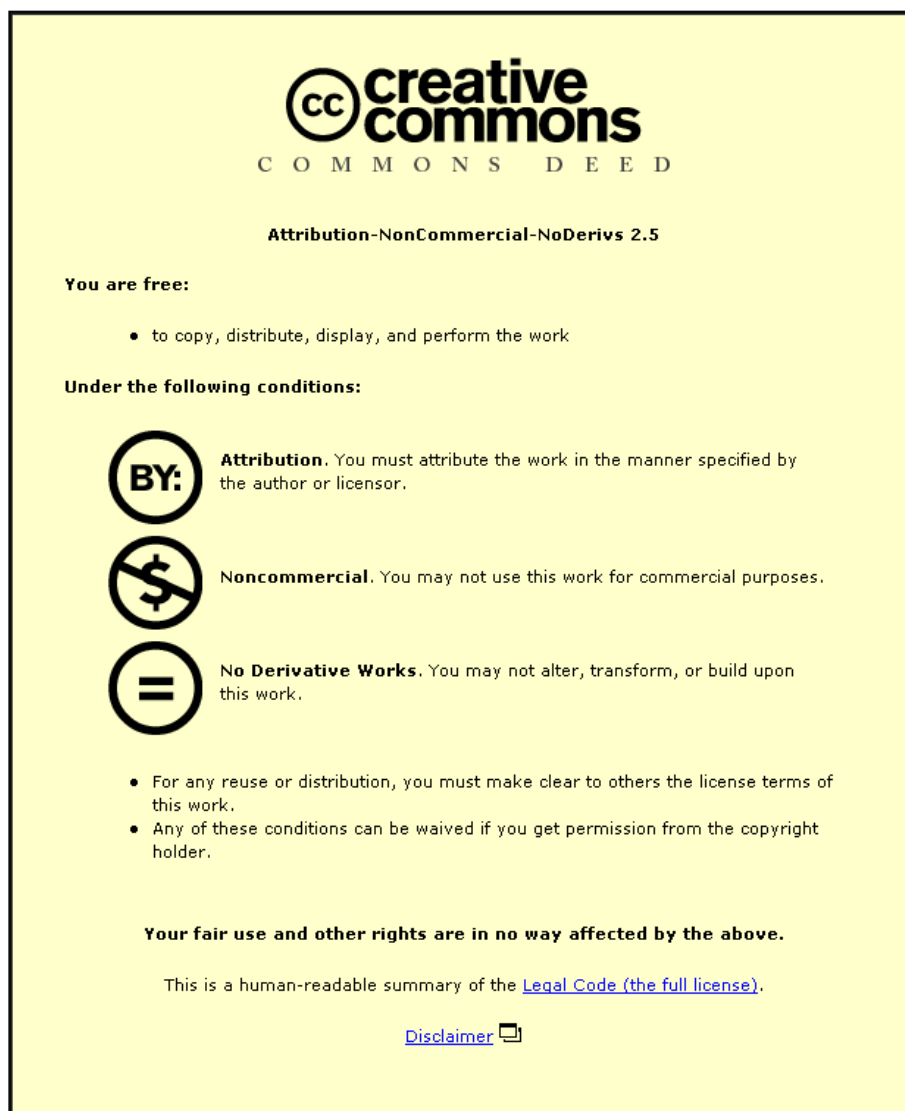


This item was submitted to Loughborough University as a PhD thesis by the author and is made available in the Institutional Repository (<https://dspace.lboro.ac.uk/>) under the following Creative Commons Licence conditions.



For the full text of this licence, please go to:
<http://creativecommons.org/licenses/by-nc-nd/2.5/>



Pilkington Library

Author/Filing Title *DNIES*

Vol. No. Class Mark *T*

**Please note that fines are charged on ALL
overdue items.**

FOR REFERENCE ONLY

0402450779



BADMINTON PRESS
UNIT 1, BROOK ST
SYSTON
LEICESTER, LE7 1GD
ENGLAND
TEL : 0116 260 2917
FAX : 0116 269 6639

STATIC, DYNAMIC AND FATIGUE CHARACTERISTICS OF HELICAL CABLES

by

Mr. Timothy James Davies, M. Eng., B. Eng.

A Doctoral Thesis submitted in partial fulfilment of the requirements for the award of
Doctor of Philosophy of Loughborough University

December 2000

© by Timothy Davies 2000

 Liverpool University	
Library	
Date	Dec 01
Class	
Acc No.	040248077

ABSTRACT

Extensive parametric studies have been carried out, using the orthotropic sheet theoretical model of Hobbs and Raoof, on a wide range of spiral strand constructions, with outside diameters, d , and lay angles, α , in the practical ranges, $16.4 \text{ mm} \leq d \leq 184 \text{ mm}$, and $11^\circ \leq \alpha \leq 24^\circ$, respectively. The effects of an external hydrostatic pressure on certain structural characteristics of sealed spiral strands, used in deep water applications, have also been studied in some detail, for water depths ranging from 0 m to 2000 m. The results, based on such theoretical parametric studies, have, for example, been used to refute claims by Jolicoeur that, by a simple modification, a significant improvement to the original orthotropic sheet model of Hobbs and Raoof had been found. In addition, using such studies, axial fatigue life design S-N curves have been developed, which cater for the effects of an externally applied hydrostatic pressure on sheathed spiral strands. Simple (hand-based) formulations have also been developed for estimating the maximum frictional axial and torsional hysteresis along with the associated $\frac{\text{axial load range}}{\text{mean axial load}}$, and $\frac{\text{range of twist}}{2}$, respectively, at which they occur, relating to both, the in-air conditions and also when a sheathed spiral strand is subjected to an external hydrostatic pressure.

The previously reported work of Raoof and his associates, in connection with the response of helical cables (spiral strands and/or wire ropes) to impact loading, has been extended to include the development of closed-form solutions for predicting the extensional-torsional wave speeds and displacements, in axially preloaded helical cables, experiencing a half-sine type of impact loading at one end, with the other end fixed. The influence of the lay angle on the response of a spiral strand to three different (i.e. unit-step, triangular and half-sine) forms of impact loading functions, has also been analysed, with much emphasis placed on the practical implications of the final results in connection with non-destructive methods of wire fracture detection under service conditions.

The bending characteristics of helical cables have been addressed in some considerable detail. The position of zero lateral deflection (i.e. the effective point of fixity) for socketed spiral strands has been shown to lie at some distance inside the

socket, and the traditional assumption of a constant effective bending stiffness, for determining the minimum radii of curvature at the points of fixity to the cables, has been shown to be a reasonable one for cases when the maximum lateral deflection is of the order of one cable diameter. A simple, but reliable method has been proposed for the experimental determination of the cable bending stiffness, which largely overcomes the shortcomings of the previously adopted techniques.

Based on a general form of Hruska's formulations, as proposed by Strzemiecki and Hobbs, and using the predictions based on a recently reported model by Raoof and Kraincanic, a simple (hand-based) method is proposed for obtaining reliable estimates of the no-slip and full-slip axial stiffnesses of wire ropes, with either independent wire rope (IWRC) or fibre cores.

The question of size effects, in connection with the determination of the axial or torsional frictional hysteresis, plus the axial fatigue life of large diameter (multi-layered) spiral strands is critically examined on a theoretical basis.

Finally, the implications of using the no-slip axial stiffnesses (as opposed to the full-slip values), as permitted by the pre-standard ENV 1993 – 2, Eurocode 3, for analysing certain characteristics of cable structures under serviceability loading conditions is addressed, in the context of the structural behaviour of a two-dimensional cable truss. To this end, the practical implications of changes in the lay angle of the supporting spiral strands (with this parameter controlling the variations in the no-slip and full-slip strand axial stiffnesses) in terms of, for example, estimates of the vertical deflections of the truss have been examined. It is theoretically demonstrated that, in view of the rather small axial load perturbations (cf. mean axial loads) under serviceability limit state conditions, use of the more appropriate no-slip stiffnesses (as opposed to the traditionally used full-slip values) leads to practically significant reductions in the estimated values of the vertical deflections of the cable truss.

ACKNOWLEDGEMENTS

I would firstly, and fore-mostly like to thank my supervisor, Professor Mohammed Raoof, whose enthusiasm for this subject is unparalleled and whose guidance, encouragement and advice has made this project, not only interesting, but, at times, extremely enjoyable.

I would also like to thank Dr. Cecil Scott of Loughborough University for his substantial input in the formulation of the half-sine impact loading equations.

Finally, I would like to thank my family, particularly my mum and dad, whose continued support and encouragement has enabled me to continue my education unhindered.

CONTENTS

ABSTRACT

ACKNOWLEDGEMENTS

CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOTATION

CHAPTER 1	INTRODUCTION	1
CHAPTER 2	LITERATURE SURVEY	
2.1	INTRODUCTION	5
2.2	MECHANICAL MODELS	6
2.1.1	Purely Tensile Models	6
2.2.2	Thin Rod Models	7
2.2.3	Semi-Continuous Models	16
2.3	STIFFNESS CHARACTERISTICS OF CABLES	20
2.4	HYSTERETIC CHARACTERISTICS OF HELICAL CABLES	23
2.5	HYDROSTATIC PRESSURE EFFECTS ON THE STIFFNESS AND HYSTERESIS OF SHEATHED SPIRAL STRANDS	27
2.6	TERMINATIONS AND ANCHORAGE SYSTEMS	29
2.7	IMPACT LOADING ON HELICAL CABLES	31
2.8	AXIAL FATIGUE CHARACTERISTICS OF HELICAL CABLES	32

2.9	BENDING OF STEEL CABLES	37
2.10	RECOVERY LENGTH OF FRACTURED WIRES	44
2.11	SUMMARY AND CONCLUSIONS	46

CHAPTER 3 SEMI-CONTINUOUS ANALYSIS OF MULTI-LAYERED SPIRAL STRANDS

3.1	INTRODUCTION	54
3.2	THEORY	57
3.2.1	Strand Radial (Clench) and Circumferential (Hoop) Contact Forces	59
3.2.2	Properties of the Orthotropic Sheet	62
3.2.3	Axial Stiffness	64
3.2.4	Torsional Stiffness	65
3.2.5	Hysteresis	67
	3.2.5.1 Axial Hysteresis	69
	3.2.5.2 Torsional Hysteresis	70
3.2.6	Axial/Torsional Coupling	71
3.2.7	Proposed Modifications to the Orthotropic Sheet Theoretical Model	74
3.3	ANALYSIS	74
3.4	RESULTS	81
3.4.1	Axial Stiffness	84
3.4.2	Torsional Stiffness	87
3.4.3	Axial Hysteresis	88
3.4.4	Torsional Hysteresis	91
3.4.5	Axial/Torsional Coupling Coefficients	92
3.4.6	Simplified Formulations for Axial/Torsional	

	Coupling Coefficients	97
	3.4.6.1 Full-Slip Case	97
	3.4.6.2 No-Slip Case	98
3.5	DISCUSSION	108
3.6	CONCLUSIONS	110

CHAPTER 4 IMPACT LOADING OF HELICAL CABLES

4.1	INTRODUCTION	112
4.2	THEORY	113
	4.2.1 Constitutive Relations For Helical Cables	113
	4.2.2 Dynamic Analysis	113
	4.2.2.1 Equations Of Motion	114
	4.2.2.2 Response to Impact Loading	115
	4.2.2.3 Speed of Axial and Torsional Waves	119
4.3	RESULTS	120
4.4	DISCUSSION	132
4.5	CONCLUSIONS	133

CHAPTER 5 RESTRAINED BENDING CHARACTERISTICS OF HELICAL CABLES AT TERMINATIONS

5.1	INTRODUCTION	135
5.2	EXPERIMENTAL OBSERVATIONS	137
5.3	THEORY	141
	5.3.1 Local Phenomenon near the Fixed End	141
	5.3.2 Calculation of Plane-Section $(EI)_{\text{eff}}$	145
5.4	RESULTS	146
5.5	DISCUSSION	148

5.6	CONCLUSIONS	148
CHAPTER 6	EXPERIMENTAL DETERMINATION OF THE CABLE BENDING STIFFNESS	
6.1	INTRODUCTION	150
6.2	EXPERIMENTAL MEASUREMENTS	151
6.3	THEORY	154
6.3.1	Distorted Shape of the Cable – Simplified Method	154
6.3.2	Distorted Shape of the Cable – Rigorous Method	155
6.3.3	Method of Solution	162
6.4	CALCULATION OF THE PLANE-SECTION BENDING STIFFNESS, (EI)	164
6.5	CALCULATION OF THE TORSIONAL STIFFNESS (GJ)	166
6.6	RESULTS	167
6.7	DISCUSSION	176
6.8	CONCLUSIONS	177
CHAPTER 7	SIMPLE DETERMINATION OF WIRE ROPE AXIAL STIFFNESS	
7.1	INTRODUCTION	179
7.2	HRUSKA'S APPROACH	181
7.3	ANALYSIS	182
7.4	RESULTS	183
7.4.1	Simple Formulations	200
7.5	DISCUSSION	201
7.6	CONCLUSIONS	202

**CHAPTER 8 HYDROSTATIC PRESSURE EFFECTS ON THE
AXIAL FATIGUE LIFE ESTIMATION OF
LARGE DIAMETER SHEATHED SPIRAL STRANDS**

8.1	INTRODUCTION	203
8.2	THEORY	205
8.2.1	Development of the S-N Curves	205
8.2.2	Effect of Mean Axial Load and Grade of Wire	206
8.2.3	Hydrostatic Pressure Effects	207
8.3	RESULTS	208
8.4	DISCUSSION	247
8.5	DESIGN EQUATIONS	249
8.6	SUMMARY AND CONCLUSIONS	251

**CHAPTER 9 SIMPLE FORMULATIONS FOR THE
DETERMINATION OF THE MAXIMUM
AXIAL AND TORSIONAL HYSTERESIS**

9.1	INTRODUCTION	253
9.2	THEORY	255
9.3	RESULTS	255
9.3.1	Axial Hysteresis	255
9.3.2	Torsional Hysteresis	271
9.3.3	Numerical Examples	285
9.4	DISCUSSION	286
9.5	CONCLUSIONS	287

**CHAPTER 10 EFFECT OF STRAND DIAMETER ON
 AXIAL FATIGUE AND FRICTIONAL
 HYSTERESIS PREDICTIONS**

10.1	INTRODUCTION	289
10.2	THEORY	290
10.3	RESULTS	291
	10.3.1 Axial Fatigue Results	293
	10.3.2 Axial Hysteresis	301
	10.3.3 Torsional Hysteresis	303
10.4	DISCUSSION	305
10.5	CONCLUSIONS	306

**CHAPTER 11 ANALYSIS OF A TWO-DIMENSIONAL
 CABLE TRUSS**

11.1	INTRODUCTION	307
11.2	THEORY	308
11.3	RESULTS	311
11.4	DISCUSSION	327
11.5	CONCLUSIONS	327

CHAPTER 12 CONCLUSIONS

12.1	INTRODUCTION	329
12.2	MAIN FINDINGS	332

REFERENCES	337
-------------------	------------

LIST OF FIGURES

Figure 1.1 - Various Components of a Spiral Strand and Wire Rope.

Figure 1.2 - Typical Wire Rope Lays: (a) Right Regular Lay; (b) Left Regular Lay; (c) Right Lang Lay; (d) Left Lang Lay; and (e) Right Alternative Lay Wire Ropes.

Figure 1.3 - Typical Examples of Cross-Sectional Constructions for Wire Ropes.

Figure 3.1 - (a) Geometry of a Layer of Spiral Strand; and (b) the Equivalent Orthotropic Cylinder.

Figure 3.2 - Pattern of Interwire Contact Forces in a Multi-Layered Spiral Strand.

Figure 3.3 - Contact Forces Acting Within a Spiral Strand.

Figures 3.4a Variation of the Parameter c_k with Changes in the Cable Mean and b - Axial Strain, S'_1 , for the 127 mm Outside Diameter Strand ($\alpha = 12^\circ$) as Calculated Using Both the RH and RH₂ Models.

Figures 3.4c Variation of the Parameter c_k with Changes in the Cable Mean and d - Axial Strain, S'_1 , for the 127 mm Outside Diameter Strand ($\alpha = 18^\circ$) as Calculated Using Both the RH and RH₂ Models.

Figures 3.4e Variation of the Parameter c_k with Changes in the Cable Mean and f - Axial Strain, S'_1 , for the 127 mm Outside Diameter Strand ($\alpha = 24^\circ$) as Calculated Using Both the RH and RH₂ Models.

Figures 3.5a Variation of the Axial Stiffness with Changes in the Axial and b - Load Range / Mean Load, as Calculated Using both the RH

and RH₂ Models at two Different Levels of Cable Mean Axial Strain: (a) $S'_1 = 0.0015$ and (b) $S'_1 = 0.0035$.

Figure 3.6 - Torque – Twist Relationship for Axially Pre-Loaded Spiral Strands, as Calculated Using Both the RH or RH₂ Models.

Figures 3.7a Variation of the Overall Frictional Axial Damping with
and b - Changes in the Axial Load Range / Mean Load, as Calculated
by: (a) RH Model, and (b) RH₂ Model.

Figure 3.8 - Variation of the Torsional Frictional Damping with Changes in
the Twist per Unit Length for the 127 mm Axially Pre-loaded
Spiral Strands, as Calculated Using both the RH and RH₂
Models.

Figures 3.9a Variation of the Overall Axial/Torsional Coupling
and b - Coefficients, A_2 and A_3 , as Calculated Using Both the RH and
RH₂ Models, with the Lay Angle, for a 127 mm Outside
Diameter Spiral Strand, Based on the No-Slip and Full-Slip
Limiting Conditions; Assuming a Strand Mean Axial Strain of:
(a) $S'_1 = 0.0015$; and (b) $S'_1 = 0.0035$.

Figures 3.10a Comparison of the Full-Slip Stiffness Coefficients k_2 and k_3 , in
and b - the Various Layers of 127 mm Diameter Spiral Strands, as
Calculated Using Both the RH and RH₂ Models.

Figure 3.11 - Comparison of the No-Slip Layer Stiffness Coefficients, k_2 and
 k_3 , with Changes in the Lay Angle, at Various Levels of Cable
Mean Axial Strain, as Calculated Using Both the RH and RH₂
Models.

Figure 3.12a - Variation of the Full-Slip Stiffness Coefficient, k_1 , with
Changes in the Lay Angle, at Various Levels of Cable Mean
Axial Strain, S'_1 , as Calculated Using both the RH and RH₂

Models.

Figure 3.12b - Variation of the No-Slip Stiffness Coefficient, k_1 , with Changes in the Lay Angle, at Various Levels of Cable Mean Axial Strain, S'_1 , as Calculated Using both the RH and RH₂ Models.

Figure 3.12c - Variation of the Full-Slip Stiffness Coefficient, k_3 , with Changes in the Lay Angle, at Various Levels of Cable Mean Axial Strain, S'_1 , as Calculated Using both the RH and RH₂ Models.

Figure 3.12d - Variation of the No-Slip Stiffness Coefficient, k_3 , with Changes in the Lay Angle, at Various Levels of Cable Mean Axial Strain, S'_1 , as Calculated Using both the RH and RH₂ Models.

Figure 3.12e - Variation of the Full-Slip Stiffness Coefficients, k_2 and k_4 , with Changes in the Lay Angle, at Various Levels of Cable Mean Axial Strain, S'_1 , as Calculated Using both the RH and RH₂ Models.

Figure 3.12f - Variation of the No-Slip Stiffness Coefficients, k_2 and k_4 , with Changes in the Lay Angle, at Various Levels of Cable Mean Axial Strain, S'_1 , as Calculated Using both the RH and RH₂ Models.

Figure 3.13a - Variation of the Full-Slip Stiffness Coefficients, $k_1 - k_4$, with Changes in the Lay Angle, at Various Levels of Cable Mean Axial Strain $0.0008 \leq S'_1 \leq 0.0035$, as Calculated Using both the RH and RH₂ Models.

Figure 3.13b - Variation of the No-Slip Stiffness Coefficients, $k_1 - k_4$, with

Changes in the Lay Angle, at Various Levels of Cable Mean Axial Strain $0.0008 \leq S'_1 \leq 0.0035$, as Calculated Using both the RH and RH₂ Models.

Figures 4.1(a- c) Comparison of the Axial and Rotational Displacements, and (d-f) - Respectively, Along the Cable at Time $t = 0.001163$ (sec), Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.

Figures 4.2(a-c) Comparison of the Axial and Rotational Displacements, and (d-f) - Respectively, at the Middle Point of the Cable $X = 5m$, as a Function of Time, Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.

Figures 4.3(a-c) Comparison of the Axial and Rotational Displacements, and (d-f) - Respectively, Along the Cable at Time $t = 0.001163$ (sec), Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.

Figures 4.4(a-c) Comparison of the Axial and Rotational Displacements, and (d-f) - Respectively, at the Middle Point of the Cable $X = 5m$, as a Function of Time, Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.

Figures 4.5(a-c) Comparison of the Axial and Rotational Displacements,

and (d-f) - Respectively, Along the Cable at Time $t = 0.001163$ (sec), Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.

Figures 4.6(a-c) Comparison of the Axial and Rotational Displacements, and (d-f) - Respectively, at the Middle Point of the Cable $X = 5\text{m}$, as a Function of Time, Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.

Figures 4.7(a-c) Comparison of the Axial and Rotational Displacements, and (d-f) - Respectively, Along the Cable at Time $t = 0.001163$ (sec), Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.

Figures 4.8(a-c) Comparison of the Axial and Rotational Displacements, and (d-f) - Respectively, at the Middle Point of the Cable $X = 5\text{m}$, as a Function of Time, Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.

Figure 5.1 - Deflected Shape of an Axially Preloaded Spiral Strand Subjected to a Single Lateral Point Load.

Figure 5.2 - The Experimentally Obtained Deflected Shapes for the 39 mm Strand at the Propped Socket Under Modes $(-\delta)$ and $(+\delta)$ of Free Bending – after Raoof (1992a).

Figure 5.3 - Net Deflection at Various Locations Along the 39 mm O. D. Spiral Strand Near the Fixed Socket, Obtained by the Differential Displacement Method – after Raoof (1992a).

Figure 5.4 - Bending Deflection of a Simple Tendon.

Figure 5.5 - The Experimental and Theoretically Obtained Deflected Shapes for the 39 mm Outside Diameter Spiral Strand at the Propped Socket Under Modes $(-\delta)$ and $(+\delta)$ of Free Bending: (a) Full-Slip Regime; (b) No-Slip Regime.

Figure 6.1 - Observed Distorted (Helical) Shape of the 164 mm Outside Diameter Strand Subjected to an Axial Force, F .

Figure 6.2 - Variations of the Ripple Amplitude with Changes in the Applied Axial Force for Both the First and Final Loading Runs.

Figure 6.3 - Planar Representation of the Helical Cable Used in the Simplified Analysis.

Figure 6.4 - Schematic Representation of the Helical Cable Under Zero Axial Load, Based on the First Loading Run.

Figure 6.5 - Schematic Representation of the Helical Cable Under an Applied Axial Load.

Figure 6.6 - External and Internal Forces and Moments Acting on the Helical Rod.

Figure 6.7 - Torque – Twist Relationship for the 164 mm Outside Diameter Axially Pre-Loaded Cable at a Mean Axial Strain $S'_1 = 0.0025$.

- Figure 6.8 - Variation, Based on the Simplified Approach, of the Bending Moment as a Function of the Parameter $\frac{1}{R}$.
- Figure 6.9 - Experimental Plus Theoretical Variations of the Ripple Amplitude with Changes in the Applied Axial Force Based on Both the Simplified and Rigorous Methods.
- Figure 6.10a - Theoretical Variations of the Parameter G' with Changes in the Applied Axial Force for Both the First and Final Loading Runs.
- Figure 6.10b - Theoretical Variations of the Parameter H with Changes in the Applied Axial Force for Both the First and Final Loading Runs.
- Figure 6.10c - Theoretical Variations of the Cable Shear Force, N' , with Changes in the Applied Axial Force for Both the First and Final Loading Runs.
- Figure 6.11 - Theoretical Variations of the Cable Axial Strain, ϵ_{hl} , with Changes in the Applied Cable Axial Force, F , for Both the First and Final Loading Runs, Based on the Rigorous Method.
- Figure 6.12 - Theoretical Variations of Δ with Changes in the Applied Cable Axial Force, F , for Both the First and Final Loading Runs, Based on the Rigorous Method.
- Figure 7.1 - Construction Details for a Typical Six-Stranded Wire Rope with an Independent Wire Rope Core (IWRC) – After Lee (1991).
- Figure 7.2a - Relationship Between the Full-Slip E-Values for Various Wire Rope Constructions with Either a Fibre Core (Case 1) or an

IWRC as Calculated Using Raoof and Kraincanic's Model and Hruska's Formula.

Figure 7.2b - Relationship Between the Full-Slip E-Values for Various Wire Rope Constructions with Either a Fibre Core (Case 2) or an IWRC as Calculated Using Raoof and Kraincanic's Model and Hruska's Formula.

Figure 7.2c - Relationship Between the Full-Slip E-Values for the Wire Ropes with IWRC as Calculated Using Raoof and Kraincanic's Model and Hruska's Formula.

Figure 7.3a - Relationship Between the Full-Slip and No-Slip E-Values for Various Wire Rope Constructions with Either a Fibre Core (Case 1) or an IWRC as Calculated Using Raoof and Kraincanic's Model.

Figure 7.3b - Relationship Between the Full-Slip and No-Slip E-Values for Various Wire Rope Constructions with Either a Fibre Core (Case 2) or an IWRC as Calculated Using Raoof and Kraincanic's Model.

Figure 7.3c - Relationship Between the Full-Slip and No-Slip E-Values for Various Wire Rope Constructions with IWRC as Calculated Using Raoof and Kraincanic's Model.

Figure 7.4a - Correlations Between the Experimental Results and the Theoretical Fitted Curves, With the Latter as Given in Fig. 7.2a.

Figure 7.4b - Correlations Between the Experimental Results and the Theoretical Fitted Curves, With the Latter as Given in Fig. 7.2b.

Figure 7.4c - Correlations Between the Experimental Results and the Theoretical Fitted Curves, With the Latter as Given in Fig. 7.2c.

Figure 7.5a - Correlations Between the Experimental Results and the Theoretical Fitted Curves, With the Latter as Given in Fig. 7.3a.

Figure 7.5b - Correlations Between the Experimental Results and the Theoretical Fitted Curves, With the Latter as Given in Fig. 7.3b.

Figure 7.5c - Correlations Between the Experimental Results and the Theoretical Fitted Curves, With the Latter as Given in Fig. 7.3c.

Figures 8.1(a - d) - Theoretical Plots of the Endurance Limit Versus Mean Axial Load for the Three Different Sheathed Spiral Strand Constructions, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (a) and (c) $K_a = 1.0$ and 0.5, Respectively, for the Fatigue Life to First Outermost Wire Fracture; (b) and (d) $K_a = 1.0$ and 0.5, Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.

Figures 8.1(e-h) - Theoretical Plots of the Endurance Limit Versus Mean Axial Load for the Three Different Sheathed Spiral Strand Constructions, at a Water Depth of 500 m, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (e) and (g) $K_a = 1.0$ and 0.5, Respectively, for the Fatigue Life to First Outermost Wire Fracture; (f) and (h) $K_a = 1.0$ and 0.5, Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.

Figures 8.1(i-l) - Theoretical Plots of the Endurance Limit Versus Mean Axial Load for the Three Different Sheathed Spiral Strand Constructions, at a Water Depth of 1000 m, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (i) and (k) $K_a = 1.0$ and 0.5, Respectively, for the Fatigue Life to First Outermost Wire Fracture; (j) and (l) $K_a = 1.0$ and 0.5, Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.

Figures 8.1(m-p) - Theoretical Plots of the Endurance Limit Versus Mean Axial Load for the Three Different Sheathed Spiral Strand Constructions, at a Water Depth of 1500 m, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (m) and (o) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (n) and (p) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.

Figures 8.1(q-t) - Theoretical Plots of the Endurance Limit Versus Mean Axial Load for the Three Different Sheathed Spiral Strand Constructions, at a Water Depth of 2000 m, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (q) and (s) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (r) and (t) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.

Figures 8.2(a-e) - Composite Plots of the Theoretical Predictions of Strand Fatigue Life to all Layer Wire Fractures for a Given Level of Mean Axial Load, $S'_1 = 0.002867$, Varying Magnitudes of Lay Angle, α , and at Varying Water Depths; (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 1.0$.

Figures 8.3(a-e) - Composite Plots of the Theoretical Predictions of Strand Fatigue Life to all Layer Wire Fractures for a Given Level of Mean Axial Load, $S'_1 = 0.002867$, Varying Magnitudes of Lay Angle, α , and at Varying Water Depths; (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 0.5$.

Figures 8.4(a-e) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 12$ degrees) Based on the Fatigue Life to First Outer Layer Wire Fracture and Subjected to a

Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 1.0$.

Figures 8.4(f-j) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 12$ degrees) Based on the Fatigue Life to First Innermost Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (f) 0 m; (g) 500 m; (h) 1000 m; (i) 1500 m; (j) 2000 m - $K_a = 1.0$.

Figures 8.5(a-e) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 18$ degrees) Based on the Fatigue Life to First Outer Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 1.0$.

Figures 8.5(f-j) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 18$ degrees) Based on the Fatigue Life to First Innermost Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (f) 0 m; (g) 500 m; (h) 1000 m; (i) 1500 m; (j) 2000 m - $K_a = 1.0$.

Figures 8.6(a-e) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 24$ degrees) Based on the Fatigue Life to First Outer Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 1.0$.

Figures 8.6(f-j) - Lower Bound S -N Curves for the 127 mm Outside Diameter

Sheathed Spiral Strand ($\alpha = 24$ degrees) Based on the Fatigue Life to First Innermost Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (f) 0 m; (g) 500 m; (h) 1000 m; (i) 1500 m; (j) 2000 m - $K_a = 1.0$.

Figures 8.7(a-e) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 12$ degrees) Based on the Fatigue Life to First Outer Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 0.5$.

Figures 8.7(f-j) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 12$ degrees) Based on the Fatigue Life to First Innermost Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (f) 0 m; (g) 500 m; (h) 1000 m; (i) 1500 m; (j) 2000 m - $K_a = 0.5$.

Figures 8.8(a-e) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 18$ degrees) Based on the Fatigue Life to First Outer Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 0.5$.

Figures 8.8(f-j) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 18$ degrees) Based on the Fatigue Life to First Innermost Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (f) 0 m; (g) 500 m; (h) 1000 m; (i) 1500 m; (j) 2000 m - $K_a = 0.5$.

- Figures 8.9(a-e) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 24$ degrees) Based on the Fatigue Life to First Outer Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 0.5$.
- Figures 8.9(f-j) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 24$ degrees) Based on the Fatigue Life to First Innermost Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (f) 0 m; (g) 500 m; (h) 1000 m; (i) 1500 m; (j) 2000 m - $K_a = 0.5$.
- Figures 8.10(a-e) - Effect of Varying the Magnitude of the Lay Angle, on the S-N Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.001$, and at Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 1.0$.
- Figures 8.10(f-j) - Effect of Varying the Magnitude of the Lay Angle, on the S-N Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.002$, and at Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 1.0$.
- Figures 8.10(k-o) - Effect of Varying the Magnitude of the Lay Angle, on the S-N Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.002867$, and at Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 1.0$.
- Figures 8.10(p-t) - Effect of Varying the Magnitude of the Lay Angle, on the S-N

Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.004$, and at Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 1.0$.

Figures 8.11(a-e) - Effect of Varying the Magnitude of the Lay Angle, on the S-N Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.001$, and at Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 0.5$.

Figures 8.11(f-j) - Effect of Varying the Magnitude of the Lay Angle, on the S-N Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.002$, and at Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 0.5$.

Figures 8.11(k-o) - Effect of Varying the Magnitude of the Lay Angle, on the S-N Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.002867$, and at Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 0.5$.

Figures 8.11(p-t) - Effect of Varying the Magnitude of the Lay Angle, on the S-N Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.004$, and at Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 0.5$.

Figures 8.12(a-d) - Comparison of Alternative Design S-N Curves Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (a) and (c) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire

Fracture; (b) and (d) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.

Figures 8.12(e-h) - Comparison of Alternative Design S-N Curves, At 0 m and 500 m Water Depth, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (e) and (g) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (f) and (h) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.

Figures 8.12(i-l) - Comparison of Alternative Design S-N Curves, At 0 m and 1000 m Water Depth, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (i) and (k) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (j) and (l) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.

Figures 8.12(m-p) - Comparison of Alternative Design S-N Curves, At 0 m and 1500 m Water Depth, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (m) and (o) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (n) and (p) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.

Figures 8.12(q-t) - Comparison of Alternative Design S-N Curves, At 0 m and 2000 m Water Depth, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (q) and (s) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (r) and (t) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the

Innermost Layer.

Figures 9.1(a-d) - Variations of the Maximum Axial Hysteresis, $(\Delta U/U)_{\max}$ with Changes in the $E_{\text{no-slip}}/E_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (a) $S'_1 = 0.0006$, (b) $S'_1 = 0.00145$, (c) $S'_1 = 0.002867$, and (d) $S'_1 = 0.00430$, Respectively - at 0 m Water Depth, as Calculated Using Methods (a) and (b).

Figures 9.1(e-h) - Variations of the Maximum Axial Hysteresis, $(\Delta U/U)_{\max}$ with Changes in the $E_{\text{no-slip}}/E_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (e) $S'_1 = 0.0006$, (f) $S'_1 = 0.00145$, (g) $S'_1 = 0.002867$, and (h) $S'_1 = 0.00430$, Respectively - at 500 m Water Depth, as Calculated Using Methods (a) and (b).

Figures 9.1(i-l) - Variations of the Maximum Axial Hysteresis, $(\Delta U/U)_{\max}$ with Changes in the $E_{\text{no-slip}}/E_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (i) $S'_1 = 0.0006$, (j) $S'_1 = 0.00145$, (k) $S'_1 = 0.002867$, and (l) $S'_1 = 0.00430$, Respectively - at 1000 m Water Depth, as Calculated Using Methods (a) and (b).

Figures 9.1(m-p) - Variations of the Maximum Axial Hysteresis, $(\Delta U/U)_{\max}$ with Changes in the $E_{\text{no-slip}}/E_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (m) $S'_1 = 0.0006$, (n) $S'_1 = 0.00145$, (o) $S'_1 = 0.002867$, and (p) $S'_1 = 0.00430$, Respectively - at 1500 m Water Depth, as Calculated Using Methods (a) and (b).

Figures 9.1(q-t) - Variations of the Maximum Axial Hysteresis, $(\Delta U/U)_{\max}$ with Changes in the $E_{\text{no-slip}}/E_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (q) $S'_1 = 0.0006$, (r) $S'_1 = 0.00145$, (s) $S'_1 = 0.002867$, and (t) $S'_1 = 0.00430$, Respectively - at 2000 m Water Depth, as Calculated Using Methods (a) and (b).

Figures 9.2(a-d) - Variations of ζ with changes in γ , at 0 m Water Depth, for a

Wide Range of Cable Mean Axial Strains: (a) $S'_1 = 0.0006$, (b) $S'_1 = 0.00145$, (c) $S'_1 = 0.002867$, and (d) $S'_1 = 0.00430$.

Figures 9.2(e-h) - Variations of ζ with changes in γ , at 500 m Water Depth, for a Wide Range of Cable Mean Axial Strains: (e) $S'_1 = 0.0006$, (f) $S'_1 = 0.00145$, (g) $S'_1 = 0.002867$, and (h) $S'_1 = 0.00430$.

Figures 9.2(i-l) - Variations of ζ with changes in γ , at 1000 m Water Depth, for a Wide Range of Cable Mean Axial Strains: (i) $S'_1 = 0.0006$, (j) $S'_1 = 0.00145$, (k) $S'_1 = 0.002867$, and (l) $S'_1 = 0.00430$.

Figures 9.2(m-p) - Variations of ζ with changes in γ , at 1500 m Water Depth, for a Wide Range of Cable Mean Axial Strains: (m) $S'_1 = 0.0006$, (n) $S'_1 = 0.00145$, (o) $S'_1 = 0.002867$, and (p) $S'_1 = 0.00430$.

Figures 9.2(q-t) - Variations of ζ with changes in γ , at 2000 m Water Depth, for a Wide Range of Cable Mean Axial Strains: (q) $S'_1 = 0.0006$, (r) $S'_1 = 0.00145$, (s) $S'_1 = 0.002867$, and (t) $S'_1 = 0.00430$.

Figures 9.3(a-d) - Variations of the Maximum Torsional Hysteresis, $(\Delta U/2U)_{\max}$, with Changes in the $(d_4)_{\text{no-slip}}/(d_4)_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (a) $S'_1 = 0.0006$, (b) $S'_1 = 0.00145$, (c) $S'_1 = 0.002867$, and (d) $S'_1 = 0.00430$ - at 0 m Water Depth.

Figures 9.3(e-h) - Variations of the Maximum Torsional Hysteresis, $(\Delta U/2U)_{\max}$, with Changes in the $(d_4)_{\text{no-slip}}/(d_4)_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (e) $S'_1 = 0.0006$, (f) $S'_1 = 0.00145$, (g) $S'_1 = 0.002867$, and (h) $S'_1 = 0.00430$ - at 500 m Water Depth.

Figures 9.3(i-l) - Variations of the Maximum Torsional Hysteresis, $(\Delta U/2U)_{\max}$, with Changes in the $(d_4)_{\text{no-slip}}/(d_4)_{\text{full-slip}}$ Ratio, as a Function of

the Cable Mean Axial Strains: (i) $S'_1 = 0.0006$, (j) $S'_1 = 0.00145$, (k) $S'_1 = 0.002867$, and (l) $S'_1 = 0.00430$ - at 1000 m Water Depth.

Figures 9.3(m-p) - Variations of the Maximum Torsional Hysteresis, $(\Delta U/2U)_{\max}$, with Changes in the $(d_4)_{\text{no-slip}}/(d_4)_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (m) $S'_1 = 0.0006$, (n) $S'_1 = 0.00145$, (o) $S'_1 = 0.002867$, and (p) $S'_1 = 0.00430$ - at 1500 m Water Depth.

Figures 9.3(q-t) - Variations of the Maximum Torsional Hysteresis, $(\Delta U/2U)_{\max}$, with Changes in the $(d_4)_{\text{no-slip}}/(d_4)_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (q) $S'_1 = 0.0006$, (r) $S'_1 = 0.00145$, (s) $S'_1 = 0.002867$, and (t) $S'_1 = 0.00430$ - at 2000 m Water Depth.

Figures 9.4(a-d) - Variations of κ with changes in λ , at 0 m Water Depth, for a Wide Range of Cable Mean Axial Strains: (a) $S'_1 = 0.0006$, (b) $S'_1 = 0.00145$, (c) $S'_1 = 0.002867$, and (d) $S'_1 = 0.00430$.

Figures 9.4(e-h) - Variations of κ with changes in λ , at 500 m Water Depth, for a Wide Range of Cable Mean Axial Strains: (e) $S'_1 = 0.0006$, (f) $S'_1 = 0.00145$, (g) $S'_1 = 0.002867$, and (h) $S'_1 = 0.00430$.

Figures 9.4(i-l) - Variations of κ with changes in λ , at 1000 m Water Depth, for a Wide Range of Cable Mean Axial Strains: (i) $S'_1 = 0.0006$, (j) $S'_1 = 0.00145$, (k) $S'_1 = 0.002867$, and (l) $S'_1 = 0.00430$.

Figures 9.4(m-p) - Variations of κ with changes in λ , at 1500 m Water Depth, for a Wide Range of Cable Mean Axial Strains: (m) $S'_1 = 0.0006$, (n) $S'_1 = 0.00145$, (o) $S'_1 = 0.002867$, and (p) $S'_1 = 0.00430$.

Figures 9.4(q-t) - Variations of κ with changes in λ , at 2000 m Water Depth, for

a Wide Range of Cable Mean Axial Strains: (q) $S'_1 = 0.0006$,
(r) $S'_1 = 0.00145$, (s) $S'_1 = 0.002867$, and (t) $S'_1 = 0.00430$.

Figure 10.1 - Effect of the Strand Outer Diameter, d , on the Theoretical Predictions of S-N Curves for the First Wire Fracture in the Innermost Layer of Spiral Strands, Assuming $K_a = 1.0$: (a) $\alpha = 12^\circ$; (b) $\alpha = 18^\circ$; and (c) $\alpha = 24^\circ$.

Figure 10.2 - Effect of the Strand Outer Diameter, d , on the Theoretical Predictions of S-N Curves for the First Wire Fracture in the Outermost Layer of Spiral Strands, Assuming $K_a = 1.0$: (a) $\alpha = 12^\circ$; (b) $\alpha = 18^\circ$; and (c) $\alpha = 24^\circ$.

Figure 10.3 - Effect of the Strand Outer Diameter, d , on the Theoretical Predictions of S-N Curves for the First Wire Fracture in the Innermost Layer of Spiral Strands, Assuming $K_a = 0.5$: (a) $\alpha = 12^\circ$; (b) $\alpha = 18^\circ$; and (c) $\alpha = 24^\circ$.

Figure 10.4 - Effect of the Strand Outer Diameter, d , on the Theoretical Predictions of S-N Curves for the First Wire Fracture in the Outermost Layer of Spiral Strands, Assuming $K_a = 0.5$: (a) $\alpha = 12^\circ$; (b) $\alpha = 18^\circ$; and (c) $\alpha = 24^\circ$.

Figure 10.5 - Effect of the Strand Outer Diameter, d , on the Theoretical Predictions of the Axial Hysteresis, Based on Methods (a) and (b), for: (a) $\alpha = 12^\circ$; (b) $\alpha = 18^\circ$; (c) $\alpha = 24^\circ$.

Figure 10.6 - Effect of the Strand Outer Diameter, d , on the Theoretical Predictions of the Torsional Hysteresis, for: (a) $\alpha = 12^\circ$; (b) $\alpha = 18^\circ$; (c) $\alpha = 24^\circ$.

Figure 11.1 - Geometrical Details of the Cable-Truss.

- Figure 11.2 - Variations of the Vertical Deflection at the Centre ($X = 40$ m) of the Cable Truss with Changes in the Externally Applied Load, for all Three Different Spiral Strand Constructions, Based on Their Full-Slip Axial Stiffnesses.
- Figure 11.3a - Variations of the Vertical Deflection Along the Span of the Cable Truss Under an Externally Applied Vertical Load of 2 kN/m, Based on Both the No-Slip and Full-Slip Axial Stiffnesses for the Top and Bottom Chords with the Vertical Hangers Always Experiencing the Full-Slip Condition.
- Figure 11.3b - Variations of the Vertical Deflection Along the Span of the Cable Truss Under an Externally Applied Vertical Load of 0.6375 kN/m, Based on Both the No-Slip and Full-Slip Axial Stiffnesses for the Top and Bottom Chords, with the Vertical Hangers Always Experiencing the Full-Slip Condition.
- Figure 11.4a - Diagram Showing the Total Force (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with No External Load Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.96 mm ($\alpha = 12$ degrees).
- Figure 11.4b - Diagram Showing the Total Force (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with No External Load Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.35 mm ($\alpha = 18$ degrees).
- Figure 11.4c - Diagram Showing the Total Force (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with No External Load Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 49.56 mm ($\alpha = 24$ degrees).

degrees).

Figure 11.5a - Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with No External Load Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.96 mm ($\alpha = 12$ degrees).

Figure 11.5b - Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with No External Load Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.35 mm ($\alpha = 18$ degrees).

Figure 11.5c - Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with No External Load Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 49.56 mm ($\alpha = 24$ degrees).

Figure 11.6a - Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with an External Load of 0.6375 kN/m Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.96 mm ($\alpha = 12$ degrees).

Figure 11.6b - Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with an External Load of 0.6375 kN/m Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.35 mm ($\alpha = 18$ degrees).

Figure 11.6c - Diagram Showing the Change in Pretension (kN) in Each

Member of the Cable Truss, Based on the Full-Slip Regime, with an External Load of 0.6375 kN/m Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 49.56 mm ($\alpha = 24$ degrees).

Figure 11.7a - Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the No-Slip Regime for the Top and Bottom Chords, with an External Load of 0.6375 kN/m Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.96 mm ($\alpha = 12$ degrees).

Figure 11.7b - Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the No-Slip Regime for the Top and Bottom Chords, with an External Load of 0.6375 kN/m Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.35 mm ($\alpha = 18$ degrees).

Figure 11.7c - Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the No-Slip Regime for the Top and Bottom Chords, with an External Load of 0.6375 kN/m Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 49.56 mm ($\alpha = 24$ degrees).

LIST OF TABLES

- Table 3.1a - Construction Details for the 16.4 mm Outside Diameter Spiral Strand.
- Table 3.1b - Construction Details for the 39 mm Outside Diameter Spiral Strand.
- Table 3.1c - Construction Details for the 41 mm Outside Diameter Spiral Strand.
- Table 3.1d - Construction Details for the 45 mm Outside Diameter Spiral Strand.
- Table 3.1e - Construction Details for the 51 mm Outside Diameter Spiral Strand.
- Table 3.1f - Construction Details for the 52 mm Outside Diameter Spiral Strand.
- Table 3.1g - Construction Details for the 63 mm Outside Diameter Spiral Strand.
- Table 3.1h - Construction Details for the 127 ($\alpha = 12^\circ$) mm Outside Diameter Spiral Strand.
- Table 3.1i - Construction Details for the 127 ($\alpha = 18^\circ$) mm Outside Diameter Spiral Strand.
- Table 3.1j - Construction Details for the 127 ($\alpha = 24^\circ$) mm Outside Diameter Spiral Strand.
- Table 3.1k - Construction Details for the 164 mm Outside Diameter Spiral Strand.
- Table 3.1l - Construction Details for the 184 mm Outside Diameter Spiral Strand.
- Table 3.2 - Values of the Full-Slip Coefficients A-D in Equation (3.58).
- Table 3.3a - Values of the No-Slip Coefficients A-D in Equation (3.58), After

Raoof and Kraincanic (1995).

Table 3.3b - Values of the No-Slip Coefficients A–D in Equation (3.58), From the Present Study.

Table 4.1 - Values of the Full-Slip and No-Slip Constitutive Constants for the Three 127 mm Outside Diameter Spiral Strands, with lay angles of 12° , 18° and 24° , as Calculated Using the Orthotropic Sheet Theory.

Table 4.2 - Numerical Results for the Axial and Torsional Wave Speeds C_1 and C_2 , the Ratios of Torsional to Extensional Oscillations R_1 and R_2 , and the Natural Frequencies ω_1 and ω_2 for the 39 mm Outside Diameter and Axially Preloaded Spiral Strand, Based on the No-Slip and Full-Slip Assumptions.

Table 4.3 - Numerical Results for the Axial and Torsional Wave Speeds C_1 and C_2 , the Ratios of Torsional to Extensional Oscillations R_1 and R_2 , and the Natural Frequencies ω_1 and ω_2 for the Three 127 mm Outside Diameter and Axially Preloaded Spiral Strands, Based on the No-Slip and Full-Slip Assumptions.

Table 6.1 - Experimental Measurements Relating the Changes in the Axial Load to Variations in the Ripple Amplitude, for the First Loading Run.

Table 6.2 - Experimental Data Regarding Variations in the Axial Load with Changes in the Ripple Amplitude, for the Final Loading Run.

Table 6.3 - Values of the Bending and Torsional Stiffnesses for the 164 mm Outside Diameter Cable.

Table 6.4 - Determination of the Ripple Amplitude and Bending Moment, for Each Given Axial Load, for the First Loading Run, Using the Simplified Method.

- Table 6.5 - Determination of the Ripple Amplitude and Bending Moment, for Each Given Axial Load, for the Final Loading Run, Using the Simplified Method.
- Table 6.6 - Determination of the Ripple Amplitude and Bending Moment, for Each Given Axial Load, for the First Loading Run, Using the Rigorous Method.
- Table 6.7 - Determination of the Ripple Amplitude and Bending Moment, for Each Given Axial Load, for the Final Loading Run, Using the Rigorous Method.
- Table 7.1a - Construction Details and Calculation Routines for Hruska's Axial Stiffness of a 33 mm Outside Diameter (IWRC) Wire Rope.
- Table 7.1b - Construction Details and Calculation Routines for Hruska's Axial Stiffness of a 40 mm Outside Diameter (IWRC) Wire Rope.
- Table 7.1c - Construction Details and Calculation Routines for Hruska's Axial Stiffness of a 55.6 mm Outside Diameter (IWRC) Wire Rope.
- Table 7.1d - Construction Details and Calculation Routines for Hruska's Axial Stiffness of a 76 mm Outside Diameter (IWRC) Wire Rope.
- Table 7.1e - Construction Details and Calculation Routines for Hruska's Axial Stiffness of a 76 mm (Comparator) Outside Diameter (IWRC) Wire Rope.
- Table 7.1f - Construction Details and Calculation Routines for Hruska Axial Stiffness of a 9.53 mm Outside Diameter (FC) Wire Rope.
- Table 7.1g - Construction Details and Calculation Routines for Hruska Axial Stiffness of a 40.5 mm Outside Diameter (FC) Wire Rope.

- Table 7.2 - Summary of the Numerical Results for the Axial Stiffnesses as Calculated Using Hruska's Simple Formula and the Model Proposed by Raoof and Kraincanic.
- Table 7.3 - Summary of the Experimentally Determined Results for the Axial Stiffnesses of the Various Wire Rope Constructions.
- Table 7.4 - Values of the Constant Coefficients A - C in Equation (7.2) for all of the Fitted Curves in Figs. 7.2a-c, Along with the Correlation Coefficients, R.
- Table 7.5 - Values of the Constant Coefficients A - C in Equation (7.3) for all of the Fitted Curves in Figs. 7.3a-c, Along with the Correlation Coefficients, R.
- Table 8.1a - Construction Details for the 127 ($\alpha = 12^\circ$) mm Outside Diameter Spiral Strand.
- Table 8.1b - Construction Details for the 127 ($\alpha = 18^\circ$) mm Outside Diameter Spiral Strand.
- Table 8.1c - Construction Details for the 127 ($\alpha = 24^\circ$) mm Outside Diameter Spiral Strand.
- Table 8.2 - Values of the Constant Parameters a and b for Sheathed Spiral Strands, as Defined in Equation (8.6).
- Table 9.1a - Values of the Coefficients, A-C, in Equation (9.1), for a Wide Range of Strand Mean Axial Strains, S'_1 , and Water Depths, Along with the Correlation Coefficients, R, as Calculated Using Method (a).
- Table 9.1b - Values of the Coefficients, A-C, in Equation (9.1), for a Wide Range of Strand Mean Axial Strains, S'_1 , and Water Depths, Along with the

Correlation Coefficients, R, as Calculated Using Method (b).

- Table 9.2 - Values of the Coefficients, A and B, in Equation (9.3), for a Wide Range of Strand Mean Axial Strains, S'_1 , and Water Depths, Along with the Correlation Coefficients, R, Based on Method (a).
- Table 9.3 - Values of the Coefficients, A, B and C, in Equation (9.6), for a Wide Range of Strand Mean Axial Strains, S'_1 , and Water Depths, Along with the Correlation Coefficients, R.
- Table 9.4 - Values of the Coefficients, A and B, in Equation (9.8), for a Wide Range of Strand Mean Axial Strains, S'_1 , and Water Depths, Along with the Correlation Coefficients, R.
- Table 9.5 - Calculation Procedures, Based on the Simple Methods, for Calculating the Maximum Axial Hysteresis and the Corresponding Load Range / Mean Load Ratio for a 127 mm ($\alpha = 18^\circ$) Outside Diameter Spiral Strand.
- Table 9.6 - Calculation Procedures, Based on the Simple Methods, for Calculating the Maximum Torsional Hysteresis and the Corresponding Range of Twist / 2 for a 127 mm ($\alpha = 18^\circ$) Outside Diameter Spiral Strand.
- Table 10.1a - Construction Details for the 127 ($\alpha = 12^\circ$) mm Outside Diameter Spiral Strand – Assumed Ultimate Breaking Load = 13510 kN.
- Table 10.1b - Construction Details for the 127 ($\alpha = 18^\circ$) mm Outside Diameter Spiral Strand – Assumed Ultimate Breaking Load = 13510 kN.
- Table 10.1c - Construction Details for the 127 ($\alpha = 24^\circ$) mm Outside Diameter Spiral Strand – Assumed Ultimate Breaking Load = 13510 kN.

Table 10.2 - Values of the Parameters in Equation (10.1) and the Ultimate Breaking Loads of the Different Spiral Strand Constructions.

Table 11.1a - Construction Details for the 48.96 mm ($\alpha = 12^\circ$) Outside Diameter Spiral Strand.

Table 11.1b - Construction Details for the 48.35 mm ($\alpha = 18^\circ$) Outside Diameter Spiral Strand.

Table 11.1c - Construction Details for the 49.56 mm ($\alpha = 24^\circ$) Outside Diameter Spiral Strand.

Table 11.2 - Construction Details for the 16.4 mm Outside Diameter Spiral Strand (Used as Vertical Hangers).

Table 11.3 - Magnitudes of the Chord and Hanger Axial Stiffnesses (EA).

NOTATION

A_1, A_2, A_3, A_4	Constitutive constants
A	Coupled extensional-rotational stiffness matrix for a stranded cable, duration of impact load, gross area of a layer of wires
A_{core}	Area of core (king) wire
A_{gi}	Gross steel area of layer i used in the orthotropic sheet model
A_{net}, A, A_{ni}	Net steel area of layer i
A_{wi}	Wire cross-sectional area
b	Half-width of the line-contact patch, semi-axis of the ellipse of contact
C_1	Speed of axial wave propagation
C_2	Speed of torsional wave propagation
c_k	Constant ratio (always negative) in the orthotropic sheet model
d, D	Wire diameter
$d\alpha_i$	Change in the lay angle in layer i
$d_{\epsilon h}$	Incremental change in the wire axial strain
$d\phi$	Applied twist increment per unit length
E_s, E_{steel}	Young's modulus for steel

E	Complete elliptical integral of second kind
EA	Cable axial stiffness
EI	Flexural rigidity
$EI_{\text{eff}}, EI_{\text{cable}}$	Strand effective bending rigidity
F_0, M_0	Amplitudes of external axial and torsional load disturbances, respectively
F	Strand axial force
g	Acceleration due to gravity
G	Shear modulus
G, G', H	Components of internal moment resultants in an individual helical wire
$(GJ)_{\text{full-slip}}$	Strand's full-slip torsional stiffness
H	Initial ripple wave length
H_b	Axial pretension in the bottom chord of a cable truss
H_i	Hruska's parameter for layer i
h	Length of the cable, water depth
I	Second moment of inertia for a strand, mass moment about the cable axis

I_x, I_y	Moments of inertia about the x and y axes, respectively
$k_{1i}, k_{2i}, k_{3i}, k_{4i}$	Stiffness coefficients for layer i
K	Spinning loss factor
K	Cable axial stiffness = EA
K_a, K_b	Constants ($K_a = 0.5$ or 1.0) in the axial fatigue model
K_{fs} & K_{ns}	Strand's full-slip and no-slip axial stiffnesses, respectively
K_s	Interwire stress concentration factor
ℓ	Cable span
m, M	Total bending moment on the strand, torque
m_i	Number of wires in layer i of an equal lay core
n	Number of wires in a layer of the spiral strand
n_c	Number of layers in an equal lay core construction (excluding the king wire)
N	Total number of layers in the spiral strand
N, N', T	Components of internal force resultants in a helical rod
P_o	Amplitude of the lift force per unit length, normal line-contact force
p	Normal force per unit length of a rectangular contact patch

P	Total normal force on a trellis contact patch
P_{MS}	Hoop (circumferential) contact force in a multi-layer strand
P_n	External nodal force in a cable truss
P_{RC}	Hoop (circumferential) contact force in the single layer strand with a rigid core
R	Cable outer radius
R_i	Ratios of torsional to extensional oscillations ($i = 1, 2$)
r, r_1	Initial and final depths of ripple, respectively
r, r_1	Initial and final helix radii of a wire, respectively
r_i	Helix radius of layer i in a multi-layered strand
S_1	Wire axial strain
S'_1	Cable axial strain
S_{11}	Compliance of the orthotropic sheet in the direction parallel to the wire axes
S_2	The approach strain normal to S_1 between the centres of wires in line-contact
S_{22}	Normal compliance of two cylinders in line-contact
S'_2	Radial contraction of a layer in the strand's normal cross-section due to interwire contact deformations

S'_{2C}	Total radial strain in the strand's normal cross-section (including the rigid body component)
S'_{2R}	The 'rigid body' radial strain in the strand's normal cross-section in the absence of a rigid core
S'_6	Engineering shear strain
S_{66}	Tangential compliance of two cylinders in line-contact
S_{6T}	Tensorial shear strain
s	Distance measured along centre line of a helical rod
S'	Strand endurance limit
S_e	Reduced magnitude of the strand endurance limit due to the presence of interwire contact stresses
S_i	Engineering strains referred to the axes of orthotropy
S_{ult}	Ultimate tensile strength for a wire
T'_1	Axial stress increment on the strand's normal cross-section
T'_2	Hoop stress in the strand's normal cross-section
t	Time
T	Tangential force, constant tension along the cable
T_i	Stresses referred to the axes of orthotropy

T_h	Axial force in the vertical hangers due to the pretension in the bottom chord
T'_i	Transformed stresses at an angle α to the principle axes of orthotropy
u	Longitudinal displacement of the cable
U	Energy input per cycle per unit length
X	Measured distance along the span of a cable truss
X, Y, Z	Components of external force resultants per unit length acting on the individual helical wires, coordinates of a point in the deformed cable in the base reference frame
X_H	Magnitude of the external hydrostatic pressure per unit length of the wire in the outer layer
X_{MS}	Radial force in the multi-layer strand
X_{RC}	Radial force in the single layer strand with a rigid core
α', α	Lay angles in the deformed and undeformed states, respectively
β	Angle which locates the lines of action of the line-contact forces, Helix angle
β_i	Lay angle of the 'j th' layer
δ	Tangential displacement between two wires
$\delta,$	Relative slippage of the centres of the contacting cylinders under

the action of a monotonically increasing tangential force

$\delta_{\max}$	Magnitude of relative slippage at the onset of gross sliding
δ	Logarithmic decrement
Δ	Change in the ripple wave length resulting from an applied axial force
Δ_r	Tangential relative displacement between two neighbouring wires in line-contact
$\Delta_{\max}$	Relative tangential displacement between two bodies at the onset of full-sliding
ΔE	Energy dissipation per cycle per unit length for the pure tangential loading of two cylinders in line-contact
ΔE_i	Energy dissipation per cycle per unit length of a wire in layer i
ΔU	Total energy dissipation per cycle per unit length of the strand
ε_l	Wire axial strain due to bending of a spiral strand
ε	Cable axial strain = $\partial u / \partial x$
ε_c	Strand axial strain
$\varepsilon_h, \varepsilon_{hl}$	Wire axial strain
Ψ_0	Rotation of the tangent (at a point of fixity) relative to the direction of the tensile force in a cable

κ_0, κ_0'	Initial curvatures of the centre-line of a helical wire
κ_1, κ_1'	Final curvatures of the centre-line of a helical wire
μ	Coefficient of interwire friction
ϕ	Angular rotation of the cable
ρ	Density of steel, Radius of curvature
$\overline{\sigma}'$	Nominal axial stress in the helical wires
$\overline{\sigma}'_{\max}$	Maximum effective Von-Mises stress
τ	Shear stress, twist per unit length
τ_0, τ_1	Initial and final twists, respectively
Γ	Twist per unit length = $\partial\theta/\partial x$
ν	Poisson's ratio
ω_i	Axial and torsional natural frequencies ($i = 1, 2$)

CHAPTER 1

INTRODUCTION

Due to their high axial strength to weight ratio, and low bending rigidity, spiral strands and wire ropes have many important structural applications in both fields of onshore and offshore engineering: these include stays for guyed masts, hangers for suspension bridges, the main cables of cable-stayed bridges, lift applications, overhead transmission lines and mooring systems for deep sea offshore platforms, to name a few. Due to their wide range of structural applications, and the requirement by industry for ever larger diameter steel cables, the research into this field has gained increased momentum since the mid 1970's.

Raoof (1983) gives a detailed account of the terminologies and definitions used in connection with the manufacture and design of spiral strands and wire ropes, and such a detailed description will not be repeated here. Perhaps, it suffices to say that a spiral strand is composed of a central core (which may either be a single wire or a helical assembly of individual wires with a small helix angle) around which individual metallic wires are wrapped helically in various layers, while a wire rope consists of a group of spiral strands wound helically (with the individual wires forming a double helix) around a central core, which itself maybe a small diameter fibre or independent wire rope, Fig. 1.1. A wire rope is also categorised by the way its wires are laid to form the strands and in the way its outer strands are laid around the core, Fig. 1.2: here it can be seen that the direction of the wires in the Lang's lay is the same as that of the outer spiral strands (e.g. both right hand lay), whereas in the regular lay type, the direction of the lay of the wires is opposite to that of the lay of the outer strands in a rope. Fig. 1.3 shows examples of several basic cross-sectional constructions for wire ropes, as currently offered by the manufacturers.

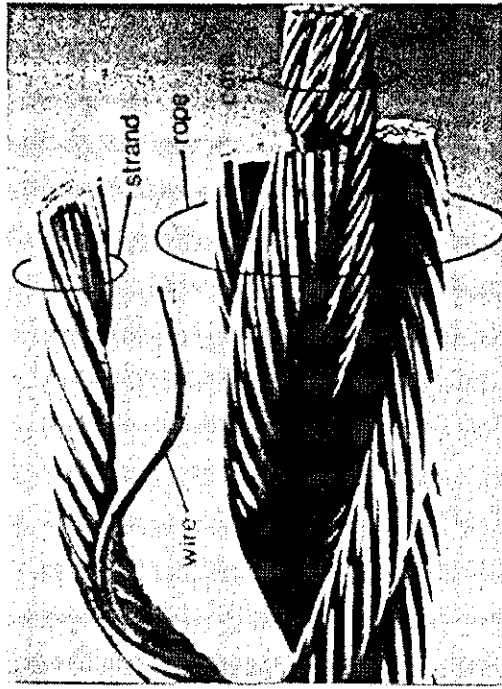


Fig. 1.1 – Various Components of a Spiral Strand and Wire Rope.

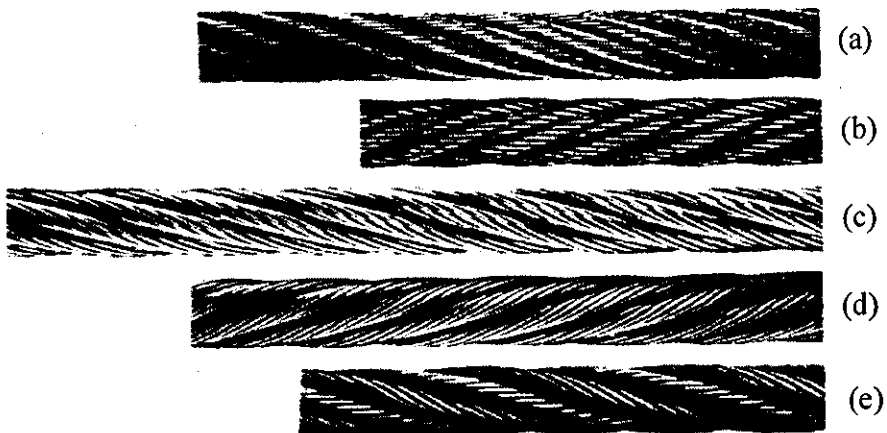


Fig. 1.2 – Typical Wire Rope Lays: (a) Right Regular Lay; (b) Left Regular Lay; (c) Right Lang's Lay; (d) Left Lang's Lay; and (e) Right Alternative Lay Wire Ropes.

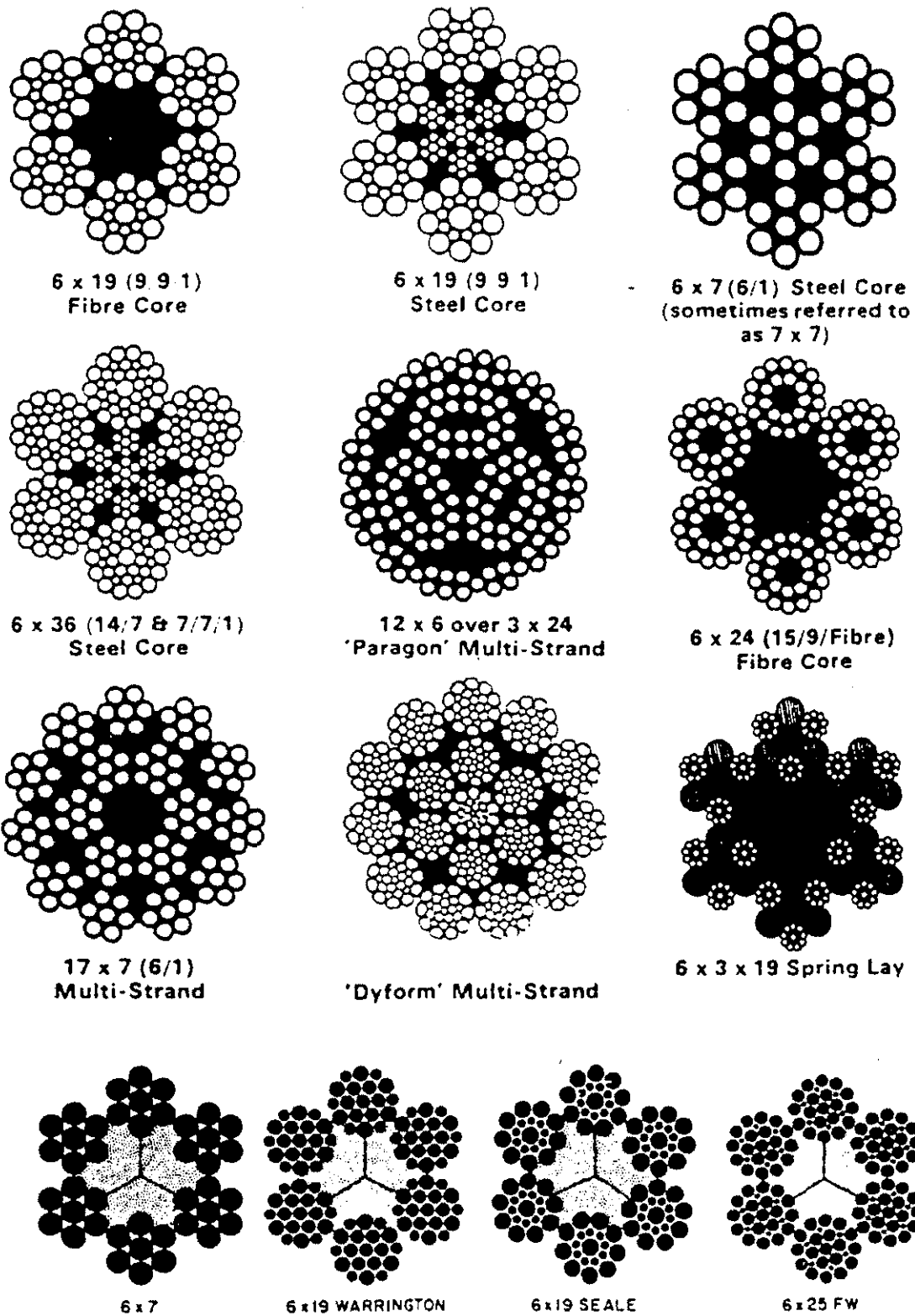


Fig. 1.3 – Typical Examples of Cross-Sectional Constructions for Wire Ropes.

In order that the numerical analyses carried out in this thesis could be done so with a good degree of accuracy and speed, it was necessary to make extensive use of FORTRAN programming. Using this language, new computer programs have been

developed, and have been used along side the original programs of Raoof (based on the orthotropic sheet model), to analyse the response of spiral strands to impact loading; to calculate the bending stiffness of spiral strands; to analyse certain effects of an external hydrostatic pressure applied to sheathed spiral strands; and to analyse the fatigue life of multi-layered spiral strands, along with some other problems of practical concern.

The next chapter is the literature review, in which a large number of publications have been critically examined. The summary and conclusions section of the literature review explains the reasoning behind the work reported in the subsequent chapters.

CHAPTER 2

LITERATURE SURVEY

2.1 INTRODUCTION

In the present chapter, a general overview of the previous research on the static and dynamic response of helical steel cables under various loading conditions is presented, and the available literature is critically examined. Over the years, particularly the last two decades, significant advances have been made in this field, and complex models have been developed which enable one to predict (with varying degrees of accuracy) the various mechanical properties of spiral strands and wire ropes. In spite of these advances, and the growing number of researchers working in this field, there are still many unresolved problems: some of these will be identified in the present literature review. It is hoped that the work presented in this thesis will go, at least some of the way, to answering these problems. This literature review is concerned with specific papers from pre 1970, but with more emphasis on the papers published in the public domain from 1970 to the present day.

In recent publications, Utting (1994a, b and 1995) surveyed the literature from 1984 – 1994, concentrating on the various mathematical modelling techniques used to predict the response of spiral strands and wire ropes to various types of external loading. The fatigue behaviour of wire ropes, wire rope terminations, and armoured cables were also covered. In addition, certain design procedures and experimental works were reviewed. Up to 1984, numerous reviews of the available literature had been carried out: for example, Sayenga (1993) has carried out a historical evaluation of high tensile steel wire ropes used in modern civil engineering applications, and reference may also be made to the works of Utting and Jones (1984), who have surveyed well over 200 articles, Weber (1975), who has reviewed the early literature, Forestier-Walker (1952), who has recorded the history of the British wire rope industry from 1830 to 1952, and Raoof (1982), Huang (1993), and Kraincanic (1995), amongst others, who have all studied the literature in considerable detail.

2.2 MECHANICAL MODELS

Over the past fifty years, various mathematical models aimed at predicting the mechanical behaviour of helical strands and ACSR electrical conductors under axisymmetric loading have been proposed. A detailed description of each model can be found in the corresponding publications and will not be repeated here. Instead, the salient features of each model along with some of the main advantages as well as their possible shortcomings will be mentioned in what follows.

2.2.1 Purely Tensile Models

The first real attempt at modelling the behaviour of spiral strands was conducted by Hruska (1951, 1952 and 1953), who was inspired by Hall (1951). Hall recognised the need to be able to calculate the stresses in small wire ropes. He considered a wire rope under the influence of an axial load by assessing the axial tensile force in each component wire, neglecting interwire friction. Hall concluded that the axial stresses in the outer wires were appreciably higher than those in the inner wires. This finding inspired Hruska to examine the conclusions of Hall.

Hruska (1951) considered three types of internal actions (i.e. tension, radial and tangential forces) when a strand is subjected to a purely tensile force, however, no bending or twisting of the helical wires were included in the analysis. In Hruska's first publication, he concentrated on the tensile force in the strand. The wire tensile stresses in each layer were found to vary in proportion to the square of the cosine of the lay angle, and were calculated assuming that the changes in the lay angle and the helix radii of the wires were negligible. In spite of these simplifying assumptions, Hruska's findings have been used as the basis for all the subsequent theoretical approaches. Hruska disagreed with the findings of Hall (1951), and concluded that, the outer wires were less stressed than the inner ones. Hruska also mentioned the importance of interwire friction: it was suggested that due to friction, a broken wire could recover and carry the full tension within a distance of only a few lay lengths. Although Hruska recognised the importance of friction, he failed to include it in his model. The second paper by Hruska (1952), considered the effect of the radial forces, and, based on wire kinematics, a relationship between the radial and tensile forces acting on the helical wires in a layer of a strand, was proposed, ignoring any slight changes in the lay angle and strand diameter. The proportion of the radial forces

resisted by the hoop stresses over the line-contact patches in the individual layers was not considered in any detail. In his third paper, Hruska (1953) analysed the influence of the tangential forces in wire ropes. It was shown how to calculate the internal moments and the changes in the stress distribution and lay angle in a helical strand or wire rope caused by a rotation, induced by an axial load.

2.2.2 Thin Rod Models

Leissa (1959) extended Hruska's theory to investigate the interwire contact stresses and the effect they have on the mode of failure of wire ropes. This was the first time Hertzian contact stress theory was used to evaluate the contact stresses in wire ropes. The analysis was related to a seven wire strand subjected to a purely tensile load, and, once again, interwire frictional effects and contact deformations were neglected.

Green and Laws (1966) developed a general thermodynamical theory for rods. The theoretical formulations were applied to three separate cases including an elastic rod, an elastic string, and the case where the rod is assumed to be inextensible. The thin rod model of Green and Laws (1966) was linearized by Ramsey (1988), who then applied it to helical constituent wires in spiral strands. The major difference between the two theories lies in their kinematic variables which quantify the deformations of the thin rod. When Ramsey's theory was used to analyse the bending of a cable in which there was sufficient interwire friction to prevent any relative movement of the constituent wires, it was found that the constituent wires exhibit a strong tendency to rotate with respect to one another. Ramsey (1990) extended his original analysis, identifying the presence of a non-zero distributed moment component in the radial direction.

Chi (1971) analysed the operating characteristics of a multi-wired strand subjected to an axial tension. Chi extended the method proposed by Hruska for the stress analysis of spiral strands and wire ropes, and catered for the ever presence of a strand's diametral contraction. Hruska had assumed that the deformations and end rotations are small, and that the diameter of the strand or rope remains constant during loading. These assumptions, are not, however, applicable to large diameter ropes and for ropes with fibre cores. Chi proposed a theoretical formulation, for the determination of the axial strains in the wires with the effect of cable diametral contraction catered for.

This method was used to gain a more rational insight into the effect of end rotation and the effect of a reduction in the diameter of the strand on the axial stress distribution among the wires. In a subsequent paper, Chi (1972) subjected the spiral strand to extension and end rotation. The importance of radial contraction was again emphasised, but no theoretical or experimental data was given to enable the degree of strand radial contraction to be quantified.

Durelli et al. (1972) measured the direct strains in the various helical wires of a seven wire strand, with one end either fixed or free to rotate, experiencing external axial, bending or torsional loading. The wire axial strains were measured using brittle coatings and electrical resistance strain gauges. The data was very dispersed, and the predictions did not agree well with the experimental results, which was thought to be due to irregularities in the strand geometry. Durelli and Machida (1973) noted that there was a difficulty in measuring the axial strains in small wires. To try and overcome this problem, a method was developed by means of which oversized epoxy models could be manufactured in an attempt to accurately measure these strains. The results of the axial strain measurements in the individual oversized wires, when compared with the theoretical values, were found to be good. In another paper, Machida and Durelli (1973) extended the model proposed by Hruska to include wire bending and twisting. They proposed certain linear expressions for the determination of the axial force, and bending plus twisting moments in the helical wires, and for the axial force and twisting moments in the rigid core of a seven wire strand subjected to axial and torsional displacements. Experimental measurements on oversized epoxy models of the strand showed reasonable agreement with the theory.

Knapp (1975) presented a procedure for the analysis of straight helical cables subjected to external tension and/or torsion. The problem of the frictionless, but geometrically non-linear strands with compressible cores was addressed. The boundary conditions assumed were that the cables were either fixed at both ends or, fixed at one end with the other one free to rotate. The so-developed non-linear equations catered for the compressibility and material non-linearity of the core. The resulting equations were complex, so simplified equations were derived, which were amenable to hand calculations. Limited experimental results on cables with only two layers of armouring wires provided reasonable support for the theoretical predictions.

The simple 'hand based' equations are, however, best employed when the cable has a rigid core and the displacements are small. In another paper, which was an extension of the theory of Machida and Durelli (1973), Knapp (1979) developed a new stiffness matrix for a straight cable. In the analysis, in which wire bending and twisting were included, the cable was treated as a composite element, and the cylindrical core element was assumed to be compressible. The resulting equations included internal geometric non-linearities associated with large deformations, which were then linearized to give a linear stiffness matrix for the coupled axial/torsional behaviour of helical strands. The agreement between the theoretical and experimental results was encouraging. As pointed out by Knapp, these theoretical formulations are only strictly applicable if the deflections of the armour wires are geometrically compatible.

Lanteigne (1985) was concerned with the mechanical behaviour of ACSR conductors under static loading conditions, and proposed a model (as he put it) of a general nature. It was found that, for small curvatures, the flexural rigidity was comparable to the upper limit accepted by ACSR users. As the curvature increases, the flexural rigidity decreases due to the development of frictional forces between various layers which (if overcome) can result in sliding. Interwire contact deformations were neglected, and no supporting experimental data was provided.

Huang (1978) dealt with the finite extension of an elastic strand with a central core (king wire) surrounded by a single layer of helical wires, subjected to axial forces and twisting moments. It was assumed that in the undeformed state the core and helical wires touch each other. When a tensile load is applied to the strand, a compressive contact force can develop between the core and the helical wires. Due to the inclusion of the contact pressure between the core and the helical wires, negative pressure was found to develop between the helical wires in line-contact. Consequently, separation between the wires within the outer layer was found to occur. It appeared that such separation between the helical wires depends upon the magnitude of the contact forces between the core and the helical wires, and the magnitude of the axial force carried by the outer wires. Huang studied two problems; namely the extension of the strand with a separation of the outer wires, and the extension of the strand in the absence of wire separation. In Huang's work, two types of end conditions were assumed, fixed-fixed

and fixed-free to rotate. It was found that the extension of the strand always causes a separation between the outer helical wires, originally in line-contact.

Costello and Phillips (1973) recognised, as Machida and Durelli (1973) had, the importance of the bending and twisting moments in each helical wire. Costello and Phillips presented a so-called exact solution for the deformed single layer strand, which allowed all the wire stresses (axial, bending, contact, and twisting) to be calculated. To achieve this, they separated the cable into thin rods and solved six geometrically non-linear equations for the bending and twisting of a thin rod subjected to axial loads. It was assumed that the cable was loaded by an axial force and twisting moment, but frictional forces between the wires were neglected. It was further assumed that, in the unloaded configuration, the wires in line-contact were just touching each other, and that the core was fully compressible, so that the radial force exerted by the core on the wires could be neglected. The presence of wire axial strain was neglected, and the strand axial strain was assumed to arise from variations in the helix angle. This last assumption was removed from a subsequent analysis by the same authors, Costello and Phillips (1976), making the wire axial strain the new independent variable. In other papers, Costello and Sinha (1977a) and Costello and Miller (1979) used the approach of Costello and Phillips (1973) to determine the geometrically non-linear behaviour of helical cables in a variety of applications.

Costello and Sinha (1977a) presented a frictionless theory for the determination of the static response of wire ropes, including those with complex cross-sections. Expressions were also presented for the determination of the strands' bending, torsional, and axial stiffnesses. The helical strands were assumed to behave like initially curved thin rods. Costello and Miller (1979) subsequently developed a theory, capable of predicting the static response of regular or Lang's lay wire rope composed of six strands with twelve wires in each strand. It was concluded (by a numerical comparison) that a Lang's lay rope under tension has practically no torsional stiffness, and should not be used when the end of the rope is free to rotate.

Kumar and Cochran (1990) developed closed-form solutions capable of determining the extension of twisted wire ropes with fibrous cores, when subjected to axial and torsional forces. The numerical results were compared with the corresponding

numerical results obtained by Costello and Phillips (1976). The theory was essentially based on Costello's model, with the assumption of small geometrical changes, so that closed-form solutions for the static elastic deformations of multi-layered spiral strands with fibrous cores could be developed. The wires in the same layer were assumed to touch each other in line-contact in an unstressed condition. Two cases were analysed: namely, when both ends are fixed against rotation, and when one end is free to rotate. It was found that the number of helical wires was of significant importance for fixed-free end conditions, and that the steel Poisson's ratio was shown to have a small effect when both ends of the cable were fixed against rotation. It was also shown that the axial stiffness of the cable was strongly affected by the lay angle of the wires.

Kumar et al. (1997) utilised the theory of Costello and Phillips (1974) to obtain closed-form expressions for the maximum line-contact stresses in a highly idealised seven wire spiral strand with a fibrous core. It was suggested that using very large lay angles, e.g. 45° , would promote a longer life span for a cable, and that for wire ropes whose main function is dissipating vibrational energy, then, lay angles as large as 30° lead to much larger contact stresses and, hence, frictional hysteresis. Cables with lay angles as large as 30° are not, as far as the present author is aware, used in practice, with the practical lay angles being within, say, the range of $11^\circ \leq \alpha \leq 24^\circ$

Lee (1991) described the double helix geometry of the wires within a rope, using Cartesian co-ordinate equations. It was found that the wire curvature and torsion functions could be related to the bending stress. The paper describes the geometry of the double helix, which was argued, will enable one to determine the deformations and strain components along a wire under operating conditions.

Costello (1983) investigated the axial and bending response of a multi-layered spiral strand. Expressions for the wire axial, torsional, and bending stresses were presented, and, not surprisingly, the largest axial tensile stress was found to occur in the centre wire. Once again, interwire friction was neglected, and it was also assumed that the central core was sufficiently large to prevent the outer wires from touching each other in line-contact. The equations developed by Costello were linearized by Velinsky et al. (1984), which enabled the 2×2 stiffness matrix for coupled axial/torsional deformations of a spiral strand to be applied (with relative ease) to wire ropes with

complex cross-sections. Also presented, is an axial load-deformation curve for the static loading of a wire rope, showing that the experimental value for the axial stiffness was slightly smaller than the theoretically predicted one.

Phillips and Costello (1985) generalised the approach developed by Velinsky et al. (1984) for any kind of wire rope with an IWRC. Expressions for the axial, bending, torsional, and line-contact loads on the individual wires of the constituent strands, in complex wire rope constructions, were developed for the situation where a rope is pulled axially and bent to a prescribed curvature. Interwire frictional effects were neglected, and it was suggested that the theory is best applicable to a rope which is well lubricated, or for a rope that is only loaded in tension, (i.e. no external bending or twisting). Interwire contact deformations were neglected, and the changes in the helix radius were assumed to be due to the Poisson's ratio contraction of the wire material in the strand. At the same time as Phillips and Costello (1985), Velinsky (1985b) was developing a theory for the analysis of a wire rope with a fibre core, subjected to both an axial force and an axial twisting moment. The fibre core was assumed to act in a linearly elastic manner under the normal contact pressure loading from the adjacent strands. The individual strands were analysed using the previously developed theory of Velinsky et al. (1984). The theoretical and experimental results for a 6×19 Seale fibre core wire rope, relating to the effective axial modulus of elasticity and the effective Poisson's ratio of the rope, compared favourably.

Cantin et al. (1993) considered a wire rope with a polypropylene core under axial/torsional loading. The tests were conducted on a 40.5 mm outside diameter six stranded wire rope. For changes in the wire geometry (such as pitch), the rotation of the rope in several sections and the torsional moment were all measured. The end conditions of the rope were either restrained or free to rotate. The results were compared with the linear theoretical model of Costello and his associates. The experimentally observed large changes in the axial modulus could not be handled theoretically, with the theoretical model assuming these to be constant.

Velinsky (1985a) developed a geometrically non-linear theoretical model, capable of analysing wire ropes with complex cross-sections. The theory was used to analyse a 6×19 IWRC Seale wire rope, in both the Lang's and regular lay configurations. The

results were compared with those from a linear theoretical model, and the differences between the results based on the two approaches were found to be negligible, at least for this particular rope construction and the practical axial load ranges.

Kumar and Cochran (1987) also proposed a linearized version of Costello's (1983) equations. Closed-form solutions were developed for predicting the elastic deformation characteristics (such as the axial and/or torsional moduli) of multi-layered strands with a metallic wire core, experiencing external tensile and/or torsional loads. The effects of the layout of the layers, the number of wires in each layer, and the direction and magnitude of the lay angles, on these elastic deformation characteristics, were investigated. Finally, these authors proposed a simple design criterion for non-rotating cables.

Conway and Costello (1991) used the linear equilibrium equations developed by Costello (1983) and Velinsky et al. (1984), based on the frictionless theory, to describe the axial response of two different strand constructions with elliptical outer wires, with the ends of the strand fixed, so as to prevent any end rotation. The first strand consisted of a centre wire with a circular cross-section and six helical outer wires with elliptical cross-sections that only made contact with the centre wire. The second strand was the same as the first, but with the six helical outer wires having circular cross-sections, except for a small flat surface which only made contact with the centre wire. It was found that (not surprisingly) the flat surface (in the second strand construction) significantly reduced the contact stresses, which were calculated ignoring the effects of wire curvature between the outer wires and the centre wire.

As earlier noted by Hobbs and Raoof (1982), LeClair (1991) also noted that many of the thin rod models (Velinsky, 1984 and 1985, Costello, 1983, and Phillips and Costello, 1973), for analysing the response of metallic strands, ignored the effect of the interwire contact deformations, which had long been recognised as an important factor in the analysis of helical cables (Hobbs and Raoof, 1982). It was noted that, particularly helical strands with non-metallic components, such as in instrumentation cables, may experience significant effects due to such contact deformations. By using Hertzian contact stress theory, the interlayer contact compliances relating to point (trellis) contact were determined by LeClair (1991), who also assumed that the helical

wires in the innermost layer are in line-contact with the central (king) wire. It was discovered that the introduction of compliant layers was an effective means of reducing the load in the signal carrying wires (not the armouring wires) of instrumentation cables, this being the case because of the compliant layer reducing the tension and the contact forces by at least an order of magnitude over the case in which the compliant layer is absent. Numerical results based on both a three and a ten layer metallic strand were presented.

Utting and Jones (1985) presented the preliminary test results conducted on a variety of seven wire spiral strands. The shortcomings of the test procedure and equipment used were highlighted, and solutions were suggested by means of which more reliable test data could be obtained. This paper essentially describes the test procedure used for obtaining the experimental data for seven wire spiral strands. Utting and Jones (1987a, and b), reported a variety of test data on seven wire strands, while similar experimental results for a nineteen wire spiral strand were reported in another paper by Utting and Jones (1988). The test results were compared with the theoretical model of Machida and Durelli (1973), which neglects interwire friction, Poisson's ratio effects, and wire flattening due to contact forces, and were found to agree reasonably well in terms of extension, and torque generated (in fixed-fixed tests), and overall strand rotation (in free-fixed tests). However, the axial strain measurements on the helical wire surfaces revealed unequal load sharing between nominally identical helical wires, particularly in the region adjacent to the end terminations, which may have significant implications in axial fatigue studies.

Utting and Jones (1987a) carried out another set of tests on seven wire spiral strands subjected to an axial load with various end restraints, in order to measure the strand extension, rotation, and torque, as well as wire tension and bending moment. The strands' lay angles ranged from 9.2° to 17° , with central core and helical wire diameters of 3.94 and 3.73 mm, respectively. The test results showed that the share of the total axial load taken by the helical wires decreased as the torsional restraint on the strand was reduced. A mathematical model was developed, using the same principles as those of Velinsky et al. (1984), to explore the changes in the helix angle under an applied axial load, due to Poisson's ratio effects in the wires, wire flattening under interwire pressure, and the effect of friction between the core and the helical wires.

Guided by a comparison of the theoretical and experimental results, Utting and Jones (1987b) argued that whereas friction and wire flattening have very little effect on estimates of the overall strand response, the deformations of the individual wires can be significantly affected by the magnitude of friction, contact forces, and the proximity to the strand terminations. Utting and Jones (1988) also performed tensile tests on a nineteen wire spiral strand with end conditions ranging from full fixity through to full rotation. The torque generated was recorded along with the strand extension and rotation over a gauge length of 600 mm. Such test results were then compared with the available theoretical models. It was concluded that the strands' response to an axial load could be predicted, with a reasonable degree of accuracy, by the discrete model of Velinsky et al. (1984). However, it was suggested that the theoretically determined deformation of the individual wires assumes a uniformity in the load distribution which, in reality, does not exist, and, hence, the deformation of a particular wire may be many times greater than that predicted by the existing discrete theories. Utting (1988) presented a model of the wire load distributions over the strand sections adjacent to the terminations. Tensile and shear forces across the wire sections were calculated in this region. The results from the model were compared with the experimental results from Velinsky et al. (1984), as well as the test data for the seven wire strands and a nineteen wire strand, previously reported by Utting and Jones (1987a and 1987b). Interesting comparisons were drawn between the stress levels at the mid-strand position and in the vicinity of the end termination under various conditions of end restraint. The main observation, based on a comparison of the experimental results and available discrete theories, was that the theoretical predictions based on a seven wire strand are more accurate than those for a nineteen wire strand.

Jolicoeur and Cardou (1991) carried out a theoretical numerical comparison of many of the above models, which in turn were compared with experimental results reported by Knapp (1979), McConnell and Zemke (1982) and Utting and Jones (1987a). The equations for each model were standardised so that the differences and similarities could be easily highlighted. It was discovered that all of the models were quite accurate as far as predicting the axial stiffness was concerned. The degree of agreement between the theoretical and experimental results varied when the coupling coefficients were calculated, where the symmetry of the stiffness matrix is a crucial

factor. It was also noted that, in terms of the global cable stiffness, all of the models yield comparable results, but they are not equivalent when it comes to the evaluation of the local effects such as interwire or interlayer pressures and non-linear behaviour. As discussed later, to address these cases, more advanced models should be used, such as the orthotropic sheet theoretical model of Hobbs and Raoof (1982).

Recently, Jiang (1995) presented general formulations (as he put it) for the non-linear and linear analysis of wire ropes. In his formulations, wires, strands, and wire ropes were considered as a kind of identical structure characterised by seven stiffness and deformation constants. The general formulations developed could then be used to analyse wire ropes consisting of various complex cross-sections as well as simple wire strands. Jiang's work was discussed by Jolicoeur (1996a), who pointed out some typographical errors and a possible sign error in the formulations. Sathikh et al. (1996) also made some comments about the lack of symmetry in the stiffness matrix. It should be noted that it would be nice to always have symmetry in the stiffness matrix to make the solution theoretically sound and consistent.

Sathikh et al. (1996) proposed a model which eliminated the asymmetry in the stiffness matrix, common to the majority of discrete models. The model was based on the Ramsey (1988 and 1990) – Wempner (1973) theory of generalised strains, which overcomes the origin of the lack of symmetry, identified as an inadequacy of the previously adopted formulations in relation to the wire twist and change in curvature. The predictions based on this approach, along with those of other thin rod models, and experimental data on a wide variety of seven wire spiral strands, were compared, yielding favourable results. Although this model does identify the origin of the lack of symmetry in the stiffness matrix, it is still a discrete model, which has been shown to provide less favourable results than semi-continuous models, as the total number of wires in a spiral strand increases to more than, say, 19.

2.2.3 Semi-Continuous Models

At the present time, there are two main modelling approaches for the analysis of multi-layered spiral strands. The models mentioned in the previous section are collectively known as discrete models, in which equilibrium and compatibility equations are established for each individual helical wire of the strand. The second

type of approach is very different to the discrete modelling concept in that a strand consisting of a core and N layers of helical wires is mathematically represented by N concentric orthotropic cylinders, whose mechanical properties are averaged as a continuum to match the behaviour of their corresponding layer of wires. With discrete modelling, the concept is easily understood in the sense that the approach follows the physical reality, but with semi-continuous models this is not so apparent, as a continuous medium is being used to model a discontinuous reality.

One of the main advantages of semi-continuous models, over the classical discrete approaches, is that as the number of wires in a strand increases (to more than, say, 19), then the accuracy of the models increase, because the properties of the strand are averaged over a greater number of wires. The other major advantage of the semi-continuous model is that the problem of interwire contact, inherent in strand modelling, is sufficiently simplified to be mathematically tractable. To effectively model a simple spiral strand, the approach adopted should take account of the point contact between the wires of consecutive layers, and the line-contact between the wires within a layer. The appropriate formulations for such contact phenomena have yet to be satisfactorily developed in a discrete modelling approach.

Currently, there are two types of semi-continuous models in use. The first was developed by Hobbs and Raoof (1982), and is known as the orthotropic sheet approach. It was assumed that the orthotropic layers were thin, and it was postulated that each layer of wires in a strand (although discontinuous) has enough wires (more than, say, 19) for its properties to be averaged, so that the layer can be treated as an orthotropic sheet. The elastic properties of the sheets, whose principal axes run parallel and perpendicular to the individual wire axes, are determined as a function of the external load perturbation, using well established results in the field of contact stress theory. Then, using the formulations of Hearmon (1961), it is a simple procedure to transform the elastic properties to values parallel and perpendicular to the strand axis. The compatibility equations are initially developed for a strand with its ends fixed against rotation, and assuming that, with zero axial load on the cable, the wires within each layer are just touching each other in line-contact. For a counter-laid construction, the stiffnesses in the hoop direction (where the wires are in line-contact) are much greater than the ones in the radial direction. It is this key property that is

used to set-up a series of non-linear compatibility equations to obtain the normal forces acting over the various contact patches throughout the structure. Using this information, interwire movements, changes in wire axial strains and the compliances for an external load perturbation of a given type and size can be found. The axial and/or torsional hysteresis in the strand can also be estimated, and from the properties of the sheets of wires, simple transformations and summation leads to estimates of the axial and torsional stiffnesses.

The second semi-continuous approach was developed by Jolicoeur and Cardou (1994 and 1996) who derived equations describing the behaviour of a system of coaxial orthotropic cylinders under bending, tensile and torsional loading. It was found that a cylinder under bending will not elongate or rotate. It was also suggested that the curvature caused by a bending moment occurs in a plane perpendicular to the axis of the applied moment. Numerical results for a simple case enabled the evaluation of the slip of one cylinder with respect to the other, under full-slip conditions, and the evaluation of the amount of friction required to prevent slipping. This model shares some similarities with the model of Hobbs and Raoof (1982), but is essentially different in many respects. Both approaches, however, are based on the principal of continuum mechanics and the elasticity of anisotropic materials. Jolicoeur and Cardou's (1994 and 1996) concept was originally developed for the analysis of ACSR conductors under bending and axial loading. In this approach, the cylinders are considered to be thick walled, as opposed to the thin walled assumption of Hobbs and Raoof, which makes the problem a tri-dimensional one, whereas the model proposed by Hobbs and Raoof is, essentially, a bi-dimensional one. Blouin and Cardou (1988) derived equations which described the behaviour of a system of coaxial cylinders under axisymmetric loading. It was recognised, by these authors, that discrete thin rod models were inadequate for the study of the bending behaviour of spiral strands, which is of paramount importance in the study of transverse vibrations and fatigue. Cardou and Jolicoeur (1997) conducted a reasonably extensive review of some discrete models; namely those proposed by Durelli and Machida (1973), McConnell and Zemke (1982), Utting and Jones (1985 and 1987b), Lutchansky (1969), Lanteigne (1985) and Costello (1983), along with the semi-continuous model of Hobbs and Raoof (1982) and Jolicoeur and Cardou (1994 and 1996). The review was restricted to the elastic behaviour, under small deformations, of simple strands. Multi-stranded

cables, as well as more complex strand characteristics, such as fatigue, were not covered in the review. Jolicoeur (1997) compared the two semi-continuous models of Hobbs and Raoof (1982) and Jolicoeur and Cardou (1996), by conducting a theoretical static stiffness comparison using a seven wire strand and multi-layered electrical overhead conductor. It was claimed that the models gave good results for tension and torsion, but the model of Jolicoeur and Cardou gave better bending stiffness results. It would have been more beneficial if the models had been compared by conducting an analysis using significantly larger diameter multi-layered (practical) structural strands with varying lay angles, with the lay angle having been found by Raoof and his associates to be the sole controlling geometrical parameter as far as various strand stiffness coefficients are concerned (Raoof, 1997). Raoof and Kraincanic (1994a) used an extensive series of theoretical parametric studies covering a wide range of cable (and wire) diameters and lay angles. It was shown that the 2×2 stiffness matrices for large diameter spiral strands, where the number of wires are in excess of nineteen, can be very different, depending on whether the classical discrete or semi-continuous modelling approach is used. By a careful comparison with previously published experimental results for 39, 41 and 127 mm outside diameter spiral strands, it was shown that the orthotropic sheet model gave much better predictions of the cable axial/torsional stiffness and hysteretic characteristics, under both static and cyclic loading regimes, than the thin rod theories. However, it was demonstrated that the thin rod theories gave more accurate predictions for seven wire strands, especially for the fixed-free end conditions.

Raoof and his associates have, over a number of years, used the orthotropic sheet theory to analyse many structural aspects of strand behaviour. In this review all of these aspects will be briefly touched upon: more in-depth information can be found in the cited publications. Raoof (1991e) has presented a summary of his work to date, where the theoretical predictions of various multi-layered helical strand properties have been compared with available experimental data, and a number of design charts and simple formulations have also been presented, with these being amenable to hand calculations, using a pocket calculator, hence, of value to busy practising engineers.

2.3 STIFFNESS CHARACTERISTICS OF CABLES

Owada (1952) calculated the axial and torsional stiffnesses of a simple (seven wire) strand by employing Kirchhoff's equations of equilibrium for thin rods. The contact forces between the wires and the core wire were calculated, neglecting interwire friction. For this single lay cable, the theoretical results appeared to be supported by the experimental results, but Owada's formulations are difficult to follow, partly because of his style of writing.

Costello and Sinha (1977b) examined the variations of the torsional stiffness of a seven wire strand with applied axial load and rotation of the cable. It was found that the variations were, for this particular cable and range of axial load and rotation, rather small, so that the torsional stiffness could very nearly be assumed to be a constant.

Strzemiecki and Hobbs (1988) conducted static and dynamic tests on a variety of multi-layered spiral strands and wire ropes, by subjecting the specimens to cyclic axial or bending load perturbations, whilst under a constant mean axial load. The tests, performed on a 40 mm outside diameter IWRC wire rope, showed that the rope effective axial stiffness is, due to the presence of interwire friction, not a constant, varying from the no-slip to the full-slip limit, as a function of the variations in the axial load range/mean load ratio.

Hobbs and Raoof (1982 and 1985) were the first to show that the axial stiffness of axially preloaded spiral strands varies between two limits (the full-slip and no-slip) as a function of the axial load perturbation. The full-slip limit was found to be a function of only the lay angle. A simple relationship between the no-slip and full-slip limits was presented in a graphical form (Hobbs and Raoof, 1985). Raoof and Hobbs (1988b) further developed their earlier model, as reported elsewhere (Hobbs and Raoof, 1982), to enable the assessment of the contact forces and the associated relative displacements between the wires, taking interwire friction fully into account, in large diameter spiral strands with their ends fixed against rotation. Due to the frictional effects, the strand axial stiffness was shown to be a non-linear function of the applied load perturbations, therefore, simplified routines were recommended, which provided a means of estimating the upper and lower bounds to the strand axial stiffness. Experimental results on various large diameter spiral strands were found to

support the predictions of the full-slip axial stiffnesses. In another paper, Raoof and Hobbs (1989) addressed the response of a multi-layered spiral strand to an externally applied torque. The theory predicts the full-slip histories over the line-contact patches, from the micro-slips on the periphery of the contact patches at low loads, to the onset of gross slip at higher loads and beyond. The no-slip torsional stiffness was found to be a function of the mean axial load, but independent of the effective coefficient of interwire friction. The theory also predicts the torsional energy dissipation quotient under continued uniform cyclic loading, with experimental verifications of the theory discussed, in some considerable detail, by Raoof and Hobbs (1988a).

Raoof (1990e) developed simple design formulations to provide a means of estimating the upper (no-slip) and lower (full-slip) bounds to the axial moduli of spiral strands, plus the associated maximum level of logarithmic decrement under continued uniform axial cyclic loading, and the corresponding axial load range/mean load ratio. Experimental data on a 127 mm outside diameter spiral strand supported the proposed formulations for the full-slip axial stiffness. Extensions to the orthotropic sheet theory enabled the prediction of the upper and lower bounds to the axial stiffness of multi-strand wire ropes to be made. Experimental results on a newly manufactured 40 mm outside diameter spiral strand supported the theoretical full-slip stiffness, which was found to be independent of the level of bedding-in and life history of the strand. The no-slip prediction provides a useful upper bound to the experimental data for newly manufactured cables. As with any simplified method, it was noted that there are inevitably some inaccuracies, but the simplified routines presented in this paper were suggested to be accurate enough for most practical purposes.

Raoof (1992d) conducted extensive theoretical parametric studies on a wide range of spiral strand constructions in order to provide simple formulations, as opposed to the complex nature of the original version of the orthotropic sheet theoretical model, for the prediction of various strand mechanical properties, which are of relevance to bridging and floating offshore platform applications. In this publication, the simple formulations for predicting reliable estimates of the strand axial and free-bending stiffnesses are presented in detail. Simple formulations are also provided for estimating the wire axial strains, interwire slippage over the line-contact patches

within each layer, and also the rotations over the trellis points of interlayer contact. Once again, the match between the theoretical and large scale experimental results was good.

Raoof and Kraincanic (1995a) used the results from an extensive series of theoretical parametric studies to propose a simple method for obtaining reliable estimates of the 2×2 stiffness matrix relating to the axial/torsional coupling of large diameter axially preloaded spiral strands. Straightforward routines were developed for obtaining the no-slip and full-slip bounds to the stiffness coefficients. As discussed later, despite the fact that, at the time of the original study, not all of the strand construction details available today were then available to these authors, the simplified polynomials developed, particularly the full-slip ones, are, indeed, very accurate.

Raoof and Kraincanic (1995b) used the orthotropic sheet theoretical model of Hobbs and Raoof (1982) for estimating the 2×2 stiffness matrix for the constituent spiral strands, using which a theoretical model for analysing large diameter steel ropes was developed, taking the effects of interwire friction into account. The model provides a reasonably simple means of predicting the axial and torsional stiffness coefficients. This model was further checked by Raoof and Kraincanic (1995c), who reported the no-slip and full-slip bounds to the rope effective stiffness coefficients, under axial/torsional coupling, for a number of large diameter wire ropes with an independent wire rope core (IWRC) or fibre core. The theoretical predictions of the axial stiffness, under full-slip conditions, were found to agree well with the available experimental data for some realistic stranded wire ropes with IWRC (33, 40 and 76 mm outside diameters) or fibre cores (9.53 and 40.5 mm outside diameters). Raoof and Kraincanic (1995d) carried out a more in-depth analysis (backed by test data from other sources) of wire ropes with fibre cores.

Using the orthotropic sheet theoretical model, Raoof (1997) provided some numerical results for certain overall strand characteristics based on three different types of 127 mm outside diameter spiral strands with lay angles of 12° , 18° and 24° , the other geometrical parameters of which were kept nominally the same. The lay angle was the only geometrical parameter which was varied, and hence, the influence of changes in the lay angle on, say, the stiffness coefficients and hysteresis under various modes

of loading could be investigated on a theoretical basis. Theoretical results were presented, which showed the practically significant effect of changing the lay angle, in various layers, on the estimated values of the axial, torsional, and plane-section bending stiffnesses, and their associated frictional hysteresis in axially preloaded spiral strands. However, as was noted by Raoof, although reducing the lay angle may increase the strand axial stiffness, at the same time, it may significantly reduce the sometimes very desirable levels of structural damping in the strands.

Kraincanic and Hobbs (1997) presented results from tests on a 76 mm outside diameter wire rope. The ends of the 8 m long test specimen were prevented from rotating and the torque was measured for various levels of axial load. The results from the tests compared favourably with the theoretically predicted values. Various available models were then used to predict the torque factors (used to represent the torque generated when loading a rope axially) for wire ropes with outside diameters of 55.6 and 76 mm, and with independent wire rope cores (IWRC). The magnitudes of the torque factors obtained from the experiments agreed closely with the theoretically predicted values, which related to the full-slip case in the present terminology. The results also showed the significant influence of the lay angle on the rope torque factor. In a later publication, Kraincanic and Hobbs (1998) provided some much needed experimental data on the no-slip and full-slip axial stiffnesses and torsional effects in a 76 mm wire rope with an IWRC. The test specimen was 7 m long and the ends were prevented from rotating. The axial stiffness was measured for both large load ranges and for small load perturbations superimposed on larger mean axial loads. The agreement between Kraincanic and Hobbs' experimental data and theoretical results based on Raoof and Kraincanic's (1995c) model for the no-slip and full-slip axial stiffnesses was very encouraging, although it was suggested that as far as the no-slip torsional stiffness is concerned, there is a need for further theoretical developments.

2.4 HYSTERETIC CHARACTERISTICS OF HELICAL CABLES

Claren and Diana (1969b) discussed certain laws governing the response of a helical cable to exciting forces, such as wind. They concentrated on the response of systems composed of a taut cable on which one or more Stockbridge dampers were installed for preventing lateral vibrations and, hence, the occurrence of cable restrained bending fatigue failures resulting from aeolian vibrations. The computed deformations were

compared with experimental values and were claimed to be sufficiently accurate for most practical purposes. It was demonstrated that neglecting the bending stiffness is feasible in describing a cable's overall response. It should, however, be noted that, what the authors do not say, is that it is clearly not feasible in the study of cable bending stresses to ignore cable bending stiffness since, in the absence of bending stiffness, there could be no bending stresses. In another (earlier) publication, Claren and Diana (1969a) showed that a correlation exists between the dynamic strains occurring in a cable span, and those occurring at the rigidly clamped ends of vibrating taut circular beams. The process by means of which wire slippage will reduce the dynamic strains and contribute to the cables' internal damping was also discussed.

Tilly (1988) reported on some axial damping test data on large diameter cables (spiral strands, locked coil ropes, and wire ropes) with outside diameters ranging from 44 mm to 70 mm. The axial damping capacity of these newly manufactured cables was found to be relatively low, with measured values of logarithmic decrement in the range of 0.01 to 0.07. The test data showed that the damping increased with increasing values of the lay angle and that, over a wide range of loading frequencies, it was independent of the loading frequency.

Hobbs and Raoof (1984) developed a method for calculating the axial hysteresis in old and fully bedded-in spiral strands. Variations in the hysteresis with axial pre-load and load range for any spiral strand construction could be predicted. The theoretical results compared favourably with experimental measurements relating to the axial energy dissipation of an old 39 mm outside diameter spiral strand, as earlier reported by Hobbs, Ghavami and Wyatt (1978). It was theoretically shown that hysteresis may most easily be increased by slightly increasing the lay angle, provided that some reduction in the strand axial stiffness is acceptable. The torsional characteristics, such as stiffness and hysteresis, could also be predicted theoretically, and the theoretical predictions were supported by the test data of Raoof and Hobbs (1988a), with their tests carried out on an old (i.e. fully bedded-in) 39 mm outside diameter spiral strand. Most importantly, their assumed value of the coefficient of friction was kept constant throughout their theoretical-experimental comparisons so that this parameter was not used as a convenient calibration (fiddle) factor. For an old strand, it was

experimentally shown that random loading could significantly increase the level of torsional hysteresis above that found for regular cyclic loading, provided that the interwire force changes are large enough to overcome the line-contact interwire friction. It was also found that, in their tests carried out on a newly manufactured 41 mm outside diameter spiral strand, hysteresis measurements on such newly manufactured strands could prove to be misleading for long term applications where the helical cable eventually becomes fully bedded-in under the action of external forces of a random nature. Raoof (1990a) has suggested that the hysteretic behaviour of a newly manufactured spiral strand is, indeed, very different from that of an old and fully bedded-in helical cable. According to Raoof, due to the gradual nature of the interwire/interlayer fretting, helical cables could need a lengthy period of bedding-in before their internal structure becomes reasonably stabilised: during this period the spiral strand's damping characteristics can vary in a very complex fashion. It was, however, suggested by Raoof (1990a) that the full-slip axial and torsional stiffnesses are not as sensitive to the degree of bedding-in.

Raoof (1991c) presented certain simplified routines which provide useful information as regards the magnitude of the interwire/interlayer movements. A simple method for estimating the maximum level of axial hysteresis and its associated axial load range/mean load ratio is also given, in addition to a hand-based method for setting up the 2×2 stiffness matrix for the full-slip case of coupled extensional-torsional deformations. In particular, unlike previous theories, the orthotropic sheet theoretical model was found to give rise to an almost symmetrical stiffness matrix over a wide range of geometrical parameters. Raoof and Huang (1992h), based on an extensive series of theoretical parametric studies, have reported simple (hand-based) methods for determining the axial stiffness and hysteresis, the free-bending stiffness and hysteresis, along with the wire kinematics and strand lateral contraction, for axially preloaded multi-layered spiral strands experiencing external axial or free-bending load perturbations.

Raoof and Huang (1991) proposed an analytical model for obtaining the upper bound to the frictional damping of axially loaded single layered spiral strands undergoing cyclic bending to a constant radius of curvature. The results of their analysis indicate that, for sufficiently large levels of radii of curvature and cable axial strain, increasing

the helix angle may lead to some (although, perhaps, not practically very significant) increases in the cable damping that may only be predicted using the no-slip to full-slip interwire/interlayer friction model. Unlike Raoof and Huang's (1991) partial slip model, the previous methods for obtaining the frictional damping had employed the Coulomb rigid-plastic friction model, which tended to grossly overestimate the cable damping for large radii of curvature. In another publication, Raoof (1991f) considered the axial damping response of a multi-layered spiral strand, for a given mean axial load. The axial specific damping under continued uniform cyclic loading was predicted using two different methods to provide a double-check. One of the methods was based upon estimates of the energy dissipation per cycle on each of the line-contact patches within the strand, summation yielded a value for the overall strand hysteresis. This value of strand hysteresis should closely match the value obtained from the overall load-deflection curve, which is used to follow the loading and subsequent unloading response of a spiral strand. It was found that the theoretical predictions were supported by experimental data on a fully bedded-in (old) 39 mm outside diameter spiral strand.

In a later publication, Raoof (1998b) offered an explanation as regards the underlying reasons for the very large discrepancies (by a factor of 10 – 100) found between his proposed theoretical and experimental results, and those reported in the literature by Roberts (1968). It was also argued that using small (seven wire) strands to predict the damping characteristics of much larger diameter multi-layered strands is not a viable approach as the frictional damping mechanisms in single and multi-layered spiral strands are very different. In addition, the main source of the discrepancies between Raoof's results and those of Roberts (1968) were attributed to the methods of testing, whereby the previously adopted experimental techniques were claimed to have been grossly affected by the hysteresis in the test rig rather than the specimens.

Seppa (1971) has also presented a method for the experimental determination of the self-damping capacity of the transverse vibrations in transmission line conductors. In his work, the self damping measurements were carried out on an ACSR Drake transmission line conductor. The results showed a good correlation with the levels observed in actual spans in service. Experimental measurements have also been carried out by Vanderveldt et al. (1973), on twelve different rope constructions, to

investigate the damping mechanism(s) in transverse vibrations. The correlations between their theoretical and experimental results were encouraging, considering that the theoretical analysis was based upon a simple model involving lumped masses.

Raoof and Huang (1993a) reported a theoretical means, based on the orthotropic sheet theoretical model of Hobbs and Raoof, for predicting the structural frictional damping of axially preloaded spiral strands undergoing lateral vibrations, due to, for example, vortex shedding. Raoof and Huang showed that the overall structural damping decreased substantially with increasing cable span and, to a lesser extent, with increasing levels of mean axial load. It is now possible, for pin-ended and axially preloaded spiral strands undergoing plane-section bending, to show significant variations of the equivalent damping ratio with the type of strand construction, length of cable and the mode of lateral vibration. This is in contrast to the traditional approaches, which invariably assumed a constant damping ratio based on rather limited experimental data. The practical implications of assuming a constant damping ratio for obtaining estimates of the maximum amplitudes of vibration, under vortex-shedding instabilities, were also briefly addressed. The proposed theoretical model was supported by some previously published empirical formulations based on large-scale experiments on overhead transmission lines. In another paper, Raoof and Huang (1993b) addressed the effect of hydrostatic pressure on the damping characteristics of axially pre-loaded sheathed spiral strands undergoing sustained lateral vibrations in deep water applications. Contrary to the widely accepted view that the maximum amplitudes of vibration under vortex shedding instabilities may be predicted by assuming a constant viscous damping coefficient, this model shows that the damping coefficient is a function of a number of cable parameters. The structural damping factor decreases substantially with increasing cable span and increasing mean axial load, as was the case in the previous publication of Raoof and Huang (1993a) which related to in-air conditions.

2.5 HYDROSTATIC PRESSURE EFFECTS ON THE STIFFNESS AND HYSTERESIS OF SHEATHED SPIRAL STRANDS

Gecha (1989) presented the results of both a theoretical and experimental study of the stresses, strains and displacements in sheathed spiral strands, under the action of axial forces, experiencing external hydrostatic pressures. The cable was assumed not to

untwist under stretching, and interwire friction was neglected. The formulae, however, took account of the radial compressibility of the cable, the increase in radial rigidity due to the interaction between the wires in a layer, the Poisson effect for the core and the wires, and the effect by which the longitudinal rigidity of the cable is reduced and the tensile load taken by these elements is redistributed due to the external hydrostatic pressure and the pressure of the outer layer of wires on the inner layers. The discrepancies between the experimental and theoretical results was found not to exceed 10%.

Raoof (1990c) presented an insight into the effects of high external hydrostatic pressures (in the presence of substantial air filled voids inside the strand) on the axial/torsional stiffnesses, hysteresis and fatigue behaviour of multi-layered sheathed spiral strands. It was shown that high external hydrostatic pressures can significantly influence the pattern of interwire/interlayer contact forces in sheathed cables. Increasing the water pressure on a sealed strand, in deep water applications, leads to significant variations in the predicted damping behaviour of the cable under both the axial and torsional modes of loading. The torsional stiffness was found to be dependent on the depth of water for all ranges of twist. It was noted that, increasing the level of hydrostatic pressure results in a substantial increase in the magnitude of interlayer contact patch stresses, which can lead to significant reductions in the cable fatigue life in long term applications. Simple formulations for estimating the bounding solutions to the strand plane-section bending stiffness, plus the associated critical radii of curvature of axially preloaded large diameter sheathed spiral strands experiencing high external hydrostatic pressures, taking the effect of interwire friction into account, were presented by Raoof and Huang (1992a). The strand plane-section bending stiffness was found to vary between the no-slip and full-slip bounds, which were found, for all practical purposes, to be independent of the magnitude of the external hydrostatic pressure. It was also found that the free-bending hysteresis under uniform cyclic bending was a function of the strand's construction details (primarily the lay angle), radius of curvature, level of mean axial load, and the magnitude of the external hydrostatic pressure. Using the previous findings of Raoof and his associates, Raoof and Kraincanic (1994b) presented a reliable and simple (hand-based) method, based on extensive theoretical parametric studies, for estimating the 2×2 no-slip and/or full-slip stiffness matrices of large diameter multi-layered sheathed spiral

strands experiencing external hydrostatic pressures. It was theoretically shown that both the no-slip and full-slip stiffness coefficients in the matrix defining the axial/torsional coupling were largely independent of the level of mean axial load, and the magnitude of the externally applied hydrostatic pressure.

2.6 TERMINATIONS AND ANCHORAGE SYSTEMS

Due to the fact that a number of fatigue failures occur at (or in the vicinity of) the end terminations, it is perhaps worth covering a few of the important points regarding the construction of these terminations, and the mechanics of the load transfer between the wire rope and the termination. Until fairly recently, wire ropes were terminated with Flemish eyes or zinc filled sockets, but more recently loops clamped with cold forged ferrules have also been used. The main disadvantages of these types of terminations is the specialist equipment and skills required to ensure that the terminations are efficient, especially if they are to be formed on site. The British Standard (BS 463 part 1, 1958) provides some guidance on the terminations to helical cables.

Gabriel and Helmes (1991) described the structural characteristics and effects of an anchorage of high strength steel wires in a cast zinc alloy cone. Due to the disadvantages of the traditional methods of terminating wire ropes, an alternative method was sought. Gatham (1979) and Dodd (1981) looked at the use of resin as a socketing medium. The concept of using resin as a socketing medium was first introduced in the 1960's by the U. S. Naval Department, amongst others, who conducted successful socket tests with several types of resins, including epoxy and urethane. Fairly recent tests have shown that resin poured sockets have some distinct advantages over zinc. The preparation of the resin for attaching a socket is relatively simple for a qualified person, requiring the mixing of only two ingredients, and as no heat is required, this eliminates the equipment and inherent hazards associated with an open flame and combustible material needed to prepare zinc poured sockets. Dodd (1981) also found that the resin socket termination has a superior fatigue performance capability. Hanzawa et al. (1982) used test specimens terminated with epoxy resin and a zinc/copper (Zn/Cu) alloy. The epoxy resin was found to fair better, with low fragility, high adhesiveness to the wires, and minor slip out under load, which was in contrast to the Zn/Cu alloy. The only major problem with the use of epoxy resin in practice, as far as this author is aware, is the lack of long term performance data.

Chaplin and Sharman (1984), seeing the increase in the use of resin, attempted to provide a detailed understanding of the mechanism by which the load is transferred from the rope into the termination. Understanding this load transfer mechanism should help to identify the causes of potential problems and improve the design of the sockets.

At around the same time that Gatham (1979) and Dodd (1981) were conducting their individual research, Matanzo and Metcalf (1981) were evaluating the efficiency of nine different types of wire rope terminations. The tests were carried out on five different Lang's and regular lay wire ropes. The wire rope terminations included the Flemish loop, wedge socket, zinc poured and resin poured sockets, to name a few. Based on the test results, it was concluded that the efficiency of a wire rope termination assembly is dependent upon the wire rope termination itself, most importantly, and secondly the interaction of the termination with the wire rope. In another publication, Metcalf and Matanzo (1980) evaluated the replacement criteria of wire rope terminations by establishing a database based on which a scientific approach could be built, to try and replace the empirical method based on past commercial experience. The test specimens were the same as those used by Matanzo and Metcalf (1981). It was found that the most significant factor affecting the strength of a wire rope termination was stress. Wire rope construction, class and diameter were found to be relatively insignificant.

Zhang and Leech (1985) used an inhomogeneous finite element method to estimate the stresses in wire rope terminations. The results proved to be of considerable use for optimising the design of wire rope terminations. Mitchell et al. (1974) conducted an axisymmetric finite element analysis of a broad class of end terminations, consisting of a metal casing joined to the fibre-reinforced plastic material by a layer of potting medium. Tests were carried out in conjunction with the analytical model and, as a consequence, new end fittings for fibre-reinforced plastic rods and ropes were developed that very nearly approached (within several percent) the full tensile strength of three commercially available glass reinforced plastic products. The results demonstrated the importance of using a relatively thick layer of low stiffness potting material in high strength potted end fittings. A finite element model of a seven wire strand was also developed by Jiang and Henshall (1999) to analyse termination

effects. The model, taking friction into account, provided a means of determining the effect of a fixed end termination on the contact forces, and the relative movements between the wires along the contact lines. In their plots, showing contact force or relative movement against distance from the fixed end, it is not clear as to where the authors consider the position of zero movements to be, i.e. at the socket face or, as will be shown in chapter 5.0 of this thesis, at some distance inside the end termination.

2.7 IMPACT LOADING ON HELICAL CABLES

Ringleb (1957) analysed the response of a cable to oblique impact loading, determining the relationship between the impact stresses and the impact velocity. The possible rotational motion of the ends of the cable was ignored, as was the energy absorption and also the diametral changes under the transient response. Ringleb's closed-form solution for the longitudinal sound velocity in an axially pre-loaded cable appears to be supported by his experimental data, for the case of transverse impact.

Samras et al. (1974) emphasized the previously neglected importance of taking the coupled extensional-torsional behaviour of a wire rope, under dynamic loading, into account. Equations of motion were derived, and the constitutive equations, relating the rope tension and torque to the rope extension and rotation, were postulated. Two types (axial and/or torsional) of waves were shown to propagate through a straight rope section. The critical frequencies which induce resonance in a rope, for a number of differing end conditions, were calculated on the basis of the coupled extensional-torsional equations of motion. The frequencies predicted by this axial/torsional coupled theory were shown to be significantly lower than those predicted by the purely extensional approaches. The findings of Samras et al. were utilised by Skop and Samras (1975) who analysed two types of wire rope, one torque balanced and the other not. The equations describing the coupled extensional-torsional oscillations in a wire rope were employed, in order to calculate the response to ocean waves of a line supporting a spherical payload - a common problem encountered during ocean salvage and construction operations. The damping of the rope itself was neglected, but the equivalent viscous damping of the payload was taken into account. The results of the calculations show the significant differences that exist between the response as predicted by the coupled theory and that predicted by the classical approach, where

axial/torsional coupling is neglected. The theory also highlights the large differences that exist between the predicted coupled responses of the two types of rope.

Phillips and Costello (1977) presented geometrically non-linear coupled equations of motion, which govern the axial and rotational displacements of a straight, single lay, twisted wire cable. It was shown that the equations could be linearized and used to predict the dynamic axial and rotational displacements of an axially impacted cable, with one end fixed against rotation and the other subjected to the impact load. The determination of the stresses and strains within the cable were based on a knowledge of the displacement gradients provided by the linearized theory. No experimental data was, however, given in support of their theoretical results.

Jiang et al. (1991) investigated the extensional-torsional forced vibrations of coupled systems. Closed-form solutions were obtained, neglecting frictional (damping) forces, for the case when the vibrations were caused by harmonic axial forces and couples. To illustrate the application of their theory, the forced vibrations of a helical spring was considered in some detail.

Raoof et al. (1993) developed closed-form solutions for predicting the extensional-torsional wave speeds and displacements in axially preloaded spiral strands, experiencing specific forms of impact loading – i.e. unit-step and/or triangular, at one end, with the other end fixed. It was shown that, for sufficiently small levels of external impact loads, the use of the traditional full-slip force-displacement stiffness matrix can be misleading (due to the presence of interwire friction) and the alternative no-slip stiffness matrix should be adopted. In another paper, Raoof et al. (1994) used the same closed-form solutions to provide a more in-depth understanding of the effect of using the alternative no-slip stiffness matrix. These findings may have significant implications in, say, non-destructive in service methods for wire fracture detection, to be discussed in considerable detail in a later chapter.

2.8 AXIAL FATIGUE CHARACTERISTICS OF HELICAL CABLES

In the 1980's, the U. S. Navy planned to moor floating platforms off the continental shelf in 3 km of water. Practical trials were started, but the cost of the trials was enormous and the range of rope conditions that it was possible to study was rather

limited. Hearle (1996) conducted a survey on the ropes used for moorings in deep water applications. The paper concentrates on the use of polyester ropes such as nylon, and addresses the fatigue resistance and the termination effects of such ropes. A number of test results are used to demonstrate the effectiveness of the ropes and a small synopsis is presented as to why fibre ropes should be chosen in preference to the more conventional steel cables. Gabriel and Nürnberger (1992) paid close attention to the chemical attack, material characteristics, production, and different types of cables, as possible parameters affecting the fatigue life. Practical recommendations concentrating on the advantages and disadvantages of various options regarding axial fatigue endurance are given.

Starkey and Cress (1959) showed, by mathematical analysis, that by far the greatest stresses in a wire rope result from Hertzian contact stresses at the points of contact of wire-on-wire. It was also argued that the usual mode of failure of a wire rope is initiated by fretting fatigue at the points of contact, which in turn initiates fatigue cracks and multiple wire breakages. In their paper, interwire friction was neglected, and all the stresses were assumed to be linearly elastic.

Knapp and Chiu (1988) proposed a numerical fatigue model capable of predicting the cycles-to-failure of helically armoured cables subjected to a fluctuating axial tension. A method was presented for calculating the maximum magnitude of the wire stresses at the points of interlayer contact, which were thought to control the strands' axial fatigue life. The model ignored interwire friction, interlayer slip and contact deformations. Encouraging agreement was found between the theory and test data.

Hanzawa et al. (1982) conducted experimental fatigue tests on 50 mm outside diameter wire ropes. The fatigue life of the wire rope was defined as 'when the wire breakage rate exceeds 5%'. They found that, when the stress range was low, the first wire fractures occurred in the outer strands, but at higher levels of stress range, the first wire breakages occurred in the inner and core strands. The fatigue strength was also found to increase with an increase in the ultimate wire tensile strength and diameter. The epoxy resin filled sockets proved to be very efficient, exhibiting low fragility and high adhesiveness. This was in contrast to the Zn/Cu alloy which exhibited high numbers of wire breakages within the socket and considerable slip-out.

Gabriel (1985) presented a statistical method for determining the fatigue strength of tension members made of cold-drawn wires, which is based on the results of fatigue tests on short specimens, and precision measurements of the cold-drawn wire.

Knapp (1989) presented a numerical model which predicts the axial fatigue life of straight strands subjected to fluctuating tension. It was theoretically shown that large Hertzian stresses can be produced due to the wire contact, with the latter occurring either between adjacent layers (radial contact) or between adjacent wires within the same layer (circumferential contact). Fairly reasonable agreement was found between the theoretical results and the experimental data, as regards the strand axial fatigue life, based on four two layer wire strands, in spite of the fact that the analysis ignored interwire friction, interwire contact deformations and relative slippage.

Mitamura et al. (1992) examined the fatigue strength of a parallel wire strand consisting of 37 galvanised wires and terminated at both ends with zinc poured sockets. The fatigue strength of the parallel wire strand was found to decrease due to wire breakages at the zinc poured sockets. The causes of this reduction in fatigue strength were investigated using a finite element model. Based on such results, the authors proposed a method by means of which the fatigue strength could be improved without changing the socket structure. The improvements suggested, included providing a gentle taper angle on the socket wall to reduce the wire stress concentration, avoiding bending of the wires, and displacing the wire splay initiation point from the area of stress concentration - i.e. the point at which the wires are separated inside the socket. It was found that using the suggested improvements could increase the fatigue strength of this particular parallel wire strand by a factor of, say, 2.

Raoof (1990d, and 1991a) developed a model, based on the fatigue results for individual wires, and the pattern of the Hertzian contact stresses over the individual contact patches, capable of predicting the cable axial fatigue life away from the terminations (i.e. in the free-field) under uniform cyclic axial loading. Experimental data on a 51 mm outside diameter spiral strand confirmed the validity of the theoretical model. It was, however, mentioned that the presence of a termination can lead to significant reductions in the observed cable fatigue life. It was also shown

(theoretically) that in relation to sheathed spiral strands, in deep water applications, the axial fatigue life will be reduced with increasing water depth. Raoof (1991b) extended the previously reported orthotropic sheet theoretical model to provide an insight into the pattern of the Hertzian stresses throughout multi-layered spiral strands (i.e. over both line- and/or trellis-contact patches). Using these results, a model, based on first principles, was developed which could predict the strand axial fatigue life for both cases of wire fractures occurring either in the free-field (i.e. away from the terminations) and/or in the vicinity of the end terminations. Numerical data, based on an extensive series of theoretical parametric studies, showed that the strand construction details and the grade of wire can have a marked influence on the shape of a strands' S-N curves. It was also found that the S-N curves are influenced by the level of mean axial load, which, in turn, significantly influences the endurance limit of the strand. The theory is capable of predicting internal wire fractures. The paper also addresses the effects of hydrostatic pressure and corrosion fatigue due to seawater. Moreover, an explanation is offered for the previously reported experimental observations, that a significant reduction in the cable fatigue life due to sea water does not occur when compared with the in-air performance.

Raoof (1992b) critically examined the API recommended practice regarding the axial fatigue life estimation of moorings. The considerable shortcomings of the API document were noted, particularly the unconservative nature of API's S-N design curves for helical steel cables.

Casey (1993) provided the results of two studies conducted to evaluate the fatigue performance of large diameter wire ropes used in offshore mooring applications. The tests were carried out on six wire ropes, multi-strand and spiral strands with outside diameters ranging from 38 mm to 127 mm. The tests included tension-tension and bending-tension fatigue modes of external loading, with the cables subjected to loads of both constant and variable amplitudes, and tension-tension fatigue tests in seawater, with the cables subjected to a constant amplitude load perturbation. The results, although fairly extensive, are presented in tabular (rather than graphical) form and are difficult to interpret. Chaplin (1993) discussed the test results of Casey's investigation in some detail with the results presented in a graphical form. Chaplin, then, using a lower bound approach, recommended what was claimed to be a generally applicable

axial design S-N curve. The combined mode of bending-tension fatigue was identified by Chaplin as an important degradation mechanism for mooring ropes operating with fairlead pulleys and, using some test data, an empirical method was proposed for evaluating the fatigue life of wire ropes under bending-tension.

Raoof and Alani (1997), and Raoof (1995b, and 1996) provided further experimental support for Raoof's axial fatigue model for large diameter spiral strands, using the test data on strands with outer diameters ranging from 25 mm – 127 mm. The test data was obtained from a variety of sources, including Imperial College, TRL, Bridon Ropes, in the U. K., and the University of Alberta in Canada, hence, providing ample support for the general reliability of the model.

The controlling role of different classes of interwire/interlayer contact patches on various cable characteristics was discussed by Hobbs and Raoof (1996). In particular, the central role of the interlayer (trellis) points of contact, in the context of cable axial and restrained bending fatigue performance, is emphasised in the light of recent theoretical and experimental findings, using well-established results in the field of contact stress theory. The variation of the axial stiffness and hysteresis with load amplitude and mean load is also described. Raoof (1998a) presented newly developed S-N curves for predicting the axial fatigue life of large diameter spiral strands, taking the strand construction details into account. The proposed S-N curves cater for the detrimental effects of the end terminations, and enable one to design against either external or internal wire fractures under uniform axial fatigue conditions. The proposed S-N curves are compared with others recommended by API, Chaplin and Tilly, which are the ones most commonly referred to. As argued by Raoof, the S-N curves proposed by API, Chaplin and Tilly are all purely empirical, based on experimental data relating to test specimens which are unlikely to have covered the full range of first order design parameters, specifically the lay angle.

Alani and Raoof (1997) used the theoretical model of Raoof to try and shed some light on the effect of the mean axial load on the axial fatigue life of large diameter steel helical strands. The theoretical parametric studies showed that the endurance limit increases with increasing levels of strand mean axial load. It was also theoretically shown that increasing the lay angle leads to a reduction in the magnitude of the

endurance limit for both the outer or inner wires in a spiral strand. Finally, the previous claim by Chaplin and Potts (1991), who had advocated the use of an equivalent load range (rather than load range) based on Goodman's transformation (in order to cater for the effect of a mean axial load) was questioned and theoretically shown to be of no practical benefit as regards producing S-N curves for steel cables.

Raoof and Hobbs (1994) addressed the most appropriate (underlying) type of statistical distribution to be used for statistically analysing axial fatigue test data for wire ropes, and, unlike previously adopted normal or weibul distributions (by others), the Gumbel distribution was shown to be the most representative one. It was argued that more attention should be paid to the design of cable fatigue tests, concentrating on the termination effects. They also addressed the question of the minimum desired length of axial fatigue test specimens, which, was suggested, should be around ten lay lengths.

2.9 BENDING OF STEEL CABLES

The available literature on the free bending of cables in the 'free-field' (i.e. away from the terminations), and in the vicinity of the end terminations, has been reviewed by Raoof and Hobbs (1984), and Raoof (1989 and 1992a), who noted that, despite the efforts of many researchers, dating back to the early part of the century, little progress had been made in this area. It was suggested that, with the previous work being almost entirely experimental in nature, the results could not be used to predict the response of other spiral strands unless a sound theoretical insight into the problem was achieved.

Chapman (1908) was amongst the first to examine the bending stresses in cables, taking the helical nature of the wires into account. A simple (although not accurate) equation describing the bending of steel helical cables was derived. Cyclic bending experiments were conducted which highlighted the significant effects of interwire friction. However, there was no method available at that time to enable Chapman to quantify his experimental observations.

Wyatt (1960) proposed an equation defining the bending deflection of a simple tendon (wire). In his formulations, Wyatt assumed that the position of zero lateral deflection

was at the point at which the cable exits the fixed end (i.e. at the cable-socket interface). As discussed later on in this thesis, the actual point of zero deflection is at some unknown distance inside the socket, and is dependent upon a number of factors, including quantifiable variables such as the strand construction and unquantifiable variables such as the skill of the craftsman making the socket. In Wyatt's work, the variations of the stresses, induced by the lateral deflection of the cable, across a section of the cable, were analysed. The influence of the clamping bands and of tensioned wire wrapping was also considered.

The pressing need to understand free-bending fatigue problems in the field of structural engineering became apparent in the early 1980's, when a report for the U. K. Department of Energy identified the necessity for a much better understanding of this problem. Free-bending fatigue problems are a source of concern in structures such as suspension bridge hangers and tethered buoyant platforms, where fatigue failures near partially restrained terminations, of various types, are of concern. Fatigue tests on large diameter strands are so exceptionally expensive and the consequences of cable failure so alarming, that the need for a theoretical model, capable of predicting the free-bending fatigue behaviour of spiral strands, based on a careful interpretation of the experimental results, was given a new urgency.

The development of early models was traditionally based on the maximum bending strain approach, which assumed that the strand bending fatigue life was governed by the maximum bending strains, which, using the concept of simple beam bending theory, occur at the extreme fibre position, which would, then, be the location where the initial wire fractures are expected to occur. In the late 1970's Hobbs and Ghavami (1982) carried out axial and free-bending fatigue experiments on two different spiral strand constructions – a 16 mm and a 39 mm outside diameter spiral strand. The 39 mm test specimens were 7.40 m in length between the socket faces. In the in-line fatigue tests, on 16 mm and 39 mm strands, the wire failures appeared to be concentrated at the socket, with the first observed wire failures occurring in the outer layer. In the bending fatigue tests on the 39 mm outside diameter spiral strand, the wire failures also appeared to be concentrated at the socket, but the first observed wire fractures were found to invariably occur at the so-called neutral axis (in terms of the simple beam bending theory). The time interval between the first and final wire

failures in the flexural case indicated that regular inspection may provide some safeguard against sudden failure under fluctuating stress conditions. The design implications of the results of Hobbs and Ghavami (1982), in relation to stays for guyed masts and suspension bridge hangers, were discussed by Hobbs and Smith (1983) who proposed a tentative design procedure against bending fatigue failure. Raoof and Hobbs (1984) proposed a theoretical model to try and explain the interesting and, at the time, perhaps surprising experimental observations of Hobbs and Ghavami in their bending fatigue tests that initial wire fractures invariably occurred at the neutral axis. The proposed model enabled one to predict the free-bending behaviour of a spiral strand, both within the zone of influence of a termination, and also in the free-field - i.e. well away from any termination effects. Their model was developed for a constant curvature imposed on an axially preloaded spiral strand fixed at one end. These authors accounted for both the geometrical and material non-linearities with the individual helical wires remaining linearly elastic and the interwire friction causing the material non-linearities. Raoof (1989) compared the predictions of this theoretical model with some large scale experimental results which provided a fundamental insight into the, perhaps, initially puzzling experimental observations of Hobbs and Ghavami (1982). The conclusion drawn was that the primary mode of failure was interwire fretting fatigue between the often counter-laid wires in the vicinity of the partially restrained termination, which was greatest at the neutral axis, and least at the extreme fibre position. Various predictions of this theoretical model were further validated by Raoof (1992a), who reported results based on large scale experiments on two 7.9 m long spiral strands with outside diameters of 39 mm and 41 mm. Numerous strain gauges were placed at the mouth of the socket on both strands (thirty on the 39 mm strand and twenty four on the 41 mm strand, with one electrical resistance strain gauge placed on each individual galvanised outer wire).

Vinogradov and Atatekin (1986) presented an analysis of the bending deformations of a cantilever single layer cable, with interwire friction and the twisting of the individual wires taken into account. The so-obtained numerical data showed that the total frictional losses in a seven wire strand depend on the helix angle and tightness (packing) of the wires.

LeClair and Costello (1988) developed a solution for the case of bending to a constant curvature of a single layer spiral strand in which the individual wires satisfied the six non-linear equilibrium equations of Love (1944). It was assumed that the axial load was sufficiently large to ensure that contact was maintained between an outer wire and the centre (King) wire when the strand was bent. Their purely theoretical model included the interwire frictional effects. Interpreting the results, it was felt that in the actual case of bending, the tensile forces due to bending were rather small compared to the tensile loads in the strand due to the externally applied axial loads and may, thus, be neglected when determining the wire stresses.

Sterren (1993) described a calculation method, based on stress measurements, to establish the bending stiffness of a wire rope in both free and forced bending. The model takes account of the wire rope geometry and internal friction. It was found that the so-obtained bending stiffness was only valid under free bending conditions, and was related to the tensile force, in that a higher tensile force gives rise to a higher bending stiffness.

Knapp (1988) investigated the helical wire stresses in bent cables using a simplified kinematic model. In his model, a number of assumptions were made: the helical wires were assumed to be linearly elastic, interwire contact stresses were neglected, and the lay angle and pitch radius were assumed to remain constant. In the course of Knapp's work, two bounding conditions were considered: a frictionless case, and also the presence of an infinite interlayer shear stiffness, with the latter corresponding to plane-section bending.

Lutchansky (1969) developed a simple rational mathematical model to study the effects of shear interaction in a single lay, helically wound, armoured cable bent to a prescribed curvature. Using the proposed model, a description of the axial stress concentration near the clamp, as well as in the free-field was obtained. It was assumed that the core suffers from no shear distortion, whilst the interaction between the outer layer wires and the core was assumed to be controlled by a constant distributed shear stiffness. Lutchansky did not provide any analytical or realistic experimental means of obtaining values of the shear stiffness, which determines the interaction shear force between the outer-layer wires and the core. Instead,

Lutchansky only quoted tentative values for the shear stiffness coefficient. The major limitation in Lutchansky's linear model is that it did not predict the occurrence of interwire slippage, which is essential in the analysis of the interwire fretting process near the terminations, as fully discussed by Raoof (1989), who extended the model of Lutchansky to cater for the occurrence of interwire/interlayer slippage (Raoof and Hobbs, 1984, and Raoof, 1990b).

Witz and Tan (1990) presented a non-linear model which described the interaction between all of the structural components of multiple layered flexible structures. The model gives the axial-torsional load displacement relationships, which were verified with experimental data. In a slightly later paper, Witz and Tan (1991) described a general analytical model capable of predicting the flexural structural behaviour of multiple layered flexible structures, which is dominated by the relative movement which occurs between the component layers. The relative slip between the component layers obviously occurs when the imposed curvature exceeds a critical value: unfortunately in the bending tests reported in this paper, the critical curvatures were very small and difficult to measure accurately. By comparison with available experimental data, it was suggested that the model provided reliable predictions of the bending stiffness for multiple layered flexible structures under conditions of pure flexure.

Raoof and his associates, in a series of publications, further developed and manipulated the orthotropic sheet theoretical model to try and fully understand the bending phenomenon. Raoof and Huang (1992c) suggested a method for obtaining reliable estimates of the interwire/interlayer shear stiffnesses in axially preloaded multi-layered spiral strands undergoing bending to a constant radius of curvature. Using this data, it was possible to obtain reliable estimates of the total wire strain, under the action of cable bending. Contrary to previous published work, it was shown that for a wide range of practical levels of axial load and imposed curvature, the maximum wire axial stress component in large diameter spiral strands is, invariably, much greater than the corresponding wire bending stress(es) about its own neutral axis.

Raoof (1992c), based on an extension of Lutchansky's (1969) linear model, proposed a theoretical means of determining the maximum interwire/interlayer slippage between the outer and penultimate layers of an axially preloaded spiral strand undergoing free-bending to a constant radius of curvature with its end clamped. Using this information, in conjunction with well established results in the field of contact stress theory, a parameter was suggested for estimating the free-bending fatigue life of spiral strands, whose initial wire fractures invariably occur at the fixed terminations. Raoof and Huang (1992d) extended the orthotropic sheet model further, by taking the effects of external water pressure exerted on sheathed spiral strands, into account. Numerical results were presented for a 102 mm outside diameter sheathed spiral strand experiencing various levels of mean axial load and external hydrostatic pressure. It was shown that, assuming plane-section bending, the upper and lower bounds to the plane-section bending stiffness are independent of the level of mean axial load and external hydrostatic pressure. The analytical model takes into account the reduction in the strand bending rigidity due to interlayer slippage. The model takes the geometrical non-linearities due to changes in the lay angle and helix radius into account. The effects of the hydrostatic pressure on sheathed spiral strands in conjunction with free-bending fatigue at the terminations was also discussed by Raoof (1993a), who conducted extensive theoretical parametric studies covering the full range of cable (and wire) diameters and lay angles in sealed spiral strands under external hydrostatic pressures, due to depths of water up to 1500 m.

Raoof and Huang (1992i) conducted a series of theoretical parametric studies (based on the orthotropic sheet concept) on a wide range of spiral strand constructions. The parametric studies provided useful information in respect of the various free-bending characteristics of an axially preloaded sheathed spiral strands experiencing external hydrostatic pressure and undergoing cyclic bending movements to a constant radius of curvature. It proved possible to propose rather simple formulations for obtaining estimates of the upper and lower bounds to the plane-section bending stiffness, and also to recommend a simple means of predicting the critical radii of curvature associated with the maximum magnitude of the specific damping. Using the results from the free-bending model, Raoof and Huang (1992e) showed that even for rather small levels of radii of curvature, the plane-section bending assumption could be violated, and interlayer slippage could occur within the helical assembly of the axially

loaded wires. By using an extended version of the orthotropic sheet model it is now possible to obtain reasonable estimates of the gradual reductions in the strand effective bending stiffness associated with changes in the radii of curvature, with geometrical non-linearities and line-contact force effects accounted for.

Hobbs and Raoof (1994) presented a detailed account of the different mechanisms of interwire fretting fatigue that operate inside helical steel cables under different modes of external cyclic loading. In the light of theoretical and experimental findings, the central role of interlayer points of contact, in the context of the cable axial and restrained bending fatigue performance, was emphasised. In addition, simple formulations were presented which enabled the appropriate levels of normal load, and relative displacements, adopted in the twin wire fretting experiments, to be determined. Raoof and Kraincanic (1995e) demonstrated the applicability of the orthotropic sheet model to locked coil cables. A simple means of predicting the axial and plane-section bending stiffnesses of locked coil cables under full-slip conditions was also presented. Raoof (1993a, b) and Raoof and Huang (1994) addressed the problem of designing against free bending fatigue at the points of fixity, with Raoof and Huang providing a contact stress-slip versus fatigue life plot (for in-air conditions) based on a wider range of cable constructions than the one previously reported by Raoof (1993b). The final formulations of Raoof (1993b), based on extensive theoretical parametric studies, provide a simple (hand-based) method for estimating the contact stress-slip parameter, with this parameter shown to control the free bending fatigue life, providing unified plots of contact stress-slip versus fatigue life. Similar design procedures in connection with the free bending fatigue life estimation of sheathed spiral strands, in deep water applications, are, on the other hand, reported by Raoof (1993a).

Costello and Butson (1982) presented a theory capable of predicting the static response of wire ropes subjected to tension, torsion and bending. The results showed that, for a wire rope, the maximum tensile stress occurs in the core wire, which receives the largest axial and bending strain. In their model, interwire friction was neglected and it was also assumed that, when the rope was subjected to an axial force and an axial twisting moment, there was no contact between the central king wire and the outer ones. Another theoretical model, based on a seven wire strand, was

proposed by LeClair (1989). The model was capable of providing an upper bound estimate of the relative motion between the wires in bending, by considering the geometry of a deformed helical wire. An upper bound to the work done by the assumed Coulomb (rigid-plastic) frictional forces in the cable bent over a sheave was determined by considering the components of wire curvature and twist. The analysis addressed the case of a strand in which the outer wires only make contact with the core wire, and the case in which the outer wires touched each other in line-contact, in the absence of any king wire. Raoof and Huang (1991) extended LeClair's model to include a more realistic interwire friction behaviour, with LeClair's simple Coulomb approach replaced by the partial-slip to full-slip contact stress model as originally proposed by Mindlin (1949). The case of a cable bent over a sheave was also analysed by Hobbs and Nabijou (1995), who examined the bending strains in the wires of a frictionless rope as it was bent over a sheave. Their theoretical analysis took account of the fact that the initial and final curvature vectors are not, in general, parallel, and also that the analysis is complicated by the doubly helical nature of the path of a wire, even in a straight rope. It was shown that the bending strain in the wires in the innermost layer was higher than in the wires of the outermost layer. It was also noted that if friction was included, the locations and values of such maximum bending strains could be significantly altered. In a subsequent paper, and by extending their initial analysis, Nabijou and Hobbs (1995a) investigated the frictional behaviour of heavily loaded wire ropes bent over relatively small diameter sheaves, using various wire rope constructions. Nabijou and Hobbs (1995b) also analysed the relative movements between the centre lines of the strands of a wire rope during bending, and the movements between the individual helical wires forming the strands. The position of greatest relative movement was identified, which was shown to coincide with the most common location of wire failures in the fatigue testing of wire ropes bent over sheaves. The highest interwire slip was found to occur between the contacting wires of adjacent strands.

2.10 RECOVERY LENGTH OF FRACTURED WIRES

Chien and Costello (1985) used an analytical model to determine values of the effective (or recovery) length of a fractured wire, where the effective length is the length measured from the fractured end of the wire to the position at which the wire is able to carry its full share of the axial load. Frictionless thin rod theory was used to

determine the magnitudes of the radial contact forces in a simple seven wire strand, and a more complex 6×25 IWRC wire rope. The recovery length of a fractured wire in a wire rope was calculated using the Coulomb type friction, in conjunction with the Saint-Venant's principle. In a seven wire straight strand, the theoretical results indicated that a broken centre wire picks up its appropriate share of the load in less than 1.25 times the pitch of the outer six helical wires. In the wire rope with an IWRC, this was reduced to 1.18 times the pitch of the strands. The effective length was, theoretically, found to be independent of the axial load on the strand. Only the broken centre wire of a strand or the outer layer wire in a rope could be handled by this method.

Raoof (1991d) reported an analytical model for determining the recovery length of a fractured wire in an axially preloaded multi-layered helical strand experiencing, say, axial fatigue loading. The model is, unlike the model of Chien and Costello (1985), capable of following the no-slip to full-slip interwire friction transition along the recovery length. A closed-form solution was developed which provided a simple means of estimating the magnitude of the recovery length as a function of the mean axial load on the cable. It was suggested that, in the absence of any other data, a recovery length of twice the spiral strand pitch should be used. Raoof and Huang (1992g) developed a closed-form solution, based on a highly idealised cable, capable of describing the full-slip to no-slip friction transition along a fractured wire in parallel wire cables experiencing external hydrostatic pressure due to prestressed wire wrapping and/or intermittent cable clamps such as those used for the main cables of suspension bridges.

Raoof and Kraincanic (1993) presented a theoretical model capable of predicting the recovery length in any internal layer of an axially preloaded sheathed spiral strand experiencing external hydrostatic pressure. Theoretical parametric studies were used to determine reliable estimates of the variations in the recovery length in air, for any inner layer of a sheathed spiral strand, with changes in the lay angle. Numerical results showed that the magnitude of the recovery length in sheathed spiral strands could be reduced substantially by subjecting the strand to high levels of external hydrostatic pressure, depending upon the lay angle, and, to a lesser extent, the strand mean axial load.

Raoof and Kraincanic (1995c), using the outcome of the orthotropic sheet theory as regards the patterns of interwire/interlayer contact forces throughout axially preloaded multi-layered spiral strands, developed a theoretical model for predicting the recovery length of a fractured wire in any internal layer of an axially preloaded spiral strand, and showed that the recovery length is a weak function of the cable mean axial load. Results from theoretical parametric studies were used to recommend a simple means of determining the variations of the recovery length with changes in the lay angle for any internal layer of an axially preloaded spiral strand.

2.11 SUMMARY AND CONCLUSIONS

A wide selection of the available literature on spiral strands and wire ropes has been discussed, and certain gaps in the literature have been highlighted. As such, it is the purpose of this section to summarise the key issues raised by the literature review which have led to the research reported in this thesis.

The models used to analyse the behaviour of spiral strands have improved markedly since the pioneering work of Hruska in the 1950's. Costello and his associates, have, over a number of years, carried out a reasonably in-depth study of the static behaviour of single layered spiral strands under tensile, bending, and torsional loading conditions, neglecting the effects of interwire friction. In later publications, they included the effects of interwire friction to study (with little success) the static response of single layered spiral strands to axial, torsional and bending loading conditions. Various other authors have proposed models of varying complexities, to analyse the response of spiral strands: namely, LeClair, Lantaigne, Machida and Durelli, Velinsky, and Kumar and Cochran, to name a few. The problem with these discrete models is that they ignore the problem of interwire contact deformations, which is of paramount importance in strand modelling, and, as a result, they can only accurately predict the behaviour of small (say, seven wire) spiral strands.

To try and overcome the inadequacies of the discrete models, Hobbs and Raoof have developed a semi-continuous approach, known as the orthotropic sheet theoretical model, in which the problem of interwire contact is sufficiently simplified to be mathematically tractable. Another advantage of this semi-continuous model is that as the number of wires in the spiral strand increases (to, say, more than 19), then, the

accuracy of the model increases, as the properties of the strands can be averaged over a greater number of wires. As already mentioned, many discrete models have been proposed to try to analyse the behaviour of spiral strands, although Raoof and his associates have shown the discrete models to be inadequate for analysing large diameter multi-layered spiral strands - hence, the practical value of the semi-continuous models. Indeed, an interesting difference between the two types of approaches is, as shown by Raoof and his associates, that the orthotropic sheet model gives rise to a very nearly symmetrical stiffness matrix over the full range of cable design parameters, whereas the discrete models tend to exhibit varying degrees of asymmetry in the stiffness matrix. Another semi-continuous model has been developed by Jolicoeur and Cardou, which is similar in some respects to the model of Hobbs and Raoof, although essentially different. Both of these semi-continuous models have been found to give superior results (when compared to the classical discrete models), for large diameter multi-layered spiral strands.

Jolicoeur, in a recent publication, compared the two semi-continuous models with various discrete models, and proposed a revision to the model of Hobbs and Raoof, which he claimed gives improved results. It is this proposed modification that forms the basis of the first chapter (chapter 3), in which extensive theoretical parametric studies are used to compare the original model of Hobbs and Raoof with the revised model proposed by Jolicoeur.

The next chapter is concerned with the impact loading of helical cables. Various authors have tried to analyse the response of helical cables to impact loading, with varying degrees of success. Many of the models have ignored the ever presence of interwire friction in the cable, such as those models proposed by Skop and Samras, and Jiang et al., and finally Ringleb who (in addition to neglecting interwire frictional effects) has ignored the possible rotational motion of the ends of the cable.

Raoof and his associates, using the coupled axial/torsional no-slip and/or full-slip matrices, as determined by the orthotropic sheet theory, have developed closed-form solutions for predicting the extensional-torsional wave speeds and displacements in a 39 mm outside diameter axially preloaded multi-layered spiral strand subjected to

specific forms of impact loading – i.e. unit-step and triangular, at one end, with the other end fixed against any rotation.

Chapter 4 takes the analysis of Raoof and his associates further, firstly by considering the response of a 39 mm outside diameter spiral strand to a half-sine impact loading function, and secondly by carrying out a complete analysis, using all the three (i.e. unit-step, triangular and half-sine) types of impact loading functions, on three different 127 mm outside diameter spiral strands with lay angles of 12° , 18° and 24° , so that the effect of variations in the lay angle (i.e. the sole first order geometrical parameter), on the response of a spiral strand to such different types of impact loading is examined in some detail.

The part of the literature survey concerned with the bending of helical cables, both in the free-field and in the vicinity of the terminations, highlighted the fact that the majority of the early works, being almost entirely experimental in nature, are of very limited predictive value as they lack generality.

Three main points have been raised through the literature review in relation to the bending of helical cables. Firstly, early models have assumed that the location of the initial wire fractures, under restrained cyclic bending, is at the extreme fibre position, but experimental measurements by Hobbs and Raoof have demonstrated that the neutral axis is the location of the initial wire fractures under restrained cyclic bending. Raoof and his associates have suggested a method for estimating the maximum interwire/interlayer slippage, which is now accepted to be the controlling factor governing wire fractures at the so-called neutral axis position. Secondly, previous models have assumed that the position of zero lateral deflection is at the cable-socket interface. No experimental or theoretical work has, however, been offered in support of this assumption. Finally, many theoretical formulations have adopted a mathematically convenient constant effective bending stiffness, using which the radii of curvature at the points of fixity have been calculated.

The relative advantages and disadvantages of the different types of terminations have also been critically addressed. It has been found that the use of resin poured sockets leads to significant practical advantages over the traditional terminations, such as zinc

poured sockets, but there is no available information, as far as this author is aware, as to the long term behaviour of the resin poured terminations.

It is the purpose of chapter 5, in conjunction with the experimental measurements of Raoof, and the simple approach proposed by Wyatt, to try and determine as to where the position of zero cable lateral deflection (i.e. the effective point of fixity) lies, as this may have significant practical implications in the theoretical analyses of the termination effects on the fatigue performance of helical cables. The question as to whether the traditional assumption of assuming a constant effective bending stiffness, for determining the minimum radii of curvature, at the points of fixity to the cables undergoing lateral deflections of the order of one cable diameter, is also examined.

Chapter 6 is related to chapter 5, in that they are both concerned with the bending of helical cables. Raoof noted that axially preloaded spiral strands undergo plane-section bending, only for sufficiently small levels of cable lateral deflections. Beyond a certain limit of cable lateral deflection, plane-sections do not remain plane and interlayer slippage takes place, causing some reduction in the strand's effective bending stiffness. In chapter 6, a simple (but reliable) method is proposed for the experimental determination of the effective bending stiffness of practical (large diameter) cables, at reasonable cost and effort, with this method overcoming the rather significant shortcomings of the previously adopted approaches.

Early models used for calculating the axial and torsional stiffnesses have largely been based on seven wire spiral strands, and, therefore, such models have to be used with a certain degree of caution, when applied to practical (large diameter) multi-layered spiral strands. Raoof and his associates have used the orthotropic sheet theoretical model to provide simple methods for estimating the upper and lower bounds to the strand axial stiffness, with the theoretical predictions supported (in a number of cases) by carefully obtained large scale experimental data. Raoof and Kraincanic recently developed a theoretical approach for analysing large diameter wire ropes with either an independent wire rope core (IWRC) or fibre core. Most importantly, encouraging correlations have been found between their theoretical predictions and a fairly large body of experimental data from other sources.

Strzemiecki and Hobbs proposed a general form of Hruska's equation, based on Hruska's theory of the 1950's, by means of which a rough idea of the axial stiffness of a wire rope could be found. The so-obtained results were then compared with the results from the orthotropic sheet theory of Hobbs and Raoof for spiral strands, and also with experimental data for both spiral strands and wire ropes with IWRC. Strzemiecki and Hobbs suggested that the method was best applied to spiral strands, with a modified version required for wire ropes.

The purpose of chapter 7 is to conduct a similar analysis to that of Strzemiecki and Hobbs, but, in relation to wire ropes with either independent wire rope cores (IWRC), or fibre cores (in both regular and Lang's lay configurations). Based on such an approach, a simple method for obtaining the no-slip and/or full-slip axial stiffnesses of the ropes, which is amenable to hand calculations, using a pocket calculator, is developed.

Since the mid-1970's there has been a significant increase in the size of spiral strands used, particularly in the offshore oil industry. The cost of conducting large scale tests is considerable, and the need for a reliable method, capable of predicting, for example, the strand axial fatigue life has (over the last two decades) gained increased importance.

Few researchers have proposed models to try to theoretically predict the axial fatigue life of spiral strands, such as Knapp and his associates, and Starkey and Cress, who ignored the effects of interwire friction. It was not until the purely empirical works of Chaplin, Tilly, and API that design charts were developed which could be used to predict the axial fatigue life of spiral strands.

Raoof conducted extensive theoretical parametric studies (based on his axial fatigue model) on a wide range of spiral strand constructions, resulting in design S-N curves. Comparing the theoretical design charts of Raoof with the purely empirical versions of Chaplin, Tilly, and API has revealed some alarming points. It is also, perhaps, worth mentioning that these authors have all used differing definitions as to what constitutes fatigue failure. In addition, the considerable shortcomings of particularly the API recommended S-N curve are highlighted by Raoof's work.

The design charts produced by Raoof are fairly comprehensive, but of use for only in-air conditions (as far as sheathed spiral strands are concerned). It was felt that conducting theoretical parametric studies to produce design S-N curves which catered for the effects of an external hydrostatic pressure, as found in deep sea applications, on sheathed spiral strands would be of some use: this is the reasoning behind the work reported in chapter 8.

Various authors have carried out experimental damping measurements on large diameter cables (spiral strands and wire ropes), such as Tilly, Roberts, Seppa, and Vanderveldt et al. However, the results from the different tests are often considerably different, in some cases by as much as a factor of even 100. Wyatt (1978), on the other hand, has carried out very careful experimental measurements of the axial energy dissipation in an old and fully bedded-in 39 mm outside diameter spiral strand. Based on the work of Raoof and Hobbs, axial hysteresis may most easily be increased by increasing the magnitude of the lay angle (within current manufacturing limits). In further tests on a newly manufactured 41 mm outside diameter spiral strand, it has been shown by Raoof, that hysteresis measurements on newly manufactured spiral strands could prove to be misleading for long term applications where the helical cable eventually becomes fully bedded-in under the action of external forces of often a random nature, hence, an explanation is offered by Raoof relating to the underlying reason for the observed discrepancies between the damping measurements of Wyatt (which, incidentally, could fairly accurately be predicted by the orthotropic sheet model) and the axial damping test results reported by others.

Based on the orthotropic sheet theoretical approach, Raoof and Hobbs have developed a method for calculating variations in the axial and torsional hysteresis with changes of the associated $\frac{\text{axial load range}}{\text{mean axial load}}$ ratio, and $\frac{\text{range of twist}}{2}$, respectively. Indeed, it has been argued that the use of the traditional Coulomb (rigid-plastic) friction model may give rise to grossly misleading damping predictions under axial, torsional, and bending uniform cyclic loading.

As the damping capacity of spiral strands used in cable-stayed bridges, as inclined hangers in suspension bridges, and as moorings in deep sea applications, contributes

considerably to the overall damping of the structure, it was felt that simple formulations capable of determining the maximum frictional axial and torsional hysteresis, along with the associated $\frac{\text{axial load range}}{\text{mean axial load}}$ ratio, and $\frac{\text{range of twist}}{2}$, respectively, at which they occur, would be of some practical use.

To this end, extensive theoretical parametric studies were carried out on a large number of spiral strands with widely different constructions, covering a wide range of strand mean axial strains and water depths, and simple methods, which are amenable to hand calculations, were developed: this forms the subject of chapter 9.

Another problem that came to light, in the course of the literature search, was the question of size effects. The cost of carrying out laboratory measurements on spiral strands is, as mentioned before, considerable, and, in view of this fact, the majority of previously reported measurements have been carried out on seven (or nineteen) wire spiral strands. The potential uncertainties with using small diameter spiral strands to predict the structural behaviour of much larger diameter cables, in the absence of any sound theoretical basis, is obvious.

By using the construction details for various 127 mm outside diameter spiral strands, Raoof studied the problem of size effects in relation to the axial fatigue life of spiral strands: in his approach, by removing various outer layers, smaller diameter spiral strands were formed, but the lay angles were kept the same. The numerical results based on such a theoretical approach demonstrated fairly small differences between the predictions of the axial fatigue life, provided that a scaling factor of 2-3 is used, with the lay angles being kept the same in the course of the scaling process. Using a theoretical approach, similar to that of Raoof, chapter 10 examines the question of the influence of size effects on the axial and torsional hysteresis, as well as extending the previous work of Raoof in connection with the axial fatigue life of multi-layered spiral strands, to cover the full range of manufacturing limits for the lay angle.

The final chapter (chapter 11) is not directly related to the literature search, but came about because of a book written by Broughton and Ndumbaro (1994), which contained a computer programme for the static analysis of two- or three-dimensional

cable structures with rigid supports. The purpose of this chapter is (by using a realistic example, as described by Irvine (1988)), to analyse the effects of variations of the lay angle (hence, changes in the strand axial stiffness) on the vertical deflection(s) of a two-dimensional cable truss, based on both the no-slip and full-slip regimes, hence, demonstrating the significant practical implications of using the no-slip axial stiffnesses (in preference to the full-slip values) for analysing various characteristics of cable structures under service conditions.

CHAPTER 3

SEMI-CONTINUOUS ANALYSIS OF MULTI-LAYERED SPIRAL STRANDS

3.1 INTRODUCTION

Spiral strands and wire ropes (helical steel cables) have many applications in the field of structural engineering, such as the pre-stressing of concrete, hangers for suspension bridges, the main cables for cable-stayed bridges and stays for guyed masts, as well as the mooring systems for floating offshore platforms. Over the years, the growing need to use larger diameter helical cables (up to, say, 184 mm outside diameter) has necessitated the re-examination of the traditional methods of cable design, which, until the 1970's, were largely based on the commercial past experience relating to the behaviour of much smaller diameter cables.

Several mathematical models are currently available for the prediction of the properties and the response of spiral strands to an external load perturbation. These, range in complexity from Hruska's simple model (1951, 1952 and 1953), in which the wires are assumed to carry only tensile loads, to the relatively more complex model of Costello et al.'s (1976 and 1985) which is based on Love's equations for curved rods. Other models have also been developed by Machida and Durelli (1973), Knapp (1979), and Kumar and Cochran (1987), who proposed mathematical models fairly similar to Costello's model, and, finally, Ramsey (1988) and Lanteigne (1985) who have made an attempt to include interwire friction in their proposed models, with these being basically different from the one proposed by Costello and his associates.

All of the above theoretical models are based on the discrete approach, in which equilibrium and compatibility equations are established for each individual helical wire of the strand. The concept of the discrete approach is relatively straightforward in the sense that the models follow the physical reality.

In 1982, Hobbs and Raoof, later followed by Jolicoeur and Cardou (1994), developed a new approach in which the N layers of helical wires are represented by N concentric orthotropic cylinders (Fig. 3.1) whose properties are chosen to match those of the corresponding layers of helical wires. This approach to spiral strand modelling is

based on a semi-continuous concept. The semi-continuous approach is a little harder to understand as a continuous medium is being used to represent a discontinuous reality. The major advantage of the semi-continuous model is that the problem of interwire contact, inherent in strand modelling, is sufficiently simplified (although hardly easy) to be mathematically tractable.

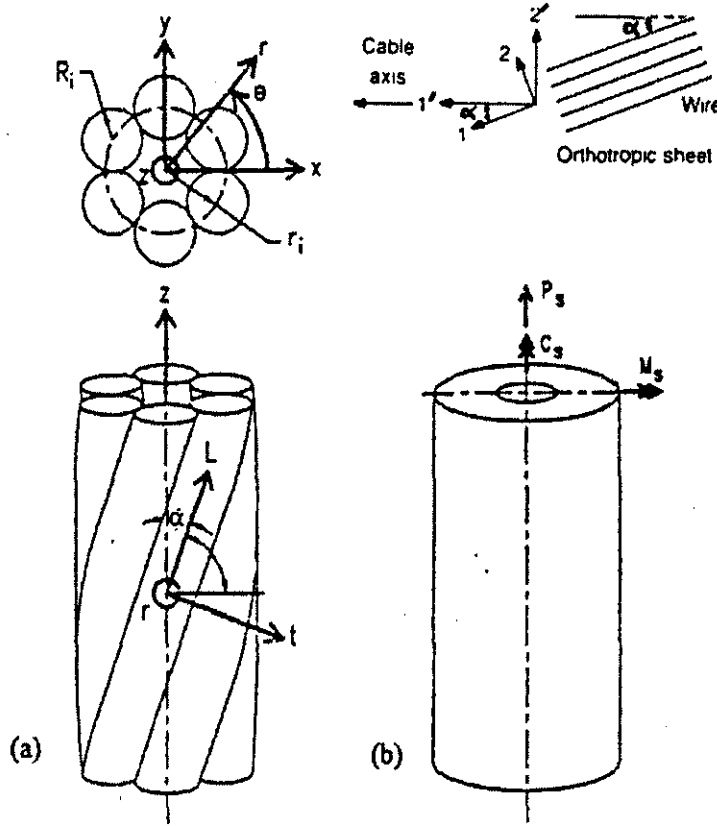


Figure 3.1 – (a) Geometry of a Layer of Spiral Strand; and (b) the Equivalent Orthotropic Cylinder

At present, there are two available semi-continuous modelling approaches for the analysis of multi-layered spiral strands. The first semi-continuous one is known as the orthotropic sheet model and was developed by Hobbs and Raoof (1982), who assumed that the orthotropic layers were thin. It was postulated that each layer of wires in a strand (although discontinuous) has enough wires (more than, say, 19) for its properties to be averaged so that the layer can be treated as an orthotropic sheet, Fig. 3.1b. The elastic properties of the sheets, whose principal axes run parallel and perpendicular to the individual wire axes, are determined (as a function of the external load perturbation) from well established results in the field of contact stress theory.

Then, using the formulations of Hearmon (1961), it is a simple procedure to transform the elastic properties to values parallel and perpendicular to the strand axis. The compatibility equations are initially developed for a strand with its ends fixed against rotation, and assuming that, with zero axial load on the cable, the wires are just touching each other in line-contact. For a counter-laid construction, the stiffnesses in the hoop direction (where the wires are in line-contact) are much greater than the ones in the radial direction. It is this key property that is used to set-up a series of non-linear compatibility equations to obtain the normal forces acting over the various contact patches throughout the structure, Fig. 3.2.

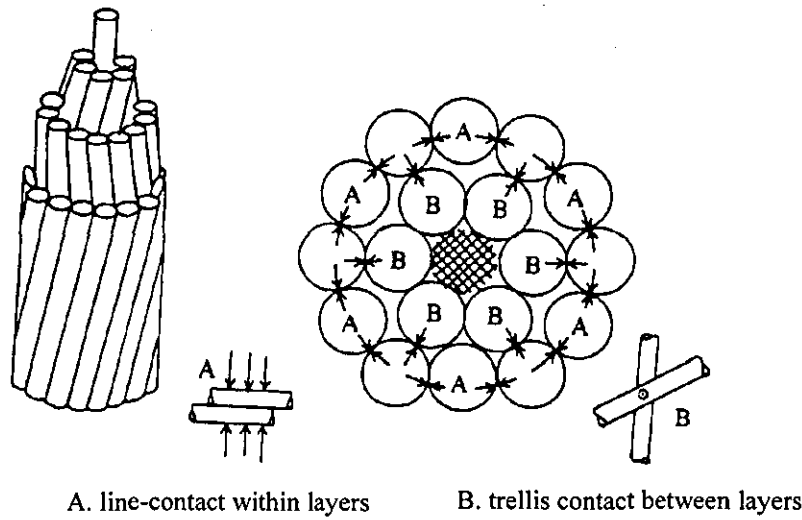


Figure 3.2 – Pattern of Interwire Contact Forces in a Multi-Layered Spiral Strand.

Using this information, interwire movements, changes in wire axial strains and the compliances for an external load perturbation of a given type and size can be found. The axial and/or torsional hysteresis in the strand can also be estimated, and from the properties of the sheets of wires, simple transformations and summation leads to estimates of the axial and torsional stiffnesses.

The second semi-continuous model was developed by Jolicoeur and Cardou (1994 and 1996) for the analysis of steel reinforced aluminium overhead electrical conductors (ACSR). The model is based on a continuum mechanics approach and the elasticity of anisotropic materials with the assumption that the cylinders are thick-walled. This

model makes the problem a tri-dimensional one, whereas the model of Hobbs and Raoof is essentially of a bi-dimensional nature.

This chapter is only concerned with the orthotropic sheet theoretical model of Hobbs and Raoof and the proposed modification to this model by Jolicoeur (1997). The model of Hobbs and Raoof has been covered in great detail elsewhere (Hobbs and Raoof, 1982, and Raoof, 1983) and will not be repeated here. Instead, the main features of the model (especially, those features pertaining to the other chapters in the thesis) will be presented here for completeness.

3.2 THEORY

The orthotropic sheet theoretical model was developed for the analysis of multi-layered spiral strands with counter-laid layers. The theory is based on the main assumption that the gaps between the wires under zero external load are small enough to be neglected. Each layer of wires in a multi-layered spiral strand is treated as a statically indeterminate orthotropic cylinder with a compliant core. The core is, however, assumed to resist the rigid body radial movement which would occur in its absence, due to a change in the lay angle, α (as the axial load changes), causing the wires to assume a closer packing formation.

The model uses a set of compatibility equations to establish the kinematics of a cylinder, and thus, to obtain the cylinder local strains when the global strains (on the strand) are known. Very briefly, these compatibility equations are as follows:

$$S'_{2R} = \left[\frac{\cos^2 \alpha \left(\cos^2 \alpha' + \tan^2 \left(\frac{\pi}{2} - \frac{\pi}{n} \right) \right)}{\cos^2 \alpha' \left(\cos^2 \alpha + \tan^2 \left(\frac{\pi}{2} - \frac{\pi}{n} \right) \right)} \right]^{\frac{1}{2}} - 1 \quad (3.1)$$

where, S'_{2R} = the rigid body radial strain in the strand cross-section, α = the lay angle in the undeformed state, α' = the lay angle after deformation, and n = the number of wires in the layer.

The total radial strain, S'_{2C} , in the cable cross-section (including the rigid body component) is, then, calculated using

$$S'_{2C} = (1 + S'_1) \frac{\tan \alpha'}{\tan \alpha} - 1 - S'_{2R} \quad (3.2)$$

where, S'_1 = the cable axial strain.

From Eqns. (3.1) and (3.2), the radial contraction of the layer in the cable's normal cross-section due to the interwire contact deformations, S'_2 , may be calculated from

$$S'_2 = S'_{2C} - S'_{2R} \quad (3.3)$$

Due to this radial strain, the wires in the corresponding layer experience a slight decrease in their axial strain ($= d\epsilon_h$), where

$$d\epsilon_h = \cos \alpha \left[1 + (1 + S'_{2R})^2 \tan^2 \alpha \right]^{\frac{1}{2}} - 1 \quad (3.4)$$

The tensorial shear strain, S_{6T} , is thus

$$S_{6T} = \frac{1}{2(1 + S_2)} [\tan \alpha (1 + S_1 - d\epsilon_h) - \tan \alpha'] \quad (3.5)$$

and, the wire axial strain, S_1 , is

$$S_1 = \frac{\cos \alpha (1 + S'_1)}{\cos \alpha'} - 1 \quad (3.6)$$

The two-dimensional element in its final deformed state may always be rotated through an angle α' , with the strains on it considered as second order tensors. Therefore

$$S_2 = -\frac{S'_1 - S'_2}{2} \cos 2\alpha' + \frac{S'_1 + S'_2}{2} \quad (3.7)$$

Moreover, for a cable with rotationally fixed ends, the cable axis is coincident with the principal axis of the element, hence

$$S_{6T} = \left(\frac{S'_1 - S'_2}{2} \right) \sin 2\alpha' \quad (3.8)$$

For a given cable axial strain, S'_1 , Eqns. (3.1)-(3.8) are solved as a set of non-linear simultaneous equations where the eight unknowns are S'_{2R} , S'_{2C} , S'_2 , d_{eh} , S_{6T} , S_1 , S_2 and α' . The equations are solved by treating S_1 as the primary unknown and using the Newton-Raphson process of iteration, with the derivatives approximated by central finite differences. Once the difference between S_{6T} as calculated by Eqns. (3.5) and (3.8) becomes negligible, then the correct value of S_1 has been found, and the kinematical equations provide a set of compatible strains in the anisotropic cylinder with a rigid core.

3.2.1 Strand Radial (clench) and Circumferential (hoop) Contact Forces

In a multi-layered spiral strand, the radial force exerted on any layer is due, in part to the radial body forces in that layer, in addition to the clenching effects of the outer layers (Hobbs and Raoof, 1982). The magnitude of the radial force grows inwards, starting with the outermost layer.

In layer i , the radial force, X_{RCi} , acting as a body force in the wires of that layer, can be found directly, assuming that the wire axial strains, $S_{1,i}$, are known (Hobbs and Raoof, 1982)

$$X_{RC,i} = \pi \frac{D_i^2}{4} E S_{1,i} \frac{\sin^2 \alpha_i}{r_i} \quad (3.9)$$

where, D_i = the wire diameter, α_i = the lay angle, and r_i = the helix radius of layer i .

The contact forces in the hoop direction in layer i , $P_{RC,i}$, are given implicitly by Hobbs and Raoof (1982) as

$$S_{2,i}D_i = \frac{4P_{RC,i}(1-\nu^2)}{\pi E} \left(\frac{1}{3} + \ln \frac{D_i}{b_i} \right) \quad (3.10)$$

where, the total width of the rectangular contact area in layer i , $2b_i$, is given by

$$2b_i = 1.6 \left(\frac{P_{RC,i}D_i(1-\nu^2)}{E} \right)^{\frac{1}{2}} \quad (3.11)$$

In the above, ν = the steel Poisson's ratio, and E = the Young's modulus for steel. The angle, β , which locates the lines of action of the line-contact forces, $P_{RC,i}$, is given by Phillips and Costello (1973) as, Fig. 3.3

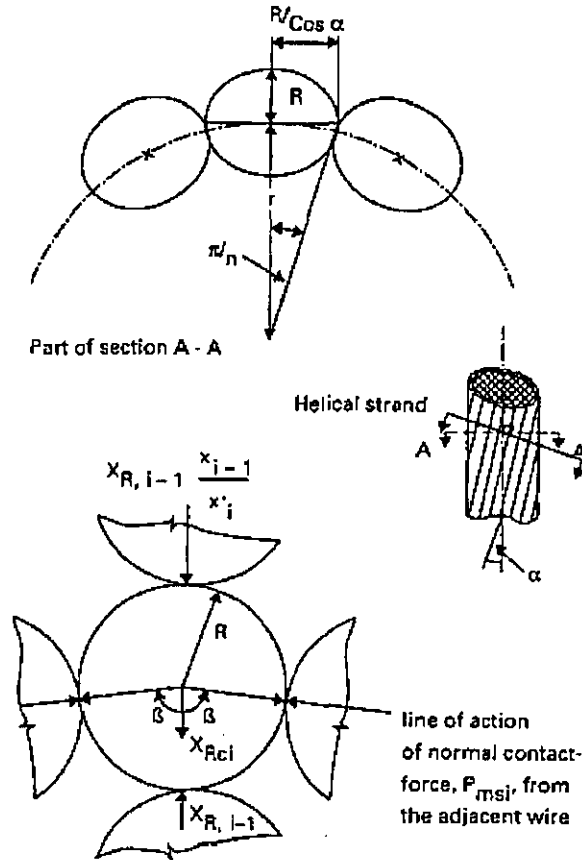


Fig. 3.3. – Contact Forces Acting Within a Spiral Strand.

$$\cos\beta_i = \frac{1}{\sin^2\alpha_i} \left\{ \left[1 + \frac{\tan^2\left(\frac{\pi}{2} - \frac{\pi}{m_i}\right)}{\cos^2\alpha_i} \right]^{\frac{1}{2}} - \left[\tan^2\left(\frac{\pi}{2} - \frac{\pi}{m_i}\right) \times \left[1 + \frac{1}{\cot^2\alpha_i \cos^2\left(\frac{\pi}{2} - \frac{\pi}{m_i}\right) \left[\cos^2\alpha_i + \tan^2\left(\frac{\pi}{2} - \frac{\pi}{m_i}\right) \right]} \right]^{\frac{1}{2}} + \cos^4\alpha_i \right]^{\frac{1}{2}} \right\} \quad (3.12)$$

For the outer layer (layer 1), the hoop forces are a function of the clench forces generated in the helical wires so that the ratios of P_{RC1}/X_{RC1} and P_{MS1}/X_{MS1} are identical, where, P_{MS1} = the line-contact forces in layer 1 (outer), and X_{MS1} = the radial force per unit length of helical wire in that layer.

For the other (inner) layers, the additional clench force provided by each wire in layer i acting on layer $j = i+1$, X_{Ri} , is given by

$$X_{Ri} = X_{MS,1} - 2P_{MS,i}\cos\beta_i \quad (3.13)$$

With the total radial force, $X_{MS,j}$, experienced by each wire in layer j given by

$$X_{MS,j} = E \frac{\pi D^2}{4} \frac{S_{l,j} \sin^2\alpha_j}{r_j} + X_{R,i} \frac{n_i}{n_j} \quad (3.14)$$

where, n_i and n_j = the number of wires in the two layers i and $j = i + 1$.

Using the previously calculated P_{RC}/X_{RC} data for layer $i+1$, it is then possible to find the corresponding values for P_{MS} and X_{MS} for layer $i+1$ (Hobbs and Raoof, 1982). The process is then repeated, moving in another layer each time, until the whole strand has been analysed.

3.2.2 Properties of the Orthotropic Sheet

The properties of the orthotropic sheet are derived to represent a layer of wires in terms of the principal axes 1 and 2, parallel and perpendicular to the wire axes, respectively, Fig. 3.1b. Using Hearmon's (1961) notation, the stresses T'_i and the engineering strains S'_i referred to axes at an angle α to the principal axes are related by

$$\begin{aligned} S'_1 &= S'_{11} T'_1 + S'_{12} T'_2 + S'_{16} T'_6 \\ S'_2 &= S'_{12} T'_1 + S'_{22} T'_2 + S'_{26} T'_6 \\ S'_6 &= S'_{16} T'_1 + S'_{26} T'_2 + S'_{66} T'_6 \end{aligned} \quad (3.15 \text{ a-c})$$

where, assuming $\cos \alpha = m$ and $\sin \alpha = n$

$$\begin{aligned} S'_{11} &= m^4 S_{11} + 2m^2 n^2 S_{12} + n^4 S_{22} + m^2 n^2 S_{66} \\ S'_{12} &= m^2 n^2 S_{11} + (m^4 + n^4) S_{12} + m^2 n^2 S_{22} - m^2 n^2 S_{66} \\ S'_{16} &= -2m^3 n S_{11} - 2mn(m^2 - n^2) S_{12} + 2mn^3 S_{22} + mn(m^2 - n^2) S_{66} \\ S'_{22} &= n^4 S_{11} + 2m^2 n^2 S_{12} + m^4 S_{22} + m^2 n^2 S_{66} \\ S'_{26} &= -2mn^3 S_{11} - 2mn(m^2 - n^2) S_{12} + 2m^3 n S_{22} - mn(m^2 - n^2) S_{66} \\ S'_{66} &= 4m^2 n^2 S_{11} - 8m^2 n^2 S_{12} + 4m^2 n^2 S_{22} + (m^2 - n^2)^2 S_{66} \end{aligned} \quad (3.16 \text{ a-f})$$

In the above equations, subscripts 1, 2 and 6 refer to the directions parallel, perpendicular, and tangential to the wire axes, respectively.

The compliance of the sheet in the direction parallel to the wire axes, S_{11} , can be expressed as the ratio of the sheet area to the wire cross-sectional area, divided by the Young's modulus of the wire material

$$S_{11} = \frac{4}{\pi E} \quad (3.17)$$

The compliance S_{12} , is

$$S_{12} = -\nu S_{11} \quad (3.18)$$

From Hobbs and Raoof (1982), the individual wires are treated as parallel cylinders in line-contact, with their normal compliances, S_{22} , given by (Roark and Young, 1975)

$$S_{22} = \frac{1}{E} \left(0.11 + 0.59 \ln \frac{ED}{P_{MS}} \right) \quad (3.19)$$

The above formula has been derived assuming a Poisson's ratio $\nu = 0.28$. The corresponding tangential compliance, S_{66} , is given by

$$S_{66} = \frac{S_{22}}{1-\nu} \left(1 - \frac{\Delta_{\ell}}{2\Delta_{\ell \max}} \right)^{-\frac{1}{2}} \quad (3.20)$$

where, $2\Delta_{\ell \max}$ is the amplitude of the sliding corresponding to uniform cyclic loading, with reversed tangential displacements taking place. The limiting displacement, $\Delta_{\ell \max}$, is equal to

$$\Delta_{\ell \max} = \frac{3\mu P_{MS}}{4(1-\nu)} S_{22} \quad (3.21)$$

with $\Delta_{\ell \max}$ denoting the tangential displacement between the centre lines of the wires in line-contact at the onset of gross sliding (full-slip).

Eqn. (3.21) is only valid for $\Delta_{\ell} \leq 2\Delta_{\ell \max}$, with S_{66} becoming infinite at the limit. For larger Δ_{ℓ} , S_{66} can be taken as infinitely large, corresponding to the case of full-slip (gross-sliding), whereas for the no-slip case $\Delta_{\ell} = 0$.

3.2.3 Axial Stiffness

Hobbs and Raoof (1982) assumed that a change in the strand axial stress, T'_1 , was associated with a perturbation of the hoop stress, T'_2 , in the same ratio as the stresses in the local co-ordinate system of the layer, such that

$$\frac{T'_2}{T'_1} = \frac{T_2}{T_1} \quad (3.22)$$

It was further assumed that, with both ends fixed against rotation, the shear strain, S'_6 , was equal to zero. This assumption leads to the desired axial flexibility of the membrane, as obtained from Eqns. 3.16a and c:

$$\frac{S'_1}{T'_1} = S'_{11} + c_k S'_{12} - \frac{S'_{16}}{S'_{66}} (S'_{16} + c_k S'_{26}) \quad (3.23)$$

where

$$c_k = \frac{T_2}{T_1} \quad (3.24)$$

with the strand axial flexibility $\frac{S'_1}{T'_1}$ being a function of the strand axial strain, S'_1 .

For a given S'_1 , c_k is equal to the slope of the T_2 versus T_1 plots. Numerical differentiation is then used to obtain the value of c_k (always negative) for a given mean axial strain on the strand, as the plots of T_2 versus T_1 are non-linear.

The calculation of the axial stiffness depends upon which limiting slip condition is assumed: for the limiting condition of full-slip, the following procedure is used (Raoof, 1983)

- (1) For a given mean axial strain, calculate S_{11} , S_{12} , S_{22} and $\Delta_{\ell \max}$ using Eqns. (3.17, 3.18, 3.19 and 3.20)
- (2) Using the following equation, calculate the tangential relative displacement between the centres of the wires in line-contact, Δ_t , where

$$\Delta_r = 2DS_{6T} \quad (3.25)$$

- (3) For each value of Δ_r , calculate the corresponding value of S_{66} , using Eqn. (3.20)
- (4) Then, for each value of S_{66} and for the various levels of strand axial strain perturbations, obtain the changes in S'_{1/T_1} , using Eqn. (3.23)
- (5) T'_1 is, finally, integrated with respect to the strand axial strain perturbation, dS'_1 , to obtain values of the axial force for each layer as a function of the changes in dS'_1 . Summation over the layers gives the strand axial force change as a function of dS'_1 .

Following the development of slip in the various layers, a different procedure is used (Raoof, 1983)

- (1) For a known strand mean axial strain, calculate c_k (Eqn. 3.24) and, hence, S_{22} (Eqn. 3.19)
- (2) For a range of strand axial strains calculate the values of S_{66} (Eqn. 3.20)
- (3) Then, for each value of S_{66} and for various levels of strand axial strain perturbations, obtain the changes in S'_{1/T_1} , using Eqn. (3.23)
- (4) T'_1 is, finally, integrated with respect to the strand axial strain perturbation dS'_1 , to obtain values of the axial force for each layer, as a function of the changes in dS'_1 . Summation over the layers gives the strand axial force change as a function of dS'_1 .

3.2.4 Torsional Stiffness

On applying a torque increment to an axially pre-loaded spiral strand, Raoof and Hobbs (1989) postulated that the changes in the axial and hoop strains were zero (i.e.

S'_1 and $S'_2 = 0$). From Eqns. (3.15 a – c), the tangential shear flexibility $\frac{S'_6}{T'_6}$ was,

then, obtained as

$$\frac{S'_6}{T'_6} = S'_{66} + \frac{2S'_{12}S'_{16}S'_{26} - S'_{22}S'^2_{16} - S'_{11}S'^2_{26}}{S'_{11}S'_{22} - S'^2_{12}} \quad (3.26)$$

If S'_1 and S'_2 are zero, the shear strain increment in the layer, S'_6 , is simply

$$S'_6 = r d\phi \quad (3.27)$$

where, $d\phi$ is the applied twist increment per unit length, and r is the helix radius of the wire under consideration, given by (Phillips and Costello, 1973)

$$r_i = \frac{D_i}{2} \left[1 + \frac{\tan^2 \left(\frac{\pi}{2} - \frac{\pi}{n_i} \right)}{\cos^2 \alpha_i} \right]^{\frac{1}{2}} \quad (3.28)$$

Integrating both sides of Eqn. (3.26) gives the total shear strain, which is the sum of the increments - i.e. $r\phi$. With reference to the wire axes, the total shear strain is

$$S' = r\phi \cos 2\alpha \quad (3.29)$$

and, hence, the tangential displacement between the wires, δ , is

$$\delta = Dr\phi \cos 2\alpha \quad (3.30)$$

The procedure for calculating the torsional stiffness is as follows (Raoof and Hobbs, 1989)

- (1) Calculate S_{11} , S_{12} , S_{22} , $\Delta_{\tau \max}$ and S'_6 (Eqns. 3.17, 3.18, 3.19, 3.21 and 3.27, respectively) by assuming τ and $d\phi = 0$
- (2) Set $\phi = \phi + d\phi$
- (3) From Eqn. (3.30) calculate δ , and from Eqns. (3.15 a – c) calculate S_{66}
- (4) Transform the compliances using Eqns. (3.16 a – f)
- (5) Using Eqn. (3.26), calculate the shear stress increment, T'_6 , and the total shear stress, using

$$\tau = \tau + T'_6 \quad (3.31)$$

(6) Return to step (2) as desired.

From the above procedure, it is, then, another simple procedure to calculate the torque generated by a layer, for a particular twist per unit length, using

$$M_i = \tau A r \quad (3.32)$$

The total torque generated by all the layers is given by

$$M = \sum_{i=1}^N M_i \quad (3.33)$$

An estimate of the axial force change induced by an externally applied torque can also be obtained. The formulations are based on the assumption that the induced axial force in a given layer is not large enough to significantly alter the line-contact normal forces in the layer resulting from the initial axial pre-load on the strand. Once again, assuming S'_1 and S'_2 are zero, Eqns. (3.15 a – c) give

$$\frac{S'_6}{T'_1} = \frac{S'^2_{26} S'_{22} - 2S'_{16} S'_{26} S'_{12} + S'^2_{26} S'_{11} + S'^2_{12} S'_{66} - S'_{66} S'_{22} S'_{11}}{S'_{16} S'_{22} - S'_{26} S'_{12}} \quad (3.34)$$

where, T'_1 is the axial stress increment.

3.2.5 Hysteresis

The procedure for calculating the hysteresis has already been given in great detail by Raoof (1983), but due to its relevance to later chapters, the main features will be repeated here.

For both, the axial and torsional hysteresis, the orthotropic sheet theoretical model of Hobbs and Raoof provides two different methods for the prediction of the frictional

energy dissipated under continued uniform cyclic axial or torsional loading. The two methods provide a mutual check.

The first method (method a) combines the skew-symmetric axial (or torsional) load-displacement diagrams for the loading and subsequent unloading phases of a uniform cyclic axial (or torsional) movement forming the hysteresis loop for a given axial pre-load on the strand.

The second method (method b) assesses the energy dissipation per cycle on each of the line-contact patches within the strand, summation yields a value for the total strand axial or torsional hysteresis. Most importantly, the overall frictional hysteresis is assumed to be as a result of the energy dissipation over the line-contact patches throughout the spiral strand, with the small contribution from the torsional hysteresis over the trellis points of interlayer contact neglected.

Method (a): Here, the load-displacement plots for the loading and subsequent unloading of a spiral strand form a skew-symmetric hysteresis loop. It follows that the energy dissipation per cycle, for each loading level, may be obtained by finding the area enclosed by the loading curves. The area enclosed by the loading and subsequent unloading curves can be found by the methods employed in section 3.2.3 for the axial case, and section 3.2.4 for the torsional case.

Method (b): For the pure tangential loading of two non-spherical bodies (cylinders) in line-contact, the energy dissipation, ΔE , for the partial slip regime, is given by

$$\Delta E = \frac{18 \mu^2 p^2}{5 (1-\nu)} S_{22} \times \left[1 - \left(1 - \frac{T^*}{\mu p} \right)^{\frac{5}{3}} - \frac{5 T^*}{6 \mu p} \right. \\ \left. \times \left\{ 1 + \left(1 - \frac{T^*}{\mu p} \right)^{\frac{2}{3}} \right\} \right] \quad (3.35)$$

where, $T^*/\mu p$ is defined by the following

$$\frac{T^*}{\mu p} = \left\{ 1 - \left(1 - \frac{\Delta_\ell}{\Delta_{\ell \max}} \right)^{\frac{3}{2}} \right\} \quad (3.36)$$

with $\Delta_{\ell \max}$ given by Eqn. (3.20).

Once $T^*/\mu p$ reaches unity (i.e. $\frac{\Delta_\ell}{\Delta_{\ell \max}} = 1$), then, gross sliding takes place and the energy dissipation, ΔE , is given by

$$\Delta E = \frac{3}{5} \frac{\mu^2 p^2}{(1-\nu)} S_{22} + 4\mu p (\Delta_\ell - \Delta_{\ell \max}) \quad (3.37)$$

3.2.5.1 Axial Hysteresis.

For the case of axial hysteresis, the slip over the line-contact patch area, Δ_ℓ , is determined by

$$\Delta_\ell = 2kDS'_1 \quad (3.38)$$

with the energy dissipation in the strand per unit length given by

$$\Delta U = \sum_i \left(\frac{n\Delta E}{\cos \alpha_i} \right) \quad (3.39)$$

and, finally, the energy input per cycle per unit length is

$$U = \frac{1}{2} \left(\frac{\text{axial load range}}{2} \right) S'_1 \quad (3.40)$$

The procedure for calculating the axial hysteresis is as follows:

- (1) For a given mean axial load, find the change in S'_1 , for any assumed axial load perturbation, using the procedure outlined in section 3.2.3

- (2) Using Eqns. (3.21) and (3.41) check to see whether $\Delta_{\ell}/\Delta_{\ell\max}$ is less than one or not.
- (3) If $\Delta_{\ell}/\Delta_{\ell\max} < 1$ (partial slip regime), calculate the energy dissipated per cycle per unit length, using Eqns. (3.35) and (3.36),
 If $\Delta_{\ell}/\Delta_{\ell\max} = 1$ (full-slip regime), then, the energy dissipated per cycle per unit length is calculated using Eqns. (3.37) and (3.38).
- (4) The energy loss ratio, as a function of the axial load range to mean axial load ratio, is obtained by using Eqns. (3.39) and (3.40).

3.2.5.2 Torsional Hysteresis

For the case of torsional hysteresis, the slip over one body is given by

$$\Delta_{\ell} = DS'_6 \cos \alpha \quad (3.41)$$

where, $S'_6 = r \frac{d\phi}{d\ell}$

with $\frac{d\phi}{d\ell}$ corresponding to half of the perturbation range, and the energy input per cycle per unit length is

$$U = \frac{1}{2} \left(\frac{M}{2} \right) \frac{d\phi}{d\ell} \quad (3.42)$$

The procedure for calculating the torsional hysteresis is:

- (1) For a known mean axial load, use the procedure in section 3.2.4 to find the change $2 \frac{d\phi}{d\ell}$ corresponding to any assumed torque perturbation, M.
- (2) Same as (2) for the axial case
- (3) Same as (3) for the axial case, but use Eqn. (3.38) instead of Eqn. (3.41)

- (4) The energy loss ratio, as a function of the twist for a given mean axial load, is obtained by using Eqns. (3.39) and (3.42).

3.2.6 Axial/Torsional Coupling

As discussed by Raoof (1991a), for the two extreme cases of either no-slip or full-slip, the constitutive equations relating the strand tension (F) and torque (M) to the cable deformations, may be postulated to be of the general linearized form

$$\frac{F}{E_s} = A_1 \varepsilon_c + A_2 \tau \quad (3.43)$$

$$\frac{M}{E_s} = A_3 \varepsilon_c + A_4 \tau \quad (3.44)$$

where, A_1 , A_2 , A_3 and A_4 are the overall (strand) stiffness coefficients, which are dependent upon both the cable material and construction, and are obtained by summation of the layer stiffnesses. In the above equations, ε = the axial strain, and τ = the twist per unit length, with

$$A_1 = \sum_{i=1}^N [A_{ni} k_{1i}] + A_{core}$$

$$A_2 = \sum_{i=1}^N A_{ni} r_i k_{2i}$$

$$A_3 = \sum_{i=1}^N A_{ni} r_i k_{3i}$$

$$A_4 = \sum_{i=1}^N A_{ni} r_i^2 k_{4i} \quad (3.45 \text{ a - d})$$

while, the stiffness coefficients $k_{1i} - k_{4i}$, for layer i , are defined by

$$k_{1i} = \frac{4}{\pi E} \left(\frac{T'_1}{S'_1} \right)_i$$

$$k_{2i} = \frac{4}{\pi E} \left(\frac{T'_1}{S'_6} \right)_i$$

$$k_{3i} = \frac{4}{\pi E} \left(\frac{T'_6}{S'_1} \right)_i$$

$$k_{4i} = \frac{4}{\pi E} \left(\frac{T'_6}{S'_6} \right)_i \quad (3.46 \text{ a - d})$$

k_1 and k_4 are always positive, irrespective of the direction of lay. However, k_2 and k_3 are negative for left hand lay, and positive for right hand lay. In the above equations, the area of the king wire, A_{core} , is given by

$$A_{\text{core}} = \frac{\pi D_{\text{core}}^2}{4} \quad (3.47)$$

or, for an equal lay core construction consisting of n_c layers (excluding the king wire) with m_i being the number of wires in layer i of the equal lay core

$$A_{\text{core}} = \frac{\pi D_{nc+1}^2}{4} + \sum_{i=1}^{n_c} m_i \frac{\pi}{4} D_i^2 \cos^3 \alpha_i \quad (3.48)$$

The net steel area for layer i in the strand cross-section, A_{ni} , on the other hand, can be calculated using the following

$$A_{ni} = m_i \pi \frac{D_i^2}{4} \sec \alpha_i \quad (3.49)$$

where, it is assumed that the wire cross-sections in the strand normal cross-section are elliptical. The corresponding gross area of the layer used in the orthotropic sheet theoretical model is thus

$$A_{gi} = \frac{4}{\pi} A_{ni} \quad (3.50)$$

From Eqns. (3.42) and (3.43), the strand stiffness matrix is

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \quad (3.51)$$

It should be noted at this point that certain previously available theoretical models lead to a significant margin of error between the two constitutive constants, A_2 and A_3 . It is encouraging to note that the orthotropic sheet model does, indeed, find that (Raouf, 1991c)

$$A_2 \cong A_3 \quad (3.52)$$

which is compatible with the Maxwell-Betti reciprocal theorem for linear elastic structures.

When the forces acting on the strand are known (but not the deformations), it is useful to calculate the inverse of the stiffness matrix, A^{-1} , which is

$$A^{-1} = \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix} \quad (3.53)$$

with the corresponding stiffness equations given as

$$\epsilon = C_1 \frac{F}{E_s} + C_2 \frac{M}{E_s} \quad (3.54)$$

$$\tau = C_3 \frac{F}{E_s} + C_4 \frac{M}{E_s} \quad (3.55)$$

3.2.7 Proposed Modifications to the Orthotropic Sheet Theoretical Model

In a recent publication, Jolicoeur (1997) proposed a slight modification to the orthotropic sheet theoretical model of Hobbs and Raoof (1982). The proposed modification involved the way in which the parameter c_k was calculated.

In the original formulations of Hobbs and Raoof [section 3.2.3, Eqns. (3.22)-(3.24)], numerical differentiation was used to obtain the (invariably, negative) value of c_k , which was equal to the slope of the T_2 versus T_1 plots at a given cable mean axial strain.

Jolicoeur (1997) stated that, in the original formulations, Hobbs and Raoof had not clarified why this relatively complex method had been used to evaluate c_k , and, thus, proposed, as he put it, a more rigorous and simpler method. It was proposed that, if the tensorial properties of the stresses are known in the layer (i.e. local) system of co-ordinates, they may be obtained in the strand system of co-ordinates by tensorial rotation. The value of c_k would, then, be calculated using the following equation (Jolicoeur, 1997)

$$c_k = \frac{T_2 \cos^2 \alpha + T_1 \sin^2 \alpha}{T_2 \sin^2 \alpha + T_1 \cos^2 \alpha} \quad (3.56)$$

In his publication, Jolicoeur does not, however, explain how the values of T_1 and T_2 are to be calculated. In the light of this omission, it will be assumed in what follows, that the values of T_1 and T_2 will be calculated using the method proposed by Hobbs and Raoof (1982) with these parameters, then, inserted into Eqn. (3.56).

3.3 ANALYSIS

In order to distinguish between the two different approaches, the original method of Hobbs and Raoof will be referred to as the RH model, and the model containing the modifications proposed by Jolicoeur will be hereafter referred to as the RH₂ model.

In his paper, Jolicoeur carried out a numerical evaluation of the two (i.e. RH and RH₂) models, amongst others, by assessing the strand axial stiffness (EA)_s, the torsional stiffness (GJ)_s, and the overall tension-torsion coupling coefficients, A₂ and A₃, for a seven wire spiral strand and a Drake 26/7 ACSR multi-layered strand with an outside diameter of 28 mm. The analysis was carried out assuming a nominal strand axial strain, S'₁, of 0.1%.

The comparative analysis conducted by the present author takes the analysis of Jolicoeur a stage further. By conducting an extensive series of theoretical parametric studies, the layer stiffness coefficients, k₁ – k₄, for a wide variety of strand constructions (Tables 3.1a - *ℓ*) and cable axial strains, S'₁, will be obtained.

In particular, the presently reported detailed analysis also covers three different 127 mm outside diameter spiral strands with lay angles of 12°, 18° and 24°, with nominally similar values for the other geometrical parameters (such as the number of wires and their diameters). By using these three spiral strands, the influence of the lay angle, which has been found to be the controlling geometrical parameter affecting a strand's behaviour (Raoof 1997), may be explored fully. Once again, the analysis compares the layer stiffness coefficients, k₁ – k₄, the overall strand axial and torsional coupling coefficients, A₂ and A₃, the axial frictional hysteresis, and the overall frictional torsional damping, as predicted by both the RH and the RH₂ models.

In the present analysis, the following values of the various parameters are used: Young's modulus for steel E_{steel} = 200 kN/mm², the coefficient of interwire friction μ = 0.12, and the steel Poisson's ratio ν = 0.28, with the various spiral strand construction details given in Tables 3.1a – *ℓ*.

Table 3.1a – Construction Details for the 16.4 mm Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A _{ni} (mm ²)	Gross Steel Area A _{gi} (mm ²)
1	12	RH	3.25	11.91	6.41	101.739	129.539
2	6	LH	3.25	11.42	3.3	50.780	64.655
Core	1	-	3.594	-	-	-	10.145
$A_{core} = 10.145 \text{ mm}^2$						$A_g = 194.194 \text{ mm}^2$	

Table 3.1b – Construction Details for the 39 mm Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	30	RH	3.54	17.74	17.73	310.010	394.717
2	24	LH	3.54	16.45	14.10	246.297	313.595
3	18	LH	3.54	15.93	10.57	184.236	234.577
4	12	RH	3.54	14.90	7.04	122.217	155.611
5	7	RH	3.54	15.42	4.19	71.469	90.997
King	1	-	5.05	-	-	-	20.030

$$A_{core} = 20.030 \text{ mm}^2$$

$$A_g = 1189.497 \text{ mm}^2$$

Table 3.1c – Construction Details for the 41 mm Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	24	RH	4.57	12.45	17.93	403.151	513.308
2	18	LH	4.57	11.96	13.45	301.805	384.270
3	12	RH	4.57	11.25	8.99	200.692	255.529
	6	RH	3.43	7.00	-	-	54.210
Core	6	RH	3.43	7.70	-	-	32.694
	6	RH	3.38	4.00	-	-	53.444
	1	-	3.38	-	-	-	8.973

$$A_{core} = 149.320 \text{ mm}^2$$

$$A_g = 1153.107 \text{ mm}^2$$

Table 3.1d – Construction Details for the 45 mm Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	54	RH	2.36	18.01	21.34	248.385	316.254
2	48	LH	2.36	18.01	18.97	220.787	281.115
3	42	LH	2.36	18.01	16.60	193.189	245.975
4	37	RH	2.36	18.01	14.63	170.190	216.693
5	31	LH	2.36	18.01	12.26	142.592	181.553
6	25	RH	2.36	18.01	9.89	114.993	146.414
7	19	LH	2.36	18.01	7.53	87.395	111.275
8	14	RH	2.28	18.01	5.37	60.104	76.527
	7	-	1.44	13.07	-	-	10.537
Core	7	-	1.88	12.20	-	-	18.144
	7	-	1.96	7.62	-	-	20.566
	1	-	2.65	-	-	-	5.515

$$A_{core} = 54.763 \text{ mm}^2$$

$$A_g = 1575.806 \text{ mm}^2$$

Table 3.1e– Construction Details for the 51 mm Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	36	RH	4.01	12.83	23.59	466.296	593.707
2	30	LH	4.01	12.63	19.65	388.274	494.365
3	24	LH	4.01	12.35	15.72	310.283	395.064
4	18	RH	4.01	11.94	11.79	232.354	295.842
5	12	RH	4.01	11.14	7.89	154.462	196.667
	6	-	2.31	7.74	-	-	24.465
Core	6	-	3.00	6.71	-	-	41.546
	6	-	2.84	4.05	-	-	37.714
	1	-	3	-	-	-	7.069

$$A_{core} = 110.793 \text{ mm}^2$$

$$A_g = 1975.646 \text{ mm}^2$$

Table 3.1f– Construction Details for the 52 mm Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	42	RH	3.38	20.93	24.20	403.476	513.721
2	36	LH	3.38	20.68	20.72	345.264	439.603
3	31	LH	3.38	18.99	17.66	294.164	374.541
4	24	RH	3.53	18.30	14.23	247.394	314.992
5	18	LH	3.53	20.25	10.81	187.767	239.073
6	15	RH	3.00	19.78	7.65	112.677	143.465
7	9	LH	3.00	24.50	4.77	69.912	89.015
8	3	RH	3.38	20.95	1.99	28.824	36.699

$$A_{core} = 0.00 \text{ mm}^2$$

$$A_g = 2151.109 \text{ mm}^2$$

Table 3.1g– Construction Details for the 63 mm Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	35	RH	5.00	13.99	28.735	708.231	901.748
2	29	LH	5.00	14.007	23.823	586.863	747.218
3	23	RH	5.00	14.006	18.912	465.441	592.618
4	17	LH	5.00	14.004	14.008	344.019	438.018
5	11	RH	5.00	14.00	9.124	222.597	283.419
6	5	LH	5.00	13.985	4.339	101.174	128.818
7	1	-	3.95	-	-	12.254	15.603

$$A_{core} = 0.00 \text{ mm}^2$$

$$A_g = 3107.442 \text{ mm}^2$$

Table 3.1h – Construction Details for the 127 ($\alpha = 12^\circ$) mm Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	56	RH	6.60	12.00	60.17	1958.671	2493.857
2	50	LH	6.60	12.00	53.73	1748.813	2226.658
3	44	LH	6.60	12.00	47.29	1538.955	1959.459
4	38	RH	6.60	12.00	40.85	1329.098	1692.260
5	32	LH	6.50	12.00	33.89	1085.581	1382.204
6	26	RH	6.50	12.00	27.56	882.034	1123.041
7	20	LH	6.50	12.00	21.23	678.488	863.878
8	14	RH	6.60	12.00	15.15	489.668	623.464
	7	-	4.00	8.57	-	-	85.051
Core	7	-	5.20	8.03	-	-	144.330
	7	-	5.20	4.92	-	-	147.023
	1	-	7.10	-	-	-	39.592

$$A_{core} = 415.996 \text{ mm}^2$$

$$A_g = 12364.821 \text{ mm}^2$$

Table 3.1i – Construction Details for the 127 ($\alpha = 18^\circ$) mm Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	54	RH	6.55	18.0	59.22	1913.307	2436.098
2	48	LH	6.55	18.0	52.64	1700.717	2165.420
3	42	LH	6.55	18.0	46.07	1488.127	1894.743
4	36	RH	6.55	18.0	39.50	1275.538	1624.065
5	31	LH	6.55	18.0	34.02	1098.380	1398.500
6	25	RH	6.55	18.0	27.46	885.790	1127.823
7	19	LH	6.55	18.0	20.90	673.200	857.145
8	14	RH	6.30	18.0	14.85	458.899	584.289
	7	-	3.90	13.07	-	-	77.289
Core	7	-	5.10	12.20	-	-	133.526
	7	-	5.25	7.62	-	-	142.554
	1	-	7.00	-	-	-	38.485

$$A_{core} = 391.854 \text{ mm}^2$$

$$A_g = 12088.082 \text{ mm}^2$$

Table 3.1j – Construction Details for the 127 ($\alpha = 24^\circ$) mm Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	54	RH	6.40	24.0	60.23	1901.575	2421.160
2	48	LH	6.40	24.0	53.54	1690.289	2152.142
3	42	LH	6.50	24.0	47.58	1525.583	1942.432
4	36	RH	6.50	24.0	40.79	1307.642	1664.942
5	30	LH	6.60	24.0	34.53	1123.489	1430.471
6	24	RH	6.60	24.0	27.64	898.791	1144.377
7	18	LH	6.80	24.0	21.38	715.567	911.088
8	14	RH	6.10	24.0	14.94	447.865	570.240
	7	-	3.90	17.89	-	-	72.069
Core	7	-	5.10	16.75	-	-	125.557
	7	-	5.25	10.58	-	-	143.935
	1	-	7.00	-	-	-	38.485

$$A_{core} = 380.045 \text{ mm}^2$$

$$A_g = 12236.851 \text{ mm}^2$$

Table 3.1k – Construction Details for the 164 mm Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	72	RH	6.50	18.00	78.34	2512.276	3198.73
2	66	LH	6.50	18.00	71.81	2302.920	2932.169
3	60	RH	6.50	18.00	65.29	2093.564	2665.608
4	54	LH	6.50	18.00	58.77	1884.207	2399.047
5	48	RH	6.50	18.00	52.24	1674.851	2132.486
6	42	LH	6.50	18.00	45.72	1465.495	1865.926
7	36	RH	6.60	18.00	39.80	1295.086	1648.955
8	30	LH	6.60	18.00	33.18	1079.238	1374.129
9	24	RH	6.60	18.00	26.56	863.391	1099.303
10	18	LH	6.60	18.00	19.95	647.543	824.477
	14	-	6.00	17.99	-	-	340.575
	7	-	3.70	12.98	-	-	69.641
Core	7	-	4.85	12.15	-	-	120.824
	7	-	4.95	7.53	-	-	131.254
	1	-	6.70	-	-	-	35.254

$$A_{core} = 697.552 \text{ mm}^2$$

$$A_g = 20140.829 \text{ mm}^2$$

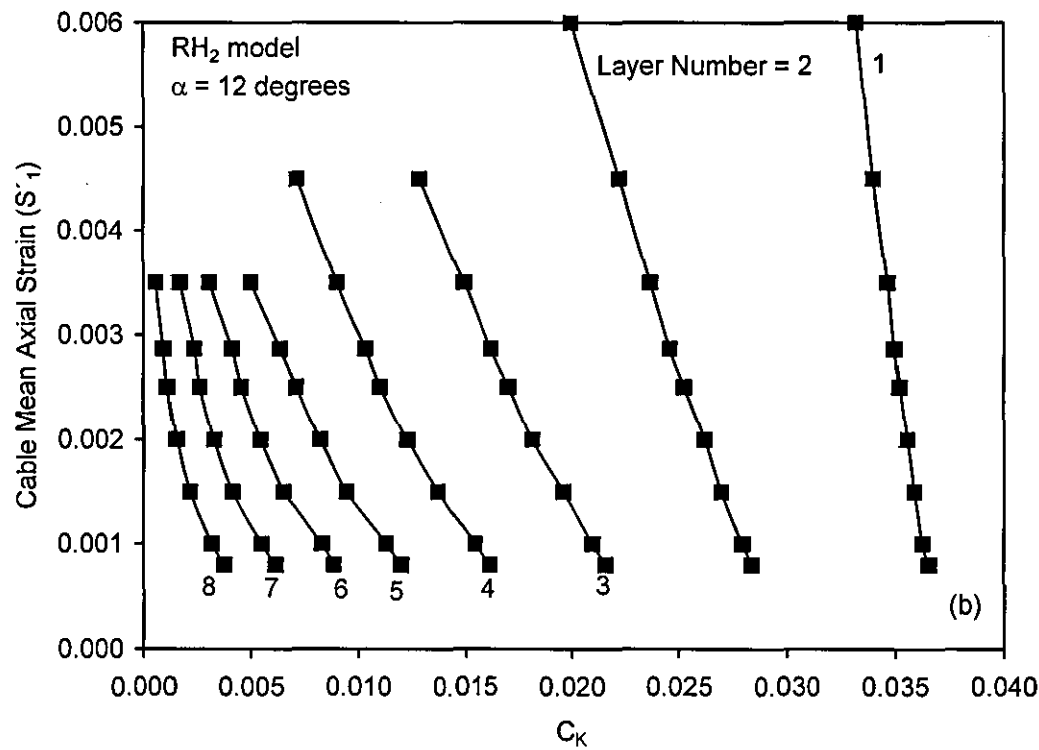
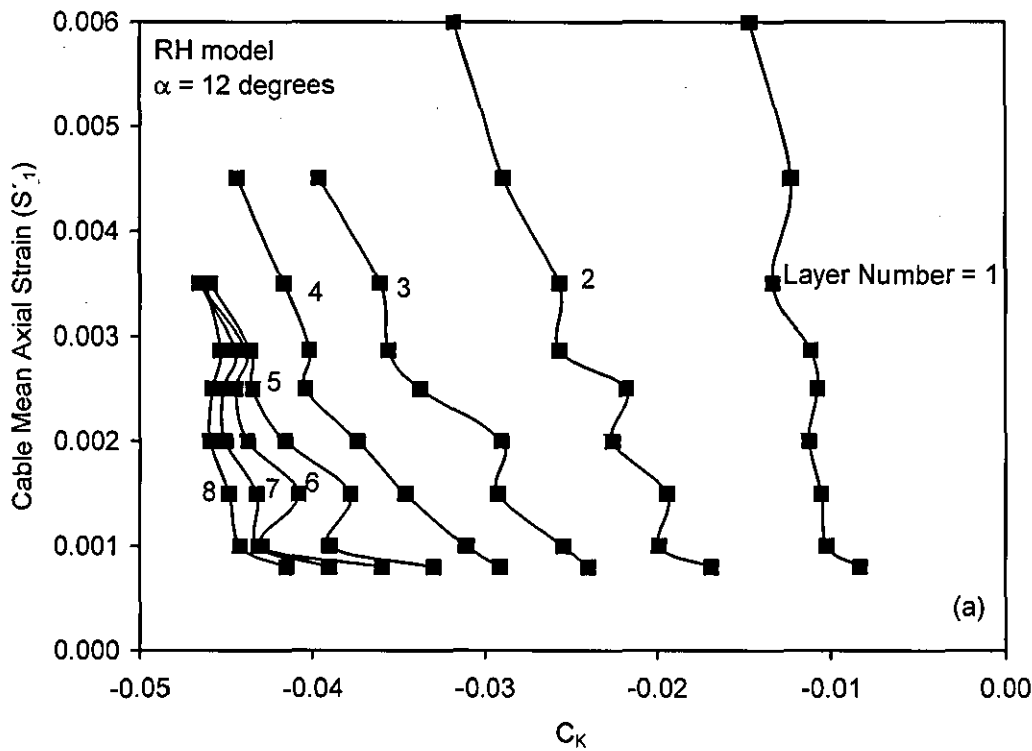
Table 3.16 – Construction Details for the 184 mm Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (deg)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	74	RH	7.10	18.0	87.945	3080.751	3922.534
2	68	LH	7.10	18.0	80.818	2830.960	3604.491
3	62	RH	7.10	18.0	73.69	2581.170	3286.447
4	56	LH	7.10	18.0	66.563	2331.379	2968.404
5	54	RH	6.60	18.0	59.668	1742.629	2473.432
6	48	LH	6.60	18.0	53.049	1726.781	2198.606
7	42	LH	6.60	18.0	46.421	1510.934	1923.780
8	37	RH	6.50	18.0	40.282	1291.031	1643.792
9	31	LH	6.50	18.0	33.763	1081.675	1377.231
10	25	RH	6.60	18.0	27.665	899.365	1145.107
11	19	LH	6.60	18.0	21.054	683.518	870.282
	14	RH	6.25	17.96	-	-	369.736
	7	-	3.90	13.02	-	-	77.336
Core	7	-	5.10	12.17	-	-	133.571
	7	-	5.20	7.57	-	-	144.804
	1	-	7.10	-	-	-	39.592

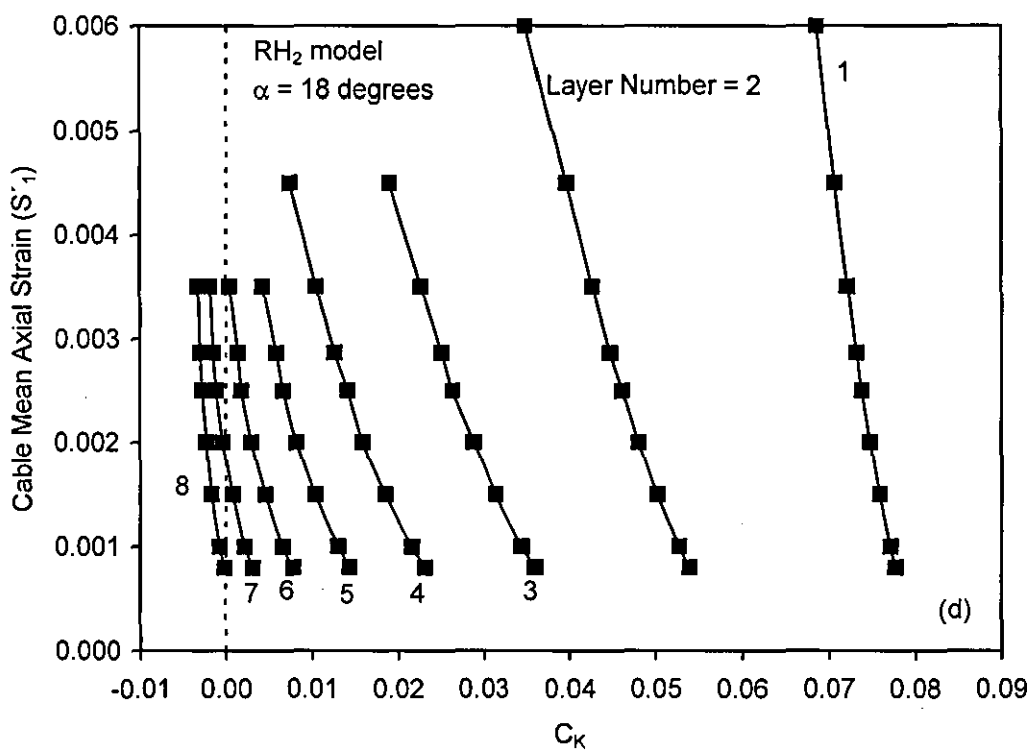
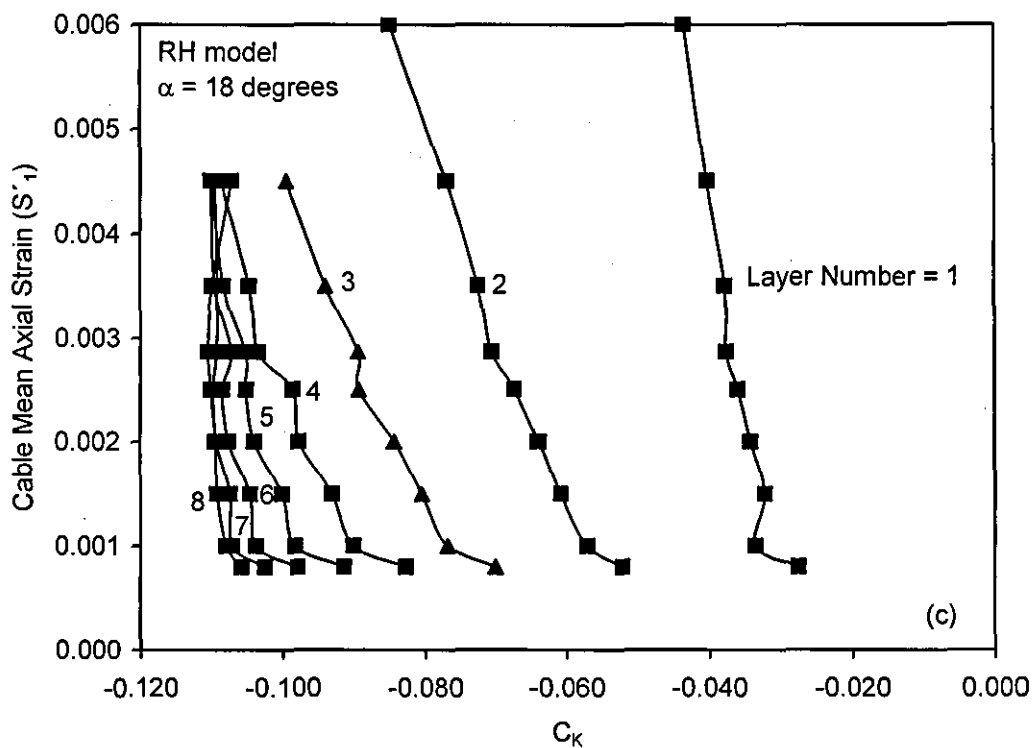
$$A_{core} = 765.040 \text{ mm}^2$$

$$A_g = 25414.106 \text{ mm}^2$$

3.4 RESULTS



Figures 3.4a and b – Variation of the Parameter c_k with Changes in the Cable Mean Axial Strain, S'_1 , for the 127 mm Outside Diameter Strand ($\alpha = 12^\circ$) as Calculated Using Both the RH and RH₂ Models.



Figures 3.4c and d – Variation of the Parameter c_k with Changes in the Cable Mean Axial Strain, S'_1 , for the 127 mm Outside Diameter Strand ($\alpha = 18^\circ$) as Calculated Using Both the RH and RH₂ Models.

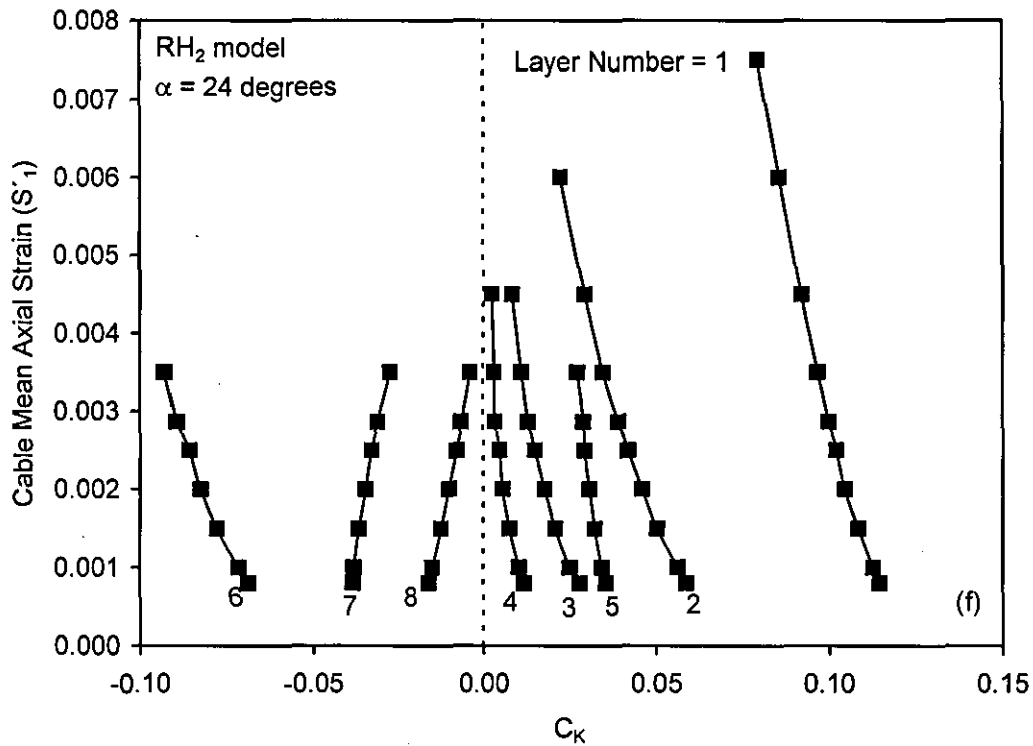
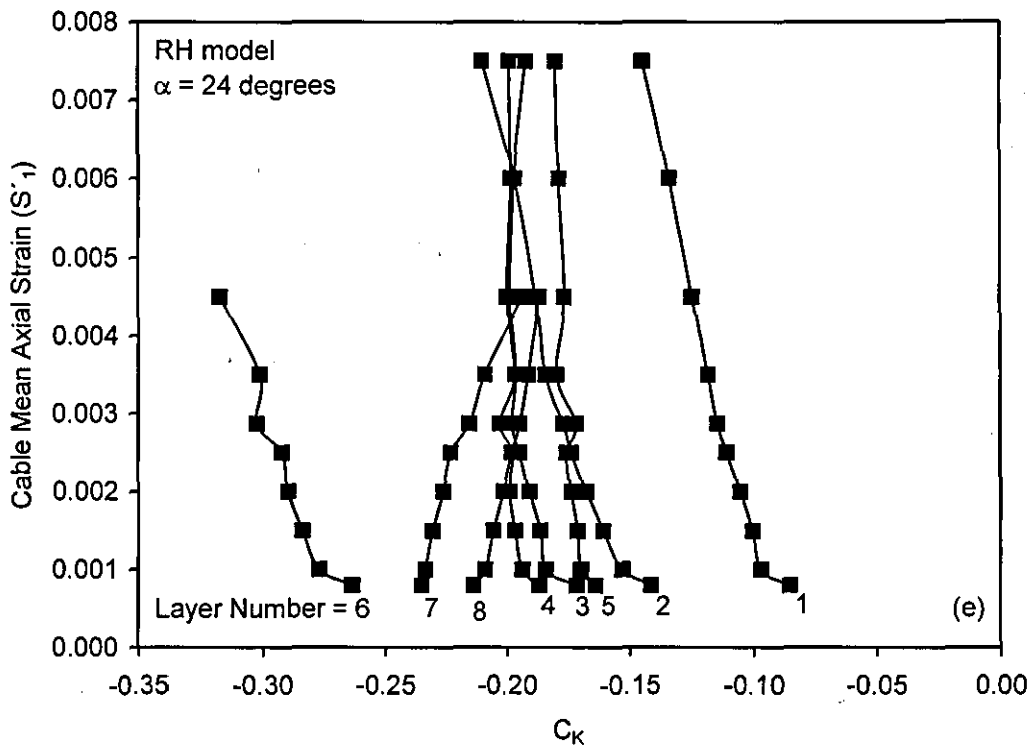


Figure 3.4e and f – Variation of the Parameter c_k with Changes in the Cable Mean Axial Strain, S'_1 , for the 127 mm Outside Diameter Strand ($\alpha = 24^\circ$) as Calculated Using Both the RH and RH₂ Models.

Figs. 3.4b, d and f show plots of c_k versus S'_1 , for the 127 mm spiral strands ($\alpha = 12^\circ$, 18° and 24°) calculated using the RH_2 model. It is interesting to note that, for a lay angle of 12° , all of the c_k values for all levels of practical cable mean axial strains, are positive. As the lay angle, α , increases, the percentage of c_k values, which are positive, decreases, until at $\alpha = 24^\circ$, associated with which approximately half of the c_k values are positive with the other half being negative. The implication of a positive value for c_k is that tensile forces are operative normal to the line-contact patches. In other words, for c_k to be positive, the hoop stress, T_2 , would have to be positive (i.e. tensile), which, in the context of a practical strand, is impossible. Plots of variations in the c_k values with changes in the strand mean axial strain, S'_1 , based on the RH model, for all the three different 127 mm outside diameter spiral strands with lay angles of 12° , 18° and 24° , are, on the other hand, presented in Figs. 3.4a, c and e, where, in all cases, the c_k values are negative. The results in the following sections will be obtained by assuming that a positive value for c_k is equal to zero: Jolicoeur has, however, not touched upon this rather crucial point and apparently fails to recognise that, according to his way of calculating c_k , in some cases the so-obtained values of c_k are positive and not compatible with the physical reality. In the present work, therefore, Jolicoeur's RH_2 model has been modified by the way of assuming a zero value for the positive cases of c_k as determined by Eqn. (3.56).

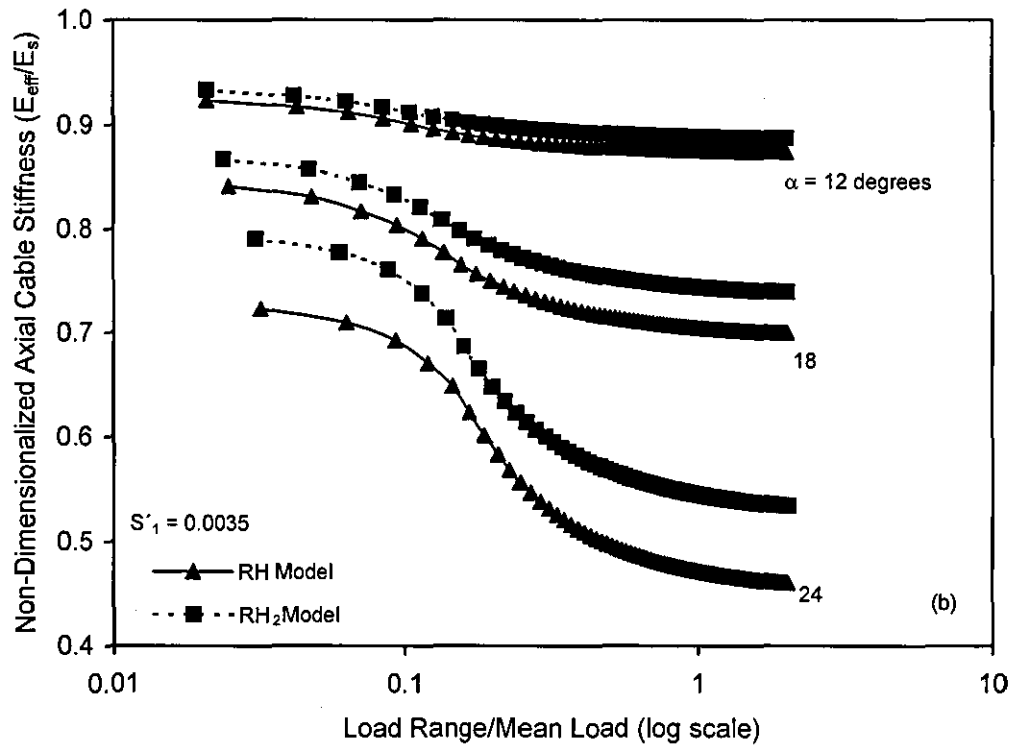
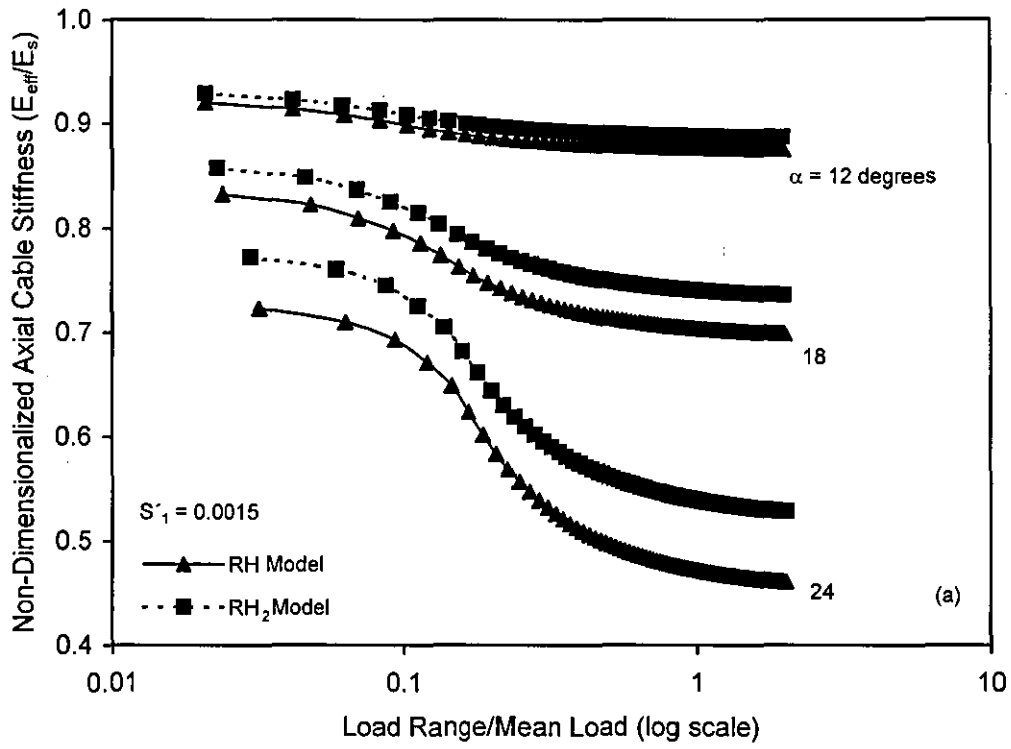
3.4.1 Axial Stiffness

The orthotropic sheet theoretical model is capable of predicting the two limiting cases of the axial stiffness (i.e. the no-slip and full-slip limits), as well as following the variations between them. Axial stiffness results for the three different 127 mm outside diameter spiral strand constructions with lay angles $\alpha = 12^\circ$, 18° and 24° (Tables 3.1h, i and j), as calculated using both the RH and RH_2 models, are presented in Figs. 3.5.

As Figs. 3.5 clearly show, for both models, increasing the lay angle leads to significant increases in the ratio of the no-slip to full-slip cable axial stiffness, where the full-slip cable axial stiffness corresponds to the lower bound limit, whereby the wires in line-contact undergo gross sliding with the interwire frictional forces becoming insignificant compared to the axial force changes within the individual helical wires. The no-slip limit corresponds to the upper bound value of the axial

stiffness, associated with the wires in line-contact effectively sticking together, behaving almost like a solid rod (with allowance given for the presence of gaps among the wires). It is, perhaps, worthwhile mentioning that cable manufacturers invariably quote the axial stiffness results which correspond to the full-slip value, in the present terminology, and relate to newly manufactured (but prestretched) strands. When the strands have been in-service for a long period of time, they become internally stable, in terms of the interwire contacts, which undergo a significant degree of fretting and bedding-in (Raoof, 1990). Raoof (1983) discovered that the mean axial load barely affects the full-slip axial and/or torsional stiffnesses, but it does have some influence on the no-slip stiffnesses (particularly the torsional one).

The differences between the plots of Figs. 3.5 are not always significant – e.g. for when the lay angle is only 12°. However, when the lay angle increases to 24°, the ratio of $\frac{E_{\text{no-slip}}}{E_{\text{full-slip}}}$ can be as much as 1.57 for the RH model, and 1.48 for the RH₂ model, which may have significant practical implications for the serviceability limit state conditions for structures supported by such strand constructions.



Figures 3.5a and b – Variation of the Axial Stiffness with Changes in the Axial Load Range / Mean Load, as Calculated Using both the RH and RH_2 Models at two Different Levels of Cable Mean Axial Strain: (a) $S'_1 = 0.0015$ and (b) $S'_1 = 0.0035$.

3.4.2 Torsional Stiffness

The torsional stiffness results are the same for both the RH and RH₂ models as the parameter c_k is not involved in the calculation of the torsional stiffness coefficient A_4 . Fig. 3.6 shows plots of torque versus twist per unit length for the three different 127 mm spiral strands with lay angles of 12°, 18° and 24°. The plots are based on two different levels of cable mean axial strain $S'_1 = 0.0015$, and 0.0035.

The plots of Fig. 3.6 clearly demonstrate that both the no-slip and full-slip torsion moduli, A_4 , for a spiral strand can increase significantly with increases in the lay angle. At lower values of lay angles the transition from the no-slip to the full-slip regime is more acute.

The torque–twist relationship is found to be non-linear, due to the frictional non-linearity of the line-contact patch, rather than the geometrical non-linearities such as those associated with changes in the lay angle. The no-slip torsional stiffness is obviously greater than the full-slip torsional stiffness as, for sufficiently high values of twist per unit length, full line-contact slippage takes place between the neighbouring wires in the individual layers. The value of the no-slip torsion modulus is independent of the friction coefficient, μ (Raoof, 1983). The magnitude of the full-slip torsion modulus appears to be insensitive to changes in the magnitude of the mean axial load, over the practical ranges of S'_1 .

As discussed by Raoof (1990a), the full-slip torsional stiffness is not dependent on the working age of the cable, once the initial constructional stretch has been taken out. However, the no-slip torsion modulus is dependent upon the degree of bedding-in, and provides an upper bound solution to the axial and torsional characteristics of the spiral strands in practice.

Finally, the magnitude of the no-slip torsion modulus is greatly influenced by the level of mean axial load, with the magnitude of the no-slip torsional stiffness, A_4 , increasing significantly with sufficiently large increases in S'_1 .

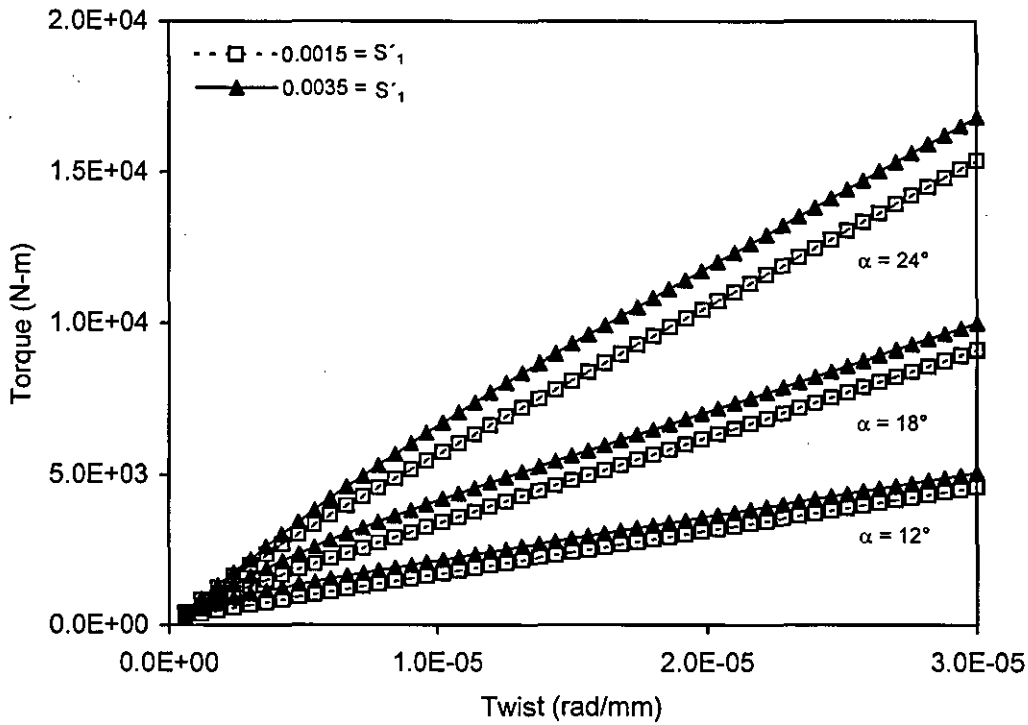


Figure 3.6 – Torque – Twist Relationship for Axially Pre-Loaded Spiral Strands, as Calculated Using Both the RH or RH₂ Models.

3.4.3 Axial Hysteresis

Figs. 3.7a and b present the variation of the overall axial frictional damping with changes in the $\frac{\text{axial load range}}{\text{mean axial load}}$ ratio for the three types of 127 mm outside diameter spiral strands as predicted by the RH and RH₂ models, respectively. The predictions of the axial hysteresis are based on the two different methods (a) and (b), as described in section 3.2.5. The close agreement between the two methods (a) and (b) appears reasonable, and the previously reported (e.g. Hobbs and Raoof, 1984) extensive theoretical parametric studies on a wide range of spiral strand constructions have confirmed this close agreement for the RH model. The two methods (a) and (b) were not expected to agree exactly, due to the differences in the simplifying assumptions regarding the cyclic variations in the line-contact normal forces in the two methods. A fairly close agreement between the predictions based on the RH and RH₂ models, over a wide range of lay angles $12^\circ \leq \alpha \leq 24^\circ$, is demonstrated in Figs. 3.7.

Figs. 3.7 clearly show significant increases in the fully bedded-in axial damping associated with modest increases in the magnitude of the lay angle: this is due to the magnitudes of the normal contact forces generated in the hoop direction within the orthotropic layer being increased with increasing α , with increases in layer lay angles also increasing the rate of relative slippage between the wires in line-contact (Raoof, 1997). If increasing the lay angle is to be used to deliberately increase the hysteresis, it should be done with caution so as to guard against other adverse effects on stiffness and axial fatigue (Raoof, 1997).

Raoof (1991f) discovered that changing the value of the coefficient of friction, μ , does not affect the shape of the axial hysteresis curves, but merely causes a lateral shift of the plot. As Raoof commented, the rather unexpected result was that the maximum specific damping, $\left(\frac{\Delta U}{U}\right)_{\max}$, appeared to be independent of the value of μ .

Figs. 3.7 also show that, over a wide range of cable mean axial strains ($0.0008 \leq S'_1 \leq 0.0035$), the variation of the maximum hysteresis $\left(\frac{\Delta U}{U}\right)_{\max}$, which occurs due to the no-slip to full-slip transition over the individual line-contact patches, with changes in the imposed strand mean axial strain, appears to be practically not very significant, despite the non-linear nature of the interwire contact problem. Although, as the lay angle increases up to 24° , there is some slight increases in the value of $\left(\frac{\Delta U}{U}\right)_{\max}$ with increasing cable mean axial strain, S'_1 , in view of a number of other uncertainties in terms of, for example, the value assigned to μ , this is not thought to be practically very significant.

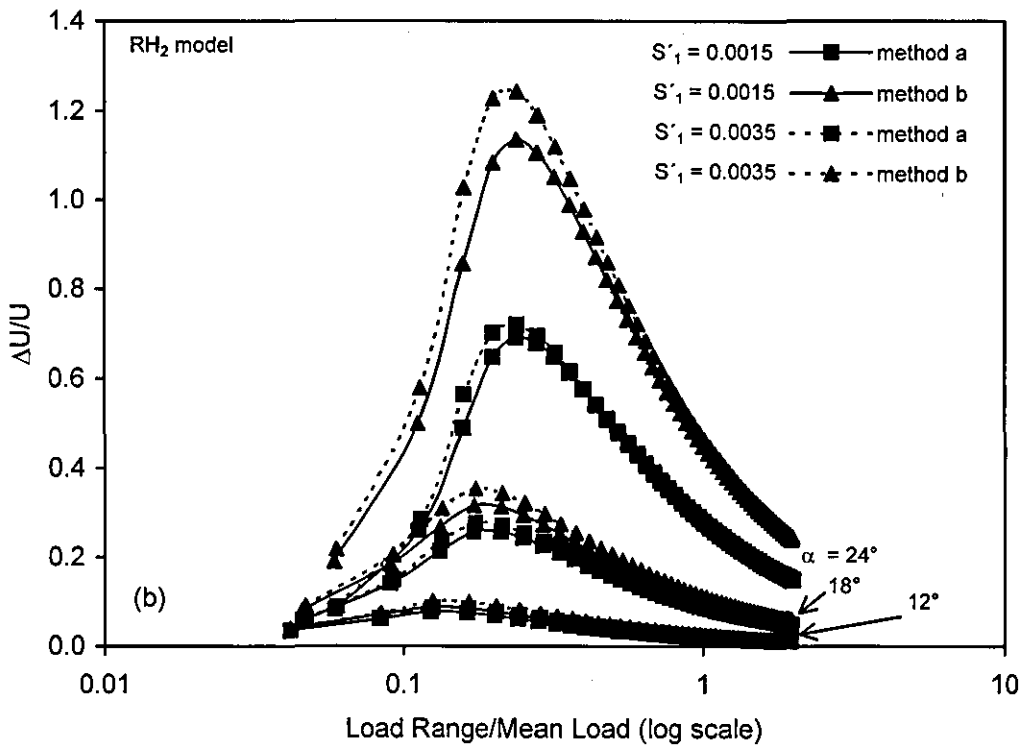
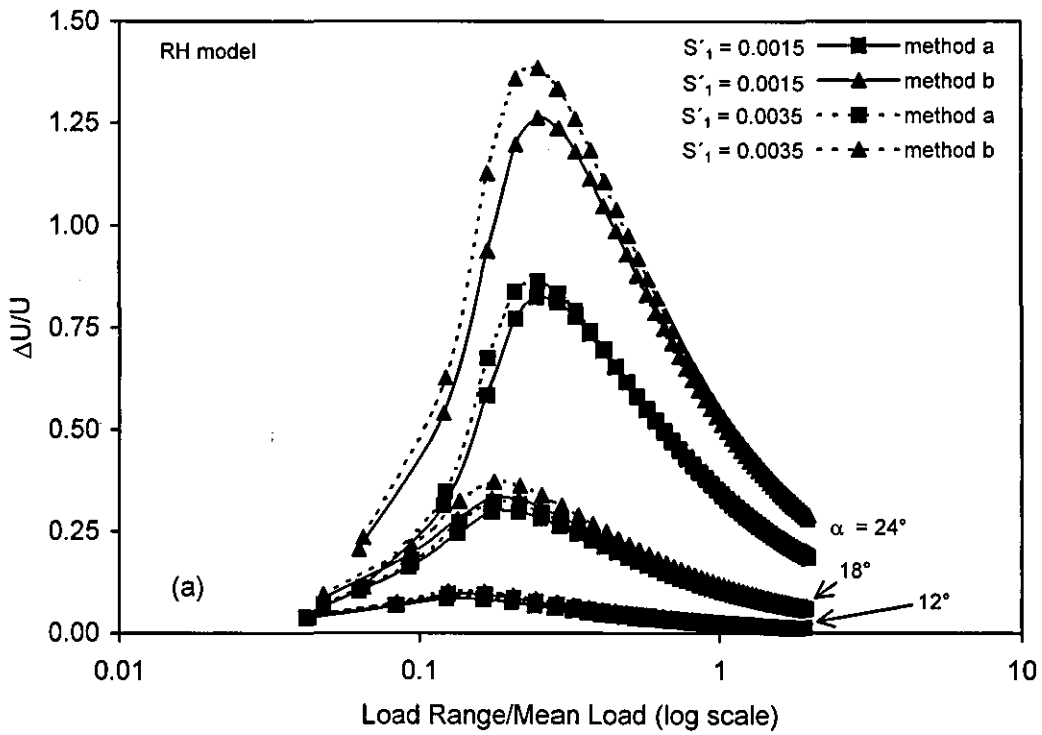


Figure 3.7a and b – Variation of the Overall Frictional Axial Damping with Changes in the Axial Load Range / Mean Load, as Calculated by: (a) RH Model, and (b) RH₂ Model

3.4.4 Torsional Hysteresis

The results for the torsional hysteresis are the same for both the RH and RH₂ models as the torsional hysteresis, associated with the stiffness coefficient A_4 , is not dependent upon the parameter c_k .

Fig. 3.8 demonstrates the dependence of the torsional hysteresis on the level of twist per unit length and the magnitude of the mean axial strain on the strand, for all the 127 mm diameter spiral strands with lay angles of 12°, 18° and 24°.

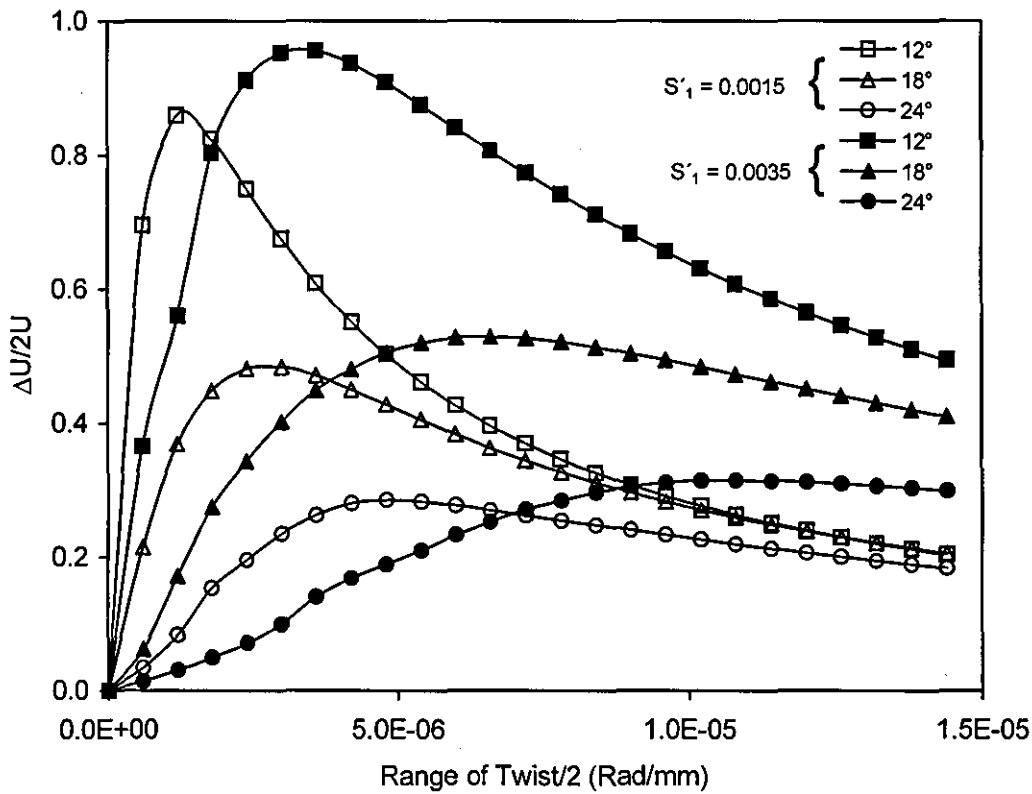


Figure 3.8 – Variation of the Torsional Frictional Damping with Changes in the Twist per Unit Length for the 127 mm Axially Pre-loaded Spiral Strands, as Calculated Using both the RH and RH₂ Models.

The torsional hysteresis data relates to fully bedded-in spiral strands, and from Fig. 3.8 it can be seen that, increasing the lay angle from 12° to 24° causes significant increases in the torsional hysteresis under uniform cyclic loading.

It should be noted that, unlike the axial case, both methods (a) and (b) have been found to give identical results for frictional torsional hysteresis. It was noted by Raoof and Hobbs (1989) that at large (and probably impractical) levels of twist the

torsional hysteresis is somewhat overestimated (cf. experimental data), whilst at rather small twists the reverse occurs: for sufficiently small twists, the overall hysteresis is not frictional, but is dominated by the intrinsic material damping, which is viscous, hence, leading to an amplitude independent (i.e. constant) logarithmic decrement.

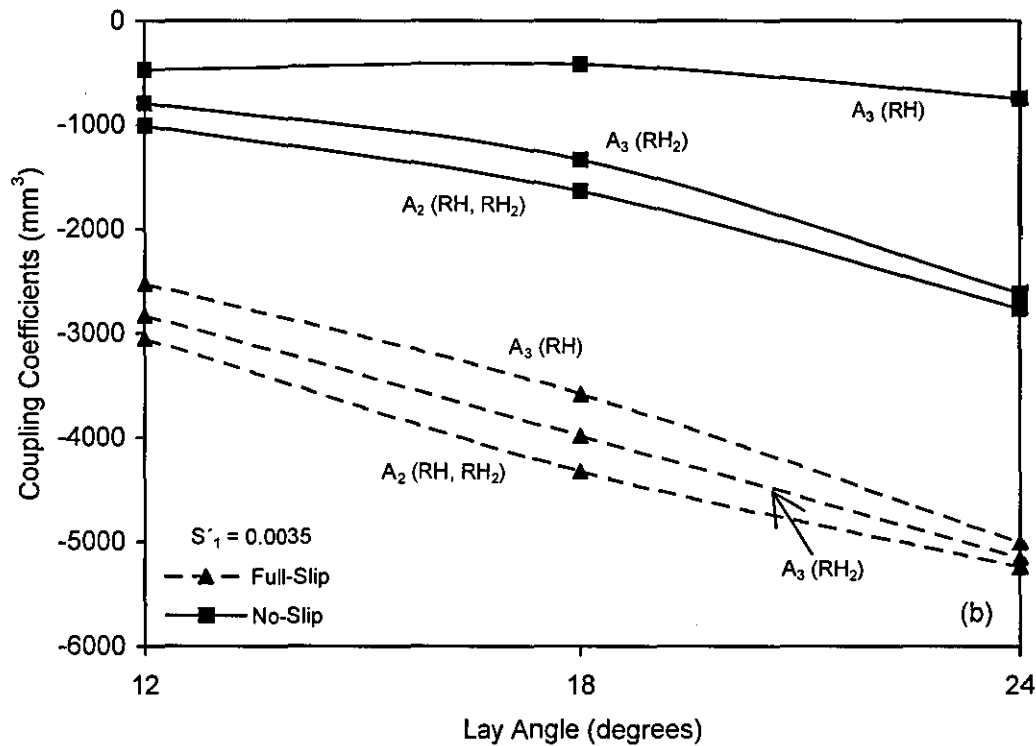
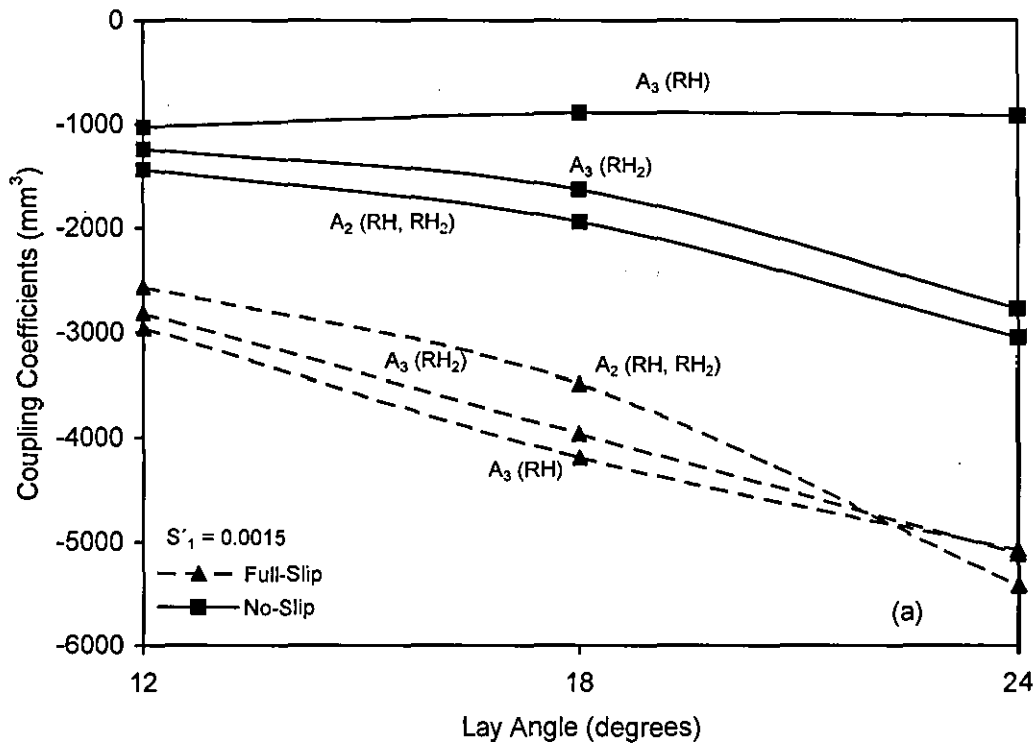
3.4.5 Axial/Torsional Coupling Coefficients

The overall coupling coefficients, A_2 and A_3 , for the three different 127 mm outside diameter spiral strands, which should be equal, are shown in Figs. 3.9a and b for both the no-slip and full-slip limiting conditions at assumed values of cable mean axial strains $S'_1 = 0.0015$ and 0.0035 , as calculated using both the RH and RH₂ models.

It can be seen that at both levels of mean axial strain and for both the no-slip and full-slip limiting conditions, the RH₂ model apparently gives a more symmetrical stiffness matrix, and the differences between the two overall coupling coefficients are more pronounced for the no-slip regime. Comparing such results relating to the overall coupling coefficients could, however, be misleading: this is due to the rather small values of these constants in the nominally torsionally balanced spiral strands in which the accumulation of small errors in the course of algebraically adding up the contributions of the counter-laid layers in order to predict the overall values for the whole strand has led to such apparent (although not practically significant) anomalies. In the circumstances, therefore, it was, instead, decided to compare the layer stiffness coefficients in order to arrive at more precise conclusions.

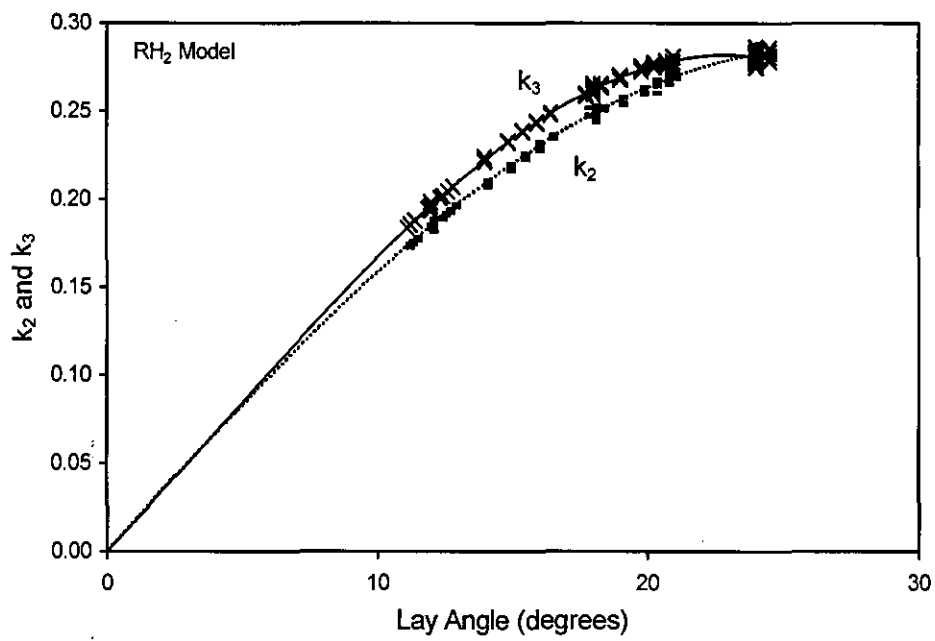
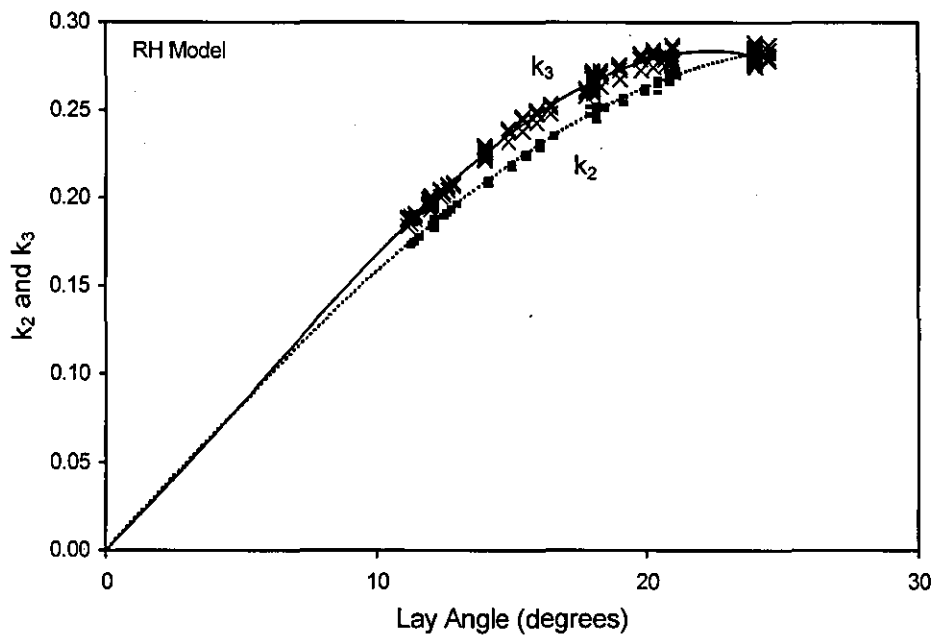
Figs. 3.10a and b compare the full-slip stiffness coefficients k_2 and k_3 , for the individual layers, which according to the Maxwell-Betti reciprocal theorem should be equal. As the plots show, for both models, k_2 is almost equal to k_3 , over the full range of manufacturing limits for the lay angle $12^\circ \leq \alpha \leq 24^\circ$.

Due to the sensitivity of the no-slip stiffness coefficients k_2 and k_3 to changes in the mean tension applied to the spiral strand (section 3.4.6.2), it was felt that the comparison of the two coefficients would be better presented for each individual cable mean axial strain, S'_1 , rather than on one chart which covers a range of cable mean axial strains. Figs. 3.11 show the comparison of the no-slip layer stiffness coefficients, k_2 and k_3 , with changes in the lay angle, covering various levels of cable

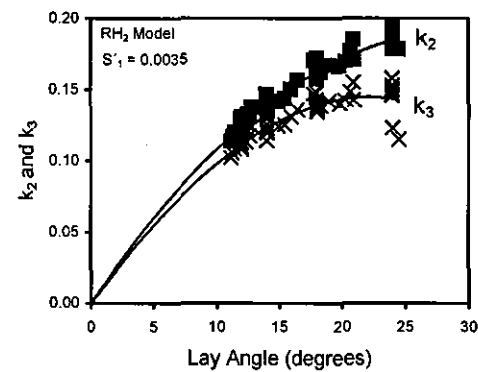
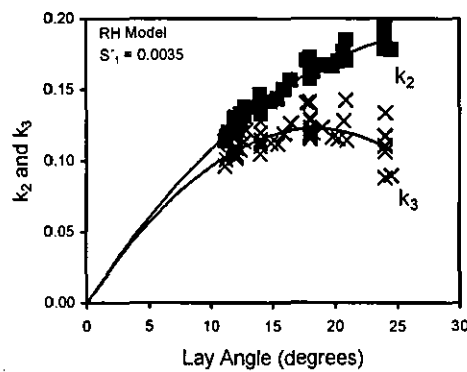
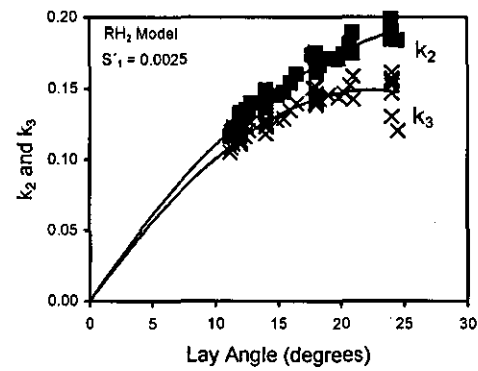
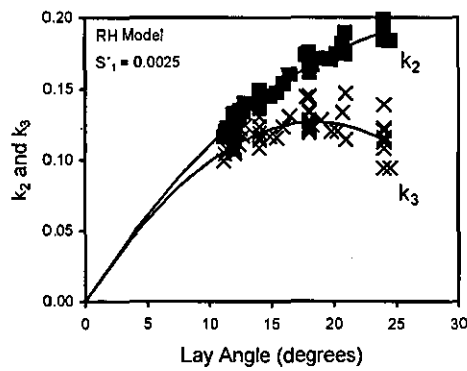
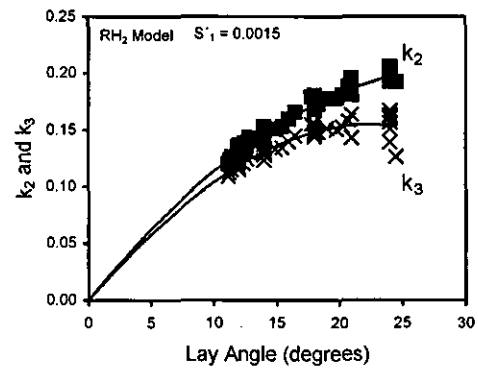
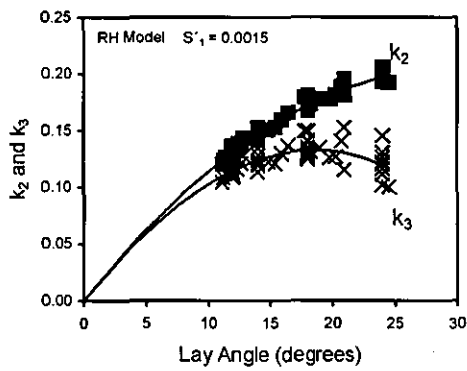
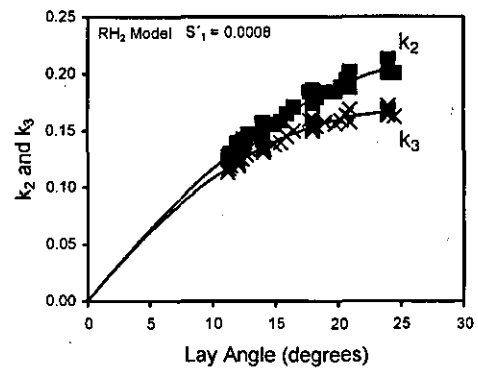
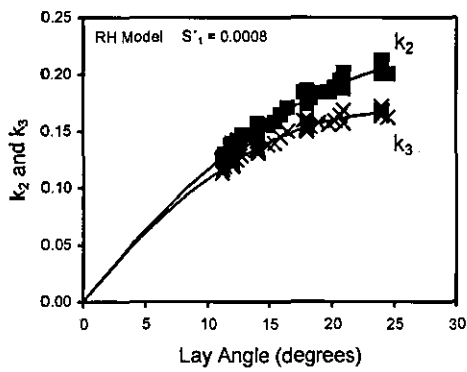


Figures 3.9a and b – Variation of the Overall Axial/Torsional Coupling Coefficients, A_2 and A_3 , as Calculated Using Both the RH and RH₂ Models, with the Lay Angle, for a 127 mm Outside Diameter Spiral Strand, Based on the No-Slip and Full-Slip Limiting Conditions; Assuming a Strand Mean Axial Strain of: (a) $S'_1 = 0.0015$; and (b) $S'_1 = 0.0035$.

mean axial strain $0.0008 \leq S'_1 \leq 0.0035$. The results clearly show that at low levels of cable mean axial strain $S'_1 = 0.0008$, the differences between the RH and RH₂ models is largely insignificant. As the cable mean axial strain increases up to $S'_1 = 0.0035$, then, the RH₂ model appears to give better predictions – i.e. for sufficiently large values of strand mean axial strain, S'_1 , k_2 and k_3 are closer to each other when predicted using the RH₂ model, than, when using the RH model. The differences are more pronounced as the lay angle increases from 20° onwards – i.e. towards the upper limit of the current manufacturing limits for the lay angle. In conclusion, for $\alpha < 20^\circ$, the differences between the predictions of no-slip k_3 (which is, unlike k_2 , dependent on the parameter c_k) using either RH or RH₂ models is not thought to be practically significant.



Figures 3.10a and b – Comparison of the Full-Slip Stiffness Coefficients k_2 and k_3 , in the Various Layers of 127 mm Diameter Spiral Strands, as Calculated Using Both the RH and RH₂ Models.



Figs. 3.11 – Comparison of the No-Slip Layer Stiffness Coefficients, k_2 and k_3 , with Changes in the Lay Angle at Various Levels of Cable Mean Axial Strain, as Calculated Using Both the RH and RH₂ Models.

3.4.6 Simplified Formulations for Axial/Torsional Coupling Coefficients

3.4.6.1 Full-Slip Case

As discussed by Raoof (1991c), the values of the coefficients, $A_1 - A_4$, may be estimated using Eqns. (3.45), (3.47) – (3.50) in conjunction with the following polynomials

$$k_1 = 1 + 0.00162\alpha - 0.00102\alpha^2 - 0.000002236\alpha^3 \quad (3.57a)$$

$$k_2 = 0.01702\alpha - 0.0000471\alpha^2 - 0.000007364\alpha^3 \quad (3.57b)$$

$$k_3 = 0.01633\alpha + 0.0002126\alpha^2 - 0.00001648\alpha^3 \quad (3.57c)$$

$$k_4 = -0.000511\alpha + 0.000429\alpha^2 - 0.00000555\alpha^3 \quad (3.57d)$$

where, α is in degrees and the layer stiffness coefficients $k_1 - k_4$ are defined by Eqns. (3.46a – d).

The theoretical parametric study conducted in the present work includes spiral strand constructions which were not widely available at the time of the original theoretical parametric studies conducted by Raoof (1991c). As a result, the simplified polynomial equations derived from the present parametric study are slightly different from Eqns. (3.57a – d). The individual constant coefficients for the polynomials obtained from the present parametric study are given in Table 3.2, with the individual coefficients A-D defining each polynomial of the form

$$k_i = A_i + B_i\alpha + C_i\alpha^2 + D_i\alpha^3 \quad (3.58)$$

Table 3.2 – Values of the Full-Slip Coefficients A-D in Equation (3.58), Along with the Correlation Coefficients, R.

Model	Coefficient	A	B ($\times 10^{-2}$)	C ($\times 10^{-3}$)	D ($\times 10^{-6}$)	R
RH	k_1	1	0.3844	-1.311	7.676	0.9874
RH & RH ₂	k_2	0	1.7320	-0.088	-5.952	0.9984
RH	k_3	0	1.5893	0.277	-18.799	0.9946
RH & RH ₂	k_4	0	0.0621	0.446	-6.210	0.9993
RH ₂	k_1	1	0.1161	0.9298	0.4358	0.9988
RH ₂	k_3	0	1.6583	0.16117	15.169	0.9990

Figs. (3.12a, c and e) show the variations of the full-slip layer coefficients $k_1 - k_4$ with the lay angle, for various levels of cable mean axial strain, as calculated using both the RH and RH₂ models, with Fig. 3.13a showing the same full-slip coefficients for all the different levels of cable mean axial strain ($0.0008 \leq S'_1 \leq 0.0035$). The plots show a very close agreement between the data for both the RH and RH₂ models.

For all of these full-slip charts, there appears to be very little difference between the RH and RH₂ models.

3.4.6.2 No-Slip Case

As discussed by Raoof and Kraincanic (1995a), the no-slip stiffness coefficient k_1 is, similar to all the full-slip stiffness coefficients $k_1 - k_4$, very nearly independent of the mean tension applied to the cable. However, the values of the no-slip coefficients $k_2 - k_4$ were found to be much more sensitive to changes in mean tension. Moreover, as was the case in the work of Raoof and Kraincanic (1995a), Figs. 3.13b also exhibit fairly significant degrees of scatter in the no-slip plots of k_3 versus lay angle.

Raoof and Kraincanic (1995) found that, for all practical purposes, for a given cable mean axial strain, it was possible to fit third order polynomials through the data, resulting in fairly reasonable scatter around the fitted curves. The minimum observed value of the correlation coefficient, R , was 0.7, which was, perhaps, sufficiently accurate for most practical purposes. These findings were confirmed by the present theoretical parametric studies (Figs. 3.12b, d and f).

Table 3.3a gives the values of the coefficients $A - D$ in Equation (3.58), along with the correlation coefficients, R , as obtained by Raoof and Kraincanic (1995). Table 3.3b gives the values of the same coefficients, for both the RH and RH₂ models, as obtained from the present parametric study.

Once again, the differences between the charts developed using the two (RH and RH₂) models appears to be minimal, with only the k_1 and k_3 stiffness coefficients being dependent on the parameter c_k , hence, being slightly different depending as to whether the RH or RH₂ model has been used.

• **Table 3.3a** – Values of the No-Slip Coefficients A–D in Equation (3.58), After Raoof and Kraincanic (1995).

Axial Strain	Coefficient k_i	A	B ($\times 10^{-2}$)	C ($\times 10^{-4}$)	D ($\times 10^{-6}$)	Correlation Factor R
0.0008	i = 1	1	0.02813	-6.392	6.618	0.9999
	2	0	1.434	-2.577	0.6247	0.99
	3	0	1.422	-3.236	-1.052	0.91
	4	0	2.046	-7.742	13.23	0.98
0.0015	i = 1	1	0.04230	-6.537	6.424	0.99
	2	0	1.417	-2.755	1.090	0.98
	3	0	1.402	-3.377	-1.681	0.79
	4	0	2.170	-8.362	14.15	0.97
0.0025	i = 1	1	0.01522	-6.191	6.407	0.999
	2	0	1.400	-2.906	1.506	0.98
	3	0	1.377	-3.534	-1.022	0.75
	4	0	2.292	-9.006	15.17	0.97
0.0035	i = 1	1	-0.004451	-5.952	6.452	0.999
	2	0	1.383	-2.976	1.704	0.98
	3	0	2.385	-9.504	15.96	0.96
	4	0	-0.01882	-5.779	6.539	0.9995

Table 3.3b – Values of the No-Slip Coefficients A–D in Equation (3.58), From the Present Study.

Axial Strain	Model	Coefficient k_i	A	B ($\times 10^{-3}$)	C ($\times 10^{-4}$)	D ($\times 10^{-6}$)	Correlation Factor R
0.0008	RH	i = 1	1	0.1580	-6.1362	8.4106	0.9952
	RH ₂	1	1	0.1580	-6.1362	8.41055	0.9953
	RH or RH ₂	2	0	13.854	-2.1515	-0.2857	0.9877
	RH	3	0	0.1388	-3.1726	1.1364	0.9771
	RH ₂	3	0	0.1388	-3.1726	1.1364	0.9771
	RH or RH ₂	4	0	21.614	-8.7813	15.5292	0.9751
0.0015	RH	i = 1	1	0.2046	-6.3499	-6.0156	0.9980
	RH ₂	1	1	-0.8622	-4.6010	4.24642	0.9954
	RH or RH ₂	2	0	0.1362	2.2792	0.09154	0.9840
	RH	3	0	0.1344	2.9531	2.3609	0.700
	RH ₂	3	0	12.965	-2.4277	-1.1597	0.9419
	RH or RH ₂	4	0	22.992	-9.5270	16.7170	0.9693
0.0025	RH	i = 1	1	0.0428	-6.0617	6.1157	0.9984
	RH ₂	1	1	-1.0146	-4.2705	4.0072	0.9945
	RH or RH ₂	2	0	0.1339	-2.3893	0.4269	0.9798
	RH	3	0	0.1317	-3.1128	-1.6452	0.681
	RH ₂	3	0	12.6545	-2.4855	-0.7851	0.9295
	RH or RH ₂	4	0	24.298	-10.244	17.873	0.9632
0.0035	RH	i = 1	1	0.2127	-5.7698	5.9556	0.9986
	RH ₂	1	1	-1.0045	-4.1519	4.1234	0.9938
	RH or RH ₂	2	0	13.147	-2.3889	0.4805	0.9768
	RH	3	0	0.1282	-3.0636	-1.5379	0.672
	RH ₂	3	0	0.1237	-2.4820	-0.6143	0.9215
	RH or RH ₂	4	0	24.411	-10.923	19.088	0.9586

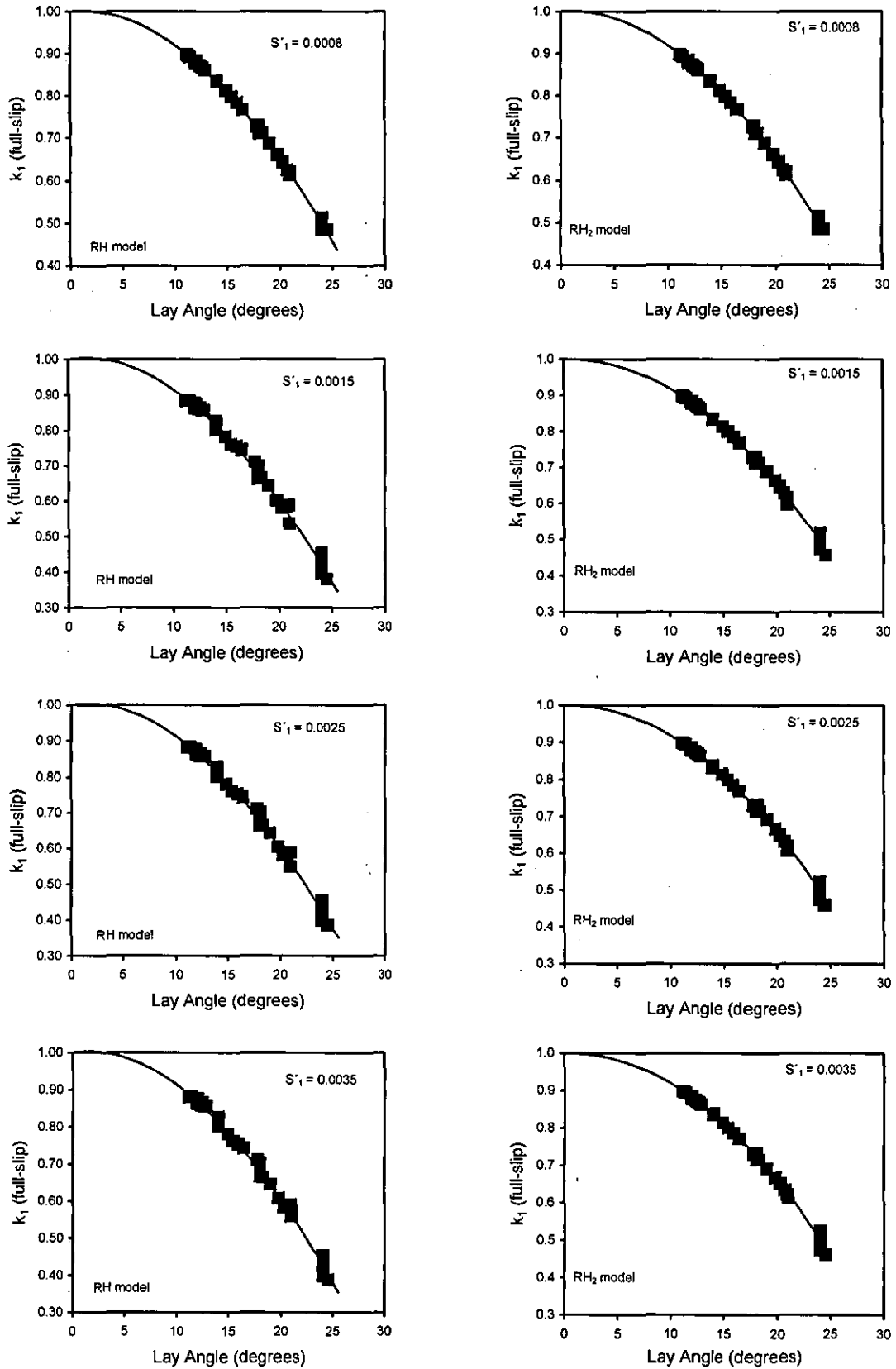


Fig. 3.12a – Variation of the Full-Slip Stiffness Coefficient, k_1 , with Changes in the Lay Angle, at Various Levels of Cable Mean Axial Strain, S'_1 , as Calculated Using both the RH and RH₂ Models.

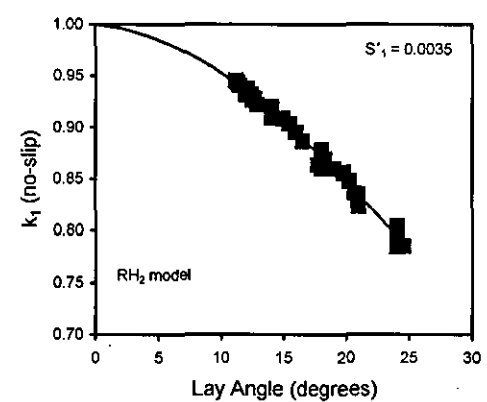
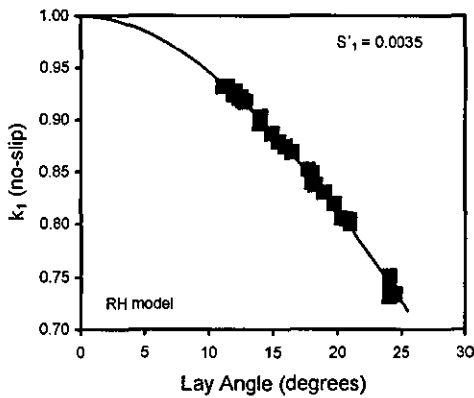
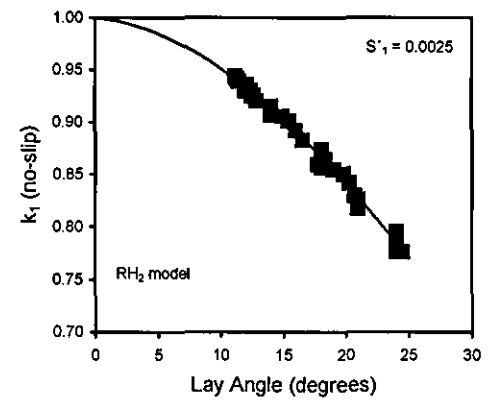
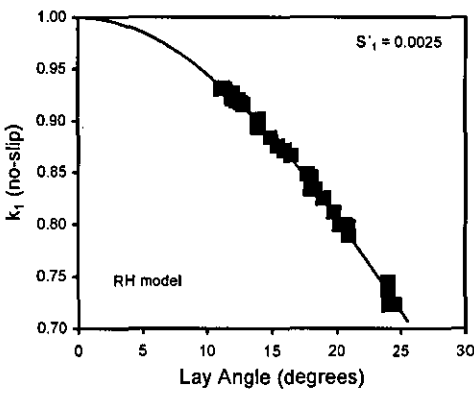
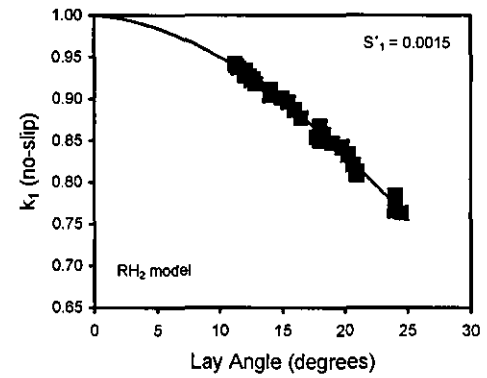
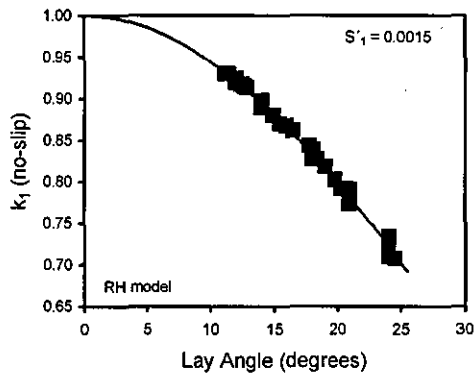
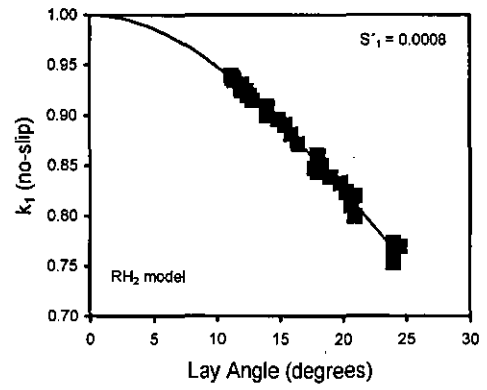
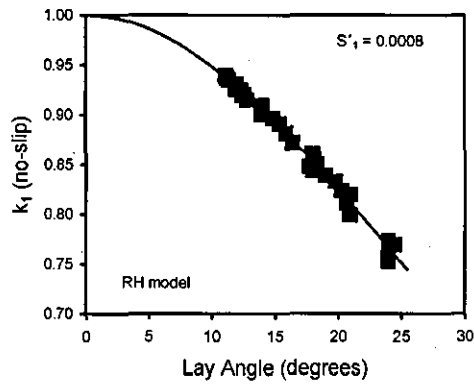


Fig. 3.12b – Variation of the No-Slip Stiffness Coefficient, k_1 , with Changes in the Lay Angle at Various Levels of Cable Mean Axial Strain, S'_1 , as Calculated Using both the RH and RH_2 Models.

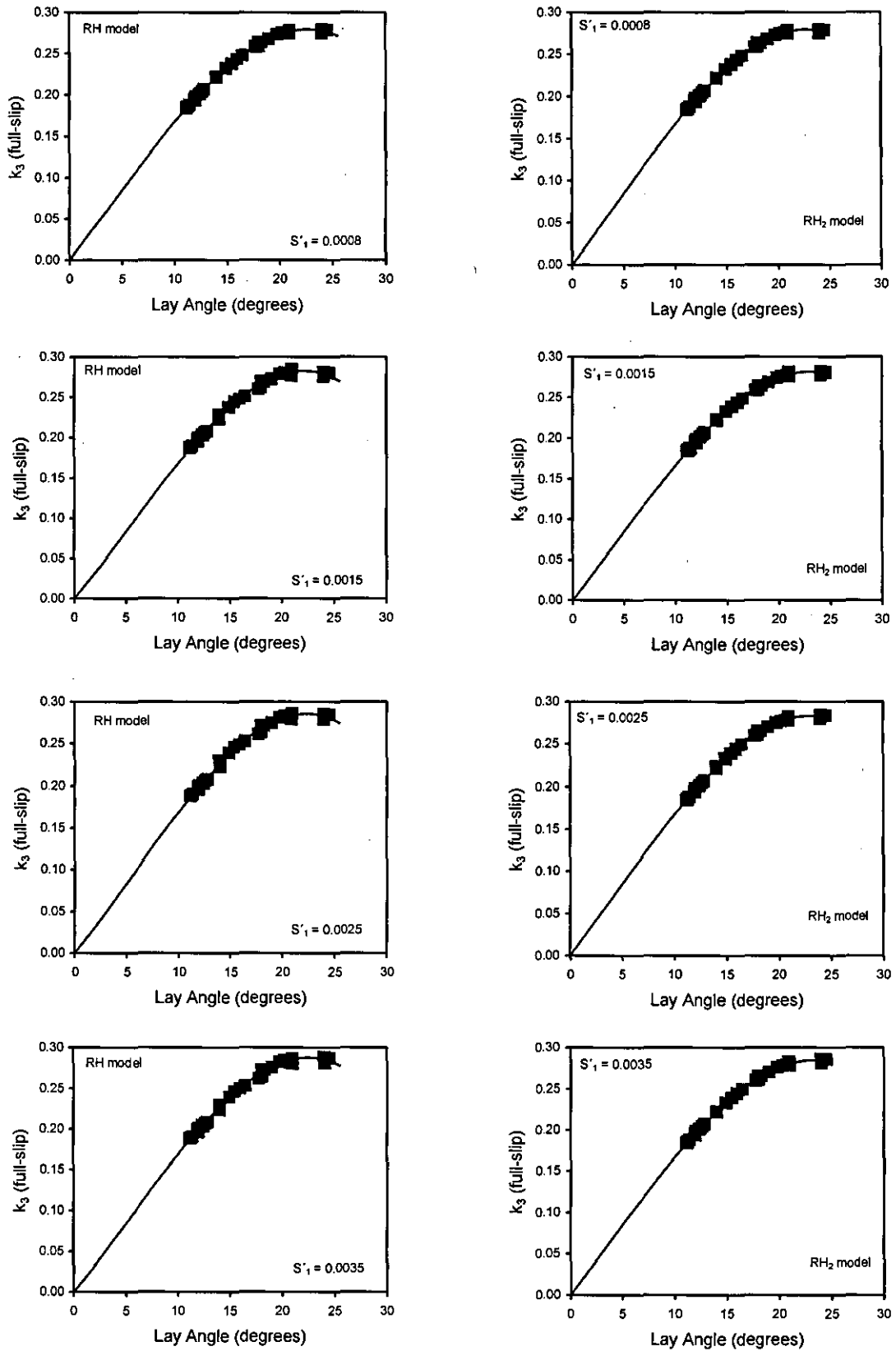


Fig. 3.12c – Variation of the Full-Slip Stiffness Coefficient, k_3 , with Changes in the Lay Angle, at Various Levels of Cable Mean Axial Strain, S'_1 , as Calculated Using both the RH and RH₂ Models.

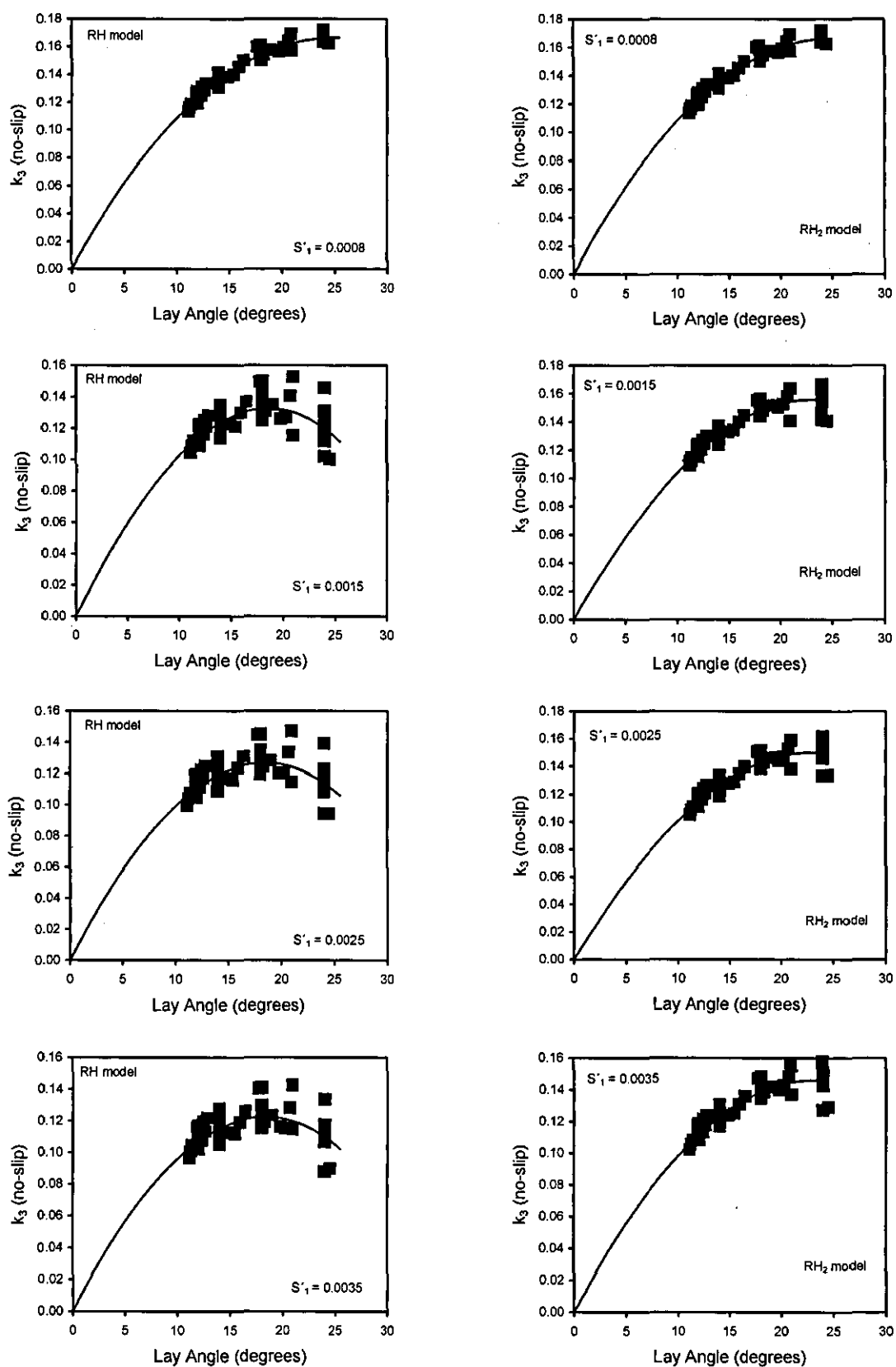


Fig. 3.12d – Variation of the No-Slip Stiffness Coefficient, k_3 , with Changes in the Lay Angle at Various Levels of Cable Mean Axial Strain, S'_1 , as Calculated Using both the RH and RH₂ Models.

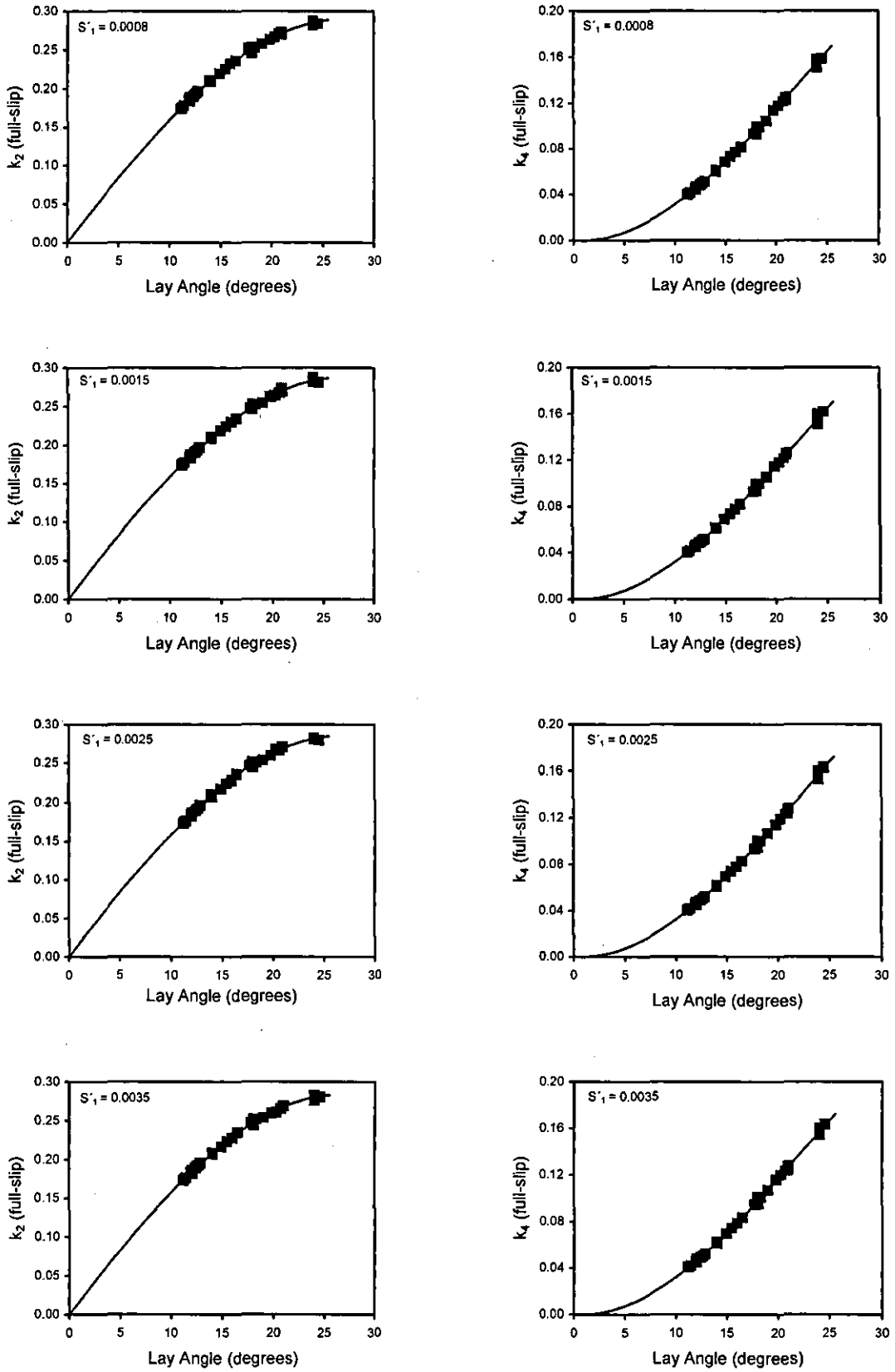


Fig. 3.12e – Variation of the Full-Slip Stiffness Coefficients, k_2 and k_4 , with Changes in the Lay Angle, at Various Levels of Cable Mean Axial Strain, S'_1 , as Calculated Using both the RH and RH₂ Models.

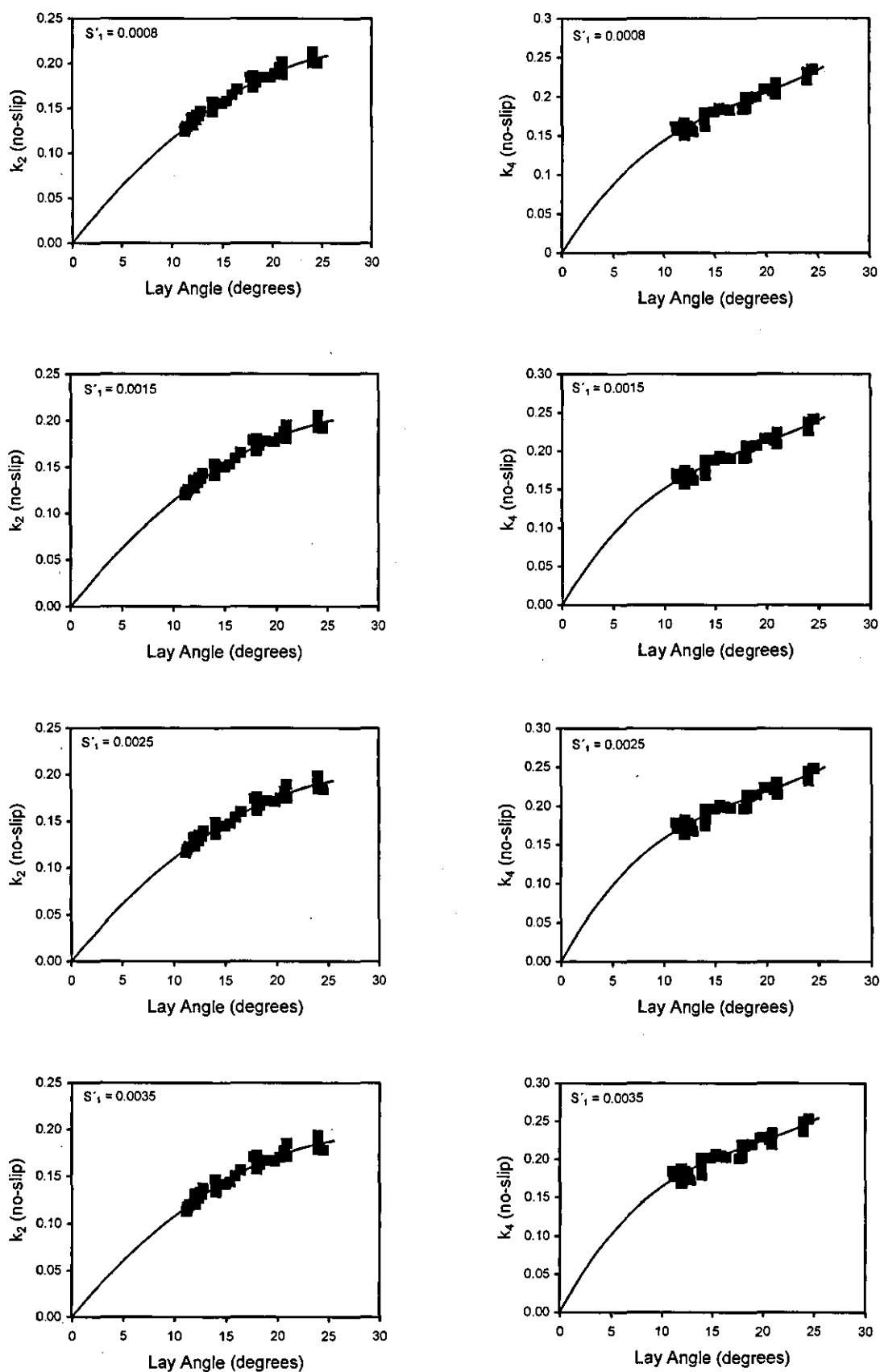


Fig. 3.12f – Variation of the No-Slip Stiffness Coefficients, k_2 and k_4 , with Changes in the Lay Angle, at Various Levels of Cable Mean Axial Strain, S'_1 , as Calculated Using both the RH and RH₂ Models.

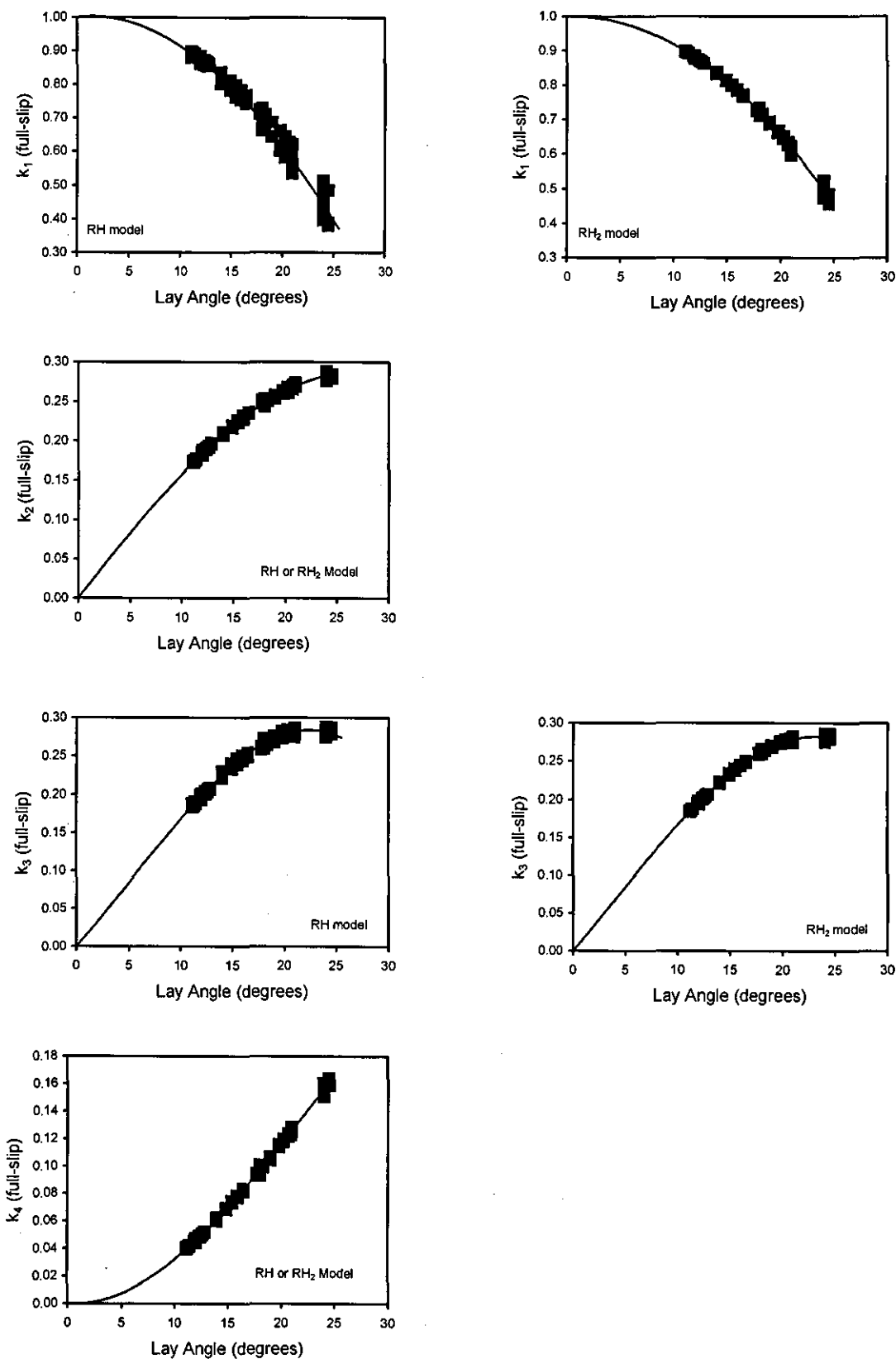


Fig. 3.13a – Variation of the Full-Slip Stiffness Coefficients, $k_1 - k_4$, with Changes in the Lay Angle, at Various Levels of Cable Mean Axial Strain $0.0008 \leq S'_1 \leq 0.0035$, as Calculated Using both the RH and RH₂ Models.

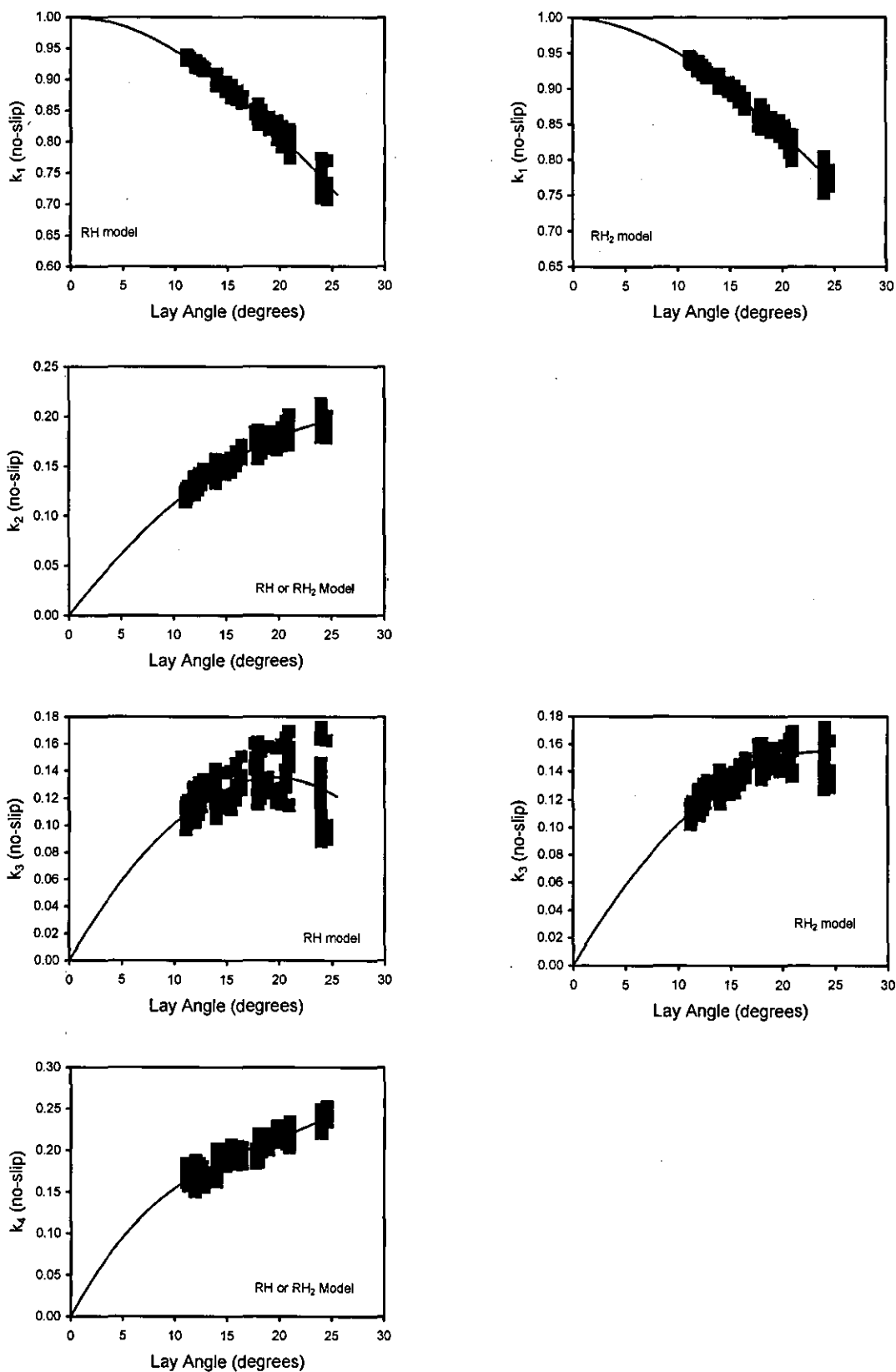


Fig. 3.13b – Variation of the No-Slip Stiffness Coefficients, $k_1 - k_4$, with Changes in the Lay Angle, at Various Levels of Cable Mean Axial Strain $0.0008 \leq S'_1 \leq 0.0035$, as Calculated Using both the RH and RH_2 Models.

3.5 DISCUSSION

The orthotropic sheet theoretical model of Hobbs and Raoof (1982) has previously been used to obtain theoretical estimates of the axial, torsional and free-field bending stiffnesses plus the associated hysteresis of axially pre-loaded spiral strands experiencing superimposed external load perturbations. The comparisons between various theoretical results and carefully obtained large scale experimental data, have been found to be very encouraging. The present comparative analysis provides theoretical estimates and, hence, comparisons between the various strand properties using both the RH and RH₂ (including the proposed assumptions of the current analysis) models, featuring the modifications proposed by Jolicoeur.

The RH₂ model gives results featuring a slight increase in the axial stiffness (Figs. 3.5a and b) and a slight decrease in the axial hysteresis results (Figs. 3.7a and b) for the different levels of cable mean axial strain, S'_1 . These changes are slight, and considering the uncertainties regarding the exact value of the Young's modulus for steel, E_{steel} , and the assumed interwire coefficient of friction, μ , the discrepancies between the RH and the RH₂ model are not believed to be practically significant.

The main difference between the results from the two (RH and RH₂) models is in the value of the coupling coefficient, A_3 . From Figs. 3.9a and b (which compare the overall strand coupling coefficients at two different levels of cable axial strain $S'_1 = 0.0015$ and 0.0035) it appears that the RH₂ model gives a more accurate prediction of A_3 (i.e. A_3 is closer to A_2), and the so-obtained stiffness matrix will be more symmetrical. The differences between the RH and RH₂ predictions of A_3 coefficients appear to be more pronounced for the no-slip case, especially as the lay angle increases from 12° to 24° . Appearances can, however, be deceptive and upon inspection of the full-slip layer stiffness coefficients (Figs. 3.10a and b) it is clear that the differences, if any, between the two models is not as significant as when comparing the overall full-slip coupling coefficients. An examination of the no-slip layer stiffness coefficients (Figs. 3.11), on the other hand, shows that, once again, the overall coupling coefficients can be misleading. Although the RH₂ model ultimately leads to a more symmetrical no-slip stiffness matrix (even for the more sensible layer stiffness coefficients), the differences between the RH and RH₂ predictions of k_3 are

not thought to be practically significant, especially at lower values of cable mean axial strain, and at practically relevant ranges of lay angles, $\alpha < 20^\circ$.

In his publication, Jolicoeur (1997), did not clarify the method used to obtain the values of the stresses T_1 and T_2 needed for the tensorial rotation. As previously stated, the only way, to the present authors knowledge, of obtaining these stresses is by numerical procedures as developed by Hobbs and Raoof (1982), and then, inserting the results into the formula proposed by Jolicoeur (Eqn. (3.56)). It is probable that, in this respect, Jolicoeur has misunderstood the formulations of the original RH model.

Jolicoeur suggested that if the tensorial properties of the stresses are known in the layer system of co-ordinates, they can be known in the strand system of co-ordinates by the process of tensorial rotation. The problem with this is that the ever present curvature of the strand in the hoop direction is not, then, taken into account. In the original formulations of the RH model, this curvature was also ignored when using Hearmon's notation to transform the stresses, but only for determining the stiffnesses, once T_1 and T_2 have been calculated. For the case of the stiffnesses it was found (by experimental – theoretical comparisons) that ignoring the effects of curvature in the hoop direction produced only minor errors in the theoretical predictions. In Jolicoeurs model, the curvature in the hoop direction has been ignored for both the stiffness calculations and the calculation of c_k . In the present work, however, it has been shown that ignoring the hoop curvature in the calculations of c_k , via Eqn. (3.56), can lead (in some cases) to positive values of c_k which, obviously, violate the physical reality for the axially preloaded spiral strand inside which the normal interwire line-contact forces, in the hoop direction, cannot be tensile.

The RH_2 model does give rise to a more symmetrical no-slip stiffness matrix and leads to very similar predictions (cf. RH model) for the axial and torsional stiffnesses and hysteresis, but considering the anomalies in the theory of the RH_2 model as highlighted above and the fact that the original RH model has been extensively verified, experimentally, over the years, the RH_2 model is believed to lack theoretical rigour, and is not an improvement over the RH model.

Finally, it should at once be noted that, unlike Jolicoeur who has based his main conclusions on the construction details relating to only a few spiral strand constructions with a very limited range of lay angles, which incidentally are closer to the manufacturing lower limit for this parameter, the present work is based on the analysis of strand constructions which (most importantly) cover the full range of manufacturing limits for this sole controlling factor – i.e. $12^\circ \leq \alpha \leq 24^\circ$, where, particularly the larger values of the lay angle, have been instrumental in highlighting any major potential differences between the RH and RH₂ models..

3.6 CONCLUSIONS

A comparative analysis was carried out using two models: the previously reported orthotropic sheet theoretical model of Raoof and Hobbs (RH), and a slightly modified version of the RH model, the RH₂ model, as recently proposed by Jolicoeur. The analysis involved conducting an extensive series of theoretical parametric studies on a wide range of multi-layered spiral strand constructions, covering a wide range of cable (and wire) diameters and lay angles. This is in stark contrast to the work of Jolicoeur who restricted his analysis to seven wire strands and one multi-layered strand, thus limiting his findings to a very narrow range of lay angles (which is the sole controlling geometrical parameter).

The proposed formulations of Jolicoeur completely ignore the presence of strand curvature, inherent in strand modelling, not only for the determination of the stiffness coefficients, but also for the calculation of the crucial parameter, c_k : it has been shown that using the formulation of Jolicoeur to calculate c_k , leads, in the majority of cases, to positive values for this parameter, which violates the physical reality for the cable.

In spite of the oversights in its formulations, the results of the RH₂ model compare favourably with those of the RH model. The differences between the predictions of the axial and/or torsional stiffnesses and associated frictional hysteresis based on either the RH or the RH₂ models, as demonstrated by the numerical results, are thought to be not practically significant. In view of the fact that the RH model has been extensively verified by experimental results, and because of the oversights in the formulations of Jolicoeur (perhaps, as a result of Jolicoeur misunderstanding the

original formulations of Hobbs and Raoof), the RH_2 model is not thought to be an improvement over the RH model

CHAPTER 4

IMPACT LOADING OF HELICAL CABLES

4.1 INTRODUCTION

The orthotropic sheet theory has previously been reported by Raoof and his associates, (Raoof and Hobbs, 1988b, and Raoof and Kraincanic, 1995b), for obtaining reliable estimates of the coupled axial/torsional stiffnesses for axially preloaded spiral strands and wire ropes, which (as discussed in chapter 3) have been found to vary between the two limiting values of full-slip and no-slip, as a function of the external load perturbations. Very briefly, the axial and torsional stiffnesses for small load changes have been shown to be significantly larger than for large load changes, because small load disturbances do not induce interwire slippage. In the presence of interwire friction, and for sufficiently small external load disturbances, the wires stick together, and the cable will effectively behave as a solid rod (with allowance being made for the presence of gaps between the individual wires): these conditions are known as the no-slip regime. When large variations in the external load take place, with its associated large changes in the interwire contact forces within the various layers of helical wires, the tangential force changes between the round wires in line-contact will be large enough to overcome interwire friction and induce sliding movements on the interwire line-contact patches: these conditions are, on the other hand, known as the full-slip regime. Obviously, a large number of axial stiffness results have traditionally been provided by the cable manufacturers based on their shop measurements, however, such results invariably relate to the full-slip axial stiffness in the present terminology.

In this chapter, analytical (i.e. closed-form) solutions will be reported for the response of large diameter and multi-layered spiral strands or wire ropes to impact loading, with a detailed analysis of the coupled extensional/torsional wave propagations (based on the no-slip and/or full-slip constitutive relations) along the cable. The numerical results presented in this chapter will emphasize the fact that the previously discussed controlling (i.e. first order) effect of the lay angle on the axial/torsional full-slip and no-slip stiffnesses has a practically significant influence on various wave propagation characteristics of axially preloaded helical cables.

4.2 THEORY

4.2.1 Constitutive Relations for Helical Cables

This topic has been addressed in considerable detail elsewhere (Raoof and Hobbs, 1988b, and Raoof and Kraincanic, 1995b) with its salient features presented in chapter 3 of this thesis, where for the extreme cases of either no-slip and/or full-slip, for the constitutive equations relating the cable tension, F , and torque, M , to the cable deformations, it has been postulated that

$$F = A_1 \epsilon + A_2 \Gamma \quad (4.1a)$$

$$M = A_3 \epsilon + A_4 \Gamma \quad (4.1b)$$

where A_1 , A_2 , A_3 and A_4 are constitutive constants dependent on both the cable material and construction. In Eqns. (4.1), ϵ = axial strain = $\partial u / \partial x$, and Γ = twist per unit length = $\partial \theta / \partial x$.

Experimental measurements have verified the postulated linear form of the constitutive equations for the full-slip case. In particular, it is shown that within experimental accuracy, $A_2 \approx A_3$, which (as discussed in chapter 3) is compatible with the Maxwell-Betti reciprocal theorem for linear elastic structures. Raoof and Kraincanic (1995a) and Raoof (1991c) provide a simple means of obtaining the no-slip and full-slip predictions of A_1 - A_4 for axially preloaded spiral strands, while Raoof and Kraincanic (1995b) give a detailed account of a theoretical model for estimating the no-slip and full-slip values of A_1 - A_4 for axially preloaded wire ropes with an independent wire rope core. As previously shown, for a given cable construction, the full-slip stiffness coefficients have been found to be (for all practical purposes) independent of the cable mean axial load, with the no-slip stiffness coefficients being a function of the mean axial tension on the helical cable.

4.2.2 Dynamic Analysis

The theory regarding the unit-step and triangular impact loading functions has been covered in great detail elsewhere (Raoof et al., 1994, Huang, 1993), but, for completeness, the theory will be repeated here, along with certain new formulations

for the half-sine impact loading function, which have not previously been reported by others.

4.2.2.1 Equations of Motion

The equations of motion for a coupled system with constitutive relations as defined in the previous section, are (Raoof et al., 1994)

$$m \frac{\partial^2 u}{\partial t^2} = A_1 \frac{\partial^2 u}{\partial x^2} + A_2 \frac{\partial^2 \theta}{\partial x^2} \quad (4.2a)$$

$$I \frac{\partial^2 \theta}{\partial t^2} = A_3 \frac{\partial^2 u}{\partial x^2} + A_4 \frac{\partial^2 \theta}{\partial x^2} \quad (4.2b)$$

where, m is the mass per unit length

$$m = A\rho \quad (4.3)$$

and where, ρ is the density of steel, and A is the area of the steel in the spiral strand (including the core wires) as defined by Equations (3.49, 3.47 and/or 3.48). I is the mass moment about the cable axis, per unit length of the structure in the unloaded configuration

$$I = 2I_{xx}\rho \quad (4.4)$$

where,

$$\begin{aligned} I_{xx} &= \sum_{i=1}^N I_{ni} \\ &= \sum_{i=1}^N \left[\frac{\pi^2}{256} \left[[2r_i + D_i]^4 - [2r_i - D_i]^4 \right] \right] \end{aligned} \quad (4.5)$$

θ is the angular rotation of the cable whose longitudinal displacement is u , and t = time.

4.2.2.2 Response to Impact Loading

Jiang et al. (1990) considered a coupled system, with one end fixed at $x = 0$, and subjected to sinusoidal forms of the excitation functions for axial force $F_0(t)$ and torque $M_0(t)$ at the other end $x = h$, assuming the following boundary conditions

$$u(0, t) = 0, \quad \theta(0, t) = 0 \quad (4.6a)$$

$$A_1 \frac{\partial u}{\partial x} + A_2 \frac{\partial \theta}{\partial x} \Big|_{x=h} = F_0(t) \quad (4.6b)$$

$$A_3 \frac{\partial u}{\partial x} + A_4 \frac{\partial \theta}{\partial x} \Big|_{x=h} = M_0(t) \quad (4.6c)$$

while at time $t = 0$, there was assumed to be no motion, with the associated boundary conditions

$$u(x, 0) = 0, \quad \theta(x, 0) = 0 \quad (4.7a)$$

$$\frac{\partial u(x, 0)}{\partial t} = 0, \quad \frac{\partial \theta(x, 0)}{\partial t} = 0. \quad (4.7b)$$

For the above boundary conditions, the solution to the equations of motion (i.e. Eqns. 4.2a, b) is given by Jiang et al. (1990) as

$$u(x, t) = \sum_{i=1}^2 [a_i F_i(x, t) + c_i M_i(x, t)] \quad (4.8a)$$

$$\theta(x, t) = \sum_{i=1}^2 [b_i F_i(x, t) + d_i M_i(x, t)] \quad (4.8b)$$

with

$$F_i(x, t) = \frac{4\omega_i}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \sin \frac{(2n+1)\pi x}{2h} \times \int_0^t F_0(t-z) \sin \frac{(2n+1)\pi z}{2\omega_i} dz \quad (4.9a)$$

and

$$M_i(x, t) = \frac{4\omega_i}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \sin \frac{(2n+1)\pi x}{2h} \times \int_0^t M_0(t-z) \sin \frac{(2n+1)\pi z}{2\omega_i} dz \quad (4.9b)$$

where

$$a_i = \frac{1}{2mh} \left[1 \pm \frac{IA_1 - mA_4}{\sqrt{(IA_1 - mA_4)^2 + 4mIA_2A_3}} \right] \quad (4.10a)$$

$$b_i = \frac{\pm A_3}{h\sqrt{(IA_1 - mA_4)^2 + 4mIA_2A_3}} \quad (4.10b)$$

$$c_i = \frac{\pm A_2}{h\sqrt{(IA_1 - mA_4)^2 + 4mIA_2A_3}} \quad (4.10c)$$

where, $i = 1, 2$, and the positive and negative signs in Equations (4.10a - c) correspond to cases with $i = 1$ and $i = 2$, respectively, and

$$d_i = \frac{1}{2Ih} \left[1 \mp \frac{IA_1 - mA_4}{\sqrt{(IA_1 - mA_4)^2 + 4mIA_2A_3}} \right] \quad (4.11a)$$

$$\omega_i^2 = \frac{h^2 \left[(IA_1 + mA_4) \mp \sqrt{(IA_1 - mA_4)^2 + 4mIA_2A_3} \right]}{2(A_1A_4 - A_2A_3)} \quad (4.11b)$$

where, $i = 1, 2$, and the negative and positive signs in Equations (4.11a, b) correspond to cases with $i = 1$ and $i = 2$, respectively.

Thus, for a given excitation - i.e. with $F_0(t)$ and $M_0(t)$ defined, the values of $u(x,t)$ and $\theta(x,t)$ may be found.

In what follows, three different types of impact loading functions of the general form

$$F_0(t) = F_0 g(t) \quad (4.12a)$$

$$M_0(t) = M_0 g(t) \quad (4.12b)$$

are considered, where F_0 and M_0 are the amplitudes of the external load disturbances. Three distinctly different cases of $g(t)$ are used. In the first instance, a unit-step function for $g(t)$ defined as

$$g(t) = \begin{cases} 1, & 0 \leq t \leq A \\ 0, & A < t < \infty \end{cases} \quad (4.13)$$

is assumed. Using Eqns. (4.12a, b) and (4.13) the following has been found by Raoof et al. (1994)

$$u(x,t) = \sum_{i=1}^2 (F_0 a_i + M_0 c_i) w_i(x,t) \quad (4.14a)$$

$$\theta(x,t) = \sum_{i=1}^2 (F_0 b_i + M_0 d_i) w_i(x,t) \quad (4.14b)$$

with

$$w_i(x,t) = \left(\frac{4\omega_i}{\pi} \right)^2 \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} \sin \frac{(2n+1)\pi A}{4\omega_i} \\ \times \sin \frac{(2n+1)\pi x}{2h} \sin \frac{(2n+1)\pi (2t-A)}{4\omega_i}, \quad i = 1, 2 \quad (4.15)$$

A is the duration of the impact loading and h is the length of the cable.

For a triangular impact function, $g(t)$ in Eqns. (4.12a, b) may be expressed in the form

$$g(t) = \begin{cases} \frac{-1}{A}(t - A), & 0 \leq t \leq A \\ 0, & A < t < \infty. \end{cases} \quad (4.16)$$

The solution for $u(x, t)$ and $\theta(x, t)$ can be expressed in the same form as those in Eqns. (4.14a, b) with the formulations for parameter $w_i(x, t)$ given as (Raoof et al., 1994)

$$\begin{aligned} w_i(x, t) = \sum_{n=0}^{\infty} \sin \frac{(2n+1)\pi x}{2h} & \left[\frac{(4\omega_i)^3 (-1)^n}{2A(2n+1)^3 \pi^3} \cos \frac{(2n+1)(2t-A)\pi}{4\omega_i} \sin \frac{(2n+1)\pi A}{4\omega_i} \right] \\ & + \frac{(4\omega_i)^2 (-1)^n}{(2n+1)^2 \pi^2} \sin \left[\frac{(2n+1)\pi A}{4\omega_i} \right] \sin \left[\frac{(2n+1)\pi}{4\omega_i} (2t-A) \right] - \frac{8\omega_i^2 (-1)^n}{(2n+1)^2 \pi^2} \\ & \times \cos \frac{(2n+1)(t-A)\pi}{2\omega_i} \end{aligned} \quad (4.17)$$

Alternatively, for a half-sine impact function, $g(t)$ in Eqns. (4.12a, b) may be expressed in the form

$$g(t) = \begin{cases} \sin \frac{\pi t}{A}, & 0 < t < A \\ 0, & A < t < \infty \end{cases} \quad (4.18)$$

The solution for $u(x, t)$ and $\theta(x, t)$ can also be expressed in the same form as those in Eqns. (4.14a, b) with the parameter $w_i(x, t)$, for this form of impact loading function in Eqns. (4.14a, b), given by

$$\begin{aligned} w_i(x, t) = \frac{8A\omega_i}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(1 - [(2n+1)A/2\omega_i]^2)} \\ \times \cos \frac{(2n+1)\pi A}{4\omega_i} \times \sin \frac{(2n+1)\pi x}{2h} \times \sin \frac{(2n+1)\pi(2t-A)}{4\omega_i} \end{aligned} \quad (4.19)$$

4.2.2.3 Speed of Axial and Torsional Waves

For the unit-step, triangular and half-sine impact loading functions, the equations defining $w_i(x, t)$, may be rewritten in alternative forms. For the unit-step impact loading function, the alternative form of Eqn. (4.19) is given as (Raoof et al., 1994)

$$w_i(x, t) = \sum_{n=0}^{\infty} \frac{2(-1)^n}{\lambda_i^2} \sin \frac{\lambda_i A}{2} \left\{ \cos \left[kx - \lambda_i \left(t - \frac{A}{2} \right) \right] - \cos \left[kx + \lambda_i \left(t - \frac{A}{2} \right) \right] \right\} \quad (4.20)$$

where

$$k = \frac{(2n+1)\pi}{2h} \quad (4.21)$$

and

$$\lambda_i = \frac{(2n+1)\pi}{2\omega_i} \quad i=1,2 \quad (4.22)$$

The speeds of the axial and torsional wave propagations, C_1 and C_2 respectively, are given by

$$C_i = \frac{\lambda_i}{k} \quad i=1,2 \quad (4.23)$$

or

$$C_i = \frac{h}{\omega_i} \quad i=1,2 \quad (4.24)$$

For the triangular impact loading function, on the other hand, Equation (4.17) may be rewritten as follows (Raoof et al., 1994)

$$w_i(x, t) = \sum_{n=0}^{\infty} \frac{(-1)^n}{\lambda_i^2} \left\{ \frac{2}{A\lambda_i} \sin \left(\frac{\lambda_i A}{2} \right) \times \left[\sin \left[kx - \lambda_i \left(t - \frac{A}{2} \right) \right] + \sin \left[kx + \lambda_i \left(t - \frac{A}{2} \right) \right] \right] \right\}$$

$$+ 2\sin\left(\frac{\lambda_i A}{2}\right) \times \left[\cos\left[kx - \lambda_i \left(t - \frac{A}{2}\right)\right] - \cos\left[kx + \lambda_i \left(t - \frac{A}{2}\right)\right] \right] - \sin\left[kx - \lambda_i (t - A)\right] + \sin\left[kx + \lambda_i (t - A)\right] \quad (4.25)$$

For the half-sine impact loading function, the alternative form of Equation (4.19) is given as

$$w_i(x, t) = \frac{4A\omega_i}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(1 - [(2n+1)A/2\omega_i]^2)} \cos\frac{\lambda_i A}{2} \times \left\{ \cos\left[kx - \lambda_i \left(t - \frac{A}{2}\right)\right] - \cos\left[kx + \lambda_i \left(t - \frac{A}{2}\right)\right] \right\} \quad (4.26)$$

As expected, the speeds of the wave propagations do not depend on the type of externally applied impact loading, and the values of C_1 plus C_2 are the same for the unit-step, triangular and half-sine impact loading functions.

4.3 RESULTS

Numerical results have been obtained for a 39 mm and three 127 mm outside diameter axially preloaded multi-layered spiral strands with the latter strand constructions having lay angles of 12°, 18° and 24°. The construction details for these four strands are given in Tables 3.1b, h, i and j, respectively.

For the present purposes, the spiral strands are all assumed to be 10 m in length, with the 39 mm spiral strand experiencing an axial pretension of 0.41 MN and the three 127 mm spiral strands experiencing a mean axial strain $S'_1 = 0.002867$, which roughly corresponds to one third of their ultimate tensile strengths. The Young's modulus for steel $E_s = 200$ kN/mm², and the corresponding Poisson's ratio $\nu = 0.28$, with the density of steel $\rho = 7850$ kg/m³.

Based on the so-called exact formulations (i.e. not the simplified version) of the orthotropic sheet theory, the full-slip stiffness matrix for the 39 mm outside diameter spiral strand, is

$$\begin{cases} \frac{F}{E_s} = 708.54\epsilon + 351.73\Gamma \\ \frac{M}{E_s} = +352.49\epsilon + 15263.3\Gamma \end{cases}$$

with the corresponding no-slip stiffness matrix, for a mean axial load of 0.41 MN, given as

$$\begin{cases} \frac{F}{E_s} = 823.4\epsilon + 267.64\Gamma \\ \frac{M}{E_s} = +235.6\epsilon + 34483.4\Gamma \end{cases}$$

where, in the above, the units of (mm²), (mm³), (mm³), and (mm⁴) have been used for the constants A₁, A₂, A₃ and A₄, respectively. The full-slip and no-slip constitutive constants for the three 127 mm outside diameter spiral strands are given in Table 4.1.

Table 4.1. – Values of the Full-Slip and No-Slip Constitutive Constants for the Three 127 mm Outside Diameter Spiral Strands, with lay angles of 12°, 18° and 24°, as Calculated Using the Orthotropic Sheet Theory.

	Lay Angle (degrees)	A ₁ (mm ²)	A ₂ (mm ³)	A ₃ (mm ³)	A ₄ (mm ⁴)
Full-Slip	12	8836.60	-2576.17	-3011.91	928782
	18	6860.21	-3602.98	-4325.86	1838043
	24	4520.34	-4889.42	-5193.44	3104408
No-Slip	12	9388.95	-1324.69	-783.62	3447433
	18	8373.84	-1769.99	-428.91	3878215
	24	7491.03	-2879.24	-826.59	4693255

It should be noted that, particularly the no-slip values of A₂ and A₃ in Table 4.1, are (at first sight) not close to each other: this is due to the rather small values of these constants in the nominally torsionally balanced spiral strands in which, although the no-slip A₂ and A₃ constants for individual layers were, indeed, found to be fairly similar, the accumulation of small errors in the course of algebraically adding up the contributions of the counter-laid layers in order to predict the overall values for the whole strand has led to such apparent (although not practically significant) anomalies.

Tables (4.2) and (4.3) give estimates of the axial and torsional natural frequencies ω_1 and ω_2 , axial and torsional wave speeds C_1 and C_2 , respectively, and the ratios of torsional to extensional oscillations R_1 and R_2 (Raoof et al., 1994), for both the no-slip and the full-slip regimes, for the 39 mm and three 127 mm outside diameter spiral strands.

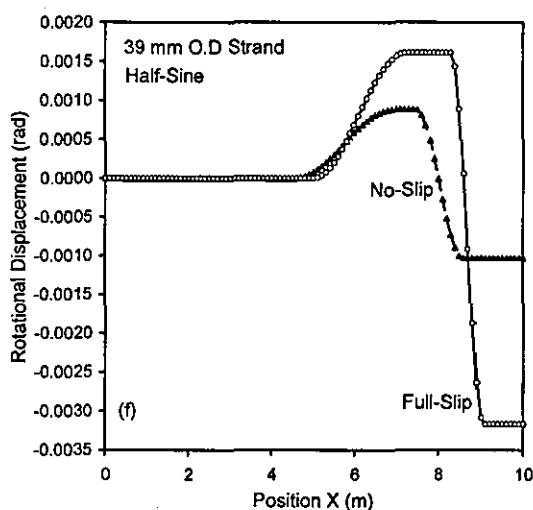
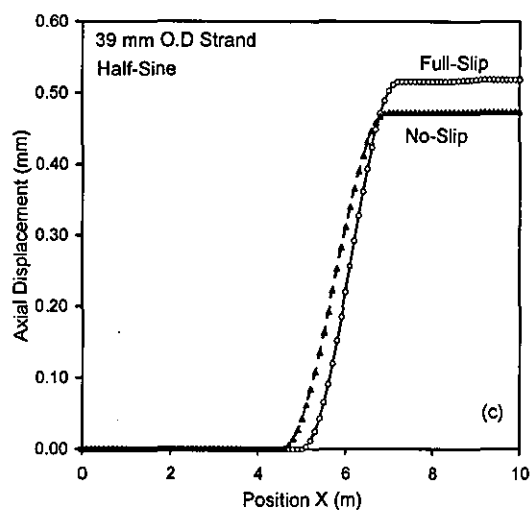
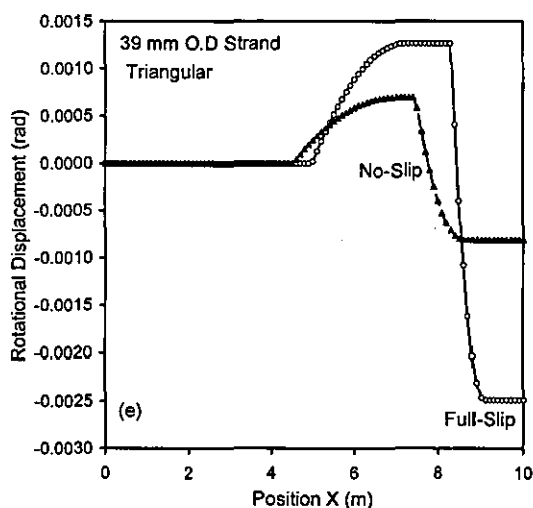
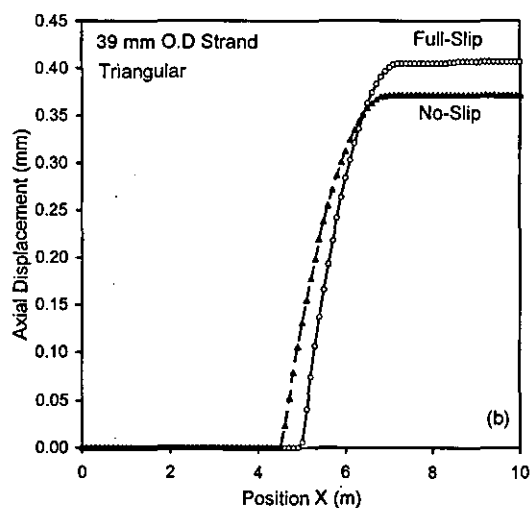
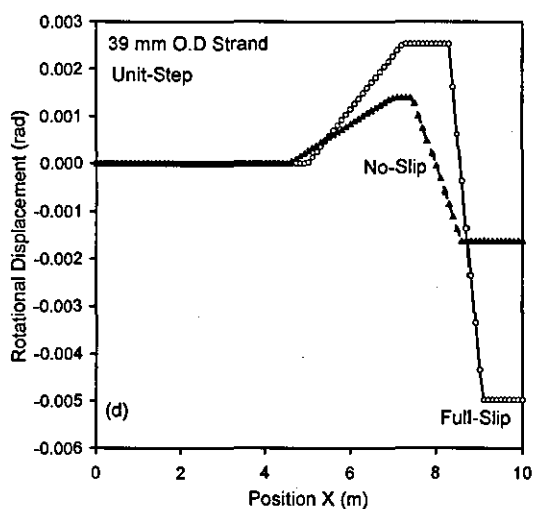
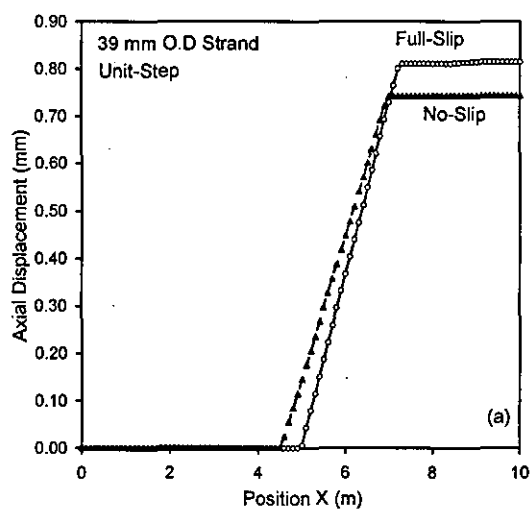
Table 4.2 - Numerical Results for the Axial and Torsional Wave Speeds C_1 and C_2 , the Ratios of Torsional to Extensional Oscillations R_1 and R_2 , and the Natural Frequencies ω_1 and ω_2 for the 39 mm Outside Diameter and Axially Preloaded Spiral Strand, Based on the No-Slip and Full-Slip Assumptions.

	No-Slip	Full-Slip
C_1 (m/sec)	4714	4374.0
C_2 (m/sec)	2186	1447.0
R_1	1.920	2.935
R_2	2414.0	1794.0
ω_1 (rads/sec)	0.002120	0.002290
ω_2 (rads/sec)	0.004570	0.006910

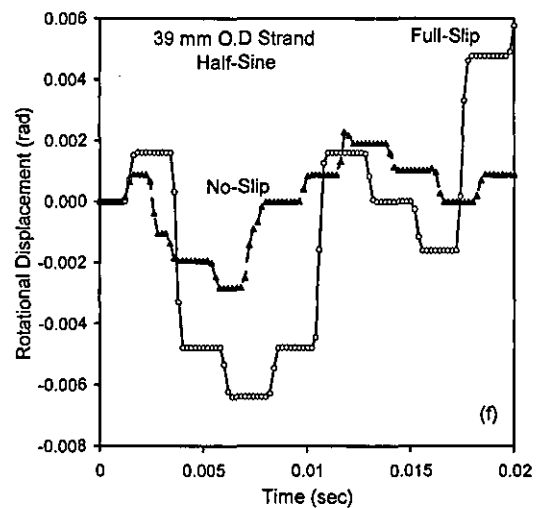
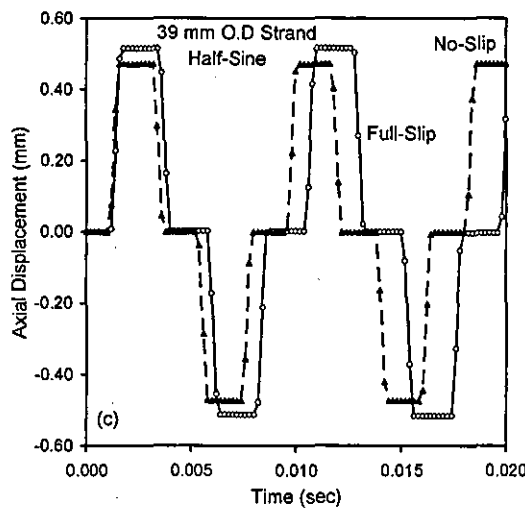
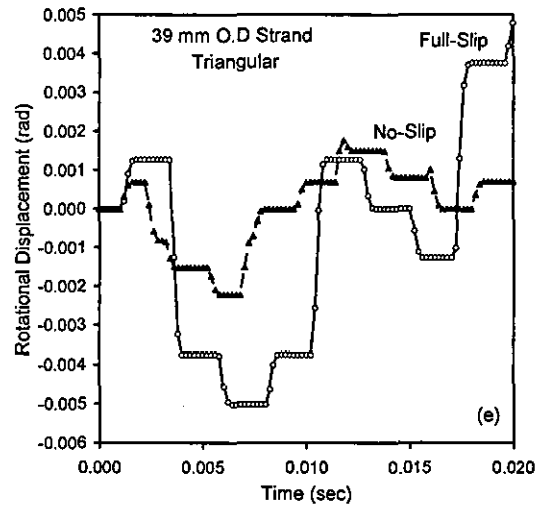
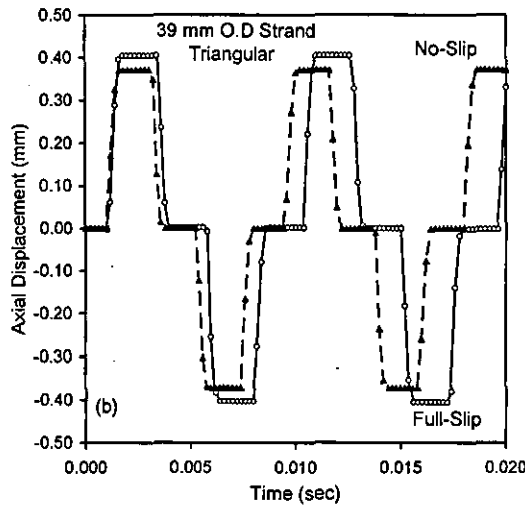
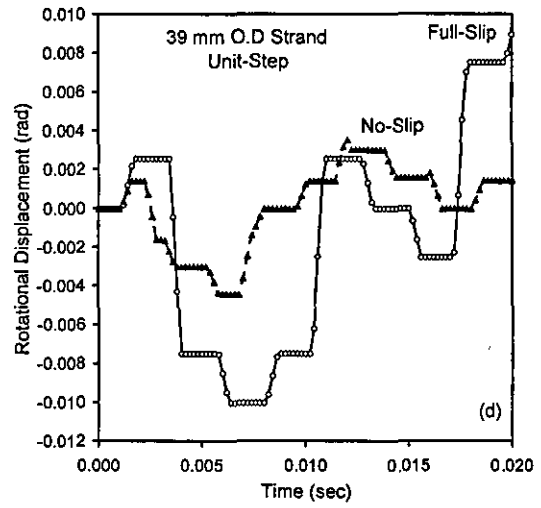
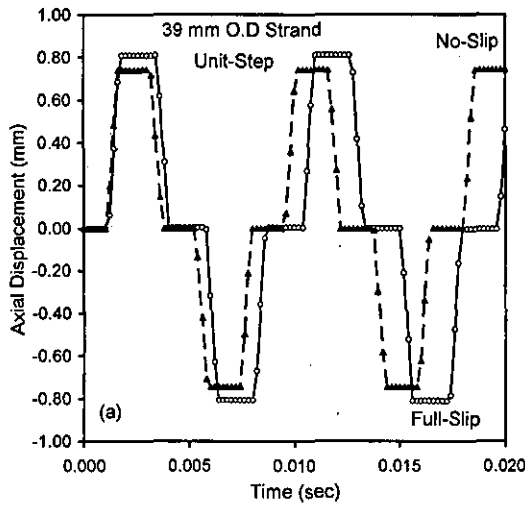
Table 4.3 - Numerical Results for the Axial and Torsional Wave Speeds C_1 and C_2 , the Ratios of Torsional to Extensional Oscillations R_1 and R_2 , and the Natural Frequencies ω_1 and ω_2 for the Three 127 mm Outside Diameter and Axially Preloaded Spiral Strands, Based on the No-Slip and Full-Slip Assumptions.

127 mm outside diameter spiral strand						
	$\alpha = 12$ degrees		$\alpha = 18$ degrees		$\alpha = 24$ degrees	
	Full-Slip	No-Slip	Full-Slip	No-Slip	Full-Slip	No-Slip
C_1 (m/sec)	4197.18	4324.20	3742.83	4134.65	3023.36	3889.41
C_2 (m/sec)	1080.25	2082.23	1567.88	2279.13	1988.52	2448.77
R_1	-0.230	-0.069	-0.502	-0.048	1.283	-0.109
R_2	3202.91	5439.06	1569.83	3293.49	524.02	1570.37
ω_1 (rads/sec)	0.002383	0.002313	0.002672	0.002419	0.003308	0.002571
ω_2 (rads/sec)	0.009257	0.004803	0.006378	0.004388	0.005029	0.004084

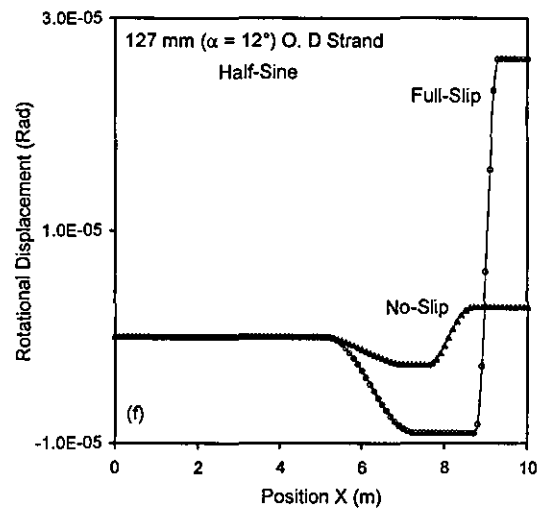
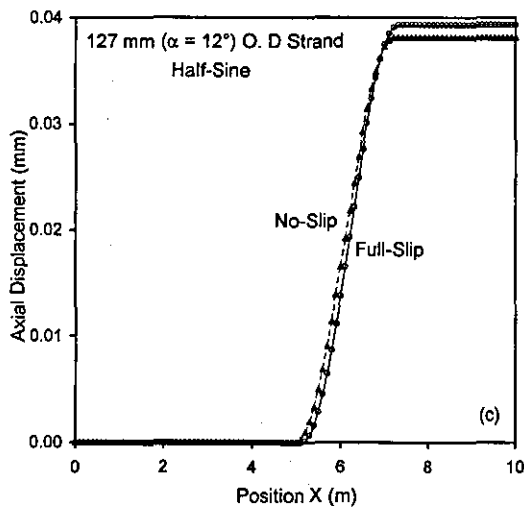
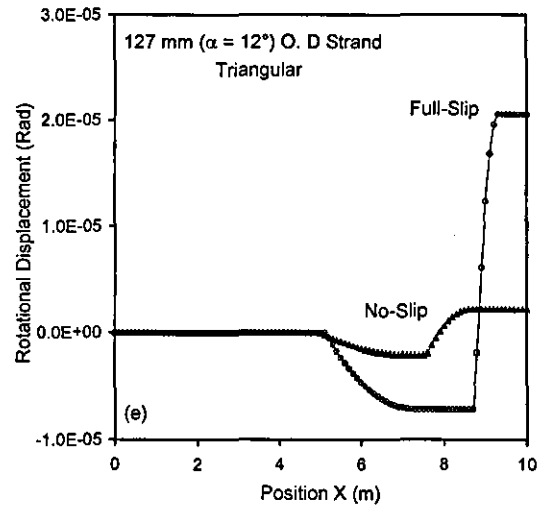
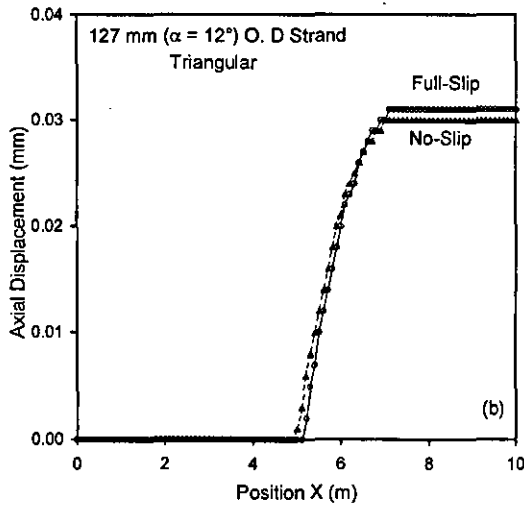
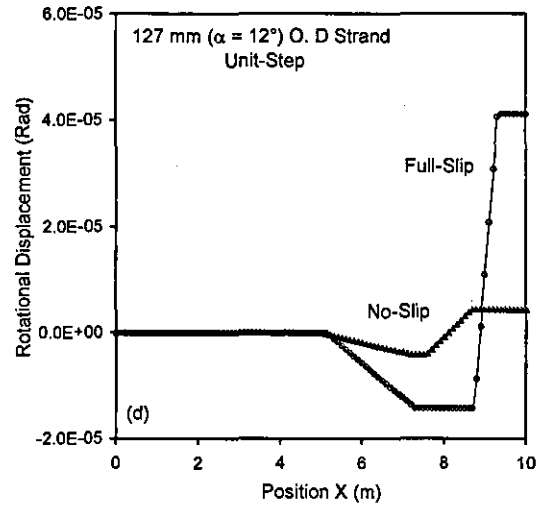
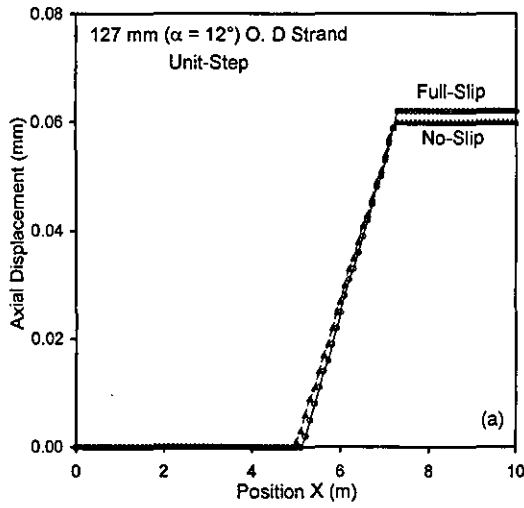
Practically significant differences are found between the no-slip and full-slip solutions in all cases, with Table (4.3) showing the rather significant extent by which certain wave characteristics (such as amplitudes and speeds) differ for $12^\circ \leq \alpha \leq 24^\circ$ depending on whether the full-slip or no-slip solution is adopted. It should be pointed out that all the axial/torsional wave characteristics given in Tables (4.2) and (4.3) are independent of the specific form of impact loading at the end of the cable.



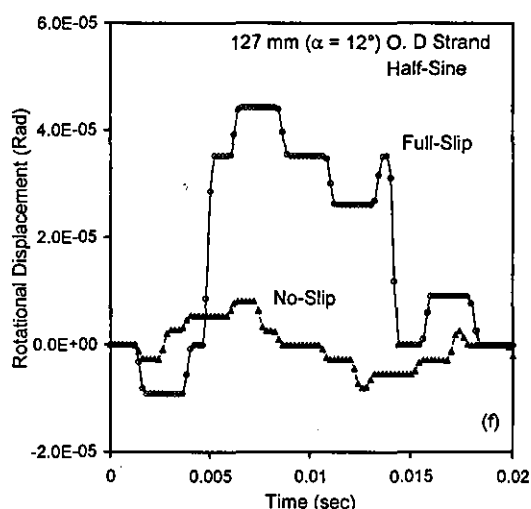
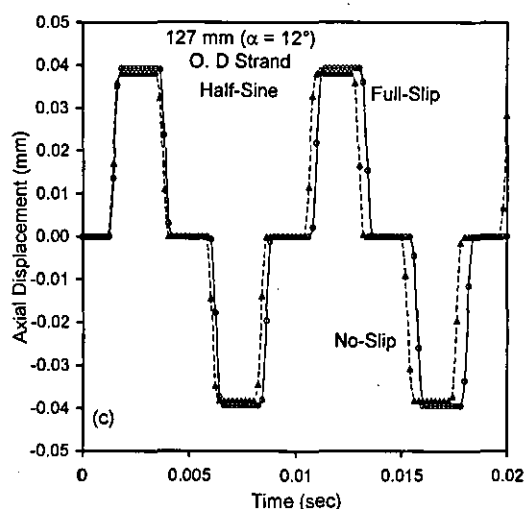
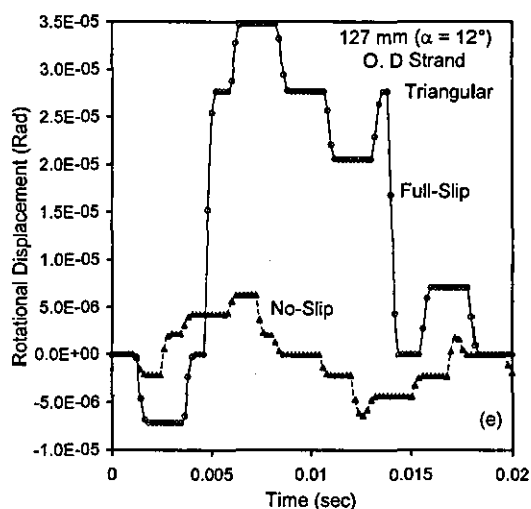
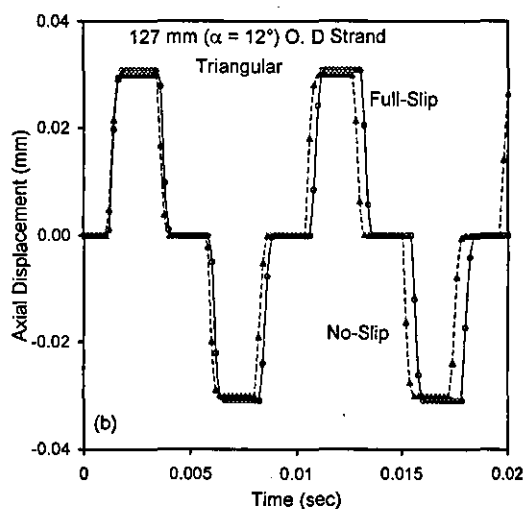
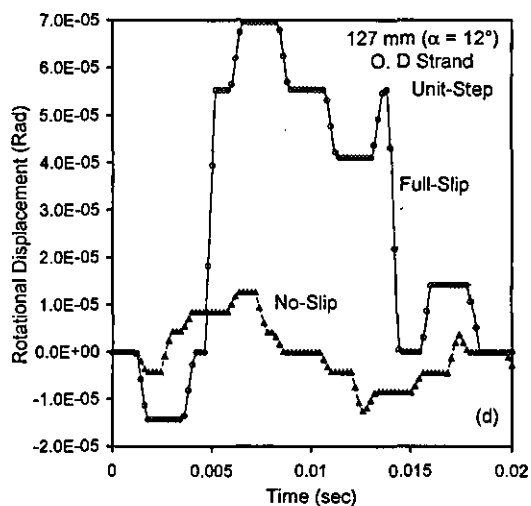
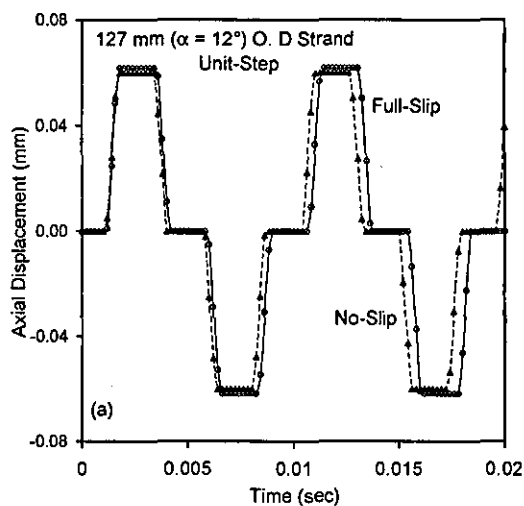
Figs. 4.1(a-c) and (d-f) - Comparison of the Axial and Rotational Displacements, Respectively, Along the Cable at Time $t = 0.001163$ (sec), Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.



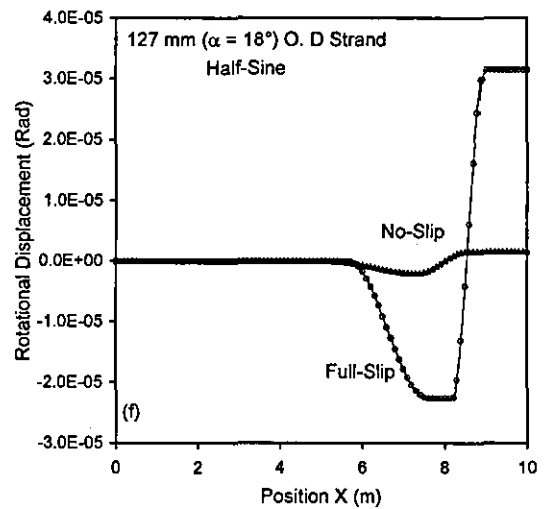
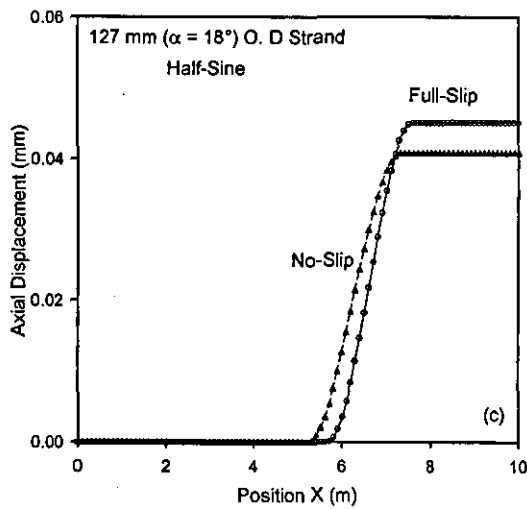
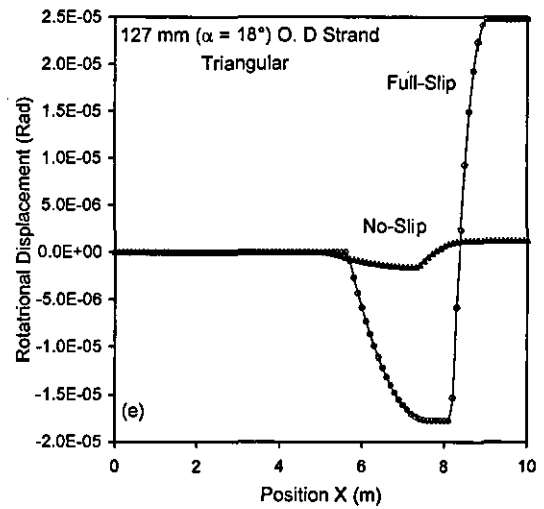
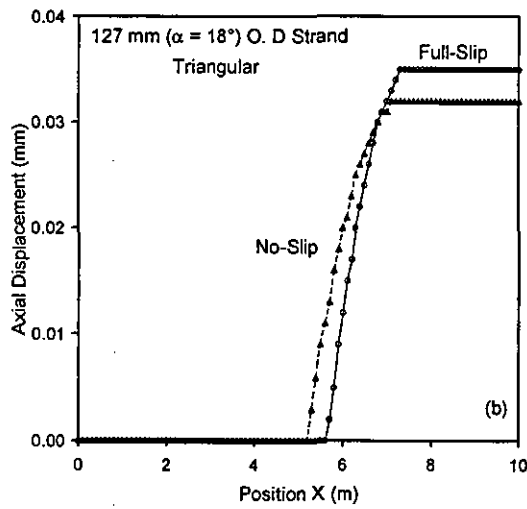
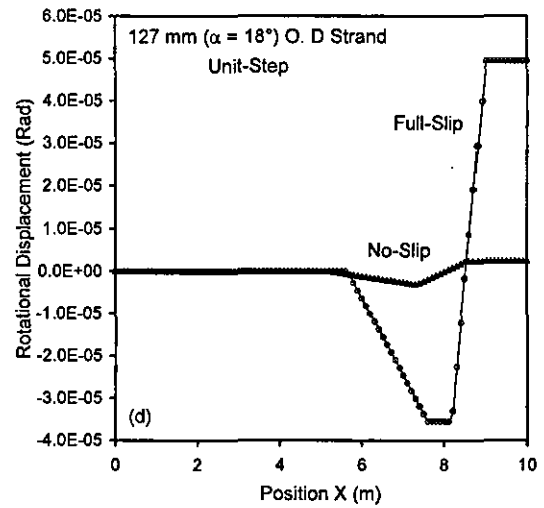
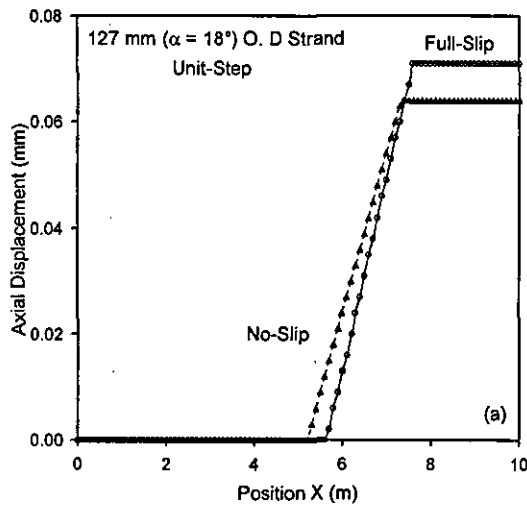
Figs. 4.2(a-c) and (d-f) - Comparison of the Axial and Rotational Displacements, Respectively, at the Middle Point of the Cable $X = 5\text{m}$, as a Function of Time, Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.



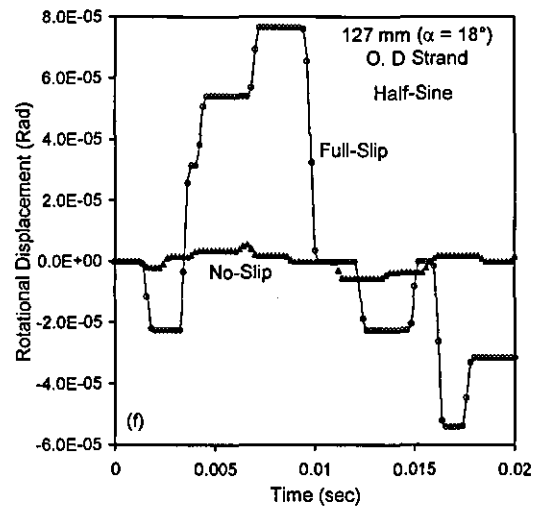
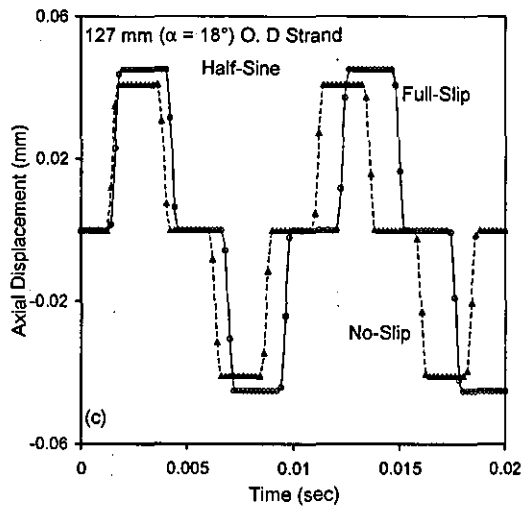
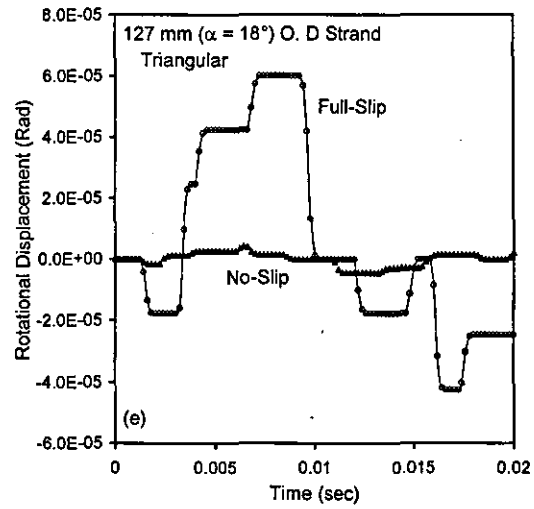
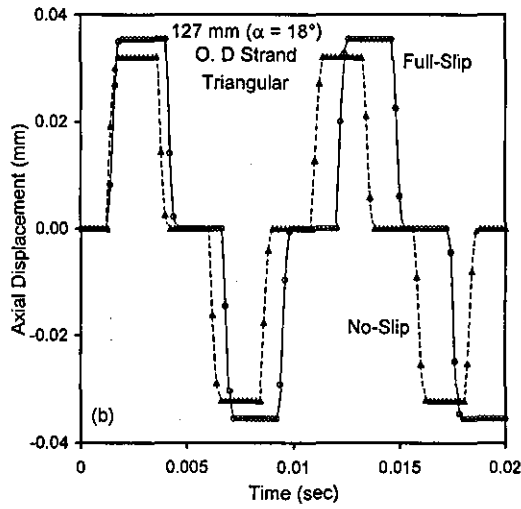
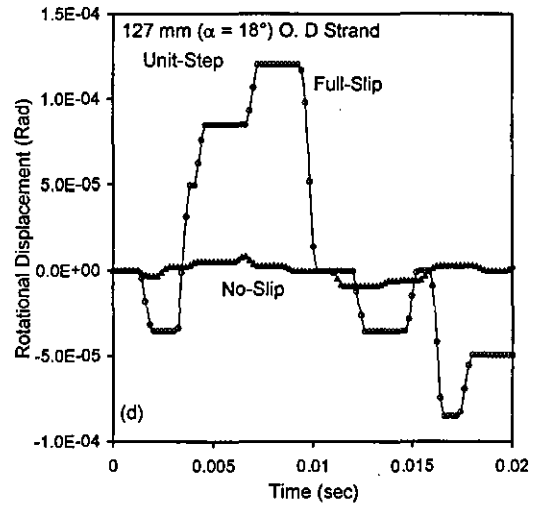
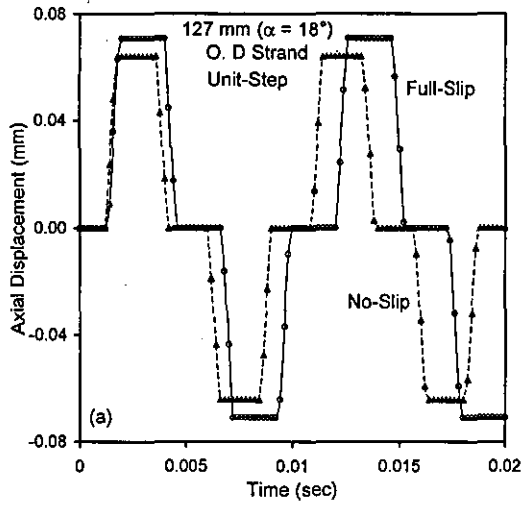
Figs. 4.3(a-c) and (d-f) - Comparison of the Axial and Rotational Displacements, Respectively, Along the Cable at Time $t = 0.001163$ (sec), Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.



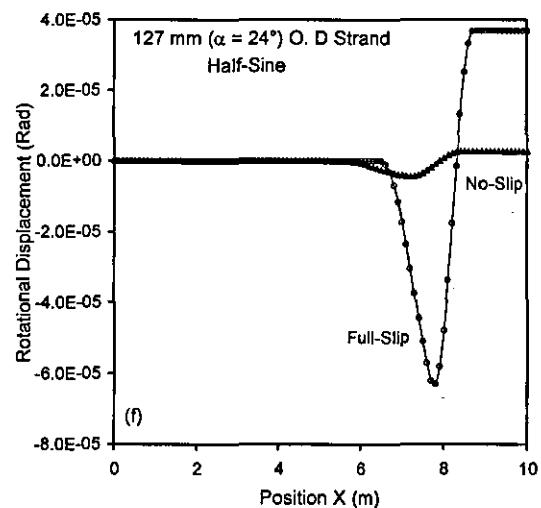
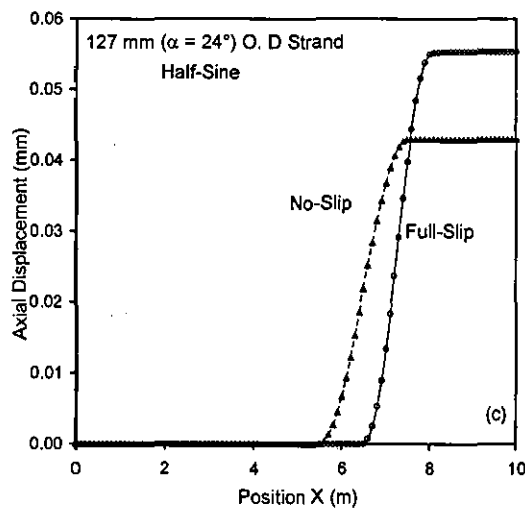
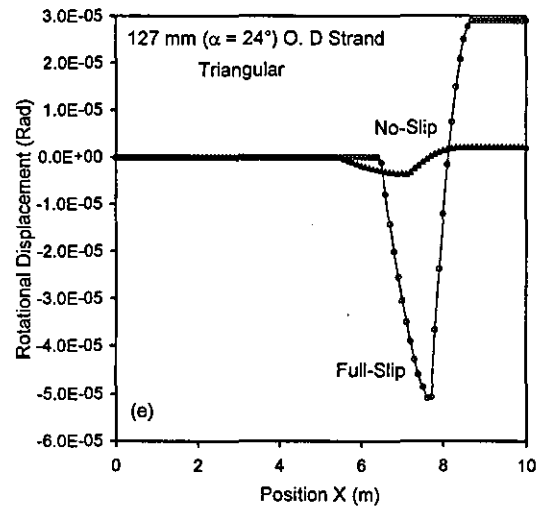
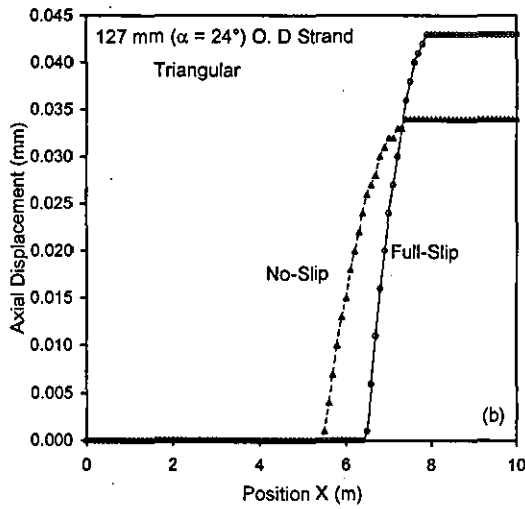
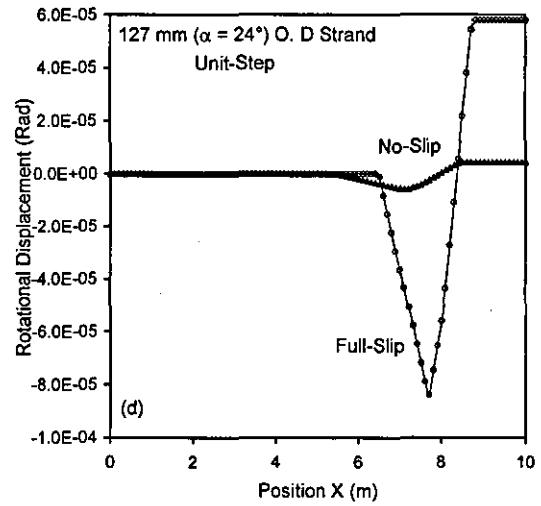
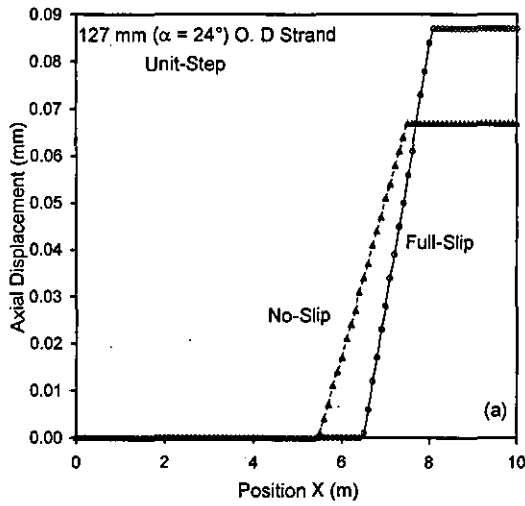
Figs. 4.4(a-c) and (d-f) - Comparison of the Axial and Rotational Displacements, Respectively, at the Middle Point of the Cable $X = 5\text{m}$, as a Function of Time, Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.



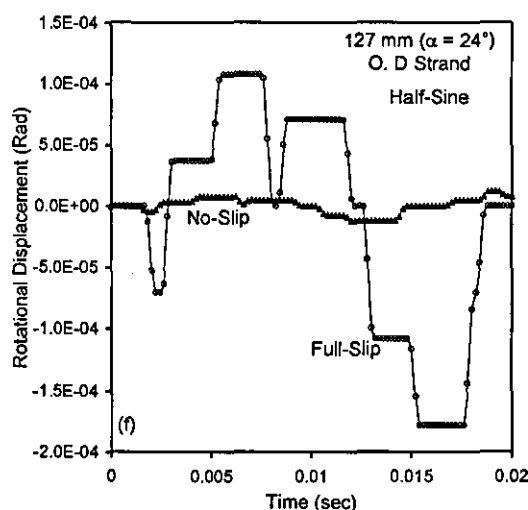
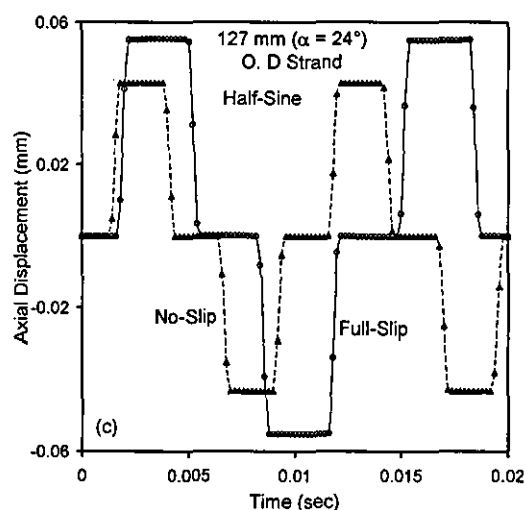
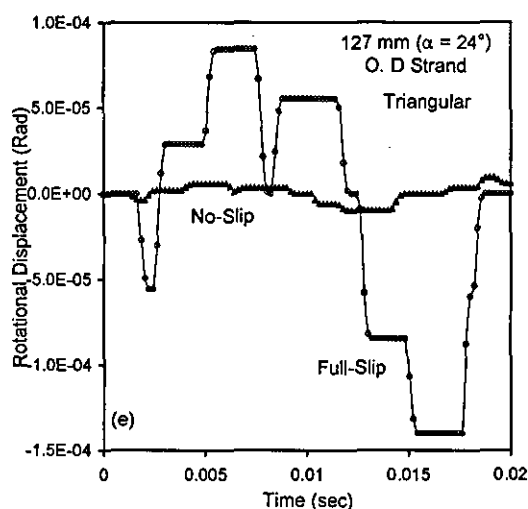
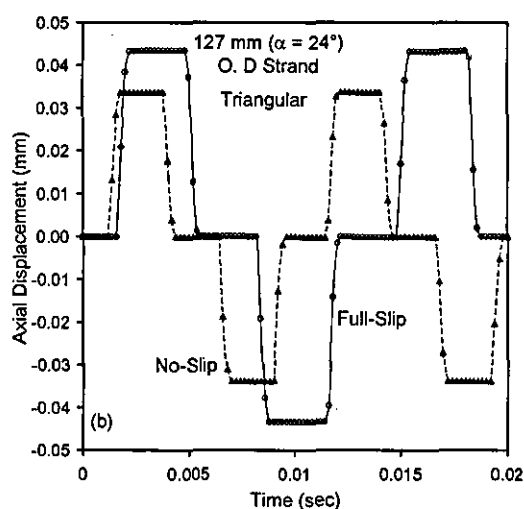
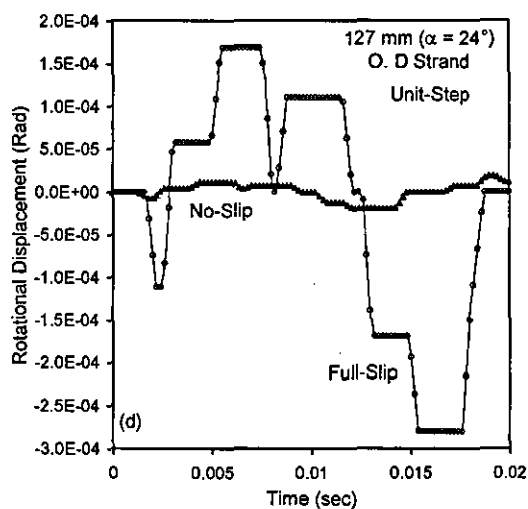
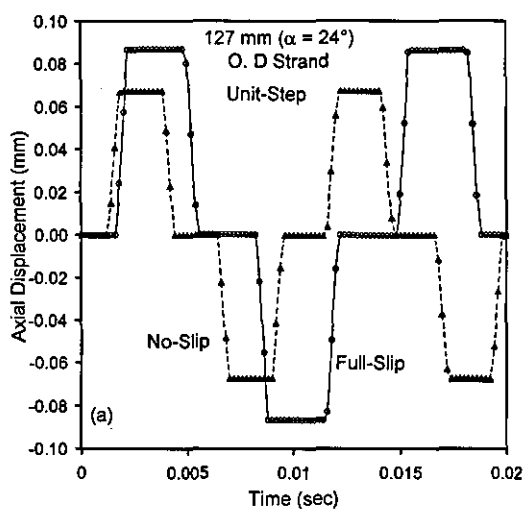
Figs. 4.5(a-c) and (d-f) - Comparison of the Axial and Rotational Displacements, Respectively, Along the Cable at Time $t = 0.001163$ (sec), Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.



Figs. 4.6(a-c) and (d-f) - Comparison of the Axial and Rotational Displacements, Respectively, at the Middle Point of the Cable $X = 5m$, as a Function of Time, Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.



Figs. 4.7a-c) and (d-f) - Comparison of the Axial and Rotational Displacements, Respectively, Along the Cable at Time $t = 0.001163$ (sec), Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.



Figs. 4.8(a-c) and (d-f) - Comparison of the Axial and Rotational Displacements, Respectively, at the Middle Point of the Cable $X = 5\text{m}$, as a Function of Time, Subjected to an Impact Load of Duration $A = 0.00052$ (sec) for the No-Slip and Full-Slip Conditions: (a, d) Unit-Step Loading Function; (b, e) Triangular Loading Function; (c, f) Half-Sine Loading Function.

Figs. 4.1(a-c) compare the variations of the axial displacements at time $t = 0.001163$ sec, for the 39 mm outside diameter spiral strand, based on the full-slip and no-slip solutions, as a function of the coordinates along the cable with the end at $x = 0$ fixed and the end at $x = 10$ m subjected to unit-step, triangular and half-sine impact loading functions, respectively, with $A = 0.00052$ sec, $F_0 = 50$ kN and $M_0 = 0$. The corresponding rotational displacements along this same cable for all three forms of loading functions are presented in Figs. 4.1(d-f). Figs. 4.2(a-c) and 4.2(d-f) present the axial and rotational displacements, respectively, at the centre of the 39 mm diameter cable, for the three types of loading functions, as functions of time, as calculated by the no-slip and full-slip constitutive matrices.

From the results presented in Figs. 4.1(a-f) and 4.2(a-f) it is apparent that there exist some rather significant differences between the no-slip and full-slip solutions.

Figs. 4.3(a-c) show the variations of the axial displacements, at time $t = 0.001163$ sec, along the length of the 127 mm ($\alpha = 12^\circ$) diameter cable for both the full-slip and no-slip regimes, with the end of the cable, at position $X = 0$, fixed and the other end of the cable at $X = 10$ m, subjected to unit-step, triangular and half-sine impact loading functions, respectively, with the duration of the impact load $A = 0.00052$ sec, $F_0 = 50$ kN, and $M_0 = 0$. Figs. 4.3(d-f) show the corresponding rotational displacements along the length of this same cable for the three loading functions, respectively, at time $t = 0.001163$ sec, based on the full-slip and no-slip regimes, as a function of the distance, X , along the cable. Figs. 4.4(a-c) compare the variations of the axial displacements, as a function of time, at the centre ($X = 5$ m) of the 127 mm ($\alpha = 12^\circ$) diameter cable for the unit-step, triangular and half-sine impact loading functions, for both the full-slip and no-slip regimes. Figs. 4.4(d-f) show the corresponding rotational displacements, as a function of time, at the centre of this same cable for the three impact loading functions, based on the full-slip and no-slip regimes.

Figs. 4.5(a-c), 4.6(a-c) and Figs. 4.5(d-f), 4.6(d-f) show the variations of the axial and rotational displacements, as a function of the distance along the cable, and as a function of time at the centre of the cable, respectively, for the 127 mm ($\alpha = 18^\circ$) diameter cable. Similarly, Figs. 4.7(a-c), 4.8(a-c) and Figs. 4.7(d-f), 4.8(d-f) show the

variations of the axial and rotational displacements, as a function of the distance along the cable, and as a function of time at the centre of the cable, respectively, for the 127 mm ($\alpha = 24^\circ$) diameter cable. In all the plots in Figs. 4.1 – 4.8(a-f), the same values of $F_0 = 50$ kN, $M_0 = 0$, $A = 0.00052$ sec, and $t = 0.001163$ sec have been assumed.

Once again, from the graphical results it is evident that some rather significant differences exist between the full-slip and no-slip wave propagation characteristics. An important observation is that as the lay angle increases, within the practical limits, the differences between the two bounding solutions of the various full-slip and no-slip wave propagation characteristics become increasingly more pronounced.

4.4 DISCUSSION

Traditional solutions have invariably adopted the full-slip assumption which, although valid for large levels of external loading and/or newly manufactured (but prestretched) helical cables (Raoof 1990a), fail to provide accurate predictions for cases when the amplitudes of externally applied impact loads on fully bedded-in cables are fairly small. Such cases occur in connection with the non-destructive methods for the in-situ detection of individual wire fractures under, say, axial fatigue loading, whereby the fracture of an individual wire sends a small but measurable shock wave(s) along the cable which is picked up by the electronic black boxes. Most importantly, the present results throw considerable doubt on the validity of the traditional methods for calibrating such so-called electronic black boxes. Very briefly, instrumentation experts calibrate their devices by picking up what they call significant effects, which are (under laboratory conditions) often simulated by deliberately fracturing a wire in a newly manufactured and axially loaded cable at the end of which the black box signals (waves) are picked up. However, in old and fully bedded-in cables, in practice, the cable structure is compacted in such a way that (with an individual wire carrying a small fraction of the total axial load on the cable) the amplitudes and speeds of the axial and torsional waves released by the fracture of an individual wire are governed (because of their small magnitudes) by the no-slip stiffnesses, which are significantly different from the full-slip stiffnesses which govern the behaviour of newly manufactured cables (Raoof 1990a) originally used for calibrating the black boxes. It is, therefore, suggested that caution should be exercised in interpreting the data obtained from such devices under service conditions using the traditional methods of

calibrations based on the full-slip behaviour of newly manufactured cables: in the above, the full-slip wave characteristics, such as amplitudes and speeds are theoretically shown to be significantly different from the corresponding no-slip wave characteristics for a number of assumed (namely, unit-step, triangular and half-sine) loading functions. It is, therefore, suggested that such electronic devices should be calibrated using well bedded-in (old) helical cables which have seen service conditions for a number of years. It is, perhaps, also worth mentioning that the exact form of the loading function relating to the sudden fracture of a wire inside the cable obviously remains unpredictable and very difficult (if at all possible) to determine, using the currently available experimental techniques. However, the use of widely different forms of loading functions, as adopted in the present work, should reasonably cover the range of possibilities, and the final results based on all three types of such loading functions have invariably supported the view that the no-slip wave propagation characteristics are, indeed, significantly different from the corresponding full-slip ones.

4.5 CONCLUSIONS

The current analysis extends the previously reported work of Raoof and his associates, who developed closed-form solutions for predicting the various characteristics of coupled extensional-torsional waves induced by various forms of impact loading at one end of steel helical cables (spiral strands and wire ropes) with the other end fixed against any movement. The analysis of the 39 mm outside diameter spiral strand was conducted for completeness of the work carried out by Raoof et al. (1994), (to include the half-sine impact loading function), who carried out a theoretical study of a spiral strand's response to a unit-step and triangular impact loading function. The theoretical analysis was taken further to provide detailed results based on three different 127 mm outside diameter spiral strands with widely varying lay angles (within current manufacturing limits) to enable the effects of variations in the lay angle on the various wave propagation characteristics to be examined.

It is argued that, due to the presence of interwire friction in axially preloaded helical cables, for sufficiently small levels of load perturbations (due to the fracture of an individual wire) applied to fully bedded-in (old) cables, one should use the no-slip version of the constitutive relations. Significant differences have been found between

a number of axial/torsional wave characteristics induced in cables subjected to unit-step, triangular and half-sine forms of impact loading functions, depending as to whether the no-slip or full-slip version of the constitutive relations are used in the analysis. It is demonstrated that the use of the no-slip version of the constitutive relations is even more critical as the lay angle increases for a given strand construction. The present findings may have significant practical implications in relation to currently adopted techniques by industry for calibrating the electronic boxes, which are subsequently used for the in-situ detection of individual wire fractures under, say, fatigue loading associated with cable supported structures.

CHAPTER 5

RESTRAINED BENDING CHARACTERISTICS OF HELICAL CABLES AT TERMINATIONS

5.1 INTRODUCTION

This chapter is concerned with certain aspects of bending effects in multi-layered spiral strands in the absence of sheaves, fairleads or other formers, so that the radius of curvature of the strand is not predetermined. These conditions will be referred to as “free bending”. Free bending problems are a source of concern (and not infrequent failures) in structures ranging from floating offshore platforms, suspension and cable-stayed bridges, and the stays for guyed masts, to electro-mechanical cables where cable fatigue failures near partially restrained terminations of various types caused by, for example, hydro- or aerodynamic loading are not uncommon.

For the free bending of long steel cables under an approximately steady mean axial load, it is common to introduce a mathematically convenient constant effective bending stiffness, $(EI)_{\text{eff}}$, for the cable, using which, the radii of curvature at the points of restraint are calculated. Traditionally, the maximum bending strains have then been found on the basis of a variety of, frankly, sweeping assumptions: these strains have further been assumed (without any sound experimental and/or theoretical justification) to govern the strand bending fatigue life.

The validity of such maximum (extreme fibre) bending stress approaches (in terms of the simple beam bending theory), however, has fairly recently been questioned by Raoof and his associates (Raoof and Hobbs, 1984, and Raoof, 1990b, 1992c, and 1994a), who have experimentally demonstrated that the neutral axis (as opposed to the extreme fibre position) is the location where the initial wire fractures under restrained cyclic bending invariably occur. Spiral strand restrained bending fatigue failures at the terminations have been shown to be controlled by interwire/interlayer fretting which is greatest, not at the extreme fibre position, but in the vicinity of the so-called neutral axis where wire bending stresses are, indeed, minimal. A simple new design method against restrained bending fatigue has thus been developed (Raoof, 1994a),

which is now included in the Prestandard ENV 1993-2, Eurocode 3: Design of Steel Structures – Part 2: Steel Bridges (October 1997).

There are, however, a number of still unresolved issues which need clarification and some aspects of these form the subject of the present chapter. Here, the practical implications of assuming a constant effective bending stiffness for determining the deflected shape of the strands in the immediate vicinity of fixed (or, rather, nominally fixed) zinc socketed end terminations for cases when axially preloaded multi-layered spiral strands undergo, say, vortex shedding instabilities with maximum lateral deflections of the order of one cable diameter, is addressed.

As regards the spiral strand effective bending stiffness, due to their peculiar construction, axially preloaded spiral strands undergo plane-section bending, only for sufficiently small levels of cable lateral deflections (Raoof, 1992a). Beyond a certain limit of lateral deflection, plane-sections do not remain plane and interlayer slippage takes place, starting from the outer layer, and spreading towards the centre of the strand, depending on the level of axial tension and imposed radius of curvature (Raoof, 1992a). Indeed, in view of the extreme values of curvature at the points of fixity to the cables, one wonders as to whether the traditional method of assuming a constant effective bending stiffness for theoretically predicting the radii of curvature at the fixed terminations is a reasonable approach, providing accurate predictions to be used as an input into the subsequent design calculations against restrained bending fatigue.

Based on carefully conducted large scale experiments on an axially preloaded 39 mm outside diameter multi-layered spiral strand, the deflected shapes of the cable at the terminations have been obtained, covering a wide range of lateral cable deflections. These results have been obtained by Raoof (1992a) who used Poffenberger's extended arm method (Poffenberger and Swart, 1965) (a well known technique in the field of overhead transmission lines) to measure the deflected shapes of the cable in the immediate vicinity of nominally fixed zinc poured sockets.

5.2 EXPERIMENTAL OBSERVATIONS

Full details of the carefully conducted large scale experiments are reported elsewhere (Raoof, 1992a), and only a few of the most relevant observations will be repeated here for completeness.

The free bending tests of Raoof (1983) were carried out on a 7.9 m, axially preloaded 39 mm outside diameter spiral strand with the construction details given in Table (3.1b). In these experiments, the specimen was subjected to a state of loading, as shown in Fig. (5.1).

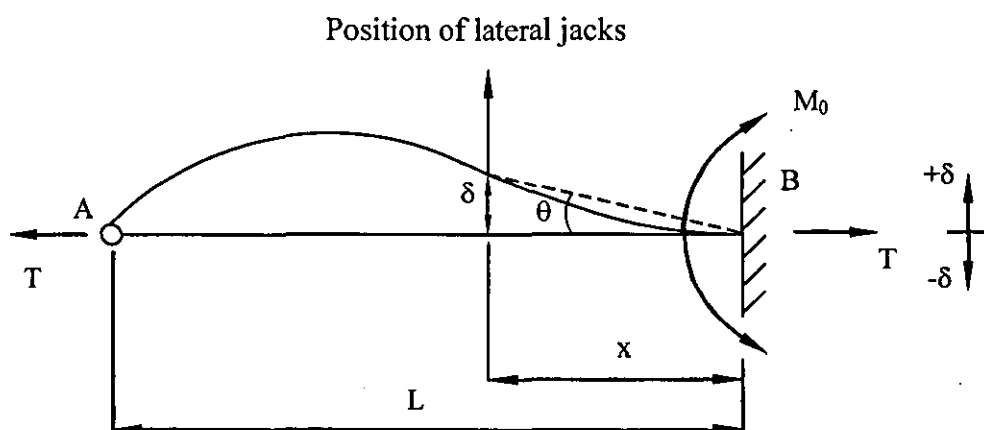


Fig. 5.1 - Deflected Shape of an Axially Preloaded Spiral Strand Subjected to a Single Lateral Point Load.

The strand was terminated at both ends with zinc poured sockets to BS 463 (1958), except for an elongated jaw. The socket at the fixed hold-down (position A in Fig. 5.1) was mounted with its pin vertical. The lateral exciter was placed at $x = 2530$ mm from the face of the other socket (whose pin was horizontal), at the cross-head position (i.e. position B). Four substantial box-sections were welded to the base plate so that the slightly flexible socket with a horizontal pin, could be propped against them and, hence, a nominally fixed end against lateral movements could be achieved. Following Poffenberger and Swart (1965), a rigid arm was extended from the socket along which a series of ± 5 mm range displacement transducers were positioned. Each transducer had a flat end which rested on a ball head mounted on the helical strand, and was positioned as close to the termination as was physically possible.

Following the arguments of Poffenberger and Swart, this arrangement was designed to provide reliable estimates of the net strand lateral deflections, free from any rigid body effects. Poffenberger and Swart formulated a mathematical solution which defined the relationship between the conductor outer-wire strain and the measured bending amplitude, called the differential displacement, of the short but critical segment of the deflected curve near the clamp. In their experiments on overhead transmission lines, the possibility of joint flexibility was taken into account by rigidly extending a displacement transducer from the suspension clamp body, on a deflection arm which rotated with the clamp. The relative (effective bending) displacement of the strand, at a point situated about 75 mm from the edge of the support, was then measured. Such data was subsequently used in a theoretical formulation which related the so-called effective bending displacement to the direct wire strains in the extreme fibre position. Limiting such strains was believed to minimise potential free bending fatigue problems at the terminations. Such an approach has, over the years, gained wide acceptance in the field of overhead transmission lines.

In the series of experiments conducted by Raoof, the test set-up enabled Raoof to obtain certain test data regarding the Poffenberger-Swart differential (effective bending) displacements in the vicinity of the socket. In these tests, the free bending modes were classified as $(-\delta)$, $(+\delta)$, and $(\pm\delta)$, where the modes $(-\delta)$ and $(+\delta)$ correspond to cases where the strand is cycled to one side, while type $(\pm\delta)$ identifies a reversed bending process. The positive sign represents the case in which the lateral strand deflection at the fixed socket is in the same direction as the lay angle in the outermost layer of wires, when looking along the strand from the cross-head (position B in Fig. 5.1) and from above.

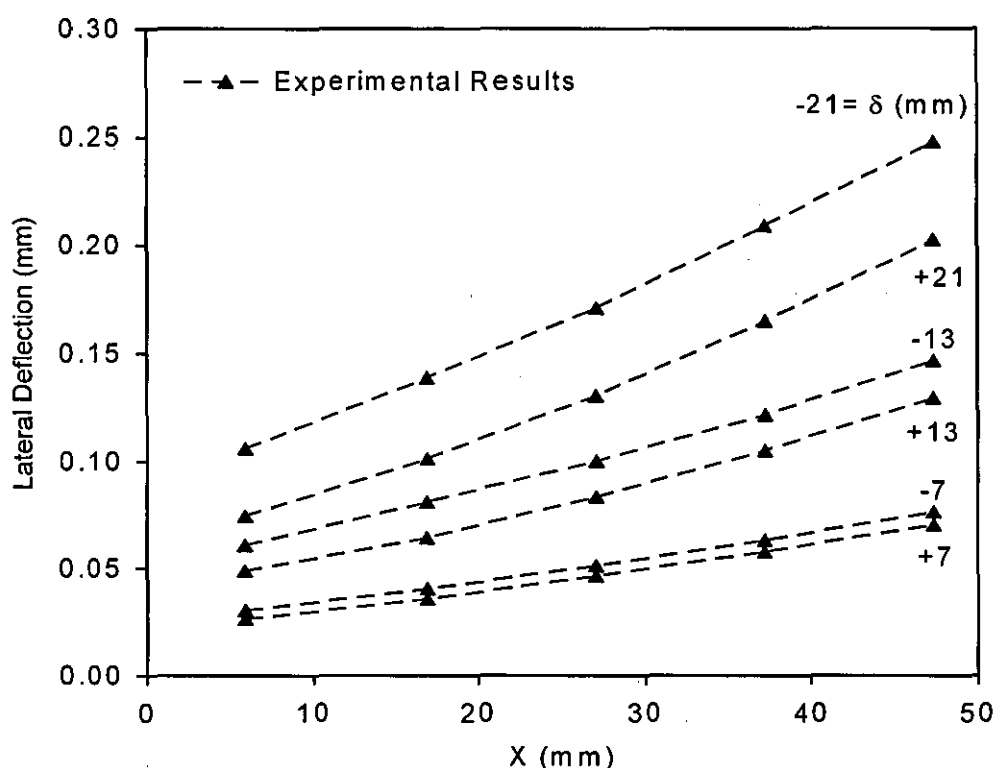


Fig. 5.2 - The Experimentally Obtained Deflected Shapes for the 39 mm Strand at the Propped Socket Under Modes ($-\delta$) and ($+\delta$) of Free Bending – after Raoof (1992a).

Fig. (5.2) shows the experimentally obtained deflected shapes for the 39 mm strand at the propped socket under modes ($-\delta$) and ($+\delta$) of free bending. The mean axial load was 0.41 MN (with the strand having an ultimate tensile strength of 1.23 MN), and, as mentioned previously, the deflected shapes were obtained using the Poffenberger-Swart differential displacement method. The lack of symmetry in these test results, is not due to significant changes in the strand's effective bending rigidity (as confirmed by the individual axial wire strain measurements fully discussed by Raoof, 1992a). Instead, it is believed to be partly due to the presence of external rigid body movements, as is demonstrated in Fig. 5.3.

Fig (5.3) shows the total deflection range of each transducer under modes ($-\delta$) and ($+\delta$) (i.e., 2δ) plotted against the total rigid body movement of the light split clamp, at the lateral jack position (i.e. at $x = 2530$ mm in Fig. 5.1). In Fig. (5.3), the corresponding points for the ($\pm\delta$) mode are also superimposed onto these. All of

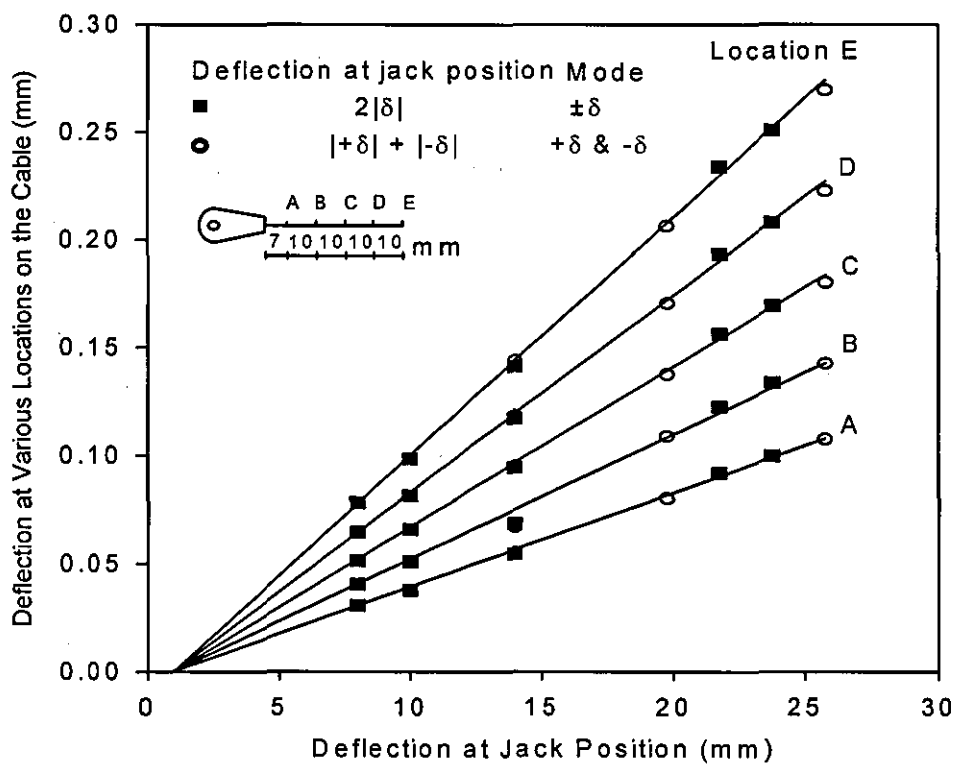


Fig. 5.3. - Net Deflection at Various Locations Along the 39 mm O. D. Spiral Strand Near the Fixed Socket, Obtained by the Differential Displacement Method – after Raoof (1992a).

these plots can be linearly extrapolated to very nearly a single point on the horizontal axis with an intercept of approximately 1 mm. In other words, there appears to be an initial offset (probably due to the eccentricity as the strand enters the cone, which is likely to be dependent upon the skill of the craftsman making the socket and other clearance problems), between the lateral strand deflection and the rigid body movement of the clamp at the lateral jack position. This, however, is not a serious pitfall and, as shown in the following, it may safely be bypassed for the present purposes. The plotted test data points in Fig. (5.2) clearly demonstrate that, irrespective of this secondary clearance problem, the strand lateral deflection at the face of the socket (i.e., at the strand-zinc interface) is not zero and, in fact, the point of zero lateral net deflection is somewhere well inside the conical housing. Taken with the rapid decrease in curvature away from the fixed point, this implies that too much should not be expected of the strain gauge readings at the face of the socket, as previously attempted by various researchers in order to verify their theoretical predictions of, say, the wire bending strains at the terminations. Rather than exact

correlations with available theories, which invariably assume a definite end fixity at, say, the face of the socket, more qualitative results can be expected: these are discussed by Raoof (1992a).

For the present purposes, the following section presents a simple semi-empirical method, based on the experimental data presented in Figs. (5.2) and (5.3), for obtaining a reasonable estimate of the location, well inside the zinc poured socket, where the position of effective zero lateral cable deflection lies. Moreover, in what follows, the validity of assuming a constant effective bending stiffness, $(EI)_{\text{eff}}$, based on the plane-section bending assumption, aimed at determining the deflected shape of the strand in the immediate vicinity of the fixed termination, will also be critically examined, assuming that the cable undergoes only small maximum lateral deflections of the order of one cable diameter.

5.3 THEORY

5.3.1 Local Phenomenon Near the Fixed End

Back in 1960, Wyatt (1960) considered the fundamental behaviour of a tendon carrying a large axial load and subjected to bending at one end with the other end fixed as shown in Fig. (5.4).

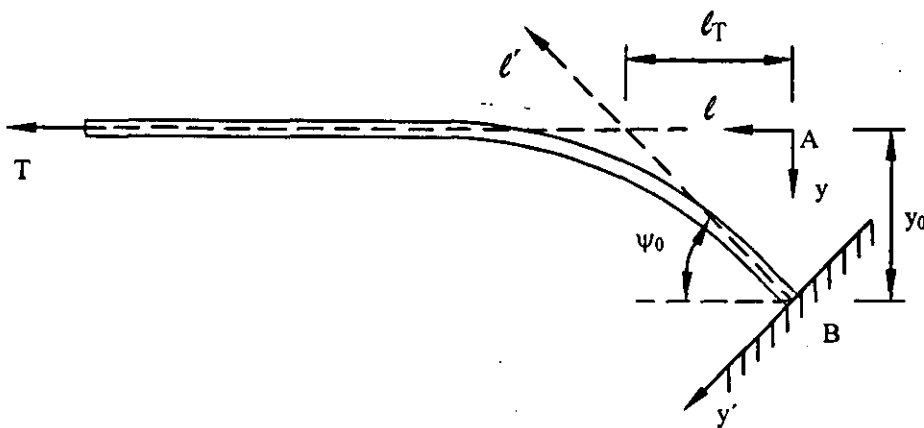


Fig. 5.4 - Bending Deflection of a Simple Tendon.

In this Figure, the angle Ψ_0 is the rotation of the tangent (say, at a clamping band) relative to the direction of the relevant tension, T . The moment equilibrium equation at any section, then, gives

$$M = (EI)_{\text{eff}} \frac{d^2 y}{d\ell^2} = Ty \quad (5.1)$$

where, $(EI)_{\text{eff}}$ = the effective bending stiffness of the cable, M = the bending moment, and ℓ denotes the dimension measured along the cable. The solution to the differential equation (5.1) is of the general form

$$y = Ae^{g\ell} + Be^{-g\ell} \quad (5.2)$$

where

$$g = \sqrt{\frac{T}{(EI)_{\text{eff}}}} \quad (5.3)$$

Based on the end conditions in Fig. (5.4), therefore, one can determine the constants A and B , and arrive at the following

$$y = \frac{\Psi_0}{g} e^{-g\ell} \quad (5.4)$$

Using Equations (5.1) and (5.4), the radius of curvature, ρ , at the fixed end, where $\ell = 0$, is given as

$$\frac{1}{\rho} = \Psi_0 \sqrt{\frac{T}{(EI)_{\text{eff}}}} \quad (5.5)$$

It should be noted that, in practice, the angle Ψ_0 is usually very small so that $\tan\Psi_0 \approx \Psi_0$. Equation (5.5) is similar to the one derived by Irvine (1992), who

considered the general solution, via the method of matched inner and outer expansions, for the problem of a beam-cable of very slight flexural rigidity subjected to lateral vibrations. In his work, Irvine provided a solution for the local condition of full fixity, in the immediate vicinity of the supports, and demonstrated that (due to the presence of the exponential functions with negative exponents), within a very short distance from the fixed end of the order of one cable diameter, the local effect of bending rigidity dies away quickly as one moves into the span. Using a numerical example based on the solutions to Equation (5.1), Wyatt (1960) also showed that for a single wire, the slope of Fig. (5.4) is reduced to 1% of the end value of Ψ_0 in a length of approximately 4 in. (101.6 mm), a distance that is very small indeed, in relation to the total span in practice.

It should, on the other hand, be noted that in the present experimental set-up, as shown in Fig. (5.1), the cable is subjected, not to a transverse distributed load, but experiences a single transverse point load: for Raoof's experimental arrangement (Raoof, 1992a), it has been shown (Hobbs and Smith, 1983) that the radius of curvature, ρ , at the fixed end may reasonably be calculated from the following

$$\frac{1}{\rho} \approx 1.1 \frac{\delta}{x} \sqrt{\frac{T}{(EI)_{\text{eff}}}} \quad (5.6)$$

where, $\frac{\delta}{x} = \tan \theta$, with the other parameters defined in Fig. (5.1).

Following the experimental observations in section 5.2, for a lateral deflection δ (at the lateral jack position) at a distance $x = 2530$ mm from the fixed end

$$\tan \theta = \frac{\delta - 1}{2530} \quad (5.7)$$

where, δ is in mm and, in Equation (5.7), its magnitude is reduced by 1 mm (hence, the factor 1 in the numerator), because of the 1 mm intercept on the horizontal axis in Fig. (5.3), in order to take out any undesired effects of eccentricity or rigid body

movements, etc., in the subsequent semi-empirical approach for determining the location of the effective point of fixity inside the socket.

Using Equations (5.5), (5.6) and (5.7), therefore, the equivalent angle of rotation, Ψ_0 , in Raoof's experiments, is

$$\Psi_0 \approx \tan^{-1} 1.1 \left(\frac{\delta - 1}{2530} \right) \quad (5.8)$$

where, as mentioned previously, δ is in mm, and Ψ_0 is in radians. For reasons that become clear later, the $y - \ell$ coordinate axes in Fig. (5.4) may be rotated through an angle Ψ_0 with the alternative coordinate axes being $y' - \ell'$, through the following relations

$$y' = \ell \sin \Psi_0 + y \cos \Psi_0 \quad (5.9a)$$

$$\ell' = \ell \cos \Psi_0 - y \sin \Psi_0 \quad (5.9b)$$

Based on Equations (5.4) and (5.9), then, one arrives at the following

$$y' = \ell \sin \Psi_0 + \left[\frac{\Psi_0}{g} e^{-g\ell} (\cos \Psi_0) \right] \quad (5.10a)$$

$$\ell' = \ell \cos \Psi_0 - \left[\frac{\Psi_0}{g} e^{-g\ell} (\sin \Psi_0) \right] \quad (5.10b)$$

Finally, for $\ell = 0$, Equation (5.10a) gives

$$y_0 = \frac{\Psi_0}{g} (\cos \Psi_0) \quad (5.11)$$

so that with the origin of the coordinate system moved from point A in Fig. (5.4) to point B in this same Figure, one gets

$$y' = \left[\ell \sin \Psi_0 + \left[\frac{\Psi_0}{g} e^{-\ell'} (\cos \Psi_0) \right] \right] - y_0 \quad (5.12)$$

with ℓ' again given by Equation (5.10b). Equations (5.12) and (5.10b) with Ψ_0 given by Equation (5.8), and g defined by Equation (5.3), then, define the deflected shape of the cable in the immediate vicinity of the fixed end, with y' and ℓ' representing the lateral deflection and longitudinal axis, respectively, of the spiral strand in the experimental set-up of Raoof (1992a), as shown in Fig. (5.1). However, it should be noted that in Equations (5.12) and (5.10b), the location of the ideal point of fixity (i.e. as to whether it is at the face of the socket or well inside it) is not exactly defined, and this will be clarified later, using the experimental results of Raoof in Fig. (5.2). Finally, it is, perhaps, worth mentioning that due to the very small magnitudes of Ψ_0 (say, $\Psi_0 \approx 2-3$ degrees) in Equation (5.10b), in practice, $\ell \approx \ell'$.

5.3.2 Calculation of Plane-Section $(EI)_{\text{eff}}$

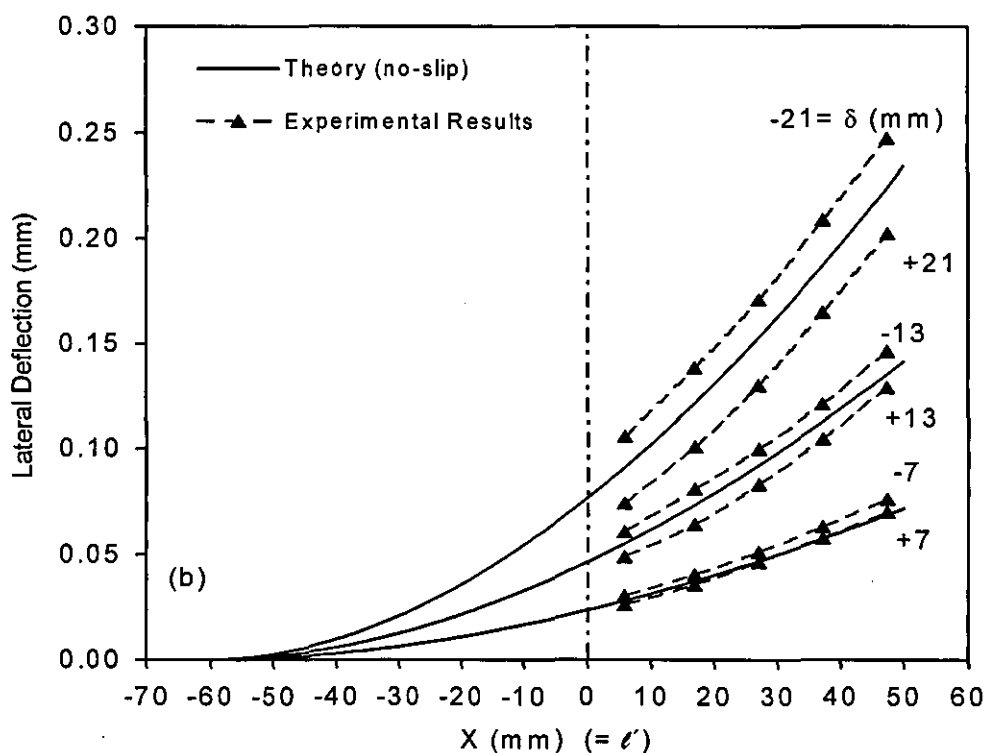
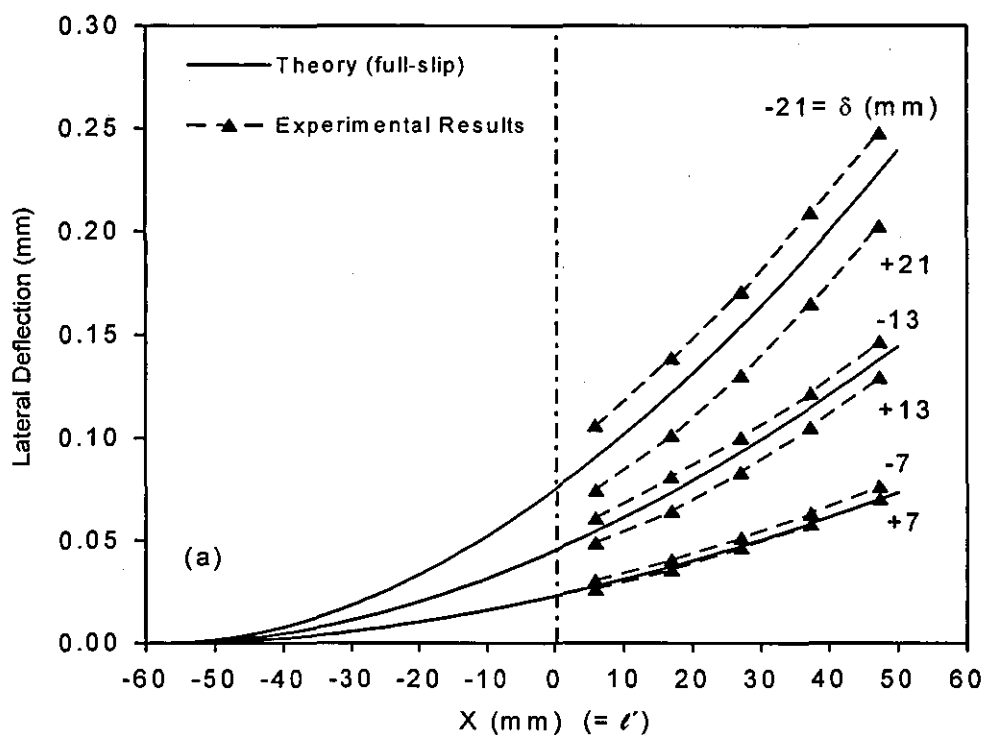
The calculation of the effective plane-section bending stiffness, $(EI)_{\text{eff}}$, has been addressed in considerable detail by Raoof and Hobbs (1984). Very briefly, similar to the axial and/or torsional stiffnesses, the $(EI)_{\text{eff}}$ in an axially preloaded multi-layered spiral strand, varies (due to the ever presence of interwire friction) between an upper (no-slip) and a lower (full-slip) bound, with the definitions for the no-slip and full-slip regimes given by Raoof and Hobbs (1984), and also, in chapters 3 and 4 of this thesis.

Raoof (1992d) deals with extensive theoretical parametric studies carried out on a number of realistic strand constructions with widely varying strand (and wire) diameters and lay angles, using the formulations based on the previously reported orthotropic sheet theory: guided by such results, a very simple method (amenable to hand calculations, using a pocket calculator) for obtaining reliable estimates of the no-slip and/or full-slip $(EI)_{\text{eff}}$ has been developed for the cases of plane-section bending. Obviously, for sufficiently small levels of radii of curvature plane-sections no longer remain plane, and interwire slippage (associated with which are drastic reductions in the bending stiffness) takes place, starting from the outermost layer and spreading

inwards, towards the centre of the strand (depending on the level of axial tension on the cable) (Raoof, 1992a). It is also, perhaps, worth mentioning that, over the current manufacturing limits, the difference between the no-slip and full-slip plane-section $(EI)_{\text{eff}}$ is about 10-25 % (with the exact value depending on the strand construction details).

5.4 RESULTS

Raoof and Huang (1992h) have reported the no-slip and full-slip values of the plane-section bending stiffnesses for the 39 mm outside diameter spiral strand (used in the presently reported experiments) as 1.513×10^{10} and 1.279×10^{10} N-mm², respectively. For a mean axial tension $T = 0.41$ MN, then, the theoretical deflected shapes of the 39 mm strand, in the vicinity of a nominally fixed zinc socket, may be obtained using the method developed in the previous section, for different values of $\delta = 7, 13$ and 21 mm, and by initially assuming the ideal point of fixity to be located at the face of the socket where $\ell' = 0$. Comparisons between the so-obtained theoretical deflection curves and the experimental plots in Fig. (5.2), however, have suggested that in order to obtain good correlations between the theoretical and experimental results, the theoretical plots should be shifted in a negative ℓ' direction until reasonable agreement is found between the theoretical predictions and the test data for all values of δ . Fig. (5.5a) presents the outcome of such an exercise: by using the full-slip value of $(EI)_{\text{eff}}$, and by moving the theoretical plots in the negative ℓ' direction by 58 mm (where $\ell' = 0$ denotes the cable-socket face) a very good match between the theory and the test data, over a wide range of $7 \leq \delta \leq 21$ mm has been obtained. Fig. (5.5b), on the other hand, presents similar results for the case when the no-slip value of $(EI)_{\text{eff}}$ has been used for producing the theoretical curves: for this latter case, the point of zero lateral deflection was found to be located at $\ell' = -61$ mm (inside the conical housing), which is quite close to the figure of $\ell' = -58$ mm associated with the corresponding theoretical full-slip $(EI)_{\text{eff}}$ case. In other words, guided by the results presented in Figs. (5.5a, b), it is concluded that the use of the plane-section bending assumption for predicting the $(EI)_{\text{eff}}$ for such an application is a reasonable one, and the ideal point of end fixity, at least for this particular socket-strand system, is located inside the conical housing, at a distance from the face of the socket of approximately 60 mm.



Figs. 5.5 - The Experimental and Theoretically Obtained Deflected Shapes for the 39 mm Outside Diameter Spiral Strand at the Propped Socket Under Modes $(-\delta)$ and $(+\delta)$ of Free Bending: (a) Full-Slip Regime; (b) No-Slip Regime.

5.5 DISCUSSION

The findings of the previous section may have significant practical implications in theoretical (including Finite Element) studies (e.g., Lutchansky, 1969) of the termination effects on, say, the fretting fatigue of steel cables which have (to date) assumed (without any previous theoretical and/or experimental justification) an ideal point of fixity located at the face of the zinc socket with the interwire fretting movements, being of significant magnitudes within an extremely limited length (Raoof and Hobbs, 1984, and Raoof, 1990b, 1992a and 1992c) of the cables in the vicinity of the ideal point of fixity. In this context, the interested reader may also refer to Ramsey (1991) and, Jiang and Henshall (1999), both of whom address the problem of interwire fretting in close proximity to the ideal points of end fixity to spiral strands experiencing axial loading. In addition, in view of the extremely short length of the zone of influence of the bending stiffness, along the deflected shape of the cable, located at the terminations, it is concluded that the previous practice, by certain researchers, who have tried to experimentally determine (in an indirect way) the magnitude of the cable effective bending stiffness by a curve fitting exercise involving the matching of theoretically determined overall cable deflected shapes (which include the effect of the bending stiffness) with the corresponding deflected shapes as determined by the tests, is fraught with difficulties and uncertainties, and is not a reasonably viable approach.

5.6 CONCLUSIONS

Using data from large scale and carefully conducted free bending experiments (as previously reported by Raoof) on an axially preloaded 39 mm outside diameter helical strand, in conjunction with a theoretical model, an insight is provided as regards certain characteristics of the laterally deflected cables in the immediate vicinity of zinc socketed end terminations. Based on such a semi-empirical approach, it is demonstrated that the spiral strand plane-section effective bending stiffness(es) may, indeed, be used for the theoretical determination of the radii of curvature at nominally fixed ends to strands undergoing, say, vortex shedding instabilities with associated maximum lateral deflections of the order of one cable diameter. Moreover, the results demonstrate that for the socket-cable system, the effective point of fixity is located not at the face of the zinc socketed termination, but well within the socket itself: for

Raoof's experimental set-up, the effective point of fixity was found to be approximately 60 mm inside the conical housing, faraway from the face of the socket. The significant practical implications of these findings, in the context of the work of others, have also been critically addressed.

CHAPTER 6

EXPERIMENTAL DETERMINATION OF THE CABLE BENDING STIFFNESS

6.1 INTRODUCTION

There are various experimental techniques, as discussed by Malinovsky (1993), available for the determination of a cable's effective bending stiffness. As mentioned by Malinovsky, in the course of numerous experiments with tensioned helical cables, the bending stiffness has been found to be heavily dependent upon the applied tension, and can vary between two limiting conditions, corresponding to either full or zero interlayer shear interaction of the wire elements in a helical cable. In the former case, the cable acts very nearly as a 'solid bar' with allowance given for the presence of helical voids; in the latter, the individual helical wires act independently and merely bend about their own neutral axes. For large diameter multi-layered spiral strands, the difference between the two limits is unacceptably large, being given approximately by the square of the strand/wire diameter ratio, Raoof (1989).

The problem with the experimental determination of the effective bending stiffness is the discrepancy between the values of this parameter as determined by the different methods, which is mainly due to the experimental conditions, and depends on the level of imposed curvature. For example, using a 34 mm outside diameter fibre-core wire rope, the bending stiffness was determined (Malinovsky, 1993) using two different methods; the frequency method, and the method of static bending, and was found to be 3000 Nm² and 534 Nm², respectively. In other words, the difference between the experimental results was rather significant, with the value of the bending stiffness determined by the method of static bending being 5.6 times less than that as determined using the frequency method.

As mentioned previously, for the bending of steel helical cables under an approximately steady mean axial load, it is common to introduce a mathematically convenient constant effective bending stiffness, $(EI)_{\text{eff}}$, for the cable, using which the radii of curvature at the points of restraint are then calculated. Due to their peculiar

construction, axially preloaded spiral strands undergo plane-section bending only for sufficiently small levels of cable maximum lateral deflections (Raoof, 1992a). Beyond a certain limit of maximum lateral deflection, plane sections do not remain plane and interlayer slippage takes place, starting from the outer layer, and spreading towards the centre of the strand, depending on the level of axial tension and the imposed radius of curvature (Raoof, 1992a).

As already discussed in Chapter 5, large scale free-bending experimental observations on axially preloaded 39 mm outside diameter spiral strands have been reported by Raoof (1989 and 1992d), who showed that, in the case of strands subjected to practical working ranges of axial load, and laterally bent with large radii of curvature such as those experienced under, for example, vortex shedding instabilities (i.e. maximum bending amplitudes of the order of one strand diameter), it may be assumed that the strand cross-section remains plane during the bending cycle.

Raoof and Hobbs (1984) have argued that even with infinite shear stiffness between the layers of an axially preloaded multi-layered spiral strand (i.e. assuming that plane-sections remain plane during bending), line-contact interwire slippage within the wires in the individual layers, and hence some (although not very significant) reductions in the strand's effective bending stiffness, $(EI)_{\text{cable}}$, may still take place.

The purpose of this chapter is to present a simple, but still more reliable method compared to those described by, for example, Malinovsky (1993), for experimentally obtaining the effective cable bending stiffness. The experimentally determined bending stiffness for a 164 mm outside diameter spiral strand will, then, be compared with the plane-section bending stiffness predictions based on the method proposed by Raoof (1992d) (as presented in section 6.4).

6.2 EXPERIMENTAL MEASUREMENTS

It was discovered, accidentally, that by wrapping a helical cable (in this case, a 164 mm outside diameter spiral strand) around a 5 m diameter bobbin (6m combined cable and bobbin diameter) for transportation purposes, when the cable was unwrapped the shape of the cable was (because of the bobbin to strand diameter ratio, D/d , being too

small) distorted, i.e. it was no longer straight, Fig. (6.1), but formed a ‘corkscrew’ shape. This happened with a bobbin to cable diameter ratio, D/d , equal to approximately 30 ($= 5000 / 164$).

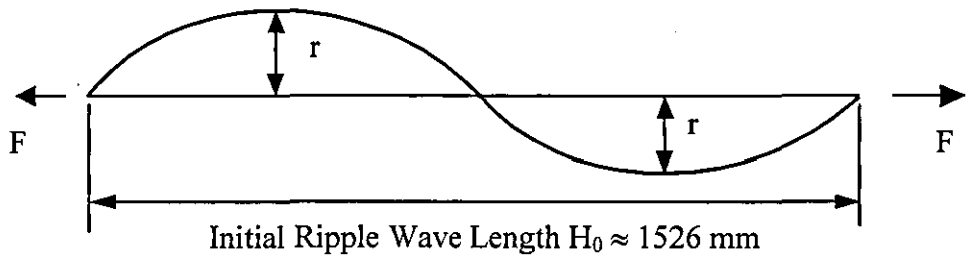


Fig. 6.1 – Observed Distorted (Corkscrew) Shape of the 164 mm Outside Diameter Strand Subjected to an Axial Force, F.

In-situ measurements related variations in the axial load on the cable, F , to changes in the ripple range, $2r$. Table (6.1) shows the relationship between the axial load on the helical cable and the ripple range for the first loading run. The load on the cable was then removed, and reapplied for a number of times, following which the relationship between the axial load on the cable and the ripple range corresponding to a final loading run, was again measured with the data presented in Table (6.2). Fig. (6.2) presents the so-obtained relationships between the axial load and the ripple range for both the first and final loading runs.

Table 6.1 - Experimental Measurements Relating the Changes in the Axial Load to Variations in the Ripple Range, for the First Loading Run.

Axial Load, F (kN)	Ripple Range, $2r$ (mm)	α (radians)	$1/R$ (m^{-1})	Bending Moment, m (kN-m)
350	29.0	0.059632	0.244948	5075.0
650	27.0	0.055528	0.228163	8775.0
1000	26.0	0.053475	0.219761	13000.0
1350	25.0	0.051422	0.211355	16875.0
1650	24.0	0.049369	0.202942	19800.0
2000	23.0	0.047315	0.194525	23000.0
2350	22.0	0.045261	0.186103	25850.0
2650	22.0	0.045261	0.186103	29150.0
3000	19.5	0.040123	0.165027	29250.0
3300	19.0	0.039096	0.160809	31350.0

Table 6.2 - Experimental Data Regarding Variations in the Axial Load with Changes in the Ripple Range, for the Final Loading Run.

Axial Load, F (kN)	Ripple Range, 2r (mm)	α (radians)	1/R (m ⁻¹)	Bending Moment, m (kN-m)
0	22.5	0.046185	0.189894	0.0
350	21.6	0.044336	0.182311	3771.3
3300	18.5	0.0338068	0.156590	30525.0

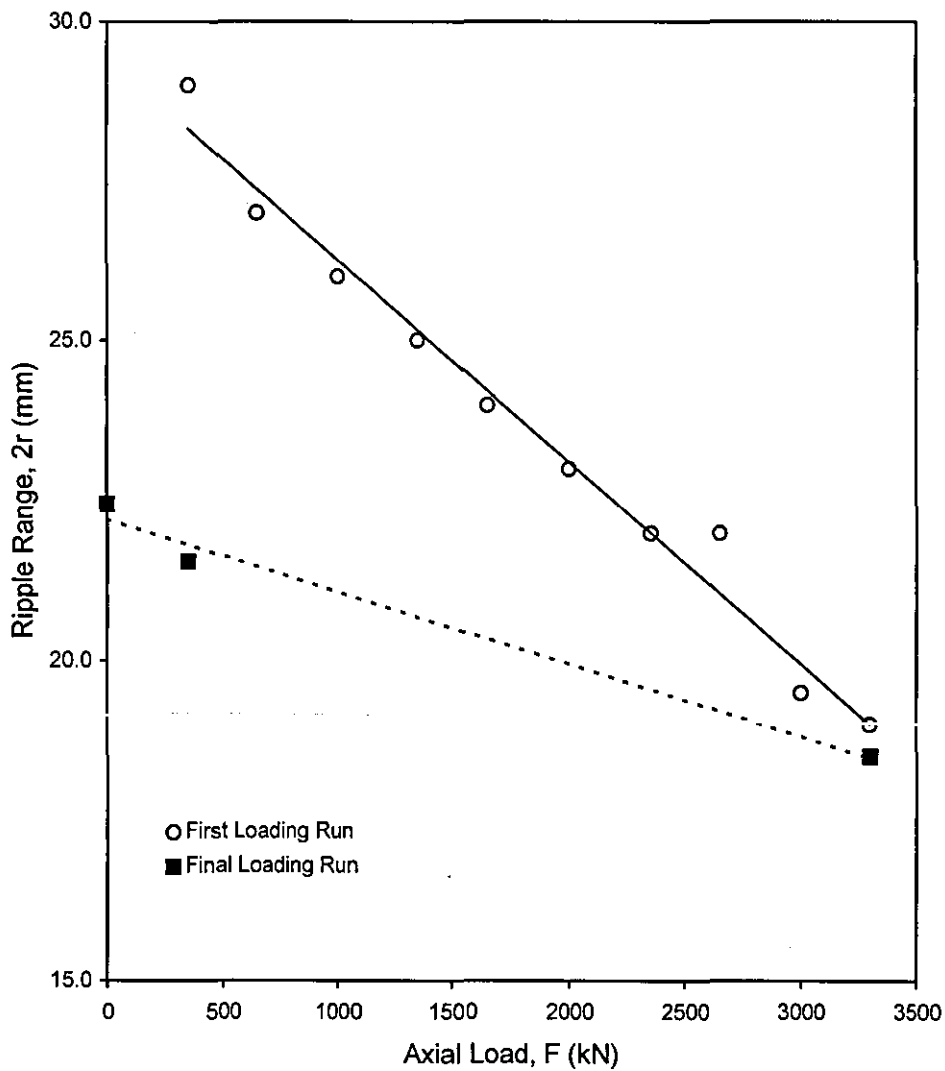


Fig. 6.2 – Variations of the Ripple Range with Changes in the Applied Axial Force for Both the First and Final Loading Runs.

6.3 THEORY

6.3.1 Distorted Shape of the Cable – Simplified Method

The simplified theory is, as the name suggests, a simple method for determining the effective bending stiffness of the cable $(EI)_{\text{cable}}$ based on the relationship between the axial force, F , and the ripple range, $2r$, Fig. (6.2). As will be explained later, this approach also enables one to reasonably extrapolate the results, such as those presented in Figure (6.2), to regions where no test data is available.

Figure (6.3) shows the planar geometry of the helical cable used in the simplified analysis, where a helix may always be unwrapped to form a right-angled triangle.

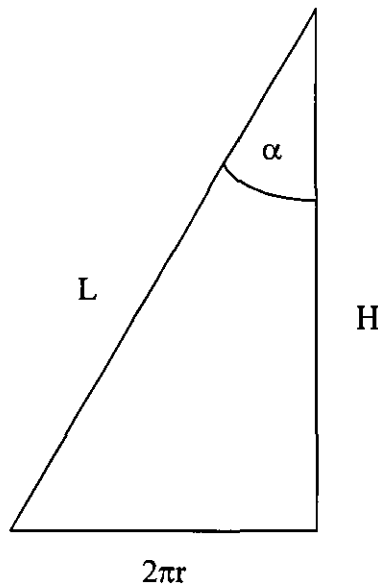


Fig. 6.3 – Planar Representation of the Helical Cable Used in the Simplified Analysis.

The lay angle, α , in Fig (6.3) is

$$\alpha = \tan^{-1}\left(\frac{2\pi r}{H}\right) \quad (6.1)$$

where, H is the measured ripple wave length, and r is the corresponding depth of the ripple in the test specimen. Once α has been calculated, the value of the radius of curvature, R , for the helix can be determined from

$$R = \frac{r}{\sin^2 \alpha} \quad (6.2)$$

as can the bending moment, m , using the following

$$m = F \times r \quad (6.3)$$

The effective bending stiffness of the cable, $(EI)_{\text{cable}}$, is, then, quite simply, the slope of the bending moment, m , against the curvature, $1/R$, plots. This simple approach, however, assumes the contributions from other internal moments and forces in the so-called helical rod to be sufficiently small to be neglected. A justification for this assumption forms the main purpose of the next section, where a more rigorous approach is presented, which takes all of the possible internal actions into account in arriving at a more exact relationship between the axial force, F , and the ripple range, $2r$. Comparisons between the F versus $2r$ plots based on the simple and the more rigorous approaches, will, then, clarify the practical implications of ignoring the possible influence of a number of internal forces and couples in the simplified approach.

6.3.2 Distorted Shape of the Cable – Rigorous Method

Love (1944) has presented certain equilibrium equations for the solution to the problem of a helical spring subjected to an axial force and a twisting moment. In Love's approach, the spring is treated as a thin curved rod which satisfies the six non-linear equations of equilibrium. As an axial force is applied to the spring, the lay angle changes, as does the spring's helix radius. Blanco and Costello (1974) used Love's equations to determine the changes in various structural characteristics of the spring as controlled by an internal cylindrical constraint with friction between the spring and the internal cylinder ignored. The formulations of Love, and Blanco and Costello are used as the basis for the following developments.

Initially, when there is zero axial load, F , on the cable, the relationship between the ripple range and the ripple wave length can be described, schematically, by Fig. (6.4),

where, for the first loading run, $r_0 = 15$ mm, Fig. (6.2). Using simple trigonometry, the following relations are established

$$\sin \alpha_0 = \frac{2\pi r_0}{L_0} \quad (6.4)$$

$$L_0 = \sqrt{(2\pi r_0)^2 + (H_0)^2} \quad (6.5)$$

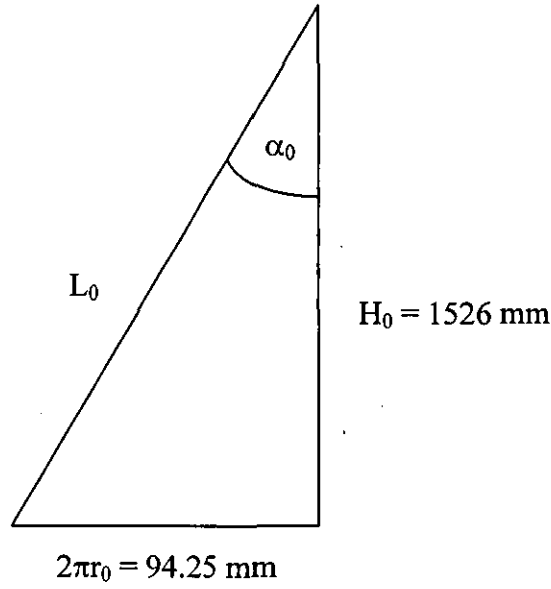


Fig. 6.4 – Schematic Representation of the Helical Cable Under Zero Axial Load, Based on the First Loading Run.

When an axial force, F , is applied to the cable as in Fig. (6.1), it deforms in such a way that the relationship between the ripple range and the ripple wave length, for the helical rod, is defined as in Figure (6.5).

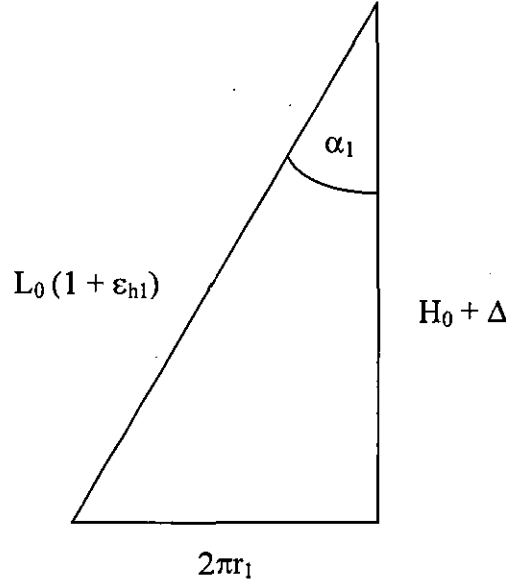


Fig. 6.5 – Schematic Representation of the Helical Cable Under an Applied Axial Load.

Once again, using simple trigonometry, the relationships between the geometrical parameters in Fig. (6.5) are established as

$$\sin \alpha_1 = \frac{2\pi r_1}{L_0(1 + \epsilon_{h1})} \quad (6.6)$$

$$L_0(1 + \epsilon_{h1}) = \sqrt{(2\pi r_1)^2 + (H_0 + \Delta)^2} \quad (6.7)$$

and, denoting the axial tension in the cable as T

$$T = E_{\text{full-slip}} \times A_{\text{net}} \times \epsilon_{h1} \quad (6.8)$$

where, ϵ_{h1} is the axial strain in the cable resulting from the applied axial force F , Δ is the change in the ripple wave length resulting from the applied axial force, r_1 is the new ripple depth, and A_{net} is the net steel area of the helical cable. The full-slip Young's modulus of the cable, $E_{\text{full-slip}}$, is given, simply, by Equations (6.44) – (6.45) (Raoof 1990e) or, more accurately, by the orthotropic sheet theoretical model of Hobbs and Raoof (1982), as summarised in chapter 3.0.

The equations of equilibrium for a thin rod are given by Love (1944), as

$$\frac{dN}{ds} - N'\tau_1 + T\kappa'_1 + X = 0 \quad (6.9)$$

$$\frac{dN'}{ds} - T\kappa_1 + N\tau_1 + Y = 0 \quad (6.10)$$

$$\frac{dT}{ds} - N\kappa'_1 + N'\kappa_1 + Z = 0 \quad (6.11)$$

$$\frac{dG}{ds} - G'\tau_1 + H\kappa'_1 - N' + K = 0 \quad (6.12)$$

$$\frac{dG'}{ds} - H\kappa_1 + G\tau_1 + N + K' = 0 \quad (6.13)$$

$$\frac{dH}{ds} - G\kappa'_1 + G'\kappa_1 + \theta = 0 \quad (6.14)$$

where, s is the distance along the centre line of the rod, κ_1 and κ'_1 are the components of the final curvatures, τ_1 is the final twist, and X , Y , Z , K , K' and θ are the components of the external forces and couple-resultants per unit length along the rod, with N , N' , T , G , G' and H being the components of the internal forces and couple-resultants of the helical rod in the normal, binormal and tangential directions, Fig. (6.6). For the helical rod, the bending and twisting couples, G , G' and H , are related to the initial curvatures κ_0 and κ'_0 and the initial twist, τ_0 , by

$$G = (EI)_{\text{full-slip}} (\kappa_1 - \kappa_0) \quad (6.15)$$

$$G' = (EI)_{\text{full-slip}} (\kappa'_1 - \kappa'_0) \quad (6.16)$$

$$H = (GJ)_{\text{full-slip}} (\tau_1 - \tau_0) \quad (6.17)$$

where, $(EI)_{\text{full-slip}}$ is the full-slip plane-section bending stiffness of the cable, being obtained for a specific cable, with a reasonably high degree of accuracy, by the method described in section 6.3.1, or less accurately, but good enough for most practical purposes, by the method proposed in section 6.4 (Equations (6.44) – (6.53)), Raoof (1992d). $(GJ)_{\text{full-slip}}$ is the full-slip torsional stiffness, and can be calculated using the method described later.

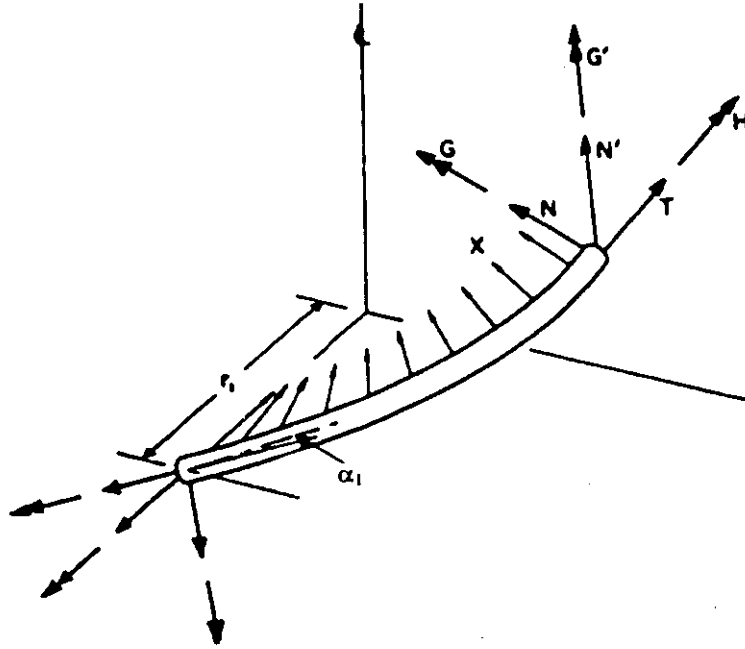


Fig. 6.6 – External and Internal Forces and Moments Acting on the Helical Rod.

For the helical rod, the initial curvatures, κ_0 and κ'_0 and twist, τ_0 , are given by

$$\kappa_0 = 0 \quad (6.18)$$

$$\kappa'_0 = \frac{\sin^2 \alpha_0}{r_0} \quad (6.19)$$

$$\tau_0 = \frac{\sin \alpha_0 \cos \alpha_0}{r_0} \quad (6.20)$$

where, α_0 and r_0 are the initial lay angle and helix radius of the rod, respectively, corresponding to when $F = 0$. When an axial force, F , is applied to the rod, Fig. (6.1), the final (deformed) curvatures and twist are given by

$$\kappa_1 = 0 \quad (6.21)$$

$$\kappa'_1 = \frac{\sin^2 \alpha_1}{r_1} \quad (6.22)$$

$$\tau_1 = \frac{\sin \alpha_1 \cos \alpha_1}{r_1} \quad (6.23)$$

where, α_1 and r_1 are the lay angle and helix radius of the deformed rod, respectively, under an applied axial load, F . It is assumed that there is zero friction along the surface of the helical rod, and that the external bending moments K and K' are both equal to zero, with $X = 0$. For a constant tension T along the length of the rod, then, Equations (6.9) – (6.14) become (Blanco and Costello, 1974)

$$-N'\tau_1 + Tk'_1 = 0 \quad (6.24)$$

$$Y = 0 \quad (6.25)$$

$$Z = 0 \quad (6.26)$$

$$-G'\tau_1 + Hk'_1 - N' = 0 \quad (6.27)$$

$$N = 0 \quad (6.28)$$

$$\theta = 0 \quad (6.29)$$

As explained next, Equations (6.8) – (6.29) provide a simple means of relating the depth of the ripple, r , to the lay angle of the rod, α .

Using Equations (6.22), (6.23) and (6.24), the following results

$$-N' \frac{\sin \alpha_1 \cos \alpha_1}{r_1} + T \frac{\sin^2 \alpha_1}{r_1} = 0 \quad (6.30)$$

Similarly, using Equations (6.22), (6.23), and (6.27)

$$N' = -G \frac{\sin \alpha_1 \cos \alpha_1}{r_1} + H \frac{\sin^2 \alpha_1}{r_1} \quad (6.31)$$

where

$$G' = (EI)_{\text{full-slip}} \left(\frac{\sin^2 \alpha_1}{r_1} - \frac{\sin^2 \alpha_0}{r_0} \right) \quad (6.32)$$

and

$$H = (GJ)_{\text{full-slip}} \left(\frac{\sin \alpha_1 \cos \alpha_1}{r_1} - \frac{\sin \alpha_0 \cos \alpha_0}{r_0} \right) \quad (6.33)$$

Using Equations (6.32), (6.33) and (6.31), one gets

$$N' = -(EI)_{\text{full-slip}} \left(\frac{\sin^2 \alpha_1}{r_1} - \frac{\sin^2 \alpha_0}{r_0} \right) \frac{\sin \alpha_1 \cos \alpha_1}{r_1} + (GJ)_{\text{full-slip}} \left(\frac{\sin \alpha_1 \cos \alpha_1}{r_1} - \frac{\sin \alpha_0 \cos \alpha_0}{r_0} \right) \frac{\sin^2 \alpha_1}{r_1} \quad (6.34)$$

From Equation (6.30), N' is given as

$$N' = T \tan \alpha_1 \quad (6.35)$$

while, from force equilibrium (Blanco and Costello, 1974)

$$F = T \cos \alpha_1 + N' \sin \alpha_1 \quad (6.36)$$

From Equations (6.35) and (6.36)

$$\cos \alpha_1 = \frac{T}{F} \quad (6.37)$$

and

$$N' = F \sin \alpha_1 \quad (6.38)$$

Equations (6.34) and (6.38), give the magnitude of the applied axial force, F , as

$$F = -(EI)_{\text{full-slip}} \left(\frac{\sin^2 \alpha_1}{r_1} - \frac{\sin^2 \alpha_0}{r_0} \right) \frac{\cos \alpha_1}{r_1} +$$

$$(GJ)_{\text{full-slip}} \left(\frac{\sin \alpha_1 \cos \alpha_1}{r_1} - \frac{\sin \alpha_0 \cos \alpha_0}{r_0} \right) \frac{\sin \alpha_1}{r_1} \quad (6.39)$$

Finally, by combining Equations (6.6), (6.8) and (6.37), r_1 is given by

$$r_1 = \frac{\sin \alpha_1 \left[L_0 \left(1 + \frac{F \cos \alpha_1}{E_{\text{full-slip}} \times A_{\text{net}}} \right) \right]}{2\pi} \quad (6.40)$$

6.3.3 Method of Solution

For given values of r_0 and H_0 (hence, L_0 and α_0 as determined by Equations (6.5) and (6.4), respectively); in order to calculate the depth of ripple, r_1 , for a given mean axial load, F , an initial value of α_1 is assumed and, from Equation (6.40), an initial value for r_1 is, then, calculated. For the present work, the initial value of α_1 has been obtained

via the simplified approach as explained in section 6.3.1. Based on the Newton-Raphson method

$$\alpha_{\text{new}} = \alpha_1 - \frac{F(\alpha_1)}{F'(\alpha_1)} \quad (6.41)$$

where, from Equation (6.40)

$$F(\alpha_1) = \frac{\sin \alpha_1 \left[L_0 \left(1 + \frac{F \cos \alpha_1}{E_{\text{full-slip}} \times A_{\text{net}}} \right) \right]}{2\pi} - r_1 \quad (6.42)$$

and

$$F'(\alpha_1) = \frac{L_0 \cos \alpha_1 + \frac{L_0 F \cos 2\alpha_1}{E_{\text{full-slip}} \times A_{\text{net}}}}{2\pi} \quad (6.43)$$

A new value of α_1 (α_{new}) may, thus, be determined. α_{new} may subsequently be inserted into Equation (6.39) to determine the updated value for r_1 (r_{new}), using the Newton-Raphson iteration process, where

$$r_{\text{new}} = r_1 + \frac{F(r_1)}{F'(r_1)} \quad (6.44)$$

and, from Equation (6.39)

$$F(r_1) = -(EI)_{\text{full-slip}} \left(\frac{\sin^2 \alpha_1}{r_1} - \frac{\sin^2 \alpha_0}{r_0} \right) \cos \alpha_1 +$$

$$(GJ)_{\text{full-slip}} \left(\frac{\sin \alpha_1 \cos \alpha_1}{r_1} - \frac{\sin \alpha_0 \cos \alpha_0}{r_0} \right) \sin \alpha_1 - (F \times r_1) \quad (6.45)$$

and

$$F'(r_1) = -(EI)_{\text{full-slip}} \left(\frac{-\sin^2 \alpha_1 \cos \alpha_1}{r_1^2} \right) + (GJ)_{\text{full-slip}} \left(\frac{-\sin^2 \alpha_1 \cos \alpha_1}{r_1^2} \right) - F \quad (6.46)$$

The iteration process is continued until the desired level of accuracy is achieved (in the present work, when $|\alpha_{n+1} - \alpha_n| \leq 0.00000001$, where the subscript n refers to the number of iterations).

6.4 CALCULATION OF THE PLANE-SECTION BENDING STIFFNESS, (EI)

The full-slip and no-slip plane-section bending stiffnesses, $(EI)_{\text{full-slip}}$ and $(EI)_{\text{no-slip}}$, maybe calculated, using the procedure developed by Raoof (1992d). For each individual layer i , including the core wire(s) of the cable, Hruska's parameter, H_i , is calculated from Equation (6.44)

$$H_i = \cos^4 \alpha_i \quad (6.47)$$

The full-slip orthotropic E-values for the individual layers i , $E_i^{\text{full-slip}}$, are given by

$$\frac{E_i^{\text{full-slip}}}{E_{\text{steel}}} = -0.26442 - 2.004046H_i + 6.5735H_i^2 - 3.3068H_i^3 \quad (6.48)$$

$$0.70 \leq H_i \leq 1.0$$

where, the Young's modulus for steel $E_s = 200 \text{ kN/mm}^2$.

The corresponding no-slip E-values for the individual layers i , may be obtained from

$$\frac{E_i^{\text{no-slip}}}{E_i^{\text{full-slip}}} = 3.998 - 7.916K_1 + 7.238K_1^2 - 2.321K_1^3 \quad (6.49)$$

$$0.35 \leq K_1 \leq 1.0$$

where

$$K_1 = \frac{E_i^{\text{full-slip}}}{E_{\text{steel}}} \quad (6.50)$$

The next stage is to calculate the second moment of area for each layer, I_{ni} , where

$$I_{ni} = \left[\frac{\pi}{4} \right] \times \left[\frac{\pi}{64} \right] \times \left[(2r_i + D_i)^4 - (2r_i - D_i)^4 \right] \quad (6.51)$$

In the above, r_i , D_i and n_i are the theoretical helix radius, (given by Equation (3.28)), the wire diameter, and the number of wires in layer i , respectively.

λ_i for each layer i is

$$\lambda_i = \frac{I_{ni}}{I_0} \quad (6.52)$$

where

$$I_0 = \sum I_{ni} \quad (6.53)$$

The effective plane-section bending stiffness $(EI)_{\text{eff}}$ for either the full-slip or no-slip limiting conditions is, finally, given by

$$(EI)_{\text{eff}} = I \sum_{i=1}^n \lambda_i E_i \quad (6.54)$$

where

$$I = \frac{\pi}{4} \left(\frac{\pi d^4}{64} \right) \quad (6.55)$$

6.5 CALCULATION OF THE TORSIONAL STIFFNESS, (GJ)

This has been covered in section 3.2.4, and will not be repeated here. Very briefly, for any twist per unit length, the layer contributes a torque, M_i , to the total torque, M , on the cable, where

$$M_i = \tau_i A_i r_i \quad (6.3.32))$$

and

$$M = \sum_{i=1}^N M_i \quad (6.3.33))$$

In the above, τ_i is the layer shear stress, r_i is the helix radius of the wires (as determined by Equation (3.28)), and A_i is the gross area of a layer of wires (as calculated by Equations (3.49) and (3.50)) with the subscripts i referring to layer i .

At large torques, the cable behaviour is dominated by large slipping movements (full-slip), whilst at small torques (in the region close to the origin) the cable behaviour is dominated by the no-slip limiting condition, Fig. (6.7). By taking a tangent to the twist per unit length against torque plot in Fig. (6.7), which is a function of the cable mean axial strain, at large torque values, the value of $(GJ)_{\text{full-slip}}$ is obtained. Alternatively by taking a tangent to the torque-twist curve at small torques (in the vicinity of the origin) the value of $(GJ)_{\text{no-slip}}$ may be determined. As mentioned previously, unlike $(GJ)_{\text{no-slip}}$, $(GJ)_{\text{full-slip}}$ is (for all practical purposes) independent of the level of axial preload on the cable.

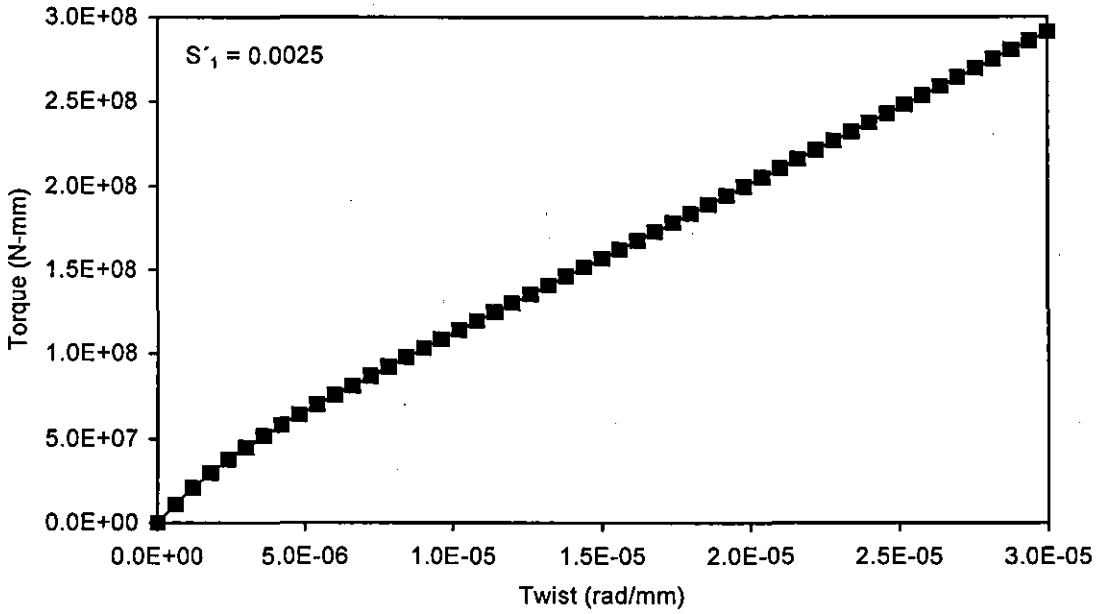


Fig. 6.7 – Torque – Twist Relationship for the 164 mm Outside Diameter Axially Pre-Loaded Cable at a Mean Axial Strain $S'_1 = 0.0025$.

6.6 RESULTS

Figure (6.8) presents the variations, based on the simplified approach, of the bending moment, m , as a function of changes in $\frac{1}{R}$, where R is the radius of curvature, over the full range of axial force, F , and the associated values of r in the experiments. The effective bending stiffness, based on the experimental data, which is the slope of the plots in Fig. (6.8), may, then, be determined. Table (6.3) shows the values of the so-obtained effective cable bending stiffnesses, $(EI)_{\text{cable}}$, for both the first and final loading runs, the theoretical plane-section bending stiffnesses, $(EI)_{\text{full-slip}}$ and $(EI)_{\text{no-slip}}$, as calculated using the method summarized in section 6.4, and the torsional stiffnesses, $(GJ)_{\text{full-slip}}$ and $(GJ)_{\text{no-slip}}$, as calculated using the method given in section 6.5, where $(GJ)_{\text{no-slip}}$ corresponds to a cable mean axial strain $S'_1 = 0.0025$.

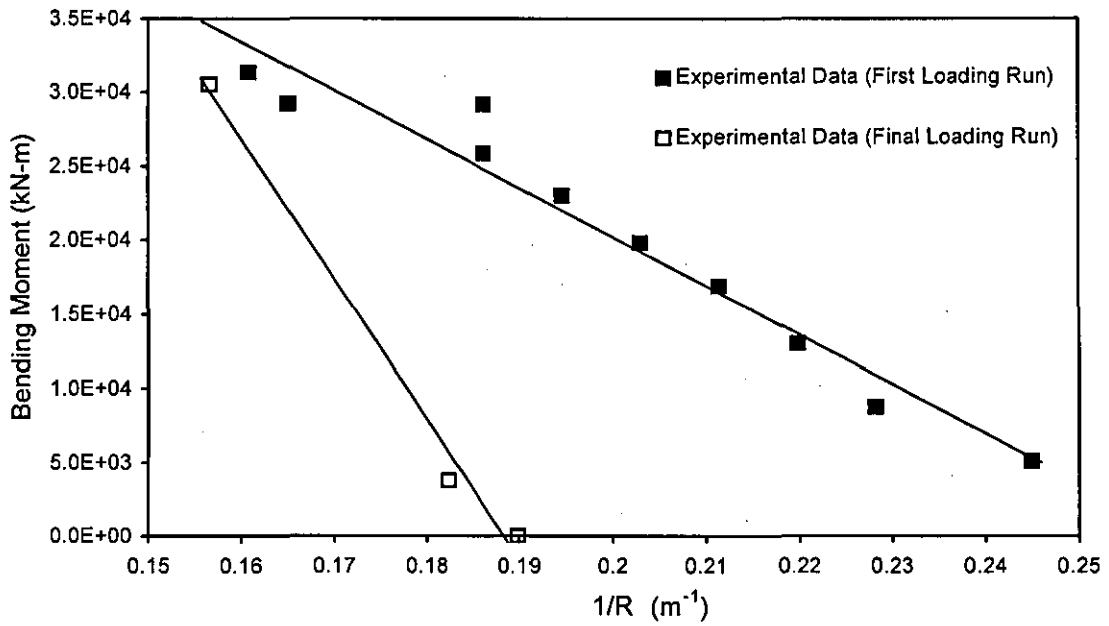


Fig. 6.8 – Variation, Based on the Simplified Approach, of the Bending Moment as a Function of the Parameter $\frac{1}{R}$.

Table 6.3 – Values of the Bending and Torsional Stiffnesses for the 164 mm Outside Diameter Cable.

First Loading Run $(EI)_{\text{cable}}$	3.611×10^{11} (N-mm ²)
Final Loading Run $(EI)_{\text{cable}}$	9.48×10^{11} (N-mm ²)
Plane-Section $(EI)_{\text{full-slip}}$	3.74×10^{12} (N-mm ²)
Plane-Section $(EI)_{\text{no-slip}}$	4.59×10^{12} (N-mm ²)
$(GJ)_{\text{full-slip}}$	8.31×10^{11} (N-mm ²)
$(GJ)_{\text{no-slip}}$	1.77×10^{12} (N-mm ²)

Using Equations (6.1) – (6.3), in conjunction with the experimentally determined values of $(EI)_{\text{cable}}$ as given in Table 6.3, predictions, based on the simplified method, of the ripple range and the curvature, $\frac{1}{R}$, outside the original experimental range, can be made for any given axial force, F . Tables (6.4) and (6.5) present such results for the first and final loading runs, respectively.

Table 6.4 – Determination of the Ripple Range and Bending Moment, for Each Given Axial Load, for the First Loading Run, Using the Simplified Method.

Axial Load, F (kN)	Ripple Range, 2r (mm)	α (radians)	1/R (m ⁻¹)	Bending Moment, m (kN-m)
0	30.0	0.061683	0.25333	0.0
350	29.0	0.059632	0.24495	5075.0
650	27.0	0.055528	0.22816	8775.0
1000	26.0	0.053475	0.21976	13000.0
1350	25.0	0.051422	0.21135	16875.0
1650	24.0	0.049369	0.20294	19800.0
2000	23.0	0.047315	0.19453	23000.0
2350	22.0	0.045261	0.1861	25850.0
2650	22.0	0.045261	0.1861	29150.0
3000	19.5	0.040123	0.16503	29250.0
3300	19.0	0.039096	0.16081	31350.0
3995	18.0	0.037040	0.15237	35951.2
4558	17.0	0.034984	0.14393	38742.8
5192	16.0	0.032927	0.13548	41535.5
5911	15.0	0.030871	0.12703	44329.4
6732	14.0	0.028814	0.11857	47124.4
7680	13.0	0.026757	0.11012	49920.3
8786	12.0	0.024700	0.10166	52717.2
10094	11.0	0.022642	0.09319	55514.9

Table 6.5 – Determination of the Ripple Range and Bending Moment, for Each Given Axial Load, for the Final Loading Run, Using the Simplified Method.

Axial Load, F (kN)	Ripple Range, 2r (mm)	α (radians)	1/R (m ⁻¹)	Bending Moment, m (kN-m)
0	22.5	0.046185	0.1898936	0.0
350	21.6	0.044336	0.1823114	3771.3
3300	18.5	0.038068	0.1565895	30525.0
3787.5	18.0	0.037040	0.1523692	34087.3
4952.1	17.0	0.034984	0.1439255	42092.6
6262.7	16.0	0.032927	0.1354782	50101.4
7748.5	15.0	0.030871	0.1270275	58113.4
9446.9	14.0	0.028814	0.1185736	66128.5
11407.2	13.0	0.026757	0.1101166	74146.5

Using Equations (6.4) – (6.43), predictions, based on the rigorous method, of the ripple range and, hence, the bending moment, m , can also be made for any given axial force, F . Tables (6.6) and (6.7) present the numerical results for the aforementioned parameters, using the rigorous method, for the first and final loading runs, respectively. It is, perhaps, worth mentioning that the results in Tables (6.6) and (6.7) are all based on the measured values of r_0 and H_0 for the first and final loading runs, respectively.

Table 6.6 – Determination of the Ripple Range and Bending Moment, for Each Given Axial Load, for the First Loading Run, Using the Rigorous Method.

Axial Load, F (kN)	Ripple Range, 2r (mm)	α (radians)	1/R (m ⁻¹)	Bending Moment, m (kN-m)
0	30.0	0.061683	0.25333	0
350	28.4	0.058337	0.23956	4966.47
650	27.1	0.055742	0.22887	8815.11
1000	25.8	0.052989	0.21754	12895.20
1350	24.6	0.050493	0.20726	16592.58
1650	23.6	0.048532	0.19919	19496.24
2000	22.6	0.046427	0.19052	22611.80
2350	21.7	0.044496	0.18256	25469.54
2650	20.9	0.042963	0.17625	27736.76
3000	20.1	0.041302	0.1694	30192.9
3300	19.5	0.039977	0.16394	32152.23
3995	18.1	0.037208	0.15253	36243.44
4558	17.2	0.035230	0.14438	39165.53
5192	16.2	0.033238	0.13617	42107.12
5911	15.2	0.031233	0.12791	45066.06
6732	14.3	0.029219	0.11961	48038.88
7680	13.3	0.027192	0.11126	51029.76
8786	12.3	0.025153	0.10285	54036.54
10094	11.3	0.023102	0.0944	57061.18

Table 6.7 – Determination of the Ripple Range and Bending Moment, for Each Given Axial Load, for the Final Loading Run, Using the Rigorous Method.

Axial Load, F (kN)	Ripple Range, 2r (mm)	α (radians)	1/R (m ⁻¹)	Bending Moment, m (kN-m)
0	22.5	0.046288	0.190739	0.0
350	22.0	0.045300	0.18581	3845.8
650	21.6	0.044486	0.182444	7015.2
1000	21.2	0.043573	0.178664	10573.2
1350	20.8	0.042695	0.175035	13989.4
1650	20.4	0.041971	0.172039	16810.9
2000	20.0	0.041156	0.168667	19985.2
2350	19.7	0.040371	0.165421	23039.9
2650	19.3	0.039722	0.162735	25568.0
3000	19.0	0.038990	0.159708	28417.5
3300	18.7	0.038384	0.157199	30778.7
3787.5	18.2	0.037437	0.153284	34464.1
4952.1	17.2	0.035353	0.14466	42581.4
6262.7	16.2	0.033266	0.136023	50710.1
7748.5	15.2	0.031176	0.127375	58849.5
9446.9	14.2	0.029083	0.118715	66999.5
11407.2	13.2	0.026988	0.110047	75157.2

Figure (6.9) shows the variation of the ripple range with changes in the applied axial force for both the experimental data and the theoretical predictions, based on both the simplified and rigorous methods. The variation of the bending and twisting moments plus the shear force along the spiral strand, G' , H and N' , respectively, with applied axial force, are shown graphically, for the first and final loading runs, in Figs. (6.10a, b and c), respectively. Fig. (6.11) shows the variation of the strand axial strain, ϵ_{h1} with changes in the applied axial force.

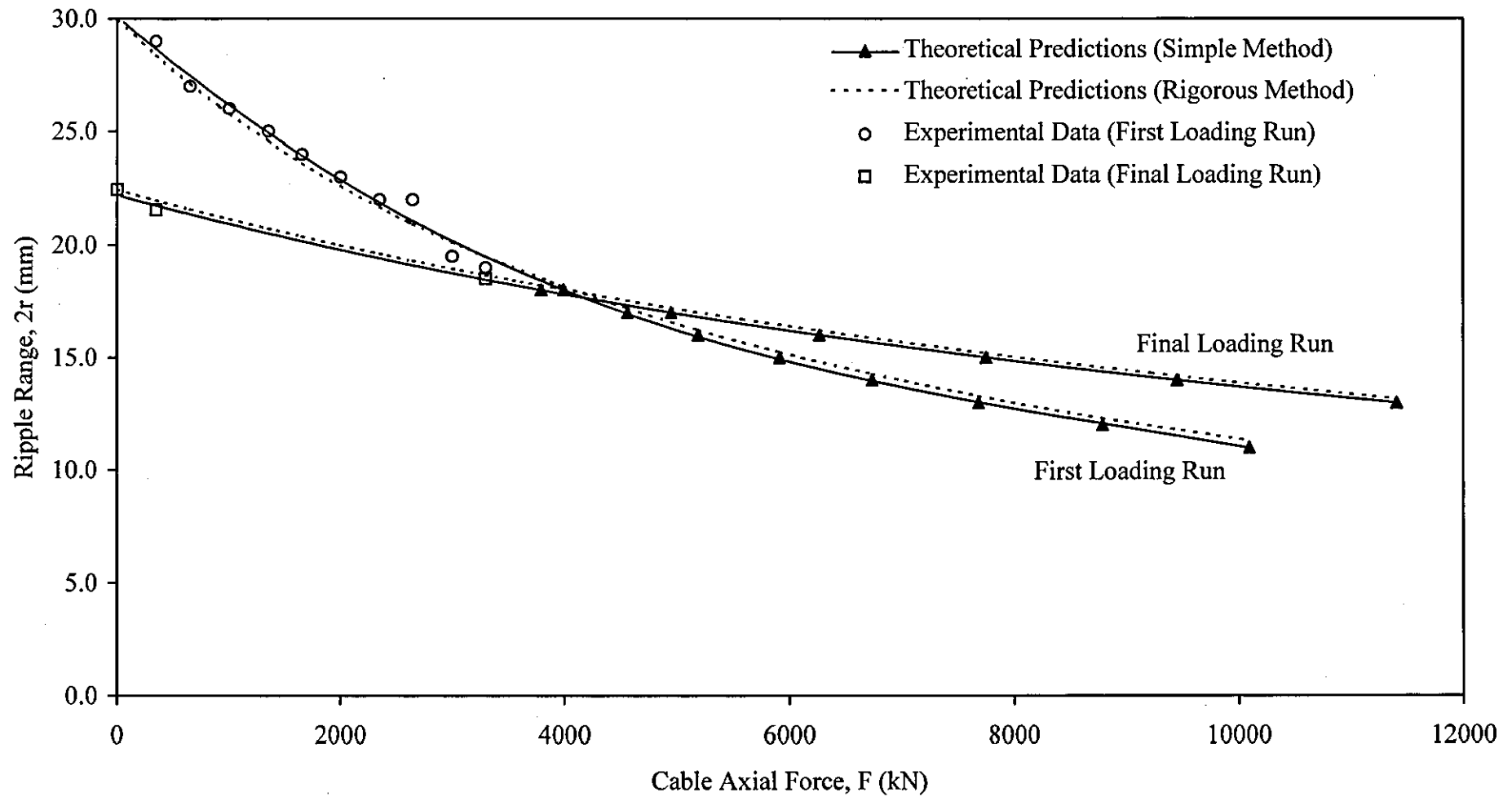


Fig. 6.9 – Experimental Plus Theoretical Variations of the Ripple Range with Changes in the Applied Axial Force Based on Both the Simplified and Rigorous Methods.

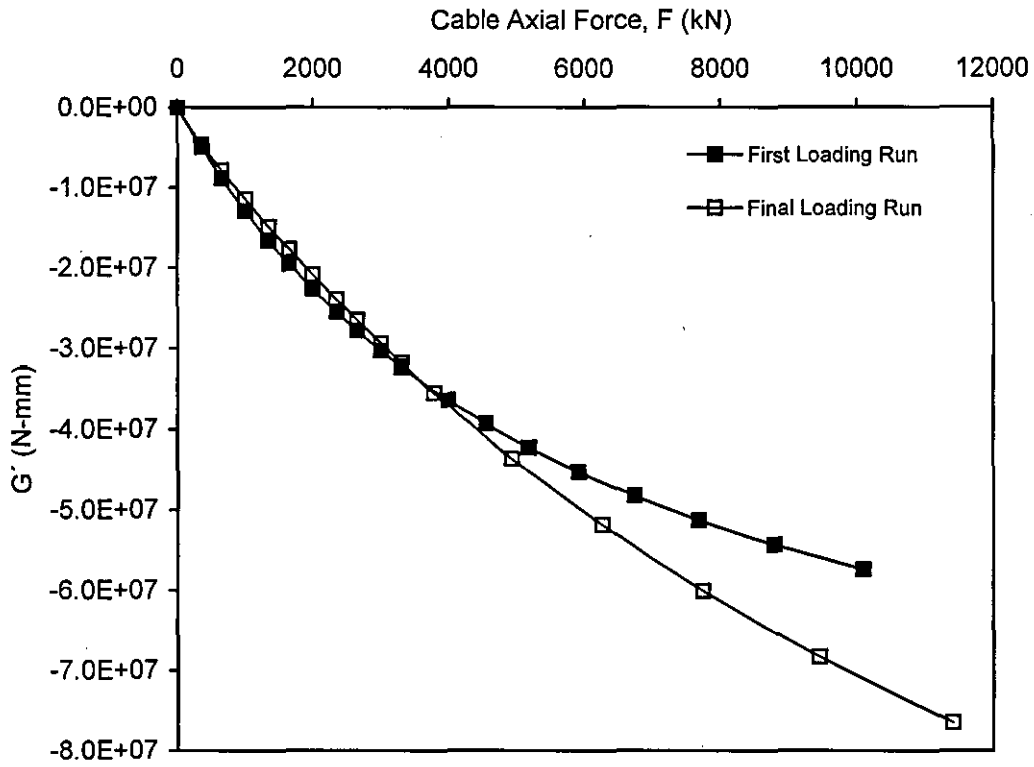


Fig. 6.10a – Theoretical Variations of the Parameter G' with Changes in the Applied Axial Force for Both the First and Final Loading Runs.

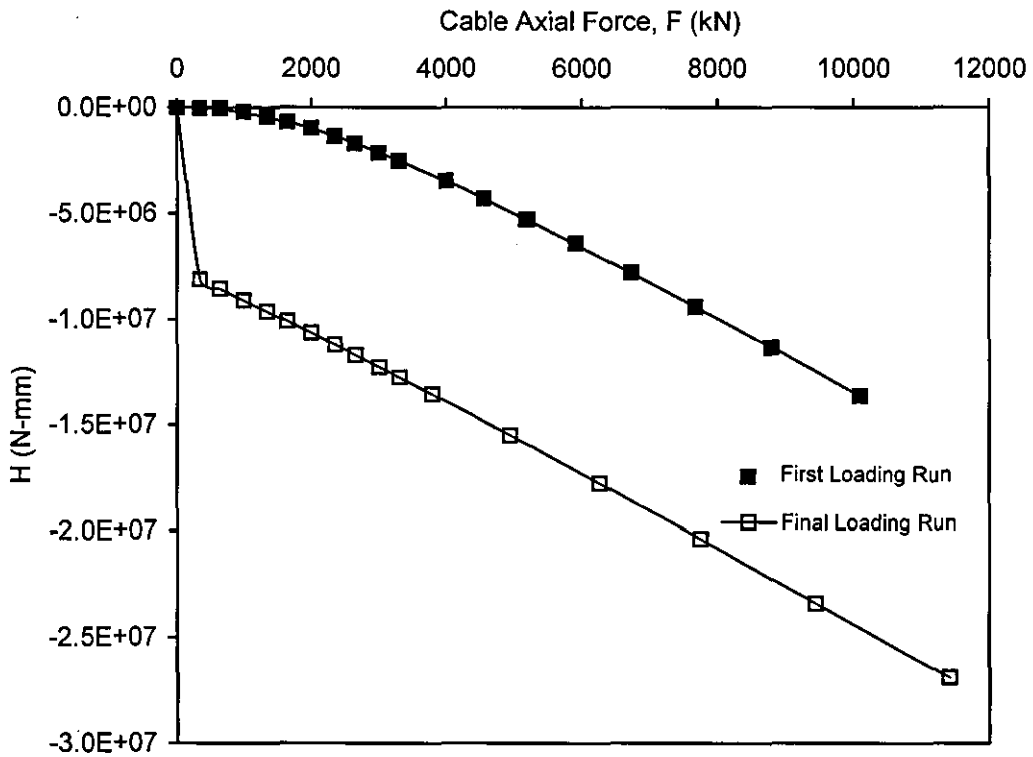


Fig. 6.10b – Theoretical Variations of the Parameter H with Changes in the Applied Axial Force for Both the First and Final Loading Runs.

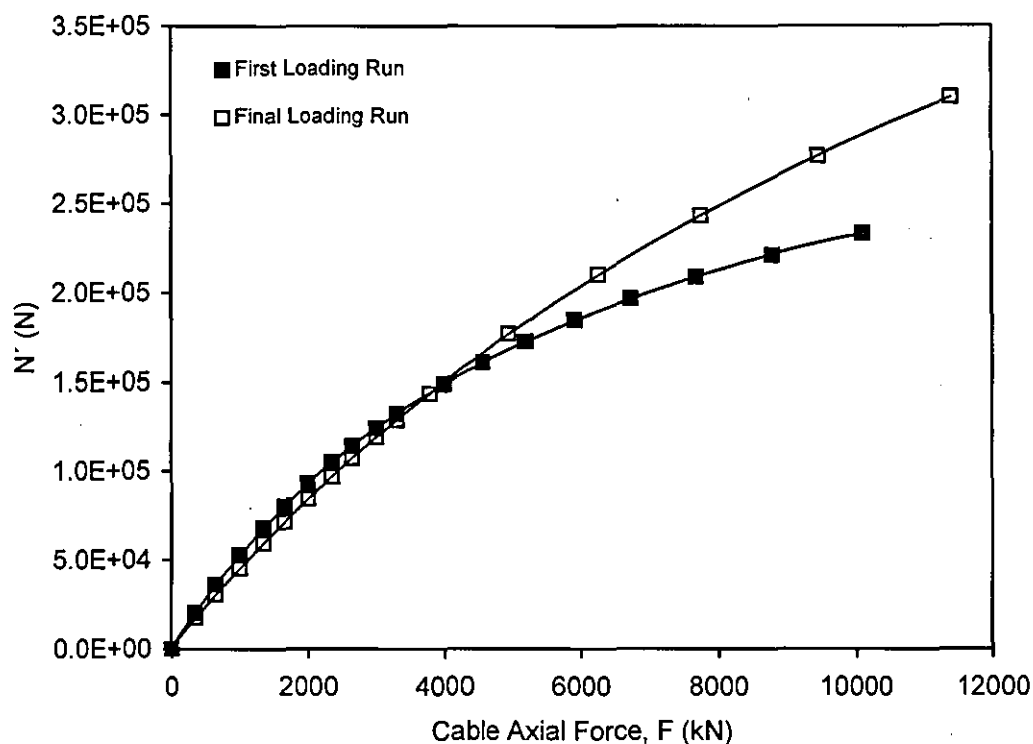


Fig. 6.10c – Theoretical Variations of the Cable Shear Force, N' , with Changes in the Applied Axial Force for Both the First and Final Loading Runs.

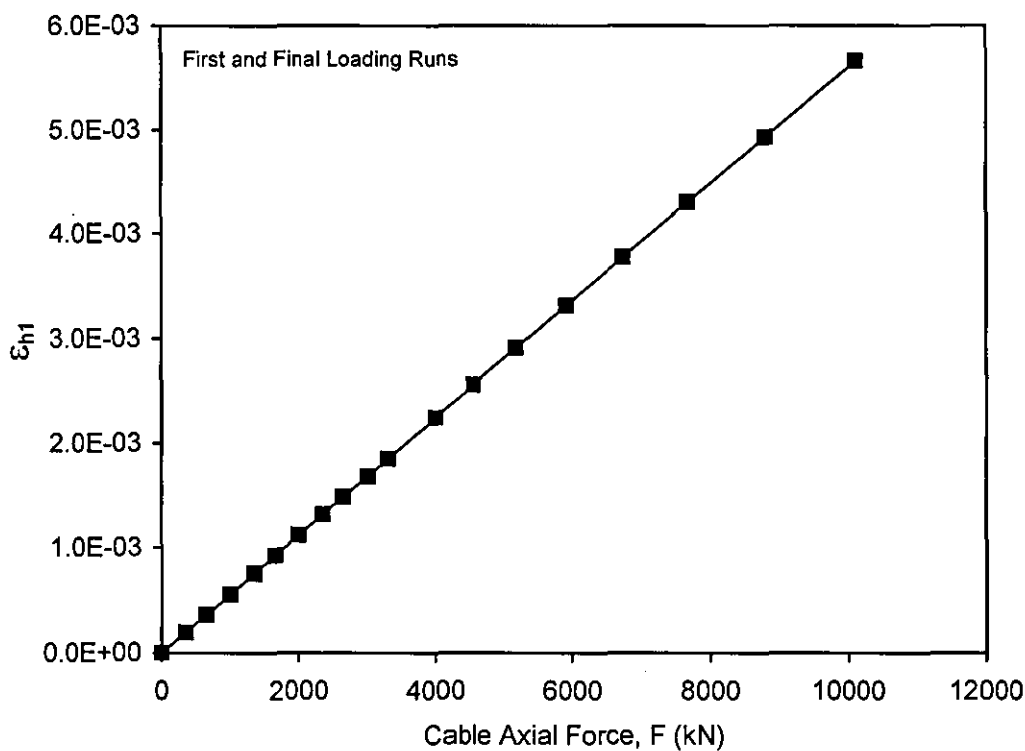


Fig. 6.11 – Theoretical Variations of the Cable Axial Strain, ϵ_{h1} , with Changes in the Applied Cable Axial Force, F , for Both the First and Final Loading Runs, Based on the Rigorous Method.

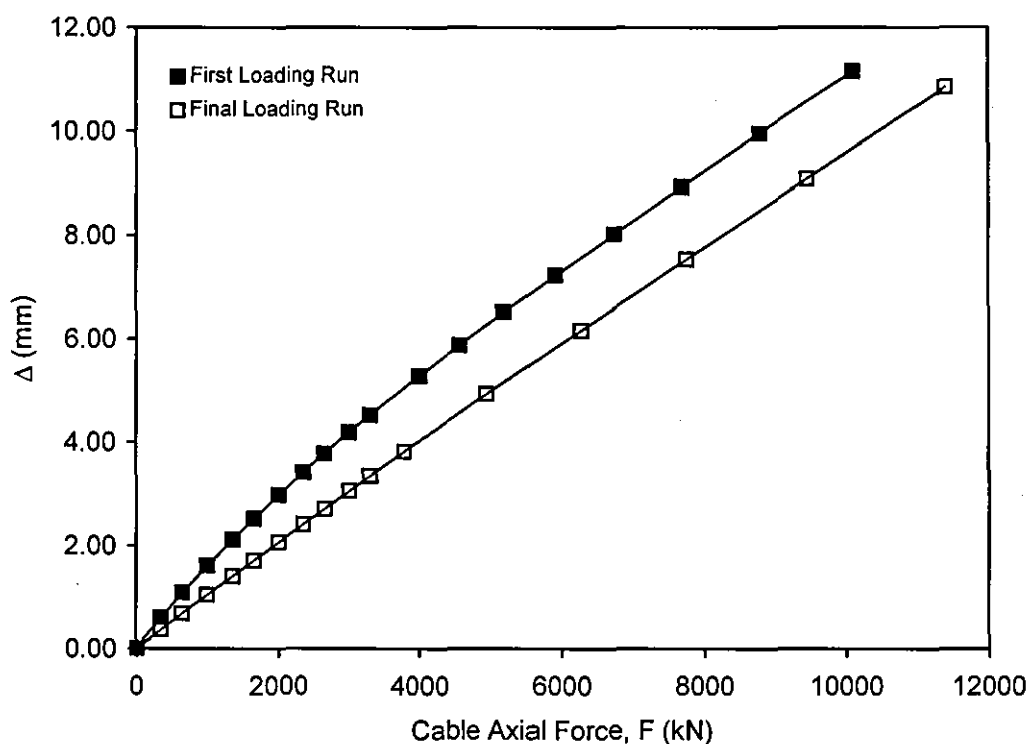


Fig. 6.12 – Theoretical Variations of Δ with Changes in the Applied Cable Axial Force, F , for Both the First and Final Loading Runs, Based on the Rigorous Method.

6.7 DISCUSSION

From Fig. (6.8) and Table (6.3) it is clear that there exists some rather significant differences between the experimentally obtained bending stiffnesses, $(EI)_{\text{cable}}$, for the first and final loading runs, the difference being a factor of 2.6. This difference is believed to be due to the gradual nature of the interwire bedding-in, due to the pre-stretching of the cable, and as such the helical cables may need (depending upon their construction) a very lengthy period of working-in for their internal structure to become reasonably stabilised (Raoof, 1990a). During this period, the bending stiffness will change in a complex way, due to its sensitivity to the degree of bedding-in.

Comparing the prediction of the full-slip plane-section bending stiffness with the experimentally obtained one for the final loading run shows a difference by a factor of 4, with the experimentally determined value being four times less than the theoretically obtained full-slip plane-section bending stiffness. Due to the lack of information concerning the number of loading runs, and, hence, the degree of bedding-in that the strand has experienced, it is felt that if the cable had been subjected to more loading runs, eventually, the measured cable bending stiffness would have, probably,

approached the theoretical plane-section limit, as the internal structure of the cable would have become more stable. It should also be noted that due to the ever presence of interwire friction, the cable effective bending stiffness is loading path dependent and the previous loading history will probably have a significant influence on the measured values of $(EI)_{\text{cable}}$.

Fig. (6.9) shows the theoretical predictions, based on both the simplified and rigorous methods, along with the experimental data for the first and final loading runs with the experimental data extrapolated to other regions of axial force, F , for which no test data is available. The correlations between the theoretical and experimental data, where available, is very encouraging, and, at all levels of cable axial force, the match between the results based on both the simplified and rigorous approaches is encouraging, reinforcing the fact that the influence of a number of parameters ignored in the simplified approach, regarding the exact form of the F versus $2r$ plots, is sufficiently small to be ignored.

Figs. (6.10a – c), Fig. (6.11) and Fig. (6.12) show the variations of the various parameters used in the rigorous theoretical model with changes in the externally applied force F . The only anomaly within the results is shown in Fig. (6.10b), for the final loading run, where a downward shift in the H versus F plot is found, with this probably being due to numerical problems (although the computer programme was run with double precision).

6.8 CONCLUSIONS

Presented in this chapter is a promising experimental method for obtaining the effective bending stiffness of a helical steel cable, the reliable experimental determination of which, until now, had largely proven to be elusive. The previously available experimental data relating to the effective bending stiffness of a cable had invariably been too dependent upon the specific experimental technique employed. The presently proposed experimental technique is believed to be a significant step forward in measuring the in-situ effective bending stiffness, for even very large diameter cables, at reasonable cost and effort, involving minimal physical interface

with the imposed cable deformations, with this having been (at least in some cases) a major obstacle in obtaining trustworthy test data.

CHAPTER 7

SIMPLE DETERMINATION OF WIRE ROPE AXIAL STIFFNESS

7.1 INTRODUCTION

During the past decade or so, considerable interest has been shown in the mechanical characteristics of wire ropes, for use in both onshore and offshore applications. At this point, it is probably worthwhile explaining the main difference between a spiral strand and a wire rope. A spiral strand is a group of wires laid helically in successive layers over a central king wire (or equal lay core), while a wire rope consists of (typically) six strands laid helically over a central core (Fig. 7.1) which may itself consist of a twisted fibre core (FC) or a smaller independent wire rope core (IWRC).

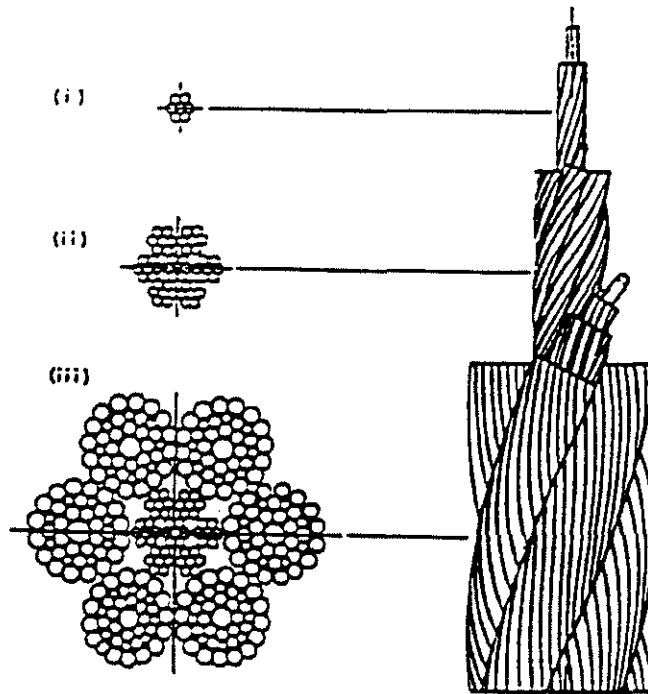


Fig. 7.1 - Construction Details for a Typical Six-Stranded Wire Rope with an Independent Wire Rope Core (IWRC) - After Lee (1991).

Helical steel cables (wire ropes and/or spiral strands) are used extensively in bridge design and as tension members for suspended and stayed structures, generally. With reference to the offshore industry, there has been a growing need for longer, and stronger cables, with increasingly larger outside diameters, for use as components in

mooring systems for, for example, oil exploration and production platforms. The decision as to whether a spiral strand or a wire rope should be used is dependent upon the intended type of application. Wire rope is a little more flexible axially than a spiral strand, but considerably more flexible in bending, which is why wire ropes are used as tractive elements over pulleys, winch drums and fairleads in mines and cable cars (amongst others).

Steel cable design and manufacture is often considered to be as much an art as an exact science. With the increasingly larger diameter cables being used by the industry, the need for a model which could accurately predict the mechanical characteristics, and lead to an understanding of the underlying phenomena of wire rope and spiral strand behaviour has (over the last two decades) been given a new urgency.

Hobbs and Raoof (1982) have developed the orthotropic sheet theoretical model, the salient features of which have already been reported in some detail in chapter 3: this concept is capable of predicting, with a good degree of accuracy, the mechanical characteristics of spiral strands. The results from the orthotropic sheet concept have later been used by Raoof and Kraincanic (1995b, d) to develop a model for analysing various characteristics of wire ropes. The problem with the theoretical model of Raoof and Kraincanic is that it is mathematically rather complex, and there is a clear need to develop simple routines which are amenable to hand calculations, so that the formulations may be used by busy practising engineers with minimum effort.

Hruska (1951) has reported a very simple formula for calculating the full-slip modulus, with this effectively being a weighted average of the cubes of the lay angles multiplied by the Young's modulus for steel. Using Hruska's simple approach, Hobbs and Raoof (1986) analysed, amongst other aspects of spiral strand behaviour, the axial stiffness characteristics of large diameter spiral strands. They showed that in repeated (cyclic) loading regimes, the effective axial stiffness of spiral strands (with their ends fixed against rotation) varied (as a function of the externally applied load perturbations) between two limits. The lower limit, the full-slip stiffness, was shown to be a function of the lay angle only. The larger (no-slip) stiffness was, on the other

hand, found to be a sole function of the full-slip axial stiffness, as predicted by the orthotropic sheet theory.

In a later publication, Raoof (1990e), based on the results from an extensive series of theoretical parametric studies, using a wide range of large diameter spiral strand constructions, in conjunction with Hruska's simple formula, presented simple routines, based on the more accurate orthotropic sheet model, for estimating the no-slip and full-slip axial moduli of axially preloaded spiral strands, with any construction details. Raoof (1990e) also showed that primarily because of the inclusion of the strands' diametral contractions in the orthotropic sheet theory, the estimates of axial stiffness based on this approach are significantly lower than those based on Hruska's formulations.

In 1988, Strzemiecki and Hobbs proposed a general form of Hruska's formulations which could be used to obtain an estimate of the full-slip axial stiffness of a wire rope. An attempt was subsequently made by Strzemiecki and Hobbs to calculate, more accurately, the full-slip and no-slip axial stiffnesses of wire ropes, using the same procedure as that adopted by Hobbs and Raoof (1986) in connection with spiral strands, with the final outcome being encouraging, although there was obviously room for improvement as demonstrated by a comparison of their final predictions with their corresponding large scale and carefully obtained experimental data.

The purpose of this chapter is, based on an extension of the work of Strzemiecki and Hobbs (1988), to develop simple formulations for calculating the full-slip and no-slip axial stiffnesses of wire ropes, with either fibre or independent wire rope cores, which are amenable to simple hand calculations.

7.2 HRUSKA'S APPROACH

The general form of Hruska's equation, proposed by Strzemiecki and Hobbs (1988), for the determination of the axial stiffness of a wire rope is

$$H = \frac{E_{\text{rope}}}{E_{\text{steel}}} = \frac{\sum_{j=1}^N \left[\sum_{i=1}^n A_{wi} \cos^3 \alpha_i \right] \cos^3 \beta_j}{\sum_{j=1}^N \left[\sum_{i=1}^n A_{wi} / \cos \alpha_i \right] / \cos \beta_j} \quad (7.1)$$

where, E_{steel} is the Young's modulus of steel, N is the number of strands in the rope, n is the number of wires in the strand, A_{wi} is the cross-sectional area of the wires, α_i is the lay angle of layer i in the strand, and β_j is the lay angle of a strand in layer j of the rope.

According to Equation (7.1), the dominant parameters controlling the rope axial stiffness are the lay angles β_j of the strands in the wire rope and also the lay angles α_i of the solid steel wires forming the individual spiral strands, with the parameter A_{wi} also playing a role.

7.3 ANALYSIS

The numerical data relating to both the no-slip and full-slip axial stiffnesses, based on the work of Raoof and Kraincanic (1995b, d), on a number of wire ropes, with fibre or independent wire rope cores, has been used in what follows. The fibre core wire ropes had outside diameters of 9.53 and 40.5 mm, and were analysed assuming both a regular lay (R L), and a Lang's lay (L L) type of construction. A Lang's lay rope is the type in which the directions of the lay of the individual wires in the outer strands, and that of the outer strands in the rope, are the same. If the lay directions of the wires and the strands are the opposite of each other, then the rope is of a regular lay type. It should be noted that the 'Hruska' stiffnesses, as calculated by Equation (7.1), are the same regardless of the type of lay. The wire ropes with independent wire rope cores had outside diameters of 33, 40, 55.6, and 76 mm. Two 76 mm outside diameter wire ropes were used in the analysis; a reasonably fully bedded-in (comparator) and a new wire rope.

The results, after Raoof and Kraincanic (1995d), for the fibre core wire ropes have been obtained assuming two different patterns (cases) of interstrand contacts:

case 1. - the strands in the wire rope are assumed to be just touching each other in line-contact, in an unstressed condition, so that the interstrand contacts in the hoop direction (with a higher normal stiffness compared to that in the radial direction) govern the diametral contraction of the rope; and

case 2. - the strands in the wire rope are assumed to be resting on the fibre core, in the presence of significant gaps between the adjacent strands, so that the wire rope experiences radial deformations due to fibre core compliance.

The exact details of the theoretical model, relating to each case of interstrand contacts in wire ropes with fibre cores, are reported by Raoof and Kraincanic (1995d)

The construction details of the wire ropes used in the present work, in conjunction with the calculation details of the Hruska axial stiffnesses, are presented in Tables 7.1a - g. These tables follow the same format as that originally used by Strzemiecki and Hobbs (1988).

7.4 RESULTS

Table 7.2 presents a summary of the final estimates of the axial stiffnesses, based on Hruska's approach, for each of the seven different wire rope constructions, as well as the numerical results for both the no-slip and full-slip axial stiffnesses, as reported by Raoof and Kraincanic and later on by Kraincanic and Hobbs (1999), based on the considerably more complex (although more accurate) model of Raoof and Kraincanic. Table 7.3, on the other hand, presents values of the corresponding experimentally determined axial stiffnesses, wherever available.

Table 7.1a - Construction Details and Calculation Routines for Hruska's Axial Stiffness of a 33 mm Outside Diameter (IWRC) Wire Rope.

1	2	3	4	5	6	7	8	9	10	11	12
Layer in the Rope	N Strand (i)	β_j	Layer in the Strand	n Wires (i)	d_{wi}	A_{wi}	α_i	$n_w A_{wi} / \cos \alpha_i$	$n_w A_{wi} \times \cos^3 \alpha_i$	$N \times \sum_i (9) / \cos \beta_j$	$N \times \sum_i (10) \times \cos^3 \beta_j$
Core (1)	1	0	Core (1)	1	1.6028	2.018	0	2.018	2.018		
King Strand			King Wire 2	6	1.4694	1.696	16.29	10.600	8.998		
TOTALS								12.618	11.015	12.618	11.015
IWRC Strands (2)	6	19.17	Core (1) 2	1 6	1.4042 1.3104	1.549 1.349	0 8.93	1.549 8.191	1.549 7.801		
TOTALS								9.740	9.350	61.869	47.274
Outer Strands (3)	6	19.76	Core (1) 2 3	1 9 9	2.9114 1.4024 2.5356	6.657 1.545 5.050	0 12.26 -21.23	6.657 14.226 48.755	6.657 12.972 36.807		
TOTALS								69.638	56.436	443.972	282.255
GRAND TOTALS										518.459	340.545

$$\left(\frac{E_{\text{rope}}}{E} \right)_{\text{full-slip}} = \frac{340.545}{518.459} = 0.657$$

Table 7.1b- Construction Details and Calculation Routines for Hruska's Axial Stiffness of a 40 mm Outside Diameter (IWRC) Wire Rope.

1	2	3	4	5	6	7	8	9	10	11	12
Layer in the Rope	N Strand (i)	β_j	Layer in the Strand	n Wires (i)	d_{wi}	A_{wi}	α_i	$n_w A_{wi} / \cos \alpha_i$	$n_w A_{wi} \times \cos^3 \alpha_i$	$N \times \sum_i (9) / \cos \beta_j$	$N \times \sum_i (10) \times \cos^3 \beta_j$
Core (1)	1	0	Core (1)	1	2	3.142	0	3.142	3.142		
King Strand			King Wire 2	6	1.84	2.659	14.88	16.508	14.402		
TOTALS								19.649	17.544	19.649	17.544
IWRC Strands (2)	6	17.92	Core (1) 2	1 6	1.84 1.68	2.659 2.217	0 14.88	2.659 13.762	2.659 12.007		
TOTALS								16.421	14.666	103.548	75.798
Outer Strands (3)	6	18.19	Core (1) 2 3 4	1 8 16 16	3.15 1.88 1.484 2	7.793 2.776 1.730 3.142	0 -18.29 -18.29 -18.29	7.793 23.389 29.147 52.940	7.793 19.009 23.688 43.026		
TOTALS								113.269	93.516	715.362	481.110
GRAND TOTALS										838.560	574.452

$$\left(\frac{E_{rope}}{E} \right)_{full-slip} = \frac{574.452}{838.56} = 0.685$$

Table 7.1c - Construction Details and Calculation Routines for Hruska's Axial Stiffness of a 55.6 mm Outside Diameter (IWRC) Wire Rope.

1	2	3	4	5	6	7	8	9	10	11	12
Layer in the Rope	N Strand (i)	β_j	Layer in the Strand	n Wires (i)	d_{wi}	A_{wi}	α_i	$n_w A_{wi} / \cos \alpha_i$	$n_w A_{wi} \times \cos^3 \alpha_i$	$N \times \sum_i (9) / \cos \beta_j$	$N \times \sum_i (10) \times \cos^3 \beta_j$
Core (1)	1	0	Core (1)	1	2.26	4.011	0	4.011			
King Strand			King Wire 2	6	2.26	4.011	10.58	24.485			
TOTALS								28.497	26.874	28.497	26.874
IWRC Strands (2)	6	16.1	Core (1) 2	1 6	2.26 2.26	4.011 4.011	0 10.58	4.011 24.485	4.011 22.862		
TOTALS								28.497	26.874	177.960	143.004
Outer Strands (3)	6	12.65	Core (1) 2 filler 4	1 6 6 12	3.96 3.86 1.63 3.71	12.136 11.702 2.087 10.810	0 -9.2 0 -17.7	12.136 71.128 12.520 136.170	12.316 67.538 12.520 112.159		
TOTALS								232.134	204.534	1427.454	1139.986
GRAND TOTALS										1633.911	1309.864

$$\left(\frac{E_{rope}}{E} \right)_{full-slip} = \frac{1309.864}{1633.911} = 0.802$$

Table 7.1d - Construction Details and Calculation Routines for Hruska's Axial Stiffness of a 76 mm Outside Diameter (IWRC) Wire Rope.

1	2	3	4	5	6	7	8	9	10	11	12
Layer in the Rope	N Strand (i)	β_j	Layer in the Strand	n Wires (i)	d_{wi}	A_{wi}	α_i	$n_w A_{wi} / \cos \alpha_i$	$n_w A_{wi} \times \cos^3 \alpha_i$	$N \times \sum_I(9) / \cos \beta_j$	$N \times \sum_I(10) \times \cos^3 \beta_j$
Core (1)	1	0	Core (1)	1	3.962	12.329	0	12.329	12.329		
King Strand			King Wire 2	6	3.607	10.218	20.76	65.567	50.127		
							TOTALS	77.896	62.456	77.896	62.456
IWRC Strands (2)	6	17.09	Core (1) 2	1 6	3.607 3.15	10.218 7.793	0 18.7	10.218 49.365	10.218 39.738		
							TOTALS	59.583	49.956	374.013	261.758
Outer Strands (3)	6	16.08	Core (1) 2	1 8	6.35 3.658	31.669 10.509	0 -8.21	31.669 84.946	31.669 81.517		
			3	8	3.302	8.563	-12.69	70.222	63.608		
			8	2.438	4.668	-12.69	38.281	34.676			
			4	16	3.785	11.252	-16.97	188.224	157.521		
								GRAND TOTALS		3032.947	2288.337

$$\left(\frac{E_{\text{rope}}}{E} \right)_{\text{full-slip}} = \frac{2288.337}{3032.947} = 0.754$$

Table 7.1e - Construction Details and Calculation Routines for Hruska’s Axial Stiffness of a 76 mm (Comparator) Outside Diameter (IWRC) Wire Rope.

1	2	3	4	5	6	7	8	9	10	11	12
Layer in the Rope	N Strand (i)	β_j	Layer in the Strand	n Wires (i)	d_{wi}	A_{wi}	α_i	$n_w A_{wi} / \cos \alpha_i$	$n_w A_{wi} \times \cos^3 \alpha_i$	$N \times \sum_i (9) / \cos \beta_j$	$N \times \sum_i (10) \times \cos^3 \beta_j$
Core (1)	1	0	Core (1)	1	3.9	11.946	0	11.946	11.946		
King Strand			King Wire 2	6	3.54	9.842	14.84	61.091	53.339		
							TOTALS	73.037	65.285	73.037	65.285
IWRC Strands (2)	6	18.11	Core (1) 2	1 6	3.34 3.1	8.762 7.548	0 14.84	8.762 46.849	8.762 40.904		
							TOTALS	55.610	49.666	351.052	255.864
Outer Strands (3)	6	18.24	Core (1)	1	5.8	26.421	0	26.421	26.421		
			2	8	3.5	9.621	-9.53	78.046	73.826		
			3	8	2.4	4.524	-13.91	37.285	33.100		
			8	3.2	8.042	-13.07	66.051	59.468			
			4	16	3.7	10.752	-18.35	181.250	147.102		
							TOTALS	389.052	339.917	2457.810	1747.257
								GRAND TOTALS		2881.899	2068.407

$$\left(\frac{E_{\text{rope}}}{E} \right)_{\text{full-slip}} = \frac{2068.407}{2881.899} = 0.7177$$

Table 7.1f - Construction Details and Calculation Routines for Hruska Axial Stiffness of a 9.53 mm Outside Diameter (FC) Wire Rope.

1	2	3	4	5	6	7	8	9	10	11	12
Layer in the Rope	N Strand (i)	β_j	Layer in the Strand	n Wires (i)	d_{wi}	A_{wi}	α_i	$n_w A_{wi} / \cos \alpha_i$	$n_w A_{wi} \times \cos^3 \alpha_i$	$N \times \sum_l (9) / \cos \beta_j$	$N \times \sum_l (10) \times \cos^3 \beta_j$
Core (1)	1		Core (1) King Wire	1	0.89	0.622	-	0	0	0	0
							TOTALS	0	0		
IWRC	6	19.5	Core (1)	9	0.432	0.147	9.29	1.337	1.268		
							TOTALS	1.337	1.268	8.508	6.372
Outer	6	19.5	Core (1) 2	9	0.762	0.456	-16.07	4.271	3.642		
							TOTALS	4.271	3.642	27.187	18.302
									GRAND TOTALS	35.695	24.674

$$\left(\frac{E_{rope}}{E} \right)_{full-slip} = \frac{24.674}{35.695} = 0.691$$

Table 7.1g - Construction Details and Calculation Routines for Hruska Axial Stiffness of a 40.5 mm Outside Diameter (FC) Wire Rope.

1	2	3	4	5	6	7	8	9	10	11	12
Layer in the Rope	N Strand (i)	β_j	Layer in the Strand	n Wires (i)	d_{wi}	A_{wi}	α_i	$n_w A_{wi} / \cos \alpha_i$	$n_w A_{wi} \times \cos^3 \alpha_i$	$N \times \sum_i (9) / \cos \beta_j$	$N \times \sum_i (10) \times \cos^3 \beta_j$
Core (1)	1		Core (1) King Wire	1	3.1	7.548	-	0	0	0	0
							TOTALS	0	0		
IWRC	6	16.86	Core (1)	8	1.85	2.688	7.5	21.690	20.957		
							TOTALS	21.690	20.957	135.984	110.214
Outer	6	16.86	Core (1) 2	8	3.53	9.787	-14.34	80.812	71.202		
							TOTALS	80.812	71.202	506.650	374.454
							GRAND TOTALS		642.633	484.669	

$$\left(\frac{E_{rope}}{E} \right)_{full-slip} = \frac{484.699}{642.633} = 0.754$$

Table 7.2 - Summary of the Numerical Results for the Axial Stiffnesses as Calculated Using Hruska's Simple Formula and the Model Proposed by Raoof and Kraincanic.

Type of Core Construction	Rope Outside Diameter (mm)	Hruska	Raoof and Kraincanic		
		$\left(\frac{E_{rope}}{E}\right)_{full-slip}$	$\frac{E_{no-slip}}{E_{steel}}$	$\frac{E_{full-slip}}{E_{steel}}$	$\frac{E_{no-slip}}{E_{full-slip}}$
IWRC	33	0.657	0.561	0.478	1.174
IWRC	40	0.685	0.623	0.537	1.158
IWRC	55.6	0.802	0.779	0.714	1.092
IWRC	76	0.754	0.708	0.646	1.096
IWRC	76 (Comparator)	0.718	0.654	0.583	1.122
FC (RL) – 1†	9.53	0.691	0.714	0.654	1.092
FC (RL) – 1†	40.5	0.754	0.778	0.729	1.068
FC (LL) – 1†	9.53	0.691	0.701	0.636	1.102
FC (LL) – 1†	40.5	0.754	0.763	0.708	1.077
FC (RL) – 2*	9.53	0.691	0.658	0.608	1.081
FC (RL) – 2*	40.5	0.754	0.732	0.689	1.063
FC (LL) – 2*	9.53	0.691	0.652	0.600	1.086
FC (LL) – 2*	40.5	0.754	0.722	0.675	1.069

$E_{steel} = 200 \text{ kN/mm}^2$

IWRC: Independent Wire Rope Core,

FC: Fibre Core,

LL: Lang's Lay,

RL: Regular Lay,

1†: Case one, and

2*: Case two.

Figs. 7.2a and b show the relationship between the full-slip axial stiffnesses as calculated using the simple formula of Hruska, and Raoof and Kraincanic's model, for all of the different wire rope constructions; relating to the results of cases 1 and 2 for the ropes with fibre cores, respectively. Fig. 7.2c shows similar comparisons, but with the data relating to the fibre core wire ropes omitted.

Figs. 7.3a and b present the relationships between the $\frac{E_{full-slip}}{E_{steel}}$ and the $\frac{E_{no-slip}}{E_{full-slip}}$ ratios, as calculated using Raoof and Kraincanic's model for ropes with fibre cores (cases 1 and 2) or IWRC, respectively. Fig. 7.3c presents similar correlations, but, with the data relating to the fibre core wire ropes omitted.

Table 7.3 - Summary of the Experimentally Determined Results for the Axial Stiffnesses of the Various Wire Rope Constructions.

Type of Core Construction	Rope Outside Diameter	Source of Results	Experimental Results		
			$\frac{E_{no-slip}}{E_{steel}}$	$\frac{E_{full-slip}}{E_{steel}}$	$\frac{E_{no-slip}}{E_{full-slip}}$
IWRC	33	Velinsky et al. (1984)	-	0.530	-
IWRC	40	Strzemiecki & Hobbs (1988)	0.625	0.545	1.146
IWRC	76	Kraincanic & Hobbs (1997)	0.685	0.653	1.049
			0.722	0.678	1.065
			0.723	0.652	1.109
			0.819	0.663	1.235
			0.738	0.662	1.115
IWRC	76 (Comparator)	Raoof & Kraincanic (1996)	-	0.566	-
FC (RL) - 1	9.53	Velinsky et al. (1984)	-	0.692	-
FC (RL) - 1	40.5	Cantin et al. (1993)	-	-	-
FC (LL) - 1	9.53	Velinsky et al. (1984)	-	-	-
FC (LL) - 1	40.5	Cantin et al. (1993)	-	0.730 - 0.794	-
FC (RL) - 2	9.53	Velinsky et al. (1984)	-	0.692	-
FC (RL) - 2	40.5	Cantin et al. (1993)	-	-	-
FC (LL) - 2	9.53	Velinsky et al. (1984)	-	-	-
FC (LL) - 2	40.5	Cantin et al. (1993)	-	0.730 - 0.794	-

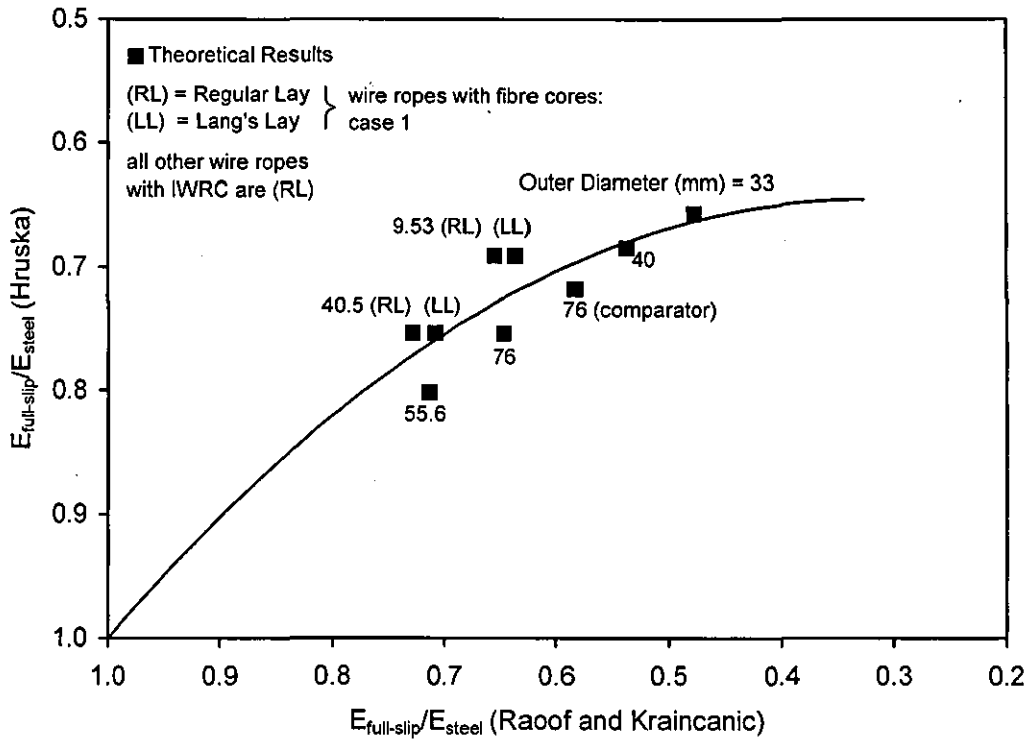


Fig. 7.2a – Relationship Between the Full-Slip E-Values for Various Wire Rope Constructions with Either a Fibre Core (Case 1) or an IWRC as Calculated Using Raoof and Kraincanic's Model and Hruska's Formula.

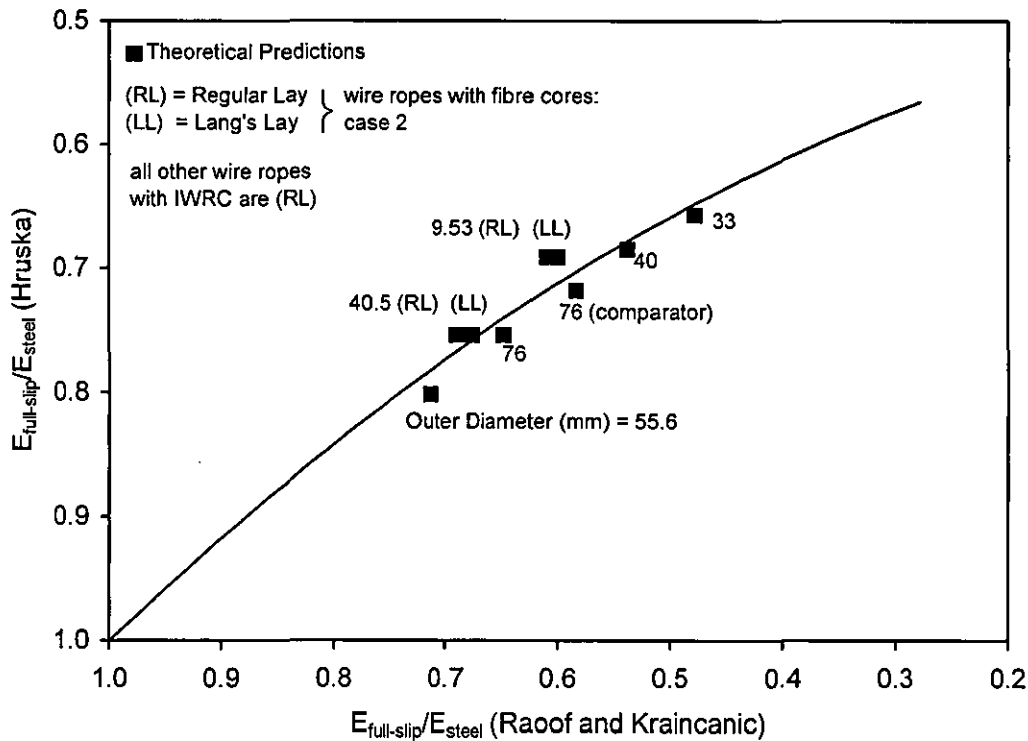


Fig. 7.2b – Relationship Between the Full-Slip E-Values for Various Wire Rope Constructions with Either a Fibre Core (Case 2) or an IWRC as Calculated Using Raoof and Kraincanic's Model and Hruska's Formula.

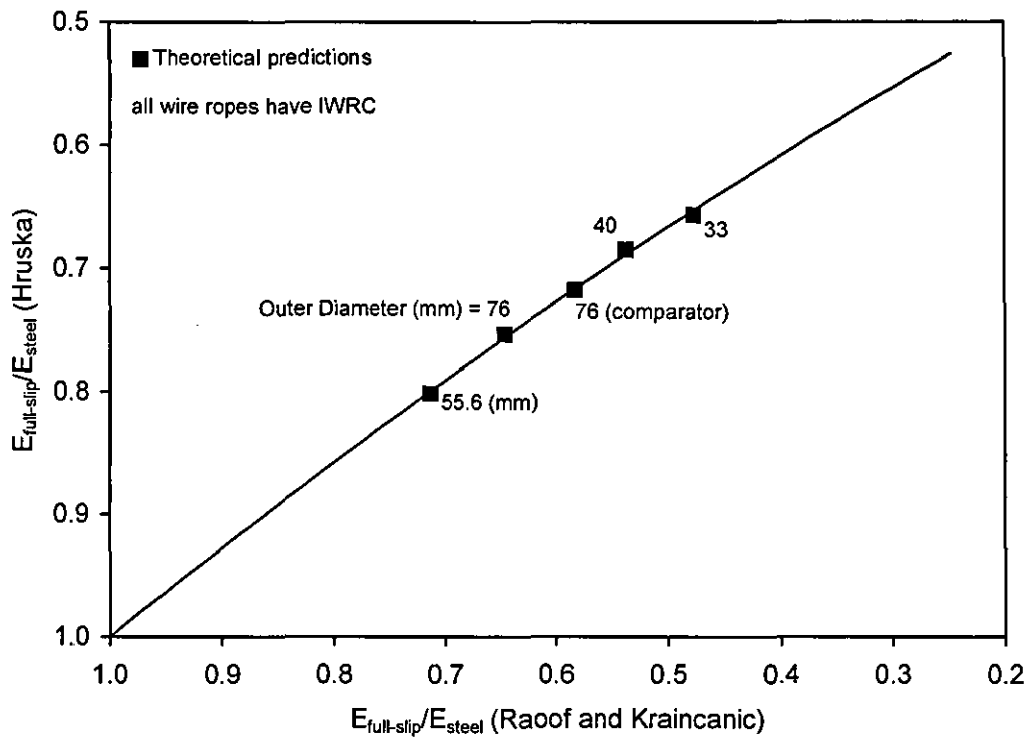


Fig. 7.2c – Relationship Between the Full-Slip E-Values for the Wire Ropes with IWRC as Calculated Using Raoof and Kraincanic's Model and Hruska's Formula.

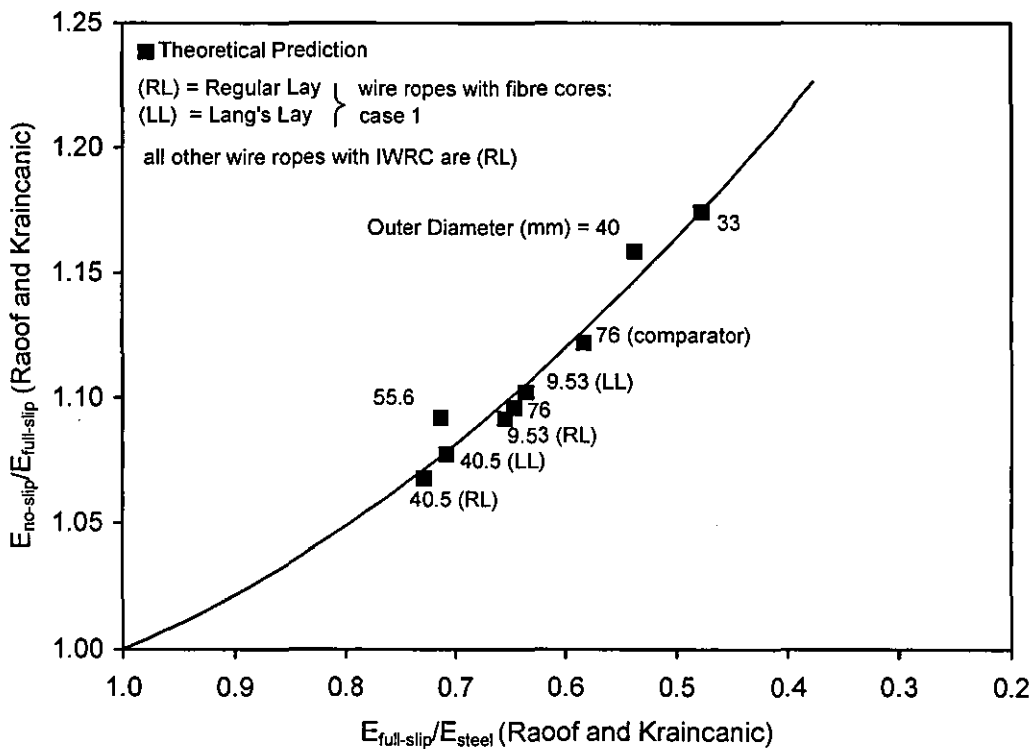


Fig. 7.3a – Relationship Between the Full-Slip and No-Slip E-Values for Various Wire Rope Constructions with Either a Fibre Core (Case 1) or an IWRC as Calculated Using Raoof and Kraincanic's Model.

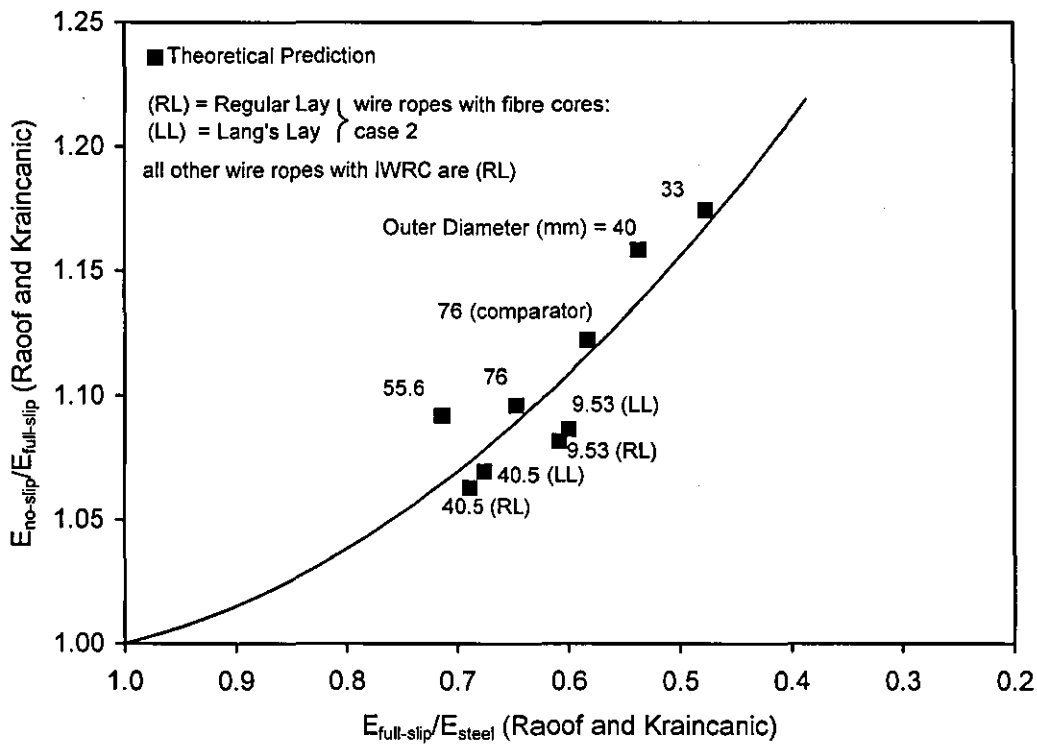


Fig. 7.3b – Relationship Between the Full-Slip and No-Slip E-Values for Various Wire Rope Constructions with Either a Fibre Core (Case 2) or an IWRC as Calculated Using Raoof and Kraincanic's Model.

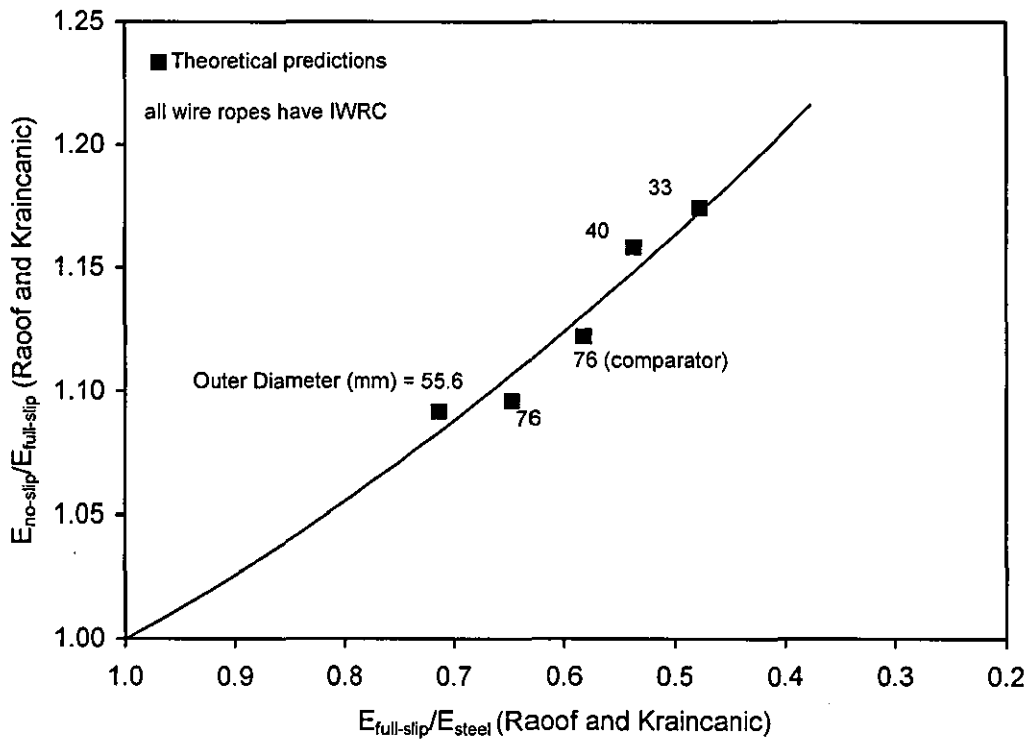


Fig. 7.3c - Relationship Between the Full-Slip and No-Slip E-Values for Various Wire Rope Constructions with IWRC as Calculated Using Raoof and Kraincanic's Model.

Finally, Figs. 7.4a-c and Figs. 7.5a-c present the correlations between the available experimental data and the theoretical fitted curves as presented in Figs. 7.2a-c and Figs 7.3a-c, respectively. It should, however, be noted that, wherever scatter was observed in the available experimental data, the plotted data in Figs. 7.4a-c and Figs. 7.5a-c relate to the average value. Moreover, in all these plots, either the Hruska's parameter is plotted against the test data, or the experimental $E_{\text{full-slip}}/E_{\text{steel}}$ value is plotted against the experimental $E_{\text{no-slip}}/E_{\text{full-slip}}$ value.

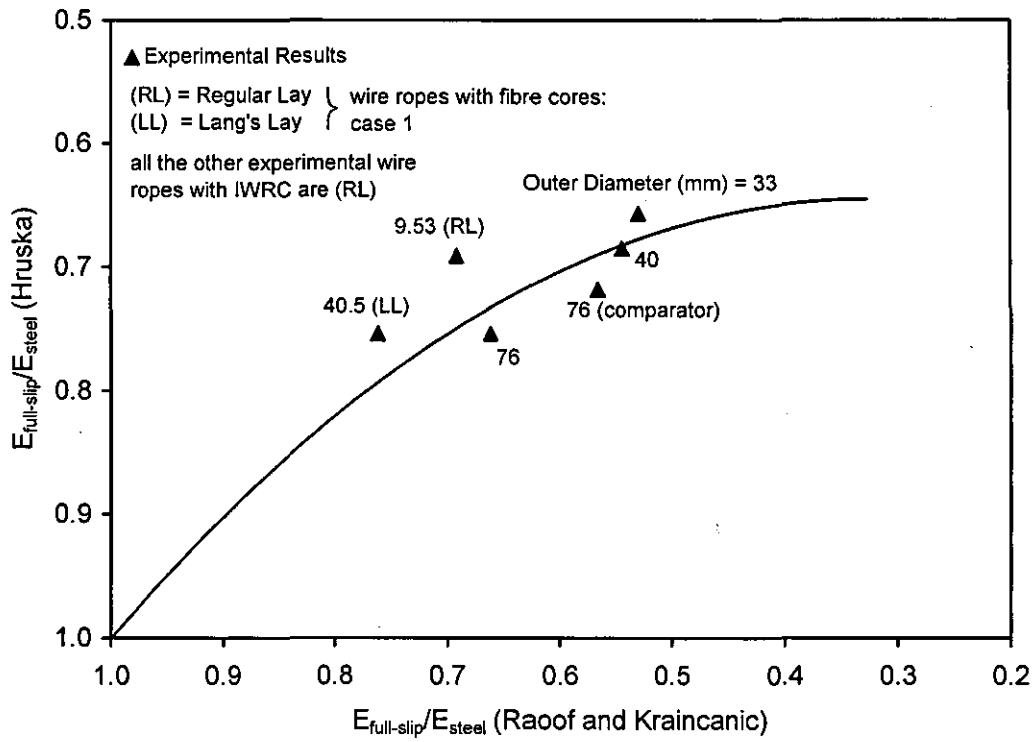


Fig. 7.4a - Correlations Between the Experimental Results and the Theoretical Fitted Curves, With the Latter as Given in Fig. 7.2a.

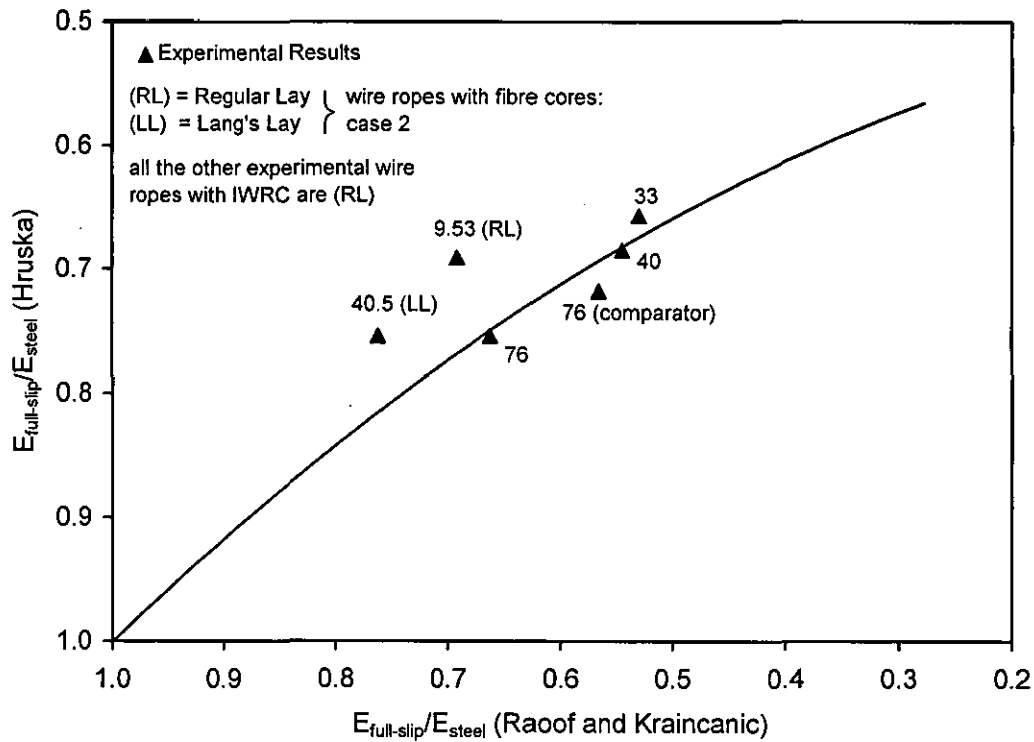


Fig. 7.4b - Correlations Between the Experimental Results and the Theoretical Fitted Curves, With the Latter as Given in Fig. 7.2b.

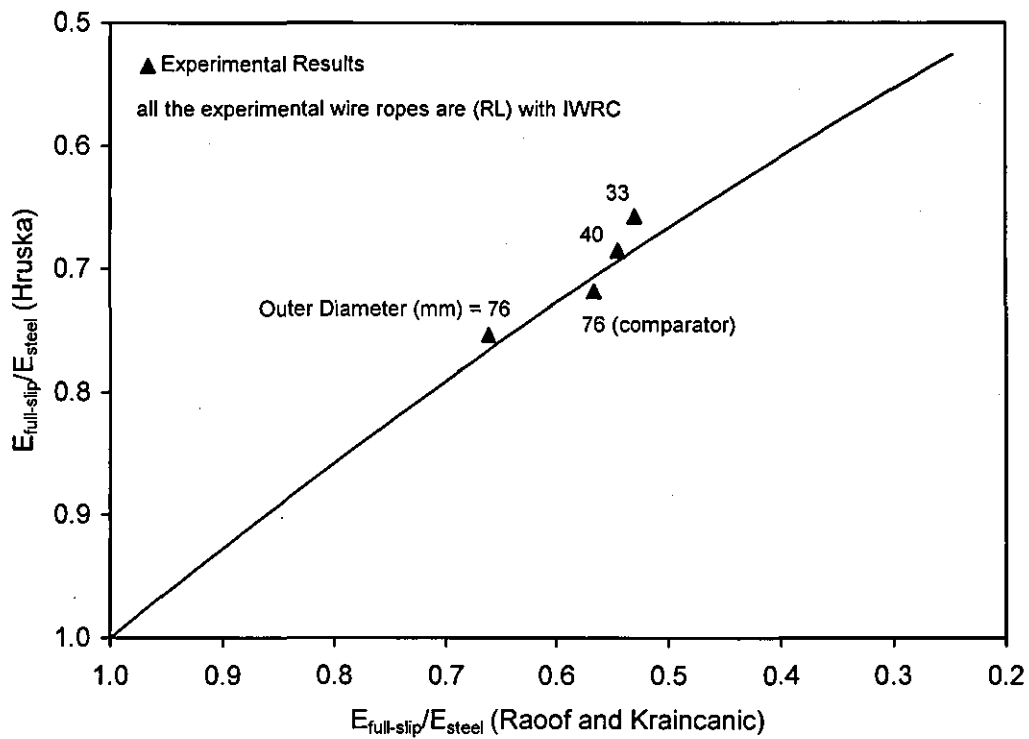


Fig. 7.4c - Correlations Between the Experimental Results and the Theoretical Fitted Curves, With the Latter as Given in Fig. 7.2c.

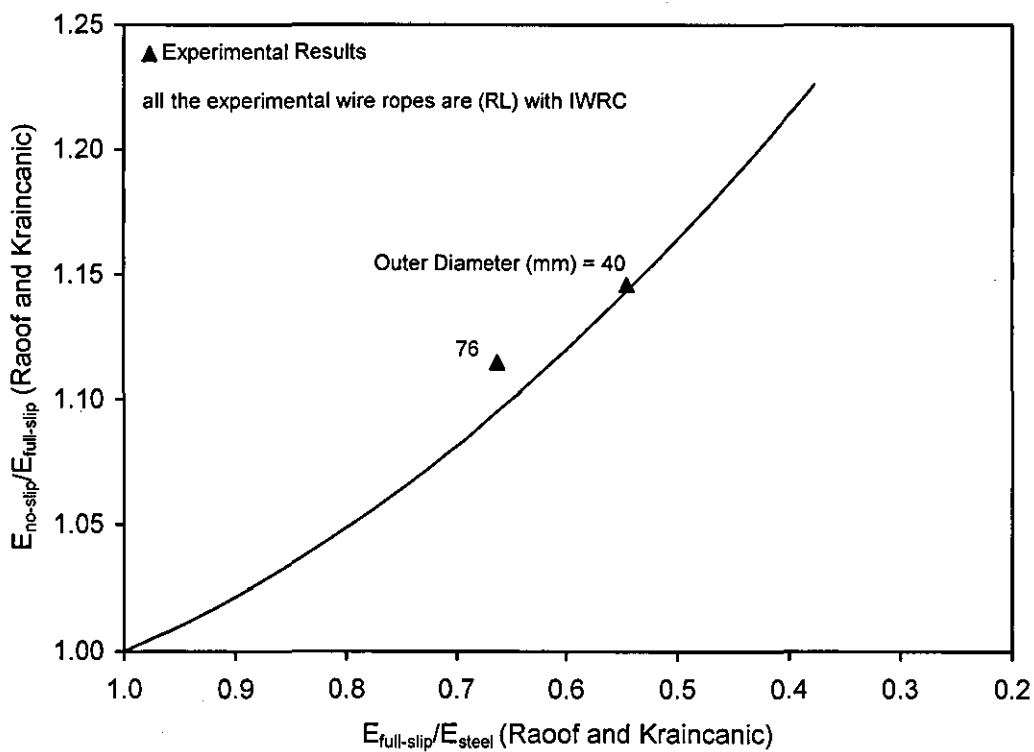


Fig. 7.5a - Correlations Between the Experimental Results and the Theoretical Fitted Curves, With the Latter as Given in Fig. 7.3a.

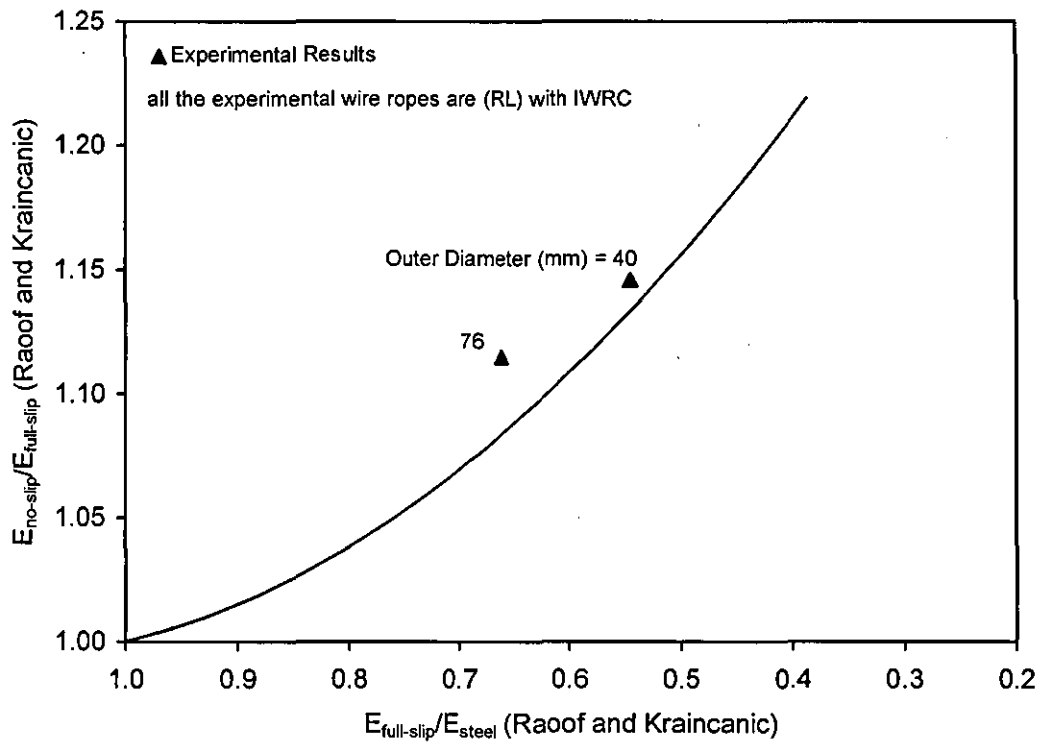


Fig. 7.5b - Correlations Between the Experimental Results and the Theoretical Fitted Curves, With the Latter as Given in Fig. 7.3b.

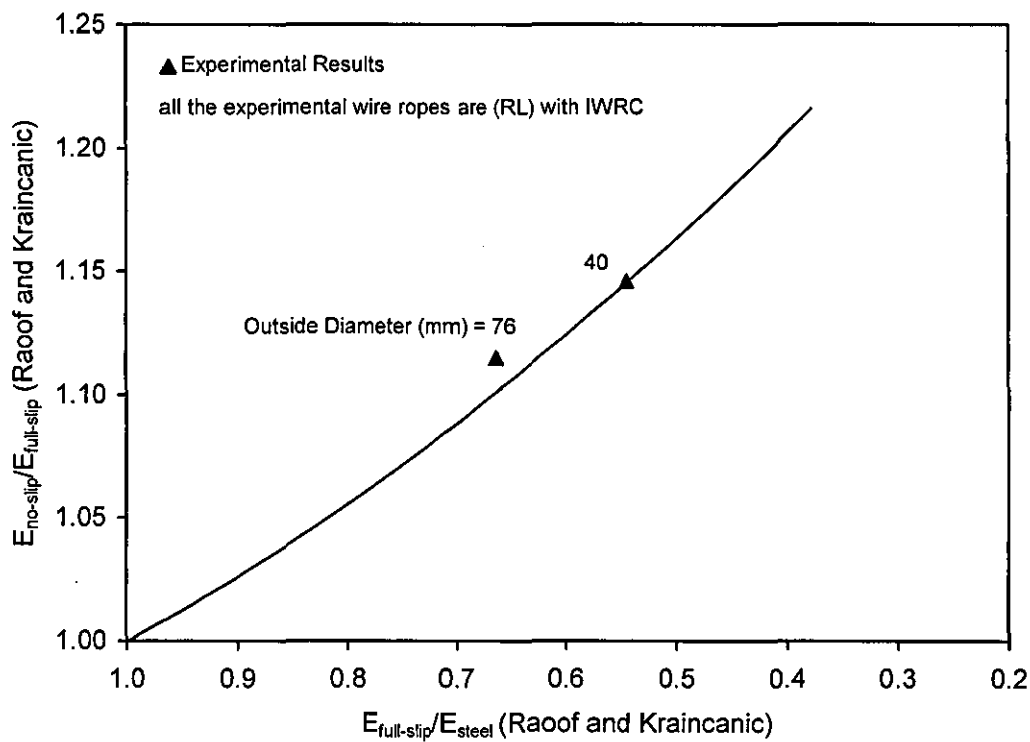


Fig. 7.5c - Correlations Between the Experimental Results and the Theoretical Fitted Curves, With the Latter as Given in Fig. 7.3c.

7.4.1 Simple Formulations

Using the results presented in the previous section, a simple method for determining the full-slip and no-slip axial stiffnesses of wire ropes with either fibre cores or IWRC can be developed by fitting various non-linear curves through the data. In Figs. 7.2a, b and c, Hruska's simple parameter H is given by Equation (7.1), as developed by Strzemiecki and Hobbs (1988). Once H is calculated, the full-slip axial stiffness, based on the more accurate model of Raoof and Kraincanic, may be found using a second order polynomial of the general form

$$\frac{E_{\text{full-slip}}}{E_{\text{steel}}} = A(H^2) + B(H) + C \quad (7.2)$$

where, the constant coefficients $A - C$ are given in Table 7.4, and correspond to the situation as to whether the fibre core wire ropes are to be included (Figs. 7.2a and b) or not (Fig. 7.2c), and, if included, which different pattern of interstrand contacts for the ropes with fibre cores (i.e. whether case 1 or 2, as defined in section 7.3) is to be considered.

Table 7.4 - Values of the Constant Coefficients $A - C$ in Equation (7.2) for all of the Fitted Curves in Figs. 7.2a-c, Along with the Correlation Coefficients, R .

Reference	A	B	C	R
Fig. 7.2a	-0.5462	2.2662	-0.72	0.820
Fig. 7.2b	-0.4156	2.1061	-0.69	0.931
Fig. 7.2c	-0.4913	2.3228	-0.83	0.998

Turning to the no-slip case, Figs. 7.3a-c show the theoretical relationships between the no-slip and full-slip moduli, with the individual numerical results having been found to be very nearly independent of the level of mean axial load on the cable (over the working load ranges). Denoting $\frac{E_{\text{full-slip}}}{E_{\text{steel}}} = k_1$, fitted curves defined by second order polynomials of the general form

$$\frac{E_{\text{no-slip}}}{E_{\text{full-slip}}} = A(k_1^2) + Bk_1 + C \quad (7.3)$$

provide a simple means of finding the no-slip axial stiffness, once the corresponding full-slip axial stiffness has been found; depending upon whether the fibre core wire ropes are to be included (Figs. 7.2a and b) in the analysis or not (Figs. 7.2c), and, if included, whether case 1 or 2 is to be adopted for the pattern of interstrand contacts in relation to the wire ropes with fibre cores. The values of the constant coefficients A - C in Equation (7.3) are given in Table 7.5, along with the correlation coefficients, R.

Table 7.5 - Values of the Constant Coefficients A - C in Equation (7.3) for all of the Fitted Curves in Figs. 7.3a-c, Along with the Correlation Coefficients, R.

Reference	A	B	C	R
Fig. 7.3a	0.2855	-0.7563	1.471	0.9763
Fig. 7.3b	0.4043	-0.9180	1.514	0.8893
Fig. 7.3c	0.4656	-0.5759	1.410	0.9909

7.5 DISCUSSION

Similar to the results for spiral strands, based on the orthotropic sheet theory, the results for the axial stiffness of wire ropes, based on Raoof and Kraincanic's model, were found to give significantly lower $E_{\text{full-slip}}$ values than those based on Hruska's simple approach, with the lay angle playing a primary (controlling) role.

Comparing Figs. 7.2a and b, it is found that the theoretical data is less scattered around the fitted curve when the fibre core wire ropes are analysed assuming that the strands in the rope are resting on the fibre core (case 2), in contrast to the situation when the strands in the wire rope are assumed to be just touching each other in line-contact (case 1). On the other hand, the opposite is found when comparing Figs. 7.3a and b, where the scatter of the data points about the fitted curves is less for case 1 (cf. case 2) of interstrand contacts in wire ropes with a fibre core.

With the data for the fibre core wire ropes omitted from the plots, the degree of scatter around the fitted curves is significantly less (refer to Fig. 7.2c and Fig. 7.3c) with the fitted mean curve(s) very nearly passing through all the theoretical data points which

cover a rather wide range of strand diameters and lay angles: this, then, suggests that the wire rope axial stiffness is determined by the lay angles of the wires in the strands and of the strands in the rope, with the other geometrical parameters having a second order effect.

7.6 CONCLUSIONS

Numerical data, based on Raoof and Kraincanic's model, has been used in relation to the upper (no-slip) and lower (full-slip) bounds to the axial stiffness of wire ropes with either fibre or independent wire rope cores. Available experimental data on a wide range of wire ropes have previously been reported to provide encouraging support for both the full-slip and no-slip predictions of this model.

The present work clearly demonstrates that, using the simple formulation of Hruska, higher values of the full-slip axial stiffness are obtained when compared to the more refined model of Raoof and Kraincanic. With this born in mind, a simple method has been proposed by means of which the full-slip and no-slip axial stiffnesses of large diameter wire ropes, with either fibre cores or IWRC, may be estimated. The proposed method is based on the remarkable correlations found between the predictions of the axial stiffnesses as obtained from Hruska's, and Raoof and Kraincanic's approaches, strongly suggesting that the lay angles (both of the wires in the strands and the strands in the rope) are the prime (controlling) parameters, with the cross-sectional areas of the individual wires also playing a role. The presently proposed method is amenable to simple hand calculations, using a pocket calculator, hence, of value to busy practising engineers.

CHAPTER 8

HYDROSTATIC PRESSURE EFFECTS ON THE AXIAL FATIGUE LIFE ESTIMATION OF LARGE DIAMETER SHEATHED SPIRAL STRANDS

8.1 INTRODUCTION

The integrity of many major cable supported structures, in both onshore and offshore applications, is strongly dependent upon the cable anchorage systems, which are usually expensive to install, and, if it becomes necessary, very costly and difficult to replace. Since the mid-1970's there has been a significant increase in the size of steel helical cables being used, particularly in the offshore industry. The cost of conducting large scale experimental tests on steel cables is considerable and, particularly in the case of fatigue tests, very time consuming. Simply scaling up the cable diameters, via extrapolation of the orthodox designs, is (in the absence of a sound theoretical understanding) a risky process. To address this, Raoof (1990d and 1991a, b) has used an extension of the previously reported orthotropic sheet theoretical model of Hobbs and Raoof (1982) (in connection with the patterns of interwire/interlayer contact forces throughout a spiral strand) to develop a theoretical model for predicting the axial fatigue life of the spiral strands to first outermost (or innermost) wire fractures, both at the fixed end and away from the detrimental effects of end terminations - i. e. in the free-field. Large scale experimental data, on eleven different spiral strands, with outside diameters ranging from 25 mm to 127 mm (Raoof, 1990d, 1991a and 1996, Alani and Raoof, 1995 and Raoof and Alani, 1997), as tested by a number of independent institutions, have provided ample support for the general validity of Raoof's theoretical predictions.

Based on Raoof's theoretical axial fatigue model, design S-N curves, which take the construction details of large diameter multi-layered spiral strands into account, have been produced (Raoof, 1998a), which are applicable to any spiral strand construction, and enable one to design against first outermost (or innermost) wire fracture at either the fixed end termination or in the free-field. These design S-N curves were produced by conducting extensive theoretical parametric studies on three different 127 mm

outside diameter spiral strands with lay angles of 12°, 18° and 24°. Raoof's design S-N curves were compared with other (purely empirical) design S-N curves which, at the time, were commonly referred to in the literature; namely those of Tilly (1988), API (1991) and Chaplin (1993). It was found that, in certain cases, particularly the API recommendations provided unduly unconservative estimates (Raoof, 1992b). In particular, it was argued by Raoof that, in the context of strand axial fatigue, the lay angle was the main (first order) geometrical parameter (Raoof, 1997).

Back in the early 1980's, cable manufacturers offered high density and supposedly impermeable polythene sheaths to protect the large diameter cables against corrosion, in deep water applications. Raoof (1990c) theoretically showed that a high external hydrostatic pressure can (in the presence of substantial air-filled voids inside the internally lubricated cables) significantly influence the patterns of interwire/interlayer contact forces in sheathed spiral strands. It was also argued that the application of an external hydrostatic pressure on a sealed spiral strand will suppress the slippage of the wires in the cable by increasing the frictional forces between them. Increasing levels of interwire/interlayer contact stresses can have a marked effect on a sealed cable's axial and free bending fatigue life. Raoof, using a realistic 39 mm outside diameter spiral strand, demonstrated, theoretically, that substantial increases in a sealed strand's trellis contact patch stresses can lead to significant reductions in its axial and/or restrained bending fatigue life in long term applications.

As will be discussed in chapter 10, Raoof (1998a) has addressed the question of size effects regarding the axial fatigue performance of spiral strands, and has made recommendations for carrying out practically sensible future axial fatigue tests on scaled down specimens, with his work having been extended, in chapter 10 of this thesis, to cover the full range of manufacturers limits (as far as the lay angle is concerned).

The purpose of this chapter is to provide a more complete set of axial fatigue design S-N curves than those reported by Raoof (1998a), which cater for the effects of an external hydrostatic pressure on sheathed spiral strands, and are believed to be of particular importance in deep water offshore platform applications: it is, perhaps,

worth mentioning that Raoof's previous work only related to in-air axial fatigue conditions.

8.2 THEORY

8.2.1 Development of the S-N Curves

The orthotropic sheet theoretical model of Raoof and Hobbs (1982) has already been reported in considerable detail in the literature, with its salient features presented in chapter 3, and, as such, will not be repeated here. The theory relating to the axial fatigue of multi-layered spiral strands has been presented on many occasions by Raoof and his associates (Raoof, 1998a, 1996 and Alani and Raoof, 1997), but due to its importance to the subsequent analysis, its salient features will be repeated in the following, along with the main contact force formulations relating to the application of an external hydrostatic pressure.

Using the orthotropic sheet theoretical model, reliable estimates of the interwire contact forces (and stresses) throughout axially loaded multi-layered spiral strands can be obtained. Experimental observations suggest that the individual wire failures are largely located over the trellis points of interlayer contact, due to the high stress concentration factors in these locations.

The stress concentration factor, K_s may, then, be calculated for a given mean axial load, once the maximum effective Von-Mises stress, $\overline{\sigma}'_{\max}$, has been calculated over the trellis points of contact, where (Knapp and Chiu, 1988)

$$K_s = \frac{\overline{\sigma}'_{\max}}{\overline{\sigma}'} \quad (8.1)$$

In the above, $\overline{\sigma}'$ is the nominal axial stress in the helical wires (Raoof and Hobbs, 1988b).

Raoof (1990d) deals with the topic of strand axial fatigue at some length. Using the values of K_s , in conjunction with axial fatigue data for single wires, a theoretical

model has been developed using which the axial fatigue life of spiral strands, under constant amplitude cyclic loading, could be predicted from first principles.

For carbon steel wires, the fatigue stress-number of cycles (S-N) plots possess an endurance limit, S' , below which no damage occurs. The magnitude of S' is traditionally (based on experimental observations by others) compared to the ultimate wire tensile strength, S_{ult} , and an approximate value of S' can be estimated from $S' = 0.27 S_{ult}$, with this relationship relating to single galvanised steel wires. The reduced magnitude of the endurance limit, S_e , which takes interwire contact and fretting, as well as size effects and surface conditions, etc., into account, may be defined as

$$S_e = K_a K_b S' \quad (8.2)$$

where, $K_b = \frac{1}{K_s}$, and K_a is a constant.

The so-obtained values of the parameter S_e , then, are used to produce the S-N curves for fatigue life to first outermost (or innermost) wire fractures, at the terminations or in the free-field, in multi-layered spiral strands.

8.2.2 Effect of Mean Axial Load and Grade of Wire

Raoof (1998a) has shown that changes in the grade (i.e. ultimate tensile strength) of wire, S_{ult} , can lead to a fairly significant degree of scatter in the axial fatigue results. However, it has been demonstrated by Raoof (1998a) that provided one non-dimensionalizes the strand's axial stress range, by dividing it by the appropriate magnitude of the ultimate breaking load (U.B.L.), variations in the grade of wire do not lead to a significant degree of scatter in the associated S-N plots, and (for all practical purposes) a very nearly unified S-N curve, for a strand construction, may be adopted, for a wide range of grades of wire. In what follows, the ultimate breaking loads (U. B. L.) of the spiral strands used in the theoretical parametric studies, have been estimated from

$$U.B.L. = A_s K S_{ult} \quad (8.3)$$

where, A_s = the net steel area, K = the spinning loss factor, and S_{ult} = the ultimate wire tensile strength (i.e. grade).

As regards the effect of the mean axial load on the S-N curves, Raoof (1998a) has shown that the previous argument by certain researchers, who have advocated the use of a modified Goodman or Gerber approach for spiral strands in order to cater for the effect of mean axial load, when presenting axial fatigue results, does not lead to any advantages when compared to the use of axial stress range: it is shown that the traditional axial stress range (as opposed to an equivalent axial load range based on the Goodman or Gerber approaches) leads to less scatter of the axial fatigue data about the fitted mean curve in the axial fatigue S-N plots, and the alternative approach of using an equivalent axial load range for plotting the S-N curves does not (despite the extra efforts involved) lead to any advantages. For the present purposes, therefore, all the following proposed S-N curves are based on the strand axial stress range.

8.2.3 Hydrostatic Pressure Effects

The hydrostatic pressure, applied externally to a sealed spiral strand, has a significant influence on the normal interwire/interlayer forces in the radial direction. As mentioned in chapter 3, in the absence of an externally applied uniform hydrostatic pressure, the relationship between the line-contact forces, P_{MS} , and radial forces, X_{MS} , in the outer layer of a multi-layered spiral strand, is identical to that between P_{RC} , and X_{RC} for that layer. However, with the hydrostatic pressure present, X_{RC} and X_{MS} for layer 1 (and, hence, the corresponding P_{RC} and P_{MS}) are no longer the same, with X_{MS} given by

$$X_{MS,1} = X_{RC,1} + X_H \quad (8.4)$$

where, X_H corresponds to the magnitude of the external hydrostatic pressure per unit length of the wires in the outermost layer - i.e.

$$X_H = \frac{2\pi R \rho g h}{n \cos \alpha_1} \quad (8.5)$$

In the above, h is the depth of water, R is the cable outer radius, ρ is the water density, g is the acceleration due to gravity, and n is the number of wires in the outer layer, the lay angle of which is α_1 . For sealed spiral strands, this alternative method for calculating $X_{MS,1}$, in the presence of an external hydrostatic pressure, replaces the method summarised in chapter 3. Otherwise, the analysis may, then, be carried out using the same equations as presented in chapter 3 in relation to in-air conditions.

8.3 RESULTS

The results were obtained by conducting extensive theoretical parametric studies on three different 127 mm outside diameter spiral strands with lay angles, α , of 12° , 18° and 24° . Tables 8.1a, b and c give the construction details for all the three different spiral strands. In what follows, the ultimate wire tensile strength is assumed to be $S_{ult} = 1520 \text{ N/mm}^2$, and the estimated ultimate breaking loads (U. B. L.) for all three strand constructions are assumed to be equal to 13510 kN. At this point, it is worth explaining that a K_a value of 1.0 is an appropriate factor for wire fractures which happen in the free field (i.e. away from the influence of end terminations), while a K_a value of 0.5 corresponds to wire fractures which are influenced by the detrimental effects of end terminations. Finally, although all the following numerical results are based on $S_{ult} = 1510 \text{ N/mm}^2$ and U. B. L. = 13510 kN, the final design S-N curves are (based on Raoof's (1998a) arguments) of general applicability, because all the axial load (or stress) ranges in the proposed design S-N curves have been non-dimensionalized with respect to the strand ultimate breaking load.

Table 8.1a – Construction Details for the 127 ($\alpha = 12^\circ$) mm Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (deg)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	56	RH	6.60	12.00	60.17	1958.671	2493.857
2	50	LH	6.60	12.00	53.73	1748.813	2226.658
3	44	LH	6.60	12.00	47.29	1538.955	1959.459
4	38	RH	6.60	12.00	40.85	1329.098	1692.260
5	32	LH	6.50	12.00	33.89	1085.581	1382.204
6	26	RH	6.50	12.00	27.56	882.034	1123.041
7	20	LH	6.50	12.00	21.23	678.488	863.878
8	14	RH	6.60	12.00	15.15	489.668	623.464
	7	-	4.00	8.57	-	-	85.051
Core	7	-	5.20	8.03	-	-	144.330
	7	-	5.20	4.92	-	-	147.023
	1	-	7.10	-	-	-	39.592

$$A_{core} = 415.996 \text{ mm}^2$$

$$A_g = 12364.821 \text{ mm}^2$$

Table 8.1b – Construction Details for the 127 ($\alpha = 18^\circ$) mm Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (deg)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	54	RH	6.55	18.0	59.22	1913.307	2436.098
2	48	LH	6.55	18.0	52.64	1700.717	2165.420
3	42	LH	6.55	18.0	46.07	1488.127	1894.743
4	36	RH	6.55	18.0	39.50	1275.538	1624.065
5	31	LH	6.55	18.0	34.02	1098.380	1398.500
6	25	RH	6.55	18.0	27.46	885.790	1127.823
7	19	LH	6.55	18.0	20.90	673.200	857.145
8	14	RH	6.30	18.0	14.85	458.899	584.289
	7	-	3.90	13.07	-	-	77.289
Core	7	-	5.10	12.20	-	-	133.526
	7	-	5.25	7.62	-	-	142.554
	1	-	7.00	-	-	-	38.485

$$A_{core} = 391.854 \text{ mm}^2$$

$$A_g = 12088.082 \text{ mm}^2$$

Table 8.1c – Construction Details for the 127 ($\alpha = 24^\circ$) mm Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	54	RH	6.40	24.0	60.23	1901.575	2421.160
2	48	LH	6.40	24.0	53.54	1690.289	2152.142
3	42	LH	6.50	24.0	47.58	1525.583	1942.432
4	36	RH	6.50	24.0	40.79	1307.642	1664.942
5	30	LH	6.60	24.0	34.53	1123.489	1430.471
6	24	RH	6.60	24.0	27.64	898.791	1144.377
7	18	LH	6.80	24.0	21.38	715.567	911.088
8	14	RH	6.10	24.0	14.94	447.865	570.240
	7	-	3.90	17.89	-	-	72.069
Core	7	-	5.10	16.75	-	-	125.557
	7	-	5.25	10.58	-	-	143.935
	1	-	7.00	-	-	-	38.485

$$A_{core} = 380.045 \text{ mm}^2$$

$$A_g = 12236.851 \text{ mm}^2$$

Figs. 8.1(a-d), (e-h), (i-l), (m-p) and (q-t) show variations of the endurance limit with changes in the mean axial load (both expressed as a percentage of the cable ultimate breaking load (U.B.L.)), at water depths of 0 m, 500 m, 1000 m, 1500 m and 2000 m, respectively, and for different values of $K_a = 1.0$ and 0.5, for all the three different 127 mm outside diameter spiral strands.

The endurance limit for a spiral strand, with the fatigue life defined as the number of cycles to first outermost (or innermost) wire fracture, depends on the type of strand construction, and, for a given strand construction, increases with an increasing level of strand mean axial strain, S'_1 . This is because of the geometrically non-linear nature of the problem of wire flattening at the trellis points of contact between the neighbouring wires of the various layers.

Figs. 8.1(b, d), (f, h), (j, l), (n, p) and (r, t) show the variation of the endurance limit with mean axial load, at various levels of water depth, for $K_a = 1.0$ and/or 0.5, with the fatigue life defined as the number of cycles to first innermost wire fracture. The important observation is that the endurance limit appears to be largely unaffected by the level of external hydrostatic pressure to which the spiral strand is subjected. As regards the plots relating to the first innermost wire fracture, the endurance limit decreases with increasing lay angle, for both $K_a = 1.0$ and 0.5, over the full range of

lay angles $12^\circ \leq \alpha \leq 24^\circ$, and for all water depths up to 2000 m. For all water depths, the so-obtained theoretical endurance limit, based on a K_a value of 0.5, is half of the corresponding theoretical endurance limit based on a K_a value of 1.0.

Figs. 8.1(a, c), (e, g), (i, k), (m, o) and (q, s) show variations of the so-obtained theoretical endurance limits with changes in the mean axial load, at various levels of water depth, and for values of $K_a = 1.0$ and 0.5, with the fatigue life defined as the number of cycles to first outermost wire fracture. The endurance limit is shown to decrease with an increase in the magnitude of the lay angle, but only up to a water depth of somewhere between 500 m and 1000 m. As the water depth increases beyond a certain level (between 500 m and 1000 m), then, the spiral strand with a lay angle, $\alpha = 24^\circ$ has a higher theoretical endurance limit than the spiral strand with a lay angle of 18° , with the spiral strand with a lay angle of 12° invariably having the highest endurance limit, regardless of the magnitude of the externally applied hydrostatic pressure. Once again, for all water depths, the so-obtained theoretical endurance limits, based on a K_a value of 0.5, are half of the corresponding theoretical endurance limits based on a K_a value of 1.0. However, it should be borne in mind that, in practice, there is (unlike such so-called exact theoretical predictions) usually a rather significant degree of scatter in the fatigue test data. Despite all such practical uncertainties, one thing is for sure: by sufficiently decreasing the lay angle (within current manufacturing limits) the endurance limit is likely to increase significantly, while the depth of water does not have a substantial influence on the endurance limit. The endurance limit in the free-field is always found to be significantly higher than that at the terminations.

Figs. 8.2(a-e) and Figs. 8.3(a-e) show the individual data points for all the layers of the three different 127 mm outside diameter spiral strands with lay angles of 12° , 18° and 24° , assuming a constant mean axial strain $S'_1 = 0.002867$, with K_a values of 1.0 and 0.5, respectively, and at various water depths ranging from 0 m to 2000 m. The fatigue life, defined as the number of cycles to first wire fracture for all the layers, ranging from the innermost to the outermost ones, is, for any given load range (expressed as a percentage of the U.B.L.), lower for an assumed value of $K_a = 0.5$ than for $K_a = 1.0$. It can be seen that for both sets of plots, a significant degree of scatter is

exhibited. It should be noted that these plots all assume a constant $S'_1 = 0.002867$, and changing S'_1 from, for example, 0.00050 to, say, 0.004 will lead to an even larger degree of scatter.

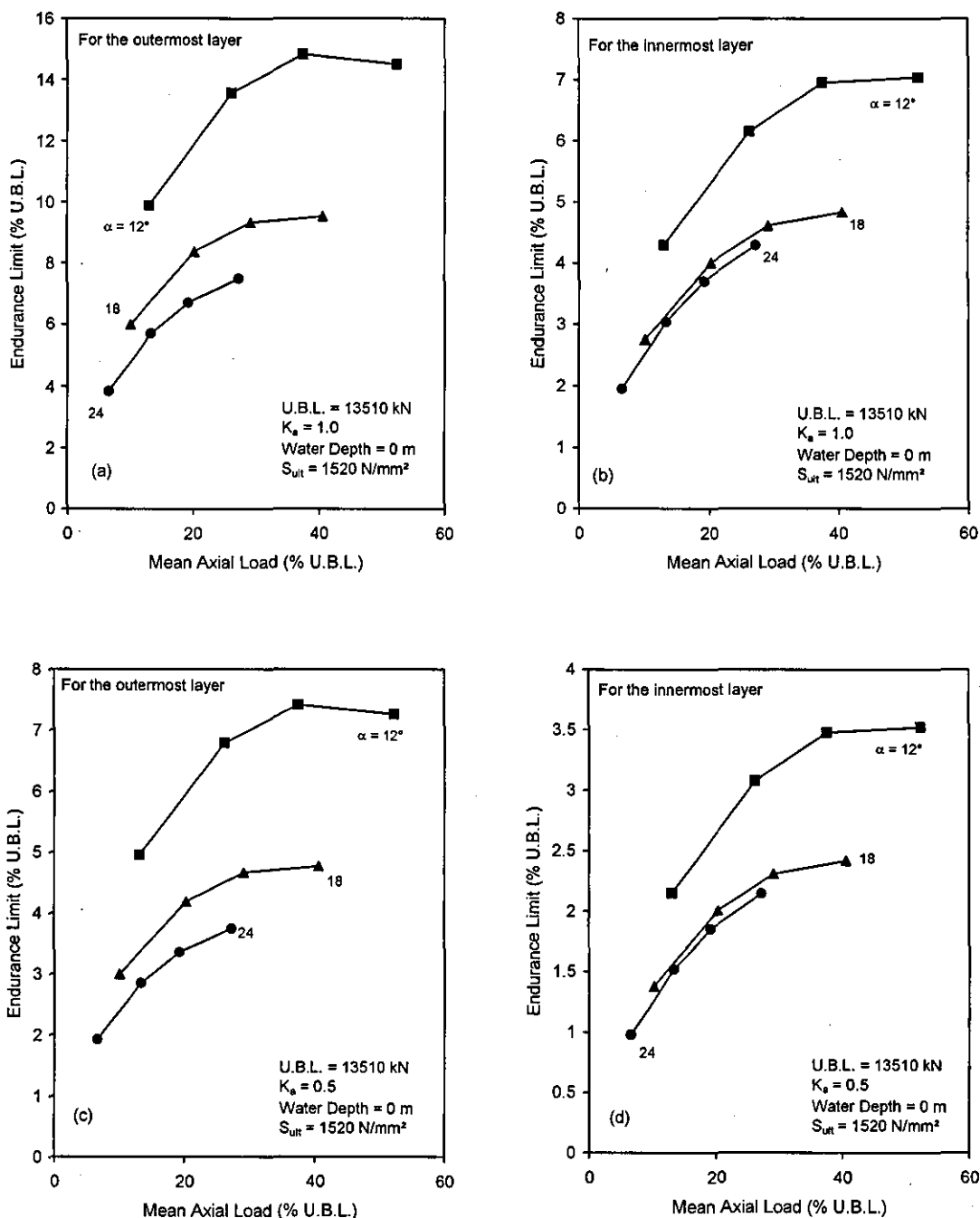
Figs. 8.4(a-e) and 8.4(f-j) present plots (in log – log scale) of the load range (expressed as a percentage of the U. B. L.) against axial fatigue life, for the 127 mm outside diameter spiral strand ($\alpha = 12^\circ$), to first outermost and innermost wire fractures, respectively, with the assumed value of K_a in these plots equal to 1.0. The plots cover a wide range of cable mean axial strains, $0.001 \leq S'_1 \leq 0.004$. Figs. 8.5(a-e) and 8.5(f-j) show similar plots for the 127 mm outside diameter spiral strand ($\alpha = 18^\circ$), with the corresponding plots for the 127 mm outside diameter spiral strand ($\alpha = 24^\circ$) being shown in Figs. 8.6(a-e) and 8.6(f-j). Figs. 8.7(a-e) and 8.7(f-j), Figs. 8.8(a-e) and 8.8(f-j) and Figs. 8.9(a-e) and 8.9(f-j) show similar plots, corresponding to the three different 127 mm outside diameter spiral strands with lay angles, $\alpha = 12^\circ$, 18° and 24° , respectively, but for an assumed K_a value of 0.5. Each figure includes a lower bound straight line to all the individual theoretical data points which are (compared to Figs. 8.2(a-e) and Figs. 8.3(a-e)) found to exhibit a much less degree of scatter. A careful examination of the plots reveals a number of interesting points. Firstly, for a given axial load range, the fatigue life to first outermost (or innermost) wire fracture is slightly less for a K_a value of 0.5 than for a K_a value of 1.0: this is more pronounced at lower load ranges (expressed as a percentage of the U. B. L.). Secondly, for all the lay angles, the fatigue life to first innermost wire fracture associated with water depths of up to somewhere between 500 m and 1000 m, produces the most conservative lower bound S-N curve. At some point between these two levels of hydrostatic pressures, the fatigue lives to first outermost and innermost wire fractures become similar. Finally, increasing the lay angle tends to give a lower S-N curve for all levels of hydrostatic pressure, for both $K_a = 1.0$ and 0.5, with the axial fatigue life defined as the number of cycles to first outermost or innermost layer wire fracture.

Figs. 8.10(a-e), (f-j), (k-o) and (p-t) show plots of the S-N curves for the outermost and innermost layers (with the S-N curves for all the corresponding intermediate layers, for a given strand construction, lying between these two limits) of the three different 127 mm outside diameter spiral strands with lay angles of 12° , 18° and 24° ,

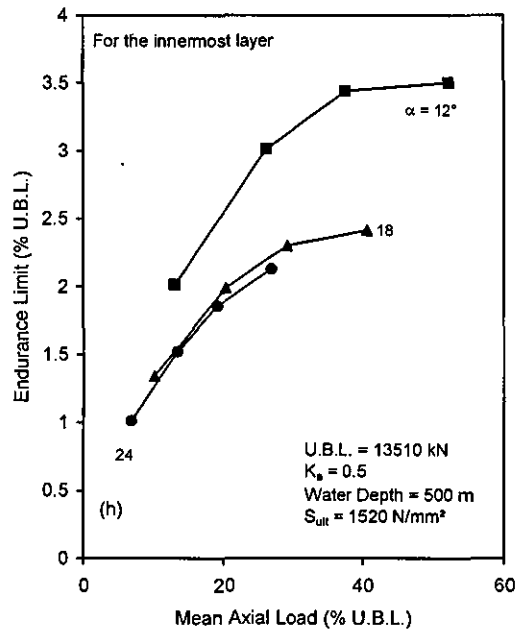
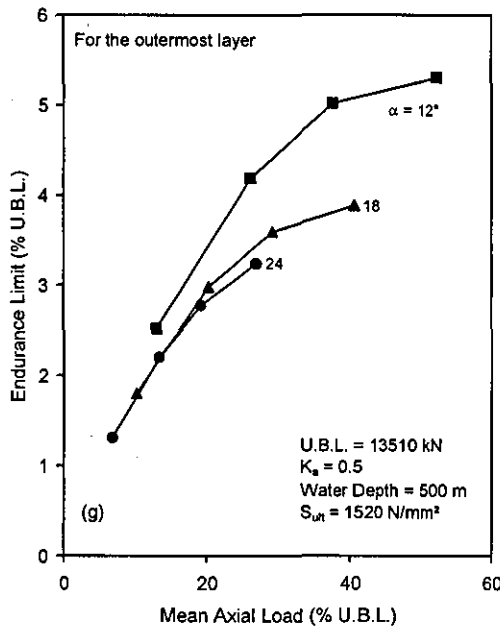
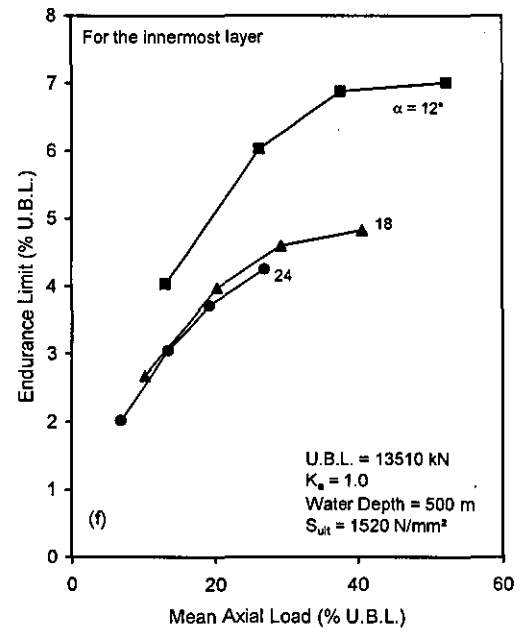
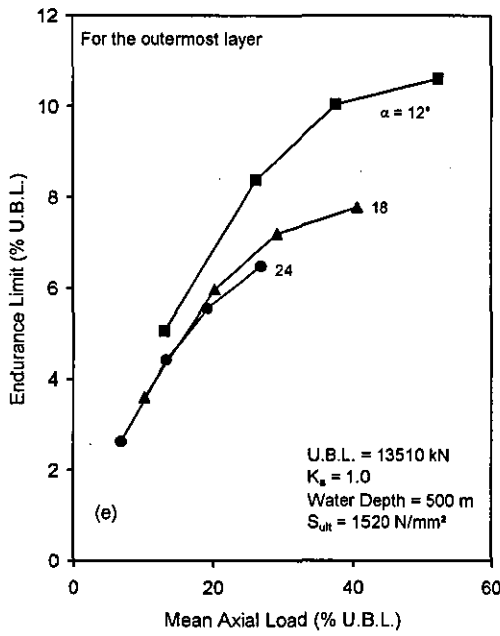
assuming a K_a value of 1.0, at various water depths, and assuming strand mean axial strains $S'_1 = 0.001, 0.002, 0.002867$ and 0.004 , respectively. Figs. 8.11(a-e), (f-j), (k-o) and (p-t) present similar plots, but for an assumed K_a value of 0.5. For both values of $K_a = 1.0$ and 0.5 , and for a given value of cable mean axial strain, S'_1 , and strand construction, increasing the level of external hydrostatic pressure on the sheathed strand brings the predictions of the fatigue life to first outermost and innermost wire fractures closer together. Another observation is that, increasing the level of cable mean axial strain on the strand, for a given level of external hydrostatic pressure, and for both values of $K_a = 1.0$ and 0.5 , in some cases brings the predictions of the fatigue life to first wire fracture (innermost and outermost), for the three 127 mm outside diameter spiral strands ($\alpha = 12^\circ, 18^\circ$ and 24°) closer together – for example, for a given water depth of 2000 m, with $K_a = 0.5$, and fatigue life defined as the number of cycles to first outer layer wire fracture, the predictions of the fatigue life in Fig. 8.11t are fairly similar for all the three different spiral strands, assuming a strand mean axial strain, $S'_1 = 0.004$, whereas for a strand mean axial strain, $S'_1 = 0.001$, the plots in Fig. 8.11e show that there is a clear distinction between the fatigue lives of the three different spiral strands.

Figs. 8.12(a-d), (e-h), (i-l), (m-p) and (q-t) present all the lower bound S-N curves, based on the present extensive theoretical parametric studies, assuming K_a values of 1.0 and 0.5, with fatigue failure defined as the number of cycles to first outermost (or innermost) wire fracture, and at various levels of external hydrostatic pressures, equivalent to water depths of 0 m, 500 m, 1000 m, 1500 m and 2000 m, respectively. In all these figures, three theoretical lower bound S-N curves, corresponding to lay angles of $12^\circ, 18^\circ$ and 24° are presented. Included in Figs. 8.12(a-d) are the purely empirical lower bound in-air S-N curves, as recommended by API (1991), Chaplin (1993) and Tilly (1988), for comparison purposes. In Figs. 8.12(a-d), the fatigue life to first innermost wire fracture provides the most conservative lower bound S-N curve for the three spiral strand constructions, for both $K_a = 1.0$ and 0.5 , but, the one with $K_a = 0.5$ is found to be even more conservative than the corresponding lower bound plots assuming $K_a = 1.0$. Most importantly, the level of external hydrostatic pressure, within the range 0 m – 2000 m, appears to have very little (if any) effect on the fatigue life to first innermost wire fracture (Figs. 8.12(b, d), (f, h), (j, l), (n, p) and (r, t))

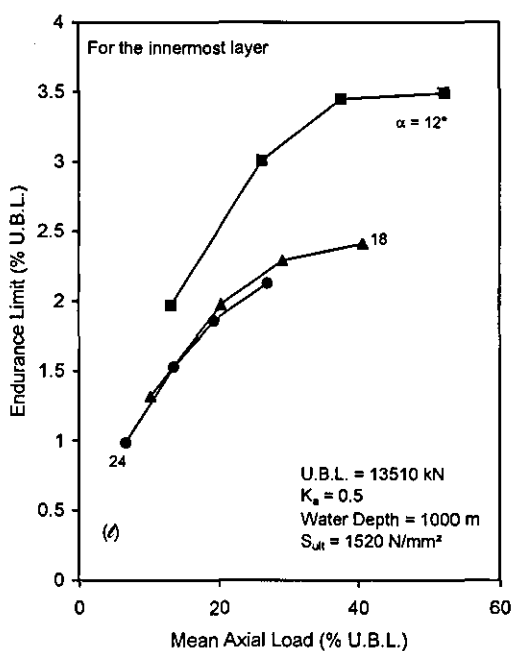
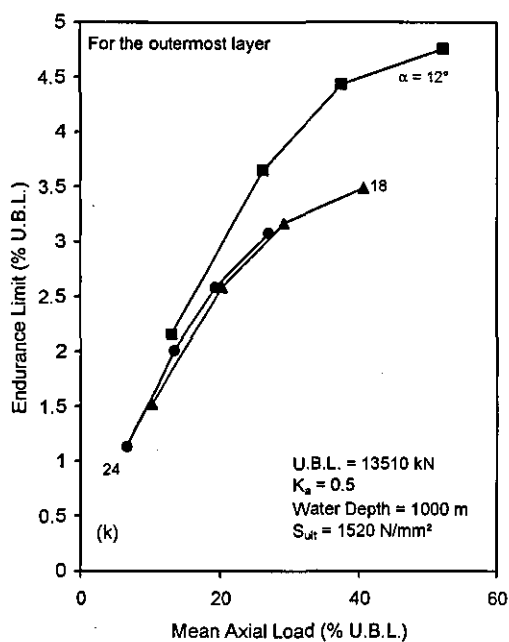
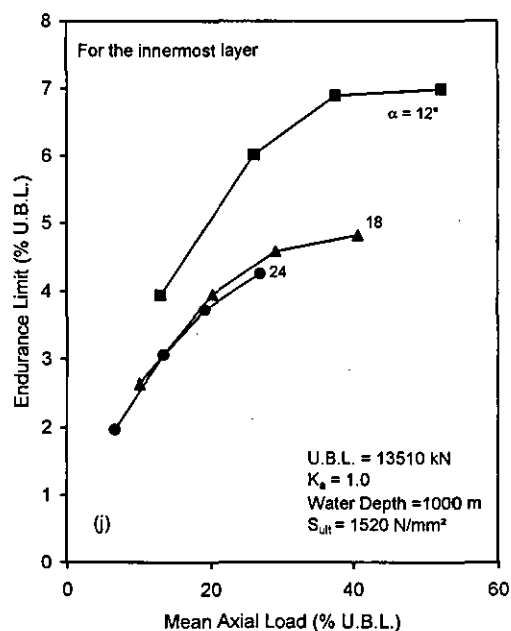
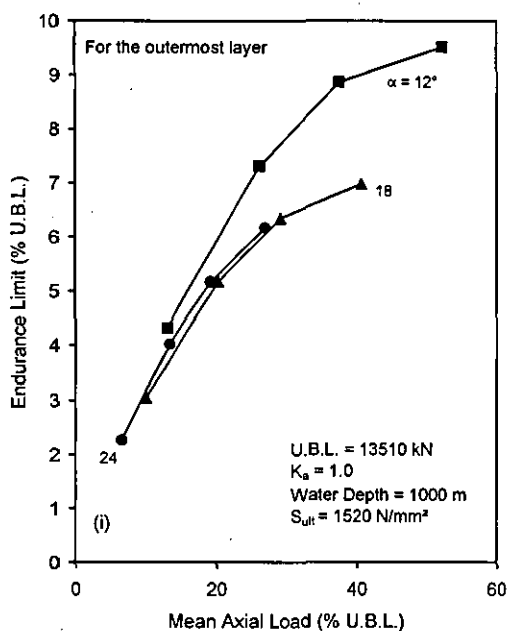
regardless of the assumed value for K_a . As regards the lower bound S-N curves with fatigue life defined as the number of cycles to first outermost wire fracture (Figs. 8.12(a, c), (e, g), (i, k), (m, o) and (q, s)), increasing the level of external hydrostatic pressure is shown to reduce the fatigue life, but only slightly, once the level of external hydrostatic pressure exceeds 500 m, although, by increasing the water depth from 0 m to 500 m, there is found to be a practically significant reduction in the fatigue life (compared to the in-air conditions).



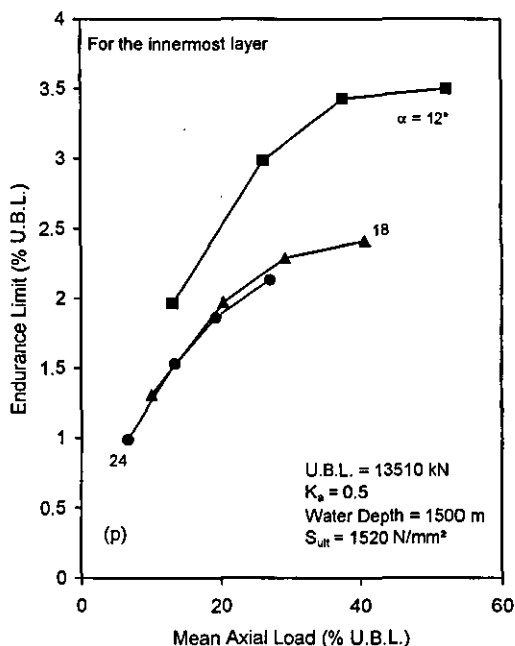
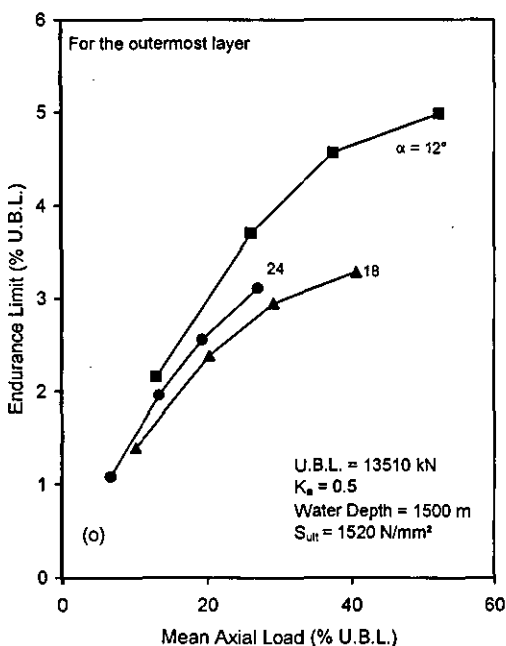
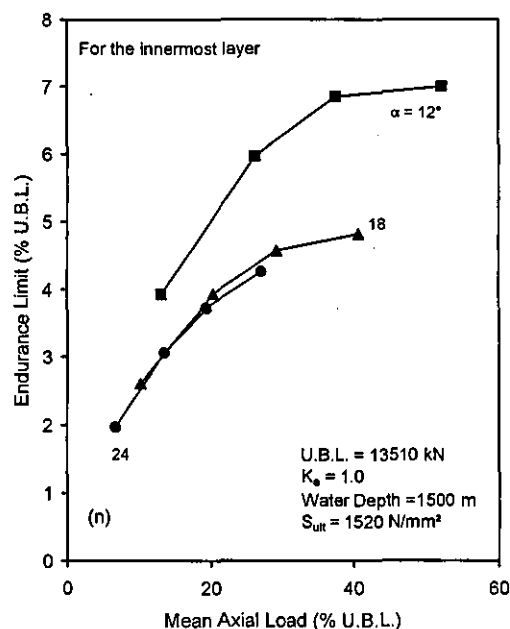
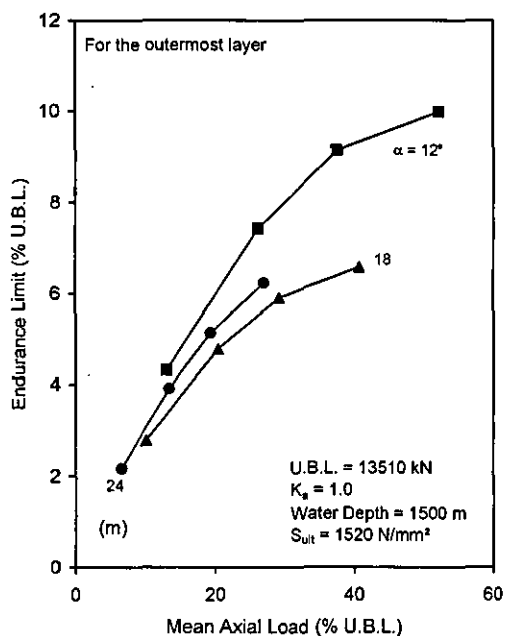
Figs. 8.1(a-d) – Theoretical Plots of the Endurance Limit Versus Mean Axial Load for the Three Different Sheathed Spiral Strand Constructions, for In-Air Conditions, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (a) and (c) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (b) and (d) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.



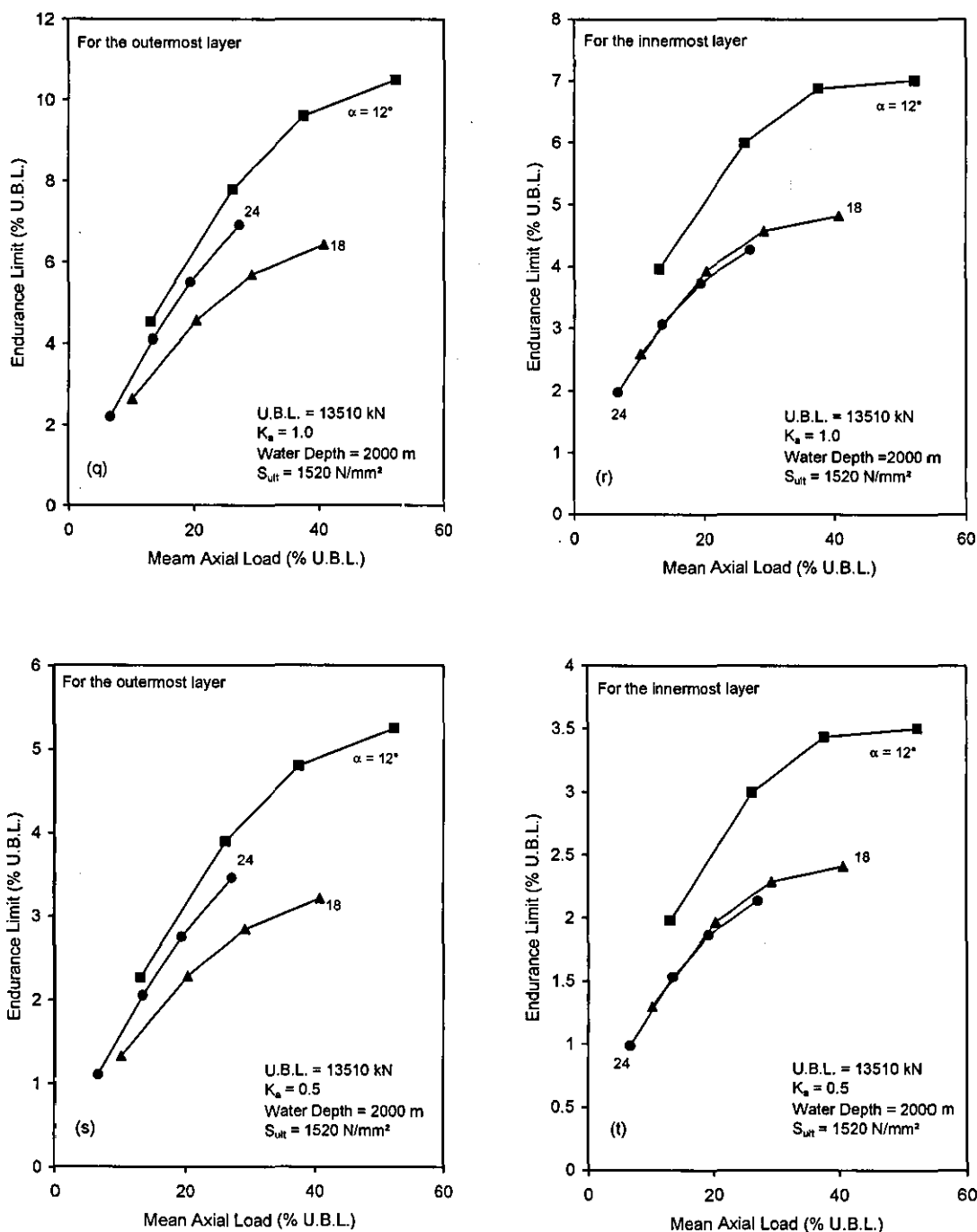
Figs. 8.1(e-h) – Theoretical Plots of the Endurance Limit Versus Mean Axial Load for the Three Different Sheathed Spiral Strand Constructions, at a Water Depth of 500 m, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (e) and (g) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (f) and (h) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.



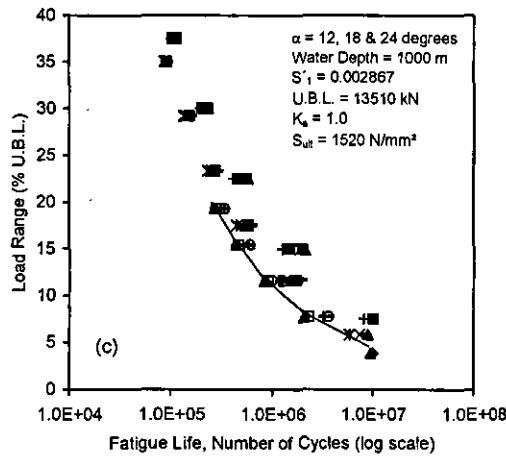
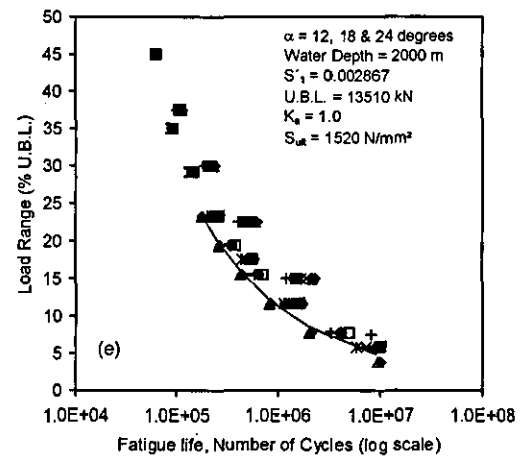
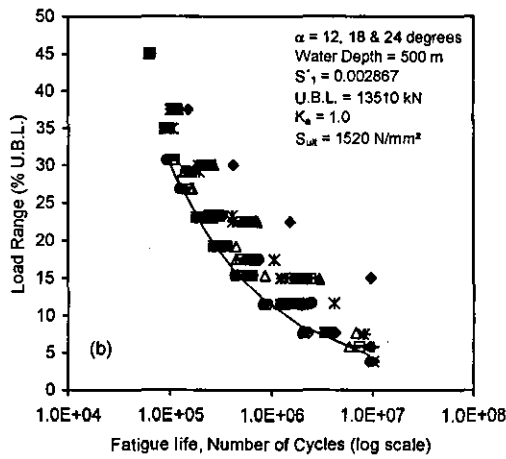
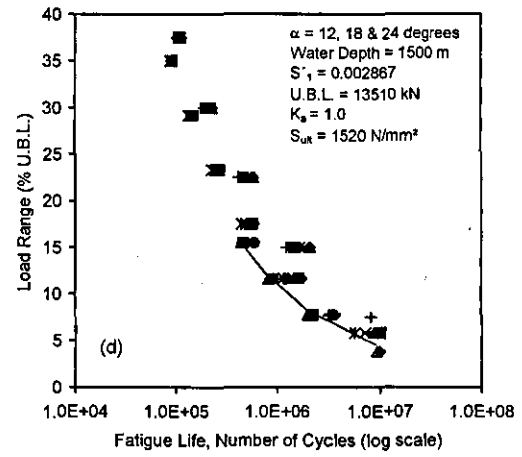
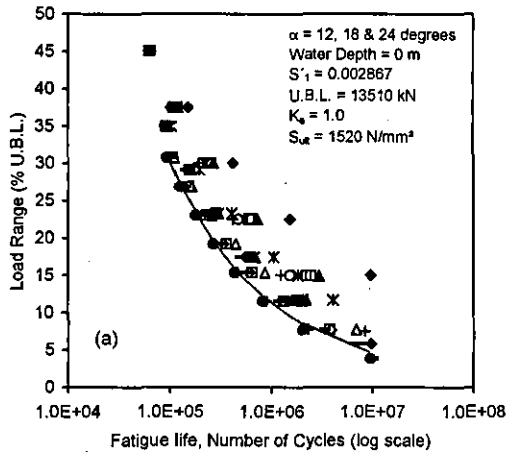
Figs. 8.1(i-l) – Theoretical Plots of the Endurance Limit Versus Mean Axial Load for the Three Different Sheathed Spiral Strand Constructions, at a Water Depth of 1000 m, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (i) and (k) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (j) and (l) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.



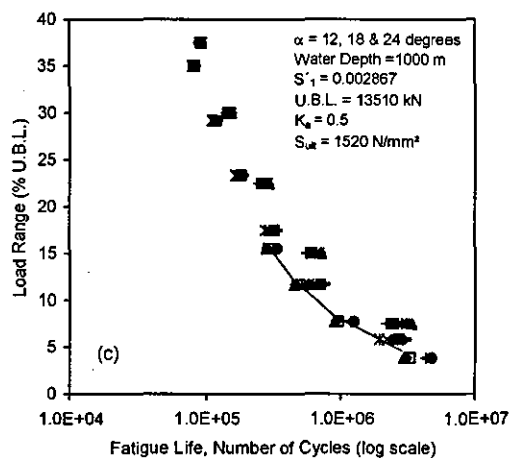
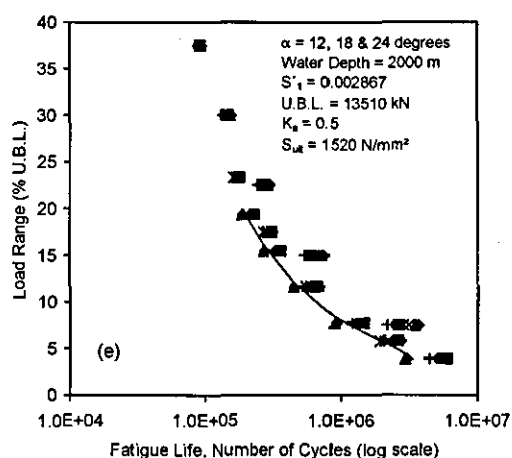
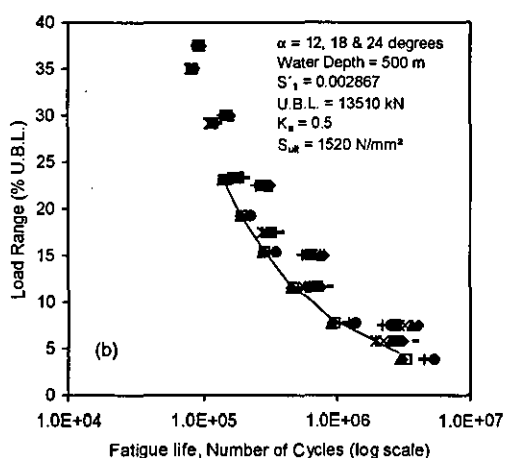
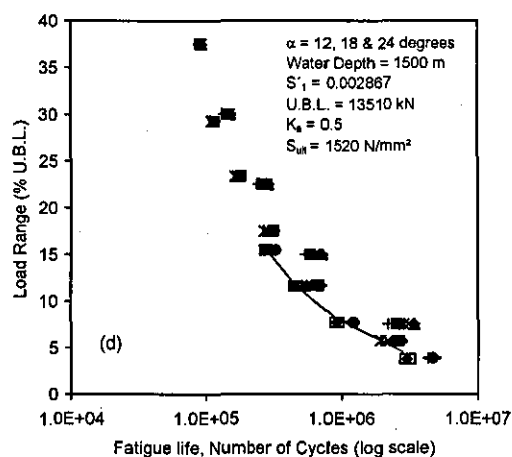
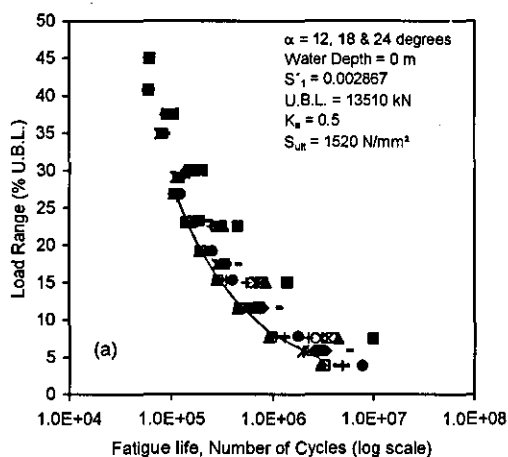
Figs. 8.1(m-p) – Theoretical Plots of the Endurance Limit Versus Mean Axial Load for the Three Different Sheathed Spiral Strand Constructions, at a Water Depth of 1500 m, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (m) and (o) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (n) and (p) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.



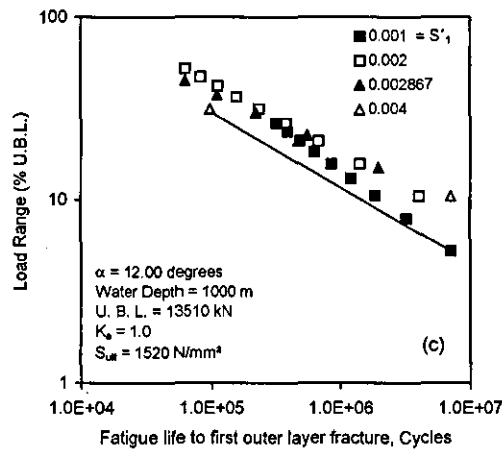
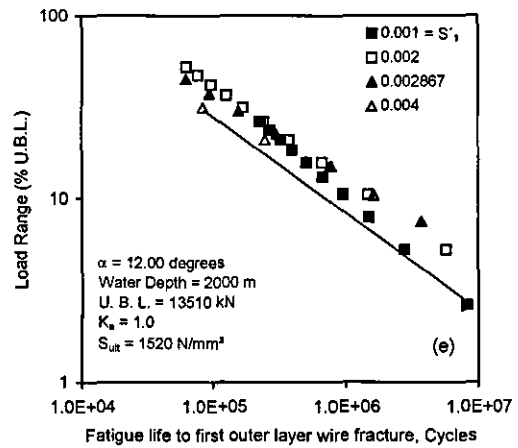
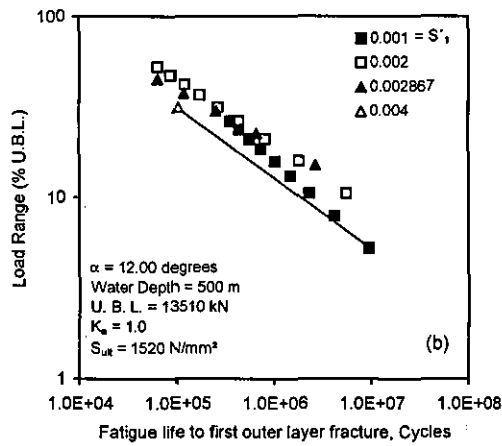
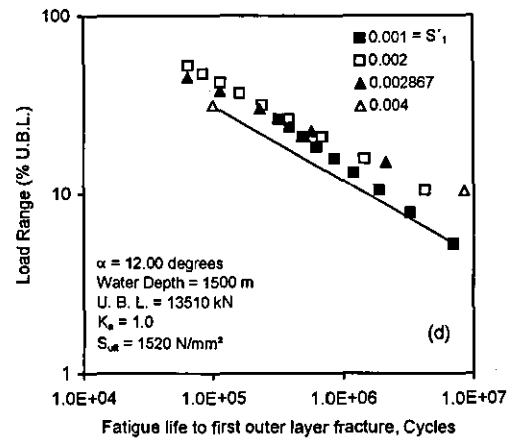
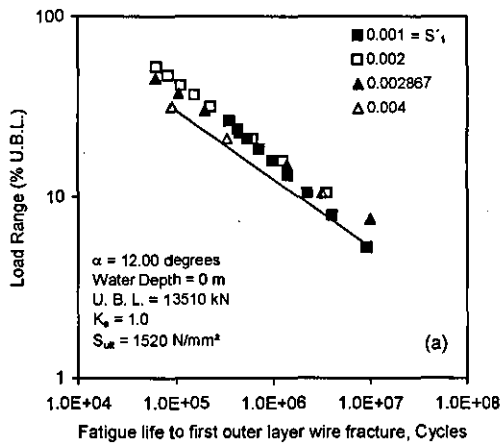
Figs. 8.1(q-t) – Theoretical Plots of the Endurance Limit Versus Mean Axial Load for the Three Different Sheathed Spiral Strand Constructions, at a Water Depth of 2000 m, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (q) and (s) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (r) and (t) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.



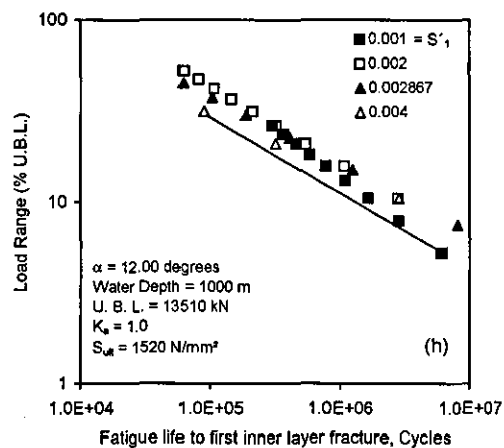
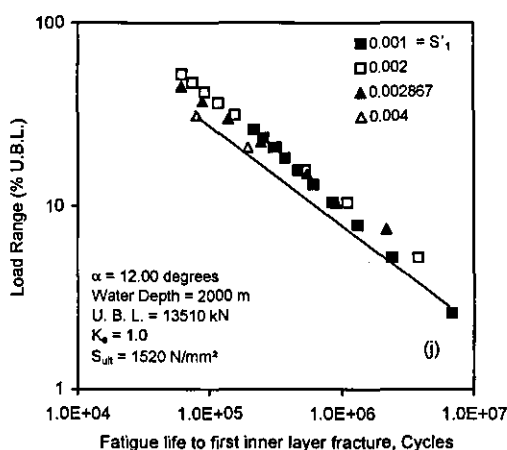
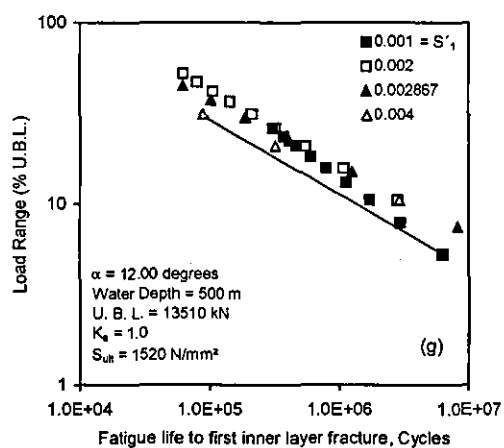
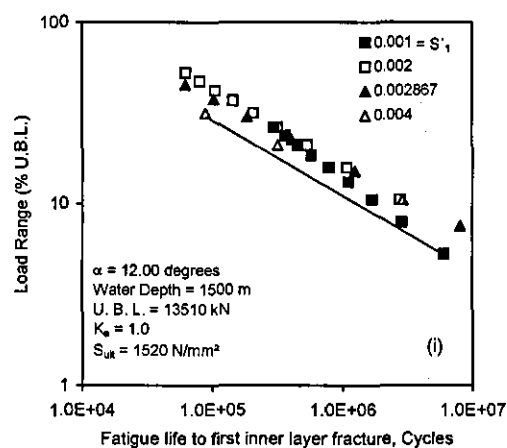
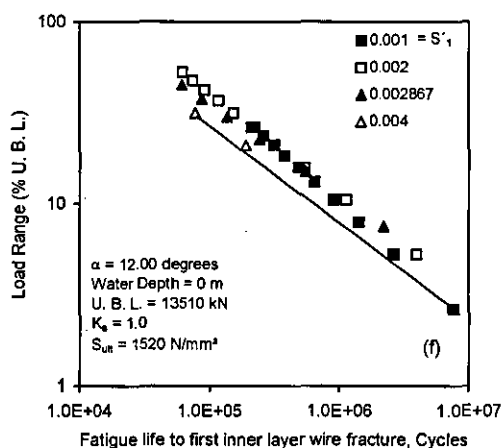
Figs. 8.2(a-e) - Composite Plots of the Theoretical Predictions of Strand Fatigue Life to First Wire Fracture for all the Layers at a Given Level of Mean Axial Strain, $S'_1 = 0.002867$, Varying Magnitudes of Lay Angle, α , and at Varying Water Depths; (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 1.0$.



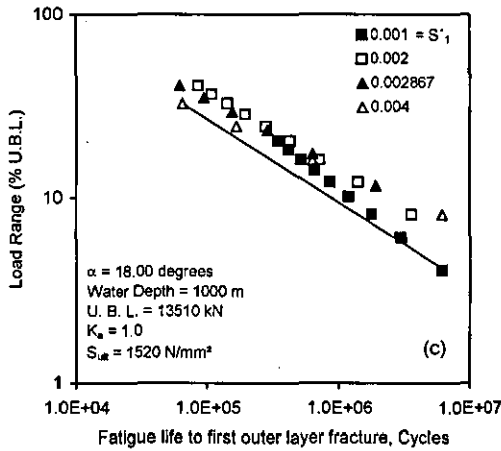
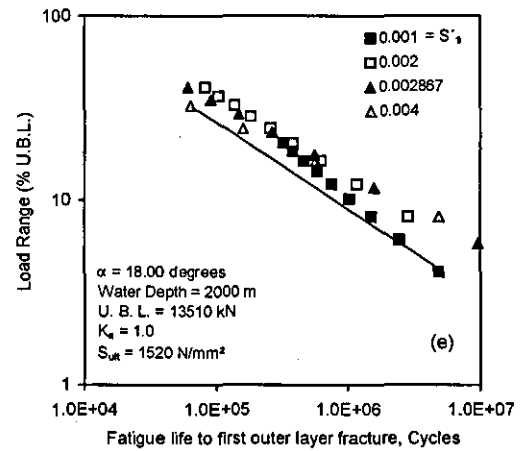
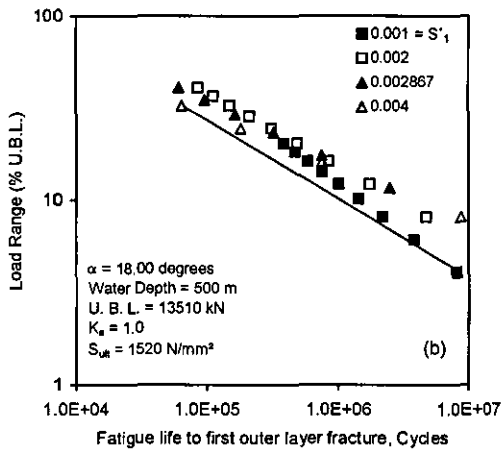
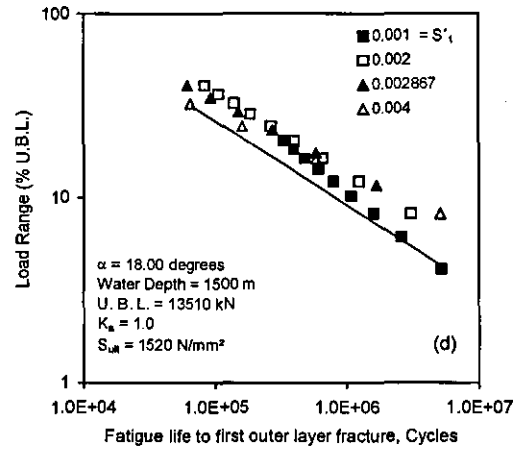
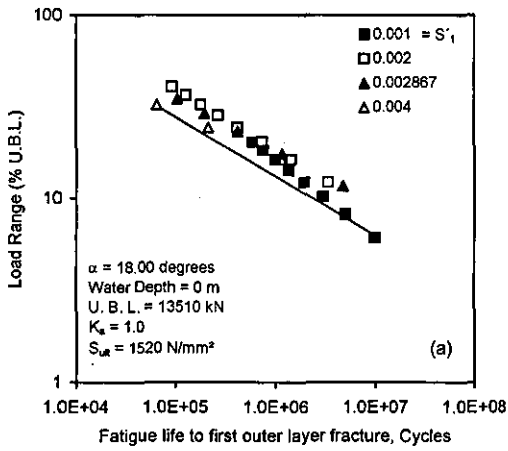
Figs. 8.3(a-e) - Composite Plots of the Theoretical Predictions of Strand Fatigue Life to First Wire Fracture for all the Layers at a Given Level of Mean Axial Strain, $S'_1 = 0.002867$, Varying Magnitudes of Lay Angle, α , and at Varying Water Depths; (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 0.5$.



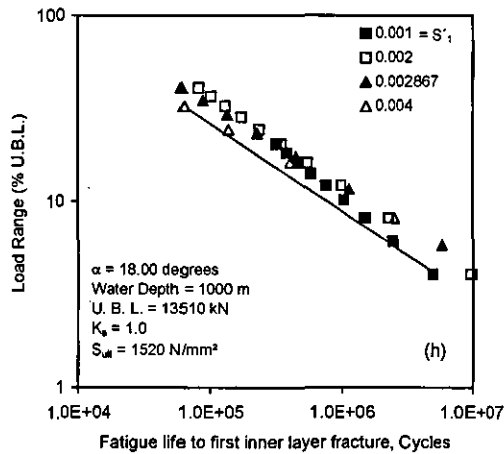
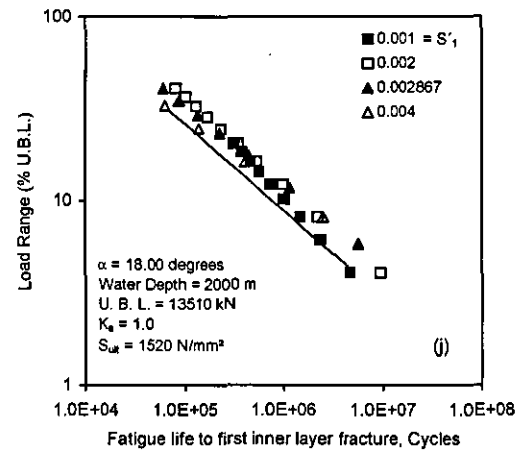
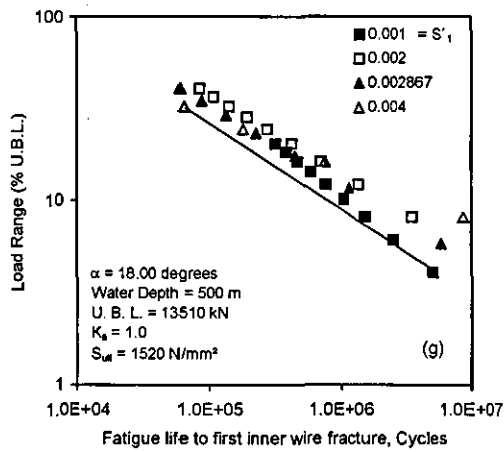
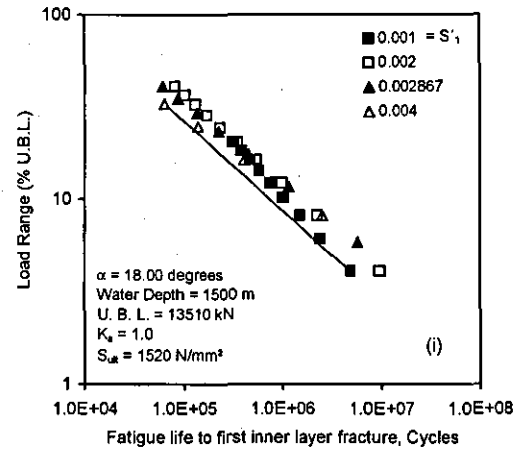
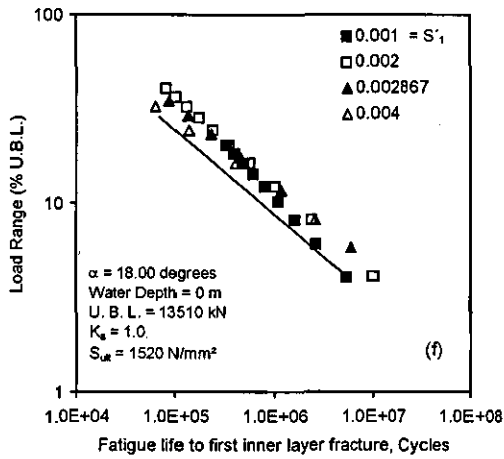
Figs. 8.4(a-e) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 12$ degrees) Based on the Fatigue Life to First Outer Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_{11} , and Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 1.0$.



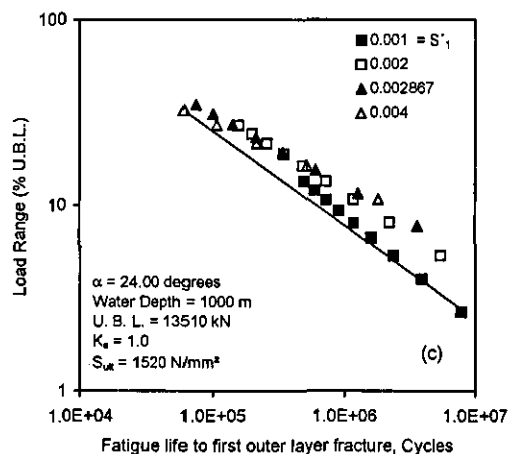
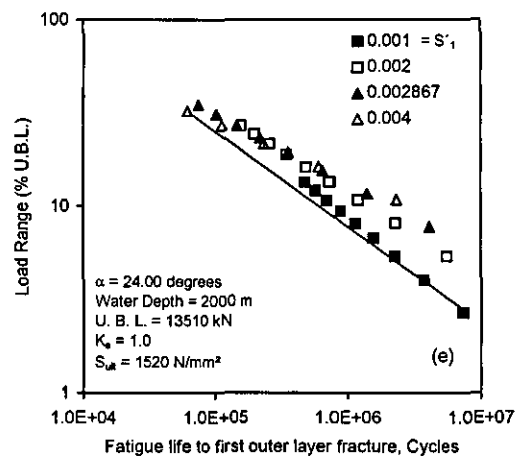
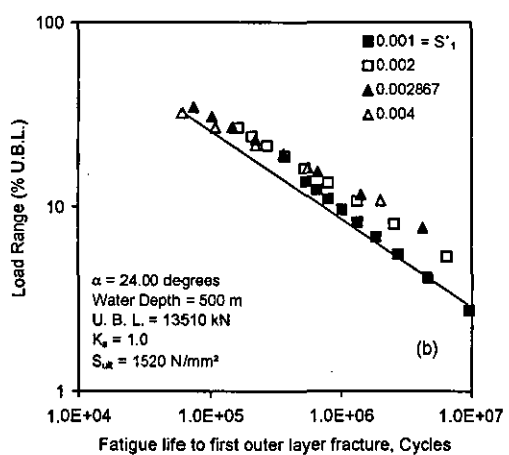
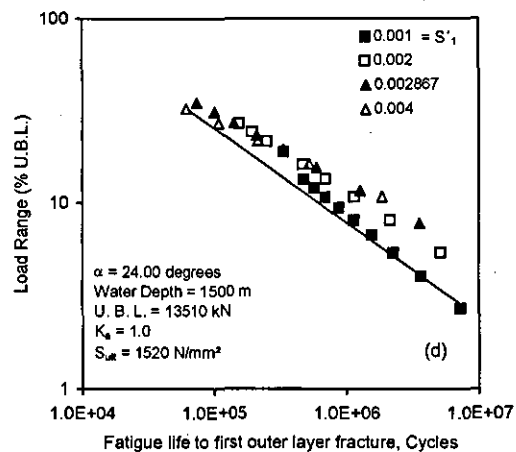
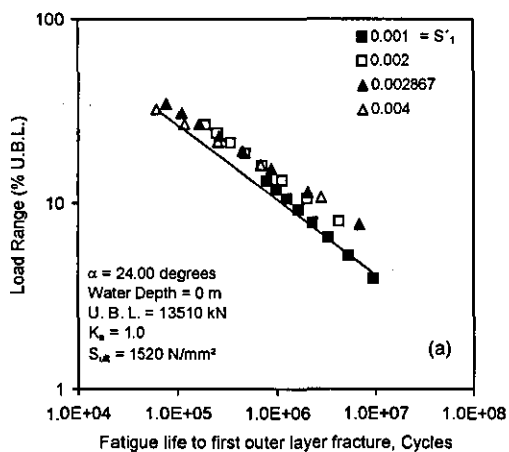
Figs. 8.4(f-j) - Lower Bound S-N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 12$ degrees) Based on the Fatigue Life to First Innermost Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (f) 0 m; (g) 500 m; (h) 1000 m; (i) 1500 m; (j) 2000 m - $K_a = 1.0$.



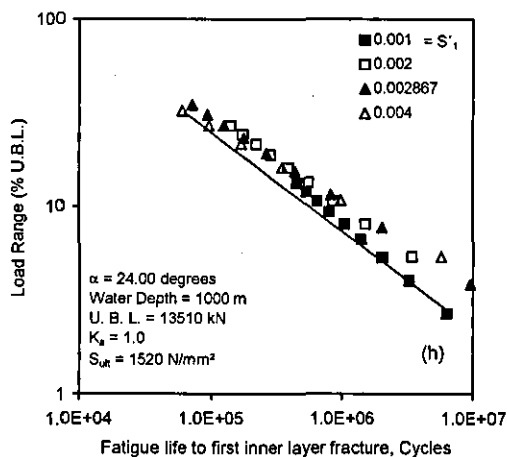
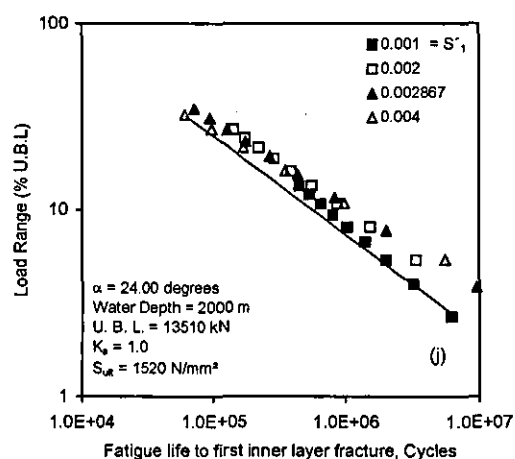
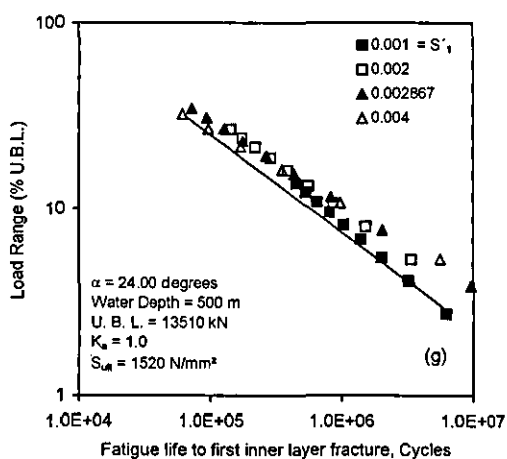
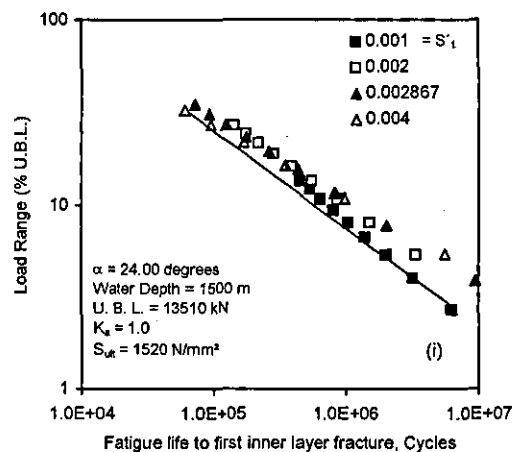
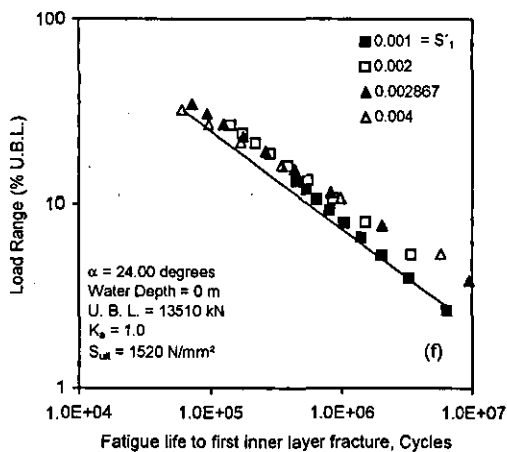
Figs. 8.5(a-e) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 18$ degrees) Based on the Fatigue Life to First Outer Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_{11} , and Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 1.0$.



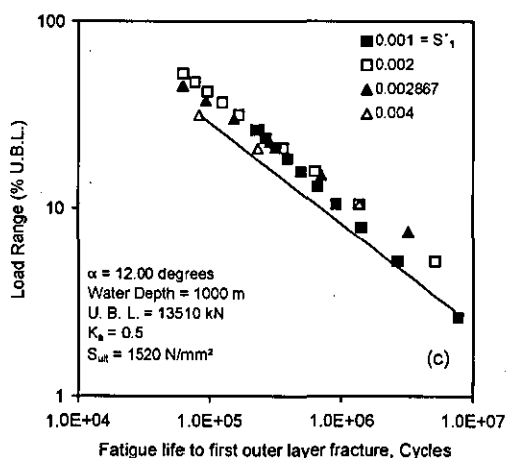
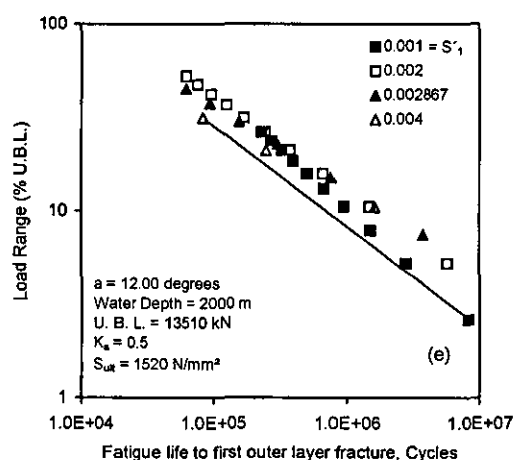
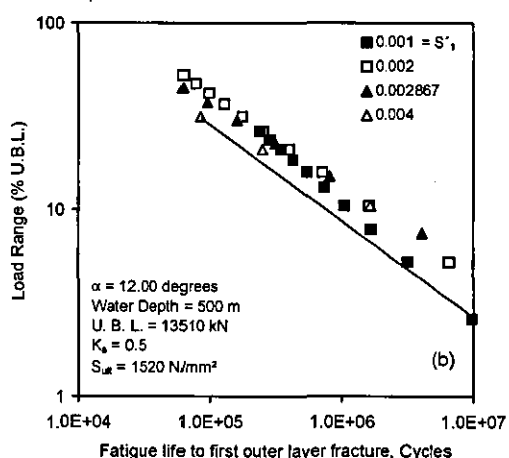
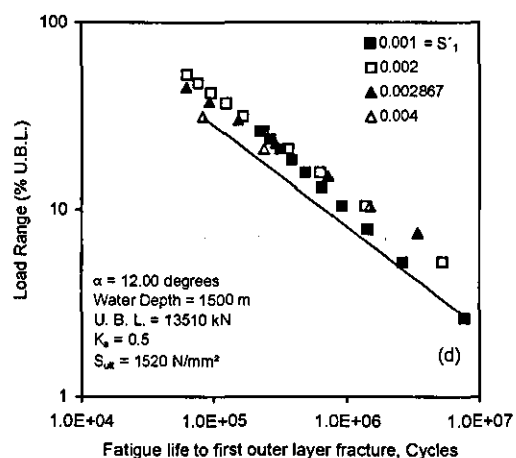
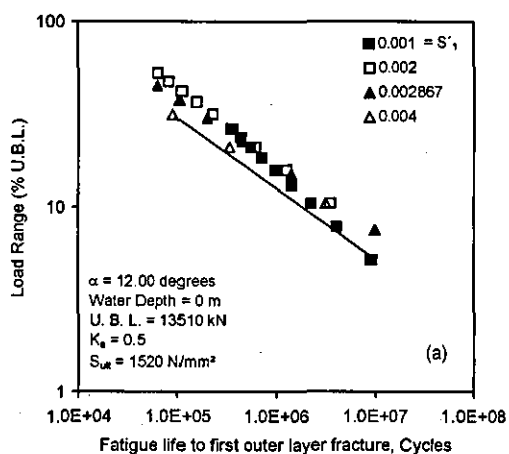
Figs. 8.5(f-j) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 18$ degrees) Based on the Fatigue Life to First Innermost Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (f) 0 m; (g) 500 m; (h) 1000 m; (i) 1500 m; (j) 2000 m - $K_a = 1.0$.



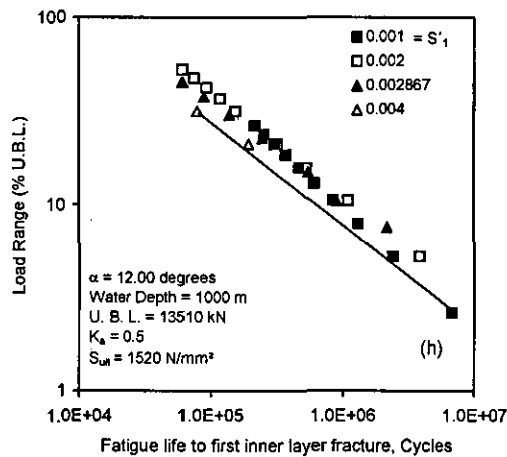
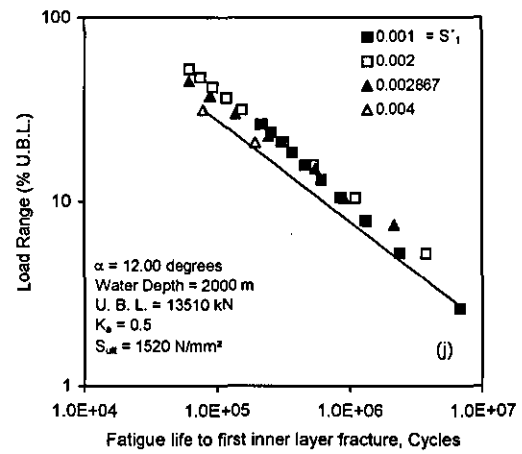
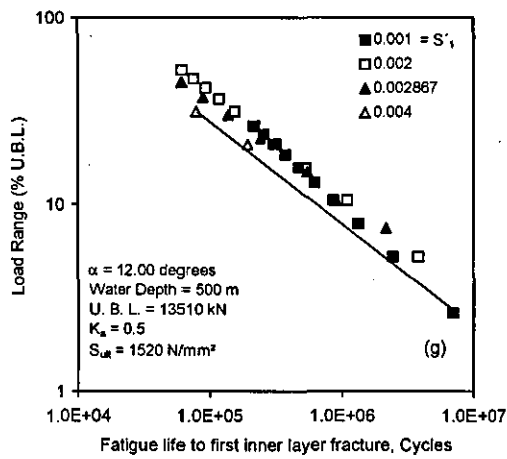
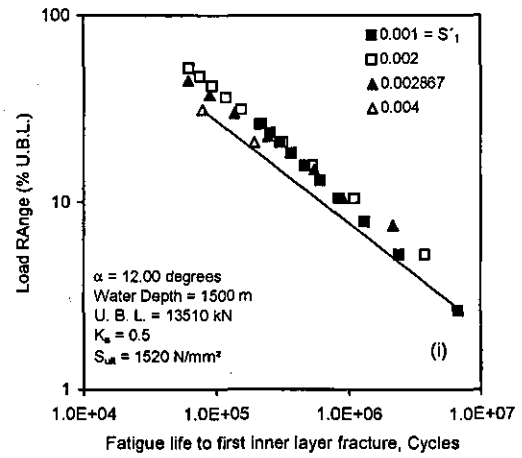
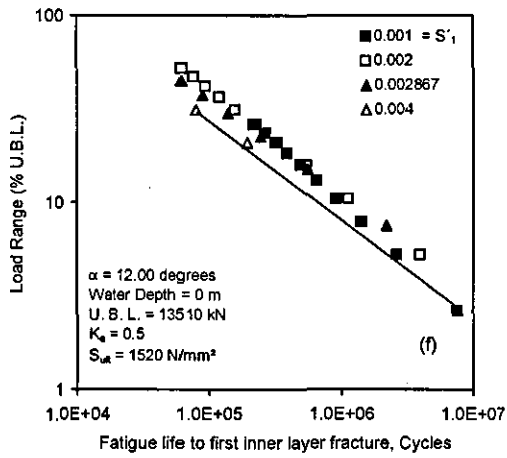
Figs. 8.6(a-e) - Lower Bound S-N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 24$ degrees) Based on the Fatigue Life to First Outer Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 1.0$.



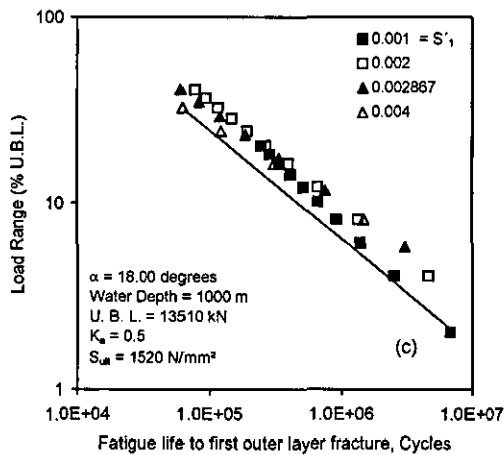
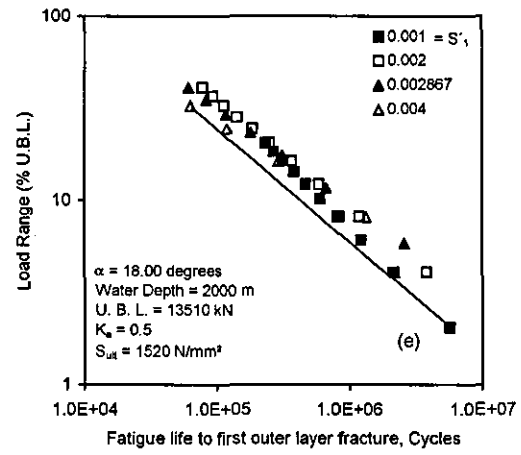
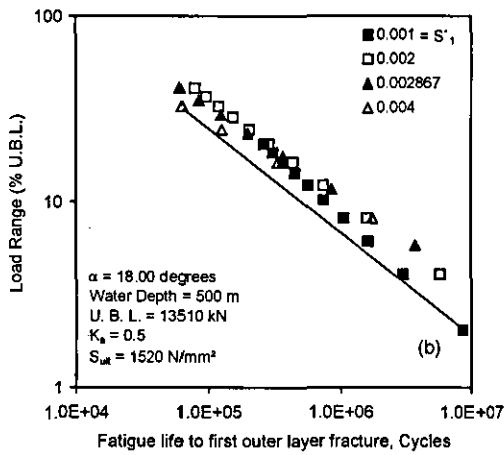
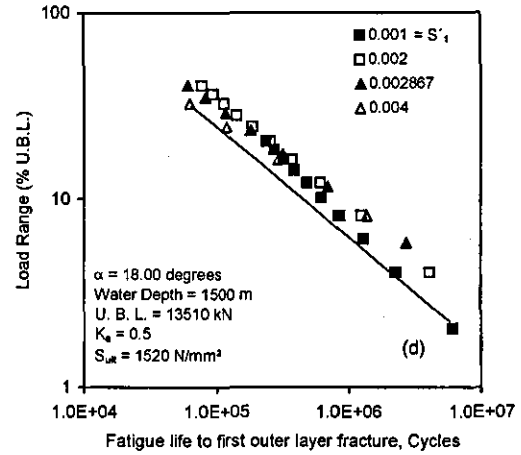
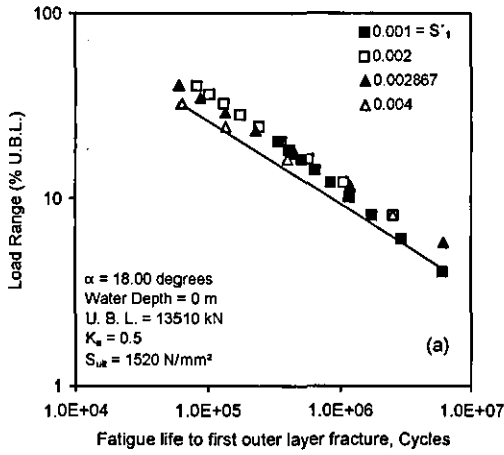
Figs. 8.6(f-j) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 24$ degrees) Based on the Fatigue Life to First Innermost Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_{1} , and Varying Levels of Water Depth: (f) 0 m; (g) 500 m; (h) 1000 m; (i) 1500 m; (j) 2000 m - $K_a = 1.0$.



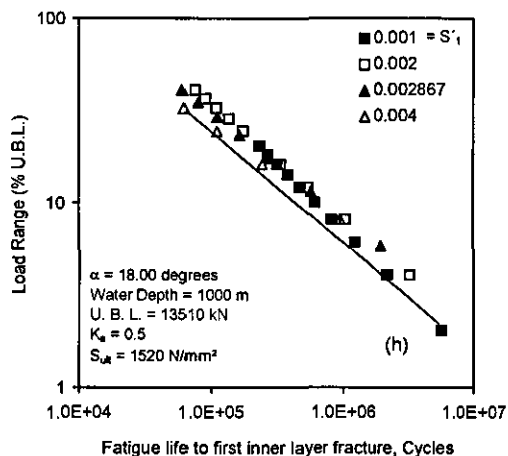
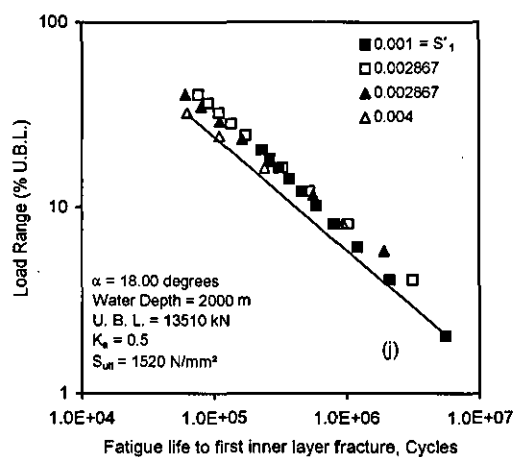
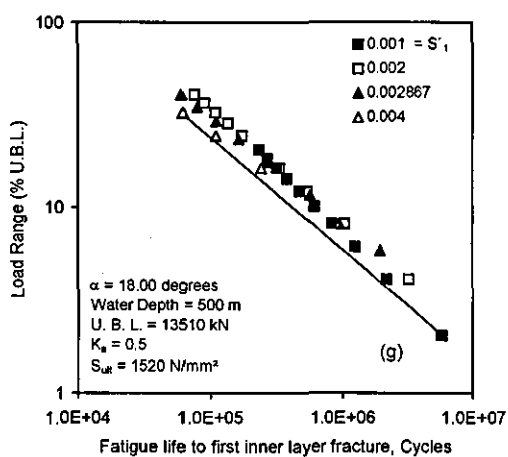
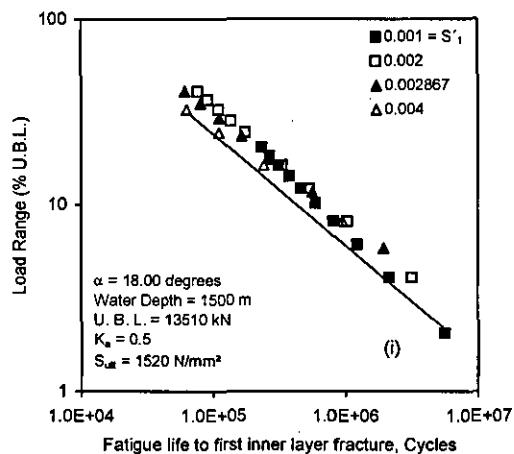
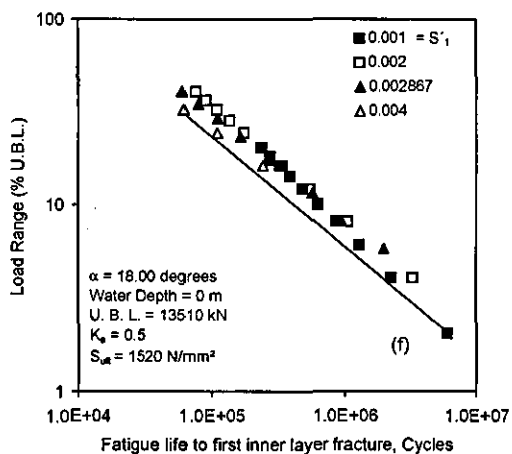
Figs. 8.7(a-e) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 12$ degrees) Based on the Fatigue Life to First Outer Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_{11} , and Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 0.5$.



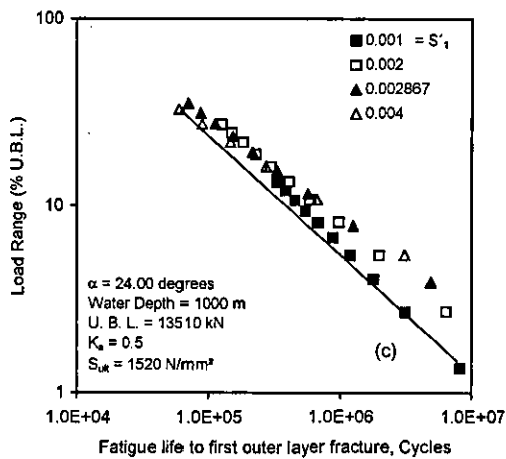
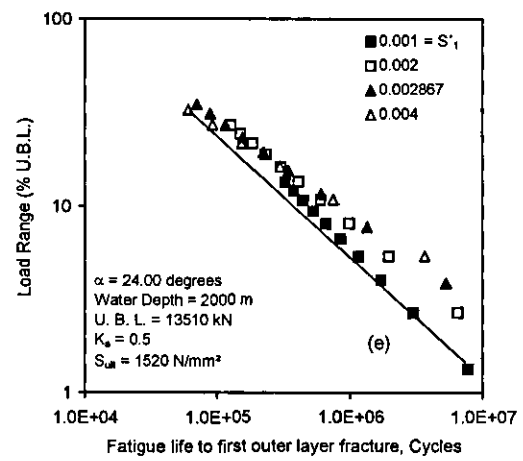
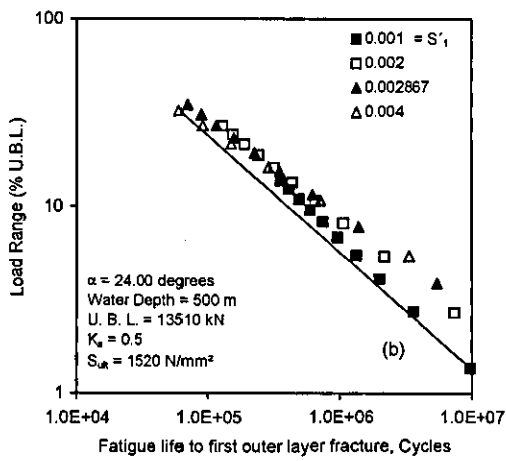
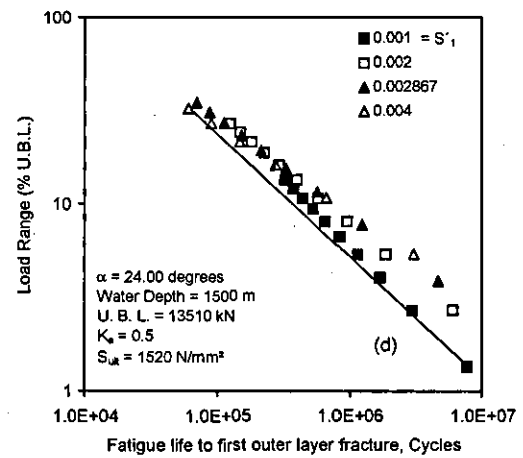
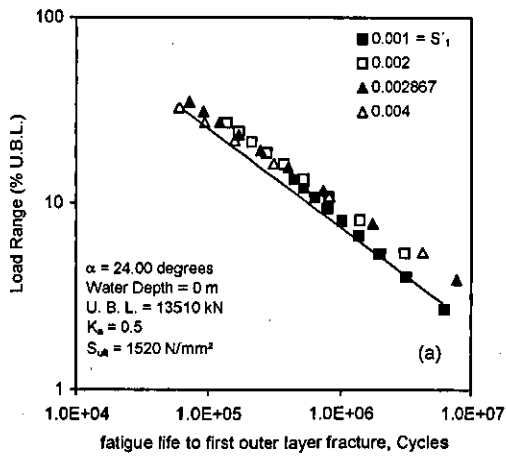
Figs. 8.7(f-j) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 12$ degrees) Based on the Fatigue Life to First Innermost Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_{11} , and Varying Levels of Water Depth: (f) 0 m; (g) 500 m; (h) 1000 m; (i) 1500 m; (j) 2000 m - $K_a = 0.5$.



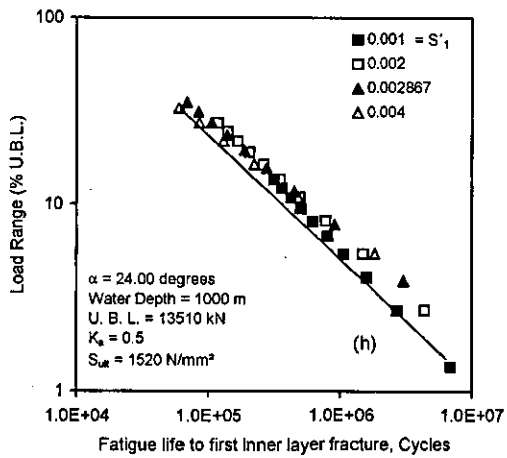
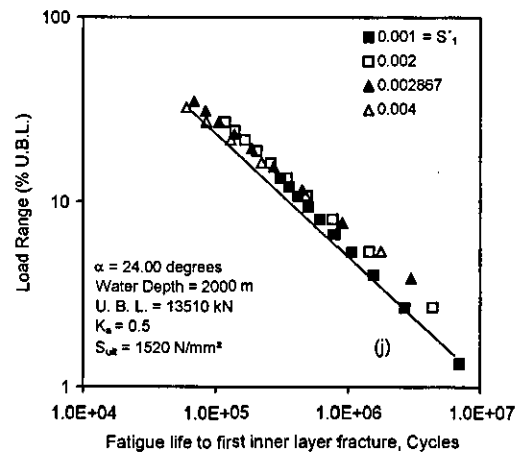
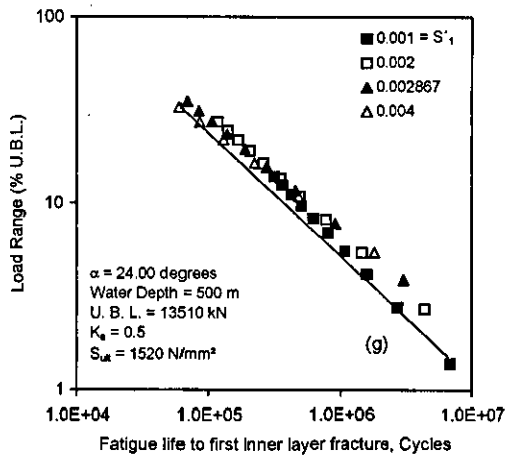
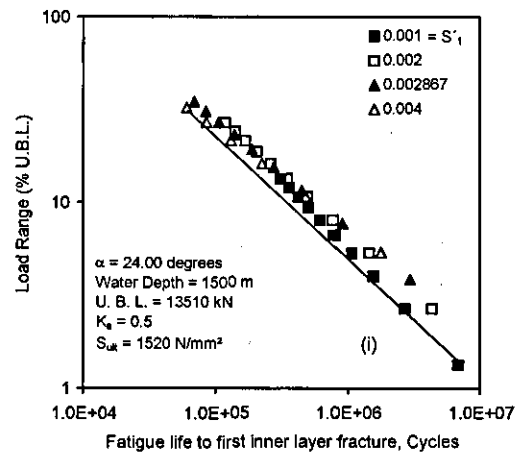
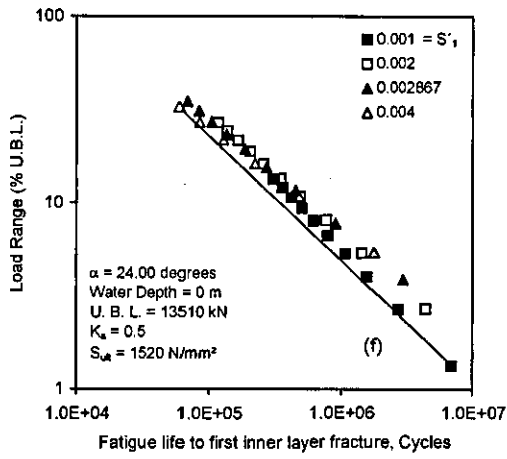
Figs. 8.8(a-e) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 18$ degrees) Based on the Fatigue Life to First Outer Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_{11} , and Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 0.5$.



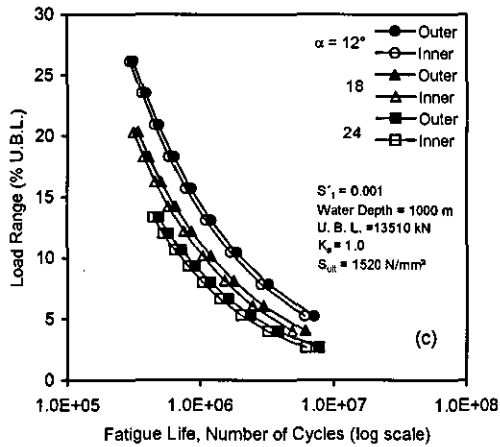
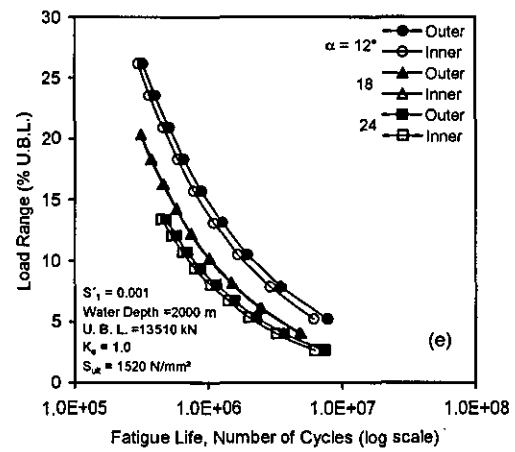
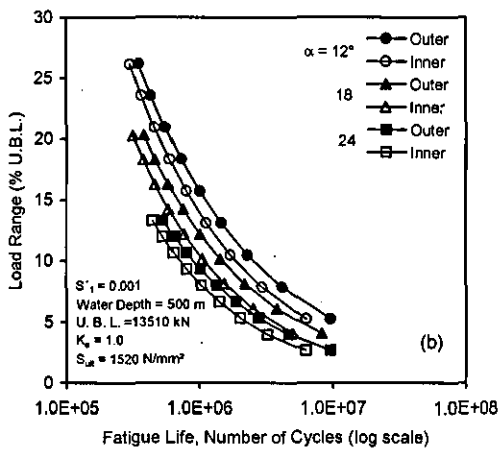
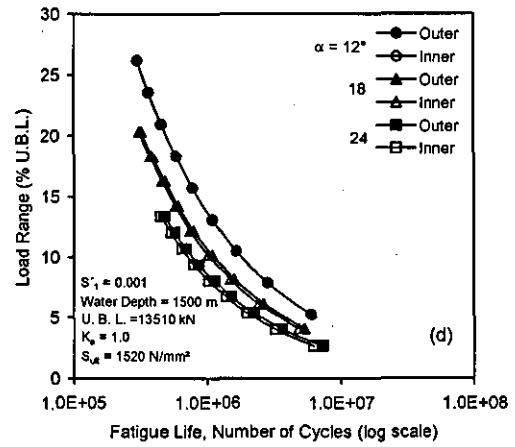
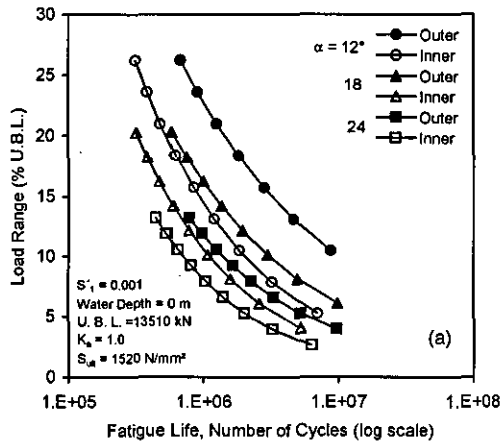
Figs. 8.8(f-j) - Lower Bound S-N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 18$ degrees) Based on the Fatigue Life to First Innermost Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (f) 0 m; (g) 500 m; (h) 1000 m; (i) 1500 m; (j) 2000 m - $K_a = 0.5$.



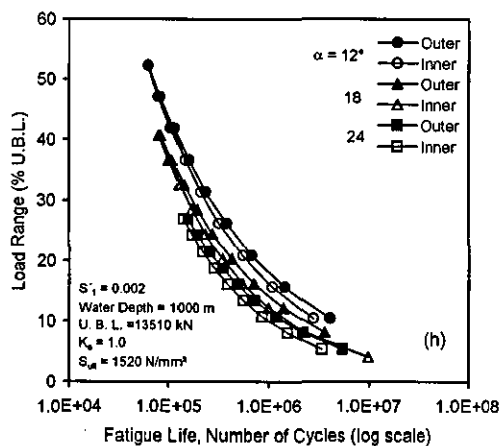
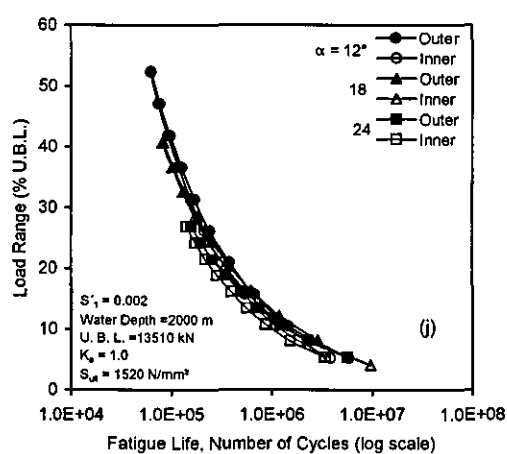
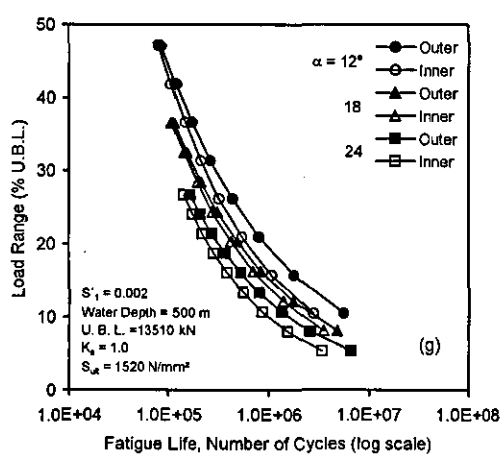
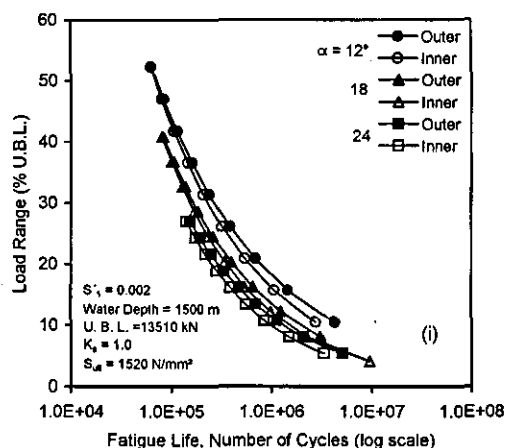
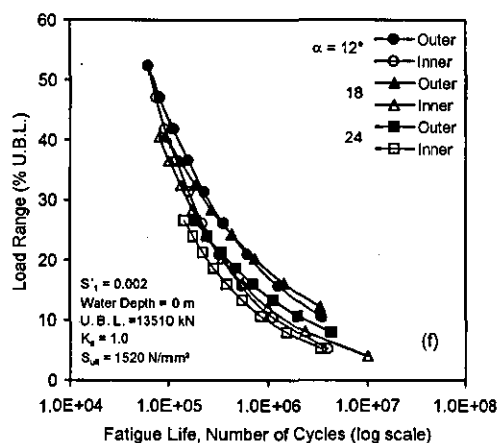
Figs. 8.9(a-e) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 24$ degrees) Based on the Fatigue Life to First Outer Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_1 , and Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 0.5$.



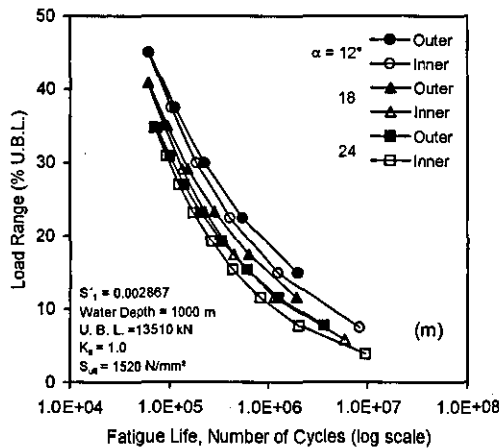
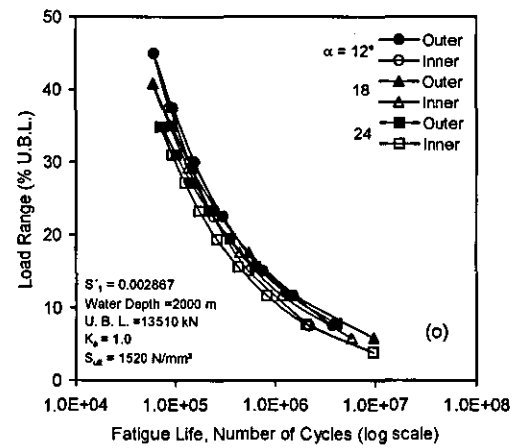
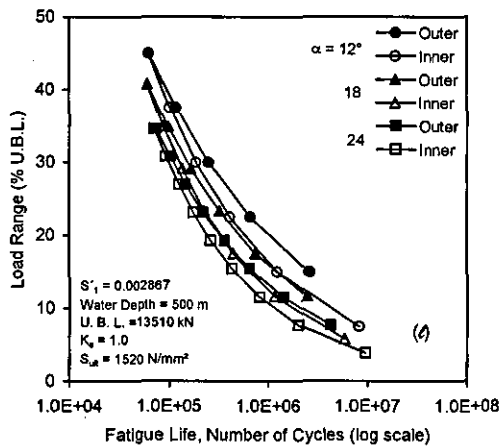
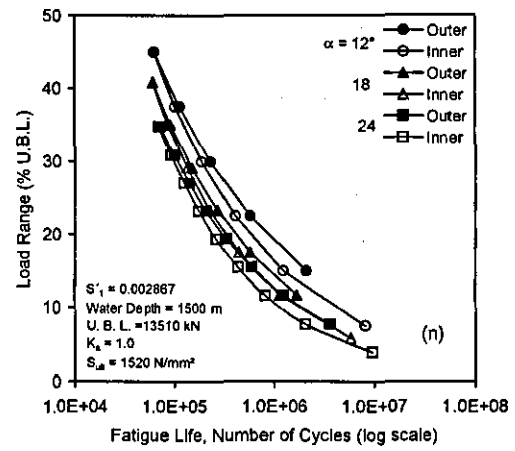
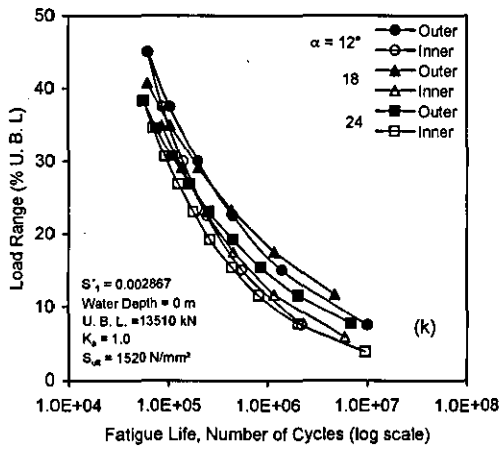
Figs. 8.9(f-j) - Lower Bound S -N Curves for the 127 mm Outside Diameter Sheathed Spiral Strand ($\alpha = 24$ degrees) Based on the Fatigue Life to First Innermost Layer Wire Fracture and Subjected to a Wide Range of Mean Axial Strains, S'_{1} , and Varying Levels of Water Depth: (f) 0 m; (g) 500 m; (h) 1000 m; (i) 1500 m; (j) 2000 m - $K_a = 0.5$.



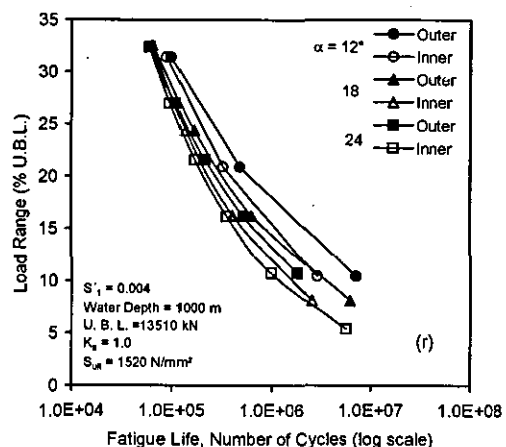
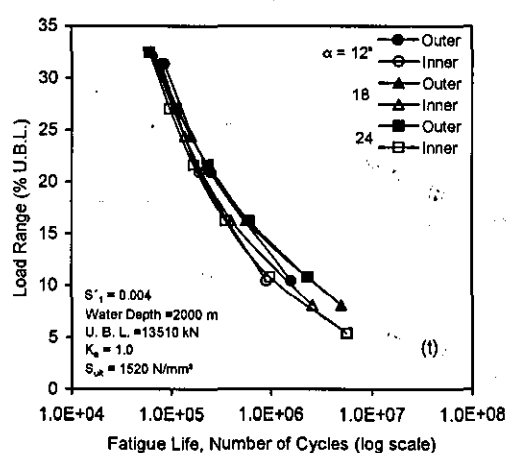
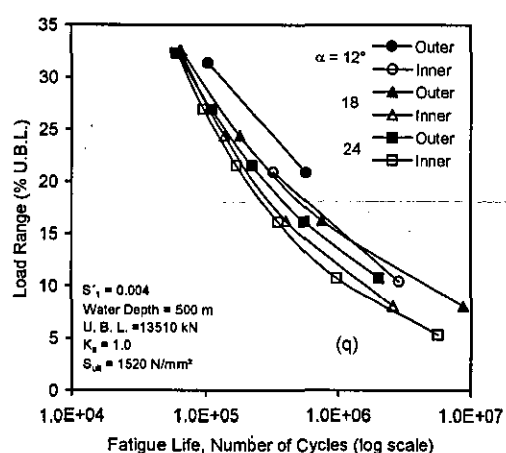
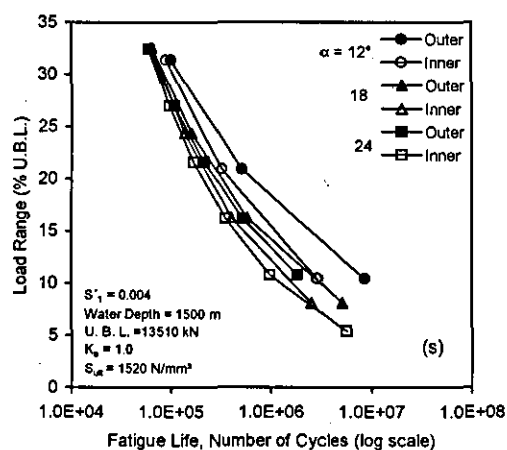
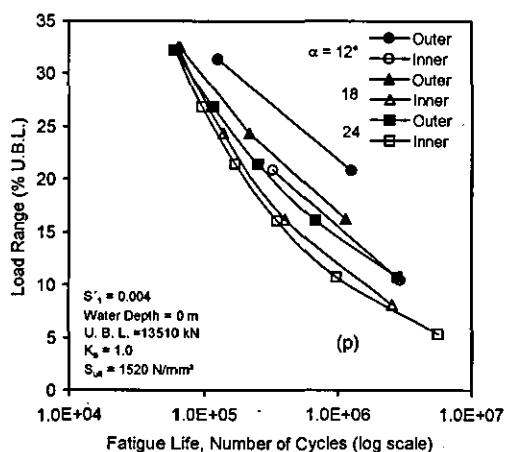
Figs. 8.10(a-e) - Effect of Varying the Magnitude of the Lay Angle, on the S-N Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.001$, and at Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 1.0$.



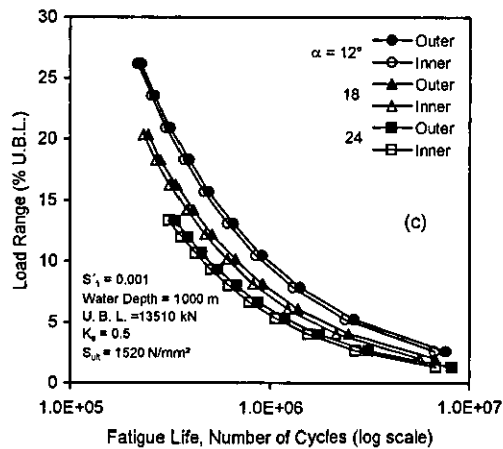
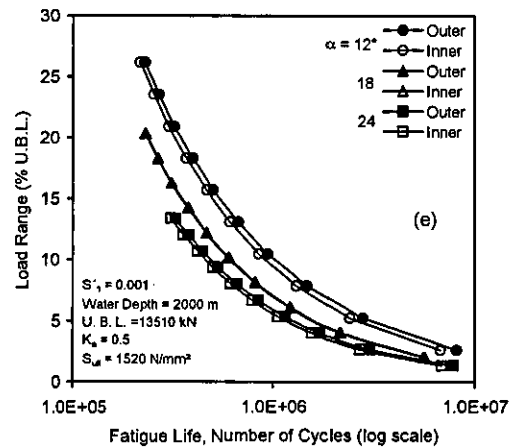
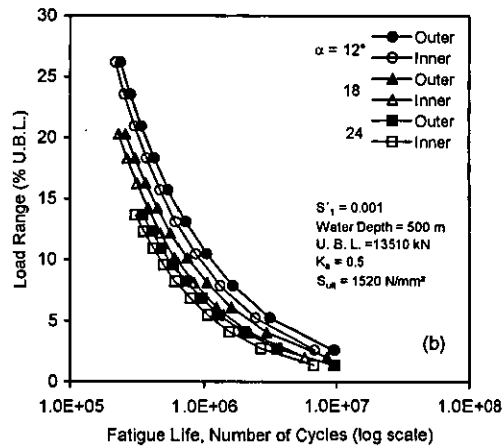
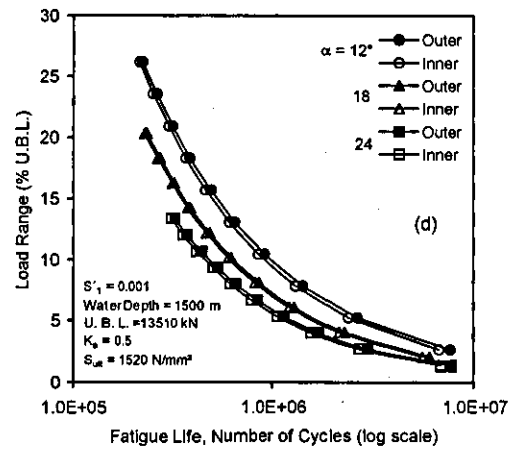
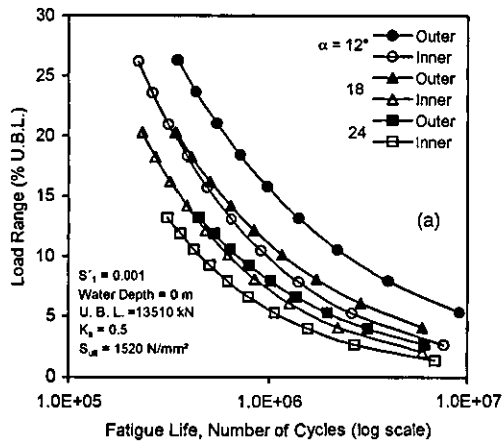
Figs. 8.10(f-j) - Effect of Varying the Magnitude of the Lay Angle, on the S-N Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.002$, and at Varying Levels of Water Depth: (f) 0 m; (g) 500 m; (h) 1000 m; (i) 1500 m; (j) 2000 m - $K_a = 1.0$.



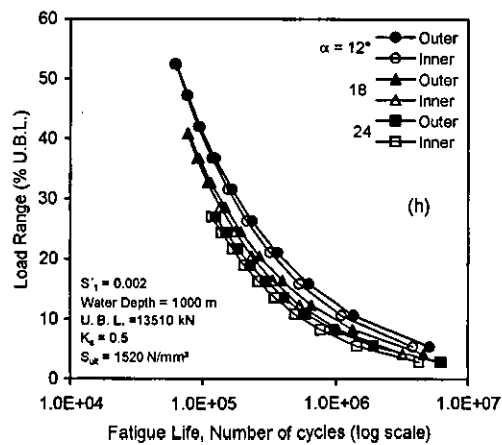
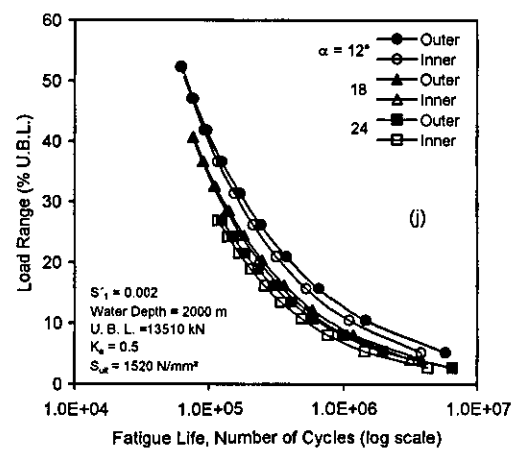
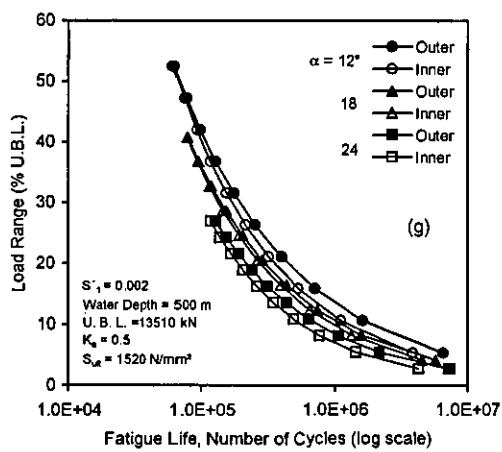
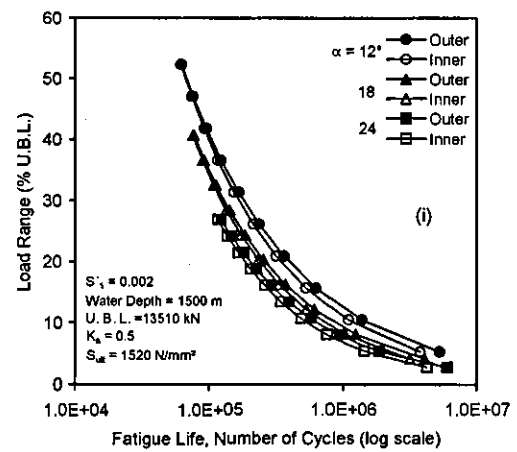
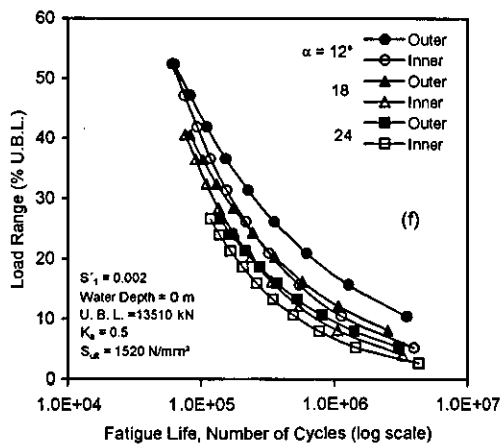
Figs. 8.10(k-o) - Effect of Varying the Magnitude of the Lay Angle, on the S-N Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.002867$, and at Varying Levels of Water Depth: (k) 0 m; (l) 500 m; (m) 1000 m; (n) 1500 m; (o) 2000 m - $K_a = 1.0$.



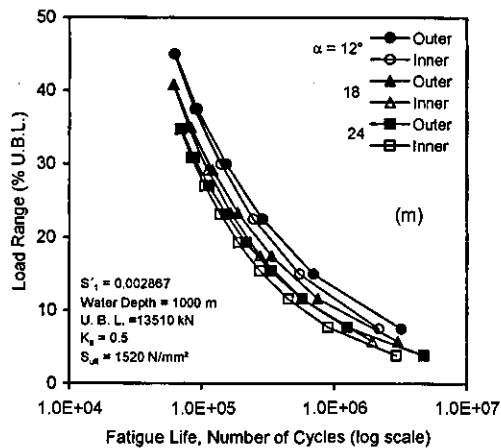
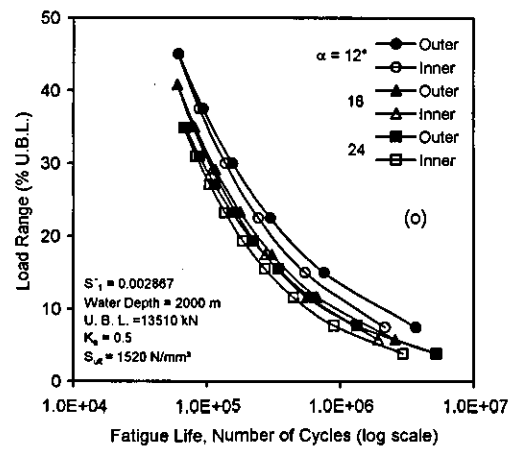
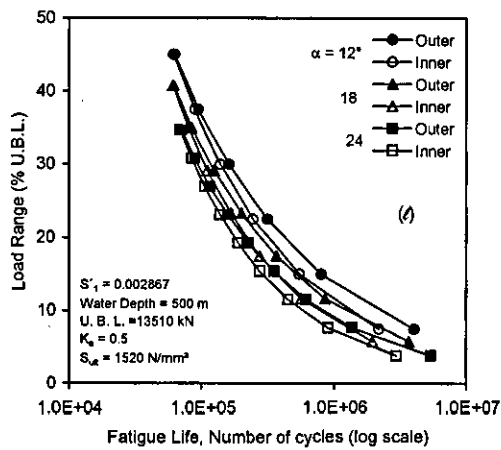
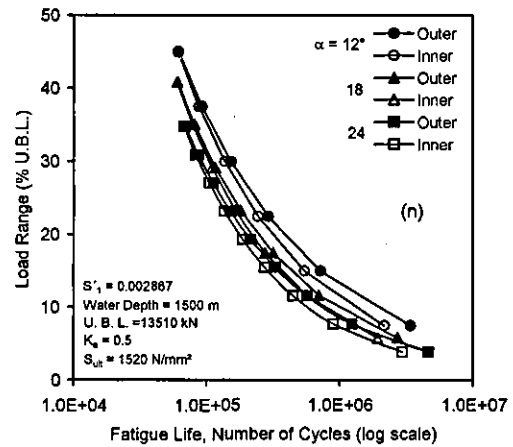
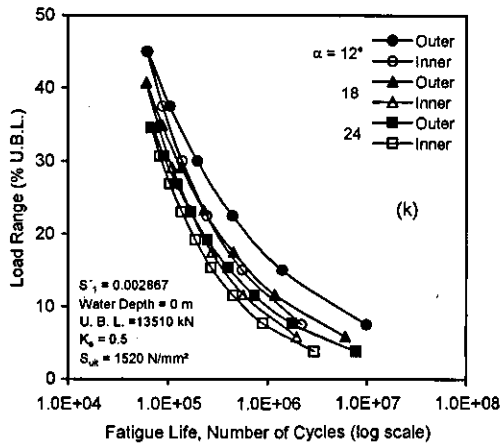
Figs. 8.10(p-t) - Effect of Varying the Magnitude of the Lay Angle, on the S-N Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.004$, and at Varying Levels of Water Depth: (p) 0 m; (q) 500 m; (r) 1000 m; (s) 1500 m; (t) 2000 m - $K_a = 1.0$.



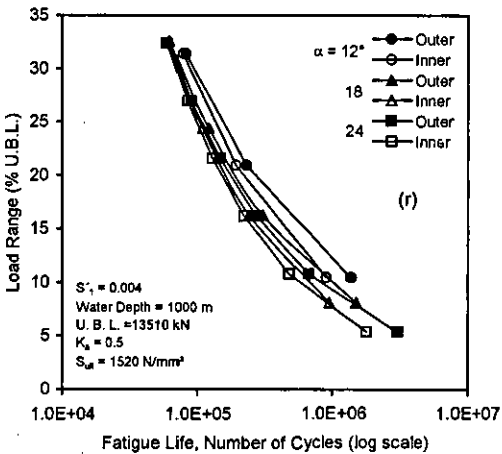
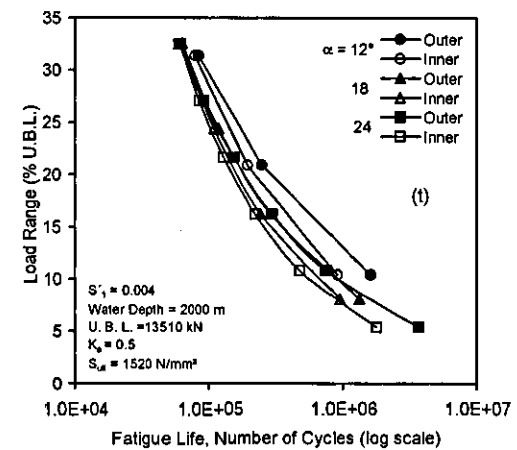
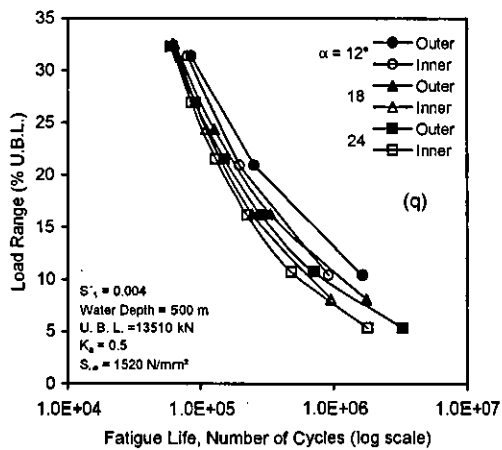
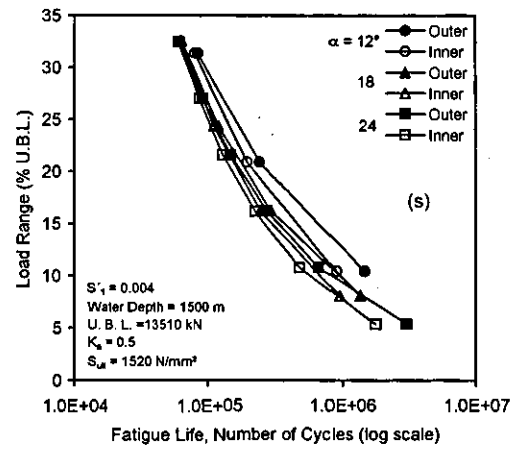
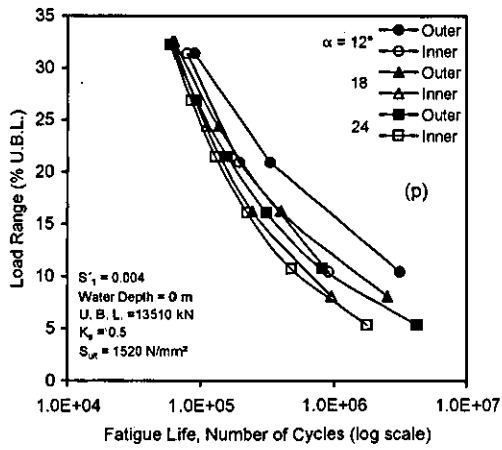
Figs. 8.11(a-e) - Effect of Varying the Magnitude of the Lay Angle, on the S-N Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.001$, and at Varying Levels of Water Depth: (a) 0 m; (b) 500 m; (c) 1000 m; (d) 1500 m; (e) 2000 m - $K_a = 0.5$.



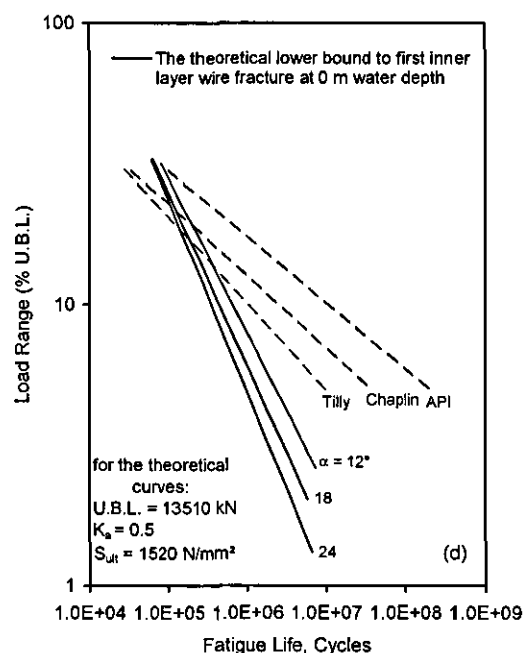
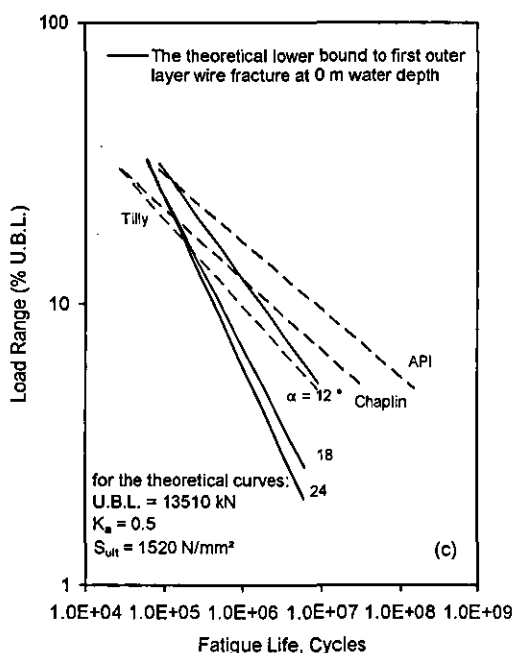
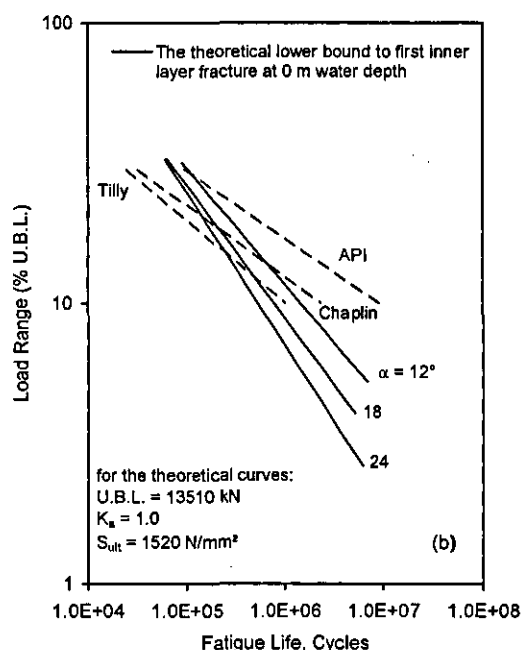
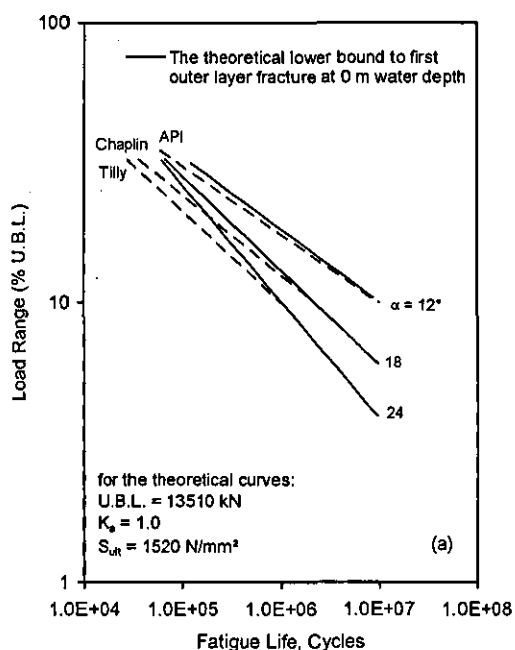
Figs. 8.11(f-j) - Effect of Varying the Magnitude of the Lay Angle, on the S-N Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.002$, and at Varying Levels of Water Depth: (f) 0 m; (g) 500 m; (h) 1000 m; (i) 1500 m; (j) 2000 m - $K_a = 0.5$.



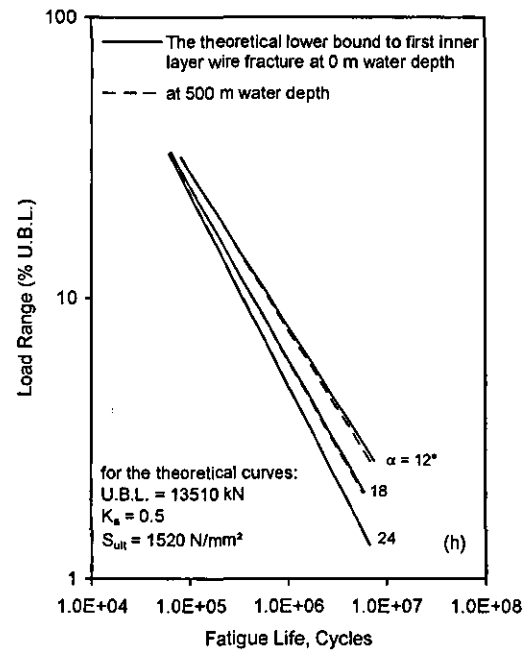
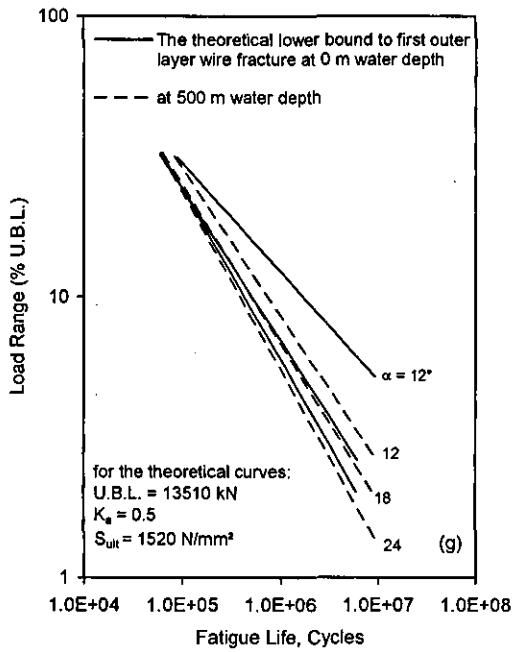
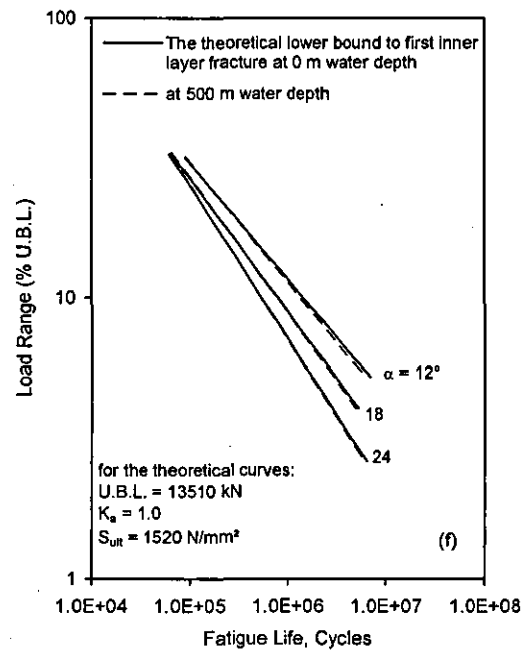
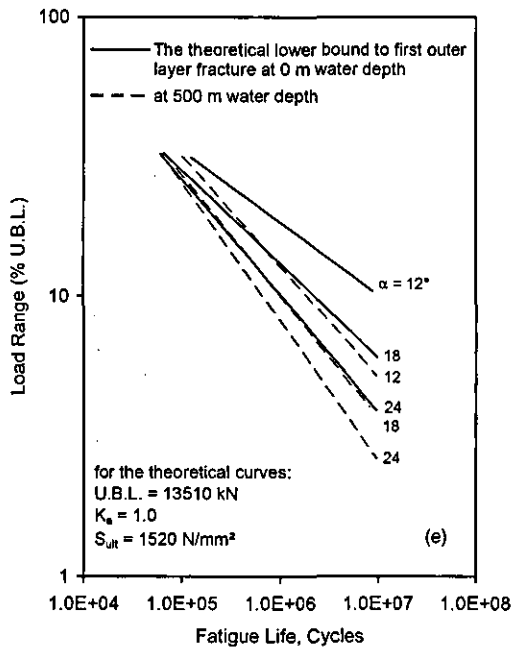
Figs. 8.11(k-o) - Effect of Varying the Magnitude of the Lay Angle, on the S-N Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.002867$, and at Varying Levels of Water Depth: (k) 0 m; (l) 500 m; (m) 1000 m; (n) 1500 m; (o) 2000 m - $K_a = 0.5$.



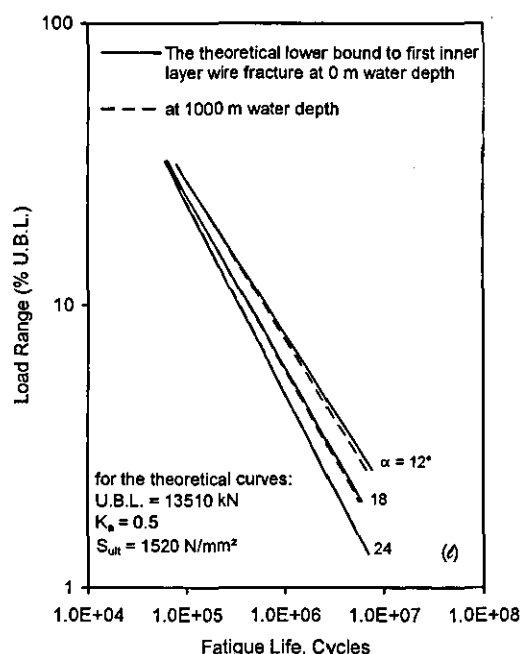
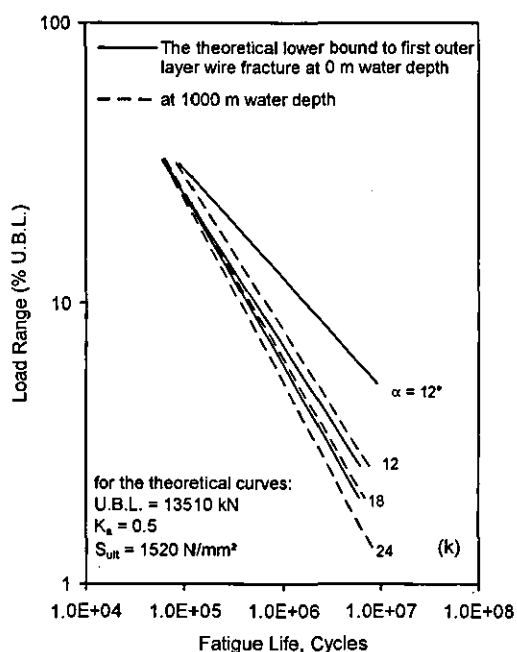
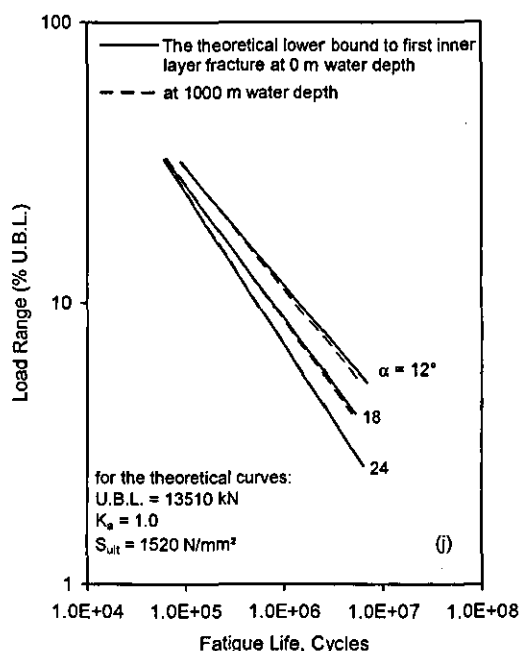
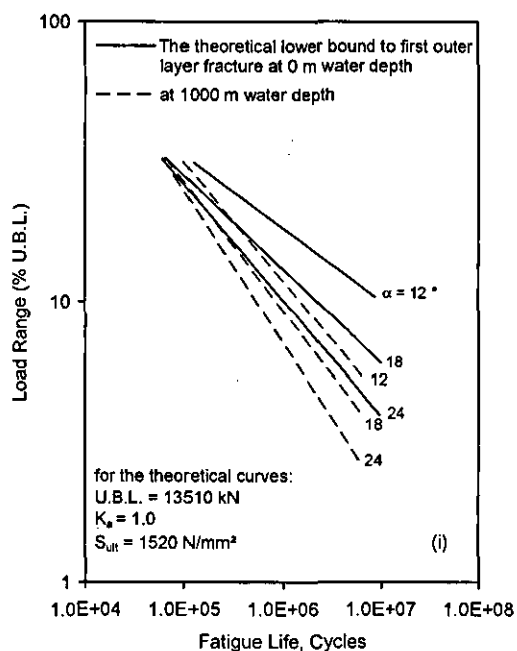
Figs. 8.11(p-t) - Effect of Varying the Magnitude of the Lay Angle, on the S-N Curves, for the Three Different 127 mm Outside Diameter Sheathed Spiral Strands at a Mean Axial Strain $S'_1 = 0.004$, and at Varying Levels of Water Depth: (p) 0 m; (q) 500 m; (r) 1000 m; (s) 1500 m; (t) 2000 m - $K_a = 0.5$.



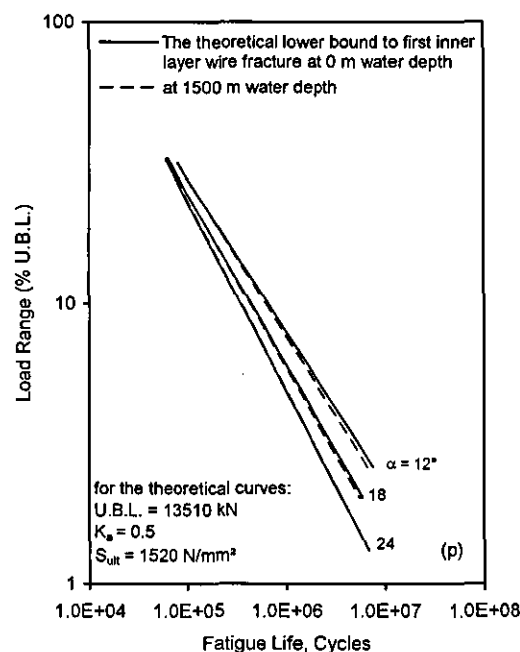
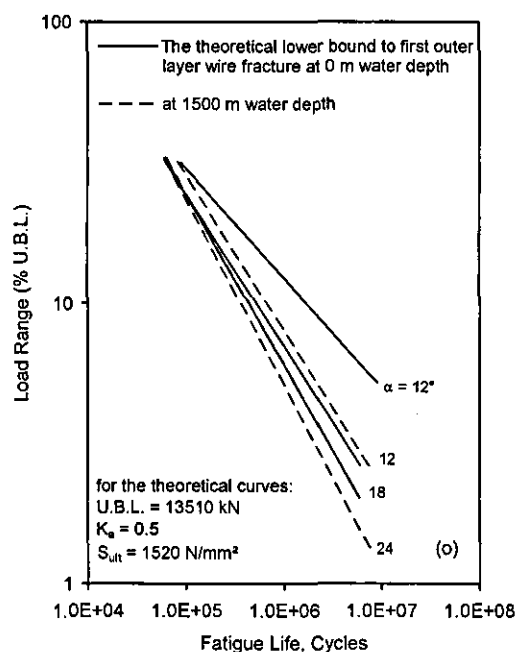
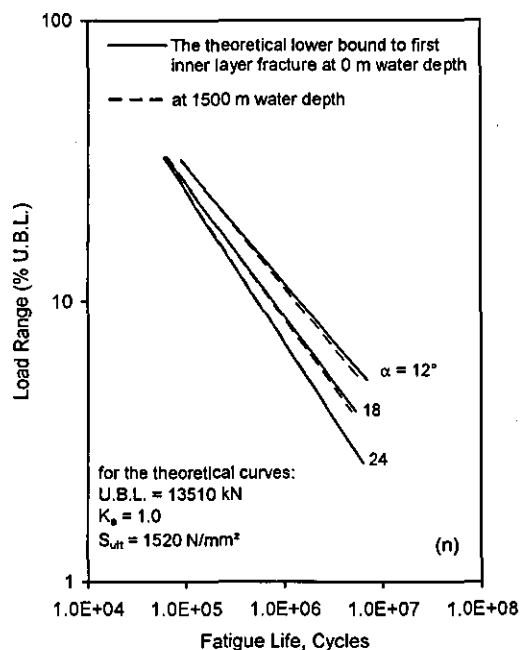
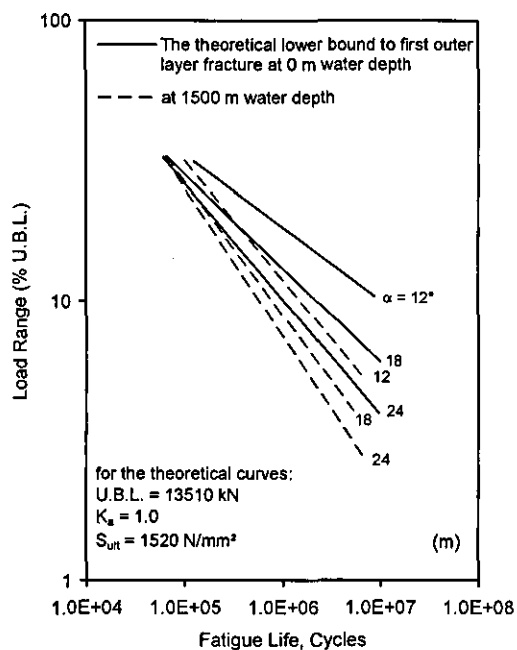
Figs. 8.12(a-d) – Comparison of Alternative Design S-N Curves, for In-Air Conditions, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (a) and (c) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (b) and (d) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.



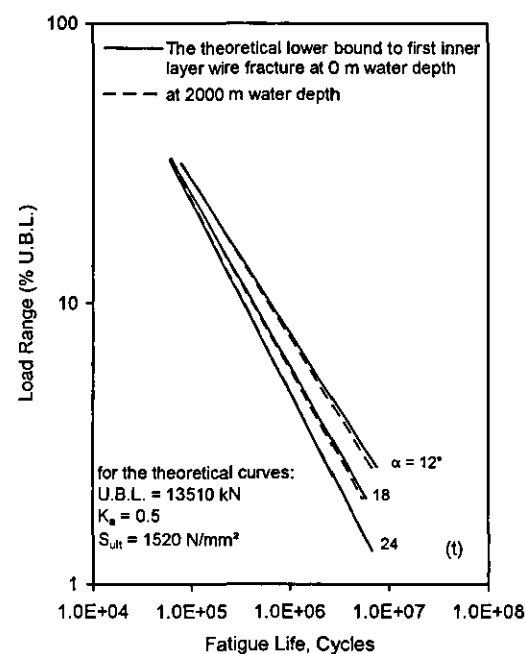
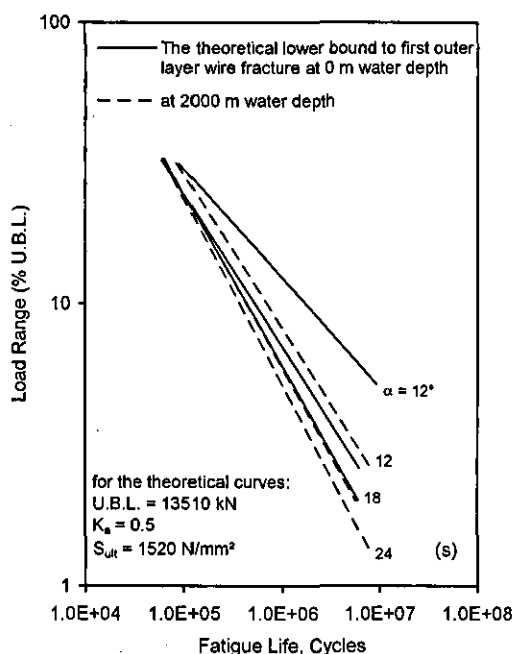
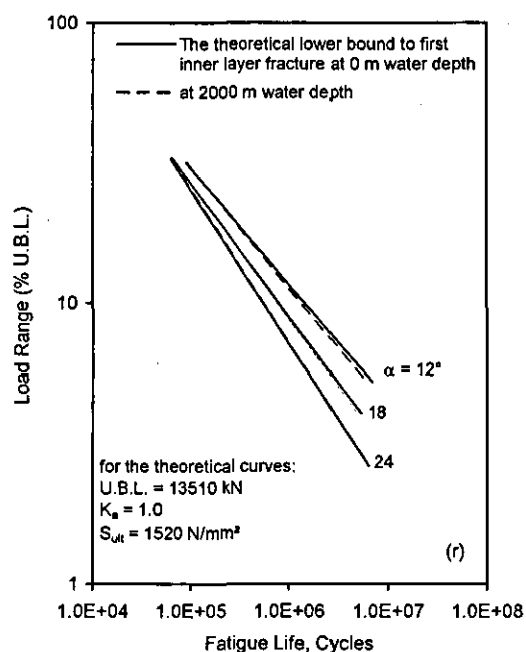
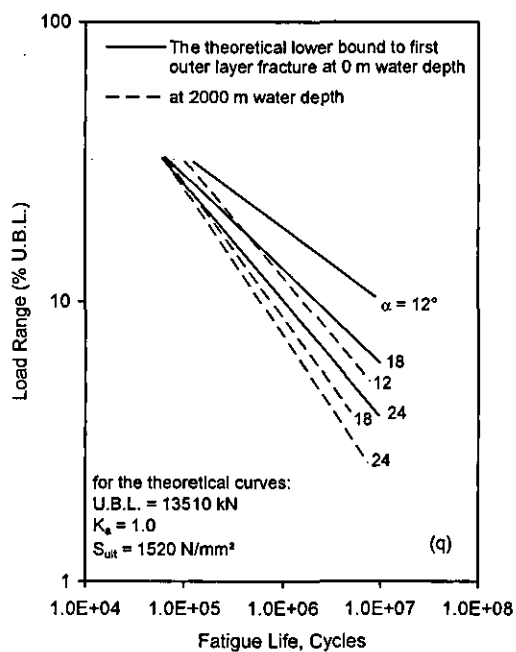
Figs. 8.12(e-h) – Comparison of Alternative Design S-N Curves, At 0 m and 500 m Water Depth, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (e) and (g) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (f) and (h) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.



Figs. 8.12(i-l) – Comparison of Alternative Design S-N Curves, At 0 m and 1000 m Water Depth, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (i) and (k) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (j) and (l) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.



Figs. 8.12(m-p) – Comparison of Alternative Design S-N Curves, At 0 m and 1500 m Water Depth, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (m) and (o) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (n) and (p) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.



Figs. 8.12(q-t) – Comparison of Alternative Design S-N Curves, At 0 m and 2000 m Water Depth, Based on Various Theoretical Criteria for Axial Fatigue Failure and Different Values of K_a : (q) and (s) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Outermost Wire Fracture; (r) and (t) $K_a = 1.0$ and 0.5 , Respectively, for the Fatigue Life to First Wire Fracture in the Innermost Layer.

8.4 DISCUSSION

As already discussed by Raoof (1998a), a spiral strand undergoing constant amplitude cyclic axial loading in air, does, indeed, possess an endurance limit. Figs. 8.1(a-d) -(q-t) show the magnitude of this theoretically determined endurance limit, not only for in-air conditions but also when the sheathed strands are subjected to external hydrostatic pressures, equivalent to water depths of between 500 m and 2000 m. The endurance limit is a 'cut off' point below which no fatigue damage is usually assumed to occur. However, in offshore platform applications, a spiral strand is exposed to the potentially detrimental effects of seawater in the form of corrosion, which is of particular concern at the points of end terminations within the water splash zone. For this reason, it was decided to assume a non existence of an endurance limit for the proposed design S-N curves. In other words, the straight line S-N curves (in log – log scale) for sheathed strands are assumed not to have any cut off point below which no fatigue damage occurs, and any level of axial stress range is assumed to cause some (although, perhaps, small) level of damage. This point may have significant practical implications in offshore platform or, indeed, bridging applications: in such cases, the small amplitude forces (from waves and/or wind) are the ones with the highest number of occurrences, and a knowledge of small amplitude/long life behaviour is of particular importance.

Figs. 8.2(a-e) and 8.3(a-e) present the individual fatigue data points for all the layers of the three different types of 127 mm outside diameter spiral strands, assuming a constant mean axial strain $S'_1 = 0.002867$, and K_a values of 1.0 and 0.5, respectively. Even for a given strand mean axial strain, the combined data for all the three different strand constructions exhibit a significant degree of scatter. Obviously, a significantly higher degree of scatter would be expected, if the magnitude of the strand mean axial strain was also varied within, say, the practical range $0.0005 \leq S'_1 \leq 0.004$. Due to the significant degree of scatter exhibited in Figs. 8.2(a-e) and Figs. 8.3(a-e), and in-line with the previous work of Raoof, it was decided to present the design S-N curves for each individual value of lay angle, as in Figs. 8.4(a-e), (f-j), Figs. 8.5(a-e), (f-j), Figs. 8.6(a-e), (f-j), Figs. 8.7(a-e), (f-j), Figs. 8.8(a-e), (f-j) and Figs. 8.9(a-e), (f-j). In this way, more sensible lower bound S-N curves were obtained, with the individual data

points, relating to each lower bound S-N curve, exhibiting reasonable degrees of scatter.

Figs. 8.12(a-d), (e-h), (i-l), (m-p) and (q-t) present all the presently proposed lower bound design S-N curves, based on extensive theoretical parametric studies, for all levels of hydrostatic pressure, and for both values of $K_a = 1.0$ and 0.5 . Considering that these plots are presented in log – log scale, the significant influence of the lay angle on the axial fatigue life of sheathed spiral strands in deep water (or, indeed, in-air) applications, is obvious.

As mentioned previously, the assumed values of the U. B. L. and grade of wire for producing the proposed theoretical lower bound S-N curves are 13510 kN and 1520 Nmm², respectively. However, because the axial load range in these plots is non-dimensionalized with respect to the U. B. L., all these theoretical lower bound S-N curves are of general applicability, irrespective of the magnitude of the U. B. L. and grade of wire. Also presented in Figs. 8.12(a-d) are the lower bound S-N curves (for in-air conditions) as recommended by API (1991), Chaplin (1993) and Tilly (1988). In producing these purely empirical S-N curves, none of these references differentiate between the various types of spiral strand (and/or rope) constructions. Moreover, different types of failure criteria were adopted by these researchers. Chaplin used the failure criteria as being the one to total collapse, while Tilly chose the number of cycles to 5 % wire failure (i.e. life to fatigue initiation), whereas the failure criteria adopted by API is not defined in the code. The lay angles of the spiral strands used for producing Tilly's design S-N curve were equal to 14°, 18° and 21°, while Chaplin's test strands had lay angles of 18°. The strand construction details used by API are not given in the publicly available literature.

The potentially unsafe nature of the previously reported lower bound S-N curves of API, Tilly and Chaplin for certain (smaller) levels of axial load range (depending upon the magnitude of the lay angle), and for using the 'in-air' S-N curves as a guide to the fatigue life estimation of sheathed spiral strands experiencing high levels of external hydrostatic pressure is particularly noteworthy. The API recommended S-N curve can be unconservative for certain practical cases. As regards Tilly's or Chaplin's

recommended S-N curves, the situation depends on the failure criteria adopted in practice, and the magnitude of the lay angles of the wires in the strands, which are to be used in a given structure. In this context, one should also decide as to whether fatigue failures are to be predicted for cases when they happen at, or in the vicinity of, the end terminations, or in the free-field (i.e. away from the detrimental effects of end terminations).

8.5 DESIGN EQUATIONS

All the lower bound design S-N curves developed in this chapter (as presented in Figs. 8.12(a-d), (e-h), (i-l), (m-p) and (q-t)) may be defined by the following simple equation

$$R = aN^b \quad (8.6)$$

where, R = the axial load range (expressed as a percentage of the ultimate breaking load (U. B. L.)), and N = the axial fatigue life in cycles, with a and b being constant parameters depending on the assumed value of K_a , water depth, and lay angle, α , plus the position of the first wire fracture within the sheathed spiral strand (i.e. as to whether it happens in the outermost or innermost layer). For a given spiral strand (irrespective of the imposed level of mean axial load), values of the parameters a and b may simply be obtained from Table 8.2, which cover external water depths ranging from 0 m to 2000 m, with the intermediate values to be obtained by interpolation.

Table 8.2 – Values of the Constant Parameters a and b for Sheathed Spiral Strands, as Defined in Equation (8.6).

Location of first wire fracture	K_a	Water depth (m)	α (degrees)					
			12		18		24	
			a	b	a	b	a	b
Outermost layer	1.0	0	644.24	-0.2577	1336.23	-0.3348	3098.44	-0.4143
		500	2999.88	-0.3953	3838.21	-0.4304	7471.70	-0.4941
		1000	3843.79	-0.4184	5131.40	-0.4569	12121.31	-0.5382
		1500	3967.71	-0.4205	6012.09	-0.4713	10253.98	-0.5226
		2000	3719.21	-0.4141	6584.26	-0.4796	10052.16	-0.5205
Outermost layer	0.5	0	2599.36	-0.3873	12574.54	-0.5420	26807.59	-0.6082
		500	11829.05	-0.5233	16716.14	-0.5648	32591.72	-0.6283
		1000	15546.55	-0.5483	22681.66	-0.5925	41212.97	-0.6495
		1500	15828.09	-0.5495	26807.59	-0.6082	45083.19	-0.6575
		2000	14633.85	-0.5422	29462.62	-0.6163	44108.60	-0.6554
Innermost layer	1.0	0	3352.52	-0.4099	5944.05	-0.4711	12137.70	-0.5387
		500	3787.16	-0.4206	6262.52	-0.4756	12137.70	-0.5387
		1000	3953.15	-0.4244	6440.13	-0.4781	12137.70	-0.5387
		1500	3974.23	-0.4249	6557.88	-0.4798	12137.70	-0.5387
		2000	3914.95	-0.4236	6622.12	-0.4807	12137.70	-0.5387
Innermost layer	0.5	0	14391.73	-0.5436	26807.59	-0.6082	53746.66	-0.6742
		500	16291.94	-0.5546	28198.53	-0.6127	53746.66	-0.6742
		1000	17020.68	-0.5585	29034.86	-0.6154	53746.66	-0.6742
		1500	17107.53	-0.5590	29590.15	-0.6171	53746.66	-0.6742
		2000	16848.00	-0.5576	29893.80	-0.6180	53746.66	-0.6742

8.6 SUMMARY AND CONCLUSIONS

Based on an extensive series of theoretical parametric studies, conducted on some very substantial (127 mm outside diameter) multi-layered spiral strands with realistic construction details, covering a wide range of lay angles $12^\circ \leq \alpha \leq 24^\circ$, a new set of design S-N curves for predicting the axial fatigue life of sheathed spiral strands to first outermost (or innermost) wire fracture, both at, or in the vicinity of, the end terminations, as well as in the free-field, experiencing a wide range of external hydrostatic pressures (equivalent to water depths of 0 m to 2000 m) have been developed.

As mentioned previously, the theoretical model of Raoof (presently used to produce the design S-N curves) has already been verified against a large number of carefully obtained large scale experimental data (under in-air conditions), using specimens with diameters ranging from 25 mm to 164 mm, with the tests carried out by a number of independent institutions, using test specimens from different cable manufacturers – hence, providing ample support for its general validity (at least for in-air conditions).

It has been shown that, at 0 m water depth (i.e. corresponding to in-air conditions), modest increases in the lay angle, within current manufacturing limits, can lead to practically significant reductions in the strand axial fatigue life. It has also been demonstrated that applying an external hydrostatic pressure to sheathed spiral strands can result in a practically significant reduction in their axial fatigue life, and that the presence of an external hydrostatic pressure only affects (cf. in-air conditions) the number of cycles to first outermost (and, not innermost) wire fracture. The application of an external hydrostatic pressure is, indeed, shown (theoretically) not to affect the fatigue life to first innermost wire fracture, over a wide range of sheathed spiral strand construction details and depths of external water.

The final numerical results have also demonstrated the important practical implications of taking the detrimental effects of the end terminations into account, with the fatigue life to outermost (or innermost) wire fractures being substantially lower than that for cases when fatigue failure (defined as the number of axial load

cycles to either outermost (or innermost) wire fractures) occurs in the free-field – i.e. away from the end termination.

Finally, the proposed S-N curves have been compared with others (for in-air conditions) recommended by API, Chaplin and Tilly, which are the ones most commonly referred to in the literature. It has been shown that, in certain cases, these purely empirical S-N curves, particularly the one presented by API, may provide unconservative results for practical applications. The implication by API that their design S-N curve, although originally produced at 0 m water depth (corresponding to in-air conditions), can be used as a guide to the fatigue behaviour of sheathed spiral strands experiencing substantial levels of external hydrostatic pressure, has been shown to be misleading. Unlike the presently proposed design S-N curves, all of the other ones (i.e. those proposed by API, Chaplin and Tilly) have been produced, based on purely empirical approaches, using test data relating to specimens which are unlikely to have covered the full range of first order design parameters (particularly the lay angle) – hence, the practical significance of the presently proposed design S-N curves which cover the full range of current manufacturing limits, as far as the most important geometrical parameter (i.e. the lay angle) is concerned.

CHAPTER 9.0

SIMPLE FORMULATIONS FOR THE DETERMINATION OF THE MAXIMUM AXIAL AND TORSIONAL HYSTERESIS

9.1 INTRODUCTION

The importance of understanding the damping capacity of light and flexible structures, such as long span highway bridges, is best illustrated by the spectacular collapse, on November 7, 1940 (one year after construction was completed) of the first Tacoma Narrows bridge, which spanned one mile over the Tacoma Narrows in Bremerton, Washington, U. S. A. The bridge was nicknamed the Galloping Gertie, because of its constant rocking and twisting in the wind. Even today, no one is absolutely sure as to what exactly caused the collapse of the bridge, but all of the theories put forward to date obviously involve the effects of aerodynamic forces.

These aerodynamic forces had previously not come into play, as the majority of earlier bridge designs involved deep trusses, which did not have the same problems, as the sheer weight and stiffness of the trusses was very effective in resisting most of the aerodynamic forces. The original designers of the Tacoma Narrows bridge did not have a good understanding of the possible dynamic forces acting on the bridge, and as a result of this failure, the need to understand the damping capacity of light and flexible structures has since then gained increased importance.

As mentioned by Wyatt (1977), the major sources of damping in conventional bridges have largely been eliminated in recent fully participating designs, which are characterised by a reduction, to undesirably low levels, in the logarithmic decrement. This did not happen in earlier designs as riveted or black bolted joints, and other potential sources of friction, particularly in the deck system, provided considerable frictional hysteresis. In modern bridges, the hysteresis, provided by the deck system in earlier designs, has been greatly reduced, due to the bridge deck being integrated with the main load bearing members and the extensive use of welding. As a result of this, the significant hysteresis in spiral strands used in cable-stayed bridges, and as hangers in suspension bridges, can form a much larger part of the total damping in recent structures.

The Severn bridge used the alternative inclined hanger system, developed to exploit the damping capacity of cables, where the truss action of the inclined hangers was intended to give more effective damping (due to larger force variations in the hangers) than would have been obtained from using the conventional vertical hangers, Raoof (1983).

By and large, the experimental and/or theoretical works on helical cables published in the public domain, mostly relate to small diameter (e.g. seven wire) spiral strands whose behaviour does not necessarily coincide with that of much larger diameter cables used in practice, and the information relating to the damping capacity of spiral strands is no exception.

The torsional hysteresis of spiral strands and electrical conductors is also of particular importance in the analysis of certain aero- or hydro-dynamic problems. Galloping, which is normally associated with some form of asymmetry in the strand cross-section, can be associated with torsional effects. By far the most common cause of galloping, in the case of overhead transmission lines, is the non-uniform accumulation of ice over the cable's cross-section.

As explained by Raoof and Hobbs (1989), when the wind is at 90° to the conductor, vertical galloping may occur for a horizontal airflow. In other cases, instability can be associated with the coupling of two or three degrees of freedom (two translations and the torsional rotation) which may, individually be quite stable. Owing to the complex movements of the section, the angle of attack changes continuously, so that the body experiences aerodynamic lift forces which tend to amplify the movement, more or less without interruption. For significant amplitudes to develop, the energy input to the system must exceed that dissipated by structural damping: in such cases, torsional hysteresis can be an important factor.

Hobbs and Raoof (1984) focused on the axial loading problem, with particular reference to axial damping and stiffness. In another publication, Raoof and Hobbs (1989) developed a method for the torsional analysis of spiral strands, which followed a similar route to that originally established for the axial case.

The purpose of the present chapter is to present simple methods for the determination of the maximum axial and torsional frictional hysteresis and the corresponding axial load range/mean axial load ratio, and range of twist/2, respectively, at which they occur. Simple routines will be developed covering a wide range of cable mean axial strains $0.0006 \leq S'_1 \leq 0.00430$, and external hydrostatic pressures (in the case of sheathed spiral strands in deep water applications) ranging from equivalent water depths of 0 m to 2000 m.

9.2 THEORY

The orthotropic sheet theoretical model, and the methods for obtaining both the axial and torsional frictional hysteresis, have already been covered in some detail in chapter 3, and will not be repeated here. Moreover, the theory relating to the effect that an externally applied hydrostatic pressure has on various structural characteristics of a sheathed spiral strand has been covered in chapter 8.

The following extensive theoretical parametric studies were carried out on a number of spiral strand constructions, the details of which have already been given in chapter 3 (Tables 3.1a – l).

9.3 RESULTS

9.3.1 Axial Hysteresis

Figs. 9.1(a-d) show the variations, for all the spiral strand constructions used in this study, of the maximum axial hysteresis, $(\Delta U/U)_{\max}$, with changes in the $E_{\text{no-slip}}/E_{\text{full-slip}}$ ratio, at zero water depth, and at various levels of cable mean axial strains: (a) $S'_1 = 0.0006$, (b) $S'_1 = 0.00145$, (c) $S'_1 = 0.002867$, and (d) $S'_1 = 0.00430$, respectively, as calculated using methods (a) and (b). Third order polynomials were fitted through the theoretical data points - one equation for method (a), and one for method (b), for each mean axial strain, where

$$\left(\frac{\Delta U}{U} \right)_{\max} = A + B(V) + C(V^2) \quad (9.1)$$

with

$$V = \frac{E_{\text{no-slip}}}{E_{\text{full-slip}}} \quad (9.2)$$

Figs. 9.1(e-h), 9.1(i-l), 9.1(m-p) and 9.1(q-t) show similar plots, but for water depths of 500 m, 1000 m, 1500 m and 2000 m, respectively. Table 9.1a gives the values of the coefficients A-C in Equation (9.1) for all the assumed strand mean axial strains, S'_1 , and for a wide range of water depths, along with the correlation coefficients, R, based on method (a), whilst Table 9.1b shows the same coefficients based on method (b).

The variations of the corresponding $\frac{\text{axial load range}}{\text{mean axial load}}$ ratios, at which the maximum frictional axial hysteresis occurs, with the corresponding changes in the $\frac{E_{\text{no-slip}}}{E_{\text{full-slip}}}$ ratios, at 0 m water depth, are shown in Figs. 9.2(a-d) for a wide range of strand mean axial strains: (a) $S'_1 = 0.0006$, (b) $S'_1 = 0.00145$, (c) $S'_1 = 0.002867$, and (d) $S'_1 = 0.00430$, respectively. Figs. 9.2(e-h), 9.2(i-l), 9.2(m-p) and 9.2(q-t) show similar plots, but at water depths of 500 m, 1000 m, 1500 m and 2000 m, respectively. Second order polynomials were fitted through the data, where

$$\zeta = A(\gamma) + B(\gamma)^2 \quad (9.3)$$

with

$$\zeta = \left(\frac{D_1 r_1 \times \frac{\text{Load Range}}{\text{Mean Load}}}{\mu} \right) \quad (9.4)$$

and

$$\gamma = D_1 r_1 \times \left(\frac{E_{\text{no-slip}}}{E_{\text{full-slip}}} \right) \quad (9.5)$$

In the above, D_1 is the wire diameter in the outer layer whose helix radius is r_1 , and μ is the coefficient of interwire friction.

Table 9.2 gives the values of the coefficients A and B in Equation (9.3) for a wide range of strand mean axial strains, S'_1 , and water depths, along with the correlation coefficients, R. It should be noted that as far as the critical values of the axial load range / mean axial load ratios, associated with the maximum axial hysteresis, are concerned, the methods (a) and (b) give very similar results (within a few percentage points), with the data in Table 9.2 and Figs. 9.2(a-d) – (q-t), based on method (a).

Table 9.1a – Values of the Coefficients, A-C, in Equation (9.1), for a Wide Range of Strand Mean Axial Strains, S'_1 , and Water Depths, Along with the Correlation Coefficients, R, as Calculated Using Method (a).

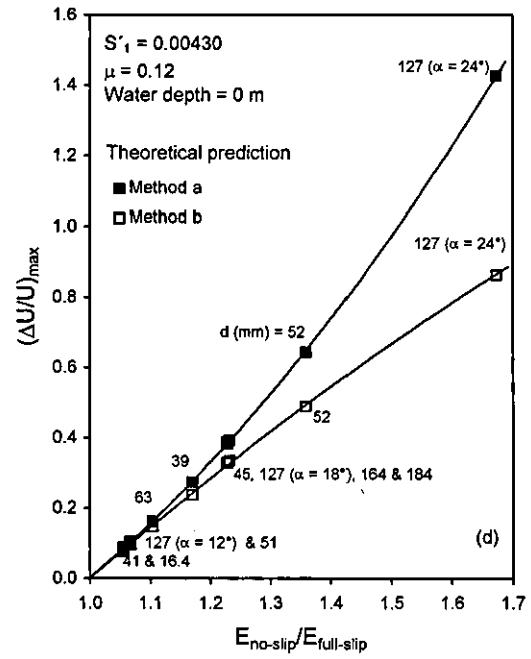
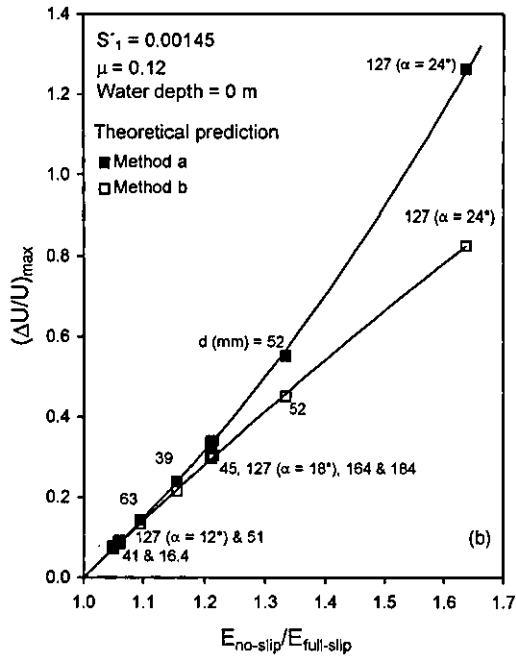
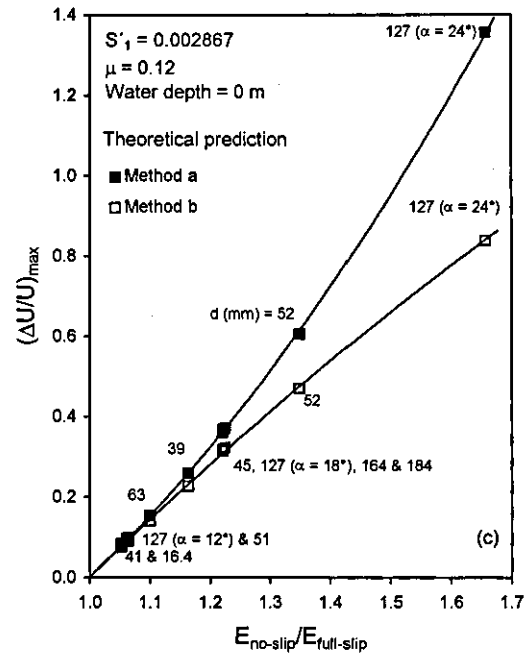
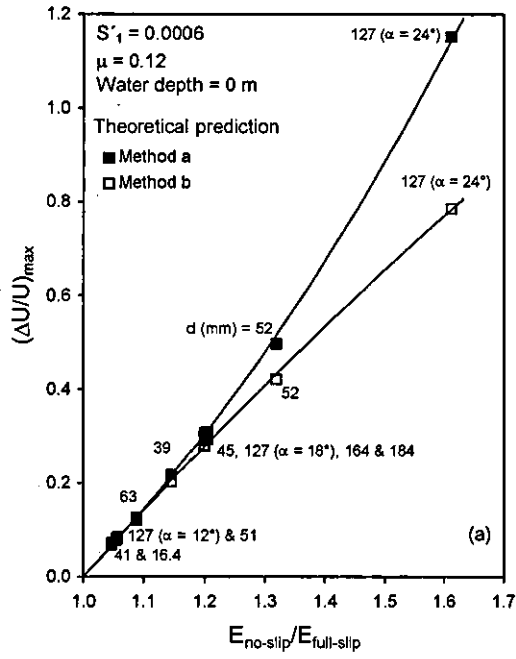
Water Depth (m)	S'_1	A	B	C	R
0	0.0006	-0.470	-0.3947	0.8659	0.9997
0	0.00145	-0.480	-0.4282	0.9091	0.9998
0	0.002867	-0.485	-0.4604	0.9465	0.9999
0	0.00430	-0.450	-0.5397	0.9925	0.9999
500	0.0006	-0.300	1.0068	0.2948	0.9982
500	0.00145	-1.050	0.4503	0.6005	0.9997
500	0.002867	-0.900	0.1513	0.7498	0.9999
500	0.00430	-0.830	0.0210	0.8103	0.9999
1000	0.0006	-1.530	1.5400	-0.0099	0.9939
1000	0.00145	-1.310	1.0107	0.2998	0.9974
1000	0.002867	-1.130	0.5846	0.5458	0.9973
1000	0.00430	-1.020	0.3564	0.6649	0.9998
1500	0.0006	-1.510	1.5644	-0.0570	0.9915
1500	0.00145	-1.400	1.2484	0.1504	0.9956
1500	0.002867	-1.310	0.9924	0.3188	0.9973
1500	0.00430	-1.180	0.6982	0.4819	0.9982
2000	0.0006	-1.320	1.2922	0.0286	0.9918
2000	0.00145	-1.220	1.0006	0.2208	0.9958
2000	0.002867	-1.335	1.1078	0.2285	0.9964
2000	0.00430	-1.360	1.1055	0.2561	0.9966

Table 9.1b – Values of the Coefficients, A-C, in Equation (9.1), for a Wide Range of Strand Mean Axial Strains, S'_1 , and Water Depths, Along with the Correlation Coefficients, R, as Calculated Using Method (b).

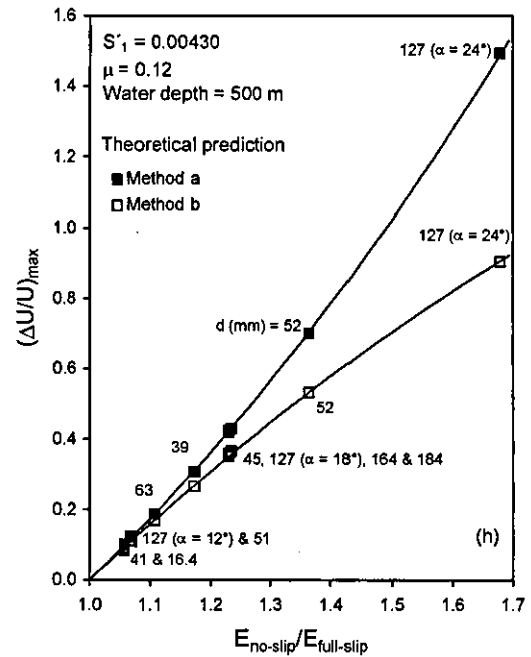
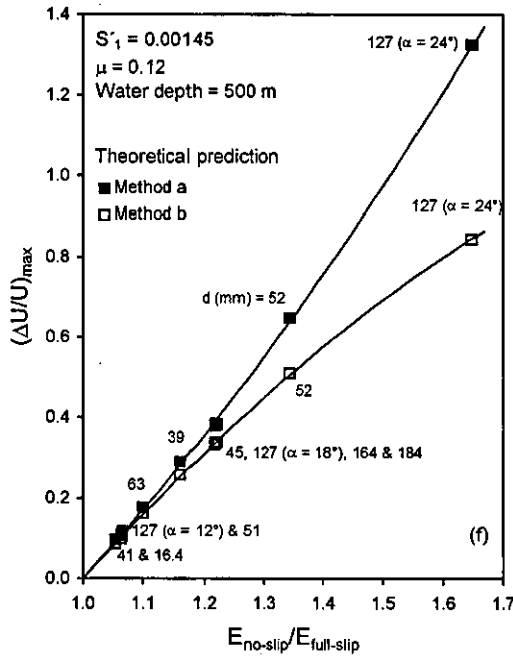
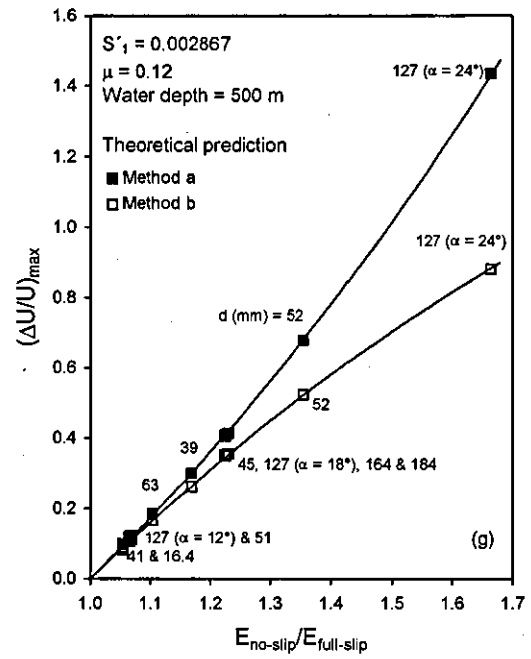
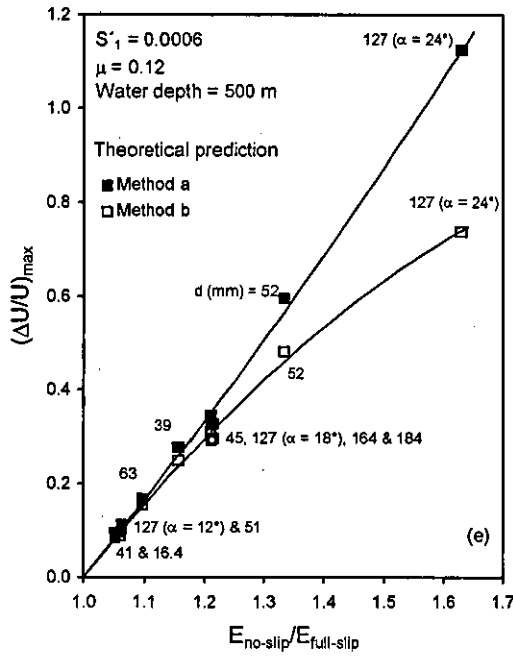
Water Depth (m)	S'_1	A	B	C	R
0	0.0006	-1.700	1.9598	-0.2598	0.9996
0	0.00145	-1.700	1.9466	-0.2471	0.9998
0	0.002867	-1.700	1.9609	-0.2579	0.9996
0	0.00430	-1.750	2.0292	-0.2783	0.9997
500	0.0006	-2.300	2.9910	-0.6907	0.9982
500	0.00145	-2.200	2.7418	-0.5421	0.9999
500	0.002867	-2.100	2.5659	-0.4649	0.9998
500	0.00430	-2.000	2.4005	-0.3983	0.9998
1000	0.0006	-2.250	3.0288	-0.7781	0.9922
1000	0.00145	-2.250	2.9205	-0.6708	0.9968
1000	0.002867	-2.200	2.7643	-0.5645	0.9994
1000	0.00430	-2.100	2.5850	-0.4824	0.9998
1500	0.0006	-2.080	2.7947	-0.7144	0.9891
1500	0.00145	-2.200	2.9055	-0.7064	0.9946
1500	0.002867	-2.200	2.8464	-0.6446	0.9968
1500	0.00430	-2.150	2.7202	-0.5689	0.9976
2000	0.0006	-1.950	2.6028	-0.6522	0.9872
2000	0.00145	-2.000	2.6062	-0.6060	0.9949
2000	0.002867	-2.150	2.8161	-0.6647	0.9957
2000	0.00430	-2.250	2.9532	-0.7031	0.9956

Table 9.2 – Values of the Coefficients, A and B, in Equation (9.3), for a Wide Range of Strand Mean Axial Strains, S'_1 , and Water Depths, Along with the Correlation Coefficients, R, Based on Method (a).

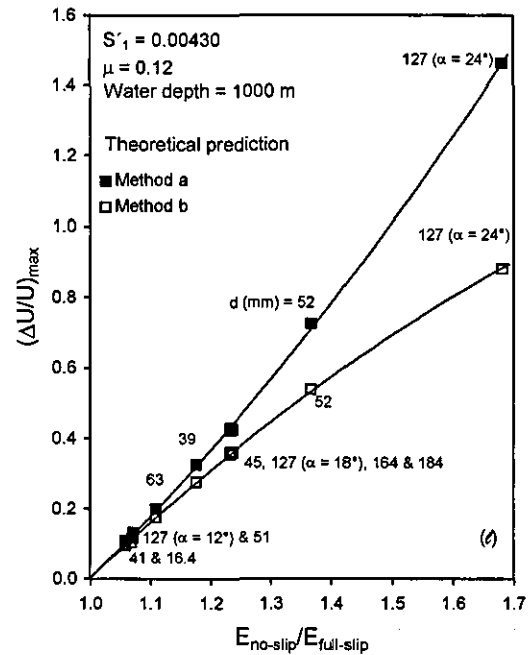
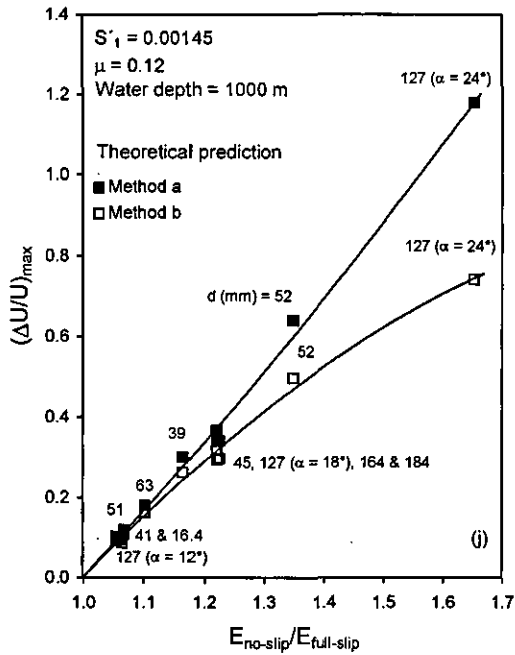
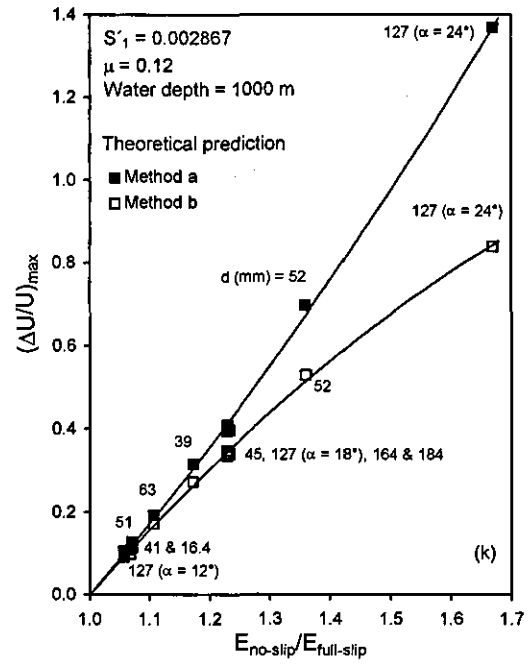
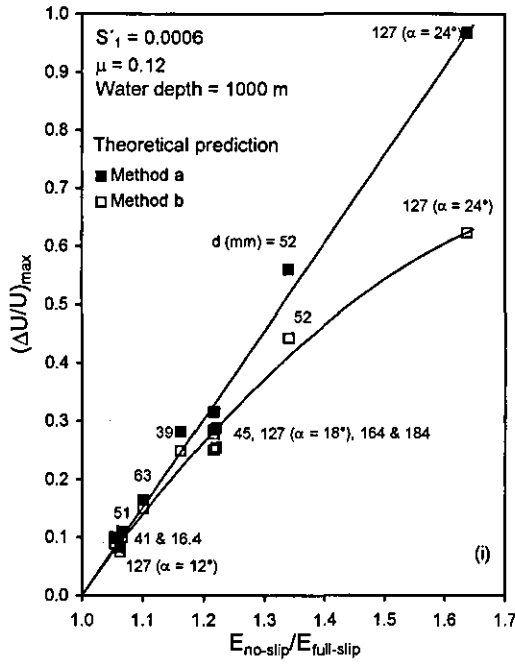
Water Depth (m)	S'_1	A	B	R
0	0.0006	0.9515	0.0007	0.9968
0	0.00145	0.9351	0.0006	0.9942
0	0.002867	0.8548	0.0006	0.9946
0	0.00430	0.8472	0.0006	0.9945
500	0.0006	3.1476	0.0007	0.9871
500	0.00145	1.7534	0.0004	0.9950
500	0.002867	1.2581	0.0005	0.9962
500	0.00430	1.1402	0.0004	0.9961
1000	0.0006	5.4386	-0.0002	0.9899
1000	0.00145	2.7977	0.0001	0.9944
1000	0.002867	1.9965	6.0×10^{-5}	0.9897
1000	0.00430	1.5056	0.0003	0.9949
1500	0.0006	6.4619	-0.0015	0.9419
1500	0.00145	3.0749	-0.0005	0.9669
1500	0.002867	2.0626	0.0002	0.9906
1500	0.00430	1.6515	0.0002	0.9957
2000	0.0006	8.0495	-0.0042	0.8982
2000	0.00145	3.7631	-0.0017	0.9335
2000	0.002867	2.3908	-0.0009	0.9651
2000	0.00430	2.1264	-0.0007	0.9641



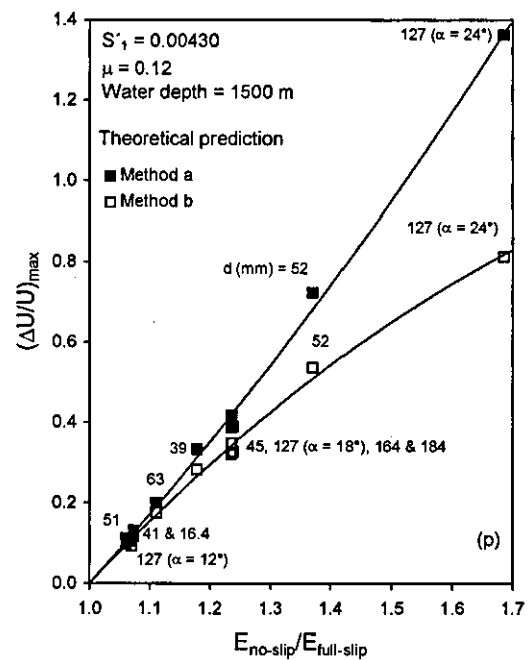
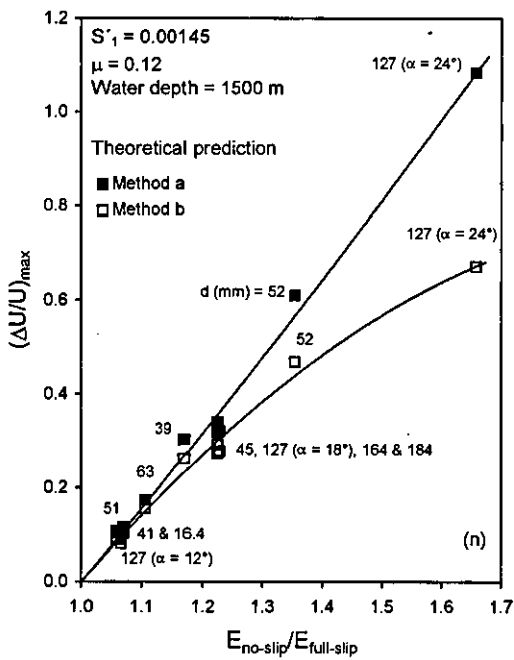
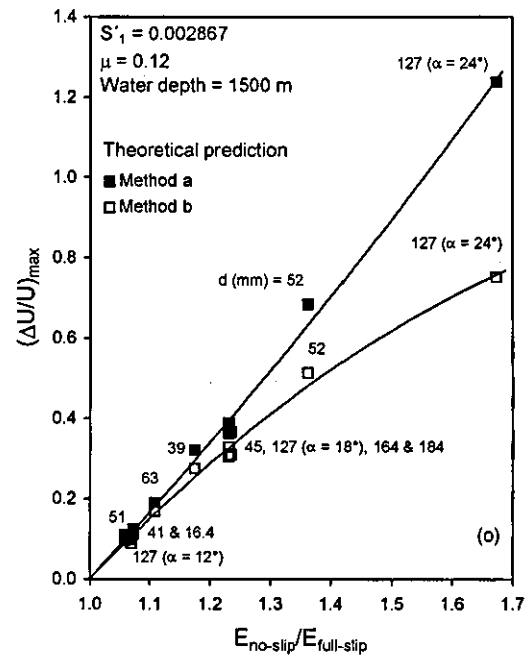
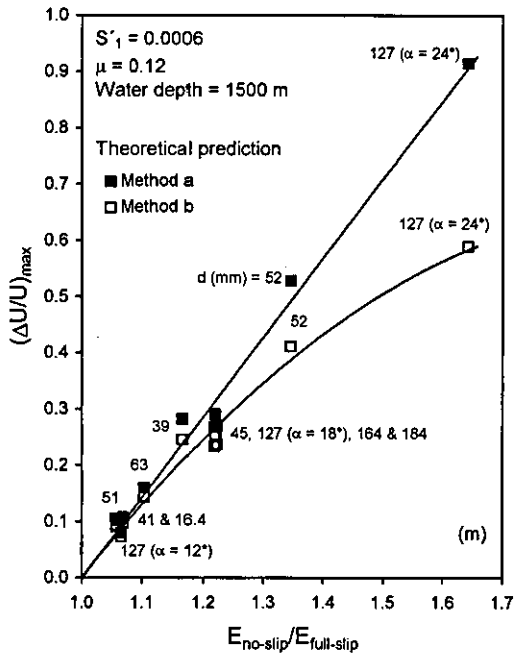
Figs. 9.1(a-d) - Variations of the Maximum Axial Hysteresis, $(\Delta U/U)_{\max}$ with Changes in the $E_{\text{no-slip}}/E_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (a) $S'_1 = 0.0006$, (b) $S'_1 = 0.00145$, (c) $S'_1 = 0.002867$, and (d) $S'_1 = 0.00430$, Respectively - at 0 m Water Depth, as Calculated Using Methods (a) and (b).



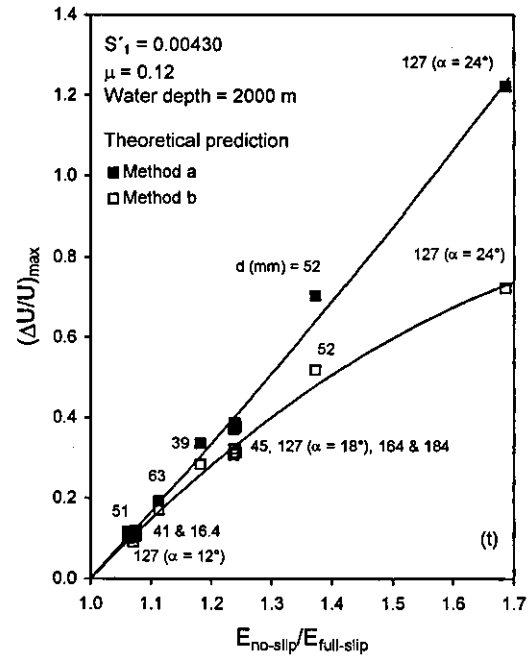
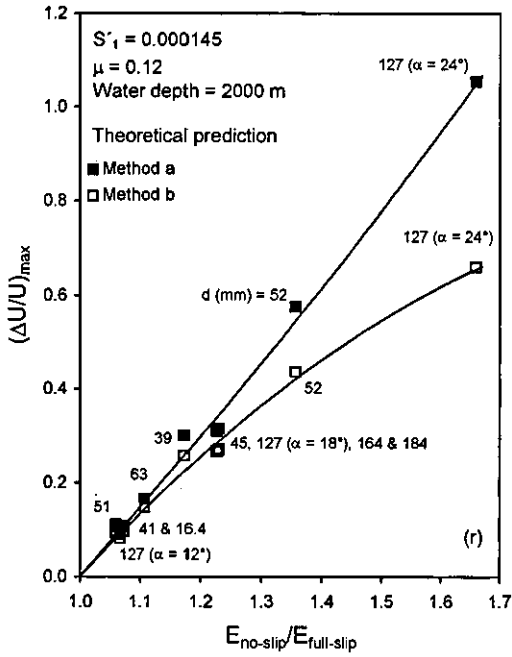
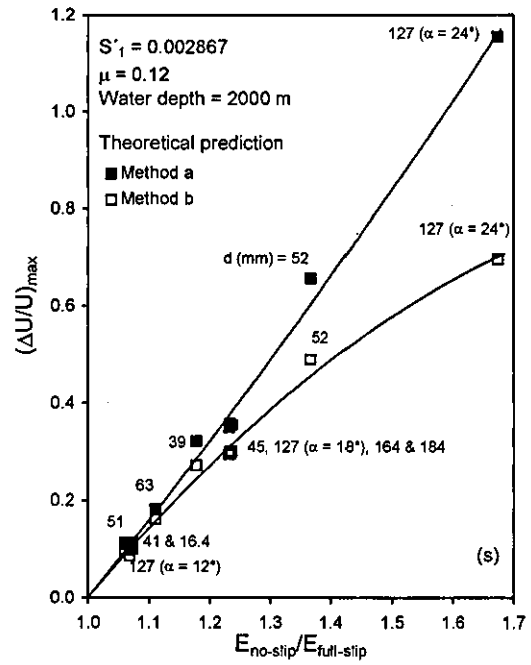
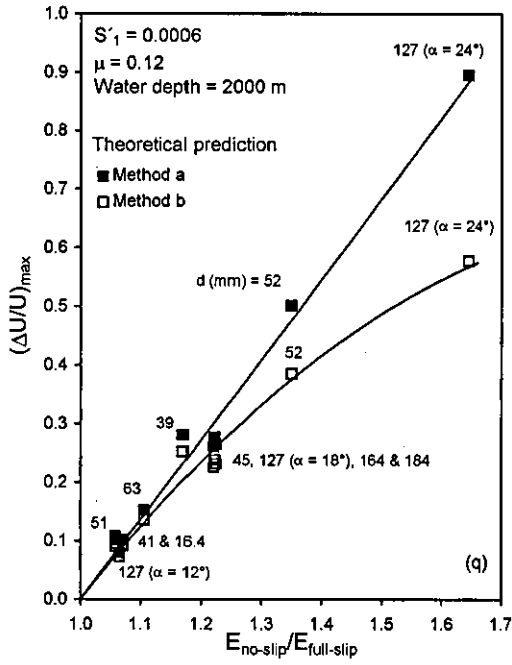
Figs. 9.1(e-h) - Variations of the Maximum Axial Hysteresis, $(\Delta U/U)_{\max}$ with Changes in the $E_{\text{no-slip}}/E_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (e) $S'_1 = 0.0006$, (f) $S'_1 = 0.00145$, (g) $S'_1 = 0.002867$, and (h) $S'_1 = 0.00430$, Respectively - at 500 m Water Depth, as Calculated Using Methods (a) and (b).



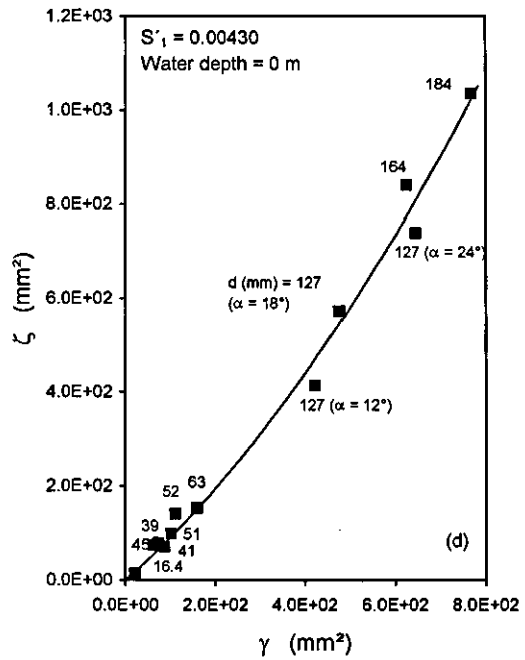
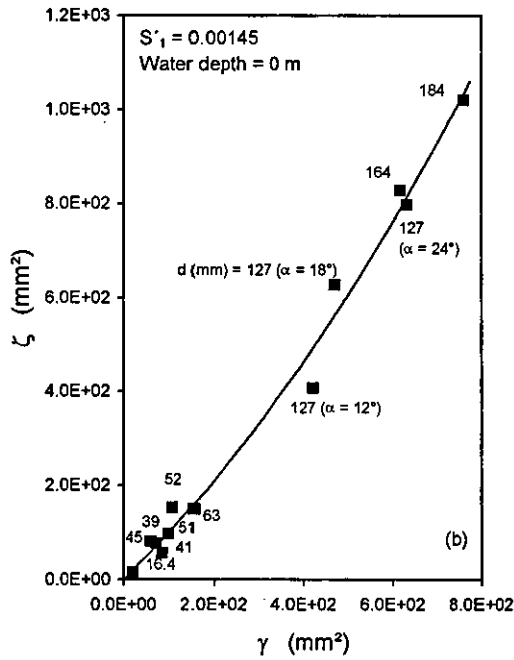
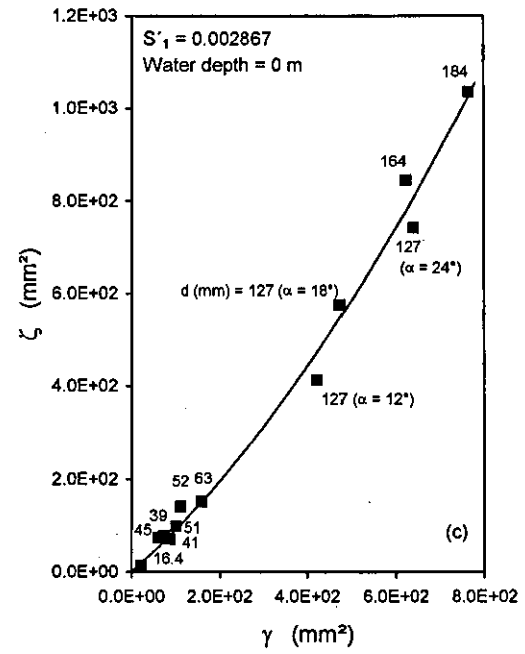
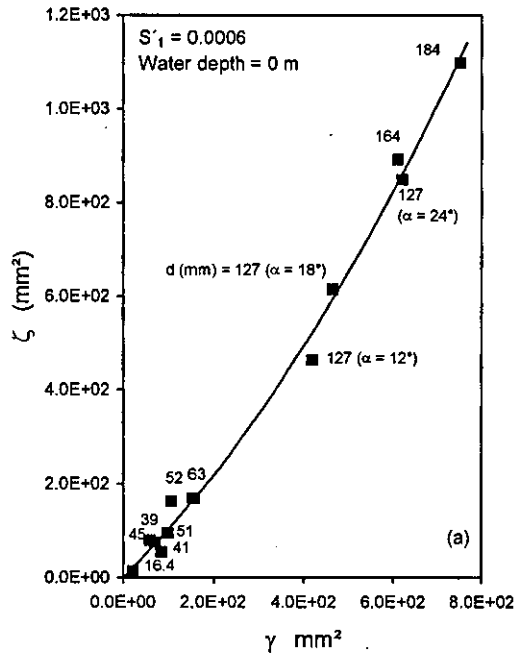
Figs. 9.1(i-l) - Variations of the Maximum Axial Hysteresis, $(\Delta U/U)_{\max}$ with Changes in the $E_{\text{no-slip}}/E_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (i) $S'_1 = 0.0006$, (j) $S'_1 = 0.00145$, (k) $S'_1 = 0.002867$, and (l) $S'_1 = 0.00430$, Respectively - at 1000 m Water Depth, as Calculated Using Methods (a) and (b).



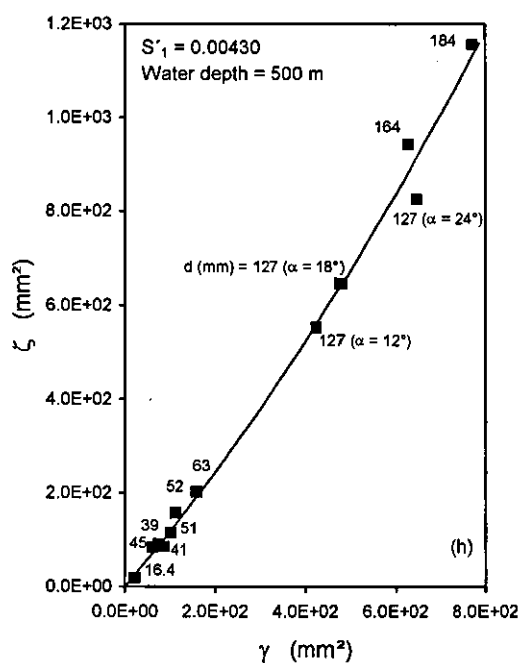
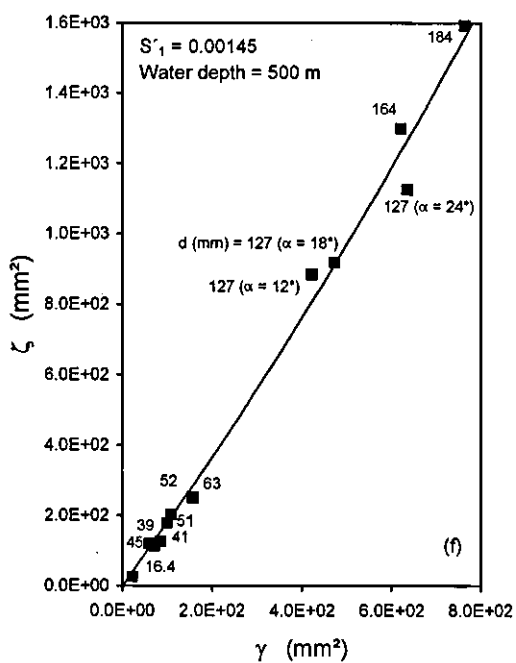
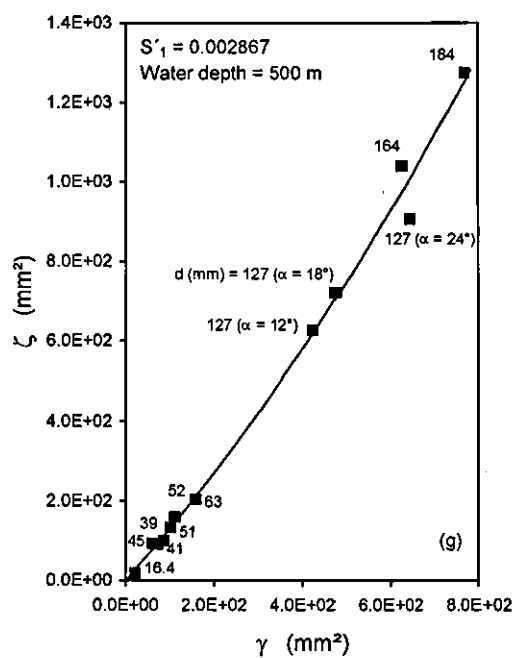
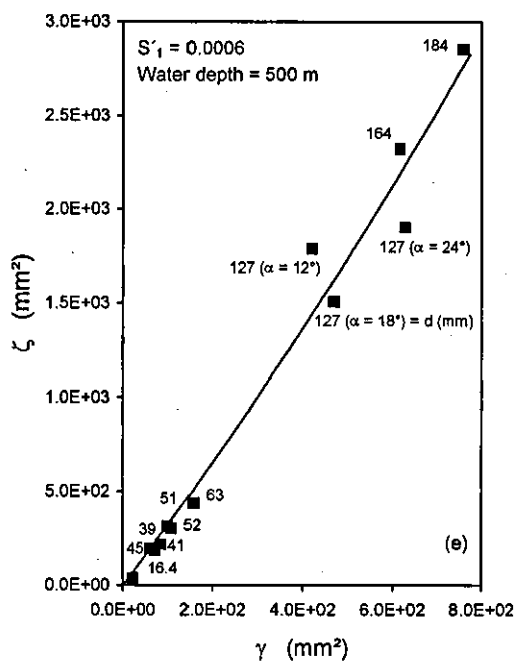
Figs. 9.1(m-p) - Variations of the Maximum Axial Hysteresis, $(\Delta U/U)_{\max}$ with Changes in the $E_{\text{no-slip}}/E_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (m) $S'_1 = 0.0006$, (n) $S'_1 = 0.00145$, (o) $S'_1 = 0.002867$, and (p) $S'_1 = 0.00430$, Respectively - at 1500 m Water Depth, as Calculated Using Methods (a) and (b).



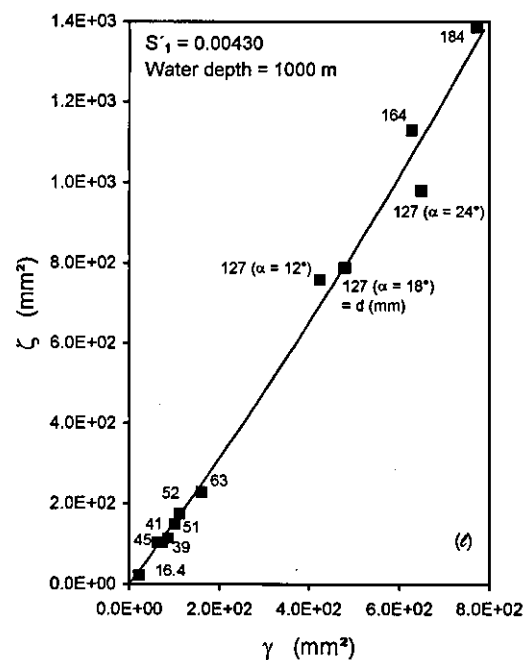
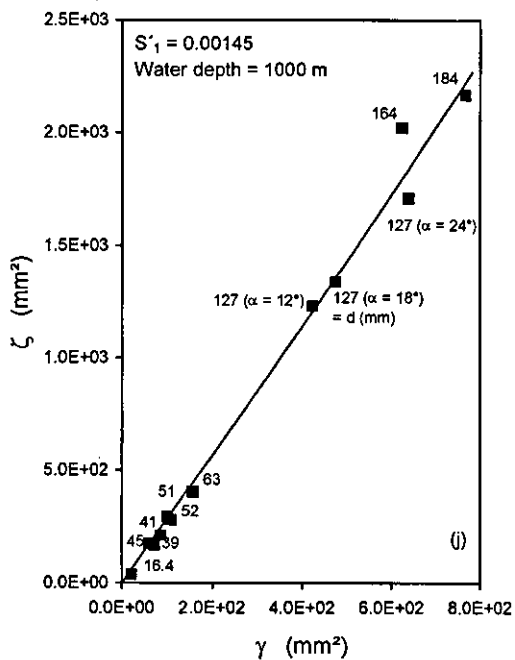
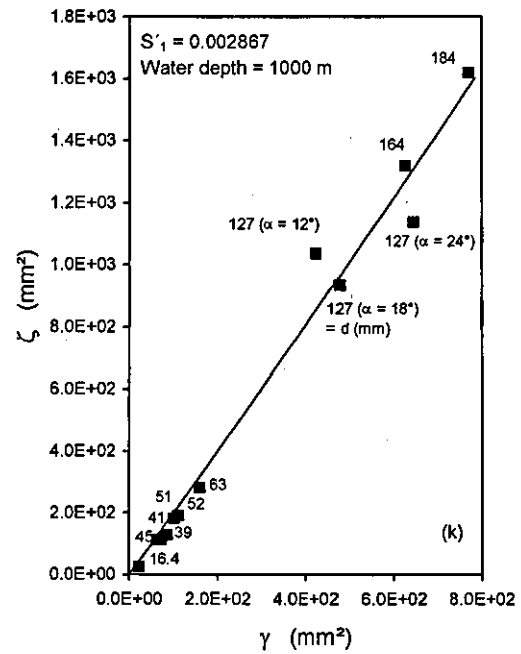
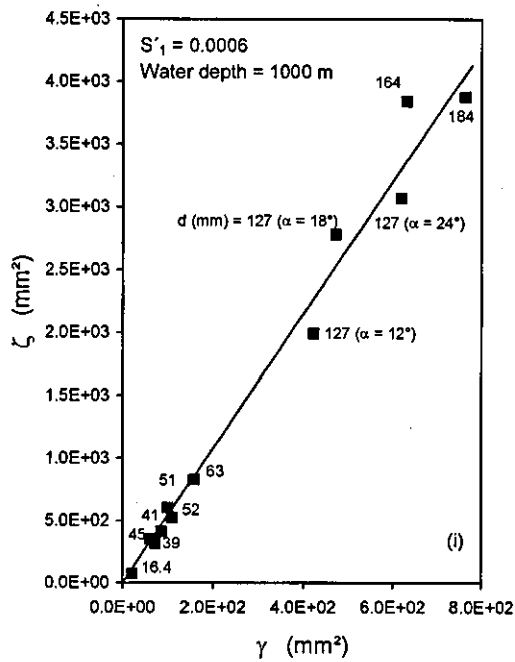
Figs. 9.1(q-t) - Variations of the Maximum Axial Hysteresis, $(\Delta U/U)_{\max}$ with Changes in the $E_{\text{no-slip}}/E_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (q) $S'_1 = 0.0006$, (r) $S'_1 = 0.00145$, (s) $S'_1 = 0.002867$, and (t) $S'_1 = 0.00430$, Respectively - at 2000 m Water Depth, as Calculated Using Methods (a) and (b).



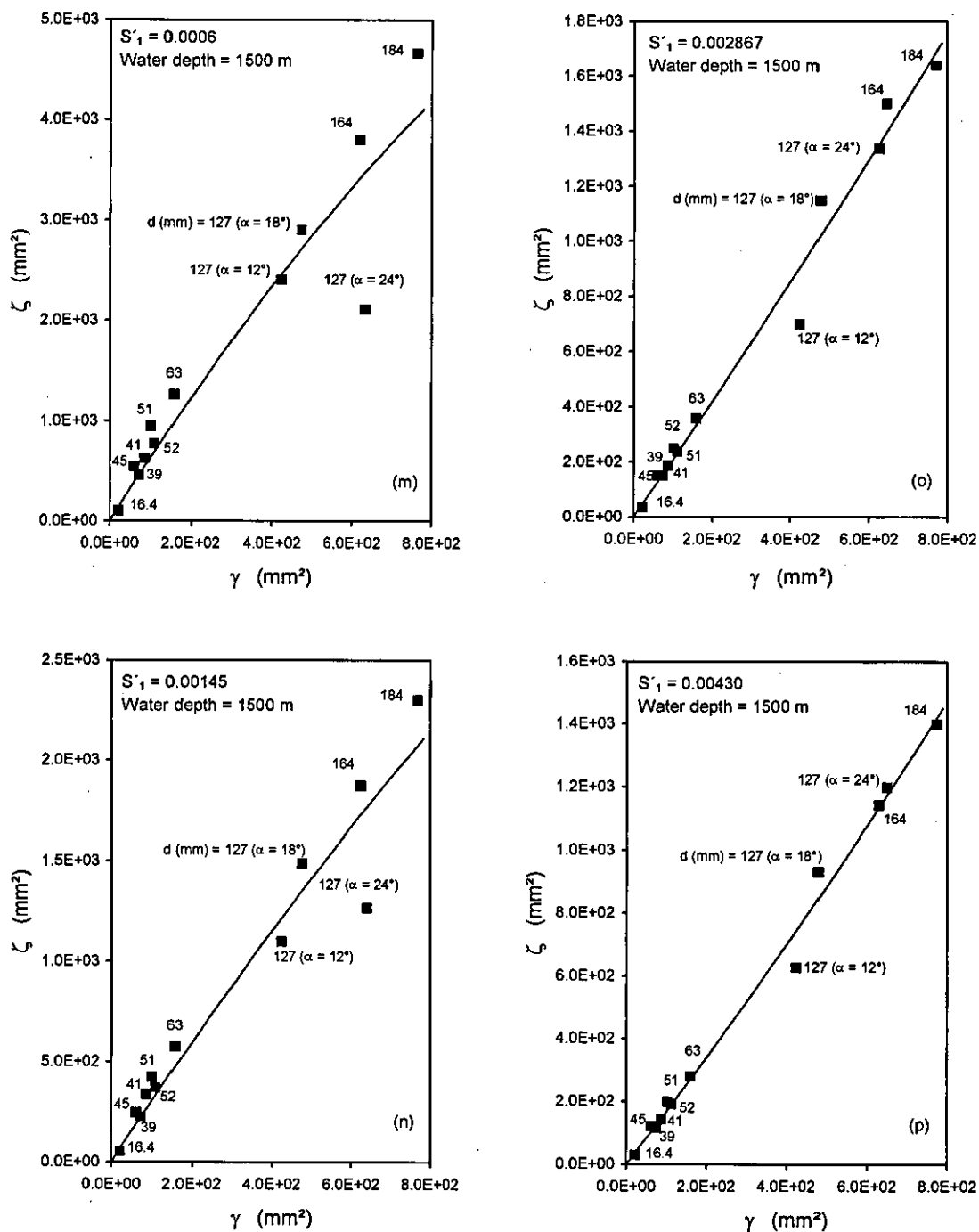
Figs. 9.2(a-d) - Variations of ζ with changes in γ , at 0 m Water Depth, for a Wide Range of Cable Mean Axial Strains: (a) $S'_1 = 0.0006$, (b) $S'_1 = 0.00145$, (c) $S'_1 = 0.002867$, and (d) $S'_1 = 0.00430$.



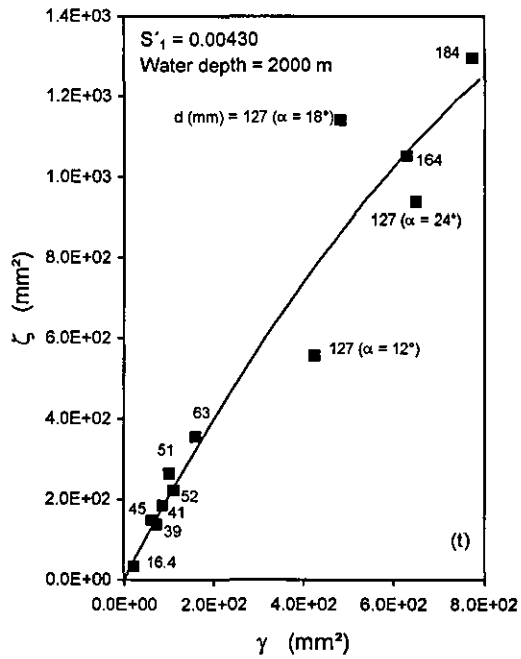
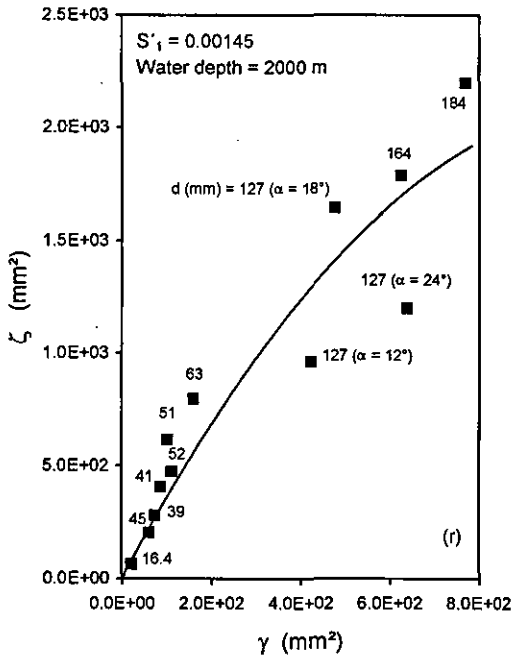
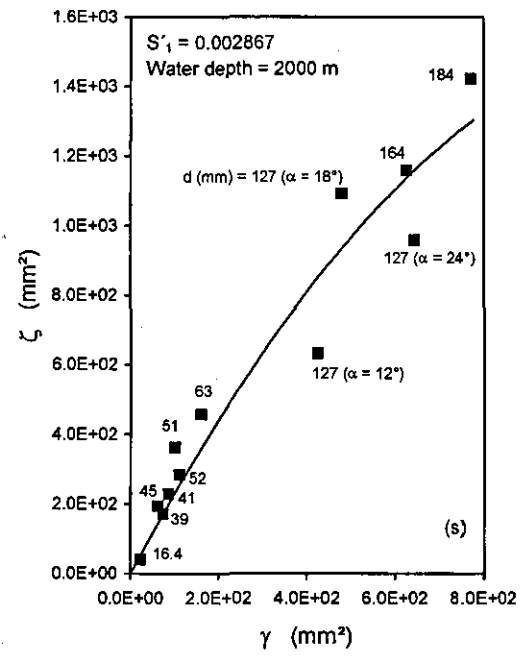
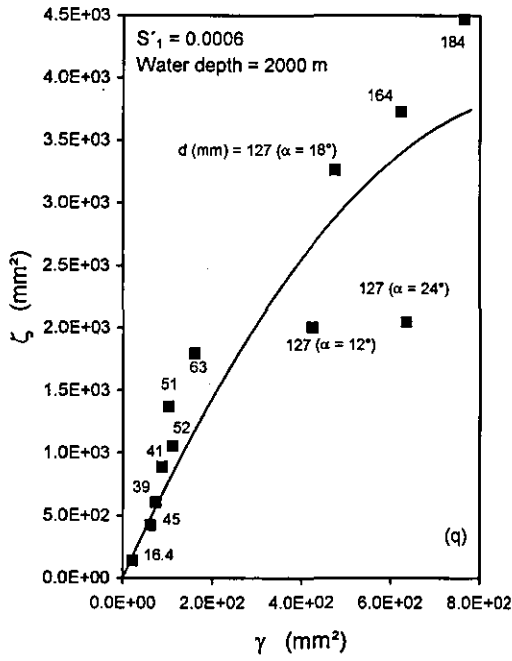
Figs. 9.2(e-h) - Variations of ζ with changes in γ , at 500 m Water Depth, for a Wide Range of Cable Mean Axial Strains: (e) $S'_1 = 0.0006$, (f) $S'_1 = 0.00145$, (g) $S'_1 = 0.002867$, and (h) $S'_1 = 0.00430$.



Figs. 9.2(i-l) - Variations of ζ with changes in γ , at 1000 m Water Depth, for a Wide Range of Cable Mean Axial Strains: (i) $S'_1 = 0.0006$, (j) $S'_1 = 0.00145$, (k) $S'_1 = 0.002867$, and (l) $S'_1 = 0.00430$.



Figs. 9.2(m-p) - Variations of ζ with changes in γ , at 1500 m Water Depth, for a Wide Range of Cable Mean Axial Strains: (m) $S'_1 = 0.0006$, (n) $S'_1 = 0.00145$, (o) $S'_1 = 0.002867$, and (p) $S'_1 = 0.00430$.



Figs. 9.2(q-t) - Variations of ζ with changes in γ , at 2000 m Water Depth, for a Wide Range of Cable Mean Axial Strains: (q) $S'_1 = 0.0006$, (r) $S'_1 = 0.00145$, (s) $S'_1 = 0.002867$, and (t) $S'_1 = 0.00430$.

9.3.2 Torsional Hysteresis

Figs. 9.3(a-d) show the variations of the maximum torsional hysteresis, $(\Delta U/2U)_{\max}$, with changes in the $(d_4)_{\text{no-slip}}/(d_4)_{\text{full-slip}}$ ratio, at 0 m water depth, for a large number of spiral strand constructions, and a wide range of strand mean axial strains: (a) $S'_1 = 0.0006$, (b) $S'_1 = 0.00145$, (c) $S'_1 = 0.002867$, and (d) $S'_1 = 0.00430$, respectively. Figs. 9.3(e-h), 9.3(i-l), 9.3(m-p) and 9.3(q-t) present similar plots, but for water depths of 500 m, 1000 m, 1500 m and 2000 m, respectively. Once again, third order polynomials may be used to describe the relationships between $(\Delta U/2U)_{\max}$, and $(d_4)_{\text{no-slip}}/(d_4)_{\text{full-slip}}$, which are of the general form

$$\left(\frac{\Delta U}{2U}\right)_{\max} = A + B(T) + C(T)^2 \quad (9.6)$$

where

$$T = \frac{(d_4)_{\text{no-slip}}}{(d_4)_{\text{full-slip}}} \quad (9.7)$$

with the torsional stiffness coefficient, d_4 , defined as $d_4 = \frac{T'_6}{S'_6}$, where $S'_6 = r_1 \frac{d\phi}{dl}$ with r_1 = the helix radius of the outer layer, and T'_6 = the shear stress in the equivalent orthotropic sheet relating to the outer layer (i.e. layer 1), based on the net steel area.

Table 9.3 gives the values of the coefficients A-C in Equation (9.6) for a wide variety of strand mean axial strains, S'_1 , and water depths, with the correlation coefficients, R, also included in the Table.

Figs. 9.4(a-d) show the variations of $\frac{\text{range of twist}}{2}$ (expressed as a function of certain other parameters as defined in Equation (9.9)) with corresponding changes in $(\Delta U/2U)_{\max}$ (also expressed as a function of certain other parameters as defined in Equation (9.10)), at 0 m water depth, and at various levels of strand mean axial strains: (a) $S'_1 = 0.0006$, (b) $S'_1 = 0.00145$, (c) $S'_1 = 0.002867$ and (d) $S'_1 = 0.00430$, respectively, for a wide range of spiral strand constructions.

Figs. 9.4(e-h), 9.4(i-l), 9.4(m-p) and 9.4(q-t) show similar plots corresponding to water depths of 500 m, 1000 m, 1500 m and 2000 m, respectively. The equation describing the fitted curves through the theoretical data, is of the general form

$$\kappa = A\lambda^B \quad (9.8)$$

where

$$\kappa = \left(\frac{\text{range of twist}}{2} \right) D_1 r_1 \quad (9.9)$$

and

$$\lambda = \frac{\left(\frac{D_1 r_1 (d_4)_{\text{full-slip}}}{J_{\text{strand}}} \right)}{\left(\frac{\Delta U}{2U} \right)_{\text{max}}} \quad (9.10)$$

with

$$J_{\text{strand}} = \frac{\pi}{4} \left(\frac{\pi d^4}{32} \right) \quad (9.11)$$

where, d is the strand outside diameter.

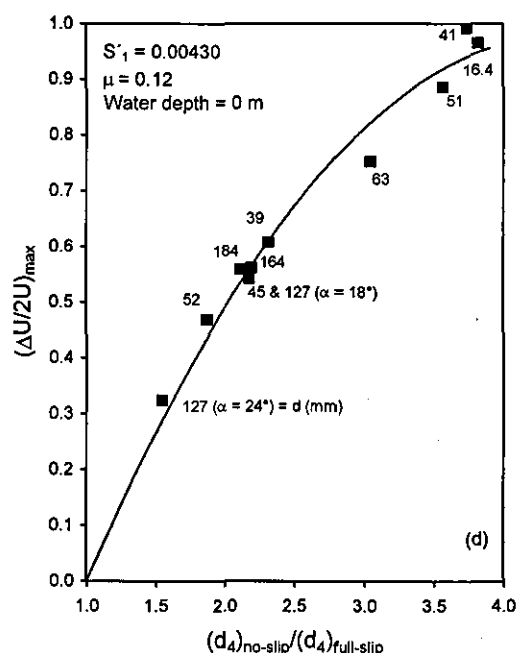
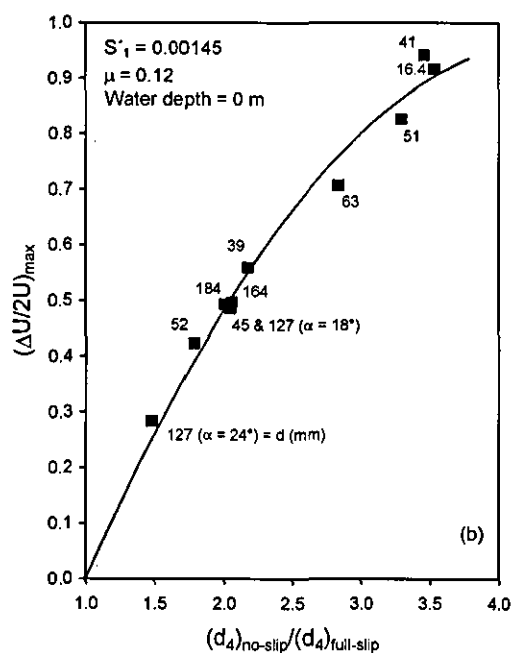
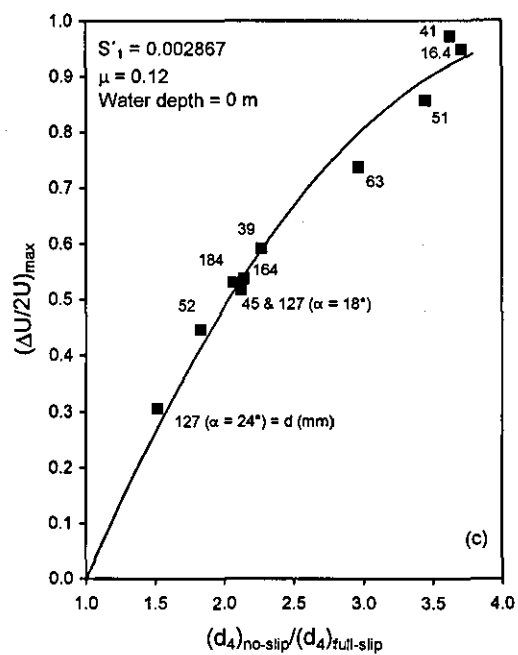
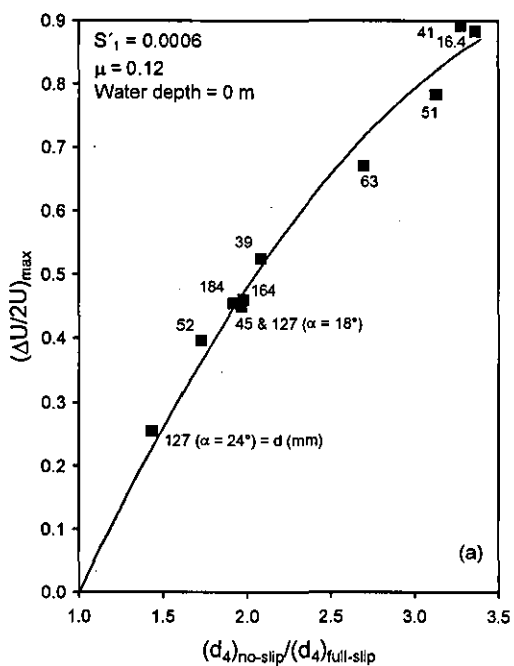
Table 9.4 shows the values of the coefficients A and B in Equation (9.8) along with the correlation coefficients, R, for a wide variety of strand mean axial strains, S'_1 , and water depths.

Table 9.3 – Values of the Coefficients, A, B and C, in Equation (9.6), for a Wide Range of Strand Mean Axial Strains, S'_1 , and Water Depths, Along with the Correlation Coefficients, R.

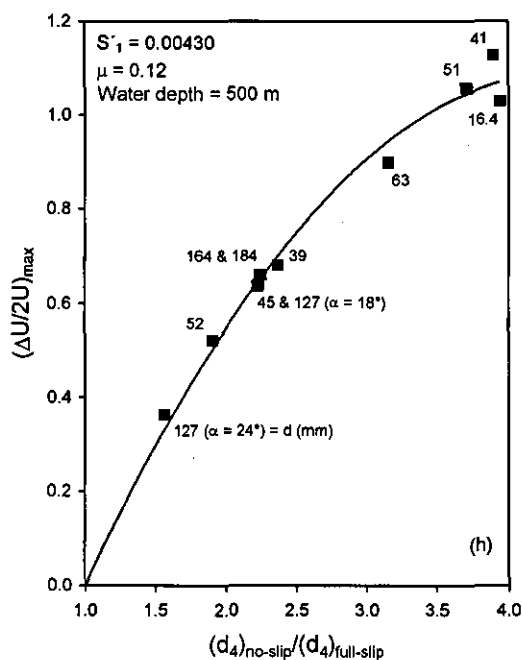
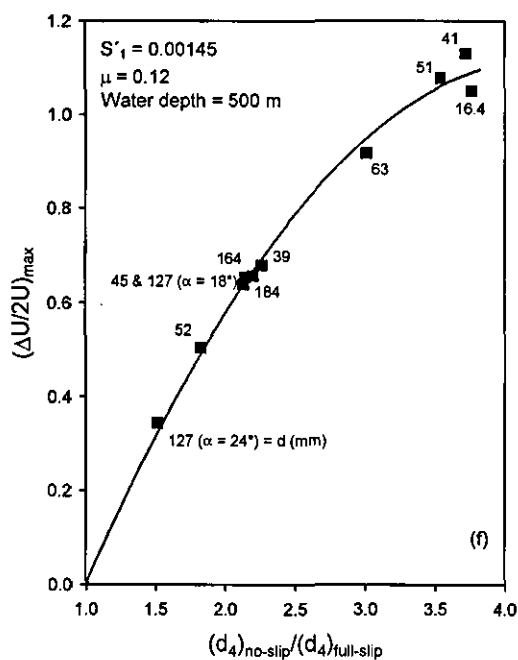
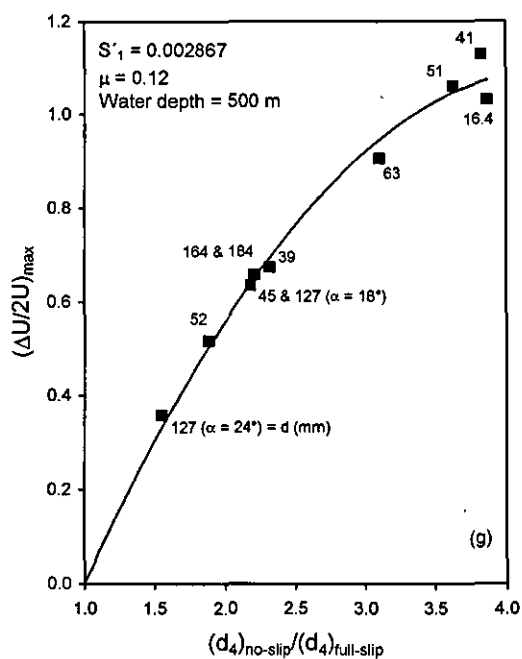
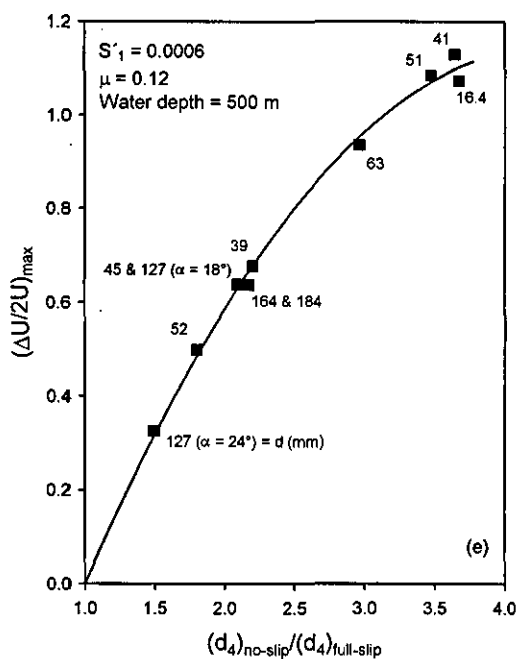
Water Depth (m)	S'_1	A	B	C	R
0	0.0006	-0.650	0.7322	-0.0837	0.9907
0	0.00145	-0.650	0.7311	-0.0825	0.9895
0	0.002867	-0.656	0.7388	-0.0839	0.9881
0	0.00430	-0.665	0.7483	-0.0854	0.9866
500	0.0006	-0.790	0.8924	-0.1028	0.9973
500	0.00145	-0.780	0.8839	-0.1029	0.9955
500	0.002867	-0.755	0.8541	-0.0985	0.9931
500	0.00430	-0.740	0.8338	-0.0951	0.9918
1000	0.0006	-0.715	0.7948	-0.0800	0.9950
1000	0.00145	-0.755	0.8456	-0.0912	0.9963
1000	0.002867	-0.765	0.8587	-0.0947	0.9960
1000	0.00430	-0.755	0.8487	-0.0935	0.9945
1500	0.0006	-0.645	0.7048	-0.0611	0.9896
1500	0.00145	-0.700	0.7731	-0.0751	0.9937
1500	0.002867	-0.730	0.8120	-0.0834	0.9951
1500	0.00430	-0.735	0.8186	-0.0850	0.9949
2000	0.0006	-0.560	0.6022	-0.0412	0.9774
2000	0.00145	-0.612	0.6646	-0.0537	0.9801
2000	0.002867	-0.660	0.7224	-0.0653	0.9841
2000	0.00430	-0.675	0.7434	-0.0697	0.9878

Table 9.4 – Values of the Coefficients, A and B, in Equation (9.8), for a Wide Range of Strand Mean Axial Strains, S'_1 , and Water Depths, Along with the Correlation Coefficients, R.

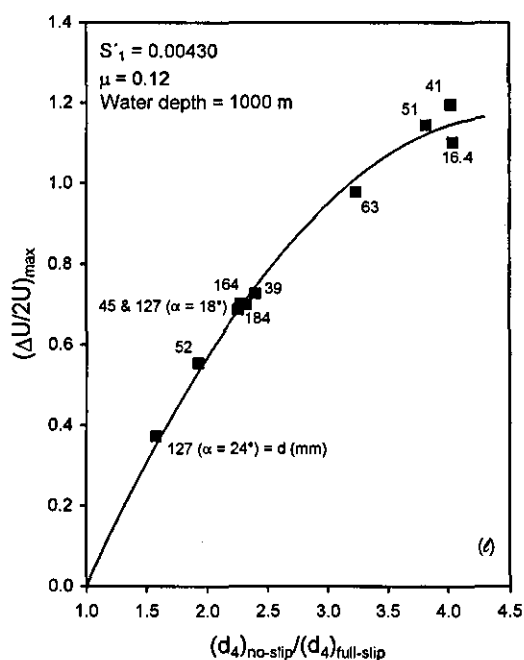
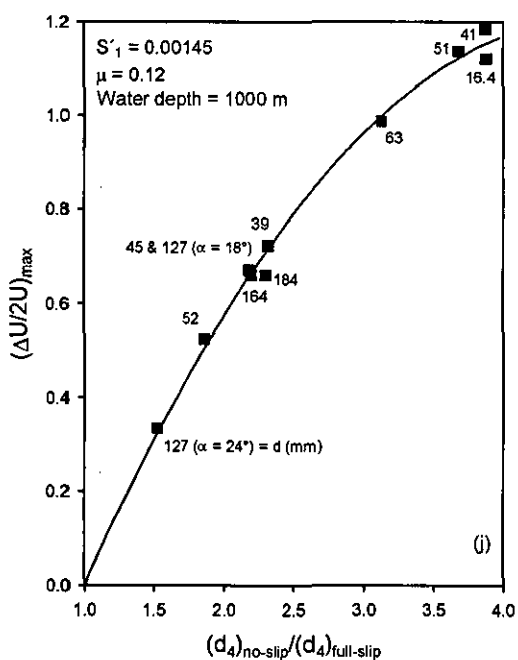
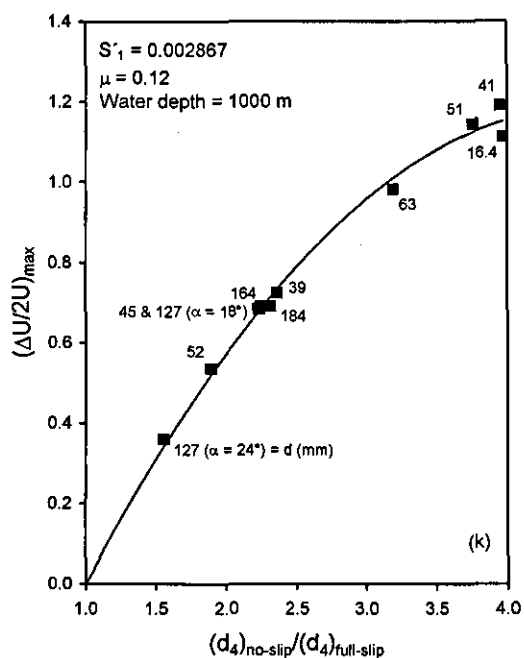
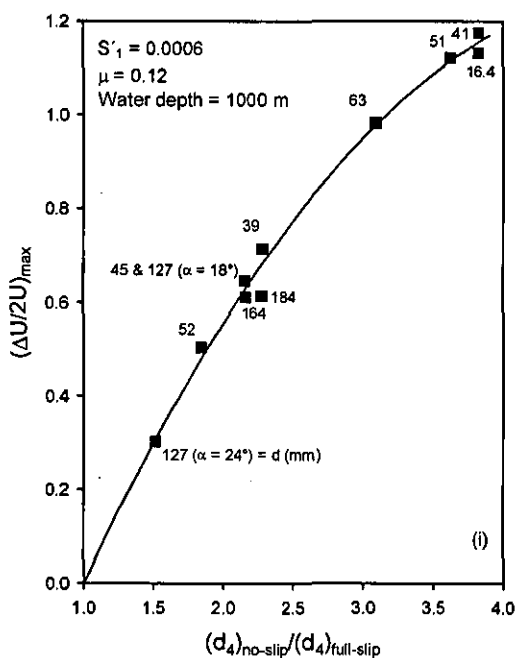
Water Depth (m)	S'_1	A	B	R
0	0.0006	4.0×10^{-8}	0.5654	0.9916
0	0.00145	1.0×10^{-7}	0.5521	0.9914
0	0.002867	3.0×10^{-7}	0.5444	0.9898
0	0.00430	6.0×10^{-7}	0.5307	0.9877
500	0.0006	1.0×10^{-6}	0.4302	0.9812
500	0.00145	2.0×10^{-6}	0.4301	0.9884
500	0.002867	2.0×10^{-6}	0.4339	0.9903
500	0.00430	4.0×10^{-6}	0.4245	0.9899
1000	0.0006	2.0×10^{-6}	0.4194	0.9734
1000	0.00145	3.0×10^{-6}	0.4215	0.9774
1000	0.002867	4.0×10^{-6}	0.4150	0.9803
1000	0.00430	6.0×10^{-6}	0.4111	0.9850
1500	0.0006	4.0×10^{-6}	0.4188	0.9708
1500	0.00145	5.0×10^{-6}	0.4123	0.9714
1500	0.002867	7.0×10^{-6}	0.4027	0.9749
1500	0.00430	8.0×10^{-6}	0.4025	0.9769
2000	0.0006	5.0×10^{-6}	0.4218	0.9672
2000	0.00145	6.0×10^{-6}	0.4148	0.9681
2000	0.002867	8.0×10^{-6}	0.4092	0.9698
2000	0.00430	9.0×10^{-6}	0.4083	0.9734



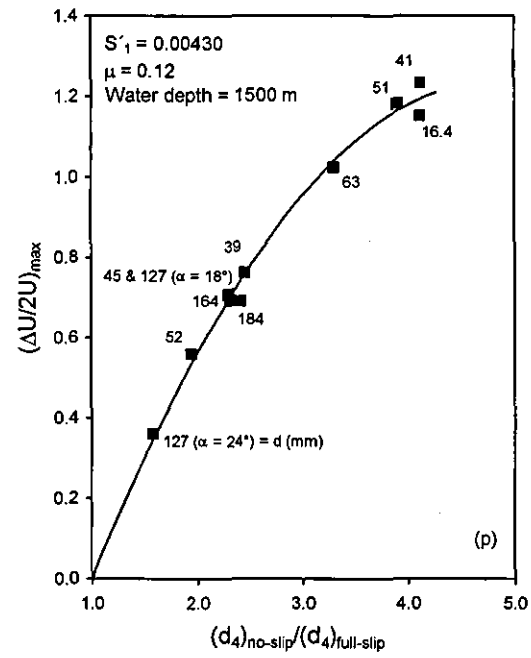
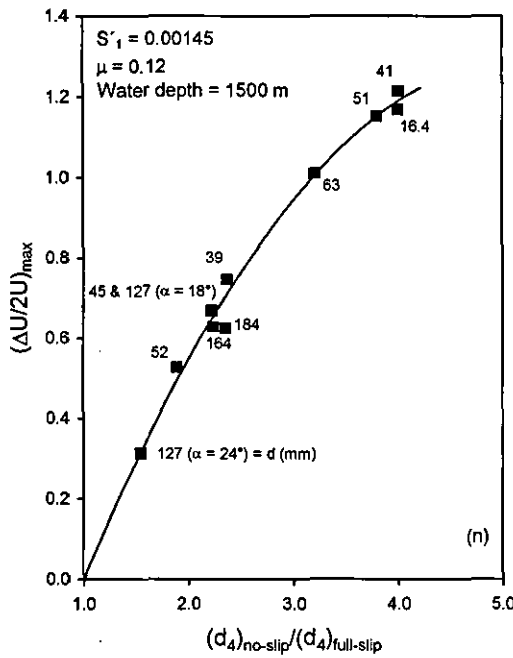
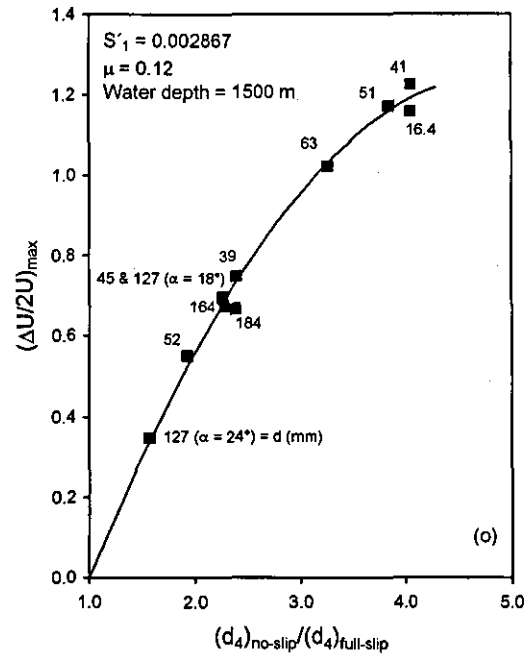
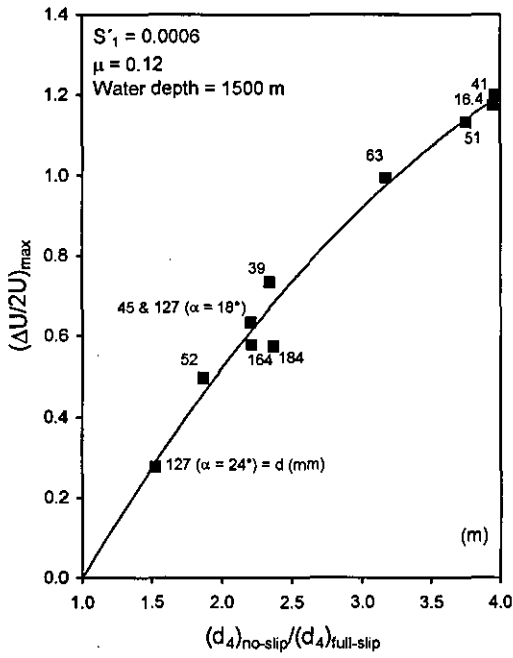
Figs. 9.3(a-d) - Variations of the Maximum Torsional Hysteresis, $(\Delta U/2U)_{\max}$, with Changes in the $(d_4)_{\text{no-slip}}/(d_4)_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (a) $S'_1 = 0.0006$, (b) $S'_1 = 0.00145$, (c) $S'_1 = 0.002867$, and (d) $S'_1 = 0.00430$ - at 0 m Water Depth.



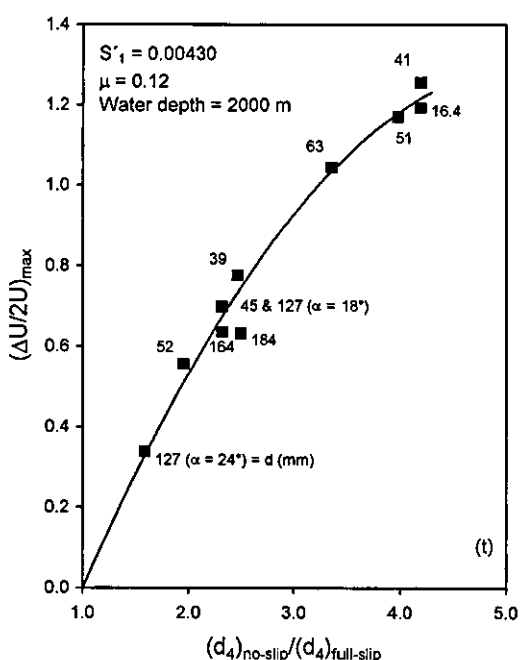
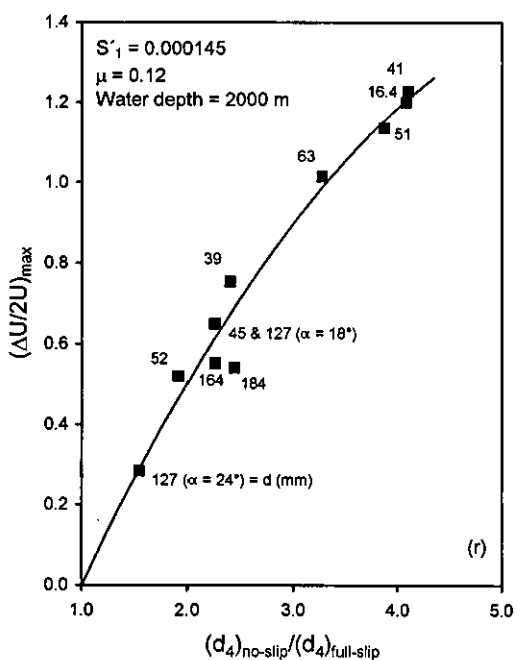
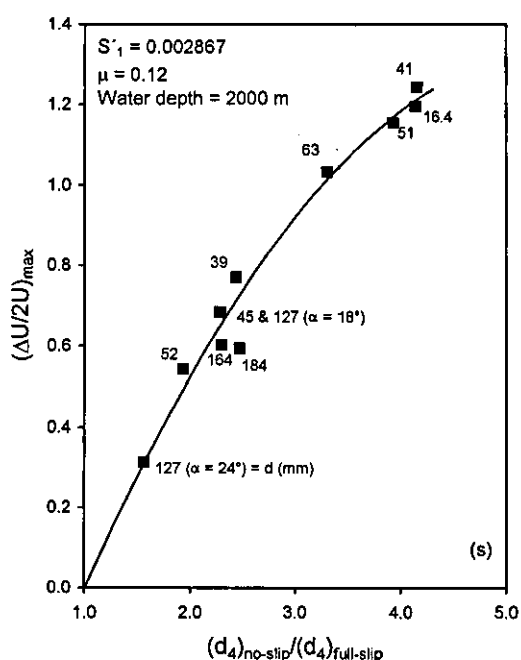
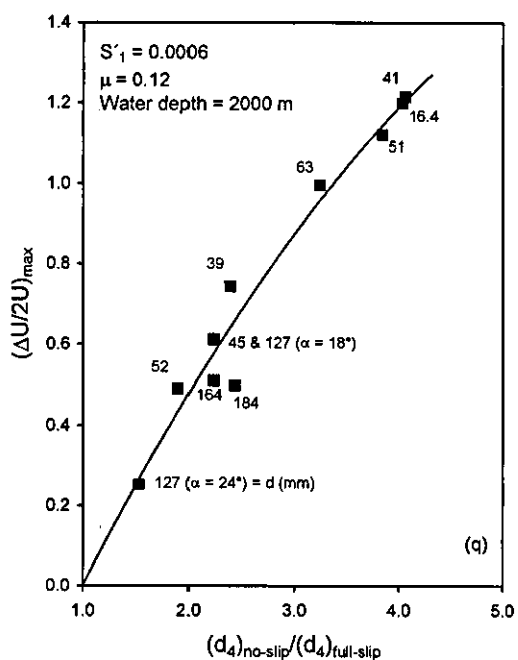
Figs. 9.3(e-h) - Variations of the Maximum Torsional Hysteresis, $(\Delta U/2U)_{\max}$, with Changes in the $(d_4)_{\text{no-slip}}/(d_4)_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (e) $S'_1 = 0.0006$, (f) $S'_1 = 0.00145$, (g) $S'_1 = 0.002867$, and (h) $S'_1 = 0.00430$ - at 500 m Water Depth.



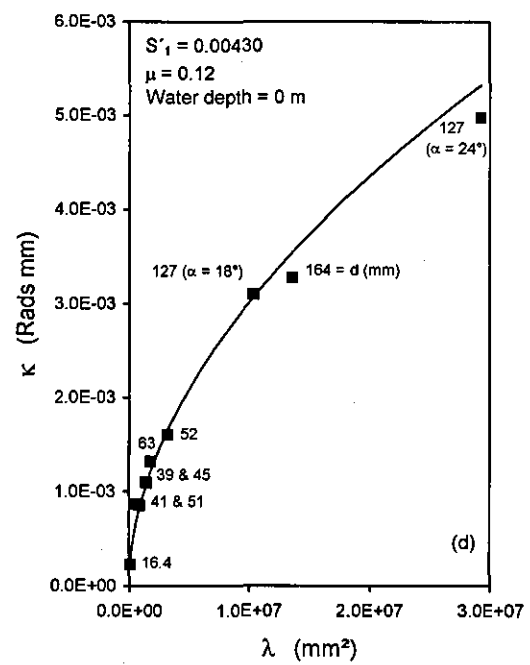
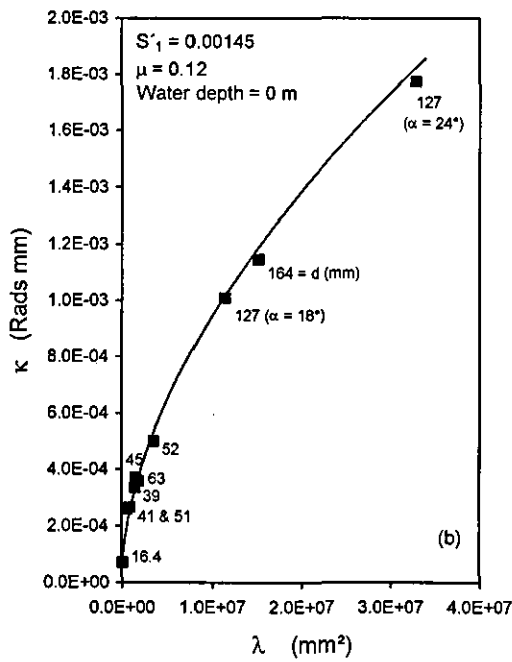
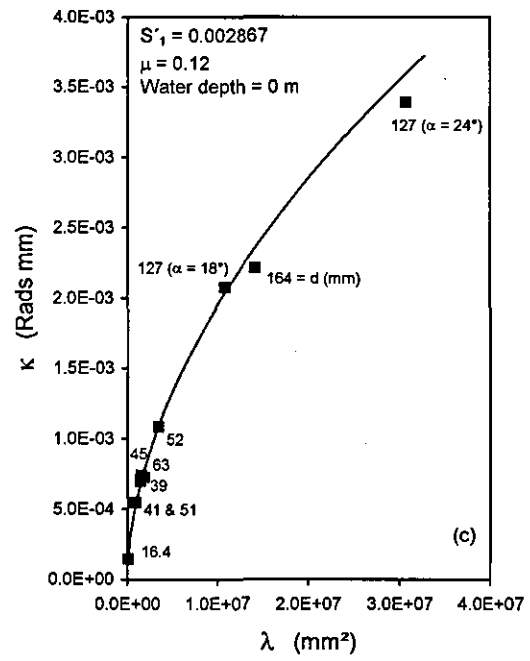
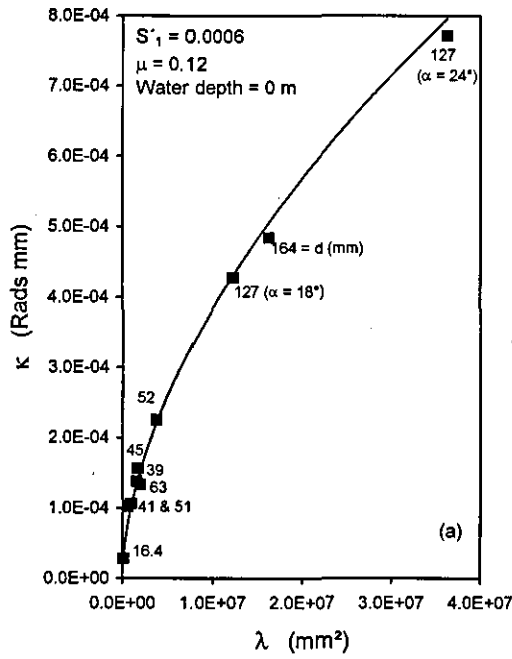
Figs. 9.3(i-d) - Variations of the Maximum Torsional Hysteresis, $(\Delta U/2U)_{\max}$, with Changes in the $(d_4)_{\text{no-slip}}/(d_4)_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (i) $S'_1 = 0.0006$, (j) $S'_1 = 0.00145$, (k) $S'_1 = 0.002867$, and (d) $S'_1 = 0.00430$ - at 1000 m Water Depth.



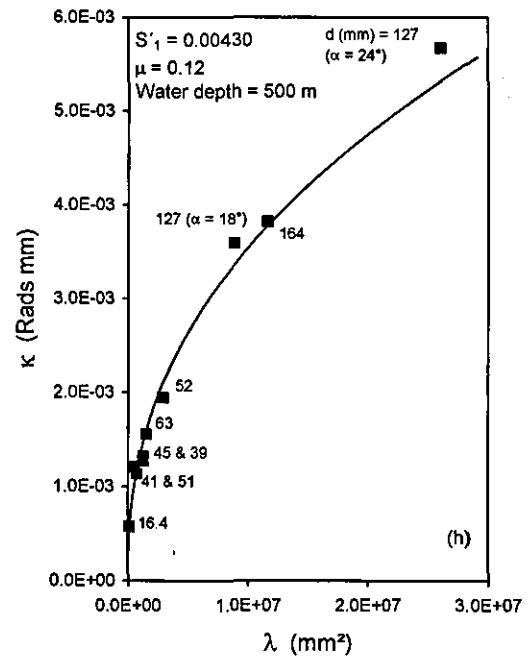
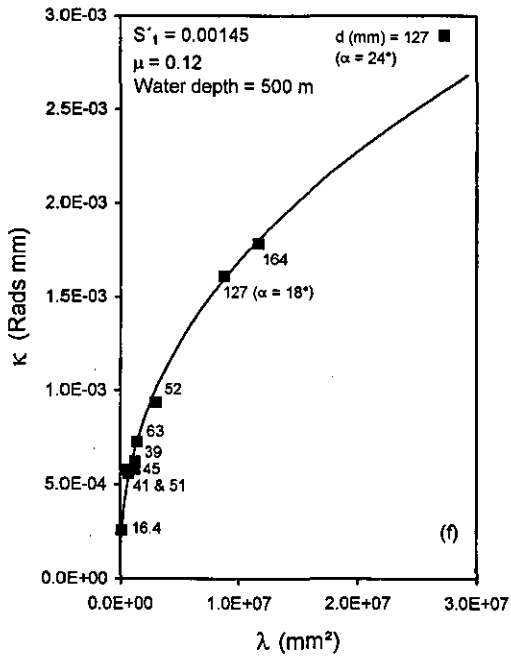
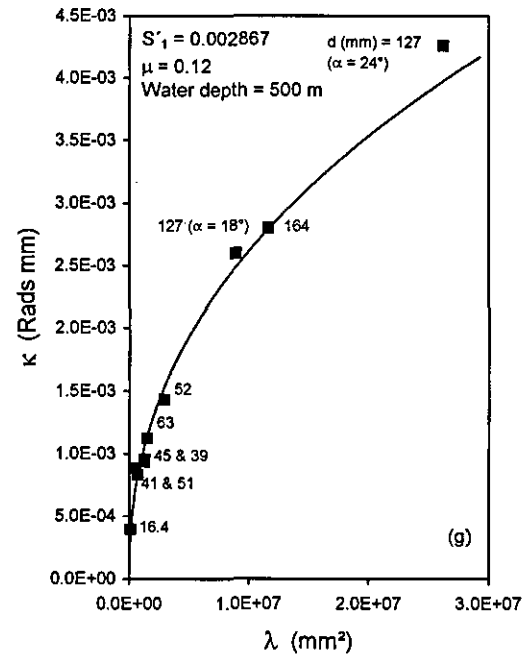
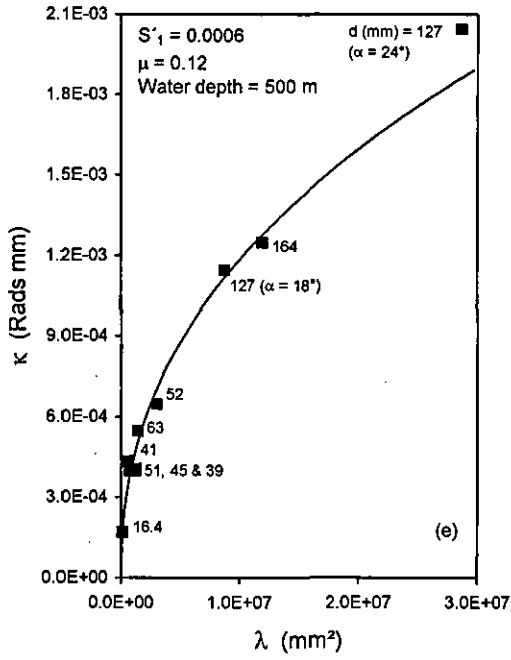
Figs. 9.3(m-p) - Variations of the Maximum Torsional Hysteresis, $(\Delta U/2U)_{\max}$, with Changes in the $(d_4)_{\text{no-slip}}/(d_4)_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (m) $S'_1 = 0.0006$, (n) $S'_1 = 0.00145$, (o) $S'_1 = 0.002867$, and (p) $S'_1 = 0.00430$ - at 1500 m Water Depth.



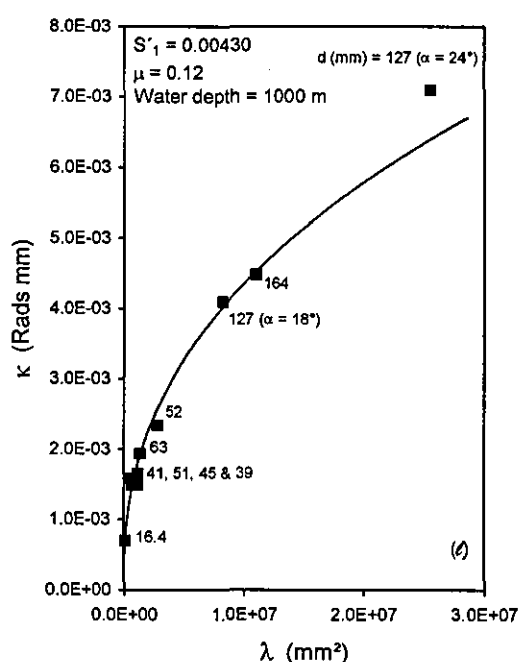
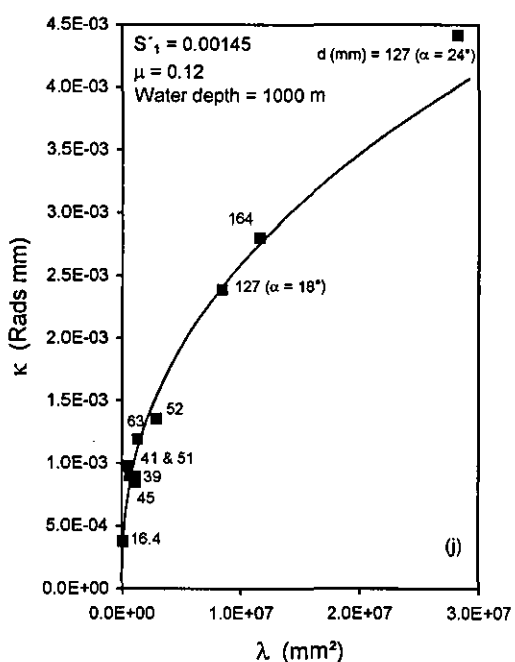
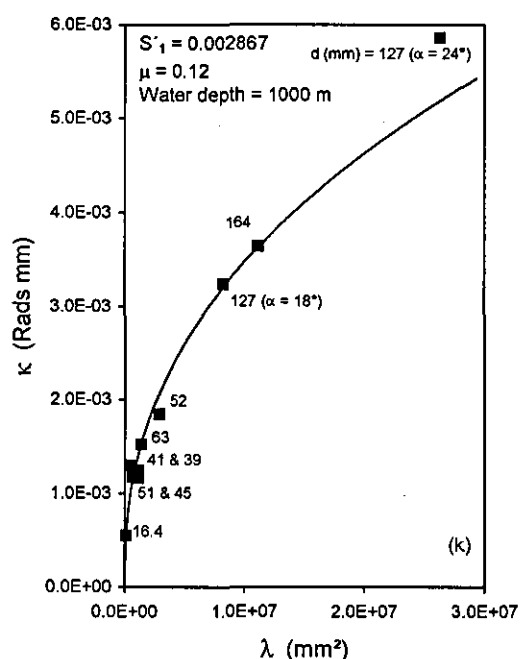
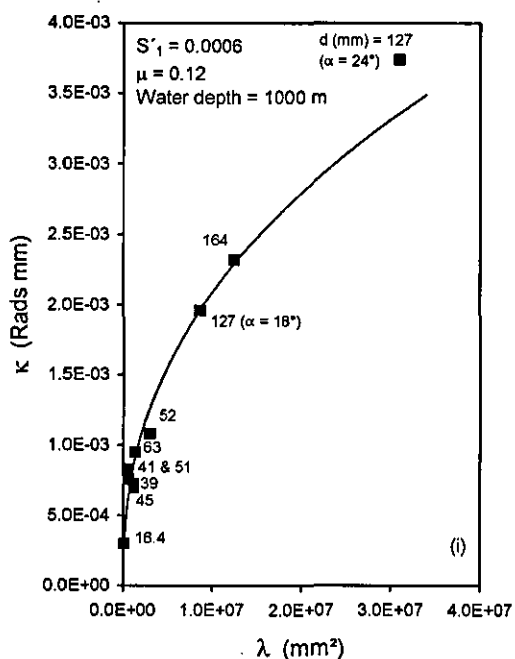
Figs. 9.3(q-t) - Variations of the Maximum Torsional Hysteresis, $(\Delta U/2U)_{\max}$, with Changes in the $(d_4)_{\text{no-slip}}/(d_4)_{\text{full-slip}}$ Ratio, as a Function of the Cable Mean Axial Strains: (q) $S'_1 = 0.0006$, (r) $S'_1 = 0.00145$, (s) $S'_1 = 0.002867$, and (t) $S'_1 = 0.00430$ - at 2000 m Water Depth.



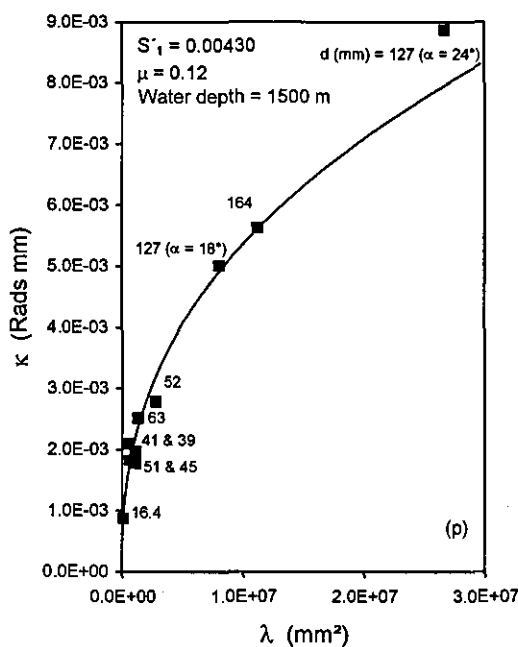
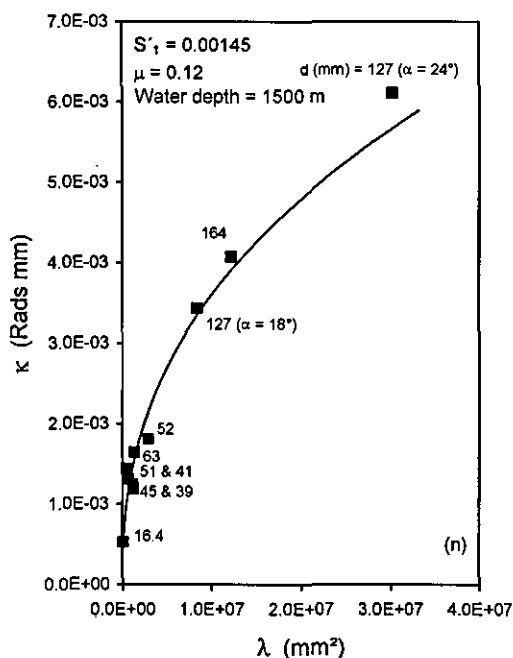
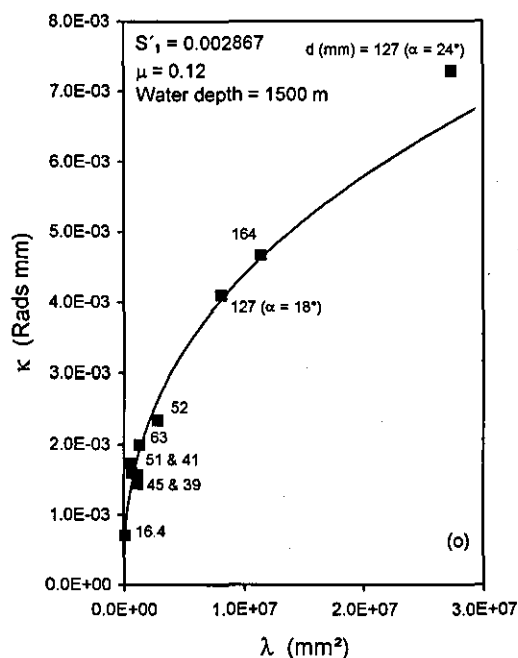
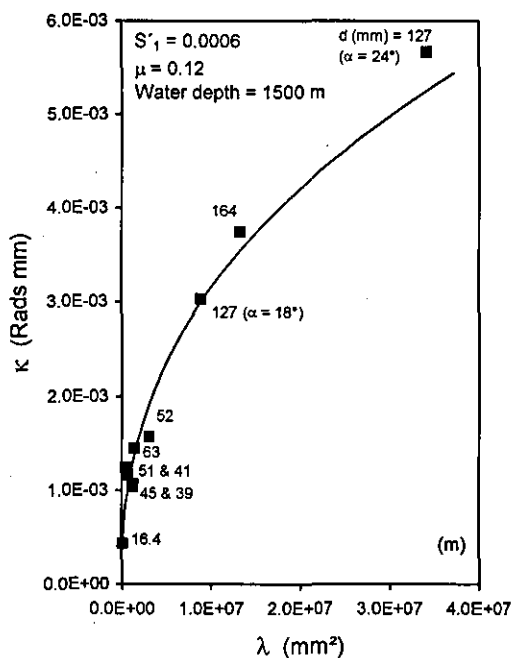
Figs. 9.4(a-d) - Variations of κ with changes in λ at 0 m Water Depth for a Wide Range of Cable Mean Axial Strains: (a) $S'_1 = 0.0006$, (b) $S'_1 = 0.00145$, (c) $S'_1 = 0.002867$, and (d) $S'_1 = 0.00430$.



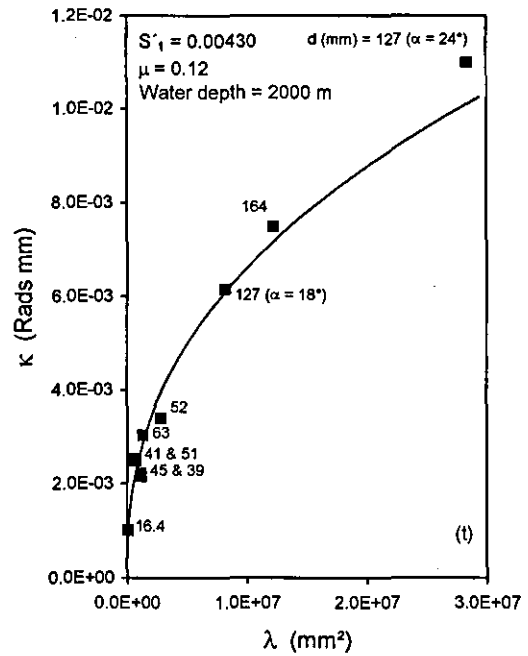
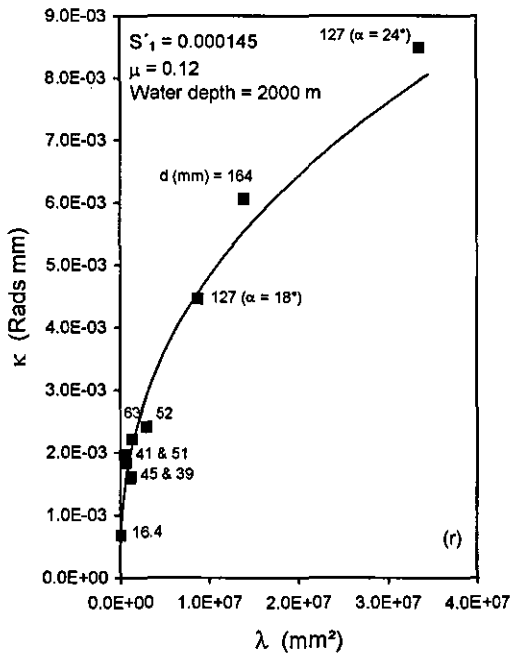
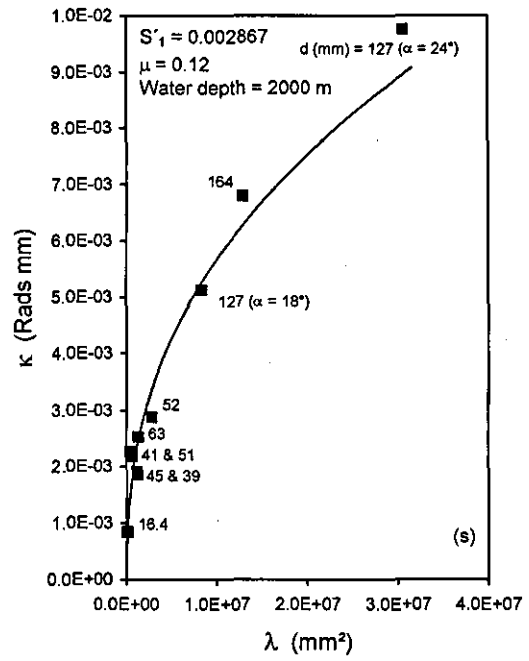
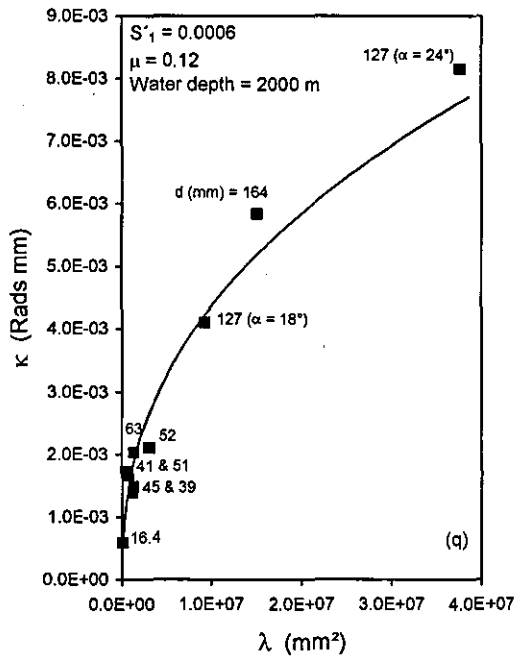
Figs. 9.4(e-h) - Variations of κ with changes in λ at 500 m Water Depth for a Wide Range of Cable Mean Axial Strains: (e) $S'_1 = 0.0006$, (f) $S'_1 = 0.00145$, (g) $S'_1 = 0.002867$, and (h) $S'_1 = 0.00430$.



Figs. 9.4(i-l) - Variations of κ with changes in λ at 1000 m Water Depth for a Wide Range of Cable Mean Axial Strains: (i) $S'_1 = 0.0006$, (j) $S'_1 = 0.00145$, (k) $S'_1 = 0.002867$, and (l) $S'_1 = 0.00430$.



Figs. 9.4(m-p) - Variations of κ with changes in λ at 1500 m Water Depth for a Wide Range of Cable Mean Axial Strains: (m) $S'_1 = 0.0006$, (n) $S'_1 = 0.00145$, (o) $S'_1 = 0.002867$, and (p) $S'_1 = 0.00430$.



Figs. 9.4(q-t) - Variations of κ with changes in λ at 2000 m Water Depth for a Wide Range of Cable Mean Axial Strains: (q) $S'_1 = 0.0006$, (r) $S'_1 = 0.00145$, (s) $S'_1 = 0.002867$, and (t) $S'_1 = 0.00430$.

9.3.3 Numerical Examples

To illustrate the accuracy of the proposed polynomials, numerical examples are given in Tables 9.5 and 9.6. The information presented in Tables 9.5 and 9.6 will also facilitate the use of the simple procedures developed in the previous sections. For the present purposes, a 127 mm ($\alpha = 18^\circ$) outside diameter spiral strand, experiencing a mean axial strain $S'_1 = 0.002867$, at 0 m water depth, with $E_{\text{steel}} = 200 \text{ kN/mm}^2$ and $\mu = 0.12$, will be used. The values of some of the geometrical parameters used in the various equations, relating specifically to this spiral strand, are: $r_1 = 59.22 \text{ mm}$, $D_1 = 6.55 \text{ mm}$, $J_{\text{strand}} = 2.006 \times 10^7 \text{ mm}^4$, with the full construction details given in Table 3.1i.

Table 9.5- Calculation Procedures, Based on the Simple Methods, for Calculating the Maximum Axial Hysteresis and the Corresponding Load Range / Mean Load Ratio for a 127 mm ($\alpha = 18^\circ$) Outside Diameter Spiral Strand.

Equation Number	Parameter	Value
O. S. T.*	$E_{\text{no-slip}} / E_{\text{full-slip}}$	1.221
9.1	Method (a): $(\Delta U/U)_{\text{max}}$	0.3639
9.1	Method (b): $(\Delta U/U)_{\text{max}}$	0.3098
9.5	$\gamma \text{ (mm}^2\text{)}$	473.6149
9.3	$\zeta \text{ (mm}^2\text{)}$	405.130
9.4	Load Range / Mean Load	0.1253
Exact Solution (O. S. T*)	Method (a): $(\Delta U/U)_{\text{max}}$	0.3607
	Method (b): $(\Delta U/U)_{\text{max}}$	0.316
	Load Range / Mean Load	0.178

* O. S. T. : Orthotropic Sheet Theory

Included in Tables 9.5 and 9.6 are also the numerical results as obtained by the so-called exact solution. A comparison of the results based on the simplified and computer-based solutions demonstrates the reasonable accuracy of the proposed simplified methods for both the axial and torsional loading regimes. Finally, it is

perhaps, worth mentioning that the numerical values of $(d_4)_{\text{no-slip}} / (d_4)_{\text{full-slip}}$ and

$(d_4)_{\text{full-slip}}$ in Table 9.6 may, alternatively, be estimated accurately by the simple (hand-based) procedures, as fully discussed by Raoof and Kraincanic (1995a), in preference to the rather involved (computer-based) method, utilising the complex orthotropic sheet formulations.

Table 9.6- Calculation Procedures, Based on the Simple Methods, for Calculating the Maximum Torsional Hysteresis and the Corresponding Range of twist / 2, for a 127 mm ($\alpha = 18^\circ$) Outside Diameter Spiral Strand.

Equation Number	Parameter	Value
O. S. T.*	$(d_4)_{\text{no-slip}} / (d_4)_{\text{full-slip}}$	2.125
O. S. T.*	$(d_4)_{\text{full-slip}} \text{ (Nmm}^{-2}\text{)}$	2.895×10^{11}
9.6	$(\Delta U/2U)_{\text{max}}$	0.5351
9.10	$\lambda \text{ (Nmm}^{-4}\text{)}$	1.0462×10^7
9.8	$\kappa \text{ (rad-mm)}$	1.9888×10^{-3}
9.9	Range of Twist / 2 (rad/mm)	5.13×10^{-6}
Exact Solution (O. S. T*)	$(\Delta U/2U)_{\text{max}}$	0.5176
	Range of Twist / 2 (rad/mm)	5.35×10^{-6}

* O. S. T. : Orthotropic Sheet Theory

9.4 DISCUSSION

An examination of the plots of maximum axial hysteresis, $(\Delta U/U)_{\text{max}}$, against the $E_{\text{no-slip}}/E_{\text{full-slip}}$ ratio, Figs. 9.1(a-d) – Figs. 9.1(q-t), is instructive: for any given water depth, increasing the value of the strand mean axial strain, S'_1 , causes an increase, although slight, in the maximum axial hysteresis, for a given cable construction. Moreover, for any given water depth and strand mean axial strain, an increase in the $E_{\text{no-slip}}/E_{\text{full-slip}}$ ratio (associated with an increase in the lay angle) results in increasing differences between the maximum axial hysteresis, as calculated using methods (a) and (b). Finally, increasing the water depth causes only minor variations in the maximum axial hysteresis.

An examination of the associated $\frac{\text{axial load range}}{\text{mean axial load}}$ and $\frac{E_{\text{no-slip}}}{E_{\text{full-slip}}}$ ratios (both expressed as functions of certain other parameters), Figs.

9.2(a-d) – Figs. 9.2(q-t), suggests that for an assumed value of γ and water depth, increasing the value of the strand mean axial strain, S'_1 , causes a reduction in the value of ζ (for a given strand construction). This reduction is more noticeable as the water depth increases. An increasing water depth also causes a greater degree of scatter of the theoretical results about the fitted curves.

For a given water depth, increasing the strand mean axial strain, S'_1 , for a given cable construction, causes a very slight increase in the maximum torsional hysteresis, $(\Delta U/2U)_{\text{max}}$, for a specific value of the $\frac{(d_4)_{\text{no-slip}}}{(d_4)_{\text{full-slip}}}$ ratio, Figs. 9.3(a-d) – Figs. 9.3(q-

t), which (considering other potential uncertainties in the problem) is not believed to be of any practical significance. In Figs. 9.4(a-d) – Figs. 9.4(q-t), for a given cable construction, and given λ , increasing the strand mean axial strain, S'_1 , is found to cause an increase in the value of κ , as does increasing the level of the external hydrostatic pressure applied to a sheathed spiral strand.

9.5 CONCLUSIONS

By using the previously reported orthotropic sheet model of Hobbs and Raoof, and based on an extensive series of theoretical parametric studies covering a wide range of cable constructions, theoretical equations have been produced which can accurately predict the values of the maximum axial and torsional frictional hysteresis, and the corresponding $\frac{\text{axial load range}}{\text{mean axial load}}$ ratio and $\frac{\text{range of twist}}{2}$, respectively, at which they occur.

It is concluded, that at all levels of strand mean axial strains and water depths, the axial hysteresis may be significantly increased by quite modest increases in the lay angle, within current manufacturing limits. For a given spiral strand construction and strand mean axial strain, however, increasing the maximum axial hysteresis (by increasing the lay angle) is associated with a reduction in the maximum levels of

torsional hysteresis. It should be pointed out that increasing the lay angle to alter the values of the axial and/or torsional hysteresis can also cause a reduction in the strand axial stiffness, which may not always be desirable.

For a given mean axial strain, increasing the level of the external hydrostatic pressure on a sheathed spiral strand causes some, although not really practically significant, reduction in the maximum value of the axial hysteresis, whilst, at the same time, slight associated increases in the maximum value of the torsional hysteresis occur.

Finally, the very encouraging values of the correlation coefficients for all the plots in Figs. (9.1) – (9.4) is noteworthy.

The results presented in this chapter should prove to be of some value in providing reliable estimates of the maximum axial and/or torsional specific damping for use in connection with certain aero- or hydro-dynamic instability calculations relating to both unsheathed and sheathed spiral strands used in, for example, bridging or deep water platform applications.

CHAPTER 10

EFFECT OF STRAND DIAMETER ON AXIAL FATIGUE AND FRICTIONAL HYSTERESIS PREDICTIONS

10.1 INTRODUCTION

The construction industry, both onshore and offshore, is always looking to push the boundaries of construction further by building larger, and more complex structures, such as the long span suspension bridges built, over the recent years, in Japan. As such, the diameters of the steel cables used in these structures are getting increasingly larger. The cost of carrying out experimental measurements on large diameter (multi-layered) spiral strands is considerable, and, in the case of fatigue measurements, very time consuming.

It has long been argued, that using the data based on small diameter spiral strands to predict the response of larger (multi-layered) spiral strands should be done so with a great deal of care. In fact, the results based on small diameter spiral strands, can, in some cases, be so different from those relating to large diameter spiral strands that it can be considered dangerous to even attempt to use them as a guide to the behaviour of the full-scale versions.

As theoretically demonstrated by Raoof (1998a), using S-N curves based on very small diameter spiral strands (e.g. with seven or nineteen wires) to predict the axial fatigue life of larger diameter (e.g. 127 mm) spiral strands is a risky process and, in some cases, can lead to unconservative estimates. However, providing that the lay angles are kept similar during the scaling process, obtaining results based on a scaling factor of, say, 2 or 3 is not unreasonable, and may be adopted in practice for experimentation as an alternative approach to full-scale testing, with considerable financial savings.

With the above comments born in mind, it is the purpose of this chapter to see if a similar scaling process can be applied to the axial and torsional hysteresis of spiral strands. Much attention will also be given to the question of axial fatigue: the

theoretical S-N curves of Raoof (1999) (in the context of size effects), although covering a wide range of strand diameters, only relate to a lay angle $\alpha = 18^\circ$. In what follows, such S-N curves will be extended to cover the full current manufacturing limits for the lay angle.

10.2 THEORY

The hysteresis analysis was carried out using the orthotropic sheet theoretical model of Hobbs and Raoof (1982), the salient features of which have already been reported in some detail in chapter 3, with the complete theory presented by Raoof (1983). The theoretical model used for predicting the axial fatigue life of spiral strands is, on the other hand, reported by Raoof (1990d, 1991a and b). It is worthwhile, at this point, mentioning the exact method used to calculate the ultimate breaking load (U. B. L.) of the different spiral strand constructions, used in what follows, as this value can have a significant influence on the shape of the S-N curves, and hence, the axial fatigue life predictions. The ultimate breaking loads of the spiral strands were calculated using the following equation

$$\text{U.B.L.} = A_s K S_{\text{ult}} \quad (10.1)$$

where, A_s is the net steel area of the strand, K is the spinning loss factor, and S_{ult} is the ultimate wire tensile strength, which for the present purposes, was assumed to be equal to 1520 N/mm^2 . It should, however, be noted that, as theoretically demonstrated by Raoof (1999), provided one non-dimensionlizes the axial load range by dividing it by the appropriate magnitude of ultimate breaking load, variations in the grade of wire do not lead to a significant degree of scatter in the associated S-N plots, and (for all practical purposes) a very nearly unified S-N curve for a strand construction maybe adopted for a wide range of grades of wire. Bearing this in mind, for all the S-N curves presented in the next section, the axial load range has been divided by the magnitude of the ultimate breaking load, for the sake of generality in terms of the grade of wire.

10.3 RESULTS

The analysis was based on three different 127 mm outside diameter spiral strands with lay angles of 12°, 18° and 24°, respectively, which were all assumed to have the same ultimate breaking load equal to 13510kN. The construction details for these spiral strands are shown in Tables 10.1a, b, and c, respectively. To analyse the effect of the strand diameter on various aspects of spiral strand behaviour, smaller diameter spiral strands were produced by removing certain outer layers from the corresponding 127 mm outside diameter spiral strands.

Table 10.1a – Construction Details for the 127 ($\alpha = 12^\circ$) mm Outside Diameter Spiral Strand
- Assumed Ultimate Breaking Load = 13510kN.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	56	RH	6.60	12.00	60.17	1958.671	2493.857
2	50	LH	6.60	12.00	53.73	1748.813	2226.658
3	44	LH	6.60	12.00	47.29	1538.955	1959.459
4	38	RH	6.60	12.00	40.85	1329.098	1692.260
5	32	LH	6.50	12.00	33.89	1085.581	1382.204
6	26	RH	6.50	12.00	27.56	882.034	1123.041
7	20	LH	6.50	12.00	21.23	678.488	863.878
8	14	RH	6.60	12.00	15.15	489.668	623.464
	7	-	4.00	8.57	-	-	85.051
Core	7	-	5.20	8.03	-	-	144.330
	7	-	5.20	4.92	-	-	147.023
	1	-	7.10	-	-	-	39.592

$$A_{core} = 415.996 \text{ mm}^2$$

$$A_g = 12364.821 \text{ mm}^2$$

Table 10.1b – Construction Details for the 127 ($\alpha = 18^\circ$) mm Outside Diameter Spiral Strand
- Assumed Ultimate Breaking Load = 13510kN.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	54	RH	6.55	18.0	59.22	1913.307	2436.098
2	48	LH	6.55	18.0	52.64	1700.717	2165.420
3	42	LH	6.55	18.0	46.07	1488.127	1894.743
4	36	RH	6.55	18.0	39.50	1275.538	1624.065
5	31	LH	6.55	18.0	34.02	1098.380	1398.500
6	25	RH	6.55	18.0	27.46	885.790	1127.823
7	19	LH	6.55	18.0	20.90	673.200	857.145
8	14	RH	6.30	18.0	14.85	458.899	584.289
	7	-	3.90	13.07	-	-	77.289
Core	7	-	5.10	12.20	-	-	133.526
	7	-	5.25	7.62	-	-	142.554
	1	-	7.00	-	-	-	38.485

$$A_{core} = 391.854 \text{ mm}^2$$

$$A_g = 12088.082 \text{ mm}^2$$

Table 10.1c – Construction Details for the 127 ($\alpha = 24^\circ$) mm Outside Diameter Spiral Strand
- Assumed Ultimate Breaking Load = 13510kN.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	54	RH	6.40	24.0	60.23	1901.575	2421.160
2	48	LH	6.40	24.0	53.54	1690.289	2152.142
3	42	LH	6.50	24.0	47.58	1525.583	1942.432
4	36	RH	6.50	24.0	40.79	1307.642	1664.942
5	30	LH	6.60	24.0	34.53	1123.489	1430.471
6	24	RH	6.60	24.0	27.64	898.791	1144.377
7	18	LH	6.80	24.0	21.38	715.567	911.088
8	14	RH	6.10	24.0	14.94	447.865	570.240
	7	-	3.90	17.89	-	-	72.069
Core	7	-	5.10	16.75	-	-	125.557
	7	-	5.25	10.58	-	-	143.935
	1	-	7.00	-	-	-	38.485

$$A_{core} = 380.045 \text{ mm}^2$$

$$A_g = 12236.851 \text{ mm}^2$$

In other words, by removing layers 1, 2 and 3, with 54, 48 and 42 wires, respectively, from, for example, the 127 mm outside diameter spiral strand with a lay angle of 18° , one is left with a strand construction which has an outside diameter of 85.55 mm with its outermost layer having 36 wires. Similarly, by further removing layers 4, 5, and 6, with 36, 31 and 25 wires, respectively, one is left with the construction details relating

to a strand with an outside diameter of 48.35 mm and only two layers of helical wires (layers 7 and 8) surrounding the equal lay core construction which in turn, would be identical for all the three alternative strand diameters.

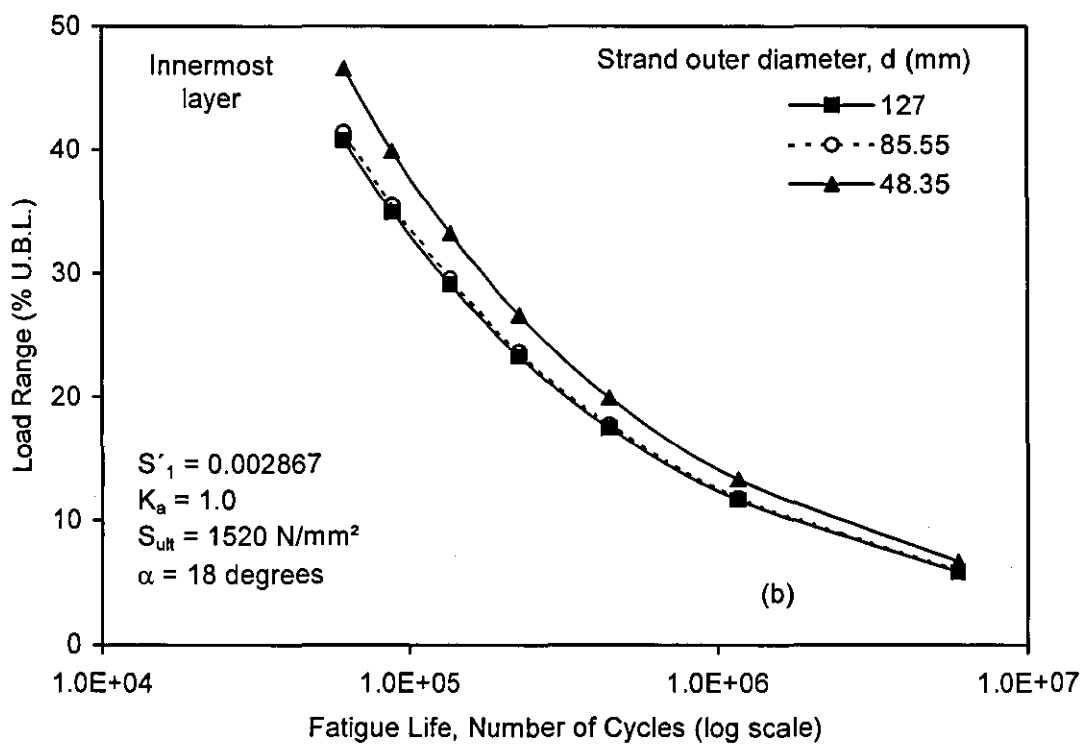
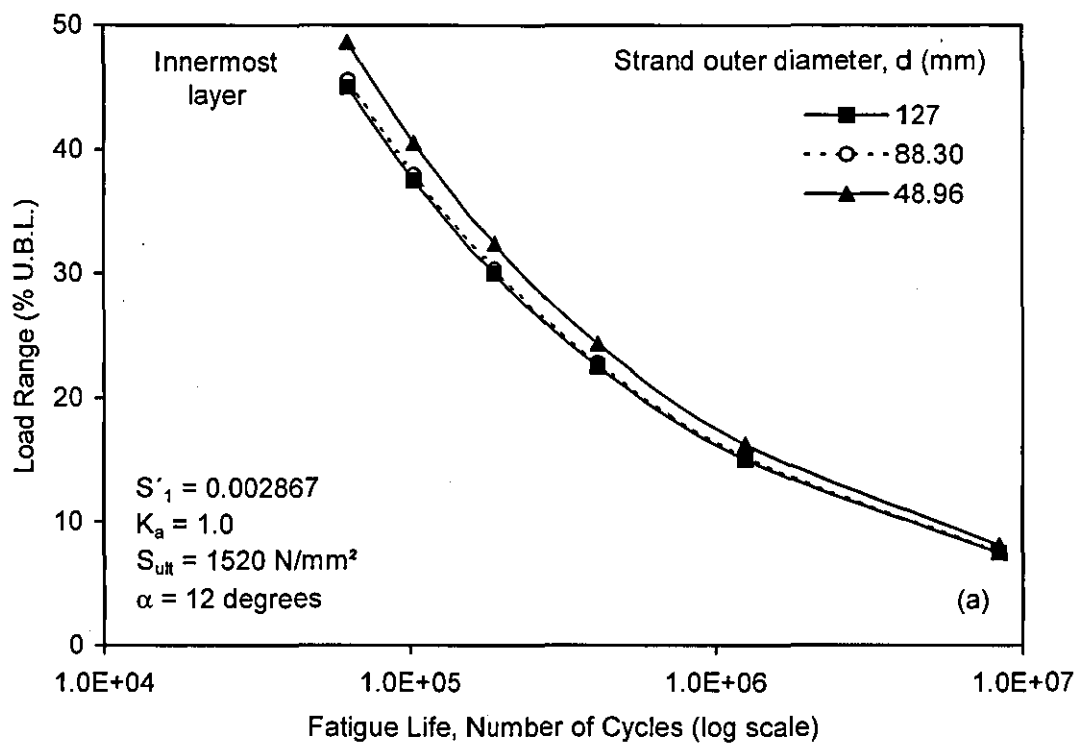
Table 10.2 shows the values of the ultimate breaking loads for all of the different spiral strand constructions, along with the values of all the other parameters in Equation 10.1.

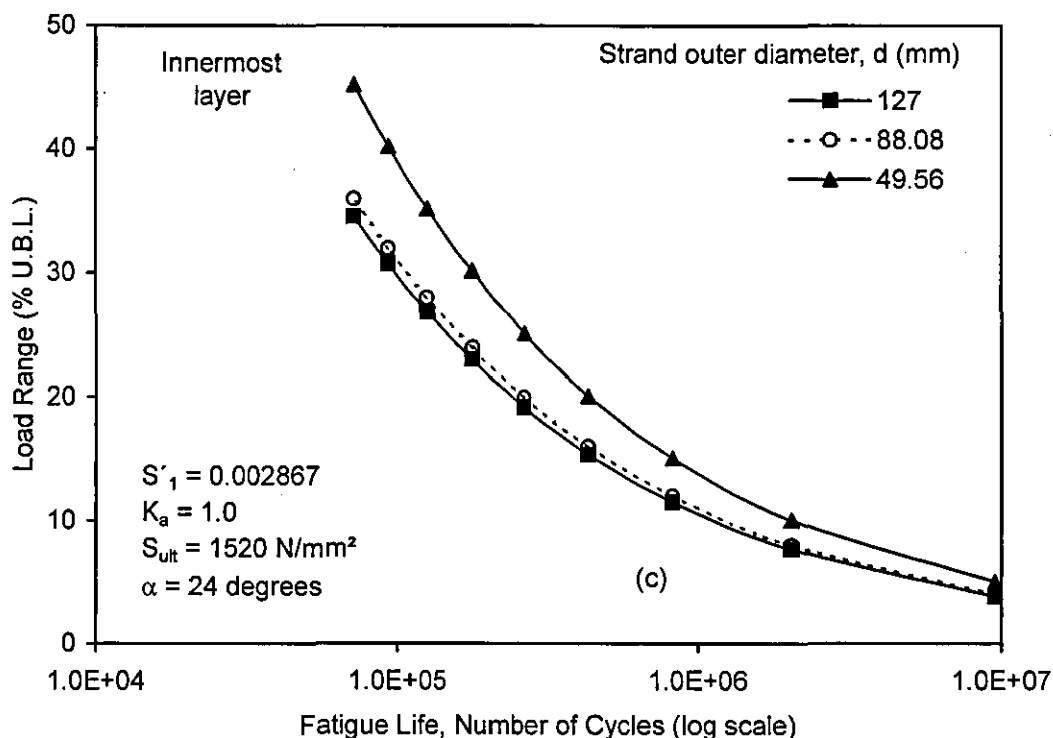
Table 10.2 - Values of the Parameters in Equation (10.1) and the Ultimate Breaking Loads of the Different Spiral Strand Constructions.

Lay Angle (degrees)	Strand Outside Diameter (mm)	Net Steel Area (A_s) (mm^2)	Spinning Loss Factor (K)	Ultimate Wire Tensile Strength (S_{ult}) (N/mm^2)	Ultimate Breaking Load (kN)
12	127	10038.03	0.89	1520	13510
12	88.30	4791.59	0.89	1520	6449
12	48.96	1494.88	0.89	1520	2012
18	127	9801.72	0.91	1520	13510
18	85.55	4699.57	0.91	1520	6478
18	48.35	1439.86	0.91	1520	1986
24	127	9909.29	0.90	1520	13510
24	88.08	4791.84	0.90	1520	6533
24	49.56	1461.92	0.90	1520	1993

10.3.1 Axial Fatigue Results

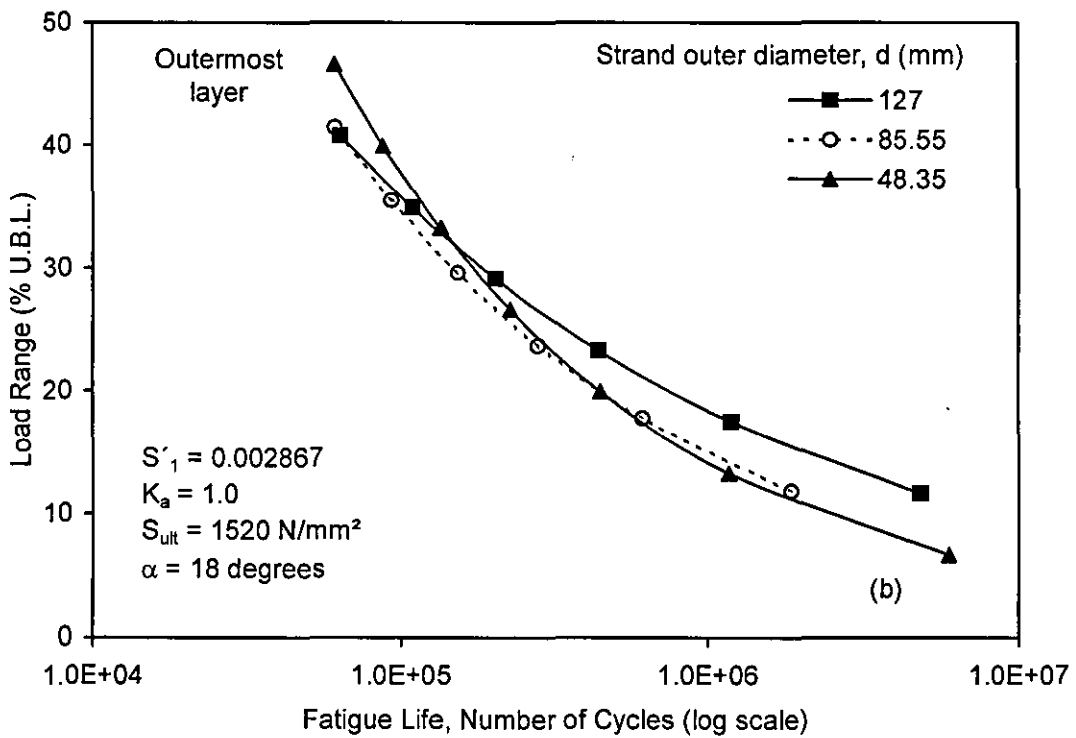
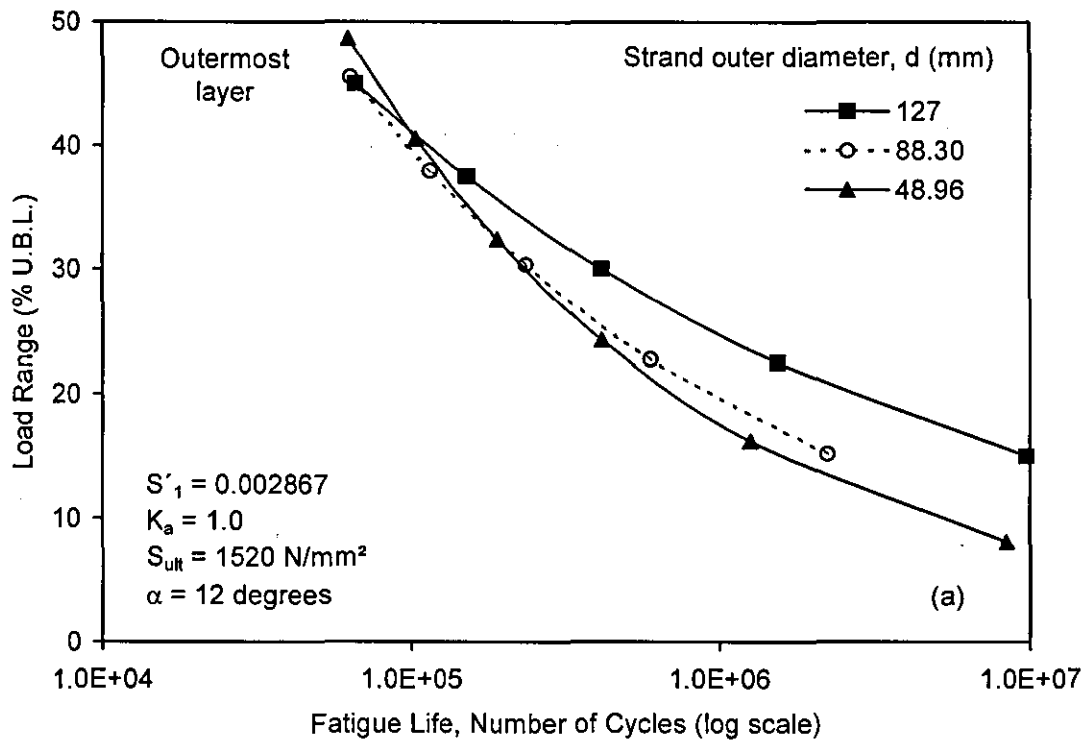
The findings of the current theoretical parametric studies, with regards to the axial fatigue life of spiral strands, are the same as those of Raoof (1998a). Figs. 10.1a, b and c, show the effect of the strand outside diameter, d , on the theoretical predictions of the S-N curves for the first wire fracture in the innermost layer of a spiral strand, assuming $K_a = 1.0$ – i.e. the first wire fractures are assumed to occur in the free-field, away from the effects of end terminations.

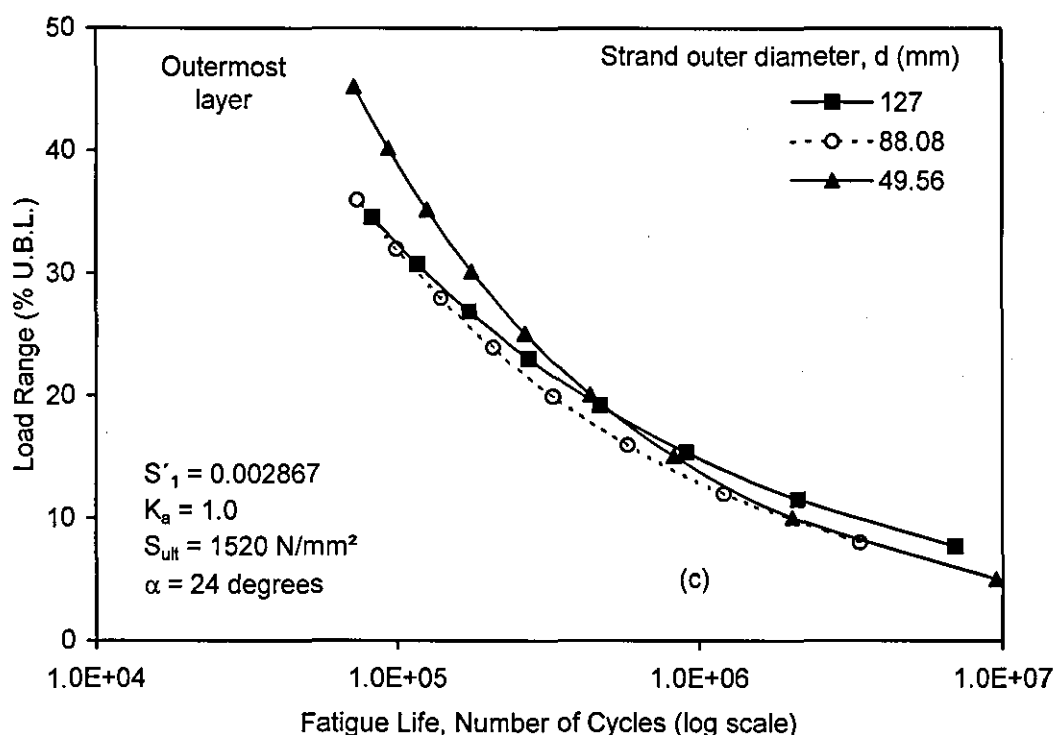




Figs. 10.1 - Effect of the Strand Outer Diameter, d, on the Theoretical Predictions of S-N Curves for the First Wire Fracture in the Innermost Layer of Spiral Strands, Assuming $K_a = 1.0$: (a) $\alpha = 12^\circ$; (b) $\alpha = 18^\circ$; and (c) $\alpha = 24^\circ$.

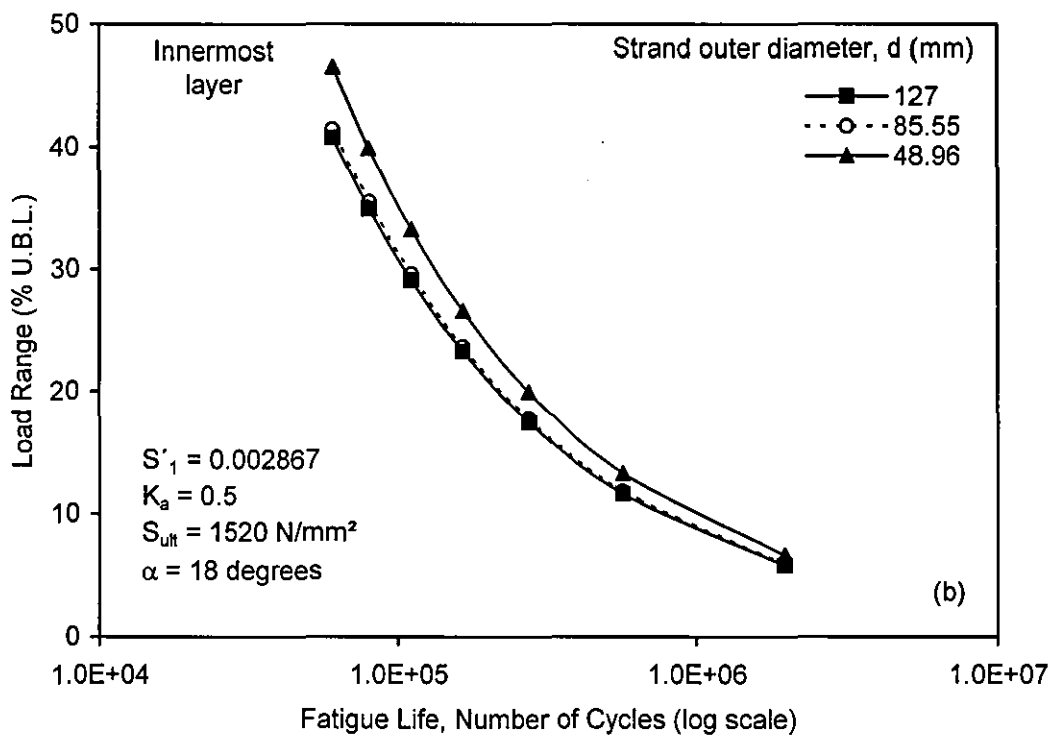
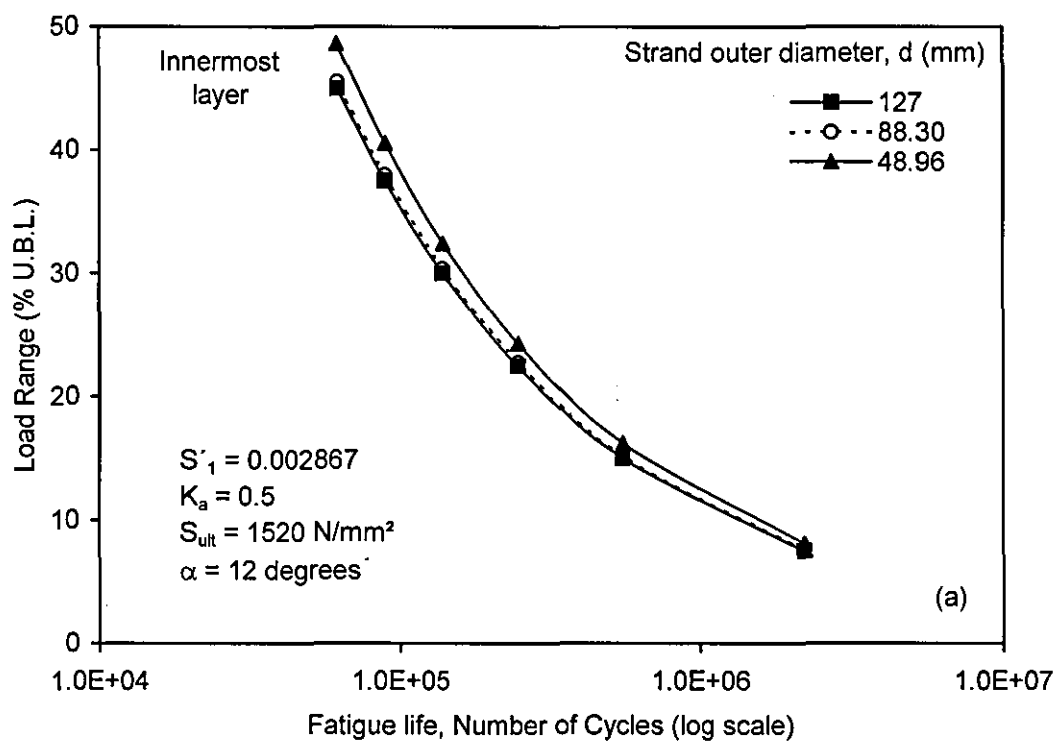
The results show that, assuming fatigue failure is defined as the number of cycles to first innermost wire fracture, for a given axial load range the larger the outside diameter of a spiral strand, the lower would be its axial fatigue life. The differences between the results only really show any significance when the spiral strand is left with only two outer layers (i.e. the smallest outside diameter spiral strand). Figs. 10.2a, b and c, on the other hand, show similar plots, but with these results relating to the occurrence of the first wire fractures in the outermost layer.

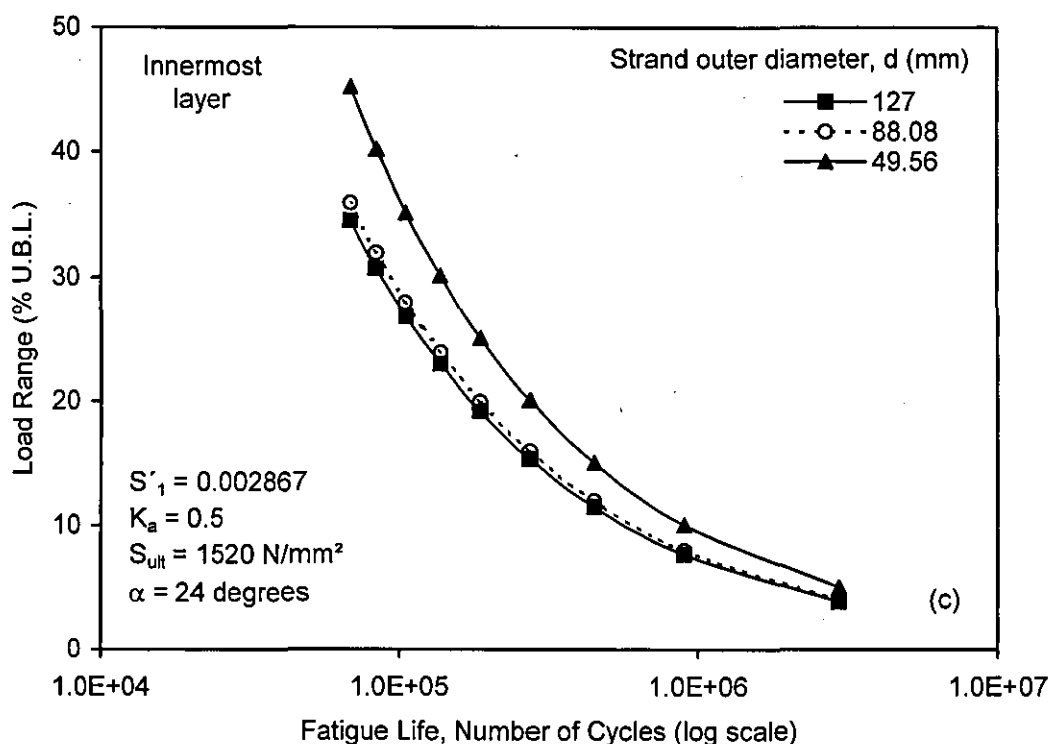




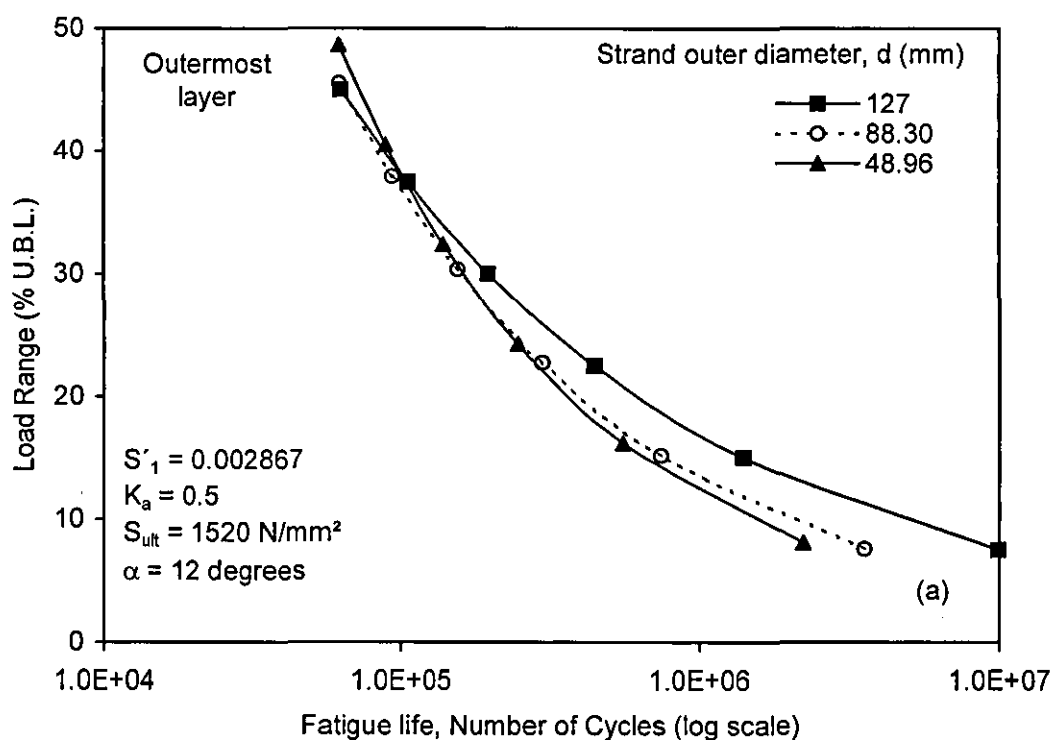
Figs. 10.2 - Effect of the Strand Outer Diameter, d, on the Theoretical Predictions of S-N Curves for the First Wire Fracture in the Outermost Layer of Spiral Strands, Assuming $K_a = 1.0$: (a) $\alpha = 12^\circ$; (b) $\alpha = 18^\circ$; and (c) $\alpha = 24^\circ$.

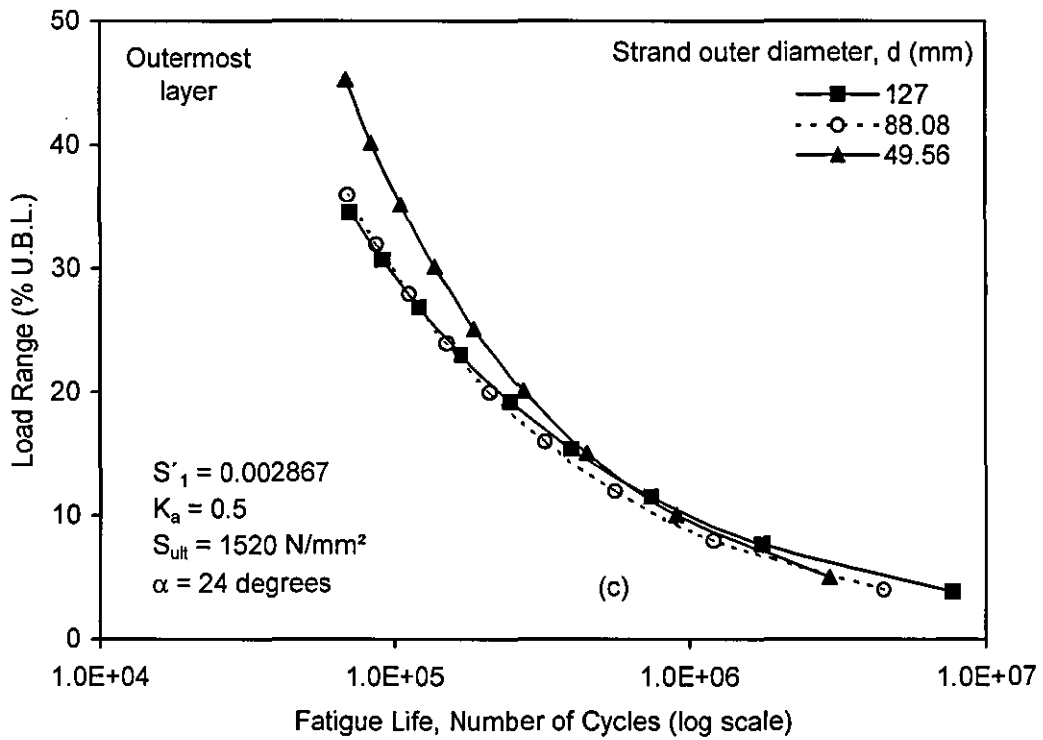
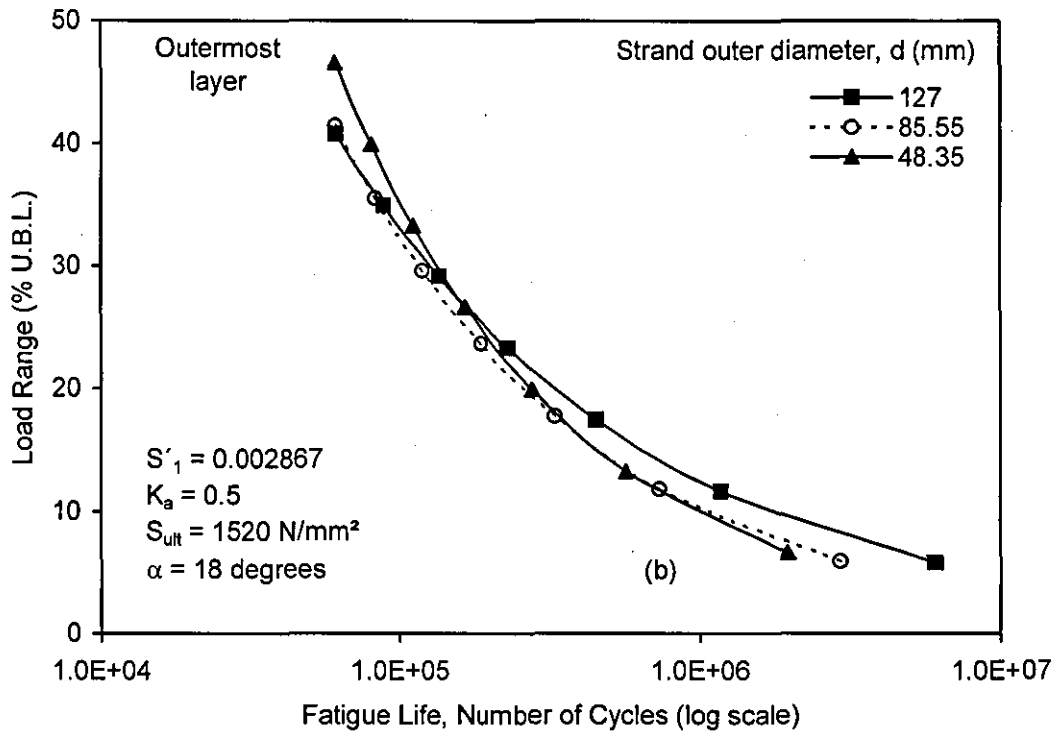
The fatigue life to first outermost layer wire fracture does not follow the same pattern as the corresponding fatigue life to first innermost layer wire fracture. The general trend appears to be that, at higher load ranges, the larger the outside diameter of the spiral strand, the lower its axial fatigue life, but at a certain (lower) load range this changes, in that a larger outside diameter spiral strand will have a higher axial fatigue life. The point at which the change over occurs is dependent upon the lay angle, in that at larger lay angles (i.e. 24°) this change takes place at a lower load range than for, say, a spiral strand with a lay angle of 12° , although the difference is not thought to be practically very significant. The next set of plots, Figs. 10.3a, b and c, are the same as those in Figs. 10.1a, b and c, but assuming $K_a = 0.5$ – i.e. the fatigue life to initial wire fracture is influenced (reduced) by the presence of end terminations.





Figs. 10.3 - Effect of the Strand Outer Diameter, d , on the Theoretical Predictions of S-N Curves for the First Wire Fracture in the Innermost Layer of Spiral Strands, Assuming $K_a = 0.5$: (a) $\alpha = 12^\circ$; (b) $\alpha = 18^\circ$; and (c) $\alpha = 24^\circ$.

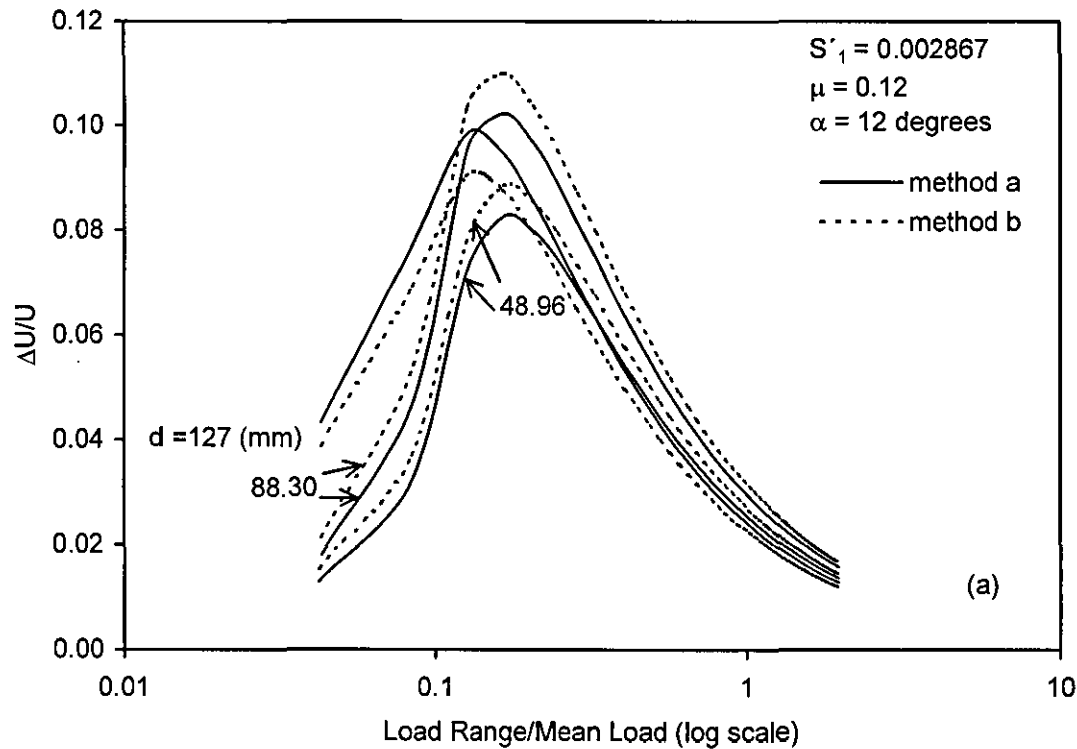


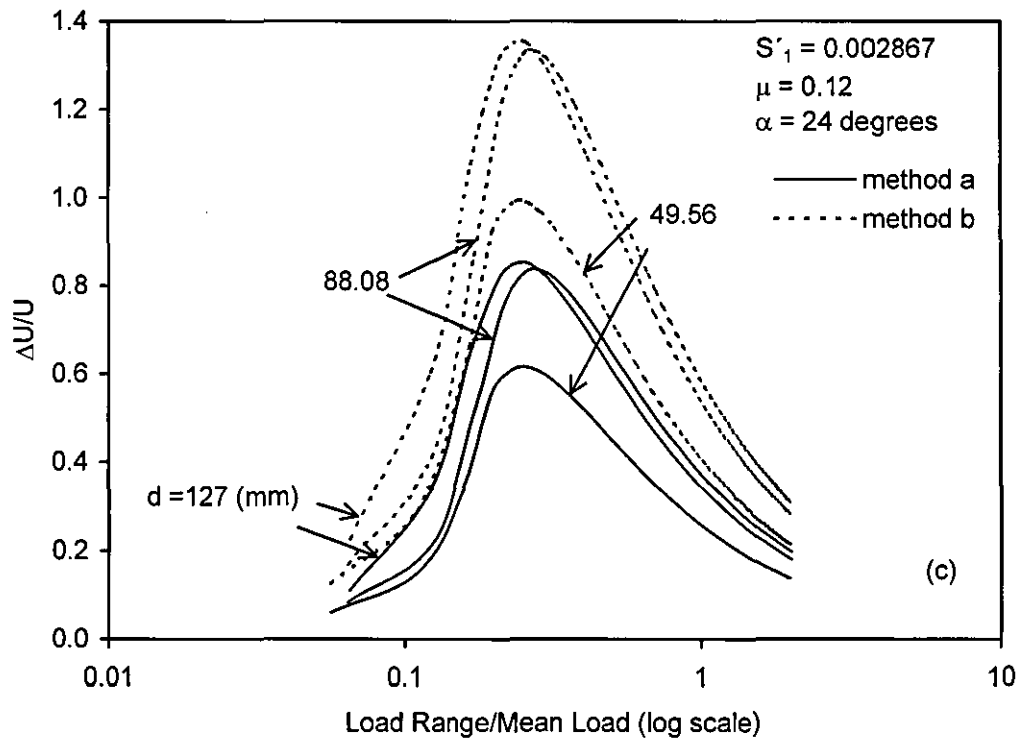
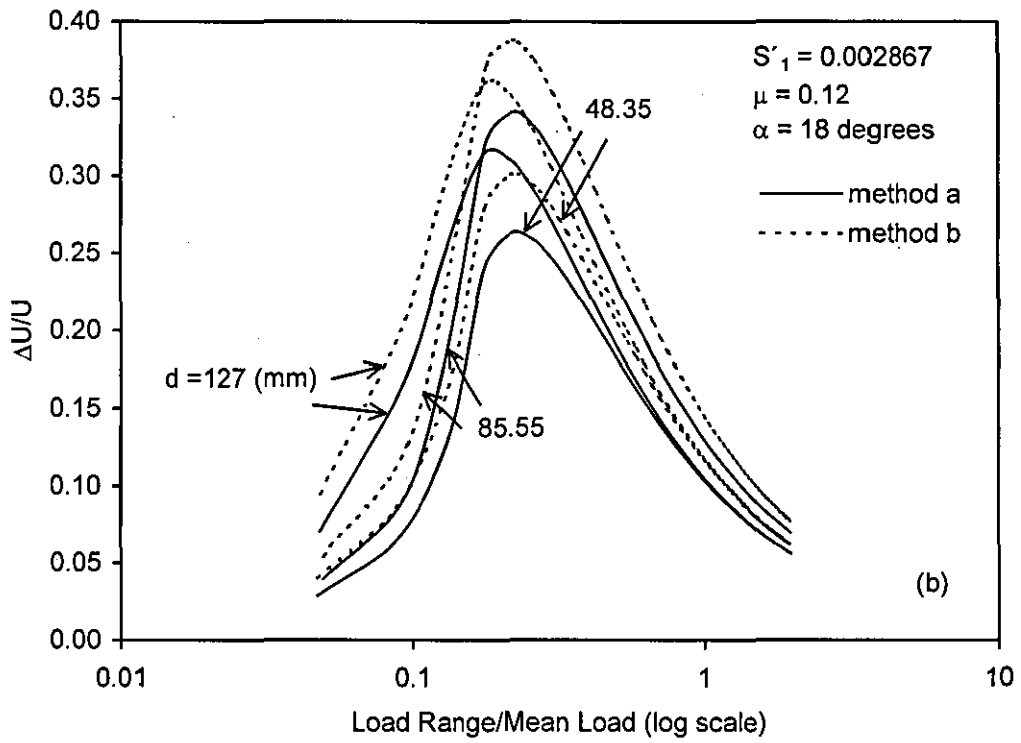


Figs. 10.4- Effect of the Strand Outer Diameter, d , on the Theoretical Predictions of S-N Curves for the First Wire Fracture in the Outmost Layer of Spiral Strands, Assuming $K_a = 0.5$: (a) $\alpha = 12^\circ$; (b) $\alpha = 18^\circ$; and (c) $\alpha = 24^\circ$.

For the fatigue life to first innermost layer wire fracture (with $K_a = 0.5$), once again, for a given load range, the larger the outside diameter of a spiral strand, the lower would be its axial fatigue life. The fatigue life to first outermost layer wire fracture (with $K_a = 0.5$) follows the same pattern as that for $K_a = 1.0$, in that at higher load ranges, the larger the outside diameter of the spiral strand, the lower its axial fatigue life, but at a certain (lower) load range this changes, in that a larger outside diameter spiral strand will have a higher axial fatigue life. The point at which the change over occurs is dependent upon the lay angle, in that at larger lay angles (i.e. 24°) this change takes place at a lower load range than for, say, a strand with a lay angle of 12° , although, once again the difference is not thought to be practically very significant.

10.3.2 Axial Hysteresis



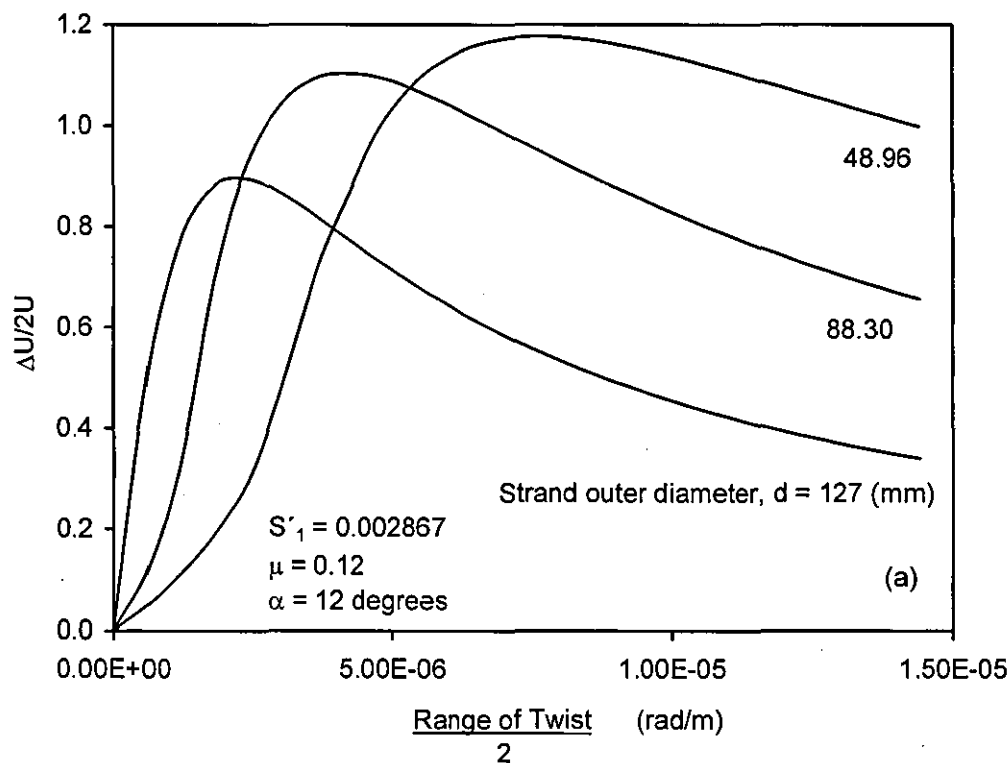


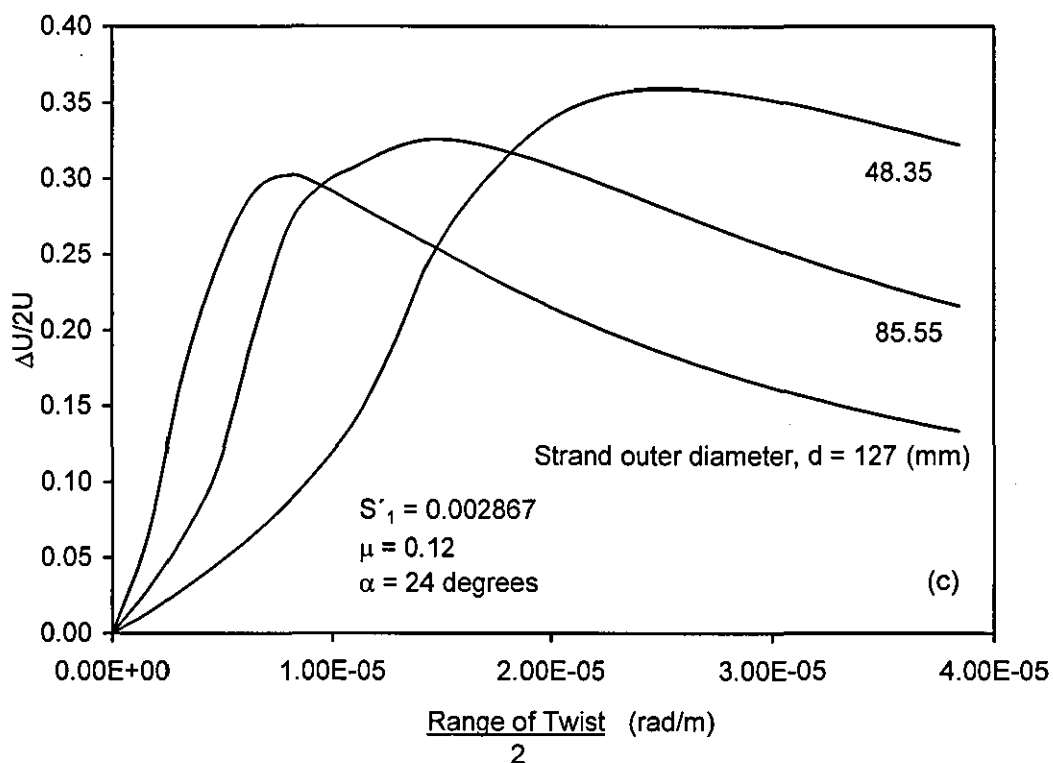
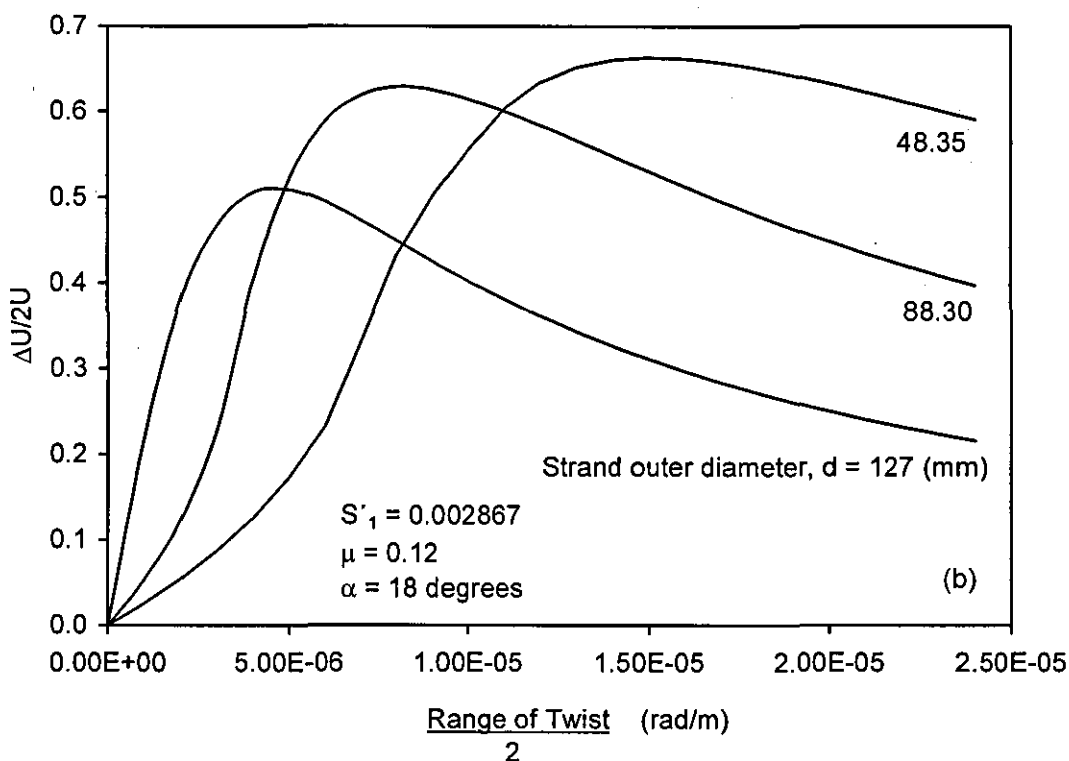
Figs. 10.5 - Effect of the Strand Outer Diameter, d , on the Theoretical Predictions of the Axial Hysteresis, Based on Methods (a) and (b), for: (a) $\alpha = 12^\circ$; (b) $\alpha = 18^\circ$; (c) $\alpha = 24^\circ$.

Figs. 10.5a, b and c show the variation of the axial hysteresis, for a mean axial strain $S'_1 = 0.002867$, with changes in the $\frac{\text{axial load range}}{\text{mean axial load}}$ ratio, for the spiral strands with lay angles of 12° , 18° and 24° , respectively, as calculated using methods (a) and (b), which have been previously described in chapter 3. For all lay angles, the smallest outside diameter spiral strand exhibits the lowest hysteresis, whilst at lower load ranges (before the maximum value of the hysteresis is reached) the larger diameter spiral strand (127 mm) has the higher hysteresis. Once the maximum hysteresis has been reached, then, the larger diameter spiral strand (127 mm) exhibits a lower hysteresis than the slightly smaller diameter spiral strand.

10.3.3 Torsional Hysteresis

Figs. 10.6(a-c) show variations of the torsional hysteresis with changes in the range of twist $\frac{\text{range of twist}}{2}$, assuming a mean axial strain $S'_1 = 0.002867$.





Figs. 10.6 - Effect of the Strand Outer Diameter, d , on the Theoretical Predictions of the Torsional Hysteresis, for: (a) $\alpha = 12^\circ$; (b) $\alpha = 18^\circ$; (c) $\alpha = 24^\circ$.

For all lay angles, at lower values of $\frac{\text{range of twist}}{2}$, the larger the outside diameter of the spiral strand, the greater the hysteresis, whilst at higher values of $\frac{\text{range of twist}}{2}$, the larger the strand outside diameter, the lower the hysteresis. Finally, for all lay angles, the smaller the strand outside diameter, the greater the value of the maximum hysteresis, and the larger the $\frac{\text{range of twist}}{2}$, at which this occurs.

10.4 DISCUSSION

As shown in Figs. 10.1(a-c) - 10.4(a-c), the differences between the S-N curves corresponding to the spiral strands with smaller outside diameters and those with larger outside diameters are not of any practical significance for most design purposes, provided that a scaling factor of 2 – 3 is used with the lay angles being kept the same in the course of the scaling process. This is particularly the case, in view of the fact that the presently adopted fatigue failure criteria corresponds to the number of cycles to first wire fracture, and, in practice, there is usually a rather significant difference between the number of cycles to the initial wire fracture and total collapse, leaving the designer with a very desirable margin of safety against total failure.

Figs. 10.5(a-c) show the theoretical results for axial hysteresis as a function of the external $\frac{\text{axial load range}}{\text{mean axial load}}$ ratio. Once again, the difference between the axial hysteresis values, based on the larger and smaller outside diameter spiral strands, is small, and not thought to be of any practical significance, especially considering the practical uncertainties such as the exact value chosen for the interwire coefficient of friction, in addition to variations in the patterns of interwire contact, and as to whether method (a) or (b) is used for predicting the axial hysteresis (amongst other factors).

Figs. 10.6(a-c) show the effect of the outside diameter on the theoretical predictions of the torsional hysteresis. The torsional hysteresis appears to be more sensitive to the effects of the strand outside diameter, with, in some cases, the difference between the torsional hysteresis for the larger and smaller outside diameter spiral strands being by

as much as a factor of 7, at low values of $\text{range of twist} / 2$, whilst differences by a factor of approximately 3, at higher levels of $\text{range of twist} / 2$, can occur. One thing to note is that the maximum values of the torsional hysteresis for the three different outside diameter spiral strands are (for all practical purposes) similar.

10.5 CONCLUSIONS

The question of size effects regarding the axial fatigue and axial plus torsional frictional hysteresis of spiral strands has been addressed on a theoretical basis, and recommendations have been made for carrying out realistic axial fatigue tests on scaled down (by a factor of 2 – 3) specimens (i.e. those with a smaller diameter, while the other geometrical parameters, particularly the lay angle, are kept nominally the same) as an alternative to the much more expensive, time-consuming and difficult option of full scale testing on very large diameter strands.

Unlike axial hysteresis in relation to which a scaling factor of 2 – 3 (with the lay angles kept the same) is a reasonable option for most practical purposes, in the case of torsional hysteresis, there does not appear to be a short cut to obtaining the torsional hysteresis of large diameter (multi-layered) spiral strands, by conducting tests on scaled down specimens, unless one is only trying to obtain a rather approximate idea as to the magnitude of the maximum torsional hysteresis.

CHAPTER 11

ANALYSIS OF A TWO-DIMENSIONAL CABLE TRUSS

11.1 INTRODUCTION

A cable truss is a counter-stressed double-cable system in which the top and bottom chords consist of continuous pre-stressed cables anchored at each end, and between which, numerous light rigid spacers are placed to provide the web members.

The cable truss has many structural advantages, particularly as a means of supporting the roofs of large spanning buildings. It is structurally very efficient and light, yet possesses considerable rigidity.

A computer program, capable of carrying out both a linear and non-linear analysis of either a two- or three-dimensional cable structure with rigid supports, has been developed by Broughton and Ndumbaro (1994). In the case of a non-linear analysis, it caters for both material and/or geometrical non-linearities. This computer program analyses the structure based on the standard Newton–Raphson technique, requiring very few inputs, which are the coordinate geometry of the structure, the axial stiffness of each element (member), the position of the loads, the boundary conditions, and finally, the value of the pretension force in each member.

In the formulations of their computer program, Broughton and Ndumbaro have ignored the relatively small weight of the cables and spacers, so that the initial free-hanging geometry is specified by the cable pretensions, the length of the spacers, and the span.

The purpose of this chapter is, using the computer program of Broughton and Ndumbaro, to assess the influence of the lay angle of the spiral strands on the vertical deflection profile of a two-dimensional cable truss under a uniformly distributed load, based on both the no-slip and full-slip regimes in relation to the axial stiffness of its constituent spiral strands.

11.2 THEORY

The nature of the theory relating to the development of the computer program is rather involved, and is presented in some considerable detail by Broughton and Ndumbaro (1994) in a style that is comparatively easy to follow. As such, the theoretical formulations behind the computer program will not be presented here.

The two-dimensional cable-truss used in the following analysis is based on an example described by Irvine (1981). The chosen example, Fig. 11.1, is a biconcave cable truss, which is typical of a parallel array used to support a rectangular roof. The top and bottom chords were assumed to be spiral strands with the same constructions, for a given cable truss, having outside diameters of either, 48.96 mm ($\alpha = 12^\circ$), 48.35 mm ($\alpha = 18^\circ$) or 49.56 mm ($\alpha = 24^\circ$), depending upon the magnitude of their lay angle. The construction details for the three different spiral strands are given in Tables 11.1a, b, and c, corresponding to lay angles of 12° , 18° , and 24° , respectively. The chords were connected by vertical hangers (spiral strands) which had outside diameters of 16.4 mm. The construction details for the 16.4 mm outside diameter spiral strand are given in Table 11.2.

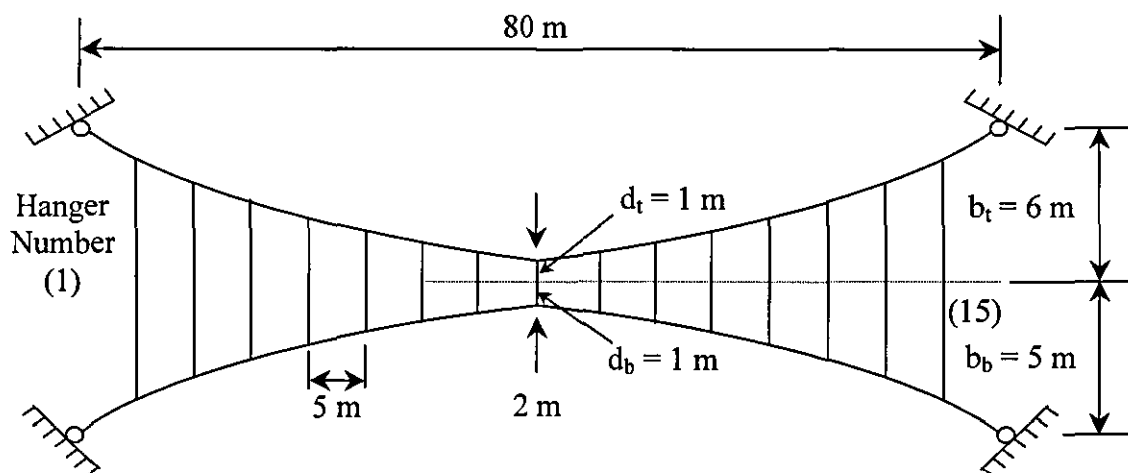


Fig. 11.1 – Geometrical Details of the Cable-Truss.

Table 11.1a – Construction Details for the 48.96 mm ($\alpha = 12^\circ$) Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
7	20	LH	6.50	12.00	21.23	678.488	863.878
8	14	RH	6.60	12.00	15.15	489.668	623.464
Core	7	-	4.00	8.57	-	-	85.051
	7	-	5.20	8.03	-	-	144.330
	7	-	5.20	4.92	-	-	147.023
	1	-	7.10	-	-	-	39.592

$$A_{core} = 415.996 \text{ mm}^2$$

$$A_g = 1487.342 \text{ mm}^2$$

Table 11.1b – Construction Details for the 48.35 mm ($\alpha = 18^\circ$) Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
7	19	LH	6.55	18.0	20.90	673.200	857.145
8	14	RH	6.30	18.0	14.85	458.899	584.289
Core	7	-	3.90	13.07	-	-	77.289
	7	-	5.10	12.20	-	-	133.526
	7	-	5.25	7.62	-	-	142.554
	1	-	7.00	-	-	-	38.485

$$A_{core} = 391.854 \text{ mm}^2$$

$$A_g = 1441.434 \text{ mm}^2$$

Table 11.1c – Construction Details for the 49.56 mm ($\alpha = 24^\circ$) Outside Diameter Spiral Strand.

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (degs)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
7	18	LH	6.80	24.0	21.38	715.567	911.088
8	14	RH	6.10	24.0	14.94	447.865	570.240
Core	7	-	3.90	17.89	-	-	72.069
	7	-	5.10	16.75	-	-	125.557
	7	-	5.25	10.58	-	-	143.935
	1	-	7.00	-	-	-	38.485

$$A_{core} = 380.045 \text{ mm}^2$$

$$A_g = 1481.328 \text{ mm}^2$$

Table 11.2 – Construction Details for the 16.4 mm Outside Diameter Spiral Strand (Used as Vertical Hangers).

Layer	Number of Wires (n)	Lay Direction	Wire Diameter (D) (mm)	Lay Angle (deg)	Pitch Circle Radius (theo) r(mm)	Net Steel Area A_{ni} (mm ²)	Gross Steel Area A_{gi} (mm ²)
1	12	RH	3.25	11.91	6.41	101.739	129.539
2	6	LH	3.25	11.42	3.3	50.780	64.655
Core	1	-	3.594	-	-	-	10.145
$A_{core} = 10.145 \text{ mm}^2$						$A_g = 194.194 \text{ mm}^2$	

The top and bottom chords were subjected to an axial pretension of 664kN, which, based on the following Equation, corresponds to approximately $\frac{1}{3}$ of the ultimate breaking load (U.B.L.) of the three cables.

$$\text{U.B.L.} = A_s K S_{ult} \tag{11.1}$$

where, A_s is the net steel area of the spiral strand, K is the spinning loss factor, and S_{ult} is the ultimate wire tensile strength, which, for the present purposes, was assumed to equal 1520 N/mm².

As a result of the axial pretension in the bottom chord, the ties connecting the top and bottom chords experience a tensile force. The magnitude of this force, T_h , is calculated using the following equation given by Irvine (1981)

$$T_h = \frac{H_b(b_b - d_b)}{8l^2} \times \text{spacing} \tag{11.2}$$

where, H_b is the axial pretension in the bottom chord, b_b and d_b are defined in Fig. 11.1, the spacing is the distance between the vertical ties, and l is the span.

The axial stiffnesses of the top and bottom chords, K , along with the hangers, are

$$K = E \times A \tag{11.3}$$

where, E is either the full-slip or no-slip modulus of the chords (spiral strands), and A is their net steel area. The presently assumed magnitudes of the axial stiffnesses, K, are given in Table 11.3.

Table 11.3 – Magnitudes of the Chord and Hanger Axial Stiffnesses (EA).

Outside Diameter (mm)	Outer Lay Angle (Degrees)	Full-Slip Axial Stiffness, K_{fs} (kN)	No-Slip Axial Stiffness, K_{ns} (kN)
16.4	Hanger	22555.3	23689.1
48.96	12	194900.9	208615.6
48.35	18	157502.1	196908.6
49.56	24	97064.2	169655.8

The cable truss was loaded along the top chord with a variety of uniformly distributed loads (U. D. L.). The loads were applied at each node along the top chord of the cable truss (Fig. 11.1). The magnitude of each nodal force, P_n , was calculated using

$$P_n = \frac{\text{U.D.L.} \times 80}{15} \quad (11.4)$$

where, 80 is the span of the cable truss in metres, and 15 is the number of nodes. The assumed external vertical loads applied to the cable truss ranged in magnitude from 0 to 5 kN/m.

11.3 RESULTS

Fig. 11.2 shows the variations of the vertical deflection at the centre ($X = 40$ m) of the cable truss with changes in the externally applied vertical load, for all three different spiral strand constructions, based on the full-slip regime. The influence of the lay angle of the spiral strands on the vertical deflection at the centre of the cable truss can be clearly seen.

Fig. 11.3a shows the variations of the vertical deflection across the span of the cable truss under an externally applied vertical load of 2 kN/m, based on both the no-slip and full-slip regimes (as regards the axial stiffnesses of the spiral strands). The

external load of 2 kN/m was chosen as this is the maximum load corresponding to which the axial strain in the three spiral strands (used as top and/or bottom chords) is equal to or less than 5×10^{-3} . The effect of the lay angle, and, of using either the no-slip or full-slip strand axial stiffnesses on the vertical deflection of the cable truss can also be clearly seen. It should be noted that, for all the plots in both Figs. 11.3a and b (and, indeed, all the results in this chapter), due to large variations in the axial loads of the vertical hangers, the full-slip axial stiffness has invariably been used in relation to the vertical hanger deformations and associated axial forces.

Fig. 11.3b presents similar plots to those in Fig. 11.3a, but based on an externally applied load of 0.6375 kN/m, which corresponds to a maximum axial load range of approximately 5% of the magnitude of the axial pretension applied to the top and bottom chords.

Figs. 11.4a - c, show the values of the total force in each member of the cable truss, based on the spiral strands with lay angles of 12° , 18° and 24° , respectively, corresponding to the full-slip regime, and in the absence of any externally applied loads.

Figs. 11.5a - c, show the changes in the pretension in each member of the cable truss, based on the full-slip axial stiffnesses for the spiral strands with lay angles of 12° , 18° and 24° , respectively, with no external load applied to the truss.

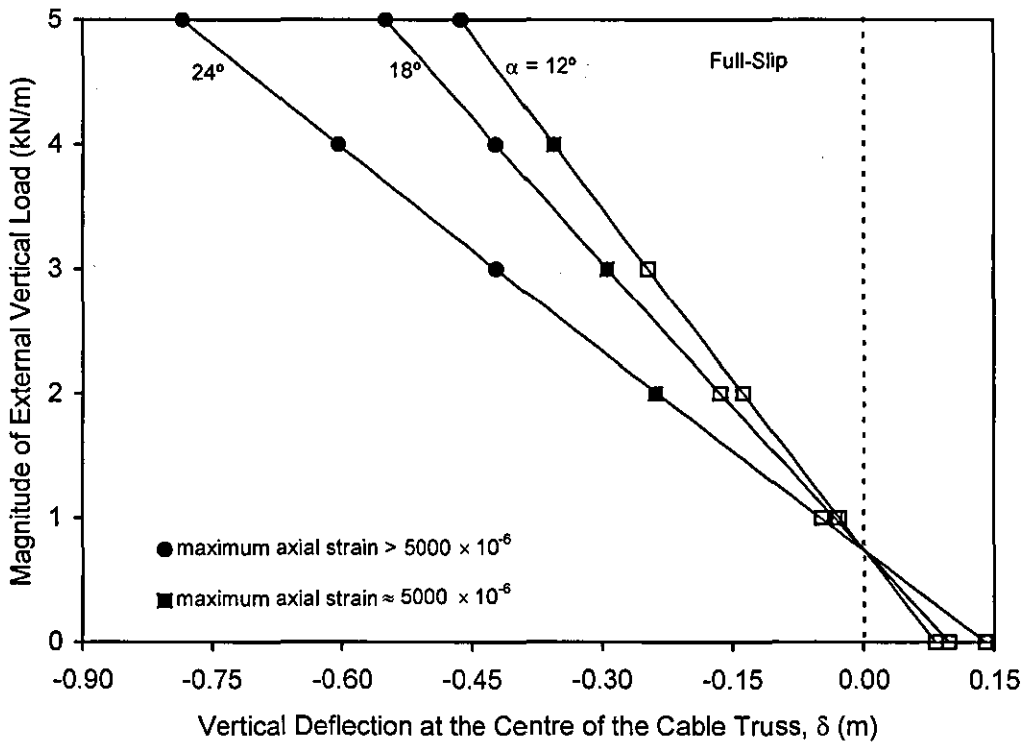


Fig. 11.2 - Variations of the Vertical Deflection at the Centre ($X = 40$ m) of the Cable Truss with Changes in the Externally Applied Load, for all Three Different Spiral Strand Constructions, Based on Their Full-Slip Axial Stiffnesses.

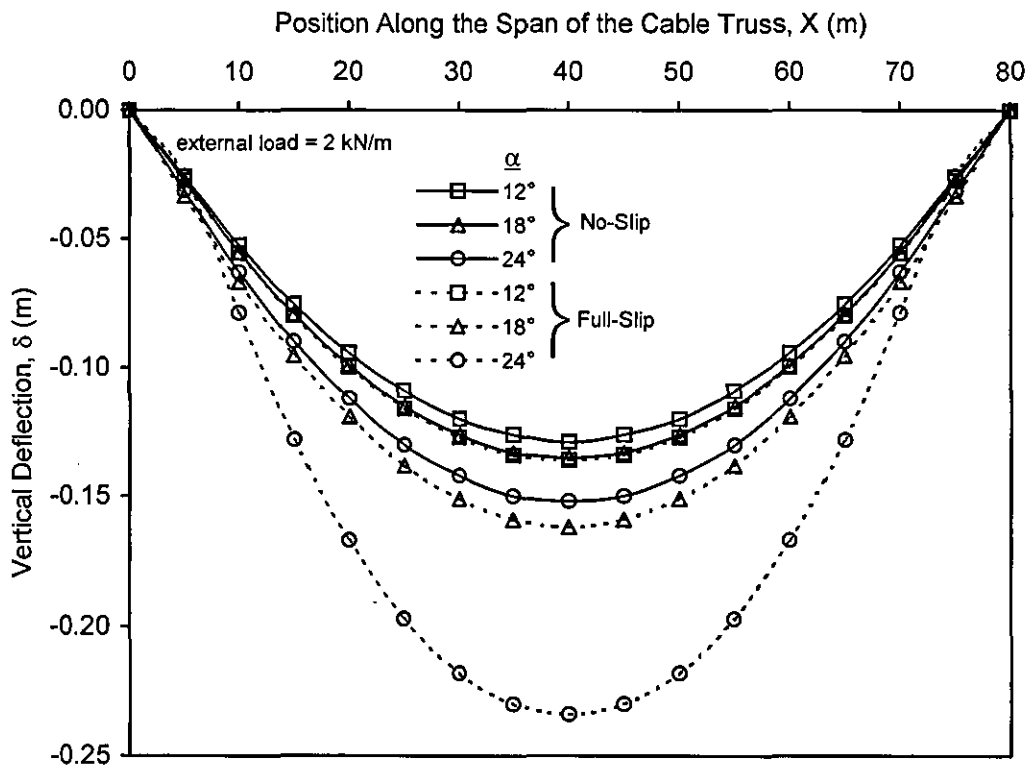


Fig. 11.3a - Variations of the Vertical Deflection Along the Span of the Cable Truss Under an Externally Applied Vertical Load of 2 kN/m, Based on Both the No-Slip and Full-Slip Axial Stiffnesses for the Top and Bottom Chords, with the Vertical Hangers Always Experiencing the Full-Slip Condition.

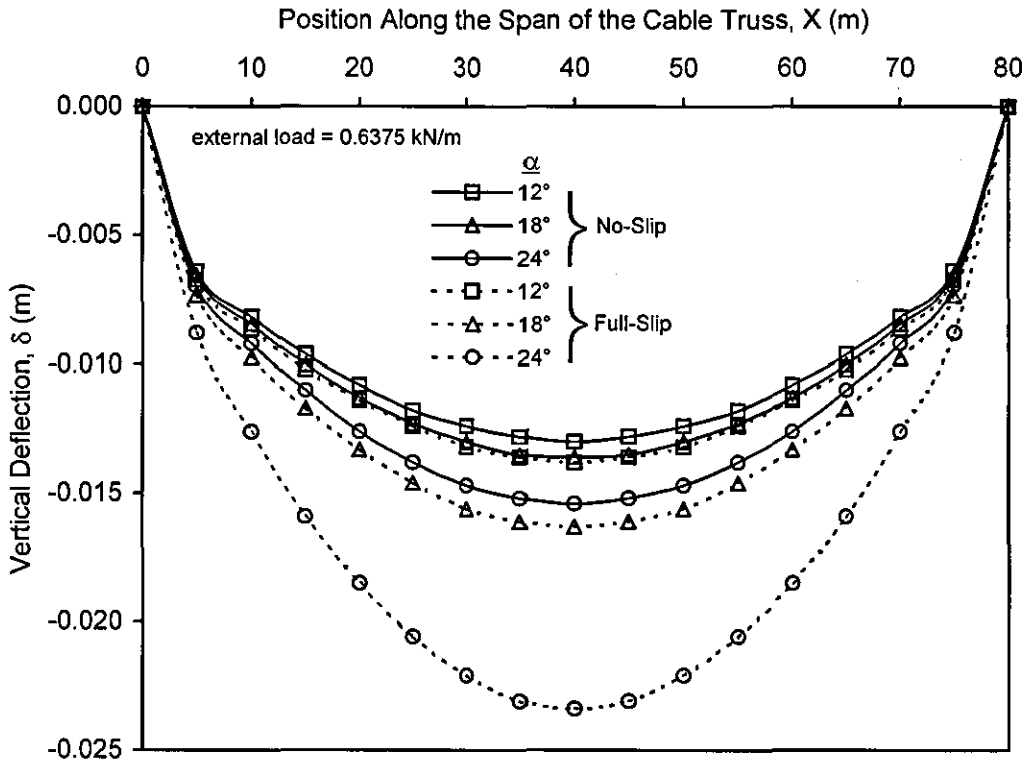


Fig. 11.3b - Variations of the Vertical Deflection Along the Span of the Cable Truss Under an Externally Applied Vertical Load of 0.6375 kN/m, Based on Both the No-Slip and Full-Slip Axial Stiffnesses for the Top and Bottom Chords, with the Vertical Hangers Always Experiencing the Full-Slip Condition.

Figs. 11.6a - c show the changes in pretension in each member of the cable truss, based on the spiral strands with lay angles of 12°, 18° and 24°, respectively, corresponding to the full-slip regime with an external load of 0.6375 kN/m applied to the cable truss. Figs. 11.7a - c show similar diagrams but based on the no-slip axial stiffnesses for the top and bottom chords, with the vertical hangers always experiencing the full-slip condition.

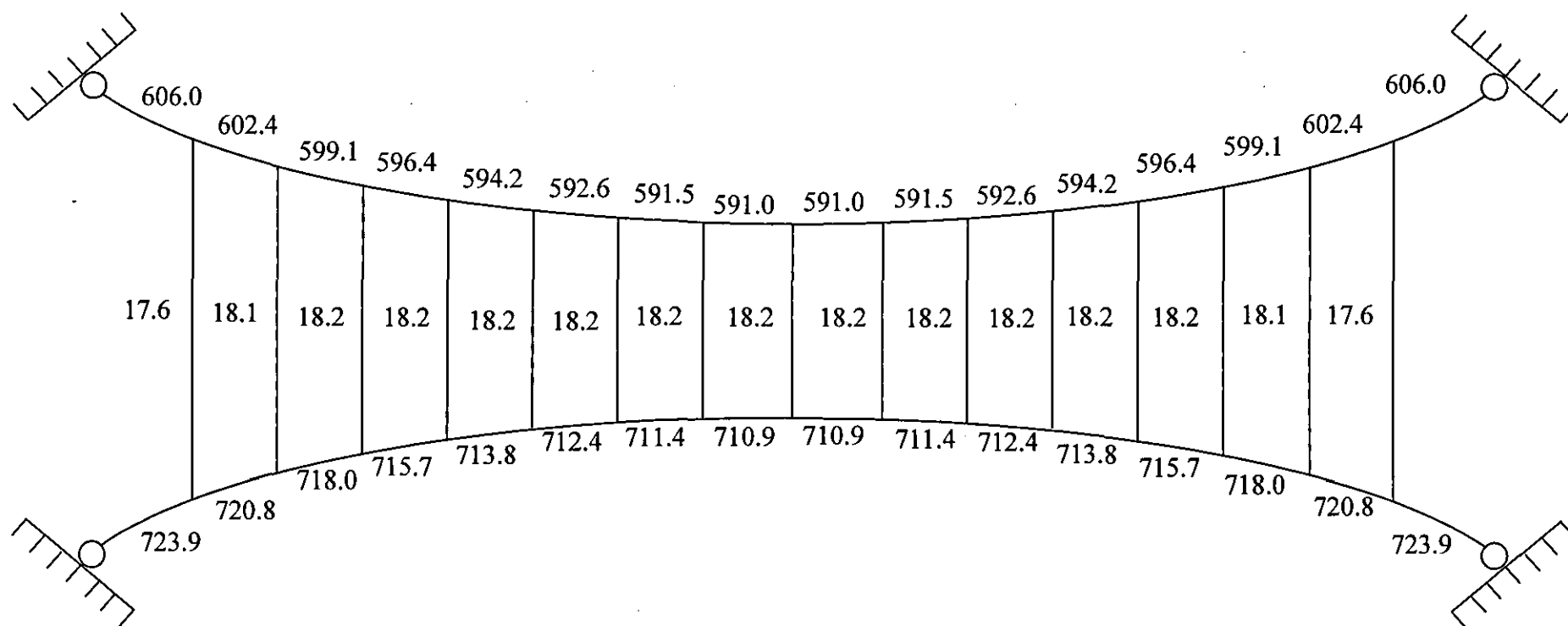


Fig. 11.4a - Diagram Showing the Total Force (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with No External Load Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.96 mm ($\alpha = 12$ degrees).

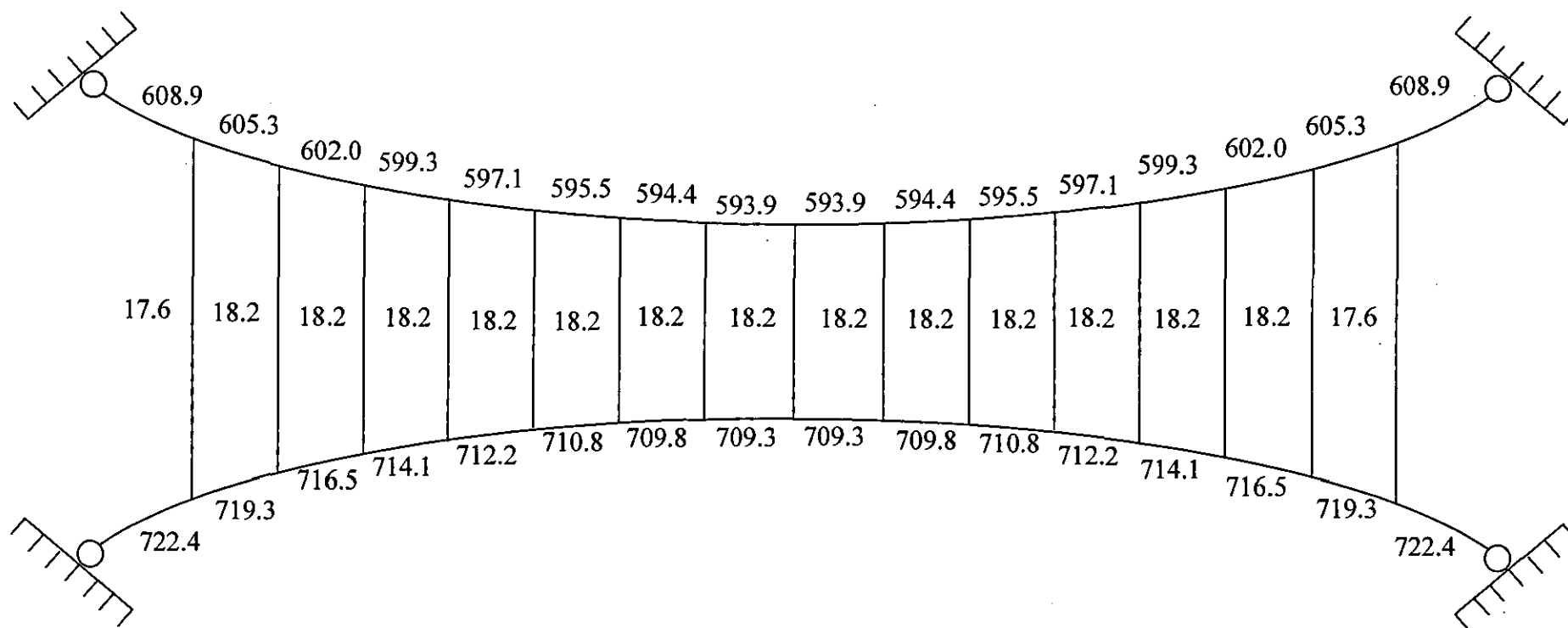


Fig. 11.4b - Diagram Showing the Total Force (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with No External Load Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.35 mm ($\alpha = 18$ degrees).

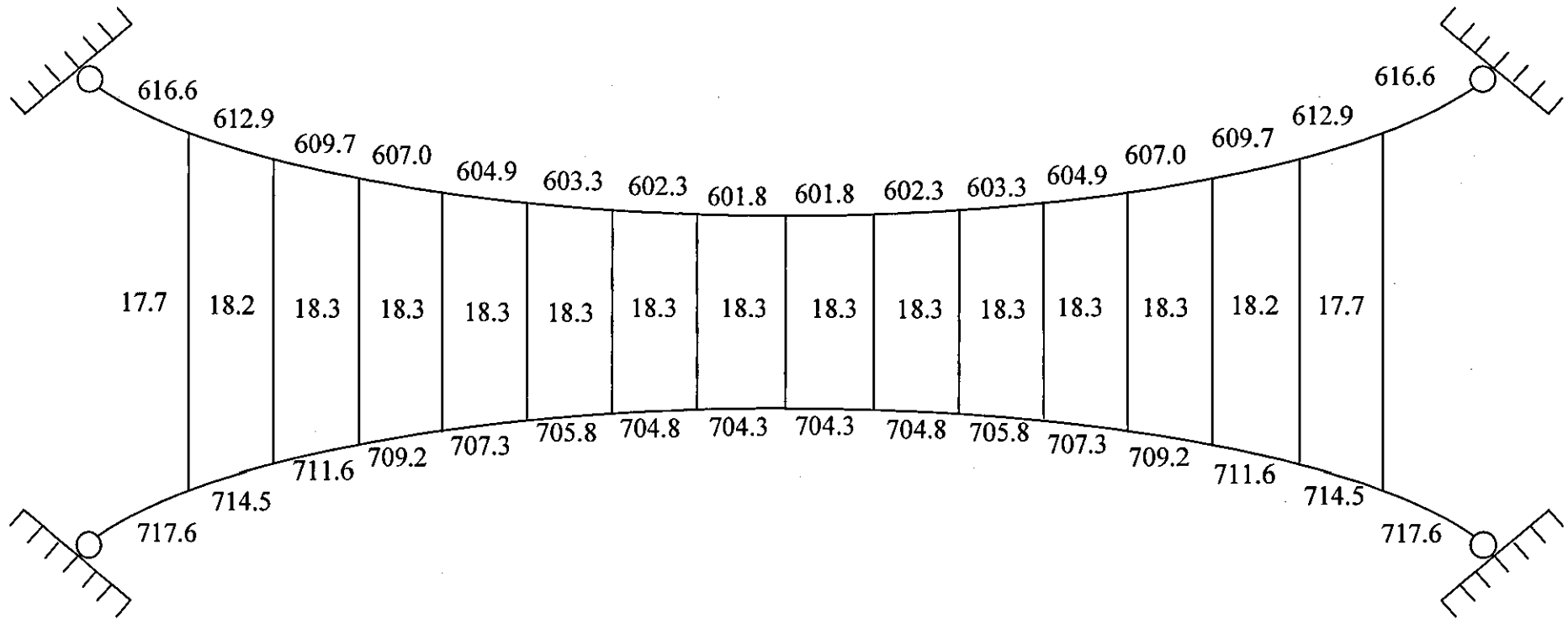


Fig. 11.4c - Diagram Showing the Total Force (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with No External Load Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 49.56 mm ($\alpha = 24$ degrees).

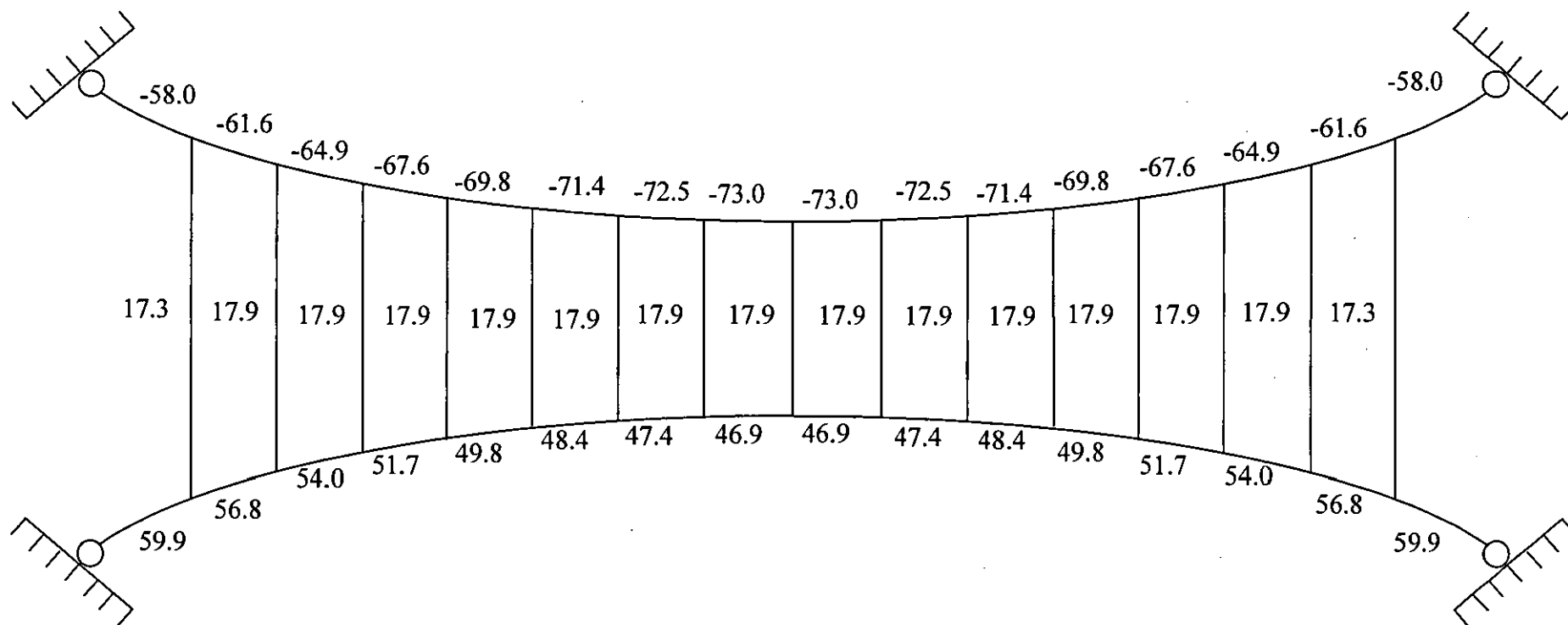


Fig. 11.5a - Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with No External Load Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.96 mm ($\alpha = 12$ degrees).

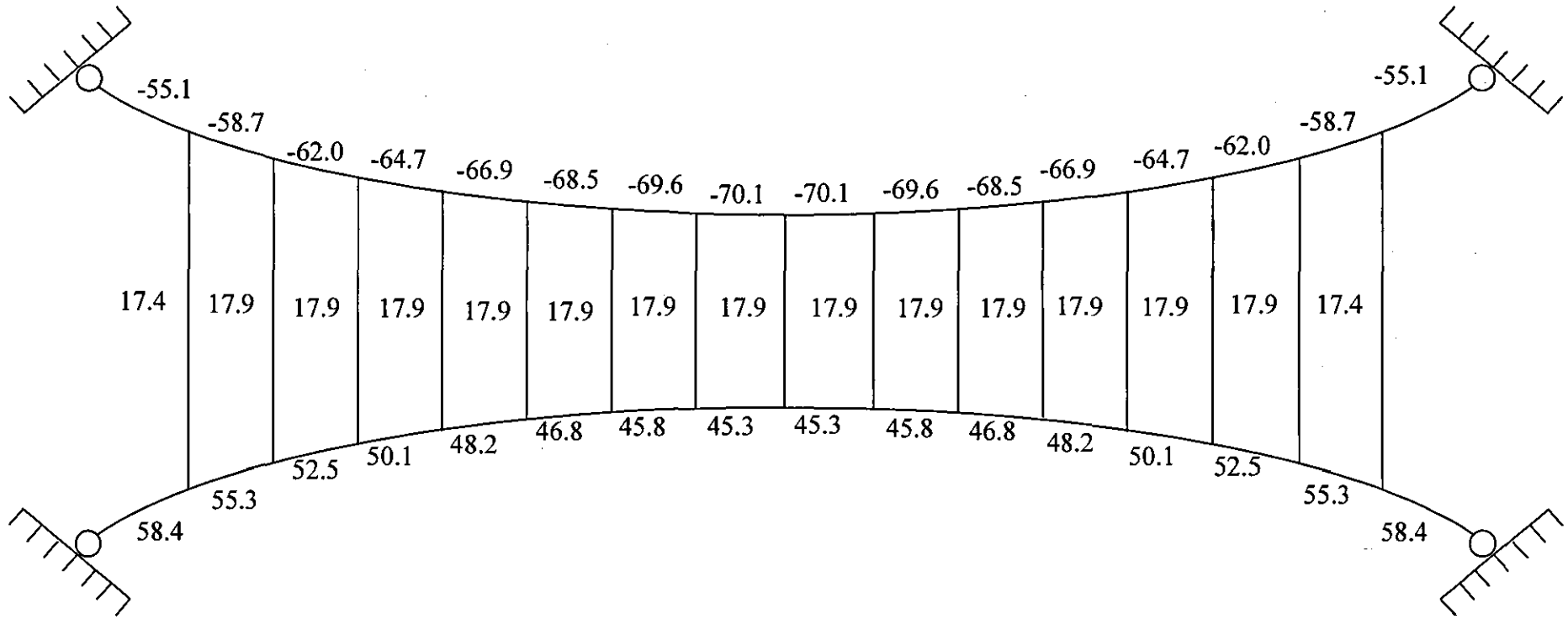


Fig. 11.5b - Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with No External Load Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.35 mm ($\alpha = 18$ degrees).

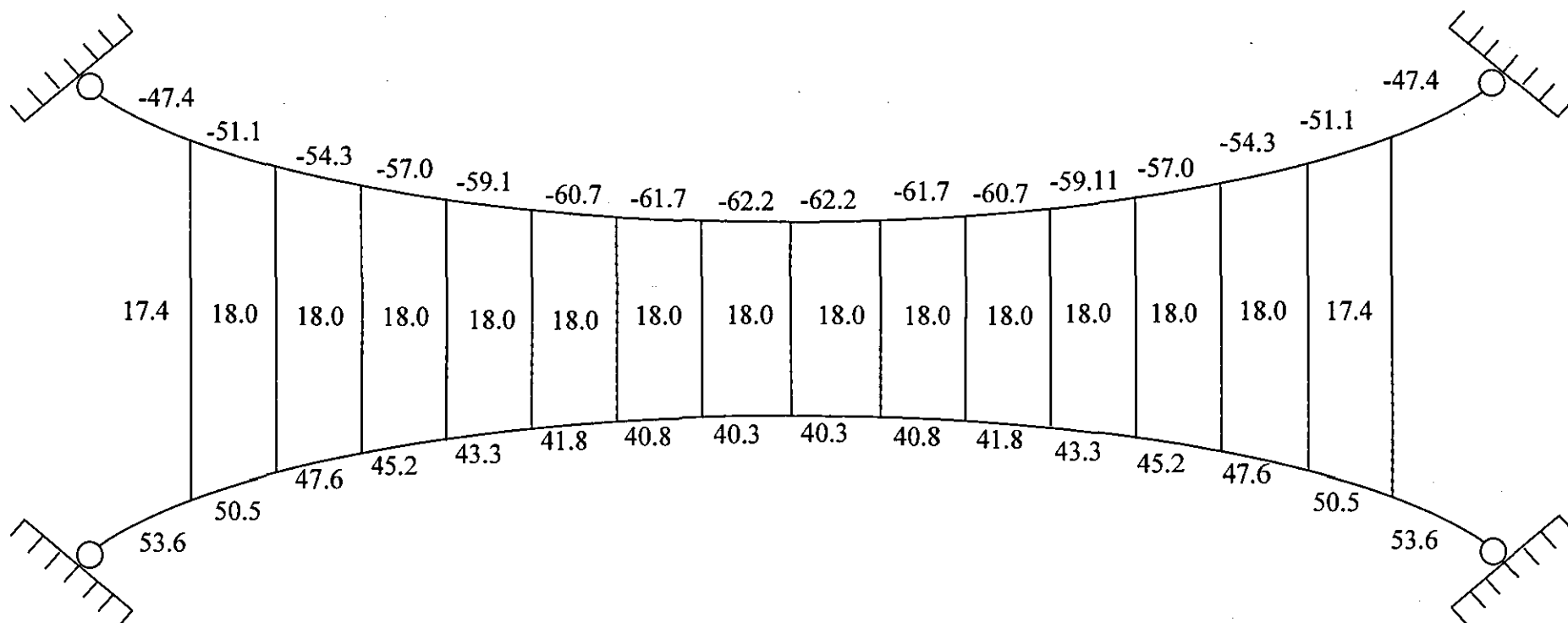


Fig. 11.5c - Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with No External Load Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 49.56 mm ($\alpha = 24$ degrees).

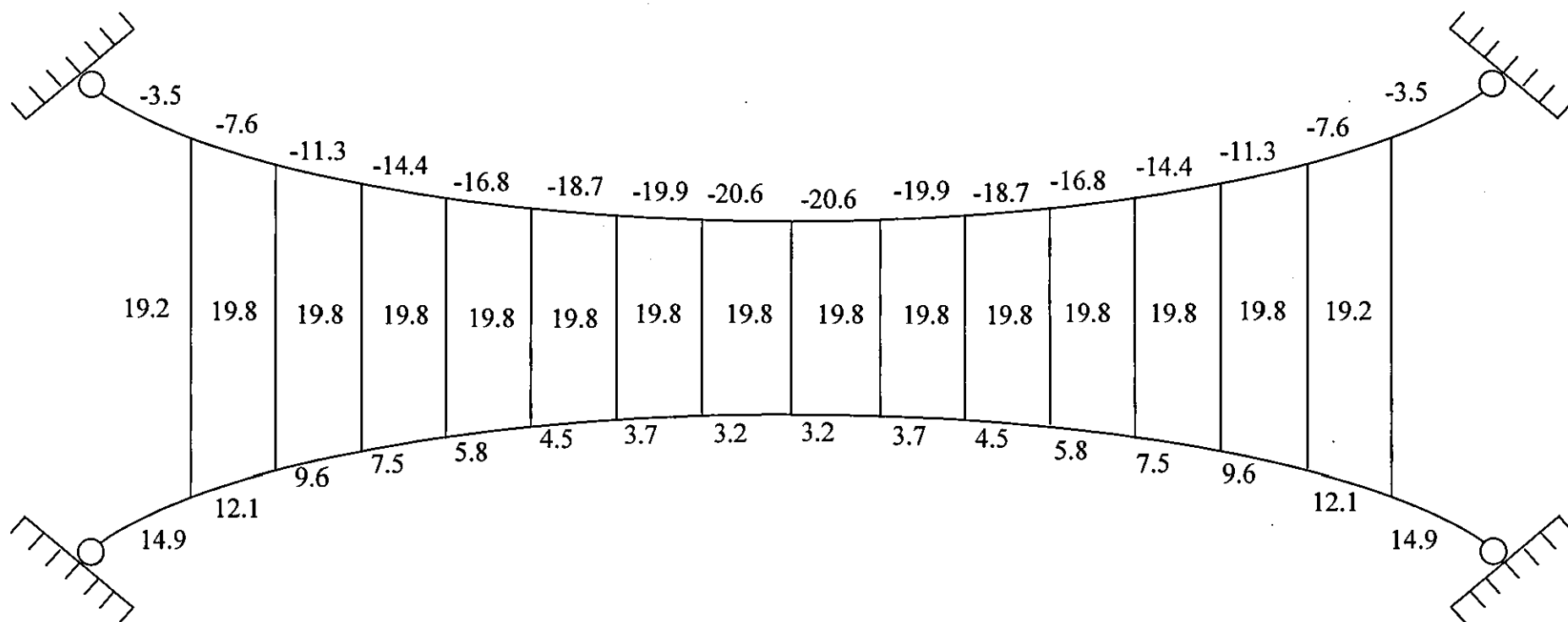


Fig. 11.6a - Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with an External Load of 0.6375 kN/m Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.96 mm ($\alpha = 12$ degrees).

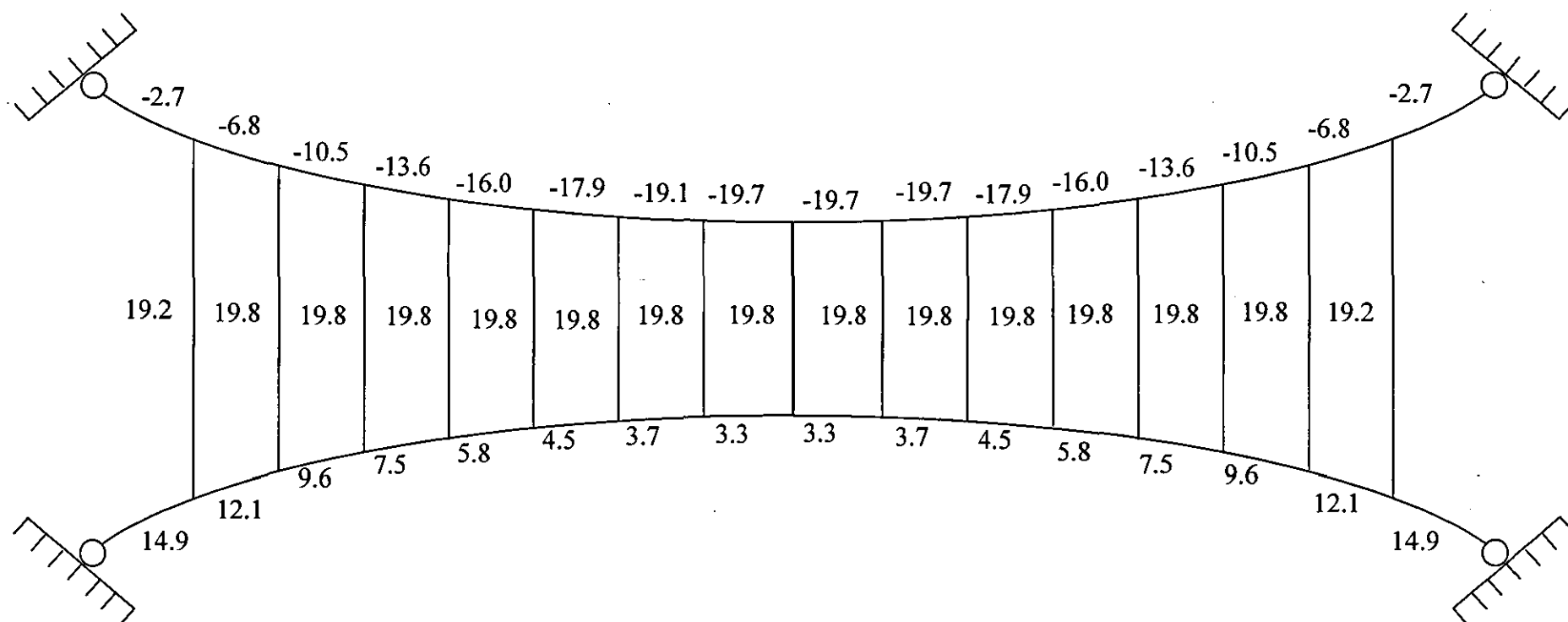


Fig. 11.6b - Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with an External Load of 0.6375 kN/m Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.35 mm ($\alpha = 18$ degrees).

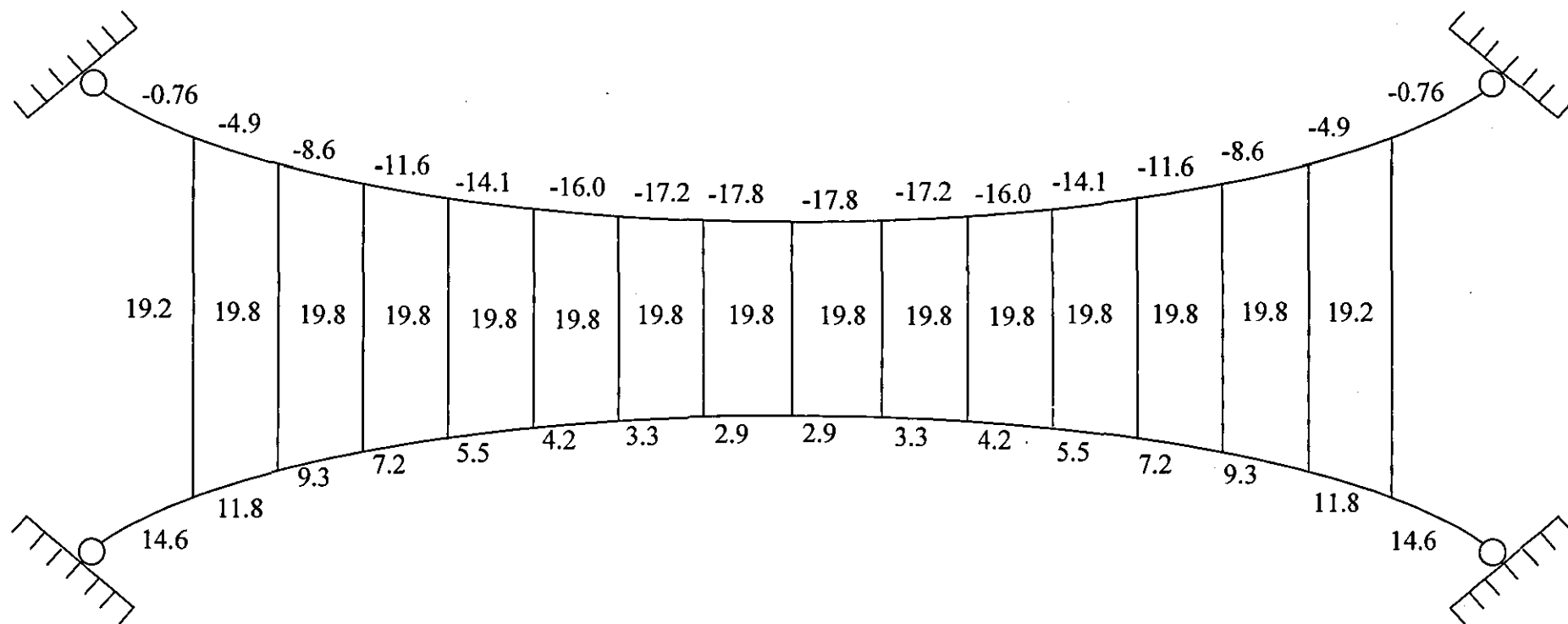


Fig. 11.6c- Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the Full-Slip Regime, with an External Load of 0.6375 kN/m Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 49.56 mm ($\alpha = 24$ degrees).

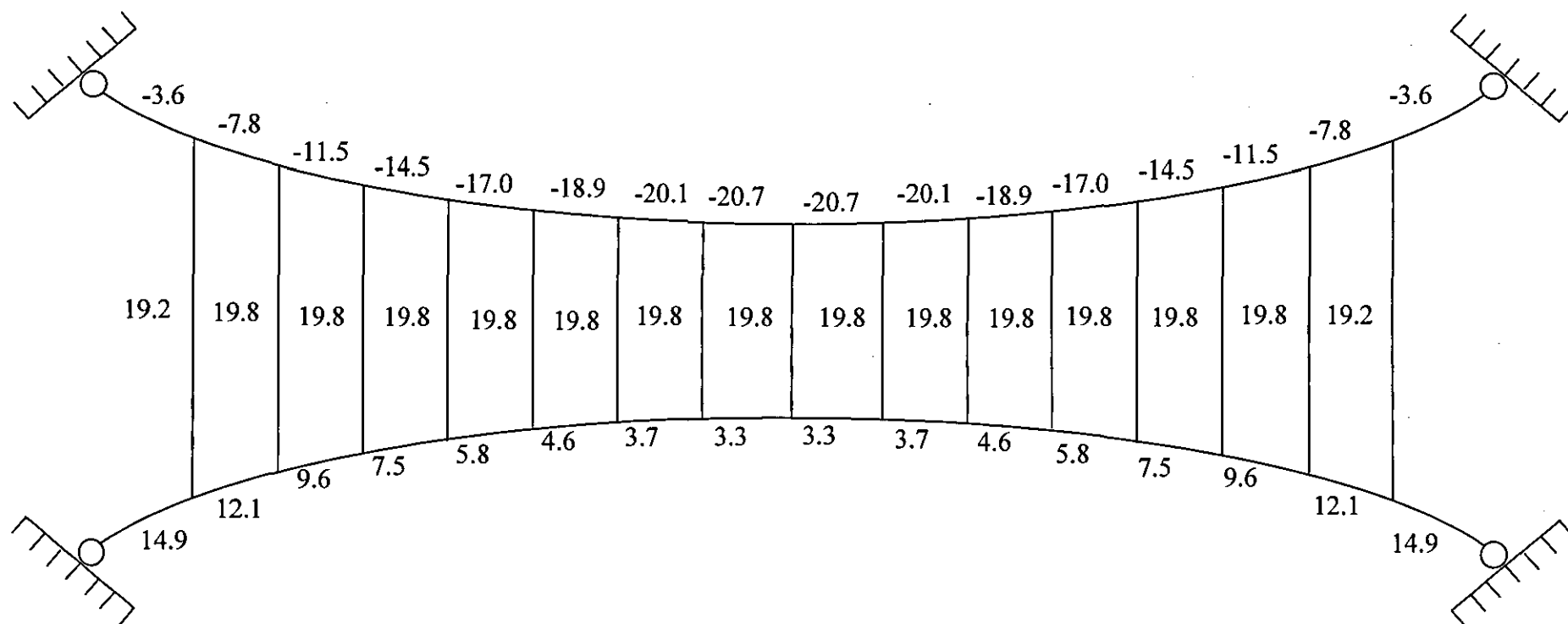


Fig. 11.7a- Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the No-Slip Regime for the Top and Bottom Chords, with an External Load of 0.6375 kN/m Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.96 mm ($\alpha = 12$ degrees).

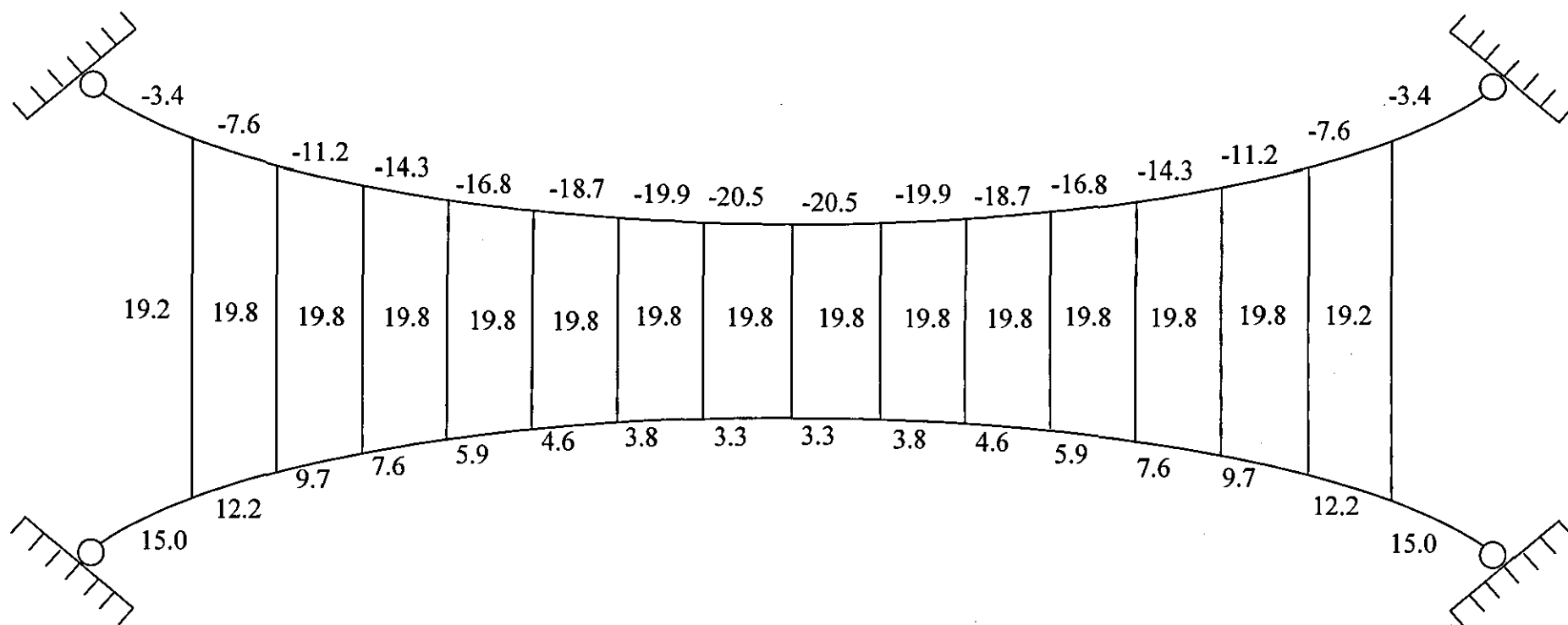


Fig. 11.7b- Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the No-Slip Regime for the Top and Bottom Chords, with an External Load of 0.6375 kN/m Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 48.35 mm ($\alpha = 18$ degrees).

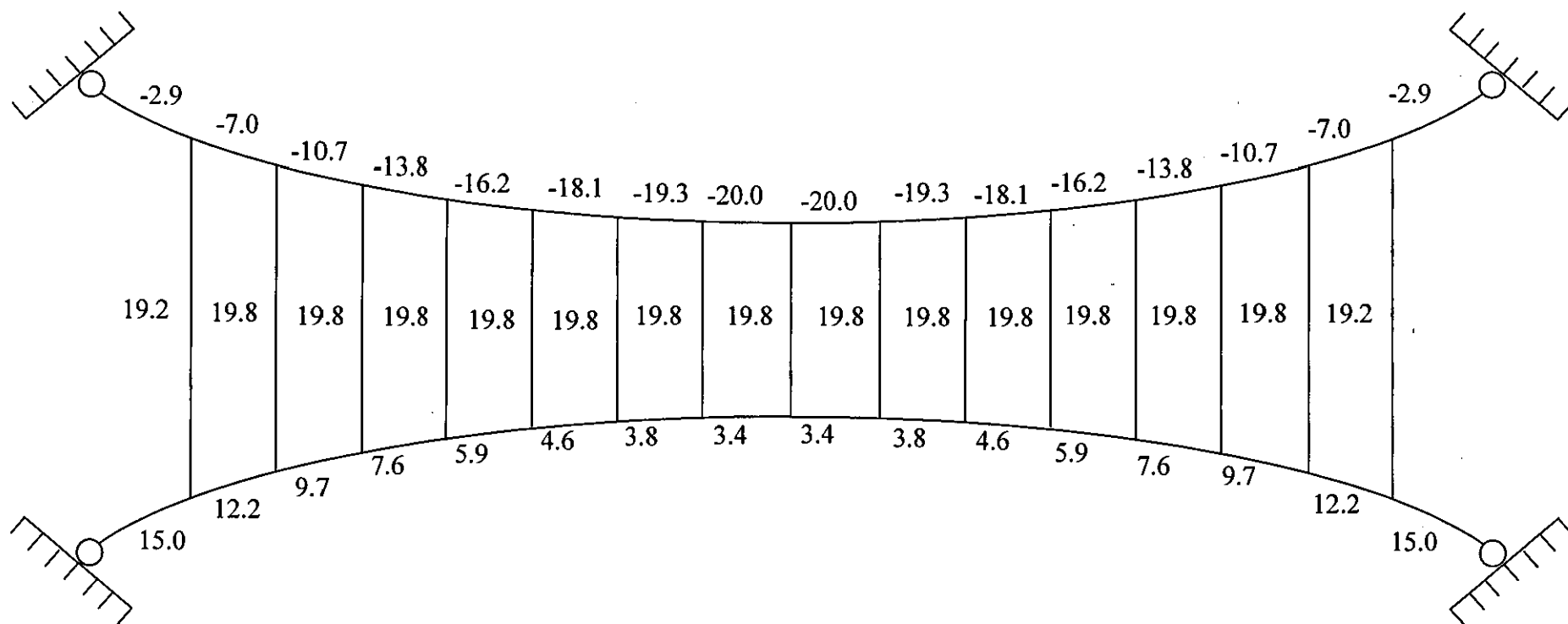


Fig. 11.7c- Diagram Showing the Change in Pretension (kN) in Each Member of the Cable Truss, Based on the No-Slip Regime for the Top and Bottom Chords, with an External Load of 0.6375 kN/m Applied to the Truss, and with the Top and Bottom Chords Having an Outside Diameter of 49.56 mm ($\alpha = 24$ degrees).

11.4 DISCUSSION

Based on the results shown in Fig. 11.2 it can be seen that, for a given externally applied load, the vertical deflection at the centre of the cable truss increases with increasing values of the lay angle. The differences between the magnitudes of the vertical deflections, between the results based on three different spiral strand constructions, increases fairly significantly as the external load increases. This is coupled with the variations of the location on the individual plots at which the strand axial strain exceeds 5×10^{-3} : for example, the external load at which the axial strain exceeds 5×10^{-3} for the 48.96 mm ($\alpha = 12^\circ$) outside diameter spiral strand is approximately twice that for the 49.56 mm ($\alpha = 24^\circ$) outside diameter spiral strand.

The effect of using either the no-slip or full-slip stiffnesses for the top and bottom chords (with the vertical hangers experiencing full-slip conditions) for calculating the deflected shape(s) of the truss is shown in Figs. 11.3a and b. The vertical deflection of the cable truss is (not surprisingly) greater, in connection with the full-slip (as opposed to no-slip) conditions. As the lay angle of the individual spiral strands increases, the differences between the magnitudes of the vertical deflection, as calculated using the no-slip and full-slip stiffness coefficients, increases sufficiently to be of practical importance.

11.5 CONCLUSIONS

Using the computer program developed by Broughton and Ndumbaro, the effect of the lay angle of the spiral strands forming the top and bottom chords of a cable truss, on the vertical deflection response of the truss, has been examined. It has been shown that the lower the lay angle of the spiral strands (and, hence, the greater their axial stiffness), for a given externally applied load, the lower would be the magnitude of the vertical deflections, with such changes being sufficiently significant to be of practical importance.

The significant practical implications of using the no-slip (as opposed to the full-slip) axial stiffnesses for the spiral strands forming the top and bottom chords of the cable truss, as regards the calculated values of the truss vertical deflections, have also been addressed in some detail. It is, however, worth mentioning that, in this context, the

behaviour of newly manufactured spiral strands which only undergo full-slip axial deformations, irrespective of the imposed $\frac{\text{axial load range}}{\text{mean axial load}}$ ratios, should be borne in mind, as, in practice, it is often newly manufactured (but prestretched) cables which are used for constructing new structures.

The present findings reinforce the soundness of the recommendations (based on the work of Hobbs and Raoof) in the Prestandard ENV 1993-2, Eurocode 3: Design of Steel Structures – part 2: Steel Bridges (October 1997) in relation to the use of the no-slip (as opposed to the full-slip) strand axial stiffnesses whenever of practical significance: as presently demonstrated, this may prove to be of practical relevance (depending on the lay angle of their helical cables) when analysing the vertical deformations of cable supported structures which have seen service-conditions for a number of years, with their helical cables having assumed a fully bedded-in condition.

CHAPTER 12

CONCLUSIONS

12.1 INTRODUCTION

The literature review highlighted the many practical uses of spiral strands and wire ropes (helical steel cables) across a wide range of industries, not only the onshore and offshore construction industries, but also the leisure and mining industries, amongst others.

It was concluded that, by and large, the early theoretical models developed for predicting the various structural characteristics of helical steel cables suffered from rather serious shortcomings. Indeed, when compared with available experimental data, it had become clear that such models could only accurately predict the behaviour of small (i.e. seven or nineteen wire) diameter strands, with such small scale models not being representative of the much larger diameter cables commonly used by the construction industry. Most importantly, the vast majority of earlier theoretical models had neglected important factors such as interwire friction and contact deformations. Hence, reliable information of practical use was rather scarce. In short, cable design and manufacture had traditionally been treated as an art, rather than a science, and as such, it was an area where the rule of thumb reigned supreme, with the previously available design and analysis methods being largely based on past commercial experience. At the same time, there has (over the last two decades) been a growing need for increasingly longer and larger diameter cables, particularly for use by the offshore industry, in connection with anchoring floating offshore platforms to the sea bed. It had, however, been a point of serious concern that using the previous knowledge on the behaviour of smaller diameter cables to try to predict the various characteristics of much larger ones, by a simple process of extrapolation, is a risky process (indeed, in many cases, unacceptable). In view of the fact that full scale testing is not only expensive, but also time consuming, there has been an urgent need for developing alternative theoretical models for the design and analysis of helical steel cables, which (once verified against carefully conducted test data) can be used for carrying out extensive theoretical parametric studies, with such an exercise leading

to the development of simple (but reliable) methods of analysis for direct practical use.

It was not until the development of the semi-continuous models, such as the orthotropic sheet model of Hobbs and Raoof, in which the layers of wires are treated as equivalent orthotropic sheets, that the problem of interwire friction and contact deformations could be analytically handled, and hence, the various structural characteristics of large diameter spiral strands could be predicted with a practically acceptable degree of accuracy. Over the last two decades, or so, the orthotropic sheet model has been used to reliably predict the overall axial, torsional and plane-section free-bending stiffnesses, and their associated frictional hysteresis under continued uniform cyclic loading, as well as interwire line-contact slippage plus interlayer torsional movements over the trellis contact patches. In a large number of cases, carefully obtained large scale test data have provided very encouraging support for the various predictions, based on the orthotropic sheet approach. Moreover, by carrying out hysteresis tests on fully bedded-in, as well as newly manufactured spiral strands, it has been shown that the hysteresis test data from newly manufactured spiral strands is very different from that of old (fully bedded-in) specimens that have seen in-service conditions for a number of years. In particular, it has been shown by Hobbs and Raoof that the orthotropic sheet model can reliably predict the frictional hysteresis of old and fully bedded-in axially preloaded spiral strands under superimposed axial and/or torsional uniform cyclic load perturbations. Moreover, by carrying out extensive theoretical parametric studies, on a wide range of spiral strand constructions, covering the full range of current manufacturing limits of strand (and wire) diameters and lay angles, simple design formulations (which are amenable to hand calculations, using a pocket calculator) have been developed for predicting the full-slip and/or no-slip axial, torsional and plane-section free-bending stiffnesses, plus the maximum hysteresis under uniform cyclic axial loading.

The results from the orthotropic sheet model, in relation to the full-slip and/or no-slip coupled axial/torsional stiffnesses, have been used to analyse the response of spiral strands to different forms of impact loads. Moreover, the estimated values of interwire/interlayer normal contact forces, based on the orthotropic sheet approach, have been used as an input into a theoretical model for determining the recovery

length of a fractured wire in any internal layer of an axially preloaded spiral strand. This is, however, by no means an exhaustive list of the theoretical and practical applications for which the orthotropic sheet theoretical model has been used: it is merely an attempt to display the versatility of this modelling approach. The question of axial and restrained bending fatigue has also enjoyed considerable attention in the available literature, by a number of researchers, including Raoof and his associates. In particular, Raoof and his associates have proposed fundamentally different theoretical models for estimating the axial and restrained bending fatigue life of spiral strands, based on first principles: Raoof's theoretical predictions of the axial and restrained bending fatigue life, which are based on the concepts of interwire stress concentration factor and fretting fatigue, respectively, have been supported by some extensive sets of large scale test data on spiral strands, including those obtained by Raoof and his associates, and also those based on specimens produced by different manufacturers and tested in a number of different institutions, both in the U. K. and abroad, hence, providing ample evidence for the general validity of both theoretical models. In particular, by carrying out extensive theoretical parametric studies, Raoof has again developed simple (hand-based) design methods, against both the axial and restrained bending fatigue failures, for use by busy practising engineers. Raoof has also extensively addressed the various structural characteristics of axially preloaded sheathed spiral strands currently used in deep-sea floating offshore platform applications, whereby he has extended the orthotropic sheet as well as the axial and restrained bending fatigue models (which were originally developed for in-air conditions), to cater for the presence of high external hydrostatic pressures exerted on sheathed spiral strands. Finally, Raoof and Kraincanic have developed a theoretical model, for predicting the no-slip and/or full-slip axial/torsional stiffness matrices for wire ropes with either independent wire rope cores (IWRC) or fibre cores, with the theoretical predictions supported by large scale test data from other sources.

A careful study of the publicly available literature suggested that, despite all the encouraging progress mentioned above, there are still a number of unresolved issues remaining in this field, some of which have formed the subject of the research reported in detail in this thesis: in the following section, the main findings of the present work will be briefly summarised. As a pre-requisite to this, however, it is, perhaps, also worth mentioning that, the vast majority of the developments reported in

this thesis are essentially extensions to the previous works of Raoof and his associates, hence, the reason behind putting much emphasis on their previous findings throughout the present work.

12.2 MAIN FINDINGS

By carrying out extensive theoretical parametric studies, on a wide range of multi-layered spiral strands, considerable doubt has been cast on the soundness of the suggestions by Jolicoeur, regarding the so-called improvements to the original orthotropic sheet model of Hobbs and Raoof. Although, the predictions based on the model containing a somewhat modified version of Jolicoeur's so-called improvements, were found to compare favourably with those based on the original model of Hobbs and Raoof, in view of certain oversights in Jolicoeur's approach, it is felt that Jolicoeur's contribution does not lead to any real improvements over the original model of Hobbs and Raoof. In his formulations, Jolicoeur has ignored the presence of strand curvature, not only for determining the orthotropic sheet stiffness coefficients, but also in calculating the crucial parameter c_k , which, using the equations proposed by Jolicoeur, turned out (in the majority of cases) to be positive, which violates the physical reality for the strand, where the wires in line-contact should experience normal compressive (and not tensile) contact forces.

Closed-form solutions for predicting the extensional-torsional wave speeds and displacements in axially preloaded helical cables experiencing a half-sine impact loading function at one end, with the other end fixed, have been developed. Using extensive theoretical parametric studies, based on these solutions, in conjunction with the closed-form solutions developed by Raoof et al. (1994) (for unit-step and triangular impact loading functions), the effects of varying the lay angle (over the full range of current manufacturing limits) on the various wave propagation characteristics have been examined. It has been argued that, due to the presence of interwire friction in axially preloaded helical cables, for sufficiently small levels of load perturbations applied to fully bedded-in axially preloaded helical cables, one should use the no-slip (rather than full-slip) version of the constitutive relations. It has also been demonstrated that the use of the no-slip version of the constitutive relations gains increasing importance as the lay angle increases. These findings may have significant practical implications, particularly in relation to the currently adopted techniques by

industry for calibrating the electronic boxes, which are subsequently used for the in-situ detection of individual wire fractures under, say, fatigue loading conditions associated with cable supported structures.

Using data from carefully conducted large scale free-bending experiments on an axially preloaded 39 mm outside diameter multi-layered spiral strand, in conjunction with a newly developed theoretical model, an insight is provided as regards certain characteristics of laterally deflected cables in the immediate vicinity of zinc socketed end terminations. It has been shown that the spiral strand plane-section bending stiffness may, indeed, be used for the theoretical determination of the radii of curvature at nominally fixed ends to strands undergoing, say, vortex shedding instabilities with associated maximum lateral deflections of the order of one cable diameter. It has also been demonstrated that for the socket-cable system, the effective point of end fixity is located, not (as commonly assumed by others) at the face of the zinc socketed termination, but well within the socket itself. The significant practical implications of this finding, in the context of the previously reported works by others, have been critically examined.

A simple (but reliable) experimental method for obtaining the effective bending stiffness of even a very large (e.g. 164 mm) diameter helical steel cable, which had, until now, proven to be elusive, has been developed. Unlike traditional methods for determining the effective bending stiffness of a helical cable, which had invariably been too dependent upon the specific experimental technique employed, the current method is believed to be a significant step forward in measuring the effective bending stiffness, for practically any cable diameter, at a reasonable cost and effort, involving minimal physical interface with the imposed cable deformations.

A simple (hand-based) formulation (based on a generalised form of Hruska's original formulations), has been developed for obtaining reliable estimates of the no-slip and/or full-slip axial stiffnesses for wire ropes with either IWRC or fibre cores. The simplified method leads to the same predictions as the original theoretical model of Raoof and Kraincanic, with the latter being mathematically too complex for use by busy practising engineers – hence, the value of the present simple approach which is amenable to hand calculations, using a pocket calculator. Based on the correlations

between the predictions of the axial stiffnesses as calculated using Hruska's simple formulation, and also, the more complex (but refined) method of Raoof and Kraincanic, it has been argued that (as far as the no-slip and/or full-slip wire rope axial stiffnesses are concerned) the lay angles (of both the wires in the strands and the strands in the rope) are the prime (controlling) geometrical parameters.

Using the axial fatigue theoretical model of Raoof, and by carrying out extensive parametric studies on a wide range of sheathed spiral strand constructions, a new set of design S-N curves for predicting the axial fatigue life to first outermost (or innermost) wire fracture, both at (or in the vicinity) of the end terminations, as well as in the free-field, experiencing a wide range of external hydrostatic pressures, have been developed. The results clearly show the first order (controlling) effect of the lay angle on the axial fatigue life of sheathed spiral strands, experiencing high levels of external hydrostatic pressure. It has been shown that, although the fatigue life to first innermost layer wire fracture is largely unaffected by the magnitude of the applied external hydrostatic pressure, the fatigue life to first outermost layer wire fracture may be significantly reduced under the influence of sufficiently high levels of external hydrostatic pressure. The practical importance of taking the detrimental influence of the end terminations (on the axial fatigue life) into account is also demonstrated, with the fatigue life to first outermost (or innermost) wire fracture at the termination being shown to be substantially less than the fatigue life to first outermost (or innermost) wire fracture in the free-field (i.e. sufficiently away from the end terminations, not to be influenced by their detrimental effects). Finally, the design S-N curves have been compared with others (corresponding to in-air conditions) which are more commonly referred to in the literature, and it has been shown that, in certain cases, these design S-N curves, particularly the one proposed by API, may provide alarmingly unconservative results for practical applications.

Extensive theoretical parametric studies, covering a wide range of sheathed spiral strand constructions and levels of external hydrostatic pressure (as in deep sea applications), have been carried out to provide simple (hand-based) formulations aimed at estimating the values of the maximum axial and/or torsional frictional hysteresis, along with the corresponding $\frac{\text{axial load range}}{\text{mean axial load}}$ ratio and

range of twist $\frac{\pi}{2}$, respectively, at which they occur. It has theoretically been shown that, at all levels of strand mean axial strain and water depth, the axial hysteresis under uniform cyclic loading may be significantly increased (with the associated maximum torsional frictional hysteresis decreasing) by quite modest increases in the lay angle, within current manufacturing limits. The magnitudes of both the maximum theoretical axial and/or torsional hysteresis in sheathed spiral strands, however, have been found to be largely unaffected by the level of external hydrostatic pressure.

Recommendations have been made by means of which reliable test data for the axial fatigue life and hysteresis of large diameter (multi-layered) spiral strands may be obtained by resorting to tests on smaller diameter (by a scaling factor of, say, 2-3) spiral strands as an alternative to the much more expensive and difficult option of full scale testing. Very briefly, it has been shown that a scaling factor of, say, 2-3, with the lay angles kept the same, may be adopted for obtaining realistic test data with considerable financial savings. With regards to the torsional hysteresis, however, testing smaller diameter spiral strands to give an indication of the torsional hysteresis of larger diameter (even with a scaling factor of 2-3) strands has been shown not to lead to practically acceptable results, over the full range of lay angles and amplitudes of torsional deformations, although, a scaling factor of, say, 2-3 (with the lay angles kept the same) has been found to provide a somewhat approximate idea of the maximum level of torsional frictional hysteresis, for a given lay angle and strand mean axial strain.

The example of a two-dimensional cable truss (made-up of axially preloaded spiral strands) has been chosen (with the truss being analysed using a computer program developed by Broughton and Ndumbaro) to establish the effect of choosing the strand no-slip axial stiffnesses (as opposed to the full-slip values) for calculating the deflection response of such a structure under serviceability loading conditions. Obviously, following the work of Raoof and his associates, the lower the lay angle of the spiral strands, the greater would be the strands' axial stiffnesses and, hence, the lower the magnitude of the vertical deflections. The so-obtained estimates of the vertical deflections, based on either the full-slip or the no-slip strand axial stiffnesses, were found to be (in some cases) sufficiently different to make a practically

significant difference. In particular, increasing the lay angle, in the supporting spiral strands, has been shown to lead to greater discrepancies (which can be sufficiently large to be of practical importance) between the estimates of the vertical deflections, as calculated using the two alternative no-slip and full-slip strand axial stiffnesses. This finding reinforces the soundness of the recommendations (based on the work of Hobbs and Raoof) in the prestandard ENV 1993-2, Eurocode 3: Design of Steel Structures Part 2: Steel Bridges (October 1997) in relation to the use of the no-slip spiral strand stiffnesses (rather than their full-slip values) wherever of practical significance.

REFERENCES

- Alani, A., and Raoof, M. (1995), "Axial Fatigue Characteristics of Large Diameter Spiral Strands", Proceedings of 5th International Offshore and Polar Engineering Conference, J. S. Chung, M. Sayed, and A. M. Gresnight (eds.), The Hague, The Netherlands, Vol. II, June, pp. 260-265.
- Alani, A., and Raoof, M. (1997), "Effect of Mean Axial Load on Axial Fatigue Life of Spiral Strands", International Journal of Fatigue, Vol. 19, No. 1, January, pp. 1-11.
- API 2FP1 (RP2FP1) (1991), "Draft API Recommended Practice for Design, Analysis, and Maintenance of Mooring for Floating Production Systems. First Edition, May.
- Arend, M. (1993), "The Load Transfer Mechanisms Anchoring High Strength Tension Elements", O.I.P.E.E.C. Round Table Conference, Delft, The Netherlands, pp. III52-III67.
- Blanco, J. A., Costello, G. A. (1974), "Cylindrical Constraint of Helical Springs", Journal of Applied Mechanics, ASME, Vol. 41, December, pp. 1138-1140.
- Blouin, F., and Cardou, A. (1989), "A Study of Helically Reinforced Cylinders Under Axially Symmetric Loads and Application to Strand Mathematical Modelling", International Journal of Solids and Structures, Vol. 25, No. 2, pp. 189-200.
- British Standards Institution (1958), "Sockets for Wire Ropes", BSI, London, BS 463, Part 1.
- Broughton, P., and Ndumbaro, P. (1994). The Analysis of Cable & Catenary Structures. Thomas Telford Services Ltd., London, U. K.
- Cantin, M. M., Cubat, D., Nguyen Xuan, T. (1993), "Experimental Analysis and Modelisation of the Stiffness in Torsion of Wire Ropes", O.I.P.E.E.C Round Table Conference, Delft, The Netherlands, pp. II67-II77.

Cardou, H., and Jolicoeur, C. (1997), "Mechanical Models of Helical Strands", *Applied Mech. Review*, ASME, Vol. 50, No. 1, January, pp. 1-14.

Casey, N. F. (1993), "The Fatigue Endurance of Wire Ropes for Mooring Offshore Structures", O.I.P.E.E.C Round Table Conference, Delft, The Netherlands, pp. 124-149.

Chaplin, C. R. (1993), "Prediction of Offshore Mooring Ropes", O.I.P.E.E.C Round Table Conference, Delft, The Netherlands, pp. 150-175.

Chaplin, C. R., and Potts, A. E. (1991), "Wire Rope Offshore – A Critical Review of Wire Rope Endurance Research Affecting Offshore Applications", University of Reading Research Report for the Department of Energy, HMSO Publication OTH91341.

Chaplin, C. R., and Sharman, P. C. (1984), "The Mechanics of Load Transfer in Resin Socketed Terminations", *Wire Industry*, Vol. 50, October, pp. 749-751.

Chapman, R. W. (1908), "The Stresses in Wire Ropes due to Bending", *The Engineering Review*, Vol. 29, July-December, pp. 258-269.

Chi, M. (1971), "Analysis of Multi-Wire Strands in Tension and Combined Tension and Torsion", Catholic University of America, Technical Report 71-9, Washington D. C.

Chi, M. (1972), "Analysis of Operating Characteristics of Strands in Tension Allowing End Rotation", Ocean Technology Division Meeting, Paper No. 73-WA/Oct-19, ASME, New York, November, pp. 372-398.

Chien, C. H., and Costello, G. A. (1985), "Effective Length of a Fractured Wire in Wire Rope", *Journal of Engineering Mechanics*, ASCE, Vol. 111, No. 7, July, pp. 952-961.

Christen, R., and Oplatka, G. (1973), "Tail Cones: Does Doubling Back of the Wire Ends Increase the Safety", *Wire*, July/August, pp. 160-164.

Claren, R., and Diana, G. (1969a), "Dynamic Strain Distribution of Loaded Stranded Cables", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-88, No. 11, pp. 1678-1686.

Claren, R., and Diana, G. (1969b), "Mathematical Analysis of Transmission Line Vibration", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-88, No.12, December, pp.1741-1771.

Conway, T. A., and Costello, G. A. (1991), "Response of a Strand with Elliptical Outer Wires", *International Journal of Solids and Structures*, Vol. 28, No. 1, pp. 33-42.

Costello, G. A. (1977), "Large Deflections of Helical Spring due to Bending", *Journal of the Engineering Mechanics Division, ASCE*, Vol. 103, No. EM3, May, pp. 481-487.

Costello, G. A. (1983), "Stresses in Multi-Layered Cables", *Journal of Energy Resources Technology, ASME*, Vol. 105, pp. 337-340.

Costello, G. A., and Butson, G. J. (1982), "Simplified Bending Theory for Wire Rope", *Journal of the Engineering Mechanics Division, ASCE*, Vol. 108, No. EM2, pp. 219-227.

Costello, G. A., and Miller, R. E. (1979), "Lay Effect of Wire Rope", *Journal of the Engineering Mechanics Division, ASCE*, Vol. 105, No. EM4, August, pp. 597-608.

Costello, G. A., and Phillips, J. W. (1974), "A More Exact Theory for Twisted Wire Cables", *Journal of the Engineering Mechanics Division, ASCE*, Vol. 100, No. EM5, October, pp. 1096-1098.

Costello, G. A., and Phillips, J. W. (1976), "Effective Modulus of Twisted Wire Cables", *Journal of the Engineering Mechanics Division, ASCE*, Vol. 102, No. EM1, February, pp. 171-181.

Costello, G. A., and Sinha, S. K. (1977a), "Static Behaviour of Wire Rope", *Journal of the Engineering Mechanics Division, ASCE*, Vol. 103, No. EM6, December, pp. 1011-1022.

Costello, G. A., and Sinha, S. K. (1977b), "Torsional Stiffness of Twisted Wire Cables", *Journal of the Engineering Mechanics Division, ASCE*, Vol. 103, No. EM4, August, pp. 766-770.

Dobyns, A. L. (1981), "Analysis of Simply Supported Orthotropic Plates Subject to Static and Dynamic Loads", *AIAA Journal*, Vol. 19, No. 5, May, pp. 642-650.

Dodd, J. M. (1981), "Resin as a Socketing Medium", *Wire Industry*, Vol. 48, May, 1981, pp. 343-344.

Durelli, A. J. (1973), "Response of Epoxy Oversized Models of Strands to Axial and Torsional Loads", *Experimental Mechanics*, Vol. 13, August, pp. 313-321.

Durelli, A. J., Machida, S., and Parks, V. J. (1972), "Strains and Displacements on a Steel Wire Strand", *Naval Engineers Journal*, Vol. 84, No. 6, December, pp. 85-93.

Feyer, K., and Schiffner, G. (1986), "Torque and Torsional Stiffness of Wire Rope-Part I", *Wire*, Vol. 36, No. 8, pp. 318-320.

Feyer, K., and Schiffner, G. (1987), "Torque and Torsional Stiffness of Wire Rope-Part II", *Wire*, Vol. 37, No. 1, pp. 23-30.

Forestier-Walker, E. R. (1952), "A History of the Wire Rope Industry in Great Britain", *British Wire Rope Manufacturers*.

Gabriel, K. (1985), "On the Fatigue strength of Wires in Spiral Ropes", *Journal of Energy Resources Technology*, ASME, Vol. 107, March, pp. 107-112.

Gabriel, K., and Helmes, F. (1991), "The Mechanics of Socketing: The Zinc Alloy Cast Cone as a Special Compound Structure", *Proceedings of the First International Offshore and Polar Engineering Conference*, Edinburgh, August, pp. 257-264.

Gabriel, K., and Nürnburger, U. (1992), "Failure Mechanisms in Fatigue", IABSE Workshop on Length Effect on Fatigue of Wires and Strands, El Poular, Madrid, September, 1992, International Association for Bridge and Structural Engineering (IABSE), Zürich, Report No. 66, pp. 51-72.

Gatham, D. W., (1979), "Resin Socketing for Wire Rope Attachments", *Wire Journal*, Vol. 12, Part 6, pp. 82-85.

Gecha, E. Ya. (1989), "Stresses and Strains in Cables Under the Action of Tensile Forces and External Hydrostatic Pressures", *Soviet Machine Science*, No. 1, pp. 30-36.

Green, A. E., and Laws, N. (1966), "A General Theory of Rods", *Proceedings of the Royal Society, London*, A283, pp. 145-155.

Hall, H. M. (1951), "Stresses in Small Wire Ropes", *Wire and Wire Products*, Vol. 26, March, pp. 228 and 257-259.

Hanzawa, M., Yokoto, H., Toda, Y., and Yokoyama, K. (1982), "Fatigue Behaviour of Large Diameter Ropes", *Society of Petroleum Engineers Journal*, Vol. 22k (3), June, pp. 420-425.

Hearle, J. W. S. (1996), "Ropes: From Ancient Craft to Modern Engineering. Pt. 2: Mooring in Deep Waters", *Textile Horizons*, Vol. 16, No. 3, June/July, pp. 28-31.

Hearmon, R. F. (1961), *Applied Anisotropic Elasticity*. Oxford, Oxford University Press.

Hobbs, R. E., and Ghavami, K., with Wyatt, T. A. (1978), "Fatigue of Socketed Cables: In Line and Transverse Tests on 38 mm Specimens; Internal Damping in 38mm Specimens", Imperial College, Civil Engineering Department, Ceslic Report SC2.

Hobbs, R. E., and Nabijou, S. (1995), "Changes in Wire Curvature as a Rope is Bent Over a Sheave", *Journal of Strain Analysis*, Vol. 30, No. 4, pp. 271-281.

Hobbs, R. E., and Raoof, M. (1982), "Interwire Slippage and Fatigue Prediction in Stranded Cables for TLP Tethers" in 'Behaviour of Offshore Structures', Chrysosostomidis, C. and Connor, J.J.(eds.) Hemisphere Publishing/McGraw- Hill, New York, Vol. 2, pp.77-99. (Proceedings, 3rd International Conference on Behaviour of Offshore Structures, 1982, M.I.T., Cambridge, MA, U.S.A.).

Hobbs, R. E., and Raoof, M. (1984), "Hysteresis in Bridge Strand", *Proceedings of the Institution of Civil Engineers, Part II*, Vol. 77, December, 1984, pp. 445-464.

Hobbs, R. E., and Raoof, M. (1986), "The Behavior of Large Spiral Strands", *Proceedings of the 5th International Offshore Mechanics and Arctic Engineering Symposium*, ASME, Tokyo, Japan, Vol. 3, pp. 543-549.

Hobbs, R. E., and Raoof, M. (1994), "Mechanism of Fretting Fatigue in Steel Cables", *International Journal of Fatigue*, Vol. 16, June, pp. 273-280.

Hobbs, R. E., and Raoof, M. (1996), "Behaviour of Cables Under Dynamic or Repeated Loading", *Journal of Constructional Steel Research*, Vol. 39, No. 1, pp. 31-50.

Hobbs, R. E., and Smith, B. W. (1983), "Fatigue Performance of Socketed Terminations to Structural Strands", Proceedings of the Institution of Civil Engineers, Part II, Vol. 75, March, pp. 35-48.

Hruska, F. H. (1951), "Calculation of Stresses in Wire Ropes", Wire and Wire Products, Vol. 26, September, pp. 766-767 and 799-801.

Hruska, F. H. (1952), "Radial Forces in Wire Ropes", Wire and Wire Products, Vol. 27, May, pp. 459-463.

Hruska, F. H. (1953), "Tangential Forces in Wire Ropes", Wire and Wire Products, Vol. 28, May, pp. 455-460.

Huang, N. C. (1978), "Finite Extension of an Elastic Strand with a Central Core", Journal of Applied Mechanics, ASME, Vol. 45, December, pp. 852-858.

Huang, Y. P. (1993), "Predictions of Static and Dynamic Performance of Multi-Layered Spiral Strands Under Various Loading Conditions", Thesis Presented to South Bank University in Partial Fulfilment of the Requirements for the Degree of Doctor of Philosophy. February.

Irvine, M. (1988). Cable Structures. Penerbit ITB. Bandung, Indonesia.

Irvine, M. (1992), "Local Bending Stress in Cables", Proceedings of the Second International Offshore and Polar Engineering Conference, San Francisco, June, Vol. 2, pp. 342-345.

Jiang, W. (1995), "A General Formulation of the Theory of Wire Ropes", Journal of Applied Mechanics, ASME, Vol. 62, September, pp. 747-755.

Jiang, W. G., and Henshall, J. L. (1999), "The Analysis of Termination Effects in Wire Strand Using the Finite Element Method", Journal of Strain Analysis, Vol. 34, No. 1, pp. 31-38.

Jiang, W., Wang, T. L., and Jones, W. K. (1990), "Forced Vibration of Coupled Extensional-Torsional Systems", *Journal of Engineering Mechanics*, ASCE, Vol. 117, No. 5, May, pp. 1171-1190.

Jolicoeur, C. (1996), "Discussion: A General Formulation of the Theory of Wire Ropes", *Journal of Applied Mechanics*, ASME, Vol. 63, December, pp 1053.

Jolicoeur, C. (1997), "Comparative Study of Two Semicontinuous Models for Wire Strand Analysis", *Journal of Engineering Mechanics*, ASCE, Vol. 123, No. 8, pp. 792-799.

Jolicoeur, C., and Cardou, A. (1991), "A Numerical Comparison of Current Mathematical Models of Twisted Wire Cables Under Axisymmetric Loads", *Journal of Energy Resources Technology*, ASME, Vol. 113, December, pp. 241-249.

Jolicoeur, C., and Cardou, A. (1994), "Analytical Solution for Bending of Coaxial Orthotropic Cylinders", *Journal of Engineering Mechanics*, ASCE, Vol. 120, No. 12, December, pp. 2556-2574.

Jolicoeur, C., and Cardou, A. (1996), "Semicontinuous Mathematical Model for Bending of Multi-Layered Wire Strands", *Journal of Engineering Mechanics*, ASCE, Vol. 122, No. 7, July, pp. 643-650.

Knapp, R. H. (1975), "Non-Linear Analysis of a Helically Armoured Cable with Nonuniform Mechanical Properties in Tension and Torsion", *Proceedings, IEEE/MTS Conference: Engineering in the Ocean Environment*, San Diego, California, pp. 155-164.

Knapp, R. H. (1979), "Derivation of a New Stiffness Matrix for Helically Armoured Cables Considering Tension and Torsion", *International Journal for Numerical Methods in Engineering*, Vol. 14, pp. 515-529.

Knapp, R. H. (1988), "Helical Wire Stresses in Bent Cables", *Journal of Offshore Mechanics and Arctic Engineering*, ASME, Vol. 110, pp. 55-61.

Knapp, R. H. (1989), "Axial Fatigue Model for Wire Rope Strand", *Proceedings of the ASCE Structures Congress, Materials and Material Behaviour*, Orlando, FLA, U.S.A., pp. 347-359.

Knapp, R. H., and Chiu, E. Y. C. (1988), "Tension Fatigue Model for Helically Armoured Cables", *Journal of Energy Resources Technology*, ASME, Vol. 110, March, pp. 12-18.

Kraincanic, I. (1995), "Analysis of the Coupled Axial/Torsional Behaviour of Spiral Strands, Wire Ropes, and Locked Coil Cables", Thesis Presented to South Bank University in Partial Fulfilment of the Requirements for the Degree of Doctor of Philosophy. October.

Kraincanic, I., and Hobbs, R. E. (1997), "Torque Induced by Axial Load in a 76 mm Wire Rope: Comparison of Experimental Results and Theoretical Predictions", O.I.P.E.E.C Round Table Conference on the application of Endurance Prediction of Wire Ropes, University of Reading, Reading, U. K., September.

Kraincanic, I., and Hobbs, R. E. (1998), "Axial Stiffness and Torsional Effects in a 76 mm Wire Rope: Experimental Data and Theoretical Predictions", *Journal of Strain Analysis*, Vol. 34, No. 1, pp. 39-51.

Kumar, K., and Cochran, J. E. (1987), "Closed-Form Analysis for Elastic Deformations of Multi-Layered Strands", *Journal of Applied Mechanics*, ASME, Vol. 54, December, pp. 898-903.

Kumar, K., and Cochran, J. E. (1990), "Analytical Estimation for Static Deformation of Wire Ropes with Fibrous Core", *Journal of Applied Mechanics*, ASME, Vol. 57, December, pp. 1000-1003.

Kumar, K., Cochran, J. E., and Cutchins, J. A. (1997), "Contact Stresses in Cables due to Tension and Torsion", *Journal of Applied Mechanics*, ASME, Vol. 64, December, pp. 935-939.

Lanteigne, J. (1985), "Theoretical Estimation of the Response of Helically Armoured Cables to Tension, Torsion, and Bending", *Journal of Applied Mechanics*, ASME, Vol. 52, June, pp. 423-432.

LeClair, R. A. (1989), "Upper Bound to Mechanical Power Transmission Losses in Wire Rope", *Journal of Engineering Mechanics*, ASCE, Vol. 115, No. 9, September, pp. 2011-2019.

LeClair, R. A. (1991), "Axial Response of Multi-Layered Strands with Compliant Layers", *Journal of Engineering Mechanics*, ASCE, Vol. 117, No. 12, December, pp. 2884-2903.

LeClair, R. A., and Costello, G. A. (1988), "Axial, Bending and Torsional Loading of a Strand with Friction", *Journal of Offshore Mechanics and Arctic Engineering*, ASME, Vol. 110, February, pp. 38-42.

Lee, W. K. (1991), "An Insight into Wire Rope Geometry", *International Journal of Solids and Structures*, Vol. 28, No. 4, pp. 471-490.

Leissa, A. W. (1959), "Contact Stresses in Wire Ropes", *Wire and Wire Products*, Vol. 34, March, pp. 372-374.

Love, A. E. H. (1944). *A Treatise on the Mathematical Theory of Elasticity*. Dover Publications, New York.

Lutchansky, M. (1969), "Axial Stresses in Armor Wires of Bent Submarine Cables", *Journal of Engineering for Industry*, ASME, Vol. 91, August, pp. 687-693.

Machida, S., and Durelli, A. J. (1973), "Response of a Strand to Axial and Torsional Displacements", *Journal of Mechanical Engineering Science*, Vol. 15, No. 4, pp. 241-251.

Malinovsky, V. A. (1993), "Bending Stiffness of Steel Wire Ropes", O.I.P.E.E.C Round Table Conference, Delft, The Netherlands, pp. II35-II49.

Matanzo, F., and Metcalf, J. T. (1981), "Efficiency of Wire Rope Terminations", *Journal of Engineering Materials and Technology*, Vol. 103, April, pp. 164-170.

McConnell, K. G., and Zemke, W. P. (1982) "A Model to Predict the Coupled Axial Torsion Properties of ACSR Electrical Conductors", *Experimental Mechanics*, Vol. 22, No. 6, pp. 237-244.

Metcalf, J. T., and Matanzo, F. (1980), "Wire Rope Terminations, Selection, and Replacement Criteria", *Proceedings of the 12th Annual Offshore Technology Conference*, Houston, Texas, May, pp. 517-524.

Mindlin, R. D. (1949), "Compliance of Elastic Bodies in Contact", *Journal of Applied Mechanics*, ASME, Vol. 16, pp. 259-268.

Mitamura, T., Sugii, K., Okukawa, A., and Tanaka, Y. (1992), "Fatigue Strength of Parallel Wire Strands in Anchorage", *IABSE Workshop, Length Effect on Fatigue of Wires and Strands El Paular*, Madrid, Report No. 66, September, pp. 179-189.

Mitchell, R. A., Woolley, R., and Halsey, N. (1974), "High-Strength End Fittings for FRP Rod and Rope", *Journal of the Engineering Mechanics Division*, ASCE, Vol. 100, No. EM4, August, pp. 687-706.

Nabijou, S., and Hobbs, R. E. (1995a), "Frictional Performance of Wire and Fibre Ropes Bent Over Sheaves", *Journal of Strain Analysis*, Vol. 30, No. 1, pp. 45-57.

Nabijou, S., and Hobbs, R. E. (1995b), "Relative Movements Within Wire Ropes Bent Over Sheaves", *Journal of Strain Analysis*, Vol. 30, No. 2, pp. 155-165.

Oplatka, G., and Roth, M. (1993), "Bending Fatigue of Locked Coil Ropes in the Neighbourhood of Cast Sockets part 2: Influence of Lubrication", O.I.P.E.E.C Round Table Conference, Delft, The Netherlands, pp. IV10-IV23.

Owada, S. (1952), "Calculation of Tensile and Torsional Stiffnesses of Single-Lay Cables", *Proceedings of the 2nd Japan National Congress for Applied Mechanics*, pp. 159-164.

Phillips, J. W., and Costello, G. A. (1973), "Contact Stresses in Twisted Wire Cables", *Journal of the Engineering Mechanics Division, ASCE*, Vol. 99, No. EM2, April, pp. 331-341.

Phillips, J. W., and Costello, G. A. (1977), "Axial Impact of Twisted Wire Cables", *Journal of Applied Mechanics, ASME*, Vol. 44, March, pp. 127-131.

Phillips, J. W., and Costello, G. A. (1985), "Analysis of Wire Ropes with Internal-Wire-Rope Cores", *Journal of Applied Mechanics, ASME*, Vol. 52, September, pp. 510-516.

Poffenberger, J. C., and Swart, R. L. (1965), "Differential Displacement and Dynamic Conductor Strains", *IEEE Transactions, Power Apparatus and Systems, PAS-84*, pp. 281-289.

Ramsey, H. (1988), "A Theory of Thin Rods with Applications to Helical Constituent Wires in Cables", *International Journal of Mechanical Science*, Vol. 30, No. 8, pp. 559-568.

Ramsey, H. (1991), "Analysis of Interwire Friction in Multi-Layered Cables Under Uniform Extension and Twisting", *International Journal of Mechanical Science*, Vol. 32, No. 8, pp. 709-716.

Raoof, M. (1983), "Interwire Contact Forces and the Static, Hysteretic and Fatigue Properties of Multi-Layered Structural Strands", Thesis Presented to the University of London in Partial Fulfilment of the Requirements for the Degree of Doctor of Philosophy, September.

Raoof, M. (1989), "Free Bending Tests on Large Spiral Strands", Proceedings of the Institution of Civil Engineers, Part II, Vol. 87, December, pp. 605- 626.

Raoof, M. (1990a), "Comparison Between the Performance of Newly Manufactured and Well- used Spiral Strands", Proceedings of the Institution of Civil Engineers, Part II, Vol. 89, March, pp. 103- 120.

Raoof, M. (1990b), "Free Bending of Spiral Strands", Journal of Engineering Mechanics, ASCE, Vol. 116, No.3, pp. 512-530.

Raoof, M. (1990c), "Effect of Hydrostatic Pressure on Strand Behaviour", Journal of Strain Analysis, Vol. 25, No. 2, pp. 75- 84.

Raoof, M. (1990d), "Axial Fatigue of Multi-Layered Strands", Journal of Engineering Mechanics, ASCE, Vol. 116, No. 10, pp. 2083- 2099.

Raoof, M. (1990e), "Simple Formulae for Spiral Strands and Multi-Strand Ropes", Proceedings of the Institution of Civil Engineers, Part II, Vol. 89, December, pp. 527- 542.

Raoof, M. (1991a), "Axial Fatigue Life Prediction of Structural Cables from First Principles", Proceedings of the Institution of Civil Engineers, Part II, Vol. 91, March, pp. 19- 38.

Raoof, M. (1991b), "Cable Fatigue Prediction in Offshore Applications", Proceedings of the 10th International Conference on Offshore Mechanics and Arctic Engineering, ASME, Stavanger, Norway, Vol. III-B, June, pp. 403-411.

Raoof, M. (1991c), "Methods for Analysing Large Spiral Strands", *Journal of Strain Analysis*, Vol. 26, No. 3, pp. 165-174.

Raoof, M. (1991d), "Wire Recovery Length in a Helical Strand Under Axial-Fatigue Loading", *International Journal of Fatigue*, Vol. 13, No. 2, pp. 127-132.

Raoof, M. (1991e), "Cable Behaviour Under Various Loading Conditions", *Structural Engineering Review*, Vol. 3, No. 2, pp. 89-99.

Raoof, M. (1991f), "The Prediction of Axial Damping in Spiral Strands", *Journal of Strain Analysis*, Vol. 26, No. 4, pp. 221-229.

Raoof, M. (1992a), "Free Bending Fatigue of Axially Preloaded Spiral Strands", *Journal of Strain Analysis*, Vol. 27, No. 3, pp. 127-136.

Raoof, M. (1992b), "A Critical Review of Draft API Recommended Practice 2FP1 Regarding Fatigue Life Estimation of Moorings", *Proceedings of the 11th International Conference on Offshore Mechanics and Arctic Engineering*, ASME, Calgary, Canada, Vol. III, Part B, June, pp. 521-532.

Raoof, M. (1992c), "Free-Bending Fatigue Life Estimation for Cables at Points of Fixity", *Journal of Engineering Mechanics*, ASCE, Vol. 118, No. 9, September, pp. 1747-1764.

Raoof, M. (1992d), "Design Recommendations for Steel Cables", *Structural Engineering Review*, Vol. 4, No. 3, pp. 223- 233.

Raoof, M. (1993a), "Design of Sheathed Spiral Strands Against Free Bending Fatigue at Terminations", *Journal of Strain Analysis*, Vol. 28, No. 3, pp. 163-174.

Raoof, M. (1993b), "Design of Steel Cables Against Free Bending Fatigue at Terminations", *The Structural Engineer*, Vol. 71, No. 10, May, pp. 171-178.

Raoof, M. (1994a), "Free Bending Fatigue Prediction of Steel Cables with the Effect of Interwire Fretting taken into Account", Fretting Fatigue, ESIS 18, (Edited by R B Waterhouse and T C Lindley), Mechanical Engineering Publications, London, pp. 267-281 (Proceedings of the International Conference on Fretting Fatigue, Sheffield, April, 1993).

Raoof, M. (1994b), "Prediction of Lateral Vibrations and Associated Fatigue Problems of Bridge Cables", Proceedings of the Deauville Conference on Cable-Stayed and Suspension Bridges, organised by the International Association for Bridge and Structural Engineering (IABSE), France, October, Vol. II, pp. 465-474.

Raoof, M. (1995a), "Characteristics of Locked Coil Strands Under Free Bending", International Journal of Offshore and Polar Engineering, Vol. 5, No. 2, June, pp. 153-160.

Raoof, M. (1995b), "Prediction of Axial Fatigue Life of Spiral Strands", in 'Structural Steel', Shanmugan, N. E., and Choo, Y. S., (eds.), Pergamon, Vol. 1, 1995b, pp. 391-397, (Proceedings, 4th Pacific Structural Steel Conference, Oct., Mandarin, Singapore, 1995).

Raoof, M. (1996), "Axial Fatigue Life Prediction of Bridge Cables", Proceedings of the Third International Conference on Bridge Management, Inspection, Assessment, and Repair, J. E., Harding, G. A. R., Parke, and M. J., Ryall (eds.), E. & F. N. Spon, University of Surrey, Guildford, Surrey, U.K., April, pp. 523-531.

Raoof, M. (1997), "Effect of Lay Angle on Various Characteristics of Spiral Strands", International Journal of Offshore and Polar Engineering, Vol. 7, No. 1, March, pp. 54-62.

Raoof, M. (1998a), "Recommendations Regarding Design of Spiral Strands Against Axial Fatigue Failure", Proceedings of 8th International Offshore and Polar Engineering Conference, Chung, J. S., Frederking, R. M. W., Saeki, H., and Moshagen, H., (eds.), Montreal, Canada, Vol. II, May, pp. 239-247.

Raoof, M. (1998b), "Damping Performance of Bridge Cables", Proceedings of Fifth Pacific Structural Steel Conference (PSSC), Seoul, Korea, Vol. 1, October, pp. 521-526.

Raoof, M., and Alani, M. (1997), "Axial Fatigue of Spiral Strands in Offshore Platform Applications", Proceedings of 7th International Offshore and Polar Engineering Conference, Honolulu, Hawaii, U.S.A., Vol. II, May, pp. 169-176.

Raoof, M. and Hobbs, R. E. (1984), "The Bending of Spiral Strand and Armoured Cables Close to Terminations", Journal of Energy Resources Technology, ASME, Vol. 106, No. 3, pp. 349-355.

Raoof, M., and Hobbs, R. E. (1988a), "Torsion Tests on Large Spiral Strands", Journal of Strain Analysis, Vol. 23, No. 2, pp. 97- 104.

Raoof, M., and Hobbs, R. E. (1988b), "Analysis of Multi-Layered Structural Strands", Journal of Engineering Mechanics, ASCE, Vol. 114, July, pp. 1166-1182.

Raoof, M., and Hobbs, R. E. (1989), "Torsional Stiffness and Hysteresis in Spiral Strands", Proceedings of the Institution of Civil Engineers, Part II, Vol. 87, Dec., pp. 501- 515.

Raoof, M., and Hobbs, R. E. (1994), "Analysis of Axial Fatigue Data for Wire Ropes", International Journal of Fatigue, Vol. 16, October, pp. 493-501.

Raoof, M., and Huang, Y. P. (1991), "Upper Bound Prediction of Cable Damping Under Cyclic Bending", Journal of Engineering Mechanics, ASCE, Vol. 117, No. 12, December, pp. 2729- 2747.

Raoof, M. and Huang, Y. P. (1992a), "Bending Stiffness and Hysteresis of Sheathed Spiral Strands", Proceedings of 2nd International Offshore and Polar Engineering Conference, M. S., Triantafyllou, J. S., Chung, K. Karal, and A. L., Tunik (eds.), San Francisco, U.S.A., Vol. II, June, pp. 369-379.

Raoof, M., and Huang, Y. P. (1992b), "Free Bending Hysteresis in Spiral Strands", Proceedings of 2nd International Offshore and Polar Engineering Conference, M. S., Triantafyllou, J. S., Chung, K. Karal, and A. L., Tunik (eds.), San Francisco, U.S.A., Vol. II, June, pp. 380- 391.

Raoof, M. and Huang, Y. P. (1992c) "Wire Stress Calculations in Helical Strands Undergoing Bending", Journal of Offshore Mechanics and Arctic Engineering, ASME, Vol. 114, No. 3, August, pp. 212- 219.

Raoof, M., and Huang, Y. P. (1992d), "Free Bending Characteristics of Sheathed Spiral Strands Under Cyclic Loading", Journal of Strain Analysis, Vol. 27, No. 4, pp. 219- 226.

Raoof, M., and Huang, Y. P. (1992e), "Free Bending Characteristics of Axially Preloaded Spiral Strands", Proceedings of The Institution of Civil Engineers, Structures and Buildings, Vol. 94, November, pp. 469- 484.

Raoof, M., and Huang, Y. P. (1992f), "Axial and Free Bending Analysis of Spiral Strands Made Simple", Journal of Engineering Mechanics, ASCE, Vol. 118, No. 12, December, pp. 2335- 2351.

Raoof, M., and Huang, Y. P. (1992g), "Wire Recovery Length in Suspension Bridge Cable", Journal of Structural Engineering, ASCE, Vol. 118, No. 12, December, pp. 3255- 3267.

Raoof, M., and Huang, Y. P. (1992h), "Simplified Methods for Analysing Steel Strands", The Structural Engineer, Vol. 70, No. 22, November, pp. 390-397.

Raoof, M., and Huang, Y. P. (1992i), "Simple Methods for Estimating Various Sheathed Spiral Strand Bending Characteristics", Proceedings of the Institution of Civil Engineers, Structures and Buildings, Vol. 94, November, pp. 485-500.

Raoof, M., and Huang, Y. P. (1993a), "Lateral Vibrations of Steel Cables Including Structural Damping", Proceedings of The Institution of Civil Engineers, Structures and Buildings, Vol. 99, May, pp. 123-133.

Raoof, M., and Huang, Y. P. (1993b), "Effect of Non-Linear Structural Damping on Cable Lateral Vibrations", Proceedings of 3rd International Offshore and Polar Engineering Conference, J. S. Chung, K. Karal, M. S. Triantafyllou, and R. W. Frederking (eds.), Singapore, Vol. II, June, pp. 435-444.

Raoof, M., and Huang, Y. P. (1993c), "Cyclic Bending Characteristics of Sheathed Spiral Strands in Deep Water Applications", International Journal of Offshore and Polar Engineering, Vol. 3, No. 3, September, pp. 189-196.

Raoof, M., and Huang, Y. P. (1994), "Estimation of Spiral Strand Bending Fatigue Life", Proceedings of the Seventh International Conference on Behaviour of Offshore Structures (BOSS '94), C. Chrysosostomidis (ed.), Pergamon, Vol. 2, M.I.T., Cambridge, MA, U.S.A., July, pp. 203-212.

Raoof, M., Huang, Y. P., and Pithia, K. (1993), "Axial-Torsional Coupling of Spiral Strands Under Impact Loading", Proceedings of the Institution of Civil Engineers, Structures and Buildings, Vol. 99, August, pp. 371-374.

Raoof, M., and Huang, Y. P., and Pithia, K. D. (1994), "Response of Axially Preloaded Spiral Strands to Impact Loading", Computers and Structures, Vol. 51, No. 2, pp. 125-135.

Raoof, M., and Kraincanic, I. (1993), "Recovery Length in Sheathed Spiral Strands in Deep-Water Platform Applications", International Journal of Fatigue, Vol. 15, No. 6, pp. 485-492.

Raoof, M., and Kraincanic, I. (1994a), "Critical Examination of Various Approaches Used for Analysing Helical Cables", Journal of Strain Analysis, Vol. 29, No. 1, pp. 43-55.

Raoof, M., and Kraincanic, I. (1994b), "Axial/Torsional Coupling of Sheathed Spiral Strands in Deep Water Applications", *Proceedings of the Institution of Civil Engineers, Structures and Buildings*, Vol. 104, November, pp. 377-390.

Raoof, M., and Kraincanic, I. (1995a), "Simple Derivation of the Stiffness Matrix for Axial/Torsional Coupling of Spiral Strands", *Computers and Structures*, Vol. 55, No. 4, pp. 589-600.

Raoof, M., and Kraincanic, I. (1995b), "Analysis of Large Diameter Steel Ropes", *Journal of Engineering Mechanics*, ASCE, Vol. 121, No. 6, June, pp. 667-675.

Raoof, M., and Kraincanic, I. (1995c), "Recovery Length in Multi-layered Spiral Strands", *Journal of Engineering Mechanics*, ASCE, Vol. 121, No. 7, July, pp. 795-800.

Raoof, M., and Kraincanic, I. (1995d), "Characteristics of Fibre-Core Wire Rope", *Journal of Strain Analysis*, Vol. 30, No. 3, pp. 217-226.

Raoof, M., and Kraincanic, I. (1996a), "Behaviour of Large Diameter Wire Ropes", *International Journal of Offshore and Polar Engineering*, Vol. 6, No. 3, September, pp. 219-226.

Raoof, M., and Kraincanic, I. (1996a), "Characteristics of Locked Coil Cables Under Coupled Axial/Torsional Loading", *Proceedings of 6th International Offshore and Polar Engineering Conference*, Los Angeles, California, U. S. A., Vol. II, May, pp. 197-203.

Ringleb, F. O. (1957), "Motion and Stress of an Elastic Cable due to Impact", *Journal of Applied Mechanics*, ASME, Vol. 24, September, pp. 417-425.

Roark, R. J., and Young, W. C. (1975). *Formulas for Stress and Strain*. 5th Edition, International Student Edition, McGraw-Hill International, New York.

Roberts, G. (1968), "Severn Bridge – Design and Contract Arrangements", Proceedings of the Institution of Civil Engineers, Vol. 41, September, pp. 1-48

Samras, R. K., Skop, R. A., and Milburn, D. A. (1974), "An Analysis of Coupled Extensional-Torsional Oscillations in Wire Rope", Journal of Engineering for Industry, ASME, Vol. 74, November, pp. 1130-1135.

Sathikh, S., Jayakumar, and Jebaraj, C. (1996), "Discussion: A General Formulation of the Theory of Wire Ropes", Journal of Applied Mechanics, ASME, Vol. 63, December, pp. 1053-1054.

Sathikh, S., Moorthy, M. B. K., and Krishnan, M. (1996), "A Symmetric Linear Elastic Model for Helical Wire Strands Under Axisymmetric Loads", Journal of Strain Analysis, Vol. 31, No. 5, pp. 389-399.

Sayenga, D. (1993), "The First Applications of High-Tensile Steel Wire Ropes for Civil Engineers", O.I.P.E.E.C., Round Table Conference, Delft, The Netherlands, pp. III2-III15.

Seppa, T. O. (1971), "Self-Damping Measurements and Energy Balance of ACSR DRAKE", IEEE Winter Power Meeting, New York, Paper No. 71-CP-161-PWR, pp. 1-8.

Skop, R. A., Samras, R. K. (1975), "Effects of Coupled Extensional-Torsional Oscillations in Wire Rope During Ocean Salvage and Construction Operations", Journal of Engineering for Industry, ASME, Vol. 97, May, pp. 485-492.

Starkey, W. L., and Cress, H. A. (1959), "An Analysis of Critical Stresses and Mode of Failure of a Wire Rope", Journal of Engineering for Industry, ASME, Vol. 81, November, pp. 307-316.

Sterren, B., G., van der (1993), "Bending Stress, Rope Curvature and Bending Stiffness", O.I.P.E.E.C. Round Table Conference, Delft, The Netherlands, pp. II50-II66.

Strzemiecki, J., and Hobbs, R. E. (1988), "Properties of Wire Rope Under Various Fatigue Loadings", Imperial College, Civil Engineering Department, CESLIC Report SC6.

Tilly, G. P. (1988), "Performance of Bridge Cables", 1st Oleg Kerensky Memorial Conference, Institution of Structural Engineers, London, June, Session 4, pp. 22-28.

Utting, W. S. (1988), "Wire Loading Near Rope Terminations", Wire Industry, March, pp. 251-257.

Utting, W. S. (1994a), "Survey of Literature on the Behaviour of Steel Wire Ropes-Part I", Wire Industry, September, Vol. 61, No. 729, pp. 633-635.

Utting, W. S. (1994b), "Survey of Literature on the Behaviour of Steel Wire Ropes-Part II", Wire Industry, November, Vol. 61, No. 731, pp. 746-748.

Utting, W. S. (1995), "Survey of Literature on the Behaviour of Steel Wire Ropes-Part III", Wire Industry, Vol. 62, No. 737, May, pp. 269-270.

Utting, W. S., and Jones, N. (1984) "A Survey of the Literature on the Behaviour of Wire Rope", Wire Industry, Vol. 51, September, pp. 623-629.

Utting, W. S., and Jones, N. (1985), "Tensile Testing of a Wire Rope Strand", Journal of Strain Analysis, Vol. 20, No. 3., February, pp. 151-164.

Utting, W. S., and Jones, N. (1987a), "The Response of Wire Rope Strands to Axial Tensile Loads-Part I. Experimental Results and Theoretical Predictions", International Journal of Mechanical Science, Vol. 29, No. 9, pp. 605-619.

Utting, W. S., and Jones, N. (1987b), "The Response of Wire Rope Strands to Axial Tensile Loads-Part II Comparison of Experimental Results and Theoretical Predictions", *International Journal of Mechanical Science*, Vol. 29, No. 9, pp. 621-636.

Utting, W. S., and Jones, N. (1987c), "Wire Tensions and Bending Moments in Axially Loaded Seven Wire Strand", *Strain*, Vol. 23, August, pp. 109-116.

Utting, W. S., and Jones, N. (1988), "Axial-Torsional Interactions and Wire Deformations in 19-Wire Spiral Strand", *Journal of Strain Analysis*, Vol. 23, No. 2, pp. 79-86.

Vanderveldt, H. H., Chung, B. S., and Reader, W. T. (1973), "Some Dynamic Properties of Axially Loaded Wire Ropes", *Experimental Mechanics*, Vol. 13, pp. 24-30.

Velinsky, S. A. (1985a), "General Non-Linear Theory for Complex Wire Rope", *International Journal of Mechanical Science*, Vol. 27, No. 78, pp. 497-507.

Velinsky, S. A. (1985b), "Analysis of Fibre-Core Wire Rope", *Journal of Energy Resources Technology*, ASME, Vol. 107, September, pp. 388-393.

Velinsky, S. A., Anderson, G. L., and Costello, G. A. (1984), "Wire Rope with Complex Cross Sections", *Journal of Engineering Mechanics*, ASCE, Vol. 110, No. 3, March, pp. 380-391.

Vinogradov, O. G., and Atatekin, I. S. (1986), "Internal Friction due to Wire Twist in Bent Cables", *Journal of Engineering Mechanics*, ASCE, Vol. 112, No. 9, September, pp. 859-873.

Webber, W. (1975), "Development and Production of Wire Rope I and II", *Wire World International*, (I) Vol. 16, 1974, pp. 286-291, (II) Vol. 17, 1975, pp. 20-24.

Wempner, G. (1973), *Mechanics of Solids with Applications to Thin Bodies*. McGraw-Hill, New York, Chapter 8.

Witz, J. A., and Tan, Z. (1990), "On the Axial-Torsional Structural Behaviour of Flexible Pipes, Umbilicals and Marine Cables", *Marine Structures*, Vol. 5 (2-3), pp. 202-227.

Witz, J. A., and Tan, Z. (1991), "On the Flexural Structural Behaviour of Flexible Pipes, Umbilicals and Marine Cables", *Marine Structures*, Vol. 5 (2-3), pp. 229-249.

Wyatt, T. A. (1960), "Secondary Stresses in Parallel Wire Suspension Cables", *Journal of the Structural Division, ASCE*, Vol. 86, No. ST7, July, pp. 37-59.

Wyatt, T. A. (1977), "Mechanics of Damping", *Symposium on Dynamic Behaviour of Bridges*, Crowthorne, Berks., Transport and Road Research Laboratory Supplementary Report 275, pp. 10-21.

Zhang, S., and Leech, C. M. (1985), "Use of Inhomogeneous Finite Elements for the Prediction of Stress in Rope Terminations", *Engineering Computations*, Vol. 2, March, pp. 56-62.

