## OIL AND GAS MARKETS IN THE UK: EVIDENCE FOR FROM A COINTEGRATING APPROACH

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## 24th October 2004

## Abstract

The paper examines the relationship between UK wholesale gas prices and the Brent oil price over the period 1996-2003. Tests for Unit Roots and Cointegration are carried out and it is discovered that a long run equilibrium relationship between UK gas and oil prices predates the opening of the UK-Mainland Europe Inter-connector. Following a recursive methodology (Hansen & Johansen 1999), it was found that the cointegrating relationship is present throughout the sample period. However, the long run solutions seem to be more volatile. Evidence is provided that the short run relationship is linear and impulse response functions are used to examine the effects that a shock in oil would have on gas.

**Keywords:** oil, gas, cointegration, nonparametric cointegration, recursive trace test, error correction, impulse response

JEL: C22, C52, O13, Q43

Acknowledgements: We would like to thank Terence C. Mills, Christian Richter and Ian Rutledge for their useful comments. The usual disclaimer applies.

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### **1. INTRODUCTION**

Between mid-1999 and the end of 2000, UK wholesale natural gas prices, which had remained at very low levels since the spot and futures markets were first established in 1995-1996, increased very substantially, with prices almost doubling. This rapid increase in the price of gas was concomitant with that occurring in the international oil market, where the price of Brent Crude oil increased by more than one hundred per cent during 1999. Between 2001 and 2003 UK gas prices continued to experience much greater volatility than previously and, by late 2002, the rising oil price was again accompanied by rising gas prices.

This development was, on the face of it, surprising, since the conventional view of the behaviour of gas prices in a liberalised market like that of the UK, is that *"the linkage with oil prices is now much less transparant."* (Barton & Vermeire, 1999, p.1). However, a report by Ilex Energy Consulting Ltd (Ilex 2001) to the UK Government in January 2001 argued that the main factor explaining the rise in the gas price through 1999-2000 was the link that had recently been established between the UK and oil-indexed Continental gas markets. The opening of the UK-Belgium inter-connector gas pipeline at the end of 1998 established a relationship between two gas markets which had previously been quite independent of one another. Ilex concluded that *"the link with oil indexed gas prices on the Continent is the most important factor in explaining the rise in UK gas prices through 2000"* (Ilex 2001, p.iii).

The inter-connector pipeline between Bacton in Norfolk and Zeebrugge in Belgium was originally built to allow UK gas to be supplied to continental markets where reserves and gas production are much lower than that of the UK. However, the flow of gas can be reversed (albeit at a smaller rate than the flow from the UK to the continent). After the opening of the inter-connector in October 1998, initially gas began to flow in this 'reverse' direction, from the Continent to the UK. However, with Continental gas prices rising sharply in 1999, it became clear to UK gas traders that arbitrage profits were available, selling UK gas, via the inter-connector, in the higher priced Continental markets. Thus gas prices traded at Bacton began to converge with gas prices in Zeebrugge<sup>1</sup>. Continental gas prices are indexed to the price of oil (either crude or fuel oil) in long-term contracts so that when oil prices rise eventually so will Continental gas prices through their contracts. Therefore, arbitrage activities between the UK and Continental gas markets that led to price convergence between the two markets also indirectly linked UK gas prices to the price of oil.

Contrasting with the argument presented by Ilex (2001), which posits a link between gas and oil prices which is fundamentally dependent upon the existence of contractual terms linking oil and gas prices in a non-liberalised gas market, Barcella (1999) discovered a relationship between oil and gas prices in the liberalised US gas market) which was based on more fundamental and long-run economic factors. The author found that crude oil and natural gas prices were highly correlated in the US, "yearly trends in crude oil and natural gas prices ...are highly correlated, with a coefficient of 0.916. The weekly gas and fuel oil prices... are less highly correlated, but are cointegrated" (Barcella, 1999, p.12) Barcella argued that the close relationship between natural gas and oil prices (both crude and fuel oil) reflected the underlying economic fact that the fuels were substitutes for one another in a large number of industrial processes. In particular she referred to "the significant inter-fuel competition in the electric power sector."

However, in the UK, prior to the opening of the inter-connector and the establishment of a direct link between the UK gas price and the oil price, the relationship between the two commodity prices certainly appeared to be

<sup>&</sup>lt;sup>1</sup> Plus the costs of transportation via the inter-connector.

minimal. In spite of the same underlying substitutability factors, natural gas prices for industrial customers in the UK fell well below the calorific equivalent oil prices between 1994 and 1998.<sup>2</sup> On the other hand this does seem to have been attributable to a 'conjunctural' factor – a substantial, but temporary, oversupply of gas in this period and consequent fierce gas-on-gas competition which drove down both spot gas prices and base prices in new contracts (IEA, 1998, p.9) Ilex (2001) conclusions were based on a descriptive analysis of the UK and Continental gas markets and networks rather an econometric analysis of the relationship between UK gas prices and oil prices. This paper investigates whether econometric analysis supports the argument presented by Ilex. Specifically, we examine the presence of a cointegrating (long-run) relationship between UK gas prices; did this emerge only after the opening of the Inter-connector (as Ilex's work would appear to suggest) or does it pre-date the Inter-connector? And what is the nature of the oil price-gas price relationship in the short-run?

Economic theory suggests that past changes in the oil price cause current changes in the price of UK gas but not vice versa. Apart from the fact that Continental gas prices are indexed to the price of oil and not vice-versa, it seems unlikely that the gas price would be able to influence the oil price. The size of the contract market and volumes traded for oil are vastly greater than for the gas market<sup>3</sup>. The Brent crude oil price is an international price determined in large part by the world supply and demand for oil, whereas the UK gas market is, by definition, only a national one. However, it is possible that expectations of an oil-gas relationship

<sup>&</sup>lt;sup>2</sup> Domestic gas prices followed a different trajectory since they were controlled by the Regulator (OFGAS and then later, OFGEM) from 1986 until 1998. However, prices to Industrial customers were excluded from regulation. Partly because they were controlled by the RPI-X method which initially was very lax, nominal domestic gas prices actually rose after privatisation and when they eventually began to decline in the mid-1990s, the decline was considerably less than that experienced by industrial gas prices.

could cause gas prices to feed back onto oil prices. To investigate these possibilities, we shall also employ Vector Error Correction Models (VECM) specifications to model the UK gas price on the one hand and impulse response functions are generated on the other to examine the consequences of a shock introduced in the oil price.

The rest of the paper is organised as follows. Section 2 presents the related literature and Section 3 provides more information about the data set. Unit root and cointegration tests are considered in Section 4 whereas the error correction models are given in Section 5. The discussion of the empirical analysis and the impulse response function analysis is provided in Sections 6 and 7 respectively. Finally, Section 8 contains some concluding remarks.

## 2. Related Literature

Most of the literature on energy price modelling in general has focused on electricity markets rather than gas markets. Electricity and gas markets are similar in the sense that these commodities have been deregulated over the last decade in many countries. Since deregulation, prices for these commodities have become more volatile. Therefore understanding the factors that drive them on the one hand and modelling them on the other could reduce uncertainty, something that is highly desirable for market participants.

Electricity prices have however, been much more volatile than gas prices, partly due to the non-storability of electricity (Huisman and Mahieu, 2001). It is, perhaps, for this reason that most of contemporary research has concentrated on modelling electricity prices, rather than gas prices.

<sup>&</sup>lt;sup>3</sup> Volumes for the Brent oil futures contract market at the IPE per day were 1,639,041 in February 2002 compared to only 43,560 for the natural gas futures contract market.

De Vany and Walls (1999) used cointegration methods to model regional US electricity markets and to assess price integration between these markets. Using a vector error correction model, the authors provide strong evidence of cointegration between eleven regional electricity markets in the US. More recently, Hendry and Juselius (2000, 2001) discuss extensively the cointegrating relations between weekly gasoline prices at different locations.

In a univariate framework, Robinson (2000) considers modelling the behaviour of electricity prices in England and Wales since the creation of their spot market in 1990. It is argued that the process is nonlinear and a logistic smooth transition model (LSTAR) is fitted.

#### **3. DATA SOURCES**

The price of wholesale natural gas in the UK is determined in several markets, the Over-the-Counter (OTC) or 'spot' market, the On the Day Commodity Market (OCM) used primarily for gas balancing purposes, and the IPE gas futures market. Price data is available for all three markets which we will use in our empirical analysis.

The most liquid market is the OTC market. Total annual volumes traded (both physical and 'paper' trades) amount to around 6 times the throughput of the UK gas pipeline system. Dealing on the OTC market is usually through bi-lateral trade or through a broker and consequently price data is confidential, or only available through commercial energy consultancies. PH Heren Ltd, produces a monthly series - the 'Heren Index'-, which is a volume-weighted average of OTC transaction prices reported during the month in pence/therm. The series commences April 1995 and PH Heren Ltd has kindly made this data available to the authors.

Another source of wholesale gas price data is the On-the-day Commodity Market (OCM) operated by EnMO. This market was established on 1<sup>st</sup> October 1999<sup>4</sup> replacing the original 'Flexibility Mechanism' operated by the UK pipeline company, Transco which had been originally introduced to provide a mechanism for daily gas balancing on the UK pipeline system. EnMO publishes cash-out price data as a daily series and the System Average Price (SAP is broadly an average trading price for wholesale gas measured in pence per kilowatt hour (p/kWh) on a particular day. Although the OCM is a relatively small market<sup>5</sup> and exhibits a high degree of volatility there is evidence to suggest that OCM cash-out prices (SAP) are strongly correlated with the OTC (spot) prices (OFGEM, 2000, p.25), and can therefore be used as a good proxy for the daily spot price in our econometric analysis.

The final source of UK wholesale natural gas price data we shall be employing in our analysis is from the International Petroleum Exchange (IPE), established in January 1997, and is the main exchange for trading gas futures contracts in the UK. The exchange was established to manage the risk in the underlying physical gas market, i.e., the OTC market. The IPE provides daily a one month forward price for wholesale natural gas prices for physical delivery within the UK natural gas grid at the National Balancing Point (NBP). As this data series is a one month forward price – although it is closely related to the spot price – it experiences less volatility than the spot or OCM price data

In Europe and throughout most of the world, gas prices are quoted per KWh. In the UK natural gas is usually priced in pence per therm, while in the international crude oil markets, oil is priced in US dollars per barrel. We have

<sup>&</sup>lt;sup>4</sup> Prior to this a different market structure known as the Flexibility Mechanism was in place.

<sup>&</sup>lt;sup>5</sup> OCM volumes are only about 8 percent of total UK pipeline throughput

converted both into pence per kilowatt-hour (p/kWh) in order to allow direct comparison between the two fuel prices<sup>6</sup>.

European Gas contracts are indexed against inland German heating and fuel oil, and this is where the link with the oil price occurs. Crude oil is benchmarked against a variety of blends; in the UK and Europe the benchmark blend is Brent. The spot price of Brent crude oil is available from several commercial energy agencies and the monthly series is obtained from *Datastream*.

#### 4. UNIT ROOT AND COINTEGRATION TESTS

The Augmented Dickey Fuller (ADF) and the Phillips-Perron (PP) unit root tests for each series are presented in Table 1. Each of the three data series for wholesale natural gas, i.e., IPE, OCM's SAP and the Heren Index series exhibit unit roots and are integrated of order one, as does the price of Brent crude oil, (see Table 1). Both the ADF and the PP tests suggest that the first differences of the series are stationary.

Having established that the wholesale natural gas data and oil price series each contain a unit root we can proceed to investigate the existence of a cointegrating relationship between the two commodity prices. As explained in the previous section we expect the two variables to be cointegrated after the opening of the Inter-connector However the possibility of a cointegration relationship predating that will also be investigated.

<sup>&</sup>lt;sup>6</sup> The kWh is a more internationally accepted unit of volume than the therm, one therm = 29.3 kilowatts an hour. One barrel of Brent crude oil is taken to be equal to 1597 kWh: we have been unable to discover a precise calorific definition for Brent, but we believe this is a reasonable approximation). Oil in US\$ barrel is converted using daily or monthly exchange rates (depending on the frequency of the gas data it is being compared to); the oil price in £/barrel can then be converted into pence by multiplying by 100 and finally to pence/kWh by dividing by 1597.

Two distinct methodologies are used to test for cointegration; the trace test developed by Johansen (1995) and the nonparametric test for cointegration proposed by Breitung (2002). The Johansen procedure, like many others, requires estimation of various structural and nuisance parameters. For example, a vector autoregressive (VAR) lag order must be specified and the lag parameters estimated. To get around this problem we employ the recently developed nonparametric test for cointegration due to Breitung (2002). No lag structure or deterministic terms need to be estimated. As Breitung (2002) notes: " there are a number of situations where the nonparametric approach may be attractive. Since the short-run component does not affect the asymptotic null distribution of the test statistic, the test is robust against deviations from the usual assumption of linear short-run dynamics" (also, see the technical appendix).

Both the Johansen and Breitung tests accept the hypothesis that a cointegrating equation exists between crude oil and the three different proxies for gas. Using the logs of the series, the outcome was further confirmed. Table 2 presents the results for all the different proxies of the gas for both the parametric (Johansen) and the nonparametric (Breitung) method. However, relationships between economic variables do not necessary remain the same throughout time. Factors like technology, innovation, political crisis (1974 for instance),, investment (inter connector) etc could influence and alter the nature of the relationship,

To test the stability of the results presented in Table 2 on the one hand and examine potential changes in the relationship between the two variables over time, we employ a recently developed econometric methodology. Following Hansen & Johansen (1999) we consider the parameter constancy in the cointegrated VAR model. This is very useful since we would like to examine whether the opening of the UK-Mainland Europe inter-connector has affected the relationship between oil and gas. The adopted methodology recursively estimates the trace test (see Johansen 1995 and Hansen & Johansen 1999) and uses the time paths of the estimated parameters as a diagnostic tool in evaluating the parameter constancy. Figures 3 and 5 present the results. We calculate the trace test statistic for each additional observation using an expanding window and then divide the test statistic by the critical value. If this fraction is greater than one then we accept that the two series are cointegrated and there is a longrun relationship. If it is below one cointegration is rejected. As it is obvious from both graphs the ratio is above unity throughout our sample for both the levels and the logs of the prices. The latter allows us to conclude firstly, that the prices were cointegrated for the whole period 1996-2003 and that the inter-connector did not change that relationship, and secondly, that the (potential) outliers did not alter this relationship in any point in time.

The long-run solution estimates for all the series are presented in Table 3. The estimates vary from 0.38 (2) to 0.855 (4). The coefficients for IPE and Heren are very close (around 0.5 for the levels and 0.8 for the logs). The long-run coefficient for SAP is the smallest in both cases (equations (2) and (5)). The next step was to investigate whether there were considerable changes in magnitudes of these coefficients throughout the period and especially whether the inter-connector has affected the relationship. In other words we ask "how long-un is our long-run solution?" We proceed with estimating the recursive  $\beta$ 's together with their standard errors (see equation 1 in the next section), using an expanding window. Each point in the graph corresponds to what an investigator at the time would have found (see Figures 4 and 6). In 1999 we observe a reduction in the Brent long-run coefficient which suggests a smaller dependency during that year and up to mid 2000. This seems to be reverted from mid 2001 up to the end of our sample (see Figure 4). The same picture emerges from the log analysis (see Figure 6). In both cases little variation appears after 2001. Note

here that the last point in Figure 6 corresponds to the long-run solutions presented in Table 3 (0.855 and (-1)\* -0.542 respectively).

To sum-up, both methodologies support the existence of a cointegrating relationship between UK gas prices and the oil price in the period prior to the opening of the Bacton - Zeebrugge gas inter-connector<sup>7</sup>. These results provide support for the theory that an equilibrium relationship between UK gas prices and the oil price has come about before the opening of the gas inter-connector. The long run coefficients decrease in the first 15 months of the operation but quickly moved upwards after that. Our initial findings, therefore, indicate the presence of a long-run equilibrium relationship but does not imply a linear short-run specification.

## 5. ERROR CORRECTION MODELS

In Johansen's (1988, 1995) notation, we write a *p*-dimensional Vector Error Correction Model (VECM) as:

$$\Delta y_{t} = \sum_{i=1}^{k-1} \Gamma_{i} \Delta y_{t-i} + \Pi y_{t-1} + \mu + \varepsilon_{t}, t = 1, \dots, T$$
(1)

where  $\Delta$  is the first difference operator,  $y_t$  is the set of I(1) variables discussed above,  $\varepsilon_t \sim niid(0,\Sigma)$ ,  $\mu$  is a drift parameter, and  $\Pi$  is a  $(p \ge p)$  matrix of the form  $\Pi = \alpha\beta'$ , where  $\alpha$  and  $\beta$  are  $(p \ge r)$  matrices of full rank, with  $\beta$  containing the rcointegrating vectors and  $\alpha$  carrying the corresponding loadings in each of the rvectors. In our approach, we set  $y_t = [Brent, Heren]'$  and  $\varepsilon_t = [\varepsilon_{Bt}, \varepsilon_{Ht}]$ .

 $<sup>^{7}</sup>$  Two periods when the inter-connector was shut were not included in the data analysis – 16-26 July 2000 and 17-27 September 2001.

The linearity assumption which characterizes equation 1 has not received support by the empirical results of the literature lately. Many tests for neglected non-linearity have been proposed in the literature. Instead of using a single statistical test, for the purposes of this paper, four different tests are considered; McLeod & Li (1983), Engle LM (1982), Tsay (1986) and the Brcok et al (BDS) (1996). All these tests share the principle that once any (linear or non-linear) structure is removed from the data, any remaining structure should be due to an (unknown) non-linear data generating mechanism. All the procedures embody the null hypothesis that the series under consideration is an *i.i.d.* process.

The McLeod & Li test looks at the autocorrelation function of the squares of the prewhitened data and tests whether corr  $(e_t^2, e_{t-k}^2)$  is non-zero for some *k* and can be considered as an LM statistic against ARCH effects (see Granger & Terasvirta 1993; Patterson & Ashley 2000). The test suggested by Engle (1982) is an LM test, which should have considerable power against GARCH alternatives (see Granger & Terasvirta 1993; Bollerslev, 1986). The Tsay (1986) test explicitly looks for quadratic serial dependence in the data and has proven to be powerful against a TAR (Threshold Autoregressive) process. The BDS test is a nonparametric test for serial independence based on the correlation integral of the scalar series,  $\{e_t\}$  (see Brock, Hsieh & LeBaron 1991 and Granger & Terasvirta 1993). This is a general linearity test where the alternative to linearity can be considered to be a stochastic non-linear model (Granger & Terasvirta 1993). The reader is also referred to the detailed discussion of these tests in the technical appendix and the simulations in Patterson & Ashley (2000).

#### **6.** Empirical Analysis

A general-to-specific approach is followed where each VECM was estimated for i= 0 to 6 and lag order was selected using the Akaike Information Criterion (AIC) and the Hannan-Quin Information criterion (HQ). The results for both the raw data and the logs are presented in Tables 4 and 5. The error correction term is

significant in both models 1 and 2 for the Heren equation<sup>8</sup> and the lagged values of both variables are found to be significant..

The residuals of these models were saved and the four tests for linearity were estimated. These tests could provide us information about any neglected nonlinearity present in the VECM on the one hand and on the other guide us into the nature of this (potential) non-linearity (McLeod & Li for ARCH, Engle for GARCH, Tsay for TAR and BDS as general linearity test). Table 6 reports the tests for residuals of models 1 and 2. The employed tests are, like most econometric procedures, only asymptotically justified. Given the limited sample available, the tests are estimated using both the asymptotic theory and the bootstrap. The values under "asymptotic theory" are based on the large sample distributions of the relevant test statistics. For the "Bootstrap" results, 1000 new samples are independently drawn from the empirical distribution of the prewhitened data. Each new sample is used to calculate a value for the test statistic under the null hypothesis of serial independence. The obtained fraction of the 1000 test statistics, which exceeds the sample value of the test statistic from the original data, is then reported as the significance level at which the null hypothesis can be rejected (for a detailed discussion on the sample size, the asymptotic theory and the bootstrap see Patterson and Ashley 2000).

Throughout the battery of the tests we can accept the null hypothesis that the residuals of the second equation in both cases are *i.i.d* (we are not interested in the residuals of the Brent equation since this is affected by other factors).

To summarise the results, we have used a vector error correction specification to capture the short run dynamics of the relationship between oil and gas prices in

<sup>&</sup>lt;sup>8</sup> We are interested in the equation where (L)Heren is the dependent variable as Brent is affected by other factors.

the light of the completion of the inter-connector pipeline which connects the oilindexed continental gas markets with the UK. We provided evidence that this mechanism is linear using four different test statistics which conclude that the employed VECM can satisfactory explain the dynamics of the series.

### 7. IMPULSE RESPONSE FUNCTIONS

Using the VECM system that has been estimated in the previous section, we extend the analysis and generate impulse response functions. A shock to the *i*th variable not only directly affects the *i*th variable but it is also transmitted to all the other endogenous variables through the dynamic (lag) structure of the VECM. An impulse response function (IRF) traces the effect of a one-time shock to one of the innovations on current and future values of the endogenous variables. If the innovations  $\varepsilon_t$  are contemporaneously uncorrelated, the interpretation of the impulse response is straightforward. The *i*th innovation  $\varepsilon_{i,t}$  is simply a shock to the *i*th endogenous variable *y*<sub>it</sub>.

The generalised IRF (GIRF) can be defined as

$$GIRF(n,\varepsilon_t,\omega_{t-1}) = E[y_{t+n}|\varepsilon_{i,t},\omega_{t-1}] - E[y_{t+n}|\sigma_{t-1}]$$
(8)

where  $y_t$  is a random vector,  $\varepsilon_{t+i}$  is a random shock,  $\varpi_{t-1}$  a specific realisation of the information set  $\Omega_{t-1}$  and n is the forecast horizon. The GIRF is a random variable given by the difference between two conditional expectations which are themselves random variables. We estimate the generalized impulses (GIRF) following Pesaran and Shin (1998). They construct an orthogonal set of innovations that does not depend on the VAR ordering. The generalized impulse responses from an innovation to the *j*th variable are derived by applying a variable specific Cholesky factor computed with the *j*th variable at the top of the Cholesky ordering [for more details see Pesaran and Shin (1998)]. It would be useful to point out that that IRF analysis can be viewed as a 'conceptual experiment'. We are interested in investigating the consequences of introducing a shock to the oil price. Figures 7 and 8 present the results of our IRF analysis. Introducing a positive shock to the oil price, we observe a negative response from gas price which dies out after 7 periods (response standard errors were calculated using 1000 Monte Carlo repetitions). However this response is not statistically different from zero. In the second graph the shock is introduced to the log prices (Model 2). Again a negative response from Gas price is observed which dies out much quicker (after four periods) but still is not significant.

#### 8. CONCLUSIONS

The sharp increase observed in UK wholesale natural gas prices during 1999-2000 was attributed to the opening of the UK-Belgium gas pipeline, the agument being that this gave opportunities to both the UK and Continental gas traders for arbitrage profits. Evidence is provided that the UK gas prices are cointegrated with oil prices using both the Johansen methodology and the recently developed Breitung nonparametric procedure. However, using recursive techniques it has also been demonstrated that cointegration is accepted throughout the whole sample period 1996-2003 and that this was not affected by the inter-connector. In the same framework, the long run coefficients reduced considerably (from 1 to 0.2) for the first 15 months but seem to converge to a 0.5 (or 0.8 for the logs) long run value. We employed a VECM specification to capture the short-run dynamics. Three tests for neglected non-linearity were estimated: the McLeod-Li and Engle test for (G)ARCH effects and the BDS test statistic as general test for linearity. Bootstrap values as well as asymptotic are generated. Strong evidence emerges to support the argument that the relationship between oil and gas is a linear one. Finally, we generated impulse response functions to investigate the

response of Gas price as a result of a shock introduced in the price of oil. Negative responses from gas seem to die out quickly.

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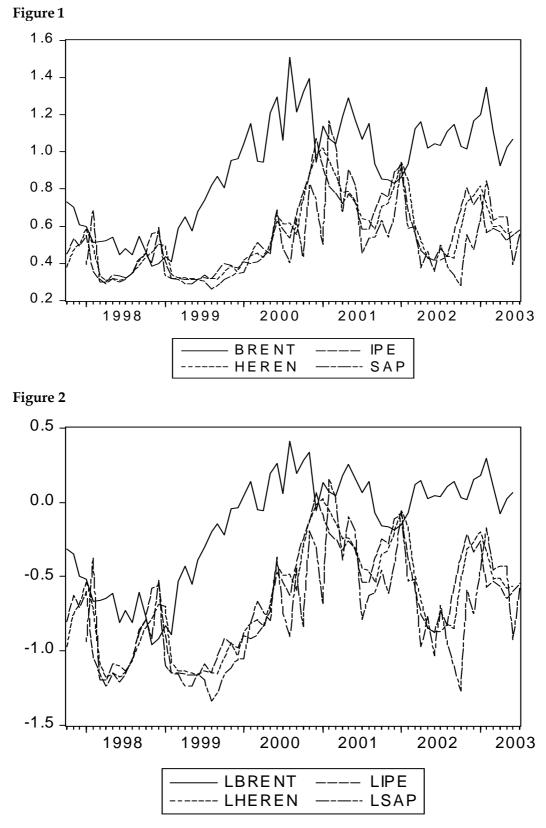
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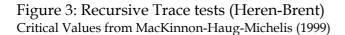
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Note: see the discussion in Section 3 for the unit of measurement.



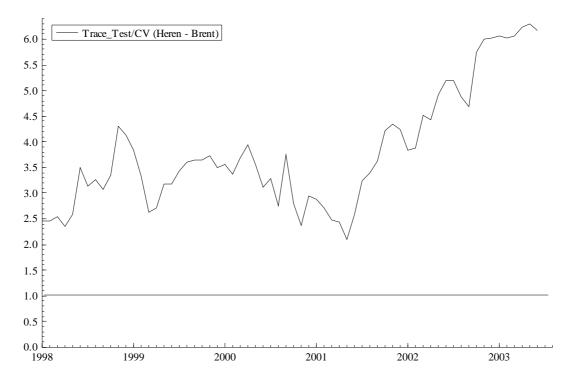


Figure 4: Recursive Beta Coefficients (Heren-Brent)

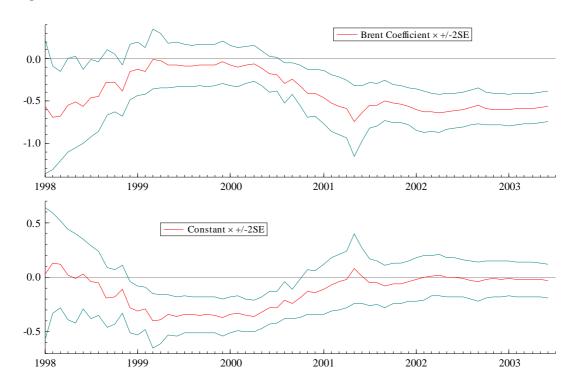


Figure 5: Recursive Trace test (LHeren – LBrent)

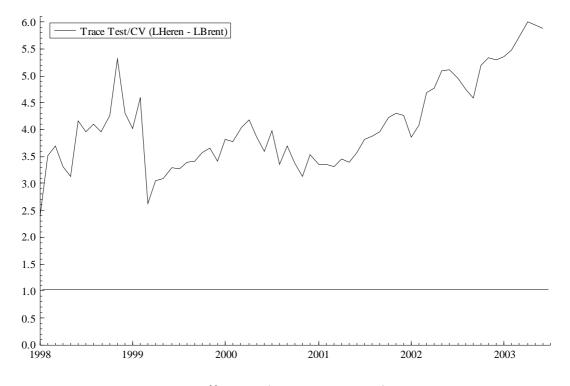


Figure 6: Recursive Beta Coefficients (LHeren - LBrent)

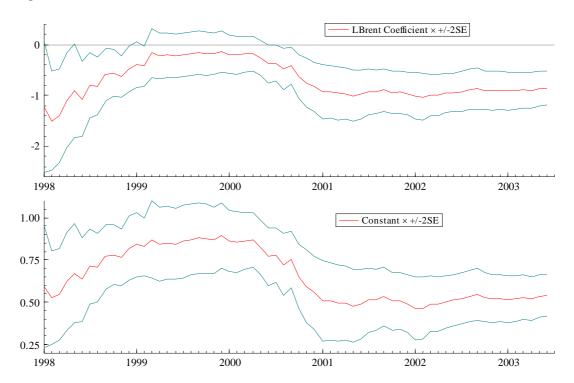


Figure 7: Impulse response Function (from Model 1). Response standard errors were calculated using 1000 Monte Carlo repetitions.

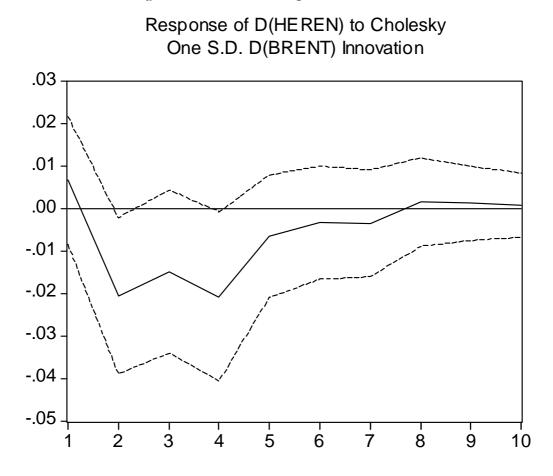


Figure 8 Impulse Response Function (from Model 2). Response standard errors were calculated using 1000 Monte Carlo repetitions.

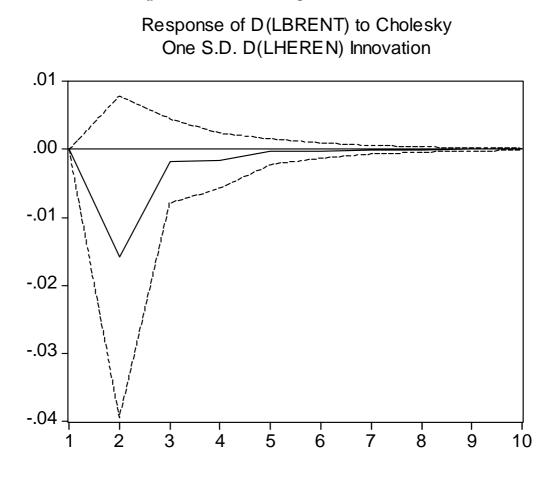


Table 1: Unit Root tests

|       | <u></u>  |   |             |        |             |         | <u> </u>    |        | FIRST DIFFE | ERENCES- |
|-------|----------|---|-------------|--------|-------------|---------|-------------|--------|-------------|----------|
|       | SAMPLE   |   | LEVE        | LS     | FIRST DIFF  | ERENCES | LEVEL       | s-Logs | Log         | S        |
|       |          |   | t-Statistic | Prob.* | t-Statistic | Prob.*  | t-Statistic | Prob.* | t-Statistic | Prob.*   |
|       | 1996:01- |   |             |        |             |         |             |        |             |          |
| Brent | 2003:6   | ADF   | -1.6102     | 0.4733 | -12.9788    | 0.0001  | -1.7962     | 0.3802 | -11.8837    | 0.0001   |
|       |          | PP  | -1.8734     | 0.3434 | -13.1087    | 0.0001  | -1.5812     | 0.4880 | -12.0483    | 0.0001   |
|       | 1996:01- |   |             |        |             |         |             |        |             |          |
| Heren | 2003:6   | ADF   | -0.7310     | 0.3972 | -7.0174     | 0.0000  | -2.7727     | 0.0660 | -7.5093     | 0.0000   |
|       |          | PP  | -0.5884     | 0.4598 | -6.7131     | 0.0000  | -2.0166     | 0.2794 | -7.2735     | 0.0000   |
|       | 1997:01- |   |             |        |             |         |             |        |             |          |
| IPE   | 2003:7   | ADF   | -1.9892     | 0.2910 | -7.2432     | 0.0000  | -2.4529     | 0.1311 | -7.0196     | 0.0000   |
|       |          | PP  | -2.1758     | 0.2167 | -7.2496     | 0.0000  | -2.1366     | 0.2312 | -7.0408     | 0.0000   |
|       | 1997:12- |   |             |        |             |         |             |        |             |          |
| SAP   | 2003:7   | ADF   | -2.3988     | 0.1462 | -9.7133     | 0.0000  | -0.8105     | 0.3610 | -8.8628     | 0.0000   |
|       |          | PP  | -0.9706     | 0.2935 | -13.2230    | 0.0000  | -1.1456     | 0.2271 | -12.8855    | 0.0000   |
|       |          | C.V.: 1% level -3.49917, 5% level -2.89155, 10% level -2.58285                  |             |        |             |         |             |        |             |          |
|       |          | *MacKinnon (1996) one-sided p-values.   |             |        |             |         |             |        |             |          |
|       |          | ADF Lag Length: (Decision based on Schwartz Info Criterion, MINLAG=0 MAXLAG=11) |             |        |             |         |             |        |             |          |
|       |          | PP Bandwidth selection based on Newey-West                                      |             |        |             |         |             |        |             |          |

Note: ADF is the Augmented Dickey-Fuller test statistic, PP is the Phillips-Perron test statistic, C.V. Critical Values

| Table | e 2: ' | Testing | for ( | Cointegrat | tion |
|-------|--------|---------|-------|------------|------|
|       |        |         |       | 0          |      |

|                | H0:    | _          | _        |                      | _     |       | Simulated        | <u> </u>       |
|----------------|--------|------------|----------|----------------------|-------|-------|------------------|----------------|
|                | rank<= | Trace Test | [Prob]   | <b>Breitung Test</b> | 10%CV | 5% CV | <i>p</i> -values | Data           |
| 1Heren-Brent   | 0      | 23.67      | 0.015**  | 454.66**             | 261   | 329.9 | 0.0154**         | 1996:1-2003:6  |
|                | 1      | 2.2        | 0.737    | 18.06                | 67.89 | 95.6  | 0.5361           |                |
| 2SAP-Brent     | 0      | 28.41      | 0.002*** | 330.06**             | 261   | 329.9 | 0.0487**         | 1997:12-2003:6 |
|                | 1      | 4.06       | 0.415    | 16.63                | 67.89 | 95.6  | 0.5808           |                |
| 3IPE-Brent     | 0      | 23.94      | 0.013**  | 396.15**             | 261   | 329.9 | 0.0251**         | 1997:1-2003:6  |
|                | 1      | 2.08       | 0.759    | 15.11                | 67.89 | 95.6  | 0.654            |                |
| 4LHeren-LBrent | 0      | 22.55      | 0.022**  | 369.93**             | 261   | 329.9 | 0.0368**         | 1996:1-2003:6  |
|                | 1      | 2.45       | 0.69     | 18.48                | 67.89 | 95.6  | 0.5265           |                |
| 5LSAP-LBrent   | 0      | 26.07      | 0.006*** | 288.34*              | 261   | 329.9 | 0.0771*          | 1997:12-2003:6 |
|                | 1      | 3.94       | 0.433    | 15.28                | 67.89 | 95.6  | 0.6411           |                |
| 6LIPE-LBrent   | 0      | 19.98      | 0.053*   | 366.04**             | 261   | 329.9 | 0.0358**         | 1997:1-2003:6  |
|                | 1      | 2.22       | 0.734    | 14.78                | 67.89 | 95.6  | 0.6568           |                |

\*\*, \* denotes rejection at the 1%, and 5% significance level respectively (critical values from Doornik, 1998). Trace Test is the cointegration test proposed by Johansen (1995). Breitung test is the nonparametric cointegration test suggested by Breitung (2002). The simulated *p*-values are based on 10000 replications of Gaussian random walks with length n = 90.

|     | ženi i konstručio i |              |                  |
|-----|--|--------------|------------------|
|     |  | Costant s.e. | Coefficient s.e. |
| (1) | Heren= 0.033 + 0.567 Brent   | (0.078)      | (0.089)          |
| (2) | SAP = 0.179 + 0.38 Brent   | (0.067)      | (0.071)          |
| (3) | IPE = 0.07 + 0.54 Brent  | (0.064)      | (0.07)           |
| (4) | LHeren = -0.542 + 0.855 LBrent   | (0.061)      | (0.1698)         |
| (5) | <i>LSAP</i> = -0.61 + 0.592 <i>LBrent</i>  | (0.042772)   | (0.11204)        |
| (6) | <i>LIPE</i> = -0.51 + 0.808 <i>LBrent</i>  | (0.047)      | (0.12)           |

Table 3: Long-run Cointegrating Equations (Johansen)

| Table 4: VECM for Brent – Heren (Model 1) |           |                                  |  |  |  |
|---|-----------|----------------------------------|--|--|--|
|   | D(BRENT)  | D(HEREN)                         |  |  |  |
| D(BRENT(-1))                              | -0.366263 | -0.202569                        |  |  |  |
|   | [-3.145]  | [-2.849]                         |  |  |  |
| D(BRENT(-2))                              | -0.159369 | -0.137813                        |  |  |  |
|   | [-1.323]  | [-1.874]                         |  |  |  |
| D(BRENT(-3))                              | 0.136781  | -0.175990                        |  |  |  |
|   | [ 1.198]  | [-2.524]                         |  |  |  |
| D(HEREN(-1))                              | -0.151838 | 0.386844                         |  |  |  |
|   | [-0.941]  | [ 3.926]                         |  |  |  |
| D(HEREN(-2))                              | 0.029989  | 0.033838<br>[ 0.320]<br>0.007904 |  |  |  |
|   | [ 0.173]  |                                  |  |  |  |
| D(HEREN(-3))                              | 0.271467  |                                  |  |  |  |
|   | [ 1.600]  | [ 0.076]                         |  |  |  |
| С   | 0.004508  | 0.003308<br>[ 0.446]             |  |  |  |
|   | [ 0.371]  |                                  |  |  |  |
| CV1(-1)                                   | -0.054633 | -0.278945                        |  |  |  |
|   | (0.09937) | (0.06067)                        |  |  |  |
|   | [-0.550]  | [-4.598]                         |  |  |  |
| R-squared                                 | 0.195086  | 0.317301                         |  |  |  |
| Adj. R-squared                            | 0.125526  | 0.258302                         |  |  |  |
| S.E. equation                             | 0.113984  | 0.069594                         |  |  |  |
| F-statistic                               | 2.804560  | 5.378101                         |  |  |  |
| Log likelihood                            | 71.18688  | 115.0975                         |  |  |  |
| Akaike AIC                                | -1.419930 | -2.406685                        |  |  |  |
| Schwarz SC                                | -1.196232 | -2.182987                        |  |  |  |
| Log likelihood                            |           | 186.6995                         |  |  |  |
| Akaike information criterion              |           | -3.835943                        |  |  |  |
| Schwarz criterion                         |           | -3.388548                        |  |  |  |

| Table 5 VECM for LBrent – LHeren (Model 2) |           |           |  |  |  |  |
|--|-----------|-----------|--|--|--|--|
|  | D(LBRENT) | D(LHEREN) |  |  |  |  |
| D(LBRENT(-1))                              | -0.227250 | -0.144515 |  |  |  |  |
|  | [-2.112]  | [-1.249]  |  |  |  |  |
| D(LHEREN(-1))                              | -0.120214 | 0.335327  |  |  |  |  |
|  | [-1.324]  | [ 3.435]  |  |  |  |  |
| С  | 0.006452  | 0.004084  |  |  |  |  |
|  | [ 0.494]  | [ 0.291]  |  |  |  |  |
| CV2(-1)                                    | -0.014647 | -0.211189 |  |  |  |  |
|  | (0.04810) | (0.05171) |  |  |  |  |
|  | [-0.304]  | [-4.084]  |  |  |  |  |
| R-squared                                  | 0.074198  | 0.220152  |  |  |  |  |
| Adj. R-squared                             | 0.041523  | 0.192628  |  |  |  |  |
| Sum sq. resids                             | 1.287229  | 1.487766  |  |  |  |  |
| S.E. equation                              | 0.123060  | 0.132299  |  |  |  |  |
| F-statistic                                | 2.270777  | 7.998548  |  |  |  |  |
| Log likelihood                             | 62.22290  | 55.78002  |  |  |  |  |
| Akaike AIC                                 | -1.308380 | -1.163596 |  |  |  |  |
| Schwarz SC                                 | -1.196531 | -1.051747 |  |  |  |  |
| Mean dependent                             | 0.004942  | 0.005692  |  |  |  |  |
| S.D. dependent                             | 0.125698  | 0.147238  |  |  |  |  |
| Determinant resid covariance               | 0.000263  |           |  |  |  |  |
| Determinant resid covariance               | 0.000240  |           |  |  |  |  |
| Log likelihood                             |           | 118.3787  |  |  |  |  |
| Akaike information criterion               |           | -2.480420 |  |  |  |  |
| Schwarz criterion -2.256722                |           |           |  |  |  |  |

Standard Errors in () and t-statistics in [] CV1 and CV2 are the error correction terms.

### Table 4: VECM for Brent – Heren (Model 1)

| $\epsilon_{\rm H}$ (from Model 1) $\epsilon_{\rm LH}$ (from Model 2) |           |            |           |            |  |  |  |
|--|-----------|------------|-----------|------------|--|--|--|
|  | •         | ,          | •         |            |  |  |  |
|  | BOOISTRAP | ASYMPTOTIC | BOOISTRAP | ASYMPTOTIC |  |  |  |
| MCLEOD-LI TEST   |           |            |           |            |  |  |  |
| USING UP TO LAG 20   | 0.895     | 1.00       | 0.829     | 1.00       |  |  |  |
| USING UP TO LAG 24   | 0.882     | 1.00       | 0.753     | 1.00       |  |  |  |
|  |           |            |           |            |  |  |  |
| ENGLE TEST   |           |            |           |            |  |  |  |
| USING UP TO LAG 1  | 0.703     | 0.777      | 0.772     | 0.827      |  |  |  |
| USING UP TO LAG 2  | 0.782     | 0.891      | 0.774     | 0.891      |  |  |  |
| USING UP TO LAG 3  | 0.920     | 0.972      | 0.894     | 0.968      |  |  |  |
| USING UP TO LAG 4  | 0.967     | 0.993      | 0.961     | 0.992      |  |  |  |
|  |           |            |           |            |  |  |  |
| TSAY TEST  | 0.631     | 0.721      | 0.658     | 0.726      |  |  |  |
|  |           |            |           |            |  |  |  |
| BDS  |           |            |           |            |  |  |  |
| Dimension  |           |            |           |            |  |  |  |
| 2  | 0.1108    | 0.0649     | 0.3412    | 0.3031     |  |  |  |
| 3  | 0.1348    | 0.0942     | 0.4062    | 0.4004     |  |  |  |
| 4  | 0.1810    | 0.1477     | 0.6064    | 0.6783     |  |  |  |
| 5  | 0.1804    | 0.1523     | 0.7102    | 0.8391     |  |  |  |
| 6  | 0.2938    | 0.3109     | 0.6266    | 0.7477     |  |  |  |

Table 6: Linearity Tests for the Residuals of Models 1 and 2 (only from the Heren and LHeren equations)

Note: only *p*-values are reported

#### **TECHNICAL APPENDIX**

#### **BREITUNG NONPARAMETRIC TEST FOR COINTEGRATION**

Breitung's unit roots and cointegration test employ a variance ratio as the test statistic. As noted this approach can eliminate the problem of the specification of the short run dynamics and the estimation of nuisance parameters. If  $\{y_t\}_1^T$  denotes an observable process that can be decomposed as  $y_t = \delta' d_t + x_t$ , where  $\delta' d_t$  is the deterministic part ( $d_t$ =1 or [1,t]'), and  $x_t$  is the stochastic part. If we do not assume the deterministic part, then  $y_t$  is consistent with  $x_t$ . The null hypothesis is that  $x_t$  is I(1), if  $T \to \infty$ ,  $T^{-1/2}x_{[aT]} \Rightarrow \sigma W(a)$ , where  $\sigma$ >0 represents the constant (long-run variance), and W(a) denotes a Brownian motion, [] is the integer part. The expression of  $x_t$  makes possible the application of a general data generating process. Asymptotically, to construct a consistent estimate which does not require the specification in short run dynamics and an estimate of  $\sigma$ , Breitung has proposed the following test statistic

$$\hat{\rho} = \frac{T^{-4} \sum_{t=1}^{T} \hat{U}_{t}^{2}}{T^{-2} \sum_{t=1}^{T} \hat{u}_{t}^{2}}$$
(1)

where  $\hat{u}_t$  is the OLS residuals that  $\hat{u}_t = y_t - \hat{\delta}' d_t$ , and  $\hat{U}_t$  is the partial sum process that  $\hat{U}_t = \hat{u}_1 + ... + \hat{u}_t$ . If  $y_t$  is I(0), the test statistic  $\hat{\rho}_T$  converges to 0.Breitung shows that the variance ratio test has favourable small sample properties using Monte Carlo simulations.

We could proceed and test for cointegration by the generalasation of the nonparametric unit roots test on the assumption that the process can be decomposed into a *q*-dimensional vector of stochastic trend components  $\xi_t$  and a (*n*-*q*)-dimensional vector of transitory components of  $v_t$  where *n* is the number of

variables. Asymptotically,  $\xi_t$  and vt is  $T^{-1/2}\xi_{[aT]} \Rightarrow W_q(a)$  and  $T^{-2}\sum_{t=1}^T v_t v_t' = o_p(1)$ , respectively, where  $W_q(a)$  denotes a *q*-dimensional Brownian motion with unit covariance matrix. The dimension of  $\xi_t$  is related to the cointegration rank. In addition, it assumes that the variance of  $\xi_t$  diverges with a faster rate than  $v_t$ instead if assuming the stationarity of  $v_t$ . From the assumption, the transitory component denoting the cointegration relationship can be generated by any process.

To test the number of cointegrating vectors, Breitung has proposed the following problem about the  $n \ge n$  matrix  $A_t$ ,  $B_t$ .

$$\left|\lambda_{j}B_{T}-A_{T}\right|=0\tag{2}$$

where  $A_T = \sum_{t=1}^T \hat{u}_t \hat{u}'_t$ ,  $B_T = \sum_{t=1}^T \hat{U}_t \hat{U}_t'$ , and  $\hat{U}_t = \sum_{j=1}^t \hat{u}_t$  represent the *n*-dimensional partial sum concerning  $\hat{u}_t$ . The problem is equilvelant to solving the eigenvalue of  $R_T = A_T B_T^{-1}$ . The solution of equation (1) is  $\lambda_j = (\eta'_j A_T \eta_j)/(\eta'_j B_T \eta_j)$  where  $\eta_j$  is the eigenvalue of  $\lambda_j$ . If the vectors of the stochastic trends are less than q,  $T^2 \lambda_j$  diverges to infinity. In that case, since stochastic trends are linked with each other, a cointegrating vector exists. Hence, the test statistic is the following.

$$\Lambda_q = T^2 \sum_{j=1}^q \lambda_j ,$$

where  $\lambda_1 \leq \lambda_2 \leq ... \leq \lambda_n$  is the ordered eigenvalues of  $R_T$ . The idea of cointegration rank behind the approach is similar to Johansen's idea. The statistic tests whether a *q*-dimensional stochastic component is rejected at the significance level.

#### **BDS** TEST FOR RANDOMNESS

A powerful test used for independence -and, under certain circumstances, for non-linear dependencies- was developed by Brock, Dechert, and Scheinkman (1996) and is based on the correlation integral. The BDS statistic tests the null hypothesis that the elements of a time series are independently and identically distributed (IID). For a time series which is IID, the distribution of the statistic:

$$W_{m}(\varepsilon) = \frac{\sqrt{n} \left\{ C_{m}(\varepsilon) - C_{1}(\varepsilon)^{m} \right\}}{\sigma_{m}(\varepsilon)}$$
(1)

is asymptotically  $N(0,1).W_m(\varepsilon)$  is known as the BDS statistic.  $C_m(\varepsilon)$  denotes the fraction of *m*-tuples in the series, which are within a distance of each other and  $\sigma_m(\varepsilon)$  is an estimate of the standard deviation under the null hypothesis of *IID*. The test statistic is asymptotically standard normal under the null of whiteness. The null is rejected if the test statistic is absolutely large, (say greater than 1.96). If the null hypothesis of *IID* cannot be accepted this implies that the residuals contain some kind of hidden structure, which might be non-linear – or even chaotic. Following the recommendation by Brock, Hsieh & LeBaron (1991, p169) and the suggestions by Brooks & Heravi (1999), we set  $\varepsilon/\sigma = 0.5$  to 2, and m = 2 to 4.

### MCLEOD AND LI TEST

The McLeod and Li test (McLeod and Li, 1983) can be used as a portmanteau test of non-linearity. To test for non-linear effects in time series data McLeod and Li have proposed the statistic:

$$Q(m) = \frac{n(n+2)}{n-k} \sum_{k=1}^{m} r_a^2(k)$$
(2)

where

$$r_a^2(k) = \frac{\sum_{t=k+1}^n e_t^2 e_{t-k}^2}{\sum_{t=1}^n e_t^2} \qquad k = 0, 1, \dots, n-1$$
(3)

are the autocorrelations of the squared residuals,  $e_t^2$ , obtained from fitting a model to the data. If the series  $e_t$  is independently and identically distributed (*IID*) then the asymptotic distribution of Q(m) is  $\chi^2$  with *m* degrees of freedom.

### ENGLE LM TEST

This test was suggested by Engle (1982) to detect ARCH disturbances. Bollerslev (1986) suggests that it should also have power against GARCH alternatives. Since it is a Lagrange Multiplier test, the test statistic itself is based on the  $R^2$  of an auxiliary regression, which in this case can be defined as:

$$e_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_k e_{t-i}^2 + v_t$$
 (4)

Under the null hypothesis of a linear generating mechanism for  $e_t$ ,  $NR^2$  for this regression is asymptotically  $\chi^2(p)$ .

## TSAY TEST

The Tsay (1986) test is a generalisation of the Keenan (1985) test. It explicit looks for quadratic serial dependence in the data.

Let K=k(k-1)/2 column vectors  $V_1, ..., V_k$  contain all of the possible cross-products of the form  $e_{t-i}e_{t-j}$ , where  $i \in [1,k]$  and  $j \in [i,k]$ . Thus,  $v_{t,1} = e_{t-1}^2$ ,  $v_{t,2}=e_{t-1}e_{t-2}$ ,  $v_{t-3}=e_{t-1}e_{t-3}$ ,  $v_{t,k+1}=e_{t-2}e_{t-3}$ ,  $v_{t,k+2}=e_{t-2}e_{t-4}$ ,...,  $v_{t,k} = e_{t-k}^2$ . And let  $\hat{v}_{t,j}$  denote the projection of  $v_{t,i}$  on the subspace orthogonal  $e_{t-1}, ..., e_{t-k}$ , (i.e. the residuals from a regression of  $v_{t,j}$  on  $e_{t-1}, ..., e_{t-k}$ .

The parameters  $\gamma_1, ..., \gamma_k$  are then estimated by applying OLS to the regression equation

$$e_{t} = \gamma_{0} + \sum_{i=1}^{K} \gamma_{i} \hat{v}_{t,i} + \eta_{t}$$
(7)

Note that the *jth* regressor in this equation is  $\hat{v}_{t,j}$ , the period *t* fitting error from a regression of  $v_{t,j}$  on  $e_{t-1}, \dots, e_{t-k}$ . So long as *p* exceeds *K*, this projection is unnecessary for the dependent variable  $\{e_t\}$  if it is pre-whitened using an AR(*p*) model. The Tsay test statistic then is just the usual *F* statistic for testing the null hypothesis that  $\gamma_1, \dots, \gamma_k$  are all zero.