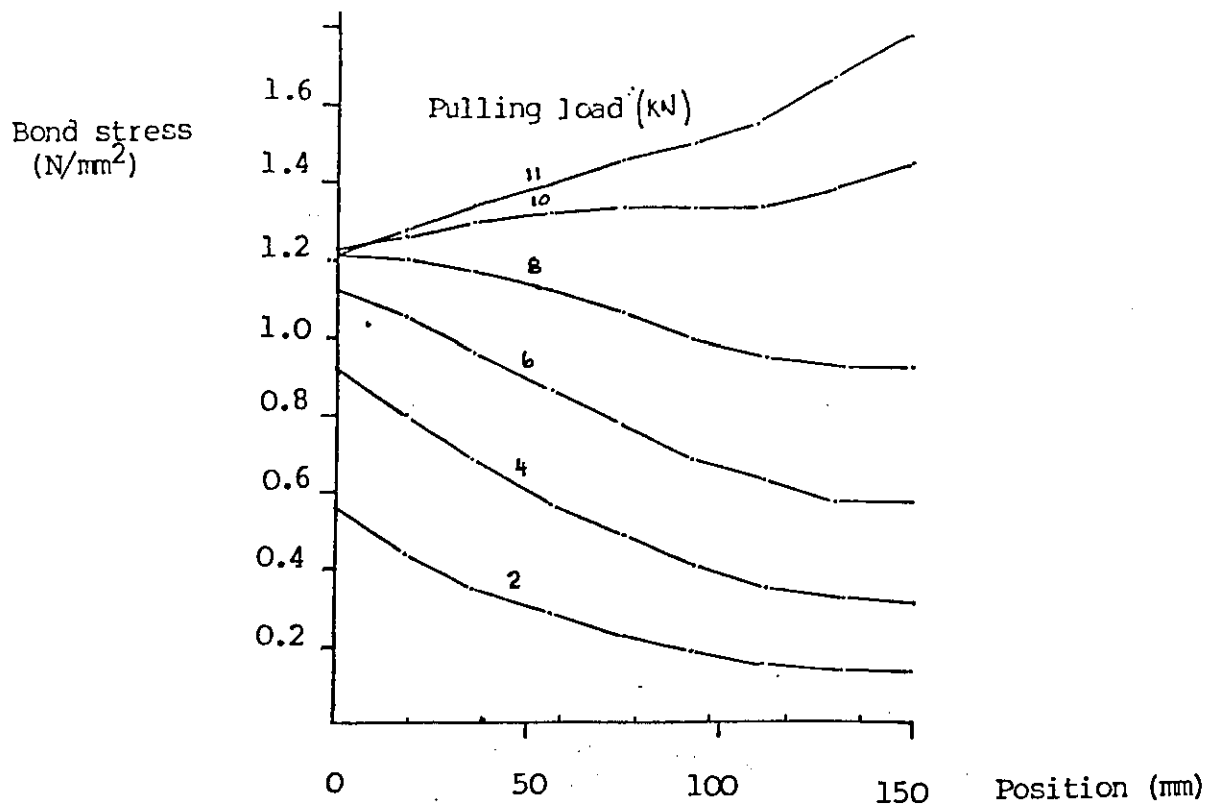


FIGURE 7.4 ANALYTICAL BOND STRESS AND CONCRETE STRESSES NEAR THE BAR IN THE PULLOUT TEST

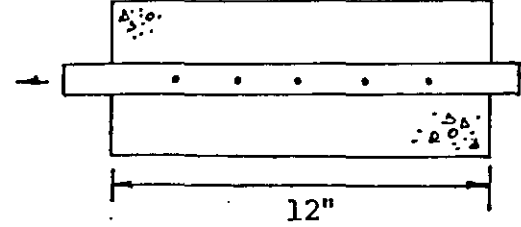
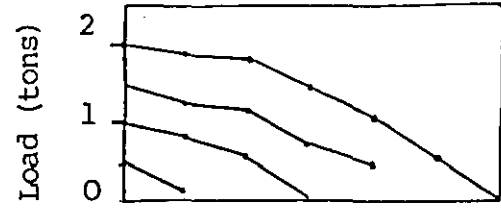
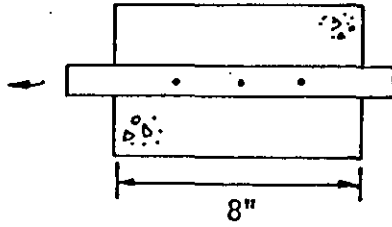
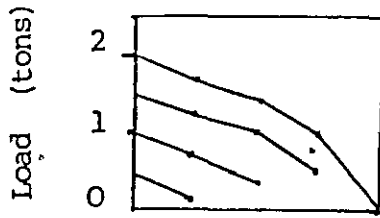
analytical result. The bond model reasonably accurately predicts the changing nature of the bar stress distribution, from an exponential type of curve to a convex or gradual S-shape. The analytical bond stress distribution (Figure 7.4) clearly shows how the bond stress distribution must change so that the gradual change from an exponential to a convex or gradual S-shape in the bar stress distribution is obtained. The bond stress at or near the pulling end must reach a peak value and then reduce to a value less than the allowable bond stress along the rest of the bar. It may also be observed that very large changes in the bond stress distribution must occur before there is a substantial change in the bar stress distribution.

Some representative examples of the load distributions in the pull-out test obtained by other researchers are shown in Figure 7.5 and the same changes in the shape of the load distributions are observed.

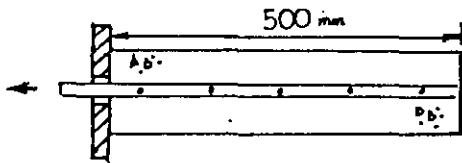
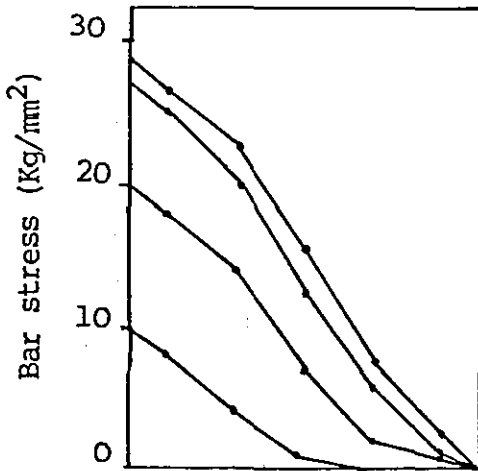
### 7.3.2 Standish (1982) tests.

For these analyses the bond parameters used are tabulated in Table 7.1 with the  $q_0$  parameter equal to  $3 \text{ N/mm}^2$ . The diameter of the steel bar was 12mm. Where the boundary of the concrete cube was subjected to lateral loading the full lateral load was applied in conjunction with the first increment of pulling load. For the lateral stresses of zero and  $12 \text{ N/mm}^2$  the pulling load was applied in increments of 2 kN except close to failure where it was reduced to increments of 1 kN. For the lateral stress of  $24 \text{ N/mm}^2$  the pulling load was applied in increments of 5 kN up to 25 kN and then in increments of 2kN thereafter.

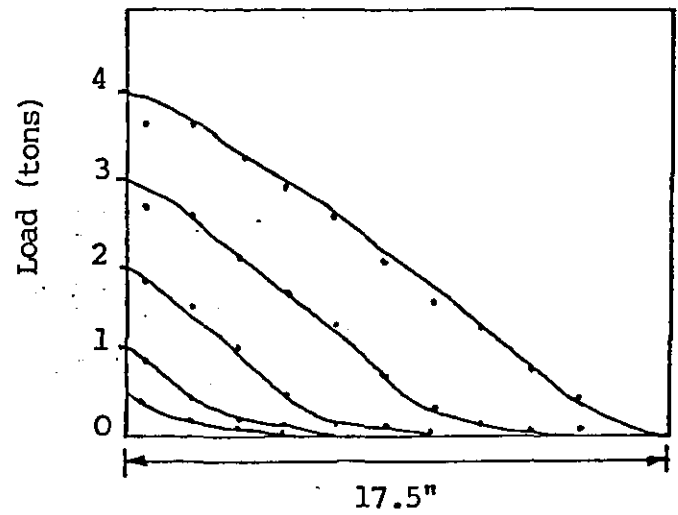
The analytical load distributions for each value of lateral stress are shown in Figure 7.6, however there are no experimental load distributions against which to compare them. The analytical values of



WILKINS (1951)



PARLAND (1957)



6" dia. Concrete specimen

PEATIE AND POPE (1950)

FIGURE 7.5 LOAD DISTRIBUTIONS IN THE ORDINARY PULLOUT TEST FOR PLAIN ROUND BARS OBTAINED BY OTHER INVESTIGATORS

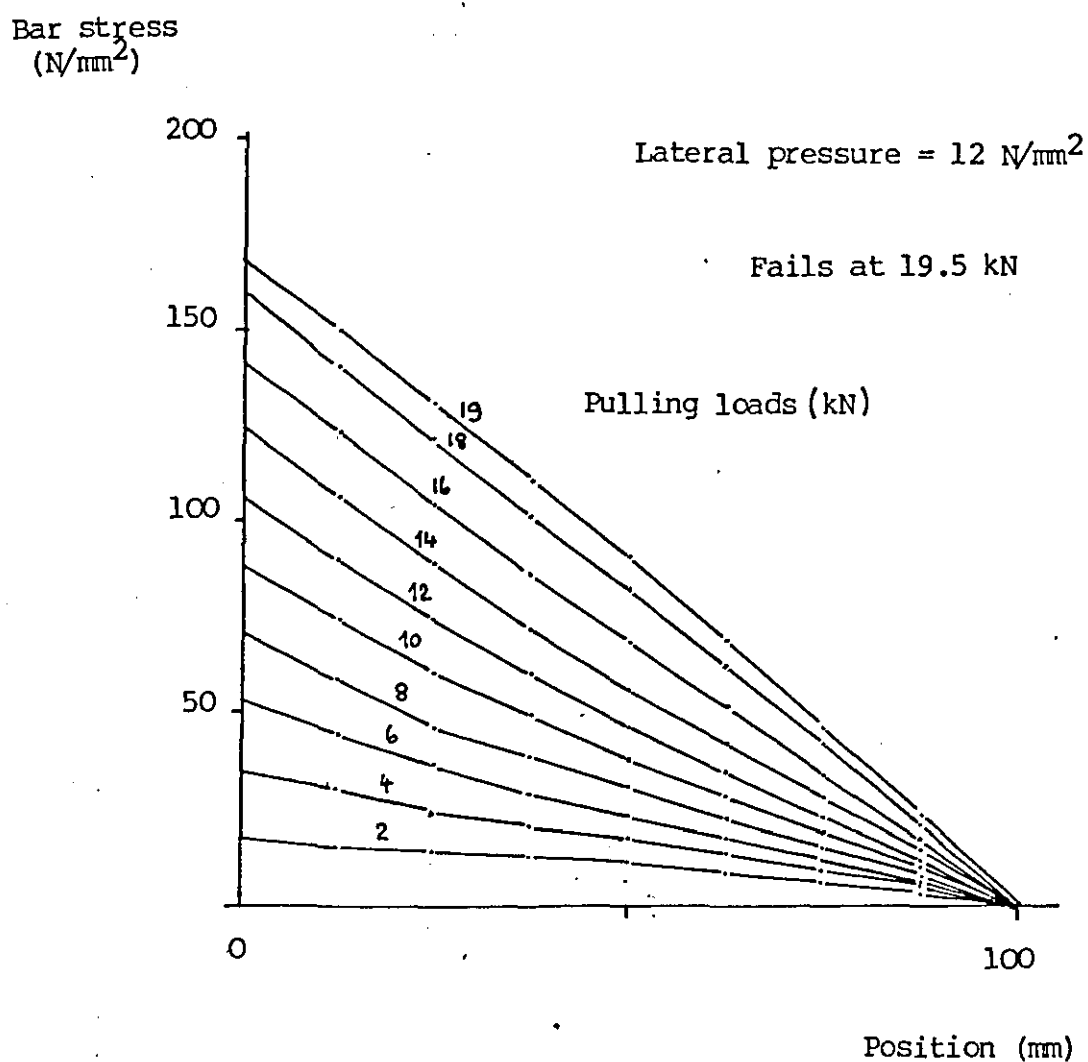
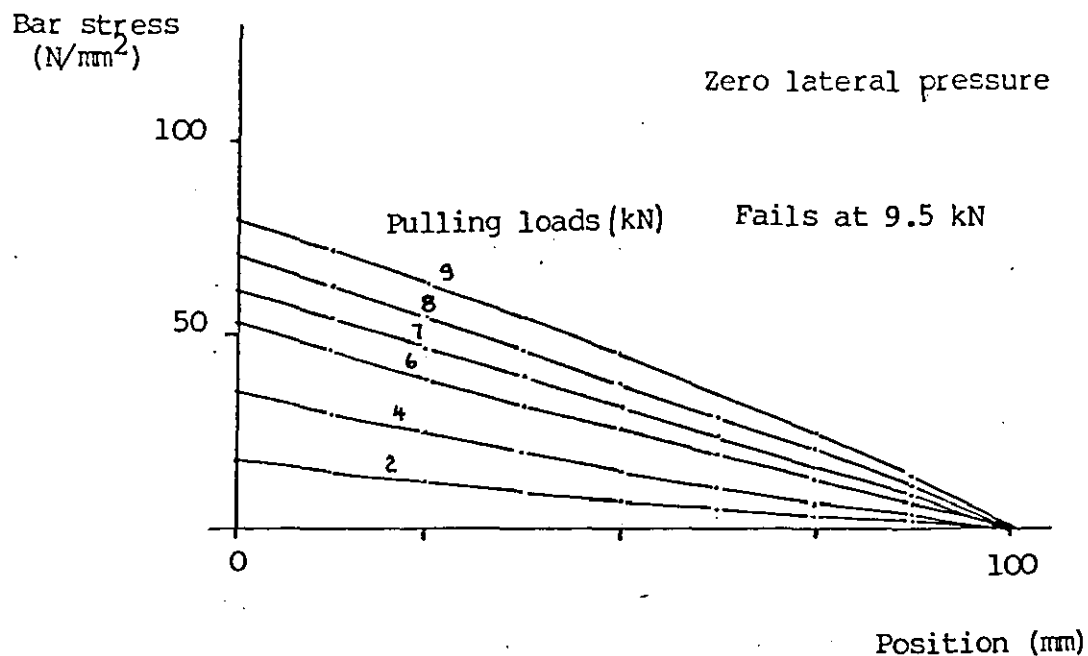


FIGURE 7.6 ANALYTICAL BAR STRESS DISTRIBUTIONS IN THE PULLOUT TEST  
WITH DIFFERENT LATERAL PRESSURES APPLIED ACROSS THE CUBE

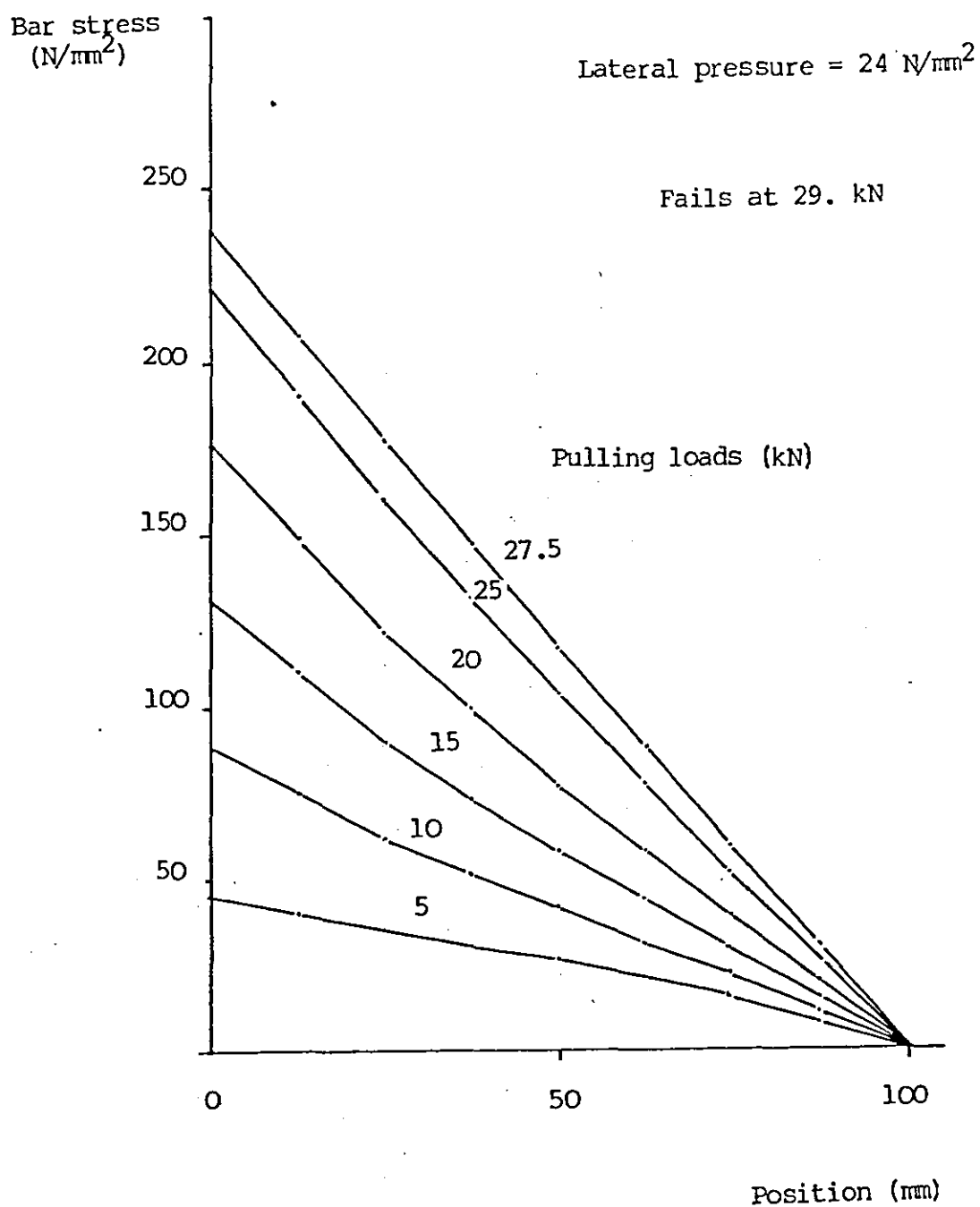


FIGURE 7.6 continued

pullout load against lateral stress are compared with the experimental results in Figure 7.7 and the analytical model very accurately predicts the variation in pullout load with lateral stress.

The free end slips obtained from the analyses, after corrections for the longitudinal expansion of the concrete when laterally loaded, are shown in Figure 7.8 against the free end slips obtained by Standish. The free end slips compare reasonably well with the experimentally observed behaviour. In Figure 7.8 the predicted pullout load for zero lateral pressure is lower than the actual value of pullout measured for this particular cube specimen and this accounts for the difference in the two curves at each load level.

The analytical longitudinal and lateral concrete stresses at the bar-concrete interface at the failure pullout load are shown in Figure 7.9. The magnitude of concrete stresses at this load are very low with the maximum lateral stress only 11 percent of the tensile failure strength.

### 7.3.3 How the bond parameters effect the bar stress distributions in the ordinary pullout test.

This section illustrates how the values of the bond parameters in the bond model affect the bar stress distribution in the ordinary pullout test.

#### (i) $R_0$ Initial bond stress-slip modulus

If the  $R_0$  value is increased from  $200 \text{ N/mm}^3$ , the tendency at low loads is to increase the amount of force being transferred by bond near the pulling end and thereby the load in the bar is transferred more rapidly to the concrete. This is indicated by the increased steepness

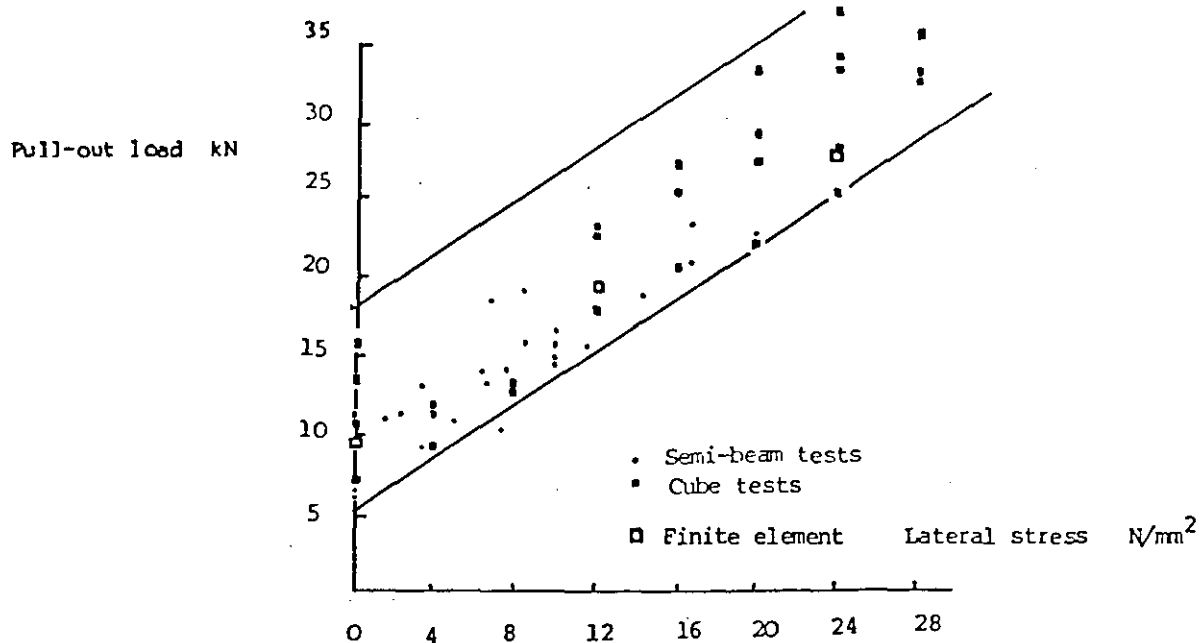


FIGURE 7.7 EFFECT OF LATERAL PRESSURE ON PULLOUT LOAD IN THE PULLOUT TEST. ANALYTICAL AND EXPERIMENTAL COMPARISON

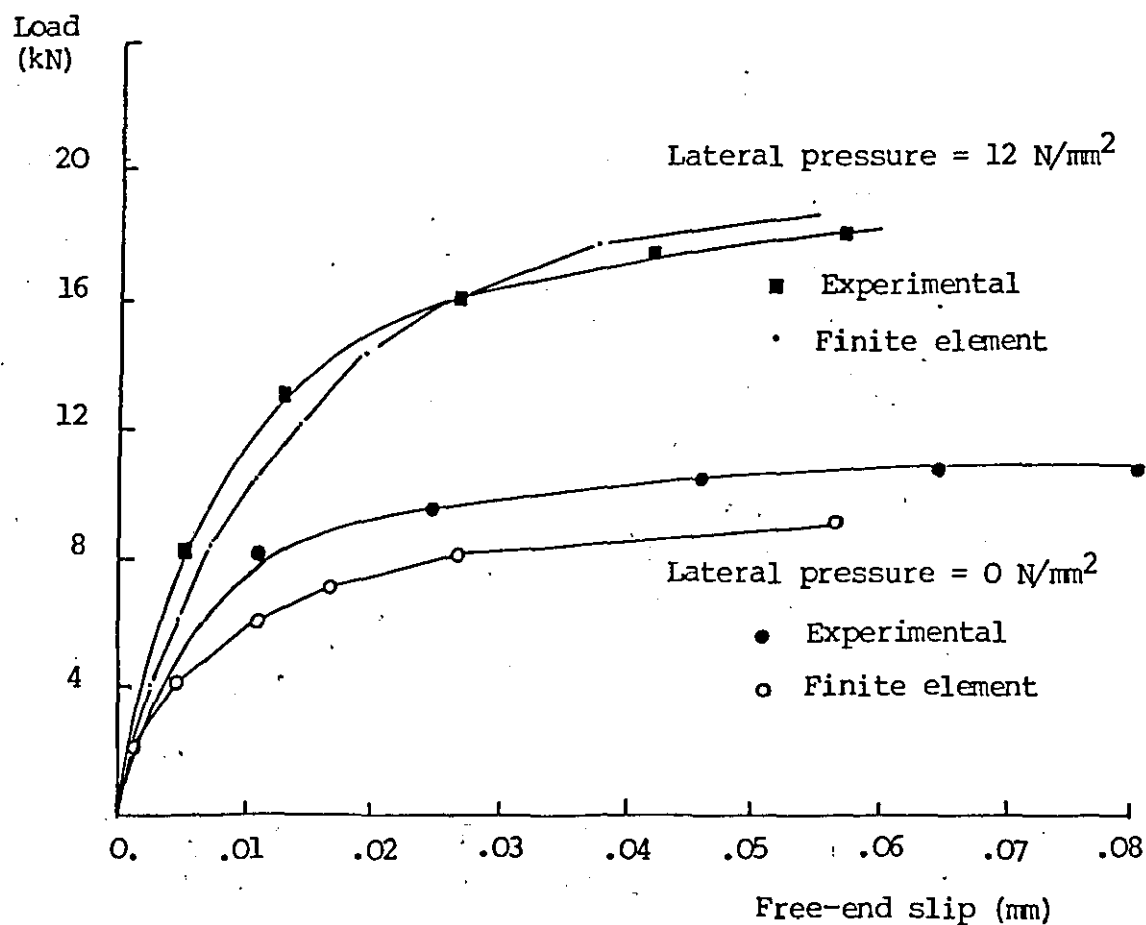


FIGURE 7.8 EXPERIMENTAL AND ANALYTICAL LOAD FREE-END SLIPS FOR THE PULLOUT TEST

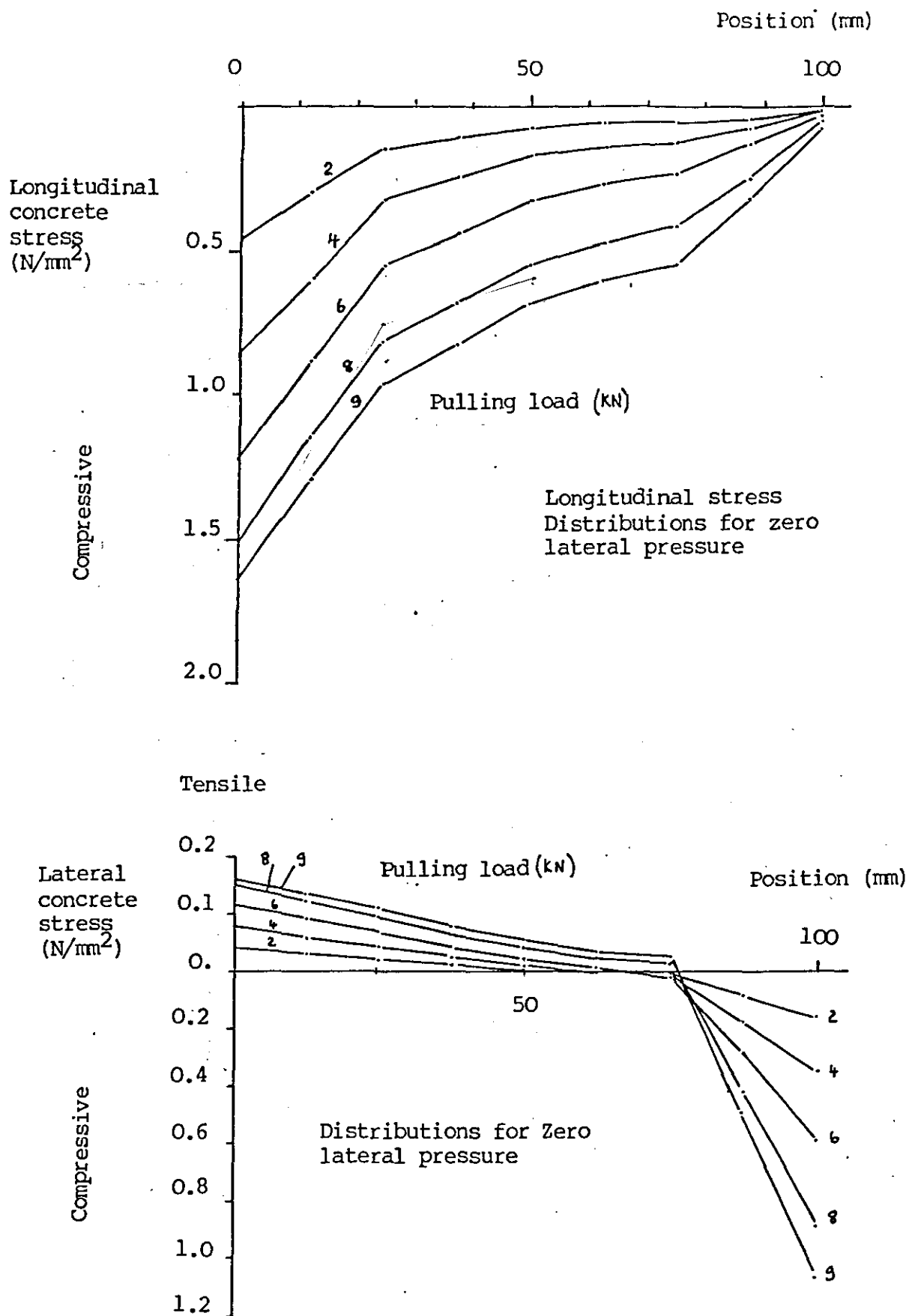


FIGURE 7.9 ANALYTICAL CONCRETE DISTRIBUTION NEAR THE BAR IN THE ORDINARY PULLOUT TEST

in the bar stress curve and corresponding increased bond stress at the pulling end at the same load level as shown in Figure 7.10a .

(ii)  $q_0$  parameter .

The value of the  $q_0$  parameter corresponds to the local ultimate bond stress due to shrinkage alone. However the magnitude of  $q_0$  determines the value of the pullout load and is approximately proportional to the pullout load.

(iii)  $\mu$  parameter

This parameter effects the value of local ultimate bond stress and the bond distribution at failure (known as the bond failure envelope) in the pullout test. If  $\mu$  equals zero then the bond failure envelope is horizontal (Figure 7.10b). Lateral pressure and radial pressure effects draw the failure envelope curve down as shown in Figure 7.10b. This phenomenon has the effect of producing the convex shape of the bar stress distribution.

(iv)  $\Delta_u$  parameter

The effect of the size of  $\Delta_u$  and  $R_0$  on the local bond stress-slip curve is illustrated in Figure 7.10c. The pullout load is approximately proportional to  $\Delta_u$  . A large  $\Delta_u$  has the effect that failure is gradual as the bond stress and slip increase.

(v)  $\beta$  parameter

This parameter can have the most dramatic effect on the bar stress distribution. Depending on the relative values of the other parameters the value of  $\beta$  can have very little effect or a marked effect on the bar stress distribution. For a low  $R_0$  and high  $\Delta_u$  combination  $\beta$  will have little effect on the bar stress distribution as once bond failure starts at the pulling end the reduction in bond stress by

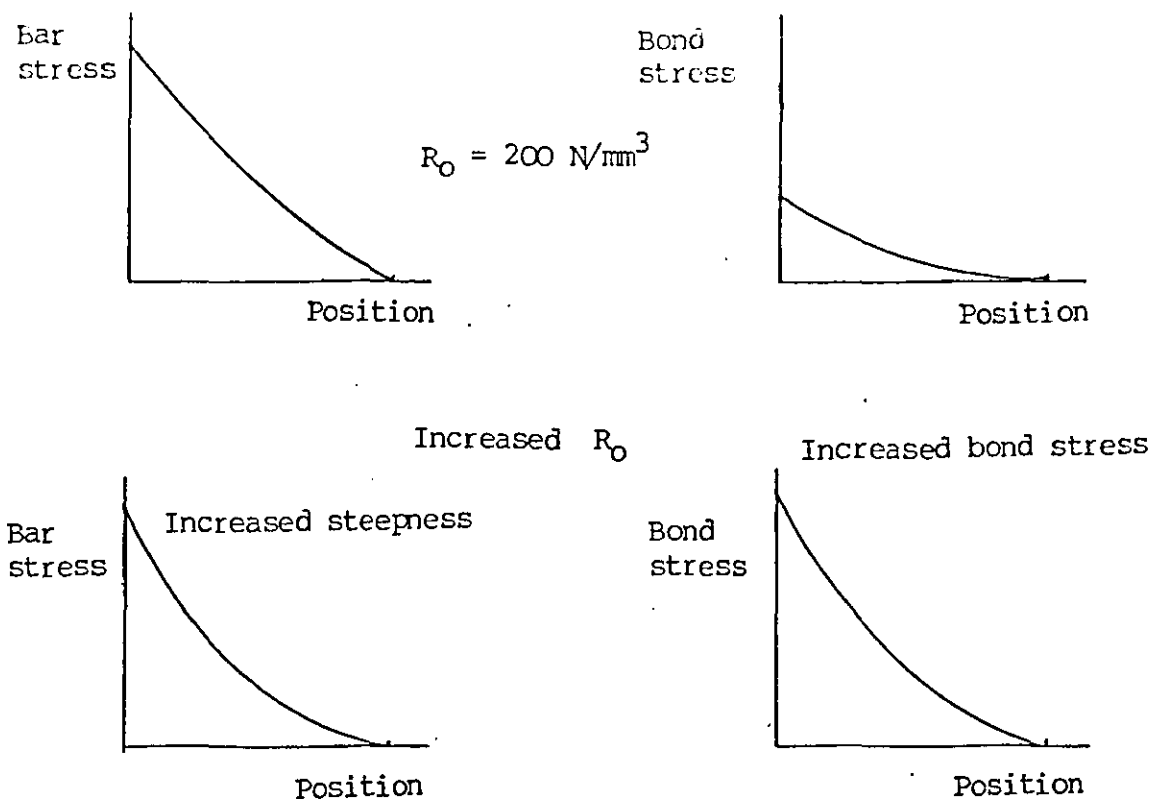


FIGURE 10a EFFECT OF  $R_0$  PARAMETER ON BAR AND BOND STRESS DISTRIBUTIONS  
AT LOW LOADING IN THE PULLOUT TEST

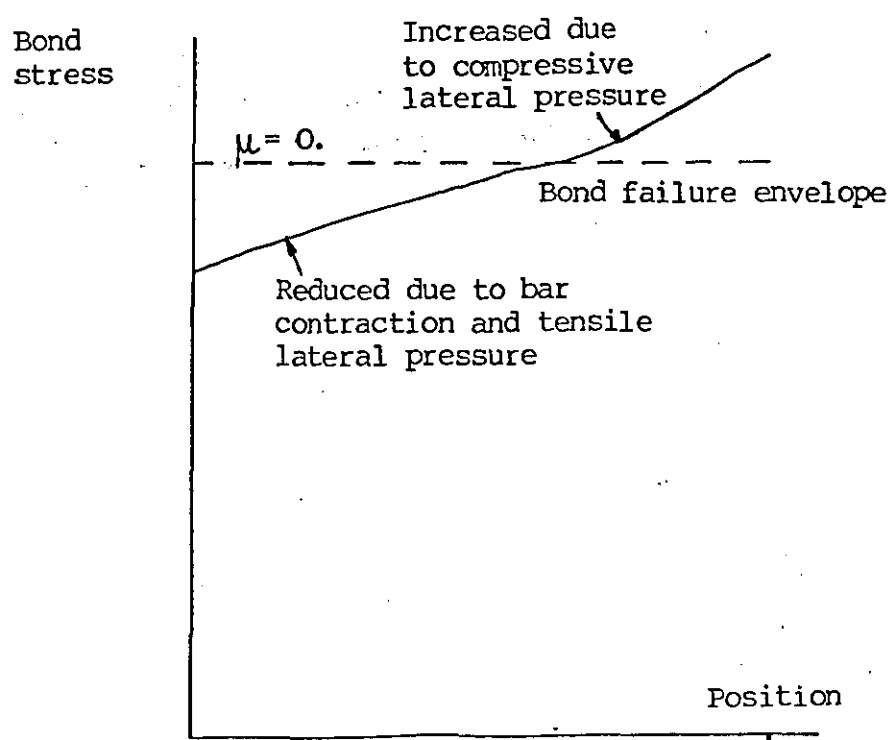


FIGURE 10b EFFECT OF  $\mu$  PARAMETER ON BOND FAILURE ENVELOPE  
IN THE PULLOUT TEST

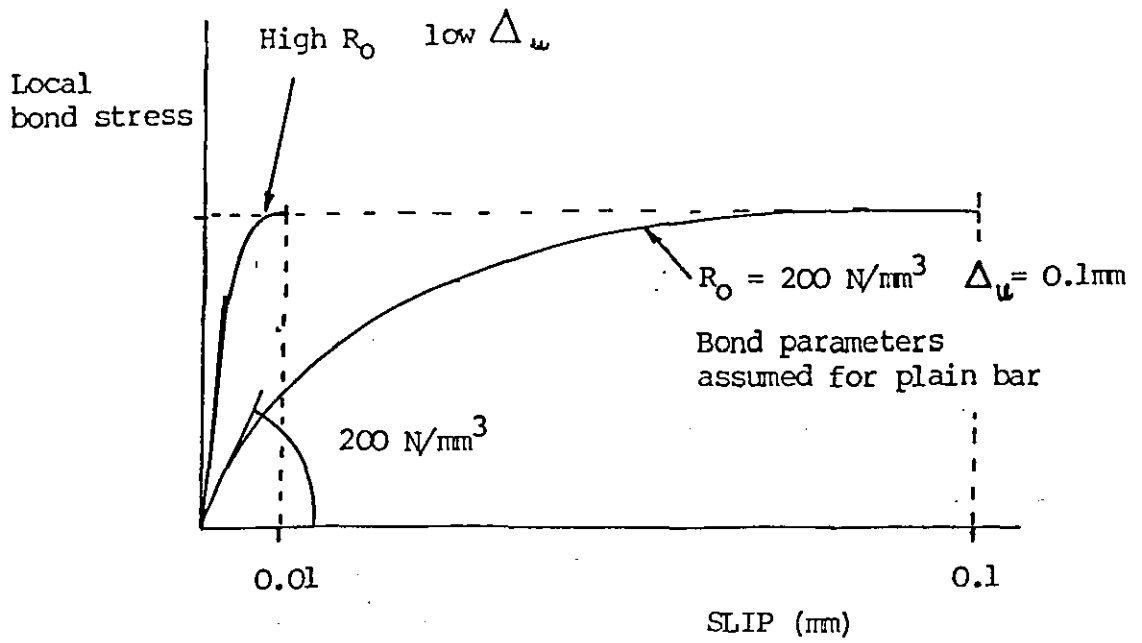


FIGURE 10c COMPARISON OF  $R_O$  AND  $\Delta_u$  PARAMETERS ON THE LOCAL BOND STRESS-SLIP RELATIONSHIP

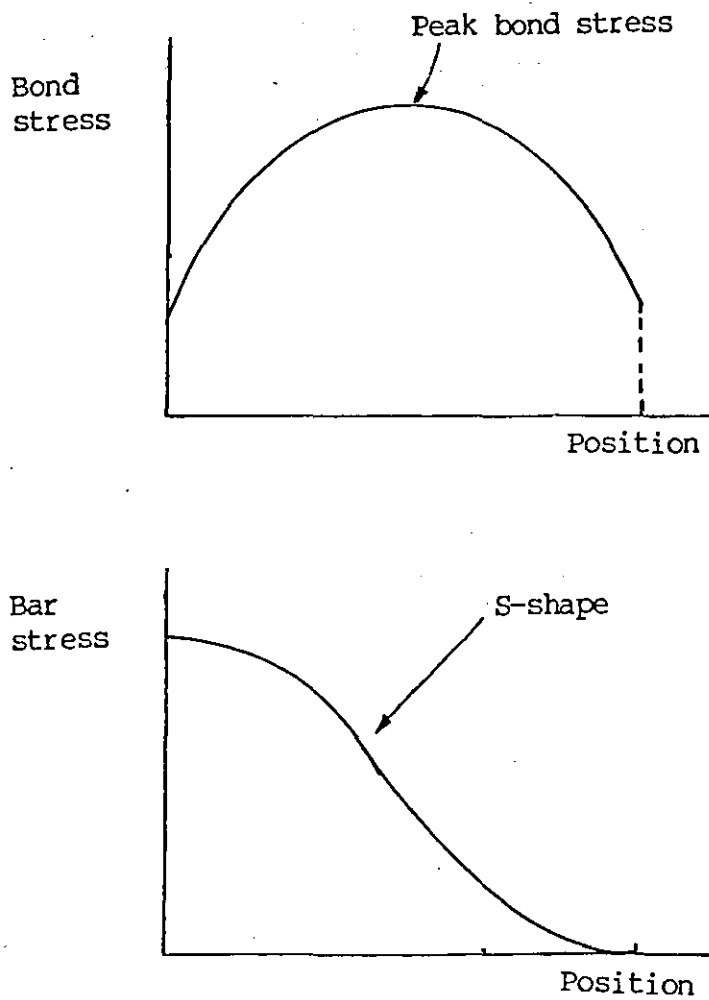


FIGURE 10d S-SHAPE BAR LOAD DISTRIBUTION IN THE PULLOUT TEST

$\beta$  will cause rapid bond failure along the rest of the bar. With a high  $R_o$  and low  $\Delta_u$  the effect is to produce the 'n' shape in the bond stress distribution (Figure 7.10d) and the associated S-shape load distribution. Complete bond failure in this instance is slower to develop after the onset of bond failure at the pulling end.

(vi) Further considerations - Elastic modulus of the concrete.

Throughout these analyses the radial pressure due to Poisson's radial contraction of the bar is based on a fixed value of concrete elastic modulus of  $30 \text{ kN/mm}^2$  in Equation 3.8 (Chapter 3) and the value of radial pressure is approximately proportional to the magnitude of the concrete elastic modulus. The concrete elastic modulus changes with stress state and the effect on the radial pressure relief due to Poisson's contraction is currently not accounted for. Although the effect on the radial relief would be quite marked, as very large changes in the bond stress distribution are required, there will be little change in the bar stress distributions.

#### 7.4 MAINS PULLOUT TEST.

Mains (1951) measured the load distributions in his pullout tests using an internal gauging technique. The concrete specimens were 21 in. long x 12 in. deep x 8 in. wide with the bars 2.5 in. from the bottom and additional stirrup reinforcement was used. The steel bars were 7/8 in. nominal plain round. Only the concrete crushing strength is known with an average value of 3790 psi. ( $26.1 \text{ N/mm}^2$ ).

The test has been modelled by the finite element mesh shown in Figure 7.11 and the bond parameters used are tabulated in Table 7.1 with  $q_o$  equal to  $3. \text{ N/mm}^2$ . The load was applied in increments of 8.896 kN (2 kips) up to 44.48 kN (10 kips) and then in smaller increments of

15 Concrete elements  
6 Steel elements  
5 Bond elements  
75 Nodes

203mm thick

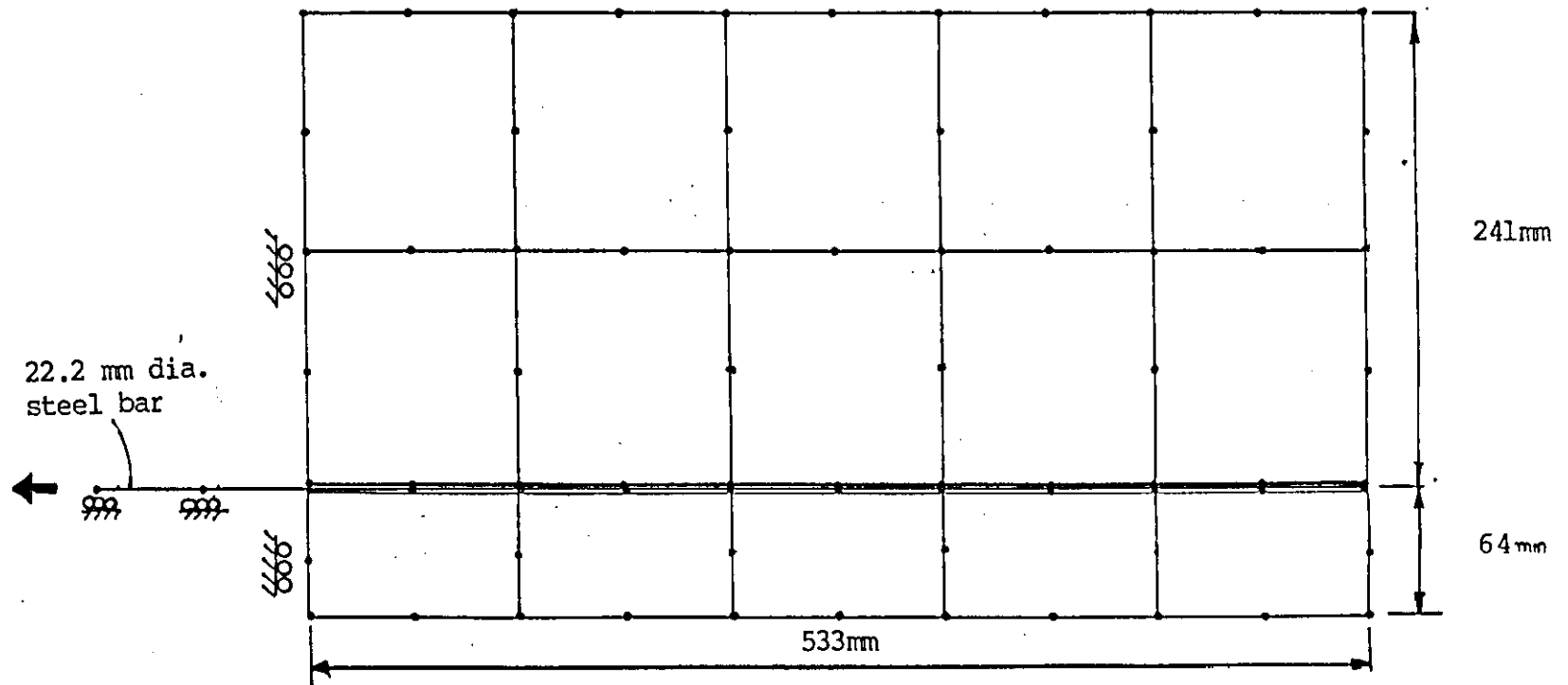
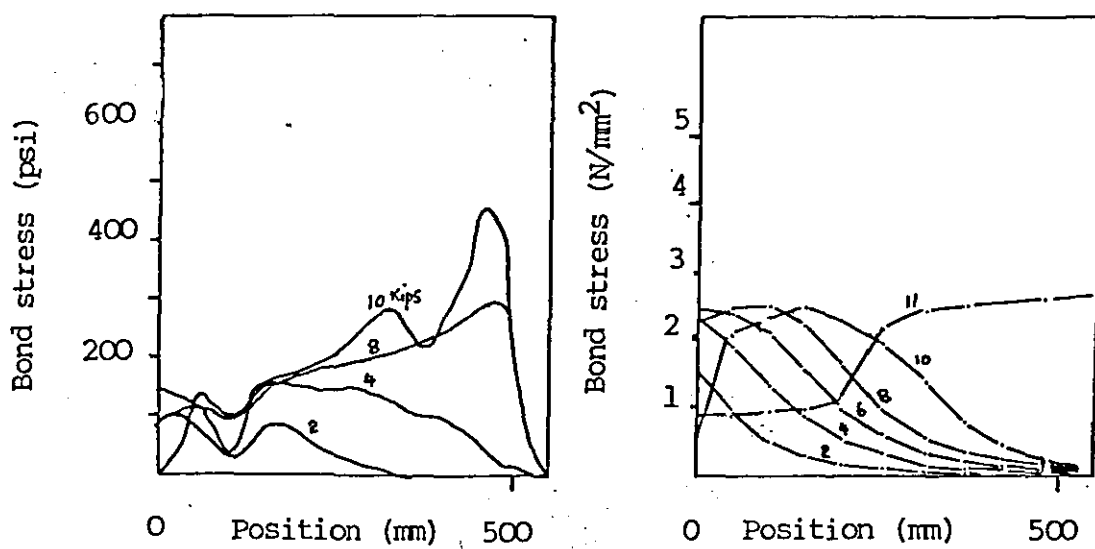
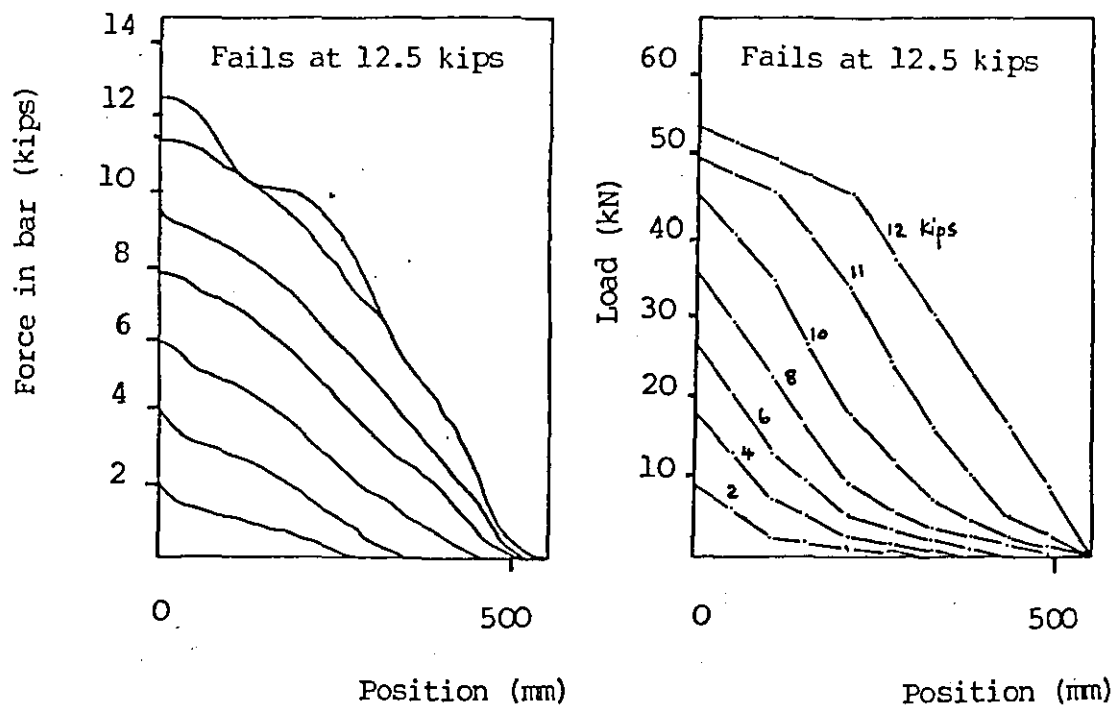


FIGURE 7.11 FINITE ELEMENT MESH AND BOUNDARY CONDITIONS FOR  
MAINS ECCENTRIC PULLOUT TEST



Experimental results

Finite element results

FIGURE 7.12 EXPERIMENTAL AND ANALYTICAL COMPARISON OF BAR LOAD AND BOND STRESS DISTRIBUTIONS IN THE MAINS ECCENTRIC PULLOUT TEST

4.448 kN (1 kip) and 2.224 (0.5 kips).

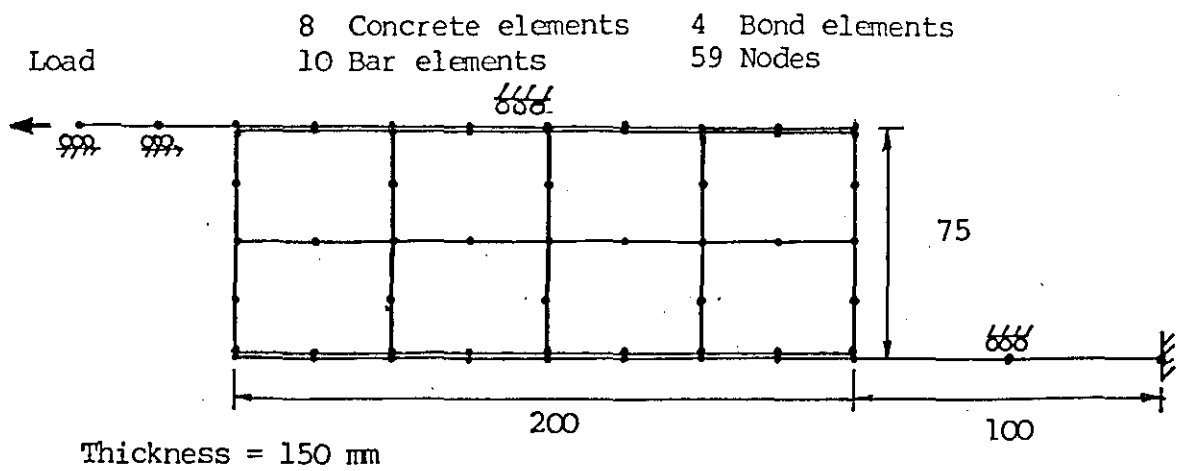
The bar force and bond stress distributions are shown in Figure 7.12 and compared with the experimental results. There is very good agreement with the value of the pullout load at failure and reasonable agreement of the bar load distributions. The analytical bar load distributions tend to show a slightly greater S-shape than the experimental results.

An important feature of this test, which is correctly modelled in the analysis, is the progressive transfer of peak bond stress through the specimen as failure is reached. As excessive slip occurs at the loaded end (i.e. slip greater than 0.1mm ) the local maximum bond stress is reduced by the  $\beta$  parameter. The shift in load towards the unloaded end causes a progressive failure in which the point of maximum bond stress moves along the bar. This phenomenon has also been observed in experimental pullout tests by Perry and Thompson (1966).

#### 7.5 DOUBLE-ENDED PULLOUT TEST.

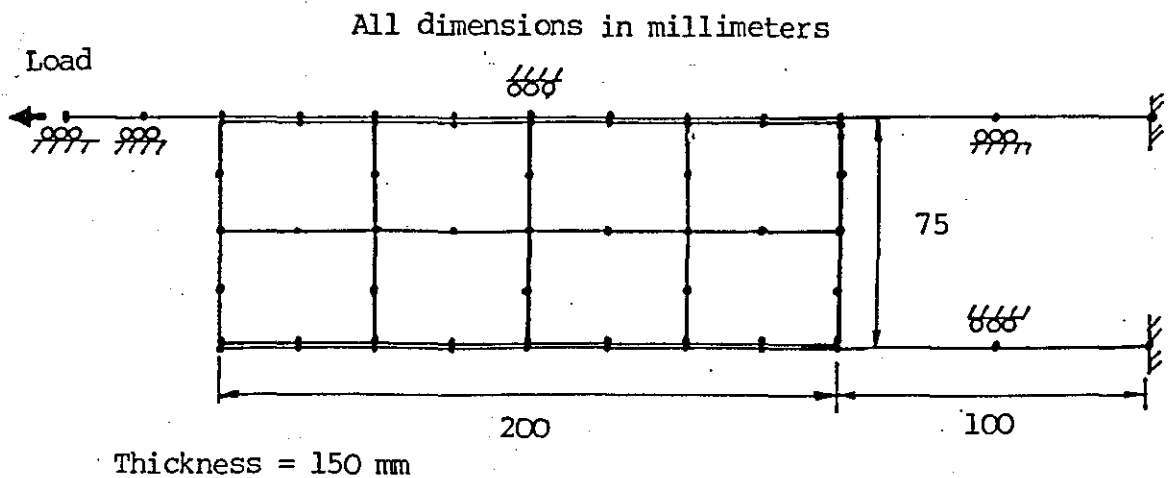
Load distributions have been obtained for this test by the author (Chapter 6). The concrete blocks were 200mm long x 150mm wide x 150mm deep and the embedded bright drawn steel round bars were 16mm in diameter.

Advantage of the symmetry has been taken and only one quarter of the complete test has been modelled using the finite element mesh illustrated in Figure 7.13. The bond parameters used are tabulated in Table 7.1 with  $q_0$  equal to  $2.0 \text{ N/mm}^2$ . The load was applied in increments of 2.5 kN up to 12.5 kN and then in smaller increments up to the predicted failure load of 14.75 kN .



All dimensions in millimeters

FIGURE 7.13 FINITE ELEMENT MESH AND BOUNDARY CONDITIONS FOR THE DOUBLE ENDED PULLOUT TEST



8 Concrete elements      4 Bond elements  
11 Bar elements      61 Nodes

FIGURE 7.14 FINITE ELEMENT MESH AND BOUNDARY CONDITIONS FOR THE TRANSFER TEST

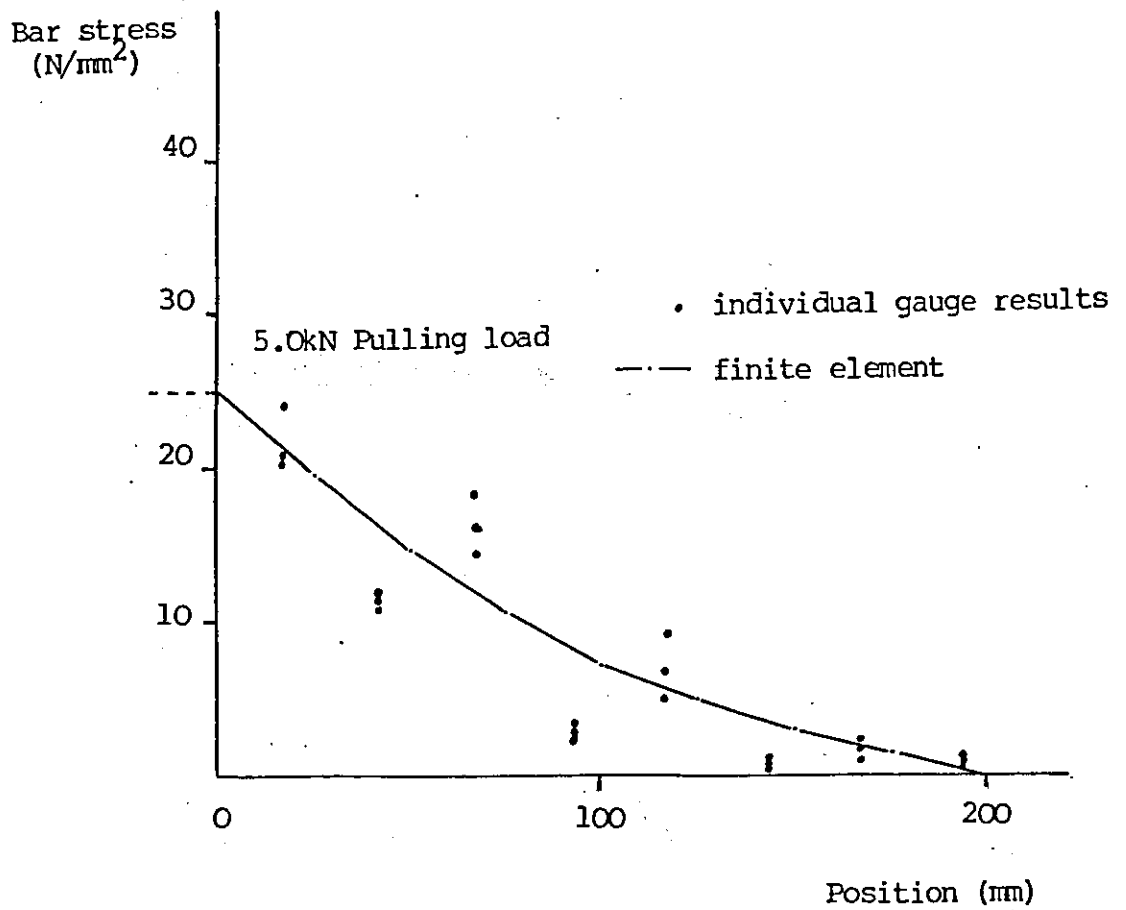
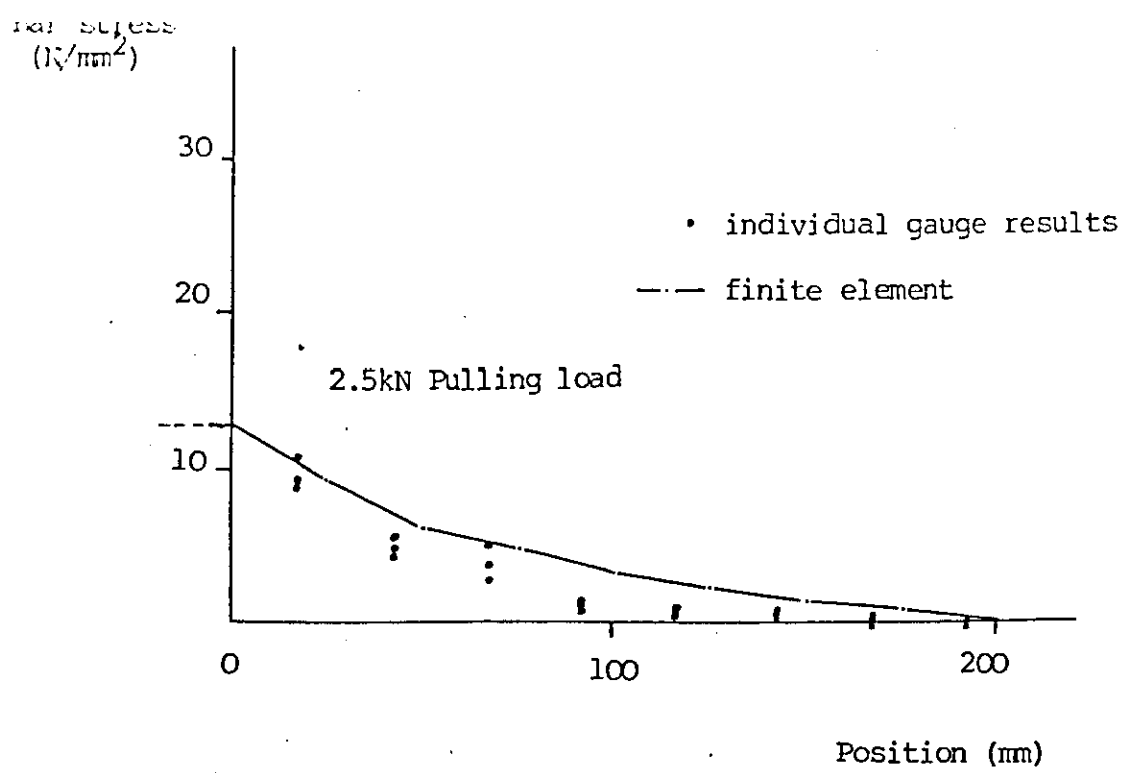


FIGURE 7.15 EXPERIMENTAL AND ANALYTICAL BAR STRESS DISTRIBUTIONS  
IN THE DOUBLE ENDED PULLOUT TEST

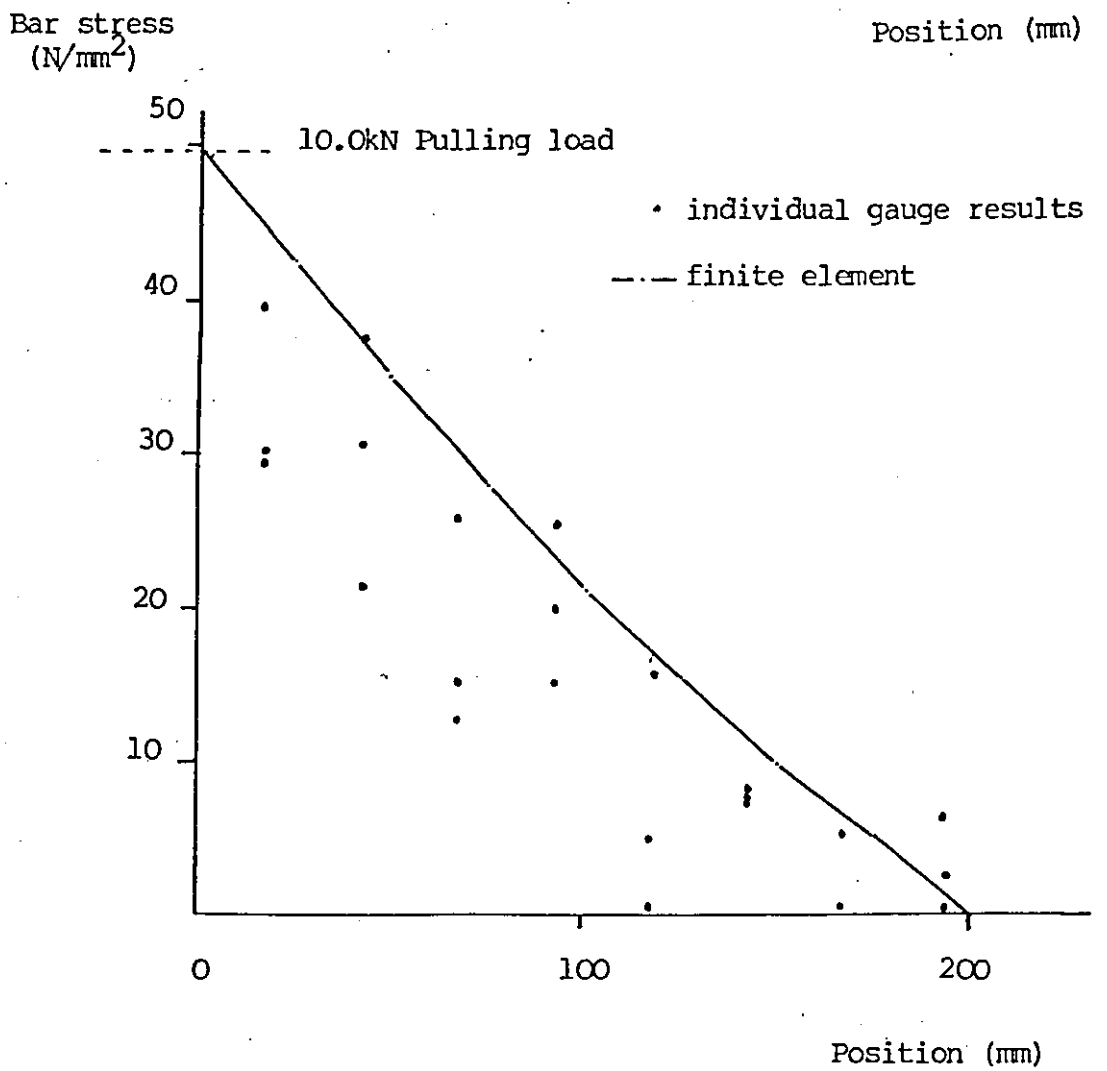
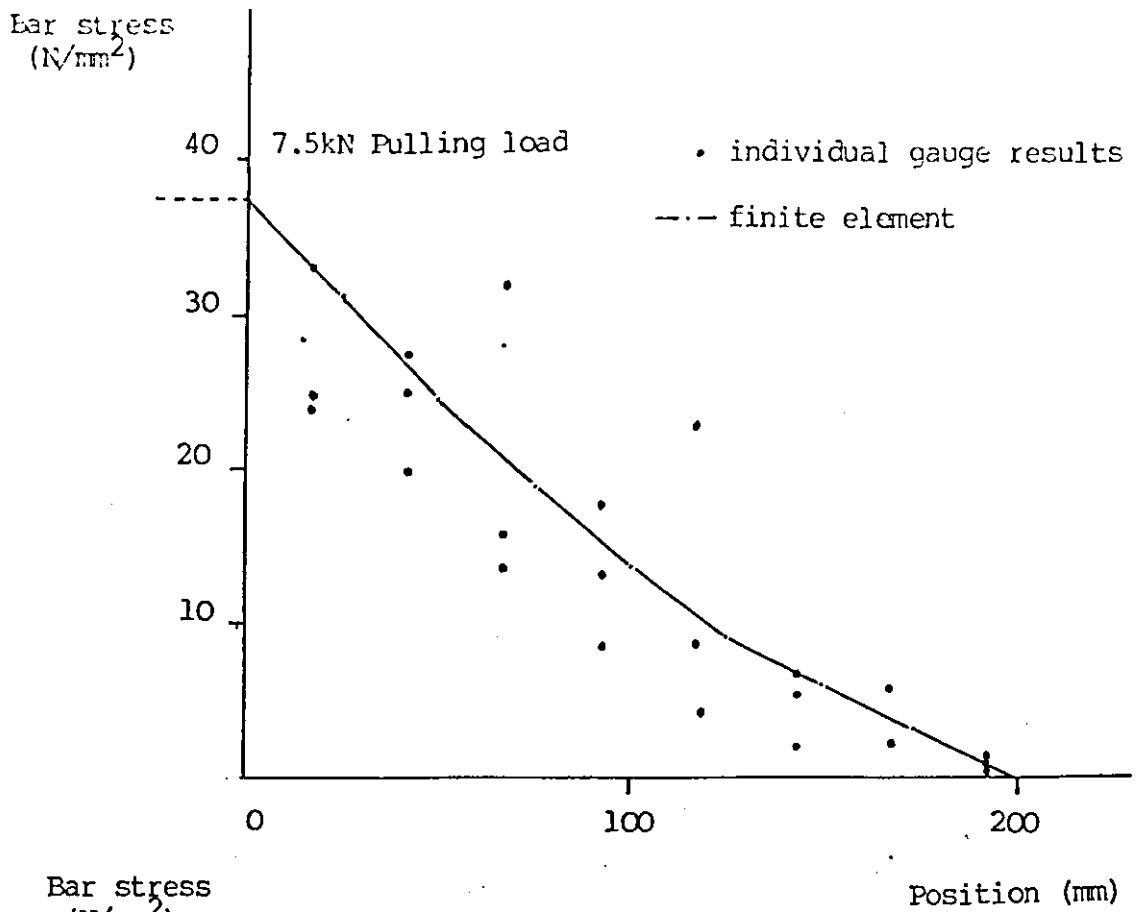


FIGURE 7.15 continued

The analytical load distributions are compared with the author's experimental results in Figure 7.15. The results show that at the lowest load of 2.5 kN the analytical curve generally underestimates the bar stress whereas at 5. kN and 7.5 kN the analytical curve is approximately in the middle of the test data. At the highest load of 10 kN the analytical curve is almost linear and over estimates the experimental bar stress distribution. At this load level the experimental results do not tend towards an S-shape but there is too much scatter within the data to conclude any definite match or lack of match between the analytical and experimental results.

#### 7.6 TRANSFER TEST.

Load distributions have been obtained for this test by the author (Chapter 6). The concrete blocks were of the same dimensions in the double ended pullout test and the bright drawn steel bars were 16mm in diameter.

Advantage of the symmetry has been taken and only one quarter of the specimen was modelled by the finite element mesh illustrated in Figure 7.14. Two analyses were performed using the bond parameters as given in Table 7.1 with the  $q_0$  parameter equal to 2  $\text{N/mm}^2$  and 3  $\text{N/mm}^2$ . The loads were applied in increments of 10 kN up to 80 kN. The analytical load distributions for both cases are compared with the experimental results in Figure 7.16. There is a lack of agreement between the experimental results and both sets of analytical results and serves to illustrate several points. The analytical load distributions tend to curve the wrong way compared to the experimental results. In the experimental test most of the pulling force which is to be transferred to the concrete occurs within the first 50mm of bar embedment. The analytical

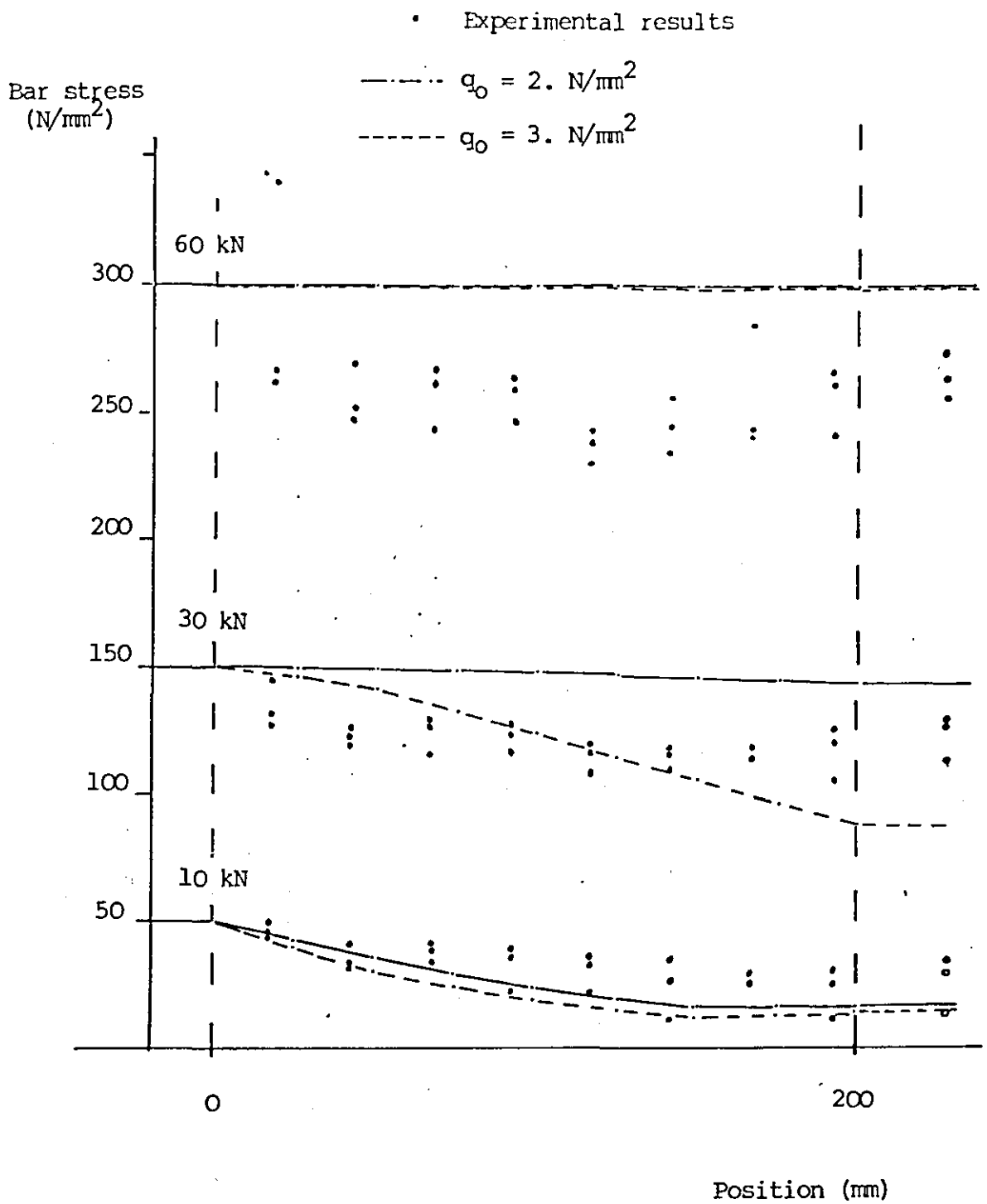


FIGURE 7.16 EXPERIMENTAL AND ANALYTICAL LOAD DISTRIBUTIONS  
IN THE TRANSFER TEST

model produces a shape of bar stress distribution similiar to that predicted in the ordinary pullout test (i.e convex between the pulling load at one end and the load at the other end of the concrete block). There is therefore a very marked difference between the analytical and experimental bar stress distributions.

## 7.7 SUMMARY.

The bond model developed in Chapter 3 has been used to analyse bond tests with embedded plain round bars. The tests which have been analysed are the ordinary pullout test, the eccentric pullout of Mains (1951), the double ended pullout test and the transfer test. Experimental evidence in the pullout test of the bar stress distributions and the effect of lateral stress across the concrete cube on the pullout load is available from the author's own tests and the tests of Standish (1982) respectively. In the ordinary pullout test and the Mains eccentric pullout test the analytical bar stress distributions compare favourably with the observed distributions. The bond model very accurately predicts the increased pullout load in the ordinary pullout test due to lateral stress across the concrete cube. Further the model accurately predicts the changing nature of the bar stress distribution with increased pulling load.

For the double ended pullout test the comparison between the model and the experimental results is less conclusive due mainly to the large scatter in the experimental bar stress distribution at loads close to pullout.

In the transfer test there is a very marked difference between the analytical and experimental bar stress distributions. This test requires further study to confirm the experimentally measured distributions and may require further study and modification of the bond model.

## CHAPTER 8    NUMERICAL EXAMPLES OF THE BOND MODEL APPLIED TO CONCRETE REINFORCED WITH DEFORMED BARS

### 8.1    INTRODUCTION.

This chapter is concerned with the application of the bond model to reinforced concrete structures with embedded deformed bars. The theoretical problems associated with applying the bond model developed in Chapter 3 to deformed bars are discussed.

By assuming a frictional bond mechanism for deformed bars and using the available experimental evidence to estimate the parameters involved, the current bond model has been applied to structures with embedded deformed bars. The model has been used to analyse the ordinary pullout test, the Mains (1951) pullout test and a beam-column intersection.

For the beam-column intersection, models have been tested by the author and the experimental details are reported in Chapter 6 but for the ease of comparison the experimental results are reported here before the analytical results.

### 8.2    APPLICATION OF THE BOND MODEL TO DEFORMED BARS

Since deformed bars rely mainly on the mechanical interlock of the ribs bearing against the surrounding concrete, it would at first appear that the frictional bond model developed for plain bars is unlikely to adequately model the bonding of deformed bars to concrete. For deformed bars the bond failure values in pullout tests depend on the bursting of the concrete cover or the concrete shear strength. The local bond is highly dependent on the local cracking in the vicinity of the ribs and the separation of the concrete from the bar as the concrete slides up the rib face and moves away from the bar, as shown by Goto

(1971), Figure 2.6 (Chapter 2). The local ultimate bond stress is therefore unlikely to be governed by a frictional bond mechanism as with plain bars.

Despite this, the observed bond stress-slip behaviour of deformed bars in bond tests where there is adequate confinement of the reinforced concrete is similiar in certain aspects to the behaviour of plain bars. The local bond stress-slip relationships obtained by Dorr (1978) and Nilson (1972), (Figures 3.6 and 3.7) show that this relationship is :

- (a) non-linear,
- (b) that some finite slip occurs for a very small bond stress and
- (c) that a peak bond stress is reached.

Therefore the bond stress-slip curves for deformed bars can be modelled in a similiar fashion to the plain bars using the currently adopted bond stress-slip relationship based on the Saenz (1964) curve. However the local maximum bond stress is governed by different criteria from the frictional mechanisms, the over-riding criterion being the initiation of cracking in the concrete at or very close to the bar surface.

The experimental results of Robins and Standish (1982), Figure 3.4, however indicate that for the tests that they performed that there is an increase in the ultimate pullout load with increasing lateral stress, up to  $15 \text{ N/mm}^2$ . The author suggests that this observed behaviour has more to do with a cracking criterion rather than a frictional bond mechanism. This cracking criterion is probably dependent on the dimensions of the concrete cube and in particular the amount of concrete cover. For higher lateral stresses (greater than  $15 \text{ N/mm}^2$ ) the bars failed by the concrete shearing and there was no apparent increase in pullout load with lateral pressure.

In the opinion of the author the best approximation that could currently be made is to model the bond behaviour of deformed bars by assuming a frictional bond mechanism. It is then important to observe whether failure in the pullout test occurs by bursting of the concrete rather than the local ultimate bond stress being reached all along the bar with the consequent failure by gross local bond slip.

### 8.3 BOND AND CONCRETE PARAMETERS TO BE USED.

#### 8.3.1 Experimental evidence for the bond parameters.

The experimental data of Robins and Standish (1982), Figure 3.4, can be used to approximate the bond behaviour of deformed bars in the bond model by assuming a frictional bond mechanism for both the splitting and shearing types of failure. The behaviour of deformed bars that Robins and Standish observed in their pullout tests was probably dependent on the type of bond test and in particular on the amount of concrete cover. Dorr (1978) has observed an increase in maximum bond stress with increasing confining pressure up to  $20 \text{ N/mm}^2$  and Untrauer and Henry (1965) have observed the same effect for uniaxial lateral pressures up to  $16 \text{ N/mm}^2$ , Figure 3.5. The evidence suggests that the local ultimate bond is increased by lateral pressure but there are other criteria such as the splitting of the concrete cover and the concrete shear strength which will directly affect the local bond failure value .

#### 8.3.2 Bond parameters - estimates and assumed values

The following values for the bond parameters have been chosen and represent the two distinct regions obtain in the pullout tests of Robins and Standish (1982):

Set (A) represents the concrete shear failure region (Figure 3.4).

Set (B) represents the splitting region (Figure 3.4)

The assumed sets of bond parameters (A) and (B) are tabulated in Table 8.1.

(i)  $R_0$  - Initial bond stress-slip modulus.

In comparison to plain bars this value should be much higher to reflect the greater enhanced bonding of deformed bars. Initial estimates for the initial bond stress-slip modulus are about  $1500 \text{ N/mm}^3$  from Dorr's (1978) results and  $500 \text{ N/mm}^3$  from Nilson's (1972) results.

For both sets of parameters (A) and (B) the initial bond stress-slip value is assumed to be  $1000 \text{ N/mm}^3$ .

(ii)  $q_0$  and  $\mu$  parameters.

The experimental results of Robins and Standish (1982) indicate two distinct modes of failure and two possible sets of values for both of these parameters. Initial estimates of the values for  $q_0$  and  $\mu$  which are to be used in the bond model are for the splitting region  $q_0$  equals  $9.5 \text{ N/mm}^2$  and  $\mu$  equals 1.05 and for the shearing region  $q_0$  equals  $11.4 \text{ N/mm}^2$  and  $\mu$  equals zero.

(iii)  $\Delta u$  parameter

Local bond stress-slip relationships for deformed bars have been obtained by Nilson (1972) and Dorr (1978). The results of Dorr, Figure 3.5, suggests that the tolerance slip at which the maximum bond stress occurs increases with confining pressure. The results of Dorr (1978), Nilson (1972) and Untrauer and Henry (1965) indicate that the tolerance slip lies within the range 0.01 to 0.1 mm. A value of 0.1 mm has been assumed for both sets of parameters (A) and (B).

(iv)  $\beta$  parameter

For slips in excess of the tolerance slip the maximum bond stress is likely to be maintained or slightly reduced from the ultimate bond

TABLE 8.1 BOND PARAMETERS FOR DEFORMED BARS USED IN ANALYSES

| PARAMETER                                                                              | SYMBOL     | VALUE                             |                                   |
|----------------------------------------------------------------------------------------|------------|-----------------------------------|-----------------------------------|
|                                                                                        |            | SET A<br>SHEARING<br>REGION       | SET B<br>SPLITTING<br>REGION      |
| INITIAL BOND STRESS-SLIP MODULUS                                                       | $R_0$      | 1000 N/mm <sup>3</sup>            | 1000 N/mm <sup>3</sup>            |
| SLOPE OF LOCAL ULTIMATE BOND STRESS-RADIAL PRESSURE LINE                               | $\mu$      | 0.                                | 1.05                              |
| TOLERANCE SLIP                                                                         | $\Delta_u$ | 0.1 mm                            | 0.1 mm                            |
| RATIO OF MAXIMUM BOND STRESS TO ULTIMATE BOND STRESS FOR SLIPS GREATER THAN $\Delta_u$ | $\beta$    | 1.0                               | 1.0                               |
| BOND MODULUS ORTHOGONAL TO THE BAR                                                     | $R_n$      | 10 <sup>5</sup> N/mm <sup>3</sup> | 10 <sup>5</sup> N/mm <sup>3</sup> |
| ULTIMATE LOCAL BOND STRESS DUE TO SHRINKAGE                                            | $q_0$      | 11.4 N/mm <sup>2</sup>            | 9.5 N/mm <sup>2</sup>             |
|                                                                                        |            | (Robins and Standish, 1982)       |                                   |

TABLE 8.2 LOAD INCREMENTS USED IN THE PULLOUT TESTS

| BOND PARAMETER SET AND LATERAL PRESSURE |                 |                 |                                |                 |                      |                 |  |
|-----------------------------------------|-----------------|-----------------|--------------------------------|-----------------|----------------------|-----------------|--|
| SET A $q_0 = 11.4$ $\mu = 0.$           |                 |                 | SET B $q_0 = 9.5$ $\mu = 1.05$ |                 |                      |                 |  |
| ZERO and 12 N/mm <sup>2</sup>           |                 |                 | ZERO                           |                 | 12 N/mm <sup>2</sup> |                 |  |
| INCR. NO.                               | INCR. SIZE (kN) | TOTAL LOAD (kN) | INCR. SIZE (kN)                | TOTAL LOAD (kN) | INCR. SIZE (kN)      | TOTAL LOAD (kN) |  |
| 1                                       | 10              | 10              | 5                              | 5               | 10                   | 10              |  |
| 2                                       | 10              | 20              | 2.5                            | 7.5             | 10                   | 20              |  |
| 3                                       | 5               | 25              | 2.5                            | 10              | 5                    | 25              |  |
| 4                                       | 5               | 30              | 2.5                            | 12.5            | 5                    | 30              |  |
| 5                                       | 5               | 35              | 2.5                            | 15              | 5                    | 35              |  |
| 6                                       | 5               | 40              | 2.5                            | 17.5            | 5                    | 40              |  |
| 7                                       | 2               | 42              | 1.5                            | 19              | 2.5                  | 42.5            |  |
| 8                                       |                 |                 | 1                              | 20              |                      |                 |  |
| 9                                       |                 |                 | 1                              | 21              |                      |                 |  |
| 10                                      |                 |                 | 0.25                           | 21.25           |                      |                 |  |

stress value. An initial estimate for this parameter is unity.

(v)  $R_n$  bond modulus orthogonal to the bar

Similar to the plain bar the value assumed for this parameter is arbitrarily high to ensure compatibility of the steel and concrete in the direction orthogonal to the bar and the value assumed is  $10^5 \text{ N/mm}^3$ .

8.3.3 Concrete parameters

The concrete parameters as were used in the plain bar problems are assumed and are tabulated in Table 7.2.

8.4 ORDINARY PULLOUT TEST.

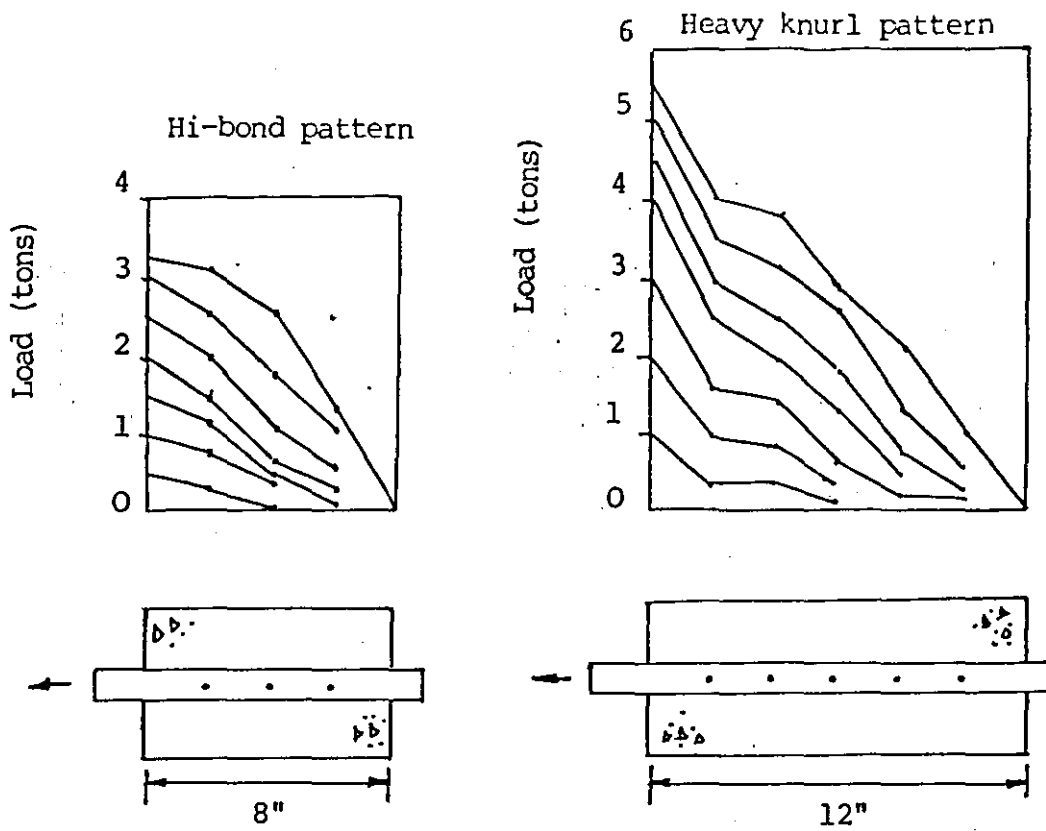
8.4.1 General.

The finite element <sup>analysis</sup> will be compared against the experimental results reported by Standish (1982). For this ordinary pullout test there was no additional shear reinforcement in the concrete cubes. The same finite element mesh as was used to model the plain bars in the ordinary pullout test has been used (Figure 7.1). The concrete cube is assumed to be  $100 \times 100 \times 100 \text{ mm}$  and the reinforcement has a nominal diameter of 12 mm.

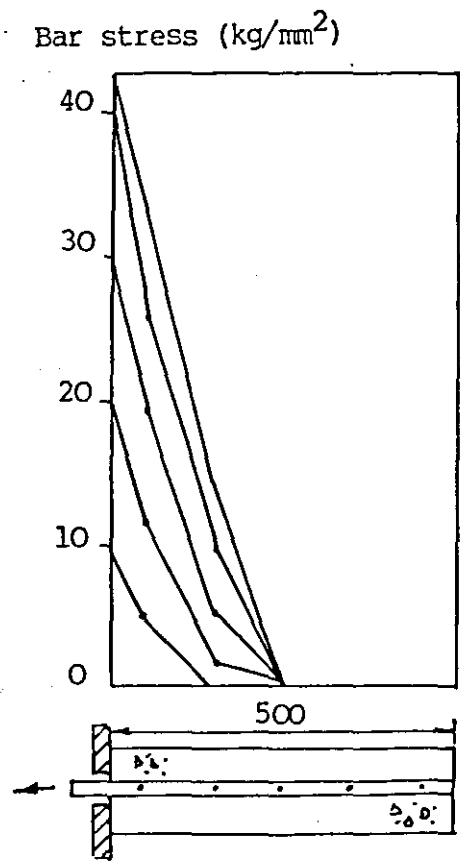
Two different sets of bond parameters were used to analyse this problem and are given in Table 8.1. In total, four analyses were performed using two different sets of bond parameters and two lateral pressures (zero and  $12 \text{ N/mm}^2$ ). The increments of load used in each analysis are given in Table 8.2. For comparison the results from other experimental investigations with zero lateral pressure are shown in Figure 8.1.

8.4.2 Analysis with bond parameter set (A).

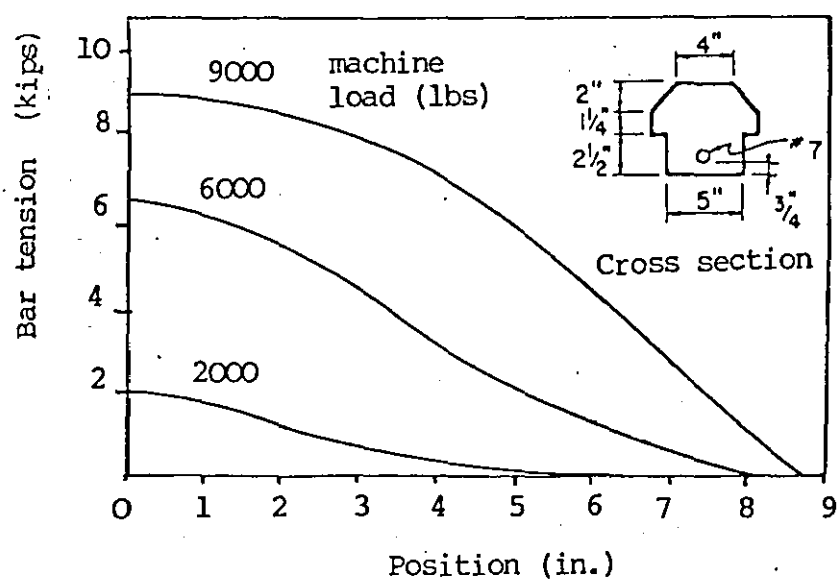
The load distributions obtained for the analyses with lateral



WILKINS (1951)



PARLAND (1957)



PERRY AND THOMPSON (1966)

FIGURE 8.1 LOAD DISTRIBUTIONS IN THE ORDINARY PULLOUT TEST FOR DEFORMED BARS OBTAINED BY OTHER INVESTIGATORS

pressures of zero and  $12 \text{ N/mm}^2$  are shown in Figures 8.2 and 8.3 respectively. The free-end slips against pulling load for both cases of lateral pressure are shown in Figure 8.4.

For both lateral pressures the load distributions are very similar since the local ultimate bond stress in both instances is fixed for all pressure conditions at the interface as  $11.4 \text{ N/mm}^2$ . Any differences in the analyses occur because of the differing longitudinal expansion of the concrete due to the lateral compressive stress, however the failure load for both lateral pressures is the same. In both analyses, failure occurs when the bond stress at each point along the embedded reaches the local ultimate bond stress of  $11.4 \text{ N/mm}^2$  and occurs in each case at a pulling load of 42 to 43 kN. Since the bond stress at failure all along the embedded bar was a fixed value the load distribution at failure is a linear reduction from the pulling load value to zero at the free end. For both lateral pressures the load distributions at low loads are steeply 'exponential' in shape but with increased pulling load gradually change and straighten out until the straight line at failure is obtained, (Figure 8.2 and 8.3).

The analytical free-end slips against pulling load for both lateral pressures are almost identical as shown in Figure 8.4. In comparison with the results obtained by Standish (1982) the slips match reasonably well up to about 25 kN and then under estimate the slip as failure is approached.

The analytical longitudinal and lateral concrete stresses for both lateral pressures are shown in Figures 8.5 and 8.6 and the magnitude of the stresses is well below the concrete tensile failure stress.

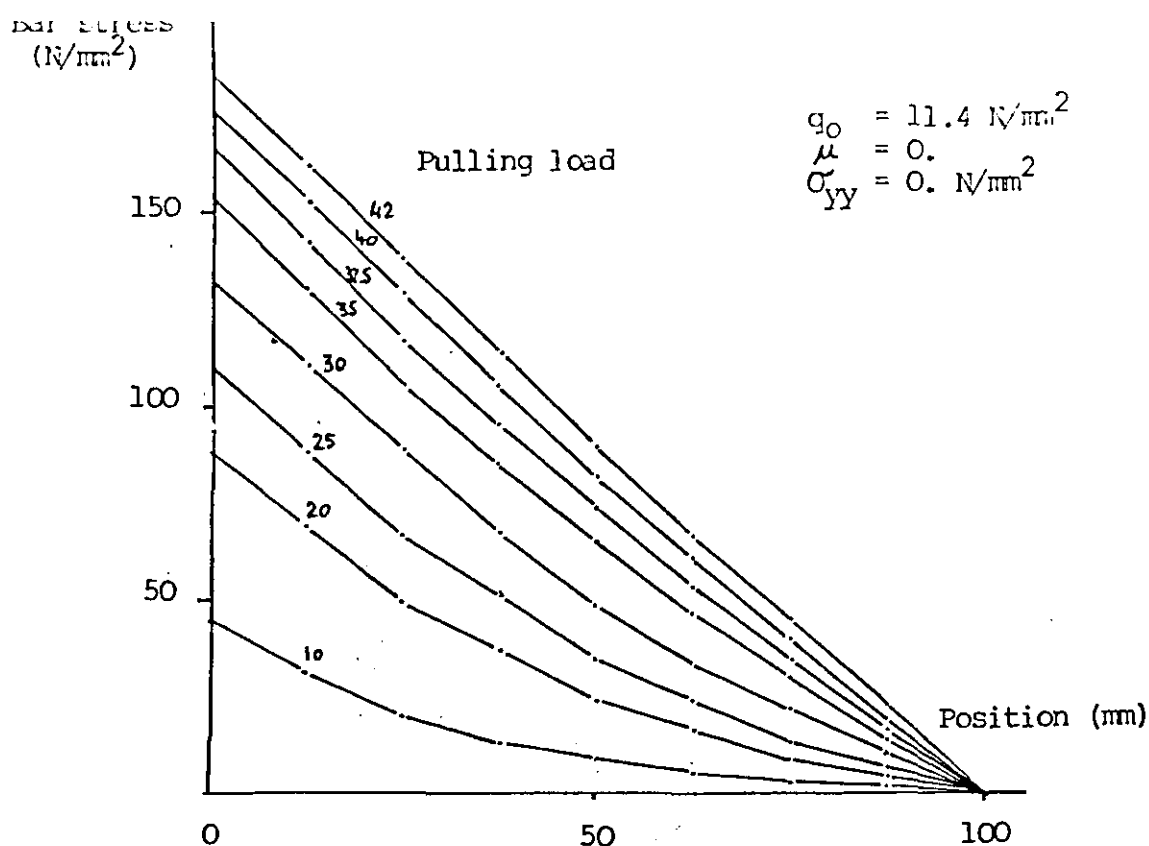


FIGURE 8.2 ANALYTICAL BAR STRESS DISTRIBUTIONS IN THE PULLOUT TEST  
NO LATERAL PRESSURE, BOND PARAMETERS (A)

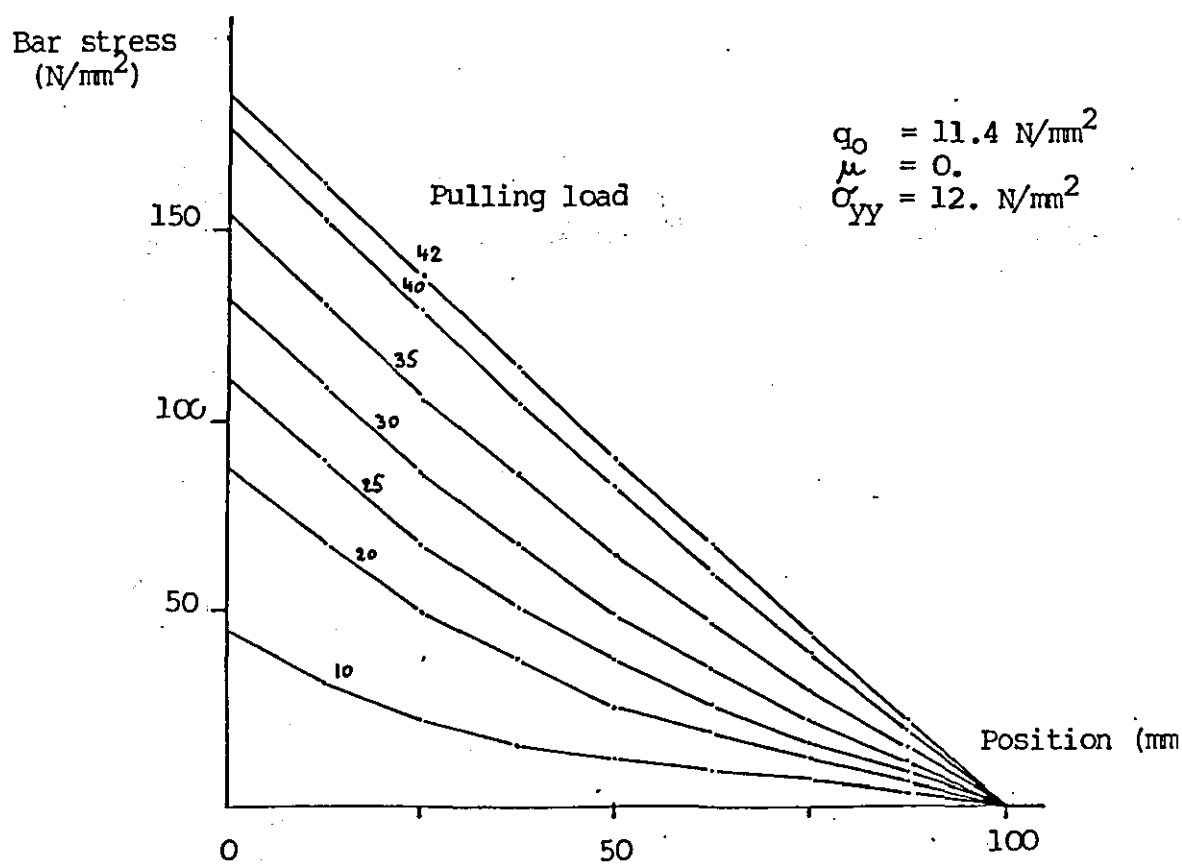


FIGURE 8.3 ANALYTICAL BAR STRESS DISTRIBUTIONS IN THE PULLOUT TEST  
WITH A LATERAL PRESSURE OF  $12 \text{ N/mm}^2$ , BOND PARAMETERS (A)

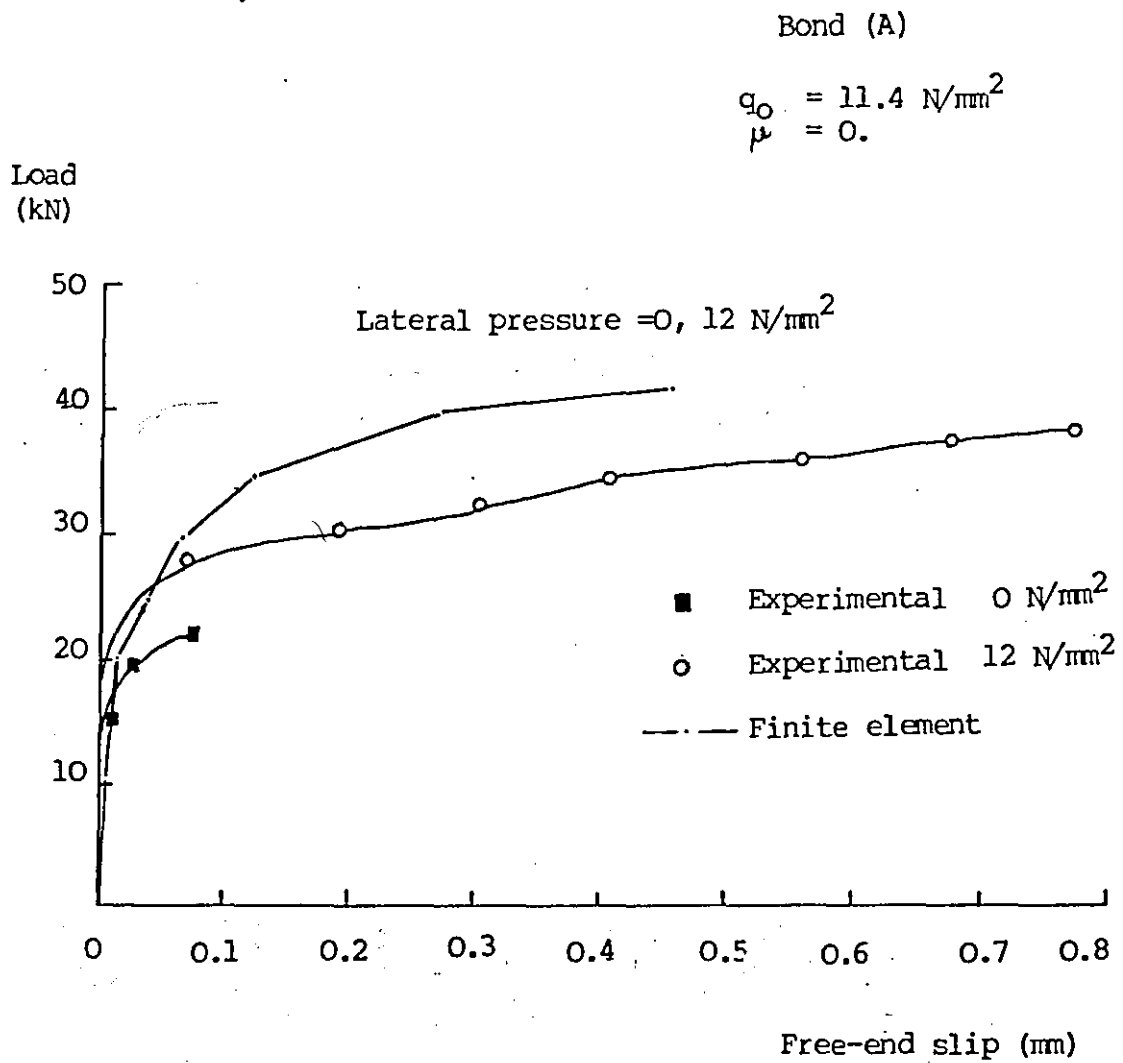


FIGURE 8.4 EXPERIMENTAL AND ANALYTICAL LOAD FREE-END SLIPS  
 FOR DEFORMED BARS IN THE PULLOUT TEST

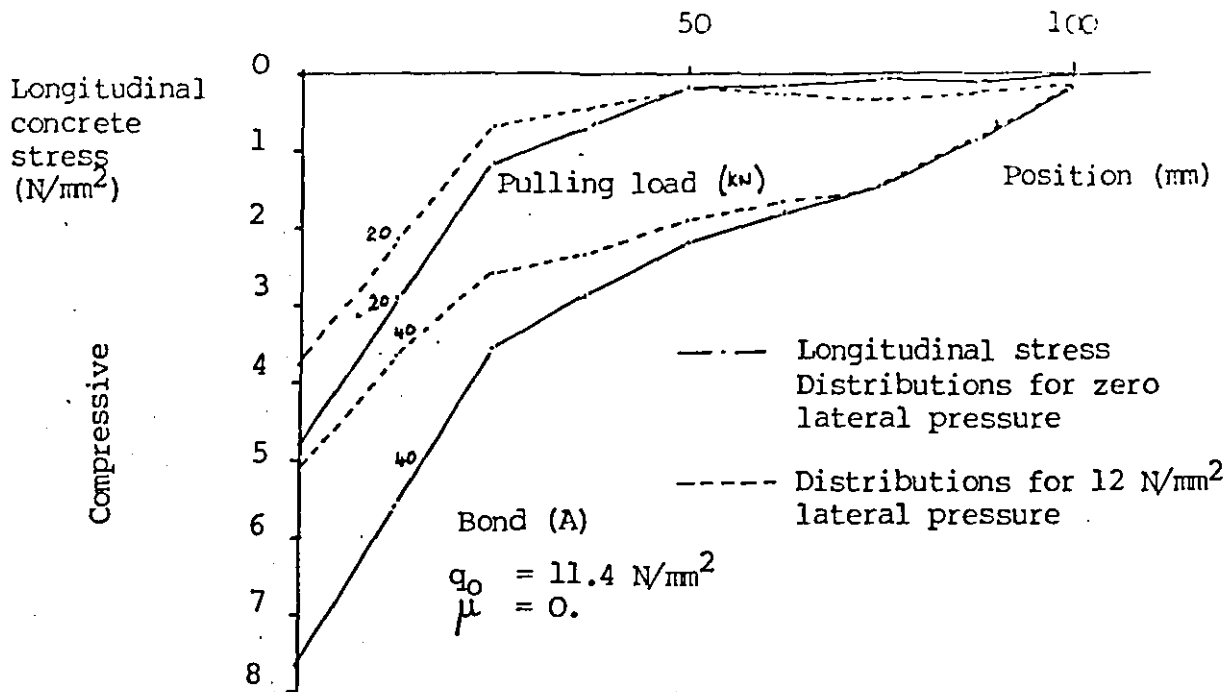


FIGURE 8.5 ANALYTICAL LONGITUDINAL CONCRETE STRESS AT BAR LEVEL IN THE ORDINARY PULLOUT TEST

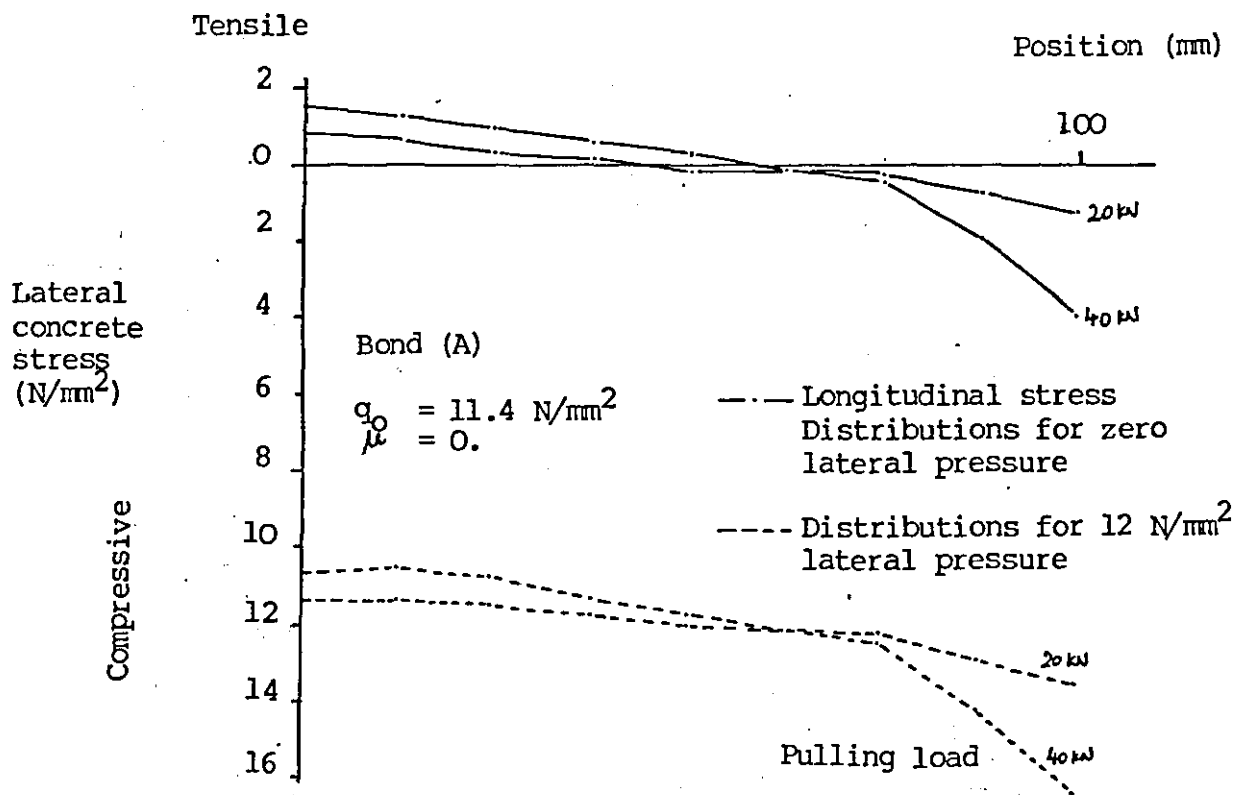


FIGURE 8.6 ANALYTICAL LATERAL CONCRETE STRESS AT BAR LEVEL IN THE ORDINARY PULLOUT TEST

#### 8.4.3 Analysis with bond parameter set (B).

The load distributions obtained for the analyses with lateral pressures of zero and  $12 \text{ N/mm}^2$  are shown in Figures 8.7 and 8.8 respectively. The failure loads in each case were 20.5 kN and 42.5 kN respectively. For the zero lateral pressure case the load distribution curves at low loads are steep and 'exponential' and with increasing load gradually change to an approximate straight line distribution at about 17.5 kN (85 percent failure load) and thereafter up to the failure load become increasingly convex. This shape in the load distributions is slightly more marked than in the corresponding analyses of the plain bar embedded in the pullout test.

For the lateral pressure of  $12 \text{ N/mm}^2$  the load distribution exhibits a reversed S-shape at low loading which changes to a concave shape on increased loading. The initial shape of the distribution is caused by the lateral loading which has a direct and marked effect on the local ultimate bond stress and bond stress-slip relationship with bond parameter set (B).

The predicted free-end slips against pulling load are illustrated for both lateral pressure cases in Figure 8.9. For zero lateral pressure the analysis through all load stages over-estimates the slip. For a lateral pressure of  $12 \text{ N/mm}^2$  the predicted slip is reasonably accurate up to about 25 kN (60 percent failure load) and thereafter under estimates the amount of slip.

The analytical longitudinal and lateral concrete stresses for both lateral pressure cases are shown in Figures 8.10 and 8.11 and the magnitude of concrete stresses are all well below the concrete tensile failure strength.

Bar stress  
( $\text{N/mm}^2$ )

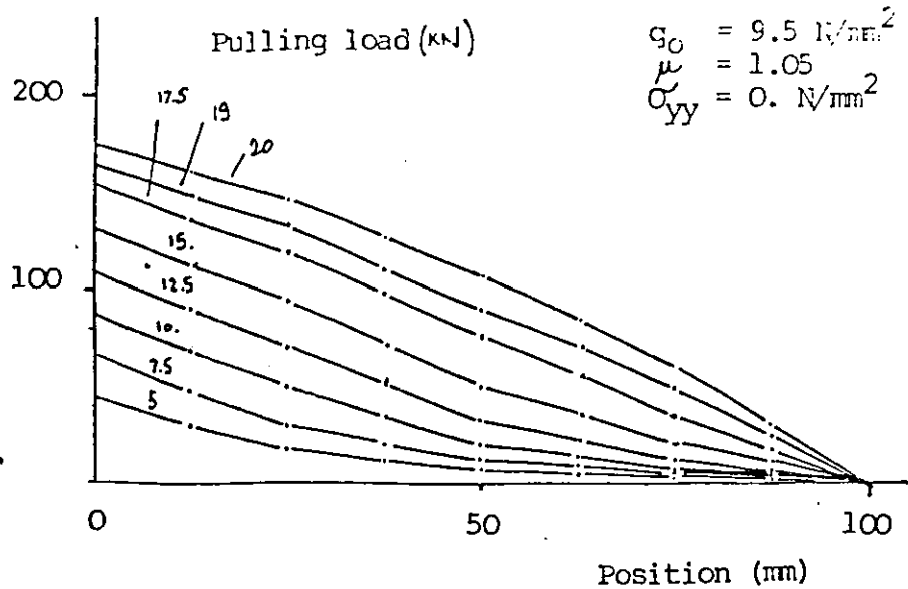


FIGURE 8.7 ANALYTICAL BAR STRESS DISTRIBUTIONS IN THE PULLOUT TEST  
NO LATERAL PRESSURE, BOND PARAMETERS (B)

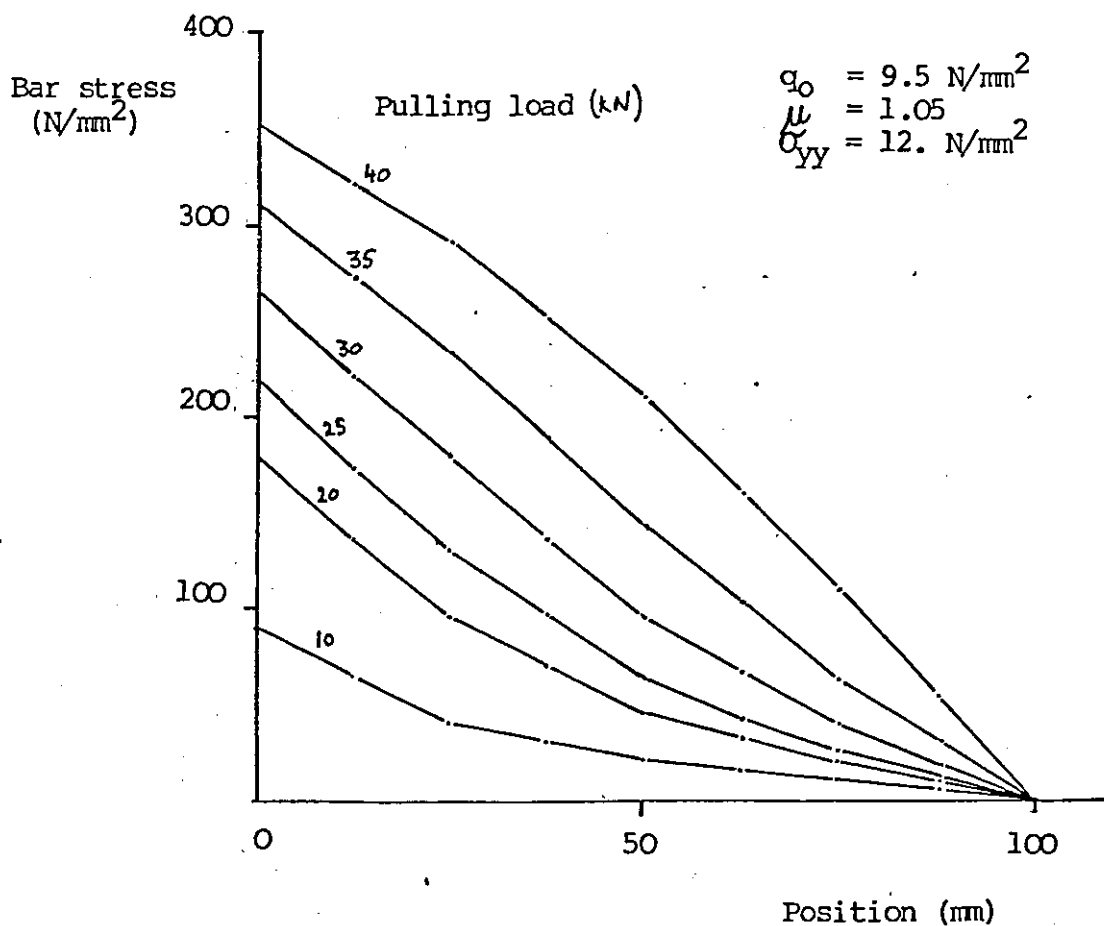


FIGURE 8.8 ANALYTICAL BAR STRESS DISTRIBUTIONS IN THE PULLOUT TEST  
WITH A LATERAL PRESSURE OF  $12 \text{ N/mm}^2$ , BOND PARAMETERS (B)

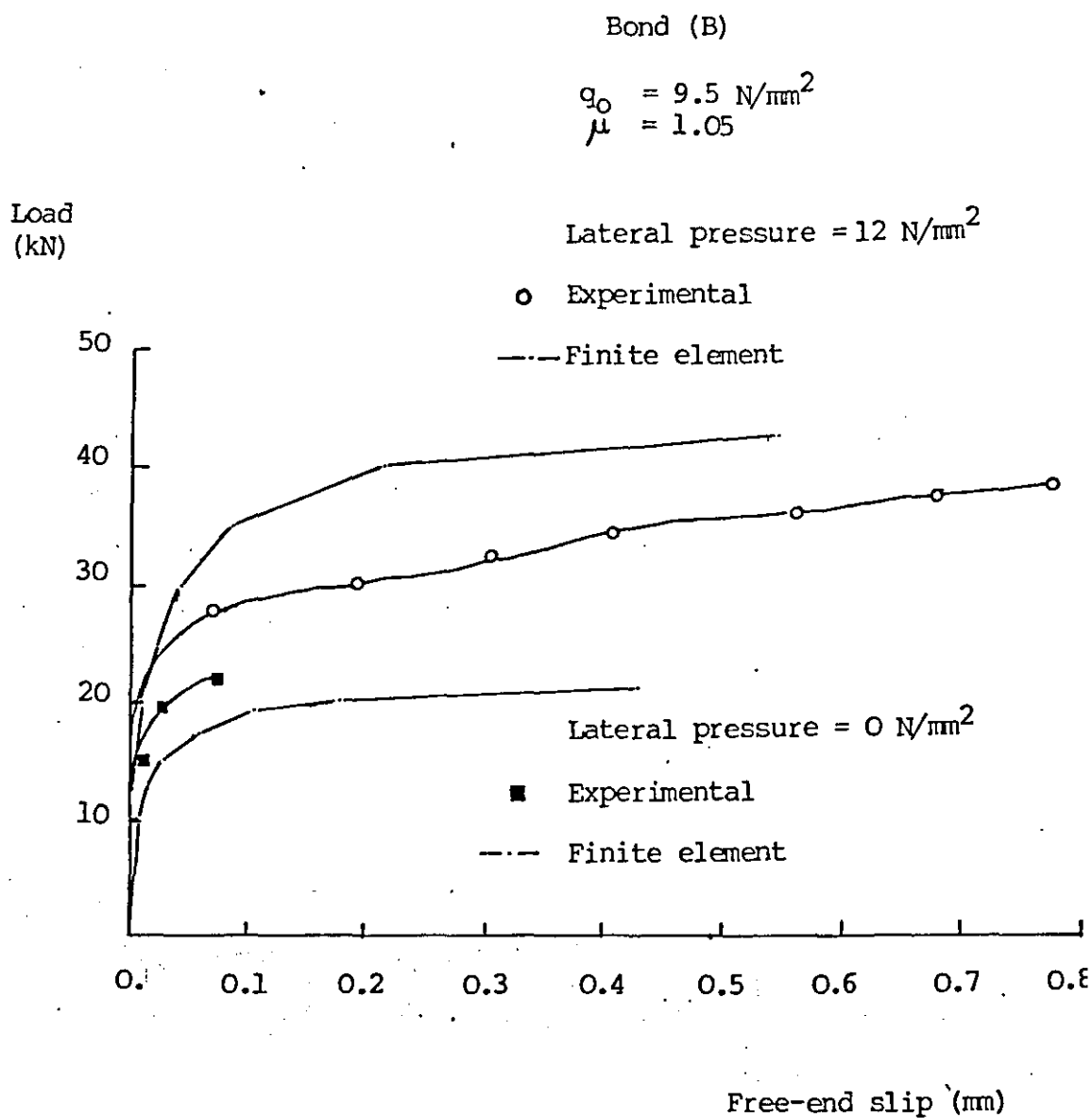


FIGURE 8.9 EXPERIMENTAL AND ANALYTICAL LOAD FREE-END SLIPS  
 FOR DEFORMED BARS IN THE PULLOUT TEST

Longitudinal  
concrete  
stress  
(N/mm<sup>2</sup>)

Compressive

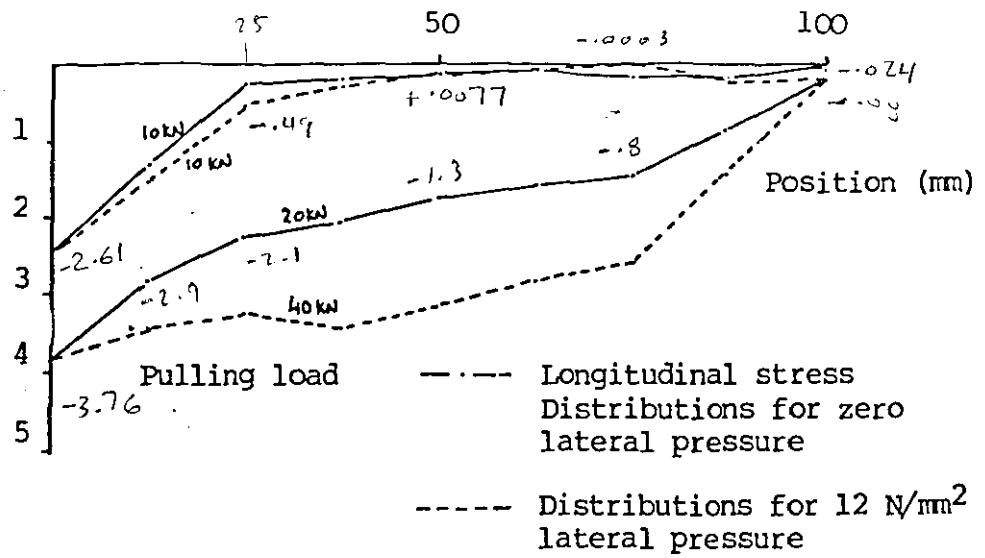


FIGURE 8.10 ANALYTICAL LONGITUDINAL CONCRETE STRESS AT BAR LEVEL IN THE ORDINARY PULLOUT TEST FOR LATERAL STRESSES OF ZERO AND 12 N/mm<sup>2</sup>. BOND PARAMETERS (B)

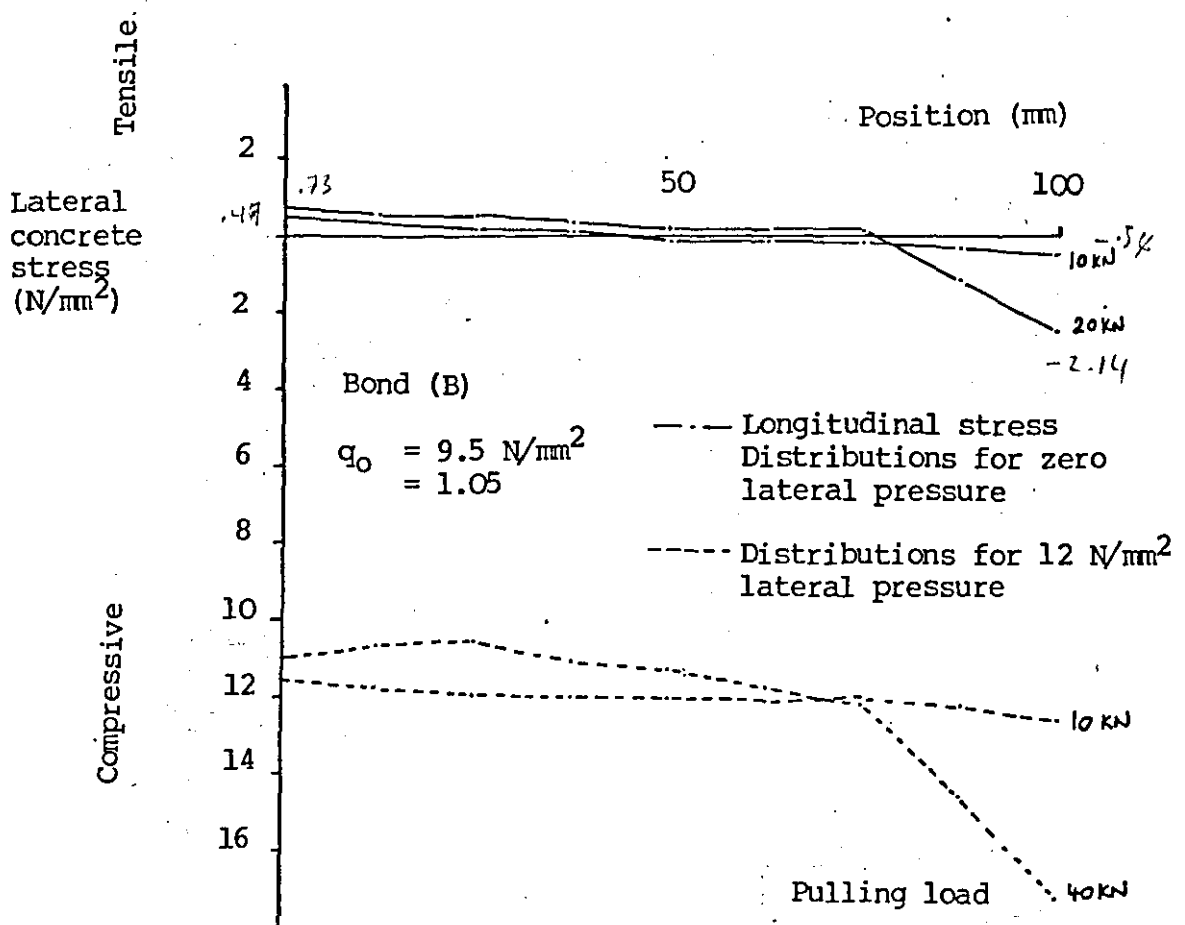


FIGURE 8.11 ANALYTICAL LATERAL CONCRETE STRESS AT BAR LEVEL IN THE ORDINARY PULLOUT TEST

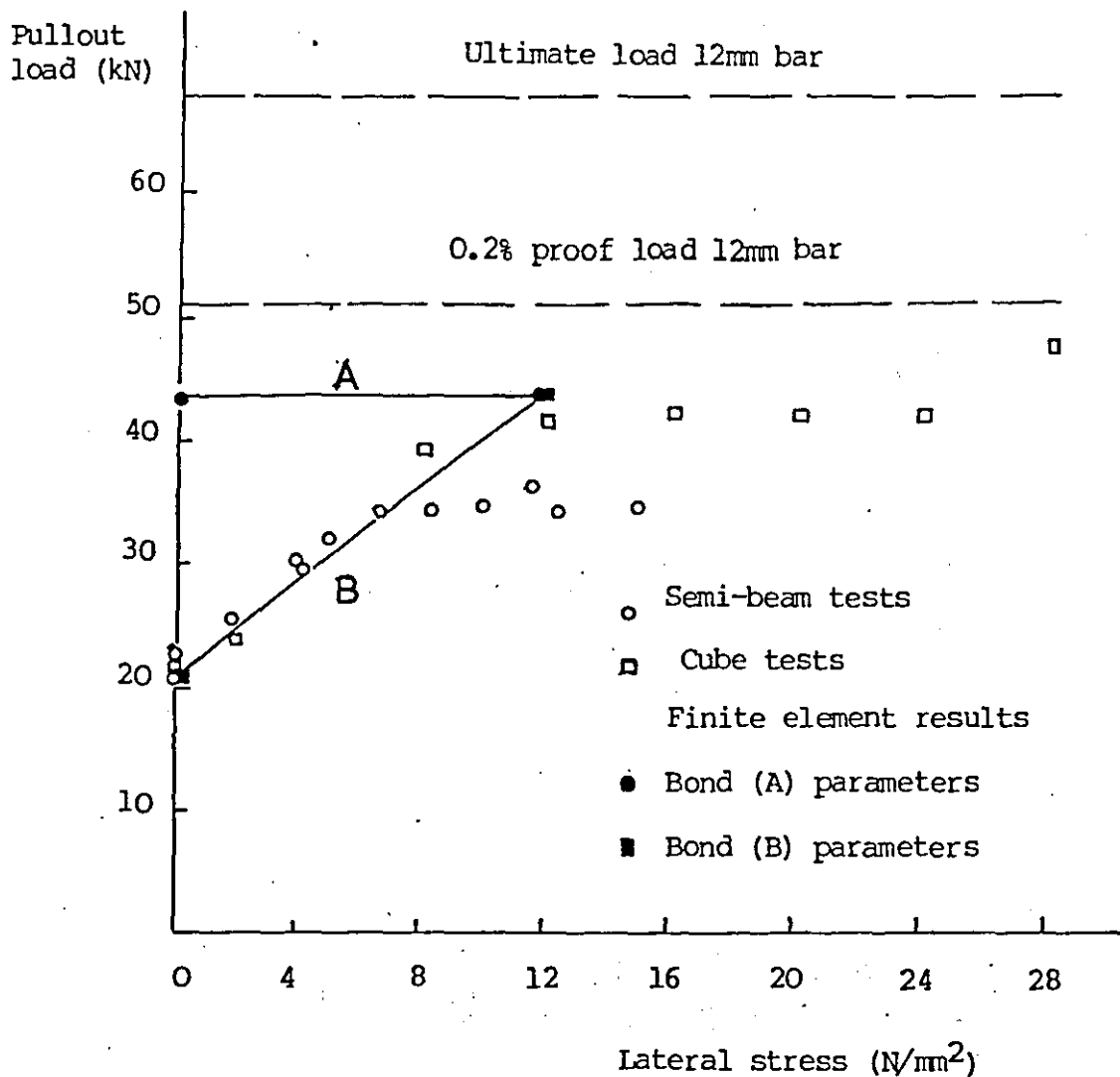


FIGURE 8.12 EFFECT OF LATERAL STRESS ON PULLOUT LOAD.

EXPERIMENTAL AND ANALYTICAL RESULTS FOR DEFORMED BARS

The predicted failure loads against lateral stress for both sets of bond parameters are compared with the test results of Robins and Standish (1982) in Figure 8.12. Bond parameter set (A) gives the same pullout for both lateral stresses whereas with bond parameter set (B) the pullout loads agree reasonably well with the observed behaviour.

#### 8.4.4 Discussion of both bond parameter sets.

Failure for all analyses occurred when all the points along the embedded bar had slipped by more than 0.1 mm, i.e. the failure was by a frictional bond slipping mechanism whereas in the experimental tests, failure occurred at zero lateral pressure by the concrete splitting and at pressures greater than  $12 \text{ N/mm}^2$  by the concrete shearing. For the zero lateral pressure case using either of the bond parameter sets there is no likely indication of splitting cracks occurring in the concrete. The maximum tensile radial concrete stress is about 54 percent of the concrete tensile failure strength.

The author considers that it may be possible to adjust the bond parameters such that a better match is obtained to the free end slips particularly for the higher lateral pressure region where failure is by concrete shearing and is therefore likely to be governed by a frictional-sliding type of mechanism. The mode of failure for zero lateral pressure however is incorrect and using the current bond model the analysis is unable to predict the splitting type of failure. Further discussion of this important phenomenon and the development of the bond model for use with deformed bars is discussed in greater detail in Chapter 9.

#### 8.5 MAINS PULLOUT TEST (1951).

Mains measured the load distribution of deformed bars in his pullout tests using an internal gauging technique (Chapter 2). The

deformed bars were 7/8 in. nominal diameter and the concrete specimens were 21 in. x 12 in. x 8 in. with the bars embedded 2.5 in. from the bottom. Additional stirrup reinforcement was used. The tests have been modelled by the same mesh as was used to model the plain bar case (Figure 7.1). Two analyses were performed using the two sets of bond parameters (A) and (B) as given in Table 8.1. The load in both cases was applied in increments of 17.792 kN (4 kips). The load distributions for bond parameter set (A) are shown in Figure 8.13 and for bond parameter set (B) in Figure 8.14.

For the analyses with parameter set (A) the load distributions are too steep when compared with the observed behaviour. Mains found the 0.002 percent yield stress occurred at a bar load of 43 kips and the ultimate yield stress at 61 kips. The analysis was therefore stopped at 177.92 kN (40 kips) which corresponds to stress of 459 (N/mm<sup>2</sup>). No attempt in the analytical model was made to analyse the strain hardening behaviour of the steel which occurred in the actual test.

The (A) set of bond parameters gives no reduction in bond stress due to radial pressure effects and consequently the analysis does not predict the limiting bond stress at the pulling end corresponding to about 750 psi (5.17 N/mm<sup>2</sup>).

For the analysis with parameter set (B) at low pulling loads the load distributions are too steep when compared with the behaviour. With increased loading the S-shape of curve is developed at about 20 kips. The bond parameters tend to produce a more exaggerated S-shape of load distribution than the actual behaviour. The bond stress at the pulling end is reduced too much when compared with that as observed. In the finite element analysis bond failure by excess slipping at all points along the bar occurs at a pulling load of 24 kips.

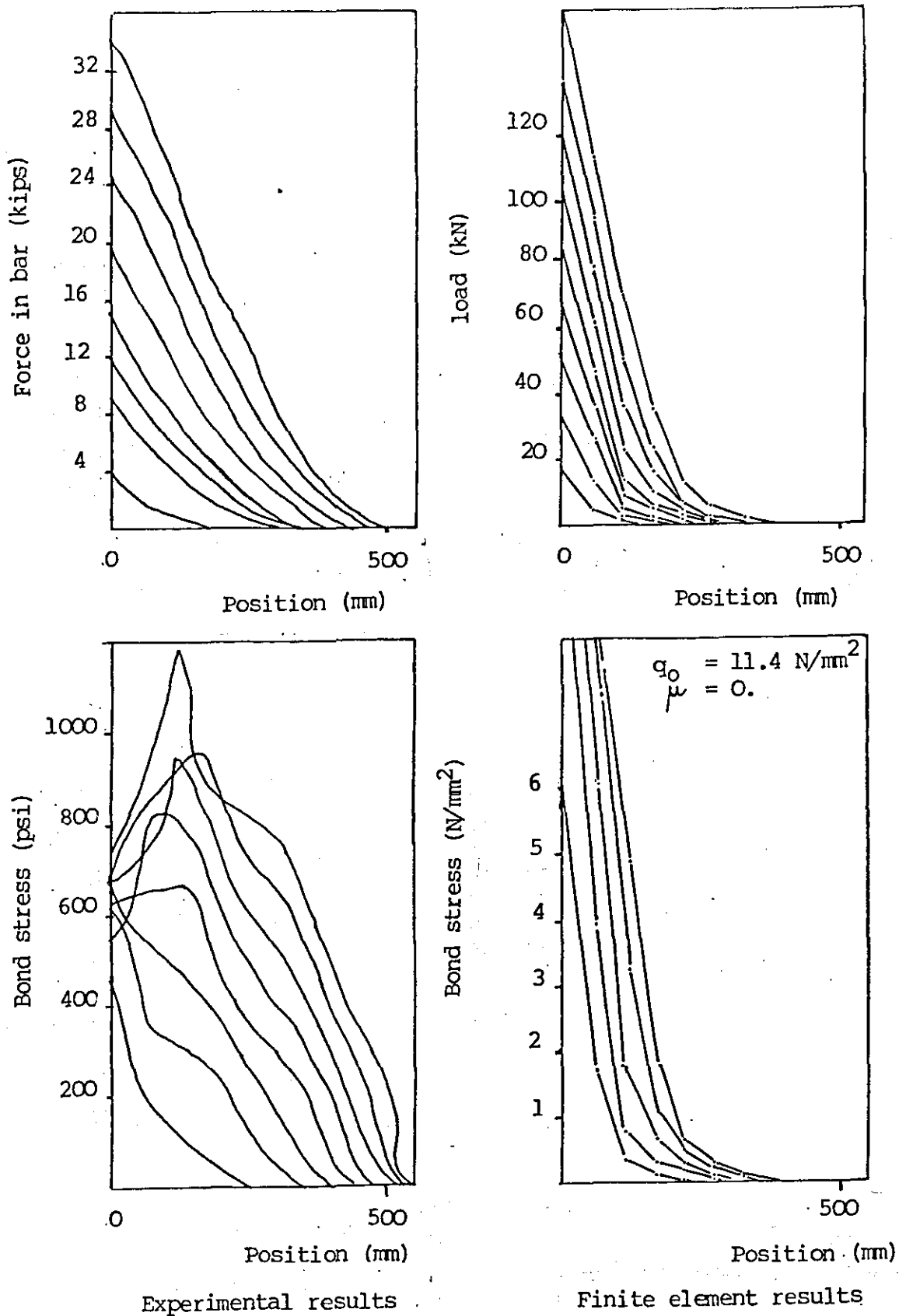


FIGURE 8.13 EXPERIMENTAL AND ANALYTICAL COMPARISON OF BAR LOAD AND BOND STRESS DISTRIBUTIONS IN THE MAINS ECCENTRIC PULLOUT TEST WITH BOND PARAMETERS (A)

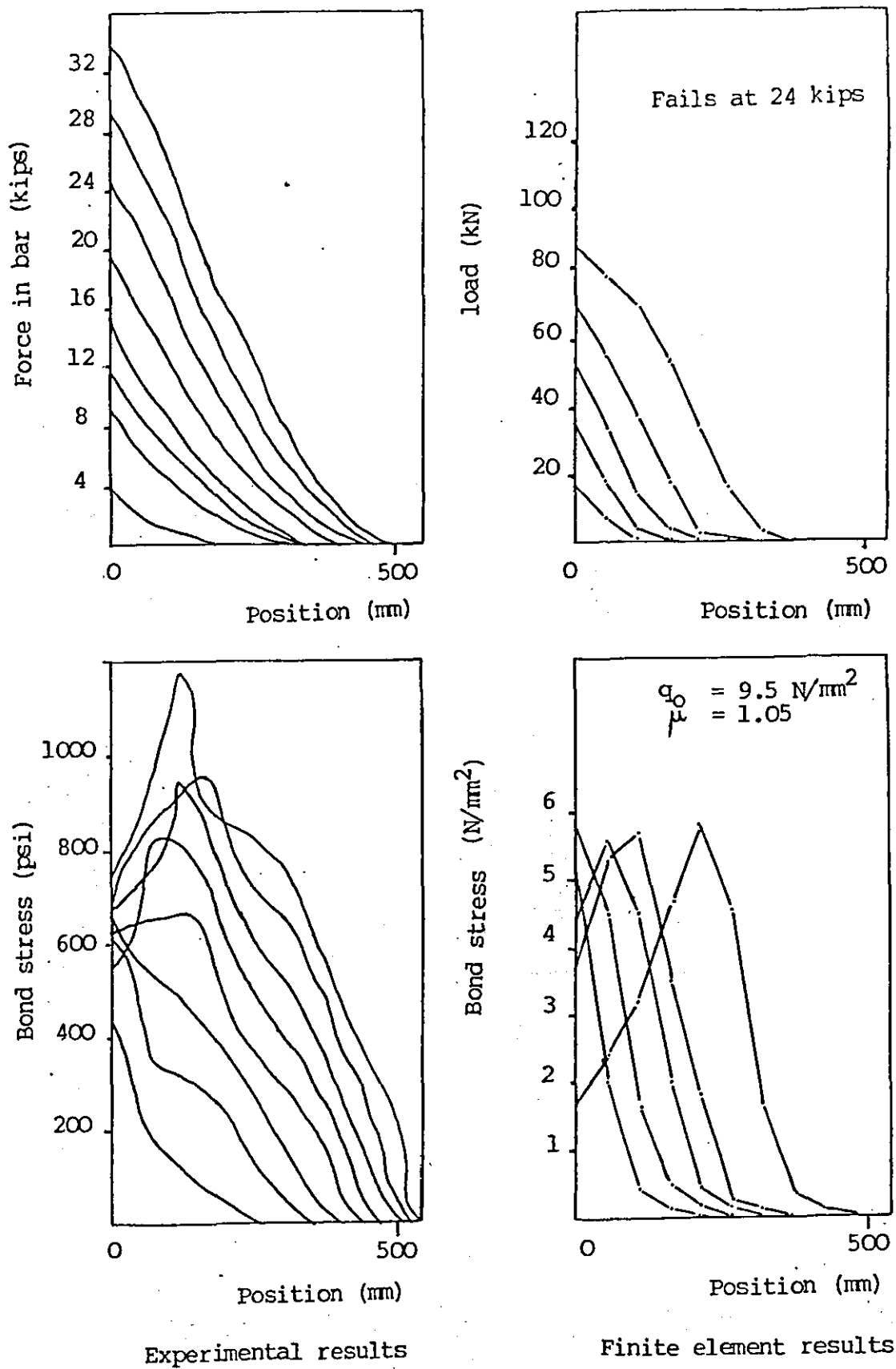


FIGURE 8.14 EXPERIMENTAL AND ANALYTICAL COMPARISON OF BAR LOAD AND BOND STRESS DISTRIBUTIONS IN THE MAINS ECCENTRIC PULLOUT TEST WITH BOND PARAMETERS (B)

For both analyses the load distributions are too steep and suggest that the initial bond stress-slip modulus is too high. For this test the concrete cover was sufficient to prevent failure by cracking and in this instance a cracking criterion was not the dominant feature. The author suggests that it may be possible to adjust the bond parameters to more accurately match the experimental load distributions by using a lower initial bond stress-slip modulus and reduced values of  $q_0$  and  $\mu$ .

## 8.6 BEAM-COLUMN INTERSECTION.

Load distributions along the main reinforcing bars have been obtained by the author and details of the beam-column models and test method are described in Chapter 6. For the ease of comparison with the analytical results, the experimental results are now described.

### 8.6.1 Experimental results.

The bar stress distributions within the column width obtained from the two tested models (I) and (II) are illustrated in Figures 8.15 and 8.16 respectively and the corresponding crack distributions at the failure loads are shown in Figures 8.17 and 8.18. In both cases the bar stress distribution within the column width were found to be U-shaped distributions, but not smooth as reported by Allwood (1980). The local variations in the bar stress distribution was probably due to the formation of major cracks within the column width. These effects on the bar stress distribution are similar to those observed by Mains (1951) of cracks on the bar stress distributions in beam specimens. Considerable variation between the results of each test of the bar stress distribution within the column in a region close to the column face was also probably due to the formation of large cracks near the corner of the top of the beam and column. There is also a considerable difference between the results

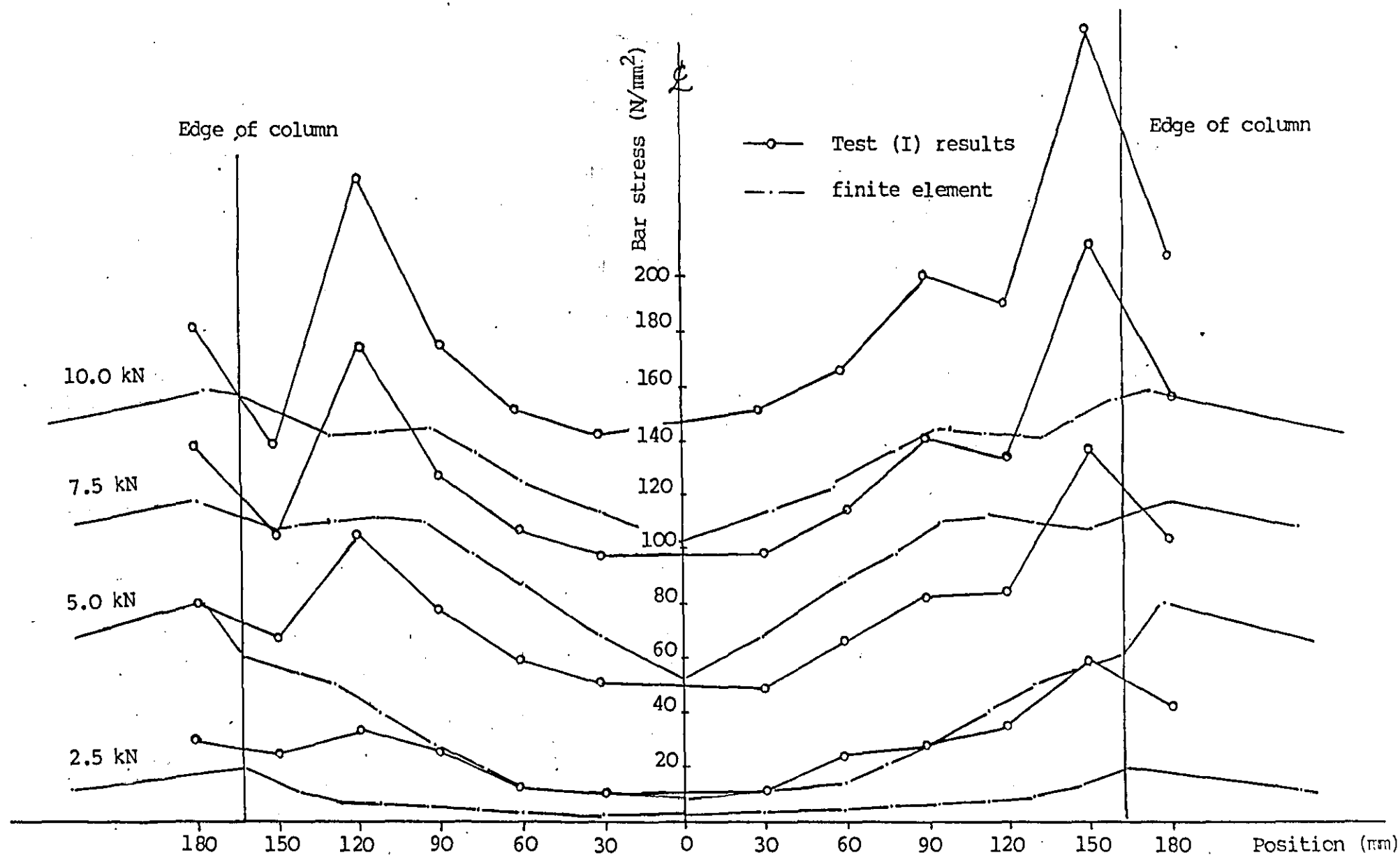


FIGURE 8.15 LOAD DISTRIBUTIONS IN THE BEAM-COLUMN INTERSECTION

EXPERIMENTAL TEST (I) AND ANALYTICAL RESULTS

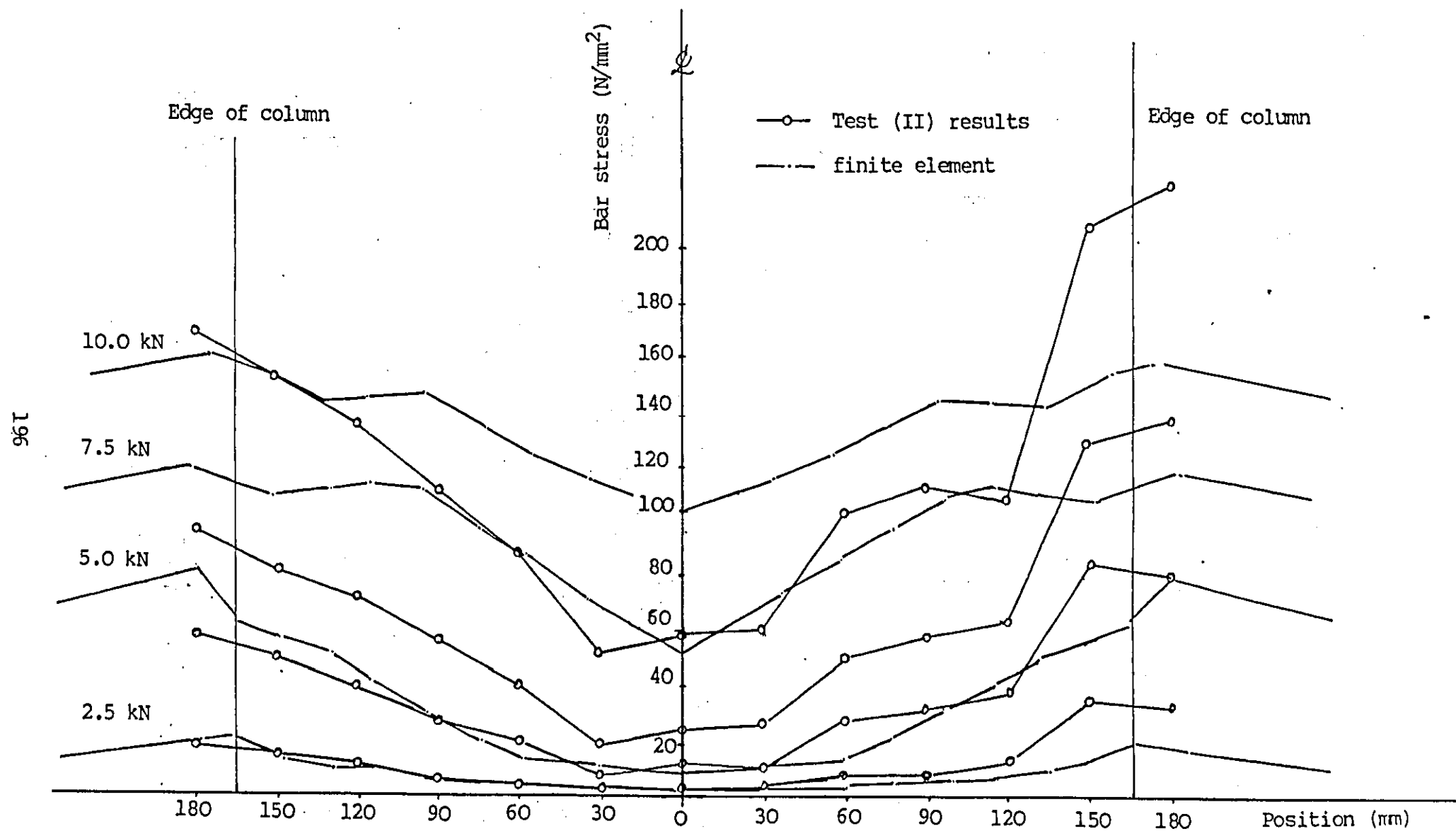


FIGURE 8.16 LOAD DISTRIBUTIONS IN THE BEAM-COLUMN INTERSECTION

EXPERIMENTAL TEST (II) AND ANALYTICAL RESULTS

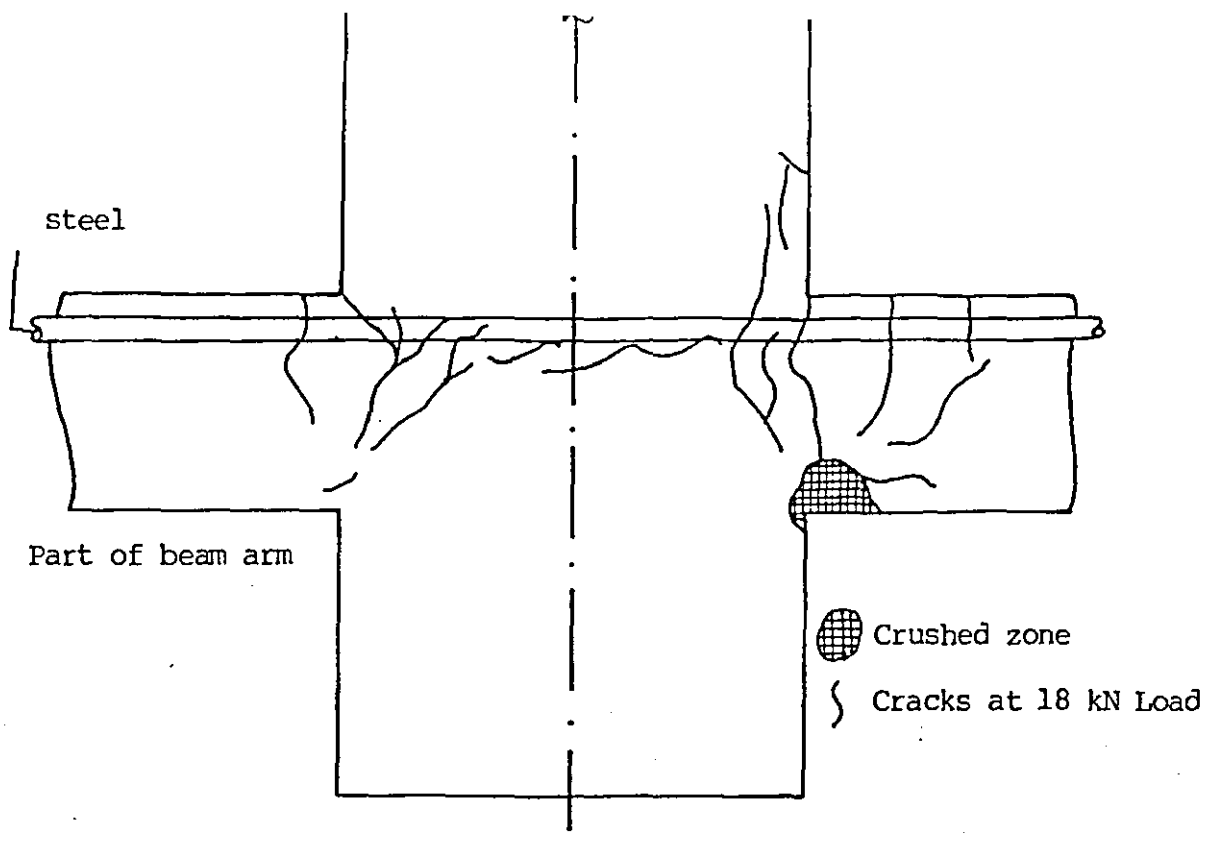


FIGURE 8.17 EXPERIMENTAL CRACK DISTRIBUTION IN THE BEAM-COLUMN TEST (I)

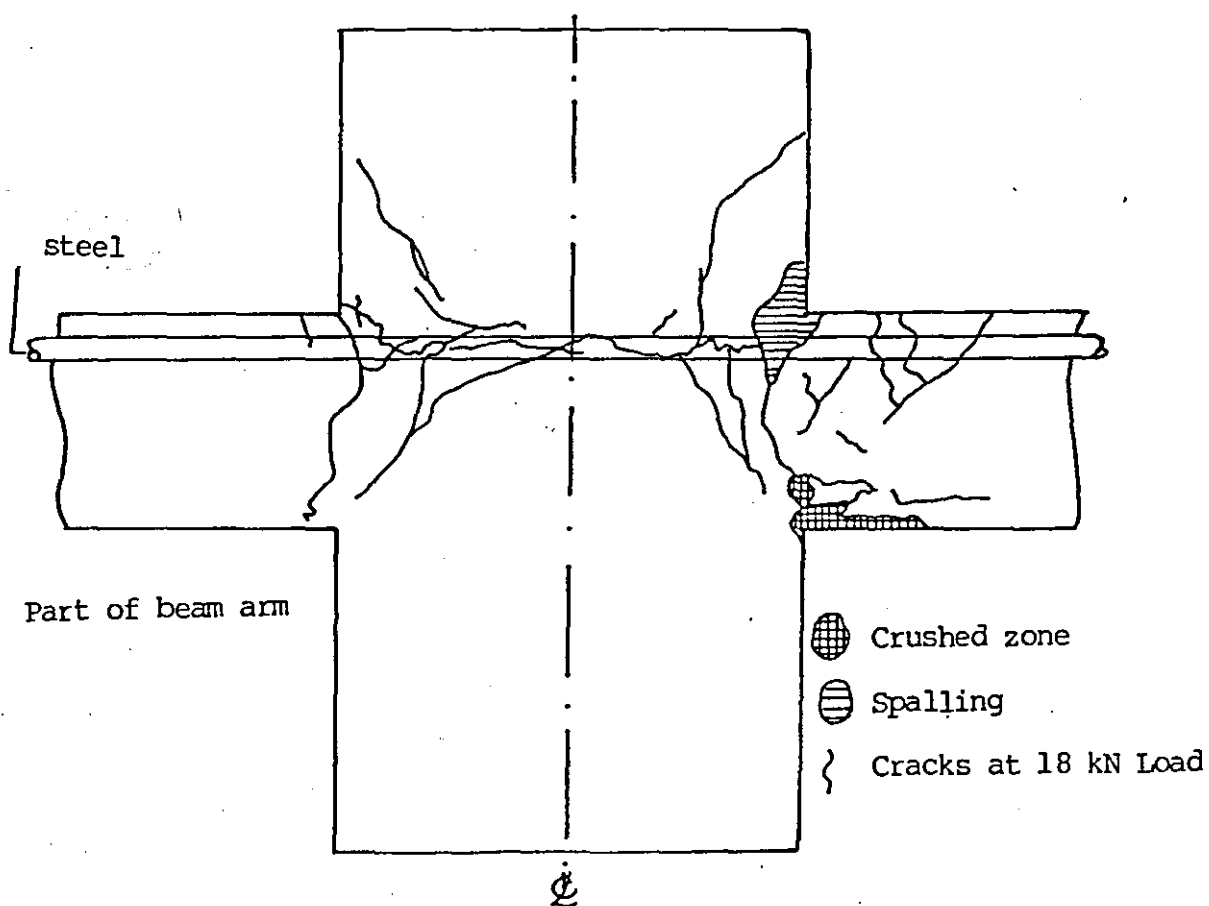


FIGURE 8.18 EXPERIMENTAL CRACK DISTRIBUTION IN THE BEAM-COLUMN TEST (II)

(I) and (II) in the magnitude of the bar stresses within the column width although the general shape of the distribution is similar. The results of test (I) are similar to the results of Allwood (1980).

#### 8.6.2 Analytical results.

Advantage of the symmetry has been taken and only one half of the beam-column has been modelled with the finite element mesh shown in Figure 8.19. There are 56 QUAD8SM elements representing the concrete, 9 BAR3 elements representing the reinforcement and 9 BOND6 elements representing bond between the steel and concrete. A single analysis up to a total beam arm load of 10 kN was performed with the load applied in increments of 2.5 kN. The analysis was performed using bond parameter set (B). During the analysis transverse cracking of the concrete took place; this is the first occasion during all of the analyses that this phenomenon has occurred and the amount of cracking produced in the analytical model was extensive. The total number of affected concrete Gauss points at the end of each load increment and the total number of crack release phases required for each load increment are given in Table 8.3. A considerable amount of computing effort was involved in analysing this particular structure. By comparison, the Mains pullout analysis using bond parameters (A) with 10 load increments and a total number of iterations of 52 took 32.5 minutes of c.p.u time, whereas the beam-column with 4 load increments and a total number of iterations of 766 took 1460<sup>3 min</sup> minutes of c.p.u. time. (All calculations were performed on a PRIME 750 machine, using the Genesys System)

The load distributions along the main reinforcing bar at beam arm loads of 2.5, 5.0, 7.5 and 10 kN are compared with the experimental results of tests (I) and (II) in Figures 8.15 and 8.16 respectively. The crack patterns at the end of each load increment at loads of 2.5, 5.0, 7.5 and 10 kN are shown in Figures 8.20, 8.21, 8.22 and 8.23 respectively.

FIGURE 8.19 FINITE ELEMENT MESH AND BOUNDARY CONDITIONS FOR  
THE BEAM-COLUMN INTERSECTION

TABLE 8.3 DETAILS OF THE ANALYSIS OF THE BEAM COLUMN

| LOAD<br>INCREMENT<br>NUMBER | SIZE<br>OF THE<br>INCREMENT<br><br>(kN) | TOTAL<br>LOAD<br><br>(kN) | TOTAL NUMBER<br>OF CRACKED<br>GAUSS POINTS | NUMBER OF CRACK<br>RELEASE SEQUENCES<br>REQUIRED | TOTAL<br>NUMBER OF<br>ITERATIONS |
|-----------------------------|-----------------------------------------|---------------------------|--------------------------------------------|--------------------------------------------------|----------------------------------|
| 1                           | 2.5                                     | 2.5                       | 9                                          | 7                                                | 51                               |
| 2                           | 2.5                                     | 5.0                       | 65                                         | 9                                                | 114                              |
| 3                           | 2.5                                     | 7.5                       | 144                                        | 13                                               | 358                              |
| 4                           | 2.5                                     | 10.0                      | 203                                        | 7*                                               | 243*                             |
| TOTAL NUMBER OF ITERATIONS  |                                         |                           |                                            |                                                  | <hr/> 766 <hr/>                  |

\* Still cracks to be released

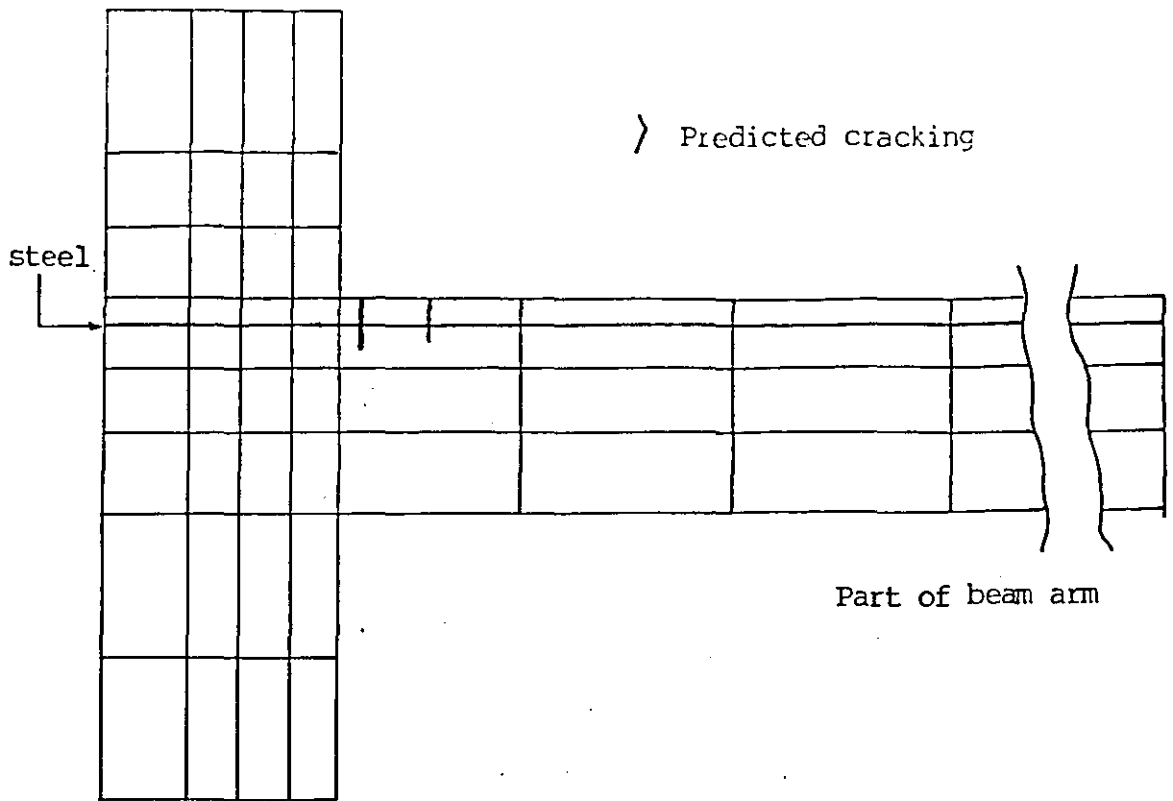


FIGURE 8.20 ANALYTICAL CRACK DISTRIBUTION IN  
THE BEAM-COLUMN AT 2.5 kN LOAD

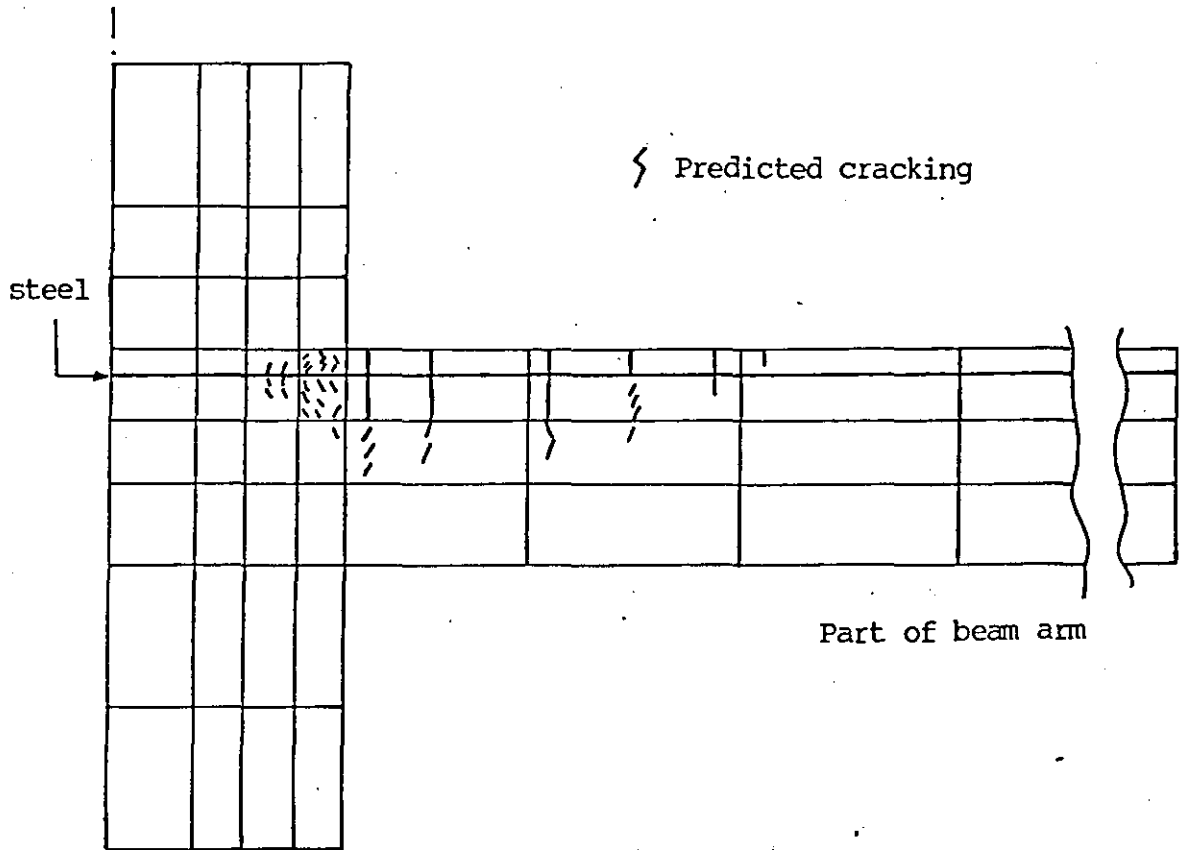


FIGURE 8.21 ANALYTICAL CRACK DISTRIBUTION IN  
THE BEAM-COLUMN AT 5.0 kN LOAD

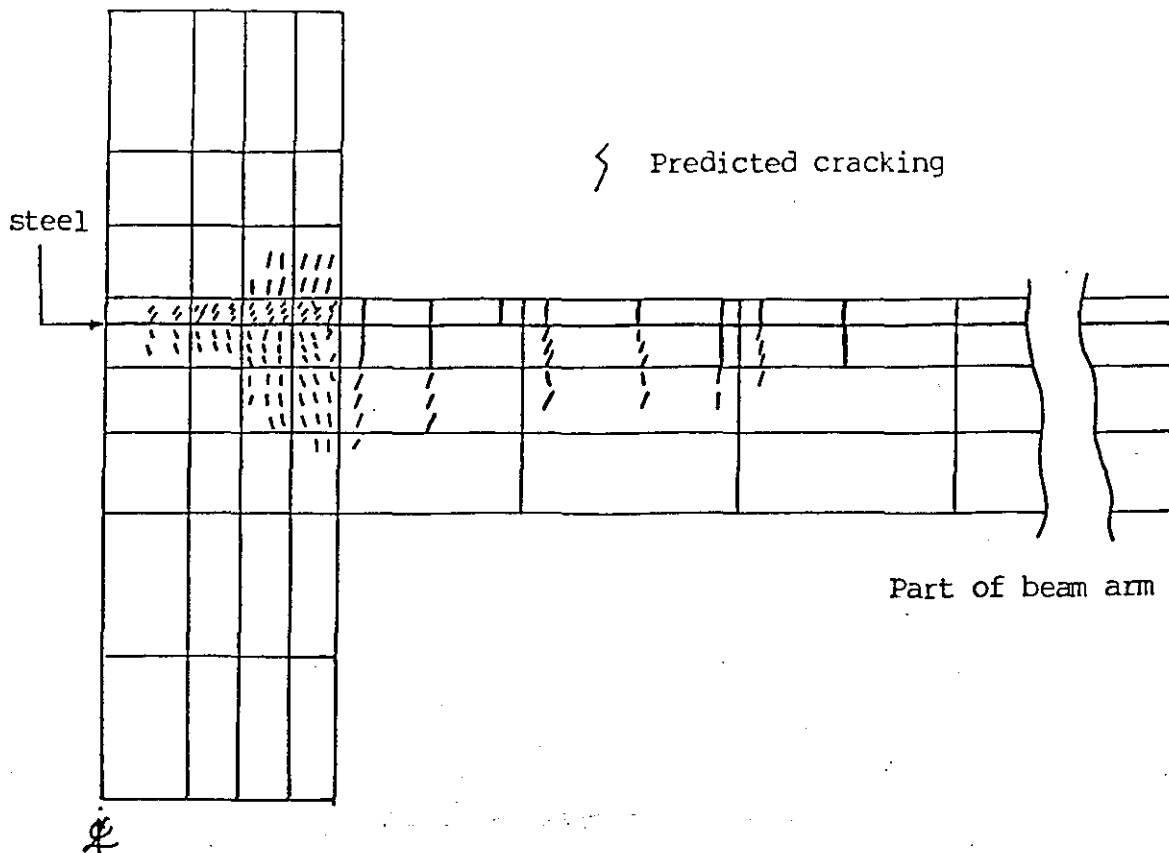


FIGURE 8.22 ANALYTICAL CRACK DISTRIBUTION IN  
THE BEAM-COLUMN AT 7.5 kN LOAD

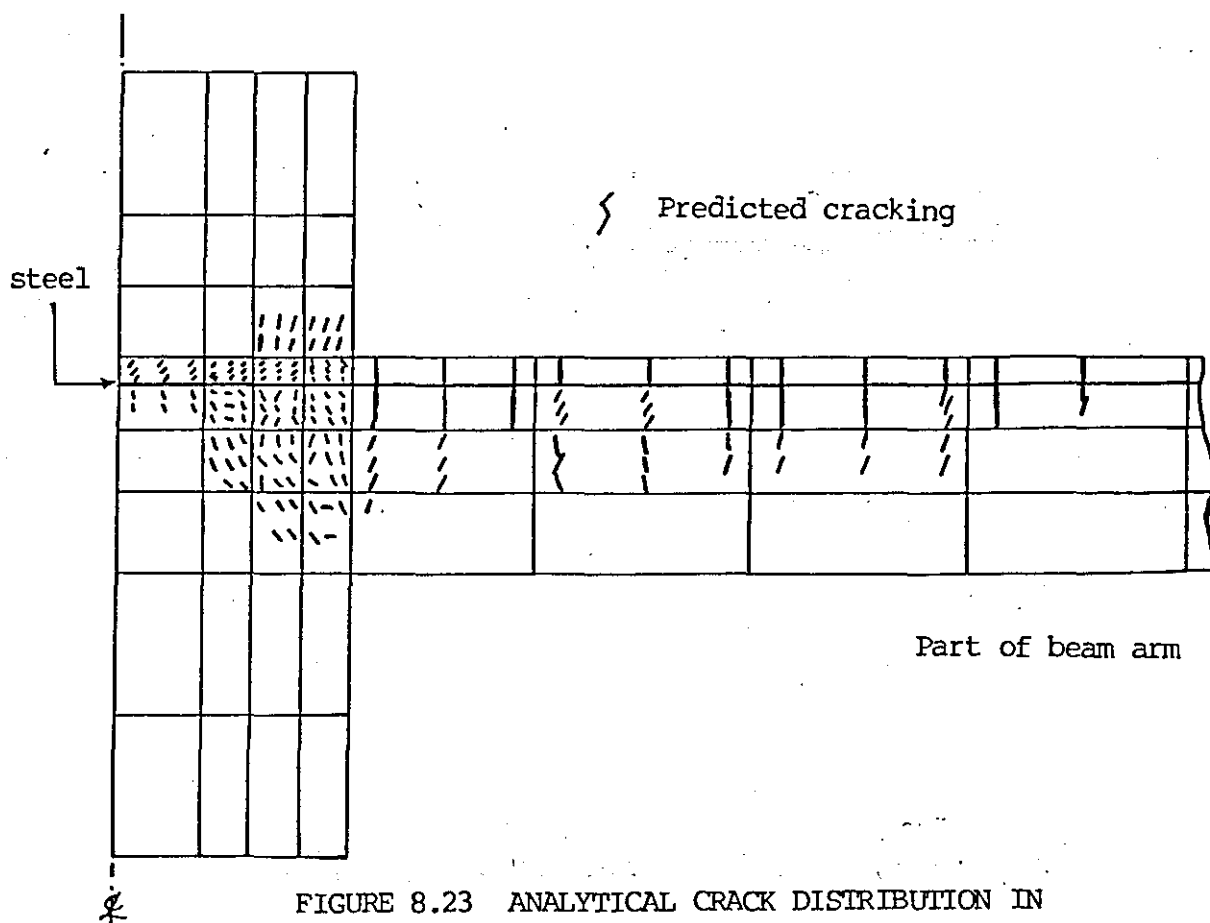


FIGURE 8.23 ANALYTICAL CRACK DISTRIBUTION IN  
THE BEAM-COLUMN AT 10.0 kN LOAD

At 2.5 kN the analytical result compares favorably with test (II), (Figure 8.16) whereas against test (I), (Figure 8.15) it grossly underestimates the actual test results. At the edge of the column the analytical bar stress is about 50 percent of the theoretically evaluated bar stress assuming a cracked concrete section and a parabolic concrete stress distribution in the uncracked part (CP110:1972). This result occurs as the analytical cracked concrete only penetrates 15-20 mm in depth.

At 5 kN the analytical bar stress distribution compares favourably with test (II) but is well below the results of test (I).

At 7.5 kN the analytical result predicts a fairly constant bar stress for the first 40-50 mm into the column width due to the extensive cracking predicted near the bar in this region. Compared with test (II) the bar stress distribution is too high since in the test the load drops off immediately on entry to the column, whereas in comparison to test (II) the analytical result underestimates the bar stresses.

At 10 kN the analytical result predicts a slight drop from  $150 \text{ N/mm}^2$  in the bar stress on entry to the column and is then nearly constant for about one quarter of the column width before decreasing to about  $100 \text{ N/mm}^2$ . The bar stress at the edge of the column is still slightly less than the theoretical stress by cracked section analysis. Compared with test (II) the analytical bar stress on entry to the column is below that measured however does not decrease as rapidly as in the test. Initially for the left hand side of the column width the results agree very well but the analytical result gives less reduction further into the column. Compared with test (I) the analytical bar stress distribution remains below that measured throughout the column width. Overall the results of this analysis are promising but require further investigation.

The early development of transverse cracks in the beam arm of the specimen corresponds reasonably well with the expected behaviour. With increasing load however, the crack development in the column width is more extensive than expected. At 10 kN load the zone of cracked concrete is a strip parallel with the bar. The author would have anticipated the development of cracks in a fan shape spreading at about 45 degrees both top and bottom of the bar. There is a 'domino' effect in the crack development, which can be attributed to three effects either: (1) the bond stress-slip modulus is too high and therefore there is too rapid a transference of force from the bar to the concrete, thereby causing high tensile stresses in the surrounding concrete; (2) the uncracked concrete is unable to carry high shear forces without tensile cracking occurring, or (3) the cracked concrete is unable to carry shear forces without cracking occurring in neighbouring uncracked elements. The shear carrying capacity of the cracked concrete is of great importance and the shear modulus for the cracked concrete may be too low. The shear retention factor (Equation 4.8) is about 0.62 for the cracked concrete, however the actual value is directly related to the current elastic modulus parallel to the cracking and the effective shear modulus may be considerably lower. Khouzam (1977) noted the importance of the magnitude of the shear modulus of the cracked concrete in her analyses of a tensile bond specimen and that too low a value of shear modulus would give incorrect prediction of the real behaviour.

The shear capacity of the concrete and the bond stiffness need further investigation. Cracking in this analysis develops too rapidly and is too extensive. Although there was extensive transverse cracking the development of the cracks occurred in a logical order and in this respect the model appears to work correctly.

Very little can be inferred about the bond model except for the magnitude of the initial bond stress-slip modulus which appears to be too high. By comparison, Allwood (1980) in his simple piecewise elastic analysis of the beam-column used an initial bond stress-slip modulus of  $78 \text{ N/mm}^3$ , which gave good prediction of the bar stress distributions in the column width.

## 8.7 SUMMARY

It was considered unlikely that the bond model developed for plain bars based on a frictional mechanism would adequately model the behaviour of deformed bars since deformed bars rely mainly on the mechanical interlock of the ribs with the surrounding concrete. The experimental data of Robins and Standish (1982) indicates that in the ordinary pullout test the pullout load increases with lateral pressure up to  $15 \text{ N/mm}^2$  and that failure occurs by splitting of the concrete. For higher lateral pressures there is no apparent increase in the pullout load, above that for a lateral pressure of  $15 \text{ N/mm}^2$ , and failure occurs by the shearing of the concrete across the tops of the ribs. This suggests that for the lower lateral pressure region, there is an overriding criterion which determines the onset of the concrete splitting.

The best approximation the author could make was to assume a frictional bond mechanism for deformed bars and estimate the parameters in the model from the experimental data. The model was applied to the ordinary pullout test, Mains eccentric pullout test, and a beam-column intersection.

For the ordinary pullout test the experimental pullout load-lateral stress relationship can be reproduced, however it required modelling the the splitting and shearing regions of failure separately

with different bond parameters. Failure for both sets of bond parameters and for all lateral pressures considered was by excessive slip all along the bar. For zero lateral pressure, splitting of the concrete was not predicted with either of the bond parameter sets used.

In the Mains pullout test the specimen does not fail by cracking and analytical load distributions using two different bond parameters straddle the observed behaviour.

The beam-column required a considerable amount of computing effort, mainly to deal with the transverse cracking which was predicted. Cracking of the concrete was the dominant feature and the shear modulus of the uncracked and cracked concrete is thought to be of great importance in the analysis. Although cracking occurred in a logical order the number of cracks was far too extensive in comparison with the observed behaviour. Little could be inferred with respect to the bond model but the important parameter was the bond stress-slip modulus and the assumed value of  $1000 \text{ N/mm}^3$  for the deformed bar was too high.

9.1    INTRODUCTION.

The major inaccuracy in the present bond model applied to deformed bars is that it is unable to predict the splitting failure of the concrete, as in the case of the ordinary pullout test with no lateral loading. The plane stress model is unable to accurately model the radial transmission of force from the bar to the surrounding concrete and any local concentration in concrete stress close to the bar surface and so predict concrete splitting cracks.

This chapter considers ways of more accurately modelling these radial forces both by reference to the method of analysis and by modifications to the present bond model, so that the concrete splitting might be predicted. Further the effect of splitting or transverse cracking on the local bond stress-slip relationships of deformed bars is also considered.

9.2    CONSIDERATION OF THE METHOD OF ANALYSIS.

9.2.1    Axi-symmetric or three-dimensional analyses ?

The author's approximation of three dimensional structures as a plane stress model assumes the concrete stresses to be uniform over the thickness of the specimen. Using this model the bars may be considered to be effectively external to the concrete but transferring forces according to the bond model at the level of the interface as shown in Figure 9.1.

For the ordinary pullout test with deformed bars using the current bond model then even by allowing for a local concentration of radial and tangential pressure surrounding the bar, the calculated concrete stresses do indicate the likelihood of cracking failure. The

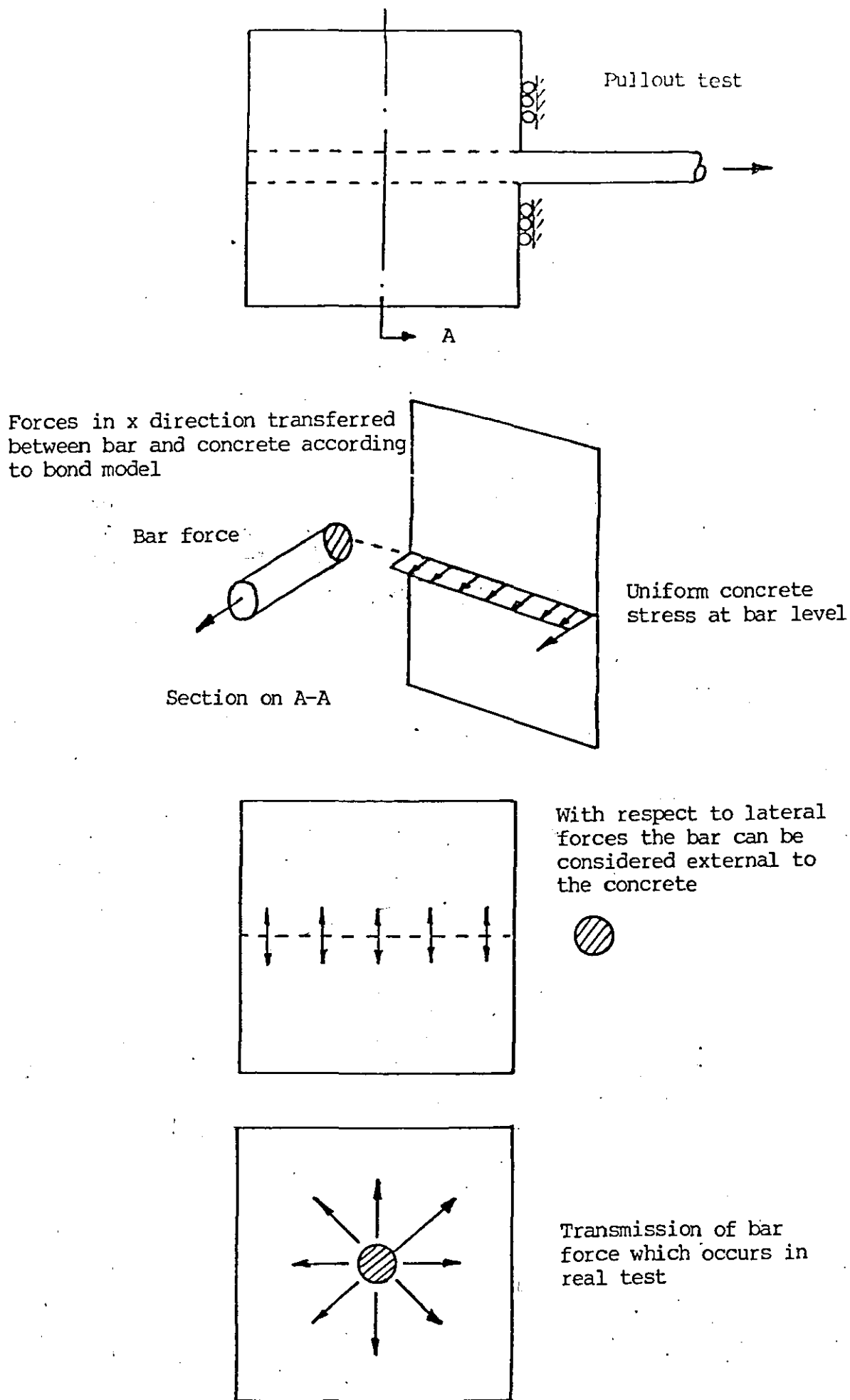


FIGURE 9.1 CONCRETE STRESSES IN THE TWO DIMENSIONAL PLANE STRESS MODEL OF THE PULLOUT TEST

action by which bond forces are radially transmitted to the surrounding concrete cannot be adequately modelled by the plane stress model. Pullout tests would be more accurately modelled by an axi-symmetric analysis and the radial forces modelled accordingly, however more complex reinforced concrete structures might only be more realistically modelled by a three-dimensional analysis.

With an axi-symmetric analysis a further advantage would be the use of solid elements to represent the steel rather than bar (axial force only) elements. From axi-symmetric analyses using solid elements for the steel the computed radial and tangential stresses of the concrete elements would then be inclusive of the forces of bond action, the bar radial contraction and the effect of a confining pressure (if applied). The radial pressure values of concrete elements near the bar could be used directly in the bond model (Equation 3.5).

Simple elastic three-dimensional analyses would be possible, however nonlinearity would be prohibited by the amount of additional computational time involved and would require a very powerful and faster computer processor than at present.

#### 9.2.2 Prediction of splitting cracks.

Within the current plane stress type of analysis of the ordinary pullout test splitting cracks will not be predicted due to the low stress levels in the longitudinal direction. Arbitrary rules might be used however to predict these cracks.

According to Tepfers (1979) the bond stresses develop radial and tangential pressures. The radial pressure is  $\gamma \tan \alpha$  where  $\alpha$  is equal to the angle of radiation and the tangential pressure calculated from

thick walled cylinder theory assuming an internal pressure of  $\gamma \tan \alpha$  is

$$\sigma_t = \gamma \tan \alpha \frac{(c + d/2)^2}{(c + d/2)^2 - (d/2)^2} + \frac{(d/2)^2}{(d/2)^2} \quad (9.1)$$

where  $\gamma$  = bond stress

$d$  = nominal diameter of the bar

$c$  = concrete cover (distance to surface of concrete cylinder)

$f_t$  = tensile strength of the concrete

When  $\sigma_t$  exceeds the tensile strength of the concrete cracking develops at the inside surface of the concrete. The associated bond stress value is a lower bound solution and further estimates of the bond stress at which bursting occurs may be made by assuming the surrounding concrete cylinder to be either fully plastic or partly cracked. An estimate for the partly cracked stage is that the failure bond stress is:

$$\gamma = f_t \frac{(c + d/2)}{1.664 d} \quad (9.2)$$

and for the fully plastic stage:

$$\gamma = f_t \frac{2 c}{d} \quad (9.3)$$

From experimental evidence Tepfers found that splitting cracks for pullout test occurred within the bounds given by Equations (9.2) and (9.3).

Within the plane stress analysis or axi-symmetric analysis where bar elements are used the above Equations (9.2) and (9.3) might be used to predict the onset of splitting cracks in the concrete. A simple criterion of a fixed bond stress to predict when cracking occurs could

be modified to allow for lateral pressure effects. From the experimental results of Robins and Standish (1982) the value of bond stress at which cracking occurs would be increased by lateral pressure. The new governing equation might be of the form:

$$\tau_{\text{crack}} = \tau_{\text{Tepfers}} + m \sigma_{\text{lateral stress}} \quad (9.4)$$

where  $\tau_{\text{crack}}$  = local bond stress at which splitting occurs

$\tau_{\text{Tepfers}}$  = bond stress at which splitting occurs for zero lateral stress

$m$  = coefficient

$\sigma_{\text{lateral}}$  = lateral stress

A more sophisticated approach would be to consider the tangential pressures at the steel/concrete interface due to the effects of concrete shrinkage, bar radial contraction, lateral pressure and bond action. The total tangential pressure is given by the following equation :

$$\sigma_t = P_r(\text{shrinkage}) - \nu P_r(\text{bar contr.}) + \text{Coeffl } P(\text{lateral}) + \sigma_t(\text{bond}) \quad (9.5)$$

where  $\sigma_t$  = tangential stress

$P_r(\text{shrinkage})$  = radial pressure due to shrinkage

$P_r(\text{bar})$  = radial pressure due to bar contraction

$\text{Coeffl}$  = coefficient relating lateral to radial pressure

$P(\text{lateral})$  = lateral stress

$\sigma_t(\text{bond})$  = tangential stress due to bond action (Equation 9.1)

$\nu$  = Poisson's ratio

Cracking of the concrete would be initiated at the bar/concrete interface when  $\sigma_t$  exceeds the tensile strength of the concrete. For all these criteria governing the onset of concrete splitting cracks the concrete elements are unlikely to be already cracked or near to cracking.

Therefore the analysis must be over-ridden and cracks forced into concrete elements adjacent to the bar. If the concrete is deemed to have cracked then the nearest concrete element will also be deemed to have cracked and it's properties will have to be suitably modified.

### 9.3 THE EFFECT OF CRACKING ON THE LOCAL BOND STRESS-SLIP RELATIONSHIP OF A DEFORMED BAR.

Within the present bond model no account is taken of the effect of transverse or splitting cracks (if they occurred) on the local bond stress-slip relationships, except for the changes in radial pressure at the surface of the bar. Labib (1976) observed that bond slip reversal occurred when primary cracks were formed in the analysis of tensile bond specimens. He assumed that the unloading path is linear passing back through the origin and meets the bond stress-slip curve for bond slip in the opposite sense from the original slip (Figure 2.9). In the current bond model the loading history does not effect the local bond stress-slip relationship.

Ciampi, Eligenhausen et al.(1981) and Edwards and Yannopoulos (1978) with tests on cyclic and repeated loading of pullout specimens have observed a stiff 'unloading branch' in the bond stress-slip relationship and that on returning to zero bond stress some permanent bond slip results. The effect of transverse cracking on the local bond stress-slip relationship may be considered to be similiar to the 'unloading branch' in cyclic loading. The author suggests that the bond stress-slip path is probably similiar to that observed by Ciampi, Eligenhausen et al. for cyclic loading and the resultant bond stress-slip path should be very stiff.

The effect of splitting cracks on the local bond stress-slip

relationship has not been observed mainly as sudden failure occurs in the concrete specimen. Labib (1976) assumed that a gradual reduction in bond stress occurs with increasing bond slip. Nilson (1968) assumed no bond capacity for that part of a deformed bar close to exit from the concrete block (exterior bond links) when the local maximum bond stress was reached and the concrete was cracked in a cone shape. The author suggests that a rapid reduction in bond stress is likely to occur with increasing slip when splitting cracks occur.

Within the current bond model a simple way of taking into account the effect of splitting cracks would be to modify the value of the  $\beta$  parameter. The maximum bond stress value corresponds to the onset of concrete cracking near the bar surface and is the amount of bond that can exist just before cracking takes place. The  $\beta$  parameter now takes on a new meaning and is the bond capacity for a bar in concrete which has splitting cracks. If we assume that there is no capacity to transfer force from the bar to the cracked concrete, i.e. no bond then  $\beta$  equals zero and the Nilson (1968) 'exterior' bond link bond stress-slip relationship is obtained.

The combination of a splitting crack criterion based on the local bond stress, the  $\beta$  parameter equal to zero, and the forcing of cracks through the concrete elements adjacent to the bar might adequately model the splitting crack phenomenon.

The main objective of the work described in this thesis was to develop a bond model for plain bars and to evaluate its applicablity. Initial studies were conducted into using the model for deformed bars. The bond model has been incorporated into the author's finite element program which uses the initial stress method for non-linear analysis. The bond model was used to analyse bond tests with plain bars and also deformed bars in some bond tests and a beam-column intersection.

Some experimental work was conducted to obtain additional information as to the nature of bar stress distributions in bond tests with plain bars.

There are three main areas within the work from which conclusions have been drawn, namely : the experimental work, application of the bond model to plain bars and the application of the bond model to deformed bars. Recommendations for further experimental work and for modifications to the bond model for the modelling of deformed bars are made.

#### 10.1     EXPERIMENTAL WORK:- PLAIN BAR BOND TESTS AND BEAM-COLUMN TEST.

Bond tests with embedded plain bars were conducted by the author and were namely, the ordinary pullout test, the double ended pullout test, and the transfer test. In addition two beam-column intersection models with deformed bars were tested. In all the tests the steel bars were strain gauged so that steel strains could be measured inside the concrete specimens. Three different methods of strain gauging were tried : the 'recess and cap' method, the 'recess and fill' method and the split bar method.

The conclusions made from these experiments are as follows.

- (1) In the ordinary pullout test with plain round bars the changing shape of the bar stress distribution with increasing pulling load as observed by others was confirmed . The results are similiar to those observed by Mains (1951), Peattie and Pope (1956) and Parland (1957) , where the bar stress distribution changes from and 'exponential' type of shape at low loads to a gradual convex or S -shape near the failure load.
- (2) In the double ended pullout test the bar stress distribution remained of an 'exponential' shape with increasing load. There was no indication that the distribution was changing to an S-shape.
- (3) In the transfer test most of the load which was to be transferred from the centre bar to the outer bars through the concrete occurred within a short distance of the specimen (about one-eighth of the length of the specimen).
- (4) The general shape of the bar stress distributions in the column width of the beam-columns was U-shaped, however the effect of cracking causes a local disturbance to be superimposed on this general distribution. This effect is similiar to the effect of cracks on the bar stress distribution in beam specimens observed by Mains (1951).
- (5) The split bar method of internally gauging the bars was the most successful method utilised, however this method is restricted to bar lengths of less than about one metre.
- (6) The 'recess and cap' method of strain gauging worked reasonably well, however the steel caps were very difficult to fabricate and extra care was required to waterproof and electrically insulate the strain gauges.

- (7) The 'recess and fill' method, although a much quicker and easier method than the 'recess and cap' method was not suitable once the bars were embedded in concrete. The strain gauges underneath the resin were susceptible to lateral pressure effects and the resin material was found occasionally to uplift and bulge with large slip movements.

## 10.2 APPLICATION OF THE BOND MODEL TO PLAIN BARS.

The proposed bond model for plain bars is based on a non-linear bond stress-slip relationship up to an ultimate bond stress at a tolerance slip. The ultimate bond stress is a function of the radial pressures exerted at the bar/concrete interface by the initial concrete shrinkage, the bar radial contraction and concrete lateral pressures. For slips in excess of the tolerance slip the maximum bond stress is a fixed proportion of the ultimate bond stress.

The bond model was incorporated into the author's finite element program and used to analyse the ordinary pullout test with or without lateral pressure, the Mains (1951) eccentric pullout test, the double ended pullout test and the transfer test.

The conclusions made from these analyses are as follows .

- (1) The frictional bond model incorporated into the finite element method can accurately predict the distribution of load in the ordinary pullout test.
- (2) The bond model also accurately predicts the failure load and the increase due to lateral pressure.
- (3) There is reasonable agreement between the analytical and experimental free-end slips in the ordinary pullout test.

- (4) In the ordinary pullout test the load distributions change from an 'exponential' shape at low loads to a straight line then to convex shape or S-shaped distribution. Suitable variations in the bond parameters will adjust the degree of S-shape in the bar stress distribution and the ultimate pullout load. Very marked changes in the bar stress distribution will only occur with very substantial changes in the bond stress distribution. For the particular cube pullout tests analysed marked changes in the shape of the bar stress distribution will only occur if in the current bond model the initial bond modulus is relatively large, the tolerance slip is small and  $\beta$  is much less than unity.
- (5) The model reasonably predicts the load distribution and the failure load for the eccentric pullout test of Mains (1951). Further the model reasonably accurately predicts the progressive bond failure and the associated shifting of the peak bond stress along the bar.
- (6) Due to the scatter of the experimental points in the double ended pullout test the comparison between analytical and experimental results is inconclusive for or against agreement.
- (7) There is a marked difference between the analytical results and the experimental results in the transfer test. In the analytical model substantial bond forces are transferred over the entire embedment length, whereas in the experimental tests bonding appears to occur over only a short distance with most of the force transferred from bar to concrete over this length. This test requires further investigation.

### 10.3 APPLICATION OF THE BOND MODEL TO DEFORMED BARS

The bond model based on a frictional bond mechanism was used to analyse the ordinary pullout test and the Mains (1951) eccentric pullout test. Two different sets of bond parameters were used which are based on the splitting and shearing regions of the experimental results of Robins and Standish (1982). A beam-column intersection was analysed using only the bond parameters which are based on the splitting region.

Conclusions from using this model for deformed bars in bond tests and the beam-column are as follows.

- (1) The finite element model is unable to predict the failure mechanism of splitting in the concrete in the ordinary pullout test for lateral pressures less than  $15 \text{ N/mm}^2$ .
- (2) The current plane stress model is unable to give realistic stresses in the concrete surrounding the bar. In the ordinary pullout test the magnitude of concrete Y-Y stresses near the bar are very low. The concrete stresses are average values of stress over the thickness of the concrete specimen and therefore the plane stress model is unable to accurately model the radial and tangential stresses close to the bar.
- (3) Using the current bond and the parameters assigned to represent deformed bars only fair approximations of the bar stress distributions in the ordinary pullout test and the Mains pullout test can be made. For the Mains pullout test where there are no splitting cracks a better match between the analytical and experimental bar stress might be made using the current bond model and by adjustment of the current bond parameters.
- (4) In the analysis of the beam-column intersection transverse

cracking was predicted and at low loads the crack pattern was as expected. With increasing load, cracking in the beam arms was as expected, however cracking in the column width was far too extensive. The order in which the cracks developed was logical and in this respect the model appears to work quite well.

The high value of initial bond stress-slip modulus and the magnitude of the shear modulus of the cracked (in particular) and uncracked concrete are thought to be the major factors producing the considerable degree of cracking.

- (5) The finite element program and developed bond model is a useful tool for further investigation of the bonding of plain bars and deformed bar problems.

#### 10.4 RECOMMENDATIONS FOR FURTHER WORK.

The first step which the author recommends is to re-analyse the ordinary pullout test and the double ended pullout test using an axisymmetric model and investigate the magnitude of concrete stresses close to the bar. From these analyses it should be possible to establish whether concrete splitting cracks will develop and if they are predicted whether their onset is related to the level of bond stress.

Using the plane stress model, investigate the use of a splitting crack criterion based the local bond stress and lateral pressure. Further model the concrete splitting within the bond model with the  $\beta$  parameter equal to zero and then force the splitting cracks into the adjacent concrete elements.

Additional tests on the modelling of transverse cracks should be performed and a suitable test for comparisons is the tensile bond specimen

of Broms (1965) who gives details of the distribution of cracks. Further consideration should be given to the effect of bond reversal and the effects of cracking on the local bond stress-slip relationship as outlined in Chapter 9.

Initial work on the beam-column is promising and the author recommends re-analysing this structure with a lower initial bond stress-slip modulus and a higher shear modulus for cracked and uncracked concrete.

Further experimental studies on the transfer test may yield additional information on the bonding of steel to concrete. Additional tests using the superior split bar method of gauging plain bars in ordinary pullout with lateral pressure should be conducted to provide further bar stress distributions for comparisons with the analytical results.

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## APPENDIX A

### THE STRESS DISTRIBUTION AROUND AN ELASTIC CIRCULAR DISC INSERTED INTO AN ELASTIC PLATE WHICH IS SUBJECTED TO A UNIAXIAL STRESS FIELD

The distribution of tangential and radial pressures at the bar-concrete interface may be estimated using the formulae derived by Muskhelishvili (1956). For the problem illustrated in Figure A.1, the following assumptions are made :

- (1) The concrete body is an infinitely large elastic isotropic plate containing a circular hole of radius  $R$  ;
- (2) The steel bar is a circular elastic inclusion inserted into the hole ; and
- (3) The disc is joined perfectly to the plate.

The radial and tangential stress within the concrete body are given by the formulae :

$$\sigma_r = \frac{p}{2} \left[ 1 - \gamma \frac{R^2}{r^2} + \left( 1 - 2\beta \frac{R^2}{r^2} - 3\delta \frac{R^4}{r^4} \right) \cos 2\theta \right] \quad (A.1)$$

$$\sigma_t = \frac{p}{2} \left[ 1 + \gamma \frac{R^2}{r^2} - \left( 1 - 3\delta \frac{R^4}{r^4} \right) \cos 2\theta \right] \quad (A.2)$$

where  $\sigma_r$  = radial stress

$\sigma_t$  = tangential stress

$p$  = stress in the uniaxial direction

$R$  = radial co-ordinate in the concrete body

$r$  = radius of the elastic inclusion

For the assumptions given the parameters  $\gamma, \beta, \delta$  are as follows

$$\beta = -2 \frac{(\mu_0 - \mu)}{\mu + \mu_0}$$

$$\delta = \frac{\mu_0 - \mu}{\mu + \mu_0}$$

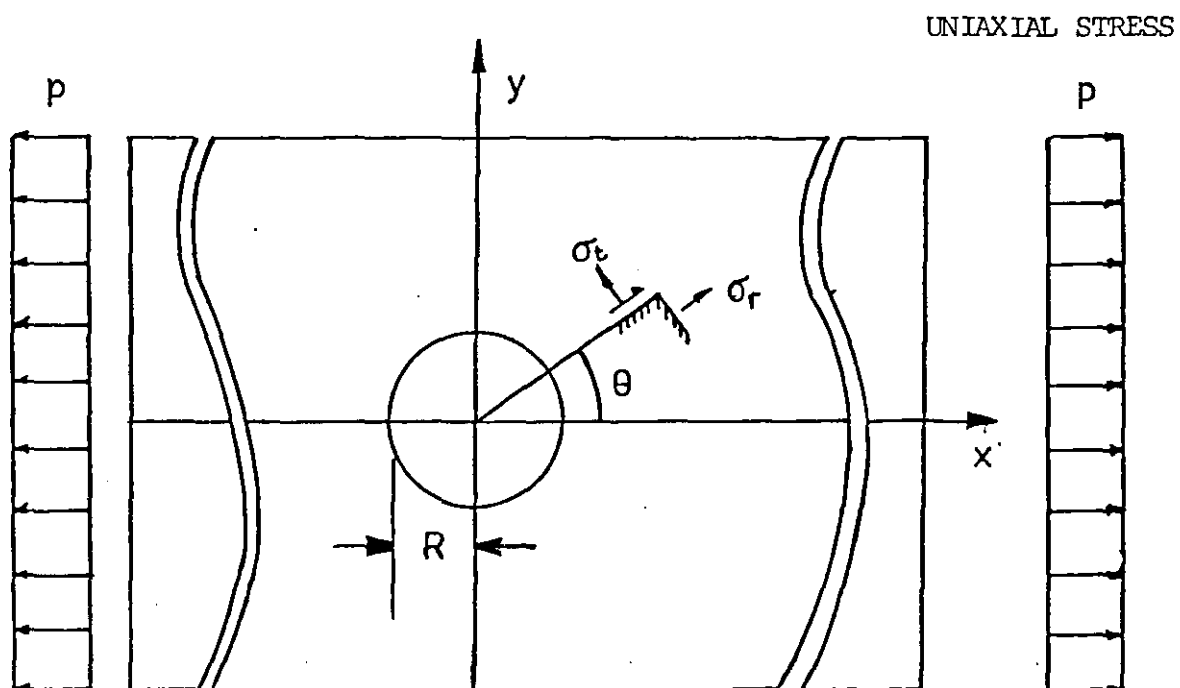


FIGURE A.1 UNI-AXIAL TENSION APPLIED TO A PLATE CONTAINING A  
CIRCULAR HOLE INTO WHICH AN ELASTIC DISC IS INSERTED

$$\gamma = \frac{\mu (x_0 - 1) - \mu_0 (x - 1)}{2 \mu_0 + \mu (x_0 - 1)}$$

$$\text{where } x_0 = \frac{3 - v_0}{1 + v_0} \quad x = \frac{3 - v}{1 + v} \quad \mu_0 = \frac{E_0}{2(1 + v_0)} \quad \mu = \frac{E}{2(1 + v)}$$

where  $E_0, E$  = modulus of elasticity for steel and concrete respectively  
 $v_0, v$  = Poisson's ratio for steel and concrete

For the steel concrete problem and assuming  $E_0 = 200\,000 \text{ N/mm}^2$ ,  
 $E = 28\,000 \text{ N/mm}^2$ ,  $v_0 = 0.3$  and  $v = 0.2$  then the radial and tangential pressures at the interface with  $r = R$  are given by :

$$\sigma_r = \frac{p}{2} [1.54081 + 1.34139 \cos 2\theta] \quad (\text{A.3})$$

$$\sigma_t = \frac{p}{2} [0.45918 + 0.02415 \cos 2\theta] \quad (\text{A.4})$$

The values of  $\sigma_r$ ,  $\sigma_t$  are tabulated for values of

| $\theta^\circ$ | $\sigma_r/p$ | $\sigma_t/p$ |
|----------------|--------------|--------------|
| 0              | 1.44109      | 0.24166      |
| 15             | 1.35124      | 0.24000      |
| 30             | 1.10575      | 0.23562      |
| 45             | 0.77040      | 0.22959      |
| 60             | 0.43505      | 0.22355      |
| 75             | 0.18956      | 0.21913      |
| 90             | 0.09971      | 0.21751      |

TABLE A.1 VALUE OF RADIAL AND TANGENTIAL PRESSURE  
 COEFFICIENTS FOR ANGLE  $\theta$

The average interfacial radial pressure is given by :

$$\sigma_{av} = \frac{p}{2} \int_0^{90} \sigma_r d\theta$$

Evaluating the integral using the Trapezoidal integration rule the following relationship is obtained :

$$\underline{\sigma_{av} = 0.7704 p}$$

## APPENDIX B

### GUIDE TO THE AUTHOR'S PROGRAMS.

#### B.1 INTRODUCTION.

The software package used by the author is the Genesys System 2.7\*. This software allows subsystems to be developed that are specific to the users requirements. The author has used and expanded a subsystem from the GENESYS library known as the LUFE system (Loughborough University Finite Element Subsystem). The use of the LUFE subsystem to the finite element solution of plane stress and axi-symmetric stress problems is described in LUFE 'A guide to using the program' (1979) to which the reader is referred.

The general form of a computer run (known as an engineering job) is a declaration of data tables (e.g. giving nodal co-ordinates, boundary conditions, nodal loads, material parameters etc. followed by a master or controlling segment. The master segment consists of a list of problem orientated commands which controls the order in which the data tables are to be read and the order in which the relevant subprograms within the subsystem are performed. The master segment may also contain statements such as DO- and IF-. Examples of input data are given in Section B.2.6

For more information the reader is referred to the Genesys Centre Reference Manual (1972) and Chapter 11 of Finite Element Techniques in Structural Mechanics (1970).

\* Genesys Ltd.

## B.2 GENERAL INFORMATION AND DATA PREPARATION.

### B.2.1 UNITS.

The user must use the standardised units which are :

|         |                   |
|---------|-------------------|
| FORCE   | NEWTONS (N)       |
| LENGTH  | MILLIMETRES (mm)  |
| STRAINS | MILLISTRAINS (mS) |

and combinations thereof :

|                                      |
|--------------------------------------|
| STRESS (N/mm <sup>2</sup> )          |
| ELASTICITY (N/mm <sup>2</sup> )      |
| SLIP (mm)                            |
| BOND MODULUS (N/mm <sup>2</sup> /mm) |

This is important since the concrete model is not independent of the units used.

### B.2.2 SIGN CONVENTION.

The sign convention used is that tension is positive.

### B.2.3 LIMITATIONS AND RESTRICTIONS.

#### i) BOND6 element

Horizontal bond elements only are allowed. The finite element mesh must be orientated so that the bond elements are parallel with the global X-axis.

#### ii) RENUMBER

Maximum number of nodes for renumbering is 300.

### B.2.4 DATA PREPARTION

Examples of data input are given in Section B.2.6 , however for more details the reader is referred to the LUFÉ 'A Guide to using the program' (1979) which shows how the tables of data and elements are input.

The following elements are used to represent the various phases :

BOND6      elements for BOND  
BAR3        elements for the STEEL  
QUAD8SM    elements for the CONCRETE

The preparation of data for these elements is given in Tables B.1 and B.2.

### B.3      PROBLEM ORIENTATED COMMANDS.

The following is a list of problem orientated commands that the author has added to the LUFE subsystem. The list of commands complements those commands given in the LUFE 'A Guide to using the Program' (1979) and are presented here in a similiar format.

#### B.3.1

PROBLEM TYPE IS 'type'

This command causes certain initialisation routines to be performed and ensures that the correct overlays are entered. The 'type' must be 'CONCRETE' for 2-D plane stress reinforced concrete analysis.

#### B.3.2

START      ↗ GAUSS STRESSES FOR ELEMENTS 't', 't', ...  
            → DISPLACEMENTS 's'  
            ↘ EQUIVALENT UNIAXIAL STRAINS

All three commands are required to initialise a reinforced concrete problem. Elements are identified by the titles 't' of the tables used for input. The START GAUSS STRESSES FOR ELEMENTS 't' command initialises and sets the accumulated Gauss point stresses for the elements 't' to zero. The elements type 't' must be from BOND6, QUAD8SM

TABLE B.1    ELEMENTS AVAILABLE    BAR3 AND QUAD8SM

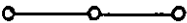
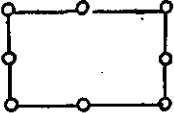
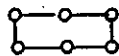
| ELEMENT<br>NAME                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | NUMBER<br>OF NODES | MATERIAL<br>DATA | SECTION<br>DATA                                 | STRESSES |         |       |    |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|------------------|-------------------------------------------------|----------|---------|-------|----|
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                    |                  |                                                 | ONLY     | ELEMENT | NODAL | NO |
| BAR3<br>(reinforcement)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 3                  | $E_o$            | A                                               | *        | *       | *     | *  |
| <p>NOTES</p>  <p>Isoparametric 3 noded bar element taking axial load only.<br/> Nodes numbered left to right.<br/> <math>E_o</math> = Young's modulus (<math>N/mm^2</math>)<br/> A = Cross-sectional area (<math>mm^2</math>)</p>                                                                                                                                                                                                                                                                                                                                                                  |                    |                  |                                                 |          |         |       |    |
| QUAD8SM                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 8                  | $f_{cu}, E_o$    | $\epsilon_{cu}, \epsilon_{tu}$<br>$f_t, \nu, t$ | *        | *       | *     | *  |
| <p>NOTES</p>  <p>Isoparametric 8 noded membrane element.<br/> Nodes in anti-clockwise order.</p> <p> <math>f_{cu}</math> = Cube strength (<math>N/mm^2</math>) (given positive)<br/> <math>E_o</math> = Initial Young's modulus in compression (<math>N/mm^2</math>)<br/> <math>\epsilon_{cu}</math> = Failure strain in uniaxial compression (mS) (given positive)<br/> <math>\epsilon_{tu}</math> = Failure strain in uniaxial tension (mS)<br/> <math>f_t</math> = Failure strength in uniaxial tension (<math>N/mm^2</math>)<br/> <math>\nu</math> = Poisson's ratio<br/> t = Thickness </p> |                    |                  |                                                 |          |         |       |    |

TABLE B.2      ELEMENT AVAILABLE BOND6

| ELEMENT<br>NAME | NUMBER<br>OF NODES | MATERIAL<br>DATA | SECTION<br>DATA                                     | STRESSES |         |       |    |
|-----------------|--------------------|------------------|-----------------------------------------------------|----------|---------|-------|----|
|                 |                    |                  |                                                     | ONLY     | ELEMENT | NODAL | NO |
| BOND6           | 6                  | ITYPE            | $\Sigma u, R_t, R_n$<br>$q_o, \mu, \Delta_u, \beta$ | *        | *       | *     | *  |

NOTES



6 noded shearing element.  
Must be parallel with global O-X axis.  
Nodes in anti-clockwise order steel to concrete.

$\Sigma u$  = Total perimeter of bars (mm)  
 $R_t$  = Initial bond modulus parallel to bar (N/mm<sup>3</sup>)  
 $R_n$  = Initial bond modulus orthogonal to bar (N/mm<sup>3</sup>)  
 $q_o$  = Ultimate bond stress due to concrete shrinkage (N/mm<sup>2</sup>)  
 $\mu$  = Slope of ultimate bond stress-radial pressure line  
 $\Delta_u$  = Slip at which maximum bond stress occurs (mm)  
 $\beta$  = Factor by which maximum bond stress is reduced when slip exceeds  $\Delta_u$ .

ITYPE

- |   |                                      |    |                                           |
|---|--------------------------------------|----|-------------------------------------------|
| 1 | RJA type bond                        | 6  | Saenz curve - varying ultimate bond       |
| 2 | Nilson's relationship                | 7  | Labib and Edwards approximation to curves |
| 3 | Quadratic Bond 1                     | 8- | Linear                                    |
| 4 | Quadratic Bond 2                     |    |                                           |
| 5 | Desayi curve - varying ultimate bond |    |                                           |

| ITYPE | u | Rt    | Rn   | q <sub>o</sub> | μ | Δ <sub>u</sub> | β |
|-------|---|-------|------|----------------|---|----------------|---|
| 1     | * | 78.   | *    | 0              | 0 | 0              | 0 |
| 2     | * | 979.  | *    | 0              | 0 | 0              | 0 |
| 3     | * | 100.  | *    | 0              | 0 | 0              | 0 |
| 4     | * | 100.  | *    | 0              | 0 | 0              | 0 |
| 5     | * | 500.  | *    | *              | * | *              | * |
| 6     | * | *     | *    | *              | * | *              | * |
| 7     | * | 1432. | 268. | 0              | 0 | 0              | 0 |
| 8-    | * | *     | *    | 0              | 0 | 0              | 0 |

\* Value must be given  
0 Give as zero

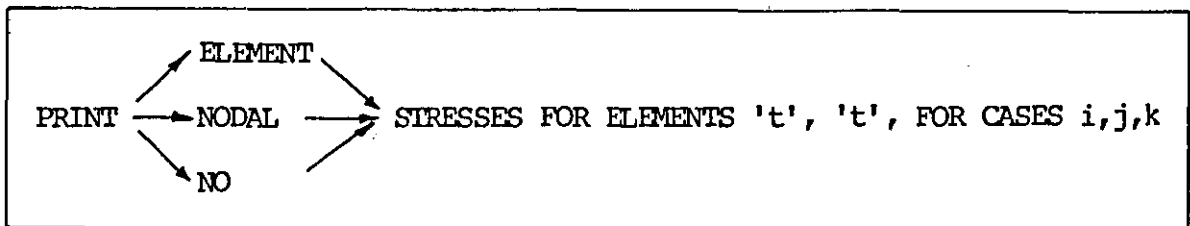
All material and section properties must be included. Some values must be specific for ITYPE, whilst others can be set to zero. The table above indicates.

and BAR3.

The START DISPLACEMENTS 's' commands initialises the vector of accumulated nodal displacements from the table 's', which is normally a table of zero displacements.

The START EQUIVALENT UNIAXIAL STRAINS command initialises the accumulated concrete equivalent uniaxial strains at each Gauss point of the concrete elements and sets them to zero.

### B.3.3



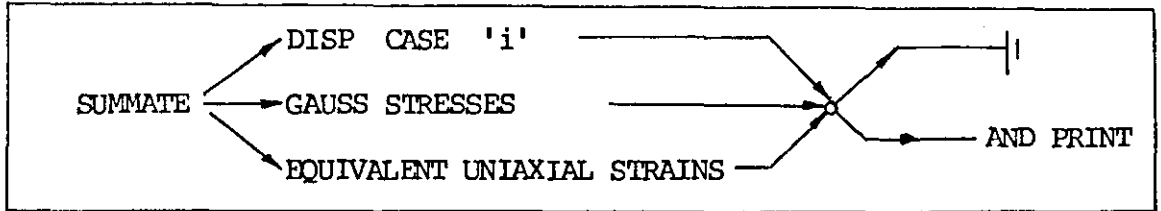
These commands have different implications from those used in the ordinary plane stress or plain strain 2-D analysis. The element types 't' may be from BCND6, QUAD8SM, BAR3. Using the PRINT ELEMENT STRESSES command the stresses due to the incremental load are calculated at the Gauss points of the elements type 't' and printed out fully.

The PRINT NO STRESSES command does exactly the same as the PRINT ELEMENT STRESSES command except there is NO print-out.

The PRINT NODAL STRESSES command takes the accumulated Gauss point stresses of each element of element type 't' and calculates the extrapolated nodal values of stress and nodal averages for the same element type 't'.

The PRINT STRESSES command does exactly the same as PRINT NODAL STRESSES except there is NO print-out.

#### B.3.4



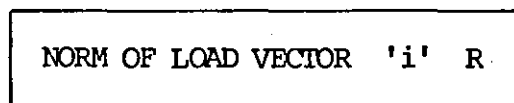
The command SUMMATE DISP CASE accumulates nodal displacements adding the displacement vector due to load case 'i' to the accumulated displacement vector.

The SUMMATE GAUSS STRESSES command accumulates Gauss point stresses for all element types 't' listed in the START GAUSS STRESSES command. The incremental Gauss point stresses must be calculated prior to the use of this command i.e PRINT ELEMENT STRESSES ... or PRINT NO STRESSES ... used before SUMMATE GAUSS STRESSES.

The SUMMATE EQUIVALENT UNIAXIAL STRAINS command adds the equivalent uniaxial strains due to an increment of load to the accumulated equivalent uniaxial strains at each Gauss point of the concrete elements. The PRINT ELEMENT STRESSES ... or PRINT NO STRESSES ... command must precede this SUMMATE command.

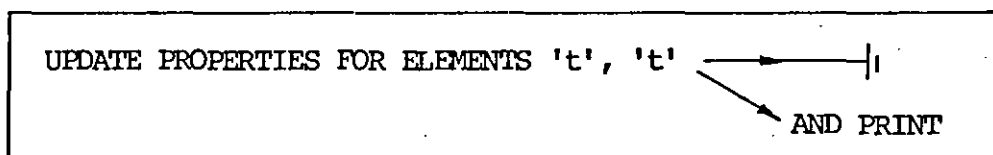
Printing is optional with all these commands.

#### B.3.5



The Euclidean norm of the load vector, load case i is calculated and given to the variable R .

#### B.3.6



The material properties of the elements 't' are updated at the Gauss points of the respective elements. The element types 't' may be either BOND6 or QUAD8SM. Overall element properties are used for the bar elements and the UPDATE command cannot be used on BAR3 elements. The material properties updated at the respective Gauss points of the concrete and bond elements are :

'QUAD8SM' - concrete elements

YM<sub>1</sub> = Young's modulus in  $\sigma_1$  direction

YM<sub>2</sub> = Young's modulus in  $\sigma_2$  direction

PR = Poisson's ratio

THETA = Angle anti-clockwise from the O-X axis to  $\sigma_1$

'BOND6' - bond elements

RT = Bond stress-slip modulus parallel to the bar

RN = Bond stress-slip modulus orthogonal to the bar

Printing is optional but when the print option is used the following information is provided :

'QUAD8SM'

element no., Gauss point no., accumulated stresses  $\sigma_1, \sigma_2$ , total equivalent uniaxial strains, THETA, YM1, YM2

'BOND6'

element no., bar global node numbers, Gauss point number, concrete lateral pressure, combined radial pressure value, local ultimate bond stress, bond slip, RT, RN

Concrete failure

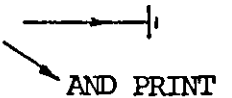
When a concrete Gauss point has failed by cracking or crushing the following information is output regardless of whether the print option was specified or not.

Failure message

Accumulated stresses, tensile strength or compressive strength

Element no., Gauss point no., accumulated stresses, accumulated equivalent uniaxial strains, THETA, current strains

#### B.3.7

|                                                  |                                                                                     |
|--------------------------------------------------|-------------------------------------------------------------------------------------|
| RESIDUAL FORCES FOR ELEMENTS 't', 't',... CASE I |  |
|--------------------------------------------------|-------------------------------------------------------------------------------------|

For each element of type 't' the residual stresses are calculated at the respective Gauss points . Elements type 't' may be BOND6 or QUAD8SM. Contributions to the residual nodal forces are accumulated in the dummy load vector case I (I must be 2). Printing is optional but when the print option is used the following information is output :

'QUAD8SM'

element no., Gauss point no., principal stresses, THETA, residual stresses, accumulated equivalent uniaxial strains, IFAIL1, IFAIL2

Similar to the UPDATE command when the concrete fails a message is output whether the print option was specified or not. The failure message has the same format as before

'BOND6'

element no., Gauss point no., accumulated bond stress, theoretical bond stress, residual bond stress, concrete lateral pressure, bar radial pressure, local ultimate bond stress.

#### B.3.8

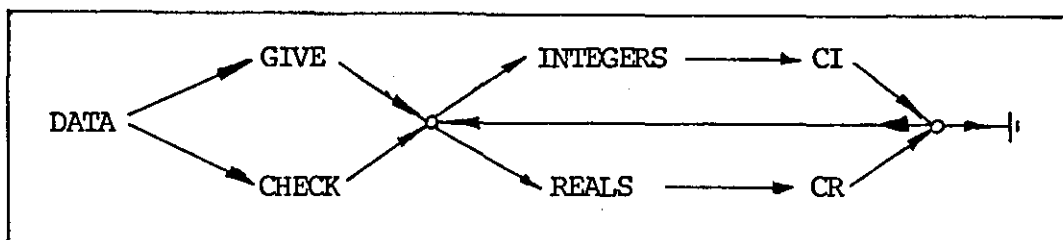
|                            |   |
|----------------------------|---|
| NUMBER OF CRACKS DEVELOPED | R |
|----------------------------|---|

With this command the number of 'failed' concrete Gauss points (either by cracking or crushing) since the command was last invoked (start of the job if not used before) is obtained and given to the variable R. Every Gauss point of each concrete element is interrogated to find out whether it has failed by cracking or crushing. If the Gauss point has failed then it is flagged.

IFAIL1 = 0 means no failure. =1 means failure in  $\sigma_1$  direction

IFAIL2 = 0 means no failure. =1 means failure in  $\sigma_2$  direction

### B.3.9



Variables used in the \*MASTER segment of an engineering job are lost when the job is finished or saved. The DATA GIVE command allows such variables to be stored in Public arrays and when the \*SAVE command is used the variables are stored on the 'data file'. When a new engineering job is restarted from the 'data file' the stored variables may be retrieved using the DATA CHECK command. The variables are stored in Public arrays RDATA () for reals and IDATA () for integers.

### B.4 EXAMPLE OF INPUT DATA

The example problem is that of the ordinary pullout test with lateral loading and the input data is shown in Section B.4.1. This input data is to initialise the problem, to apply the first load increment and iterate to a solution. The input data for further load increments is shown in Section B.4.1. The finite element mesh for this problem is shown in Figure 7.1 (Chapter 7)

# B.4.1 EXAMPLE OF INITIAL INPUT DATA

\*GENESYS

\*START 'LIFE' FILING AS 'SDP.PULL1'

JOB NON LINEAR PULLOUT TEST

\*TABLES

'COORDS'

| NODE | COORDS   | REP | ADDN | FINALXY   |
|------|----------|-----|------|-----------|
| 1    | 0,0      | 8   | 1    | 150,0     |
| 10   | 0,18.75  | 4   | 1    | 150,18.75 |
| 15   | 0,37.5   | 8   | 1    | 150,37.5  |
| 24   | 0,56.25  | 4   | 1    | 150,56.25 |
| 29   | 0,75     | 8   | 1    | 150,75    |
| 38   | -37.5,75 | 10  | 1    | 150,75    |

Topology details

'QUAD8SM'

| NODES                   | MATERIAL     | SECTION                  |
|-------------------------|--------------|--------------------------|
| 1,2,3,11,17,16,15,10    | 32.062,32958 | 2.16,0.0909,2.96,0.2,150 |
| 3,4,5,12,19,18,17,11    | =            | =                        |
| 5,6,7,13,21,20,19,12    | =            | =                        |
| 7,8,9,14,23,22,21,13    | =            | =                        |
| 15,16,17,25,31,30,29,24 | =            | =                        |
| 17,18,19,26,33,32,31,25 | =            | =                        |
| 19,20,21,27,35,34,33,26 | =            | =                        |
| 21,22,23,28,37,36,35,27 | =            | =                        |

Concrete elements

'BAR3'

| NODES    | MATERIAL | SECTION   | REP | ADDN |
|----------|----------|-----------|-----|------|
| 38,39,40 | 200000   | 100.53097 | 4   | 2    |

Bar elements

'BOND6'

| NODES             | MATERIAL | SECTION      | REP | ADDN |
|-------------------|----------|--------------|-----|------|
| 40,41,42,31,30,29 | ITYPE    | XPERIM,RT,RN | 3   | 2    |
| A,GRAD,DELTA,BETA |          |              |     |      |

Bond elements

'SUPPORTS'

| NODES      | SPRINGS |
|------------|---------|
| 29         | -1,-1   |
| J=30,37) J | 0,-1    |
| I=38,48) I | 0,-1    |
| 1,10,15,24 | -1,0    |

Boundary conditions

'LOAD'

| NODES | VALUES       |
|-------|--------------|
| 38    | -XLOAD*0.5,0 |
| 1     | 0,1.*PRESS   |
| 2     | 0,4.*PRESS   |
| 3     | 0,2.*PRESS   |
| 4     | 0,4.*PRESS   |
| 5     | 0,2.*PRESS   |
| 6     | 0,4.*PRESS   |
| 7     | 0,2.*PRESS   |
| 8     | 0,4.*PRESS   |
| 9     | 0,1.*PRESS   |

Nodal loads

'DUMMY'

| NODES     | VALUES |
|-----------|--------|
| J=1,48) J | 0,0    |

Dummy load case

'IDISP'

| NODES     | VALUES |
|-----------|--------|
| L=1,48) L | 0,0    |

Initialise nodal displacements

```

*MASTER
  XPERIM=50.26548*0.5
  XLOAD=2500.
  ITYPE=6
  RT=200.
  RN=1.E5
  A=2.
  GRAD=0.4
  DELTA=0.1
  BETA=0.5
  PRESS=0.*10000./24.
  PROBLEM TYPE IS 'CONCRETE'
  USE 'COORDS','SUPPORTS','QUAD8SM','BAR3','BOND6'
  USE 'LOAD' AS CASE 1
  START GAUSS STRESSES FOR ELEMENTS 'QUAD8SM','BAR3','BOND6'
  START EQUIVALENT UNIAXIAL STRAINS FOR ELEMENTS ...
  'QUAD8SM'
  START DISP 'IDISP'
  K=1
  MESSAGE
  MESSAGE 'LOAD INCREMENT = ',K
  MESSAGE
  CALC K
  RENUMBER
  ASSEMBLE AND REDUCE D
  SOLVE FOR CASE 1 D
  PRINT NO STRESSES FOR ELEMENTS 'QUAD8SM','BAR3','BOND6' ...
  FOR CASE 1
  SUMMATE GAUSS STRESSES
  SUMMATE EQUIVALENT UNIAXIAL STRAINS
  PRINT DISPLACEMENTS FOR CASE 1
  SUMMATE DISP CASE 1
  DO 10 I=1,8
  USE 'DUMMY' AS CASE 2
  MESSAGE
  MESSAGE 'ITERATION NO. ',I
  MESSAGE
  PRINT STRESSES FOR ELEMENTS 'QUAD8SM','BAR3' FOR CASE 1
  RESIDUAL FORCES FOR ELEMENTS 'QUAD8SM' CASE 2
  RESIDUAL FORCES FOR ELEMENTS 'BOND6' CASE 2 AND PRINT
  NORM OF LOAD VECTOR 2 R
  MESSAGE
  MESSAGE 'NORM OF LOAD VECTOR 2 = ',R
  MESSAGE
  IF(I.NE.1) GOTO 20
  TOLER=R/100.
  MESSAGE 'TOLERANCE VALUE = ',TOLER
  MESSAGE
  20 SOLVE FOR CASE 2 D
  PRINT NO STRESSES FOR ELEMENTS 'QUAD8SM','BAR3',...
  'BOND6' FOR CASE 2
  SUMMATE GAUSS STRESSES
  SUMMATE EQUIVALENT UNIAXIAL STRAINS
  IF(R.LT.TOLER)GOTO 30
  SUMMATE DISP CASE 2
  10 CONTINUE
  MESSAGE 'AFTER 8 ITERATIONS TOLERANCE IS ',R
  GOTO 100

```

Declaration of variables  
used in tables above

Initialise  
problem

Calculate global stiffness matrix

Solve for load increment

Accumulate Gauss point stresses  
Accumulate concrete uniaxial strain

Initialise dummy load vector

Calculate  
residual  
force vector &  
Euclidean  
Norm

Apply residual  
load vector and  
calculate stresses

Accumulate stresses  
and strains

```

30 SUMMATE DISP CASE 2 AND PRINT
   PRINT NODAL STRESSES FOR ELEMENTS 'QUAD8SM',... Nodal values of
   'BOND6','BAR3' FOR CASE 1 stress
UPDATE PROPERTIES FOR ELEMENTS 'QUAD8SM' AND PRINT
UPDATE PROPERTIES FOR ELEMENTS 'BOND6' AND PRINT
100 CONTINUE
1000 CONTINUE
*SAVE
*EXIT

```

#### B.4.2 EXAMPLE OF INPUT DATA TO APPLY THE NEXT LOAD INCREMENT

```

*GENESYS
*RESTART 'LIFE' FROM 'SDP.PULL1' FILING AS 'SDP.PULL2'
JOB NON-LINEAR PULLOUT TEST
*TABLES
  'LOAD'
    NODES      VALUES
    38         -XLOAD*0.5,0
*MASTER
  XLOAD=2500.                                Size of the load increment
  USE 'LOAD' AS CASE 1
  MESSAGE
  TLOAD=2.5                                    Previous total load
  TINC=2.5                                    Size of the increment
  K=2                                         Increment number
  TLOAD=TLOAD+TINC
  MESSAGE
  UPDATE PROPERTIES FOR ELEMENTS 'QUAD8SM' AND PRINT  Update properties
  UPDATE PROPERTIES FOR ELEMENTS 'BOND6' AND PRINT
  MESSAGE 'LOAD INCREMENT = ',K,' TOTAL LOAD = ',TLOAD
  MESSAGE
  CALC K                                         New stiffness matrix
  RENUMBER
  ASSEMBLE AND REDUCE D
  SOLVE FOR CASE 1 D                           Solve for increment
  PRINT NO STRESSES FOR ELEMENTS 'QUAD8SM','BAR3','BOND6' ...
    FOR CASE 1
  SUMMATE GAUSS STRESSES                       Accumulate stresses
  SUMMATE EQUIVALENT UNIAXIAL STRAINS          and strains and
  PRINT DISPLACEMENTS FOR CASE 1              displacements
  SUMMATE DISP CASE 1
  200 DO 10 I=1,8
  USE 'DUMMY' AS CASE 2                        Iterate
  MESSAGE
  MESSAGE 'ITERATION NO. ',I
  MESSAGE
  PRINT STRESSES FOR ELEMENTS 'QUAD8SM','BAR3' FOR CASE 1
  RESIDUAL FORCES FOR ELEMENTS 'QUAD8SM' CASE 2
  RESIDUAL FORCES FOR ELEMENTS 'BOND6' CASE 2 AND PRINT
  NORM OF LOAD VECTOR 2 R
  MESSAGE
  MESSAGE 'NORM OF LOAD VECTOR 2 = ',R
  MESSAGE
  IF(I.NE.1) GOTO 25
  TOLER=R/100.
  RMAX=R

```

```

MESSAGE 'TOLERANCE VALUE = ',TOLER
MESSAGE
  25 IF(R.GT.RMAX)GOTO 500
  19 RMAX=R
  20 SOLVE FOR CASE 2 D
PRINT NO STRESSES FOR ELEMENTS 'QUAD8SM','BAR3','BOND6' ...
  FOR CASE 2
SUMMATE GAUSS STRESSES
SUMMATE EQUIVALENT UNIAXIAL STRAINS
  IF(R.LT.TOLER)GOTO 30
SUMMATE DISP CASE 2
  10 CONTINUE
MESSAGE 'AFTER 8 ITERATIONS TOLERANCE IS ',R
GOTO 1000
30 SUMMATE DISP CASE 2 AND PRINT
  PRINT NODAL STRESSES FOR ELEMENTS 'QUAD8SM','BOND6','BAR3' ...
  FOR CASE 1
GOTO 100
500 MESSAGE 'ITERATION PROCESS IS DIVERGING'
  MESSAGE 'ITERATION ',I-1,'NORM =',RMAX
  MESSAGE 'ITERATION ',I,'NORM =',R
GOTO 19
100 CONTINUE
1000 CONTINUE
*SAVE
*EXIT

```

## APPENDIX C

### PROGRAMMERS GUIDE TO THE AUTHOR'S PROGRAMS

#### C.1 INTRODUCTION.

This appendix briefly explains the form of a subsystem and the particular additions to the LUFE subsystem made by the author. The programs are briefly explained here and listed in Appendix D. More information on the Genesys System subsystem and the programming language GENTRAN can be obtained from the Genesys Reference manual (1972).

#### C.2 GENESYS SUBSYSTEM.

Each subsystem of one or more 'overlays' and each 'overlay' is a collection of subprograms (similiar to subroutines in FORTRAN). When a particular problem orientated command is used in the \*MASTER segment of an engineering job the required 'overlay' is performed. The command not only dictates which 'overlay' is to be invoked but also the entry point to the 'overlay'. Within the 'overlay' the entry subprograms are declared at the beginning. Data can be passed between 'overlays' by declaring the required variables in a PUBLIC block similiar to the COMMON block in FORTRAN.

#### C.3 ADDITIONS TO LUFE MADE BY THE AUTHOR

The author has added three 'overlays' to the LUFE subsystem and these are now stated indicating the subprograms contained within them.

| overlay title : | CONC1    | CONC2    | CONC3    |         |
|-----------------|----------|----------|----------|---------|
| subprograms :   | STIFF *  | PREP1    | NORM *   | SHP3    |
|                 | PREP1    | PREP2    | PREP3    | RESID * |
|                 | PREP2    | PREP3    | PREP4 *  | RBOND   |
|                 | PREP3    | STRS *   | UPDATE * | RQUAD   |
|                 | QUAD8    | SHP8     | DARWIN   | SAENZ   |
|                 | GAUSS    | CONSX8   | STRAIN   | BOND    |
|                 | SHP8     | GAUSS    | CRACK *  | ULTBND  |
|                 | BAR3     | BAR3S    | EMOD     | LSQFIT  |
|                 | ISTRES * | SUMMAT * | SHP8     |         |
|                 | BOND6    | BND6S    | GAUSS    |         |

\* Entry points

'CONC1' deals with the initialisation of a number of arrays and the calculation of the elemental stiffness matrices for all element types.

'CONC2' deals with the calculation and the accumulation of Gauss point stresses and strains of each element type and the accumulation of nodal displacements.

'CONC3' deals with updating material properties monitored at the Gauss points of each element type. The calculation of the theoretical stresses, residual stresses and residual nodal forces for use in the initial stress method of force correction. The bond and concrete models are contained in this part of the program.

#### C.4 EXPLANATION OF THE SUBPROGRAMS.

All the subprograms written by the author are now described and are listed in Appendix D. A dictionary of variable names is given in Appendix E.

##### C.4.1 Subprogram STIF

This subprogram is mainly a steering routine to calculate the local element stiffness matrices and accumulate in the global stiffness matrix (part of VALS (,,)). The available elements for reinforced concrete are defined in TABLE (,) along with the number of nodes and material properties per element. The number of elements and the number of dimensions are obtained from MARK ( ). Subprogram PREP1 is called to find the names of the elements and how many of each type. A loop is invoked for each element of a particular type (IT) and subprogram PREP2 is called to find the local node numbers, co-ordinates and geometrical properties. The correct stiffness matrix subprogram is called which returns the elemental stiffness matrix in vector ST ( ).

#### C.4.2 Subprogram PREP1

This subprogram finds the string name of an element ITYPE (IT) and identifies this type of element in NICK ( ). The arrays ELEMS (,,) and VALS (,,) are redefined to accommodate the element type if it has not been previously called.

#### C.4.3 Subprogram PREP2

For each element J of element type IT this subprogram finds the number of local nodes NN and the co-ordinates of the local nodes ELCO (,) from COORDS (,). When this subprogram is called from STIF, L=0, and VALS (,,) is redefined to accommodate the elemental stiffness matrix.

#### C.4.4 Subprogram PREP3

This subprogram finds the element type number KT for element string NAME. The element type number KT corresponds to the order the elements were invoked in the START GAUSS STRESSES... command.

#### C.4.5 Subprogram QUAD8

This subprogram calculates the element stiffness matrix for an 8-noded isoparametric quadrilateral element using 3x3 Gauss point integration with two degrees of freedom per node. The stiffness ES (,) is initially set to zero. The Gauss points are numbered (1 to 8) in an anti-clockwise order from the one nearest the local element node number 1 and the central Gauss point numbered 9. A loop is invoked through all the Gauss points and the subprogram SHP8 is called to obtain the components of the [ B ] matrix, AA(,) and the determinant of the Jacobian DET. The components of the [ D ] matrix are obtained (Equation 4.8) and the contributions to the stiffness matrix for the Gauss point are calculated and accumulated in ES (,). Since the elemental stiffness

matrix is symmetric the lower triangular part of the stiffness matrix is calculated and then these values copied into the upper triangle.

#### C.4.6 Subprogram GAUSS

For the appropriate order of Gauss integration (either 2 or 3 point rule) this subprogram sets up the Gauss point co-ordinates GP ( ), the weighting factors HG ( ) and a control vector, KONTR0L ( ) which holds the standard Gauss reference numbers for 2x2 or 3x3 pattern of points.

#### C.4.7 Subprogram SHP8

This subprogram calculates the Cartesian derivatives of the nodal shape functions ,i.e. [ B ] matrix and the determinant of the Jacobian (DET). for the 8-noded isoparametric element. ZE and ET are the  $\xi, \eta$  co-ordinates of a point respectively and ZI ( ) and EI ( ) are the  $\xi, \eta$  co-ordinates of the local nodes. The derivatives of the nodal shape functions with respect to the local axes are held in A ( ). ACOB is the Jacobian and BACO contains the inverse of the Jacobian. If DET is zero an error message is printed and the error trap MARK (12) is set to 1 .

#### C.4.8 Subprogram BAR3

This subprogram calculates the elemental stiffness matrix for a 3-noded axial force bar element. The overall length of the element (S) is calculated from the co-ordinates of the nodes and the coefficient (F)  $EA / 3 S^3$ . A loop is invoked to calculate all the terms of the stiffness matrix directly.

#### C.4.9 Subprogram ISTRES

This subprogram is essentially for initialising the reinforced concrete problem and is called to carry out one of three functions either :

- (a) allocates zero Gauss point stresses to the arrays AGAUSS (,,) and GSTRESS (,,) and initialises the properties monitored at the Gauss points of the elements called in the START GAUSS STRESSES ... command. or
- (b) allocates zero Gauss point equivalent uniaxial strains to STRAIN (,) and ASTRAN (,) when called using START EQUIVALENT UNIAXIAL STRAINS. or
- (c) allocates nodal displacements to the vector DISPL (,) as read from a table of initial nodal displacements.

The clause number of the command START is obtained to find which function of (a), (b), (c) is invoked and the appropriate coding of the subroutine entered.

(a) The elements called in the command are checked against the standard elements in TABLE (,) and if there are any errors MARK (12) is set to 1. TABLE (,) also specifies for each element type, the number of Gauss points for each element for storing stresses and the number of material properties monitored at each Gauss point. The order of the names of the elements invoked in the command is stored in KNAME (). The arrays AGAUSS (,,) , GSTRESS (,,) and GPROPS (,) are redefined to accommodate the Gauss point stresses and material properties. The material properties at each Gauss point for elements QUAD8SM and BOND6 are initialised from ELEMS (,,).

The properties monitored at each Gauss point are for :

#### QUAD8SM

|     |                  |    |       |                  |     |
|-----|------------------|----|-------|------------------|-----|
| YM1 | initially set at | EO | PR    | initially set at | PR  |
| YM2 | ditto            | EO | THETA | ditto            | 0.0 |

#### BOND6

|    |                  |       |
|----|------------------|-------|
| RT | initially set as | $R_0$ |
| RN | ditto            | $R_n$ |

The QUAD8SM nodal and BAR3 nodal stress vectors CSIGMA ( ) and BSIGMA ( )

respectively are defined and set to zero.

(b) The element type in ITYPE corresponding to QUAD8SM is found and the equivalent uniaxial strain arrays STRAIN (,) and ASTRAN (,) are re-defined. The array NSTATE (,) is also set up and is a flag which gives the current status of the concrete Gauss points as to whether they have failed or not in the principal stress directions.

(c) The name of the displacement table TAB is obtained and the vector DISPL (,) is defined to be of length 2NP where NP is the number of nodal points and set to zero. The table TAB is read and an error message output if the data is not of the form : node no., x-displacement, y-displacement. If the read node number is greater than NP an error message is given.

#### C.4.10 Subprogram BOND6

This subprograms calculates the elemental stiffness matrix for a 6-noded shearing element. The stiffness matrix ES (,) is initially set to zero and the length of the bond element (X) obtained. The Gauss co-ordinates and weighting factors for a 3 point Gauss integration rule are set up as GP ( ) and HG ( ) respectively. A loop is invoked through all the Gauss points and at each Gauss point the nodal shape functions are evaluated, the current bond moduli (RT, RN) obtained from GPROPS (,) and the contributions to the stiffness matrix for the integrating point are calculated and accumulated in ES (,).

#### C.4.11 Subprogram STRS

This subprogram is mainly a steering routine to calculate stresses for the various elements and is similiar to the subprogram STIF. The elements for which stresses may be calculated is given in TABLE (,). A loop is invoked for each element type and for each element

the local node no's., geometrical properties and nodal displacements are obtained. The subprogram corresponding to the element type is called for each element to calculate the required stresses which may be either (a) Gauss point stresses due to the load increment or (b) nodal values of stress extrapolated from the accumulated values of stress at the Gauss points. ISTRS determines whether (a) or (b) is calculated in the sub-program call. If ISTRS is equal to 0 or 4 then nodal average stresses for each element type are calculated. Nodal average values of stress for the concrete elements are put into CSIGMA ( ) and nodal average values for the steel into BSIGMA ( ).

#### C.4.12 CONSX8

This subprogram calculates values of stress for an 8-noded iso-parametric quadrilateral element which represents the concrete. Calculations are either: (a) 3x3 Gauss point stresses due to a load increment or (b) nodal values of stress extrapolated from the accumulated values of stress held at the Gauss points in AGAUSS (,). For (a) strains are initially evaluated at the 2x2 Gauss points. A call to subprogram GAUSS sets up the Gauss points GP ( ) and the weighting factors HG ( ). The nodal displacements due to the incremental load are held in DE ( ). A loop is invoked through all 4 Gauss points and the subprogram SHP8 is called to obtain the components of the [ B ] matrix held in AA (,) and the contributions to the strains calculated and accumulated in STRN2 (,). The strains at all the 2x2 Gauss points are then extrapolated in a bi-linear fashion to the 3x3 Gauss points. AH ( ) contains the weighting factors for the extrapolation and the 3x3 Gauss point strains are held in STRN3 (,). Stresses at the 3x3 Gauss points are then calculated from the strains held at these points by the simple calculation of :

$$[ \sigma ] = [ D ] [ \epsilon ]$$

These values of stress at the 3x3 Gauss points due

to the increment of load are then held in GSTRESS (,). For the dummy load case only the equivalent uniaxial strains are calculated (Equation 4.14) from the Gauss point stresses and held in STRAIN (,).

For (b), the accumulated Gauss point stresses are in AGAUSS (,) and a bi-linear extrapolation is performed on the stresses at the four outer 3x3 Gauss points to the nodes. The weighting factors are held in AH ( ) and the final nodal values of stress are held in SIGMA ( ).

#### C.4.13 Subprogram BAR3S

This subprogram calculates bar stresses for the 3 noded axial force bar element representing the steel. Calculations are either:

- (a) 2 point Gauss bar stresses due to the increment of load or
- (b) nodal values of stress extrapolated from the accumulated values held at the 2 Gauss points. For (a) the nodal displacements due to the load increment are held in DE ( ) and the bar stresses at the Gauss points GS1 and GS2 are calculated directly using the exact [ B ] matrix coefficients C1, C2, C3 and these incremental stresses are held in GSTRESS. For (b) the accumulated values of bar stress at the 2 Gauss points are held in AGAUSS (,) and the coefficient for a linear expansion to the nodes are C1 and C2. The nodal values of bar stress and the equivalent bar radial pressure (Equation 3.8) are held in SIGMA ( ). An error message is printed if any of the bar stresses exceed  $460 \text{ N/mm}^2$  (taken to be the yield stress).

#### C.4.14 Subprogram BND6S

This subprogram calculates the bond stresses for 6-noded bond shearing element. Calculations are either : (a) bond stresses evaluated at the 3 Gauss points due to an increment of load or (b) nodal values of bond stress extrapolated from the accumulated values of stress held

at the 3 Gauss points. The Gauss coefficients are held in GP ( ), the weighting factors in HG ( ), and the bond slip parallel and normal to the bar at the nodes are held in U ( ) and V ( ). A loop is invoked through all the Gauss points , A ( ) holds the evaluated nodal shape functions and the current bond moduli at the Gauss points are held in R ( ). Contributions to the bond stress are calculated and accumulated in GS ( ). The final values of Gauss point bond stress are held in GSTRESS (,). For (b) the coefficients for an extrapolation to the nodes using the nodal shape functions from the Gauss points are C1, C2, C3. The accumulated values of bond stress at the 3 Gauss points are held in AGAUSS (,) and the nodal values calculated from the Gauss point values are held in SIGMA ( ).

#### C.4.15 Subprogram SUMMAT

This subprogram has several purposes namely :

- (a) adds the nodal displacements due to a load increment to the accumulated nodal displacements held in DISPL (,) or
- (b) adds the Gauss point stresses for each element type due to an increment of load to the accumulated values of stress held in AGAUSS (,) or
- (c) adds the equivalent uniaxial strains at the Gauss points of the concrete elements due to the load increment to the accumulated values of strain held in ASTRAN (,).

For (a) the nodal displacements due to the load case LDCSE are extracted from VALS (,,) and held in vector D ( ). A loop is invoked for all nodes (NP) and the components of D ( ) are added to DISPL (,). For (b) a loop is invoked through all element types and then for each element. The Gauss point values of stress held in GSTRESS (,) are added to the accumulated values of stress held in AGAUSS (,). For (c) a loop is invoked through all concrete elements. The incremental values of equivalent uniaxial

strain in STRAIN (,) are added to the accumulated values in ASTRAN (,). Variables K and IPRINT are flags for printing and equal to 3 means print.

#### C.4.16 Subprogram NORM

This subprogram calculates the Euclidean norm of the load vector load case ILOAD. The load vector corresponding to case ILOAD is extracted from VALS (,,) and held in RELOAD. A loop is invoked and RNORM is the accumulated value of the components squared. The square root of RNORM gives the Euclidean norm and is given back to the problem orientated command using the PUT statement.

#### C.4.17 Subprogram PREP4

This subprogram either stores or retrieves the data variables stored in Public arrays IDATA ( ) and RDATA ( ). The clause number ICASE is obtained which is 1 for storing data and 2 for retrieving data. KCASE is the clause number corresponding to whether integer or reals need to stored or retrieved. In the case of storing the data variables they are read from the list in the command and stored in RDATA ( ) for reals and IDATA ( ) for integers. A suitable message notes the stored values. Retrieved data variables are given to the variables in the problem orientated command.

#### C.4.18 Subprogram UPDATE

This subprogram for an element type (either BOND6 or QUAD8SM) calculates the new properties at each Gauss point of each element. For the BOND6 elements new bond moduli are calculated which depend on the amount of slip, concrete lateral pressure and bar radial pressure effects. The bond moduli are the gradients of the tangents to the local bond stress-slip relationships. For the concrete elements (QUAD8SM) the new Young's

moduli of elasticity are the tangent slopes of the stress-strain curves which depend on the accumulated stresses and accumulated equivalent uniaxial strains. The element types to be updated are read from the problem orientated command into NAME ( ). NUM is the number of element types . Depending on the element type one of two sections of code is enacted.

#### BOND6

A loop is invoked through all elements and for each element the local node numbers NN ( ), accumulated nodal displacements UN ( ), nodal concrete lateral pressures CSIGMA ( ) and the nodal equivalent bar radial pressures BSIGMA ( ) are obtained. Subprogram LSQFIT is called to fit linear functions by least squares to the nodal values of concrete lateral pressure against position and bar radial pressures against position. Values of pressure at the 3 Gauss points are obtained from these regression lines. The bond slip at each Gauss point is evaluated using the nodal values of displacement for the concrete and bar nodes (UN) and the nodal shape functions by calling subprogram SHP3. The new bond moduli RT and RN are calculated with the subprogram BOND. IPRINT is a flag to print or not and equal to 1 means print.

#### QUAD8SM

A loop is invoked through all the elements and for each element the local node numbers NN ( ), accumulated nodal displacements UN ( ), global co-ordinates of the nodes ELCO (,) and the concrete properties FCU, EO, FT, ECU, ETU are obtained. A loop is invoked through all the Gauss points of an element and NSTATE (,) is checked to see whether the point has failed. If the concrete has not failed the accumulated stresses AGAUSS (,) are transformed into principal stresses P ( ). If the point has failed the direction of the failure plane is obtained (FI) and the accumulated stresses are transformed to stresses SNN, SCC, normal and parallel to the

plane of failure. The accumulated equivalent uniaxial strains are extracted ASTRAN (,) and the subprogram DARWIN is called to calculate the predicted failure stress and strains (FSTRES (2), FSTRAN (2)) and obtain NFAIL. The subprogram EMOD calculates the updated Young's moduli, angle FI and these are stored in GPROPS (,). KPRINT is a flag to print or not.

#### C.4.19 Subprogram DARWIN

This subprogram is the author's interpretation of the Darwin and Pecknold (1974) concrete model, with Poisson's ratio constant and the parameter R equal to 3. S1 and S2 are the current principal stresses and E (1) and E (2) are the current accumulated equivalent uniaxial strains. S1 and S2 are assigned to XMAX and XMIN where XMAX is greater than XMIN. The coding for the appropriate stress state is entered (i.e. either the stresses XMAX,XMIN are biaxial tension, biaxial compression or tension-compression.) The predicted failure stresses from the Kupfer failure envelope are given by FSTRES ( ) and the corresponding equivalent uniaxial failure strains by Darwin and Pecknold's model as FSTRAN ( ). NFAIL is a flag to indicate whether failure has occurred for the current stresses or strains. For each region the stresses are first checked whether they lie outside the biaxial failure envelope so indicating a stress failure. The predicted failure stresses and strains are then evaluated. A check on the strains is then performed and failure by strain flagged or not. If FT has been set at a value greater than  $10. \text{ N/mm}^2$  then the concrete is treated as an artificial material within no tension failure.

#### C.4.20 Subprogram STRAIN

This subprogram checks the compatibility of the current equivalent uniaxial strains E ( ) with the predicted failure values of equivalent

uniaxial strain FSTRAN ( ). If in each principal stress direction the two values of strain from E ( ) and FSTRAN ( ) are not of the same sign in each principal stress direction then a suitable error message is output. If the current strain is greater in magnitude than the corresponding failure strain then the flag IFAIL is increased by one.

#### C.4.21 Subprogram CRACK

This subprogram checks all the concrete elements and their Gauss points to see if they have failed. The total number of new failed points is returned to the problem orientated command (IFAIL). For each concrete element a loop is invoked through all Gauss points. The current status of the point is obtained ISTATE (,) from NSTATE (,). If the point has failed in both directions no further calculations are necessary otherwise the accumulated values of stress are obtained. If the point has failed in one direction only, then these stresses are transformed to the crack plane to obtain SNN and SCC, otherwise principal stresses P (1) and P (2) are calculated. The accumulated equivalent uniaxial strains E ( ) are obtained and the subprogram DARWIN called to check on the failure of the concrete. If any new failures are indicated then the flag NSTATE (,) is changed and IFAIL the number of new failures increased by one.

#### C.4.22 Subprogram EMOD

This subprogram calculates the current values of Young's modulus in the two principal stress directions given the failure stresses FSTRES ( ) and strains FSTRAN ( ) and the current uniaxial strains E ( ) in each direction. For a compressive stress the Saenz curve is fitted and for a tensile stress a straight line to the failure stress and strain. For strains greater in magnitude than the failure strain Young's modulus is assumed to be zero.

#### C.4.23 Subprogram SHP3

This subprogram evaluates the nodal shape functions  $A ( )$  for a 3-noded isoparametric line element for local co-ordinate ZE.

#### C.4.24 Subprogram RESID

This subprogram is mainly a steering routine to calculate the contributions to the consistent nodal force vector from the residual stresses at the Gauss points of either QUAD8SM or BOND6 elements. For the element type NAME (I) a loop is invoked for each element J and the co-ordinates of the nodes ELCO (,) and the accumulated nodal displacements UN ( ) for this element obtained. For a bond element the nodal average values of concrete lateral pressure (LP1, LP2, LP3) and equivalent bar radial pressure (RS1, RS2, RS3) obtained. The subprogram LSQFIT is called to obtain the linear least squares line to these nodal values. The relevant subprogram is called to obtain the residual forces at the nodes for this element RLOAD ( ) and the values are added to VALS (,ILOAD,2) where ILOAD is the dummy load case.

#### C.4.25 Subprogram ROUAD

This subprogram calculates the consistent nodal force vector for a QUAD8SM element. The concrete parameters FCU, EO, THICK, ECU, ETU, FT are obtained for the element J from array ELEMS (,,). A loop is invoked through all Gauss points and the accumulated values of stress SXX, SYX, TXY are obtained and the current status of failure ISTATE ( ). If the Gauss point has failed the stresses SXX, SYX, TXY are transformed on to the plane of the crack at angle FI to obtain stresses SNN and SCC. For uncracked concrete the principal stresses P (1) and P (2) are calculated. The accumulated values of equivalent uniaxial strain are obtained E (1) and E (2). Subprograms DARWIN and SAENZ are called to evaluate the

theoretical stresses in the current principal stress directions THEOP ( ). The residual stresses are calculated RP ( ) and transformed back to the global axes as RSXX, RSY, RXY. The residuals are then deducted from the accumulated stresses AGAUSS (,,). Finally a loop is invoked through all Gauss points and the contributions to the residual load vector RELOAD calculated from the residuals.

#### C.4.26 Subprogram RBOND

This subprogram calculates the consistent nodal force vector for a BOND6 element. The bond type KGEOM is obtained from ELEMS (9,J,IT). The Gauss co-ordinates GP ( ) and weightings HG ( ) are set up. For each Gauss point (IG) the slips SLIP and SLIPN are evaluated from the nodal values of slip and the values of concrete lateral pressure (LATPR) and equivalent bar radial pressure (RADPR) calculated using the coefficients of regression ALP, BLP and ARS, BRS respectively. The subprogram BOND is called to find the theoretical stress THSTRS. The residual bond stress RSTRSS (IG) is calculated and deducted from the accumulated bond stress AGAUSS (,,). Finally a loop is invoked and for each Gauss point its contributions to the residual force vector RLOAD ( ) are calculated.

#### C.4.27 Subprogram SAENZ

This subprogram calculates the theoretical principal stress THEOP ( ) for the current equivalent uniaxial strain in each of the principal stress directions given the ratio of current principal stresses. If the concrete has failed in one of the principal stress directions (given by ISTATE (,)) then the theoretical stress is set to zero. For a compressive stress the Saenz curve (Chapter 4) is fitted to the predicted failure stress, obtained from the Kupfer failure envelope and the predicted equivalent uniaxial failure strain obtained from the Darwin

model. The initial tangent modulus is taken as  $E_0$ . For a tensile stress, a straight line is fitted from zero to the failure stress and strain. If the current strain is greater in magnitude than the failure strain the theoretical stress is deemed to be the failure stress.

#### C.4.28 Subprogram BOND

This subprogram calculates the theoretical bond stress or tangent bond modulus for a bond element parallel and normal to the bar axis. The bond parameters : YINTER, GRAD, DELTA, BETA are extracted from ELEMS (,,). KGEOM directs which bond model is being used. S is the theoretical bond stress parallel to the bar axis, RT is the bond modulus in this direction and RN is the bond modulus orthogonal to RT. D and DN are the slips parallel and orthogonal to the bar. Bond models available are :

- (1) RJA Type
- (2) Nilson's relationship (1972)
- (3) Quadratic bond stress-slip curve 1
- (4) Quadratic bond stress-slip curve 2
- (5) Desayi curve - varying ultimate bond stress
- (6) Saenz curve - varying ultimate bond stress
- (7) Approximations to the Labib and Edwards curves (1976)
- (8) - Linear

#### C.4.29 Subprogram ULTBND

This subprogram calculates the ultimate local bond stress (UBMAX) from the following parameters : concrete lateral pressure (LATPR), the equivalent bar radial pressure (RADPR) and the coefficients of the bond model (Chapter 3), the  $q_0$  parameter (YINTER) and the slope of the ultimate bond stress-radial pressure line (GRAD). UBMAX is always greater than or equal to zero.

#### C.4.30 Subprogram LSOFIT

This subprogram calculates the coefficients (A,B) of the least squares fit of  $y = A + Bx$  to the data points  $X_i, Y_i$  ( $i = 1$  to  $N$ ) held in XYCORDS (,). N is the total number of data points.

# APPENDIX D LISTING OF THE AUTHOR'S PROGRAMS

## D.1 OVERLAY CONCL

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*GENESYS
*GENTRAN 'LUF'
AT FINISH,ERRORS STATISTICS
COMPILE OVERLAY 'CONCL' AS 33
*OVERLAY
    'CONCL'
SUBPROGRAMS STIF,PREP1,PREP2,PREP3,QUAD8,GAUSS,SHP8,...
    BAR3,ISTRES,BOND6
ENTRY STIF,ISTRES
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    SUBROUTINE STIF
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    PUBLIC MARK(20),KT(20),VALS(,,),NS(,,)
    LOCAL NE(10),GEOM(10),ELCO(10,3)
    DIMENSION TABLE(3,10)
    INTEGER TABLE
    NTAB=3
    TABLE(1,1)='QUAD8SM',8,7
    TABLE(1,2)='BAR3',3,2
    TABLE(1,3)='BOND6',6,8
    N=MARK(11)
    NDIMEN=MARK(2)
    NFREE=MARK(10)
    DO 100 I=1,N
        IT=KT(I)
        CALL PREP1(IT,NAME,M,NICK)
        NREF=1
        DO 300 J=1,NTAB
            IF(NICK.EQ.TABLE(1,J)) GO TO 301
300 NREF=NREF+1
        MESSAGE 'NO STIFFNESS ROUTINE FOR ELEMENTS TYPE ',NAME
        MARK(12)=1
        GO TO 100
301 NNCH=TABLE(2,NREF)
    NPCH=TABLE(3,NREF)
    DO 200 J=1,M
        CALL PREP2(J,IT,NN,NE,NP,GEOM,ELCO,0)
        IF(NN.EQ.NNCH.AND.NP.EQ.NPCH) GO TO 202
        MESSAGE 'ELEMENT TYPE ',NAME,' NO. ',J,' HAS EITHER ;-'
        MESSAGE 'WRONG NO. OF NODES OR PROPERTIES'
        MARK(12)=1
        GO TO 200
202 EQUATE 199 (VALS(,J,IT),ST)
    GO TO (1,2,3,4,5,6,7,8,9,10),NREF
    1 CALL QUAD8(GEOM,ELCO,ST(3),J)
    GOTO 99
    2 CALL BAR3(GEOM,ELCO,ST(3),NE)
    GOTO 99
    3 CALL BOND6(GEOM,ELCO,NE,ST(3),J,IT)
    GOTO 99
    4 CONTINUE
    5 CONTINUE
    6 CONTINUE
    7 CONTINUE

```

```

      8 CONTINUE
      9 CONTINUE
     10 CONTINUE
     99 N2=NN*NFREE
        ST(1)=N2,N2
        N3=N2*N2+4
        ST(N3-1)=NN
        ST(N3)=NFREE
        DO 199 K=1,NN
          ST(N3+K)=NE(K)
     199 CONTINUE
        RELEASE VALS(,J,IT)
        K=NN+1
        REDEFINE (NS(,J,IT),K)
        EQUATE 200 (NS(,J,IT),NNQ)
        NNQ(1)=NN
        DO 201 K=1,NN
     201 NNQ(K+1)=NE(K)
     200 CONTINUE
     100 CONTINUE
        RETURN
        END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  SUBROUTINE PREP1(IT,NAME,M,NICK)
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  PUBLIC ITYPE(,),VALS(,,),NS(,,),ELEMS(,,)
  DIMENSION ISTR(70)
  FIX 10 ISTR
  NAME=ITYPE(IT)
  EXPLODE NAME (ISTR(1),NCS)
  DO 5 J=1,NCS
    I=J-1
    IF(ISTR(J).EQ.'/') GO TO 6
  5 CONTINUE
    I=NCS
  6 IMplode NICK (ISTR(1),I)
    LENGTH (ELEMS(,,IT),M)
    LENGTH (VALS(,,),K)
    IF(IT.GT.K) REDEFINE (VALS(,,),IT),(NS(,,),IT)
    REDEFINE (VALS(,,IT),M),(NS(,,IT),M)
  10 CONTINUE
    RETURN
    END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  SUBROUTINE PREP2(J,IT,NN,NE,NP,GEOM,ELCO,L)  ''L=0 FOR STIFFNESS
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  PUBLIC ELEMS(,,),COORDS(,),MARK(20),VALS(,,)
  LOCAL NE(10),GEOM(10),ELCO(10,3)
  EQUATE 1 (ELEMS(,J,IT),X)
  NN=X(1)+0.1
  NP=X(2)+0.1
  DO 3 K=1,NN
  3 NE(K)=X(2+K)+0.1
  DO 4 K=1,NP
  4 GEOM(K)=X(2+K+NN)
  1 CONTINUE
    NDIMEN=MARK(2)
    NFREE=MARK(10)

```

```

RELEASE ELEMS(,J,IT)
DO 5 K=1,NDIMEN
EQUATE 5 (COORDS(,K),A)
DO 5 K1=1,NN
K2=NE(K1)
5 ELCO(K1,K)=A(K2)
IF(L.NE.0) RETURN
K=(NN*NFREE)**2+NN+4
REDEFINE (VALS(,J,IT),K)
RETURN
END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
SUBROUTINE PREP3(NAME,KT)
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
PUBLIC KNAME()
LENGTH (KNAME(),LEN)
DO 10 KT=1,LEN
IF(KNAME(KT).EQ.NAME) GOTO 20
10 CONTINUE
20 RETURN
END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
SUBROUTINE QUAD8(GEOM,ELCO,ES,JT)
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
PUBLIC MARK(20),GPROPS(,,)
LOCAL ELCO(10,3),GEOM(7),ES(16,16)
LOCAL KONTRL(9),AA(2,8),GP(3),HG(3)
DIMENSION EI(8),ZI(8)
FIX 40 EI,ZI
ZI(1)=-1,0,1,1,1,0,-1,-1
EI(1)=-1,-1,-1,0,1,1,1,0
LL=0
DO 1 I=1,16
DO 1 J=1,16
1 ES(I,J)=0.0
NG=3
CALL GAUSS(NG,GP,HG,KONTRL)
CALL PREP3('QUAD8SM',KT)
LENGTH(GPROPS(,1,KT),NGPROP)
NPROPS=NGPROP/NG/NG+0.1
DO 10 IG=1,NG ''G.P.S EXECUTED IN UNSTANDARD ORDER
ZE=GP(IG)
DO 10 JG=1,NG
ET=GP(JG)
LL=LL+1
N=KONTRL(LL) '' FINDS STANDARD G.P. REFERENCE
CALL SHP8(ZE,ET,ZI(1),EI(1),ELCO,AA,DET)
IF(DET.LE.0.0) RETURN
C=DET*GEOM(7)*HG(IG)*HG(JG)
'' EVALUATE D MATRIX FOR N TH GAUSS POINT
EQUATE 11 (GPROPS(,JT,KT),G)
N=(N-1)*NPROPS+1
E1=G(N)
E2=G(N+1)
PR=G(N+2)
11 THETA=G(N+3)
C=C/(1.-PR*PR)
D1=E1*(COS(THETA))**2+E2*(SIN(THETA))**2

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```

D2=PR*SQRT(E1*E2)
D3=0.5*(E1-E2)*COS(THETA)*SIN(THETA)
D4=E1*(SIN(THETA))**2+E2*(COS(THETA))**2
D5=D3
D6=0.25*(E1+E2-2.*D2)
12 DO 30 I=1,8
    BI=AA(1,I)
    CI=AA(2,I)
    DO 30 J=I,8
        BJ=AA(1,J)
        CJ=AA(2,J)
        K=2*I-1
        L=2*J-1
        ES(K,L)=ES(K,L)+C*(BI*(D1*BJ+D3*CJ)+CI*(D3*BJ+D6*CJ))
        ES(K,L+1)=ES(K,L+1)+C*(BI*(D2*CJ+D3*BJ)+CI*(D5*CJ+D6*BJ))
        ES(K+1,L)=ES(K+1,L)+C*(CI*(D2*BJ+D5*CJ)+BI*(D3*BJ+D6*CJ))
30 ES(K+1,L+1)=ES(K+1,L+1)+C*(CI*(D4*CJ+D5*BJ)+BI*(D5*CJ+D6*BJ))
10 CONTINUE
    DO 40 J=1,16
    DO 40 I=J,16
40 ES(I,J)=ES(J,I)
    RETURN
    END
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    SUBROUTINE GAUSS (NG,GP,HG,KONTRL)
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    LOCAL GP(3),HG(3),KONTRL(9)
    GOTO(2,2,3),NG
    2 Z=0.577350269189626
    GP(1)=Z
    GP(2)=Z
    HG(1)=1.
    HG(2)=1.
    KONTRL(1)=3,2,4,1
    GOTO 100
    3 Z=0.774596669241483
    GP(1)=Z
    GP(2)=0.
    GP(3)=Z
    HG(1)=0.5555555555555556
    HG(2)=0.8888888888888889
    HG(3)=HG(1)
    KONTRL(1)=5,4,3,6,9,2,7,8,1
100 RETURN
    END
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    SUBROUTINE SHP8(ZE,ET,ZI,EI,ELCO,AA,DET)
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    LOCAL EI(8),ZI(8),ELCO(10,3),AA(2,8),ACOB(2,2),BACO(2,2),A(2,8)
    PUBLIC MARK(20)
    DO 11 I=1,7,2
        ZEI=ZI(I)
        ETI=EI(I)
        ZEO=ZEI*ZE
        ETO=ETI*ET
        A(1,I)=ZEI*(1.0+ETO)*(2.0*ZEO+ETO)/4.0
11 A(2,I)=ETI*(1.0+ZEO)*(2.0*ETO+ZEO)/4.0
    DO 12 I=2,6,4

```

```

      ETI=EI(I)
      ETO=ETI*ET
      A(1,I)=-ZE*(1.0+ETO)
12  A(2,I)=ETI*(1.0-ZE*ZE)/2.0
      DO 13 I=4,8,4
      ZEI=ZI(I)
      ZEO=ZEI*ZE
      A(1,I)=ZEI*(1.0-ET*ET)/2.0
13  A(2,I)=-ET*(1.0+ZEO)
      DO 20 I=1,2
      DO 20 J=1,2
      SUM=0.0
      DO 21 K=1,8
21  SUM=A(I,K)*ELCO(K,J)+SUM
20  ACOB(I,J)=SUM
      DET=ACOB(1,1)*ACOB(2,2)-ACOB(1,2)*ACOB(2,1)
      IF (DET.LE.0.0) GO TO 25
      BACO(1,1)=ACOB(2,2)/DET
      BACO(1,2)=-ACOB(1,2)/DET
      BACO(2,1)=-ACOB(2,1)/DET
      BACO(2,2)=ACOB(1,1)/DET
      DO 22 I=1,2
      DO 22 J=1,8
22  AA(I,J)=BACO(I,1)*A(1,J)+BACO(I,2)*A(2,J)
      RETURN
25  MESSAGE 'A QUAD8 ELEMENT HAS NODES TANGLED, COORDS ARE ...'
      DO 26 I=1,8
26  PRINT 100,ELCO(I,1),ELCO(I,2)
100 FORMAT(2F10.3)
      MARK(12)=1
      RETURN
      END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  SUBROUTINE BAR3(GEOM,ELCO,ES,NE)
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  PUBLIC ITYPE(),STRESS(,)
  LOCAL GEOM(2),ELCO(10,3),ES(6,6),NE(3)
  X=ELCO(3,1)-ELCO(1,1)
  Y=ELCO(3,2)-ELCO(1,2)
  S2=X*X+Y*Y
  S=SQRT(S2)
  LENGTH(ITYPE(),LEN)
  DO 10 I=4,LEN
  IF(ITYPE(I).EQ.'BAR3')GOTO 20
10  CONTINUE
20  J=NE(1)*3-2
    K=NE(2)*3-2
    L=NE(3)*3-2
30  F=GEOM(1)*GEOM(2)/(S2*S*3.)
    A=X*X*F
    B=Y*Y*F
    C=X*Y*F
    DO 100 I=1,5,2
    DO 200 K=1,5,2
    CONST=-8.
    IF(I.EQ.K)GOTO 50
    IF(I.EQ.5.AND.K.EQ.1)CONST=1.
    IF(I.EQ.1.AND.K.EQ.5)CONST=1.

```

```

      GOTO 60
50  CONST=7.
    IF (I.EQ.3) CONST=16.
60  ES(I,K)=A*CONST
    ES(I+1,K)=C*CONST
    ES(I,K+1)=C*CONST
    ES(I+1,K+1)=B*CONST
200 CONTINUE
100 CONTINUE
    RETURN
    END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  SUBROUTINE ISTRES
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    ' ALLOCATES ZERO GAUSS POINT STRESSES TO
    ' AGAUSS(,,) AND GSTRESS(,,) AND
    ' STARTS G.P. PROPERTIES, GPROPS(,,)
    ' ALLOCATES ZERO GAUSS POINT STRAINS TO
    ' STRAIN(,,) AND ASTRAN(,,)
    ' ALLOCATES DISPLACEMENTS TO DISPL()
    ' AS READ FROM AN INITIAL DISPL. TABLE
    PUBLIC MARK(20),AGAUSS(,,),GSTRESS(,,),DISPL(),PRNCPL(,,),...
      NSTATE(,,),ITYPE(),ELEMS(,,),STRESS(,,),KNAME(),GPROPS(,,)
    PUBLIC CSIGMA(),BSIGMA(),STRAIN(,,),ASTRAN(,,)
    DIMENSION V(20),TABLE(3,3),NAME(10)
    INTEGER TAB,NAME,TITLE,TABLE
    NP=MARK(8)
    GET(0,0) N1
    GOTO(100,200,400),N1
100  GET(2,2) TAB
    READ(TAB,0) L,M
    ND=2*NP
    REDEFINE(DISPL(),ND)
    DO 110 I=1,ND
110  DISPL(I)=0.0
    DO 120 J=1,M
    READ(TAB,L) N, (V(1),NV)
    IF(NV.NE.2) MESSAGE 'EXPECTING 2 DISPLACEMENTS AT NODE ',N,...
      ' NOT ',NV,' IN TABLE ', $TAB
    IF(N.LE.NP) GOTO 130
    MESSAGE 'DISPLACEMENTS GIVEN FOR NODE ',N,...
      ' GREATER THAN MAX. NODE ',NP
    GOTO 120
130  DISPL(2*N-1)=V(1)
    DISPL(2*N)=V(2)
120  CONTINUE
    RETURN
    ' INITIALIZE GAUSS POINT STRESSES IN AGAUSS(,,) ,GSTRESS(,,)
200  MARKER=0
    DEFINE(KNAME(),3)
    TABLE(1,1)='QUAD8SM',9,4
    TABLE(1,2)='BAR3',2,0
    TABLE(1,3)='BOND6',3,2
    GET(3,5) (NAME(1),NUM)
    LENGTH(ITYPE(),LEN)
    DO 210 I=1,NUM
    DO 220 K=4,LEN
    IF(ITYPE(K).EQ.NAME(I)) GOTO 230

```

```

220 CONTINUE
    MESSAGE 'ELEMENT TYPE ', $NAME(I), ' DOES NOT EXIST'
    MARKER=1
230 DO 240 J=1,3
    IF (NAME(I).EQ.TABLE(1,J)) GOTO 250
240 CONTINUE
    MESSAGE $NAME(I), ' IS NON-STANDARD TYPE'
    MARKER=1
    GOTO 210
250 IF (MARKER.EQ.0) GOTO 260
    MARK(12)=1
    RETURN
260 NGP=TABLE(2,J)+0.1
    NPROPS=TABLE(3,J)+0.1
    NGPROP=NPROPS*NGP
    KNAME(I)=ITYPE(K)
    LENGTH(ELEMS(,K),NEL)
    NGAUS=NGP*3
    REDEFINE(AGAUS(,,),NUM),(GSTRESS(,,),NUM),...
    (GPROPS(,,),NUM)
    REDEFINE(AGAUS(,,I),NEL),(GSTRESS(,,I),NEL),...
    (GPROPS(,,I),NEL)
    DO 270 JJ=1,NEL
    REDEFINE(AGAUS(,JJ,I),NGAUS),(GSTRESS(,JJ,I),NGAUS),...
    (GPROPS(,JJ,I),NGPROP)
    IF (NGPROP.EQ.0) GOTO 300
    DO 280 LL=1,NGP
    KC=(LL-1)*NPROPS+1
    IF (KNAME(I).EQ.'QUAD8SM') GOTO 290
    GPROPS(KC,JJ,I)=ELEMS(11,JJ,K)
    GPROPS(KC+1,JJ,I)=ELEMS(12,JJ,K)
    GOTO 280
290 GPROPS(KC,JJ,I)=ELEMS(12,JJ,K)
    GPROPS(KC+1,JJ,I)=ELEMS(12,JJ,K)
    GPROPS(KC+2,JJ,I)=ELEMS(16,JJ,K)
    GPROPS(KC+3,JJ,I)=0.
280 CONTINUE
300 DO 270 LL=1,NGAUS
    GSTRESS(LL,JJ,I)=0.
270 AGAUS(LL,JJ,I)=0.
210 CONTINUE
    '' INITIALIZE CONCRETE AND BAR3 NODAL STRESS VECTORS
    NODS=MARK(8)
    NDSTR=NODS*3
    REDEFINE(CSIGMA(),NDSTR)
    REDEFINE(BSIGMA(),NDSTR)
    DO 470 I=1,NDSTR
    BSIGMA(I)=0.
470 CSIGMA(I)=0.
    RETURN
    '' INITIALIZE GAUSS POINT UNIAXIAL STRAINS IN STRAIN(,,),ASTRAN(,,)
400 NGP=9
    GET(4,6)NAME(1)
    IF (NAME(1).EQ.'QUAD8SM') GOTO 420
    MARK(12)=0
    RETURN
420 LENGTH(ITYPE(),LEN)
    DO 440 K=4,LEN

```

'' FIND K ELEMENT='QUAD8SM'

```

      IF (NAME(1).EQ.ITYPE(K))GOTO 450
440  CONTINUE
450  LENGTH(ELEMS(,K),NEL)  '' NO OF ELEMENTS
      REDEFINE(NSTATE(,NEL),(STRAIN(,,NEL),(ASTRAN(,,NEL)
      DO 460 I=1,NEL
      REDEFINE(STRAIN(,I),NGP),(ASTRAN(,I),NGP)
      NGP2=NGP*2
      REDEFINE(NSTATE(,I),NGP2)  '' NSTATE=STATUS OF GP IN 2 PRINC. DIRN
      DO 460 J=1,NGP
460  REDEFINE (STRAIN(,J,I),2),(ASTRAN(,J,I),2)
      RETURN
      END
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE BOND6(GEOM,ELCO,NE,ES,JT,IT)
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      PUBLIC GPROPS(,,)
      LOCAL GEOM(8),ELCO(10,3),ES(12,12),NE(6)
      DIMENSION GP(7),HG(7),A(6)
      X1=ELCO(1,1)
      X2=ELCO(2,1)
      X=ABS(X1-X2)
      CALL PREP3('BOND6',KT)
      DO 1 I=1,12
      DO 1 J=1,12
1  ES(I,J)=0.
      NG=3
      Z=0.7745966692
      Y=0.
      GP(1)=-Z
      GP(2)=Y
      GP(3)=Z
      HG(1)=0.5555555555
      HG(2)=0.8888888888
      HG(3)=HG(1)
      DO 10 IG=1,NG
      ZE=GP(IG)
      A(1)=ZE/2.*(ZE-1.)
      A(2)=(1.+ZE)*(1.-ZE)
      A(3)=ZE/2.*(ZE+1.)
      A(4)=-A(3)
      A(5)=-A(2)
      A(6)=-A(1)
      RT=GPROPS(IG*2-1,JT,KT)
      RN=GPROPS(IG*2,JT,KT)
      VKH=RT*X*GEOM(2)*HG(IG)
      VKV=RN*X*GEOM(2)*HG(IG)
      DO 20 I=1,6
      BI=A(I)
      DO 20 J=1,6
      BJ=A(J)
      K=2*I-1
      L=2*J-1
      ES(K,L)=ES(K,L)+VKH*BI*BJ
20  ES(K+1,L+1)=ES(K+1,L+1)+VKV*BI*BJ
10  CONTINUE
      RETURN
      END
*EXIT

```

## D.2 OVERLAY 'CONC2'

```

*GENESYS
*GENTRAN 'LUFÉ'
AT FINISH,ERRORS STATISTICS
COMPILE OVERLAY 'CONC2' AS 35
*OVERLAY
    'CONC2'
SUBPROGRAMS PREP1,PREP2,PREP3,STRS,SHP8,...
    CONXS8,GAUSS,BAR3S,SUMMAT,BND6S
ENTRY STRS,SUMMAT
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    SUBROUTINE PREP1(IT,NAME,M,NICK)
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

    This subprogram is identical to PREP1 in 'CONC1'

    END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    SUBROUTINE PREP2(J,IT,NN,NE,NP,GEOM,ELCO,L)

    This subprogram is identical to PREP2 in 'CONC1'

    END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    SUBROUTINE PREP3(NAME,KT)

    This subprogram is identical to PREP3 in 'CONC1'

    END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    SUBROUTINE STRS
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    '' CORRESPONDS TO STIF BUT HANDLES
    '' STRESS SUBROUTINES
    PUBLIC MARK(20),LOAD(),VALS(,,),KT(20),ITYPE(),SIGMA()
    PUBLIC CSIGMA(),BSIGMA(),ISTRS,NUM(),COORDS(,)
    DIMENSION LC(20),DE()
    DIMENSION LMS(20),ISTR(70)
    LOCAL NE(10),GEOM(10),ELCO(10,3)
    INTEGER TABLE(2,10)
    FIX 100 LMS,ISTR
    NTAB=3
    TABLE(1,1)='QUAD8SM',1
    TABLE(1,2)='BAR3',1
    TABLE(1,3)='BOND6',1
    GET(5,3) (LC(1),NL)
    GET(4,4) (LMS(1),NLSENT)
    NODS=MARK(8)
    NDSTR=3*NODS
    REDEFINE (SIGMA(),NDSTR),(NUM(),NODS)
    FIX 100 NUM
    NLTS=MARK(11)*2
    NFREE=MARK(10)
    DO 100 NC=1,NL
    I=LC(NC)
    LCASE=I
    K=LOAD(I)

```

```

      EQUATE 100 (VALS(,I,3),D)
      DO 100 IPS=1,NLSPNT
      NAME=LMS(IPS)
      DO 110 I=1,NLTS
      IT=I+3
      IF(NAME.EQ.ITYPE(I+3)) GO TO 111
110 CONTINUE
      MESSAGE 'ELEMENT TYPE', $NAME, ' NOT USED'
      GO TO 100
111 CONTINUE
      EXPLODE NAME (ISTR(1),NCS)
      DO 105 J=1,NCS
      I=J-1
      IF(ISTR(J).EQ.'/') GO TO 106
105 CONTINUE
      I=NCS
106 IMplode NICK (ISTR(1),I)
      LENGTH (VALS(,IT),M)
      NREF=1
      DO 320 J=1,NTAB
      IF(NICK.EQ.TABLE(1,J)) GO TO 321
320 NREF=NREF+1
      MESSAGE 'NO STRESS ROUTINE FOR ELEMENT TYPE ', $NAME
      GO TO 100
321 IF(TABLE(2,NREF).GT.0) GO TO 322
      MESSAGE
      MESSAGE 'NODAL STRESSES NOT POSSIBLE WITH ELEMENT TYPE ', $NAME
      ISTRS=1
322 MESSAGE
      MESSAGE 'STRESSES FOR ELEMENTS ', $NAME, ' LOAD CASE ', $K
      IF(ISTRS.EQ.1) MESSAGE 'ELEMENT      —STRESSES—'
      MESSAGE
      DO 300 I=1,NDSTR
300 SIGMA(I)=0.
      DO 301 I=1,NODS
301 NUM(I)=0.
      DO 200 J=1,M
      CALL PREP2(J,IT,NN,NE,NP,GEOM,ELCO,1)
      LDE=NN*NFREE
      REDEFINE (DE(),LDE)
      FIX 200 DE
      N3=1
      DO 202 NI=1,NN
      NODE=NE(NI)
      NOFF=(NODE-1)*NFREE+2
      DO 202 NJ=1,NFREE
      DE(N3)=D(NOFF+NJ)
202 N3=N3+1
      GO TO (1,2,3,4,5,6,7,8,9,10),NREF
      1 CALL CONSX8(GEOM,ELCO,DE(1),SIGMA,NUM,NE,ISTRS,J,IT,LCASE)
      GOTO 99
      2 CALL BAR3S(GEOM,ELCO,DE(1),SIGMA,NE,NUM,J,IT,ISTRS)
      GOTO 99
      3 CALL BND6S(GEOM,ELCO,DE(1),NE,J,IT,NUM,SIGMA,ISTRS)
      GOTO 99
      4 CONTINUE
      5 CONTINUE
      6 CONTINUE

```

"DO FOR ALL ELEMENTS  
 "PUT DEFLECTIONS FOR  
 "THIS ELEMENT INTO DE

```

7 CONTINUE
8 CONTINUE
9 CONTINUE
10 CONTINUE
99 CONTINUE
200 CONTINUE
    IF(ISTRS.EQ.1) GO TO 100
    IF(ISTRS.EQ.3) GOTO 100
    MESSAGE 'NODAL AVERAGE STRESSES ', $NAME
    KPRINT=0
    IF(NAME.EQ.'QUAD8SM') KPRINT=1
    IF(ISTRS.EQ.0) GOTO 405
    MESSAGE
    IF(KPRINT.EQ.1) PRINT 401
    IF(KPRINT.EQ.0) PRINT 403
    MESSAGE
405 N=1
    DO 400 I=1,NODS
        X=NUM(I)
        IF(X.EQ.0.0) GOTO 400
        SIGMA(N)=SIGMA(N)/X
        SIGMA(N+1)=SIGMA(N+1)/X
        SIGMA(N+2)=SIGMA(N+2)/X
        IF(NAME.NE.'QUAD8SM') GOTO 410
        CSIGMA(N)=SIGMA(N)
        CSIGMA(N+1)=SIGMA(N+1)
        CSIGMA(N+2)=SIGMA(N+2)
410 IF(NAME.NE.'BAR3') GOTO 411
        BSIGMA(N)=SIGMA(N)
        BSIGMA(N+1)=SIGMA(N+1)
        BSIGMA(N+2)=SIGMA(N+2)
411 CONTINUE
        IF(ISTRS.EQ.0) GOTO 400
        IF(KPRINT.EQ.1) GOTO 412
        PRINT 404, I, SIGMA(N), SIGMA(N+1), COORDS(I,1), COORDS(I,2)
        GOTO 400
412 CN=(1/GEOM(1))
        STRX=CN*(SIGMA(N)-GEOM(2)*SIGMA(N+1))
        STRY=CN*(SIGMA(N+1)-GEOM(2)*SIGMA(N))
        GXY=SIGMA(N+2)*2.*(1+GEOM(2))/GEOM(1)
        A=SQRT((SIGMA(N)-SIGMA(N+1))**2/4. + SIGMA(N+2)**2)
        P1=(SIGMA(N)+SIGMA(N+1))/2.+A
        P2=P1-2.0*A
        FI=0.
        T1=0.-2.*SIGMA(N+2)
        T2=SIGMA(N)-SIGMA(N+1)
        IF(T1.EQ.0.0.AND.T2.EQ.0.0) GOTO 101
        FI=ATAN2(T1,T2)*28.64788976
101 CONTINUE
        PRINT 402, I, SIGMA(N), SIGMA(N+1), SIGMA(N+2), P1, P2, FI
400 N=N+3
100 CONTINUE
401 FORMAT(' NODE,          SXX,          SY,          TX,          P1,          P2',...
' ANGLE(DEGS)')
402 FORMAT(15,6F10.3)
403 FORMAT(' NODE,          SXX,          COORDINATES')
404 FORMAT(15,4F10.3)
    RETURN

```

```

      END
      'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
        SUBROUTINE CONSX8(GEOM,ELCO,DE,SIGMA,NUM,NE,ISTRS,JP,IT,LCASE)
      'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
        PUBLIC MARK(20),ANGLE(,),AGAUSS(,),GSTRESS(,),ITYPE(,),GPROPS(,,)
        PUBLIC STRAIN(,,)
        LOCAL GEOM(7),ELCO(10,3),DE(16),NE(10),SS(4,3),SN(8,3)
        LOCAL KONTRL(9)
        DIMENSION SIGMA(,),NUM(,),ZI(8),EI(8)
        HIGH DET,SUM,EXX,EYY,GXY,BI,CI,UI,VI,GE,GZ
        HIGH GP(3),HG(3),AA(2,8),AH(7),STRN2(4,3),STRN3(9,3)
        FIX 1 ZI,EI
        ZI(1)=-1.,0.,1.,1.,1.,0.,-1.,-1.
        EI(1)=-1.,-1.,-1.,0.,1.,1.,1.,0.
        CALL PREP3('QUAD8SM',KT)
        IF(ISTRS.EQ.2.OR.ISTRS.EQ.0)GOTO 1000
        IF(ISTRS.LE.1) PRINT 8
        8 FORMAT(15X,'3 X 3 GAUSS POINT STRAINS')
        LENGTH(GPROPS(,1,KT),NGPROP)
        NPROPS=NGPROP/9+0.1
        ' EVALUATE STRAINS AT THE 2 X 2 GAUSS POINTS
        '
        NG=2
        CALL GAUSS(NG,GP,HG,KONTRL)
        LL=0
        DO 100 IG=1,2
          GZ=GP(IG)
          DO 100 JG=1,2
            GE=GP(JG)
            LL=LL+1
            CALL SHP8(GZ,GE,ZI(1),EI(1),ELCO,AA,DET)
            EXX=0.
            EYY=0.
            GXY=0.
            DO 200 K=1,8
              BI=AA(1,K)
              CI=AA(2,K)
              UI=DE(2*K-1)
              VI=DE(2*K)
              EXX=EXX+BI*UI
              EYY=EYY+CI*VI
            200 GXY=GXY+CI*UI+BI*VI
            N=KONTRL(LL)
            STRN2(N,1)=EXX
            STRN2(N,2)=EYY
            100 STRN2(N,3)=GXY
            ' BILINEAR EXTRAPOLATION OF INCREMENTAL
            ' 2 X 2 GAUSS POINT STRAINS TO 3 X 3 GAUSS POINTS
            AH(1)=-.2,0.02917961734,-.2,1.370820466
            AH(5)=AH(1),AH(2),AH(3)
            DO 19 J=1,3
              K=3
              DO 19 L=1,7,2
                SUM=0.0
                DO 22 I=1,4
                  22 SUM=SUM+AH(I+K)*STRN2(I,J)
                  K=K-1
                19 STRN3(L,J)=SUM

```

```

DO 30 J=1,3
DO 31 I=1,5,2
31 STRN3(I+1,J)=(STRN3(I,J)+STRN3(I+2,J))/2.0
   STRN3(8,J)=(STRN3(7,J)+STRN3(1,J))/2.0
30 STRN3(9,J)=(STRN2(1,J)+STRN2(2,J)+STRN2(3,J)+STRN2(4,J))/4.
   IF(ISTRN.EQ.3)GOTO 99
   PRINT 35
35 FORMAT(10X,'EXX',7X,'EYY',7X,'GXY')
DO 40 I=1,9
   PRINT 41,I,STRN3(I,1),STRN3(I,2),STRN3(I,3)
41 FORMAT(I5,3E15.8)
40 CONTINUE
   CONTINUE
   MESSAGE
   MESSAGE '                SXX                SYY                TXY'
99 CONTINUE
DO 500 I=1,9  ''GP EXECUTED IN STANDARD ORDER
'' EVALUATE D MATRIX FOR I TH GAUSS POINT
'' CALCULATE STRESSES FOR I TH GAUSS POINT
'' PROPERTIES E1,E2,PR,THETA
EQUATE 3 (GPROPS(,JT,KT),G)
N=(I-1)*NPROPS+1
E1=G(N)
E2=G(N+1)
PR=G(N+2)
3 THETA=G(N+3)
C=1/(1.-PR*PR)
D1=E1*(COS(THETA))**2+E2*(SIN(THETA))**2
D2=PR*SQRT(E1*E2)
D3=.5*(E1-E2)*COS(THETA)*SIN(THETA)
D4=E1*(SIN(THETA))**2+E2*(COS(THETA))**2
D5=D3
D6=.25*(E1+E2-2.*D2)
EXX=STRN3(I,1)
EYY=STRN3(I,2)
GXY=STRN3(I,3)
SXX=C*(D1*EXX+D2*EYY+D3*GXY)
SYY=C*(D2*EXX+D4*EYY+D5*GXY)
TXY=C*(D3*EXX+D5*EYY+D6*GXY)
CONTINUE
ALPHA=SQRT((SXX-SYY)**2/4.+TXY**2)                '' PRINCIPAL STRESSES
KOUNT=I*3-2
GSTRESS(KOUNT,JT,KT)=SXX                        '' STORE GP STRESSES
GSTRESS(KOUNT+1,JT,KT)=SYY
GSTRESS(KOUNT+2,JT,KT)=TXY
SIGMA1=(SXX+SYY)/2.+ALPHA
SIGMA2=SIGMA1-2.*ALPHA
IF(LCASE.NE.2)GOTO 5
FI2=ANGLE(I,JT)                                ''FOR DUMMY LOAD CASE ONLY
CENTRE=(SXX+SYY)*.5
RADIUS=(SXX-SYY)*.5
SIGMA1=CENTRE+RADIUS*COS(FI2)+TXY*SIN(FI2)
SIGMA2=CENTRE-RADIUS*COS(FI2)-TXY*SIN(FI2)
5 STRAIN(1,I,JT)=0.
   STRAIN(2,I,JT)=0.                            '' CHECK IF E1 OR E2 =0
   IF(E1.NE.0.) STRAIN(1,I,JT)=SIGMA1/E1
   IF(E2.NE.0.) STRAIN(2,I,JT)=SIGMA2/E2
   IF(ISTRN.EQ.3)GOTO 90

```

```

      PRINT 95,JT,I,SXX,SYX,TXY
95  FORMAT(2I5,3E18.10)
90  CONTINUE
500 CONTINUE
      IF(ISTRN.NE.3)MESSAGE ' '
      RETURN
      ' '      BILINEAR EXTRAPOLATION OF ACCUMULATED
      ' '      GAUSS POINT STRESSES TO NODES USING OUTER 4 GPS
1000 AH(1)=-.166666667,.02116943699,-.166666667,1.312163849
      AH(5)=AH(1),AH(2),AH(3)
      DO 1019 J=1,3      ' ' J=1 MEANS SXX
      K=3
      DO 1019 L=1,7,2
      SUM=0.0
      DO 1022 I=1,4
      II=2*I-1
      KOUNT=II*3+J-3
      ' 'FIND OUTER 4 GAUSS POINTS
1022 SUM=SUM+AH(I+K)*AGAUSS(KOUNT,JT,KT)
      K=K-1
      SN(L,J)=SUM      ' ' THIS LEAVES CORNER NODE STRESSES IN SN( )
1019 CONTINUE
      DO 1030 J=1,3
      DO 1031 I=1,5,2
1031 SN(I+1,J)=(SN(I,J)+SN(I+2,J))/2.0
1030 SN(8,J)=(SN(7,J)+SN(1,J))/2.0
      IF(ISTRN.EQ.0)GOTO 1050
      PRINT 1035
1035 FORMAT(10X,'SXX',7X,'SYX',7X,'TXY',7X,'P1',7X,'P2',7X,'ANGLE',...
      5X,'COORDINATES')
      DO 1040 I=1,8
      A=SQRT((SN(I,1)-SN(I,2))**2/4.0+SN(I,3)**2)
      P1=(SN(I,1)+SN(I,2))/2.0+A
      P2=P1-2.0*A
      IF(ABS(SN(I,3)).LT.0.00001)GOTO 1070
      FI=ATAN2((0.0-2.0*SN(I,3)),(SN(I,1)-SN(I,2)))*28.6479
      GOTO 1060
1070 FI=0.
1060 PRINT 1041,NE(I),SN(I,1),SN(I,2),SN(I,3),P1,P2,FI,ELOC(I,1),...
      ELOC(I,2)
1041 FORMAT(15,9F10.3)
1040 CONTINUE
1050 CONTINUE
      DO 1 I=1,8
      K=(NE(I)-1)*3
      SIGMA(K+1)=SIGMA(K+1)+SN(I,1)
      SIGMA(K+2)=SIGMA(K+2)+SN(I,2)
      SIGMA(K+3)=SIGMA(K+3)+SN(I,3)
      N=NE(I)
      NUM(N)=NUM(N)+1
1  CONTINUE
      IF(ISTRN.EQ.2)MESSAGE
      RETURN
      END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  SUBROUTINE GAUSS (NG,GP,HG,KONTROL)
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

```

This subprogram to GAUSS in 'CONCL' except for the following line :

HIGH GP(3),HG(3)

END

```
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
SUBROUTINE SHP8(ZE,ET,ZI,EI,ELCO,AA,DET)
```

This subprogram is identical to SHP8 in 'CONCL' except for the following lines :

```
HIGH AA(2,8),ACOB(2,2),BACO(2,2),A(2,8)
HIGH ZE,ET,SUM,ZEO,ZEI,ETI,ETO,DET
```

END

```
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
SUBROUTINE BAR3S(GEOM,ELCO,DE,SIGMA,NE,NUM,JT,IT,ISTRS)
```

```
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
```

```
PUBLIC ITYPE(),GSTRESS(,),AGAUSS(,)
LOCAL GEOM(2),ELCO(10,3),DE(6),NE(3)
DIMENSION NUM(),SIGMA(),SN(3)
```

PR=0.3

'' POISSONS RATIO FOR STEEL

E=GEOM(1)

'' YOUNGS MODULUS

X=ELCO(3,1)-ELCO(1,1)

Y=ELCO(3,2)-ELCO(1,2)

S2=X\*X+Y\*Y

J=NE(1)

K=NE(2)

L=NE(3)

CALL PREP3('BAR3',KT)

IF (ISTRS.EQ.2.OR.ISTRS.EQ.0) GOTO 100

'' EXTRAPOLATION

CONST=GEOM(1)/S2

'' E/(L\*L)

U1=DE(1)

'' NODAL DISPLACEMENTS

V1=DE(2)

U2=DE(3)

V2=DE(4)

U3=DE(5)

V3=DE(6)

'' EXACT B MATRIX COEFFICIENTS

C1=2.154700517654419

C2=2.309401035308838

C3=0.154700517654419

GS1=CONST\*((-C1\*U1+C2\*U2-C3\*U3)\*X+(-C1\*V1+C2\*V2-C3\*V3)\*Y)

GS2=CONST\*((C3\*U1-C2\*U2+C1\*U3)\*X+(C3\*V1-C2\*V2+C1\*V3)\*Y)

DO 150 KK=1,6

150 GSTRESS(KK,JT,KT)=0.

'' STORE GP STRESSES

GSTRESS(1,JT,KT)=GS1

GSTRESS(4,JT,KT)=GS2

IF (ISTRS.EQ.3) RETURN

PRINT 160,J,K,L

160 FORMAT('ELEMENT ',3I3,' AXIAL STRESS')

MESSAGE ' '

PRINT 170,GS1

PRINT 170,GS2

MESSAGE

170 FORMAT(17X,F10.3)

RETURN

'' EXTRAPOLATION OF STRESSES TO NODES + STORE IN SIGMA()

100 NUM(J)=NUM(J)+1

NUM(K)=NUM(K)+1

```

NUM(L)=NUM(L)+1
J=J*3-2
K=K*3-2
L=L*3-2
C1=1.366025447845459
C2=0.3660253882408142
GS1=AGAUSS(1,JT,KT)
GS2=AGAUSS(4,JT,KT)
SN(1)=C1*GS1-C2*GS2
SN(3)=-C2*GS1+C1*GS2
SN(2)=(SN(3)+SN(1))/2
SIGMA(J)=SIGMA(J)+SN(1)
SIGMA(K)=SIGMA(K)+SN(2)
SIGMA(L)=SIGMA(L)+SN(3)
COEFF=4.74E-5
SIGMA(J+1)=SIGMA(J)*PR/E/COEFF          '' RADIAL PRESSURE
SIGMA(J+2)=0.0
SIGMA(K+1)=SIGMA(K)*PR/E/COEFF          '' RADIAL PRESSURE
SIGMA(K+2)=0.0
SIGMA(L+1)=SIGMA(L)*PR/E/COEFF          '' RADIAL PRESSURE
SIGMA(L+2)=0.0
IF(ISTR.EQ.0)GOTO 230
PRINT 200
200 FORMAT(10X,' AXIAL STRESS')
MESSAGE ' '
DO 210 KK=1,3
PRINT 220,NE(KK),SN(KK)
IF(SN(KK).GE.460.)PRINT 211
211 FORMAT(5X,'*** YIELD STRESS EXCEEDED ***')
220 FORMAT(15,5X,F10.3)
210 CONTINUE
230 CONTINUE
RETURN
END
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
SUBROUTINE SUMMAT
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
''ACCUMULATES NODAL DISPLACEMENTS OR
''ACCUMULATES STRESSES FOR AN ELEMENT TYPE
''OR ACCUMULATES GAUSS POINT STRESSES FOR ALL ELEMENTS
PUBLIC MARK(20),SIGMA(),STRESS(),DISPL(),VALS(,,),...
LDCSE,PRNCPL(),NUM(),ITYPE(),AGAUSS(,,),GSTRESS(,,)
PUBLIC STRAIN(,,),ASTRAN(,,)
INTEGER NAME
NP=MARK(8)                                ''NP=NO. OF NODES
GET(0,0)N1
IF(N1.EQ.2)GOTO 100
IF(N1.EQ.4)GOTO 200
IF(N1.EQ.5)GOTO 300
100 GET(2,3)LDCSE
GET(2,0)K
ND=2*NP
EQUATE 110 (VALS(,LDCSE,3),D)
DO 110 I=1,ND
110 DISPL(I)=DISPL(I)+D(I+2)
IF(K.NE.3)RETURN
MESSAGE
MESSAGE' ACCUMULATED NODAL DISPLACEMENTS:'

```

```

MESSAGE
MESSAGE
MESSAGE '  NODE          U          V'
MESSAGE
DO 120 I=1,NP
120 PRINT 130,I,DISPL(2*I-1),DISPL(2*I)
130 FORMAT(I5,2E18.10)
RETURN
'' ACCUMULATES GAUSS POINT STRESSES FOR ALL ELEMENTS
200 KK=4
GET(4,0) IPRINT
LENGTH(AGAUSS(,,),NETYPE)
LENGTH(ITYPE(),LEN)
DO 250 I=1,NETYPE
DO 210 K=KK,LEN
IF(ITYPE(K).EQ.'QUAD8SM')GOTO 215
IF(ITYPE(K).EQ.'BAR3')GOTO 215
IF(ITYPE(K).EQ.'BOND6')GOTO 215
210 CONTINUE
215 KK=K+1
NAME=ITYPE(K)
LENGTH(AGAUSS(,,I),NEL)
LENGTH(AGAUSS(,I,I),NGAUS)
IF(IPRINT.NE.3)GOTO 230
MESSAGE 'ACCUMULATED GAUSS POINT STRESSES FOR ELEMENTS ', $NAME
MESSAGE ' '
MESSAGE '          GAUSS POINT          SXX          SYX          TXY'
230 DO 250 JJ=1,NEL
IF(IPRINT.NE.3)GOTO 240
MESSAGE 'ELEMENT ',JJ
MESSAGE ' '
240 NGP=NGAUS/3+0.1
EQUATE 250 (AGAUSS(,JJ,I),A),(GSTRESS(,JJ,I),S)
DO 250 LL=1,NGP
MM=LL*3-2
A(MM)=A(MM)+S(MM)
A(MM+1)=A(MM+1)+S(MM+1)
A(MM+2)=A(MM+2)+S(MM+2)
IF(IPRINT.NE.3)GOTO 250
PRINT 255,LL,A(MM),A(MM+1),A(MM+2)
255 FORMAT(15X,I6,3X,3E18.10)
250 CONTINUE
RETURN
'' ACCUMULATES G.P. EQUIVALENT UNIAXIAL STRAINS
300 GET(5,0) IPRINT '' PRINT OR NOT
LENGTH(ASTRAN(,,),NEL)
LENGTH(ASTRAN(,,1),NGP)
IF(IPRINT.NE.3)GOTO 330
MESSAGE
MESSAGE 'ACCUMULATED EQUIVALENT UNIAXIAL STRAINS'
MESSAGE
330 DO 350 J=1,NEL
IF(IPRINT.NE.3)GOTO 340
MESSAGE 'ELEMENT ',J
MESSAGE
340 DO 350 K=1,NGP
EQUATE 350 (ASTRAN(,K,J),A),(STRAIN(,K,J),S)
A(1)=A(1)+S(1)

```

```

      A(2)=A(2)+S(2)
      IF(IPRINT.NE.3)GOTO 350
      PRINT 355,K,A(1),A(2)
355  FORMAT(15X,I6,2E18.10)
350  CONTINUE
      RETURN
      END

```

```

'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  SUBROUTINE BND6S(GEOM,ELCO,DE,NE,JT,IT,NUM,SIGMA,ISTRS)
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  PUBLIC STRESS(,),ELEMS(,,),LDCSE,ITYPE(),GSTRESS(,,),AGAUSS(,,)
  PUBLIC GPROPS(,,)
  LOCAL GEOM(6),ELCO(10,3),DE(12),NE(6)
  DIMENSION NUM(),SIGMA(),SN(6),GP(7),HG(7),A(6),GS(7),U(3),V(3)
  J=NE(1)
  K=NE(2)
  L=NE(3)
  NG=3
  ZE=0.7745966692
  Y=0.
  GP(1)=-ZE
  GP(2)=Y
  GP(3)=ZE
  HG(1)=0.5555555555
  HG(2)=0.8888888888
  HG(3)=HG(1)
  CALL PREP3('BOND6',KT)
  IF(ISTRS.EQ.2.OR.ISTRS.EQ.0)GOTO 100
  EQUATE 40 (GPROPS(,JT,KT),R)
  DO 10 IG=1,NG
    GS(IG*2-1)=0.
    GS(IG*2)=0.
    ZE=GP(IG)
    A(1)=ZE/2.*(ZE-1.)
    A(2)=(1.+ZE)*(1.-ZE)
    A(3)=ZE/2.*(ZE+1.)
    A(4)=-A(3)
    A(5)=-A(2)
    A(6)=-A(1)
    DO 20 I=1,3
      JJ=2*I-1
      BI=A(I)
      KK=12-JJ
      U(I)=DE(JJ)-DE(KK)
      V(I)=DE(JJ+1)-DE(KK+1)
      GS(IG*2)=GS(IG*2)+V(I)*BI
20  GS(IG*2-1)=GS(IG*2-1)+U(I)*BI
      GS(IG*2-1)=R(IG*2-1)*GS(IG*2-1)
      GS(IG*2)=R(IG*2)*GS(IG*2)
10  CONTINUE
      DO 50 KK=1,9
50  GSTRESS(KK,JT,KT)=0.
40  GSTRESS(1,JT,KT)=GS(1)
      GSTRESS(2,JT,KT)=GS(2)
      GSTRESS(4,JT,KT)=GS(3)
      GSTRESS(5,JT,KT)=GS(4)
      GSTRESS(7,JT,KT)=GS(5)
      GSTRESS(8,JT,KT)=GS(6)

```

```

      IF (ISTRS.EQ.3) RETURN
      PRINT 60,J,K,L
60  FORMAT('ELEMENT ',3I3,' BOND STRESS')
      MESSAGE ' '
      PRINT 70,GS(1),GS(2)
      PRINT 70,GS(3),GS(4)
      PRINT 70,GS(5),GS(6)
70  FORMAT(17X,2F10.3)
      MESSAGE ' '
      RETURN
      ' ' EXTRAPOLATION OF BOND STRESSES TO NODES
100 NUM(J)=NUM(J)+1
      NUM(K)=NUM(K)+1
      NUM(L)=NUM(L)+1
      J=J*3-2
      L=L*3-2
      K=K*3-2
      C1=(ZE+1.)/2./ZE/ZE
      C2=(ZE+1.)*(ZE-1.)/ZE/ZE
      C3=(1.-ZE)/2./ZE/ZE
      GS(1)=AGAUSS(1,JT,KT)
      GS(2)=AGAUSS(2,JT,KT)
      GS(3)=AGAUSS(4,JT,KT)
      GS(4)=AGAUSS(5,JT,KT)
      GS(5)=AGAUSS(7,JT,KT)
      GS(6)=AGAUSS(8,JT,KT)
      SN(1)=C1*GS(1)+C2*GS(3)+C3*GS(5)
      SN(2)=C1*GS(2)+C2*GS(4)+C3*GS(6)
      SN(5)=C3*GS(1)+C2*GS(3)+C1*GS(5)
      SN(3)=GS(3)
      SN(4)=GS(4)
      SN(6)=C3*GS(2)+C2*GS(4)+C1*GS(6)
      SIGMA(J)=SIGMA(J)+SN(1)
      SIGMA(K)=SIGMA(K)+SN(3)
      SIGMA(L)=SIGMA(L)+SN(5)
      SIGMA(J+1)=SIGMA(J+1)+SN(2)
      SIGMA(J+2)=0.
      SIGMA(K+1)=SIGMA(K+1)+SN(4)
      SIGMA(K+2)=0.
      SIGMA(L+1)=SIGMA(L+1)+SN(6)
      SIGMA(L+2)=0.
      IF (ISTRS.EQ.0) GOTO 230
      PRINT 200
200  FORMAT(10X,'BOND STRESS')
      MESSAGE ' '
      DO 210 KK=1,3
      PRINT 220,NE(KK),SN(KK*2-1),SN(KK*2)
220  FORMAT(I5,5X,2F10.3)
210  CONTINUE
230  CONTINUE
      RETURN
      END
*EXIT

```

### D.3 OVERLAY CONC3

```

*GENESYS
*GENTRAN 'LIFE'
AT FINISH,ERRORS STATISTICS
COMPILE OVERLAY 'CONC3' AS 36
*OVERLAY
    'CONC3'
SUBPROGRAMS NORM,PREP3,PREP4,UPDATE,DARWIN,STRAIN,CRACK,EMOD,SHP8,...
    GAUSS,SHP3,RESID,RBOND,RQUAD,SAENZ,BOND,ULTBND,LSQFIT
ENTRY NORM,PREP4,UPDATE,RESID,CRACK
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    SUBROUTINE NORM
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
        ' CALCULATES THE EUCLIDEAN NORM
        ' OF THE GLOBAL LOAD VECTOR LOADCASE I
        PUBLIC MARK(20),VALS(,,)
        NP=MARK(8) 'NP=NO. OF NODES
        GET(0,5) ILOAD
        EQUATE 100 (VALS(, ILOAD, 2), RELOAD)
        ND=NP*2
        RNORM=0.
        DO 100 I=1,ND
100    RNORM=RELOAD(I+2)*RELOAD(I+2)+RNORM
        RNORM=SQRT(RNORM)
        PUT(0,6) RNORM                ' GIVES VALUE OF NORM TO MASTER
        RETURN
    END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    SUBROUTINE PREP3(NAME,KT)

        This subprogram is identical to PREP3 in 'CONC1'

    END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    SUBROUTINE PREP4
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
        ' GENERAL DATA VARIABLES STORED IN PUBLIC
        PUBLIC IDATA(),RDATA()
        GET(0,0) ICASE
        IF(ICASE.EQ.2) GOTO 200
        GET(1,0) KCASE
        IF(KCASE.EQ.4) GOTO 400
        REDEFINE(IDATA(),100)
300    GET(3,2) (IDATA(1),ILENI)
        REDEFINE(IDATA(),ILENI)
        DO 10 I=1,ILENI
            MESSAGE 'IDATA',I,' ',IDATA(I)
10    CONTINUE
        GET(3,0) KCASE
        IF(KCASE.EQ.0) GOTO 1000
400    REDEFINE(RDATA(),100)
        GET(4,2) (RDATA(1),ILENR)
        REDEFINE(RDATA(),ILENR)
        DO 5 I=1,ILENR
            MESSAGE 'RDATA',I,' ',RDATA(I)
5    CONTINUE
        GET(4,0) KCASE

```

```

        IF (KCASE.EQ.0) GOTO 1000
        GOTO 300
200  GET(2,0) KCASE
        IF (KCASE.EQ.4) GOTO 600
700  LENGTH(IDATA(), ILEN1)
        PUT(3,2) (IDATA(1), ILEN1)
        GET(3,0) KCASE
        IF (KCASE.EQ.0) GOTO 1000
600  LENGTH(RDATA(), ILENR)
        PUT(4,2) (RDATA(1), ILENR)
        GET(4,0) KCASE
        IF (KCASE.EQ.0) GOTO 1000
        GOTO 700
1000 CONTINUE
        RETURN
        END

```

'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX'

# SUBROUTINE UPDATE

'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX'

```

        '' GENERAL SUBROUTINE WHICH
        '' UPDATES MODULI/PROPERTIES FOR
        '' EACH ELEMENT TYPE AT THE GAUSS POINTS
PUBLIC NSTATE(,), AGAUSS(,), KNAME(), MARK(20), ITYPE(), GPROPS(,),
PUBLIC CSIGMA(), BSIGMA(), ELEMS(,), ASTRAN(,), DISPL(), COORDS(,)
LOCAL A(6), ELCO(10,3), ZI(8), EI(8), AA(2,8), FSTRES(2), FSTRAN(2)
        LOCAL P(2), XYCORD(3,2), E(2), UN(16)
        DIMENSION NAME(3), MODUL(2), GP(9)
        REAL LATPR, LP1, LP2, LP3, MODUL
        GET(0,0) IPRINT
        GET(0,5) (NAME(1), NUM)
        LENGTH (AGAUSS(,), NETYPE), (ITYPE(), LEN)
        MARKER=0
        DO 100 I=1, NUM                                ''NO OF ELEMENT TYPES TO UPDATE
        DO 110 KT=1, NETYPE
        IF (NAME(I).EQ.KNAME(KT)) GOTO 120
110  CONTINUE
        MESSAGE 'ELEMENT TYPE ', $NAME(I), ' DOES NOT EXIST'
        MARKER=1
        GOTO 100
120  IF (NAME(I).EQ.'BAR3') GOTO 100
        LENGTH(GPROPS(,KT), NEL)                                ''NO OF ELEMENTS
        LENGTH(GPROPS(,1,KT), NGPROP)                            ''TOTAL NO OF PROPERTIES
        DO 130 IT=4, LEN
        IF (NAME(I).EQ.ITYPE(IT)) GOTO 140
130  CONTINUE
140  IF (NAME(I).EQ.'QUAD8SM') GOTO 500
        ''
        ''      UPDATE BOND6 ELEMENT
        ''
        NPROPS=2                                '' NO OF PROPERTIES
        NGP=NGPROP/NPROPS+0.1                    ''NO OF GAUSS POINTS
        XYCORD(1,1)=1.
        XYCORD(2,1)=0.
        XYCORD(3,1)=1.
        MESSAGE
        MESSAGE 'UPDATED BOND6 PROPERTIES'
        IF (IPRINT.EQ.0) GOTO 31
31  DO 30 K=1, NEL                                '' FOR EACH ELEMENT

```

```

      IF (IPRINT.EQ.1) MESSAGE K
      KGEOM=ELEMS(9,K,IT)+0.1
      EQUATE 40 (ELEMS(,K,IT),X)
      NN=X(1)+0.1
      DO 50 L=1,NN
      NE=X(2+L)+0.1
      KK=L*2
      UN(KK-1)=DISPL(NE*2-1)
50 UN(KK)=DISPL(NE*2)
      NE1=X(8)
      NE2=X(7)
      NE3=X(6)
      NB1=X(3)
      NB2=X(4)
      NB3=X(5)
      LP1=CSIGMA(NE1*3-1)
      LP2=CSIGMA(NE2*3-1)
      LP3=CSIGMA(NE3*3-1)
      RS1=BSIGMA(NB1*3-1)
      RS2=BSIGMA(NB2*3-1)
      RS3=BSIGMA(NB3*3-1)
      IF (IPRINT.EQ.0) GOTO 33
      MESSAGE NE1,NE2,NE3
      MESSAGE' GP LPR RADPR UB SLIP RT',...
      RN'
33 N=3
      XYCORD(1,2)=RS1
      XYCORD(2,2)=RS2
      XYCORD(3,2)=RS3
      CALL LSQFIT(N,XYCORD,ARS,BRS)
      XYCORD(1,2)=LP1
      XYCORD(2,2)=LP2
      XYCORD(3,2)=LP3
      CALL LSQFIT(N,XYCORD,ALP,BLP)
      ZE=0.774596669241483
      GP(1)=ZE
      GP(2)=0.
      GP(3)=ZE
      DO 40 IG=1,NGP
      ZE=GP(IG)
      SLIP=0.
      SLIPN=0.
      CALL SHP3(ZE,A)
      DO 56 II=1,3
      JJ=2*II-1
      BI=A(II)
      KK=12-JJ
      V=UN(JJ+1)-UN(KK+1)
      SLIPN=SLIPN+V*BI
      U=UN(JJ)-UN(KK)
56 SLIP=SLIP+U*BI
      LATPR=ALP+ZE*BLP
      RADPR=ARS+ZE*BRS
      CALL BOND(KGEOM,K,IT,SLIP,SLIPN,S,RT,RN,LATPR,RADPR,UBMAX)
      GPROPS(IG*2-1,K,KT)=RT
      GPROPS(IG*2,K,KT)=RN
      IF (IPRINT.EQ.1) PRINT 57,IG,LATPR,RADPR,UBMAX,SLIP,RT,RN
57 FORMAT(I5,4F8.4,2F10.3)

```

```

40 CONTINUE
   IF(IPRINT.EQ.1)MESSAGE
30 CONTINUE
   GOTO 100
   ''
   '' UPDATE QUAD8SM -CONCRETE
   ''
500 NPROPS=4                                ''NO OF GP PROPERTIES
   NGP=NGPROP/NPROPS+.1
   MESSAGE
   MESSAGE 'UPDATED CONCRETE PROPERTIES'
   IF(IPRINT.EQ.0)GOTO 32
   MESSAGE
   MESSAGE 'EL GP SIGMA1    SIGMA2    STR1    STR2    ANGLE(DEG)',...
   '    YM1        YM2'
   MESSAGE
32 DO 230 K=1,NEL                            '' FOR EACH ELEMENT
   EQUATE 160 (Elems(,K,IT),X)
   NN=X(1)+.1                                '' NO OF NODES
   FCU=X(11)                                '' UNIAXIAL COMPRESSIVE STRENGTH
   E0=X(12)                                '' INITIAL YOUNGS MODULUS IN COMPRESSION
   FT=X(15)                                '' UNIAXIAL TENSILE STRENGTH
   ECU=X(13)                                '' UNIAXIAL COMPRESSIVE FAILURE STRAIN
   ETU=X(14)                                '' UNIAXIAL TENSILE FAILURE STRAIN
   DO 150 L=1,NN
   NE=X(2+L)+.1
   KK=L*2
   UN(KK-1)=DISPL(NE*2-1)                    ''UN() = ACCUMULATED NODAL DISP.
   UN(KK)=DISPL(NE*2)
   ELCO(L,1)=COORDS(NE,1)
150 ELCO(L,2)=COORDS(NE,2)
   DO 160 IG=1,NGP
   KPRINT=IPRINT
   SXX=AGAUSS(3*IG-2,K,KT)
   SYY=AGAUSS(3*IG-1,K,KT)
   TXY=AGAUSS(3*IG,K,KT)
   II=4*(IG-1)+2
   ISTAT1=NSTATE(IG*2-1,K)
   ISTAT2=NSTATE(IG*2,K)
   IF(ISTAT1.EQ.1.AND.ISTAT2.EQ.0)GOTO 300
   ALPHA=SQRT((SXX-SYY)**2/4.+TXY**2)
   P(1)=(SXX+SYY)/2.+ALPHA                    '' PRINCIPAL STRESSES
   P(2)=P(1)-2.*ALPHA                        '' P1 AND P2
   DO 10 IP=1,2
   P1=P(IP)
   P1=ABS(P1)
   IF(P1.LT.0.000001)P(IP)=0.
10 CONTINUE
   IF(ABS(TXY).LT.0.000001)GOTO 180
   SMOD=ABS(SXX-SYY)
   FI=ATAN2((2.*TXY),(SMOD))*0.5
   GOTO 181
180 FI=0.
181 IF(SYY.GT.SXX)FI=1.57079632679489-FI
   GOTO 310
300 MESSAGE 'CRACKED CONCRETE'
   KPRINT=1
   FI=GPROPS(II+2,K,KT)

```

```

      FI2=FI*2.
      CENTRE=(SXX+SYX)*0.5
      RADIUS=(SXX-SYX)*0.5
      SNN=CENTRE+RADIUS*COS(FI2)+TXY*SIN(FI2)
      SCC=CENTRE-RADIUS*COS(FI2)-TXY*SIN(FI2)
      P(1)=SNN
      P(2)=SCC
310  E(1)=ASTRAN(1,IG,K)
      E(2)=ASTRAN(2,IG,K)
      CALL DARWIN(P(1),P(2),E,FCU,FT,E0,ECU,ETU,FSTRES,FSTRAN,...
                NFAIL)
      IF(NFAIL.NE.0)KPRINT=1
      CALL EMOD(FSTRES,FSTRAN,P,E,E0,YM1,YM2)
      IF(YM1.EQ.0.AND.NFAIL.EQ.0)YM1=E0
      GPROPS(II,K,KT)=YM2
      GPROPS(II+2,K,KT)=FI
      GPROPS(II-1,K,KT)=YM1
195  FORMAT(9X,3F15.6)
      EXX=EXX*1000.
      EYY=EYY*1000.
      GXY=GXY*1000.
      E(1)=E(1)*1000.
      E(2)=E(2)*1000.
      DEGFI=FI*57.29578
      E1=E(1)*1000.
      E2=E(2)*1000.
      IF(KPRINT.EQ.1)PRINT 194,K,IG,P(1),P(2),E1,E2,...
      DEGFI,YM1,YM2
194  FORMAT(2I2,2F10.6,2F7.1,F10.3,2F10.2)
160  CONTINUE
230  CONTINUE
100  CONTINUE
      IF(MARKER.EQ.1)MARK(12)=1
      RETURN
      END
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE DARWIN(S1,S2,E,FCU,FT,E0,ECU,ETU,FSTRES,...
                FSTRAN,NFAIL)
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      ''  NONLINEAR CONCRETE MODEL BASED ON DARWIN/PECKNOLD
      ''  CONCRETE MODEL JULY 1974
      ''  POISSONS RATIO = CONSTANT ,R=3.
      ''
      LOCAL FSTRES(2),E(2),FSTRAN(2)
      KSWAP=0
      NFAIL=0
      IF(S1.LT.S2)GOTO 40
      XMAX=S1
      XMIN=S2
      GOTO 50
40  XMAX=S2
      XMIN=S1
      KSWAP=1
50  XK=10000.
      C=10000.
      IF(XMAX.NE.0.)C=XMIN/XMAX
      IF(XMIN.NE.0.)XK=XMAX/XMIN
      IF(XMIN.GE.0.)GOTO 100
      '' TENSION-TENSION

```

```

IF(XMAX.GT.0.)GOTO 200                                '' TENSION-COMPRESSION
''
''MESSAGE      COMPRESSION - COMPRESSION ZONE
''
A=(1.+3.65*XK)*(-FCU)/(1.+XK)**2
F=A-XMIN
IF(F.LT.0)GOTO 310
MESSAGE 'CONCRETE HAS FAILED BY CRUSHING'
PRINT 350,S1,S2,FCU
350 FORMAT(3F10.6)
NFAIL=3
310 FSTRES(2)=A
FSTRES(1)=FSTRES(2)*XK
IF(-FSTRES(1).LT.FCUGOTO 305
FSTRAN(1)=ECU*(-FSTRES(1)/FCU*3.-2.)
GOTO 306
305 A=1.6*(FSTRES(1)/FCU)**3
B=2.25*(FSTRES(1)/FCU)**2
C=-0.35*(FSTRES(1)/FCU)
FSTRAN(1)=ECU*(A+B+C)
306 FSTRAN(2)=ECU*(-FSTRES(2)/FCU*3.-2.)
IF(NFAIL.EQ.3)GOTO 900
CALL STRAIN(E,FSTRAN,KSWAP,IFAIL)
E1=E(1)*1000.
IF(E1.GT.ETU)IFAIL=IFAIL-1
IF(E1.GT.0.0.AND.E1.LT.0.01)GOTO 370
IF(IFAIL.LE.0)GOTO 900
FSTR2=FSTRAN(2)
E2=E(2)*1000.
NFAIL=2
IF(E2.GT.FSTR2.AND.E1.GT.0.01)GOTO 900
MESSAGE 'CONCRETE FAILED BY CRUSHING - STRAINS'
PRINT 360,E(1),E(2),ETU,FSTRAN(1),FSTRAN(2)
360 FORMAT(5F10.6)
NFAIL=3
GOTO 900
370 MESSAGE 'NUMERICAL PROBLEM WITH SMALL STRAINS'
MESSAGE 'DEEMED NOT TO HAVE FAILED'
GOTO 900
''
200 CONTINUE
''MESSAGE      COMPRESSION - TENSION ZONE
''
ALPHA=-0.65*FCU
IF(FT.GE.10.)GOTO 260                                ''ARTIFICIAL MATERIAL
BETA=FT/XK
IF(BETA.LE.ALPHA)GOTO 202
F=XMAX-FT
IF(F.LT.0.)GOTO 210
MESSAGE 'CONCRETE HAS FAILED IN THE COMP-TENSION ZONE'
PRINT 250,S1,S2,FCU
250 FORMAT(3F10.4)
NFAIL=2
210 FSTRES(2)=FT/XK
IF(XMIN.GT.-FCU)GOTO 201
MESSAGE 'FAILED IN BOTH DIRECTIONS'
NFAIL=3
GOTO 201

```

```

202 F=(1.+3.28*XK)*(-FCU)/(1.+XK)**2-XMIN
   IF (F.LT.0.) GOTO 205
   MESSAGE 'CONCRETE HAS FAILED IN COMP-TENSION ZONE'
   PRINT 250, S1, S2, FCU
   NFAIL=2
205 FSTRES(2)=(1.+3.28*XK)*(-FCU)/(1.+XK)**2
   IF (XMIN.GT.-FCU) GOTO 201
   MESSAGE 'FAILED IN BOTH DIRECTIONS'
   NFAIL=3
201 FSTRES(1)=FT
   FSTRAN(1)=ETU
   A=1.6*(FSTRES(2)/FCU)**3
   B=2.25*(FSTRES(2)/FCU)**2
   C=-0.35*(FSTRES(2)/FCU)
   FSTRAN(2)=-ECU*(A+B+C)
   EMOD2=FSTRES(2)/FSTRAN(2)*2000.
   IF (EMOD2.GT.E0) FSTRAN(2)=FSTRES(2)/E0*2000.
   IF (NFAIL.EQ.3) GOTO 900
   CALL STRAIN(E,FSTRAN,KSWAP,IFAIL)
   IF (IFAIL.EQ.0) GOTO 900
   MESSAGE 'CONCRETE FAILED IN COMP-TEN STRAINS',IFAIL
   PRINT 250, S1, S2, FCU
   NFAIL=2
   IF (IFAIL.EQ.2) NFAIL=3
   GOTO 900
   ''
   '' ARTIFICIAL MATERIAL NO TENSION FAILURE
   ''
260 FSTRES(1)=FT
   FSTRES(2)=-FCU
   FSTRAN(1)=ETU
   FSTRAN(2)=-ECU
   GOTO 900
100 CONTINUE
   ''MESSAGE TENSION - TENSION ZONE
   ''
   F=XMAX-FT
   IF (F.LT.0.) GOTO 110
   MESSAGE 'CONCRETE HAS FAILED BY CRACKING'
   PRINT 150, S1, S2, FT
150 FORMAT(3F10.4)
   NFAIL=1
   IF (XMIN.GT.FT) NFAIL=4
110 FSTRES(1)=FT
   FSTRES(2)=FSTRES(1)
   FSTRAN(1)=ETU
   FSTRAN(2)=ETU
   IF (NFAIL.NE.0) GOTO 900
   CALL STRAIN(E,FSTRAN,KSWAP,IFAIL)
   IF (IFAIL.EQ.0) GOTO 900
   MESSAGE 'CONCRETE HAS CRACKED - STRAINS'
   PRINT 150, S1, S2, FT
   NFAIL=1
900 FSTRAN(1)=FSTRAN(1)/1000.
   FSTRAN(2)=FSTRAN(2)/1000.
   IF (KSWAP.EQ.0) RETURN
   S=FSTRES(2)
   FSTRES(2)=FSTRES(1)

```

```

      FSTRES(1)=S
      TEMP=FSTRAN(2)
      FSTRAN(2)=FSTRAN(1)
      FSTRAN(1)=TEMP
      RETURN
      END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE STRAIN(E,FSTRAN,KSWAP,IFAIL)
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      LOCAL E(2),FSTRAN(2)
      IFAIL=0
      E1=E(1)*1000.
      E2=E(2)*1000.
      IF(KSWAP.EQ.0)GOTO 100
      TEMP=E2
      E2=E1
      E1=TEMP
100  DO 200 I=1,2
      TEMP=E(I)*FSTRAN(I)
      IF(TEMP.LT.0.)PRINT 300
200  CONTINUE
      D1=ABS(E1)-ABS(FSTRAN(1))
      D2=ABS(E2)-ABS(FSTRAN(2))
      IF(D1.GT.0.00001)IFAIL=IFAIL+1
      IF(D2.GT.0.00001)IFAIL=IFAIL+1
300  FORMAT('STRAINS INCOMPATIBLE PROBABLE FAILURE')
      RETURN
      END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE CRACK
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      ' FINDS THE NUMBER OF FAILED ZONES WHICH HAVE DEVELOPED
      ' DURING THE LAST ITERATION SEQUENCE
      PUBLIC ITYPE(),ASTRAN(,,),NSTATE(,),AGAUSS(,,),ELEMS(,,)
      PUBLIC GPROPS(,,)
      LOCAL ISTATE(2),E(2),FSTRES(2),FSTRAN(2)
      DIMENSION P(2)
      IFAIL=0
      CALL PREP3('QUAD8SM',KT)
      LENGTH(AGAUSS(,1,KT),NGAUS)
      NGP=NGAUS/3+0.1
      LENGTH(ITYPE(),LEN)
      DO 5 IT=4,LEN
      IF(ITYPE(IT).EQ.'QUAD8SM')GOTO 20
5  CONTINUE
20  LENGTH(ELEMS(,,IT),NEL)
      DO 100 K=1,NEL
      EQUATE 300 (ELEMS(,K,IT),X)
      FCU=X(11)
      E0=X(12)
      FT=X(15)
      ECU=X(13)
      ETU=X(14)
      DO 150 IG=1,NGP
      ISTATE(1)=NSTATE(IG*2-1,K)
      ISTATE(2)=NSTATE(IG*2,K)
      IPRINT=0
      IF(ISTATE(1).EQ.1.AND.ISTATE(2).EQ.1)IPRINT=1
      ' FOR EACH ELEMENT
      ' UNIAXIAL COMPRESSIVE STRENGTH
      ' INITIAL YOUNGS MODULUS IN COMPRESSION
      ' UNIAXIAL TENSILE STRENGTH
      ' UNIAXIAL COMPRESSIVE FAILURE STRAIN
      ' UNIAXIAL TENSILE FAILURE STRAIN
      ' CURRENT STATUS OF GP
      ' FAILED

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```

      IF (IPRINT.EQ.1) MESSAGE K, IG, 'FAILED BOTH DIRECTIONS'
      IF (IPRINT.EQ.1) GOTO 150
      SXX=AGAUSS(3*IG-2,K,KT)
      SYY=AGAUSS(3*IG-1,K,KT)
      TXY=AGAUSS(3*IG,K,KT)
      IF (ISTATE(1).EQ.1) GOTO 500
      ALPHA=SQRT((SXX-SYY)**2/4.+TXY**2)
      P(1)=(SXX+SYY)/2.+ALPHA
      P(2)=P(1)-2.*ALPHA
      IF (ABS(TXY).LT.0.00001) GOTO 180
      SMOD=ABS(SXX-SYY)
      FI=ATAN2((2.*TXY),(SMOD))*0.5
      GOTO 181
180 FI=0.
181 IF (SYY.GT.SXX) FI=1.57079632679489-FI
      GOTO 250
500 MESSAGE 'CRACKED CONCRETE'
      IPRINT=1
      II=4*(IG-1)+4
      FI=GPROPS(II,K,KT)
      FI2=FI*2.
      CENTRE=(SXX+SYY)*0.5
      RADIUS=(SXX-SYY)*0.5
      SNN=CENTRE+RADIUS*COS(FI2)+TXY*SIN(FI2)
      SCC=CENTRE-RADIUS*COS(FI2)-TXY*SIN(FI2)
      P(1)=SNN
      P(2)=SCC
250 DO 10 IP=1,2
      P1=P(IP)
      P1=ABS(P1)
      IF (P1.LT.0.000001) P(IP)=0.
10 CONTINUE
      E(1)=ASTRAN(1,IG,K)
      E(2)=ASTRAN(2,IG,K)
      CALL DARWIN(P(1),P(2),E,FCU,FT,E0,ECU,ETU,FSTRES,FSTRAN,...
                  NFAIL)
      IF (NFAIL.EQ.0.AND.IPRINT.EQ.1) MESSAGE K, IG
      IF (NFAIL.EQ.0) GOTO 150 'NO FAILURE
      MESSAGE K, IG
      IF (NFAIL.EQ.1) GOTO 200
      IF (NFAIL.EQ.2) GOTO 200
      NSTATE(IG*2,K)=1
      NSTATE(IG*2-1,K)=1
      IFAIL=IFAIL+1
      MESSAGE 'ELEMENT ',K,' GP',IG,' TO BE RELEASED'
      GOTO 150
200 IF (ISTATE(1).EQ.1) GOTO 150
      II=4*(IG-1)+4
      OLDFI=GPROPS(II,K,KT)
      MESSAGE OLDFI,FI
      GPROPS(II,K,KT)=FI
      NSTATE(IG*2-1,K)=1
      IFAIL=IFAIL+1
      MESSAGE 'ELEMENT ',K,' GP',IG,' TO BE RELEASED'
150 CONTINUE
300 CONTINUE
100 CONTINUE
      PUT(0,5) IFAIL

```

' GIVES NUMBER OF FAILED POINTS TO MASTER

```

      RETURN
      END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE EMOD(FSTRES,FSTRAN,P,E,E0,YM1,YM2)
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      LOCAL P(2),FSTRES(2),FSTRAN(2),E(2),YMOD(2)
      ' UPDATES YOUNGS MODULI IN PRINCIPAL STRESS
      ' DIRECTIONS USING THE SAENZ CURVE AND
      ' ACCUMULATED EQUIVALENT UNIAXIAL STRAINS
      DO 100 I=1,2
      ' MESSAGE FSTRES(I),FSTRAN(I),P(I),E(I)
      IF(P(I).NE.0)GOTO 50
      MESSAGE 'POSSIBLY STRAINS AND STRESSES INCOMPATABLE'
      AE=ABS(E(I))
      IF(AE.GT.0.000001)GOTO 250      ' FAILED RELEASED STRESSES
50  EIU=0.-E(I)                      ' CHANGE SIGNS
      EIC=0.-FSTRAN(I)
      AEIU=ABS(EIU)
      AEIC=ABS(EIC)
      DIFF=AEIU-AEIC
      IF(DIFF.GE.0.000001)GOTO 250   ' FAILED UNRELEASED STRESSES
      IF(AEIC.LT.0.000001)GOTO 200
      ES=FSTRES(I)/FSTRAN(I)
      IF(EIC.LT.0.)GOTO 150
      A=(1.-(EIU/EIC)**2)
      B=(E0/ES-2.)*EIU/EIC
      C=(EIU/EIC)**2
      B=(1.+B+C)**2
      YMOD(I)=E0*A/B
      GOTO 100
150  YMOD(I)=ES
      GOTO 100
200  YMOD(I)=E0
      GOTO 100
250  YMOD(I)=0.
100  CONTINUE
      YM1=YMOD(1)
      YM2=YMOD(2)
      RETURN
      END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE SHP8(ZE,ET,ZI,EI,ELCO,AA,DET)

      This subprogram is identical to SHP8 in 'CONC1'

      END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE GAUSS (NG,GP,HG,KONTRL)

      This subprogram is identical to GAUSS in 'CONC1'

      END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE SHP3(ZE,A)
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      ' SHAPE FUNCTIONS FOR A
      ' 3-NODED ISOPARAMETRIC LINE ELEMENT
      LOCAL A(6)

```

```

      A(1)=ZE/2.*(ZE-1.)
      A(2)=(1.+ZE)*(1.-ZE)
      A(3)=ZE/2.*(ZE+1.)
      A(4)=-A(3)
      A(5)=-A(2)
      A(6)=-A(1)
      RETURN
      END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE RESID
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      ' THIS SUBROUTINE CALCULATES THE CONSISTENT NODAL
      ' FORCE VECTOR FROM THE RESIDUAL GAUSS POINT
      ' STRESSES FOR EITHER A QUAD8 OR BOND6 ELEMENT
      PUBLIC MARK(20),ANGLE(,),AGAUSS(,,),VALS(,,),ELEMS(,,),ITYPE()
      PUBLIC CSIGMA(),BSIGMA(),DISPL(),COORDS(,)
      DIMENSION NE(10),NAME(3)
      LOCAL UN(16),XYCORD(3,2),ELCO(10,3),RLOAD(400)
      REAL LATPR,LP1,LP2,LP3
      GET(0,0)IPRINT
      GET(0,7)ILOAD
      LENGTH(VALS(,,2),LENVAL)
      IF(ILOAD.LE.LENVAL)GOTO 5
      MESSAGE 'DUMMY LOADCASE ',ILOAD,' DOES NOT EXIST'
      MARK(12)=1
      RETURN
5  GET(0,5)(NAME(1),NUM)
   DO 100 I=1,NUM
   IF(NAME(I).EQ.'BOND6')GOTO 10
   IF(NAME(I).EQ.'QUAD8SM')GOTO 10
   MESSAGE 'NO ROUTINE AVAILABLE ** ERROR **'
   MARK(12)=1
   RETURN
10 LENGTH(ITYPE(),LEN)
   DO 20 IT=4,LEN
   IF(ITYPE(IT).EQ.NAME(I))GOTO 30
20 CONTINUE
30 CALL PREP3(NAME(I),KT)
   XYCORD(1,1)=-1.
   XYCORD(2,1)=0.
   XYCORD(3,1)=1.
   LENGTH(AGAUSS(,1,KT),NGAUS)
   NGP=NGAUS/3+0.1
   LENGTH(ELEMS(,IT),NEL)
   MESSAGE
   MESSAGE $NAME(I),'STRESSES DURING THE ITERATION'
   DO 40 J=1,NEL
   IF(NAME(I).NE.'QUAD8SM')GOTO 70
   REDEFINE(ANGLE(,),NEL)
   REDEFINE(ANGLE(,J),NGP)
70 CONTINUE
   EQUATE 40 (ELEMS(,J,IT),X)
   NN=X(1)+0.1
   NP=X(2)+0.1
   DO 50 K=1,NN
   NE(K)=X(2+K)+0.1
   KK=K*2
   KJ=NE(K)*2

```

```

      KI=NE(K)
      ELCO(K,1)=COORDS(KI,1)
      ELCO(K,2)=COORDS(KI,2)
      UN(KK-1)=DISPL(KJ-1)
50  UN(KK)=DISPL(KJ)
      IF (NAME(I).EQ.'BOND6') GOTO 80
      CALL RQUAD(NGP,IT,KT,J,UN,ELCO,RLOAD,IPRINT)
      GOTO 110
80  NE1=X(8)
      NE2=X(7)
      NE3=X(6)
      NB1=X(3)
      NB2=X(4)
      NB3=X(5)
      LP1=CSIGMA(NE1*3-1)
      LP2=CSIGMA(NE2*3-1)
      LP3=CSIGMA(NE3*3-1)
      RS1=BSIGMA(NB1*3-1)
      RS2=BSIGMA(NB2*3-1)
      RS3=BSIGMA(NB3*3-1)
      N=3
      XYCORD(1,2)=RS1
      XYCORD(2,2)=RS2
      XYCORD(3,2)=RS3
      CALL LSQFIT(N,XYCORD,ARS,BRS)
      XYCORD(1,2)=LP1
      XYCORD(2,2)=LP2
      XYCORD(3,2)=LP3
      CALL LSQFIT(N,XYCORD,ALP,BLP)
      CALL RBOND(NGP,IT,KT,J,UN,ELCO,RLOAD,ALP,ARS,BLP,BRS,IPRINT)
110 CONTINUE
      IF (IPRINT.EQ.1) MESSAGE 'DUMMY LOAD'
      DO 60 K=1,NN
      N=NE(K)
      JJ=K*2-1
      II=N*2+1
      VALS(II,ILOAD,2)=VALS(II,ILOAD,2)+RLOAD(JJ)
      VALS(II+1,ILOAD,2)=VALS(II+1,ILOAD,2)+RLOAD(JJ+1)
      IF (IPRINT.EQ.1) PRINT 200,N,RLOAD(JJ),RLOAD(JJ+1)
200 FORMAT(I3,3X,2E18.10)
      60 CONTINUE
      40 CONTINUE
100 CONTINUE
      RETURN
      END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  SUBROUTINE RBOND(NGP,IT,KT,J,UN,ELCO,RLOAD,ALP,ARS,BLP,BRS,IPRINT)
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      PUBLIC ELEMS(,,),AGAUSS(,,)
      LOCAL A(6),ELCO(10,3),UN(16),RLOAD(400)
      DIMENSION RSTRSS(3),GP(3),HG(3)
      REAL LATPR
      KGEOM=ELEMS(9,J,IT)
      GEOM=ELEMS(10,J,IT)
      X1=ELCO(1,1)
      X2=ELCO(2,1)
      X=ABS(X1-X2)
      EQUATE 30 (AGAUSS(,J,KT),S)

```

```

ZE=0.774596669241483
GP(1)=ZE
GP(2)=0.
GP(3)=ZE
HG(1)=0.5555555555555555
HG(2)=0.8888888888888888
HG(3)=HG(1)
IF(IPRINT.EQ.1)MESSAGE 'EL. GP ACCUM. THEO. RESID. ',...
' SLIP LPR RPR UB'
DO 30 IG=1,NGP
ZE=GP(IG)
SLIP=0.
SLIPN=0.
CALL SHP3(ZE,A)
DO 20 I=1,3
JJ=2*I-1
BI=A(I)
KK=12-JJ
V=UN(JJ+1)-UN(KK+1)
SLIPN=SLIPN+V*BI
U=UN(JJ)-UN(KK)
20 SLIP=SLIP+U*BI
LATPR=ALP+ZE*BLP
RADPR=ARS+ZE*BRS
CALL BOND(KGEOM,J,IT,SLIP,SLIPN,THSTRS,RT,RN,LATPR,RADPR,UBMAX)
II=3*IG-2
RSTRSS(IG)=S(II)-THSTRS
IF(IPRINT.EQ.1)PRINT 200,J,IG,S(II),THSTRS,RSTRSS(IG),SLIP,...
LATPR,RADPR,UBMAX
200 FORMAT(2I3,7F8.4)
AGAUSS(II,J,KT)=AGAUSS(II,J,KT)-RSTRSS(IG)
30 CONTINUE
DO 50 I=1,12
50 RLOAD(I)=0.
DO 60 IG=1,NGP
ZE=GP(IG)
CALL SHP3(ZE,A)
CONST=RSTRSS(IG)*HG(IG)*X*GEOM
DO 70 I=1,6
BI=A(I)
K=2*I-1
70 RLOAD(K)=RLOAD(K)+CONST*BI
60 CONTINUE
RETURN
END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
SUBROUTINE RQUAD(NGP,IT,KT,J,UN,ELCO,RLOAD,IPRINT)
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
PUBLIC NSTATE(,),ELEMS(,),ANGLE(,),ASTRAN(,),AGAUSS(,)
PUBLIC GPROPS(,)
LOCAL ELCO(10,3),FSTRES(2),FSTRAN(2),AA(2,8),ZI(8),EI(8)
LOCAL ISTATE(2),UN(16),RLOAD(400)
DIMENSION RP(2),RSXX(9),RSYY(9),RTXY(9),P(2)
LOCAL E(2),THEOP(2)
LOCAL GP(3),HG(3),KONTRL(9)
HIGH DELTA,BETA,GAMMA
EQUATE 30(AGAUSS(,J,KT),S)
FCU=ELEMS(11,J,IT)

```

```

      EQ=ELEMS(12,J,IT)          '' INITIAL YOUNGS MODULUS
      THICK=ELEMS(17,J,IT)       '' THICKNESS OF CONCRETE
      ECU=ELEMS(13,J,IT)        '' UNIAXIAL COMP. FAILURE STRAIN
      ETU=ELEMS(14,J,IT)        '' UNIAXIAL TENSILE FAILURE STRAIN
      FT=ELEMS(15,J,IT)         '' TENSILE STRENGTH
      LL=0
      ZI(1)=-1.,0.,1.,1.,1.,0.,-1.,-1.
      EI(1)=-1.,-1.,-1.,0.,1.,1.,1.,0.
      NG=SQRT(NGP+1.)
      CALL GAUSS(NG,GP,HG,KONTRL)
      IF(IPRINT.EQ.0)GOTO 31
      MESSAGE'EL GP SIGMA1 SIGMA2 ANGLE(DEG) RESID1 RESID2',...
      ' STR1 STR2 FAIL'
      MESSAGE
31 DO 30 IG=1,NG
      ZE=GP(IG)
      DO 30 JG=1,NG
      KPRINT=IPRINT
      ET=GP(JG)
      LL=LL+1
      N=KONTRL(LL)
      ANGLE(N,J)=0
      CALL SHP8(ZE,ET,ZI(1),EI(1),ELCO,AA,DET)
      EXX=0.
      EYY=0.
      GXY=0.
      DO 170 L=1,8
      BI=AA(1,L)
      CI=AA(2,L)
      UI=UN(2*L-1)
      VI=UN(2*L)
      EXX=EXX+BI*UI
      EYY=EYY+CI*VI
170 GXY=GXY+BI*VI+CI*UI
      ISTATE(1)=NSTATE(N*2-1,J)
      ISTATE(2)=NSTATE(N*2,J)
      SXX=S(3*N-2)
      SYX=S(3*N-1)
      TXY=S(3*N)
      IF(ISTATE(1).EQ.1.AND.ISTATE(2).EQ.0)GOTO 300
      ALPHA=SQRT((SXX-SYX)**2/4.+TXY**2)
      P(1)=(SXX+SYX)/2.+ALPHA
      P(2)=P(1)-2.*ALPHA
      DO 10 I=1,2
      P1=P(I)
      P1=ABS(P1)
      IF(P1.LT.0.000001)P(I)=0. ''-VE ZERO P(I) PROBLEM
10 CONTINUE
      FI2=ATAN2((2.*TXY),(SXX-SYX))
      FI=FI2*0.5
      IF(ABS(TXY).LT.0.00001.AND.SXX.GT.SYX)FI2=0.
      IF(ABS(TXY).LT.0.00001.AND.SXX.LT.SYX)FI2=3.1415926535897824
      ANGLE(N,J)=FI2
      GOTO 181
300 MESSAGE 'CRACKED CONCRETE'
      KPRINT=1
      II=4*(N-1)+2
      FI=GPROPS(II+2,J,KT)

```

```

      FI2=FI*2.
      CENTRE=(SXX+SYI)*0.5
      RADIUS=(SXX-SYI)*0.5
      SNN=CENTRE+RADIUS*COS(FI2)+TXI*SIN(FI2)
      SCC=CENTRE-RADIUS*COS(FI2)-TXI*SIN(FI2)
      P(1)=SNN
      P(2)=SCC
181  E(1)=ASTRAN(1,N,J)
      E(2)=ASTRAN(2,N,J)
      CALL DARWIN(P(1),P(2),E,FCU,FT,E0,ECU,ETU,FSTRES,FSTRAN,...
                  NFAIL)
      IF(NFAIL.NE.0)KPRINT=1
      CALL SAENZ(ISTATE,FSTRES,FSTRAN,E,E0,THEOP)
      RP(1)=P(1)-THEOP(1)
      RP(2)=P(2)-THEOP(2)
      ALPHA=(RP(1)-RP(2))*COS(FI2)/2.
      RSXX(N)=(RP(1)+RP(2))*0.5+ALPHA
      GAMMA=RSXX(N)
      BETA=ALPHA*2.0
      DELTA=GAMMA-BETA
      RSYI(N)=DELTA
      RTXI(N)=(RP(1)-RP(2))*SIN(FI2)/2.
195  CONTINUE
      E1=E(1)*1000000.
      E2=E(2)*1000000.
      DEGFI=FI*57.29578
      IF(KPRINT.EQ.1)PRINT 205,J,N,P(1),P(2),DEGFI,RP(1),...
      RP(2),E1,E2,ISTATE(1),ISTATE(2)
205  FORMAT(2I2,2F10.6,F10.3,2F10.6,2F7.1,2I2)
      ' MESSAGE
      AGAUSS(3*N-2,J,KT)=S(3*N-2)-RSXX(N)
      AGAUSS(3*N-1,J,KT)=S(3*N-1)-RSYI(N)
      AGAUSS(3*N,J,KT)=S(3*N)-RTXI(N)
30   CONTINUE
      DO 50 I=1,16
50    RLOAD(I)=0.
      LI=0
      DO 60 IG=1,NG
      ZE=GP(IG)
      DO 60 JG=1,NG
      ET=GP(JG)
      LI=LI+1
      N=KONTRL(LI)
      CALL SHP8(ZE,ET,ZI(1),EI(1),ELCO,AA,DET)
      IF(DET.EQ.0.)RETURN
      CONST=DET*THICK*HG(IG)*HG(JG)
      DO 70 I=1,8
      BI=AA(1,I)
      CI=AA(2,I)
      K=2*I-1
      RLOAD(K)=RLOAD(K)+CONST*(BI*RSXX(N)+CI*RTXI(N))
70   RLOAD(K+1)=RLOAD(K+1)+CONST*(CI*RSYI(N)+BI*RTXI(N))
60   CONTINUE
      RETURN
      END
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  SUBROUTINE SAENZ(ISTATE,FSTRES,FSTRAN,E,E0,THEOP)
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

```

```

'' FINDS THE THEORETICAL STRESSES IN THE TWO
'' PRINCIPAL DIRECTIONS GIVEN THE FAILURE
'' VALUES AND THE ACCUMULATED EQUIVALENT
'' UNIAXIAL STRAINS USING THE SAENZ CURVE
LOCAL ISTATE(2),FSTRES(2),FSTRAN(2),E(2),THEOP(2)
DO 190 K=1,2
  IF(ISTATE(K).EQ.1)GOTO 184
  EIU=E(K)
  EIC=FSTRAN(K)
  AEIU=ABS(EIU)
  AEIC=ABS(EIC)
  DIFF=AEIU-AEIC
  IF(DIFF.GE.0)GOTO 200
  ASTRES=FSTRES(K)
  ASTRES=ABS(ASTRES)
  IF(ASTRES.LT.0.000001)GOTO 184
  ES=FSTRES(K)/FSTRAN(K)
  IF(EIC.GT.0)GOTO 182
  B=(E0/ES-2.)*EIU/EIC
  C=(EIU/EIC)**2
  B=1.+B+C
  A=E0/B
  GOTO 183
182 A=ES
183 THEOP(K)=EIU*A
  GOTO 185
184 THEOP(K)=0.
  GOTO 185
200 THEOP(K)=FSTRES(K)
185 CONTINUE
190 CONTINUE
  RETURN
  END
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  SUBROUTINE BOND(KGEOM,J,IT,D,DN,S,RT,RN,LATPR,RADPR,UBMAX)
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
'' VARIOUS BOND MODELS I.E. BOND STRESS
'' VERSUS SLIP RELATIONSHIPS
PUBLIC ELEMS(,,)
REAL LATPR
RN=ELEMS(12,J,IT)
MARKER=0
MARKN=0
YINTER=ELEMS(13,J,IT)
GRAD=ELEMS(14,J,IT)
DELTA=ELEMS(15,J,IT)
BETA=ELEMS(16,J,IT)
IF(DN.GT.0.)GOTO 16
DN=-DN
MARKN=1
16 IF(D.GT.0)GOTO 30
D=-D
MARKER=1
30 GOTO(1,2,3,4,5,6,7,8,9,10),KGEOM
''RJA TYPE BOND
1 IF(D.GT.0.47436)GOTO 15
S=78.*D
RT=78.

```

```

      GOTO 20
15  RT=11.1
      S=3.7+(D-0.047436)*11.1
      GOTO 20
      ''NILSONS RELATIONSHIP
2   IF(D.GE.0.0113948)GOTO 25
      S=979*D-57241.*D*D+835627.*D*D*D
      RT=979-114482*D+2506881.*D*D
      GOTO 20
25  S=4.9595
      RT=0.
      GOTO 20
      '' QUADRATIC BOND CURVE
3   IF(D.GE.0.2)GOTO 35
      S=-250.*D*D+100.*D
      RT=-500.*D+100
      GOTO 20
35  S=10.
      RT=0.
      GOTO 20
      '' QUADRATIC CASE 2
4   IF(D.GE.0.1)GOTO 45
      S=-500.*D*D+100.*D
      RT=-1000.*D+100.
      GOTO 20
45  S=5.
      RT=0.
      GOTO 20
      '' DESAYI CURVE - VARYING ULTIMATE BOND STRESS
5   DMAX=0.1
      CALL ULTBND(LATPR,UBMAX,RADPR,YINTER,GRAD)
      IF(D.GE.DMAX)GOTO 55
      RU=UBMAX/DMAX
      S=2.*RU*D/(1.+100.*D*D)
      A=2.*RU*(1.-100.*D*D)
      B=1.+100.*D*D
      RT=A/B/B
      GOTO 20
55  S=UBMAX
      RT=0.
      GOTO 20
      '' SAENZ TYPE CURVE - VARYING ULTIMATE BOND STRESS
      '' DEVELOPMENT OF BOND STRESS-SLIP CURVE DEPENDENT
      '' ON CONCRETE LATERAL PRESSURE
6   DMAX=DELTA '' MAX BOND STRESS AT DMAX SLIP
      CALL ULTBND(LATPR,UBMAX,RADPR,YINTER,GRAD)
      IF(D.GE.DMAX)GOTO 65
      IF(UBMAX.EQ.0)GOTO 66
      RSEC=UBMAX/DMAX
      RZERO=ELEMS(11,J,IT)
      DRATIO=D/DMAX
      B=1.+(RZERO/RSEC-2.)*DRATIO+DRATIO*DRATIO
      S=RZERO*D/B
      RT=RZERO*(1.-DRATIO*DRATIO)/B/B
      GOTO 20
65  S=UBMAX*BETA
      RT=0.
      GOTO 20

```

```

66 RT=0.
   S=0.
   RN=0.
   GOTO 20
   '' LABIB AND EDWARDS CURVES
7  RT=0.
   S=4.962
   IF(D.GT.0.0762) GOTO 75
   RT=1432.
   S=RT*D
   IF(D.LT.0.00254) GOTO 75
   RT=1000.
   S=RT*(D-0.00254)+3.609
   IF(D.LT.0.002931) GOTO 75
   RT=500.
   S=RT*(D-0.002931)+4.
   IF(D.LT.0.003931) GOTO 75
   RT=6.39
   S=RT*(D-0.003931)+4.5
75  RN=268.
   IF(DN.GT.0.01) RN=0.
   GOTO 20
8  CONTINUE
9  CONTINUE
   '' ELASTIC CASE
10 RT=ELEMS(11,J,IT)
   S=RT*D
20 IF(MARKER.EQ.1) S=-S
   RETURN
   END
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   SUBROUTINE ULTBND(LATPR,UBMAX,RADPR,YINTER,GRAD)
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   REAL LATPR,RLAT
   COEFF=0.7704
   RLAT=LATPR*COEFF+RADPR
   IF(RLAT.GE.0.) GOTO 100 '' TENSILE LATERAL PRESSURE
   UBMAX=-GRAD*RLAT+YINTER
   ''MESSAGE 'BOND ',UBMAX,LATPR,RADPR,RLAT
   RETURN
100 UBMAX=-GRAD*RLAT+YINTER
   IF(UBMAX.LT.0.) UBMAX=0.
   RETURN
   END
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   SUBROUTINE LSQFIT(N,XCOORD,A,B)
''XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
   LOCAL XCOORD(3,2)
   ''
   '' LINEAR LEAST SQUARES FIT
   ''
   SXX=0.
   SXY=0.
   SX=0.
   SY=0.
   DO 2 I=1,N
   X=XCOORD(I,1)
   Y=XCOORD(I,2)

```

```

      SX=SX+X
      SY=SY+Y
      SXX=SXX+X*X
      SXY=SXY+X*Y
2     CONTINUE
      AN=N
      B=(SX*SY-AN*SXY)/(SX*SX-SXX*AN)
      A=(SY-B*SX)/AN
      RETURN
      END
*EXIT

```

## APPENDIX E

### DICTIONARY OF VARIABLE NAMES

|                       |                                                                                                                          |
|-----------------------|--------------------------------------------------------------------------------------------------------------------------|
| A (NDIMEN,NN)         | Partial derivatives of the nodal shape functions                                                                         |
| AA (NDIMEN,NN)        | Partial Cartesian derivatives of the nodal shape functions                                                               |
| ACOB (NDIMEN,NN)      | Jacobian matrix for a concrete element                                                                                   |
| AGAUSS (NGAUS,NEL,KT) | Accumulated Gauss point stresses for all element types used                                                              |
| AH (7)                | Weightings applied to Gauss point values of a concrete element for extrapolation purposes                                |
| ANGLE (NGP,NEL)       | Angle anticlockwise to $O_1$ from the O-X axis at each concrete Gauss point                                              |
| ARS, ALP              | Coefficient in linear least squares fit to nodal values of equivalent bar radial pressure and concrete lateral pressure  |
| ASTRAN (2,NGP,NEL)    | Accumulated concrete equivalent uniaxial strains                                                                         |
| BACO (NDIMEN,NDIMEN)  | Inverse of the Jacobian matrix                                                                                           |
| BETA                  | Ratio of maximum bond stress to peak bond stress for bond slips greater than DELTA                                       |
| BSIGMA (NP)           | Nodal values of bar radial pressure based on average nodal axial stresses extrapolated from the Gauss point values       |
| BRS, BLP              | Coefficients in linear least squares fit to nodal values of equivalent bar radial pressure and concrete lateral pressure |
| CENTRE                | Value of stress at the centre of the Mohr's circle of stress transformation                                              |
| COORDS (NP,NDIMEN)    | Cartesian co-ordinates of the global nodes                                                                               |
| CSIGMA (3NP)          | Average nodal concrete stresses                                                                                          |
| C1, C2,... C          | Coefficients (general)                                                                                                   |
| DE (NRCS)             | Nodal displacements due to the load increment for an element                                                             |
| DEGFI                 | Angle anticlockwise to $O_1$ from O-X axis (degrees)                                                                     |
| DELTA                 | Value of bond slip at which peak bond stress occurs                                                                      |
| DET                   | Determinant of the Jacobian matrix                                                                                       |

|                                 |                                                                                                                                                                                                      |
|---------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| DISPL (ND)                      | Accumulated nodal displacements                                                                                                                                                                      |
| D1, D2,... D6                   | Terms in the concrete [ D ] matrix                                                                                                                                                                   |
| E(2), E1, E2                    | Intermediate variables for values of strain at the concrete Gauss points                                                                                                                             |
| ECU                             | Concrete equivalent uniaxial compressive failure strain                                                                                                                                              |
| EI (NN)                         | The $\eta$ -co-ordinate of the nodes of a concrete element                                                                                                                                           |
| EIU                             | Failure value of equivalent uniaxial strain for the current principal stress ratio                                                                                                                   |
| EIC                             | Accumulated value of equivalent uniaxial strain                                                                                                                                                      |
| ELOC (NN,NDIMEN)                | The Cartesian co-ordinates of an element                                                                                                                                                             |
| ELEMS (2+NN+NPARAMS,<br>NEL,IT) | Value of node number and material parameters for each element :<br><br>1 gives NN ;<br>2 gives NPARAMS ;<br>3 to 2+NN gives the node numbers ;<br>3+NN to 2+NN+NPARAMS gives the material parameters |
| ES (NRCS,NRCS)                  | Stiffness matrix for an element                                                                                                                                                                      |
| ETU                             | Concrete equivalent uniaxial tensile failure strain                                                                                                                                                  |
| EO                              | Initial tangent modulus of elasticity for the concrete                                                                                                                                               |
| FCU                             | Concrete compressive strength                                                                                                                                                                        |
| FI, FI2                         | Angle (radians) anti-clockwise to $\sigma_1$ from O-X axis and $FI2 = 2 \times FI$                                                                                                                   |
| FSTRES (2)                      | The concrete failure stresses for the current principal stress ratio                                                                                                                                 |
| FSTRAN (2)                      | The equivalent uniaxial failure strains for the current principal stress ratio                                                                                                                       |
| FT                              | Concrete tensile failure strength                                                                                                                                                                    |
| GE                              | The $\eta$ -co-ordinate of Gauss point                                                                                                                                                               |
| GEOM (8)                        | The initial material parameters for an element                                                                                                                                                       |

Bar element

1 = Young's modulus of elasticity  
2 = cross-sectional area of the bar

Bond element

|                |            |
|----------------|------------|
| 1 = bond type  | 5 = YINTER |
| 2 = Perimeter  | 6 = GRAD   |
| 3 = initial RT | 7 = DELTA  |
| 4 = initial RN | 8 = BETA   |

Concrete element

|         |                     |
|---------|---------------------|
| 1 = FCU | 5 = FT              |
| 2 = EO  | 6 = Poisson's ratio |
| 3 = ECU | 7 = THICK           |
| 4 = ETU |                     |

GP (NGP)

The  $\eta$  or  $\xi$  co-ordinates of a Gauss point

GPROPS (i,NEL,KT)

The current values of material parameters monitored at the Gauss points

Bond elements

i = 2xNN - 1  
i gives current RT  
i+1 gives current RN

Concrete elements

i = 4xNN - 3  
i gives YM1  
i+1 gives YM2  
i+2 gives PR  
i+3 gives OLDFI

GRAD

The slope of the local ultimate bond stress-radial pressure line

GS (3)

Intermediate variable for Gauss point stresses

GSTRESS (NGAUS,  
NEL,KT)

Values of Gauss point stresses for a load increment for all element types

HG (NGP)

Weighting factors for the appropriate Gauss rule

IDATA (ILENI)

Integer data variables

IG

Counter from 1 to NG

ILENI

Number of integer data variables in problem orientated command

ILENR

Number of real data variables in problem orientated command

IFAIL

Flag to indicate whether the current uniaxial strains in each principal stress direction exceed in magnitude the corresponding failure strains.  
= 0 means both less  
= 1 means one greater  
= 2 means both greater

ILOAD

Load case number corresponding to dummy load

IPRINT

Flag for printing or not. =1 means print otherwise 0

|                               |                                                                                                                                                                                                                                                                                                                                                                                                                        |
|-------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| ISTATE (2), ISTAT1,<br>ISTAT2 | Flag for current status of a concrete Gauss point<br>in each principal stress direction<br>= 0 means no failure<br>= 1 means failed                                                                                                                                                                                                                                                                                    |
| ISTRS (70)<br>ISTRS           | Vector used to store a name string<br>Flag to indicate which type of stresses are to be<br>calculated and printed. A value of :<br><br>0 means extrapolate accumulated Gauss point<br>stresses to nodes only<br><br>1 means calculate incremental Gauss point stresses<br><br>2 means extrapolate accumulated Gauss point<br>stresses and print<br><br>3 means calculate incremental Gauss point stresses<br>and print |
| IT                            | Element type number used in ELEMS ()                                                                                                                                                                                                                                                                                                                                                                                   |
| ITYPE ( )                     | Name strings of all element types used in an<br>analysis. The order of the elements corresponds<br>to the order in which the tables were read                                                                                                                                                                                                                                                                          |
| JG                            | Counter from 1 to NG                                                                                                                                                                                                                                                                                                                                                                                                   |
| KNAME (NETYPE)                | Name strings of all element types used in an<br>analysis. The order corresponds to the order read<br>in the START GAUSS STRESSES command                                                                                                                                                                                                                                                                               |
| KONTROL (NGP)                 | Gives the standard local Gauss point number as the<br>DO-loop will yield them in an unstandard order                                                                                                                                                                                                                                                                                                                   |
| KPRINT                        | Flag for printing or not. Similiar to IPRINT                                                                                                                                                                                                                                                                                                                                                                           |
| KT                            | Element type number used in KNAME (NETYPE)                                                                                                                                                                                                                                                                                                                                                                             |
| KT (20)                       | Array holding the element type values (IT) for<br>the current element types contributing to the<br>global stiffness matrix                                                                                                                                                                                                                                                                                             |
| LATPR                         | Concrete lateral pressure                                                                                                                                                                                                                                                                                                                                                                                              |
| LC, LCASE                     | Load case number                                                                                                                                                                                                                                                                                                                                                                                                       |
| LCSE (20)                     | Array used to store the load case numbers given<br>in a problem orientated command                                                                                                                                                                                                                                                                                                                                     |
| LEN                           | Length of an vector in an array (general)                                                                                                                                                                                                                                                                                                                                                                              |
| LL                            | Counter for the Gauss points 1 to NGP                                                                                                                                                                                                                                                                                                                                                                                  |
| LMS (20)                      | Array containing the name strings of the element<br>types for which stresses are to be calculated                                                                                                                                                                                                                                                                                                                      |
| LOAD (NLC)                    | Name strings of each load case title                                                                                                                                                                                                                                                                                                                                                                                   |

|                          |                                                                                                                                                                                                                                                            |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
|--------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|----------------------|------------------|---------------|--------------------|--------------|-------------|--------------|--------------|---|--|--|--|
| LP1, LP2, LP3            | Values of concrete lateral pressure at the Gauss points of a bond element                                                                                                                                                                                  |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| MARK (20)                | Testing facility and error trap                                                                                                                                                                                                                            |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| 1                        | gives the problem 'type'                                                                                                                                                                                                                                   |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| 2                        | gives the number of dimensions (NDIMEN)                                                                                                                                                                                                                    |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| 3=1<br>4=1<br>5=1<br>6=1 | <table><tr><td rowspan="4">}</td><td>if a</td><td>{ supports }</td><td>has been read</td></tr><tr><td>table of</td><td>{ elements }</td><td>otherwise 0</td></tr><tr><td></td><td>{ loads }</td><td></td></tr><tr><td></td><td></td><td></td></tr></table> | }            | if a                 | { supports }     | has been read | table of           | { elements } | otherwise 0 |              | { loads }    |   |  |  |  |
| }                        | if a                                                                                                                                                                                                                                                       |              | { supports }         | has been read    |               |                    |              |             |              |              |   |  |  |  |
|                          | table of                                                                                                                                                                                                                                                   |              | { elements }         | otherwise 0      |               |                    |              |             |              |              |   |  |  |  |
|                          |                                                                                                                                                                                                                                                            |              | { loads }            |                  |               |                    |              |             |              |              |   |  |  |  |
|                          |                                                                                                                                                                                                                                                            |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
|                          | if the assembled global stiffness matrix has been reduced, otherwise 0                                                                                                                                                                                     |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| 7=1                      | if the nodes have been renumbered, otherwise 0                                                                                                                                                                                                             |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| 8                        | gives the number of nodes (NODS)                                                                                                                                                                                                                           |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| 9                        | gives the number of supported nodes                                                                                                                                                                                                                        |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| 10                       | gives the number of degrees of freedom per node                                                                                                                                                                                                            |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| 11                       | gives the number of types of elements                                                                                                                                                                                                                      |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| 12=1                     | if an error occurred in calculating the global stiffness matrix, otherwise 0                                                                                                                                                                               |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| 13=1<br>14=1<br>15=1     | <table><tr><td rowspan="3">}</td><td>if an error occurred</td><td>{ co-ordinates }</td><td>other-</td></tr><tr><td>during the reading</td><td>{ supports }</td><td>wise</td></tr><tr><td>of any table</td><td>{ elements }</td><td>0</td></tr></table>     | }            | if an error occurred | { co-ordinates } | other-        | during the reading | { supports } | wise        | of any table | { elements } | 0 |  |  |  |
| }                        | if an error occurred                                                                                                                                                                                                                                       |              | { co-ordinates }     | other-           |               |                    |              |             |              |              |   |  |  |  |
|                          | during the reading                                                                                                                                                                                                                                         |              | { supports }         | wise             |               |                    |              |             |              |              |   |  |  |  |
|                          | of any table                                                                                                                                                                                                                                               | { elements } | 0                    |                  |               |                    |              |             |              |              |   |  |  |  |
| 16                       | gives the largest node difference of any element                                                                                                                                                                                                           |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| 17=1                     | if an error whilst reading load table, otherwise 0                                                                                                                                                                                                         |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| 18                       | gives the largest node number in data                                                                                                                                                                                                                      |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| 19= - ; and              |                                                                                                                                                                                                                                                            |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| 20= 1                    | if test output is required otherwise 0                                                                                                                                                                                                                     |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| MARKER                   | Flag for error in reading element type from START GAUSS STRESSES command<br>=1 if an error, otherwise 0                                                                                                                                                    |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| NAME                     | Name of an element                                                                                                                                                                                                                                         |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| NCL                      | Clause number                                                                                                                                                                                                                                              |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| ND                       | Total number of displacements (NP x NFREE)                                                                                                                                                                                                                 |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| NDIMEN                   | Number of dimensions                                                                                                                                                                                                                                       |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| NDSTR                    | Total number of nodal stresses in an analysis                                                                                                                                                                                                              |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| NE (NN)                  | Global node numbers of an element                                                                                                                                                                                                                          |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| NEL                      | The number of elements of an element type                                                                                                                                                                                                                  |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |
| NETYPE                   | The number of element types used in an analysis                                                                                                                                                                                                            |              |                      |                  |               |                    |              |             |              |              |   |  |  |  |

|                   |                                                                                                                                                                                  |
|-------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| NFAIL             | Flag in the concrete model for type of failure<br>0 means no failure<br>1 means failed in biaxial tension<br>2 means failed in compression-tension<br>3 means failed by crushing |
| NFREE             | Number of degrees of freedom per node                                                                                                                                            |
| NG                | Number of Gauss points in one direction                                                                                                                                          |
| NGAUS             | Total number of stresses for an element being monitored at the Gauss points                                                                                                      |
| NGP<br>NGP2       | Total number of Gauss points for an element<br>NGP x 2                                                                                                                           |
| NGPROPS           | Total number of properties for an element<br>(NGPxNPROPS)                                                                                                                        |
| NICK              | Name string of an element type                                                                                                                                                   |
| NLC               | Number of separate load cases in an analysis                                                                                                                                     |
| NPROPS            | Number of properties per Gauss point                                                                                                                                             |
| NN                | Number of local nodes                                                                                                                                                            |
| NP, NODS          | Total number of nodes in an analysis                                                                                                                                             |
| NRCS              | Number of rows or columns of an element stiffness matrix (NNxNFREE)                                                                                                              |
| NREF              | Counter from 1 to NTAB                                                                                                                                                           |
| NS (NN+1,NEL,KT)  | Global node numbers of all the elements<br>1 gives the number of nodes of an element (NN)<br>2 to NN+1 gives the global node numbers                                             |
| NSTATE (NGP2,NEL) | Flag for current status of all concrete Gauss points similar to ISTATE(2)                                                                                                        |
| NTAB              | Number of element types available                                                                                                                                                |
| NUM (NP)          | Number of elements contributing nodal stresses to each node                                                                                                                      |
| OLDFI             | THETA value at end of the previous load increment                                                                                                                                |
| P(2), P1, P2      | Principal stresses                                                                                                                                                               |
| PR                | Poisson's ratio                                                                                                                                                                  |
| RADIUS            | The radius of Mohr's circle of stress                                                                                                                                            |
| RADPR             | Equivalent radial pressure due to bar radial contraction                                                                                                                         |

|                       |                                                                                       |
|-----------------------|---------------------------------------------------------------------------------------|
| RDATA (ILENR)         | Real data variables used in problem orientated command                                |
| RLAT                  | Combined concrete and bar radial pressures                                            |
| RLOAD (NRCS)          | Residual load vector for an element                                                   |
| RN                    | Bond modulus normal to the bar                                                        |
| RNORM                 | The Euclidean norm of the load vector case                                            |
| RP (2)                | The concrete residual stresses in the two principal stress direction at a Gauss point |
| RS1, RS2, RS3         | Equivalent bar pressures due to radial contraction                                    |
| RSXX, RSY, RTXY       | Concrete residual stresses transformed into x-y co-ordinates                          |
| RSTRSS (NGP)          | Bond residual stresses at each Gauss point of an element                              |
| RT                    | The current bond modulus parallel with the bar                                        |
| RSEC, RU              | Secant value of bond modulus at peak bond stress                                      |
| RZERO                 | Initial tangent value of bond modulus                                                 |
| SIGMA ( )             | Nodal values of stress (general)                                                      |
| SLIP, SLIPN           | Bond slip parallel and normal to the bar                                              |
| SN (40)               | Extrapolated nodal values of stress for an element                                    |
| SNN, SCC              | Stresses in the direction of a crack and orthogonal to the crack                      |
| STRAIN (NGP2,NEL)     | Accumulated equivalent uniaxial strains for each Gauss point of all concrete elements |
| STRESS (NGAUS,NEL,KT) | Accumulated Gauss point stresses for all element types                                |
| STRN2 (4,3)           | Concrete strains at the 2x2 Gauss points                                              |
| STRN3 (9,3)           | Concrete strains extrapolated to the 3x3 Gauss points                                 |
| SXX, SYY, TXY         | $\sigma_{xx}$ , $\sigma_{yy}$ , $\tau_{xy}$ stresses                                  |
| TABLE (3,4)           | Table of data relevant to each element type                                           |

In STRS

1 = 'element name'  
2 = 1 means nodal stresses possible

In STIFF

1 = 'element name'  
2 = no. of local nodes  
3 = no. of initial material properties

|                                           |                                                                                                                                                                                                                                                                                                                                                             |
|-------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| THEOP (2)                                 | The theoretical principal concrete stresses for the current values of accumulated uniaxial strain                                                                                                                                                                                                                                                           |
| THETA                                     | Angle to $\sigma_1$ from O-X axis during the update of the concrete properties                                                                                                                                                                                                                                                                              |
| THICK                                     | The thickness of the concrete element                                                                                                                                                                                                                                                                                                                       |
| THSTRS                                    | Theoretical bond stress for (SLIP)                                                                                                                                                                                                                                                                                                                          |
| UBMAX                                     | Local ultimate bond stress                                                                                                                                                                                                                                                                                                                                  |
| U, V, U(), V()                            | Values of displacement or slip (general)                                                                                                                                                                                                                                                                                                                    |
| VALS (ND+2,NLC,2)                         | Array containing load vectors<br>1 gives ND<br>2 gives the value 1 indicating a vector is held<br>3 to ND+2 gives the components of the load vector                                                                                                                                                                                                         |
| VALS (ND+2,NLC,3)                         | As above but holds the displacements for each load case                                                                                                                                                                                                                                                                                                     |
| VALS (NRCS <sup>2</sup> +NN+4,<br>NEL,KT) | Array containing the stiffness matrix and node numbers of all the elements. Locally<br>1 and 2 gives NRCS<br>3 to NRCS <sup>2</sup> +2 gives the values of the element stiffness matrix column by column<br>NRCS <sup>2</sup> +3 gives NN<br>NRCS <sup>2</sup> +4 gives NFREE<br>NRCS <sup>2</sup> +5 to NRCS <sup>2</sup> +NN+4 gives element node numbers |
| VKH, VKV                                  | Bond stiffness values                                                                                                                                                                                                                                                                                                                                       |
| XYCORD (NGP,2)                            | Pairs of data points for least squares analysis                                                                                                                                                                                                                                                                                                             |
| YINTER                                    | The $q_0$ parameter in the bond model (Chapter 3)                                                                                                                                                                                                                                                                                                           |
| YMOD(2), YM1, YM2                         | Young's moduli for the concrete in the principal stress directions                                                                                                                                                                                                                                                                                          |
| ZI (NN)                                   | The $\xi$ -co-ordinate of the nodes of an element                                                                                                                                                                                                                                                                                                           |

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