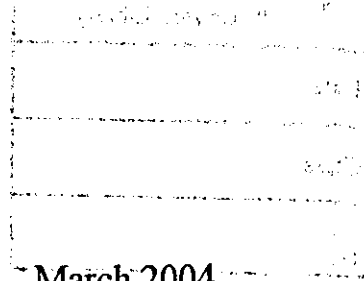# Class/Room/Mathematics: A Praxiology of Early Education

By

Geraldo Gomes

A Doctoral Thesis
Submitted in partial fulfilment of the requirements for the award of
Doctor of Philosophy of Loughborough University

March 2004

Abstract

This thesis is a semiotic and praxiological study of classroom mathematics in early education in Brazil. The classes observed range from pre-school literacy to 4th grades, and the mathematical contents include sets and numbers, the decimal system, and, of course, arithmetic. By 'semiotic' and 'praxiological' I mean to indicate descriptions according to which mathematics learning is analysed not as a feature of a learner's individual cognitive processes, but in terms of how signs are put into use in practical circumstances, as teachers and pupils identify, problematise, and (dis) agree upon their significance and application. It also argues in favour of the idea that the 'learner', the cognising mind, the foundational subject of psychological and sociological studies is another sign, or an effect, of this semiotic-practical formation.

The thesis is designed, firstly, as a criticism to those same psychological and sociological studies – including conversational and discursive ones – for being unable to observe the socially and materially heterogeneous formation of classrooms. I suggest that not only different forms of agency ('class' versus 'subject') are involved in managing diverse activities, but that the material association between gestures, discourse, material objects, structured interfaces, drawings, conventional scientific notation, etc., is the very basis on which mathematics teaching occurs, and that a focus on discourse or cognition alone necessarily alienate some of those semiotic objects. Therefore, the study of classroom mathematics is also the study of the mathematics classroom. I show that: (1) The 'class' is constituted as an agent vis-à-vis the teacher; (2) that the *blackboard* is an important representational interface, and is used to hold the collective as a single witnessing agency; (3) that the content that is being taught is a crucial technology of analysis of the setting for the participants, in a par – and in conflict – with some professional analytic renditions of what is going on; and (4) that individual pupils are *interpellated as* mathematical 'subjects', and that those subjects are temporally dispersed in relation to the 'class'.

Key words: classroom, mathematics, technologies of learning, praxiology, semiotics, ethnomethodology, discourse analysis.

Acknowledgements

To Derek Edwards, my first supervisor, for helping me to start understanding the 'problems' I was searching for, and for sharing his brilliant discursive-analytical skills. As a student, I have always found Derek's eye for detail both surprising and inspiring.

To Malcolm Ashmore, my second supervisor, for having accepted me as his student towards the end of my staying in Loughborough, and having provided me with fascinating discussions about knowledge and science. For me, Malcolm was not only intellectually challenging and committed, in spite of the circumstances, but also a patient and compassionate person to work with.

To my friends Cristian and Lora Tileaga, Jovan Byford, Sidnei Souza, Edirle Menezes, Alberto Cortes-Ramos, Angelo Durandegui, and especially Patricia Roberts, whose support I can never thank enough. Together, all these people made not only this work possible, but my life richer, more interesting, more intelligent, more loving; we shared many conversations, laughs, drinks, films, and I really feel highly indebted to them for their friendship and support.

Many thanks to Deirdre Lombard, a lovely, fun, and direct person without whom I do not know how I would have managed my staying here.

Thanks to all the academic authors that feature in the text, including those I disagree with, for being part of this. A special thanks to one who *does not* feature in the text: Oswald de Andrade, Brazilian critic and scholar whom I have rediscovered so many years away from my school time; As early as in the 1920s Oswald taught us all about the dangers of epistemic sclerosis, and the wonders of cultural and intellectual anthropophagy. It has been a pleasure to meet him again, as well as a reinstatement of identity.

Thanks to CAPES, my sponsor.

Contents

CHAPTER 1

Introduction

Can mathematics be social?

Mathematics may represent a challenging, sometimes inappropriate topic of investigation for the social sciences. Radical theories in sociology, for example, particularly the sociology of scientific knowledge (SSK), have used mathematics to provoke a breakdown in naturalistic conceptions of truth and knowledge, claiming it as a legitimate object for sociological and historical analysis (see Bloor, 1976). Some may still see it as superficial to say that mathematics is social *just* because it has to be available in a publicly accountable language or notational system to any person interested in it, and that ultimately its core logic is transpersonal (Bunge, 2000). What such positions convey is an idea of *social* (and technical, for that matter) that is in contrast to the rational actors of the Enlightenment, of the thinking mind who deals with 'reality' as an *epistemological* problem. Like science, mathematics can be seen as one of the last refuges for 'objective knowledge', and like science, it constitutes a polemic, problematic, 'hard case' for the scrutiny of contemporary constructionist approaches, especially in respect to such notions as *truth* and *objectivity*. Nevertheless, mathematics has to be learned, is learnable, and is designed to be so. The sociological analysis therefore turns out to be a way of observing and describing how the inspectable features of such knowledge are made visible for the relevant parts. Again like science, in order to understand how such knowledge is accomplished *in situ*, methodology sections in academic textbooks are not sufficient or even truthful descriptions (Kuhn, 1970; Gilbert and Mulkay, 1984). As 'instructional matter' (Livingston, 2000), mathematics and its 'internal' workings are more complex than the established philosophical views allows us to see. One should also note

1

that such discussions are highly oriented to the description of *academic* mathematics *as* mathematics itself. On the other hand, the *use* of mathematics as a cultural tool can be observed in many professional and mundane activities, such as the modelling of dynamic equations in physics and chemistry, statistics, in children's education and in the routine everyday affairs of counting and quantifying.

In this study, I am specifically interested in young children's mathematics education in classrooms. I am interested in how the mathematics classroom – and the studies on classroom mathematics – organise its resources, including analytical ones, and delegate knowledge and learning. Traditionally, this kind of study is organised around two distinct fields, those of 'mathematics' and 'education'. The first is discussed in chapter 2, where we examine mathematics' special place amongst the sciences. The timelessness of mathematics has covered it with a prestige that has justified, since Pythagoras, its place as the foundation of 'objective' knowledge, surpassing naïve sensorial empiricisms and rhetoric. Our rational compulsion to see mathematics as disentangled from history and contingency had been the ultimate obstacle to a radical conceptualisation of knowledge and science as products of active *association* between semiotic and technological actors in culture. Semiotics, the science of signs, has ventured into such a project, not by questioning the truth of mathematics, but its extra- and pre-semiotic nature. It shows how mathematical signs operate formally and how such operations – including the totality of representations at hand, patterns of inference, and systems of proof – are what mathematics is all about. The philosopher Ludwig Wittgenstein also contributed significantly to the analysis of mathematical knowledge as practice, as a set of 'language games'. Wittgenstein shows how mathematics is conventionalised, and departs from nowhere else than the very fact of its 'learnability', that its inference routines are 'impressed' techniques that need to be learned as such and such, and that those ways are 'games' that we learn to play inexorably. Finally, ethnomethodology also constitutes a big player in the discussion: Eric Livingston (1986; 1987), above all, has produced a detailed study on the reasoning procedures that constitute the 'lived work' of

mathematics proving, for which its final work, a complete mathematical 'proof-account', stands as its best representation.

If we turn now to the 'education' side of the coin, we observe that almost all the works mentioned above, with the exception of Wittgenstein's, fail to do justice to particular ways and instances in which mathematics is (designed to be) learned. I want to qualify this. Livingston's work, for example, is about such practical methods and designs, but it does not contain a single case of analysis of practices other than his own interpretative and documentary practices of developing the sense of a proof. Although he deals with the relevance of every single dot and line drawn in the course of writing mathematically, and refuses importing theoretical actors into his analysis, his project has a sense of formalism proper to semiotics. No classrooms in sight! Classrooms are present in Wittgenstein's work, but only indirectly: they, as well as the figure of the 'child' and other identities at the limits of human adult language (Peters, 2001), feature as constant presences in a kind of philosophical 'thought experiment': what is it like having to infer in a certain language if you do not know what it is to start with? It is worth remembering that Wittgenstein himself spent time as a schoolteacher, and that his experiences influenced him to consider the problem of 'socialisation' as philosophically foundational (Bloor, 1983); if we can observe how a 'child' is positioned so that it can learn a certain semiotic design or literary technology (e.g. mathematics), as well as the 'resistance' around it (and the forces that settle it, that bring it to closure), then we can understand what kind of knowledge mathematics is.

The solution coming from the scholars interested in the operative conditions of the classroom and its influences over cognitive activity turn out not to be a solution at all! Instead of producing a method to understand the axiomatics of mathematics, as in the cases discussed above, such studies have produced the axiomatics of the classroom, and in some cases, the axiomatics of classroom 'discourse'. As we will see in chapter 3, the latter dispenses with the theorisation and analysis of the classroom as a social, cognitive and epistemic mechanism for producing and validating 'knowledge' and 'subjects',

consisting of nothing more than a discursive version of the social contract, a reiterative activity of holding people together in the 'same' world as an accomplishment of conversational intersubjectivity. In this case, the 'reality', the 'facts', the 'mathematics', are all irrelevant in explanatory terms. They count insofar as they 'count as' in people's formulations of their 'lived' world. As I show in chapter 4 and 5, that does not live up to its promise either, and the use of conversational analytical schemes in contexts like the laboratory and the classroom are condemned to triviality if they cannot include other, heterogeneous elements to the analysis: it is primordial to understand how features of the turn-taking system, for example, delegate and reflect specific forms of epistemic agency (the 'class', chapter 3), are mediated by representational affordances of classroom technologies (the 'blackboard', chapter 4, which is also a way of taking 'room' seriously), and how the 'content', the mathematics, is constituted as an analytic discipline on a par with the sociological descriptions we can make of it (chapter 5). Perhaps another way of saying this demands considering the two fields of 'visibility' referred to earlier as having only 'heuristic' value, as a description one should commit to in weak terms. In practice, the movement in which knowledge is accountably produced as self-contained and at the same time designed to be visible, to be grasped by witnessing and active participants is less pronounced, or accomplished as an *effect* inside the very practices at stake; the objectivity and necessity of mathematics and the competence of the learning subject are effects of how the complexity of the classroom is 'installed'. What I am suggesting is to treat such practices as 'mathematical-educational'.

Although a so-called 'mathematics education' field now represents an extensive body of literature in cognitive and developmental research and constitutes one of the more developed areas in the psychology of instruction it has virtually failed to describe the way participants orient to the forms in which mathematical concepts and practices in classroom education are taught and discussed. Rather, these studies have focused either on children's spontaneous knowledge of mathematics or on a so-called mathematical *language*, that is, the description of mathematics as a set of grammatical, axiomatic values. Even those studies that investigate mathematical practices *in*

the making (Carraher et al., 1988; Lave, 1988) can be said to fall short of a more radical social and praxiological approach as they rely on importing explanatory concepts into their descriptions, as well as designed methodologies of research (e.g. experiments, problem-solving testing). Furthermore, they tend to ignore the idea that, as they write about the objects of their inquiry, they formulate an alternative, participatory, judgmental way of dealing with the things 'observed'. It is worth remarking at this point that the field of 'mathematics education' was primarily constituted as a field for psychological investigation, and therefore a large part of the discussions ahead consist of discussions with psychologists and a way to respecify some of the interests in the field. Some of the conceptual difficulties regarding the 'mainstream' conceptualisations of the problems discussed above are:

- The status of mathematics as a non-indexical, 'decontextualised' type of knowledge. Studies in *comparative, cross-cultural psychology* show that it is rather a 'Western' practice to conceive of mathematics as a purely axiomatic, operational reasoning process (e.g. Cole et al., 1971). To that I would add that most studies give very little attention to the material and literary set-ups in which the teaching of mathematics finds its place.

- Mathematical understanding seems to be a 'hard-case' for psychologists and educationalists in general to conceive of in terms of 'socialisation'. Although they can refer to mathematical notation systems as cultural and historical means and outcomes, it is generally accepted that children *construct* the core logic of mathematics in their spontaneous, natural, adaptative mental development. In this sense, one can talk about *unmediated* understanding. To 'mimic' mathematical practices and mathematical language does not mean one 'understands' it, they would say. This attitude is related to the pervasive influence of Jean Piaget's theory of cognitive growth, in which logic and mathematics are the very basis of socialised thinking (Nunes, 2000). It also leads to known psychological claims about the decontextualised,

abstract nature of mature human thinking. According to this view, we cannot account for mental development through public criteria, such as *discourse*, for the mind is the continuous outcome of *inner* regulatory processes; perhaps we could do the opposite move, and explain how discourse is allowed to take place because individual (mature, non-egocentric) minds can already communicate.

- Educational research usually starts from some scientifically established version of knowledge (e.g., grammar, maths, and physics) in order to determine how far children's understandings are from them. This is partly because most researchers are involved themselves as participants, as education-makers. The consequence is that they often have a 'theory' about children's minds and how it can assimilate 'knowledge' (which they also know already *as* such-and-such).

- Most of the available 'critical' psychological approaches challenging these cognitive assumptions have not been able to overcome many of the limitations cited above. There is a sense that some of their most accomplished outcomes are good speculative theories, rather than well-grounded descriptive analyses. Besides, they still carry much of the same *prescriptive* concerns of traditional developmental theory. Here I am concerned with how understanding is accomplished interactionally for all practical purposes (Garfinkel, 1967), therefore being methodologically indifferent to pre-defined views of mind or reality.

The language and culture of mathematics

It seems that a comparison between the rigid *grammar* of mathematics and the pervasive *indexicality*[1] of natural language was one of the main interests of the logicians and mathematicians of the late XIX century until Ludwig

---

[1] Harold Garfinkel (1967) emphasized in his ethnomethodological program the property of words and actions by which they take their *sense* from their occasions of use; that is, they are *indexed* by the local conditions and resources of particular interactional events.

Wittgenstein's *Tractatus Logico-Philosophicus* (1972). Logic and mathematics were envisioned as the perfect language, able to overcome the imperfections and distortions of everyday discourse, a fact largely demonstrated by the description of *indexical* and *deictic* terms, such as pronouns (Garfinkel and Sacks, 1990). Mathematics is seen as inexorable, therefore *true*, as context-free, and like science it offers similar problems with regard to the analysis of knowledge's description and accountability *in situ*. The study of scientific knowledge, as a limit case for the study of the social construction of human knowledge, represents a paradigmatic case for mathematics education research: just as science has been shown to be based on assembling materials, techniques, spaces, argument, as well as text-constructing practices, the production of mathematical knowledge can also be seen as a set of *indexical*, *instruction*-saturated, *accountable* practices.

A common idea is to conceive of mathematics as *language*, whose grammar is the mathematics textbook. *Grammars* are not inappropriate or misguiding 'analytical' objects per se; rather, I would say they do not necessarily pose the adequate problems for the social analysis of mathematical learning. Saying it another way, the activity of learning how to inspect and find mathematical entities as a domain of practices is not to be accounted for necessarily *mathematically* (Lynch, 1993). After Vygotsky, psychologists describe those grammars as *mediational tools* and have tried to explain the processes of internalisation of those systems into 'mental functions', like logical reasoning and memory. One of the main objectives of that kind of research is to explain how human beings come to master those systems through the everyday use of language. The idea here is to argue for a case in which the learning of mathematics can be analysed as activity, and such a strategy implies not assuming a radical separation between mathematical 'language' and 'discourse' as in traditional psycholinguistics (Saporta, 1961). Rather, what counts as 'competent' is an accountable feature of those same practices.

This view on the distinction between mathematical *language* and *discourse* is highly reflected in mathematics education research. Hughes (1986), for example, argues that children's difficulties with mathematics can be accounted

for in terms of their encounter with 'the language of mathematics' (Hughes, 1986: 45). In his comparison between mathematics and language (and subsequent formulation of mathematics *as* a language), Pimm (1987) suggests that 'knowing a language' is describable in terms of (1) structural, and (2) communicative competence, the latter involving how to use language in 'context', unmistakably echoing the propositional/pragmatic dichotomy mentioned above. Walkerdine (1988) pointed out that the formal mathematical reasoning conveyed in schools can be described as pure *metonymic* (as opposed to *metaphoric*) language, one that undermines multiple signification, reflecting structural linguistics' accounts of the dynamics of language. In anthropology, Pinxten (1994) offered a similar linguistic view. Drawing from B. L. Whorf's *linguistic relativity theory*, he argues that mathematical language is a 'typical' western language with all its *atomistic* features (e.g., ontological opposition of subject, action and object). By taking *language structure* as a unit of analysis, the anthropologist/psychologist glosses on thinking *via* grammar description.

A short counter-example will suffice to set up the discussion on a different basis. From an ethnomethodological point of view, Gephart (1988) studied how the very production and use of *statistical* data in social research can be analysed as a form of organised social practice without postulating, in principle, a strong distinction between competence and performance. Gephart addresses three fundamental activities in social research concerning the use of statistics: (1) Producing raw data; (2) Statistics at work or decision-making; and (3) Rhetoric.

With regard to the production of 'hard data' Gephart cites a study by MacKay (1974) in which the latter 'sought to determine if intelligence tests measure the skills they purport to measure' (Gephart, 1988:21), which is the well-known formulaic way to refer to the validity of psychological tests. 'Official' answers to the tests were contrasted with respondents' accounts for their own answers. For example, children were presented with the item "I went for a ride" and then were asked to select an appropriate picture among a 'boy swimming, a boy walking and an auto[mobile]' (Ibid: 21). The procedure assumes that the

designated correct answer ('the auto') is a non-ambiguous, non-indexical, semantic implication of the sentence "I went for a ride". It also takes-for-granted the relevance *in principle* of "ride" in determining what children (should) do with the sentence. Gephart reports that MacKay pointed out that "went" suggests that the ride is not happening now and therefore if children orient to the *current* activities as correct answer (e.g., "I went for a ride, but now I'm swimming/walking"), any of the other two alternatives can be considered correct. This is important in the sense that it shows the common sense grounds on which psychologists construct 'raw data' to be given statistical treatment. One of the remarkable characteristics of this procedure is how it fails to acknowledge ambiguity and variability as part of the phenomena.

Secondly, producing statistical outputs is not free from assumptions or decision-making processes. This kind of issue can be seen, for instance, in the use of *parametrical* tests in psychological measurement, which assumes that the phenomenon under investigation is fit for *numerical* measurement. Qualitative approaches to cognitive development, for example, have long rejected the notion that a difference of '1' in a school exam is due to a difference of '1' in intelligence, the difference being rather attributable to the (absence of) certain competences or notions, and we acknowledge that in relation to the work of no one less than Jean Piaget. Gephart also remarks that 'conventional methodologies implicitly assume a model of the subject or actor as a rational being capable of assigning quantitative to qualitative phenomena consistently and reliably using the criteria or meanings desired by the social scientist' (Ibid. 35). This does not concern the dispute over the *rational* versus *non-rational* nature of the respondents; rather, it is a concern about the straightforward assumption of common knowledge between respondents and researchers. Finally, the way data and arguments are assembled and presented in a *text* constitute quite an interesting subject matter for *rhetorical* analysis. The use of a particular literary style in accounting for a factual reality through statistics can be appreciated in the way statistical values (e.g. correlation scores, such as .9 or .5) are attached to 'commentaries' or 'evaluative' words concerning their strength, like 'substantial' or 'relatively small'. Gephart

comments that 'terms that are similar were observed to be applied to different numeric values and similar values were linked to different descriptive terms. Indeed the value one author considered "substantial" was often the same or less than .1 from what another author labelled "relatively small"' (p.60).

In the field of studies in mathematics education, the last two decades or so has seen an increasing number of researchers and scholars turned their attention to mathematical forms of knowledge as cultural objects, rather than as some kind of universal, logically necessary, decontextualised ways of thinking (Carraher et al., 1988; Cole et al., 1971; Lave, 1988; Pinxten, 1994; Saxe; 1991), as well as to the ways in which mathematical rationality is appropriated in formal schooling (Lerman, 1994; Pimm, 1987; Walkerdine, 1988). Although many of those works have addressed important questions with regard to the relation between mathematical learning and cultural processes, they have not paid close attention to how those things get done methodically in settings designed to contain the work of learning as such-and-such documentary practices. Instead, most studies have focused, as I pointed out earlier, on mathematical language itself, be it as a framework with which children's learning is to be compared, a reproduction of Western atomistic cosmology in terms of classes of *objects* as separated from *actions*, or as a language that alienates *metaphor* and *narrative* from signification (Walkerdine, 1988). In this sense, while researchers in this area are highly committed to a particular view of mathematics and its teaching, they pay little attention to particular details of the children's socialisation towards such forms of knowledge. The emphasis on participants' settings, concepts and methods by sociologists of practice, social psychologists and semioticians implies that the investigation must be careful about making sense of the data in terms of an ordered, pre-defined explanatory system, thus losing sight of what is going on in the participants' own terms.

The data on which this study is based was gathered in the cities of João Pessoa, Recife and Fortaleza, in the Northeast of Brazil, and therefore feature originally in Brazilian Portuguese (see Appendix). The schools are part of the public system of education. The whole of the data comprise recordings from

several grades, ranging from pre-school to 4th grade, with no specific grade or mathematical topic previously selected. The lessons were recorded using video and audio equipment, and no arrangements were made with teachers towards modification of the regular proceedings. The approach I take here is largely *semiotic* and *praxiological* (Lynch, 1993), to define it in broader terms. Because I analyse interactive situations, much of the discussion is oriented by conversation analysis (CA) (Sacks, 1992; Hutchby and Wooffitt, 1998), discursive psychology (Edwards, 1997), and especially ethnomethodology (Garfinkel, 1967; Hester and Francis, 2000). I am also interested in the work of Foucault (1977) and his analysis of 'disciplines', and actor-network theory and the pursuit of 'actors' in sociotechnical practices (Latour, 1987; Law and Hassard, 1999). The latter found notoriety in the field of sociology of scientific knowledge, which is also a major inspiration here (Bloor, 1976; Lynch and Jordan, 1995; Shapin and Schaffer, 1985). Many aspects concerning those approaches will be developed as the argument unfolds.

Although the tone here is largely ethnomethodological, I do not claim this thesis to be an exercise in Ethnomethodology; yet neither do I claim it not to be. I think it is primarily ethnomethodological in attitude, and it has been clearly inspired by ethnomethodological discourse and discussions. Moreover, it is full of ethnomethodological references. It sometimes – but not always – intentionally avoids being ethnomethodological in (1) the use of jargon, or in the ways of framing the problems at stake; in (2) not making accountability claims of neutrality and non-mediation; and (3) in recognising that this discussion is constructive in itself and that this thesis was produced to be read by experts. In that sense, I do not share the view that ethnomethology, CA or analytic philosophy work better as a 'therapeutic' (in Wittgenstein's sense) way out of philosophy or constructive thinking than anything else, including constructive thinking. Although the nature of the game is different, technical language, jargon, commonplaces, orthodoxies, canons and literary technologies of reporting are overwhelmingly present in such disciplines and are as much a source of 'neurosis' as any other use of vernacular language in the social sciences. Some of my difficulties with the tenets of the aforementioned disciplines are more clearly expressed whenever the critique

of conversation analysis is at stake. If my critical concerns about CA can be considered coherent with other expressions of ethnomethodological thinking than I want to be believe that, philosophically, my assumptions are significantly ethnomethodological. Indeed, I want to suggest that some of the ideas developed here can be understood as a critical dialogue with conversation analysis. I suggest some ways in which I think CA 'fails' as an alternative to psychological explanations of classroom-based cognition, each of which will be the basis for the empirical chapters.

CHAPTER 2

Mathematics in so many words: reason, perception, discourse and social practices

2.1. The disputed nature of mathematical knowledge

One of the most fascinating things about mathematics is its timeless appeal. Such 'objective' and 'unmediated' quality to mathematical knowledge begs the question concerning its nature and scope, where adjectives like 'universal', 'eternal' and 'abstract' promptly come to mind. While the features of natural language, for example, are often conceived of in terms of their constitutive *arbitrariness*[2] and *indexicality*, mathematics has long been cultivated as the ultimate 'hard' knowledge, a depository of transcendental truths that are *necessary* and *general* in applicability.

It was an accomplishment of the Ancient Greeks to have translated it into a formal and coherent field of studies in the form of a system of *demonstration* and *deduction* (Devlin, 1998), as opposed to a collection of inductive techniques of measurement, suggesting that since the advent of its highly 'abstract' maturity, mathematics has been ahead of its applications (Freudenthal, 1973). Mathematics was also to become averse to the mechanisms of rhetorical dispute present in the Greek tradition, and the nature of its objects to be established as necessarily true, 'a fact anonymous to its authorship, available for endless inspection, established for all time – and this as a required feature of the demonstration itself (Livingston, 1986: ix).

---

[2] Linguistics, as originally formulated by Ferdinand de Saussure (1974), was predicated on the independence of the linguistic *sign* both from the intentions and motivations of the individual speaker and the correspondence with a 'referent', a 'real' object. The way *signs* afford *signification* is to be accounted for, according to Saussure, in terms of how they relate to other signs, either by substitution – metaphor – or by association – metonymy – in a structural system, such as English grammar or algebra (see also Jakobson, 1971).

Although the relation of (universal) mathematical entities to (local, immediate, contingent) 'reality', or to writing, scribbling, drawing, etc., was and still is a matter of dispute, it is fairly accepted in professional circles that 'mathematical signs do not code, record, or transcribe anything extramathematical: mathematical items evoke and mean what they mean, what they are to signify, directly and not as intermediates for something else' (Rotman, 1993: 25)[3]. Umberto Eco (1991) called 'monoplanar' such semiotic apparatuses, in which content and form are mutually assimilated.

Mathematics can be regarded as the very form of the 'Creation's work'[4], setting the background for the ordered, rational, objective knowledge of the sciences. It is said to 'reveal' the language of nature itself. Heidegger (1978) referred to 'modern' forms of observation that amount to the 'sciences'[5] as a 'mathematical projection', that, more than just dealing with numbers, reflects and anticipates the form of the outcomes scientists *want* to learn from 'things', i.e. their *axiomatic* nature. Galileo, for example, famously expressed that 'the great book of nature can be read only by those who now the language in which it was written. And this language is mathematics' (in Devlin, 1998: 10). Unmistakably indebted to Greek philosophical rationalism, this statement projects the image of a world of ideal forms of which our 'lived', contingent one is a mere deformation, an illusion. For Plato, knowing is 'reminiscing' such a place, from where all souls come[6]. In this 'purified' world, which stands beyond our acknowledgement of everyday appearances – and of social

---

[3] In Chapter 3 we see how this feature of philosophical mathematical discourse is somehow subverted in the classroom, as it tries to establish the relevance and worldly character of knowledge.

[4] 'Mystical doctrines as to the relation of time to eternity are also reinforced by pure mathematics, for mathematics objects, such as numbers, if real at all, are eternal and not in time. Such eternal objects can be conceived as God's thoughts. Hence Plato's doctrine that God is a geometer, and Sir James Jean belief that He is addicted to arithmetic (...) The combination of mathematics and theology, which began with Pythagoras, characterized religious philosophy in Greece, in the Middle Ages, and in modern times down to Kant' (Russell, 2001: 56).

[5] Here I follow Bruno Latour and several other writers in the sociology of scientific knowledge who, while deferent to scientific accomplishments, refuse to describe as 'scientific' in principle the mediational activities and technologies that produce accountably 'scientific' outcomes.

[6] The dialogue between Socrates and the slave boy in the *Meno*, in which the former accountably *elicits* from the latter a geometrical understanding he *already* knew, is a classic example, and has been extensively used in educational, psychological and sociological discussions (Edwards, 1997; Mercer, 1995; Macbeth, 2000; Bruner, 1971).

practices of conceptual and technological 'mediation' (Latour, 1998) – a realm of pure rationality and beauty, of concepts and timeless forms allegedly stands.

In modern days, the widespread use of mathematics and measurement in most aspects of our everyday lives and 'institutions' [7], has arguably backfired the conception that 'everything is mathematics'; 'abstract patterns', some would say, 'are the very essence of thought, of communication, of computation, of society, and of life itself' (Devlin, 1998: 10). Mathematics has acquired a primary importance in most spheres of knowing activity (most notably in 'scientific' forms of inquiry, see note 5), continuing a tradition for which it has even assumed, with the Greeks and Western thinkers after them, aesthetic and religious undertones (Devlin, 1998; Gould, 1998; Russell, 2001). Greek mathematics has also helped to establish the primacy of thought over the senses as the model for inquiry to be followed by the sciences. Such is its predominance in 'Galilean' scientific spirit that it has been written, for example, that the sciences continuously advance by their last decimal number (Kaplan, 1964). 'It is the goal of a Galilean science to use mathematics to discover the inherent structure of the world; an inherent structure that is *always* and *already* mathematical' (Lynch, 1993: 81).

All of this points out to a 'tendency' or rational 'compulsion' (Bloor, 1987) according to which mathematics is to be accounted for as something that will be there long after all the notebooks, blackboards, chalks, pencils, gestures, and local routines of inference once used to make it learnable and intelligible in classrooms, and else, have worn out. Partly, the task set for the present work is to render notebooks, blackboards, chalks, pencils, gestures, local routines of inference and classrooms important in understanding what mathematics is. On a more ironic note it is curious – given Platonism and its idea of reminiscence

---

[7] Counting and measuring practices seem to be integral to the facet of modern scientific societies by which *invariant* and *mobile* literary technologies (e.g. mathematical notation) shape our activities of perception and reasoning in diverse ways (Latour, 1990). Nowhere this equation is better seen than in Jean Piaget's experiments in cognitive development, in which carefully calibrated objects and experimental conditions work as a template for the interpretation of *endogenous* mechanisms of perception and cognition. Piaget's mistake was not realising that we perceive and reason as we do, mathematically speaking, *because* of such calibrated measures, that our cognitive powers are always and already immersed in this 'metrological' order.

of the world of ideal forms – to find how difficult mathematics reportedly is for people in general, and for kids at schools, in particular. The arid and sometimes defeating character of mathematics is acknowledged by semiotician Brian Rotman when he says that 'one need, it seems, to have been inside the dressing room in order to make much sense of the play' (2000: 1). Seymour Papert (1979), commenting on the distinguished philosopher and mathematician Henri Poincaré's belief on an innate aesthetic quality of some minds to appreciate mathematical beauty, credits 'our culture' for the idea that 'the experience of mathematical pleasure are accessible only to a minority, perhaps a very small minority, of the human race' (p. 105). As we shall see later, in children's experience at schools, the immediacy of mathematics is, at least, problematic.

Some of the most enduring discussions about mathematics and the attempt to define its nature have also help defining the field of inquiry known as epistemology or theory of knowledge. Logical and mathematical forms of knowledge have always pushed the epistemological investigation to its limits, and the quest for its origins has arguably driven such investigations to their metaphysical solutions. Empiricists such as John Locke and David Hume had ventured into supra-empirical descriptions of 'secondary qualities', 'abstract ideas' and 'causality' as products of the mind. Kant differentiated between synthetic, synthetic *a priori*, and analytical forms of knowledge[8]. Logical empiricism used the logico-mathematical analysis of statements to declare about their validity, etc. The point in relation to the theory of knowledge is that mathematics 'is so often invoked as an exception, prohibitive limit, or clear counterinstance to the more radical reaches of such theory: to post Nietzschean epistemology, for example, or to post-Kuhnian history and sociology of science' (Smith and Plotnitsky, 1995: 373). Epistemology's primary concern had been that of how it is 'possible' to know, putting into perspective the relations between 'knowledge', 'reality' and the knowing 'subject'. The problem at stake is that of whether mathematics – a form of analytical

---

[8] Synthetic knowledge is predicated on experiential claims, that is, knowledge claims based on the intuition of the *real* (as opposed to logical, or deductive, claims). The novelty of Kant's philosophy was the formulation of 'synthetic *a priori*' (e.g. causality, number, being) forms of knowledge as basic attributes of the 'epistemic subject'.

knowledge (as Kant described it) – belongs somehow to a conceivable mathematical reality 'out-there' (in which case the connections with the human mind are to be established), to the workings of the human mind (in which case the connections with reality are to be established), or even to the isomorphism between both: 'The absolute certainty of a mathematical proof and the indefinitely enduring nature of mathematical truth are reflections of the deep and fundamental status of the mathematician's patterns in both the human mind and the physical world' (Devlin, 1998: 9). The solutions, more often articulated in terms of the two first questions – the primacy of 'reality' or the primacy of 'mental operations' – gave rise to diverse Platonist and intuitionist philosophies of mathematics (see this section and section 2.2. See section 2.3 on Piaget functionalist theory of knowledge for an example of a more 'isomorphic' solution).

The focus on the 'conditions' of possibility of knowledge has marked epistemology as a *normative* enterprise, an effort to define – and therefore constrain – what valid knowledge count as (Popper, 1959). Such a task has alienated much of the contemporary scholarly work on the history and the social production of science, and more specifically the sociology of scientific knowledge (SSK) (Bloor, 1976). In the SSK tradition, 'truth' is as socially mediated and constructed 'error' (the principle of 'symmetry'), and factual statements are analysed as artefactually assembled in installed sites that afford their proliferation through a set of technological and representational devices, such as the laboratory and all sorts of inscriptions and notations (Latour and Woolgar, 1979; Lynch, 1985). In SSK accounts, epistemology had become pretentiously legislative, rather than 'analytical'. In the context of science studies, the appeal to 'true' conditions of knowledge reflects the commitment to the rhetoric of 'representation', whereby language and notational systems in general are said to map onto objective conditions in nature (Lynch and Woolgar, 1990). Representation was classically linked to the idea of the *referent* of a sign, and therefore is based on the notion of a pre-existing, ontological reality (Foucault, 1995). In that sense, language was seen as nothing more than a means to send representation to its origins, to the duplication of nature's surface (Sinha, 1988). In the case of mathematics, the

17

duplication of truth's surface!

American philosopher Charles Sanders Peirce, a mathematician himself, was one of the pivotal figures in the respecification of Kant's rationalist critique in terms of the sign-saturated nature of the 'mind'. In the 18th century, Kant had taken the theory of knowledge to a new rationalist and metaphysical turn. Like other philosophers before him, Kant placed the possibility of knowledge as situated between *intuition* (phenomenal representation) and *judgement* (general concept). Unlike them, he dismissed the idea that the latter could be derived from the former by mere observation and association. Kant asserted that non-tautological forms of *a priori* judgement, that is, knowledge predicated on structures previous to experience, such as being, space, time, causality and number, account for the basis of our *understanding* of reality. Although Peirce sets the scope for a philosophical or formal semiotics in the wake of Kant's epistemology – still away from the social analysis of sign use in practices – we can already envisage in his work the rejection of the analysis of knowledge purely in terms of *representation* at the same time it constitutes the material and creative dimension to Kant's subject.

The old semiotics of representation is essentially dualist, describing the relation between the represented and a representing entity, like a mental image. Peirce introduces the concept of *interpretant*, by which he proceeds to disentangle semiotics from Kant's transcendental analysis. The interpretant is the *third* element in a dynamics that comprises also the *sign* and the *object*. In fact, the interpretant mediates the relation between sign and object, being itself an extension of the sign. Ultimately, the interpretant is another sign, a means of multiplying the meaning of the object *ad infinitum*. The interpretant, therefore, translates the sign and becomes consecutively a new sign, for which a new intrepretant is to be mobilised in a process of 'ilimited semiosis' (Eco, 1974). In Peircean mathematical prose the symbol 1, for example, does not designate a *token*, an object analysable in terms of its 'optic' properties (i.e. size, shape, colour), or a mere instrument of similitude, of representation. Rather, it is a *type*, an 'abstract pattern of writing' (Rotman, 2000: 22), whose meaning is an operation inside a code; together they constitute the *sign* '1'

(Ibid.). Thus, the notion of interpretant liberates semiotics from the metaphysics of the *referent* insofar as it declares its independence from an object with a determined truth-value; its potency is to be found in relation to other signifiers (de Saussure, 1974), to language games (Wittgenstein, 1967), to culture (Geertz, 1973). However foundational, the original sign (e.g. a pictorial or mental image) is nothing more than a 'condition of possibility' for knowledge, not its formal or operative aspect. In that sense Peirce, Kant and Piaget are in agreement. But whereas Kant and Piaget found their line of argumentation in abstract cognoscent subjects, semiotic analysis is committed, at its best, to following the practical and material trajectories signs and its delegated actors take to establish meaning (Latour, 1987; Rotman, 2000).

The semiotic thesis establishes the fact that mathematical imagination is grounded on the manipulation of *inscriptions*. It advocates a 'signifier-driven' conception that captures mathematics special relation to *writing*. Rotman (1987; 1993; 2000) proposes that the task for a semiotics of mathematics is to put into question the 'naturalness' of mathematical knowledge. That would imply examining the accountably transcendent nature of its conceptual building blocks, such as the *integers*, or 'natural' numbers (e.g. 1, 2, 3, ... n). 'What does it mean to say of these numbers that they are infinite, that they form a progression which is endless? In what sense are they natural, that is to say, before, independent, and outside of us?'(Rotman, 1993: pp. ix-x).

Calculation are experiments, Rotman says, and although we often take for granted the physical and social mediation of their production (see chapter 6, on the accountability of 'knowing'), they constitute conventional routines to manipulate objects and symbols (Bloor, 1983). Following Wittgenstein (see section 2.2), Rotman rejects realist assumptions for which mathematical grammar warrants the conclusion that the results of all mathematical operations are already in place; for Platonism, mathematical rules and their accomplished application co-exist at any moment in time: 25×25 is already 625 (Bloor, 1983: 84). He also rejects the intuitionist thesis according to which the objects of mathematics are reducible to mental constructions, whereas its

written syntax would be nothing more than a secondary nature. Intuitionists have attacked syntax as primary under the argument that it affords a false mathematics of the infinite, whereas no concrete mental constructions of the real can be identified with it. The origins of mathematics, they argued, has to be linked to the finite states of affairs natural language describes (Rotman, 2000), mathematical notations being a secondary phenomenon in relation to the mind's intuitions. Rotman critically points out a hierarchy within 'Western thought' which operates as to consider the activity of mathematical 'writing' as secondary to 'thinking', an abbreviated transcription of it: 'first meanings then notations, objects then names, ideas than expressions, numbers then numerals' (1993: 33)[9]. Bruno Latour (1998) has also argued that the Western modernity can be characterised in terms of how thinking proceeds by purifying the discourses on the *natural* and *social* orders from their constituent 'mediation', from the assembling of those such same objects in heterogeneous conditions of production, involving hard technologies, inscriptions, discourses, etc. Such an intellectual habit needs to be overcome, says Rotman. The effect of this hierarchy is to rank such mental abstractions as primary in relation to writing, as well as the best explanation of it (Livingston, 1986; Lynch, 1995; see also chapters 5 and 6).

The question addresses what would be left for the 'philosophical' explanation of mathematics once the material and social forces in the form of objects, written inscriptions, agreements on observables and inference rules, etc., have been taken into account. The semiotic model proposed by Rotman 'identifies mathematical reasoning in its entirety – proofs, justifications, validation, demonstrations, verifications – with the carrying out of chains of *imagined actions* that detail the step-by-step realization of a certain kind of symbolically instituted, mentally experienced narrative' (Rotman, 1993: 66, emphasis added). The way the model works simulates 'thought experiments' which 'carry out' a 'narrative' whose grammar employs all the operational resources listed in the quoted above, intending to leave nothing to Platonic or intuitionist explanations. By conferring privilege to pre-semiotic objects, those theories

---

[9] See chapter 5 for a discussion of how such distinctions operate in the teaching-learning practices in the mathematics classroom.

have ultimately failed to answer satisfactorily how we (humans) possibly came to *know* objects outside space and time:

> 'It is simply not plausible – either historically or conceptually – to ignore the way notational systems, structures and assignments of names, syntactical rules, diagrams, and modes of representation are constitutive of the very "prior" signifieds they are supposedly describing' (Ibid: 33).

Rotman's theory about the different mathematical 'agencies' that allow the model to be run is an interesting and complex take on the Peircean semiotics of the *interpretant*. He observes that solely as a set of abstract formulae, mathematics hides from sight several of its constitutive features; at the same time it suppresses others in order to maintain its (supra) rational and general character:

> 'Two crucial features of formal mathematics – call it the official *Code* – stand out. First, every text written in the Code is riddled with *imperatives*, with commands and exhortations such as "multiply items in *w*", "integrate *x*", "prove *y*", "enumerate *z*", detailing precise procedures and operations that are to be carried out. Second, the Code is completely without *indexical* expressions, those fundamental and universal elements of natural languages whereby such terms as "I", "you", "here", "this", as well as tensed verbs, tie the meaning of messages to the physical context of their utterance' (Rotman, 1993: 7).

Rotman contrasts the 'official' *Code* with an informal, meta-*Code*. Meta-Codes are pragmatic, part of the user's take on the formal *Code*, and demand the understanding of natural language. Rotman proposes that in order to understand the *semiosis* of mathematical expressions, three semiotic agencies have to be considered: (1) the mathematical *Subject*, a reflexive 'self', as Peirce called it (Rotman, 2000), a transcendental reader/writer of *correct* mathematical meanings; (2) the *Agent*, an automaton who carries out primitive orders like 'count', 'add', 'reverse', and so on; and (3) the *Person*, a socio-historical individual immersed in the indexicality of natural language, and bearer of the meta-*Codes* needed to interpret mathematical expressions. The *Agent* is strictly linked to the *Subject*, but whereas the *Agent* carries out the

execution of he code at a 'sub-coded' level (p. 8), the *Subject* 'understands' concepts, definitions, proofs, etc. 'In terms of the distinction between imperatives, it is the Subject who carries out inclusive demands to "consider" and "define" certain worlds and to "prove" theorems n relation to these, and it is his Agent who executes the actions within such fabricated worlds, such as "count", "integrate", and so on, demanded by exclusive imperatives' (Rotman, 2000: 20). The Agent is a 'skeleton diagram' (Ibid: 14) of the Subject who reflects and commands actions within fictional worlds, actions that the *Subject* itself cannot carry out since he 'can only manipulate very small finite sequences of written signs' (Ibidem); the *Agent*, on the other hand, is the actor imagined by the Subject to travel everywhere, to carry out the distances and potential infnity of mathematics. The command to 'to add 2'in the sequence of 2, 4, 6, 8 … etc., projects such infinity, for which the Agent, who is blind to experiential or logical feasibility, is called. Finally, the Person represents the individual, the 'I' of natural language, the existential version of the (transcendental) Subject, and is ruled out of mathematical discourse. I will come back to these distinctions in chapter 6, when is argue that 'knowing' in the mathematics classroom projects the accountability of pupil-as-*Subject*.

One of the problems with the semiotic theory proposed by Rotman is that it implies that the way his model operates simulate all the interesting things we have to know about in our quest for the *learnability* of mathematics, or of how its observables come to be used and in which way. It is as if his 'thought experiments' have the power to exhaust the question, with no need for empirical investigation about concrete experiments (e.g. educational) in mathematical thinking/writing. His formulation has a strong need of a theoretical definition of 'mathematics-as-language' (Rotman, 1993: 7), which has troubled also the effort into describing mathematics cognition and education in the terms of discourse analysis (see section 2.4). Therefore, it is not clear to what extent the 'experiments' that can be derived from the model still nod or not to logical formalism in the sense that their actions are reducible to possibilities already contained in the system, that is, to how the 'lived-work' of mathematics' production is logically and necessarily tied to its formal 'proof-accounts' (see Livingston, 1986).

2.2. Beyond reason: mathematics as social practice

A major obstacle for the social study of mathematics is directly related to notions of 'truth' and 'objectivity'. What do we possibly mean when we say that mathematics is social? Do we mean that if we apply our *base 10* numerical system somewhere else it won't work? Or possibly that other numerical systems, such as that of the Oksapmin of Papua New Guinea, in which a number system without basis, used only to count based on the correspondence between objects and body parts (Saxe, 1991), have no logic or invariant properties? Or do we mean that the logic of street sellers in Brazil, when they report to ground their arithmetical operations on mental strategies that separate numbers into smaller 'blocks' in order to calculate the change to be returned to their clients, is altogether different from the disengaged truth expected from problem-solving activities in the classroom (Carraher et al., 1985)? None of these answers have been satisfactorily accepted before, and that several aspects of mathematical practices are culturally-bounded could not retreat social, psychological and educational analysis from a compromise that maintains mathematics' status as *transcendent* at the same time it assimilates the variation of its representation and communication characteristics. (Vergnaud, 1991; Carraher et al., 1988; Nunes, 1992).

In psychology, important *socio-cultural* approaches to cognition subscribe to the correspondent view in which notions of *propositional* and *pragmatic* come across as distinctly separate forms of knowledge. 'Conceptual fields' (Vergnaud, 1991) and 'scientific concepts' (Vygotsky, 1987) are invoked as if they were independent from, and prior to, language and social practices. Some cognitive anthropologists (see Schweder, 1984, for a review) are likely to argue that the way cultural participants orient to conceptual knowledge is different from the meaning of the conceptual knowledge 'itself'. There is the almost inevitable consequence of turning the analysis of cultural knowledge into a kind of marginal analysis of 'values', 'patterns of transmission', 'social constraints', and so on, which obviously hints at the question of whether mathematics can ever be said *social* at all.

On the eyes of 'classical theory', one of the problems with the cultural analysis of science, in general, and of mathematics, in particular, is that the accountably general validity of mathematical knowledge consistently undermines their *relativism* and sometimes 'solipsistic' concern with discourse (Smith and Plotnitsky, 1995). Here, I am not questioning whether mathematics is 'true' or not! I agree with Lynch when he says that 'it is pointless to argue whether or not mathematical theorems are "true". In a sense, they are everything we knew about disengaged truth. This is not because there is a disengaged, objective position from which to evaluate them' (1991: 86). So, depending on the philosophical footing of the debate the answer to the question of truth and objectivity is 'yes, mathematics is objective', or, 'yes, mathematics is objective, but ...' where 'but' opens up for a whole new set of questions, including, as Lynch put it, that concerning the way the 'objective' and 'neutral' footings from which to evaluate it are produced. For example, it seems to be a central notion to the analysis of mathematics as public accomplished the notion that cultural members use shared understandings in order to evaluate what *counts as* an instance of, say, a 'proof'; the construction by ancient mathematicians of the 'sameness' of different geometrical figures, e.g. triangles, on the basis of abstract, conceptual definitions (Nunes, 2000) is the kind of accomplishment that opens up space for the kind of social, cultural analysis, discussed here.

Latour (1993) refers to a certain 'Archimedes' *coup d'état* as the movement by which the great philosopher and mathematician, after putting mathematics in the map of politics by enlisting physical and geometrical principles in the production of military technology for the King Hiero of Syracuse (thereby making technology and political representation 'commensurable', p. 110), retreats from producing any form of official knowledge about how such relations were made possible, mathematical knowledge being kept *incommensurable* with the 'vulgar' needs of the State. 'The balance sheet is doubly positive: Hiero defends Syracuse with the machine whose dimensions we know how to calculate through proportions, and the collective also grown proportionally; but the origin of this variation in scale, of this commensurability, disappears for ever, leaving the empyrean of mathematics

as a resource of fresh forces, always available, never visible' (Latour, 1993: 111). The question is not, Latour suggests, that mathematics *per se* can be read in terms of 'interests' – '*Geometry* and *Statics* are *State*-oriented' – nor that Syracuse society was in a mathematical straightjacket from then on, as some form of twisted 'cultural' argument would have us believe; rather, we see how 'Archimedes procured a different principle of composition for the Leviathan by transforming the relation of political representation into a relation of mechanical proportion' (p. 110), that is, how 'non-human' actors were enlisted to organise and enlarge the scale of the State, at the same that 'social' and 'epistemic' were subsequently divorced. However, the question stands regarding how to describe mathematical objects and relations in terms that challenge traditional philosophical accounts; furthermore, the question stands regarding how it can be grasped.

Wittgenstein was amongst the most significant contemporary scholars to reject Platonism and intuitionism in mathematics (Bloor, 1983). His *Remarks on the Foundations of Mathematics* (2001) appeared for the first time in 1956 and shares the same analytic spirit of his well-known *Philosophical Investigations*. Indeed, of all sources, Wittgenstein provides one of the most complete foundations for the discussion on the conventionalisation of mathematics; more than that, his writings point to the investigation of the nature of mathematical knowledge in open reference to how it is *learned* in the first place! Such is the importance of learning and its practical set-ups in Wittgenstein later work that Bloor (1983) declares, in his discussion about mathematics as an 'anthropological phenomenon', that the meaning of 'foundations' in Wittgenstein's work on mathematics is closely related to his experiences as a schoolteacher. It is worth noting, for example, that Wittgenstein repeatedly use of the figure of 'the child' in his writing in order to substantiate his philosophical queries (see chapter 5). The 'child' is a compelling example of membership at the limits of *our* language, of the language of 'reason' (Edwards, 1997; Walkerdine, 1984, 1988). That means that the child – as well as the madman and the foreign, not to mention animals and machines – are potential sources of trouble as far as the *visibility* of even

our most basic premises of cultural knowledge are concerned, e.g. the rules for 'counting'[10].

Wittgenstein did not start with a version of what kind of knowledge mathematics is; he advocated the search for one in the analysis of the techniques by which a mathematical formula can stand as a representation of the 'process' it is a formula of. Unlike Piaget (see section 2.3), Wittgenstein considered that a formal rendition of knowledge does not provide the parameters for how the 'mind' should be characterised; the foundational question is pursued as an effect of the question of *learnability*, not vice-versa. In consonance with the ethos of the *Investigations*, he focuses on the sense of mathematical inference patterns as a matter of convention, of 'agreement':

> '"The way the formula is meant determines which steps are
> to be taken." What is the criterion for the way the formula is
> meant? Presumably the way we always use it, the way we
> were taught to use it' (2001: 36, §2).

Without denying the general applicability of mathematical formulas (a strategy that avoids the pitfalls of contemporary cultural relativism – Latour, 1993), Wittgenstein chooses to clarify the practices of inference by which mathematical formulas come to establish their truthfulness and universal applicability. The inexorability of mathematics is realised through its very (reiterable) occasions of use. As Lynch (1991) puts it, evoking an ethnomethodological solution, 'a mathematical theorem does not depend on the particulars of their situated actions. Yet, somehow, the disengaged adequacy of their work practices is the achievement of those selfsame, local and occasioned practices' (p. 87)'. Truth and rigour are to be treated, then, as accountable matters. To invoke the ethnomethodological *ethos* once more, the rules of mathematical inference under study must be seen as both constitutive of conceptual entities and their operations as well as exhibits of the decontextualised truth of mathematics. In Wittgenstein, the inexorability of logical inference is equivalent to the mechanical application of the rule, to

---

[10] In an anecdotal but very illuminating example, Lynch (1993) cites the case of a child who turns her back on the adult who asks her to count 'backwards' after a first round of counting up to 'five'.

'impressing a technique' with exactitude (Bloor, 1983), and Wittgenstein recommends that we look at which language games are in operation in order to understand, in each case, what inferring or calculating consists of.

Inferring, counting, calculating, he says, are procedures in a given language game. When we count objects it is not a trivial matter that we use the sequence of numerals as we have always done; to start a counting sequence with 8, 5, 2, ..., say, as if we are using the techniques of correspondence between numerals and sets, is to undermine one of our most important institutions, one that several activities in our daily lives depends on, Wittgenstein would say. The mercilessness with which we count the same way all over again denounces the importance of such 'procedure'. 'But is this counting only a use, then; isn't there also some truth corresponding to this sequence?" The truth is that counting has proved to pay. – "Then do you want to say that 'being true' means: being usable (or useful)?" – No, not that; but that it can't be said of the series of natural numbers – any more than of our language – that it is true, but: that it is usable, and, above all, it is used"'(Wittgenstein, 2001 [1956]: §4, pp. 37-38). Counting as we do is 'true' because it pays off.

Perhaps one way to understand Wittgenstein's analytical commitment is to say – advancing aspects of the ethnomethodological discussion – that he was interested in how practices come to have specific 'values' attached to them, e.g. 'mathematical'. It is well known that he repeatedly addressed, in opposition to the logicists like Frege and Russell, that logic and mathematics were different language games: 'Logic is a kind of ultra-physics, the description of the 'logical structure' of the world, which we perceive through a kind of ultra-experience (with the understanding e.g.)' (Wittgenstein, 2001: 40, §8). Mathematics, on its turn, refers not only to academic, high-concept thinking, but accountably to activities as diverse as classifying, counting, measuring, quantifying, etc., which share a strong 'family resemblance'. The attempts to conceptualise those activities in terms of logical factors cannot detract us from the observation that its everyday grammar is deployed in complex ways. For example: 'What does it mean for me to say e.g.: this number can be got by multiplying these two numbers? This is a rule telling us

that we must get this number if we multiply correctly; and we can obtain this rule by multiplying the two numbers, or again in a different way (though any procedure that leads to this result might be called 'multiplication'). Now I am said to have multiplied when I have carried out the multiplication 265 X 463, and also when I say: "twice four is eight", although here no calculating procedure led to the product (which, however, I could also have worked out). And so we also say conclusion is drawn, where it is not calculated' (Ibid: 40, §7). To paraphrase Bruno Latour (1993), although we can recognise a set of practices as 'mathematics', social and anthropological analysis should make little concession to its adjective use ('mathematical') and even less to its adverbial use ('mathematically').

Wittgenstein had little to offer to theoretical discussions concerning the characterisation of knowledge as 'mathematical', and of actions as 'mathematically'. If he discusses the canons of academic logic and mathematics in the *Investigations* and in the *Remarks* is to show their marked status within the field of epistemology and their conventional nature. Such a philosophical attitude undoubtedly points to the need for the investigation of the reflexive constitution of knowledge *in situ* later developed by the ethnomethodologists, but have been a permanent problem for 'constructive' sociology (Garfinkel, 1967; Lynch, 1995) and even semioticians. Rotman, for example, remarks that 'though sharp, interesting, and unfailingly provocative, Wittgenstein's fragmentary and idiosyncratically unsystematic dicta do not address the *theoretical* question of mathematical language and discourse in any direct or even indirect way. Nor do they indicate how one might do so' (p. 17). Nevertheless, Wittgenstein did not intend to solve theoretical questions, but, as it is known, to analyse them. The theoretical question Rotman asks for projects mathematics as a kind of discourse that semioticians (and mathematicians and philosophers) can, in advance, identify and clarify as such-and-such. Although Rotman's semiotic model questions the naturalness of mathematics, he does not go deep into the problems concerning how the way the *Code* operates constitute a matter of social convention, how *action* and *rule* mutually constitute each other, how a proposition is derivative from

another by means of a rule only if we can establish that anything is, by means of some rule (Wittgenstein, 2001).

Wittgenstein's irreductionism projected him as one of the main adversaries of Platonism and classical epistemological theory. Platonism is indissociable from the idea of 'discovery' in mathematics. 'Arithmetical propositions are true because they correspond to facts about entities called 'numbers'' (Bloor, 1983: 84). While a more social interpretation will look for the way mathematics is conventionalised,

> 'Wittgenstein's position is that we don't really know what we are saying when we glibly refer to the unknown and uncomputed parts of an infinite sequence of numbers of this kind. We are transferring intuitions derived from finite sequences of numbers, and are assuming that they apply without difficulty to the infinite case' (Ibid: 88).

Bloor makes clear that Wittgenstein's rejection of Platonism is connected with his finitism. 'The number series does not exist in advance of our use of it: its reality extends no further than our actual practice' (Ibid: Ibidem). The fact that we use such routines in mechanical ways helps establishing its pre-existing appeal. However, the finitism of intuitionists like Brouwer, and his statement that 'intuitionist mathematics is an essentially languageless activity of the mind having its origin in the perception of a move in time' (in Rotman, 2000: 28), is also strange to Wittgenstein's investigations, once it portrays the primacy of meaning over language, and the 'after-the-fact' character of convention. For Wittgenstein, mathematics forms a 'network of norms' (Wittgenstein, in Bloor, 1983: 91), and *normative* understandings are, at least originally, agreed upon (although they can be forced upon!). Mathematics was invented, not discovered. Wittgenstein had no analytical regard for the 'fundamental laws' of mathematics as transcendental, either in the form of eternal reality or of mental operations. Besides, he understood that such laws, thus formulated, do not contain an account of their own inference procedures and its fundaments. Lately, some currents in sociological thinking, especially ethnomethodology, have addressed this problem by proposing a turn to empirical analysis of how the 'lived-work' of mathematical actions and

accounts is integral to the constitution of mathematical objects, e.g. proofs, (Livingston, 1986, 1987, 2000; Lynch, 1993; Sharrock and Ikeya, 2000).

Livingston (1986) distinguishes between a 'proof-account' and the 'lived-work' of a mathematical proof. The lived-work, largely unnoticed, supports the production of the proof-account as disengaged. The details of the *in situ* production of a mathematical object, it is claimed, answer for the transcendental qualities of mathematics that seem to surpass them. Livingston sets out to examine the 'natural accountability' of a proof in relation both to its public character within the community of mathematicians and to the fact that 'they are done recognizably as proofs; they are produced as the objects that they recognizably are' (1986: 23); the development of a proof is *witnessed* and *witnessable* as such as mathematicians work out its adequacy.

Livingston's purpose is nothing less than demonstrate how the transcendental rigour and adequacy of mathematical proofs is thoroughly accomplished at the mathematics' 'work-site', say, at the blackboard; that is, how matters of ontology can be appreciate once the witnessable adequacy of the 'local' is worked out step-by-step, publicly (Bloor, 1987). In chapter 4 we see how the work of scribbling at the blackboard, for example, enlists witnessing and collective agency in order to establish the necessary character of the knowledge at stake.

With outstanding mathematical knowledge and ability, Livingston goes on to conduce the reader through the step-by-step application and operational accountability of each single arbitrary mark on the page, regarding two classical mathematical problems: 'Godel's theorem', by which the axiomatics of arithmetic can be proven, itself, passive to arithmetic description; and a theorem of Euclidean geometry, about the measure of an inscribed angle at the circumference, and its relation (1/2) to the angle of its intercepted arch (Livingston, 1986: 2). I do not mean to pursue the technicalities of Livingston's argument (which are many), but to highlight a few characteristics of his work regarding the problem of 'learnability'. Because his work – which represents the major ethnomethodological contribution to the debate on

mathematics – never openly elaborates (practical) questions regarding *agency* in mathematical reasoning, or the *Subject* of (or subjection to) mathematics, or even more surprisingly, regarding practical, particular 'work-sites' (e.g. classrooms), some criticisms have been levelled at his argumentative strategies and ultimately to whether he accomplished the task he set out to do, namely, that of elucidating the compelling status of mathematics' extra-human ontology through the analysis of local practices.

Livingston's analysis of the character of the local work of mathematical proving has, according to Bloor (1987), four characteristics: (1) coherently with the emphasis on locality, the author largely takes for granted – or refuses defining – the ontological status of things like circles, lines and angles (Bloor, 1987), taking that we can at least recognise those objects; 'the more important claim being made is that all that needs to be known will be supplied in the course of the proof' (Ibid: 345). The demonstration exhibits and exhausts the set of actions and resources needed to deploy and understand the development of the proof; (2) a plea is made to consider a tendency to focus, as proofs unfold, on object that are already there, in the *future* of the proof, 'to see beyond the developing sequence of activities by which the proof is presented' (Ibid: 346); Livingston calls it a 'projected gestalt' (1986: 10), and it would be responsible for the temporal sequence of proving *seeming*, at any given point, somehow inessential, (3) we hold mathematical writing – a given angle (a) on a piece of paper – to be the representative of a 'class', the class of angles, and therefore arbitrary; and (4) the final formulation of the proof elaborates a new temporal order of thinking in relation to the activities of producing the proof through writing and inference trials in the first place.

The topics are sound and undoubtedly in line with a praxiological understanding of what a research program oh mathematics should be interested in, although, as Bloor observes, Livingston himself discourages, a programmatic follow-up to his 'uniquely adequate' studies. However, it is possible to trace the historical pedigree of his arguments in the work of mathematicians and logicians such as Russell, Lakatos and Wittgenstein, which in Bloor's review has the sense of undermining the incommensurability proposed between ethnomethodology and

other 'non-classical' studies of mathematical reasoning. More importantly, Bloor contests that Livingston has solved the problem of the gap between the local and the universal that has puzzled the philosophers of mathematics since the Greeks: 'If we examine his account carefully we see that it is made up of claims about what 'a prover may realize', and of 'what would follow from the original case'. We are told that the prover is looking for 'an extractable method' – that is, one which can be extracted from the contingencies surrounding particular cases. This method then 'offers those three cases as making up all the possible cases' (...) As far as I can see, this description of the 'work' of theorem-proving makes use of our usual, unselfconscious way of talking about mathematical discoveries' (Bloor, 1987: 349). According to Bloor we are back at the philosophical problematic, ground zero. The nature of the prover's 'compulsion' in practical work-sites, such as the classroom or the scientific laboratory is the (rejected) problem that would 'illuminate what goes on when we 'realize' something in the course of a mathematical proof' (p. 349). Because of his preferred choice of investigation, Livingston's definition of a 'proof' is strikingly reminiscent of semioticians' definition of a 'sign', the coupling of reflexively constituted signifier/signified-types, practical extensions of each other:

> 'In contrast to this, a proof – the pairing of proof-account and lived-work – is a whole subject – in fact, a social object [...]
> In the presence of the social object – the proof – a prover can examine its proof-account to see if, practically, everything is in it and if it is a proper and practically precise description of the lived-work of the proof' (p. 104).

Livingston' method leaves no room to any possibility other than that the prover (in his book, himself) not only can, but will, determine the adequacy of the work of proving by means that are local yet rationally compelling, worked out yet powerful enough to send us to the a-theoretical, original mathematical scene. The problem again is that such a solution seems to imply that the terms that constitute the proof are reducible to each other, that the adequacy of their mutual reference is *necessary*, not conventionalised; in order to study the latter we need to 'follow actors' and consider how 'reducing' something to something else is a matter of 'translation' (Latour, 1987, 1988). Unfortunately, Livingston' plea for the observation of the 'work' of producing a proof does

not accompany the empirical investigation of a work-site for the production of such activities. We are taken on board only to follow Livingston's minutely elaborated relation between graphic displays and accounts, but no actual problem-solving process with other social actors are involved in his accounts; no observable, witnessable learning installations are at display apart from the author's own thinking strategies and specialist musings. 'The pure abstraction, the 'work-site', is preferred to any of the concrete instances that might fall under it' (Bloor, 1987: 352). In the next two sections, we see how psychology, while still largely committed to a naturalised understanding of mathematics, pursued some of these problems in other terms.

2.3. Piaget and the genesis of number in the child

Historical research on the study of mathematical thinking in psychology and pedagogy would necessarily cover, amongst other works, Gestalt psychologist Max Wertheimer's *Productive Thinking* (1961) and educationalist Z.P. Dienes' (1960) 'new mathematics', a child-centred educational proposal that privileged learning through games and 'constructive' activities. Dienes popularised the use of concrete activities with material 'blocks' that embodied mathematical structures, that is, which could be so arranged and combined as to be read into *as* mathematics. The most influential work on children's development of mathematical concepts was, however, Jean Piaget and Alina Szeminska's *The Child's Construction of Number* (originally published in 1941). This book consolidates Piaget's line of argumentation on the 'operatory'[11] organisation of abstract thinking (Sugarman, 1987), replacing his previous interests on sensory-motor adaptation and the emergence of verbal cognition in the first two years of a

---

[11] Operatory thinking, as Piaget meant it, proceeds according to tautological rules of reasoning in which actions can be logically undone by applying an equivalent 'reverse' procedure. Such actions are abstract and above the level of experience; they are pure operations, not sensorial data. As Piaget pointed out in the foreword to the 'number' book, 'it now remains, in order to discover the mechanisms that determine thought, to investigate how the sensory-motor schemata of assimilating intelligence are organised in operational systems on the plane of thought. Beyond the child's verbal constructions, and in line with his practical activity, we now have to trace the development of the operations which give rise to number and continuous quantities, to space, time, speed, etc.' (1971: vii).

child's life. For Piaget, the construction of the number – and therefore of logic – is akin to the use of language and practical activity. It is in that sense that the 'number' book is crucial in the development of the new 'operatory' theory.

Piaget's work has established several of the main themes further appropriated by researchers interested in the cognitive and educational investigation on mathematical thinking, although Piaget himself has rarely addressed the question of mathematics education[12]: the conservation of continuous and discontinuous quantities, relations of class and order, the understanding of the cardinal number, additive and multiplicative structures, etc., were among his preferred topics. In that volume, Piaget argued that the construction of *number* is correlative to the construction of *logic* by children, resulting from the synthesis between *class inclusion* (A+A'= B; B [A+A']+B'= C, etc.) and *order* (n+1), or in other words, from the 'reciprocal assimilation' of these logical schemata (Kamii, 1986). As a matter of functional psychology, the logical intuition of the real is 'constructed' as the cognising subject acts over its environment, recognising, comparing, classifying. Number is a matter of abstracting the relations between 'quantities', and therefore is originally grounded on the activity over a quantifiable reality. In that sense, Piaget's solution is largely one of 'isomorphism' between the structure of the world and the structure of the mind.

He stressed 'that a pre-numerical period corresponds to the pre-logical level' (Piaget: viii), although it is evident that at the 'pre-logical' stage children already know and use numbers (Hughes, 1986, Sinclair, 1991). Such logical structures would be implied in children's real understanding of the numerical system and its operations, and were to be rendered visible through Piaget's

---

[12] From a critical point of view, Hughes (1986) comments on the relevance and status of Piaget's approach amongst educationalists, and observes the loss of influence Piagetian-based pedagogy had over British primary schools. Hughes argues that Piaget's epistemological approach does not have a clear relation with the actual teaching and learning of mathematics as they are practiced in primary schools, and that an (counterintuitive) implication of his theory would be that mathematics is 'easy', since it depends, at some extent, on the spontaneous character of an individual's learning.

method of interviewing children, known as 'clinical method'[13]. Piaget implies the mutual relation between logic and mathematics at the same he dissociates them from language and social conditions in general. *Actual* understanding is clearly not to be confused with the 'mere' use of symbols. For Piaget and Szeminska, the use of numerical signifiers by children cannot be taken as a sign of the mastery of number, but simply as 'recitation' without 'meaning'. Logical structures are on the basis of what can properly be called operational knowledge and are constituted out of what Piaget called 'reflexive abstraction'.

Piaget describes two fundamental kinds of abstraction, that is, of *representation* of experiential objects. First, there is the *empirical abstraction*, a reproduction of sensorial data; the representation of size, shape and colour, for instance, is an outcome of this 'function'. The second kind of abstraction concerns the logical properties that can be derived from the relations between objects. Piaget called this kind of representation 'reflexive' because its *referent* does not exist as a physical object, but as a 'mental' invention. 'Number' is a good example of what a reflexive abstraction represents. It is an ideal object, a synthesis that is 'itself numerical, because it turns into new properties strange to the initial groupings: the most important is the replacement of the tautology A+A=A by the iteration A+A=2A' (Piaget and Szeminska, 1971: 15) [14].

Kamii (1986) points out that Piaget has distinguished three forms of knowledge. She refers to the domains of physical, logico-mathematical and social knowledge. 'Physical knowledge is the knowledge of objects of external reality' (Kamii, 1986: 14, my translation). The 'method' used in this kind of knowledge is 'observation' or 'empirical abstraction', as I referred previously.

---

[13] Piaget's clinical method, best represented in his famous tests of conservation of volume, consisted of a cross-examination of sorts, where children where ranked in a non-conservative/conservative continuum, with changing conditions in the interviewing determining where she should be placed in the results (e.g. a child answers question A in a way that shows conservation till question B troubles her).

[14] My translation. It is worth saying that the English translation from the original French omits this part from the foreword. It is, therefore, a translation from my Portuguese version.

Logico-mathematical knowledge is 'relational'. Logical 'relations' are not 'things' themselves and, therefore, cannot be assimilated by empirical observation. For example, when someone refers to an object as being 'larger than' another, we can reasonably consider that 'if the person would not put the objects within this relation, for him/her the difference would not exist' (Ibid: 14).

'Social' knowledge refers to the set of conventions used by the members of a common culture, like language and mathematical notation. Thus, *um, dois, três*, for Portuguese speakers, *one, two, three*, for English speakers, and *une, deux, trois*, for French speakers are labels conventionally attached to their (accountably universal) meanings. Kamii makes two observations with regard to this sort of knowledge: (1) social conventions are arbitrary, that is, it does not maintain any logical or necessary relation to whatever object it represents, whereas the 'concept' of number does; (2) transmission of social conventions cannot be taken as an explanatory principle for the acquisition of number concept, since it cannot help cognitively immature children; children who have not abstracted the principle behind numbers cannot be taught through the language of numerals alone, whereas 'the child that has already constructed logical-mathematical knowledge about the seven or eight is able to represent this idea with symbols or signs' (Ibid: 15).

The 'construction' of number deploys the skill of abstracting the idea of 'unit'. The 'first act of abstraction' (Steffe et al., 1983: 1), that is, the sensorial 'knowledge' of figural patterns, is the beginning of what leads to reflexive abstraction. While this argument is not unreasonable (see Rotman, 1987, for a comment on the de-*iconisation* of numerical symbols and the (problematic) emergence of the meaning of *zero* [0]), it seems to confound formal, historical and psychological levels of analysis. Piagetian constructivism quite often portrays children development and the history of scientific knowledge as 'isomorphic' (see Piaget and Garcia, 1983; Garcia, 1996).

Piaget's assumption of a functional continuity between sensory-motor activity, on one side, and symbolic cognition, on the other, has been largely influential

on the subsequent theorisation of the acquisition of mathematical skills. Indeed, this has been one of the main implications of the explanatory use of the concept of 'reflexive abstraction'. It presupposes the projection of an axiomatic basis underlying children's physical and perceptual actions, at the same time those basis reflects properties of the real world, or to use Piagetian language, a capacity to 'accommodate' them. For example, much has been told about infants' 'knowledge' of *numerosities* before they can properly understand number. Bryant (1974) discusses the role of perceptual judgements in four-year-olds' evaluation of quantities in experimental situations. His question was whether children would be able to react to numerical differences prior to the mastery of counting, in a sort of perceptual estimation sometimes referred to as *subitizing* (Von Glasersfeld, 1982; Durkin, 1993; Fayol, 1996). One of the early supporters for a positive answer to this question was Alfred Binet, the pioneer of intelligence tests, who suggested that 'before knowing how to count a child accustoms himself to the idea of numbers. He knows what it is to have many marbles or very few of them. He, therefore, makes use of an instinctive and probably unconscious numbering system before becoming acquainted with verbal numbering which we are charged with teaching him' (Binet in Bryant, 1974: 109). Bryant addresses the 'unconscious/conscious' distinction made by Binet in terms of the difference between a *relative* and an *absolute* code in the context of the discrimination tasks. He suggests that in using a *relative* code children can 'realise' the numerical difference between two sets of objects without being able to report such difference in terms of its actual numerical values; this would indicate the presence of a primitive *estimation* skill grounded on visual perception. The *absolute* code, on the other hand, would refer to numerical competence *strictu sensu*, like counting, which entails applying rules of class inclusion, sequencing and cardinality. In this case, children should be able to compare two sets of objects, and then estimate the exact difference between them. It is not my purpose here to question Bryant's results in the experimental situation. I want to note the remarkable accomplishment of the perceptualist assumptions of Bryant's work that the notion of 'code' itself does not have any specificity in terms of *system* of *method* for coding: either 'children' *perceive* something, in which case they do not really know, or they *know* it, in which case they are

only using the *knowledge* in question as *we* know it (and as nature *is!*). Bryant has asked 'if he [the child] is aware of this difference in number, is he aware of it in a relative or in an absolute manner?' (1974: 109). There is no bodily, material or symbolic mediation to be considered in the way the question is framed. *What* actual code or codes (or methods) are being deployed? *How* are they being used? No doubt such trends in developmental psychology are (optimistically) complimentary of human capacities to *realise* things, but 'being aware' either way is also a nod to a realist conception according to which the very essence of mathematics escapes the work of 'fixing' its force and universal appeal. In the following quote the quality of 'number' is portrayed as in the object itself:

> 'Numbers - that is to say, whole numbers - arises from the recognition of patterns in the world around us: the pattern of 'oneness', the pattern of 'twoness', the pattern of 'threeness', and so on. To recognize the pattern we call 'threeness' is to recognize what a collection of three apples, three children, three footballs, and three rocks have in common' (Devlin, 1998: 13).

Gelman and Galistel (1978) offer a similar 'perceptualist' argument. The authors state that 'when the young child encounters numerosities that he can represent numerically, he can bring to bear reasoning principles of surprising sophistication. But these reasoning principles do not come into play in the absence of specific numerosities. Furthermore, the numerosities (sets of one or more objects) must be small enough for the child to determine the number that represents them. A set of 57 toy mice, although indisputably a numerosity, does not lead to numerical reasoning in the 3-year-old child' (Gelman and Galistel, 1978: 51). Gelman and Galistel treat 'numerosities' as real objects out there. Again, despite the emphasis on the *activity* of the knowing subject, Piaget's constructivism treats the world as 'knowable' (Walkerdine, 1988), which led some scholars to dub him 'psychologist of the real' (Rotman, 1977).

A major problem with the Piagetian approach is the 'role' attributed to social context in cognitive development, or in fact the absence of one. For example, the classroom, as a mediating device, has no place in Piaget's explanatory

system. In the same way, language and other social activities do not find way as active developmental forces in his theory (Walkerdine and Sinha, 1978). In the last 25 years or so a growing body of literature in educational psychology and cognitive development has readdressed the balance towards the social and the cultural in the analysis of mathematics learning by privileging more interactional, and especially discursive, approaches.

2.4. The discourse of mathematics education research

Parting company with Piaget, in this section I highlight some trends in the psychological and educational analysis of mathematics 'cognition' for which the concepts of *operatory* and *semiotic* go hand in hand. Piaget's theory has purified their applications, considering the former 'meaningful' and the latter 'representational'. This 'fallacy' has found support, in the last 15 years or so, under the argument that the same mathematical 'invariants' (cardinality, order, reversibility) can be found across diverse activities and cultures for which Members use divergent strategies and representations to cope with 'problem-solving' activities (Carraher et al., 1988; Nunes, 1992; Vergnaud, 1991)[15]. I do not mean to categorically deny that assertion –the possibility of its validity according to some rule – but to address its implication that whatever mathematical representations are at stake we have two processes to explain: a *formal* one and a *cultural* one. A non-metaphysical analysis of mathematics as part of society's fabric must, then, be able to deconstruct the assumptions behind the 'gatekeeping logic' for which the formal part is a dividend of the 'aesthetic' qualities of the human mind (McHoul, 1996). Most particularly, I want to focus on approaches that have taken the notions of culture and discourse into their agendas.

Piaget had focused on the child's first years of life in order to argue that the use of numerical symbols cannot be translated into real understanding of the

---

[15] It has been claimed, for example, that *illiterate* persons make use of the same mathematical (and linguistic) invariants in their variably structured and represented activities as literate persons (Labov, 1972; Nunes, 1992).

concept of number; the latter is grounded on the action over the real, not on the generalised use of linguistic categories. It is certain that activities involving numbers and operations are part of everyday life in the most diverse contexts. Classroom mathematics might put new 'cognitive' demands on schoolchildren's side, but in no way constitute the beginning of their mathematical 'enculturation'; numerical signs – spoken and written – are already part of the experience of pre-school children: reference to time, hour, age, measure, house and telephone numbers, etc., are all bound to the objects surrounding them, where the parents or caretakers work as organisers of the ordered use of such representations (Saxe et al., 1984). In that context, the analysis of the language used between teachers or caretakers and learners constitute one the most interesting features concerning the 'visibility' of mathematics, or, as we have seen before, of the reflexive self that Brian Rotman describes as the mathematical *Subject*. For children arguably are at the limits of the semiotic comprehension attributed to the *Subject* of mathematics, they face all sorts of linguistic obstacles while learning the 'language of mathematics', such as ambiguities and absence of trivial uses of metaphor (Durkin e Shire, 1991; Walkerdine, 1988).

Voigt (1992; 1993) address the pragmatics of mathematics learning in the classroom as a question of 'negotiation of meaning'. From a practical, educational point-of-view it is clear that the communication between teachers and young learners cannot be organised and regulated on the basis of inferential criteria of the highest order, of the sort that the *Subject* contemplates previously to sending the *Agent* to blindly carry out his deliberations on the proper course of action, in the same way that children do not learn how to talk by learning first of all how to conceive the 'grammar' of their mother tongue; rather, it is assuming that children can participate and respond in language-like, meaningful ways, that parents allow them to enter such a symbolic order (Bruner, 1983; Rommetveit, 1985). Opposite to the Piagetian theoretical recommendations, it is becoming more and more accepted amongst educationalists that the *discursive* negotiation of actions and meanings are the primary source of mathematical knowledge for children (not secondary, or merely 'representational'). Development-wise, it corresponds to

saying that the *Agent* and the *Subject* are created at the same time, with no primacy conferred to the *Subject* over the *Agent*. Voigt argues that school mathematics presupposes, therefore, *two* rational bases: (a) the mathematical *modeling* of the experience from the relations between real objects, and (b) the inference rules within a particular mathematical model. As *modeling* faces the interactional problems mentioned above, the potential conflicts between social actors cannot be disambiguated by calling on more strict inferential rules, but only 'negotiated' in ways that establish accountably 'shared' new grounds for subsequent teaching.

Such grounds are routinely accomplished, 'for all practical purposes' (Garfinkel, 1967), as can be attested by the fact that classroom lessons and particular instruction sequences are skillfully brought to closure every school day (Macbeth, 2000). The terms in which it is done is precisely what is open to theory and analysis. Pimm (1987) argued that the teacher is not only a mediator between the pupils and mathematics, but a 'native speaker', a linguistic model to be copied in order communicate and discuss mathematics. In an overview of their work on mathematics education (and on the concept of number, more particularly), Perret-Clermont, Perret and Bell (1989) conclude that pupils seem to respond to specific commands in instructions sequences that allow them to go on according to what is acceptable in the classroom, looking for efficient strategies to avoid failure; rather than controlling any underlying principle, their knowledge of the numerical system seem to be pragmatic, constituted by a set of 'action principles' (p. 21) that does not necessarily correspond to the logicised type of knowledge expected from teacher and curriculum makers. Still in the wake of the pragmatic turn in mathematics education research, Cobb et al. (1995) talk about 'socio-mathematical' norms in the classroom. Their 'socioconstructivist 'analysis focus, like in Voigt's, in the establishment of consensual meanings, trying to encompass theoretically both the 'cultural' and the 'individual'. As Lerman puts it: 'Learning is about becoming, it is about participation in practices (…) But people react differently in those practices, and perform their own trajectories through them. In arguing that people are *discursively* constituted the individual does not disappear; instead, the notion of individuality requires a

reinterpretation' (2001: 88, emphasis added). Despite a remaining conceptual interest in the contribution of the individual learner to the process of 'meaning-making' in mathematics as *endogenous*, these approaches highlight the importance of social and pragmatics factors in a field marked by the overdetermination of ideal, transcendental principles.

The focus on *intersubjectivity* has been proposed by those who advocate a thoroughly socio-cultural analysis of mathematical learning. Following Vygotsky and his plea for the analysis of development as the internalisation of cultural systems of symbols (see chapter 3), Lerman (1996) criticises the accountability of academic mathematics and of the research on mathematics cognition as *reified*, responsible for the epistemological underpinnings of educational and developmental studies whereby children's competences are defined negatively, in relation to forms of knowldge they still do not possess, or to how distant they are from such knowledges. Intersubjectivity pressuposes the creation of dialogical spaces of communication and public accountability, where participation can distributed and sanctioned.

The emphasis on the control of the participation in the mathematics classroom have joined, to a large extent, the critique to the epistemological conceptions of knowldege, and raised the question of cognitive activity as the entry into the symbolic orders of society through the participation in organisational, technical and power relations. Piaget had developed a rather cryptic theory of the continuty between the sensorimotor adaptation during the first two years of life and the emergence of the symbolic function (1951), that is, of how the action schemes that constitute the former are converted into the possibility of performing mental representations. The plea for the investigation of discourse and social interaction as constructive (Edwards and Mercer, 1987) confer a remarkably discontinuous character to symbolic cognition, and questions the very basis on which developmental psychology is built as a 'science', that of the figure of the 'child'(Walkerdine, 1984; 1988).

In *The Mastery of Reason* (1988) Valerie Walkerdine investigates the mathematics classroom as a discursive practice, observing how different styles

of *subjectivity* are produced in different, and sometimes conflictive, practices of 'signification'such as in the household and the school – where different relations between objects, words and actions are established. Walkerdine draws on post-structuralist lingustics, pychoanalysis and Foucault's theory of power relations to produce one of the most radical works in contemporary developmental psychology. She proposes that 'mathematical' rationality is accomplished at the cost of the exclusion of multiple signification in language, by the supression of the 'vertical' axis of linguistic substitution by which new possibilities of translation and meaning are open, in a word, of *metaphor*. It is worth nothing that Walkerdine's semiotics, following psychoanalyst Jacques Lacan (1986), emphasises the primacy of the 'signifier'over the 'signified', regarded as an effect of the way signifier 'chains' work. This is a step further from both Peirce's semiotics (in which a signifier is *part* of a 'sign') and Saussure's semiology (in which the *signified* was regarded as primary, even if it is not to be confounded with a realist notion of 'referent'). As our immediate contact with 'signs'can only be through their signifiers, whose continuous substituion afford the emergence of signified 'areas', the semiotic chain is arguably a 'signifier chain' (Eco, 1991). For Walkerdine, formal school mathematics supresses this combinatory axis in favour of a purely contiguous and axiomatic discourse, that is, that of 'metonymy'. Walkerdine argues that while sentences such as 'two plus two makes four' exhibit the built-in character of natural language by which interpretation and practical epistemologies (e.g. 'makes' as physically assembling, or creating something new, as opposed to establishing logical *equivalence*) can run freely, its convertion into things like '2+2 = 4'cannot, as they establish the primarily abstract, detached character of the sentence *as* mathematics. The latter, the mathematical sentence, purports to mean strictly an *operation* (Otte, 1991).

Post-structuralist work such as Walkerdine's has questioned not only research on developmental psychology, but all the conceptual apparatus of psychology as a science; concepts such as 'rationality' and 'development' have been aimed at and described as 'fantasies' of control, a reflection of 'phalocentric' society (Henriques et al., 1984; Walkerdine, 1988). Its arguments revolve around the historical formation of discourses and power relations, in which different

'subject positions'are constructed as signs (e.g. the child, the pupil, etc.), and have furnished the basis for the critical disenchantment and dispair with the so-called 'Modernity' (Latour, 1993).

Walkerdine takes the case of formal, academic mathematics as an example of the production of a powerful 'regime of truth'(Foucault, 1972), a discourse shut to multiple signfication that underpins the the production of 'scientific' rationality as we have been discussing all along this chapter. According to her, mathematical forms are devoided of meaning, which is precisely what allows mathematical 'discourse' to be superimposed to and read from other discourses (Walkerdine, 1988). However despite of its political and critical accountability, some aspects of Walkerdine's analysis have incurred in solutions that are, in terms of analytical scope, much similar to those of Rotman's semiotic analysis. Above all, in a pre-defined conception of language as a closed semiotic system, whose variations help defining which aspects of 'society' – thus defined by the workings of language – are in operation.

As we will see when I criticise the assumptions behind 'conversation analysis' of classroom education in the following chapters, such a conception constitute a technology of analysis of language that is either 'realist' or simply refuses applying its constructivist philosophy onto itself, which would imply, symmetrically, dealing with how this particular form of social analysis is a 'construction', not more or less real than the scientific truths it analyses (Latour, 1993); or rather coming into terms with the way the rhetoric of language studies came into play (Billig, 1999). Because of this profound asymmetry, that is, because the fields of discourse studies, on one hand, and mathematics, on the other, are established academic disciplines with defined jargons and set of observables, the observations made about the latter (mathematics) through the former (discourse) seem to be condemned to triviality, in the sense that it shows nothing more than the application of a 'method' to *another* topic. In a critical editorial to a volume on the use of discourse analysis in mathematics education research, Morgan (2000) recognises that 'It is difficult to see in many of the chapters how

communication in mathematics education is distinct from communication in other subject areas other than in the apparently incidental topics being discussed in the classroom' (p. 96). Besides, Walkerdine's work is fully immersed in the psychological debate, albeit a critical one, and rather than giving up developmental psychology as an 'illusion', she uses critical sources to make that discipline her own; the 'truth' of development is simply displaced from the traditional regulatory mechanisms that scholars like Piaget – whose work Walkerdine knows well and disputes – talk about, to the regulations of 'discourse'.

An activity-based, cultural psychology of mathematics learning derived from the work of Vygotsky (1978) is another major approach to dominate the publications in the area. *Discourse* is recontextualised in terms of language use and social interaction, rather than self-contained sign systems. 'Developments in the last 25 years with regard to mathematics education (...) reinforced the call for a more discursive approach, taking into account the pupils' own understandings of a mathematical problem (...) as well as doing justice to the fact that mathematics is a cultural activity that emerges out of sociocultural practices of a community' (Van Oers, 2001: 66). Also, the study of problem-solving abilities in the laboratory was slowly substituted, as Van Oers indicate, by an interest in more spontaneous, naturalistic situations, not only in the classroom but also in various cultural contexts of mathematical activity (Carraher et al., 1988; Lave, 1988; Saxe, 1991; Scribner and Cole, 1981), emphasizing how the performance of the same invariant cognitive functions can show a marked performance in non-school situations (Sfard, Forman and Kieran, 2001).

During the last 15 years or so, a more pronounced focus on language and discourse has taken place amongst educationalists and psychologists interested in the nature of mathematics learning. 'In the domain of mathematics education, the term discourse seems these days to be on everyone's lips' (Sfard, 2001: 13). Not rarely, this interested have turned into a review of the debate of the nature of mathematics itself, and therefore to characterise mathematics *as* a language (Pimm, 1987; 1994; Pinxten, 1994; Walkerdine,

45

1988). The comparison between the rigid *grammar* of mathematics and the fuzziness of natural language had been one of the core interests of the logicians and mathematicians from the late 19th century to Wittgenstein's *Tractatus*. Logic and mathematics were envisioned as the perfect philosophical language, able to overcome the ambiguous and indexical character of everyday discourse (Garfinkel and Sacks, 1990).

Following Vygotsky, this emergent tradition has addressed the questions concerning the way inference-making processes in mathematics are mediated by discursive activity and the negotiation of meaning in social interaction (Edwards and Mercer, 1987; Lerman, 1996; Mercer, 1995; Sfard, 2001; Voigt, 1992). Most of the researchers in this area have one way or another subscribed to a program of investigations for which the distinction and interrelations between propositional and pragmatic knowledge, mentioned above, is a basic guideline for social and psychological research. It has been suggested, for example, that the specialty of mathematical discourse rests on (1) its reliance on symbolic artifacts as mediators for public communication, and (2) the conventionalisation of 'meta-rules' to regulate mathematical communication (Sfard, 2001). 'The meta-rules are the observer's construct and they usually remain tacit for the participants of the discourse' (Ibid: 13). It is worth noting that Vygotsky himself had distinguished two different functions of speech, 'representational' and 'communicative'. While the former designate symbolic 'tools' that constitute the topic of one's 'thoughts', the latter refers to ways used to go about the topic in always that are publicly intelligible and acceptable. 'While tools are the shapers of the content, that is, of the object-level aspects of discourse (...) meta-discursive rules are the molders, enablers and navigators of the communicational activities' (Sfard, 2001: 28). There is, of course, a marked rationalist thrust in Vygotsky's idea of a 'representational' function of discourse, as we have seen when discussing the realism of classical philosophical thinking, for which mathematics has a pre-semiotic nature.

Socio-cultural studies in mathematics education suffer from putting very little emphasis on analysis. The articulation between the levels mention above is rarely articulated empirically, and the few empirical studies there

are quite happy to give *ad hoc* support to the theory (à la Piaget) that mathematical invariants are hardly constituted directly either by the arbitrary symbols used to convey them or by the meta-rules used to convey their sense to other people (Carraher et al., 1988; Saxe, 1991). The latter become the proper arena of discursive studies, in the form of the mechanisms of interaction by which cultural knowledge can be shared and internalised by the subject, and if Piaget misses the *social* from his explanatory accounts, socio-cultural psychologists can be said to miss the *technical* from theirs! In other words, just like the philosophical, sociological and psychological accounts of mathematics and mathematics learning often 'miss' the social and material technologies and trajectories of its production, the studies of the 'pragmatics' surrounding it are at risk of turning into an 'axiomatics' of social interaction stripped from its living topics and concrete semiotic actors. How such a purely interactional, and particularly educational, axiomatics can be envisaged is the topic of the next chapter.

CHAPTER 3

Order installed: The classroom as an analytical object in psychological and social research

3.1. The early origins of the modern classroom: moral economy, monitorial schoolrooms and disciplinary practices

The classroom is such a commonplace in our modern schooling systems that is easy for researchers interested in learning to take it for granted, even to naturalise it. Not doing so would mean to engage seriously with its historical set up as a valuable source of information about what kinds of things classrooms are, or were designed to be. Of course, 'what classrooms are' can be dealt with in different ways, such as asking about its economic and ideological origins (Hamilton, 1980), its relations with society at large, particularly with the reproduction of the class system (Bernstein, 1971), and its workings as a disciplinary 'machine', a technology of control that mobilises space, objects, knowledge and people (Foucault, 1977), etc.

Theoretical psychology, for example, has been 'analytically' interested in the classroom since Vygotsky and his program for the study of the cultural and educational basis of development in 1920s and 1930s (1978; 1987), in which the classroom featured as a somewhat 'explanatory' developmental factor (see section 3.2). Such move has naturalised it as a causal factor of sorts, a social 'opportunity' for people with different experience and knowledge to engage in 'intersubjective' exchanges. In research terms, this has implied a strong 'technical' separation between the historical, social and psychological 'facts' of schooling. In this chapter, I go on to suggest that whether one starts investigating the classroom as the constitution of disciplinary uses of the body, time and space (Foucault), a site for a specialised form of talk-in-interaction

(CA) or as a context for the emergence of new forms of 'intersubjective' activity (Vygotsky), it is undeniable that the study of the classroom 'machinery' in its own right has much to inform the study of 'learning' as a public domain of practices; even more interestingly, we can use the same reasoning in later chapters to discuss how the classrooms studied found and maintain 'epistemic' contracts between its members, that is, how it deals with questions of knowledge and cognition.

Here, the interest in such ideas, as well as in some liberal, 'Enlightened' educational reforms of the 18th and 19th centuries is due to the extent in which the school, via the creation of a 'classroom system' (Hamilton, 1980), was formulated as a site for the multiplication of discipline and learning technologies directed not at one, but at several (classed) groups of pupils at the same time. Inquiry on non- or pre- classroom educational models becomes then necessary in order to fully grasp the classroom's meaning as a modern social technology. As Hamilton rightly points out, 'the widespread penetration of the classroom system had another important ideological effect. It obscured the fact that, before about 1800, schooling had been organised around a quite different vocabulary, and quite different assumptions, resources and practices' (Ibid: 282). Such an inquiry points to the fact that psychological and pedagogic epistemic models of learning or views on 'mankind', e.g. cognitive, can be respecified as specific socio-historical contracts that go back and beyond its normative formulations in social and psychological research. They are, themselves, models of/for social order.

Take, for example, the much-heralded psychological literature on 'situated cognition' in the late 80's and early 90's, and its pedagogical counterpart, the study of learning in terms of 'apprenticeship' (Brown et al., 1989; Carraher et al., 1988; Lave, 1988; Lave and Wenger, 1991). Such studies often emphasizes the differences between 'street' and 'classroom' forms of learning, where the former would display all the characteristics of relevant, usable knowledge and the latter the sterile, useless recourse to formulaic, high-cultured, scientific discourse detached from actual usage in worldly practices. Macbeth (1996) analyses the claims of the pioneer program on 'situated cognition' devised by

Brown, Collins and Duguid (1989) and its plea for 'authenticity' in schooling. This program is predicated on the assumption that classroom activities are (or should be) a form of cognitive apprenticeship, an (to be) 'authentic' enterprise able to juxtapose 'classroom practices to real wordly practices, and classroom learning to informal and/or apprentice structure of learning' (Macbeth, 1996: 273). However, in order to understand how 'apprenticeship' can constitute a practical (as opposed to a 'normative', or 'epistemological') model of learning, one which is at the same time previous and/or external to current formal practice of schooling, we should be able to trace its work in specific learning contexts. Michel Foucault (1977) contrasts the organization of schools in the 17th Century based on 'apprenticeship' practices with the modern classroom. He points out that:

> 'We find here the characteristics of guild apprenticeship: the relation of dependence on the master that is both individual and total; the statutory duration of the training, which is concluded by a qualifying examination, but which is not broken down according to a precise programme; an overall exchange between the master who must give his knowledge and the apprentice who must offer his services, his assistance and often some payment. The form of domestic service is mixed with a transference of knowledge' (p. 156).

We can subvert many of the characteristics above in order to find out (or to find again) crucial aspects about the way the modern classroom operates, such as the accountably 'facilitating' task of the teacher (as opposed to dependent, authoritative, and transferable knowledge production), the overwhelming presence of the examination and the correlative invention of the child's mind as a field of studies and, of course, the existence of progression through the classing system itself. Part of the criticism towards programs like Brown et al.'s stems from its calling as a 'critical', interventionist, prescriptive enterprise, and to its analytical vocabulary as the (unformulated) formulation of 'moral orders' (Macbeth, 1996). As we saw above, apprenticeship-based learning, with its appeal to 'situatedness' and its centripetal, 'periphery-to-center' character (Lave and Wenger, 1991), is a socio-historical feature of learning practices that is strange to the practice one observes in the modern classroom. Macbeth reiterates that in that (critical) context of Brown et al's

text, 'situatedness comes into view as a potential and ignored resource for leveraging a long-standing program of educational research and reform' (p. 273).

Another example is that of Lave and Wenger's (1991) concept of 'legitimate peripheral participation'. One of the central and much repeated points in their ethnographic research is the relevance that participating and, therefore, acting and taking responsibility, has as a central role in the practices they analyse. Like Brown et al. the authors argue in favour of a non-personal, non-contemplative, non-empiricist concept of learning as 'apprenticeship', that is, as 'master-apprentice' relation. They see this notion as a powerful epistemological principle: 'participation in the cultural practice in which any knowledge exists is an epistemological principle of learning' (Lave and Wenger, 1991: 98). Again, the characteristics of a suggested 'moral order' act as an epistemic threshold on the basis of which *all* learning is to be investigated. Like in the paper by Brown et al. discussed by Macbeth, Lave and Wenger's analysis deliver a sense of frustration with the 'moral orders' of schooling, which are, according to them, predicated on a mistaken *view* of learning; namely, 'on claims that knowledge can be decontextualised' (Lave and Wenger, 1991: 40). Lave and Wenger's statement above is, at the same time, a critique and a prescription.

Historian of education David Hamilton reports that the term 'classroom' appeared for the first time in English sources in a minute of the Faculty of Glasgow University in the late 18th century (Hamilton, 1983), and that the ideas of then contemporary Glaswegian scholar Adam Smith particularly are pivotal in understanding some of the ideological underpinnings of the classroom system as we know today. Renowned for his work on political economy ('The Wealth of Nations', originally published in 1776) Smith had also developed a detailed treatise on 'ethics' that can be seen as a company to his more prestiged book. Hamilton goes on to argue that Smith's 'A Theory of Moral Sentiments' (originally published in 1759) constitutes the ethical and philosophical basis on which the Glasgow school will develop its educational proposals into the 19th century, with names like George Jardine and Robert

Owen, and in some aspects, with the 'monitorial' reformists like Adam Bell and Joseph Lancaster.

Hamilton's argument, focusing on the link between the classroom system and Smith's formulation of a theory of ethics, points out to the case made for the benefits of mass teaching, and can be specified as stating that 'the production and distribution of educational 'goods' or 19th-century popular schooling' (Ibid: 282) describe a new kind of 'moral economy'. According to Hamilton, Smith's system of ethics was built around the principle of 'sympathy' or 'fellow feeling' (Ibid: 283), a concept that finds echo in his markedly liberal economic cosmology, for which self and collective interest are regarded as intrinsically related. It also became a natural legitimation for later developments of simultaneous instruction. Smith considered sympathy to be the moral bond between all members of society, but unlike some philosophers, who saw it as an 'essence', he saw it as belonging in a kind of moral economy. 'In Smith's revised usage, sympathy became something that is shared, like common property (...) To the extent, therefore, that individuals were in sympathy with each other, they could be regarded, in Smith's terms, as morally equal (cf. the presumed economic equality of buyer and seller under conditions of free trade)' (p. 289). What Hamilton is arguing is that the ideas on 'division of labour' to be found in *The Wealth of Nations*, and on 'fellow-felling', developed in *A Theory of Moral Sentiments*, were to support the rationale found in the arguments for 'classing' in education. Smith's 'collective' yet 'market oriented' philosophy (Ibid: 288) implied that the ranking and clustering of individuals in a classroom facilitated learning just as the coupling of moral sympathy and division of labour contributed to the advancement of society.

Other Glaswegian scholars, such as George Jardine and Robert Owen, set in place the continuation of the case for 'simultaneous instruction' based on Smith's precepts. The instructional blueprint for the Glasgow scholars was a variation of the medieval lecture. The format of the lecture was the 'dictation', a markedly authoritative ('catechist') discursive design based on repetition; besides, it was given in Latin, as opposed to vulgar, street-bound language.

Jardine' methods represented a commitment between lecturing and tutorial procedures towards the use of vernacular language (for example, the use of a 'extempore system of questioning', in English, Ibid: 291), or "easy dialogue' between a teacher and a group of 'not more than thirty or forty students" (Ibid: 292). On the other hand, scholars like Robert Owen started promoting the relevance of 'learning' (by opposition to 'virtue' and 'discipline') to the elementary education of the lower classes. Owen's radical political philosophy embraced the argument, in the wake of Smith's ethics, that the happiness of the individual and that of the collectivity is part of the same social project. However, Owen vividly discussed the distribution of 'rationality' as an educational good for the lower classes in a way that was new, where an emphasis on rational 'understanding' of such moral and social contracts was placed. 'Unlike the conservatives of the day who assumed that the virtue of the working class could be assured through forms of bodily discipline, the philosophic radicals claimed that a more 'durable' character would be formed when, in Owen's words, 'the mind fully understands that which is true" (Ibid: 293). Interestingly, Hamilton reports that *because* of Owen's defence of rationality as an educational goal, mentalist concepts such as 'understanding', 'perception', 'attention', etc., found their own way into his educational rhetoric. We shall go back to the topic of 'understanding' and 'rational' accountability in learning in chapter 6.

If the discussion above can be traced back at least to Adam Smith's conception of a moral theory of 'sympathy' between society's members, one of the most well known educational reforms of modern times is that of the 'monitorial system' proposed by Joseph Lancaster and Andrew Bell in the early 19th century. The monitorial system comes chronologically after the development of the Glaswegian Enlightenment, and it is usually considered less progressive (Hamilton, 1980), having had little regard, like in the case of Robert Owen for example, for questions concerning the distribution of 'knowledge' to the public at large. It allegedly originated as a form of 'charity schooling' when Andrew Bell, a superintendent of East India Company's Orphanage in Madras, started employing pupils as teaching assistants (Ibid). By the end of the 18th Century, charity schools (originally, as Hamilton states, part of the 'craft' economy of

the 16th and 17th centuries) were forced to cope with new urban demands, largely due to industrialization and its wider economic consequences, such as wage labour and unemployment (with a consequent large contingent of indigent children), and were forced to find ways of keeping financially viable. Bell's contribution and his subsequent defence of the new school system in the language of the 'division of labour' finds echo in that context (Ibid.).

At the heart of Lancaster's and Bell's proposals was the principle according to which the school should be able to multiply its teaching resources by enlisting 'monitors', selected and rewarded according to a system of meritocracy. The monitorial system placed meritocracy and competition at the centre of its pedagogical practices, and is somehow indebted, ideologically, to Adam's Smith's conception of good competition (or 'emulation', Hamilton, 1980) as an integral part of his moral economy. The system had also assimilated the modern abandonment of 'catechism' language for the current use of vernacular discourse and the efficient disciplining of 'collectivities'. Lancaster wrote about the 'economy of time' necessary to the teaching of young children and the poor, and that the 'practical evidence' pointed to the fact that 'a very large number of children may be superintended by one master; and that they can be self-educated by their own exertions, under his care' (Lancaster, 1805). The school is to be divided into many classes, each of which has a monitor appointed to it. The duties of monitors are many; besides teaching, monitors were to be charged with tasks as varied as supervising cleanliness, enquiring after absentees, distributing books and collecting them after the lesson is over, etc. 'The word monitor, in this institution, means, any boy that has a charge either in some department of tuition or of order' (Ibid.). Arguably, all the attributions delegated to the monitor attest to the rising preoccupations with efficient governability, in which disciplinary practices such as mass education – as well as its ideological basis (Hamilton, 1980) – were modeled according to the one or other feature of the 'division of labour' system; in it, a relation of docility-utility of behaviour and its compartmentalisation is produced and monitored, and consequentially mechanised (Foucault, 1977; Walkerdine, 1988).

54

According to Lancaster's recommendations, the school's organisation into classes and the presence of the whole monitorial network was accountably central in facilitating learning. Another important recommendation for the work of teaching to take place was to level the proficiency standard of the students put together under the same tutoring. These themes will return in section 3.2 when we discuss Vygotsky's use of formal education as a theoretical and practical tool in developmental psychology. For now it suffices to say that Lancaster's (however unsophisticated) 'psychological' assumptions about the work of composing a classroom are thoroughly modern and away from the craft apprenticeship economy of the workhouses of the pre-industrial era (Hamilton, 1980). They also have a striking resonance with aspects of 20th Century pedagogical psychology (e.g. Vygotsky), so as to convey, for example, the notion that 'if the number of boys studying the same lesson, in any school, should amount to six, their proficiency will be nearly doubled by being classed, and studying in conjunction' (Lancaster, 1805). Vygotsky's notion of 'zone of proximal development', and its use in class composition, will rest on similar principles.

Lancaster is not entirely clear about how the methods for teaching can be effective and about what they consist of; how studying together may be of benefit for each individual member of the group and whether the class has a specific form of 'agency' or 'responsiveness' in its own right, as opposed to the individual pupil. In words others, he does not have a 'moral cosmology' as complex and elaborated as that of Adam Smith and his followers. In general, very little is described in relation of a system for *assessment* as well, although is worth remembering that neither psychology nor pedagogy were part of the regulative practices of education at the time. However, his interest in the teaching of the 'collective', and his support of it as a distinctive learning mechanism, is quite remarkable. Also, his support of emulation and merit in the context of classroom technologies set up for the collective participation are notable. There are a few suggestions on the work of organising the distribution of tasks, participatory turns and merit in the classroom that are worth mentioning. For example:

'Another method of teaching the alphabet is, by a large sheet of pasteboard suspended by a nail on the school wall, twelve boys, form the sand class, are formed into a circle round this alphabet, standing in their numbers, 1, 2, 3, &c. to 12 [...] the best boy stands in the first place; he is also decorated with a leather ticket, gilt, a lettered merit, as a badge of honour. He is always the first boy questioned by the monitor who points to a particular letter in the alphabet, 'What letter is that'. If he tells readily, what letter it is, all is well, and he retains his place in the class; which he forfeits, together with number and ticket, to the next boy who answers the question, if he cannot' (Lancaster, 1805).

Here we see some of the distinctive features of the classroom-as-machinery, or, as it will be conventionalised later in this chapter, as 'installation' (Macbeth, 2000). As mentioned before, the distribution of tasks, participatory rights and assessment reveal, as Macbeth puts it, the 'clockwork' aspect of instructional activities in the classroom. The quote above also suggests how contemporary discourse in the classroom involving the use of 'extempore' practices of conversation (as opposed to the 'lecture' model) was elaborated as the ideas on class teaching developed. It also shows how such classroom orders relate to the classroom as a 'site', as a composed space; in the classroom, material and literary technologies, as well as togetherness and spatiality, are operative conditions.

These aspects feature prominently in Michel Foucault's (1977) analysis of the classroom as a historical form of social technology. Foucault's rendition of the classroom in the context of the emergence of disciplinary practices in the 19th century is more familiar with Lancaster's monitorial system than with the Scottish *ideologues* of the 18th century portrayed by Hamilton. Therefore, Foucault places the analytical emphasis on time and spatial organisation in the classroom. He repeatedly describes those as integral to the composition of a 'learning machine', where the 'apprentice-master' model of the workhouses was superseded by the classroom's 'sympathy of numbers' (Hamilton, 1980; Macbeth, 2000) and its relation with the redistribution of space and instruction methods:

'By assigning individual places it made possible the

supervision of each individual and the simultaneous work of all. It organized a new economy of the time of apprenticeship. It made the educational space function like a *learning machine,* but also as a machine for supervising, hierarchizing, rewarding' (Foucault, 1977: 147, emphasis added).

He refers later again to the classroom as 'machinery': 'The disciplines, which analyse space, break up and rearrange activities must also be understood as *machinery* for adding up and capitalizing time' (p. 157, emphasis added). 'Discipline', as a set of ordering techniques, operates through the constitution of 'docility' (by use of observational, 'panoptical' power – therefore the primacy of space) and 'utility' (the economical 'mechanisation' of individual behaviour). In Foucault's account, disciplinary regimes of power have produced a new dimension to behaviour; in effect, the body was 'technicised', so as to produce, literally, efficient work. The worlds of discipline (e.g. military training, prisons, hospitals, classrooms, etc.) articulate subjects-in-cohorts (educational, military, administrative) as a composed, maximising, efficiency-driven force. 'The individual body becomes an element that may be placed, moved, articulated on others (p. 164). On the then mechanised classroom, he said that 'the school became a machine for learning, in which each pupil, each level and each moment, if correctly combined, were permanently utilized in the general process of teaching' (p. 165).

Under disciplinary distribution of time and space new 'interactional' habits were formed, and classroom training was forever alienated from the time and space of the 'master', its members ranked in terms of class and stage and assessed through standard examinations set in place in the wake of a new ideology of individual rational progress. 'A whole analytical pedagogy was being formed, meticulous in its detail (it broke down the subject being taught into its simplest elements, it hierarchized each stage of development into small steps) and also very precocious in its history (...)' (p. 159). Foucault notes that questions of assessment, i.e. the accountability of learning in terms of mental predicates, and the emergence of the 'child' as a field of scholarly observation are major symptoms of the classroom system. He suggests that a kind of 'uninterrupted examination' (p. 186), a fundamental variation of the newly

found use of vernacular language in the schoolrooms, as we saw before, 'duplicated along its entire length the operation of teaching' (Ibid: Ibidem). This is a profound historical and sociological observation that can be used to read, in practical terms, the debate on the *nature* of social order, *where* it can be observed and *how* it is distributed (see section 3.3.).

The establishment of the classroom examination as a permanent factor in the reproduction of new teaching-learning techniques pointed to the emergence of such a phenomenon of order. It signalled the creation of a complex order epitomised in a well-known Foucaultian dictum: 'Another power, another knowledge' (1977: 226). This sentence can be used to describe the movement in which pedagogy (as well as other disciplinary spaces) eludes its *technological* nature to enter a universe of *epistemic* inquiry and justification, where a system of 'power' constitutes its analytical capabilities into a new field of 'knowledge', a science. The examination has a pivotal role in this process (see section 3.2, in which the role of standardised measures of intelligence in the origins of Vygotsky's 'pedagogical psychology' is discussed). Here, I quote Foucault at lenght: 'The examination enabled the teacher, while transmitting his knowledge, to transform his pupils into a whole field of knowledge. Whereas the examination with which an apprenticeship ended in the guild tradition validated an acquired aptitude – the 'master-work' authenticated a transmission of knowledge that had already been accomplished – the examination in the school was a constant exchanger of knowledge; it guaranteed the movement of knowledge from the teacher to the pupil, but it extracted from the pupil a knowledge destined and reserved for the teacher. The school became the place of elaboration for pedagogy' (Ibid: 187).

It is in that context that Foucault refers to psychology and pedagogy as 'strange sciences' (p. 226). Their links with the disciplines have shaped them in several ways, the educational use of psychology being a case in point, one which came to be an important element in pedagogical practice and educational reform (Walkerdine, 1984). The implication in Foucault's reading on the formation of such power relations is that the 'objectification' provided by the new sciences, e.g. psychology (by means of which an 'epistemically' informed description of

the 'pupil' is taken) is extensively used as a mechanism of 'subjection' through (and in the name of) a system of governability that is seen not as arbitrary, but primarily rational. When 'truth' and 'scientific' arguments are produced in the contemporary classroom, an apparatus of rationality and epistemic argumentation must necessarily follow (Walkerdine, 1988), and the multiplication of control must also accompany the multiplication of rational resources, of which public agreement and literary observation in the classroom through 'cohort' activity is seen a part (Payne and Hustler, 1983; see also section 3.4 in this chapter).

3.2. Vygotsky and the classroom as a developmental factor

Lev S. Vygostky was, amongst other things, an educationalist concerned with the problem of disadvantaged children (e.g. blind, deaf, retarded), which prompted his interest in the practical application of psychological research (Luria, 1992). In Foucaultian prose, we could say that Vygotsky was one of the main architects of a new field of 'knowledge' – developmental psychology, pedagogy – stemming from a new field of disciplinary power – mass education, in which the child's 'mind' and learning activities are becoming a 'scientific' object; more than that, in which the child in becoming 'the child' (Aries, 1962; Walkerdine, 1984). His experience with educational matters seems to have influenced him on the view that school learning constitutes one of the pillars of children's mental development, a missing link in learning and developmental theory, as we saw last chapter when discussing Piaget's approach to cognition. In a paper titled 'Interaction between learning and development' (Vygotsky, 1978), he directly discussed the 'effects' of schooling on mental growth.

According to Vygotsky, saying that 'learning' and 'development' are related processes means, on one hand, not to identify one with the other, and not to portray them as mutually exclusive, on the other. He classified several major theories on the subject and subsequently rejected them. This is the case for behaviourism, for which learning and development were implied to be one and

the same. Piaget is also criticised, on the basis that his theory not only sees both as clear-cut different things, but also conceived of the latter as imposing the limits of the former's possibilities. Vygotsky accepted Gestalt psychologist Kurt Koffka's thesis about learning and development as interrelated phenomena with distinct temporal cycles, with learning as a 'structuring' factor. However, Vygotsky insisted, Koffka omitted the role of semiotic and interactional mechanisms in the emergence of mental 'gestalts', that is, qualitatively new aspects of development. He believed that the two processes generate mutual effects and that school learning provides for new contexts of development.

The concept used in order to address the issues of development and school learning – for Vygotsky, the privileged field for *scientific* notions and systems of representation – is that of 'zone of proximal development' (or ZPD), defined as:

> 'The distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers' (Vygotsky, 1978: 86).

Thus formulated this notion seems to imply a measure of mental capacity. According to Wertsch (1984) the concept of ZPD was coined around 1930, although it has interested Western psychologists only recently. He asserts that the place of the notion in Vygotsky's work takes off as a means to discuss schooling and its relations with the use of standardised psychological tests, widely used in Western Europe at the time. Vygotsky himself defended the pedagogical use of I.Q. tests and it is in that context that one is to find the origins of his renowned notion of 'proximal' development.

One of Vygotsky's concerns was the compatibility between the school curricula and the children's level of development, that is, whether the children were mature enough to learn what the curriculum proposed to be taught at a certain stage. Vygotsky defined such a developmental stage as the capacity to

solve determined 'problems' independently, as a result of well-established (cognitive) foundations. He referred to that as the level of 'actual' development, which could be diagnosed by standardised intelligence tests, such as the I.Q. However, Vygotsky argued that the fact that what a child is capable of 'solving' alongside a more experienced partner is also indicative of his/her mental development, emphasising the relevance of 'embryonic' skills, i.e. coping with situations and resources that have not yet been 'internalised', as in the case of 'real' development. To this capacity he called level of 'potential' (or proximal) development.

In a well-known example, Vygotsky describes a situation in which two children with the same level of real development (in this case, the mental age of 8-years according to a I.Q. Test) perform differently when monitored by an adult or more experienced partner. One of the children is able, in the assisted task, to solve standardised problems for the mental age of 9-years whereas the other scores the equivalent to the mental age of 12-years. Vygotsky's 'problem' was to explain such results.

Vygotsky saw those differences as related to the individual skills of the two children, that is, they would present different intellectual dynamics that could not be diagnosed from the competencies they had internalised as a result of previous learning, or their level of real development. Those 'zones of proximal development', visible when in collaboration with an adult, not only explained the difference in the results referred to above but, in Vygotsky's view, they could predict the subsequent course of the children's success at school.

According to Van der Veer e Valsiner (1996), Vygotsky was interested in the problem of the homogeneous composition of classrooms by intellectual level, an enterprise for which the notion of ZPD had a fundamental role. Coherent with his judgement that "good learning' is that which is in advance of development' (1978: 89), Vygotsky considered that the most profitable composition, from the point of view of learning, should be based on the determination of an 'ideal' mental age for a given class, which would correspond to the level of exigency relative to the mental age required to deal

independently with the curricular activities proposed. Consequently, The 'ideal' age was to be somewhere above the line of the 'real' age as a 'composition' criterion. For example, the instruction design for children with real mental age of 6-years should be planned for a mental age superior to 6-years. But how to determine an ideal age that would suit a whole group of students in order to guarantee some degree of success? In Van der Veer and Valsiner's (1996) account, Vygotsky's investigations demonstrated that the most 'favourable' difference between actual and ideal mental ages was the ZPD, that is, a child whose measure of intellectual development in a collaborative situation was two years above his/her mental age measured independently, should be classed with others in a context that offers challenges for a mental age two years above his/her independent scores.

Vygotsky and his students also observed in several researches a phenomenon of 'regression to the average' of the measures of intellectual performance during schooling: children who had high scores tended to score lower at a later stage whereas the ones who had low scores tended to improve their classification in I.Q. tests. Such observations came to be important in his pedagogical thinking and were on the basis of research questions designed to answer why I.Q. high-scoring children did not seem to take much advantage out of instruction in early schooling. The relevant thing about this fact, that Vygostky called the 'levelling effect of schooling' (Van der Veer e Valsiner, 1996), was its direct influence on his ideas about class composition, using the level of proximal development and the ideal mental age as analytical and technological (prescriptive) tools. Instruction would favour 'delayed' children, constituting new 'proximal zones', whilst more gifted students would likely get stuck within heterogeneous compositions. Vygotsky explained that the latter cannot be the greatest beneficiaries of such traditional methods of class composition, since the extension in which their 'potential zones' are to be challenged by the curriculum is smaller than in the case of less developed children.

One of problems with the approach outlined above is, though, related to the concept of 'development' itself. Vygotsky intended not to confound it with a

sum of partial 'learning' events, but did not offer a thorough definition of the concept. Although several notions used by him can be seen as developmental units, it is plausible that the term development can be more properly applied to what Vygotsky (1978) called 'higher mental functions', such as categorical reasoning, logical memory and self-reflection. Van der Veer e Valsiner (Ibid.) argue that Vygotsky had in mind that the 'contents' of school activities such as reading, writing and arithmetic create a particularly relevant developmental difficulty for the child, namely, that of the conscious monitoring of his/her own activity. Such skill takes place, in Vygotsky's view, as a non-planned consequence of instruction, as a new mental resource. If the child was until then arguably non-*conscious* of his/her own cognitive resources, he/she is put in a position to *topicalise* them in the classroom, and therefore, influence the appropriate, normal, accountable features of his/her own reasoning and conduct. To use a current expression at the time, the movement is 'dialectical': the 'activity' precipitates the conditions for the emergence of new (non-curricular) 'functions' (as opposed to 'contents') at the same time it continuously engage and bases itself on the work of such functions (remember that 'class composition' depended on a set of mental attributions and measurements of such 'functions').

Vygotsky's theory does not explain clearly how several partial learning moments can produce something like a 'general concept', but it is quite clear that for him development and learning describe different psychological dynamics even though they are highly interconnected. In this line of reasoning the emphasis on the classroom was analytically residual, but nevertheless important as it helps to argue that the *repercussions* of instruction – especially in what is to be found in the interactions with more experienced co-participants – will always be broader than the *contents* of instruction. By doing that, Vygotsky was able to put the classroom and its social, instructional mechanisms in the map of psychological research as a developmental factor in its own right to be taken into account.

In the light of the concept of ZDP such as it was originally formulated, Vygotsky's take on cognitive development gives room to a series of

contradictions concerning his 'dialectical' thinking, as observed by Van der Veer and Valsiner (1996). First, the idea of ZPD and its pedagogical applications assumes that development is markedly linear, such that a trivial implication of Vygotsky's position is that a difference of two years between the two different measures of competence (actual and ideal) would disappear in two years (Ibid.). Secondly, the social 'context' or 'environment' in which development takes place is left unanalysed; instead, it is treated as a general notion of 'external' support. The quality of such external support – the assistance by adults in problem solving tasks, for example – is rather presumed (equivalent, in the case of the example given above) than analysed. Besides, the prediction that the independent performance of a social actor within the performance limits of its group(s) of membership, suggesting as a logical implication the impossibility of newcomers overcoming older contemporaries had critics accusing Vygotsky's socio-cultural theory of being narrow and 'transmissionist' (Ibid.).

However, later readings of his pedagogical concepts, especially with his rediscovery in the West, seem to point to new understanding of the zone of proximal development as a public, interactionally structured interface for 'cognitive' performance. Rogoff and Wertsch (1984) suggest that learning itself occurs *in the* 'zone', a spatial metaphor that indicates the public, accessible, observable character of learnability, in which identities, expertise and meanings are negotiated. Wertsch (1984) argues that the potential level of development does not guarantee the quality of the intersubjective workings that will take place in any particular interaction; he accuses Vygotsky of not having described sufficiently the meaning and psychological effects of the interaction between partners in different levels.

That means that the analysis of social interaction (or the interpsychological aspect) cannot be reduced to a measure of 'competence'. Bruner (1985), for example, suggested the notion of 'scaffolding' as an explanation for the process of language acquisition by children, that is, that such a process should be analysed in terms of the 'formats' of adult-child interaction and its correspondent mechanisms of mediation in that they progressively empower

the child into the mastery of their parents' language, or as Lave and Wenger (1991) put it, from the *periphery* to the *centre* of practices. The point is that Vygotsky's insightful ideas about the classroom context as an engine of development were not matched, analytically, by a detailed exploration of the ordering of interactional, cultural or institutional events. More recently, some researchers have been turning to the notion of ZDP as 'discourse' (Candela, 1997) or as a 'symbolic space' (Meira and Lerner, 2001), that is, to the emergence of forms of intelligibility and participation through the language of/for the 'other'. In the next section, a point is made for the analysis of such processes in their own right.

3.3. An ethnomethodological reading: order at all points and the classroom-as-installation

One of the interesting things about the work of Vygotsky and his concept of 'zone of proximal development' is that at the same time it was conceivably used to furnish the basis of new scientific regimes of truth, extensive to the development of mass schooling and its technologies of instruction and discipline (e.g. class composition), it took psychological analysis into the level of a more dynamic, socio-interactionist conception of mind and development, a conception that currently finds echo in the activities of several scholars, organisations and publications in psychology. Socio-cultural psychology, activity theory, situated cognition are amongst the approaches indebted to Vygotsky's writings. His work also rendered formal education in the classroom an accountable aspect of the modern, urban, industrial psychological world, in a way that he could not detect amongst the peasants in the remote areas of Uzbekistan, for whom syllogistic reasoning and geometrical conceptions of space were not readily available in standardised situation of testing (Luria, 1976; Vygotsky and Luria, 1993).

One thing Vygotsky did not do, though, was to carefully analyse the interactional dynamics that afforded movement in the 'zones' of development, or how the material and literary resources of the classroom constituted, for all

practical purposes, the cultural activities of teaching and learning. Rather, those things assumed static, taken-for-granted status in his writings, a sort of causal factor to be dealt with; 'more experienced peers' are taken to represent particular interactional contexts for children, and formal (professional) descriptions of 'cultural systems of signs' (e.g. algebra) are used as the basis on which the entry into literary culture is discussed. Even less likely in Vygotsky's work was the implication that the study of such interactional and conceptual technologies can be set up by taking those *objects*, and the *practices* which they are part of, in their own right as containing observable instances of how development, learning and cognition feature as a 'problem' in the first place, instead of using them as causal influences of some sort. In any case, cognition is something to be explained as an island of individual development, surrounded by interactional opportunities, expertise, knowledge, tools, etc. In what follows, I draw on Harvey Sacks at first to sketch a more social approach to the 'zones' of instruction Vygotsky talked about, viewing them from the perspective of how public ordering is invoked and maintained, even how it can even be considered a primary aspect of a non-causal approach to socialisation. I finish the section with a discussion that expands Sacks' approach to the problem of order into some relevant features of ethnomethodological work that are often 'forgotten' by conversation analysis.

As we saw before, 'disciplines' (Foucault, 1977) work continuously through/for the ordering of its constituent parts, and in the case of modern pedagogy, through a relentless, all-encompassing teaching-examination routine. The ordering of the relations between persons or social interaction is, amongst others, one the most relevant features of this process, and under the heading 'intersubjectivity', or the mutual intelligibility between 'minds', it stands as one of the foundational topics in philosophy and the social and human sciences. One of the proponents of a non-psychological, more public and interactional conception of 'intersubjective' activity was Harvey Sacks, who went on to establish the discipline of conversation analysis. He has developed an understanding of social order that contrasts with that of 'traditional', 'constructive' sociological and psychological approaches, especially with those keen to translate such an understanding into a

methodological orthodoxy that reduces sociological intelligibility to the use of *ad hoc* explanatory schemes (Cicourel, 1964; Lynch, 1993). The reason why this is so is because such forms of investigation have considered social phenomena and social categories as naturally occurring entities, whose characteristics follow a 'distribution' in a specific population[16]. Thus, for example, following this 'aggregate' view of social order (Schegloff, 1992), a factor analysis or a t-test are the appropriate tools to describe which forms of knowledge, beliefs or attitudes are related to which specific social groups. Accordingly, social collectivities, thus described as such-and-such by the researcher, 'own' (Sharrock, 1974) certain types of power, worldviews, attribution biases, and so on.

Sacks' contrastive take on 'social order' is indebted to major works in the sociology of social interaction, such as those developed by Goffman (1959) and Garfinkel (1967), especially in what they refer to the constitutive role of language in social life. Garfinkel has developed his 'ethnomethodological' programme in response to a classical 'axiom' in sociology, due to Durkheim, according to which the objectivity of social facts, such as knowledge, beliefs, rituals, kinship relations, religious canons, etc, are the basis of collective life and the 'glue' that hold individuals into collectivities. Garfinkel conducted his studies under the assumption, diametrically opposed to that of Durkheim, that is precisely 'society' what is glued together by the countless 'ethnomethods' – publicly accountable ways of naming, identifying, pointing to, recognising, counting, justifying, repairing – that members use to conduct whatever affairs they are concerned about.

Sacks analysed conversations[17] as a paradigm of social order, and in doing so paved the way to the discipline of conversation analysis. He was convinced

---

[16] 'Now, for whatever reasons there were, the social sciences tended to grow such that the important theories tended to have a view that if you look at a society as a piece of machinery, then what you want to consider is the following: There are relatively few orderly products of it. There is, them a big concern for finding "good problems"; that is, to find data that which is generated which *is* orderly, and then attempt to construct the machinery necessary to give to you those results' (Sacks, 1992: 483).

[17] Conversations can be regarded as forms of talk exchange between two or more parts in which no pre-allocation of turns at talk in terms of its form, length and content can be identified. The use of the expression 'talk-in-interaction' seems to encompass a broader range

that sociology should be an 'analytic' discipline, and in arguing for the 'usability of conversational data for doing sociology' (Sacks, 1972), tried to established a programme for the study of social order in alignment with the basic tenets of the ethnomethodological programme of Harold Garfinkel; for Sacks' analytical sociology of conversation, order is to be found not as distributed or aggregate societal logic, but 'at all points' in naturally occurring, unadulterated phenomena of social interaction (Sacks, 1992; Schegloff, 1992b).

'Order at all points' stands as a challenging argument for conversation analysis and for interactional studies in general. It implies that analysts and participants alike systematically encounter, establish and orient to morally and epistemically normative criteria in conducting their situated affairs. Note that this is not only a methodological precept for conversation analysis[18], i.e. how to go about doing research, but a integral aspect of the social fabric itself, by which new members – e.g. children, pupils, foreigners – can 'see' and 'learn' about culture, reality, identity, and their place in it; in that sense, 'order' operates in a way that is designed for it to be 'seen' and 'learned' in practice, case by case, situation by situation, within and across particular activities (Edwards, 1997; Sacks, 1992; Schegloff, 1992).

In the 1950s with Chomsky's 'generative grammar', *talk* was considered to be random 'performance', a topic better described in terms of its infinite behavioural variability, thus hardly amenable to scientific scrutiny. The only aspect of language that can be describable in formal terms is, according to that theory, its *structure*, and consequently, the universal linguistic 'competence' to form grammatical sentences by users of a natural language. One of the questions championed by Chomsky in its well-known critique of B. F. Skinner's behaviourist account of 'verbal behaviour' (Skinner, 1957)

---

of phenomena, as to include, for example, discursive activities analysed under the rubric of 'institutional talk' (Drew and Heritage, 1992; Thornborrow, 2001), for which the definition above is inappropriate.

[18] In its origins, the argument *had* to do with methodological questions, and more specifically, with the debate on the relevance of data sampling (Schegloff, 1992). Sacks take on problem was to reject the idea that social order is primarily exhibited on an 'aggregate level' (Ibid: xlvi). See, for example, Sacks's (1992) lecture 33 on 'sampling and subjectivity'.

concerned how would it be possible for young children to generate an indefinite number of grammatical sentences given the limited amount of linguistic 'input' they have received from their caretakers (Chomsky, 1959). In the early 1960s, Harvey Sacks started investigating talk as orderly phenomena in its own right. His enterprise undertook, in a somewhat symmetrical way, a description of an 'apparatus' that could account for certain 'hearings', an 'inference-making machinery' that is part and parcel of the sequential order of a conversation (Sacks, 1992; Schegloff, 1992)[19], and 'socialisation' – to allude to the 'input' problem above – as the process by which such phenomena of order is made visible and usable for newcomers.

Sacks also remarked that it is rather difficult *not* to find order in the material one has to look at, given that it is such an important resource for a culture (p. 485), and that the way members are 'built' since an early age is not by being set as sampling machines, where order could be identified *before* sampling and have its importance decided upon at a later stage; rather, the detailed moral order they – members – encounter in the structure of publicly accountable behaviour and their 'general' application can alternatively account for the 'socialisation' problem discussed above in relation to Chomsky's 'linguistic input' problem. Despite the limited and random 'sample' of order any new member is exposed to he/she 'comes out in many ways pretty much like everybody else, and be able to deal with pretty much anyone else' (Sacks, 1992: 485). The main implication of this 'holographic' view of social order (McHoul and Rapley, 2001; Schegloff, 1992) is the idea that 'culture' can be described formally, as a set of general 'methods' (Eglin, 1980) by which interactions are carried on, and upon which actors rely to 'generate' jointly oriented states of affairs. In this way of thinking, a casual conversation is expected to contain the foundations of the whole 'culture' within it[20]. It is said,

---

[19] Schegloff (1992) points out that the kind of problematics that set out some of the most well-known developments of CA, especially recognisable in Sacks's paper on 'the analysability of stories by children' (1972), share formal similarities with the type of questions posed by Chomsky for the description of a universal grammar, in terms of the generation of a descriptive 'apparatus' and the reliance of the analyst on his/her own (cultural) expertise on devising the answers (p. xxi).

[20] This argument is somehow similar to Vygotsky's (1987) idea, in psychology, of taking the 'word meaning' as the unit of analysis of 'consciousness', whose 'higher functions' – logical reasoning, memory – would reflect a basic semantic dynamics revolving around the

for example, that the specialized forms of talk-in-interaction that came to be studied as 'institutional' (Drew and Heritage, 1992) derive from the basic structures of conversation as described by Sacks and his associates (e.g., Sacks et al., 1974), exhibiting all along variations of its basic properties (Drew and Heritage, 1992; Edwards, 1997; Schegloff, 1992b).

According to Schegloff (1992b), Sacks' idea of a culture organized interactionally and at all points questions directly the Chomskyan axiom according to which 'language acquisition' must be accounted for in terms of innate cognitive structures once the analyst has established that 'degenerate' samples of talk cannot produce the grammar 'devices' responsible for the ability to generate sentences, as discussed above. Sacks' interest in the organisation of a culture at all points implies, consequently, that the tasks of language learning and socialisation are resourcefully built around the induction from the limited interactional 'events' to which learners have been exposed. The formal properties of order that members find at every situation (e.g. conversational norms for turn-taking) and the fact that order is reflexively designed so as to be grasped by members[21], so that 'things are so arranged as to permit him to', in Sacks's words (p. 485), are the main sources for the systematic observability and acquisition of culture in interaction. I would suggest, at this point, that the organization of such a form of public, visible, graspable culture for its members relies largely on the structure of talk-in-interaction, but also in competencies in which talk is associated to other modalities of semiotic mediation, including forms of written representation and material technologies (see next chapter for a more detailed discussion). Notice that the problem here is (and has been all along) one of *using* and *performing* 'culture', not necessarily of *representing* it. So, the question is how is the classroom designed to be usable, that is, to afford 'learning' activities? And how do those features come together in the way 'knowledge' is presented and conventionalised in the classroom?

---

development and stabilisation of word meaning. Vygotsky's argument is itself analogous to Marx's use of 'value' as the 'cell' of the capitalist mode of economic reproduction.

[21] Garfinkel's central recommendation for ethnomethodological studies of order consist of the observation that 'activities whereby members produce and manage settings of organized everyday affairs are identical with members' procedures for making those settings "accountable"' (1967: 1)

70

Douglas Macbeth (2000) has suggested the idea that classrooms are 'installations', that is, contexts for documenting 'order' saturated by relevant ways of analysing space, objects and action. Museums and art galleries come to mind immediately (Hemmings et al., 1997). In such places objects are so arranged that the visitors can *find* orderly 'things' (e.g. pre-historical Britain, early engine technology, French impressionism, etc.) in a specific disposition and within an ordered, graspable flux. Such 'findings' are analytical through and through; they allow for a form of 'reading' in a competent world of knowing action, and even though they do not *determine* the courses of behaviour to be taken within the confines of the installation, they make its resources (e.g. proximity of display, textual information) accountably relevant in cluing and understanding the mutual relation between the objects-in-installation at display. They also often make a case for their marked 'out-there-ness', their existence previous and beyond the installation (and its 'descriptive' apparatus), their 'documental' character. Macbeth argues that the classroom, likewise, displays an accountable 'worldliness' that purports to reach outside and beyond the classroom itself, that is, that the modern social technology of the classroom system delivers educational goods that are to be accounted for in terms of a relation to 'orders' that operate in the world-at-large. In chapter 5, I discuss how the mathematics classroom entangles 'language games' of inference to a sense of logical 'necessity' that goes beyond its visible, semiotic orders, as a form of 'representation' (Edwards, 1997).

As far as 'knowledge' and 'competence' are concerned, the two defining features of the classroom-as-'installation' are (1) the emphasis on knowing action, or its 'analytical' thrust, in which several kinds of resources are mobilised (material, conceptual and interactional), so as to imply that 'the teacher is engaged in pulling a world-for-remark into view, and the students are not simply responding, they are finding the world she is pointing to' (Macbeth, 2000: 26); and (2) its philosophically 'representational' policy, its epistemic commitment to a conception of the world as 'knowable', and knowledge as 'caused' and as 'usable'. The classroom's technologies install and simulate the world-for-remark it describes. The sociological analysis of such practices is at

best incomplete without the reference to the material, organisational and conceptual-analytical technologies that constitute a classroom-in-session. Macbeth argues that:

> 'If we allow that teaching's work is foundationally interactional, then teaching's 'objects' – its curriculum, instruction, and object lessons – are interactional too. In the classroom, the teaching of grammar or math entails organizing grammatical and arithmetic fields of questions, answers, objects and relations. For these tasks, cohort organization is a material resource; it organizes interactional contextures for seeing the instruction, producing the lesson, and attaching its objects to the room in public and inspectable ways' (2000: 30).

CA's use of an holographic view of order in the way discussed before, as consisting of the reproduction, at all points, of the properties of conversation as described by formal analysis, often bypasses the detailed constitution of settings in terms of their 'perspicuous' properties (Garfinkel and Wieder, 1991). Burns (1997), for example, analyses a law school classroom focusing on its 'professionally- oriented' character, in which classroom actions are organised as to make reference to, and simulate, the outside, 'worldly' activity of law practicing. The interaction between professor and students is remarkably oriented to the simulation of a set of courtroom procedures in which the professor uses the emerging understandings of the students during the 'staging' of the legal tasks (e.g. objections, amendments, offers of evidence, etc.) as the blueprint for pedagogical interchange. Few studies in the CA tradition have openly addressed questions that go beyond the 'binary' inference machines Sacks talked about in the form of 'adjacent' structures, so as to address issues of relations between commonsense knowledge, categorical descriptions and formal structures (McHoul and Watson, 1984), or the identity of the 'class' as opposed to individual students (Lerner, 1995; Payne and Hustler, 1980). The latter, which returns us to the original quest for a moral economy in which the 'collective' becomes a legitimate actor in the classroom, is the topic of the next section.

3.4. Conversation analysis and the 'class' as a participatory framework

It is a commonplace in the conversational and discursive analysis of the classroom that the organization of participation between teacher and pupils describes a three-part sequence consisting of an 'initiation' by the teacher, a 'reply' to her questions in the second position, and then a third, evaluative rejoinder by the teacher, sometimes alluded to in the literature as the distinctive feature that separates classroom talk from everyday conversations (Mehan, 1985, 1986). In this section I am interested, to put it in CA terms, in the nature of the 'reply', therefore, the second position of the aforementioned sequence[22], and how it implies the work of an accountably collective agency, a witnessing and corroborating audience (Shapin and Schaffer, 1985) whose co-presence and concerted actions were, as we saw it earlier, part of Adam Smith's formulation of a moral educational economy (Hamilton, 1983; Macbeth, 2000).

I want to explore the argument that 'the pupils as a collectivity are overwhelmingly constituted as one party to the talk *vis à vis* the teacher as the other party' (Payne and Hustler, 1980: 56). I treat the way the actors are brought into play and 'morally' implied in their participation as an integral feature of the 'classroom-as-installation'. In the next chapter, I refer to the 'class' and how it is implied and used in the set up of classroom's literary technologies (such as running a representation on the blackboard), and how implying a collective – or allowing and encouraging it to take place – can be glossed as an important epistemic mechanism, one which grants the rational basis and democratic governability of knowledge (Walkerdine, 1988).

It is interesting that one of the major CA papers on (turn-taking in) the classroom, McHoul's 'The organization of formal talk in the classroom' (1978), displays no sensitivity to the problem of the identity of the class. For the most part, in the CA literature the 'reply' is seen indiscriminately as a

---

[22] In a very interesting paper, Schegloff (1992) has suggested that 'repair' in the third position is the arena where intersubjectivity is formulated and disputed. Here I am interested in how the dynamics of the adjacency pair constituted by 'question-answer', or 'initiation-reply', in founding the horizon for 'class' identity, arguably an important social mechanism in the '*class-room*'.

logical consequence, that is, as an *answer* that is relative to, and demanded by, a *question*. Without questioning such a proposition, I want to argue that there is more to how this reply works as a social mechanism than CA research let us see. CA's version of 'knowledge' in the classroom is indebted to the 'Wittgensteinian' philosophical argument that knowledge is 'public'. For instance: 'Instead of seeing children's knowledge as private and internal states, as a personal possession, an interactional view of teaching and learning recommends seeing knowledge as *public* property, *social* constructions, assembled jointly by teachers and students that become visible in social contexts' (Mehan, 1986: 101). It is worth remarking that the question I am trying to address here concerns the nature and legitimation of the 'public property' mentioned above; what if this 'public' nature is not only a philosophical assumption but also a 'statutory' condition that can be analysed? Instead of taking the 'public' character of such knowledges as a philosophical truism, to focus on how a specific kind of agency (the class) can 'know' and 'agree upon' the world, that is, establish a footing for epistemic claims, can be a more promising answer to his question.

Ethnographer Martyn Hammersley (see, for example, Hammersley, 1990) was one first to pay attention to the organization of participation in the classroom. He was concerned with the description of formal aspects of discursive order (e.g. turn-taking system, question-answer sequences) in the classroom even before conversation analysts directly turned their attention to the issue. I wish to comment on some aspects of Hammersley's original take on the issue of classroom participation and identity of the speakers, and then contrast it with other works in the CA tradition to finally offer my own take on the issue.

Hammersley analyses instances of what he calls 'whole-class questions', which he then identifies with 'traditional' pedagogies. Such events consist of the teachers' question addressing not a specific pupil, but by the opening of a problematic next-turn to be occupied by a self-selected speaker. 'The teacher tries to reduce classroom interaction to a two-party format, with himself as one speaker and one or another pupil acting as the other' (Ibid: 16). So far, no news. Strangely enough, Hammersley uses a kind of normative understanding

about how opportunities in talk are to be allocated when he states that 'problems necessarily arise since only one slot is provided for the participation of a large number of pupils. Potentially, some 17 speakers are competing for one answer slot. From the point of view of teachers, the classroom encounter as an interaction system, focused on and co-ordinated from the front, can just as easily disintegrate as a result of 'over-participation' by pupils as it can by escalating inattention' (p. 16).

The quote above is quite unusual 'analytically' and I want to argue, false. For the teacher to do what Hammersley is arguing against (or wondering about) is precisely the point, the epistemic implications of which are further elaborated in the next chapter, where we get a sense of how the orientation to the graphisms and written formalisms of mathematics and its interpretation uses 'sympathy' and the accountable nature of joint observation and participation to carry on with the 'business' of conventionalising (otherwise arbitrary) knowledge as 'necessary'. The way different knowledges (e.g. mathematics, grammar, biology, and so on) may invoke different technologies of observability and consensus production, and therefore of social organisation, is not, I suggest, a question for most of the CA literature.

For instance, Hammersley have difficulty recognising that classroom activities are, at times, constituted to be seen and attended to by all pupils at the same time:

> 'The teachers demand participation and differentiate pupils
> on the basis of the 'quality' of that participation, yet the form
> which official participation must take is highly restricted and
> there are only limited opportunities, given the number of
> pupils in a class. It is not surprising, therefore, that
> considerable unofficial participation occurs' (p. 19).

In a rather secondary way, Hammersley recognises that one of the implications of the teacher's discourse is that it is a logical path to a 'generalised second turn'. The problem is, as we have identified, that he sees it as a problem! Why would the teacher do that in the first place? In several cases, the practice of transcription itself allows for the relevancy of the problem. Mehan (1986)

makes a distinction, in some of his transcripts, between 'all' and 'many' (p. 92), although they are neither specified nor differentiated in terms of their social implications. For example, it might be that the allocated opportunities for the turns occupied by 'all' and by 'many' (not all) are precisely the same, the actual number being, in this case, inferentially irrelevant. Lerner (1993) uses the term 'class' (p. 117), accompanied by the description 'mostly in unison', adding that 'other than an individual student answer is implicated (...) In Excerpt 5 the teacher addresses the class as a association of students, and this is done in a manner that establishes the relevance of a response by the members of the class as a team or ensemble' (p. 117). A 'slot' of the kind that constitutes the incomplete turn-constructional units Lerner talks about can invite a 'choral' answer.

Lerner (1993, 1995) offers an account of collaborative sentence construction in the classroom that takes into consideration the work of what he calls 'collectivities in action'. By focusing on how those collectivities are afforded by particular ways in which the teacher designs 'turn-constructional units', Lerner gives a step forward from Hammersley's distinct use of CA into a more 'situated' understanding of the constitution of the class as a responsive agent. The fact that the 'class' is of interest here does not mean that the structuring of talk in the classroom cannot, and does not, dispense with 'class' conversation; my take is that they are open to 'class participation' as legitimate, as an usable possibility of the turn-taking system for instruction purposes. The witnessing 'other', the responsibly implied third part, in a word, the 'collective', is nevertheless a constitutive feature of the classroom's epistemic nature and even individual answers are accountably class-relevant (Payne and Hustler, 1980). Lerner (1995), for example, speaks of a mechanism of 'incomplete' turns in structuring what comes next as the competent way of occupying the 'reply' position. I come back to this issue in more detail in chapter 4, when I talk about the way the class is continuously invoked in the activity of performing a calculation on the blackboard. The interest in Lerner's argument rests in considering how such incomplete turns open the avenues of the turn taking system in 'classroom'-relevant ways. 'I show how each turn type can include an unfinished turn-constructional component that invites completion by its

recipients' (p. 115). Less interesting aspects of his argument include precisely an active 'denial' of how research projects the issue of the relation between the turn-taking opportunities and the identity of the class as a sociologically relevant feature of the activities analysed:

> 'Though I have selected a site of talk-in-interaction for analysis, I do not make an automatic inference that one or another characterization of the identities of the participants or the site of this interaction as a classroom-in-session has an ongoing relevance or consequentiality for the organization of the features of talk-in-interaction I describe' (Lerner, 1995: 114).

This is where I would depart from most CA research on the topic. One of the most 'naturalised' truisms in the CA tradition is that those settings are primarily sites of 'talk-in-interaction', where one should be cautious in making inferences about the 'identities of the participants' and the 'site of this interaction' (above) and their possible 'consequentiality'. Of course, there are a set of rhetorical assumptions in the quote above, the most prominent being that (a) whereas talk is local and organisational, other features do not lend themselves to *analysis*, i.e. as ordering devices, but only in terms of their 'causal' power, or 'consequentiality', and (b) that we potentially 'know' (or are willing to do so, in a glossing way) what the 'identities of the participants' or the 'site of this interaction' mean, and therefore should necessarily conclude that they are external features to whatever we can bring to the analysis, that they are 'theory'; that is, we must avoid being 'constructive'. However, is it not reasonable to say that everyday school practice at the same time presumes, works with and arrives at different positions and identities, from its architecture and dressing codes to its ascription of development and competence, and that is no less true of its turn-taking system and assessment practices themselves? Such 'identities' and the way the communicative organisation goes in instructional settings such as the ones I analyse in this work are inseparable, and many conversation analysts have acknowledged before that the turn-taking system is a system of power under the teacher's control (Thornborrow, 2001) and that institutional settings, in general, are mediated by identities and tasks (Hutchby and Wooffitt, 1998). That those identity features are locally

formulated (represented) in so many words – if and when they are – is an empirical question that all analysts are obviously and rightfully willing to accept; that they are not, does not do away with the empirical, analytical question on the taken-for-granted features of everyday activities, on the things that are needed to hold such documentary practices together. In section 3.1 we saw how those identities were at the origin of the formulation of such a social technology in the first place. It is in that sense that I have suggested that Sacks' extraordinary solution to the problem of order should be coupled with the analysis of what Douglas Macbeth (2000) have called 'installations', a material as well as literary template for the work of simulation of accountably 'worldly' models of order, which allows the analysts to attend to the perspicuous (along with the 'formal') character of such settings at the same time it links them to society at large. The identity of this 'site' is not an external feature of this 'pure' talk-in-interaction contract; it is integral to what they are doing and how they are doing it. In that sense, Lerner's *excuse* for having 'selected a site of talk-in-interaction for analysis' is totally unnecessary; moreover, it must be noticed that he leaves the subject open as he states that 'I do not make an automatic inference that one or another characterization (...) has an ongoing relevance or consequentiality for the organization of the features of talk-in-interaction I describe'. He does not deny its consequentiality, but refuses dealing with it analytically, when his own description is, as is mine, to deal with it 'symmetrically' (Ashmore, 1989), also one or another characterization (Billig, 1987; Edwards, 1997). In chapter 2, we saw how Bloor's (1987) critique of Livingston's work on the foundations of mathematics rests in part of the arbitrary and unexplained choice of not formulating the subject.

In an often-ignored paper from the ethnomethodological front, Payne and Hustler (1980) offer a very useful analysis of the practical relevance and management of the classroom 'cohort'. I will shortly discuss some of its central aspects. The following passage is in line with the argument I am trying to put across here:

> 'On the basis of our observations of teachers conducting their
> lessons we wish to suggest that one general strategy a teacher

uses to handle the pupils in his lesson is to constitute them as a class, as a collectivity, as a cohort. Further we argue that the constitution of the pupils as a cohort is a feature of the occasion of the lesson which is made available to us in part through the organisation of the talk. *It is at least in part through the ways in which the talk is organised that the collective pupil identity is defined, maintained and managed'* (p. 50, emphasis added).

The italicised passage is fundamental in conceiving the possibility of analysing the identity of the 'class' in consonance with the formal description of talk, and their mutual implication. 'To deal with one of those modes of organisation only, would be to ignore how they may interrelate to provide for participants and the researcher a particular sense of social structure' (Ibid: 53). It also makes Hammersley's question sounds absurd, since it is precisely by constituting the class as an agent that the teacher can manage a large group of pupils. For Payne and Hustler, the cohort is a mechanism that reflexively projects the activity as 'classroom' activity. In the same vein, it is the context in which a specific kind of agent is implied in relation to the 'teacher', the single individual position without which such reflexive cannot take place.

The authors even speak of a preference for the class as a relevant second part (the 'reply'). The most striking thing about their paper, though, is that in practice – unlike Mehan's and Lerner's papers – Payne and Hustler's do not show any exhibit of the concerted actions of the cohort, only of individuals; there are overlaps, but does not describes cohort action necessarily, especially if the pupils were not responding the same thing (Lerner, 1995). For example, they use a transcript in which a question is repeatedly thrown back at the class as an evidence for class preference. In Payne and Hustler's sense then, the 'class' means that the teacher did not select anyone in particular. 'The talk is between the teacher and the class; he has turns and they, as a collectivity, have turns. So when an individual pupil talks he is taking *their* turn, i.e. the turn of the class' (p. 60). While I agree with this characterisation, which emphasises the 'logical' identity of the 'class turn', I think that it would profit from the demonstration that 'class action', or the *concrete* use of the logical identity of the class, can and do follow the allocation of such turns; besides, I think that a

more complex characterisation is in place. In terms of the organisation of talk-in-interaction, we can say that the way opportunities to talk are allocated in the classroom describe the cohort as a logical, accountable entity, and at the same time allows us to see it as a current and acceptable method by not resorting to repair or indication of transgression of the turn-taking system. As far as 'conversation' is concerned, I suggest that a few distinctive characteristics allow us to speak of a 'class'; I shall use those and the general arguments developed so far as a license to use the term 'class' in the following chapters.

The characteristics I am suggesting are: (a) *the way the pupils are addressed, or the 'logical' implications of it.* This is the sense of Payne and Hustler's argument about the 'turn of the class', even for the cases in which only one pupil respond at a turn. For example, turns in which no particular pupil is selected can signal the open character of the turn to more than one respondent as valid. In the following, a cascading sequence of answers by different pupils can be observed to a particular question:

Extract (1) 3rd grade, EE, 3:

```
1.      T:    to take the real proof of this division (1.0)
2.            now (.) this divisor is working as what now?
3.      P:    °multipli(    )°=
4.      P:    =multiplied
5.      P:    >multiplier<
6.      P:    multiplier::=
7.      T:    =multiplier now it turned into ↑the-
8.            multiplier wasn't it ain't I going to work now with the
9.            multiplication? (.) it turned into the
10.           multiplier (.) and the quotient now turned ↑into=
11.     PP:   =multiply::[ing
12.     T:               [multiplying (.) so now we have
13.           four by four?
14.     PP:   six[tee::n
```

Four consecutive answers follow the original question before a rejoinder by the teacher in line 7. However, in lines 11 and 14 we can see a concerted action consisting of the answer of more than one pupil. *More than one pupil* is the

operative description and the practical meaning of the PP in the transcription, a practical solution that points to the identity of the class as it avoids the insoluble – however unproblematic for CA scholars – problems of transcription involving the description of the cohort, such as used by Mehan, Lerner, and Payne and Hustler. This indicates my second point: (b) *the fact that more than one pupil do actually answer at a time, or, the effective taking up of the implications of the logical aspects of turn-taking.* Payne and Hustler avoided the empirical demonstration of this fact, rather restricting themselves to the logical argument. Notice also that in line 12 no problem is raised by the teacher regarding the lack of appropriateness of such a concerted action. To the contrary, the teacher partially overlaps the answer with a paraphrase of the same response and extends it into the next question, to which, again, a joint response can be seen (line 14). This is the third criterion: (c) *the fact that they get away with it, which works as a 'confirmation' of its appropriateness.* Last, the epistemic importance of multiplying observation, participation and 'sympathy' (agreement?): (d) that the class is not only a conversational agent, but rather that it may be conversational in the first place because it works as a social mechanism for the constitution of literary technologies of 'witnessing' (mathematical) knowledge, of fixing its public character. This hypothesis introduces the following chapter.

CHAPTER 4

Artefacts, knowledge and discursive practices: the blackboard and the social order of the mathematics classroom

4.1. Acting with artefacts

One of the missing links in interactional studies of classroom education relates to the technological affordances of the classroom as an 'installation' (Macbeth, 2000), in which phenomena of order are distributed alongside the (heterogeneous) lines of open lessons, local instructions, orders, question-answer sequences, written inscriptions, demonstrations, spatial arrangements, material devices, etc. Instead, as I have indicated previously, the analysis of classroom practices has been by and large the study of classroom *discourse* (Edwards and Mercer, 1987; Hammersley, 1990). The idea I want to pursue here is that in order to describe classroom activities as a particular form of 'culture' (Edwards, 1997), as an observable, publicly available set of practices for the persons involved, the analyst should enlist not only talk (for members enlist heterogeneous elements in composing their actions), but the relations between the discursive and other practical and representational resources in order to account for what doing 'classroom' stuff consists of. As I will discuss later, rather than advancing an account of how those 'technologies' determine such practices or how discourse 'constructs' material artefacts and their technicality as 'text', I make a case for how in such an 'installation', and in particular aspects of it, discourse and 'the other things' share, indeed constitute, a complex semiotic economy[23].

---

[23] I use the term 'economy' here in contrast to other available dynamic metaphors, such as 'mechanics' or 'mechanism', for example. The advantage of the 'economy' metaphor is that it

Here, 'technology' refers to material and representational objects bound to, and integral to the accomplishment of classroom activities, that is, to *hard* as well as *literary* technologies (Shapin and Schaffer, 1985). In this chapter I look at the blackboard and how its use is designed to support 'lessons' as a joint accomplishment of teacher and class, as discussed in the previous chapter (Macbeth, 2000; Payne and Hustler, 1980). I also intend to explore the idea of using the blackboard *as* doing mathematics, a discipline highly dependent on graphisms and written inscriptions (Rotman, 2000), despite its 'Platonic', decontextualised appeal. I loosely borrow the term 'affordances' from the use Hutchby (2001) makes of it, drawing on James Gibson's (1979) account of the psychology of visual perception. In general terms, Hutchby's argument goes on to show how diverse technologies such as the telephone and the Internet ('technologies for communication'), computer-aided systems, laboratory equipment, etc., participate, through the actions they *afford*, in the shaping of the various activities and objects of practice, at the same time that they are constituted by those same activities. The relation I want to explore here is one of 'association' (Latour; 1986; Law, 1986) between those different media, rather than one of 'technological determinism' or 'social representation' of technology (Hutchby, 2001).

During the last thirty years or so, the investigations about the relations between culture and social practices, on one hand, and knowledge and cognition, on the other, have sprung considerably in the social sciences. Despite marked differences regarding the conceived role of theories and methods in social research, interpretative anthropology, sociocultural psychology, activity theory, social constructionism, discourse and conversation analysis, ethnomethodology, post-structuralism, actor-network theory, are amongst the approaches to suggest novel ways to look at such a complex relationship. Those studies can be gathered together around a common 'technological' interest in culture and cognition: instead of conceiving of those as naturally

---

does not dispense with the attribution of 'agency', as well as allowing us to focus on how action can 'capitalise' from the heterogeneous resources of the worksite, instead of 'theoretically' commensurable ones. The implications of the 'mechanics' metaphor are widely known in psychology and cannot be sustained even in the present, more sociological, argument (e.g. the mechanics of talk).

occurring, transcendental entities that determine human relations, they are now seen as constituted by a set of symbolic and material mediation devices in an open-ended, continuous process of 'construction'. That is to say, in other words, that society and mind are treated as actively brought into being in local practices as such resources are used and shared[24].

However, the convergence amongst those approaches is only partial, the 'technological' one being the most prominent, albeit controversial itself. Edwards (1997), for example, points out that opposite to cultural psychology, discourse analysis has a preference for the observation of 'epistemological construction', that is, people's representations, in talk*ing*, of what count as mind, reality, society, etc. On the other hand, cultural psychologists pursue what Edwards (Ibid.) calls 'ontological constructionism', or the idea that culture and language construct, or 'engineer', actual psychological and social functions there were not at work before. One of the implications of this is that while cultural psychology dispute explanatory grounds of 'where' and 'how' psychological phenomena takes place, discursive psychology founds its programme on the analysis of 'cognitive' accountability in discourse (see also Edwards and Potter, 1992). In that sense, the focus on talk in discursive psychology represents an effort to disarticulate the classical linguistic philosophy of 'representation' within various forms of (social psychological) research, that is, to treat discourse as active and artefactual, hence 'technological', rather than to bring in material and technical (non-discursive) set-ups into the analysis of action[25]. Later, I shall press the point further that

---

[24] Lev Vygotsky and his followers that have laid the foundations, as early as in the 1920s and 1930s, for a (Marxist) 'cultural psychology' by claiming the prominent role of language and social interaction in the emergence of typically human 'higher mental functions' such as conceptual reasoning, logical memory and self-reflection (Vygostky, 1978; 1987; Wertsch, 1985). Those functions are, in Vygotsky's view, occasioned by the internalization of social relations and their practical, detailed interactional routines, especially through talk; he also wrote about the importance of activities with objects in the emergence of consciousness and culture (Vygotsky and Luria, 1993). One of Vygotsky's students, Leontiev (1981), took further that task by focusing on the concept of 'activity' and 'tools' as the mediators of the relation between persons and external reality. In Marx's dialectical perspective, Leontiev argued, the organization of human activity by the means of material tools is seen as the vehicle for changes in the human kind's 'objective' living conditions, and therefore as preparing the means for the transformation of man himself. He argued, with Marx, that in production the individual is objectified and in the consumption the object is subjectivised (p. 46).

[25] Brown and Middleton (2001) argue, from a Heideggerian point-of-view, that taking the original sense of technology as *techne*, as tool use and craft as well as art of the mind (p. 128),

the presence of those same technical objects has a direct relevance for the way language is used (see sections 4.2 and 4.3).

More recently, interactional studies of work activities, as well as the sociology of scientific knowledge, have focused on how technology is brought into play by social actors in diverse contexts and how those same objects constitute a site for 'practical reasoning' (Garfinkel, 1967) in organisational settings such as the laboratory (Collins, 1986; Latour and Woolgar, 1979; Lynch, 1985), the classroom (Hall and Stevens, 1995; Macbeth, 2000) architectural design and engineering calculations (Henderson, 1995; Suchman, 2000), control in transport networks such as airports and railways, etc. (Heath, Hindmarsh and Luff, 1999; Suchman, 1992), etc. The emergence of the so-called 'workplace studies' has placed technological resources at the heart of the understanding of organizational activity. As Heath and Luff (2000) have put it, 'it would seem unfortunate to rest with a sociology which treats as epiphenomenal the socially organised competencies and reasoning on which personnel rely in using technologies, whatever they might be, as part of their daily work' (pp. 7-8). For some, this implies a considerably different analytical and theoretical task, radical even; it has been argued that *we* not only 'use' artifacts, but act alongside them, 'extend' ourselves through them (at the same time that we constitute them), sometimes interact with them (Suchman, 1987)[26]. Drawing from insights in ethnomethodology and the sociology of scientific knowledge, I am interested in looking at the ways in which various technical artifacts and representational resources enter the fabric of order production in the workplace, as well as in scientific and educational practices. The question is that of how technological and materially organised settings afford what they do – scientific discovery, learning, management and control, as well as various forms of 'cognitive' activity.

---

it is possible to understand how discourse analysis in psychology has excelled in the study of the latter, that is, of how 'psychological' phenomena are worked up in social interaction, while failing to explain its former sense, related to how equipment and skill come into the constitution of several contexts in which psychological topics such as knowledge and memory are at stake.

[26] Suchman's critique of cognitive models of planned action, and a consequent defence of a 'contingency' model, is a support to the idea of technological affordances and interaction with hardware.

85

In Hutchby's (2001) account, there are at least two broad ways of conceiving of the relations between technology and human actions: (1) 'technological determinism', according to which new technologies *cause* the emergence of new activities and new social relations between their users, as in the technological revolutions of agriculture and industry and (2) various research programmes on the 'social shaping of technology' (Ibid: 16). The latter include several forms of interactionism and constructionism, in which technology and materiality are better conceived of as relating to the 'social' (human actions, communication, society's structure, etc.) under the auspices of 'social order'. In that context, technological artifacts and networks are sometimes conceptualized as 'text', as a conventionalized interpretation of its users (Grint and Woolgar, 1997); or, as in the 'ethnomethodological' and 'conversation analytic' tradition, technology is 'made at home' (Sacks, 1992) by users through 'ordinary' methods of practical reasoning.

In more radical approaches in the SSK tradition, such as the 'actor-network theory', the interaction between 'humans' and 'non-humans' actors are seen as part and parcel of the constitution of socio-technical settings (Latour, 1988a; Law, 1991). Human action and cognition are to be understood within the relations or associations between heterogeneous actors (or *actants*) and represent scaled down, project-*able* 'technological' outcomes themselves in relation to the practices they are a part of. One of the direct implications of the actor-network analysis is that the distinction between *technical* and *social* (or *mental*, for that matter) is, at best, arbitrary, and that 'agency' is no longer seen as a 'property' of an independent human actor. Acting 'with' artifacts assumes the double sense of using technology, but also interacting alongside it, and being constituted by it. The old ontology of subject-object does not work here; rather, the problem is seen as one of how the properties of 'enunciation' about reality and knowledge delegate the quality of being 'actants' to different things (Latour, 1988). It is a description policy, a research commitment: analytically, social and technological are treated symmetrically, so as reveal how each one is implied in the constitution of the other. Whereas 'we' design technological *things*, they can allow for schemes of action that not necessarily precede their

use[27], and often offer a template for the representation of 'our' own abilities (Brown and Middleton, 2001); technology's affordances mean 'permission and promise' (Latour, 2002). To use the jargon of actor-network theory, the problem becomes one of describing how 'human' and 'non-human' actors are built simultaneously (Latour, 1993; Law, 1991).

Those ideas give us a sense of how 'technological' readings of cognitive topics are at the heart of how the sociology of scientific knowledge, to mention one field, has been able to respecify the traditional concerns about the nature of knowledge from an epistemological to a *praxiological* perspective. Artefacts constitute a context for cognitive affordances and analysability[28], in a way that allows, for instance, equipment such as counters to make possible the observation of radiation; telescopes to make astronomical phenomena inspectable; or industrially calibrated beakers to be the basis on which Piaget defined the criteria for his renowned conservation tests (Piaget, 1974), and without which Piaget himself would not be able to 'conserve' volume as intended (Latour, 1990). Producing, à la Piaget, analytical 'axiomatics' of various kinds (conversational, mental or epistemological) in detriment of looking at the interaction with those technical, 'metrological', elements, have conceptual implications in the direction of reducing the phenomena into a disciplined 'social' (or 'psychological') object for analysis. For social and critical researchers to ignore the scale in which technological 'mediation' is embedded in modern life's organisations and is an integral agent in the constitution of (several kinds of) order means being at risk of falling on 'mentalisms' or 'sociologisms' of various sorts, a task that never went past the project of the description of a 'transcendental subject' (or transcendental cultures) (Latour, 1998). Latour (1990) formulates the problem as follows:

> 'It seems to me that the only way to escape the simplistic
> relativist position is to avoid both "materialist" and
> "mentalist" explanations at all costs and to look instead for

---

[27] For such an analysis regarding technologies for communication like the telephone and the Internet, see Hutchby, 2000.
[28] Even tough mediators are largely suppressed in the way facts are reported; see Latour and Woolgar, 1979; Latour, 1997; Suchman, 1990),

more parsimonious accounts, which are empirical through and through, and yet able to explain the vast effects of science and technology' (p. 21).

A most interesting case in point is that of 'visual cognition' (Goodwin, 1997, 2000; Henderson, 1995; Latour, 1990; Latour and Woolgar, 1979; Lynch, 1990; Suchman, 1993), a field of studies that has demonstrated how 'technicised' or 'professional' vision represents an accomplishment that cannot be said to exist as an ordinary form of practical reasoning without the apparatuses at stake. The observation of the 'microscopic' is inextricably tied to the affordances of the microscope. It is the case of material techniques constituting the appearance of the reality, a condition referred to elsewhere as 'phenomenotechnique' (Bachelard, 1973; Latour and Woolgar, 1979). Within the field of conversation analysis, Goodwin has produced relevant work on how talk is bounded together with divergent forms of semiotic mediation, like gaze, movement and material resources for action (Goodwin, 1995; 1996; 1997). Together, they constitute 'practical fields of activity', and are irreducible to the determination by a superordinate conversational order. Goodwin's question is, ultimately, one of range: what aspects of human action is analysis to render relevant? Only talk? Or is talk embedded in a chain of order production that associates various heterogeneous elements? Goodwin (and this work) follows the second alternative. The point is to show that human action displays a greater range of (analysable) semiotic resources than what is allowed by traditional (conversational) analytical approaches. Those multiple semiotic fields mutually implicate and elaborate each other, and make possible to constitute particular fields of practical reasoning, such as scientific observation and mathematical calculations. In Goodwin's words 'the construction of action through talk within situated interaction is accomplished through the temporally unfolding juxtaposition of quite different kinds of semiotic resources' (1997: 3).

At this point, we can establish, with Goodwin and others, that the main aspect of the analysis of the classroom-as-installation consist in asking how unique semiotic devices are juxtaposed to more general structures of action of the kind

elucidated by conversation analysis. The argument 'centres upon a complex interplay between the *normative structures* of conversational interaction and the *communicative affordances* offered by different forms of technology' (Hutchby, 2001: 13). It constitutes less a question of imposing 'explanatory burden' (Latour, 1990: 23) on the material configurations of settings – as well as on imagery and formal notations – than one of elucidating how their association produce particular semiotic complexities. Therefore, I take that the analysis of the translations between different semiotic media is totally compatible with looking at how events unfold in sequence and implicate each other. Differently from Hutchby, though, my use of conversational analytical research is not concerned with producing the description of the 'normative structures'.

4.2. Mediating mathematical educational order

As I have indicated earlier in the discussion on the nature of mathematical knowledge, the material practices by which mathematical arguments are conveyed are often overlooked, analytically, on behalf of descriptions of how the 'mind' comes to engage with 'pure' mathematical logic and aesthetics (Papert, 1979). It is as if there is no 'mediation' to be considered: unlike 'synthetic' knowledge[29], to use Kant's terminology, mathematical concepts are not to be found at the end of experimental workings neither are they to describe our everyday experiences, like the narratives. The growth of the mind and the growth of the sciences are structurally similar, both under the driving force of formal logic, Piaget would say. In recent years, with the growing interest in culture and discourse in psychology, the investigation of mathematical learning and understanding came to include new actors, such as diverse representational systems and varying cultural ways of using mathematics to structure public activities (Carraher et al., 1988; Cole and Scribner, 1981; Lave, 1988; Saxe, 1991). Under the influence of the later

---

[29] Knowledge derived from experience, rather than from pure deduction, as in the case of 'analytical', or tautological, knowledge. Those categories are important in Kant's philosophy, insofar as they are used to establish the existence of *a priori* synthetical judgements, that is, non-deductive knowledge that is nevertheless previous to experience, e.g. space and time.

Wittgenstein, also the role of conventionalisation and the inexorability of formal, mathematical 'language games' have constituted new forms of readership to the problem of the relation between 'mind' and mathematical knowledge.

Nevertheless, the ways in which those features are accomplished in practice, in face-to-face (educational) interaction, are still less known, despite the existence of a so-called 'mathematics education' research field. The reason why this is so is because that field has grown out of an explanatory interest in individual cognitive processes, rather than complex 'practices', or 'distributed' forms of cognition (Hutchins, 1991), and is historically linked to theoretical orthodoxies in developmental psychology, especially Piaget's and Vygotsky's. Here, I am interested in the new, 'cultural' form of readership to the question of mathematical learning and teaching in particular. The mathematics classroom constitutes a resourceful semiotic context, the 'staging' of which counts on – and arrives at – several 'knowing' objects, linguistic and other. The nature of this relation, between linguistic resources, on the one hand, and 'other', non-linguistic, on the other, is the point of this chapter. For example, in transcribing the language used in the classroom lessons that constitute the basis for this study I frequently came across examples like this, in which teacher and pupils are starting to perform a 'division':

Extract (1) 3rd grade, EE, 3:

```
T:          look here in in the letter a (0.8) we're seeing (.)
            erm:: (0.4) which is the- the dividend?
            (0.4)
PP:         ninety ↑[two
T:                  [ninety two ((clears throat))
            ninety two that is going to be divided ↑by=
PP:         =four
```

In the following, the 'class' has just divided 9 by 2, and obtained 4 as an answer. They then apply a 'reverse' operation, multiplying 4 by 2, resulting in 8, a number smaller than the integer they have divided:

Extract (2) 3rd grade, EE, 3:

```
T:        ↑but I have nine (.) to get to nine still how many
          left?
PP:       O:::NE=
T:        one left (0.8) now what what am I going to do?
PP:       [take the two
P:        [bring down the two
T:        hm? (.) the two. you observe that here in the
          division (.) I started dividing ↑by the ten >wasn't it?<
P:        °it [was°
P:            [was
```

The problem opens up at first as an attempt at transcribing as well as a call for clarifying 'indexicals' (Garfinkel and Sacks, 1990), to later reveal itself as the question of how talk is linked in a documentary chain with other media in 'wired' settings such as schools, laboratories, professional spaces, etc.

As the analysis of these transcripts goes, there are a few intriguing and, perhaps, potentially obscure passages as far as 'readership' is concerned. To mention a few: in Example 1, the language being used brings the attention of the class to a location, 'here', and more precisely, 'here in the letter a'. There is no such a conventionalised physical space in this classroom called 'the letter a', so that it must refer to a rather temporally relevant feature of this particular 'lesson', to an 'object' that constitutes and is constituted by this ongoing event. Still, 'the letter a' is happening *somewhere* to be seen, and that place is attended to by everybody in a coordinated move that dispenses with the formulation, in so many words, of what such a place is and where to find it. Further indicative of its co-ordination with its significant surroundings, the language provides for the assumption, attestation or obligation of this orientation *as* shared ('we're seeing'), and is accountably consequential for what is to be found next, projecting it as 'see-*able*'.

Bearing only the transcript in sight, it can be said that that display of joint orientation rightly and properly guides the witnesses of the 'letter a' and its workings as much as it potentially conceals from the readers of these words.

That is because 'here' is a *deictic* word, which presumes, demands and constitute co-orientation in order to acquire a sense, and the sentence 'we're seeing' excludes the readers from participation (just as it includes multiple witnesses in context), for there is nothing to be seen here, in these pages so far, that can be identified with the properties of the actors' visual field (so as to potentially include the reader as part of the 'we'-group). The visual field in case is crucially the same for both teacher and class, instead of each single individual in the classroom having to find a display of 'letter a' in their notes or textbooks, for which the word 'here' would probably be inadequate or too vague an instruction, and in which case a more specific or extensive instruction would be needed.

In the sequence, the teacher asks 'which is the- the dividend?', for which she gets the answer 'ninety two' from the *class*[30]. Keeping track of our 'guess the transcript' exercise we can conclude that 'dividend' and 'ninety two' are related to the observable 'letter a', previously referred to in the sequence, therefore distributed in *some* space, and that the pupils *find* 'ninety two' *where* they find the 'dividend' (for which the former is a determined numerical value). The joint orientation to the same 'field' of action is, again, the unremarkable basis on which this is so effortlessly accomplished. Without it, the appropriate answers to a then potentially nonsensical question ('which's the- the dividend?') [31] could range from 'I don't know', 'how could I know?', 'the value to be divided', to 'the dividend is the thing that always come on the top left'![32]. We can reasonably say that that way of questioning is accountable for providing a 'findable', 'discoverable' answer, indeed, for projecting and 'eliciting' it as part of its own design (Edwards and Mercer, 1987). 'Dividends'

---

[30] See the discussion related to the use of the category 'class' in the previous chapter.

[31] In that sense it is relevant to observe that the original Portuguese word in the teacher's question ('*quanto é o- o dividendo?*') translates literally into 'how much', not into 'which', as the analysis and the English translation suggests. My choice for translation was due to the view that the question instructs the search for an identifiable discrete unit, instead of instructing a calculation procedure, as the choice of 'how much' might suggest. The question is designed in such a way as to unequivocally get a 'number' for an answer.

[32] In Brazilian classrooms the notation for 'divisions' goes as follows:

```
e.g. 92 ∟ 4          where          dividend ∟ divisor
     12   23                        (remainder)  quotient
     (0)
```

are things that are written, objects to be inspected *in* and *as* 'school mathematics' in literary form, as other competent ways of performing a division outside school clearly indicate (Carraher et al., 1988). The point is that the question in extract 1 is itself contextualised and made relevant by its boundaries with other representational media; its 'material' surroundings are crucial for its design, and at this point we can start seeing how the transcript is incomplete as a 'cultural gloss' (Ochs, 1979).

Example 2 presents a case for how the contingent efforts of the actors in the classroom are continuously referred to, translated, interpreted, accounted for, in relation to the memorable records against which knowledge claims (and disclaims) can be checked and conventionalised (or erased), in a sort of 'documentary' relationship (Garfinkel, 1967). The use of indexical terms such as 'here' and the definite article preceding the numeral 'two' ('the two') runs in a way similar to the first example, and arguably works as to index some form of 'optical' record (Henderson, 1995). We can see that at the end of the extract the teacher calls the class to 'observe' that in the calculation on display she started dividing by the 'ten', and the subsequent question tag asks for a confirmation of that as recognizable, amounting to a public sense of known or 'given' information (Edwards and Mercer, 1987). The teacher's injunction at the end of the extract ('you[33] observe that here in the division (.) I started dividing ↑by the ten >wasn't it?<) works as an account for 'bringing down the two' from the 'dividend', to be added to the *remainder* of the division, where (4), the divisor, has already divided the first integer (9) of the original intended dividend (92). The first integer is 'the ten' referred to by the teacher ('I started to divide ↑by the ten >wasn't it?<'), who thereby introduces a distinction into the visibility of the 'place value' of a number in a numerical expression (Walkerdine, 1988)[34]. In practical terms, instead of getting down to the

---

[33] 'Vocês' in Portuguese, therefore plural.

[34] I take it as more or less irrelevant, as far as the present demonstration is concerned, to establish whether this is the first time 'division' is being conceptualised in this classroom (it is not, by the way!). Although classrooms trade on formulations of 'common knowledge', or what is already 'given' information (Edwards and Mercer, 1987), those are interwoven with the reiterated formulation, every now and again, of a reportable order. So even though this is not the first lesson on division, the 'division order', and its sequential analysability, is to be found 'at all points' (Sacks, 1992).

business of having a calculation done, the teacher makes observable, inspectable, reportable, the documentary relevance of having 'the two' brought down as the 'unit' in relation to the 'ten'. The relevance of this for the present argument stems from the particular material display of the lesson and its presentation in conducting the activities of teaching and learning, by making artefacts and representational conventions an integral part of the documentary processes that constitute the classroom and the accountability of knowledge (re)production in it.

These two examples present a number of interesting passages that suggest a joint orientation to the co-presence of an 'activity field' (Goodwin, 1997) that can be seen and acted upon by the teacher and the students alike. The displays I am referring to are, of course, the inscriptions on the *blackboard*. Those inscriptions consist of numbers, lines, diagrams, etc., and they are introduced, shaped, conventionalised as the lesson unfolds. Having said that, I would remark that these observations did not intend to exhaust the analysis of such passages. Their function is to bring the blackboard and its often *analytically* unremarkable presence into *analytic* attention. The ubiquitous presence of the blackboard in the classroom has made it a rather 'uninteresting', 'invisible' object for the analysis of classroom activities, usually confined within the description of the talk between speakers and hearers as the totality of the relevant resources, just as the 'telephone', as a form of technology, was barely taken into account in the early formulations of conversation analytical problems (Hutchby, 2001).

It does not take much of a 'thought experiment' to get a sense of how different 'lessons' could turn out to be without the support of (or association with) the (analytically) 'invisible' blackboard and other technological-representational devices. At this point, such a thought experiment is suggestive, especially if, like in the sciences, classrooms can be taken as contexts for production of literary practices of 'representation' and 'observability' (Lynch and Woolgar, 1990; Shapin and Schaffer, 1985). Such an experiment can take the form of the questions: How would the relationship between teacher and class be structured? Would there be such a relationship, that is, the two-part exchange

system between the teacher and pupils? If so, what aspects of such a relationship would be affected by the absence of the blackboard? What would be the intelligible design of the things teacher and pupils talk *about*? Would scientific (verbal) formulations document anything but themselves? Is the use of the blackboard just a tool for representing talk, or is there a finer interrelation between them?

So far I have tried to address the curious 'transparency' of the blackboard (that is, the fact that we, as analysts, do not see *it*, do not see how at points it mediates a paradigm of classroom order) by analysing the 'discourse' *around* it. Thus, the practical relevance of the blackboard can be established by tracking the language associated with its use in a (mathematical) lesson. I would suggest, as an starting assumption and provisory answer to the questions brought forward by the thought experiment above, that the blackboard allows the 'class', as a group, as first-hand observers of the 'knowledge' at stake to be (taken as) responsive (in relation to a field of mutual orientation) and responsible (accountable for 'observing' it, that is, as 'learners'), as a 'witnessing' and 'validating' resource/audience, at the same time it embodies and enacts the very activity of *doing* mathematics[35], of setting up mathematical representations and inference routines. To put it the other way round, the *activities* on and around the blackboard mediate the process of holding the (mathematics) classroom collective accountable as a design for *any* observer-participant, to be potentially graspable to whoever would enlist themselves amongst the pupils sitting down facing the blackboard with the incumbency to 'learn'[36].

---

[35] I mean this as opposed to just *representing* the activity of doing mathematics. Whereas it is true that mathematics often operates in the classroom by establishing relationships with 'extra-mathematical' entities (which means, for the sake of the present argument, 'extra-writing', like mathematising pedagogical materials such as building blocks), it is also relevant to say that in professional, academic contexts, its developed, 'pure' grammar accountably operates as an entity in its own right, and is generally taken as the best account of the work of its own proving (Livingston, 1986). In those circles, sketching on inscribable surfaces is sufficient condition for the lived-work of mathematical deduction and proving. It is in that sense, as I will discuss later, that 'screening' surfaces like the blackboard afford mathematical activity and provide the basis for its enactment, in a way that the non-tautological sciences cannot afford, although the distinction, as I pointed out, is not to be taken as a radical one, at least as far as educational settings are concerned.

[36] The 'teaching' is also happening for the analyst, as a potential classroom subject, as it is happening for any third part concerned with watching and following the lesson.

The previous exercise holds some interest not only because it allows us to go 'analytical' by bracketing off an embodied understanding of the situation, in this case involving the use of an artefact such as the blackboard, but because it shows that language returns us to co-presence and local action in two ways, both by showing its commitments to address *reference* (in the form of technical objects, space and people), and, at the same time, by allowing us to investigate ways in which particular associations with those spatial and material 'signifieds' shape the economy of its use for the contextual competencies of the here-and-now. The relation between grammar, embodied activity and graphical representations are configured in a complex way in the ongoing work of 'demonstrating' knowledge. In the literature on talk-in-interaction this has been discuss to show, for example, how physicists' arguments at the blackboard, especially the use of pronouns and predicates of 'motion', work to construct the identity of physical referents, its character as animate/inanimate actors and the cogency of subject/object distinctions, and that this process is integral to shared understanding in the site of demonstration (Ochs, Gonzales and Jacoby, 1996).

Extract (3) 3rd grade, EE, 2:

```
1.   P:    miss can it be done, four hundred divided by three?
2.   T:    four hundred divided by three? (turns to the
3.         blackboard)
4.   P:    uh huh.
5.   T:    ↑for sure (.) there you have a normal division. You
6.         do four hundred (.) divided ↑by (.) three (1.0)
```

At the beginning of the extract, the teacher repeats out loud the pupil's question back to the class (line 2: 'four hundred divided by three?'), turns to the blackboard and then answers affirmatively ('for sure'), adding that 'there you have a normal division'. Returning questions and answers to the public domain of the classroom as a whole, accountably as a responsive, 'sympathetic' agent on its own right, indicates that 'lesson talk' is

meticulously designed for third parties, indeed for a participatory audience (as opposed to a 'overhearing' one); it plays a role not only as a simple response, but as a 'validating' mechanism, one by which the teacher casts the moral and rational force of the agreement, of her delivery of knowledge as publicly observable and understandable. This is integral to the constitution of the *class* as a public body. We can propose, perhaps, that this kind of repetition is a strategy used in contexts designed for third parts, in which contributions to the talk attend to an accountably public and witnessable 'morality', for which questions, instructions, clarifications are to become everybody's, and emanates from 'reason' rather than from authority. I am suggesting that the blackboard is central in mediating this feature of classroom instruction.

It is interesting to address the (practical) relation between the reference to 'normal division' in line 6 and those shared forms of public witnessing. Although we do not need as yet to worry about the semantics of it, accountably 'normal' forms of division can comprise the production of (1) whole numbers (integers, with no remainder) – e.g. 9:3; (2) Divisions with decimals, that is, which quotient is not a whole number – e.g. 9:4; (3) Negative numbers, or less than zero. Can we decide, on the basis of the extract above, which one is the logical framework in which 'normal' is an appropriate description? It seems that an alternative way to go about it is to consider that here 'normal' projects both (1) an appeal to common knowledge, 'normal' being one-of-those-things-has-been-solved-before, that is, a already recognizable and workable problem (notice that the opening question also includes 'too'), and (2) relevantly, a foundation or account for the act of *demonstration* on the board; the blackboard affords 'normal' to be heard as 'doable'. Without focusing now in detail on the merits of what the teacher *meant* by 'normal', it is certain that she treats the 'normality' of the problem as a demonstrable matter. The point here is to show, from the extract above, how some aspects of the appeal of such demonstrability is interactionally organised.

From lines 2-3, we learn that the teacher turns to the board as the 'arena' in which the answer is to be demonstrated, even before answering it! It is as if addressing the board and the demonstrability of the answer as its very

condition describes a hearer's maxim (as referred to by Harvey Sacks) made relevant by the question. It shows that the question's design, in its apparent (and perhaps logical) prospects for a yes/no answer, would be incomplete at best without addressing the construction of the answer as a step-by-step written demonstration. The activity at the blackboard, and its design of visibility for a larger audience, embodies a quality of mathematical work that is vital, but nevertheless non-documentary (Suchman, 1990): that of carrying out the actions and operations that extend the simple acknowledgement of a 'mathematical' object into the realisation of the predictive experiments that mathematical assertions open up, but do not evidence *per se* (Rotman, 2000). As we saw briefly in chapter 2, Rotman refers to this as the work of a mathematical 'Agent', an automaton that represents a 'skeleton diagram' of the subjectivity that sees and interprets mathematical objects and their appropriate applications, and whose function is to 'execute' the operations whose outcomes will be selected and judged as a corroboration of the prediction made by a mathematical assertion. In chapter 6, we recapitulate some of those distinctions in order to discuss the attribution of agency in learning mathematics.

Whereas it is the case that these extracts contain much simpler mathematics than those we could find in algebra, geometry, or dynamic equations in physics, they nevertheless represent teaching efforts for which the procedures and expected outcomes are taken to be invisible for the young learners. In that sense, the demonstration of these simple mathematical notions and operations rely on what can be taken as the work of Rotman's 'Agent' as part of a signified 'skeleton', so to say. The former gain its local intelligibility, in part, from the second, and that is why the blackboard is analytically relevant. Of course, as the automaton's work is not necessarily or predominantly verbal the question subsists as to what the analyst should include in the transcript. Not to include non-verbal details is not an option, as I hope to have argued. Consider this from extract 3 (the underlined symbols represent the inscriptions written on the blackboard simultaneously to the words above them):

```
T:      ↑for sure (.) there you have a normal division. You
```

```
do four hundred (.) divided ↑by (.) three (1.0)
     400              L            3
```

As can be seen, there is an improvement in relation to extract 3, consisting of the underlined sequence of 4, L, and 3. This line shows the concomitant production of words and written symbols. The remarkable work it does is to indicate that lines 5-6 (above: you do four hundred divided ↑by three) are not redundant in relation to line 2 (four hundred divided by three?) and its subsequent corroboration ('four sure'), as a verbal-only transcript might suggest. Moreover, it shows that 'you do' (5-6) takes its sense and place as a kind of running commentary on the actions being taken on the blackboard, both verbal and written, the primacy or basic character of any of them being a false question that denounces the crucial incompleteness of verbal-only transcripts; 'you do' = [doing it], and rather than representing the redundancy I referred to it performs something new within the 'paraphrases' of the original question.

It remains an additional problem. 4, L, and 3 constitute entries to a structured medium that has a certain configuration in space, the positions of which will determine how each item operates. While the increment in transcription provided above was important, it produces only the visibility of temporal associations without addressing the question of what configuration the pupils see on the board. Although mathematically trained adults can most certainly infer how the writing goes from the very suggestion of a calculation, I suggest that representations (ultimately pictures or video of the lesson) of spatial distribution on the blackboard are needed insofar as the pedagogical problem is precisely that of how children will be able to recognize and simulate those activities all over again[37]. In the following, PR means 'partial representation' (of a running algorithm):

```
T:    ↑for sure (.) there you have a normal division. You
      do four hundred (.) divided ↑by (.) three (1.0)
         400              L            3
```

---

[37] I am aware that in British schools, for example, the shape of the algorithm is not the same as the one I have been discussing (see footnote 11).

Having started from a critical examination of the language used, as opposed to simply registering the co-presence of the blackboard, has allowed us then to convey a sense of the importance of that co-presence and how it affords language-in-interaction to be designed in specific ways due to its association with it. The discourse used suggests, in this case, the relevance of 'unpacking' its associates, or not bracketing off reference, as well as of 'following' all the relevant actors in these activities of seeing, writing, demonstrating (Latour, 1987).

I would suggest that in the case of the contexts under analysis here, one productive way to go about describing the practices they amount to is to treat the blackboard as a device that mediate (mathematical) knowledge and classroom education (that is, it is 'mathematical educational'), so as to produce (1) classroom-bound mathematical order, designed-to-be-learned stuff, on one hand, and (2) mathematically oriented, subject matter-accountable classroom competence, on the other. There is no order outside these multiple associations; no formal, pre-packaged description of social order (and the teachers for sure do not use them in accounting for competence!) can adequately account for the actual development of those events. The mathematics taught in the classroom describes and analyses a form of social order, that is, it projects the kinds of competences it requires in order for that context to be accomplished as a, say, mathematics 'lesson', and vice-versa: the ad hoc, contingent public dynamics by which mathematical representations are constituted, e.g. the way notations and inference routines are negotiated and made conventional, can be understood as integral to mathematics (see chapter 5; see also Livingston's (1986) account of the 'lived-work' of mathematical proving, discussed in chapter 2).

Here, I take the effort of pointing to some of the 'affordances' of the blackboard in the classroom, understood as a mediator in an installation, or learning machinery (Foucault, 1977; Macbeth, 2000). Lucy Suchman (1993)

has designed a research question on the 'affordance' of technological settings in a way that has great relevance here:

> 'I am concerned here with the following problem: how is it that the material practice of a possible class of work sites, and of one site specifically, constitute those sites as *centres* for the coordination of human activity, in two senses: first, as concerned with the production of a coherent *temporal* relation between prescribed and observable-reportable events; and, second, as constituting spatial centres within an extended system of distributed activity' (p. 113).

I would like to suggest that the blackboard is a centre for coordination of several activities, ranging form question-answer strategies, through extensive indexical work involving gesture and written representations. I shall translate Suchman analytical interest for my purposes by focusing on two 'mediating' roles of the board in the classroom:

(1) The use of the blackboard supports the ways of addressing the 'class', being designed so as to establish relations of participation and accountability, that is, with holding and distributing the interactional floor and with producing normative and consensual orientations. However, while some authors focus on the 'ideological' basis for this 'sympathy of numbers' (Hamilton, 1980; Macbeth, 2000), I prefer to focus on the 'technological' aspects of that process.

(2) The activities at the blackboard embody and stage mathematical 'demonstration' through construction of (erasable) records that are 'concrete', that is, visible, inspectable, and, at the same time, 'conceptual' (Suchman, 1990); it makes (mathematical) knowledge 'intelligible' for all practical purposes (Garfinkel, 1967).

4.3. The blackboard as a social and representational device

The blackboard constitutes one of the oldest known educational technologies and its use is such a commonplace that it hardly 'seen' as a technology at all

(de Beaugrande, 2003). It is likely to have served well educational needs to transmit more 'formal' kinds of instruction, becoming a 'trivial' piece of classroom furniture only in the 19th century, as 'lecture'-like teaching have won over the 'catechism' as a more popular method for the transmission of knowledge (Hamilton, 1980). At the same time that lectureship was emerging out of the more independent circles of protestant preaching activities in 16th century England (Hill, 1964), books like Comenius' *Orbis Sensualis Pictus* (1658) were advancing progressive educational recommendations that stressed the coupling of knowledge and pictorial representation in founding children's understanding of the world. In 1630 Comenius had written that pictures 'in books and on walls please them [children], so they ought not to be denied; rather, one ought to take pains to provide and point out such things to them' (Comenius, cited in Bowen, 1967). Comenius designed his famous *Orbis Pictus* and its picture/explanation order to the effect of embodying his principles of 'learning by doing, with simple concrete experiences and seeing the whole world as a classroom' (Ibid: 26).

The idea of a 'blackboard' has been recently used in studies of human-computer interaction to designate a 'context architecture' that centralises the posting of messages to a common server, as opposed to more 'distributed' architectures, for which there is not only one standard communication link for each component (Winograd, 2001). In the classroom context, the practical relevance of the blackboard is constituted by and for the actors as a field of joint activity, affording, on the other hand, for this activity to take peculiar turns. The theoretical-analytical relevance of the blackboard for a 'praxiological' description of educational activity stems from the fact that it is one of the links to the production of that 'local educational order' (Hester and Francis, 2000). By that I mean to emphasize a point I have made earlier about the inadequacy of rendering a context's 'unique possibilities' intelligible as a variation of a single formal pattern of social interaction. Thus, although the moral order of 'conversation' and its variations constitute the unmistakable basis by which the actors live their (educational) experience, it does not preclude the observations that (1) there are other semiotically recognizable phenomena of order and competence that are nevertheless non-conversational;

and (2) that if 'conversation' means anything to the way we understand what participants are doing when learning mathematics on the blackboard, then its analyzability has much to gain, at most points, if seen in conjunction with other forms of competence.

There are cases, of course, in which a focus on discourse alone has produced a number of interesting empirical findings, especially in relation to the distribution of the rights to speak in the classroom (McHoul, 1978; Mehan, 1985). The most well known are those in which exchanges consisting of a 'question-answer-evaluation' sequence, where the teacher occupy the first and third position, are discussed. In those studies, whether a lesson is about mathematics, grammar, geography, chemistry, physics, etc., is more or less irrelevant; the analytic footing is on the organization of the interactional format between teacher and class. It is relevant, though, that much knowledge has been produced about classroom interaction from data consisting of 'beginnings' and 'in-betweens' of classroom lessons, as well as 'telling-offs' (Payne and Cuff, 1982; Danby and Baker, 2000). Those are clearly classroom-bound phenomena, but ones that deliberately 'miss' the things the lessons are about. Moreover, it could be argued, if the analyst wants to study power, control, and identities in the classroom how would mathematics, say, be of any relevance?[38] Despite all of this, there are a few interesting studies that have discussed the relations between sequential organization of interaction and the process of making visible a subject matter's own technology of analyzability (Heap, 2000; Hemmings et al., 2000; McHoul and Watson, 1984; Macbeth, 2000; see also chapter 2).

Lessons relate to the blackboard as a field of demonstrability and, consequentially, of joint 'observability', in which the 'cohort' (Cuff and Payne, 1982) is addressed as a 'whole'. The members of the same linguistic community can recognise the teacher's words during such lessons but

---

[38] See Valerie Walkerdine's works, especially *Counting Girls Out: Girls and Mathematics* (1989), for an affirmative answer.

nevertheless know little about what she is talking[39], and in that sort of discovery-oriented cognitive niche that 'representation' finds its place (Woolgar and Lynch, 1990). Classroom discourse is accountably discourse that *represents* something else, worldly sates of affairs (Macbeth, 2000). Similarly to the sciences, whose accomplishments inform its practices, the classroom's 'overridingly epistemic focus' (Lynch, 1993) is constituted by producing relations between *signifiers* and *signifieds* of various sorts. However, this operates not in a general, foundational semiotic design, but rather in a way that allows participants to 'see' both sides of the link. It operates by a principle, again common to the sciences, according to which a relation between 'represented' and 'representing' is to be established as a visible pair (Garfinkel et al., 1981). In the data I analyse, the interactional economy by which one arrives at the object through the interplays between represented and representing is, nevertheless, *classroom* business, designed for educational purposes, accountable to and in accordance with educational 'ideologies'. Research has quite often forgotten that lessons cannot, or have not, proceed(ed) without mediators of this kind.

Suchman (1990), in one of the few references to the use of the (white) board – in this case in computational design studies – refers to it as a 'technology for representation', one upon which practice is 'duplicated', 'transformed', reflected on. I have mentioned earlier in this chapter that this qualification of the blackboard can be contrasted with other forms of technological mediation, most prominently the so-called 'technologies for communication', such as the telephone (Hutchby, 2001). Suchman poses the problem arguing that

> 'A common technology for representation in our laboratory is the "whiteboard". We begin with the observation, due to Livingston (1978), that the inscriptions on a whiteboard – lists, sketches, lines of code, lines of text and the like – are produced through activities that are not themselves reconstructable from these "docile records"' (p. 314).

---

[39] See Goodwin's (1997) and Lynch's (1993) similar observations about their observer's positions in scientific laboratories. See also next chapter on 'transparency' of actions and accounts.

It is important to note that the use of the blackboard is not to be seen as a formal description of the activity of writing itself, as in general accounts of writing, or as a systematic, logical depiction of the written symbols being used, but rather as how those activities and representations are produced as accountably relevant in/for a context of practices; of how they produce a set of actions and its inspectable features *as* mathematics (Livingston, 2000; see also chapter 5). It is tempting for the sociologist to proceed so as to take Suchman's observation (above) as an opportunity to engage analytically with 'social' things, such as of 'writing' and 'reading', as reducible to discursive negotiations. Rather, I want to explore the idea of the 'classroom-as-installation' (Macbeth, 2000) as 'materially heterogeneous' (Law, 1987; Suchman, 2000), some aspects of mathematical 'agency' and intelligibility being inextricably linked to the activity of writing on the board. It is in that sense that a looking at the associations between formulating knowledge and engaging the participants in *doing* things with 'hard' technologies and representations is of great analytic relevance. As we shall see in the next chapter when we look at 'instructed actions', although sociological description has established, at least as far as conversation analysis is concerned, the functional independence between an 'explaining' and a 'doing' machinery of social action (Sacks, 1990), taking the former as its proper object of study, the public orientation to the learnable, visible, graspable 'sense' of technological-representational devices and their use in the classroom (and other locations, such as the laboratory) warrants its place as a field of competence and, consequentially, as a relevant analytical and theoretical object.

The blackboard does not stand out at first as a form of 'technology'. It is not a technology for communication (Hutchby, 2001), like the telephone, neither a machine nor a program one communicates with, nor it is an expert's system, working out a built-in operational system or axiomatics of any kind. It stands there, hung on the wall, behind the teacher's *space* and widely visible to the pupils placed in several rows facing the teacher. The blackboard, one might say, is not that much different from 'just' a piece of paper. However, it all depends on how keen one is to argue that a piece of paper and what it 'affords' in terms of 'literate' action, historically for human-kind, and situatedly for a

specific context of practices, is better described as 'just' that (Bruner, 1990; Havelock, 1963; Goody, 1978; Olson, 1994).

Suchman's project (1990) is, again, a valuable source of inspiration. She establishes as an assumption of her work that the board 'both supports and is organized by the structure of face-to-face interaction. On that assumption, our analysis is aimed at uncovering the relationship between (i) the organization of face-to-face interaction, (ii) the collaborative production of the work at hand and (iii) the use of the whiteboard as an interactional and representational resource' (p. 314). In the present analysis, I try to uncover some of these topics. In her discussion, the author establishes a set of eight conjectures about the representational workings of the (white) board (pp. 315-317), some of which are of relevance here. Some of those conjectures, drawn from her observations, are:

1.   The board as a medium for the construction of 'concrete conceptual objects' (p. 315), by which represented objects can be 'run', that is, can be structurally inspected and developed;

2.   The board organises shared orientation to a common interactional space, where the status of objects as 'incomplete, problematical, satisfactory and the like' (p. 315) are continuously negotiated not only through spoken discourse, but also by other signifying displays such as bodily position and gestures;

3.   Writing on the board and the discourse associated with it are systematically organised, so that 'the board provides a second interactional floor, co-extensive and sequentially interleaved with that of talk' (p. 316). Also, working on the board can be conversationally relevant, insofar it allows for the writer to take and hold the floor; moreover, the activity of writing document the talk, translating the writer's understandings of it into other forms of representation, extending his/her previous turn, or projecting relevant features of the subsequent ones;

4.   The order of the writing on the board reflects the conceptual as well as the local, sequential order of its production. The arrangements of the items on the surface depends both on how next entries are to be fixed spatially in relation to previous ones, as well as on its conceptual and structural relevance;

5.   'Items entered on the whiteboard may or may not become records of the event' (p. 317). Here, Suchman calls the attention to the flexibility of the writing on the blackboard, and the potential for erasing marks that will not constitute part of the formal structures and accounts of the activities and contingencies of producing them. Thus, some of those 'ghost' entries 'may be communicative without being documentary' (Ibid: 317).

For Suchman, the methods of work at the (white) board must be described in terms of how we understand the role of 'representations' in diverse practices, such as the laboratory. In her account, the use of the board amalgamates a phenomenon familiar to the sociologist of science according to which the contingent relations between 'shop talk' and technology use in scientific environments are noticeably absent from public scientific statements and artefacts. In order not to take this observation as an 'irony', an evaluative critique of science, her recommendation is that representations are better understood as products and resources of situated practices, and that its technological devices are the taken-for-granted basis of scientific reasoning. In classrooms, representational devices are the product of the joint effort between teacher and cohort as they establish the uniqueness of their real-time practices as 'mathematical educational', and they are resources to the extent that their presence is instrumental in affording a complex and heterogeneous semiotic economy of discourse, writing, gesture, and material artefacts.

Eric Livingston's work on the ethnomethodological foundations of mathematics (see Chapter 3) can also be mentioned as a site in which the production of mathematical proofs (its lived-work), as a field of practical action, can be inspected for its 'indexical' features, even though his study is not 'observational'. In attending, and drawing our attention to, the details of a temporally and spatially organized circumstance of geometrical proving, he

notes that 'in presenting this material in a lecture, a prover uses his embodied presence to the blackboard and to the audience to achieve the exhibited precision of his work and talk' (1986: 191). Although Livingston argument is organized around an 'abstract' work-site of mathematical proving (Bloor, 1987), in the Introduction to his major work on the topic one finds a text saturated with deictic terms such as 'here', 'this', 'this line', 'this case', 'this triangle', and so on, seemingly designed so as to guide the reader through the local and graphical demonstrability of the mathematical argument at stake. The text amounts to a sense of what is to be literally 'seen' – diagrams, angles, lines, arrows representing movement and change, some of which will disappear from the final 'proof-account' of that mathematical entity – in producing the structuring of our attention to the relevant local circumstances. It could be argued that the analysis in the text somehow simulates a blackboard-like device (the Introduction of which is the transcription of a talk!); it makes (mathematical) order inspectable, classifiable, visible, in the details of the local determination of a proof, reaching the 'precision of his work and talk', as referred to above. Livingston's style aims to deliver a sense of proving as practical action, one in which a proof stands there not despite its representational devices and other 'social' contingencies, but *precisely* because somebody is proving it, with all that entails, including establishing the basis on which the observer is to be positioned as the witness of something that has a 'necessary', 'axiomatic', 'foundational' character. Even though a common assumption in the 'natural accountability' of mathematics is that of the independence of a proof from its material demonstration (in the sense that the proof is not 'made up'), those indexical features 'voice', for the written text, the lively process of 'sketching' the argument on a surface for its continuous inspection, or, following Rotman (2000), they describe an automaton that operates as an extension of the (mathematical) 'subject' that understand that circumstance and its steps as a 'proof'. In learning mathematics, the 'Subject', the one who understands, and the 'Agent' (the automaton in Rotman's words) who carries out actions and manipulations of symbols are inseparable.

Next, we see how an arithmetical calculation is 'run' on the blackboard for a group of third-graders. The transcript, also from a lesson on 'division',

introduces an event in which sentences are peculiarly interwoven, mostly as to complement each other, having the blackboard as a 'signified' template (the symbols for 'writing' and 'pointing at' stand in a temporally contingent relation with the spoken words. The simultaneous writing features in non-numbered lines with the symbols written on the blackboard reproduced and underlined under the concomitant word; the simultaneous pointing is marked with ▲. T = Teacher; P = Pupil; PP = More than one pupil; PR: Partial representation on the blackboard):

Extract (4) 3rd grade, EE, 2 (continuation of 3):

```
7.    T:    can I divide four by three?
                     ▲         ▲


      PR:   400 ∟3


8.    P:    you can
9.    P:    you can
10.   P:    °you can't°
11.   T:    can because the four is bigger than the three (.)
                            ▲                      ▲
12.         >so I say< four divided by three ↑makes=
                            ╱
                            ─
                            ▲                   ▲


      PR:   4'00 ∟3


13.   PP:   [=ONE::
      T:    [ 1


      PR:   4'00 ∟3
                1


14.   T:    [one times three?
                 ▲       ▲
15.   P:    four
```

```
16.  P:    °four°
17.  PP:   [THREE:::=
18.  T:    [↑one times three?
19.  PP:   THREE:::
20.  T:    =to four how many lacking?
                  ▲
21.  PP:   [ONE:::
22.  T:    [and now?
              1
```

PR: 4'00 ⌊3
       1   1

```
23.  P:    bring down the zero
24.  P:    [bring down the zero
25.  T:    [I also ↑bring down the zero (.) now how many do I have?
                          ╱                              0
```

PR: 4'0'0 ⌊3
       10   1

```
26.  PP:   ten:::=
27.  T:    =to divide by three how much is that?
                          ▲
28.        (2.0)
29.  P:    three
30.  P:    [three
31.  P:    [three
32.  T:    (turns to a pupil behind her) five? ↑it is=
33.  PP:   three
     T:       3
```

PR: 4'0'0 ⌊3
       10   13

```
34.  T:    three times three=
```

```
                    ▲              ▲
35.  PP:    NI::NE
36.  T:     to ten ↑lacks=
                    ▲

37.  PP:    ONE::=


38.  T:     =one. now I ↑bring down=
                1_____  ,



     PR:    4'0'0 ⌐3
            10     13
             1



39.  PP:    =ZE:RO
40.  T:     zero. ten divided by three makes=
                0
                        ▲              ▲



     PR:    4'0'0' ⌐3
            10     13
            10



41.  PP:    THREE:
     T:         3



     PR:    4'0'0' ⌐3
            10     133
            10



42.  T:     and three times three?
                        ▲          ▲
43.  PP:    NI:NE=
44.  T:     =to ten?
                        ▲
45.  PP:    ONE::
                1
```

111

```
PR:     4'0'0' ∟3
        10      133
        10
        (1)
```

```
46.  T:    so the result was one hundred and thirty three=
47.        ((Blackens the second 3 with the chalk)) and the
48.        [remainder=
49.  P:    [=miss miss
50.  P:    miss?
```

The extract above represents a fairly common event in the mathematics classrooms I have observed. Some of these routines consist in 'running' written representations on the blackboard. The nature and implications of how such activity takes place is what concerns me here. Most of the exercise is taken with the teacher addressing the whole class (no choice of next speaker, no repairing of multi-voiced, unison-like, multi-speaker turns; see chapter 3). One of the most remarkable features of extract 3 in relation to this is the grammatical abbreviation of its turns. The grammatical contribution from each part's turn is rather short, contained, and the transcript reads at first as noticeably 'truncated'. That might suggest that the representation of classroom events in conversational 'transcripts' is continuously in search for a 'scenario', and I have tried to offer some elements of a more 'embodied' gloss in previous extracts in this chapter. From the outset, and most notably from line 12 onwards, we can notice a question-answer activity constituted by very short injunctions from each part, attending concomitantly to the way an algorithm has been written and developed on the board (400 ∟ 3). Let's call this talk, with Wittgenstein (1967), 'elliptical'.

The question-answer sequences observed are accountable to this semiotically structured board, and from it they bear their very own relevance. It is interesting to consider that, contrary to standard analytical assumptions, the talk itself lends very little to the 'formulation' of the board, to its constitution

112

as 'text'. However, as chapter 3 indicated, partly the analysis of classroom education is the analysis of resources that have evolved as part of regulatory practices that favoured the invisibility of technique and power in support of the accountable governability by 'reason' (Foucault, 1977; Walkerdine, 1988). Just what is 'social' about the blackboard is a question to be tackled analytically: is the overwhelming use of this object in the classroom analysable? In what terms? Is it represented in talk? Does it matter? Could it have any implications for the way knowledge is accounted for and identities are constituted in the classroom?

In line 7 the teacher uses the '4' from '400' written on the board in order to divide it by '3'. This obeys, as we saw in Example 2, the logic of the algorithm in dividing the first integer in the dividend (left) that is larger than the divisor (right). In the same line 'can I' makes the next action accountable, instead of simply performing it for the sake of the teacher's authority. What does the verb 'can' address in this case? Permission, as in 'can I go home now'? No. 'Four' as a multiple with no leftover? No. Bigger number? That seems to be the answer. Moreover, the original question in line 7 projects the 'formulation' (see chapter 5) in line 11 that introduces and justifies the task; it does so it in order to establish its non-arbitrary character, or the invisibility of its pedagogical mechanisms (Walkerdine, 1988). Line 11 culminates in an inference technique according to which numbers divide integers bigger than themselves, or to say it in relation to the dispositions on the blackboard, the numeral on the left side ('4') has to be bigger that the one on the right ('3') of the L shape. The move is important, since it rules out several other inference rules, including the (mathematically valid) alternative according to which anything can be divided by anything. 'Four' is then selected as relevant for the work of analysing and accounting for the procedure used. When it is finally established that four can be divided by three (line 11), a frame is set up at line 12 ('so') for a cascading, elliptical structure of joint talk in which the grammatical terms of the question are omitted, furthermore including the assumption that each step is to be carried on without having to be accounted for like the first one.

This elliptic pattern runs successfully from lines 12 to 45, affording few breakdowns, repairs or assessments (exceptions are lines 18 and 32, in which hearable wrong answers are addressed by the teacher without much explanation; note that in the case of the first one she just repeats the question!). It is composed mostly by *paraphrases*, repetitions that appropriate what has been previously said and recycle it in a new questioning proceeding (lines 14, 25, 34, 38, 40 and 42). It is also populated with utterances whose design points to the *completion* of what has been said before, that is, implies a previous expression as part of the construction of the subsequent action (lines 20, 27, 36, and 44). In the elliptical dialogue answers are paraphrased or simply used as starters for the teacher's subsequent questioning in an artful, integrative way. These methods help to compose a rolling 'commentary' that mirrors the way order is to be found sequentially in the way the notations on the blackboard are run. The point about the elliptical talk in extract 4 is that it potentialises the maximum possible agreement between teacher and pupils while seemingly dispensing with the need for explanatory activity; there are no 'whys' or 'justifications' in the way the answers are made public. Rather, several mathematical competences seem to be taken for granted, both in performing partial calculations and understanding which steps were to be taken in relation to the algorithm being written (appropriately so, if we consider, for example, how effortless the decision-making in line 23 is). These distinctions will come to life once more in chapter 6, when we explore other relevant frames and attributions of agency. For now, it suffices to say that in the interactional format we observe here the kinds of subjective accountability in discourse studied by socio-interactionists seem to be out of the question. As long as this activity at the blackboard lasts there is no sign of the 'subject' of learning.

Two other important aspects of this extract are:

(1) How the 'blackboard activities' enter the shape of linguistic choices, which address once more the question of discourse and its transcription; and (2) the constitution of the 'class' as an 'agent' (chapter 3).

If we look closer we can see how talk, pointing and writing constitute a complex, albeit apparently simple and effortless, action of mutually indexing and documenting each other. For example, the use of *definite article* to designate the numbers in line 11 ('the four'; 'the three') is supported by the reference to the numerals inscribed on the blackboard, although they had been used without resource to definite articles in line 7. Those strategies are clearly classroom-bound, 'arithmetic-relevant activity' instead of 'activity-relevant arithmetic' (Lave, 1988), and questions are designed to refer to general things, as opposed to *this* 'four' or *this* 'three'. Answers, on the other hand, are relevant in terms of their local demonstrability, while questions *per se* are not, although they need to be justified, as in line 11. Note, however, that in both cases, they are mentioned as the teacher points to the numerals.

Pupils' answers are also projected onto the blackboard, as can be seen in lines 13, 33 and 41. Those are pupils' conversational turns, and an exclusively verbal transcript would miss on the fact that the teacher is engaged in making such answers written 'documents' that not only can be used again, but become a record in the changing algorithm. On the blackboard, children's answers are objectified and put to analysis. The blackboard stabilises knowledge and representation for all practical purposes; it displays a checkable summary of the knowledge being negotiated, as well as effectively builds a communication channel that goes form the class to teacher, with no need of resorting to third-parts. Also note that the use of tenses reflects what is currently being done at the blackboard, that is, it points to a 'see-able' action, one that is being at the same time commented upon (line 25: 'I also bring down the zero', which is of course accompanied by writing; see also lines 12, 36 and 38).

Partly, it seems that children are learning how to operate a machine or screen – the blackboard. Actually, the transcript is quite reminiscent of Vygotsky's (1987) description of 'inner speech' in problem-solving situations, in which grammar is notably abbreviated, involving mainly short verbal commands which associate themselves with *subject* and *object* of action in ways that are diverse from those used to communicate. In other words, this 'strange' form of dialogue (from a 'transcriptional' point of view) is similar, in its totality, to a

model of *thinking*. If the analyst wants to describe how a particular scientific subject matter is taught in the classroom, so this kind of detail is vital, because it shows how, in each case, models of verbal thinking are socialized in relation to the technical conceptual features of a setting. Turning away from it would be the equivalent in SSK, for example, to not showing the technical equipment scientists use. The technical equipment, however, is at the very heart of the demonstration because it translates the 'facts' that have to be raised to sustain a general, abstract representation (Latour, 1990). The elliptic talk that mirrors the steps of the operation involving writing, talk and gesture, from line 14 onwards, is incomprehensible without the blackboard format and its role in instruction. Can any of the features of the so-called 'institutional talk' be sustained without the socio-technical objects available and their use in documentary routines? Can we imagine the modern classroom without a blackboard, and if the answer were affirmative, would we have the same interactional resources at hand? Can the two-part conversation between the teacher and the class, and even learning accountability, exist without this unremarkable object?

I have already established, in chapter 3, the logical and practical existence of the 'class'. That was related, then, with the way turn-taking is sometimes designed in the classroom and was primarily understood in the relevant literature as being at the service of a 'managerial' vocation of the classroom: how teachers can deal with so many students at the same time? (Hammersley, 1990). This question finds a natural niche in conversation analysis since that discipline has generally no analytical regard for the content being studied. It is as if CA is only interested in things like beginnings, endings and other administrative activities; one has, indeed, a palpable sense that the managerial question and the selective analytical agenda of CA are inextricably related.

Logically, then, the class is delegated by the allocation of turn-taking, as in chapter 3. If the class was not a class *to speak* there would be no reason for it to exist. However, such system is not 'natural'; in the jargon of CA itself, it constitutes an 'institutional' variation of the classical model of conversation, a deviant case (Drew and Heritage, 1992). However, the class, as *class-that-*

speaks, is not an overhearing, obligation-free agent like in other studies of such 'deviant cases' (Atkinson, 1984). It is *necessarily* active, it is the second in a two-part conversation with the teacher, and it is summoned to 'see' and 'speak' on that basis. At the same time we have also observed that teacher-class conversations are rarely, if ever, the proper context in which the accountability of the behaviour and cognition of individuals is at stake. I want to suggest that such a 'conversation' seems to be, above all, a method of producing shared witnessing and agreement.

Such a consideration, and the reflections made in chapter 3, on the origins and mechanics of the classroom as 'machinery', allows us to go beyond conclusions to be taken on terms that are 'conversational', or even consider conversation as primary. Indeed, the nature of this witnessing agent is best described as an 'epistemic' contract, rather than conversational. It has interesting – potentially derivative philosophically– parallels with the legacy of English experimentalism in the 17th century as described by Shapin and Schaffer (1985): the literary technologies by which witnesses can be multiplied, can observe collectively, and virtually, through specialised authoritative sources on 'matter-of-fact' issues (Shapin and Schaffer, 1985). Hard technologies such as the air-pump, in the case of Shapin and Schaffer's study, which conditioned the discussion over the existence of the *vacuum*, operate alongside *literary* technologies that instruct non-observers (e.g. readers) on how to 'observe' phenomena, and *social* technologies 'that incorporated the conventions experimental philosophers should use in dealing with each other and considering knowledge-claims' (Ibid: 25). Robert Boyle, experimentalist and the main character in this story, was convinced that not only the performance of the instruments and their calibration was crucial in the production of matters-of-fact, but 'the assurance of the relevant community that they had been so performed' (Ibid: 55). Knowledge had to be witnessed and that was to be, according to Boyle, a collective act: 'In natural philosophy, as in criminal law, the reliability of testimony depended upon its multiplicity' (Ibid. 56). In this legal analogy, the multiplication of witness meant delegating to nature its own signature, instead of that of God or the State (Latour, 1993). In the case of the classroom, this epistemology presumes that its technologies

operate within a frame of visual perception and factual description, in which equipment, conceptual resources, phenomenal fields and spaces are crucial to observability. This brings us back to Foucault and the analysis of space in disciplining collective agents (chapter 3). The class, vis-à-vis the teacher, is as much a physical as a discursive phenomenon:

A classroom on the Lower East Side in New York City during the so-called progressive period of American Education (1880-1920) (Extracted from the University of Notre Dame Website – www.nd.edu)

In the picture above, a group with no less than twenty pupils observe the teacher writing on the blackboard. Their bodies are all positioned in the same direction, facing the teacher and the board. The board, as a template for the documentation of the session via the production of stable signifiers is visible to everyone: not only is the class/pupils compulsorily responsive in the face of this alignment, they are responsible, insofar as their answers become entries in the board, as we have seen in lines 13, 33 and 41. Lines 15 and 16 in extract 4, for example, are confronted with line 17 in the sequence, in which a group of pupils give the right answer out loud:

```
T:          [one times three?
                 ▲           ▲
P:          four
P:          °four°
PP:         [THREE:::=
T:          [↑one times three?
PP:         THREE:::
```

The responsibility is that of learning *objective* knowledge: it is the responsibility of participating in the modern public order of matters-of-fact, of delegating reality, of replacing a judiciary cause by a scientific one (Latour, 1993)[40]. The format above is also designed for agreement: the way the teacher guides the step-by-step reasoning process with the help of pointing, simultaneous comment, and (indirect) forms of 'repair' as we can see above, with overwhelming use of paraphrases and completion (as opposed to the assessment of the mistakes so dear to teaching, and performing cognitive assessment; Macbeth, 2000) are a strong indication of this. The 'class' is guided all the way through observation and response, becoming a *witnessing* and *validating* resource for a mathematical reasoning procedure carried on without the need for *justification* at all points, and that multiplies, for all practical purposes, the number of participants that share the teacher's perspective on the setting. It enlists pupils to play the same language game, at the same time it amounts to the 'dispreferred' character of disagreement. The class is a kind of mechanical mind that nevertheless is necessary for the work of individual assessment, as we will see in chapter 6. There, I show how the pupil, or the 'the 'subject' is a significant other to the class. The point is that the 'logical', turn-taking frame of 'teacher-class' subsists for the maintenance of the classroom's *epistemic focus* (Lynch, 1993) and is hold together, as much as it holds back, by the social, technical and material-spatial features of the installation. The whole apparatus of the classroom, including space, materials and literary forms of action, is needed in order to raise documents of 'learning' and 'knowledge'.

4.4. A few remarks on technology and conversation

This section concludes with a few comments on the analysis of language, especially on the conversational-analytical literature, which presents varied answers to the question of the relations between discourse and technology and the analytical burden it can carry. Needless to say, for CA talk is given priority

---

[40] In chapter 5, the question of witnessing takes a more radical and compelling shape, as mathematics is *formulated* in the context of its association with 'empirical' matters.

in the analysis of action, or rather, it *is* the action. Alongside more 'open' works like Goodwin's and Hutchby's on the affordances of technologies for communication, rests a more 'conservative' line of argumentation, one in which technology is assimilated – in a rather 'Piagetian' sense – to conversation. The latter can take as a standpoint for critique the rejection to 'technological determinism':

> 'As Sacks (1992b) suggested, rather than thinking of how such technologies radically transform our dealings with others, it may be worthwhile to reflect on how technological innovations are "made at home in the world " (...) and embedded within our ordinary ways of working, talking, and interacting' (Heath and Luff, 2000: 338).

Although the study, on the use of radio in transport control activities (ibid.), pay attention to the way various competencies are put into play around the specific conventions of the 'practicalities at hand', as for instance, 'one party only, the controller, begins talk on the radio' (p. 346), the analytic footing is that of 'making things at home' by means of familiar communicational skills (e.g. recognizing who controls turns at talk), the 'ordinary ways of working, talking, and interacting' quoted above. The emphasis is on how technologies are interactionally constituted, more than how the social orders they analyse are technology-bound. In reading this particular approach (Heath and Button, 2002; Luff and Heath, 2002) one cannot help but notice how the topic of 'intersubjectivity' or 'conversational intelligibility' is given priority in a way that is sometimes circular, self-referential, and, as I expect to have indicated empirically, at risk of missing the phenomena it claims to describe, by resorting to a rhetorical reliance on its 'ordinariness' (Billig, 1999). I take the ordinariness of conversational phenomena, that is, the fact that it occurs, for granted, but I do not consider its formalised description and deviant conditions to be sufficient in order to describe how mathematics is taught and learned in classrooms. The aforementioned circularity of the argument stems from a standard critique: (a) the study of public order is the study of inter-personal, 'inter-subjective' relations; (b) those relations are lived through and saturated with 'ordinary' talk; (c) talk is what is studied as such-and-such (e.g. sequential analysis); (d) the proper study of culture, society and cognition is

the study of formal, sequential analysis. Without denying the impressive practical accomplishments of conversation analysis, it is clear that there is a set of assumptions about 'social action' informing its research programme. Besides, the technologies that inspire those studies and the practices they are related to, and are constitutive of, the very variations from the classic conversational model as described by Sacks and his colleagues (Sacks et al., 1974), variations sometimes called 'institutional' (Drew and Heritage, 1992; Watson, 1992). But institutions, broadly speaking, are more than institutional 'talk'. Schools, art galleries and factories appeal to us differently, and their affordances as installations for action and understanding are related to how different collectivities of things and people are mobilised (teachers, pupils, blackboards, 'scientific' notations; maps, visitors, classified collections, 'art', paintings, sculptures, videos; division of labour, 'work', task distribution, mass production, machines, etc.). Diverse technologies of order production (Whalen, 1995), from printed forms or computer interfaces to laboratorial instruments, allow for complex documentary relations with talk, so that the variations from 'at home' practices that account for institutional talk might as well be related to their link with those resources. Of course, I am not suggesting that normative accounts of technology set up the assumptions for the study of socio-technical settings. That must be an empirical task. As an implication of such an analytic strategy is fair to say that the 'boundaries between language, cognitive processes and structure in the material world dissolve' (Goodwin, 1997: 43) and 'rather than locating a homogeneous field for analysis the notion of embodiment encompasses many different kinds of phenomena' (ibid: 46). Extract 4 shows that in relation to the blackboard in greater detail.

If Garfinkel's (1967) programmatic call to the investigation on the 'incarnate', reflexive relation between members' accounts and the way a 'setting' is produced to be *observed* and *recognised* is right, then it is necessary to look at the unique features of a setting for which such accounts are accounts of/for. Rather than considering such accounts as the best 'representation' of the settings they help to constitute, Garfinkel sees them as 'reflexively and essentially tied for their rational features to the socially organized occasions of their use for they are *features* of the socially organized occasions of their use'

(1967: 4). From that we can conclude that in the mathematics classroom the very rationality of a mathematical account is reflexively tied to its localised demonstration[41]. The organised 'occasion' of their use, to paraphrase Garfinkel, includes the writing of notations such as numbers, algorithms, diagrams, graphics, and the like, on surfaces such as paper, blackboard, computer screens and their particular representational affordances, the bodily orientations of the 'demonstrator' towards them and the instructional techniques on how to find order in their particular 'spatiotemporal' features. This also implies to say that verbal accounts are not the only 'social' feature of such settings, and that the very notion of 'social' needs to be re-addressed. Although this recommendation is not a departure from the 'order-at-all-points' analytic strategy (described in Chapter 3), it does not subscribe to the homogeneous, 'monadic' view of exhibition of the whole in each and every of its parts, especially when specific organisational 'installations' such as the classroom are concerned. That would make describing 'talk' as equivalent to describing what those organizations and their documentary resources are about. Where it is certainly the case that the concepts of language *tell us* ultimately what those things are, that is, language is what 'represents' practice for academic readership, I would like to argue that language is also designed, in practical contexts, in *relation to* such things, to be used alongside them, so that a focus on how talk *represents* the world 'outside' language (Edwards et al., 1995) is a rather specific and narrow analytic enterprise (Hutchby, 2001).

It is worth mentioning Sacks's early investigations on recorded phone calls to a Suicide Prevention Center in Los Angeles. These original investigations, together with Schegloff's well-known analysis of phone calls as interactional summons (Schegloff, 1968), and Schegloff and Sacks's (1973) work on

---

[41] I would like to point out that an account of the sociological fact of the demonstration itself, which includes, in this case, a description of certain ways in which blackboards are used – but not *talked about* – by members, and its relation to members' accounts, is the analyst's. I take the view that this analytical exercise is partly 'creative', in the sense that it reflexively engages with the material it refers to continuously, rather than being determined by it, offering a view of an 'endogenous organisation' that is nevertheless saturated with a technology of analysability that does not belong to the original practice, producing a description of a different order than that of members' themselves. I take this to be a condition of the business of describing that is neither undesirable nor invalidating, at least not as long as the analysis is not concerned with disputing the 'truth' or 'falsity' of participant's accounts, but with formulating their organisation and use (Lynch, 1993).

'opening up closings', to select a few important ones, form the core of the early conversation-analytical corpus, and are the basis of the elaboration of the 'simplest systematics for turn-taking in conversation' (Sacks et al., 1974). It is worth remarking that from the early paradigm of telephone conversation, the relevant methodical aspects of social order are to be 'heard' (as opposed to 'seen', for example), by participants and analysts alike, in the course of 'disembodied' interactions taking place over the telephone. In such occasions, 'complicating' features of face-to-face talk such as gaze and gesture do not count as interactional resources or play into the actors' joint phenomenal field, therefore allowing for a description of a 'pure' economy of conversation.

However, in the case of telephone conversations – on which there is a great deal of conversation analytical research – the relation between the normative orientations of speakers to each other's turns at talk and the emergence of new interactional patterns 'afforded' by the use of the telephone has not often been taken into account in conversation analysis (see overview in Hutchby, 2001). Hutchby argues that since the introduction of the telephone, new communicative practices were afforded by its use. He mentions particularly what he calls 'intimacy at a distance' and the work of 'the categories of social identity that the telephone makes available for speakers to assign themselves, or be assigned, to' (Ibid: 81). He then goes on to report research which account for the early uses of telephones as related to functional, instrumental purposes, particularly business-oriented. Its widespread appeal and use as a 'sociability' device came at a later stage, when the companies realised the potential of the telephone for intimacy and social relationships, largely due to the women's particular use of it (pp. 82-83; Hooper, 1993). At the same time, the new demands of telephone communication precipitated novel forms of self-presentation and identity ascription. However, as far as sociological analysis was concerned, the telephone remained mundane and invisible (Hutchby, 2001).

Hutchby refers, though, to a remark by Sacks (1992) regarding the telephone as an 'institution' and the 'unique possibilities' of things such as institutions: 'It may well be that institutions could get examined [by members] for their

unique possibilities, and when their unique possibilities are found, they're employed' (Sacks, 1992, vol. 2: 162). The classroom-as-installation, I would argue, is just one of those possibilities, comprising a set of discursive and material resources that together afford 'unique possibilities' such as showing, demonstrating, pointing, mutually witnessing, and structuring talk in particular ways. Shapin and Shaffer (1985), for example, have shown how a taken-for-granted modern institution such as the 'experiment' was established and given rational and compelling character by producing simultaneously knowledge objects (vacuum), material technologies (the air-pump) and literary practices of 'observation'. Intersubjectivity, the world we agree and act upon as the same, was, in the case of experimental physical sciences, a technology developed to domesticate observation, and at a later stage, to multiply 'virtual' witness beyond the boundaries of the laboratories through the dissemination of scientific 'reports' (Ibid.). The invention of 'intersubjectivity', however, is inextricably linked to the material and representational objects installed to give access to competent knowledge.

That suggests that however grounded and analytically applicable notions like 'adjacency pair' may be, they cannot alone tell the ongoing 'story' of those institutional activities and their unique organisational possibilities, their unique adequacy (Garfinkel and Wieder, 1991). If we want to take the empirical analysis of institutions and their organisational order seriously, we must look at the practical implications of an array of resources and how they mutually relate in/for the constitution of such a setting. There is no doubt that a number of variations related to the basic formal properties of speech-exchange systems as described by Sacks and his colleagues (e.g. Sacks et al. 1974) are at the core of, and indeed are constitutive of, teaching-learning practices (McHoul, 1978, 1990; Mehan, 1985; Sinclair and Coulthard, 1975). But to describe the classroom as classroom talk alone, at moments when demonstrably talk is designed to attend to other forms of practical competency with, through or around material artefacts or other forms of representation is to apply invariably a model for the sake of a methodological argument, rather than its relevance. This relevance, and the orientation to those employable 'unique' aspects of institutions is, of course, to be analysed rather than presumed in each case.

I follow the implications of Sacks's recommendation for the investigation of order at all points in social research[42], but I also intend bringing into the picture some things that have been relegated to a secondary status by scholarly work on the analysis of talk-in-interaction, mainly for methodological reasons, but that nevertheless constitute 'technologies of order production' (Whalen, 1995). The question, then, is: how the technologies and artefacts found in the classroom productively co-construct a structure of 'order at all points'? Here, to paraphrase Sacks and his idea of 'hearer's maxims', the use of the blackboard encapsulates 'doer's maxims', ways of performing and acting out, as well as ways of seeing in common with others. In socio-technical networks, displays of knowledge are often counted in relation to displays of 'practical subversion' (Suchman, 2001), of 'knowingly' acting with, on and around a setting's unique possibilities. Sacks's conception of 'order at all points' does not warrant translating into 'conversation at all points', even less so into 'only conversation at all points'. Conversation is but one observable paradigm of order and its intelligibility in sociotechnical contexts is tied to other forms of competency and analysability (Goodwin, 1997; Lynch, 1993; Suchman, 2000).

---

[42] My use of it tend to be more reminiscent of a 'distributed' conception, than of a holographic (McHoul and Rapley, 2001) one, though.

CHAPTER 5

Talking Mathematics: Formulations, situated actions and transparency in instructional activities

5.1. Conversation analysis and knowledge visibility

In the previous chapters I have argued that the technical vocabulary of conversation analysis (CA) can constitute a *necessary*, but by no means *sufficient* condition for the understanding of classroom practices. Arguably, CA's very own branch of 'thick' description (Geertz, 1973), powerful as it is in portraying the emergent, local-yet-normative, orientation of society's members to each other's talk, turns into a 'flat' way of looking at some of the most complex settings of the social fabric, where space, timing, materiality, and particular technologies of accountability[43] (Suchman, 1993) are concerned. Here, I bear in mind that classroom 'talk' is only part of what I am calling classroom 'practices', and that any description of such talk is part of our own 'interested' and 'selective' practices of describing. I go on to suggest that this is the very problem the teacher has at hand in making inspectable the lesson's instruction design.

The question is important insofar as most of the research on classrooms coming from the CA tradition has, given a few exceptions (McHoul and Watson, 1984; Macbeth, 2000), failed to do justice to the classroom's *aboutness*, that is, to how classroom constitute sites or installations for the performance of the competent worlds of knowledge. Here, I want to focus on classroom talk as it mobilises and formulates the objects of its own practice, and how the situated actions that hold the possibility of such work accountably

---

[43] Space, time, materiality and technologies of accountability are not to be confounded with 'nature', or *a priori*, necessary conditions of knowledge. Those are sociotechnical objects put to circulation in practices, forging and mediating new social links, and constitute integral parts of the 'society made durable' (Latour, 1991), or 'culture' (Pickering, 1992).

constitute a practical extension of them (Lynch and Jordan, 1995). How do forms of representation and situated activity come together in order to open classroom practices and technologies to inspection, that is, how do they design the transparency of a (school) competent mathematical world?

In the following extract, teacher and pupils (3rd graders) are involved in a task concerning the arithmetic table for addition[44]. In this 'non-blackboard' exercise (see previous chapter), conducted through questioning alone, pupils are required to pay attention to the unfolding of the task[45], but are each asked individually about the table:

Extract (1) 3rd grade, EE, 1:

```
1.    T:        erm:: you. tell me how much two plus three makes
2.              (1.0)
3.    P1:       six
4.              (2.0)
5.    T:        ( ) how much does two plus three make?
6.    P2:       °five°
7.    T:        °five° (.) and:: three plus two?
8.    P2:       five
9.    T:        five
10.             (1.0)
11.   T:        how much is it now say now how much two plus three
12.             makes
13.   P1:       °five°
14.   T:        °five° (.) and three plus two?
15.   P1:       °five°
16.   T:        oh yes now you learned (.) now you that laughed how
17.             much does
18.             erm: two plus seven make?
19.   P:        ( )=
20.   T:        = and: five plus two?
21.   P:        seven
```

---

[44] This consists of tables for the calculation of all combinations between integers from 1 to 9, regarding addition and multiplication. So the multiplication table for 9 is: 9x1; 9x2; 9x3 ... 9x9.

[45] The teacher had just requested 'silence' from the class, explaining that 'when we are going to ask the reading of the table in a classroom erm:: (.) we have to make silence mainly because the answer of the first ones go on to hel(h)p by the time we get to the last ones they have already lear(h)ned'. The laughter symbols in the transcript relate to the fact that there has been some trouble about *who* the first and the last ones are going to be!

22.   (3.0)

I want to turn my focus on the resources that the teacher uses to make the task 'inspectable' or to open access to the 'content' of classroom activity. I would like to suggest from the outset that the task, and indeed classroom tasks more generally, are both (1) designed to be inspected and (2) related to things accountably external and more general than the task itself[46], i.e. content, knowledge, reality, to the effect that the 'actions' that amount to the accomplishment of the task can be considered as 'legitimate' extensions of such things, and differentiated aspects of reality vis-à-vis other (social) agents.

At the beginning of the extract the teacher selects a pupil and asks her 'how much two plus three makes', for which she (wrongly) replies 'six'[47]. The teacher does not repair the error directly but rather opens the next turn for a new candidate answer (by the same token constituting the previous one as 'wrong'). As I have argued in the previous chapter, it is possible that relying on third parties has the effect of securing the public character of the teacher's intended outcomes as 'witnessable' or 'inferable'. The table has previously been a topic for the class, and this occasion is raised to test their knowledge on the subject, an exercise that had been arranged earlier in the week. That the teacher starts the exercise with direct questions about it assumes that the students are both knowledgeable and responsible. For example, after the error in line 3, the teacher hands over the slot to the next pupil before going back to the P1 in line 11 without ever mentioning a way (e.g. counting) to find the answer. The adequacy of the knowing agency assumed by the teacher is · checked against a procedure in which a set of 'conservation' principles can be seen at work. That is the meaning of the strategy for 'reversing' the order of the numbers in the way the calculation is framed, an act of manipulating 'order' itself that takes full significance a little further down the line (line 23 onwards, below). The procedure is repeated with the pupil 'that laughed' (line

---

[46] This point is explored analytically to more detail in section in section 5.4.
[47] Although it is not centrally relevant here, note that it is suggestive that 'two', 'three' and 'six' *do* go together in the sentence 'two *times* three makes six'.

16). The teaching that comes next builds up from this methodically engineered phenomenon of order.

Given that the mathematics lesson shown above goes on to inspect its procedural basis for arriving at certain conclusions in terms other than 'the conversational', i.e. 'the mathematical', such technologies of accountability or indigenous sociologies (Lynch and Jordan, 1995) constitute analytical grounds on a par with, and in potential contrast to *formal*, (1) 'external' sociologies, including sociologies of 'methods' (e.g. CA), and (2) philosophical accounts of the intelligibility of mathematical knowledge.

This is important insofar as it re-addresses the question of where the ground of analysis lies, a topic championed by CA in terms of the concept of 'participant's orientations'. Thus, according to the CA canon, people's 'answers' to 'questions' show an orientation to the fact that the latter produces the former as conditionally relevant to whatever participants are doing. That is a universal social technology, and like others of this kind, e.g. invitation-refusal sequences (Drew, 1984), can be seen to populate our interactions with each other. These mechanisms are also 'context-sensitive', and the study of their variations is at the heart of CA's research programme. So, although a lesson 'looks' very different from a telephone conversation, it can be studied in terms of the legacy of the latter's formal analysis: 'openings', 'closings', 'adjacency pairs', 'repair', and so on. What constitutes the trouble is that these topics are understood as seen and recognised by participants themselves! Harvey Sacks' compelling insights on this matter rely on the idea that people *see* 'activities', that they see *gestalts*, and that they produce conduct so that it can be seen as such-and-such (Sacks, 1992). However, the visibility of such practices is accounted for by participants in vernacular terms, and it is on that basis that the classroom is made open for inspection[48]. If members' analytical glosses can be considered legitimate accounts of their own activities, CA may as well 'disattend' participant's orientations, what they talk about (Billig,

[48] The selection of data is relevant here: I am not *as* interested in the sequential analysis of 'pauses' *as* I am in the way in the 'translations' that reflexively formulate the 'content'.

1999). It implies that CA accounts might be in competition not only with other academic accounts, but also with those of the participants' themselves[49].

One way to see the 'elliptical' pattern of talk by the blackboard analysed in the previous chapter is to identify it as a 'question-answer' sequence. Another way is to see it as the production of a cogent set of methods for which a mathematical object 'X' is produced as an adequate account. Questions and answers are still in place, they are left unaffected as descriptors of some sort (just as is the natural language supporting them!). But that is beside the point. As far as participants' understandings are concerned, if those methods were to be made analytically visible as 'question-answer sequences', that would imply that 'X' is the adequate description of 'question answer-sequences'! But of what kind, and about what, and bound to which resources? That is partly why the blackboard itself, and what it affords as a 'cognitive' site was so important to my analysis. The activities on and around the blackboard are produced as the activities for which 'X' is an adequate account, as extensions of mathematical knowledge and competence.

A case in point is the teacher's use of the category 'question' below, continuing the previous extract. This brings us back to where we left the analysis.

Extract (2) 3rd grade, EE, 1 (continuation of 1):

```
23.  T:      now you have already heard that I asked the three
24.          girls over there didn't I?
25.  P:      yes
26.  T:      I already asked the three of them and I'd like you to
27.          tell me (.) what did you notice ↑something↓ in the
28.          form of the question that I've asked them?
29.  PP:     (        )
30.  T:      CALM DOWN one at a time.
31.  P:      °(        )°
```

---

[49] A further and important implication relates to the way analytical practices (CA, the teacher's discourse, this thesis) find the legitimacy and adequacy of (inevitable) glossing practices, for which a plea for 'reflexivity' is welcomed (Ashmore, 1989).

```
32.  PP:        (pupils speak at the same time)
33.  P:         miss you've=
34.  PP:        (pupils speak at the same time)
35.  T:         WAIT calm down folks
36.  P:         =applied the commutative property
37.  T:         I'VE APPLIED she noticed see? ((addressing another
38.              pupil)) she- I've applied- you knew- you- know what
39.              you were saying but you're not managing to explain
40.              yourself. ↑I've applied the property=
41.  PP:        =commuta[tive
42.  T:                 [commutative (.) ah:: (.) how is the
43.              commutative property applied?
44.  PP:        (undistinguishable; noisy)
45.  T:         (change) the order of the items and doesn't alter=
46.  P:         the factors
47.  T:         =the sum
48.  P:         the factors
49.  T:         [no love=
50.  P:         [the factors
51.  T:         = (          ) (.) now you. how much does erm:: two
52.              plus eight
53.              make?
54.              (3.0)
55.  P:         ten.
56.  T:         and eight plus two?
57.              (1.0)
```

In lines 23-24 (above), a set of relevant features can be identified in relation to the classroom analytical procedures. First, the teacher formulates the frame in which *individual*, as opposed to *class*, questioning has been taking place; she also uses it to go from one to the other, from 'the three of them' to 'you'[50]. Secondly, it is arguable that 'question' (line 28) is as important an analytical category in the discourse above as it is for professional analysis. I want to explore the idea that one of the analysable features of teaching discourse is that it proceeds as to deliver a thorough analysis of the setting, its objects, regularities, points of opening and closure, etc., and that the use of the category 'question' is an indication of such a thing. As a minor (speculative) example,

---

[50] In the original in Portuguese the teacher refers to 'vocês' (plural) and not to 'você' (singular). Such a clarification is necessary since the English language does not differentiate between 'you' as singular or plural. That means that, as far as the use of Portuguese is concerned, the 'cohort' was designated *grammatically*.

note the use of 'now' in lines 11, 16, and 23 of extracts 1 and 2. Without extending the argument much, it seems that at those particular junctures 'now' documents a discontinuity between instructional events, so as to project the relevant 'inclusiveness' of what comes after. So in lines 23-24 the episodes with the 'three girls' are recounted and enlisted as the experiential basis on which the teacher 'would like' the students to answer next (line 26). In particular, her 'question' (line 28), and notably its 'form' (again line 28), appears to be of interest, and here we have the scope for a practical frame of 'orientation'.

In traditional discourse and conversation analytical research on classrooms, 'questions' are treated as a general sequential object, for which replies are logical consequences. I am interested in how the teacher's questioning constructs a template for mathematical accountability, rather than as a formal account of 'institutional' conversational techniques consisting of mechanisms such as 'insertion sequence', 'receipt', 'revised question', etc., (Antaki, 2002). Even though Antaki (Ibid.) rightly points out, in a brief comparison between classroom activities and interviewing methods, that there are basic procedural differences in the projects of teachers and interviewers, his analysis has no bearing on the argument I am suggesting here, namely, that a questioning trajectory can be recollected (as the teacher seems to suggest) in vernacular terms as a set of accountably 'specific', subject-matter related, extensive displays of (mathematical) 'knowledge' in the instructional sequence at stake. That 'for the teacher, the important thing seems to be that the pupil be seen to get at an answer by independent reasoning' (Ibid: 421) is only part of the story, because it shows no regard for what the 'reasoning' is about, and to how that competent world is simulated in the classroom. Rather than drawing from CA and trying to render instructional activities intelligible in terms of any given combination of the things Antaki presents, why don't we (analysts) learn from the teacher's own account the sense of the trajectory she offered as a legitimate extension of the mathematics curriculum? The most interesting thing about analysing instructions in the classroom is that the teacher is telling us, analysts, where to look at (the questions the three girls were asked), and how to look back (the *form* of the question); in a sense, she's teaching us as well. Just think

of this: if we were to record a lesson and suddenly decide that our mathematics is not very good these days and that we would wish to learn a bit more, or to be reminded of it, we would immediately be caught out of our analyst's position. As we, analysts of interaction, never seem to be interested in being direct beneficiaries of (that) 'learning', many of our reports are 'critical', 'disappointed', and tend to consider the classroom 'artificial'. The pupils seem to understand how to follow the teacher's recommendations much better, and although they may get caught in the wrong trials, they do it through the selection of relevant features of the setting's 'reasoning' resources. That is how the 'commutative property' (lines 40-43), by which one '(change) the order of the items and doesn't alter = [...] =the sum, lines 45-47), comes to light as the adequate description of the activities in lines 1-19. Those activities, by their turn, actively display the commutative property, in a relation of proper documentation. If the kids had said that they noticed in the 'form' of the 'question' that it was the first part in an 'adjacency pair' structure that makes the second conditionally relevant, they would have been plainly wrong!

CA phraseology has it that the classroom is 'language-saturated' (McHoul and Watson, 1984; Watson, 1992), a formulation which (1) is true, (2) justifies an analytical strategy and its technology (the transcript) to take place, and (3) suppresses the analysis of features that work alongside discourse to hold together the order of classroom activities and actions. These three characteristics can be summed up by saying that while there is no denying the fact that discursive interaction (especially in the form of question-answer sequences controlled by the teacher) are readily inspectable phenomena in classrooms, its formalised description (as 'context-sensitive' variation of a 'context-free' structure; see Lerner, 1995) is (wrongly, in my view) portrayed as circumscribing what there is to know about that setting *as* classroom, all packed in a general 'theory' of social order that is reflexively tied to a set of 'professional hearings' (Ashmore and Reed, 2000) afforded by the disciplined transcription of audio-taped materials.

For instance, CA has little to say about how the situated practices of attending to, and taking distance from, particular technologies and frames of reference in

133

a 'work' environment set up the basis for 'folk' epistemologies of 'concrete' versus 'abstract' reasoning (Latour, 1990; Lave and Wenger, 1991). It is, of course, less a question of misrepresenting those than producing a selected set of 'observables' (often dubbed *the* phenomena against 'constructive', sociological research[51]). Conversational analytical work concerned with the 'other things' have proceeded by a most interesting broadening in scope – although not necessarily in analytical footing – giving visibility to 'heterogeneous' elements in their analysis of social practices and dealing, in one way or another, with the boundaries of the discipline (Button, 1993; Goodwin, 1995, 1997; Heath and Luff, 1993; Hutchby, 2001; Whalen, 1995). Investigations on scientific and professional activities have made that turn more prominently before, and while recognising the central role of language and discourse for the reproduction and investigation of the way settings are organised and competencies are located, they have pushed the discussion further to include several other 'actors', material, technical, representational (Callon, 1986; Hacking, 1992; Latour, 1991; Law, 1991; Lynch, 1990; Lynch and Woolgar, 1990; Suchman, 1993). Instead of bracketing off those 'entities' by taking on board the often-troublesome intellectual notion that 'language is all there is', those projects have rendered them visible and analysable, and have (re) specified their interplays with talk-in-interaction.

Lave and Wenger (1991) argue that because the technologies of practice (its contents and disciplined procedures, instruments, axiomatic 'inscriptions', machinery, etc.) carry significant aspects of that practice's heritage, they constitute one of the most important means of access to its inspectability; they connect actual use, understanding and history. Lave and Wenger refer to the opening of techniques and practices for inspection as *transparency*.

---

[51] Whereas I agree with the policy of avoiding 'social explanation' by means of mobilising and importing 'big issues' (Sacks, 1992) and other 'external' actors, I also think that that is neither a prerogative of CA within the field of social studies nor a warrant for the idea that CA tools – or a focus on discourse alone – are a neutral way of describing participant's own orientation in practices (Billig, 1999; Stokoe and Smithson, 2001). The idea that is possible to describe action from 'one's point of view' – the 'cognitive' and 'culturalist' resonance of which CA tries to avoid with the concept of 'orientation' – has a long intellectual pedigree and is an integral part of approaches such as symbolic interactionism and Piagetian psychology, as for the whole movement of *Versthehen* sociology (Law and Lodge, 1984). Piaget, it is worth remarking, turned children's 'wrong' answers to cognitive tasks into a 'visible' field of analysis, by claiming that they are systematic and related to the ways children's cognitive life is structured at different stages of development.

'Transparency in its simplest form may just imply that the inner workings of an artifact are available for the learner's inspection: the black box can be opened, it can become a "glass box"' (Ibid: 102). Settings and technologies can also be *opaque*, that is, the way they afford access and understanding to learners is rather limited. A more refined view on transparency is, according to Lave and Wenger, to conceive it as 'dually' characterised by *invisibility* and *visibility*. The former refers to the 'unproblematic' assimilation of *know how* into the course of an activity, or what is taken to be (or to become) 'tacit knowledge' (Collins, 1985). 'Telling' how to make a piece of furniture, such as a chair, is different, for carpenters, from the lived work of managing tools and learning how to use them over raw materials, so that 'talk' about shapes and angles do not contain exhaustive, detailed instructions on how to use a hammer, or which one is most suitable for which task or materials (Gherardi, 2000). The latter ('visibility') takes the form of 'extended access to information' (Lave and Wenger, 1991: 103), to 'understanding', and has been identified with a more 'abstract', 'mediated' or 'conscious' manifestation of cognitive activity, a point I will touch upon again later in this work. However, as we shall see, rather than being categorical opposites, the visible and invisible workings of transparency are combined in order to produce the accountable outcomes of mathematics education. Despite the fact that Lave and Wenger's analysis was designed to attend to practices of 'apprenticeship' (see chapter 3), in which members move from 'legitimate peripheral participation' towards the centre of masterly practices, some aspects of their analysis can be retained here, especially the idea that as the methodical organisation of access, and most notably to 'meaning', the notion of transparency 'does not apply to technology only, but to all forms of access to practice' (Ibid: 102).

As far as the history of such practices is concerned, those sociotechnical artefacts are intertwined with the establishment and circulation of new 'facts', concepts and technologies of analysis. That the construction and circulation of an air-pump in 17th Century England could mobilise at the same time the observability of the vacuum, the logic of experimental thought and replicability, and the whole edifice of metaphysics, with consequences to the

modern representation of nature (in opposition to *political* representation) is truly exceptional (Latour, 1993; Shapin and Schaffer, 1985). A closer focus on the production of the 'contents' of practices, and on how they can be 'social', opens up access to the historical dimension of discourse (Edwards, 1989), something CA work has been accused of disattending to[52] (Billig, 1989), and is the basis of the contemporary, empirical 'critique' of knowledge known as the sociology of scientific knowledge (SSK) (Bloor, 1976; Latour, 1999; Pickering, 1992). The problem was somehow addressed by Wittgenstein when asking about whether our language is 'complete', or 'whether it was so before the symbolism of chemistry and the notation of the infinitesimal calculus were incorporated in it; for those are, so to speak, suburbs of our language' (Wittgenstein, 1967: 8e). Those 'suburbs' have different ages, and they are assimilated into 'town' under different times and conditions; some of the houses have items from various periods; some others look brand new, says Wittgenstein.

5.2. Training and naming

In his *Philosophical Investigations* (1967 [1953]), Wittgenstein remarks that the notion of a language as 'meaning' or 'reference' points either to (1) a primitive conception of language and, or, (2) a the 'idea of a language more primitive than ours' (p. 3e). The first is fully expressed in his critique of St. Augustine's conception of language as a system of representation and description, according to which its concepts have the role or function to map onto an 'external' reality. On that, his philosophy is in alignment with, and indeed sets the tone for, the 'pragmatic' turn in philosophy and in the social

---

[52] While it is clear that it has not been designed to do so, such a limitation in scope tend to produce both a highly successful technical accomplishment, and also a 'conservative' formalism, a disciplined phraseology and way of going around 'data', a network that is unable to 'expand' and 'capitalise' (Latour, 1993) from non-conversational aspects of practices. In the previous chapter, it was my intent to show some links between 'conversational' and 'non-conversational' in the classroom.

sciences. The latter, however, points to the activity – amongst others – of doing precisely that: naming, representing, describing:

> 'Let us imagine a language for which the description given by Augustine is right. The language is meant to serve for communication between a builder A and an assistant B. A is building with building-stones: there are blocks, pillars, slabs and beams. B has to pass the stones, and that in the order in which A needs them. For this purpose they use a language consisting of the words "block", "pillar", "slab", "beam". A calls them out; – B brings the stone which he has learnt to bring at such –and-such call. – Conceive this a complete primitive language' (Wittgenstein, 1967: 3e).

Wittgenstein concedes that that is an appropriate description of a 'circumscribed region' of language, or of a 'language game'. For *that*, 'the description given by Augustine is right'. To use his celebrated idea of 'games', an all-encompassing definition of language as *representation*, or any other for that matter (e.g. conversation), is bound to fail on the basis of the analogy that only some games, but not others, are played on a 'board' (e.g. chess). Some games are played simultaneously by a large group of people kicking a ball on a field; some, again with a ball, forbid it to be kicked; some others are played using only the bodies of the players, and so on. In the previous chapter I showed how a specific (asymmetric, topical) configuration of talk-in-interaction is put together alongside writing, gestures and visual orientation to produce a 'form of life' literally on and around a board. However, while the constitution of speakers' identities and tasks in that case is akin to the practices of 'conversation', it hardly fits the example of the 'game' between the builder and his assistant, although it is reminiscent of it at points, not least because the use of material devices are at the core of both tasks.

The word 'task' is appropriate here and, indeed, to describe the scenario portrayed in the quote above as one of 'representation', in an Augustinian way, is misguiding. Wittgenstein notices that the purpose of the builder is not to 'evoke images' in the assistant, but to get from him the items that the words refer to; to get things *done*. He also suggests that this is arguably the way children's learning of language comes about: by being trained in language

games such as naming and performing with the words, as well as responding to them accordingly, for the 'commands' they are (such as in the example quoted above). 'Here, the teaching of language is not explanation, but training' (Wittgenstein, 1967: 4e).

The point is one about 'ostensive teaching' of words. By that Wittgenstein meant to explore the practices by which words and objects are associated. So, to use a classical philosophical portrait, the contingent spatio-temporal 'presentation' of an apple together with the uttered word 'apple' can create a 're-presentation' of the original object. Wittgenstein accepts this connection as a practical matter, something that *can* be the purpose of using words at certain points, but he then takes his discussion further to another place. To understand a word or an order, says the philosopher, is to act upon it in certain ways (such as biting the apple), ways that are brought about by particular forms of 'training', as he puts it[53]. The crucial question is that it is perfectly imaginable (and potentially analysable) that the canons of 're-presentation' can be established and maintained constant (as well as invoked to legislate upon genuine cases and deviations from it!) despite divergences in 'training'[54].

---

[53] It occurs to me that the first half of this sentence, as I put it, has a remarkable similarity with what Piaget and his followers have suggested to be the earlier sources of 'meaning': 'action'. Of course, the nature of each philosophy is radically different. Piagetian 'semiotics' has it that the progressive abstraction from actions' 'formal' properties is on the basis of understanding. Language, on its turn, is supposed to give action a public face, a 'symbol', and a kind of focused mobility (Piaget, 1982). Piaget understood language as representation (Maier, 1996), an arbitrary 'superstructure' to 'motivated' developing cognitive structures. This theory portrays the relations between language and thinking as an 'additive' structure: the possibilities include (1) language + thinking; (2) thinking – language; and (3) language – thinking. Different combinations produce phenomenon that ranges from mimic to highly abstract mental processes. Wittgenstein saw 'thinking' as bound to public forms of life, to language games. It is worth quoting one of the definitions Wittgenstein provided for the latter concept: 'I shall also call the whole, consisting of language and the actions into which it is woven, the "language-game"' (1967: 5e). The terms are strikingly similar to Piaget's, but they imply different things. A given language game consists, then, of *words* and *actions*. Although they do not represent the same, those concepts also echo the ethnomethodological distinction between 'formulations' and 'ad hoc actions' (Garfinkel and Sacks, 1970), which is one the main concerns of this chapter. Situated, ad hoc actions are part and parcel of the work of *indexicality* in sociotechnical settings (Goodwin, 1997).

[54] Piaget comes to mind again. As psychology students will know, 'conservation', or the capacity to keep constant the perception of (logical) objects through (physical) dislocation and transformation, is on the basis of all his studies and constitute the very proof of 'acquisition' of a notion. It is relevant to say that Piaget saw this as a natural, fundamental ability, not as an effect of the demands of (school) education. In Latourian terms (Latour, 1987; 1988) conservation is afforded by the 'metrological' capabilities of actor-networks, not of 'minds' or 'subjects', which discipline delegated observers into apparatuses able to send their (superimposable) work back to be combined and coordinated in 'centres of calculation'.

Training is variable in its detail, situated, done in real time. By its turn, representational canons assume – as it is conventionally understood – the status of 'abstract' entities, beyond and prior to its significant relations with 'concrete' phenomena, and are usually conceived of as the best description of the latter. Mathematics stands out as an exemplary case of this relation.

It is relevant in this context that Wittgenstein is often interested in the 'foundational' question of learning and teaching as one of the basis of his philosophical arguments[55] (Peters, 2001), something referred to earlier as the 'socialisation problem' (Edwards, 1997; Schegloff, 1992). Peters (2001), for instance, writes about how scholarly work on Wittgenstein's investigations, such as that by the American philosopher Stanley Cavell, is dominated by the opening of the *Investigations*, in which the figure of the child-learner is mobilised. Cavell had noticed that is not a trivial matter that Wittgenstein begins his major study by showing how Augustine makes an account of his own childhood experiences and his learning of language the pivotal point where (his) philosophy begins. The philosophical question is turned into a question of development and genealogy, the child being, in Cavell's own admission, an unlikely actor in philosophical texts. The instructional contexts of 'culture' (e.g. classrooms), or its 'forms of life', provide the settings that 'house' the concepts and practices that children will have to 'master' towards accountably rational adulthood (Walkerdine, 1988)[56].

But how do (mathematics) teaching and learning take place, or, how are those forms of life so arranged? That they are is a starting point here, not, as I have argued, as some kind of psychological quality, but as the organisation of a state of affairs, through complex processes of 'translation' between different objects

---

[55] David Bloor (1983) points out the difference between Wittgenstein's interest in the 'foundations' of mathematics as the problem of 'learnability', and the philosophical tradition on foundations, related to the efforts to establish normative epistemological footing for theory and analysis. Bloor associates, and indeed locates, this feature of Wittgenstein's philosophical thinking with (in) his experiences as a schoolteacher. Some suggest that that is such an important link that his philosophy can be seen as construed as pedagogy (Peters, 2001; Spector, 2001).
[56] In Peters (2001) account, some of the other voices that Wittgenstein mobilises in order to portray action at the limit of (adult) language and thought include the 'madman', the 'foreigner' and 'animals' (p. 133).

(Callon, 1986) and 'documentary' relations between situated actions and normative understandings (Garfinkel, 1967), as well as the 'delegation' of knowing agency and accountability[57] (Edwards, 1997). Classrooms are constituted, amongst other things, by a set of activity frames and their respective sequential work (Lemke, 1990). That the actors enact this cycle repeatedly is a simplest criterion for the argument that teaching and learning, and their multiple activity frames (e.g. teacher-class interaction on the blackboard; teacher-pupil direct instruction; pupil-pupil joint tasks, etc.) reach (practical) closure (Macbeth, 2000). Just *how* they do it is the important thing. Some of the ways by which this task is conducted through local instructed actions (Wittgenstein's 'training'), on the one hand, and formulations or rules that account for the knowledge and actions at stake, on the other, is the point of this chapter.

5.3. Formulations and instructed actions

The local production of a set of activities *in* and *as* a specific domain of practices, so as to include renditions of those same contexts as describable and explainable, in a word, as 'rational', is a common feature of many educational, professional and scientific practices (Amerine and Bilmes, 1990; Lynch, 1993; Suchman, 1987; Livingston, 2001). Be it in the context of apprenticeship-based learning 'contracts' (Lave and Wenger, 1991) or in the effort to replicate technological accomplishments based on published guidelines (Collins, 1985), the translation between setting-defining 'rules' for action and their application is one of the central concerns of the analysis of practices since Wittgenstein. This relates to the ways states of affairs in a 'work' location are put together, and how they can be 'understood'. Classically, in psychology and cognitive studies, as well as in communication theory, it is thought that such states of affairs are produced as effects of rule-caused or rule-governed processes, such as 'logic' generating 'reasoning' and 'grammar' generating 'speech' (Edwards, 1997).

---

[57] That will be the point of chapter 6.

'Formulating', or 'accounting for', in instructional actions can be translated as the work of making inspectable, establishing relevance and visibility, 'making sense' in/of a setting; it constitutes the 'intension' of a given class of objects and practices so assembled (Law and Lodge, 1984). In a word, formulating is related to naming, selecting, classifying, theorizing, etc. (Garfinkel and Sacks, 1990; Goodwin, 1997; Lynch, 1993):

Extract (3) pre-school, EL, 1:

```
T:      The number eleven has one ten and one unit
```

Extract (4) pre-school, EL, 1:

```
T:      Ten tens are one hundred units
```

The propositions above are hardly of the kind we are likely to hear in most (non-school) everyday contexts. In our daily life, the use of numbers rarely, if ever, has the level of thematic awareness and sophistication observed in extracts 3 and 4. Our common uses of number and number words display 'reason' and adequacy throughout, but seldom 'justification' (Wittgenstein, 1967). In extract 3, 'eleven' is qualified as a 'number' and dismembered into a given quantity of 'tens' and 'units'. This reflexive relation between quantities and analytical categories can be seen again in extract 4, and goes on to establish the 'place value' of a number in two digit-plus (written) numerals, where the knowledge of the relative positions of units, tens, hundreds, etc., are to be built into the skills for performing with arithmetical algorithms.

In what follows, the arrow lines indicate generalizations that are usually conceived of as *immediate* or *non-mediated* mathematical knowledge:

Extract (5) pre-school, EL, 2:

```
T:      a pair is formed by two things, Dani (.) here there is one pair
        of leaves (.) I want two pairs
```

Extract (6) pre-school, EL, 2:

```
T: →   yes (.) two is even (.) and this one left? If it there is
       something left it cannot be even (.) so three is=
P:     odd.
```

The important point here is that, at least as far as the presumed competence of the pupils involved is concerned, the intelligibility of such mathematical propositions is tied to a set of resources that establish its demonstrability in sequentially organised ways. In extract 5, the teacher's injunction is a formulation used to identify and repair the misunderstanding of a previous request. It implies, consequentially, that (1) the order is not obvious, and (2) that recognising 'right' from 'wrong' cases of compliance is in order. It also indicates that mathematics' power as a kind of 'ultra-physics' (Wittgenstein, 2001), a logicised, abstract, synthetic domain of knowledge whose concepts do not recognize phenomenal extension but are rather a reflection of *relations* is, in the classroom practice, the result of a set of physical manipulations, grammatical translations and techniques for 'observing', that is, of 'infra-physics' (Latour, 1988b). In the example, an explanation of the concept of 'pair' is offered, bringing mathematical ('two') and mathematisable ('things') together as part of a definition. The teacher had just asked the pupils to bring her, from the school *patio*, two pairs of leaves. Dani had brought back two leaves, instead of four, the correct answer. The occasion is raised as a proper 'teaching' one, since even though correct answers are targeted as projected outcomes (Amerine and Bilmes, 1990), the work of teaching relies fundamentally on taking on board and working up the incorrect ones (Macbeth, 2001). Instead of being made to stand on its own, the definition is used as part of a running commentary that exposes and diagnoses the work of producing it. This feature of classroom teaching can be seen again in extract 6, where the question on the 'one left', *that* one left, resurfaces as part of the more general statement that 'if there is something left it cannot be *even*'. The instruction revolves precisely around a method to find '*even*'. What had gone on before the event I have isolated consisted of Mateus – one of the pupils – answering that '*three*' was '*even*' (as opposed to '*odd*'), drawing a circle, for the effect of

demonstration, around two out of three objects drawn on the blackboard. We join the action at the point the teacher takes up from that. Relevant features are then selected, classified, named, technicised by her (e.g. 'this one left') and featured into a general insight that oversees the work of its sequential assembly (just to recuperate it again, under the auspices of disciplined 'cognition', such as in lines 48-52 in extract 2). Note also the use of the conditional 'if' and the substitution of 'and *this one* left' by '*something* left', suggesting the passage from a grammar of 'seeing' to one of conditionality and absence of paradigmatic, substitutable chains of signification (Walkerdine, 1988).

In the context of science studies, Collins (1985) has referred to the 'algorithmical model of learning' as the view on scientific method according to which formal accounts (of experiments) can 'duplicate', or set up a kind of 'virtual witnessing' (Shapin and Schaffer, 1985; Lynch and Bogen, 1994) for the situated actions that constitute, *in situ*, the work of the experimental setting. Rather, Collins advocates an 'enculturation model', to which 'skills', as opposed to 'formal instruction', gain a central place in the analysis – and teaching – of scientific activity and its reproduction. Thus, the idea is that 'individuals knowledge must be acquired by contact with the relevant community rather than by transferring programmes of instruction' (1985: 159). In his study of the replication of a laser by professional scientists, Collins noticed that none of the practitioners observed were able to build the laser device in question based only on published information, having to obtain knowledge from personal communication with successful laboratories; that none was successful where the informant was someone who had not built the device himself; and that even in the case of having established communication with 'first hand' informants, success would depend on extended periods of communication with them, although that was not a total guarantee of success. Collins shows that formal accounts of laser building fail to account for the success of replication, in a context in which effective transfer of knowledge was personal, skill-bound, and 'invisible', so that the relevant expertise could only be seen in trying to build the device, and sometimes, despite training via the 'proper' channels, could not be seen at all.

The ways instructed actions and formulations are interwoven are multiple and complex, and are open to empirical investigation. For example, generally formulated instructions can be a built-in aspect of an interactant's performance in a way that interpelates its counterpart's performance as 'structured' or 'planned' (Suchman, 1987). Rule-oriented accounts of practices and behaviour would have us believe that general instructions, accounts, recipes, etc., pave a clear way for the work of their realisation. In practice, things are more complex than that. More generally in instruction sequences, seemingly 'abstract' categories are inserted at the end of framing stories or scenarios so as to deliver a sense of the required task, or at the end of the sequence of practical manipulation of the task's resources, as a gist on what has been previously done and said (Heritage and Watson, 1979):

Extract (7) 3rd grade, EA, 2:

```
T:      can I divide four by three?
P:      you can
P:      you can
P:      °you cannot°
T: →    I can because the four is bigger than the three
```

Notice, though, that mathematical 'abstractions' of the kind we see in the arrowed line above are never quite like 'gists' (Heritage and Watson, 1979); they are, more convincingly, particular kinds of 'translation' that expand the scope of the situated actions that antecede them at the same time that they, formally, take distance from them. In this microscopic moment of a division exercise (400 divided by 3) the teacher selects 4 as the number to be divided. She then asks if she can do that. She gets different answers, at the end of which she replies that she can '*because* the four is bigger that the three'. This justification is, in fact, projected in the sequence by the question 'can I divide four by three?'. Instead of *just* carrying out the division through any form of authoritative competence, the teacher problematises the reasonable character of what they are doing by punctuating it with 'rational' justification. The fact that 'the four is bigger than the three' may be a later item in the sequential organization of that local problem, but stands as its logical 'condition'

('because' ...). As Garfinkel, Lynch, Livingston and others have shown, 'abstract' propositions – particularly written inscriptions – while taking their sense from the local conditions of their production, seem to elude them completely and to become intelligible objects of their own (Garfinkel, et. al, 1981; Latour and Woolgar, 1979; Livingston, 1987; Lynch, 1993).

Extract (8) 4th grade, EF, 1:

```
T:    look I want you (0.4) to give me a result (.) solving the
      expression (.) and to show me ↑too (.) erm::: how these boxes
      were piled in accordance with the numerical expression that the::
      the- the- employee made up
```

In the extract above the teacher demands that not only a result is in order, but a certain kind of result, in 'accordance' with a model. Just how to accomplish that was the point of the 'exemplary' sequence that preceded the transcript above. We join extract 8 when a pupil has just said he did not understand 'very well' what he had to do according to an earlier instruction. The teacher offers a reminder on what she had previously asked in terms of '*solving* the expression' and '*showing* X in accordance with numerical expressions'. I want to focus on the trajectory of the teacher's discourse in delivering a task. I would suggest that the organisation of the task is complex in that in trying to exclude 'authoritative' sources of meaning, it implies the 'necessary' documentary link between a story, the establishment of a optical and material consistency, and knowing or explanatory orders. The exercise was previously set up – on the blackboard – as follows, beginning with a story of a shop employee who is faced with the task of counting how many boxes there were in 'stock'. The teacher reads the story:

Extract (9) 4rd grade, EF, 1:

```
1.    T:    (…) in the deposit of a shop there were many boxes(.)
2.          piled in many ways (.) you know what piled means, don't
3.          you?
4.    P:    yes ((makes gestures with hands, simulating piling))
5.    T:    you're seeing what she's doing? ok they were piled in many
```

145

```
6.              ways. Luis employee of that shop needed to check (.) how
7.              many boxes (.) were there and he set out registering (.)
8.              he was very smart >he knew numerical expressions< (.) and
9.              he set out registering (1.0) how he was seeing the >piling
10.             of the boxes<. So there is a register he did like that
11.             look
12.             [(9.0)

                [writes on the blackboard: (1°) 9 ÷ 3 X 3 + 5 =
13.    T:       this was the situation that he observed. (2.0) right? The
14.             second situation he observed (.) of piling of boxes=
15.    P:       =you've missed the little line
16.    T:       ah?
17.    P:       below
18.             [(8.0)
19.             [writes on the blackboard: (2°) (3 + 1) X 4 =
20.    T:       the second situation was this one (.)
```

According to the teacher, as the employee 'was very smart' and knew 'numerical expressions', he set out to organise his activity of 'registering' according to the way the boxes were piled. That produced a mathematically organised phenomenal field of action (Suchman, 2000). The teacher goes on to ask the pupils to (1) solve the employee's mathematical translation of the 'piles' and (2) produce a 'graphic' representation of the piles, given the 'numerical expressions' available. Here, *numerical expression* designates the use of arithmetic to reflect, or rather model, perceived regularities in the environment. It is also a 'prospective account' (Amerine and Bilmes, 1990), whose adequacy is to be backed against the work of arriving at it, as seen in the extract above; what we saw in extract 4 was the concepts being used once again, in a rather self-explanatory way.

The story 'above' hints at the 'mathematisable' all along; indeed, it contains a vernacular account of actions that make inspectable the employee's work as the work of producing mathematical descriptions. It does so by enrolling a set of characters, actions and scenarios that are constitutive of a matrix for calculations and transactions: employee, deposit, stock, check, count, piled, 'many' boxes. Although those words do not strike us as necessarily 'mathematical', what the shop sells or which goods were stocked in that particular deposit, or why Luis, the employee, 'needed' to count them is

irrelevant. That is a 'story' of what Luis observed with mathematical 'eyes', of how he made a collection of 'things' to become a 'register'. What Luis *saw*, though, is to be represented through a combination of heterogeneous resources to produce the adequacy of the story's description for instruction purposes. Seeing and registering are then re-combined to 'solve' and 'embody' just what Luis saw.

'Seeing' is a 'phenomenotechnical' achievement in the classroom, as well as in scientific laboratories (Latour and Woolgar, 1979). From line 11 onwards, the teacher offer a series of displays that correspond to the 'situations' that Luis 'observed': '9 ÷ 3 x 3 + 5 =' and '(3 + 1) x 4 =' (lines 12 and 19). Such representations consist of algorithms that model the 'piles'. The matter becomes even more compelling as those algorithms are used to deduce the 'observations', rather than standing as the deductive product of the mathematisation of the 'real'. A few moments later during the same instruction sequence, the teacher explains how to go about fulfilling the current task:

Extract (10) 4rd grade, EF, 1:

```
21.   T:     (.) you're going to take the back sheet of the squared
22.          notebook (.) let's pay attention  (.) you're going to copy
23.          those expressions=
24.   P:     =the (        )?
25.   T:     calm. no. You're going to copy >the way I put in the
26.          board< situation one two and three=
27.   P:     =can it be done with pen?
28.   T:     it can. You're going to show me in two ways (3.0) you're
29.          going to solve (1.0) the expression that Luis made and
30.          you're going to show me (.) how those boxes were piled in
31.          the situation one (.) how those boxes were piled in the
32.          situation two and how they were piled in the situation
33.          three. leave it ↑well displayed because I am to pass by
34.          now to see your stocks (.) do I need to repeat the
35.          orientation?
36.   P:     ↑no
37.   T:     do I?
38.   P:     no.
```

The set of numerical techniques used for *registering* observations is also used as a template for *producing* (generating) the same observations. In some ways, this 'reversibility' is granted by the way the story is framed. As I suggested before, the story already constitutes a field of mathematical activity, as to equate, for example, the piles with a given numerical set as the 'observations' go, the '*many ways*' of which are inherently numerical: otherwise boxes (as anything) are *piled* by the '*single*' feature of putting one on the top of the other! (The point, of course, is not to defy semantics). For the sake of the task, the numerical items in the register reverse into the production of the 'boxes' intended, represented by a set of small wooden cubes distributed by the teacher, and vice-versa. The children are thus asked to copy the numerical expressions from the blackboard into their notebooks in order to come up with the 'sum' of the boxes and then produce a material version of the numerical expression with the small cubes. This is the instruction delivery, or the practical 'orientation' (line 35; also, see discussion of extract 2) that I tried to describe rather retrospectively from the problematic stance that culminates in extract 8. As I suggested, the interplays between formulating actions and executing tasks seem to reveal, in the case of mathematics at least, not only how the latter is sequentially positioned and vernacularly formulated *as* the legitimate work of the former, but also how its conceptual resources are made to disentangle authoritative agencies – in favour of the rational, or necessary – from the work of its production: the story at the beginning of the sequence prospectively account for what teacher and pupils are doing, fixes the sense of the task yet to come, validates its 'worldliness', etc.

This strategy seems to suggest that a greater number of semiotic delegates are needed in order to secure the 'logical', instead of 'arbitrary', standpoint of the different conceptual resources at hand, so that the mathematics immanent in 'piles' makes Luis' day work 'storyable', at the same time the story makes the task 'meaningful'. Above we saw a narrative with various relevant characters and actions, forms of 'graphic' representation and numerical symbolism. The very 'representational' *morals* embodied in the story, with its portrayal of *signifiers* and *signifieds* concerning Luis' reasoning process, reflects as well as constitute the accountably 'principled' vein of this particular lesson.

Following a range of traditions in the contemporary critique of social action, I have suggested that there is no intrinsic (transcendental) relationship between the formulation of a rule and its application and that the role of the analyst is to follow the intricate relationships between how settings are produced and accounted for, rather than looking elsewhere (e.g. culture, society, mind, supernatural, etc.) for the 'determination' of action. Here, I treat the relations between accounts and *ad hoc* actions as mutually implicative states of affairs particular to their circumstances of use. That has implications for research:

> 'A recitation of formal structures can not count as an adequate sociological description, when no account is given of the local production of *what* those structures describe' (Lynch and Jordan, 1995: 227).

This topic is at the very heart of ethnomethodology's celebrated disputes with 'constructive' approaches (Garfinkel, 1967; Heritage, 1984) on the nature of 'sociological description' (Sacks, 1990). This relates to the former's refusal to rely on vernacular, commonsensical formulations of 'social facts' as *resources* to investigate and explicate society's assemblage of actors and activities. A 'social fact', etnomethodologically speaking, is the reflexive, implicative production of actors and circumstances. Thus, a 'set' (see section 5.4.) is a sociotechnical 'fact' itself to be established, despite the 'gatekeeping logic' (McHoul, 1996) that vigilantly refuses to regard mathematics as 'made up'. The relevant question here is: can mathematics and mathematical inscriptions account for the work of producing mathematics and mathematical inscriptions? Is there any determinate, functional relation between 'the situated production of actions' and the set of rules or formulations that can figure as an account of those same actions? The default ethnomethodological answer – which I tend to follow here – is no, although that does not mean that the links between actions and formulations are not accountably relevant as the documentary work of producing something *as* mathematics. It is not the case, I would argue, that the 'formulations' are mathematical whereas the '*ad hoc* actions' are not; their 'rational' and 'ordered' character are established locally, ethnomethodologists would say, where they mutually support and refer to each other. Instead of pursuing a sociology *of* mathematics teaching, we are recommended to

describe the 'indigenous sociologies' *in* mathematics teaching (Livingston, 2000), that is, the field of classroom instruction and practical action *in* and *as mathematics* (Lynch and Jordan, 1995). Basically, by sociology *in* mathematics I mean to indicate the difference between traditional accounts of mathematical activity, on the one hand, and the local, accountably rational properties of the organization of an activity and its *report–able* features, on the other. The latter, of course, is related to the work of ethnomethodology, according to which, as I pointed out before, accounts are reflexively tied to their contexts of use.

The expression also alludes to a discussion raised by Harvey Sacks as to how we can find the activities by which a scientific practice is reproduced through the availability of a range of technical and ordinary instructions and formulations; Sacks reasoned that to describe such methods and its applications or variants in everyday phenomena of order production would be one of the tasks of sociology (Ibid.). 'Sociology' in mathematics and mathematical practices is, therefore, members' phenomena, part of their encounter with, and production of, *epistemic* issues.

What Sacks was pointing to was how 'methods' – as an integral part of science – constitute an inspectable arena of knowing action (Lynch and Bogen, 1994); how a set of scientific *propositions*, to put it differently, cannot explicate on their own the methods used to replicate scientific observations. Gherardi (2000) suggests that sharing 'forms of life' – what Wittgenstein termed 'primitive 'language games', such as ordering and instructing – is the prerequisite for teaching and learning 'propositional knowledge', that is, knowledge rules and formulations (as we have been calling them). Indeed, it is suggested that these are the very basis of common understanding, although, as I have pointed out, the question can be expanded by asking how such primitive mechanisms of mutual intelligibility are accountably practical extensions of the fields they 'formulate' (Lynch, 1993). Conversely, in 'cognitivist' studies, (knowledge of) rules are taken as the generative basis of psychological and communicative processes (Edwards, 1997). 'Disembedded' knowledge is then depicted as the basis of embedded, situated action. This 'betrayal' of the logic

of practice introduces a reflexive logic through distance and separation between subjective and objective 'poles'; in that move, knowledge 'in' and 'about' a setting is articulated (Gherardi, 2000).

With reference to the previous examples, it is appropriate to say that formulations are places that lessons continuously arrive at. That is to say that is precisely the situated work of assembling demonstrations that sets up the conditions under which formulations can subsist as 'unique' mathematical accounts, as 'telling-order-designs' of such exhibits (Morrison, 1981). So far, I have tried to put 'abstract' formulations in the lessons into (inter-) action perspective. By doing so, are we at risk of reducing the 'content' to a lateral, epiphenomenal aspect of sequential interaction? In order to avoid this, I want to consider how the situated, ad hoc actions in the classroom are part of content's accountable orders.

5.4. Opacity, transparency and the work of representation

The general idea of bringing 'invisible' worlds into a perceptive surface is not unknown to mathematicians themselves. Devlin (1998) directly suggests that mathematics 'makes the invisible visible', that is, that formal mathematical knowledge allows us to see 'reality' in a way that would be impossible otherwise. Mathematical notations are *the* apparatus that makes the task of *knowing* possible, not least because mathematical entities and relations are the building blocks of reality itself. Drawing from a (Platonic) hyper-realistic rhetoric, the author considers mathematical representations to be a 'lens' into an independent mathematical reality: 'mathematical notation no more *is* mathematics than musical notation *is* music [...] the symbols on a page are just a representation of the mathematics' (p. 5). His point is, at a second level, that of mathematics as a kind of cognitive 'organ':

> 'For most of its history [...] the only way to appreciate mathematics was to learn how to 'sight-read' the symbols [...] Mathematics can be 'seen' only with the 'eyes of the mind'. It is as if we had no sense of hearing, so that only someone able

to sight-read musical notation would be able to appreciate the patterns and harmonies of music' (pp. 6-7).

Most likely with the eyes of 'agreement', Wittgenstein would say (Bloor, 1983). In a more 'contractual' view, seeing as intended, or in accordance with some rule, demands that certain inference techniques, and not others, are impressed and sanctioned in relation to the formal inscriptions that stand as their projected result and adequate description, e.g. an algorithm. There are 'wrong' and 'right' ways of performing arithmetic (Lynch, 1993; see section 5.4.), as the analysis of classroom assessment can easily show, and although there is no possibility of compiling a formal record of all the practical and material trials that constitute the 'right' cases, there must be acknowledgement of the 'felicitous' conditions under which particular cases can count as 'proper', as members examine the links between their results and procedures. The call for looking at 'conventionalisation', 'socialisation', 'enculturation', and so on, is, thus, a call to look at how 'visibility' is produced for 'non-members', 'newcomers', the 'incompetent', for those who are accountably at the limits of language, like the 'child' in Wittgenstein's *Investigations* (Peters, 2001).

The practical sites in which those processes live are part and parcel of the 'knowledge' orders they mediate; they harbour and inhabit them at the same time. Lynch and Jordan (1995) argue, in the case of molecular biology, that the specificity of the field must not be forgotten as an implication of (sociological) analytical rejection for 'abstract, quasi-causal conceptions of information and instructions' (ibid: 241). The field of molecular biology is not to be described in terms of a general sociological standpoint, e.g. talk-in-interaction. In a way which supports my argument for the mathematics classroom as a 'perspicuous setting' (Garfinkel and Wieder, 1991), Lynch and Jordan make a strong case for molecular biology as an accountable scientific order characterised by the use of procedures *in* and *as* a field of practical activity. The authors go on to identify a range of 'instructional' problematics indigenous to molecular biology/sociology, such as how accounts of instructed actions are placed within the *doings* of intracellular order itself, i.e. how cells 'instruct' their

reproduction, and how those molecular orders are appropriated, domesticated, 'co-opted' by the methods that describe them. They also affirm that 'such a science of practical actions is nothing more, and nothing less, than a practical achievement of molecular biology itself' (Lynch and Jordan, 1995: 241). That is the reason why I have been reluctant in giving analytic priority to a disciplined 'vision' of social order as a starting point. For example, as far as a 'conversation' is concerned, one needs at least two things to be able to 'operate' as a conversationalist: (1) know the language in which the conversation is taking place, that is, its grammar and vocabulary (English, Portuguese, Japanese, and so on); and (2) to display a sensitivity and orientation to its economy and morality, that is, to how a conversation, as a piece of interactional technology, is achieved and sustained. These include actively displaying 'understanding' or 'hearing' of the other part and taking turns properly, such as respecting rights for holding the interactional floor – of the kind 'one person speaks at a time' (Sacks et al., 1974). Variations of those basic properties can be observed across very disparate 'cultures' (Moerman, 1988)[58] and all organisational contexts of social life. Nevertheless, it does not take much to acknowledge as a simple truism that, despite that being the case, we are not fluent at 'everything'.

By 'everything' I mean the countless fields of expertise and practical activity whose access is not transparent to 'novices' or 'just plain folks' (Lave, 1988; 1991), and in which membership and inspectability are two faces of the same coin. At the time children join school, they are already fluent at major aspects of 'conversation' (Baker and Freebody, 1989) but not at, say, 'mathematics' (neither at lesson methods) even though number words and some primary language games with numbers (e.g. counting) may be already 'suburbs' of their language (Hughes, 1985; Sinclair, 1991). Despite a common 'stock of knowledge' covering conversation and mathematics, teacher and pupils operate in a setting in which 'school mathematics' is an 'opaque' field, so to speak, whose visibility is yet to be established. Both terms, 'school' and

---

[58] Conversational universals cannot be, for that reason, the basis of the anthropological understanding of the notion of 'culture'. Although they are undoubtedly 'cultural', they are unable to account for social differences along the lines of family, kinship, marriage, religion, education, philosophy, economical transactions and value, etc.

'mathematics', have an importance of their own here because they reflect differences in practices prior to kids' introduction to the classroom, and at the same time are irreducible to a general account of social order. At *school*, pupils and teachers are constituted as interactants, amongst other things, through discursive practices that, while relying on the same mechanisms found in interaction everywhere, produce different implications (Drew and Heritage, 1992), like the so-called 'question-with-known-answer', whose function is not to ask for 'information' the questioner does not have (Mehan, 1986; Macbeth, 2000). 'Mathematics', in its turn, becomes a topic in its own right and, as Valerie Walkerdine (1988) shows, is put to usages that depart from those of 'home' practices. For example, in the classroom 'relational' terms such as 'more' can establish very different 'paradigmatic' relations with other terms, and while it is highly likely that in the school logicised use it is often contrasted with the term 'less', on other occasions, such as at the dinner table, it can stand as the semantic counterpart of 'no more' (Ibid.). Doubt about whether the children's compliance with the classroom routines is explainable in terms of their 'understanding' of the content of the lesson[59], does not imply that the lesson is not a 'mathematics' one, or that the compliance terms are not accountably 'mathematical' situated actions. Mathematics is a *specific* domain of knowledge and the situated production of accountable mathematical orders cannot be reduced to a general theory of action (Lynch and Jordan, 1995). The fact that the opacity and transparency of such setting live through sequences of questions and answers that distribute rights to speak and found primitive

---

[59] Some ethnomethodological texts display a sense of 'disappointment' regarding the procedures for, and outcomes of, instructional activities and its relation to the curriculum: 'As we shall see, for children, the translation from instructions to performance is particularly hazardous, engendering diverse, unforeseen, and quaint difficulties. The result is not that the children do not learn, but that they learn something rather different from what the "experiment" is designed to teach' (Amerine and Bilmes, 1990: 324-325). They authors openly say that instead of learning 'science' in the lessons they observed (p. 326), pupils face a rather different cognitive task: that of how to learn following instructions. Their solution to this critical quandary is to imply that following instructions *is* the curricular design for the early school grades they are commenting upon. There also seems to be a sense of critique in Lucy Suchman's brilliant study of human-machine communication (Suchman, 1987), which shows that emergent interactional troubles for one of the parts (the human) can be accounted for in terms of the other's (the machine) built-in design for communication as a form of calculation. Such a design works to select and make relevant only a few visible aspects of its interlocutor's behaviour, interpellating his/her conduct as 'planned'. Software application designers seem to have taken notice of Suchman's 'critique' (Hutchby, 2001).

inference routines – as well as through the use of a natural language does not mean that the setting's visibility can be reduced to them. Such visibility, that is, the fact that we are 'build' to 'see' things as such-and-such (e.g. phenomena, activities, abstract categories) is a pervasive question, and is the point of psychological and social theories as diverse as those of Jean Piaget and Harvey Sacks. What is common to both, though, is that the way we 'see' things is not to be accounted for in terms of our physical apparatus.

In Piaget's cognitive-developmental theory, overcoming opacity is equivalent to developing notions of logical necessity (e.g. reversibility and conservation of logical operations) involving a process he called 'reflexive abstraction'. Reflexive abstraction describes the internalisation of the formal properties of one's practical actions, rather than the sensorial assimilation of the objects' qualities, which Piaget referred to as 'empirical' (abstraction). These two modes correspond roughly to Locke's *primary* and *secondary* qualities: colour and shape, for example, are 'empirical', whereas 'number' and relations such as 'bigger than' are not in the objects themselves, but are the product of one's *acting* upon them; it is in this sense that they are *reflexive*. Piaget's thesis describes the functional continuity between sensory-motor action and representational thinking: 'Symbols' and 'signs' come to be part of the 'cognitive' repertoire during early imitative activity and to a large extent are 'created' spontaneously by the growing organism. What is peculiar in Piaget's theory is that a sign's *meaning* is grounded in individual development and, ultimately, in the 'schematism' of adaptative action, but, as I said before, not in the sensorial apparatus itself.

In the early 1960s, Harvey Sacks started investigating talk as an orderly phenomenon in its own right. His enterprise undertook the description of an 'apparatus' that could account for certain 'hearings', an 'inference-making machinery' that is part and parcel of the sequential order of a conversation (Sacks, 1992; Schegloff, 1992)[60], and 'socialisation' as the process by which

---

[60] Schegloff (1992) points out that the kind of problematic that set out some of the most well-known developments of CA, especially recognisable in Sacks's paper on 'the analysability of stories by children' (1972), share formal similarities with the type of questions posed by Chomsky for the description of a universal grammar, in terms of the generation of a

such phenomena of order is made visible and usable for newcomers. The formal properties of order that members find at every situation (e.g. conversational norms for turn-taking) and the fact that order is reflexively designed so as to be grasped by members[61], so that 'things are so arranged as to permit him to', in Sacks's words (p. 485), are the main sources for the systematic observability and acquisition of culture in interaction.

Problems of opacity and transparency are, then, at the heart of the 'acquisition of culture'. However, it seems that in the case of science learning the description of conversational competencies in their own right miss crucial aspects of how settings are manipulated and accounted for, and to which the attribution of learning is bounded (Piaget makes the same mistake by producing version of *'cognition* in its own right'). Few conversation analysts have faced the issue of how different kinds of competence are put to work in producing and accounting for 'knowledge'. In that respect, Goodwin observes that:

> 'This sense of basic, recognizable interactive organization running smack into a opaque wall, a domain of phenomena which seems absolutely crucial to what the participants are doing, but which I don't understand simply by speaking the same language or living in the same country, is what has struck me almost every time I've done fieldwork in a new professional or scientific workplace' (Goodwin, 1997: 24).

The activity Goodwin is observing and referring to is one of determining the characteristics of a land site (e.g. its 'age') in an archaeological excavation through the use of a colour chart. He notes that the simple and straightforwardly visible conversational interaction between two scientists at work does very little to assist his understanding of a scene whose phenomena, activity and its technological intermediaries remain non-transparent. For Goodwin, as for Latour and Woolgar (1979), it is not the lack of general

descriptive 'apparatus' and the reliance of analysts on his/her own (cultural) expertise on devising the answers to the problem he/her posed to him/herself (p. xxi).
[61] Garfinkel's central recommendation for ethnomethodological studies of order consists of the observation that 'activities whereby members produce and manage settings of organized everyday affairs are identical with members' procedures for making those settings "account-able"' (1967: 1).

familiarity with a new setting that accounts for the feeling of opaqueness regarding the situated actions that take place within it. Instead, it is member's orientations to resources the outsider does not have access to that constitute the gap.

In the following extract, the students (5 year-olds) are sitting down on the floor so as to form a 'circle', inside of which the next task's 'materials' (*cards* depicting diverse signs, e.g. = , ≠; numerals, e.g. 2, 7; and diverse objects, e.g. 'stars', 'fruits', 'matches') are to be made available and activity to take place (M = Mateus):

Extract (11) pre-school, EL, 2:

```
1.    T:    for us to say that one thing equals another we do not
2.          always need to use the word equal (.) we can use a sign
3.          (.) I am going to show (.) who knows the sign for
4.          equality?
5.    P:    I know.
6.    P:    (     )
7.    T:    I am asking when I mean this word equals that one (.) I am
8.          going to show you (.) look (.) the little sign we use to
9.          say that one thing equals another ((shows a card with the
10.         sign = )) (.) this is the little sign of equal (.) I mean
11.         (.) Daniel, look (.) house equals house ((simultaneously
12.         shows three cards that make [CASA] [=] [CASA] together))
13.         (.) this is the little sign that says that things are
14.         equal (.) each one of you is going to get now a little
15.         sign (…)

      (a few seconds later)

16.   T:    everybody now have the little sign of equality (.) I'm
17.         going to place it here in the centre of the circle ( ) (.)
18.         you're going to see that here is this kind of material
19.         with various objects in various shapes, various colours
20.         (.) here are the numerals (.) I want you to form two sets
21.         of equal things, or two equal sets
22.   M:    I've already formed
23.   T:    have you formed a set?
24.   M:    (( shows [7] [=] [7] with the cards ))
```

```
25.   T:    seven equals seven (.) is this a set? (.) what's
26.         Lacking to form a set there, Mateus?
27.   M:    one more
28.   T:    look (.) this is a numeral, the numeral seven (.) where
29.         are the things to signify that there are seven things
30.         here? (.) I'm not seeing anything look I'm only seeing
31.         numerals (.) a numeral seven and another numeral seven (.)
32.         I want a set.
```

It can be said that this event immediately precedes the work of producing a state of affairs that complies with, and is accountable in terms of, the formulations it represents. It delivers a task for which the appropriate courses of action are not yet specified. I want to call the attention to the arrowed lines at the beginning and the end of the extract as performing the work of 'formulating'. In the first one (lines 1-2), an investment in the worldly sense of the task (and consequently, of the mathematical concept at stake in the curriculum) is made by stating that 'equal', also a vernacular term that 'we say', can be represented otherwise. 'Equal' is a 'word' that 'we' 'use' (line 2), an arbitrary thing, as semioticians would argue, since we do not have 'always' to display it to convey what we 'mean'. The word 'equal' is less than its meaning, for which the 'equal sign' can be another representation, equally able to 'say' it (lines 8-9). The meaningfulness and worldliness of the lesson are reinforced in line 7, when the teacher makes use of direct speech ('I'm asking when I mean *this word equals that one*') in a kind of 'animated' footing (Goffman, 1981), as someone saying (or thinking) that; note that '*when I mean*' stands as a potentially recognisable, or simulate-*able*, ordinary action scenario. The second case (lines 20-21) establishes what is to be done next ('I want you to form to sets of equal things, or two equal sets').

As we saw earlier, the teacher proceeds to exemplify her initial proposition. For that, she manipulates a set of cards. [HOUSE] [=] [HOUSE], she says, means[62] 'house equals house', which, grammatical oddness notwithstanding as

---

[62] There is a sense in which an understanding of 'equality' is already assumed in the interaction, for to 'mean' (lines 7, 10) can be seen as a part and parcel of the ongoing argument on 'equal'. In that sense, 'mean' means 'equals', an operator in the vernacular accounts of reasoning that purportedly gives sense to the task. Such an account is a paradigm of the more formal phrasing constituted later by the assembly of cards; the idea is more or less the

the latter, 'vernacular' one is concerned (e.g. absence of articles or demonstrative pronouns[63]), has the role of translating the ordinariness of 'meaning' onto a written code. In the sequence, she introduces a set of materials to the children, again cards, containing the 'equal sign' (=), and, as it is claimed in the transcript above, 'numerals' (line 20) and 'things' (line 21), and asks them to form 'two sets of equal things, or two equal sets'. Mateus, one of the pupils, promptly comes up with a candidate answer: [7] [=] [7] (line 24). The teacher's injunction following the answer consists in a 'repair' strategy that hands correction over to the pupil, confirming a preference for self–repair in classroom discourse (McHoul, 1990). The coupling of the questions 'is this a set?' and 'what's lacking in order to form a set?' (lines 25-26) implies, without saying, that Mateus' answer is to count as inappropriate, incomplete, 'lacking' at best. Arguably, the notion the teacher seems to be pursuing is that a 'set' – or any other notion for that matter, such as 'equal' – *means* something, that is, it is irreducible to the representation device used to convey it. The task at hand is then to show, to establish indeed, a 'meaningful' semiotic link between *signifieds* (things, events, vernacular meanings) and *signifiers* (formal inscriptions). It is important to remark that this observation does not relate, at this point, to a general conception of a semiological process underlying the shared use of language in the classroom. What I am suggesting is that the knowledge being put together in the above transcript is the accountable outcome of the activities and methods of/for associating diverse elements, of forging 'visible' links; semiotic 'translation', 'modelisation', and the travels from one competent world (e.g. narrative, vernacular understanding) to another (logic, mathematics) then becomes the very topic of classroom teaching and inspectability.

---

following: ('by [this word] [I mean] [this word]') → ('(I) ['mean' '=' or or 'equals']') → ('[X] [=] [X]').

[63] This is a question that can be addressed again in terms of how the prominent role of the referent in making order visible 'affords' grammatical alternatives. That the sentences 'a house is equal to another house', or 'this house is equal to that house' do not feature as 'less strange' utterances in the exchange between teacher and pupils can be accounted for as a case for language being designed not only to comment upon, but to map onto the formal limits of the referent (the cards), to become yet *another language*, i.e. mathematics; Ironically, the aim is precisely to suppress the production of meaning in relation to referential or metaphoric dimensions of language use (Walkerdine, 1988).

Again, the contested answer in lines 25-26 helps to make the point, and here it is easy to go 'critical' in relation to the job the teacher has done. Besides the opaque, indeterminate nature of following an instruction properly, the indexicality of which the unfolding, ad hoc technologies of training are the means, the very example set at the beginning of the task ([HOUSE] [=] [HOUSE]) fails to constitute a legitimately followable paradigm, a model for a 'correct' answer. The way Mateus' intervention (line 22) arguably reflects the immediacy and availability of that model is quite interesting ('I've *already* formed'), an economy of activity that is soon questioned. Notice that Mateus' answer to the request to form two equal sets is analogous to that in the teacher's exemplary case: the repetition of a term at each extreme of the expression, separated by an equal sign indicating their equivalence:

$$[HOUSE] [=] [HOUSE]$$
$$[7] [=] [7]$$

Apart from that example, no clue had been offered on how to go about solving this apparently simple exercise. 'Materials' (line 18), of course, had been made available to be used, and they consisted not only of the 'numerals' that composed the pupil's answer. 'Various objects in various shapes, various colours' (line 19) were presented to the pupils, as categorically distinct and sequentially prior to, 'numerals' (line 20). Retrospectively, it is easier to see that those distinctions, which operate in the request form 'I want you to form two sets of equal *things*' (lines 20-21), work as a 'prospective account' (Amerine and Bilmes, 1990), an account that comes to life in the teacher's contestation of Mateus' answer, as discussed above. The explanation gains further elaboration when after Mateus' failure to address the teacher's question on what was 'wrong' with his answer by saying 'one more' (line 27) – arguably orienting to the problem of 'lacking' as a *numerical* one – the extent of previously introduced, or alluded to, categorical distinctions are used both as an account of the pupils' reasoning and as a 'remedial' measure into the task's completion, that is, it 'indexes' the activity's projected outcome (lines 28-30: 'this is a *numeral*, the numeral seven (.) where are the *things* to mean that there are seven things here?).

The equivalence between the classes of objects separated by the equal sign cannot, however, be warranted by the existence of the sign itself, as if it had, by virtue of its presence alone, some legislative power over the meaning of 'equality'. So, although the 'inappropriate' character of Mateus' (mathematically correct) answer allowed the teacher to make room for the curricular demands placed upon the meaning of 'sets' and 'equivalence', it is not the case that any two given sets of 'things' (as opposed to numerals) will be automatically considered 'equal' if they have the equal sign [=] placed between them. Thus, the 'things' that form sets in the task arranged by the teacher are to be interpellated not in their 'thing-ness', but in terms of their 'numerical' equivalence, or more precisely, bi-univocal correspondence[64], so that the relevance of any term in the expression is accountable in terms of the others' presence. Having set up the accountable relevance of the 'things' that numerals represent, the teacher can pursue on which basis is 'equality' to be reasoned about. For example, in the following short extract the teacher (T) is talking to Ivo (I), who had just completed his task, forming 'two equal sets' with numerically equivalent, but element-different, sets:

Extract (12), pre-school, EL, 2:

```
T:     look at this set and tell me why this set is equal to that one.
I:     ((Ivo points to the equal sign))
T:     because of the sign? But are they equal or different?
I:     different
```

In this case the pupil has apparently been misled by the teacher's questioning method, which contradicts the implication, in his pointing the equal sign, that *it* (the *sign*) was the *cause* of the 'equality' between the sets. Although the speculation on what Ivo *really* thought is beyond the analytical recommendations embodied here (Edwards, 1993), it is fascinating that his 'interpretation' of the teacher's request – as understood by the teacher – is a

---

[64] The concept of 'bi-univocal correspondence' must be familiar to whoever have come across Piaget's studies, his interest in set theory and his renowned tests for 'conservation of quantities', and consists of the principle that a set is equal to another if, and only if, to each element of the first set corresponds one, and one only, element of the second. It is not unlikely that the task under analysis here and its 'ideological' features draw from the influence of Piagetian psychology and its explorations of logico-mathematical thinking.

frontal alternative to her 'theory' of meaning, or the distribution of different 'agencies' to different categorical entities (*things – sets – numerals; saying – meaning*), as we discussed above. It seems to level the *notion* of equality to something ultimately arbitrary, even possibly under the teacher's will, an undesirable learning outcome which is, at the same time, an artefact of teaching.

The pupil not only 'looks at the set' in order to *report* his grasp of the problematics raised by the controlling adult; he finds it (the *'why'* of the request) *precisely* there, in matter and space, for which pointing serves him well as a way of 'telling'. The *reason* is *there*, it subsists as a visible *mark* on the ground! The teacher replies with two questions, the second of which represents a complex interactional device. After *interpreting* what Ivo had just *done* ('because of the sign'?) in a way that prefaces the possibly problematical character of his answer, the teacher asks a second question whose centrepiece – the conjunction *but* – generates a set of 'hearable' possibilities. The first one has a *retrospective* effect, and makes room for the idea that it has not been established from the beginning that the sets were, in fact, equal, which the original question somehow suggested. Retrospectively, the request to 'look at this set and tell me why this set is equal to that one', does not now necessarily say 'those two sets are equal, now tell me why', as it potentially asks 'they are not equal, and I can see it; are you *still* to think that they are equal, considering that you can look at it again?'. The reason for that is that is that questions are 'heard' in the classroom not as requests for *information* but rather as *confirmations* or *negations*, that is, as performing assessment and repair (Mehan, 1986). It is an integral part of the skills pupils develop in an educational setting to be able to see the very format of the lesson (e.g. turn-taking system), and its followable paths, unfolding before them (Amerine and Bilmes, 1990; Macbeth, 2000). By using the conjunction *'but'* the teacher effectively builds a clue, a hearable classroom maxim into the 'negation' reasoning that Ivo attends so effortlessly in the sequence. However, it is difficult to accept that the teacher was pursuing to deny that Ivo's *numerically equivalent sets* were equal. This brings us to a possible use of *'but'* that is alternative to the straightforward use of that grammatical item to indicate

contrast. The teacher's question makes sense only if heard against the particular accountability of the semiotic agents she is pursuing. It could be rewritten as follow: 'the mark [=] you have pointed to notwithstanding, are the sets *per se*, in their mathematical value, equal? Do they *mean* the same mathematically (for which the local adequacy of conventional representations would constitute another problem)?'. In Latourian mode, we could say that the teacher is disciplining her delegates.

One could ask at this point whether there is a reason for the accountable minutiae of 'set theory' to be present in the curricular activities of pre-school children (in the case of extracts 11 and 12). In the case of school mathematics, that has traditionally reflected a call for 'understanding', rather than 'reproducing' instruction. Seemingly, educational reforms in mathematics have assimilated the *logicised* views of modern mathematics made relevant in set theory[65], and given visibility in psychology and epistemology by Jean Piaget and his followers as 'developmental' criteria (although Piaget held different views from logicians regarding the explanatory role of symbolic formalism). Its instructional counterpart, seems to count both on an interpretation of language as a site for reference and representation, and conversely, on an educational appropriation of such linguistic philosophy of meaning and reference into a child-friendly, Augustinian pedagogics, where 'meaning' has precedence over 'symbolisation' or 'representation'. The question is important insofar as it reflects the search for the kind of enlightened, non-authoritative, de-individualised 'necessary knowledge' (Smith, 1993) that Piaget championed. In extract 13, we join the teacher and the pupils when they have just divided 92 by 4, and are to engage in a proof procedure.

Extract (13) 3rd grade, EE, 3:

```
1.    T:    I know I am sure we are sure that this division is right
2.          isn't it?
3.    P:    it is=
```

---

[65] Set theory is generally associated with the names of mathematicians and philosophers like Cantor, Russell, Frege and the collective known as Bourbaki, that in the late 19th Century stared to establish the philosophical and logical basis of arithmetic and the concept of number.

```
4.    T:    =we did it correctly didn't we?
5.    PP:   °yes°
6.    T:    but (.) ↓to be surer I'm going to take the real proof (.)
7.          the real proof of multiplication- of of: division that is
8.          done with the=
9.    P:    °multipli[cation°
10.   T:            [multiplication (.) isn't it? (.) so In order to
11.         do this multiplication what I'm I going to do? How am I
12.         going to start?
13.         (2.0)
14.   P:    (with the table?)
15.         (1.4)
16.   P2:   multiplying the re[sult=
17.   P:                     [multi[plying the divi-
18.   P2:                         [=by four
19.   P3:   the four times the twenty three
20.   T:    how's that?
21.         (0.8)
22.   P3:   multiply four times the twenty three
23.   T:    ((turns to the board)) so I multi↑ply four I'm going to
24.         place the: divisor: I'm going to multiply four >which is
                                                      ▲
25.         the divisor< by: twenty three. that is I'm going to multiply
                                                ▲
26.         the divisor by the =
27.   PP:   quo[tien:t
28.   T:       [quotient. And I'm going to find ↑the =
29.   PP:   REMAIN:DER=
30.   T:    remainder?
31.   PP:   ↑divid[en:d
32.   T:          [dividend I'm going to find ↑the dividend
```

In what preceded the extract above, after writing the algorithm '92 ⌐ 4' on the blackboard and established that 92 was to be divided by 4, the teacher proposes that 'I'm going to say that: >this room has ninety two pupils< (0.4) I am going to divide this room ↑in (.) four groups. We're going to discover now how many pupils are going to be in each group'. Again, a property of enunciation in school mathematics by which a 'scenario' conveys the *meaning* of an activity and its extension, its aggregates, the 'represented', is seen as part of the accountable orders of teaching-learning. The 'reversibility' of logical operations (Piaget, 1972) stands, in the interactional designs for mathematics

'learnability' (from which we can de-construct the 'black box' of mathematical representation) not only a practical task, but also a resource that allows the shifting out to yet another level of 'reasoned' representation[66].

After the calculation is performed, the teacher goes on to establish a method of proof to determine whether the procedure was followed correctly. The point of the method of proof is to produce the 'visible' adequacy of intermediate values and procedures, by applying them in 'reverse' operations, in order to reach the starting point of the calculation, or the original number to be divided ('dividend', lines 32-33). It is interesting to follow how the proof method elaborates the 'sociology' and 'semiotics' of the division. It does so by differentiating a number of categories and telling how they are instructed to 'act'. From lines 1-9 we are given the description of a 'shared' state of affairs and the 'reason' for the actions that *are going to* be performed: although 'we are sure' that the division is right (note that the teacher repair herself in line 1, after starting with 'I know'), 'to be surer' (line 6) is in order, and that is something that can be done by taking the 'real proof'. The problem is: why is it a question? Why do we need to be surer?

In establishing how the 'real proof' of the division is done with the 'multiplication' (initially in lines 7-10) the teacher takes the analytical vein of

---

[66] I also observed this extraordinary passage from the same activity (again, before the 'proof method' applied in extract 13):

T: I have nine I want to divide it by four it makes=
PP: =two:::::
T: ↑why two?
PP: because two times four is eight

This passage imply a few things: (1) answers have 'reasons', they are not arbitrary, or they can be interpellated as such; (2) the reasons are invoked as accountable procedures; (3) those procedures are related to the project of the activity. The local rationale being displayed here is as follows: 2 is the result of 9 divided by 4, and this is so because 2 times 4 is eight! We must have missed something: 9 divided by 4 is 2.25, and 8 is the inadequate number as the 'dividend' in the reversed procedure, where the 9 apparently disappeared. Of course, the intended division is 92 by 4, and the representation is still being 'run' at this point. The point I wanted to observe is that in these collectively readily accessible answers a 'whole number' is placed as the partial right answer in the ongoing project of a demonstrable outcome, and that the 'reversibility' action is reflexively tied to the result, rather being seen as external to it.

the 'proof method' a step further into the visibility of the activity of 'division':
her questions in lines 11-12 and the answers in 14-19 describe the performance
of an 'inference-making machine' (Sacks, 1992) in the mathematics classroom.
Its significance takes the form of the question: how should the pupils 'hear' the
question in line 11? What kind of answer displays the competence the teacher
is after? Similarly to the discussion of extract 2, I suggest that the problem can
be translated as that of making inspectable the lesson's instruction design in
terms that are 'integrally' *analytical*, and that the analysis describe and
constitute the activity *in* and *as* mathematics. As Eric Livingston puts it: 'One
of the aims of instruction is to supply, through demonstrations and practical
exercises, accessible settings that open technical practice for inspection' (2000:
245).

The answers in lines 14-19 orient to the horizon of a mathematically relevant
analysis in complying with the teacher's question, that is, they acknowledge
the 'inference rich' aspect of the questioning (which is reformulated in lines
11-12, from '*what* am I going to *do*?' to the suggestive '*how* am I going to
start?'). The teacher takes from the (adequate) answers of one of the pupils,
interwoven in a sequence of tentative answers (lines 16-19), and translates
them in a way that bears on other examples we have seen before in that they
expose the *semiotics* required to inspect the task. Such analytical design
includes not only the 'numerals', but what do they do depending on where you
find them (as the actions of pointing in lines 24 and 25 shows), and to what
kind of 'entity' they correspond to; thus:

'Multiply four times the twenty three' (line 22)

translates into

'I'm going to multiply <u>four</u> >which is the divisor< <u>by</u>: twenty three.' (lines 23-
25)

Which translates into

'I'm going to multiply the divisor <u>by</u> the= =quo[tien:t' (lines 25-27)

The passage from one kind of enunciation to the other, and their logical place
in a sequence of documentary observations, is more than an exercise being

made here; it was made there and then, as the presence of grammatical items operating their 'translation' shows: 'so' (line 23, taking up from 22); 'which is' (line 24), upgrading 'four' into 'divisor'; and 'that is' (line 25), which identify 'twenty three' with 'quotient' and summarize the 'proof' instruction. 'So' we have:

$$4 \times 3, \text{ 'which is' } \dots$$
$$\dots \text{ divisor} \times 3, \text{ 'that is' } \dots$$
$$\dots \text{ divisor} \times \text{quotient} = \text{dividend}$$

We have seen other examples of paradigmatic substitutions in the 'analysis' of the situation. In extract 1 and 2, we have something like:

'Two plus three *makes* five'

↓

'Three plus two *makes* five'

↓

'Something in the *form* of the question'

↓

'Applied the commutative property'

↓

'The order of the items doesn't alter the sum'

↓

'Two plus eight *makes* ten'

↓

'Eight plus two *makes* ten'

As far as extracts 9 and 10 are concerned, we could re-write them as follows:

'Many boxes'

↓

'Piles'

↓

'Register'

↓

'Numerical expressions'

↓

'Situations 1, 2, 3'
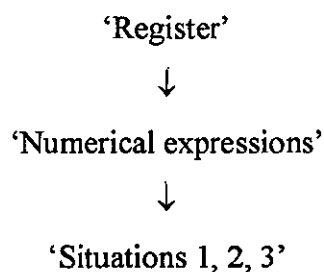
The 'semiotic' extract *par excellence* was, however, extract 11, where the very categorie s used evoked a sort of analysis of representational levels:

'"saying" one thing equals another'

↓

'a sign' (=)

↓

'[CASA] [=] [CASA]'

And:

'Little sign of equality'

↓

'Materials in various shapes, various colours'

↓

'Numerals'

↓

'Things'

↓

'Two equal sets'
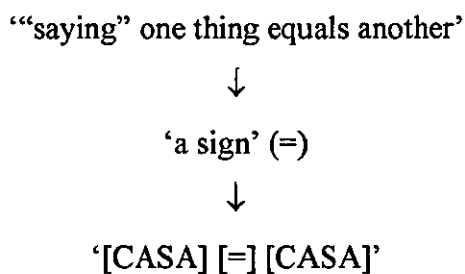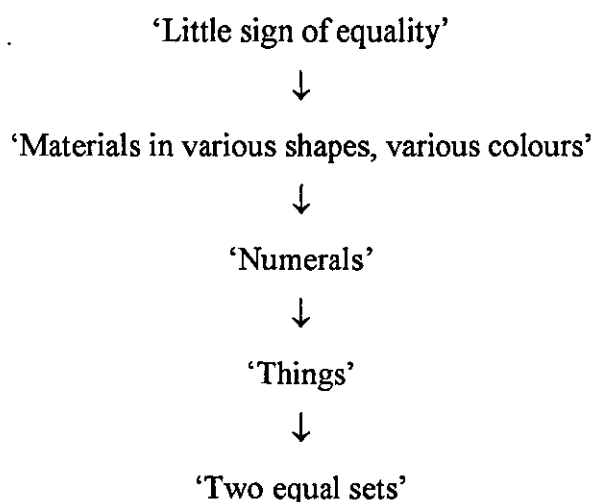
The way in which the sense of the task is justified at the outset in many of classroom exercises is an indication of how the accountable work of 'inspectability' rests on non-arbitrary basis. The call for rational 'justification' in extract 7, the many shifts between storylines and mathematisation in extracts 8-10, the suggested scenario for the 'animation' of mathematical reasoning as 'meaning' and the multi-referential relations between different entities (number, numeral, things, sets) in extract 11, and the interpellation of the

'equal sign' as powerful in extract 12 are some of things I am trying to point at. What I am suggesting is that this is an aspect of the mathematics produced for the 'classroom', for 'teaching'. All these theoretical 'levels' (meaning, reference, signs, etc.) constitute different forms of semiotic 'delegation', or 'agencies', as they shift 'in' and 'out' frames of reference (Latour, 1988b), in the unfolding of the naturally occurring production of the work for which certain rules and instructions are 'uniquely adequate' descriptions. The function of this movement seems to be eluding the 'arbitrariness' of sources of 'meaning' or 'knowledge'. Thus, Piaget's conception of number is that (1) number is true to (2) the logical structures of (a) class and (b) order that are true to (3) reflective abstraction from physical manipulations of objects that, on its turn, are true to (4) the patterns of accommodation to the environment typical of primitive sensorial and motor schemes, that are ultimately linked to (6) the organism's built-in reflexes. It is an astonishing belief in cognitive and developmental continuity that refuses to concede to logical symbolism or 'intuitionism' a foundational status. The classroom discourse we have observed so far uses similar mechanisms. Its rhetoric suppresses the visibility of a (arguably inappropriate) third, 'democratic', 'political', 'ideological', 'anarchic', 'opinionated', source of *episteme*; it seems that the teacher's analytical stand quite often produces the tension between logical necessity and the arbitrariness of authority as part of the accountable work of using and understanding mathematics.

We are not far from concluding that the teacher is teaching semiotics theory! Not in the sense that she is teaching things that are learned in a chain of substitution (which would be a general, quasi-explanatory statement), but that now and again the 'chain' itself is made visible, as part of the work of entangling 'knowledge' to 'non-arbitrary' sources. Meaning, representation, reference, shifting in and out levels are offered as the 'telling orders' of the actions performed (Morrison, 1981). Such semiotic *ethos* in the (mathematics) classroom denotes the opening, for analysis, of 'knowledge events and what might constitute an exhibit of their understanding' (Ibid: 245). It is, though, a 'semiotics' whose source of meaning is logical *necessity*, or identity, the (accountably) could-*not*-be-otherwise link between activities, empirical

phenomena, graphic representations and mathematical symbols. It is a discourse without a future, without *metaphor* (Walkerdine, 1988), based on the reiterable work of mutual reference between its terms. Its abstract features can be found as an 'observable', as the 'maths of the lesson' (Macbeth, 2000) in the sequential organization of the instruction, as we saw in many examples. Such organization is replete with 'impressing techniques' that allow the translation between those intermediaries to stand as a proper compliance with a formal rule.

Rather than framing the question in terms of 'power' or the 'teacher's control' (Thornborrow, 2001), perhaps the classroom can legitimately be seen as one of the sites that is part and parcel of an rational of (economic) liberalism (Hamilton, 1980), where power has increasingly become invisible (Walkerdine, 1988; Bernstein, 1971) or detached from a personal source. While the agenda is unmistakably the teacher's, the extracts we have seen operate 'enlightened' routines that accountably frees the local social contract of 'inspection' from the over-determination by a 'sovereign' teacher to found it again in 'reason', in the universality of the 'transcendental pupil', in the witnessability of scientific knowledge (or in simulations of order that lend 'meaningfulness' to classroom activities). Pupils' participation in open teacher-class instruction sequences is kept to a minimum, mainly at the minimalist format of teacher-class dialogue, while the trajectories of the reasoning to which they respond is designed to project 'witnessability' to a maximum, as we can see in extracts 1 and 2. In such occasions, pupils are positioned to 'realise' rather than to 'produce' knowledge. The way pupils are *interpellated* as subjects of knowledge is the topic of the next chapter.

CHAPTER 6

Accountability, human agency and the other things: the learner in relation to the installation

6.1. Discourse and cognition

I have previously argued that conversational and discursive studies of classroom education, as well as the sociocultural psychology of mathematics learning, have taken up the problem of 'intersubjectivity' as their main analytic agenda. This seems to be a direct consequence of a theoretical and methodological inclination in the context of the (social) sciences' division of labour, namely, that of aligning 'commensurable', 'homogeneous' objects for the effect of analysis: thus, co-presence and talk belong together; the *content* of a science, on the other hand, belongs elsewhere, as does technology (Law, 1991). Issues of 'agency' versus 'structure', for example, have divided diverse sociologies, and few systematic efforts have been made to overcome this double agenda (Giddens, 1984), and even less to include the non-human 'masses' into the equation (Latour, 1992). In the conversation-analytical perspective such interest in 'inter-subjectivity' was translated into a concern with shared discursive routines in the classroom, and especially with how rights to speak are distributed; how inferential reasoning is conventionalised in relation to the 'machinery' of social interaction; and how participants 'repair' each other's understanding of the ongoing talk. The *transcript* is the main trace and mediator of such an 'inclination' – it is arranged so as to *perform* the machinery of conversation and its sequential and implicative structure, or, to take it further, that of social order itself.

Similarly, in the sociocultural psychology inspired by the work of Vygotsky and his associates, learning is conceptualised as an 'internalised' version of

intersubjectivity, or social interaction, with *thinking* standing as an abbreviated form of conversation, as talk to oneself; according to this viewpoint, it ultimately consists of the *speaker* being also the *recipient* of talk. In chapter 3, we saw not only the historical basis of the classroom system in the reflections of the ideologues of the 18th and 19th century, as well as in the development of Vygotsky's pedagogical psychology in Russia, but how such a purely social machinery, a mundane, unremarkable, reiterable performance of the 'social contract' in face-to-face interaction has been central in the way several 'cognitive' topics have been re-shaped in the social sciences; more specifically, we saw how the ethnomethodological and conversation analysis of classroom practices does away with the analysis of the Cartesian 'subject' and constitutes an alternative to the psychological explanation of learning by tackling the question of (inter) subjectivity as a public object. The agenda is to recontextualise, redescribe, the resources of the cognitive sciences and everyday discourse in which the Cartesian subject is evoked in order to address the public accountability of knowledge and action.

The way we talk about those approaches here as if they share the same kind of analytical and theoretical foothold is two-fold: (a) it recognises that they proceed by enlisting homogeneous elements for analysis, as I mentioned previously; (b) it addresses their bearing on an argument that has been central in the modern studies of human sociality, present in philosophies as divergent as those of Piaget and Wittgenstein: the foundational problem of the child's mind (or absence of it!). Piaget, as we know, was interested in a functional psychology of consciousness not as an end in itself; his programme encompassed the historical study of the physical sciences with respect to how, in their successive historical stages, such sciences developed a set of *observables* and principles of *conservation* (Piaget and Garcia, 1983). Wittgenstein was, as I mentioned before, aware of the importance of the child's voice in his investigations (Peters, 2001), and conversation analytical research on children's activities and socio-cognitive competencies make notable 'concessions' to the standard CA's anti-Cartesianism (Hutchby, 2002; Wootton, 1997). Such 'concession' seems to be a warrant to investigate the child as 'child' in the first place: 'One of the main things we have learned from

various studies of children of this age is that they have a phenomenal capacity to detect orderliness in the information with which they are presented. The absence of overt rule-like instruction with regard to such things as the properties of objects, the referential relationships of words to things, the rules of grammar and moral concerns does not normally appear to hinder the emergence of an impressive order of competence in all such spheres' (Wootton, 1997: 3).

The sociological argument put forward by Wootton (above) rests on the undermining of classic sociological corollaries about the determination of action and cognition by social 'structures'. In this context, the study of children's actions finds a central place in the rhetoric of the social sciences, including various sorts of analytical philosophies and discursive perspectives (Peters, 2001; Walkerdine, 1988). Schegloff (1992), for example, argues that the 'socialisation problem' was widely used in Harvey Sacks' 1964-65 lectures as a resource for analysis. Sacks was interested in the kinds of competencies socialisation need to produce in order to render activities observable and reproducible as such-and-such (e.g. learning). 'A problem for a sociology interested in describing socialization will consist in large part of how it is that a human gets built who will produce his activities such that they're graspable in this way. That is to say, how that is that he'll behave such that these machines can be used to find out what he's up to' (Sacks, 1992: 119). The message is to think about how pervasive is children's encounter with social order and how urgent it is to live *with*, *as* and *for* it, despite the potentially limited number of contexts they participate in.

Wootton (1997) takes the 'conversational' problem as having direct relevance for the discussion on children's cognitive development, disputing terrain with major works in the field of cognitive growth and its dynamic mechanisms: '[...] if my observations were correct then rather than the conflict emphasized within the Piagetian tradition it was agreement that played the more pivotal role' (p. x).

In general, the argument goes against the simple view on the 'transfer' of

173

culture during socialisation, by which children's actions would be shaped by external societal norms. The simple use of notions such as 'everyday' and 'institutional' are unable to take into account the complexities in the form of actors, activities and settings in which children actively play a part and by which they are interpellated *qua* children or pupils (or *in*competent Members, if you like). Wootton's theory of intersubjectivity explores conversation analysis to a clear developmental edge, to claim that practical agreements between child and adult – what he calls 'understandings' – take the shape of public preferences for action, building for the children a 'working capacity in taking into account knowledge of other people's minds' (p. 13). This process sees, as a consequence, the emergence of significant cognitive abilities. According to Wootton, a revolutionary aspect of the child's mental development relates to his/her capacity to articulate understandings that are designed for the parents to agree with (p. 24). That is a compelling sign of how cognitive 'processes' operate in reference to the sequential organization of talk and co-presence. The child comes to recognize that other people act on understandings, and a new order of intersubjectivity is observed in her ability to act accordingly and in consistently agreeing (and agreeable with) ways.

That is not far, again, from Vygotsky's concept of 'zone of proximal development', as seen in chapter 3, and studies like Wootton's can be seen as constituting a fascinatingly 'operational' or 'empirical' version of Vygotsky's work. Wootton rightly recognises the obvious differences, and necessary implications for the logic of research, of studying young children; the apparent divergence in 'understandings' between young children and their adult counterparts in interaction pose all kinds of problems for analysis, and virtually turn socio-interactional studies into a relevant subfield of developmental studies, that is, have researchers approaching the 'child' as a natural category.

Wootton also mentions the notion of 'repair' (Schegloff, 1993) as a critical 'developmental' skill. The notion of repair addresses precisely the potential 'breakdown' in intersubjectivity, and therefore constitutes a practical terrain for the analysis of the latter. Schegloff has defended a 'procedural sense' of intersubjectivity, stating that:

'A substantial body of work in CA can be appreciated for its bearing on the interface between cognition and interaction. Much CA work brings general concerns with the *methodical* underpinnings (the *how*) of ordinary shared knowledge and skilled practice to a defined focus in the conduct of everyday interaction, accessible to empirical enquiry' (p. 153).

Schegloff argues that the structures of talk-in-interaction – such as turn-taking and repair – should not be considered as external to those cognitive and intersubjective understandings psychologists and linguists talk about; the organisation of conversation is not a neutral medium for the transmission of messages nor a bureaucratic system for ritualistic face-to-face interaction, but as we have seen with Vygotsky and Wootton, the most important mediator in the emergence of autonomous forms of human action and thinking[67]. This recommendation strongly counterpoints, amongst others, 'Piagetian' and 'Chomskyan' kinds of cognitive theorisation and their 'representational' and 'communicative' views on language (Edwards, 1997).

Repair, understood as a 'organized set of practices by which parties to talk-in-interaction can address problems in speaking, hearing, and understanding the talk' (Schegloff, 1993: 155), is a compelling entry to the study of socially shared cognition as embedded in the visible, learnable, procedural features of social interaction. The implication of such an empirical and theoretical project can be far-reaching; Schegloff, for example, speculates on the evolutionary designs in terms of shared understanding and the maintenance of face-to-face interaction through flexible mechanisms: 'the kinds of language components from which it [interaction] is fashioned –sounds, words, and sentences– have the character they do and are formed the way they are in part because they are designed to inhabit an environment in which the apparatus of repair is available [...] In like manner, our articulatory apparatus and our practices of articulation and hearing may have develop the way they did in part because repair is available to catch such problems in speaking and hearing as may arise. Similar considerations apply to other aspects of natural language' (Ibid: 155).

---

[67] In an article on 'Sacks and psychology', Derek Edwards (1995a) informs us that Harvey Sacks had approvingly read the work of Vygotsky, and it is worth noting that his own 'discursive psychology' – largely influenced by Sacks and CA – have been discussed at points in relation to the socio-cultural school of psychology (Edwards, 1995b, Valsiner, 1999).

In the lesser level of moment-to-moment distribution of repair opportunities, cognition and shared understanding constitute a practical problem insofar as members display 'hearing' problems, and *repair* is said to the very production of the sequential organisation of talk. In the classroom, the 'third position repair', that is, an action of repair in third turn that addresses the project of a first ('question') and the relevance of its appropriation ('answer') in second turn, has been portrayed by Schegloff as the 'the last systematically provided opportunity to catch (among other problems) divergent understandings that embody breakdowns of intersubjectivity' (Ibid: 158). This has been referred to in the literature on classroom interaction as the 'feedback' or 'evaluation' (Coulthard, 1977; McHoul, 1978; Mehan, 1985), as mentioned in chapter 3. There, our attention was aimed at the second part in the structure constituted by the pair 'question/answer', in order to determine how the 'class' is delegated as an agent in its own right, whose mediation through the activities around the blackboard was also shown in chapter 4.

The study of learning and cognition as placed in organisations has, of course, a place in the CA tradition, but the theoretical and analytical focus on intersubjectivity and homogeneous elements forgets that the learning mind is constituted as an independent actor in relation to things such as 'reality', 'technologies', 'knowledge', 'social factors', etc.:

> 'Interaction and talk-in-interaction are structured environments for action and cognition, and they shape both the constitution of the actions and utterances needing to be "cognized" and the contingencies for solving them. To bring the study of cognition explicitly into the arena of the social is to bring it home again' (Schegloff, 1993: 168).

I take Schegloff to be right, but his description to be incomplete, and that is the reason why, I insist, conversation analytical reports consistently ignore the documentary relations between talk-in-interaction and other forms of representation, including members' topics, material devices, spaces and the body. My point is that is theoretically futile, and analytically improbable (although the technology used for transcription manage to afford just that), to determine beforehand that the structure of talk-in-interaction should be given

analytical priority in relation to the analysis of installation for the performance of competent worlds of knowledge (e.g. mathematics). As we saw before, the use of cognitive vocabulary in educational philosophy can be traced back to Robert Owen's radical political philosophy and its plea to the distribution of 'knowledge' (as opposed to 'domestication') amongst the lower classes, and Michel Foucault analyses how continuous assessment (third position repair?) became part and parcel of school activities, turning the pupil into a scientific and administrative object in its own right. The kinds of cognitive accountability studied by discourse analysts might be in need of expanding their connection with other descriptions, not only historical but also local. That is the point of this chapter, which also concludes the present work.

The discourse about children's abilities and competencies to *mentally* construct the orderliness of their world seems to be a natural place to start talking about the importance of cognition, or cognitive ideas, in social life. As Lucy Suchman (1987) pointed out, the 'mind' and its qualities feature as an important aspect of the accountability of public activities, although we should consider its relations with the latter contingent and pragmatic, instead of being their best representation. Such public activities (e.g. classroom education) might eventually project the importance of accounting for one's actions in general, theoretical, normative terms (which are then subverted and seen as their cause, as Suchman argues). In the classroom, issues of *agency* ascription are relevant topics for local analysability of activities and settings, just as the *content* of the lesson performs the analysis of the installation and action within it, making the invisible visible (see chapter 5). In other words, classrooms are settings for the delegation of the observation and assimilation of knowledge, i.e. 'learning'. Sociology and the social psychology of social interaction will have a say on the topic insofar as the use of categories to identify human actors are treated as topics, instead of resources. Such principle is a major ethnomethodological recommendation. This strategy – together with the widespread use of the notion of 'discourse' in the social sciences – have shown to pay off, although it has done little to challenge effectively the division of labour within it: the new actors are of the order of 'society-as-discourse', 'culture-as-discourse', 'psychology-as-discourse', 'gender-as-discourse', etc.

For this characterisation still begs the question of the existence of 'putative' social or cognitive objects (see Coulter, 1999), it has disputed space alongside approaches which are effectively about how those categories feature as discourse topics, so that we have 'society talk', 'culture talk', 'mind talk', 'gender talk', etc.

Our previous discussions have critically revolved around a proposal for a sociology of learning, showing that (1) it is possible to conceive the *public* character of knowledge not only as a assumption or philosophical truism (i.e. 'there's no private language, then ...'), but in relation to a collective agent ('the class') that underlies the historical formulation of the modern classroom system and its 'division of labour' order (chapter 3); (2) that it is little parsimonious, if not impossible, to understand certain forms of talk and action without the appropriate reference to a few *material* and *literary* technologies integral to the constitution of the classroom (chapter 4); and (3) that most interactional studies have a practical way of 'forgetting' that classroom lessons are *about* something, with activities often framed and made accountable in relation to that (chapter 5). All those themes – which are also forms of designating 'agencies' in research (Latour, 1988b) – can be conceived of as significant others to the idea of an individual learner and his cognitive processes: the 'class', the 'room' and the 'mathematics', all differentiated from the individual mental experience, in explanatory terms. As see saw in chapter 2, the sociological analysis of mathematical knowledge often sees itself required to somehow cope with 'psychological' problems such as the 'rational compulsion' that account for one seeing mathematical inscriptions as arbitrary and representational, and proving activities as teleological (Bloor, 1987; Livingston, 1986). Of course, the solution is not to offer another problematic *ad hoc* psychological theory, but to readdress the question of the subject of (mathematics) learning in ways that can render its relation to its others vital. What of the 'subject' – with whom our discussion was inevitably tied in chapter 2 and who is a pivotal part of the curriculum and the educational system as a whole – if instead of considering that 'social' and 'technical' matrixes of mediation are external aspects of learning, we see it as the very things that structure, bond, contain and make 'learning' (or its attribution)

possible? Is there a place for subjectivity amidst the clockwork engines of classroom activity, of the classroom apparatus? How is the subject placed within the installation? Is it a problem for the actors involved, and if it is so, how do they address it?

I want to analyse some ways in which the 'construction' of the *subject* takes place in the classroom. I will start with a negative argument or demonstration, though. In the following (see extended extract 1 in the Appendix), I find an example where an instructional frame operates by mobilising the collective agent I have called the 'class' vis-à-vis the teacher (as seen in chapter 4). The important thing is that in the whole extension of the sequence the *subject*, or the individual learner as a differentiated semiotic agency, is not available. The question is even more compelling if we consider that the classroom is primarily a setting for *teaching* and *learning*, for staging *learnable* science, rather than for producing new, unexpected knowledge (Atkinson and Delamont, 1977; Collins, 1986). After all, the classroom constitutes a set of practices in which the pupils themselves are a field of observation and influence, and above all, assessment (Foucault, 1977). As I said in chapter 3, since Vygotsky, the classroom has been seen as a primer for the investigation of psychological processes. However, psychological accountability within it seems to be very restricted – and particularly framed for some activities.

I will not analyse the transcript in detail, for some of its compelling analytical features were described in chapter 4. For that reason only I am including nothing more than the 'conversational' transcript. My intention in using it is to make a simpler point, and in order to represent it I have reproduced this segment of the lesson to the extent I did, although I have not reproduced the entire segment here: These 288 lines represent only part of a sequential format that extends itself further. Nevertheless, a sense of its extension is crucial for a contrast with the activities in which the *subject* is invoked. What format is that then, or rather, what am I trying to frame by calling the attention to a counterexample?

In extract 1 teacher and pupils perform a calculation and then establish its

relevance through a set of *proof* procedures that reverse the order in which the relevant symbols were manipulated, and the nature of the operation at stake. More importantly for the present discussion, it enlists the *class* throughout as an agent of mathematical reasoning. I do not want to suggest that the *class* and the classroom were developed primarily by a drive to secure an epistemological ground for teaching and learning that could capitalise on the multiplication of witnesses; nonetheless, they can be seen effectively as machines, mediators for that. In Chapter 3, for example, we saw briefly how the constitution of these technologies and techniques followed the economical and ideological defense of mass instruction, and how its particular arrangements had both financial and learning-oriented tenets and limitations; thus, Lancaster's education of the poorest within the 'industrious classes' of British society had produced a sense of how learning might benefit from class composition within a system ('monitorial') that was developed as a response to changing social and economic conditions.

Our reflection in chapters 3 and 4 pointed to the emergence of the 'class' amidst a set of economical, social and epistemic conditions, and to its practical existence in relation to the classroom-as-installation. In hindsight we can also observe that what we have called, with Schegloff, 'third position repair' is rarely an option when the 'class' is constituted as the operative responsive agent vis-à-vis the teacher. The strategies by which the collective is interpellated as *class* minimize breakdowns in the dialogue; at the same time it elicits answers that can be used as starters or complements in the subsequent course of interaction, as we have seen previously.

Also, the transcript barely contains (if any) psychological observations, accounting or phrasing. It might be that grasping whether someone 'know' something, or has 'learned', is managed differently across situations inside the classroom (or rather, does not apply to 'cohort' activity). There are plenty of injunctions by the teacher regarding 'observability', though. The class is commanded to 'observe', 'see', 'find out', 'pay attention here', attest about what has been 'carried out' (see, for example, lines 1, 11, 31, 48, 78, 79, 94, 97, 123). It is as if all the elements for performing the natural accountability of

the mathematical knowledge (Livingston, 1987) are 'out there', assembled externally to be witnessed, and is by constituting that kind of 'agent' (collective) that this very process is afforded. I want to argue that in extract 1 nobody is interpellated individually as *Subject* (Althusser, 1971; Law and Moser, 1999; Rotman, 2000), but collectively as *witnesses*. The role of the *witness* blurs the strict distinctions between the accountability of the subject and that of the object; what *witnessing* warrants is an understanding of the 'real', the *accountability* of the real, at the same time it opens up to the investigation of its own constitution (of the collective of witnesses) as bearers of the real.

No particular pupil is positioned so as to place himself in relation to the installation, or regarding its mastery of the curricular content at stake. In line 31, we can see the use of a generalised 'you' (*vocês*, plural in the original), and in line 203, the relevant agent is addressed again, 'folks' ('gente', in the original); The transcript suggest – if I can demonstrate that what goes on there is alternative to other forms of delegating agency – that the analysis of the way activities are framed and practically organised have consequences for the ways cognitive accountability is organised in the classroom. The framing of the activity distribute several resources for the accountability work of 'epistemic' subjects. This idea goes beyond the methodological recommendations of CA and discursive psychology to show that other 'sequential' structures can be observed to be involved in the distribution of agencies in the classroom. The example we analysed in chapter 4 and in extract 1 in this chapter show that the 'class', an actor perfectly graspable in terms of the turn-taking system (chapter 3) is consistently enlisted for extended periods in time and that the agenda under which this is done – and shifted – is the teacher's . The correlative idea, that of the teacher vis-à-vis the class is also observable in practice: the abundant use of the pronoun I in extract 1 – in opposition to the extracts in which individual pupils are called to account for their actions (see it in later sections) – positions the teacher as a particular, differentiated mathematical 'agent'. Here, I recall Brian Rotman's *agencies* (chapter 2): I suggest that while the pupils, as 'class', are interpellated as 'Subjects' (in Rotman's sense) and share such a entitlement with the teacher herself (line 1, 'we're seeing ...

the dividend'; line 11, 'we're going to find out now ...'; lines 45-46, '*I* know ... *we* are sure that this division is correct isn't it?', and various generalised uses of 'I' as a common observer, e.g. line 79, 'if I don't find it', suggesting a theoretical, decision-making observer), indicating a capacity to *comprehend* mathematical concepts, the teacher is the one that represents the 'Agent', or the automaton that carry out actions and transformations locally (lines 15-16; 25-26; 67-70; 83; 108; 115).

In the remainder of this chapter I indicate where we may find a contrast between the extract discussed above and examples where accounting for the *individual* activities of 'learning' and 'problem solving' becomes relevant:

Extract (2) 4th grade, EF, 2:

```
1.    T:     for what I read in your answers it would be (       )
2.           erm: very different. Apart from Pedro who already spoke
3.           (    ) at once, another person wants to say which was
4.           the ( ) that made the (        )
5.    P:     (raises hand)
```

The example is quite revealing. In this case, the performance of learning is to be composed by individual 'answers'. The 'class' is being dispensed with: *Pedro* is one of the relevant actors here, as is 'another person' but him. Observe also that Pedro is not allowed to speak again, that is, he has already occupied a conversational slot and therefore cannot purport to represent the *class* or the totality of 'agencies' projected by the teacher's turn design.

6.2. Agency and semiotic delegation

One of the most relevant problems for the contemporary study of learning and of mental processes relates to the question of where, and in which form, one can find the 'mind' that learns (Cobb, 1999). The theories we have seen previously have one way or another rejected explanatory principles based on elementary (behavioural) or metaphysical assumptions, on behalf of more

social and materialistic explanations. Their solutions can roughly be grouped in: (1) the mind as the use of cultural means, or tools (the Vygostkian solution); (2) the mind as 'distributed' into heterogeneous materials, its differentiated character being an effect within certain practices, and ultimately dissolved into the concepts of 'technical' and 'social'; this is what I understand to be the 'actor-network' position, as well as the 'distributed cognition' one (Hutchins, 1991); and (3) the 'mind' as a *category* in the moral accountability of face-to-face interaction. That is the conversation and discourse analytical take on the problem. I am interested in aspects of the solutions (2) and (3). As I see them, they are not incompatible philosophically and the latter's mandatory methodology for the analysis of the *homogeneous*, or the 'social' (as *talk*), prove to be unsatisfactory in face of the question of how settings are framed, mobilised, displaced, reorganised and commented upon in order to keep the mind as an invariant condition, a point emphasised by the actor-network approach.

The empirical programme of discursive psychology, for example, was developed as a critique to the approach to language as *representation* of mental phenomena, an assumption underlying most of modern *cognitive psychology*. Unlike cognitive theory, discursive psychology is concerned with the detailed analysis of language in its own right, as social activity. Also, its explanatory scope is not *cognition*, but *discourse*. In this sense, it also departs from developmental psychology's explanatory interests and, of course, from its various concerns with language as a developmental *factor*.

Drawing on conversation analysis, discursive psychology advocates the analysis of mental categories and predicates in talk as 'the prime focus of theory and investigation' (Edwards, 1995: 57). It treats discourse as analysable in its own right, examining how it works *in* and *for* social practices, rather than as a second-best methodological choice. According to them, there are at least two good reasons for that. The first concerns 'reflexivity': discourse is the outcome of our own academic practices, practices that categorise, compare, explain, that is, bring to light the very objects of our enquiries. The question is whether or not we can convey a categorical distinction between the 'objects'

we analyse and our own 'texts'. As Edwards pointed out, "This is because the world beyond the text, the world that includes cultural practices, activities, cognitive development and mind, is precisely what the texts are all about. In producing the culture of cultural psychology, we understand the nature of those non-textual things through each other's writings" (Ibid: 56).

This immediately poses the question of which relations should our analytic practices have with the materials we analyse. Both academics and non-academics are users of descriptions of what 'mind' and the 'outside world' are like. Categories such as 'human' versus 'mechanical', 'natural' versus 'artificial', 'intelligent' versus 'automatic' are available to a greater range of practices than professional psychology, so "the relevance of a discourse-based perspective broadens to include all the categorical distinctions of cultural psychology itself" (Edwards, 1995: 57).

Like in CA, discourse analysts are cautious in taking psychological categories (and sociological as well) as starting points or explanatory resources for analysis. For instance, they are not concerned about making their work on talk and text intelligible in terms of 'social class', 'ethnicity', 'cognitive schemata', and the like. It does not dispute over nor advocates a particular view of human nature. It is *methodologically indifferent* in this regard. What it does is to analyse people's deployment of categories and accounts of mind and reality. Such orientations are publicly displayed, discursively managed formulations of context and cognition, which construct, assess, repair, resist previous actions and provide the basis for subsequent ones.

These methodological precepts avoid at least three potential shortcomings of the analysis: (1) it avoids starting from the analyst's own moral/ideological standards, thus avoiding being judgmental about people the work is supposed to be *analysing*. (2) This framework also avoids the theoretical agenda of the notion of 'representation'. If you conceive of your analytical materials not as representations of the external or internal reality of society and mind, but rather as a set of public resources for *action*, which are available and accountable for members of a 'common' culture, you can take a rather

different look at your data, one that focus in the production of society – including representations – as a practical accomplishment of its members; (3) It avoids what sociologist of science Michael Mulkay has called *vassalage* (see Potter, 1996). The concept implies sociologists' and psychologists' taking of dominant views in other areas in order to establish *a priori* normative criteria onto the reading of their data (e.g. developmental psychologists deploying scientific and logical criteria in order to understand how far children are from 'mature' cognitive development). Of course, from a discursive point of view that would mean text being reading through text, not by means of external, *objective* criteria. The important thing is that precisely those texts are responsible for saying what reality is about, and therefore are analysable *as* texts.

The *socialisation* into some form of practice and membership is viewed as including ways of accounting for action in terms of knowledge, memory, motive, opinion and reality, as well as the interplay between them. The pragmatic, interaction-oriented appeal of such insights has been proved analytically relevant in some way or another by scholars such as psychologist Lev Vygotsky and conversation analyst Harvey Sacks, in arguing for some of the essentials features of the 'socialisation problem' (Edwards, 1997). Sacks (1992), for example, addressed the question of how parents' assessments of children are design to recognise children's behaviour as such-and-such action, and how this can be learned and *subverted* by children for controlling adult inferences. Other recent studies have shown the relevance of discursive and conversational-analytical methods for describing structured developmental contexts and children's competence as social participants (Candela, 1995; Gardner, 1998; Hutchby and Moran-Ellis, 1998; Turnbull and Carpendale, 1999).

There is, however, a sense of disappointment with the kind of explanatory burden not only constructive theories can carry, but also other *objects*, be it in material or representational form. While Vygotsky based his investigations on the assumption that material and symbolic resources were tools through which cognitive 'functions' emerge in the first place, discursive psychology has

popularised a methodological claim for the primacy of discourse, under which the 'material' can be reduced to 'textual' and 'reflexive' in a defense of relativism that seeks to undermine the rhetoric of 'realist' arguments (Edwards, Ashmore and Potter, 1995). In that sense, discursive psychology is well attuned with the philosophy of intersubjectivity we have been discussing. My restrictions to it –which are the same I have directed towards CA – are two-fold: (1) the mandate for the study of talk in its own right as answering the question of order, or ordering, is based both on a philosophical truism originated by Wittgenstein's *Investigations*, more relevantly – with the consequent critique of 'representation' – and a 'technological' pay-off, in the form of the *transcript* and the original CA reports on telephone conversations (Hutchby, 2001); (2) in relation to that, the analysis of the classroom's documentary practices cannot dispense with the analysis of the 'classroom-as-installation' (Macbeth, 2000), as I have argued in chapters 3 and 4. The social and representational affordances of the blackboard, the manipulations by which several actions are conventionalised as *mathematical*, etc., are not 'theory'; they are constitutive of the very possibility of Membership, and of the subject. The classroom is one of the mediators in a configuration that in order to produce a 'learning' document, or an individual who has learned X, raises a whole educational system.

Bruno Latour (1990) offers a related way of looking at cognitive issues by addressing some problems regarding the relations between 'visual perception' and 'cognition'. In traditional developmental theory the former is considered as a precondition for the latter, and in cognitive, information-processing psychology the basis under which cognitive schemata can perform generalised, normative understandings of reality (Edwards, 1997). Latour focus on 'technical' ways of looking with printed images and texts of various sorts in scientific activities, in a way that is reminiscent of our discussion on the 'affordances' of social and technical artefacts in chapters 3 and 4. 'Scientists start seeing something once they stop look at nature and look exclusively and obsessively at prints and flat inscriptions' (p. 39; see also Latour and Woolgar, 1979).

Latour regards Piaget's tests as a practical site for establishing relations between abstract, *immutable* and yet *mobile* entities that are displaced through new resources without being modified. This is at the heart at the so-called 'conservation experiments', in which children are asked to follow physical 'transformations' and to detect their 'invariant' features (e.g. quantity, volume). The assumption is that depending on the 'stage' of development the child is in, the abstract, general, invariant features of objects and relations are 'observable'. This is, as Latour points out, a non-*technicised* view on perception and cognition. In laboratories, for example, the *measure* upon which to read conservation would be an immediately relevant issue; the shift from nonconserving to conserving is then explained not in cognitive, but in technical terms. 'Volume' would mean little without the resorting to industrially calibrated beakers:

> 'In others words, Piaget is asking his children to do a laboratory experiment comparable in difficulty to that of the average Nobel Prize winner (...) So again, most of what we grant a priori to "higher cognitive functions" might be concrete tasks done with new calibrated, graduated, and written objects (...) What Piaget takes as the logic of the psyche, is the very logic of mobilization and immutability which is so peculiar to our scientific societies, when they want to produce hard facts to dominate on a large scale. No wonder that all these "abilities" to move fast in such a world get better with schooling!' (p. 51).

Latour's preferred strategy is to leave nothing to cognitive explanations, if sociotechnical accounts can extend the description as far as to dispense with the import of *third* actors from external, theoretical schemes. The sociotechnical analysis proposed by Latour looks at the rise of new forms of 'inscription' and the social contracts and resources it mobilises. Social effects (knowldege, power), alliances (witnesses) and technologies (inscriptions) are intimately related in this view; the analysis of the 'inscriptional work' of science and technology sites rejects the divide between *mental* (cognitive) and *technical*.

Latour has also been a pivotal figure in the development of actor-network

theory (Law and Hassard, 1999), an important approach to the sociology of scientific knowledge that declared that for matters of investigation it was irrelevant to distinguish between human and non-human actors as differentiated *in principle*. Human actors, they claimed, are to be seen rather as an *effect* of differentiation within the activities under scrutiny, just as non-humans are to be considered as agents that act upon others; scientific practices, after all, have populated our lives with non-human actors: bacteria, DNA, atoms, gravity, electromagnetism, infinity, markets, society, culture, minds, etc., and it is useless to presume their exclusively constructed and discursive character vis-à-vis humans (which I will call *subjects* in this chapter, for the purpose of distinguishing them from non-human *actors*, or *actants*), whom, by their turn, would be *real* and a *foundational*. All those *actors act*. In our descriptions, bacteria *act*: They multiply, spread, cause diseases, kill, and society has built institutions to deal with their action, namely bacteriology and medicine, to mention two of them (Latour, 1988). In our descriptions, humans sometimes feature as passive, victims, empty signs within larger structures, etc.

Every semiotic practice is faced then with the issue of delegating actors, or *actants* (Latour, 1988b), in order to describe and explain certain observable effects. Derek Edwards (1997) has rightly observed that 'cognition' and 'reality' are categories often implied, if not formulated, simultaneously, where the displacements of one are used to show the invariant character of the other: in the experimental sciences, the invariant character of the *object* co-exists with the manipulation of conditions under which *subjectivity* is no longer a source of interference; in Piaget's experimental settings, as well as in classrooms, mind, knowledge and reality are artefacts symmetrically created, in order to analyse the effects of one over the other. Let's go back briefly to extract 1 from chapter 5:

Extract (3) 3rd grade, EE, 1:

```
1.   T:     erm:: you. tell me how much two plus three makes
2.          (1.0)
3.   P1:    six
4.          (2.0)
```

```
5.    T:      (        ) how much does two plus three make?
6.    P2:     °five°
7.    T:      °five° (.) and:: three plus two?
8.    P2:     fiv_
9.    T:      five
10.           (1.0)
11.   T:      how much is it now say now how much two plus three makes
12.   P1:     °five°
13.   T:      °five° (.) and three plus two?
14.   P1:     °five°
15.   T:      oh yes now you learned (.) now you that laughed how much
16.           does erm: two plus seven make?
17.   P:      (        )=
18.   T:      = and: five plus two?
19.   P:      seven
20.           (3.0)
```

As far as the distribution of agencies is concerned, in extract 3 the subject is to be kept constant through a procedure of manipulating knowledge: Such a manipulation embodies and makes visible a mathematical principle, that of the 'reversibility' of operations (chapter 2), but also serve to prove the constancy of cognition; a specific pupil (line 1:'you', P1) is called to respond. He gets a wrong answer and the question is returned back to the class, being taken up by one of the pupils in line 6. After the success of P the question – and curiously the 'same' question – is returned to P1, who then succeeds. Mathematical content and the attribution of learning negotiate a differentiating line as part of the same curricular move; the attribution of learning of this kind does not happen often in other practical contexts (Carraher et al., 1988). For example, in line 15 'now you learned' reach closure through psychological accountability, which nonetheless, as I hope to have argued, requires a skilled practical set-up. Finally, the command 'now you who laughed' carries on the procedure by projecting as a third *person* to be interpellated by the teacher in sequence.

In the case of extract 3 we observed how a set of actors are being questioned as individual respondents, potentially accountable in terms of their private learning processes, distributed to the terrain of the psycho-social, differentiated from the object. I am referring to *interpellation* here from the use Law (2000),

borrowing from Louis Althusser, make of it. Law reflects on Althusser's formulation of the problem: 'he says that there are moments of recognition, moments when we recognize ourselves because we have been addressed, called out to, in a particular way. "Hey! You!" And round we turn to face the policemen, the head-teacher, the priest. At those moments we become, as he puts it, subjects because we are subjected to an authority, a Subject with a capital S. We are located, in relation to that Subject, as biddable small *s* subjects precisely because we recognize ourselves, and (this is crucial) *because we have no choice*' (Law, 2000: 14). Later in this chapter I go on to suggest the *Subject* of classroom mathematics is interpellated, in a way similar to that observed in Piaget's conservation experiments, as what is accountably not displaced in such conservation trials. From that we conclude that a special relation with the installation and the distribution of participation is crucial in determining the pupil as *Subject*. The important thing here is the analysis of 'relations', 'including the relational formation of the distribution between the knowing subject and the object that is known' (Ibid: 13). Then, the disembodied subject of psychology, the bearer of decontextualised mathematical thinking, is *embodied* and *participates* in activities and rituals for which he/she is *interpellated* as an abstract mathematical consciousness. Nevertheless, and that is the point here, that cannot be done without raising heterogeneous resources and distributing the scenarios under which *agency* and *objectivity* fall into place as distinctive objects in the classroom. In this view, not only 'knowledge' is an effect, but also the 'person', or 'member'. Law and Moser (1999) argue that the 'person' is assembled within the ordering principles of heterogeneous settings.

Let's consider an initial problem. In chapter 5 we have established that classroom mathematics is also for doing 'analysis', for interpreting or modeling the world. Just how that can be conventionalized in the classroom is a fascinating question, and a common alternative in the literature refers to so-called 'word problems' (Durkin and Shire, 1991; Sajlo and Wyndhamn, 1993). Those portray situations described so as to elicit mathematical solutions to problems one could eventually face in real life: counting, calculating, doing commercial transaction, measuring, etc. Of course, the nature of this activity in

the classroom ultimately produces the problems *per se* as irrelevant, mere blueprints for abstract, mathematical sentences (Carraher et. al, 1988; Walkerdine, 1988). However, in order to do so the use of word problems needs to delegate agents that will produce documentary relations between situations and abstract patterns within the story at stake. The stories not rarely create fictional human agents that behave on behalf of mathematical translations. The problem is not incidental; an *interested* subject does help to establish the empirical, modelling-oriented character of classroom mathematics, a task for which all sorts of pedagogic materials have their use warranted. In chapter 5, extract 9, we saw one of these cases. In the story, Luis, an employee in a shop, need to produce, or 'register', a certain mathematical representation. Sometimes the pupils themselves are projected as characters in fictional situations:

Extract (4), 1st grade, ER, 1:

```
1.    T:    Renan got eight balls as a gift from his father (.) only
2.          that Samuel got there and they both went to play, and
3.          you know what happened? Two of the balls were blown.
4.    P:    Seven, seven, seven
5.    T:    wait (.) how many did he get?
6.    P:    five.
7.    T:    no, how many balls did you get from your father?
8.          (to Renan)
9.    P:    eight.
10.   T:    eight balls (.) Samuel got there and blew two without
11.         intention
12.   PP:   SIX:
13.   T:    how many left?
14.   PP:   six.
15.   T:    six balls (.) well done.
```

Extract (5), 1st grade, ER, 1:

```
1.    T:    (reading) the teacher asked each child to bring an
2.          ingredient for the G4 to make a cake for mommy (.) how many
3.          eggs did Helio bring?
4.    PP:   six.
5.    T:    six eggs (.) let's make believe that three of them broke
```

```
6.                (.) how many eggs left?
7.     R:         three.
8.     T:         come on Yuri (.) how many eggs left?
9.     PP:        three.
10.    Hélio:     it wasn't three
11.    P:         it was.
12.    Hélio:     it wasn't (.) I brought one more then it was six
13.    T:         no, love (.) make believe that three broke.
14.    P:         yes (.) three eggs left
15.    Hélio:     it didn't break
16.    T:         Helinho (.) in this task in reality what happened was
17.                that Helinho brought six (.) only one broke, wasn't it?
18.                But here in the task we are making believe that three
19.                broke (.) how many eggs left?
20.    Hélio:     three.
```

In extract 4, one of the pupils, Renan, is enlisted in the story as a character. So is Samuel, also a pupil. In lines 2-3, and then again in 10-11, we learn that Samuel unintentionally 'blew' two out of the eight balloons Renan's father had given him in the imagined scenario. Samuel's presence translates a 'real' event of subtraction, an exemplar of (however 'unintentional') mathematical operator. One of the pupils anticipate the imminent testing situation in line 4, understanding that the teacher is not simply telling a story, treating the listing of the relevant actors and events as a 'transition relevant place'(Hutchby and Wooffitt, 1998). In practice, the pupil answers to a question that was *not* asked in line 3, although it indicates a practical understanding of what the teacher wanted next. Nevertheless, the wrong answer in line 4 – and subsequently in line 6 – sees the teacher recasting the question in line 5, 7, 10 and 13, and shaping it into a more polished mathematical question: the 'gift' aspect (line 1) is gone, and so is the fact that Samuel went there to play in the first place and therefore the unexpected negative consequences of playing were 'unintentional'.

The task is called 'oral calculation', and can also be observed in extract 5. The name – which is curricular – is already a gloss on the capacities to be assessed in comparison to the performance during 'class' instruction (see extract 1) and guided problem-solving using representational and materially-structured

resources; 'you have to do the calculation in the head', says the teacher, just before from where we take in extract 5. The children are supposed to reason about the problems suggested by the teacher without pen and paper or any other device and then answer individually in the presence of others. In extract 5 the teacher's narrative mixes up actual past events and imagined scenarios much to one of the pupil's protest. Helio is involved in an argument, both with the teacher and his colleagues (lines 10, 12 and 15), as to whether he broke *three* eggs when the teacher asked the class to bring some ingredients to bake a cake for 'mother's day'. Actually, an event such as Helio breaking an egg has indeed happened, but involved only *one* egg, which is the main source of trouble in extract 5. The teacher wants him to 'make believe' that *three* eggs broke, as she had already proposed to the whole class in line 5, and which constitute the background for the questioning of Yuri in line 8. However I want to suggest that there is a perhaps 'incomplete' mediation in the instruction to 'make believe', or a 'rational compulsion' to believe that the correspondent (right) answer to the order to make believe is straightforward: why the suggestion to make believe in line 5 should necessarily be followed by 'three left' as the right answer to the question in lines 6 and 8? Or rather, how can it possibly not to? Why is not Helio rationally compulsed?

The teacher is sensitive to the two different registers and addresses them in lines 16-19. It is almost as if, from Helio's (practical) perspective, one could make believe *something* and at the same time that has nothing to do with 'reality', not even for the task's sake. Those are two different things! By the way, that this is a thing called 'task' and warrants powers to *make believe* to interfere with *reality* is something to be indexed, and effectively is in the teacher's explanation. We saw with Wittgenstein in chapter 2 and 5, in relation to the child's voice and its socialisation, the 'unnaturalness' of inference procedures. I am not suggesting this as a specific developmental condition, but as an alternative explanation based on how the links for producing inference techniques might be tied, or socialised. Other pupils attended properly to the make believe call earlier than Helio, and the fact that the story uses him as a character in a potentially embarrassing situation might have increased his stakes in not making it even worse! Nevertheless, the explanation by the

teacher towards the end of the extract is forced to deal with the differentiation between matters of 'task' and 'reality'; the latter, containing Helio's preferred version, is readdressed in line 17, and overcome by the reassurance in a technology of reasoning that can *reshape* it for practical purposes. At the end it works; at the end Helio 'agrees' with the 'public' preference, and the mediation is 'incomplete' (as I refer to earlier) only insofar as it does not contain the clarification of its own rhetoric, as it does not reveal its potential alternatives, such as 'making believe' at one moment and coming back to strictly 'empirical' activities on the next.

Walkerdine (1988) argued that the 'imaginary' order of fantasy in the classroom constitute a thoroughly different order form that demanded by formal, abstract mathematical reasoning. The author observed the use of money in 'pretend play' in the classroom to induce the understanding of numerical relations and concluded that insofar as the context creates strong symbolic, social and gendered positions, mathematical rationality – metonymic, purely axiomatic – is relegated to a secondary post. School mathematics interpellates schoolchildren as the *subject* of mathematics, instead of 'play' or 'reality', or other possibilities. Säljö e Wyndhamn (1993) focused on a similar problem in terms of 'communication premises'. They observed that pupils from different classes – social studies and mathematics – in Sweden tended to answer differently to a problem-solving situation involving the value of postages in relation to their weight, although the Swedish post operates with a table that attributes fixed prices to certain weight ranges. For example, between 1 and 15 grams all letters cost precisely the same. They also observed that the student in the social studies class tended to perform better, using the Post Office's very own practical rules. The ones in the mathematics classroom tended to design complicated calculations to determine the value of each letter, even though they had access to the same information the others had, that is, to the table and its fixed values; they tried to construct mathematical models to sort out a highly trivial task (with the table providing all the answers), for example, by triangulating values: if 15 grams costs, say, 10, how much is 8 grams? The right answer is 'exactly the same', but not for the mathematical subjects! The authors interpreted the results in terms of communicative

premises, or meta-rules of inference, that is, they saw the pupils orienting to a virtual quality of' what-has-to-done-in-a-mathematics-classroom'.

6.3. How many subjects?

Even a superficial look at the literature in psychology and the social sciences reveal a commitment to a notion of *subjectivity*, or of the *subject*. Perhaps rightly so, but in either case it shows how pervasive the topic is. Talk of the latter is preferable to the former, because it does not imply necessarily the formulation of cognitive states or the determination of action by inner mental processes. The subject of a grammatical sentence, for example, is distinct both from his/her own action and from the object of his action, and at the same time seen as the objective carrier of the action in relation to the object (Whorf, 1956; Pinxten, 1994). No 'subjectivity' strings attached. These questions are also at the heart of those sciences in the form of the old problem of *agency* versus *structure*, or *self*-determination (persons as locus of action) versus *external* determination (persons as means, and/or victims, of structures' action).

In the field of mathematics learning it is not difficult to perceive how diverse forms of *human* agency have been specially distributed: while Oksapmin mathematics is *culture* (Saxe, 1991), designed and enacted by *cultural* subjects, 'Greek' mathematics is *mathematics*, ever-existing and reminisced by the rational mind; while children are treated as a sort of character in a psychology play, facing new problems and acquiring reasoning powers, the teacher is portrayed as a sociological force setting up language games for the others, the psychological beings. Don't teachers solve problems, or reason? Don't children express themselves within the boundaries of public language and accountability? Aren't the ancient Greeks as cultural as anyone else? How those links are created and stabilized is the question we should be looking at. As such, those are the schemes that make the anthropology of the 'modern world' impossible (Latour, 1993). Recent critical approaches have other ideas. Structuralist analysis in linguistics and psychoanalysis has seen abundant

reference to the 'subject' – if not to the 'agent' – in the form of the so-called 'subject of language', for instance. The psychoanalysis of Jacques Lacan explored it consistently, arguing in terms of a person *subjected* to the antics of metaphor and metonymy, the very mechanisms of the 'unconscious structured as a language'. Brian Rotman (chapter 2), distinguished between the *Subject* and the *Agent* of mathematics: while the first is referred to as a transpersonal mathematical consciousness, the latter is seen as an automaton that carries out the Subject's orders into distant places (e.g. infinity), but only comes into existence by means of a third form of embodied agency, which Rotman called the *Person*. Piaget hardly distinguished between those three, but his theory can be seen as a formulation of how forms of structured subjectivity emerge out of the proactive assimilation of reality, even if there is at any given point, as Piaget would argue, the need to elaborate on *a priori* adaptative mechanisms (e.g. reflexes). Respecification notwithstanding, not even ethnomethodology and conversation analysis escape from having its foundational vocabulary addressing one way or the other such queries: The assimilation of the real (as 'public') is described in terms of what a 'Member' do, a Member that is not a 'cultural dope', in opposition to Parsons' notion of how social norms are internalised in the process of 'socialisation'.

As Bruno Latour, Michel Callon, John Law and others in actor-network theory have argued, the problem arises as a consequence of the need for putting together *a priori* descriptions which concern only homogeneous elements, such as 'actions': talk, gesture, gaze, co-presence. 'Actions' here means 'human' actions, and psychology and sociology, amongst others, find their place amongst the sciences precisely by 'purifying' (Latour, 1993) the distribution of agency and subjectivity amongst several objects in the world. That is the reason why I have argued that this discourse is mainly foundational: the knowing subject or the Member is a starting assumption, a foundational truism. It uses Wittgenstein's argument for the impossibility of a private language or Sacks' ideas on the rules underlying Members' 'observables', that is, that activities and actors are designed to *look like* the things they purport to be, as basis for the argument that whatever analysis of verbal transcripts comes up with, then that is Members' competence! That is the reason why I have

argued for the constitution of the 'class' as an agent in its own right, vis-à-vis the teacher and the knowledge at interest; rather, *for* the knowledge at interest. In my analysis, the blackboard and related activities mediate the delegation of agency to the collective, to virtual witnesses, and the standard transcript used by conversation analysts, let alone their analytical focus, is considered incomplete and as being far from 'unmediated', as some want to believe. In the case of the classroom, the transcript almost arbitrarily lose sight of the fact the 'teacher-class' frame is only possible within the 'class' and the 'room', a conjunction of material, technical, social and cognitive attributes that distributed within that installed world of competence *as* and *for* education (Lynch and Jordan, 1995).

The discussion concerning the 'subject' (of knowledge, learning) can take a few different shapes in the relevant literature. I want to suggest a way to read at least two of them, in consonance with our previous discussions. The question here is relevant insofar the constant assessment in the classroom interpellates schoolchildren at the intersection of some of these 'theoretical' forms of agency. We can say that questions of accountability in the classroom address the social production of 'children', and 'children-as-learners', or learning *Subjects*, taking from Brian Rotman's categories. As I argued previously, in developmental research both 'classroom' and 'mathematics' have independently been used to accomplish such a description; the classroom, for Vygotsky (see chapter 3); the mathematics, for Piaget (see chapter 2). That is, while Vygotsky raised the whole logic of intersubjectivity in education to observe the 'subject', Piaget designed a set of experiments in which the direct manipulation of mathematical building blocks would reveal it. The relevant readings include the kind of agency delegation that has been on the basis of the distinction between psychology and sociology and other social disciplines: (1) the *transcendental*, foundational subject of Kant, which is also the developmental subject of Piaget's theory, with functional overtones; (2) the *social* or *cultural* subject, as portrayed by Vygotsky and social scientists more generally.

Let's consider the notion of 'culture' in relation to psychology. A great deal of

research in psychology has insisted on the cultural nature of cognitive activity, turning to public spaces such as the mathematics classroom in order to investigate basic reasoning processes and even to infer about the origins of private mental processes. The focus, then, is on the 'sociogenesis' of consciousness, that is, its links with, and origins from, language and social practices (Chaiklin and Lave, 1993; Lave, 1988; Lave and Wenger, 1991; Newman, Griffin and Cole, 1991; Wertsch, Del Rio and Alvarez, 1995). Generally speaking, the idea that processes such as learning and memory take place *inside* individuals has been widely questioned, suggesting that human psychology is firmly anchored in culture and society (Bruner, 1990; Wertsch and Tulviste, 1996).

Vygotsky defined 'consciousness' as the 'experience of the experience' (Lee, 1985), or the faculty of having experiences as objects (stimuli) for other experiences (Ibid.). The important question turns out to be the way such experiences are *codified*. It is important to notice that differently from more sociological and formalised approaches to signs, Vygotsky's own brand of developmental semiotics clearly acknowledge the fact that children explore a number of (random as well as organised) physical and social experiences before mastering natural language, and therefore it is natural that Vygotsky does not start by declaring that the sign is the very fundament of experience, although his argumentation will inevitably take to the conclusion that 'cultural signs' become the most immediate cause of behaviour for culturally developed 'subjects'. His quest is precisely to explain what lies in the apparent discontinuity between the development of the individual reflexes and the so-called higher mental functions; in a word, Vygotsky was interested in explaining the bridging of the gap between nature and culture.

The solution offered focused on the way individuals 'internalise' cultural systems of signs, such as the natural language and the number system. For Vygotsky, such system themselves constitute forms of cognitive functioning, so that different representational mechanisms would correspond to different – and differently valued – styles of cognitive performance; in the sociocultural approach, it could not be explained otherwise the differences between

'primitive' and 'modern' thinking, that is, the cultural evolution (Van der Veer and Valsiner, 1996). Latour (1999) argues that traditional social sciences have replaced the Kantian philosophy of 'mind-in-a-vat' for one of 'minds-in-a-vat', constrained by the orientation of their group(s) of membership. In that sense, as far as culture is concerned there are as many subjects as there are cultures that can be differentiated in terms of the basic functions of their representational systems. The focus on 'culture' multiplies the numbers of subjects available. However, it seems to me that there is a trade-off between theory and empirical analysis in sociocultural research, so that the more we move towards culture in Vygotskian research, the more we come close to cultural and literary technologies and lose track of the *subject* as it continuously assessed in the classroom; whereas the more we move towards the classroom the more we approach developmental and epistemological concerns, less culture-oriented. In that sense, Vygotsky's 'cultural' subject is secondary to his own education-bounded 'developmental' *subject* (chapter 3).

Some authors prefers to focus, following Foucault, in the emergence of forms of subjectivity amidst the machinery of control founded by the disciplines and the way their discourses constitute 'subject positions'. For Walkerdine (1988), mathematical 'subjectivity', or 'reason' for short (p. 212), is achieved at some cost for the subject of natural language. Walkerdine argues that the transcendental subject, the subject of development is an artifact of systems of governability: 'My claim is that the child is an object of pedagogic and psychological discourses. It does not exist and yet is proved to be real everyday in classrooms and laboratories the world over [...] That discourse claims to tell the truth about the universal properties of 'the child' which 'has concepts' (p. 202).

The 'child' is a signifier produced in the classroom, it is a 'fact' produced within the regulatory practices of family life and formal education. Discourse is one of such regulatory mechanisms, and it proceeds, according to Walkerdine, by means of a language that dispenses with metaphoric signification (see chapter 2) at the same it opposes contingency and logical necessity (see also my chapter). In Foucaultian terms, pedagogy became

'invisible', 'covert', the claims to reason being disentangled from plain authority and experience. Walkerdine puts the question eloquently:

> 'At first, power was visible, vested in the presence of the authority of the teacher, but within child-centred pedagogy power became diffuse and subversive. The 'free' child was more highly observed, regulated, and monitored than ever before (...) Yet this child was not to recognize the criteria for regulation. Power then became the overthrow of the Other in rational argument – the teacher's claim to know. I submit that such power is essential to the new profession of reasoners and that the fantasy inherent in 'Reason's Dream', an idealized and calculable universe, is part and parcel of the dream of rational government. The dream, therefore, is not just a wild and crazy dream of playing God, but a fantasy invested in current attempts to govern through bourgeois democracy. Its concomitant is the rise in the 'caring professions' to render the governed governable' (p. 214).

In a reflection somewhat similar to my own, the author states that 'it will be remembered that 'the class' forms a signifier in contrastive opposition to 'the child' (p. 205). However, Walkerdine's semiotic analysis has little regard for the kinds of facts and settings that are produced for display and that constitute relevant others in relation to the 'child', that is, to the one who learns. The classroom-as-installation, the constitution of the class as an epistemic mechanism by which the 'multiplication of others' can be secured, or the reflexive and documentary character of instructional matters between discourse and other things have little explanatory interest in her work. I have been arguing that the classroom, not only the formal features of discourse, can be understood as 'machines'. How does such a complex machine produce the subject?

6.4. Socialising the mathematical subject

Although the classroom is designed to be a site in which the assessment of competence is continuously present, just how competence is translated and made visible in terms of 'psychological' features remains problematic. Is the classroom-in-session a perspicuous setting for the detailed, theoretical,

elaboration of 'self' or 'mind' processes? Yes and no. In several moments its aims, resources and topics lie elsewhere. To a large extent, the 'class' frame is averse to psychological accountability and carefully avoids breakdowns in the two-part dialogue between the teacher and the class, with third-position repair being a rare accomplishment. In that context, the classroom boundaries between mental, situated, technical and material are very problematic. I am trying to say that although commonsense might say the contrary, the use of psychological language in the classrooms I have observed seemed to be quite rare. So where can we find a *person* being interpellated in the classroom as a knowing *subject*? I want to suggest a few examples that, outside the boundaries of the 'class frame', indicate the attribution of the subject as a differentiated agent. The examples seem to point to the fact that:

- The subject is interpellated as the element in the distribution of 'agencies' which is not displaced in *conservation* trials;
- The subject is addressed in *repair*, when 'right' versus 'wrong' answers are at stake; usually, matters of *how* one 'finds' an answer are relevant here;
- And finally, the subject is formulated in *assessment*, in the attribution of competence and learning; in this case, 'psychological' words offer a clue as to how such a distribution of agency takes place.

The 'contexts' above are interrelated and are considered separately only for heuristic reasons. In practice, those aspects of classroom practice cannot be separated, and from now on our examples will deal with differences between them only insofar as they are operative in the extracts observed. For example, psychological words are not necessarily used at the end of sequences as summary of what supposedly happened in the pupils' 'heads'; they can be placed as indexes that account for as well as introduce action alternatives in 'teacher-pupil' instruction sequences. In what follows we see a few passages from a lesson in which the teacher is trying to convey a representation of the decimal system, focusing on the concepts of *ten* and *unit*. The task set by the teacher involves several materials, such as little boxes representing the place

values for 'tens' and 'units' in numerical expressions (e.g. 26 → 2 x 10=20,

plus 6), and grains of beans and corns representing the values of *one* (unit) and

ten, respectively. The task consists in:

1. The pupil has to collect a certain number of beans from a pile;

2. *Ten* beans are to be exchanged for a grain of corn, *ten* being its 'value';

3. The corn, which is worth *ten*, is to be placed inside a little box with 'ten'
written on it;

4. Any remainders whose value adds up to a sum below *ten* must be placed
inside the little box with 'unit' written on it.

Extract (6), pre-school, EL, 1 (D: Daniel; M: Mateus):

```
1.    T:    you boys there (.) Daniel how many tens does the number
2.          fifteen have?
3.    P:    five
4.    P2:   ten
5.    D:    five
6.    T:    before answering I want you to think a little (.) a ten is
7.          worth how much? how many units does a ten have?
8.    M:    a ten has two units
9.    T:    Mateus how many tens- how many units - how many beans do I
10.         need to exchange for a corn? How many beans do I need?
11.   M:    ten
12.   T:    so a ten has how many units?
13.   PP:   ten
```

In line 1 Daniel is chosen as the respondent, and is asked about the number of

tens[68] contained in the number *fifteen*. He follows up in line 5 – after two other

pupils – by answering 'five', wrongly. The whole sequence of answers have a

'paradigmatic' resonance within the context of the task and in relation to the

values at stake, and could be seen as answers to potential questions within the

same frame (e.g. 'how many units in a ten?'; 'how many units left in the

---

[68] In the original transcript in Portuguese, two words are being used: *dez* and *dezena*. The former corresponds to the English numeral *ten*, as in 'ten oranges', while the later represent a grouping of ten, rather like the word *dozen*, and is relevant in the context of analysing the position of numbers in a based 10 numerical system, by which multiples of ten can be referred to.

fifteen apart from the ten?', etc.). The way the teacher deals with it points less to their absurdity than to the fact that the pupils have not given it enough consideration, that they have to 'think a little' (line 6); it is as if they had answered on the basis of something they had *looked at*, not thought through. 'Think' undoubtedly perform the work of accountability within the frame in which individual pupils are questioned, but just *how* to think, or to recognize it as such, is explained subsequently, where the procedural aspects of getting the right answer are laid down through a series of questions. 'To think' is as much as explanation as it is the name of the game they are going to play next.

'A ten is worth how much?', asks the teacher in lines 6-7, for which an index is created in the question 'how many units does a ten have?' (line 7). Mateus' mistake in line 9 prompts a new index that has the teacher elaborating on 'procedural' aspects of the game with boxes, those concerning the relations betweens *beans* and *corns* (lines 9-10: 'how many beans do I need to exchange for a corn?'). I call it 'procedural' for the following reason: arguably, there are two questions where there seems to be, or is implied to be, one. Again, like in the example of Helio in extract 5, two different competencies are being put together at the service of using one of them to document the other continuously, without much detailed explanation about what would link them together. One is the question 'how many tens does the number fifteen have?' (lines 1-2), the other is 'how many beans do I need to exchange for a corn?' (line 10). The first question demands from the pupils the understanding of the notions of like 'ten' (ten units) and 'unit', and how they can be found in relation to the number fifteen; the second is intended as a physical 'embodiment' of the number, in which different material devices represent aspects of our base 10 numerical system. Although it is not my intention to argue about the difference between those two registers as a *principled* one, I take that the former (1) is the 'curricular' one, (2) describe a set of *resources* already used in the pupils 'vocabulary' but put to trial in this context as *topic*, and (3) that the index created by the latter is rhetorically alternative to other conventions, materials and instructional strategies.

Actually, the effort is put into making them equivalent. The interesting aspect

of the last segment is how it is organised in a 'if-so'-like structure that implicates the discourse about 'tens-units' and 'corns-beans' as being the same. I am referring especifically to lines 10 and 12, in which the objects of the question are replaced sequentially and so designed as to *conserve* the same answers in lines 11 and 13 ('ten') as logically coherent, avoiding to displace the respondent, or the obviousness of the answer for the respondent. 'So' (line 12) helps linking the two answers by suggesting that the 'ten-unit' relation is contained in the 'bean-corn'relation in the question in line 10, and so, I suggest, is the nature of the cognitive work that performs the latter.

In the following, we have another example of the practical difficulties and resolutions involved in manipulating two different registers in the dialogue between the teacher and Gilvan. The teacher had just asked to the pupils to represent the number *twenty*. We join extract 7 when Gilvan starts counting a group of beans when approached by the teacher:

Extract (7) Teacher and Gilvan, pre-school, EL, 1:

```
1.    P:    one two three four five six seven eight nine ten=
2.    T:    YEAH ((raises hand)) ten are you going to exchange for a
3.          little corn or not?
4.    P:    I am ((takes a grain of corn))
5.    T:    where are these ten going to?
6.    P:    these ten goes to- here ((takes the UNITS box))
7.    T:    these ten you are doing the exchange Gilvan (.)you
8.          remember that these ten units you put here in order to
9.          exchange them for a corn which is worth ten little beans
10.         (.) where is this ten going?
11.   P:    ((takes the TEN box and puts the corn grain inside it))
12.   T:    look at how many were left there
13.   P:    ((counts, pointing at the grains)) NINE TEN
14.   T:    and now what are you going to do?
15.   P:    now here ((takes the remaining beans to the UNIT box))
16.   T:    is it?
17.   P:    ((takes the ten beans back and is taking them to the other
18.         amount of beans on the table))
19.   T:    how much is a little corn worth?
20.   P:    ten
21.   T:    here there are ten beans (.) are you going to change it or
```

```
22.          not for the little corn?
23.    P:    I am
24.    T:    don't be afraid go do what you think you have to do
25.    P:    ((put the beans in the UNIT box))
26.    T:    how many units do you have inside here?
27.    P:    ten
28.    T:    you have here ten units (.) can you exchange them or not?
29.    P:    no ((shakes his head))
30.    T:    how many units do you have here? ((open the UNIT box))
31.    P:    ten
32.    T:    a ten is worth how many units?
33.    P:    ten
34.    T:    you're telling me that a little corn is worth ten beans
35.          (.) you told me that a ten has ten units (.) can you
36.          exchange these ten beans for this corn that is worth ten?
37.    P:    I can
38.    T:    so do it (.) how do you exchange it?
39.    P:    ((put the ten beans that were on the UNIT box back at the
40.          large pile of beans on the table))
41.    T:    where's the corn going to?
42.    P:    ((open the UNIT box))
43.    T:    read the name of the box
44.    P:    unit
45.    T:    this corn is what (.) a unit or a ten?
46.    P:    ten
47.    T:    and it is going to what box?
48.    P:    ((takes the TEN box))
49.    T:    Gilvan the corn is worth a ten it is worth ten (.) the
50.          little bean is  the unit (.) each little is worth only one
51.          but a corn is worth ten ten beans
```

In this dialogue between Gilvan and the teacher, we can note a few interesting features concerning joint activity, assessment, and the predication of 'learning'. In lines 2-3 we see the teacher interrupting Gilvan at the count of *ten*, and then asking (suggesting?) if he is going to take the procedure of exchanging the beans for a corn, that is, a 'ten'. Needless to say that it is the right procedure to be taken; that is the reason why the question is framed the way it is, and Gilvan goes along with that preference for action effortlessly (line 4). His inappropriate response in line 6, though, has the teacher put him in a position to 'remember' what the right action of 'exchange' is. 'Remember' here does a similar job to that of 'think' in the last extract. 'Remember' comes

with what is *remembered*. It not only offers a way to account for action that is predicated on 'psychological' lexicon, but it offers an action alternative that instantiates that predication. It also seems that, as such a 'psychological' vocabulary is widely involved in situations of *repair* (extracts 3, 5, 6 and 7, above), its uses are connected to the assessment of pupils *qua* 'subjects'. As I have observed before, the 'class frame' – in   which the elliptical talk we observed in extract 1 is present – is designed to avoid the kinds of cognitive accountability that are the stuff psychologists talk about, and to reduce disagreement as a conversational preference; on the other hand, the opportunities for repair are not only accepted but integrated into dialogue as a curricular demand over teaching (Macbeth, 2000), in the 'subject frame'.

Similarly to the use of 'remember', 'look at' (line 12) is followed by a counting procedure, a kind of 'active' witnessing, of 'looking at' by 'doing'. Again, contrarily to the 'blackboard' stuff, here the pupil plays Rotman's *Agent*, a form of consciousness that *executes* things, while in the case of the blackboard/class-related activities such a role was a teacher's prerogative. That can be seen in other *doings* in the extract 7: lines 15, 17-18, 25, 39-40, 42, and 48. Note that in these cases the actions constitute legitimate *turns* at interaction, which can be seen in the teacher's correspondent replies to the same actions. It is as if Gilvan is following a 'doer's maxim', to paraphrase Harvey Sacks, in attending to teacher's questions: if he had not acted over the materials given to him in the way he did, he would have been accountable for not *demonstrating* his knowledge, not being able to make available his reasoning processes. In lines 21-22 the teacher asks Gilvan if he is going to exchange 'ten beans' for 'one corn', to which he answers affirmatively, 'I am'. The question is framed as to allow for the observation of Gilvan's decision-making, although it clearly establishes the relevance of the procedure described in the question itself. Because the pupil does not go on to take a more active response in accordance to the method suggested, the teacher joins in suggesting 'don't be afraid go do what you think you have to do' (line 24). Also, observe that in line 37 he answers simply 'I can' to a teacher's question, to which the latter replies 'so do it (.) how do you exchange it' in line 38. The logical aspect of Gilvan's answer is considered insufficient in both passages!

Significantly, the pupil's mind is 'phrased' in line 34, when the teacher says 'you're telling me that a little corn is worth ten beans', attributing a rationally accountable character to Gilvan's actions. Edwards and Mercer (1987) observed that teachers elicit the right answers from pupils through discourse strategies of 'cueing', and then attribute the origin of the reasoning processes to them. This is an example in a which a series of displacements involving material (boxes, beans, corns) and conceptual (ten, units) techniques are used to infer about a conserving, non-displaced subject that controls the invariant aspects of a changing phenomenal world in 'cognitive' terms. The pattern can be found again in extract 8:

Extract (8) pre-school, EL, 1:

```
1.    T:       how many tens does numeral eleven have? (turning to
2.             Gilvan).
3.    Diego:   ten tens.
4.    T:       open the box (.)look at how many corn grains there's
5.             inside
6.    Gilvan:  one.
7.    T:       in the number eleven there's one ten (.) only one (.)
8.             and this corn grain is worth what? How many beans is it
9.             worth?
10.   PP:      ten.
11.   T:       ten (.) how many units is there in the number eleven?
12.            (.) open the unit box and look (.) don't look without
13.            thinking
14.   Diego:   there are ten tens here.
15.   T:       how many units are there? (.) how may units are there in
16.            the number eleven?
17.   Mateus:  one.
18.   P:       (draws a square in the blackboard with the number 11
19.            written within it) the number eleven has one ten and one
20.            unit.
```

This is, no doubt, an overwhelmingly *concrete* conception of 'number'. Diego's (wrong) answer in line 3 to the 'numerical' question (line 1) can be disambiguated by looking inside a box! This assessment is even more

compelling when we read lines 15-16 and the question about the quantity of units in the 'number' eleven: it is suggested that eleven has one unit, not eleven units! In this representational game, the teacher is clearly using 'ten' and 'unit' as mutually exclusive notions, not in the terms of the concept of 'class inclusion' that we saw with Piaget, but distributed in space, in concrete representational devices. It is on that basis that the children are asked to observe, not 'without thinking', in lines 12-13. Again, Rotman's *Agent* and *Subject* are implied at the same time in the teacher's questioning routines; only that here the capacity to perform as *Agent* is primary in allowing for the visibility of the capacity to 'understand' in the first place. The execution of the task's prescribed methods is taken as the better proof that the *Subject* conserves and understands.

The extracts above open again a discussion that we started in chapter 2 and continued in chapter 5, on instructed actions; I am referring to the links between abstract concepts and the rules for their manipulation and the nature of the activities by which those rules are followed. An interesting debate on this issue between ethnomethodologist Michael Lynch and sociologist of science David Bloor has introduced some interesting issues in relation to the delegation of the *respondent* or the *subject* as a mediator between a rule and the way *competent* actors follow it (Bloor, 1992; Lynch, 1993). The debate dates back to a famous example by Wittgenstein, referred to elsewhere as that of the 'awkward student' (Collins, 1986): suppose a teacher ask a schoolchild to produce a numerical sequence based on the rule 'n+2', for which as a result he/she gets the answer 2, 22, 222, 2222, and so on.

Lynch's take on the problem does away with any analytical tendency to formulate the 'student', considering the example simply as a case of violation of the rule. Lynch considers that there is an intrinsic, mutually referential relation between the intelligibility of a rule and its proper application in contexts in which the applicability of the rule is relevant. The awkward student's answer is simply' wrong', and to try to 'explain' it, or its underlying logic, is equivalent to try to explain how the rule determines its 'right' application; for Lynch that is besides the point: the rule *is* its right application,

and vice versa! The author states that this makes no more sense than the question 'how does this side of the coin determine the other side as its obverse?' (Lynch, 1993: 173). Other solutions, like the one he attributes to Bloor, tend to search for 'third', explanatory actors, like socialised tendencies, interpretative schemata, cognition, culture, etc (Ibid.). For Lynch, the scope of the student's answer is failure to obey the norm; it is not a potential interpretation, it is not an interpretation at all. The rule is not open to such relativistic interpretations, but that does not mean that it is transcendental, either. It is conventional, only that its conventional application is inexorable! 'It is a rule in, of, and as counting by twos. The formulation of the rule does not cause its extension, nor does the meaning of the rule somehow cast a shadow over all the actions carried out in accord with it. The indefinite series of actions sustains the rule's intelligibility "blindly" without pause for interpretation, deliberation, or negotiation. Although this is nothing other than a *social* phenomenon, it does not call for an explanation using concepts proper to a particular social science discipline' (Ibid: 180).

What is the place of 'cognition' in Lynch's account then? None, to be more exact. It is clear that, on one hand, Lynch questions the legitimacy of cognitive and 'constructive' sociological studies; on the other, he is not interested in how 'cognition' is 'distributed' when members formulate epistemic orders such as those of the classroom, with its apparatus, ways of representing and making knowledge claims, and, of course, the *subject*. However, teachers *do* that when they pursue the accountability of knowledge in relation to the cognitive projects of individuals, or the accountability of individuals and their underlying competence in relation to their methodic operations, as we have seen in some of the examples above, and in relation to I have been calling the 'class frame'. Here, I am concerned with the delegation of agency. It seems to me important to address this question since it is at the heart of professional psychology and classroom teaching. I agree with most of Lynch's analysis, but it seemingly sets out ethnomethodology as interested, critical and (de) constructionist in relation to 'cognitive' studies! It is as if all the practices in the world are treated as legitimate knowledge-building resources (as 'practical accomplishments'), but sociology and cognitive studies (and teaching!) in

general. I would like to make a 'weak' suggestion on the basis of the discussions and analyses so far, one that points to the fact that Lynch's foundational language must be grounded empirically. For example, observe the following passage: '"Inference" and "cognition" are implicated only as secondary products or analytic reconstructions of how particular assemblages of acts must have been produced. The speed of the assembly outstrips any effort to reason abstractly about it' (Lynch, 1993: 222). Apart from the semi-cognitive hypothesis on the 'speed' of reasoning (outstripped by the speed of the 'assemblages of acts'), it is not clear by *whom* inference and cognition are implicated 'only' as 'secondary', not even how they are *not* implicated as 'primary', an assessment that runs against commonsensical assumptions on the predication of psychology as a discipline, and the constant evaluation of pupil's competence (Foucault, 1977). The point is that Lynch seems to be right in relation to what I have been referring to as the 'class frame': as I have discussed in extracted 1 in this chapter, all the relevant features are accountably external to the human players themselves; they are things to be seen and checked out, and in relation to procedures in which certain actions, but not others, are 'carried out'. The orientation is clearly non-cognitive, but that is part of the local social contract between teacher and cohort. The 'subject frame', however, seems to open up for some of Bloor's favourite themes.

If the class frame is a special case of the Lynch argument, as well as Livingston's (chapter 2), the discussions in this chapter are more likely to respecify some of David Bloor's questions, such as we saw in chapter 2 in relation to Livingston's ethnomethodological account of 'proofs'. There are already indications there of a failure to conceive the topic in relation to the 'gestalts' by which provers come to project the next steps in a sequence of proving, a topic advanced by Livingston himself, as well as an overall failure to conceive any thoughts, sometimes at the risk of looking arbitrary (Bloor, 1987), in relation to a mathematical 'subjectivity'. It is not the case of advancing a psychological theory, and I hope that is not what I am doing in this chapter; rather, it means to treat the subject praxiologically, as an accountably relevant aspect of what people do in some contexts, such as the classroom. If the calculations by the blackboard, due to the class frame, take

for granted the psychological reality of the actors involved (Bloor, 1983), instead of representing it, the interpellation of pupils as subjects may take the contrary route and presume calculation within the context of an all-important *cognitive* procedure. In the following, some of the ways in which *cognitive* accountability might have seem irrelevant or inconsistent with Livingston's investigations can be observed:

Extract (9) pre-school, EL, 1:

```
1.   P:        (...) now let's see (.) whose pens are these?
2.   AA:       Marília.
3.   P:        Marília (.) these are Marília's crayons (.) let's see
4.             how  many there are
5.   Marília:  it is broken miss.
6.   P:        hum? Let's see (.) shall we count? (.) one two three
7.             four five six seven eight nine ten eleven twelve (.)
8.             let's see if any did break Marília.
9.   AA:       it did.
10.  P:        how many did break folks? (.) how many?
11.  AA:       three.
12.  P:        three broke (.) there are twelve pens, three broke (.)
13.            how many left? (.) try to make in the head.
```

The request at line 13 is also a 'prospective account' (Amerine and Bilmes, 1990) of a relevant aspect of this exercise: that of making a calculation without the help of other devices than the 'head'. Although the assessment of any pupil in particular is likely to have (and effectively has) a practical, observable procedure recounted for which to 'make in the head' (line 13) is a proper account, it is not necessarily true that those are 'analytic reconstructions', as Lynch have suggested: they can be as early in the process of 'construction' and 'design' of tasks as the 'curriculum' itself. In Brazil, the National Curricular Parameters for Mathematics (1995), has tried to reassess the heritage of the so-called Modern Mathematics of the 60s and 70s, which privileged the formalisation of concepts and the language of set theory (Ibid: 2), in the light of the cognitive paradigms of problem-solving in mathematics, as well as the emphasis on the learner's own activity in the 80s and 90s (Carraher et al., 1988). One of their programmatic objectives was to lead the learner to

'appreciate the character of mathematics as intellectual game, recognising it as an aspect that stimulates the interest, the curiosity, the spirit of investigation, and the development of the ability to solve problems' (1995: 12). The document emphasises the variety of situations and procedures in which one finds mathematics, and makes no excuse in recommending the teaching/learning not only of written strategies of problem-solving, but also 'mental' ones and by the use of 'calculators' (Ibid: 16). Although I analyse no cases involving the use of calculators in the classroom, it seems fascinating as an alternative to the sociologies of learning and mathematics we have discussed so far. It is fascinating because it allows us to imagine a scenario in which the question of cognition, or that of how someone came to 'think' or 'know' of something (see below), cannot straightforwardly use aspects of mathematical practical action and argumentation as the problem-solving project of an individual learning subject, but neither can it count on the 'lived-work' as described by Livingston (1986). In the following the teacher has asked the pupils to calculate how many 'eggs' would be the result of Helio's 'six eggs' plus Naia's 'six eggs', in an exercise that we saw partly in extract 5:

Extract (10) Teacher and Samuel, pre-school, EL, 1:

```
1.    T:    how did you do it? (.) how did you think it was
2.          twelve? how did you do it?
3.    S:    ((counts the drawings on his sheet))
4.    T:    but how did you do that (.) how is that- so many
5.          eggs (.) how did you find it out? (.) where are
6.          Naia's eggs?
7.    S:    ((counts on the sheet))
8.    T:    so you took Naia's eggs and added to Hélio's and
9.          you got=
10.   S:    =twelve
11.   T:    ah::
```

Note that in lines 1-2 'how did you do it' and 'how did you think' are used interchangeably' as being indexes to one another. The answer, however, is not a verbal, cognitive account, but a procedure, the only response – from the pupil's 'point-of-view' – that can attest to the appropriateness of how the pupil 'thought'. The answer is rather problematic, since it does not recount the terms

of the problem: the 'how' on which the teacher insists consistently in lines 4-6, reaches a resolution in line 8-9, where the teacher implies the pupil's strategy in line 7 ('so') as possessing the features of the right answer: the one who adds up two different sets of objects, Helio's and Naia's eggs! The teacher even displays an indication of comprehension, of insight, in line 11 ('ah:::'), after Samuel completes her sentence in line 10.

Extract (11) Teacher and Marília, pre-school, EL, 1:

```
1.    T:    tell me Marilia how did you find twelve?
2.    M:    ((points to the written [12] on her sheet. Then
3.          starts counting with her fingers))
4.    T:    how did you get that? (.) how did you do it if you
5.          put it in your hands? (.) you said you find twelve
6.          and how many fingers do hands have?
7.    M:    ten
8.    T:    then how was it?
9.    M:    I put down ten but I thought it wasn't ten then I
10.         put down twelve
11.   T:    but how did you find twelve, hein Marilia? (.)
12.         where are Naia's? (.) Marilia you copied from your
13.         classmate (.) aren't you understanding? (.) we need
14.         to understand you see?
```

In the case of the extract above, there are remarkable features involving the use of psychological language in accounting for 'assemblages of actions' (Lynch, 1993: 222), requiring one as a logical condition of having the problem solved, and then denying that there was one at work in the first place. Similarly to the previous example with Samuel, the teacher frames the problem of learning or of knowing as a question of 'how', not unlike Lynch and Livingston themselves. Note, however, that this is a different kind of 'how'. While a simply wrong answer would make Lynch dismissive of ever trying to explain how to analyse it, and a right answer created interested in showing the internal relation of between a rule and its lived-work (Livingston, 1986), the exercise above, as it is set up, makes the 'how', as the moment-to-moment building of such relation unavailable for the teacher herself. The teacher has not seen these pupils performing anything that has led to the (correct) answers they have

given. The whole point of the exercise is precisely to make these things come up in the pupil's accounts. But is it right that the teacher is after a realistic account of 'how' (exactly, ethnomethodologically) they got the correct answer, for which Samuel's first answer (by counting) could be a truthful one? If the relation between a rule and its application is intrinsic and mutually referential (Lynch's point), that is far from the nature of the teacher's injunctions in the extract above, and the reason why that is so, is that in this context there is a need to distribute agency to something other than the mathematics, or rather, to make mathematics to be like something other than itself: reason. So far, the rule ('add six to six') and the answers ('twelve') are fine; what is the problem, then?

Marilia has found the correct answer; it is written in a sheet of paper in front of her. She also counts with her fingers, a procedure similar to that used by Samuel, but for which she can only count with ten units, according to an argument used by the teacher to dismiss the procedure as valid (lines 4-6). In lines 9-10, Marilia skillfully subverts the use of the psychological language – as we saw with Samuel – from proactive and physical, to casual and 'internal'; if not 'internal', at least 'mediating': note that the 'thought' comes in between a sequence in which two different things are 'put down' to paper! That, of course, is not enough for the teacher, who insists on the 'how' question, this time qualifying it by suggesting the lack of important representational terms ('where's Naia's [eggs]?'), since Marilia's answers obviously fail to attend to the discrete semiotic entities that compose the problem. That Marilia got the correct answer warrants the conclusion that she most likely was not 'understanding' (lines 13-14), but might have followed a *rational* procedure anyway, by copying from one of her classmates (line 12). She didn't find it rationally, but used the rationally inferable method of copying from someone who rationally found it. The same kind of rational delegation can be seen in the dialogue with Renan:

Extract (12) Teacher and Renan, pre-school, EL, 1:

1.   T: how did you think of it? (.) how did you find

```
2.              twelve?
3.      P(R):   I've thought (.) I've already thought twelve
4.      T:      no you didn't (.) you can't already think
5.              twelve (.) how did you find that? (.) where are
6.              Naia's eggs?
7.      P(R):   ((points to the drawings on his sheet))
8.      T:      ah:: so you took Naia's eggs and added to Hélio's?
9.      P(R):   ((nods))
10.     T:      it was that that I wanted to know (.) the way
11.             you thought
```

What is at stake here, again, is just *how* he found the solution. Like Marilia, he claims to have 'already thought twelve'. 'No you didn't (.) you can't already think twelve', says the teacher in reply. In relation to Marilia, he has the advantage of having represented Naia's through drawings in his notebook, in which case the same procedure of 'cued elicitation' initiated with Samuel applies. His compliance and nod at line in line 9 is then summarised as to make the point for having mathematics there in the first place, mathematics for 'learning': 'it was that that I wanted to know (.) the way you thought'. In this case 'thought' is used as an after-fact construction that has denied its use in 'merely' mental ways in the first place.

In our search for a social and semiotic standpoint, we have talked about the conditions for the emergence of a public subject that can witness and agree collectively upon a rational state of affairs as part of the epistemic requisites of the installation. This chapter, as I said before, is about the *installation*'s, or the class' significant other. This significant other, the *subject*, can only be studied in relation to the installation. For some, such a conclusion is a natural consequence of the conceived material heterogeneity of organisations, of which individual human actors are a constitutive and constituted part (Law and Moser, 1999). The 'pupil' is therefore an effect, a performance. The (accepted) psychological fact of the child's 'learning mind' is formulated at the expense of *mediation*, although it is made possible by its very presence, be it in the form of material technologies or instructed actions. It is even arguable that without the classroom the fact of the child's mind would not be such an important public object (Foucault, 1977). While the 'class' enlists the whole

installation, the accountability of the individual pupil formulates its 'epistemics' as disengaged, and therefore 'denies' the installation.

CONCLUSION

This study focused on how the materiality of the classroom, as well as the delegation of knowing agents, affords 'learning'. That is to say, on how the stabilization of its knowledge, scope and methods depend on the constitution and circulation of spaces, objects, inscriptions, composable surfaces, etc., on one hand, and on the disciplining of observation and negotiation of public identities (e.g. witnesses) on the other. Moreover, these two poles act as to set up a peculiar semiotic economy, where the accumulation of one depends on the other. Thus, the possibility of collective agency (e.g. the 'class', Chapter 3 and 4) depends on the possibility of joint witnessing and responsivity afforded by accountably 'analytical' spaces.

Foucault's work on the rise of 'disciplinary' practices is an undeniable source of inspiration here. His analysis point to the simultaneous creation of power and episteme or of how new social technologies could have counterparts in terms of new objects of knowledge. The interplay between the assessment of normative knowledge in the classroom, on one hand, and pupil's competence, on the other, or, between the accountability of the knowledge and the accountability of individual learning was the subject of this chapter. Foucault offers some indications of how differentiation might have taken place when considering the technologies of 'examination' in the classroom:

> 'The seriation of successive activities makes possible a whole investment of duration by power: the possibility of a detailed control and a regular intervention (of differentiation, correction, punishment, elimination) in each moment of time; the possibility of characterizing, and therefore of using individuals according to the levels in the series that they are moving through; the possibility of accumulating time and activity, of rediscovering them, totalised and usable in a final result, *which is the ultimate capacity of an individual.* Temporal dispersal is brought together to produce a profit, thus mastering a duration that would otherwise elude one's grasp. Power is articulated directly onto time; it assures its

217

control and guarantees its use' (1977: 160, emphasis added).

Partly, my analysis of different frames in the classroom detects and offers an alternative way of understanding the 'temporal dispersal' that is brought together through distributing agencies in order to make a 'profit': such a profit is 'epistemic', as well as 'governmental' and 'administrative'. The relation with (mathematical) knowledge brings this distribution into account, and furnishes a way in which the profit can be 'rationally' established in its dialogue with another important actor in society, science. Conversation analysis, a discipline with which I have dialogued intensely, and that have helped me to understand some practical aspects of managing the temporal dispersal referred above, has also been the blueprint for a critique that recognizes it as omitting at least four crucial aspects in relation to the mathematics classroom: (1) the 'class', (chapter 3), (2) the 'room', especially the blackboard as a social mediator that helps to holds the class together as an agent in its own right (chapter 4); (3) the 'mathematics', which accountably set the need for the reflection on the analytical force of actions and accounts *in*, *of*, and *as* mathematics; and (4) the 'subject', the rationally-governed actor/victim of the disciplines, the pay-off of a whole educational system that in real time practices designs a 'temporal dispersal'(within a conceptual and technological frame) that affords its existence. Such omissions might set my own analysis as irreconcilable with CA.

The technologies with which the classroom engine is made to work are, I insist, a decisive factor in understanding cognition. Thus, in chapter 4 the centrality of the blackboard as a medium for the 'calibration' of joint activity and for the ascription of identity to a collective – with a clear epistemic function – was emphasized, as it was shown, in chapter 5, how documentation practices which make possible, through the coupling of action and language, to render the very setting intelligible as a 'mathematical lesson'. Despite their seemingly unremarkable character, those technologies (of centralising inscribable surfaces and discursive glosses or formulations) constitute powerful 'secondary causes' (Latour, 1990) that put into effect all the so-called 'important' issues, or primary causes, of mind, knowledge and society.

Such 'primary causes' represent the purified objects of the (human) sciences (Latour, 1993), such as 'cognition' and 'society', and because of their prior and primary status in relation to the other things in the world with which they come into contact – which we have analysed before as the semiotics of the 'homogeneous' – they have been the source of the common mistake the ethnomethodologists have long identified as the confusion between *topic* and *resource* (Garfinkel, 1967), that is, they are at the same time the object and the means of inquiry. Furthermore, it has always been a problem for human and social scholars to identify by which powers and associations all those facts of thinking and culture come into efficiency. Some of the literature's preferred topics, or allegedly *primary* causes of 'order', which can be potential points of contact, however respecified, between the present work and traditional approaches, include (a) *shared learning and cognition*, and the way it is realised in the classroom through the use of mediational technologies; however, rather than considering such technologies as intermediaries between an individual consciousness and the real, I focused on how they help to constitute the agents themselves, vis-à-vis other agents and the 'real' (chapter 4); (b) The issue of *concrete* versus *abstract* thinking. Vygotsky addressed this question as central to the relations between development (as conceived by psychology) and learning in social contexts. He then distinguished between children's *spontaneous* versus *scientific* concepts, the latter referring to the kinds of formal representations learned at school. Here, the question turns out to be how members orient to such issues *as* differentiated, *as* some sort of *collection*, how those things *index* each other, and what kinds of practical or conceptual work they perform (chapter 5); and (c) The use of cognitive language to account for the *individual mind* in problem-solving activities. Here, the problem of 'how' (pervasive in the kind of analysis that discursive psychology carries on) becomes a member's problem, as they openly have to justify and account for their procedures and methods in the learning setting (chapter 6). This work has been an attempt to overcome part of such defficiencies and set out to respecify a few 'psychological' topics while doing justice to the associations of many objects, concepts, actions, and places in bringing (cognitive) order together.

Appendix

Original Transcripts in Brazilian Portuguese

Chapter 3 – Order Installed: the classroom as an analytical object in psychological and social research:

Extract (1) 3rd grade, EE, 3:

```
P:    pra tirar a prova real dessa divisão (1.0)
      agora (.) esse divisor tá sendo o quê agora?
A:    °multiplica( )°=
A:    =multiplicado
A:    >multiplicador<
A:    multiplicador::=
P:    =multiplicador agora ele passou a ser ↑o-
      multiplicador não é que eu não vou trabalhar agora
      com a multiplicação? (.) ele passou a ser
      multiplicador (.) e o quociente agora passou a ↑ser=
AA:   multiplican::[do
                    [multiplicando (.) então agora tem
           quatro vezes quatro?
AA:   dezess[ei::s
```

Chapter 4 – Artefacts, Knowledge and Discursive Practices: the blackboard and the social order of the mathematics classroom:

Extract (1) 3rd grade, EE, 3

```
P:    olhe aqui na na letra a (0.8) nós estamos vendo
      (.) é:: (0.4) quanto é o- o dividendo?
      (0.4)
AA:   noventa e ↑do[is
P:                 [noventa e dois ((limpa a garganta))
      noventa e dois que vai ser dividido ↑por=
AA:   =quatro
```

Extract (2) 3rd grade, EE, 3

```
P:      ↑mas eu tenho nove (.) pra chegar em nove inda
             falta quanto?
AA:     UM:::=
P:      falta um (0.8) agora que que eu- vou fazer?
AA:     [pegar o dois
A:      [abaixar o dois
P:      hã (.) o dois. vocês observem que aqui na
        divisão (.) eu comecei a dividir ↑pela dezena
        >não foi isso?<
A:      °fo[i°
A:         [foi
```

Extract (3) 3rd grade, EE, 2

```
A:      tia também pode fazer quatrocentos dividido
        por três?
P:      quatrocentos dividido por três? ((dirige-se ao
        quadro negro))
A:      uh huh.
P:      ↑com certeza (.) aí é é uma divisão normal. Você faz
        quatrocen:tos (.) dividido ↑por (.) três (1.0)
```

Extract (4) 3rd grade, EE, 2 (continuation of 3)

```
P:          posso dividir quatro por três?
                       ▲          ▲


PR:         400 ⌊3

A:          pode
A:          pode
A:          °pode não°
P:          pode porque o quatro é maior que o três (.) >então
                       ▲              ▲


            eu falo< quatro dividido por três ↑dá=
                '
                ‾
```

```
                    ▲                 ▲


PR:    4'00 ∟3


AA:        [=UM::
P:         [ 1



PR:        4'00 ∟3
                   1


P:         [uma vez três?
               ▲        ▲
A:         quatro
A:         °quatro°
AA:        TRES:::=
P:         ↑uma vez três?
AA:        TRÊS:::
P:         =para quatro quanto falta?
                     ▲
AA:        [UM:::
P:         [e agora?
            1



PR:        4'00 ∟3
           1    1



A:         abaixa o zero
A2:        [abaixa o zero
P:         [eu ↑baixo mais o zero (.) agora eu tenho quanto?
                         ˙                              0

PR:        4'0'0 ∟3
           10    1



AA:        dez:::=
P:         =pra dividir por três quanto dá?
                         ▲

           (2.0)
A:         três
A:         [três
A:         [três
```

P: *((volta-se a um aluno atras dela))* cinco? ↑dá=

AA: três

P: <u>3</u>

PR: 4'0'0 ∟3

            10   13

P: três vezes três=
            ▲   ▲

AA: NO::VE

P: para dez ↑falta=
                        ▲

AA: UM::=

P: =um. agora eu ↑baixo=
            <u>1   </u>'

PR: 4'0'0 ∟3

            10   13

            1

AA: =ZE:RO

P: zero. dez dividido por três dá=
            <u>0</u>
                    ▲   ▲

PR: 4'0'0' ∟3

            10   13

            10

AA: TRÊS:

P: <u>3</u>

PR: 4'0'0' ∟3

            10   133

            10

P: e três vezes três?
                ▲   ▲

AA: NO:VE=

```
P:          =para dez?
                     ▲
AA:         UM::
P:          1


PR:         4'0'0' ∟3
            10      133
            10
             (1)


P:          então o resultado foi cento e trinta e três=
            ((escurece o segundo 3 com o giz)) e o [resto=
A:                                                 [=tia ô tia
P:          tia?
```

Chapter 5 - Talking mathematics: Formulations, situated actions and transparency in instructional activities:

Extract (1) 3rd grade, EE, 1

```
P:          é:: você. me diga aí quanto é que dá dois mais
            três
            (1.0)
A(1):       seis
            (2.0)
P:          (      ) quanto é que dá dois mais três?
A:          °cinco°
P:          °cinco° (.) e:: três mais dois?
A:          cinco
P:          cinco
            (1.0)
P:          quanto é- agora diga agora quanto é que dá dois
            mais três
A(1):       °cinco°
P:          °cinco° (.) e três mais dois?
A:          °cinco°
P:          ah sim agora aprendeu (.) agora você que riu quanto
            é que dá é: dois mais      cinco?
A:          (      )=
P:          = e: cinco mais dois?
```

224

```
A:          sete
            (3.0)
```

Extract (2) 3rd grade, EE, 1 (continuation of 1)

```
P:          agora vocês já escutaram que eu perguntei a três
            meninas ali não foi isso?
A:          foi
P:          já perguntei a elas três e eu gostaria que vocês me
            dissessem (.) que é que vocês notaram ↑alguma
            coisa↓ na forma da pergunta que eu fiz pra ela
AA:         (            )
P:          PERAÍ CALMA um de cada vez.
A:          (            )
AA:         (alunos falando ao mesmo tempo)
A(2):       tia a senhora=
AA:         (alunos falando ao mesmo tempo)
P:          PERAÍ calma gente
A(2):       =aplicou a propriedade comutativa
P:          EU APLIQUEI ela notou tá vendo? ela- eu apliquei-
            você soube- você- sabe o que é que você tava
            dizendo só que você nao tá sabendo se explicar. ↑eu
            apliquei a propriedade=
AA:         =comuta[tiva
P:                 [comutativa (.) e:: (.) como é que se aplica
            a propriedade comutativa?
AA:         (inaudível; muito barulho)
P:          (trocou) as ordens das parcelas e não altera=
A:          os fatores
P:          =a soma
A:          os fatores
P:          não meu amor=
A:          os fatores
P:          = (            ) (.) agora você. quanto e que dá é::
            dois mais oito?
            (3.0)
A:          dez
P:          e oito mais dois?
```

Extract (3) pre-school, EL, 1:

```
P:      o numero onze tem uma dezena e uma unidade
```

Extract (4) pre-school, EL, 1:

```
T:      dez dezenas são cem unidades
```

Extract (5) pre-school, EL, 2:

```
P:   um par é formado por duas coisas, Dani (.) aqui tem um par de
     folhas (.) eu quero dois pares
```

Extract (6) pre-school, EL, 2:

```
T: →   sim (.) dois é par (.) e esse que sobrou? se sobrou algum não
        pode ser par (.) então três é=
P:      ímpar.
```

Extract (7) 3rd grade, EA, 2:

```
P:      eu posso dividir quatro por três?
A:      pode
A:      pode?
A:      °pode não°
P:      pode porque o quatro é maior que o três (.)
```

Extract (8) 4th grade, EF, 1:

```
P:      =olhe eu quero que você (0.4) me dê um
        resultado (.) resolvendo a expressão (.) e que
        me mostre também (.) é::: como estavam
        empilhadas estas caixas de acordo com a
        expressão numérica que o:: o- o- empregado fez
```

Extract (9) 4rd grade, EF, 1:

```
P:      no depósito de uma loja (.) estavam várias é: Sílvio (.)  precisa
        ouvir tá? (.) no depósito de uma loja estavam várias caixas (.)
        empilhadas de várias maneiras (.) sabe o que é empilhada né?
```

A: é

P: tão vendo o que (   ) pronto estavam empilhadas de <u>várias</u>
    maneiras. <u>Luis</u> empregado dessa loja precisou conf<u>erir</u> (.) quantas
    caixas (.) estavam lá e ele saiu regis<u>trando</u> (.) ele era muito
    sabido >ele sabia expressões numéricas< (.) e ele saiu
    registrando (1.0) como ele tava vendo os >empilhamentos das
    caixas<. Então tem um registro que ele fez assim ó
    [(9.0)
    [[<u>escreve no quadro-negro</u>: (1°) 9 ÷ 3 X 3 + 5 =

P: isso daqui foi a situacão que ele observou (2.0) certo? a segunda
    situação que ele observou (.) de empilhamento de caixas=
A: faltou o tracinho
P: ah?
A: embaixo
    [(8.0)
    [[<u>escreve no quadro-negro</u>: (2°) (3 + 1) X 4 =

P: a segunda situação foi essa (.)

Extract (10) 4rd grade, EF, 1:

P: só um pouco viu amor (.) vocês vão pe<u>gar</u> a folhinha de trás do
    caderno quadriculado (.) vamo prestar atenção (.) vocês vão
    co<u>piar</u> essas expressões=
A: =os (   )?
P: calma. não. vão co<u>piar</u> >do jeito que eu botei no quadro< situação
    um dois e três=
A: pode ser de caneta?
P: pode. E vocês vão mostrar pra mim de dois jeitos (3.0) vocês vão
    <u>resolver</u> (1.0) a expressão que Luís fez e vocês vão me <u>mostrar</u>
    (.) como estas caixas estavam empilhadas na situação um (.) como
    estas caixas estavam empilhadas na situação dois e como estavam
    empilhadas na situação três. deixem ↑bem arrumadinhos porque eu
    vou passar agora pra olhar o estoque de vocês (.) preciso repetir
    a orientação?
A: ↑não
P: preciso?
A: não.

Extract (11) pre-school, EL, 2:

P: Pra gente dizer que uma coisa é igual à outra nem sempre a gente
    precisa usar a palavra igual (.) a gente pode usar um sinal (.)
    eu vou mostrar (.) quem é que conhece o sinal de igualdade?

A: Eu sei.

A2: (   )

P: Eu tô perguntando quando eu quero dizer essa palavra é igual a
    essa (.) vou mostrar pra você (.) olha (.) o sinalzinho que a
    gente usa para dizer que uma coisa é igual à outra *(mostra uma
    cartolina com o sinal "=")* (.) esse é o sinalzinho de igual (.)
    eu quero dizer (.) Daniel, olha (.) casa igual a casa *(mostra
    três pedaços de cartolina que fazem CASA = CASA)* (.) esse   é o
    sinalzinho que diz que as coisas são iguais (.) cada um de vocês
    vai receber agora um sinalzinho *(inaudível na seqüência)*

    (Distribue materiais aos estudantes)

    (.) todo mundo já tem agora o sinalzinho de   igualdade (.) eu
    vou colocar aqui no centro do círculo *(inaudível na seqüência)*
    (.) vocês vão ver que aqui tem esse tipo de material com vários
    objetos de várias formas, várias cores (.) aqui tem os numerais
    (.) eu quero que vocês formem dois conjuntos de coisas iguais, ou
    dois conjuntos iguais.

M: Já formei

P: você formou um conjunto?

M: *(mostra '7=7' com as cartolinas).*

P: Sete é igual a sete (.) isso é um conjunto? (.) o
    que é que falta pra formar um conjunto aí, Mateus?

M: ais um!

P: olha (.) isso é um numeral, o numeral "sete" (.)
    cadê as coisas pra significar que são sete coisas
    aqui? (.) eu não tô vendo nada, olha, só tô vendo
    numeral (.) um numeral sete e outro numeral sete (.)
    eu quero um conjunto.

Extract (12), pre-school, EL, 2:

P: Olhe pra esse conjunto e diga porque esse conjunto é igual a
    esse. *(sobre dois conjuntos iguais numericamente, mas diferentes
    em seus elementos)*

I: *(Ivo aponta o sinal de igualdade)*

P: Por causa do sinal? Mas eles são iguais ou diferentes?

I: Diferentes

Extract (13) 3rd grade, EE, 3:

```
P:    =porque três vezes quatro são: doze (1.0) é::
      eu sei (     ) nós temos certeza que essa divisão
      está correta não está?
A:    tá=
P:    =nós efetuamos corretamente não foi?
AA:   °foi°
P:    mas (.) ↓para ter mais certeza eu vou tirar a prova real (.) a
      prova real da multiplicação-    da da: divisão que é feita com a=
A:    °multipli[cação°
P:              [multiplicação (.) não é isso? (.) então    pra eu
      fazer essa multiplicação que é que eu vou fazer? eu vou começar
      como?
      (1.0)
A:    (com a tabuada?)
      (1.4)
A2:   multiplicando o resulta[do=
A:                          [multi[plica o divi-
A2:                               [=por quatro
A:    (o quatro o três e o dividendo)
P:    como é?
      (0.8)
A:    multiplica quatro vezes o dividendo
P:    então eu multi↑plico quatro eu vou colocar o: divisor: vou
      multiplicar quatro >que é o divisor< por: vinte e três. ou seja
      eu vou multiplicar o divisor pelo=
AA:   quocie[n:te
P:          [quociente. e vou encontrar ↑o=
AA:   RES:TO=
P:    resto?
AA:   divi↑d[en:do
P:           [dividendo eu vou encontrar ↑o dividendo
```

Chapter 6 - Accountability, human agency and the other things: the learner in relation to the installation

Extract (1) 3rd grade, EE, 3:

```
1.    T:    look here in in the letter a (0.8) we're seeing (.)
2.          erm:: (0.4) what's the- the dividend?
```

229

```
3.              (0.4)
4.    PP:      ninety ↑[two
5.    T:               [ninety two ((clears throat))
6.             ninety two that is going to be divided (by=
7.    PP:      =four
8.    T:       let's see when it is possible to- to I'm going t-
9.             going (.) I'm going to say tha:t >this room has
10.            ninety two pupils.< (0.4) I'm going to divide this
11.            room (in (.) four groups. we're going to find out now
12.            how many pupils are going to stay in each group. can
13.            I divide ni:ne by four?
14.   PP:      yes::
15.   T:       I have nine I want to divide them by four it is going
16.            to make=
17.   PP:      tw::o
18.   T:       (why two?
19.   PP:      because two times two is [eight
20.   P:                                [it is eight
21.   T:       two times four is=
22.   PP:      [=eight
23.   T:       [eight is going to make two (.) two times four=
24.   PP:      eight.
25.   T:       (but I have nine (.) to get to nine still how many
26.            left?
27.   PP:      O:::NE=
28.   T:       one left (0.8) now what what am I going to do?
29.   PP:      [take the two
30.   P:       [bring down the two
31.   T:       hm? (.) the two. you observe that here in the
32.            division (.) I started to divide (by the ten
33.            >wasn't it?<
34.   P:       °ye[s°
35.   P:          [yes
36.   T:       now is that I'm going to divide the=
37.   PP:      =u[nit
38.   T:         [unit (.) instead of ninety two now I (have=
39.   PP:      =twel::ve
40.   T:       to di(vide by four I wonder how much will it make?
41.   PP:      THREE:::
42.   T:       (why?
43.   PP:      BECAUSE THREE TIMES FOUR IS TWELVE=
44.   T:       =because three times four is twelve: twelve (1.0)
45.            hm:: I know (      ) we are sure that this
46.            division is correct isn't it?
47.   P:       it is=
```

230

```
48.  T:    =we carried it out correctly didn't we?
49.  PP:   °yes°
50.  T:    but (.) (to be more sure I am going to take the real
51.        proof (.) the real proof of the multiplication- of
52.        the of the: division is done with the=
53.  P:    °multipli[cation°
54.  T:             [multiplication (.) isn't that so? (.) so in
55.        order for me to do this multiplication what am I
56.        going to do? How I am going to start?
57.        (1.0)
58.  P:    (with the table?)
59.        (1.4)
60.  P2:   multiplying the re[sult=
61.  P:                     [multi[ply the divi-
62.  P2:                         [=by four
63.  P:    (the four the three and the dividend)
64.  T:    how is this?
65.        (0.8)
66.  P:    multiply four times the dividend
67.  T:    so I multiply four I'm going to put the:
68.        divisor: I am going to multiply four >which is the
69.        divisor< by: twenty three. that is I'm going to
70.        multiply the divisor by the=
71.  PP:   quotie[n:t
72.  T:          [quotient. and I'm to find (the=
73.  PP:   REMAIN:DER=
74.  T:    remainder?
75.  PP:   divi(d[en:d
76.  T:          [dividend I'm going to find (the dividend
77.        here putting the four I'm going to- erm: now to
78.        carry out (a multiplication. (0.8) four ti- >now see<
79.        (if I don't find it( you said that I am going to
80.        multiply and I'm going to find the dividend (.) and if I
81.        don't find the dividend?
82.  PP:   the calculation is wrong
83.  T:    the calculation is wrong.  So I do four times three?=
84.  PP:   TWEL::VE
85.  T:    goes?=
86.  PP:   ONE::
87.  T:    four times two?=
88.  PP:   EI::GHT
89.  T:    plus one?=
90.  PP:   NI::NE
91.  T:    so is it right or wrong?
92.  PP:   RI::GHT
```

```
93.          (0.4)
94.  T:      let's now >pay attention< here in the letter a we
95.          only did hm: unit and ten (.) isn't that so? In the
96.          letter b we have (unit ten and hundred( (.) so I'm
97.          going to start carrying out this division by the- by
98.          the unit >isn't that so<?
99.  PP:     °no°
100. P:      no miss the hundred=
101. T:      =I'm going to start by the hundred (.) now I have
102.         four can I divide four by four?
103. PP:     YES::
104. T:      if I have (1.0) four pencils (   ) to divide with
105.         four pupils here in the room how many pencils I'm
106.         going to give to each one?
107. PP:     O::NE
108. T:      I am going to give one. (  ) there are four I'm
109.         going to divide four by four it's going to be?
110. PP:     one:
111. T:      one times four?
112. PP:     four=
113. T:      =to four?
114. PP:     nothing
115. T:      I'm going to drop now=
116. PP:     =one::
117. T:      that is now I dropped the ten wasn't it?
118. P:      yes
119. T:      now there's only one (.) to divide by four
120. P:      drop [the zero
121. P:           [dro[p the zero
122. PP:              [DROP THE ZERO
123. T:      look pay attention here in this case (.) when here we
124.         have a number that cannot be divided by the divisor
125.         (.) that is (smaller than the divisor so what am I
126.         going to do I am going to put zero in the quotient
127.         <zero in the quotient> so that that I can drop (the
128.         (.) following number that is seven (.) now how many
129.         do I have?
130. PP:     seventeen:
131. T:      now I have seventeen to divide (by=
132. PP:     four:
133. T:      I wonder how many is going to make here?
134. P:      fo:[ur
135. P:         [fo:[ur
136. PP:           [fo:[ur
137. P:               [four
```

232

```
138. T:    four? so (.) four times four?
139. PP:   SIXTEEN::
140. T:    to seventeen?
141. PP:   O:::NE
142. T:    one (left) (1.6) in this division who is the:::
143.       quotient?
144.       (0.4)
145. P:    >one hundred [and four<
146. PP:              [one hundred and four
147. T:    one hundred and four and what is quotient?
148. P:    the re[sult of the divi::sion
149. PP:        [it is the result=
150. T:    =it is the result of (the=
151. PP:   =divi::sion
152. T:    isn't that so? this- division is (.) exact or inexact?
153. PP:   inexact
154. T:    it is inexact why?
155.       (0.2)
156. P:    because one was [left
157. PP:                  [because one was left:
158.       (19.0)
159. T:    it is inexact because it has (left (.) remainder. of how
160.       many is the remainder?
161. PP:   °o::ne°
162. T:    of one. Now <I'm going to take the proof> (.) putting the
163.       divisor isn't that so?
164.       (0.6)
165. T:    the divisor >if I took the divisor from here I am going to
166.       put it here< to take the real proof of this division (1.0)
167.       now (.) this divisor is being what now?
168. P:    °multip(      )°=
169. P:    =multiplied
170. P:    >multiplying<
171. P:    multiplier::=
172. T:    =multiplier now it turned into (the-
173.       multiplier isn't it because ain't I going to work out the
174.       multiplication now? (.) it turned into the multiplier (.)
175.       and the quotient now turned (into=
176. PP:   multiplying
177. T:                      [multiplying (.) so now it is four times
178.       four?
179. PP:   sixteen::
180. T:                  [sixteen  goes?
181. PP:   o::ne
182. T:    four times nothing?
```

```
183.  PP:    no:thing
184.  T:     then I drop?
185.  PP:    o::ne
186.         (0.8)
187.  T:     four times one?
188.  PP:    four
189.  T:     I found- the- (0.8) divisor?
190.  PP:    NO::
191.  T:     no I (didn't find the divisor yet (.) by the way the- the-
192.         dividend why hadn't I found the dividend yet?
193.  P:     ↑miss because you hadn't added yet the=
194.  P:     =because you hadn't added [yet
195.  T:                               [because it I still have to
196.         add ↑the- remainder (.) here I'm going to use now ↑a sum
197.         (1.2) I'm going to put the remainder (.) six plus one?
198.  PP:    SE::ven
199.  T:     seven here I drop=
200.  PP:    one::
201.  T:     four (.) I'm going to drop [four] isn't it?
202.  PP:                               [four]
203.  T:     can you understand folks?
204.  PP:    yeah:
205.         (0.6)
206.  T:     that uh- this these >one hundred and seventeen is not part
207.         of this (fifteen)< right?
208.  P:     °yeah°
209.         (0.4)
210.  T:     so now I've found ↑the (.) dividend (.) isn't that so?
211.         (0.8)
212.  T:     now we have the letter c (.) in which I have unit ten
213.         ↑hundred=
214.  P:     =and u[nit of thousand
215.  P2:          [and unit of [thousand
216.  T:                        [unit of thousand (.) <now I have
217.         till the unit of thousand> in that case it is going to be
218.         divided ↑by (.) [six
219.  P:                     [six
220.  T:     here we have till the ten multip- erm di↑vided (.) by
221.         (thousand) (.) here we have till the hundred divided ↑by=
222.  PP:    four=
223.  T:     unit isn't it? And here by unit too. ((to a pupil)) look
224.         ahead you're going to find (          ) (.) divided by
225.         ten
226.         (7.0)
227.  T:     but here I have one thousand eight hundred and twenty four
```

```
228.          to divide by six (1.0) if I have one thousand eight
229.          hundred and twenty four to divide by six (.) I'm going to
230.          have to divide one by six isn't it?
231. P:       n[o::
232. P:        [no:
233. PP:       (              )=
234. T:       =ah:: I'm going to divide eighteen I am going to take
235.          eighteen to divide ↑by=
236. PP:      =[six
237. T:        [six. how nay is it going to make?
238. PP:      THREE::
239. T:       why?
240. PP:      BECAUSE THREE TIMES SIX IS EIGHTEEN=
241. T:       =because three times six are eighteen here it's going to
242.          be three times six eighteen (to eighteen=
243. PP:      NO:thing
244. T:       nothing (.) and now I (drop=
245. PP:      TWO::
246. T:       right. (1.0) now I only have two to divide by six [I'm not
247.          going to be able
248. P:                                                          [drop-
249.          drop=
250. PP:      ((many pupils speak at the same time))=
251. T:       =here I put zero in the quotient and I drop=
252. PP:      the four
253.          (0.8)
254. T:       now it is what?
255. PP:      twenty four=
256. T:       =twenty four divided by six?
257. P:       fo[ur
258. PP:        [four
259. T:       why four ?
260. PP:      BECAUSE FOUR TIMES SIX IS TWENTY FOUR
261. T:       four times six is twenty four here I put the four. four
262.          times six is >twenty four< to twenty four?
263. PP:      nothing
264. T:       now I'm going to take=
265. P:       pro[of
266. P2:         [proof
267. P3:      (      ) the parenthesis [miss
268. P4:                              [the parenthesis miss
269. P5:      the parenthesis
270.          (1.0)
271. T:       let's go. Six times four?
272. PP:      TWENTY FOUR=
```

```
273.  T:    =four- (1.0) to twen- (.) >six times four twenty four<
274.        goes?
275.  PP:   two=
276.  T:    =how many is six times nothing?
277.  PP:   nothing=
278.  T:    =I put?
279.  PP:   two
280.        (0.6)
281.  T:    six times three?
282.  P:    ei[gh-
283.  PP:      [eighteen
284.  T:    I found here the dividen- the quot- the dividend?
285.  PP:   yeah::
286.  T:    I found the dividend that is of?
287.        (0.8)
288.  PP:   ONE THOUSAND EIGHT HUNDRED AND TWENTY FOUR=
289.  T:    <one thousand eight hundred and twenty four>
```

Extract (1) 3rd grade, EE, 3 (original)

```
P:    olhe aqui na na letra a (0.8) nós estamos vendo
      (.) é:: (0.4) quanto é o- o dividendo?
      (0.4)
AA:   noventa e ↑do[is
P:                 [noventa e dois ((limpa a garganta))
      noventa e dois que vai ser dividido ↑por=
AA:   =quatro
P:    vamos ver quando é que dá pra- pra eu vou t-
      vou (.) vou dizer que: >essa sala tem noventa e
      dois alunos.< (0.4) eu vou dividir essa sala ↑em
      (.) quatro grupos. a gente vai vai descobrir
      agora quantos alunos vai ficar em cada grupo. Eu
      posso dividir no:ve por quatro?
AA:   po::de
P:    eu tenho nove quero dividir por quatro vai dar=
AA:   do::is
P:    ↑por que dois?
AA:   porque duas vez quatro é oi[to
A:                               [é oito
P:    duas vezes quatro são=
AA:   [=oito
P:    [oito vai dar dois (.) duas vezes quatro=
AA:   oito.
```

```
P:      ↑mas eu tenho nove (.) pra chegar em nove inda
        falta quanto?
AA:     UM:::=
P:      falta um (0.8) agora que que eu- vou fazer?
AA:     [pegar o dois
A:      [abaixar o dois
P:      hã (.) o dois. vocês observem que aqui na
        divisão (.) eu comecei a dividir ↑pela dezena
        >não foi isso?<
A:      °fo[i°
A:         [foi
P:      agora que eu vou dividir a=
AA:     =uni[dade
P:          [dade (.) ao invés de noventa e dois agora eu
        ↑tenho=
AA:     =do::ze
P:      pra divi↑dir por quatro quanto será que vai dar?
AA:     TRÊ:::S
P:      por↑quê?
AA:     PORQUE TRÊS VEZ QUATRO SÃO DOZE=
P:      =porque três vezes quatro são: doze (1.0) é:: eu
        sei (       ) nós temos certeza que essa divisão
        está correta não está?
A:      tá=
P:      =nós efetuamos corretamente não foi?
AA:     °foi°
P:      mas (.) ↓para ter mais certeza eu vou tirar a
        prova real (.) a prova real da multiplicação- da
        da: divisão que é feita com a=
A:      °multipli[cação°
P:               [multiplicação (.) não é isso? (.) então
        pra eu fazer essa multiplicação que é que eu vou
        fazer? eu vou começar como?
        (1.0)
A:      (com a tabuada?)
        (1.4)
A2:     multiplicando o resulta[do=
A:                            [multi[plica o divi-
A2:                                 [=por quatro
A:      (o quatro o três e o dividendo)
P:      como é?
        (0.8)
A:      multiplica quatro vezes o dividendo
P:      então eu multi↑plico quatro eu vou colocar o:
        divisor: vou multiplicar quatro >que é o divisor<
```

```
            por: vinte e três. ou seja eu vou multiplicar o
            divisor pelo=
AA:     quocie[n:te
P:              [quociente. e vou encontrar ↑o=
AA:     RES:TO=
P:      resto?
AA:     divi↑d[en:do
P:              [dividendo eu vou encontrar ↑o dividendo
            aqui colocando o quatro eu vou- é: agora realizar
            ↑uma multiplicação. (0.8) quatro ve- >agora ó<
            ↑se eu não encontrar↓ vocês disseram que eu vou
            multiplicar e vou encontrar o dividendo (.) e se
            eu não encontrar o dividendo?
AA:     a conta tá errada
P:      a conta está errada. então eu faço quatro vezes
            três?=
AA:     DO::ZE
P:      vai?=
AA:     UM::
P:      quatro vezes dois?=
AA:     OI::TO
P:      mais um?=
AA:     NO::VE
P:      então ela está certa ou errada?
AA:     CER::TA
            (0.4)
P:      vamos agora >preste atenção< aqui na letra a nós
            só fizemos é: unidade e dezena (.) não é isso? Na
            letra b nós temos uni↑dade dezena e centena↓ (.)
            então eu vou começar a efetuar essa divisão
            pelas- pela unidade >não é isso<?
AA:     °não°
A:      não tia a centena=
P:      =eu vou começar pela centena (.) agora eu tenho
            quatro eu posso dividir quatro por quatro?
AA:     PO::DE
P:      se eu tiver (1.0) quatro lápis ( ) pra dividir
            com quatros alunos aqui na sala eu vou dar
            quantos lápis pra cada um?
AA:     UM::
P:      eu vou dar um. (              ) tem quatro vou dividir
            quatro por quatro que vai dar?
AA:     um:
P:      uma vez quatro?
AA:     qua:tro=
```

```
P:    =para quatro?

AA:   na:da

P:    eu vou baixar agora=

AA:   =um::

P:    ou seja eu baixei agora a dezena não foi isso?

A:    foi

P:    agora só tem um (.) pra dividir por quatro

A:    abaixa [o zero

A:           [ab[aixa o zero

AA:            [ABAIXA O ZERO

P:    olhe presta atenção aqui nesse ca:so (.) quando
      aqui nós temos um numero que não possa ser
      dividido pelo divisor (.) que é me↑nor que o
      divisor então o que é que eu vou fazer eu vou
      colocar zero no quociente <zero no quociente> pra
      poder poder baixar ↑o (.) número seguinte que ↓é
      é sete (.) agora quanto é que eu tenho?

AA:   dezesse:te

P:    agora eu tenho dezessete pra dividir ↑por=

AA:   qua:tro

P:    quanto será que vai dar aqui?

A:    qua:[tro

A:        [qu[a:tro

AA:         [qua:[tro

A:               [quatro

P:    quatro? então (.) quatro vezes quatro?

AA:   DEZESSEIS::

P:    para dezessete?

AA:   UM:::

P:    (falta) um (1.6) nessa divisão quem é o:::
      quociente?
      (0.4)

A:    >cento [e quatro<

AA:          [cento e qua:tro

P:    cento e quatro e o que é quociente?

A:    o re[sultado da divisã::o

AA:       [é o resulta::do=

P:    =é resultado ↑da=

AA:   =divisã::o

P:    não é isso? essa- divisão é (.) exata ou inexata?

AA:   inexa:ta

P:    ela é inexata porquê?
      (0.2)

A:    porque sobrou [um

AA:                 [porque sobrou um:
```

```
       (19.0)
P:     ela é inexata porque ela dei↑xou (.) resto. de quanto
       é o resto?
AA:    °um::°
P:     de um. agora eu <vou ti rar a prova> (.) colocando o
       divisor não é isso?
       (0.6)
P:     o divisor >se eu tirei o divisor daqui vou colocar
       aqui< pra tirar a prova real dessa divisão (1.0)
       agora (.) esse divisor tá sendo o quê agora?
A:     °multiplica( )°=
A:     =multiplicado
A:     >multiplicador<
A:     multiplicador::=
P:     =multiplicador agora ele passou a ser ↑o-
       multiplicador não é que eu não vou trabalhar agora
       com a multiplicação? (.) ele passou a ser
       multiplicador (.) e o quociente agora passou a ↑ser=
AA:    multiplican::[do
P:                  [multiplicando (.) então agora tem
       quatro vezes quatro?
AA:     dezess[ei::s
P:            [seis vai?
AA:    um::
P:     quatro vezes nada?
AA:    na:da
P:     aí eu baixo?
AA:    um::
       (0.8)
P:     quatro vezes um?
AA:    qua::tro
P:     eu encontrei- o- (0.8) divisor?
AA:    NÃ::O
P:     não eu ↑não encontrei o divisor ainda (.) aliás o- o-
       o- dividendo eu ainda não encontrei o dividendo por
       quê?
A:     t↑ia porque você ainda não somou o=
A:     =porque a senhora não somou [ainda
P:                                 [porque tá faltando eu
       somar com ↑o- resto (.) aqui eu vou utilizar agora
       ↑uma adição (1.2) vou colocar ↑o resto (.) seis mais
       um?
AA:    SE::te
P:     sete aqui eu baixo=
AA:    um::
```

```
P:    quatro (.) vou baixar [quatro] né?
AA:                         [quatro]
P:    dá pra vocês entender dá gente?
AA:   dá:
      (0.6)
P:    que e- essa esses >quatrocentos e dezessete não faz
      desse (quinze) não< viu?
A:    °é°
      (0.4)
P:    então agora eu encontrei ↑o (.) dividendo (.) não é
      isso?
      (0.8)
P:    agora nós temos a letra c (.) que eu tenho unidade
      dezena cen↑tena=
A:    =e unida[de de milhar
A2:           [e unidade de mi[lhar
P:                            [unidade de milhar (.) <agora
      eu tenho até a unidade de milhar> sendo que ali é vai
      ser dividido ↑por (.) [seis
A:                          [seis
P:    aqui nós temos até a dezena multip- é divi↑dido (.)
      por (milhar) (.) aqui nós temos até a centena
      dividido ↑por=
AA:   quatro=
P:    unidade né? e aqui também por unidade. ((para um
      aluno)) ó lá na frente tu vai- encontrar (     )
      (.) dividido por dezena
      (7.0)
P:    mas aqui eu tenho mil oitocentos e vinte e quatro pra
      dividir por seis (1.0) se eu tenho mil oitocentos
      e vinte e quatro pra dividir por seis (.) eu vou ter
      que dividir um por seis não é?
A:    nã[o::
A:      [não:
AA:   (               )=
P:    =ah:: eu vou dividir dezoito eu vou pegar dezoito pra
      dividir ↑por=
AA:   =[seis
P:     [seis. quanto será que dá?
AA:   TRÊ::S
P:    por quê?
AA:   PORQUE TRÊS VEZ SEIS É DEZOI::TO=
P:    =porque três vezes seis são dezoito aqui vai dar três
      três vezes seis dezoito ↑para dezoi↓to=
AA:   NA:da
```

```
P:    nada (.) e agora eu ↑baixo=
AA:   DO::IS
P:    pronto. (1.0) agora eu só tenho dois pra dividir por
      seis eu [não vou poder
A:            [baix- baixa=
AA:   ((muitos falam ao mesmo tempo))=
P:    =aqui eu coloco zero no quociente e baixo=
AA:   o qua:tro
      (0.8)
P:    agora deu quanto?
AA:   vinte e qua:tro=
P:    =vinte e quatro dividido por seis?
A:    qua[tro
AA:      [quatro
P:    quatro por quê?
AA:   PORQUE QUATRO VEZ SEIS É VINTE E QUATRO
P:    quatro vezes seis são vinte e quatro aqui eu coloco o
      quatro quatro vezes seis >são vinte e quatro< para
      vinte e quatro?
AA:   na::da
P:    agora eu vou tirar=
A:    pro[va
A2:      [prova
A3:   (      ) os parêntese [tia
A4:                         [os parêntese tia
A5:   os parêntese
      (1.0)
P:    vamo lá. seis vezes quatro?
AA:   VINTE E QUA:TRO=
P:    =quatro- (1.0) para vi- (.) >seis vezes quatro vinte
      e quatro< vão?
AA:   DOIS=
P:    =quanto é seis vezes nada?
AA:   NA:DA=
P:    =eu coloco?
AA:   do:is
      (0.6)
P:    seis vezes três?
A:    oi[t-
AA:     [DEZOITO
P:    eu encontrei aqui o dividen- o quoci- o- dividendo?
AA:   ENCONTROU::
P:    encontrei o dividendo que é de?
      (0.8)
AA:   MIL OITOCENTOS E VINTE E QUATRO=
```

```
P:      <mil oitocentos e [vinte e quatro>
```

Extract (2) 4th grade, EF, 2

```
P:      pelo que eu li nas respostas de vocês seria (  ) é bem
        diferente. sem ser Pedro que já  falou (      ) de uma vez
        outra coisa pessoa quer dizer qual foi qual foi a (  ) que deu
        os (três)?
A:      (levanta a mão)
```

Extract (3) 3rd grade, EE, 1

```
P:          é:: você. me diga aí quanto é que dá dois mais  três
            (1.0)
A(1):       seis
            (2.0)
P:          (      ) quanto é que dá dois mais três?
A:          °cinco°
P:          °cinco° (.) e:: três mais dois?
A:          cinco
P:          cinco
            (1.0)
P:          quanto é- agora diga agora quanto é que dá dois
            mais três
A(1):       °cinco°
P:          °cinco° (.) e três mais dois?
A:          °cinco°
P:          ah sim agora aprendeu (.) agora você que riu quanto
            é que dá é: dois mais cinco?
A:          (      )=
P:          = e: cinco mais dois?
A:          sete
            (3.0)
```

Extract (4), 1st grade, ER, 1:

```
P:      Renan ganhou oito bolas de presente do pai dele (.) só que
        Samuel chegou lá e os dois foram brincar, e sabe o que
        aconteceu? Duas bolas estouraram.
```

```
A:        Sete, sete, sete...
P:        Peraí (.) ele ganhou quantas?
A:        Cinco.
P:        Não, quantas bolas você ganhou do seu pai?    (dirigindo-se a
          Renan)
Renan:    Oito.
P:        Oito  bolas (.) Samuel chegou lá e estourou duas sem
          querer.
AA:       Seis!
P:        Quantas ficou?
AA:       Seis.
P:        Seis bolas (.) muito bem.
```

Extract (5), 1st grade, ER, 1:

```
P:        ((lendo)) A titia pediu para cada criança trazer um
          ingrediente para o G4 fazer pão da mamãe (.) Hélio trouxe
          quantos ovos?
AA:       Seis.
P:        Seis ovos (.) faz de conta que quebraram três (.) Quantos
          ovos ficaram?
A:        Três.
P:        Vai Yuri (.) quantos ovos ficaram?
AA:       Três.
Hélio:    Não ficou três!
A:        Ficou.
Hélio:    Ficou não! Eu trouxe mais um e aí ficou seis!
P:        Não, amor (.) faz de conta que quebraram três.
A:        É (.) ficou três ovos.
Hélio:    Quebrou não!
P:        Helinho (.) nesta tarefa, na realidade o que aconteceu é
          que Helinho trouxe seis (.) quebrou um só né? Mas aqui na
          tarefa a gente tá fazendo de conta que quebrou três (.)
          Quantos ovos ficaram?
Hélio:    Três.
```

Extract (6), pre-school, EL, 1 (D: Daniel; M: Mateus):

```
P:        Meninos aí (.) Daniel, quantas dezenas tem o número
          quinze?
A:        Cinco.
```

A2:	Dez.
Daniel:	Cinco.
P:	Antes de responder eu quero que vocês pensem um pouquinho (.) uma dezena vale quanto? (.) Quantas unidades tem uma dezena?
Mateus:	Uma dezena tem duas unidades.
P:	Mateus, quantas dezenas- quantas unidades- quantos feijões eu preciso pra trocar por um milho? de quantos feijões eu preciso?
Mateus:	Dez.
P:	Então uma dezena tem quantas unidades?
AA:	Dez.

Extract (7) Teacher and Gilvan, pre-school, EL, 1:

A:	um dois três quatro cinco seis sete oito nove dez=
P:	Epa! ((ergue a mão em sinal de pare)) (.) dez você vai trocar por um milhinho ou não?
A:	((prontamente pega um milho)) Vou.
P:	Pra onde é que vai os dez?
A:	O dez vai pra- (.) aqui. ((pega a caixa das UNIDADES))
P:	Esse dez você tá fazendo a troca Gilvan!/ você se lembra que essas dez unidades coloca pra cá ((junto dos outros feijões)) pra trocar por um milho que vale dez feijõezinhos (.) pra onde vai essa dezena?
A:	((pega a caixa das DEZENAS e guarda o grão de milho))
P:	Vê quantos sobraram aí
A:	((conta, apontando os grãos)) NOVE, DEZ
P:	e agora (.) o que é que você vai fazer?
A:	Agora aqui ((Gilvan leva os feijões restantes até a caixa UNIDADES))
P:	É?
A:	((leva os dez feijões ao outro montante que está em cima da mesa))
P:	Quanto vale um milhinho?
A:	Dez
P:	Aqui tem dez feijões (.) vai trocar ou não pelo milhinho?
A:	Vou
P:	Não tenha medo não (.) vá (.) faça o que você acha que tem que fazer.
A:	((põe os dez feijões na caixa UNIDADES))
P:	Você tem aqui dentro quantas unidades?
A:	Dez

P: Você tem aqui dez unidades (.) pode ou não trocar?

A: Não. ((balança a cabeça negativamente)).

P: Quantas unidades você tem aqui? ((abre a caixa UNIDADES))

A: Dez

P: Uma dezena vale quantas unidades?

A: Dez.

P: Você está me dizendo que um milhinho vale dez feijões/
        você me disse que uma dezena tem dez unidades/ você
        pode trocar esses dez feijões por esse milho que vale dez?

A: Pode.

P: Então troque (.) como é que você troca?

A: ((põe os dez feijões que estavam na caixa "unidades" juntos
        com o restante))

P: Pra onde é que vai o milho?

A: ((abre a caixa UNIDADES))

P: Leia o nome da caixa!

A: Unidade.

P: Esse milho é o quê? (.) uma unidade ou uma dezena?

A: Dezena.

P: E pra que caixa ele vai?

A: ((pega a caixa DEZENAS))

P: Gilvan, o milho vale dezena, vale dez (.) o feijãozinho é a
        unidade (.) cada feijãozinho só vale um, mas um milho
        desses vale dez, dez feijões.

Extract (8) pre-school, EL, 1:

P: Quantas dezenas tem o numeral onze? (dirigindo-se a
            Gilvan).

Diego: Dez dezenas.

P: Abra a caixa (.) olhe quantos milhos tem aí dentro.

Gilvan: Um.

P: No número onze existe uma dezena (.) somente uma (.) e
esse   milho aí vale quanto? Vale quantos feijões?

AA: Dez.

P: Dez (.) quantas unidades tem no número onze? (.) abre a
            caixinha da unidade e olha (.) não responde sem pensar.

Diego: Aqui tem dez dezenas.

P: Quantas unidades tem? (.) quantas unidades tem no
            número onze?

Mateus: Um.

P: (desenha no quadro um quadrado com o número 11 escrito
            dentro) O número onze tem uma dezena e uma unidade.

Extract (9) pre-school, EL, 1:

P: (...) agora vamos ver (.) de quem são esses lápis?
AA: Marília.
P: Marília (.) aqui é os lápis de cêra de Marília (.) vamos
            ver quanto contém.
Marília: Tá quebrado tia.
P: Hã? Bora ver (.) vamos contar? (.) um dois três quatro
            cinco seis sete oito nove dez onze doze (.) vamos ver se
            quebrou algum de Marília.
AA: Quebrou.
P: Quebraram quantos gente? (.) Quantos quebraram.
AA: Três.
P: Três quebraram (.) são doze lápis, três quebraram (.)
            quantos que ficaram? (.) Tentar fazer na cabecinha.

Extract (10) Teacher and Samuel, pre-school, EL, 1:

P: Como você fez? (.) como você achou que tinha doze? como
            você fez isso?
Samuel: ((conta os elementos desenhados no papel))
P: Mas como você fez (.) como é que- tanto ovo? (.) como foi
            que você descobriu? (.) cadê os ovos de Naia?
Samuel: ((conta no papel))
P: então você pegou os ovos de Naia juntou com os de Hélio e
            deu=
Samuel: =Doze.
P: ah::

Extract (11) Teacher and Marília, pre-school, EL, 1:

P: Fala Marília como você encontrou doze?
Marília: ((tinha o numeral "12" escrito em sua folha de papel.
            Conta nos dedos para responder à professora))
P: Como você conseguiu (.) como você fez isso se você colocou
            na mão (.) você disse que era doze e na mão tem quantos
            dedos?
Marília: Dez.
P: E como foi?
Marília: Eu botei dez mas achei que não era dez e botei doze.

P: Mas como você achou doze, hein Marília? (.) onde estão os
            de Naia? (.) Marília, você copiou do colega (.) você não
            está entendendo? (.) agente tem que entender, viu?

Extract (12) Teacher and Renan, pre-school, EL, 1:

P: Como você pensou? (.) como você achou doze?

Renan: Eu pensei (.) eu pensei já doze.

P: Não, pensou não (.) não pode pensar já doze (.) como foi
            que você achou? (.) cadê os ovos de Naia?

Renan: ((Renan mostra desenhos no papel))

P: ah:::: então você pegou os ovos de Naia e juntou com os de
            Hélio?

Renan: ((acena afirmativamente))

P: Era isso que titia queria saber (.) como você pensou

References

Althusser, L. (1971) Ideology and Ideological State Apparatuses (Notes towards an investigation). In *Lenin and Philosophy and Other Essays*. London: New Left Books

Amerine, R., & Bilmes, J. (1990). Following instructions. In M. Lynch & S. Woolgar (Eds.) *Representation in scientific practice*. Cambridge, MA: MIT Press.

Antaki, C. (2002) Personalized revision of 'failed' questions. *Discourse Studies* 4 (4), 411-428.

Aries, P. (1962). *Centuries of childhood: A social history of family*. New York: Alfred A. Knopf.

Ashmore, M. (1989). *The reflexive thesis: Wrighting sociology of scientific knowledge*. Chicago: University of Chicago Press.

Ashmore, M. and Reed, D. (2000) Innocence and Nostalgia in Conversation Analysis: The Dynamic Relations of Tape and Transcript.' *Forum Qualitative Sozialforschung / Forum: Qualitative Social Research* [Online Journal], 1 (3). Available at: http://qualitative-research.net/fqs-texte/3-00/3-00ashmorereed-e.htm

Atkinson, J.M. (1984b). Public speaking and audience responses: Some techniques for inviting applause. In J.M. Atkinson & J.C. Heritage (Eds.), *Structures of social action: Studies in conversation analysis*. Cambridge: Cambridge University Press.

Atkinson, R. and Delamont, S. (1977) Mock-ups and cock-ups: the stage management of guided discovery instruction. In P. Woods and M. Hammersley (Eds.) *School Experience: explorations in the sociology of education*. London: Croom Helm.

Bachelard, G. (1973) *Le Nouvel Esprit Scientifique*. Paris: Presses Universitaires de France.

Baker, C. and Freebody, P. (1989) *Children's First School Books: Introduction to the culture of literacy*. Oxford: Basil Blackwell

Bernstein, B. (1971). *Class, codes and control.* Volume 1. London: Routledge & Kegan Paul.

Billig, M. (1999) Whose terms? Whose ordinariness? Rhetoric and ideology in conversation analysis, *Discourse & Society* 10, 543-58

Bloor, D. (1976). *Knowledge and Social Imagery.* London: Routledge & Kegan Paul.

Bloor, D. (1983). *Wittgenstein: A social theory of knowledge.* New York: Columbia University Press.

Bloor, D. (1987) The living foundations of mathematics. *Social Studies of Science* 17, 337-358.

Bloor, D. (1992) Left and right Wittgensteinians. In A. Pickering (Ed.) *Science as Practice and Culture.* Chicago: University of Chicago Press.

Bowen, J. (1967) Introduction to J. A. Comenius' *Orbis Sensualis Pictus: Facsimile of the Third London Edition 1672.* Sydney: Sydney University Press.

Brown, J., Collins, A. and Duguid, P. (1989) Situated Cognition and the Culture of Learning. *Educational Researcher.* 18 (1), 32-42.

Brown, S. and Middleton, D. (2001) Performing the past in electronic archives: interdependencies in the discursive and non-discursive ordering of institutional rememberings. *Culture & Psychology* 7 (2), 123-144.

Bruner, J.S. (1971). *The relevance of education.* Harmondsworth: Penguin.

Bruner, J.S. (1983a). *Child's talk.* Oxford: Oxford University Press.

Bruner, J.S. (1985). Vygotsky: a historical and conceptual perspective. In J.V. Wertsch (Ed.) *Culture, communication and cognition: Vygotskian perspectives.* Cambridge: Cambridge University Press.

Bruner, J.S. (1990). *Acts of meaning.* Cambridge, MA: Harvard University Press.

Bryant, P. (1974) *Perception and Understanding in Young Children: an experimental approach.* New York: Basic Books.

Bunge, M. (2000) Philosophy from the outside. *Philosophy of the Social Sciences* 30 (2), 227-245.

Burns, S. (1997). Practicing law: A study of pedagogic interchange in a law school classroom. In M. Travers & J.F. Manzo (Eds.), *Law in action: Ethnomethodological and conversation analytic approaches to law*. Aldershot: Ashgate Publishing.

Button, G. (1993). *Technology in working order: Studies of work, interaction, and technology*. London: Routledge.

Callon, M. (1986). The sociology of an actor-network: The case of the electric vehicle. In M. Callon, J. Law, & A. Rip (Eds.) *Mapping the dynamics of science and technology*. London: MacMillan.

Candela, A. (1995) Classroom science: rhetoric and discourse of the pupils. Unpublished doctoral dissertation, Loughborough University.

Candela, A. (1997) The discursive construction of argumentative contexts in science education. In C. Coll and D. Edwads (Eds.) *Teaching, Learning and Classroom Discourse*. Madrid: Fundacio Infancia y Aprendizage.

Carraher, T., Carraher, D., & Schliemann, A. (1985). Mathematics in the streets and in schools. *British Journal of Developmental Psychology* 3, 21-29.

Carraher, T. Carraher, D. e Schliemann, A. (1988) *Na Vida Dez, Na Escola Zero*. São Paulo: Cortez.

Chaiklin, S. e Lave, J. (Eds.) (1993) *Understanding Practice: perspectives on activity and context*. Cambridge: Cambridge University Press

Chomsky, N. (1959). A review of 'Verbal Behaviour' by B.F. Skinner. *Language* 35, 26-58.

Cicourel, A. (1964) *Method and Measurement in Sociology*. New York: The Free Press of Glencoe.

Cobb, P. (1999) Where is the mind? In P. Murphy (Ed.) *Learners, Learning and Assessment*. London: Paul Chapman Publishing.

Cobb, P. Gravemeijer, K., Yackel, E., Mc Clain, K., and Whitenack, J. (1995) Mathematizing and Symbolizing: the emergence of chains of

signification in the first-grade classroom. In D. Kirshner e J. A. Whitson (Eds.) *Situated Cognition Theory: social, semiotic, and neurological perspectives*. Hillsdale, NJ: Erlbaum.

Cole, M., Gay, J., Glick, J., & Sharp, D.W. (1971). *The cultural context of learning and thinking*. New York: Basic Books.

Collins, H.M. (1985). *Changing order: Replication and induction in scientific practice*. London: Sage.

Comenius, J. A. (1967) *Orbis Sensualis Pictus: Facsimile of the Third London Edition 1672*. Sydney: Sydney University Press.

Coulter, J. (1999). Discourse and mind. *Human Studies* 22, 163-181.

Coulthard, M. (1977). *An introduction to discourse analysis*. London: Longman

Danby, S. and Baker, C. D. (2000) Unravelling the Fabric of Social Order in Block Area. In Hester, S. and Francis, D. (Eds.) *Local Education Order: Ethnomethodological studies of knowledge in action*. Amsterdam: John Benjamins.

De Beaugrande, R. (2003) Cognition and technology in education: Knowledge and information – language and discourse. Available at: http://beaugrande.bizland.com/CognitionTechnologyEducation.htm

de Saussure, F. (1974). *Course in general linguistics*. London: Fontana.

Devlin, K. (1998) *The Language of Mathematics: Making the Invisible Visible*. New York: W. H. Freeman and Company.

Dienes, Z. P. (1960) *Building Up Mathematics*. London: Hutchinson.

Drew, P. (1984). Speakers' reportings in invitation sequences. In J.M. Atkinson and J.C. Heritage (Eds.), *Structures of social action: Studies in conversation analysis*. Cambridge: Cambridge University Press.

Drew, P., & Heritage, J.C. (Eds.), (1992). *Talk at work: Interaction in institutional settings*. Cambridge: University of Cambridge Press.

Durkin, K. (1993) The representation of number in infancy and early childhood. In C. Pratt e A. Garton (Eds.) *Systems of Representation in Children: development and use*. New York: John Wiley & Sons.

Durkin, K. e Shire, B. (1991) Lexical ambiguities in mathematical contexts. In: K. Durkin e B. Shire (Eds.) *Language in Mathematical Education. Research and Practice*, Milton Keynes and Philadelphia: Open University Press.

Eco, U. (1974) *As Formas do Conteúdo*. São Paulo: Perspectiva/Edusp.

Eco, U. (1991) *Semiótica e Filosofia da Linguagem*. São Paulo: Ática.

Edwards, D. (1993). But what do children really think? Discourse analysis and conceptual content in children's talk. *Cognition and Instruction* 11 (3/4), 207-225.

Edwards, D. (1995a) Sacks and psychology. *Theory and Psychology* 5 (3), 579-597.

Edwards, D. (1995) A commentary on discursive and cultural psychology, *Culture & Psychology* 1 (1), 55-65.

Edwards, D. (1997). *Discourse and cognition*. London: Sage.

Edwards, D., & Mercer, N.M. (1987). *Common knowledge: The development of understanding in the classroom*. London: Routledge.

Edwards, D., & Potter, J. (1992). *Discursive psychology*. London: Sage.

Edwards, D., Ashmore, M., & Potter, J. (1995). Death and furniture: The rhetoric, politics, and theology of bottom line arguments against relativism. *History of the Human Sciences* 8 (2), 25-49.

Eglin, P. (1980) Culture as method: location as an interactional device. *Journal of Pragmatics* 4, 121-35

Fayol, M. (1996) *A Criança e o Número*. Porto Alegre: Artes Médicas.

Foucault, M. (1972). *The archaeology of knowledge.* London: Tavistock/Routledge.

Foucault, M. (1995) *As Palavras e as Coisas.* São Paulo: Martins Fontes.

Foucault, M. (1977) *Discipline and Punish: The birth of the prison.* London: Allen Lane.

Freudenthal, H. (1973) *Mathematics as an Educational* Task. Dordrecht: Reidel.

Garcia, R. (1996) Criar para compreender: a concepção piagetiana do conhecimento. In: E. Martí (Ed.) *Cem Anos com Piaget: Temas Fundamentais em Psicologia e Educação.* São Paulo: Artes Médicas.

Garfinkel, H. (1967). *Studies in ethnomethodology.* Englewood Cliffs, NJ: Prentice-Hall.

Garfinkel, H., & Sacks, H. (1990). On formal structures of practical actions. In J. Coulter (Ed.), *Ethnomethodological sociology.* Aldershot, UK: Edward Elgar.

Garfinkel. H., Lynch, M., & Livingston, E. (1981). The work of a discovering science construed with material from the optically discovered pulsar. *Philosophy of the Social Sciences* 11, 131-158.

Garfinkel, H., & Wieder, D.L. (1991). Two incommensurable, asymmetrically alternate technologies of social analysis. In G. Watson & R. Seiler (Eds.), *Text in context: Contributions to ethnomethodology.* London and Beverly Hills, CA: Sage.

Geertz, C. (1973). *The interpretation of cultures.* New York: Basic Books.

Gelman, R., & Gallistel, C.R. (1978). *The child's understanding of number.* Cambridge, MA: Harvard University Press.

Gephart, R.P. (1988). *Ethnostatistics: Qualitative foundations for quantitative research.* London and Beverly Hills, CA: Sage.

Gherardi, S. (2000) Practice-based theorizing on learning and knowing in organizations. *Organization* 7(2), 211-223.

Gibson, J.J. (1979). *The ecological approach to visual perception.* Boston, Mass.: Houghton Mifflin.

254

Giddens, A. (1984). *The constitution of society: Outline of the theory of structuration*. Cambridge: Polity.

Gilbert, G.N., & Mulkay, M. (1984). *Opening Pandora's box: A sociological analysis of scientists' discourse*. Cambridge: Cambridge University Press.

Goffman, E. (1959). *The presentation of self in everyday life*. Garden City, NY: Doubleday.

Goodwin, C. (1995). Seeing in depth. *Social Studies of Science* 25, 237-274.

Goodwin, C. (1996). Transparent vision. In E.Ochs, E.A. Schegloff, & S.A. Thompson (Eds.), *Interaction and grammar*. Cambridge: Cambridge University Press.

Goodwin, C. (1998) Action and embodiment within situated human interaction. UCLA, Mimeo.

Goody, J. (Ed.) (1978) *Literacy in Traditional Societies*. London: Cambridge University Press.

Gould, S. J. (1998) *Questioning the Millennium: A rationalist's guide to a precisely arbitrary countdown*. London: Vintage.

Grint, R. and Woolgar, S. (1997) *The Machine at Work*. Cambridge: Polity.

Hacking, I. (1992) The self-vindication of the laboratory sciences. In A. Pickering (Ed.), *Science as practice and culture*. Chicago: University of Chicago Press.

Hall, R. and Stevens, R. (1995) Making space: a comparison of mathematical work in school and professional design practices. In Star, S. L. (Ed.) *The Cultures of Computing*. Oxford: Blackwell.

Hamilton, D. (1980) Adam Smith and the moral economy of the classroom system. *Journal of Curriculum Studies* 12 (4), 281-298.

Hammersley, M. (1990) *Classroom Ethnography: Empirical and methodological essays*. Buckingham: Open University press.

Havelock, E. (1963). *Preface to Plato*. Cambridge MA: Harvard University Press.

Heap, J. (2000) The Boundaries of Writing: Paying Attention to the Local Educational Order. In Hester, S. and Francis, D. (Eds.) *Local Education Order: Ethnomethodological studies of knowledge in action*. Amsterdam: John Benjamins.

Heath, C. and Button, G. (2002) Editorial Introduction. *British Journal of Sociology* 53 (2), 157-161.

Heath, C., Hindmarsh, J. and Luff, P. (1999) Interaction in isolation: The dislocated world of the London Underground train driver. *Sociology* 33 (3), 555-575.

Heath, C., & Luff, P. (1993). Disembodied conduct: Interactional asymmetries in video-mediated communication. In G. Button (Ed.), *Technology in working order*. London: Routledge.

Heath, C. and Luff, P. (2000) *Technology in Action*. Cambridge: Cambridge University Press

Heidegger, M. (1978) Modern Science, Metaphysics and Mathematics. In D. F. Krell (Ed.) *Martin Heidegger: Basic Writings*. London and Henley: Routledge & Kegan Paul.

Hemmings, T., Randall, D., Marr, L. and Francis, D. (2000) Task, Talk and Closure: Situated Learning and the Use of an 'Interactive' Museum Artefact. In Hester, S. and Francis, D. (Eds.) *Local Education Order: Ethnomethodological studies of knowledge in action*. Amsterdam: John Benjamins.

Henderson, K. (1995) The visual culture of engineers. In Star, S. L. (Ed.) *The Cultures of Computing*. Oxford: Blackwell.

Henriques, J., Hollway, W., Urwin, C., Venn, C., & Walkerdine, V. (1984). *Changing the subject: Psychology, social regulation and subjectivity*. London: Methuen.

Heritage, J. (1984) *Garfinkel and Ethnomethodology*. Cambridge: Polity Press

Heritage, J.C., & Watson, D.R. (1979). Formulations as conversational objects. In G. Psathas (Ed.), *Everyday language: Studies in ethnomethodology*. New York: Irvington.

Hester, S. & Francis, D. (Eds.) (2000). *Local educational order: Ethnomethodological studies of knowledge in action*. Amsterdam: John Benjamins.

Hill, C. (1969) *Society and Puritanism in Pre-Revolutionary England*. London: Panther Books.

Hopper, R. (1993). *Telephone conversation*. Bloomington, IA: Indiana University Press.

Hughes, M. (1985) *Children and Number: difficulties in learning mathematics*. Oxford: Blackwell.

Hutchby, I. (2002) Resisting the incitement to talk in child counselling: aspects of the utterance I don't know. *Discourse Studies* 4(2), 147-168.

Hutchby, I. (2001). *Conversation and technology: From the telephone to the internet*. Cambridge, UK: Polity.

Hutchby, I., & Wooffitt, R. (1998). *Conversation analysis: Principles, practices and applications*. Oxford: Polity.

Hutchby, I. & Moran-Ellis, J. (Eds.) (1998) *Children and Social Competence: Arenas of action*. London, Washington, D.C.: The Falmer Press

Hutchins, E. (1991). The social organization of distributed cognition. In L.B. Resnick, J.M. Levine, & S.D. Teasley (Eds.), *Perspectives on socially shared cognition* (pp. 283-307). Washington DC: American Psychological Association.

Jakobson, R. (1971). *Selected writings*. The Hague: Mouton.

Kamii, C. (1986) *A Criança e o Número*. Campinas: Papirus.

Kaplan, A. (1964) *The Conduct of Inquiry: Methodology for Behavioural Science*. London: Chandler.

Kuhn, T. S. (1970). *The Structure of Scientific Revolutions*. Chicago: University of Chicago Press.

Labov, W. (1970). The logic of nonstandard English. In F. Williams (Ed.) *Language and poverty*. Chicago: Markham

Lacan, J. (1986) *O Seminário: livro 1: os escritos técnicos de Freud*, Rio de Janeiro: Zahar.

Lancaster, J. (1805) *Improvements in Education, as It Respects the Industrious Classes of the Community*. First printed in London, 1805; excerpts reproduced by the San Diego Sate University Website. Available at: http://edweb.sdsu.edu/people/Dkitchen/TE655/lancaster_text.htm

Latour, B. (1986). The Powers of Association. In J. Law (Ed.) *Power, Action and Belief: a New Sociology of Knowledge?*. London, Boston and Henley, Routledge and Kegan Paul.

Latour, B. (1987). *Science in Action.* Milton Keynes: Open University Press.

Latour, B. (1988). Mixing humans and nonhumans together: The sociology of a door-closer. *Social Problems* 35 (3), 298-310.

Latour, B. (1988). *The Pasteurization of France.* Cambridge, MA: Harvard University Press.

Latour, B. (1988b) A relativistic account of Einstein's relativity. *Social Studies of Science* 18 (1), 3-44.

Latour, B. (1990). Drawing things together. In M. Lynch & S. Woolgar (Eds.), *Representation in scientific practice.* Cambridge, MA: MIT Press

Latour, B. (1991). Technology is society made durable. In J. Law (Ed.), *A sociology of monsters.* London: Routledge.

Latour, B. (1992). Where are the missing masses? The sociology of a few mundane artifacts. In W. Bijker & J. Law (Eds.), *Shaping technology/ building society: Studies in sociotechnical change.* Cambridge, MA: MIT Press.

Latour, B. (1993). *We have never been modern.* Hemel Hempstead, UK: Harvester Wheatsheaf.

Latour, B. (1999) Pandora's Hope: Essays on the reality of science studies. Cambridge, MA: Harvard University Press.

Latour, B. (2002) Morality and technology: the end of the means. *Theory, Culture & Society* 19(5/6), 247-260.

258

Latour, B., & Woolgar, S. (1979). *Laboratory Life: The social construction of scientific facts.* London: Sage.

Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life.* Cambridge: Cambridge University Press.

Lave, J. and Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* Cambridge: Cambridge University Press.

Law, J. (Ed.) (1986) *Power, Action and Belief: a New Sociology of Knowledge?.* London, Boston and Henley, Routledge and Kegan Paul.

Law, J. (1987). Technology and heterogeneous emgineering: The case of Portuguese expansion. In W.E. Bijker, T. Hughes, & T. Pinch (Eds.), *The social construction of technological systems.* Cambridge, MA: MIT Press.

Law, J. (1991). Introduction: Monsters, machines, and sociotechnical relations. In J. Law (Ed.) *A Sociology of Monsters.* London: Routledge.

Law, J. (2000). On the Subject of the Object: Narrative, Technology and Interpellation. *Configurations* 8, 1-29.

Law, J. and Hassard, J. (Eds.), (1999) *Actor Network Theory and After.* Oxford and Keele: Blackwell.

Law, J., & Lodge, P. (1984). *Science for social scientists.* London: Macmillan.

Law, J. and I. Moser (1999). Managing, Subjectivities and Desires. *Concepts and Transformation* 4(3), 249-279.

Lee, B. (1985). Intellectual origins of Vygotsky's semiotic analysis. In J.V. Wertsch (Ed.) *Culture, communication and cognition: Vygotskian perspectives.* Cambridge: Cambridge University Press.

Lemke, J. (1990). *Talking science: Language, learning, and values.* Norwood, NJ: Ablex.

Leontiev, A.N. (1981). The problem of activity in psychology. In J.V. Wertsch (Ed.) *The concept of activity in Soviet psychology.* Armonk, NY: Sharpe, 1981.

Lerman, S. (Ed.) (1994) *Cultural Perspectives on the Mathematics Classroom.* Dordrecht: Kluwer.

Lerman, S. (1996) Intersubjectivity in mathematics learning: a challenge to the radical constructivist paradigm? *Journal for Research in Mathematics Education* 27, 133-150.

Lerman, S. (2001) Cultural, discursive psychology: a sociocultural approach to studying the teaching and learning of mathematics. *Educational Studies in Mathematics 46, 87–113.*

Lerner, G.H. (1993). Collectivities in action: Establishing the relevance of conjoined participation in conversation. *Text* 13 (2), 213-245.

Lerner, G.H. (1995). Turn design and the organization of participation in instructional activities. *Discourse Processes, 19,* 111-131.

Livingston, E. (1986). *The Ethnomethodological foundations of mathematics.* London: Routledge & Kegan Paul.

Livingston, E. (1987). *Making Sense of Ethnomethodology.* London: Routledge.

Livingston, E. (2000) The Availability of Mathematics as an Inspectable Domain of Practice through the Use of Origami. In Hester, S. and Francis, D. (Eds.) *Local Education Order: Ethnomethodological studies of knowledge in action.* Amsterdam: John Benjamins.

Luff, P. and Heath, C. (2002) Broadcast Talk: initiating calls through a computer-mediated technology. *Research on Language and Social Interaction* 35 (3), 337-366.

Luria, A. (1976) *Cognitive Development: Its Cultural and Social Foundations.* Cambridge, MA: Harvard University Press.

Lynch, M. (1985). *Art and artifact in laboratory science: A study of shop work and shop talk in a research laboratory.* London: Routledge & Kegan Paul.

Lynch, M. (1991) Method: measurement ordinary and scientific measurement as ethnomethodological phenomena. In G. Button (Ed.) *Ethnomethodology and the Human Sciences.* Cambridge: Cambridge University Press.

Lynch, M. (1993). *Scientific practice and ordinary action: Ethnomethodology and social studies of science.* Cambridge: Cambridge University Press.

Lynch, M., and Bogen, D. (1994) Harvey Sacks's primitive natural science. *Theory, Culture and Society* 11, 65-104.

Lynch, M. and Jordan, K. (1995) Instructed actions in, of and as molecular biology. *Human Studies* 18, 227-244.

Lynch, M. and Woolgar, S. (Eds.) (1990). *Representation in scientific practice.* Cambridge, MA: MIT Press.

Macbeth, D (1996) The discovery of situated worlds: analytic commitments, or moral orders? *Human Studies* 19, 267-87

Macbeth, D. (2000) Classrooms as installations: Direct instruction in the early grades. In Hester, S. and Francis, D. (Eds.) *Local Education Order: Ethnomethodological studies of knowledge in action.* Amsterdam: John Benjamins.

MacKay, R. (1974) Standardized tests: objective/objectified measures of 'competence'. In A.V. Cicourel, K.H. Jennings, S. H. M. Jennings, K. C. W. Leiter, R. MacKay, H. Mehan, and David R. Roth, *Language Use and School Performance.* New York: Academic Press.

Maier, R. (1996) Knowledge and social interaction. *Culture & Psychology* 2 (3), 333-342.

McHoul, A.W. (1978). The organization of turns at formal talk in the classroom. *Language in Society* 7, 183-213.

McHoul, A. W. (1990). The organization of repair in classroom talk. *Language in Society* 19, 349-377.

McHoul, A. (1996) *Semiotic Investigations: Towards an effective semiotics.* Lincoln: University of Nebraska Press.

McHoul, A. and Watson, D.R. (1984) Two axes for the analysis of "commonsense" and "formal" geographical knowledge in classroom talk. *British Journal of the Sociology of Education* 5, 281-302.

McHoul, A. and Rapley, M. (2001) 'Ghost: Do not forget; this visitation / Is but to whet thy almost blunted purpose — Culture, Psychology and "Being Human"'. *Culture and Psychology* 7 (4), 433–451.

Mehan, H. (1985). The structure of classroom discourse. In T. van Dijk (Ed.) *Handbook of discourse analysis*, Volume 3: *Discourse and dialogue*. London: Academic Press.

Mehan, H. (1986). 'What time is it Denise?': Asking known information questions in classroom discourse. In M. Hammersley (Ed.) *Case Studies in Classroom Research: A Reader*. Milton Keynes: Open University Press.

Meira, L. and Lerman, S. (2001) The zone of proximal development as a symbolic space. Social Science Research Papers, Faculty of Humanities and Social Science, South Bank University, London. Available at: http://www.lsbu.ac.uk/fhss/research/reports/stevelerman.shtml

Mercer, N. (1995) *The Guided Construction of Knowledge: Talk amongst teachers and learners*. Clevedon: Multilingual Matters

Moerman, M. (1988). *Talking culture: Ethnography and conversation analysis*. Philadelphia, PA: University of Pennsylvania Press.

Morgan, C. (2000) Language in Use in Mathematics Classrooms: Developing Approaches to a Research Domain. *Educational Studies in Mathematics* 41, 93–99.

Morrison, K. (1981) Some properties of "Telling Order Designs" in didactic inquiry. *Philosophy of the Social Sciences* 11, 245-62

Newman, D., Griffin, P., & Cole, M. (1989) *The Construction Zone*. Cambridge: Cambridge University Press.

Nunes, T. (1992) Cognitive invariants and cultural variation in mathematical concepts. *International Journal of Behavioral Development* 15 (4), 433-453.

Nunes, T. (2000) Mathematics learning as the socialization of the mind. *Mind, Culture, and Activity* 6 (1), 33-52.

Ochs, E. (1979). Transcription as theory. In E. Ochs & B. Schieffelin (Eds.), *Developmental pragmatics*. New York: Academic Press.

Olson, D. R. (1994) *The World on Paper*. Cambridge & New York: Cambridge University Press

Otte, M. (1991) *O Formal, o Social e o Subjetivo: Uma Introdução à Filosofia e à Didática da Matemática*. São Paulo: Edusp.

Papert, S. (1979) The mathematical unconscious. In J. Wechsler (Ed.) *On Aesthetics in Science*. Cambridge, MA: The MIT Press.

Parametros Curriculares Nacionais – Matemática: Versao Preliminar (National Curricular Parameters for Mathematics) (1995). Ministerio da Educação do Brasil.

Payne. G. and Cuff, E.C. (Eds.) (1982) *Doing teaching: the practical management of classrooms*. London: Batsford Academic and Educational Ltd.

Payne, G. and Hustler, D. (1980) Teaching the class: the practical management of a cohort. *British Journal of Sociology of Education* 1(1), 49-66.

Perret-Clermont, A-N., Perret, J-F., e Bell, N. (1989) The social construction of meaning and cognitive activity in elementary school children. *Paper presented at the Conference on Socially Shared Cognition*, University of Pittsburgh.

Peters, M. (2001) Wittgensteinian pedagogics: Cavell on the figure of the child in the *Investigations*. *Studies in Philosophy and Education* 20, 125-138.

Piaget, J. (1951). *Play, dreams and imitation in childhood*. London: Routledge & Kegan Paul.

Piaget, J. (1974) *The Child and Reality: problems of genetic psychology*. London: Frederick Muller.

Piaget, J. (1982) *Seis Estudos de Psicologia*. Rio de Janeiro: Forense.

Piaget, J. and Garcia, R. (1983) *Psychogenèse et histoire des sciences*. Paris, Flammarion.

Piaget, J. And Szeminska, A. (1971) *A Gênese do Número na Criança*. Rio de Janeiro: Zahar.

Pickering, A. (1992). From science as knowledge to science as practice. In A. Pickering (Ed.) *Science as Practice and Culture*. Chicago: University of Chicago Press.

Pimm, D. (1987) Speaking Mathematically: communication in mathematics classrooms. London: Routledge and Kegan Paul.

Pinxten, R. (1994) Anthropology in the mathematics classroom? In S. Lerman (Ed.) *Cultural Perspectives on the Mathematics Classroom*. Dordrecht: Kluwer.

Popper, K. (1959). *The logic of scientific discovery*. London: Hutchinson.

Rogoff, B., & Wertsch, J.V. (Eds.) (1984). *Children's learning in the 'zone of proximal development'*. San Francisco: Jossey-Bass.

Rommetveit, R. (1985). Language acquisition as increasing linguistic structuring of experience and symbolic behaviour control. In J.V. Wertsch (Ed.) *Culture, communication and cognition: Vygotskian perspectives*. Cambridge: Cambridge University Press.

Rotman, B. (1977) *Jean Piaget: Psychologist of the Real*. Bristol: The Harvester Press.

Rotman, B. (1987) *Signifying Nothing: The semiotics of zero*. New York: St. Martin's Press.

Rotman, B. (1993) The Ghost in Turing's Machine: Taking God Out of Mathematics and Putting the Body Back In. Stanford, CA: Stanford University Press.

Rotman, B. (2000) *Mathematics as Sign: Writing, Imagining, Counting*. Stanford: Stanford University Press.

Russell, B. (2001) *History of Western Philosophy*. London: Routledge.

Sacks, H. (1972) An initial investigation of the usability of conversational data for doing sociology. In: D. Sudnow (Ed.) *Studies in Social Interaction*. New York: Free Press.

Sacks, H. (1990). Sociological description. In J. Coulter (Ed.), *Ethnomethodological sociology*. Aldershot, UK: Edward Elgar.

Sacks, H. (1992). *Lectures on conversation* (Vols. I & II, edited by G. Jefferson). Oxford: Basil Blackwell.

Sacks, H., Schegloff, E.A., & Jefferson, G. (1974). A simplest systematics for the organization of turn-taking for conversation. *Language 50* (4), 696-735.

Säljo, R. e Wyndhamn, J. (1993) Solving everyday problems in the formal setting: an empirical study of the school as context for thought. In: S. Chaiklin e J. Lave (Eds.) Understanding Practice: perspectives on activity and context, Cambridge: Cambridge University Press.

Saporta, S. (1961) *Psycholinguistics: a Book of Readings*. New York: Holt, Rineheart and Winston.

Saxe, G. (1991) Culture and Cognitive Development: studies in mathematical understanding, Hillsdale: Lawrence Erlbaum Associates.

Saxe, G.B., Gearhart, M., & Guberman, S.R. (1984). The social organization of early number development. In B. Rogoff & J.V. Wertsch (Eds.), *Children's learning in the 'zone of proximal development'* (pp. 19-30). San Francisco: Jossey-Bass.

Schegloff, E.A. (1968). Sequencing in conversational openings. *American Anthropologist 70*, 175-195.

Schegloff, E.A. (1991a). Conversation analysis and socially shared cognition. In L.B. Resnick, J.M. Levine, & S.D. Teasley (Eds.), *Perspectives on socially shared cognition* (pp. 150-171). Washington DC: American Psychological Association.

Schegloff, E.A. (1992b). Introduction. In G. Jefferson (Ed.), *Harvey Sacks, Lectures on conversation, Volume 1 Fall 1964-Spring 1968* (pp. ix-lxii). Oxford: Basil Blackwell.

Schegloff, E.A., & Sacks, H. (1973) Opening up closings. *Semiotica 7*, 289-327.

Schweder, R. & Levine, R. A. (Eds.) (1984) *Culture Theory: Essays on Mind, Self, and Emotion.* Cambridge: Cambridge University Press.

Scribner, S and Cole, M. (1981) *The Psychology of Literacy*. Cambridge, MA: Harvard University Press.

Sfard, A. (2001) There is more to discourse than meets the ears: looking at thinking as communication to learn more about mathematical learning. *Educational Studies in Mathematics* 46, 13–57.

Sfard, A., Forman, E. and Kieran, C. (2001) Learning Discourse: Sociocultural approaches to research in mathematics education. *Educational Studies in Mathematics* 46, 1–12.

Shapin, S., & Schaffer, S. (1985). *Leviathan and the air-pump*. Princeton, NJ: Princeton University Press.

Sharrock, W.W. (1974). On owning knowledge. In R. Turner (Ed.), *Ethnomethodology* (pp. 45-53). Harmondsworth, UK: Penguin.

Sharrock, W. and Ikeya, N. (2000). Instructional Matter: Readable Properties of an Introductory Text in Matrix Algebra. In Hester, S. and Francis, D. (Eds.) *Local Education Order: Ethnomethodological studies of knowledge in action* . Amsterdam: John Benjamins.

Sinclair, A. (1991) Children's production and comprehension of written numerical representations. In: K. Durkin e B. Shire (Eds.) *Language in Mathematical Education. Research and Practice.* Milton Keynes and Philadelphia: Open University Press.

Sinclair, J. and Coulthard, R.M. (1975). *Towards an analysis of discourse: The English used by teachers and pupils.* London: Oxford University Press.

Sinha, C. (1988) *Language and Representation: a socio-naturalistic approach to human development.* Great Britain: Harvester-Wheatsheaf.

Skinner, B.F. (1957). *Verbal Behavior.* New York: Appleton-Century-Crofts.

Smith, L. (1993) *Necessary Knowledge: Piagetian perspectives on constructivism.* Hove: Lawrence Erlbaum Associates.

Smith, B. H. and Plotnitsky, A. (1995) Introduction: networks and symmetries, decidable and undecidable. *The South Atlantic Quarterly* 94 (2), 371-388.

Steffe, L., Von Glasersfeld, E., Richards, J., e Cobb, P. (1983) *Children's Counting Types: philosophy, theory and application.* New York: Praeger.

Stokoe, E.H., & Smithson, J. (2001). Making gender relevant: Conversation analysis and gender categories in interaction. *Discourse and Society* 12, 217-244.

Suchman, L.A. (1987). *Plans and situated actions: The problem of human-machine interaction*. Cambridge: Cambridge University Press.

Suchman, L.A. (1988). Representing practice in cognitive science. In M. Lynch & S. Woolgar (Eds.), (1990). *Representation in scientific practice*. Cambridge, MA: MIT Press.

Suchman, L.A. (1992) Technologies of accountability: of lizards and airplanes. In: G. Button (Ed.) *Technology in working order: studies of work, interaction and technology*. London: Routledge.

Suchman, L.A. (2000) Organizing alignment: a case of bridge-building. *Organization* 7 (2), 311-327.

Sugarman, S. (1987) *Piaget's Construction of Child Reality*. Cambridge: Cambridge University Press.

Thornborrow, J. (2001) *Power Talk: Language and Interaction in Institutional Discourse*. London: Longman.

Turnbull, W. & Carpendale, J. (1999) A social pragmatic model of talk: Implications for research on the development of children's social understanding. *Human development* 42, 328-55.

Valsiner, J. (1999). *Quo vadis* discourse analysis? *Derek Edwards' Discourse and cognition. Culture & Psychology* 5 (3), 357-361.

van der Veer, R., & Valsiner, J. (1991). *Understanding Vygostsky: A quest for synthesis*. Oxford: Basil Blackwell.

Van Oers, B. (2001) Educational forms of initiation in mathematical culture. *Educational Studies in Mathematics* 46, 59–85.

Vergnaud, G. (1991) El Niño, las Matemáticas y la realidad: problemas de la enseñanza de las matemáticas en la escuela primaria. Mexico: Trillas.

Von Glasersfeld, E. (1982) Subitizing: the role of figural patterns in the development of numerical concepts. *Archives de Psychologie* 50, 191-218.

Voigt, J. (1992) Negotiation of mathematical meaning in classroom processes: social interaction and learning mathematics, *Paper presented to the Theory Working Group*, ICME 7, Quebec.

Voigt, J. (1993) Ascribing mathematical meaning to empirical phenomena: custom and dynamics of classroom discourse, Hamburg, *Mimeo*.

Vygotsky, L.S. (1978). *Mind in society: The development of higher psychological processes*. London: Harvard University Press.

Vygotsky, L.S. (1987a). *Thought and language*. Edited by Alex Kozulin. Cambridge, MA: MIT press.

Vygotsky, L. S. and Luria, A.R. (1993) *Studies on the History of Behavior: Ape, Primitive, and Child*. Edited and translated by V. I. Golod and J. E. Knox. Hillsdale, NJ: Erlbaum.

Walkerdine, V. (1984). Developmental psychology and the child-centred pedagogy: the insertion of Piaget into early education. In J. Henriques, W. Hollway, C. Urwin, C. Venn, & V. Walkerdine, *Changing the subject*. London: Methuen.

Walkerdine, V. (1988). *The mastery of reason: Cognitive development and the production of rationality*. London: Routledge.

Walkerdine, V. (1989) *Counting Girls Out*. London: Virago.

Walkerdine, V. e Sinha, C. (1978) The internal triangle: language, reasoning and social context. In: I. Marková (Ed.) *The Social Context of Language*. New York: John Wiley & Sons.

Watson, D.R. (1992) Ethnomethodology, conversation analysis and education: An overview. *International Review of Education* 38 (3), 257-74.

Wertheimer, M (1961) *Productive Thinking*. London: Tavistock.

Wertsch, J.V. (1984). The zone of proximal development: Some conceptual issues. In B. Rogoff & J.V. Wertsch (Eds.), *Children's learning in the 'zone of proximal development'*. San Francisco: Jossey-Bass.

Wertsch, J.V. (1985). *Vygotsky and the social formation of mind*. Cambridge, MA: Harvard University Press.

Wertsch, J.V., del Rio, P., & Alvarez, A. (Eds.) (1995) *Sociocultural studies of mind*. Cambridge: Cambridge University Press.

Wertsch, J. V. e Tulviste, P. (1996) L. S. Vygotsky and contemporary developmental psychology. In: H. Daniels (Ed.) An Introduction to Vygotsky, London: Routledge.

Whalen, J. (1995). A technology of order production: Computer-aided dispatch in public safety communications. In P. ten Have & G. Psathas (Eds.), *Situated order: Studies in the social organization of talk and embodied activities*. Washington, DC: University Press of America.

Whorf, B.L. (1956). *Language, thought and reality: Selected writings of Benjamin Lee Whorf*. J.B. Carroll (Ed.), Cambridge, MA: MIT Press.

Winograd, T. (2001) Architectures for Context. *Human-Computer Interaction* 16, 401-419.

Wittgenstein, L. (1972). *Tractatus Logico-philosophicus*. London: Routledge & Kegan Paul.

Wittgenstein, L. (1967). *Philosophical investigations*. Oxford: Blackwell.

Wittgenstein, L. (2001). *Remarks on the foundations of mathematics*. Oxford: Blackwell.

Wootton, A.J. (1997). *Interaction and the development of mind*. Cambridge: Cambridge University Press.