Human capital and the ambiguity of the Mankiw-Romer-Weil model [*]

T.Huw Edwards

Dept of Economics, Loughborough University and CSGR Warwick UK
Tel (44)01509-222718
Fax 01509-223910
T.H.Edwards@lboro.ac.uk

December 2004

Abstract

Mankiw, Romer and Weil's (1992) finding of a cross-country relationship between savings rates, school enrolment and income levels is highly ambiguous. Their interpretation that it is consistent with an augmented Solow model depends on the implausible assumption that educational productivity is vastly higher in advanced countries than poor ones. On the alternative assumption of constant educational productivity, their model is very close to an AK-type, but with rising educational costs producing a degree of conditional convergence.

KEYWORDS: Growth, Human Capital, Endogenous Growth..

JEL Classification: O11, O41

---

1 Introduction.

Mankiw, Romer and Weil (henceforth MRW) claimed in 1992 that widening Solow's neo-classical growth model to include human capital overturned the conventional wisdom that it could not explain cross-country income differentials. Consequently they challenged the need for endogenous growth models (e.g. Romer, 1986 , Lucas, 1988).

This note does not cover criticisms of MRW's estimation or econometric methodology: rather I concentrate on the point that simply finding a 'conditional convergence' relationship with sensible-looking parameters does not of itself mean an augmented Solow model has been identified. Conditional convergence could be generated by a braod range of models, and MRW's neoclassical interpretation depends upon a highly questionable treatment of the cost of acquisition of human capital. In fact, on plausible parameter values, but with an alternative treatment of human capital costs, the model estimated shares many of the main features of an endogenous growth model, rather than the neoclassical-type models MRW believed they were resurrecting.

2 Outline of the Mankiw-Romer-Weil approach

MRW argued that the familiar Cobb-Douglas formulation of Solow's growth model should be extended to include human capital H as well as physical capital K. This would imply an underlying aggregate production function of the form

$$Y_{ct} = K_{ct}^{\alpha} H_{ct}^{\beta} (A_{ct} L_{ct})^{1-\alpha-\beta}, \tag{1}$$

where Y is total income, L is the labour supply and A is a technology parameter, with L growing at an annual rate n and A growing at rate g.

Following Solow, MRW rewrite income, physical and human capital in (1) in terms of quantities per unit of effective labour, $y_t = Y_t/A_tL_t$ etc.The changes over time in physical and human capital per unit effective labour are

$$k'_t = s_k y_t - (n + g + \delta)k_t, \tag{2}$$

$$h'_t = s_h y_t - (n + g + \delta)h_t, \tag{3}$$

where δ is proportionate depreciation for both physical and human capital. Savings rates for physical and human capital, s_k and s_h respectively, are assumed to be constant over time, though not across countries. Solving for steady-state solutions k^* and h^*, MRW derive an equation for steady-state income growth

$$\ln Y_t = \ln A_0 + gt - ((\alpha + \beta)/(1 - \alpha - \beta)) \ln(n + g + \delta)$$

$$+ (\alpha/(1 - \alpha - \beta)) \ln s_k + (\beta/(1 - \alpha - \beta)) \ln s_h. \tag{4}$$

The physical capital savings rate, s_k, was approximated by the investment share in GDP, while the human capital savings rate s_h was measured by the proportion of the working age population at any one time enrolled in secondary school - 'SCHOOL' in the MRW estimated equations. Estimation on cross-section samples of 98 and 75 countries respectively in 1985 yielded greatly improved fit compared to the Solow model excluding human capital, and

the parameter restrictions implied in equation (4) were not rejected statistically, while the implied income shares of physical and human capital, both around 0.3, were judged to be plausible.[1]

3   The treatment of human capital

Arguably equations (2) and (3) should both contain measures of the costs of acquiring physical and human capital, since these may differ between countries. Prices of both types of capital may change relatively to those of consumer goods as income levels alter. More formal analysis should include separate production functions for both capital goods[2]. However, I suggest that it may not be too bad an approximation here to equate physical capital and consumer goods prices.

With human capital the problem is much more serious. Equation (3) measures the volume of human capital in terms of income foregone during education - meaning a year of schooling would be around 40 times more valuable in terms of units of human capital acquired in Norway (1985 GDP per adult $19,723) than in Chad (GDP $462 per adult). This assumption almost certainly results in MRW seriously overestimating the difference in stocks of human capital per head.[3]

To take account of this, I suggest a slightly modified model, where education is a separate

---

[1] A third regression, on 22 OECD countries, did not perform well.

[2] Thanks to Neil Rankin for making this point.

[3] Note that a doctor or engineer trained in Norway would not earn 40 times the wage paid to a colleague who had migrated from Chad.

sector and the total potential workforce, $\overline{L}$ is split into proportions θ being educated and $(1 - \theta)$ working. Ignoring unemployment, the ratio of those being educated per worker is therefore $(\theta/(1 - \theta))$. θ, which I take as exogenous, is essentially the same variable MRW used to proxy s_k.

Further, assume human capital accumulation is a linear function of years of schooling, so that the change in average human capital per unit of 'augmented' labour is

$$h'_t = \eta(\theta/(1 - \theta)) - (n + g + \delta)h_t, \tag{5}$$

where η is a scale parameter. MRW implicitly assume that η is proportional to total factor productivity across the whole economy. I suggest this is unrealistic. In this paper, I investigate a more general model where educational labour productivity is related to average labour productivity in the rest of the economy with a uniform elasticity

$$\eta_c = \overline{\eta} y_c{}^\phi \tag{6}$$

for each country c. Since education is a service sector, the Balassa-Samuelson literature would suggest $0 \leqslant \phi \leqslant 1$. Of particular interest is the case where educational productivity is constant across all countries ($\phi = 0$).

As a minor simplification, it is assumed that the resources employed in education (mostly the people being educated) are not measured in official GDP. We will also add to the model a random fluctuation in total factor productivity outside education, ϵ_c. so that

$$y_{ct} = \epsilon_c k_{ct}^\alpha h_{ct}^\beta, \tag{7}$$

and it follows that the equilibrium conditions for h and k for each country (denoted now with ^{*AS} to denote the augmented Solow model equilibrium) are then

$$h_{ct}^{*AS} = y_c^{*AS\phi}(\theta/(1-\theta))/(n+g+\delta), \tag{8}$$

$$k_{ct}^{*AS} = s_k y_{ct}^{*AS}/(n+g+\delta) \tag{9}$$

Substituting for h^{*AS} and k^{*AS} into (9) we therefore obtain y^{**AS}, which can be written in logs as

$$\ln Y_c^{*AS} = \ln A_0 + gt$$

$$-((\alpha+\beta)/(1-\alpha-\beta\phi))\ln(n+g+\delta) + (\alpha/(1-\alpha-\beta\phi))\ln s_{kc}$$

$$+(\beta/(1-\alpha-\beta\phi))\ln\theta_c - (\beta/(1-\alpha-\beta\phi))\ln(1-\theta_c)) + (1/(1-\alpha-\beta\phi))\ln\epsilon_c. \tag{10}$$

While this contains the same parameters as equation (4) it can be seen that the parameter restrictions are different, reflecting a different underlying model. Nevertheless, the key coefficients on $ln(n+g+\delta), \ln s_k$ and $\ln s_h$ are still in the same relative proportions.

It follows that the only differences is that the terms in equation (10) apart from lnA_0 and gt are just those in equation (4) scaled up by $(1-\alpha-\beta)/(1-\alpha-\beta\phi)$, and that there is the one extra term $-(\beta/(1-\alpha-\beta\phi))\ln(1-\theta_c))$. In fact, however, the extra term will not greatly change the regression, since $\ln\theta_c - \ln(1-\theta_c)$ is very nearly approximated by a linear function of $\ln\theta_c$ with a very small intercept and a slope coefficient only slightly less than 1.[4] A_0 is just a constant scalar. When the model is estimated over a cross-section

---

[4]On data points of $\theta = 2\%, 5\%, 10\%, 15\%, 20\%$ the fitted slope coefficient is 0.92.

sample in a single year only, the differences in coefficients on gt become irrelevant.We can therefore approximately relate the models in (10) and (4):

$$Y_c^{*AS} \approx (Y_c^{*MRW})^{(1-\alpha-\beta\phi)/(1-\alpha-\beta)}. \tag{11}$$

3.1 The model fitted by Mankiw et al

In this note, I concentrate on the steady-state cross-country version of the MRW model, and ignore the later set of estimates based upon changes in income 1960-85 using a modified partial adjustment version of the model.[5] Mankiw et al fitted first an unrestricted and then a restricted version of equation (4)/(10). Their key results were that the coefficient on $ln(s_k)$ and that on θ (which I argue could quite easily be a proxy for $\theta/(1-\theta)$ with virtually no effect on fit) are both very close to unity, while that on $(n+g+\delta)$ is approximately minus 2.

To understand the ambiguity of these results, consider the interpretation of a rough version of their estimated cross-country equation

$$\ln Y = CONSTANT + \ln sk + \ln SCHOOL - 2\ln(n+g+\delta) + residual. \tag{12}$$

To fit this, α and β would have to satisfy approximately the following equations:

---

[5]The results of the dynamic equations were somewhat less plausible, giving a larger coefficient for physical capital and smaller for human capital than the static equation. The loglinear adjustment model MRW use for off-steady-state convergence (based on a Taylor approximation around the steady-state point) may well be too approximate to apply to growth rates over a 25 year period.

$$\alpha/(1 - \alpha - \beta\phi) = 1; \tag{13}$$

$$\beta/(1 - \alpha - \beta\phi) = 1; \tag{14}$$

$$(\alpha + \beta)/(1 - \alpha - \beta\phi) = 2. \tag{15}$$

(15) is just linear combination of the other two. (12)-(13) will be satisfied by values

$$\beta = \alpha = 1/(2 + \phi). \tag{16}$$

Working from data on factor income shares in GDP, MRW express a prior expectation that α and β should both be close to $1/3$. But those are exactly the values implied by (16) when $\phi = 1$. Therefore, if one were to accept that educational productivity is directly proportional to GDP, the implied factor shares in an augmented Solow model would be very close to the fitted coefficients of their restricted regression. However, for values of $\phi < 1$, the fitted regression can only be satisfied by values of α and β greater than $1/3$, which would be inconsistent with a neoclassical model, at least within a Cobb-Douglas framework and given observed income shares. Hence, on a more plausible model of education, the MRW model is not consistent with an augmented Solow model.

Nevertheless, a broader class of models does fit the restricted MRW equation: namely models with technological spillovers external to the firm. Say

$$y = k^{\widehat{\alpha}} h^{\widehat{\beta}}, \tag{17}$$

where the fitted values of $\widehat{\alpha}$ and $\widehat{\beta}$ still need to satisfy

$$\widehat{\beta} = \widehat{\alpha} = 1/(2 + \phi), \tag{18}$$

but that now these can be decomposed into

$$\widehat{\alpha} = \alpha + \gamma \tag{19}$$

and

$$\widehat{\beta} = \beta + \delta, \tag{20}$$

where $\alpha = \beta = 1/3$ and γ and δ represent external technological spillovers. In this case, (12) would be satisfied by

$$\delta = \gamma = (1 - \phi)/(6 + 3\phi). \tag{21}$$

This shows that, to fit (12), technological spillovers are only zero when $\phi = 1$. When $\phi = 0$, so that educational productivity is constant across countries, $\widehat{\beta} = \widehat{\alpha} = 1/2$. In this case, the long-run steady state levels of income per capita are given by

$$y^{**} = k^{**1/2} h^{**1/2}. \tag{22}$$

This is an AK-type model, with non-educational output homothetic in terms of a Cobb-Douglas aggregate of capital. However, while this model produces much slower conditional convergence than an augmented Solow model, it does nevertheless produce some convergence due to rising educational costs in advanced countries. Only where productivity in education, too, rises proportionately with income will an AK model produce totally en-

dogenous growth.

4   Implications

MRW's estimated model is consistent with a general class of models, not just the augmented
Solow model they favour. All of these models produce roughly the same convergence
pattern as documented by MRW. However, unless educational attainment per hour study
time is vastly higher in rich than in poor countries, the augmented Solow model cannot be
supported by their result. By contrast, a model with technical spillovers, but which shows
convergence due to the lack of difference in educational sector productivity is more credible
for several reasons: not least the observation that skilled labour tends to flow from poor
countries to richer ones, where it tends to earn more despite its abundance.[6] While the
two models may have similar convergence properties in a closed economy, increasing trade,
capital and labour flows between rich and poor countries since 1985 suggest the two models
may have very different predictions today.

It is also worth noting that the existence or non-existence of technological spillovers has
important implications in terms of optimal economic policies. For this reason alone, the
ambiguity of MRW's result should cause people to treat their findings with due caution.

---

[6]High returns to education in poor countries, as noted by MRW, are because education is cheap, not
because human capital is well paid.
See also Easterly and Levine's 'stylized facts' (2001).

References

[1] Easterly, W. and R.Levine (2001): 'Its Not Factor Accumulation: Stylised Facts and Growth Models'. World Bank Economic Review 15(2); pp 177-219.

[2] Lucas, R.E. (1988): 'On the Mechanics of Economic Development', Journal of Monetary Economics 22, pp 3-42.

[3] Mankiw, N.G., D.Romer and D.N.Weil (1992): 'A Contribution to the Empirics of Economic Growth,' Quarterly Journal of Economics, 107 pp 407-437.

[4] Romer, P.M. (1986): 'Increasing Returns and Long-Run Growth', Journal of Political Economy 94, pp 1002-38.

[5] Solow, R.M. (1956): 'A Contribution to the Theory of Economic Growth', Quarterly Journal of Economics 70, pp 65-94.