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EVALUATION OF THE UK AND USA CODES OF PRACTICE FOR REINFORCED CONCRETE SLAB DESIGN

by

JASSIM MOHAMMAD HASSAN, B.Sc.

A Master's Thesis

submitted in partial fulfilment of the requirements

for the award of

Master of Philosophy of the Loughborough University of Technology

June 1989

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DEDICATION

For my wife Thawra and my children Adhra, Mohammad and Mayada.

ACKNOWLEDGMENTS

I am indebted to the Iraqi Ministry of Higher Education and Scientific Research for sponsoring the research programme in this thesis.

I owe Professor L L Jones and Mr J G Dickens, my supervisors, a debt of gratitude for their guidance during this work. Such guidance has provided inspiration, not only for this project, but for any further research work I may undertake. The numerous discussions held with them and their invaluable suggestions and constructive criticism contributed greatly to my research experience and the work in this thesis.

Thanks are also due to Dr R J Allwood, the Director of Research, for his encouragement during the research period.

I would like to express my gratitude to my colleague Mr J E Ogendo, research assistant in the Department of Transport Technology, for his help in discussing my ideas and opinions on the research work.

Thanks are also due to the following staff of the Computer Centre for their help: Mr B Kelly, Mr G Harris and Mr B Scott.

I am also grateful to Mrs D C Boyd for her kindness and general secretarial assistance throughout this work and to Mrs J Billing for her assistance.

Finally, special thanks go to my wife Thawra and my children Adhra, Mohammad and Mayada for their patience during the period of this research, and to my parents for their continuing support.

DECLARATION

No portion of the research referred to in this thesis has been submitted in support of an application for another degree or qualification at this or any other university or other institution of learning.

SUMMARY

After an introductory Chapter on slabs, the broad design provisions of the British and American Codes of Practice are set out. A historical review of elastic and ultimate load methods of slab design together with examples is then followed by a discussion on loads, load factors, material factors, patterns of loading and the division of slabs into various strips.

Three extensive chapters with examples on the use of the Codes of Practice examine and discuss the provisions and behaviour of slabs on rigid and semi-rigid supports and flat slabs supported by columns. The results of an extensive elastic finite element investigation are compared with the various methods available for the design of the three types of slabs under both serviceability and ultimate conditions.

In Chapter 5 on rigidly supported slabs it is concluded that for the British Code the ultimate load recommendations are satisfactory but that in general the moment coefficients recommended require considerable negative moment redistribution and in some cases by considering the finite element results the steel must almost be yielding under the serviceability loads. With one exception the American code is better from the serviceability condition aspect but the simply supported slab bending moment coefficients would cause premature failure.

Chapter 6 on slabs on semi-rigidly supported slabs indicates the British code is sadly deficient on design information for this type of slab while the American code gives proposals which give answers which are broadly in agreement with the finite element analysis.

Chapter 7 on flat slabs shows that both the British and American codes are reasonably satisfactory both from the serviceability and ultimate conditions.

The final Chapter highlights areas which need attention and make some suggestions for further study.

CONTENTS

Read in addition

Read in addition

CHAPTER 1

INTRODUCTION

Reinforced concrete slabs are one of the commonest structural elements, and although large numbers of them are designed and built, their elastic and plastic behaviour is not always fully understood. This occurs, in part at least, because of the mathematical complexities involved when dealing with the equations that govern the elastic behaviour of plates.

Since the theoretical analysis of slabs and plates is less widely known than the analysis of elements such as beams, the provisions in codes of practice generally provide both design criteria and methods of analysis for slabs, while only criteria are provided for most other elements. However the elastic methods of analysis given by the codes are necessarily approximate since the so-called "exact" elastic analysis methods would be difficult to formulate. Failure criteria are also another necessary inclusion.

The provisions of a code are based on many years of research and field experience which should therefore result in the provision of practical and simplified methods for analysis and design. Design offices typically prefer to follow simplified methods rather than use more exact solutions which would often involve the use of computers. However codes are always changing as knowledge and experience of materials, construction practices and analysis techniques improve, and as a consequence the scientific reason for a particular rule or specific value for a coefficient in a code may not always be clear.

Not unnaturally different countries have developed different codes and the aim of this thesis is to examine the basis of and to compare the recommendations for the design of reinforced concrete slabs in the codes of practice for concrete work in the United Kingdom and the United States of America.

In the UK the relevant code is BS8110 [1], 'The Structural Use of Concrete, 1985', and in the USA the codes ACI 318-83 [2] and ACI 318-63 [3], 'American Concrete Institute, Building Code Requirements for Reinforced Concrete', are used.

Both codes include simplified methods based on recommended coefficients to evaluate the bending moments at customary critical sections and the basis of comparison will mainly be through these recommended coefficients and the various design procedures. In making comparisons different factors have been taken into account such as loading patterns, load factors, ratios of dead to live load, factors for materials and the method of structural analysis used and serviceability and ultimate states. It should be noted that only lateral uniformly distributed load and rectangular solid slabs will be considered.

The codes require designs to satisfy both serviceability and ultimate conditions and for the former it will be necessary to examine available elastic analysis techniques such as the direct solution of the plate equation and methods for numerical solution based on finite element techniques.

For ultimate conditions, failure theories are used, the best known methods for the plastic analysis of slabs being the upper-bound yield-line method and the lower-bound Hillerborg method. The solutions obtained from these elastic and collapse theories will be examined in relation to the code recommendations.

In order to determine the steel requirements for slabs, the yield criterion will need to be established and therefore the Wood-Armer reinforcement rules, which are a function of the field bending and twisting moments, will be examined.

For convenience in this study slabs have been classified by their support conditions, namely rigidly supported, semi-rigidly supported and flat slabs.

A rigid support is one which is assumed not to deflect vertically along its length. Generally this type of support will be provided by brick or concrete walls, or a beam which can be regarded as having infinite flexural stiffness.

Semi-rigidly supported slabs are slabs which are supported by beams arranged, for purposes of this thesis, in a rectangular grid supported by columns at the intersection of the beams. The stiffness of the beams relative to the stiffness of the slab varies from zero to infinity corresponding, respectively, to the type of slab known as a flat slab where there are no beams, to slabs on rigid supports. The range of slabs between these two types are considered as semi-rigidly supported slabs and are an intermediate type between slabs on rigid supports and beamless slabs (flat slabs).

A flat slab is a reinforced concrete slab, generally without beams or girders to transfer the loads to supporting members. The slab may be of constant thickness throughout or may be thickened as a drop panel in the area of the column. The column may also be of constant section or it may be flared to form a column head or capital. The work in this thesis is confined to flat slabs without drop panels or flared heads to the column.

After this introduction a brief historical review of slab design and development is given followed by an introduction to the British and American Codes of Practice and a chapter on the various factors which are thought to influence the various moment coefficients used in these codes. A chapter each is then devoted to the study of rigidly supported, semi-rigidly supported and flat slabs which include typical calculations by different techniques and comments on the code recommendations. The thesis is concluded with numerous comments on the two codes which have become apparent during the study.

CHAPTER 2

THE BROAD PROVISIONS OF THE UK AND USA REINFORCED CONCRETE CODES OF PRACTICE

2.1 Introduction

This chapter describes in broad terms the design methods in the two codes for:

(a) rigidly supported slabs;

(b) semi-rigidly supported slabs; and

(c) flat slabs.

Each method is later given in considerable detail in Chapters 5, 6 and 7. The design recommendations may be broadly divided into various types, namely:

(a) simplified approaches, based on bending moment coefficients;

(b) the equivalent frame method; and

(c) alternative methods, employing elastic analysis in various forms or ultimate load methods such as the yield-line analysis or Hillerborg's strip method. When these alternative methods are used it must be ensured that other limit state requirements are met.

2.2 BS 8110

2.2.1 Rigidly supported slabs

For rigidly supported rectangular two-way spanning slabs, BS 8110 gives a simplified method which tabulates bending moment coefficients to enable the maximum moments at the critical sections to be calculated in each of the two principal directions. The coefficients are tabulated for different types of slabs, taking into account different boundary conditions and aspect ratios of the panel (i.e. the ratio of length to width of the slab). The major set of moment coefficients are for restrained slabs which have adequate provision to resist torsion at the corners and are also prevented from lifting at the corners. Coefficients are also given for simply supported slabs which do not have adequate provision to resist torsion at the corners and are not prevented from lifting.

To design such rectangular slabs therefore the designer merely consults the Tables, extracts the relevant coefficient and calculates his bending moments based on a suitably factored load and the relevant dimensions.

The designer is also allowed to use elastic analysis or collapse methods, though as will be shown later there is a relationship between these methods and the tabulated coefficients.

2.2.2 Semi-rigidly supported slabs

BS 8110 does not give a separate method for semi-rigidly supported slabs, such as slabs supported on beams, but allows the designer to treat them as slabs on rigid supports. The coefficients and methods used in the previous section are used for this class of slab.

2.2.3 Flat slabs

BS 8110 gives two principal methods for designing flat slabs which are supported on columns positioned at the intersection of rectangular grid lines for slabs where the aspect ratio is not greater than 2.

The first method is based on simple moment coefficients at critical sections. This can be used where the lateral stability is not dependent on the slab-column connections and is subject to the following provisions:

(a) the single load case is considered on all spans; and

(b) there are at least three rows of panels of approximately equal spans in the direction being considered.

The second approach is the equivalent frame method, which as the name suggests, involves subdividing the structure into sub frames and the use of moment distribution or similar analysis techniques to obtain the forces and moments at critical sections.

Other methods for designing flat slabs are again also acceptable, such as on yield-line analysis, Hillerborg's 'advanced' strip method and finite element analysis.

2.3 ACI Code

2.3.1 General

According to the ACI 318-83 code all two-way reinforced concrete slab systems, including rigidly supported, semi-rigidly supported, and flat slabs, should be analysed and designed by unified approaches such as the Direct Design Method (DDM) or Equivalent Frame Method (EFM).

Briefly, the direct design method is restricted to slabs loaded by a uniformly distributed vertical load and which are supported on equally (or nearly so) spaced columns. The method uses a procedure that involves computing the total factored static moment M_0 for all spans in each direction. This total static moment M_0 is then distributed to negative factored moment $\overset{-}{M}_u$ at the critical section at the support and positive factored moment $\overset{+}{M}_u$ at the critical section near the mid span using bending moment coefficients provided by the code. These moments at the critical sections are then distributed between column and middle strips using a Table of coefficients given in the code.

In contrast, the equivalent frame method (EFM) has a wider scope of application. Thus, the EFM does not place a limit on the column spacing and allows for both distributed and point loads in the vertical and/or horizontal directions. The technique employs an analysis of a strip of slab and associated columns where these are modelled as a rigid frame. Moments are distributed to critical sections by an elastic analysis rather than by the use of factors such as bending moment coefficients. Patterns of loading must be considered if the live load is greater than 0.75 x the dead load. This loading case is beyond the scope of the DDM. The positive and negative moments at critical sections obtained by the EFM are then distributed to column and middle strips in the same manner as for the DDM using the same table of coefficients. More details of both these methods are to be found in Chapters 6 and 7.

The complexity of the generalized approach, particularly for systems that do not meet the requirements for analysis by the DDM in the present code, has led many engineers to continue using the design method of the older ACI 381-63 code for the simple cases of two-way slabs supported on four sides by rigid supports [4]. An example of the two methods is given in Chapter 5.

As with the British Code plastic and elastic methods of analysis are also permitted provided other limit conditions are satisfied.

2.3.2 Rigidly supported slabs

For this class of slab the designer may therefore use the bending moment coefficients given in the older ACI 318-63; or the Equivalent Frame Method; or the Direct Design Method; or plastic and elastic methods.

As far as the Tables of bending moment coefficients are concerned there is a similarity between the two codes though for various reasons explained later the bending moment coefficients at first sight appear quite different.

2.3.3 Semi-rigidly supported slabs

For this class of slab the Direct Design or Equivalent Frame Method can be used as can plastic and elastic methods. Perhaps wisely in view of the variability of the rigidity of side supports the ACI code does not permit the use of the bending moment coefficients used for rigidly supported slabs as is the case with the British code.

2.3.4 Flat slabs

Although the ACI code deals with slabs supported on columns with drops, this work is restricted to flat plates i.e. slabs supported on columns without drops. Again the ACI code gives two principal design approaches, the DDM and EFM. In the recommended equivalent frame method the designer may use either the moment distribution method to obtain forces and moments at critical sections or any suitable elastic method. The ACI code also permits finite element analysis and other theoretical approaches such as yield-line analysis and the Hillerborg method, provided that strength and serviceability requirements are met.

2.4 Comparison of the Two Codes

The major difference between the two codes for the process of calculating moments is that the DDM and EFM can be used for all classes of slab in the American code whereas the EFM and equivalent DDM method is only used for flat slabs in the British code.

Simplified bending moment coefficients may be used for rigidly supported slabs in both codes and these same coefficients can be used for semi-rigidly supported slabs in the British code but not in the American code unless relatively stiff beams are used. These coefficients are virtually the only reference to the design of semi-rigidly supported beams in the British code and it is woefully deficient from this aspect. Conversely the ACI code specifically states the stiffness requirements of the beams if the coefficients are to be used and gives the DDM method as a simple alternative.

There are also several other differences between the two codes. First the load factors are different and in the British code factors of safety are applied to the materials whilst in the American code they are not, but this latter code has a structural type factor which is absent from the British code.

One of the difficulties of the comparison therefore is to establish a common base from which the two codes can be compared. This process of establishing a common base is discussed later. It is also however intended to examine elastic and plastic methods of slab design to see if any relationship exists between these methods and the code recommendations and to establish any implications of such relationships.

CHAPTER 3
METHODS FOR SLAB ANALYSIS

3.1 Introduction

There are two main sections in this chapter. The first presents a brief summary of the historical development of elastic and plastic rectangular solid slab theories and the second describes the available approaches for each method.

3.2 Historical Development of Slab Theories

3.2.1 Elastic theory

3.2.1.1 Slabs on rigid supports

The behaviour of plates spanning in two directions and loaded perpendicularly to their planes was first investigated at the beginning of the nineteenth century. The differential equation of bending was derived by Lagrange in 1811 and in 1820 Navier [5] presented the solutions for a simply supported rectangular plate subjected to a uniformly distributed load or with a load concentrated at the centre.

Towards the end of the nineteenth century, shipbuilders began using steel plates in place of wood and this created a need for analytical solutions of plate problems [6]. In 1921, Westergaard and Slater published their classical work on the analysis and design of slabs [7]. This paper included a sound demonstration of the theory of plates, ingenious projections of the available theoretical solutions to solve practical problems, and a comprehensive study of the implications of the then available tests on flat slabs and two-way slabs. In 1926, Westergaard [8] published a paper proposing a method of design for two-way slabs. This paper contained moment coefficients for slabs and supporting beams. The coefficients were based on the analysis of continuous plates on rigid supports providing no torsional restraint.

Prior to 1950 most of the elastic solutions of plate problems were solved analytically using the direct solution of the appropriate governing differential equations or by energy methods. These methods were successfully employed to solve single, rigidly supported, rectangular plates with free, simply supported or fully fixed boundaries. However, when the boundary conditions of a plate are more complex, the analysis becomes increasingly tedious and even impossible. In such cases numerical and approximate methods are the only practical approach. Fortunately with the advent of computers, numerical techniques, such as finite differences and finite elements, have been used increasingly to obtain solutions to such problems.

3.2.1.2 Slabs on semi-rigid supports

In the case of beam and slab construction, the early solutions considered only the interactive vertical force between the beams and the plate and the eccentric connection of the plate and beam (L-beam action) and torsional restraint from the beams was not considered. Later however researchers analysed models which reflect the elastic behaviour of actual structures, in particular, the effect of beam flexural stiffness and eccentric beam-slab connection (T- or L-beam).

In 1953 Sutherland, Goodman and Newmark [9] published a solution for a rectangular interior panel with simple beams (no T-beam action) of varying flexural rigidity. The solution was obtained using the Ritz energy approach. Wood [10], in 1955, gave the boundary conditions for full composite action between a slab and an edge beam which included the effect of eccentric connection and torsional stiffness of the edge beam. He then went on to use the finite difference method to solve the problems of a square single panel and a square interior panel with flexible beams, although in these solutions the effects of eccentric connection and torsion were not considered. Generally speaking however the research on this complex subject has been somewhat limited.

3.2.1.3 Flat slabs

The first flat slab was constructed by Turner in 1906 but it was not until 1914 that Nichols [11] published the first simple analysis of a flat slab. Nielsen had obtained a finite difference solution for a square interior panel on point supports and Timoshenko and Woinowsky-Krieger [5] give some solutions of rectangular interior panels on point supports and square interior panels on square supports which had also been confirmed by Nadai and Woinowsky, using the classical approach [5]. Again as the loading and boundary conditions become more complex the fewer are the classical solutions. Since the strict mathematic solutions became more difficult the alternative was to approximate the problem with the result that the total structure was subdivided into substructures often simplified as with the common equivalent frame method.

However with the advent of computers and finite element programmes such simplification can be avoided if desirable.

3.2.2 Collapse theories

Collapse theories, as the name implies, attempt to predict the load at which failure will occur. It is now well known that for a mathematically correct failure solution three conditions need to be satisfied, namely

a. the mechanism condition;

b. the equilibrium condition; and

c. the yield condition.

If a slab merely satisfies condition (a) then the solution is unsafe or an upper bound since there may be places other than along yield lines where the yield condition has been reached. If condition (b) is satisfied at all points and (c) satisfied at a single or several points, the load is a lower bound solution since sufficient yield may not have taken place to form a collapse mechanism. In this thesis upper bound solutions will be restricted to yield-line analysis and for lower bound solutions the main emphasis will be restricted to Hillerborg's work or elastic moment fields reinforced in accordance with the Wood-Armer reinforcement rules.

3.2.2.1 Yield-line theory

The first recorded instance of collapse loads being calculated for rectangular slabs is attributed to Ingerslev [12] in 1923 who used a method which was later realised to be an intuitive application of yield-line theory.

Yield-line theory was extended and advanced by a Danish engineer, Johansen, who published his doctoral thesis on the subject in 1943 [13]. The early literature on yield-line theory was mainly in Danish and in 1953 Hognestad [14] produced the first summary of this work in English. By the 1960's, yield-line theory had been extensively treated in publications by Wood [15], Jones [16], Wood and Jones [17], Kemp [18], Morley [19] and numerous other authors. Yield-line theory which is based on a mechanism collapse of the slab is an upper bound for the collapse load value. The method is applicable to rigidly supported, semi-rigidly supported or flat slabs.

3.2.2.2 Hillerborg's strip method

In 1956 Hillerborg [20] introduced his simple strip method which calculates a lower bound to the slab strength, and is thus an inherently safe value of the collapse load. Hillerborg's simple strip method has however limitations and is generally only suitable for rigidly supported or semi rigidly supported slabs where the semi-rigid supports are beams. In 1959 however Hillerborg [21] extended his method and developed his 'advanced' strip method. This method is little known in this country and is suitable for flat slabs. The method has recently been extended by Jones and Wood [22].

3.2.2.3 Other lower bound theories

Any method which satisfies the conditions of equilibrium and yield is a lower bound solution. Later it will be shown that the yield condition generally in use is that due to Wood and Armer though Hillerborg predates their more rigorous approach. Any set of equations which satisfy equilibrium therefore constitutes a lower bound theory. Unquestionably the commonest method used is the calculation of the moment field by elastic techniques, usually finite element analysis, and these field moments are used in conjunction with the Wood-Armer reinforcement rules. This technique forms the basis of most modern day computer programs.

3.3 Elastic Analysis

3.3.1 Basic slab theory

This sub-section introduces the terminology and theory employed for the elastic analysis of homogeneous and isotropic plate-like structures. In the next sub-section the problem of applying this basic theory to reinforced concrete is discussed.

The governing differential equation of elastic, homogeneous, isotropic plates subject to lateral load is

$$\frac{\partial^4 w}{\partial x^4} + \frac{2\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \tag{3.1}$$

where w = deflection of plate in direction of loading at point (x,y)

q = vertical loading imposed on plate per unit area

D = flexural rigidity of plate

$$= \frac{Eh^3}{12(1 - \mu^2)} \tag{3.2}$$

E = Young's modulus of plate material

h = plate thickness

μ = Poisson's ratio

The expression for the moments, using the co-ordinate axis system shown in Fig. 3.1, are:

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \frac{\mu\partial^2 w}{\partial y^2}\right) \tag{3.3}$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \frac{\mu\partial^2 w}{\partial x^2}\right) \tag{3.4}$$

$$M_{xy} = -D(1 - \mu)\frac{\partial^2 w}{\partial x \partial y} \tag{3.5}$$

The derivation of the equations above can be found elsewhere [5, 23].

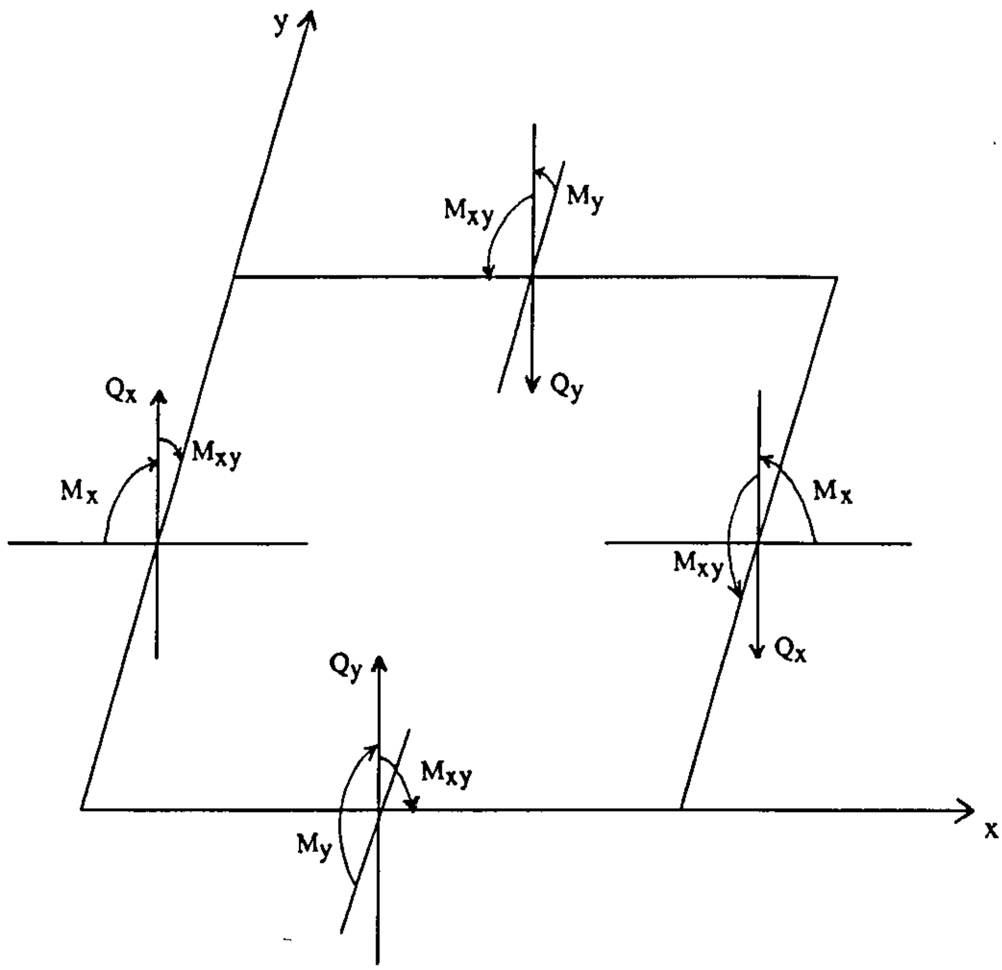


Fig. 3.1 System of axes and sign conventions

The general form of the governing equation 3.1 was first established by Lagrange in 1811 and it is often referred to as Lagrange's equation.

Lagrange's equation is actually an approximation to the true governing differential equation of a plate. This is because the effect of shear deformation on the deflection of the plate is ignored in its derivation. The true governing differential equation would be of the sixth order [5]. For plates which are not thick in relation to their span the solution of equation 3.1 gives results which are sufficiently accurate.

Most elastic analysis methods consider the slab stiffness to be uniform throughout the slab. This is true for a uniform steel plate but it is not true for a constant thickness reinforced concrete slab. Generally speaking, the slab reinforcement is varied throughout the slab and between the top layer and bottom layer. The variation results in different Young's modulus throughout the slab and must affect the slab stiffnesses. Similarly the value of Poisson's ratio for a slab suffering different degrees of cracking could expect to vary.

Therefore it has been concluded that when using these equations for reinforced concrete the values of E, h and μ are subjects which would in themselves be sources of extensive study. In the absence of any convincing argument to the contrary and accepting the limitations the E value will be taken as that for concrete, h the total slab thickness and μ as 0.2.

3.3.2 Methods of elastic slab analysis

3.3.2.1 Direct solution

The direct solution is obtained by solving the differential equation of the plate directly, using analytical methods of pure mathematics to find the internal forces and moments. Exact solutions for plates are difficult to find. Although some simple plate cases have been solved, others for different cases are extremely difficult using the classical solution for plates.

The first method of dealing with rectangular plates was developed by Navier in 1820, using double trigonometric series to transform the differential equations into a series of algebraic equations. Solutions to a considerable number of isotropic plate problems were produced in the first half of the present century, and an excellent survey has been presented by Timoshenko and Woinowsky-Krieger [5]. Particular results for a simply supported square plate and for a square plate fixed at all edges are reproduced in Table 3.1 and are discussed later.

In the past, bending moment coefficients, for orthotropic reinforcement in reinforced concrete slab design, were based on these exact solutions with some modification in light of experimental tests. The modification was due to the difficulty of incorporating the effect of torsion field moments in these coefficients mathematically. Westergaard was the first to propose coefficients for design; these coefficients were modified in the light of tests carried out by Slater.

It is noted that the direct solutions are of use only for simple slab problems; they contributed to slab design by providing design coefficients with the aid of experimental tests.

3.3.2.2 Grillage analysis

The plate problem can be solved by a numerical approach called a grillage analysis. In this approach the plate is modelled as a grillage of interconnected longitudinal and transverse beams. In this model the slab's longitudinal stiffness is concentrated in the longitudinal beams while the transverse stiffness is concentrated in the transverse beams. This approach is based in part on the physical resemblance between an interconnected grillage of beams and a plate. The flexural and torsional stiffnesses of the grillage members are determined so that as close an approximation to the behaviour of a slab is obtained. The accuracy of a solution is largely dependent on the aptness of this structural modelling. This method can give good predictions, and has been used reliably on a wide variety of slab bridge decks. Literature discussing grillage analysis and its application can be found in publications by Morice [24] and Hambly [25], however this form of analysis has not been used in this thesis.

3.3.2.3 The finite difference method

For many plate problems of considerable practical interest, an analytical solution of the governing differential equations cannot be found. Fortunately, the numerical treatment of differential equations can yield approximate results that are acceptable for most practical purposes. The finite difference method is one of these numerical techniques. In the method of finite difference, a slab is first covered by a grid of stations. Where possible, a regular grid of equally spaced stations is employed. The derivatives in the differential equation 3.1 are then replaced by difference quantities at the intersection points (stations) of the grid. This is readily done manually through the use of a difference equation operator at each point. One equation is written for each point at which the deflection is unknown, and the group of equations is then solved simultaneously for the unknown deflections. Once the deflections have been found, the moments and shear are found using the appropriate relationship between deflections of groups of points. The derivation of the finite difference operators and the determination of internal forces are covered by Timoshenko [5].

The finite difference method has two main disadvantages: it requires (to a certain extent) mathematically trained operators; and certain boundary conditions are difficult to handle.

For slab analysis however it is now common practice to employ the well established and the more flexible finite element method, for which numerous computer programs have been written and which method is described next.

3.3.2.4 The finite element method

Today elastic analysis for complex structural cases are usually carried out by the finite element method. It is the most powerful and versatile of the numerical techniques currently available for structural analysis and can handle slab design involving orthotropy, varying depth, edge beams and practical boundary conditions.

In the finite element method, the actual continuum comprising the structure to be analysed, e.g. a concrete slab modelled as a uniform plate, is replaced by an equivalent idealized structure composed of discrete elements referred to as finite elements. The elements are bounded by intersecting straight or curved lines, and are connected together at a number of nodes. All material properties of the original plate are retained. The finite elements themselves take many and varied forms depending on the shape they are supposed to represent. For example, to represent flat plates, the choice of finite elements will usually be of triangular or of quadrilateral shape, whilst for solids, the finite elements will usually appear in the form of tetrahedrons or cubes. One of the many attractive features of the method is that the analysis is not constrained to using one type of element for the analysis of the complete structure. For example, slabs supported on beams and columns can be modelled by two-dimensional elements (e.g. a plate finite element) for the slab and one-dimensional elements (e.g. simple engineering beam) for the beams. However, the resulting substitute structure of the assemblage of finite elements should be chosen in such a manner that close similarities between the displacement patterns of the original and substitute structure are retained. In practice, since the displacements of the structure of interest are not known, the choice of substitute structure is based on engineering judgement and experience. Thus, for instance, if the actual structure is considered to have largely plate-like characteristics then it should be modelled by the appropriate plate finite elements.

A critical operation in the finite element method is the generation of element stiffness matrices, which are intimately linked to the compatibility of the deformations within the element as well as between the adjacent elements. Having found the individual stiffness matrices for the finite elements, the elements are then combined in an assembly procedure to form the global stiffness matrix that represents the stiffness characteristic of the structure at the nodal interconnections of its individual idealized elements. The global stiffness matrix k is related to the nodal forces and displacement by the matrix equation $\overline{P} = k\,\overline{\delta}$, where $\overline{P}$ is the vector of nodal forces, $\overline{\delta}$ is the vector of nodal deformations. Literature discussing finite element theory, practice and application can be found in many publications [26, 27, 28].

The object of this thesis is not only to compare the two specific Codes of Practice but also to comment on their validity. It will therefore be necessary to carry out an elastic analysis of various structures so that comparisons can be made. Because of the complexity of the structures examined it was decided that this analysis would be carried out using the finite element technique since direct solutions were not available for many of the cases and no suitable finite difference package was available. The finite element package that was used was that produced by PAFEC though some modifications had to be carried out to the basic package to make it suitable for use for reinforced concrete slabs.

3.3.2.4.1 PAFEC finite element analysis for slabs

The structural analysis of slabs in this thesis was performed using a general purpose finite element package known as PAFEC - Programme for Automatic Finite Element Calculation which is available at Loughborough University of Technology. Details on the use of this package can be found in appropriate manuals [29, 30, 31].

For reinforced concrete design we are particularly concerned with the field moments M_x, M_y and M_{xy}. Regrettably the PAFEC package outputs stresses and therefore it was first necessary to modify the package to convert the principal stress output into moments.

In addition to obtain the required reinforcement M_x^+, M_x^-, M_y^+, M_y^- it is necessary to apply the Wood-Armer yield condition rules so that additional modifications to the package were necessary before the package could be applied to reinforced concrete.

3.3.2.4.2 Modification of PAFEC for stress to moment output

The results from PAFEC for plate bending analysis is in a stress format. They include the principal stresses and their directions on three main levels of the plate section at each node of each element. These stress results had to be modified to field moments at the same nodes using the equations:

$$M_x = (\sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta)Z \tag{3.8}$$

$$M_y = (\sigma_1 \sin^2\theta + \sigma_2 \cos^2\theta)Z \tag{3.9}$$

$$M_{xy} = [(\sigma_1 - \sigma_2) \sin\theta \cos\theta)]Z \tag{3.10}$$

where σ_1, σ_2 are principal stresses and

θ the angle of the principal plane, in radians, measured as positive from the element x-axis in an anticlockwise sense

Z is the section modulus

M_x, M_y, M_{xy} are field moments

This modification to the PAFEC package was written in Fortran 77 and is included as Appendix 3A.

The moments obtained are the average of the moments at the common nodes of the meeting elements.

3.3.2.4.3 Wood-Armer reinforcement rules

Generally, reinforcing bars are placed at right angles in the x and y directions because it is impractical for the bars to follow the curvilinear directions of the principal stresses over the slab as shown in Figs. 3.2 and 3.3. The determination of the ultimate resisting moments required for a general design moment field M_x, M_y, and M_{xy} presents a problem if the torsional moment M_{xy} is present. Generally, designers have ignored the torsional moment M_{xy}, because of lack of a method to account for it, but clearly this is unsafe, particularly where twists are high, such as in the corner regions of slabs. The ultimate resisting moments required for a general design moment field including torsion are considered by applying the rules given by Wood and Armer [32]. The basic rules are as follows.

At any point in a slab where the field moments have been determined, the "ultimate resisting moment" provided by orthotropic reinforcement can be calculated by:
Bottom reinforcement (M_x^+, M_y^+):

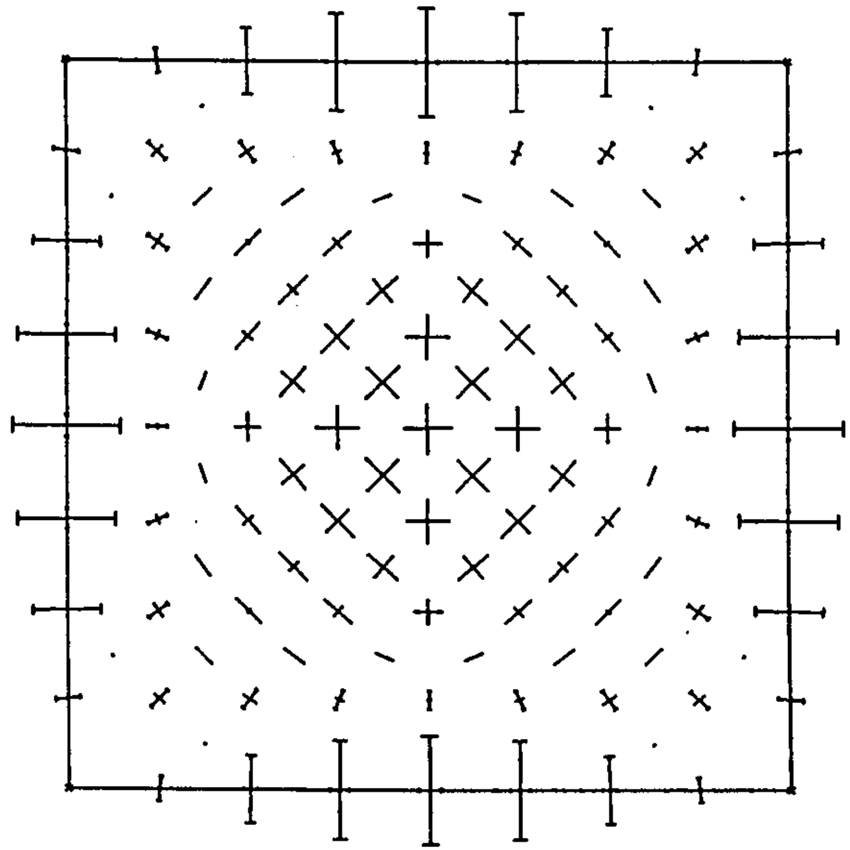


Fig. 3.2 Principal stress orientation at bottom surface of a single clamped edges panel of slab

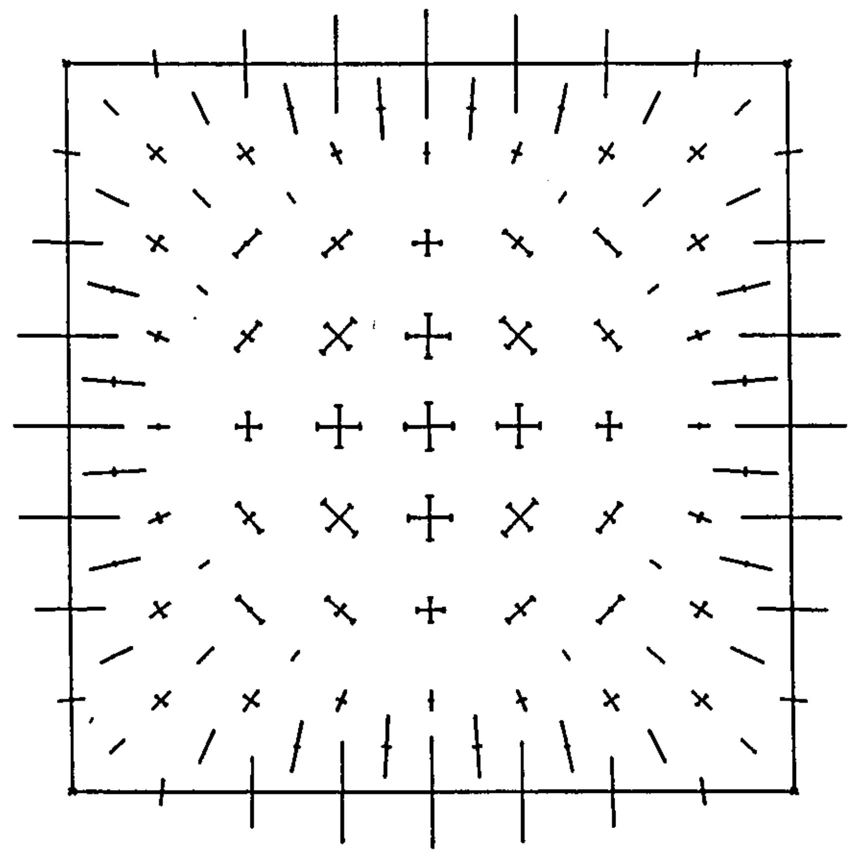


Fig. 3.3 Principal stress orientation at top surface of a single clamped edges panel of slab

$$M_x^+ = M_x + k\left|M_{xy}\right| \tag{3.11}$$

$$M_y^+ = M_y + \frac{1}{k}\left|M_{xy}\right| \tag{3.12}$$

where k is positive and arbitrary . It should be noted that the least quantity of reinforcement at any point is given when k = 1.

If both M_x^+ and M_y^+ are found to be negative, no bottom bar is needed in either direction. If either M_x^+ or M_y^+ is found to be negative, then the moments change to:

either

$$M_x^+ = M_x + \left|\frac{M_{xy}^2}{M_y}\right| \text{ with } M_y^+ = 0 \tag{3.13}$$

or

$$M_y^+ = M_y + \left|\frac{M_{xy}^2}{M_x}\right| \text{ with } M_x^+ = 0 \tag{3.14}$$

If negative M_x^+ or M_y^+ still occurs, no bottom bars are needed.

For the top reinforcement (M_x^-, M_y^-) the equations become

$$M_x^- = M_x - k\left|M_{xy}\right| \tag{3.15}$$

$$M_y^- = M_y - \frac{1}{k}\left|M_{xy}\right| \tag{3.16}$$

Again k must be positive but need not have the same value as that used for the bottom reinforcement.

If both M_x^- and M_y^- are found to be positive, no top bar is needed in either direction. If either M_x^- or M_y^- is found to be positive, then the moments change to either

$$M_x^- = M_x - \left|\frac{M_{xy}^2}{M_y}\right| \text{ with } M_y^- = 0 \tag{3.17}$$

or

$$M_y^- = M_y - \left|\frac{M_{xy}^2}{M_x}\right| \text{ with } M_x^- = 0 \tag{3.18}$$

If positive M_x^- or M_y^- still occurs, no top bars are needed.

The Wood-Armer rules have been included in the amended computer program and are included as Appendix 3B.

3.3.2.4.4 Assessment of number of finite elements required

When using any finite element package the accuracy of the results is highly dependent on the number of elements used in order to model the structure. In order to determine what might be considered a reasonable number of elements the author tried various numbers of elements at the start of the more extended analysis and compared the results for two types of slabs where a classical solution existed.

Timoshenko and Woinowsky have tabulated results which were found by classical solutions for some plate problems with simply supported or clamped edges and subjected to uniformly distributed loads. These problems were therefore solved using PAFEC and the two sets of results are tabulated for comparison in Table 3.1. The PAFEC plate element type No. 44200 was used which is a four-noded quadrilateral element with six degrees of freedom per node when assembled into the global stiffness matrix. The element formulation allows for combined membrane action and plate bending.

The analysis of the trial slabs was performed with two different numbers of elements for each case. The first employed a 4 x 4 element mesh and the second an 8 x 8 element mesh. It was found that the 4 x 4 element mesh results were quite inaccurate when compared with Timoshenko's results. In Table 3.1 the figures in brackets are the ratios of the finite element analysis results divided by Timoshenko's values. There was a 26% error in deflection in the centre of the panel for the simply supported slab and errors of 9% for the moment values; for the fixed edge slab the errors were 21% and 17% respectively. In contrast the results using the 8 x 8 mesh showed a maximum deflection error of 3% and moment error of 4%. Whilst adopting an even finer mesh

Type of panel	$\frac{L_y}{L_x}$	μ	Finite Element Mesh	Deflection at centre of panel ($\div wL_x^4/D$)		Moments at centre of panel ($\div wL_x^2$)				Moments at centre of fixed edges ($\div wL_x^2$)			
						M_x		M_y		M_x		M_y	
				F.E.A.	Timo.	F.E.A.	Timo.	F.E.A.	Timo.	F.E.A.	Timo.	F.E.A.	Timo.
Simply supported at four edges	1.0	0.3	4x4	0.003 (0.74)	0.00406	0.052 (1.09)	0.0479	0.052 (1.09)	0.0479	-	-	-	-
			8x8	0.00412 (1.01)	0.00406	0.0489 (1.02)	0.0479	0.0489 (1.02)	0.0479	-	-	-	-
Clamped at four edges	1.0	0.2	4x4	0.0010 (0.79)	0.00126	0.0250 (1.17)	0.0213	0.0250 (1.17)	0.0213	0.0474 (0.92)	0.0513	0.0474 (0.92)	0.0513
			8x8	0.00130 (1.03)	0.00126	0.0222 (1.04)	0.0213	0.0222 (1.04)	0.0213	0.0502 (0.98)	0.0513	0.0502 (0.98)	0.0513

Note: Values between brackets show the ratio of Finite Element Analysis (F.E.A.) to Timoshenko's results.

Table 3.1 Comparison of Finite Element Analysis with Timoshenko's results

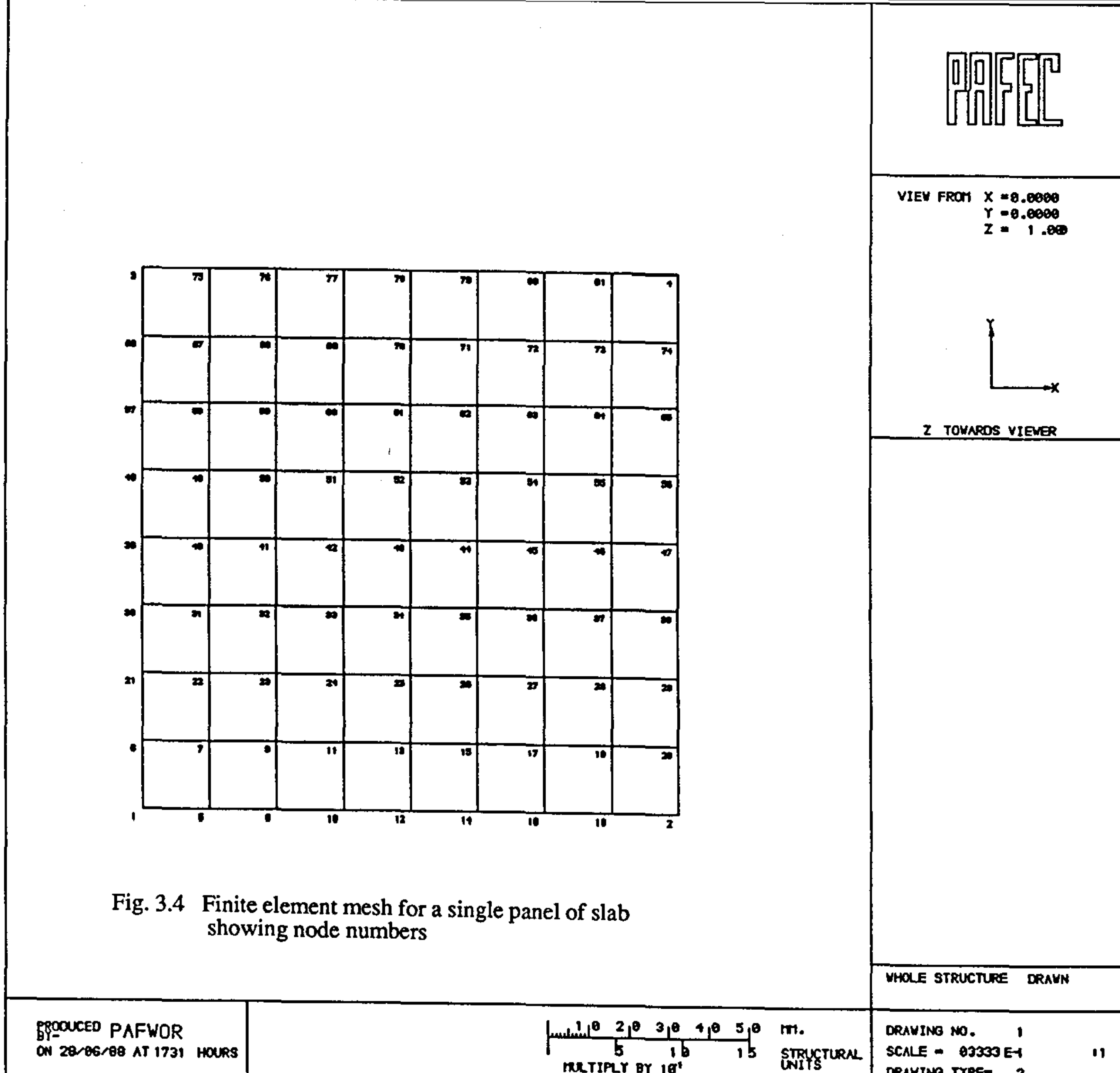


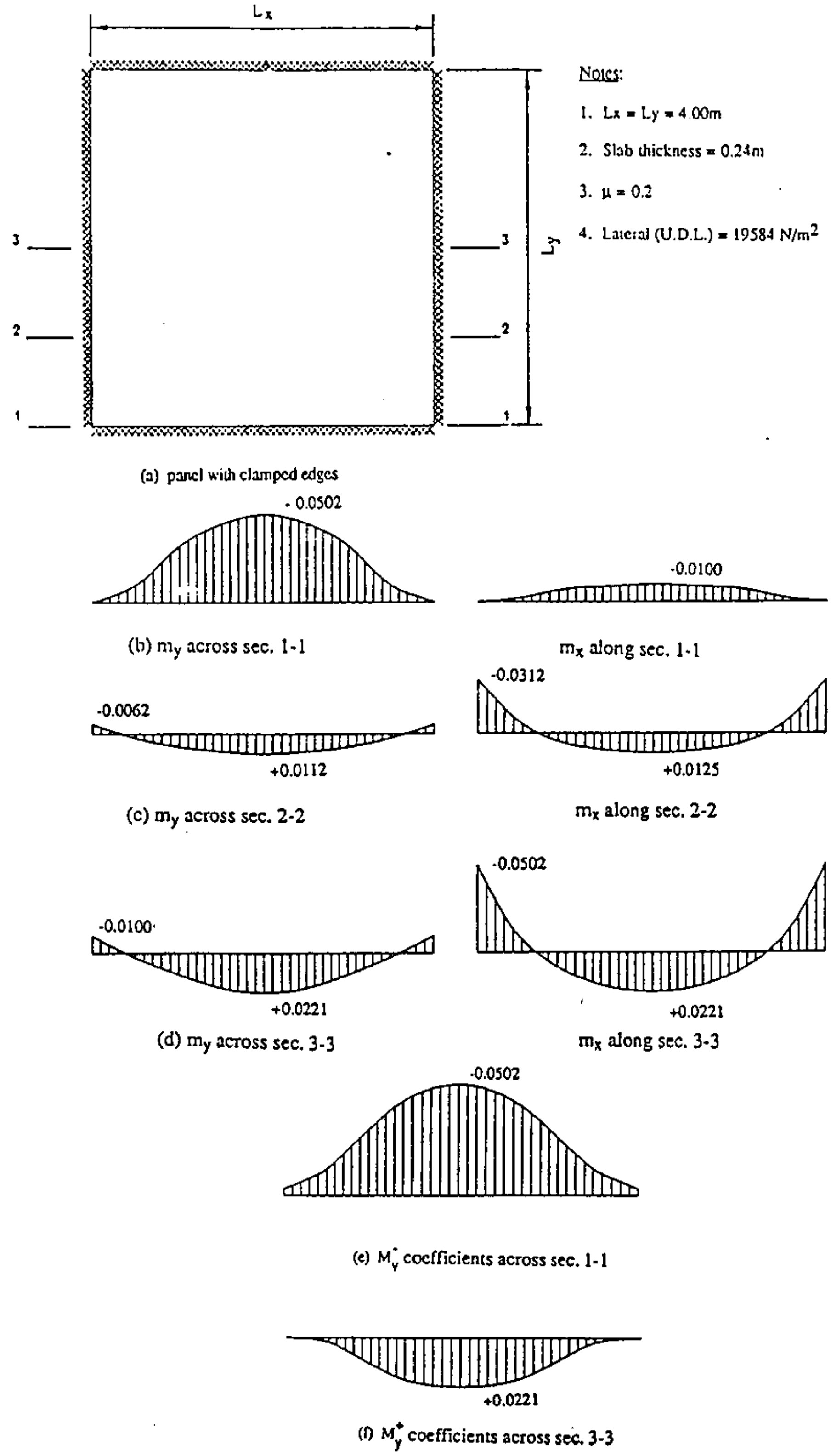
Fig. 3.4 Finite element mesh for a single panel of slab showing node numbers

would have reduced this error still further, in some cases analysed later there are 12 slabs with 64 elements each and the resulting output and computation time was likely to be excessive. It was considered therefore in view of all the other assumptions that this discrepancy in comparison with the exact solution was sufficiently accurate and an 8 x 8 mesh was therefore adopted for all panels in subsequent analysis. Fig. 3.4 shows the element mesh and the node numbers for the plate used in this analysis.

3.3.2.4.5 Finite element example

Prior to analysing the different cases given in the code which entails multipanel slabs with pattern loading, it was decided to analyse the simple case of a single panel with clamped edges to establish the general procedure (Fig. 3.5a).

The dimensions of the plate considered were 4.00 x 4.00 m with Poisson's ratio as 0.2 (as used in BS 8110). The chosen load is uniformly distributed with the ratio of characteristic imposed load to characteristic dead load set at 1.25. This ratio of live to dead load has been introduced in order to facilitate the use pattern of loading for multipanel systems later on. The ultimate load n will therefore be = 1.4 D.L + 1.6 x 1.25 D.L = 3.4 D.L. For the Finite Element Analysis the plate is divided into 8 x 8 elements as in Fig. 3.4. The results from the PAFEC basic program were converted to field moments M_x, M_y, M_{xy} for each node using equations 3.8, 9, 10 and these values were then introduced into equations 3.11 to 3.18 in order to determine the equivalent reinforcement bending moments using the Wood-Armer rules. The values at the 81 nodes are given in Table 3.2 and these values are then divided by nL^2 to give the coefficient form in Table 3.3. Fig. 3.5b, c, d shows the variation of the moment coefficient $m_x = M_x/nL_x^2$, $m_y = M_y/nL_y^2$ at different sections of plate. For the purpose of reinforcement, the Wood-Armer rules for practical reinforcement can be applied in the x and y directions by finding (M_x^+, M_x^-, M_y^+, M_y^-) at each node; (M_x^+, M_y^+) and (M_x^-, M_y^-) are used for bottom and top reinforcement respectively. The actual moments have been divided by nL_x^2 to obtain the moment coefficients. It is particularly interesting to



Note: m_{xy} is too small to draw

Fig. 3.5 Field and reinforcing coefficient moment diagrams for slab with clamped edges

Table 3.2 Field moments and reinforcing moments values in slab with clamped edges

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
1	0.0000	0.0000	1046.3999	1046.3999	-1046.3999	1046.3999	-1046.3999
2	0.0000	0.0000	-1046.3999	1046.3999	-1046.3999	1046.3999	-1046.3999
3	0.0000	0.0000	-1046.3999	1046.3999	-1046.3999	1046.3999	-1046.3999
4	0.0000	0.0000	1046.3999	1046.3999	-1046.3999	1046.3999	-1046.3999
5	-712.8855	-3562.9546	-310.7227	0.0000	-1023.6083	0.0000	-3873.6777
6	-3562.9546	-712.8855	-310.7227	0.0000	-3873.6777	0.0000	-1023.6083
7	-758.4001	-758.4001	-2648.9160	1890.5159	-3407.3164	1890.5159	-3407.3164
8	-1957.6433	-9783.1562	-123.2482	0.0000	-2080.8916	0.0000	-9906.4062
9	402.2370	-1511.0374	-2644.7192	3046.9561	-2242.4824	1133.6819	-4155.7568
10	-2838.5024	-14191.8984	-29.7234	0.0000	-2868.2261	0.0000	-14221.6230
11	570.7563	-2418.7568	-1512.5046	1516.5603	-941.7483	0.0000	-3931.2617
12	-3146.5381	-15727.0625	0.0000	0.0000	-3146.5381	0.0000	-15727.0625
13	560.7345	-2802.5747	-0.0221	560.7345	0.0000	0.0000	-2802.5752
14	-2838.5024	-14191.8984	29.7234	0.0000	-2868.2261	0.0000	-14221.6230
15	570.7563	-2418.7568	1512.5042	1516.5598	-941.7478	0.0000	-3931.2612
16	-1957.6433	-9783.1562	123.2482	0.0000	-2080.8916	0.0000	-9906.4062
17	402.2370	-1511.0374	2644.7187	3046.9556	-2242.4819	1133.6814	-4155.7568
18	-712.8855	-3562.9546	310.7226	0.0000	-1023.6082	0.0000	-3873.6772
19	-758.4001	-758.4001	2648.9150	1890.5149	-3407.3154	1890.5149	-3407.3154
20	-3562.9546	-712.8855	310.7226	0.0000	-3873.6772	0.0000	-1023.6082
21	-9783.1562	-1957.6433	-123.2482	0.0000	-9906.4062	0.0000	-2080.8916
22	-1511.0374	402.2370	-2644.7192	1133.6819	-4155.7568	3046.9561	-2242.4824
23	2160.3599	2160.3599	-2723.6069	4883.9668	-563.2471	4883.9668	-563.2471
24	3579.7051	3173.8940	-1599.8511	5179.5557	0.0000	4773.7451	0.0000
25	3924.5386	3501.0601	0.0000	3924.5386	0.0000	3501.0601	0.0000
26	3579.7056	3173.8936	1599.8506	5179.5557	0.0000	4773.7441	0.0000
27	2160.4795	2160.4795	2723.4858	4883.9648	-563.0063	4883.9648	-563.0063
28	-1511.0374	402.2370	2644.7187	1133.6814	-4155.7568	3046.9556	-2242.4819
29	-9783.1562	-1957.6433	123.2482	0.0000	-9906.4062	0.0000	-2080.8916
30	-14191.8984	-2838.5024	-29.7234	0.0000	-14221.6230	0.0000	-2868.2261
31	-2418.7568	570.7563	-1512.5046	0.0000	-3931.2617	1516.5603	-941.7483
32	3173.8940	3579.7051	-1599.8511	4773.7451	0.0000	5179.5557	0.0000
33	5535.5996	5535.5996	-951.5447	6487.1436	0.0000	6487.1436	0.0000
34	6163.2827	6208.7168	1.1711	6164.4531	0.0000	6209.8877	0.0000
35	5535.5996	5535.5986	951.5687	6487.1680	0.0000	6487.1670	0.0000
36	3173 8940	3579.7056	1599.8506	4773.7441	0.0000	5179.5557	0.0000
37	-2418.7568	570.7563	1512.5044	0.0000	-3931.2612	1516.5601	-941.7480
38	-14191.8984	-2838.5024	29.7234	0.0000	-14221.6230	0.0000	-2868.2261
39	-15727.0625	-3146.5381	0.0000	0.0000	-15727.0625	0.0000	-3146.5381
40	-2802.5728	560.9724	0.0000	0.0000	-2802.5728	560.9724	0.0000
41	3501.0601	3924.5386	0.0000	3501.0601	0.0000	3924.5386	0.0000
42	6209.9072	6164.4912	0.0000	6209.9072	0.0000	6164.4912	0.0000
43	6940.7988	6940.7988	0.0000	6940.7988	0.0000	6940.7988	0.0000
44	6208.9258	6163.0723	-1.1832	6210.1084	0.0000	6164.2549	0.0000
45	3501.0601	3924.5386	0.0000	3501.0601	0.0000	3924.5386	0.0000
46	-2802.5747	560.7344	0.0221	0.0000	-2802.5752	560.7344	0.0000
47	-15727.0625	-3146.5381	0.0000	0.0000	-15727.0625	0.0000	-3146.5381
48	-14191.8984	-2838.5024	29.7234	0.0000	-14221.6230	0.0000	-2868.2261
49	-2418.7568	570.7563	1512.5042	0.0000	-3931.2612	1516.5598	-941.7478
50	3173.8936	3579.7056	1599.8508	4773.7441	0.0000	5179.5557	0.0000
51	5534.6064	5536.5918	951.5565	6486.1621	0.0000	6488.1475	0.0000
52	6163.2812	6208.7168	-1.1711	6164.4521	0.0000	6209.8877	0.0000
53	5535.5986	5535.5996	-951.5690	6487.1670	0.0000	6487.1680	0.0000
54	3173.8936	3579.7031	-1599.8508	4773.7441	0.0000	5179.5557	0.0000
55	-2418.7568	570.7563	-1512.5046	0.0000	-3931.2617	1516.5603	-941.7483
56	-14191.8984	-2838.5024	-29.7234	0.0000	-14221.6230	0.0000	-2868.2261
57	-9783.1562	-1957.6433	123.2482	0.0000	-9906.4062	0.0000	-2080.8916
58	-1511.0374	402.2370	2644.7187	1133.6814	-4155.7568	3046.9556	-2242.4819
59	2160.4795	2160.4795	2723.4858	4883.9648	-563.0063	4883.9648	-563.0063
60	3579.7056	3173.8940	1599.8506	5179.5537	0.0000	4773.7441	0.0000
61	3924.5391	3501.0601	0.0000	3924.5391	0.0000	3501.0601	0.0000
62	3579.7031	3173.8936	-1599.8511	5179.5537	0.0000	4773.7441	0.0000
63	2160.4795	2160.4795	-2723.4868	4883.9658	-563.0073	4883.9658	-563.0073
64	-1511.0374	402.2370	-2644.7192	1133.6819	-4155.7568	3046.9561	-2242.4824
65	-9783.1562	-1957.6433	-123.2482	0.0000	-9906.4062	0.0000	-2080.8916
66	-3562.9546	-712.8855	310.7226	0.0000	-3873.6772	0.0000	-1023.6082
67	-758.4001	-758.4001	2648.9150	1890.5149	-3407.3154	1890.5149	-3407.3154
68	402.2370	-1511.0374	2644.7187	3046.9556	-2242.4819	1133.6814	-4155.7568
69	570.7563	-2418.7568	1512.5044	1516.5601	-941.7480	0.0000	-3931.2612
70	560.7344	-2802.5747	0.0221	560.7344	0.0000	0.0000	-2802.5752
71	570.7563	-2418.7568	-1512.5046	1516.5603	-941.7483	0.0000	-3931.2617
72	402.2370	-1511.0374	-2644.7192	3046.9561	-2242.4824	1133.6819	-4155.7568
73	-758.4001	-758.4001	-2648.9160	1890.5159	-3407.3164	1890.5159	-3407.3164
74	-3562.9546	-712.8855	-310.7227	0.0000	-3873.6777	0.0000	-1023.6083
75	-712.8855	-3562.9546	310.7226	0.0000	-1023.6082	0.0000	-3873.6772
76	-1957.6433	-9783.1562	123.2482	0.0000	-2080.8916	0.0000	-9906.4062
77	-2838.5024	-14191.8984	29.7234	0.0000	-2868.2261	0.0000	-14221.6230
78	-3146.5381	-15727.0625	0.0000	0.0000	-3146.5381	0.0000	-15727.0625
79	-2838.5024	-14191.8984	-29.7234	0.0000	-2868.2261	0.0000	-14221.6230
80	-1957.6433	-9783.1562	-123.2482	0.0000	-2080.8916	0.0000	-9906.4062
81	-712.8855	-3562.9546	-310.7227	0.0000	-1023.6083	0.0000	-3873.6777

Table 3.3 Field and reinforcing coefficient moment values in slab with clamped edges

MOMENT COEFFICIENTS FOR:

NODE NO.	MX	MY	MXY	MX+	MX-	MY+	MY-
1	0.0000	0.0000	0.0033	0.0033	-0.0033	0.0033	-0.0033
2	0.0000	0.0000	-0.0033	0.0033	-0.0033	0.0033	-0.0033
3	0.0000	0.0000	-0.0033	0.0033	-0.0033	0.0033	-0.0033
4	0.0000	0.0000	0.0033	0.0033	-0.0033	0.0033	-0.0033
5	-0.0023	-0.0114	-0.0010	0.0000	-0.0033	0.0000	-0.0124
6	-0.0114	-0.0023	-0.0010	0.0000	-0.0124	0.0000	-0.0033
7	-0.0024	-0.0024	-0.0085	0.0060	-0.0109	0.0060	-0.0109
8	-0.0062	-0.0312	-0.0004	0.0000	-0.0066	0.0000	-0.0316
9	0.0013	-0.0048	-0.0084	0.0097	-0.0072	0.0036	-0.0133
10	-0.0091	-0.0453	-0.0001	0.0000	-0.0092	0.0000	-0.0454
11	0.0018	-0.0077	-0.0048	0.0048	-0.0030	0.0000	-0.0125
12	-0.0100	-0.0502	0.0000	0.0000	-0.0100	0.0000	-0.0502
13	0.0018	-0.0089	0.0000	0.0018	0.0000	0.0000	-0.0089
14	-0.0091	-0.0453	0.0001	0.0000	-0.0092	0.0000	-0.0454
15	0.0018	-0.0077	0.0048	0.0048	-0.0030	0.0000	-0.0125
16	-0.0062	-0.0312	0.0004	0.0000	-0.0066	0.0000	-0.0316
17	0.0013	-0.0048	0.0084	0.0097	-0.0072	0.0036	-0.0133
18	-0.0023	-0.0114	0.0010	0.0000	-0.0033	0.0000	-0.0124
19	-0.0024	-0.0024	0.0085	0.0060	-0.0109	0.0060	-0.0109
20	-0.0114	-0.0023	0.0010	0.0000	-0.0124	0.0000	-0.0033
21	-0.0312	-0.0062	-0.0004	0.0000	-0.0316	0.0000	-0.0066
22	-0.0048	0.0013	-0.0084	0.0036	-0.0133	0.0097	-0.0072
23	0.0069	0.0069	-0.0087	0.0156	-0.0018	0.0156	-0.0018
24	0.0114	0.0101	-0.0051	0.0165	0.0000	0.0152	0.0000
25	0.0125	0.0112	0.0000	0.0125	0.0000	0.0112	0.0000
26	0.0114	0.0101	0.0051	0.0165	0.0000	0.0152	0.0000
27	0.0069	0.0069	0.0087	0.0156	-0.0018	0.0156	-0.0018
28	-0.0048	0.0013	0.0084	0.0036	-0.0133	0.0097	-0.0072
29	-0.0312	-0.0062	0.0004	0.0000	-0.0316	0.0000	-0.0066
30	-0.0453	-0.0091	-0.0001	0.0000	-0.0454	0.0000	-0.0092
31	-0.0077	0.0018	-0.0048	0.0000	-0.0125	0.0048	-0.0030
32	0.0101	0.0114	-0.0051	0.0152	0.0000	0.0165	0.0000
33	0.0177	0.0177	-0.0030	0.0207	0.0000	0.0207	0.0000
34	0.0197	0.0198	0.0000	0.0197	0.0000	0.0198	0.0000
35	0.0177	0.0177	0.0030	0.0207	0.0000	0.0207	0.0000
36	0.0101	0.0114	0.0051	0.0152	0.0000	0.0165	0.0000
37	-0.0077	0.0018	0.0048	0.0000	-0.0125	0.0048	-0.0030
38	-0.0453	-0.0091	0.0001	0.0000	-0.0454	0.0000	-0.0092
39	-0.0502	-0.0100	0.0000	0.0000	-0.0502	0.0000	-0.0100
40	-0.0089	0.0018	0.0000	0.0000	-0.0089	0.0018	0.0000
41	0.0112	0.0125	0.0000	0.0112	0.0000	0.0125	0.0000
42	0.0198	0.0197	0.0000	0.0198	0.0000	0.0197	0.0000
43	0.0222	0.0222	0.0000	0.0222	0.0000	0.0222	0.0000
44	0.0198	0.0197	0.0000	0.0198	0.0000	0.0197	0.0000
45	0.0112	0.0125	0.0000	0.0112	0.0000	0.0125	0.0000
46	-0.0089	0.0018	0.0000	0.0000	-0.0089	0.0018	0.0000
47	-0.0502	-0.0100	0.0000	0.0000	-0.0502	0.0000	-0.0100
48	-0.0453	-0.0091	0.0001	0.0000	-0.0454	0.0000	-0.0092
49	-0.0077	0.0018	0.0048	0.0000	-0.0125	0.0048	-0.0030
50	0.0101	0.0114	0.0051	0.0152	0.0000	0.0165	0.0000
51	0.0177	0.0177	0.0030	0.0207	0.0000	0.0207	0.0000
52	0.0197	0.0198	0.0000	0.0197	0.0000	0.0198	0.0000
53	0.0177	0.0177	-0.0030	0.0207	0.0000	0.0207	0.0000
54	0.0101	0.0114	-0.0051	0.0152	0.0000	0.0165	0.0000
55	-0.0077	0.0018	-0.0048	0.0000	-0.0125	0.0048	-0.0030
56	-0.0453	-0.0091	-0.0001	0.0000	-0.0454	0.0000	-0.0092
57	-0.0312	-0.0062	0.0004	0.0000	-0.0316	0.0000	-0.0066
58	-0.0048	0.0013	0.0084	0.0036	-0.0133	0.0097	-0.0072
59	0.0069	0.0069	0.0087	0.0156	-0.0018	0.0156	-0.0018
60	0.0114	0.0101	0.0051	0.0165	0.0000	0.0152	0.0000
61	0.0125	0.0112	0.0000	0.0125	0.0000	0.0112	0.0000
62	0.0114	0.0101	-0.0051	0.0165	0.0000	0.0152	0.0000
63	0.0069	0.0069	-0.0087	0.0156	-0.0018	0.0156	-0.0018
64	-0.0048	0.0013	-0.0084	0.0036	-0.0133	0.0097	-0.0072
65	-0.0312	-0.0062	-0.0004	0.0000	-0.0316	0.0000	-0.0066
66	-0.0114	-0.0023	0.0010	0.0000	-0.0124	0.0000	-0.0033
67	-0.0024	-0.0024	0.0085	0.0060	-0.0109	0.0060	-0.0109
68	0.0013	-0.0048	0.0084	0.0097	-0.0072	0.0036	-0.0133
69	0.0018	-0.0077	0.0048	0.0048	-0.0030	0.0000	-0.0125
70	0.0018	-0.0089	0.0000	0.0018	0.0000	0.0000	-0.0089
71	0.0018	-0.0077	-0.0048	0.0048	-0.0030	0.0000	-0.0125
72	0.0013	-0.0048	-0.0084	0.0097	-0.0072	0.0036	-0.0133
73	-0.0024	-0.0024	-0.0085	0.0060	-0.0109	0.0060	-0.0109
74	-0.0114	-0.0023	-0.0010	0.0000	-0.0124	0.0000	-0.0033
75	-0.0023	-0.0114	0.0010	0.0000	-0.0033	0.0000	-0.0124
76	-0.0062	-0.0312	0.0004	0.0000	-0.0066	0.0000	-0.0316
77	-0.0091	-0.0453	0.0001	0.0000	-0.0092	0.0000	-0.0454
78	-0.0100	-0.0502	0.0000	0.0000	-0.0100	0.0000	-0.0502
79	-0.0091	-0.0453	-0.0001	0.0000	-0.0092	0.0000	-0.0454
80	0.0062	-0.0312	-0.0004	0.0000	-0.0066	0.0000	-0.0316
81	-0.0023	-0.0114	-0.0010	0.0000	-0.0033	0.0000	-0.0124

compare the effect of using the Wood-Armer rules on the bottom reinforcement by comparing m_y and M_y^+ in Fig. 3.5d and f.

3.4 Yield-Line Analysis

3.4.1 Simple theory

The yield-line method for slabs is the earliest and the most successful application of plasticity to stuctural concrete. It enables numerous shapes of slab to be analysed which had never been attempted by traditional elastic analysis.

To find the collapse load, a collapse mechanism, composed of rigid portions of the slab separated by lines of plastic hinges must first be postulated. The ultimate load is calculated by stipulating the deflection of one point in the slab and using the virtual work method, in which the work done in the yield lines is equated to the loss of work due to the load deflection i.e. the external work $\Sigma(w\delta)$ is equated to the energy dissipated in the yield lines due to the rotation of the rigid regions (internal virtual work $= \Sigma\int(M\theta)$. The pattern may initially be defined by variable geometric parameters and differentiation of the work equation may be necessary to establish the most critical value of the unknown geometrical parameters and hence find the most critical load.

Generally but not necessarily in the yield-line method the reinforcement is initially imagined to be placed uniformly across the whole width of the slab. The conventional representation of reinforcement bending strength/unit length in the yield-line method is as shown in Fig.3.6, where m is a uniform positive bending strength/unit length across the short span, μm is a uniform bending strength/unit length across the long span. The strength -im and -iμm similarly represent the value of the bending strength due to the (negative) top steel.

The parameters μ and i are very important in reinforced concrete slab design. The parameter μ is the relative proportion of the resistance moment of long to short span. For suitable serviceability behaviour the value of μ used in design should not be too dissimilar to that found by elastic behaviour.

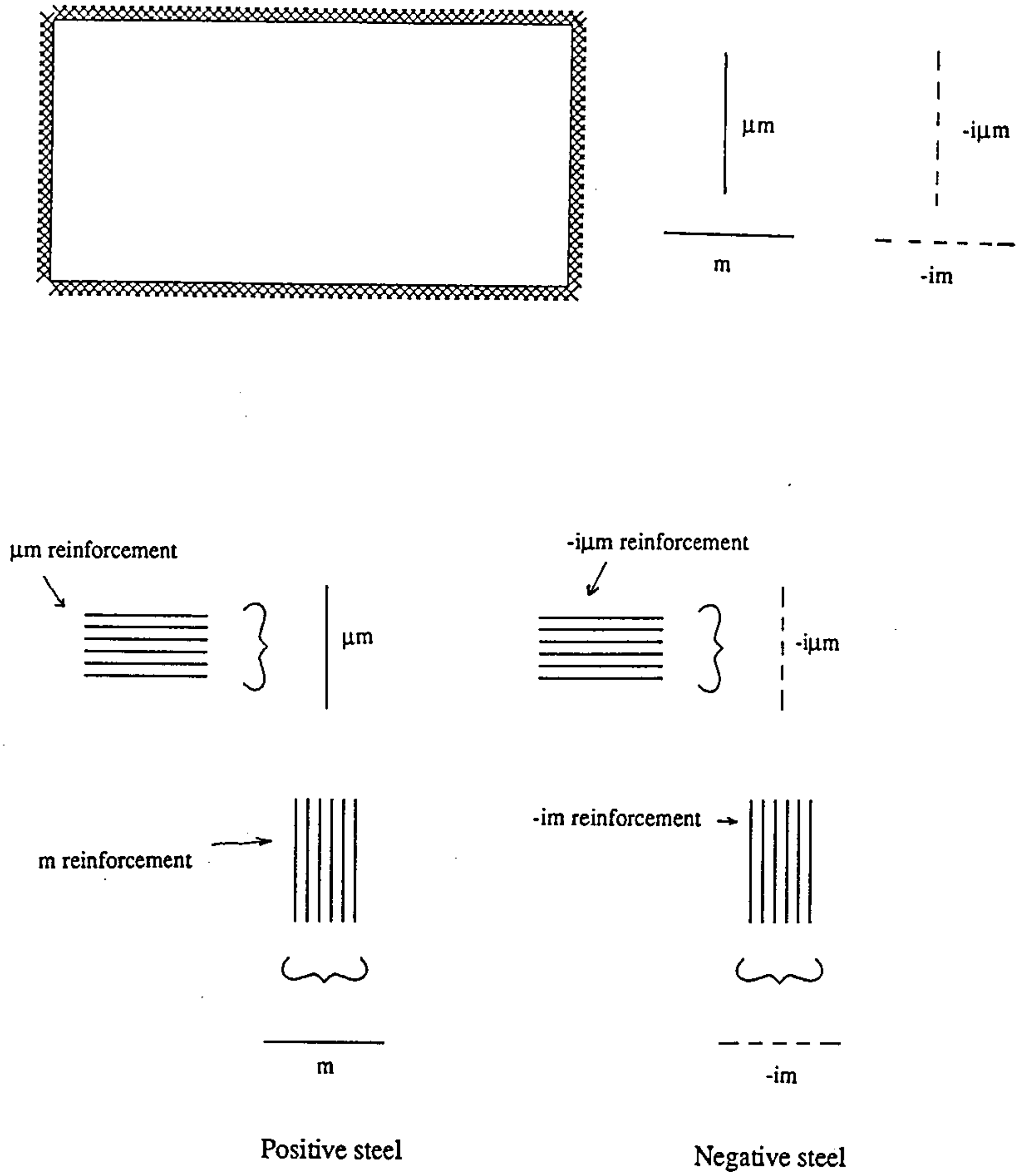


Fig. 3.6 Conventional representation of reinforcement bending moment in the Yield-Line method

The other parameter, i, is the relative proportion of the negative resistance moment to the positive resistance moment and again a suitable i value used in design should be not dissimilar to that obtained by the elastic behaviour, in order to avoid excessive redistribution of moments.

3.4.2 Example of yield-line analysis

For comparison with the previous clamped square problem using a finite element solution consider the square slab in Fig. 3.7 with the yield-line pattern shown. For unit deflection of the centre $\Sigma(w\delta) = wL^2/3$ and $\Sigma\int(M\theta) = m(1 + i)8$ and hence

$$m(1 + i) = \frac{wL^2}{24} \tag{3.19}$$

If the ratio of i was chosen in the same proportion as the maximum elastic negative and positive moments reinforcement coefficients in Fig. 3.5e and f we would have $i = 0.0502/0.0221 = 2.27$. The application of this factor in the equation would give $m = 0.0127\ wL^2$ with $im = 0.029\ wL^2$. These are of the order of 58% of the maximum values indicated by elastic analysis. It would however be quite permissible to have had the steel in the centre half 3 times that in the edge quarter spans which would not change significantly the answer since we are integrating along the yield line and retaining the same average. This would then lead to the moment distribution of $m = 0.0191\ wL^2$ and $im = 0.0434\ wL^2$ in the centre of the span and edges which would have been 86.5% of the elastic values and therefore requiring little redistribution. The values in the edge strips would be one-third of those values. The banded steel distribution compared with the elastic distribution is shown in Figure 3.12. In the central region as stated both the positive and negative steel is of the order of 86% of the peak elastic moment. Where the yield-line distribution cuts into the elastic distribution yielding will take place first with the moments being redistributed to those places where the yield-line distribution is in excess of the elastic distribution.

The banding of the original uniform steel is regarded as a feature not emphasised sufficiently. Both the banded and uniform steel cases will fail at the same

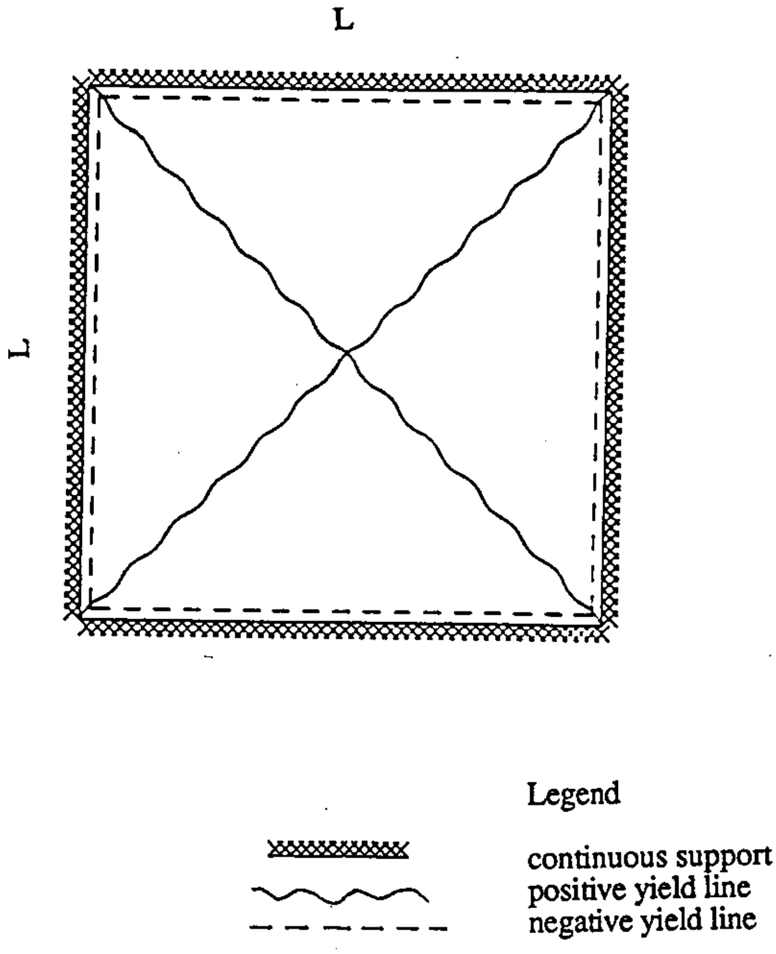


Fig. 3.7 Yield-Line pattern for a square clamped edge slab

value but cracking would occur much earlier at about 58% of the ultimate load without banding. It is therefore essential that designers have a good knowledge of elastic distribution even when using yield-line analysis. For this simple example it can be seen that yield-line analysis carried out originally with uniform steel can then have this banding but of the same quantity as a uniform distribution and this can given answers not too dissimilar to the elastic values.

3.4.3 Corner levers

For more accuracy in applying yield-line theory corner effects (corner levers) should be taken into account. For the purpose of simplicity, it is usually assumed that the positive yield-line, in rectangular rigidly supported slabs, goes right into the corners as shown in Fig. 3.8a. In fact, if the corner is not held down, it tends to lift up, due to strong torsional moments in the corner regions, causing modification in the yield-line pattern as shown in Fig. 3.8b.

If the corner is held down and no top steel is provided cracks will appear on the top surface as shown in Fig. 3.8c. Line ab is then a yield line of zero strength. If some top steel is provided and the corner is held down, the yield-line pattern in Fig. 3.8c will form with ab as a yield line with some negative moment strength. If an adequate area of steel is provided at the top and the corner prevented from lifting, the corner yield line of Fig. 3.8a will develop. Similarly, in continuous slabs the yield-line pattern near the corner will be as shown in Fig. 3.8d.

If the corner yield-line patterns of Fig. 3.8b or c are taken into account, the ultimate load of the slab will be lower than for the pattern in Fig. 3.8a with a single line entering the corner. The reduction is greater when circular fans, Fig. 3.8e, rather than triangular segments form in the corner. As an approximation with slabs supported on 4 sides the effect of corner levers is to require an increase in the moment by about 10% if no top steel is provided at the corner.

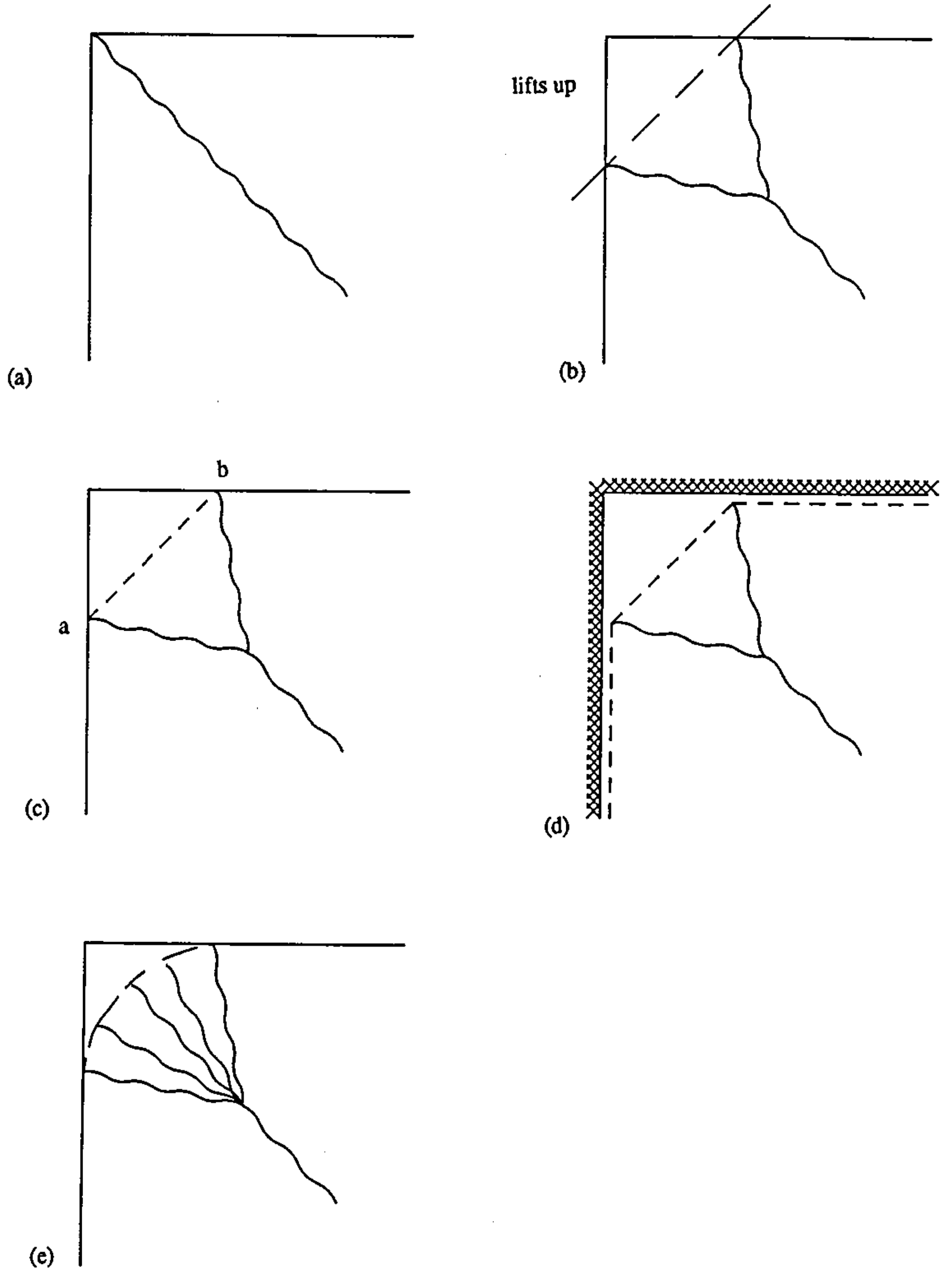


Fig. 3.8 Yield Lines in the corner of a slab

3.4.4 Slabs with beams

Yield-line analysis presents no particular difficulties when dealing with slabs on semi rigid supports and where a slab is supported by beams. The slab reinforcement can be calculated assuming the beams do not fail and the beam strength can be calculated assuming a combined failure of slab and beam.

Thus in Figures 3.9a and b, the slab steel obtained from Fig. 3.9a would be m = $wL^2/24$ and the work equation for Fig. 3.9b would be

$$\frac{wL^2}{2} = 4m + \frac{8M}{L} \qquad (3.20)$$

which for a minimum value of $m = wL^2/24$ gives $M = wL^3/24$. The designer has in fact a choice of beam strength M between $wL^3/24$ and 0. The latter case would be for a flat slab, i.e. no beams in which case equation 3.20 rightly then gives $m = wL^2/8$.

3.4.5 Flat slabs

When yield-line analysis is applied to flat slabs it is necessary to consider extensive patterns involving large sections of the slab and in addition local patterns around the columns. Typical patterns necessary to consider are shown in Figure 3.10. As with slabs on rigid supports the calculated uniform steel can be banded into columns and middle strips, as is shown in detail in Chapter 7.

3.5 Hillerborg's strip method

3.5.1 General

Another approach to the calculation of the ultimate load are the lower bound techniques in which theoretically the calculated ultimate load is either too low or correct. Thus it gives a safe solution. For a lower bound solution a slab with a given loading must have a moment field which satisfies the governing equilibrium equation at all points and must not violate the yield criterion. The requirement of equilibrium of moments for a slab element such as that shown in Fig. 3.1 is expressed:

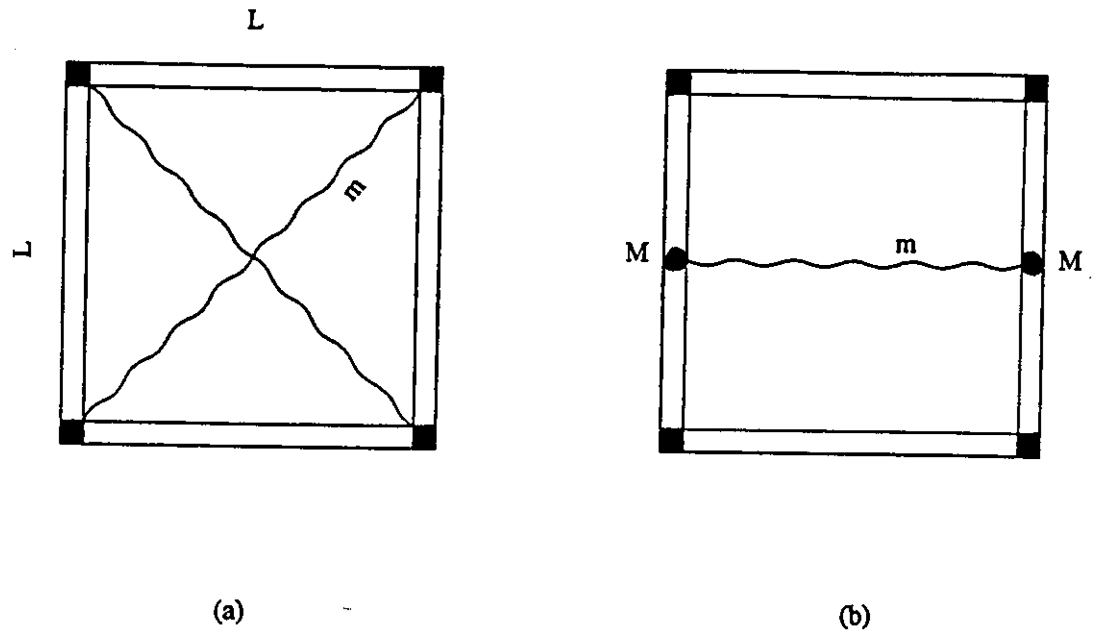


Fig. 3.9 Alternative modes of collapse for a beam-supported slab

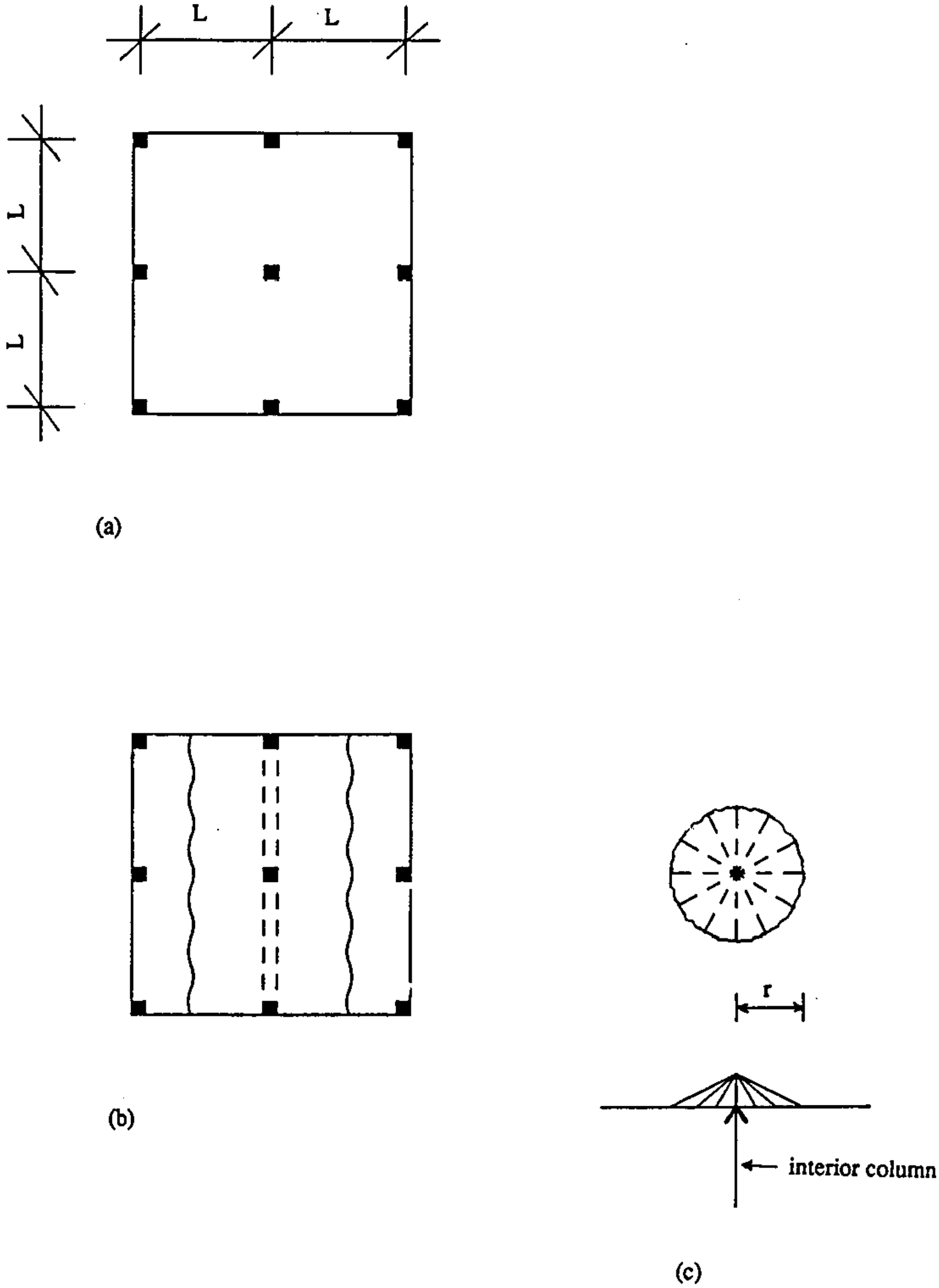


Fig. 3.10 Typical patterns of flat slab failure
(a) Floor plan of flat slab
(b) Folding failure pattern
(c) Local failure pattern in interior column

$$\frac{\partial^2 M_x}{\partial x_2} - 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -w \tag{3.21}$$

To obtain a lower bound solution, the load is apportioned between the terms:

$$\frac{\partial^2 M_x}{\partial x^2}, \quad \frac{\partial^2 M_y}{\partial y^2}, \quad \frac{2\partial^2 M_{xy}}{\partial x \partial y}$$

and the values of these must of course satisfy the boundary conditions. The load can be carried by a suitable combination of slab bending and/or twisting in the two directions. Therefore, the determination of a lower bound solution is often not as simple as the upper bound analysis, especially in the case of odd shaped slabs and awkward boundary conditions.

3.5.2 Hillerborg's 'simple' strip method

The simple strip lower bound method suggested by Hillerborg in 1956 assumed the load to be carried by bending only, i.e. the twisting moment M_{xy} is made zero. This simple method can only be applied to rigid or semi-rigidly supported slabs. The moments are determined by dividing the load into parts which are carried by a system of strips running in the x and y direction, which are designed as beams. Equation 3.21 can be replaced by two equations which represent twistless strip action.

$$\frac{\partial^2 M_x}{\partial x^2} = -\alpha w \tag{3.22}$$

and

$$\frac{\partial^2 M_y}{\partial y^2} = -(1 - \alpha)w \tag{3.23}$$

The load distribution factor α is arbitrary, and is not even confined to the range $0 \leqslant \alpha \leqslant 1$. The theory leads to a simple direct solution giving the distribution of moments over the entire slab from which the reinforcement can easily be calculated since $M_x^+ = M_x$ etc because M_{xy} in the Wood-Armer Rules is zero.

3.5.3 Example of Hillerborg's simple strip method

There is no requirement to keep the distribution of the load in the x and y directions the same and these can be varied as appropriate. If the clamped square slab is considered the simplest division of the load would be as shown in Fig. 3.11a i.e. w/2 in either direction.

The free bending moment along an x strip ab would give a maximum central moment of $wL^2/16$. Just as with yield-line analysis the designer has a free choice of continuity moments and if the ratio of 2.27 as used previously is assumed this would give a uniform maximum negative edge moment of $2.27/3.27 \times wL^2/16 = 0.0434\ wL^2$ and internal positive moment of $0.0191\ wL^2$. These values are not at all dissimilar to the maximum values $0.0502\ wL^2$ and $0.0221\ wL^2$ respectively found by elastic analysis in Fig. 3.5 but it must be remembered the negative values are constant along the whole edge so that there would be no decrease towards the corners as in Fig. 3.5. The positive moments could if required be decreased towards the edges.

If as in the previous yield-line solution we wished to make the moments in the edge strip one-third of those in the centre strip the more complex load distribution in Fig. 3.11b would achieve this to give a free bending moment diagram of $wL^2/13.33$ in the centre i.e. edge and central moments of $0.0521\ wL^2$ and $0.0229\ wL^2$. The moments on the edge strips would be one-third of these values. If the slab is reinforced for these maximum moments we get the distribution shown in Fig. 3.12.

The major results of the three examples shown in Fig. 3.12 are highly instructive. First it demonstrates that even when one chooses the same ratios of positive to negative moments yield-line analysis always requires less steel. Principally this is because the steel has to be banded and therefore from elastic or the strip method one reinforces for the maximum values and therefore includes more steel than if one could reinforce variably. Second it is clear that it is quite possible to choose a pattern of reinforcement when using either yield-line analysis or Hillerborg which is not too

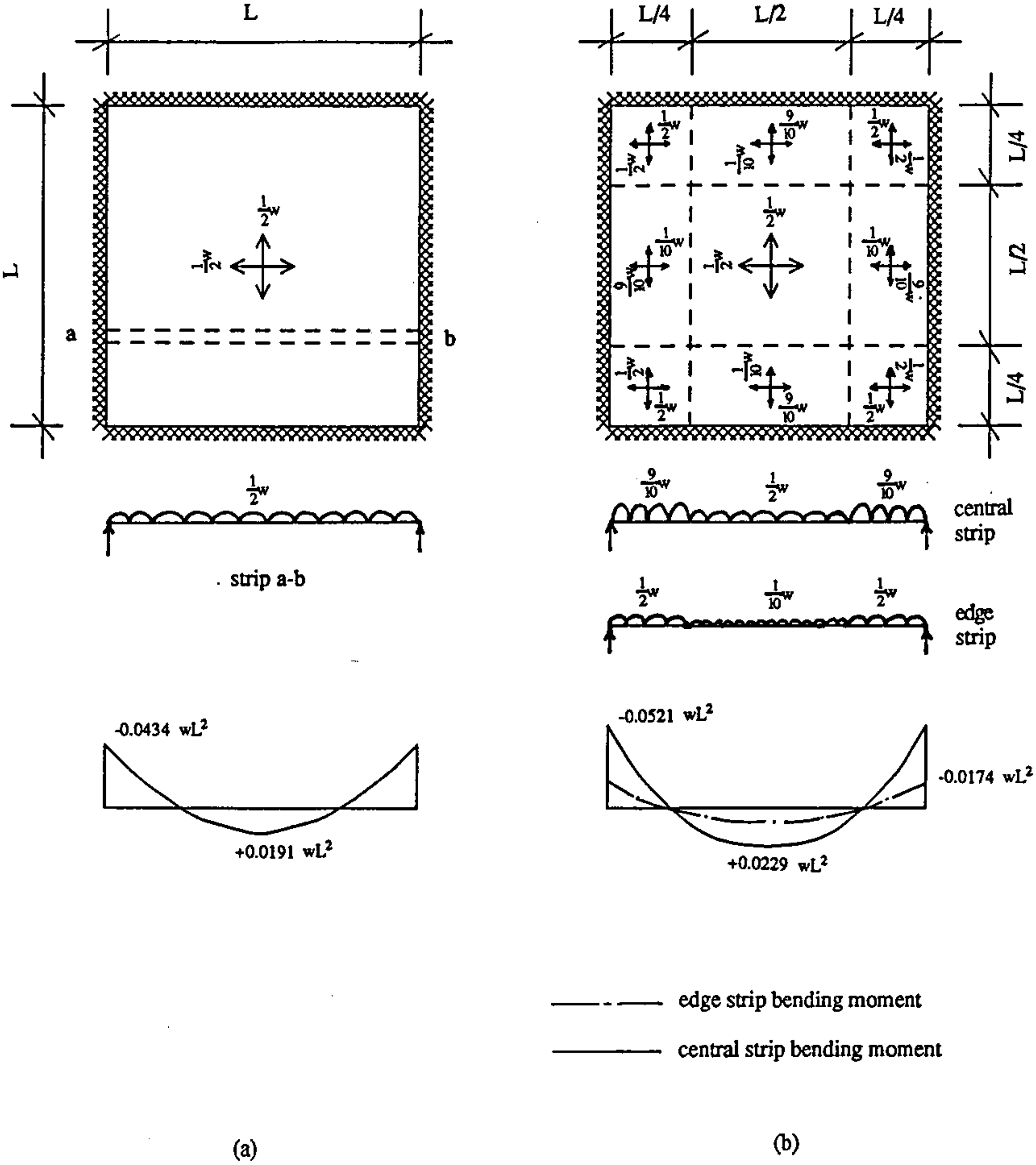
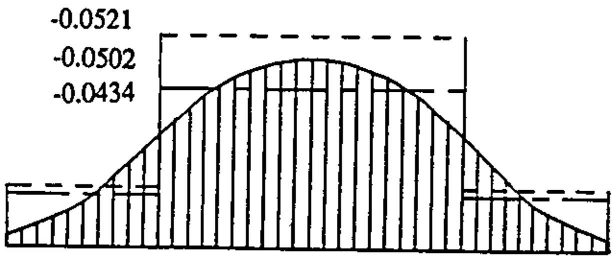
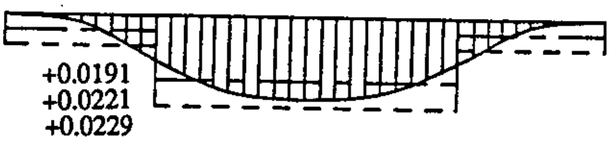


Fig. 3.11 Example of Hillerborg's simple strip method



(a) M_y^- coefficient across the support.



(b) M_y^+ coefficient across the midspan.

– – – – Hillerborg
——— Elastic
—— - —— Yield-Line

Fig. 3.12 Analysis results of bending moment coefficient of square clamped edges slab by elastic analysis, Yield-Line analysis and Hillerborg method, with the central moment 3 times the edge moments dissimilar to the elastic maximum values and therefore does not require too much redistribution of moment.

The examples also show that whether using yield-line analysis or Hillerborg in spite of the latter being a lower bound solution considerable redistribution in the relation to the elastic values may have to occur if an unwise load distribution pattern is chosen. An excellent example of this would be to design the square slab with the distribution all in one direction and for simplicity no continuity. Hillerborg would require $wL^2/8$ in the x direction and zero in the y direction. The yield line in Fig. 3.13 would also give $wL^2/8$ and therefore this would be the exact collapse load, i.e. a coincidental upper and lower bound. The design would however be disastrous with cracking along edges ab and cd at extremely low loads. This merely acts as a demonstration that even an exact mathematic collapse solution may not be a good design and again emphasises the importance of knowledge of elastic distribution.

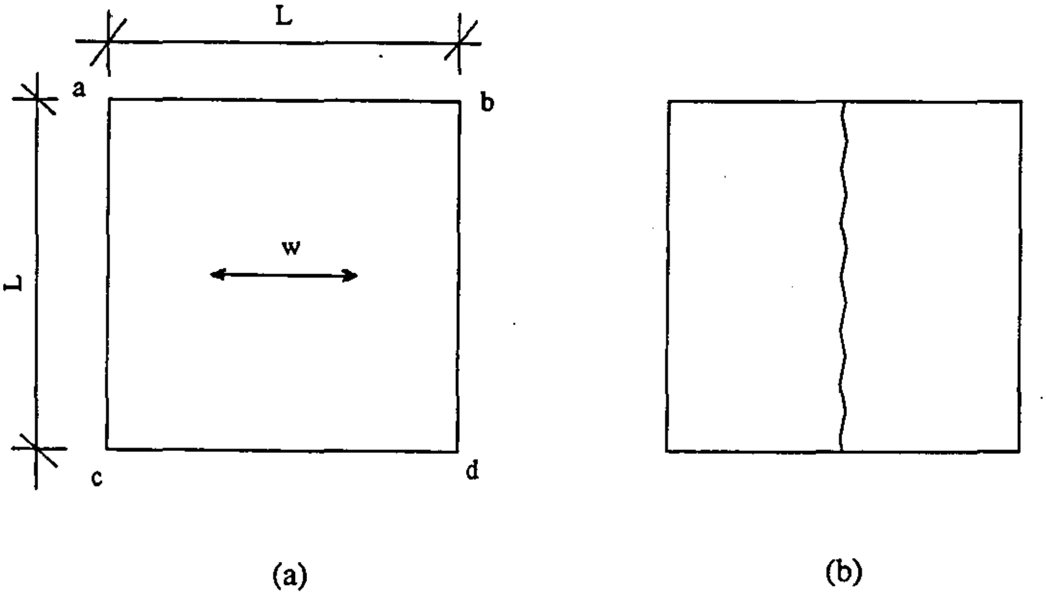


Fig. 3.13 Square slab
(a) Load distribution in one-way
(b) Yield-Line failure

3.5.4 Hillerborg's advanced method

The simple strip method cannot deal with openings, re-entrant corners, and beamless slabs with column supports without the use of strong bands to help distribute the load to the supports.

To extend the scope of his original method to flat slabs, Hillerborg developed his 'advanced' strip method, which employs combinations of complex moment fields and variable k values in the Wood-Armer reinforcement rules. The simplicity and directness of his original simple strip concept has therefore been somewhat clouded as a consequence and Hillerborg [33] himself admits that the complex theoretical derivation of the advanced strip method is probably one of the reasons it is not often used. If one accepts Hillerborg's derivation it is only necessary for design purposes to specify the average edge moments along the edges of his advanced elements and he guarantees these moments will not be exceeded within the element. When designing, therefore, a slab is divided into elements bounded by lines of zero shear force and zero twisting moment, the positions of which may be determined by using elastic continuous beam theory as a rough guide. These zero shear lines occur at the positions of maximum sagging and hogging bending moments, i.e. the element boundaries. Any element supported by a column marked 2 in Fig. 3.14 is treated as an advanced element whilst for the others marked 1 there is simple strip action . The advanced elements type 2, with their special moment fields in effect permits the concentrated column load to be dissipated as a uniformly distributed load and allows one way strip action to be considered in the adjoining elements. It is felt no useful purpose would be served in this thesis by restating Hillerborg's proofs [34], to which reference can easily be made. Instead it is intended to accept his statements that if the size of the advanced elements are determined by assuming quasi-beam supports between columns and choosing a strip moment distribution using the whole load w in both directions, then the moments so determined at the edges of the elements will not be exceeded inside the element. It needs to be emphasised that Hillerborg places restrictions on the values of these edge

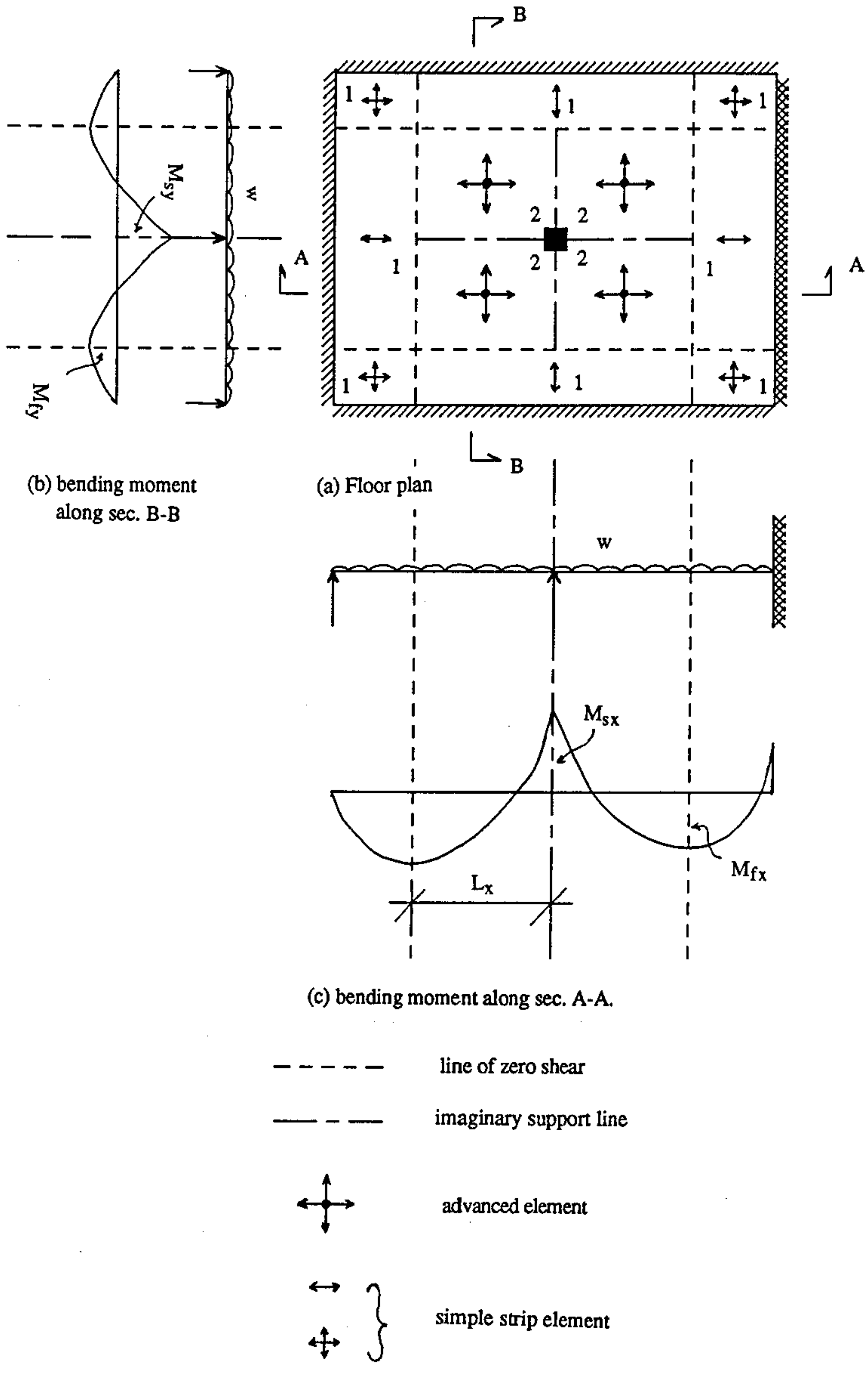
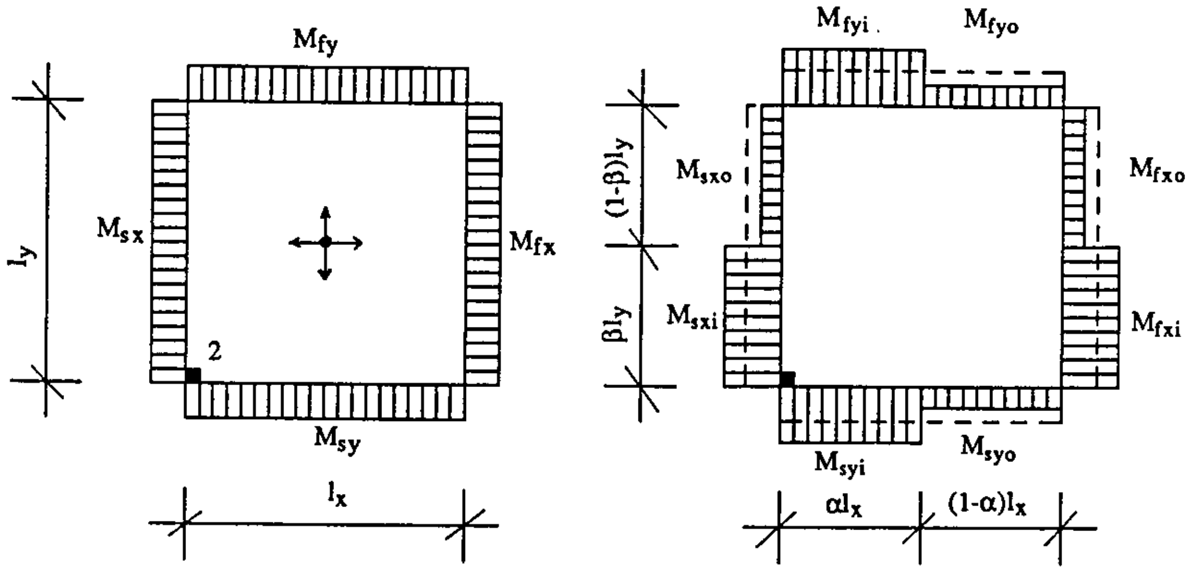


Fig. 3.14 Rectangular slab with column supports, showing different types of slab elements



(a) average edge moment
from beam theory

(b) suggested edge moment
adjustment

M_s : support moment

M_f : field moment

Fig. 3.15 Corner supported element (type 2)

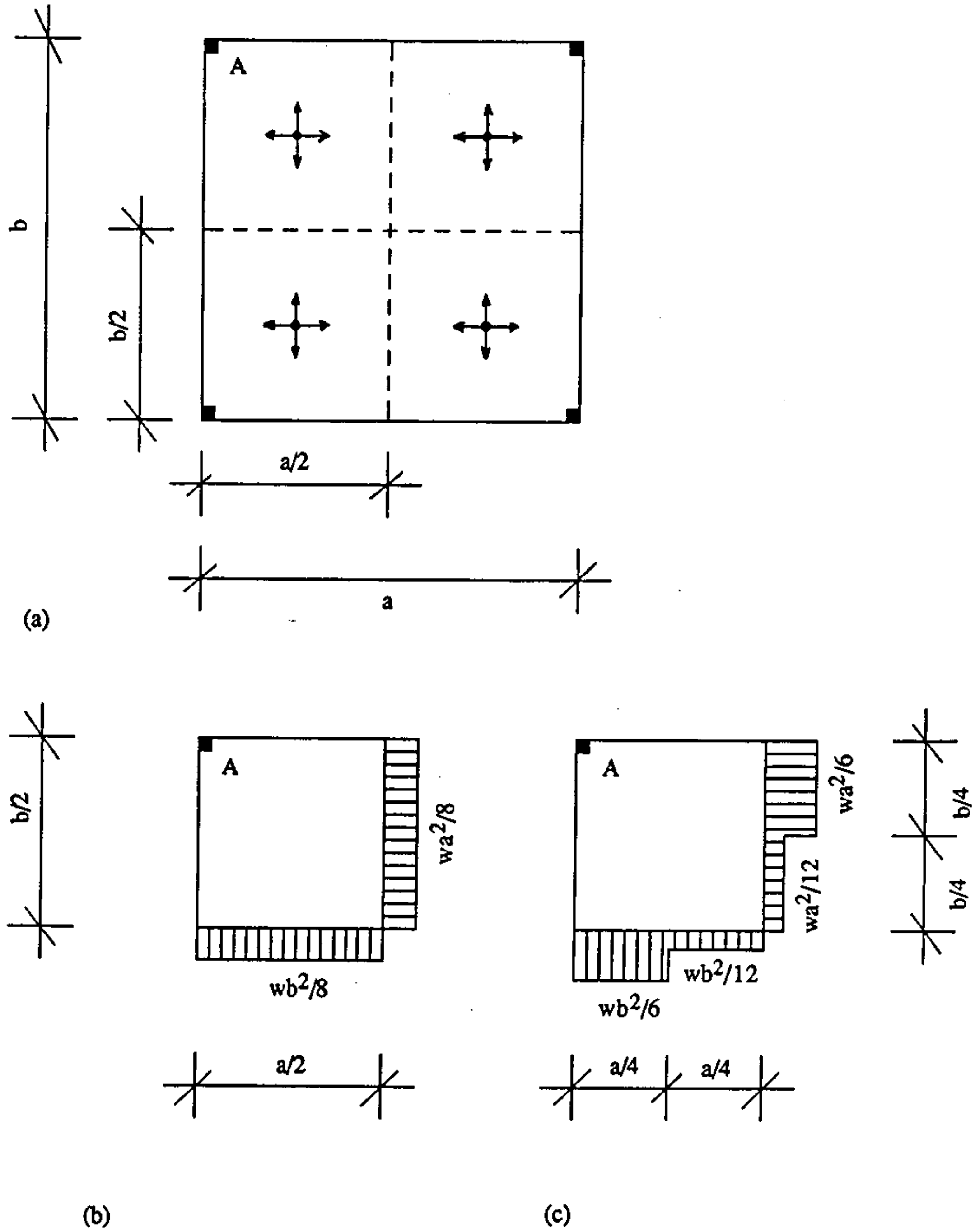


Fig. 3.16 Example of Hillerborg's advanced method

moments which must be observed otherwise the interior moments will exceed the edge moments. These restrictions can lead to difficulties as is now explained.

The average span and support moments along the edges of type 2 (corner supported) element obtained from the beam theory become the average edge moments for the corner supported element designed by Hillerborg's advanced strip method.

Figure 3.15a shows the initial average edge moments for the corner supported element assuming simple strip action.

The distribution of these average moment then has to be adjusted to satisfy certain constraints set by Hillerborg but he proves that if the corner supported elements are reinforced, initially across the whole element, for these adjusted edge moments then the yield condition will not be exceeded within the elements.

Hillerborg introduces two parameters κ_x and κ_y which place restraints on the adjustment of the average edge moments. These coefficients indicate what proportion of the total static moments ($\frac{1}{2}wl_x^2$ and $\frac{1}{2}wl_y^2$) on the element is carried by the difference in moment between the inner and outer parts of the edges. The κ-values can theoretically vary between zero, corresponding to a constant moment along the whole edge, and 1, corresponding to the case where the whole static moment is carried by the part of the edge closest to the point support.

Thus in Figure 3.15b for the general set of edge moments with α and $\beta = \frac{1}{2}$ Hillerborg defines κ_x as

$$\frac{(M_{sxi} + M_{fxi}) - (M_{sxo} + M_{fxo})}{wl_x^2} = \kappa_x \qquad (3.24)$$

with a similar expression for κ_y.

For practical design, Hillerborg calculates the limits for κ as

$0.3 \leqslant \kappa \leqslant 0.75.$

The extent of the reinforcement for his advanced elements are simple and are as follows:

a) For the positive field moment, the reinforcement is carried across the full width and through the whole corner-supported element.

b) The negative reinforcement must be anchored more than 0.6 of the element length from the column.

Whilst the method seems complicated in use, it is relatively easy except for certain special cases. Thus consider the slab supported on 4 columns in Figure 3.16.

The initial chosen bending moment diagrams give zero edge moments and central moments of $wa^2/8$ and $wb^2/8$. The average edge moments for element A therefore as shown in Fig. 3.16b. With constant edge moments of zero and a uniform field moment from equation 3.24 $\kappa = 0$ which is outside the range. If m per unit length represents the increase in the inner moment then to be satisfactory

$$\frac{\frac{1}{2}(m+m)}{\frac{1}{2}\frac{wa^2}{4}} \nless 0.33$$

$$\text{i.e.} \quad m \nless \frac{wa^2}{24}$$

i.e. a distribution of $\frac{wa^2}{8} + \frac{wa^2}{24} = \frac{wa^2}{6}$ on the inner edge

and $\frac{wa^2}{8} - \frac{wa^2}{24} = \frac{wa^2}{12}$ on the outer edge

A similar process could be carried out for the y direction.

If this steel is carried across the whole slab the design is satisfactory.

In general as can be seen the method is easy to use but the major difficulty with Hillerborg's advanced elements arises with elements where it is not possible to stay within his κ limits without adjusting the loading distribution on adjoining simple elements. While this is possible it makes the design process rather more complex.

Recent extensions by Jones and Wood [22] have overcome this problem albeit at the cost of additional reinforcement around the columns.

3.6 General Comments

In this chapter after a historical review of elastic and collapse theories it has been indicated that any subsequent elastic analysis will be carried out using finite element analysis. This technique will use the PAFEC package together with two additional modifications which have had to be included.

The basic theory of yield-line analysis and Hillerborg's strip method have also been outlined and simple examples given. It is now intended to use these various methods to examine the advice given in the British and American Codes of Practice and to draw conclusions from this examination.

APPENDIX 3A

Computer program to modify PAFEC principal stresses to field moments

APPENDIX 3A

Computer program to modify PAFEC principal stresses to field moments

```
CCC     PROGRAME NO. 1
        DIMENSION X(12)
             CHARACTER*32 FNAME
        REAL MXT,MYT,MXYT,MXB,MYB,MXYB
        PARAMETER (PI=3.14159265)
          WRITE(1,'(''  ENTER SOURCE FILE NAME '')')
          READ(1,'(A)')FNAME
        OPEN (7,FILE=FNAME,STATUS='OLD')
           WRITE(1,'(''  ENTER RESULTS FILENAME '')')
           READ(1,'(A)')FNAME
        OPEN (8,FILE=FNAME,STATUS='NEW')
        H=0.24
        READ(7,'(//)')
10      READ(7,*,END=100)I1,I2,I3,(X(I),I=4,12)
        X(1)=I1
        X(2)=I2
        X(3)=I3
        Z=(H**2)/6.0
        X(6)=X(6)*PI/180.0
        X(12)=X(12)*PI/180.0
        MXT=(X(4)*(COS(X(6)))**2+X(5)*(SIN(X(6)))**2)*Z
        MYT=(X(4)*(SIN(X(6)))**2+X(5)*(COS(X(6)))**2)*Z
        MXYT=((X(4)-X(5))*SIN(X(6))*COS(X(6)))*Z
        MXB=(X(10)*(COS(X(12)))**2+X(11)*(SIN(X(12)))**2)*Z
        MYB=(X(10)*(SIN(X(12)))**2+X(11)*(COS(X(12)))**2)*Z
        MXYB=((X(10)-X(11))*SIN(X(12))*COS(X(12)))*Z
        WRITE(8,'(2I8,3X,6F12.4)')I1,I3,MXT,MYT,MXYT,MXB,MYB,MXYB
        GO TO 10
100     CLOSE (7)
        CLOSE (8)
        STOP
        END
```

APPENDIX 3B

Computer program to determine reinforcing moments according to Wood-Armer rules

APPENDIX 3B

Computer program to determine reinforcing moments according to Wood-Armer rules

```
CCC    PROGRAME NO. 2
C
C
C
       CHARACTER*70 INPFIL, OUTFIL
       INTEGER IOS, NODENO
C
C
       REAL VMXNEG, VMYNEG, VMXPOS, VMYPOS
       REAL VMX, VMY, VMXY
C
C
10     CONTINUE
C
       PRINT20
20     FORMAT(/,1X,'Please enter the input filename', /)
       READ(*,'(A)') INPFIL
C
       OPEN(7, FILE=INPFIL, STATUS='OLD', IOSTAT=IOS)
       IF(IOS.NE.0) THEN
         PRINT22, INPFIL
22     FORMAT(//,1X,'****error**** on attempting to open the file :',
     1       /,1x,'"',A,'"',
     2       //,1X,'Possibly because it does not exist or is already ',
     3       'in use',
     4       /,1X,'Please try again',/)
         GOTO 10
       END IF
C
C
25     CONTINUE
C
       PRINT30
30     FORMAT(/,1X,'Please enter the output filename', /)
       READ(*,'(A)') OUTFIL
       OPEN(8, FILE=OUTFIL, STATUS='NEW', IOSTAT=IOS)
       IF(IOS.NE.0) THEN
         PRINT40, OUTFIL
40     FORMAT(//,1X,'****error**** on attemting to open the file :',
     1       /,1X,'"',A, '"',
     2       //,1X,'Possibly because it already exists or is in use ',
     3       /,1X,'Please try again', /)
        GOTO 25
       END IF
C
C
       WRITE(8, 50)
50     FORMAT(1X,' NODE', T16, 'MX', T32, 'MY', T48, 'MXY', T63, 'MX+',
     1       T78, 'MX-', T95, 'MY+', T110, 'MY-',/)
C
C
       NODENO = 0
       PRINT*
       PRINT*, 'Processing ... '
       PRINT*
60     CONTINUE
C
       READ(7, *, END = 80) VMX, VMY, VMXY
```

```
CCC    PROGRAME NO. 2


      NODENO = NODENO + 1
C
C      TREAT CASE FOR THE   MX- AND  MY-
C
      VMXNEG = VMX - ABS(VMXY)
      VMYNEG = VMY - ABS(VMXY)
C
      IF( (VMXNEG.GE.0.0) .AND. (VMYNEG.GE.0.0) ) THEN
         VMXNEG = 0.0
         VMYNEG = 0.0
      ELSE IF( (VMXNEG.GT.0.0) .AND. (VMYNEG.LT.0.0) ) THEN
         VMXNEG = 0.0
         VMYNEG = VMY - ABS(VMXY*VMXY/VMX)
         IF(VMYNEG.GT.0.0) VMYNEG = 0.0
      ELSE IF( (VMXNEG.LT.0.0) .AND. (VMYNEG.GT.0.0) ) THEN
         VMXNEG = VMX - ABS(VMXY*VMXY/VMY)
         VMYNEG = 0.0
         IF(VMXNEG.GT.0.0) VMXNEG = 0.0
      ELSE

C
C      DO NOTHING
C
      END IF

C
C      TREAT THE CASE FOR  MX+  AND MY+
C
      VMXPOS = VMX + ABS(VMXY)
      VMYPOS = VMY + ABS(VMXY)
C
      IF( (VMXPOS.LE.0.0) .AND. (VMYPOS.LE.0.0) ) THEN
         VMXPOS = 0.0
         VMYPOS = 0.0
      ELSE IF( (VMXPOS.LT.0.0) .AND. (VMYPOS.GT.0.0) ) THEN
         VMXPOS = 0.0
         VMYPOS = VMY + ABS(VMXY*VMXY/VMX)
         IF( VMYPOS.LT.0.0) VMYPOS = 0.0
      ELSE IF( (VMXPOS.GT.0.0) .AND. (VMYPOS.LT.0.0) ) THEN
         VMXPOS = VMX + ABS(VMXY*VMXY/VMY)
         VMYPOS = 0.0
         IF(VMXPOS.LT.0.0) VMXPOS = 0.0
      ELSE
C
C      DO NOTHING
C
      END IF
C
C
      WRITE(8, 70) NODENO, VMX, VMY, VMXY, VMXPOS, VMXNEG, VMYPOS,
     1             VMYNEG
 70   FORMAT(1X, I6, T11, F13.4, T27, F13.4, T43, F13.4, T59, F13.4,
     1       T75, F13.4, T91, F13.4, T107, F13.4)
C
C
      GOTO 60
C
C
```

```
CCC   PROGRAME NO. 2


C     END PROGRAM
C
80    CONTINUE
C
      CLOSE(7)
      CLOSE(8)
C
      PRINT*
      PRINT*,  'Job Done. '
      PRINT*
      END
```

CHAPTER 4

FACTORS INFLUENCING COMPARISON OF MOMENT COEFFICIENTS

4.1 Introduction

The simplified methods, of both codes, are based on moment coefficients and these appear to be quite different in each code even for the same slab cases. The values of the coefficients depend on a number of factors which must be taken into account while using each of the methods to find the final moments. These items include the loading factors of the characteristic dead and live load values, partial factors of safety either on materials or the type of structure, load patterns and the width of the slab to which the coefficients apply.

4.2 Characteristic Loads

The two codes differ in their recommended characteristic dead and live loads for different types of occupancy. For use with the British code, these values are given in part 1 of BS 6399:1984 - Code of Practice for Dead and Imposed loads [35], and for the ACI code these can be found in 'Minimum design loads for buildings and other structures', American National Standards Institute Standard A58.1-1982 [36].

Table 4.1 shows some typical values of loading used in the USA and UK for different types of buildings.

The suggested values differ slightly in the two codes and they are generally higher in the UK than in the USA. However it seems likely that except for assembly areas with fixed seats which may be due to seating regulations the difference has come mainly from converting from pounds/sq ft to kN/m^2 than for any other reason.

Occupancy or use	UK kN/m^2	USA psf	Ratio UK/USA
1. Assembly areas and theatres			
Fixed seats	4.0	60 (2.874 kN/m^2)	1.39
Stage Floors	7.5	150 (7.185 kN/m^2)	1.04
2. Dance halls and ballrooms	5.0	100 (4.79 kN/m^2)	1.04
3. Office buildings			
Offices	2.5	50 (2.395 kN/m^2)	1.04

Table 4.1: Some typical live loadings in UK and USA for different types of buildings.

4.3 Partial Safety Factors

Partial safety factors are used in the codes to try to ensure that designs have an acceptably low probability of failure. The concepts of partial safety factors however differ in the two codes, so some rationalisation is required before comparison between them can be made.

In BS 8110, two partial safety factors are used, one for loads and the other for material strengths. For loads, the partial safety factors differ for dead and live loads and may vary according to the type of applied load (e.g. vertical loads, wind loads, ... etc.). The interest here is, of course, vertical loads. The partial safety factor is 1.4 for dead load and 1.6 for imposed load. The latter is higher because there is less likelihood of assessing accurately the imposed load than for the dead load which can be predicted more accurately. In the ACI code, the partial safety factor for dead load is also 1.4, and for live load is 1.7. The reason for the difference in these values is the same as that given for BS 8110.

It is seen that both codes employ the same partial safety factor for dead load (1.4) but different values for the live load (BS 8110 use 1.6, ACI use 1.7). These differences will therefore yield slightly different final moments even for the same loading.

Other partial safety factors are taken into account in each code. BS 8110 introduces partial safety factors for the material strengths (γ_m) with the following explanation ..."The characteristic strengths of materials are based on results of many tests, and the characteristic value selected is that strength under which not more than 5% of the results fall. Concrete strength (f_{cu}) is based on the 28 day compressive strength as determined from cube tests while for reinforcement the characteristic strength (f_y) is based on the yield or 0.2% of proof stress. Partial safety factors (γ_m) are used with these characteristic strengths, to allow for the possible differences between the strength of laboratory samples and the strength of material of the actual structure. The reasons behind this are that workmanship and quality control differ between laboratory or factory and site of work." Generally, in BS 8110, a partial safety factor of 1.5 is used for concrete and 1.15 for reinforcement. It can be observed that the partial safety factor for concrete is higher than that for reinforcement. This is due to the greater variability in concrete in comparison to steel. Laboratory tests on flexural bending indicates that the compressive strength of concrete in bending is lower than the strength predicted by cube test at 28 days. In the light of this BS 8110 specifies that 0.67 of the cube value is used. Therefore the average design stress for concrete in compression is given by

$$\frac{\text{characteristic concrete strength}}{\text{partial safety factor}} \text{ x compressive strength factor}$$

$$\text{namely } \frac{f_{cu}}{1.5} \text{ x } 0.67 = 0.446\, f_{cu} \simeq 0.45\, f_{cu}$$

The design for reinforcement in tension is expressed as

$$\frac{\text{characteristic reinforcement strength in tension}}{\text{partial safety factor}}$$

$$\text{which is } \frac{f_y}{1.15} = 0.87\, f_y$$

The total factor against failure will be a combination of load factors and material factors. In slabs we are primarily concerned with bending. The bending strength is a function of the steel area, yield stress and lever arm. If the concrete stress is factored this will cause a decrease in the lever arm but with lowly reinforced slabs this is not likely to be significant and certainly would be similar to any reductions in the American Code.

It would not therefore be significantly wrong to assume the global safety factor against failure caused by the tensile yielding of steel reinforcement is calculated from the expression

(steel partial safety factor) x (load partial safety factor)

which results in the following values:

$1.15 \times 1.4 = 1.61$ for dead load

and $1.15 \times 1.6 = 1.84$ for live load

In practice the global safety factor employed will be between these, depending on the relative proportions of dead load to live load.

In contrast, the ACI code does not use material strength safety factors, γ_m, but employs another type of safety factor which is called the strength reduction factor ϕ. This factor varies according to the nature of the behaviour of the member in the structure, e.g. a value of 0.9 for bending moments. In order to determine a suitable global safety factor, ACI requires that the partial safety factor for characteristic loads should be divided by the strength reduction factor ϕ. Thus for a strength reduction factor of 0.9 the values for use in determining the global safety factors are

$$\frac{1.4}{0.9} = 1.555 \quad \text{for dead load}$$

and $$\frac{1.7}{0.9} = 1.88 \quad \text{for live load}$$

Thus the global factors for the British and American Codes are 1.61 and 1.555 for dead and 1.84 and 1.88 for live loads, respectively.

The variation of the global safety factor with the ratio of live load to dead load has been calculated for both BS 8110 and the ACI code, based on the above figures, and the results are shown in Table 4.2. It can be seen that over a practical live/dead ratio of 0.5 to 2, the global factor is virtually the same.

Table 4.2 Global safety factor according to BS8110 and ACI codes

L.L./D.L.	UK	USA	UK/USA
0.5	1.686	1.663	1.014
0.6	1.696	1.677	1.011
0.7	1.705	1.689	1.009
0.8	1.712	1.699	1.008
0.9	1.719	1.709	1.006
1.0	1.725	1.718	1.004
1.1	1.731	1.725	1.003
1.2	1.735	1.732	1.002
1.3	1.740	1.739	1.001
1.375	1.743	1.743	1.000
1.4	1.744	1.745	0.999
1.5	1.748	1.750	0.999
1.6	1.752	1.755	0.998
1.7	1.755	1.760	0.997
1.8	1.758	1.764	0.997
1.9	1.761	1.768	0.996
2.0	1.763	1.772	0.995

4.4 Load Patterns

The probability of some panels being loaded while others are not certainly cannot be ignored and does cause a significant difference in the bending moments at critical sections.

Most of the floors in the multipanel structures are assumed to have all the panels loaded uniformly. However the probability of a certain class of patterns of loading occurring which give rise to higher moments at the critical sections should be considered. Thus the patterns of loading considered in this thesis are shown in Figure 4.1. The shaded panels are loaded with the live plus dead loads, while the unshaded panels carry only the dead load. The checkerboard loadings usually produce maximum moments in panels which are rigidly supported and continuous on some or all four sides while the strip loadings generally produce maximum moments in panels on semi-rigid support or flat slab [37]. In addition, the ratio of live load to dead load is very important in determining the effect of pattern loads. Pattern loads are obviously of much greater potential importance in a structure in which the live load is several times the dead load than in a structure in which the live load is only a fraction of the dead load.

Generally, BS 8110 simplifies the loading to a single load case of the maximum design load on all panels. However, for structures designed for storage or where the ratio of the characteristic live load to the characteristic dead load exceeds 1.25 the pattern load must be considered.

In contrast, when using the coefficients in ACI 318-63, a limit for the ratio of L.L. to D.L. is not given since the coefficients have taken into account the effect of loading patterns and they are used separately for dead load and live load.

The other methods recommended by ACI 318-83, namely the EFM or DDM require that when the loading pattern is known, the structure should be analysed for that load. If the pattern is not known then all panels should be loaded with the factored live and dead load provided that the unfactored live load does not exceed 0.75 of the

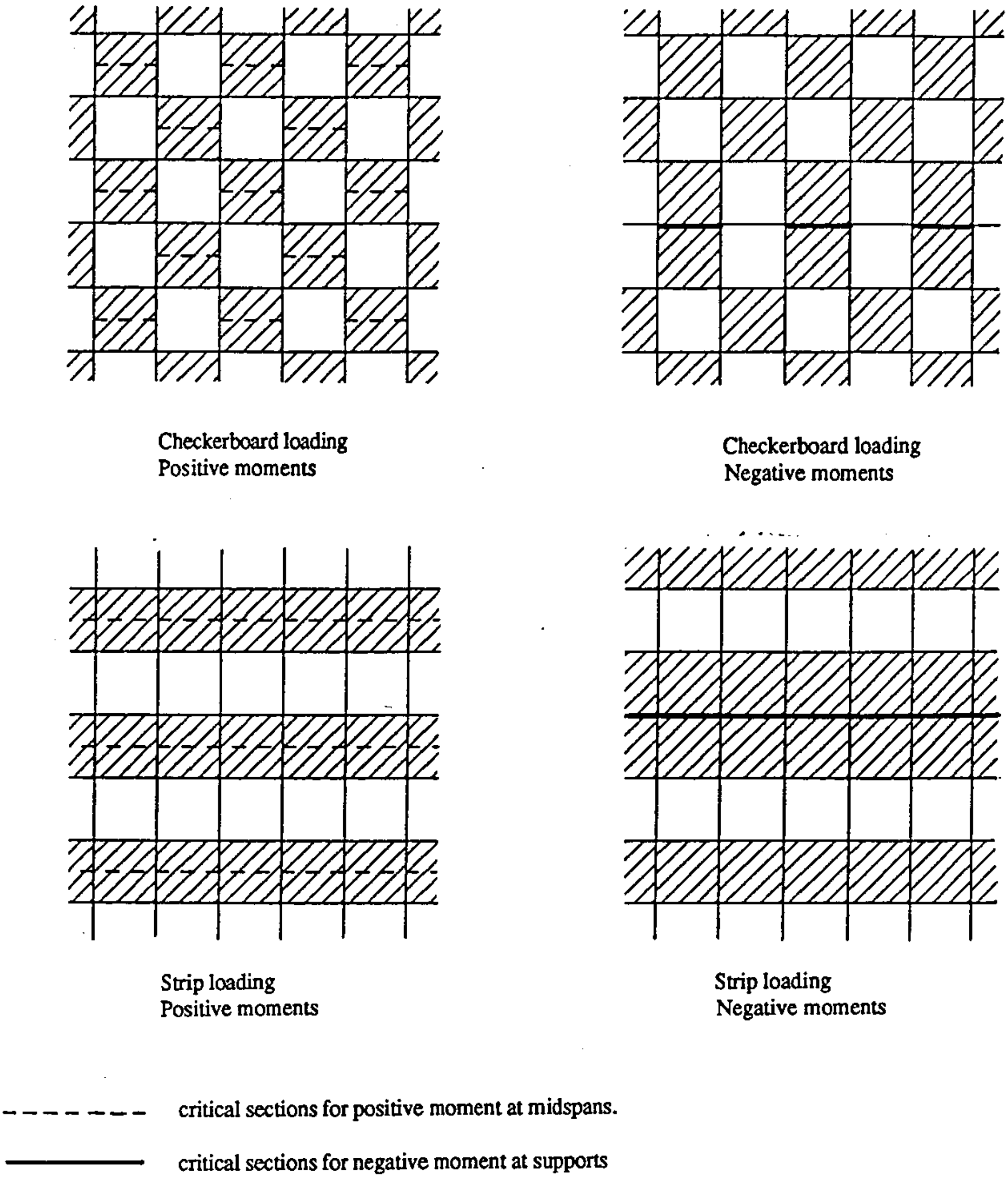


Fig. 4.1 Examples of classes of loading patterns that give rise to moments at critical sections on a multipanel floor.

unfactored dead load. If this limit is exceeded then pattern loading needs to be considered as shown in Figure 4.1.

4.5 Width of slab over which the coefficients are applied

For design purposes codes usually divide slab panels into middle and edge or column strips, and both the codes investigated use such a system. In BS 8110 for rigid and semi rigidly supported slabs the middle strip is three-quarters of the width while for flat slabs the centre strip is half the width. In the ACI code the centre strip is always half the width for all types of slabs.

The moment coefficients for slabs on rigid support in BS 8110 are for the middle strips only with minimum steel being required with edge strips. Whilst in ACI 318-63 the coefficients are for middle strips and 2/3 of the coefficient values are used for column strips. The strip width and extent of the moment coefficients for rigidly supported slabs are shown in Figures 4.2 and 4.3.

4.6 Conclusions

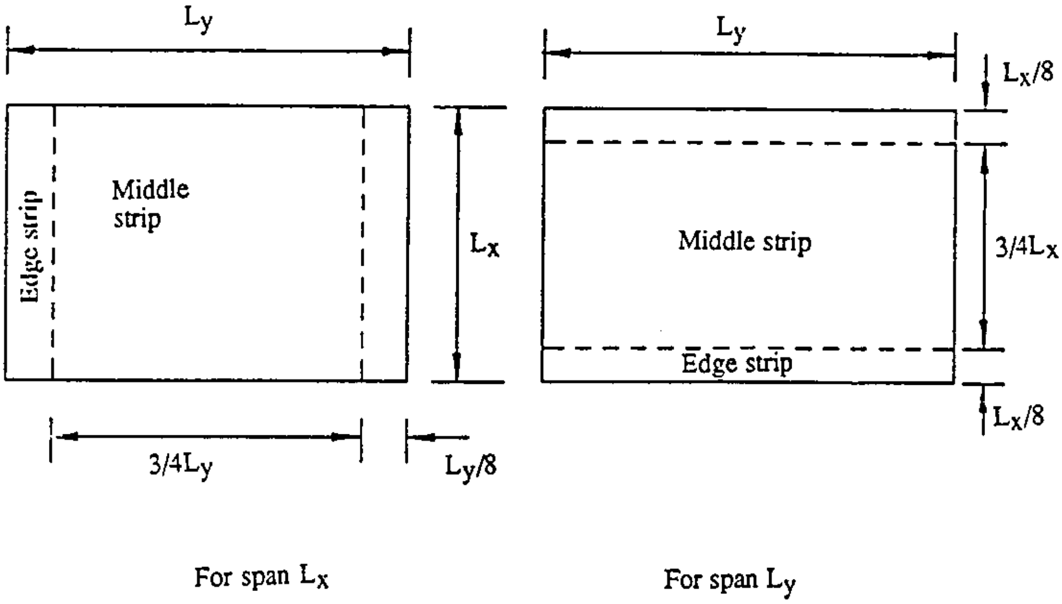
a) Since the ACI and British characteristic loads in section 4.2 are quite similar no account will be taken of this and the same loads will be used in typical calculations or as multipliers on bending moment coefficients.

b) Table 4.2 indicates that the global load factor hardly varies over the whole range of dead/live load so that this may be assumed to be constant over the whole range.

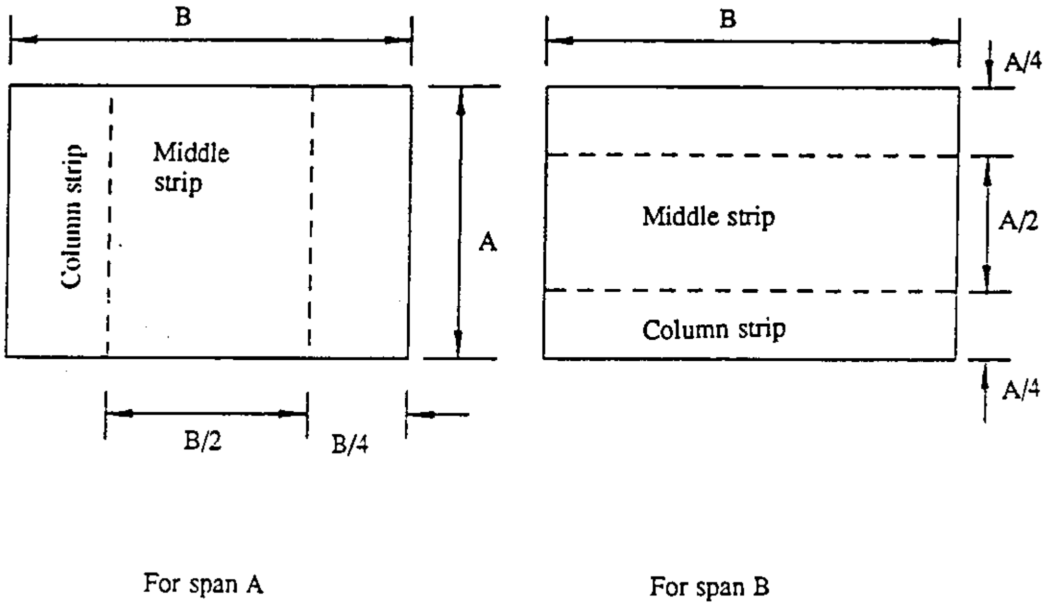
c) The loading patterns may not however be ignored since this leads to significant changes in the maximum moments.

d) Finally, the British code regards its middle strip as 3L/4 while the ACI code uses L/2. For rigidly supported slabs with their simplified moment coefficient since the ACI code requires 2/3 of the central coefficient in the edge strips the equivalent length is 5L/6 or 0.83L with the British code value at 0.75L with minimum steel used in the edge zones. The resulting equivalent length is similar and therefore the coefficients themselves only will be compared, though in typical calculations the recommended values are used.

For semi-rigid and flat slabs the differences will need to be taken into account where necessary.



(a) BS 8110 for rigidly supported slabs.

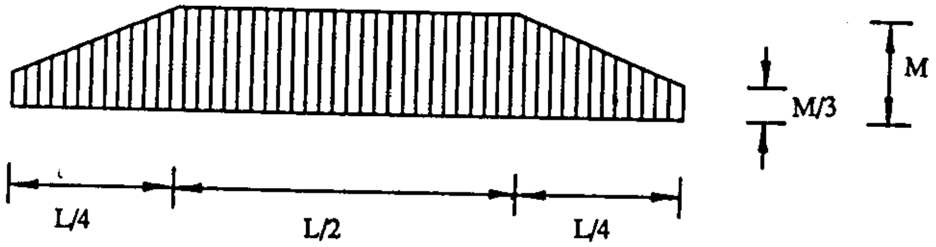


(b) ACI all slabs and BS 8110 for flat slab.

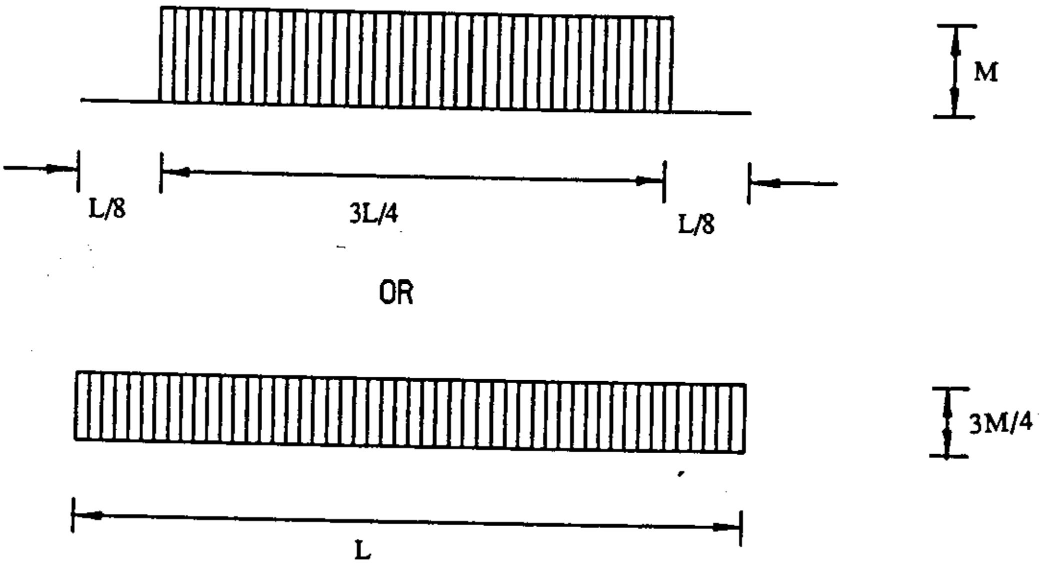
Fig. (4.2) : Division of slab into strips according to

(a) BS 8110 (rigid supports)

(b) ACI all slabs and BS 8110 flat slabs.



(a) ACI



(b) BS8110

Fig. (4.3) : Bending moment diagram across rigidly supported panels due to the coefficients according to

(a) ACI 318-63

(b) BS 8110.

CHAPTER 5
SLABS ON RIGID SUPPORTS

5.1 Introduction

The purpose of this chapter is two-fold. The first is to study the provisions of BS 8110 and ACI as applied to slabs on rigid supports with a view to identifying their similarities, or differences, origins and any anomalies.

The second purpose is to investigate the codes in more detail in order to assess their derivation and by examining the various factors during both the elastic and plastic phases to comment on whether they are considered satisfactory.

The first section involves a presentation of the codes of practice including the basic terminology employed by the national codes of practice for concrete works in the UK and USA. This is then followed by a description of the provisions and design procedure embodied in the separate codes and an example of the design of a simple but realistic slab system using both codes.

The second section, in which the moment coefficients given in BS 8110 and ACI are examined in detail, is structured as follows:

a) types of rigidly supported panels considered in BS 8110 and the ACI code;

b) an examination of the derivation of the moment coefficients used in both codes;

c) the evaluation of moment coefficients for different loading pattern and aspect ratios during the elastic phase using finite element analysis;

d) comments and comparisons of the results obtained from (b) and (c); and

e) conclusions.

5.2 Terminology used in the Codes of Practice

The terminology used in the codes of practice of relevant interest involves loads, strengths of materials and divisions of slab panels. These are summarized in Table 5.1.

BS 8110 (U.K.)		ACI (U.S.A.)	
Loads		Loads	
Characteristic dead load	g_k	Dead load	D.L.
Characteristic imposed load	q_k	Live load	L.L.
Ultimate design load where $n = 1.4\ g_k + 1.6\ q_k$	n,	Ultimate factored design load where $w_u = 1.4$ D.L. + 1.7 L.L.	w_u,
Characteristic strength of concrete, based on cube test	f_{cu}	Specified compressive strength of concrete, based on cylinder test	f_c
Span	L	Span	L,
		shall be considered as the centre to centre distance between supports or the clear span plus twice the thickness of slab, whichever is the smaller	
Middle strip L is span of direction being considered	3L/4,	Middle strip L is span of direction being considered	L/2
Edge strip	L/8	Column* strip (ACI 318-63)	L/4

* Although there is no physical column in the structure, the ACI uses the term 'column strip'.

Table 5.1: Terminology in the British and American Codes of Practice

5.3 BS 8110 The Structural Use of Concrete

5.3.1 Moment coefficients

BS 8110 gives moment coefficients, in Table 5.2, for rectangular slabs with any combination of continuous or simply-supported edges, provided that all four corners are held down and suitable provisions are made for torsion.

In BS 8110 slabs are considered to be divided in each span direction into middle strips and edge strips as shown in Figure 4.2, the middle strip being three-quarters of the width and each edge strip one-eighth of the width.

BS 8110 requires, firstly, that the characteristic dead and imposed loads on adjacent panels be approximately the same. Secondly, the span of adjacent panels in the direction perpendicular to the line of the common support should be approximately the same as the span of the panel considered in that direction.

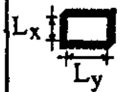
In addition to the above requirements the code rules that the maximum design moments calculated in the light of the code's moment coefficients, and equations apply only to the middle strips and no further redistribution should be made.

Before proceeding further it should be pointed out that there are a number of minor anomalies in Table 5.2. When a slab has an L_y/L_x ratio of 1 then the short and long span coefficients should be the same in cases of symmetry or interchangeable where x and y are interchanged. Thus in case 1 the first and last values of the negative moment should not be 0.031 and 0.032 but the same. Similarly the long span coefficients in case 2, namely 0.037 and 0.028, should be the same as the first values for case 3 which are 0.039 and 0.030. Similar slight differences occur in case 4, between cases 5 and 6, and between 7 and 8, and finally case 9. Where coefficients have been used for square slabs later in the thesis usually the higher value has been taken if the values are slightly different.

5.3.2 Sequence of slab design

The analysis and design steps for rigidly supported restrained slabs where the corners are prevented from lifting, and provision for torsion is made, are as follows.

Table 5.2 Bending moment coefficients for rectangular panels supported on four sides with provision for torsion at corners (adapted from BS8110 Table 3.15)

Cases	Moments Considered	Short span coefficients, β_{sx} Values of L_y/L_x								Long span coefficients, β_{sy}, for all values of L_y/L_x
		1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
Case 1	Neg. Mom. at Cont. Edge	0.031	0.037	0.042	0.046	0.050	0.053	0.059	0.063	0.032
	Pos. Mom. at Midspan	0.024	0.028	0.032	0.035	0.037	0.040	0.044	0.048	0.024
Case 2	Neg. Mom. at Cont. Edge	0.039	0.044	0.048	0.052	0.055	0.058	0.063	0.067	0.037
	Pos. Mom. at Midspan	0.029	0.033	0.036	0.039	0.041	0.043	0.047	0.050	0.028
Case 3	Neg. Mom. at Cont. Edge	0.039	0.049	0.056	0.062	0.068	0.073	0.082	0.089	0.037
	Pos. Mom. at Midspan	0.030	0.036	0.042	0.047	0.051	0.055	0.062	0.067	0.028
Case 4	Neg. Mom. at Cont. Edge	0.047	0.056	0.063	0.069	0.074	0.078	0.087	0.093	0.045
	Pos. Mom. at Midspan	0.036	0.042	0.047	0.051	0.055	0.059	0.065	0.070	0.034
Case 5	Neg. Mom. at Cont. Edge	0.046	0.050	0.054	0.057	0.060	0.062	0.067	0.070	-
	Pos. Mom. at Midspan	0.034	0.038	0.040	0.043	0.045	0.047	0.050	0.053	0.034
Case 6	Neg. Mom. at Cont. Edge	-	-	-	-	-	-	-	-	0.045
	Pos. Mom. at Midspan	0.034	0.046	0.056	0.065	0.072	0.078	0.091	0.100	0.034
Case 7	Neg. Mom. at Cont. Edge	0.057	0.065	0.071	0.076	0.081	0.084	0.092	0.098	-
	Pos. Mom. at Midspan	0.043	0.048	0.053	0.057	0.060	0.063	0.069	0.074	0.044
Case 8	Neg. Mom. at Cont. Edge	-	-	-	-	-	-	-	-	0.058
	Pos. Mom. at Midspan	0.042	0.054	0.063	0.071	0.078	0.084	0.096	0.105	0.044
Case 9	Neg. Mom. at Cont. Edge	-	-	-	-	-	-	-	-	-
	Pos. Mom. at Midspan	0.055	0.065	0.074	0.081	0.087	0.092	0.103	0.111	0.056

Note: A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unhatched edge indicates the discontinuous edges.

(a) Estimate the effective depth d of the slab from span/effective depth ratio given in Table 3.10 of BS 8110.

(b) Size up the total slab thickness h by adding to d the radius of reinforcement bars to be used and the appropriate amount of cover needed.

(c) Check that the section complies with requirements for fire resistance (BS 8110, Table 3.5 and Figure 3.2).

(d) Check that the reinforcement cover and concrete grade comply with requirements for durability (BS 8110, Table 3.4).

(e) Having chosen the appropriate live load q_k, calculate the ultimate load n using the equation

$$n = 1.4\, g_k + 1.6\, q_k \tag{5.1}$$

where g_k is characteristic dead load, and

q_k is characteristic imposed load.

(f) Calculate the bending moments as follows.

(i) Determine the aspect ratio for slab panel (L_y/L_x) and select the slab case from Table 5.2 (BS 8110 Table 3.15) which has the appropriate boundary conditions.

(ii) Select the moment coefficients (β_{sx}, β_{sy}) for the positive and negative moments which correspond to the case and aspect ratio being considered and calculate the moment/unit width using the equations

$$M_{sx} = \beta_{sx}\, nL_x^2 \tag{5.2}$$

$$M_{sy} = \beta_{sy}\, nL_x^2 \tag{5.3}$$

where M_{sx}, M_{sy} are the maximum moments at midspan on strips of unit width spanning L_x and L_y respectively.

(iii) In a multispan situation the support moments calculated for adjacent panels may differ significantly. In order to maintain equilibrium at a support where this occurs the moments should be regarded as fixed end moments and distributed according to the relative stiffnesses of adjacent spans, to give new support and midspan moments.

(g) Reinforcement calculation

The area of steel required is assessed as follows:

(i) Middle strip

Determine M/bd^2 and hence find the value of area of steel required using design aids (graphs), tables or equations. If there is less than the minimum defined by 0.0013 bh in the case of high yield steel, or 0.0024 bh in the case of mild steel then this minimum area must be used.

In spite of the lack of tabulated negative moment coefficients in Table 3.15 BS 8110 for discontinuous edges, the code recommends the use of half the midspan moment in the same direction at discontinuous edges, if any.

(ii) Edge strip

The reinforcement in an edge strip, parallel to the edge, need not exceed the minimum stated in the previous section.

(h) Torsion reinforcement

Torsion reinforcement must be provided at any corner contained by edges over which the slab is not continuous. Both top and bottom reinforcement must be provided, each level containing bars placed parallel to the sides of the slab and extending in these directions for a distance of one-fifth of the shorter span, as shown in Figure 5.1(a). The total area of the bars in each of the two layers, per unit width of slab, should be 3/4 of the area required for the maximum midspan moment in the slab. Torsion steel equal to half the above amount should be provided at corners in which only one edge is discontinuous. No torsion steel need be provided at corners contained by edges over both of which the slab is continuous.

5.4. ACI Code

5.4.1 Moment coefficients

The moment coefficients used in ACI 318-63 had been used in Europe for a long time prior to their introduction to the American Code. The method is based on a procedure for the analysis of continuous slabs developed by Marcus [38] and introduced to the USA by Rogers [39] who also developed the method as given in its present form.

The moment coefficient Tables are reproduced in Tables 5.3, 5.4 and 5.5. It should be noted that the cases for which the coefficients are tabulated are the same as those in BS 8110 and include all combinations of fixed or simply supported edges. The edges which are fixed are marked with hatching (see footnote to Tables).

In the ACI code the slabs are considered as divided in each direction into middle strips and edge strips as shown in Figure 4.2(b), namely a middle strip is one-half of a panel in width, symmetrical about the panel centre line and extending through the panel in the direction in which moments are considered.

A column strip is one-half of a panel in width, occupying the two quarter-panel areas outside the middle strip. Where the ratio of short to long span (m) is less than 0.5, the slab shall be considered as a one-way slab.

Critical sections for moment calculations are located at:

(a) for negative moments along the edges of the panel at the faces of the supports, and

(b) for positive moments, along the centre lines of the panels.

The bending moments for the middle strips shall be computed by the use of Tables 5.3, 5.4 and 5.5 from

$$M_A = C_A \, wA^2 \tag{5.4}$$

and

$$M_B = C_B \, wB^2 \tag{5.5}$$

Table 5.3 Coefficients for negative moments in slabs according to ACI (ACI 318-63 Method 3 - Table 1)

$M_{A\,neg} = C_{A\,neg} \times w \times A^2$) where w = total uniform dead plus live load
$M_{B\,neg} = C_{B\,neg} \times w \times B^2$)

Ratio $m = \frac{A}{B}$		Case 1 B A	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{A\,neg}$	0.045	0.061	0.033	0.050	0.075	-	0.071	-	-
	$C_{B\,neg}$	0.045	0.033	0.061	0.050	-	0.076	-	0.071	-
0.95	$C_{A\,neg}$	0.050	0.065	0.038	0.055	0.079	-	0.075	-	-
	$C_{B\,neg}$	0.041	0.029	0.056	0.045	-	0.072	-	0.067	-
0.90	$C_{A\,neg}$	0.055	0.068	0.043	0.060	0.080	-	0.079	-	-
	$C_{B\,neg}$	0.037	0.025	0.052	0.040	-	0.070	-	0.062	-
0.85	$C_{A\,neg}$	0.060	0.072	0.049	0.066	0.082	-	0.083	-	-
	$C_{B\,neg}$	0.031	0.021	0.046	0.034	-	0.065	-	0.057	-
0.80	$C_{A\,neg}$	0.065	0.075	0.055	0.071	0.083	-	0.086	-	-
	$C_{B\,neg}$	0.027	0.017	0.041	0.029	-	0.061	-	0.051	-
0.75	$C_{A\,neg}$	0.069	0.078	0.061	0.076	0.085	-	0.088	-	-
	$C_{B\,neg}$	0.022	0.014	0.036	0.024	-	0.056	-	0.044	-
0.70	$C_{A\,neg}$	0.074	0.081	0.068	0.081	0.086	-	0.091	-	-
	$C_{B\,neg}$	0.017	0.011	0.029	0.019	-	0.050	-	0.038	-
0.65	$C_{A\,neg}$	0.077	0.083	0.074	0.085	0.087	-	0.093	-	-
	$C_{B\,neg}$	0.014	0.008	0.024	0.015	-	0.043	-	0.031	-
0.60	$C_{A\,neg}$	0.081	0.085	0.080	0.089	0.088	-	0.095	-	-
	$C_{B\,neg}$	0.010	0.006	0.018	0.011	-	0.035	-	0.024	-
0.55	$C_{A\,neg}$	0.084	0.086	0.085	0.092	0.089	-	0.096	-	-
	$C_{B\,neg}$	0.007	0.005	0.014	0.008	-	0.028	-	0.019	-
0.50	$C_{A\,neg}$	0.086	0.088	0.089	0.094	0.090	-	0.097	-	-
	$C_{B\,neg}$	0.006	0.003	0.010	0.006	-	0.022	-	0.014	-

Note: A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unhatched edge indicates a support at which torsional resistance is negligible.

Table 5.4 Coefficients for dead load positive moments in slabs according to ACI (ACI 318-63 Method 3 - Table 2)

$M_{A\,pos\,DL} = C_{A\,DL} \times w \times A^2$) where w = total uniform dead load
$M_{B\,pos\,DL} = C_{B\,DL} \times w \times B^2$)

Ratio $m = \frac{A}{B}$		Case 1 B A 	Case 2 	Case 3 	Case 4 	Case 5 	Case 6 	Case 7 	Case 8 	Case 9 
1.00	$C_{A\,DL}$	0.018	0.023	0.020	0.027	0.027	0.018	0.033	0.027	0.036
	$C_{B\,DL}$	0.018	0.020	0.023	0.027	0.018	0.027	0.027	0.033	0.036
0.95	$C_{A\,DL}$	0.020	0.024	0.022	0.030	0.028	0.021	0.036	0.031	0.040
	$C_{B\,DL}$	0.016	0.017	0.021	0.024	0.015	0.025	0.024	0.031	0.033
0.90	$C_{A\,DL}$	0.022	0.026	0.025	0.033	0.029	0.025	0.039	0.035	0.045
	$C_{B\,DL}$	0.014	0.015	0.019	0.022	0.013	0.024	0.021	0.028	0.029
0.85	$C_{A\,DL}$	0.024	0.028	0.029	0.036	0.031	0.029	0.042	0.040	0.050
	$C_{B\,DL}$	0.012	0.013	0.017	0.019	0.011	0.022	0.017	0.025	0.026
0.80	$C_{A\,DL}$	0.026	0.029	0.032	0.039	0.032	0.034	0.045	0.045	0.056
	$C_{B\,DL}$	0.011	0.010	0.015	0.016	0.009	0.020	0.015	0.022	0.023
0.75	$C_{A\,DL}$	0.028	0.031	0.036	0.043	0.033	0.040	0.048	0.051	0.061
	$C_{B\,DL}$	0.009	0.007	0.013	0.013	0.007	0.018	0.012	0.020	0.019
0.70	$C_{A\,DL}$	0.030	0.033	0.040	0.046	0.035	0.046	0.051	0.058	0.068
	$C_{B\,DL}$	0.007	0.006	0.011	0.011	0.005	0.016	0.009	0.017	0.016
0.65	$C_{A\,DL}$	0.032	0.034	0.044	0.050	0.036	0.054	0.054	0.065	0.074
	$C_{B\,DL}$	0.006	0.005	0.009	0.009	0.004	0.014	0.007	0.014	0.013
0.60	$C_{A\,DL}$	0.034	0.036	0.048	0.053	0.037	0.062	0.056	0.073	0.081
	$C_{B\,DL}$	0.004	0.004	0.007	0.007	0.003	0.011	0.006	0.012	0.010
0.55	$C_{A\,DL}$	0.035	0.037	0.052	0.056	0.038	0.071	0.058	0.081	0.088
	$C_{B\,DL}$	0.003	0.003	0.005	0.005	0.002	0.009	0.004	0.009	0.008
0.50	$C_{A\,DL}$	0.037	0.038	0.056	0.059	0.039	0.080	0.061	0.089	0.095
	$C_{B\,DL}$	0.002	0.002	0.004	0.004	0.001	0.007	0.003	0.007	0.006

Note: A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unhatched edge indicates a support at which torsional resistance is negligible.

Table 5.5 Coefficients for live load positive moments in slabs according to ACI (ACI 318-63 Method 3 - Table 3)

$M_{A\ pos\ LL} = C_{A\ LL} \times w \times A^2)$ where w = total uniform live load
$M_{B\ pos\ LL} = C_{B\ LL} \times w \times B^2)$

Ratio $m = \frac{A}{B}$		Case 1 (B, A)	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{A LL}$	0.027	0.030	0.028	0.032	0.032	0.027	0.035	0.032	0.036
	$C_{B LL}$	0.027	0.028	0.030	0.032	0.027	0.032	0.032	0.035	0.036
0.95	$C_{A LL}$	0.030	0.032	0.031	0.035	0.034	0.031	0.038	0.036	0.040
	$C_{B LL}$	0.025	0.025	0.027	0.029	0.024	0.029	0.029	0.032	0.033
0.90	$C_{A LL}$	0.034	0.036	0.035	0.039	0.037	0.035	0.042	0.040	0.045
	$C_{B LL}$	0.022	0.022	0.024	0.026	0.021	0.027	0.025	0.029	0.029
0.85	$C_{A LL}$	0.037	0.039	0.040	0.043	0.041	0.040	0.046	0.045	0.050
	$C_{B LL}$	0.019	0.020	0.022	0.023	0.019	0.024	0.022	0.026	0.026
0.80	$C_{A LL}$	0.041	0.042	0.044	0.048	0.044	0.045	0.051	0.051	0.056
	$C_{B LL}$	0.017	0.017	0.019	0.020	0.016	0.022	0.019	0.023	0.023
0.75	$C_{A LL}$	0.045	0.046	0.049	0.052	0.047	0.051	0.055	0.056	0.061
	$C_{B LL}$	0.014	0.013	0.016	0.016	0.013	0.019	0.016	0.020	0.019
0.70	$C_{A LL}$	0.049	0.050	0.054	0.057	0.051	0.057	0.060	0.063	0.068
	$C_{B LL}$	0.012	0.011	0.014	0.014	0.011	0.016	0.013	0.017	0.016
0.65	$C_{A LL}$	0.053	0.054	0.059	0.062	0.055	0.064	0.064	0.070	0.074
	$C_{B LL}$	0.010	0.009	0.011	0.011	0.009	0.014	0.010	0.014	0.013
0.60	$C_{A\ neg}$	0.058	0.059	0.065	0.067	0.059	0.071	0.068	0.077	0.081
	$C_{B\ neg}$	0.007	0.007	0.009	0.009	0.007	0.011	0.008	0.011	0.010
0.55	$C_{A\ neg}$	0.062	0.063	0.070	0.072	0.063	0.080	0.073	0.085	0.088
	$C_{B\ neg}$	0.006	0.006	0.007	0.007	0.005	0.009	0.006	0.009	0.008
0.50	$C_{A\ neg}$	0.066	0.067	0.076	0.077	0.067	0.088	0.078	0.092	0.095
	$C_{B\ neg}$	0.004	0.004	0.005	0.005	0.004	0.007	0.005	0.007	0.006

Note: A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unhatched edge indicates a support at which torsional resistance is negligible.

where C_A and C_B are the moment coefficients as given in Tables 5.3, 5.4 and 5.5. A and B are the lengths of the short and long spans respectively. For negative moments Table 5.3 is used and w is the total factored dead load plus live load. For positive moments, the factored dead load is used with Table 5.4 and the factored live load with Table 5.5. The total positive moment is the sum of the two.

The reason for the different coefficients for dead and live load in the positive moments is to allow for pattern loading, though it would seem to have been more logical to have a Table for positive and negative moments due to dead load which cannot be pattern loading and a Table which increased both the positive and negative moments due to live load to allow for pattern loading.

The bending moments in the column strips should be gradually reduced from the full value M_A and M_B from the edge of the middle strip to one-third of these values at the edge of the panel.

5.4.2 Sequence of slab design

The sequence of design follows a similar pattern to the British Code but with somewhat different rules which are as follows.

(a) The slab thickness h is determined and should not be less than $3\frac{1}{2}$ in. nor less than the perimeter of the slab divided by 180.

(b) Having chosen an appropriate live load the negative moments at continuous edges are calculated from Table 5.3 from the equations

$$M_A = C_A wA^2$$

or

$$M_B = C_B wB^2$$

where $$w = 1.4\,D.L + 1.7\,L.L \tag{5.6}$$

and D.L is the dead load and L.L is the live load.

The positive moment at midspan is determined in two parts. Firstly, due to dead load only using Table 5.4, using the equations

$$M_A = C_A wA^2$$

or

$$M_B = C_B wB^2$$

where w is 1.4 D.L.

Secondly, due to live load only using Table 5.5 from the equations

$$M_A = C_A wA^2$$

or

$$M_B = C_B wB^2$$

where w = 1.7 L.L.

The total positive moment is the sum of the D.L. and L.L. positive moments.

Negative moments at discontinuous edges must be allowed for at a value of one-third of the positive moments in the same direction to cater for any partial fixity.

(c) In a multispan case the support moments calculated for adjacent panels, may differ significantly and where the negative moment on one side of a support is less than 80 percent of that on the other side, the difference must be distributed in proportion to the relative stiffness of the slab for each side.

(d) Reinforcement calculation

(i) Middle strip

Determine for both the positive and negative steel

$$\frac{M_u}{\phi bd^2}$$

where $\phi = 0.9$ is the strength reduction factor, b is unit width and d is the effective depth and hence find the steel area by using design aids (graphs), tables or equations. The minimum area of steel required is given in Table 5.6.

Table 5.6 Minimum percentages of temperature and shrinkage reinforcement in slabs

Slabs where Grade 40 or 50 deformed bars are used	0.0020
Slabs where Grade 60 deformed bars or welded wire fabric (smooth or deformed) are used	0.0018
Slabs where reinforcement with yield strength exceeding 60,000 psi measured at yield strain of 0.35 percent is used	$(0.0018 \times 60{,}000)/f_y$

The area of steel for discontinuous edges is one-third of positive moment in its direction.

(ii) Column strip

Reinforcement in column strip should be assumed to be two-thirds the maximum moment at middle strip in the same section.

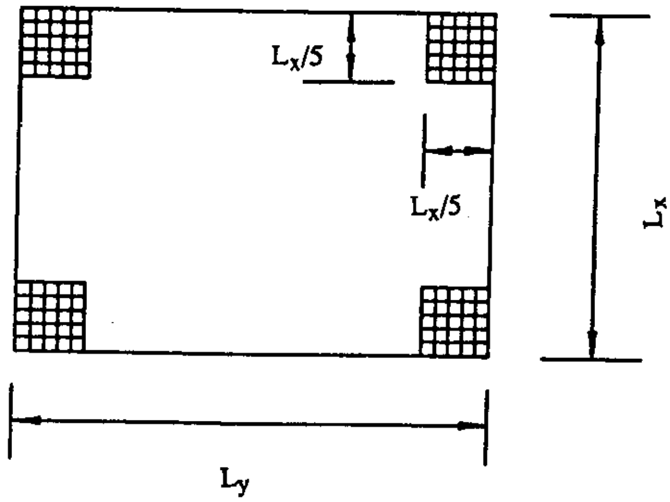
(e) Torsion reinforcement

Torsion reinforcement in both top and bottom of slab must be provided equal to the maximum positive moment in the slab.

The direction of the moment may be assumed to be parallel to the diagonal or parallel to the sides of the slab. It must be provided for a distance in each direction from the corner equal to one-fifth the longer span as shown in Figure 5.1b.

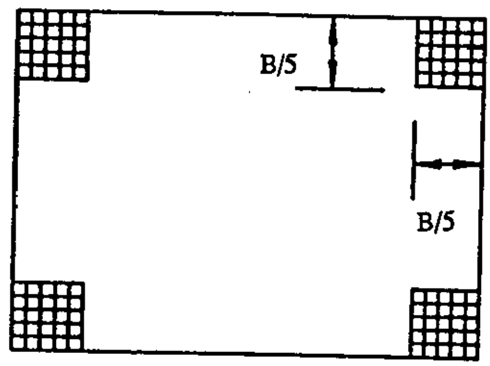
5.4.3 Typical Design Calculations using BS8110 and the ACI Code

In order to demonstrate the application of the previous design provisions, a numerical example is now given, first following the British then the American Codes. The example shows the steps required in each code for a multi-panelled floor three-spans in either direction as shown in Figure 5.2. The same service loads and slab thickness are used for both codes and the calculations shown as they might be prepared



At corner (A_s) for each layer=3/4(A_s) required for max. midspan moment.

(a)



B - Longer span

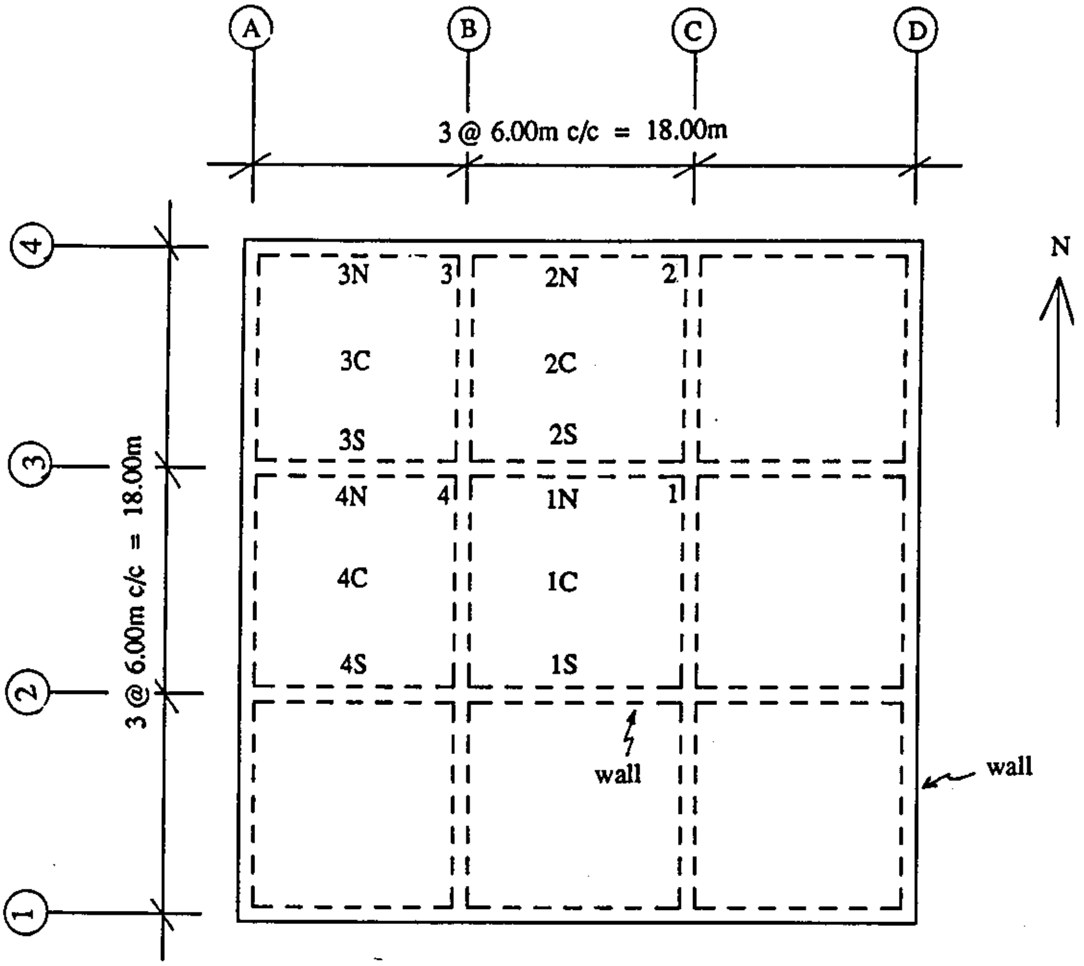
At corner (A_s) for each layer = (A_s) required for max. midspan moment.

(b)

Note: All edges for slabs above are discontinuous.

Fig. 5.1 : Corner reinforcement according to
(a) BS 8110
(b) ACI.

in a design office. The solution is restricted to the north-south direction. Actual loads have been chosen which give a dead/live load ratio of approximately 1 which is the mean of the 0.75 American and 1.25 British recommendation for checking pattern loading.



Floor plan

Notes:

1. Slab thickness = 200mm
2. All supporting walls are 240mm thick
3. Fire resistance requirements = 1 hr.
4. Exposure conditions = severe (external) and mild (internal)
5. f_{cu} = 30 N/mm2
6. f_y = 460 N/mm2
7. Calculations will be carried out in the North-South direction. For identification the position at the top of panel 1 will be termed 1N and that at the bottom 1S while that in the centre 1C. Similarly for other panels.

Fig. 5.2 Structural Summary Sheet

5.4.3.1 BS 8110

CALCULATIONS	Comments
DURABILITY AND FIRE RESISTANCE	
Min. cover for mild exposure = 25mm	Cover 25mm
Max. fire resistance of 200mm slab	
with 24mm cover = 2 hours	Therefore fire resistance OK
LOADING	
Self-weight of 200mm; 0.20 x 24 = 4.8	
Finishings = 1.0	
Characteristic dead load = 5.8	$g_k = 5.8$ kN/m^2
Imposed load = 5.0	
Partitions = 1.0	
Characteristic imposed load = 6.0	$q_k = 6.0$ kN/m^2
Design load $n = 1.4\ g_k + 1.6\ q_k$	
$= 1.4 \times 5.8 + 1.6 \times 6.0 = 17.72$ kN/m^2	$n = 17.72$ kN/m^2
ULTIMATE BENDING MOMENTS	
Panel 1 (interior panel, Table 5.2 Case 1)	
$L_x = 6.0$ m; $L_y = 6.0$ m; $L_y/L_x = 1.0$	
N→ S Initial values	
U.B.M. at edge (1N) $= -0.031 \times 17.72 \times 6^2$	
$= -0.331 \times 637.92$	
$= -19.775$ kNm/m	-22.684 kNm/m (after later adjustment)
U.B.M. at midspan (1C) $= 0.024 \times 637.92$	
$= 15.31$ kNm/m	+12.401 kNm/m (after later adjustment)

Panel 2 (edge panel, Table 5.2 Case 3)	
$L_x = 6.0$ m; $L_y = 6.0$ m; $L_y/L_x = 1.0$	
N $\rightarrow$ S Initial values	
U.B.M. at edge (2S) = -0.039 x 637.92	
= -24.879 kNm/m	-22.684 kNm/m (after later adjustment)
U.B.M. at midspan (2C) = +0.030 x 637.92	
= 19.14 kNm/m	+21.336 kNm/m (after later adjustment)
Panel 3 (corner panel, Table 5.2 Case 4)	
$L_x = 6.0$ m; $L_y = 6.0$ m; $L_y/L_x = 1.0$	
N $\rightarrow$ S Initial values	
U.B.M. at edge (3S) = -0.047 x 637.92	
= -29.982 kNm/m	-29.982 kNm/m
U.B.M. at midspan (3C) = +0.036 x 637.92	
= +22.965 kNm/m	+22.965 kNm/m
Panel 4 (edge panel, Table 5.2 Case 2)	
$L_x = 6.0$ m; $L_y = 6.0$ m; $L_x/L_y = 1.0$	
N $\rightarrow$ S Initial values	
U.B.M. at edge (4N) = -0.039 x 637.92	
= -24.879 kNm/m	-24.879 kNm/m
U.B.M. at midspan (4C) = +0.029 x 637.92	
= +18.50 kNm/m	+18.50 kNm/m

Support moments adjustment between panels 1 and 2.

	Panel 2	Panel 1	
	$3k\theta$	$4k\theta$	
	0.43	0.57	Distribution coefficient
(2S)	-24.879	+19.775	(1N)
	+2.195	+ 2.909	
	-22.684	+22.684	Final support moment

Midspan moment adjustment:

Panel 1

The sum of support and midspan moments before the above support adjustment, was 35.085 kN m/m, therefore midspan moment after that adjustment becomes 35.085 - 22.684 = 12.401 kN m/m

Panel 2

Before the support adjustment, the sum of support and midspan moments was 44.02 kN m/m, therefore midspan moment after that adjustment becomes 44.02 - 22.684 = 21.336 kN m/m.

Final Moment Values

1N = 22.684 kN m/m

1C = 12.401 kN m/m

2C = 21.336 kN m/m

MAIN REINFORCEMENT

Assuming the use of max. 12 mm bars:

Since the panels are square Ly/Lx = 1.0,

let d = the average d for upper and lower bars in mesh.

d = 200 - 25 - 12 = 163 mm

Min. reinforcement = 0.13/100 x 1000 x 163

= 211.9 mm^2/m

Therefore min. reinforcement = 211.9 mm^2/m

Panel 1 at midspan (1C)

$$\frac{M}{bd^2} = \frac{12.401 \times 10^6}{10^3 \times 163^2} = 0.46$$

$$\text{Therefore } \frac{100A_s}{bd} = 0.13$$

Therefore A_s = 0.13/100 x 1000 x 163

= 211.9 mm^2 = min. reinf. OK

A_s = 211.9 mm^2/m

at edges (1N)

$$\frac{M}{bd^2} = \frac{22.684 \times 10^6}{10^3 \times 163^2} = 0.854$$

Therefore 100 A_s/bd = 0.225

Therefore A_s = 0.225/100 x 1000 x 163

= 366.75 > min. reinf. OK

A_s = 366.75 mm^2/m

Panel 2 at midspan (2C)

$$\frac{M}{bd^2} = \frac{21.336 \times 10^6}{10^3 \times 163^2} = 0.779$$

Therefore 100A_s/bd = 0.21

Therefore A_s = 0.21/100 x 1000 x 163

= 342.3 > min. reinf. OK

A_s = 342.3 mm^2/m

at cont. edge (2S).

$$\frac{M}{bd^2} = \frac{22.684 \times 10^6}{10^3 \times 163^2} = 0.854$$

Therefore 100A_s/bd = 0.225

Therefore A_s = 0.225/100 x 1000 x 163

= 366.75 > min. reinf. OK

A_s = 366.75 mm^2/m

at discont. edge (2N)

A_s = 50% of midspan reinforcement

Therefore A_s = 342.3/2

= 171.15 < min. reinf.

Therefore use min. reinf.

A_s = 211.9 mm^2/m

Panel 3 at midspan (3C)

$$\frac{M}{bd^2} = \frac{22.965 \times 10^6}{10^3 \times 163^2} = 0.864$$

Therefore 100A_s/bd = 0.23

Therefore $A_s = 0.23/100 \times 1000 \times 163$

$= 374.9 >$ min. reinf. OK

$A_s = 374.9\ mm^2/m$

at cont. edge (3S)

$$\frac{M}{bd^2} = \frac{29.982 \times 10^6}{10^3 \times 163^2} = 1.128$$

Therefore $100A_s/bd = 0.29$

Therefore $A_s = 0.29/100 \times 1000 \times 163$

$= 472.7 >$ min. reinf. OK

$A_s = 472.7\ mm^2/m$

at discont. edge (3N)

$A_s = 50\%$ of midspan reinforcement

Therefore $A_s = 374.9/2$

$= 187.45 <$ min. reinf.

Therefore use min. reinf.

$A_s = 211.9\ mm^2/m$

Panel 4 at midspan (4C)

$$\frac{M}{bd^2} = \frac{18.50 \times 10^6}{10^3 \times 163^2} = 0.720$$

Therefore $100A_s/bd = 0.18$

Therefore $A_s = 0.18/100 \times 1000 \times 163$

$= 293.4 >$ min. reinf.

$A_s = 293.4\ mm^2/m$

At cont. edge of panel 4 the moment is -24.879 kN m/m while that of panel 3 is -29.9. The greater value will be used.

$A_s = 472.7\ mm^2/m$

TORSION REINFORCEMENT

At corner of panel 3:

A_s req = 3/4 x 374.9 = 281.175 mm^2/m

For top and bottom

$A_s = 281.175\ mm^2/m$

At corners between panels 2 and 3, and 3 and 4

A_s req = 3/8 x 374.9 = 140.587 mm^2/m

For top and bottom

$A_s = 140.587\ mm^2/m$

DEFLECTION

Basic span/effective ratio = 26 max.

$$\frac{M}{bd^2} = \frac{22.965 \times 10^6}{10^3 \times 163^2} = 0.864$$

$$f_s = \frac{5 \times 460 \times 374.9}{8 \times 375.9} = 287.5\ N/mm^2$$

$$\text{Modification factor} = 0.55 + \frac{(477 - f_s)}{120(0.9 + \frac{M}{bd^2})} < 2.0$$

$$= 0.55 + \frac{(477 - 287.5)}{120(0.9 + 0.60)}$$

$$= 1.6$$

Therefore allowable span/effective depth ratio

= 26 x 1.6 = 41.6

Actual span/effective depth ratio

= 6000/163 = 36.81

Therefore L/d ratio OK

5.4.3.2 ACI

CALCULATIONS	Comments
THICKNESS	
$h_{min} = \dfrac{2(6.00 + 6.00)}{180}$	
$= 0.133$ m < 0.20m OK	Therefore h = 0.20m
LOADING	
Self-weight of 0.20; 0.20 x 24 = 4.8 kN/m^2	
Finishings = 1.0 kN/m^2	
Therefore total dead load (D.L) = 5.8 kN/m^2	D.L = 5.8 kN/m^2
Live load = 5.0 kN/m^2	
Partitioning = 1.0 kN/m^2	
Therefore total live load (L.L) = 6.0 kN/m^2	L.L = 6.0 kN/m^2
The factored loads on which the design is to be based are:	
D.L = 1.4 x 5.8 = 8.12 kN/m^2	1.4 D.L = 8.12 kN/m^2
L.L = 1.7 x 6.0 = 10.20 kN/m^2	1.7 L.L = 10.20 kN/m^2
w (total) = 18.32 kN/m^2	w_{total} = 18.32 kN/m^2
ULTIMATE BENDING MOMENTS	
Coefficients from Tables 5.3, 5.4 & 5.5)	
Panel 1 (interior panel,)	
A = 6.0m; B = 6.0m; m = A/B = 1.0	
N⟶S Initial values	
M_{neg} (at 1N) = 0.045 x 18.32 x 6^2	
= -29.678 kN m/m	-25.167 kN m/m (after later adjustment)

$M_{pos,d.L}$ (at 1C)	$= +0.018 \times 8.12 \times 6^2$	
	$= +5.262$ kN m/m	
$M_{pos,L.L}$ (at 1C)	$= +0.027 \times 10.2 \times 6^2$	
	$= +9.914$ kN m/m	
$M_{pos,Tot.}$ (at 1C)	$= +15.176$ kN m/m	+19.687 kN m/m (after later adjustment)

Panel 2 (exterior panel, case 3)

A = 6.0m, B = 6.0m; m = 1.0

N ⟶ S Initial values

M_{neg} (at 2S)	$= -0.033 \times 18.32 \times 6^2$	
	$= -21.764$ kN m/m	-25.167 kN m/m (after later adjustment)
$M_{pos,d.L}$ (at 2C)	$= +0.020 \times 8.12 \times 6^2$	
	$= +5.846$ kN m/m	
$M_{pos,L.L}$ (at 2C)	$= +0.028 \times 10.2 \times 6^2$	
	$= +10.282$ kN m/m	
$M_{pos,Total}$ (at 2C)	$= +16.128$ kN m/m	+12.725 kN m/m (after later adjustment)
M_{neg} (at 2N)	= -1/3 x positive moment	
	$= -(16.128)/3$	
	$= -5.376$ kN m/m	-4.242 kN m/m (after later adjustment)

Panel 3 (corner panel, case 4)

A = 6.0m; B = 6.0m; m = 1.0

N ⟶ S Initial values

M_{neg} (at 3S)	$= -0.05 \times 18.32 \times 6^2$	
	$= -32.976$ kN m/m	-32.976 kN m/m
$M_{pos,d.L}$ (at 3C)	$= +0.027 \times 8.12 \times 6^2$	
	$= +7.893$ kN m/m	
$M_{pos,L.L}$ (at 3C)	$= +0.032 \times 10.2 \times 6^2$	
	$= +11.750$ kN m/m	

$M_{pos,Total}$ (at 3C)	= +19.643 kN m/m	+19.643 kN m/m
Mneg (at 3N)	= -1/3 x positive moment	
	= -1/3(19.643)	
	= -6.548 kN m/m	-6.548 kN m/m

Panel 4 (exterior panel, case 2)

A = 6.0m; B = 6.0m; m = 1.0

N$\longrightarrow$S Initial values

M_{neg} (at 4N)	= $-0.061 \times 18.32 \times 6^2$	
	= -40.230 kN m/m	-40.230 kN m/m
$M_{pos,d.L}$ (at 4C)	= $=0.023 \times 8.12 \times 6^2$	
	= +6.723 kN m/m	
$M_{pos,L.L}$ (at 4C)	= $+0.030 \times 10.2 \times 6^2$	
	= +11.016 kN m/m	
$M_{pos,Total}$ (at 4C)	= +17.739 kN m/m	+17.739 kN m/m

Support moments adjustment between panels

1 and 2

	Panel 2	Panel 1	
	$3k\theta$	$4k\theta$	
	0.43	0.57	Distribution coefficient
(2S)	-21.764	+29.678	(1N)
	-3.403	- 4.511	
	-25.167	+25.167	Final support moment

Midspan moment adjustment:

Panel 1

The sum of support and midspan moments

before the above support adjustment was

44.854 kN m/m

Therefore new midspan positive moment

= 44.854 - 25.167

= 19.687 kN m/m

+19.687 kN m/m

Panel 2

The sum of moments was 37.892

Therefore new midspan positive moment

= 37.892 - 25.167

= 12.725 kN m/m

+12.725 kN m/m

Accordingly the discontinuous negative moment (2N) will be 1/3(12.725)

= 4.242 kN m/m

-4.242 kN m/m

MAIN REINFORCEMENT

Assuming the use of maximum 12mm bars; since the panels are square A/B = 1.0, let d = the average d for upper and lower bars in mesh.

Use cover 25mm.

$f_y = 460 \text{ N/mm}^2 = 66715.01 \text{ psi}$

Min. ratio of reinforcement

$$= \frac{0.0018 \times 60000}{f_y}$$

$$= \frac{0.0018 \times 60000}{66715.02}$$

$$= 0.0016.$$

Min. reinforcement $= 0.0016 \times 1000 \times 163$

$= 260.8 \text{ mm}^2/\text{m}$

Min. reinforcement = $260.8 \text{ mm}^2/\text{m}$

Panel 1 at midspan (1C)

Assume the stress block depth a = 5.3

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})}$$

$$A_s = \frac{19.687 \times 10^6}{0.9 \times 460(163 - \frac{5.3}{2})} = 296.56 \text{ mm}^2/\text{m}$$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$= \frac{296.56 \times 460}{0.85 \times 30 \times 1000} = 5.34 \quad \text{OK}$$

A_s = 296.56 mm²/m > min. reinf. OK | A_s = 296.56 mm²/m

At continuous edge (1N)

assume the stress block depth a = 6.9 mm

$$A_s = \frac{25.167 \times 10^6}{0.9 \times 460(163 - \frac{6.9}{2})} = 381.01$$

$$a = \frac{381.01 \times 460}{0.85 \times 30 \times 1000} = 6.87 \quad \text{OK}$$

A_s = 381.01 mm²/m > min. reinf. OK | A_s = 381.01 mm²/m

Panel 2

At midspan (2C)

Assume the stress block depth a = 3.4 mm.

$$A_s = \frac{12.725 \times 10^6}{0.9 \times 460(163 - \frac{3.4}{2})} = 190.56$$

$$a = \frac{190.56 \times 460}{0.85 \times 30 \times 1000} = 3.43 \quad \text{OK}$$

A_s = 190.56 mm²/m < min. reinforcement

Therefore use min. reinforcement = 260.8 mm²/m | A_s = 260.8 mm²/m

At continuous edge (2S)

use same as for (1N)

$A_s = 381.01 \text{ mm}^2/\text{m}$ | $A_s = 381.01 \text{ mm}^2/\text{m}$

and at discontinuous edge (2N)

= 1/3 x midspan value

= 1/3(260.8)

= $86.933 \text{ mm}^2/\text{m}$ | $A_s = 86.933 \text{ mm}^2/\text{m}$

Panel 3

At midspan (3C)

Assume the stress block depth a = 5.3mm.

$$A_s = \frac{19.643 \times 10^6}{0.9 \times 460(163 - \frac{5.3}{2})} = 295.90$$

$$a = \frac{295.90 \times 460}{0.85 \times 30 \times 1000} = 5.34 \quad \text{OK}$$

$A_s = 295.90 \text{ mm}^2/\text{m} >$ min. reinf. | $A_s = 295.90 \text{ mm}^2/\text{m}$

At continuous edge (3S)

Assume the stress block depth a = 9.0mm

$$A_s = \frac{32.976 \times 10^6}{0.9 \times 460(163 - \frac{9.0}{2})} = 502.54$$

$$a = \frac{502.54 \times 460}{0.85 \times 30 \times 1000} = 9.06 \quad \text{OK}$$

$A_s = 502.54 \text{ mm}^2/\text{m} >$ min. reinf. | $A_s = 502.54 \text{ mm}^2/\text{m}$

At discontinuous edge (3N)

= 1/3(295.90)/3

= $98.63 \text{ mm}^2/\text{m}$ | $A_s = 98.63 \text{ mm}^2/\text{m}$

Panel 4

At midspan (4C)

Assume the stress block depth a = 4.8mm

$$A_s = \frac{17.739 \times 10^6}{0.9 \times 460(163 - \frac{4.8}{2})} = 266.80$$

$$a = \frac{266.80 \times 460}{0.85 \times 30 \times 1000} = 4.81 \quad \text{OK}$$

A_s = 266.80 > min. reinforcement	A_s = 266.80 mm^2/m
At continuous edge (4N) use same as for (3S) A_s = 502.54 mm^2/m	A_s = 502.54 mm^2/m
TORSION REINFORCEMENT At corner of panel (3) A_s req = midspan positive steel = 295.90 mm2/m	A_s = 295.90 mm2/m
At corner between panels 2 and 3 A_s req = 295.90 mm^2/m	A_s = 295.90 mm2/m
At corner between panels 3 and 4 A_s req = 295.90 mm^2/m	A_s = 295.90 mm2/m

DEFLECTION

Since slab thickness = 200mm > 90mm

and since

$$\frac{2 \times (6000 + 6000)}{180} = 133.33\text{mm} < 200\text{mm}$$

Therefore deflection control OK.

Table 5.7 Steel reinforcement quantities for a sample design employing ACI and BS8110 code requirements

Panel number and type	Location of section	Code	A_s mm^2/m (in middle strip)	A_s per middle strip	A_s per two half column strip	Total A_s in the entire width of critical section	Total A_s in the critical sections of north south direction
1 interior panel	midspan	ACI	296.56	889.68	593.12	1482.8	ACI = 5292.9
		BS8110	211.9	953.55	317.85	1271.40	BS8110 = 5207.85
	cont. edge	ACI	381.01	1143.03	762.02	1905.05	
		BS8110	366.75	1650.375	317.85	1968.225	
2 edge panel	midspan	ACI	260.8	782.4	521.6	1304.00	ACI = 4513.05
		BS8110	342.3	1540.35	317.85	1858.20	BS8110 = 5097.825
	cont. edge	ACI	381.01	1143.03	762.02	1905.05	
		BS8110	366.75	1650.375	317.85	1968.225	
	disc. edge	ACI	260.8	782.4	521.6	1304.00	
		BS8110	211.9	953.55	317.85	1271.40	
3 corner edge	midspan	ACI	295.90	887.7	591.8	1479.50	ACI = 5296.2
		BS8110	374.9	1687.05	317.85	2004.9	BS8110 = 5721.3
	cont. edge	ACI	502.54	1507.62	1005.08	2512.7	
		BS8110	472.7	2127.15	317.85	2445.00	
	disc. edge	ACI	260.8	782.4	521.6	1304.00	
		BS8110	211.9	953.55	317.85	1271.40	
4 edge panel	midspan	ACI	266.80	800.4	533.6	1334.00	ACI = 6359.4
		BS8110	293.4	1320.30	317.85	1638.15	BS8110 = 6528.15
	cont. edge	ACI	502.54	1507.62	1005.08	2512.7	
		BS8110	472.7	2127.15	317.85	2445.00	

5.4.3.3 Conclusions on calculations

No major comments need be made on the calculatons and as will have been seen the procedures are very similar and relatively straightforward.

The results of the calculations in terms of areas of steel at critical sections are shown in Table 5.7.

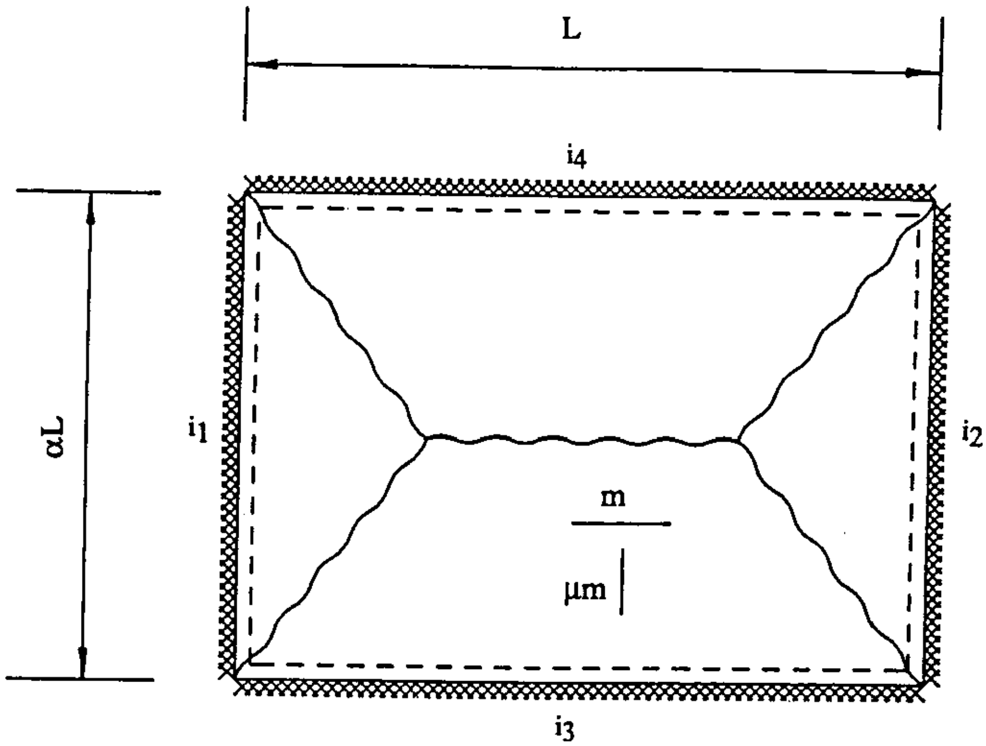
Although in most cases the BS8110 coefficients are less than the ACI values except for panel 1 BS8110 requires more steel than the ACI code which seems to be a contradiction. The main reason is that the outside quarter strip of the British code while having a zero moment coefficient requires the minimum amount of steel which is not insignificant. Thus for panel 1, of the 5207.85 mm^2 some 635 mm^2 is minimum steel without which the British code would require much less than the ACI code. For panel 2 the extra steel for this requirement, less the ACIdifference at a discontinuous edge, is 430 mm^2. Reductions of the same order occur for panels 3 and 4 which if they were disregarded would in fact make the British code requirement lead to less steel than the ACI code which is consistent with lower moment coefficients.

No major conclusions can therefore be drawn from these calculations except to say the process is similar and broadly leads to approximately the same quantity of steel at the critical sections.

5.5 Derivation of BS8110 moment coefficients

The moment coefficients in Table 3.15 in BS 8110 have been attributed to Taylor et al. [40] using yield-line analysis in which the pattern in Figure 5.3 was considered. The effects of corner levers have been ignored, which is acceptable if torsion reinforcement is included. Taylor's calculation was based on the assumption that the resisting bending moment was uniform across the width of the panel.

The solution for the pattern in Figure 5.3 is well established and given in references [16, 17]. The general bending moment equation will be:



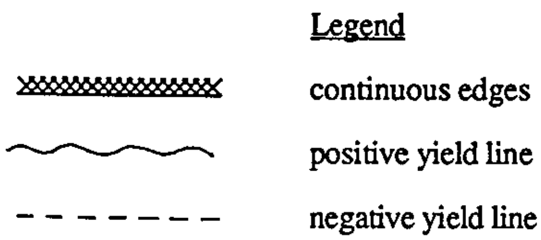


Fig. 5.3 Pattern of Yield-Line

$$m = \frac{w\alpha^2 L^2}{6\gamma_{34}^2} \left\{ \sqrt{\left[3 + \mu\left(\frac{\alpha\gamma_{12}}{\gamma_{34}}\right)^2\right]} - \frac{\alpha\gamma_{12}\sqrt{\mu}}{\gamma_{34}} \right\} \quad (5.7)$$

where

$$\gamma_{12} = \sqrt{(1 + i_1)} + \sqrt{(1 + i_2)}$$

$$\gamma_{34} = \sqrt{(1 + i_3)} + \sqrt{(1 + i_4)}$$

m is the bending moment/unit length for the short span,

w is the uniform distributed load,

α is the ratio of short span to long span,

L is the long span,

μ is the ratio of positive bending moment in the long span to the positive bending moment at short span, and

i_1, i_2, i_3 and i_4 are the ratio of negative moments at the supports to the positive moment at midspan.

At this stage the steel is assumed across the whole width of the slab though the code concentrates it in the middle band. The code states that the ratio of negative to positive steel is $\frac{4}{3}$ so that i where applicable is always this value.

The value of μ is not constant and has to be taken as that calculated from the code values. The values for all 9 cases have been calculated from equation 5.7 and the moment coefficient calculated. This in turn was then multiplied by 4/3 since the uniform steel is compressed into the 3/4 middle strip. These are summarised in Table 5.8 and compared with the code values.

Since the negative moments are always 4/3 times the positive moments, only the positive moments are compared in the Table.

The bracketted figures in Table 5.8 are the ratio of the code value/yield-line value and the closeness to unity for most cases shows that the code's coefficients are almost identical to the yield-line solutions given by equation 5.7. Thus the values in

Table 5.8 Positive moments in slab panels

Cases	Short span L_y/L_x 1.0	Short span L_y/L_x 2.0	Long span
Case 1 	0.024 0.024 (1.00)	0.048 0.049 (0.98)	0.024 0.024 (1.00)
Case 2 	0.029 0.029 (1.04)	0.050 0.051 (0.98)	0.028 0.028 (1.00)
Case 3 	0.030 0.032 (0.94)	0.067 0.064 (1.05)	0.028 0.032 (0.88)
Case 4 	0.036 0.035 (1.03)	0.070 0.068 (1.03)	0.034 0.035 (0.97)
Case 5 	0.034 0.035 (0.97)	0.053 0.052 (1.02)	0.034 0.035 (0.97)
Case 6 	0.034 0.033 (1.03)	0.100 0.088 (1.14)	0.034 0.033 (1.03)
Case 7 	0.043 0.043 (1.00)	0.074 0.073 (1.01)	0.044 0.043 (1.02)
Case 8 	0.042 0.043 (0.98)	0.106 0.101 (1.05)	0.044 0.043 (1.02)
Case 9 	0.055 0.056 (0.98)	0.111 0.107 (1.04)	0.056 0.056 (1.00)

Notes:
1. The top line shows code value and the second shows the yield-line theory solution.
2. Values in brackets show the ratio of code value to yield-line solution value.

Table 5.8 confirm the British code values are based on yield-line analysis with the i value 4/3 and the total steel compressed into 3/4 of the span width.

Thus as far as the ultimate condition is concerned the amount of steel provided is satisfactory but no comment at this stage can be made about the serviceability conditions.

5.6 Derivation of moment coefficient of ACI 318-63

As mentioned in section 5.4, the moment coefficients used in ACI 318-63 are based on a procedure for the analysis of two-way slabs initially developed by Marcus [38] and introduced into the USA and developed into its present form by Rogers [39].

The purpose of this section is to compare the moment coefficient quoted by ACI 318-63 for use in two-way slab design with those that would be computed using the procedure for two-way slab design initially developed by Marcus.

The section is begun by quoting the basic expressions developed by Marcus to calculate the moments in both main directions of two-way slabs. These equations are then applied to the cases of two-way slab design in ACI 318-63 in order to make the necessary comparisons.

Marcus derived simple expressions for calculating the moments in the long- and short-span directions of two-way slabs by equating the maximum deflection of simple strips in two perpendicular directions. The deflections are based on the elastic behaviour of the simple strips with a modification factor which supposedly allows for twisting moments. The basic expressions derived are as follows.

$$M_A = m_A(1 - \phi_A)$$

$$M_B = m_B(1 - \phi_B)$$

where M_A indicates the final moment in the A direction;

M_B indicates the final moment in the B direction;

m_A is the value of the moment obtained by loading the strip in the A direction with its supposed loading proportion w_A;

m_B is the value of the moment obtained by loading the strip in the B direction with w_B.

($w_A + w_B = w$ = total uniformly distributed load per unit square area); and

$$\phi_A = \frac{5}{6} \times \frac{A^2}{B^2} \times \frac{m_{A\,max}}{M_{OA}}; \quad \phi_B = \frac{5}{6} \times \frac{B^2}{A^2} \times \frac{m_{B\,max}}{M_{OB}}$$

where M_{OA} and M_{OB} are the values of the respective bending moments on strips of unit width, simply supported and loaded with the full load of w per linear ft.

The value of ϕ_A and ϕ_B varies with the type of supports but a typical example from Rogers' [39] paper is set out below.

"Case - Slab freely supported on all four edges.

w = Load per square foot of slab (D.L. + L.L.).

w_A and w_B = portions of w in directions A and B respectively ($w_A + w_B = w$).

A and B = Spans in directions A and B respectively.

Maximum deflection of one foot wide middle-strip in A direction:

$$\frac{5}{32} \times \frac{w_A \times A^4}{E \times t_1^3}$$

Maximum deflection of one foot wide middle-strip in B direction:

$$\frac{5}{32} \times \frac{w_B \times B^4}{E \times t_1^3}$$

The maximum deflection occurs at the middle-span, where both deflections are equal, or

$$w_A \times A^4 = w_B \times B^4, \text{ and thus}$$

$$w_A = \frac{w \times B^4}{A^4 + B^4} \text{ amd } w_B = \frac{w \times A^4}{A^4 + B^4}$$

$$m_{A\,max} = \frac{w \times A^2}{8} \frac{B^4}{A^4 + B^4}; \quad M_{OA} = \frac{w \times A^2}{8},$$

$$\text{and}\ \frac{m_{A\,max}}{M_{OA}} = \frac{B^4}{A^4+B^4};\quad m_{B\,max} = \frac{w \times B^2}{8}\ \frac{A^4}{A^4+B^4};$$

$$M_{oB} = \frac{w \times B^2}{8},\ \text{and}\ \frac{m_B\ \text{max}}{M_{oB}} = \frac{A^4}{A^4+B^4}$$

$$\phi_A = \phi_B = \phi = \frac{5}{6} \times \frac{A^2 \times B^2}{A^4+B^4};\ \text{or}\ (1-\phi) = v_a =$$

$$1 - \left(\frac{5}{6} \times \frac{A^2 \times B^2}{A^4+B^4}\right),\ \text{and finally}$$

$$M_{A\,max} = \frac{1}{8} \times w_A \times A^2 \times v_a;\ \text{and}\ M_{B\,max} = \frac{1}{8} \times w_B \times B^2 \times v_a;$$

For a square plate, the value of v_a is equal to 0.583 which certainly is a substantial reduction of Moment-value."

The last sentence infers that Marcus' method reduces the mid bending moment value to 0.583 ($wL^2/16$) from $wL^2/16$ which is to be expected from twistless strips. This therefore gives a central moment coefficient of 0.036 wL^2. No comment on this value is made at this stage since only the derivations of the values are being considered at the moment. Six different edge supported cases in all have been considered for aspect ratios of 1:1 and 1:2, namely those shown in Table 5.9. By rotating some of these through 90° the other three code cases can be obtained and therefore are sufficient for comparison. Using a similar technique to that used for the simply supported case the various ϕ_A and ϕ_B values can be determined and lead to the values in Table 5.9. For each case the top unbracketted figure is that arrived at using Marcus' method, while the bracketted figure is the code value either for positive moments used for dead loads, Table 5.4, or for the negative moments taken from Table 5.3. The figure in the third row is the code value/Marcus value.

It can be seen that for all the positive moments the results are virtually identical and that for the negative value with one exception the code values are larger by 7 to 9 per cent, the variation clearly being due to rounding.

Table 5.9 Values of ACI coefficients and those obtained from Marcus' method

Aspect ratio and type of moments / Case type	1.0		2.0	
	Positive moment	Negative moment	Positive moment	Negative moment
	0.0364 (0.036) 1.00	- -	0.0946 (0.095) 1.00	- -
	0.018 (0.018) 1.00	0.042 (0.045) 1.07	0.0366 (0.037) 1.00	0.0783 (0.086) 1.09
	0.0334 (0.033) 1.00	0.089 (0.071) 0.80	0.0607 (0.061) 1.00	0.122 (0.097) 0.80
	0.0266 (0.027) 1.00	0.0694 (0.075) 1.08	0.038 (0.039) 1.00	0.0823 (0.090) 1.09
	0.027 (0.027) 1.00	0.0625 (0.050) 0.80	0.059 (0.059) 1.00	0.117 (0.094) 0.80
	0.0226 (0.023) 1.00	0.055 (0.061) 1.09	0.0377 (0.038) 1.00	0.0808 (0.088) 1.09

Note: The top line shows Marcus' value, the second line shows the code value and the third line shows the ratio of code to Marcus' value

At the end of his paper Rogers discusses patterned loading and indicates when two adjacent panels are loaded that the negative coefficient will be higher than for uniform loading and the factor used appears to be an increase of about 8%. Dead loading cannot give a checker-board load pattern but live loading can. He therefore postulates that if all panels are loaded with $p_1 = 1/2$ L.L. giving coefficients half of the Marcus values, then on a checker-board layout panels are loaded with $p_2 = 1/2$ L.L. or $p_3 = -1/2$ L.L. and assuming simple supports for all later loadings the combination of the two loading sets give the same effect as full load and zero load in a checker-board fashion. On this basis therefore the positive moment coefficients for live loading will be half the sum of the positive moment coefficient for the case in question and the simply supported case. Thus for a fully fixed slab the positive live load coefficient would be $1/2(0.018 + 0.036) = 0.27$, namely half the sum of the first top two code figures in column 1 of Table 5.9. All the ACI values given in Table 5.5 have been checked and they are indeed based on this hypothesis.

The one exception to a remarkably consistent set of results is the third case considered where the negative moment value for both the 1:1 and 1:2 aspect ratio is some 20% less than the Marcus value. The original equations have been checked carefully though such an error is unlikely with both aspect ratios. No compensating relief has been given to the positive moment if some redistribution had been allowed. This is of course an asymmetrical case and it may be an allowance was made since the maximum deflections in one direction are not at the centre. No explanation can be found in the literature and two printing errors are unlikely. The difference remains unresolved.

With this exception it can be concluded that the ACI moment coefficients are obtained as follows:

(i) the negative moment coefficients in Table 5.3 are based on Marcus' method factored up by about 8% to allow for patterned loading;

(ii) the dead load positive moment coefficients are based on Marcus' method; and

(iii) the live load positive moment coefficients are half the sum of the particular case and the simply supported case again from Marcus' coefficients.

Discussion of the code values will not be made at this stage since values obtained from the finite element analysis need to be incorporated into the discussion.

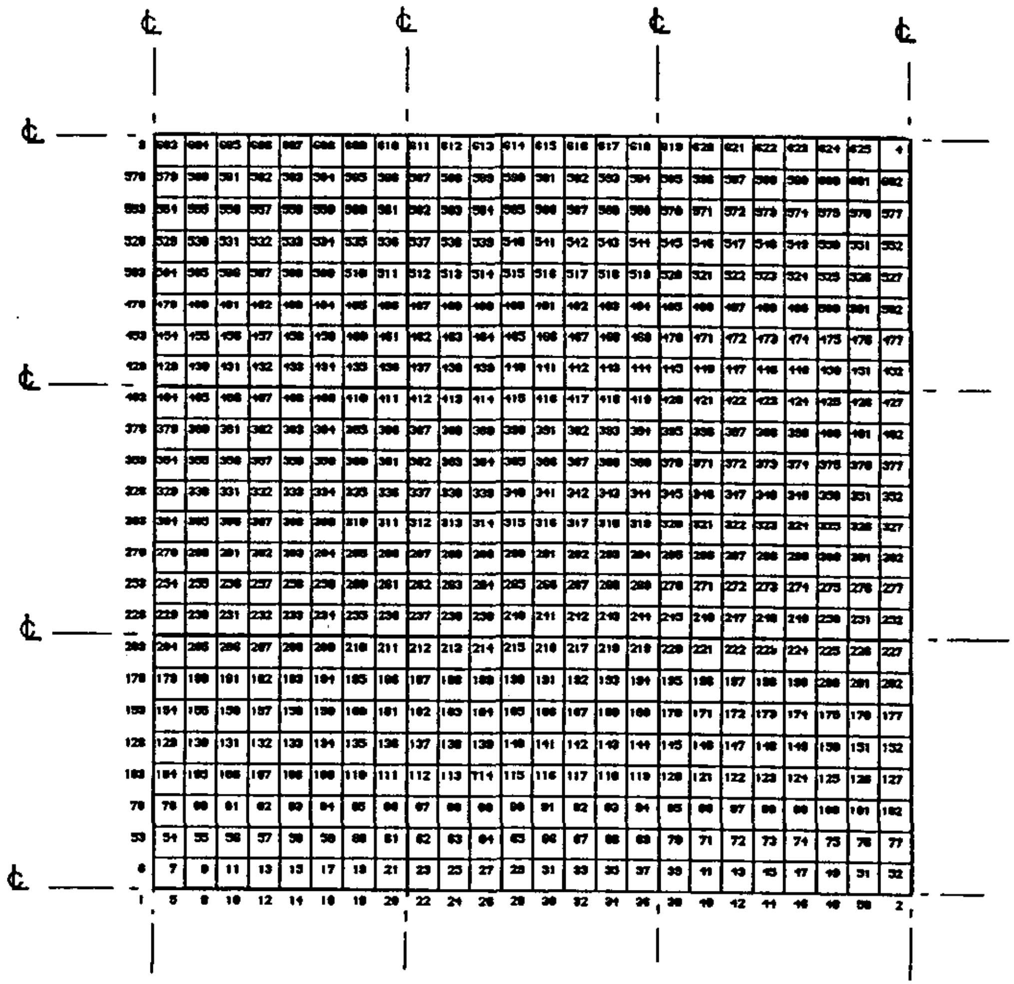
5.7 Finite Element Analysis

An extensive finite element analysis was carried out in order to calculate the maximum amount of steel that would be required from the elastic field moment values of M_x, M_y and M_{xy} and then applying the Wood-Armer reinforcement rules. The analysis covered all the nine different cases of support conditions given in the code for slabs with a 1:1 and 2:1 aspect ratio. For ease of recognition these are numbered case 1 to 9 corresponding to the sequence in BS8110, Table 5.2. The analysis covered what would be regarded as all likely loading patterns.

The object of the analysis was to determine the elastic moments to compare in broad terms the elastic moment coefficients found with the recommended code values and to check whether any elastic moment at the serviceability condition would cause the steel recommended in the code to yield under this condition.

In all the cases that follow the slab which is being examined in detail is marked S in any relevant Figures.

The configuration of slabs that has been chosen is that from which the worst cases are likely to occur. Usually this involves 3 or 4 different configurations and loading patterns. The loading patterns chosen were all slabs loaded, or a mixture of some slabs loaded with dead load only and others with a load of 1.4 times the dead load plus 1.6 x the live load. The live load was set at 1.25 the dead load which is the recommended limit in BS8110 if pattern loading is not required to be examined. As recommended in an earlier section each slab was divided into 8 x 8 finite elements which means on average that some 576 elements were used for each analysis since that slab configuration usually consisted of some 9 connected panels. The slab thickness in all cases was 0.24m and in all spans 4m.



Note : centre-lines show the positions of supports.

Fig. 5.4 Finite element mesh for a nine panels of slab, for patterns 1,2 and 3 in case 1, showing node numbers.

PAFEC

VIEW FROM X =0.0000
Y =0.0000
Z = 1.000

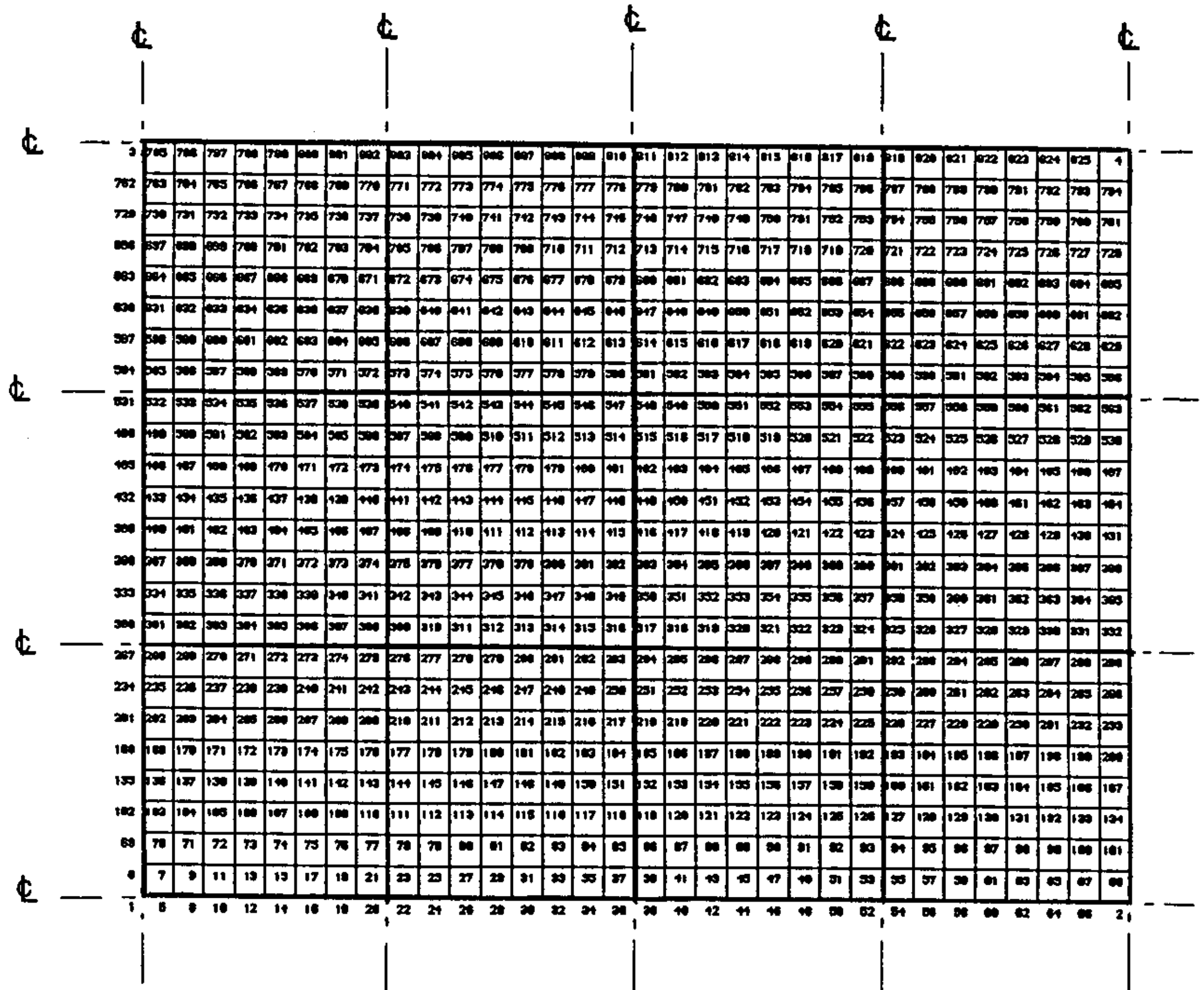
Z TOWARDS VIEWER

WHOLE STRUCTURE DRAWN

PRODUCED BY- PAFWOR
ON 28/05/88 AT 10 9 HOURS

10 20 30 40 50 MM.
5 15 25 35 STRUCTURAL UNITS
MULTIPLY BY 10'

DRAWING NO. 1
SCALE = 01200 E-1
DRAWING TYPE= 2



Note : centre-lines show the positions of supports.

Fig. 5.5 Finite element mesh for a twelve panels of slab, for pattern 4 in case 1, showing node numbers.

VIEW FROM X =0.0000
Y =0.0000
Z = 1.000

Z TOWARDS VIEWER

WHOLE STRUCTURE DRAWN

PRODUCED BY- PAFWOR
ON 11/06/88 AT 10 2 HOURS

MULTIPLY BY 10[1] STRUCTURAL UNITS

DRAWING NO. 1
SCALE = 01200 E-1 11
DRAWING TYPE= 2

The node numbering scheme where 9 or 12 connected slabs were examined is shown in Figures 5.4 and 5.5, and a similar scheme was adopted when fewer slabs were used. Clearly the output for all the cases examined was extensive and only a small portion of this is contained here. The remainder is lodged with the Department of Civil Engineering.

5.7.1 Case 1, slab restrained on four sides; aspect ratio 1:1

The case number, the 4 chosen panel layouts and loading patterns are shown in Figure 5.7 and for pattern 1 the values of M_x, M_y, M_{xy} and the steel requirements based on the Wood-Armer rules M_x^+, M_x^-, M_y^+ and M_y^- at the 81 nodes of the slab marked S only are given in Table 5.10.

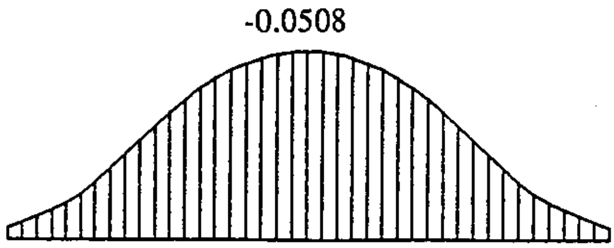
The individual values of this large set of data have been converted into moment coefficients by dividing by nL^2 (= 313344) and of these the moment coefficients at the critical centre and edge sections have been plotted (see Fig. 5.6). The output for loading patterns 2, 3 and 4 are given in Tables 5.11, 5.12 and 5.13. These in turn have been divided by nL^2 and the highest single positive or negative coefficient in both the north-south and east-west directions have been abstracted and given in Table 5.14a. The loading patterns relevant to these cases are shown above Table 14a in Fig. 5.7.

The first interesting point to observe from Table 5.14a is that the worst loading condition for the negative moment coefficient of slab S is pattern 4 on its eastern side; when two adjacent slabs are loaded the value is 0.0618. The worst pattern for the positive moment is pattern 2 with a value of 0.0296 when the two adjacent spans are unloaded. This is part confirmation of the common practice to consider adjacent spans loaded for negative bending moments and the span only loaded for positive moments when designing multispan beams. The patterned loading causes 22% and 34% increase respectively in the negative and positive moments compared with the uniformly loaded slab pattern 1. This is a significant increase and it is therefore questionable whether the British code should allow the value of the live load to be as high as 1.25 the dead load before patterned loading is taken into account. The ACI code limits the value to 0.75 of

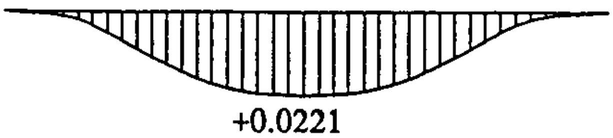
Table 5.10 Field moments and reinforcing moment values in slab S in pattern 1 of case 1 shown in Fig. 5.7

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
211	162 0000	162.0000	-176.3961	338.3961	-14.3961	338.3961	-14.3961
212	-660.5906	-3375.4897	20.1226	0.0000	-680.7133	0.0000	-3395.6128
213	-2004.1094	-9686.2910	110.8309	0.0000	-2114.9404	0.0000	-9797.1230
214	-2883.3516	-14334.2500	91.9682	0.0000	-2975.3198	0.0000	-14426.2187
215	-3201.1943	-15917.2090	0.0000	0.0000	-3201.1943	0.0000	-15917.2090
216	-2883.3516	-14334.2500	-91.9682	0.0000	-2975.3198	0.0000	-14426.2187
217	-2004.1091	-9686.2910	-110.8309	0.0000	-2114.9404	0.0000	-9797.1230
218	-660.5905	-3375.4902	-20.1226	0.0000	-680.7133	0.0000	-3395.6133
219	162.0000	162.0000	176.3961	338.3961	-14.3961	338.3961	-14.3961
236	-3375.4897	-660.5906	20.1226	0.0000	-3395.6128	0.0000	-680.7133
237	-712.8000	-712.8000	-2574.9019	1862.1018	-3287.7021	1862.1018	-3287.7021
238	423.6486	-1489.2488	-2629.6699	3053.3184	-2206.0215	1140.4211	-4118.9189
239	533.1813	-2445.9814	-1505.5559	1459.8843	-972.3746	0.0000	-3951.5376
240	500.9047	-2853.6255	-0.0233	500.9047	0.0000	0.0000	-2853.6260
241	533.1813	-2445.9814	1505.5554	1459.8835	-972.3741	0.0000	-3951.5371
242	423.6486	-1489.2488	2629.6689	3053.3174	-2206.0205	1140.4202	-4118.9180
243	-712.8002	-712.8000	2574.9014	1862.1011	-3287.7017	1862.1013	-3287.7017
244	-3375.4897	-660.5906	-20.1226	0.0000	-3395.6128	0.0000	-680.7133
261	-9686.2910	-2004.1094	110.8309	0.0000	-9797.1230	0.0000	-2114.9404
262	-1489.2490	423.6486	-2629.6699	1140.4209	-4118.9189	3053.3184	-2206.0215
263	2150.3999	2150.3999	-2735.9922	4886.3916	-585.5923	4886.3916	-585.5923
264	3544.3179	3158.8813	-1609.5701	5153.8877	0.0000	4768.4512	0.0000
265	3875.3760	3478.2231	0.0000	3875.3760	0.0000	3478.2231	0.0000
266	3542.9839	3157.8154	1608.3770	5151.3604	0.0000	4766.1924	0.0000
267	2150.6401	2150.6392	2735.7510	4886.3906	-585.1108	4886.3896	-585.1118
268	-1486.9114	421.3110	2630.5337	1143.6223	-4117.4453	3051.8442	-2209.2231
269	-9686.2910	-2004.1094	-110.8309	0.0000	-9797 1230	0.0000	-2114.9404
286	-14334.2500	-2883.3516	91.9682	0.0000	-14426.2187	0.0000	-2975.3198
287	-2445.9814	533.1813	-1505.5559	0.0000	-3951.5376	1459.8843	-972.3746
288	3158.8813	3544.3179	-1609.5701	4768.4512	0.0000	5153.8877	0.0000
289	5509.1992	5509.1992	-958.7444	6467.9434	0.0000	6467.9434	0.0000
290	6131.2490	6185.5498	0.0160	6131.2646	0.0000	6185.5654	0.0000
291	5509.1992	5509.1992	958.7443	6467.9434	0.0000	6467.9434	0.0000
292	3157.8149	3542.9839	1608.3772	4766.1914	0.0000	5151.3604	0.0000
293	-2445.9814	533.1813	1505.5554	0.0000	-3951.5371	1459.8835	-972.3741
294	-14334.2500	-2883.3521	-91.9682	0.0000	-14426.2187	0.0000	-2975.3203
311	-15917.2090	-3201.1943	0.0000	0.0000	-15917.2090	0.0000	-3201.1943
312	-2853.6274	500.6670	0.0000	0.0000	-2853.6274	500.6670	0.0000
313	3478.2231	3875.3760	0.0000	3478.2231	0.0000	3875.3760	0.0000
314	6185.7139	6131.0840	-0.0157	6185.7295	0.0000	6131.0996	0.0000
315	6914.3994	6914.3994	0.0000	6914.3994	0.0000	6914.3994	0.0000
316	6185.5947	6131.2041	0.0000	6185.5947	0.0000	6131.2041	0.0000
317	3478.2231	3875.3760	0.0000	3478.2231	0.0000	3875.3760	0.0000
318	-2853.6274	500.6671	0.0000	0.0000	-2853.6274	500.6671	0.0000
319	-15917.2090	-3201.1943	0.0000	0.0000	-15917.2090	0.0000	-3201.1943
336	-14334.2500	-2883.3516	-91.9682	0.0000	-14426.2187	0.0000	-2975.3198
337	-2445.9814	533.1813	1505.5554	0.0000	-3951.5371	1459.8835	-972.3741
338	3157.8154	3542.9839	1608.3770	4766.1924	0.0000	5151.3604	0.0000
339	5509.1992	5509.1992	958.7443	6467.9434	0.0000	6467.9434	0.0000
340	6131.0400	6185.7588	-0.0317	6131.0713	0.0000	6185.7900	0.0000
341	5509.1992	5509.1992	-958.7444	6467.9434	0.0000	6467.9434	0.0000
342	3157.8154	3542.9839	-1608.3774	4766.1924	0.0000	5151.3613	0.0000
343	-2445.9814	533.1813	-1505.5559	0.0000	-3951.5376	1459.8843	-972.3746
344	-14334.2500	-2883.3516	91.9682	0.0000	-14426.2187	0.0000	-2975.3198
361	-9686.2910	-2004.1091	-110.8309	0.0000	-9797.1230	0.0000	-2114.9404
362	-1489.2488	423.6486	2629.6689	1140.4202	-4118.9180	3053.3174	-2206.0205
363	2150.5200	2150.5200	2735.8711	4886.3906	-585.3511	4886.3906	-585.3511
364	3544.3174	3158.8813	1609.5696	5153.8867	0.0000	4768.4502	0.0000
365	3875.3760	3478.2236	0.0000	3875.3760	0.0000	3478.2236	0.0000
366	3542.9839	3157.8149	-1608.3774	5151.3613	0.0000	4766.1924	0.0000
367	2150.6401	2150.6392	-2735.7520	4886.3916	-585.1118	4886.3906	-585.1128
368	-1486.9111	421.3111	-2630.5342	1143.6230	-4117.4453	3051.8452	-2209.2231
369	-9686.2910	-2004.1091	110.8309	0.0000	-9797.1230	0.0000	-2114.9404
386	-3375.4902	-660.5906	-20.1226	0.0000	-3395.6133	0.0000	-680.7133
387	-712.8000	-712.8002	2574.9014	1862.1013	-3287.7017	1862.1011	-3287.7017
388	423.6486	-1489.2488	2629.6694	3053.3179	-2206.0210	1140.4207	-4118.9189
389	533.1813	-2445.9814	1505.5554	1459.8835	-972.3741	0.0000	-3951.5371
390	500.6649	-2853.6250	0.0179	500.6649	0.0000	0.0000	-2853.6255
391	533.1813	-2445.9814	-1505.5559	1459.8843	-972.3746	0.0000	-3951.5376
392	423.6486	-1489.2488	-2629.6694	3053.3179	-2206.0210	1140.4207	-4118.9189
393	-712.8002	-712.8002	-2574.9019	1862.1016	-3287.7021	1862.1016	-3287.7021
394	-3375.4902	-660.5906	20.1226	0.0000	-3395.6133	0.0000	-680.7133
411	162.0000	162.0000	176.3961	338.3961	-14.3961	338.3961	-14.3961
412	-660.5906	-3375.4897	-20.1226	0.0000	-680.7133	0.0000	-3395.6128
413	-2004.1094	-9686.2910	-110.8309	0.0000	-2114.9404	0.0000	-9797.1230
414	-2883.3521	-14334.2500	-91.9682	0.0000	-2975.3203	0.0000	-14426.2187
415	-3201.1943	-15917.2070	0.0000	0.0000	-3201.1943	0.0000	-15917.2070
416	-2883.3521	-14334.2500	91.9682	0.0000	-2975.3203	0.0000	-14426.2187
417	-2004.1091	-9686.2910	110.8309	0.0000	-2114.9404	0.0000	-9797.1230
418	-660.5906	-3375.4897	20.1226	0.0000	-680.7133	0.0000	-3395.6128
419	162.0000	162.0000	-176.3961	338.3961	-14.3961	338.3961	-14.3961

Note: To get moment coefficients the values above should be divided by nl_x^2 which is 313344N.



Negative moment coefficients at the support



Positive moment coefficients at mid-span

Fig. 5.6 Variation of moment coefficients along the critical sections of slab S in pattern 1 of case 1 shown in Fig. 5.7

Table 5.11 Field moments and reinforcing moment values in slab S in pattern 2 of case 1 shown in Fig. 5.7

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
211	104.4000	104.4000	-3939.6006	4044.0005	-3835.2007	4044.0005	-3835.2007
212	-395.2610	-2035.9395	-3408.4604	3013.1992	-3803.7217	1372.5210	-5444.4004
213	-1256.0239	-6080.6572	-2392.1724	0.0000	-3648.1963	0.0000	-8472.8301
214	-1820.8547	-9063.1465	-1214.7075	0.0000	-3035.5625	0.0000	-10277.8555
215	-2028.5669	-10115.4336	0.0000	0.0000	-2028.5669	0.0000	-10115.4336
216	-1820.8547	-9063.1465	1214.7073	0.0000	-3035.5620	0.0000	-10277.8555
217	-1256.0442	-6080.6611	2392.1631	0.0000	-3648.2075	0.0000	-8472.8242
218	-395.2610	-2035.9395	3408.4600	3013.1987	-3803.7212	1372.5205	-5444.3994
219	104.4000	104.4000	3939.5996	4043.9995	-3835.1997	4043.9995	-3835.1997
236	-2035.9395	-395.2610	-3408.4609	1372.5215	-5444.4004	3013.1997	-3803.7222
237	580.7999	580.7999	-4722.8311	5303.6309	-4142.0312	5303.6309	-4142.0312
238	1938.1196	869.8801	-3926.2070	5864.3262	-1988.0874	4796.0869	-3056.3271
239	2257.5347	698.0646	-2129.9868	4387.5215	0.0000	2828.0513	-1311.5806
240	2309.3779	581.1821	0.0000	2309.3779	0.0000	581.1821	0.0000
241	2257.5312	698.0685	2129.9873	4387.5186	0.0000	2828.0557	-1311.5808
242	1934.6948	873.3046	3926.6694	5861.3643	-1991.9746	4799.9736	-3053.3652
243	580.7999	580.7999	4722.8301	5303.6299	-4142.0303	5303.6299	-4142.0303
244	-2035.9395	-395.2610	3408.4600	1372.5205	-5444.3994	3013.1987	-3803.7212
261	-6080.6572	-1256.0239	-2392.1724	0.0000	-8472.8301	0.0000	-3648.1963
262	869.8801	1939.1196	-3926.2070	4796.0869	-3056.3271	5864.3262	-1988.0874
263	4315.5596	4315.5596	-3412.4297	7727.9893	0.0000	7727.9893	0.0000
264	5692.0928	5715.1055	-1904.3040	7596.3965	0.0000	7619.4092	0.0000
265	6041.8701	6174.1270	0.0317	6041.9014	0.0000	6174.1582	0.0000
266	5690.2959	5716.9023	1904.3069	7594.6025	0.0000	7621.2090	0.0000
267	4315.5596	4315.5596	3412.4292	7727.9883	0.0000	7727.9883	0.0000
268	869.8801	1938.1196	3926.2061	4796 0859	-3056.3262	5L64.3252	-1988.0864
269	-6080.6533	-1256.0039	2392.1802	0.0000	-8472.8340	0.0000	-3648.1841
286	-9063.1463	-1820.8547	-1214.7075	0.0000	-10277.8555	0.0000	-3035.5625
287	696.1666	2259.4331	-2129.3071	2825.4736	-1310.5088	4388.7402	0.0000
288	5715.1055	5692.0928	-1904.3040	7619.4092	0.0000	7596.3965	0.0000
289	7868.3984	7868.3984	-1076.3645	8944.7617	0.0000	8944.7617	0.0000
290	8457.9765	8615.6211	0.0755	8458.0508	0.0000	8615.6953	0.0000
291	7869.6113	7869.5869	1075.1643	8944.7754	0.0000	8944.7500	0.0000
292	5716.9199	5690.2793	1904.3066	7621.2266	0.0000	7594.5859	0.0000
293	698.2269	2257.6128	2129.8735	2828.1001	-1311.1350	4387.4863	0.0000
294	-9063.1465	-1820.8547	1214.7073	0.0000	-10277.8555	0.0000	-3035.5620
311	-10115.433A	-2028.5669	0.0000	0.0000	-10115.4336	0.0000	-2028.5669
312	581.1621	2309.3779	0.0000	581.1821	0.0000	2309.3779	0.0000
313	6174.0410	6041.9570	0.0000	6174.0410	0.0000	6041.9570	0.0000
314	8615.8223	8457.7754	0.0000	8615.8223	0.0000	8457.7754	0.0000
315	9285.3594	9285.8379	0.0004	9285.3594	0.0000	9285.8379	0.0000
316	8615.8926	8457.7051	0.0000	8615.8926	0.0000	8457.7051	0.0000
317	6174.2139	6041.7842	0.0000	6174.2139	0.0000	6041.7842	0.0000
318	581.6620	2309.3779	0.0000	581.6620	0.0000	2309.3779	0.0000
319	-10115.4336	-2028.5669	0.0000	0.0000	-10115.4336	0.0000	-2028.5669
336	-9063.1465	-1820.8547	1214.7073	0.0000	-10277.8555	0.0000	-3035.5620
337	698.2289	2257.6104	2129.8745	2828.1030	-1311.1372	4387.4844	0.0000
338	5715.7197	5689.0791	1905.5068	7621.2266	0.0000	7594.5859	0.0000
339	7868.3974	7868.3994	1076.3643	8944.7637	0.0000	8944.7637	0.0000
340	8457.7402	8615.8574	·0.0131	8457.7520	0.0000	8615.8691	0.0000
341	7060.3994	7068.3904	-1076.3645	0944.7637	0.0000	0944.7617	0.0000
342	5716.9169	5690.2793	-1904.3071	7621.2256	0.0000	7594.5859	0.0000
343	698.3911	2257.6885	-2129.7627	2828.1538	-1310.6943	4307.4512	0.0000
344	-9063.1465	-1820.8547	-1214.7075	0.0000	-10277.8555	0.0000	-3035.5625
361	-6080.6611	-1256.0442	2392.1631	0.0000	-8472.8242	0.0000	-3648.2075
362	869.8803	1938.1196	3926.2061	4796.0859	-3056.3262	5864.3252	-1988.0864
363	4315.6797	4315.6797	3412.3091	7727.9883	0.0000	7727.9883	0.0000
364	5690.2793	5716.9199	1904.3066	7594.5859	0.0000	7621.2266	0.0000
365	6041.8711	6174.1270	-0.0317	6041.9023	0.0000	6174.1582	0.0000
366	5691.4961	5718.1025	-1903.1072	7594.6025	0.0000	7621.2090	0.0000
367	4315.6797	4315.6797	-3412.3096	7727.9893	0.0000	7727.9893	0.0000
368	869.8800	1939.1196	-3926.2070	4796 0869	-3056.3271	5864.3262	-1988.0874
369	-6080.6572	-1256.0239	-2392.1724	0.0000	-8472.8301	0.0000	-3648.1963
386	-2035.9392	-395.2610	3408.4600	1372.5208	-5444.3994	3013.1987	-3803.7212
387	580.7799	580.7999	4722.8301	5303.6299	-4142.0303	5303.6299	-4142.0303
388	1938.1196	869.8801	3926.2061	5864.3252	-1988.0864	4796.0859	-3056.3262
389	2257.6109	698.2289	2129.8745	4387.4854	0.0000	2828.1030	-1311.1367
390	2309.3779	581.4220	·0.0025	2309.3804	0.0000	581.4244	0.0000
391	2257.6109	698.2289	-2129.8750	4387.4854	0.0000	2828.1035	-1311.1377
392	1934.6948	873.3047	-3926.6704	5861.3652	-1991.9756	4799.9746	-3053.3657
393	580.7999	580.7999	-4722.8311	5303.6309	-4142.0312	5303.6309	-4142.0312
394	-2035.9392	-395.2610	-3409.4604	1372.5212	-5444.4004	3013.1992	-3803.7217
411	104.4000	104.4000	3939.5996	4043.9995	-3835.1997	4043.9995	-3835.1997
412	-395.2610	-2035.9392	3408.4600	3013.1987	-3803.7212	1372.5208	-5444.3994
413	-1256.0239	-6080.6572	2392.1714	0.0000	-3648.1953	0.0000	-8472.8301
414	-1820.8545	-9063.1465	1214.7073	0.0000	-3035.5620	0.0000	-10277.8555
415	-2028.5669	-10115.4336	0.0000	0.0000	-2028.5669	0.0000	-10115.4336
416	-1820.8547	-9063.1465	-1214.7075	0.0000	-3035.5625	0.0000	-10277.8555
417	-1256.0442	-6080.6602	-2392.1636	0.0000	-3648.2080	0.0000	-8472.8242
418	-395.2610	-2035.9392	-3408.4604	3013.1992	-3803.7217	1372.5212	-5444.4004
419	104.4000	104.4000	-3939.6006	4044.0005	-3835.2007	4044.0005	-3835.2007

Table 5.12 Field moments and reinforcing moment values in slab S in pattern 3 of case 1 shown in Fig. 5.7

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
275	103.2000	103.2000	-3576.0005	3679.2002	-3472.8008	3679.2002	-3472.8008
276	-399.3741	-2051.0264	-2822.7793	2423.4048	-3222.1538	771.7529	-4873.8057
277	-1262.9680	-6121.8330	-1686.2024	0.0000	-2949.1704	0.0000	-7808.0361
278	-1839.1509	-9140.8516	-476.8484	0.0000	-2315.9995	0.0000	-9617.7012
279	-2049.3994	-10207.4023	660.6830	0.0000	-2710.0825	0.0000	-10868.0859
280	-1858.9856	-9267.4160	1619.1760	0.0000	-3478.1616	0.0000	-10886.5937
281	-1304.0667	-6321.2139	2185.5342	0.0000	-3489.6011	0.0000	-8506.7480
282	-443.6158	-2256.8643	1958.7930	1256.4729	-2402.4092	0.0000	-4215.6572
283	105.3479	117.6121	0.1190	105.4677	0.0000	117.7319	0.0000
308	-1686.1594	-329.8409	-3080.4644	1394.3049	-4766.6240	2750.6230	-3410.3057
309	753.1725	518.8262	-4195.6348	4948.8076	-3442.4614	4714.4609	-3676.8086
310	1980.9392	647.0604	-3294.3213	5275.2598	-1313.3821	3941.3813	-2647.2612
311	2164.2300	302.7296	-1471.9680	3636.1978	0.0000	1774.6975	-698.4070
312	2023.1353	-1.1355	582.8944	2606.0293	0.0000	581.7588	-169.0758
313	1658.7676	-60.3680	2472.9790	4131.7461	-814.2114	2412.6108	-2533.3472
314	810.8180	-30.8182	3723.8711	4534.6885	-2913.0532	3693.0527	-3754.6895
315	-1373.2263	-352.3721	3436.0215	2062.7930	-4809.2500	3083.6494	-3788.3936
316	-6167.6367	-1253.1636	0.0000	0.0000	-6167.6367	0.0000	-1253.1636
341	-5449.2285	-1128.1880	-2142.4146	0.0000	-7591.6436	0.0000	-3270.6025
342	1186.0759	1840.3237	-3528.4419	4714.5176	-2342.3662	5368.7656	-1688.1182
343	4397.9385	3951.6606	-2937.5161	7335.4541	0.0000	6889.1768	0.0000
344	5529.3193	5059.4795	-1410.6873	6940.0059	0.0000	6470.1660	0.0000
345	5534.5771	5215.0215	434.8816	5969.4580	0.0000	5649.9023	0.0000
346	4620.5742	4463.4248	2150.1094	6770.6836	0.0000	6613.5342	0.0000
347	2291.6655	2025.1338	3245.8335	5537.4990	-954.1680	6070.9668	-420.6997
348	-2726.1187	362.1179	2952.2290	226.1104	-5678.3477	3314.3467	-2590.1113
349	-13093.1738	-2674.8276	0.0000	0.0000	-13093.1738	0.0000	-2674.8276
374	-8245.2539	-1661.9465	-1085.1758	0.0000	-9330.4297	0.0000	-2747.1226
375	1111.1973	2136.4824	-1918.2158	3029.4131	-611.0503	4054.6982	-0.0000
376	5824.4434	5232.3545	-1649.8120	7474.2549	0.0000	6882.1660	0.0000
377	7664.1963	7045.4014	-811.5668	8475.7617	0.0000	7856.9678	0.0000
378	7799.2949	7404.7031	230.8149	8030.1094	0.0000	7635.5176	0.0000
379	6482.1934	6295.4053	1205.8479	7688.0410	0.0000	7501.2529	0.0000
380	3091.5942	3927.6050	1804.6599	4896.2539	0.0000	5632.2646	0.0000
381	-3968.0449	318.1244	1602.6772	0.0000	-5570.7227	965.4392	-1284.5530
382	-17649.2500	-3533.1543	0.0000	0.0000	-17649.2500	0.0000	-3533.1543
407	-9220.5352	-1047.0667	0.0000	0.0000	-9220.5352	0.0000	-1049.0669
408	1028.5107	2173.0889	0.0000	1028.5107	0.0000	2173.0889	0.0000
409	6289.7197	5551.8789	-0.3074	6290.0264	0.0000	5552.1855	0.0000
410	8389.0332	7570.9648	0.0000	8389.0332	0.0000	7570.9648	0.0000
411	8577.3301	7987.4678	-0.2589	8577.5879	0.0000	7987.7266	0.0000
412	7114.3770	6767.2217	0.1432	7114.5195	0.0000	6767.3643	0.0000
413	3332.1802	4052.6191	0.0293	3332.2095	0.0000	4052.6484	0.0000
414	-4446.2002	256.2796	0.0000	0.0000	-4446.2002	256.2796	0.0000
415	-19184.8281	-3835.9741	0.0000	0.0000	-19184.8281	0.0000	-3835.9741
440	-8245.2539	-1661.9465	1085.1753	0.0000	-9330.4297	0.0000	-2747.1221
441	1111.1973	2136.4824	1918.2153	3029.4126	-611.0493	4054.6978	0.0000
442	5824.4434	5232.3555	1649.8118	7474.2549	0.0000	6882.1670	0.0000
443	7663.0420	7044.1562	812.9108	8475.7512	0.0000	7857.0664	0.0000
444	7799.2744	7404.7236	-230.9013	8030.2549	0.0000	7635.7041	0.0000
445	6482.1934	6295.4053	-1205.8481	7688.0410	0.0000	7501.2529	0.0000
446	3091.5942	3827.6050	-1804.6604	4896.2539	0.0000	5632.2646	0.0000
447	-3968.0449	318.1244	-1602.6777	0.0000	-5570.7227	965.4396	-1284.5535
448	-17649.2500	-3533.1538	0.0000	0.0000	-17649.2500	0.0000	-3533.1538
473	-5449.2285	-1128.1880	2142.4141	0.0000	-7591.6426	0.0000	-3270.6021
474	1186.0762	1840.3235	3528.4409	4714.5166	-2342.3647	5368.7637	-1688.1174
475	4397.9385	3951.6606	2937.5156	7335.4541	0.0000	6889.1758	0.0000
476	5529.3193	5059.4795	1410.6868	6940.0059	0.0000	6470.1660	0.0000
477	5534.2656	5215.3320	-435.0067	5969.2715	0.0000	5650.3379	0.0000
478	4620.5742	4463.4248	-2150.1099	6770.6836	0.0000	6613.5342	0.0000
479	2291.7949	2025.2441	-3245.7148	5537.5098	-953.9199	6070.9590	-420.4707
480	-2726.1182	362.1179	-2952.2300	226.1118	-5678.3486	3314.3477	-2590.1123
481	-13093.1738	-2674.8276	0.0000	0.0000	-13093.1738	0.0000	-2674.8276
506	-1686.1594	-329.8408	3080.4634	1394.3040	-4766.6230	2750.6226	-3410.3042
507	753.1736	518.8262	4195.6338	4948.8066	-3442.4604	4714.4600	-3676.8076
508	1980.9392	647.0604	3294.3203	5275.2588	-1313.3811	3941.3804	-2647.2603
509	2164.2803	302.9187	1471.8694	3636.1499	0.0000	1774.7881	-698.0602
510	2023.1550	-0.7952	-582.8132	2605.9683	0.0000	582.0179	-168.6871
511	1658.7676	-60.3680	-2472.9795	4131.7471	-814.2119	2412.6113	-2533.3477
512	810.8181	-30.8182	-3723.8716	4534.6895	-2913.0537	3693.0532	-3754.6899
513	-1373.2280	-352.3721	-3436.0220	2062.7939	-4809.2500	3083.6499	-3788.3940
514	-6167.6367	-1253.1636	0.0000	0.0000	-6167.6367	0.0000	-1253.1636
539	103.2000	103.2000	3576.0000	3679.1997	-3472.8003	3679.1997	-3472.8003
540	-399.3741	-2051.0264	2822.7788	2423.4043	-3222.1533	771.7524	-4873.8057
541	-1262.9680	-6121.8330	1686.2019	0.0000	-2949.1699	0.0000	-7808.0352
542	-1839.1509	-9140.8496	476.8483	0.0000	-2315.9995	0.0000	-9617.6992
543	-2049.3999	-10207.4023	-660.6831	0.0000	-2710.0830	0.0000	-10868.0859
544	-1858.9856	-9267.4160	-1619.1765	0.0000	-3478.1621	0.0000	-10886.5937
545	-1304.2776	-6321.2432	-2185.4561	0.0000	-3489.7339	0.0000	-8506.6992
546	-443.6158	-2256.8647	-1958.7930	1256.4727	-2402.4092	0.0000	-4215.6582
547	105.3479	117.6121	-0.1198	105.4677	0.0000	117.7319	0.0000

Table 5.13 Field moments and reinforcing moment values in slab S in pattern 4 of case 1 shown in Fig. 5.7

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
211	102.0000	102.0000	-3594.0000	3696.0000	-3492.0000	3696.0000	-3492.0000
212	-399.7905	-2036.2100	-2850.0957	2450.3052	-3249.8862	813.8857	-4886.3057
213	-1256.9976	-6088.9238	-1718.0847	0.0000	-2975.0825	0.0000	-7807.0088
214	-1826.9021	-9078.6992	-505.9677	0.0000	-2332.8701	0.0000	-9584.6680
215	-2031.5149	-10114.8867	632.8789	0.0000	-2664.3940	0.0000	-10747.7656
216	-1828.4175	-9098.7832	1600.0112	0.0000	-3428.4287	0.0000	-10698.7949
217	-1264.5586	-6138.4824	2198.9146	0.0000	-3463.4731	0.0000	-8337.3984
218	-408.9124	-2096.6880	2019.6438	1536.5181	-2428.5566	0.0000	-4116.3320
219	104.3451	117.1749	134.7414	239.0865	-30.3963	251.9163	-17.5665
236	-1704.8423	-327.9579	-3089.2983	1384.4561	-4794.1406	2761.3403	-3417.2563
237	749.6187	529.5812	-4211.4258	4961.0439	-3461.8071	4741.0068	-3681.8447
238	1990.5088	675.8910	-3306.0449	5296.5537	-1315.5361	3981.9355	-2630.1943
239	2179.7925	349.0871	-1481.8738	3661.6660	0.0000	1830.9607	-658.3253
240	2047.1140	64.2207	581.7512	2628.8652	0.0000	645.9719	-101.1020
241	1696.7021	12.0975	2478.9126	4175.6143	-782.2104	2491.0098	-2466.8154
242	854.8585	42.7413	3738.8022	4593.6602	-2883.9438	3781.5435	-3696.0610
243	-1348.5920	-293.0081	3449.1328	2100.5405	-4797.7256	3156.1245	-3742.1411
244	-6096.4971	-1235.5044	6.4300	0.0000	-6102.9277	0.0000	-1241.9346
261	-5467.8154	-1131.0103	-2146.5459	0.0000	-7614.3613	0.0000	-3277.5562
262	1182.5713	1843.8284	-3532.9370	4715.5078	-2350.3657	5376.7646	-1689.1086
263	4403.8926	3967.3057	-2941.4805	7345.3730	0.0000	6908.7861	0.0000
264	5546.8916	5085.1064	-1408.9487	6955.8398	0.0000	6494.0547	0.0000
265	5560.6729	5244.1250	439.4177	6000.0898	0.0000	5683.5420	0.0000
266	4655.2979	4493.5010	2163.2388	6818.5361	0.0000	6656.7393	0.0000
267	2314 80 6	2850.2295	3257.6733	5572.4824	-942.8638	6107.9023	-407.4438
268	-2724.5620	372.5618	2954.0688	229.5068	-5678.6309	3326.6304	-2581.5073
269	-13093.3125	-2674.6895	-54.7003	0.0000	-13148.0137	0.0000	-2729.3901
286	-8266.3359	-1662.4651	-1082.1208	0.0000	-9348.4570	0.0000	-2744.5859
287	1106.0625	2138.7368	-1918.7205	3024.7827	-615.2751	4057.4570	0.0000
288	5836.9297	5239.0703	-1649.2688	7486.1982	0.0000	6888.3389	0.0000
289	7684.4258	7053.9717	-811.5795	8496.0039	0.0000	7865.5508	0.0000
290	7832.5469	7414.6523	237.2173	8069.7637	0.0000	7651.8691	0.0000
291	6513.2021	6297.9971	1211.8770	7725.0791	0.0000	7509.8740	0.0000
292	3105.7251	3815.8740	1813.3999	4919.1250	0.0000	5629.2734	0.0000
293	-3994.7295	294.6492	1598.2856	0.0000	-5593.0156	934.1210	-1303.6365
294	-17744.1562	-3558.2476	-49.5190	0.0000	-17793.6758	0.0000	-3607.7666
311	-9247.7129	-1854.6880	0.0000	0.0000	-9247.7129	0.0000	-1854.6880
312	1023.5283	2178.0713	0.0000	1023.5283	0.0000	2178.0713	0.0000
313	6299.2871	5551.9111	0.0000	6299.2871	0.0000	5551.9111	0.0000
314	8413.6270	7575.1709	-0.3595	8413.9863	0.0000	7575.5303	0.0000
315	8609.5078	7993.6904	0.0000	8609.5078	0.0000	7993.6904	0.0000
316	7144.9180	6760.6797	0.0000	7144.9180	0.0000	6760.6797	0.0000
317	3346.8911	4030.7085	0.0000	3346.8911	0.0000	4030.7085	0.0000
318	-4488.6221	217.5812	0.0000	0.0000	-4488.6221	217.5812	0.0000
319	-19376.6797	-3874.5254	0.0000	0.0000	-19376.6797	0.0000	-3874.5254
336	-8266.3379	-1662.4651	1082.1206	0.0000	-9348.4590	0.0000	-2744.5859
337	1106.0625	2138.7368	1918.7200	3024.7822	-615.2742	4057.4565	0.0000
338	5836.9287	5239.0693	1649.2683	7486.1963	0.0000	6888.3369	0.0000
339	7683.9375	7054.4609	811.8556	8495.7930	0.0000	7866.3164	0.0000
340	7832.5469	7414.6514	-237.2173	8069.7637	0.0000	7651.8682	0.0000
341	6513.2021	6297.9971	-1211.8774	7725.0791	0.0000	7509.8740	0.0000
342	3104 2490	3017.3501	-1813.0730	4917.3213	0.0000	5630.4229	0.0000
343	-3994.7295	294.6492	-1598.2861	0.0000	-5593.0156	934.1213	-1303.6370
344	-17744.1562	-3558.2476	49.5190	0.0000	-17793.6758	0.0000	-3607.7666
361	-5467.8193	-1131.0303	2146.5361	0.0000	-7614.3555	0.0000	-3277.5664
362	1182.5713	1843.8284	3532.9360	4715.5068	-2350.3647	5376.7637	-1689.1077
363	4405.0137	3968.5854	2940.2822	7345.2959	0.0000	6908.8672	0.0000
364	5546.8916	5085.1074	1408.9482	6955.8398	0.0000	6494.0557	0.0000
365	5559.5625	5242.8359	-440.7280	6000.2900	0.0000	5683.5635	0.0000
366	4653.4639	4495.3350	-2163.3008	6816.7646	0.0000	6658.6357	0.0000
367	2314.9390	2850.3403	-3257.5547	5572.4932	-942.6157	6107.8945	-407.2144
368	-2724.5620	372.5617	-2954.0698	229.5078	-5678.6318	3326.6313	-2581.5083
369	-13093.3105	-2674.6895	54.7003	0.0000	-13148.0117	0.0000	-2729.3901
386	-1704.8423	-327.9579	3089.2979	1384.4556	-4794.1406	2761.3398	-3417.2559
387	749.6185	529.5812	4211.4248	4961.0430	-3461.8066	4741.0059	-3681.8437
398	1990.5085	675.8910	3306.0444	5296.5527	-1315.5359	3981.9351	-2630.1538
389	2179.9585	349.6409	1481.5703	3661.5288	0.0000	1831.2112	-657.2822
390	2047.1145	64.2276	-581.7499	2628.8643	0.0000	645.9774	-101.0943
391	1696.7024	12.0975	-2478.9131	4175.6152	-782.2107	2491.0103	-2466.8159
392	854.8585	42.7413	-3738.8032	4593.6611	-2883.9448	3781.5444	-3696.0620
393	-1348.5920	-293.0081	-3449.1338	2100.5415	-4797.7266	3156.1255	-3742.1421
394	-6096.4971	-1235.5042	-6.4300	0.0000	-6102.9277	0.0000	-1241.9343
411	102.0000	102.0000	3594.0000	3696.0000	-3492.0000	3696.0000	-3492.0000
412	-399.7905	-2036.2100	2850.0952	2450.3047	-3249.8857	813.8853	-4886.3057
413	-1256.9976	-6088.9229	1718.0842	0.0000	-2975.0820	0.0000	-7807.0078
414	-1826.9023	-9078.6992	505.9676	0.0000	-2332.8701	0.0000	-9584.6680
415	-2031.5149	-10114.8867	-632.8792	0.0000	-2664.3940	0.0000	-10747.7676
416	-1828.4172	-9098.7832	-1600.0117	0.0000	-3428.4292	0.0000	-10698.7949
417	-1264.5586	-6138.4824	-2198.9150	0.0000	-3463.4736	0.0000	-8337.3984
418	-408.9124	-2096.6880	-2019.6443	1536.5190	-2428.5571	0.0000	-4116.3330
419	104.3451	117.1749	-134.7414	239.0865	-30.3963	251.9163	-17.5665

the dead load which will of course limit the increase relative to a uniform load before it has to be taken into account.

Comparisons of these maximum elastic values with the British code is somewhat pointless since the British coefficients are based on yield-line analysis. It might however be noted that for the four patterns the ratios of negative to positive moment coefficients are 2.29, 1.11, 2.23 and 2.24 whilst the chosen yield-line ratio is 1.33 indicating a considerable allowance for redistribution, indeed perhaps even an excessive amount and the consequences of this are discussed later when considering serviceability.

The ACI code is however based on Marcus' quasi-elastic technique which Rogers states gives almost an exact solution. It would not be unreasonable therefore to compare the two sets of elastic values. For case 1 the ACI negative coefficient is 0.045 but the worst value in Table 5.14a is 0.0618, which is 37% larger but in reality because of its position, namely next to an edge panel, the 0.045 coefficient should be reduced to 0.0395 since the negative coefficient for the edge panel common edge is 0.033. Thus the actual ratio would be 56% higher. Conversely the positive moment coefficient allowing for the live load ratio would initially be 0.0243 compared with the maximum value of 0.0296, i.e. 21% higher, but after redistributon of the negative moment the design positive coefficient would rise to 0.0314 which is actually higher than the actual value. If the initial panel coefficient values are assumed and the larger value negative moment taken then these are closer to the actual maximum values and certainly better than the British code which might be expected since it is not based on elastic values.

As an interesting guide the finite element elastic values were measured and averaged over the centre three-quarters of the panel both for the fully loaded case and the worst loading case and the values are given in Table 5.32 in columns 3 and 6 respectively. The moment coefficient values after taking into account the two different negative values at the common edge are given in columns 1 and 2 of this Table. If the values are compared with the average for the fully loaded case they exceed the value as indicated by the ratios in brackets. However for the worst case of patterned loading they are insufficient as can be seen from the bracketted figures in columns 4 and 5 where they are less than 1. It is interesting to note that the common negative coefficients for the British code are 0.031 for the centre panel and 0.039 for the edge panel, thus the British code value increases for panel S if the moments are redistributed. Conversely the ACI values are 0.045 and 0.033 which causes a decrease. If therefore a rule were instituted that at a common boundary the larger of the two negative moments be taken the British code value would be 0.039 compared with the worst average of 0.043 and the ACI value would be 0.045 which is slightly larger than the worst average value.

For the positive moments redistribution decreases the value while the ACI value increases. If the values were to remain as given in the code the British value would be 0.024 and the ACI value 0.0242 which compares with the worst value of 0.0196. For this case it would therefore seem that this would be a sensible rule to incorporate.

In this comparison it is the average elastic moment over the middle three-quarters that is being examined. Table 5.14a shows that the maximum values are 0.0618 for the negative moment and 0.0296 for the positive moment which are well above the average values found for the full, loaded case. Indeed since at present redistribution is permitted it is worth examining the possibility that the yield might be exceeded at the serviceability condition. The finite element analysis was not carried out for dead and live load only but the coefficients can be obtained from the previous results.

Pattern 1 is for the fully loaded case for which the maximum negative moment coefficient is 0.0508 and the equivalent moment will be 0.0508 x 3.4 DL^2 where D is the dead load. If we find the worst other maximum negative coefficient which is case 4 and call this coefficient 4 shortened to C_4, this represents a moment of C_4 x 3.4 DL^2. We can therefore deduct the dead load moment from that to find that due to 0.4D + 1.6 x 1.25D.

Therefore 3.4 DL^2 (C_4max - C_1max/3.4) represents 1.92 x live load.

The moment coefficient due to dead + live load expressed in terms of 3.4 DL^2 will therefore be

$$\left[\frac{3.4}{1.92}(C_4\max - \frac{C_1\max}{3.4}) + C_1\max\right] \div 3.4$$

$$= \frac{1}{1.92}(C_4\max + \frac{0.92}{3.4} \times C_1\max)$$

which for C_1 of 0.0508 and C_4 of 0.0618 from Table 5.14a gives 0.0393.

It should be noted this is a coefficient of the full load and that it is higher than the redistributed negative British coefficient, the ratio being 0.9. The material factor of 0.87 has not been taken into account but this strictly is an allowance on the materials. What this means is that if the ratio of code coefficient/service load coefficient falls below 0.87 (assuming full strength material) then the steel will yield at the service load. This assumption of course presumes that the section behaves elastically up to the steel yield condition which is not strictly true. In reality the concrete stress strain curve is not linear and therefore some redistribution of moments will actually take place. The ratio of the code coefficient/service load coefficient has been calculated for both negative and positive moments for the British and ACI codes and is given in Table 5.34. It can be observed that BS8110 comes close to yield at the supports for this condition, indicating yet again since the positive ratio is higher, that the 4/3 ratio of negative to positive steel is too low.

5.7.2 Other cases 2 - 9; aspect ratio 1

Similar finite element analyses were carried out for all the other cases and the results are given in Figures 5.8 - 5.15 and Tables 5.15 to 5.22a and b.

The same conclusions can be made concerning loading patterns, namely that the worst negative coefficient occurs when the slabs on either side of the common boundary are loaded and the worst positive moment when the adjoining slabs are unloaded.

In every case involving negative and positive moments the ratio of worst negative to positive is of the order of 2.25.

The code values in the north-south direction (assumed to be the short span) are summarised after allowing for the redistribution of the negative moment in Table 5.32. These in turn have been divided by the average coefficient from the finite element analysis for the fully loaded case and the worst pattern loading case. As might be expected in all cases the ratio is equal to or worse for pattern loading. The negative moment value in cases 6 - 9 is interesting where the support is a simple one which shows the effect of the twisting moment requiring negative steel.

Again as for case 1 the redistribution of the negative moment coefficient often reduces the value to be taken at a support. If the higher value is taken then in most cases with the ACI code, which is the only reasonable comparison, taking the higher value would ensure the code coefficient is closer to the worst finite element value. The ratio of the code value to service load coefficient value is given in Table 5.34. It can be observed for the British code that the negative moment cases 1 - 4 in particular are extremely low indicating that yield is almost occurring at the service load. In all cases no such problem occurs with the positive moments. An exceptionally low value occurs in the ACI code for the negative moment in case 3 and this clearly requires revision.

5.7.3 Cases 1 - 9; aspect ratio 1:2

Finite element analyses were carried out for all 9 cases with slabs of aspect ratio 1:2 and the main results summarised in Figure 5.16 - 24 and Tables 5.23 - 5.31a and b.

These results again have been compared with the average finite element values for the fully loaded case and worst patterned case in Table 5.33 and the serviceability ratios compared in Table 5.35.

The pattern that emerges is generally quite similar to the analysis for slabs with an aspect ratio of 1:1 except for the British code for case 1 and 2 where the positive steel is quite low at the serviceability condition. This however is not a feature of the original coefficient but because due to redistribution of the negative moment for case 1, for example the positive coefficient has been reduced from 0.048 to 0.0332. This is a further example of allowing the higher value of the moment to be retained and not to redistribute the negative moment but in this case it has a bad effect on the positive moment.

5.7.4 Failure condition

Since the British code is based on yield-line analysis clearly the coefficients should satisfy the failure conditions. The ACI code is supposedly based on quasi-elastic values though the finite element check has shown that Marcus' values certainly do not reach the worst elastic distribution nor indeed in some cases the uniform loading case.

Table 5.36 shows the moment coefficients for both codes in the short direction on the assumption that the live load is 1.25 the dead since this influences the positive moment coefficients. The 1:1 aspect cases are considered first. One might suspect difficulty satisfying the ultimate load condition where the ACI values are less than the British ones. For case 3 both the ACI values are less than the British but the moments in the east-west direction are those for case 2 where the negative coefficient for the ACI code is much higher. Again case 6 is compensated by case 5. Case 8 is low but case 7 probably just compensates. For case 9 however the value is too low and with simple supports there is no compensation. For a square slab, since torsion steel is included the yield-line solution is $wL^2/24$, i.e. the coefficient is 0.0416 compared with 0.0373. The ACI value is worsened since this value is only effectively over 5L/6 reducing the net coefficient to 0.031. If the factor on bending of 0.9 is introduced the value increases to 0.0345 but this is only 83% of what is needed. Thus for this case the ACI code would cause failure at a lesser load than the factored load. Case 9 for the 1:2 ratio is also on the borderline. The failure to meet the ultimate condition is serious and needs correction and it is further suggested that cases 5 - 8 also need checking.

5.8 Conclusions

(a) The finite element analysis confirms the well established practice with beams that the highest negative moment at a support occurs when the two adjacent panels are loaded and the maximum positive moment when the panel itself is loaded.

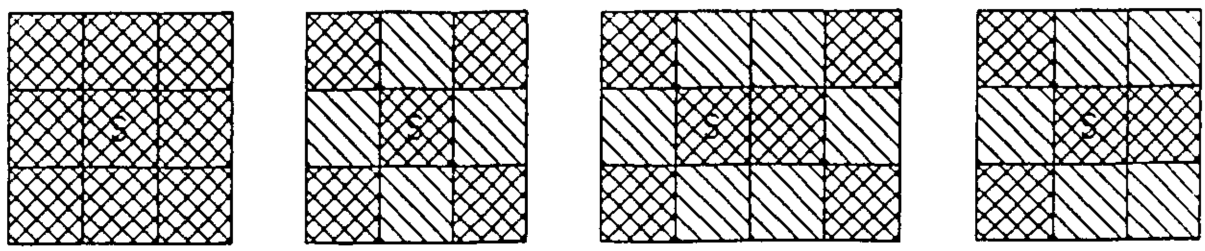
(b) Since all the code bending moment coefficients are less than the worst values found both for uniform loading and patterned loading it is recommended that no redistribution of the two different negative coefficients at a boundary are redistributed since this practice makes one of the values even worse. It is suggested the higher value is taken and no distribution carried out.

(c) Because BS8110 is based on yield-line analysis with the negative/positive moments always set at the ratio 4/3 whilst the elastic ratio is of the order of 2.25 then in support cases 1 - 4 the negative steel is almost at the yield at the serviceability condition. A higher moment at the centre of the support is required. This could be achieved by slightly increasing the ratio to 5 say or alternatively since minimum steel is always required in the edge zones by increasing the centre value and having say half this amount in the edge zones.

(d) In the ACI code for case 3 the negative moment coefficient seems to have a low value so that the steel is in danger of yielding at the serviceability condition. This needs revising.

(e) Whilst all the BS8110 values are safe for the ultimate condition in the ACI code case 9 in particular, namely simply supported slabs are unsafe at the ultimate condition. Cases 7 and 8 also need checking over the whole range of aspect ratios since they also appear to be on the borderline for safety at the ultimate condition.

(f) Both codes are relatively easy to use and the total steel required is of the same order. It is however recommended where there are two different negative moment coefficients at a support that the higher value is used and that the difference is not redistributed.



Pattern: (1) (2) (3) (4)

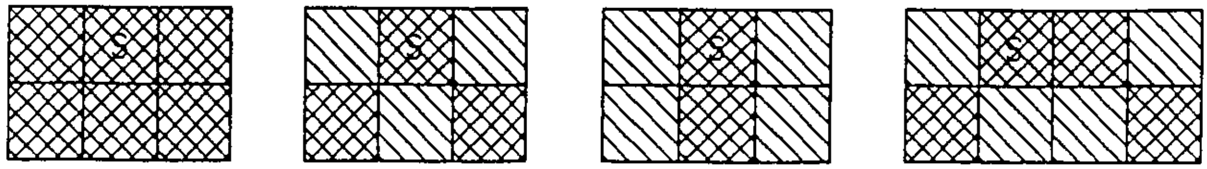
Fig. 5.7 Case 1: Loading patterns for maximum moments considered in case of interior panel, $L_y/L_x = 1.0$

Table 5.14a Maximum elastic moment coefficients at critical sections of Fig. 5.7

Pattern No	W (West)⟶E (East)		S (South)⟶N (North)	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0508 (W & E)	0.0221	0.0508 (S & N)	0.0221
2	0.0328 (W & E)	0.0296	0.0328 (S & N)	0.0296
3	0.0612 (E)	0.0274	0.0347 (S & N)	0.0255
4	0.0618 (E)	0.0275	0.0343 (S & N)	0.0255

Table 5.14b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W (West)⟶E (East)		S (South)⟶N (North)	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0324 (W & E)	0.0127	0.0324 (S & N)	0.0127
2	0.0267 (W & E)	0.0196	0.0267 (S & N)	0.0196
3	0.0353 (E)	0.0191	0.0259 (S & N)	0.0157
4	0.0430 (E)	(0.0192)	0.0256 (S & N)	0.0156



Pattern: (1) (2) (3) (4)

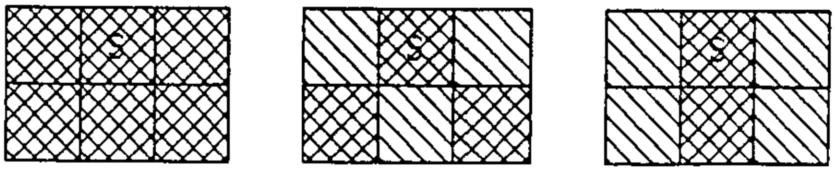
Fig. 5.8 Case 2: Loading patterns for maximum moments considered in case of one short edge discontinuous, $L_y/L_x = 1.0$

Table 5.15a Maximum elastic moment coefficients at critical sections of Fig. 5.8

Pattern No	W ⟶ E		S ⟶ N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0633 (W)	0.0271	0.0520 (S)	0.0212
2	0.0452 (W)	0.0332	0.0348 (S)	0.0293
3	0.0429 (W)	0.0288	0.0624 (S)	0.0269
4	0.0422 (W)	0.0311	0.0368 (S)	0.0254

Table 5.15b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W ⟶ E		S ⟶ N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	(0.0405) (W)	(0.0183)	0.0327 (S)	0.0126
2	0.0307 (W)	0.0234	0.0282 (S)	0.0200
3	0.0296 (W)	0.0190	0.0429 (S)	0.0194
4	0.0311 (W)	0.0229	0.0275 (S)	0.0161



Pattern: (1) (2) (3)

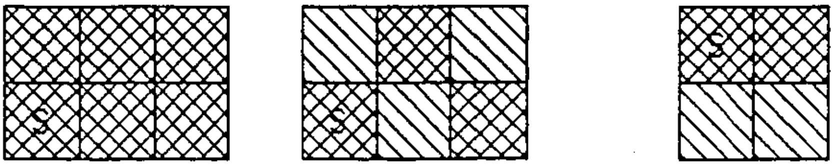
Fig. 5.9 Case 3: Loading patterns for maximum moments considered in case of one long edge discontinuous, $L_y/L_x = 1.0$

Table 5.16a Maximum elastic moment coefficients at critical sections of Fig. 5.9

Pattern No	W ⟶ E		S ⟶ N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0633 (W)	0.0271	0.0520 (S)	0.0212
2	0.0452 (W)	0.0332	0.0348 (S)	0.0293
3	0.0429 (W)	0.0288	0.0624 (S)	0.0269

Table 5.16b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W ⟶ E		S ⟶ N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0405 (W)	0.0183	0.0327 (S)	0.0126
2	0.0307 (W)	0.0234	0.0282 (S)	0.0200
3	0.0296 (W)	0.0190	(0.0429) (S)	(0.0194)



Pattern: (1) (2) (3)

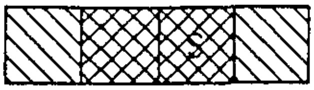
Fig. 5.10 Case 4: Loading patterns for maximum moments considered in case of two adjacent edges discontinuous, $L_y/L_x = 1.0$

Table 5.17a Maximum elastic moment coefficients at critical sections of Fig. 5.10

Pattern No	W → E		S → N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0633 (E)	0.0306	0.0680 (N)	0.0308
2	0.0458 (E)	0.0351	0.0485 (N)	0.0354
3	0.0720 (E)	0.0329	0.0449 (S)	0.0310

Table 5.17b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W → E		S → N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0454 (E)	0.0214	0.0479 (N)	0.0220
2	0.0366 (E)	0.0256	0.0381 (N)	0.0261
3	(0.0524) (E)	0.0249	0.0353 (S)	0.0211



Pattern: (1) (2) (3)

(4)

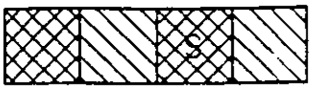
Fig. 5.11 Case 5: Loading patterns for maximum moments considered in case of two short edges discontinuous, $L_y/L_x = 1.0$

Table 5.18a Maximum elastic moment coefficients at critical sections of Fig. 5.11

Pattern No	W ⟶ E		S ⟶ N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0759 (E)	0.0327	0.0143 (S)	0.0212
2	0.0493 (E)	0.0366	0.0195 (S)	0.0292
3	0.0732 (W)	0.0345	0.0173 (S)	0.0250
4	0.0784 (E)	0.0344	0.0173 (S)	0.0246

Table 5.18b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W ⟶ E		S ⟶ N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0598 (E)	0.0248	0.0099 (S)	0.0112
2	0.0424 (E)	0.0276	0.0128 (S)	0.0184
3	0.0562 (W)	0.0269	0.0114 (S)	0.0145
4	(0.061) (E)	0.0270	0.0113 (S)	0.0143



Pattern: (1) (2)

Fig. 5.12 Case 6: Loading patterns for maximum moments considered in case of two long edges discontinuous, $L_y/L_x = 1.0$

Table 5.19a Maximum elastic moment coefficients at critical sections of Fig. 5.12 -

Pattern No	W ⟶ E		S ⟶ N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0759 (E)	0.0327	0.0143 (S & N)	0.0212
2	0.0493 (E)	0.0366	0.0195 (S & N)	0.0292

Table 5.19b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W ⟶ E		S ⟶ N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0598 (E)	0.0248	0.0099 (S & N)	0.0112
2	0.0424 (E)	0.0276	(0.0128) (S & N)	0.0184

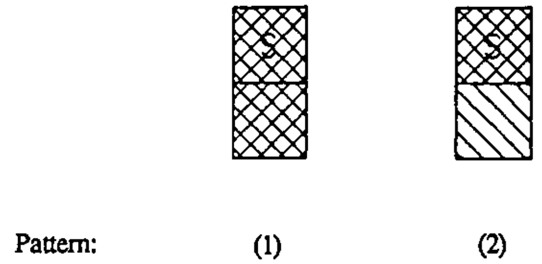


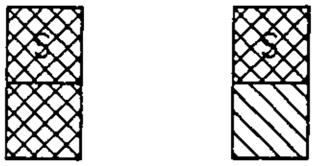
Fig. 5.13 Case 7: Loading patterns for maximum moments considered in case of three edges discontinuous (one long edge continuous), $L_y/L_x = 1.0$

Table 5.20a Maximum elastic moment coefficients at critical sections of Fig. 5.13

Pattern No	W ⟶ E		S ⟶ N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0251 (W & E)	0.0320	0.0830 (S)	0.0379
2	0.0273 (W & E)	0.0360	0.0542 (S)	0.0402

Table 5.20b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W ⟶ E		S ⟶ N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0142 (W & E)	0.0206	(0.0631) (S)	0.0314
2	0.0156 (W & E)	0.0248	0.0471 (S)	0.0321



Pattern: (1) (2)

Fig. 5.14 Case 8: Loading patterns for maximum moments considered in case of three edges discontinuous (one short edge continuous), $L_y/L_x = 1.0$

Table 5.21a Maximum elastic moment coefficients at critical sections of Fig. 5.14

Pattern No	W ⟶ E		S ⟶ N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0251 (W & E)	0.0320	0.0830 (S)	0.0379
2	0.0273 (W & E)	0.0360	0.0542 (S)	0.0402

Table 5.21b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W ⟶ E		S ⟶ N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0142 (W & E)	0.0206	(0.0631) (S)	0.0314
2	(0.0156) (W & E)	(0.0248)	0.0471 (S)	0.0321

Pattern: (1) 

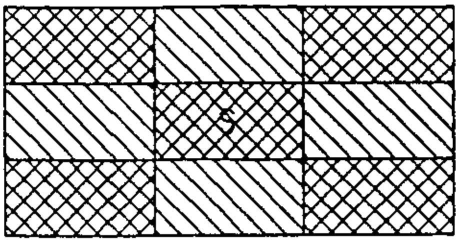
Fig. 5.15 Case 9: Loading patterns for maximum moments considered in case of four edges discontinuous, $L_y/L_x = 1.0$

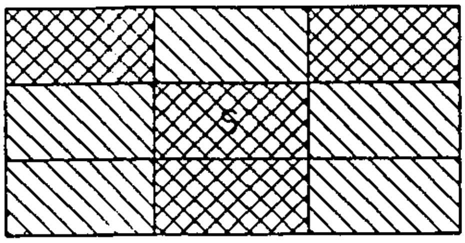
Table 5.22a Maximum elastic moment coefficients at critical sections of Fig. 5.15

Pattern No	W ⟶ E		S ⟶ N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0317 (W & E)	0.0452	0.0317 (S & N)	0.0452

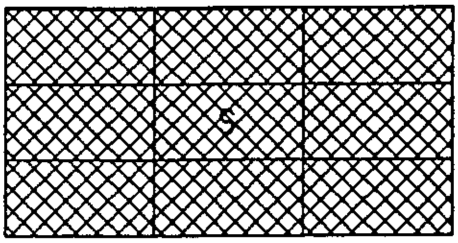
Table 5.22b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W ⟶ E		S ⟶ N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0184 (W & E)	0.0334	0.0184 (S & N))	0.0334





Pattern: (1) (2)



(3)

Fig. 5.16 Case 1: Loading patterns for maximum moments considered in case of interior panel, $L_y/L_x = 2.0$

Table 5.23a Maximum elastic moment coefficients at critical sections of Fig. 5.16

Pattern No	W ⟶ E		S ⟶ N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0322 (W & E)	0.0177	0.0611 (S & N)	0.0582
2	0.0331 (W & E)	0.0123	0.1057 (S)	0.0469
3	0.0465 (W & E)	0.0075	0.0958 (S & N)	0.0362

Table 5.23b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W ⟶ E		S ⟶ N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0272 (W & E)	0.0097	0.0524 (S & N))	0.0467
2	0.0245 (W & E)	0.0061	(0.0876) (S)	(0.0394)
3	0.0279 (W & E)	0.0028	0.0958 (S & N)	0.0288

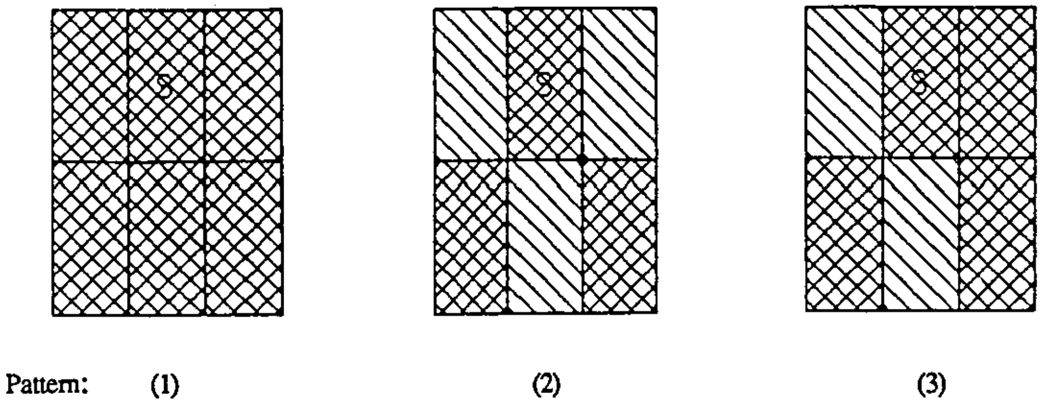


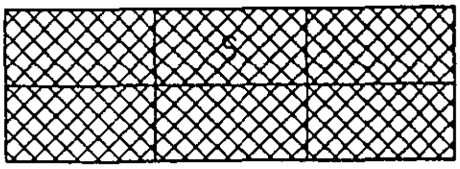
Fig. 5.17 Case 2: Loading patterns for maximum moments considered in case of one short edge discontinuous, $L_y/L_x = 2.0$

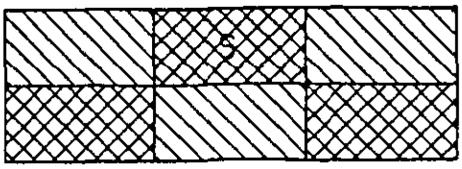
Table 5.24a Maximum elastic moment coefficients at critical sections of Fig. 5.17

Pattern No	W ⟶ E		S ⟶ N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0982 (W)	0.0350	0.0462 (S)	0.0064
2	0.0644 (W)	0.0576	0.0319 (S)	0.0170
3	0.1075 (E)	0.0471	0.0254 (S)	0.0116

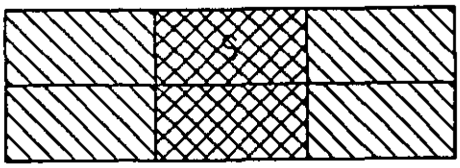
Table 5.24b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W ⟶ E		S ⟶ N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0844 (W)	0.0297	0.0276 (S)	0.0023
2	0.0563 (W)	0.0474	0.0270 (S)	0.0094
3	(0.0906) (E)	(0.0409)	0.0203 (S)	0.0057





Pattern (1) (2)



(3)

Fig. 5.18 Case 3: Loading patterns for maximum moments considered in case of one long edge discontinuous, $L_y/L_x = 2.0$

Table 5.25a Maximum elastic moment coefficients at critical sections of Fig. 5.18

Pattern No	W ⟶ E		S ⟶ N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0787 (W & E)	0.0222	0.1137 (S)	0.0571
2	0.0565 (W & E))	0.0264	0.0739 (S)	0.0720
3	0.0524 (W & E)	0.0212	0.1158 (S)	0.0585

Table 5.25b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W ⟶ E		S ⟶ N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0538 (W & E))	0.0134	0.0865 (S)	0.0451
2	0.0440 (W & E))	0.0175	0.0625 (S)	0.0574
3	0.0405 (W & E)	0.0127	(0.0920) (S)	(0.0481)

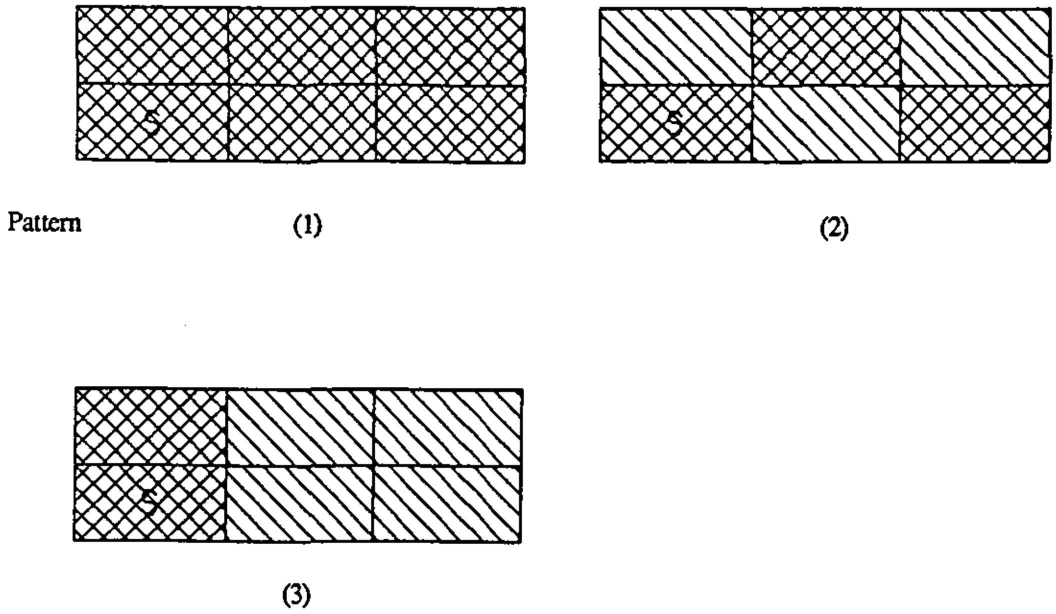


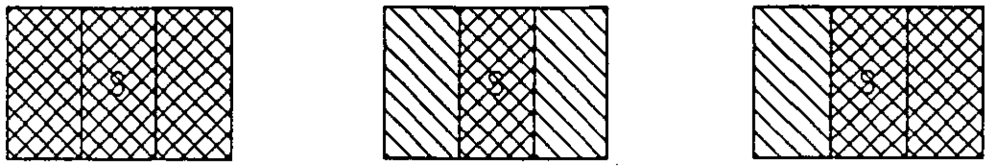
Fig. 5.19 Case 4: Loading patterns for maximum moments considered in case of two adjacent edges discontinuous, $L_y/L_x = 2.0$

Table 5.26a Maximum elastic moment coefficients at critical sections of Fig. 5.19

Pattern No	W → E		S → N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0787 (E)	0.0211	0.1173 (N)	0.0595
2	0.0565 (E)	0.0257	0.0783 (N)	0.0739
3	0.0524 (E)	0.0205	0.1180 (N)	0.0602

Table 5.26b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W → E		S → N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0538 (E))	0.0133	0.0950 (N)	0.0496
2	0.0440 (E))	0.0174	0.0680 (N)	0.0605
3	0.0405 (E)	0.0127	(0.0977) (N)	(0.0512)



Pattern (1) (2) (3)

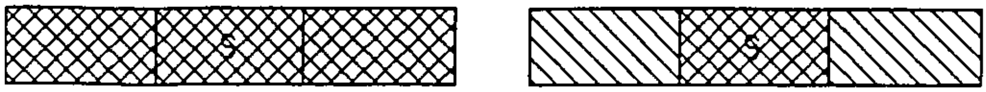
Fig. 5.20 Case 5: Loading patterns for maximum moments considered in case of two short edges discontinuous, $L_y/L_x = 2.0$

Table 5.27a Maximum elastic moment coefficients at critical sections of Fig. 5.20

Pattern No	W ⟶ E		S ⟶ N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0993 (W)	0.0337	0.0118 (S & N)	0.0053
2	0.0639 (W)	0.0570	0.0217 (S & N)	0.0162
3	0.1079 (W)	0.0457	0.0175 (S & N)	0.0108

Table 5.27b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W ⟶ E		S ⟶ N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0904 (W)	0.0306	0.0078 (S & N)	0.0018
2	0.0599 (W)	0.0481	0.0146 (S & N)	0.0088
3	(0.0959) (W)	(0.0407)	0.0114 (S & N)	0.0052



Pattern (1) (2)

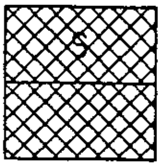
Fig. 5.21 Case 6: Loading patterns for maximum moments considered in case of two long edges discontinuous, $L_y/L_x = 2.0$

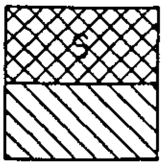
Table 5.28a Maximum elastic moment coefficients at critical sections of Fig. 5.21

Pattern No	W ⟶ E		S ⟶ N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.1185 (W & E)	0.0398	0.0341 (S & N)	0.0858
2	0.0776 (W & E)	0.0389	0.0362 (S & N)	0.0913

Table 5.28b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W ⟶ E		S ⟶ N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0891 (W & E)	0.0294	0.0209 (S & N)	0.0597
2	0.0666 (W & E))	0.0287	(0.0205) (S & N)	0.0672





Pattern (1) (2)

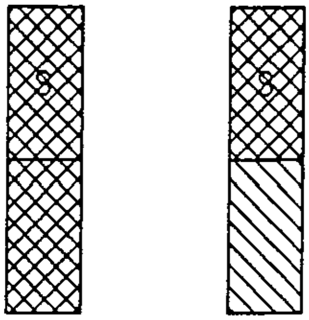
Fig. 5.22 Case 7: Loading patterns for maximum moments considered in case of three edges discontinuous (one long edge continuous), $L_y/L_x = 2.0$

Table 5.29a Maximum elastic moment coefficients at critical sections of Fig. 5.22

Pattern No	W ——→ E		S ——→ N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0294 (W & E)	0.0194	0.1201 (S)	0.0613
2	0.0351 (W & E)	0.0246	0.0789 (S)	0.0749

Table 5.29b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W ——→ E		S ——→ N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0166 (W & E)	0.0114	(0.1033) (S)	(0.0541)
2	0.0202 (W & E)	0.0161	0.0734 (S)	0.0635



Pattern (1) (2)

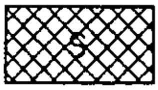
Fig. 5.23 Case 8: Loading patterns for maximum moments considered in case of three edges discontinuous (one short edge continuous), $L_y/L_x = 2.0$

Table 5.30a Maximum elastic moment coefficients at critical sections of Fig. 5.23

Pattern No	W ⟶ E		S ⟶ N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0387 (W & E)	0.0944	0.1197 (S)	0.0383
2	0.0393 (W & E)	0.0962	0.0783 (S)	0.0380

Table 5.30b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W ⟶ E		S ⟶ N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0205 (W & E)	0.0705	0.0898 (S)	0.0311
2	(0.0201) (W & E)	(0.0743)	0.0673 (S)	0.0298



Pattern (1)

Fig. 5.24 Case 9: Loading patterns for maximum moments considered in case of four edges discontinuous, $L_y/L_x = 2.0$

Table 5.31a Maximum elastic moment coefficients at critical sections of Fig. 5.24

Pattern No	W ⟶ E		S ⟶ N	
	Maximum negative moment at edge	Maximum positive moment at midspan	Maximum negative moment at edge	Maximum positive moment at midspan
1	0.0461 (W & E)	0.0368	0.0399 (S & N)	0.1014

Table 5.31b Average elastic moment coefficients over $\frac{3}{4}$ width

Pattern No	W ⟶ E		S ⟶ N	
	Average negative moment at edge	Average positive moment at midspan	Average negative moment at edge	Average positive moment at midspan
1	0.0273 (W & E)	0.0273	(0.0194) (S & N)	0.0816

Table 5.32 Comparison of moment coefficients given in BS8110 and ACI codes with the finite element analysis for slabs under fully loaded and worst pattern of loading; aspect ratio is 1.0

Cases	Moments Considered	FULLY LOADED BS8110 (BS/EF)	FULLY LOADED ACI (ACI/EF)	FULLY LOADED Average Elastic Moment (EF)	WORST PATTERN (BS/EF)	WORST PATTERN (ACI/EF)	WORST PATTERN Average Elastic Moment (EF)
Case 1	Neg. Mom. at Cont. Edge (-)	0.0356	0.0395	0.0324			0.0430
		(1.099)	(1.219)		(0.828)	(0.919)	
	Pos. Mom. at Midspan (+)	0.0194	0.0314	0.0127			0.0196
		(1.528)	(2.472)		(0.990)	(1.602)	
Case 2	Neg. Mom. at Cont. Edge	0.0436	0.0567	0.0405			0.0405
		(1.077)	(1.400)		(1.077)	(1.400)	
	Pos. Mom. at Midspan	0.0244	0.0347	0.0183			0.0234
		(1.333)	(1.896)		(1.043)	(1.483)	
Case 3	Neg Mom. at Cont. Edge	0.039	0.0342	0.0327			0.0429
		(1.193)	(1.046)		(0.909)	(0.797)	
	Pos. Mom. at Midspan	0.030	0.0257	0.0126			0.0200
		(2.381)	(2.040)		(1.5)	(1.285)	
Case 4	Neg. Mom. at Cont. Edge	0.0436	0.0567	0.0479			0.0524
		(0.910)	(1.184)		(0.832)	(1.082)	
	Pos. Mom. at Midspan	0.036	0.0311	0.0220			0.0261
		(1.636)	(1.414)		(1.379)	(1.192)	
Case 5	Neg. Mom. at Cont. Edge	0.0523	0.0777	0.0598			0.061
		(0.875)	(1.299)		(0.857)	(1.274)	
	Pos. Mom. at Midspan	0.034	0.0311	0.0248			0.0276
		(1.371)	(1.254)		(1.232)	(1.127)	
Case 6	Neg. Mom. at Cont. Edge	-	-	0.0099			0.0128
		-	-		-	-	
	Pos. Mom. at Midspan	0.034	0.0243	0.0112			0.0184
		(3.036)	(2.170)		(1.848)	(1.321)	
Case 7	Neg. Mom. at Cont. Edge	0.057	0.0736	0.0631			0.0631
		(0.903)	(1.166)		(0.903)	(1.166)	
	Pos. Mom. at Midspan	0.043	0.0355	0.0314			0.0321
		(1.369)	(1.130)		(1.340)	(1.106)	
Case 8	Neg. Mom. at Cont. Edge	-	-	0.0142			0.0156
		-	-		-	-	
	Pos. Mom. at Midspan	0.042	0.0311	0.0206			0.0248
		(2.039)	(1.510)		(1.694)	(1.254)	
Case 9	Neg. Mom. at Cont. Edge	-	-	0.0184			0.0184
		-	-		-	-	
	Pos. Mom. at Midspan	0.055	0.0373	0.0334			0.0334
		(1.647)	(1.117)		(1.647)	(1.117)	

Note: A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unhatched edge indicates the discontinuous edges.

Table 5.33 Comparison of moment coefficients given in BS8110 and ACI codes with the finite element analysis for slabs under fully loaded and worst pattern of loading; aspect ratio is 2.0

Cases	Moments Considered	FULLY LOADED BS8110 (BS/EF)	FULLY LOADED ACI (ACI/EF)	FULLY LOADED Average Elastic Moment (EF)	WORST PATTERN (BS/EF)	WORST PATTERN (ACI/EF)	WORST PATTERN Average Elastic Moment (EF)
Case 1	Neg. Mom. at Cont. Edge (-)	0.0778	0.0909	0.0781			0.0876
		(0.996)	(1.164)		(0.888)	(1.038)	
	Pos. Mom. at Midspan (+)	0.0332	0.0547	0.0288			0.0467
		(1.153)	(1.899)		(0.711)	(1.171)	
Case 2	Neg. Mom. at Cont. Edge	0.0818	0.0948	0.0844			0.0906
		(0.969)	(1.123)		(0.903)	(1.046)	
	Pos. Mom. at Midspan	0.0352	0.0539	0.0297			0.0474
		(1.185)	(1.815)		(0.743)	(1.137)	
Case 3	Neg Mom. at Cont. Edge	0.089	0.0923	0.0865			0.0920
		(1.029)	(1.067)		(0.967)	(1.003)	
	Pos. Mom. at Midspan	0.067	0.0706	0.0451			0.0574
		(1.486)	(1.565)		(1.167)	(1.230)	
Case 4	Neg. Mom. at Cont. Edge	0.093	0.0975	0.0950			0.0977
		(0.979)	(1.026)		(0.952)	(0.998)	
	Pos. Mom. at Midspan	0.070	0.0724	0.0496			0.0605
		(1.411)	(1.460)		(1.157)	(1.197)	
Case 5	Neg. Mom. at Cont. Edge	0.086	0.0975	0.0904			0.959
		(0.951)	(1.079)		(0.897)	(1.017)	
	Pos. Mom. at Midspan	0.037	0.0537	0.0306			0.0481
		(1.209)	(1.755)		(0.769)	(1.116)	
Case 6	Neg. Mom. at Cont. Edge	-	-	0.0209			0.0205
		-	-		–	–	
	Pos. Mom. at Midspan	0.100	0.0879	0.0597			0.0672
		(1.675)	(1.472)		(1.488)	(1.308)	
Case 7	Neg. Mom. at Cont. Edge	0.098	0.1006	0.1033			0.1033
		(0.977)	(0.974)		(0.977)	(0.974)	
	Pos. Mom. at Midspan	0.074	0.0739	0.0541			0.0635
		(1.368)	(1.366)		(1.165)	(1.164)	
Case 8	Neg. Mom. at Cont. Edge	-	-	0.0205			0.0201
		-	-		–	–	
	Pos. Mom. at Midspan	0.105	0.0941	0.0743			0.0743
		(1.413)	(1.266)		(1.413)	(1.266)	
Case 9	Neg. Mom. at Cont. Edge	-	-	0.0194			0.0194
		-	-		–	–	
	Pos. Mom. at Midspan	0.111	0.0985	0.0816			0.0816
		(1.360)	(1.207)		(1.360)	(1.207)	

Note: A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unhatched edge indicates the discontinuous edges.

Table 5.34 Ratio of the code coefficient/service load coefficient for both positive and negative moments for the BS8110 and ACI codes; the aspect ratio is 1.0.

Cases	Moments considered	Code coefficient and ratio of code coefficients (after redistribution) divided by service load coefficient; slab aspect ratio 1:1		Worst finite element coefficient at service load
		BS8110	ACI	(L.L. + D.L.)
Case 1	Neg. Mom. at Cont. Edge	0.0356 (0.9)*	0.0395 (1.005)	0.0393
	Pos. Mom. at Midspan	0.0194 (1.05)	0.0314 (1.70)	0.0185
Case 2	Neg. Mom. at Cont. Edge	0.0436 (0.89)*	0.0567 (1.16)	0.049
	Pos. Mom. at Midspan	0.0244 (1.16)	0.0347 (1.64)	0.0211
Case 3	Neg. Mom. at Cont. Edge	0.039 (0.98)*	0.0342 (0.86)*	0.0398
	Pos. Mom. at Midspan	0.030 (1.64)	0.0257 (1.41)	0.0182
Case 4	Neg. Mom. at Cont. Edge	0.0436 (0.94)*	0.0567 (1.22)	0.0464
	Pos. Mom. at Midspan	0.036 (1.58)	0.0311 (1.37)	0.0227
Case 5	Neg. Mom. at Cont. Edge	0.0523 (1.02)	0.0777 (1.50)	0.0515
	Pos. Mom. at Midspan	0.034 (1.43)	0.0311 (1.31)	0.0237
Case 6	Neg. Mom. at Cont. Edge	- -	- -	-
	Pos. Mom. at Midspan	0.034 (1.88)	0.0243 (1.34)	0.0181
Case 7	Neg. Mom. at Cont. Edge	0.057 (1.036)	0.0736 (1.34)	0.0550
	Pos. Mom. at Midspan	0.043 (1.63)	0.0355 (1.34)	0.0263
Case 8	Neg. Mom. at Cont. Edge	- -	- -	-
	Pos. Mo. at Midspan	0.042 (1.80)	0.0311 (1.33)	0.0233
Case 9	Neg. Mom. at Cont. Edge	- -	- -	-
	Pos. Mom. at Midspan	0.055 (1.84)	0.0373 (1.25)	0.0299

Note: The values without brackets are the coefficients and the bracketted figures are the ratio of the coefficients divided by the worst finite element coefficients

Table 5.35 Ratio of the code coefficient/service load coefficient for both positive and negative moments for the BS8110 and ACI codes; the aspect ratio is 2.0.

Cases	Moments considered	Code coefficient and ratio of code coefficients (after redistribution) divided by service load coefficient; slab aspect ratio 1:2		Worst finite element coefficient at service load
		BS8110	ACI	(L.L. + D.L.)
Case 1 	Neg. Mom. at Cont. Edge Pos. Mom. at Midspan	0.0778 (1.14) 0.0332 (0.94)*	0.0909 (1.33) 0.0547 (1.55)	0.0685 0.0354
Case 2 	Neg. Mom. at Cont. Edge Pos. Mom. at Midspan	0.0818 (1.17) 0.0352 (1.00)	0.0948 (1.36) 0.0539 (1.54)	0.698 0.0349
Case 3 	Neg. Mom. at Cont. Edge Pos. Mom. at Midspan	0.089 (1.17) 0.067 (1.47)	0.0923 (1.20) 0.0706 (1.55)	0.0763 0.0455
Case 4 	Neg. Mom. at Cont. Edge Pos. Mom. at Midspan	0.093 (1.19) 0.070 (1.49)	0.0975 (1.25) 0.0724 (1.54)	0.0780 0.0468
Case 5 	Neg. Mom. at Cont. Edge Pos. Mom. at Midspan	0.086 (1.23) 0.037 (1.07)	0.0975 (1.39) 0.0537 (1.56)	0.0702 0.0344
Case 6 	Neg. Mom. at Cont. Edge Pos. Mom. at Midspan	- - 0.100 (1.68)	- - 0.0879 (1.47)	- 0.0596
Case 7 	Neg. Mom. at Cont. Edge Pos. Mom. at Midspan	0.098 (1.23) 0.074 (1.55)	0.1006 (1.27) 0.0739 (1.55)	0.0794 0.0476
Case 8 	Neg. Mom. at Cont. Edge Pos. Mo. at Midspan	- - 0.105 (1.66)	- - 0.0941 (1.48)	- 0.0634
Case 9 	Neg. Mom. at Cont. Edge Pos. Mom. at Midspan	- - 0.111 (1.66)	- - 0.0985 (1.47)	- 0.067

Note: The values without brackets are the coefficients and the bracketted figures are the ratio of the coefficients divided by the worst finite element coefficients

Table 5.36 Relevant coefficients of BS8110 and ACI codes for rigidly supported slabs for the short span only in two different aspect ratios of 1.0 and 2.0

Cases	Moments Considered	$L_y/L_x = 1.0$		$L_y/L_x = 2.0$	
		BS	ACI	BS	ACI
Case 1	Neg. Mom. at Cont. Edge	0.031	0.0466	0.063	0.0891
L_x  L_y	Pos. Mom. at Midspan	0.024	0.0243	0.048	0.0565
Case 2	Neg. Mom. at Cont. Edge	0.039	0.0632	0.067	0.0912
	Pos. Mom. at Midspan	0.029	0.0282	0.050	0.0575
Case 3	Neg. Mom. at Cont. Edge	0.039	0.0342	0.089	0.0923
	Pos. Mom. at Midspan	0.030	0.0257	0.067	0.0706
Case 4	Neg. Mom. at Cont. Edge	0.047	0.0518	0.093	0.0975
	Pos. Mom. at Midspan	0.036	0.0311	0.070	0.0724
Case 5	Neg. Mom. at Cont. Edge	0.046	0.0777	0.070	0.0933
	Pos. Mom. at Midspan	0.034	0.0311	0.053	0.0579
Case 6	Neg. Mom. at Cont. Edge	-	-	-	-
	Pos. Mom. at Midspan	0.034	0.0243	0.100	0.0879
Case 7	Neg. Mom. at Cont. Edge	0.057	0.0736	0.098	0.1006
	Pos. Mom. at Midspan	0.043	0.0355	0.074	0.0739
Case 8	Neg. Mom. at Cont. Edge	-	-	-	-
	Pos. Mom. at Midspan	0.042	0.0311	0.105	0.0941
Case 9	Neg. Mom. at Cont. Edge	-	-	-	-
	Pos. Mom. at Midspan	0.055	0.0373	0.111	0.0985

Note: A crosshatched edge indicates that the slab continues across, or is fixed at, the support; an unhatched edge indicates the discontinuous edges.

CHAPTER 6
SLABS ON SEMI-RIGID SUPPORTS

6.1 Introduction

It is suggested in BS8110 slabs on semi-rigid support, i.e. supported by beams, be designed by the same simplified method given for slabs on rigid supports. The ACI code values for beams on rigid supports are only recommended if the beams are about 3 times the slab depth and cater for shallower beams by requiring the use of the EFM or DDM method. The Direct Design Method (DDM) is used in this Chapter which is one which it will be seen takes into consideration the effect of the stiffness of the supporting beams.

The purpose of this Chapter therefore is to describe the requirements of the ACI DDM method in detail, and then to apply and compare the moments and moment coefficients so calculated for a specific design example with those obtained using BS8110. In addition the elastic solution derived from a finite element analysis will be carried out and these results will be examined and compared with the two sets of code values.

6.2 BS8110 Code Requirements

BS8110 does not give a separate method for slabs on semi-rigid supports, but infers that they be treated as slabs on rigid supports. Therefore, the recommended moment coefficients for rigidly supported slabs, given in its simplified method, will be used for semi-rigidly supported slabs.

6.3 ACI - The Direct Design Method (DDM)

6.3.1 Description of Direct Design Method

In broad terms if one considers the layout of a typical bay of a slab system (see Fig. 6.1) the Direct Design Method first assumes that the static loading condition is fulfilled, which in the north-south direction is

$$\frac{1}{2}(M_{ab} + M_{cd}) + M_{ef} = \frac{1}{8}\,wL_2L_{1n}^2 = M_o$$

where L_{1n} is the clear span in the L_1 direction.

For the east-west direction this condition is

$$\frac{1}{2}(M_{gh} + M_{ij}) + M_{cd} = \frac{1}{8}\,wL_1L_{2n}^2 = M_o'$$

where L_{2n} is the clear span in the L_2 direction.

Generally the total end moments will differ depending on whether it is an interior or end span. If the total static moment generally is defined by

$$M_o = \frac{wL_2L_n^2}{8}$$

the code recommendations for an interior span are that the

negative moment $^-M_u = 0.65\ M_o$ and the (6.1)

positive moment $^+M_u = 0.35\ M_o$ (6.2)

For an external span the moment proportions will depend on the edge condition restraint as shown in Fig. 6.2 and the relevant proportions to be assumed are given in Table 6.1. It can be observed the exterior negative moment increases with increasing restraint and the positive moment reduces accordingly.

The actual moment/unit width across sections such as ab, ef and cd are not of course constant but vary in the general form shown in Fig. 6.1b, and for design purposes the total moment is subdivided between the column and middle strips as shown by the broken lines in Fig. 6.1b.

The proportions of the moment carried by the column strip are given in Table 6.2 and are dependent on the coefficients L_2/L_1, α and β_t where

L_1 is length of span in the direction that moments are being determined, measured centre-to-centre of supports;

L_2 is length of span in the direction perpendicular to L_1, measured centre-to-centre of supports;

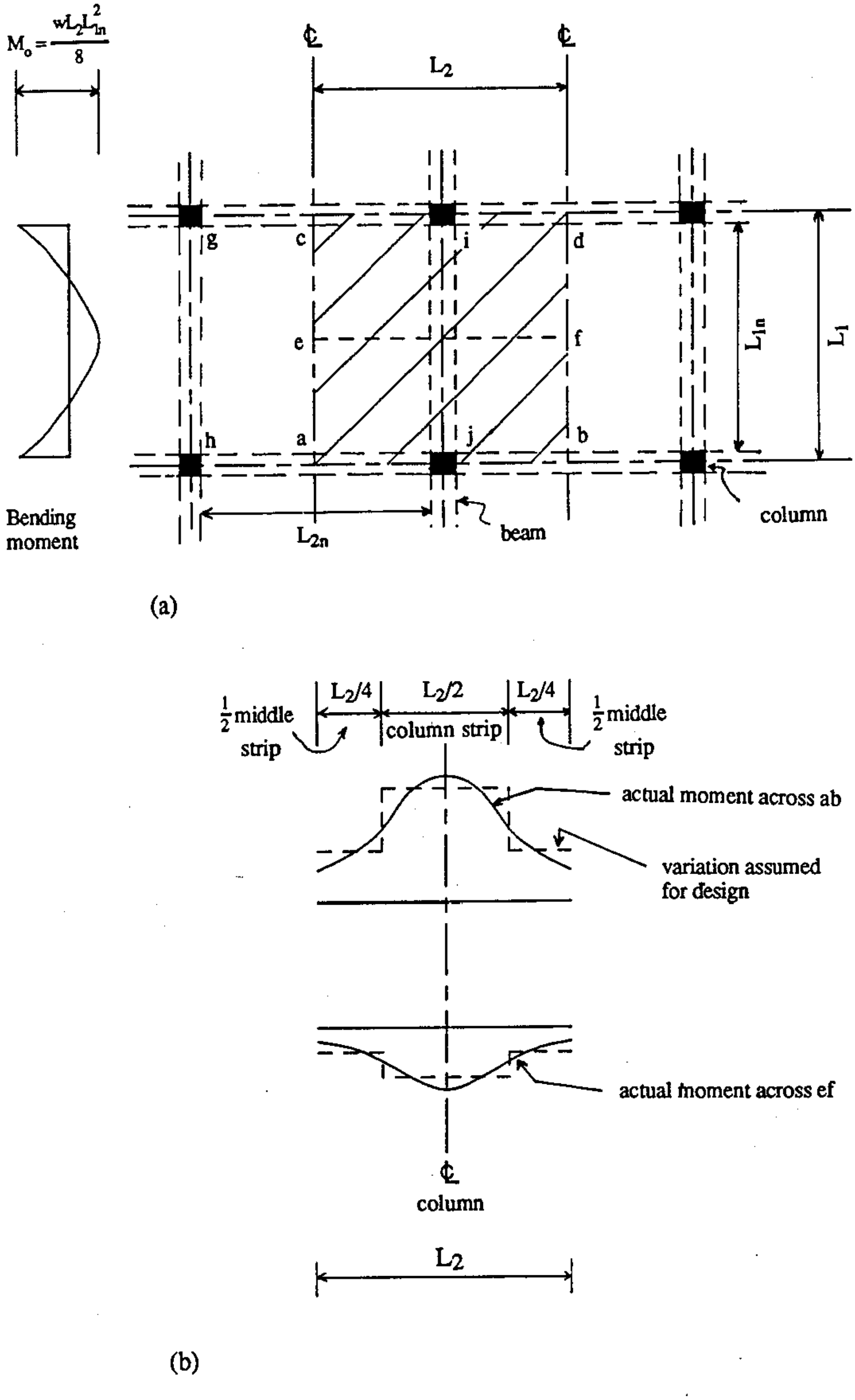


Fig. 6.1 Layout of a typical bay of a slab system

a) Total static moment for L_1 direction
b) Moment variation across width of critical sections

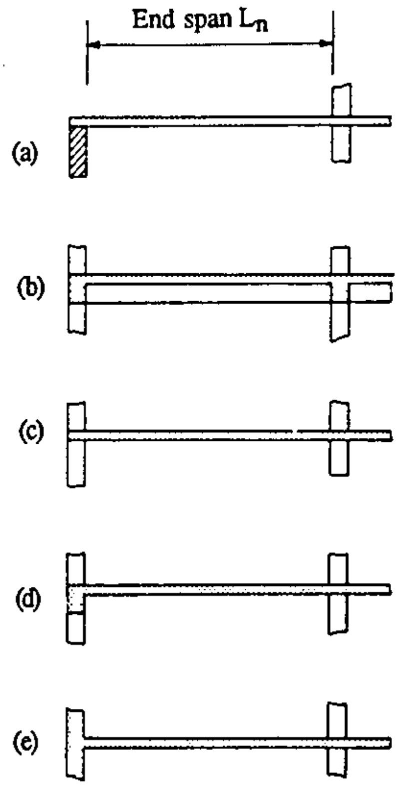


Fig. 6.2 Conditions of edge restraint considered in distributing total static moment M_0 to critical sections in an end span:

a) exterior edge unrestrained, e.g. supported by masonry wall;
b) slab with beams between all supports;
c) slab without beams, i.e. flat plate;
d) slab without beams between interior supports but with edge beam;
e) exterior edge fully rstrained, e.g. by monolithic concrete wall.

Table 6.1 Distribution factors applied to static moment M_0 for positive and negative moments in end span

Conditions of edge restraint / Moment considered	(a) Exterior edge unrestrained	(b) Slab with beams between all supports	(c) Slab without beams between interior supports: Without edge beam	(d) Slab without beams between interior supports: With edge beam	(e) Exterior edge fully restrained
Interior negative moment	0.75	0.70	0.70	0.70	0.65
Positive moment	0.63	0.57	0.52	0.50	0.35
Exterior negative moment	0	0.16	0.26	0.30	0.65

Table 6.2 Column-strip moment, percent of total moment at critical section

Moment considered \ Aspect ratio		L_2/L_1 = 0.5	1.0	2.0
Interior negative moment				
$\alpha_1 L_2/L_1 = 0$		75	75	75
$\alpha_1 L_2/L_1 \geq 1.0$		90	75	45
Exterior negative moment				
$\alpha_1 L_2/L_1 = 0$	$\beta_t = 0$	100	100	100
	$\beta_t > 2.5$	75	75	75
$\alpha_1 L_2/L_1 \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t > 2.5$	90	75	45
Positive moment				
$\alpha_1 L_2/L_1 = 0$		60	60	60
$\alpha_1 L_2/L_1 \geq 1.0$		90	75	45

α is the relative stiffness of the beam to the slab spanning the same direction of that beam;

β_t is the relative restraint provided by the torsional resistance of the effective transverse edge beam.

If a beam parallel to the slab span is present, 85% of the moment in the column strip is taken by the beam if $\alpha_1 L_2/L_1 > 1.0$. For values of $\alpha_1 L_2/L_1$ between 1.0 and 0, the proportion of moment distributed to the beam is assumed to vary linearly between 85% (corresponding to $\alpha_1 L_2/L_1 = 1$) and 0 (corresponding to $\alpha_1 L_2/L_1 = 0$).

It should be noted that the negative and positive factored moments may be modified by 10%, provided the total moments are not less than the total static moment for a panel in the direction considered.

These various stages and coefficients define the design process and can be used provided the following limitations are not exceeded.

a) There must be a minimum of three continuous spans in each direction.

b) Panels shall be rectangular and have aspect ratios that are 2:1 or less.

c) Span lengths may differ by up to one-third of the length of the longer span.

d) Columns may not be offset by more than 10% of the span in the direction of the offset from either axis between centrelines of successive columns.

d) All loads shall be due to gravity only and uniformly distributed over an entire panel.

e) The live load shall not exceed three times the dead load.

f) For a panel with beams between supports on all sides, the relative stiffness of the beams in the two perpendicular directions must be in the range given by

$$0.2 < \frac{\alpha_1 L_2^2}{\alpha_2 L_1^2} < 5.0$$

g) The slab thickness shall not be less than

$$h = \frac{L_n(800 + 0.005\, f_y)}{36000 + 5000\, \beta\left[\alpha_m - 0.5\,(1 - \beta_s)(1 + 1/\beta)\right]}$$

or

$$h = \frac{L_n(800 + 0.005\, f_y)}{36000 + 5000\, \beta(1 + \beta_s)}$$

where

L_n = clear span in long direction, in inches

α_m = average value of α (α is ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centrelines of adjacent panels, if any, on each side of the beam) for all beams on edges of panel

β_s = ratio of length of continuous edges to total perimeter of slab panel

β = ratio of the clear spans in the long span and short span directions

In addition, the thickness h must not be less than 3.5 in (90 mm).

However, the thickness need not be more than

$$h = \frac{L_n(800 + 0.005\, f_y)}{36000}$$

6.3.2 Summary of DDM steps

The following sequence of steps is followed in the design process.

a) Estimate slab thickness.

b) Calculate ultimate factored design load, w_u.

c) Compute total factored static moments M_o for all spans.

d) Distribute M_o to negative support moment ^-M_u and midspan positive moment ^+M_u for each panel in accordance with Table 6.1.

e) Distribute ^-M_u and ^+M_u laterally at their associated critical sections into column and middle strips of panels as described in Table 6.2.

f) Distribute column strip moments found in step (e) above between the edge beam support (if any) and the slab.

g) Redistribution of moments between critical sections up to 10% may be used if thought necessary.

6.4 Application of Codes to a Typical Sample Design

In order to demonstrate an application of the respective code provisions, a worked numerical example is given in Appendix 6A following both the British and American codes. Fig. 6A1 in Appendix 6A shows a plan and cross-sectional view of the example for analysis. It is a multi-panelled floor with three spans in both directions, namely the minimum required by the ACI code DDM method. All the panels are supported by beams, cast monolithically with the slabs and all the beams are assumed to be continuous over pin supports at their points of intersection. The same slab thickness and beam depth are used for both codes and the solution is restricted to the north-south direction.

Although both codes start with the same sizes and loads, their designs diverge slightly at the beginning of the calculations due to different partial safety factors for their characteristic loads.

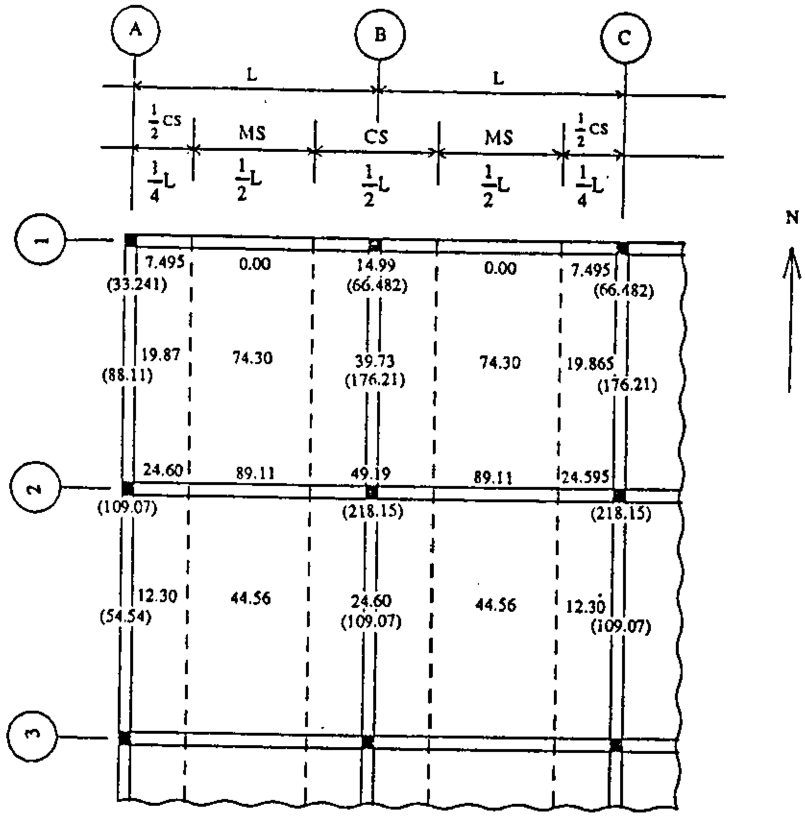
BS8110 gives no real guidance for slabs on semi-rigid supports and the calculation has therefore been carried out using the coefficients for slabs supported on four sides as detailed in Chapter 5 Table 5.2. The supporting beams have been designed to carry the slab load in accordance with BS8110 clause 3.5.3.7 as shown in Appendix 6A and to be over three spans with continuity over the middle two supports.

The technique suggested in the ACI code is the DDM which makes allowance for beams of different stiffnesses and the calculations follow the recommendations.

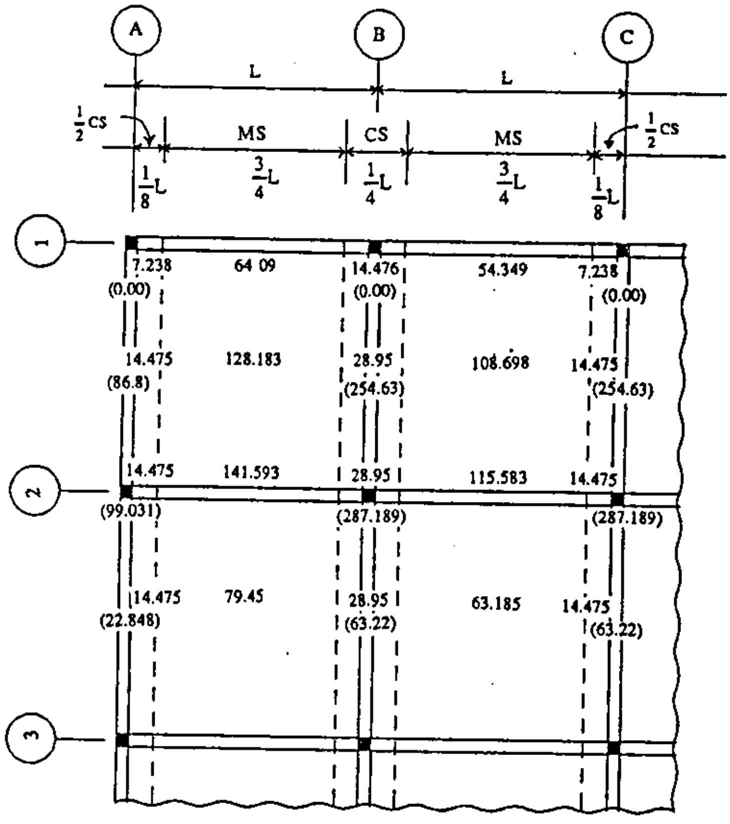
The various moments/unit width or beam moments calculated by these methods in Appendix 6A have been summarized in Fig. 6.3, and the following observations can be made.

6.4.1 Comparison of Code Designs

If the comparison is started by considering the moments in the beams down column row B, Fig. 6.3, it will be noted the ACI code includes a negative value at the exterior column support. The value for the British code is recorded as zero since pinned supports were assumed. In reality there will be continuity into the column as



(a)



(b)

Legend
CS : Column Strip
MS : Middle Strip

Note: values in brackets show the beam moments

Fig. 6.3 Various moments in slab and beams of typical sample design using

a) ACI
b) BS 8110

some moment would exist and this could have been allowed for by a simple end model. However the ACI practice of ensuring negative end reinforcement in the beam is certainly sensible in view of the variability of possible end restraint and a set of values such as the last row of Table 6.1 would seem good practice provided the column can transmit the moment.

If the positive moment in the end span and the negative moment at the first interior support in the two codes are compared then the British code values are both considerably higher, namely 254.63 compared with 176.21 and 287.189 compared with 218.15. One contribution to this is that the ACI code allows an end moment but the main reason for this is that the British code treats the beams as rigid supports with the slab parallel to the beam making no contribution. Indeed in the British code there is zero in the column strip except for the requirements of minimum steel. This situation is clearly wrong. As the beam bends it will take the slab with it and there will unquestionably be bending moments in the slab parallel and close to the beam and reinforcement is therefore vital. To disregard the contribution of the middle strip steel is also questionable.

Conversely the ACI code accepts a proportion of the static moment is carried by both the column strip and the beam. The proportions carried by each depends on the ratio of the stiffnesses of the slab and beam. The weaker the beam the less is proportioned to the beam. This clearly is structurally what happens. Thus the British code with moment coefficients has only the two extremes, namely rigid supports or a flat slab with no intermediate values between. This omission in the code is considered a matter which requires rectifying.

If the positive beam moment at an interior span is considered the value from BS8110 is only some 58% of the ACI value. This is mainly because the support moment is extremely high and if the designer chose to redistribute some 45 kNm from the support moment then this would give the same value as the ACI code and would only require a 15% redistribution which is well within the code limits.

If one considers the slab moments or moment coefficients in the column strip there is no comparison. The British code value is merely that for minimum steel. The ACI values are a fixed proportion determined by the beam/slab stiffness of the moment at any section. In this case the proportion is approximately 18.4% of the total moment. The code itself gives no guide as the I value of a downstand beam and Winter [4] used the gross section area then factored this up by 2 since he regards it as a T beam which of course has a higher value. However at the support this assumption cannot be realistic. In this particular case with the factor of 2 applied the stiffness ratios were almost unity indicating the beam carried 85% of the total moment. If the factor of 2 had not been applied the proportion would have been approximately 43%, i.e. a half with an appropriate increase in the slab strip moment. Thus the beam moment is extremely dependent on the stiffness ratio α, though the total strip moment is only slightly dependent on its value. For comparison purposes later the moment coefficients from the ACI code can be calculated for this example and are as follows.

Positive exterior span	=	0.018
Negative first interior span	=	0.022
Positive interior span	=	0.011

If now the moment coefficients in the middle strip are considered at the extreme edge of the slabs along row 1 the British code gives zero coefficients but another clause recommends that at a discontinuous edge the negative steel be half the positive value. While it could not be found it is likely that a similar statement exists in the ACI code although Table 6.2 specifically states that for low values of β_t (the measure of torsion connection) that the column strip carries 100% of the moments. At the middle of the first interior span the moment coefficients can be calculated and both codes have a value of approximately 0.033. At the first interior support the British value is 0.0355 while the ACI value can be calculated to be 0.0403 which again is very similar. At the centre of an interior span the moment coefficients are 0.0194 and 0.0202 which again are similar.

Having compared and commented on the values at specific points a comment is needed on the total moments. The ACI code is based on the assumption of the total static moment and the total moments summed across various sections will not depart from this too much. For the British code we ignore the obligatory steel at the edge along row 1. Then for an exterior span the sum of the positive moments plus half the negative value is

Slab positive moment	128.183 + 28.95	=	157.133
Positive beam moment	86.8 + 254.63/2	=	214.115
Slab negative moment	141.593 + 28.95	=	170.54
Beam negative moment	0.5(99.031 + 287.189/2)	=	121.297
			663.085

The static moment is $20.064 \times 6^3/8 = 541.728$. Thus the actual provision is 22% more than the static moment. The reason for this of course is that the slab steel is calculated as though it is supported on rigid supports and the beam steel carries the whole load through the slab reactions. The slab steel is completely ignored in the strength calculation which is grossly conservative.

The major conclusions to this section therefore are that because the British code does not easily cater for the composite beam and slab action of this type of construction

(i) the slab column strip steel is inadequate;

(ii) by treating the slab and beams as separate elements the total steel used is excessive;

(iii) perhaps fortuitously the slab middle strip moment coefficients are similar to the ACI code which does recognize composite action; and

(iv) consideration for an allowance due to composite action should be included in the British code.

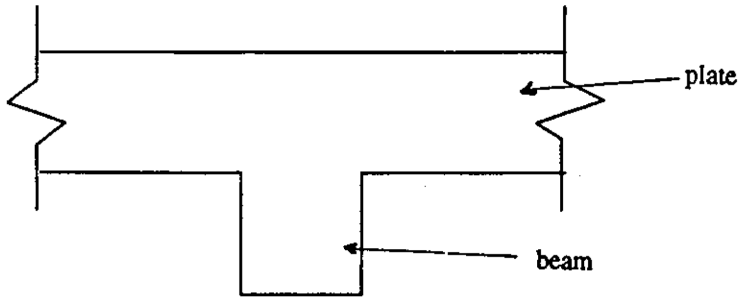
6.5 Finite Element Analysis of Slabs on Semi-Rigid Supports

A number of slab panels of aspect ratio 1.0 with different boundary conditions were analysed using the finite element method to calculate the various moment coefficients. The sample panels used were supported on a rectangular grid of elastic beams. All panels were assumed to be of the type that are cast monolithically with their beams, and all the beams were continuous over pin supports at their points of intersection. All panels were assumed to be fully loaded and pattern loading was not considered.

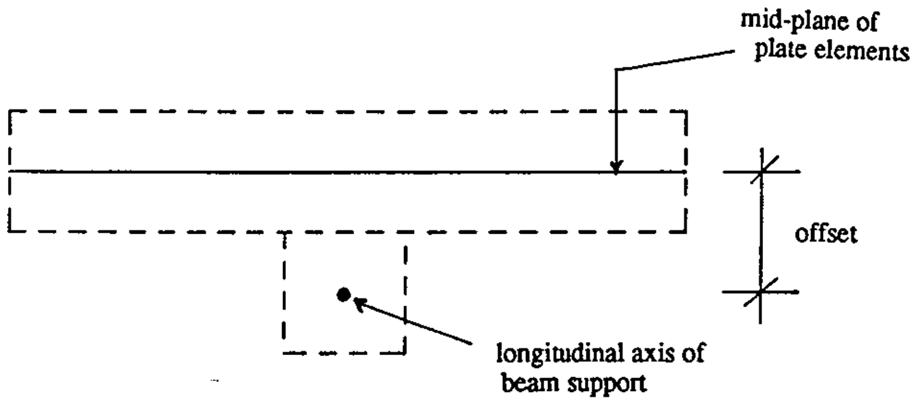
Two different beam depths were considered, giving increasing beam stiffnesses in order to examine this effect and compare the finite element results with those for the same panels but calculated using the simplified BS8110 and ACI code methods.

The slabs on semi-rigid supports were modelled as a beam-plate system and analysed using the general purpose finite element package PAFEC. This engineering problem which consists of slabs on elastic beam supports involves essentially two types of structural members, the plate and the supporting beams which are cast monolithically as shown in Fig. 6.4a. For the purpose of finite element analysis each panel was idealized by an assemblage of flat plate finite elements. The plate finite element (PAFEC reference number 44200) used was a four-noded quadrilateral element suitable for problems involving combined plate bending and membrane (in-plane) effects. The mesh size for each panel was a uniform grid of 8 x 8 elements. Since the PAFEC element library does not have a stiffened plate element the monolithic beam-plate connection was modelled using the offset beam element (PAFEC reference number 34200). This element is a simple engineering beam which may be applied with its centroid offset from, for the sample problem here, the middle axis of the remainder of the structure as shown in Fig. 6.4b.

The offset beam element possesses four nodes and is shown in Fig. 6.5. In this Figure nodes 1 and 2 are conventional nodes which define the longitudinal elastic axis of the engineering beam. Nodes 1 and 2 are attached to nodes 3 and 4 respectively and the latter two nodes are used to attach the beam element to the remainder of the structure and so provide the desired offset.



(a) actual construction - monolothically cast panel
and beam.



(b) idelalized finite element beam-plate model of beam support
monolothically cast panel and beam.

Fig. 6.4 Beam-plate representation of monolothically cast panel
and beam.

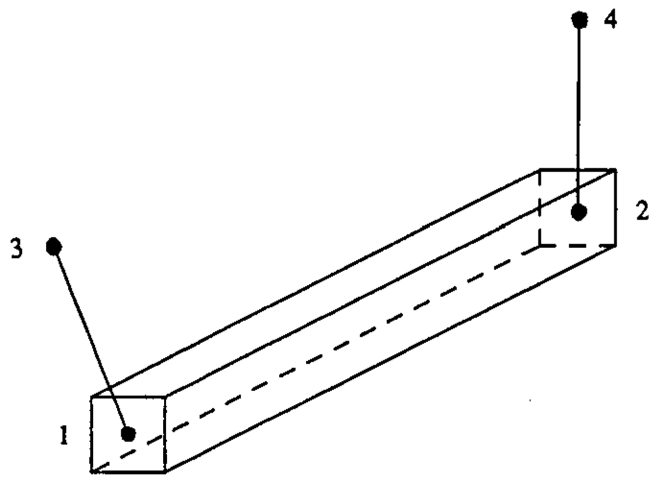


Fig. 6.5 Offset beam element (PAFEC reference number 34200).
Nodes 1 and 2 define beam elastic axis, nodes 3 and 4 are offset nodes.

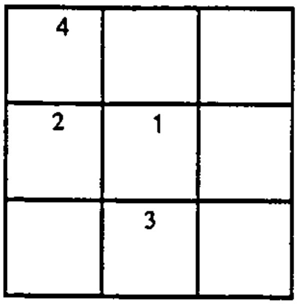
The results from PAFEC include principal stresses and their direction on the top, middle and bottom surfaces of the plate section at each node of each plate element. These have been modified to obtain the equivalent normal stresses in the main directions (global x and y directions) at the same nodes. In general, each element meeting at a node will give different stresses, therefore at every node the average stress due to all the contributing elements was used. These stresses include the effect of both bending and membrane stresses. The effect of the membrane stress was allowed for and the resulting pure bending stresses were then used to assess the bending moment and moment coefficients for the panel case under consideration.

In all three different slab configurations were considered, namely those shown in Fig. 6.6. These were chosen since they correspond to the nine different edge restraint cases given in the two codes for slabs supported on rigid supports.

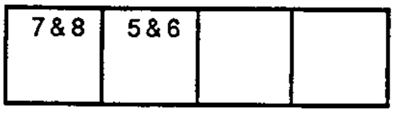
All panels were of the same uniform thickness of 0.24m, assumed to be isotropic, with a Poisson's ratio value of 0.2. The panels were subject to the same uniform distributed load of 19.584 kN/m^2.

This load corresponds to a live load of 1.25 times the dead load so that if necessary comparisons could be made with the same assumptions in other Chapters. Two different edge beam depths were used, the first with a downstand equal to the slab depth (D) of 0.24 m, the second with a downstand depth of 2D (0.48 m). In addition in the next chapter flat slabs are analysed, i.e. slabs with beams of zero downstand. The output from the next chapter, this section and Chapter 5 therefore correspond to supporting beam depths of 1, 2 and 3 and infinity times the slab thickness, namely four different beam stiffnesses.

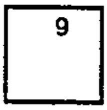
In this section in order to obtain the moment coefficients m_x^+, m_x^-, m_y^+, m_y^- which when multiplied by wL^2 give the equivalent steel moments/unit length, the following five steps were involved.



Slab configuration (a)



Slab configuration (b)



Slab configuration (c)

Note : numbers represents the case numbers.

Fig. 6.6 Three slab configurations which together cover all the different cases analysed for panels on edge beams.

(i) The PAFEC output for principal stress results was edited so that only the numerical values for the stresses at each node of the panel structure remained on file.

(ii) The file resulting from step (i) above was provided as input to PROGRAM 3 (see Appendix 6B). This program converted principal stresses to normal stress in the global axis set for each node of the panel. The output from the program was edited for use in the next step.

(iii) The modified output file from step (ii) was used as input to PROGRAM 4 (see Appendix 6C). This program determined the average direct stress at each node and the associated moment. The output file from the program was edited for use in the next step.

(iv) The modified output file from step (iii) was provided as input to PROGRAM 5 (see Appendix 6D). This program calculated the average nodal moment at each node due to the different elements meeting at the node. The output from this program was edited for use in the next step.

(v) The modified output file from step (iv) was provided as input to PROGRAM 2 (see Appendix 3B). This program uses the Wood and Armer rules to determine the reinforcement moment at each node.

In reality the output from this section alone if examined in total detail could virtually have been a thesis in its own right. The examination was therefore restricted to a detailed examination for an interior slab, i.e. panel 1 for the slab configuration (a) in Fig. 6.6 and the assessment of the average value of the slab moment coefficients for other edge conditions. These limited results are however in themselves quite interesting.

6.5.1 Examination of the Finite Element Results for an Interior Panel

The depth of the supporting beams will be expressed as a proportion of the slab depth, D, the width being constant at D. A flat slab therefore is regarded as being of depth D, that with a downstand of D being of depth 2D, and that of downstand 2D of depth 3D.

The negative and positive moment coefficients across the slab at the column line and midspan for beams of depth 2D and 3D determined in this Chapter are shown in Fig. 6.7(a) and (b). To these have been added the results from Chapter 5 with rigid supports representing infinite stiffness and those from Chapter 7 for flat slabs. It should be noted that these coefficients are from the slab only. At the extremities 0 and L the total moment over the beam width would need to have added the effect of the downstand. In addition near to the beam the slab would be acting as the flange of a T or L beam and in this region the axis of zero stress would not be the middle plane of the slab. This effect as can be seen from Fig. 6.7 seems to be beginning at approximately L/8 or 0.5m from the beam centreline which corresponds to a half flange width of 2D for beams of depth 2D and 3D. It is most marked for the negative moments where the slab and beam are acting as an inverted T beam. After some consideration it was decided to use the full width of slab as a measure of the average slab moment since the downstand respresents the 'extra' that has to be added to create a beam.

If the negative moments are considered first it can be seen that the behaviour is quite different depending on the stiffness of the beam. For infinite stiffness, a rigid support, the moment at the supported edge is zero whilst the value increases considerably as the stiffness is reduced. The lowest value at the centre is with the least stiff beams and the highest for rigid beams. Exactly the same pattern can be observed for the positive moments though the increase at the supports is not as marked as with the negative moment coefficients. This is probably due to the influence of the column which is unyielding and therefore must attract peak values. However the difference in behaviour of the beam action for positive moments and inverted T beam for negative moments also must have some effect.

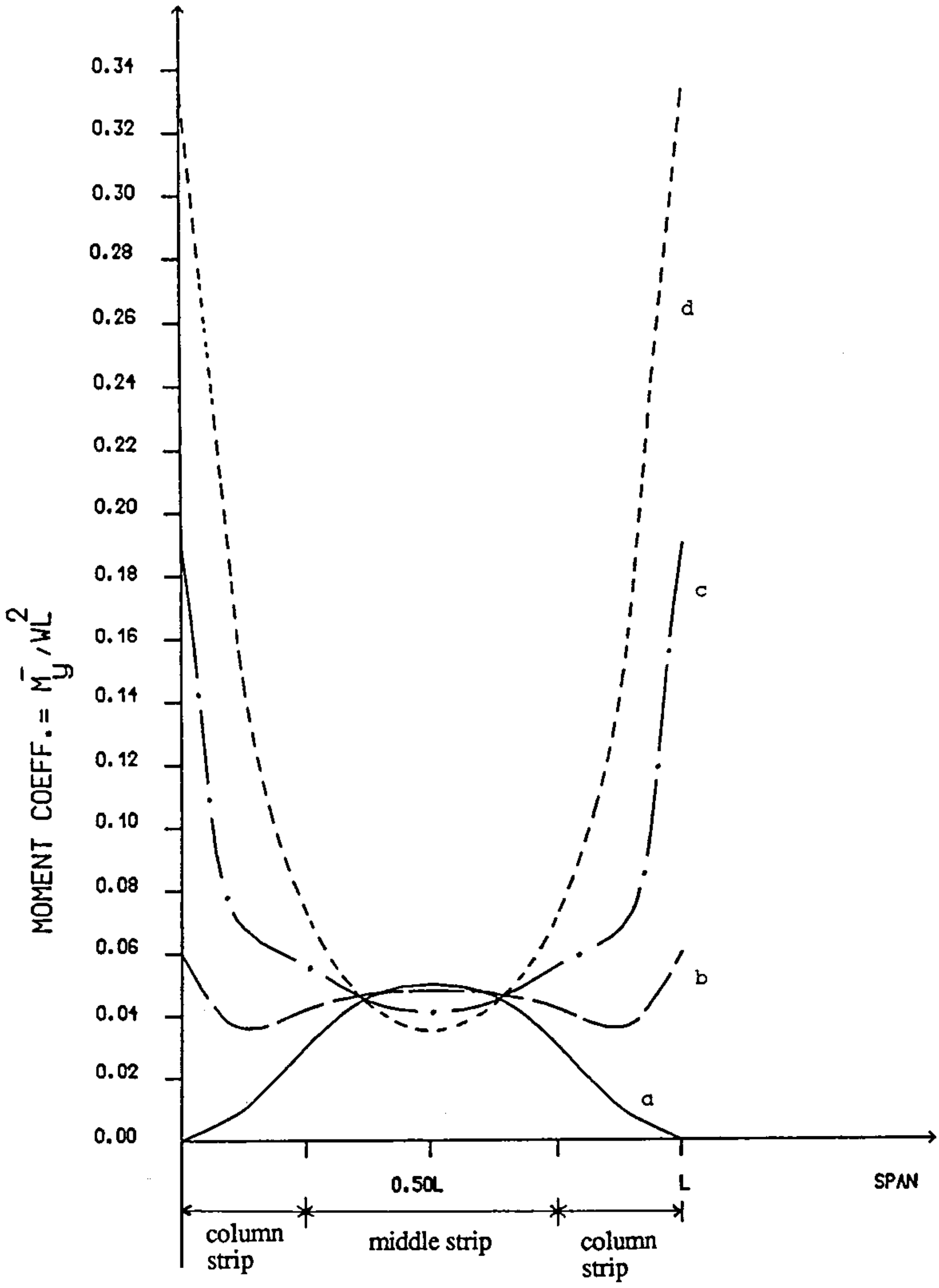


Fig. 6.7(a) Negative bending moment coefficient diagram at a supported edge of an interior panel and supported by each of

a) on rigid support $= \alpha$ ————————

b) on elastic beam of depth $= 3D$ —— —— ——

c) on elastic beam of depth $= 2D$ ——·——·——

d) as flat slab $= D$ – – – – – –

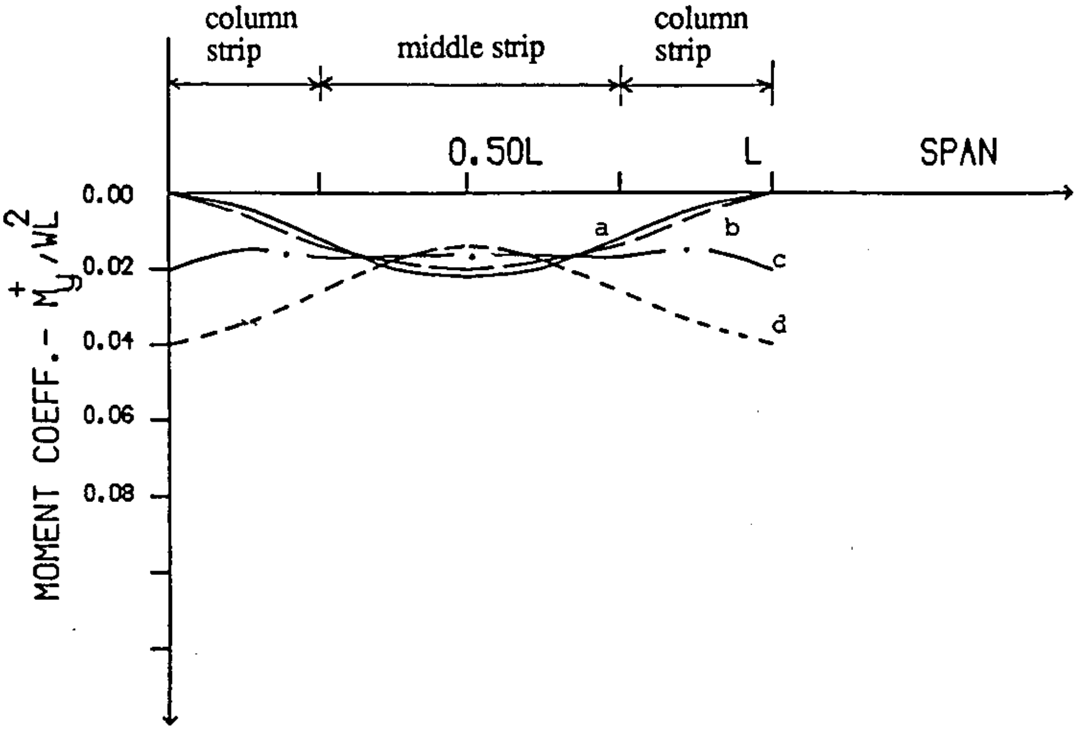


Fig. 6.7(b) Positive bending moment coefficient diagram at the midspan of an interior panel supported by each of

a) on rigid support of total depth = α
b) on elastic beam of total depth = 3D
c) on elastic beam of total depth = 2D
d) as flat slab of total depth = D

Table 6.3 Effect of different types of supports on the moment coefficients of an interior panel

Type and location of moment \ Support type of panel	Rigid support infinite beam depth	Edge beam of total depth 3D	Edge beam of total depth 2D	No beam, total depth D
Negative moment coefficient at continuous edge	0.028	0.0495	0.067	0.104
Positive moment coefficient at mid-span	0.011	0.012	0.0175	0.025
Sum of positive and negative coefficients	0.039	0.0615	0.0845	0.129
% of static moments	31.2	49.2	67.6	103.2

The areas under the curves were measured and average values found and these are shown in Table 6.3. Since this is an interior panel the total contribution to the static moment $wL_2L_1^2/8$ will be the sum of the positive and negative moment coefficients multiplied by $wL_2L_1^2$ and the proportion of the static moment will be that product divided by the static moment. These are also shown in Fig. 6.3. The trend is exactly as might be expected namely that the stiffer the beam the less the proportion carried by the slab. Further, as again might be expected, the stiffer the beam becomes both the negative and positive coefficients decrease though not in the same proportion. The positive moments vary between an average value of 0.011 and 0.025 which is about a twofold increase, whilst the negative coefficients increase from 0.028 to 0.104 which is about a fourfold increase. This increase is somewhat misleading since the 'supporting' beam is carrying a lesser proportion.

It is therefore interesting to examine the middle strip which is taken to be half the width. The average values were calculated from the areas under the curves and are shown in Table 6.4.

It can be seen these negative coefficients are reasonably constant with slightly higher values for the stiffer beams. The positive moments are however virtually constant.

Table 6.4 F.E. average moment coefficients in middle strips

Support type of panel / Type and location of moment	Rigid support infinite beam depth	Edge beam of total depth 3D	Edge beam of total depth 2D	No beam, total depth D
Negative moment coefficient at continuous edge	0.043	0.046	0.046	0.048
Positive moment coefficient at mid-span	0.018	0.017	0.017	0.019

From these figures or by taking the areas under the curve we can also find the average coefficient in the column strip as shown in Table 6.5.

Table 6.5 F.E. average moment coefficients in edge strips

Support type of panel / Type and location of moment	Rigid support infinite beam depth	Edge beam of total depth 3D	Edge beam of total depth 2D	No beam, total depth D
Positive moment coefficient at mid-span	0.004	0.006	0.017	0.034
Negative moment coefficient at continuous edge	0.013	0.043	0.096	0.18

6.5.2 Comparison of finite element results for an interior panel with codes of practice

Since BS8110 really does not give a method for slabs on semi-rigid supports no comparison is really worthwhile except to note from Fig. 6.7 that with a slab with beams of total depth 3D there is little difference between the results and those for one with fully rigid supports. Based on the gross cross section area of the slab a beam of depth 3D and width D gives for this slab an α stiffness ratio of 1.62. When the beam depth is reduced to 2D it can be seen from Fig. 6.7 both the positive and negative moments begin to depart from zero at the edges and indeed are reasonably constant across the section. This beam section gives an α stiffness ratio of 0.48. An α value of 1 would require a beam of total depth of 2.55, i.e. a downstand of 1.55D, and this would give a value in Fig. 6.7 between lines (b) and (c) and would probably be quite satisfactory for use with the BS8110 coefficients. It might therefore be worthwhile introducing a clause into BS8110 to the effect that the moment coefficients are only valid provided the ratio of the beam to slab stifnesses (α) is greater than or equal to unity. This is consistent with findings that Winter [4] notes that the American code coefficients should only be used with beams where the total depth is about 3 times the slab depth.

In the ACI code using the ACI DDM method for an interior span the negative moment contribution is 0.65 M_0 and the positive moment 0.35 M_0 and for an interior span the values attributed to the column strip is 75% of the total moment. Thus 25% is always attributed to the middle strip which in effect is stating that the value both for the positive and negative moments is irrespective of the beam stiffness. The values in Table 6.4 confirm this assumption of a constant value is quite valid.

If 25% of 0.35 M_0 is attributed to the middle strip then working on full span lengths this corresponds to a total moment of $0.25 \times 0.35\ wL_2L_1^2/8 = 0.0109\ wL_2L^2$. This however is carried by a width of $L_2/2$ so the moment coefficient is 0.0218 (wL_1^2). The value found by finite element analysis in Table 6.4 is 0.018. These are already sufficiently close to confirm the value assumed, but in fact in the ACI code the effective span is $0.96L_1$ hence the actual equivalent coefficient in terms of L_1^2 is 0.0218 x $(0.96)^2$ = 0.02 which is even closer to the finite element result.

If the negative moments are now examined the total moment in the middle strip is 0.25 x 0.65 $wL_2L_1^2/8$ = 0.0203 $wL2L_1^2$ which over a length of $L_2/2$ gives a coefficient of 0.0406. This is slightly less than row 1 in Table 6.4 but certainly close. We can therefore conclude that the ACI DDM method for the middle strip of an interior slab is totally consistent with regard to distribution and magnitude of moments with the finite element results for the whole range of beam stiffnesses.

Nothing further need be said about the middle strip but the DDM method distributes the column strip moment to the beam and slab. When the beam to slab stiffness ratio α is >1 then the beam is attributed with 85% of the moment with the percentage decreasing linearly to 0 as $\alpha \to 0$.

For an edge beam of total depth 3D and of width D which is 0.24m then based on gross cross-sectional areas $\alpha = 0.24 \times 27D^3/4.0 \times D^3 = 1.62$ which is > 1. Therefore DDM would attribute 85% of the column strip moment to the beam. The total positive moment column strip value is 0.75 x 0.35 M_o = 0.75 x 0.35 $wL_2L_1^2/8$ = 0.0328 w $L_2L_1^2$ and of this 15% is attributed to the slab which over a length of L/2 would give a slab strip moment coefficient of 0.0098 corresponding to the figure in Table 6.5 of 0.006. This value is not too dissimilar in view of their small value which is actually less than the minimum allowed. A better comparison might be to examine the beam moment which is 0.85 x 0.0328 $wL_2L_1^2$ = 0.0279 $wL_2L_1^2$. The average value of the finite element positive slab moment from Table 6.3 is 0.012 and the total positive moment is assumed to be 0.35 $wL_2L_1^2/8$ = 0.0438 $wL_2L_1^2$ which if 0.012 $wL_2L_1^2$ for the full slab width is deducted gives for the beam 0.0318 $wL_2L_1^2$ compared with ACI value of 0.0279 $wL_2L_1^2$, which values are certainly extremely similar.

If the negative moments are now considered the total moment recommended is 0.75 x 0.65 x $wL_2L_1^2/8$ = 0.0609 $wL_2L_1^2$, of which 15% is attributed to the slab strip of width L/2 giving a slab column strip moment coefficient of 0.0183 wL_1^2. This value is to be compared with line (b) on Fig. 6.7 which in the edge strip from Table 6.5 has an average value of 0.043 wL_1^2. With this particular beam it would therefore appear that the slab negative strip moment coefficient is far too low. If the beam moments are compared, the total negative beam moment recommended is 0.85 x 0.0609 $wL_2L_1^2$ = 0.0518 $wL_2L_1^2$. The average negative slab moment from Table 3 is 0.0495 $wL_2L_1^2$. The total moment is 0.65 $wL_2L_1^2/8$ leaving 0.0318 $wL_2L_1^2$ to be carried by the beam. This is consistent with the previous result that too much has been attributed to the beam giving a smaller value and hence lower coefficient for the slab itself.

If we now consider the slab supported by a beam of total depth 2D then on gross cross-sectional areas only $\alpha = 0.24 \times 8D^3/4D^3 = 0.48$. Winter [4] recommends since there is T beam action that this value should be multiplied by 2 to give an α value of 1 hence the moment to be attributed by the ACI code would virtually be the same as in the previous set of comparisons. As we have noted before the middle strip values are virtually the same as the finite element analysis. The positive column strip moment is 0.75 x 0. 35 $wL_2L_1^2/8$ = 0.0328 $wL_2L_1^2$ as before which with 85% distributed to the beam gives a beam moment of 0.0279 $wL_2L_1^2$ and a slab edge moment coefficient again of 0.0098 which is too low compared with the value of 0.096 in Table 6.5. If Winter's multiplier of 2 is ignored then α = 0.48 and the beam moment would be 0.48x 0.0279 $wL_2L_1^2$ = 0.0134 $wL_2L_1^2$. This leaves (0.0328 - 0.0134)$wL_2L_1^2$ to be taken by the column strip of width $L_2/2$ giving a coefficient of 0.039 which is now too high compared with the 0.017 value.

If the negative moments are examined this time using $\alpha = 0.48$ then the beam moment would be 0.48 x 0.85 x 0.0609 $wL_2L_1^2$ = 0.0248 $wL_2L_1^2$ which leads to a slab strip moment coefficient of 0.072 which comapres favourably with the value of 0.096 in Table 6.5. Had the value of $\alpha = 1$ been taken we would again have had a strip moment coefficient of 0.0183 which is far too low. This clearly shows that the method by which α is calculated does have a significant influence on the way the moment is carried by the slab column stirp or beam.

6.6 Conclusions related to interior spans only

6.6.1 BS8110

(i) If the British Code of Practice is considered first the most general conclusion that can be reached is that a simple method for beams on semi-rigid supports is not provided and the coefficients for beams on rigid supports should only be used in cases where the total beam depth exceeds about 2.5 times the slab depth.

(ii) With beams of a lesser depth than this the slab column strip moments are significant whereas the code actually regards them as zero and only minimum steel would normally be provided.

(iii) Because the beams are designed on the basis of the reaction from the slab and the slab strength ignored then the beams are overdesigned.

(iv) It is strongly recommended that clauses on beams on semi-rigidly supported beams be included in the British code in future.

6.6.2 ACI code

(i) With one or two reservations the ACI DDM method does seem to be reasonably consistent with beam depths which vary from flat slabs to fully rigid supports.

(ii) The values recommended for both the positive and negative moments in the middle strip agree extremely well with the finite element results.

(iii) If the value of the ratio of beam/slab ratio α is based on the gross cross-sectional areas then the positive beam and column strip moments are in good agreement with the finite element results.

(iv) It would seem that in the column strip the negative moment attributed to the beam is too high with a consequent low value of the slab moment coefficient.

6.6.3 General

It is emphasized that these conclusions are based on the analysis of an interior slab only and some of the conclusions may not be valid for end spans.

It is suggested that a more detailed study with a great number of beam depths needs to be carried out before more quantitative conclusions can be drawn.

APPENDIX 6A

Typical sample design

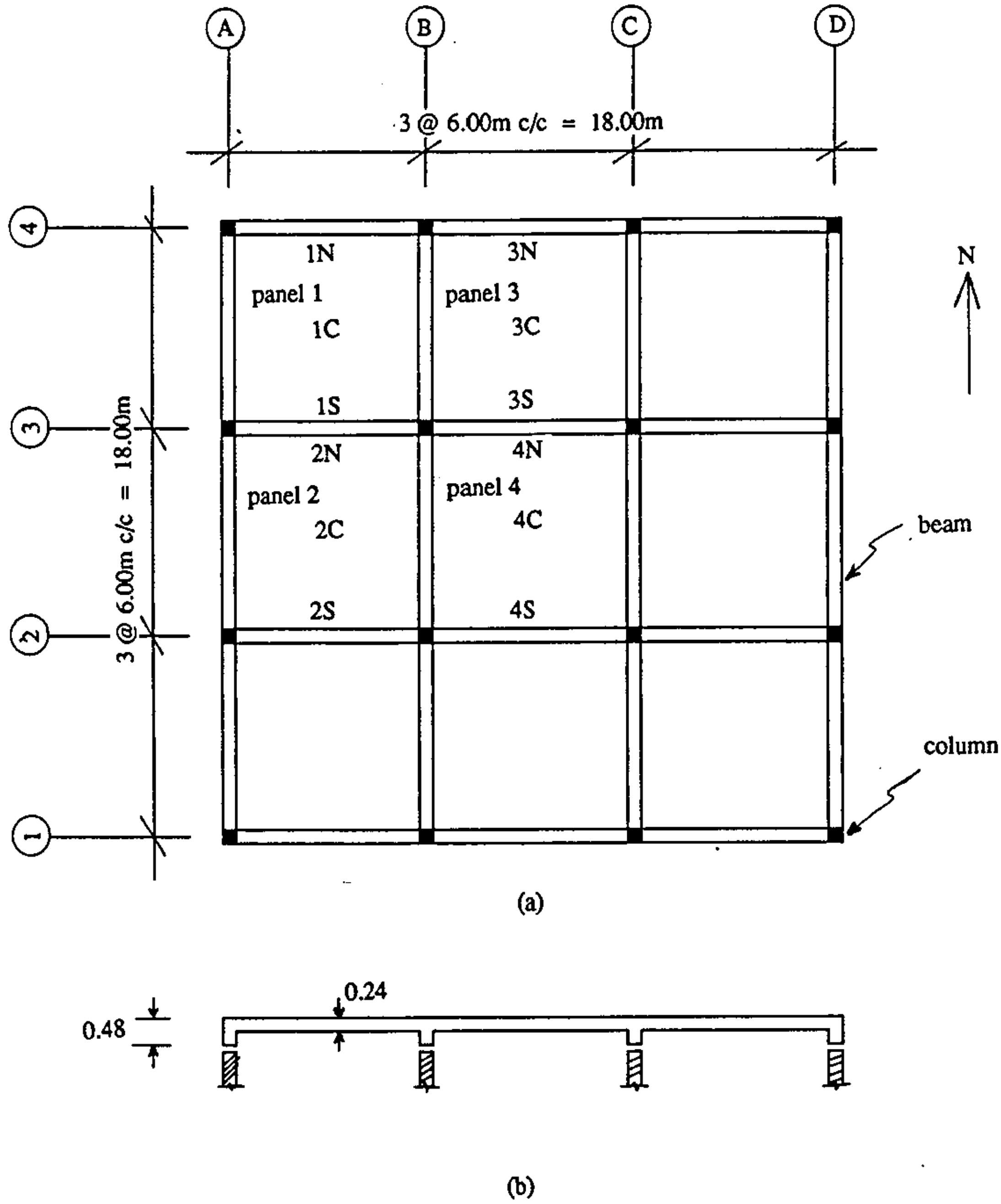


Fig. 6A1 Nine panels supported by beams

a) plan
b) cross-section

BS8110

CALCULATIONS	Comments
LOADING	
Self weight of 0.24m; 0.24 x 24 = 5.76 kN/m^2	
Others = 4.00 kN/m^2	
Therefore	
Characteristic dead load g_k = 9.76 kN/m^2	$g_k = 9.76\ kN/m^2$
Characteristic imposed load q_k = 4.00 kN/m^2	$q_k = 4.00\ kN/m^2$
Design load $n = 1.4g_k + 1.6q_k$	
$= 1.4(9.76) + 1.6(4.00)$	
$= 20.064\ kN/m^2$	
SLAB ULTIMATE BENDING MOMENTS	
Panel 1 (corner panel)	
$L_x = 6.0m$; $L_y = 6.0m$; $L_y/L_x = 1.0$	
N⟶S	
U.B.M. at cont. edge (1S) $= -0.047 \times 20.064 \times 6.0^2$	
$= -33.95$ kN.m/m	
U.B.M. at midspan (1C) $= +0.036 \times 20.064 \times 6.0^2$	
$= +26.00$ kN.m/m	
Panel 2 (edge panel)	
$L_x = 6.0m$; $L_y = 6.0m$; $L_y/L_x = 1.0$	
N⟶S	
U.B.M. at cont. edge (2N) $= -0.039 \times 20.064 \times 6.0^2$	
$= -28.17$ kN.m/m	
U.B.M. at midspan (2C) $= +0.029 \times 20.064 \times 6.0^2$	
$= +20.95$ kN.m/m	
Support moment adjustment between panels 1 and 2	

	Panel 1	Panel 2	
	3kθ	4kθ	
	0.43	0.57	Distribution coefficient
(IS)	-33.95	+28.17	(2N)
	+ 2.485	+ 3.295	
	-31.465	+31.465	

IS = 31.465 kN.m/m

which corresponds to a moment coefficient of 0.0435

Coefficient = 0.0435

Midspan moment adjustment for panel 1.

The sum of support and midspan moments before the above support adjustment, was 59.95 kN.m/m, therefore midspan moment after that adjustment becomes

59.95 - 31.465 = 28.485 kN.m/m

1C = 28.485 kN.m/m

which corresponds to a moment coefficient of 0.0394

Coefficient = 0.0394

The discontinuous edge must be provided with negative steel of one-half the positive value, i.e. 14.243 kN.m/m.

For panel 2 before the support adjustment, the sum of support and midspan moments was 49.12 kN.m/m, therefore midspan moment after that adjustment becomes

49.12 - 31.465 = 17.655 kN.m/m

2C = 17.655 kN.m/m

which corresponds to a moment coefficient of 0.0244.

Coefficient = 0.0244

Therefore the total moment in the middle strip at the critical sections is

Panel 1

Edge negative moment = 4.5 x 14.243 = 64.09 kN.m

Positive moment = 4.5 x 28.485 = 128.18 kN.m

Negative internal moment = 4.5 x 31.465 = 141.59 kN.m

Panel 2

Negative moment = 4.5 x 31.465 = 141.59 kN.m

Positive moment = 4.5 x 17.655 = 79.45 kN.m

For panel 3 (edge panel)

$L_x = 6.0m$; $L_y = 6.0m$; $L_y/L_x = 1.0$

N⟶S

U.B.M. at cont. edge (3S) $= -0.039 \times 20.064 \times 6.0^2$

$= -28.170$ kN.m/m

U.B.M. at midspan (3C) $= +0.030 \times 20.064 \times 6.0^2$

$= +21.670$ kN.m/m

Panel 4 (interior panel)

$L_x = 6.0m$; $L_y = 6.0m$; $L_y/L_x = 1.0$

N⟶S

U.B.M. at cont. edge (4N) $= -0.031 \times 20.064 \times 6.0^2$

$= -22.391$ kN.m/m

U.B.M. at midspan (4C) $= +0.024 \times 20.064 \times 6.0^2$

$= +17.335$ kN.m/m

Support moment adjustment between panels 3 and 4

	Panel 3	Panel 4		
	$3k\theta$	$4k\theta$		
	0.43	0.57	Distribution coefficient	
(3S)	-28.170	+22.391	(4N)	
	+ 2.485	+ 3.294		
	-25.685	+25.685	Final support moment	3S = 25.685 kN.m/m

which corresponds to a moment coefficient of 0.0355 — Coefficient = 0.0355

Midspan moment adjustment for panel 3

The sum of support and midspan moments before the above support adjustment, was 49.84 kN.m/m, therefore midspan moment after that adjustment becomes

49.84 - 25.685 = 24.155 kN.m/m — 3C = 24.155 kN.m/m

which corresponds to a moment coefficient of 0.0334 — Coefficient = 0.0334

The discontinuous edge must be provided with negative steel of one-half the positive value, i.e. 12.08 kN.m/m.

For panel 4 before the support adjustment, the sum of support and midspan moments was 39.726 kN.m/m, therefore midspan moment after that adjustment becomes

39.726 - 25.685 = 14.041 kN.m/m — 4C = 14.041 kN.m/m

which corresponds to a moment coefficient of 0.0194 — Coefficient = 0.0194

Therefore the total moment in the middle strip at the critical section is

Panel 3

Edge negative moment = 4.5 x 12.08 = 54.35

Positive moment = 4.5 x 24.155 = 108.698 kN.m

Negative internal moment = 4.5 x 25.685 = 115.583 kN.m

Panel 4

Negative moment = 4.5 x 25.685 = 115.583 kN.m

Positive moment = 4.5 x 14.041 = 63.185 kN.m

For column strip use minimum reinforcement:

Assuming the use of max. 12mm bars;

since the panels are square $L_y/L_x = 1.0$,

let d = the average d for upper and lower bars in mesh

d = 240 - 25 - 12 = 203mm

Min. reinforcement = (0.13/100) x 1000 x 203

= 263.9 mm^2

N.B. for the purpose of comparison with the ACI method, the column strip moment according to the minimum reinforcement will be assessed

$$M = A_s(0.87f_y)(0.9d)$$

$$= \frac{263.9}{1000 \times 1000} \times (0.87 \frac{460}{1000} \times 1000 \times 1000)(0.9 \times \frac{203}{1000})$$

$$= 19.30 \text{ kN.m/m}$$

The equivalent moment coefficient is 0.0267.

The total moment in a half column strip is

19.30 x 0.75 = 14.475 kN.m

BEAM ULTIMATE BENDING MOMENT

Beams on column line B

To assess the bending moments in the beam between panels 1 and 3 the loads from the two panels are (using Table 3.16 BS8110)

$= 0.36\ nL_x + 0.40\ nL_x$

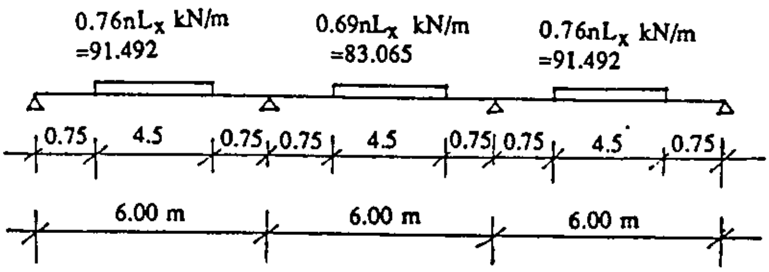
$= 0.76\ nL_x$ kN/m

and between panels 2 and 4 are

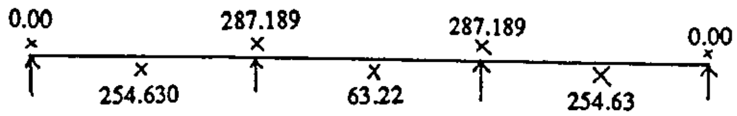
$= 0.33\ nL_x + 0.36\ nL_x$

$= 0.69\ nL_x$ kN/m

According to the recommendation in the code the distribtuion of these loads on the beam supporting two-way slabs will be as follows.

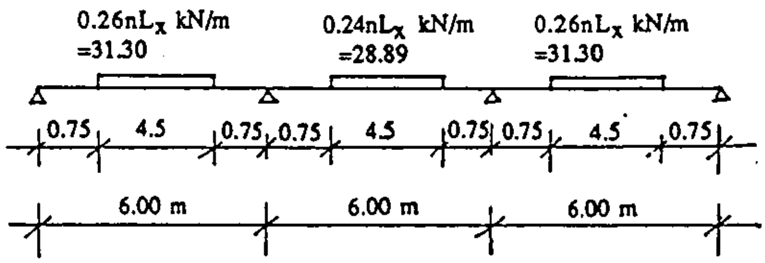


These loads lead to the following bending moments.

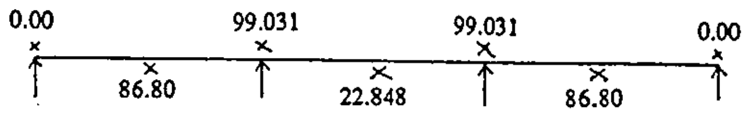


Beams on column line A.

Using the same procedure for edge beams on column line A, the distribution of loads will be as follows



which results in the following bending moments



SUMMARY

We may therefore calculate the moments at the various sections as follows.

Column line A and edge strip

Exterior negative beam moment = 0.00

Exterior negative edge strip moment = 7.238 kN.m

First span positive beam moment = 86.80 kN.m

First edge strip positive moment = 14.475 kN.m

First interior negative edge column beam moment = 99.031 kN.m

First interior negative edge strip moment = 14.475 kN.m

Interior edge beam positive moment = 22.848 kN.m

Interior edge strip positive moment = 14.475 kN.m

Middle strip between column lines A and B

Exterior negative edge moment = 0.5 x 28.485 x 4.5 = 64.09 kN.m

First interior positive moment = 28.485 x 4.5 = 128.183 kN.m

First interior negative moment = 31.465 x 4.5 = 141.593 kN.m

Interior positive moment = 17.655 x 4.5 = 79.45 kN.m

Column line B and column strip

Exterior negative beam moment = 0.00

Exterior negative column strip moment for slab only = 7.238 x 2 = 14.475 kN.m

First span positive beam moment = 254.630 kN.m

First column strip positive moment for slab only = 14.475 x 2 = 28.95 kN.m

First interior negative beam moment = 287.189 kN.m

First interior negative column strip moment for slab only = 14.475 x 2 = 28.95 kN.m

Interior positive moment for the beam = 63.22 kN.m

Interior positive column strip moment for slab only = 14.475 x 2 = 28.95 kN.m

Middle strip between column lines B and C

Exterior negative edge moment = 0.5 x 24.155 x 4.5 = 54.35 kN.m

First interior positive moment = 24.155 x 4.5 = 108.70 kN.m

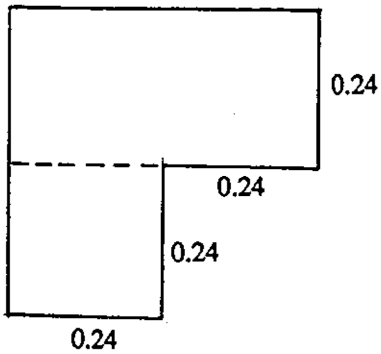
First interior negative moment = 25.685 x 4.5 = 115.583 kN.m

Interior positive moment = 14.041 x 4.5 = 63.185 kN.m

ACI Code

CALCULATIONS	Comments
loading	
$L_1 = 6.00m$	
$L_2 = 6.00m$	
$L_n = 5.76m$	
$h = 0.24m$	
Self weight of 0.24m slab = 0.24 x 24 = 5.76 kN/m^2	
Others = 4.00 kN/m^2	
Therefore total dead load (D.L) = 9.76 kN/m^2	D.L = 9.76 kN/m^2
Live load (L.L) = 4.00 kN/m^2	L.L = 4.00 kN/m^2
Ultimate factored load (w_u) = 1.4 D.L + 1.7 L.L	
=1.4(9.76) + 1.7(4.00)	
= 20.464 kN/m^2	w_u = 20.464 kN/m^2
Calculation of Beam and Strip Method	
Therefore total static moment (M_o)	
$= \frac{w_u L_2 L_n^2}{8}$	
$= \frac{20.464 \times 6.00 \times (5.76)^2}{8}$	
= 509.21 kN.m	M_o = 509.21 kN.m
For the slab of width 4m	
$I_s = \frac{bh^3}{12}$	
$= \frac{4.00 \times (0.24)^3}{12}$	
= 4.608 E - 03	

$$C = \sum \left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3}$$



$$C_1 = \left(1 - 0.63\,\frac{0.24}{0.48}\right)\frac{(0.24)^3(0.48)}{3}$$

$$= 1.5151\ \mathrm{E} - 03$$

$$C_2 = \left(1 - 0.63\,\frac{0.24}{0.24}\right)\frac{(0.24)^3(0.24)}{3}$$

$$= 4.0919\ \mathrm{E} - 04$$

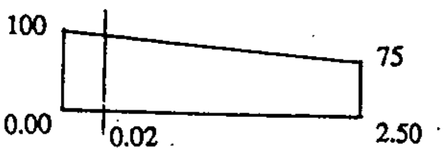
Therefore $C = 1.9243\ \mathrm{E} - 03$

$$\beta_t = \frac{C}{2I_s}.$$

$$= \frac{1.9243\ \mathrm{E} - 03}{2 \times 4.608\ \mathrm{E} - 03}$$

$$= 0.2088$$

using interpolation of the values from Table 6.2



Therefore dist. ratio of moment to column strip = 99.8%

use 100%

The edge restraint is judged to be of type (b) in Table 6.1 hence the column strip will carry $0.16\ M_o = 81.474$ kN.m

For interior beam

$$I_b = \frac{1}{12} \times 0.24 \times 0.48^3 \times 2$$

Factor 2 suggested by Winter for T beam

$$= 4.42368\text{ E - }03$$

$$I_s = \frac{1}{12} \times 4.00 \times 0.24^3$$

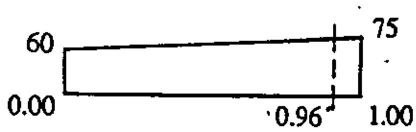
$$= 4.608\text{ E - }03$$

$$\alpha = \frac{I_b}{I_s}$$

$$= \frac{4.42368\text{ E - }03}{4.608\text{ E - }03}$$

$$= 0.96$$

For the positive moment in the exterior span from Table 6.1 the total moment is 0.57 M_0 (case b) and from Table 6.2 using interpolation



The column strip will carry 74.4% of this.

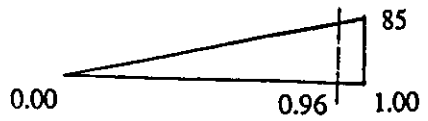
At the first interior column the total moment is 0.7 M_0 of which 75% is taken by the column strip.

For the first interior span from equation 6.1 the total moment is 0.65 M_0 but the value of 0.70 M_0 is larger therefore this will be taken. The total positive moment is 0.35 M_0 again of which 75% is taken by the column strip.

Column strip moment distribution between the beam and the slab. The factor

$$\alpha_1 \frac{L_2}{L_1} = 0.96$$

Therefore by interpolation



The beam will take 81.6% of moment with 1 - 0.816 = 0.184 for the slab in column strip. This applies to all moments for an interior beam. For the exterior beam the ratio is near to 85% but the same ratio will be used.

SUMMARY

We may therefore calculate the moments at the various sections as follows.

<u>Column line A and edge strip</u>

Exterior negative beam moment = 0.16 x (509.21/2) x 0.816 = 33.241 kN.m

Exterior negative edge strip moment = 0.16 x (509.21/2) x 0.184 = 7.50 kN.m

First span positive beam moment = 0.57 x (509.21/2) x 0.744 x 0.816 = 88.11 kN.m

First edge strip positive moment = 0.57 x (509.21/2) x 0.744 x 0.184 = 19.87 kN.m

First interior negative edge column beam moment

= 0.70 x (509.21/2) x 0.75 x 0.816 = 109.07 kN.m

First interior negative edge strip moment

= 0.70 x (509.21/2) x 0.75 x 0.184 = 24.60 kN.m

Interior edge beam positive moment = 0.35 x (509.21/2) x 0.75 x 0.816 = 54.54 kN.m

Interior edge strip positive moment = 0.35 x (509.21/2) x 0.75 x 0.184 = 12.30 kN.m

<u>Middle strip between column lines A and B</u>

Exterior negative edge moment = 0.0

First interior positive moment = 0.57 x 509.21 x 0.256 = 74.30 kN.m

First interior negative moment = 0.70 x 509.21 x 0.25 = 89.11 kN.m

Interior positive moment = 0.35 x 509.21 x 0.25 = 44.56 kN.m

Column line B and column strip

Exterior negative beam moment = 0.16 x 509.21 x 0.816 = 66.482 kN.m

Exterior negative column strip moment for slab only

= 0.16 x 509.21 x 0.184 = 14.99 kN.m

First span positive beam moment = 0.57 x 509.21 x 0.744 x 0.816 = 176.21 kN.m

First column strip positive moment for slab only

= 0.57 x 509.21 x 0.744 x 0.184 = 39.73 kN.m

First interior negative beam moment = 0.70 x 509.21 x 0.75 x 0.816 = 218.15 kN.m

First interior negative column strip moment for slab only

= 0.70 x 509.21 x 0.75 x 0.184 = 49.19 kN.m

Interior positive moment for the beam = 0.35 x 509.21 x 0.75 x 0.816 = 109.07

Interior positive column strip moment for slab only

= 0.35 x 509.21 x 0.75 x 0.184 = 24.60 kN.m

Middle strip between column lines B and C

Exterior negative edge moment = 0.0

First interior positive moment = 74.30 kN.m

First interior negative moment = 89.11 kN.m

Interior positive moment = 44.56 kN.m

APPENDIX 6B

Computer program to convert principal stresses to normal stress in the global axis set for each node of the panel

APPENDIX 6B

Computer program to convert principal stresses to normal stress in the global axis set for each node of the panel.

```
CCC    PROGRAME NO. 3
       DIMENSION X(12)
            CHARACTER*32 FNAME
       REAL SXT, SYT, SXYT, SXB, SYB, SXYB
       PARAMETER (PI=3.14159265)
         WRITE(1,'(''  ENTER SOURCE FILE NAME '')')
         READ(1,'(A)')FNAME
       OPEN (7,FILE=FNAME,STATUS='OLD')
          WRITE(1,'(''  ENTER RESULTS FILENAME '')')
          READ(1,'(A)')FNAME
       OPEN (8,FILE=FNAME,STATUS='NEW')
       READ(7,'(//)')
10     READ(7,*,END=100)I1, I2, I3,(X(I),I=4,12)
       X(1)=I1
       X(2)=I2
       X(3)=I3
       X(6)=X(6)*PI/180.0
       X(12)=X(12)*PI/180.0
       SXT=X(4)*(COS(X(6)))**2+X(5)*(SIN(X(6)))**2
       SYT=X(4)*(SIN(X(6)))**2+X(5)*(COS(X(6)))**2
       SXYT=(X(4)-X(5))*SIN(X(6))*COS(X(6))
       SXB=X(10)*(COS(X(12)))**2+X(11)*(SIN(X(12)))**2
       SYB=X(10)*(SIN(X(12)))**2+X(11)*(COS(X(12)))**2
       SXYB=(X(10)-X(11))*SIN(X(12))*COS(X(12))
       WRITE(8,'(2I8,3X,6F14.4)')I1,I3,SXT,SYT,SXYT,SXB,SYB,SXYB
       GO TO 10
100    CLOSE (7)
       CLOSE (8)
       STOP
       END
```

APPENDIX 6C

Computer program to determine the average direct stress
at each node and the associated moment

APPENDIX 6C

Computer program to determine the average direct stress at each node and the associated moment.

```
CCC     PROGRAME NO. 4
        DIMENSION X(12)
             CHARACTER*32 FNAME
         REAL AVE1,AVE2,AVE3
          WRITE(1,'(''  ENTER SOURCE FILE NAME '')')
          READ(1,'(A)')FNAME
        OPEN (7,FILE=FNAME,STATUS='OLD')
           WRITE(1,'(''  ENTER RESULTS FILENAME '')')
           READ(1,'(A)')FNAME
        OPEN (8,FILE=FNAME,STATUS='NEW')
        H=0.24
        READ(7,'(//)')
10      READ(7,*,END=100)I1,I2,(X(I),I=3,8)
        X(1)=I1
        X(2)=I2
        Z=(H**2)/6.0
         AVE1=(ABS(X(3))+ABS(X(6)))/2.0
        IF(X(6).LT.0.0)THEN
        AVE1=(-1.0)*AVE1
        END IF
        AVE1=AVE1*Z
        AVE2=(ABS(X(4))+ABS(X(7)))/2.0
        IF(X(7).LT.0.0)THEN
        AVE2=(-1.0)*AVE2
        END IF
        AVE2=AVE2*Z
        AVE3=(ABS(X(5))+ABS(X(8)))/2.0
        IF(X(8).LT.0.0)THEN
        AVE3=(-1.0)*AVE3
        END IF
        AVE3=AVE3*Z
        WRITE(8,'(2I8,3X,3F14.4)')I1,I2,AVE1,AVE2,AVE3
        GO TO 10
100     CLOSE (7)
        CLOSE (8)
        STOP
        END
```

APPENDIX 6D

Computer program to calculate the average nodal moment at each node due to the different elements meeting at the node

APPENDIX 6D

Computer program to calculate the average nodal moment at each node due to the different elements meeting at the node.

```
CCC    PROGRAME NO. 5
            CHARACTER*32 FNAME
       REAL A(5),B(5),C(5)
       INTEGER NUMNOD(2000)
       REAL AV1(2000),AV2(2000),AV3(2000)
       LOGICAL FIRST
       DATA COL1/0.0/COL2/0.0/COL3/0.0/
       FIRST=.TRUE.
       ICNT=1
     -   WRITE(1,'('' ENTER SOURCE FILE NAME '')')
         READ(1,'(A)')FNAME
       OPEN (5,FILE=FNAME,STATUS='OLD')
          WRITE(1,'('' ENTER RESULTS FILENAME '')')
          READ(1,'(A)')FNAME
       OPEN (6,FILE=FNAME,STATUS='NEW')
 10    READ(5,*,END=99)IDUM,I,A(ICNT),B(ICNT),C(ICNT)
       IF(FIRST) THEN
        ITEMP=I
        FIRST=.FALSE.
        ICNT=ICNT+1
        GOTO 10
       ELSE
           IF(I.NE.ITEMP) THEN
             LAST=ITEMP
             NUM=ICNT-1
             DO 20 K= 1,NUM
             COL1=COL1+A(K)
             COL2=COL2+B(K)
             COL3=COL3+C(K)
 20          CONTINUE
             AV1(ITEMP)=COL1/NUM
             AV2(ITEMP)=COL2/NUM
             AV3(ITEMP)=COL3/NUM
             NUMNOD(ITEMP)=NUM
             A(1)=A(ICNT)
             B(1)=B(ICNT)
             C(1)=C(ICNT)
             DO 30 K=1,NUM
             COL1=0
             COL2=0
             COL3=0
 30          CONTINUE
             ICNT=2
              ITEMP=I
             GOTO 10
          ELSE
             ICNT=ICNT+1
             GOTO 10
          ENDIF
        ENDIF
 99    DO 100 I = 1,LAST
       WRITE(6,999)I,NUMNOD(I),AV1(I),AV2(I),AV3(I)
 100   CONTINUE
       CLOSE(5)
       CLOSE(6)
 999   FORMAT('NODE = ',I4,' NO OF POINTS = ',I3,/,10X,3(2X,F12.4))
       STOP
       END
```

CHAPTER 7
FLAT SLABS

7.1 Introduction

A flat slab is a reinforced concrete slab, without beams to transfer the loads to supporting members. The slab may be of constant thickness throughout or it may be thickened with a drop panel in the area of the column. The column may also be of constant section or it may be flared to form a column head or capital.

The work reported in this chapter is confined to flat slabs without a drop panel or flared head to the column.

The purpose of this chapter is to describe and demonstrate the principal steps of the procedures for the main methods recommended in both codes, namely the equivalent frame method, the simplified coefficient method, finite element analysis and yield-line analysis, and to compare the various results obtained.

7.2 BS8110 Code Requirements

7.2.1 Introduction

BS8110 gives provisions for designing flat slabs with aspect ratios not greater than 2 and supported on columns positioned at the intersection of rectangular grid lines. These provisions include a simplified method based on coefficients, the equivalent frame method and other methods such as yield-line, Hillerborg or elastic finite element analysis techniques.

7.2.2 Simple coefficient method

The simple method based on bending moment and shear force coefficients is subject to the following conditions:

(a) there are at least three rows of panels of approximately equal spans in the direction being considered; and

(b) the single load case of the maximum design load on all spans only is considered.

The coefficients for use with the simplified (Direct Design) method are reproduced in Table 7.1. These coefficients will be compared later in this chapter, with the resulting coefficients due to the EFM and finite element analysis. It should be noted when using the simplified code coefficients with the case of a single load on all spans, the resulting moment should not be redistributed, since the coefficients given already allow for redistribution.

	Outer support Column	Outer support Wall	Near centre of first span	First interior support	Centre of interior span	Interior support
Moment	-0.04FL*	-0.02FL	+0.083FL*	-0.063FL	+0.071FL	0.055FL
Shear	0.45F	0.4F	-	0.6F	-	0.5F
Total col. moments	0.04FL	-	-	0.022FL	-	0.022FL

*The design moments in the edge panel may have to be adjusted to comply with BS8110 3.7.4.3.

NOTE 1. F is the total design ultimate load on the strip of slab between adjacent columns considered (i.e. $1.4G_k + 1.6Q_k$).

NOTE 2. L is the effective span = $L_1 - 2h_c/3$.

NOTE 3. The limitations of BS8110 section 3.7.2.6 need not be checked.

NOTE 4. These moments should not be redistributed.

Table 7.1 Bending moment and shear force coefficients for flat slabs of three or more equal spans (Table 3.19 BS8110)

The division of these total moments between the column and middle strips is the same as in the EFM method (Table 7.2).

7.2.3 Equivalent frame method

7.2.3.1 Frame representation

The first stage in the analysis by the equivalent frame method is the representation of the actual three-dimensional structure containing flat slabs, as floors for instance, by a number of equivalent frames, as shown in Fig. 7.1. These equivalent frames consist of a row of columns and strips of supported slabs. Each strip is

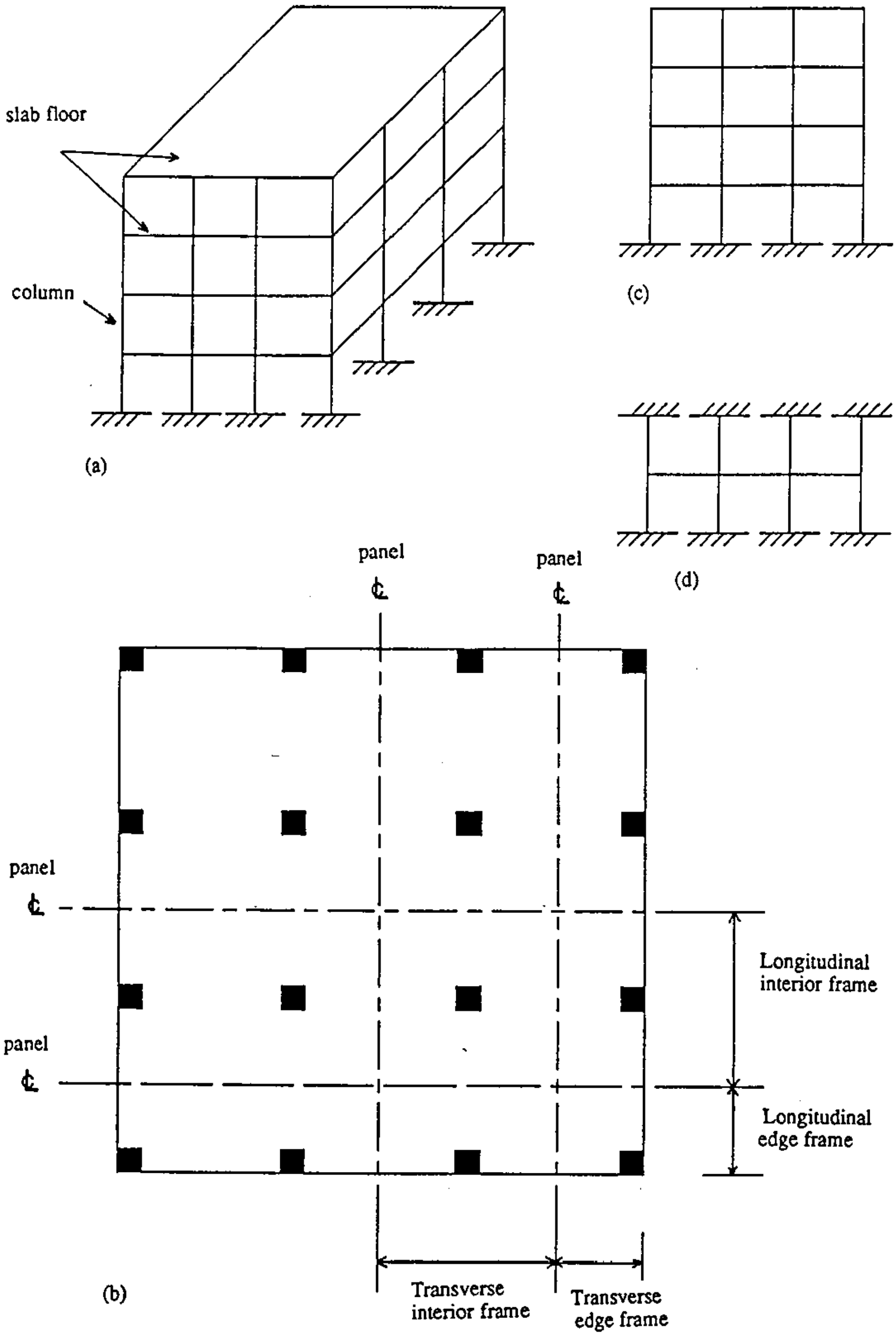


Fig. 7.1 Equivalent frame :

(a) Three dimensional multi-bay multi-story building.
(b) Plan of equivalent frames.
(c) Elevation of equivalent frame.
(d) Elevation of sub-frame.

bounded laterally by the centre line of the panel on either side of the centre line of the columns. The equivalent frames are taken longitudinally and transversely across the building.

In order to determine the effect of vertical loading on the floor slab it is sufficiently accurate to consider the sub-frame of Fig. 7.1(d) with the columns above and below the floor under consideration fixed at their far ends.

The stiffness of the columns in the equivalent frame is equal to the stiffnesses of the actual columns and the stiffness of the beams in an equivalent frame is equal to the stiffnesses of the $\frac{1}{2}$widths of slab on either side of the column as shown in Fig. 7.1(b). When a structure is subjected to horizontal loading it is necessary to consider the full frame of Fig. 7.1(c) in order to determine the effect of the loading on the floor slab.

7.2.3.2 Load arrangement and design moment

When using the BS8110-based EFM, considerable simplification in loading arrangements can be made if the imposed load is not greater than the dead load and if the area of a bay exceeds 30 m^2. In such cases, it is only necessary to consider the single load case of the maximum ultimate design load on all spans. Where this single load case has been assumed in design by the equivalent frame method, the support moments may be reduced by 20%, with a resulting increase in the span moments.

For the more general case of loading, the code recommends the application of the following two arrangements of loading:

(a) alternate spans loaded with the maximum ultimate design load ($1.4\ G_k + 1.6\ Q_k$) and all other spans loaded with the minimum dead load ($1.0\ G_k$); and

(b) all spans loaded with the maximum design ultimate load ($1.4\ G_k + 1.6\ Q_k$).

Since the EFM models columns by centre-lines, the thickness of a column needs to be borne in mind when considering the design moment to apply to it. Thus, BS8110 specifies that the negative moment to be applied to the column is that at a distance $h_c/2$ from the centre line of the column (where h_c is the effective diameter of a column). This procedure should be done providing the sum of the maximum positive design moment (M_3) and the average of the negative design moments ($(M_1 + M_2)/2$) in any one span of the slab for the whole panel width is not less than

$$M_o = \frac{nL_2}{8}(L_1 - \frac{2h_c}{3})^2 \qquad (7.1)$$

or

$$(\frac{M_1 + M_2}{2}) + M_3 \nless \frac{nL_2}{8}(L_1 - \frac{2h_c}{3})^2 \qquad (7.2)$$

When the above condition is not satisfied, the negative design M_1 and M_2 moments should be increased in their ratio to this value.

7.2.3.3 Panel division and their apportionments

A flat slab panel shall be considered as consisting of strips in each direction. BS8110 gives different consideration for the edge or corner panels and interior panels. Interior panels are divided as shown in Fig. 7.7a. In the case of panels with different dimensions meeting at a common support, the division of the panels into strips over the region of the common support should be taken as that calculated for the panel giving the wider column strip.

Having analysed the equivalent frame for design moments, the moments at critical sections should be apportioned between the column strip and middle strip, as given in Table 7.2.

Table 7.2: Division of design moments at critical sections between strips comprising the panel

	Column strip	Middle strip
Negative moment	75%	25%
Positive moment	55%	45%

In the case of an edge or corner panel the positive design moments in the span and negative design moments over interior edges should be apportioned and designed exactly as for an internal panel,using the same definition of column and middle strips as for an internal panel.

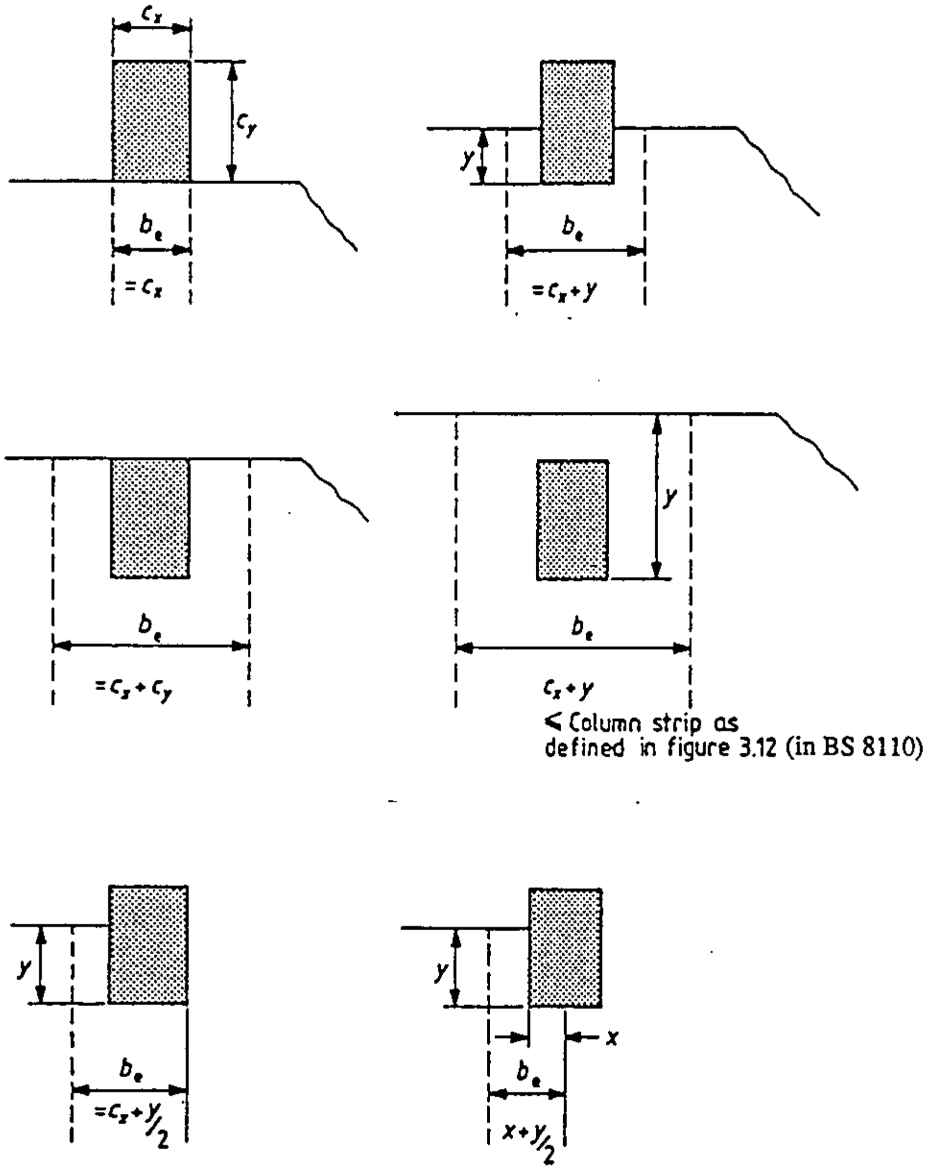
Particular care is given for design moment transfer between a slab and edge or corner columns by ensuring a satisfactory breadth of a moment transfer strip. This is necessary because since there is no marginal beam, column strips needed near the edge or corner columns are generally narrower than that appropriate for an internal panel. The breadth of this strip (or moment transfer strip), b_e, for various typical cases is shown in Fig. 7.2. The value of b_e should never be taken as greater than the column strip width appropriate for an interior panel.

The maximum design moment, $M_{t,max}$, which can be transferred to the column by this moment transfer strip is:

$$M_{t,max} = 0.15\, b_e d^2 f_{cu} \tag{7.3}$$

where d is the effective depth of top reinforcement.

$M_{t,max}$ must exceed 50% of the design moment, as obtained by an analysis based on the equivalent frame method, otherwise the structural arrangements should be changed. Having obtained an acceptable value for $M_{t,max}$, the design edge moment in the slab should be reduced to a value not greater than $M_{t,max}$ and the positive design moments in the span adjusted accordingly. In the middle strip at the edge of an edge panel, reinforcement for negative design moments is only needed in the cases when there is a moment arising from loading on the extension of the slab beyond the column centre line and top reinforcement at least equal to the recommended minimum reinforcement should be provided and extending into the span.



NOTE. y is the distance from the face of the slab to the innermost face of the column.

Fig. 7.2 Definition of breadth of effective moment transfer strip, b_e, for various typical cases (Figure 3.13 BS 8110).

7.2.3.4 Reinforcement layout

Generally, bending moments change throughout the slab and the magnitude of the bending moments at critical sections decrease at locations away from these sections. The area of bending reinforcement may therefore be reduced by curtailing bars where they are no longer required. Naturally, each curtailed bar should extend beyond the point at which it is no longer needed so that it may be anchored into the concrete.

The BS8110 code gives simplified rules for curtailing bars, as shown in Fig. 7.3.

It is recommended in the code to place the area of the negative reinforcement apportioned to the column strip nearer to the column's centre line. This is done by using two-thirds of the reinforcement area apportioned for the column strip and placing it on the half-column strip nearer to the column's centre line, leaving the other half-column strip with the rest of the reinforcement.

7.3 ACI Code Requirements

The ACI code describes two general approaches, the Direct Design Method (DDM) and the Equivalent Frame Method (EFM), which can be used for the design of flat slabs. As stated earlier in section 2.3 the EFM will be used to demonstrate the design of a flat slab. The preferred method recommended by ACI is the equivalent frame method and this requires the use of either the moment distribution method or any suitable elastic method to obtain forces and moments at critical sections. The ACI code also permits the use of the finite element analysis and other approaches such as yield-line analysis and the Hillerborg method, provided that strength and serviceability requirements are met.

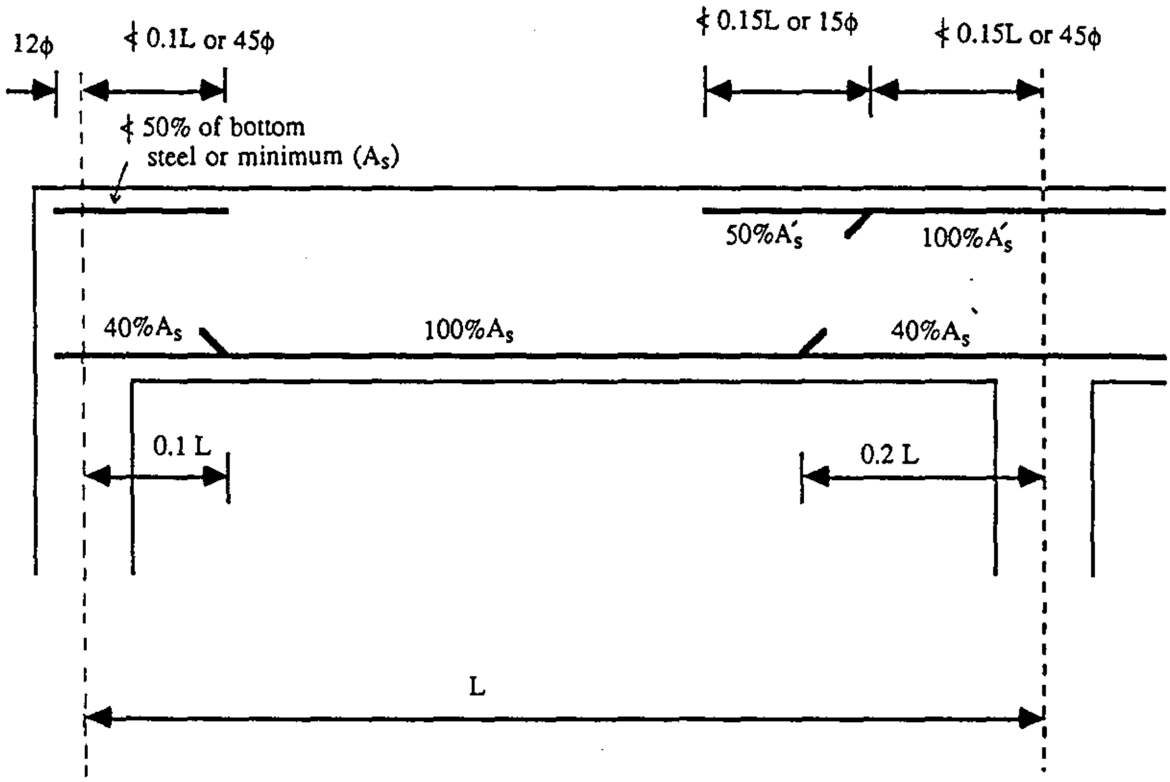


Fig. 7.3 BS 8110 simplified rules for curtailment of bars in flat slab (adapted from BS 8110).

7.3.1 The Direct Design Method

This method has been described in Chapter 6 and therefore will not be repeated here; it can be regarded as a slightly more sophisticated version of the BS8110 simple coefficient method.

7.3.2 Equivalent Frame Method

7.3.2.1 Frame representation

The same idealization used in BS8110 to divide the structure into equivalent frames in longitudinal and transverse direction, is adopted in the ACI code.

In multi-storey multi-bay buildings the equivalent frame may be analysed in its entirety or each floor separately by using the sub-frames.

To establish the slab load and slab stiffness, the slab width considered is one-half of the panel width on each side of the column in question. The stiffness is based on gross concrete area.

Unlike BS8110, the ACI code uses the equivalent column stiffness K_{ec} in its analysis. This equivalent column stiffness is due to the consideration that some part of the slab behaves as a torsional member and requires the introduction of a torsional stiffness effect to the system. Figure 7.4 shows an equivalent column which represents the column above and below a slab plus an attached torsional member transverse to the direction in which moments are being determined and extending to the bounding lateral panel centre lines on each side of the column.

The flexibility (inverse of the stiffness) of the equivalent column is taken as the sum of the flexibility due to the actual columns (i.e. $1/\Sigma K_c$) and the flexibility of the torsional member ($1/K_t$), namely

$$\frac{1}{K_{ec}} = \frac{1}{\Sigma K_c} + \frac{1}{K_t} \tag{7.4}$$

The stiffness K_t of the torsional member is calculated from the definition of its cross-section as shown in Fig. 7.5 and is expressed by the following

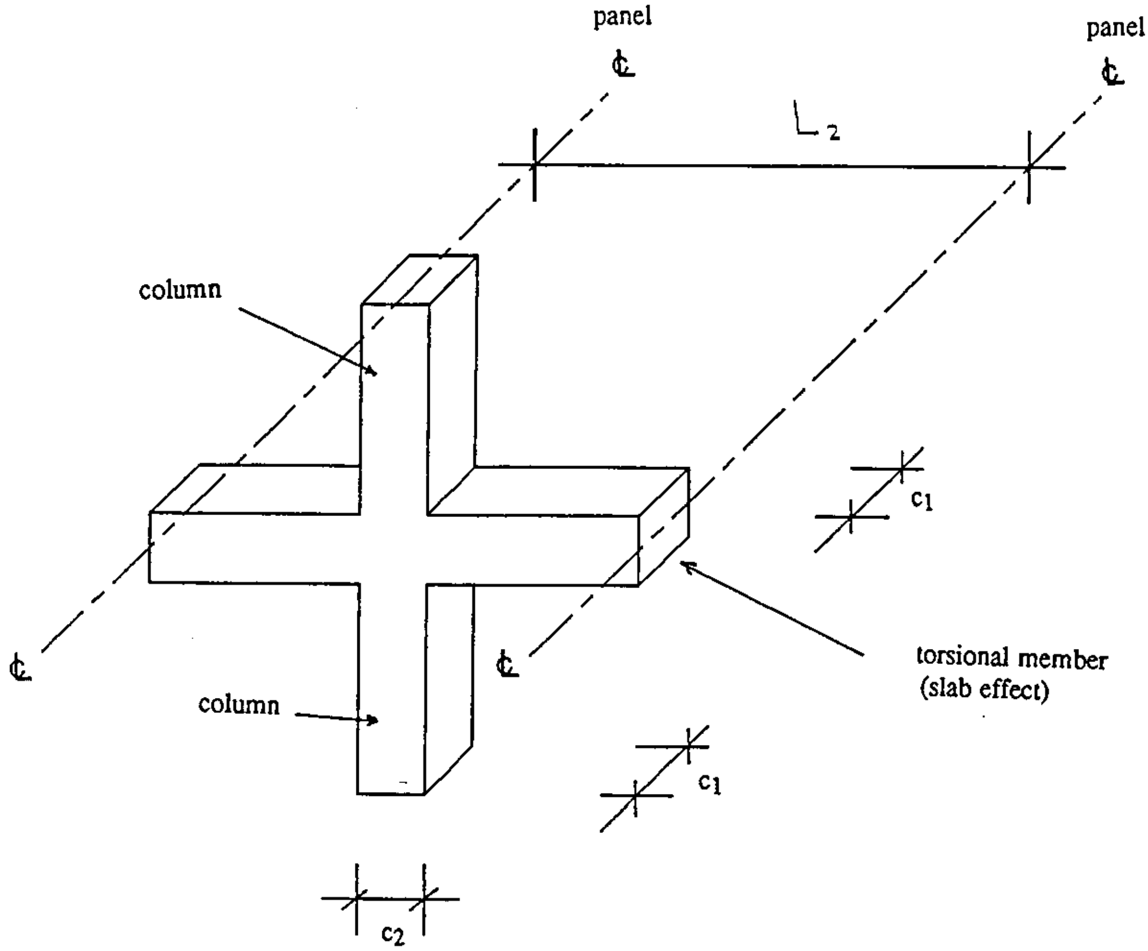
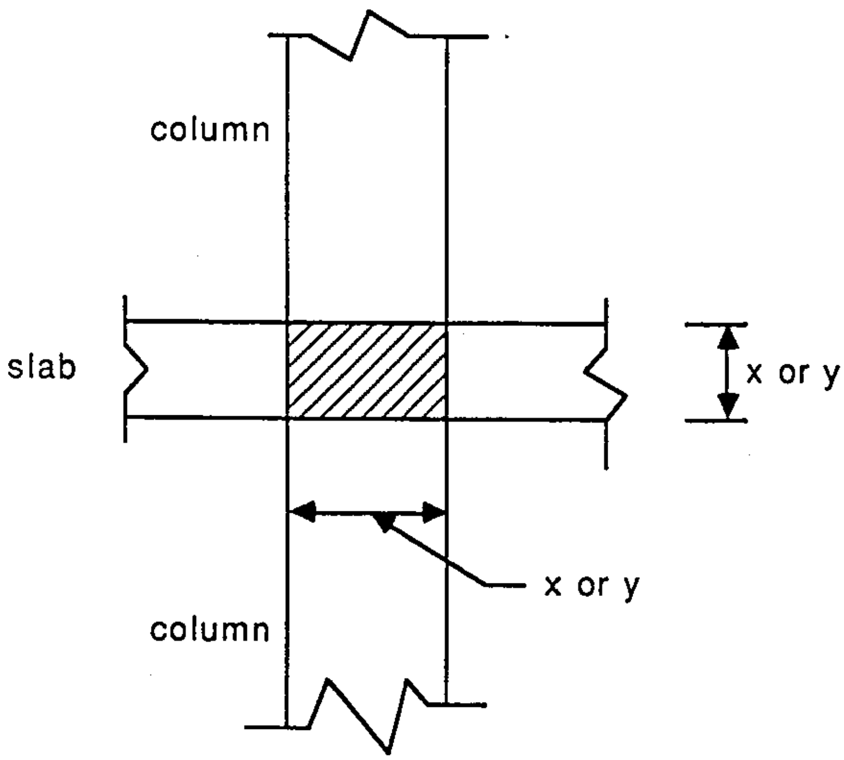


Fig. 7.4 The equivalent column concept.



x is the smaller dimension

Fig. 7.5 Effective cross section (shown shaded) of the torsion arm.

$$K_t = \sum \frac{9\,E_{cs}\,C}{L_2(1 - \frac{C_2}{L_2})^3} \tag{7.5}$$

where C is given by

$$C = \sum (1 - 0.63\,\frac{x}{y})\,\frac{x^3 y}{3} \tag{7.6}$$

where E_{cs} = modulus of elasticity of slab concrete

C_2 = size of rectangular column, capital, or bracket in direction L_2

C = cross-sectional constant

x = smaller dimension of the torsional member

and y = larger dimension of the torsional member.

The summation in equation 7.5 applies to the typical case in which there are edge beams (or torsional member in flat slab) on both sides of the column.

While the summation in equation 7.6 is for the general purpose of the EFM which cover the slab supported by beams. The torsioned constants for edge beams (L-shaped) and interior beams (T-shaped) are due to the summation of the rectangular parts of each shape.

It is noted that the introduction of the torsional stiffness effect in the equivalent column concept is suitable when using moment distribution or other hand calculation procedures of analysis.

7.3.2.2 Load arrangement and design moment

In situations where the pattern of loading is known, the structure should be analysed for that load system. If the loading pattern is not known ACI specifies the following procedure.

When the unfactored live load does not exceed three-quarters of the unfactored dead load, or the nature of the live load is such that all panels will be loaded simultaneously, the maximum moments may be assumed to occur at all sections with full factored live and dead load on all spans of the system (see Fig. 7.6a).

If the unfactored live load exceeds three-quarters of the unfactored dead load then pattern loadings need to be considered as follows.

(a) For the maximum positive moment, factored dead load on all spans and 0.75 times the full factored live load on the panel in question and on alternate panels, see Fig. 7.6b.

(b) For the maximum negative moment at an interior support, factored dead load on all panels and 0.75 times the full factored live load on the two adjacent panels, see Fig. 7.6c.

For these pattern loadings the final design moments shall not be less than for the case of full factored dead and live load on all panels, (as in Fig. 7.6a).

Structural analysis employing the Equivalent Frame Method gives moments at the centre of the joint where ends of the members meet. In order to allow for the thickness of the supports the ACI code permits the moments at the face of the (equivalent) rectangular cross section of the support to be used in the design of the slab reinforcement. If the support does not have a rectangular cross section then the ACI code specifies that it should be treated as a square section support of the same area. The code specifies that for columns extending more than 0.175 L_1 from the face of the support, the moments can be reduced to the values existing at 0.175 L_1 from the centre of the joint. Since these moments that the ACI code requires to be used for the supports are not those calculated by the EFM for the centre of the joint where the supports meet, it is necessary to check for static equilibrium. Therefore if the total of the design moments (i.e. the positive moment plus the average of the negative end moments) obtained for a particular span is greater than $M_o = w_u L_2 L_n^2/8$ (which is the required value for static equilibrium), the code permits a reduction in those moments proportionately so that their sum does equal M_o.

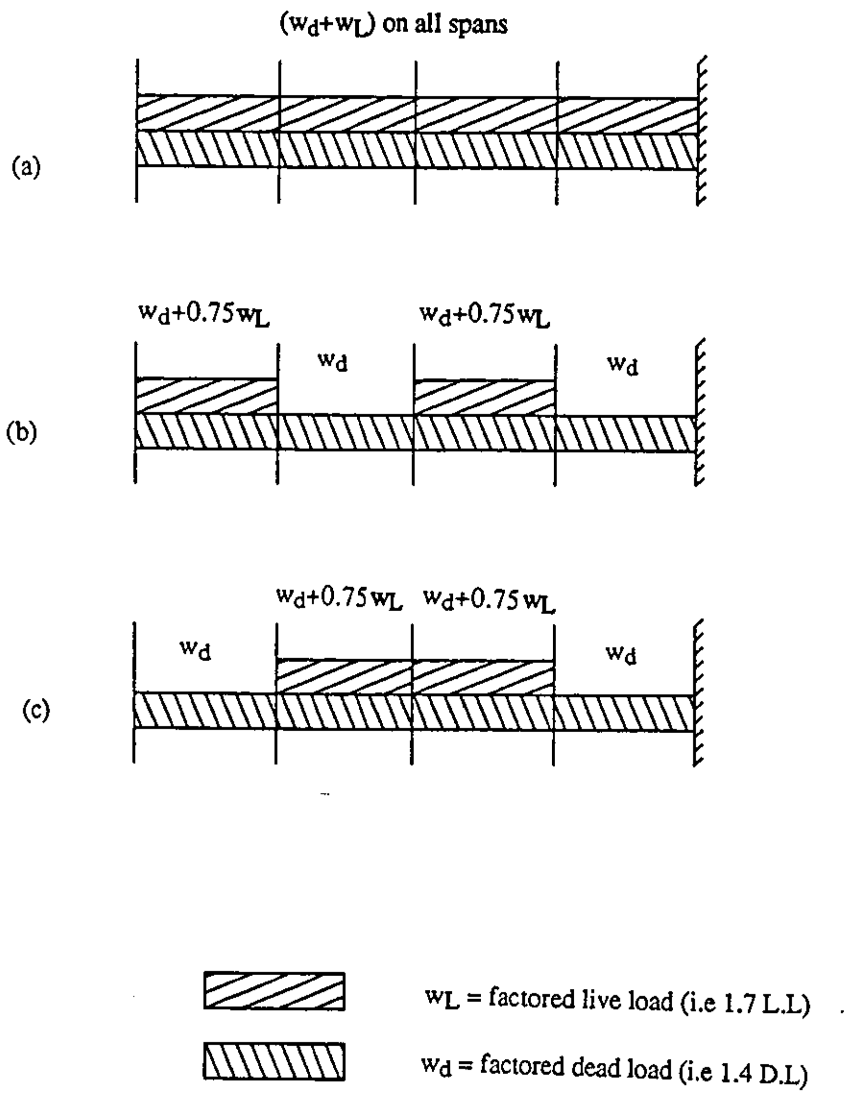


Fig. 7.6 ACI loading arrangement

(a) Full factored dead and live load on all spans.
(b) Alternative span loading for maximum positive moment at midspan.
(c) Adjacent span loading for maximum negative moment at the support.

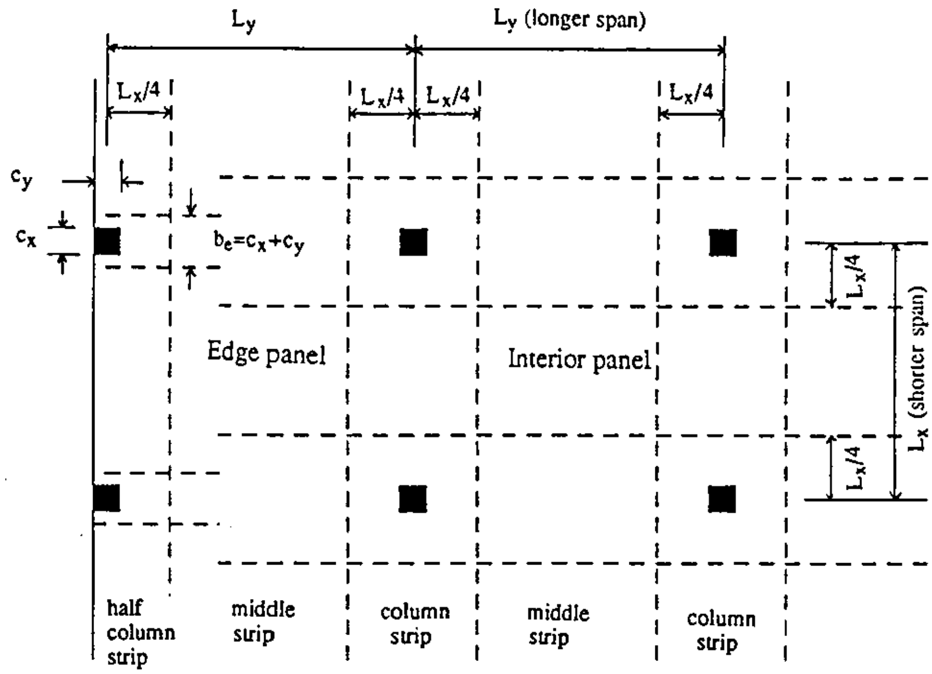
However, it is important to mention that the above adjustment for static equilibrium is not needed if the analysis is done by the moment distribution method. This is because the analysis based on the moment distribution method gives only the moments at the ends of members and the positive moment at midspan will be derived by subtracting the average negative end-moments from the total static moment. As a result, the sum of this positive midspan moment and the average of negative moments at the face of the supports, rather than the average of the maximum negative moments at the joints (which will have larger magnitudes than at the faces of the supports), will be less than the total static moment of the span, i.e. there is no need for the ACI adjustment for static equilibrium mentioned above.

Negative and positive factored moments may be modified by 10% in case of all spans loaded with full factored load, provided the total static moment for a panel in the direction considered is not less than that required by $(w_u L_2 L_n^2)/8$. In the case of using pattern loading no redistribution is needed since the factored live load will be multiplied by 0.75.

7.3.2.3 Panel division and their apportionments

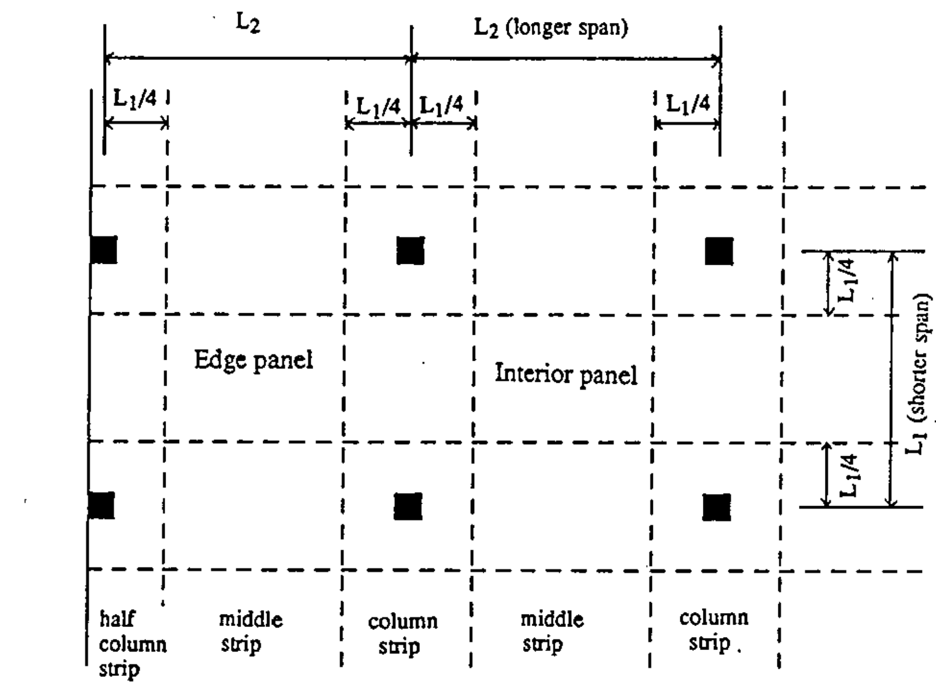
A flat slab shall be considered as consisting of strips in each direction as shown in Fig. 7.7b.

Once the negative and positive moments have been determined for each equivalent frame, these are distributed to column and middle strips of the flat slab in accordance with the apportionment given in Table 7.3, which has been extracted from the ACI code's Tables in clause 13.6.4 [2] for general two-way slabs with or without beams.



b_e : breadth of effective moment transfer strip (see fig. 7.2)

(a) BS 8110



L_1 assumed to be the shorter span.

(b) ACI

Fig. 7.7 Division of panels in flat slab.

Type of moment \ Type of strip	Column strip	Middle strip
Negative moments (interior support)	75%	25%
Positive moments	60%	40%
Negative moment (exterior support)	100%	-

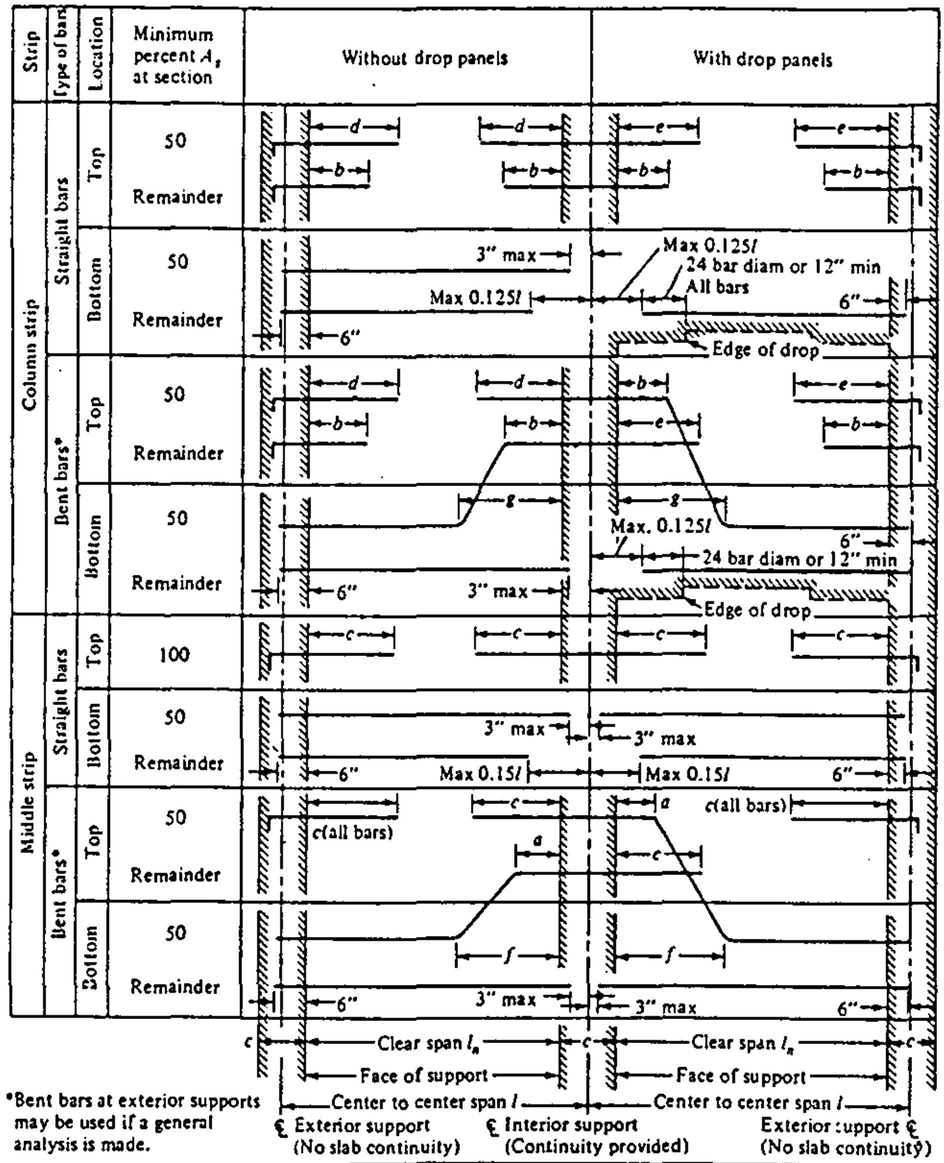
Table 7.3: Division of design moments at critical sections between strips comprising a panel

7.3.2.4 Reinforcement layout

For slabs designed by the EFM, the ACI allows the bars to be curtailed as shown in Fig. 7.8. When adjacent spans have unequal lengths, the extension of the negative moment bars past the face of the support is based on the length of the longer span. Fig. 7.8 shows two options, the first using straight bars and the second bent up bars. Nowadays, straight bar systems are almost exclusively used to simplify the placing of the bars and avoid the cost of bending..

7.4 Application of Codes' EFM to a sample flat slab

In this section a numerical example is discussed in which a sample structure is designed and compared when using the EFM procedures of both the ACI code and BS8110. The example uses the floor slabs shown in Fig. 7.9a and it is assumed there is a floor above and below it. This means that the example floor slab has columns both above and below it and thus allows the example to investigate the straightforward use of the equivalent column concept of ACI. The storey height of each level is 3.00m. In addition to its self-weight the slab carries an imposed load of 4.00 kN/m^2. The example is restricted to a consideration of a vertical loading on an interior equivalent frame in the West-East direction.



	Bar length from face of support						
	Minimum					Maximum	
Mark	a	b	c	d	e	f	g
Length	$0.14l_n$	$0.20l_n$	$0.22l_n$	$0.30l_n$	$0.33l_n$	$0.20l_n$	$0.24l_n$

Fig. 7.8 Minimum bend-point locations and extensions for reinforcement in slabs without beams (from the ACI code).

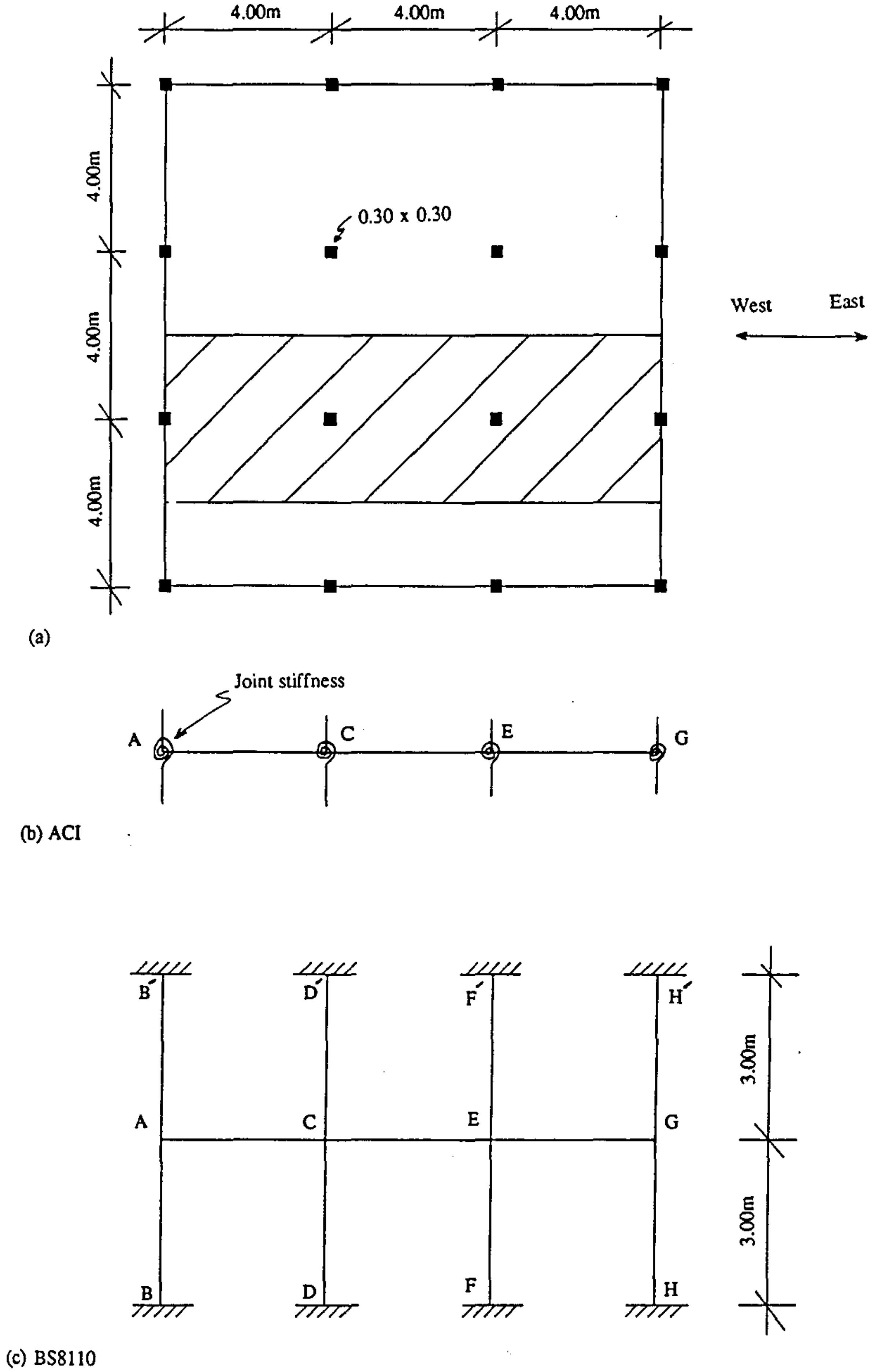


Fig. 7.9 Sample flat slab (representing a floor between floors) and its equivalent sub-frames according to ACI and BS8110

The equivalent analyses for the design moment have been carried out for both codes using the moment distribution method. This is to be able to incorporate the torsional effect of the slab into the equivalent column concept of the ACI code.

In applying the ACI EFM a sub-frame has been idealized using the equivalent column concept and is represented by Fig. 7.9b while for the BS8110 EFM the sub-frame was as idealized using the actual columns as shown in Fig. 7.9c. Details of the calculations performed and resulting moment distributions are given in Appendix 7A. In this example the torsional flexibility effect in the ACI-based EFM was found to be negligible in comparison to the flexibility of the columns and thus contributes little to the final results. This is not surprising since in practice the torsional effect is clearly more significant when beams are present, for example when the edge of a slab is supported by a beam. However if this example is typical for flat slabs generally then a statement in the code could be made to this effect.

Before discussing the results three points are noted:

(a) BS8110 recommends an allowance of 20% redistribution reduction when all spans are loaded on the initial moments, while the ACI makes a 10% redistribution just before the final stage. It is noted that when applying these redistributions to a slab section, BS8110 requires that the reductions made at all the sections where negative moments exist possess a moment resistance of not less than 80% of their previous value (ACI uses a 10% criterion rather than a 20% criterion). A consequence of this is that the conventional moment distribution diagram for a slab section obtained by an elastic analysis (see Fig. 7.11a) takes a "discontinuous" form such as that shown in Fig. 7.11b. In Fig. 7.11a adjacent portions of the negative and positive bending moment curves meet at the point of contraflexure where the moment is zero. On applying the correction, these adjacent curves no longer meet at the same point and result in the discontinuity. Thus the negative portion of the bending moment curve reduces to zero at the same point of contraflexure mentioned above. The positive portion of the bending moment curve is offset toward the supports.

(b) The need for adjustment to the design moments of a panel to ensure equilibrium required by both codes has been checked. ACI recommends the use of negative moments at the face of rectilinear supports, in the case of this example this is at 0.15m from the centre of the support. In contrast, BS8110 recommends the use of negative moments at as distance of $h_c/2$ from the centre of support (where h_c is the effective diameter of the support) and in this example the required distance is 0.169m from the centre of the support.

(c) In this example two different values of negative moments for adjacent spans occurred at the interior supports, but following the code rules, the larger of the two values control the design and was employed.

The results of the bending moments (before their distribution to column and middle strips) obtained by the EFM procedures of both codes, for the flat slab of Fig. 7.9a, are shown in Figs. 7.10d and 7.11d. The moments at the critical sections from both codes are now compared.

The negative moments using both code methods are similar in value, but the positive moments in BS8110 are higher than in the ACI code. The reason for these observed differences include:

- the codes use different bending moment redistribution ratios;
- the codes employ their individual criteria for determining the location of the faces of supports in a design. Thus, for a square section support, BS8110 employs an equivalent circular section while ACI uses the dimensions of the square section itself.
- the codes' methods of adjusting a critcal section moment for an equilibrium check differ. BS8110 specifies that if the sum of the maximum positive design moment (M_3) and the average of the negative design moments ($(M_1 + M_2)/2$) in any one span of the slab for the whole panel width is less than

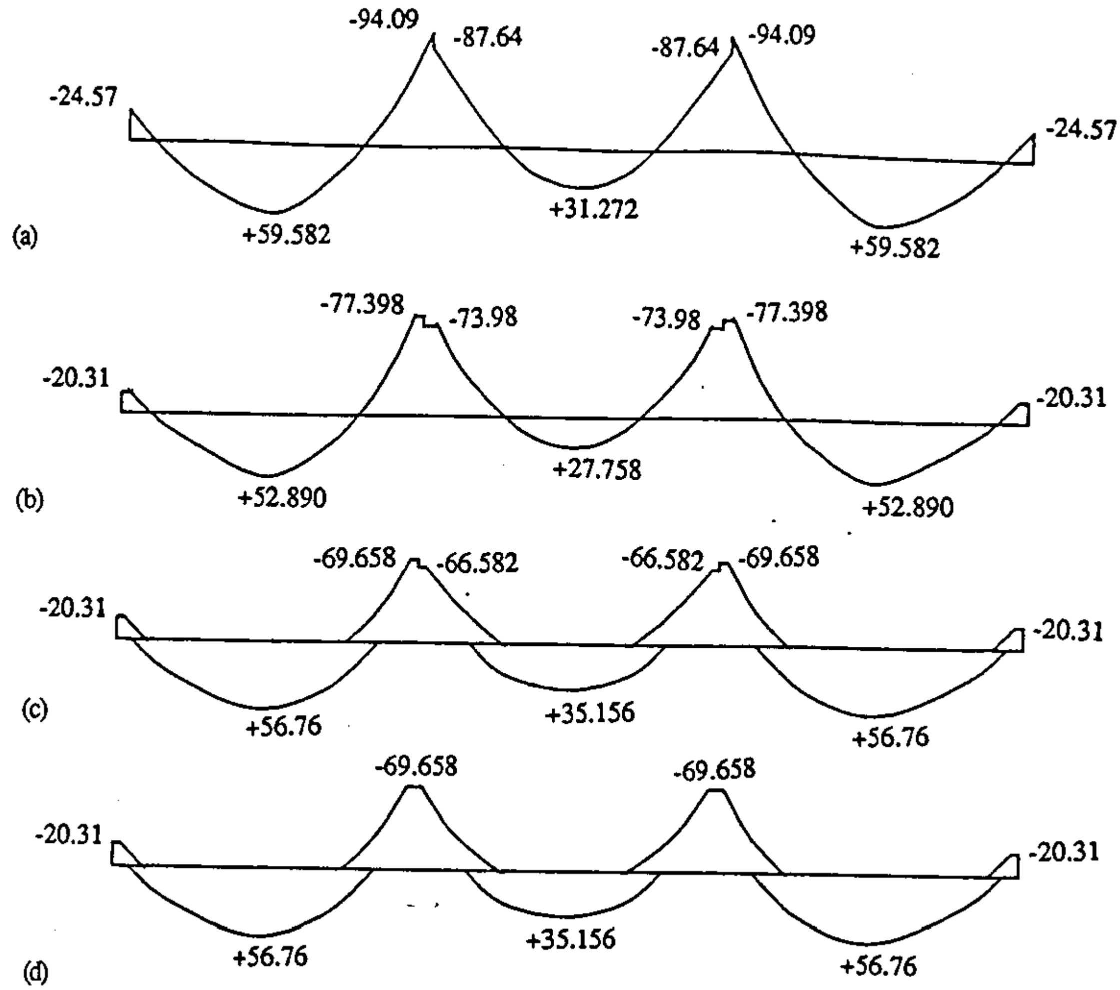


Fig. 7.10 Bending moment diagram derived from the ACI equivalent frame method

a) elastic analysis by moment distribution
b) bending moments at the faces of supports (after adjustment $\not> M_o$)
c) 10% redistribution
d) largest negative moment on interior support is controlled

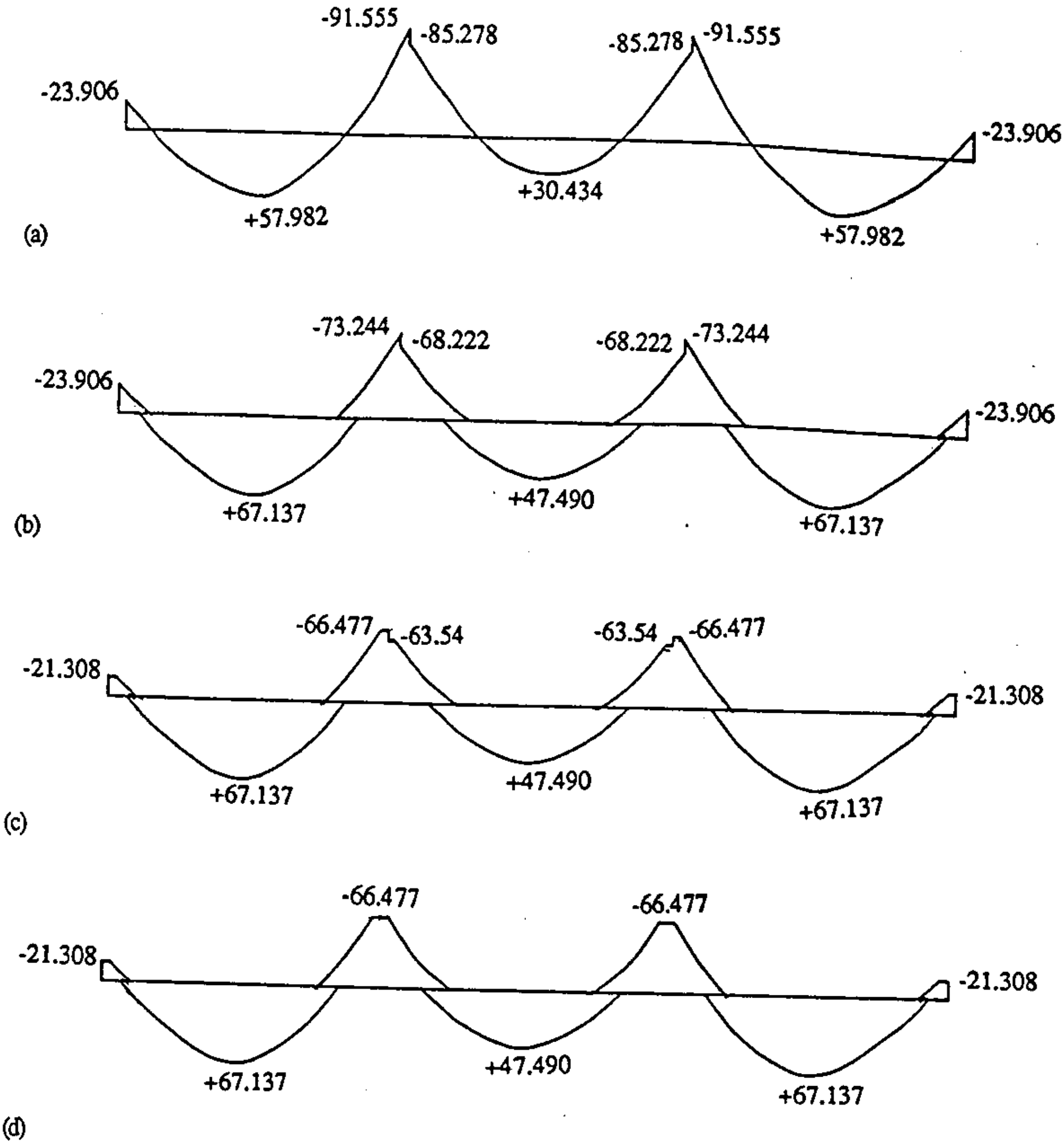


Fig. 7.11 Bending moment diagram derived from the BS8110 equivalent frame method

a) elastic analysis by moment distribution
b) 20% redistribution
c) bending moments at the faces of supports
d) largest negative moment on interior support is controlled

$$M_o = \frac{nL_2}{8}(L_1 - \frac{2h_c}{3})^2$$

then the design moments at the critical sections should be increased. However, ACI requires that if the sum of the maximum positive design moment (M3) and the average of the negative design moments ($(M_1 + M_2)/2$) exceeds

$$M_o = w_u L_2 L_n^2/8$$

then an adjustment downwards of the section moments must be made.

The values of the moments assigned to the column and middle strips at critical sections for the slab according to the code's provisions are shown in Fig. 7.12. An important observation from this Figure are the criteria with which the negative moments at exterior and interior supports have been assigned. Two points are noted for this observation. Firstly, at the exterior supports in the ACI code all the moments are assigned to a column strip, while in BS8110 they are all assigned to the effective moment transfer strip, b_e, leaving the space between the effective moment transfer strips of the edge panel to be furnished with the minimum reinforcement. Secondly, at the interior support, the same values for the ratios, for moment assignment at critical sections, are used in both codes for column and middle strips. However BS8110 does not apply the moment assigned to the column strip uniformly over the column strip as ACI does, but apportions two-thirds of the assigned moment to the middle half of the column strip nearest the column, the remainder of the assigned moment is allocated to the other half of the column strip.

7.5 The simple coefficient method and Equivalent Frame method of BS8110

The moment values obtained by the Equivalent Frame method can be used to derive the moment coefficients which can then be compared with the coefficients in the code's simple method given in Table 7.1. Details of the calculations used to derive for these coefficients are given in Appendix 7B. The main results from this Appendix are shown in terms of moment coefficients in Fig. 7.13. It can be seen from this Figure

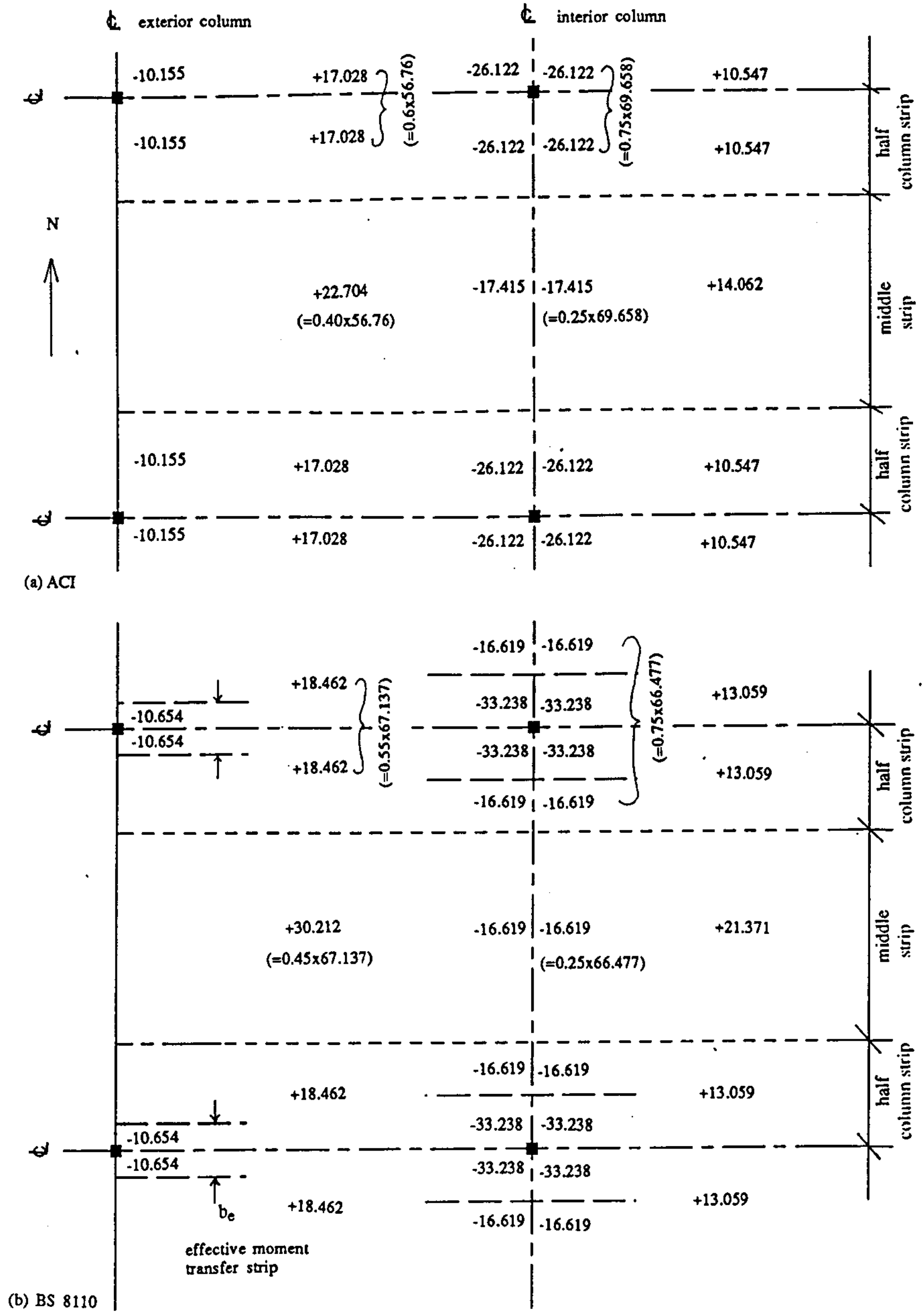


Fig. 7.12 Moments assigned to the column and middle strips at critical sections (in the East-West direction) for the slab for ACI and BS8110

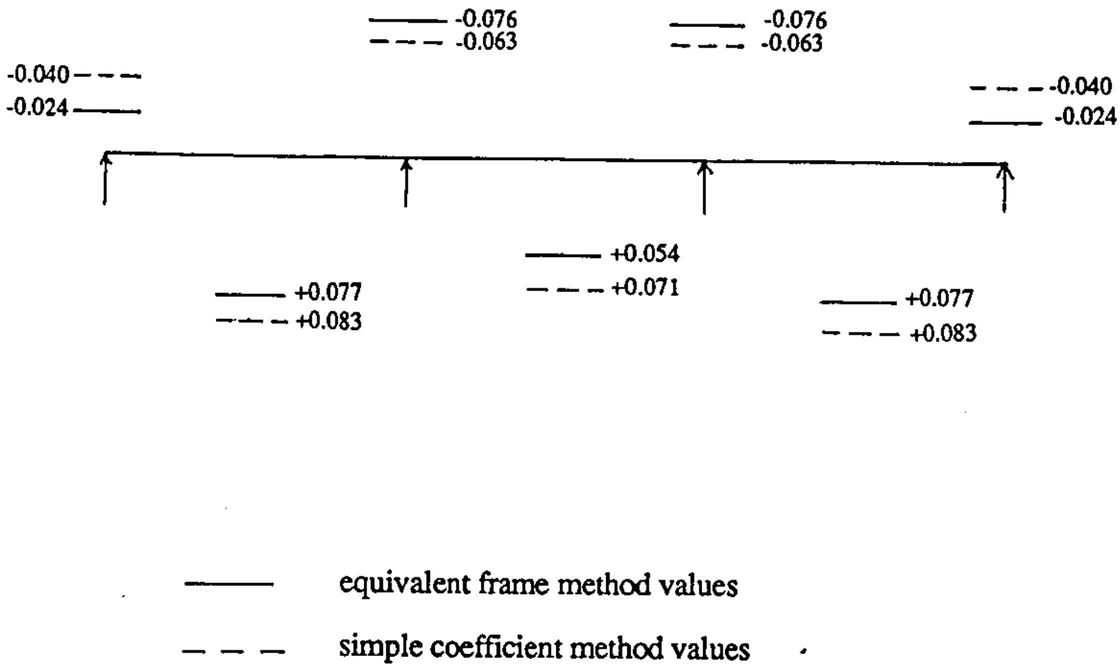


Fig. 7.13 Moment coefficients calculated by the equivalent frame method of BS8110 and as supplied for the simple coefficient-based method in BS8110

that the negative moment coefficient at the interior support obtained by the equivalent frame method is higher than that given by the simple coefficient-based method and the positive moment coefficient from the equivalent frame method at mid-span is lower than that of the simple method.

At all points except the first interior column the simple coefficient method is safer. It appears the simple coefficient method has allowed a greater redistribution at the interior columns but overall the method is simpler and safer, at least in this case than the EFM.

7.6 Finite Element Analysis of Flat Slabs

The finite element analysis method was used to analyse the same flat slab sample structure described and analysed in section 7.4 by the equivalent frame method (see Fig. 7.9).

Again the sample structure is a floor (with a floor above and below it) of a multi-bay multi-storey building and analysed for vertical loads only. In Fig. 7.9 the floor is modelled using equivalent two-dimensional transverse and longitudinal frames. The finite element method allows the floor to be represented as a three-dimensional model (comprising the entire flat slab and the columns above and below it) and this latter approach has been adopted in the analysis described below.

The general purpose finite element package PAFEC was used and for the purpose of the analysis each panel was idealized by an assemblage of flat plate four-noded elements (PAFEC reference number 44200). Each panel was subdivided into a uniform grid of 8 x 8 elements.

The simple engineering beam finite element (PAFEC reference number 34000) was used to idealize the columns of the flat slab system. This element for the simple engineering beam has the customary two nodes and six degrees of freedom per node. The finite element model for the floor slab and associated column is shown in Fig. 7.14. The model has 576 flat plate elements, 32 beam elements.

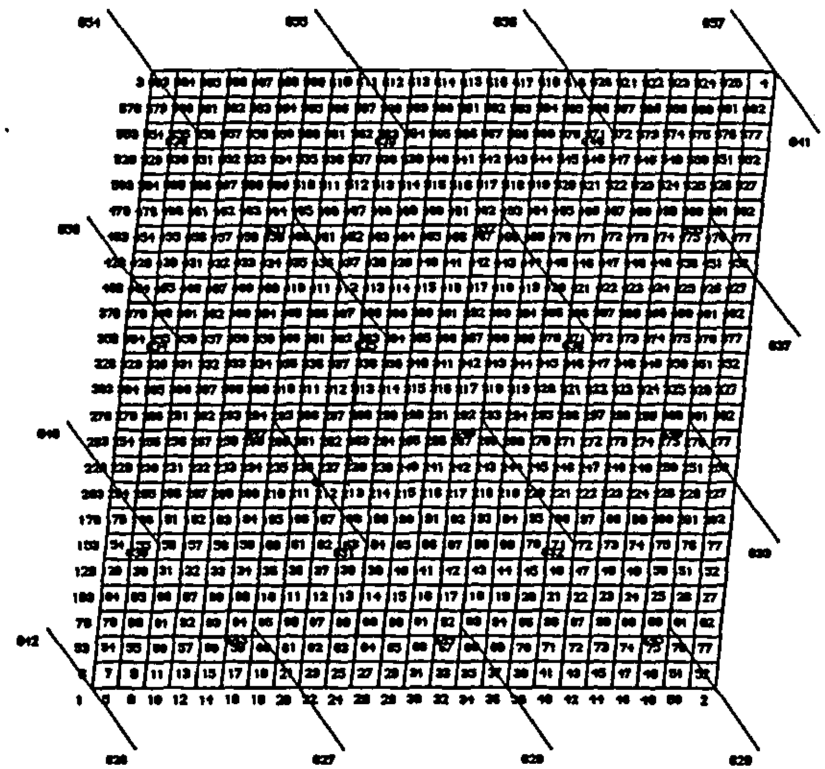


Fig. 7.14 Finite element mesh for a nine panel flat slab floor showing node numbers

The results for the principal stresses from PAFEC were modified by the computer programs used in Chapter 6, i.e. the principal stresses were converted to equivalent directional stresses which were then used with the Wood-Armer rules to obtain reinforcement moments. The output from the program is given in Appendix 7C.

The moments along the four critical sections in the East-West direction of the sample structure shown in Fig. 7.15 are presented in Figures 7.16 - 7.18.

7.6.1 Moment coefficients

The diagrams of effective moments using the Wood-Armer rules derived from the finite element analysis were converted to moment coefficient form by the following procedure.

a) The area under the curve in Fig. 7.18 was evaluated for the appropriate section bounded by the centre lines of the exterior and interior span.

b) This area was divided by the distance between these centre lines to find the average moment over this width.

c) The average moment was divided by the ultimate design value of wL^2 on the span to obtain the coefficient.

For the negative moment at the exterior support, section 1.1, Fig. 7.18

$$\text{the moment coefficient} = \frac{14296.8}{313344} = 0.0456$$

For the positive moment at the first midspan, section 2.2, Fig. 7.18

$$\text{the moment coefficient} = \frac{21699.4}{313344} = 0.069$$

For the negative moment at the first interior support, section 3.3, Fig. 7.18

$$\text{the moment coefficient} = \frac{30684.1}{313344} = 0.098$$

For the positive moment at the mid interior span, section 4.4, Fig. 7.18

$$\text{the moment coefficient} = \frac{11953.4}{313344} = 0.038$$

The negative moment coefficients calculated above are based on the assumption that the columns are point supports whereas in reality they have a finite width. It is

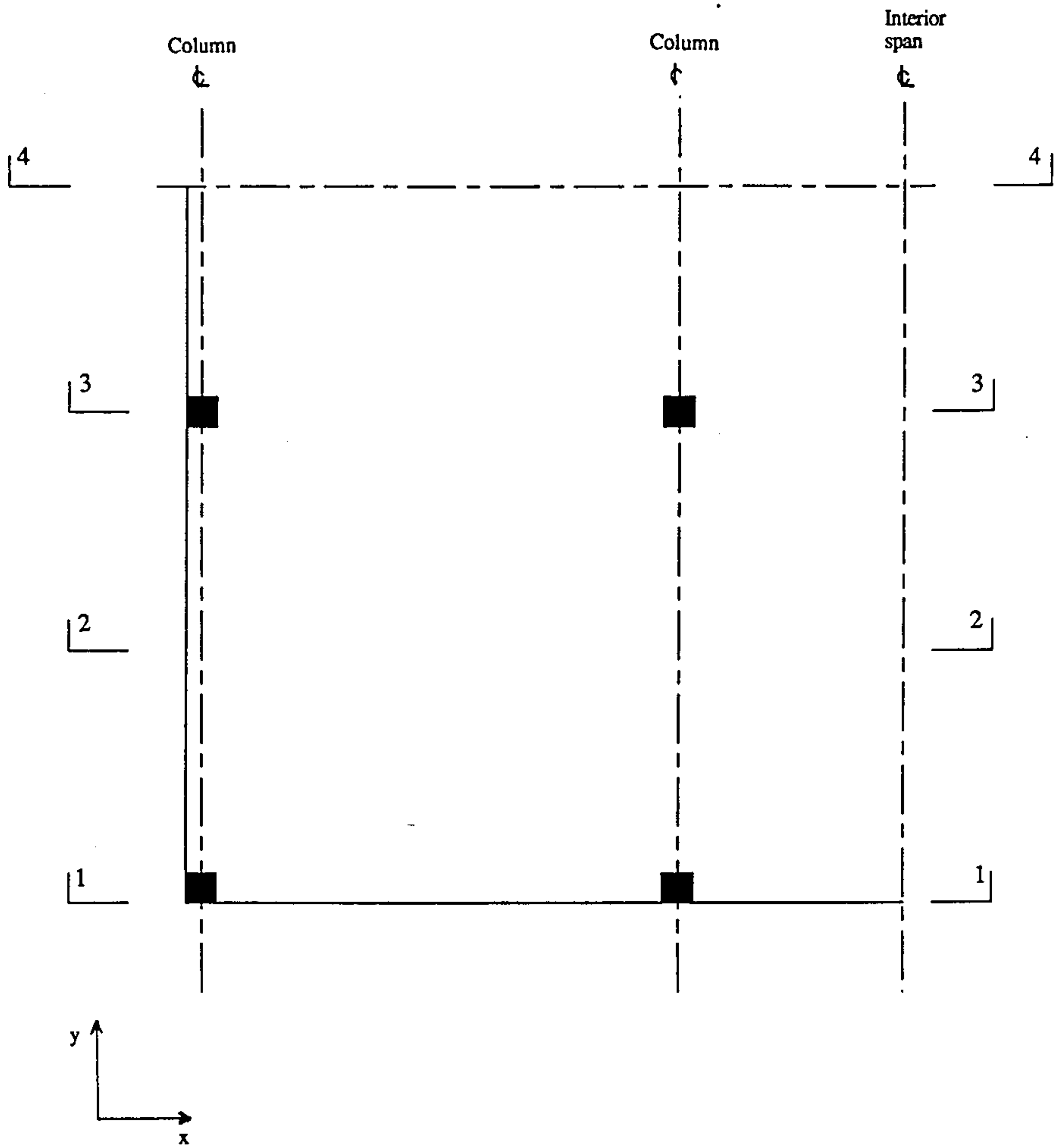


Fig. 7.15 Plan showing a quarter of the structure detailed in Fig. 7.14 (The distribution of moments along sections 1-1, 2-2, 3-3, and 4-4 are given in Figures 7.16 to 7.18)

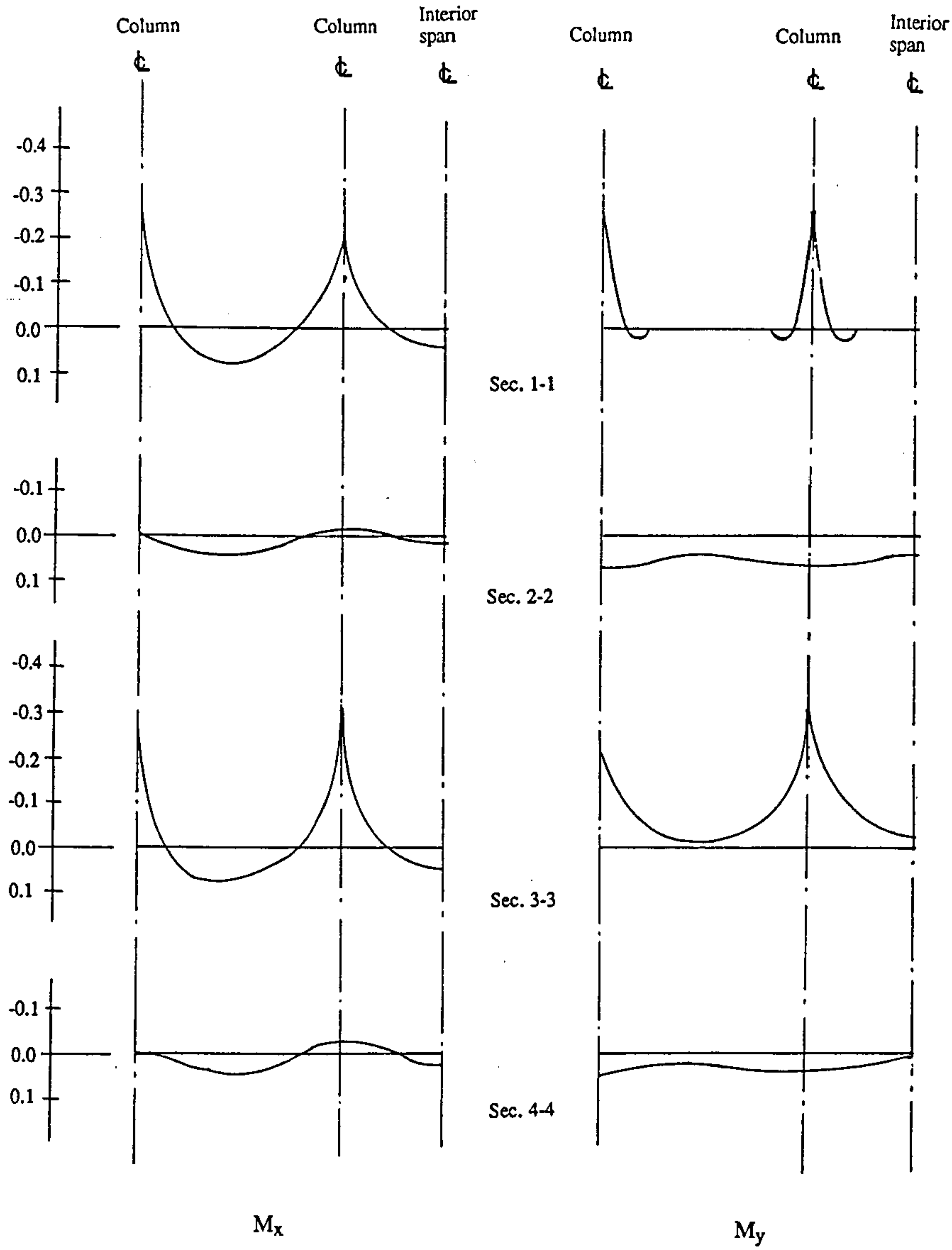


Fig. 7.16 Distribution of bending moment coefficients corresponding to M_x and M_y for the sections specified on Fig. 7.15

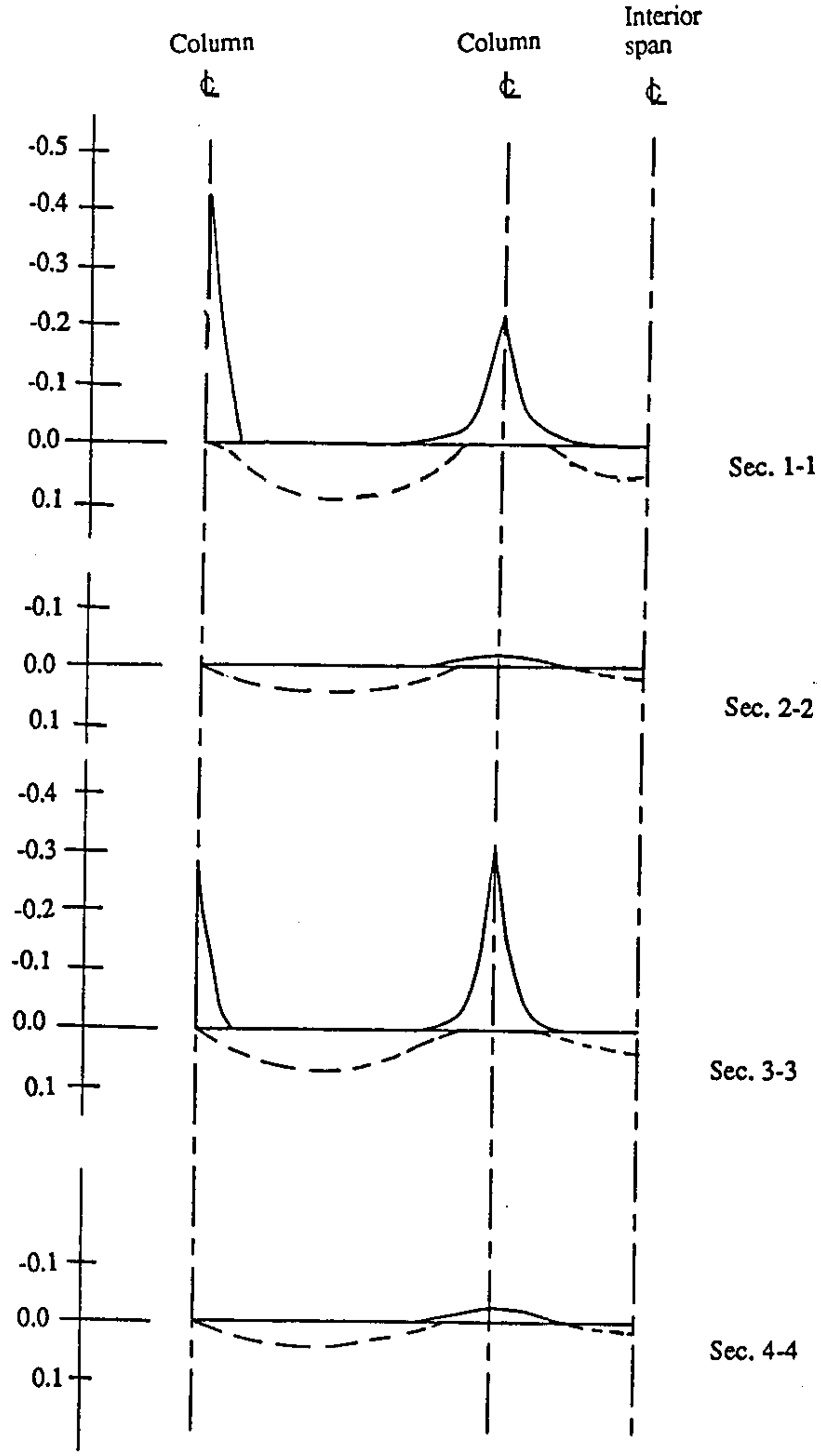


Fig. 7.17 Distribution of the Wood-Armer moment coefficient values corresponding to M_x^- and M_x^+ for the sections specified on Fig. 7.15

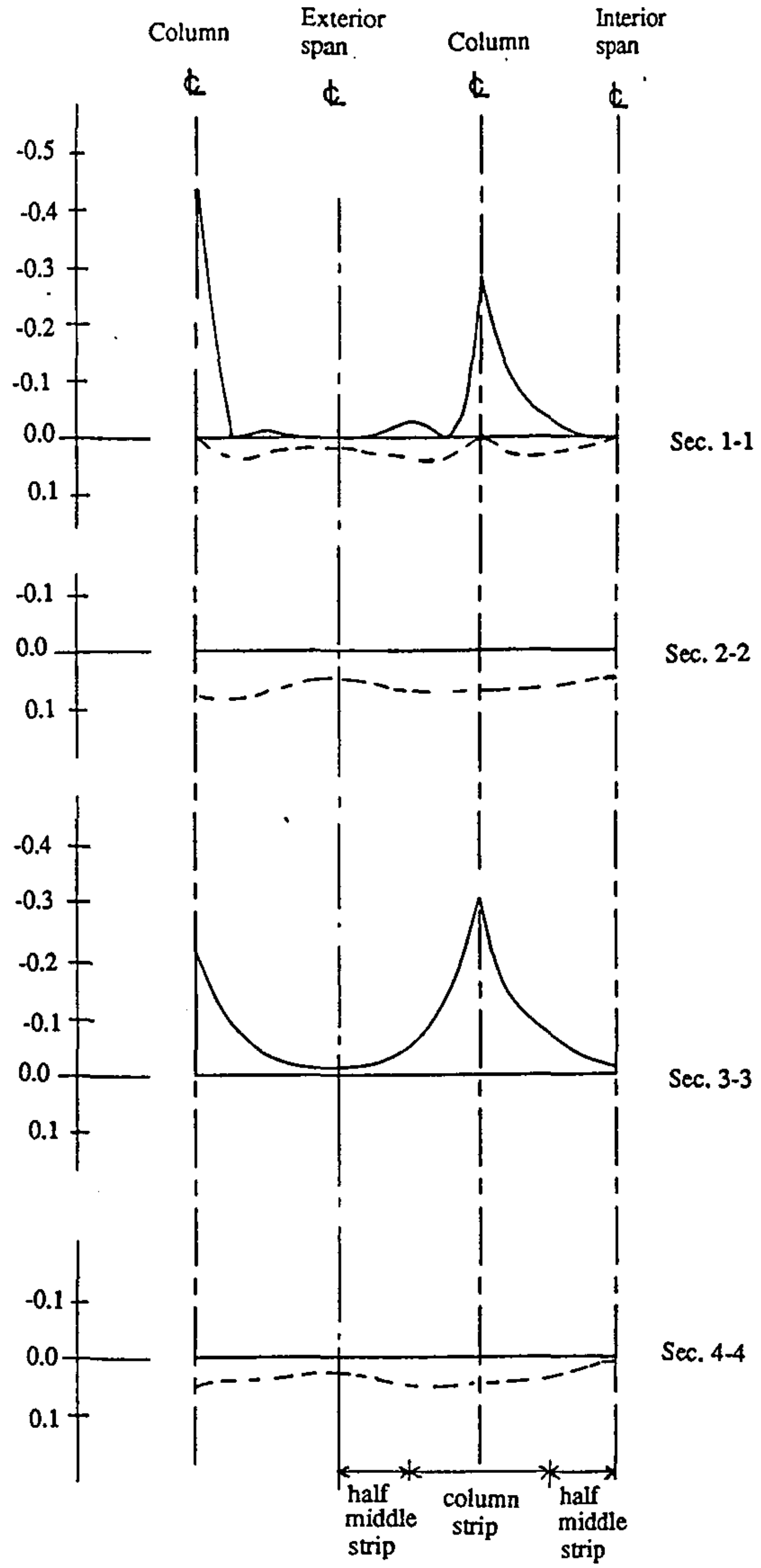


Fig. 7.18 Distribution of the Wood-Armer moment coefficient values corresponding to M_y^- and M_y^+ for the sections specified on Fig. 7.15

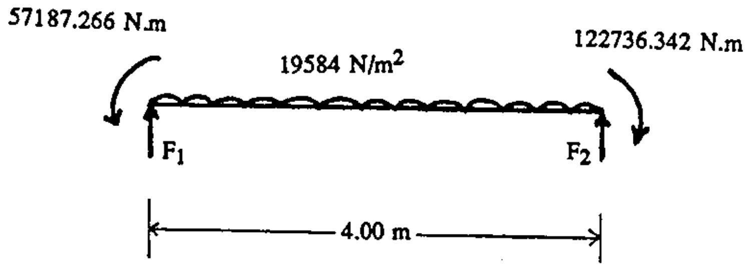
therefore necessary to reduce these to the value that exists at the face of the column in order that direct comparison can be made with the code.

These calculations are as follows.

$$F_2(4) + 57187.266 - 122736.342 - 19584\,(4)(4)\frac{4}{2} = 0$$

$F_2 = 173059.269$

$F_1 = 140284.731$

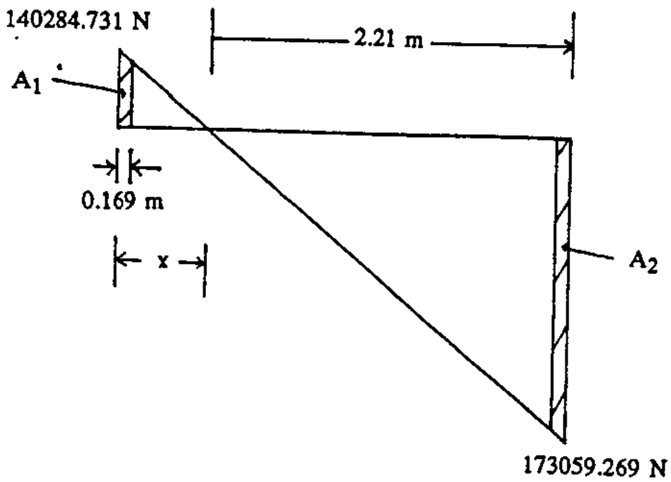


and hence the distance x to the point of zero shear by similar triangles is

$$\frac{313344}{4} = \frac{140284.731}{x}$$

or

$x = 1.79$



If y is the shear value at the column edge

$$\frac{y}{1.621} = \frac{140284.731}{1.79}$$

therefore $y = 127039.972$

and $A_1 = 22588.9374$

Similarly for the shear at F_2

$$\frac{y_2}{2.041} = \frac{173059.269}{2.21}$$

therefore $y_2 = 159825.3249$

and $A_2 = 28128.75$

The moments at the faces of supports are therefore

Exterior support: $57187.266 - 22588.9374 = 34598.8286$

Average = 34598.3286/4 = 8649.582

Coefficient = 8649.582/313344 = 0.028

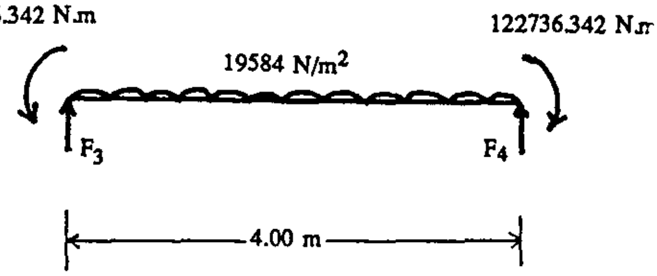
Moment at exterior face of interior support: 122736.34 - 28128.75 = 94607.59

Average = 94607.59/4 = 23651.898

Coefficient = 23651.898/313344 = 0.075

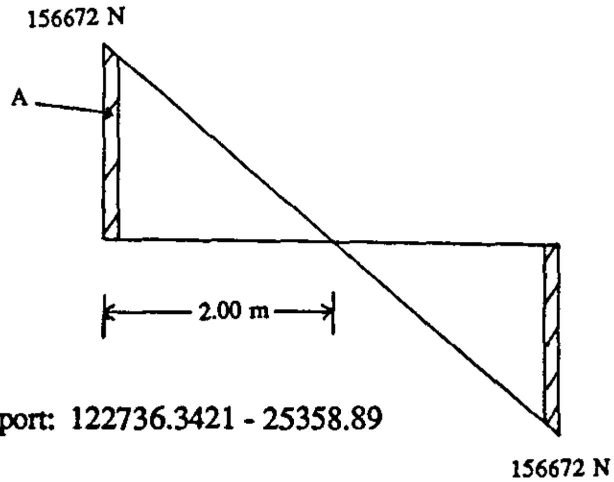
Similarly for the interior span

$$F_3 = F_4 = \frac{313344}{2} = 156672$$



from which it is found that

A = 25358.89



$$\frac{156672}{2} = \frac{y}{1.831}$$

y = 143433.216

Therefore moment at interior face of interior support: 122736.3421 - 25358.89

= 97377.452

Average = 97377.452/4 = 24344.363

Coefficient = 24344.363/313344 = 0.078

7.6.2 Comments on finite element results

The values of the moment coefficients which have been calculated in section 7.6.1 are shown in Fig. 7.19 together with the values from the other techniques that have been used. The values at all the critical positions are remarkably similar, with the positive moment coefficient being smaller and the negative ones larger than the simplified and equivalent frame method. In both these methods however redistribution at the first interior support has been allowed which would account for this. If the finite element values at the first interior support were reduced by 0.008 to a value of 0.070, i.e. midway between the simplified and equivalent frame value, the positive values would increase to 0.073 and 0.046 in the first and second spans respectively. These values confirm that both code approaches as far as the moment coefficients are concerned along an interior column line are a good reflection of the actual moments.

A similar check cannot reasonably be made along an exterior column line since the moment value at the first node point away from the column on two cases is zero so that the form of the peaks at the columns in section 1.1 of Fig. 7.18 could only be estimated roughly. This plot however does confirm that the negative moment is essentially confined to a very narrow band at the support and the requirement to confine the column strip in width at an external column is clearly very sensible.

The next points that can be checked are the suggested British code division of the positive and negative moments into the column and middle strips as given in Table 7.2. The coefficients are the same in the ACI code for negative moments but there a 60% and 40% division of the positive moments is suggested. For the positive moments if section 2-2 in Fig. 7.18 is considered, and the values under the curve measured, then the proportions that are found are 55% and 45% while at section 4-4 they are 58% and 42%. These values are therefore totally consistent with the code recommendations.

For the negative moments for an outside column line, section 1-1, Fig. 7.18 this confirms the recommendation that the negative moment be confined solely to the column strip. Again by measuring the area under the curve in section 3-3 the proportions in the column and middle strips are 82% and 18%. This value is slightly different to the code values of 75% and 25% but it should be noted that a point support has been considered. If this peak value is reduced to that at the column faces then the proportions would be closer to the code values.

The values shown in Fig. 7.17 are also interesting. These are not moment envelopes through different loadings but the positive and negative steel requirement due to a single loading case. The overlapping is of course due to the twisting moments M_{xy} in the Wood-Armer rules.

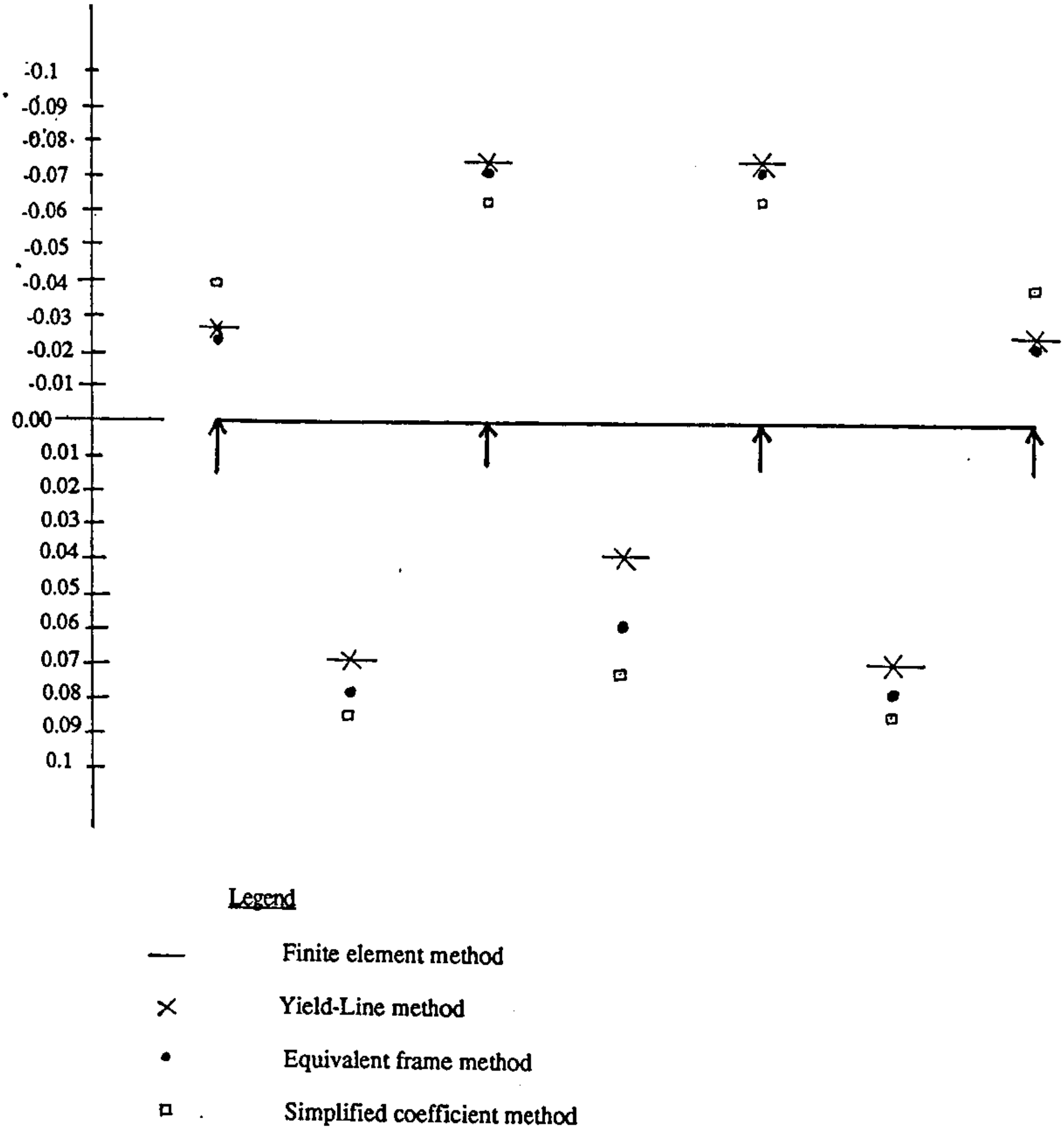


Fig. 7.19 Moment coefficients calculated by different recommended methods

Clearly in any future work a finer mesh should be used in the finite element program around the column so that more detailed results can be obtained in an area where the moments are changing rapidly. Nevertheless these results confirm that the two methods proposed in the codes are quite satisfactory.

7.7 Yield-line Analysis

Yield-line analysis was also used to analyse the same flat slab sample structure described and analysed in section 7.4 by the equivalent frame method (see Fig. 7.9). Using yield-line analysis, several trial modes of failure were examined. Basically, two types of yield-line pattern arise in beamless floors; one involves overall failure and the general folding of the floor and the other involves local collapse around the columns (i.e. fan or partial fan mechanisms).

7.7.1 Overall failure patterns

There are essentially two possible overall failure modes as shown in Fig. 7.20 as modes 1 and 2. Since the average ratio, in the two codes, of the positive moment in an exterior span to the moment at the first interior support is approximately 1 then this ratio was initially chosen. No edge restraint on the exterior columns was initially allowed and the span was taken as the full span of 4m.

The analysis corresponding to modes 1 and 2 for overall failure is given in Appendix 7D sections (a) and (b) with the failure modes given in Fig. 7.20.

This analysis gives moment coefficients of zero at the outside column and positive and negative moment coefficients in the first span of 0.086 (wL^2) and 0.039 (wL^2) for the positive moment on an interior span. For the outside span these values are higher than any of the previous methods which is odd in view of the fact that this is an upper bound technique which should given lower moment values. The explanation is relatively simple but the original calculations have been left since it is a warning that yield-line analysis needs using with discretion. For modes 1 and 2 the yield lines only involve the maximum moments and no additional load can be picked up by the slab having to form yield lines where the steel is excessive in order to obtain a simple steel ?

layout. For this reason any factors taken into account in the previous methods need to be included in the yield-line analysis. Thus the significant restraint of the outer columns and yield lines forming at the face of interior columns can make a considerable difference. Indeed when the column restraint moment is added if the yield-line moments are taken in the same proportions as the average finite element values in Fig. 7.19 we find in Appendix 7D section (c) that the average yield-line moments are virtually identical with the average finite element results which is what is expected. The values are plotted as crosses in Fig. 7.19.

The yield-line values do not of course need to be kept at these constant values and can be redistributed into middle and column strip in the proportions given in Table 7.2. This is shown in Appendix 7D(d). The distribution into the middle and column strip can of course be any value chosen but a choice consistent with the elastic ratios is sensible.

7.7.2 Local column failure

So far the calculations have dealt with overall failure but local failure around the columns needs to be considered. An example of a local fan mechanism calculation is shown in Appendix 7D(e). From this calculation it can be seen that this fan mechanism is just safe but only if the design observes the rule to have 2/3 of the negative strip steel in the middle half of the column strip. Other mechanisms for corner and edge columns would also need to be checked.

7.7.3 Yield-line conclusions

The calculation shows how easy it is to use yield-line analysis both for overall and local failures. It needs to be emphasised however that had the chosen ratios of the positive to negative moments not been in the broad proportions of the elastic analysis and a further division into column and middle strips not been carried out then the design might have necessitated considerable redistribution.

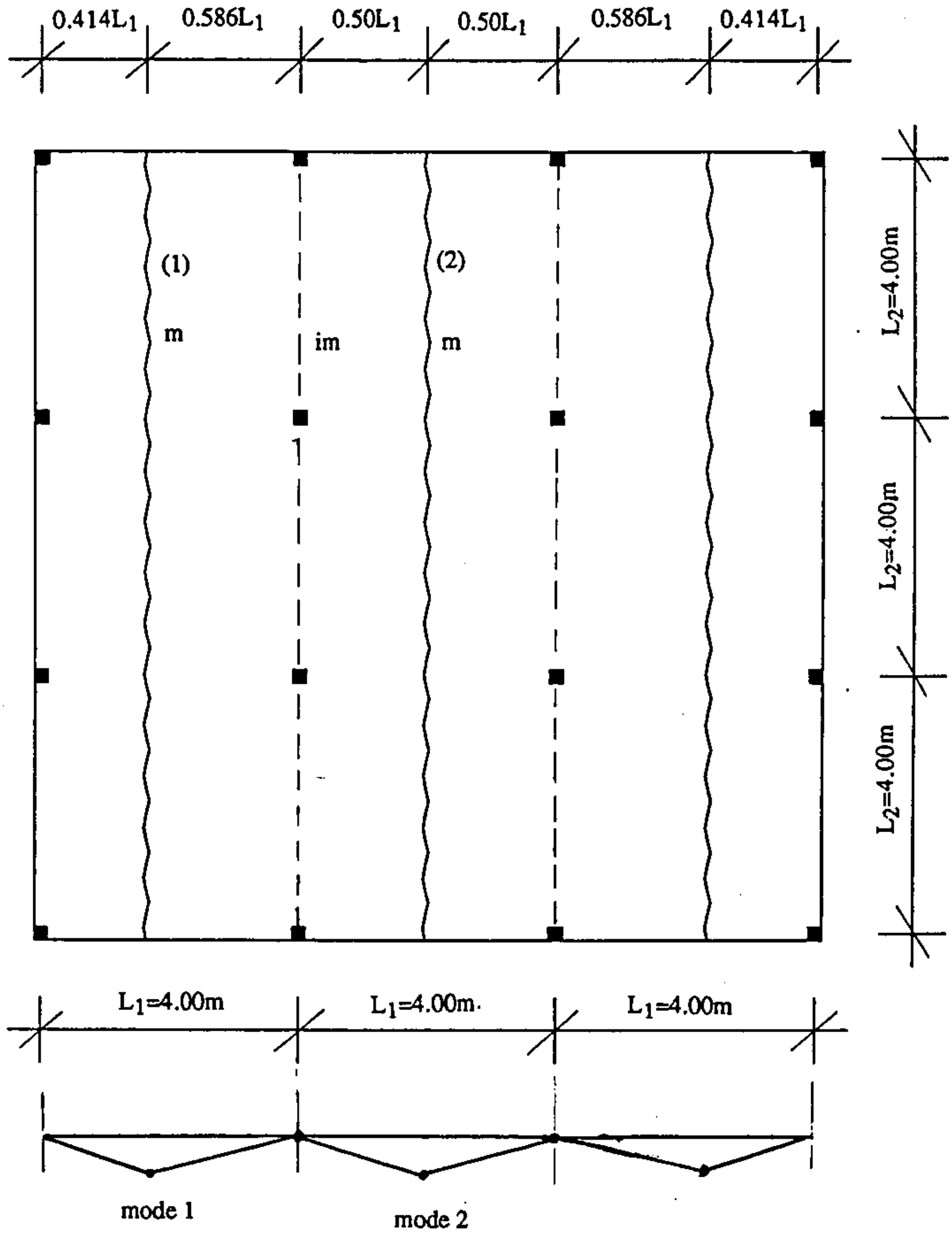


Fig. 7.20 Uniformly loaded flat slab with folding yield-line patterns

7.8 Conclusions

For the simplified coefficient method, equivalent frame methods of both codes, the finite element analysis and yield-line analysis it is possible to conclude the following.

(i) All three code methods are in good agreement with the average moment values found by finite element analysis.

(ii) The distribution of the positive moments into the column and middle strips of 55 - 45% in the British code and 60 - 40% in the ACI code are substantiated by the finite element analysis which yielded an average between the two.

(iii) The distribution of the negative moments into the column and middle strips of 75 - 25% used in both codes was not quite consistent with the finite element value of 82 - 18%. However this value assumed a point support column and the ratio would have been closer to the code values if the actual column size is allowed for.

(iv) The finite element analysis confirmed the code recommendation that at an exterior column the whole negative moment be confined to the middle strip.

(v) The yield-line analysis of a local failure at a column showed the necessity to concentrate more of the column strip steel into the centre half of the strip.

(vi) In broad terms all the techniques suggested in the codes appear to be quite satisfactory.

APPENDIX 7A

Sample design using the EFM to ACI and BS8110

APPENDIX 7A.

Sample design using the EFM to ACI and BS8110

1. ACI

The slab thickness has been assumed to be 240mm.

$$D.L. = 0.24 \times 24 = 5.76\ kN/m^2$$

$$L.L. = 4.00\ kN/m^2$$

It is noted that L.L. < 0.75 D.L. and it is therefore not necessary to apply pattern loading.

$$w_u = 1.4\ D.L. + 1.7\ L.L.$$

$$= 1.4(5.76) + 1.7(4.00)$$

Therefore $w_u = 14.864\ kN/m^2$

Stiffness for the slab (K_s)

$$= \frac{4\,E_c I_s}{L}$$

$$= \frac{4\,E_c (4.00 \times 0.24^3)}{12 \times 4.00}$$

Therefore $K_s = 4.608 \times 10^{-3}\ E_c$

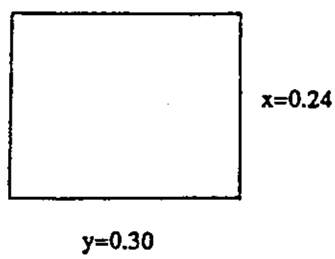
Stiffness for the column (K_c)

$$= \frac{4\,E_c I_c}{L}$$

$$= \frac{4\,E_c (0.30 \times 0.30^3)}{12 \times 3.00}$$

Therefore $K_c = 9 \times 10^{-4}\ E_c$

Torsional constant (C) $= (1 - 0.63\,\frac{x}{y})\,(x^3y)/3$



$$= (1 - 0.63\,\frac{0.24}{0.30})\,\frac{0.24^3(0.3)}{3}$$

-- 1.00

$$K_t = \sum \frac{9\,E_c C}{L_2 (1 - C_2/L_2)^3}$$

$$= 2\left[\frac{9\,E_c\,(1.00)}{4.00\,(1 - \frac{0.30}{4.00})^3}\right]$$

$$= 5.686\,E_c$$

$$\frac{1}{K_{ec}} = \frac{1}{\Sigma K_c} + \frac{1}{K_t}$$

$$\frac{1}{K_{ec}} = \frac{1}{2 \times 9 \times 10^{-4} E_c} + \frac{1}{5.686\,E_c}$$

$$\frac{1}{K_{ec}} = \frac{5.686\,E_c + 2 \times 9 \times 10^{-4} E_c}{2 \times 9 \times 10^{-4} \times 5.686\,E_c^2}$$

$$\frac{1}{K_{ec}} = \frac{5.688\,E_c}{0.0102\,E_c^2}$$

Therefore -

$$K_{ec} = 1.793 \times 10^{-3} E_c$$

Therefore distribution factor (D.F.) for moment distribution calculation at exterior joint:

for slab

$$= \frac{4.608 \times 10^{-3} E_c}{1.793 \times 10^{-3} E_c + 4.608 \times 10^{-3} E_c} = \frac{4.608 \times 10^{-3} E_c}{6.401 \times 10^{-3} E_c}$$

$$= 0.720$$

for the equivalent column

$$= \frac{1.793 \times 10^{-3} E_c}{6.401 \times 10^{-3} E_c} = 0.28$$

D.F. for interior joint:

for slab

$$= \frac{4.608 \times 10^{-3} E_c}{1.793 \times 10^{-3} E_c + 2(4.608 \times 10^{-3} E_c)} = \frac{4.608 \times 10^{-3} E_c}{11.009 \times 10^{-3} E_c}$$

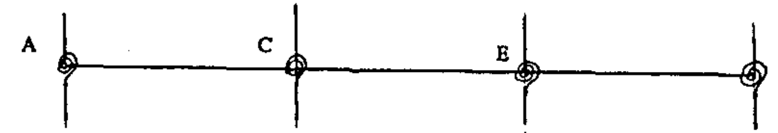
$$= 0.42$$

for the equivalent column

$$= \frac{1.793 \times 10^{-3} E_c}{11.009 \times 10^{-3} E_c} = 0.16$$

See moment distribution Table 7A.1.

Table 7A.1 moment distribution solution



Joint	A		C			E			G	
Member	equivalent column at A	AC	CA	equivalent column at C	CE	EC	equivalent column at E	EG	GE	equivalent column at G
D.F.	0.28	0.72	0.42	0.16	0.42	0.42	0.16	0.42	0.72	0.28
F.E.M.		-79.275	+79.275		-79.275	+79.275		-79.275	+79.275	
cyc. 1 (Bal.	+22.197	+57.078							-57.078	-22.197
(c.o.			+28.539					-28.539		
cyc. 2 (Bal.			-11.986	-4.567	-11.986	+11.986	+4.567	+11.986		
(c.o.		-5.993			+5.993	-5.993			+5.993	
cyc. 3 (Bal.	+1.678	+4.315	-2.517	-0.959	-2.517	+2.517	+0.959	+2.517	-4.315	-1.678
(c.o.		-1.2585	+2.1575		+1.2585	-1.2585		-2.1575	+1.2585	
cyc. 4 (Bal.	+0.3524	+0.9061	-1.4347	-0.5466	-1.4347	+1.4347	+0.5466	+1.4347	-0.9061	-0.3524
(c.o.		-0.7174	+0.4531		+0.7174	-0.7174		-0.4531	+0.7174	
cyc. 5 (Bal.	+0.2009	+0.5165	-0.4916	-0.1873	-0.4916	+0.4916	+0.1873	+0.4196	-0.5165	-0.2009
(c.o.		-0.2458	+0.2583		+0.2458	-0.2458		-0.2583	+0.2458	
cyc. 6 (Bal.	+0.0688	+0.1770	-0.2117	-0.0806	-0.2117	+0.2117	+0.0806	+0.2117	-0.1770	-0.0688
(c.o.		-0.1058	+0.0885		+0.1058	-0.1058		-0.0885	+0.1058	
cyc. 7 (Bal.	+0.0296	+0.0762	-0.0816	-0.0311	-0.0816	+0.0816	+0.0311	+0.0816	-0.0762	-0.0296
(c.o.		-0.0408	+0.0381		+0.0408	-0.0408		-0.0381	+0.0408	
Σ moments	+24.5267	-24.5675	+94.0869	-6.3716	-87.6363	+87.6363	+6.3716	-94.0869	+24.5675	-24.5267

where D.F. is distribution factor, F.E.M. is fixed end moment, Bal. is Balance, c.o. is carry over factor and cyc. is the cycle.

In order to assess the positive moments at mid-span:

Total static moment

$$= \frac{w_u L_2 L_1^2}{8}$$

$$= \frac{14.864(4.00)(4.00)^2}{8}$$

$$= 118.912$$

Therefore positive moment at first span $= 118.912 - \frac{1}{2}(24.57 + 94.09)$

$= 59.582$

Positive moment at interior span $= 118.912 - \frac{1}{2}(87.64 + 87.64)$

$= 31.272$

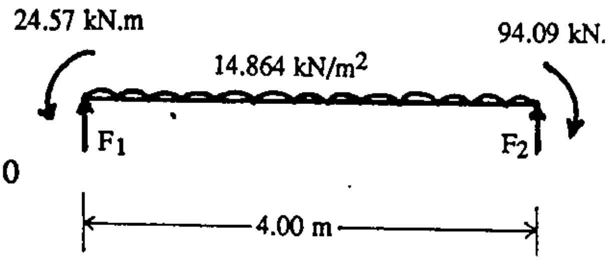
See the analysis results in Fig. 7.11a.

Negative moment at face of supports will be needed for design then,

$$F_2(4) + 24.57 - 04.09 - 14.864(4)\frac{(4)}{2} = 0$$

Therefore $F_2 = 47.108$

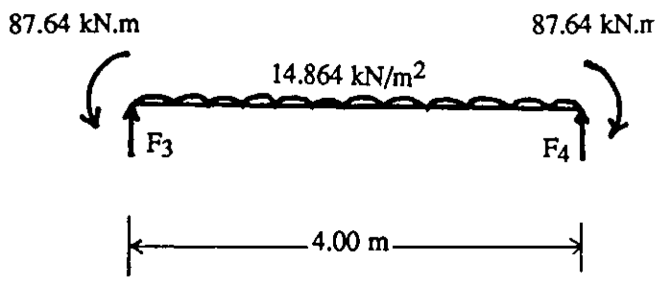
Therefore $F_1 = 12.348$

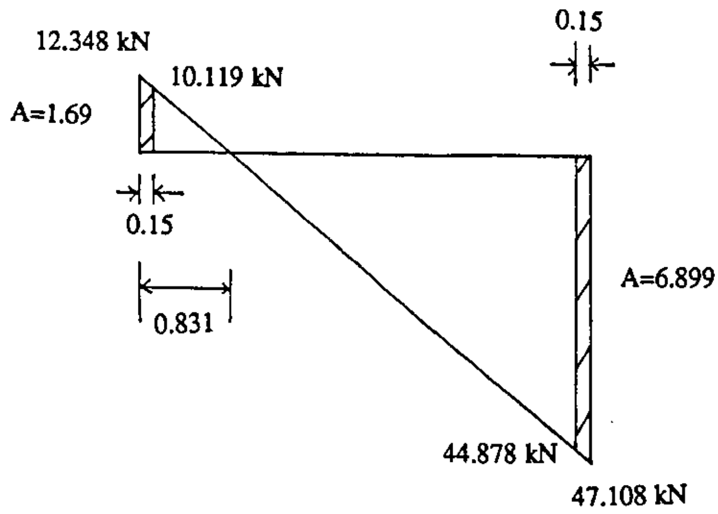


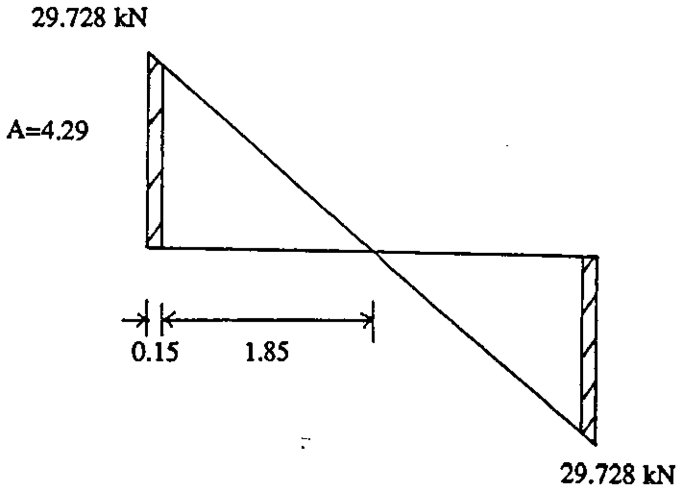
$$F_4(4) + 87.64 - 87.64 - 14.864(4)\frac{(4)}{2} = 0$$

Therefore $F_4 = 29.728$

Therefore $F_3 = 29.728$







Therefore negative moments at faces of supports are as follows:

At exterior column	= 24.57 - 1.69
	= 22.88
At interior column face for the first span	= 94.09 - 6.899
	= 87.191
At interior column face for interior span	= 87.64 - 4.29
	= 83.35

Adjustment:

$$\frac{M_1 + M_2}{2} + M_3 > M_o = \frac{w\,L_2\,L_n^2}{8}$$

This is a requirement of the ACI code and is discussed in section 7.3.2.2.

$$M_o = \frac{14.864(4.00)(3.70)^2}{8}$$

$$= 101.744$$

$$\frac{M_1 + M_2}{2} + M_3 \text{ for first span is}$$

$$\frac{22.88 + 87.191}{2} + 59.582$$

$$= 114.618 > M_o \quad \text{not O.K.}$$

Therefore needs adjustment

$$\frac{22.88}{2 \times 114.618} = 9.981\%$$

$$\frac{87.191}{2 \times 114.618} = 38.035\%$$

$$\frac{59.582}{114.618} = 51.984\%$$

$$114.618 - 101.744 = 12.874$$

$$22.88 - \left[12.874 \times \frac{9.981 \times 2}{100}\right] = 20.31$$

$$87.191 - \left[12.874 \times \frac{38.035 \times 2}{100}\right] = 77.398$$

$$59.582 - \left[12.874 \times \frac{51.984}{100}\right] = 52.890$$

$$\frac{20.31 + 77.398}{2} + 52.890$$

$$= 101.744 = M_o \quad \text{O.K.}$$

$$\frac{M_1 + M_2}{2} + M_3 \quad \text{for interior span is}$$

$$\frac{83.35 + 83.35}{2} + 31.272$$

$$= 114.622 > M_o \quad \text{not O.K.}$$

Therefore needs adjustment.

$$\frac{83.35}{2 \times 114.622} = 36.36\%$$

$$\frac{31.272}{114.622} = 27.28\%$$

$$114.622 - 101.744 = 12.878$$

$$83.35 - \left[12.878 \frac{36.36 \times 2}{100}\right] = 73.98$$

$$31.272 - \left[12.878 \frac{27.28}{100}\right] = 27.758$$

$$\frac{73.98 + 73.98}{2} + 27.758$$

$$= 101.738 \simeq M_o = 101.744$$

Redistribution:

ACI recommends 10% redistribution when selecting reinforcement in order to make it more practical.

First span:

$$77.398 - 0.10(77.398) = 69.658$$

$$52.890 + \frac{0.10(77.398)}{2} = 56.76$$

Check total static moment after the redistribution.

$$\frac{M_1 + M_2}{2} + M_3 = \frac{20.31 + 69.658}{2} + 56.76$$

$= 101.744$ equal to total static moment before redistribution O.K.

Interior span:

$$73.98 - 0.10(73.98) = 66.582$$

$$27.758 + 0.10(73.98) = 35.156$$

Check total static moment after the redistribution.

$$\frac{M_1 + M_2}{2} + M_3 = \frac{66.582 + 66.582}{2} + 35.156$$

$= 101.738$ equal to total static moment before redistribution O.K.

The largest negative moments at the interior supports are controlled on both faces of that support.

The bending moments at the various stages of the calculations are set out in Fig. 7.10.

Finally moments at critical sections are distributed according to the ratios given in ACI for column and middle strips, see Fig. 7.13a.

2. BS8110 calculations

G_k = $0.24 \times 24 = 5.76\ kN/m^2$

Q_k = $4.00\ kN/m^2$

n = $1.4\ Gk + 1.6\ kN/m^2$

= $1.4(5.76) + 1.6(4.00)$

= $8.064 + 6.4$

= $14.464\ kN/m^2$

Stiffness for the slab $(K_s) = \dfrac{4E_c I_s}{L}$

$$= \frac{4E_c(4.00 \times 0.24)}{12 \times 4.00}$$

Therefore $K_s = 4.608 \times 10^{-3} E_c$

Stiffness for the upper and lower columns (K_c)

$$= \frac{4E_c I_c}{L}$$

$$= \frac{4E_c(0.30 \times 0.30^{3)}}{12.\ 3.00}$$

Therefore $K_c = 9 \times 10\ E_c$

$$\text{D.F. for slab} = \frac{4.608 \times 10^{-3} E_c}{9 \times 10^{-4} E_c + 9 \times 10^{-4} E_c + 4.608 \times 10^{-3} E_c}$$

$$= \frac{4.608 \times 10^{-3} E_c}{1.8 \times 10^{-3} E_c + 4.608 \times 10^{-3} E_c}$$

= 0.720

$$\text{For upper column} = \frac{9 \times 10^{-4} E_c}{1.8 \times 10^{-3} E_c + 4.608 \times 10^{-3} E_c}$$

= 0.14

For lower column = 0.14.

However, it seems the slab factor is the same as in moment distribution in ACI. The analysis in the ACI calculations will therefore be used after some modification for the difference in ultimate load considered in both codes, see Table 7.A2 for results.

Negative moment at edge column obtained from the equivalent frame analysis should be checked by:

$M_{t,max} = 0.15\, b_e\, d^2\, f_{cu}$

f_{cu} is assumed $= 30$ N/mm^2

$d = 0.24 = 0.03 = 0.21$ m

$b_e = C_x + C_y$

$= 0.30 + 0.30$

$= 0.60$ m

Therefore $M_{t,max} = 0.15(600)(210)^2 \times 30$

$= 11907 \times 10^4$ N.mm

$= 119$ kN.m which is greater than the moment (23.906 kN.m) obtained by EFM analysis

Table 7.A2 Results of moment distribution for the slab considered in BS8110 after modification from ACI analysis in Table 7A.1.

Joint	A		C			E			G	
Member	AB	AC	CA	CD	CE	EC	EF	EG	GE	GH
Moment = ACI result x $\frac{14.464}{14.864}$	+23.867	-23.906	-91.555	-6.200	85.278	+85.278	+6.200	-91.555	+23.906	-23.867

Positive midspan moments:

First span:

$$\text{Total static moment} = \frac{nL_2 L_1^2}{8}$$

$$= \frac{14.464(4.00)(4.00)^2}{8}$$

$$= 115.712$$

$$\text{Midspan moment} = 115.712 - \frac{1}{2}[23.906 + 91.555]$$

$$= 57.982$$

Interior span:

Total static moment = 115.712

Mid-span moment $= 115.712 - \frac{1}{2}[85.278 + 85.278]$

$= 30.434.$

Redistribution:

First span

$$91.555 - 0.20(91.555) = 73.244$$

$$57.982 + \frac{0.20(91.555)}{2} = 67.137$$

Check total static moment after redistribution

$$\frac{73.244 + 23.906}{2} + 67.137 = 115.712 \quad \text{O.K.}$$

Interior span:

$85.278 - 0.20(85.278) = 68.222$

$30.434 + 0.20(85.278) = 47.490$

Check total static moment after redistribution

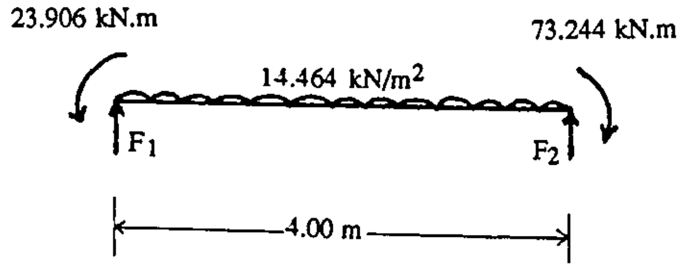
$$\frac{68.222 + 68.222}{2} + 47.490 = 115.712 \quad \text{O.K.}$$

Then to find negative moment at a distance $h_c/2$ from the centre line of the support:

$h_c = 1.128a$ where a = side of square column

$h_c = 1.128\,(0.30)$

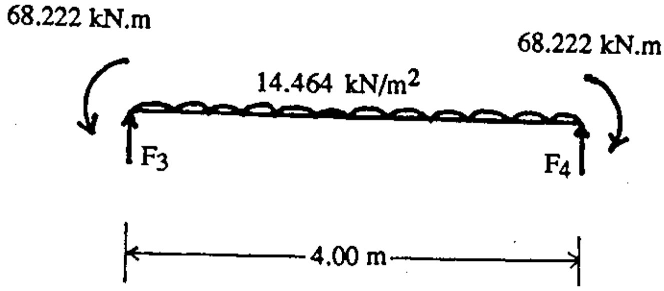
$h_c = 0.338$



$$F_2(4) + 23.906 - 73.244 - 14.464(4)\frac{(4)}{2} = 0$$

Therefore $F_2 = 41.263$

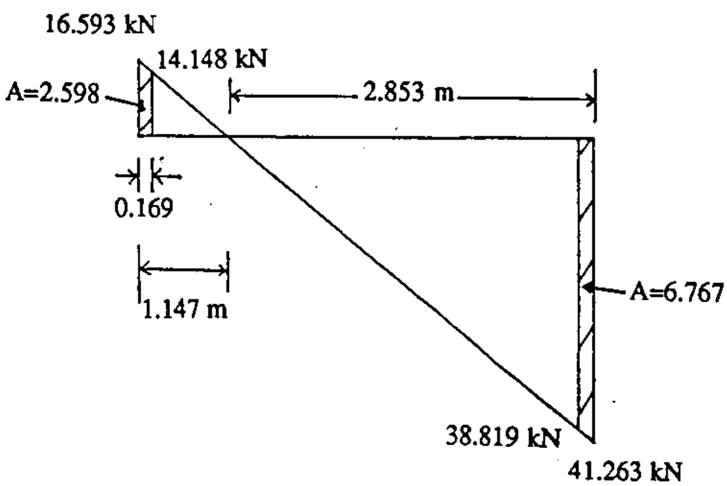
$F_1 = 16.593$

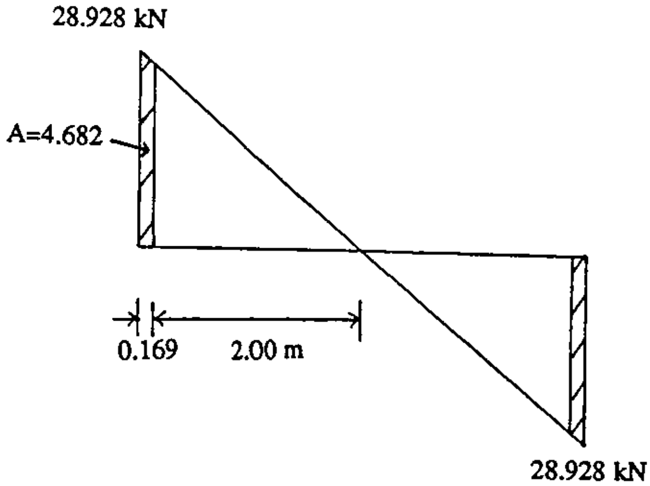


$$F_4(4) + 68.222 - 68.222 - 14.464(4)\frac{(4)}{2} = 0$$

Therefore $F_4 = 28.928$

$F_3 = 28.928$





Therefore negative moments at faces of equivalent support are as follows:

At exterior column $= 23.906 - 2.598$

$= 21.308$

At interior column for

the first span $= 73.244 - 767$

$= 66.477$

At interior column for

the interior span $= 68.222 - 4.682$

$= 63.54$

Adjustment:

$$\frac{M_1 + M_2}{2} + M_3 < M_o$$

$$M_o = \frac{n\, l_2}{8}\left(l_1 - \frac{2h_c}{3}\right)^2$$

h_c = 1.128a where a = side of square column

h_c = 1.128(0.30)

h_c = 0.338

$$M_0 = \frac{14.464(4.00)}{8}(4.00 - \frac{2(0.338)}{3})^2$$

$$= 103.042$$

First span:

$$\frac{M_1 + M_2}{2} + M_3 = \frac{21.308 + 66.477}{2} + 67.137$$

$$= 111.03 > M_o \quad \text{O.K.}$$

The moment values at the various calculation stages are shown in Fig. 7.11.

APPENDIX 7B

Assessment of moment coefficient due to equivalent frame method in BS8110 for comparison purposes with the code simplified coefficient method.

APPENDIX 7B

Assessment of moment coefficient due to equivalent frame method in BS8110 for comparison purposes with the code simplified coefficient method.

$n \quad = 14.464$

$F \quad = 14.464\,(4.00 \times 4.00)$

$\quad = 231.424$

$$L = L_1 - \frac{2h_c}{3}$$

$h_c \quad = 0.338$

$$L = 4.00 - \frac{2(0.338)}{3}$$

$\quad = 3.775$

$M \quad = C F L$

$$C = \frac{M}{FL}$$

For outer support:

$$C = \frac{21.308}{231.424(3.775)} = 0.024$$

For interior support:

$$= \frac{66.477}{231.424(3.775)} = 0.076$$

Near centre of first span:

$$= \frac{67.137}{231.424(3.775)} = 0.077$$

Centre of interior span:

$$= \frac{47.490}{231.424(3.775)} = 0.054$$

APPENDIX 7C

Finite element output for flat slab analysis

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
1	-87840.0156	-87840.0156	-44640.0078	0.0000	-132480.0312	0.0000	-132480.0312
2	-87840.0156	-87840.0156	44640.0000	0.0000	-132480.0312	0.0000	-132480.0312
3	-87840.0156	-87840.0156	44640.0000	0.0000	-132480.0312	0.0000	-132480.0312
4	-87840.0156	-87840.0156	-44640.0078	0.0000	-132480.0312	0.0000	-132480.0312
5	6681.1133	12504.4844	-4411.5850	11092.6973	0.0000	16916.0664	0.0000
6	12504.4844	6681.1133	-4411.5850	16916.0664	0.0000	11092.6973	0.0000
7	1056.4875	1057.9116	6810.5703	7867.0576	-5754.0830	7868.4814	-5752.6592
8	18383.3320	-1218.5334	6768.5195	25151.8516	0.0000	5549.9854	-3710.6201
9	16375.8516	7979.3467	4899.6250	21275.4766	0.0000	12878.9707	0.0000
10	25006.4453	900.1119	2168.7070	27175.1523	0.0000	3068.8188	0.0000
11	21956.7461	8129.6504	1840.6831	23797.4258	0.0000	9970.3320	0.0000
12	25451.8516	427.8261	-1803.9331	27255.7812	0.0000	2231.7588	0.0000
13	22386.9336	8359.4609	-1440.5969	23827.5273	0.0000	9800.0566	0.0000
14	20784.5625	935.9171	-5729.5303	26514.0898	0.0000	6665.4473	-643.5011
15	17735.9180	8162.4766	-4730.3926	22466.3086	0.0000	12892.8691	0.0000
16	9376.3516	-1528.3521	-10308.2187	19684.5703	-931.8672	8779.8652	-11836.5723
17	8072.5254	7895.9434	-7794.8184	15867.3437	0.0000	15690.7617	0.0000
18	-10581.1973	13317.1934	789.9902	0.0000	-10628.0605	13376.1738	0.0000
19	-10036.4297	-117.4927	-9718.1934	0.0000	-19754.6250	9292.5547	-9835.6875
20	-69061.2031	-88810.8437	2806.8789	0.0000	-71868.0937	0.0000	-91617.7344
21	-33511.8047	15290.9922	1314.9111	0.0000	-33624.8828	15342.5840	0.0000
22	-10968.6348	12792.6328	-1837.5303	0.0000	-11232.5781	13100.4648	0.0000
23	-9305.0449	-455.1353	9176.0742	0.0000	-18481.1211	8593.7539	-9631.2109
24	6568.8350	-1538.4353	8614.9082	15183.7422	-2046.0732	7076.4727	-10153.3437
25	5153.3369	7796.8184	6729.0713	11882.4102	-654.2090	14525.8887	0.0000
26	15431.4551	920.0323	4004.9448	19436.3984	0.0000	4924.9766	-119.3759
27	12313.9102	8239.6855	3472.8096	15786.7187	0.0000	11712.4941	0.0000
28	17947.6719	413.2683	0.0000	17947.6719	0.0000	413.2883	0.0000
29	14731.4160	8514.9805	0.0000	14731.4160	0.0000	8514.9805	0.0000
30	15431.3613	918.5902	-4005.3135	19436.6719	0.0000	4923.9033	-121.0157
31	12297.2793	8227.5156	-3463.0171	15760.2949	0.0000	11690.5312	0.0000
32	6568.8350	-1538.4353	-8614.9121	15183.7461	-2046.0771	7076.4766	-10153.3477
33	5154.6123	7797.7051	-6728.0137	11882.6250	-650.4502	14525.7187	0.0000
34	-10968.6348	12792.6328	1837.5283	0.0000	-11232.5781	13100.4648	0.0000
35	-9314.5840	-444.8875	-9170.9687	0.0000	-18485.5547	8584.6797	-9615.8574
36	-69061.2031	-88810.8437	-2806.8828	0.0000	-71868.0937	0.0000	-91617.7344
37	-33511.8047	15290.9922	-1314.9121	0.0000	-33624.8828	15342.5840	0.0000
38	-10581.1973	13317.1934	-789.9922	0.0000	-10628.0625	13376.1738	0.0000
39	-10037.1465	-119.8962	9716.8887	0.0000	-19754.0352	9286.9531	-9836.7852
40	9376.3516	-1528.3521	10308.2168	19684.5664	-931.8652	8779.8633	-11836.5703
41	8072.7031	7896.0537	7794.6709	15867.3730	0.0000	15690.7246	0.0000
42	20784.4961	935.0214	5729.7695	26514.2656	0.0000	6664.7900	-644.5338
43	17735.5742	8160.4219	4731.2324	22466.8047	0.0000	12891.6543	0.0000
44	25451.8320	424.0048	1804.1965	27256.0273	0.0000	2228.2012	0.0000
45	22386.9531	8361.8398	1440.3757	23827.3281	0.0000	9802.2148	0.0000
46	25006.4219	902.0558	-2168.3335	27174.7539	0.0000	3070.3892	0.0000
47	21932.9062	8122.2842	-1838.9932	23771.8984	0.0000	9961.2773	0.0000
48	18382.7852	-1222.7881	-6770.0449	25152.8281	0.0000	5547.2568	-3716.0723
49	16375.9395	7979.2588	-4899.5439	21275.4805	0.0000	12878.8027	0.0000
50	6681.1133	12504.4844	4411.5830	11092.6953	0.0000	16916.0664	0.0000
51	1059.8394	1059.8396	-6812.4990	7872.3379	-5752.6602	7872.3379	-5752.6602
52	12504.4844	6681.1133	4411.5830	16916.0664	0.0000	11092.6953	0.0000
53	-1222.7881	18382.7852	6770.0430	5547.2549	-3716.0708	25152.8281	0.0000
54	7979.2568	16375.9395	4899.5420	12878.8008	0.0000	21275.4805	0.0000
55	15144.0000	15144.0000	3743.8057	18887.8047	0.0000	18887.8047	0.0000
56	19869.7109	14570.2832	1511.0691	21380.7773	0.0000	16081.3516	0.0000

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
57	19996.0625	14611.9336	-989.0555	20985.1172	0.0000	15600.9883	0.0000
58	15654.8105	14945.1855	-3570.1704	19224.9805	0.0000	18515.3555	0.0000
59	6712.5225	16015.4746	-5725.5439	12438.0664	0.0000	21741.0156	0.0000
60	-5827.4912	18516.2852	-5722.3545	0.0000	-7595.9531	24135.3984	0.0000
61	-12636.8105	20412.8008	103.6119	0.0000	-12637.3379	20413.6484	0.0000
62	-7204.1621	18203.3594	5726.8896	0.0000	-9005.8770	22755.9023	0.0000
63	3386.4751	15830.3223	5259.0752	8645.5488	0.0000	21089.3945	0.0000
64	9981.5215	14997.6758	2824.9727	12806.4941	0.0000	17822.6484	0.0000
65	12121.9961	14902.0000	1.2249	12123.2207	0.0000	14903.2246	0.0000
66	9987.6035	15015.5918	-2835.5752	12823.1777	0.0000	17851.1641	0.0000
67	3390.7729	15826.0254	-5264.5596	8655.3320	0.0000	21090.5820	0.0000
68	-7208.7646	18207.9570	-5715.6816	0.0000	-9002.9824	22739.8008	0.0000
69	-12660.7305	20412.7227	-104.9915	0.0000	-12661.2715	20413.5898	0.0000
70	-5825.1904	18516.3828	5721.8750	0.0000	-7593.3467	24136.7734	0.0000
71	6713.2920	16009.9062	5731.4014	12444.6934	0.0000	21741.3047	0.0000
72	15657.6035	14942.3906	3569.8999	19227.5000	0.0000	18512.2891	0.0000
73	20019.2617	14588.7324	995.1849	21014.4453	0.0000	15583.9160	0.0000
74	19868.5859	14571.4082	-1513.4099	21381.9922	0.0000	16084.8164	0.0000
75	15144.0000	15144.0000	-3743.8071	18887.8047	0.0000	18887.8047	0.0000
76	7983.1992	16376.7988	-4897.7217	12880.9199	0.0000	21274.5195	0.0000
77	-1222.7881	18382.7852	-6770.0449	5547.2568	-3716.0723	25152.8281	0.0000
78	899.9128	25006.4062	2168.5239	3068.4365	0.0000	27174.9297	0.0000
79	8124.6338	21932.9609	1838.6458	9963.2793	0.0000	23771.6055	0.0000
80	14547.9727	19868.0234	1517.9902	16065.9629	0.0000	21386.0117	0.0000
81	18493.2969	18490.6953	707.9553	19201.2500	0.0000	19198.6484	0.0000
82	18434.5234	16261.4687	-383.7853	18818.3086	0.0000	18645.2539	0.0000
83	14126.5723	19233.4219	-1542.6472	15669.2187	0.0000	20776.0664	0.0000
84	6445.9805	21319.6133	-2389.0107	8834.9902	0.0000	23708.6211	0.0000
85	-1852.0554	23956.0508	-2001.1067	149.0513	-2019.2129	25957.1562	0.0000
86	-5842.0176	24766.0117	214.5771	0.0000	-5843.8770	24773.8906	0.0000
87	-3552.8062	23753.6016	2287.7969	0.0000	-3773.1523	25226.8047	0.0000
88	2751.8042	21116.1914	2418.9341	5170.7383	0.0000	23535.1250	0.0000
89	8183.6172	19209.9766	1375.9927	9559.6094	0.0000	20585.9687	0.0000
90	10198.4258	18553.5664	0.0000	10198.4258	0.0000	18553.5664	0.0000
91	8181.2734	19209.9219	-1376.3606	9557.6328	0.0000	20586.2812	0.0000
92	2747.0713	21116.1250	-2419.5039	5166.5752	0.0000	23535.6289	0.0000
93	-3550.5654	23756.1602	-2275.3955	0.0000	-3768.5059	25214.3555	0.0000
94	-5839.1064	24789.5000	-201.8268	0.0000	-5840.7500	24796.4727	0.0000
95	-1853.6543	23957.6523	1989.8926	136.2383	-2018.9324	25947.5430	0.0000
96	6443.6436	21319.5508	2389.3921	8833.0352	0.0000	23708.9414	0.0000
97	14148.2559	19235.7383	1535.5593	15683.8145	0.0000	20771.2969	0.0000
98	18434.4648	16261.5234	383.7491	18818.2109	0.0000	18645.2695	0.0000
99	18491.9961	18491.9961	-707.9464	19199.9414	0.0000	19199.9414	0.0000
100	14571.4062	19868.5859	-1513.4099	16084.8164	0.0000	21381.9922	0.0000
101	8129.3516	21933.0391	-1838.0251	9967.3750	0.0000	23771.0625	0.0000
102	893.9652	25010.1914	-2147.7798	3041.7446	0.0000	27157.9687	0.0000
103	424.0048	25451.8320	-1804.1970	2228.2017	0.0000	27256.0273	0.0000
104	8361.8398	22386.9531	-1440.3765	9802.2148	0.0000	23827.3281	0.0000
105	14588.7324	20019.2617	-995.1853	15583.9160	0.0000	21014.4453	0.0000
106	18261.3125	18434.6797	-384.1856	18645.4961	0.0000	18818.8633	0.0000
107	18096.7500	18095.2422	287.9999	18384.7461	0.0000	18383.2383	0.0000
108	13662.0098	19145.9844	844.6213	14506.6309	0.0000	19990.6055	0.0000
109	6181.0791	21356.5156	1060.1582	7241.2373	0.0000	22416.6719	0.0000
110	-1230.5613	23627.3594	816.0985	0.0000	-1258.7498	24168.5859	0.0000
111	-4858.6465	24509.8437	232.0354	0.0000	-4860.8437	24520.9219	0.0000
112	-3080.1265	23386.5195	-370.4280	0.0000	-3085.9941	23431.0664	0.0000
113	2284.9146	20951.8789	-644.5527	2929.4673	0.0000	21596.4297	0.0000
114	7493.2383	18805.9570	-462.2880	7955.5254	0.0000	19268.2422	0.0000

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
115	9539.4863	18019.7109	0.1379	9539.6230	0.0000	18019.8477	0.0000
116	7498.0186	18805.9766	462.0880	7960.1064	0.0000	19268.0625	0.0000
117	2287.3125	20951.8828	644.4982	2931.8105	0.0000	21596.3789	0.0000
118	-3080.1245	23386.5195	370.3984	0.0000	-3085.9912	23431.0586	0.0000
119	-4863.4385	24533.8320	-231.6962	0.0000	-4865.6270	24544.8672	0.0000
120	-1228.1633	23627.3594	-816.0864	0.0000	-1256.3511	24169.6289	0.0000
121	6183.4707	21356.5273	-1060.0122	7243.4824	0.0000	22416.5391	0.0000
122	13638.1504	19145.8437	-846.4575	14484.6074	0.0000	19992.3008	0.0000
123	18096.3711	18095.6172	-288.0016	18384.3711	0.0000	18383.6172	0.0000
124	18262.2695	18433.7227	383.6592	18645.9258	0.0000	18817.3789	0.0000
125	14612.3848	20019.6094	992.3225	15604.7070	0.0000	21011.9297	0.0000
126	8361.8398	22386.9531	1440.3757	9802.2148	0.0000	23827.3281	0.0000
127	423.0497	25451.8281	1804.2651	2227.3145	0.0000	27256.0898	0.0000
128	935.0214	20784.4961	-5729.7715	6664.7920	-644.5348	26514.2656	0.0000
129	8158.3828	17735.2148	-4732.0918	12890.4746	0.0000	22467.3047	0.0000
130	14942.3906	15657.6035	-3569.9014	18512.2891	0.0000	19227.5039	0.0000
131	19233.4219	14126.5723	-1542.6472	20776.0664	0.0000	15669.2187	0.0000
132	19145.9844	13662.0098	844.6213	19990.6055	0.0000	14506.6309	0.0000
133	14244.0000	14244.0000	3107.9893	17351.9883	0.0000	17351.9883	0.0000
134	5348.2139	16009.3848	4510.0879	9858.3008	0.0000	20519.4727	0.0000
135	-4124.6797	18335.0742	3612.4224	0.0000	-4836.4082	21498.8555	0.0000
136	-8647.5332	19124.3281	194.5949	0.0000	-8669.5137	19128.6953	0.0000
137	-5896.8682	17952.0625	-3110.6465	0.0000	-6435.8662	19592.9531	0.0000
138	1490.3403	15311.0977	-3811.2446	5301.5850	0.0000	19122.3398	0.0000
139	7979.7578	13418.6406	-2308.2075	10287.9648	0.0000	15726.8477	0.0000
140	10362.8555	12797.1406	-4.4465	10367.3008	0.0000	12801.5859	0.0000
141	7983.4150	13417.3809	2309.6812	10293.0957	0.0000	15727.0605	0.0000
142	1492.1079	15311.2500	3810.7236	5302.8311	0.0000	19121.9727	0.0000
143	-5896.8691	17952.0664	3110.6279	0.0000	-6435.8604	19592.9375	0.0000
144	-8662.7383	19124.3320	-194.5114	0.0000	-8664.7168	19128.6992	0.0000
145	-4124.6777	18335.0742	-3612.4404	0.0000	-4836.4141	21498.8906	0.0000
146	5352.9854	16031.0117	-4516.8447	9869.8301	0.0000	20547.8555	0.0000
147	14243.9980	14243.9980	-3107.9902	17351.9883	0.0000	17351.9883	0.0000
148	19145.9844	13662.0098	-844.6216	19990.6055	0.0000	14506.6309	0.0000
149	19233.4219	14126.5723	1542.6467	20776.0664	0.0000	15669.2187	0.0000
150	14945.1855	15654.8105	3570.1689	18515.3516	0.0000	19224.9766	0.0000
151	8162.4990	17735.8945	4730.4141	12892.9121	0.0000	22466.3086	0.0000
152	933.2301	20784.3672	5730.2490	6663.4785	-646.5992	26514.6133	0.0000
153	-1528.3521	9376.3516	-10308.2187	8779.8652	-11836.5723	19684.5703	-931.8672
154	7895.1143	8071.8311	-7795.5781	15690.6914	0.0000	15867.4082	0.0000
155	16015.0117	6710.5869	-5726.4922	21741.5039	0.0000	12437.0781	0.0000
156	21319.6172	6445.9775	-2389.0029	23708.6172	0.0000	8834.9805	0.0000
157	21356.5078	6181.0859	1060.2078	22416.7148	0.0000	7241.2930	0.0000
158	16009.3828	5348.2139	4510.0879	20519.4687	0.0000	9858.3008	0.0000
159	4863.5986	4863.5986	7472.3994	12335.9980	-2608.8008	12335.9980	-2608.8008
160	-10676.7910	7021.5879	7394.3662	0.0000	-18071.1602	12142.6621	-372.7783
161	-18476.4102	7786.8008	192.6451	0.0000	-18481.1797	7788.8086	0.0000
162	-12214.8926	6378.0889	-6844.5781	0.0000	-19059.4727	10213.4258	-466.4893
163	1180.0068	3919.9927	-6606.0059	7786.0127	-5425.9990	10525.9980	-2686.0132
164	9764.5215	4013.8774	-3534.2700	13298.7910	0.0000	7548.1475	0.0000
165	12476.0586	4319.1406	0.0000	12476.0586	0.0000	4319.1406	0.0000
166	9740.7207	4011.2764	3528.6655	13269.3848	0.0000	7539.9414	0.0000
167	1180.9368	3921.4624	6607.1729	7788.1094	-5426.2363	10528.6348	-2685.7104
168	-12214.6562	6380.2529	6845.2910	0.0000	-19059.9492	10216.4629	-465.0381
169	-18500.3281	7789.1221	-194.0582	0.0000	-18505.1641	7791.1572	0.0000
170	-10676.5566	7023.7539	-7395.0811	0.0000	-18071.6406	12145.9316	-371.3271
171	4863.5996	4863.5996	-7472.4014	12336.0000	-2608.8018	12336.0000	-2608.8018
172	16030.7363	5350.8613	-4517.6084	20548.3437	0.0000	9868.4687	0.0000

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
173	21356.5000	6178.6934	-1060.3420	22416.8398	0.0000	7239.0352	0.0000
174	21319.6680	6448.3271	2388.6587	23708.3242	0.0000	8836.9844	0.0000
175	16010.7930	6717.2041	5729.5410	21740.3320	0.0000	12446.7441	0.0000
176	7896.5137	8073.0840	7794.2520	15690.7656	0.0000	15867.3359	0.0000
177	-1528.3521	9376.3516	10308.2168	8779.8633	-11836.5703	19684.5664	-931.8652
178	13317.1934	-10581.1973	789.9902	13376.1738	0.0000	0.0000	-10628.0605
179	-119.2412	-10036.8418	-9717.3398	9288.7656	-9836.5820	0.0000	-19754.1836
180	18516.1719	-5829.7783	-5722.8613	24134.0742	0.0000	0.0000	-7598.5645
181	23956.0508	-1852.0554	-2001.1067	25957.1562	0.0000	149.0513	-2019.2129
182	23627.3594	-1228.1614	816.0818	24169.6211	0.0000	0.0000	-1256.3486
183	18332.0625	-4121.6660	3622.2192	21515.3555	0.0000	0.0000	-4837.3779
184	7023.7539	-10676.5566	7395.0781	12145.9277	-371.3242	0.0000	-18071.6367
185	-18570.0078	-18570.0078	11296.0059	0.0000	-29866.0156	0.0000	-29866.0156
186	-41633.3359	-7938.6709	-814.7994	0.0000	-42448.1406	0.0000	-8753.4707
187	-18488.2578	-19141.3477	-10544.7285	0.0000	-29032.9883	0.0000	-29686.0781
188	3397.3691	-11449.3730	-6490.9316	7077.2373	-3093.5625	0.0000	-17940.3047
189	12011.3066	-5800.1113	-2799.8389	13362.8477	0.0000	0.0000	-6452.7549
190	14625.7969	-3955.4004	0.0000	14625.7969	0.0000	0.0000	-3955.4004
191	11990.4727	-5800.8760	2788.5454	13330.9570	0.0000	0.0000	-6449.3896
192	3399.5298	-11449.1348	6491.6484	7080.2881	-3092.1187	0.0000	-17940.7852
193	-18488.2578	-19141.3477	10544.7246	0.0000	-29032.9844	0.0000	-29686.0742
194	-41633.3359	-7938.6709	814.7990	0.0000	-42448.1406	0.0000	-8753.4707
195	-18571.2852	-18571.1250	-11294.8125	0.0000	-29866.0977	0.0000	-29865.9375
196	7013.8135	-10671.4160	-7401.2422	12147.0000	-387.4287	0.0000	-18072.6602
197	18335.1367	-4122.3428	-3612.0498	21500.0586	0.0000	0.0000	-4833.9229
198	23627.3594	-1228.1614	-816.0820	24169.6250	0.0000	0.0000	-1256.3486
199	23957.6523	-1851.2588	1989.7629	25947.4141	0.0000	138.5042	-2016.5154
200	18516.1719	-5829.7783	5722.8584	24134.0703	0.0000	0.0000	-7598.5625
201	-117.3289	-10036.3535	9718.3008	9292.9980	-9835.6309	0.0000	-19754.6562
202	13317.1934	-10581.1973	-789.9922	13376.1738	0.0000	0.0000	-10628.0625
203	-88810.8437	-69061.2031	2806.8789	0.0000	-91617.7344	0.0000	-71868.0937
204	15290.9922	-33511.8047	1314.9111	15342.5840	0.0000	0.0000	-33624.8828
205	20412.8047	-12636.8105	103.6119	20413.6523	0.0000	0.0000	-12637.3379
206	24765.5234	-5836.7305	201.1359	24772.4531	0.0000	0.0000	-5838.3643
207	24533.8398	-4858.6406	231.6584	24544.8828	0.0000	0.0000	-4860.8281
208	19124.3320	-8665.1387	194.4987	19128.6953	0.0000	0.0000	-8667.1172
209	7793.9121	-18500.3164	194.0745	7795.9473	0.0000	0.0000	-18505.1523
210	-7938.6709	-41633.3437	-814.7994	0.0000	-8753.4707	0.0000	-42448.1484
211	-98016.0312	-98016.0312	372.3215	0.0000	-98388.3594	0.0000	-98388.3594
212	-9586.8574	-40095.5547	1277.0464	0.0000	-10863.9043	0.0000	-41372.6016
213	3983.2700	-19117.6758	249.7751	3986.5332	0.0000	0.0000	-19133.3398
214	12686.6348	-10526.6387	151.6081	12688.8164	0.0000	0.0000	-10528.4512
215	15336.9180	-8007.3232	-0.0001	15336.9180	0.0000	0.0000	-8007.3232
216	12686.6348	-10526.6387	-151.6083	12688.8164	0.0000	0.0000	-10528.4512
217	3985.6689	-19117.6758	-250.0215	3988.9385	0.0000	0.0000	-19133.3633
218	-9560.7461	-40097.6641	-1292.0276	0.0000	-10852.7754	0.0000	-41389.6953
219	-98016.0312	-98016.0312	-372.3232	0.0000	-98388.3594	0.0000	-98388.3594
220	-7936.2822	-41633.3281	814.9701	0.0000	-8751.2539	0.0000	-42448.3047
221	7789.1182	-18500.3242	-194.0165	7791.1523	0.0000	0.0000	-18505.1602
222	19124.3320	-8665.1387	-194.4988	19128.6953	0.0000	0.0000	-8667.1172
223	24509.8398	-4858.6455	-232.1485	24520.9297	0.0000	0.0000	-4860.8447
224	24813.4883	-5841.4951	-201.4247	24820.4336	0.0000	0.0000	-5843.1309
225	20412.8047	-12636.8105	-103.6122	20413.6523	0.0000	0.0000	-12637.3379
226	15290.9922	-33511.8047	-1314.9119	15342.5840	0.0000	0.0000	-33624.8828
227	-88810.8437	-69061.2031	-2806.8828	0.0000	-91617.7344	0.0000	-71868.0937
228	12792.6328	-10968.6348	-1837.5303	13100.4648	0.0000	0.0000	-11232.5781
229	-454.7729	-9304.8789	9176.3203	8594.7637	-9631.0937	0.0000	-18481.1992
230	18203.3594	-7204.1621	5726.8896	22755.9023	0.0000	0.0000	-9005.8770

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
231	23756.1367	-3552.9390	2275.6611	25213.6992	0.0000	0.0000	-3770.9307
232	23362.5508	-3080.1567	-369.5490	23406.8867	0.0000	0.0000	-3086.0024
233	17952.0625	-5896.8682	-3110.6465	19592.9531	0.0000	0.0000	-6435.8662
234	6380.2832	-12214.6875	-6845.2490	10216.4375	-464.9658	0.0000	-19059.9375
235	-19141.3477	-18488.2578	-10544.7285	0.0000	-29686.0781	0.0000	-29032.9883
236	-40097.9922	-9584.4199	1289.2458	0.0000	-41387.2422	0.0000	-10873.6660
237	-19114.8047	-19114.8086	10738.1758	0.0000	-29852.9805	0.0000	-29852.9844
238	2595.7700	-13148.5742	6776.7891	6088.5342	-4181.0195	0.0000	-19925.3633
239	11325.1895	-7813.9941	2960.4780	12446.8203	0.0000	0.0000	-8587.8828
240	13956.1094	-6084.1133	0.0000	13956.1094	0.0000	0.0000	-6084.1133
241	11325.1914	-7813.9941	-2960.4795	12446.8242	0.0000	0.0000	-8587.8848
242	2595.8433	-13148.6465	-6776.7051	6088.5010	-4180.8623	0.0000	-19925.3516
243	-19114.8047	-19114.8086	-10738.1797	0.0000	-29852.9844	0.0000	-29852.9883
244	-40097.9922	-9584.4180	-1289.2466	0.0000	-41387.2422	0.0000	-10873.6660
245	-19141.3477	-18488.2578	10544.7227	0.0000	-29686.0703	0.0000	-29032.9805
246	6378.0869	-12214.8945	6844.5762	10213.4238	-466.4873	0.0000	-19059.4727
247	17928.3008	-5897.1045	3108.2729	19566.6211	0.0000	0.0000	-6435.9941
248	23386.5156	-3082.5229	370.4863	23431.0430	0.0000	0.0000	-3088.3926
249	23729.7617	-3550.5684	-2285.7246	25201.2266	0.0000	0.0000	-3770.7368
250	18208.0625	-7206.4668	-5715.1973	22740.5859	0.0000	0.0000	-9000.3691
251	-454.6580	-9304.8262	-9176.4004	8595.0879	-9631.0586	0.0000	-18481.2266
252	12792.6328	-10968.6348	1837.5283	13100.4648	0.0000	0.0000	-11232.5781
253	-1538.4353	6568.8350	8614.9082	7076.4727	-10153.3437	15183.7422	-2046.0732
254	7796.7734	5153.3848	6729.0801	14525.8535	0.0000	11882.4648	-654.2119
255	15826.2754	3392.9219	5263.8223	21090.0977	0.0000	8656.7441	0.0000
256	21116.1523	2749.4448	2419.2446	23535.3945	0.0000	5168.6895	0.0000
257	20951.8828	2287.3135	-644.5026	21596.3828	0.0000	2931.8159	0.0000
258	15311.2148	1491.9033	-3810.8179	19122.0312	0.0000	5302.7207	0.0000
259	3919.9927	1180.0068	-6606.0059	10525.9980	-2686.0132	7786.0127	-5425.9990
260	-11448.8281	3401.6245	-6492.4512	0.0000	-17941.2812	7083.3916	-3090.8267
261	-19117.6719	3985.6665	249.6872	0.0000	-19133.3164	3988.9272	0.0000
262	-13169.5488	2590.3462	6784.2861	0.0000	-19953.8359	6085.2676	-4193.9404
263	-124.8000	-124.8000	6520.7988	6395.9980	-6645.5996	6395.9980	-6645.5996
264	8480.2734	-353.8763	3465.2744	11945.5469	0.0000	3111.3979	-1769.8833
265	11276.4473	-173.9030	-0.0107	11276.4473	0.0000	0.0000	-173.9030
266	8481.4023	-352.6035	-3471.4463	11952.8477	0.0000	3118.8428	-1773.4697
267	-117.4299	-129.7701	-6519.5908	6402.1602	-6637.0215	6389.8203	-6649.3613
268	-13170.4180	2591.2144	-6783.2695	0.0000	-19953.6875	6084.8574	-4192.0557
269	-19117.6758	3983.2700	-249.7753	0.0000	-19133.3398	3986.5332	0.0000
270	-11449.1289	3399.5269	6491.6514	0.0000	-17940.7812	7080.2900	-3092.1245
271	3919.0176	1178.5818	6607.1826	10526.1992	-2688.1650	7785.7637	-5428.6016
272	15333.6973	1490.5408	3817.0195	19150.7148	0.0000	5307.5596	0.0000
273	20975.8555	2289.7402	645.2639	21621.1172	0.0000	2935.0039	0.0000
274	21139.7891	2747.4058	-2422.3130	23562.1016	0.0000	5169.7187	0.0000
275	15830.3184	3386.4805	-5259.0840	21089.4023	0.0000	8645.5645	0.0000
276	7797.3906	5154.2070	-6728.3721	14525.7617	0.0000	11882.5781	-651.7080
277	-1541.7654	6567.3643	-8617.1230	7075.3574	-10158.8887	15184.4863	-2049.7588
278	917.4220	15431.2812	4005.6177	4923.0391	-122.3473	19436.8984	0.0000
279	8233.1250	12315.6719	3473.1714	11706.2949	0.0000	15788.8418	0.0000
280	14997.2207	9979.5762	2825.9136	17823.1328	0.0000	12805.4883	0.0000
281	19209.9219	8181.2734	1376.3555	20586.2773	0.0000	9557.6289	0.0000
282	18805.9766	7498.0225	-462.0920	19268.0664	0.0000	7960.1143	0.0000
283	13421.1582	7977.2393	-2305.6563	15726.8145	0.0000	10282.8945	0.0000
284	4011.2036	9740.7949	-3528.6099	7539.8135	0.0000	13269.4043	0.0000
285	-5800.1113	12011.3066	-2799.8389	0.0000	-6452.7549	13362.8477	0.0000
286	-10526.6387	12686.6348	151.6081	0.0000	-10528.4512	12688.8164	0.0000
287	-7813.9941	11325.1695	2960.4780	0.0000	-8587.8828	12446.8203	0.0000
288	-353.0921	8477.0898	3469.9941	3116.9019	-1773.4922	11947.0840	0.0000

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
289	6344.3994	6344.3984	2057.9692	8402.3672	0.0000	8402.3672	0.0000
290	8847.5937	5629.2041	0.0885	8847.6816	0.0000	5629.2920	0.0000
291	6346.7988	6346.7988	-2057.9321	8404.7305	0.0000	8404.7305	0.0000
292	-353.8509	8480.2500	-3465.3096	3111.4585	-1769.8906	11945.5586	0.0000
293	-7813.9961	11325.1914	-2960.4580	0.0000	-8587.8750	12446.8066	0.0000
294	-10502.6660	12686.6621	-152.4036	0.0000	-10504.4980	12688.8730	0.0000
295	-5802.7695	12013.9648	2792.1807	0.0000	-6451.7041	13357.5078	0.0000
296	4011.9722	9764.0254	3535.2397	7547.2119	0.0000	13299.2637	0.0000
297	13417.3516	7983.4473	2309.7266	15727.0781	0.0000	10293.1738	0.0000
298	18805.9609	7493.2344	462.2631	19268.2227	0.0000	7955.4971	0.0000
299	19209.9219	8181.2734	-1376.3562	20586.2773	0.0000	9557.6289	0.0000
300	15019.0508	9988.9453	-2831.5820	17850.6328	0.0000	12820.5273	0.0000
301	8238.5137	12317.4805	-3470.0493	11708.5625	0.0000	15787.5293	0.0000
302	916.3412	15431.2109	-4005.8955	4922.2363	-123.5770	19437.1055	0.0000
303	411.3687	17947.6680	-0.0001	411.3688	0.0000	17947.6680	0.0000
304	8512.5840	14731.4141	0.0461	8512.6289	0.0000	14731.4590	0.0000
305	14901.7187	12122.2773	0.0000	14901.7187	0.0000	12122.2773	0.0000
306	18553.5664	10198.4258	0.0000	18553.5664	0.0000	10198.4258	0.0000
307	18019.7187	9541.8789	0.0000	18019.7187	0.0000	9541.8789	0.0000
308	12796.2539	10339.7422	0.0000	12796.2539	0.0000	10339.7422	0.0000
309	4323.9365	12476.0625	0.0000	4323.9365	0.0000	12476.0625	0.0000
310	-3950.6025	14625.7988	0.0000	0.0000	-3950.6025	14625.7988	0.0000
311	-8012.1182	15336.9141	-0.0001	0.0000	-8012.1182	15336.9141	0.0000
312	-6084.1133	13956.1094	0.0000	0.0000	-6084.1133	13956.1094	0.0000
313	-174.2866	11276.4473	0.0000	0.0000	-174.2866	11276.4473	0.0000
314	5629.2979	8852.2988	0.0000	5629.2979	0.0000	8852.2988	0.0000
315	7941.5908	7941.6074	-0.0001	7941.5908	0.0000	7941.6074	0.0000
316	5633.5957	8850.4004	-1.1422	5634.7373	0.0000	8851.5410	0.0000
317	-173.2088	11276.4473	0.0042	0.0000	-173.2088	11276.4473	0.0000
318	-6084.1104	13956.1074	-0.0001	0.0000	-6084.1104	13956.1074	0.0000
319	-8012.1191	15336.9141	-0.0001	0.0000	-8012.1191	15336.9141	0.0000
320	-3955.4058	14625.8008	0.0000	0.0000	-3955.4058	14625.8008	0.0000
321	4319.1426	12476.0547	0.0000	4319.1426	0.0000	12476.0547	0.0000
322	12796.2539	10339.7422	0.0000	12796.2539	0.0000	10339.7422	0.0000
323	18019.7187	9541.8789	0.0000	18019.7187	0.0000	9541.8789	0.0000
324	18553.5664	10198.4258	0.0000	18553.5664	0.0000	10198.4258	0.0000
325	14902.2793	12121.7148	0.0000	14902.2793	0.0000	12121.7148	0.0000
326	8514.9805	14731.4160	0.0000	8514.9805	0.0000	14731.4160	0.0000
327	412.8004	17947.6680	0.0074	412.8160	0.0000	17947.6719	0.0000
328	916.8350	15431.2441	-4005.7656	4922.6006	-123.0137	19437.0078	0.0000
329	8231.1660	12298.4316	-3460.9687	11692.1348	0.0000	15759.4004	0.0000
330	14997.9922	9983.6035	-2824.1562	17822.1484	0.0000	12807.7598	0.0000
331	19209.9219	8181.2725	-1376.3606	20586.2812	0.0000	9557.6328	0.0000
332	18805.9609	7493.2344	462.2840	19268.2422	0.0000	7955.5176	0.0000
333	13421.3711	7979.4277	2304.9736	15726.3437	0.0000	10284.4004	0.0000
334	4011.2764	9740.7227	3528.6655	7539.9414	0.0000	13269.3867	0.0000
335	-5800.8760	11990.4727	2768.5449	0.0000	-6449.3896	13330.9551	0.0000
336	-10526.6387	12686.6348	-151.6083	0.0000	-10528.4512	12688.8164	0.0000
337	-7813.9941	11325.1914	-2960.4795	0.0000	-8587.8848	12446.8242	0.0000
338	-353.0921	8477.0698	-3469.9956	3116.9033	-1773.4934	11947.0840	0.0000
339	6345.5908	6345.6074	-2056.7700	8402.3594	0.0000	8402.3770	0.0000
340	8847.5937	5629.2041	0.0886	8847.6816	0.0000	5629.2920	0.0000
341	6348.0127	6347.9863	2056.7310	8404.7422	0.0000	8404.7168	0.0000
342	-351.4763	8482.6739	3465.2749	3113.7983	-1767.0830	11947.9473	0.0000
343	-7815.9316	11327.1270	2952.1294	0.0000	-8585.3301	12442.1641	0.0000
344	-10526.6465	12686.6426	151.7336	0.0000	-10528.4629	12688.6281	0.0000
345	-5800.8760	11990.4707	-2768.5679	0.0000	-6449.4004	13330.9766	0.0000
346	4007.9043	9744.0937	-3526.2310	7534.1348	0.0000	13270.3242	0.0000

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
347	13417.3457	7983.4512	-2309.7349	15727.0801	0.0000	10293.1855	0.0000
348	18805.9609	7493.2344	-462.2842	19268.2422	0.0000	7955.5186	0.0000
349	19209.9180	8181.2754	1376.3679	20586.2852	0.0000	9557.6426	0.0000
350	15016.0469	9989.5488	2834.6333	17850.6797	0.0000	12824.1816	0.0000
351	8232.9434	12299.0527	3459.9160	11692.8594	0.0000	15758.9687	0.0000
352	918.7708	15431.3730	4005.2671	4924.0371	-120.8103	19436.6367	0.0000
353	-1541.7654	6567.3643	-8617.1230	7075.3574	-10158.8887	15184.4863	-2049.7588
354	7797.1797	5153.9385	-6728.6094	14525.7891	0.0000	11882.5469	-652.5430
355	15826.0254	3390.7729	-5264.5596	21090.5820	0.0000	8655.3320	0.0000
356	21116.1992	2751.7954	-2418.9072	23535.1055	0.0000	5170.7021	0.0000
357	20951.8828	2287.3125	644.4982	21596.3789	0.0000	2931.8105	0.0000
358	15311.1484	1490.7698	3811.0918	19122.2383	0.0000	5301.8613	0.0000
359	3919.0171	1178.5818	6607.1816	10526.1973	-2688.1646	7785.7627	-5428.6006
360	-11448.8281	3401.6250	6492.4482	0.0000	-17941.2773	7063.3887	-3090.8232
361	-19117.6719	3985.6665	-249.6875	0.0000	-19133.3164	3968.9272	0.0000
362	-13169.3047	2592.5020	-6785.0156	0.0000	-19954.3203	6068.2393	-4192.5137
363	-123.6000	-123.6000	-6522.0020	6398.4014	-6645.6025	6398.4014	-6645.6025
364	8477.2422	-352.0434	-3469.5972	11946.8379	0.0000	3117.5537	-1772.0930
365	11276.4453	-174.3353	0.0071	11276.4453	0.0000	0.0000	-174.3353
366	8481.4004	-352.6035	3471.4448	11952.8437	0.0000	3118.8413	-1773.4690
367	-123.6000	-123.6000	6519.5771	6395.9766	-6643.1777	6395.9766	-6643.1777
368	-13170.1191	2593.3149	6784.0605	0.0000	-19954.1797	6087.8525	-4190.7461
369	-19117.6719	3985.6665	249.6872	0.0000	-19133.3164	3968.9272	0.0000
370	-11449.1621	3399.5586	-6491.6201	0.0000	-17940.7852	7080.2754	-3092.0615
371	3919.9922	1180.0068	-6606.0049	10525.9961	-2686.0127	7786.0117	-5425.9980
372	15333.7891	1491.8870	-3816.6670	19150.4531	0.0000	5308.5537	0.0000
373	20951.8828	2287.3101	-644.4608	21596.3398	0.0000	2931.7705	0.0000
374	21139.8750	2752.1226	2421.6870	23561.5586	0.0000	5173.8096	0.0000
375	15826.3301	3392.8672	5263.7578	21090.0859	0.0000	8656.6250	0.0000
376	7797.8125	5154.7461	6727.8926	14525.7051	0.0000	11882.6387	-650.0273
377	-1538.4353	6568.8350	8614.9082	7076.4727	-10153.3437	15183.7422	-2046.0732
378	12792.6328	-10968.6348	1837.5283	13100.4648	0.0000	0.0000	-11232.5781
379	-454.5098	-9304.7578	-9176.5020	8595.5039	-9631.0137	0.0000	-18481.2617
380	18203.3984	-7204.2031	-5726.8047	22755.7812	0.0000	0.0000	-9005.8613
381	23756.1445	-3552.9468	-2275.6084	25213.6367	0.0000	0.0000	-3770.9282
382	23362.5508	-3080.1572	369.5488	23406.8867	0.0000	0.0000	-3086.0029
383	17952.0664	-5896.8691	3110.6279	19592.9375	0.0000	0.0000	-6435.8604
384	6380.2832	-12214.6875	6845.2461	10216.4336	-464.9629	0.0000	-19059.9336
385	-19141.3477	-18488.2578	10544.7246	0.0000	-29686.0742	0.0000	-29032.9844
386	-40097.9922	-9584.4199	-1289.2466	0.0000	-41387.2422	0.0000	-10873.6680
387	-19114.8086	-19114.8047	-10738.1797	0.0000	-29852.9883	0.0000	-29852.9844
388	2595.7705	-13148.5742	-6776.7910	6088.5361	-4181.0205	0.0000	-19925.3672
389	11325.1895	-7813.9941	-2960.4795	12446.8223	0.0000	0.0000	-8587.8848
390	13956.1094	-6084.1133	-0.0001	13956.1094	0.0000	0.0000	-6084.1133
391	11327.1426	-7815.9473	2952.0859	12442.1445	0.0000	0.0000	-8585.3223
392	2595.8433	-13148.6465	6776.7031	6088.4990	-4180.8604	0.0000	-19925.3516
393	-19113.6055	-19113.6055	10739.3750	0.0000	-29852.9805	0.0000	-29852.9805
394	-40097.9922	-9584.4199	1289.2458	0.0000	-41387.2422	0.0000	-10873.6660
395	-19142.5781	-18489.4297	-10543.5293	0.0000	-29686.1094	0.0000	-29032.9609
396	6378.0869	-12214.8926	-6844.5791	10213.4277	-466.4902	0.0000	-19059.4727
397	17928.3555	-5894.7607	-3107.8984	19566.9336	0.0000	0.0000	-6433.5186
398	23386.5195	-3080.1250	-370.4279	23431.0664	0.0000	0.0000	-3085.9927
399	23729.7617	-3550.5688	2285.7231	25201.2227	0.0000	0.0000	-3770.7368
400	18207.9609	-7208.7646	5715.6787	22739.8008	0.0000	0.0000	-9002.9805
401	-443.3186	-9313.6211	9172.1934	8589.5937	-9615.5137	0.0000	-18485.8164
402	12792.6328	-10968.6348	-1837.5303	13100.4648	0.0000	0.0000	-11232.5781
403	-88810.8437	-69061.2031	-2806.8828	0.0000	-91617.7344	0.0000	-71868.0937
404	15290.9922	-33511.8047	-1314.9124	15342.5840	0.0000	0.0000	-33624.8828

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
405	20412.7227	-12660.7305	-104.9915	20413.5898	0.0000	0.0000	-12661.2715
406	24766.0117	-5842.0176	-214.5939	24773.8906	0.0000	0.0000	-5843.8779
407	24533.8359	-4858.6406	-231.6586	24544.8789	0.0000	0.0000	-4860.8281
408	19124.3281	-8665.1367	-194.6286	19128.6992	0.0000	0.0000	-8667.1191
409	7789.1328	-18500.3398	-193.9794	7791.1660	0.0000	0.0000	-18505.1719
410	-7938.6699	-41633.3359	814.7988	0.0000	-8753.4687	0.0000	-42448.1406
411	-98016.0312	-98016.0312	-372.3232	0.0000	-98388.3594	0.0000	-98388.3594
412	-9582.0312	-40097.9766	-1289.4138	0.0000	-10871.4453	0.0000	-41387.3906
413	3983.2700	-19117.6758	-249.7753	3986.5332	0.0000	0.0000	-19133.3398
414	12686.6621	-10502.6660	-152.4036	12688.8730	0.0000	0.0000	-10504.4980
415	15336.9141	-8012.1191	-0.0001	15336.9141	0.0000	0.0000	-8012.1191
416	12686.6348	-10526.6387	151.6081	12688.8164	0.0000	0.0000	-10528.4512
417	3985.6689	-19117.6758	250.0212	3988.9385	0.0000	0.0000	-19133.3633
418	-9560.7461	-40097.6641	1292.0269	0.0000	-10852.7734	0.0000	-41389.6953
419	-98016.0312	-98016.0312	372.3213	0.0000	-98388.3594	0.0000	-98388.3594
420	-7933.8838	-41633.3281	-815.0251	0.0000	-8748.9102	0.0000	-42448.3594
421	7789.1182	-18500.3242	194.0162	7791.1523	0.0000	0.0000	-18505.1602
422	19124.3281	-8665.1367	194.6284	19128.6992	0.0000	0.0000	-8667.1191
423	24509.8437	-4858.6465	232.0354	24520.9219	0.0000	0.0000	-4860.8437
424	24789.9922	-5841.9961	215.3055	24797.9258	0.0000	0.0000	-5843.8662
425	20412.7227	-12660.7324	104.9912	20413.5898	0.0000	0.0000	-12661.2734
426	15293.3809	-33511.7891	1315.0740	15344.9863	0.0000	0.0000	-33624.8750
427	-88810.8437	-69061.2031	2806.8789	0.0000	-91617.7344	0.0000	-71868.0937
428	13317.1934	-10581.1973	-789.9922	13376.1738	0.0000	0.0000	-10628.0625
429	-119.5686	-10036.9941	9717.1113	9287.8535	-9836.6816	0.0000	-19754.1055
430	18493.2969	-5830.9023	5717.7861	24100.1602	0.0000	0.0000	-7598.7363
431	23957.6523	-1853.6543	1989.8926	25947.5430	0.0000	136.2383	-2018.9324
432	23627.3672	-1228.1682	-815.9944	24169.5117	0.0000	0.0000	-1256.3496
433	18328.3125	-4117.9150	-3632.0425	21531.8086	0.0000	0.0000	-4837.6611
434	7025.8711	-10676.2734	-7395.8564	12149.2578	-369.9854	0.0000	-18072.1328
435	-18571.1250	-18571.2852	-11294.8105	0.0000	-29865.9375	0.0000	-29866.0977
436	-41633.3437	-7938.6699	814.7990	0.0000	-42448.1484	0.0000	-8753.4707
437	-18488.2578	-19141.3477	10544.7227	0.0000	-29032.9805	0.0000	-29686.0703
438	3397.3687	-11449.3711	6490.9297	7077.2354	-3093.5610	0.0000	-17940.3008
439	12014.0137	-5800.4170	2791.8457	13357.7793	0.0000	0.0000	-6449.1934
440	14625.7988	-3955.4009	0.0000	14625.7988	0.0000	0.0000	-3955.4009
441	11990.4238	-5803.2295	-2788.8813	13330.6875	0.0000	0.0000	-6451.9023
442	3401.6245	-11448.8281	-6492.4512	7083.3916	-3090.8267	0.0000	-17941.2812
443	-18488.2578	-19141.3477	-10544.7205	0.0000	-29032.9883	0.0000	-29686.0781
444	-41633.3437	-7938.6709	-814.7996	0.0000	-42448.1484	0.0000	-8753.4707
445	-18572.4863	-18572.3281	11293.6074	0.0000	-29866.0977	0.0000	-29865.9375
446	7013.8135	-10671.4160	7401.2393	12146.9961	-387.4258	0.0000	-18072.6562
447	18335.0742	-4124.6777	3612.4395	21498.8867	0.0000	0.0000	-4836.4131
448	23627.3633	-1225.7693	815.9438	24170.5000	0.0000	0.0000	-1253.9470
449	23957.6523	-1853.6541	-1989.8933	25947.5430	0.0000	136.2393	-2018.9321
450	18516.2656	-5827.4707	-5722.3994	24135.4844	0.0000	0.0000	-7595.9619
451	-117.6563	-10036.5059	-9718.0820	9292.1035	-9835.7402	0.0000	-19754.5898
452	13317.1934	-10581.1973	789.9902	13376.1738	0.0000	0.0000	-10628.0605
453	-1528.3521	9376.3516	10308.2168	8779.8633	-11836.5703	19684.5664	-931.8652
454	7895.3809	8072.1045	7795.3057	15690.6855	0.0000	15867.4102	0.0000
455	16010.7930	6717.2051	5729.5410	21740.3320	0.0000	12446.7461	0.0000
456	21319.6172	6445.9775	2389.0015	23708.6172	0.0000	8834.9785	0.0000
457	21356.5000	6178.6934	-1060.3423	22416.8398	0.0000	7239.0352	0.0000
458	16009.6641	5350.3330	-4509.3184	20518.9805	0.0000	9859.6504	0.0000
459	4863.5996	4863.5996	-7472.4014	12336.0000	-2608.8018	12336.0000	-2608.8018
460	-10676.7930	7021.5889	-7394.3691	0.0000	-18071.1641	12142.6660	-372.7803
461	-18476.3828	7791.5752	-192.6993	0.0000	-18481.1523	7793.5840	0.0000
462	-12214.8945	6378.0889	6844.5762	0.0000	-19059.4727	10213.4238	-466.4873

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
463	1177.6436	3917.5557	6606.0098	7783.6533	-5428.3662	10523.5645	-2600.4541
464	9764.0254	4011.9722	3535.2397	13299.2637	0.0000	7547.2119	0.0000
465	12476.0586	4321.5400	-0.0335	12476.0918	0.0000	4321.5732	0.0000
466	9743.7402	4005.8579	-3527.0903	13270.8301	0.0000	7532.9482	0.0000
467	1177.1118	3918.0874	-6608.3535	7785.4648	-5431.2422	10526.4395	-2690.2661
468	-12215.1191	6375.9150	-6843.8770	0.0000	-19058.9961	10210.3965	-467.9619
469	-18500.3281	7789.1230	194.0579	0.0000	-18505.1641	7791.1582	0.0000
470	-10671.4160	7013.8135	7401.2393	0.0000	-18072.6562	12146.9961	-387.4258
471	4863.5986	4863.5986	7472.3984	12335.9961	-2608.7998	12335.9961	-2608.7998
472	16005.3613	5349.8359	4515.2969	20520.6562	0.0000	9865.1328	0.0000
473	21356.5156	6181.0801	1060.1621	22416.6758	0.0000	7241.2422	0.0000
474	21319.6016	6445.9941	-2389.0503	23708.6484	0.0000	8835.0430	0.0000
475	16010.7930	6717.2051	-5729.5430	21740.3359	0.0000	12446.7480	0.0000
476	7896.1787	8072.7715	-7794.5781	15690.7559	0.0000	15867.3496	0.0000
477	-1528.3521	9376.3516	-10308.2187	8779.8652	-11836.5723	19684.5703	-931.8672
478	935.0214	20784.4961	5729.7695	6664.7900	-644.5338	26514.2656	0.0000
479	8158.3799	17735.2148	4732.0879	12890.4668	0.0000	22467.3008	0.0000
480	14945.1855	15654.8105	3570.1689	18515.3516	0.0000	19224.9766	0.0000
481	19233.4219	14126.5723	1542.6467	20776.0664	0.0000	15669.2187	0.0000
482	19145.8437	13638.1504	-846.4575	19992.3008	0.0000	14484.6074	0.0000
483	14243.9980	14243.9980	-3107.9902	17351.9883	0.0000	17351.9883	0.0000
484	5348.0156	16009.5820	-4509.8516	9857.8672	0.0000	20519.4336	0.0000
485	-4118.3125	18328.7070	-3631.6975	0.0000	-4837.9863	21531.6367	0.0000
486	-8662.7383	19124.3320	-194.5659	0.0000	-8664.7187	19128.6992	0.0000
487	-5897.1045	17928.3008	3108.2729	0.0000	-6435.9941	19566.6211	0.0000
488	1491.9033	15333.7734	3816.6934	5308.5967	0.0000	19150.4648	0.0000
489	7979.7568	13418.6387	2308.2061	10287.9629	0.0000	15726.8437	0.0000
490	10339.7422	12796.2539	0.0000	10339.7422	0.0000	12796.2539	0.0000
491	7977.3750	13421.0215	-2305.8203	10283.1953	0.0000	15726.8418	0.0000
492	1492.7788	15333.8594	-3816.4209	5309.1992	0.0000	19150.2773	0.0000
493	-5894.5234	17952.1211	-3110.2705	0.0000	-6433.3896	19593.2656	0.0000
494	-8667.5332	19124.3281	194.5615	0.0000	-8669.5137	19128.6953	0.0000
495	-4124.6777	18335.0742	3612.4395	0.0000	-4836.4131	21498.8867	0.0000
496	5356.9229	16027.0762	4521.5166	9878.4395	0.0000	20548.5898	0.0000
497	14244.0000	14244.0000	3107.9893	17351.9883	0.0000	17351.9883	0.0000
498	19145.8437	13638.1504	846.4570	19992.3008	0.0000	14484.6074	0.0000
499	19235.7383	14148.2559	-1535.5598	20771.2969	0.0000	15683.8145	0.0000
500	14945.1855	15654.8105	-3570.1704	18515.3555	0.0000	19224.9805	0.0000
501	8160.4346	17735.5625	-4731.2471	12891.6816	0.0000	22466.8086	0.0000
502	936.3657	20784.5937	-5729.4121	6665.7773	-642.9849	26514.0039	0.0000
503	424.0048	25451.8320	1804.1965	2228.2012	0.0000	27256.0273	0.0000
504	8361.8398	22386.9531	1440.3757	9802.2148	0.0000	23827.3281	0.0000
505	14588.7324	20019.2617	995.1849	15583.9160	0.0000	21014.4453	0.0000
506	18260.6250	18435.3672	383.8826	18644.5039	0.0000	18819.2461	0.0000
507	18095.1172	18096.8750	-288.0165	18383.1328	0.0000	18384.8906	0.0000
508	13661.0742	19122.9219	-839.9714	14501.0449	0.0000	19962.8906	0.0000
509	6181.0791	21356.5156	-1060.1589	7241.2373	0.0000	22416.6719	0.0000
510	-1223.3674	23627.3633	-815.9231	0.0000	-1251.5437	24171.5391	0.0000
511	-4858.6465	24509.8398	-232.0355	0.0000	-4860.8437	24520.9180	0.0000
512	-3082.5264	23386.5195	370.4067	0.0000	-3088.3931	23431.0273	0.0000
513	2287.3135	20951.8828	644.5023	2931.8154	0.0000	21596.3828	0.0000
514	7493.2383	18805.9570	462.2879	7955.5254	0.0000	19268.2422	0.0000
515	9544.2695	18019.7266	0.1379	9544.4062	0.0000	18019.8633	0.0000
516	7495.6182	18805.9766	-462.0882	7957.7061	0.0000	19268.0625	0.0000
517	2284.9136	20951.8828	-644.5487	2929.4619	0.0000	21596.4297	0.0000
518	-3082.5244	23386.5195	-370.4070	0.0000	-3088.3911	23431.0273	0.0000
519	-4863.4434	24509.8359	232.0730	0.0000	-4865.6416	24520.9062	0.0000
520	-1228.1633	23627.3594	816.0862	0.0000	-1256.3511	24169.6289	0.0000

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
521	6183.4707	21356.5273	1060.0117	7243.4824	0.0000	22416.5391	0.0000
522	13638.1523	19145.8437	846.4570	14484.6094	0.0000	19992.3008	0.0000
523	18096.1250	18095.8711	288.0002	18384.1250	0.0000	18383.8711	0.0000
524	18261.7930	18434.1992	-383.2476	18645.0391	0.0000	18817.4453	0.0000
525	14612.3848	20019.6094	-992.3229	15604.7070	0.0000	21011.9297	0.0000
526	8366.5918	22387.0000	-1439.8982	9806.4883	0.0000	23826.8945	0.0000
527	424.9597	25451.8398	-1804.1284	2229.0879	0.0000	27255.9648	0.0000
528	897.2991	25010.2187	-2147.4849	3044.7837	0.0000	27157.7031	0.0000
529	8119.9141	21932.8789	-1839.2590	9959.1719	0.0000	23772.1367	0.0000
530	14549.5801	19866.4141	-1520.2927	16069.8711	0.0000	21386.7031	0.0000
531	18491.8281	18492.1641	-707.9404	19199.7656	0.0000	19200.1016	0.0000
532	18433.7695	18262.2227	383.7755	18817.5430	0.0000	18645.9961	0.0000
533	14124.9180	19235.0742	1540.3789	15665.2969	0.0000	20775.4531	0.0000
534	6448.3076	21319.6875	2388.5991	8836.9062	0.0000	23708.2852	0.0000
535	-1849.9180	23932.3164	1998.4885	148.5706	-2016.8035	25930.8047	0.0000
536	-5839.0986	24813.4922	-201.3285	0.0000	-5840.7324	24820.4336	0.0000
537	-3550.5757	23729.7695	-2285.6875	0.0000	-3770.7368	25201.1836	0.0000
538	2749.4390	21116.1562	-2419.2212	5168.6602	0.0000	23535.3750	0.0000
539	8181.2734	19209.9219	-1376.3562	9557.6289	0.0000	20586.2773	0.0000
540	10198.0215	18553.9727	-3.6639	10201.6836	0.0000	18557.6328	0.0000
541	8183.6172	19209.9766	1375.9973	9559.6133	0.0000	20585.9727	0.0000
542	2749.7734	21139.8242	2422.0337	5171.8066	0.0000	23561.8555	0.0000
543	-3552.7959	23753.5937	2287.8374	0.0000	-3773.1499	25226.8555	0.0000
544	-5839.1064	24789.5000	201.8266	0.0000	-5840.7500	24796.4727	0.0000
545	-1853.6541	23957.6523	-1989.8933	136.2393	-2018.9321	25947.5430	0.0000
546	6445.9941	21319.6016	-2389.0503	8835.0430	0.0000	23708.6484	0.0000
547	14148.2559	19235.7383	-1535.5598	15683.8145	0.0000	20771.2969	0.0000
548	18434.2109	18261.7812	-383.1645	18817.3750	0.0000	18644.9453	0.0000
549	18491.9961	18491.9961	707.9159	19199.9102	0.0000	19199.9102	0.0000
550	14571.4102	19868.5859	1513.4092	16084.8184	0.0000	21381.9922	0.0000
551	8129.3516	21933.0430	1838.0244	9967.3750	0.0000	23771.0664	0.0000
552	897.2991	25010.2187	2147.4839	3044.7827	0.0000	27157.6992	0.0000
553	-1222.7881	18382.7852	-6770.0449	5547.2568	-3716.0723	25152.8281	0.0000
554	7977.2891	16380.3086	-4895.2666	12872.5547	0.0000	21275.5742	0.0000
555	15144.0000	15144.0000	-3743.8071	18887.8047	0.0000	18887.8047	0.0000
556	19869.7109	14570.2812	-1511.0698	21380.7773	0.0000	16081.3496	0.0000
557	19996.0625	14611.9316	989.0551	20985.1172	0.0000	15600.9863	0.0000
558	15667.9395	14956.0566	3582.1157	19250.0547	0.0000	18538.1719	0.0000
559	6712.5176	16015.4805	5725.5391	12438.0566	0.0000	21741.0195	0.0000
560	-5827.5039	18516.3008	5722.3252	0.0000	-7595.9463	24135.3437	0.0000
561	-12636.8105	20412.8047	-103.6122	0.0000	-12637.3379	20413.6523	0.0000
562	-7208.7227	18207.9180	-5715.7734	0.0000	-9003.0020	22739.9336	0.0000
563	3386.4751	15830.3223	-5259.0771	8645.5508	0.0000	21089.3984	0.0000
564	9985.0547	15018.1426	-2833.2432	12818.2969	0.0000	17851.3828	0.0000
565	12121.9961	14901.9980	-1.2250	12123.2207	0.0000	14903.2227	0.0000
566	9987.6016	15015.5937	2835.5737	12823.1738	0.0000	17851.1641	0.0000
567	3395.0103	15826.5859	5263.0156	8658.0254	0.0000	21089.6016	0.0000
568	-7208.7646	18207.9609	5715.6787	0.0000	-9002.9805	22739.8008	0.0000
569	-12660.7324	20412.7266	104.9912	0.0000	-12661.2734	20413.5937	0.0000
570	-5827.4580	18516.2539	-5722.4258	0.0000	-7595.9668	24135.5391	0.0000
571	6719.1914	16011.2031	-5728.6367	12447.8281	0.0000	21739.8398	0.0000
572	15654.8105	14945.1836	-3570.1704	19224.9805	0.0000	18515.3516	0.0000
573	20019.6094	14612.3848	-992.3229	21011.9297	0.0000	15604.7070	0.0000
574	19868.5859	14571.4082	1513.4092	21381.9922	0.0000	16084.8164	0.0000
575	15132.0000	15132.0000	3731.8906	18863.8906	0.0000	18863.8906	0.0000
576	7981.3252	16376.2715	4898.7119	12880.0371	0.0000	21274.9805	0.0000
577	-1222.7881	18382.7852	6770.0430	5547.2549	-3716.0708	25152.8281	0.0000
578	12504.4844	6681 1133	4411.5830	16916.0664	0.0000	11092.6953	0.0000

NODE	MX	MY	MXY	MX+	MX-	MY+	MY-
579	1059.8396	1059.8391	-6812.4980	7872.3369	-5752.6592	7872.3369	-5752.6592
580	16375.8496	7979.3477	-4899.6270	21275.4766	0.0000	12878.9746	0.0000
581	21956.7422	8129.6494	-1840.6838	23797.4258	0.0000	9970.3320	0.0000
582	22386.9492	8361.8437	1440.3965	23827.3437	0.0000	9802.2402	0.0000
583	17735.9180	8162.4756	4730.3896	22466.3047	0.0000	12892.8652	0.0000
584	8074.1367	7897.6211	7793.1738	15867.3105	0.0000	15690.7949	0.0000
585	-10036.1250	-116.8374	9718.6367	0.0000	-19754.7617	9294.3535	-9835.4746
586	-33511.8047	15290.9941	-1314.9119	0.0000	-33624.8828	15342.5859	0.0000
587	-9304.4785	-453.0120	-9177.1699	0.0000	-18481.6484	8598.5898	-9630.1836
588	5154.0576	7797.3008	-6728.4873	11882.5449	-652.1230	14525.7871	0.0000
589	12313.9102	8239.6855	-3472.8110	15786.7207	0.0000	11712.4961	0.0000
590	14731.4160	8514.9805	0.0000	14731.4160	0.0000	8514.9805	0.0000
591	12299.0527	8232.9434	3459.9160	15758.9687	0.0000	11692.8594	0.0000
592	5155.1816	7798.0977	6727.5410	11882.7227	-648.7725	14525.6387	0.0000
593	-9313.6191	-443.3186	9172.1934	0.0000	-18485.8125	8589.5957	-9615.5137
594	-33511.8047	15290.9922	1314.9111	0.0000	-33624.8828	15342.5840	0.0000
595	-10036.2051	-118.1980	-9718.1582	0.0000	-19754.3633	9291.9922	-9836.3574
596	8073.0654	7896.4365	-7794.3027	15867.3672	0.0000	15690.7383	0.0000
597	17735.8945	8162.4980	-4730.4170	22466.3086	0.0000	12892.9141	0.0000
598	22387.0000	8366.5898	-1439.8982	23826.8945	0.0000	9806.4863	0.0000
599	21933.0391	8129.3516	1838.0244	23771.0625	0.0000	9967.3750	0.0000
600	16376.2715	7981.3252	4898.7119	21274.9805	0.0000	12880.0371	0.0000
601	1060.0793	1060.0793	6812.2559	7872.3350	-5752.1768	7872.3350	-5752.1768
602	12504.4844	6681.1133	-4411.5850	16916.0664	0.0000	11092.6973	0.0000
603	6681.1133	12504.4844	4411.5830	11092.6953	0.0000	16916.0664	0.0000
604	18383.3320	-1218.5334	-6768.5215	25151.8516	0.0000	5549.9873	-3710.6216
605	25002.6328	902.4680	-2189.7710	27192.4023	0.0000	3092.2588	0.0000
606	25451.8359	425.4395	1804.1089	27255.9414	0.0000	2229.5483	0.0000
607	20784.5625	935.9154	5729.5273	26514.0898	0.0000	6665.4424	-643.5011
608	9376.3516	-1528.3521	10308.2168	19684.5664	-931.8652	8779.8633	-11836.5703
609	-10581.1973	13317.1934	-789.9922	0.0000	-10628.0625	13376.1738	0.0000
610	-69061.2031	-86810.8437	-2806.8828	0.0000	-71868.0937	0.0000	-91617.7344
611	-10968.6348	12792.6328	1837.5283	0.0000	-11232.5781	13100.4648	0.0000
612	6568.8350	-1538.4353	-8614.9121	15183.7461	-2046.0771	7076.4766	-10153.3477
613	15431.2812	917.4700	-4005.6123	19436.8906	0.0000	4923.0820	-122.2964
614	17947.6680	412.3285	0.0150	17947.6797	0.0000	412.3434	0.0000
615	15431.3809	918.8583	4005.2397	19436.6172	0.0000	4924.0977	-120.7081
616	6567.3643	-1541.7654	8617.1211	15184.4844	-2049.7568	7075.3555	-10158.8867
617	-10968.6348	12792.6328	-1837.5303	0.0000	-11232.5781	13100.4648	0.0000
618	-69061.2031	-86810.8437	2806.8789	0.0000	-71868.0937	0.0000	-91617.7344
619	-10581.1973	13317.1934	789.9902	0.0000	-10628.0605	13376.1738	0.0000
620	9376.3516	-1528.3521	-10308.2187	19684.5703	-931.8672	8779.8652	-11836.5723
621	20784.2695	931.8873	-5730.6123	26514.8789	0.0000	6662.4990	-648.1498
622	25451.8359	424.4815	-1804.1577	27255.9922	0.0000	2228.6392	0.0000
623	25002.6406	903.4402	2189.6836	27192.3242	0.0000	3093.1235	0.0000
624	18382.7852	-1222.7881	6770.0430	25152.8281	0.0000	5547.2549	-3716.0708
625	6681.1133	12504.4844	-4411.5850	11092.6973	0.0000	16916.0664	0.0000

APPENDIX 7D

Yield-Line Analysis

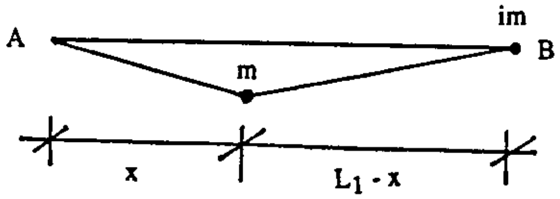
APPENDIX 7D

Yield-Line Analysis

(a) Mode 1 Exterior span

Assume hinge at x from A.

The work equation is



$$mL_2\left(\frac{1}{x}+\frac{1}{L_1-x}\right)+imL_2\frac{1}{L_1-x} = \frac{wL_1L}{2}$$

which leads to

$$m = \frac{wL_1}{2}\left\{\frac{L_1x-x^2}{L_1+ix}\right\}$$

This is maximum when

$$\frac{L_1x-x^2}{L_1+ix} = \frac{L_1-2x}{i}$$

or

$$iL_1x-ix^2 = L_1^2+iL_1x-2xL_1-2ix^2$$

giving

$$x = \frac{-2L_1\pm\sqrt{4L_1^2+4iL_1^2}}{2i}$$

If it is assumed i is 1

$$x = \frac{-2L_1+\sqrt{4L_1^2+4L_1^2}}{2}.$$

Therefore

$$x = \frac{-2L_1 + 2\sqrt{2}\,L_1}{2} = 0.414L_1$$

$$m = \frac{wL_1}{2}\left\{\frac{L_1 - 2x}{i}\right\}$$

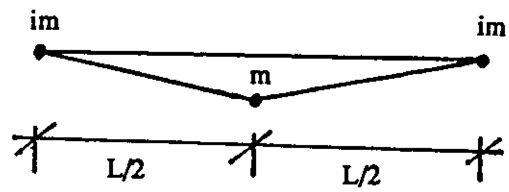
$$= \frac{wL_1}{2}\left\{\frac{L_1 - 0.828L_1}{1}\right\}$$

$$m = 0.086\ wL_1^{\ 2}$$

therefore

$$im = 0.086\ wL_1^{\ 2}$$

(b) For Mode 2



$$im + m = \frac{wL^2}{8}$$

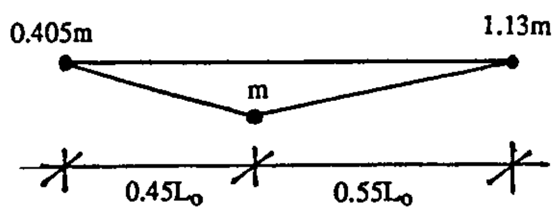
If we make no reduction in i for the interior support taking im as for first interior support

$$m = 0.125\ wL^2 - 0.086\ wL^2$$

$$m = 0.039\ wL^2$$

(c) Mode 1 assuming end column restraint and yield line forming outside line of first interior column. The average finite element moment coefficients at the outside column and first interior column are -0.028 and -0.078 and positive span moment coefficient is 0.069 (Fig. 7.19). If the positive moment is m then the exterior column moment is 0.406 m and the interior moment 1.13 m.

For end restraints of 0.406 m and 1.13 m assume hinge in span at 0.45 L_o



$$\Sigma(M\theta) = \frac{1.406m}{0.45L_o} + \frac{2.13m}{0.55L_o}$$

$$= \frac{7m}{L_o}$$

$$\Sigma(W\delta) = \frac{wL_o}{2}; \text{ hence } m = \frac{wL_o^2}{14}, \text{ with } L_o = 4 - 0.15 = 3.85$$

The total moment over a 4 metre width is therefore

$$M = \frac{2wL_o^2}{7}$$

If this is expressed as a coeficient in the form CFL' then

$$\frac{2w(3.85)^2}{7} = Cw\ 16 \times 3.775$$

giving C = 0.07 compared with the value of 0.069 from the finite element average, i.e. virtually identical. The other coefficients in the first span will tehrefore also be the same as the finite element values since the same original proportions were assumed.

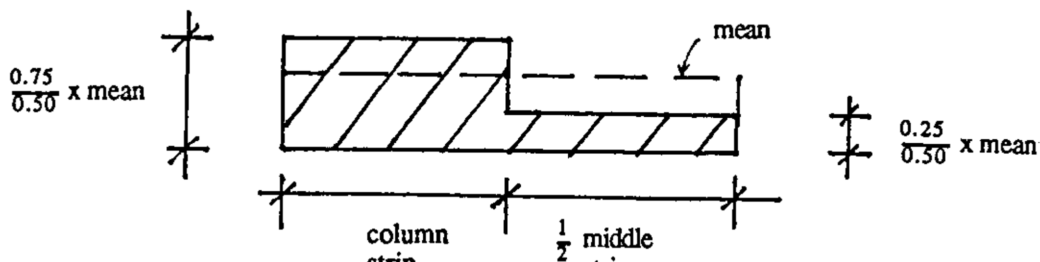
For mode 2 the work equation is

$$(m + im) = \frac{wL_o^2}{8} \text{ and the total moment over a 4 metre width will be } \frac{wL_o^2}{2}$$

If the total moment coefficient is equated to this.

(d) Distribution into column and middle strips

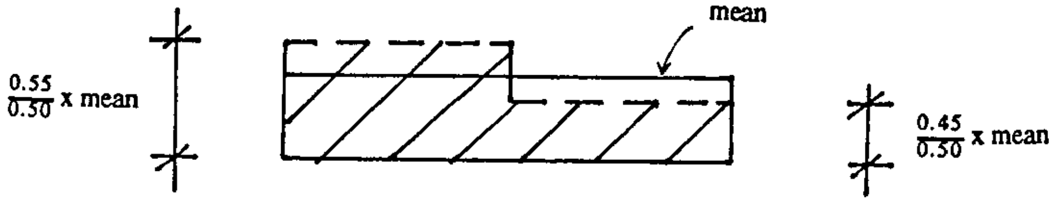
For distribution as in the code (Table 7.2)



Therefore at first interior column

$$\text{Neg. mom. at column strip} = 0.078\ wL^2 \times 1.5$$
$$= 0.117\ wL^2$$
$$\text{Neg. mom. at middle strip} = 0.078\ wL^2 \times 0.5$$
$$= 0.039\ wL^2$$

Exterior span, positive moment



$$\text{Pos. mom. at column strip} = 0.069\ wL^2 \times \frac{0.55}{0.50}$$
$$= 0.076\ wL^2$$

$$\text{Pos. mom. at middle strip} = 0.069\ wL^2 \times \frac{0.45}{0.50}$$
$$= 0.062\ wL^2$$

Interior span

The negative moment for the interior column will be the same as that found for first interior column above.

For the positive moment distribution in the interior span

$$\text{CFL} = \frac{wL_o^2}{2}$$

or

$$C = \frac{w(3.75)^2}{w(2)(16)(3.775)} = 0.116$$

but the coefficient at the first interior column is 0.078 hence the positive moment coefficient is

$$= 0.116 - 0.078$$

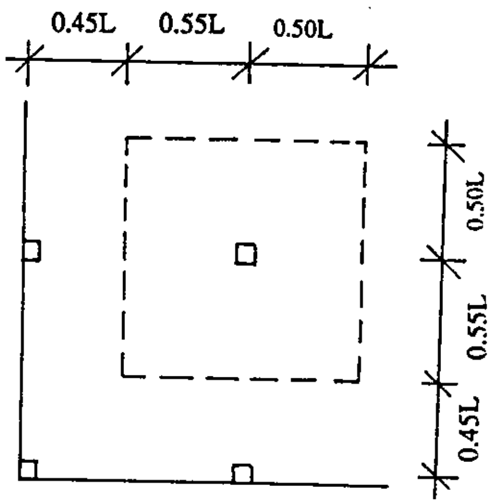
$$= 0.038$$

which again is identical with the average finite element value.

$$\text{Pos. column} = 0.038 \times \frac{55}{50} = 0.042\,wL^2$$

$$\text{Pos. middle} = 0.038 \times \frac{45}{50} = 0.034\,wL^2$$

(e) Local failure



First interior column

For genuine interior column the load will be $P = wL^2$.

For first interior column however the load will be

$$P = w(1.05)L^2 = 1.1\ wL^2$$

which is a worse case to consider.

For a full fan failure we have $P = 2\pi(m + im)$

In column strip top steel is $0.117\ wL^2$

however clause 3.7.3.1 of BS8110 requires 2/3 of the column steel to be placed in the central half of the column strip so that over the column

$$im = 0.117(wL^2) \times \frac{0.67}{0.50} = 0.156(wL^2)$$

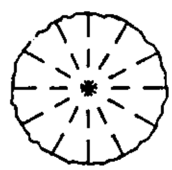
In column strip bottom steel would be curtailed to 40% of the mean of first and second span, i.e.

$$m = \frac{1}{2}(0.076 + 0.042) \times 0.4 = 0.024$$

Therefore

$$2\pi(m + im) = 2\pi(0.156 + 0.024)wL^2$$

$$= 1.13\ wL^2$$



full fan

which just exceeds the column load $P = 1.1\ wL^2$ and is therefore safe from load failure.

CHAPTER 8

CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

8.1 Rigidly Supported Slabs

consideration?

(a) After considering, using finite element analysis, the worst case of patterned loading for the load case of live/dead load equal to 1.25 it can be seen from Table 5.34 for support cases 1 to 4 in BS8110 that the negative reinforcement is very close to yielding at the serviceability condition.

This conclusion is based on the assumption of linear behaviour which is not strictly true due to concrete's non linear stress strain curve. The problem could be relieved if the code coefficients had been calculated using a slightly higher value for the negative/positive steel ratio of 1.5 instead of 4/3 in the yield-line calculation which is the basis of the code coefficients. Alternatively since the coefficients only apply to the middle 3/4 of the width and minimum steel is always required in the outer edge it might be better to redistribute the total amount of steel calculated on the 4/3 ratio to 1.5 times the mean in the central region and half the mean in the outer edges.

(b) The present practice in the British code of redistributing the negative moment at a common edge where the values are different should be reconsidered since inevitably this must reduce even more the value of the negative steel in relation to one of the slabs.

(c) Generally the amount of negative steel in the ACI code is higher than the British code and therefore is better from the serviceability aspect. There is however one exception, namely for a slab restrained on 3 sides and simply supported on the other where the negative moment on the edge parallel to the simply supported edge would yield at the serviceability condition for the worst pattern loading found using a live/dead load ratio of 1.25.

This ratio is above that of 0.75 where the ACI code requires patterned loading to be taken into account.

There is however no guide as to how the coefficients in one direction can be adjusted due to a reduction in fixity in the direction at right angles to this. This is an area which might be investigated in the future.

(d) For the case of simply supported slabs the coefficients given in the ACI code are unsafe at the ultimate condition. The British code value for a square slab which is based on yield-line analysis is 0.055 while the ACI code value is 0.036, with an average value of 0.030 allowing for the reduction in the edge zone. The yield-line solution is $wL^2/24$, i.e. a coefficient of 0.0417.

(e) The use of the coefficients in design practice is extremely easy as is demonstrated in the specimen calculations.

8.2 Semi-rigidly Supported Slabs

(a) The British code makes no specific reference to these slabs but covers 'slabs supported by beams or walls' and flat slabs. There is no apparent lower limitation for the stiffness of the beams. The finite element analysis showed conclusively that the negative moments vary over a wide range as the beam stiffness reduces. It also showed, with all the provisions included in section 8.1, that if the supporting beams have an overall depth of 2.5 to 3 times the slab thickness (with a breadth of the slab thickness) then the coefficients given in BS8110 are reasonably satisfactory. For lower stiffnesses than this the method is not satisfactory for evaluating the negative moments.

It is considered that this is a deficiency in the code which needs to be addressed.

(b) The Direct Design Method given in the ACI code gives answers which are in reasonable agreement with the moment distribution found by the finite element analysis. However, while the total values attributed to the middle and column strips are satisfactory the proportion attributed to the beam appears to be too high. A slightly cautious tone is used for this statement since the rate of change in moment near the beam is significant and a finer mesh in the finite element analysis needs to be used to be more accurate in this area.

The proportion of the moment carried by the beam for both the positive and negative moments is certainly an area which requires more detailed study.

8.3 Flat Slabs

(a) For flat slabs both the simplified coefficient method and the equivalent frame method gave total moments at the critical sections which were not too dissimilar to the finite element analysis.

(b) The recommended distribution of the positive moments to the column and middle strips was remarkably consistent with the finite element results which gave an average split of 57 - 43% whereas the British code recommends 55 - 45% and the ACI code 60 - 40%.

(c) The recommended distribution of 75 - 25% between the column and middle strips in both codes compared with the 82 - 18% found by finite element analysis assuming point column supports. With a finer mesh around an actual column it is believed because of the reduction in the peak moment that the 75 - 25% is likely to be more realistic.

(d) A yield-line solution using the same ratios as the total moment at the critical sections found in the finite element analysis confirmed the values of the total moments. In addition a local fan mechanism check on an interior column confirmed the need recommended in BS8110 to concentrate 2/3 of the negative steel in the column strip in the middle half.

8.4 General

Both codes refer to yield-line analysis and Hillerborg's strip method as being acceptable design methods. While it is accepted that phrases such as 'other limit states need to be satisfied' are used it is felt that the need for a more specific statement that the moments obtained by these methods need to be roughly in the proportions of the elastic moments needs adding to ensure that redistribution is not excessive.

8.5 Finite Element Analysis

Though not a conclusion it should be noted that two modifications to the PAFEC finite element stress program which transform the PAFEC principal stress results into reinforcement moments in accordance with the Wood-Armer rules are now available from the Department of Civil Engineering.

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