0402699491

Business and Growth Cycle Analysis for the Euro Area

by

Xiaoshan Chen

A Doctoral Thesis

Submitted in partial fulfilment of the requirements

for the award of

Doctor of Philosophy of Loughborough University

15-06-2009

© by Xiaoshan Chen (2009)

Abstract

This thesis revisits the issue of business cycle synchronisation in the euro area by utilising time-series models that may overcome some of the drawbacks in the existing literature. Two major contributions are made to the existing literature of evaluating cycle synchronisation.

First, instead of identifying turning points from individual macroeconomic time-series, as carried out in most studies, this thesis obtains turning points from multivariate information. It is hoped that including more variables containing business cycle information in the dating process may produce more accurate turning points and, in turn, improve the accuracy of measuring cycle correlation. In doing so, both parametric and non-parametric business cycle dating procedures are used. These include the quarterly Bry-Boschan (BBQ) algorithm, a single dynamic factor model and the Markov-switching dynamic factor model.

Second, unlike the traditional approach that measures growth cycle synchronisation in the euro area by calculating pairwise cycle correlations, this thesis analyses the degree of growth cycle comovement within a multivariate setting by using a VAR model with cointegration. The advantages of this approach are two-fold. Firstly, it does not require prior filtering or decomposition of data. Secondly, as it is based on a VAR, dynamic interactions between the variables can be modelled. The common trends and cycles in the output of seven major euro area countries are investigated. A number of hypotheses on various types of common/codependent cycles among the seven national GDP series are tested using canonical correlation-based tests, GMM and likelihood ratio statistics. The number of common/codependent cycles indicates the level of growth cycle synchronisation. The multivariate Beveridge-Nelson (BN) decomposition with common trend and cycle restrictions imposed are then utilised to provide a detailed investigation of the trend and cyclical movements in the output series.

Overall, the results obtained from measuring business cycle and growth cycle synchronisation in the euro area contradict the Optimum Currency Area (OCA) criterion that members of a monetary union should share a high degree of cycle synchronisation. Furthermore, variations in economic performance are observed across the euro area, which may lead to diverging monetary policy requirements, and may consequently reduce the appropriateness of having a common monetary policy.

Parts of the thesis also focuses on the euro area wide economy by looking at three issues concerning the aggregate euro area output gap using the multivariate unobserved components model. The reliability of the different output gap measures obtained from various unobserved components models is assess according to three criteria: the size of subsequent revisions to the data, the unbiasedness of the filtered estimates, and inflation forecasting. Results show that the models with the damped slope output trend imposed provide the best fit to the data and give relatively reliable output gap estimates. These models are then used to analyse the degree of business cycle moderation and the impact that changes in real interest rates have on output and inflation.

JEL classification: C32; C52; E32, E52.

Keywords: Synchronisation; Turning points; Markov-switching; Common factor; Common cycle; Codependent cycle; The output gap; Unobserved components; State-space model; Augmented Kalman filter.

Acknowledgement

I would like to thank my supervisor, Terence Mills, for his support throughout writing this thesis. His knowledge of time series econometrics and excellent supervision has been a huge help over the past three years. I also wish to thank the Department of Economics for the financial assistance and facilities they have provided and all the staff and PhD students in the department for their support.

My sincere thanks also go to a number of academics, including Tommaso Proietti, Franz Palm, Tara Sinclair, Maximo Camacho, Soon Yip Wong, Jennifer Castle, Marcelo Sánchez, Efthymios Pavlidis, Qingwei Wang and Sai Ding, for giving me constructive suggestions on my conference papers.

Finally, I would like to thank my parents and my partner for their patience when the thesis was being finalised.

Loughborough University
Pilkington Library
Date 07\10
Class
Acc. No. 0402699491

Statement of Publication and Wold Count

Parts of this thesis have been published or have been accepted for publication. Chapter 4, 'Evaluating Growth Cycle Synchronisation in the EU', has been published in Economic Modelling, 2009, Volume 26(2), pp. 342-351. Parts of Chapter 2 and Chapter 3 have been accepted for a special issue of the Revue Française d'Economie.

The papers that comprise the thesis have been presented at various seminars and conferences. Chapter 5, 'Analysing Output Gaps and Monetary Policy Transmission within a State-Space Framework: An Application to the Euro Area', has been presented at the Annual Conference of the Royal Economic Society, April 2009; and the Annual Conference of the Scottish Economic Society, April 2009. Chapter 4, 'Evaluating Growth Cycle Synchronisation in the EU', has been presented at the 5th Eurostat Colloquium on Modern Tools for Business Cycle Analysis, September 2008. Finally, an early version of the paper, 'Evaluating Synchronisation of the Euro Area Business Cycles using Multivariate Coincident Macroeconomic Indicators', produced from Chapters 2 and 3, has been presented at the Loughborough University Department of Economics Seminar Series, November 2007 and at the 2nd All China Economics International Conference, December 2007.

Several applications were used to produce this thesis. The single dynamic factor and the Markov-switching dynamic factor models used in Chapters 2 and 3 are based on the Gauss codes written by Chang-Jin Kim and Charles R. Nelson, available at http://www.econ.washington.edu/user/cnelson/SSMARKOV.htm. In Chapter 4, a RATS code developed by Estima was used to produce the univariate Beveridge-Nelson decomposition; STAMP was used for the Harvey and Trimbur's (2003) structural time series model; The FIML procedure in PcGIVE was used to estimate the various models in Chapter 4; and the multivariate Beveridge-Nelson decomposition was based on the Gauss code that replicates the results of Proietti (1997). Finally, Chapter 5 is based on portions of the Ox code provided by Tommaso Proietti, which was used to produce Proietti (2007, 2008).

Word Count: The body of this thesis comprises 232 pages and proximately 51,142 words.

Contents

List of Tables

List of Figures

Introduction

Economic and monetary union (EMU) in Europe has resulted in an increase in research into business cycle synchronisation at both the national and regional level. It has been argued by Optimum Currency Area (OCA) theory that a high degree of business cycle synchronisation between member states is crucial for the smooth functioning of the EMU. As the EMU permanently removes national monetary and exchange independence and constrains the use of national fiscal policy, as set out in the Stability and Growth Pact (SGP), a common monetary policy acts to stabilise output fluctuations and inflation rates in the euro area as a whole. Highly synchronised national business cycles will facilitate the implementation of monetary policy, as the stance and timing of a common policy can be clearly defined. However, de-synchronisation will complicate the operation of the EMU, as countries in different phases of their business cycle have different monetary requirements. Business cycle synchronisation is not static and instead is evolving over time. As such, the endogenous OCA theory argues that the EMU may by itself spur the emergence of a common euro area business cycle due to economic and financial integration and more coordinated policies. However, the "Krugman hypothesis" holds the opposite view that the EMU will lead to cross-country specialisation and therefore less synchronisation.

Since evaluating business cycle synchronisation is mainly an empirical issue, a large number of studies have been undertaken to measure the degree of business cycle synchronisation in the euro area. These studies can be broadly divided into two groups, according to whether the aim is to evaluate the synchronisation of classical cycles or growth cycles. Considerably more studies focus on growth cycles than classical cycles. This is, in part, because when growth cycles are separated from trend growth they are stationary series, and most measures of synchronisation require stationary series as inputs. However, disagreement remains over how the trend should be identified and estimated. A range of parametric and non-parametric approaches, such as various structural time-series models and the Hodrick-Prescott (HP) and band-pass filters, are extensively applied to macroeconomic variables to obtain their trend and cyclical components. However, the estimated cyclical components vary depending on the decomposition methodologies used (Canova, 1998). This may, in turn, give mixed results when measuring cycle correlations. The second group of studies avoid trend and cycle decomposition and define classical cycles in terms of turning points of the original data series. Two fundamentally different approaches, the Markov-switching model and variants of the Bry and Boschan (1971) algorithm, are widely used in the literature to date turning points. The pros and cons of these approaches are discussed in Harding and Pagan (2003) and Hamilton (2003). In addition, the use of different measures of synchronisation contributes to the mixed results on cycle correlation. Different cycle measures are used to indicate the degree of business cycle synchronisation, including the bivariate correlation coefficient (Artis and Zhang, 1997, 1999), an aggregated correlation coefficient (Camacho *et al.*, 2006), a dispersion measure proposed by Artis, Marcellino and Proeitti (2004a), the various concordance indices in Harding and Pagan (2002), and so on. Furthermore, in addition to the different methodologies used to identify business cycles and various measures of synchronisation, the choice of economic variables can also have a significant impact on the conclusions drawn when measuring cycle synchronisation. The literature survey presented in Chapter 1 concludes that there is no consensus on whether business cycle synchronisation in the euro area has reached a level where a common monetary policy can benefit all member states. There is also no consensus on whether fixed exchange rates or the introduction of the euro have had an effect on increasing cycle synchronisation.

This thesis revisits the issue of business cycle synchronisation in the euro area by bringing time-series models that may overcome some of the drawbacks in the existing literature of evaluating cycle synchronisation. Two major contributions are made to the existing literature of evaluating cycle synchronisation. First, most studies, including Harding and Pagan (2002), Garnier (2003) and Artis, Marcellino and Proietti (2004a), measure classical cycle synchronisation using turning points identified from individual macroeconomic series, such as industrial production and real GDP. However, analysing univariate economic variables is not optimal for dating business cycle turning points. This is not only because there is no variable that can well represent aggregate economic activity, but also because the comovement of many economic variables throughout the cycle cannot be analysed in a univariate framework. Therefore, Chapters 2 and 3 employ the alternative dynamic factor (DF) models, proposed by Stock and Watson (1989, 1991, 1993) and Diebold and Rudebusch (1996), to construct a composite index that is a weighted average of a number of coincident macroeconomic variables. It is hoped that including more relevant variables may produce more accurate turning points and, in turn, improve the accuracy of measuring cycle correlation.

Second, most studies of growth cycle synchronisation in the euro area have calculated the correlation between each pair of estimated cycles. Univariate trend-cycle decomposition methodologies, such as the HP and band-pass filters, are extensively used to extract cycles from industrial production, real GDP or expenditure categories of GDP. These filters are known to produce spurious cycles for non-stationary data and, as such, the results are sensitive to the decomposition methodology used (Canova, 1988; Cogley and Nason, 1995; Murray 2003; Doorn, 2006). To overcome this issue, Chapter 4 evaluates the degree of growth cycle synchronisation using a multivariate framework which does not require prior filtering or decomposition of the GDP series. The common trends and cycles in the GDP series of seven major EMU countries are investigated. A number of hypotheses on various types of common cycles and codependent cycles are tested using canonical correlation-based tests, Generalised method of moments (GMM) and likelihood ratio statistics (Vahid and Engle, 1993, 1997; Hecq *et al.* 2000, 2006; Schleicher, 2007). The number of common and codependent cycles between countries provides an indication of the level of growth cycle synchronisation. Furthermore, Chapter 4 employs the multivariate Beveridge-Nelson decomposition with common trend and cycle restrictions proposed by Proietti (1997) and Hecq *et al.* (2000) to decompose the seven GDP series into their trend and cyclical components simultaneously. The advantages of this decomposition are two-fold. Firstly, dynamic interactions between the variables can be modelled, as the technique is based on a VAR. Secondly, if common feature restrictions are imposed correctly, the estimation efficiency and forecasting ability of a model will improve. Comparing out-of-sample forecasting performance across different models suggests that more parsimonious models, with additional common cycle restrictions imposed, outperform the less restricted model, with only common trend restrictions imposed, by producing smaller forecast errors.

The outline of the thesis is as follows. Chapter 1 first reviews the methodologies used to date classical cycles and to identify trends and cycles in macroeconomic variables, as these are the foundations for any business cycle related analysis. Various measures of synchronisation are then discussed, ranging from simple correlation coefficients to complex model-based approaches. Finally, Chapter 1 takes stock of the existing literature on evaluating business cycle and growth cycle synchronisation between previous members of the Exchange Rate Mechanism (ERM) and members of the EMU. The main objective of Chapter 1 is to identify avenues of research that have not been sufficiently explored in the existing literature.

The empirical analysis of this thesis begins in Chapter 2 and ends in Chapter 5. Chapters 2, 3 and 4 evaluate the synchronisation of classical cycles and growth cycles among EMU member states. Chapter 5 focuses on the euro area economy as a whole by analysing three aspects of the aggregate euro area output gap.

In particular, Chapter 2 evaluates the degree of business cycle synchronisation between the aggregate euro area and its member countries from the 1970s to the 2000s. Three non-EMU countries (the UK, the US and Canada) are also included in the analysis so as to benchmark the changes of synchronisation that have occurred in the euro area. Instead of using individual economic variables to represent aggregate economic activity, Stock and Watson's (1989, 1991, 1993) single DF model is applied to four quarterly coincident macroeconomic variables, including real GDP, industrial production and unemployment, to estimate a composite index. Business cycle turning points of this index can then be identified using Harding and Pagan's (2000, 2001, 2002) BBQ algorithm. The concordance of turning points and the similarity of business cycle phases are used to measure the degree of synchronisation. Overall, Chapter 2 does not find a common tendency for euro area members to become either more or less synchronised over time. Furthermore, variations in economic performance observed across the euro area may lead to diverging monetary policy requirements and, consequently, reduce the appropriateness of having a common monetary policy for all members.

Chapter 3 employs Diebold and Rudebusch's (1996) Markov-switching dynamic factor (MSDF) model, based on Kim's (1994) approximate maximum likelihood estimation, to date business cycle turning points for the same countries that were analysed in Chapter 2. The MSDF model combines Stock and Watson's single DF model and Hamilton's (1989) univariate Markov-switching model, and can thus incorporate two stylised facts of the business cycle; the comovement of economic variables throughout the cycle and the asymmetry between business cycle phases. Compared to the dating procedure used in Chapter 3, the MSDF model produces a composite index as well as smoothed regime probabilities to indicate business cycle turning points. This model has been extensively applied to US data (Kim and Yoo, 1995; Chauvet, 1998; Kim and Nelson, 1999c), but less often applied to the euro area countries.[1] The results obtained in Chapter 3 indicate that the MSDF model is more successful in distinguishing different regimes for larger economics (i.e., Germany, France and the US) whose business cycle phases were of roughly constant magnitudes over the sample periods analysed. However, Belgium, Italy and the Netherlands show greater volatility during the 1970s and early 1980s than in more recent years. MSDF models which lack a mechanism to account for business cycle moderation fail to produce reasonable parameter estimates and smoothed regime probabilities for these countries. Therefore, structural breaks are introduced to the intercepts of the MSDF model to reduce the impact that large recessionary and expansionary phases have on the model's parameter estimates.

Chapter 4 investigates growth cycle synchronisation in seven major euro area countries, Austria, Belgium, France, Germany, Italy, the Netherlands and Spain, during the period 1980Q1 to 2007Q3. Two univariate structural time-series models, the Beveridge-Nelson (1981) decomposition and Harvey and Trimbur's (2003) unobserved component model, are first used to identify the trend and cyclical components for each GDP series. The two approaches yield starkly different results. This confirms the argument in Canova (1998) that the use of different trend-cycle decomposition methodologies may influence the results obtained. The main focus of Chapter 4 is to evaluate synchronisation using a multivariate approach that identifies the number of common trends and cycles in the seven national GDP series. The number of common trends, as determined by Johansen's (1995) cointegration test, constrains the maximum number of common and codependent cycles in the seven GDP series. Adhering to these constraints, the presence of three types of common cycles (i.e., strong, weak and mixed forms) and codependent cycles are examined. The multivariate BN decomposition with common trend and common cycle restrictions imposed is then used to analyse the trend

[1] Chauvet and Yu (2006) proposed a MSDF model with a filter that minimises the occurrence of false turning points. They applied this modified MSDF to both the aggregate measures of G7 and OECD countries and individual G7 countries including Germany, France and Italy.

and cyclical movements in the GDP series. The relative importance of permanent and transitory shocks to the variance of total output is assessed using the forecast error variance decomposition proposed by Issler and Vahid (2001). The results indicate that, in the short-term, the majority of the output variance can be attributed to cyclical fluctuations rather than the trend components. Over longer time periods it is the trend components of output that explain the majority of output fluctuations.

Chapter 5 analyses the output gap for the EMU as a whole by looking at the reliability of output gap estimates, the degree of business cycle moderation and the effectiveness of monetary policy transmission through the interest rate channel. The output gap, as a proxy of excess demand, is widely used but is notoriously difficult to measure as it is a latent variable. Estimates of the output gap can vary significantly depending on the decomposition methodology used. Therefore, it is important to set up criteria to judge which model provides the most reliable output gap estimate. Chapter 5 compares the reliability of various output gap estimates obtained from multivariate unobserved components models that incorporate output decomposition with other economic variables carrying relevant business cycle information, such as the inflation rate and the unemployment rate. A bivariate model of output and inflation and a trivariate model of output, inflation and unemployment are estimated. Both models are imposed with four alternative output trend specifications: damped slope (DS), local linear trend (LLT), random walk (RW) and the Hodrick-Prescott (HP) trend. Estimates of the output gap are assessed against three criteria; the size of subsequent revisions to the data, the unbiasedness of the filtered estimates, and inflation forecasting. The results show that the bivariate model of output and inflation outperforms the univariate model of output decomposition. However, including the unemployment rate in the analysis does not significantly improve output gap estimates according to the three criteria used. Different specifications of output trend can have a significant impact on both a model's goodness of fit and the reliability of its output gap estimates. The bivariate and trivariate models with the DS output trend imposed provide the best fit to the data and give relatively reliable output gap estimates. However, models with the HP restrictions imposed are strongly rejected due to strong autocorrelation in the residuals. The bivariate and trivariate models with the DS output trend imposed are found to be the most appropriate specifications in the analysis. These models are then used to investigate business cycle moderation and the effectiveness of the interest rate channel for the euro area.

Finally, the Epilogue concludes this thesis, drawing together the research from the substantive chapters. Further research prospects are also illustrated. This thesis mainly aims to improve the existing literature on evaluating euro area cycle synchronisation by applying time-series models that may overcome some of the drawbacks in the existing literature. The responsiveness of the output gap and inflation rates to changes in real interest rates are also assessed for the euro area wide economy by using the most appropriate unobserved components models in the analysis.

Chapter 1 - A Survey of the Literature

1.1 Introduction

The optimality and sustainability of EMU has been frequently challenged by both academics and policymakers. Since monetary integration permanently removed national monetary independence, member states are unable to use monetary policy to protect themselves from asymmetric shocks. Given the rigidities observed in most European labour markets, a greater use of national fiscal policy is required to compensate for this lack of national monetary instruments. However, within EMU, the use of fiscal policies is also constrained by the Stability and Growth Pact (SGP)[2]. As such, there remains a concern as to whether a common monetary policy can stabilise output fluctuations and inflation rates for the euro area as a whole.

Optimal Currency Area (OCA) theory is frequently used to provide a theoretical foundation for analysing the suitability of a currency union formed by a group of sovereign countries who forego monetary and exchange rate independence. The theory of OCA, which was established by Mundell (1961), McKinnon (1963) and Kenen (1969), defines conditions and properties under which a currency union can operate smoothly. One key criterion highlighted in the OCA theory is that member countries should share a high degree of business cycle synchronisation to ensure that a common monetary policy can stabilise area wide economic fluctuations and inflation.[3] Divergent business cycles across members of a monetary union will lead to different monetary requirements. For example, a country in the downward phase of its business cycle would require an expansionary monetary policy to stimulate economic growth, whereas a country in the upward phase of its cycle would prefer tighter monetary policy to

[2] The SGP was adopted in 1997 and requires that national government budget deficits are not higher than 3% of GDP and that national government debt should be lower than 60% of GDP.

[3] Other criteria proposed in the OCA theory include labour mobility (Mundell, 1961), economic openness (McKinnon, 1963), industrial diversification (Kenen, 1969), price and wage flexibility (Fleming, 1971, Corden, 1972), inflation rate similarity (Haberler, 1970; Fleming, 1971; Ishiyama, 1975) and fiscal and political integration (Haberler, 1970; Mintz, 1970; Tower and Willett, 1976; Cukierman et al., 1992). It is widely accepted that if countries share these properties, their output fluctuations and inflation rates can be stabilised through those mechanisms, instead of via national monetary and exchange rate adjustments.

prevent the risk of increasing inflation. The establishment of EMU has further stimulated the debate surrounding endogenous OCA theory, which states that operating in a monetary union helps member countries eventually become optimal members of that monetary union even if they were not before (Frankel and Rose, 1998). If this theory is true, we would expect EMU by itself will generate a greater degree of business cycle synchronisation in member countries.

Many articles have been written on the issue of whether business cycles in the euro area have become more synchronised, as argued by endogenous OCA theory. Theoretical arguments in this field remain between endogenous OCA theory (the European Commission, 1990; Frankel and Rose, 1998) and the "Krugman hypothesis" (Krugman and Venables, 1993). The former suggests that the operation of monetary union should generate synchronised cycle comovements between members due to greater trade intensity and financial market integration. However, the latter argues that cycle divergence might occur as the result of further economic integration. This is based on trade theory and economies of scale and agglomeration effects. As further integration may induce greater specialisation, sector-specific shocks may eventually become region-specific shocks, causing diverging business cycles in the European countries.

Given different theoretical viewpoints, various empirical analyses have been undertaken to measure the degree of business cycle synchronisation in the euro area, but no consensus has been reached. Differences in the results can be attributed to the use of various macroeconomic variables over different sample periods and different methodologies to identify business cycles and measure cycle synchronisation. It is worth noting that another strand of empirical studies attempts to identify the potential determinants of cyclical convergence. Many factors have been suggested that may drive business cycle synchronisation. These include international trade, financial market integration, exchange rate regimes, technology spillovers, and economic structures. However, it is fair to say that no agreement has yet been reached on this issue either.

As the main focus of this thesis is to provide business cycle measures and to evaluate the degree of cycle synchronisation in the euro area, this chapter provides a review of the literature on dating classical cycle turning points and modelling trends and cycles in macroeconomic time-series. This is essential since, before any empirical analysis of business cycles can take place, economists need to define both what is meant by the term 'business cycle' and how business cycle information can be identified from historical data. Studies measuring euro area business cycle synchronisation are also surveyed in this chapter.

The remainder of this chapter is organised as follows. Section 1.2 discusses two broad definitions of business cycles. Models used to identify business cycle turning points are reviewed in Section 1.3. Section 1.4 outlines the main trend and cycle decomposition methodologies for growth cycle studies. Section 1.5 discusses measures of synchronisation. The main findings in the literature of business cycle and growth cycle synchronisation in the euro area are summarised in section 1.6. Section 1.7 concludes.

1.2 Classical business cycles and growth cycles

A distinction has to be made between (classical) business cycles and growth (deviation) cycles. The empirical analysis of business cycles has a long intellectual history. The conventional technique used to identify business cycles was developed by researchers at the National Bureau of Economic Research (NBER) (Mitchell, 1927; Burns and Mitchell, 1938; Burns and Mitchell, 1946). In the influential study by Burns and Mitchell (1946), the following definition of the business cycle is proposed:

A cycle consists of expansions occurring at about the same time in many economic activities followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next business cycle; This sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; They are not divisible into shorter cycles of similar character with amplitudes approximating their own (Burns and Mitchell, 1946, P. 3).

This definition explains what constitutes a business cycle and the duration of a cycle. However, questions remain over how to identify business cycle turning points from historical data, how to quantify the co-movement of a specific time-series with the aggregate business cycle fluctuations, and what are the most appropriate economic variables to use in identifying the business cycle. Given the definition quoted above, the NBER business cycle dating committee produced a set of statistical measures, known as *specific cycles and reference cycles*, to identify the US business cycle turning points from movements in output, income, employment and trade volumes. The periods of expansions and recessions can be highlighted once turning points are identified. A peak indicates the end of an expansion and the beginning of a recession, and *vice versa* for a trough. Recessions are the periods of absolute declines in output and other measures of economic activity. Recessions are characterised with the so-called 'three Ds', which denotes that a recession should be sufficiently long (duration), involve a substantial decline in economic activity (depth), and be spread widely across the economy (diffusion). However, the NBER business cycle dating approach is often criticised due to its lack of theoretical foundation (Koopmans, 1947).

Unlike the conventional business cycle definition outlined above, more recent studies have analysed fluctuations in economic time-series around their long-run trend growth. Deviations from the long-run trend are defined as the growth cycle (the output gap). In this context, expansions and recessions are periods of increasing and decreasing growth (Zarnowitz, 1985). Compared to business cycle phases, where the average recession is considerably shorter than expansions due to underlying trend growth, growth expansions and recessions have approximately the same duration. The growth cycle is usually considered to be an indicator of inflationary pressures. As central banks' main objective is to keep inflation stable, knowing the growth cycle provides them with important information on the build-up of inflationary pressures in the economy.

In contrast to the 'measurement without theory' statistical methods used to analyse the business cycle, growth cycle studies are directly derived from business cycle theories. The debate as to whether fluctuations in macroeconomic time-series are dominated by short-term cyclical fluctuations or long-term trend growth has profound methodological implications. Both traditional Keynesian and Monetarist theories hold the view that fluctuations in output are driven by demand shocks, and are temporary deviations from potential output growth. This view is the foundation for traditional trend-cycle decomposition methodologies, which use linear or polynomial deterministic regression equations to eliminate the trend component in a series. However, the use of a linear time trend is challenged by Klein and Kosobud (1961), as it seems implausible that economic growth can be well approximated by a constant deterministic trend given the presence of structural changes, varying rates of production factor accumulation and technical innovations. It is therefore reasonable to consider breaks in the trend or even period-by-period random or stochastic trends. The presence of trend variability is supported by Granger (1966), who shows that the typical spectral shape of a macroeconomic series is monotonically decreasing, implying that fluctuations in a series are dominated by very low frequency components (the trend component), rather than business cycle frequency components. Furthermore, Beveridge and Nelson (1981) and Nelson and Plosser (1982) initiated the debate on whether macroeconomic time-series are trend-stationary (TS) or difference-stationary (DS). Beveridge and Nelson (1981) propose a trend-cycle decomposition methodology by assuming that the decomposed series is generated by a DS process. Nelson and Plosser (1982) employ the Dickey-Fuller (1979) test to examine whether a set of US macroeconomic time-series are indeed TS or DS, and find the evidence favours the latter. This study was followed by Campbell and Mankiw (1987a, b) and Stock and Watson (1988a), who all support the view that output is best modelled as a DS process. The presence of stochastic trends confirms the finding of Granger (1966) that fluctuations in macroeconomic time-series are mainly attributed to fluctuations of the long-term trend. More importantly, this result supports the Real Business Cycle (RBC) theory over traditional Keynesian and Monetarist theories, as the transitory (monetary) shocks are considered to be less important in determining the movement of the series than permanent shocks. However, not all researchers are convinced by the stochastic trend view of macroeconomic dynamics. Rudebusch (1992) and DeJong and Whiteman (1991), who re-examine this issue by applying alternative techniques to the Nelson and Plosser data set, conclude that the failure to reject the null hypothesis of DS against the alternative of TS is due to the low power of the conventional integration tests.[4] In addition, Perron (1989) shows that the DS hypothesis may be rejected if the trend is modelled as a nonlinear function of time, where shifts in the deterministic trend are caused by infrequent permanent shocks.

[4] Rudebusch (1992) calculates small-sample distributions for various unit root test statistics and concludes that the unit root tests often fail to distinguish between DS and TS. This finding is consistent with DeJong and Whiteman (1991), who adopt a Bayesian perspective to identify the priors needed for the DS and TS representations and find that for most macroeconomic series the TS hypothesis is generally supported.

Controversy also remains over whether the trend and cycle components are correlated. Separation of trend and cycle components is only justifiable if one can clearly distinguish between the factors determining long-run growth and those determining cyclical fluctuations. The RBC theory attempts to remove the dichotomy between trend and cycle, as it suggests that long-run economic growth and short-run cyclical fluctuations are accounted for by the same productivity shock. On the other hand, the trend-cycle dichotomy is supported by most Keynesian and rational expectation models. For example, sophisticated Keynesian macroeconomic models, such as the Fair model (Fair, 1994), incorporate a production function to model output trend growth. In addition, rational expectations with misperception models, such as Lucas (1973) and Okun (1980), have monetary impulses that move output temporarily away from the potential trend growth. In these models, aggregate demand shocks are thought to temporarily drive the economy from its natural growth rate, while this natural growth rate is determined by the capital stock, the labour force and technology in long-run equilibrium.

Given the controversy that remains over the major business cycle theories, various trend and cycle decomposition methodologies have been proposed, in which the trend components are modelled in either a stochastic or deterministic manner, and are either correlated or uncorrelated with the cycle. In the next section, the major methodologies used to identify business cycle turning points are reviewed. Section 1.4 then surveys the main techniques used in growth cycle studies.

1.3 Dating business cycle turning points

Both parametric and non-parametric methods of dating classical business cycles have been proposed in the literature. The most commonly used parametric method is the Markov-switching (MS) model popularised by the seminal work of Hamilton (1989, 1990). The non-parametric approach often refers to the Bry and Boschan (BB) (1971) dating algorithm and its quarterly extension developed by Harding and Pagan (2000, 2001, 2002), known as the BBQ algorithm. A comparison of these two methods has been made by Harding and Pagan (2003) and Hamilton (2003).

1.3.1 The regime switching models

In an influential article, Hamilton (1989) developed a Markov-based, regime-switching method to model time-series subject to abrupt, non-linear regime changes. He applied a two-regime Markov-switching model to US quarterly real GNP growth over the period of 1953-1984. Real GNP growth is modelled as an AR (4) process:

$$\left(\Delta Y_t - \mu_{s_t}\right) = \phi_1\left(\Delta Y_{t-1} - \mu_{s_{t-1}}\right) + \phi_2\left(\Delta Y_{t-2} - \mu_{s_{t-2}}\right) + \ldots + \phi_4\left(\Delta Y_{t-4} - \mu_{s_{t-4}}\right) + e_t, \quad (1.1)$$
$$\mu_{s_t} = \mu_0\left(1 - S_t\right) + \mu_1 S_t,$$

where $e_t \sim NID(0,\sigma^2)$. The idea behind regime-switching is to allow the mean growth rate, μ_{s_t}, to take on different values depending on the latent state variable (denoted as S_t). The likelihood of switching from one regime to the other is determined by the transition probabilities:

$$\Pr(S_t = 1 | S_{t-1} = 1) = p,\ \Pr(S_t = 0 | S_{t-1} = 1) = 1 - p,$$
$$\Pr(S_t = 0 | S_{t-1} = 0) = q,\ \Pr(S_t = 1 | S_{t-1} = 0) = 1 - q.$$

For each regime, the probability rule which governs the likelihood of various observations is the normal density function, with different mean growth rates for recessions and expansions. The normal density function of ΔY_t based on past information is given by

$$f\left(\Delta Y_t | \psi_{t-1}, S_t, S_{t-1}, S_{t-2}, S_{t-3}, S_{t-4}\right)$$
$$= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(- \frac{\left\{\left(\Delta Y_t - \mu_{s_t}\right) - \phi_1\left(\Delta Y_{t-1} - \mu_{s_{t-1}}\right) - \ldots - \phi_4\left(\Delta Y_{t-4} - \mu_{s_{t-4}}\right)\right\}^2}{2\sigma^2} \right)$$

Therefore, the log likelihood function is

$$\ln L = \sum_{t=1}^{T} \ln\left(f\left(\Delta Y_t | \psi_{t-1}, S_t, S_{t-1}, S_{t-2}, S_{t-3}, S_{t-4}\right)\right).$$

Maximum likelihood estimation (MLE) provides the parameter estimates $\{\mu_0, \mu_1, \sigma, p, q\}$. The MLE parameter estimate of a particular regime mean is the sample mean of the series corresponding to the periods when the series was in that regime. The parameter estimates of the transition probabilities are given by frequency counts of the pattern of known regime switches. The parameter estimates obtained in Hamilton (1989) characterise the business cycle in two ways. Firstly, the mean growth rate appears negative in recessions and positive in expansions. Secondly, transition probabilities indicate that the average duration of recessions is much shorter than expansions. More importantly, he obtains a time-series of recession probabilities. This provides a quarterly chronology of US business cycles which successfully replicates the NBER official business cycle dates over 1953-1984. Furthermore, in order to avoid the singularity problem[5] which may occur in the unrestricted MLE used in Hamilton (1989), Hamilton (1991) proposed a quasi-Bayesian MLE approach, in which the likelihood function incorporates a priori information about the unknown means and variances[6].

Despite the initial success of Hamilton's (1989) model in identifying US business cycle turning points over 1953-1984, it is widely accepted that this model often fails to provide reasonable parameter estimates and regime probabilities when an extended sample period or different datasets are used. Therefore, there have been a large number of subsequent extensions and refinements to this model. Modifications have been made to Hamilton's model by introducing structural breaks in the model's parameters, by bringing in regime-dependent volatilities, intercepts or AR parameters, and by including additional regimes to model business cycle dynamics.[7] In addition, since Hamilton (1989) did not examine the statistical significance of his model, Hansen (1992) tests the null hypothesis of a linear AR(4) against the alternative of Hamilton's MS model using standardised likelihood-ratio (LR) test statistics, and fails to reject the null hypothesis.

[5] This problem occurs when a small subset of sample observations appears to be tightly clustered together. Unrestricted MLE results in this cluster being interpreted as constituting drawings from one normal distribution with very small variance and the rest of sample as coming from a second normal distribution with a much larger variance.

[6] Prior means and variances were set to be the sample means and variances of the negative growth rate observations for regime 1 and positive growth rate observations for regime 2.

[7] Another strand of papers relaxes the assumption of fixed transition probabilities by modelling with time-varying or duration dependent transition probabilities: for example, Diebold *et al*. (1999), Filardo (1994), Filardo and Gordon (1998), Lahiri and Wang (1994), Durland and McCurdy (1994) and Layton and Smith (2007).

However, he finds strong evidence that a simple switching model fits the data better than both the AR(4) and Hamilton's model.[8]

In order to account for the significant moderation of US business cycles observed during recent decades, Kim and Nelson (1999a) introduce a one-time break in the mean growth rates and the residual variance of Hamilton's model. The means and variances are thus specified as

$$\mu_{0t} = \mu_0^* + \mu_{00}^* D_t,\ \mu_0^* < \mu_0^* + \mu_{00}^* \tag{1.2}$$
$$\mu_{1t} = \mu_1^* + \mu_{11}^* D_t,\ \mu_1^* > \mu_1^* + \mu_{11}^*$$
$$\sigma_t^2 = (1 - D_t)\sigma_0^2 + D_t \sigma_1^2,\ \sigma_0^2 > \sigma_1^2 .$$

where D_t is the one-time parameter which shifts from zero to unity at an unknown date τ. The likelihood of D_t switching from zero to unity is governed by the transition probabilities:

$$\Pr(D_t = 0 | D_{t-1} = 0) = q_{00},\ \Pr(D_t = 1 | D_{t-1} = 0) = 1 - q_{00},$$
$$\Pr(D_t = 1 | D_{t-1} = 1) = 1,\ \Pr(D_t = 0 | D_{t-1} = 1) = 0 .$$

Under the null hypotheses that $\mu_{00}^* = \mu_{11}^* = 0$ and $\sigma_0^2 = \sigma_1^2$, this model collapses to Hamilton's model. Four competing models are constructed to examine various null and alternative hypotheses:

1. Hamilton's model with no structural break $[\mu_{00}^* = \mu_{11}^* = 0, \sigma_0^2 = \sigma_1^2]$;
2. A model with a structural break in the mean growth rates $[\mu_{00}^* \neq \mu_{11}^* \neq 0, \sigma_0^2 = \sigma_1^2]$;
3. A model with a structural break in the variance $[\mu_{00}^* = \mu_{11}^* = 0, \sigma_0^2 \neq \sigma_1^2]$;
4. A model with structural breaks in both mean growth rates and the variance $[\mu_{00}^* \neq \mu_{11}^* \neq 0, \sigma_0^2 \neq \sigma_1^2]$.

[8] The simple switching model allows for the intercept and the second AR parameter to randomly shift between two values. Therefore there is no persistence in the states and the sum of the transition probabilities p and q is restricted to one.

A Bayesian model selection procedure is employed to compare the above models. A comparison of the marginal likelihoods obtained using Gibbs sampling suggests that Hamilton's model is clearly dominated by the other three. This indicates that the moderation of US business cycles comes from two sources of stabilisation: a narrowing gap between the mean growth rates and a decline in the volatility of real output growth. Mills and Wang (2003a) apply the Kim and Nelson (1999a) model to other G7 countries. As with the US, a decline in output growth volatility is observed among other G7 countries. Narrowing growth differentials between recessionary and expansionary regimes are found in five of the seven countries, with the UK and Germany being the exceptions.

Additional phases in business cycle dynamics have also been introduced to the Hamilton model, including the three-regime model used in Sichel (1994), Boldin (1996) and Clements and Krolzig (1998), and the 'bounce back' model proposed by Kim *et al.* (2005). Boldin (1996) examines the robustness of Hamilton's model by applying a three-regime model to a revised version of Hamilton's data and also to an extended sample period. Three local maxima in the likelihood function are found when the revised data are used. One set of parameters, corresponding to a local maximum, is almost identical to the coefficients reported by Hamilton (1989). However, the other two local maxima fail to provide reasonable parameter estimates. Moreover, there is no longer a local maximum which can reproduce Hamilton's parameter estimates when the extended sample period is used. More positive results are obtained when a three-regime MS model is applied to capture recessions, post recession rapid-recoveries and moderate growth periods. A single local maximum is found, which is robust across different sample periods. The NBER official dates are also captured fairly well by the smoothed recession probabilities obtained in the three-regime model. As with Boldin (1996), Clements and Krolzig (1998) also adapt Hamilton's model to different sample periods of US GNP and fail to obtain adequate results. However, by applying a three-regime MS model with a regime-dependent intercept rather than mean growth rate, they obtain a business cycle chronology which corresponds closely to the NBER official dates.

Unlike Boldin (1996) and Clements and Krolzig (1998), Kim *et al.* (2005) assume that the post-recession recovery is strongly correlated with the length and severity of the preceding recession. They therefore propose a 'bounce-back' model as follows:

$$\phi(L)\left(\Delta Y_t - \mu_0 - \mu_1 S_t - \lambda \sum_{j=1}^{m} S_{t-j}\right) = e_t, \tag{1.3}$$

where μ_0 is the underlying growth rate. If $S_t = 1$ and $\mu_0 + \mu_1 < 0$, the economy is in a recessionary regime. When $\lambda > 0$, the summation term $\sum_{j=1}^{m} S_{t-j}$ measures the 'bounce-back' effect, suggesting that the growth rate will be above μ_0 for the first m periods of an expansion. The summation term increases each period up to the length of the preceding recession and peaks after the recession ends. It then diminishes as the expansion persists until it reaches zero. Applying this model, they find a large 'bounce-back' effect in the US recovery phase, and this effect is robust to allowing for a one-time break in business cycle volatility in the mid-1980s or to relating the size of the 'bounce-back' effect to the depth of the previous recession.

The assumption made in Kim *et al.* (2005) is consistent with the 'Plucking model' proposed by Friedman (1964, 1993), in the sense that each recession is assumed to be of the same amplitude as the succeeding expansion. In a plucking model, output cannot exceed a ceiling level but is plucked downwards by recessionary shocks at irregular intervals. The severity of recessions varies over time, but output always returns to the ceiling level. Based on Friedman's plucking model, Kim and Nelson (1999b) propose an unobserved-component model, in which output, Y_t, can be decomposed into a stochastic trend component, τ_t, and a cyclical component, c_t, which has both plucking and asymmetric features. The stochastic trend is modelled as a local linear trend, with a regime-dependent variance on the level disturbance:

$$\tau_t = g_{t-1} + \tau_{t-1} + v_t,\ v_t \sim NID\left(0, \sigma^2_{v,S_t}\right), \tag{1.4}$$

$$g_t = g_{t-1} + w_t,\ w_t \sim NID\left(0, \sigma^2_w\right),$$

$$\sigma^2_{v,S_t} = \sigma^2_{v,0}\left(1 - S_t\right) + \sigma^2_{v,1} S_t.$$

The cycle component is given by

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \pi_{S_t} + u_t,\ u_t \sim NID\left(0, \sigma^2_{u.S_t}\right), \tag{1.5}$$
$$\pi_{S_t} = \pi S_t,\ \pi < 0,$$
$$\sigma^2_{u.S_t} = \sigma^2_{u.0}\left(1 - S_t\right) + \sigma^2_{u.1} S_t,$$

Innovations to the cycle are assumed to be a mixture of two types of shock: a usual symmetric shock u_t and a discrete asymmetric shock, π_{S_t}, that is dependent upon S_t. $\sigma^2_{u.S_t}$ is the regime-dependent variance of the cycle disturbance. If $\sigma^2_{u.S_t} = 0$ and $\pi_{S_t} < 0$, this implies the presence of a ceiling level for output, as suggested in Friedman's plucking model. Thus, when $S_t = 1$ the aggregate demand shocks 'pluck' output downward away from the ceiling level. Kim and Nelson find that the US real GDP data can be well characterised by the plucking model. However, this model is less successful when output data for other G7 members is used. Mills and Wang (2002) show that, although negative asymmetric shocks influence the cyclical fluctuations, the presence of a ceiling level is only found in the UK, France and Italy.

Sinclair (2008) modifies Kim and Nelson's plucking model by allowing for the correlation between the innovations of the trend and cycle components. As such, the negative correlation between a symmetric transitory shock and the permanent innovations is revealed and the symmetric transitory shock can be interpreted primarily as the adjustment to permanent shocks. Moreover, the trend component appears more variable than in Kim and Nelson's 'plucking' model with zero-correlation between the trend and cycle innovations. This suggests that US real GDP may experience more permanent fluctuations than previously explained by the conventional 'plucking' model. There may be different types of recessions with different underlying causes, rather than only asymmetric transitory shocks. As such, the smoothed probabilities of asymmetric transitory shocks fail to identify the recessions which occurred during the 1970s, as supply shocks played a major part in these recessions.

1.3.2 The Bry and Boschan method and its quarterly applications

Unlike the MS models discussed above, Bry and Boschan (1971) developed a non-parametric algorithm which successfully replicates the NBER reference dates regardless of the sample periods used. Due to its reliability, the BB algorithm has been used by the Centre for International Business Cycle Research (CIBCR) to identify business cycle chronologies for the US and eleven other countries. The BB algorithm operates in three steps. Firstly, major cycle movements in a time-series are identified. Secondly, neighbourhoods of peaks and troughs are established. Finally, the peaks and troughs are determined by narrowing the search to these neighbourhoods. Three constraints are imposed in the third step when identifying the final turning points. Firstly, a full cycle should have a minimum duration of 15 months in order to separate it from any seasonal movements. Secondly, no phase can be less than 5 months in duration. Thirdly, no turning point is declared within 6 months of the beginning or end of the series. The third restriction could be a potential problem for the BB algorithm, as it may take some time to recognise phase changes that have already occurred. As highlighted by the CIBCR researchers, it can take as long as 12 months to identify a turning point when using the BB algorithm. Adhering to these constraints, turning points are identified and refined sequentially using three different filters with a decreasing degree of smoothness; a centred, unweighted, 12-month moving average is first used, then a Spencer filter (i.e., a centred, weighted moving average), then finally a short-span moving average. At each step, potential peaks and troughs are identified as the highest and lowest values within a window width containing the previous five and the next five months. The turning points in the original series are finally determined within neighbourhoods of peaks and troughs obtained from the short-span moving average. If these dates satisfy the duration constraints, they are recognised as the final peaks and troughs for the series.

Harding and Pagan (2000, 2001, 2002) adapt the BB algorithm to quarterly data, where a peak (trough) occurring at time t is the maximum (minimum) value within $t \pm 2$ quarters. The resulting algorithm is known as the BBQ algorithm. The censoring rules are also adjusted to allow the minimum duration of a phase to be two quarters and a complete cycle to last at least 5 quarters.

It is worth noting one related technique proposed by Artis, Marcellino and Proietti (2004a). This methodology extends the BBQ algorithm, using the theory of Markov chains, to implement the minimum duration constraints and to enforce the alternation of peaks and troughs. The minimum duration of a full cycle determines the order of the Markov chain, whereas the minimum duration of a phase indicates the number of possible states. Applying this method to the euro area aggregate macroeconomic time-series, they obtain turning points which mimic those in Harding and Pagan (2001).

1.3.3 Measuring business cycles using multivariate information

It should be noted that both the parametric and non-parametric methods discussed above are based on a univariate framework. This means that the comovement among individual economic variables throughout the business cycle cannot be modelled. In the analysis of Burns and Mitchell (1946), the historical concordance of hundreds of series, including income, interest rates and prices, were investigated. The choice of which economic time-series should be used for business cycle dating is not straightforward. The standard measure is real GDP. However, this variable is unsuitable for gaining a timely and accurate insight into the current state of the economy due to its lagged publication and frequent revisions. Industrial production is also frequently used in the literature, but represents less than 20% of total output in the euro area. Rather than looking for a particular economic time-series to represent aggregate economic activity, an alternative approach is to construct the underlying common fluctuations among several time-series. The dynamic factor (DF) models follow this line of thinking. These models have long been used in cross-sectional analysis, and their generalisation to dynamic environments is set out in Sargent and Sims (1977), Geweke (1977), and Watson and Engle (1983). More recent examples include Forni *et al.* (2000, 2001), Stock and Watson (2002a, 2002b, 2005), Reichlin *et al.* (2006, 2007) and Jungbacker and Koopman (2008). The most prominent example is the single common factor model, proposed by Stock and Watson (1989, 1991, 1993) to estimate composite coincident and leading indices. They assume that one common dynamic factor drives the comovement of several economic time-series, and reflects the state of the overall economy. This model is applied in Chapter 2 to obtain composite indices for euro area countries. It is worth noting that, as the series used by Stock and Watson were not cointegrated, their model was therefore estimated in first-differences. However, the null hypothesis of no cointegration is rejected for several of the countries analysed in Chapter 2. Therefore, a two-step procedure is used where the error correction term is estimated independently using a vector error correction model (VECM) and then included in the subsequent model estimation. It is also possible to set up this model with variables in levels. The unknown parameters in the model can be estimated by maximum likelihood using an augmented Kalman filter initiated with a diffuse prior (De Jong, 1989, 1991).

In a slight variation of Stock and Watson's (1989, 1991, 1993) DF model, Diebold and Rudebusch (1996) propose the Markov-switching dynamic-factor (MSDF) model. This model incorporates nonlinear dynamics into the common factor extraction by combining the DF model with Hamilton's model. The MSDF model reflects two stylised features of the business cycle put forward by Burns and Mitchell (1946): the comovement among individual economic series through the cycle, and the asymmetry of business cycle phases. Kim and Yoo (1995), Chauvet (1998) and Kim and Nelson (1999c) all successfully implemented the MSDF using Kim's (1994) approximate MLE that combines the Kalman filter and Hamilton's filter, along with appropriate approximations.[9] To avoid these approximations, Kim and Nelson (2001) propose an alternative solution by using Gibbs sampling. The MSDF model has been applied by Chauvet (1998), Kim and Yoo (1995), Kim and Nelson (1999c, 2001), and Mills and Wang (2003b) to construct the coincident economic indicator and business cycle turning points for the US and the UK. Chapter 3 applies this method to the euro area countries. It performs fairly well in identifying business cycle turning points for large economies, such as Germany and France, where recessions and expansions are of roughly constant amplitude over the entire sample. However, it was less satisfactory at dating business cycles for countries, such as Belgium and the Netherlands, whose recessions were deeper during the 1970s and 1980s than in recent decades. Therefore, structural breaks are introduced into the model's parameters to improve parameter estimates and smoothed regime probabilities.

[9] Detailed discussions of Kim's (1994) filtering algorithm are presented in Appendix A3.

Another important extension to the univariate Hamilton model is the Markov-switching vector autoregression (MS-VAR) proposed by Krolzig (1997a, 1997b). This model is designed to detect common business cycle turning points among multivariate time-series without constructing any composite indices. In the general case, a MS-VAR of order p with M regimes, $\text{MS}(M)\text{-VAR}(p)$, can be expressed as follows:

$$\Delta Y_t = \mu(S_t) + \sum_{i=1}^{p} A_i(S_t)\Delta Y_{t-i} + \varepsilon_t(S_t),\ \varepsilon_t(S_t) \sim NID\left(0, \sum(S_t)\right), \quad (1.6)$$

where ΔY_t is an $N \times 1$ vector of the observed variables; μ is an $N \times 1$ vector of intercept terms; A_i is an $N \times N$ matrix of the autoregressive coefficients at lag i; and ε_t is an $N \times 1$ vector of disturbance terms. In this specification, both the intercepts, autoregressive parameters and disturbances are conditional on the unobserved state variable $S_t = 1,2,\ldots M$. Krolzig (1997b) also considers the case where time-series are cointegrated. Therefore, a Markov-switching vector error correction model (MS-VECM) is proposed:

$$\Delta Y_t = \mu(S_t) + \sum_{i=1}^{p-1} D_i \Delta Y_{t-i} + \Pi Y_{t-p} + \varepsilon_t(S_t),\ \varepsilon_t(S_t) \sim NID\left(0, \sum(S_t)\right), (1.7)$$

where $D_i = -\left(I_n - \sum_{j=1}^{i} A_j\right)$, $\Pi = I_n - \sum_{i=1}^{p} A_i = A(1) = -\alpha\beta^{\mathsf{T}}$, β is the $r \times N$ cointegrating matrix and α is the $N \times r$ corresponding loading matrix. To estimate the above model, Krolzig proposes a two-step procedure. In the first stage, Johansen's (1995) maximum likelihood procedure is used to determine the cointegrating rank and to estimate the cointegrating matrix, β. In the second stage, conditional on the estimated β, the remaining parameters of the MS-VECM are obtained using the EM algorithm.

Harding and Pagan (2006) develop a non-parametric procedure to replicate the NBER reference cycle by using the four time-series most frequently used by NBER researchers, non-farm employment, industrial production, trade sales and disposable income, over the period of 1951M1-2002M12. The BB algorithm is used to identify specific turning points from each series. Prior to constructing the reference cycle using these specific turning points, they propose heteroscedasticity and autocorrelation consistent tests to examine whether there exists a common cycle in the four series. Given the presence of a common cycle, a three-step non-parametric algorithm is then applied to consolidate specific turning points into a set of common turning points.

In the first step, the distances in months from time t to the closest peak (trough) in each series are found, denoted DP_{it} (DT_{it}). This gives a vector of dimension four. The median of the elements in this vector is then found, denoted DP_t (DT_t). In the second step, the potential common peaks (troughs) are picked out as local minima in DP_t (DT_t) with a window-width of δ centred at time t:

$$M^P = \{DP_{t+\Delta t} \geq DP_t\}, \text{ for all } |\Delta t| \leq \delta . \tag{1.8}$$

As such, M^P (M^T) is a vector containing the central dates of the cluster of peaks (troughs). Once the central dates are located, whether a specific peak belongs to a particular central date m_j^P or m_k^P is defined in the third step:

$$C(m_j^P) = \{d(m_j^P, t_{jt}) < d(m_k^P, t_{jt})\}, \text{ for all } j \neq k \text{ and } d(m_j^P, t_{jt}) \leq \bar{d}, \tag{1.9}$$

where $C(m_j^P)$ represents the cluster of peaks centred at m_j^P. $\bar{d}$ defines the maximum width of a cluster and is usually set to be 24 for monthly data and 8 for quarterly data. Clusters of troughs can be defined in a similar way. This algorithm can be considered to be a formalisation of the procedures used by the NBER. By applying this algorithm, Harding and Pagan (2006) produce a chronology very similar to the NBER reference cycle.

1.4 Modelling trends and cycles in macroeconomic time-series

This section discusses the main statistical techniques used to identify trends and cycles in macroeconomic time-series. As controversy remains over whether macroeconomic time-series are more appropriately represented as DS or TS processes, and whether there is interaction between trend growth and cyclical fluctuations, a wide variety of decomposition methods have been proposed, based on different assumptions about the variability and exogeneity of the trend. Depending on the methodology used, the resulting cyclical components may differ significantly in terms of cycle duration, amplitude and spectrum shape. Such sensitivity has been discussed by Canova (1998).

1.4.1 Structural model-based trend and cycle decompositions

This section begins with two model-based, univariate decomposition methodologies that produce starkly different results: the Beveridge-Nelson (BN) (1981) decomposition and the Unobserved-component (UC) models introduced by Clark (1989), Harvey and Jaeger (1993) and Harvey and Trimbur (2003). The BN decomposition typically yields a volatile trend and a small, noisy, cycle. In contrast, the UC models usually produce a smooth trend component and a highly persistent cycle.

The BN decomposition provides a measure of trend and cycle for an integrated time-series. It shows that any ARIMA $(p,1,q)$ process can be decomposed into the sum of a random walk plus a drift and a stationary component. Consider the Wold (1938) representation of a stationary first-differenced series $\Delta y_t = \beta + \lambda(L)\varepsilon_t = \beta + \sum_{j=0}^{\infty} \lambda_j L^j \varepsilon_t$, where β is the long-run mean of the $\{\Delta y_t\}$ process and ε_t are uncorrelated innovations with zero mean and variance σ^2. The BN trend component is defined as the infinite forecast of the time-series $\{y_t\}$ less the deterministic drift:

$$\mu_t = \lim_{k\to\infty}\left(\hat{y}_{t+k|t} - k\beta\right) = y_t + \left(\sum_{i=1}^{\infty}\lambda_i\right)\varepsilon_t + \left(\sum_{i=2}^{\infty}\lambda_i\right)\varepsilon_{t-1} + \ldots, \qquad (1.10)$$

where $\hat{y}_{t+k|t}$ is the k-step ahead linear predictor of y_{t+k} based on information at time t. The cycle component of $\{y_t\}$ is the difference between the trend and the value of y_t:

$$c_t = -\left(\sum_{i=1}^{\infty}\lambda_i\right)\varepsilon_t - \left(\sum_{i=2}^{\infty}\lambda_i\right)\varepsilon_{t-1} - \ldots \qquad (1.11)$$

The trend and cycle components can alternatively be given by

$$\mu_t = (1-L)^{-1}\beta + \lambda(1)(1-L)^{-1}\varepsilon_t, \tag{1.12}$$

$$c_t = (1-L)^{-1}[\lambda(L)-\lambda(1)]\varepsilon_t. \tag{1.13}$$

Thus the innovations of the BN trend and cycle are perfectly correlated. The trend component can also be expressed as $\mu_t = \frac{\lambda(1)}{\lambda(L)}y_t$. This is a one-sided trend estimator in the sense that only current and past observations are used in its construction. Proietti and Harvey (2000) further propose a two-sided BN estimator, $\mu_{t|T} = \frac{[\lambda(1)]^2}{\lambda(L)\lambda(L^{-1})}y_t$, by incorporating future observations.

Compared with the BN decomposition, where y_t is only decomposed into its trend and cycle components, the UC model allows for additional components to be separated from y_t. A univariate UC model may be written as

$$y_t = \mu_t + c_t + \gamma_t + v_t + \varepsilon_t,\ \varepsilon_t \sim NID(0,\sigma_\varepsilon^2), \tag{1.14}$$

where γ_t is the seasonal component, v_t is a first-order autoregressive component and ε_t is the irregular term. The stochastic trend component is given by

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t,\ \eta_t \sim NID(0,\sigma_\eta^2), \tag{1.15}$$
$$\beta_t = \beta_{t-1} + \xi_t,\ \xi_t \sim NID(0,\sigma_\xi^2),$$

where μ_t is the level of the trend and β_t is the slope of the trend. The slope parameter β_t allows the trend to change smoothly, but in the special case where $\sigma_\xi^2 = 0$ the trend reduces to a random walk with a drift, which is consistent with the BN trend. A variety of trend specifications can be obtained by imposing restrictions on the variance parameters σ_ε^2, σ_η^2 and σ_ξ^2. These are presented in the following table reported in Koopman *et al.* (2006).

Table 1.1 Some special level and trend specifications

Level	σ_ε^2	σ_η^2	
constant term	*	0	
local level (LL)	*	*	
random walk (RW)	0	*	
Trend	σ_ε^2	σ_η^2	σ_ξ^2
deterministic	*	0	0
LL with fixed slope	*	*	0
RW with fixed drift	0	*	0
local linear (LLT)	*	*	*
smooth trend	*	0	*
second differencing	0	0	*
Hodrick-Prescott	*	0	$1/1600\sigma_\varepsilon^2$

Note: * indicates any positive value.

With regard to the cycle, Clark (1989) specifies a stationary cyclical component as a finite autoregression. A trigonometric specification of the cyclical component is introduced by Harvey and Jaeger (1993), which is given by

$$\begin{bmatrix} c_t \\ c_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} c_{t-1} \\ c_{t-1}^* \end{bmatrix} + \begin{bmatrix} k_t \\ k_t^* \end{bmatrix}, \tag{1.16}$$

where k_t and k_t^* are two mutually uncorrelated white noise disturbances with zero mean and common variance σ_k^2. The parameter ρ is known as the damping factor with $0 < \rho \leq 1$; λ_c is the frequency in the range $0 < \lambda_c \leq \pi$. The stochastic cycle becomes a first-order autoregressive process if λ_c is 0 or π.

A higher order cycle is introduced by Harvey and Trimbur (2003), where an $i-th$ order stochastic cycle is specified as

$$\begin{bmatrix} c_{1,t} \\ c_{1,t}^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} c_{1,t-1} \\ c_{1,t-1}^* \end{bmatrix} + \begin{bmatrix} k_t \\ 0 \end{bmatrix}, \text{ and} \tag{1.17}$$

$$\begin{bmatrix} c_{i,t} \\ c_{i,t}^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} c_{i,t-1} \\ c_{i,t-1}^* \end{bmatrix} + \begin{bmatrix} c_{i-1,t} \\ 0 \end{bmatrix}, \text{ for } i = 2,\ldots n$$

The value of i determines the smoothness of the cycle. When $i = 6$, this model-based filter has a similar gain function to Baxter and King's (1999) filter, while in the case of $i = 1$, the generalised cycle reduces to Harvey and Jaeger's (1993) trigonometric specification.

A seasonal component is introduced by Harvey and Jaeger (1993) to avoid the potentially distorting effects of seasonal adjustment procedures. It can be modelled as the trigonometric form $\gamma_t = \sum_{j=1}^{[s/2]} \gamma_{j,t}$, where $\gamma_{j,t}$ is given by

$$\begin{bmatrix} \gamma_{j,t} \\ \gamma_{j,t}^{*} \end{bmatrix} = \begin{bmatrix} \cos\lambda_j & \sin\lambda_j \\ -\sin\lambda_j & \cos\lambda_j \end{bmatrix} \begin{bmatrix} \gamma_{j,t-1} \\ \gamma_{j,t-1}^{*} \end{bmatrix} + \begin{bmatrix} \omega_{j,t} \\ \omega_{j,t}^{*} \end{bmatrix}, \; j = 1, \ldots [s/2], \tag{1.18}$$

where $\lambda_j = 2\pi j/s$ is the frequency, and ω_t and ω_t^* are both $NID(0, \sigma_\omega^2)$.

It is worth noting that, in contrast to the BN decomposition, in which the innovations to the trend and cycle are perfectly correlated, the disturbances in the UC model that drive the unobserved components are mutually independent. Morley *et al.* (2003) demonstrate that, once the orthogonal restriction imposed on the trend and cyclical components in Clark's (1989) UC model is relaxed, it gives the same trend-cycle decomposition as the BN decomposition.

1.4.2 Filters

This subsection reviews three widely applied linear filters, by Hodrick and Prescott (HP) (1997), Baxter and King (BK) (1999) and Christiano and Fitzgerald (CF) (2003), that are based on the theory of spectral analysis. As defined by Burns and Mitchell (1946), the conventional definition of the business cycle considers fluctuations in the series associated with periodicities within the business cycle duration of 6 to 32 quarters. This corresponds to a business cycle frequency range of $\omega_{c1} = 2\pi/6$ to $\omega_{c2} = 2\pi/32$. An ideal band-pass filter, which gives a frequency response of unity in the band $\omega_{c1} \leq |\omega| \leq \omega_{c2}$ and zero elsewhere, is a useful tool for extracting the business cycle frequency components. It can be constructed as the difference between two ideal low-pass filters with cut-off frequencies ω_{c1} and ω_{c2}. The impulse response coefficients of an ideal band-pass filter are given by

$$a_B(L) = \frac{\omega_{c2} - \omega_{c1}}{\pi} + \sum_{j=1}^{\infty} \frac{\sin(\omega_{c2} j) - \sin(\omega_{c1} j)}{\pi j} \left(L^j + L^{-j}\right) \tag{1.19}$$

In practice, an approximation of this ideal filter is needed as the filter requires an infinite-order moving average that, in turn, requires a filtered series of infinite length. The BK and CF filters are based on the same ideal band-pass filter, but their approximation methods differ in two ways. Firstly, the BK filter assumes that the filtered time-series are independent and identically distributed, while the CF filter presumes that they follow a random walk. Secondly, the BK filter assumes symmetric weights whereas the CF filter does not. These two differences in assumptions lead to divergent cyclical components. The CF filter puts more weight on lower frequencies, whilst the BK filter places equal weight on all business cycle frequency components. As a consequence, the CF filter can produce more accurate low frequency business cycle components than the BK filter, while the BK filter estimates the ideal filter more accurately for shorter business cycle frequencies. In addition, the trend component in the filtered series is automatically removed by the BK filter as it assumes symmetric weights. However, the CF filter does not make this assumption and so the trend must be removed before applying this filter. Moreover, the assumption of symmetric weights results in a loss of observations at the beginning and end of the filtered series when the BK filter is used. Therefore, if the focus of the research is on cycles towards the end of the sample, it is advisable to employ the CF filter.

Unlike the above filters, the HP filter was designed to minimise fluctuations in the cyclical component, subject to a penalty for variation in the second-difference of the trend component:

$$\min\{\mu_t\}_{t=1}^{T} \sum_{t=1}^{T} \left\{(y_t - \mu_t)^2 + \lambda\left[(\mu_{t+1} - \mu_t) - (\mu_t - \mu_{t-1})\right]^2\right\}. \tag{1.20}$$

where λ is a Lagrangean multiplier that controls the smoothness of the trend. The higher the value of λ, the smoother the trend. In the limit, as λ approaches infinity, μ_t becomes a linear trend, while if λ tends to zero, the trend is equivalent to the original data series. Although many early studies fixed λ_Q at 1,600 for quarterly data, the optimal values for λ_Q lie between 1,000-1,050. For monthly and annual data, Mills (2003c) recommends $80{,}000 < \lambda_M < 160{,}000$ and $5 < \lambda_A < 10$, respectively.

The finite sample HP trend extraction filter, $\mu_T = (\lambda F + I_T)^{-1} y_T$, is given by the first-order conditions of equation (1.20), where y_T is the $(T \times 1)$ vector of the original series, and F is a Toeplitz matrix with diagonal band $[1,-4,6,-4,1]$, initial and end conditions $F_{11} = F_{TT} = 1$, $F_{22} = F_{T-1,T-1} = 5$ and $F_{12} = F_{21} = F_{T,T-1} = F_{T-1,T} = -2$, and zeros elsewhere. As with an ideal high-pass filter, the transfer function of the HP cycle filter, $c_T = (\lambda F + I_T)^{-1} \lambda F y_T$, is zero at zero frequency and approaches unity at π radians. However, since the transfer function increases gradually, a large proportion of low frequency components pass through the filter. This phenomenon is pronounced when a filtered series is integrated. This issue is discussed in more detail by Cogley and Nason (1995).

Harvey and Jaeger (1993) show how the HP trend filter may be rationalised as the optimal estimator of the trend component in a UC model, $y_t = \mu_t + c_t + \xi_t$, where μ_t is specified as a local linear trend with restrictions $\lambda = \sigma_\varepsilon^2 / \sigma_\xi^2$, $\sigma_\eta^2 = 0$ and $c_t = 0$ imposed.

1.4.3 Multivariate extensions of structural models

It is often the case that we wish to model trends and cycles of a group of time-series. For example, King *et al.* (1991) examine the implication of neoclassical growth theory by looking at whether US consumption, investment and output share a common stochastic trend. Mills and Harvey (2005) further analyse common trends, common/codependent cycles and common non-linearities in output, consumption and investment for the G7 countries. In addition, Vahid and Engle (1993) and Carlino and Sill (2001) investigate the existence of common trends and cycles among per capita incomes of US regions. Barillas and Schleicher (2005) further extend the multivariate analysis to Canadian sectoral output data. Furthermore, Rünstler (2002) and Camba-

Méndez and Rodriguez-Palenzuela (2001) find that multivariate UC models which incorporate output and other cyclical indicators, such as the inflation rate, capacity utilisation and factor inputs, can produce more reliable output gap estimates. Therefore, the univariate structural models and linear filtering discussed above are often extended to multivariate settings. The focus of this subsection is to outline the multivariate structural models, as these types of models are applied in the subsequent empirical chapters. In particular, Chapter 4 employs the multivariate BN decomposition, with common factor restrictions imposed, to provide a detailed insight into the trend and cyclical movements in the GDP series of the major euro members. Chapter 5 then utilises the multivariate UC model which combines statistical decomposition with macroeconomic relations to estimate various output gap measures for the aggregate euro area.

For the multivariate BN decomposition, common factor restrictions include both long-run restrictions imposed by the presence of common trends (Engle and Granger 1987, Stock and Watson, 1988b), and short-run restrictions imposed by common cycles (Vahid and Engle, 1993). The model is the Wold representation of a vector of differenced time-series. However, in practice, empirical studies are based on finite order VAR representations. Chapter 4 outlines a number of test statistics proposed by Vahid and Engle (1993, 1997), Hecq *et al.* (2000, 2006) and Schleicher (2007) to determine the number of common and codependent cycles among a set of stationary time-series. Chapter 4 also presents the multivariate BN decomposition proposed by Proietti (1997) and Hecq *et al.* (2000), which takes into account common trend and cycle restrictions.

A straightforward multivariate extension to the univariate UC model is the seemingly unrelated time-series equations (SUTSE) model (Harvey, 1989). It has a similar form to the univariate version, except that Y_t is an $N \times 1$ vector of observations, depending on unobserved components that are also vectors. Consider the case where Y_t can be decomposed into vectors of trends, cycles and irregular terms as follows

$$Y_t = \mu_t + \psi_t + \varepsilon_t,\ \varepsilon_t \sim [0, \Sigma_\varepsilon], \tag{1.21}$$

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t, \eta_t \sim [0, \Sigma_\eta], \tag{1.22}$$

$$\beta_t = \beta_{t-1} + \xi_t, \xi_t \sim [0, \Sigma_\xi], \tag{1.23}$$

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \left\{ \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \otimes I_n \right\} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} k_t \\ k_t^* \end{bmatrix}, \tag{1.24}$$

$$E(k_t k_t^{\mathsf{T}}) = E(k_t^* k_t^{*\mathsf{T}}) = \Sigma_k, \text{ and } E(k_t k_t^{*\mathsf{T}}) = 0 \tag{1.25}$$

Since all components and disturbances are $N \times 1$ vectors, the variances Σ_ε, Σ_η, Σ_ξ and Σ_k become $N \times N$ diagonal matrices. Strong restrictions are imposed so that the cycle components of different variables are assumed to have the same damping factor ρ and frequency λ_c. This implies that these cycles have the same properties, such as common autocovariance functions and spectra.

If N variables in Y_t are cointegrated, having r cointegrating vectors, there exist $N-r$ common trends. These common trends may arise through common levels, common slopes or both. To allow comparisons with the multivariate BN decomposition with common trend and cycle restrictions imposed, the slope parameter is fixed so that $\Sigma_\xi = 0$ and $\beta_t = \beta_{t-1} = \bar{\beta}$. In this case, $\mu_t = \Theta_\mu \bar{\mu}_t + \mu_0$, where $\bar{\mu}_t$ is a vector of $N-r$ common trends, μ_0 has zero for its first $N-r$ elements while its last r elements are unconstrained constants, and Θ_μ is an $(N \times (N-r))$ factor loading matrix. The model then becomes

$$Y_t = \Theta_\mu \bar{\mu}_t + \mu_0 + \psi_t + \varepsilon_t, \ \varepsilon_t \sim [0, \Sigma_\varepsilon], \tag{1.26}$$

$$\bar{\mu}_t = \bar{\mu}_{t-1} + \bar{\beta} + \bar{\eta}_t, \ \bar{\eta}_t \sim [0, D_\eta], \tag{1.27}$$

where $\beta = \Theta_\mu \bar{\beta}$, $\eta_t = \Theta_\mu \bar{\eta}_t$ and $\Sigma_\eta = \Theta_\mu D_\eta \Theta_\mu^{\mathsf{T}}$.

Similarly, the existence of common cycles implies that $\psi_t = \Theta_\psi \bar{\psi}_t$, where $\bar{\psi}_t$ is the $((N-s) \times 1)$ vector of common cycles and Θ_ψ is the $(N \times (N-s))$ factor loading matrix such that $\Sigma_\psi = \Theta_\psi D_\psi \Theta_\psi^{\mathsf{T}}$. Therefore, a model with common levels and cycles imposed can be written as

$$Y_t = \Theta_\mu \hat{\mu}_t + \mu_0 + \Theta_\psi \hat{\psi}_t + \varepsilon_t, \tag{1.29}$$

$$\hat{\mu}_t = \hat{\mu}_{t-1} + \hat{\beta} + \hat{\eta}_t, \tag{1.30}$$

$$\begin{bmatrix} \hat{\psi}_t \\ \hat{\psi}_t^* \end{bmatrix} = \left\{ \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \otimes I_n \right\} \begin{bmatrix} \hat{\psi}_{t-1} \\ \hat{\psi}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \hat{k}_t \\ \hat{k}_t^* \end{bmatrix}. \tag{1.31}$$

To ensure the factor loading matrices Θ_μ and Θ_ψ are identifiable, they are restricted to lower triangular matrices, and the variance matrices D_η and D_ψ are set to be diagonal to ensure that common factors are uncorrelated. The low triangularity restrictions imposed on Θ_μ and Θ_ψ are merely for estimation purposes. Once the model parameters are estimated, the common trends and cycles can then be premultiplied by any orthogonal matrix. This allows the transformed common factors to be more easily interpreted.

More sophisticated multivariate extensions of the UC model are used in Chapter 5 to estimate the unobserved features of the euro area economy. These multivariate UC models typically combine output decomposition with potentially useful information about the supply side of the economy and the macroeconomic relations containing business cycle information, such as the Phillips curve and Okun's law. The advantages of the multivariate UC model are two-fold. Firstly, its flexibility outperforms any other decomposition model, including those listed above. Applying this model reveals not only the unobserved quantities of an economy, such as the output gap, core inflation and the natural rate of unemployment, but also allows for the rich dynamic interactions which occur between the unobserved and observed quantities to be modelled in specific ways according to the objectives of the research. This is demonstrated by the bivariate and trivariate specifications used in Chapter 5. Secondly, this model is preferable to the purely statistical decomposition methodologies as it can provide more reliable output gap measures (Camba-Méndez and Rodriguez-Palenzuela, 2001; Rünstler, 2002). Chapter 5 provides a short literature review of this type of multivariate UC model.

1.5 Measuring business cycle synchronisation

Once business cycle or growth cycle measures for members of a currency union are obtained, synchronisation of these cyclical components can be evaluated using certain statistical measures. The correlation coefficient is frequently used in the literature to measure contemporaneous and lead/lag correlations for each pair of cycles (Artis and Zhang 1997, 1999; Agresti and Mojon, 2001; Darvas and Szapáry, 2004). More sophisticated measures of correlation have also been developed, such as the concordance index of Harding and Pagan (2002), the dispersion measure of Artis, Marcellino and Proietti (2004a), and multivariate UC models with dynamic converging mechanisms proposed in Luginbuhl and Koopman (2004) and Koopman and Azevedo (2008).

The index of concordance proposed by Harding and Pagan (2002) measures the fraction of time which two series spend in the same business cycle phase. This index for countries i and j can be constructed as follows

$$I_{ij} = T^{-1}\sum_{t=1}^{T}\{S_{jt}S_{it}+(1-S_{jt})(1-S_{it})\} \tag{1.32}$$

where S_{jt} and S_{it} are binary variables obtained from regime classification with unity denoting expansions and zero indicating recessions. Under the assumption that S_{jt} and S_{it} are independent, the estimate of the expected value of the concordance index is $E(I_{ij}) = 1 + 2\overline{S}_j\overline{S}_i - \overline{S}_j - \overline{S}_i$. Subtracting this from I_{ij} gives the mean corrected concordance index:

$$I_{ij}^{*} = 2T^{-1}\sum_{t=1}^{T}\{(S_{it}-\overline{S}_i)(S_{jt}-\overline{S}_j)\}. \tag{1.33}$$

Artis, Marcellino and Proietti (2004a) propose a test statistic based on a standardised concordance index. I_{ij}^{*} is divided by a consistent estimate of its standard error under the null hypothesis of independence, which is the square root of

$$\hat{\sigma}^2 = \hat{\gamma}_i(0)\hat{\gamma}_j(0) + 2\sum_{\tau=1}^{l}\frac{T-\tau}{T}\hat{\gamma}_i(\tau)\hat{\gamma}_j(\tau) \tag{1.34}$$

where $\hat{\gamma}_i(\tau)$ is the lag τ sample autocovariance of S_{it} and l is the truncation parameter.

Harding and Pagan (2002) also propose a linear regression approach to test the independence of a national business cycle, S_{it}, from the reference cycle, S_{jt}. This regression equation is given by

$$S_{jt} = \alpha + \beta S_{it} + \varepsilon_t. \tag{1.35}$$

The Newey-West estimator of the standard error is used to obtain the heteroscedasticity and autocorrelation consistent (HAC) t-statistic for the null hypothesis that $\beta = 0$.

Unlike various concordance indices which compute bilateral correlations in fixed sample periods, the dispersion measure proposed by Artis, Marcellino and Proietti (2004a) can provide a measure of synchronisation within a group of binary series at each point of time. This dispersion measure is constructed using the diffusion index. This index measures how diffuse business cycle fluctuations are across a group of binary series on a 0-1 continuous scale. If there are N binary series, the diffusion index is given by

$$D_t = \sum_{i=1}^{N} w_i S_{it},\ t = 1,\ldots,T,\ \sum_{i=1}^{N} w_i = 1. \tag{1.36}$$

The dispersion measure can then be computed as $D_t(1-D_t)$. It has a maximum value of 0.25 when $D_t = 0.5$, and a minimum value of zero when all series are in the same phase of their business cycles. A measure is also proposed to evaluate the dispersion within a group of growth cycles using the following weighted variance of the individual cycles, ψ_{it}, from the average cycle, $\bar{\psi}_t$: $\frac{1}{\sum_i w_i}\sum_{i=1}^{N} w_i(\psi_{it} - \bar{\psi}_t)^2$, $\bar{\psi}_t = \frac{1}{\sum_i w_i}\psi_{it}$.

In addition to analysing bilateral correlations, Camacho *et al.* (2006) compute the combined correlation of several bilateral correlation coefficients. The Fisher transformation of an individual bilateral correlation coefficient, $\varsigma_i = \tanh^{-1}(r_i) = 0.5(\ln(1+r_i) - \ln(1-r_i))$, is used to calculate the combined correlation of N bilateral coefficients as follows:

$$\varsigma = \tanh^{-1}(r) = \frac{1}{T_1 + T_2 + \ldots + T_N}(T_1\varsigma_1 + T_2\varsigma_2 + \ldots + T_N\varsigma_N) \tag{1.37}$$

where T_i denotes the size of sample i. The aggregate correlation coefficient can then be recovered from ς as $r = \tanh(\varsigma)$.

Based on frequency domain analysis, Croux *et al.* (2001) propose a measure known as dynamic correlation to evaluate synchronisation of two time-series across different frequencies. The dynamic correlation of two series x_t and y_t within the business cycle frequency band is specified as

$$\rho_{xy}(\Lambda^+) = \frac{\int_{\Lambda^+} C_{xy}(\lambda)d\lambda}{\sqrt{\int_{\Lambda^+} S_x(\lambda)d\lambda \int_{\Lambda^+} S_y(\lambda)d\lambda}}, \tag{1.38}$$

where Λ^+ denotes the business cycle frequency band. The spectrum of x and y are denoted as $S_x(\lambda)$ and $S_y(\lambda)$, and the cross spectrum between x and y is $S_{xy}(\lambda)$. $C_{xy}(\lambda) = \text{real}(S_{xy}(\lambda))$ is the cospectrum between x and y. For infinite time-series this measure is identical to the static correlation between two band-pass filtered series. However, for finite economic time-series this equality does not hold as both the band-pass filter and the dynamic correlation are estimated imperfectly. Croux *et al.* (2001) also construct a measure of cohesion based on the weighted average of dynamic correlations, which provides a summary measure of comovement within a group of variables. The cohesion within the business cycle frequency band Λ^+ is given by

$$coh_x(\Lambda^+) = \frac{\sum_{i \neq j} w_i w_j \rho_{x_i x_j}(\Lambda^+)}{\sum_{i \neq j} w_i w_j}, \tag{1.39}$$

where w_i, $i = 1,2,\dots N$, are the positive weights associated with the variables x_i, $i = 1,2,\dots N$. Croux *et al.* calculate the cohesion of the output series for the US and the euro area, and conclude that the former is higher than the latter across all frequencies.

Rather than computing statistical indices, recent research, including Camacho and Perez-Quiros (2006), Luginbuhl and Koopman (2004) and Koopman and Azevedo (2008), proposes model-based approaches to evaluate the degree of synchronisation. Camacho and Perez-Quiros (2006), for example, use bivariate MS models to investigate the unobserved states of two business cycles. There are two extreme cases of business cycle correlation, where two business cycles are completely independent (two independent Markov processes are hidden in the bivariate model), or where they are perfectly synchronised (only one Markov process for both variables). As the actual correlation observed between two cycles will lie somewhere between these two extremes, Camacho and Perez-Quiros model the data generating process as a linear combination of these two extreme situations. The distance between each pair of cycles is then measured as the distance from the case of perfect correlation. Furthermore, they simulate 100 pairs of output growth series with perfectly synchronised business cycles and 100 pairs with completely independent cycles. The BBQ algorithm and the univariate MS model are applied to these simulated series to identify turning points. Bilateral correlations among these 200 pairs of cycles are then evaluated using the concordance index. In addition, they compute their measure of business cycle distance. Camacho and Perez-Quiros conclude that the bivariate MS model outperforms the conventional univariate approaches used by Harding and Pagan (2002) and Guha and Banerji (1998), as the univariate approaches often find a low level of synchronisation, especially where two business cycles are perfectly synchronised.

Luginbuhl and Koopman (2004) use the standard SUTSE model, outlined in equations (1.21)-(1.31), imposed with time-varying rank-reduction mechanisms, to analyse the convergence of per capita GDP between five euro area countries. They define convergence as a reduction in the ranks of covariance matrices associated with the disturbance vectors driving trends, cycles and volatilities. The trend and cycle components are driven by the variance matrices, Σ_ξ and Σ_k, which can be decomposed as $\Sigma_\xi = \Theta_\mu D_\xi \Theta_\mu^\top$ and $\Sigma_\psi = \Theta_\psi D_\psi \Theta_\psi^\top$ using Cholesky decompositions. If convergence occurs in the cycle components, the number of nonzero diagonal elements of D_ψ reduces. A gradual movement of a particular nonzero diagonal element $d_{\psi,i}$ towards zero can be modelled using the following logit function

$$d^*_{\psi,i,t} = d_{\psi,i} \exp\left(s_{\psi,i,t}\right) / \left\{1 + \exp\left(s_{\psi,i,t}\right)\right\}, \quad s_{\psi,i,t} = s_{\psi,i} \times \left(t - \tau_{\psi,i}\right). \qquad (1.40)$$

The parameter $s_{\psi,i}$ determines how quickly the function $s_{\psi,i,t}$ approaches zero and $\tau_{\psi,i}$ determines the mid-point of the change. A similar logit function is also introduced into the diagonal matrix D_ξ to account for gradual convergence in the trend components.

Koopman and Azevedo (2008) further explore how multivariate UC models can be used to investigate growth cycle relations among the real GDP series of six euro area countries and the US. In particular, the multivariate UC model with time-varying phase shifts and time-varying relations between cycles is proposed. As the main focus of their study is the cycle components, the CF filter is used to isolate business cycle frequency fluctuations. The multivariate cycle model proposed in Rünstler (2004) is then fitted to the CF filtered series to account for phase shifts between cycles:

$$y_t = diag\left\{\cos\left(\lambda d_\xi\right)\right\}\psi_t + diag\left\{\sin\left(\lambda d_\xi\right)\right\}\psi_t^+, \qquad (1.41)$$

where y_t is a vector of CF filtered time-series. The vector $d_\xi = \left(0, \xi_2, \ldots \xi_N\right)$ has its first element restricted to zero, so that the phase of the first cycle in ψ_t is the reference for phase shifts of the other cycle elements. The phase shift between y_{it} and y_{jt} is measured by $\xi_i - \xi_j$ for $i, j = 1, \ldots, N$. As with Luginbuhl and Koopman (2004), the time-varying phase shifts are modelled using the logit function

$$\xi_{i,t} = \xi_i \times \exp\left(s_{\xi,i,t}\right) / \left\{1 + \exp\left(s_{\xi,i,t}\right)\right\}, \quad s_{\xi,i,t} = s_{\xi,i} \times \left(t - \tau_{\xi,i}\right), \qquad (1.42)$$

for $i = 1, \ldots, N$, and where $s_{\xi,i}$ determines the shape of the logit function and $\tau_{\xi,i}$ determines the mid-point of the change. They define convergence to be when phase shifts between two cycles turn towards zero, such as $\xi_{i,t} - \xi_{j,t} = 0$. Their UC specification also incorporates the time-varying relation feature. The covariance matrix of the cycle disturbance vector can be specified as $\Sigma_k = CRC$, where C is a diagonal matrix of standard deviations and R contains contemporaneous correlations:

$$R = \begin{bmatrix} 1 & \rho_{k,2,1} & \cdots & \rho_{k,N,1} \\ \rho_{k,2,1} & 1 & & \rho_{k,N,2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{k,N,1} & \rho_{k,N,2} & \cdots & 1 \end{bmatrix}, \tag{1.43}$$

Similar logit functions are also imposed on $\rho_{k,i,j}$ to capture changes in the relationships between cycles.[10] These may have two distinct periods: in the first period cycles are correlated, $|\rho_{k,i,j}| < 1$, whereas in the second period the cycles are perfectly correlated, $\rho_{k,i,j} = 1$.

1.6 Business cycle and growth cycle synchronisation in the euro area

Various studies have examined the synchronisation of cyclical indicators aross the euro area countries. These studies can broadly be divided into two groups according to whether they analyse growth cycle or business cycle synchronisation. A large number of studies have focused on growth cycles because cyclical fluctuations separated from trend growth are usually stationary, and most statistical analyses of synchronisation require stationary series as inputs. In addition, since classical business cycles occur much less frequently than growth cycles, analysing the latter will provide more information on the comovement of cyclical fluctuations.

Artis and Zhang (1997) is an example of the type of growth cycle synchronisation analysis that has been undertaken. The HP filter is used to obtain the cyclical components of industrial production indices for the ERM countries. By computing contemporaneous correlation coefficients between Germany and the other ERM member countries both before and after the introduction of ERM, they show that growth

[10] $\rho_{k,i,j} = \pm\left|1-(1-b)\times\exp(s_{k,i,j,t})\{1+\exp(s_{k,i,j,t})\}^{-1}\right|$, $s_{k,i,j,t} = s_{k,i,j}\times(t-\tau_{k,i,j})$. The coefficient b ensure that the correlation, $\rho_{k,i,j}$, is between b and one.

cycles in the ERM countries become more synchronised with Germany in the post-ERM period. This suggests that fixed exchange rate regimes may have a positive impact on growth cycle synchronisation.

In contrast to Artis and Zhang (1997), Inklaar and de Haan (2001) do not find a clear connection between exchange rate regimes and growth cycle synchronization. They apply the HP filter to an updated sample period. Unlike Artis and Zhang (1997), who split the whole sample period into two subsamples, Inklaar and de Haan evaluate changes in growth cycle correlations over four sub-samples (1960:1-1971:3, 1971:4-1979:4, 1979:4-1987:9 and 1987:10-1997:12), as they believe that exchange rate stability is not uniform across each of these periods. They find that there is an increase in cycle correlation (with the German cycle) in 1971-1979, but this was reversed in 1979-1987. This finding contradicts the assertion that fixed exchange rate regimes increase the degree of growth cycle synchronisation.

Massmann and Mitchell (2004) seek to resolve this dispute by using a series of rolling windows, rather than just two or four windows of fixed width as utilised by Artis and Zhang (1997) and Inklaar and de Haan (2001). In addition, Massmann and Mitchell use a number of univariate trend-cycle decomposition methodologies, including the BN decomposition, the UC model, and the BK and HP filters, to identify growth cycle measures from industrial production indices for 12 euro area countries from 1960M1 to 2001M8. The BB algorithm is also used to identify classical business cycle turning points in the monthly industrial production indices. The minimum duration of a phase and of a full cycle are restricted to be at least 6 and 15 months, respectively. Rather than focusing on individual bilateral correlation coefficients, Massmann and Mitchell construct the mean and variance of all bilateral correlation coefficients between the 12 countries for each rolling window. Cycle convergence is defined to be when the estimated mean correlation coefficients tend towards unity and the variances tend towards zero over time. Although they confirm Canova's (1998) conclusion that the cycles identified vary significantly depending on the methodology used, these differences do not translate to the measures of cycle convergence, as they find periods of common convergence and divergence. The mean correlation coefficient appears to follow an upward trend until the mid-1970s, when this process is reversed and the mean falls to zero in the mid to late 1980s. This is consistent with Inklaar and de Haan's

(2001) finding that the growth cycle correlations of EU countries (with the German cycle) are higher in 1971-1979 than 1979-1987. There is also evidence of increasing cycle convergence in the run-up to EMU, as the mean coefficients increase and the variances fall over this period. This finding is confirmed by Angeloni and Dedola (1999), who also conclude that, in the pre-EMU period from 1993Q1 to 1997Q1, bilateral correlation coefficients between Germany and other ERM countries increased across all growth cycle fluctuations in prices as well as in real economic performance, such as real GDP and industrial production indices. This may suggest that monetary integration contributes to greater growth cycle synchronisation between member countries. This finding is supported by Darvas and Szapáry (2004) but rejected by Camacho *et al.* (2006).

Some studies classify euro area countries into core and peripheral groups. The core members generally refer to Germany, France, Italy, Belgium, the Netherlands and Austria. The first five countries were the original founding members of the EU and the sixth, Austria, had a fixed exchange rate with the Deutsche Mark from the 1960s. Compared to the core members, the peripheral countries (Finland, Ireland, Portugal, Spain and Greece) joined the EU much later, and hence joined the common market at a later date. It is generally found that the core euro area countries exhibit a higher degree of growth cycle synchronisation with the euro area as a whole compared to those in the peripheral group. This may suggest that monetary integration has had some influence on national growth cycles, and thus explains the weaker linkages found for the 'latecomers'. However, the high correlation between the core group and the euro area as a whole may simply reflect the large weights assigned to the core countries when constructing aggregate euro area data series. Agresti and Mojon (2001) is a typical example of the core-periphery literature. They compare the contemporary and lead/lag correlation coefficients of 10 euro area growth cycles with the aggregate euro area cycle. The BK filter is used to identify cyclical fluctuations in real GDP, consumption, investment and short-term interest rates for individual countries and the aggregate euro area. They conclude that large and core countries exhibit a greater degree of growth cycle synchronisation with the euro area than do peripheral countries. Darvas and Szapáry's (2004) findings are broadly consistent with Agresti and Mojon (2001). In order to check the robustness of their results, Darvas and Szapáry use the HP and BK filters to identify the cyclical fluctuations in real GDP and its components for 10 euro area countries and the aggregate euro area. The evolution of cycle correlations between individual countries and the euro area cycle is evaluated using five measures (cycle correlation, leads/lags, volatility, persistence of the cycle and a measure of impulse-response) across four non-overlapping five-year subsamples between 1983-2002. In addition to concluding that the core countries on average share a greater degree of synchronisation than the peripheral countries with the euro area cycle, they also find that synchronisation has significantly increased during the last two subsample periods, 1993-1997 and 1998-2002.

Darvas and Szapáry (2004), Artis, Marcellino and Proietti (2004b) and Camacho *et al.* (2006) further analyse synchronisation between eight Central and Eastern European countries (CEECs)[11] and the euro area. Darvas and Szapáry (2004) conclude that, apart from Hungary, Poland and Slovenia, synchronisation between the other CEECs and the euro area remains low. Rather than using static measures of cycle correlation as in Darvas and Szapáry (2004), Artis, Marcellino and Proietti (2004b) compute bilateral dynamic correlations between band-pass filtered industrial production indices of eight CEECs with the major euro area countries. They again observe that, with the exceptions of Hungary, Poland and Slovenia, the other countries are diverging from the euro area. Furthermore, they identify an even lower level of growth cycle synchronisation between the euro area and the CEECs compared to the previous accession countries at the time when they were about to participate in the EU.[12] This finding may raise concerns as to the appropriateness of most CEEC countries adopting the euro in the near future. Camacho *et al.* (2006) consider three different measures of cycle comovements that have been proposed in the recent literature. These include the correlations of the VAR forecast errors (den Haan, 2000), dynamic correlation defined by Croux *et al.* (2001), and the linear regression approach proposed in Harding and Pagan (2002). In addition to analysing pairwise correlations across countries, Camacho *et al.* (2006) study the comovements both within and between the euro area and the CEECs. To do so, bilateral correlation coefficients are aggregated using the approach given in equation (1.37). Despite the heterogeneity of these measures, the paper

[11] The eight CEECs include Estonia, Czech Republic, Hungary, Latvia, Lithuania, Poland, Slovak Republic and Slovenia.

[12] They compute dynamic correlation estimates between the earlier accession countries, including Ireland, the UK, Greece, Spain, Portugal, Austria, Finland and Sweden, and a set of EU member countries (Germany, Italy and France).

concludes that synchronisation among euro area members is much higher than between the euro area and the CEECs. However, unlike Darvas and Szapáry (2004) and Massmann and Mitchell (2004), Camacho *et al.* conclude that the establishment of the EMU has not significantly increased synchronisation across the euro area, and that the synchronisation among member countries emerged prior to the introduction of the euro. This is because the aggregate correlation within the euro area is higher during 1962-1975 than the period of 1990-2003.

Other studies compare growth cycle synchronisation within the euro area with synchronisation in the US. Since the US regions and states operate in a fixed exchange rate system, the experience of the US may indicate whether fixed exchange rates produce more synchronised growth cycles. Wynne and Koo (2000) compare growth cycle correlations between EU countries with correlations between 12 Federal Reserve districts in the US. The growth cycle fluctuations in real GDP, employment and prices are recovered using the BK filter. The pairwise correlation coefficients and the associated standard errors obtained using GMM estimates are used to assess the degree of growth cycle synchronisation within the EU and the US. They identify a significantly greater degree of cycle correlation within the US Federal Reserve districts than between European countries. They conclude that, if EMU member countries stay inside the EMU for a very long time, their cyclical fluctuations may become more synchronised like the US regions. This view is also supported by Croux *et al.* (2001), who compare the level of cohesion among 51 US states, 8 US regions, 17 European countries and 11 EMU members. As expected, they find that the US regions have the highest degree of cohesion at all frequencies, followed by US states, EMU members and European countries. The difference in cohesion is large between Europe and the US at business cycle frequencies, but is much smaller at lower frequencies.

Recent examples, including Luginbuhl and Koopman (2004) and Koopman and Azevedo (2008), apply multivariate UC models with time-varying mechanisms to account for gradual changes in cycle correlations. These models have an advantage over conventional approaches in that they automatically capture the changes in cycle correlation, thus avoiding the need to arbitrarily split the sample period into several subsamples. However, a major drawback of these models is that changes in cycle correlation can only be revealed to increase or decrease. As a consequence, these models cannot show periods of cycle convergence and divergence across a given sample period, as in Inklaar and de Haan (2001) and Massmann and Mitchell (2004). By applying the multivariate UC model imposed with time-varying rank-reduction mechanisms, Luginbuhl and Koopman (2004) find that both slope and cycle components in per capita GDP of five European countries began converging after the introduction of the ERM. In particular, the cyclical components of the GDP series for Italy, the Netherlands, and Spain converged to the German and French cycles at the beginning of the 1990s. Koopman and Azevedo (2008) further extend the multivariate UC model to incorporate time-varying phase shifts and time-varying cycle relations. The bivariate specification is used to analyse growth cycle comovements between euro area countries and the aggregate euro area. In addition to finding a high degree of growth cycle synchronisation for France and Germany with the euro area across the sample period, the Italian and Spanish growth cycles are also found to become more synchronised with the euro area over time. Italy displays a more synchronised growth cycle with the euro area from 1980 onwards, which may suggest that the formation of the ERM had a positive impact on raising growth cycle synchronisation. An increase in cycle correlation of Spain with the euro area occurred in the 1990s, and roughly coincided with the introduction of the Common Market in 1993.

In short, studies analysing growth cycle synchronisation, as outlined above, have obtained very mixed results. Artis and Zhang (1997) and Darvas and Szapáry (2004) find evidence of greater growth cycle synchronisation after countries joined a currency arrangement or a monetary union. However, others, including Camacho *et al.* (2006), do not. Instead of studying comovement of growth cycles, another strand of research evaluates the concordance of business cycle turnings points identified using the MS model-based approaches or variants of the BB algorithm.

Artis, Krolzig and Toro (2004), for example, investigate whether there exists a common European business cycle using industrial production indices for nine EU countries from 1970 to 1996. With the exception of Germany, three-regime MS models are applied to the other countries analysed instead of the two-regime model as proposed in Hamilton (1989). Therefore, this approach distinguishes between three regimes: recessions, moderate growth and fast growth periods, rather than just expansions and recessions. Synchronisation is evaluated using pairwise correlation coefficients between the smoothed recession probabilities and Pearson contingency coefficients between binary variables over the entire sample period. Since both indices indicate a reasonable level of business cycle synchronisation, they conclude that business cycles in the nine EU countries may be driven by a common underlying factor. Therefore, the MS-VAR approach proposed by Krolzig (1997a, 1997b) is employed to identify common regime shifts among the nine industrial production indices. As a result, common recessions are identified during 1974-1975, 1979-1982 and 1990-1992, which roughly coincide with the two oil price shocks and the ERM crisis. Furthermore, based on the estimated MS-VAR model, they analyse the impulse response of each industrial production index to a shift in regimes. They find that the industrial production indices of France, Germany, the Netherlands, Belgium and the UK respond in a similar manner, in terms of timing and magnitude, when the common regime shifts from moderate growth to recession. They also observe that Spain, Portugal and France react the most strongly when the regimes shift to fast growth periods. As with Artis, Krolzig and Toro (2004), Altavilla (2004) also uses the univariate MS model, but with two regimes, to date business cycle turning points in real GDP series for five euro area members, the aggregate euro area and the US between 1980 and 2001. The comovement of each national business cycle with the reference cycles, the euro area or the US cycle, is studied by computing the concordance index and correlation coefficient over two subsamples: the pre-Maastricht period (1980-1991) and the post-Maastricht period (1992-2002). They find a moderate increase in business cycle synchronisation between member countries and the aggregate euro area over these two periods. A three-regime MS-VAR model with regime-dependent intercept and heteroscedasticity is fitted to the real GDP series of the five euro area members to detect common turning points among these series. Three common recessions are identified during 1980, 1992 and 2001.

In contrast to Artis, Krolzig and Toro (2004) and Altavilla (2004), who rely on business cycle indicators obtained from individual series using the univariate MS approach, Camacho and Perez-Quiros (2006) analyse business cycle synchronisation of the G7 countries, based on the bivariate MS model. They use the parameter estimate of the distance from the full dependence case to indicate the degree of divergence between each pair of business cycles. Their results suggest that three euro area countries, Germany, France and Italy, share more synchronised business cycle dynamics with each other but nonsynchronised cycles with English speaking countries (the UK, the US and Canada).

Harding and Pagan (2001), Garnier (2003) and Artis, Marcellino and Proietti (2004a) apply modified versions of the BB algorithm to identify business cycle turning points. Harding and Pagan (2001) use the quarterly BB algorithm to obtain turning points in the real GDP series for six euro area countries and the aggregate euro area. Various concordance indices are computed to indicate synchronisation between national business cycles and the aggregate euro area. They conclude that synchronisation between euro area business cycles remains low. Garnier (2003) compares four variants of the BB algorithm to date turning points in industrial production indices for a large number of countries, including 12 euro area members, three EU but non-EMU countries (Denmark, Sweden and the UK), and two non-EU countries (the US and Japan) over the period of 1962-2001. Pearson correlation coefficients and mean corrected concordance indices of each business cycle with German and US reference cycles are computed over two subsamples, pre-EMU and post-EMU. Although mixed results are obtained when measuring synchronisation between individual euro area cycles and the German cycle, both measures indicate that euro area business cycles are increasingly independent of the US cycle.[13] Last but not least, Artis, Marcellino and Proietti (2004a) extend the BBQ algorithm using the theory of Markov chains to date turning points in the industrial production indices for 12 euro area countries. The degree of business cycle synchronisation is evaluated using a dispersion measure. They conclude that there is no clear tendency for either convergence or divergence.

In general, studies which assess both growth cycle and business cycle synchronisation conclude that synchronisation is found to be weaker in business cycles than growth cycles. These include Artis, Marcellino and Proietti (2004a), Altavilla (2004) and Harding and Pagan (2002).

[13] Pearson coefficients indicate that, on average, synchronisation between individual euro area cycles and the German cycle has declined, while mean corrected concordance indices suggest the opposite.

1.7 Conclusions

This chapter illustrates some of the controversies and difficulties incurred when trying to identify business cycle information from historical data. It provides a literature review of dating classical business cycle turning points and extracting trends and cycles from macroeconomic time-series. Furthermore, various measures of synchronisation are discussed, ranging from simple correlation-coefficient to complicated model-based approaches. The existing literature on evaluating the synchronisation of classical cycles and growth cycles in the euro area is reviewed. There is still no consensus on whether fixed exchanges rate regimes or monetary union have resulted in increased cycle synchronisation. The thesis aims to provide a fresh look at business cycle synchronisation in the euro area by introducing time-series models that may overcome some of the problems in the literature. In the subsequent empirical chapters, the synchronisation of classical business cycles is evaluated using turning points identified using multivariate information, rather than just from individual economic variables as used in most previous studies (i.e., Harding and Pagan, 2002; Garnier, 2003; Artis, Marcellino and Proietti, 2004a). In addition, a multivariate approach is used to analyse growth cycle synchronisation for seven major euro area countries. One benefit of this approach is that it does not require prior filtering or decomposition of the GDP series and, in turn, avoids the sensitivity problem encountered by using different decomposition methodologies. Instead of analysing individual EMU member states, Chapter 5 focuses on the euro area economy as a whole by assessing three issues concerning the aggregate euro area output gap using multivariate unobserved component models. The impact that changes in real interest rates have on the output gap and inflation rates is evaluated using the most appropriate models as suggested by the criteria used in the analysis.

Chapter 2 - Evaluating the Synchronisation of Euro Area Business Cycles: An Application of Nonparametric Business Cycle Dating Methodology

2.1 Introduction

Chapter 1 surveyed a number of empirical studies that evaluate the degree of classical business cycle synchronisation by comparing the concordance of turning points. Among recent examples, Beine *et al.* (2003) and Artis, Krolzig and Toro (2005) find a high degree of synchronisation among business cycles of euro area members. However, Harding and Pagan (2001) and Altavilla (2004) conclude that the level of business cycle synchronisation remains relatively low compared to growth cycle synchronisation. Moreover, Camacho *et al.* (2006) conclude that the introduction of the euro has not significantly increased synchronisation across the euro area, and that the synchronisation among member countries occurred prior to the formation of EMU. Another strand of the literature, which includes Camacho *et al.* (2008), Artis, Marcellino and Proietti (2005) and Krolzig and Toro (2005), evaluates synchronisation by examining the similarities and differences of business cycle characteristics. The average business cycle duration, amplitude and shape are compared across countries and notable differences are found. Camacho *et al.* (2008) also analyse the evolution of business cycle characteristics over two subsamples (1962-1989 and 1990-2004). They find that variances in business cycle characteristics increase over time.

To date, much of the empirical literature, including that mentioned above, has measured business cycle synchronisation using cycles identified from individual macroeconomic time-series, such as industrial production and real GDP. However, only analysing univariate time series may not be optimal for dating business cycles. First, although real GDP is the broadest output variable, it is less cyclical and subject to more frequent revisions than other macroeconomic indicators. In contrast, industrial production appears more volatile than GDP but represents less than 20% of the total output of the euro area. More importantly, a large number of studies in dating classical business cycles, including Burns and Mitchell (1946), Stock and Watson (1989, 1991, 1993, 1999) and Hamilton (2003), have highlighted the comovement of many macroeconomic variables as being a key feature of a business cycle. This feature cannot be analysed by looking at individual variables as previous studies have done.

It is also believed that the accuracy of business cycle identification improves when more variables are included in the cycle dating analysis. However, there are no fixed rules as to which economic variables should be used, how many should be included, and what data frequency should be considered for the analysis. The NBER business cycle dating committee dates US business cycle turning points by using four monthly variables; real income, industrial production, volume of sales and employment. These variables are known as coincident macroeconomic variables, as they share a common cycle with the unobserved state of the economy. Data limitations mean that such analysis cannot be replicated for the euro area. Instead, a committee founded by the Centre for Economic Policy Research (CEPR) suggests analysing a number of quarterly coincident macroeconomic variables, such as real GDP, industrial production, gross fixed capital formation and employment, to date business cycles for the aggregate euro area and for individual member states.

The main objective of this chapter is to evaluate classical cycle synchronisation using turning points identified from multivariate information. To do this, Stock and Watson's (1989, 1991, 1993) single dynamic factor (DF) model is employed to estimate a composite index of a number of coincident macroeconomic variables. Harding and Pagan's (2000, 2001, 2002) quarterly extension of the Bry and Boschan (1971) procedure, the BBQ algorithm, is then used to identify turning points in this index. The two-step business cycle dating strategy used in this chapter is discussed in Harding and Pagan (2001), who defined this approach as locating turning points in an aggregated index.

The rest of this chapter is organised as follows. Section 2.2 presents the BBQ algorithm and Stock and Watson's DF model. The properties of the data, along with a modified DF model incorporating error correction terms, are discussed in section 2.3. In section

2.4, the business cycle turning points for the aggregate euro area and individual countries are reported. The concordance of turning points and the similarities of cycle characteristics are evaluated in sections 2.5 and 2.6, respectively. Finally, section 2.7 concludes.

2.2 Locating turning points in the composite index using the BBQ algorithm

The business cycle dating strategy utilised in this study involves two steps. First, the single DF model is used to estimate a composite index which is a weighted average of four coincident macroeconomic variables. Turning points of this index are then identified using the BBQ algorithm. By applying the BBQ algorithm a peak (trough) at time t is defined as the maximum (minimum) value during the period from $t-k$ to $t+k$, where $k=2$ for quarterly data. This algorithm is expressed in the following two equations,

$$peak\ at\ t = \{(y_{t-2}, y_{t-1}) < y_t > (y_{t+1}, y_{t+2})\}, \tag{2.1}$$

$$trough\ at\ t = \{(y_{t-2}, y_{t-1}) > y_t < (y_{t+1}, y_{t+2})\}, \tag{2.2}$$

where y_t is the composite index. A recession is declared if y_t declines for two consecutive quarters. Recessions and expansions can be highlighted once turning points are located. A recession starts from one quarter after a peak to the following trough, whilst an expansion is the period from one quarter after a trough to the subsequent peak.

Stock and Watson's single DF model assumes the existence of an underlying common dynamic factor, which drives the comovements of individual coincident economic variables. As the variables used by Stock and Watson were integrated of order one, but not cointegrated, their model is estimated in first differences.[1] The growth rate of each variable consists of a common factor and an idiosyncratic component:

[1] The four coincident variables used by Stock and Watson (1989, 1991, 1993) are industrial production, real personal income less transfer payments, real manufacturing and trade sales and employee hours in non-agricultural establishments.

$$\Delta Y_{it} = D_i + \gamma_i(L)\Delta C_t + e_{it},\ i = 1,2,\ldots,4, \tag{2.3}$$

where ΔC_t determines the comovement of different economic variables and is orthogonal to $D_i + e_{it}$, which capture the idiosyncratic fluctuations of each variable. $\gamma_i(L)$ is a polynomial in the lag operator, L, and contains the parameter estimates of current and lagged values of ΔC_t, which reflect the sensitivity of each variable to the common factor. As employment data may slightly lag the common factor rather than being an exact coincident variable, $\gamma_i(L)$ is set to be $\gamma_i(L) = \gamma_{i0} + \gamma_{i1}L + \ldots + \gamma_{i\tau}L^{\tau}$ when employment growth is the dependent variable, otherwise it is set as $\gamma_i(L) = \gamma_{i0}$. The data generating structures of ΔC_t and e_{it} are modelled as stationary autoregressive processes:

$$\phi(L)\Delta C_t = \delta + v_t,\ \ v_t \sim NID(0,\sigma_v^2), \tag{2.4}$$

$$\psi_i(L)e_{it} = \varepsilon_{it},\ \varepsilon_{it} \sim NID(0,\sigma_i^2). \tag{2.5}$$

This model is completed by assuming that the innovations, ε_{it} and v_t, are mutually and serially uncorrelated. Stock and Watson estimated the above model using first-differenced variables, standardised to have zero mean and unit variance. An advantage of using demeaned variables is to solve the over identification problem by removing two components, D_i and δ, from the model estimation. Therefore, equations (2.3)-(2.5) become

$$\Delta y_{it} = \gamma_i(L)\Delta c_t + e_{it},\ i = 1,2,\ldots,4, \tag{2.6}$$

$$\phi(L)\Delta c_t = v_t,\ v_t \sim NID(0,\sigma_v^2), \tag{2.7}$$

$$\psi_i(L)e_{it} = \varepsilon_{it}, \varepsilon_{it} \sim NID(0,\sigma_i^2), \tag{2.8}$$

with $\Delta c_t = \Delta C_t - \delta$. To implement the Kalman filter (KF)[2], equations (2.6)-(2.8) are recast in state-space representation as

$$\Delta y_t = H\beta_t, \tag{2.9}$$

[2] The Kalman filter is discussed in Appendix B2.

$$\beta_t = F\beta_{t-1} + \varepsilon_t,\ \varepsilon_t \sim NID(0,Q). \tag{2.10}$$

Equations (2.9) and (2.10) are the measurement and transition equations of the state-space model, respectively. $\Delta y_t = [\Delta y_{1t}, \Delta y_{2t}, \Delta y_{3t}, \Delta y_{4t}]^{\mathrm{T}}$ is the vector containing four coincident economic variables. $\beta_t = [\Delta c_t, \Delta c_{t-1}, \Delta c_{t-2}, \Delta c_{t-3}, e_{1t}, e_{1t-1}, \ldots, e_{4t}, e_{4t-1}]^{\mathrm{T}}$ contains the current and lagged values of the common factor and innovation terms. The time-invariant matrices F, H and Q, contain the hyperparameters:

$$H = \begin{bmatrix} \gamma_1 & 0 & 0 & 0 & 1 & 0 & \ldots & 0 & 0 \\ \gamma_2 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ \gamma_3 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ \gamma_{40} & \gamma_{41} & \gamma_{42} & \gamma_{43} & 0 & 0 & \ldots & 1 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi_{11} & \psi_{12} & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & \psi_{41} & \psi_{42} \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 1 & 0 \end{bmatrix}$$

$$\text{and } Q = \begin{bmatrix} \sigma_v^2 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_1^2 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & \sigma_4^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \end{bmatrix}.$$

MLE is used to estimate the model's hyperparameters: $\{\phi_i, \psi_i, \gamma_i, \sigma_i\}$, based on the prediction error decomposition. Given the MLE parameter estimates, the unobserved vector β_t is calculated, with the first element being Δc_t.

2.2.1 Calculating the mean growth rate $\hat{\delta}$

Since the BBQ algorithm identifies turning points as the local maxima and minima in the level of economic activity, the level of the common factor needs to be found from the first differences:

$$C_{t|t} = C_{t|t-1} + \Delta c_{t|t} + \hat{\delta} , \tag{2.11}$$

where $\hat{\delta}$ is the estimated mean growth rate of ΔC_t given by

$$\begin{aligned} \hat{\delta} = E[\Delta C_{t|t}] &= E[W(L)\Delta Y_t \,] \\ &= W(1)E(\Delta Y_t) \\ &= W(1)\Delta \bar{Y} . \end{aligned} \tag{2.12}$$

$W(1)$ can be computed by iterating the KF as follows (where K_t denotes the Kalman gain)

$$\begin{aligned} \beta_{t|t} &= \beta_{t|t-1} + K_t(\Delta y_t - H\beta_{t|t-1}) \\ &= F\beta_{t-1|t-1} + K_t\Delta y_t - K_t HF\beta_{t-1|t-1} \\ &= (I - K_t H)F\beta_{t-1|t-1} + K_t\Delta y_t , \end{aligned} \tag{2.13}$$

As t approaches infinity, K_t approaches the steady-state Kalman gain, K. If K_t is plotted for $t = 1,2,...,T$, it becomes apparent that K_t converges to a steady-state value reasonably fast. Once the steady state is reached, the equalities $K_t = K$ and $\beta_{t|t} = \beta_{t-1|t-1}$ are found. Thus, equation (2.13) can be rewritten as

$$\beta_{t|t} = (I - (I - KH)FL)^{-1} K\Delta y_t \tag{2.14}$$

$W(1)$ is the first row of $(I - (I - KH)F)^{-1}K$, where $K = K_T$ is obtained from the last iteration. Given the value of $W(1)$, $\hat{\delta}$ can easily be calculated. By setting the initial value of $C_{t|t}$ (i.e., $C_{0|0}$) to be zero, the time series of $C_{t|t}$ is obtained. The scale of $C_{t|t}$ only reflects the speed of a country's economic growth during the studied period, rather than the size of the economy.

2.3 The properties of the data and modified models

Five coincident macroeconomic variables: real GDP, industrial production (IP), gross fixed capital formation (GFCF), retail trade (Sales) and civilian employment, are collected for the aggregate euro area and the following member states: Germany, France, Italy, Austria, Belgium, the Netherlands, Spain and Finland.[3] In the literature, these countries are usually divided into two groups: the core (Germany, France, Italy, Austria, Belgium and the Netherlands) and peripheral countries (Spain and Finland), based on their exchange rate behaviour against the Deutsche Mark (DM) in the past. Three non-EMU countries (the UK, the US and Canada) are also included in the analysis to benchmark the degree of cycle synchronisation which has occurred in the euro area. All series are seasonally adjusted, quarterly observations and expressed in logarithms (times 100). Real GDP and GFCF are taken from the OECD Quarterly National Accounts database. IP and Sales are taken from the OECD Main Economic Indicators database. Employment data for most countries are taken from the OECD Labour Force Statistics.[4]

Four of the five variables mentioned above are chosen to estimate the composite index for each country analysed. Although there are no set rules specifying which variables should be used, real GDP and employment data are preferred. The former is the broadest measure of output and the latter can provide an indication of labour market flexibility, which is an important criterion when judging the optimality of a monetary union. The time series of all the logged variables used are plotted in Figure A2.1 in Appendix A2. Pronounced outliers are observed in some of the series. To reduce the degree of non-normality in the residuals detected by the Jarque-Bera test, dummy variables are used for the affected time periods.

[3] The sales data are not available for the aggregate euro area and Spain.

[4] Employment data for France, Belgium and the Netherlands are taken from Datastream, with the series codes of FROCFEMPO, BGOCFETNO and NLOCFETNO, respectively. The aggregate euro area employment data is taken from the AWM database constructed at the ECB by Fagan et al. (2001) and updated using Datastream, data code EMEMPTOTO.

To examine whether the model outlined above is appropriate for the data used in this chapter, the Augmented Dickey-Fuller (ADF) (Dickey and Fuller, 1979) and Johansen's (1995) cointegration tests are conducted, with the test statistics reported in Tables A2.1 and A2.2 in Appendix A2. The ADF tests are unable to reject the null hypothesis of a unit root in the levels of the variables but reject the null when first differences are used.[5] However, the Johansen cointegration tests indicate the presence of one cointegrating vector among the four variables used for the aggregate euro area, France, Belgium, Italy, the UK, the US and Canada, and two for the Netherlands and Spain. Therefore, the measurement equation is modified as follows:

$$\Delta y_t = H\beta_t + A \times ECM_{t-1}, \tag{2.15}$$

where ECM_{t-1} is a vector of error correction terms and A is a matrix of corresponding adjustment parameters. A two-step estimation procedure is proposed. In the first stage, ECM_{t-1} is estimated independently from the VECM. In the second stage, conditional on the ECM_{t-1}, the remaining parameters are obtained.

2.4 Empirical results

The common factor for each country analysed is estimated using the DF factor model outlined above. The parameter estimates are reported in Tables 2.1-2.12. The time path of $C_{t|t}$, along with recessions identified using the BBQ algorithm, are plotted in Panels 1-12 of Figure 2.1.[6] These results will be discussed over the next a few sections, divided up by core, peripheral and non-EMU countries.

[5] The ADF tests with a constant and a linear time trend included indicate that the IP data for the aggregate euro area, the Netherlands and Canada, and the GDP data for Belgium and US may be trend stationary.
[6] The common factor growth rate for each country analysed is plotted in Figure A2.2, Appendix A2.

2.4.1 The aggregate euro area and core EMU countries

The aggregate euro area. The four coincident macroeconomic variables used to estimate the unobserved common factor for the aggregate euro area are real GDP, GFCF, IP and employment during the period 1975Q3-2006Q4. As one cointegrating vector is identified by the trace and eigenvalue statistics, the modified DF model including one error correction term is used, with the parameter estimates reported in Table 2.1. The common factor for the aggregate euro area is modelled as an AR(2) process. The value of $\phi_1 + \phi_2 = 0.58$ indicates the persistency of this common factor. Positive estimates of γ_i suggest that all four variables follow pro-cyclical patterns with respect to the common factor. The size of γ_i determines the response of individual variables to the common factor fluctuations. γ_1 is the largest followed by γ_2, γ_3 and γ_{40}. This indicates that real GDP responds the most to the common factor fluctuations, while employment responds the least. The fact that γ_{41} is statistically significant also suggests that employment lags the common factor. Unlike the other idiosyncratic terms, the idiosyncratic fluctuations of employment follows an AR(4) process. This may also imply slow adjustment in the euro area labour market. A significantly positive adjustment parameter, α_{13}, in the IP equation confirms the presence of a long-run relationship among the variables. The mean growth rate of the common factor, δ, is estimated to be 1.03 and is equivalent to a trend growth of 4.1 per cent per annum.

The time series of $C_{t|t}$ for the aggregate euro area is plotted in Panel 1 of Figure 2.1. The BBQ algorithm identifies three recessions over the period: 1980Q2-1981Q1, 1982Q2-1982Q4 and 1992Q2-1993Q2. Although these are not all consistent with the cycle dates produced by the CEPR business cycle dating committee[7], the fact that no recessions are detected during the 2000s is in line with the committee's findings.

[7] The three recessions identified by the committee since 1970 are 1974Q3-1974Q1, 1980Q1-1982Q3, and 1992Q1-1993Q3.

Table 2.1: Parameter estimates for DF model for the euro area

Common Factor									
ϕ_1	0.247 (0.111)*								
ϕ_2	0.336 (0.108)**								
Idiosyncratic Components									
ΔGDP	γ_1	-	-	-	ψ_{11}	ψ_{12}	-	α_{11}	σ_1^2
	0.794**				-0.058	-0.001		0.102	0.187**
	(0.080)				(0.200)	(0.006)		(0.148)	(0.055)
ΔGFCF	γ_2	-	-	-	ψ_{21}	ψ_{22}	-	α_{12}	σ_2^2
	0.664**				-0.346**	0.023		-0.083	0.280**
	(0.072)				(0.129)	(0.124)		(0.126)	(0.054)
ΔIP	γ_3	-	-	-	ψ_{31}	ψ_{32}	-	α_{13}	σ_3^2
	0.647**				0.132	0.273*		0.299*	0.389**
	(0.086)				(0.158)	(0.125)		(0.160)	(0.070)
ΔEMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	ψ_{41}	ψ_{42}	ψ_{44}	α_{14}	σ_4^2
	0.234**	0.237**	0.071	0.086	0.144	0.112	0.342**	-0.169	0.310**
	(0.061)	(0.062)	(0.064)	(0.063)	(0.094)	(0.093)	(0.102)	(0.141)	(0.043)

Long run growth rate: $\delta = 1.029$

Error correction term

$$\underset{}{\text{GDP}_{t-1}} = 18.598 - \underset{(0.211)}{0.932} \times \text{GFCF}_{t-1} + \underset{(0.273)}{2.726} \times \text{IP}_{t-1} - \underset{(0.329)}{0.163} \times \text{EMP}_{t-1}$$

Log-likelihood: -588.894

Diagnostics	**Q(4)**	**Jarque-Bera**
ΔGDP	6.390	4.338
ΔGFCF	5.048	4.650
ΔIP	8.529	26.622**
ΔEMP	6.525	9.747**

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for the euro area were estimated using data from 1975Q3-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: ΔGDP: 0.0054, 0.0047; ΔGFCF: 0.0056, 0.014; ΔIP: 0.0043, 0.0098; ΔEMP: 0.0014, 0.0028.

Germany. Unlike the aggregate euro area, no cointegration was found among the four variables used for Germany. Therefore, the DF model is simply specified in first differences. As a break in the employment data is observed in 1980Q1, a corresponding dummy variable is included in the employment equation to reduce the degree of non-normality in the residuals detected by the Jarque-Bera test. In contrast to the data generating structure of the common factor obtained for the aggregate euro area, the German common factor follows a white noise process with both ϕ_1 and ϕ_2 being small and insignificant, as illustrated in Table 2.2. As suggested by the parameter estimates of γ_i, and in line with the findings for the aggregate euro area, employment is the least responsive, among the four variables analysed, to movements in the common factor for Germany. It also lags the common factor fluctuations, with γ_{41}, γ_{42} and γ_{43} being positive and significant. These results partly reflect the comparative rigidity of the German labour market. In addition, the idiosyncratic fluctuations of real GDP, GFCF and employment all follow AR(4) data generating structures. The estimated mean growth rate appears to be rather low, only around 1.3 per cent per annum. This is largely due to the slow growth in German employment.[8]

In total, six recessions (i.e. 1973Q2-1975Q2, 1977Q2-1977Q3, 1980Q2-1982Q4, 1992Q2-1993Q2, 1995Q3-1996Q1 and 2001Q2-2003Q2) are found over the sample, as shown in Panel 2 of Figure 2.1. The recessions which occurred in the 2000s appear shallow compared to previous downturns, reflecting the increased moderation of the German business cycle.

France. One cointegration vector was found among the four variables used for France. Therefore the modified model is used with one error correction term included in the system equations. The adjustment parameters α_{11} and α_{12}, in Table 2.3, are significant. This confirms a long-run relationship among the four variables. The value of $\phi_1 + \phi_2 = 0.77$ indicates that the common factor is more persistent for France than for the aggregate euro area. Pro-cyclical patterns between individual variables and the common factor are also revealed by the positive estimates of γ_i. As with the aggregate euro area and Germany, employment again exhibits the smallest response to changes in the common factor and also lags the common factor fluctuations. This probably stems from the relative rigidity of French labour markets and the historically strong labour unions.

The time path of $C_{t|t}$, and the three recessions: 1980Q2-1981Q1, 1982Q3-1984Q2 and 1992Q2-1993Q4, are plotted in Panel 3 of Figure 2.1. It can be seen that, although no recessions are detected during the 2000s, the French economy has slowed down after a period of comparatively strong economic growth at the end of 1990s. The estimated long-run growth rate for the French common factor is about 4 per cent per annum.

[8] The quarterly mean growth rate of German employment is 0.05 percentage points, compared to 0.14 percentage points for the aggregate euro area.

Table 2.2: Parameter estimates for DF model for Germany

				Common Factor					
ϕ_1	0.073 (0.110)								
ϕ_2	0.158 (0.112)								
				Idiosyncratic Components					
Δ **GDP**	γ_1	-	-	-	-	ψ_{11}	ψ_{12}	ψ_{14}	σ_1^2
	0.811**					-0.034	-0.109	0.634**	0.167**
	(0.081)					(0.146)	(0.104)	(0.113)	(0.065)
Δ **GFCF**	γ_2	-	-	-	-	ψ_{21}	ψ_{22}	ψ_{24}	σ_2^2
	0.609**					-0.254*	-0.016	0.207*	0.473**
	(0.086)					(0.097)	(0.012)	(0.102)	(0.071)
Δ **IP**	γ_3	-	-	-	-	ψ_{31}	ψ_{32}	-	σ_3^2
	0.738**					0.002	0.020		0.427**
	(0.086)					(0.118)	(0.129)		(0.087)
Δ **EMP**	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D80q1	ψ_{41}	ψ_{42}	ψ_{44}	σ_4^2
	0.208**	0.268**	0.168*	0.118*	5.648**	0.094	-0.077	0.253*	0.435**
	(0.066)	(0.064)	(0.063)	(0.065)	(0.685)	(0.092)	(0.092)	(0.091)	(0.056)
Long run growth rate: δ = 0.374									
Log-likelihood: -642.009									
Diagnostics				**Q(4)**			**Jarque-Bera**		
Δ **GDP**				2.072			5.831		
Δ **GFCF**				5.002			4.448		
Δ **IP**				4.863			4.980		
Δ **EMP**				5.543			66.704**		

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%.The parameter estimates for Germany were estimated using data from 1970Q1-2004Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0056, 0.0098; Δ GFCF: 0.0038, 0.0278; Δ IP: 0.0037, 0.0157; Δ EMP: 0.0005, 0.0051.

Italy. One cointegration relationship was identified by the Johansen cointegration test among the four variables analysed for Italy. The presence of this long-run relationship is further confirmed by the significant adjustment parameters α_{11} and α_{13}. A dummy variable is included to catch the sudden drop in employment during 1992Q1, with the parameter estimate of this dummy found to be negative and significant. The Italian common factor follows an AR(2) process, but ϕ_2 has a negative sign. Although the Italian employment exhibits contemporaneous movements with the common factor, the value of γ_{40} appears to be rather low. As the long-run growth rate is estimated to be 0.72, this is equivalent to 2.9 per cent per annual.

Compared to the other countries analysed, the Italian economy appears to be very unstable with nine recessions identified over the sample period: 1974Q4-1975Q2, 1977Q2-1977Q3, 1982Q2-1983Q2, 1984Q4-1985Q1, 1992Q2-1993Q4, 1996Q2-1996Q4, 2001Q2-2001Q4, 2003Q1-2003Q2 and 2004Q4-2005Q1.

Table 2.3: Parameter estimates for DF model for France

				Common Factor				
ϕ_1	0.419(0.138)**							
ϕ_2	0.349(0.125)**							
				Idiosyncratic Components				
Δ GDP	γ_1	-	-	-	ψ_{11}	ψ_{12}	α_{11}	σ_1^2
	0.640**				-0.008	0.000	0.359*	0.378**
	(0.073)				(0.145)	(0.001)	(0.140)	(0.075)
Δ IP	γ_2	-	-	-	ψ_{21}	ψ_{22}	α_{12}	σ_2^2
	0.685**				0.023	0.161	0.669**	0.278**
	(0.080)				(0.189)	(0.163)	(0.162)	(0.073)
Δ Sales	γ_3	-	-	-	ψ_{31}	ψ_{32}	α_{13}	σ_3^2
	0.229**				-0.294**	0.015	0.007	0.808**
	(0.067)				(0.093)	(0.095)	(0.095)	(0.105)
Δ EMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	ψ_{41}	ψ_{42}	α_{14}	σ_4^2
	0.361**	0.066	0.104	0.136*	0.247*	0.207*	0.037	0.290**
	(0.069)	(0.096)	(0.077)	(0.070)	(0.112)	(0.106)	(0.177)	(0.044)

Long run growth rate: δ = 0.982

Error correction term

$$\underset{}{GDP_{t-1}} = 32.360 + \underset{(0.188)}{2.324} \times IP_{t-1} + \underset{(0.294)}{0.667} \times Sales_{t-1} - \underset{(0.679)}{3.152} \times EMP_{t-1}$$

Log-likelihood: -568.881

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	3.719	3.764
Δ IP	4.278	4.035
Δ Sales	0.856	12.576**
Δ EMP	3.164	1.837

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for France were estimated using data from 1975Q4-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0053, 0.0044; Δ IP: 0.0035, 0.0115; Δ Sales: 0.0037, 0.0104; Δ EMP: 0.0013, 0.0026.

Table 2.4: Parameter estimates for DF model for Italy

Common Factor

ϕ_1	0.719 (0.128)**
ϕ_2	-0.129 (0.046)**

Idiosyncratic Components

Δ GDP	γ_1	-	-	-	-	ψ_{11}	ψ_{12}	-	α_{11}	σ_1^2
	0.697**					-0.097	-0.002		0.267*	0.184**
	(0.076)					(0.190)	(0.009)		(0.133)	(0.066)
Δ GFCF	γ_2	-	-	-	-	ψ_{21}	ψ_{22}	-	α_{12}	σ_2^2
	0.403**					-0.052	0.007		0.001	0.714**
	(0.069)					(0.088)	(0.088)		(0.101)	(0.089)
Δ IP	γ_3	-	-	-	-	ψ_{31}	ψ_{32}	-	α_{13}	σ_3^2
	0.540**					-0.471**	0.002		0.355**	0.319**
	(0.058)					(0.118)	(0.107)		(0.110)	(0.063)
Δ EMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D92q4	ψ_{41}	ψ_{42}	ψ_{43}	α_{14}	σ_4^2
	0.160*	0.046	-0.074	0.093	-4.171**	-0.076	-0.212*	0.229**	-0.148	0.665**
	(0.089)	(0.119)	(0.123)	(0.097)	(0.797)	(0.091)	(0.086)	(0.086)	(0.114)	(0.080)

Long run growth rate: $\delta = 0.725$

Error correction term

$$\underset{}{GDP_{t-1}} = 2.645 + \underset{(0.105)}{0.091} \times GFCF_{t-1} + \underset{(0.086)}{1.142} \times IP_{t-1} + \underset{(0.243)}{0.521} \times EMP_{t-1}$$

Log-likelihood: -683.465

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	0.722	3.921
Δ GFCF	4.797	5.315
Δ IP	8.904	27.577**
Δ EMP	0.703	8.419*

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for Italy were estimated using data from 1970Q1-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0056, 0.0079; Δ GFCF: 0.0042, 0.0181; Δ IP: 0.0043, 0.0223; Δ EMP: 0.0012, 0.0064.

Austria. Upon plotting the four time series used for Austria, it became apparent that there were large breaks in the real GDP, GFCF and employment data series. Consequently dummy variables were inserted in the quarters when these breaks occurred. It is also worth noting that real GDP and GFCF appear significantly smoother from 1988 onwards due to different methodologies being used to deseasonalise these series pre-1988 and post-1988. As with Germany, both the trace and eigenvalue statistics cannot reject the null that the four variables used for Austria are not cointegrated. As such, the DF model is specified in first differences. One striking result observed in Table 2.5 is that γ_{40} appears negative and insignificant, while γ_{41} is positive and significant. This suggests that, although the one-period lagged Austrian employment variable is pro-cyclical to the common factor, its contemporaneous movement is found to be anti-cyclical. This may reflect the shortcomings of centralised wage bargaining in Austria, which is aimed at long run equity objectives and job preservation at the expense of greater labour market rigidity. The annual growth rate of the Austrian common factor is estimated to be 3.2 per cent.

The time path of $C_{t|t}$ is plotted in Panel 5 of Figure 2.1. Five recessions are highlighted during 1974Q2-1975Q2, 1980Q2-1982Q4, 1984Q1-1984Q2, 1992Q3-1992Q4 and 2001Q2- 2002Q4.

Table 2.5: Parameter estimates for DF model for Austria

				Common Factor					
ϕ_1	0.274 (0.136)*								
ϕ_2	0.234 (0.136)*								
				Idiosyncratic Components					
Δ **GDP**	γ_1	-	-	-	D78q1	-	ψ_{11}	ψ_{12}	σ_1^2
	0.573**				-6.095**		-0.106	-0.003	0.308**
	(0.078)				(0.759)		(0.162)	(0.009)	(0.071)
Δ **GFCF**	γ_2	-	-	-	D78q1	D03q1	ψ_{21}	ψ_{22}	σ_2^2
	0.476**				-6.993**	3.819**	-0.022	0.098	0.252**
	(0.067)				(0.649)	(0.586)	(0.133)	(0.136)	(0.053)
Δ **IP**	γ_3	-	-	-	-	-	ψ_{31}	ψ_{32}	σ_3^2
	0.498**						-0.007	0.192*	0.656**
	(0.089)						(0.096)	(0.094)	(0.096)
Δ **EMP**	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D82q1	D04q1	ψ_{41}	ψ_{42}	σ_4^2
	-0.031	0.316**	-0.157	0.082	4.593**	-3.921**	-0.119	0.174	0.571**
	(0.086)	(0.095)	(0.100)	(0.090)	(0.779)	(0.771)	(0.096)	(0.093)	(0.078)
Long run growth rate: δ =0.808									

Log-likelihood: -615.155		
Diagnostics	**Q(4)**	**Jarque-Bera**
Δ **GDP**	1.827	9.336*
Δ **GFCF**	5.193	7.588*
Δ **IP**	1.603	1.883
Δ **EMP**	2.964	27.36**

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for Austria were estimated using data from 1970Q1-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0058, 0.0087; Δ GFCF: 0.0085, 0.0155; Δ IP: 0.0050, 0.0221; Δ EMP: 0.0019, 0.0077.

The Netherlands. The trace statistic indicates one cointegrating vector among the four variables used for the Netherlands, while the eigenvalue statistic suggests two. As the two error correction terms estimated from the VECM appear stationary, they are both included in the DF model. The inference provided by the eigenvalue statistic is supported by the significant adjustment parameters, α_{12}, α_{13}, α_{22} and α_{23}. The Dutch common factor is the most persistent among the countries analysed with $\phi_1 + \phi_2 = 0.915$. One striking feature observed in the Dutch results is that the γ_i are small but ψ_{11}, ψ_{21} and ψ_{41} are large. This may imply that the four variables analysed exhibit less comovement but more individual fluctuations.

As revealed in Panel 6 of Figure 2.1, the growth of the Dutch common factor was steady after the severe recession which occurred in the early 1980s. This strong performance was boosted by the wide-ranging structural and regulatory reforms undertaken in the 1980s and by the fast growth in foreign trade. Four complete recessionary periods are identified in 1974Q4-1975Q3, 1980Q2-1983Q2, 1992Q3-1993Q2 and 2002Q1-2004Q1. The recession triggered by the ERM crisis in the early 1990s was short lived. However, the Dutch economy struggled during 2002 and 2003, due to rising labour costs and weak domestic demand. A sustained recession is identified during this period, reflecting the breakdown in the country's previously strong economic performance. The estimated long-run growth for the Netherlands is around 6.5 per cent per annum. It is significantly higher than the euro area average.

Belgium. One error correction term is included in the DF model for Belgium, with the adjustment parameter, α_{13}, found to be significant. The estimated common factor also appears very persistent with $\phi_1 + \phi_2 = 0.86$. Unlike most of the countries analysed above, the Belgian employment data appears more responsive to the common factor, with γ_{40} being the largest among all the estimated γ_i. Six recessions are identified for the Belgian economy during 1974Q4-1975Q3, 1977Q1-1977Q2, 1980Q2-1983Q2, 1991Q1-1991Q3, 1992Q3-1993Q4 and 2001Q2-2002Q1. The estimated mean growth rate for the Belgian common factor is around 4.8 per cent per annum.

Table 2.6: Parameter estimates for DF model for Netherlands

					Common Factor					
ϕ_1	1.417 (0.123)**									
ϕ_2	-0.502 (0.087)**									
					Idiosyncratic Components					
Δ GDP	γ_1	-	-	D79q1	-	ψ_{11}	ψ_{12}	α_{11}	α_{21}	σ_1^2
	0.103**			-3.797**		-0.301**	-0.023	-0.506	-0.681	0.663**
	(0.032)			(0.878)		(0.098)	(0.015)	(0.548)	(0.558)	(0.082)
Δ Sales	γ_2	-	-	D78q1	D94q1	ψ_{21}	ψ_{22}	α_{12}	α_{22}	σ_2^2
	0.079**			2.614**	-4.695**	-0.307**	-0.024	-1.171*	-1.330*	0.635**
	(0.024)			(0.815)	(0.776)	(0.095)	(0.015)	(0.505)	(0.510)	(0.076)
Δ IP	γ_3	-	-	-	-	ψ_{31}	ψ_{32}	α_{13}	α_{23}	σ_3^2
	0.091**					-0.131	-0.004	2.176**	1.830**	0.747**
	(0.034)					(0.100)	(0.007)	(0.675)	(0.672)	(0.091)
Δ EMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D96q1	ψ_{41}	ψ_{42}	α_{14}	α_{24}	σ_4^2
	0.073	0.056	0.019	0.141*	-1.703**	0.775**	-0.150	0.366	0.425	0.041**
	(0.058)	(0.072)	(0.079)	(0.056)	(0.188)	(0.281)	(0.109)	(0.337)	(0.422)	(0.009)

Long run growth rate: $\delta = 1.638$

Error correction terms

$GDP_{t-1} = -8.524 - \underset{(0.708)}{3.444} \times IP_{t-1} + \underset{(0.629)}{4.156} \times EMP_{t-1}$ $\qquad$ $Sales_{t-1} = 63.86 + \underset{(3.871)}{24.946} \times IP_{t-1} - \underset{(3.442)}{19.337} \times EMP_{t-1}$

Log-likelihood: -560.705

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	4.756	34.106**
Δ Sales	4.568	17.841**
Δ IP	4.092	1.072
Δ EMP	2.037	11.294**

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for the Netherlands were estimated using data from 1970Q1-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0064, 0.0121; Δ Sales: 0.0018, 0.0192; Δ IP: 0.0041, 0.0181; Δ EMP: 0.0029, 0.0063.

Table 2.7: Parameter estimates for DF model for Belgium

				Common Factor					
ϕ_1	1.255 (0.101)**								
ϕ_2	-0.393 (0.155)*								
				Idiosyncratic Components					
Δ **GDP**	γ_1	-	-	D80q1	ψ_{11}	ψ_{12}	ψ_{14}	α_{11}	σ_1^2
	0.253**			4.467**	0.035	0.023	-0.224*	0.262	0.525**
	(0.083)			(0.743)	(0.155)	(0.102)	(0.103)	(0.143)	(0.093)
Δ **GFCF**	γ_2	-	-	-	ψ_{21}	ψ_{22}	-	α_{12}	σ_2^2
	0.222**				-0.079	0.114		0.038	0.720**
	(0.047)				(0.089)	(0.086)		(0.109)	(0.088)
Δ **IP**	γ_3	-	-	-	ψ_{31}	ψ_{32}	-	α_{13}	σ_3^2
	0.295**				-0.200	-0.010		0.583**	0.518**
	(0.047)				(0.143)	(0.014)		(0.132)	(0.085)
Δ **EMP**	γ_{40}	γ_{41}	γ_{42}	γ_{43}	ψ_{41}	ψ_{42}	-	α_{14}	σ_4^2
	0.306**	0.075	-0.123*	0.132**	1.066**	-0.284**		-0.041	0.067*
	(0.057)	(0.062)	(0.068)	(0.047)	(0.151)	(0.080)		(0.052)	(0.039)

Long run growth rate: δ = 1.207

Error correction term

$$\underset{}{\mathrm{GDP}_{t-1}} = 15.805 - \underset{(0.080)}{0.220} \times \mathrm{GFCF}_{t-1} + \underset{(0.080)}{1.477} \times \mathrm{IP}_{t-1} - \underset{(0.312)}{1.499} \times \mathrm{EMP}_{t-1}$$

Log-likelihood: -604.932

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	15.277*	4.113
Δ GFCF	4.021	6.012
Δ IP	1.606	14.502**
Δ EMP	3.287	4.364

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for Belgium were estimated using data from 1970Q1-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0059, 0.0074; Δ GFCF: 0.0044, 0.0209; Δ IP: 0.0044; 0.0209; Δ EMP: 0.0009, 0.0027.

2.4.2 The Periphery EMU Countries: Spain and Finland

Spain. The trace statistic indicates the existence of two cointegrating vectors, whilst the eigenvalue statistic suggests that there is only one. As with the Netherlands, two cointegrating vectors are included in the DF model for Spain. The significance of α_{13} confirms the presence of the first long-run relationship among three of the four variables used for Spain. The parameter estimate of α_{23} becomes significant when α_{24} is restricted to zero, suggesting a second long-run relationship. The Spanish common factor is also persistent with $\phi_1 + \phi_2 = 0.88$. Employment is again the least responsive and lags the common fluctuations. In total five recessions are found for Spain: 1974Q4-1976Q1, 1978Q2-1979Q2, 1980Q2-1981Q4, 1983Q3-1984Q2 and 1992Q2-1993Q4. No recessions are identified from 1994Q1 onwards, which may be a consequence of the funding received from the European Region Development Fund and the strong growth observed in the construction sector. In contrast to Germany and the Netherlands, who both suffered from recessions and sluggish growth in recent years, Spanish economic growth has accelerated. The estimated mean growth rate for Spain is 4.75 per cent per annum over the period studied.

Finland. As no cointegration is found among the four variables used for Finland; the DF model is thus specified in first differences. The Finnish common factor follows an AR(2) process with both ϕ_1 and ϕ_2 positive and significant. Unsurprisingly, Finnish employment lags behind the common factor fluctuations, with γ_{40} being insignificant, and γ_{41} and γ_{43} being positive and significant. To some extent, this reflects the rigidities present in the Finnish labour market, stemming from the generous unemployment benefits and high employment protection provided. Five recessions are identified, during 1975Q2-1975Q4, 1977Q1-1977Q2, 1980Q4-1981Q1, 1990Q3-1993Q2 and 2001Q2-2001Q3. The most severe recession was observed during the early 1990s, triggered by the collapse of exports to the Soviet Union and later exacerbated by the ERM crisis. The annual growth rate of the Finnish common factor is estimated to be 3.3 per cent.

Table 2.8: Parameter estimates for the DF model for Spain

Common Factor

ϕ_1	0.397 (0.133)**
ϕ_2	0.441 (0.130)**

Idiosyncratic Components

Δ GDP	γ_1	-	-	-	-	-	ψ_{11}	ψ_{12}	ψ_{14}	α_{11}	α_{21}	σ_1^2
	0.467**						-0.460**	-0.107	-0.200*	-0.168	0.405	0.382**
	(0.060)						(0.120)	(0.116)	(0.088)	(0.260)	(0.262)	(0.064)
Δ GFCF	γ_2	-	-	-	-	-	ψ_{21}	ψ_{22}	-	α_{12}	α_{22}	σ_2^2
	0.586**						0.168	0.022		0.228	-0.084	0.262**
	(0.072)						(0.172)	(0.162)		(0.335)	(0.349)	(0.067)
Δ IP	γ_3	-	-	-	-	-	ψ_{31}	ψ_{32}	-	α_{13}	α_{23}	σ_3^2
	0.431**						-0.256**	0.195*		0.972**	-0.498*	0.550**
	(0.065)						(0.101)	(0.099)		(0.314)	(0.234)	(0.078)
Δ EMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D76q1	D76q3	ψ_{41}	ψ_{42}	-	α_{14}	α_{24}	σ_4^2
	0.185**	0.147**	0.090	0.050	2.789**	3.664**	0.254**	0.219*		-0.214	-	0.166**
	(0.048)	(0.055)	(0.052)	(0.051)	(0.412)	(0.419)	(0.100)	(0.094)		(0.173)		(0.023)

Long run growth rate: δ = 1.180

Error correction terms

$$GDP_{t-1} = \underset{}{6.403} + \underset{(0.167)}{2.054} \times IP_{t-1} - \underset{(0.179)}{0.121} \times EMP_{t-1} \qquad GFCF_{t-1} = -1.157 + \underset{(0.473)}{3.098} \times IP_{t-1} - \underset{(0.507)}{0.027} \times EMP_{t-1}$$

Log-likelihood: -539.934

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	0.164	20.741**
Δ GFCF	1.638	34.936**
Δ IP	3.033	14.456**
Δ EMP	1.386	12.524**

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The adjustment parameter a_{23} becomes significant when a_{24} is restricted to be zero. The parameter estimates for Spain were estimated using data from 1972Q3-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0069, 0.0074; Δ GFCF: 0.0069, 0.0074; Δ IP: 0.0049, 0.0172; Δ EMP: 0.0035, 0.0082.

Table 2.9: Parameter estimates for DF model for Finland

				Common Factor				
ϕ_1	0.333(0.137)**							
ϕ_2	0.349(0.128)**							
				Idiosyncratic Components				
Δ **GDP**	γ_1	-	-	-	-	ψ_{11}	ψ_{12}	σ_1^2
	0.573**					-0.622**	-0.097*	0.316*
	(0.150)					(0.144)	(0.045)	(0.130)
Δ **Sales**	γ_2	-	-	-	-	ψ_{21}	ψ_{22}	σ_2^2
	0.453**					-0.002	-0.087	0.655**
	(0.123)					(0.005)	(0.094)	(0.102)
Δ **IP**	γ_3	-	-	-	-	ψ_{31}	ψ_{32}	σ_3^2
	0.414**					-0.170	0.109	0.698**
	(0.121)					(0.091)	(0.094)	(0.106)
Δ **EMP**	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D76q1	ψ_{41}	ψ_{42}	σ_4^2
	0.025	0.333**	0.098	0.325**	5.234**	-0.096	-0.242	0.280*
	(0.097)	(0.085)	(0.149)	(0.075)	(0.737)	(0.139)	(0.252)	(0.119)
Long run growth rate: $\delta = 0.827$								
Log-likelihood: -665.342								
Diagnostics			**Q(4)**			**Jarque-Bera**		
Δ GDP			11.126*			17.706**		
Δ Sales			10.702*			19.807**		
Δ IP			4.317			17.197**		
Δ EMP			1.447			1.033		

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for Finland were estimated using data from 1970Q1-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0069, 0.0121; Δ Sales: 0.0054, 0.0184; Δ IP: 0.0093, 0.0215; Δ EMP: 0.0011, 0.0081.

2.4.3 The non-EMU countries: the UK, the US and Canada

United Kingdom. The four variables used for the UK are also cointegrated. Therefore, one error correction term is included in the DF model. A long-run relationship is confirmed by the significant adjustment parameter, α_{14}. Dummy variables are included in the GDP and IP equations to remove the pronounced outliers caused by the frequent industrial strikes during the 1970s. A dummy is also used to capture the break in the employment series during 2002Q3. Analysing the values of γ_i presented in Table 2.10 reveals that employment is again the least responsive and lags the estimated UK common factor. This runs contrary to the general view that the UK has a more flexible labour market than France and Germany. However, this finding can be explained in part by the fact that the labour market deregulation, which only commenced in the early 1980s, will not be fully reflected when estimating the sample average. Overall, four recessions are identified during, 1973Q3-1974Q1, 1974Q4-1975Q3, 1979Q3-1981Q1 and 1990Q3-1991Q3. It can be seen from Panel 10 of Figure 2.1 that, in contrast to the core EMU countries presented above, the UK economy has performed consistently well since the mid-1990s. The mean growth rate for the UK is estimated to be 2.5 per cent per annum, which is relatively low compared to the other non-EMU countries analysed.

United States. Instead of using the monthly data analysed by the NBER business cycle dating committee, four quarterly variables are used for the US to ensure consistency with the other countries analysed. One cointegration relationship is found among these variables, so that one error correction term is included in the DF model. The adjustment parameter, α_{12}, is found to be significant, which confirms the presence of a long-run relationship among the four variables used. Although the US employment series lags the common factor fluctuations, the value of γ_{40} is larger than the corresponding values for the other countries analysed, perhaps reflecting the greater flexibility of the deregulated US labour market. The five recessions identified, 1974Q1-1975Q1, 1980Q2-1980Q3, 1981Q4-1982Q4, 1990Q3-1991Q1 and 2001Q1-2001Q3, replicate the cycle dates pronounced by the NBER business cycle dating committee[9]. The mean growth rate of the US common factor is computed to be 4.6 per cent per annum.

[9] Five peaks announced by the committee are November 1973, January 1980, July 1981, July 1990 and March 2001; five troughs are March 1975, July 1980, November 1982, March 1991 and November 2001.

Table 2.10: *Parameter estimates for DF model for the UK*

Common Factor

ϕ_1	0.207 (0.122)
ϕ_2	0.170 (0.124)

Idiosyncratic Components

ΔGDP	γ_1	D73q1	D79q2	-	-	ψ_{11}	ψ_{12}	α_{11}	σ_1^2
	0.688**	3.501**	1.498*			-0.386*	-0.037	-0.066	0.168**
	(0.073)	(0.581)	(0.641)			(0.154)	(0.030)	(0.129)	(0.057)
ΔGFCF	γ_2	-	-	-	-	ψ_{21}	ψ_{22}	α_{12}	σ_2^2
	0.322**					-0.157	-0.006	-0.015	0.886**
	(0.089)					(0.098)	(0.008)	(0.100)	(0.110)
ΔIP	γ_3	D72q1	D72q2	D74q1	D74q2	ψ_{31}	ψ_{32}	α_{13}	σ_3^2
	0.621**	-2.443**	2.485**	-2.347**	3.887**	0.464**	-0.054	-0.006	0.145**
	(0.064)	(0.517)	(0.535)	(0.547)	(0.532)	(0.164)	(0.038)	(0.126)	(0.049)
ΔEMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D02q3	ψ_{41}	ψ_{42}	α_{14}	σ_4^2
	0.203**	0.168**	0.194**	0.092	-4.949**	0.339**	0.055	-0.229*	0.328**
	(0.057)	(0.063)	(0.066)	(0.064)	(0.559)	(0.098)	(0.099)	(0.107)	(0.043)

Long run growth rate: $\delta = 0.615$

Error correction term

$$\mathrm{GDP}_{t-1} = \underset{}{35.186} + \underset{(0.123)}{1.056} \times \mathrm{GFCF}_{t-1} + \underset{(0.156)}{0.674} \times \mathrm{IP}_{t-1} - \underset{(0.450)}{2.824} \times \mathrm{EMP}_{t-1}.$$

Log-likelihood: -584.199

Diagnostics	**Q(4)**	**Jarque-Bera**
ΔGDP	7.795	32.807**
ΔGFCF	3.968	3.977
ΔIP	3.289	9.306**
ΔEMP	0.855	3.417

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for the UK were estimated using data from 1970Q1-2005Q1. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: ΔGDP: 0.0057, 0.0096; ΔGFCF: 0.0064, 0.0272; ΔIP: 0.0028, 0.0186; ΔEMP: 0.0011, 0.0054.

Table 2.11: Parameter estimates for DF model for the US

				Common Factor				
ϕ_1	0.654 (0.103)**							
ϕ_2	-0.107 (0.034)**							
				Idiosyncratic Components				
ΔGDP	γ_1	-	-	D78q2	ψ_{11}	ψ_{12}	α_{11}	σ_1^2
	0.666**			2.042**	-0.510**	-0.065	-0.028	0.168**
	(0.061)			(0.578)	(0.155)	(0.040)	(0.106)	(0.047)
ΔSales	γ_2	D74q4	-	D80q2	ψ_{21}	ψ_{22}	α_{12}	σ_2^2
	0.410**	-2.149*		-1.960*	-0.196*	-0.010	0.185*	0.475**
	(0.058)	(0.760)		(0.750)	(0.095)	(0.009)	(0.081)	(0.060)
ΔIP	γ_3	D74q4	D75q1	D80q2	ψ_{31}	ψ_{32}	α_{13}	σ_3^2
	0.574**	-1.680**	-3.170**	-1.185*	0.294*	0.085	-0.155	0.185**
	(0.058)	(0.531)	(0.544)	(0.540)	(0.125)	(0.120)	(0.105)	(0.035)
ΔEMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	ψ_{41}	ψ_{42}	α_{14}	σ_4^2
	0.429**	0.163*	0.042	0.126*	0.082	-0.002	-0.207	0.389*
	(0.068)	(0.076)	(0.078)	(0.064)	(0.101)	(0.004)	(0.125)	(0.052)

Long run growth rate: $\delta = 1.160$

Error correction term

$$\mathrm{GDP}_{t-1} = \underset{(0.071)}{0.004} + \underset{(0.072)}{0.545} \times \mathrm{Sales}_{t-1} - 0.097 \times \mathrm{IP}_{t-1} + \underset{(0.197)}{0.322} \times \mathrm{EMP}_{t-1}$$

Log-likelihood: -598.771

Diagnostics	**Q(4)**	**Jarque-Bera**
ΔGDP	0.053	10.908**
ΔSales	1.198	2.322
ΔIP	2.975	14.547*
ΔEMP	4.194	7.429

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for the US were estimated using data from 1970Q1-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: ΔGDP: 0.0078, 0.0082; ΔSales: 0.0054, 0.0148; ΔIP: 0.0069, 0.0149; ΔEMP: 0.0043, 0.0048.

Canada. One cointegrating vector is also determined by the Johansen cointegration test for Canada. The adjustment parameters, α_{12} and α_{13}, appear significant, again confirming the presence of a long-run relation among the four variables used for Canada. As with the US, the Canadian labour market exhibits a certain degree of flexibility, with the value of γ_{40} found to be larger than the corresponding values for the EMU countries. As plotted in Panel 12 of Figure 2.1, four recessions are identified, during 1974Q4-1975Q1, 1981Q3-1982Q4, 1990Q2-1991Q1 and 2001Q1-2001Q3. These dates are closely correlated with the US business cycle turning points, illustrating the close economic link between the two countries. Finally, the annual growth rate for Canada is estimated to be 4.9 per cent.

Table 2.12: Parameter estimates for DF model for Canada

				Common Factor				
ϕ_1	0.799 (0.109)**							
ϕ_2	-0.160 (0.044)**							
				Idiosyncratic Components				
Δ GDP	γ_1	-	-	-	ψ_{11}	ψ_{12}	α_{11}	σ_1^2
	0.581**				-0.303**	-0.023	-0.085	0.273**
	(0.063)				(0.116)	(0.018)	(0.112)	(0.050)
Δ Sales	γ_2	-	-	D81Q1	ψ_{21}	ψ_{22}	α_{12}	σ_2^2
	0.297**			4.793**	-0.215*	0.126	0.179*	0.539**
	(0.051)			(0.758)	(0.092)	(0.091)	(0.079)	(0.067)
Δ IP	γ_3	-	-	-	ψ_{31}	ψ_{32}	α_{13}	σ_3^2
	0.586**				0.188	0.095	0.214*	0.272**
	(0.063)				(0.115)	(0.105)	(0.117)	(0.049)
Δ EMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	ψ_{41}	ψ_{42}	α_{14}	σ_4^2
	0.365**	0.266**	-0.075	-0.035	0.133	0.039	-0.205	0.341**
	(0.075)	(0.093)	(0.091)	(0.072)	(0.108)	(0.106)	(0.137)	(0.048)

Long run growth rate: $\delta = 1.241$

Error correction term

$$GDP_{t-1} = 4.586 + 0.261 \times Sales_{t-1} + 0.399 \times IP_{t-1} + 0.632 \times EMP_{t-1}$$

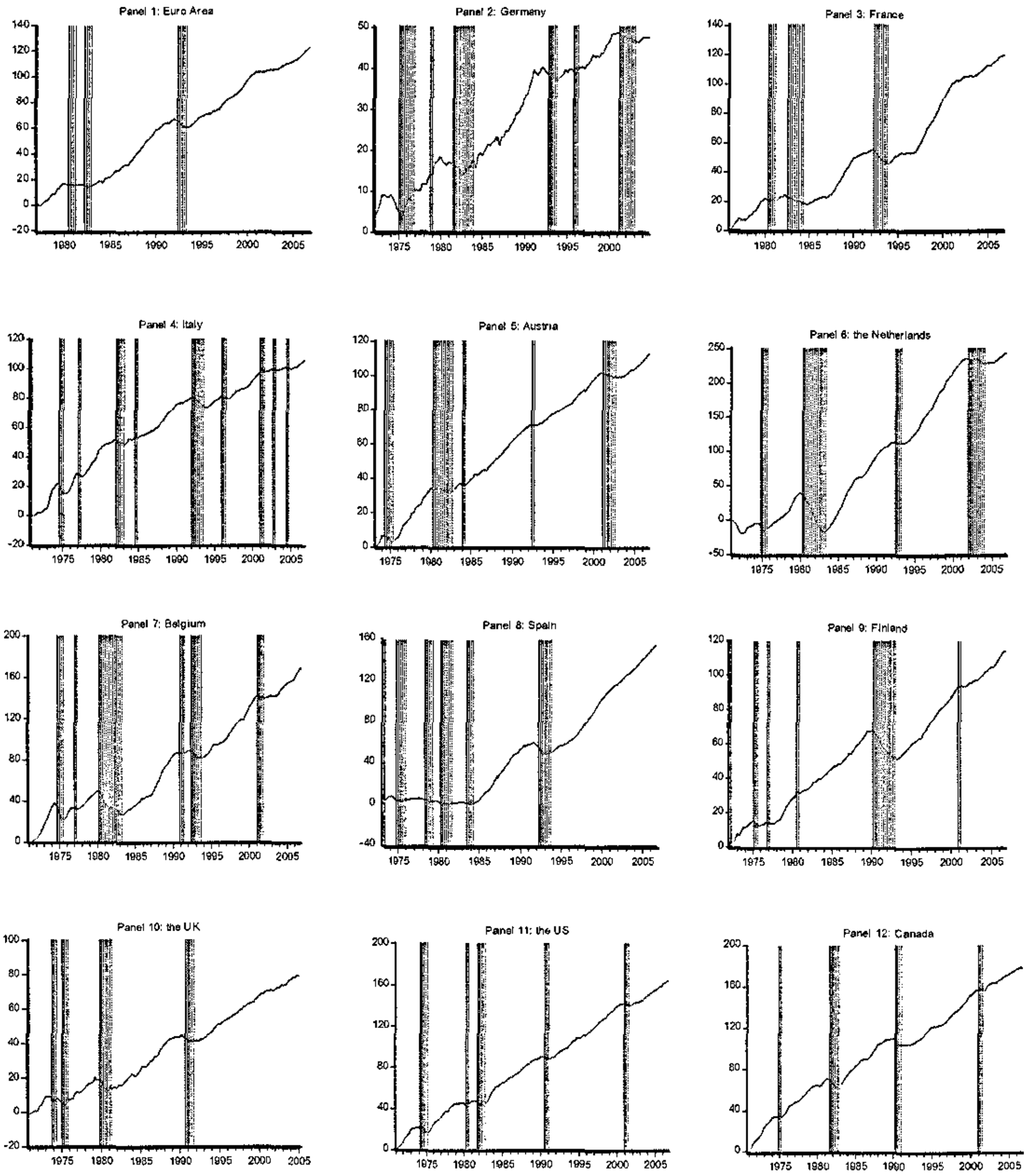
(0.045) (0.046) (0.047)

Log-likelihood: -628.861

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	7.842	6.601*
Δ Sales	0.472	0.148
Δ IP	4.947	2.141
Δ EMP	9.500	25.465**

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for Canada were estimated using data from 1970Q1-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0077, 0.0081; Δ Sales: 0.0079, 0.0168; Δ IP: 0.0064, 0.0168; Δ EMP: 0.0050, 0.0058.

Figure 2.1: Composite indices and recessions



Note: the index lines indicate the periods of recession.

2.5 Evaluating the coincidence of business cycle turning points

Having identified business cycle turning points for each country, the binary variable, S_t, can be constructed to indicate the states of an economy, with unity denoting expansions and zero indicating recessions. The pairwise cycle correlation can then be evaluated using the correlation coefficient and the various concordance indices proposed in Harding and Pagan (2002).

2.5.1 Multidimensional mapping of business cycle distance

The correlation coefficient is the most commonly used index to measure the coincidence between two variables. In this subsection, the correlation coefficients between binary variables are calculated and reported in Table 2.13. However, given the number of countries in this analysis, it is difficult to interpret all the information simply by observing the individual bilateral correlations presented in Table 2.13. Therefore, a multidimensional mapping technique, *Sammon mapping* (Sammon, 1969), is employed to reveal a geometrical picture of the interdependencies among the business cycles.[10] Countries that have non-synchronised business cycles are plotted far away from each other in the picture. Applying this approach produces a two-dimensional map for 66 pairs of business cycle distances among 12 countries by minimising the following error function, which is often referred to as *Sammon's stress*.

$$E = \frac{1}{\sum_{i<j}^{N} d_{ij}} \sum_{i<j}^{N} \frac{\left(d_{ij} - \hat{d}_{ij}\right)^2}{d_{ij}}, \tag{2.16}$$

where $N = 66$ and d_{ij} is the business cycle distance between countries i and j, obtained by subtracting the pairwise correlation coefficient from one. $\hat{d}_{ij}$ is the approximate distance between their projections on the map, given by the following equation:

[10] A function package for Matlab is used to conduct this procedure.

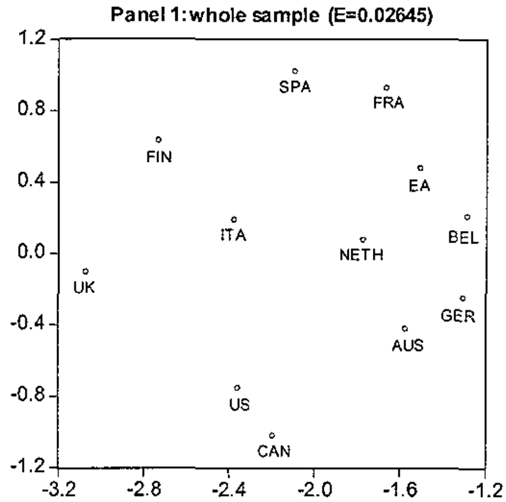
$$\hat{d}_{ij} = \left(\left\|z_i - z_j\right\|^2\right)^{1/2} = \left[\sum_{k=1}^{K}\left(z_{ik} - z_{jk}\right)^2\right]^{1/2}, \tag{2.17}$$

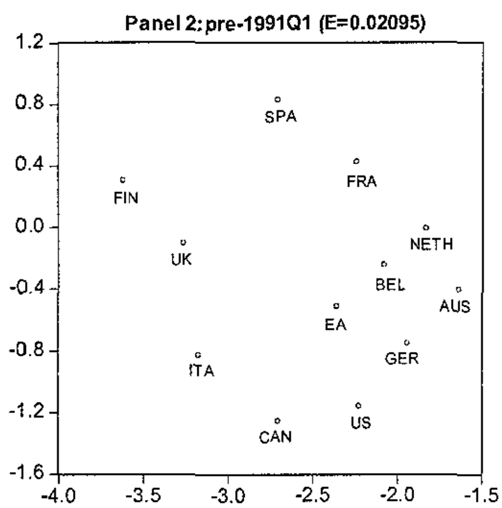
As $K = 2$, z_i and z_j are the two-dimensional projection of countries i and j, and z_{ik} and z_{jk} are the two dimensions of each country.

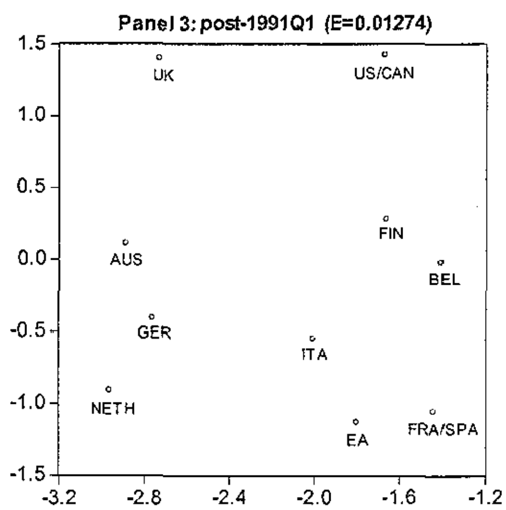
Maps of business cycle distances are shown in Panels 1-3 of Figure 2.2 over the whole sample and two subsamples with midpoint of 1991Q1. It is important to emphasise that the units denoted on the axes of the *Sammon maps* are, in themselves, meaningless, and the orientation of the picture is arbitrary; the only thing that matters is the distances between objects in the map (Kruskal and Wish, 1977). Therefore, the only aim of using *Sammon mapping* is to graphically illustrate the level of business cycle synchronisation between different groups of countries. It is also important to keep in mind that, when looking at a map that has non-zero stress, the distances among objects imperfectly represent the relations given by the data, and the greater the stress, the larger the distortion. It can be seen from Panel 1 of Figure 2.2 that, over the whole sample, the core EMU countries exhibit a higher degree of synchronisation within themselves and with respect to the euro area aggregate, compared to those in the peripheral and non-EMU groups. This is particularly true for Germany, Austria, Belgium and the Netherlands who appear close to each other and the aggregate euro area. This is partly because Austria and the Netherlands previously fixed their exchange rates against the Deutsche Mark (DM) and their economies were highly integrated with Germany.

The above relations can also be illustrated numerically by calculating the mean correlation of several pairwise correlations. The mean correlation for the core EMU countries with respect to the aggregate euro area is 0.52, compared to the corresponding values of 0.41 and 0.24 for the periphery and non-EMU countries. In addition, a even higher mean correlation of 0.57 is found within the cluster of four core EMU countries mentioned above. It is also worth noting that the highest correlation is between the US and Canada, followed by the correlation between Germany and Austria, reflecting the close economic links between these two pairs of countries.

Figure 2.2: Multidimensional mapping of business cycle distance







Note: two pairs of countries, the US and Canada, France and Spain, have perfectly synchronised turning points in the second subsample.

The changes in business cycle correlations over time are also analysed by evaluating the cycle correlations of two subsamples: pre-1991Q1 and post-1991Q1. The reason for choosing 1991Q1 as the breakpoint is because a number of important events happened around this time, including German reunification in October 1990, the ERM crisis in 1992-1993, and the adoption of the Maastricht treaty in November 1993. These events are all expected to have had significant influences on the cycle correlations between the countries analysed. A closer look at Panels 2 and 3 of Figure 2.2 reveals a few striking features. First, the tight cluster of four core EMU countries composed of Germany, Austria, Belgium and the Netherlands, who all had close distances to the euro area in the first subsample, deviated from the aggregate euro area and each other in the second subsample. However, the other two core members, France and Italy, who only had moderate correlation with the aggregate euro area, have shown an increase in synchronisation with the latter. Second, a catching-up process of business cycle convergence is also observed between the two periphery countries, Spain and Finland, and the aggregate euro area over time. Third, a picture of diverging non-EMU business cycles from the euro area as a whole can clearly be observed. This is also broadly in line with Garnier (2003), who found that euro area business cycles are increasingly independent of the US cycle. Finally, two pairs of perfectly synchronised binary variables are found between France and Spain, and the US and Canada, in the second subsample.

The above changes in cycle correlations, revealed in the maps, are again confirmed by the mean correlations of different groups (i.e., the core, periphery and non-EMU countries) with respect to the aggregate euro area. The mean correlation between the four core EMU members with the euro area almost halved over the two subsamples, with the mean correlation reducing from 0.67 to 0.37. In contrast, the mean correlation between the two periphery countries and the euro area aggregate increased dramatically from 0.25 to 0.72. A moderate increase in synchronisation is also found between the French and Italian business cycles and the aggregate euro area, with their mean correlation rising from 0.40 to 0.67. Finally, the mean correlation between non-EMU countries and the aggregate euro area decline sharply from 0.35 to -0.08, implying anti-cyclical correlations between the aggregate euro area and these countries in the second subsample.

Table 2.13: Correlation Coefficients

Whole Sample Period

	EMU	GER	FRA	ITA	AUS	BEL	NETH	SPA	FIN	UK	US
EMU											
GER	0.59										
FRA	0.57	0.30									
ITA	0.39	0.42	0.39								
AUS	0.45	0.67	0.39	0.23							
BEL	0.60	0.50	0.52	0.48	0.59						
NETH	0.51	0.48	0.44	0.27	0.62	0.54					
SPA	0.37	0.23	0.64	0.16	0.37	0.43	0.37				
FIN	0.45	0.20	0.27	0.23	0.14	0.49	0.16	0.27			
UK	0.19	0.20	0.16	-0.05	0.16	0.33	0.14	0.19	0.35		
US	0.34	0.41	0.15	0.18	0.51	0.40	0.22	0.06	0.14	0.34	
CANA	0.18	0.28	0.00	0.23	0.41	0.42	0.22	0.04	0.18	0.20	0.76

Pre-1991Q1

Post-1991Q1	EMU	GER	FRA	ITA	AUS	BEL	NETH	SPA	FIN	UK	US	CANA
EMU		0.79	0.40	0.40	0.58	0.72	0.58	0.09	0.41	0.27	0.52	0.27
GER	0.44		0.34	0.37	0.73	0.64	0.47	0.23	0.12	0.39	0.6	0.41
FRA	0.83	0.30		0.26	0.55	0.48	0.55	0.43	0.11	0.26	0.26	0.02
ITA	0.50	0.47	0.60		0.26	0.51	0.35	-0.06	0.06	0.05	0.27	0.34
AUS	0.20	0.61	0.13	0.26		0.67	0.74	0.45	0.07	0.23	0.61	0.47
BEL	0.42	0.32	0.56	0.49	0.45		0.75	0.35	0.35	0.31	0.44	0.46
NETH	0.42	0.50	0.30	0.21	0.45	0.20		0.41	0.18	0.23	0.36	0.39
SPA	0.83	0.30	1.00	0.60	0.13	0.56	0.30		0.2	0.22	0.05	0.05
FIN	0.60	0.27	0.46	0.34	0.25	0.64	0.13	0.46		0.41	0.03	0.08
UK	-0.07	-0.17	-0.09	-0.15	-0.10	0.43	-0.13	-0.09	0.46		0.35	0.17
US	-0.09	0.09	-0.10	0.13	0.26	0.34	-0.15	-0.10	0.36	0.24		0.68
CANA	-0.09	0.09	-0.10	0.13	0.26	0.34	-0.15	-0.10	0.36	0.24	1.00	

2.5.2 The concordance and mean corrected concordance indices

The index of concordance (IC) proposed by Harding and Pagan (2002) is utilised in this section to measure the length in quarters that two business cycles spend in the same phase. Let S_{it} and S_{jt} denote the binary variables for countries i and j. The IC can be calculated as follows, where T is the sample size

$$I_{ij} = T^{-1}\sum_{t=1}^{T}\{S_{jt}S_{it}+(1-S_{jt})(1-S_{it})\} \tag{2.18}$$

The value of I_{ij} equals one when two binary variables are perfectly pro-cyclical. I_{ij} equals zero when two binary variables are exactly counter-cyclical. The relation between the IC and the correlation coefficient can be seen by rewriting equation (2.18) as

$$\begin{aligned} I_{ij} &= 1+\frac{2}{T}\sum_{t=1}^{T}S_{jt}S_{it} - \bar{S}_j - \bar{S}_i \\ &= 1+2\hat{\sigma}_{ij} + 2\bar{S}_j\bar{S}_i - \bar{S}_j - \bar{S}_i \\ &= 1+2\hat{\rho}_{ij}(\bar{S}_j(1-\bar{S}_j))^{1/2}(\bar{S}_i(1-\bar{S}_i))^{1/2} + 2\bar{S}_j\bar{S}_i - \bar{S}_j - \bar{S}_i \end{aligned} \tag{2.19}$$

where $\hat{\sigma}_{ij}$ denotes the estimated covariance and $\hat{\rho}_{ij}$ is the estimated correlation coefficient between S_{it} and S_{jt}. When I_{ij} is one, the corresponding value of the correlation coefficient is one, and when I_{ij} equals zero, the correlation coefficient is -1. A problem occurs when the value of the correlation coefficient equals zero, which results in equation (2.19) becoming

$$E(I_{ij}) = 1 + 2\bar{S}_j\bar{S}_i - \bar{S}_j - \bar{S}_i \tag{2.20}$$

In this case, even if two binary variables are independent of each other, I_{ij} may be a high value simply because a large fraction of time is spent in expansions. Therefore, Harding and Pagan (2002) further proposed a mean corrected index of concordance (MCIC) to cross-check the values of I_{ij}. The MCIC is given by

$$I_{ij}^{*} = 2T^{-1}\sum_{t=1}^{T}\left\{\left(S_{it} - \bar{S}_i\right)\left(S_{jt} - \bar{S}_j\right)\right\} \tag{2.21}$$

where $\bar{S}_i = T^{-1}\sum_{t=1}^{T} S_{it}$, and $\bar{S}_j = T^{-1}\sum_{t=1}^{T} S_{jt}$. If I_{ij}^{*} is negative, this indicates a counter-cyclical relationship between the two binary series, S_{it} and S_{jt}. However, the value of I_{ij}^{*} is harder to interpret than I_{ij}, as the maximum value of I_{ij}^{*} varies across each pair of binary variables compared to I_{ij}, which has a definite maximum value of one.

The upper panel of Table 2.14 reports the values of I_{ij} and I_{ij}^{*} calculated using the whole sample. The values of I_{ij} suggest that the aggregate euro area is more in synch with Finland and US than countries such as Germany and the Netherlands. This contradicts what we have observed in Panel 1 of Figure 2.2. To cross-check the results, I_{ij}^{*} for each pair of binary variables is computed, and the results are more in line with those provided by correlation coefficients in the previous subsection. The mean I_{ij}^{*} between the core EMU countries and the aggregate euro area is 0.15, compared to the corresponding values of 0.11 and 0.05 for the peripheral and non-EMU countries, respectively. Furthermore, an average I_{ij}^{*} of 0.21 is found among the four highly synchronised core EMU business cycles.

The values of I_{ij} and I_{ij}^{*} calculated using the two subsamples are presented in the mid and lower panels of Table 2.14. Upon comparing these values, changes in cycle correlations over time are again observed across countries. Both I_{ij} and I_{ij}^{*} suggest that French, Spanish and Finnish business cycles became more synchronised with the aggregate euro area in the second subsample. However, the cluster of the four core EMU countries all deviated from the euro area as a whole. Moreover, the desynchronisation process between the non-EMU countries and the euro area, shown in Panels 2 and 3 of Figure 2.2, is only revealed when comparing values of I_{ij}^{*} across two subsamples. The corresponding values of I_{ij} provide a different story and this reflects exactly the problem with I_{ij} when a large fraction of time is spent in expansions, as for the non-EMU countries and the aggregate euro area less recessions are identified in the second than the first subsample.

Table 2.14: IC (Upper Triangle) and MCIC (Lower Triangle)

Whole Sample Period

	EMU	GER	FRA	ITA	AUS	BEL	NETH	SPA	FIN	UK	US	CANA
EMU		0.84	0.89	0.82	0.85	0.87	0.85	0.83	0.87	0.82	0.86	0.83
GER	0.19		0.75	0.77	0.86	0.80	0.79	0.69	0.70	0.71	0.77	0.74
FRA	0.14	0.10		0.81	0.83	0.85	0.83	0.90	0.81	0.80	0.81	0.77
ITA	0.11	0.16	0.11		0.75	0.83	0.74	0.72	0.77	0.69	0.77	0.79
AUS	0.12	0.26	0.11	0.08		0.86	0.87	0.79	0.75	0.75	0.86	0.84
BEL	0.17	0.20	0.16	0.16	0.20		0.83	0.80	0.83	0.79	0.82	0.83
NETH	0.14	0.19	0.13	0.09	0.21	0.19		0.78	0.74	0.72	0.75	0.76
SPA	0.10	0.09	0.18	0.06	0.12	0.15	0.13		0.08	0.75	0.72	0.73
FIN	0.11	0.07	0.07	0.07	0.05	0.15	0.05	0.02		0.83	0.79	0.81
UK	0.04	0.06	0.04	-0.01	0.05	0.10	0.04	0.05	0.09		0.85	0.82
US	0.07	0.13	0.03	0.05	0.14	0.11	0.06	0.02	0.03	0.08		0.95
CANA	0.04	0.08	0.00	0.06	0.10	0.11	0.06	0.02	0.04	0.04	0.15	

Pre-1991Q1

	EMU	GER	FRA	ITA	AUS	BEL	NETH	SPA	FIN	UK	US	CANA
EMU		0.93	0.80	0.82	0.86	0.89	0.86	0.66	0.84	0.79	0.86	0.79
GER	0.27		0.78	0.78	0.89	0.86	0.79	0.67	0.70	0.78	0.85	0.79
FRA	0.13	0.11		0.78	0.85	0.82	0.85	0.78	0.77	0.78	0.78	0.72
ITA	0.12	0.12	0.07		0.75	0.84	0.76	0.60	0.77	0.73	0.80	0.84
AUS	0.19	0.29	0.18	0.08		0.87	0.90	0.77	0.70	0.72	0.86	0.82
BEL	0.25	0.24	0.16	0.15	0.26		0.90	0.73	0.79	0.76	0.81	0.83
NETH	0.19	0.19	0.18	0.11	0.27	0.29		0.76	0.75	0.71	0.76	0.78
SPA	0.03	0.10	0.15	-0.02	0.18	0.14	0.16		0.70	0.69	0.63	0.64
FIN	0.10	0.04	0.03	0.01	0.02	0.10	0.05	0.06		0.83	0.75	0.79
UK	0.08	0.14	0.07	0.01	0.08	0.10	0.08	0.08	0.11		0.80	0.76
US	0.15	0.20	0.07	0.07	0.21	0.14	0.12	0.02	0.01	0.10		0.91
CANA	0.08	0.12	0.01	0.08	0.15	0.13	0.12	0.02	0.02	0.05	0.18	

Post-1991Q1

	EMU	GER	FRA	ITA	AUS	BEL	NETH	SPA	FIN	UK	US	CANA
EMU		0.75	0.97	0.81	0.84	0.84	0.84	0.97	0.89	0.86	0.86	0.86
GER	0.12		0.71	0.77	0.82	0.71	0.79	0.71	0.70	0.61	0.66	0.66
FRA	0.14	0.09		0.84	0.81	0.88	0.81	1.00	0.86	0.82	0.83	0.83
ITA	0.11	0.20	0.16		0.75	0.81	0.72	0.84	0.77	0.65	0.73	0.73
AUS	0.04	0.21	0.03	0.08		0.84	0.84	0.81	0.80	0.79	0.86	0.86
BEL	0.09	0.13	0.14	0.17	0.13		0.75	0.88	0.89	0.82	0.83	0.83
NETH	0.09	0.20	0.08	0.08	0.13	0.07		0.81	0.73	0.72	0.73	0.73
SPA	0.14	0.09	0.19	0.16	0.03	0.14	0.08		0.86	0.82	0.83	0.83
FIN	0.13	0.10	0.12	0.12	0.07	0.21	0.05	0.12		0.84	0.84	0.84
UK	-0.01	-0.04	-0.01	-0.03	-0.02	0.08	-0.02	-0.01	0.08		0.91	0.91
US	-0.01	0.02	-0.01	0.03	0.04	0.07	-0.03	-0.01	0.07	0.03		1.00
CANA	-0.01	0.02	-0.01	0.03	0.04	0.07	-0.03	-0.01	0.07	0.02	0.12	

2.5.3 Binary Variables as Regressand and Regressor

According to Harding and Pagan (2002), the null hypothesis that an individual business cycle, S_{it}, is independent of a reference cycle, S_{jt}, can be tested using the following regression equation:

$$S_{jt} = \alpha + \beta S_{it} + \varepsilon_t \qquad (2.22)$$

β indicates the relationship between S_{jt} and S_{it}. S_{jt} is independent from S_{it} if β is insignificantly different from zero. The Newey-West estimators of the standard errors are used to obtain the heteroscedasticity and autocorrelation consistent (HAC) t-statistics for the null hypothesis that $\beta = 0$.

Table 2.15: Test for synchronisation: whole sample period

	EMU	GER	FRA	US
EMU	-	-	-	0.308
	-	-	-	(0.188)
GER	0.456**	-	0.249	0.299*
	(0.143)	-	(0.138)	(0.111)
FRA	0.529**	0.362*	-	0.125
	(0.179)	(0.186)	-	(0.118)
ITA	0.321*	0.475**	0.341*	0.146
	(0.136)	(0.130)	(0.163)	(0.098)
AUS	0.393*	0.766**	0.366*	0.435**
	(0.160)	(0.082)	(0.154)	(0.129)
BEL	0.478**	0.546**	0.448**	0.321**
	(0.118)	(0.114)	(0.147)	(0.101)
NETH	0.419**	0.497**	0.390*	0.167
	(0.153)	(0.137)	(0.158)	(0.110)
SPA	0.319	0.260	0.585**	0.049
	(0.171)	(0.162)	(0.165)	(0.084)
FIN	0.431*	0.251	0.276	0.128
	(0.190)	(0.172)	(0.184)	(0.146)
UK	0.215	0.264	0.190	0.336**
	(0.219)	(0.173)	(0.222)	(0.107)
US	0.219	0.549**	0.173	-
	(0.205)	(0.146)	(0.157)	-
CANA	0.195	0.412*	0.001	0.828**
	(0.959)	(0.196)	(0.125)	(0.077)

Notes: Independence of individual business cycles is tested against four reference cycles, the EMU, German, French and the US business cycles. Standard errors are in parentheses. ** denotes significance at 1% and * at 5%.

Business cycle correlations for individual countries, S_{it}, with respect to the aggregate euro area, Germany, France and the US are investigated, with their binary variables used as the dependent variable, S_{jt}. Table 2.15 reports the estimates of β obtained over the whole sample. As shown in column 2 of Table 2.15, the core EMU countries all exhibit cycle correlation with the aggregate euro area, with β being statistically significant in all cases. In contrast, no business cycle synchronisation is shown between non-EMU countries and the aggregate euro area. As suggested by the estimates of β reported in columns 3 and 4, both the German and French business cycles appear correlated with the other core EMU countries. Moreover, business cycle comovements are also identified between France and Spain, and Germany and two non-EMU countries (the US and Canada). Finally, the Canadian cycle appears highly synchronised with the US cycle, with the estimated β being the largest among all parameter estimates.

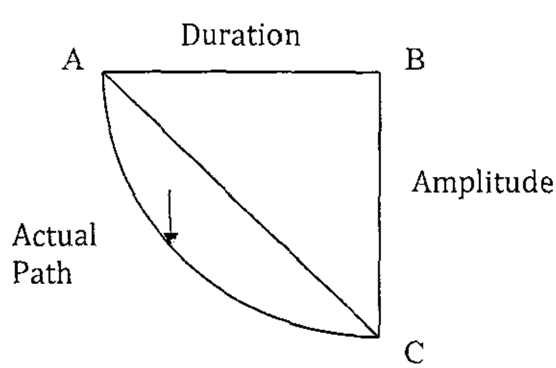
Table 2.16: Test for Synchronisation: Pre-1991Q1 and Post-1991Q1

	EMU	GER	FRA	US	EMU	GER	FRA	US
	Pre-1991Q1				Post-1991Q1			
EMU	-	-	-	0.479*	-	-	-	-0.068
	-	-	-	(0.210)	-	-	-	(0.046)
GER	0.746**	-	0.334	0.511**	0.263	-	0.209	0.051
	(0.150)	-	(0.199)	(0.143)	(0.178)	-	(0.171)	(0.081)
FRA	0.386	0.354	-	0.229	0.714**	0.429	-	-0.070
	(0.234)	(0.228)	-	(0.163)	(0.061)	(0.233)	-	(0.047)
ITA	0.428*	0.461*	0.288	0.284	0.294*	0.488*	0.412*	0.075
	(0.196)	(0.180)	(0.234)	(0.154)	(0.016)	(0.190)	(0.203)	(0.094)
AUS	0.546**	0.776**	0.530**	0.555**	0.168	0.787**	0.131	0.186
	(0.182)	(0.119)	(0.194)	(0.153)	(0.182)	(0.079)	(0.187)	(0.160)
BEL	0.642**	0.675**	0.444*	0.392**	0.288	0.360	0.442	0.211
	(0.138)	(0.123)	(0.190)	(0.137)	(0.190)	(0.206)	(0.235)	(0.123)
NETH	0.546**	0.468**	0.530*	0.304**	0.288	0.560**	0.249	-0.078
	(0.164)	(0.176)	(0.205)	(0.140)	(0.218)	(0.191)	(0.210)	(0.053)
SPA	0.075	0.225	0.378	0.040	0.714**	0.429	1.000**	-0.070
	(0.202)	(0.187)	(0.195)	(0.118)	(0.061)	(0.233)	(0.000)	(0.047)
FIN	0.319	0.172	0.148	0.040	0.417	0.311	0.378	0.231
	(0.175)	(0.171)	(0.260)	(0.211)	(0.226)	(0.250)	(0.219)	(0.160)
UK	0.296	0.440**	0.288	0.328*	-0.093	-0.358**	-0.130	0.278
	(0.246)	(0.157)	(0.249)	(0.125)	(0.068)	(0.106)	(0.083)	(0.184)
US	0.560*	0.706**	0.288	-	0.181	0.173	-0.117	-
	(0.199)	(0.137)	(0.197)	-	(0.140)	(0.222)	(0.075)	-
CANA	0.295	0.527*	0.026	0.746**	-0.083	0.173	-0.117	1.000**
	(0.253)	(0.218)	(0.178)	(0.106)	(0.062)	(0.222)	(0.075)	(0.000)

The estimates of β obtained using the two subsamples are reported in Table 2.16. As with the above findings, synchronisation between French and Spanish cycles and the aggregate euro area increased remarkably, with the null of $\beta = 0$ strongly rejected in the second subsample. On the other hand, the four core EMU business cycles are found to move out of sync with the aggregate euro area, with their estimates of β becoming insignificant. Finally, business cycle divergence between the US and the euro area is also observed by comparing the estimates of β reported in the last column of the left and right panel of Table 2.16.

2.6 Average Cycle Characteristics

The four key business cyclical characteristics; length, amplitude, steepness and welfare gains, proposed by Harding and Pagan (2000), are analysed in this section. The similarity of these characteristics across euro area business cycles is relevant in determining the effectiveness of common monetary policies. For example, if cycle steepness differs between members, a common monetary policy cannot meet the requirements of countries with deep cycles and those with mild cycles. Harding and Pagan use a triangle approximation to describe a business cycle phase, in which the height of the triangle is the amplitude and the base is the duration of the cycle. This is illustrated for a stylised recessionary phase in the diagram below. Knowledge of the height and base enables one to compute the area of the triangle to approximate the cumulated losses (gains) in one particular recession or expansion.



Instead of analysing the characteristics of one particular recessionary or expansionary phase, this section measures the average business cycle characteristics over the whole sample and the two subsample periods. The first feature considered is the average duration of recessions (expansions), which measures the total time spent in recessions (expansions) over the number of troughs (peaks),

$$D_{TP} = \frac{\sum_{t=1}^{T} S_t}{\sum_{t=1}^{T-1} (1 - S_{t+1}) S_t} \text{ and } D_{PT} = \frac{\sum_{t=1}^{T} (1 - S_t)}{\sum_{t=1}^{T-1} (1 - S_t) S_{t+1}} \tag{2.23}$$

where PT (TP) denotes the phase from peak (trough) to trough (peak).

The second feature measures the average amplitude of recessions (expansions) as:

$$AMP_{TP} = \frac{\sum_{t=1}^{T} S_t \Delta C_t}{\sum_{t=1}^{T-1} (1 - S_{t+1}) S_t} \text{ and } AMP_{PT} = \frac{\sum_{t=1}^{T} (1 - S_t) \Delta C_t}{\sum_{t=1}^{T-1} (1 - S_t) S_{t+1}} \tag{2.24}$$

where ΔC_t is the common factor growth rate calculated in the DF model.

The steepness of business cycle phases is the third feature considered. It is expressed as the ratio of the amplitude to the duration:

$$STEEP_{TP} = \frac{AMP_{TP}}{D_{TP}} = \frac{\sum_{t=1}^{T} S_t \Delta C_t}{\sum_{t=1}^{T} S_t} \tag{2.25}$$

and $$STEEP_{PT} = \frac{AMP_{PT}}{D_{PT}} = \frac{\sum_{t=1}^{T} (1 - S_t) \Delta C_t}{\sum_{t=1}^{T} (1 - S_t)}$$

This statistic describes the amount of welfare that is lost (gained) in each quarter spent in recessions (expansions). It also measures the speed at which an economy falls into and emerges from a recession. This figure tends to be big when a large portion of output loss (gain) occurs during a short period of time.

The cumulative movement of recessions (expansions), which measures the overall welfare loss (gain) during recessions (expansions), is the last feature analysed in this section. It is calculated as follows

$$CM_i = 0.5(AMP_i * D_i), \quad i = TP, PT \tag{2.26}$$

The results for the four cyclical characteristics outlined above are reported in Table 2.17 for the whole sample period. One of the stylised features of business cycles is revealed by comparing columns 2 and 3, with expansions being much longer than recessions in all the countries analysed. Significantly asymmetric business cycle phases are observed in the three non-EMU countries, with expansions lasting over six times longer than recessions. On the other hand, expansions are found to be relatively short in Germany and Italy, with recessions occurring much more frequently than in the other countries studied. Among the countries analysed, France experienced the longest average expansionary phase of 35 quarters, while the longest average recessionary phase occurred in the Netherlands, lasting for 7.2 quarters.

The asymmetric nature of business cycles is also observed in columns 4 and 5, with the amplitude of expansions being significantly larger than those for recessions. Again, the three non-EMU countries stand out, with their average expansionary amplitude being eight times larger than their recessionary amplitude. In contrast, the corresponding values for the peripheral and core EMU countries are six times and five times larger, respectively. Large business cycle amplitude is observed in the Dutch results. This reflects the severity of the recession which occurred during the early 1980s and the fast economic growth thereafter. In addition, large expansionary amplitudes are also observed in the French and Canadian business cycles, whilst the German cycle exhibits the smallest expansionary amplitude.

The steepness of expansions and recessions is reported in columns 6 and 7. Once more, both the steepest recessionary and expansionary phases are found in the Netherlands. In contrast, the German business cycle had the mildest expansionary phase. In general, the non-EMU countries and small EMU countries had steeper recessionary and expansionary phases than the other countries. To some extent, such volatility is caused by the more deregulated labour and product markets in these countries.

The cumulative movements of expansions and recessions are reported in the last two columns of Table 2.17. For all countries, the welfare gains achieved in expansions appear to be considerably higher than the losses suffered from recessions. On average, the non-EMU countries had the largest gains during the expansionary phase and the smallest losses in recessions. Again, the Dutch business cycle experienced the largest welfare gain and loss out of all the countries analysed, while the German business cycle shows the smallest welfare gain during expansions.

Table 2.17: Average Business Cycle Features

	Duration (quarters)		Amplitude (%)		Steepness (%)		Cumulated (%)	
	Expansion	Recession	Expansion	Recession	Expansion	Recession	Expansion	Recession
EMU	26.00	4.00	33.22	-2.33	1.28	-0.58	431.92	-4.67
GER	14.00	6.83	9.13	-3.49	0.65	-0.51	63.90	-11.93
FRA	35.00	6.33	49.18	-4.09	1.41	-0.65	860.73	-12.95
AUS	18.00	5.40	21.28	-2.41	1.18	-0.45	191.55	-6.51
BEL	18.67	5.33	36.94	-8.49	1.98	-1.59	344.80	-22.63
ITA	12.78	3.22	14.25	-2.65	1.12	-0.82	91.05	-4.27
NETH	27.00	7.20	84.80	-18.94	3.14	-2.63	1144.74	-68.19
SPA	21.00	5.80	36.34	-5.17	1.73	-0.89	381.52	-15.00
FIN	23.60	4.20	27.23	-4.36	1.15	-1.04	321.31	-9.16
UK	29.75	4.75	24.97	-4.96	0.84	-1.04	371.37	-11.77
US	25.20	3.60	36.57	-3.78	1.45	-1.05	460.78	-6.80
CANA	32.50	3.75	49.64	-4.83	1.53	-1.29	806.64	-9.06
Core	*20.91*	*5.72*	*35.93*	*-6.68*	*1.58*	*-1.11*	*449.46*	*-21.08*
Periphery	*22.30*	*5.00*	*31.79*	*-4.77*	*1.44*	*-0.97*	*351.42*	*-12.08*
Non	*29.15*	*4.03*	*37.06*	*-4.52*	*1.27*	*-1.13*	*546.26*	*-9.21*

Note: Core, Periphery and Non denote the mean of the cyclical characteristics for the core, peripheral and non- EMU countries.

As with the changes in cycle concordance, the evolutions in business cycle similarities and differences are also evaluated to examine the validity of the endogenous OCA theory. Table 2.18 reports the business cycle characteristics for the two subsamples: pre-1991Q1 and post-1991Q1. The standard deviations of business cycle characteristics are calculated across all the EMU countries. These figures suggest that mixed progress has been made towards cyclical synchronisation. Differences in the recessionary phase appear to diminish, while variations in the expansionary phase increase. This, again, reveals the unbalanced economic growth which has occurred across EMU members. Germany and Italy have been characterised by short and mild expansions, while French, Spanish and Finnish expansionary phases appear longer lasting, with larger amplitudes and greater welfare gains.

Comparing the mean statistics of all cyclical characteristics over the two subsamples shows that differences in business cycle characteristics among the core, periphery and non-EMU countries become more distinctive over time. On average, both recessions and expansions in the core EMU countries exhibit shorter durations and smaller amplitudes during the second subsample. The two periphery countries, particularly Spain, achieved remarkable welfare gains during their sustained expansions. Finally, the non-EMU countries also experienced long lasting expansions with moderate welfare gains and little losses suffered in recessions.

Table 2.18: Evolution of Business Cycle Features

	Duration (quarters)		Amplitude (%)		Steepness (%)		Cumulated (%)	
	Expansion	Recession	Expansion	Recession	Expansion	Recession	Expansion	Recession
				Pre-1991Q1				
EMU	22.50	3.67	32.55	-2.42	1.45	-0.66	366.19	-4.43
GER	20.33	7.33	15.58	-11.66	0.77	-1.59	158.43	-42.75
FRA	24.00	6.00	33.69	-3.70	1.40	-0.62	404.23	-11.11
AUS	17.67	6.00	25.65	-8.79	1.45	-1.47	226.60	-26.38
BEL	15.25	6.33	32.20	-39.63	2.11	-6.26	245.53	-125.48
ITA	17.00	3.00	21.82	-12.50	1.28	-4.17	185.50	-18.75
NETH	28.50	7.67	95.31	-86.55	3.34	-11.29	1358.19	-331.77
SPA	12.00	5.50	18.17	-14.51	1.51	-2.64	109.04	-39.90
FIN	16.50	3.00	18.44	-8.13	1.12	-2.71	152.14	-12.19
STD.	**5.24**	**1.76**	**26.18**	**27.83**	**0.79**	**3.49**	**415.13**	**109.74**
UK	16.25	5.33	15.38	-18.08	0.95	-3.39	125.00	-48.22
US	16.50	4.67	26.70	-16.24	1.62	-3.48	220.28	-37.90
CANA	23.33	5.50	40.94	-15.91	1.75	-2.89	477.58	-43.76
Core	**20.46**	**6.06**	**37.38**	**-27.14**	**1.73**	**-4.23**	**429.75**	**-92.71**
Periphery	**14.25**	**4.25**	**18.31**	**-11.32**	**1.32**	**-2.68**	**130.59**	**-26.05**
Non	**18.69**	**5.17**	**27.67**	**-16.74**	**1.44**	**-3.25**	**274.29**	**-43.29**
				Post-1991Q1				
EMU	59.00	5.00	67.80	-6.91	1.15	-1.38	2000.03	-17.28
GER	9.25	6.33	4.29	-9.29	0.46	-1.47	19.83	-29.42
FRA	57.00	7.00	80.18	-8.56	1.41	-1.22	2285.19	-29.97
AUS	27.50	4.50	25.37	-3.27	0.92	-0.73	348.86	-7.35
BEL	25.50	4.33	46.42	-11.30	1.82	-2.61	591.92	-24.48
ITA	9.40	3.40	8.19	-11.34	0.87	-3.34	38.51	-19.28
NETH	25.50	6.50	74.28	-8.16	2.91	-1.26	947.08	-26.53
SPA	57.00	7.00	108.98	-11.36	1.91	-1.62	3106.01	-39.76
FIN	52.00	6.00	63.12	-13.69	1.21	-2.28	1641.03	-41.06
STD.	**19.92**	**1.37**	**37.02**	**3.14**	**0.77**	**0.86**	**1124.83**	**10.87**
UK	54.00	3.00	38.33	-1.75	0.71	-0.58	1034.83	-2.62
US	60.00	2.00	76.04	-2.65	1.27	-1.33	2281.34	-2.65
CANA	60.00	2.00	75.75	-3.42	1.26	-1.71	2272.59	-3.42
Core	**25.69**	**5.34**	**39.79**	**-8.65**	**1.40**	**-1.77**	**705.23**	**-22.84**
Periphery	**54.50**	**6.50**	**86.05**	**-12.53**	**1.56**	**-1.95**	**2373.52**	**-40.41**
Non	**58.00**	**2.33**	**63.37**	**-2.61**	**1.08**	**-1.21**	**1862.92**	**-2.90**

Note: STD. denotes the standard deviation of the cyclical characteristics for the EMU countries.

2.7 Conclusions

This chapter identifies business cycle turning points for both the aggregate euro area and individual countries. Instead of just analysing GDP series, a composite index of four multivariate coincident macroeconomic variables is derived using the DF model. The BBQ algorithm is then applied to identify possible business cycle turning points in this index, with a binary variable constructed to indicate the periods of recession and expansion.

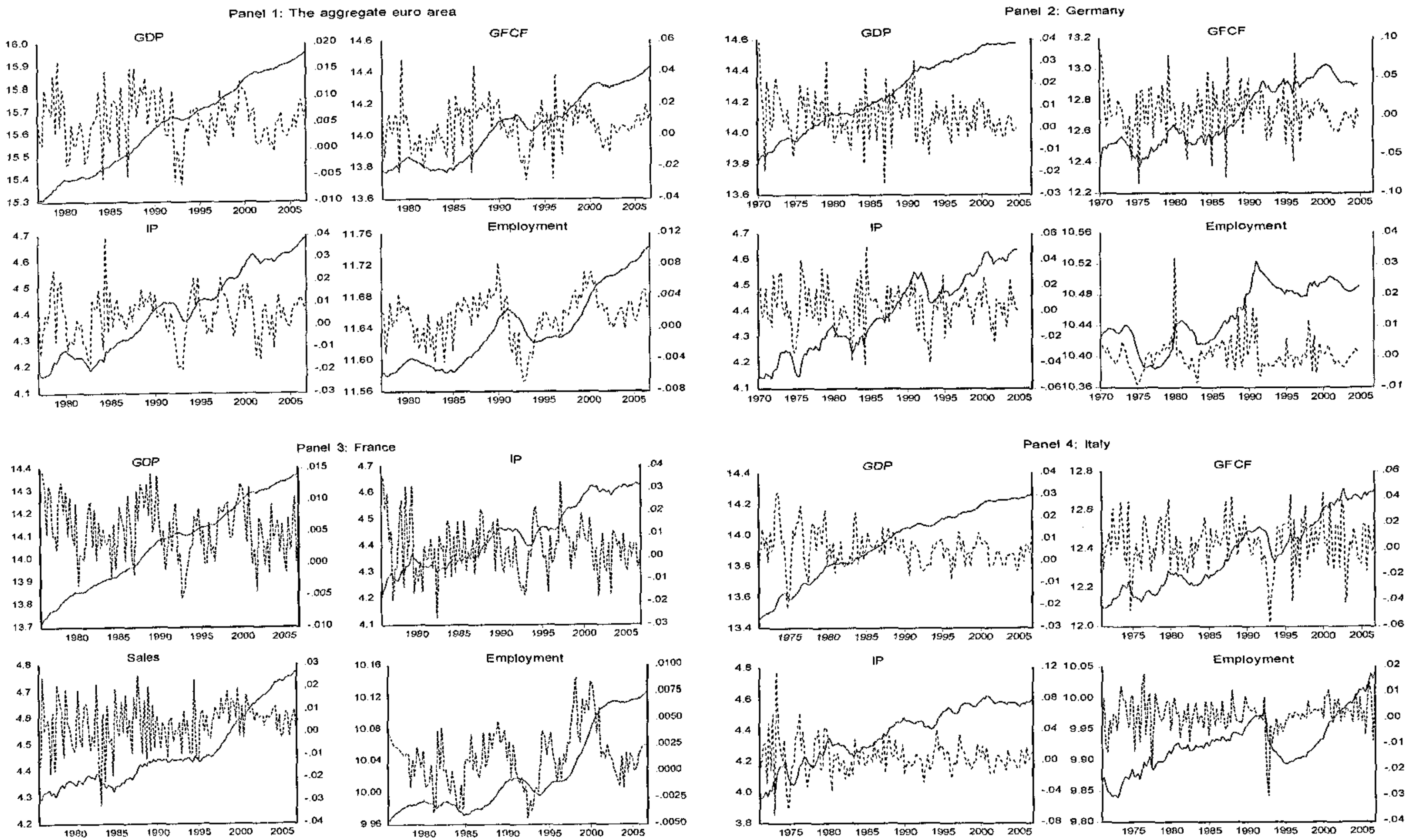
Two criteria are used to evaluate the degree of business cycle synchronisation: the concordance of business cycle turning points and similarity of business cycle characteristics. Overall, the core EMU countries share more synchronised business cycles with the aggregate euro area than with the peripheral and non-EMU countries. However, it is worth noting that the high cycle correlations with some countries may reflect the large weights they are assigned when constructing the aggregate euro area data. The evolution of business cycle correlations over time is also analysed by breaking the sample period into two subsamples, with the midpoint being 1991Q1. The business cycles between the EMU countries and the three non-EMU countries are found to be unsynchronised. This is in line with the results obtained in Stock and Watson (2003), Camacho and Perez-Quiros (2006) and Garnier (2003). However, no further business cycle convergence is observed between the aggregate euro area and four of the core EMU countries: German, Austria, Belgium and the Netherlands. This could be due to the recessions experienced by these economies during the early 2000s which were not observed in the euro area as a whole. Over the same period, France and the two peripheral countries, Spain and Finland, show a significant increase in business cycle comovements with the aggregate euro area, and have experienced robust growth during recent years.

Finally, this chapter investigated four, sample averaged, cyclical characteristics concerning cycle duration, amplitude, steepness and welfare gains. These results suggest that there exist significant differences in the business cycle phases among the EMU countries, and that the differences across expansionary phases have increased over time. This, again, confirms the unbalanced economic performance observed across the euro area. The short and mild expansions observed in Germany and Italy led to sluggish economic growth. However, the steep and long lasting Spanish expansionary phase brought huge welfare gains to the Spanish economy.

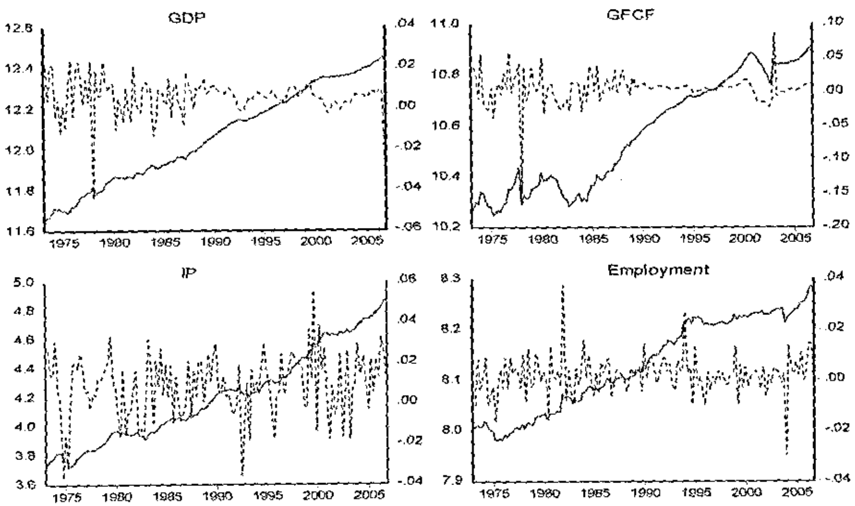
Overall, the results obtained in this chapter contradict the argument proposed in the endogenous OCA theory that operating a monetary union will increase the degree of business cycle synchronisation in terms of both concordance and similarity. Furthermore, variations in economic performance observed across the euro area will lead to diverging monetary policy requirements and, consequently, will reduce the appropriateness of having a common monetary policy for all members.

Appendix A2

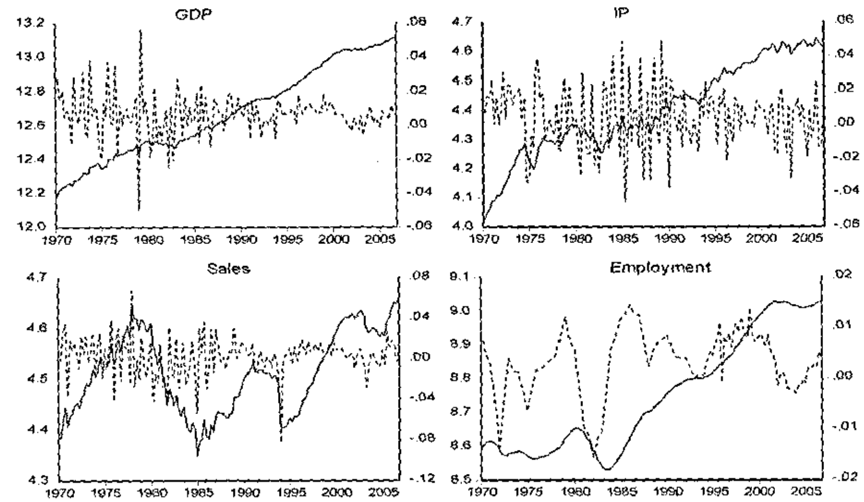
Figure A2.1 Coincident macroeconomic time series



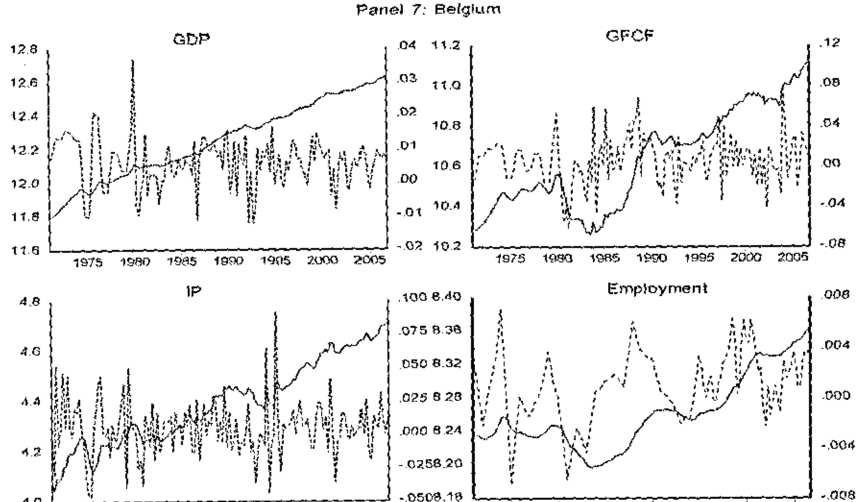
Panel 5: Austria



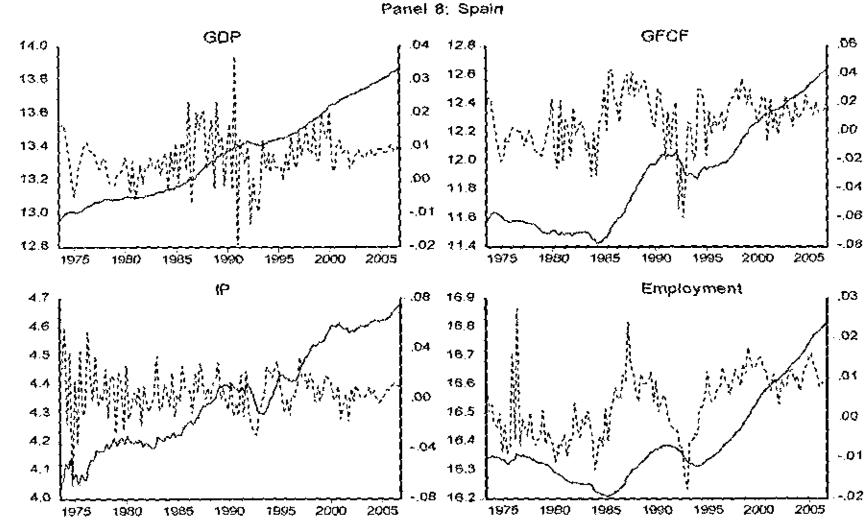
Panel 6: The Netherlands



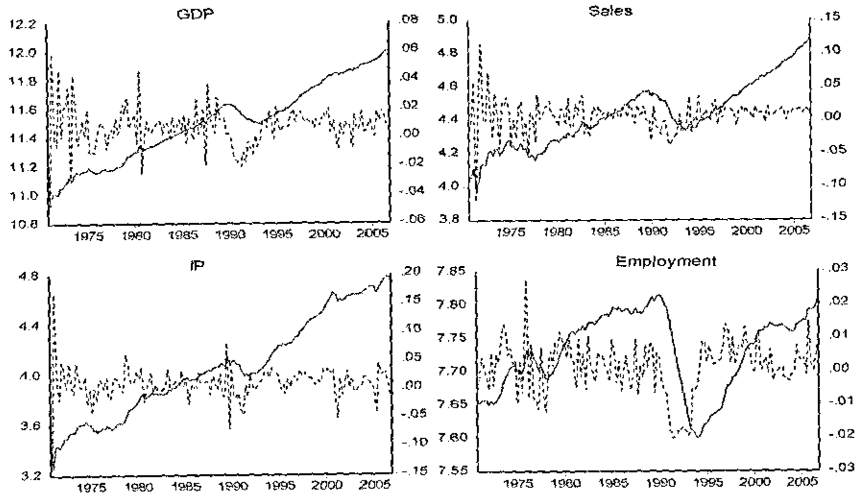
Panel 7: Belgium



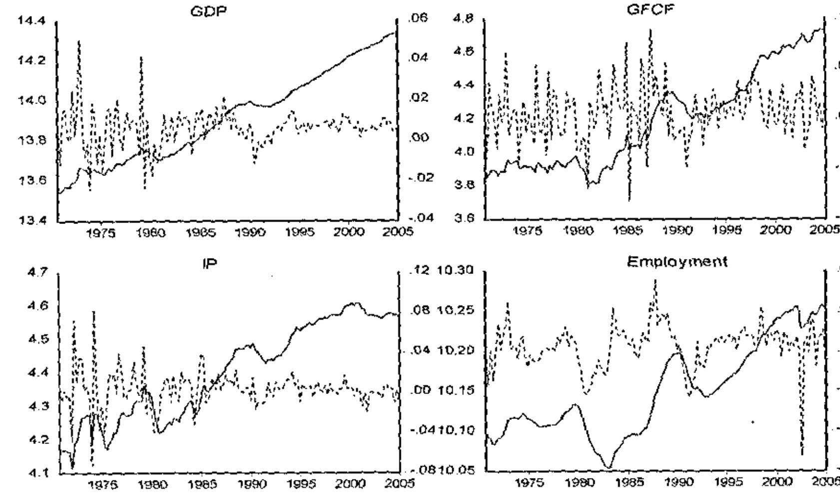
Panel 8: Spain



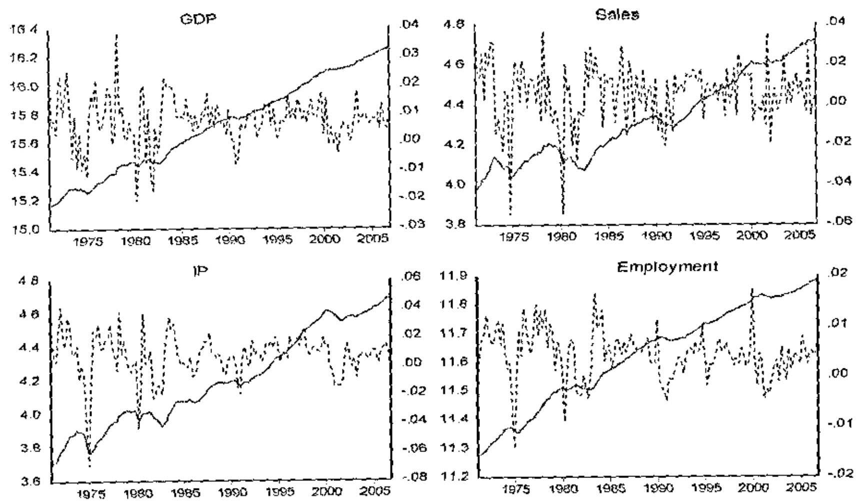
Panel 9: Finland



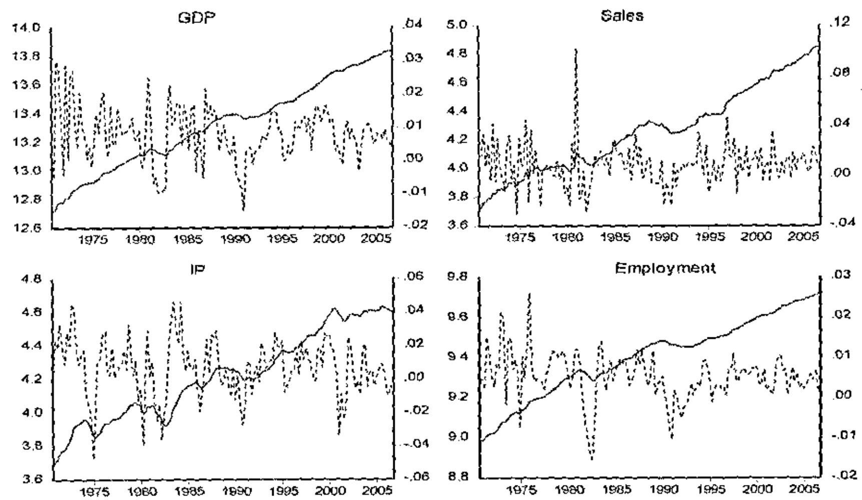
Panel 10: The UK



Panel 11: The US



Panel 12: Canada



Note: levels of data are plotted in solid lines (left axis), and the corresponding first differenced data are plotted in dashed lines (right axis).

Figure A2.2 Growth rates of common factors, Δc_t

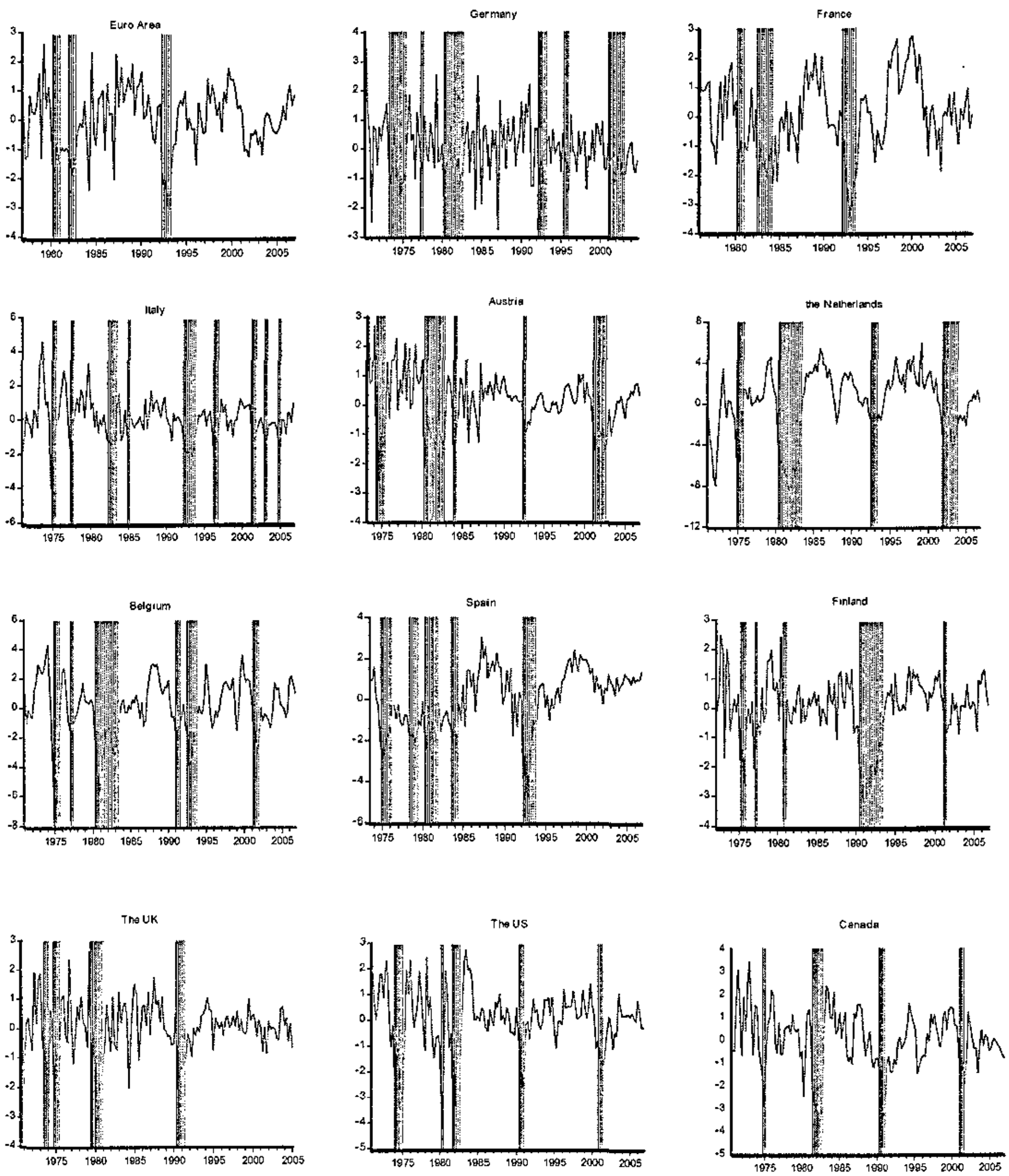


Table A2.1: Coincident Macroeconomic Variables

	Sample periods	Variables	Augmented Dickey-Fuller		
			Level		*First-differenced*
			Constant	*Constant+trend*	*Constant*
EMU	1975Q3-2006Q4	GDP	-0.334	-2.314	-5.747**
			(0.915)	(0.423)	(0.000)
		GFCF	0.266	-3.237	-5.341**
			(0.976)	(0.082)	(0.000)
		IP	-0.064	-3.543*	-5.011**
			(0.950)	(0.039)	(0.000)
		EMP	0.158	-1.854	-2.652*
			(0.969)	(0.672)	(0.056)
GER	1970Q1-2004Q4	GDP	-2.297	-1.970	-11.952**
			(0.175)	(0.612)	(0.000)
		GFCF	-1.012	-2.630	-14.361**
			(0.748)	(0.268)	(0.000)
		IP	-0.764	-3.078	-9.552**
			(0.826)	(0.116)	(0.000)
		EMP	-0.934	-3.409	-8.564**
			(0.775)	(0.054)	(0.000)
FRA	1975Q4-2006Q4	GDP	-1.416	-2.964	-6.005**
			(0.573)	(0.147)	(0.000)
		SALES	2.111	-0.235	-13.722**
			(1.000)	(0.992)	(0.000)
		IP	0.086	-2.095	-2.991*
			(0.964)	(0.543)	(0.039)
		EMP	-0.786	-2.626	-5.322**
			(0.819)	(0.270)	(0.000)
ITA	1970Q1-2006Q4	GDP	-2.081	-2.097	-6.985**
			(0.253)	(0.543)	(0.000)
		GFCF	-0.244	-2.114	-10.711
			(0.929)	(0.533)	(0.000)
		IP	-1.778	-2.869	-11.190**
			(0.390)	(0.176)	(0.000)
		EMP	0.569	-1.453	-5.002**
			(0.988)	(0.841)	(0.000)
AUS	1973Q1-2006Q4	GDP	-1.264	-2.486	-11.862**
			(0.645)	(0.335)	(0.000)
		GFCF	-0.469	-2.273	-13.136**
			(0.892)	(0.445)	(0.000)
		IP	1.644	-0.834	-5.745**
			(1.000)	(0.959)	(0.000)
		EMP	0.189	-2.279	-14.500**
			(0.971)	(0.442)	(0.000)
NETH	1970Q1-2006Q4	GDP	-0.723	-1.972	-15.556**
			(0.837)	(0.611)	(0.000)
		SALES	-1.327	-1.485	-13.993**
			(0.616)	(0.831)	(0.000)
		IP	0.134	-4.146**	-3.895**
			(0.967)	(0.007)	(0.003)
		EMP	-0.723	-1.972	-15.556**
			(0.837)	(0.611)	(0.000)

Note: ** denotes significance at 1% and * at 5%.

Table A2.1: Coincident Macroeconomic Variables (Continued)

BEL	1970Q1-2006Q4	GDP	-1.304	-4.004*	-8.418**
			(0.627)	(0.011)	(0.000)
		GFCF	-0.228	-1.904	-5.983**
			(0.931)	(0.647)	(0.000)
		IP	-0.662	-3.260	-12.368**
			(0.852)	(0.077)	(0.000)
		EMP	0.546	-1.523	-3.451*
			(0.988)	(0.817)	(0.011)
SPA	1972Q3-2006Q4	GDP	1.178	-1.953	-4.016**
			(0.998)	(0.621)	(0.002)
		GFCF	0.771	-2.224	-4.200**
			(0.993)	(0.472)	(0.001)
		IP	-0.554	-3.261	-6.276**
			(0.876)	(0.077)	(0.000)
		EMP	0.574	-1.162	-2.512
			(0.989)	(0.913)	(0.115)
FIN	1970Q1-2006Q4	GDP	-0.152	-2.376	-4.475**
			(0.940)	(0.391)	(0.000)
		SALES	0.050	-1.313	-13.318**
			(0.961)	(0.881)	(0.000)
		IP	-0.178	-2.331	-14.098**
			(0.937)	(0.414)	(0.000)
		EMP	-2.109	-2.157	-3.990**
			(0.242)	(0.510)	(0.002)
UK	1970Q1-2005Q1	GDP	-2.081	-2.097	-6.985**
			(0.253)	(0.543)	(0.000)
		GFCF	-0.244	-2.114	-10.711**
			(0.929)	(0.533)	(0.000)
		IP	-1.778	-2.869	-11.190**
			(0.390)	(0.176)	(0.000)
		EMP	0.569	-1.453	-5.002**
			(0.988)	(0.841)	(0.000)
US	1970Q1-2006Q4	GDP	-0.651	-3.863*	-9.423**
			(0.854)	(0.016)	(0.000)
		SALES	-0.261	-1.669	-11.140**
			(0.927)	(0.760)	(0.000)
		IP	-0.954	-3.175	-6.834**
			(0.768)	(0.094)	(0.000)
		EMP	-1.744	-2.553	-6.547**
			(0.407)	(0.303)	(0.000)
CAN	1970Q1-2006Q4	GDP	-1.228	-2.824	-8.164**
			(0.661)	(0.191)	(0.000)
		SALES	-0.070	-1.786	-12.835**
			(0.950)	(0.707)	(0.000)
		IP	-1.561	-3.865*	-6.283**
			(0.500)	(0.016)	(0.000)
		EMP	-1.824	-2.711	-6.197**
			(0.368)	(0.234)	(0.000)

Note: ** denotes significance at 1% and * at 5%.

Table A2.2: Cointegrating Test (Johansen 1995)

Country	Null	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$
EMU	Max-Eigen stat.	32.845**	15.539	3.744	1.713
		(0.010)	(0.253)	(0.885)	(0.191)
	Trace stat.	53.841*	20.996	5.457	1.713
		(0.012)	(0.358)	(0.759)	(0.191)
GER	Max-Eigen stat.	19.375	16.242	6.599	1.769
		(0.386)	(0.211)	(0.538)	(0.183)
	Trace stat.	43.986	24.611	8.369	1.769
		(0.110)	(0.176)	(0.427)	(0.183)
FRA	Max-Eigen stat.	32.981**	19.400	4.060	0.668
		(0.009)	(0.086)	(0.853)	(0.414)
	Trace stat.	57.109**	24.128	4.728	0.668
		(0.005)	(0.195)	(0.837)	(0.414)
ITA	Max-Eigen stat.	30.894*	17.256	6.086	1.216
		(0.018)	(0.160)	(0.602)	(0.270)
	Trace stat.	55.451**	24.557	7.302	1.216
		(0.008)	(0.178)	(0.543)	(0.270)
AUS	Max-Eigen stat.	47.570*	21.236	6.601	2.451
		(0.053)	(0.343)	(0.624)	(0.117)
	Trace stat.	26.334	14.635	4.150	2.451
		(0.072)	(0.315)	(0.843)	(0.117)
NETH	Max-Eigen stat.	40.326**	24.666*	3.779	0.459
		(0.001)	(0.015)	(0.882)	(0.498)
	Trace stat.	69.229**	28.904	4.238	0.459
		(0.000)	(0.063)	(0.883)	(0.498)
BEL	Max-Eigen stat.	37.478**	16.043	6.209	0.245
		(0.002)	(0.222)	(0.587)	(0.621)
	Trace stat.	59.974**	22.497	6.454	0.245
		(0.002)	(0.272)	(0.642)	(0.621)
SPA	Max-Eigen stat.	36.238**	20.022	10.543	0.111
		(0.003)	(0.071)	(0.179)	(0.740)
	Trace stat.	66.913**	30.675*	10.653	0.111
		(0.000)	(0.040)	(0.234)	(0.740)
FIN	Max-Eigen stat.	39.248	19.510	6.131	1.178
		(0.251)	(0.457)	(0.680)	(0.278)
	Trace stat.	19.738	13.379	4.953	1.178
		(0.360)	(0.418)	(0.748)	(0.278)
UK	Max-Eigen stat.	32.804**	13.730	7.851	2.519
		(0.010)	(0.388)	(0.394)	(0.113)
	Trace stat.	56.903**	24.099	10.370	2.519
		(0.006)	(0.196)	(0.253)	(0.113)
US	Max-Eigen stat.	27.489*	16.928	5.713	0.772
		(0.051)	(0.176)	(0.650)	(0.380)
	Trace stat.	50.903*	23.413	6.485	0.772
		(0.025)	(0.226)	(0.638)	(0.380)
CAN	Max-Eigen stat.	44.115**	10.240	5.621	0.359
		(0.000)	(0.722)	(0.662)	(0.549)
	Trace stat.	60.335**	16.220	5.980	0.359
		(0.002)	(0.697)	(0.698)	(0.549)

Note: the default option in EViews is used in the above Johansen tests in which an intercept is included in both the cointegration equation and the differenced form of the VAR. ** denotes significance at 1% and * at 5%.

Appendix B2: Kalman filter

The measurement and transition equations utilised in this chapter are

$$\Delta y_t = H\beta_t , \tag{B2.1}$$

$$\beta_t = F\beta_{t-1} + \varepsilon_t , \varepsilon_t \sim NID(0,Q) . \tag{B2.2}$$

The filtering recursive equations are as follows,

$$\beta_{t|t-1} = F\beta_{t-1|t-1} , \qquad P_{t|t-1} = FP_{t-1|t-1}F^{\top} + Q ,$$

$$\eta_{t|t-1} = y_t - H\beta_{t|t-1} , \qquad f_{t|t-1} = HP_{t|t-1}H^{\top} ,$$

$$\beta_{t|t} = \beta_{t|t-1} - P_{t|t-1}H^{\top} f_{t|t-1}^{-1} \eta_{t|t-1} ,$$

$$P_{t|t} = \left(I - P_{t|t-1}H^{\top} f_{t|t-1}^{-1} H\right)P_{t|t-1} .$$

The smoothing process for the vector β_t contains two equations:

$$\beta_{t|T} = \beta_{t|t} + P_{t|t}F^{\top}P_{t+1|t}^{-1}\left(\beta_{t+1|T} - \beta_{t+1|t}\right), \tag{B2.3}$$

$$P_{t|T} = P_{t|t} + P_{t|t}F^{\top}P_{t+1|t}^{-1}\left(P_{t+1|T} - P_{t+1|t}\right){P_{t+1|t}^{-1}}^{\top} FP_{t|t}{}^{\top} , \tag{B2.4}$$

where $\beta_{T|T}$ and $P_{T|T}$ are the initial values of the smoothing. They are obtained from the last iteration of the basic filter.

Chapter 3 - Parametric Business Cycle Dating Procedure: The Markov-switching dynamic factor model

3.1 Introduction

The seminal research of Burns and Mitchell (1946) has highlighted two dominant features of business cycles: comovement among economic variables and asymmetry between expansions and recessions. Following the non-parametric business cycle dating strategy outlined in the previous chapter, the comovements among variables can be modelled using the DF model, in which multivariate information is used to derive a composite index representing aggregate economic activity. However, this methodology cannot incorporate business cycle asymmetries as the estimated index has the same data generating process during recessions and expansions.

Hamilton (1989) utilises the Markov-switching (MS) model to capture the asymmetry between business cycle phases. This model allows the mean of an autoregression of US real GNP growth to switch between low-growth and high-growth states by following a first-order Markov process. On applying this model, Hamilton successfully replicates the NBER reference dates over the period 1953-1984. However, the model fails to identify the more recent US recessions when an extended sample period is used, primarily because of the business cycle moderation observed during recent decades. One-time breaks in both the mean growth rates and residual variances have therefore been allowed in the Hamilton model to overcome this problem (Kim and Nelson, 1999a). Mills and Wang (2003a) apply this modified model to the G7 countries, and find evidence of moderated business cycles across all member countries. In addition, Hansen (1992) extended the Hamilton model to allow for regime switching in the autoregressive parameters and residual variance. Furthermore, additional phases have been introduced into business cycle dynamics to capture periods of fast economic growth, such as the three-regime MS model used in Sichel (1994), Boldin (1996) and

Clements and Krolzig (1998), and the "bounce back" model proposed by Kim *et al.* (2005). It should be noted that the models summarised above are based on a univariate framework, so that the comovements of many macroeconomic variables through the cycle cannot be modelled. Another important extension to the Hamilton model is the Markov-switching Vector Autoregression (MS-VAR) proposed by Krolzig (1997a; 1997b). This model is designed to detect common business cycle turning points in multivariate time-series. A number of studies, including Krolzig (1997a), Krolzig and Toro (2005) and Artis, Krolzig and Toro (2004), have applied this model to the national GDP and IP series of OECD and EMU countries to identify common business cycles across countries. Krolzig (1997b) further proposes a Markov-switching vector error correction model (MS-VECM) when the time-series analysed are cointegrated. Krolzig (2001) applies a three-regime MS-VECM to detect the common regime shifts in the mean growth rates of US employment and output over the period 1960-1997, as the two series were found to have long-run dynamics when a time trend is included. In addition, Krolzig *et al.* (2002) employ this model to analyse the UK labour market using output, employment, labour supply and real earnings, which are characterised as having two cointegrating vectors. The MS-VECM is also applied to disaggregated UK industrial production data by Krolzig and Sensier (2000) to investigate the common cycle shared by six major manufacturing sectors.

Finally, the Markov-Switching Dynamic-Factor (MSDF) model, first proposed in Diebold and Rudebusch (1996), was applied by Chauvet (1998), Kim and Yoo (1995), Kim and Nelson (1999c, 2001) and Mills and Wang (2003b) to US and UK data. This model applies the Hamilton model to a multivariate setting by combining the DF model with the MS framework. Unlike the MS-VAR model, the MSDF model derives not only common business cycle turning points, but also a composite coincident index representing aggregate economic activity.

One of the objectives of this chapter is to compare the cycle dates produced by the MSDF model with those obtained in the previous chapter.[1] Differences in results are expected as the two approaches are based on fundamentally different mechanisms. Recessions identified using the BBQ algorithm are based on the rule that recessions are

[1] The pros and cons of both the MS models and the BBQ algorithm have been debated in a series of papers, including Harding and Pagan (2002) and Hamilton (2003).

defined to be the absolute fall in the level of economic activity in two consecutive quarters. However, there are shortcomings with this rule. For example, if Economy A grows by 2% in the first quarter but then declines by 0.5% in each of the following two quarters, it is deemed to be in recession. However, if Economy B contracts by 2% in the first quarter, increases by 0.5% in the second quarter, and then falls by 2% in the third quarter no recession is identified, even though Economy B is weaker than Economy A. This example illustrates that the severity of an economic downturn should also be considered when defining a recession.[2] The MSDF model seems better suited to this task. Instead of using the "two-quarter" rule, it makes inferences on the unobserved regimes of an economy using the smoothed regime probabilities. If a downturn appears to be deep, the smoothed recession probabilities will be close to unity. Likewise, if a decline is shallow, the smoothed recession probabilities are close to zero. A recession is declared only if the smoothed recession probabilities are above 0.5. Therefore, the depth of recessions can be revealed using the MSDF model.

The rest of the chapter is organised as follows. Section 3.2 presents the baseline MSDF model. Modifications of the baseline model are discussed in section 3.3. Section 3.4 describes the parameter estimates and cycle dates obtained using the MSDF model. Cycle synchronisation is evaluated using pairwise correlations of both binary variables and smoothed recession probabilities in section 3.5. Finally, section 3.6 concludes.

3.2 Baseline model specification

Kim and Nelson's (1999c, 2001) MSDF model with a two-state MS mean is specified as follows

$$\Delta Y_{it} = D_i + \gamma_i(L)\Delta C_t + e_{it}, \tag{3.1}$$

$$\phi(L)(\Delta C_t - \mu_{S_t} - \delta) = v_t,\ v_t \sim NID(0,\sigma_v^2), \tag{3.2}$$

$$\psi_i(L)e_{it} = \varepsilon_{it},\ \varepsilon_{it} \sim NID(0,\sigma_i^2), \tag{3.3}$$

$$\mu_{S_t} = \mu_0(1-S_t) + \mu_1 S_t,\ S_t = \{0,1\}. \tag{3.4}$$

[2] This is consistent with the business cycle dating approach used by the NBER, who define a recession as a significant decline in economic activity spread across the economy lasting more than a few months, rather than two consecutive quarters of decline in real GDP.

As with Stock and Watson's (1989, 1991, 1993) DF model, the growth of each macroeconomic variable, ΔY_{it} , consists of two stochastic components: a linear combination of the current and lagged values of the common factor, $\gamma_i(L)\Delta C_t$, and an individual component, $D_i + e_{it}$. In a slight variation of the DF model, the common factor is modelled as a nonlinear AR process with MS deviations from its constant long-run growth rate, δ. The value of μ_{S_t} depends on whether the economy is in a recession ($S_t = 0$) or an expansion ($S_t = 1$). $\mu_0 + \delta$ and $\mu_1 + \delta$ represent the mean growth rates during recessions and expansions, respectively, with $\mu_0 < 0 < \mu_1$. Transitions between regimes are controlled by four constant transition probabilities:

$$\Pr[S_t = 0|S_{t-1} = 0] = p_{00},\ \Pr[S_t = 1|S_{t-1} = 0] = 1 - p_{00},$$

$$\Pr[S_t = 1|S_{t-1} = 1] = p_{11},\ \Pr[S_t = 0|S_{t-1} = 1] = 1 - p_{11}.$$

The logged variables in first differences are standardised to have zero mean and unit variance. This allows equations (3.1) and (3.2) to be replaced by

$$\Delta y_{it} = \gamma_i(L)\Delta c_t + e_{it}, \tag{3.5}$$

$$\phi(L)(\Delta c_t - \mu_{S_t}) = v_t, v_t \sim NID(0, \sigma_v^2) \tag{3.6}$$

where $\Delta c_t = \Delta C_t - \delta$. By recasting equations (3.3), (3.4) (3.5) and (3.6) into state-space representation, Kim's (1994) approximate MLE[3] can be applied to estimate the hyperparameters: $\{\phi_i, \psi_i, \gamma_i, \sigma_i, p_{00}, p_{11}, \mu_0, \mu_1\}$. The state-space representation of the above model is given by

$$\Delta y_t = H\beta_t, \tag{3.7}$$

$$\beta_t = M_{\phi(L)S_t} + F\beta_{t-1} + \varepsilon_t, \varepsilon_t \sim NID(0, Q). \tag{3.8}$$

[3] Kim's (1994) filtering and smoothing algorithms, together with the approximate MLE, are discussed in Appendix A3.

Equations (3.7) and (3.8) are the measurement and transition equations, respectively. The vectors Δy_t and β_t, and the time-invariant matrices H and F, are specified as the corresponding components in the DF model. $M_{\phi(L)S_t} = [\phi(L)S_t, 0, 0 ... 0]$ contains the regime-switching mean for the common factor. The equation $\phi(L)S_t = \mu_{S_t} - \phi_1\mu_{S_{t-1}} - \phi_2\mu_{S_{t-2}}$ brings the current and two lagged two-state MS variables (i.e., S_t, S_{t-1} and S_{t-2}) into the state-space form. Therefore, a total of 2^3 states should be included at each stage of the KF iteration (Kim, 1994).[4] As with Kim and Yoo (1995), Chauvet (1995) and Kim and Nelson (1999c, 2001), this chapter assumes that the intercept of the common factor, rather than the mean of the common factor, is MS. Therefore, equation (3.6) is modified as

$$\phi(L)\Delta c_t = \mu_{S_t} + v_t . \tag{3.9}$$

Only S_t is included in the state-space representation, thus reducing the number of states to 2^2 at each iteration.

As discussed in Chapter 2, Section 2.2.1, the mean growth rate of the linear common factor is calculated as $\hat{\delta} = W(1)\Delta\overline{Y}$, where $W(1)$ is the first row of $(I-(I-KH)F)^{-1}K$. K is the steady-state Kalman gain. In the MSDF model, K is calculated as the weighted average over 2^2 $K_T^{(i,j)}$, $i,j = 1,2$, at the last iteration.[5]

3.3 Modifications to the baseline model

The two-regime baseline model presented above is first considered for all countries in the analysis. However, given the properties of the data analysed, three modifications have been made to the baseline model. First, as discussed in section 2.3, Johansen cointegration tests reject the null of no cointegration among the four variables used for the aggregate euro area, France, Italy, Belgium, the Netherlands, Spain and the three non-EMU countries. Therefore, the measurement equation (3.7) should be modified as

---

[4] Kim (1994) suggests that when current and r lagged M state MS variables are included in a state-space form, at least M^{r+1} states should be included at each stage of the KF iteration.

[5] Please see Appendix A3 for details.

$$\Delta y_t = H\beta_t + A \times ECM_{t-1}, \tag{3.10}$$

where ECM_t contains the error correction terms which are estimated independently using the VECM. A is a matrix containing the corresponding adjustment parameters.

Second, the fluctuations of the French common factor are observed to have three phases rather than two: recessions, moderate-growth and high-growth periods. As with Sichel (1994), Boldin (1996) and Clements and Krolzig (1998), who all include an additional regime in the Hamilton model to capture the phases of rapid recovery in the US business cycle dynamics, this chapter modifies the data generating process of the French common factor to have a three-regime intercept. Equation (3.4) becomes

$$\mu_{S_t} = \mu_0 S_{0t} + \mu_1 S_{1t} + \mu_2 S_{2t}, \tag{3.11}$$

The transition probabilities for France are modified accordingly:

$$p_{ij} = \Pr\left[S_t = j \middle| S_{t-1} = i\right], \; \sum_{j=1}^{3} p_{ij} = 1, \tag{3.12}$$

Finally, structural breaks are introduced into the MS intercept for Italy, the Netherlands, Belgium and Spain. This is because the baseline model fails to provide reasonable parameter estimates for these countries, and thus fails to produce satisfactory inferences on the probabilities of recessions or expansions. One potential reason is that the baseline model assumes that intercepts during recessions and expansions are constant over the entire sample. As the magnitude of the recessions for these four countries appear to vary significantly, this assumption may not be valid. The recessions during the early 1970s and mid-1980s in Italy, Belgium and the Netherlands were notably deeper than the more recent recessions, while the ERM recession in Spain during the early 1990s appears to be more pronounced than the other downturns. The presence of severe recessions over the sample period results in the recession intercept being significantly negative. As such, the smoothed recession probabilities only capture the pronounced turndowns but neglect the others. A solution to this problem is to introduce dummy variables into the intercepts to reduce the impact that large recessions and expansions have on the model's parameter estimates.[6] Therefore, (3.4) is extended as follows

$$\mu_{S_t} = \mu_{0t}(1 - S_t) + \mu_{1t} S_t,\ S_t = \{0,1\}, \tag{3.13}$$

$$\mu_{0t} = \mu_0 + \mu_{00} D_{1t},\ \mu_{00} < 0 \tag{3.14}$$

$$\mu_{1t} = \mu_1 + \mu_{11} D_{2t},\ \mu_{11} > 0 \tag{3.15}$$

where D_{1t} and D_{2t} are dummy variables. In order to define these dummy variables accurately, Bai and Perron's (2003) multiple structural break test is used to detect the significant changes in the common factor growth rates, estimated using the DF model, for the above four countries.[7] The standard likelihood ratio test is then used to cross-check the presence of these structural breaks.

3.4 Empirical results for the Markov-switching dynamic factor model

Having established the state-space representation of equations (3.3), (3.4) (3.5) and (3.9), the filtering and smoothing algorithms, proposed in Kim (1994), can be used to obtain the hyperparameters of the MSDF model, and to calculate the smoothed inferences on the unobserved states of the economy. For consistency, the data set used in Chapter 2 is also used in Chapter 3. The MLE of models' hyperparameters for the countries analysed are reported in Tables 3.1-3.12. Apart from the parameter estimates associated with the common factor, the parameters of the idiosyncratic components in the MSDF model are broadly consistent with the corresponding values in the DF model. Therefore, the analysis of this chapter focuses on the MS parameters of the common factor. The MS common factors, along with the smoothed recession probabilities, are plotted in Panels 1-12 of Figure 3.1.[8]

[6] We have also tried a more complex solution to the above problem by introducing another latent state into the intercepts of the MSDF model. This is more in line with Kim and Nelson (1999a) and Mills and Wang (2003a) than the solution presented in the main text. However, the results are unsatisfactory as adding an additional latent state causes problems for Kim's (1994) approximate MLE. More discussions are included in Appendix E3.

[7] The identified break dates and confident intervals are reported in Appendix C3.

[8] Growth rate of the MS common factor for each country analysed is plotted in Figure B3.1, Appendix B3.

3.4.1 The Aggregate Euro Area and Core EMU Economies

The Aggregate Euro Area. The parameter estimates of the MSDF model for the aggregate euro area are reported in Table 3.1. Both ϕ_1 and ϕ_2 are insignificant, suggesting that the common factor is generated by a random walk with a MS drift. Since the long run growth of the common factor, δ, is estimated to be 1.097, the mean growth rate switches between $u_0 + \delta = -0.175$ and $u_1 + \delta = 1.598$, with the transition probabilities associated with these two regimes being $p_{00} = 0.821$ and $p_{11} = 0.933$. These estimates imply that recessions (regime 0) have a duration of $(1 - p_{00})^{-1} = 5.6$ quarters, while expansions (regime 1) have a duration of $(1 - p_{11})^{-1} = 14.9$ quarters. The findings that $|u_0| > |u_1|$ and $p_{00} < p_{11}$ support the asymmetric feature of business cycle phases, with recessions being steeper and shorter than expansions.

The estimated MS common factor and the smoothed recession probabilities are plotted in Panel 1 of Figure 3.1. The smoothed probabilities can be considered as the optimal inference of the regime at time t using the full sample information ψ_T: $\Pr(S_T = j|\psi_T)$.[9] The time path of the smoothed recession probabilities is used to date the business cycle turning points. As defined by Hamilton (1989), in the case of two regimes, an observation is classified into regime 0 when $\Pr(S_t = 0|\psi_T) > 0.5$, and classified into regime 1 when $\Pr(S_t = 0|\psi_T) < 0.5$. Based on this rule, three recessions can be identified, during 1980Q2-1982Q4, 1992Q2-1993Q3 and 2001Q2-2003Q2. Although the first two recessions are consistent with the cycle dates produced by the CEPR, no recessions are reported by the committee in the early 2000s. A few brief downturns are also identified by the MSDF model during the mid-1980s. This perhaps reflects the frequent realignments of the EMS central exchange rate in this period.

[9] See Appendix A3.3 for more discussion on the smoothed regime probabilities.

Table 3.1: Parameter estimates of MSDF model for the euro area

Common Factor

ϕ_1	ϕ_2	u_0	u_1	p_{00}	p_{11}
0.074	-0.001	-1.272*	0.501*	0.821**	0.933**
(0.184)	(0.007)	(0.541)	(0.264)	(0.123)	(0.060)

Idiosyncratic Components

Δ **GDP**	γ_1	-	-	-	ψ_{11}	ψ_{12}	-	α_{11}	σ_1^2
	0.684**				-0.141	-0.005		0.033	0.175*
	(0.097)				(0.247)	(0.018)		(0.136)	(0.080)
Δ **GFCF**	γ_2	-	-	-	ψ_{21}	ψ_{22}	-	α_{12}	σ_2^2
	0.563**				-0.378*	-0.036		-0.144	0.293**
	(0.083)				(0.137)	(0.026)		(0.115)	(0.054)
Δ **IP**	γ_3	-	-	-	ψ_{31}	ψ_{32}	-	α_{13}	σ_3^2
	0.581**				0.081	-0.002		0.134	0.398**
	(0.095)				(0.135)	(0.006)		(0.137)	(0.064)
Δ **EMP**	γ_{40}	γ_{41}	γ_{42}	γ_{43}	ψ_{41}	ψ_{42}	ψ_{44}	α_{14}	σ_4^2
	0.198**	0.186**	0.053	0.067	0.166	0.134	0.333**	-0.219*	0.319**
	(0.058)	(0.058)	(0.054)	(0.056)	(0.092)	(0.092)	(0.100)	(0.112)	(0.039)

Long run growth rate: $\delta = 1.097$

Error correction term

$$\mathrm{GDP}_{t-1} = 18.598 - \underset{(0.211)}{0.932} \times \mathrm{GFCF}_{t-1} + \underset{(0.273)}{2.726} \times \mathrm{IP}_{t-1} - \underset{(0.329)}{0.163} \times \mathrm{EMP}_{t-1}$$

Log-likelihood: -592.248

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ **GDP**	3.654	2.857
Δ **GFCF**	3.691	4.847
Δ **IP**	8.588	16.508**
Δ **EMP**	5.121	18.103**

Notes: *Standard errors are in parentheses. ** denotes significance at 1% and * at 5%.* The parameter estimates for the euro area were estimated using data from 1975Q3-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0054, 0.0047; Δ GFCF: 0.0056, 0.014; Δ IP: 0.0043, 0.0098; Δ EMP: 0.0014, 0.0028.

Germany. As with the aggregate euro area, the data generating process of the German common factor follows a random walk with a MS drift. As reported in Table 3.2, both ϕ_1 and ϕ_2 are insignificant. Compared to the aggregate euro area, the average recessionary phase in Germany is much deeper whilst the average expansionary phase appears to be much milder. Mean growth rates in Germany during recessions and expansions are $u_0 + \delta = -0.758$ and $u_1 + \delta = 0.847$, respectively. The transition probabilities associated with the two regimes are $p_{00} = 0.816$ and $p_{11} = 0.914$, suggesting an average duration of the recessionary phase of

$(1-p_{00})^{-1} = 5.4$ quarters and an average duration of the expansionary phase of $(1-p_{11})^{-1} = 11.6$ quarters.

The smoothed recession probabilities, plotted in Panel 2 of Figure 3.1, highlight five recessions, during 1973Q3-1975Q2, 1980Q2-1982Q4, 1992Q2-1993Q2, 1995Q3-1996Q1 and 2001Q2-2003Q2. These cycle dates correspond closely with those produced by the BBQ algorithm in Chapter 2.

Table 3.2: Parameter estimates of MSDF model for Germany

Common Factor									
ϕ_1	ϕ_2		u_0	u_1	p_{00}			p_{11}	
-0.155	-0.006		-1.108*	0.497*	0.816**			0.914**	
(0.139)	(0.010)		(0.523)	(0.286)	(0.123)			(0.065)	
Idiosyncratic Components									
Δ **GDP**	γ_1	-	-	-	-	ψ_{11}	ψ_{12}	ψ_{14}	σ_1^2
	0.674**					-0.016	-0.096	0.633**	0.183*
	(0.114)					(0.140)	(0.100)	(0.110)	(0.080)
Δ **GFCF**	γ_2	-	-	-	-	ψ_{21}	ψ_{22}	ψ_{24}	σ_2^2
	0.510**					-0.266**	-0.018	0.212*	0.468**
	(0.097)					(0.098)	(0.013)	(0.102)	(0.053)
Δ **IP**	γ_3	-	-	-	-	ψ_{31}	ψ_{32}	-	σ_3^2
	0.624**					0.004	0.016		0.413**
	(0.100)					(0.119)	(0.130)		(0.069)
Δ **EMP**	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D80q1	ψ_{41}	ψ_{42}	ψ_{44}	σ_4^2
	0.171**	0.227**	0.145*	0.104*	5.633**	0.088	-0.083	0.250*	0.431**
	(0.061)	(0.060)	(0.055)	(0.055)	(0.685)	(0.091)	(0.091)	(0.090)	(0.042)
Long run growth rate: $\delta = 0.350$									
Log-likelihood : -641.807									
Diagnostics			**Q(4)**				**Jarque-Bera**		
Δ GDP			1.125				23.155**		
Δ GFCF			4.492				1.567		
Δ IP			2.122				5.914		
Δ EMP			2.410				64.179**		

Notes: The parameter estimates for Germany were estimated using data from 1970Q1-2004Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0056, 0.0098; Δ GFCF: 0.0038, 0.0278; Δ IP: 0.0037, 0.0157; Δ EMP: 0.0005, 0.0051. Standard errors are in parentheses. ** denotes significance at 1% and * at 5%.

France. As with the other countries analysed, a two-state MSDF model was first fitted to the French data. The parameter estimates of this model are presented in Table 3.3(a). The intercepts of recessions and expansions are $u_0 = -1.101$ and $u_1 = 1.064$, with the transition probabilities $p_{00} = 0.740$ and $p_{11} = 0.745$. Since the long run growth rate, δ, is 1.298, mean growth rates of recessions and expansions are calculated to be $u_0 + \delta = 0.197$ and $u_1 + \delta = 2.362$, respectively. This implies that the smoothed recession probabilities plotted in Panel 3(a) of Figure 3.1 identify growth cycles rather than business cycles. In contrast to the other countries analysed, the French economy appears to have experienced three business cycle phases (recessions, moderate-growth and high-growth) rather than the two phases (recessions and expansions) which are generally identified. This requires the use of a three-state MS model to clearly distinguish these different regimes.

The parameter estimates of a three-state MS model are presented in Table 3.3(b). The estimated long-run growth rate of 0.598 implies that growth rates of the three regimes are $u_0 + \delta = -1.483$, $u_1 + \delta = 0.598$ and $u_2 + \delta = 2.648$. The common factor switches among the three regimes, governed by the following nine transition probabilities: $p_{00} = 0.777$, $p_{01} = 0.223$, $p_{02} = 0$, $p_{10} = 0.077$, $p_{11} = 0.893$, $p_{12} = 0.03$, $p_{20} = 0$, $p_{21} = 0.099$ and $p_{22} = 0.901$.[10] The transition probabilities $p_{02} = 0$ and $p_{20} = 0$ imply that the French economy cannot switch directly between the recessionary and the high-growth regimes. This differs from the dynamics observed in the US where recessions are directly followed by periods of fast-recovery and then moderate growth (Clements and Krolzig, 1998; Kim *et al.*, 2005). Differences in the business cycle dynamics of the two countries may stem from the levels of rigidity observed in their economies. As the French economy is characterised by strong union power, companies may find it more costly to lay off workers during recessions and consequently may be unwilling to hire additional workers when recessions ends. This may hold back the French economy from the rapid recovery observed in the US, which has much more deregulated labour and product markers. The estimated transition probabilities for France imply that the

[10] As the parameter estimates of u_1 and p_{20} are insignificantly different from zero, they are set to zero. In total, seven additional parameters are estimated in the MSDF model, including u_0, u_2, p_{00}, p_{01}, p_{10}, p_{11} and p_{21}. The other transition probabilities are calculated as $p_{i2} = 1 - p_{i0} - p_{i1}$.

average duration of recessions, moderate-growth and high-growth periods are $(1-p_{00})^{-1}=4.5$ quarters, $(1-p_{11})^{-1}=9.3$ quarters and $(1-p_{22})^{-1}=10.1$ quarters, respectively.

The smoothed recession probabilities, plotted in Panel 3(b) of Figure 3.1, indicate four recessions, during 1980Q2-1981Q1, 1982Q3-1985Q1, 1986Q4-1987Q1 and 1992Q2-1994Q1. In addition, the smoothed high-growth probabilities capture two high-growth periods: 1987Q4-1989Q4 and 1997Q2-2001Q1.

Table 3.3 (a): Parameter estimates of MSDF model for France

Common Factor

ϕ_1	ϕ_2	u_0	u_1	p_{00}	p_{11}
0.191	0.384**	-1.101**	1.064*	0.740**	0.745**
(0.173)	(0.105)	(0.467)	(0.533)	(0.177)	(0.128)

Idiosyncratic Components

Δ GDP	γ_1	-	-	-	ψ_{11}	ψ_{12}	α_{11}	σ_1^2
	0.451**				-0.054	-0.001	0.297*	0.355**
	(0.093)				(0.156)	(0.004)	(0.125)	(0.065)
Δ IP	γ_2	-	-	-	ψ_{21}	ψ_{22}	α_{12}	σ_2^2
	0.463**				0.017	0.139	0.583**	0.307**
	(0.105)				(0.172)	(0.151)	(0.143)	(0.061)
Δ Sales	γ_3	-	-	-	ψ_{31}	ψ_{32}	α_{13}	σ_3^2
	0.158**				-0.290**	0.023	-0.018	0.808**
	(0.054)				(0.093)	(0.095)	(0.089)	(0.058)
Δ EMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	ψ_{41}	ψ_{42}	α_{14}	σ_4^2
	0.263**	0.015	0.053	0.104*	0.276**	0.221*	-0.065	0.285**
	(0.067)	(0.060)	(0.052)	(0.051)	(0.107)	(0.106)	(0.138)	(0.043)

Long run growth rate: $\delta = 1.298$

Error correction term

$$\underset{}{\mathrm{GDP}_{t-1}} = 32.360 + \underset{(0.188)}{2.324} \times \mathrm{IP}_{t-1} + \underset{(0.294)}{0.667} \times \mathrm{Sales}_{t-1} - \underset{(0.679)}{3.152} \times \mathrm{EMP}_{t-1}$$

Log-likelihood: -567.525

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	3.444	2.591
Δ GFCF	3.385	23.899**
Δ Sales	0.922	15.503**
Δ EMP	2.998	1.856

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for France were estimated using data from 1975Q4-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0053, 0.0044; Δ IP: 0.0035, 0.0115; Δ Sales: 0.0037, 0.0104; Δ EMP: 0.0013, 0.0026.

Table 3.3 (b): Parameter estimates of MSDF model for France

Common Factor

ϕ_1	ϕ_2	u_0	u_1	u_2	p_{00}	p_{01}	p_{10}	p_{11}	p_{20}	p_{21}
-0.039	0.064	-2.081**	0	2.050**	0.777**	0.223*	0.077*	0.893**	0	0.099
(0.119)	(0.091)	(0.492)	-	(0.490)	(0.101)	(0.101)	(0.046)	(0.052)	-	(0.065)

Idiosyncratic Components

Δ **GDP**	γ_1	-	-	-	ψ_{11}	ψ_{12}	α_{11}	σ_1^2
	0.700**				-0.052	-0.001	0.349**	0.238**
	(0.069)				(0.139)	(0.004)	(0.105)	(0.107)
Δ **IP**	γ_2	-	-	-	ψ_{21}	ψ_{22}	α_{12}	σ_2^2
	0.464**				-0.045	0.069	0.569**	0.498**
	(0.066)				(0.103)	(0.103)	(0.103)	(0.045)
Δ **Sales**	γ_3	-	-	-	ψ_{31}	ψ_{32}	α_{13}	σ_3^2
	0.204**				-0.305**	-0.023	0.015	0.803**
	(0.054)				(0.091)	(0.014)	(0.085)	(0.057)
Δ **EMP**	γ_{40}	γ_{41}	γ_{42}	γ_{43}	ψ_{41}	ψ_{42}	α_{14}	σ_4^2
	0.232**	0.151**	0.098*	0.104**	0.190*	0.206*	0.099	0.340**
	(0.048)	(0.048)	(0.045)	(0.044)	(0.096)	(0.101)	(0.117)	(0.038)

Long run growth rate: $\delta = 0.598$

Error correction term

$$\underset{}{GDP_{t-1}} = 32.360 + \underset{(0.188)}{2.324} \times IP_{t-1} + \underset{(0.294)}{0.667} \times Sales_{t-1} - \underset{(0.679)}{3.152} \times EMP_{t-1}$$

Log-likelihood: -575.339

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	7.837	3.255
Δ GFCF	4.033	19.088**
Δ Sales	0.927	14.544**
Δ EMP	1.154	1.634

Note: Please see the notes given underneath Table 3.3 (a).

Italy. Global factors, such as the introduction of floating exchange rate regimes and soaring oil prices, combined with internal events, notably weak government coalitions and rising labour costs, all had a negative impact on Italian economic performance in the 1970s. This is reflected by the significant volatility observed in the country's GDP and IP growth during this period. As a consequence, the baseline model, which assumes there are constant intercepts during recessions and expansions over the entire sample, fails to provide reasonable parameter estimates. As a result of the parameter estimates $u_0 = -5.850$ and $p_{00} = 0$ obtained in the baseline model, the smoothed recession probabilities fail to identify any recessions over the studied period as shown in Panel 4(a) of Figure 3.1. This could be because the gap between recessions and expansions has narrowed significantly in recent decades as the economy has stabilised.

Table 3.4 (a): Parameter estimates of MSDF model for Italy

Common Factor

ϕ_1	ϕ_2	u_0	u_1	p_{00}	p_{11}
0.773**	-0.149**	-5.850**	0.039	0.000	0.993**
(0.123)	(0.048)	(1.458)	(0.085)	(0.000)	(0.007)

Idiosyncratic Components

Δ **GDP**	γ_1	-	-	-	-	ψ_{11}	ψ_{12}	-	α_{11}	σ_1^2
	0.600**					-0.167	-0.007		0.332*	0.202**
	(0.071)					(0.175)	(0.015)		(0.134)	(0.067)
Δ **GFCF**	γ_2	-	-	-	-	ψ_{21}	ψ_{22}	-	α_{12}	σ_2^2
	0.357**					-0.044	0.005		0.040	0.706**
	(0.063)					(0.088)	(0.088)		(0.103)	(0.052)
Δ **IP**	γ_3	-	-	-	-	ψ_{31}	ψ_{32}	-	α_{13}	σ_3^2
	0.468**					-0.479**	-0.016		0.402**	0.316**
	(0.055)					(0.117)	(0.107)		(0.111)	(0.051)
Δ **EMP**	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D92q4	ψ_{41}	ψ_{42}	ψ_{43}	α_{14}	σ_4^2
	0.144*	-0.020	0.027	0.025	-4.423**	-0.080	-0.215*	0.210*	-0.143	0.667**
	(0.078)	(0.116)	(0.119)	(0.090)	(0.853)	(0.090)	(0.086)	(0.091)	(0.116)	(0.049)

Long run growth rate: $\delta = 0.815$

Error correction term

$$\underset{}{GDP_{t-1}} = 2.645 + \underset{(0.105)}{0.0911} \times GFCF_{t-1} + \underset{(0.086)}{1.1424} \times IP_{t-1} + \underset{(0.243)}{0.521} \times EMP_{t-1}$$

Log-likelihood: -679.46 **Diagnostics**	**Q(4)**	**Jarque-Bera**
Δ GDP	1.276	3.425
Δ GFCF	3.869	3.156
Δ IP	7.227	15.832**
Δ EMP	0.492	5.343

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for Italy were estimated using data from 1970Q1-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0056, 0.0079; Δ GFCF: 0.0042, 0.0181; Δ IP: 0.0043, 0.0223; Δ EMP: 0.0012, 0.0064.

As discussed in section 3.3, structural breaks were added to the model's intercepts as specified in equations (3.13)-(3.15), where dummy variables $D_{1t} = 1$ during 1974Q4-1975Q1 and $D_{2t} = 1$ during 1973Q2-1973Q4, 1976Q3-1976Q4 and 1979Q4 capture the steep recessionary and expansionary phases.[11] The parameter estimates of the modified model are reported in Table 3.4(b). The growth rates during recessions and expansions are estimated to be $u_0 + \delta = -0.816$ and $u_1 + \delta = 1.463$, respectively. These values

[11] The five break dates detected by Bai and Perron's (2003) procedure are at 1973Q2, 1973Q4, 1974Q3, 1975Q2 and 1980Q2. Dummy variables, D_{1t} and D_{2t}, are set to be consistent with these break dates or within their confidence intervals, apart from the period 1976Q3-1976Q4. However, setting $D_{2t} = 1$ during this period yields a higher log-likelihood value and more reasonable smoothed recession probabilities than leaving this period out.

increase to $u_0 + u_{00} + \delta = -5.885$ and $u_1 + u_{11} + \delta = 6.228$ when the corresponding dummy variables are set to one. The transition probabilities associated with recessions and expansions are $p_{00} = 0.662$ and $p_{11} = 0.852$ These estimates imply that the average duration of recessions is $(1 - p_{00})^{-1} = 2.9$ quarters and the average duration of expansions is $(1 - p_{11})^{-1} = 6.7$ quarters. Compared with the other countries analysed, Italy has the shortest average business cycle phases, with recessions occurring much more frequently. The null hypothesis of $u_{00} = u_{11} = 0$ is strongly rejected by the likelihood ratio statistic $\chi^2(2) = 44.06$. This indicates the presence of structural breaks in the intercepts.

Overall, the smoothed recession probabilities detect eight recessions: 1974Q4-1975Q2, 1977Q1-1977Q3, 1982Q1-1983Q2, 1992Q2-1993Q4, 1996Q1-1996Q4, 2001Q2-2001Q4, 2002Q4-2003Q2 and 2004Q3-2005Q1. These dates are close those obtained using the BBQ algorithm.

Table 3.4 (b): Parameter estimates of MSDF model for Italy

Common Factor

ϕ_1	ϕ_2	u_0	u_1	u_{00}	u_{11}	p_{00}	p_{11}
0.152	-0.006	-1.716**	0.563*	-5.069**	4.765**	0.662**	0.852**
(0.115)	(0.009)	(0.587)	(0.256)	(1.224)	(0.991)	(0.112)	(0.065)

Idiosyncratic Components

Δ **GDP**	γ_1	-	-	-	-	ψ_{11}	ψ_{12}	-	α_{11}	σ_1^2
	0.431**					-0.043	-0.001		0.246**	0.155*
	(0.078)					(0.228)	(0.005)		(0.090)	(0.070)
Δ **GFCF**	γ_2	-	-	-	-	ψ_{21}	ψ_{22}	-	α_{12}	σ_2^2
	0.253**					-0.037	0.030		-0.009	0.699**
	(0.052)					(0.088)	(0.088)		(0.085)	(0.052)
Δ **IP**	γ_3	-	-	-	-	ψ_{31}	ψ_{32}	-	α_{13}	σ_3^2
	0.329**					-0.499**	-0.061		0.330**	0.339**
	(0.057)					(0.110)	(0.106)		(0.073)	(0.047)
Δ **EMP**	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D92q4	ψ_{41}	ψ_{42}	ψ_{43}	α_{14}	σ_4^2
	0.083*	0.011	0.008	0.007	-4.453**	-0.081	-0.214*	0.206*	-0.179	0.656**
	(0.045)	(0.061)	(0.066)	(0.054)	(0.859)	(0.091)	(0.086)	(0.091)	(0.099)	(0.049)

Long run growth rate: $\delta = 0.900$

Error correction term

$$\underset{}{GDP_{t-1}} = 2.645 + \underset{(0.105)}{0.0911} \times GFCF_{t-1} + \underset{(0.086)}{1.1424} \times IP_{t-1} + \underset{(0.243)}{0.521} \times EMP_{t-1}$$

Log-likelihood: -657.430

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	5.465	6.053
Δ GFCF	3.247	3.090
Δ IP	6.514	14.935**
Δ EMP	0.560	4.907

Note: Please see the notes given underneath Table 3.4 (a).

Austria. The parameter estimates of the MSDF model for Austria are presented in Table 3.5. The mean growth rate is $u_0 + \delta = -1.548$ during recessions and $u_1 + \delta = 1.257$ during expansions. Given the transition probabilities $p_{00} = 0.826$ and $p_{11} = 0.958$, the average duration of recessions and expansions are calculated to be $(1 - p_{00})^{-1} = 5.7$ quarters and $(1 - p_{11})^{-1} = 23.8$ quarters, respectively. Three recessions are highlighted by the smoothed recession probabilities during 1974Q2-1975Q2, 1980Q2-1982Q4 and 2001Q1-2003Q2. In contrast to the cycle dates produced by the BBQ algorithm, the MSDF model does not identify any recessions during the ERM crisis, as it appears much milder than the others downturns observed.

Table 3.5: Parameter estimates of MSDF model for Austria

Common Factor

ϕ_1	ϕ_2	u_0	u_1	p_{00}	p_{11}
-0.068	0.099	-2.172**	0.633**	0.826**	0.958**
(0.150)	(0.117)	(0.473)	(0.196)	(0.089)	(0.024)

Idiosyncratic Components

ΔGDP	γ_1	-	-	-	D78q1	-	ψ_{11}	ψ_{12}	σ_1^2
	0.384**				-6.112**		0.026	0.000	0.372**
	(0.064)				(0.740)		(0.118)	(0.002)	(0.064)
ΔGFCF	γ_2	-	-	-	D78q1	D03q1	ψ_{21}	ψ_{22}	σ_2^2
	0.376**				-6.701**	4.008**	-0.166	-0.007	0.198**
	(0.057)				(0.587)	(0.653)	(0.159)	(0.013)	(0.047)
ΔIP	γ_3	-	-	-	-	-	ψ_{31}	ψ_{32}	σ_3^2
	0.334**						0.016	0.205*	0.697**
	(0.072)						(0.092)	(0.090)	(0.098)
ΔEMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D82q1	D04q1	ψ_{41}	ψ_{42}	σ_4^2
	-0.024	0.238**	-0.123	0.078	4.716**	-3.892**	-0.122	0.170	0.558**
	(0.062)	(0.069)	(0.072)	(0.066)	(0.770)	(0.771)	(0.096)	(0.094)	(0.076)

Long run growth rate: $\delta = 0.624$

Log-likelihood: -607.692

Diagnostics	**Q(4)**	**Jarque-Bera**
ΔGDP	2.705	11.844**
ΔGFCF	7.918	11.998**
ΔIP	0.843	0.986
ΔEMP	2.474	24.655**

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for Austria were estimated using data from 1970Q1-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: ΔGDP: 0.0058, 0.0087; ΔGFCF: 0.0085, 0.0155; ΔIP: 0.0050, 0.0221; ΔEMP: 0.0019, 0.0077.

The Netherlands. The Dutch recessions which occurred during the 1970s and early 1980s were much more severe than the country's more recent downturns. Therefore, a one-time break is introduced to the recession intercept, u_0, to capture the changes in the severity of recessionary phases over the sample period. D_{1t} is set to one in the subsample from 1970Q4 to 1983Q2 and to zero for the latter subsample.[12] The intercept during expansions is assumed to be constant with $D_{2t} = 0$ over the entire sample. The parameter estimates are presented in Table 3.6.[13] Compared to the above countries, the first difference of the Dutch common factor appears relatively persistent with $\phi_1 + \phi_2 = 0.635$. Before the structural break, the mean growth rates are $u_0 + u_{00} + \delta = -4.795$ and $u_1 + \delta = 2.587$ during recessions and expansions, respectively. After the break, the growth rate dramatically increases to $u_0 + \delta = -0.156$ during recessions but remains the same during expansions. It is worth noting that the intercept $u_0 = -0.979$ is insignificantly different from zero, which may suggest that the turning points identified after the break are for growth cycles rather than business cycles. In addition, in the second sample period it is found that $|u_0| < |u_1|$, which contradicts one of the stylised facts of business cycles, that recessions are steeper than expansions. As discussed in Chapter 2, this may reflect the comparatively strong economic growth observed in the Netherlands since the structural reforms undertaken in the 1980s.

The smoothed recession probabilities are plotted in Panel 6 of Figure 3.1, they indicate five economic downturns: 1974Q2-1975Q1, 1980Q1-1982Q4, 1987Q4-1988Q1, 1991Q1-1993Q4 and 2001Q2-2005Q3.

[12] Both the sequential procedure and critical values identify the significant structural break in the Dutch common factor at 1983Q2. Therefore, a one-time break is introduced to the recession intercept at this date. The break identified at 2001Q1 coincides with the peak of the sustained recession that occurred during 2001Q1-2005Q2.

[13] For the Netherlands, the baseline model failed to converge, therefore only parameter estimates of the modified model are presented, in which a one-time structural break is included in the recession intercept.

Table 3.6: Parameter estimates of MSDF model for Netherlands

Common Factor

ϕ_1	ϕ_2	u_0	u_1	u_{00}	p_{00}	p_{11}
0.792**	-0.157*	-0.979	1.764	-4.639**	0.896**	0.930**
(0.163)	(0.065)	(0.970)	(1.055)	(2.057)	(0.053)	(0.033)

Idiosyncratic Components

Δ GDP	γ_1	-	-	D79q1	-	ψ_{11}	ψ_{12}	α_{11}	α_{21}	σ_1^2
	0.060			-3.551**		-0.342**	-0.029	-0.288	-0.446	0.637**
	(0.041)			(0.847)		(0.098)	(0.017)	(0.505)	(0.505)	(0.078)
Δ Sales	γ_2	-	-	D78q1	D94q1	ψ_{21}	ψ_{22}	α_{12}	α_{22}	σ_2^2
	0.043			2.553**	-4.721**	-0.307**	-0.024	-1.051*	-1.187*	0.640**
	(0.029)			(0.816)	(0.782)	(0.096)	(0.015)	(0.496)	(0.493)	(0.077)
Δ IP	γ_3	-	-	-	-	ψ_{31}	ψ_{32}	α_{13}	α_{23}	σ_3^2
	0.056					-0.163	-0.007	2.228**	1.898**	0.720**
	(0.038)					(0.101)	(0.008)	(0.641)	(0.631)	(0.088)
Δ EMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D96q1	ψ_{41}	ψ_{42}	α_{14}	α_{24}	σ_4^2
	0.015	0.075	0.007	0.053	-1.671**	0.977**	-0.239**	0.428	0.506	0.048**
	(0.026)	(0.058)	(0.023)	(0.044)	(0.174)	(0.135)	(0.066)	(0.274)	(0.321)	(0.014)

Long run growth rate: $\delta = 0.823$

Error correction terms

$\mathrm{GDP}_{t-1} = -8.524 - \underset{(0.708)}{3.444} \times \mathrm{IP}_{t-1} + \underset{(0.629)}{4.156} \times \mathrm{EMP}_{t-1}$

$\mathrm{Sales}_{t-1} = 63.86 + \underset{(3.871)}{24.946} \times \mathrm{IP}_{t-1} - \underset{(3.442)}{19.337} \times \mathrm{EMP}_{t-1}$

Log-likelihood**:** -547.257

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	1.932	51.180**
Δ Sales	4.179	14.535**
Δ IP	2.963	0.175
Δ EMP	0.701	31.971**

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for the Netherlands were estimated using data from 1970Q1-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0064, 0.0121; Δ Sales: 0.0018, 0.0192; Δ IP: 0.0041, 0.0181; Δ EMP: 0.0029, 0.0063.

Belgium. As with the Netherlands, the Belgian economy appears to be very volatile during the 1970s and the early 1980s. The parameter estimates of $u_0 = -2.784$ and $p_{00} = 0.546$, obtained by applying the baseline model to the data, imply that the recessionary phase is steep and brief. A one-time structural break has been introduced to the recession intercept, with $D_{1t} = 1$ from 1970Q4 to 1981Q1 and $D_{1t} = 0$ for the rest of the sample period. Although the choice of 1981Q1 is ad hoc, the severity of the recessionary phases decreases dramatically after this date.[14] The parameter estimates of the modified model are presented in Table 3.7(b). Before the break, the mean growth

[14] Both the sequential procedure and BIC identify breakpoints at 1974Q2 and 1983Q3. However, there is no improvement in the log-likelihood value and the smoothed recession probabilities when a one-time break is introduced at either date. However, when the one-time break data is set at 1981Q1, we obtained reasonable smoothed recession probabilities and this date is supported by the likelihood ratio statistic.

rate was $u_0 + u_{00} + \delta = -1.444$ during recessions and $u_1 + \delta = 1.705$ during expansions. After the break, the mean growth rate is $u_0 + \delta = 0.817$ during recessions while remaining unchanged for expansions. This, again, may suggest that the turning points identified after 1981Q1 are for growth cycles. Given the transition probabilities $p_{00} = 0.863$ and $p_{11} = 0.942$, the average duration of recessions and expansions are $(1 - p_{00})^{-1} = 7.3$ quarters and $(1 - p_{11})^{-1} = 17.2$ quarters, respectively. The null hypothesis of $u_{00} = 0$ is again rejected by the likelihood ratio statistic of $\chi^2(1) = 3.674$ at the marginal significance level of 5%.

The smoothed recession probabilities, plotted in Panel 7(b) of Figure 3.1, capture four major downturns during 1974Q2-1975Q1, 1980Q2-1983Q2, 1991Q1-1993Q4, and 2001Q2-2002Q1.

Table 3.7 (a): Parameter estimates of MSDF model for Belgium

Common Factor

ϕ_1	ϕ_2	u_0	u_1	p_{00}	p_{11}
1.233**	-0.380**	-2.784**	0.204*	0.546**	0.968**
(0.102)	(0.063)	(0.498)	(0.102)	(0.192)	(0.017)

Idiosyncratic Components

Δ GDP	γ_1	-	-	D80q1	ψ_{11}	ψ_{12}	ψ_{14}	α_{11}	σ_1^2
	0.133**			4.411**	0.170	0.075	-0.277**	0.123	0.591**
	(0.041)			(0.745)	(0.097)	(0.086)	(0.085)	(0.116)	(0.073)
Δ GFCF	γ_2	-	-	-	ψ_{21}	ψ_{22}	-	α_{12}	σ_2^2
	0.165**				-0.065	0.096		0.054	0.726**
	(0.034)				(0.089)	(0.086)		(0.105)	(0.090)
Δ IP	γ_3	-	-	-	ψ_{31}	ψ_{32}	-	α_{13}	σ_3^2
	0.213**				-0.078	-0.002		0.628**	0.593**
	(0.036)				(0.121)	(0.005)		(0.137)	(0.079)
Δ EMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	ψ_{41}	ψ_{42}	-	α_{14}	σ_4^2
	0.283**	0.075	-0.160**	0.111**	1.263**	-0.399**		-0.019	0.019
	(0.032)	(0.044)	(0.034)	(0.025)	(0.181)	(0.114)		(0.038)	(0.015)

Long run growth rate: $\delta = 1.315$

Error correction term

$$GDP_{t-1} = 15.805 - \underset{(0.080)}{0.220} \times GFCF_{t-1} + \underset{(0.080)}{1.477} \times IP_{t-1} - \underset{(0.312)}{1.499} \times EMP_{t-1}$$

Log-likelihood: -597.854

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	4.199	7.233*
Δ GFCF	4.106	2.656
Δ IP	1.043	2.191
Δ EMP	3.961	9.111*

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for Belgium were estimated using data from 1970Q1-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0059, 0.0074; Δ GFCF: 0.0044, 0.0209; Δ IP: 0.0044; 0.0209; Δ EMP: 0.0009, 0.0027.

Table 3.7 (b): Parameter estimates of MSDF model for Belgium

Common Factor									
ϕ_1	ϕ_2	u_0		u_1		u_{00}		p_{00}	p_{11}
1.097**	-0.301**	-0.482		0.406*		-2.261**		0.863**	0.942**
(0.106)	(0.058)	(0.353)		(0.166)		(0.573)		(0.084)	(0.037)
Idiosyncratic Components									
Δ **GDP**	γ_1	-	-	D80q1	ψ_{11}	ψ_{12}	ψ_{14}	α_{11}	σ_1^2
	0.172**			4.406**	0.092	0.054	-0.248**	0.206	0.548**
	(0.047)			(0.740)	(0.109)	(0.090)	(0.092)	(0.119)	(0.074)
Δ **GFCF**	γ_2	-	-	-	ψ_{21}	ψ_{22}	-	α_{12}	σ_2^2
	0.175**				-0.090	0.099		0.049	0.700**
	(0.036)				(0.088)	(0.086)		(0.102)	(0.087)
Δ **IP**	γ_3	-	-	-	ψ_{31}	ψ_{32}	-	α_{13}	σ_3^2
	0.219**				-0.161	-0.006		0.588**	0.546**
	(0.038)				(0.121)	(0.010)		(0.126)	(0.076)
Δ **EMP**	γ_{40}	γ_{41}	γ_{42}	γ_{43}	ψ_{41}	ψ_{42}	-	α_{14}	σ_4^2
	0.269**	0.037	-0.107*	0.104**	1.063**	-0.283**		-0.043	0.057*
	(0.040)	(0.045)	(0.045)	(0.033)	(0.143)	(0.076)		(0.048)	(0.025)

Long run growth rate: $\delta = 1.299$

Error correction term

$$\underset{}{GDP_{t-1}} = 15.805 - \underset{(0.080)}{0.220} \times GFCF_{t-1} + \underset{(0.080)}{1.477} \times IP_{t-1} - \underset{(0.312)}{1.499} \times EMP_{t-1}.$$

Log-likelihood: -596.018

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	5.459	4.314
Δ GFCF	2.362	4.546
Δ IP	2.235	1.777
Δ EMP	6.313	7.574*

Note: Please see the notes given underneath Table 3.7 (a)

3.4.2 Periphery countries: Spain and Finland

Spain. For Spain, the MSDF model seems to provide the least satisfactory results among all the countries analysed. The parameter estimates of the baseline model are presented in Panel 8(a) of Figure 3.1. The mean growth rates are $u_0 + \delta = -2.203$ and $u_1 + \delta = 1.908$ during recessions and expansions. The transition probabilities of $p_{00} = 0.444$ and $p_{11} = 0.964$ imply that the average duration of recessions and expansions are 1.8 and 27.8 quarters, respectively. It is not surprising that the smoothed probabilities only pick up a couple of the severe recessions but neglect the other mild downturns. Since the recession triggered by the ERM crisis appears much steeper than the others, the recession intercept is biased downwards to -3.855. Therefore, allowing for structural breaks in this intercept by setting $D_{1t} = 1$ during 1992Q2-1993Q4 captures the most severe recession over the entire sample.[15] As a result, the average duration of recession increases from 1.8 quarters in the baseline model to 18.2 quarters in the modified model. The smoothed recession probabilities, plotted in Panel 8(b) of Figure 3.1, identify a prolonged recessionary period during 1974Q2-1984Q2. However, it may be more appropriate to mark this period as below trend growth, as the recession mean is $u_0 + \delta = 0.034$ during this period and it then decreased to $u_0 + u_{00} + \delta = -3.567$ during 1992Q2-1993Q4. The likelihood ratio statistic of $\chi^2(1) = 11.936$ rejects the null of $u_{00} = 0$.

Table 3.8 (a): Parameter estimates of MSDF model for Spain

Common Factor

ϕ_1	ϕ_2	u_0	u_1	p_{00}	p_{11}
0.286**	0.525**	-3.855**	0.256*	0.444*	0.964**
(0.097)	(0.090)	(0.837)	(0.104)	(0.204)	(0.020)

Idiosyncratic Components

Δ GDP	γ_1	-	-	-	-	ψ_{11}	ψ_{12}	ψ_{14}	α_{11}	α_{21}	σ_1^2
	0.342**					-0.511**	-0.177	-0.181*	0.087	0.265	0.355**
	(0.049)					(0.112)	(0.109)	(0.084)	(0.291)	(0.279)	(0.048)
Δ GFCF	γ_2	-	-	-	-	-	ψ_{21}	ψ_{22}	α_{12}	α_{22}	σ_2^2
	0.415**						0.165	0.036	0.512	-0.251	0.275**
	(0.060)						(0.140)	(0.131)	(0.369)	(0.365)	(0.053)
Δ IP	γ_3	-	-	-	-	-	ψ_{31}	ψ_{32}	α_{13}	α_{23}	σ_3^2
	0.319**						-0.281*	0.191*	1.193**	-0.614*	0.521**
	(0.052)						(0.101)	(0.099)	(0.343)	(0.319)	(0.050)
Δ EMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D76q1	D76q3	ψ_{41}	ψ_{42}	α_{14}	α_{24}	σ_4^2
	0.122**	0.099*	0.053	0.040	2.755**	3.716**	0.291**	0.214*	-0.145	-0.268	0.176**
	(0.037)	(0.039)	(0.038)	(0.037)	(0.413)	(0.418)	(0.096)	(0.093)	(0.147)	(0.300)	(0.028)

Long run growth rate: $\delta = 1.652$

Error correction terms

$\mathrm{GDP}_{t-1} = 6.403 + \underset{(0.167)}{2.054} \times \mathrm{IP}_{t-1} - \underset{(0.179)}{0.121} \times \mathrm{EMP}_{t-1}$ $\quad$ $\mathrm{GFCF}_{t-1} = -1.157 + \underset{(0.473)}{3.098} \times \mathrm{IP}_{t-1} - \underset{(0.507)}{0.027} \times \mathrm{EMP}_{t-1}$

Log-likelihood: -530.881

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	4.632	9.843**
Δ GFCF	0.663	1.526
Δ IP	0.415	1.358
Δ EMP	1.582	17.020**

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for Spain were estimated using data from 1972Q3-2006Q4. Logarithms of variables were used, each variables was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0069, 0.0074; Δ GFCF: 0.0069, 0.0074; Δ IP: 0.0049, 0.0172; Δ EMP: 0.0035, 0.0082.

[15] This recession is identified by the BBQ algorithm in Chapter 2. Following the results obtained from the Bai and Perron procedure, several alternative specifications of D_{1t} have also been explored in this study, such as setting $D_{1t} = 1$ during 1990Q4-1994Q1 or allowing a one-time break from 1985Q2 onwards. However, these specifications fail to provide reasonable parameter estimates and smoothed recession probabilities.

Table 3.8 (b): Parameter estimates of MSDF model for Spain

Common Factor

ϕ_1	ϕ_2	u_0	u_1	u_{00}	p_{00}	p_{11}
0.001	0.403**	-1.108**	0.876**	-3.601**	0.945**	0.973**
(0.113)	(0.097)	(0.310)	(0.221)	(0.826)	(0.035)	(0.021)

Idiosyncratic Components

Δ **GDP**	γ_1	-	-	-	-	ψ_{11}	ψ_{12}	ψ_{14}	α_{11}	α_{21}	σ_1^2
	0.334**					-0.396**	-0.047	-0.199*	-0.196	0.390	0.414**
	(0.052)					(0.106)	(0.106)	(0.086)	(0.287)	(0.270)	(0.048)
Δ **GFCF**	γ_2	-	-	-	-	-	ψ_{21}	ψ_{22}	α_{12}	α_{22}	σ_2^2
	0.438**						0.198	-0.010	0.248	-0.132	0.242**
	(0.062)						(0.153)	(0.015)	(0.373)	(0.356)	(0.057)
Δ **IP**	γ_3	-	-	-	-	-	ψ_{31}	ψ_{32}	α_{13}	α_{23}	σ_3^2
	0.312**						-0.254*	0.193*	0.969**	-0.529*	0.563**
	(0.053)						(0.099)	(0.097)	(0.337)	(0.212)	(0.051)
Δ **EMP**	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D76q1	D76q3	ψ_{41}	ψ_{42}	α_{14}	α_{24}	σ_4^2
	0.146**	0.113**	0.062	0.039	2.798**	3.702**	0.225*	0.201*	-0.157	-0.074	0.160**
	(0.035)	(0.038)	(0.039)	(0.037)	(0.406)	(0.412)	(0.100)	(0.096)	(0.337)	(0.317)	(0.028)

Long run growth rate: $\delta = 1.142$

Error correction terms

$\mathrm{GDP}_{t-1} = 6.403 + \underset{(0.167)}{2.054} \times \mathrm{IP}_{t-1} - \underset{(0.179)}{0.121} \times \mathrm{EMP}_{t-1}$ $\quad$ $\mathrm{GFCF}_{t-1} = -1.157 + \underset{(0.473)}{3.098} \times \mathrm{IP}_{t-1} - \underset{(0.507)}{0.027} \times \mathrm{EMP}_{t-1}$

Log-likelihood: -524.913

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	3.856	9.476**
Δ GFCF	1.219	1.677
Δ IP	2.458	17.756**
Δ EMP	2.597	28.047**

Note: *Please see the notes given underneath Table 3.8 (a).*

Finland. In comparison with the BBQ algorithm, fewer recessions are identified by the MSDF model for Finland. The smoothed recession probabilities, plotted in Panel 9, pick out three recessionary periods, during 1975Q2-1975Q4, 1980Q4-1981Q1 and 1991Q2-1993Q1. Mean growth rates during recessions and expansions are computed to be $\mu_0 + \delta = -1.588$ and $\mu_1 + \delta = 1.089$. The average duration of recessions and expansions are $(1 - p_{00})^{-1} = 4.5$ quarters and $(1 - p_{11})^{-1} = 31.3$ quarters, respectively.

Table 3.9: Parameter estimates of MSDF model for Finland

Common Factor

ϕ_1	ϕ_2	u_0	u_1	p_{00}	p_{11}
0.014	0.108	-2.340**	0.337**	0.780**	0.968**
(0.121)	(0.119)	(0.454)	(0.141)	(0.135)	(0.026)

Idiosyncratic Components

Δ GDP	γ_1	-	-	-	-	ψ_{11}	ψ_{12}	σ_1^2
	0.555**					-0.720**	-0.130*	0.223**
	(0.088)					(0.150)	(0.054)	(0.088)
Δ Sales	γ_2	-	-	-	-	ψ_{21}	ψ_{22}	σ_2^2
	0.430**					-0.002	-0.085	0.615**
	(0.075)					(0.004)	(0.097)	(0.055)
Δ IP	γ_3	-	-	-	-	ψ_{31}	ψ_{32}	σ_3^2
	0.384**					-0.170	0.117	0.676**
	(0.074)					(0.091)	(0.091)	(0.055)
Δ EMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D76q1	ψ_{41}	ψ_{42}	σ_4^2
	0.045	0.288**	0.137	0.263**	5.379**	-0.069	-0.162	0.335**
	(0.062)	(0.066)	(0.074)	(0.062)	(0.744)	(0.123)	(0.137)	(0.055)

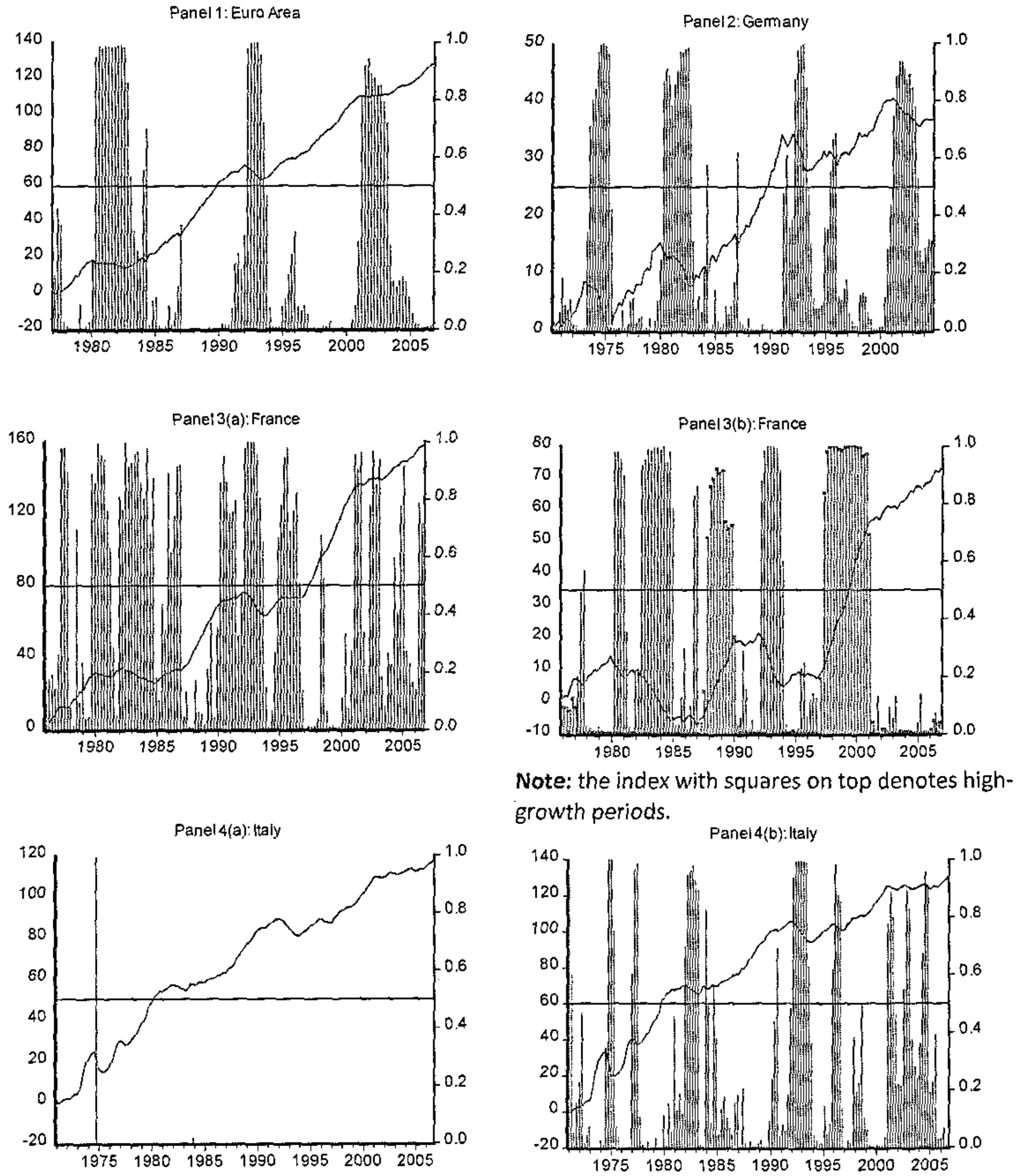
Long run growth rate: $\delta = 0.752$

Log-likelihood: -657.672

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	9.559*	21.142**
Δ Sales	8.839	25.148**
Δ IP	4.369	26.895**
Δ EMP	2.262	1.364

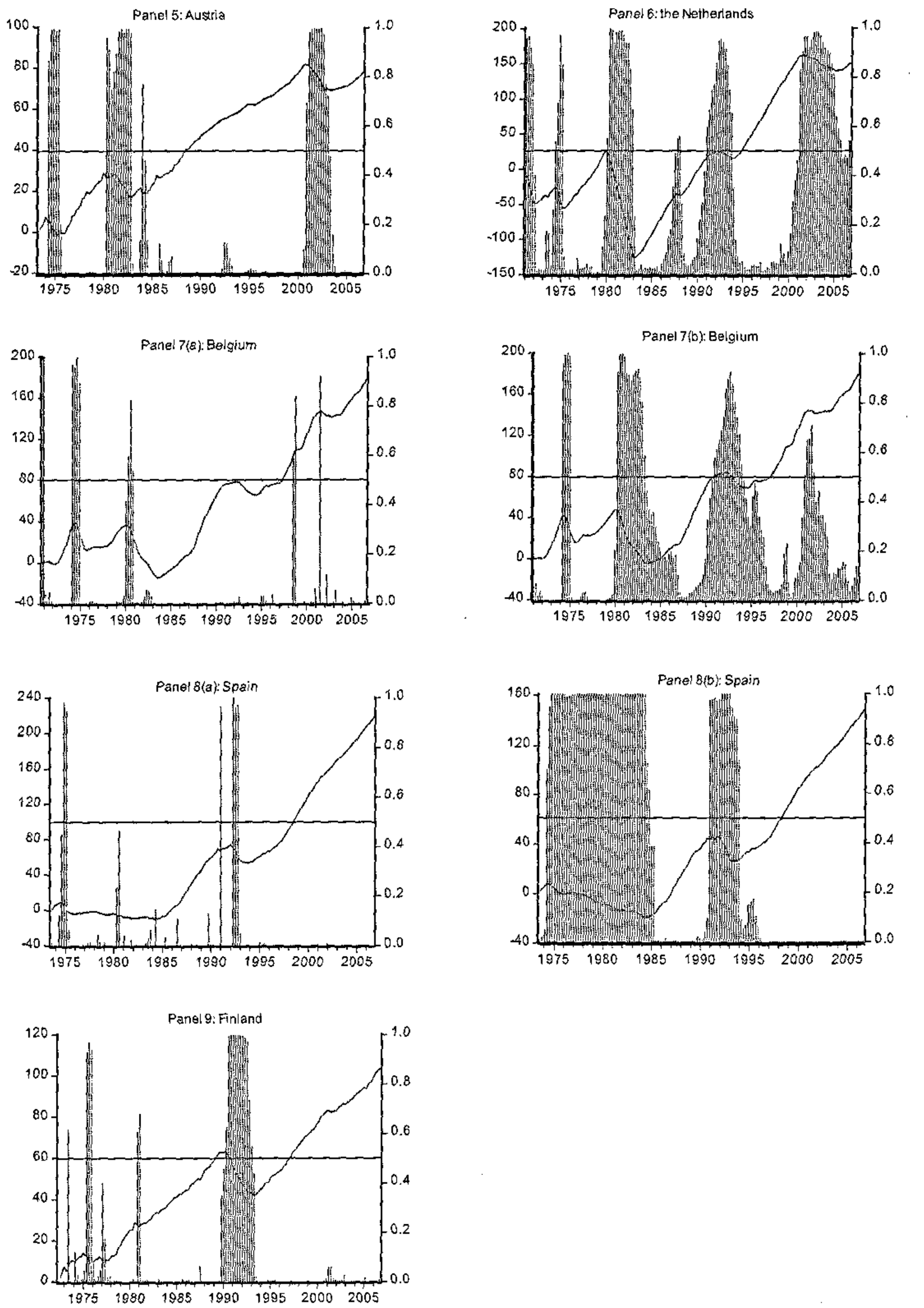
Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for Finland were estimated using data from 1970Q1-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0069, 0.0121; Δ Sales: 0.0054, 0.0184; Δ IP: 0.0093, 0.0215; Δ EMP: 0.0011, 0.0081.

Figure 3.1: MS composite indices and smoothed recession probabilities



Note: *the index with squares on top denotes high-growth periods.*

Figure 3.1: MS composite indices and smoothed recession probabilities (Continued)



Notes: The horizontal line in each figure marks the threshold value of 0.5. If the smoothed recession probability at time t is greater than 0.5, the observation at time t is classified as a recession.

3.4.3 Non-EMU Countries: The UK, The US and Canada

United Kingdom. The parameter estimates of the MSDF model for the UK are reported in Table 3.10. The mean growth rate is $u_0 + \delta = -1.785$ during recessions and $u_1 + \delta = 1.262$ during expansions. The average duration of recessions and expansions are calculated to be $(1 - p_{00})^{-1} = 4$ and $(1 - p_{11})^{-1} = 20$ quarters, respectively. Compared with the corresponding estimates in Mills and Wang (2003b), this study obtains a shorter recessionary phase and a longer expansionary phase. This is partly due to the use of different sample periods. The sample period used by Mills and Wang (2003b) included the 1960s when several brief recessions occurred. The data used in this chapter is from 1970 onwards with no recessions being picked out in the 2000s.

In total, four recessions are identified by the smoothed recession probabilities, during 1973Q3-1974Q1, 1974Q4-1975Q3, 1979Q3-1981Q1 and 1990Q3-1991Q3. This replicates the cycle dates produced in Chapter 2.

United States. The results for the US are summarised in Table 3.11. The asymmetric nature of business cycles is again supported by the parameter estimates. The mean growth rate is $u_0 + \delta = -0.184$ during recessions and $u_1 + \delta = 1.629$ during expansions. Given the estimated transition probabilities, the average duration of recessions and expansions are 4.4 quarters and 19.2 quarters, respectively. The US common factor, along with the smoothed recession probabilities, are plotted in Panel 11. Five recessions are identified, during 1973Q3-1975Q1, 1979Q1-1980Q2, 1981Q2-1982Q3, 1990Q2-1991Q1 and 2000Q4-2001Q3. These cycle dates do not correspond as closely to the NBER dates as those obtained by the BBQ approach.

Table 3.10: Parameter estimates of MSDF model for the UK

Common Factor

ϕ_1	ϕ_2	u_0	u_1	P_{00}	P_{11}
-0.192*	-0.009	-2.625**	0.422**	0.752**	0.950**
(0.102)	(0.010)	(0.398)	(0.121)	(0.108)	(0.023)

Idiosyncratic Components

ΔGDP	γ_1			D79q2	D79q2	ψ_{11}	ψ_{12}	α_{11}	σ_1^2
	0.544**			3.772**	1.671**	-0.555*	-0.077	-0.051	0.081
	(0.059)			(0.523)	(0.545)	(0.207)	(0.057)	(0.056)	(0.046)
ΔGFCF	γ_2	-	-	-	-	ψ_{21}	ψ_{22}	α_{12}	σ_2^2
	0.230**					-0.145	-0.005	-0.014	0.891**
	(0.064)					(0.097)	(0.007)	(0.083)	(0.109)
ΔIP	γ_3	D72q1	D72q2	D74q1	D74q2	ψ_{31}	ψ_{32}	α_{13}	σ_3^2
	0.411**	-2.472**	2.503**	-2.227**	3.681**	0.384**	-0.037	-0.019	0.203**
	(0.048)	(0.515)	(0.533)	(0.540)	(0.520)	(0.117)	(0.022)	(0.070)	(0.040)
ΔEMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D02q3	ψ_{41}	ψ_{42}	α_{14}	σ_4^2
	0.144**	0.120**	0.143**	0.072	-4.969**	0.351**	0.062	-0.230*	0.334**
	(0.040)	(0.043)	(0.045)	(0.042)	(0.558)	(0.090)	(0.096)	(0.090)	(0.041)

Long run growth rate: $\delta = 0.840$

Error correction term

$$\mathrm{GDP}_{t-1} = 35.186 + \underset{(0.123)}{1.0569} \times \mathrm{GFCF}_{t-1} + \underset{(0.156)}{0.674} \times \mathrm{IP}_{t-1} - \underset{(0.450)}{2.824} \times \mathrm{EMP}_{t-1}$$

Log-likelihood: -571.109

Diagnostics	**Q(4)**	**Jarque-Bera**
ΔGDP	6.445	7.902*
ΔGFCF	2.172	2.772
ΔIP	3.525	7.255*
ΔEMP	1.527	5.218

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for the UK were estimated using data from 1970Q1-2005Q1. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: ΔGDP: 0.0057, 0.0096; ΔGFCF: 0.0064, 0.0272; ΔIP: 0.0028, 0.0186; ΔEMP: 0.0011, 0.0054.

Table 3.11: Parameter estimates of MSDF model for the US

Common Factor

ϕ_1	ϕ_2	u_0	u_1	p_{00}	p_{11}
0.431**	-0.047	-1.474**	0.339*	0.771**	0.948**
(0.120)	(0.026)	(0.359)	(0.148)	(0.106)	(0.026)

Idiosyncratic Components

Δ GDP	γ_1	-	-	D78q2	ψ_{11}	ψ_{12}	α_{11}	σ_1^2
	0.560**			2.026**	-0.472**	-0.056	-0.122	0.173**
	(0.060)			(0.574)	(0.156)	(0.037)	(0.099)	(0.048)
Δ Sales	γ_2	-	D74q4	D80q2	ψ_{21}	ψ_{22}	α_{12}	σ_2^2
	0.345**		-1.723**	-3.087**	-0.197*	-0.010	0.128	0.476**
	(0.052)		(0.531)	(0.556)	(0.096)	(0.009)	(0.078)	(0.060)
Δ IP	γ_3	D74q4	D75q1	D80q2	ψ_{31}	ψ_{32}	α_{13}	σ_3^2
	0.486**	-1.178*	-2.171**	-1.966*	0.282*	0.094	-0.234**	0.185**
	(0.054)	(0.548)	(0.763)	(0.753)	(0.128)	(0.122)	(0.101)	(0.037)
Δ EMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	ψ_{41}	ψ_{42}	α_{14}	σ_4^2
	0.356**	0.141*	0.027	0.100	0.072	-0.001	-0.306**	0.389**
	(0.060)	(0.065)	(0.066)	(0.054)	(0.101)	(0.004)	(0.115)	(0.052)

Long run growth rate: $\delta = 1.290$

Error correction term

$$\text{GDP}_{t-1} = 0.004 + \underset{(0.071)}{0.545} \times \text{Sales}_{t-1} - \underset{(0.072)}{0.097} \times \text{IP}_{t-1} + \underset{(0.197)}{0.322} \times \text{EMP}_{t-1}$$

Log-likelihood: -589.215

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	1.251	7.628*
Δ Sales	1.734	3.584
Δ IP	3.961	20.577**
Δ EMP	5.050	5.510

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for the US were estimated using data from 1970Q1-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0078, 0.0082; Δ Sales: 0.0054, 0.0148; Δ IP: 0.0069, 0.0149; Δ EMP: 0.0043, 0.0048.

Canada. Mean growth rates during recessions and expansions are estimated to be $u_0 + \delta = -0.652$ and $u_1 + \delta = 1.437$. The average duration of recessions and expansions are $(1 - p_{00})^{-1} = 3.5$ quarters and $(1 - p_{11})^{-1} = 47.6$ quarters, respectively. Two recessions are identified by the smoothed recession probabilities, during 1981Q3-1982Q3 and 1990Q2-1991Q1.

Table 3.12: Parameter estimates of MSDF model for Canada

Common Factor					
ϕ_1	ϕ_2	u_0	u_1	p_{00}	p_{11}
0.609**	-0.093*	-1.948**	0.141	0.714**	0.979**
(0.125)	(0.038)	(0.611)	(0.100)	(0.175)	(0.017)

Idiosyncratic Components

Δ **GDP**	γ_1	-	-	-	ψ_{11}	ψ_{12}	α_{11}	σ_1^2
	0.530**				-0.298*	-0.022	-0.030	0.280**
	(0.059)				(0.118)	(0.018)	(0.097)	(0.048)
Δ **Sales**	γ_2	-	-	D81Q1	ψ_{21}	ψ_{22}	α_{12}	σ_2^2
	0.277**			4.758**	-0.229*	0.113	0.205**	0.534**
	(0.046)			(0.755)	(0.092)	(0.091)	(0.074)	(0.046)
Δ **IP**	γ_3	-	-	-	ψ_{31}	ψ_{32}	α_{13}	σ_3^2
	0.530**				0.231*	0.105	0.267*	0.276**
	(0.060)				(0.117)	(0.105)	(0.106)	(0.049)
Δ **EMP**	γ_{40}	γ_{41}	γ_{42}	γ_{43}	ψ_{41}	ψ_{42}	α_{14}	σ_4^2
	0.344**	0.234**	-0.052	-0.036	0.111	0.024	-0.158	0.332**
	(0.065)	(0.082)	(0.079)	(0.064)	(0.109)	(0.106)	(0.125)	(0.041)

Long run growth rate: $\delta = 1.296$

Error correction term

$$\mathrm{GDP}_{t-1} = \underset{}{4.586} + \underset{(0.045)}{0.261} \times \mathrm{Sales}_{t-1} + \underset{(0.046)}{0.399} \times \mathrm{IP}_{t-1} + \underset{(0.047)}{0.632} \times \mathrm{EMP}_{t-1}$$

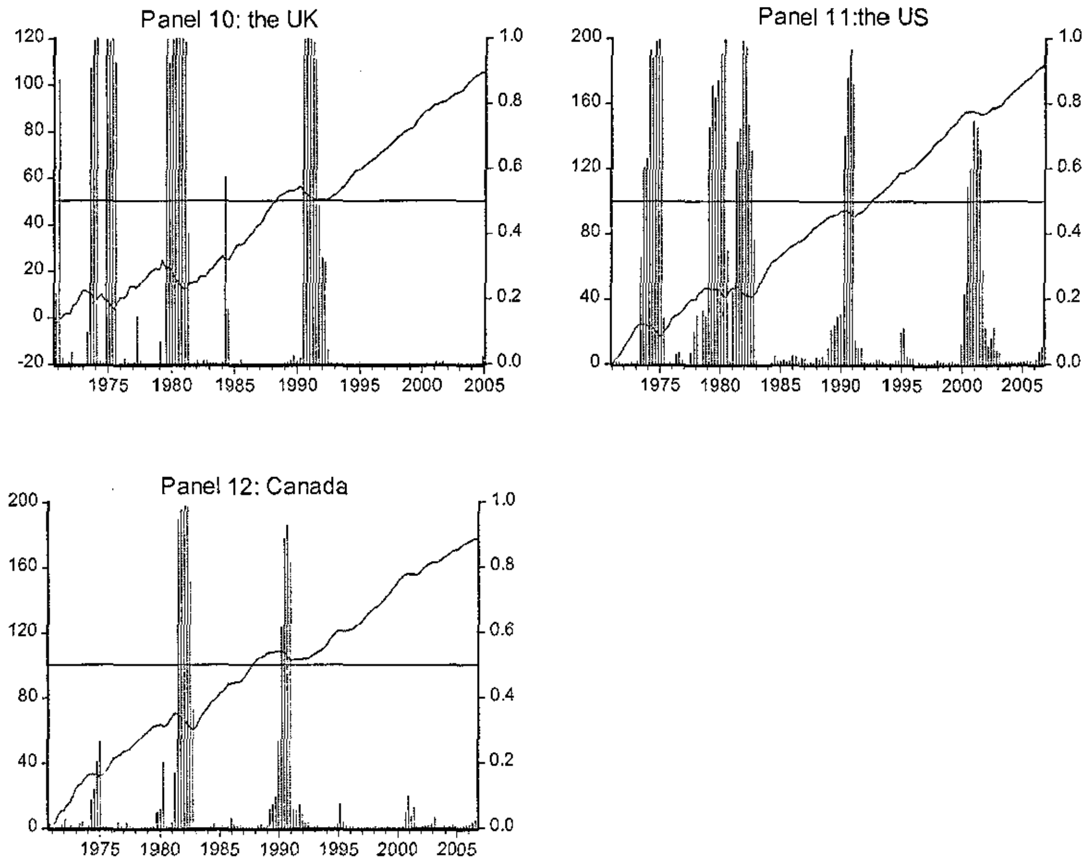
Log-likelihood: -626.231

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ **GDP**	8.723	14.589**
Δ **Sales**	0.239	0.702
Δ **IP**	6.145	0.951
Δ **EMP**	8.494	53.852**

Notes: Standard errors are in parentheses. ** denotes significance at 1% and * at 5%. The parameter estimates for Canada were estimated using data from 1970Q1-2006Q4. Logarithms of variables were used, each variable was standardised to growth rates with mean zero and unit variance prior to estimation. The sample means and standard deviations for the growth rates of the original series are: Δ GDP: 0.0077, 0.0081; Δ Sales: 0.0079, 0.0168; Δ IP: 0.0064, 0.0168; Δ EMP: 0.0050, 0.0058.

Summary. In general, the cycle dates produced by the MSDF model are broadly in line with those obtained by the BBQ algorithm used in Chapter 2. The parameter estimates obtained using the MSDF model confirms the asymmetric feature of business cycles, that recessions are steeper and shorter than expansions. The MSDF model seems more successful in distinguishing between different regimes for large economies (i.e., Germany, France, the UK and the US), whose recessions and expansions are of roughly constant magnitude over the period studied. Smaller economies exhibited greater volatility during the 1970s and early 1980s. The recession intercept for these countries is biased downwards by the severe recessions experienced during this period. As a consequence, the smoothed probabilities fail to identify the mild recessions which occurred in the later years and instead classify them as expansions. Introducing structural breaks in the intercepts seems to improve the results for Belgium, Italy and the Netherlands. For Spain, the MSDF model appears to deliver the worst fit, as the recession probabilities either capture a few brief recessions or pick out too many economic downturns.

Figure 3.1: MS composite indices and smoothed recession probabilities (Continued)



Notes: The horizontal line in each figure marks the threshold value of 0.5. If the smoothed recession probability at time t is greater than 0.5, the observation at time t is classified as a recession.

3.5 Evaluating the coincidence of regime shifts

In this section, the concordance of regime shifts is evaluated using pairwise correlations of binary variables and of the smoothed recession probabilities. The binary time-series are constructed with one denoting expansions and zero indicating recessions. For France, where three regimes are identified, the value one is assigned to both the moderate-growth and high-growth regimes and zero is assigned to recessions.

3.5.1 Multidimensional mapping of business cycle distance

The correlations of binary variables for the whole sample and two subsamples with the midpoint of 1991Q1 are given in Table 3.13. Sammon mapping, introduced in Chapter 2, is also used in this section to provide a pictorial representation of these correlations. In order to compare the results shown in Chapter 2 with those obtained in this section, Figure 2.2 is represented in Panels 1(a)-3(a) of Figure 3.2, while Panels 1(b)- 3(b) of Figure 3.2 present the three maps produced using binary series indicated by the smoothed recession probabilities over the whole sample and the two subsamples. It can be seen from both Panels 1(a) and 1(b) that, over the whole sample, the conclusions are robust using either the BBQ or the MSDF approach. More synchronised business cycles are observed between the core EMU countries and the aggregate euro area, compared to those in the peripheral and non-EMU groups. A cluster of four core EMU countries, composed of Germany, Austria, Belgium and the Netherlands, who had highly synchronised business cycles, are shown in both Panels 1(a) and 1(b).

Panels 2(a) and 2(b) of Figure 3.2 reveal that the four core EMU countries mentioned above all had closer distances to the euro area aggregate than France, Italy, the peripheral and non-EMU groups during the first subsample. This is more obviously observed in Panel 2(b), where binary series are produced by the MSDF model. However, conflicting results are revealed when comparing Panels 3(a) and 3(b) of Figure 3.2. Although both figures show that the tight cluster of four core EMU countries are driven away from each other and the desynchronisation process between Austria, Belgium, the Netherlands and the aggregate euro area are notable over the second subsample, an increase in cycle correlation between France, two peripheral countries and the euro area aggregate, which is shown in Panel 3(a), cannot be observed in Panel 3(b). This conflict can be explained by the identification of recessions during the early 2000s for the aggregate euro area but not for France and the peripheral countries when the MSDF model is used, which reduces the correlation between these countries and the euro area as a whole. Finally, a picture of diverging non-EMU business cycles from the euro area as a whole can clearly be observed from both 3(a) and 3(b).

Table 3.13: Correlation Coefficients of Binary Variables

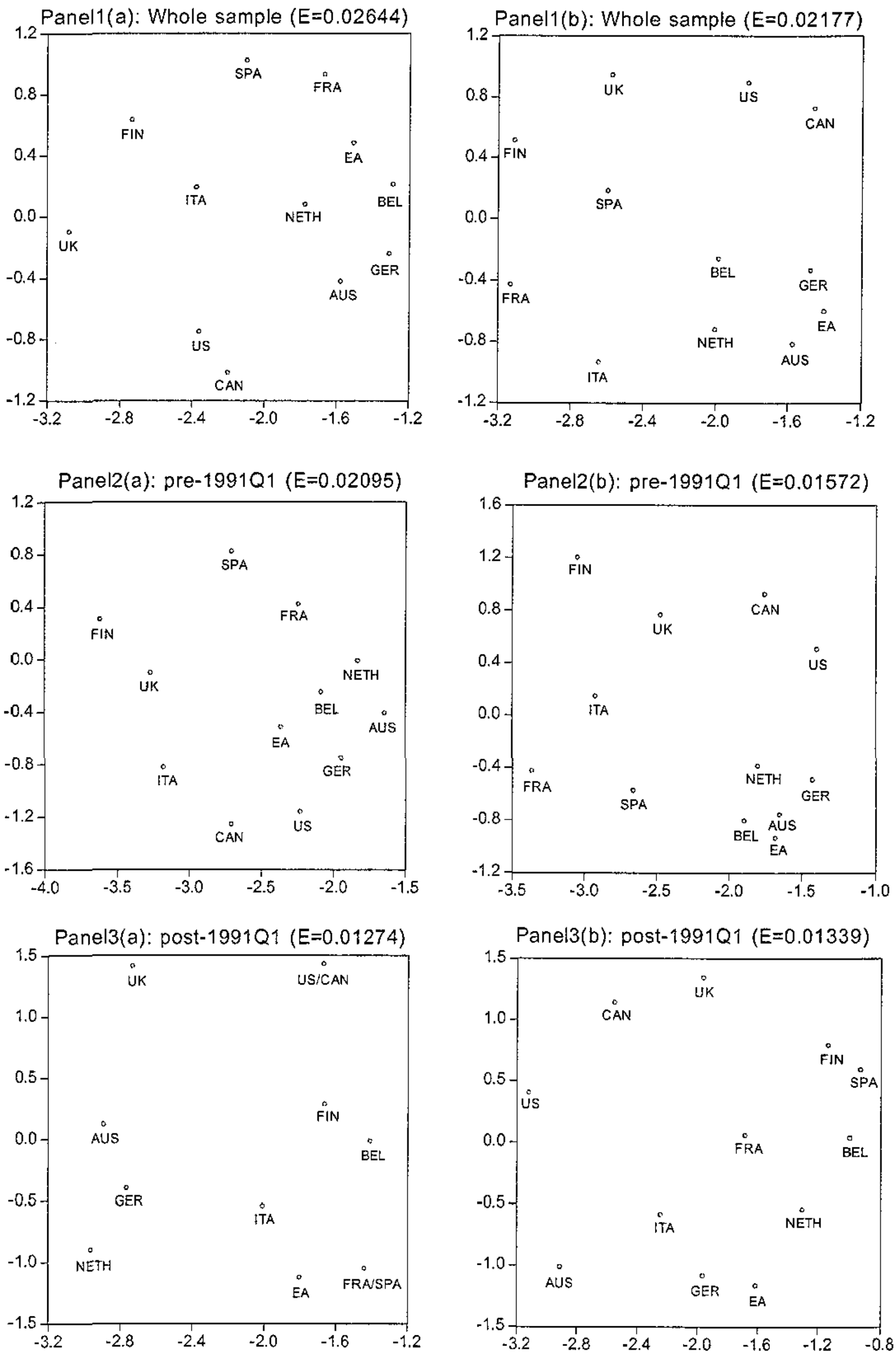
Whole Sample Period

	EMU	GER	FRA	ITA	AUS	BEL	NETH	SPA	FIN	UK	US
EMU											
GER	0.80										
FRA	0.41	0.27									
ITA	0.44	0.30	0.27								
AUS	0.79	0.68	0.12	0.28							
BEL	0.65	0.53	0.43	0.32	0.55						
NETH	0.61	0.59	0.15	0.33	0.55	0.61					
SPA	0.36	0.21	0.33	0.12	0.27	0.45	0.22				
FIN	0.16	0.13	0.19	0.07	-0.03	0.32	0.21	0.35			
UK	0.11	0.26	0.14	-0.04	0.15	0.17	0.14	0.32	0.39		
US	0.20	0.29	-0.13	0.02	0.38	0.34	0.17	0.25	0.02	0.39	
CANA	0.21	0.14	-0.07	0.11	0.24	0.25	0.16	0.16	0.25	0.13	0.53

Pre-1991Q1

		EMU	GER	FRA	ITA	AUS	BEL	NETH	SPA	FIN	UK	US	CANA
Post-1991Q1	EMU		0.85	0.40	0.30	0.90	0.85	0.71	0.54	0.11	0.27	0.30	0.35
	GER	0.76		0.36	0.19	0.79	0.72	0.68	0.30	0.06	0.43	0.50	0.25
	FRA	0.46	0.25		0.22	0.31	0.36	0.15	0.21	0.07	0.20	-0.21	-0.15
	ITA	0.56	0.41	0.44		0.36	0.33	0.23	0.30	0.02	0.09	0.09	0.23
	AUS	0.68	0.55	-0.16	0.25		0.85	0.81	0.48	0.08	0.23	0.41	0.32
	BEL	0.47	0.31	0.60	0.30	0.21		0.71	0.48	0.00	0.16	0.43	0.34
	NETH	0.57	0.52	0.30	0.39	0.36	0.51		0.36	-0.02	0.20	0.37	0.26
	SPA	0.33	0.22	0.58	0.21	-0.20	0.73	0.47		0.03	0.14	0.19	0.03
	FIN	0.20	0.20	0.39	0.10	-0.17	0.65	0.42	0.89		0.39	0.03	0.27
	UK	-0.14	0.00	-0.10	-0.18	-0.11	0.35	0.24	0.48	0.54		0.41	0.02
	US	0.08	0.00	-0.12	0.00	0.30	0.28	0.01	0.00	0.02	0.18		0.54
	CANA	-0.07	-0.10	-0.05	-0.09	-0.05	0.20	0.13	0.28	0.31	0.57	0.39	

Figure 3.2: Multidimensional mapping of business cycle distances



Note: two pairs of countries, the US and Canada, France and Spain, have perfectly synchronised turning points in the second subsample when the BBQ algorithm is used.

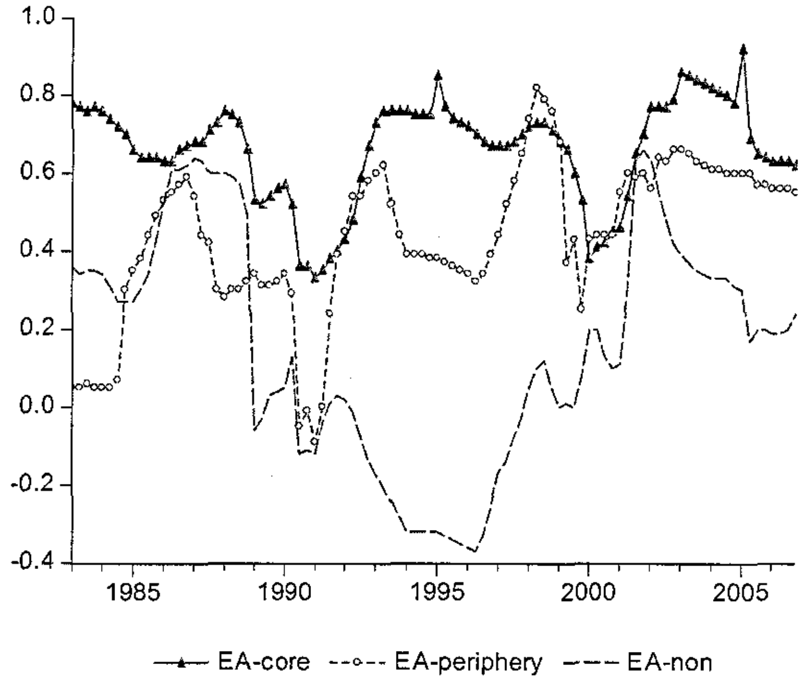
3.5.2 Rolling sample correlation of smoothed recession probabilities

In addition to analysing changes in synchronisation using two fixed subsamples[16], bilateral correlations of the smoothed recession probabilities are also computed using a series of rolling samples, with a window width of six years.[17] The mean correlation coefficients of the core, periphery and non-EMU countries with respect to the euro area aggregate are calculated for each window and are plotted in Figure 3.3. There are a few points worth noting. First, a significant decline in synchronisation between both core and periphery EMU business cycles and the aggregate euro area is observed during the 1980s. This is broadly in line with Inklaar and de Hann (2001) and Massmann and Mitchell (2004), who found desynchronised euro area growth cycles during this period. Second, a dramatic increase in cycle convergence is observed during the 1990s, specifically for the two periphery countries, whose mean correlation coefficient with respect to the euro area aggregate exceeded the core countries at the end of 1990s, but this was reversed during the 2000s. This, in part, reflects unbalanced economic performance across EMU member states after the introduction of the euro. A number of core EMU countries, such as Germany and Italy, who have large weights assigned to them when constructing aggregate euro area data series, suffered recessions and sluggish growth for several years during the 2000s, while two periphery countries, particular Spain, maintained strong economic growth. Finally, the cycle correlation between non-EMU and the euro area aggregate only increased during global downturns, such as during the early 1980s and at the beginning of 2000s, but completely diverged from the euro area during the ERM period.

[16] Pairwise correlations of the smoothed recession probabilities over the whole sample and the two subsamples are reported in Appendix D3. The focus of this subsection is to evaluate the changes in correlations using a rolling sample approach.

[17] We understand that the results are sensitive to the choice of window length. Long windows tend to smooth out important medium-term changes in synchronisation, while short windows are more sensitive to short- to medium-term deviations. The window length of six years is commonly used in the literature, see, for example, Gayer (2007).

Figure 3.3: Rolling correlations of the smoothed regime probabilities



3.6 Conclusions

In this chapter, one of the parametric business cycle dating approaches, the MSDF model, is applied to the same data used in Chapter 2 to date business cycle turning points. The cycle dates obtained using the MSDF model are broadly in line with those produced by the BBQ approach. One exception is the turning points obtained for the aggregate euro area. The smoothed recession probabilities indicate a period of recession during 2001Q1-2003Q2, while this contradicts the conclusion, drawn from the BBQ algorithm and the CEPR business cycle dating committee, that no recessions occurred during the early 2000s. As such, an increase in cycle correlation between France, two peripheral countries and the euro area aggregate, which is shown in Chapter 2, cannot be found here.

In general, the baseline MSDF model was successful at detecting turning points for large economies (i.e., Germany and the UK and the US), where the magnitude of recessionary and expansionary phases appears to be constant over the entire sample. It was less successful at producing reasonable parameter estimates and smoothed probabilities for the other countries. Therefore, adjustments had to be made for Belgium, the Netherlands, Italy and Spain to account for changes in the model's intercepts. For France, an additional regime is included to distinguish between the three phases of the French business cycle: recessions, moderate-growth and high-growth. The MS type of models were criticised by Harding and Pagan (2002) for producing significantly different results when different models and sample periods are used. It is true that, compared to the BBQ algorithm, the MS models appear to be less transparent and more dependent on the particular properties of the data. However, this is also described as an advantage over the BBQ approach by Hamilton (2003), as the MS approach can provide a specific model for the object of interest and can derive the optimal inference about it.

Appendix A3

A3.1 Specification of the MSDF Model

The MSDF model utilised in this chapter is specified as

$$\Delta y_t = H\beta_t, \tag{A3.1}$$

$$\beta_t = M_{S_t} + F\beta_{t-1} + \varepsilon_t,\ \varepsilon_t \sim NID(0,Q) \tag{A3.2}$$

where $\Delta y_t = [\Delta y_{1t}, \Delta y_{2t}, \Delta y_{3t}, \Delta y_{4t}]^{\mathrm{T}}$ is the vector containing four coincident economic variables. $\beta_t = [\Delta c_t, \Delta c_{t-1}, \Delta c_{t-2}, \Delta c_{t-3}, e_{1t}, e_{1t-1}, \ldots, e_{4t}, e_{4t-1}]^{\mathrm{T}}$ contains the current and lagged values of the common factor and innovation terms. The time-invariant and regime-independent matrices F, H and Q, contain the model's hyperparameters:

$$H = \begin{bmatrix} \gamma_1 & 0 & 0 & 0 & 1 & 0 & \ldots & 0 & 0 \\ \gamma_2 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ \gamma_3 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ \gamma_{40} & \gamma_{41} & \gamma_{42} & \gamma_{43} & 0 & 0 & \ldots & 1 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi_{11} & \psi_{12} & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & \psi_{41} & \psi_{42} \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 1 & 0 \end{bmatrix}$$

$$\text{and } Q = \begin{bmatrix} \sigma_v^2 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_1^2 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & \sigma_4^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 \end{bmatrix}$$

$M_{S_t} = \lfloor \mu_{S_t}, 0,0,0,0,0,0,0,0,0,0,0 \rfloor$ contains the MS intercept of the common factor, where the estimate of μ_{S_t} depends on the current and lagged two-state MS variables, S_t and S_{t-1}.

A3.2 The Filtering Algorithm

Kim's (1994) filter is designed for the state-space model with MS, and provides MLEs of the model's unknown parameters. It combines the KF and the Hamilton filter, along with appropriate approximations. This filter is started by the KF iterations, conditional on the current and lagged state variables, $S_t = j$ and $S_{t-1} = i$, $i, j = 1,2$. The recursive equations are as follows,

$$\beta_{t|t-1}^{(i,j)} = \widetilde{\mu}_j + F\beta_{t-1|t-1}^{i}, \qquad P_{t|t-1}^{(i,j)} = FP_{t-1|t-1}^{i}F^{\top} + Q,$$

$$\eta_{t|t-1}^{(i,j)} = y_t - H\beta_{t|t-1}^{(i,j)}, \qquad f_{t|t-1}^{(i,j)} = HP_{t|t-1}^{(i,j)}H^{\top},$$

$$\beta_{t|t}^{(i,j)} = \beta_{t|t-1}^{(i,j)} - P_{t|t-1}^{(i,j)}H^{\top}\left[f_{t|t-1}^{(i,j)}\right]^{-1}\eta_{t|t-1}^{(i,j)},$$

$$P_{t|t}^{(i,j)} = \left(I - P_{t|t-1}^{(i,j)}H^{\top}\left[f_{t|t-1}^{(i,j)}\right]^{-1}H\right)P_{t|t-1}^{(i,j)},$$

where $\beta_{t-1|t-1}^{i}$ is an inference on β_{t-1} based on information up to time $t-1$, conditional on $S_{t-1} = i$. $\beta_{t|t-1}^{(i,j)}$ is an optimal forecast of β_{t-1} based on all information up to time $t-1$, given $S_t = j$ and $S_{t-1} = i$. $P_{t|t-1}^{(i,j)}$ is the mean squared error matrix of $\beta_{t|t-1}^{(i,j)}$. $\eta_{t|t-1}^{(i,j)}$ is the prediction error and $f_{t|t-1}^{(i,j)}$ is the variance of $\eta_{t|t-1}^{(i,j)}$. Since the above KF produces a 2-fold increase in the number of $\beta_{t|t}^{(i,j)}$ and $P_{t|t}^{(i,j)}$ at the end of each iteration, approximations to $\beta_{t|t}^{j}$ and $P_{t|t}^{j}$ are needed to make the KF operable. Kim reduces the 2×2 posteriors ($\beta_{t|t}^{(i,j)}$ and $P_{t|t}^{(i,j)}$) to 2 posteriors ($\beta_{t|t}^{j}$ and $P_{t|t}^{j}$) by using two approximation equations:

$$\beta_{t|t}^{j} = \frac{\sum_{i=1}^{2} \Pr(S_t = j, S_{t-1} = i, |\psi_t)\beta_{t|t}^{(i,j)}}{\Pr(S_t = j|\psi_t)} \quad , \tag{A3.3}$$

$$P_{t|t}^{j} = \frac{\sum_{i=1}^{2} \Pr(S_t = j, S_{t-1} = i, |\psi_t) \left\{ P_{t|t}^{(i,j)} + \left(\beta_{t|t}^{j} - \beta_{t|t}^{(i,j)}\right)\left(\beta_{t|t}^{j} - \beta_{t|t}^{(i,j)}\right)^{\mathrm{T}} \right\}}{\Pr(S_t = j|\psi_t)} . \qquad \text{(A3.4)}$$

The Hamilton filtering algorithm is then utilised to calculate the probabilities $\Pr(S_t = j|\psi_t)$ and $\Pr(S_t = j, S_{t-1} = i, |\psi_t)$ in equations (A3.3) and (A3.4):

$$\Pr(S_t = j, S_{t-1} = i, |\psi_{t-1}) = \Pr(S_t = j|S_{t-1} = i)\Pr(S_{t-1} = i, |\psi_{t-1}), \qquad \text{(A3.5)}$$

$$f(y_t|\psi_{t-1}) = \sum_{j=1}^{2}\sum_{i=1}^{2} f(y_t|S_t = j, S_{t-1} = i, \psi_{t-1})\Pr(S_t = j, S_{t-1} = i, |\psi_{t-1}), \qquad \text{(A3.6)}$$

$$\Pr(S_t = j, S_{t-1} = i, |\psi_t) = \frac{f(y_t|S_t = j, S_{t-1} = i, \psi_{t-1})\Pr(S_t = j, S_{t-1} = i, |\psi_{t-1})}{f(y_t|\psi_{t-1})}, \qquad \text{(A3.7)}$$

$$\Pr(S_t = j|\psi_t) = \sum_{i=1}^{2} \Pr(S_t = j, S_{t-1} = i, |\psi_t), \qquad \text{(A3.8)}$$

$\Pr(S_t = j|S_{t-1} = i)$ is the transition probability. $f(y_t|\psi_{t-1})$ and $f(y_t|S_t = j, S_{t-1} = i, \psi_{t-1})$ are the marginal and conditional density of y_t, respectively. $\Pr(S_t = j|\psi_t)$ gives the filtered probabilities for regime j.

A3.3 The Smoothing Algorithm

Compared to the filtering process described above, which aims to produce optimal estimates of β_t and S_t based on information up to time t, $\beta_{t|t}$ and $\Pr(S_t = j|\psi_t)$, the smoothing algorithm takes into account information available after time t. The smoothed estimates of β_t and S_t, conditional on the complete sample, are written as $\beta_{t|T}$ and $\Pr(S_t = j|\psi_T)$. Since the smoothed estimates are based on more information than their filtered counterparts, in general they provide more accurate inferences.

The smoothed probability that $S_t = j$ based on full information can be derived by the following equations (see Kim, 1994; Kim and Nelson, 1998):

$$\Pr(S_t = j, S_{t+1} = k|\psi_T) = \Pr(S_{t+1} = k|\psi_T)\Pr(S_t = j|S_{t+1} = k, \psi_T)$$

$$\approx \Pr(S_{t+1}=k|\psi_T)\Pr(S_t=j|S_{t+1}=k,\psi_t)$$

$$=\frac{\Pr(S_{t+1}=k|\psi_T)\Pr(S_t=j|\psi_t)\Pr(S_{t+1}=k|S_t=j)}{\Pr(S_{t+1}=k|\psi_t)}, \quad \text{(A3.9)}$$

and

$$\Pr(S_t=j|\psi_T)=\sum_{k=1}^{2}\Pr(S_t=j,S_{t+1}=k|\psi_T), \quad \text{(A3.10)}$$

where $\Pr(S_T=j|\psi_T)$ is the initial value for smoothing, which is obtained from the last iteration of Kim's (1994) filter.

The smoothing algorithm for the vector β_t can be derived as follows, given $S_t=j$ and $S_{t+1}=k$:

$$\beta_{t|T}^{(j,k)}=\beta_{t|t}^{j}+P_{t|t}^{j}F^{\mathsf{T}}\left(P_{t+1|t}^{(j,k)}\right)^{-1}\left(\beta_{t+1|T}^{k}-\beta_{t+1|t}^{(j,k)}\right),$$

$$P_{t|T}^{(j,k)}=P_{t|t}^{j}+P_{t|t}^{j}F^{\mathsf{T}}\left(P_{t+1|t}^{(j,k)}\right)^{-1}\left(P_{t+1|T}^{k}-P_{t+1|t}^{(j,k)}\right)\left(P_{t+1|t}^{(j,k)}\right)^{-1\mathsf{T}}FP_{t|t}^{j\,\mathsf{T}},$$

where $\beta_{t|T}^{(j,k)}$ is the estimate of β_t based on the full sample and $P_{t|T}^{(j,k)}$ is the mean squared error matrix of $\beta_{t|T}^{(j,k)}$. $\beta_{t|t}^{j}$ and $P_{t|t}^{j}$ are obtained in equations (A3.3) and (A3.4).

The smoothed probabilities $\Pr(S_t=j,S_{t+1}=k|\psi_T)$ and $\Pr(S_T=j|\psi_T)$ can be used to approximate $\beta_{t|T}^{j}$ and $P_{t|T}^{j}$, using the following equations:

$$\beta_{t|T}^{j}=\frac{\sum_{k=1}^{2}\Pr(S_t=j,S_{t+1}=k,|\psi_T)\beta_{t|T}^{(j,k)}}{\Pr(S_t=j|\psi_T)}, \quad \text{(A3.11)}$$

$$P_{t|T}^{j}=\frac{\sum_{k=1}^{2}\Pr(S_t=j,S_{t+1}=k,|\psi_T)\left\{P_{t|T}^{(i,k)}+\left(\beta_{t|T}^{j}-\beta_{t|T}^{(j,k)}\right)\left(\beta_{t|T}^{j}-\beta_{t|T}^{(j,k)}\right)^{\mathsf{T}}\right\}}{\Pr(S_t=j|\psi_t)}. \quad \text{(A3.12)}$$

By taking a weighted average over the states at time t, $\beta_{t|T}$ is obtained as

$$\beta_{t|T}=\sum_{j=1}^{2}\Pr(S_t=j|\psi_T)\beta_{t|T}^{j}$$

A3.4 Approximate MLE

The conditional density $f\left(y_t \middle| S_t = j, S_{t-1} = i, \psi_{t-1}\right)$ in equation (A3.6) is based on the prediction error decomposition:

$$f\left(y_t \middle| S_t = j, S_{t-1} = i, \psi_{t-1}\right) = (2\pi)^{-\frac{N}{2}} \left| f_{t|t-1}^{(i,j)} \right|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2} \eta_{t|t-1}^{(i,j)\top} f_{t|t-1}^{(i,j)-1} \eta_{t|t-1}^{(i,j)} \right\}$$

The marginal density of y_t conditional on past information, $f\left(y_t \middle| \psi_{t-1}\right)$, is obtained from equation (A3.6). By summing the logged values of $f\left(y_t \middle| \psi_{t-1}\right)$ at the end of each iteration, the approximate log likelihood function is given by

$$LL = \ln\left[f\left(y_1, y_2, \ldots, y_T\right) \right] = \sum_{t=1}^{T} \ln\left[f\left(y_t \middle| \psi_{t-1}\right) \right].$$

A3.5 The mean growth rate $\hat{\delta}$ of ΔC_t

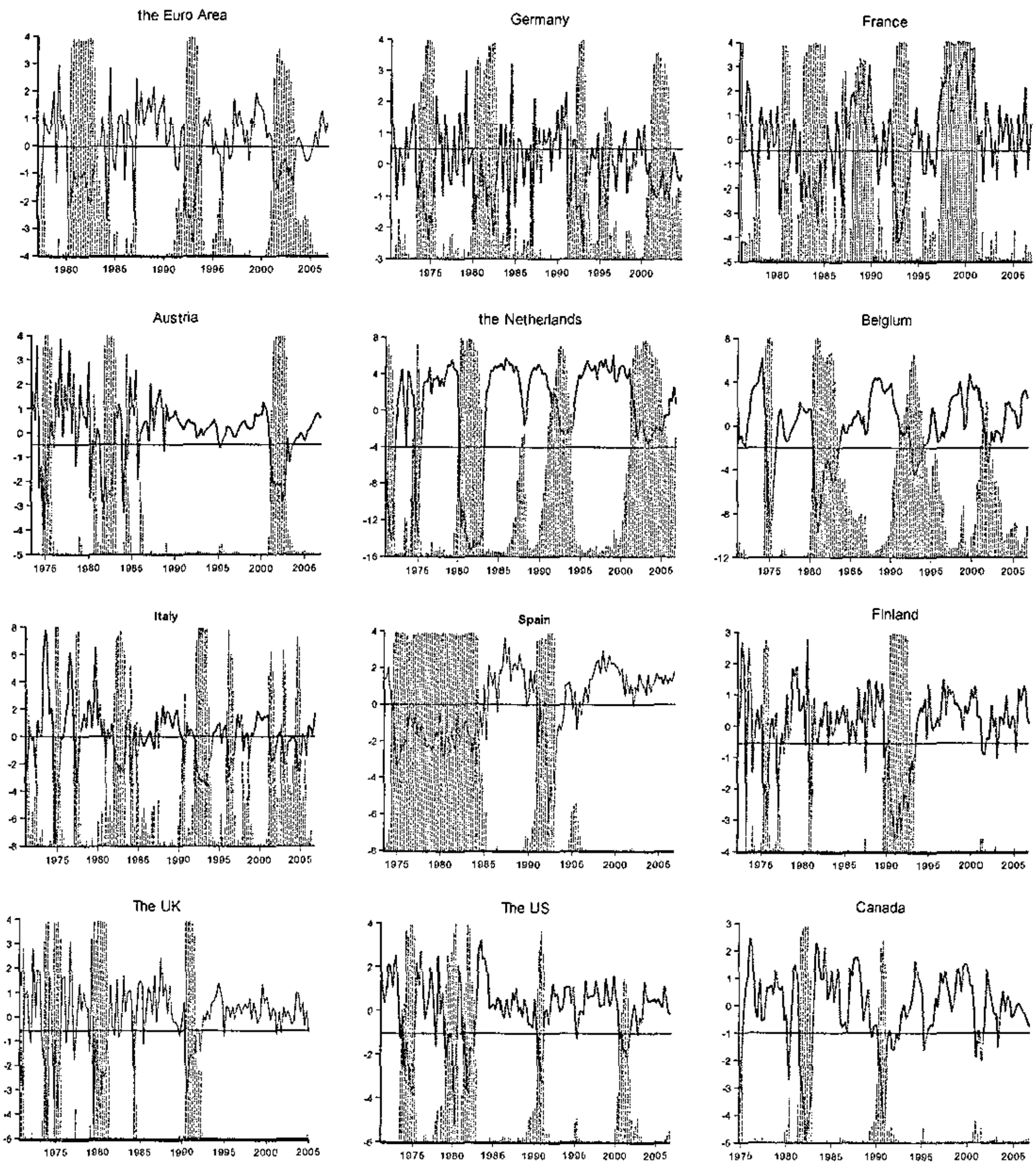
The mean growth rate of the MS common factor is computed as the weighted average of the mean growth rates of the constituent variables, $\hat{\delta} = W(1)\Delta\overline{Y}$, where $W(1)$ is the first row of $(I-(I-KH)F)^{-1}K$. $K = K_T = P_{T|T-1} H^\top f_{T|T-1}^{-1}$ is the Kalman gain at the last iteration. $P_{T|T-1}$ and $f_{T|T-1}$ are the weighted average of over $P_{T|T}^{(i,j)}$ and $f_{T|T-1}^{(i,j)}$, which are calculated as follows

$$P_{T|T-1}^{j} = \frac{\sum_{i=1}^{2} \Pr(S_T = j, S_{T-1} = i, |\psi_T) P_{T|T-1}^{(i,j)}}{\Pr(S_T = j|\psi_T)}, \quad P_{T|T-1} = \sum_{j=1}^{2} \Pr(S_T = j|\psi_T) P_{T|T-1}^{j},$$

$$f_{T|T-1}^{j} = \frac{\sum_{i=1}^{2} \Pr(S_T = j, S_{T-1} = i, |\psi_T) f_{T|T-1}^{(i,j)}}{\Pr(S_T = j|\psi_T)}, \quad f_{T|T-1} = \sum_{j=1}^{2} \Pr(S_T = j|\psi_T) f_{T|T-1}^{j}.$$

Appendix B3

Figure B3.1 Growth rates of the MS common factors, Δc_t



Note: the horizontal line in each figure marks the threshold value of 0.5.

Appendix C3

Table C3.1: Bai and Perron's multiple structural changes test

	Italy	The Netherlands		Belgium		Spain		
$\mathrm{Sup}F_T(1)$	3.748	29.611**		15.173**		66.815**		
$\mathrm{Sup}F_T(2)$	17.655**	44.037**		14.545**		87.494**		
$\mathrm{Sup}F_T(3)$	15.016**	43.548**		23.146**		60.096**		
$\mathrm{Sup}F_T(4)$	12.894**	39.403**		18.565**		52.899**		
$\mathrm{Sup}F_T(5)$	23.858**	31.878**		22.635**		65.126**		
$UD\max$	23.858**	44.037**		23.146**		87.494**		
$WD\max(5\%)$	34.343*	69.953*		38.358*		110.363*		
$WD\max(1\%)$	41.273**	79.793**		45.411**		130.654**		
$\mathrm{Sup}F(2\|1)$	7.469	61.330**		17.599**		13.871**		
$\mathrm{Sup}F(3\|2)$	11.375*	13.473*		9.825		44.641**		
$\mathrm{Sup}F(4\|3)$	24.509**	28.461**		9.094		22.883**		
Number of breaks selected								
BIC	5	2		5		5		
LWZ	0	2		0		3		
Sequential	0	4		2		4		
Break dates and confidence intervals								
	BIC	BIC	Seq.	BIC	Seq.	BIC	LWZ	Seq.
$\hat{T}_1$	73:2	83:2	83:2	74:2	74:2	85:2	85:2	85:2
	(72:4-73:3)	*(82:4-86:1)*	*(83:1-85:3)*	*(71:1-75:3)*	*(72:3-75:1)*	*(84:4-85:4)*	*(84:4-85:4)*	*(84:4-85:4)*
$\hat{T}_2$	73:4	01:1	90:1	80:1	83:3	90:4	90:4	90:4
	(73:3-74:1)	*(00:2-02:2)*	*(89:1-91:3)*	*(78:4-84:2)*	*(81:4-88:4)*	*(89:4-91:1)*	*(89:4-91:1)*	*(89:2-91:2)*
$\hat{T}_3$	74:3		95:2	83:3		94:1	94:1	96:4
	(74:2-74:4)		*(94:3-96:2)*	*(83:1-84:1)*		*(93:4-96:1)*	*(93:4-95:2)*	*(96:3-98:1)*
$\hat{T}_4$	75:2		01:1	90:2		97:2		00:4
	(74:4-75:3)		*(00:3-01:2)*	*(89:3-91:1)*		*(97:1-97:4)*		*(00:2-01:3)*
$\hat{T}_5$	80:2			93:4		00:4		
	(78:4-84:4)			*(92:2-94:4)*		*(00:2-01:2)*		

Note: three criteria used to estimate the number of breaks are the Bayesian Information criterion (BIC) suggested by Yao (1988), a modified Schwarz criterion (LWZ) proposed by Liu et al (1997), and the sequential procedure proposed by Bai and Perron. The latter is based on the sequential application of the $SupF(\ell|\ell+1)$ test using the subsequent estimates of the breaks.

Appendix D3

Table D3.1: Correlation Coefficients of Smoothed Recession Probabilities

Whole Sample Period

	EMU	GER	FRA	ITA	AUS	BEL	NETH	SPA	FIN	UK	US
EMU											
GER	0.94										
FRA	0.52	0.36									
ITA	0.56	0.46	0.43								
AUS	0.81	0.77	0.20	0.34							
BEL	0.83	0.73	0.56	0.46	0.63						
NETH	0.73	0.74	0.16	0.42	0.61	0.66					
SPA	0.42	0.25	0.41	0.15	0.27	0.39	0.11				
FIN	0.19	0.14	0.16	0.15	-0.06	0.28	0.20	0.35			
UK	0.15	0.28	0.16	-0.04	0.16	0.22	0.19	0.37	0.47		
US	0.26	0.37	-0.06	0.02	0.46	0.38	0.28	0.31	0.04	0.48	
CANA	0.28	0.23	0.00	0.16	0.36	0.42	0.29	0.24	0.23	0.19	0.61

Pre-1991Q1

		EMU	GER	FRA	ITA	AUS	BEL	NETH	SPA	FIN	UK	US	CANA
Post-1991Q1	EMU		0.96	0.51	0.46	0.94	0.90	0.80	0.59	-0.01	0.29	0.37	0.45
	GER	0.94		0.44	0.34	0.87	0.76	0.66	0.36	0.02	0.45	0.59	0.41
	FRA	0.58	0.44		0.45	0.41	0.55	0.20	0.30	-0.03	0.20	-0.13	-0.08
	ITA	0.66	0.57	0.57		0.45	0.43	0.31	0.34	0.11	0.14	0.13	0.34
	AUS	0.67	0.65	-0.12	0.26		0.85	0.80	0.47	0.00	0.27	0.48	0.47
	BEL	0.74	0.69	0.68	0.47	0.28		0.69	0.39	0.00	0.24	0.52	0.55
	NETH	0.71	0.73	0.31	0.51	0.53	0.48		0.26	0.01	0.32	0.47	0.47
	SPA	0.32	0.33	0.42	0.17	-0.16	0.66	0.32		0.02	0.16	0.24	0.10
	FIN	0.33	0.30	0.40	0.18	-0.14	0.66	0.37	0.96		0.39	0.06	0.22
	UK	-0.04	0.02	-0.03	-0.17	-0.16	0.34	0.19	0.71	0.78		0.48	0.07
	US	0.15	0.08	-0.15	-0.08	0.40	0.23	0.13	0.05	0.08	0.22		0.60
	CANA	-0.05	-0.11	-0.02	-0.13	-0.02	0.19	0.07	0.37	0.41	0.61	0.57	

Appendix E3

As illustrated in section 3.3, significant variation in the magnitude of recessionary and expansionary phases over the sample period means that the baseline model fails to produce reasonable parameter estimates and smoothed recession probabilities. One solution to this problem, used in Chapter 3, is to introduce dummy variables into the intercepts of the MSDF model in order to reduce the impact that large business cycle phases have on the model's parameter estimates. Applying this approach improves the log-likelihood values, parameter estimates and smoothed recession probabilities and so produces more reasonable turning points for Belgium, Italy, the Netherlands and Spain.

An alternative approach considered in this section is to introduce a latent state variable into the intercepts of the MSDF model. This approach is broadly in line with Kim and Nelson (1999a) and Mills and Wang (2003a), who introduced an additional latent state into the mean and residual variance of the Hamilton model to detect the unknown dates of business cycle moderation. To reduce the number of states at each stage of the KF iteration, we allow D_t in equations (3.14) and (3.15) to evolve independently of their past values. As such, eight states appear at each stage of the KF iteration. The recursive equations are as follows

$$\beta_{t|t-1}^{(i,j,k=0)} = \tilde{\mu}_j + F\beta_{t-1|t-1}^{i}, \qquad P_{t|t-1}^{(i,j,k=0)} = FP_{t-1|t-1}^{i}F^{\top} + Q,$$

$$\beta_{t|t-1}^{(i,j,k=1)} = \tilde{\mu}_j + \mu_{jj} + F\beta_{t-1|t-1}^{i}, \qquad P_{t|t-1}^{(i,j,k=1)} = FP_{t-1|t-1}^{i}F^{\top} + Q,$$

$$\eta_{t|t-1}^{(i,j,k=0)} = y_t - H\beta_{t|t-1}^{(i,j,k=0)}, \qquad f_{t|t-1}^{(i,j,k=0)} = HP_{t|t-1}^{(i,j,k=0)}H^{\top},$$

$$\eta_{t|t-1}^{(i,j,k=1)} = y_t - H\beta_{t|t-1}^{(i,j,k=1)}, \qquad f_{t|t-1}^{(i,j,k=1)} = HP_{t|t-1}^{(i,j,k=1)}H^{\top},$$

$$\beta_{t|t}^{(i,j,k=0)} = \beta_{t|t-1}^{(i,j,k=0)} - P_{t|t-1}^{(i,j,k=0)}H^{\top}\left[f_{t|t-1}^{(i,j,k=0)}\right]^{-1}\eta_{t|t-1}^{(i,j,k=0)},$$

$$\beta_{t|t}^{(i,j,k=1)} = \beta_{t|t-1}^{(i,j,k=1)} - P_{t|t-1}^{(i,j,k=1)}H^{\top}\left[f_{t|t-1}^{(i,j,k=1)}\right]^{-1}\eta_{t|t-1}^{(i,j,k=1)},$$

$$P_{t|t}^{(i,j,k=0)} = \left(I - P_{t|t-1}^{(i,j,k=0)}H^{\top}\left[f_{t|t-1}^{(i,j,k=0)}\right]^{-1}H\right)P_{t|t-1}^{(i,j,k=0)},$$

$$P_{t|t}^{(i,j,k=1)} = \left(I - P_{t|t-1}^{(i,j,k=1)}H^{\top}\left[f_{t|t-1}^{(i,j,k=1)}\right]^{-1}H\right)P_{t|t-1}^{(i,j,k=1)}, \text{ where } i,j = 1,2.$$

The Hamilton filtering algorithm that calculates the probabilities $\Pr(S_t = j|\psi_t)$ and $\Pr(S_t = j, S_{t-1} = i,|\psi_t)$ can then be modified as follows

$$\Pr\left(S_t = j, S_{t-1} = i, D_t = k|\psi_{t-1}\right) = \Pr\left(S_t = j|S_{t-1} = i\right)\Pr\left(S_{t-1} = i|\psi_{t-1}\right)\Pr\left(D_t = k|\psi_{t-1}\right),$$

$$f\left(y_t|\psi_{t-1}\right) = \sum_{j=1}^{2}\sum_{i=1}^{2}\sum_{k=1}^{2} f\left(y_t|S_t = j, S_{t-1} = i, D_t = k, \psi_{t-1}\right)\Pr(S_t = j, S_{t-1} = i, D_t = k|\psi_{t-1}),$$

$$\Pr(S_t = j, S_{t-1} = i, D_t = k|\psi_t) = \frac{f\left(y_t|S_t = j, S_{t-1} = i, D_t = k, \psi_{t-1}\right)\Pr\left(S_t = j, S_{t-1} = i, D_t = k|\psi_{t-1}\right)}{f\left(y_t|\psi_{t-1}\right)},$$

$$\Pr(S_t = j|\psi_t) = \sum_{k=1}^{2}\sum_{i=1}^{2}\Pr(S_t = j, S_{t-1} = i, D_t = k|\psi_t),$$

where $\Pr\left(D_t = k|\psi_{t-1}\right) = \Pr\left(D_t = k\right)$ and $\sum_{k=1}^{2}\Pr\left(D_t = k\right) = 1$.

Since the MSDF model is a combination of the KF and the Hamilton filter, appropriate approximations are needed at each iteration to make the above KF operable. However, unlike Kim's (1994) filter, illustrated in Appendix A3.2, where the KF produces a 2-fold increase in the number of $\beta_{t|t}^{(i,j)}$ and $P_{t|t}^{(i,j)}$ at the end of each iteration, the KF presented here gives a 4-fold increase in the number of $\beta_{t|t}^{(i,j,k)}$ and $P_{t|t}^{(i,j,k)}$.

$$\beta_{t|t}^{j} = \frac{\sum_{k=1}^{2}\sum_{i=1}^{2}\Pr(S_t = j, S_{t-1} = i, D_t = k|\psi_t)\beta_{t|t}^{(i,j,k)}}{\Pr(S_t = j|\psi_t)},$$

$$P_{t|t}^{j} = \frac{\sum_{k=1}^{2}\sum_{i=1}^{2}\Pr(S_t = j, S_{t-1} = i, D_t = k|\psi_t)\left\{P_{t|t}^{(i,j,k)} + \left(\beta_{t|t}^{j} - \beta_{t|t}^{(i,j,k)}\right)\left(\beta_{t|t}^{j} - \beta_{t|t}^{(i,j,k)}\right)^{\mathrm{T}}\right\}}{\Pr(S_t = j|\psi_t)}.$$

However, reducing the $2 \times 2 \times 2$ posteriors ($\beta_{t|t}^{(i,j,k)}$ and $P_{t|t}^{(i,j,k)}$) down to 2 posteriors ($\beta_{t|t}^{j}$ and $P_{t|t}^{j}$) requires a weighted average over $\beta_{t|t}^{(i,j,k=0)}$ and $\beta_{t|t}^{(i,j,k=1)}$, and $P_{t|t}^{(i,j,k=0)}$ and $P_{t|t}^{(i,j,k=1)}$. This may cause problems in the subsequent iterations when large variations are found between severe and normal business cycle phases. This leads to significant increases in volatility of the common factor as both $\tilde{\mu}_j$ and μ_{jj} are larger than those obtained using the dummy variable approach. Due to this problem, this approach fails to produce reasonable parameter estimates and smoothed recession probabilities for Belgium and the Netherlands. Although it gives acceptable results for Italy and Spain, the log-likelihood values in Tables E3.1 and E3.2 are lower than those in Tables 3.4(b) and 3.8(b). The dummy variable approach is thus preferred and used in Chapter 3.

Table E3.1: Parameter estimates of MSDF for Italy

Common Factor										
ϕ_1	ϕ_2	u_0		u_1	p_{00}	p_{11}		u_{00}	u_{11}	p_d
0.693**	-0.120*	-2.427*		0.756	0.542**	0.816**		-12.736*	7.646*	0.041*
(0.152)	(0.053)	(0.970)		(0.819)	(0.117)	(0.116)		(5.117)	(3.401)	(0.022)
Idiosyncratic Components										
Δ **GDP**	γ_1	-	-	-	-	ψ_{11}	ψ_{12}	-	α_{11}	σ_1^2
	0.237**					-0.231	0.013		0.439**	0.236**
	(0.085)					(0.138)	(0.016)		(0.120)	(0.062)
Δ **GFCF**	γ_2	-	-	-	-	ψ_{21}	ψ_{22}	-	α_{12}	σ_2^2
	0.142**					-0.054	0.010		0.102	0.712**
	(0.052)					(0.092)	(0.088)		(0.098)	(0.053)
Δ **IP**	γ_3	-	-	-	-	ψ_{31}	ψ_{32}	-	α_{13}	σ_3^2
	0.188**					-0.512**	-0.055		0.483**	0.303**
	(0.071)					(0.115)	(0.108)		(0.099)	(0.055)
Δ **EMP**	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D92q4	ψ_{41}	ψ_{42}	ψ_{43}	α_{14}	σ_4^2
	0.075	-0.060	0.068	-0.020	-4.416**	-0.065	-0.212*	0.198*	-0.134	0.653**
	(0.043)	(0.053)	(0.061)	(0.041)	(0.850)	(0.095)	(0.087)	(0.093)	(0.126)	(0.050)

Long run growth rate: $\delta = 1.327$

Error correction term

$$\mathrm{GDP}_{t-1} = \underset{(0.105)}{2.645} + \underset{(0.086)}{0.0911} \times \mathrm{GFCF}_{t-1} + \underset{(0.243)}{1.1424} \times \mathrm{IP}_{t-1} + 0.521 \times \mathrm{EMP}_{t-1}$$

Log-likelihood: -672.199

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	1.771	1.061
Δ GFCF	3.342	3.379
Δ IP	7.779	17.412**
Δ EMP	0.357	6.596

Figure E3.1: Italy

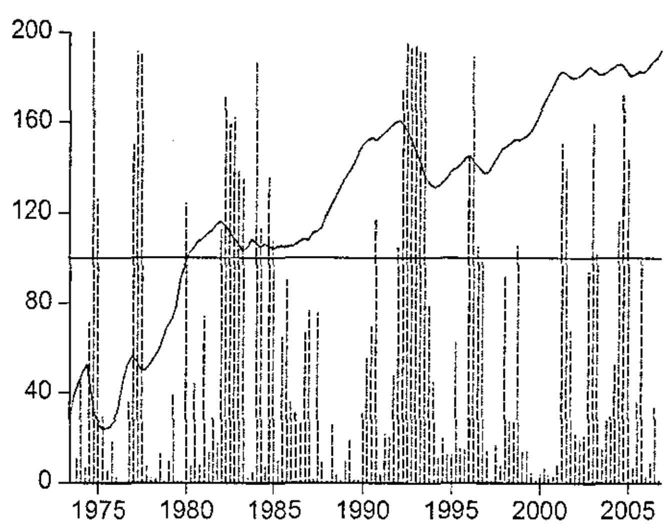


Table E3.2: Parameter estimates of MSDF for Spain

Common Factor

ϕ_1	ϕ_2	u_0	u_1	p_{00}	p_{11}	u_{00}	p_d
0. 291*	0.428**	-0.614*	0.982**	0.970**	0.976**	-6.042**	0.101*
(0.112)	(0.098)	(0.291)	(0.248)	(0.024)	(0.018)	(1.292)	(0.043)

Idiosyncratic Components

Δ GDP	γ_1	-	-	-	-	ψ_{11}	ψ_{12}	ψ_{14}	α_{11}	α_{21}	σ_1^2
	0.254**					-0.460**	-0.118	-0.226*	-0.255	0.845*	0.375**
	(0.051)					(0.110)	(0.107)	(0.086)	(0.301)	(0.302)	(0.048)
Δ GFCF	γ_2	-	-	-	-	-	ψ_{21}	ψ_{22}	α_{12}	α_{22}	σ_2^2
	0.307**						0.085	-0.002	0.088	0.466	0.320**
	(0.061)						(0.119)	(0.005)	(0.373)	(0.382)	(0.048)
Δ IP	γ_3	-	-	-	-	-	ψ_{31}	ψ_{32}	α_{13}	α_{23}	σ_3^2
	0.247**						-0.248*	0.208*	0.928**	-0.090	0.526**
	(0.051)						(0.102)	(0.100)	(0.357)	(0.345)	(0.049)
Δ EMP	γ_{40}	γ_{41}	γ_{42}	γ_{43}	D76q1	D76q3	ψ_{41}	ψ_{42}	α_{14}	α_{24}	σ_4^2
	0.095**	0.090**	0.029	0.053	2.764**	3.779**	0.245*	0.185	-0.242	0.409	0.166**
	(0.030)	(0.031)	(0.031)	(0.029)	(0.407)	(0.412)	(0.098)	(0.094)	(0.320)	(0.329)	(0.027)

Long run growth rate: $\delta = 1.853$

Error correction terms

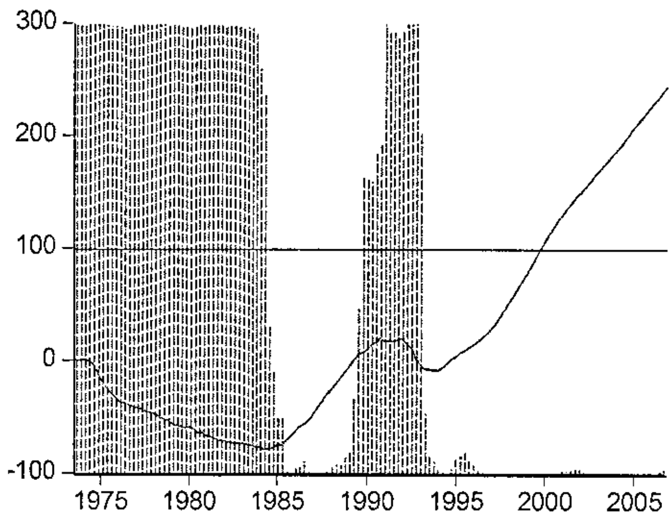
$\mathrm{GDP}_{t-1} = 6.403 + \underset{(0.167)}{2.054} \times \mathrm{IP}_{t-1} - \underset{(0.179)}{0.121} \times \mathrm{EMP}_{t-1}$ $\quad \mathrm{GFCF}_{t-1} = -1.157 + \underset{(0.473)}{3.098} \times \mathrm{IP}_{t-1} - \underset{(0.507)}{0.027} \times \mathrm{EMP}_{t-1}$

Log-likelihood: -525.945

Diagnostics	**Q(4)**	**Jarque-Bera**
Δ GDP	4.769	7.538*
Δ GFCF	0.337	0.792
Δ IP	3.216	0.643
Δ EMP	3.209	21.450**

Note: the latent state variable is only included in the recession intercept to capture the severe downturns during the ERM period.

Figure E3.2: Spain



Chapter 4 - Evaluating Growth Cycle Synchronisation in the EU

4.1 Introduction

Chapters 2 and 3 assessed the synchronisation of business cycle turning points identified from multivariate coincident economic time series using parametric and non-parametric approaches. Although more synchronised business cycles are found between the aggregate euro area and the core EMU countries than the peripheral and non-EMU countries, this may reflect the large weights assigned to the core countries when constructing the aggregate euro area data. Overall, no common increase in cycle correlations is found between the aggregate euro area and EMU countries over two subsamples: pre-1991Q1 and post-1991Q1. Furthermore, the unbalanced growth observed among EMU member states, even after the establishment of EMU, may reduce the appropriateness of having a common monetary policy for all countries.

Given this concern, the research in this chapter is undertaken to identify and analyse the growth cycles of seven major European countries, Austria, Belgium, France, Germany, Italy, the Netherlands and Spain, during the period of 1980Q1 to 2007Q3. These countries were previously members of the European Monetary System (EMS) and have been members of EMU since 1 January 1999. There has been an ongoing debate as to whether these countries, and the other members of EMU, have actually benefitted from adopting a common monetary policy. In macroeconomics, the growth cycle is consistent with the output gap, which is associated with inflationary pressures. As similar inflation rates are one important criterion for ensuring the optimality of EMU as defined in OCA theory (Fleming, 1962; Haberler, 1970), measuring the degree of growth cycle synchronisation is a relevant issue in analysing the optimality of EMU and its common monetary policy.

The endogenous OCA theory, supported by Frankel and Rose (1998) and the European Commission (1990), argues that the operation of a monetary union in itself would generate greater synchronisation. Given this assertion, this chapter examines the presence of a unique common cycle among the seven member countries mentioned above, given the operation of EMU and the quasi-union of the Exchange Rate Mechanism (ERM) of the EMS over the past thirty years.

European growth cycle synchronisation is generally evaluated by analysing bilateral correlations between estimated cycles. Univariate trend-cycle decomposition methodologies, such as the Hodrick-Prescott and band-pass filters, are widely used to extract the cyclical component from industrial production indices, real GDP or its components. There is still no consensus on whether fixed exchange rate regimes or a monetary union generate synchronised cyclical fluctuations. Artis and Zhang (1997) and Darvas and Szapáry (2004) find evidence of greater growth cycle synchronisation after countries joined a currency arrangement or a monetary union. However, Inklaar and de Haan (2001) contradict this assertion as they identify periods of convergence and divergence during the ERM period. Moreover, Camacho *et al.* (2006) conclude that the establishment of the EMU has not significantly increased synchronisation across the euro area, and that the synchronisation among member countries occurred prior to the introduction of the euro. These conflicting results are due to the use of different data, decomposition methodologies and measures of synchronisation.

More recent studies have tended to assess the degree of growth cycle synchronisation within a multivariate setting. Harvey and Carvalho (2005) decompose real GDP per capita for five core euro area countries into their trend and cycle components simultaneously by using the seemingly unrelated time-series equations (SUTSE) model, in which all cycle components are restricted to have the same damping factor and frequency. The level of cycle correlation is measured by the cross-correlations between estimated cycle components. Luginbuhl and Koopman (2004) and Koopman and Azevedo (2008) introduce various time-varying mechanisms to the SUTSE models to account for gradual changes in cycle correlations between euro area countries. Sinclair and Mitra (2008) further apply the unobserved component (UC) model, which allows for non-zero correlations between the trend and cycle components, to analyse cross-country relationships between members of the G7. An alternative approach used to assess growth cycle synchronisation in a multi-county analysis is to test for the presence of common and codependent cycles using vector autoregressive (VAR) models with cointegration. Since this methodology is based on a VAR representation, it has the advantage of allowing dynamic interactions between variables to be modelled. It is well known that cointegration between a set of I(1) variables indicates the presence of common trends (Johansen, 1988; Stock and Watson, 1988b). More recent studies, including Engle and Kozicki (1993), Vahid and Engle (1993, 1997), Hecq *et al.* (2000, 2006) and Schleicher (2007), propose test statistics to determine the number of common and codependent cycles among a set of stationary time-series. It is believed that, if the common feature restrictions are imposed correctly, estimation efficiency and the forecasting ability of a model will improve. Once appropriate models have been constructed, the Beveridge-Nelson (BN) decomposition methodology with common trend and cycle restrictions imposed, as proposed by Proietti (1997) and Hecq *et al.* (2000), can be used to calculate the trend and cyclical components for each of the variables simultaneously. This approach has been used by Vahid and Engle (1993) and Carlino and Sill (2001) to identify the number of common trends and cycles in real GDP per capita among the US states. Hecq (2005) adapts this model to analyse GDP series of Latin American countries. Beine *et al.* (2000) also apply this approach to assess the optimality of a monetary union consisting of five core European countries (i.e., Germany, Belgium, the Netherlands, Austria and France) and a restricted monetary union composed of Germany, Belgium and the Netherlands using monthly industrial production indices between 1975M1 to 1997M4. Their results indicate that even the restricted monetary union could face a stabilisation cost as one common cycle emerges with a delay of adjustment of five months.

In this chapter, both univariate and multivariate trend-cycle decomposition methodologies are applied. The BN decomposition and the unobserved component model proposed by Harvey and Trimbur (2003) are used to extract the cyclical component from individual real GDP series. Correlation coefficients between estimated cycles are then calculated to examine the degree of cycle comovement. More importantly, the common cyclical features of seven national GDP series are analysed using VAR models with cointegration. The multivariate BN decomposition incorporating trend and cycle restrictions, which has not previously been utilised in this context, is also used to provide a detailed investigation of the trend and cyclical movements in the GDP series.

The rest of the chapter is organised as follows. A brief introduction to the univariate BN decomposition and the unobserved component model proposed in Harvey and Trimbur (2003) are given in section 4.2. Section 4.3 outlines the common trend and cycle approach within the VAR framework. The trend-cycle decomposition methodology developed in Proietti (1997) and Hecq at al. (2000) is then presented in section 4.4. In section 4.5 the empirical results obtained from the multivariate approach are discussed. Section 4.6 concludes.

4.2 Beveridge-Nelson and Unobserved-component Decompositions

Two model-based trend-cycle decomposition methodologies are used to extract the cyclical components from real GDP data for the aggregate euro area and the seven EMU member states during the period 1980Q1 to 2007Q3.[1] All series are seasonally adjusted observations expressed in logarithms. The first approach used in this section was proposed by Beveridge and Nelson (1981), who demonstrated that any ARIMA $(p,1,q)$ process can be decomposed into a unique stochastic trend plus a transitory component,

$$x_t = \tau_t + c_t, \quad t = 1,...,T \tag{4.1}$$

where the trend component is defined as the infinite forecast of the series x_t, adjusted for its mean growth rate,

$$\tau_t = x_t + \lim_{k\to\infty} \sum_{i=1}^{k} \left[\Delta\widetilde{x}_{t+i|t} - E(\Delta x_t)\right] \tag{4.2}$$

where $\Delta\widetilde{x}_{t+i|t}$ is the i-step ahead linear predictor of Δx_t based on information at time t. The cycle component of $\{x_t\}$ is the difference between the trend and the value of x_t:

$$c_t = -\lim_{k\to\infty} \sum_{i=1}^{k} \left[\Delta\widetilde{x}_{t+i|t} - E(\Delta x_t)\right] \tag{4.3}$$

[1] The real GDP data are taken from the OECD Quarterly National Account database.

The BN trend has the structure of a random walk with drift, and the BN cycle is simply the deviation from the trend. The innovations of the BN trend and cyclical components are perfectly correlated.

The second approach was introduced by Harvey and Trimbur (2003). It is based on an unobserved component model consisting of stochastic trend, cycle and irregular components. This approach is closely linked to the use of Butterworth filters and can produce smoother trend and cycle components than conventional structural time-series models, such as the BN decomposition and the model introduced by Harvey and Jaeger (1993). As presented in Harvey and Trimbur (2003), this model is specified as

$$x_t = \tau_{m,t} + c_{n,t} + \varepsilon_t, \quad t = 1,\ldots,T \tag{4.4}$$

where $\tau_{m,t}$ denotes the m-th order stochastic trend, defined as

$$\begin{aligned} \Delta\tau_{m,t} &= \tau_{m-1,t} + \eta_t, \quad \eta_t \sim \mathrm{NID}\left(0, \sigma_\eta^2\right), \\ \Delta^{m-1}\tau_{m-1,t} &= \xi_t, \quad \xi_t \sim \mathrm{NID}\left(0, \sigma_\xi^2\right), \end{aligned} \tag{4.5}$$

for $m = 2,3\ldots$, where $\Delta^{m-1} = (1-L)^{m-1}$. The trend component with $\sigma_\eta^2 = 0$ and $m = 2$ produces a smooth trend. Moreover, when $m > 2$, the estimated trend component can be compared with a Butterworth filter. For most economic time series, m is usually set to be two or three. Higher values of m will give a nonlinear forecast function and the estimated trend may become more responsive to shorter-term movements in the filter series (Harvey and Trimbur, 2003, p.12).

An n-th order stochastic cyclical component is given by

$$\begin{bmatrix} c_{1,t} \\ c_{1,t}^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} c_{1,t-1} \\ c_{1,t-1}^* \end{bmatrix} + \begin{bmatrix} k_t \\ 0 \end{bmatrix} \tag{4.6}$$

and

$$\begin{bmatrix} c_{n,t} \\ c^*_{n,t} \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} c_{n,t-1} \\ c^*_{n,t-1} \end{bmatrix} + \begin{bmatrix} c_{n-1,t} \\ 0 \end{bmatrix}, \text{ for } n = 2,3\ldots, \tag{4.7}$$

where $k_t \sim \text{NID}(0, \sigma_k^2)$. The parameter ρ is the damping factor and λ_c is the frequency, which satisfy $0 < \rho \leq 1$ and $0 < \lambda_c \leq \pi$. A higher order for n leads to more concentration on a particular frequency band, and thus results in smoother cycle components than when $n = 1$. Finally, it is worth noting that innovations to the trend and cycle, denoted η_t, ξ_t and k_t, are assumed to be serially and mutually uncorrelated in the unobserved component model. This assumption is in contrast to the BN decomposition framework, where the innovations of the BN trend and cyclical components are perfectly correlated.

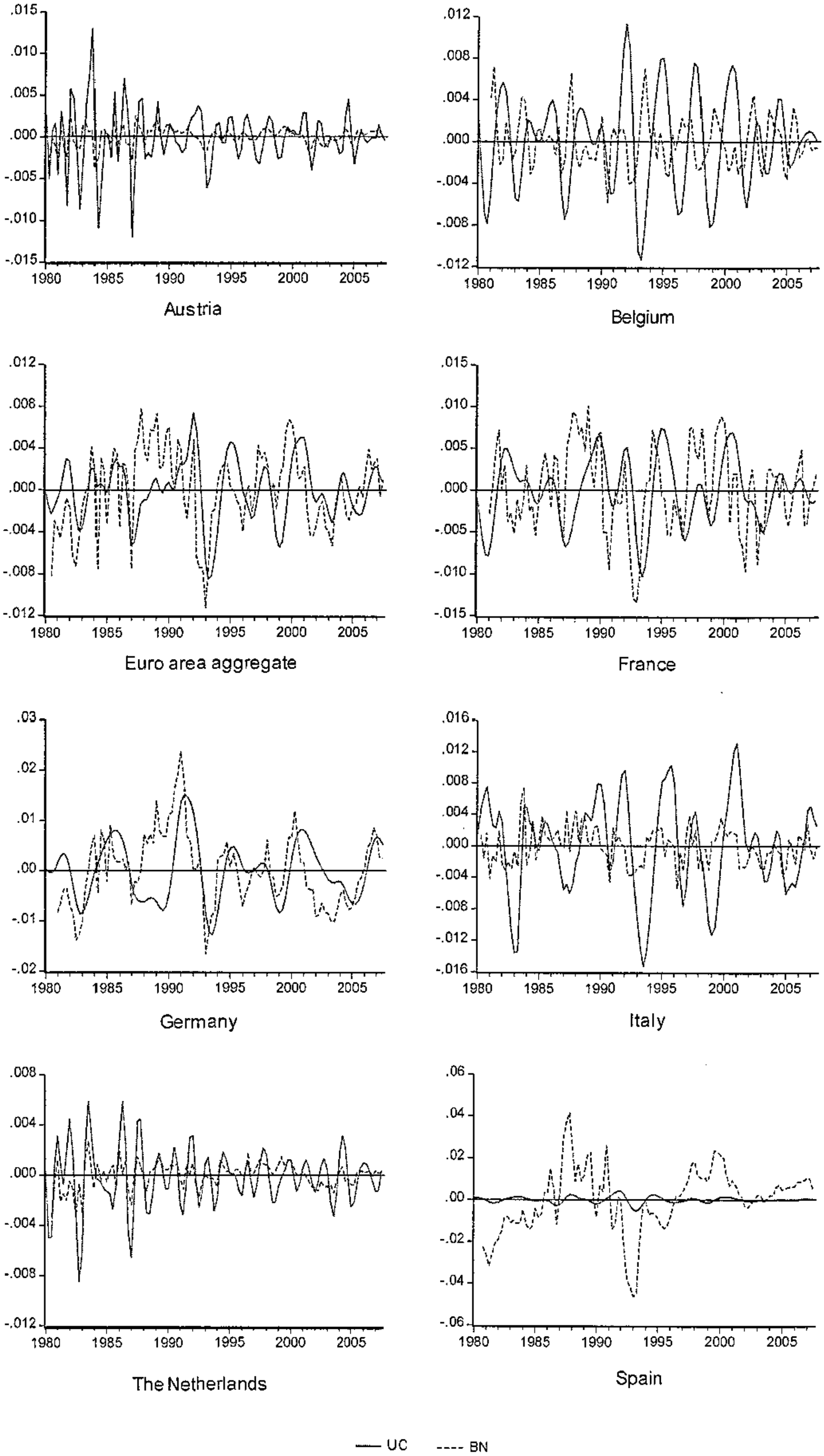
Four different models for each country are estimated and reported in Table A4.2, Appendix A4. σ_η^2 is set to zero for all models and the cycle period, $2\pi / \lambda_c$, is restricted to 20 quarters when λ_c is estimated to be near zero. The model which yields the smallest standard error and the largest log-likelihood value is preferred and highlighted in bold in Table A4.2. The model consisting of a smoothed second-order trend, a generalised cycle of order two and an irregular component is preferred for most countries. The exceptions are France and Spain for which a generalised cycle of order three is preferred.

Parameter estimates of the two methodologies are presented in Appendix A4. The estimated cyclical components are plotted in Figure 4.1. In general, the cycles obtained from the BN decomposition appear to be noisy, while the cycles estimated from Harvey and Trimbur's approach are smooth and highly persistent. In addition, as shown in Table 4.1, cycle correlations calculated from the BN cycles are, on average, smaller than the corresponding values calculated using the cycles extracted from Harvey and Trimbur's approach, especially for Belgium. The average cycle correlation between the Belgian BN cycle and the other BN cycles is 0.04, compared to the corresponding value of 0.51 calculated using cycles extracted from the unobserved component model.

Table 4.1: Cross-Correlations

The BN cycles							
	EURO	AUS	BEL	FRA	GER	ITA	NETH
EURO	1.00						
AUS	0.48	1.00					
BEL	0.06	0.17	1.00				
FRA	0.69	0.29	0.12	1.00			
GER	0.65	0.30	-0.25	0.25	1.00		
ITA	0.61	0.30	0.11	0.50	0.40	1.00	
NETH	0.48	0.35	0.12	0.26	0.36	0.29	1.00
SPA	0.66	0.32	-0.07	0.50	0.36	0.33	0.37
The UC cycles							
	EURO	AUS	BEL	FRA	GER	ITA	NETH
EURO	1.00						
AUS	0.32	1.00					
BEL	0.79	0.26	1.00				
FRA	0.71	0.21	0.61	1.00			
GER	0.76	0.11	0.43	0.29	1.00		
ITA	0.83	0.19	0.59	0.55	0.64	1.00	
NETH	0.32	0.50	0.29	0.12	0.12	0.21	1.00
SPA	0.61	0.24	0.58	0.40	0.45	0.41	0.11

Figure 4.1: BN and UC Cycles



4.3 VAR Representation with Common Trend and Cycle Restrictions

This section generalises the univariate decomposition approach to a multivariate setting. The multivariate BN decomposition with common trend and cycle restrictions is used to extract the trend and cycle components simultaneously from each output variable. The advantage of this approach is that it allows for dynamic interactions between variables and the identification of innovation sources (Lippi and Reichlin, 1993; Quah, 2002). This multivariate approach is based on a VAR representation.[2] Thus, consider a p-th order VAR with N elements in X_t:

$$X_t = m + \sum_{i=1}^{p} \Pi_i X_{t-i} + \varepsilon_t,\ \ \varepsilon_t \sim iid(0,\Sigma) \tag{4.8}$$

where $\Pi_i, i = 1,...,p$, are matrices of lag coefficients. Since some variables in X_t are I(1), the roots of $|\Pi(L)| = 0$ are either on or outside the unit circle, where $\Pi(L) = I_N - \sum_{i=1}^{p} \Pi_i L^i$. As demonstrated in Johansen (1995), the VAR in equation (4.8) can be rewritten as an (unrestricted) vector error correction model (VECM) by decomposing $\Pi(L)$ into $\Pi(1)L + \Gamma(L)(1-L)$, where $\Gamma(L) = I_N - \sum_{i=1}^{p-1} \Gamma_i L^i$, $\Gamma_i = -\sum_{j=i+1}^{p} \Pi_j$, for $i = 1,...,p-1$, to obtain

$$\Delta X_t = m - \Pi(1) X_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t \tag{4.9}$$

In equation (4.9), the long-run matrix, $\Pi(1)$, can be factored as $-\alpha\beta^{\mathsf{T}}$ if $\Pi(1)$ has rank $r < N$. In this case the matrix β contains r cointegrating vectors and α is the matrix of corresponding adjustment coefficients. There are thus r linear combinations of the variables in X_t that yield stationary series and X_t is said to have $k = N - r$ common

[2] The literature, which includes King et al. (1991), Vahid and Engle (1993, 1997), Proietti (1997), Hecq et al. (2000, 2006) and Schleicher (2007), provides various illustrations of the common trend and cycle assumptions imposed on the VAR models.

trends. It is therefore clear that the VECM imposes common trend restrictions on the VAR if $\Pi(1)$ has reduced rank.

4.3.1 Common Cycles

As with common trends, the presence of common cycles imposes additional restrictions on the VAR (Vahid and Engle, 1993). In this case, linear combinations of the first differences ΔX_t should remove all the serial correlation in these first differences. To illustrate this, premultiply both sides of equation (4.9) by an $s \times N$ cofeature matrix, denoted $\phi^{\mathsf{T}} = \begin{bmatrix} I_s & \phi^{*\mathsf{T}}_{s\times(N-s)} \end{bmatrix}$, to yield an s-dimensional vector of white noise,

$$\phi^{\mathsf{T}} \Delta X_t = \phi^{\mathsf{T}} \varepsilon_t \tag{4.10}$$

Adding the remaining $N-s$ reduced form VECM equations to (4.10) gives the 'pseudo-structural' model,

$$\begin{bmatrix} I_s & \phi^{*\mathsf{T}}_{s\times(N-s)} \\ 0_{(N-s)\times s} & I_{N-s} \end{bmatrix} \Delta X_t = \begin{bmatrix} 0_{s\times N} & \cdots & 0_{s\times N} & 0_{s\times r} \\ \Gamma_1^* & \cdots & \Gamma_p^* & \alpha^* \end{bmatrix} \begin{bmatrix} \Delta X_{t-1} \\ \vdots \\ \Delta X_{t-p} \\ \beta^{\mathsf{T}} X_{t-1} \end{bmatrix} + v_t \tag{4.11}$$

where the error term is

$$v_t = \begin{bmatrix} I_s & \phi^{*\mathsf{T}}_{s\times(N-s)} \\ 0_{(N-s)\times s} & I_{N-s} \end{bmatrix} u_t$$

It is clear that common cycle restrictions require $\phi^{\mathsf{T}}\Pi(1) = 0$ and $\phi^{\mathsf{T}}\Gamma_j = 0$, so that $\phi^{\mathsf{T}}\Delta X_t$ is independent of ε_{t-j}. Compared to the (unrestricted) VECM model, the restricted model in equation (4.11) eliminates $s(Np+r) - s(N-s)$ additional parameters and is parsimoniously nested in the VECM.

It is worth noting that the number of cofeature combinations, s, is constrained by the dimension of the VECM and the presence of cointegrating vectors. The maximum value of s is $N-r$. The test statistic proposed in Tiao and Tsay (1989) can be applied to determine the actual number of cofeature combinations,

$$C_S(s) = -(T-p-1)\sum_{j=1}^{s}\ln\left(1-\ell_j^2\right),\ s=1,\dots,n-r \tag{4.12}$$

where the ℓ_j^2, for $j=1,\dots,s$, are the s smallest estimated squared canonical correlations between ΔX_t and $W_{t-1}=\left\{\Delta X_{t-1}^{\top},\dots,\Delta X_{t-p}^{\top},X_{t-1}^{\top}\beta\right\}^{\top}$.[3] Under the null, this statistic has an asymptotic χ^2 distribution with $s(Np+r)-s(N-s)$ degrees of freedom, where p is the lag order of the VECM.

The common cycle framework outlined above corresponds to the definition of strong form reduced rank structure (SF) introduced in Hecq *et al.* (2000, 2006). In addition, they also introduce two further common cycle models; the weak and mixed form reduced rank structures (WF and MF). Under WF, there exists a cofeature matrix $\tilde{\phi}^{\top}=\left(I_s \quad \tilde{\phi}_{s\times(N-s)}^{*\top}\right)$, where $\tilde{\phi}^{\top}(\Delta X_t-\alpha\beta^{\top}y_{t-1})=\tilde{\phi}^{\top}\varepsilon_t$. The pseudo-structural model then has the structure,

$$\begin{bmatrix} I_s & \tilde{\phi}_{s\times(N-s)}^{*\top} \\ 0_{(N-s)\times s} & I_{N-s} \end{bmatrix}\Delta X_t = \begin{bmatrix} 0_{s\times N} & \cdots & 0_{s\times N} & \alpha_1^* \\ \Gamma_1^* & \cdots & \Gamma_p^* & \alpha_2^* \end{bmatrix}\begin{bmatrix} \Delta X_{t-1} \\ \vdots \\ \Delta X_{t-p} \\ \beta^{\top}X_{t-1} \end{bmatrix}+v_t \tag{4.13}$$

Thus WF only imposes the restriction that $\tilde{\phi}^{\top}\Gamma_j=0$ for all j. In other words, under WF, linear combinations of the first differenced variables should reduce to white noise processes after adjusting for long-run effects.

---

[3] By defining Y as the vector of observations on ΔX_t and W_{-1} as the matrix of observations on W_{t-1}, the ℓ_j^2, for $j=1,\dots,s$, are the s smallest eigen values of $\left(Y^{\top}Y\right)^{-1}Y^{\top}W_{-1}\left(W_{-1}^{\top}W_{-1}\right)^{-1}W_{-1}^{\top}Y$.

As with SF, the test statistic used to determine the number of WF cofeature combinations is similarly specified as

$$C_W(s) = -(T-p-1)\sum_{j=1}^{s}\ln\left(1-\tilde{\ell}_j^2\right),\ s=1,\ldots,n-1 \tag{4.14}$$

where $C_W(s)$ is distributed as χ^2 with $sNp - s(N-s)$ degrees of freedom and the $\tilde{\ell}_j^2$ denote the s smallest squared canonical correlations between ΔX_t and $W_{t-1}^* = \left\{\Delta X_{t-1}^{\mathsf{T}},\ldots,\Delta X_{t-p}^{\mathsf{T}}\right\}^{\mathsf{T}}$. In the WF case, the rank of the cofeature matrix, s, may be greater than $N-r$ but has an upper bound of $N-1$. For each value of $s \le N-r$, one can compare SF against the nesting alternative of a WF. As proved by Hecq *et al.* (2006), the existence of s WF cofeature combinations with $s > r$ already implies the presence of $s-r$ SF cofeature combinations. Therefore, it is advisable to compare the two for values of $s = \max[1, s_{WF} - r + 1]$ up to $s = \min[s_{WF}, N-r]$, where s_{WF} denotes the rank of the WF cofeature matrix. Hecq *et al.* (2006) also propose a test statistic that allows SF to be tested against the WF alternative, for $s = \max[1, s_{WF} - r + 1]\ldots s = \min[s_{WF}, N-r]$, which is given by

$$C_{SW}(s) = -(T-p-1)\sum_{j=1}^{s}\ln\left\{\frac{\left(1-\ell_j^2\right)}{\left(1-\tilde{\ell}_j^2\right)}\right\} \tag{4.15}$$

where ℓ_j^2 and $\tilde{\ell}_j^2$ are defined as above. $C_{SW}(s)$ is χ^2 with degrees of freedom equal to the rs parametric restrictions under the null of SF.

In the case of MF, an $s \times N$ cofeature matrix is specified as $\hat{\phi}^{\mathsf{T}} = (\hat{\phi}_1^{\mathsf{T}}, \hat{\phi}_2^{\mathsf{T}})$, where $\hat{\phi}_1^{\mathsf{T}}$ and $\hat{\phi}_2^{\mathsf{T}}$ are $s_1 \times N$ and $(s-s_1)\times N$ full rank matrices, with $\max(0, s-r) < s_1 < N-r$ and $s < N-1$. Under the MF assumptions, the following equations should hold

$$\hat{\phi}_1^{\mathsf{T}}\Delta X_t = \hat{\phi}_1^{\mathsf{T}}\varepsilon_t \tag{4.16}$$

$$\hat{\phi}_2^{\mathsf{T}}(\Delta X_t - \alpha\beta^{\mathsf{T}}y_{t-1}) = \hat{\phi}_2^{\mathsf{T}}\varepsilon_t \tag{4.17}$$

and the pseudo-structural form is constructed as

$$\begin{bmatrix} I_{s_1} & \hat{\phi}^{*\top}_{1(s_1\times(N-s_1))} \\ 0_{s_2\times s_1} & \hat{\phi}^{*\top}_{2(s_2\times(N-s_1))} \\ 0_{(N-s)\times s_1} & A_{(N-s)\times(N-s_1)} \end{bmatrix} \Delta X_t = \begin{bmatrix} 0_{s_1\times N} & \cdots & 0_{s_1\times N} & 0_{s_1\times r} \\ 0_{s_2\times N} & \cdots & 0_{s_2\times N} & \alpha_2 \\ \Gamma_1^* & \cdots & \Gamma_p^* & \alpha_3 \end{bmatrix} \begin{bmatrix} \Delta X_{t-1} \\ \vdots \\ \Delta X_{t-p} \\ \beta^\top X_{t-1} \end{bmatrix} + v_t, \tag{4.18}$$

where $\hat{\phi}^{*\top}_{2(s_2\times(N-s_1))} = (I_{s_2} \quad \hat{\phi}^{**\top}_{2(s_2\times(N-s))})$ and $A_{(N-s)\times(N-s_1)} = (0_{(N-s)\times s_2} \quad I_{N-s})$. Since MF is a hybrid of WF and SF, a test cannot be formulated in terms of canonical correlations. However, the MF with s_1 SF vectors and s_2 WF vectors can be tested against the nesting WF with $s = s_1 + s_2$ using the likelihood ratio approach. Under the null of MF, an LR test statistic is asymptotically χ^2- distributed with degrees of freedom equal to the number of additional parameter restrictions, $s_1 r - s_2 s_1$.

4.3.2 Codependent Cycles

Most economic time-series are not perfectly synchronised due to structural rigidities or adjustment costs. Therefore, models which consider the lead and lag relationships among variables appear more appropriate for time-series analysis. Vahid and Engle (1997) extend the common cycle assumptions to a more general setting, thus defining codependent cycles. This allows some variables in ΔX_t to lag others by a short period of, say, q lags.

Consider a Wold representation of an N - dimensional vector ΔX_t of I(0) time-series:

$$\Delta X_t = \varepsilon_t + \sum_{j=1}^{\infty} C_j \varepsilon_{t-j} = C(L)\varepsilon_t \tag{4.19}$$

and suppose there exists a $s \times N$ cofeature matrix $\phi_q^\top$ that satisfies

$$\phi_q^{\mathsf{T}} C_j \begin{cases} \neq 0 & j = q \\ = 0 & j > q \end{cases} \tag{4.20}$$

This implies that s linear combinations $\phi_q^{\mathsf{T}} \Delta X_t$ are expected to have MA(q) representations rather than being white noise. Equation (4.20) is defined as a scalar component model of order q (SCM $(0, q)$).[4]

The test statistic used to determine the number of cofeature combinations for an $\mathrm{SCM}(0, q)$ is specified as follows (Tiao and Tsay, 1989; Vahid and Engle, 1997):

$$C(0,q) = -(T-(p-1)-q)\sum_{j=1}^{s} \ln\left(1 - \frac{\ell_j^2(q)}{d_j(q)}\right) \tag{4.21}$$

Here the $\ell_j^2(q)$, for $j = 1,...,s$, are the s smallest estimated squared canonical correlations between ΔX_t and $W_{t-1-q} = \left\{\Delta X_{t-1-q}^{\mathsf{T}}, ..., \Delta X_{t-p-q}^{\mathsf{T}}, X_{t-1-q}^{\mathsf{T}} \beta\right\}^{\mathsf{T}}$, and

$$d_j(q) = 1 + 2\sum_{v=1}^{q} \hat{\rho}_v(\hat{\varsigma}^{\mathsf{T}} \Delta X_t)\hat{\rho}_v(\hat{\lambda}^{\mathsf{T}} W_{t-1-q}) \tag{4.22}$$

$\hat{\rho}_v(\alpha_t)$ is the lag-v sample autocorrelation coefficient of the process α_t, and $\hat{\varsigma}$ and $\hat{\lambda}$ are the canonical variates corresponding to $\ell_j^2(q)$.[5] The test statistic $C(0,q)$ is asymptotically distributed as $\chi^2(s(Np+r) - s(N-s))$. As with common cycle models, the number of codependent cycles is constrained by the dimension of the VECM and the number of cointegrating vectors. There can be at most $(N-r)/(q+1)$ linearly independent cofeature combinations that yield an $\mathrm{SCM}(0,q)$ and the maximum order of an SCM cofeature is $\bar{q} = N - r - 1$ (Schleicher, 2003).

[4] Tiao and Tsay (1989) propose a more general class of scalar component models, SCM ($\bar{p}, \bar{q}$), which have an ARMA ($\bar{p}, \bar{q}$) representation. The common cycle framework is a special case of SCM $(0, q)$ when $q = 0$.

[5] $\hat{\varsigma}$ and $\hat{\lambda}$ are the eigenvectors of $(Y^{\mathsf{T}}Y)^{-1} Y^{\mathsf{T}} W (W^{\mathsf{T}} W)^{-1} W^{\mathsf{T}} Y$ and $(W^{\mathsf{T}} W)^{-1} W^{\mathsf{T}} Y (Y^{\mathsf{T}} Y)^{-1} Y^{\mathsf{T}} W$ corresponding to $\ell_j^2(q)$, where W is the matrix of observations on W_{t-1-q}.

Imposing codependence cycle restrictions on a VECM is not straightforward as these restrictions are non-linear. Therefore, Vahid and Engle (1997) suggest using Generalised method of moments (GMM), which provides reasonable estimates of ϕ_q. Under an $\mathrm{SCM}(0,q)$, the orthogonality conditions and their sample estimates can be constructed as

$$E\left[\left(\phi_q^{\top}\Delta X_t\right)\otimes W_{t-1-q}\right]=0 \tag{4.23}$$

and

$$g_T(\phi_q)=\frac{1}{T^*}\sum_{t=p+q+1}^{T}\left(\phi_q^{\top}\Delta X_t\right)\otimes W_{t-1-q}=\frac{1}{T^*}\left(I_s\otimes W^{\top}Y\right)vec\left(\phi_q\right) \tag{4.24}$$

where $T^*=T-p-q$. The matrices $W=\left[\Delta W_p^{\top},\Delta W_{p+1}^{\top},\ldots,\Delta W_{T-q-1}^{\top}\right]^{\top}$ and $Y=\left[\Delta X_{p+q+1}^{\top},\Delta X_{p+q+2}^{\top},\ldots,\Delta X_T^{\top}\right]^{\top}$ are W_{t-1-q} and ΔX_t stacked though time. It is important that ϕ_q can be normalised as $\phi_q^{\top}=\left(I_s,-\varphi_{s\times(n-s)}^{\top}\right)$ and $Y=\left[Y_1,Y_2\right]$, such that the sample moment conditions becomes

$$g_T(\varphi)=\frac{1}{T^*}\left[\left(I_s\otimes W^{\top}Y_1\right)vec\left(I_s\right)-I_s\otimes(W^{\top}Y_2)vec(\varphi)\right]. \tag{4.25}$$

The GMM estimate can be derived as

$$vec(\hat{\varphi}_{GMM})=\left[\left(Y_2^{\top}W\right)\otimes I_sP_TI_s\otimes\left(W^{\top}Y_2\right)\right]^{-1}\left[\left(Y_2^{\top}W\right)\otimes I_sP_TI_s\otimes\left(W^{\top}Y_1\right)\right]vec\left(I_s\right) \tag{4.26}$$

by minimising the quadratic form $S_T(\varphi)=g_T^{\top}P_Tg_T$. P_T is a symmetric positive-definite weighting matrix, estimated using a 2-step procedure: the initial estimate is given by $P_{T,1}=\left(I_s\otimes W^{\top}W\right)^{-1}$, which is then updated as

$$P_{T,2}=\left(\sum_{i=-q}^{q}\Gamma_{i,1}^{\phi\Delta x}\otimes\Gamma_i^{W}\right)^{-1} \tag{4.27}$$

where $\Gamma_{i,1}^{\phi\Delta x}$ and Γ_i^{W} denote estimates of the i^{th} sample-autocovariances of $\phi_q^{\top}\Delta X_t$ and W_{t-1-q}, respectively, and $\phi_q^{\top}$ is the first-step GMM estimate obtained using $P_{T,1}$.

Compared to the common cycle model, the codependence framework appears to be a more appropriate approach for studying the comovement of European growth cycles. Since countries vary significantly in their labour market institutions, economic structures and openness to trade, their output data are less likely to respond simultaneously to a common shock.

4.4 Trend and Cycle Decomposition

In this section, the methodology proposed by Proietti (1997) and Hecq *et al.* (2000) is presented. This work provides a multivariate extension of the BN decomposition which takes into account the common trend and cycle restrictions. It calculates the trend and cyclical components of X_t from the parameters of the reduced form VECM. Proietti (1997) shows that a VECM can be written in state-space form as

$$\Delta X_t = Zf_t \tag{4.28}$$

$$f_t = m + Tf_{t-1} + Z^{\top}\varepsilon_t \tag{4.29}$$

where f_t is the $(Np+r)$ dimensional state vector and T is the $(Np+r)\times(Np+r)$ transition matrix, specified respectively as

$$f_t = \begin{bmatrix} \Delta X_t \\ \Delta X_{t-1} \\ \vdots \\ \Delta X_{t-p} \\ \beta^{\top} X_{t-1} \end{bmatrix}, \qquad T = \begin{bmatrix} \Gamma_1 + \alpha\beta^{\top} & \Gamma_2 & \cdots & \Gamma_p & \alpha \\ I_N & 0_{N\times N} & \cdots & 0_{N\times N} & 0_{N\times N} \\ 0_{N\times N} & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ \beta^{\top} & 0_{r\times N} & \cdots & \cdots & I_r \end{bmatrix}$$

In addition, $Z = [I_N, 0_{N\times N}, \ldots, 0_{N\times r}]$ is an $N \times (Np + r)$ matrix, and $m = [\mu^\top, 0_{1\times N}, \ldots, 0_{1\times r}]$ is a vector of dimension $Np + r$.

Consider the case when m is zero. The BN cycle in equation (4.3) is then given by $c_t = -\lim_{k\to\infty} \sum_{i=1}^{k} \Delta\widetilde{x}_{t+i|t} = -\lim_{k\to\infty} \sum_{i=1}^{k} ZT^i f_{t|t}$. If the stability condition is met, $\lim_{k\to\infty} \sum_{i=1}^{k} T^i$ converges to $(I_{Np+r} - T)^{-1} T$ and the cycle is calculated as

$$c_t = -Z(I_{Np+r} - T)^{-1} Tf_{t|t} \tag{4.30}$$

As proved by Proietti (1997), if the state-space representation is stable, BN decomposes X_t into cyclical and trend components expressed as

$$\begin{aligned} c_t &= -(I_N - P)(\Gamma(1) - \alpha\beta^\top)^{-1} \Gamma^*(L) \Delta X_t + PX_t \\ &= \psi_{2t} + \psi_{1t} \end{aligned} \tag{4.31}$$

$$\tau_t = (I_N - P)(\Gamma(1) - \alpha\beta^\top)^{-1} \Gamma(L) X_t \tag{4.32}$$

where $P = (\Gamma(1) - \alpha\beta^\top)^{-1} \alpha[\beta^\top(\Gamma(1) - \alpha\beta^\top)^{-1}\alpha]^{-1} \beta^\top$ and $\Gamma^*(L) = \Gamma_0^* + \Gamma_1^* L + \ldots + \Gamma_{p-1}^* L^{p-1}$ with $\Gamma_j^* = \sum_{i=j+1}^{p} \Gamma_i$.

However, the upward drift observed in the GDP data result in a non-zero m. Successive substitution of equation (4.29) yields the expected value of the constant, $m^* = \lim_{k\to\infty} \sum_{i=1}^{k} T^i m = (I_{Np+r} - T)^{-1} m$. m can be removed from the transition equation to the measurement equation by writing

$$\Delta X_t = Zf_t^* + Zm^* \tag{4.33}$$

$$f_t^* = Tf_{t-1}^* + Z^\top \varepsilon_t \tag{4.34}$$

where $f_t^* = f_t - m^*$. The cycle is then given by

$$c_t = -Z(I_{Np+r} - T)^{-1} T f_{t|t}^* \tag{4.35}$$

Correspondingly, a zero-mean cyclical component can be obtained by replacing ΔX_t by $\Delta X_t - Zm^*$ in equation (4.31), and also by subtracting $\left(\Gamma(1) - \alpha\beta^{\mathsf{T}}\right)^{-1}\alpha\left[\beta^{\mathsf{T}}\left(\Gamma(1) - \alpha\beta^{\mathsf{T}}\right)^{-1}\alpha\right]^{-1}\mathrm{E}\left(\beta^{\mathsf{T}}X_{t-1}\right)$ from equation (4.31), where

$$\mathrm{E}(\Delta X_t) = Zm^* = (I_N - P)\left(\Gamma(1) - \alpha\beta^{\mathsf{T}}\right)^{-1} m \tag{4.36}$$

$$\mathrm{E}\left(\beta^{\mathsf{T}} X_{t-1}\right) = -\left[\beta^{\mathsf{T}}(\Gamma(1) - \alpha\beta^{\mathsf{T}})^{-1}\alpha\right]^{-1}\beta^{\mathsf{T}}(\Gamma(1) - \alpha\beta^{\mathsf{T}})^{-1} m \tag{4.37}$$

Furthermore, Hecq *et al.* (2000) demonstrate that the above decomposition can be applied to a VECM with common cycle restrictions. However, it is worth noting that the decomposition differs under SF and WF. To illustrate this, premultiply the cyclical component, $c_t = \psi_{2t} + \psi_{1t}$, with the matrix $\phi(\phi^{\mathsf{T}}\phi)^{-1}\phi^{\mathsf{T}} + \phi_{\perp}(\phi_{\perp}^{\mathsf{T}}\phi_{\perp})^{-1}\phi_{\perp}^{\mathsf{T}} = I_n$ to yield $c_t = \psi_{2A,t} + \psi_{1A,t} + \psi_{2B,t} + \psi_{1B,t}$, where ϕ is the cofeature matrix and $\phi^{\mathsf{T}}\phi_{\perp} = 0$.[6] If SF holds, then X_t consists of a trend component, τ_t, and a common cycles component, $c_t = \psi_{2B,t} + \psi_{1B,t}$. However, under the WF assumptions, X_t is the sum of three components: τ_t, c_t, and an additional transitory component $\psi_{2A,t} + \psi_{1A,t}$, which is nonzero due to the long-run predictability of the linear combination of the variables in first differences.

[6] $c_t = \phi(\phi^{\mathsf{T}}\phi)^{-1}\phi^{\mathsf{T}}\psi_{2t} + \phi(\phi^{\mathsf{T}}\phi)^{-1}\phi^{\mathsf{T}}\psi_{1t} + \phi_{\perp}(\phi_{\perp}^{\mathsf{T}}\phi_{\perp})^{-1}\phi_{\perp}^{\mathsf{T}}\psi_{2t} + \phi_{\perp}(\phi_{\perp}^{\mathsf{T}}\phi_{\perp})^{-1}\phi_{\perp}^{\mathsf{T}}\psi_{1t}$
$= \psi_{2A,t} + \psi_{1A,t} + \psi_{2B,t} + \psi_{1B,t}$

4.5 Empirical Results

This section evaluates the degree of growth cycle synchronisation among the seven European countries analysed above. Since the aggregate euro area output data is simply the weighted sum of its member countries output, it is excluded from this analysis to avoid multicollinearity. The empirical model is based on a VAR with four lags, where the lag length was chosen according to the LR test statistic at the 5% level. To cross-check whether enough lags were included, the autocorrelation LM test was conducted, with no serial correlations being identified in the model residuals. This analysis starts by determining whether the seven output series are cointegrated. If they have cointegrating relationships, the series share common stochastic trends, and the VECM can be estimated. The ADF and Johansen cointegration tests were conducted. ADF test statistics reported in Table 4.2 were not able to reject the null of a unit root in the level of real GDP for each country but did reject the null when first differences of the series were used. As shown in Table 4.3, the trace statistic from the Johansen cointegration test indicates the existence of five cointegrating vectors, whilst the eigenvalue statistic suggests that there are two at the 5% level. Since three of the five error correction terms estimated from the VECM with three lags and five cointegrating vectors are found to be nonstationary, the inference provided by the max eigenvalue statistic is preferred.

Table 4.2: The Augmented Dickey-Fuller Test Statistics

	Level		First Differenced
	Constant	**Constant + Trend**	**Constant**
AUS	0.400	-2.747	-9.057**
	(0.982)	(0.221)	(0.000)
BEL	1.323	-3.549*	-9.033**
	(0.999)	(0.039)	(0.000)
FRA	-0.587	-2.610	-4.518**
	(0.868)	(0.277)	(0.000)
GER	-0.446	-1.288	-11.005**
	(0.896)	(0.886)	(0.000)
ITA	-1.157	-1.189	-7.590**
	(0.691)	(0.907)	(0.000)
NETH	0.997	-3.487*	-11.526**
	(0.996)	(0.046)	(0.000)
SPA	0.481	-2.554	-3.308*
	(0.985)	(0.302)	(0.017)

Notes: the numbers in parentheses are p-values. ** denotes significance at 1% and * at 5%. The ADF tests with a constant and a time trend included indicate that the Belgian and Dutch GDP series are trend stationary.

Table 4.3: Cointegrating Test (Johansen 1995)

Null	Eigenvalue	Max-Eigen stat.	Trace stat.
$r = 0$	0.46	66.45** (0.000)	226.32** (0.000)
$r \leq 1$	0.46	65.20** (0.000)	159.87** (0.000)
$r \leq 2$	0.26	32.24 (0.078)	94.67** (0.000)
$r \leq 3$	0.23	27.51 (0.051)	62.44** (0.001)
$r \leq 4$	0.18	21.05 (0.051)	34.93* (0.012)
$r \leq 5$	0.12	13.84 (0.058)	13.88 (0.086)
$r \leq 6$	0.00	0.05 (0.831)	0.05 (0.831)

Note: the default option in EViews is used in which an intercept is included in both the cointegration equation and the differenced form of the VAR. ** denotes significance at 1% and * at 5%.

Having established the order of the VECM and the number of cointegrating vectors, and obtained estimates of the corresponding cointegrating parameters, Tiao and Tsay's (1989) canonical correlation-based tests for common and codependent cycles outlined in section 4.3 are conducted. These tests are performed in the following order. First, the test statistics for SF and WF are calculated to determine the number of cofeature vectors. Next, SF is tested against the alternative of WF for $s = \max[1, s_{WF} - r + 1] \ldots s = \min[s_{WF}, N - r]$. Finally, the presence of codependent cycles is examined sequentially, starting from SCM (0,1) and then incrementally increasing the order of the SCM.

The results of the common and codependent cycle tests are presented in Table 4.4. The Tiao and Tsay tests do not reject the null of at least two SF or the null of four WF cofeature vectors at the 5% level. In other words, there are five SF or three WF common cycles among the seven output series, rather than a unique common cycle. Since the finding of four WF cofeature vectors already implies the presence of two SF cofeature vectors, the null of SF is tested against the alternative WF for $s = 3$ and $s = 4$.[7] As a result, both statistics strongly reject. This may suggest that there is not a MF structure.

[7] The existence of s WF cofeature combinations $(s > r)$ implies the presence of $s - r$ SF cofeature combinations.

Table 4.4: Canonical correlation test for common and codependent cycles

Null	Common cycles			Codependent cycles			
	$C_S(s)$	$C_W(s)$	$C_{SW}(s)$	$SCM(0,1)$	$SCM(0,2)$	$SCM(0,3)$	$SCM(0,4)$
$s \geq 1$	17.34	13.64		14.24	7.46	7.16	8.89
	(0.43)	(0.55)	-	(0.65)	(0.98)	(0.98)	(0.94)
$s \geq 2$	45.35	31.21		28.60	19.70	18.59	19.70
	(0.14)	(0.51)	-	(0.81)	(0.99)	(0.99)	(0.94)
$s \geq 3$	78.02*	55.99	22.88**	47.33	44.66	37.42	41.99
	(0.03)	(0.29)	(0.00)	(0.82)	(0.88)	(0.98)	(0.93)
$s \geq 4$	127.99**	89.42	40.07**	87.80	73.60	62.63	68.69
	(0.00)	(0.08)	(0.00)	(0.26)	(0.68)	(0.92)	(0.81)
$s \geq 5$	197.83**	132.86**	-	139.07**	117.99	112.99	101.69
	(0.00)	(0.01)		(0.01)	(0.18)	(0.28)	(0.57)
$s \geq 6$	306.57**	202.64**	-	215.31**	203.24**	169.68**	161.16*
	(0.00)	(0.00)		(0.00)	(0.00)	(0.01)	(0.04)

Note: The numbers in parentheses are p-values. ** denotes significance at 1% and * at 5%. The null of SF is tested with the WF alternative for $s = \max[1, s_{WF} - r + 1]$ up to $s = \min[s_{WF}, N - r]$, where $s_{WF} = 4$.

The test statistics for codependent cycles with $q \leq 4$ are also presented in Table 4.4.[8] The results show that the null of $s_1 \geq 4$, $s_2 \geq 5$, $s_3 \geq 5$ and $s_4 \geq 5$ cannot be rejected at the 5% level. This implies that there are five cofeature combinations, two SCM(0,0), two SCM(0,1) and one SCM(0,2).[9] However, from Hecq (1998) and Mills and Harvey (2005), the use of seasonally adjusted data may induce size distortions and low power in the common cycle testing. Therefore, GMM was used to confirm the above results, with the estimates of the cofeature matrix ϕ_q^{T}, where $q = 2$ and $s = 5$, given in Table 4.5. The sample autocorrelation functions are calculated to cross-check the data generating process of the linear combinations implied by the GMM estimates. However, the result indicates that there are one MA(0), three MA(1) and one MA(4) linear combinations.

[8] As proved in Schleicher (2003), the maximum order of an SCM cofeature is $\bar{q} = N - r - 1$. In this case $N = 7$ and $r = 2$, so the maximum order of an SCM cofeature is 4.

[9] The SF common cycle is a special case of SCM(0, q) when $q = 0$.

Table 4.5: GMM Estimates of Cofeature Vectors: $\phi_q^{\mathsf{T}} = \left(I_5, -\varphi_{(5\times2)}^{\mathsf{T}}\right)$

AUS	BEL	FRA	GER	ITA	NETH	SPA	
1	0	0	0	0	-0.367**	-0.391**	MA(1)
					(0.12)	(0.10)	
0	1	0	0	0	-0.262	-0.473**	MA(1)
					(0.16)	(0.16)	
0	0	1	0	0	0.112	-0.720**	MA(1)
					(0.11)	(0.11)	
0	0	0	1	0	-0.339**	-0.330**	MA(4)
					(0.13)	(0.13)	
0	0	0	0	1	-0.177	-0.412**	MA(0)
					(0.13)	(0.09)	

Note: Standard errors in parentheses.

Schleicher (2003) performs a Monte Carlo experiment to demonstrate that LR tests based on full information maximum likelihood (FIML) estimation of the restricted VECMs have considerably higher power than the GMM and Tiao-Tsay tests. Therefore, the LR test statistics have been calculated by estimating a VECM subject to the cross equation restrictions imposed by common cycles. Another advantage of this full system estimation is that it allows us to compute the trend-cycle decomposition and the forecast error variance decomposition. In the following tests, five models were estimated by FIML using PcGive, which includes an unrestricted VECM (model 1) and three restricted VECMs obtained by imposing the assumptions of two SF, four WF and an MF, respectively (models 2 to 4). Granger causality tests suggest that, in the Dutch equation, the lagged values of the other countries' output data are jointly insignificant. In addition, in the Spanish equation, only the lagged Belgian output data appears significant at the 5% level. Therefore, model 5 imposes extra restrictions on the Dutch and Spanish equations used in model 2 by assuming that the parameters associated with these lagged values are equal to zero. It is worth noting that models 2 to 5 are estimated by a two-step procedure. The cointegrating vectors are estimated independently from the unrestricted VECM and then included in the simultaneous equations as explanatory variables, along with the lagged first differences of the output series.

Table 4.6: The Likelihood-Ratio Tests

	Model 1	Model 2	Model 3	Model 4	Model 5
No. Parameters	168	132	96	93	99
Log-likelihood	3038.02	3016.27	2988.52	2984.01	2993.00
AIC	-53.64	-53.91	-54.07	-54.04	-54.09
HQ	-51.94	-52.57	-53.09	-51.71	-53.09
SC	-49.45	-50.61	-51.67	-53.09	-51.62
Likelihood Ratio test					
M2 vs. M1	$\chi^2(36) \sim 43.50[0.183]$				
M3 vs. M1	$\chi^2(72) \sim 98.99[0.019]*$				
M4 vs. M1	$\chi^2(75) \sim 108.02[0.007]**$				
M4 vs. M3	$\chi^2(3) \sim 9.02[0.029]*$				
M5 vs. M1	$\chi^2(69) \sim 90.04[0.045]*$				
M5 vs. M2	$\chi^2(33) \sim 46.54[0.059]$				

Notes: Model 1 is the (unrestricted) VECM with two cointegrating vectors; Model 2 is restricted with two SF cofeature vectors; Model 3 is restricted with four WF cofeature vectors; Model 4 has a MF structure with three SF vectors and one WF vector; Model 5 is obtained by imposing additional restrictions on the Dutch and Spanish equations in model 2.

The full parameter estimates of the reduced forms of the five models discussed above, along with the Granger causality test statistics, are reported in Appendix A4 in Tables A4.4-A4.8. The log-likelihood and information criteria for these models are presented in Table 4.6, providing criteria for model selection. It is worth noting that all the restricted models can be tested against the unrestricted VECM. In addition, model 4 can be tested against model 3, and model 5 can be tested against model 2. However, the other pairs of models are not nested, so that the LR test statistics are only calculated for the nested models. As the results show, the LR test statistics do not reject the null of model 2 against the alternative of model 1, and model 5 cannot be rejected against model 2 at the 5% level.

Once the appropriate models are chosen, the trend and cyclical components of each output series can be calculated simultaneously from the reduced form structure using the multivariate BN decomposition. Two sets of decompositions, calculated from models 2 and 5, are presented. The cyclical components obtained from these two models, along with the recessionary periods identified from the national GDP series using the BBQ algorithm (Harding and Pagan, 2000, 2001, 2002), are plotted in Figure 4.2. It appears that there are more growth cycles than business cycles over the period studied. In addition, the profound downturns observed in the growth cycles coincide with the recessions in business cycles. Compared to the cycles obtained from univariate BN decompositions, the multivariate extension produces highly persistent cycles with large amplitudes.

4.5.1 Out-of-Sample Forecasts

The out-of-sample forecasting performance of the restricted models is now compared to the (unrestricted) VECM model. This comparison can be used to cross-check whether the more parsimonious models obtained by imposing the common cycle restrictions fit the data better. It is believed that if the restrictions are placed appropriately, the forecasting capability of a model will improve (Hecq *et al.*, 2006; Schleicher, 2007).

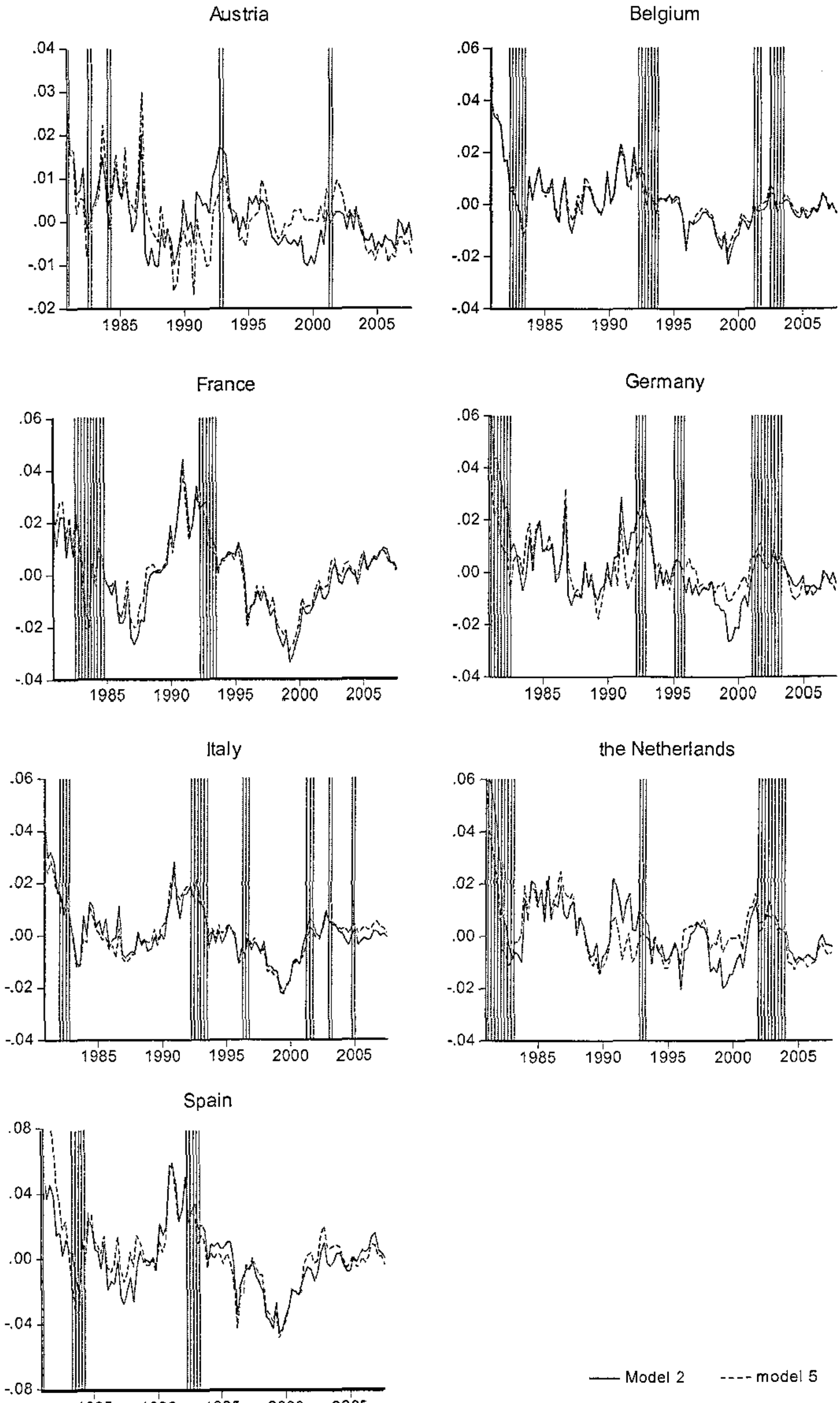
To investigate this assertion, the models were re-estimated for the period 1980Q1 to 2005Q3, with the last eight quarters of the sample used to evaluate forecast accuracy. Table 4.7 shows the root-mean-squared errors (RMSEs) for horizons from 1 to 8 quarters. Three striking features are revealed in Table 4.7. First, the restricted models considerably outperform the unrestricted VECM over all forecast horizons for all countries. Second, for most countries, except Austria and Spain, restricted VECMs with SF assumptions imposed yield smaller RMSEs. Among the countries for which this holds, model 2 is preferred for Belgium and France, whilst model 5 is preferred for Germany and Italy. Model 5 is also preferred for the Netherlands over the longer forecasting horizon. Finally, the models imposed with WF and MF restrictions are preferred for the Austrian and Spanish output data, respectively. This implies a strong predictability between the long-run relationship and linear combinations of short-run dynamics.

Table 4.7: Out-of-Sample Forecasts: Root-Mean-Squared Errors (10^{-2})

		h=1	h=2	h=3	h=4	h=6	h=8
AUS	**Model 1**	0.327	0.264	0.272	0.234	0.252	0.261
	Model 2	0.309	0.241	0.257	0.228	0.232	0.240
	Model 3	**0.249**	**0.204**	**0.222**	**0.169**	**0.176**	**0.177**
	Model 4	0.258	0.225	0.242	0.193	0.197	0.201
	Model 5	0.283	0.248	0.250	0.191	0.194	0.196
BEL	**Model 1**	0.247	0.189	0.198	0.217	0.214	0.216
	Model 2	0.228	**0.185**	**0.178**	**0.198**	**0.198**	**0.201**
	Model 3	0.272	0.246	0.252	0.249	0.246	0.248
	Model 4	**0.207**	0.190	0.202	0.204	0.199	0.202
	Model 5	0.236	0.191	0.220	0.215	0.215	0.215
FRA	**Model 1**	0.308	0.321	0.279	0.272	0.286	0.287
	Model 2	0.289	**0.297**	**0.267**	**0.265**	**0.278**	**0.274**
	Model 3	0.289	0.311	0.302	0.284	0.289	0.280
	Model 4	0.290	0.306	0.298	0.279	0.286	0.278
	Model 5	**0.266**	0.301	0.288	0.270	0.280	0.275
GER	**Model 1**	0.379	0.362	0.427	0.380	0.361	0.350
	Model 2	0.402	0.367	0.379	0.366	0.352	0.352
	Model 3	0.362	0.355	0.394	0.401	0.393	0.385
	Model 4	0.329	0.311	0.344	0.343	0.342	0.332
	Model 5	**0.304**	**0.308**	**0.321**	**0.308**	**0.308**	**0.300**
ITA	**Model 1**	0.412	0.402	0.413	0.407	0.413	0.413
	Model 2	0.394	0.378	0.380	0.378	**0.366**	0.365
	Model 3	0.324	0.364	0.393	0.416	0.421	0.416
	Model 4	0.321	0.338	0.361	0.374	0.388	0.382
	Model 5	**0.306**	**0.325**	**0.358**	**0.367**	0.377	**0.373**
NETH	**Model 1**	0.518	0.462	**0.417**	0.456	0.451	0.469
	Model 2	0.506	0.500	0.443	0.504	0.484	0.485
	Model 3	**0.396**	0.420	0.423	0.460	0.465	0.464
	Model 4	0.399	**0.415**	0.422	0.456	0.461	0.460
	Model 5	0.420	0.444	0.449	**0.448**	**0.447**	**0.447**
SPA	**Model 1**	0.313	0.274	0.302	0.343	0.349	0.355
	Model 2	0.296	0.268	0.288	0.334	0.338	0.345
	Model 3	0.216	0.246	0.277	0.291	0.286	0.291
	Model 4	**0.212**	0.229	**0.257**	**0.275**	**0.275**	**0.280**
	Model 5	0.245	**0.224**	0.262	0.284	0.288	0.289

Note: the numbers in bold are the smallest RMSE of each set of models.

Figure 4.2: Multivariate BN Cycles



4.5.2 Variance Decomposition

Decomposing each output series into trend and cyclical components raises the question of whether the cyclical innovations explain a significant proportion of the forecast errors of output fluctuations. To examine this, the relative importance of permanent and transitory shocks in explaining the variance of output is assessed using a forecast error variance decomposition. Consider an innovation ε_t , which is the sum of its permanent and transitory components:

$$\varepsilon_t = \varepsilon_{trend,t} + \varepsilon_{cycle,t} \tag{4.38}$$

Since $\varepsilon_{trend,t}$ and $\varepsilon_{cycle,t}$ are found to be correlated in most cases, it is necessary to orthogonalise them. The orthogonalisation procedure proposed by Issler and Vahid (2001) was used, in which it is assumed that the two innovations have the structure

$$\begin{bmatrix} \varepsilon_{trend,t} \\ \varepsilon_{cycle,t} \end{bmatrix} \sim N\left(0, \begin{bmatrix} \sigma_{tt} & \sigma_{tc} \\ \sigma_{tc} & \sigma_{cc} \end{bmatrix}\right) \tag{4.39}$$

The variance of ε_t can be decomposed into two orthogonal components as

$$\begin{aligned} VAR(\varepsilon_t) &= VAR(\varepsilon_{trend,t}) + VAR(\varepsilon_{cycle,t}) \\ &= \left\{\left(1 + \frac{\sigma_{tc}}{\sigma_{tt}}\right)^2 \sigma_{tt}\right\} + \left\{\sigma_{cc} + \frac{\sigma_{tc}^{\ 2}}{\sigma_{tt}}\right\} \end{aligned} \tag{4.40}$$

This procedure is comparable to a Cholesky factorisation and is sensitive to the ordering of the components. Here, $\varepsilon_{trend,t}$ is placed first in the decomposition, since the innovations to productivity are assumed to cause both trend and cycle movements in real business cycle models. One-step-ahead innovations for the trends are obtained by taking the first differences of the estimated trends. H -step-ahead trend innovations are then given by the sum of the one-step-ahead trend innovations. First-quarter cycle innovations are the residuals from a regression of the estimated cycles on the variables on the right-hand side of the VECM. For longer horizons, H- step-ahead cycle innovations were obtained by shifting the data set backwards. The variance decomposition results for models 2 and 5 are presented in Table 4.8, since they outperform the other models in forecasting, as was shown in section 4.5.1. Each cell in Table 4.8 contains two numbers. The first represents the relative importance of a trend/cyclical component to the total output variance estimated in model 5. The second number (in parentheses) is the corresponding value calculated from model 2.

Table 4.8: Forecast Error Variance Decomposition

Forecast horizons (quarters)	h=1		h=5		h=9		h=11	
	Trend	Cycle	Trend	Cycle	Trend	Cycle	Trend	Cycle
AUS	0.28	0.72	0.95	0.05	0.98	0.02	0.99	0.01
	(0.29)	(0.71)	(0.96)	(0.04)	(0.99)	(0.01)	(0.99)	(0.01)
BEL	0.43	0.57	0.97	0.03	0.99	0.01	1.00	0.00
	(0.33)	(0.67)	(0.96)	(0.04)	(0.99)	(0.01)	(0.99)	(0.00)
FRA	0.09	0.91	0.92	0.08	0.97	0.03	0.98	0.02
	(0.08)	(0.92)	(0.91)	(0.09)	(0.96)	(0.04)	(0.08)	(0.02)
GER	0.47	0.53	0.96	0.04	0.99	0.01	0.99	0.01
	(0.31)	(0.69)	(0.94)	(0.06)	(0.98)	(0.02)	(0.99)	(0.01)
ITA	0.51	0.49	0.96	0.04	0.99	0.01	0.99	0.01
	(0.42)	(0.58)	(0.97)	(0.03)	(0.99)	(0.01)	(0.99)	(0.01)
NETH	0.62	0.38	0.96	0.04	0.98	0.02	0.99	0.01
	(0.37)	(0.63)	(0.95)	(0.05)	(0.98)	(0.02)	(0.99)	(0.01)
SPA	0.12	0.88	0.94	0.06	0.98	0.02	0.99	0.01
	(0.09)	(0.91)	(0.94)	(0.06)	(0.98)	(0.02)	(0.99)	(0.01)

Notes: the first number in each cell represents the relative importance of trend/cyclical movements to the total output variance which is calculated using model 5; the numbers in parentheses are the corresponding values calculated from model 2.

In general, both models indicate that, for short period forecasts, transitory movements contribute more to total output variance than permanent components, whilst over longer time periods it is the permanent components which make the greatest contribution. The transitory movements in French and Spanish output are very significant in the first-quarter forecast. The cyclical fluctuations for these two countries also appear to be more persistent than those for other countries. For example, 91% (92%) of the total variance in French output in the first-quarter forecast can be attributed to transitory movements, whilst four quarters later 8% (9%) can.[10] The results imply that France and Spain would benefit greatly from stabilising their cyclical fluctuations. In contrast, permanent shocks explain a larger proportion of the output variance in the other

[10] The first percentage is that obtained using model 5, the percentage in parentheses is that from using model 2.

countries, especially Italy and the Netherlands. The cyclical components only account for 49% (58%) of the output variance in Italy and 38% (63%) for the Netherlands in the first-quarter forecast and their effects disappear quickly.

The contributions of the cyclical components estimated from model 5 appear smaller than those from model 2, especially for the Netherlands. One possible explanation is that model 5 imposes additional restrictions on the Dutch equation that reduce the level of noise in the Dutch cyclical component.

4.6 Conclusion

This chapter has studied the growth cycles of seven European countries since 1980. Both univariate and multivariate trend-cycle decomposition methodologies based on structural time-series models were applied to extract trend and cycle components from real GDP data.. The cyclical components estimated from the two univariate approaches vary significantly in cycle period and amplitude. The average cycle correlation between the BN cycles appears to be smaller than the corresponding correlation between the cycles estimated from the unobserved component model. This confirms the argument in Canova (1998) that the use of different trend-cycle decomposition methodologies may change the results obtained.

In the multivariate framework, five SF and three WF common cycles are found among the seven output series rather than a unique common cycle. In addition, the inference from the canonical tests shows that two codependent cycles can be identified when $q = 2$. This indicates an adjustment delay of two quarters between countries. However, GMM results are more disappointing, suggesting the delay will last one year, as an MA(4) data generating structure is found in the linear combination of the first differenced data. Overall, the presence of heterogeneous and codependent cycles identified in the multivariate approach contradicts the OCA criterion that members of a monetary union should share a high degree of growth cycle synchronisation. Furthermore, it can clearly be observed from Figure 4.2 that the seven European countries were at different stages in their growth cycle even after entering the euro area. In more recent years, Germany, Austria and the Netherlands have been characterised by below trend growth, while Spanish and French economic growth appeared to be relatively steady after the global economic downturn which occurred in 2000. These variations in economic performance lead to diverging monetary requirements, causing difficulties in defining the appropriate timing and stance of a common monetary policy. Therefore, the demands that have been made in some countries for promoting economic growth, for example Germany, while simultaneously preventing the risk of rising inflation, as in Spain, may pose challenges for the ECB and its common monetary policy.

Appendix A4

Table A4.1: Parameter Estimates for Univariate BN Models

	Beveridge Nelson Decomposition							
Dependent Variables	**Δ EURO**	**Δ AUS**	**Δ BEL**	**Δ FRA**	**Δ GER**	**Δ ITA**	**Δ NETH**	**Δ SPA**
	Constant	Constant	Constant	Constant	Constant	Constant	Constant	Constant
	0.003**	0.005**	0.005**	0.002**	0.003**	0.003**	0.005**	0.004**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
	Δ EURO_1	Δ AUS_1	Δ BEL_1	Δ FRA_1	Δ GER_1	Δ ITA_1	Δ NETH_1	Δ SPA_1
	0.205*	0.172*	0.068	0.297**	-0.092	0.302**	-0.090	-0.199*
	(0.095)	(0.091)	(0.097)	(0.095)	(0.094)	(0.092)	(0.096)	(0.092)
	Δ EURO_2		Δ BEL_2	Δ FRA_2	Δ GER_2		Δ NETH_2	Δ SPA_2
	0.187*		0.224*	0.248**	0.014		0.181*	0.383**
	(0.094)		(0.097)	(0.092)	(0.093)		(0.095)	(0.085)
			Δ BEL_3		Δ GER_3			Δ SPA_3
			0.055		0.096			0.350**
			(0.095)		(0.093)			(0.091)
			Δ BEL_4		Δ GER_4			
			-0.213*		0.323**			
			(0.093)		(0.092)			
log-likelihood	437.182	417.576	398.274	450.759	359.941	417.762	365.175	389.701
DW	2.032	1.943	1.923	2.039	1.987	2.040	1.917	1.923

Notes: DW is the Durbin-Watson statistic. Standard errors in parentheses.

Table A4.2: Parameter Estimates for Harvey and Trimbur's (2003) decomposition

			Unobserved Components model									
Series	(m,n)	Restrictions	σ_η^2	σ_ζ^2	σ_κ^2	σ_ε^2	ρ	$2\pi/\lambda_c$	R_D^2	σ	Log-likelihood	DW
EURO	**(2,2)**	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	**0**	**1.505**	**2.164**	**4.229**	**0.688**	**20**	**0.110**	**4.341**	**591.518**	**1.931**
	(2,3)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	1.524	0.674	4.914	0.624	20	0.107	4.350	591.280	1.881
	(3.2)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	0.029	2.108	4.590	0.742	20	0.049	4.509	579.453	1.911
	(3.3)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	0.029	0.674	5.239	0.662	20	0.043	4.523	579.095	1.860
AUS	**(2,2)**	$\sigma_\eta^2=0$	**0**	**2.096**	**8.030**	**0.739**	**0.409**	**6.297**	**0.066**	**5.391**	**567.705**	**1.912**
	(2,3)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	1.740	15.831	0.014	0.314	20	0.051	5.436	566.850	1.876
	(3,2)	$\sigma_\eta^2=0$	0	0.096	12.513	0	0.384	8.660	-0.032	5.696	554.212	1.882
	(3,3)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	0.050	0.917	12.539	0.624	20	-0.236	6.231	544.232	1.479
BEL	**(2.2)**	$\sigma_\eta^2=0$	**0**	**1.835**	**0.980**	**9.117**	**0.786**	**21.073**	**0.077**	**5.944**	**556.980**	**2.011**
	(2,3)	$\sigma_\eta^2=0$	0	1.527	0.771	9.144	0.652	12.618	0.077	5.946	556.966	2.017
	(3,2)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	0.015	6.250	7.932	0.715	20	0.023	6.145	545.300	2.045
	(3,3)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	0.015	2.676	9.016	0.620	20	0.029	6.125	545.634	2.022
FRA	(2.2)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	1.499	1.669	2.726	0.767	20	0.214	3.860	604.391	1.958
	(2,3)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	**0**	**1.628**	**0.560**	**3.054**	**0.674**	**20**	**0.217**	**3.853**	**604.583**	**1.931**
	(3,2)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	0.033	1.941	2.862	0.784	20	0.155	4.023	592.070	1.972
	(3,3)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	0.034	0.646	3.283	0.693	20	0.156	4.021	592.097	1.933
GER	**(2,2)**	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	**0**	**1.772**	**1.462**	**30.128**	**0.815**	**20**	**-0.004**	**8.700**	**515.279**	**1.834**
	(2,3)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	1.892	0.377	30.620	0.731	20	-0.006	8.710	515.165	1.826
	(3,2)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	0.023	1.851	30.223	0.826	20	-0.065	9.005	503.596	1.844
	(3,3)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	0.020	0.407	30.978	0.756	20	-0.066	9.007	503.493	1.831
ITA	**(2,2)**	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	**0**	**1.105**	**6.432**	**4.834**	**0.685**	**20**	**0.071**	**5.322**	**569.159**	**1.957**
	(2,3)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	1.142	3.186	5.910	0.580	20	0.069	5.328	569.020	1.935
	(3,2)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	0.017	6.541	5.037	0.705	20	0.012	5.512	557.290	1.941
	(3,3)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	0.017	3.024	6.203	0.604	20	0.009	5.523	557.064	1.912
NETH	(2,2)	$\sigma_\eta^2=0$	0	3.207	2.227	22.349	0.613	6.480	0.069	8.396	519.218	1.932
	(2,3)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	2.843	8.072	23.537	0.345	20	0.057	8.452	518.540	1.867
	(3,2)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	0.060	15.629	19.482	0.516	20	-0.014	8.060	506.494	1.932
	(3,3)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	0.061	8.970	22.821	0.436	20	-0.015	8.808	506.462	1.891
SPA	(2,2)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	2.475	0.798	16.554	0.730	20	0.194	6.630	545.111	2.142
	(2,3)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	**0**	**2.462**	**0.435**	**16.564**	**0.610**	**20**	**0.195**	**6.628**	**545.135**	**2.149**
	(3,2)	$\sigma_\eta^2=0, 2\pi/\lambda_c=20$	0	0.062	1.504	16.699	0.734	20	0.146	6.859	533.794	2.148
	(3,3)	$\sigma_\eta^2=0$	0	0.060	1.305	16.654	0.601	28	0.145	6.860	533.776	2.161

Notes: σ_ζ^2 is the slope variance; σ_κ^2 is the cycle variance; σ_ε^2 is the variance of the irregular term; The variance parameters are multiplied by 10^6; ρ is the damping factor; $2\pi/\lambda_c$ is the cycle frequency; R_D^2 is the adjusted r-squared; σ is standard error of regression; DW is the Durbin-Watson statistic.

Table A4.3: Granger-Causality Test

Dependent variable	Δ NETH	Δ SPA	Δ AUS	Δ BEL	Δ FRA	Δ GER	Δ ITA
Excluded Variables	Chi-square	Chi-square	Chi-square	Chi-square	Chi-square	Chi-square	Chi-square
Δ AUS	1.523	5.893		3.673	11.087*	2.929	2.722
	(0.677)	(0.117)		(0.299)	(0.011)	(0.403)	(0.437)
Δ BEL	1.960	8.067*	8.176*		14.906**	6.377	7.306
	(0.581)	(0.045)	(0.043)		(0.002)	(0.095)	(0.063)
Δ FRA	1.800	0.564	5.422	3.297		3.125	0.998
	(0.615)	(0.905)	(0.143)	(0.348)		(0.373)	(0.802)
Δ GER	3.725	2.402	11.088*	1.279	3.347		10.660*
	(0.293)	(0.493)	(0.011)	(0.734)	(0.341)		(0.014)
Δ ITA	1.051	4.385	8.495*	7.112	2.960	9.432*	
	(0.789)	(0.223)	(0.037)	(0.068)	(0.398)	(0.024)	
Δ NETH		4.667	21.040**	2.298	2.072	5.288	6.287
		(0.198)	(0.000)	(0.513)	(0.558)	(0.152)	(0.099)
Δ SPA	1.054		20.641**	4.966	2.839	8.656*	2.671
	(0.788)		(0.000)	(0.174)	(0.417)	(0.034)	(0.445)
All Variables	16.792	24.101	74.108**	26.993	54.584**	43.280**	35.146**
	(0.538)	(0.152)	(0.000)	(0.079)	(0.000)	(0.001)	(0.009)

Note: p-values in parentheses.

Table A4.4: Reduced form of Model 1

Cointegrating Vectors							
AUS_1	BEL_1	FRA_1	GER_1	ITA_1	NETH_1	SPA_1	Constant
1	0	1.555	-0.289	-0.311	-0.829	-0.739	-4.961
0	1	2.651	0.427	-1.240	-1.347	-0.928	-8.674

Lagged Variables	Dependent Variable						
	Δ AUS	Δ BEL	Δ FRA	Δ GER	Δ ITA	Δ NETH	Δ SPA
Constant	0.002*	0.002*	0.001	0.000	0.001	0.005**	0.005**
	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)
Δ AUS_1	0.022	0.074	0.267**	0.300	-0.217	0.112	0.298
	(0.114)	(0.146)	(0.082)	(0.210)	(0.132)	(0.203)	(0.161)
Δ AUS_2	-0.411**	-0.106	-0.030	-0.154	-0.013	-0.073	-0.191
	(0.103)	(0.132)	(0.074)	(0.190)	(0.120)	(0.183)	(0.145)
Δ AUS_3	-0.138	-0.187	0.092	0.079	-0.099	-0.141	0.030
	(0.108)	(0.138)	(0.077)	(0.199)	(0.125)	(0.191)	(0.152)
Δ BEL_1	-0.041	-0.188	0.250**	0.033	0.323*	0.036	0.350*
	(0.104)	(0.133)	(0.075)	(0.191)	(0.120)	(0.184)	(0.146)
Δ BEL_2	-0.283*	0.038	0.233**	-0.264	0.103	-0.078	0.102
	(0.108)	(0.138)	(0.078)	(0.199)	(0.125)	(0.191)	(0.152)
Δ BEL_3	-0.010	0.095	0.089	0.279	0.062	0.204	0.249
	(0.111)	(0.143)	(0.080)	(0.205)	(0.129)	(0.197)	(0.157)
Δ FRA_1	0.265	0.237	-0.099	-0.315	-0.007	0.183	0.131
	(0.156)	(0.200)	(0.112)	(0.287)	(0.181)	(0.276)	(0.219)
Δ FRA_2	0.114	-0.223	-0.021	-0.058	0.100	-0.285	-0.076
	(0.146)	(0.186)	(0.105)	(0.268)	(0.169)	(0.258)	(0.205)
Δ FRA_3	0.151	-0.100	0.222*	-0.299	0.118	-0.052	0.029
	(0.138)	(0.176)	(0.099)	(0.254)	(0.160)	(0.244)	(0.194)
Δ GER_1	-0.221*	-0.034	-0.105*	-0.338*	0.089	0.184	0.073
	(0.081)	(0.104)	(0.058)	(0.149)	(0.094)	(0.144)	(0.114)
Δ GER_2	0.023	0.003	-0.030	-0.212	0.223*	0.193	0.066
	(0.074)	(0.095)	(0.053)	(0.137)	(0.086)	(0.132)	(0.104)
Δ GER_3	0.010	0.077	-0.018	-0.107	0.199*	0.181	0.139
	(0.064)	(0.083)	(0.046)	(0.119)	(0.075)	(0.114)	(0.091)
Δ ITA_1	0.145	0.099	0.018	0.595**	0.075	0.128	-0.264
	(0.108)	(0.138)	(0.077)	(0.198)	(0.125)	(0.191)	(0.152)
Δ ITA_2	-0.033	0.127	0.035	-0.020	-0.153	0.046	0.154
	(0.108)	(0.139)	(0.078)	(0.199)	(0.126)	(0.192)	(0.152)
Δ ITA_3	-0.247*	-0.310*	-0.120	0.160	-0.122	0.142	-0.083
	(0.101)	(0.130)	(0.073)	(0.187)	(0.118)	(0.180)	(0.143)
Δ LNETH_1	0.285**	0.103	-0.029	0.112	0.045	-0.285*	-0.136
	(0.067)	(0.086)	(0.048)	(0.123)	(0.078)	(0.119)	(0.094)
Δ NETH_2	0.222*	0.123	-0.071	0.283*	0.045	-0.089	-0.175*
	(0.074)	(0.095)	(0.053)	(0.137)	(0.086)	(0.132)	(0.105)
Δ NETH_3	0.053	0.083	-0.008	0.010	-0.153*	-0.183	-0.180*
	(0.071)	(0.091)	(0.051)	(0.131)	(0.082)	(0.126)	(0.100)
Δ SPA_1	0.194*	0.110	0.007	0.423*	0.121	0.118	-0.378**
	(0.081)	(0.104)	(0.058)	(0.149)	(0.094)	(0.144)	(0.114)
Δ SPA_2	0.359**	0.242*	0.093	0.266	-0.050	0.036	0.250*
	(0.086)	(0.110)	(0.062)	(0.158)	(0.099)	(0.152)	(0.121)
Δ SPA_3	-0.013	0.079	0.003	-0.041	-0.051	-0.100	0.158
	(0.082)	(0.105)	(0.059)	(0.152)	(0.096)	(0.146)	(0.116)
Error Correction Terms							
CointEq1	-0.214**	0.028	-0.031	-0.175	0.124	0.441**	0.369**
	(0.072)	(0.092)	(0.052)	(0.133)	(0.084)	(0.128)	(0.101)
CointEq2	0.197**	-0.032	-0.045	0.158	-0.098	-0.199*	-0.266**
	(0.053)	(0.068)	(0.038)	(0.097)	(0.061)	(0.094)	(0.074)
Log-likelihood and Criteria							
No. Parameters	168		AIC	-53.6452		SC	-49.449
Log-Likelihood	3038.017		HQ	-51.9439		FPE	-9.827e-025

Table A4.5: Reduced form of Model 2

Cointegrating Vectors							
AUS_1	BEL_1	FRA_1	GER_1	ITA_1	NETH_1	SPA_1	Constant
1	0	1.555	-0.289	-0.311	-0.829	-0.739	-4.961
0	1	2.651	0.427	-1.240	-1.347	-0.928	-8.674

Lagged Variables	Dependent Variables						
	Δ AUS	*Δ BEL*	*Δ FRA*	*Δ GER*	*Δ ITA*	*Δ NETH*	*Δ SPA*
Constant	0.002*	0.002*	0.001	-0.001	0.001	0.006**	0.005**
	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)
Δ AUS_1	0.026	0.191	0.261**	-0.010	-0.030	-0.092	0.348*
	(0.104)	(0.133)	(0.070)	(0.161)	(0.109)	(0.154)	(0.150)
Δ AUS_2	-0.392**	-0.087	-0.011	-0.203	0.007	-0.031	-0.209
	(0.095)	(0.121)	(0.064)	(0.150)	(0.100)	(0.143)	(0.136)
Δ AUS_3	-0.111	-0.116	0.114	-0.108	0.002	-0.166	0.026
	(0.098)	(0.125)	(0.066)	(0.147)	(0.101)	(0.141)	(0.141)
Δ BEL_1	-0.033	-0.216	0.262**	0.108	0.274**	0.121	0.326*
	(0.097)	(0.125)	(0.068)	(0.166)	(0.108)	(0.158)	(0.138)
Δ BEL_2	-0.304**	-0.006	0.215**	-0.149	0.044	-0.079	0.109
	(0.099)	(0.127)	(0.067)	(0.156)	(0.105)	(0.149)	(0.142)
Δ BEL_3	0.020	0.146	0.116	0.146	0.128	0.227	0.233
	(0.101)	(0.128)	(0.067)	(0.149)	(0.103)	(0.143)	(0.145)
Δ FRA_1	0.203	0.125	-0.155	-0.019	-0.156	0.147	0.161
	(0.141)	(0.181)	(0.095)	(0.213)	(0.146)	(0.204)	(0.204)
Δ FRA_2	0.124	-0.257	-0.008	0.031	0.041	-0.182	-0.105
	(0.132)	(0.169)	(0.088)	(0.198)	(0.136)	(0.190)	(0.190)
Δ FRA_3	0.063	-0.195	0.136	-0.048	0.010	-0.228	0.104
	(0.125)	(0.160)	(0.084)	(0.191)	(0.130)	(0.183)	(0.180)
Δ GER_1	-0.232**	-0.096	-0.111*	-0.174	-0.005	0.257*	0.058
	(0.074)	(0.095)	(0.051)	(0.119)	(0.079)	(0.113)	(0.107)
Δ GER_2	-0.012	-0.092	-0.059	0.036	0.090	0.227*	0.069
	(0.068)	(0.087)	(0.046)	(0.105)	(0.071)	(0.101)	(0.097)
Δ GER_3	-0.002	0.009	-0.024	0.073	0.095	0.263**	0.123
	(0.059)	(0.075)	(0.040)	(0.091)	(0.062)	(0.087)	(0.085)
Δ ITA_1	0.160	0.179	0.026	0.384*	0.196	0.037	-0.246
	(0.099)	(0.127)	(0.067)	(0.157)	(0.105)	(0.150)	(0.142)
Δ ITA_2	-0.028	0.155	0.037	-0.094	-0.110	0.010	0.162
	(0.098)	(0.125)	(0.066)	(0.146)	(0.101)	(0.140)	(0.141)
Δ ITA_3	-0.206*	-0.198	-0.085	-0.134	0.036	0.099	-0.087
	(0.092)	(0.118)	(0.062)	(0.141)	(0.096)	(0.135)	(0.133)
Δ LNETH_1	0.292**	0.069	-0.018	0.201*	-0.012	-0.195*	-0.161*
	(0.062)	(0.080)	(0.043)	(0.105)	(0.068)	(0.099)	(0.089)
Δ NETH_2	0.238**	0.132	-0.054	0.261**	0.050	-0.037	-0.194*
	(0.069)	(0.088)	(0.047)	(0.113)	(0.074)	(0.107)	(0.098)
Δ NETH_3	0.046	0.099	-0.016	-0.031	-0.124	-0.239*	-0.164
	(0.065)	(0.083)	(0.044)	(0.104)	(0.070)	(0.099)	(0.093)
Δ SPA_1	0.170*	0.132	-0.021	0.364*	0.170*	-0.021	-0.335**
	(0.076)	(0.097)	(0.053)	(0.129)	(0.084)	(0.122)	(0.108)
Δ SPA_2	0.360**	0.258**	0.093	0.223	-0.024	0.009	0.257*
	(0.079)	(0.102)	(0.055)	(0.131)	(0.086)	(0.125)	(0.113)
Δ SPA_3	0.013	0.081	0.031	-0.045	-0.062	0.002	0.124
	(0.074)	(0.095)	(0.050)	(0.110)	(0.076)	(0.105)	(0.107)
				Error Correction Terms			
CointEq1	-0.212**	-0.001	-0.026	-0.100	0.078	0.502**	0.353**
	(0.068)	(0.088)	(0.048)	(0.122)	(0.078)	(0.115)	(0.097)
CointEq2	0.192**	-0.007	-0.052	0.092	-0.056	-0.264**	-0.248**
	(0.050)	(0.064)	(0.035)	(0.086)	(0.056)	(0.081)	(0.070)

Log-likelihood and Criteria					
No. Parameters	132	**AIC**	-53.911	**SC**	-50.614
Log-Likelihood	3016.268	**HQ**	-52.575	**FPE**	-3.129e-024

Table A4.6: Reduced form of Model 3

Cointegrating Vectors							
AUS_1	BEL_1	FRA_1	GER_1	ITA_1	NETH_1	SPA_1	Constant
1	0	1.555	-0.289	-0.311	-0.829	-0.739	-4.961
0	1	2.651	0.427	-1.240	-1.347	-0.928	-8.674

Lagged Variables	Dependent Variables						
	Δ AUS	Δ BEL	Δ FRA	Δ GER	Δ ITA	Δ NETH	Δ SPA
Constant	0.002*	0.003**	0.002*	0.000	0.002*	0.006**	0.004**
	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)
Δ AUS_1	0.055	0.182*	0.268**	-0.009	-0.078	-0.034	0.361**
	(0.092)	(0.085)	(0.064)	(0.134)	(0.096)	(0.106)	(0.102)
Δ AUS_2	-0.311**	-0.096	-0.057	-0.297	-0.054	0.069	-0.007
	(0.081)	(0.065)	(0.056)	(0.109)	(0.081)	(0.079)	(0.081)
Δ AUS_3	-0.031	0.032	0.126*	-0.015	0.028	0.040	0.170*
	(0.081)	(0.053)	(0.056)	(0.099)	(0.079)	(0.059)	(0.072)
Δ BEL_1	-0.006	-0.068	0.298**	0.196	0.367**	0.239	0.343*
	(0.088)	(0.096)	(0.062)	(0.142)	(0.096)	(0.124)	(0.111)
Δ BEL_2	-0.217*	0.016	0.189**	-0.194	0.000	0.093	0.296*
	(0.086)	(0.077)	(0.060)	(0.124)	(0.089)	(0.096)	(0.094)
Δ BEL_3	-0.026	-0.003	0.100*	0.016	0.073	0.060	0.128
	(0.084)	(0.052)	(0.057)	(0.100)	(0.081)	(0.056)	(0.072)
Δ FRA_1	0.065	0.028	-0.114	-0.002	-0.108	-0.094	-0.148
	(0.117)	(0.073)	(0.081)	(0.141)	(0.113)	(0.079)	(0.101)
Δ FRA_2	0.173	-0.017	0.033	0.240	0.159	0.034	-0.015
	(0.110)	(0.070)	(0.075)	(0.133)	(0.106)	(0.076)	(0.095)
Δ FRA_3	0.125	0.060	0.189*	0.169	0.120	0.045	0.210*
	(0.104)	(0.067)	(0.072)	(0.127)	(0.101)	(0.074)	(0.091)
Δ GER_1	-0.296**	-0.155*	-0.100*	-0.235	0.027	0.104	-0.081
	(0.066)	(0.065)	(0.046)	(0.100)	(0.070)	(0.082)	(0.077)
Δ GER_2	-0.077	-0.120*	-0.042	0.016	0.139*	0.099	-0.063
	(0.059)	(0.053)	(0.041)	(0.084)	(0.061)	(0.065)	(0.064)
Δ GER_3	-0.076	-0.092*	-0.022	-0.003	0.105*	0.082	-0.031
	(0.051)	(0.043)	(0.035)	(0.070)	(0.052)	(0.052)	(0.052)
Δ ITA_1	0.191*	0.012	-0.049	0.195	0.049	-0.042	-0.112
	(0.082)	(0.058)	(0.057)	(0.104)	(0.081)	(0.067)	(0.076)
Δ ITA_2	-0.055	0.078	0.038	-0.132	-0.143	-0.057	0.085
	(0.083)	(0.058)	(0.057)	(0.104)	(0.081)	(0.066)	(0.076)
Δ ITA_3	-0.194*	-0.133*	-0.075	-0.125	0.068	0.095	-0.074
	(0.079)	(0.062)	(0.055)	(0.105)	(0.079)	(0.074)	(0.078)
Δ NETH_1	0.312**	0.097	-0.028	0.260*	-0.017	-0.118	-0.096
	(0.058)	(0.064)	(0.041)	(0.094)	(0.063)	(0.083)	(0.074)
Δ NETH_2	0.234*	0.041	-0.089**	0.198*	-0.012	-0.095	-0.164*
	(0.061)	(0.059)	(0.042)	(0.091)	(0.064)	(0.074)	(0.069)
Δ NETH_3	0.096	0.108*	-0.037	-0.015	-0.172*	-0.132*	-0.041
	(0.058)	(0.056)	(0.040)	(0.086)	(0.061)	(0.070)	(0.066)
Δ SPA_1	0.210**	0.003	-0.092*	0.208*	0.041	-0.058	-0.171*
	(0.065)	(0.057)	(0.045)	(0.092)	(0.066)	(0.071)	(0.069)
Δ SPA_2	0.294**	0.166*	0.107*	0.224*	-0.042	-0.112	0.093
	(0.070)	(0.068)	(0.049)	(0.105)	(0.074)	(0.086)	(0.080)
Δ SPA_3	-0.008	0.048	0.036	-0.045	-0.063	-0.028	0.060
	(0.062)	(0.039)	(0.043)	(0.075)	(0.060)	(0.043)	(0.054)
Error Correction Terms							
CointEq1	-0.270**	0.008	-0.005	-0.028	0.123	0.422**	0.235*
	(0.066)	(0.083)	(0.047)	(0.117)	(0.076)	(0.108)	(0.093)
CointEq2	0.231**	-0.025	-0.063	0.055	-0.099	-0.198*	-0.182**
	(0.048)	(0.058)	(0.034)	(0.083)	(0.054)	(0.077)	(0.066)

Log-likelihood and Criteria							
No. Parameters	96		AIC	-54.066		SC	-51.668
Log-Likelihood	2988.519		HQ	-53.094		FPE	1.015e-023

Table A4.7: Reduced form of Model 4

Cointegrating Vectors							
AUS_1	BEL_1	FRA_1	GER_1	ITA_1	NETH_1	SPA_1	Constant
1	0	1.555	-0.289	-0.311	-0.829	-0.739	-4.961
0	1	2.651	0.427	-1.240	-1.347	-0.928	-8.674
Lagged Variables	**Dependent Variables**						
	Δ AUS	Δ BEL	Δ FRA	Δ GER	Δ ITA	Δ NETH	Δ SPA
Constant	0.002*	0.003**	0.002*	0.000	0.002*	0.006**	0.004**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Δ AUS_1	0.043	0.245**	0.274**	0.067	-0.025	-0.046	0.346**
	(0.092)	(0.084)	(0.063)	(0.126)	(0.091)	(0.102)	(0.098)
Δ AUS_2	-0.319**	-0.075	-0.050	-0.283*	-0.045	0.064	-0.001
	(0.081)	(0.069)	(0.056)	(0.106)	(0.079)	(0.079)	(0.080)
Δ AUS_3	-0.035	0.056	0.127*	0.018	0.052	0.034	0.162*
	(0.082)	(0.058)	(0.057)	(0.094)	(0.075)	(0.057)	(0.070)
Δ BEL_1	-0.010	-0.045	0.296**	0.232	0.391	0.230	0.335**
	(0.089)	(0.098)	(0.062)	(0.141)	(0.095)	(0.124)	(0.110)
Δ BEL_2	-0.231*	0.078	0.198**	-0.129	0.043	0.079	0.290**
	(0.086)	(0.078)	(0.060)	(0.117)	(0.085)	(0.095)	(0.091)
Δ BEL_3	-0.033	0.018	0.106*	0.032	0.083	0.054	0.132*
	(0.084)	(0.059)	(0.058)	(0.096)	(0.077)	(0.056)	(0.070)
Δ FRA_1	0.072	0.005	-0.120	-0.021	-0.120	-0.088	-0.152
	(0.118)	(0.082)	(0.082)	(0.135)	(0.108)	(0.079)	(0.099)
Δ FRA_2	0.181	-0.046	0.024	0.219	0.146	0.040	-0.019
	(0.111)	(0.079)	(0.076)	(0.128)	(0.102)	(0.077)	(0.094)
Δ FRA_3	0.129	0.068	0.180*	0.201	0.143	0.042	0.190*
	(0.105)	(0.074)	(0.072)	(0.121)	(0.097)	(0.072)	(0.089)
Δ GER_1	-0.296**	-0.159*	-0.101*	-0.240*	0.023	0.104	-0.079
	(0.067)	(0.067)	(0.047)	(0.099)	(0.069)	(0.082)	(0.076)
Δ GER_2	-0.072	-0.141*	-0.047	-0.004	0.125	0.102	-0.061
	(0.060)	(0.055)	(0.042)	(0.083)	(0.060)	(0.066)	(0.063)
Δ GER_3	-0.075	-0.101*	-0.022	-0.017	0.095	0.082	-0.025
	(0.051)	(0.045)	(0.035)	(0.068)	(0.050)	(0.052)	(0.052)
Δ ITA_1	0.191*	-0.009	-0.045	0.154	0.020	-0.038	-0.095
	(0.082)	(0.061)	(0.057)	(0.098)	(0.077)	(0.064)	(0.073)
Δ ITA_2	-0.061	0.107	0.043	-0.101	-0.121	-0.062	0.080
	(0.083)	(0.063)	(0.057)	(0.100)	(0.078)	(0.066)	(0.075)
Δ ITA_3	-0.191*	-0.146*	-0.079	-0.135	0.061	0.097	-0.074
	(0.079)	(0.066)	(0.055)	(0.102)	(0.077)	(0.074)	(0.077)
Δ NETH_1	0.317**	0.072	-0.029	0.228*	-0.039	-0.112	-0.090
	(0.058)	(0.064)	(0.040)	(0.093)	(0.062)	(0.082)	(0.073)
Δ NETH_2	0.238**	0.011	-0.088*	0.153	-0.043	-0.088	-0.150*
	(0.061)	(0.058)	(0.042)	(0.086)	(0.061)	(0.072)	(0.067)
Δ NETH_3	0.095	0.113*	-0.034	-0.013	-0.170**	-0.131*	-0.041
	(0.058)	(0.058)	(0.041)	(0.086)	(0.060)	(0.071)	(0.066)
Δ SPA_1	0.214**	-0.030	-0.091*	0.157*	0.005	-0.051	-0.156*
	(0.064)	(0.057)	(0.045)	(0.086)	(0.063)	(0.067)	(0.066)
Δ SPA_2	0.293**	0.173*	0.108*	0.233*	-0.036	-0.112	0.090
	(0.070)	(0.071)	(0.049)	(0.104)	(0.073)	(0.086)	(0.080)
Δ SPA_3	-0.012	0.066	0.040	-0.028	-0.051	-0.031	0.060
	(0.062)	(0.044)	(0.043)	(0.072)	(0.057)	(0.042)	(0.053)
Error Correction Terms							
CointEq1	-0.257**	-0.045	-0.012	-0.085	0.085	0.437**	0.240*
	(0.066)	(0.080)	(0.046)	(0.113)	(0.073)	(0.106)	(0.091)
CointEq2	0.220**	0.019	-0.058	0.103	-0.067	-0.212**	-0.186**
	(0.047)	(0.055)	(0.033)	(0.078)	(0.051)	(0.074)	(0.063)
Log-likelihood and Criteria							
No. Parameters	93		**AIC**	-54.038		**SC**	-51.7145
Log-Likelihood	2984.012		**HQ**	-53.096		**FPE**	3.40E-23

Table A4.8: Reduced form of Model 5

			Cointegrating Vectors				
AUS_1	BEL_1	FRA_1	GER_1	ITA_1	NETH_1	SPA_1	Constant
1	0	1.555	-0.289	-0.311	-0.829	-0.739	-4.961
0	1	2.651	0.427	-1.240	-1.347	-0.928	-8.674
Lagged			Dependent Variables				
Variables	Δ AUS	Δ BEL	Δ FRA	Δ GER	Δ ITA	Δ NETH	Δ SPA
Constant	0.002*	0.003*	0.001*	0.000	0.002*	0.007**	0.004**
	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)
Δ AUS_1	-0.026	0.101	0.309**	0.177	-0.096		
	(0.089)	(0.097)	(0.066)	(0.144)	(0.096)		
Δ AUS_2	-0.329**	-0.083	-0.019	-0.298*	-0.028		
	(0.079)	(0.072)	(0.057)	(0.109)	(0.073)		
Δ AUS_3	-0.086	-0.049	0.159*	0.062	0.034		
	(0.082)	(0.070)	(0.059)	(0.108)	(0.071)		
Δ BEL_1	-0.091	-0.176	0.252**	0.144	0.356**		0.227*
	(0.085)	(0.101)	(0.061)	(0.140)	(0.093)		(0.101)
Δ BEL_2	-0.292*	-0.010	0.210**	-0.133	0.030		0.133
	(0.087)	(0.091)	(0.061)	(0.121)	(0.082)		(0.101)
Δ BEL_3	-0.051	-0.011	0.080	0.009	0.049		0.060
	(0.088)	(0.084)	(0.060)	(0.100)	(0.072)		(0.099)
Δ FRA_1	0.131	0.124	-0.171*	-0.058	-0.098		
	(0.119)	(0.099)	(0.085)	(0.152)	(0.100)		
Δ FRA_2	0.160	-0.094	0.018	0.205	0.158		
	(0.111)	(0.089)	(0.079)	(0.137)	(0.090)		
Δ FRA_3	0.058	-0.063	0.187*	0.227*	0.101		
	(0.105)	(0.088)	(0.075)	(0.134)	(0.089)		
Δ GER_1	-0.290**	-0.131*	-0.150**	-0.344**	0.027		
	(0.063)	(0.069)	(0.047)	(0.104)	(0.068)		
Δ GER_2	-0.076	-0.134*	-0.102*	-0.100	0.109*		
	(0.058)	(0.057)	(0.042)	(0.086)	(0.056)		
Δ GER_3	-0.083*	-0.101*	-0.083*	-0.105	0.073		
	(0.050)	(0.047)	(0.036)	(0.072)	(0.047)		
Δ ITA_1	0.232*	0.058	0.003	0.202*	0.020		
	(0.082)	(0.067)	(0.059)	(0.104)	(0.068)		
Δ ITA_2	-0.063	0.103	0.044	-0.067	-0.129*		
	(0.083)	(0.068)	(0.059)	(0.105)	(0.069)		
Δ ITA_3	-0.177*	-0.131*	-0.053	-0.159	0.074		
	(0.078)	(0.067)	(0.056)	(0.103)	(0.067)		
Δ NETH_1	0.333**	0.130*	-0.025	0.184	0.020	-0.138	
	(0.054)	(0.063)	(0.041)	(0.101)	(0.062)	(0.082)	
Δ NETH_2	0.295**	0.100*	-0.074*	0.207*	-0.023	0.047	
	(0.059)	(0.061)	(0.043)	(0.101)	(0.062)	(0.080)	
Δ NETH_3	0.128*	0.168*	0.016	0.073	-0.136*	0.007	
	(0.056)	(0.057)	(0.040)	(0.096)	(0.058)	(0.075)	
Δ SPA_1	0.199*	-0.082	-0.060	0.217*	-0.062		-0.265*
	(0.067)	(0.076)	(0.047)	(0.099)	(0.068)		(0.089)
Δ SPA_2	0.345**	0.274**	0.104*	0.273*	-0.004		0.199*
	(0.070)	(0.081)	(0.050)	(0.110)	(0.074)		(0.088)
Δ SPA_3	0.039	0.179*	0.039	-0.040	0.018		0.238*
	(0.067)	(0.071)	(0.046)	(0.090)	(0.063)		(0.085)
			Error Correction Terms				
CointEq1	-0.270**	-0.052	-0.061	-0.185	0.080	0.312**	0.276**
	(0.062)	(0.077)	(0.045)	(0.114)	(0.072)	(0.083)	(0.066)
CointEq2	0.236**	0.029	-0.025	0.171*	-0.066	-0.116*	-0.189**
	(0.045)	(0.054)	(0.032)	(0.079)	(0.050)	(0.053)	(0.045)
			Log-likelihood and Criteria				
No. Parameters	99		AIC	-54.093		SC	-51.620
Log-Likelihood	2992.999		HQ	-53.091		FPE	1.302e-023

Chapter 5 - Analysing the Euro Area Output Gap within a State-Space Framework

5.1 Introduction

In the previous chapter, the multivariate BN decomposition, and two univariate trend-cycle decomposition methodologies, were used to extract the cyclical component of GDP for each country under analysis. This chapter continues to analyse growth cycles by investigating three topics concerning the euro area output gap by utilising the multivariate unobserved-component (UC) model, which incorporates a statistical output decomposition along with macroeconomic models such as the Phillips curve and Okun's law.

A fundamental objective of monetary and fiscal policy is to dampen economic fluctuations by keeping output and unemployment close to their natural rates. To do this, economists need to identify accurately the unobserved features of an economy, such as potential output (trend output), the output gap (growth cycle) and the Non-Accelerating Inflation Rate of Unemployment (the NAIRU), from observables such as real GDP and the unemployment rate. Potential output is the maximum level of output that the economy can produce at a stable inflation rate, which should be accompanied by an unemployment rate that is consistent with the NAIRU. Deviations in output from this potential are defined as the output gap, which is usually used as an indicator of inflationary pressures. When the output gap is positive, output is above its potential, the inflation rate starts rising and tighter monetary policy is needed to curb demand and inflationary pressures. Conversely, when the output gap is negative, the inflation rate is below expectations and expansionary monetary policies are required to stimulate economic growth. Moreover, from a fiscal policy perspective, by knowing the output gap, the cyclical adjusted budget deficit can be calculated. This is important as it provides a measure of the health of the underlying public finances.

In short, the output gap plays a central role in determining the stance of monetary and fiscal policies. A macroeconomic policy based on accurate output gap estimates can help to mitigate the adverse effects associated with recessions and below trend growth, and provide sustainable economic growth. Conversely, basing economic policy on unreliable output gap estimates can damage the economy. For example, the surge in inflation during the 1970s was due in part to monetary policy underestimating the size of the output gap.

In the euro area, the European Central Bank (ECB) implements a two-pillar monetary policy strategy to maintain price stability. Over the medium to long run, the objective is for monetary growth to match the euro area's potential output growth. In the short run, the output gap, along with unit labour costs, exchange rates and asset prices, are used to indicate inflationary pressures. Output gap estimates are also used to calculate the cyclical adjusted budget deficit for member states to ensure that they achieve a medium term budget balance as defined in the Stability and Growth Pact (SGP). Finally, the NAIRU indicates the degree to which labour market reforms should be undertaken.

Three aspects of the euro area output gap are investigated in this chapter. First, the reliability of output gap estimates obtained from various multivariate UC models is assessed. Second, the degree of business cycle moderation in the euro area is analysed. Third, the effectiveness of monetary policy transmission through the interest rate channel is examined. The results show that changes in real interest rates have a significant impact on the euro area output gap in the run-up to EMU and thereafter.

The rest of the chapter is organised as follows. Section 5.2 presents a brief survey of frequently used trend-cycle decomposition methodologies, along with the literature on assessing the reliability of output gap estimates. Section 5.3 presents bivariate and trivariate models for estimating the output gap, core inflation and the NAIRU. The reliability of the output gap estimates obtained from these models is assessed in section 5.4. In section 5.5, business cycle moderation is studied by allowing for time-varying variances in the models' disturbances. Section 5.6 investigates the effectiveness of the interest rate channel for the aggregate euro area by examining the response of output gap estimates to changes in real interest rates. Finally, section 5.7 concludes.

5.2 Literature reviews

Potential output, the output gap and the NAIRU are unobservable quantities that need to be estimated from observed data. Many procedures have been proposed to estimate these unobservable features, and these can broadly be divided into three groups. The first group relies on purely statistical specifications, such as the Beveridge-Nelson (1981) decomposition, Harvey and Jaeger's (1993) UC model, the Hodrick-Prescott (1997) filter, and so on. These models simply 'let the data speak' and do not include potentially useful information about the supply side of the economy and business cycle information contained in macroeconomic variables other than aggregate output. The second group employs the production function approach (PFA), which has been widely used by international institutions, such as the OECD (2001), the International Monetary Fund (de Masi, 1997) and the European Commission (McMorrow and Roeger, 2001). Unlike the first group of univariate statistical approaches, the PFA is a multivariate method that obtains potential output from the levels of its structural determinates, such as productivity and factor inputs. A potential advantage of the PFA approach over the first group of statistical approaches is that it utilises a broad range of economic data. The third group of techniques incorporate a statistical output decomposition along with macroeconomic relations, including the Phillips curve, Okun's law, and other indicators such as output capacity utilisation and factor inputs. The method used in this chapter belongs to the third group.

Various UC specifications have been used by the third group. These models allow the rich dynamic interactions that occur between the observed and unobserved features of an economy to be modelled in specific ways according to the objectives of the research. An early example is the UC model proposed by Clark (1989), who estimated a bivariate model of US output and unemployment based on Okun's law. Subsequently, Kuttner's (1994) bivariate specification combined Watson's (1986) output decomposition with the Phillips curve to relate changes in inflation to the output gap. A trivariate model of output, inflation and unemployment was proposed by Apel and Jansson (1999) to systematically estimate the NAIRU and potential output for the UK, US and Canada. Rünstler (2002) extended Kuttner's (1994) bivariate model by including capacity utilisation and factor inputs to estimate the euro area output gap. A recent example of this approach is given by Proietti (2008), who estimated a multivariate model of the US economy using mixed frequency data including quarterly GDP and monthly industrial production, unemployment and CPI inflation.

All the models discussed above contain an element of measurement error. As such, it is important to assess the degree of uncertainty surrounding estimates of the output gap. The reliability of output gap estimates has been discussed in detail from both policy and academic standpoints, as it has significant implications for defining optimal monetary policy (Orphanides and van Norden, 1999; Orphanides, 2001; Camba-Méndez and Rodriguez-Palenzuela, 2001; Rünstler, 2002; Proietti *et al.*, 2007). Orphanides and van Norden (1999) highlight the disadvantages of basing monetary policy on real-time estimates of the output gap. These include the uncertainty surrounding these estimates and their susceptibility to future revisions. They propose a minimum requirement for assessing the reliability of the output gap, that initial estimates should not be significantly affected by subsequent statistical revisions caused by the arrival of additional information at a later date. In their study, four univariate trend-cycle decomposition models are examined.[1] Four output gap estimates are obtained from each model, including real-time, quasi-real, quasi-final and final estimates. By doing so, they successfully identify the extent to which revisions in the output gap estimates are caused by data revisions, statistical revisions and model uncertainty. They conclude that statistical revisions rather than data revisions appear to be the primary source of changes to estimates. Given this conclusion, Camba-Méndez and Rodriguez-Palenzuela (2001) assess the statistical reliability of the output gap estimates according to three criteria: the consistency of the initial estimates with their subsequent revisions, the output gap estimates' ability to forecast future inflation, and the positive correlation between gap estimates and capacity utilisation. Unlike the univariate models estimated in Orphanides and van Norden (1999), Camba-Méndez and Rodriguez-Palenzuela's analysis is based on multivariate models of output, unemployment and inflation for the US and the euro area. They find that the multivariate UC models give reasonably satisfactory output gap estimates. Rünstler (2002) also investigates the uncertainty of output gap estimates caused by subsequent statistical revisions. In doing so, the reliability of the output gap estimates from various UC models is assessed in terms of three criteria; standard errors, unbiasedness and inflation forecasting. The results

[1] The four types of models analysed in Orphanides and van Norden (1999) are deterministic trends, the HP filter, the BN decomposition and the UC models.

suggest that the multivariate models, with factor inputs and capacity utilisation included, significantly reduce the uncertainty in the filtering process.

5.3 Model specification

This section utilises various multivariate UC models to produce output gap estimates. These models combine aggregate output with other macroeconomic variables that provide information about the output gap. A bivariate model of output and inflation and a trivariate model of output, inflation and unemployment, are estimated in this study.

5.3.1 Bivariate model of the output gap and inflation

We begin with an UC model containing two variables, output and inflation, that incorporates a Phillips curve relationship. The Phillips curve establishes a relationship between nominal prices and excess demand, typically proxied by estimates of the output gap. The output equation follows Harvey and Jaeger's (1993) decomposition, in which the output series, y_t, is decomposed into a trend component, μ_t, and a cyclical component, ψ_t:

$$y_t = \mu_t + \psi_t, \quad t = 1,...,T. \tag{5.1}$$

The cyclical component is specified as a second-order autoregressive process

$$\psi_t = \phi_1 \psi_{t-1} + \phi_2 \psi_{t-2} + k_t, \; k_t \sim \text{NID}(0, \sigma_k^2) \tag{5.2}$$

with $\phi_1 = 2\rho\cos\lambda_c$ and $\phi_2 = -\rho^2$. The parameters ρ and λ_c are the damping factor and cycle frequency, respectively, and satisfy $0 \le \rho < 1$ and $0 < \lambda_c < \pi$. If $\rho = 0$, the cycle reduces to a Gaussian white noise process, $\psi_t = k_t$. The stochastic trend component is given by

$$\mu_t = \mu_{t-1} + \beta_{t-1} + m + \eta_t, \; \eta_t \sim \text{NID}(0, \sigma_\eta^2), \tag{5.3}$$

$$\beta_t = \phi\beta_{t-1} + \xi_t, \;\; \xi_t \sim \text{NID}(0,\sigma_\xi^2),$$

where β_t is the slope of the trend, m is a constant drift and ϕ is the damping factor. The disturbances k_t, η_t and ξ_t are assumed to be mutually and serially independent. As proposed by Proietti *et al.* (2007), four alternative specifications of the trend component are obtained by imposing restrictions on equation (5.3). First, by setting $m = 0$, $\sigma_\xi^2 = 0$ and $\phi = 1$, the trend component reduces to a random walk with drift (RW). Second, if $|\phi| < 1$ the trend component has a damped slope (DS). Third, if $m = 0$ and $\phi = 1$ the trend becomes a local linear trend (LLT). Finally, Hodrick and Prescott's (1997) restrictions (i.e., $\sigma_k^2 = 1600\sigma_\xi^2$, $\sigma_\eta^2 = 0$ and $\rho = 0$) can also be imposed on the LLT model, yielding a smooth trend component and a white noise process. Hence, σ_k^2 is the only variance parameter to be estimated in the HP trend.

The following inflation equation is based on the Gordon (1997) triangle model, where the inflation rate is a function of inertia, excess demand and supply shocks, such as changes in the euro's nominal effective exchange rate ($\Delta Neer_t$) and commodity prices ($\Delta Compr_t$).[2]

$$\Delta P_t = \tau_t + \delta_C(L)\Delta Compr_t + \delta_N(L)\Delta Neer_t + \varepsilon_t, \; \varepsilon_t \sim \text{NID}(0,\sigma_\varepsilon^2), \quad (5.4)$$
$$\tau_t = \tau_{t-1} + \theta_\psi(L)\psi_t + \eta_{\pi}, \;\; \eta_{\pi} \sim \text{NID}(0, \sigma_\tau^2).$$

The inflation rate, ΔP_t, calculated as the logged difference of the CPI, is driven by its core component, τ_t, the supply shock proxies mentioned above, and an irregular term, ε_t. Entering the current and lagged values of output gap estimates into the core inflation equation allows the level effect of the output gap on inflation to be separated from the changes effect. The lag polynomial $\theta_\psi(L) = \theta_{\psi 0} + \theta_{\psi 1}L$ can therefore be written as $\theta_\psi(L) = \theta_\psi(1) - \theta_{\psi 1}\Delta$. If $\theta_\psi(1) = 0$, the output gap has only a transitory

[2] The Gordon triangle model used in Chapter 5 reflects backward looking inflation expectations. However, more recent literature, such as Doménech and Gómez (2006) and Berger (2008), uses the New Keynesian approach of a forward looking Phillips curve, as inflation depends on future inflation expectations.

impact on core inflation. Finally, the model is completed by assuming that the disturbances ε_t and η_π are mutually and serially uncorrelated and also orthogonal to disturbances in the output equations.

Once equations (5.1), (5.2), (5.3) and (5.4) have been recast in a state-space representation, the hyperparameters $(\phi_1, \phi_2, \theta_{\psi 0}, \theta_{\psi 1}, \phi, m, \sigma_\eta^2, \sigma_\xi^2, \sigma_k^2, \sigma_\varepsilon^2, \sigma_\tau^2)$ in the model can be estimated by MLE using a KF initiated with a diffuse prior, and the unobserved components $(\mu_t, \beta_t, \psi_t, \psi_{t-1}, \tau_t)$ are then estimated using a smoothing algorithm proposed by De Jong (1989, 1991).[3]

5.3.2 Trivariate model of the output gap, inflation and unemployment

The bivariate model is extended to include a third set of equations concerning the decomposition of the unemployment rate, denoted as u_t. The unemployment rate is decomposed into the NAIRU, u_{ut}, and a cyclical unemployment component, $\theta_{u0}\psi_t + \theta_{u1}\psi_{t-1} + \psi_{ut}$, that consists of the weighted output gap estimates, $\theta_{u0}\psi_t + \theta_{u1}\psi_{t-1}$, and an idiosyncratic cycle, ψ_{ut}:

$$
\begin{aligned}
u_t &= u_{ut} + \theta_{u0}\psi_t + \theta_{u1}\psi_{t-1} + \psi_{ut}, \qquad (5.5)\\
u_{ut} &= u_{ut-1} + \beta_{ut-1} + \eta_{ut},\ \eta_{ut} \sim \mathrm{NID}(0, \sigma_{u\eta}^2),\\
\beta_{ut} &= \beta_{ut-1} + \xi_{ut},\ \xi_{ut} \sim \mathrm{NID}(0, \sigma_{u\xi}^2),\\
\psi_{ut} &= \phi_{u0}\psi_{ut-1} + \phi_{u1}\psi_{ut-2} + k_{ut},\ k_{ut} \sim \mathrm{NID}(0, \sigma_{uk}^2).
\end{aligned}
$$

The NAIRU for the euro area aggregate is modelled as a LLT due to the presence of a time-varying slope in the unemployment rate. The unemployment rate rose from 2% in 1973-1974 to 11% by the mid-1990s and did not return to the levels observed in the early 1970s even when inflation stabilised at a low level. This suggests that the NAIRU has risen.

[3] The state-space form for equations (5.1), (5.2), (5.3) and (5.4) is presented in Appendix A5, together with the augmented Kalman filter iterations introduced by De Jong (1989, 1991).

The trivariate model containing equations (5.1), (5.2), (5.3), (5.4) and (5.5) can also be put into state-space form, with the KF and ML approach then applied to estimate the model's hyperparameters $\left(\phi_1, \phi_2, \theta_{\psi 0}, \theta_{\psi 1}, \theta_{u0}, \theta_{u1}, \phi_{u0}, \phi_{u1}, \phi, m, \sigma_\eta^2, \sigma_\xi^2, \sigma_k^2, \sigma_\varepsilon^2, \sigma_\tau^2, \sigma_{u\eta}^2, \sigma_{u\xi}^2, \sigma_{uk}^2\right)$ and the unobserved state components $(\mu_t, \mu_{ut}, \beta_t, \beta_{ut}, \psi_t, \psi_{t-1}, \psi_{ut}, \psi_{ut-1}, \tau_t)$.[4] In addition, the four alternative output trend specifications discussed above are also applied to the trivariate model.

5.4 Criteria to assess the reliability of output gap estimates

Given the variety of models used in this chapter to estimate the output gap, it is essential to have some criteria to judge which model provides the most reliable estimates. Due to the data limitations in the euro area, this chapter focuses on analysing the statistical revisions which have occurred over time. The output gap estimates obtained from the bivariate and trivariate models are assessed against three criteria, the size of the revisions, the unbiasedness of the filtered estimates, and their ability to forecast future inflation rates.

5.4.1 Size of revisions

In the UC models, the filtered output gap, $\psi_{t|t}$, and its smoothed estimates, $\psi_{t|T}$, obtained using the final data release and the full-sample parameter estimates, are defined as the quasi-final and final estimates by Orphanides and van Norden (1999). The difference between these two estimates reflects the revisions caused by uncertainty in the filtering process. It is believed that this uncertainty declines as the information set is expanded, such as by using longer sample periods or including additional variables in the filtering process (Rünstler, 2002). This chapter examines this assertion by evaluating the Root-Mean-Squared Error (RMSE) of $\psi_{t|t}(\hat{\Xi}) - \psi_{t|t+s}(\hat{\Xi})$, where $s = 8$ and $s = 12$, based on the full-sample parameter estimates, $\hat{\Xi}$.[5] The estimates of $\psi_{t|t}$

[4] The state-space form for the trivariate model is also presented in Appendix A5.

[5] It is important to stress that the revisions analysed in this study are consistent with Rünstler (2002), but are fundamentally different from the revisions discussed in Camba-Méndez and Rodriguez-Palenzuela

and $\psi_{t|t+s}$ are produced using both the bivariate and trivariate models analysed above, together with a univariate model, specified as just equations (5.1), (5.2) and (5.3). As with the multivariate models[6], four alternative output trend specifications are also applied to the univariate model.[7] The model which provides the smallest RMSE is considered to be the most appropriate for estimating the output gap.

5.4.2 Unbiasedness of filtered estimates

In this section a test proposed by Rünstler (2002) is used to examine the unbiasedness of the filtered estimates, $\psi_{t|t}$, to subsequent revisions, $\psi_{t|t+s} - \psi_{t|t}$. The variables $\psi_{t|t}$ and $\psi_{t|t+s}$ are the minimum mean square estimates of ψ_t, conditional on information available up to and including t and $t+s$, respectively (Harvey, 1989). These estimates are unbiased in the sense that the expectation of the estimation error is zero, i.e., $\mathrm{E}(\psi_t - \psi_{t|t}) = 0$ and $\mathrm{E}(\psi_t - \psi_{t|t+s}) = 0$. This property and the linearity of the expectation operator imply that $\mathrm{E}(\psi_{t|t+s} - \psi_{t|t}) = 0$. In addition, according to the law of iterated expectations (Gourieroux and Monfort, 1989), the property that $\mathrm{E}((\psi_{t|t+s} - \psi_{t|t})\psi_{t|t}) = 0$ can be derived. Since

$$\mathrm{E}(\psi_{t/t+s}|t) = \mathrm{E}(\mathrm{E}(\psi_t|t+s)|t) = \mathrm{E}(\psi_t|t) = \psi_{t/t},$$

it holds that $\mathrm{E}(\psi_{t|t+s} - \psi_{t|t}|t) = 0$, implying $\psi_{t|t}$ is the orthogonal projection of $\psi_{t|t+s}$ conditional on the information set at time t. Therefore, the following equation holds

$$\mathrm{E}((\psi_{t|t+s} - \psi_{t|t})\psi_{t|t}) = \mathrm{E}(\mathrm{E}(\psi_{t|t+s} - \psi_{t|t}|t)\psi_{t|t}) = \mathrm{E}(0|\psi_{t|t}) = 0.$$

The test for unbiasedness is designed as the regression of revisions on the filtered estimates,

(2001). In this latter study the revisions are caused by both filtering uncertainty and recursive estimation of the model hyperparameters.

[6] The multivariate models used here and later refer to the four bivariate and four trivariate models analysed in this study.

[7] The parameter estimates for the univariate models are reported in Appendix B6.

$$\psi_{t|t+s} - \psi_{t|t} = b_0 + b_1\psi_{t|t} + \varepsilon_t$$

with unbiasedness requiring $b_0 = b_1 = 0$. The Newey-West (1987) estimators of the standard errors are used to obtain the HAC t-statistics for the individual null hypotheses $b_0 = 0$ and $b_1 = 0$.

5.4.3 Inflation forecast

In macroeconomics, the output gap is considered to be an indicator of inflationary pressure and thus should provide an indication of future inflation. Several studies (Gerlach and Svensson, 2003; Rünstler, 2002; Proietti *et al.*, 2007) have used estimates of the output gap to forecast future inflation rates. In this section, the ability of the output gap to forecast future inflationary pressures is used as an important criterion in judging the reliability of output gap estimates. The variable to be forecast is the quarterly inflation rate. Since this variable is very volatile, the average out-of-sample forecasting performances of the bivariate and trivariate models are evaluated against two benchmark models.

The first benchmark is the univariate model of inflation. Unlike equation (5.4), estimates of the output gap do not enter the core inflation equation. Instead, core inflation simply follows a random walk process,

$$\Delta P_t = \tau_t + \delta_C(L)\Delta Compr_t + \delta_N(L)\Delta Neer_t + \varepsilon_t,\ \varepsilon_t \sim \text{NID}(0, \sigma_\varepsilon^2), \quad (5.6)$$
$$\tau_t = \tau_{t-1} + \eta_\pi,\ \eta_\pi \sim \text{NID}(0, \sigma_\tau^2).$$

The second model is an AR(1) specification of the first-differences of the inflationary process,

$$\Delta^2 P_t = \phi\Delta^2 P_{t-1} + \delta_C(L)\Delta^2 Compr_t + \delta_N(L)\Delta^2 Neer_t + \upsilon_t,\ \upsilon_t \sim \text{NID}(0,\ \sigma_\upsilon^2). \quad (5.7)$$

A rolling sample approach is used, with the full-sample period first divided into a pre-forecasting period from 1971Q3 to 1999Q4 and a forecasting period from 2000Q1 to 2005Q4. The pre-forecasting sample moves forward quarter by quarter and the model's hyperparameters are re-estimated at each step until the end of the sample is reached. This approach calculates unconditional forecasts of inflation. This means that forecasts of the future inflation rate are based on the predicted future output gap. The h-step-ahead forecast of inflation can thus be obtained by iterating the transition and measurement equations in the state-space form.[8] In total, 25 one-quarter-ahead forecasts and 17 eight-quarter-ahead forecasts are calculated.

The Modified Diebold and Mariano (MDM) test proposed by Harvey *et al.* (1997) is used to examine whether the differences in forecasting ability between the multivariate models and the two benchmark models are significant. Since this test statistic corrects for size distortions in small samples, it is more appropriate than the test statistics introduced in Diebold and Mariano (1995). The MDM statistic is specified as

$$MDM = \left[\frac{N+1-2K+N^{-1}K(K-1)}{N}\right]\frac{\bar{d}}{\sqrt{V(\bar{d})}}$$

where N and K denote the number of forecasts and the lag length respectively. $\bar{d}$ is the mean of a differential loss function, defined as $\bar{d} = N^{-1}\sum(e_{1t}^2 - e_{2t}^2)$, where e_{1t} and e_{2t} are the h-step-ahead forecast errors obtained from models 1 and 2. The variance of $\bar{d}$ is estimated using the heteroskedastic-autocorrelation consistent (HAC) estimator

$$\hat{V}(\bar{d}) = N^{-1}\left[\gamma_0 + 2N^{-1}\sum_{K=1}^{h-1}(N-K)\gamma_K\right].$$

Therefore, $\dfrac{\bar{d}}{\sqrt{V(\bar{d})}}$ is simply a HAC t-statistic. The HAC estimate of the standard error is calculated using the Bartlett kernel with a lag length of $h-1$ when the forecast horizon is h periods.

[8] The procedure of unconditional forecasting is discussed in Appendix A5.4.

5.5 Data and empirical results

The empirical analysis in this chapter is based on logged quarterly data for the aggregate euro area from 1970Q1 to 2005Q4. The data is taken from the AWM database (Fagan *et al.*, 2001).[9] All the series are seasonally adjusted except for the CPI and commodity prices index. Therefore, the Census X-12 procedure (US Bureau of the Census, 1999) is used to deseasonalise these two series. The ADF test statistics reported in Table 5.1 suggest that real GDP, the unemployment rate, commodity prices and nominal effective exchange rates are I(1) series, while the CPI appears to be an I(2) variable. The main data are plotted in Figure 5.1.

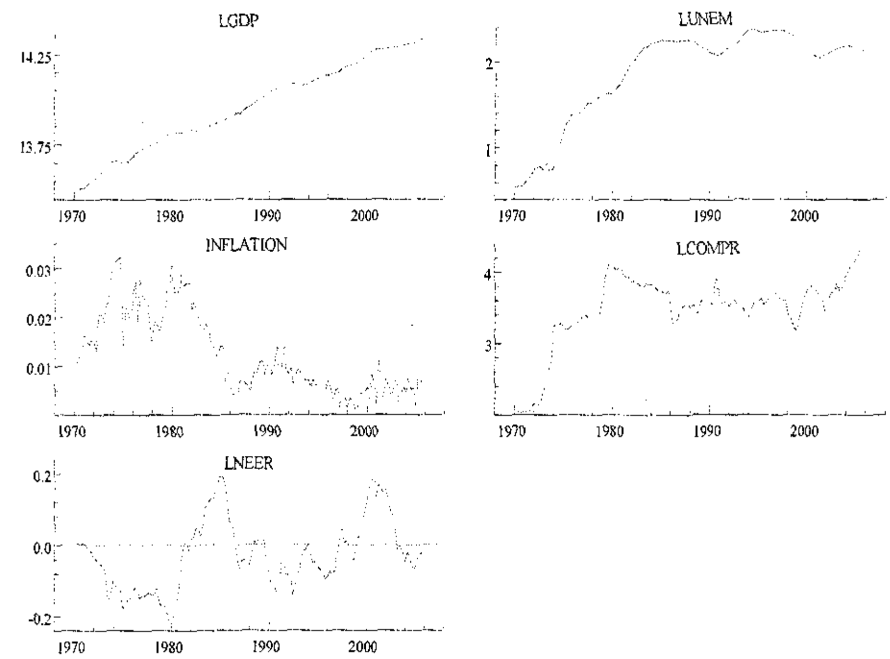
Table 5.1: The Augmented Dickey-Fuller Test Statistics

	Level Variables		First Differenced Variables
	Constant	Constant + Trend	Constant
GDP	-1.946	-2.730	-8.493**
	(0.310)	(0.226)	(0.000)
CPI	-3.928**	-3.048	-0.916
	(0.002)	(0.123)	(0.781)
UNEM	-2.950*	-1.529	-4.258**
	(0.042)	(0.815)	(0.001)
COMPR	-2.477	-2.507	-8.522**
	(0.123)	(0.324)	(0.000)
NEER	-2.333	-2.582	-8.557**
	(0.163)	(0.289)	(0.000)

Notes: p-values are in parentheses. * denotes significance at 5% and ** at 1%. UNEM = the Unemployment Rate, COMPR = Commodity Prices Index and NEER = Nominal Effective Exchange Rates.

[9] The data is taken from an updated version of the previous AWM database constructed by Fagan et al. (2001). This new version contains data prior to 1996 drawn from the previous version and extended data to 2005Q4 adjusted for the latest changes in the national accounts, including the introduction of chained volume measures.

Figure 5.1: Data



The parameter estimates for the four bivariate models are reported in Table 5.2. Two dummy variables are included in the model to capture the sudden drop in GDP observed during 1974Q4-1975Q1, and in inflation during 1975Q1. One striking result is that the model with the HP restrictions imposed is strongly rejected, as the residuals show strong autocorrelation patterns. Of the three other models, that incorporating the DS output trend provides the best fit to the data. Therefore, the following interpretation focuses primarily on this model. The fluctuations in the output gap estimates appear to be persistent as the sum of the parameter estimates ϕ_1 and ϕ_2 is close to one. The potential output growth follows an AR(1) process with the coefficient equal to 0.83. The significance of $\theta_{\psi 0}$ and $\theta_{\psi 1}$ at the 1% level implies that the output gap makes a significant contribution to core inflation. Although the null hypothesis of long-run neutrality of the output gap for core inflation is rejected by the Wald test statistic, the change effect of the output gap has a dominating impact on inflation. For instance, the change effect, measured by $-\theta_{\psi 1}$, is 0.21, while the level effect is only $\theta_{\psi 0}+\theta_{\psi 1}=0.061$ in the DS model. Finally, the parameter estimates of the dummy variables and the current supply shock variables are all found to be statistically significant. Potential output, the output gap and core inflation estimated from the bivariate models with the DS and HP output trend are plotted in Figure 5.2.

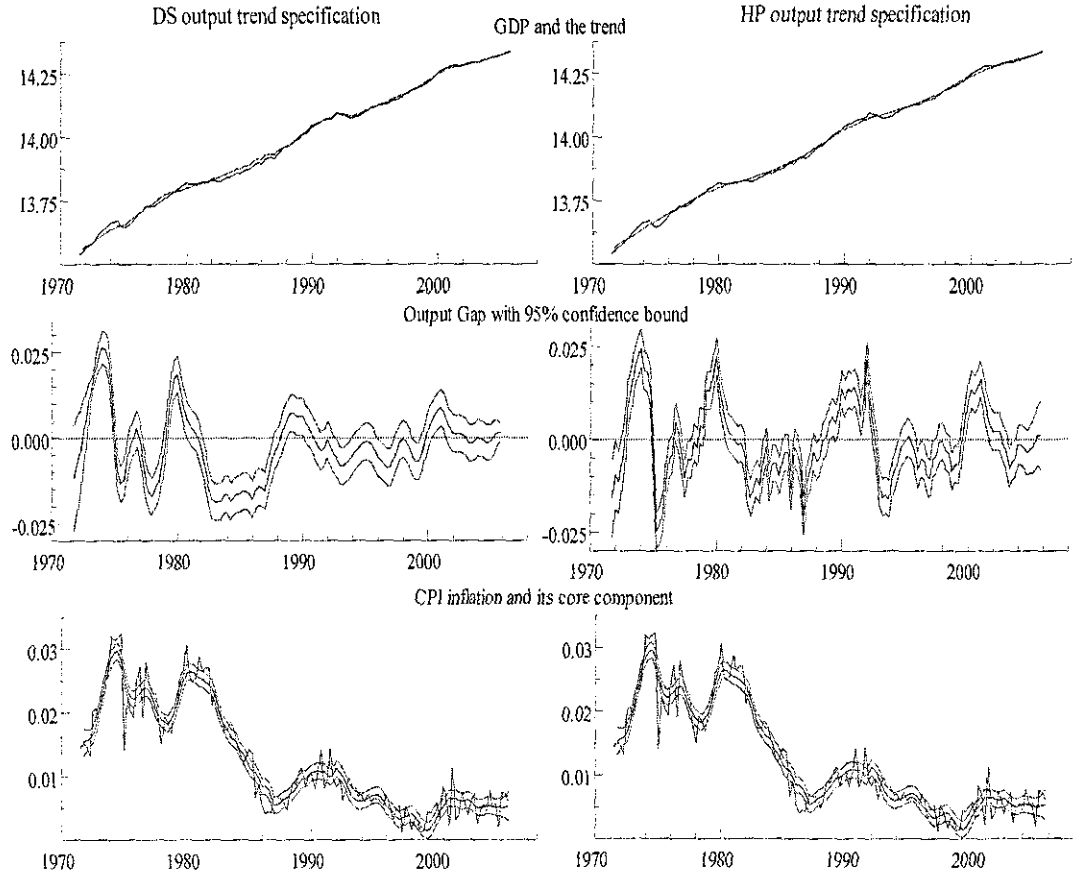
Unsurprisingly, the HP trend appears smoother than the DS trend, while the HP cycle is more volatile compared to the DS cycle.

Table 5.2: Parameter Estimates and Diagnostics for Bivariate Model

	RW	DSlope	LLT	HP
Output equation				
σ_η^2	165.280	137.040	148.700	0
σ_ξ^2	0	11.730	3.832	0.930
σ_k^2	65.455	69.325	63.923	1488.100
ϕ	1	0.828	1	1
m	0	0.006	0	0
ϕ_1	1.658**	1.550**	1.549**	-
	(0.019)	(0.029)	(0.026)	-
ϕ_2	-0.734**	-0.661**	-0.663**	-
	(0.034)	(0.051)	(0.046)	-
ρ	0.856	0.813	0.814	-
$2\pi / f_c$	24.747	20.508	19.980	-
Dum75Q1	-0.012*	-0.011*	-0.011*	-0.019
	(0.005)	(0.005)	(0.005)	(0.018)
Dum74Q4	-0.011**	-0.011**	-0.011**	-0.012**
	(0.003)	(0.003)	(0.003)	(0.004)
Inflation equation				
σ_e^2	31.067	35.388	35.247	30.250
σ_τ^2	7.385	7.942E-05	0.026	55.379
$\theta_{\psi 0}$	0.221**	0.276**	0.273**	0.177**
	(0.076)	(0.080)	(0.079)	(0.043)
$\theta_{\psi 1}$	-0.185**	-0.214**	-0.206**	-0.127**
	(0.068)	(0.074)	(0.072)	(0.044)
δ_{N1}	0.022*	0.023*	0.023*	0.024*
	(0.010)	(0.010)	(0.010)	(0.010)
δ_{N2}	0.013	0.015	0.015	0.015
	(0.010)	(0.010)	(0.010)	(0.010)
δ_{C1}	0.006*	0.006*	0.006*	0.007*
	(0.003)	(0.003)	(0.003)	(0.003)
δ_{C2}	0.004	0.004	0.004	0.005*
	(0.003)	(0.003)	(0.003)	(0.003)
Dum75Q1	-0.011*	-0.011*	-0.011*	-0.020
	(0.005)	(0.005)	(0.005)	(0.013)
Wald test for long run neutrality				
$\theta_\psi(1) = 0$	3.553	6.057*	7.186**	10.269**
Diagnostics and goodness of fit				
Log-likelihood	1148.162	1155.184	1150.570	1018.054
Q(4) y_t:	4.524	2.765	2.152	233.362**
Q(4) Δp_t:	5.782	5.946	5.578	40.758**
Normality y_t:	4.311	4.630	5.890	13.622**
Normality Δp_t:	1.634	2.014	2.134	4.161

Notes: Standard errors are provided in parentheses. The variance parameters are multiplied by 10^7. Q(4) is the Ljung-Box Q-statistic for residual autocorrelation using four autocorrelations. Normality of the residuals is checked using the Jarque-Bera test statistic. * denotes significance at the 5% level; ** denotes significance at the 1% level.

Figure 5.2: Trend, Cycle and Core Inflation for the Bivariate Models with the DS and HP output trend



The trivariate model outlined in the previous section is also estimated. The Ljung-Box Q-statistic at four lags indicates that there is significant autocorrelation in the residuals of the inflation and unemployment equations. The autocorrelation functions of these residuals were therefore examined and suggested the presence of third-order autocorrelation in the former and fourth-order autocorrelation in the latter. Therefore, moving average terms at lags three and four are included in the corresponding equations to eliminate the autocorrelation pattern in the residuals. The estimation results for the four trivariate models are reported in Table 5.3. Corresponding to the sudden decline observed in the output and inflation variables, the adverse effect of the first oil price shock also led to a dramatic increase in the unemployment rate between 1974Q4 and 1975Q2. Therefore three dummy variables are used for this period. As with the bivariate models, the model with the HP restrictions is again rejected due to strong autocorrelation in the residuals. The model with the DS output trend outperforms the other models. Both the output gap and potential output appear to be more persistent than in the bivariate models. The output gap has a cycle period of around 28 quarters and the potential output growth evolves with a slope coefficient of 0.93. However, parameter estimates of $\theta_{\psi 0}$ and $\theta_{\psi 1}$ are smaller than the corresponding values in the bivariate models, with a level effect of $\theta_{\psi 0}+\theta_{\psi 1}=0.025$ and a change effect of $-\theta_{\psi 1}=0.149$ in the DS model. The null of long-run neutrality cannot be rejected by the Wald test of the restriction $\theta_{\psi}(1)=0$ at the 5% level for all models. This implies that the output gap has only a transitory effect on inflation. The parameter estimates of θ_{u0} and θ_{u1} are all negative, suggesting that the unemployment rate is anti-cyclical with the output gap. Moreover, only θ_{u1} appears statistically significant, which may reflect adjustment delays in the labour market. It is also important to note that, although $\phi_{3\varepsilon}$ and ϕ_{4u} are only marginally significant at the 10% level, they nevertheless successfully eliminate the autocorrelation in the residuals.

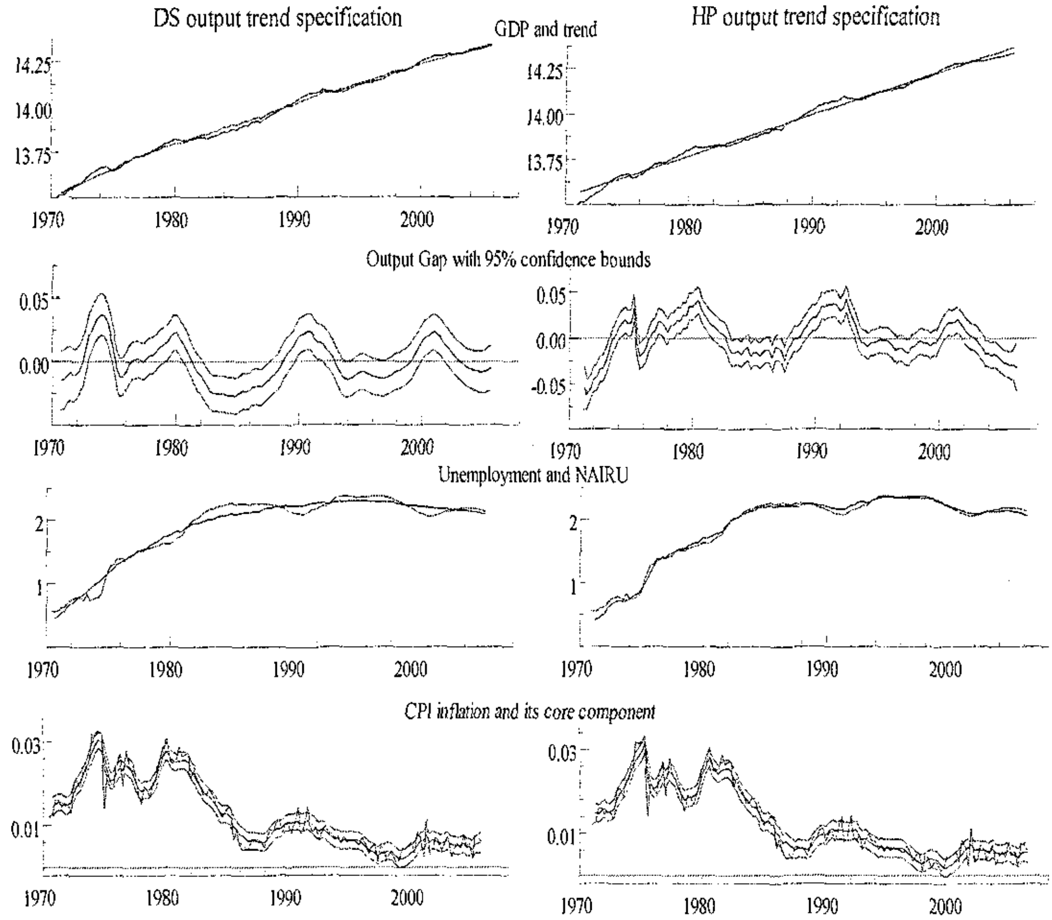
Figure 5.3 presents the time path of potential output, the output gap, the NAIRU and core inflation estimated from the trivariate models with the DS and HP output trend specifications imposed. The HP output trend again appears very smooth, being very close to a linear time trend, while the HP output cycle is more volatile than the DS output cycle. The estimate of the NAIRU from the model with the DS output trend appears less volatile than when it is obtained from the model imposed with the HP restrictions. This is because, when potential output is modelled as an HP trend, a highly volatile output gap is produced and enters into the unemployment equation. This may lead to a large variance in the NAIRU slope disturbance.

Table 5.3 Parameter Estimates and Diagnostics for Trivariate Model

	RW	DSlope	LLT	HP
Output equation				
σ^2_{η}	165.920	150.850	163.030	0
σ^2_{ξ}	0	4.883	0.176	0.936
σ^2_{κ}	49.151	48.944	49.048	1497.100
ϕ	1	0.934	1	1
m	0	0.006	0	0
ϕ_1	1.740**	1.756**	1.747**	-
	(0.066)	(0.049)	(0.063)	-
ϕ_2	-0.808**	-0.810**	-0.799**	-
	(0.108)	(0.090)	(0.116)	-
ρ	0.898	0.900	0.935	-
$2\pi / f_c$	28.398	28.443	29.173	-
Dum74Q4	-0.010	-0.010	-0.010	-0.023
	(0.006)	(0.006)	(0.006)	(0.015)
Dum75Q1	-0.012*	-0.012*	-0.012*	-0.019
	(0.006)	(0.006)	(0.006)	(0.022)
Inflation equation				
σ^2_{ε}	24.422	24.406	24.867	27.539
σ^2_{τ}	16.008	16.017	15.346	61.136
$\theta_{\psi 0}$	0.174*	0.174*	0.174*	0.185**
	(0.066)	(0.069)	(0.074)	(0.053)
$\theta_{\psi 1}$	-0.149*	-0.149*	-0.148*	-0.140*
	(0.065)	(0.067)	(0.072)	(0.053)
δ_{N1}	0.016	0.016	0.016	0.022
	(0.012)	(0.012)	(0.012)	(0.012)
δ_{N2}	0.015	0.015	0.015	0.019
	(0.012)	(0.012)	(0.012)	(0.012)
δ_{C1}	0.004	0.004	0.004	0.004
	(0.004)	(0.004)	(0.004)	(0.004)
δ_{C2}	0.005	0.004	0.005	0.006
	(0.004)	(0.004)	(0.004)	(0.004)
$\phi_{3\varepsilon}$	0.399	0.399	0.392	0.370*
	(0.248)	(0.252)	(0.285)	(0.219)
Dum75Q1	-0.010**	-0.010**	-0.010**	-0.011*
	(0.003)	(0.003)	(0.003)	(0.005)
Unemployment equation				
$\sigma^2_{u\eta}$	185.030	240.350	200.080	192.030
$\sigma^2_{u\beta}$	62.259	62.900	57.385	1379.600
σ^2_{uk}	0.005	0.011	0.005	0.004
θ_{u0}	-1.873	-1.914	-1.937	-1.079*
	(1.675)	(1.094)	(3.017)	(0.531)
θ_{u1}	-4.529**	-4.461**	-4.537*	-1.152*
	(1.425)	(1.468)	(2.481)	(0.457)
ϕ_{4u}	0.485	0.496	0.482	0.503*
	(0.320)	(0.328)	(0.375)	(0.303)
Dum74Q4	0.095**	0.094**	0.095**	0.106**
	(0.020)	(0.020)	(0.021)	(0.027)
Dum75Q1	0.098**	0.097**	0.097**	0.114**
	(0.026)	(0.026)	(0.028)	(0.040)
Dum75Q2	0.088**	0.088**	0.088**	0.090**
	(0.021)	(0.021)	(0.022)	(0.032)
Wald test for long run neutrality				
$\theta_{\psi}(1)=0$	2.871	2.777	3.053	4.991*
Diagnostics and goodness of fit				
Log-likelihood	1533.808	1541.251	1534.337	1386.907
Q(4) y_t :	3.407	0.661	0.823	210.763**
Q(4) u_t :	6.492	5.293	5.822	26.814**
Q(4) Δp_t :	2.093	2.225	2.201	57.623**
Normality y_t :	5.862	5.360	5.954	11.577**
Normality u_t :	11.381**	11.170**	11.970**	9.722**
Normality Δp_t	3.805	4.113	3.829	4.328

Note: Please see notes underneath Table 5.2.

Figure 5.3: Trend, Cycle, the NAIRU and Core Inflation for the Trivariate Models with the DS and HP output trend



The three criteria discussed in section 5.4 are now used to assess the reliability of the output gap estimates. The size of the revisions, as measured by the RMSE, is reported in the top panel of Table 5.4. On average, the bivariate models yield the smallest revisions, followed by the trivariate and univariate models. This suggests that the model embodying the Phillips curve relation improves the reliability of the output gap estimates. However, the RMSE becomes larger when the unemployment rate is included in the model. This contradicts the assertion given by Rünstler (2002) that output gap estimates can potentially be improved by including additional information in the filtering process. One possible explanation for this is that cyclical fluctuations in the unemployment rate may not be closely correlated with the output gap due to rigid labour market institutions. It is also worth noting that the DS output trend specification gives the smallest RMSE for the univariate and trivariate models. Although it is the LLT which provides the smallest RMSE for the bivariate model, the DS trend yields the second smallest value among the bivariate models. On the other hand, the models with the HP restrictions perform the worst, with considerably larger revisions compared to the other three trend specifications.

The results of the unbiasedness test, presented in the lower panel of Table 5.4, show that all univariate and bivariate models fail to produce unbiased filtered estimates. The size of b_1 in the bivariate models is significantly reduced compared to the corresponding estimates in the univariate models when the filtered estimates are tested against 12-quarter ahead estimates. However, the coefficients b_1 in bivariate models are all significantly positive, suggesting that basing monetary policy on these estimates may introduce pro-cyclical bias into policy making, as discussed in Orphanides (2001) and Ross and Ubide (2002). Revisions are found to be orthogonal to filtered output gap estimates when the trivariate models are estimated with the output trend specified as RW, DS and LLT. One striking result is that the null of $b_1 = 0$ is rejected at the 1% level for all the models where the HP restrictions are imposed. This again suggests that these restrictions are inappropriate.

Table 5.5 reports the results of the mean out-of-sample forecasts. The h-step-ahead forecast error is calculated as $\Delta P_{T+h|T} - \Delta P_{T+h}$. With the exception of the AR model, the mean errors produced by the UC models appear to be negative, suggesting that these models may have a tendency to underestimate future inflation rates. The forecasting ability across all the models is evaluated in terms of the RMSE. The model that yields the smallest RMSE provides the most accurate forecasts. The multivariate models with the output trend specified as either RW, DS or LLT tend to produce smaller values of the RMSE than the two benchmark models, especially in the early periods of forecasting. However, the models with the HP restrictions have considerably larger RMSEs than other models. This again suggests that these restrictions are not appropriate for the data analysed in this chapter. The MDM statistics in parentheses are calculated for the null hypothesis that the multivariate model is equivalent in forecasting ability to the benchmark model. The alternative hypothesis varies depending on the sign of the MDM statistic. If the statistic is positive, the alternative hypothesis is that the multivariate model is better than the benchmark model in terms of forecasting accuracy. Alternatively, if the MDM is negative, the alternative hypothesis is inverted. The MDM test statistic is compared with the critical values of Student's t-distribution with $N-1$ degree of freedom. The null of equality in forecasting performance between the multivariate models and the AR model cannot be rejected across all reported forecasting periods. However, the output gap estimates obtained from the multivariate models with the RW, DS and LLT restrictions imposed appear to be marginally preferable in the first and second step-ahead forecasts compared to the univariate model of inflation.

Table 5.4: Revision: RMSE and Bias Tests

RMSE(*100)					
Univariate Models		**RW**	**DSlope**	**LLT**	**HP**
$\widehat{\psi}_{t\|t+8}-\widehat{\psi}_{t\|t}$		0.907	0.534	0.835	1.020
$\widehat{\psi}_{t\|t+12}-\widehat{\psi}_{t\|t}$		1.120	0.630	1.011	1.127
Bivariate Models		**RW**	**DSlope**	**LLT**	**HP**
$\widehat{\psi}_{t\|t+8}-\widehat{\psi}_{t\|t}$		0.531	0.486	0.475	0.744
$\widehat{\psi}_{t\|t+12}-\widehat{\psi}_{t\|t}$		0.552	0.487	0.477	0.815
Trivariate Models		**RW**	**DSlope**	**LLT**	**HP**
$\widehat{\psi}_{t\|t+8}-\widehat{\psi}_{t\|t}$		0.605	0.508	0.618	0.765
$\widehat{\psi}_{t\|t+12}-\widehat{\psi}_{t\|t}$		0.680	0.655	0.694	0.832
Bias tests					
Univariate Models		**RW**	**DSlope**	**LLT**	**HP**
$\widehat{\psi}_{t\|t+8}-\widehat{\psi}_{t\|t}$	b_0	-0.006**	-0.001	-0.004*	0.000
		(0.002)	(0.001)	(0.002)	(0.001)
	b_1	-0.149	-0.069	-0.092	0.714**
		(0.132)	(0.077)	(0.135)	(0.096)
$\widehat{\psi}_{t\|t+12}-\widehat{\psi}_{t\|t}$	b_0	-0.009**	-0.001	-0.005**	-0.001
		(0.002)	(0.002)	(0.002)	(0.002)
	b_1	-0.308*	-0.562*	-0.240	0.726**
		(0.137)	(0.281)	(0.151)	(0.111)
Bivariate Models		**RW**	**DSlope**	**LLT**	**HP**
$\widehat{\psi}_{t\|t+8}-\widehat{\psi}_{t\|t}$	b_0	-0.000	0.001	0.001	0.000
		(0.001)	(0.001)	(0.001)	(0.001)
	b_1	0.117*	0.208**	0.216**	0.422**
		(0.049)	(0.056)	(0.059)	(0.120)
$\widehat{\psi}_{t\|t+12}-\widehat{\psi}_{t\|t}$	b_0	-0.001	0.001	0.001	-0.001
		(0.001)	(0.001)	(0.001)	(0.001)
	b_1	0.104*	0.214**	0.227**	0.399**
		(0.051)	(0.056)	(0.058)	(0.125)
Trivariate Models		**RW**	**DSlope**	**LLT**	**HP**
$\widehat{\psi}_{t\|t+8}-\widehat{\psi}_{t\|t}$	b_0	0.000	0.001	0.001	0.000
		(0.001)	(0.001)	(0.001)	(0.001)
	b_1	0.014	0.017	0.001	0.435**
		(0.074)	(0.067)	(0.074)	(0.096)
$\widehat{\psi}_{t\|t+12}-\widehat{\psi}_{t\|t}$	b_0	-0.000	0.001	0.001	-0.001
		(0.001)	(0.001)	(0.001)	(0.001)
	b_1	-0.042	-0.026	-0.061	0.414**
		(0.085)	(0.079)	(0.085)	(0.131)

Notes: Standard errors are in parentheses. * denotes significance at the 5% level; ** denotes significance at the 1% level.

Table 5.5: Out-of-Sample Forecasts: Mean Errors and RMSE (*100)

Mean Errors

	H=1	H=2	H=3	H=4	H=5	H=6	H=7	H=8
				Benchmark Models				
AR model	0.045	0.040	0.028	0.044	0.047	0.039	0.079	0.066
Univariate	-0.060	-0.092	-0.111	-0.093	-0.093	-0.103	-0.071	-0.086
				Bivariate Models				
RW	-0.060	-0.073	-0.090	-0.091	-0.089	-0.113	-0.079	-0.095
DSlope	-0.065	-0.078	-0.097	-0.100	-0.101	-0.123	-0.094	-0.110
LLT	-0.064	-0.079	-0.099	-0.104	-0.106	-0.132	-0.100	-0.117
HP	-0.045	-0.052	-0.065	-0.061	-0.055	-0.070	-0.032	-0.050
				Trivariate Models				
RW	-0.050	-0.061	-0.077	-0.080	-0.079	-0.109	-0.074	-0.094
Dslope	-0.051	-0.062	-0.078	-0.081	-0.080	-0.109	-0.075	-0.094
LLT	-0.049	-0.060	-0.076	-0.078	-0.077	-0.106	-0.072	-0.092
HP	-0.040	-0.040	-0.062	-0.059	-0.049	-0.073	-0.034	-0.052

Root-Mean-Squared Errors

	H=1	H=2	H=3	H=4	H=5	H=6	H=7	H=8
				Benchmark Models				
AR model	0.202	0.232	0.242	0.249	0.249	0.251	0.248	0.245
Univariate	0.223	0.247	0.261	0.251	0.256	0.262	0.260	0.262
				Bivariate Models				
RW	0.181	0.193	0.210	0.219	0.223	0.228	0.231	0.229
	[1.016]	*[1.688]*	*[-0.308]*	*[0.342]*	*[0.362]*	*[-0.040]*	*[0.191]*	*[0.298]*
	[1.839]*	[1.878]*	[0.303]	[0.653]	[0.943]	[1.002]	[-4.637e-04]	[0.684]
DSlope	0.182	0.193	0.210	0.218	0.222	0.227	0.230	0.227
	[0.945]	*[1.225]*	*[-0.367]*	*[0.715]*	*[0.148]*	*[-0.061]*	*[0.486]*	*[0.331]*
	[1.861]*	[1.728]*	[0.178]	[0.790]	[0.986]	[0.970]	[0.129]	[0.742]
LLT	0.181	0.193	0.209	0.219	0.223	0.229	0.231	0.228
	[1.040]	*[1.583]*	*[-0.244]*	*[0.191]*	*[0.242]*	*[-0.156]*	*[0.415]*	*[0.287]*
	[1.881]*	[1.898]*	[0.400]	[0.755]	[1.040]	[0.995]	[0.083]	[0.759]
HP	0.188	0.219	0.239	0.253	0.261	0.270	0.273	0.277
	[-0.194]	*[-0.271]*	*[-0.814]*	*[-0.675]*	*[-0.541]*	*[-0.530]*	*[-0.450]*	*[-0.286]*
	[1.246]	[0.475]	[-0.689]	[-0.856]	[-0.504]	[-0.559]	[-1.286]	[0.311]
				Trivariate Models				
RW	0.170	0.199	0.216	0.227	0.237	0.244	0.246	0.242
	[1.186]	*[1.036]*	*[-0.489]*	*[0.069]*	*[-0.434]*	*[-0.294]*	*[-0.059]*	*[0.162]*
	[1.986]*	[1.566]	[0.115]	[0.619]	[0.221]	[0.448]	[-0.144]	[0.421]
DSlope	0.171	0.200	0.217	0.228	0.238	0.245	0.247	0.243
	[1.156]	*[1.018]*	*[-0.491]*	*[0.040]*	*[-0.412]*	*[-0.309]*	*[-0.075]*	*[0.133]*
	[1.975]*	[1.549]	[0.121]	[0.608]	[0.227]	[0.419]	[-0.152]	[0.409]
LLT	0.170	0.199	0.216	0.227	0.237	0.244	0.246	0.242
	[1.172]	*[1.056]*	*[-0.492]*	*[0.050]*	*[-0.409]*	*[-0.295]*	*[-0.075]*	*[0.160]*
	[1.974]*	[1.594]	[0.111]	[0.614]	[0.816]	[0.451]	[-0.153]	[0.421]
HP	0.181	0.227	0.248	0.259	0.272	0.279	0.281	0.285
	[-0.051]	*[-0.726]*	*[-0.923]*	*[-0.643]*	*[-0.717]*	*[-0.536]*	*[-0.416]*	*[-0.321]*
	[1.492]	[0.153]	[-0.843]	[-1.062]	[-0.859]	[-0.577]	[-1.137]	[0.134]

Notes: Numbers in italic (roman) are the MDM statistics for the null that the multivariate model is equivalent in forecasting ability to the AR (univariate) model. The MDM statistics are compared with a student t-distribution with N-1 degrees of freedom. As a one-sided test is performed, the critical value associated with 5% significance levels for N-1 =24 and N-1 =23 are 1.708 and 1.711, respectively. * denotes significance at the 5% level.

5.6 Business cycle moderation

In this section, the multivariate models outlined above are modified to consider one of the most striking changes in business cycles of industrialised countries, known as the 'great moderation'. It is generally accepted that the volatility observed in economic fluctuations has declined in most industrialised countries over the past two decades. A significant body of research has been undertaken to identify the date and the possible causes of this stabilisation. In the literature, 1984 is often cited as the point at which this stabilisation occurred in the US. Several possible causes for the moderation of the US business cycle have been put forward, including changes in economic structure, improved monetary policy and the absence of major supply side shocks (Kim and Nelson, 1999a; McConnell and Perez Quiros, 2000; Stock and Watson, 2002c, 2003). Blanchard and Simon (2001), van Dijk *et al.* (2002), Mills and Wang (2002, 2003a) and Doyle and Faust (2005) have also found declines in output growth fluctuations in the other G7 members, although the magnitude and dates differ across countries.

Identifying the possible causes of business cycle moderation is outside the scope of this research. Instead, the objective of this section is to identify the degree to which output fluctuations have declined in the euro area and to date the time at which this moderation began. The model applied in this section was proposed by Proietti (2008), who estimates a bivariate UC model of US output and inflation with time-varying variances in the level and cycle disturbances. A binary variable, S_t, is constructed to indicate the degree of output fluctuations, with 1 denoting high volatility and 0 low volatility. S_t can be set either deterministically or modelled as a first-order Markov chain. This section proposes a two-step procedure. In the first step, the break in the volatility of euro area output growth is determined by the two-regime Markov-switching (MS) volatility model using the MSVAR Ox package (Krolzig, 1998). Once the date of the break is obtained, S_t is set deterministically in the multivariate UC models. Output growth is modelled as the following AR(2) process with a time-varying disturbance variance.

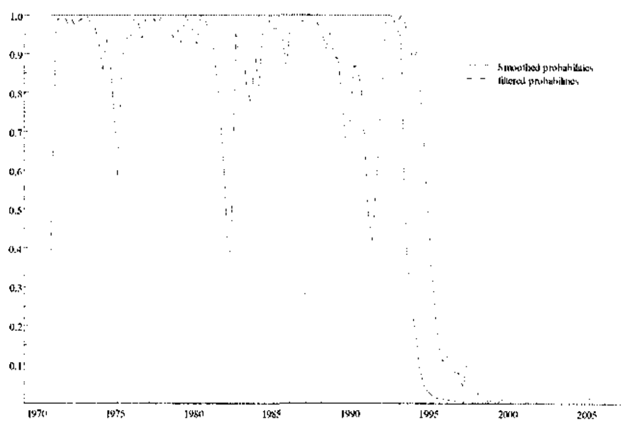
$$\Delta y_t = \beta_0 + \beta_1 \Delta y_{t-1} + \beta_2 \Delta y_{t-2} + \gamma_1 dum74q4 + \gamma_2 dum75q1 + e_t \,, \qquad (5.8)$$

$$e_t \sim NID(0, (S_t \sigma_{ea}^2 + (1 - S_t)\sigma_{eb}^2)) \,,\ S_t = \{0,1\}$$

$$\Pr[S_t = 1 | S_{t-1} = 1] = p_{11} \,,\ \Pr[S_t = 0 | S_{t-1} = 0] = p_{00} \,.$$

The lag length is selected using the Box and Jenkins (1976) strategy. Two dummy variables are also used to capture the effect of the first oil price shock. The magnitude of the disturbance variance depends on S_t. If $S_t = 1$, output growth is in the high volatility state; likewise, if $S_t = 0$ it is in the low volatility state. The transition between states follows a first-order Markov-switching process, governed by the constant transition probabilities, p_{00} and p_{11}. The smoothed probabilities of high volatility estimated at time t using the full-sample, plotted as a solid line in Figure 5.4, suggest a high volatility state before 1993Q3 and a low volatility state thereafter. This is consistent with the unstable macroeconomic situation observed in the euro area during the earlier period. However, from 1993Q3 onwards, most countries had recovered from the ERM crisis and the ratification of the Maastricht treaty further encouraged economic integration among members.

Figure 5.4: Smoothed and filtered probabilities of high volatility regime



The parameter estimates of this model are reported in Table 5.6: σ_{ea}^2 is twice the value of σ_{eb}^2. The presence of state shifts is also supported by the likelihood ratio statistic, as the null hypothesis that $\sigma_{ea}^2 = \sigma_{eb}^2$ is strongly rejected.

Table 5.6: Markov-Switching Volatility Model

Parameters estimates			
β_0	0.003** (0.001)	σ_{ea}	0.006** (0.000)
β_1	0.291** (0.083)	σ_{eb}	0.003** (0.000)
β_2	0.157* (0.077)		
$dum74q4$	-0.022** (0.006)		
$dum75q1$	-0.009 (0.006)		
Transition probabilities and regimes			
p_{11}	0.99	Regime 1	1971:4 - 1993:2
p_{00}	1.00	Regime 2	1993:3 - 2005:4
AIC: -7.766	**HQ:** -7.689	**SC:** -7.578	
Log-likelihood: 556.503			
LR statistic ($\sigma_{ea}^2 = \sigma_{eb}^2$): $\chi^2(1) = 16.996$**			

Note: Standard errors are in parentheses.

Given that the estimated date of the break is 1993Q3, S_t is set to be 1 before 1993Q3 and 0 from then onwards. The time-varying variance modification is only applied to multivariate models with the DS trend imposed, as these models provide the best fit to the data compared with the nested models, as shown in Tables 5.2 and 5.3. In addition, since fluctuations in slope estimates appear to be considerably smaller than variances in the level and cycle estimates, the time-varying variance is only considered for the level and cycle disturbances, specified as

$$\eta_t \sim NID\left(0, \left(S_t \sigma_{\eta a}^2 + (1 - S_t)\sigma_{\eta b}^2\right)\right),$$

$$k_t \sim NID\left(0, \left(S_t \sigma_{ka}^2 + (1 - S_t)\sigma_{kb}^2\right)\right).$$

The parameter estimates of the bivariate and trivariate models with time-varying variances are reported in Tables 5.7 and 5.8. In the bivariate model $\sigma_{\eta a}^2$ and σ_{ka}^2 are about six times larger than $\sigma_{\eta b}^2$ and σ_{kb}^2, respectively. However, in the trivariate model, $\sigma_{\eta a}^2$ is about four times larger than $\sigma_{\eta b}^2$, while σ_{ka}^2 is about twelve times the size of σ_{kb}^2. Therefore, it is not surprising that the likelihood ratio statistics of the null hypotheses that $\sigma_{\eta a}^2 = \sigma_{\eta b}^2$ and $\sigma_{ka}^2 = \sigma_{kb}^2$ strongly reject for both models. The persistency of the output gap does not alter significantly by introducing a one-time break in the variances of the bivariate specification. However, allowing for the break in the variances of the trivariate model does notably increase persistency, with $\phi_1 + \phi_2$ increasing to 0.96 and the cycle period increasing from 28.4 to 35.5 quarters.

Table 5.7: Parameter Estimates and Diagnostics for Bivariate Model with Time-Varying Variance

Output equation		Inflation equation	
$\sigma^2_{\eta a}$	211.240	σ^2_{ε}	34.663
$\sigma^2_{\eta b}$	36.044	σ^2_{τ}	4.141E-05
σ^2_{ξ}	10.792	$\theta_{\psi 0}$	0.284**
σ^2_{ka}	92.373		(0.090)
σ^2_{kb}	14.953	$\theta_{\psi 1}$	-0.220**
ϕ	0.823		(0.083)
m	0.006	δ_{N1}	0.021*
ϕ_1	1.545**		(0.010)
	(0.386)	δ_{N2}	0.014
ϕ_2	-0.664**		(0.010)
	(0.137)	δ_{C1}	0.006*
ρ	0.814		(0.003)
$2\pi / f_c$	19.463	δ_{C2}	0.004
Dum75Q1	-0.012*		(0.003)
	(0.006)	Dum75Q1	-0.011**
Dum74Q4	-0.011*		(0.003)
	(0.006)		

Wald test for long run neutrality: 6.551*

Log-likelihood: 1175.992

Q(4) y_t : 1.196 **Normality** y_t : 0.237

Q(4) Δp_t : 13.378* **Normality** Δp_t : 0.349

LR statistic ($\sigma^2_{\eta a} = \sigma^2_{\eta b}$ and $\sigma^2_{ka} = \sigma^2_{kb}$): $\chi^2(2) = 41.615$**

Note: Standard errors are provided in parentheses.

Table 5.8: Parameter Estimates and Diagnostics for Trivariate Model of Output, Unemployment and Inflation.

Output equation		**Inflation equation**		**Unemployment equation**	
$\sigma^2_{\eta a}$	275.66	σ^2_{ε}	23.137	$\sigma^2_{u\eta}$	6.841E-07
$\sigma^2_{\eta b}$	63.763	σ^2_{τ}	16.598	$\sigma^2_{u\beta}$	47.394
σ^2_{ξ}	0.561	$\theta_{\psi 0}$	0.169*	σ^2_{uk}	0.003
σ^2_{ka}	64.564		(0.093)	θ_{u0}	1.123
σ^2_{kb}	5.310	$\theta_{\psi 1}$	-0.140*		(2.760)
ϕ	0.940		(0.085)	θ_{u1}	-8.159**
m	0.005	δ_{N1}	0.016		(2.026)
ϕ_1	1.794**		(0.012)	Dum74Q4	0.108**
	(0.100)	δ_{N2}	0.015		(0.025)
ϕ_2	-0.830**		(0.012)	Dum75Q1	0.112**
	(0.169)	δ_{C1}	0.004		(0.032)
ρ	0.911		(0.004)	Dum75Q2	0.099**
$2\pi / f_c$	35.488	δ_{C2}	0.005		(0.026)
Dum75Q1	-0.010		(0.004)		
	(0.008)	$\phi_{3\varepsilon}$	0.426		
Dum74Q4	-0.011		(0.253)		
	(0.008)	Dum75Q1	-0.010**		
			(0.003)		

Wald test for long run neutrality: 2.245

Log-likelihood: 1577.065

Q(4) y_t: 3.218 **Normality** y_t : 0.571

Q(4) u_t: 3.473 **Normality** u_t: 1.513

Q(4) Δp_t: 1.024 **Normality** Δp_t: 5.366

LR statistic ($\sigma^2_{\eta a} = \sigma^2_{\eta b}$ and $\sigma^2_{ka} = \sigma^2_{kb}$): $\chi^2(2) = 71.628$**

Note: Standard errors are provided in parentheses.

5.7 Output gaps and monetary policy

Understanding monetary transmission mechanisms for the euro area is crucial for stimulating economic growth in member states and maintaining stability in the euro area. It is a wide ranging topic that has attracted the attention of policymakers and academics alike. There has been a growing body of literature analysing the effectiveness of alternative monetary transmission mechanisms for individual countries and for the euro area as a whole (Mojon and Peersman, 2001; van Els *et al.*, 2002; Angeloni *et al.*, 2002; Angeloni and Ehrmann, 2003). Different transmission mechanisms, for example through the banking sector and financial markets, have been discussed extensively. This section focuses on evaluating monetary transmission through the interest rate channel (IRC), as this is the conventional way through which monetary policy operates in large and relatively closed economies, such as the US and the euro area. The IRC is characterised as the impact that changes in short-term interest rates have on components of aggregate demand, the output gap and, in turn, on prices through the Phillips curve relationship. The response of the output gap to policy controlled interest rates plays an important role in achieving the ultimate objective of price stability. Therefore, Taylor (1993) proposed a simple monetary policy rule that the federal funds rate should rise by 1.5 percentage points in response to a 1 percentage point increase in the inflation rate, and by 0.5 percentage points in response to a 1 percentage point increase in GDP above its potential. The empirical reaction functions estimated by Clarida *et al.* (1998) suggest that central banks in the US, Germany and Japan all partly respond to the output gap. In a model estimated for an aggregate of five EU countries, Peersman and Smets (1999) demonstrate that, even if the central banks' sole objective is to stabilise inflation, an effective Taylor rule will include a strong response to the output gap due to its influence on future inflation. However, Orphanides and van Norden (1999) highlighted the risk of implementing inappropriate policies when using real-time output gap estimates, as these are subject to significant alterations due to data and statistical revisions.

As stable prices can significantly reduce the level of uncertainty in an economy and promote the efficient allocation of resources, the European Central Bank's (ECB) primary objective is to maintain price stability, i.e., to ensure an annual increase in the Harmonised Index of Consumer Prices (HICP) of below 2%. In order to achieve this goal the ECB implements a two-pillar monetary policy strategy to forecast and analyse inflation rates at different time horizons. One pillar is 'economic analysis', which assesses the short to medium term determinants of price developments, such as the output gap, unit labour costs, exchange rates and asset prices. The second pillar, 'monetary analysis', considers the medium to long run link between money and prices, and provides a long-term cross-check for the first pillar.

In this chapter, the responsiveness of the output gap to changes in real interest rates is evaluated using the multivariate UC models with the DS output trend imposed, as these models are able to produce relatively reliable output gap estimates and provide the best fit to the data. In doing so, the first-differences of the real interest rate are inserted into the output equation of the bivariate and trivariate models:

$$y_t = \mu_t + \psi_t, \tag{5.9}$$

$$\psi_{t+1} = \phi_1\psi_t + \phi_2\psi_{t-1} + \lambda\Delta(i_t - \pi_t^e) + k_{t+1},\ \ k_t \sim \text{NID}(0,\sigma_k^2),$$

$$\mu_{t+1} = \mu_t + \beta_t + m + \eta_{t+1},\ \ \eta_t \sim \text{NID}(0,\sigma_\eta^2),$$

$$\beta_{t+1} = \phi\beta_t + \xi_{t+1},\ \xi_t \sim \text{NID}(0,\sigma_\xi^2).$$

The real interest rate is obtained by subtracting ex ante inflation expectations from the nominal interest rate. i_t is the three-month interest rate on an annualised basis, and π_t is the annual rate of inflation, calculated as $\ln P_t - \ln P_{t-4}$. π_t^e is expected inflation at time t. Two proxies for π_t^e are used. First, π_t^e is calculated as the average value of the previous four quarters inflation rates, on the assumption that agents have adaptive expectations of future inflation rates. Second, π_t^e is simply set to be the observed inflation at time t, with agents assumed to have rational expectations.

The bivariate and trivariate models are re-estimated over the full-sample and two subsamples (pre-1993Q3 and post-1993Q3), with the estimates of λ measuring the sensitivity of the output gap to changes in the real interest rate. The choice of 1993Q3 is consistent with the date when business cycle moderation was first observed in the euro area. Parameter estimates of bivariate and trivariate models with the DS output trend for the full-sample and two subsamples are reported in Appendix B5. The trend components in the second subsample are more persistent than those in the first subsample. As the estimated slope coefficients of potential output growth are almost equal to 1, the output trends are specified as LLTs in the second subsample. It should be recalled that the second subsample is very short, which may reduce the accuracy of the parameter estimates. Moreover, strong autocorrelation is found in the residuals. The analysis in this section focuses on the parameter estimates λ and $\theta_\psi(L)$, presented in Table 5.9, that measure the effectiveness of monetary transmission through the IRC. If changes in the real interest rate have an impact on the output gap, λ is expected to be negative and statistically significant. Although all the estimates of λ have the correct sign, it is unsurprising to find that estimates of λ are insignificant in the models estimated using the full-sample and the first subsample. This is due to the diverging inflation rates caused by the relatively volatile macroeconomic situation during the 1970s and 1980s. However, parameter estimates of λ become significantly negative during the second subsample. This is broadly in line with Angeloni and Ehrmann (2003), who find that the IRC across EMU member countries became more homogenous as a result of increased comovement of national interest rates after 1995. Focusing on parameter estimates over the second subsample, when agents have rational expectations of future inflation rates, it is found that a 1 percentage point increase in the real interest rates reduces the output gap by about 0.2% in the next quarter. The output gap has only a transitory impact on inflation, suggesting that a 1 percentage point increase in the output gap raises the inflation rate by 0.1-0.15%. The values of the corresponding parameter estimates are slightly lower when agents are assumed to have adaptive expectations. Parameter estimates for $\theta_{\psi 0}$ and $\theta_{\psi 1}$ are found to be less significant in the trivariate model estimated over the two subsamples.

Table 5.9: Selected parameter estimates

Bivariate Model with DSlope					
		λ	$\theta_{\psi 0}$	$\theta_{\psi 1}$	$\theta_{\psi}(1)$
RE	1971Q3-2005Q4	-0.098	0.214**	-0.184*	0.030*
		(0.118)	(0.081)	(0.077)	[6.528]
	1971Q3-1993Q2	-0.069	0.266**	-0.211**	0.055
		(0.081)	(0.078)	(0.073)	[2.461]
	1993Q3-2005Q4	-0.243**	0.170**	-0.155*	0.015
		(0.082)	(0.074)	(0.069)	[3.586]
AE	1972Q3-2005Q4	-0.047	0.267**	-0.189*	0.078*
		(0.089)	(0.084)	(0.076)	[4.486]
	1972Q3-1993Q2	-0.107	0.242**	-0.175*	0.067
		(0.114)	(0.091)	(0.089)	[3.409]
	1993Q3-2005Q4	-0.202*	0.147*	-0.137*	0.010
		(0.103)	(0.072)	(0.068)	[1.289]
Trivariate Model with DSlope					
		λ	$\theta_{\psi 0}$	$\theta_{\psi 1}$	$\theta_{\psi}(1)$
RE	1971Q3-2005Q4	-0.046	0.172*	-0.148*	0.024
		(0.082)	(0.067)	(0.066)	[2.719]
	1971Q3-1993Q2	-0.031	0.191*	-0.161	0.030
		(0.094)	(0.090)	(0.089)	[1.568]
	1993Q3-2005Q4	-0.233**	0.144	-0.132	0.012
		(0.082)	(0.080)	(0.076)	[1.825]
AE	1972Q3-2005Q4	-0.061	0.152*	-0.127*	0.025
		(0.064)	(0.065)	(0.064)	[3.432]
	1972Q3-1993Q2	-0.047	0.161	-0.131	0.030
		(0.085)	(0.095)	(0.093)	[1.674]
	1993Q3-2005Q4	-0.175*	0.122	-0.108	0.053
		(0.085)	(0.070)	(0.068)	[1.170]

Notes: Standard errors are in parentheses; Wald test statistics are reported in squared-parentheses; AE = Adaptive Expectation, RE= Rational Expectation.

5.8 Conclusions

In this chapter, methodologies which combine a statistical trend-cycle decomposition with macroeconomic relations are used to estimate potential output and the output gap for the aggregate Euro Area. The first model used is a bivariate specification of output and CPI inflation, in which the inflation equation is based on the Gordon triangle model of inflation. The second model is a trivariate specification of output, CPI inflation and unemployment. Following Proietti *et al.* (2007), four alternative output trend specifications (i.e., RW, DS, LLT and HP) are applied to both the bivariate and trivariate models, giving eight specifications and, in turn, eight output gap estimates. Three criteria are used to analyse the reliability of the output gap estimates: the size of the revisions, the unbiasedness of the filtered output gap, and inflation forecasting. The results show that the bivariate model of output and inflation outperforms the univariate model of output decomposition. However, including the unemployment rate in the analysis does not significantly improve output gap estimates according to the three criteria used. Different specifications of trend output can have a significant impact on both a model's goodness of fit and the reliability of output gap estimates. The bivariate and trivariate models with the DS output trend imposed provide the best fit to the data and give relatively reliable output gap estimates. However, the models with the HP restrictions imposed are strongly rejected due to pronounced autocorrelation in the residuals. These models also produce less satisfactory output gap estimates.

Once the models with the DS output trend imposed are identified as being the most appropriate specifications, they are then used to investigate business cycle moderation. To do this, time-varying variances are introduced to both level and cycle disturbances, with the structural break set to be 1993Q3, which is detected by a two-regime MS volatility model. The likelihood ratio statistics for the null hypothesis of time-invariant disturbance variances are strongly rejected in both the bivariate and trivariate models.

We then examine the effectiveness of monetary policy transmission through the interest rate channel for the aggregate Euro Area. The output gap estimates obtained from multivariate models with the DS output trend imposed exhibit a significant response to changes in real interest rates over the second subsample 1993Q3 to 2005Q4. This suggests that the monetary policy pursued by the ECB may have had an impact on stabilising euro area wide economic fluctuations and inflation rates through the interest rate channel in the run-up to EMU and thereafter.

Appendix A5

A5.1 State-space Form of Bivariate and Trivariate Models

For the estimation process, the bivariate unobserved-component model of equations (5.1), (5.2), (5.3) and (5.4) is specified in state-space form, which consists of a measurement and a transition equation. The measurement equation relates the observed variables, Y_t, to the state vector of unobserved components, a_t. The transition equation defines the dynamic behaviour of a_t.

$$Y_t = Za_t + \delta X_t + Ge_t, \quad e_t \sim NID(0,I) \tag{A5.1}$$

$$a_t = Ta_t + Hu_t, \quad u_t \sim NID(0,I) \tag{A5.2}$$

where $Y_t = (y_t, \Delta P_t)^{\mathrm{T}}$ and $X_t = (\Delta Neer_t, \Delta Neer_{t-1}, \Delta Compr_t, \Delta Compr_{t-1})^{\mathrm{T}}$ are known. The state vector $a_t = (\mu_t, \beta_t, \psi_t, \psi_{t-1}, \tau_t)^{\mathrm{T}}$ contains unobserved components. $e_t = \varepsilon_t / \sigma_\varepsilon$ and $u_t = [\eta_t/\sigma_\eta, \xi_t/\sigma_\xi, k_t/\sigma_k, \eta_\pi/\sigma_\pi]^{\mathrm{T}}$ are disturbances. Z, T, G and H are time-invariant matrices containing the model's hyperparameters,

$$Z = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \; T = \begin{bmatrix} T_\mu & 0_{2\times2} & 0_{2\times1} \\ 0_{2\times2} & T_\psi & 0_{2\times1} \\ 0_{1\times2} & T_p^{\mathrm{T}} & 1 \end{bmatrix}, \; T_\mu = \begin{bmatrix} 1 & 1 \\ 0 & \phi \end{bmatrix}, \; T_\psi = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix},$$

$$T_p = \begin{bmatrix} \theta_{\psi0}\phi_1 + \theta_{\psi1} \\ \theta_{\psi0}\phi_2 \end{bmatrix}, \; G = \begin{bmatrix} 0 \\ \sigma_\varepsilon \end{bmatrix}, \; H = \begin{bmatrix} \sigma_\eta & 0 & 0 & 0 \\ 0 & \sigma_\xi & 0 & 0 \\ 0 & 0 & \sigma_k & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \theta_{\psi0}\sigma_k & \sigma_\pi \end{bmatrix}.$$

$a_0 = W_0\delta + H_0 u_0$ is the initial condition of the state vector, where $W_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and

$\delta = (\mu_0, \beta_0, \tau_0)$, containing non-stationary elements in a_0, that has a diffuse prior. The unconditional distribution of the cyclical components is initiated with $H_0\eta_0$.

In the trivariate model, observed and unobserved components are contained in the vectors $Y_t = (y_t, u_t, \Delta P_t)^{\mathrm{T}}$ and $a_t = (\mu_t, \mu_{ut}, \beta_t, \beta_{ut}, \psi_t, \psi_{t-1}, \psi_{ut}, \psi_{ut-1}, \tau_t)^{\mathrm{T}}$, respectively. The disturbances in the measurement and transition equations are $e_t = \varepsilon_t/\sigma_\varepsilon$ and $u_t = [\eta_t/\sigma_\eta, \eta_{ut}/\sigma_{u\eta}, \xi_t/\sigma_\xi, \xi_{ut}/\sigma_{u\xi}, k_t/\sigma_k, k_{ut}/\sigma_{uk}, \eta_\pi/\sigma_\tau]^{\mathrm{T}}$. The time-invariant matrices Z, T, G and H are specified as

$$Z = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \theta_{u0} & \theta_{u1} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \; T = \begin{bmatrix} T_\mu & 0_{4\times2} & 0_{4\times2} & 0_{4\times1} \\ 0_{2\times4} & T_\psi & 0_{2\times2} & 0_{2\times1} \\ 0_{2\times4} & 0 & T_{u\psi} & 0_{2\times1} \\ 0_{1\times4} & T_p^{\mathrm{T}} & 0_{1\times2} & 1 \end{bmatrix},$$

$$T_\mu = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & \phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; T_\psi = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix}, \; T_{u\psi} = \begin{bmatrix} \phi_{u0} & \phi_{u1} \\ 1 & 0 \end{bmatrix}, \; T_p = \begin{bmatrix} \theta_{\psi0}\phi_1 + \theta_{\psi1} \\ \theta_{\psi0}\phi_2 \end{bmatrix}, \; G = \begin{bmatrix} 0 \\ 0 \\ \sigma_\varepsilon \end{bmatrix},$$

$$H = \begin{bmatrix} \sigma_\eta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{u\eta} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_\xi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{u\xi} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_k & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{uk} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \theta_{\psi0}\sigma_k & 0 & \sigma_\pi \end{bmatrix}.$$

A5.2 The Augmented Kalman Filter

The Kalman Filter (KF) is a recursive algorithm that computes the minimum mean squared error (MSE) estimate of a_t, together with its MSE matrix, based on available information up to and including time t. The KF consists of two steps: prediction and updating. Initially, the optimal estimates $Y_{t|t-1}$ and $a_{t|t-1}$ are obtained, based on the previous information set at $t-1$. Once Y_t is observed at time t, $a_{t|t-1}$ is updated using the prediction error, $v_t = Y_t - Y_{t|t-1}$, following the equation: $a_{t|t} = a_{t|t-1} + K_t v_t$, where K_t is known as the Kalman gain. Depending on the information set used, the filtered and smoothed estimates of a_t are obtained, defined as $a_{t|t}$ and $a_{t|T}$. $a_{t|t}$ is estimated based on information available up to time t, while $a_{t|T}$ is based on the full-sample period from 1 to T.

The augmented KF is used in this chapter as the state components, such as μ_t, β_t and τ_t, are non-stationary. The recursive equations of the augmented KF are as follows,

$$
\begin{aligned}
v_t &= Y_t - Za_{t|t-1}, & K_t &= TP_{t|t-1}Z^{\top}F_t^{-1},\\
F_t &= ZP_{t|t-1}Z^{\top} + GG^{\top}, & V_t &= -ZA_{t|t-1},\\
q_t &= q_{t-1} + v_t^{\top}F_t^{-1}v_t, & A_{t+1|t} &= TA_{t|t-1} + K_tV_t,\\
a_{t+1|t} &= Ta_{t|t-1} + K_tv_t, & (s_t, S_t) &= (s_{t-1}, S_{t-1}) + V_t^{\top}F_t^{-1}(v_t, V_t),\\
P_{t+1|t} &= TP_{t|t-1}T^{\top} + HH^{\top} - K_tF_tK_t^{\top},
\end{aligned}
$$

for $t = 1,\ldots,T$ and with $A_{1|0} = -W_0$, $q_0 = 0$ and $(s_0, S_0) = 0$. The one-step-ahead prediction errors of the observation and state vectors and their corresponding MSE matrices are given by,

$$
\begin{aligned}
\hat{v}_t &= v_t - V_tS_{t-1}^{-1}s_{t-1},\\
\hat{F}_t &= F_t + V_tS_{t-1}^{-1}V_t^{\top},
\end{aligned}
$$

$$\hat{a}_{t|t-1} = a_{t|t-1} - A_{t|t-1} S_{t-1}^{-1} s_{t-1},$$
$$\hat{P}_{t|t-1} = P_{t|t-1} + A_{t|t-1} S_{t-1}^{-1} A_{t|t-1}^{\mathsf{T}}$$

The updated (filtered) estimates of the state vector and its covariance matrix are obtained as,

$$a_{t|t} = a_{t|t-1} - A_{t|t-1} S_t^{-1} s_t + P_{t|t-1} Z^{\mathsf{T}} F_t^{-1} \left(v_t - V_t S_t^{-1} s_t\right),$$
$$P_{t|t} = P_{t|t-1} - P_{t|t-1} Z^{\mathsf{T}} F_t^{-1} Z P_{t|t-1} + \left(A_{t|t-1} + P_{t|t-1} Z^{\mathsf{T}} F_t^{-1} V_t\right) S_t^{-1} \left(A_{t|t-1} + P_{t|t-1} Z^{\mathsf{T}} F_t^{-1} V_t\right)^{\mathsf{T}}$$

The smoothed estimates, $a_{t|T}$, are considered to provide a more accurate estimate of a_t than the filtered ones, since they use the full-sample of observations. The following equations can be iterated backwards for $t = T-1, T-2, \ldots, 1$ to obtain the smoothed estimates, with the initial values $r_T = 0$, $R_T = 0$ and $N_T = 0$ (Bryson and Ho, 1969, de Jong, 1989).

$$r_{t-1} = L_t^{\mathsf{T}} r_t + Z^{\mathsf{T}} F_t^{-1} v_t, \qquad R_{t-1} = L_t^{\mathsf{T}} R_t + Z^{\mathsf{T}} F_t^{-1} V_t,$$
$$N_{t-1} = L_t^{\mathsf{T}} N_t L_t + Z^{\mathsf{T}} F_t^{-1} Z,$$
$$a_{t|T} = a_{t|t-1} - A_{t|t-1} S_T^{-1} s_T + P_{t|t-1} \left(r_{t-1} - R_{t-1} S_T^{-1} s_T\right),$$
$$P_{t|T} = P_{t|t-1} - P_{t|t-1} N_{t-1} P_{t|t-1} + \left(A_{t|t-1} + P_{t|t-1} R_{t-1}\right) S_T^{-1} \left(A_{t|t-1} + P_{t|t-1} R_{t-1}\right)^{\mathsf{T}}$$

where $L_{t-1} = T - K_t Z^{\mathsf{T}}$. The smoothed estimates of the disturbances are given by

$$Hu_t = HH^{\mathsf{T}} \left(r_{t-1} - R_{t-1} S_T^{-1} s_T\right),$$
$$Ge_t = GG^{\mathsf{T}} \left[F_t^{-1} \left(v_t - V_t S_T^{-1} s_T\right) - K_t^{\mathsf{T}} \left(r_t - R_t S_T^{-1} s_T\right)\right]$$

A5.3 The Diffuse Likelihood Function

The discussion above assumes that the model's parameters are known. However, in most cases, these parameters are unknown and need to be estimated. Given these estimates, the unobserved components in the state vector, a_t, can then be computed iteratively. Since the initial condition, a_0, and the disturbances, $\{e_t, u_t\}$, are assumed to have multivariate normal distributions, the distribution of Y_t, conditional on past information, is also assumed to be normal. The likelihood function can be obtained from the augmented KF using ν_t and F_t based on the prediction error decomposition. The diffuse likelihood function is used, as δ has a diffuse distribution (de Jong, 1991),

$$l(Y_1, Y_2, \ldots, Y_T | \Xi) = -\frac{1}{2}\left(\sum_{t=1}^{T} \log|F_t| + \log|S_T| + q_T - s_T^{\mathrm{T}} S_T^{-1} s_T\right)$$

The unknown parameters in the model are stacked in a vector Ξ. For a given $\Xi = \Xi^*$, the KF calculates the log likelihood value. The maximum likelihood estimates of the parameters can be obtained by maximising the log likelihood with respect to Ξ.

A5.4 Unconditional Forecasting

The one-step-ahead forecast of the state vector, $\hat{a}_{T+1|T}$, and the corresponding MSE matrix are obtained from the augmented KF at time T,

$$\hat{a}_{T+1|T} = a_{T+1|T} - A_{T+1|T} S_T^{-1} s_T,$$

$$\hat{P}_{T+1|T} = P_{T+1|T} + A_{T+1|T} S_T^{-1} A_{T+1|T}^{\mathrm{T}}$$

Given $\hat{a}_{T+1|T}$, the unconditional H-step-ahead forecast of the inflation rate and the corresponding MSE can be obtained from iterating on the transition and measurement equations as follows, where $H = 2, 3, \ldots, h$.

$$\hat{a}_{T+h|T} = T\hat{a}_{T+h-1|T}$$

$$\hat{P}_{T+h|T} = T\hat{P}_{T+h-1|T}T^{\top} + HH^{\top}$$

$$\hat{Y}_{T+h|T} = Z\hat{a}_{T+h|T} + \delta X_{t+h}$$

$$\hat{F}_{T+h|T} = Z\hat{P}_{T+h|T}Z^{\top} + GG^{\top}$$

The second element in the vector $\hat{Y}_{T+h|T}$ is the *H*–step-ahead forecast of the inflation rate.

Appendix B5

Table B5.1: Parameter Estimates and Diagnostics for Univariate

	RW	DSlope	LLT	HP
σ_η^2	169.650	167.360	166.800	0
σ_ξ^2	0	0.001	0.225	0.929
σ_k^2	61.463	62.017	62.564	1486.800
ϕ	1	0.651	1	1
m	0	0.006	0	0
ϕ_1	1.739**	1.735**	1.715**	-
	(0.012)	(0.013)	(0.014)	-
ϕ_2	-0.762**	-0.761**	-0.743**	-
	(0.022)	(0.022)	(0.025)	-
Dum75Q1	-0.011**	-0.011**	-0.011**	-0.024
	(0.004)	(0.004)	(0.004)	(0.013)
Dum74Q4	-0.009*	-0.009*	-0.009*	-0.007
	(0.004)	(0.004)	(0.004)	(0.013)
		Diagnostics and goodness of fit		
Log-likelihood	535.573	542.489	535.880	399.769**
Q(4):	1.508	1.172	1.435	299.016 **
Normality test:	6.295*	4.110	6.562*	23.344**

Note: Standard errors are in parentheses. The variance parameters are multiplied by 10^7.

Table B5.2 Parameter Estimates for Bivariate Model $\left(\pi_t^e = \pi_t\right)$

	1971Q3-2005Q4	1971Q3-1993Q2	1993Q3-2005Q4
	Output equation with DSlope		
σ_η^2	138.42	177.520	49.718
σ_ξ^2	11.420	7.379	3.928
σ_k^2	64.808	102.201	0.021
ϕ	0.831	0.801	0.927
m	0.006	0.006	0.005
ϕ_1	1.588**	1.666**	1.947**
	(0.026)	(0.036)	(0.002)
ϕ_2	-0.685**	-0.728**	-1.000**
	(0.046)	(0.064)	(0.000)
ρ	0.828	0.853	0.999
$2\pi / f_c$	22.035	28.695	27.233
λ	**-0.069**	**-0.098**	**-0.243****
	(0.081)	**(0.118)**	**(0.082)**
Dum75Q1	-0.012*	-0.013*	-
	(0.005)	(0.006)	-
Dum74Q4	-0.011**	-0.011**	-
	(0.003)	(0.003)	-
	Inflation equation		
σ_ε^2	35.455	30.804	18.386
σ_τ^2	5.49E-04	14.920	0.000
$\theta_{\psi 0}$	**0.266****	**0.214****	**0.170****
	(0.078)	**(0.081)**	**(0.074)**
$\theta_{\psi 1}$	**-0.211****	**-0.184***	**-0.155***
	(0.073)	**(0.077)**	**(0.069)**
δ_{N1}	0.022*	0.015	0.021*
	(0.010)	(0.013)	(0.011)
δ_{N2}	0.016	0.026*	-0.013
	(0.010)	(0.013)	(0.012)
δ_{C1}	0.006*	0.004	0.012**
	(0.003)	(0.004)	(0.004)
δ_{C2}	0.004	0.005	-0.001
	(0.003)	(0.004)	(0.004)
Dum75Q1	-0.011*	-0.011*	-
	(0.005)	(0.006)	-
	Wald test for long run neutrality		
$\theta_\psi(1) = 0$	6.528*	2.461	3.586
	Diagnostics and goodness of fit		
Log-likelihood	1158.602	771.462	443.062
Q(4) y_t:	4.947	2.979	7.261
Q(4) Δp_t:	9.536	10.866	4.391
Normality y_t:	4.583	0.824	1.044
Normality Δp_t:	2.122	1.226	2.821

Table B5.3 Parameter Estimates for Bivariate Model $\left(\pi_t^e = \overline{\pi}_t\right)$

	1971Q3-2005Q4	1971Q3-1993Q2	1993Q3-2005Q4
	Output equation with DSlope		
σ_η^2	142.22	234.95	39.980
σ_ξ^2	10.414	4.0433	12.900
σ_k^2	61.078	64.118	5.317
ϕ	0.826	0.808	1
m	0.006	0.006	0
ϕ_1	1.591**	1.752**	1.880**
	(0.019)	(0.021)	(0.008)
ϕ_2	-0.733**	-0.871**	-0.955**
	(0.036)	(0.014)	(0.004)
ρ	0.855	0.933	0.977
$2\pi / f_c$	16.577	17.805	22.604
λ	**-0.047**	**-0.107**	**-0.202***
	(0.089)	**(0.114)**	**(0.103)**
Dum75Q1	-0.011*	-0.011*	-
	(0.005)	(0.007)	-
Dum74Q4	-0.011**	-0.011**	-
	(0.003)	(0.003)	-
	Inflation equation		
σ_ε^2	32.445	34.826	20.032
σ_τ^2	0.000	1.141	0.001
$\theta_{\psi 0}$	**0.267****	**0.242****	**0.147***
	(0.084)	**(0.091)**	**(0.072)**
$\theta_{\psi 1}$	**-0.189***	**-0.175***	**-0.137***
	(0.076)	**(0.089)**	**(0.068)**
δ_{N1}	0.023*	0.016	0.021*
	(0.010)	(0.013)	(0.013)
δ_{N2}	0.015	0.026*	-0.016
	(0.009)	(0.012)	(0.013)
δ_{C1}	0.007*	0.005	0.013**
	(0.003)	(0.004)	(0.004)
δ_{C2}	0.004	0.005	-0.002
	(0.003)	(0.004)	(0.004)
Dum75Q1	-0.010*	-0.010	-
	(0.005)	(0.007)	-
	Wald test for long run neutrality		
$\theta_\psi(1) = 0$	4.486*	3.409	1.289
	Diagnostics and goodness of fit		
Log-likelihood	1127.044	679.566	444.285
Q(4) y_t:	5.637	3.624	4.648
Q(4) Δp_t:	9.939	11.345	4.339
Normality y_t:	5.627	0.996	0.774
Normality Δp_t:	2.149	0.542	0.279

Table B5.4 Parameter Estimates for Trivariate Model $\left(\pi_t^e = \pi_t\right)$

	1971Q3-2005Q4	1971Q3-1993Q2	1993Q3-2005Q4
Output equation with DSlope			
σ_η^2	149.080	201.860	50.864
σ_ξ^2	4.920	8.476	1.685
σ_k^2	48.178	69.642	0.078
ϕ	0.793	0.786	1
m	0.006	0.006	0
ϕ_1	1.768**	1.742**	1.951**
	(0.071)	(0.070)	(0.002)
ϕ_2	-0.818**	-0.805**	-0.999**
	(0.129)	(0.129)	(0.000)
ρ	0.904	0.897	0.999
$2\pi / f_c$	29.439	25.894	28.481
λ	**-0.046**	**-0.031**	**-0.233****
	(0.082)	**(0.094)**	**(0.082)**
Dum74Q4	-0.010	-0.010	-
	(0.006)	(0.007)	-
Dum75Q1	-0.012*	-0.012*	-
	(0.006)	(0.007)	-
Inflation equation			
σ_ε^2	24.325	14.540	19.928
σ_τ^2	16.027	30.681	0.000
$\theta_{\psi 0}$	**0.172***	**0.191***	**0.144**
	(0.067)	**(0.090)**	**(0.080)**
$\theta_{\psi 1}$	**-0.148***	**-0.161**	**-0.132**
	(0.066)	**(0.089)**	**(0.076)**
δ_{N1}	0.016	0.005	0.022
	(0.012)	(0.015)	(0.014)
δ_{N2}	0.015	0.028*	-0.007
	(0.012)	(0.015)	(0.014)
δ_{C1}	0.004	0.002	0.008
	(0.004)	(0.004)	(0.007)
δ_{C2}	0.005	0.005	0.001
	(0.004)	(0.004)	(0.006)
$\phi_{3\varepsilon}$	0.397	0.645*	0.272
	(0.311)	(0.331)	(0.387)
Dum75Q1	-0.010**	-0.010**	-
	(0.003)	(0.003)	-
Unemployment equation			
$\sigma_{u\eta}^2$	359.700	0.025	0.000
$\sigma_{u\beta}^2$	63.706	87.954	270.990
σ_{uk}^2	0.010	0.011	0.012

Table B5.4 Parameter Estimates for Trivariate Model (Continued)

θ_{u0}	-1.995	-1.445	-0.030
	(1.344)	(1.791)	(1.287)
θ_{u1}	-4.336**	-4.934**	-2.867*
	(1.251)	(1.821)	(1.241)
ϕ_{4u}	0.515	0.583*	-0.826*
	(0.344)	(0.317)	(0.369)
Dum74Q4	0.095**	0.096**	-
	(0.021)	(0.024)	-
Dum75Q1	0.099**	0.101**	-
	(0.026)	(0.031)	-
Dum75Q2	0.090**	0.092**	-
	(0.021)	(0.026)	-
Wald test for long run neutrality			
$\theta_{\psi}(1)=0$	2.719	1.568	1.825
Diagnostics and goodness of fit			
Log-likelihood	1541.838	940.731	629.744
Q(4) y_t:	0.538	0.360	108.463**
Q(4) u_t :	4.576	3.046	93.770**
Q(4) Δp_t:	2.074	0.911	140.476**
Normality y_t:	5.492	0.516	5.927
Normality u_t :	16.946**	7.142*	4.606
Normality Δp_t	4.286	3.217	2.156

Table B5.5 Parameter Estimates for Trivariate Model $\left(\pi_t^e = \bar{\pi}_t\right)$

	1971Q3-2005Q4	1971Q3-1993Q2	1993Q3-2005Q4
	Output equation with DSlope		
σ_η^2	152.130	202.57	50.739
σ_ξ^2	4.524	8.6691	1.977
σ_k^2	43.189	67.472	1.656
ϕ	0.787	0.774	1
m	0.006	0.006	0
ϕ_1	1.846**	1.818**	1.940**
	(0.047)	(0.077)	(0.002)
ϕ_2	-0.892**	-0.875**	-0.992**
	(0.094)	(0.148)	(0.002)
ρ	0.945	0.935	0.999
$2\pi / f_c$	29.402	26.399	28.481
λ	**-0.061**	**-0.047**	**-0.175***
	(0.064)	**(0.085)**	**(0.085)**
Dum74Q4	-0.010	-0.009	-
	(0.006)	(0.008)	-
Dum75Q1	-0.012*	-0.012	-
	(0.006)	(0.007)	-
	Inflation equation		
σ_ε^2	24.533	30.290	20.140
σ_τ^2	14.501	13.595	0.000
$\theta_{\psi 0}$	**0.152***	**0.161**	**0.122**
	(0.065)	**(0.095)**	**(0.070)**
$\theta_{\psi 1}$	**-0.127***	**-0.131**	**-0.108**
	(0.064)	**(0.093)**	**(0.068)**
δ_{N1}	0.017	0.006	0.022
	(0.012)	(0.015)	(0.014)
δ_{N2}	0.014	0.027*	-0.008
	(0.012)	(0.015)	(0.015)
δ_{C1}	0.004	0.003	0.007
	(0.004)	(0.004)	(0.006)
δ_{C2}	0.005	0.005	0.000
	(0.004)	(0.004)	(0.006)
$\phi_{3\varepsilon}$	0.361	0.647	0.307
	(0.263)	(0.499)	(0.403)
Dum75Q1	-0.010**	-0.010**	-
	(0.003)	(0.003)	-
	Unemployment equation		
$\sigma_{u\eta}^2$	393.740	229.39	0.107
$\sigma_{u\beta}^2$	56.746	84.903	175.920
σ_{uk}^2	0.010	0.010	0.010

Table B5.5 Parameter Estimates for Trivariate Model (Continued)

θ_{u0}	-2.117	-1.737	1.142
	(1.614)	(1.511)	(1.134)
θ_{u1}	-4.055*	-4.303*	-5.276**
	(1.289)	(2.395)	(1.527)
ϕ_{4u}	0.530	0.594*	-0.753*
	(0.348)	(0.325)	(0.477)
Dum74Q4	0.091**	0.094**	-
	(0.020)	(0.030)	-
Dum75Q1	0.092**	0.085**	-
	(0.024)	(0.024)	-
Dum75Q2	0.084**	-0.010**	-
	(0.019)	(0.003)	-
Wald test for long run neutrality			
$\theta_{y}(1)=0$	3.432	1.674	1.170
Diagnostics and goodness of fit			
Log-likelihood	1500.266	897.831	629.443
Q(4) y_t:	0.499	0.379	45.497**
Q(4) u_t :	4.776	3.117	30.146**
Q(4) Δp_t :	1.974	2.210	122.583**
Normality y_t :	4.641	0.164	4.317
Normality u_t :	17.606**	5.522	5.737
Normality Δp_t	3.930	2.996	22.511**

Epilogue

The optimality and sustainability of the euro area have frequently been challenged by academics and policymakers. Optimal Currency Area (OCA) theory provides the theoretical foundations for analysing the appropriateness of a currency area. One important prerequisite highlighted in the OCA literature is that member states should share a high degree of business cycle synchronisation, as a common monetary policy cannot offset country-specific shocks. Evaluating synchronisation of cyclical fluctuations is mainly an empirical issue. A large number of empirical studies have been carried out to assess the level of cycle synchronisation before and after the introduction of the euro. The survey presented in Chapter 1 takes stock of the existing literature on evaluating business cycle and growth cycle synchronisation between previous members of the ERM and members of the EMU. Chapter 1 concludes that there is no consensus on whether or not national cycles are synchronised to the degree required for a common monetary policy to benefit all members. There is also no consensus as to whether or not there is a positive correlation between more synchronised cyclical fluctuations and fixed exchange rate regimes or a monetary union, as suggested by the endogenous OCA theory. Therefore, there is room for further research at an applied level. This thesis revisited the issue of evaluating business cycle synchronisation in the euro area and brought in time-series models that may overcome some of the drawbacks inherent in the approaches taken in the existing literature.

Most studies that measure business cycle synchronisation use turning points identified from individual macroeconomic series, such as industrial production and real GDP. This contradicts the classical business cycle definition proposed in Burns and Mitchell (1946) that a cycle should consist of expansions occurring at about the same time in many economic activities, followed by recessions, contractions and revivals. Therefore, Chapters 2 and 3 employ approaches that can date business cycle turning points using multivariate information. It is hoped that including more variables containing business cycle information in the dating process may produce more accurate turning points and, in turn, improve the accuracy of measuring cycle correlation. Synchronisation of business cycles is evaluated between the aggregate euro area, six core EMU countries (Austria, Belgium France, Germany, Italy and the Netherlands) and two peripheral

EMU countries (Spain and Finland). Three non-EMU countries (the UK, the US and Canada) are also included in the analysis to benchmark the evolution of synchronisation that has occurred in the euro area.

Chapter 2 applied Stock and Watson's (1989, 1991, 1993) single DF model to four coincident macroeconomic time-series, including real GDP, industrial production and civilian employment, to estimate a composite index for each country in the analysis. This index is a weighted average of four series, and is thus considered to be a more appropriate indicator of aggregate economic activity than any individual economic variable. Modifications have been made in the DF model when the series are cointegrated, where a two-step estimation procedure has been proposed. In the first step, the number of cointegrating vectors and values of the cointegrating coefficients are determined by Johansen's (1995) procedure. In the second step, conditional on the error correction terms obtained in the first step, the adjustment parameters and parameters in the DF model are estimated. Harding and Pagan's (2000, 2001, 2002) BBQ algorithm is then employed to date turning points in the composite index. These turning points highlight periods of recession and expansion for the overall economy. Synchronisation is evaluated in terms of two criteria, the concordance of turning points and the similarity of business cycle phases over the whole sample period (1970s to 2000s).[1] The changes in synchronisation are also evaluated over two subsamples, pre-1991Q1 and post-1991Q1, as a number of events occurred around this midpoint, such as German reunification in October 1990, the ERM crisis in 1992-1993 and the ratification of the Masstricht treaty in November 1993, all of which could have been expected to have a significant impact on the synchronisation of euro area business cycles. Maps of business cycle distances, various concordance indices and a linear regression approach proposed by Harding and Pagan are used to evaluate the coincidence of turning points between the countries analysed. Four cyclical features, measuring business cycle length, amplitude, steepness and welfare gains, first proposed in Harding and Pagan (2000), are computed to describe the similarities and differences of business cycle phases. The results indicate that the core EMU countries share more synchronised business cycle turning points with the aggregate euro area than with the peripheral and non-EMU countries. However, this may simply reflect the large weights core EMU countries are

[1] Sample periods vary across countries depending on data availability: see Table A2.1 in Chapter 2, Appendix A2 for details.

assigned to them when constructing the aggregate euro area data. The results also suggest that the three non-EMU countries have diverged from the euro area over the two subsamples. However, there is no common tendency for euro area members to become either more or less synchronised with the aggregate euro area. France and two of the peripheral countries, Finland and Spain, show a significant increase in synchronisation with the aggregate euro area. However, no further convergence was observed between the aggregate euro area and the other four core EMU countries (Austria, Belgium, Germany and the Netherlands). This finding is broadly in line with Camacho *et al.* (2006), who conclude that the introduction of the euro had not significantly increased synchronisation across the euro area, and that the synchronisation among member countries occurred prior to the formation of the EMU. A comparison of the four cycle features indicates that significant differences exist in business cycle phases among euro area countries, and that the differences across expansionary phases have increased over time. This reflects the unbalanced growth across euro area countries. Short and mild expansions observed in Germany and Italy led to slow economic growth while, on the other hand, the steep and long lasting expansion observed in Spain brought huge welfare gains to the Spanish economy. Overall, the result obtained in Chapter 2 contradicts the argument proposed in the endogenous OCA theory that a monetary union will result in more synchronised business cycles across member countries.

Chapter 3 considers dating business cycle turning points for the same countries analysed in Chapter 2 using the MSDF model that incorporates nonlinear dynamics into the estimation of the composite index by combining the DF model with the Hamilton (1989) MS model. Therefore, the MSDF model allows two stylised facts of the business cycle to be analysed – the comovement of economic variables throughout the cycle and the asymmetry of recessions and expansions. Three modifications were made to the MSDF model according to the properties of the data. First, when variables are found to be cointegrated, independently estimated error correction terms are included in the MSDF model. This is the same modification which is made to the DF model. Second, an additional regime is included in the MSDF model for France, as the French business cycle dynamics exhibit three phases, recession, moderate-growth and high-growth, rather than the two phases traditionally observed. In general, the MSDF model is more successful at identifying business cycle turning points for larger economies, such as the aggregate euro area, Germany, France, the UK and the US, whose recessions and expansions were of roughly constant magnitudes over the sample periods analysed. For economies, such as Italy, the Netherlands and Belgium, who exhibited greater volatility during the 1970s and early 1980s, the recession intercepts are biased downwards by the severe recessions that occurred during these periods. As a result, smoothed regime probabilities fail to identify the milder recessions which occurred in the later years. Therefore, a third modification is made for these countries by introducing structural breaks in the intercepts of the MSDF model to reduce the effect of large recessions and expansions on the model's parameter estimates. One objective of Chapter 3 is to compare cycle dates produced by the MSDF model with those obtained in Chapter 2. Although the two approaches are fundamentally different, the cycle dates obtained are broadly consistent. One exception is that a recession is identified during the 2000s for the aggregate euro area by the MSDF model but not by the BBQ algorithm. As such, an increase in cycle correlation between France, two peripheral countries and the euro area aggregate, which is shown in Chapter 2, is not identified in Chapter 3.

The empirical analysis carried out in Chapter 4 investigates growth cycle synchronisation in seven major euro area countries, Austria, Belgium, France, Germany, Italy, the Netherlands and Spain, during the period from 1980Q1 to 2007Q3. Two univariate trend-cycle decomposition methodologies, the Beveridge-Nelson (1981) decomposition and Harvey and Trimbur's (2003) unobserved component model, are used to identify the trend and cyclical components of real GDP for each country. The cycles extracted from the two univariate approaches vary significantly in both cycle period and amplitude. This confirms the argument in Canova (1998) that the use of different trend-cycle decomposition methodologies may influence the results obtained. The average correlation calculated from the BN cycles is found to be smaller than the corresponding correlation estimated using cycles extracted from the unobserved component model. The main focus of this chapter is to evaluate cycle synchronisation within a multivariate setting. The multivariate extension of the BN decomposition with common factor restrictions imposed is employed to accomplish this task. The common factor restrictions include both long-run restrictions imposed by the presence of common trends (Engle and Granger, 1987; Stock and Watson, 1988b; Johansen, 1995), and short-run restrictions imposed by common cycles (Vahid and Engle, 1993; Hecq *et al.*, 2000, 2006). The number of common trends in the seven national GDP series is determined by Johansen's (*1995*) cointegration test. Various types of common and codependent cycles among the GDP series are also investigated by using canonical correlation-based tests, GMM and likelihood ratio statistics (Vahid and Engle, 1993, 1997; Hecq *et al.* 2000, 2006; Schleicher, 2007). The number of common and codependent cycles provides an indication of the level of growth cycle synchronisation. The results produced from the multivariate approach indicate the presence of heterogeneous and codependent growth cycles. This contradicts the OCA criterion that members of a monetary union should share a high degree of growth cycle synchronisation.

The appropriateness of common cycle restrictions is further investigated by comparing out-of-sample forecasting performance between more parsimonious models imposed with additional common cycle restrictions and a less restricted model with only common trend restrictions imposed. The results show that the former outperform the latter for all countries over all forecast horizons. Finally, Chapter 4 assesses the relative importance of permanent and transitory shocks to total output variance using the forecast error variance decomposition proposed by Issler and Vahid (2001). For short-term forecasts, cyclical movements contribute more to total output variance than the trend components. Over longer time periods, however, it is the trend components that make the greatest contribution.

Chapter 5 focuses on the euro area wide economy by investigating three issues concerning the euro area output gap; the reliability of output gap estimates, business cycle moderation, and the effectiveness of the monetary policy transmission through the interest rate channel. As the output gap is unobserved, it has to be estimated from observed data, such as output or factors inputs. Given the use of different methodologies, estimated cyclical components can vary significantly in cycle length and amplitude (Canova, 1998). Therefore, it is important to have certain criteria to judge which model provides the most reliable output gap estimates. Chapter 5 first investigates the reliability of output gap estimates obtained from various multivariate UC models that combine a statistical output decomposition with macroeconomic relations, such as the Phillips curve and Okun's law. In particular, a bivariate model of output and inflation and a trivariate model of output, inflation and unemployment are estimated. Both models have four alternative output trend specifications imposed and thus eight output gap estimates are produced. The reliability of these estimates is assessed against three criteria: the size of subsequent revisions to the data, the unbiasedness of the filtered estimates, and inflation forecasting. The results indicate that including the unemployment rate in the bivariate model of output and inflation does not significantly improve output gap estimates according to the three criteria used. However, different specifications of the output trend can have a significant impact on both model goodness of fit and the reliability of output gap estimates. The bivariate and trivariate models with the damped slope (DS) output trend imposed provide the best fit to the data and give relatively reliable output gap estimates. However, the models with the Hodrick-Prescott (HP) restrictions imposed are strongly rejected due to severe autocorrelation in the residuals. These models also produce less satisfactory output gap estimates.

Once the models with the DS output trend imposed have been identified as being the most appropriate specifications, they are then used to investigate business cycle moderation. To do so, time-varying variances are introduced to both level and cycle disturbances, with the date of the structural break set at 1993Q3, which is detected by a two-regime MS volatility model. The likelihood ratio statistics for the null hypothesis of time-invariant disturbance variances strongly reject in both the bivariate and trivariate models. Finally, Chapter 5 examines the effectiveness of the interest rate channel for the euro area. The first-differences of the real interest rate are inserted into the output equation of the bivariate and trivariate models with the DS output trend imposed. Both models are re-estimated over the full-sample and two subsamples, pre-1993Q3 and post-1993Q3. The choice of 1993Q3 is consistent with the date when business cycle moderation began in the euro area. The output gap estimates exhibit a significant response to changes in real interest rates during the second subsample. This suggests that the monetary policy pursed by the ECB may have had an impact on stabilising euro area wide economic fluctuations and inflation rates through the interest rate channel in the run-up to EMU and thereafter.

The results obtained from evaluating synchronisation in the euro area raise concerns about the appropriateness of a common monetary policy. A significant degree of disparity in cyclical fluctuations still occurs across countries and there is no clear sign as to whether the differences will gradually decrease for countries who participate in the euro area. Unbalanced economic performance across EMU member states is another striking feature observed in the results. A number of the larger economies, such as Germany, who suffered more severe downturns and sluggish growth for several years during the 2000s, required more expansionary monetary policy to boost growth. On the other hand, smaller economies, such as Spain, which combined high growth and high inflation during this period, needed tighter monetary conditions. Overall, members at different phases of their business cycles have diverging monetary requirements that complicate the implementation of a common monetary policy for the euro area wide economy. The findings of this thesis suggest the need for structural reforms to introduce greater flexibility in product and labour markets so as to increase the adjustment speed of business cycle phases when member countries face economic uncertainty in the future. More effective short term use of fiscal policy is also needed. This does not necessarily have to contradict the Stability and Growth Pact (SGP), as the Pact aims to promote medium to long term fiscal stability in member states. A greater use of national fiscal policy in the short term during both expansions and contractions may help to reduce the degree of cyclical divergence. As such, a future challenge for the EMU is to balance the need to respect national sovereignties with co-ordinating policies to achieve strong economic growth and stable inflation in the euro area.

The last empirical chapter of this thesis, Chapter 5, focuses on the euro area wide economy. The responsiveness of the output gap and inflation rates to changes in real interest rates is investigated using the unobserved components framework. The results suggest that a common monetary policy had an impact on stabilising area wide output fluctuations and inflation rates. Chapter 5 sets out further research which could be undertaken to analyse monetary policy transmission for the euro area in an open economy framework, in which both interest rate and exchange rate channels are considered. It is widely accepted that, in an open economy, reductions in output and inflation induced by an increase in short-term nominal interest rates can be accelerated and amplified by the adjustment of exchanges rates. Although there has been a growing body of literature analysing monetary transmission mechanisms for individual member states and the euro area as a whole using a variety of techniques (structural models, VARs, panel estimation, DSGE models), and using data ranging from area-wide, national aggregates to disaggregate data at industrial-level (Mojon and Peersman, 2001; Van Els *et al.*, 2002; Angeloni *et al.*, 2002; Angeloni and Ehrmann, 2003), the multivariate unobserved components model has rarely been applied in the context of analysing monetary policy transmission. However, this model is well suited for this task. Applying the multivariate unobserved components model will enable us not only to analyse the impact that changes in interest rates and exchange rates have on output and inflation, but also to reveal the unobserved features of an economy, such as the output gap, core inflation and the equilibrium exchange rate. This proposed future research has been awarded an ESRC postdoctoral fellowship and will be carried out at the University of Glasgow between October 2009 and 2011.

Reference

Agresti, A.-M. and Mojon, B. (2001). Some stylised facts on the euro area business cycle. *ECB Working Paper* No. 95.

Altavilla, C. (2004). Do EMU members share the same business cycle?. *Journal of Common Market Studies* 42(5). 869–896.

Angeloni, I. and Dedola, L. (1999). From the ERM to the Euro: New Evidence on Economic and Policy Convergence among EU Countries, *ECB Working Paper* No. 4.

Angeloni, I. and Ehrmann, M. (2003). Monetary Policy Transmission in the Euro Area: Any Changes after EMU?. *ECB Working Paper 240.*

Angeloni, I. Mojon, B., Kashyap, A. K. and Terlizzese, D. (2002). Monetary Transmission in the Euro Area: Where Do We Stand?. *ECB Working Paper 114.*

Apel, M. and Jansson, P. (1999). A Theory-Consistent System Approach for Estimating Potential Output and the NAIRU. *Economics Letters*, 64(3). 271-275.

Artis, M.J. and Zhang, W. (1997). International business cycles and the ERM. *International Journal of Finance and Economics* 2(1). 1–16.

Artis, M. J. and Zhang, W. (1999). Further evidence on the international business cycle and the ERM: is there a European business cycle? *Oxford Economic Papers* 51, 120–132.

Artis, M, Krolzig, H-M. and Toro, J. (2004). The European business cycle. *Oxford Economic Papers*, 56(1), 1-44.

Artis, M. Marcellino, M. and Proietti, T. (2004a). Dating Business Cycles: A Methodological Contribution with an Application to the Euro Area. *Oxford Bulletin of Economics and Statistics*, 66(4). 537-565.

Artis, M. Marcellino, M. and Proietti, T. (2004b). Characterizing the Business Cycle for Accession Countries. *CEPR Discussion Papers* 4457.

Artis, M. Marcellino, M. and Proietti, T. (2005). Business Cycles in the new EU member countries and their conformity with the Euro area. *Journal of Business Cycle Measurement and Analysis*, 2, 7-41.

Bai, J. and Perron, P. (2003). Computation and analysis of multiple structural change models. *Journal of Applied Econometrics*, 18(1), 1-22.

Barillas, F. and Schleicher, C. (2005). Common Trends and Common Cycles in Canadian Sectoral Output. *Computing in Economics and Finance*, 214.

Baxter, M. and King, R. G. (1999). Measuring Business Cycles: Approximate Band-Pass Filters For Economic Time Series. *The Review of Economics and Statistics*, 81(4). 575-593.

Beine, M., Candelon, B. and Hecq, A. (2000). Assessing a Perfect European Optimum Currency Area: A Common Cycles Approach. *Empirica*, 27, 115-32.

Beine, M., Candelon, B. and Sekkat, K. (2003). EMU membership and business cycle phases in Europe: Markov-switching VAR analysis. *Journal of Economic Integration*,18, 214-242.

Berger, T. (2008). Estimating Europe's Natural Rates from a forward-looking Phillips curve, *Working Papers of Faculty of Economics and Business Administration*, Ghent University.

Beveridge, S. and Nelson, C. R. (1981). A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle. *Journal of Monetary Economics*, 7(2). 151-174.

Blanchard, O. and Simon, J. (2001). The Long and Large Decline in US Output Volatility. *Brookings Papers on Economic Activity*, 135-174.

Bry, G. and Boschan, C. (1971). Cyclical Analysis of Economic Time Series: Selected Procedures and Computer Programs. *NBER Working Paper*, No.20.

Bryson, A. E., and Ho, Y.-C. (1975). *Applied optimal control*. New York:Wiley.

Boldin, M. D. (1996). A Check on the Robustness of Hamilton's Markov Switching Model Approach to the Economic Analysis of the Business Cycle. *Studies in Nonlinear Dynamics and Econometrics* 1, 35-46

Box, G. and Jenkins, G. (1976). *Time Series Analysis: Forecasting and Control*, San Francisco: Holden-Day.

Burns A. F. and Mitchell, W.C. (1938). *Statistical Indicators of Cyclical Revivals*, New York: National Bureau of Economic Research.

Burns A. F. and Mitchell, W.C. (1946). *Measuring Business Cycles*, New York: National Bureau of Economic Research.

Camacho, M., Pérez-Quirós, G. and Saiz, L. (2006). Are European Business Cycles Close Enough to be Just One? *CEPR Discussion Papers* 4824.

Camacho, M., Pérez -Quirós, G. and Saiz, L. (2008). Do European business cycles look like one? *Journal of Economic Dynamics and Control*, Elsevier, 32(7), 2165-2190.

Camacho, M. and Pérez-Quirós, G. (2006). A new framework to analyze business cycle synchronization, In: Milas, C. Rothman, P. and van Dijk, D. *Nonlinear Time Series Analysis of Business Cycles.* Elsevier, 133-149.

Camba-Mendez, G. and Rodriguez-Palenzuela, D. (2001). Assessment Criteria for Output Gap estimates. *ECB Working Paper* 54.

Campbell, J.Y. and Mankiw, N. G. (1987a). Permanent and Transitory Components in Macroeconomic Fluctuations. American Economic Review, 77(2). 111-17.

Campbell, J.Y. and Mankiw, N. G. (1987b). Are Output Fluctuations Transitory?. *The Quarterly Journal of Economics*, 102(4). 857-80.

Canova, F. (1998). Detrending and Business Cycle Facts, *Journal of Monetary Economics*, 41(3). 475-512.

Carlino, G. and Sill, K. (2001). Regional Income Fluctuations: Common Trends and Common Cycles. *The Review of Economics and Statistics*, 83(3). 446-456.

Chauvet, M. (1998). An econometric characterization of business cycle dynamics with factor structure and regime switching. *International Economic Review*, 39, 969–96.

Chauvet, M. and Yu. C. (2006). International Business Cycles: G7 and OECD Countries. *Federal Reserve Bank of Atlanta Economic Review*. First Quarter: 43-54.

Christiano, L. J. and Fitzgerald, T. J. (2003). The Band Pass Filter. *International Economic Review* 44, 435-465.

Clark, P. K. (1989). Trend Reversion in Real Output and Unemployment, *Journal of Econometrics*, 40, 15-32.

Clarida, R., Gali, J. and Gertler, M. (1998). Monetary policy rules in practice Some international evidence. *European Economic Review*, 42(6), 1033-1067.

Clements, M.P. and Krolzig, H.M. (1998). A Comparison of the Forecast Performance of Markov-Switching and Threshold Autoregressive Models of US GNP, *Econometrics Journal*, 1, C47-75.

Cogley, T. and Nason, J. M. (1995). Effects of the Hodrick-Prescott filter on trend and difference stationary time series Implications for business cycle research, *Journal of Economic Dynamics and Control*, 19(1-2). 253-278.

Corden, W. (1972). Monetary Integration, Essays in International International Finance Section No. 93, Princeton University.

Croux, C. Forni, M. and Reichlin, L. (2001). A measure for comovement of economic variables: theory and empirics, *Review of Economics and Statistics*, 83, 232– 241.

Cukierman, A., Edwards, S. and Tabellini, G. (1992). Seignorage and Political Instability. *American Economic Review*, 82 (3), 537-555.

Darvas, Z. and Szapáry, G. (2004). Business cycle synchronization in the enlarged EU: comovements in the new and old members. *Central Bank of Hungary Working Paper* No. 2004/1.

De Jong, P. (1989). Smoothing and Interpolation with the State-Space Model *Journal of the American Statistical Association*, 84, 408, 1085-1088.

De Jong, P. (1991). The Diffuse Kalman Filter, *The Annals of Statistics*, 19, 1073-1083.

De Jong, D. N. and Whiteman, C. H. (1991). The Case for Trend-Stationarity Is Stronger Than We Thought. *Journal of Applied Econometrics*, 6(4). 413-21.

De Masi, P. (1997), IMF Estimates of Potential Output: Theory and Practice, *IMF Working Paper*, 177, 29, 1-16.

Den Haan, W. (2000). The comovements between output and prices. *Journal of Monetary Economics* 46, 3–30.

Dickey, D. and W. Fuller. (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of the American Statistical Association*, 74, 427-431.

Diebold, F. X. and Mariano, R. S. (1995) Comparing Predictive Accuracy. *Journal of Business and Economic Statistics*, 13(3), 253-63.

Diebold, F. Lee J-H, and Weinbach G. C. (1999). Regime switching with time-varying transition probabilities, in *Business cycles: durations, dynamics and forecasting*, Princeton University Press.

Diebold, F. X. and Rudebusch, G. D. (1996). Measuring business cycles: A Modern perspective. *Review of Economics and Statistics*, 78, pp. 67–77.

Durland, J. M. and McCurdy, T. H. (1994). Duration-Dependent Transitions in a Markov Model of US GNP Growth. *Journal of Business and Economic Statistics*, 12(3). 279-88.

Doménech, R. and Gómez, V. (2006) Estimating Potential Output, Core Inflation, and the NAIRU as Latent Variables, *Journal of Business & Economic Statistics*, 24, 354-365.

Doorn, D. (2006). Consequences of Hodrick-Prescott Filtering for Parameter Estimation in a Structural Model of Inventory Behaviour. *Applied Economics*, 38: 1863-1875.

Doyle, B. M. and Faust, J. (2005) Breaks in the Variability and Co-movement of G-7 Economic Growth. *The Review of Economics and Statistics*, 87(4), 721-740, 09.

Engle, R. F. and Granger, C. W. J, (1987). Co-integration and Error Correction: Representation, Estimation, and Testing. *Econometrica*, 55(2). 251-76.

Engle, R. F. and Kozicki, S. (1993). Testing for Common Features, *Journal of Business and Economic Statistics*, 11(4), 393-95.

European Commission (1990). One Market, One Money, *European Economy*, 44.

Fagan, G., Henry, J. and Mestre, R. (2001) An Area-Wide Model (AWM) for the Euro Area, *ECB Working Paper* 42

Fair, R. C. (1994). *Testing Macroeconometric Models*. London: Harvard University Press.

Filardo, A. J. (1994). Business-cycle phases and their transitional dynamics. *Journal of Business and Economic Statistics*, 12(3). 299-308.

Filardo, A.J. and Gordon, S.F. (1998). Business cycle durations. *Journal of Econometrics*. 85(1), 99-123.

Fleming, J. M. (1962). Domestic Financing Policies under Fixed and under Floating Exchange Rates, *IMF Staff Papers*, 9, 369-380.

Fleming, J. M. (1971). On Exchange Rate Unification. Economic Journal, 81, 467-488.

Forni, M. Hallin, M. Lippi, M. and Reichlin, L. (2000). The Generalized Dynamic-Factor Model: Identification and Estimation. *The Review of Economics and Statistics*, 82(4). 540-554.

Forni, M. Hallin, M. Lippi, M. and Reichlin, L. (2001). Coincident and Leading Indicators for the Euro Area. *Economic Journal*, 111(471), 62-85.

Frankel, J. A. and Rose, A. K. (1998). The Endogeneity of the Optimum Currency Area Criteria. *Economic Journal*, 108(449). 1009-25.

Friedman, M. (1964). Monetary Studies of the National Bureau, *The National Bureau Enters Its 45th Year*, 44th Annual Report, 7-25; Reprinted in Milton Friedman, 1969, *The Optimum Quantity of Money and Other Essays*, Chicago: Aldine, Chap.12, 261-284.

Friedman, M. (1993). The 'Plucking Model' of Business Fluctuations Revisited, Economic Inquiry 31(2). 171-177.

Garnier, J. (2003). Has the Similarity of Business Cycles in Europe Increased with the Monetary Integration Process? A Use of the Classical Business Cycle. *EUI Working Paper* No. 2003/12.

Gayer, C. (2007). A Fresh Look at Business Cycle Synchronisation in the Euro Area. *European Economy, Economic Papers* 287.

Gerlach, S. and Smets, F. (1998). The Taylor Rule and Interest Rates in the EMU Area. *CEPR Discussion Papers* 2271.

Gerlach, S., and Svensson, L.E.O. (2003). Money and inflation in the euro area: A case for monetary indicators? *Journal of Monetary Economics*, 50, 1649-72.

Geweke, J. (1977). The dynamic factor analysis of economic time series, in Aigner, D. J. and Glodberger, A. S. (eds.). *Latent Variables in Socio-Economic Models*. 365-383. Amsterdam: North-Holland.

Gordon, R. J., (1997) The Time-Varying NAIRU and Its Implications for Economic Policy. *Journal of Economic Perspectives*, 11(1), 11-32.

Gourieroux, C. and Monfort, A. (1989). *Statistics and Econometric Models*, Cambridge: Cambridge University Press.

Granger, C. W. J. (1966). The typical spectral shape of an economic variable, *Econometrica*. 34,150–161.

Guha, D. and Banerji, A. (1998). Testing for cycles: A Markov switching approach, *Journal of Economic and Social Measurement*, 25,163-182.

Haberler, G. (1970). The International Monetary System: Some Recent Developments and Discussion, in G. N. Halm (ed.), *Approaches to Greater Flexibility of Exchange Rates*. Princeton University Press, 145-150.

Hamilton, J. D. (1989). A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica*, 57(2). 357-84.

Hamilton, J. D. (1990). Analysis of time series subject to changes in regime. *Journal of Econometrics*, 45(1-2). 39-70.

Hamilton, J. D. (1991). A Quasi-Bayesian Approach to Estimating Parameters for Mixtures of Normal Distributions. *Journal of Business and Economic Statistics*, 9(1). 27-39.

Hamilton, J. D. (2003). Comment on A comparison of two business cycle dating methods. *Journal of Economic Dynamics and Control*, 27(9). 1691-1693.

Harding, D. and Pagan, A. (2000). Knowing the Cycle, in Backhouse, R. and Salanti, A. (eds.). *Macroeconomics and the Real World*. Oxford, Oxford University Press.

Harding, D. and Pagan, A. (2001). Extracting, Using and Analysing Cyclical Information. *MPRA Paper*, 15.

Harding, D. and Pagan, A. (2002). Dissecting the cycle: a methodological investigation. *Journal of Monetary Economics*, 49(2). 365-381.

Harding, D. and Pagan, A. (2003). A comparison of two business cycle dating methods. *Journal of Economic Dynamics and Control*, 27(9). 1681-1690.

Harding, D. and Pagan, A. (2006). Synchronization of cycles. *Journal of Econometrics*, 132(1). 59-79.

Hansen B.E. (1992). The likelihood ratio test under non-standard conditions: Testing the Markov-switching model of GNP. *Journal of Applied Econometrics*, 7, S61–82.

Harvey, A. C. (1989). *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge University Press.

Harvey, A. C. and Jaeger, A. (1993). Detrending, Stylized Facts and the Business Cycle. *Journal of Applied Econometrics*, 8(3). 231-247.

Harvey, A. C. and Trimbur, T. M. (2003). General Model-Based Filters for Extracting Cycles and Trends in Economic Time Series. *The Review of Economics and Statistics*, 85(2). 244-255.

Harvey, A. C. and Carvalho, V. (2005). Convergence in the trends and cycles of Euro-zone income. *Journal of Applied Econometrics* 20, 275-89.

Harvey, D. I, Leybourne, S. and Newbold, P. (1997). Testing the Equality of Prediction Mean Squared Errors. *International Journal of Forecasting*, 13(2), 281-291.

Hecq, A. (1998). Does Seasonal Adjustment Induce Common Cycles?. *Economic Letters*, 59, 289-297.

Hecq, A. (2005). Common Trends and Common Cycles in Latin America: A 2-step vs an Iterative Approach. Computing in Economics and Finance, 258, Society for Computational Economics.

Hecq, A., Palm, F. C. and Urbain, J. P. (2000). Permanent-Transitory Decomposition in VAR Models with Cointegration and Common Cycles. *Oxford Bulletin of Economics and Statistics*, 62(4), 511-32.

Hecq, A., Palm, F. C. and Urbain, J. P. (2006). Common Cyclical Features Analysis in VAR Models with Cointegration. *Journal of Econometrics*, 127(1), 117-141 .

Hodrick, R. J. and Prescott, E. C. (1997). Post-war US Business cycle: an empirical investigation, *Journal of Money, Credit and Banking*, 29, 1-16.

Inklaar, R., de Haan, J. (2001). Is there really a European business cycle? A comment. *Oxford Economic Papers* 53 (2), 215–220.

Ishiyama, Y. (1975). The theory of optimum currency areas: A survey. *IMF Staff papers*, 22, 344-383

Issler, J. and Vahid, F. (2001). Common cycles and the importance of transitory shocks to macroeconomic aggregates. *Journal of Monetary Economics*, 47 (3), 449–475.

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control*, 12(2-3), 231-254.

Johansen, S. (1995). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford University Press, Oxford.

Jungbacker, B. and Koopman, S. J. (2008). Likelihood-based Analysis for Dynamic Factor Models. *Tinbergen Institute Discussion Papers*, 08-007/4.

Kenen, D. P. (1969). The Theory of Optimum Currency Areas: an Elective View. *Monetary Problems of the Institutional Economy*, University of Chicago Press, Chicago 41-60.

King, R. G., Plosser, C. I., Stock, J. H. and Watson, M. W. (1991). Stochastic trends and economic fluctuations. *American Economic Review*, *81 (4), 819–840.*

Kim, C. J. (1994). Dynamic linear models with Markov switching. *Journal of Econometrics*, 60, 1–22.

Kim, C. J. and Nelson, C. R. (1998). Business cycle turning points, a new coincident index, and tests for duration dependence based on a Markov-switching model of the business cycle. *Review of Economics and Statistics*, 80,188–201.

Kim, C. J. and Nelson, C. R. (1999a). Has The US Economy Become More Stable? A Bayesian Approach Based on A Markov-Switching Model of the Business Cycle. *The Review of Economics and Statistics*, 81(4). 608-616.

Kim, C. J. and Nelson, C. R. (1999b). Friedman's Plucking Model of Business Fluctuations: Tests and Estimates of Permanent and Transitory Components. *Journal of Money, Credit, and Banking*, 31(3). 317-334.

Kim, C. J. and Nelson, C. R. (1999c). *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*, MIT Press.

Kim, C. J. and Nelson, C. R, (2001). A Bayesian Approach to Testing for Markov-Switching in Univariate and Dynamic Factor Models. *International Economic Review*, 42(4). 989-1013.

Kim, C. J. Morley, J. and Piger, J. (2005). Nonlinearity and the Permanent Effects of Recessions. *Journal of Applied Econometrics*, 20.

Kim, M. J. and Yoo, J. S. (1995). New index of coincident indicators: A multivariate Markov switching factor model approach. *Journal of Monetary Economics*, 36, 607–30.

King, R. G., Plosser, C. I., Stock, J. H. and Watson, M. W. (1991). Stochastic Trends and Economic Fluctuations. *American Economic Review*, 81(4). 819-40.

Klein, L.R. and Kosobud, R. F. (1961). Some Econometrics and Growth: Great Ratios of Economics. *Quarterly Journal of Economics*, 75, 173-198.

Koopmans, T. C. (1947). Measuring without theory. *Review of Economics and Statistics*, 29, 161-172.

Koopman, S. J. Harvey, A. C. Doornik, J. A. and Shephard, N. (2006). *STAMP 8: Structural Time Series Analyser and Modeller and Predictor*, London: Timberlake Consultants Ltd, 2006.

Koopman, S. J. and Azevedo, J. V. E. (2008). Measuring Synchronization and Convergence of Business Cycles for the Euro area, UK and US. *Oxford Bulletin of Economics and Statistics*, 70(1). 23-51.

Krolzig, H.-M. (1997a). International Business Cycles: Regime Shifts in the Stochastic Process of Economic Growth. *Economics Series Working Papers 99194*, University of Oxford, Department of Economics.

Krolzig, H.-M. (1997b). *Markov Switching Vector Autoregressions. Modelling Statistical Inference and Application to Business Cycle Analysis*. Berlin: Spinger.

Krolzig, H.-M. (1998). Econometric Modelling of Markov-Switching Vector Autoregressions using MSVAR for Ox. Manuscript, Oxford University.

Krolzig, H.-M. and Sensier, M. (2000). A Disaggregated Markov-Switching Model of the Business Cycle in UK Manufacturing. *Manchester School*, 68(4), 442-460.

Krolzig, H.-M, (2001). Business cycle measurement in the presence of structural change: international evidence. *International Journal of Forecasting*, 17(3), 349-368.

Krolzig, H.-M. and Toro, J. (2005). Classical and modern business cycle measurement: The European case. *Spanish Economic Review*, 7, 1-22.

Krolzig, H-M., Marcellino, M. and Mizon, G. E. (2002). A Markov-switching vector equilibrium correction model of the UK labour market. *Empirical Economics*, 27(2), 233-254.

Krugman, P. and Venables, A. J. (1993). Integration, Specialization and Adjustment. *CEPR Discussion Papers,* 886.

Kruskal, J. B., and Wish. M. (1977). Multidimensional Scaling. Sage Publications. Beverly Hills. CA.

Kuttner, K. N. (1994). Estimating Potential Output as a Latent Variable. *Journal of Business and Economic Statistics*, 12(3), 361-68.

Lahiri, K. and Wang, J. G. (1994). Predicting cyclical turning points with a leading index in a Markov switching model. *Journal of Forecasting*, 13, 245–264.

Layton, A. P. and Smith, D. R. (2007). Business cycle dynamics with duration dependence and leading indicators. *Journal of Macroeconomics*, 29(4). 855-875.

Lucas, R. E. (1973). Expectations and the Neutrality of Monetary. *Journal of Economic Theory*, 4(2). 103-124.

Luginbuhl, R. and Koopman, S. J. (2004). Convergence in European GDP series: a multivariate common converging trend-cycle decomposition. *Journal of Applied Econometrics*, 19(5). 611-636.

Liu, J., Wu, S., and Zidek, J. V. (1997). On segmented multivariate regressions. *Statistical Sinica*, 7, 497-525.

Lippi, M. and Reichlin, L., (1993). The dynamic effects of aggregate demand and supply disturbances: comment. *American Economic Review*. 83 (3), 644–652.

Massmann, M. and Mitchell, J. (2004). Reconsidering the evidence: are Eurozone business cycles converging?. *Journal of Business Cycle Measurement and Analysis* 1(3).: 275–308.

McConnell, M. M. and Perez-Quiros, G. (2000). Output Fluctuations in the United States: What Has Changed since the Early 1980's? *American Economic Review*, 90(5), 1464-1476.

McKinnon, R. I. (1963). Optimum Currency Areas. *American Economic Review*, 52(4). 717-725

McMorrow, K and Roeger, W. (2001). Potential Output: Measurement Methods, New Economy Influences and Scenarios for 2001-2010. A Comparison of the EU15 and the US. *Economy papers*, 105.

Mills, T. C. and Wang, P. (2002). Plucking Models of Business Cycle Fluctuations: Evidence from the G-7 Countries. *Empirical Economics*, 27(2). 255-276.

Mills, T. C. and Wang, P. (2003a). Have Output Growth Rates Stabilised? Evidence from the G-7 Economies. *Scottish Journal of Political Economy*, 50(3). 232-246.

Mills, T. C. and Wang, P. (2003b). Multivariate Markov switching common factor models for the U.K. *Bulletin of Economic Research*, 55, 177-193.

Mills, T. C. (2003c). *Modelling Trends and Cycles in Economic Time Series*, Palgrave Macmillan, New York.

Mills, T. C. and Harvey, D. I. (2005). Evidence for common features in G7 macroeconomic time series. *Applied Economics*, 37(2), 165-175.

Mintz, N.N. (1970). Monetary Union and Economic Integration. *The Bulletin*, New York University Graduate School of Business Administration, Institute of Finance, 64.

Mitchell, W. C. (1927). *Business Cycles: The Problem and its Setting*, New York: National Bureau of Economic Research.

Morley, J. C., Nelson, C. R. and Zivot, E. (2003). Why are Beveridge-Nelson and Unobserved Component Decompositions of GDP so Different? *Review of Economics and Stat*istics, 85 (2). 235-243.

Mojon B. and Peersman, G. (2001). A VAR Description of the Effects of Monetary Policy in the Individual Countries of the Euro Area. *ECB Working Paper* 92.

Mundell, R. A. (1961). A Theory of Optimum Currency Areas. *American Economic Review*, 51(4). 657-665.

Murray, C. J. (2003). Cyclical Properties of Baxter-King Filtered Time Series. *Review of Economics and Statistics*, 85(2), 472-76.

Nelson, C. R. and Plosser, C. I. (1982). Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications. *Journal of Monetary Economics*, 10, 139-162.

Newey, W. K. and West, K. D. (1987). A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica*, 55(3), 703-08.

Okun, A. M. (1962). Potential GNP: Its Measurement and Significance, *In proceedings of the Business and Economics Statistics Section, American Statistical Association*, 98-103

Okun, A. M. (1980). Rational-Expectations-with-Misperceptions as a Theory of the Business Cycle. *Journal of Money, Credit and Banking*, 12(4). 817-25.

Orphanides, A. and van Norden, S. (1999). The Reliability of Output Gap Estimates in Real Time, *Macroeconomics*, 9907006, EconWPA.

Orphanides, A. (2001). Monetary Policy Rules Based on Real-Time Data. *American Economic* Review, 91(4), 964-985.

OECD (2001) *OECD Economic Outlook*, 69.

Peersman, G. and Smets, F. (1999). The Taylor Rule: A Useful Monetary Policy Benchmark for the Euro Area?. *International Finance*, 2(1), 85-116.

Perron, P. (1989). The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis, *Econometrica*, 57(6). 1361-1401.

Proietti, T. (1997). Short-Run Dynamics in Cointegrated Systems. *Oxford Bulletin of Economics and Statistics*, 59(3). 405-22.

Proietti, T., (2008), Structural Time Series Models for Business Cycle Analysis. *MPRA Paper*, 6854.

Proietti, T. and Harvey, A. (2000). A Beveridge-Nelson smoother. *Economics Letters*, 67(2). 139-146.

Proietti, T., Musso, T., and Westermann, T. (2007). Estimating Potential Output and the Output Gap for the Euro Area: A Model-Based Production Function Approach. *Empirical Economics*, 33(1), 85-113

Quah, D., (1992). The relative importance of permanent and transitory components: identification and some theoretical bounds. *Econometrica*, 60 (1), 107–118.

Reichlin, L., Doz, C. and Giannone, D., (2006). A Quasi Maximum Likelihood Approach for Large Approximate Dynamic Factor Models. *CEPR Discussion Papers 5724.*

Reichlin, L., Doz, C. and Giannone, D., (2007). A Two-Step Estimator for Large Approximate Dynamic Factor Models Based on Kalman Filtering. *CEPR Discussion Papers 6043.*

Ross, K. and Ubide, A. (2002). Mind the Gap: What is the best measure of slack in the euro area? *IMF Working Paper* 01/203

Rudebusch, G. D. (1992). Trends and Random Walks in Macroeconomic Time Series: A Re-examination. *International Economic Review*, 33(3). 661-80.

Rünstler, G. (2002). The Information Content of Real-Time Output Gap Estimates: an Application to the Euro area. *ECB Working Paper* 182.

Rünstler, G. (2004). Modelling phase shifts among stochastic cycles. *Econometrics Journal*, 7, 232–248.

Sammon, J. W. (1969). A Nonlinear Mapping for Data Structural Analysis. *IEEE Transactions on Computers*, 18, 401-409.

Sargent, T. J. and Sims, C. A. (1977). Business cycle modelling without pretending to have to much a priori economic theory, in Sims, C. A. (ed.). *New Methods in Business Cycle Research*, 45-109. Minneapolis: Federal Reserve Bank of Minneapolis.

Schleicher, C. (2003). Essays on the Decomposition of Economic Time Series. PhD dissertation, University of British Columbia.

Schleicher, C. (2007). Codependence in Cointegrated Autoregressive Models. *Journal of Applied Econometrics*, 22(1). 137-159.

Sichel, D. E. (1994). Inventories and the three phases of the business cycle. *Journal of Business and Economic Statistics*, 12, 269–277.

Sinclair, T. M. (2008). Asymmetry in the Business Cycle: Friedman's Plucking Model with Correlated Innovations, Department of Economics, The George Washington University.

Sinclair, T. M. and Mitra, S. (2008). Output Fluctuations in the G-7: An Unobserved Components Approach. Department of Economics, The George Washington University.

Stock, J. H. and Watson, M.W. (1988a). Variables Trends in Economic Time Series. *Journal of Economic Perspectives,* 2(3). 147-174.

Stock, J. H. and Watson, M. W. (1988b). Testing Common Trends. *Journal of the American Statistical Association*, 83, 1097-1107.

Stock, J. H. and Watson, M. W. (1989). New indexes of coincident and leading indicators in Blanchard, O. J. and Fischer, S. (eds.). *NBER Macroeconomics Annual*, 351–93, Cambridge: MIT Press.

Stock, J. H. and Watson, M. W. (1991). A probability model of the coincident economic indicators in Lahiri K. and Moore, G. H. (eds.). *Leading Economic Indicators: New Approaches and Forecasting Records*, 63–95, Cambridge University Press.

Stock, J. H. and Watson, M. W. (1993). A procedure for predicting recessions with leading indicators: Econometric issues and recent experiences, in Stock, J. H. and Watson, M. W. (eds). *Business Cycles, Indicators, and Forecasting*, 95–156, Chicago, University of Chicago Press.

Stock, J. H. and Watson, M. (2002a). Forecasting using principal components from a large number of predictors. *Journal of American Statistical Association*, 97, 1167–79.

Stock, J. H. and Watson, M. (2002b). Macroeconomic forecasting using diffusion indexes. *Journal of Business and Economic Statistics*. 20, 147–162.

Stock, J. H. and Watson, M. W. (2002c) Has the Business Cycle Changed and Why?. *NBER Macroeconomics Annual*, 159-218

Stock, J. H. and Watson, M. W. (2003) Has the Business Cycle Changed? Evidence and Explanations. FRB Kansas City symposium, Jackson Hole, Wyoming, 28-30.

Stock, J. H. and Watson, M. (2005). Implications of dynamic factor models for VAR analysis. *NBER Working Papers* 11467.

Taylor, J. B. (1993). Discretion Versus Policy Rules in Practice. *Carnegie-Rochester Conference Series on Public Policy*, 39, 195-214.

Tiao, G. C. and Tsay, R. S. (1989). Model specification in multivariate time series. *Journal of the Royal Statistical Society*, Series B, 51 (2), 157–213.

Tower, E. and Willett, T. (1976). The Theory of Optimum Currency Areas and Exchange Rate Flexibility, *Special Papers in International Economics*, 11, May, (Princeton).

US Bureau of the Census (1999), *X-12-ARIMA Reference Manual*, US Department of Commerce, Washington, DC.

Vahid, F. and Engle, R. F. (1993). Common Trends and Common Cycles. *Journal of Applied Econometrics*, 8(4). 341-60.

Vahid, F. and Engle, R. F. (1997). Codependent Cycles. *Journal of Econometrics*, 80(2). 199-221.

van Els, P., Locarno, A., Morgan, J. and Villetelle, J. P. (2001) Monetary Policy Transmission in the Euro Area: What Do Aggregate and National Structural Models Tell Us?. *ECB Working Paper* 94.

van Dijk, D., Osborn, D. R. and Sensier, M. (2002). Changes in Variability of the Business Cycle in the G7 Countries, *Centre for Growth and Business Cycle Research Discussion Paper 16*, Economics, The University of Manchester.

Watson, M. W. (1986). Univariate Detrending Methods with Stochastic Trends. *Journal of Monetary Economics* 18, 49-75

Watson, M. W. and Engle, R. F. (1983). Alternative algorithms for the estimation of dynamic factor, mimic and varying coefficient regression models. *Journal of Econometrics*, 23(3). 385-400.

Wold, H. (1938). *A Study in the Analysis of Stationary Time Series*, 2nd edn, 1954, Almqvist and Wicksell.

Wyne, M. A. and Koo, J. (2000). Business Cycles Under Monetary Union: A Comparison of the EU and US. *Economica,* 67, 347–374.

Yao, Y.-C. (1998). Estimating the number of change-points via Schwarz' criteria. *Statistics and Probability letters*, 6,181-189.

Zarnowitz, V. (1985). Recent Work on Business Cycles in Historical Perspective: Review of Theories and Evidence. *NBER Working Papers,* 1503.