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On cost restrictions in spatial competition models with heterogeneous firms

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Abstract

This paper investigates the properties of two types of cost restrictions that guarantee the existence of an equilibrium in pure strategies in Bayesian spatial competition models with heterogenous firms.

1 Introduction

Aghion and Schankerman (2004) and Syverson (2004) were the first to investigate the properties of circular city models *à la* Salop (1979) with asymmetric costs. These two articles assume a very similar Bayesian set-up (i.e. the rivals' costs are unknown) but adopt different restrictions to deal with the fact that when cost asymmetry is large with respect to the transport cost parameter, a pure-strategy price equilibrium may not exist.[1]

On the one hand, the restriction in Syverson (2004) is such that there always exists an indifferent consumer located between two neighboring producers, i.e. a high cost firm facing tough competition from two low-cost neighbouring rivals has a positive market share in any state of the world. On the other, Aghion and Schankerman (2004, AS hereafter) use a weaker restriction, which only requires that a high-cost firm has, *on average*, a positive market share.

This paper develops a theoretical analysis aimed at identifying the economic implications associated with the use of different restrictions on cost asymmetry in spatial competition models. To this purpose, we adopt the same theoretical set-up as in AS to investigate the properties of the two types of cost restrictions we focus on. The first type arises when

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[1]Existence problems in localized competition models date back to d'Aspremont et. al. (1979), where it is shown that in the Hotelling set-up a pure-strategy equilibrium fails to exist when firms are too close.

the 'no mill-price undercutting' condition holds (Eaton and Lipsey, 1978). This condition implies that firms commit not to undercut their opponents, i.e. that even if profitable, firms would not choose to charge a price that may completely exclude a rival from the market. Our analysis is further extended by deriving the second type of cost restrictions without relying on firms' commitment, in line with the approaches in Vogel (2008) and Alderighi and Piga (2008).

Our main result is that the second restriction type imposes a stricter limit on the amount of cost asymmetry that is consistent with the existence of an equilibrium. Furthermore, we show that the restriction in AS is too weak even relative to our first type, so that in that model with positive probability a high-cost firm may have (ex-post) a negative market share. We obtain that after imposing the first restriction type, some of their policy recommendations are not supported by their analysis.

2 The set-up

We shall first recall AS's model and notations. On a circumference of unitary length, n evenly located firms produce a homogeneous good. Firms have either a high unit cost c_H or a low unit cost c_L; q denotes the probability of drawing a high-cost firm; c_H, c_L and q are common knowledge. Firms know their, but not their rivals', type. On the same circumference, a continuum of uniformly distributed consumers has mass normalized to 1. A generic consumer receives a utility $u = v - p_i - td_i$ if she buys one unit of the good from firm i, where v is the product's gross valuation, p_i is the mill price charged by firm i, t is the unit transport cost and d_i is the shortest distance of the consumer from firm i. Let p_i, p_{i-1} and p_{i+1} denote the prices charged by firm i and its two closest rivals. The marginal consumer between i and $i+1$ $(i-1)$ lies at a distance x_R (x_L) from firm i:

$$x_R\left(p_{i+1}, p_i\right) = \frac{1}{2n} + \frac{1}{2t}\left(p_{i+1} - p_i\right) \tag{1}$$
$$x_L\left(p_{i-1}, p_i\right) = \frac{1}{2n} + \frac{1}{2t}\left(p_{i-1} - p_i\right) \tag{2}$$

The expected profit of firm i is given by: $\Pi_i = \left(p_i - c_i\right)\left(Ex_L\left(p_{i-1}, p_i\right) + Ex_R\left(p_{i+1}, p_i\right)\right)$. Following AS, the expected market share of firm i, conditional on all other low- (high-) cost firms charging the same price p_L (p_H), is given by:

$$D_i\left(p_H, p_L, p_i\right) = Ex_L + Ex_R = \frac{1}{n} + \frac{1}{t}\left(qp_H + (1-q)\,p_L - p_i\right). \tag{3}$$

Symmetry and firms' profit maximizing behaviour imply that equilibrium prices and expected market shares are:[2]

$$
\begin{aligned}
p_H &= \frac{t}{n} + c_H - \frac{1}{2}(1-q)\Delta c && (4)\\
p_L &= \frac{t}{n} + c_L + \frac{1}{2} q\Delta c && (5)\\
D_H &= \frac{1}{n} - \frac{1}{2t}(1-q)\Delta c && (6)\\
D_L &= \frac{1}{n} + \frac{1}{2t} q\Delta c, && (7)
\end{aligned}
$$

where $\Delta c = c_H - c_L$ and $p_H - p_L = \Delta c/2$.

3 On cost restrictions with commitment

In this Section, we show that the condition presented in AS generally allows an excessive amount of cost asymmetry. We start by assuming that firms commit not to undercut rivals (Eaton and Lipsey, 1978):

Condition 1 *(No mill-price undercutting) $p_H - p_L < t/n$.*

To guarantee that in the AS solution both types of firms have ex-post positive market shares, i.e. $x_L, x_R \geq 0$, it is therefore necessary that the AS price solution satisfies Condition 1. Therefore, we replace (4) and (5) into Condition 1, obtaining:

$$\Delta c \leq 2\frac{t}{n}. \tag{8}$$

In AS, the relevant boundary condition ensuring that (4) and (5) represent a price equilibrium is given by $D_H \geq 0$ (see p. 804), which implies:

$$\Delta c \leq 2\frac{1}{1-q}\frac{t}{n}, \tag{9}$$

i.e. the degree of cost asymmetry that is allowed in the AS set-up is proportional to the parameter t/n, but it can be arbitrarily large when q is close to 1. Thus, (9) is weaker than (8). We delve deeper into the possible implications of adopting (9) instead of (8) by first demonstrating that $D_H > 0$ is a weighted average of positive and negative 'quantities'. Then, we show that when $x_L < 0$ and/or $x_R < 0$, firm i's 'real' market share should be

[2] In AS, the expressions equivalent to (4) and (5) are mis-typed; indeed, the subsequent equations, e.g., (6) and (7), are identical to the ones we present here.

null, and therefore $D_H > 0$ in (3) is misspecified (i.e. positive and negative quantities do not cancel out).

We begin by decomposing D_H into three parts in accordance with the number of low-cost neighbours a high-cost firm may potentially face. We focus on the parameters' range for which (9), but not (8), is satisfied, i.e.: $2\frac{t}{n} < \Delta c \leq 2\frac{t}{n}\frac{1}{1-q}$:

1. with probability $(1-q)^2$, firm i has two low-cost neighbours; therefore using (1) and (2) we obtain $x_L(p_L, p_H)$, $x_R(p_L, p_H) < 0$ and $x_L + x_R = 1/n - \Delta c/2t < 0$;

2. with probability $2(1-q)q$, firm i has one low-cost (e.g. $i-1$) and one high-cost neighbour (e.g. $i+1$), therefore $x_L < 0$ and $x_R > 0$ (or viceversa), with $x_L + x_R = (1/n - \Delta c/4t) \gtreqless 0$. (This case is represented in Figure 1)

3. with probability q^2, firm i has no low-cost neighbours and $x_L + x_R = 1/n$.

Adding up previous results yields the expected market share of a high-cost firm reported in (6):

$$D_H = (1-q)^2 \underbrace{\left(\frac{1}{n} - \frac{1}{2t}\Delta c\right)}_{<0} + 2q(1-q)\underbrace{\left(\frac{1}{n} - \frac{1}{4t}\Delta c\right)}_{\gtreqless 0} + q^2 \underbrace{\frac{1}{n}}_{>0} \geq 0. \tag{10}$$

Therefore, in AS, when firms' cost asymmetry is sufficiently large, the expected market share of a high-cost firm is the weighted average of positive and negative market shares, i.e., with positive probability some firms will be ex-post selling strictly non-positive quantities.

Furthermore, when (8) is violated, (10) generally misrepresents the corrected expected market share of the high-cost firm i. To show this, assume that (pure-strategy) equilibrium prices exist and that they are given by (4) and (5). Moveover, consider different intervals of cost asymmetry, indexed by $k = 0, 1, 2, ..$:

$$2k\frac{t}{n} < \Delta c \leq 2(k+1)\frac{t}{n},$$

where k represents the number of high-cost neighbours needed to shield firm i, i.e., to guarantee that a high-cost firm is not undercut by a low-cost firm located too close. Conditional on k high-cost firms being located at the right and k high-cost firms being located at the left of i, a $(k+1)$-step neighbour is high-cost with probability q, and it is low-cost with probability $(1-q)$. When a $(k+1)$-step neighbour is low-cost, it undercuts all the neighbours of firm i, and therefore it competes directly with firm i. Otherwise, if a $(k+1)$-step neighbour is high-cost, the 1-step neighbour of firm i has a positive market share and competition takes place beween that 1-step neighbour and firm i. Hence:

1. with probability q^{2k}, firm i is not undercut, and therefore its market share is $(1-q)\left(\frac{1+k}{n} - \frac{1}{2t}\Delta c\right) + q\frac{1}{n}$, and

2. with probability $1 - q^{2k}$, firm i is undercut and its market share is 0.

Thus, the expected market share of a high-cost firm is $q^{2k}\left((1-q)\left(\frac{1+k}{n} - \frac{1}{2t}\Delta c\right) + q\frac{1}{n}\right)$, which coincides with (10) when $k = 0$, but generally differs from it, when $k \geq 1$. Accordingly, for $k \geq 1$, (3) cannot be used to derive the expected market shares presented in (6) and (7). Therefore (4) and (5) do not necessarily represent the equilibrium prices of the game.

4 On some economic implications

Some of the economic policy implications in AS derived under (9) fail to hold when (8) is imposed.[3] The main point is given by the fact that AS analysis strongly relies on the fact that with sufficient heterogeneity, aggregate profit is increasing in the degree of competition (i.e. $1/t$). In particular, in Proposition 2.(iii), AS state that competition-enhancing policies increase aggregate profits if $\left(\frac{\Delta c}{2t}\right)^2 > \frac{1}{n^2 q(1-q)}$ (see page 820 in AS). Because $\frac{1}{q(1-q)} > 1$, this condition violates (8), which if taken into account implies that aggregate profit is always decreasing in the degree of competition.

Other propositions are affected by the erroneous boundary conditions. The argument parallels that for Proposition 2.(iii). An important implication of Proposition 3 hinges around the assumed divergence of interests between high- and low-cost firms, i.e. $\partial\Pi_H/\partial t > 0$ and $\partial\Pi_L/\partial t < 0$. However, the latter requirement is equivalent to $\Delta c > 2t/(nq)$, a condition which never holds under (8). Therefore, the possibility of a low-competition political economy trap is not sustained by the analysis. Moreover, in the analysis of entry carried out in Section 4 of AS and summarised in Proposition 6, increasing the degree of competition increases the probability of entry by a low-cost firm and reduces that of entry by a high-cost one if $\partial\Pi_L/\partial t < 0$, which, as discussed above, does not hold under (8). Finally, in the simulation of welfare effects in Section 5, AS consider some values that are out of the boundary conditions.

[3] To prove that AS results continue to hold when (8) is violated entails complex computations for deriving a new price equilibrium, that due to the discontinuity and non quasi-concavity of the pay-off functions is likely to be in mixed strategies (Dasgupta and Maskin, 1986). This is beyond the goals of this work.

5 On the boundary conditions without commitment

Previous considerations have been derived while assuming that firms are committed to Condition 1. As discussed in Alderighi and Piga (2008), if costs satisfy (8), but firms are free to deviate from Condition 1, then computed prices are not necessarily an equilibrium, since for some parameter ranges, a low-cost firm could profitably deviate by undercutting their closest rivals.

The following proposition guarantees the equilibrium existence without relying on the 'no mill-price undercutting' assumption.

Proposition 1 *When $q \in (0,1)$, equations (4) and (5) represent a symmetric Bayesian (pure-strategy) Nash price equilibrium if:*

$$\Delta c \leq \alpha(q)\frac{t}{n}, \tag{11}$$

where $\alpha(q) = \left(q + q^2 + 2 - q\sqrt{6q + q^2 + 5}\right) / \left(q - q^3 + 1\right) \in \left(4 - 2\sqrt{3}, 2\right)$ *and* $\alpha'(q) < 0$.

Proof See Appendix. ■

Proposition 1 states that it is more likely to have an equilibrium when q is small. The intuition follows from the fact that since profitable deviations mainly occur when low-cost firms try to undercut high-cost ones, when q is small a low-cost firm has a lower incentive to deviate because: first, the low-cost equilibrium price is smaller (with respect to the case where q is large) and therefore undercutting yields a lower unit margin; second, the chance of undercutting a high-cost firm and the size of its market share are smaller, so that undercutting yields a smaller rise in expected market share. This argument also clarifies why there are no existence problems in the standard circular model (Salop, 1979). In fact, when costs are symmetric ($\Delta c = 0$), a pure-strategy equilibrium always exists as there are no less-efficient firms to be undercut.

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Appendix

Proof of Proposition 1. To be an equilibrium, we check whether a player has no incentive to deviate from its strategy. We focus on the situation in which the boundary condition (8) holds, and we start analyzing the incentive for a firm to undercut its neighbours. Note that condition (8) implies that by gradually lowering its price, a deviating firm first undercuts 1-step high-cost neighbours, then 1-step low-cost neighbours, afterwards, 2-step high-cost neighbours, and so on and so forth.

A) We start by analysing the case in which firm i charges $p_i \in \left(p_L - \frac{t}{n}, p_H - \frac{t}{n}\right]$, so that it is able to undercut a 1-step neighbour only when the latter is a high-cost firm. In this case, it is necessary to extend (3) to account for undercutting. Thus, when $i+1$ is a low-cost firm, the marginal consumer between i and $i+1$ is located at the distance given by (1), and when $i+1$ is a high-cost firm, the marginal consumer is located between i and $i+2$ at the distance of:

$$x_{RR}\left(p_{i+2}, p_i\right) = \frac{1}{n} + \frac{1}{2t}\left(p_{i+2} - p_i\right).$$

The marginal consumer between i and $i-2$ is defined similarly: $x_{LL}\left(p_j, p_i\right) = x_{RR}\left(p_j, p_i\right)$, with $j = L, H$. By symmetry, the expected market share of firm i is therefore given by:

$$\begin{aligned} D_i\left(p_H, p_L, p_i\right) &= q\left(q \cdot 2x_{RR}\left(p_H, p_i\right) + (1-q) \cdot 2x_{RR}\left(p_L, p_i\right)\right) + (1-q) \cdot 2x_R\left(p_L, p_i\right) \\ &= q\left[q\left(\frac{2}{n} + \frac{p_H - p_i}{t}\right) + (1-q)\left(\frac{2}{n} + \frac{p_L - p_i}{t}\right)\right] + (1-q)\left(\frac{1}{n} + \frac{p_L - p_i}{t}\right). \end{aligned}$$

We focus on the case that i is a low-cost firm, since in this case the incentive to deviate is larger. Note that the profit function of i, $(p_i - c_L) D_i (p_H, p_L, p_i)$, is increasing in p_i, so that the maximum deviation profit is reached when the firm sets $p_D = p_H - \frac{t}{n}$, and obtains a unit margin $p_D - c_L = \frac{q+1}{2}\Delta c$. Thus, the expected demand is:

$$D_i = \frac{q+2}{n} - (1-q)(q+1)\frac{\Delta c}{2t}.$$

And the expected profit is:

$$\Pi_D = \frac{q+1}{2}\Delta c \left[\frac{q+2}{n} - (1-q)(q+1)\frac{\Delta c}{2t}\right]. \tag{12}$$

In order to have no incentive to deviate, $I_D = \Pi_D - \Pi_L < 0$. Noting that $\Pi_L = t(D_L)^2$ and after some simplifications we obtain: $I_D = -\frac{1}{t}\left(\frac{t}{n}\right)^2 + \frac{1}{2t}\left(\frac{t}{n}\right)\left(q + q^2 + 2\right)\Delta c - \frac{1}{4t}\left(1 + q - q^3\right)(\Delta c)^2$. Therefore:

$$\Delta c < \left[\frac{1}{q - q^3 + 1}\left(q + q^2 + 2 - q\sqrt{6q + q^2 + 5}\right)\right]\frac{t}{n}. \tag{13}$$

B) We now consider a case where a low-cost firm charges $p_{DD} \in \left(p_H - 2\frac{t}{n}, p_L - \frac{t}{n}\right]$, so that it is able to undercut both types of 1-step neighbour. Again, the profit function in the interval is increasing in p_{DD}, so that the maximum deviation profit is reached when the firm sets $p_{DD} = p_L - \frac{t}{n}$, and obtains a unit margin $p_{DD} - c_L = \frac{q}{2}\Delta c$. The expected market share is:

$$D_i = \frac{2}{n} + \frac{1}{t}\left(qp_H + (1-q)p_L - p_L + \frac{t}{n}\right) = \frac{3}{n} + \frac{q}{t}\frac{\Delta c}{2}.$$

The incentive to deviate is hence $I_{DD} = \Pi_{DD} - \Pi_L = \frac{q}{2}\Delta c\left(\frac{3}{n} + \frac{q}{t}\frac{\Delta c}{2}\right) - t\left(\frac{1}{n} + \frac{1}{2t}q\Delta c\right)^2$. Therefore, there is no incentive to deviate when $\Delta c \leq \frac{2}{q}\frac{t}{n}$, condition that is always satisfied when (13) holds.

C) By assuming lower deviation prices, unit margins reduce so that firms have less incentive to deviate. Conversely, it is easy to show that firms have no unilateral incentive to increase their prices. ■

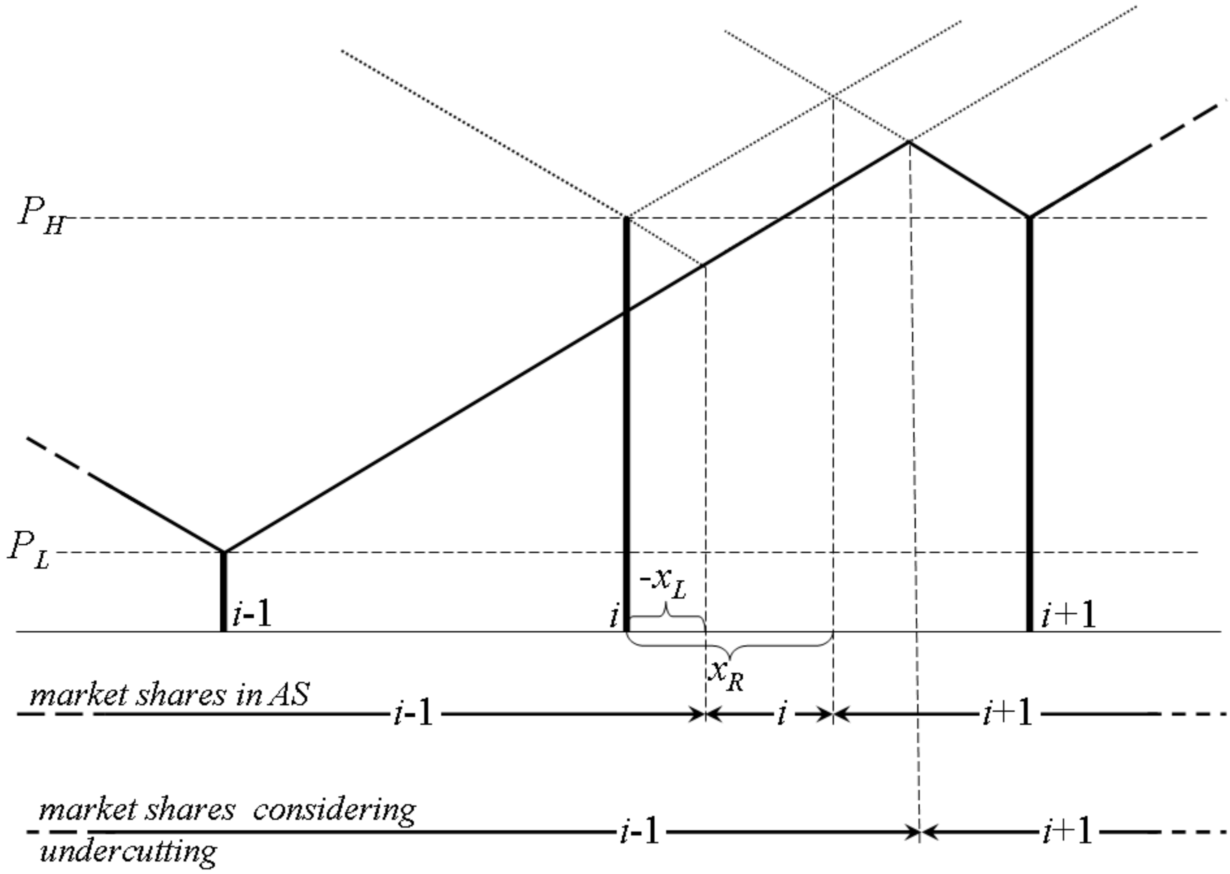

Figure 1: Market shares as computed in AS and considering *undercutting*.