# Accuracy of Low Voltage Electricity Distribution Network Modelling

by

Andrew Urquhart

A Doctoral Thesis

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#### Abstract

The connection of high penetrations of new low carbon technologies such as PV and electric vehicles onto the distribution network is expected to cause power quality problems and the thermal capacity of feeder cables may be exceeded. Replacement of existing infrastructure is costly and so feeder cables are likely to be operated close to their hosting capacity. Network operators therefore require accurate simulation models so that new connection requests are not unnecessarily constrained.

This work has reviewed recent studies and found a wide range of assumptions and approximations that are used in network models. A number of these have been investigated further, focussing on methods to specify the impedances of the cable, the impacts of harmonics, the time resolution used to model demand and generation, and assumptions regarding the connectivity of the neutral and ground conductors.

The calculation of cable impedances is key to the accuracy of network models but only limited data is available from design standards or manufacturers. Several techniques have been compared in this work to provide guidance on the level of detail that should be included in the impedance model. Network modelling results with accurate impedances are shown to differ from those using published data.

The demand data time resolution has been shown to affect estimates of copper losses in network cables. Using analytical methods and simulations, the relationship between errors in the loss estimates and the time resolution has been demonstrated and a method proposed such that the accuracy of loss estimates can be improved.

For networks with grounded neutral conductors, accurate modelling requires the resistance of grounding electrodes to be taken into account. Existing methods either make approximations to the equivalent circuit or suffer from convergence problems. A new method has been proposed which resolves these difficulties and allows realistic scenarios with both grounded and ungrounded nodes to be modelled.

In addition to the development of models, the voltages and currents in a section of LV feeder cable have been measured. The results provide a validation of the impedance calculations and also highlight practical difficulties associated with comparing simulation models with real measurement results.

#### Research outputs

The following publications have been generated from this work:

#### Journal papers

Urquhart, A.J., Thomson, M.: 'Impacts of Demand Data Time Resolution on Estimates of Distribution System Energy Losses', *IEEE Trans. Power Syst.*, 2015, **30**, (3), pp. 1483–1491.

Urquhart, A.J., Thomson, M.: 'Series impedance of distribution cables with sectorshaped conductors', *IET Gener. Transm. Distrib.*, 2015.

#### Conference paper

Urquhart, A.J.: 'Improving The Accuracy Of Cable Impedance Data For Distribution Network Modelling'. Loughborough University, 2012

#### Research dataset

Urquhart, A.J., Thomson, M.: 'Cable impedance data', DOI 10.17028/rd.lboro.2001054, https://dspace.lboro.ac.uk/dspace-jspui/handle/2134/18472

These are included here as appendices and also summarised in the body of the thesis.

#### **Citations**

The following publications cite research outputs of this work:

Frost, Thomas, Paul D. Mitcheson, and Timothy C. Green. "Power electronic voltage regulation in LV distribution networks." *Power Electronics for Distributed Generation Systems (PEDG), 2015 IEEE 6th International Symposium on*. IEEE, 2015, (Frost et al. 2015).

McKenna, E., Thomson, M.: 'High-resolution stochastic integrated thermal–electrical domestic demand model', Appl. Energy, 2016, 165, pp. 445–461. (McKenna & Thomson 2016)

Leicester, P.A., Goodier, C.I., Rowley, P.N.: 'Probabilistic analysis of solar photovoltaic self-consumption using Bayesian network models'*IET Renew. Power Gener.*, 2016, **10**, (4), pp. 448–455. (Leicester et al. 2016)

Jia, Jimmy Y., and Jason Crabtree. Driven by Demand: How Energy Gets its Power. Cambridge University Press, 2015. (Jia & Crabtree 2015)

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# LIST OF ABBREVIATIONS

CHP	Combined Heat and Power
СТ	Current Transformer
DNO	Distribution Network Operator
EV	Electric Vehicle
FE	Finite Element
GMD	Geometric Mean Distance
GMR	Geometric Mean Radius
LCNF	Low Carbon Networks Fund
LCNI	Low Carbon Network Innovation
LV	Low Voltage
MV	Medium Voltage
PME	Protective Multiple Earthing
PV	Photo Voltaic
RMS	Root Mean Square
THD	Total Harmonic Distortion
TN	Terre Neutral earthing
TN-C-S	Terre Neutral Combined and Separate earthing
TN-S	Terre Neutral Separate earthing
ТТ	Terre Terre earthing

### **1 INTRODUCTION**

#### 1.1 New Low Carbon Technologies

There is an urgent need to de-carbonise our energy supplies in order to minimise the impacts of climate change. Whilst this applies to all forms of energy use, measures to de-carbonise electricity generation are more readily available than for industrial energy use, heating, or transport.

The development of clean technologies for electricity generation places a significant emphasis on renewable sources, and particularly on photovoltaic (PV) solar energy. Renewable energy generation in industrialised countries is typically grid-connected, as shown in Figure 1-1.



Figure 1-1 – Low carbon technologies connected to the LV distribution network

The cost of solar PV is reducing at around 13% per year in European countries, such that the cost of electricity from domestic PV installation is reaching parity with the

price of electricity purchased from the grid. This situation has not yet been reached in the UK due to the lower irradiance levels but this 'grid parity' level; has been reached in many countries including Germany, Spain and southern France (CREARA 2015). Once solar PV becomes economically attractive, a significant increase in the number of installations can be expected, as in southern Germany where the low voltage (LV) grid requires reinforcement due to the number of PV installations (Braun et al. 2012).

The economic viability of residential PV also depends on regulatory policies that determine payments for imported and exported electricity, since it is generally not feasible for households to achieve 100% self-consumption without also including electricity storage. At present, high capital costs and the relatively low efficiency of the charging cycle mean that battery storage systems are not cost effective over their predicted life spans when considered solely in terms of avoiding the costs of imported electricity (McKenna & Thomson 2013). Despite this, there is increasing interest in the use of battery systems coupled with PV systems and the cost of batteries is also predicted to reduce significantly over the next few years, possibly enabling customers to 'defect' from the grid (Rocky Mountain Institute 2014).

Batteries used to maximise residential self-consumption will reduce maximum load and generation currents on the local distribution grid. However, it is also possible for home battery systems to form part of a distributed energy storage resource and used to balance supply and demand on the grid (Moixa Technology Ltd 2015). This may have the effect of increasing the peak loads on the local LV network.

The electrification of transport systems and the use of electricity in heating can also reduce the carbon intensity of primary energy requirements. This could create additional demand on the electricity system as new low carbon technologies such as heat pumps and electric vehicles are added to the distribution network.

Electric vehicles (EVs) are likely to create significant new demand on LV distribution networks, coinciding with the evening peak as vehicles are charged after their users return home. Until recently, very few EVs have been purchased but there are indications that numbers may now increase rapidly, as shown in Figure 1-2 (Department for Transport 2015). In April to June 2015, nearly 1% of new light vehicle registrations were ultra-low emission vehicles and nearly all of these are

plug-in EVs. With a typical load of 3 kW per charger, it is expected that investment in new LV network infrastructure will need to be brought forward (UK Power Networks 2014).



Figure 1-2 – Cumulative electric vehicle (cars and small van) registrations in the UK (Department for Transport 2015)

The growth in deployment of heat pumps appears less certain and there is a greater variation between high uptake and low uptake scenarios (EA Technology 2012). Based on the high update scenario the UK could expect around 7 million heat pumps to be installed by 2030, a similar number to the predicted number of electric vehicles, although the 'low' growth scenario would see only around 1 million installations.

Combined heat and power systems could also be installed on the distribution network, generating electricity at times when heating is required. Residential micro-CHP systems would then be expected to add generation to the distribution network with peak outputs in the morning and during the mid-evening (Navarro-Espinosa & Ochoa 2015; Thomson & Infield 2007). Although the evening generation from CHP systems might offset the increased load due to EVs, the predicted uptake for residential CHP installations is still uncertain. Moreover, local power flows may increase if the additional generation and loads are not co-located and connected to the same phase.

In summary, a significant number of new low carbon technology installations are being connected to the LV distribution grid and the number of these connections may soon increase rapidly. There is also considerable uncertainty over the proportions of the new types of generation or load that may occur with recent growth in the number of PV installations exceeding all projections (Solar Power Portal 2013). Most of our LV networks have been in place for many years and were not designed to accommodate these new connections. Replacement of the transformers, underground cables and overhead lines already installed is expensive and disruptive and so there is a need to ensure that the new low carbon technologies are accommodated on existing infrastructure wherever possible.

Typically, LV networks in Europe have been installed in the past with a 'fit and forget' approach and were planned using conservative worst-case estimates giving a high margin for future growth. This practice has provided spare capacity which can be utilised now in connecting low carbon technologies but with the consequence that the networks are now operated much closer to their design limits. As these limits are reached, Distribution Network Operators (DNOs) need to make judgements about the acceptability of permitting new connections onto the networks.

In order to permit the maximum possible development of low carbon technologies, there is therefore a requirement that the capacity of existing networks is well characterised such that new connections are not unnecessarily constrained. Equally, since upgrades to the network are costly, new infrastructure should not be installed earlier than necessary. Conversely, it is also necessary to ensure that the impact of new connections is recognised and that any power quality problems caused are addressed.

#### 1.2 Impacts of low carbon technologies on the LV network

Under normal operating conditions (i.e. in the absence of network faults), the DNO is required to ensure that the power quality of customer voltage supplies meets mandated requirements, with the key aspects being:

- Voltage range: to keep customer voltages within a specified band around a nominal single-phase voltage, defined as 230 V for European countries.
- Distortion: the total harmonic voltage distortion, and also the magnitudes of specific harmonic frequencies
- Unbalance: the voltage unbalance of three-phase supplies

Where additional loads are added to the network, the voltage drop along the cables is increased. The connection of distributed generation such as PV or CHP raises the voltage and would ideally counteract the voltage drop due to the loads. However, in northern Europe, the greatest demand is typically during the evenings and the greatest PV output is during the day. The LV network therefore needs to accommodate the requirements for increased load currents and also the possibility that there is a net power export and voltage rise along the feeder cable.

The customer voltage ranges are defined to have relatively wide tolerances, either ±10% for countries where EN 50160 applies or +10%/-6% based on the UK Electricity Safety, Quality and Continuity Regulations (BSI 2011b; UK Government 2002). The voltage at customer connections can be regulated by changing the transformer tap settings at the primary substation in order to maintain the customer voltages at LV connections within the desired band, as illustrated in Figure 1-3. The distribution transformers settings are not normally adaptive and so the settings at the primary substation need to allow for the voltage drops on all of the downstream distribution substations, each with different proportions of load and generation connected. A large proportion of the permitted voltage tolerance is therefore needed to allow for voltage drops in the HV network and on LV feeders that do not have any significant distributed generation. Only a small fraction of the overall tolerance band can be permitted for voltage rise.



Figure 1-3 – Voltage band management (Schmidt 2014)

In addition to meeting power quality requirements for customer connections, the DNO also needs to ensure that the currents on the LV network are within the specified limits for transformers and cables so as to maximise their lifetimes.

There is also now an increased emphasis on minimising energy losses incurred in the distribution network. In the UK, overall distribution losses are between 5% and 7% of the energy input, with a quarter of these losses arising in the LV network (Sohn Associates & Imperial College London 2014), and add costs of around 7% to domestic customer bills (Western Power Distribution 2014). There is pressure for DNOs to reduce these losses, with financial controls being applied via the Losses Incentive Mechanism (Ofgem 2014).

#### 1.3 Mitigation methods

Although the scenario shown in Figure 1-3 reflects conventional distribution networks, there are a number of methods that might be employed to provide improved voltage control at the customer connections. A simple but high cost approach is to upgrade the cables such that the voltage differences and losses are reduced. This addresses both the voltage and current constraints of the existing infrastructure.

A number of other options exist that would assist in managing the voltages. The distribution transformers can be upgraded and fitted with a dynamic tap-changing capability or additional voltage regulating equipment providing reactive power can be installed along the feeders. Similarly, the inverters of distributed generation systems or the power electronic converters of loads such as EVs could provide reactive power control in order to manage the voltage. Since these devices are distributed, this requires a common control strategy to be planned and deployed to each customer installation. Although these approaches can resolve voltage constraints, the maximum currents in the cables or transformers can be increased (also increasing losses) and so these methods are more likely to defer rather than avoid upgrades to the cabling.

More generally, demand side response concepts could be developed in order to either increase the demand at the time of peak generation or to flatten the peaks in demand by time-shifting the use of some appliances. This could also be achieved using energy storage systems, provided that the charging and discharging schedule to support the network management does not conflict with other uses of the stored energy such as in balancing the national supply and demand, or, of course, the local requirements of the customer.

Although there are various methods that can ameliorate the impacts of adding new low carbon technologies, none of these are both simple to deploy and low cost. There remains a need for DNOs to maximise the use of existing infrastructure.

#### 1.4 LV network modelling

Models of the LV network are needed for a number of different purposes, including:

- Planning new cable routes and sizing cables for new connections
- Modelling the impact of low carbon technologies added to existing networks
- Strategic planning for possible low carbon technology growth scenarios

The costs of upgrading the LV network infrastructure are high so it is important that investments are well targeted and accurate models are needed to inform the infrastructure planning process. However, until recently, modelling has been limited by the level of detail available to characterise the customer demand, even without new low carbon technologies. Since there are many customers connected to LV networks and few locations where voltage monitoring equipment can physically be attached, it has also been difficult to obtain comprehensive measurement data. Early load flow simulations were also limited by a lack of computer processing power. Such models require a description of the network topology and of the impedances of the feeder cables and the service cables that connect individual customers. Since these cables have in some cases been in the ground for many years, DNOs the information in network databases is often inaccurate or incorrect.

Some of these difficulties in modelling have now been resolved. The limitations of computing resources are less of a concern, and sophisticated monitoring equipment to record and characterise the customer demand is now available. Future network planning will also be able to make use of voltage data from smart meters. This might, for example, allow the voltage control at the primary substation to be optimised so as to allow the greatest capacity of distributed generation to be connected while ensuring that the customer voltages remain within the required limits (Leisse et al.

2013). However, although smart meters can describe the past history of the voltage supply, they clearly cannot predict the response to future proposals.

A further difficulty with LV network simulations is that many of the assumptions that have been applied in modelling higher voltage distribution networks do not remain secure when applied at low voltage. For high voltage networks, the demand is usually considered to be highly aggregated such that it has a smooth variation with time and can be predicted from mean daily profiles. It is also commonly assumed that there is minimal unbalance between the three phases. This contrasts with LV networks where the demand is highly stochastic and unbalanced. Until recently, it has also been possible to assume that the level of harmonic distortion is relatively low but, with increasing use of power electronic converters for loads and for the inverters used by distributed generation, this is no longer the case.

It also seems likely that the highly conservative approaches previously used in LV planning may unnecessarily prevent connections of new low carbon technologies. A particular example occurs in planning for PV where the worst-case scenario is considered based on the highest generation and the lowest expected demand on the feeder. However, the probability of these two extreme conditions occurring simultaneously may be very low. This was demonstrated by a UK DNO where the conventional network planning rules recommended a 300 mm<sup>2</sup> size feeder cable, but in practice the voltages were found to be within an acceptable range with smaller size cables (Western Power Distribution 2013a).

#### 1.5 Limitations of measurements

In recent years there has been a much greater interest in understanding the operation of the LV network and a number of projects have made detailed measurements. The level of detail recorded has also increased over time with monitoring at 10 minute intervals in the earlier projects (Western Power Distribution 2013b; Electricity North West Ltd 2014b) and now at 1 minute resolution for more recent measurements such as on the C2C project (Electricity North West Ltd 2014a).

However, even with 1 minute data, there are still effects that are not captured. Figure 1-4 shows an example of the recorded current in the phase L2 and neutral conductors from one of the substations monitored for the C2C measurements. The plot shows the minimum, maximum and average values, demonstrating that the maximum and minimum values differ significantly from the average. The maximum current can be up to 50% higher than the recorded minimum for the L2 conductor and up to twice as high for the neutral. Clearly there is further variation within the 1 minute periods that is hidden with this measurement resolution.



Figure 1-4 – Minimum, maximum and average currents in phase L2 conductor and neutral

A simple estimate of the neutral current can be made based on the summation of the currents in the phase conductors, assuming that these are at 50 Hz. The calculation takes account of the phase angles of the current relative to the voltage and assumes a perfect 120° relationship between the voltage phase angles. However, Figure 1-5 shows that there is a significant disparity between this estimate and the measured neutral current so clearly some of these assumptions are incorrect. The differences between these two current estimates could be due to:

- Limitations due to the time resolution of the measurement, such that the unbalance of the phase currents is not adequately represented
- Harmonics, such that the calculation of the neutral current from the phase currents (assumed to be at 50 Hz) is incorrect
- Voltage phase angle differences, such that the power factor does not provide adequate information to define the current phase angles
- Currents flowing in the ground path, such that the unbalance current does not all flow in the neutral

• Neutral currents flowing through to or from other routes, such as through a link box from another feeder



Figure 1-5 – Measured and calculated neutral current for measured substation data

#### 1.6 Research aims

There is uncertainty over the accuracy of LV network planning models and also in the more detailed analysis of measurement data that might be used in strategic studies of future low carbon technologies. To address these concerns, this research aims to investigate the accuracy of LV network modelling methods and to provide improved techniques where commonly used approximations are found to cause errors in the results.

This approach therefore differs from many previous studies where the objective has been to determine the hosting capacity for one or more types of low carbon technologies. Rather, the outcomes from this work will enable future evaluations of hosting capacities of low carbon technologies to be made with greater certainty in the models.

The key objectives of this work are therefore to:

• identify the assumptions and approximations used in LV network modelling

- quantify the impact of these approximations and provide perspective on the need for more accurate methods
- develop improved methods that provide more accurate results
- validate simulation results against practical network measurements

#### 1.7 Thesis structure

The thesis is organised as follows:

<u>Chapter 2</u> reviews the academic literature and reports of recent LV network studies to identify the assumptions and approximations that are commonly made in network models. In order to prioritise this work for this thesis, assumptions relating to cable impedances, harmonics, demand data time resolution, and the modelling of currents in the neutral and ground conductors are selected for detailed investigation.

<u>Chapter 3</u> considers the theory relating to the definition of the cable impedance. This chapter returns to the first principles of the analysis in order to define a solid basis for the use of the impedance data within the network models.

<u>Chapter 4</u> presents impedance models cables that are typically specified for new LV networks and a comparison of modelling methods with varying levels of detail and accuracy. An improved finite element method for modelling cable impedances is described and guidance given on when the greater level of detail provided by this method is needed, or when simpler approximation methods are adequate.

<u>Chapter 5</u> describes the results of measurements of a section of an LV feeder cable on the Loughborough campus. There are few studies that undertake this practical verification task so this provides a novel contribution by demonstrating the correspondence between the simulated and measured cable impedances. A range of practical difficulties that affect LV network measurements are also discussed.

<u>Chapter 6</u> describes a second set of impedance measurements in which the time varying waveforms were captured in order to provide a more reliable measurement of the phase angles of the current and voltage sinusoids.

<u>Chapter 7</u> uses the high resolution measured data to investigate how the customer voltage or the cable losses were affected by current harmonics, since the addition of

new LCTs to the network is expected to increase the level of distortion. This chapter also consider how simulation results may be inaccurate if harmonics are present in practice but omitted in the model. A method is proposed that DNOs could use to compensate for the impact on losses.

<u>Chapter 8</u> considers the impacts of the time resolution used for modelling the demand. This particularly affects loss estimates but also impacts on calculations of customer voltages in relation to the interpretation of power quality standards. An interpolation method is recommended that DNOs could use to improve loss estimates from measured data.

<u>Chapter 9</u> addresses the methods used to calculate the current in the neutral conductors and the ground and proposes a new formulation of the forward/backward sweep method for use in a four-wire simulation model. In order to take account of the interactions between ground electrodes, a novel combination of ground resistance theory and the forward/backward sweep method is developed.

<u>Chapter 10</u> applies the simulation methods developed in chapter 9 in order to quantify the impacts of different approximations for a representative urban LV network. Recommendations are given for the modelling of ground connections in LV feeders, suggesting scenarios in which the ground path need not be included in the model.

Chapter 11 presents conclusions from the thesis and proposes further work.

### 2 LITERATURE REVIEW

This chapter reviews the literature relating to network simulation models of LV networks. The aim here is to identify the assumptions and approximations that are applied in order to characterise real networks, together with their connected demand and generation, such that they can be represented in a model.

The review considers three main areas:

<u>Demand and generation</u>: the characteristics of the attached loads and generators, varying over time and in accordance with the supplied voltage.

<u>Network connectivity</u>: the topology of the feeder cables and the connection to the substation.

<u>Cable models</u>: assumptions and approximations used in defining the impedance models. The theory relating to the cable impedances is explored more thoroughly in Chapter 3 and several methods of modelling the impedances are also presented. However, this level of detail is not typically available in simulation studies and so the literature review here considers the approximations that are commonly used.

The assumptions in these three areas, sometimes with some overlap, are then summarised using a risk assessment approach. This summary has been used to identify assumptions with a common theme for further investigation.

#### 2.1 Demand and generation characteristics

Models of network voltages and losses are, of course, dependent on the accuracy of the data used to model the currents that the network must supply to customers. The current at customer connections is typically specified by a demand model which is normally calibrated in some way against measured data, either by using measurements directly or by scaling synthesised data against a known mean demand. The demand model allows for the variation of the demand over time, and with respect to voltage.

#### 2.1.1 Demand profiles

The variation of demand in time can be characterized according to standard load profiles, such as those from the UK Energy Research Centre (Elexon Ltd 1997). This

data provides the mean demand for each half-hour within the day based on customers from all regions of the UK. Load profiles have then been used to define the peak demand through the use of the Velander correlation function (Mihet-Popa et al. 2013). This uses an empirical relationship to predict the peak load, based on the total annual consumption. Profiles such as these might be used directly to define the demand but, since they are based on the aggregated demand of many customers, they do not reflect the stochastic variation of individual customer loads.

The sensitivity of power-flow simulation results to the assumed demand profile has been investigated, comparing results using UKERC data with results using profiles from the UKGDS project (Frame et al. 2012). This highlighted differences in the extent of voltage unbalance between the two sets of profiles, therefore giving different voltage ranges for the same mean demand.

The study by Frame used a statistical distribution in order to create samples that follow the required demand profiles. The demand was assumed to follow a normal distribution with the standard deviation defined such that 99.7% of the samples are within 15% of the mean. This makes a further assumption that customers local to one area are all equally represented by the national profiles. Samples for each customer and for each time period are also assumed to be independent, such that behaviour patterns that are characteristics of residents in one specific area are not represented.

An alternative approach is taken in the CREST demand model where the samples of the demand for a single domestic customer are generated from statistics relating to active occupancy of the building and from data describing typical appliances (Richardson et al. 2010). The demand per appliance was scaled in order that the samples generated by the model were consistent with a demand profile, which in this case was selected to reflect regional variations.

This 'bottom-up' method is very flexible as it allows for the model to be modified so that additional connections of new appliances or of future low carbon technologies can be incorporated (Navarro-Espinosa & Ochoa 2015).

#### 2.1.2 Correlation between customer loads

Neither of the approaches noted above for generating individual customer demand profiles include a means to represent the differences in daily routines of different customers. Since each customer-day is considered as an independent simulation, there is no direct means to specify a different correlation between the demand profiles of separate customers to the correlation between the profiles for the same customer on different days. The demand model could be configured such that a customer retains the same appliances from day to day, but the occupancy patterns are either fully independent, or are identical for each day (Richardson & Thomson 2011). The model was validated against measurement data and shown to provide good agreement in general although the low and high extremes of the average demand per customer were under-estimated (Richardson et al. 2010).

Assuming that all customers are 'average' neglects the fact that some may have different occupancy habits (such as shift workers or retired people) or that people have different attitudes to energy use. This may under-estimate the worst-case range since high or low demand behaviours are randomised between customers.

#### 2.1.3 Correlation between distributed generation

Studies of the hosting capacity for PV on LV networks often assume that the irradiance is equal across the covered area (Navarro-Espinosa & Ochoa 2015). This assumption that the irradiance is completely correlated is needed in the absence of detailed information to allow for the variation of irradiance between locations. However, on days with patchy cloud, the short term irradiance may vary across the area covered by a primary substation, all of which is subject to the same voltage set point (Wirth et al. 2011). Although this may not appear to represent the worst-case, PV output may be higher on days with patchy cloud since module temperatures are low, increasing efficiency, and the irradiance may be increased due to cloud reflections.

#### 2.1.4 Phase balance

If unbalanced demand and distributed generation are modelled as being balanced, simulations risk under-estimating neutral currents and losses within the phase conductors. Voltage extremes and losses may be under-represented if the currents are averaged between phases, and cabling and transformer assets are not utilised to their maximum capacity (Beharrysingh 2014).

The customer connections to the network may be single-phase (generally the case in the UK) or three phase (as with many customers in Germany). Where single-phase connections are provided, the time varying characteristics of loads and generation cause the currents in the three-phase mains to be unbalanced. Where customers are provided with three-phase service connections, heating loads can be connected to all three phases and so there is less current unbalance due to short term usage of high power appliances. However, the majority of lower-powered appliances are still connected to single-phase circuits and so the potential for unbalance remains.

Where distributed generation is installed on properties with three-phase customers, smaller size PV inverters are still likely to have single-phase operation. All three phases are available at the house and so the randomness of the phase allocations depends on decisions made by the installer who may not be aware of the phases selected already for other installations. Where there are many installations along a feeder cable, the extent to which the aggregated demand is balanced depends on the evenness of these connection decisions. The risk of voltage rise is increased with single-phase customer connections, and it has been proposed that network operators should aim to install three-phase connections where PV systems are connected to weak grids (Mihet-Popa et al. 2013).

In addition to short term unbalance due to appliance or generation activity, the mean demand from customers on each phase may be unbalanced, for example if networks may supply a mixture of residential and commercial customers with different demand profiles (Northcote-Green et al. 2011). This has been demonstrated by recent LV substation monitoring results in the UK where the phase with the highest loading has 29% higher demand than the mean of the three phases (Bale 2012).

Earlier simulations of LV networks assumed balanced loads as computation resources were a concern (Das 1994), but this assumption is still required for simulations where the power-flow algorithms do not allow for unbalance (Electricity North West Ltd 2014c).

Unbalanced power-flow simulations have been described for studies with large numbers of connected customers are described in (Thomson & Infield 2007; Richardson et al. 2009). In these studies the authors assigned customers sequentially to the three phases, imposing a balanced mean demand. Alternatively, the generators and loads have been randomly assigned to the phases and the results were compared with those assuming a balanced model (Reese & Hofman 2011). The unbalanced model has higher energy losses than the balanced case and also a higher probability that the permitted voltage range will be exceeded.

The results for the balanced case are not necessarily incorrect, as this scenario could arise, but the network monitoring results noted above suggest that modelling should also include scenarios in which demand models or customer phase allocations are significantly unbalanced, for which greater voltage ranges, increased losses and higher neutral currents would be expected.

#### 2.1.5 Time resolution

Probabilistic simulations typically use a time step approach in which each sample represents the demand or generation over a fixed time interval. A high time resolution is needed in order to represent 'spiky' demand characteristics. If currents are averaged over too long a period, short term voltage deviations will not be represented. Since power dissipated is proportional to the square of the current, losses are under-estimated if calculated using an average current, rather than as sum of losses over the same time period due to a variable current.

Where the power delivered to customers is required to conform to EN 50160, voltage magnitude, unbalance and harmonic distortion are considered in respect of a 10 minute RMS averaging period (BSI 2011b). This differs from the UK regulations which do not define any specific RMS averaging period (UK Government 2002) although the 10 minute time averaging of EN 50160 has been applied when comparing measured voltages to the UK regulations (Western Power Distribution 2013d). The impact of time resolution on the estimation of voltage drop/rise, and on losses has been explored by a UK DNO where it was concluded that a 5 minute resolution should be used for simulation models, although voltages are averaged to 10 minute intervals in accordance with the standard (Electricity North West Ltd

2014b). This study also demonstrates an example of the calculated losses, which are seen to increase as the time resolution of the demand data is improved.

Other simulation models have used a wide range of time resolutions, including 1 minute (Thomson & Infield 2007; Stetz et al. 2012), 15 minutes (Matrose et al. 2012), 30 minutes (Frame et al. 2012) and hourly (Ferreira et al. 2012).

The impact of selecting different time intervals has been reviewed for periods of 1 to 30 minutes (McQueen et al. 2003). For a single customer, the maximum demand taken at a 99% confidence level with 30 minute averaging was 16% below that for 1 minute samples. When the demand from 16 customers is aggregated together (and demand is more balanced), there is only a 6% difference.

The required time resolution can be considered with respect to the typical on-times of appliances. Thermostatically controlled electric heating has been identified as having the most significant impact amongst typical domestic appliances due to the high power required and the short periods when the appliance is active. An example was provided of a cooker hob on a low heat setting, modelled as a 2 kW load with a duty cycle of 30 seconds on then 120 seconds off (Newborough & Augood 1999). Clearly these switching times are much shorter than typical simulation study time resolutions, such that the time resolution chosen for the model may affect the simulation outcomes.

The proportion of energy imported for a house with a hypothetical constant generation source is also under-estimated if the demand is averaged over longer periods (Wright & Firth 2007). If the demand is 'flattened' by 30 minute averaging then it appears that on-site generation may meet a greater proportion than if the actual peaks in demand are represented by 1 minute demand averages. However, this source of error is not always taken into account (Spertino et al. 2015).

Where data is provided by smart meters, the time resolution of the model is limited by the meter configuration with typical meters having a resolution in their monitoring data between 15 minutes and 1 hour (Ferreira et al. 2012). A recent trial has been conducted with customer voltages being collected from meters on a test feeder reporting at 10 second intervals (Leiße 2013). The meters needed to be polled in order to obtain data readings, with the consequent difficulty that not all readings were sampled at the same time. The feedback of data for collection was also asynchronous, creating some uncertainty over the network state reported. In the test network, it was also only possible to instrument a subset of the customer nodes. However, such measurements represent a considerable step forward compared to earlier studies.

#### 2.1.6 Variation with voltage

Simulations of network voltages and currents depend on the voltage vs. current characteristic of loads and generators being modelled accurately.

For typical domestic appliances, the demand has been found to be a mixture of constant impedance and constant power loads (Collin et al. 2010). During most of the day, the loads were approximately 20% constant impedance and 80% constant power. In the evening, the proportion with constant impedance rose to 40% due to resistive heating loads. Simulations of a large group of residential loads, each with a constant power demand and short term peaks of power into resistive loads, give an overall characteristic closer to a constant current model (Tsagarakis et al. 2012). At night, when the resistive power peaks are absent, the characteristic reverts to a constant power model. Another study in Ireland has suggested lower exponent values, with demand reducing by 0.5% to 1% for a 1% reduction in voltage (Diskin et al. 2012).

The load model characteristics have also been studied for a large group of customers on a UK LV network, with the active power demand for residential customers varying according to an exponential load model with factor  $K_P = 1.3$  (Electricity North West Ltd 2015a). This study also noted that the load model for the reactive power is different, with a factor  $K_Q = 6.0$ . These results differ potentially from the previous studies as the voltage reduction periods were shorter (varying for periods of less than an hour). The focus of the Electricity North West study was the change in load power whereas the study in Ireland was considering the reduction in delivered energy. Reductions that apply to short term fluctuations are not necessarily sustained over longer periods. For models with high time resolution, there may therefore be a need to consider higher exponential factors in the load model exponent, but then to modify the demand based on the past history of the voltage variation so that a different load model applies over a longer term. For example, the

assumption that resistive heating loads have a constant power characteristic implies the use of thermostatic control mechanism. These loads might be considered as constant power when represented by 30 minute average demand samples (longer than the thermostatic on/off times) but could be considered as constant impedance loads when represented by 1 minute time resolution.

Although there are a range of figures used to characterise the load model, it seems likely that the assumption of constant power loads is not representative of real appliances.

In many simulation studies, the load model is not defined, and it is assumed that loads and generators have constant power (Thomson & Infield 2007; Matrose et al. 2012; Alam et al. 2012) but other simulation studies have assumed a constant current load model (Canova et al. 2009; McQueen et al. 2003) or a combination of constant power and constant impedance loads (Liu et al. 2008).

The neutral currents and voltage unbalance for the constant power model have been found to be doubled compared to results with the constant impedance model (Ciric et al. 2003). The end node customer voltage also varied by up to 7%, confirming that the simulation outcomes are sensitive to assumptions regarding the load model.

Generators driven by renewable energy are commonly assumed to provide a power output dependent on the renewable resource available, such that the power output does not depend on the network voltage (Canova et al. 2009). The same assumption is less certain for load models.

#### 2.1.7 Harmonics

Power-flow analysis, based on a phasor representation of the voltages and currents, often assumes that the system operates only at the fundamental AC frequency and harmonics are not taken into account. This neglects the impacts on voltage drops due to increased reactance at higher frequency, and assumes a greater cancellation of three-phase currents in the neutral than will occur in practice.

Harmonics from individual domestic appliances have been modelled in order to estimate the current distortion for the aggregated demand of a domestic customer (Collin et al. 2010). This model included 3<sup>rd</sup> and 5<sup>th</sup> harmonics at levels of 20% and 8% relative to the fundamental.
Harmonic distortion due to distributed generation has been found to be at a lower level to the distortion from residential loads. PV inverters operating near rated power were shown to have current Total Harmonic Distortion (THD) between 2.1% and 4.8% (Du et al. 2013). However, the distortion increased, both in absolute and relative terms, for operation at lower power. The current distortion from three-phase inverters has been shown to increase (depending on the control algorithm) if voltage distortion is also present (Castilla et al. 2013). However, in a contrasting example distributed generation from CHP units was found to slightly improve the power quality for gridconnected operation, reducing both the current and voltage distortion (Ciric et al. 2010).

Monitoring at a UK distribution substation on a network with a high penetration of PV connected has shown voltage THD up to 3.5% and current harmonics equivalent to 30% THD (Bale 2012). This monitoring also showed that the current harmonics vary between the three phases relative to the fundamental.

In simulations, the use of a sinusoidal model is sometimes explicitly stated (Richardson et al. 2009), but is assumed for power-flow studies in general unless harmonics are described. A simulation study of the impact of harmonics in (Sunderman et al. 2008) considered an unbalanced load with 3<sup>rd</sup> harmonic current at 8% of the fundamental. When the 3<sup>rd</sup> harmonic was added, the impact on overall losses was small but losses in the neutral were more than doubled. The neutral to earth voltage was also raised.

#### 2.1.8 Power factor

The power factor describes the ratio of active power delivered to the maximum power that could be delivered if the voltage and current waveforms were fully aligned in phase. It therefore includes components relating to distortion (since the average power delivered is zero if voltage and current have different frequencies) and to the phase angle of currents with respect to voltage.

Most simulation studies use a single frequency model and so implement the power factor entirely as a phase displacement between current and voltage.

Simulations models typically assume a power factor of unity for generators, where this is required by grid connection regulations (Canova et al. 2009; Matrose et al.

2012). Clearly, different assumptions are made where the impact of the power factor itself is the subject of the investigation (Stetz et al. 2012).

Loads could be assumed to have a constant power factor throughout, e.g. a value of 0.9 (Canova et al. 2009). More recent work suggests that the power factor of demand aggregated at an LV substation has a median value that is closer to unity, but also that there is a considerable range with values as low as 0.8 having been measured (Navarro-Espinosa et al. 2015). Alternatively the power factor could be specified for each customer connection individually by assigning an appropriate power factor for each appliance that contributes to the overall load (Richardson et al. 2010). This approach would represent the variation in power factor during the day as the proportion of demand due to appliances with induction motors reduces and as the proportion of demand due to heating increases.

## 2.1.9 Non-metered demand

Additional non-metered demand could be present due to street lighting or other highway equipment. The demand due to these forms of load is mostly neglected (McQueen et al. 2004; M. Thomson & Infield 2007). Data derived from customer smart meter readings would not include this non-metered demand (Western Power Distribution 2013d).

It is important to include this non-metered demand in models considering losses, since power delivered to these loads may otherwise be incorrectly attributed to technical losses, or considered as theft. The non-metered demand could also affect load allocation calculations where customers are allocated a proportion of the load measured at a substation (Shirek et al. 2012).

#### 2.1.10 Customer voltages

Different metrics can be used to report the voltage ranges at customer connections. As noted above, European countries use EN 50160 to define the permitted tolerances on customer voltages and this differs from the UK regulations (UK Government 2002; BSI 2011b). Both define the same nominal voltage of 230 V but they differ in several respects.

EN 50160 allows tolerances of +10% and -10% for 95% of a period of one week, and overall tolerances of +10% and -15%. This allows for a 5% probability that voltage

drop will cause customer voltages in the range -10% to -15%. In contrast, the UK regulations allow a range of +10% and -6%.

EN 50160 also bases the calculations of the voltage on 10 minute RMS measurements. This allows for some smoothing of the peak voltage excursions due to peaks in the demand variation. The UK regulations define the voltage as being *"calculated by taking the square root of the mean of the squares of the instantaneous values of a voltage during a complete cycle".* This interpretation has been clarified with the statement that *"the declared values and tolerances are an absolute requirement, and any variation beyond the voltage limits (apart from exceptional circumstances) without the agreement of the consumer would be treated by DTI as a breach of this regulation" (Department of Trade and Industry 2002).* No provision is made in these documents for a longer RMS averaging period.

The standards can therefore be considered to have three key differences: i) the tolerance limits; ii) the use of the 95% confidence interval, and iii) the use of a 10 minute RMS average. These differences are cited to some extent in a study by Western Power Distribution where the main concern is the 6% tolerance specified in the UK regulations which may affect the scope for energy saving via conservation voltage reduction (Western Power Distribution 2013c). The impact of voltage variation on the customers' perception of power quality was investigated in the Electricity North West "Changing Standards" project where the differences in standards with respect to tolerance limits and the use of a 95% confidence limit were highlighted (Electricity North West Ltd 2015b). The impact of the RMS averaging was not discussed in detail, although a 10 minute RMS was adopted for consistency with EN 50160.

As a consequence of these differences in standards, the results of studies investigating the hosting capacity of low carbon technologies on the LV distribution networks may not be directly transferable to areas with different voltage limits. For studies relating to UK networks, the dependency of the simulation results on the time resolution of the data needs to be taken into account.

Simulation results would also be very different if the UK regulations were taken literally and no RMS averaging period was applied. Differences between results with and without RMS averaging are shown later in Chapter 10.

# 2.2 Network connectivity

Simulations of LV networks require a model of the connectivity of the feeders and service cables. This includes the network topology and the lengths and types of each section of cable or overhead line. A more detailed model will also include details of the connections between the neutral and wire earth conductors and the body of the earth (referred to here as ground).

Assumptions relating to the network connectivity, particularly regarding grounding, are also dependent on the model used to calculate the cable impedances. There is therefore some overlap between the assumptions discussed here and those described in Section 2.3.

## 2.2.1 Database accuracy

A basic requirement in modelling real networks is that the network database accurately reflects the actual network installed. This is highlighted by (Shirek et al. 2012), where it is noted that the asset database records are as critical as the electrical model of the individual components such as the cables.

Typical concerns are that cable types in the ground may not be as recorded in the database, or that there are differences in the cable routes (and therefore lengths) and positions and connectivity of junctions. There is relatively high confidence in the database for HV networks, but the accuracy of data describing the LV network is less certain.

The phase allocation of customer service cable connections to the three-phase feeders is likely to be a key area of difficulty for the modelling of unbalanced demand and the current LCNF Smart Street project has identified this as a limitation within the LV modelling work (Electricity North West Ltd 2014c).

# 2.2.2 Service cables

Data to describe the service cables between feeder mains and the customer premises may not be readily available and the routing and connection points may need to be approximated based on street maps (Richardson et al. 2009).

Where customers are provided with single-phase service connections, there is also scope for records of phase allocations to be missing or incorrect. Although this

information is now routinely recorded, data is not always available for connections made in the past (Western Power Distribution 2013b).

To simplify the network model, it has been assumed that the customer voltage is approximated to being the voltage at the point where the service cable is attached to the main feeder, rather than at the customer end of the service cable (McQueen et al. 2003; Frame et al. 2012; Karmacharya et al. 2012; Mihet-Popa et al. 2013). Clearly this neglects the voltage drop in the service cable. Whilst this could be assumed to be minimal for customers in modern urban areas, the impact may be for greater for older installations in rural areas.

## 2.2.3 Mesh or radial topology

In many areas of the UK, LV feeders have a radial topology. For redundancy, pairs of feeders from the same secondary substation may terminate at a link box. In the case of a fault, service to customers may be provided by installing temporary connections within the link box such that the feeder without the fault also serves customers on the feeder with the fault.

Recent studies have considered the benefits of adopting a meshed topology in normal operation, initially with the relatively simple means of joining two radial feeders together at the link box. This has been shown to provide additional capacity for low carbon technologies such as PV and heat pumps to be added to the LV network, with the current being shared between the two feeders (Navarro-Espinosa et al. 2014). Where the additional demand is unequally shared between the two feeders, there is less impact on the feeder with hosting the new connections, but a corresponding increase in currents on the other feeder via the link box. A subsequent study, based on real LV network topologies, considered the potential benefits of connecting pairs of LV feeders in a mesh (Aydin et al. 2015). This demonstrated that the improvement in hosting capacity for PV varied from 0% up to 40%, with the greatest improvements where the number of customers on one feeder was twice the number on the other feeder.

To simplify the requirements for network protection, link boxes between feeders are often operated as normally open points, with the phase conductor branches removed unless there is a fault. However, the neutral and sheath conductors may remain connected through (Schneider-Electric 2009). This creates loops within the neutrals or sheaths and allows circulating currents to flow. Since the forward/backward sweep algorithm does not readily accommodate this, previous studies have made the assumption that the neutrals and sheaths can be treated as being disconnected, allowing the network to be simplified to a radial structure (Thomson & Infield 2007; Liu et al. 2008). However, the impact of this assumption is not clear.

## 2.2.4 Connectivity of neutrals, sheath and ground

The following section discusses the connections between neutral conductors, the sheath (or concentric neutral) and the earth. Where the currents in the three phases are unbalanced or include triplen harmonics, the sum of the currents in the phase conductors will be non-zero. The accuracy of simulations of the network voltages therefore depends on the modelling of the routes available for this residual current to flow back to the sub-station.

As described below in Section 3.5, it is common practice to simplify the cable impedance matrix to a  $3 \times 3$  form. This requires assumptions to be made about the connections between the neutral and ground conductors. In particular, the Kron reduction requires the assumption that the voltage between neutrals and the ground is zero at each end of a line segment (Kersting 2012).

In European networks, it is common for the LV side of the distribution transformer to have a wye configuration with the neutral point connected to ground. The ground connection is provided by earth electrodes, designed so as to ensure a low earth potential rise in the presence of fault currents. Where the LV distribution is via underground cables, the metallic sheaths and/or concentric neutrals are also connected to ground at the substation.

At the customer meter point connection (the end of the service cable), different ground configurations can be provided, according to the regulations governing the earthing system (Cronshaw 2005). In summary, these are as follows:

- TN-S: Terre (earth) and Neutral separate
- TN-C-S: Earth and neutral combined but provided separately for the customer
- TT: Earth at the substation and separate earth at the customer connection

For a TN (*Terre* and Neutral) configuration, the distribution network provides neutral and an earth conductor and so customers do not normally install an earth electode.

For a TN-C-S configuration, the neutral and earth are combined, but with TN-S, there is no connection between neutral and earth. However, for both TN configurations, the earth conductor provided by the distribution network is connected to water and gas pipes at the customer meter point. This is primarily intended to create an equipotential zone within the customer's premises, but may also provide a connection to ground if the pipes are metallic (Werda et al. 2008).

For TT earthing, there is no protective earth provided along the feeder and so the customer must install a ground electrode (Cronshaw 2005). The neutral conductor is connected to a ground electrode at the substation, but the connection is for protection only and so the neutral conductor is isolated from the ground electrodes.

In the United States, Medium Voltage (MV) lines are more widely used for local distribution, with low power LV transformers serving small groups of customers (Lakervi & Holmes 1995). An example of this configuration is shown in (Sunderman et al. 2008). In this case, the neutral at the transformer pole is connected to ground, as are the neutrals provided with the connection to each house.

LV cable mains may include junctions between different cable types. Based on normal UK practice, junction box designs allow for continuity of the cable cores and concentric neutral or sheath, but there is no connection between the neutral cores and the sheath, or between these and the ground. Similarly, there are no neutral or sheath connections to the ground where service cables are attached to mains.

However, where TN-C-S earthing is used and the customer is provided with a combined neutral/earth conductor, it is important for safety that the earth connection does not become broken. Additional ground electrodes are added at nodes within the LV feeder network, known in the UK as Protective Multiple Earthing (PME). These ground electrodes may be installed at feeder cable joints and at the ends of feeder mains.

There is therefore a wide variety of earthing configurations in use, each with different requirements for connection between neutral and sheath or concentric neutral, or between these and the earth. The detail of neutral to earth connections is also likely to be unknown in many places. This is usually considered carefully in regard to safety and fault conditions but simplifications to models are assumed for power flow

studies. Provided the network loads are balanced and have negligible harmonics, there is no concern, but the consequences for more representative real networks require investigation.

Network designs require that LV sub-station earth electrodes may have an impedance that is up to 20  $\Omega$  (E.ON Central Networks 2006) and PME earths elsewhere with resistances of up to 100  $\Omega$  (East Midlands Electricity 2001). Similarly, a neutral to ground resistances of 100  $\Omega$  has been assumed for North American overhead line poles (Sunderman et al. 2008).

The sensitivity of simulation results to the grounding impedance is tested in (Sunderland & Conlon 2012). The model assumes a neutral to ground resistance of either 0.1  $\Omega$ , 0.2  $\Omega$  or open circuit at distribution pillars and either 5  $\Omega$  or 15  $\Omega$  at the customer connection. The neutral to ground voltages at the distribution pillars are shown to double when varying between the high and low grounding resistances. However these voltages are still relatively low and it is not yet clear how significant this is in the context of the delivered customer voltages.

In discussions relating to the asset database (Shirek et al. 2012) no mention is made of the inclusion of grounding electrode details into this data. In this paper, Carson's equations are employed with the assumption of a perfectly grounded neutral, but the locations and resistances of the actual implementation of this grounding are not considered.

# 2.2.5 Substation voltage

LV distribution transformers do not generally have voltage regulation, but have fixed ratios that might be set at installation and adjusted very infrequently. Voltage regulation to respond daily to demand variations is provided by transformers at higher voltage levels. Typically the primary substation would include an On Load Tap Changer to maintain the MV voltage supplied to the distribution feeders at a defined set point.

A power-flow analysis typically requires a reference node at which the source voltage is defined (Kersting 2012). If the scope of the LV model is extended to include multiple distribution transformers and their impact on the MV feeder, it could be assumed that the secondary side of the primary substation transformer is

represented by a perfect voltage source. It can then be assumed that the voltage source for the one MV feeder of interest is independent of current flowing in the other MV feeders (Thomson & Infield 2007).

Other models consider the LV network alone and so apply a similar assumption for the source voltage at the LV distribution transformer. Reflecting the fact that the voltage is not regulated at this point, the work in (Richardson et al. 2009) employed a daily profile to define the LV secondary voltage. This makes the assumption that this voltage is dependent on currents in the other LV mains attached to the same HV feeder, but not on the currents in the LV feeder being modelled.

An alternative approach is simply to assume that there is no significant variation in the voltage supplied to the LV distribution transformer so that there is a constant LV source voltage (McQueen et al. 2003). This approach was also adopted in (Matrose et al. 2012) where the objective was to evaluate the need for voltage control at secondary substations, and where LV main voltages were presented on a per unit basis relative to a voltage of 1.0 at the distribution transformer.

The tap changing operation at the primary substation depends on currents in all of the MV feeders connected. It might be assumed that the ratio the between demands in each feeder is approximately constant, such that the tap changes are equally valid for all feeders. However this assumption might not be valid for models with PV generation, since the generators connected to one feeder network may have a different solar irradiation that those connected to other feeders at the same primary substation.

The on load tap changer maintains the voltage within a specified bandwidth, for example 2%, of the required set point (E.ON Central Networks 2006). When the bandwidth is exceeded, the voltage is adjusted by tap steps, typically 1.25% to 1.43% of the transformer turns ratio (Thomson 2000). To reduce the frequency of step changes in response to transient conditions, step changes only occur if the bandwidth is exceeded for a defined time period, typically 30 to 60 seconds (Harlow 1996). If this functionality were to be included in a simulation model, the currents from all MV feeders connected would need to be taken into account. As a simplifying assumption, the regulation has been assumed to be ideal at the primary substation secondary winding (Thomson & Infield 2007).

# 2.3 Cable models

The development of the theory relating to the calculation of cable impedance models is considered in detail in Chapter 3. The discussion here aims to review the way in which the cable impedance data is applied in simulation models.

## 2.3.1 Impedance model

An accurate simulation of the network voltages and currents depends on the model of the impedances of the cables or overhead lines between each network node. A full model of these conductors would include the series impedance and shunt admittance, plus the lumped impedances of neutral or sheath connections to ground.

Simulation models typically assume that the series impedance for any branch of the network can be modelled in terms of the impedance per unit length of the cable (Kersting 2012). This implies that the impedance does not depend on the actual length of the cable, and so any 'end effects' due to the finite length of each branch are neglected. In reality, the magnetic field from current in any conductor causes a flux linkage with all of the other branches. However, if the sum of currents within a branch is zero, and if each of the conductors in the cable are close to each other, relative to the spacing between feeders, then the total flux linkage with other branches due to the sum of the currents in the cable is cancelled out.

The shunt admittance of LV cables is generally assumed to be negligible and not taken into account (Das 1994; Sunderland & Conlon 2012; Ciric et al. 2003). It is usually assumed that the conductance can be neglected and calculations have shown little impact due to the capacitance, at least for an example with overhead line at 12.47 kV (Kersting 2012). Kersting noted that the capacitance for underground cables may be more significant although not necessarily at low voltages.

# 2.3.2 Carson's equations

Carson's equations provide a matrix  $\hat{z}$ , containing the self-impedance and mutual impedances for each conductor in a circuit with a ground return path (Carson 1926). The equations include terms to define the impedance of a conductor with a perfectly conducting ground return, and also terms to define a correction for the finite conductivity of the earth. The correction involves several terms, each with an infinite summation, and so an approximation is generally used in which only the first

resistive term and the first two reactive terms are retained and the infinite series are truncated to their leading terms (Kersting 2012). This simplification is also described as the Carson-Clem formulae (Albano et al. 2006).

Comparison between impedance matrices obtained with the full Carson's equations and with the simplified version suggests that little accuracy is lost due to the approximation, at least for the fundamental mains frequency (Kersting & Green 2011).

Carson's original equations were derived for widely spaced overhead line conductors and use a concept of image conductors reflected at the ground surface in order to model the electromagnetic fields due to the current in the ground. It is therefore not immediately clear that Carson's equations would be applicable for use with LV cables, where the conductors are underground (and so beneath the reflecting surface) and where the adjacent cores are closely spaced. Despite this, the modified Carson's equations are commonly applied to underground cables without further explanation.

Separately, an analytical solution for underground cables was developed by Pollaczek in 1934. Although the original paper is not readily available the derivation has since been repeated in English (Yin 1990). Carson's equations are shown to be approximately equivalent to Pollaczek's equations at frequencies up to the order of 1 kHz (Srivallipuranandan 1986). Above this frequency range, the assumptions used in Carson's equations no longer hold.

Results using Pollaczek's equations were shown to be consistent with finite element (FE) modelling results for cables with circular cores (Yin 1990). An FE model has also been demonstrated where the impedance at 50 Hz of cables with sector-shaped conductors are shown to be consistent with the results from the modified Carson's equations (Urquhart 2012). Deviations between the FE model and the results from Carson's equations at higher frequencies were due to the effects of eddy currents within the sector cores.

# 2.3.3 Ground resistivity

A ground resistivity of  $\rho = 100 \ \Omega m$  is typically used when calculating the impedance of conductors in a circuit with ground return path (Kersting 2012). Kersting reviewed

the impact of varying resistivity from 10 to 1000  $\Omega$ m for a scenario based on IEEE Node Test Feeder 34, showing negligible difference to line voltage (Kersting & Green 2011). However, it would also be useful to consider the impact for underground cables and with a greater unbalance than in the above test case.

Ciric suggests that the ground resistivity can vary due to humidity and on a daily basis (Ciric et al. 2004). For a long cable, Carson's equations imply that the current in the ground is distributed over a significant depth of up to 1 km (see Section 4.2.4), and it might be assumed that the conditions remain relatively constant. Where currents enter the ground at a single point, such as under fault conditions, the impact of ground resistivity variations near the surface could be significant.

#### 2.3.4 Earth current paths

It appears that different assumptions are considered for the flow of currents in the ground in the case of fault currents and for calculating the impedance of a circuit with ground return. When the earth potential rise is calculated for fault currents, the potential is assumed to vary approximately in inverse proportion to the distance from the point where the fault current enters the ground (Tagg 1964). The direction from the source of the fault current is not considered. Conversely, when Carson's equations are used to calculate the circuit impedance of a cable or overhead line, the earth return is assumed to run in parallel with the circuit conductors, such that the potential difference in the earth would vary linearly along the route of the circuit.

For LV distribution branches of a few hundred metres, it is not clear which of these assumptions is most appropriate. The physical route taken by LV mains could have various bends and turns, such that the shortest return path via earth is not necessarily along the line of the cables. As noted below in the discussion of ground resistivity, the assumption that the current distribution has a 2-D planar cross-section is questionable, based on the implied depth of the currents in the ground, compared to the length of the cable.

#### 2.3.5 Sum of currents assumed to be zero

Carson's equations are based on the assumption that the sum of currents in the cable or overhead lines equals zero. This is a secure assumption for radial networks with no ground connection.

Modelling assumptions

However, as noted in Section 2.2.4, meshed paths can exist for the neutral conductors due to connections at link boxes. Where networks are not fully radial, with loops in neutral or sheath conductors introduced by link boxes, additional circulating currents may need to be taken into account.

As noted above, connections to ground introduce the possibility that earth currents follow a different route. This might occur, for example, where two feeders are nominally isolated in terms of the electrical network topology but physically located at the same place, and where there is a different local ground potential due to current flow in each branch. The conductive path in the ground is, of course, continuous throughout the network and so the differing potentials could create a circulating current in the ground. There may also be other metallic pipework that provides additional conductive paths for which modelling is only possible with detailed GIS data to describe the infrastructure installed underground (Sunderman et al. 2008).

The impact of metal water pipes on the earth loop impedance has been studied for an Australian LV distribution network (Werda et al. 2008). This describes a configuration in which a line and neutral service connection is provided by singlephase overhead lines to each house. To provide an earth path in case of interrupt to the neutral line, this is also bonded to water pipes. For houses built after 1976, the neutral is also bonded to a local earth electrode. The study found that the earth loop impedance at the houses was increased significantly (by a factor of 3) if either the water main or the connecting service pipes were replaced by non-conductive alternative materials. Where the pipes were metallic, they were found to carry around 20% of the current returning to the substation.

This study highlights two limitations of the existing LV models. Firstly, the current is carried on conductors that are not considered in the modelling of the cable and the ground impedance. Although the sum of currents delivered to the customer must be zero, the sum of currents in the cable and ground is non-zero. Secondly, the current in the pipes is likely to follow a different route to the electricity cables so, even if the pipe were included in the model, the representative of the total set of conductors by a 2-D planar cross-section would not be valid. Although the study by Werda has been used in subsequent works (Alam et al. 2013; Alam et al. 2015) and used to provide values for the grounding resistances, the equivalent circuit model considered

for the simulation work does not appear to take account of the different current paths that the resistances imply.

A key assumption in Carson's equations is that the currents density sums to zero across a 2-D plane that is normal to the longitudinal axis of the cable, and the electric field is assumed to occur only in the axial direction. The possibility that current follows paths through other conductors already raises concerns with this assumption. A further concern arises due to the implied depth of the current within the ground, as noted in Section 2.3.4. Based on a purely longitudinal model of the current distribution, Carson's equations imply that the current is distributed within the ground to a depth on a kilometre scale. It therefore seems implausible that this current distribution could be established this between ground electrodes for an LV network that are only a few hundred metres apart. The assumption that the current flow is only longitudinal therefore seems invalid.

#### 2.3.6 Calculations of earth voltages

One of the assumptions noted in the review described above refers to the use of Carson's equations to calculate local earth voltages. This has considered in greater depth, with the work described in detail in Appendix B of this report.

Many network models have represented the network cables using a 3x3 representation of the three-phase impedances, or an equivalent 3x3 sequence impedance matrix. Using assumptions that the sum of currents equals zero, and a second assumption that neutrals are grounded (the Kron reduction), it is possible to represent the neutral and ground conductors within these 3x3 impedance matrices.

However, several authors have recently developed 4x4 impedance models in which the neutral is represented explicitly or5x5 impedance models where the ground is considered as an additional conductor (Ciric et al. 2003; Sunderland & Conlon 2012). There appears to be no concern with the 4x4 approach, but the 5x5 analysis requires equations to be developed for the impedance of the earth conductor. Although it is possible to avoid making the two assumptions noted above, further assumptions are introduced in deriving the earth conductor impedance. The assumptions implied by this method are discussed further in Section 3.4.

#### 2.3.7 Sequence impedance approximations

Power-flow simulations with unbalanced demand require an impedance matrix that defines the mutual coupling between conductors. For cables that are assumed to have symmetrical three-phase conductors, this can be specified in terms of the positive and zero sequence impedances (Kersting 2012). The mutual sequence impedances are then assumed to be zero. Since this sequence matrix contains no coupling between sequence modes, the corresponding phase impedance matrix is fully balanced. If the cable being modelled is not symmetrical in reality, the model is equivalent to making an approximation that the three phases are transposed at intervals along the cable.

The impact of this approximation was reviewed with unbalanced loads in (Kersting & Phillips 1995) and shown to have minimal impact on voltage magnitudes. However, there was a greater impact on voltage unbalance (Kersting 2011). Errors were also introduced into the results for energy losses. Although the approximation made little difference to the total loss, assuming transposed cables introduced significant error into the loss calculations for individual conductors, if the loads were unbalanced (Kersting 2011; Kersting & Phillips 1995).

Data for the zero sequence impedance is not readily always readily available but it has been found that that this can be approximated by applying a multiplying factor to the positive sequence impedance. A scaling factor of between 2.5 and 3.5 is recommended with values towards 3.5 for lines without a ground return (Nagrath 2008). Thomson & Infield used with different multipliers for the resistance for the reactance and found that the power-flow results were relatively insensitive to the choice of scaling factor (Thomson & Infield 2007). However, in a study modelling high penetrations of electric vehicles (EV) on the LV network with much greater levels of unbalance, varying the scaling factor over a range of 3 to 5 significantly affected the proportion of voltage range and unbalance constraint violations (Frame et al. 2012).

The impact of approximating the impedance matrix by the positive sequence value along (effectively using the phase conductor impedance alone, and assuming that the neutral conductor impedance is zero) has been shown to introduce considerable error into voltage calculations (Kersting & Phillips 1995).

#### 2.3.8 Impedance variations

Given that the impedance of individual installed cables in live circuits cannot be characterised, it is generally necessary to rely on the data provided by manufacturers or on cable construction standards.

Cables in the ground may also be subject to ageing effects. However, the impacts noted in a review of installed LV cables relate mainly to the insulation and sheath and so affect the shunt admittance and the probability of cable breakdown, rather than the series impedance (De Clerck et al. 2013). The effect of moisture ingress into LV cables with oil impregnated paper insulation has been experimentally investigated, again finding that the concern relates to cable failure rather than an impact on the series impedance (Rowland & Wang 2008).

Cables with aluminium conductors are subject to degradation if the outer insulation becomes damaged, either on installation or due to other ground works during the life of the cable (Cinquemani et al. 2001). Where the aluminium is exposed to moisture from the soil, hydrous aluminium oxide builds up due to the electrolysis that occurs as leakage current enters the soil (Lawson & Kong 1989). This corrosion further exposes the conductors, increasing the rate of electrolysis and eventually resulting in an open circuit since the aluminium oxide layer is non-conductive. Cables can fail catastrophically over a long period, as in the study by Cinquemani .

Copper cables are also subject to oxidation on exposes surfaces, but the copper oxide layer is conductive and does not lead to cable failure, as with aluminium (Boone 2015).

# 2.4 Risk assessment of assumptions and approximations

A summary of the modelling assumptions and approximations is presented below in Table 2-1. A shorter version of this has been published and is expanded in the discussion here (Urquhart & Thomson 2013).

In this table, the first two columns present the list of the assumptions and a short description to indicate the likely effects or risks.

The assumptions have also been categorised in terms of the risk that they present to the accuracy of network simulations. Clearly this is a subjective assessment and so a risk analysis approach has been followed in order to define a means of categorising the assumptions. The risk is determined as a numerical product of separate categories describing the uncertainty associated with the assumption, and the impact that inaccuracies might have on the results.

For each assumption, the uncertainty is rated between 1 and 3, with a high rating indicating greater uncertainty.

- 1. Assumptions that can easily be avoided, or for which the impact is well categorised
- 2. Assumptions for which the impact on LV networks is unclear, but where there is some evidence available in the literature
- 3. Assumptions for which there is little information available

The impact is rated between 1 and 4, with a high rating indicating greater impact to network performance metrics such as voltage drop/rise, unbalance, distortion or losses.

- 1. Assumptions for which there is no evidence of impact
- 2. Assumptions with possible impact in some scenarios
- 3. Assumptions that either affect neutral currents continuously, or phase currents occasionally
- 4. Assumptions that affect phase currents or voltages continuously

These are combined to show a risk score between 1 and 12.

Assumption	Uncertainty	Impact	Risk
Customer demand modelled using national mean profiles, neglecting	1	4	4
regional variations, and differences between individual customers.			
(Sections 2.1.1, 2.1.2)			
Individual customer demands synthesised from statistical distributions,	2	4	8
potentially under-estimating the phase unbalance. (Section 2.1.1)	2		
All PV installations on the LV network are subject to the same	2	2	4
irradiance. (Section 2.1.3).	2	2	4
Commercial demand is not modelled. (Section 2.1.4.)	2	4	8
Demand on each phase is modelled as being balanced.	1	4	4
(Sections 2.1.2, 2.1.4 and 2.1.5.)			
Mean demand on each phase modelled as being balanced although	2	3	6
unbalance exists due to stochastic demand variations. (Section 2.1.5.)	2		
Loads and generation represented by arithmetic mean averaged		4	8
demand samples. Short-term voltage deviations and losses will be	2		
under-estimated. (Section 2.1.5.)			
Loads modelled as constant power vs. voltage. Risks inaccuracies in	2	4	8
node voltages, and losses. (Section 2.1.6.)	2		
Generators modelled with a constant power output vs. voltage	2	1	2
variation. (Section 2.1.6.)	2		
Network approximated as operating at the fundamental frequency with	2	4	8
no harmonics. (Section 2.1.7.)	2		
A constant power factor is assumed for loads, for example 0.9.	2	3	6
(Section 2.1.8.)			
A constant power factor is assumed for generators, typically unity. This	1	1	1
may be defined by grid connection regulations. (Section 2.1.8.)			
Non-metered demand such as due to street lighting is neglected. Risks	1	4	4
under-estimating voltage drop and losses. (Section 2.1.9.)			
The distribution transformer (or primary substation, if the MV feeder is	3	2	6
included) is modelled as a constant voltage source, neglecting impacts			
on the load current. (Section 2.2.5.)			
Cable types, routes and connectivity assumed to be as described in	3	4	12
the network database. (Sections 2.2.1, 2.2.2.)			
Phase allocations for customers with single-phase supplies assumed	2	3	6
to be correctly known. (Section 2.2.2.)			
Service cables are omitted from the model. This impact of this could	2	4	8
be significant for models of rural networks. (Section 2.2.2.)			

#### Table 2-1 – Network modelling assumptions and approximations

Neutral conductors, concentric neutral or sheath assumed to be	2	3	6
grounded at each node. (Section 2.2.4.)	2	5	U
Neutral and earth connections at link boxes are ignored	3	3	9
(Section 2.2.3.)			
Neutral to ground impedance assumed to be zero. (Section 2.3.3.)	2	3	6
Shunt admittance is neglected (Section 2.3.1.)	2	2	4
The modified Carson's equations used for cable and overhead line	2	3	6
impedances rather than full equations. (Section 2.3.2.)			
Ground resistivity is assumed to be constant, e.g. 100 $\Omega$ m.	2	3	6
(Section 2.3.5)			
End effects are neglected in calculating cable impedances for LV	3	3	9
cables. (Sections 2.3.3, 2.3.5.)			
Carson's equations are partitioned to calculate separate conductor and	2	3	6
earth voltages. (Section 2.3.2.)			
Conductor impedances approximated to a 3x3 form using the Kron	2	3	6
reduction. (Section 2.2.4.)			
Impedances defined only by positive and zero sequence impedance	1	3	3
values. (Section 2.3.7.)			
Zero sequence impedance can be estimated to be a multiple of the	2	3	6
positive sequence impedance (Section 2.3.7.)			
Conductors represented by phase and neutral conductor impedances	1	3	3
(Section 2.3.7.)	÷	,	Ĵ

Table 2-1 highlights several assumptions that are of particular concern:

- <u>Cable types, routes and connectivity are assumed to be as described in the</u> <u>network database</u>. Addressing this concern would need resources to undertake cable tracing, or review of records held by network operators.
- <u>Neutral and earth connections at link boxes are ignored</u>. This could be investigated by measurement of neutral currents at link boxes or by simulation.
- End effects are neglected in calculating cable impedances for LV cables. This
  could be investigated by measurement of currents on a network with PME
  electrodes. Simulation methods could also be developed to include the
  ground electrodes in the network model.

Based on the weighting values assigned in Table 2-1, the accuracy of the network database is the greatest risk (this was also found to be a concern for the

measurements undertaken for this project, as described in Chapter 5). This risk is more appropriately investigated by the network operators and so was considered outside the scope of this PhD project.

The second and third highest risk assumptions are both related to the accuracy of the equivalent circuit model of the neutral and ground conductors and the path taken by the currents that result from unbalanced loads. The impact of these assumptions therefore depends on the likelihood that currents in the neutral and ground will occur. Many of the assumptions listed in Table 2-1 that relate to the mechanisms that cause neutral and ground currents to occur are also categorised as being high risk. A number of these aspects relating to the currents in the neutral and ground are inter-dependent, and the assumptions made in modelling them therefore need to be applied in combination. Summarising from Table 2-1, the key aspects are:

<u>Time resolution</u>: currents in the neutral and ground are caused by unbalance in the phase conductors. By averaging the demand data over longer periods, the currents will appear less variable and so short term unbalance conditions will appear to be smoothed out. The losses in the phase conductors and the unbalance current will be under-represented.

<u>Harmonics</u>: balanced currents that are at harmonic frequencies may either combine or cancel in the neutral, depending on their harmonic number.

<u>Neutral to ground connectivity</u>: many simulation studies assume connections between the neutral and ground that do not necessarily exist in practice.

<u>Cable impedances</u>: the assumptions required to model cable impedances are less secure at harmonic frequencies, and where the impact of connections between the neutral and the ground is considered. Although the use of Carson's equations to model the line impedance is well established, the assumptions required in applying this to LV feeders with short branch lengths need further investigation.

These risks affect estimates of the hosting capacity for low carbon technologies (LCTs) on LV networks but apply equally to network planning and design for conventional demand following practices developed over many years. These design methods typically include safety margins based on practical experience such that uncertainties in the calculations are not generally a concern. There is also little

awareness amongst residential customers of the power quality at their connection point.

With the introduction of LCTs, the risks identified here are likely to become more critical. The practices used in network planning are based on conventional demands and so will not necessarily have the correct safety margins for the different characteristics of embedded generation or new types of loads. There is a desire to operate networks in conditions that are closer to the rated limits, so as to avoid or defer costly upgrades, and also a greater emphasis on minimising losses. The risk factors in the network modelling are therefore particularly relevant to DNO planning regarding LCTs. In order for DNOs to make 'before' and 'after' assessments of the impact of introducing LCTs, accurate results are needed for both cases.

More specifically, the true voltage ranges with conventional loads may be much wide than shown in calculations with low time resolution demand data, and losses may therefore be greater. Similarly, the actual voltage ranges may differ from those calculated with approximated cable impedances. These concerns therefore need to be addressed so that an assessment of the change in power quality due to the introduction of LCTs can be obtained.

It is also important to note that the risks identified here act in combination with the assumptions in predicting the demand, and that this is likely to have much greater uncertainty than the uncertainty in the network model. While the uncertainty in predicting the demand is not easily addressed, at least by engineering analysis, it is possible to address the uncertainty in the network models, as in this thesis.

# 2.5 Conclusions

This chapter has identified that many assumptions and approximations are commonly used in simulation models of LV networks. Although some of these have been investigated previously there are uncertainties over many others, particularly where these are applied in combination.

Using a risk assessment approach, particular areas of concern have been identified. The accuracy of the network database is likely to be a critical factor in achieving accurate modelling results. Clearly, if models are created with an incorrect topology, cable lengths or with insufficient detail to represent the service cable connections then simulations cannot represent the real network. However, further details such as the locations and resistances of ground connections at customer premises may also have an impact. These grounding connections are not under the control of the network operator and so unlikely to be included in the records of the installed circuits.

A number of the assumptions identified with high risk relate to the currents in the neutral and ground. Four key areas have been identified that will be investigated here to determine how the assumptions made in modelling and in measurements would affect the outcome of network models. These are:

- The time resolution of the demand data
- The impact of harmonics, and of neglecting them in the model
- The circuit model of the neutral and ground conductors and the connections between them
- The models used to determine the impedances of the cables, and the assumptions made to include the neutral and the conductive path through the ground.

Since the impact of many of these assumptions is unclear, there is a risk that network planning and design models used by DNOs are not providing accurate results. This is increasingly a concern as DNOs plan for new low carbon technologies on their networks, with feeders operated with load and generation currents that differ from the demand characteristics for which the networks were planned. Although there is significant uncertainty relating to the future demand and generation, it is possible to address some of the risks relating to the network model.

# **3 CABLE IMPEDANCE THEORY**

# 3.1 Introduction

The following section discusses the basic theory relating to series impedance matrices and the assumptions involved in each stage of the development. The theory itself is already covered in various standard texts but the development of the impedance equations depends on several assumptions and approximations that are not always made clear (Kersting 2012; Glover et al. 2008). The following section therefore aims to highlight these and also provides examples of cases where these assumptions may affect modelling results.

These impedance definitions are also discussed in the paper presented as Appendix A (Urquhart & Thomson 2015b), but a fuller description is provided here.

The discussion here concentrates on the series impedance but the shunt admittance is also considered in Section 4.3 for Waveform cables.

# 3.2 Conductor impedances

The cable can be modelled as a set of conductors with associated self- and mutual impedances, as in Figure 3-1. This shows three phase conductors and the neutral and also allows for current flowing through the ground. The voltages are specified relative to a reference potential which could, for example, represent the neutral terminal of the transformer at the substation. The branch shown in Figure 3-1 does not necessarily connect to the transformer and so  $V_g$  and  $V'_g$  shown here are local ground potentials with respect to the reference potential.

The voltage drop along the cable is given by:

$$\begin{bmatrix} \Delta V_{a} \\ \Delta V_{b} \\ \Delta V_{c} \\ \Delta V_{n} \\ \Delta V_{g} \end{bmatrix} = \begin{bmatrix} V_{a} \\ V_{b} \\ V_{c} \\ V_{n} \\ V_{g}' \end{bmatrix} - \begin{bmatrix} V_{a}' \\ V_{b}' \\ V_{c}' \\ V_{n}' \\ V_{g}' \end{bmatrix} = \begin{bmatrix} \bar{z}_{aa} & \bar{z}_{ab} & \bar{z}_{ac} & \bar{z}_{an} & \bar{z}_{ag} \\ \bar{z}_{ba} & \bar{z}_{bb} & \bar{z}_{bc} & \bar{z}_{bn} & \bar{z}_{bg} \\ \bar{z}_{ca} & \bar{z}_{cb} & \bar{z}_{cc} & \bar{z}_{cn} & \bar{z}_{cg} \\ \bar{z}_{na} & \bar{z}_{nb} & \bar{z}_{nc} & \bar{z}_{nn} & \bar{z}_{ng} \\ \bar{z}_{ga} & \bar{z}_{gb} & \bar{z}_{gc} & \bar{z}_{gn} & \bar{z}_{gg} \end{bmatrix} \begin{bmatrix} I_{a} \\ I_{b} \\ I_{c} \\ I_{n} \\ I_{g} \end{bmatrix}$$
(1)

where  $\bar{z}_{ij}$  is the conductor impedance between conductors *i* and *j*.





If only the voltage difference is considered, then the circuit can be re-drawn with a short-circuit at the second terminal, as shown in Figure 3-2, and following Kersting's method (Kersting 2012).





The conductor impedances then need to be defined. In defining these impedances, it is common practice to assume that eddy currents can be neglected. This is assumed for the discussion here but the impact of eddy currents will be revisited in Section 3.7.

Based on the assumption that eddy currents can be neglected and with uniform material properties across the conductor, the current density will then also be

uniform. The resistances can then be calculated based on their cross-sectional area and resistivity.

The self-inductance values are more complicated and include components due to the flux linkage within the conductor, and also the flux linkage due to the magnetic field that is external to the conductor.

In Glover's analysis, the conductors are modelled as being infinitely long, such that flux linkage from points within a cross-section through the conductor depends only on the magnetic field within that plane (Glover et al. 2008). The flux linkage between points in the plane due to currents further along the cable is then exactly cancelled by flux linkage from currents at the same longitudinal distance in the other direction along the cable. Consequently, only the flux linkage within the plane needs to be considered. If the conductor is circular, and again assuming a uniform current density with no eddy currents, then the inductance due to internal flux linkage is a  $5 \times 10^{-8}$  H regardless of the conductor radius.

When calculating the external flux linkage, the magnetic field is considered from the radius of the conductor out to the radial distance to a point P. The total flux linkage for each conductor includes the internal flux linkage plus contributions to the magnetic field due to current in each of the conductors, so that the total flux linkage with conductor *i* as far as distance *P* is given by:

$$\lambda_{iP} = \frac{\mu_0}{2\pi} \sum_{j=1}^{N_{\text{cond}}} I_j \ln \frac{D_{Pj}}{D_{ij}}$$
(2)

where  $N_{\text{cond}}$  is the number of conductors,  $D_{ij}$  is the geometric mean distance (GMD) between conductors *i* and *j*, and  $D_{Pj}$  is the distance from conductor *j* to the point *P*.

In the case that i = j, distance  $D_{ii}$  is then the geometric mean radius of conductor i, given by  $D_{ii} = e^{-1/4}R$  for circular conductors with uniform current density, where R is the physical radius.

Ideally, for the calculation of the self- and mutual inductance terms of  $\bar{z}_{ij}$ , we would obtain an expression that does not depend on the point *P*. However, as *P* tends to infinity, the integration of the magnetic field, varying in inverse proportion to the radius from the conductor, does not provide a proper integral. The term  $\ln(D_{Pj}/D_{ij})$ 

does not converge to a finite value for  $P = \infty$ , and so the magnetic field is effectively truncated in (2) at radius *P* from the conductor. Fortunately this can be resolved, as shown below, if an additional constraint is applied that the sum of currents in all of the conductors equals zero.

And erson presented a second approach to this problem in which the inductance in  $\Omega/m$  of a conductor is given directly as:

$$L_{ij} = \frac{\mu_0}{2\pi} \left( \ln \frac{2s}{D_{ij}} - 1 \right) \tag{3}$$

where *s* is the length of the conductor (Anderson 1995). This equation has been given previously as the inductance of a straight, but finite, cylindrical conductor (Attwood 1932). The total flux linkage is calculated by integrating the flux linkage for a small segment of the conductor, allowing for the magnetic field due to currents elsewhere along the conductor. This yields a proper integral, provided that the integration includes flux linkage contributions from a finite length of conductor, even allowing for the integration of the magnetic field to an infinite distance radius.

This analysis was presented earlier by Rosa, together with a more detailed discussion on the limitations of the method (Rosa 1908). Rosa had applied the Biot-Savart law but noted that this is

"not experimentally verified for unclosed circuits; but the self-inductance of an unclosed circuit simply means its self-inductance as part of a closed circuit".

It has also been observed that the conceptual model of the finite conductor length takes no account of the route taken by the current to enter and to leave the conductor (Healy 2015). No magnetic field contributions are included for these currents when the total flux linkage is calculated. In effect, this method truncates the magnetic field at the end of the finite length of the conductor.

Both of these methods therefore involve some requirement to truncate the magnetic field. Anderson's method is widely referenced (Ciric et al. 2003) as it appears to provide a finite expression for the inductance of a single conductor. However, Anderson only uses these expressions in the context of a closed circuit.

There is also a practical difficulty with (3) when the finite length of a segment of cable is selected. The inductance per unit length of the conductor then depends on the length selected, giving different results if the conductor is considered as multiple short sections, to the results for one long section.

## 3.3 Circuit impedances

It is generally assumed that the conductors belong to circuits where the sum of currents in the cable and the ground is zero, such that:

$$\sum_{j=1}^{N_{\text{cond}}} I_j = 0 \tag{4}$$

This is a valid assumption provided that the feeder branches are completely radial although, as noted in Section 2.2.3, this may not always be the case. Even where the installed conductors are completely radial, the current in the ground may take a different route to that in the cable. However, the analysis proceeds here on that basis that the impedances equations are applied to a feeder where the sum of currents does equal zero.

Re-arranging (1) to give the voltage difference relative to the local ground at the second terminal, the voltage differences can then be described as:

$$\begin{bmatrix} \Delta V_{ag} \\ \Delta V_{bg} \\ \Delta V_{cg} \\ \Delta V_{ng} \\ 0 \end{bmatrix} = \begin{bmatrix} \Delta V_a - \Delta V_g \\ \Delta V_b - \Delta V_g \\ \Delta V_c - \Delta V_g \\ \Delta V_n - \Delta V_g \\ \Delta V_g - \Delta V_g \end{bmatrix} = \begin{bmatrix} \bar{z}_{aa} & \bar{z}_{ab} & \bar{z}_{ac} & \bar{z}_{an} & \bar{z}_{ag} \\ \bar{z}_{ba} & \bar{z}_{bb} & \bar{z}_{bc} & \bar{z}_{bn} & \bar{z}_{bg} \\ \bar{z}_{ca} & \bar{z}_{cb} & \bar{z}_{cc} & \bar{z}_{cn} & \bar{z}_{cg} \\ \bar{z}_{na} & \bar{z}_{nb} & \bar{z}_{nc} & \bar{z}_{nn} & \bar{z}_{ng} \\ \bar{z}_{ga} & \bar{z}_{gb} & \bar{z}_{gc} & \bar{z}_{gn} & \bar{z}_{gg} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \\ I_g \end{bmatrix}$$
(5)

This can be simplified to:

$$\begin{bmatrix} \Delta V_{ag} \\ \Delta V_{bg} \\ \Delta V_{cg} \\ \Delta V_{ng} \end{bmatrix} = \begin{bmatrix} \bar{z}_{aa} - \bar{z}_{ga} & \bar{z}_{ab} - \bar{z}_{gb} & \bar{z}_{ac} - \bar{z}_{gc} & \bar{z}_{an} - \bar{z}_{gn} & \bar{z}_{ag} - \bar{z}_{gg} \\ \bar{z}_{ba} - \bar{z}_{ga} & \bar{z}_{bb} - \bar{z}_{gb} & \bar{z}_{bc} - \bar{z}_{gc} & \bar{z}_{bn} - \bar{z}_{gn} & \bar{z}_{bg} - \bar{z}_{gg} \\ \bar{z}_{ca} - \bar{z}_{ga} & \bar{z}_{cb} - \bar{z}_{gb} & \bar{z}_{cc} - \bar{z}_{gc} & \bar{z}_{cn} - \bar{z}_{gn} & \bar{z}_{cg} - \bar{z}_{gg} \\ \bar{z}_{na} - \bar{z}_{ga} & \bar{z}_{nb} - \bar{z}_{gb} & \bar{z}_{nc} - \bar{z}_{gc} & \bar{z}_{nn} - \bar{z}_{gn} & \bar{z}_{ng} - \bar{z}_{gg} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \\ I_g \end{bmatrix}$$
(6)

Substituting  $I_g = -(I_a + I_b + I_c + I_n)$  then gives:

$$\begin{bmatrix} \Delta V_{ag} \\ \Delta V_{bg} \\ \Delta V_{cg} \\ \Delta V_{ng} \end{bmatrix} = \begin{bmatrix} \bar{z}_{aa} - \bar{z}_{ga} - \bar{z}_{ag} + \bar{z}_{gg} & \bar{z}_{ab} - \bar{z}_{gb} - \bar{z}_{ag} + \bar{z}_{gg} & \bar{z}_{ac} - \bar{z}_{gc} - \bar{z}_{ag} + \bar{z}_{gg} & \bar{z}_{an} - \bar{z}_{gn} - \bar{z}_{ag} + \bar{z}_{gg} \\ \bar{z}_{ba} - \bar{z}_{ga} - \bar{z}_{bg} + \bar{z}_{gg} & \bar{z}_{bb} - \bar{z}_{gb} - \bar{z}_{bg} + \bar{z}_{gg} & \bar{z}_{bc} - \bar{z}_{gc} - \bar{z}_{bg} + \bar{z}_{gg} & \bar{z}_{bn} - \bar{z}_{gn} - \bar{z}_{bg} + \bar{z}_{gg} \\ \bar{z}_{ca} - \bar{z}_{ga} - \bar{z}_{cg} + \bar{z}_{gg} & \bar{z}_{cb} - \bar{z}_{gb} - \bar{z}_{cg} + \bar{z}_{gg} & \bar{z}_{cc} - \bar{z}_{gc} - \bar{z}_{cg} + \bar{z}_{gg} & \bar{z}_{cn} - \bar{z}_{gn} - \bar{z}_{cg} + \bar{z}_{gg} \\ \bar{z}_{na} - \bar{z}_{ga} - \bar{z}_{ng} + \bar{z}_{gg} & \bar{z}_{nb} - \bar{z}_{gb} - \bar{z}_{ng} + \bar{z}_{gg} & \bar{z}_{nc} - \bar{z}_{gc} - \bar{z}_{ng} + \bar{z}_{gg} & \bar{z}_{nn} - \bar{z}_{ng} - \bar{z}_{ng} + \bar{z}_{gg} \end{bmatrix} \times \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix}$$
(7)

The line to ground voltage difference can then be defined as:

where  $\hat{z}_{ij}$  gives the circuit impedance terms, defined as

$$\hat{z}_{ii} = \bar{z}_{ii} - \bar{z}_{gi} - \bar{z}_{ig} + \bar{z}_{gg}$$
 (9)

$$\hat{z}_{ij} = \bar{z}_{ij} - \bar{z}_{gj} - \bar{z}_{ig} + \bar{z}_{gg}$$
 (10)

The equivalent circuit is now as shown in Figure 3-3.





The circuit impedances can now also be specified in terms of their resistance and reactance. Following Glover's method, as the distance to point *P* in (2) tends to infinity, the total magnetic field then tends to zero since  $\ln(D_{Pj}/D_{Pi}) \rightarrow 0$ . The terms in (2) relating to distance  $D_{Pj}$  then cancel and the total flux linkage is

$$\lambda_{iP} = \frac{\mu_0}{2\pi} \sum_{j=1}^{N_{\text{cond}}} I_j \ln\left(\frac{1}{D_{ij}}\right) \tag{11}$$

Taking the self- or mutual inductance of a conductor as the component of (11) that is proportional to the corresponding current, Kersting then partitions the total flux linkage so as to give inductances for the individual conductors (Kersting 2012), such that:

$$L_{ij} = \frac{\mu_0}{2\pi} \ln\left(\frac{1}{D_{ij}}\right) \tag{12}$$

Forming this into the overall impedance then gives conductor impedances as:

$$\bar{z}_{ii} = r_i + j\omega \frac{\mu_0}{2\pi} \cdot \ln\left(\frac{1}{D_{ii}}\right)$$
(13)

$$\bar{z}_{ij} = j\omega \frac{\mu_0}{2\pi} \cdot \ln\left(\frac{1}{D_{ij}}\right)$$
(14)

where  $r_i$  is the resistance of conductor *i* in  $\Omega/m$ , and  $\omega$  is the angular frequency.

In decomposing (11) to give the individual inductances in (12), any value could have been used in place of unity in the terms  $(1/D_{ij})$  since these factors will all cancel out when the circuit impedances of (9) and (10) are formed. Although (12) is a useful step in forming (13) and (14), such that the circuit impedances can be calculated, the inductances in (12) are not a unique result following from (11).

Equations (13) and (14) are presented by Kersting as properties of individual conductors, in advance of the assumption that the currents sum to zero. However, it appears that they are more correctly contributions from each conductor to the total impedance of a circuit.

With the constraint that the sum of currents equals zero, the circuit impedances are then given by:

$$\hat{z}_{ii} = r_i + r_g + j\omega \frac{\mu_0}{2\pi} \ln\left(\frac{D_{gi}D_{ig}}{D_{ii}D_{gg}}\right)$$
(15)

$$\hat{z}_{ij} = r_g + j\omega \frac{\mu_0}{2\pi} \ln\left(\frac{D_{gj}D_{ig}}{D_{ij}D_{gg}}\right)$$
(16)

Section 3.2 also discussed Anderson's alternative approach to defining conductor impedances. If the inductances from (3) are used to form circuit impedances with (9) and (10), then the term s relating to the finite conductor length also cancels out, giving the same result as in (15) and (16).

Although the conductor impedances of (13) and (14) involve the use of an arbitrary constant, it is nonetheless possible to calculate the voltage differences  $\Delta V_i$  according to (1), provided that the currents sum to zero. Voltage  $\Delta V_i$  can be calculated by resistance and the total flux linkage from (2) to give:

$$\Delta V_i = r_i I_i + j\omega \lambda_{iP} = r_i I_i + j\omega \frac{\mu_0}{2\pi} \sum_{j=1}^{N_{\text{cond}}} I_j \ln \frac{D_{Pj}}{D_{ij}}$$
(17)

Provided that the conductor is in a circuit with the sum of currents equal to zero, it is possible substitute (4) and also using  $\ln(D_{Pj}/D_{Pi}) \rightarrow 0$ , the voltage difference along the conductor is given by:

$$\Delta V_i = r_i I_i + j\omega \frac{\mu_0}{2\pi} \sum_{j=1, j \neq i}^{N_{\text{cond}}} I_j \ln \frac{D_{ii}}{D_{ij}}$$
(18)

The same result could have been obtained from (1) using (13) and (14). In doing so, any constant could be substituted in place of the unit value in the terms  $(1/D_{ij})$  in (13) and (14). Similarly, voltage  $\Delta V_i$  could be calculated using Anderson's equation (3), such that:

$$\Delta V_i = r_i I_i + j\omega \sum_{j=1}^{N_{\text{cond}}} I_j L_{ij}$$
<sup>(19)</sup>

As with the circuit impedance calculation, the term *s* cancels out.

This analysis confirms that the problem observed in defining the conductor impedance, such that the flux linkage includes the magnetic field as far as some form of boundary, is overcome when the impedance is calculated for a closed circuit and where the sum of currents in all paths is equal to zero.

Kersting has then used the circuit impedance, which include terms relating to each of the conductors, to propose individual impedances for each conductor. However, it must be remembered that these are contributions to the total impedance and not inherent properties of the conductors individually.

# 3.4 Carson's equations

Equations (15) and (16) cannot directly be implemented where one of the conductors is the ground, since the GMR of the ground conductor  $D_{gg}$  is unknown, as is the GMD between conductors in the cable and ground  $D_{ig}$ . This is resolved through the use of Carson's equations (Carson 1926), in which the ground currents are modelled as images of the conductors in the cable, and the impedance solved by considering the fields between the cable conductor and the images. The conceptual model was developed for overhead lines, as shown in Figure 3-4. This model assumes that the

conductors are small relative to their spacing, and that there is a uniform current distribution within the conductors of the cable. The current distribution in the ground is not directly specified, but it is assumed that the ground is infinite in radius and depth, such that the currents are not constrained by a limited cross-sectional area. Within this cross-sectional area, the current density is determined by considering the electric and magnetic fields between the ground and the overhead line conductors.

In Figure 3-4, distance  $D_{ij}$  is the GMD between conductors (equal to the centre-tocentre spacing if this is large relative to the conductor radius) and  $S_{ij}$  is the GMD between conductors and their images in the ground. The overhead line conductors are at a height  $h_i$  above the ground.



Figure 3-4 – Carson's ground conductor image model

Kersting defines a modified form of Carson's equations with a reduced number of terms for simplification (Kersting 2012). These use Imperial units of length, as follows:

$$\hat{z}_{ii,\Omega/\text{mile}} = r_{i,\Omega/\text{mile}} + 4\omega P_{ii}G + j\left(X_i + 2\omega G \cdot \ln\frac{S_{ii,\text{ft}}}{R_{i,\text{ft}}} + 4\omega Q_{ii}G\right)$$
(20)

$$\hat{z}_{ij,\Omega/\text{mile}} = 4\omega P_{ij}G + j\left(2\omega G \cdot \ln\frac{S_{ij,\text{ft}}}{D_{ij,\text{ft}}} + 4\omega Q_{ij}G\right)$$
(21)

where:

$$X_i = 2\omega G \cdot \ln \frac{R_{i,\text{ft}}}{D_{ii,\text{ft}}}$$
(22)

$$P_{ij} = \frac{\pi}{8} \tag{23}$$

$$Q_{ij} = -0.0386 + \frac{1}{2} \cdot \ln \frac{2}{k_{ij}}$$
(24)

$$k_{ij} = 8.565 \times 10^{-4} \cdot S_{ij,\text{ft}} \sqrt{f/\rho}$$
(25)

and  $R_i$  is the radius of the conductor,  $G = 0.1609347 \times 10^{-3} \Omega$ /mile, and  $\rho$  is the ground resistivity in  $\Omega$ m.

The equations can be re-written in SI units giving impedances in  $\Omega/m$ , substituting  $S_{ij,\text{ft}} = S_{ij} \times 1000/(25.4 \times 12)$  to give:

$$\hat{z}_{ii} = r_i + \frac{\mu_0 \omega}{8} + j\omega \frac{\mu_0}{2\pi} \left( \ln \frac{S_{ii}}{D_{ii}} - 0.0386 \times 2 \right)$$

$$+ \ln \frac{2}{8.565 \times 10^{-4} \cdot S_{ii} \times 1000/(25.4 \times 12) \sqrt{f/\rho}}$$

$$\hat{z}_{ij} = \frac{\mu_0 \omega}{8} + j\omega \frac{\mu_0}{2\pi} \left( \ln \frac{S_{ij}}{D_{ij}} - 0.0386 \times 2 \right)$$

$$+ \ln \frac{2}{8.565 \times 10^{-4} \cdot S_{ij} \times 1000/(25.4 \times 12) \sqrt{f/\rho}}$$

$$(26)$$

$$(27)$$

In (26), terms relating to the conductor radius  $R_i$  cancel out.

It is also possible to cancel  $S_{ij}$ , but these terms are retained here for use in the discussion below.

However, simplifying the equations where possible, the modified Carson's equations in SI units are:

$$\hat{z}_{ii} = r_i + \frac{\mu_0 \omega}{8} + j \omega \frac{\mu_0}{2\pi} \cdot \ln\left(\frac{658.9}{D_{ii}\sqrt{f/\rho}}\right)$$
 (28)

$$\hat{z}_{ij} = \frac{\mu_0 \omega}{8} + j\omega \frac{\mu_0}{2\pi} \cdot \ln\left(\frac{658.9}{D_{ij}\sqrt{f/\rho}}\right)$$
(29)

Conveniently, although these equations are derived from a concept where the conductors are overhead lines and have images in the ground, the modified forms do not depend on the distances to these images  $S_{ij}$  or the angles between them  $\Theta_{ij}$ .

The equations are frequently used for underground cables (where the cable 'height above ground' is actually a distance into the ground), and where the distance between conductors is of a similar magnitude to the conductor size with minimal error relative to the full Carson's equations that include the terms relating to the geometry (Kersting & Green 2011).

Although Carson's equations provide the circuit impedance, this total impedance has been considered in the literature to be the sum of individual conductor impedances.

For example, Anderson equated the modified Carson's equations from (28) and (29) with the circuit impedance equations (15) and (16) in order to provide information about the effective dimensions of the ground conductor (Anderson 1995). This assumes that:

$$\hat{z}_{ii} = r_i + \frac{\mu_0 \omega}{8} + j\omega \frac{\mu_0}{2\pi} \cdot \ln\left(\frac{658.9}{D_{ii}\sqrt{f/\rho}}\right) = r_i + r_g + j\omega \frac{\mu_0}{2\pi} \ln\left(\frac{D_{gi}D_{ig}}{D_{ii}D_{gg}}\right)$$
(30)

$$\hat{z}_{ij} = \frac{\mu_0 \omega}{8} + j\omega \frac{\mu_0}{2\pi} \cdot \ln\left(\frac{658.9}{D_{ij}\sqrt{f/\rho}}\right) = r_{\rm g} + j\omega \frac{\mu_0}{2\pi} \ln\left(\frac{D_{\rm gj} D_{ig}}{D_{ij} D_{\rm gg}}\right) \tag{31}$$

Taking the real parts of (30) gives the resistance of the ground conductor  $r_{\rm g}$  as:

$$r_{\rm g} = \frac{\mu_0 \omega}{8} \tag{32}$$

As noted above, resistance  $r_g$  should more correctly be described as a contribution to the impedance of a circuit from the ground conductor, rather than an inherent property of the ground conductor itself. From (32), the resistance is proportional to frequency. However, since the resistivity of the ground is constant, an increasing resistance indicates that the current is spread over a lower cross-sectional area. For a circuit with currents flowing outward in the cable and returning via the ground, this concentration of the current is consistent with the proximity effect whereby the current density in a conductor increases with the proximity to a second conductor in which the current flows in the opposite direction. Conversely, if the ground conductor was considered completely in isolation, then the current would disperse over the widest cross-sectional area available regardless of the AC frequency. If the end effects that allow for current entering and leaving the ground are neglected then the ground conductor resistance would be close to zero. (In practice, these end effects are significant, as described in Chapter 9, but all of the analysis under discussion here assumes an infinitely long cable.) Resistance  $r_g$  should therefore not be considered as an independent property of the ground conductor in the same manner that resistance  $r_i$  represents the conductor in the cable.

Taking the imaginary parts of (30) with  $D_{gi} = D_{ig}$  gives:

$$D_{ig} = \sqrt{\frac{658.9}{\sqrt{f/\rho}}} D_{gg} \tag{33}$$

Similarly, (31) gives:

$$D_{gj} = \frac{658.9}{\sqrt{f/\rho}} \frac{D_{gg}}{D_{ig}} = \sqrt{\frac{658.9}{\sqrt{f/\rho}}} D_{gg} = D_{ig}$$
(34)

Equation (33) defines the GMD between the ground and any of the cable conductors in terms of the GMR of the ground conductor  $D_{gg}$ . However, since GMR  $D_{gg}$  is unknown, an arbitrary unit length is selected, such that  $D_{gg} = 1$  (Anderson 1995). This is then combined with the inductance calculation from (3) to give selfimpedances and mutual impedances for the ground conductor as:

$$\bar{z}_{gg} = r_g + j\omega \frac{\mu_0}{2\pi} \left( \ln \frac{2s}{D_{gg}} - 1 \right)$$
(35)

$$\bar{z}_{ig} = j\omega \frac{\mu_0}{2\pi} \left( \ln \frac{2s}{D_{ig}} - 1 \right)$$
(36)

When (35) and (36) are combined into circuit impedances and the sum of currents is zero, the arbitrary value assigned to  $D_{gg}$  cancels out. However, a problem arises if (35) and (36) are used to calculate the voltage drop along the individual conductors. i.e. the voltages  $\Delta V_i$  and  $\Delta V_g$  rather than the differential line to ground voltage  $\Delta V_{ig}$ . Substituting (33) into (18) then gives  $\Delta V_i$  and  $\Delta V_g$  as:

$$\Delta V_{i} = r_{i}I_{i} + j\omega \frac{\mu_{0}}{2\pi} \sum_{j=1, j\neq i, g}^{N_{\text{cond}}} I_{j} \ln \frac{D_{ii}}{D_{ij}} + j\omega \frac{\mu_{0}}{2\pi} I_{g} \ln D_{ii} \sqrt{\frac{(1/D_{\text{gg}})\sqrt{f/\rho}}{658.9}}$$
(37)

$$\Delta V_{\rm g} = r_{\rm g} I_{\rm g} + j\omega \frac{\mu_0}{2\pi} \sum_{j=1, j\neq \rm g}^{N_{\rm cond}} I_j \ln \sqrt{\frac{D_{\rm gg}\sqrt{f/\rho}}{658.9}}$$
(38)

There is therefore a term  $\sqrt{D_{gg}}$  which contribute to increasing  $\Delta V_g$  and to decreasing  $\Delta V_i$ . Depending on the value selected for  $D_{gg}$  then more or less of the difference in line to ground voltage  $\Delta V_{ig}$  will appear as part of the line voltage  $\Delta V_i$  or in the ground voltage  $\Delta V_g$ .

The local ground potential can therefore not be determined without this arbitrary definition of  $D_{gg}$ . If  $D_{gg}$  had instead been defined as a unit length of 1 foot rather than 1 metre, then the values of  $\Delta V_i$ . and  $\Delta V_g$  would be different.

A similar issue arises in Ciric's method, developed to allow for grounding resistances between the neutral and ground conductors at each node (Ciric et al. 2003; Ciric et al. 2004; Sunderland & Conlon 2012). The method makes use of conductor impedances based on Carson's equations in order to find the local ground voltages with respect to the ground at the substation. In order to allow for the ground conductor impedance, Carson's equations are partitioned such that some terms within (28) and (29) are considered to be related to the self-impedance of the ground, and others relate to the mutual impedance between the ground and other conductors.

The self-impedance equation is developed from (26), substituting  $S_{ii} = 2h_i$ . For a conductor *a*, this gives:

$$\hat{z}_{aa} = r_a + \frac{\mu_0 \omega}{8}$$

$$+ j\omega \frac{\mu_0}{2\pi} \left( \ln \frac{2h_a}{D_{aa}} - 0.0386 \times 2 + \ln \frac{2}{5.6198 \times 10^{-3}} + \ln \frac{1}{h_a \sqrt{f/\rho}} \right)$$
(39)

This circuit impedance equation is then partitioned back to conductor impedances, according to (9). The self-impedance for the conductors in the cable includes a reactance contribution from an idealised return path (with no resistance) given by a perfect wire located at the image depth, giving:

$$\bar{z}_{aa} = r_a + j\omega \frac{\mu_0}{2\pi} \ln \frac{2h_a}{D_{aa}}$$
(40)

The remaining terms from the circuit impedance are then:

$$\bar{z}_{gg} - 2\bar{z}_{ag} = \frac{\mu_0 \omega}{8} - j\omega \frac{\mu_0}{2\pi} 0.0386 \times 2 + j\omega \frac{\mu_0}{2\pi} \ln \frac{2}{5.6198 \times 10^{-3}} + j\omega \frac{\mu_0}{2\pi} \ln \frac{1}{h_a \sqrt{f/\rho}}$$
(41)

These are partitioned such that the terms which are only frequency-dependent are considered as part of the self-impedance of the ground conductor, giving:

$$\bar{z}_{gg} = \frac{\mu_0 \omega}{8} - j\omega \frac{\mu_0}{2\pi} 0.0386 \times 2 + j\omega \frac{\mu_0}{2\pi} \ln \frac{2}{5.6198 \times 10^{-3}}$$
(42)

This leaves the mutual impedance between the ground and cable conductors as:

$$\bar{z}_{ag} = j\omega \frac{\mu_0}{4\pi} \ln \frac{h_a}{\sqrt{\rho/f}}$$
(43)

However, this partitioning is subject to an arbitrary arrangement of the numeric constants, and frequency-dependent factors. Equation (41) could have been re-written as:

$$\bar{z}_{gg} - 2\bar{z}_{ag} = \frac{\mu_0 \omega}{8} - j\omega \frac{\mu_0}{2\pi} 0.0386 \times 2 + j\omega \frac{\mu_0}{2\pi} \ln \frac{1}{\sqrt{f}} + j\omega \frac{\mu_0}{2\pi} \ln \frac{2}{5.6198 \times 10^{-3} \times h_a \sqrt{1/\rho}}$$
(44)

This would suggest a different partitioning of frequency-dependent factors to represent the ground conductor impedance. Also, the logarithmic ratios, such as  $\ln h_a / \sqrt{\rho/f}$  in (43) are not dimensionless and so the calculation is dependent on the system of units.

If (40), (42) and (43) are used in the context of a combined circuit impedance, then the assumptions made in partitioning Carson's equations are cancelled out. However, if they are used to predict the individual conductor voltages then the results are dependent on the assumptions used in separating out the terms.

A further difficulty arises in partitioning the mutual circuit impedance from (27) into conductor impedances. Following Circ's method, the distances between conductors

and images are  $D_{ij} = \sqrt{d_{ij}^2 + (h_i - h_j)^2}$  and  $S_{ij} = \sqrt{d_{ij}^2 + (h_i + h_j)^2}$ , where  $d_{ij}$  is the horizontal spacing between the conductors. For conductors *a* and *b*, this gives:
$$\hat{z}_{ab} = \frac{\mu_0 \omega}{8} + j \omega \frac{\mu_0}{2\pi} \left( \ln \frac{\sqrt{d_{ab}^2 + (h_a + h_b)^2}}{\sqrt{d_{ab}^2 + (h_a - h_b)^2}} - 0.0386 \times 2 + \ln \frac{2 \times 2}{5.6198 \times 10^{-3} \cdot \sqrt{d_{ab}^2 + (h_a + h_b)^2} \sqrt{f/\rho}} \right)$$
(45)

This circuit impedance equation is then partitioned back to conductor impedances, according to (10). The mutual conductor impedances between a and b are then:

$$\bar{z}_{ab} = j\omega \frac{\mu_0}{2\pi} \ln \frac{\sqrt{d_{ab}^2 + (h_a + h_b)^2}}{\sqrt{d_{ab}^2 + (h_a - h_b)^2}}$$
(46)

Using the same definition for  $\bar{z}_{gg}$  as in (42), then gives:

$$\bar{z}_{gb} + \bar{z}_{ag} = j\omega \frac{\mu_0}{4\pi} \ln \frac{\left(d_{ab}^2 + (h_a + h_b)^2\right)}{4(\rho/f)}$$
(47)

However, we already have  $\bar{z}_{ag}$  from (43), and  $\bar{z}_{bg}$  could be defined in the same manner. Substituting these into (47) gives:

$$4h_a h_b = d_{ab}^2 + (h_a + h_b)^2 \tag{48}$$

Equation (48) is true if  $d_{ab} = 0$  and  $h_a = h_b$  but this would effectively mean that the two conductors are co-located. There are no real-valued solutions to (48) for  $d_{ab} > 0$  so this would imply that the partitioned equations are inconsistent with the original assumptions about the geometry.

Both the Ciric and Anderson methods of partitioning Carson's equations into conductor impedances are therefore shown to rely on arbitrary assumptions. Models that use these partitioned equations are therefore presenting one possible solution that is consistent with Carson's equations, but not a unique solution. Four-wire simulation studies in which the local ground potential is calculated using this approach are therefore unreliable. However, there is no concern where Carson's equations are applied as circuit impedances (as in the following chapters of this thesis) and the voltages are expressed relative to the local ground potential.

## 3.5 Phase impedances

If the cable is not connected to ground then the ground path might be excluded from the model. This would be the case in a network with TN-S grounding, where there is a separate earth or sheath conductor included in the cable bundle (Cronshaw 2005). Unless there is a fault, and neglecting eddy currents, there is no current flowing in the sheath and so this may be excluded from the impedance matrix. The cable is typically buried and so it may also be assumed that there are no additional impedance contributions due to eddy currents induced in the ground. Making these assumptions, the circuit impedance matrix from (8) reduces to a  $3 \times 3$  form, with impedance terms corresponding to the three phases. This can then be considered as a phase impedance matrix  $z_{abc}$ .

If the neutral conductor is connected to the ground via a grounding electrode or by bonding onto earthed metal, then the ground path is included in the impedance matrix. This would be the case in a TN-C-S network using protective multiple earthing. In a network simulation, the currents in the phase conductors are determined by the line to neutral voltage in the loads, but further assumptions are needed to calculate the neutral and ground currents. A common method of resolving this uses the Kron reduction, where a perfect short-circuit is assumed between the neutral and ground conductors. This is applied to the circuit impedance matrix  $\hat{z}$  to give the 3×3 phase impedance matrix  $z_{abc}$  and the neutral current  $I_n$ , where:

$$\boldsymbol{z}_{abc} = \hat{\boldsymbol{z}}_{ij} - \hat{\boldsymbol{z}}_{in} \hat{\boldsymbol{z}}_{nn}^{-1} \hat{\boldsymbol{z}}_{nj}$$

$$\tag{49}$$

$$I_{\rm n} = -\hat{\boldsymbol{z}}_{\rm nn}^{-1} \hat{\boldsymbol{z}}_{\rm nj} I_{\rm abc} \tag{50}$$

Matrix  $\hat{z}$  is partitioned so that  $\hat{z}_{ij}$  is the 3 × 3 sub-matrix containing the selfimpedances and mutual impedances of the phase conductor circuits,  $\hat{z}_{in}$  is the 3 × 1 sub-matrix of mutual impedances between phase conductor circuits and the neutral (Kersting 2012). Similarly,  $\hat{z}_{nj}$  is the 1 × 3 sub-matrix of mutual impedances between the neutral and the phase circuits, and  $\hat{z}_{nn}$  is the self-impedance of the neutral circuit. The method can also be extended to accommodate multiple neutrals.

Where the phase currents are balanced, clearly there is no current in either the neutral or ground conductors. For unbalanced phase currents, such that the sum of the phase currents is non-zero, the return current in grounded networks is shared

between the neutral and the ground conductors but there are also significant reactive currents, as shown in Table 3-1. This takes the example of 3-core 95 mm<sup>2</sup> Waveform cable (see below in Section 4.2.1) with unbalanced phase currents. The neutral currents can be calculated from (50) and the ground currents from (4). In both of the unbalanced cases, the real part of the neutral and ground currents are seen to share the net return current from the phase conductors, but there is also a significant current circulating in a loop formed by the neutral and ground conductors which are shorted together at both ends of the cable. In the ground, the reactive component is larger than the share of the unbalance current from the phase conductors.

Table 3-1 – Neutral and ground currents for unbalanced phase currents at 50 Hz with 3-core 95 mm<sup>2</sup> Waveform cable

Phase currents	Neutral current	Ground current
$\boldsymbol{I_{abc}} = \begin{bmatrix} 1 \times e^{0} \\ 1 \times e^{-j \times 2\pi/3} \\ 1 \times e^{-j \times 4\pi/3} \end{bmatrix}$	$I_{\rm n}=0$	$I_{\rm g}=0$
$I_{abc} = \begin{bmatrix} 1\\1\\1 \end{bmatrix}$	$I_{\rm n} = -2.4302 - j1.0775$	$I_{\rm g} = -0.5698 + j1.0775$
$I_{abc} = \begin{bmatrix} 1\\0\\0 \end{bmatrix}$	$I_{\rm n} = -0.8101 - j0.3592$	$I_{\rm g} = -0.1899 + j0.3592$

The assumption that the neutral can be perfectly grounded neglects the local resistance of the interface between the electrode and the earth, which is discussed in greater detail in Chapter 9. Although there may be multiple grounding electrodes along the feeder (as with protective multiple earthing), the impedance of these grounding connections is high compared to that of the cable (Sunderland & Conlon 2012).

Even where the network has grounding electrodes at some nodes, there are likely to be junctions without grounding electrodes, such as where two different cable types are joined together, or where a service cable connects on to the main feeder. This creates a problem with the use of the Kron reduction which assumes that the neutral is grounded throughout the network. This is explored in a simple example, as in Figure 3-5, which shows a junction between two cables, each specified by their circuit impedance matrix  $\hat{z}_1$  and  $\hat{z}_2$ . No loads are connected at the junction node.

If there is a neutral to ground connection at the junction of branches 1 and 2, then the Kron reduction can be applied separately to both branches and the total phase impedance of the two branches is given by:

$$z_{abc} = \left(\hat{z}_{ij,1} - \hat{z}_{in,1}\hat{z}_{nn,1}^{-1}\hat{z}_{nj,1}\right) + \left(\hat{z}_{ij,2} - \hat{z}_{in,2}\hat{z}_{nn,2}^{-1}\hat{z}_{nj,2}\right)$$
(51)

However, if the central grounding connection does not exist, then the Kron reduction is applied to the combined circuit impedances, giving:

$$z_{abc} = (\hat{z}_{ij,1} + \hat{z}_{ij,2}) - (\hat{z}_{in,1} + \hat{z}_{in,2})(\hat{z}_{nn,1} + \hat{z}_{nn,2})^{-1}(\hat{z}_{nj,1} + \hat{z}_{nj,2})$$
(52)

If  $\hat{z}_1 = k\hat{z}_2$ , for example if the cables are of the same type but the branches have different lengths, then (51) and (52) are equivalent. However, if the cables have different impedances per unit length, then the Kron reduction will give erroneous results if ground connections are assumed where they are not physically present.



Figure 3-5 – Kron reduction applied to junction of two cable types

## 3.6 Sequence impedances

The development of sequence impedances is included here for completeness, although this is fully covered in the literature (Kersting 2012).

The sequence impedance matrix  $z_{012}$ , is given by:

$$z_{012} = A_s^{-1} \cdot z_{abc} \cdot A_s \tag{53}$$

where

$$\boldsymbol{A}_{s} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_{s}^{2} & a_{s} \\ 1 & a_{s} & a_{s}^{2} \end{bmatrix}$$
(54)

and  $a_s = e^{j2\pi/3}$ . The resulting sequence impedance matrix is of the form:

$$\mathbf{z_{012}} = \begin{bmatrix} z_{00} & z_{01} & z_{02} \\ z_{10} & z_{11} & z_{12} \\ z_{20} & z_{21} & z_{22} \end{bmatrix}$$
(55)

where  $z_{00}$ ,  $z_{11}$ , and  $z_{22}$  are the zero sequence, positive sequence, and negative sequence impedances.

For a cable with rotational symmetry between phases, or if the phases are transposed (such that each phase impedance is the average of all three, and therefore identical), then all of the off-diagonal terms of  $z_{012}$  are zero and  $z_{11} = z_{22}$ , so the impedances are fully represented by the zero and positive sequence impedances.

The transformation of phase impedances to sequence impedances therefore involves no further assumptions about the network, and the reverse transformation could be used in order to specify the cable in terms of the sequence impedance matrix. However, if the phase conductors are not fully symmetrical, specifying the impedance using only the zero and positive sequence terms omits the additional detail that would be provided by the off-diagonal terms and so is an approximation.

An example of an asymmetrical cable is given later in Section 4.4. The impedance matrix, allowing for the asymmetry of the conductors, is then:

$$\hat{\boldsymbol{z}}_{012,\text{full}} = \begin{bmatrix} 0.785 + j0.314 & 0.027 - j0.018 & -0.029 - j0.014 \\ -0.029 - j0.014 & 0.196 + j0.078 & -0.014 + j0.009 \\ 0.027 - j0.018 & 0.015 + j0.007 & 0.196 + j0.078 \end{bmatrix}$$
(56)

If the impedances were to be derived from the corresponding positive and zero sequence impedances from (56), the sequence impedance matrix becomes:

$$\hat{\boldsymbol{z}}_{012,\text{transposed}} = \begin{bmatrix} 0.785 + j0.314 & 0 & 0\\ 0 & 0.196 + j0.078 & 0\\ 0 & 0 & 0.196 + j0.078 \end{bmatrix}$$
(57)

As an example to demonstrate the impact of this approximation, the voltage vector difference is calculated for a balanced current of 50 A over a cable of length 100 m. The voltage difference calculation requires the phase impedance matrix, found using

the inverse of the amplitude of the voltage vector difference using the (53). Using the matrix from (56) gives a voltage difference with amplitude:

$$|\Delta V_{\mathbf{full}}| = \begin{pmatrix} 1.24\\ 1.09\\ 0.85 \end{pmatrix}$$
(58)

Using the approximation from (57) gives a voltage difference with amplitude:

$$\left|\Delta V_{\text{transposed}}\right| = \begin{pmatrix} 1.06\\ 1.06\\ 1.06 \end{pmatrix}$$
(59)

By approximating the impedance matrix in terms of the positive and zero sequence impedances alone, there is therefore a 17% error in the voltage calculations for this example with a balanced current.

Cable manufacturers typically provide impedance data as a 'phase impedance' that is assumed here to be the positive sequence impedance. For a balanced threephase current, and if the impedance matrix is transposed (such that the mutual sequence impedances are zero), this value is also equal to the self-impedance of the phase impedance matrix. In this special case, this gives:

$$\begin{bmatrix} \Delta V_{a} \\ \Delta V_{b} \\ \Delta V_{c} \end{bmatrix} = z_{11} \begin{bmatrix} I_{0} \\ a_{s}^{2} \times I_{1} \\ a_{s} \times I_{2} \end{bmatrix}$$
(60)

More generally, the positive sequence impedance might be taken to be the conductor impedance for the phase conductors. It could also be assumed that the conductor impedance for the neutral is the same as that for the phase conductors. For LV cables, this is often the case for the resistance and, if the neutral conductor has the same geometry as the phase conductors (e.g. for a 4-core sector-shaped cable), then this will also be the case for the reactance. For a cable with no connection to the ground (and all unbalanced load current returning through the neutral), then these conductor self-impedances can be used to form a  $4 \times 4$  conductor impedance matrix. This conductor impedance matrix is not a unique solution, but is a possible solution that is consistent with the specified positive sequence impedance. Since there is no information to specify the unbalance within

the cable, this matrix also represents a transposed cable (not modelling the asymmetry between the four conductors). This forms a conductor impedance matrix:

$$\bar{\boldsymbol{z}} = \begin{bmatrix} z_{11} & 0 & 0 & 0\\ 0 & z_{11} & 0 & 0\\ 0 & 0 & z_{11} & 0\\ 0 & 0 & 0 & z_{11} \end{bmatrix}$$
(61)

Applying (9) and (10) to (61) then gives the circuit impedances, and also the phase impedances since there are only three circuits in this case. The sequence impedance matrix from (53) then gives the result that the zero sequence impedance is exactly 4 times the positive sequence impedance.

This example would be applicable to many 4-core cables with separate neutral and earth conductors and where the neutral and the ground are isolated, as for the cable discussed in Section 4.4. Where the ground is also connected to the neutral, the ground provides a parallel path for the unbalance current and the zero sequence resistance would be expected to be lower as for the cable discussed in Section 4.2.

The positive and zero sequence impedances are used throughout this thesis as a means of summarising the impedance matrix in two complex values. As noted above, this omits the additional detail provided by the off-diagonal terms in the matrix but gives a useful metric that allows impedance matrices to be compared.

# 3.7 Conclusions

This chapter has examined the theory that is widely used to define cable impedances, considering the underlying assumptions in detail. Some of these assumptions are stated in the literature but others are often unstated or unrecognised.

The discussion has drawn a distinction between circuit impedances and conductor impedances, where the latter are contributions to the impedance of a completed circuit that are associated with a particular conductor. While equations for the inductance or reactance of individual conductors are presented in standard texts (Kersting 2012; Anderson 1995), these are shown to require either an arbitrary truncation of the magnetic field that is considered or an arbitrary partitioning of the overall circuit impedance.

Both Kersting and Anderson then apply these equations in the context of a circuit in which the sum of currents equals zero, such that the arbitrary factors are cancelled out, but without clearly stating that the conductor impedance equations cannot be used independently of this assumption.

However, the conductor impedances can be used to calculate the voltage difference along any of the conductors, provided that the dimensions and separations of the individual conductors are known, and if the sum of currents equals zero. The arbitrary factors also cancel in this case.

If the ground conductor is included, the dimensions of the conductive path are unknown and the impedance is typically found using Carson's equations. However, these provide the total circuit impedance, rather than individual conductor impedances. Several approaches have been made to partition Carson's equations into the separate impedances of a conductor in the cable and an equivalent conductor in the ground but these again require arbitrary factors to be selected. When the voltage along an individual conductor is calculated, these arbitrary factors do not cancel out. Although Carson's equations can be used to calculate the difference in line to ground voltage along a cable, the partitioned equations do not allow the difference in either the line voltage ground voltage to be calculated independently.

The circuit impedances are commonly reduced to phase impedances by assuming a multi-grounded neutral, in which the neutral has a zero impedance connection to ground at every node in the network. In practice, the ground electrodes are only installed at a sufficient number of nodes to satisfy the requirements for PME. Calculating the neutral and ground currents using the Kron reduction leads to errors if some of the ground connections do not exist.

Given these various definitions, it can be unclear how to interpret impedance data provided by cable manufacturers as the corresponding circuit configuration is not usually specified. It is assumed here that the resistance and reactance figures typically represent the positive sequence impedance. This is also equal to the 'phase impedance' in a special case where the currents and voltages are balanced and the cable is transposed. A conductor impedance matrix can be configured that is consistent with the provided positive sequence data. For an ungrounded cable, and if the neutral circuit has the same impedances as the phase circuits, then the zero sequence impedance is 4 times the positive sequence impedance. This provides a more specific 'rule of thumb' than the approximate ratios referenced in 2.3.7. The ratio between the zero and sequence impedances for networks with a multi-grounded neutral is considered further in Chapter 4.

Some previous studies have used an approximation to the sequence impedance matrix in which only the positive and zero sequence impedances are specified. This approximation to the impedance matrix can introduce significant errors into the voltage drop calculations. For the example cable described later in Section 4.4, the amplitude of the voltage difference has errors of up to 17% for balanced currents since the approximated impedance matrix does not represent the asymmetry of the cable.

# 4 CABLE IMPEDANCE MODELLING

# 4.1 Introduction

Although the accuracy of network models relies on the accuracy of the cable impedance data, the literature review in Section 2.2.5 has identified that there are a number of different assumptions and approaches taken to defining this data. This chapter therefore considers several methods of calculating the series impedance of a commonly used Waveform LV cable, comparing several analytical methods with a finite element simulation. The impacts on the impedance of manufacturing tolerances and temperature variations are also investigated.

The shunt admittance is then considered, again with an analytical technique in comparison with a finite element model.

Finally, the impedance matrix for a 4-core BS 5467 cable is modelled in order to provide impedance data can be used in Chapters 5 and 6 to compare with impedances estimated from measured data. The FE simulations of this cable type require a model of a stranded conductor, and for the impacts of eddy currents in the ferromagnetic armour and the ground to be considered.

# 4.2 Waveform cable series impedance

## 4.2.1 Cable description

Waveform type cables are commonly employed in UK low voltage networks and are the default specification for new installations (E.ON Central Networks 2006; Scottish Power 2012). The cable consists of either 3 or 4 aluminium sector conductors surrounded by a copper concentric neutral/earth conductor, as described in Table 4-1 and Figure 4-1. In the 4-core Waveform cable assemblies, the neutral is provided by one of the sector conductors, and the concentric strands provide a separate earth conductor. For clarity, the concentric strand conductors are therefore referred to here as the concentric earth, although they function as a combined neutral and earth in the 3-core assembly types.

The following description is a summary of published work conducted as part of this research project which is included here as Appendix A (Urquhart & Thomson 2015b).

The nominal cable design is standardised, with the sector conductor shapes specified in standard BS 3988 (BSI 1970) and the construction of the overall cable in BS 7870 (BSI 2001; BSI 2011a). In measured samples of cables, the insulation thickness was found to be greater than the specified minimum, thereby allowing for manufacturing tolerances while maintaining compliance to the standards. The more recent version of BS 7870 requires fewer copper strands in the concentric sheath but these have increased diameter to maintain the overall resistance (BSI 2011a). Due to these differences, installed cables (possibly several decades in age) may differ from those in the product datasheets.



Figure 4-1 - Sample cross-section of 3-core 95 mm<sup>2</sup> cable and dimensions

Cable size, mm <sup>2</sup>	95	185	300
Sector area, $a_s$ , mm <sup>2</sup> (BSI 1970)	92.14	179.5	295.
Sector radius, b, mm (BSI 1970)	10.24	14.17	18.01
Corner radius, c, mm (BSI 1970)	1.02	1.41	1.80
Sector width, w, mm (BSI 1970)	15.76	21.97	28.14
Sector angle, $\phi$ , degrees (BSI 1970)	119	119	119
Sector depth, s, mm (BSI 1970)	9.14	12.78	16.44
Sector lay length, mm (BSI 2011a)	>800	>1200	>1600
Sector resistance at 20 °C, Ω/km (BSI 2001)	0.32	0.164	0.1
Sector resistivity temperature coefficient, 20 °C (BSI 2006)	0.00403	0.00403	0.00403
Insulation thickness, t, mm (BSI 2001)	1.1	1.6	1.8
Number of strands, N <sub>S</sub> (BSI 2001)	30	42	42
Earth strand radius, $R_{\rm S}$ , mm (BSI 2001)	0.79	0.935	0.935
Sheath thickness, mm (BSI 2001)	2.1	2.5	2.8
Earth strand concentric radius, $R_N$ , mm	14.36	19.515	23.865
Earth strand resistance at 20 °C, $\Omega$ /km (BSI 2001)	0.32	0.164	0.164
Outer radius, R <sub>0</sub> , mm (BSI 2001)	17.25	22.95	27.6
Earth strand lay length, mm (BSI 2011a)	>250	>300	>440
Insulation dielectric constant (Kersting 2012)	2.3	2.3	2.3
Earth strand resistivity temperature coefficient, 20 °C (BSI	0.00393	0.00393	0.00393
2006)			

#### Table 4-1 – Parameters for 3-core Waveform cable

## 4.2.2 Analytical impedance calculations methods

Four analytical methods of calculating the impedances are presented in Appendix A. These are summarised as follows, with comparisons for the example of the 3-core 95 mm<sup>2</sup> cable type:

## Approximating sector shapes as circular

In the simplest analytical approach the sector shapes are approximated as circular conductors having the same area as the actual sector shape. The current distribution is assumed to be uniform, and the rotation of the cable lay is not taken into account. The cable is modelled separately from the ground with the unbalance current (the sum of the three phase currents) returning through the concentric earth.

## Modelling sectors using multiple sub-conductors

In this second method, the sector area is divided into sub-conductors in order to represent the geometry more accurately. This results in a 14% decrease in the zero sequence reactance and a 7% increase in the positive sequence reactance. The current density is still assumed to be uniform (Urquhart 2012).

### Analytical corrections for AC resistance

In the third method, the analysis was then refined by adding AC resistance corrections from IEC 60287 to allow for the skin effect and proximity effect caused by induced eddy currents. These corrections do not calculate the eddy currents directly (only their impact on the resistance) and so the estimated reactance is unaffected. For the 95 mm<sup>2</sup> cable size, the AC resistance corrections have minimal impact, although when the comparison was repeated for the 300 mm<sup>2</sup> cable size (larger compared to the skin depth), there was a 6% increase in the resistance.

### Including the ground path

The fourth method includes the impact of the ground path. The cable is considered to be buried underground and Carson's equations (28) and (29) were used to calculate the impedances. A perfect multi-grounded neutral is assumed, and the Kron reduction is applied to calculate the phase impedances. Compared to the cable with return path only through the neutral, adding the ground conductor gives a 14% reduction in the zero sequence resistance and increases the zero sequence reactance by a factor of 4. These effects are consistent with the examples presented in Section 3.5 showing the reactive currents flowing in a loop around the neutral and ground conductors. The impedance results from this analysis are presented in Table 4-2. The impacts of this additional ground conductor on the voltage difference and losses are explored further in Chapter 10.

## 4.2.3 Finite element model

For comparison with the analytical methods, the impedances were also calculated using a finite element modelling technique. This uses the same sector-shape geometry as in approaches 2 to 4, and also includes the ground conductor. However, the FE method models the eddy currents in detail and so this allowed the impacts on both the resistance and reactance to be taken into account. The finite element modelling technique uses the freely available software package FEMM (Meeker 2013) and was initially described work from a previous MSc project (Urquhart 2012). This method has since been developed further to allow for the rotation of the concentric earth strands relative to the sector-conductors due to the waveform pattern cable lay. In this improved method, the FE simulation model is solved separately for each strand of the concentric earth conductor, rather than assuming that they are all connected in parallel within the FE model. This allows the rotation of the concentric earth relative to the sector conductors due to the cable lay to be taken into account. Each strand has an equal probability of being at any angle relative to the sector conductor impedance is the mean impedance for any angle around the circle of strands. Since there are many strands in the concentric earth, this mean is calculated by taking the mean of the individual strand conductor impedances, each at a different angle. A Kron reduction is then applied to calculate the impedance of a single concentric earth conductor with all of the strands connected in parallel, as described in Appendix D.

As discussed in Section 3.3, the conductor impedances are contributions to the impedance of a closed circuit. For the analytical approaches, assumptions are required such that the magnetic field included in the flux linkage calculation is truncated. Similar arbitrary limits apply to the FE model since the solved cross-sectional area is subject to a finite boundary, and since this boundary has an arbitrary assignment that the magnetic vector potential equals zero. These factors do not impact on the circuit impedances formed when it is assumed that the sum of currents equals zero

The impedances for the 3-core 95 mm<sup>2</sup> Waveform cable are then given as in Table 4-2. At 50 Hz, the results from the FE simulation agree very closely to those based on the analytical techniques using the AC resistance corrections.

Analysis using AC resistance corrections from IEC 60287				
Circuit impedance, Ω/km	$\hat{\mathbf{z}} = \begin{bmatrix} 0.372 + j0.770 & 0.049 + j0.710 & 0.049 + j0.710 & 0.049 + j0.696 \\ 0.049 + j0.710 & 0.372 + j0.770 & 0.049 + j0.710 & 0.049 + j0.696 \\ 0.049 + j0.710 & 0.049 + j0.710 & 0.372 + j0.770 & 0.049 + j0.696 \\ 0.049 + j0.696 & 0.049 + j0.696 & 0.049 + j0.696 & 0.369 + j0.696 \end{bmatrix}$			
Phase impedance, assuming Kron reduction, Ω/km	$\hat{\mathbf{z}}_{abc} = \begin{bmatrix} 0.582 + j0.189 & 0.259 + j0.128 & 0.259 + j0.128 \\ 0.259 + j0.128 & 0.582 + j0.189 & 0.259 + j0.128 \\ 0.259 + j0.128 & 0.259 + j0.128 & 0.582 + j0.189 \end{bmatrix}$			
Sequence impedance, Ω/km	$\hat{\mathbf{z}}_{012} = \begin{bmatrix} 1.101 + j0.445 & 0 & 0\\ 0 & 0.323 + j0.061 & 0\\ 0 & 0 & 0.323 + j0.061 \end{bmatrix}$			
	Finite element modelling			
Circuit impedance, Ω/km	$\hat{\mathbf{z}} = \begin{bmatrix} 0.368 + j0.757 & 0.047 + j0.697 & 0.047 + j0.697 & 0.047 + j0.684 \\ 0.047 + j0.697 & 0.368 + j0.757 & 0.047 + j0.697 & 0.047 + j0.684 \\ 0.047 + j0.697 & 0.047 + j0.697 & 0.368 + j0.757 & 0.047 + j0.684 \\ 0.047 + j0.684 & 0.047 + j0.684 & 0.047 + j0.684 & 0.367 + j0.683 \end{bmatrix}$			
Phase impedance, assuming Kron reduction, Ω/km	$\hat{\boldsymbol{z}}_{abc} = \begin{bmatrix} 0.580 + j0.190 & 0.258 + j0.130 & 0.258 + j0.130 \\ 0.258 + j0.130 & 0.580 + j0.190 & 0.258 + j0.130 \\ 0.258 + j0.130 & 0.258 + j0.130 & 0.580 + j0.190 \end{bmatrix}$			
Sequence impedance, Ω/km	$\hat{\boldsymbol{z}}_{012} = \begin{bmatrix} 1.096 + j0.450 & 0 & 0 \\ 0 & 0.322 + j0.060 & 0 \\ 0 & 0 & 0.322 + j0.060 \end{bmatrix}$			

#### Table 4-2 – Impedances for the 3-core 95 mm<sup>2</sup> Waveform cable at 50 Hz, including ground circuit return

As the frequency increases, the results from the two methods diverge, as demonstrated by Figure 4-2. For the 300 mm<sup>2</sup> cable size at 450 Hz, the positive sequence impedance from the FE simulation is 33% lower than in the analytical model, and the zero sequence impedance has 6% lower resistance and 28% lower reactance.

This impedance data is available for download from, together with the corresponding circuit impedance and phase impedance data (Urquhart & Thomson 2014).



Figure 4-2 – Positive (z<sub>11</sub>) and zero (z<sub>00</sub>) sequence impedances for 3-core 95 mm<sup>2</sup> and 300 mm<sup>2</sup> cables

## 4.2.4 Ground current distribution

The work in Appendix A also highlights the large cross-sectional area needed to represent the ground conductor. For models with ground resistivity of 100  $\Omega$ m, a semi-circular boundary radius of 3 km was needed in the finite element model such that variations in this radius caused minimal change to the impedance data. The impacts of this truncation of the ground conductor are now considered here in further detail. It should be noted that truncation effect differs from that discussed in Section 3.2 (relating to the distance from a conductor that is needed for integration of the magnetic field) as it affects the dimensions of the ground conductor itself.

This surprisingly large distance is also predicted by Carson's equations since the term of  $\mu_0 \omega/8$  in (28) and (29) represents the resistive contribution of the ground

(Urquhart 2012). As discussed in Section 3.4 it is unclear how much of this resistance is due to the ground conductor impedance, or to the mutual impedance between the conductor and the ground. However, by comparing (28) and (29) with (15) and (16), it is clear that this contribution to the total circuit resistance occurs in the ground rather than in the cable conductor.

For 50 Hz, the term  $\mu_0 \omega/8$  gives a ground resistance of 0.0494  $\Omega$ /km. With ground resistivity of 100  $\Omega$ /m, the equivalent conductor requires a cross-sectional area of 2 km<sup>2</sup>. This is equivalent to a semi-circular conductor of radius 1.14 km, suggesting a similar scale for the ground conductor to that indicated by the FE modelling.

The FE model can be used to plot the current density in the ground conductor, as shown in Figure 4-3. For this case, the boundary radius was increased to 30 km so as to avoid edge effects at the previously used boundary radius of 3 km. The model was also re-configured so that each phase conductor carried a current with amplitude of 1 A and a phase angle of 0°, thereby creating the current distribution for the zero sequence mode.

The plot shows the current density along a profile starting 1 m below the cable and with increasing depth into the ground. A lateral profile is also shown for a line parallel to the surface at a depth of 0.5 m. Both curves show that the current density initially decreases rapidly with distance, but there is a reduced for distances greater than about 100 m, the current density reduces exponentially with the distance. For the lateral profile, the current density at 3 km radius is still approximately 5% of the current density at a point 0.5 above the cable.

These predictions of the ground current density, both from Carson's equations and in the FE simulation, arise from models in which the conductors are assumed to have infinite length and so can be characterised by considering a 2-D planar cross-section. For typical LV cables, the ground electrode at the substation and at the end of the feeder may be only a few hundred meters apart. Clearly the 'end effects' as the current enters and leaves the ground at the electrodes also need to be taken into account. This is addressed in Chapter 9.



Figure 4-3 – Ground current density for 3-core 95 mm<sup>2</sup> cable in the ground, zero sequence mode

## 4.2.5 Manufacturing tolerances

The standards that specify the cable types also provide an indication of the tolerances that apply to the geometry and to the conductor resistances. Samples of 95 mm<sup>2</sup> and 185 mm<sup>2</sup> cable have also been obtained and measured. Based on this practical experience and on also on permitted tolerances defined by standards, a set of possible variations to the conductor dimensions can be defined, as in Table 4-3.

Parameter	Tolerance	Comment
Sector width and depth	+0.5% / -3%	Based on maximum and minimum values (BSI 1970)
Sector area	+1% / -6%	Consistent with width and depth scaling
Sector resistance	3%	From (BSI 1970)
Neutral resistance	3%	Assuming the same value as for the sector
Insulation thickness	+50%	Based on measured samples
Sheath thickness	+20% / -20%	From (BSI 2011a), although the -20% tolerance
		should not occur around the full circumference as the
		nominal value in Table 4-1 is specified as the
		minimum average

Table 4-3 – Conductor dimension variations for 3-core Waveform c
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These tolerances have been applied as followed to test their impact on the sequence impedances of the 3-core 95 mm<sup>2</sup> cable.

<u>Sector area</u>: the sector geometry was scaled such that the shape was maintained, with width and depth scaled accordingly. The conductor resistance was unchanged so an increase in area corresponds to a reduction in current density. The gap between sectors is determined by the insulation thickness so an enlargement of the sector also implies a translation away from the cable centre.

<u>Conductor resistance</u>: although the resistance of the cable assembly is more significantly affected by temperature, individual conductors can also vary from the nominal dimensions.

<u>Insulation thickness</u>: increasing the thickness of the insulation causes the sector conductors to be moved further apart from each other. The outer diameter is unchanged so the sectors become closer to the concentric neutral. The value in Table 4-1 is specified as a minimum value, and so actual values are likely to be greater than this, due to the need to set a nominal value in manufacturing that ensures the minimum thickness will be achieved. Measured samples have indicated that the insulation thickness can be up to 50% higher than the required minimum. This increased thickness was found to be less on newer samples, indicating that manufacturing tolerances may have improved.

<u>Sheath thickness</u>: increasing the sheath thickness with the outer diameter unchanged causes the concentric neutral to become closer to the sector conductors. The thickness of the filler material between the sector conductor insulation and the concentric neutral strands is not specified in the standards, but is implied by the specification of the cable outer radius, the sheath thickness, and the neutral strand diameter. An increase in the sheath thickness with a constant outer diameter is therefore also representative of the effect of reducing the filler thickness.

Since all of these variations preserve the 120° rotational symmetry of the 3-core assembly, the impedance is fully described by the zero and positive sequence impedances, with the mutual sequence impedances being zero.

Impedance variations corresponding to the tolerances described above are presented for the 3-core 95 mm<sup>2</sup> cable in Table 4-4. Since Appendix A demonstrates that the analytical method gives results that are very close to those from the finite element modelling, the simpler analytical approach has been used here. Results are

shown for the zero sequence impedance where the neutral is perfectly grounded and also for the case where the neutral is isolated from the ground.

	Impedances, Ω/km		Percentage error		
Variation	Resistance	Reactance	Resistance	Reactance	
Positive sequence					
Nominal	0.323	0.061	-	-	
Insulation thickness x1.5	0.323	0.066	0.10%	9.16%	
Sheath thickness x1.2	0.323	0.061	0.04%	0.00%	
Sheath thickness x0.8	0.323	0.061	-0.03%	0.00%	
Sector area x1.01	0.323	0.060	0.00%	-0.10%	
Sector area x0.94	0.323	0.061	-0.03%	0.60%	
Sector resistance x1.03	0.332	0.061	2.97%	0.00%	
Neutral resistance x1.03	0.323	0.061	-0.02%	0.00%	
Zero sequence with multi-grounded neutral					
Nominal	1.101	0.445	-	-	
Insulation thickness x1.5	1.101	0.434	0.03%	-2.50%	
Sheath thickness x1.2	1.102	0.438	0.09%	-1.40%	
Sheath thickness x0.8	1.100	0.451	-0.09%	1.36%	
Sector area x1.01	1.101	0.444	0.00%	-0.19%	
Sector area x0.94	1.101	0.450	-0.01%	1.15%	
Sector resistance x1.03	1.111	0.445	0.87%	0.00%	
Neutral resistance x1.03	1.116	0.461	1.35%	3.78%	
Zero sequence with neutral isolated from ground					
Nominal	1.283	0.100	-	-	
Insulation thickness x1.5	1.283	0.088	0.02%	-11.15%	
Sheath thickness x1.2	1.283	0.094	0.01%	-5.81%	
Sheath thickness x0.8	1.283	0.105	-0.01%	5.64%	
Sector area x1.01	1.283	0.099	0.00%	-0.83%	
Sector area x0.94	1.283	0.105	-0.01%	5.13%	
Sector resistance x1.03	1.292	0.100	0.75%	0.00%	
Neutral resistance x1.03	1.311	0.100	2.24%	0.00%	

The tolerances relating to the cable geometry are seen to have very little effect on the of the sequence mode resistances. Variations in the resistance of the sector conductor cause an almost proportional variation in the positive sequence resistance (not exact, due to the AC resistance effects) but also cause an increase in the zero sequence resistance. The reactance is most sensitive to variations in the thickness of the conductor insulation, with up 9% to 11% changes for the thickness variations considered. This thickness variation was considered here with the outer diameter held constant and so an increased positive sequence reactance is accompanied by a decrease in the zero sequence reactance. This is due to the reduced spacing between the sectors and the concentric neutral if the insulation thickness increases and the sectors are moved further apart.

In general, although manufacturing tolerances have some effect on the cable impedances, the variations considered here would not be expected to significantly affect the accuracy of network models based using this cable type since the reactance is low relative to the resistance.

### 4.2.6 Temperature variation

The impedance values presented above have been calculated assuming that the conductors are at a nominal 20 °C temperature.

The impact of temperature variation for the 3-core 95 mm<sup>2</sup> cable is shown in Figure 4-4 and Figure 4-5, plotted for a range of 0 °C to the maximum rated temperature of the cable at 90 °C (Prysmian 2009). These plots show positive and zero sequence resistance values calculated using the FE simulation method.

For a cable with grounded neutral, a temperature rise from 20 °C to 30 °C causes the positive sequence resistance to rise by 4% and the zero sequence resistance by 3%. Since this change in resistance affects the proportion of unbalance current shared between the ground and the neutral, the zero sequence reactance also changes, increasing by 5%.

For a cable with the neutral isolated from the ground, a temperature rise from 20 °C to 30 °C causes an increase in both the zero and positive sequence resistance of 4%. In this case the resistance change has no impact on the reactance.

These calculations suggest that the impacts of temperature variations are more significant than the effects of manufacturing tolerances discussed in Section 4.2.5 and need further consideration.

The temperature variation might be also expected to affect the positive sequence reactance, since the skin depth increases in proportion to the square root of the conductor resistivity. However, for this case, the conductor dimensions are already relatively small compared to the skin depth and so there is negligible impact.

A network simulation could potentially be developed in which the impedance is varied in accordance with the dissipated losses, and with a thermal model to define the heat transfer into the ground. Such a simulation could require impedance data for the cables calculated according to the operational temperature, significantly increasing the complexity of the modelling. A simpler alternative would be to determine the circuit impedance at 20 °C and then scale the resistive terms in proportion to the increase in conductor resistivity. The revised sequence impedances can then be calculated using (49) and (53). For a cable with a grounded neutral, it would be incorrect to scale the resistance terms of the phase or sequence impedances 95 mm<sup>2</sup> cable at 50 Hz, this approximation approach introduces errors of less than 1%.



Figure 4-4 – Resistance variation with temperature for 3-core 95 mm<sup>2</sup> Waveform cable at 50 Hz



Figure 4-5 – Reactance variation with temperature for 3-core 95 mm<sup>2</sup> Waveform cable at 50 Hz

Chapters 5 and 6 of this thesis describe measurement work in which the series impedance of a feeder cable is estimated based on the observed current and voltage drop. This impedance estimation makes the assumption that the impedance is constant throughout the measurement period. Whilst this would likely be valid for the reactance, it is possible that the resistance would change due to the heating impact of losses.

In Chapter 6 the individual tests have a duration of 5 minutes. Assuming a worst case scenario in which there is no transfer of heat from the conductor into the surrounding insulation, a simple estimate can be made of the corresponding heat rise for this test period. It is also assumed that the current in the period before the test was zero, such that the measured currents represent a change of state, rather than a continuation of the previous load conditions.

For a cable with 95 mm<sup>2</sup> conductors, each conductor has a volume of 0.095 m<sup>3</sup>/km. For copper with a density of 8940 kg/m<sup>3</sup> (The Engineering Toolbox 2015a), this is equivalent to a conductor mass of 849 kg/km. If this conductor carries a current of 84 A (the highest RMS current observed in the set of test cases),and has a nominal resistance of 0.193  $\Omega$ /km (BSI 2008), then the corresponding losses are 1362 W/km. Over a 5 minute period, the total dissipated energy is 409 kJ/km. For copper with a specific heat capacity of 0.39 kJ/kgK (The Engineering Toolbox 2015b), this dissipated energy would cause a temperature rise of 1.2 °C. Based on the temperature coefficient of resistance for copper of 0.00393 (BSI 2006), this temperature change would increase the resistance by 0.5%. The change is proportional to the time period so the same demand current would give a 1% change in the resistance over the 10 minute period typically used when considering the RMS voltage.

These changes are small compared to the measurement inaccuracies and so it is assumed that the impedance can be considered constant over a 5 minute measurement period. It is also reasonable to consider the resistance as constant in the relation to discussions regarding the time resolution of the demand currents, where the measurement time intervals are of the order of minutes, or shorter.

Calculations for much longer periods would require a thermal model that allows for the conduction and radiation of heat away from the conductor.

## 4.3 Waveform cable shunt admittance

## 4.3.1 Analytical model

The shunt admittance is often ignored in network modelling, although it is noted that it cannot always be neglected for underground cables (Kersting 2012). It is typically assumed that the shunt conductance is zero, and so the admittance is determined by the capacitive susceptance.

The analysis below is based on Kersting's approach and developed here to allow for a cable with multiple sector conductors (Kersting 2012). The cable has  $N_{\text{sector}}$  sector conductors and  $N_{\text{strand}}$  neutral strands. In total there are  $N_{\text{cond}} = N_{\text{sector}} + N_{\text{strand}}$  conductors.

The voltage between two conductors can be determined based on the charges on each conductor. In this analytical model, it is assumed that the conductors are circular in shape and that the charge density on each conductor is uniform. This implies that the charge density is not affected by the proximity of other conductors and so allows the total field to be calculated as by the superposition of the fields due to individual conductors. In practice, the charge density would depend both on the shape and on the proximity of charges on the other conductors.

This assumption is therefore analogous to the assumption of a uniform current density when modelling the series impedance, which implies that there are no

inducted eddy currents. The magnetic field of each conductor is therefore assumed to be unaffected by the presence of the other conductors, and the total flux linkage is calculated by superposition of the flux linkage from each conductor. In practice, the induced eddy currents would cause the magnetic field of each conductor to be dependent on the proximity of the other conductors.

The voltage between conductors *i* and *j*, within a total set of  $N_{cond}$  conductors, is then given by:

$$V_{ij} = \frac{1}{2\pi\epsilon} \sum_{m=1}^{N_{\text{cond}}} q_m \ln \frac{D_{mj}}{D_{mi}}$$
(62)

where  $q_m$  is the charge on conductor m, and  $D_{mj}$  is the centre to centre distance between conductors m and j. When m = j, the distance is the radius of conductor m. If one of the neutral strands d is taken as a reference, then  $N_{pd}$  potential differences can be defined as the voltage between this strand and the other conductors, with  $N_{pd} = N_{cond} - 1$ . Assuming that the total charge sums to zero, then

$$q_d = -\sum_{m=1}^{N_{\rm pd}} q_m \tag{63}$$

Combining (62) and (63), the voltage between conductors i and d is then

$$V_{id} = \frac{1}{2\pi\epsilon} \sum_{m=1}^{N_{\rm pd}} q_m \ln \frac{D_{md} D_{di}}{D_{mi} D_{dd}}$$
(64)

The voltage can then be considered to depend on the charges on each conductor, scaled by a set of potential coefficients, so that (64) can be written in matrix form as

$$V_{id} = \sum_{m=1}^{N_{\rm pd}} \hat{P}_{im} q_m \tag{65}$$

where

$$\hat{P}_{im} = \frac{1}{2\pi\epsilon} \ln \frac{D_{md} D_{di}}{D_{mi} D_{dd}}$$
(66)

The vector describing voltages on all conductors with respect to the reference neutral strand can then be written in matrix form as  $V_{in} = \hat{P} q$ , where  $\hat{P}$  has the

dimensions  $N_{pd} \times N_{pd}$ . Since the potential of all the neutral strands is equal, the voltage between the other strands and the reference is zero and the Kron reduction can be applied to give a  $N_{sector} \times N_{sector}$  potential coefficient matrix

$$\boldsymbol{P}_{\text{sector}} = \boldsymbol{\hat{P}}_{ij} - \boldsymbol{\hat{P}}_{in} \boldsymbol{\hat{P}}_{nn}^{-1} \boldsymbol{\hat{P}}_{nj}$$
(67)

where  $\hat{P}$  is partitioned so that  $\hat{P}_{ij}$  is the  $N_{\text{sector}} \times N_{\text{sector}}$  sub-matrix,  $\hat{P}_{in}$  is the  $N_{\text{sector}} \times N_{\text{strand}}$  sub-matrix,  $\hat{P}_{nj}$  is the  $N_{\text{strand}} \times N_{\text{sector}}$  sub-matrix, and  $\hat{P}_{nn}$  is the  $N_{\text{strand}} \times N_{\text{strand}}$  sub-matrix.

If the cable has three sectors, this gives the phase impedance matrix, so that  $P_{abc} = P_{sector}$ . For a cable with a fourth sector (neutral), a further Kron reduction stage could be applied if there is a zero voltage between the neutral sector and the concentric neutral, following the same steps as outlined for the series impedance in (49). However, if the cable is installed with TN-S earthing (Cronshaw 2005) then the neutral sector may be taken as the reference in (64), and the neutral strands of the sheath are ignored (since there is no current flow from the neutral sector to the sheath other than in fault conditions). This gives a 3 × 3 matrix directly, so that:

$$P_{\rm abc} = \widehat{P}.$$

The 3  $\times$  3 capacitance matrix is then  $C_{abc} = P_{abc}^{-1}$  and the phase admittance is then

$$\boldsymbol{Y}_{abc} = 2\pi f \mathbf{j} \cdot \boldsymbol{C}_{abc} \tag{68}$$

A sequence admittance matrix can be derived in the same manner as for the series impedance, giving

$$Y_{012} = A_s^{-1} \cdot Y_{abc} \cdot A_s \tag{69}$$

### 4.3.2 Finite element simulation

The FE model for the shunt capacitance calculation uses a similar approach to that described in Appendix A for calculating the series impedance. The same representation of the cable conductors is used but the intermediate space between the conductors is now defined as being a dielectric material.

The Waveform cable geometry was entered into the FEMM solver, as shown in Figure 4-6 for the 3-core 95 mm<sup>2</sup> cable.

For the results presented here it is assumed that the concentric neutral contains the electric field and so there is no interaction with the ground. To validate this assumption, an alternative configuration with the ground included has also been tested and found to give equivalent results. The cable is therefore surrounded by a vacuum space in the FEMM model, with a boundary condition of zero voltage specified at a radius of 20 times the outer radius of the cable.

The simulations use the electrostatic mode of the FEMM software and so the results are solved as a DC system (Meeker 2013). The model assumes that charges on the conductors will separate the greatest extent possible, such that all of the charge appears on the of the conductor surfaces. The internal space within each conductor is therefore defined as a vacuum.



Figure 4-6 – FEMM model of Waveform 3-core 95 mm<sup>2</sup> cable

A separate FE simulation was required for each conductor in the model, in each case with one conductor m 'active' and having a charge of 1 Coulomb, and all of the others 'inactive' with zero applied charge. Each of these simulations then provides a set of potential differences  $\bar{P}_{im}$  between each of the conductor i and the boundary condition, defined as having a potential of zero. The voltage difference between a

conductor and the boundary for any combination of charges can then be found by superposition, giving:

$$V_{i,\text{boundary}} = \sum_{m=1}^{N_{\text{cond}}} \bar{P}_{im} q_m \tag{70}$$

The potential difference between two conductors *i* and *d* can be determined from the conductor potentials, where  $V_{id} = V_{i,\text{boundary}} - V_{d,\text{boundary}}$ , giving

$$V_{id} = \sum_{m=1}^{N_{\text{cond}}} \bar{P}_{im} q_m - \sum_{m=1}^{N_{\text{cond}}} \bar{P}_{dm} q_m$$
(71)

Substituting from (63) gives:

$$V_{id} = \sum_{m=1}^{N_{pd}} \bar{P}_{im} q_m - \sum_{m=1}^{N_{pd}} \bar{P}_{nm} q_m - \bar{P}_{in} \sum_{m=1}^{N_{pd}} q_m + \bar{P}_{nn} \sum_{m=1}^{N_{pd}} q_m$$
(72)

which can be simplified to the same form as (65) where:

$$\widehat{P}_{im} = \overline{P}_{im} - \overline{P}_{dm} - \overline{P}_{id} + \overline{P}_{dd}$$
(73)

This gives potential coefficients from simulations that are comparable to those from the analytical approach, as in (66). As in Section 4.3.1, this gives a matrix with dimensions  $N_{pd} \times N_{pd}$  which can be reduced to a 3x3 form by applying the Kron reduction, since all of the neutral strands are at the same potential. The capacitance, phase admittance and sequence admittance can then be derived by applying (67), (68), and (69) as for the analytical method.

#### 4.3.3 Capacitance for Waveform cable

The analytical and simulation methods outlined in Sections 4.3.1 and 4.3.2 have been applied to the example case of Waveform cable, defined according to the parameters in Table 4-1.

Using the analytical approach from Section 4.3.1, the capacitance could be determined with the sector shapes approximated as being circular, with the conductor radius defined in order to provide an equivalent cross-sectional area, given by

$$D_{ii} = \sqrt{a_{\rm s}/\pi} \tag{74}$$

The distances between sectors are calculated relative to a nominal centre at distance m from the cable axis, as shown in Figure 4-1 given by

$$m = b - s/2 + \delta \tag{75}$$

The centre of rotation of the sector arcs may be displaced slightly from the centre axis of the cable assembly. This increases the gaps between sectors to allow for the thickness of their insulation sleeves. The offset  $\delta$  is given by

$$\delta \simeq t/\sin(\theta_{12}/2) - b + s + c(1/\sin(\phi/2) - 1)$$
(76)

where  $\theta_{12}$  is the angular separation between two adjacent sectors (120° for a 3-core cable).

The distance between sectors is given by

$$D_{ij} = m \sqrt{\left(1 - \cos \theta_{ij}\right)^2 + \sin^2 \theta_{ij}}$$
(77)

The sequence admittance can then be found from (69), giving the values in Table 4-5.

Table 4-5 – Shunt admittance at 50 Hz for 3-core Waveform cable from analytical method

Cable size, mm <sup>2</sup>	95	185	300
Positive sequence admittance, $y_{11}$ , $\mu$ S/km	113	118	132
Zero sequence admittance, $y_{00}$ , $\mu$ S/km	26	27	29

For comparison, the admittance was also calculated according to the FE method described in Section 4.3.2. This model represents the sector geometry more accurately and also allows for the charge density to vary across the conductor surfaces.

A plot of the electric field from the FEMM simulation is shown in Figure 4-7, showing the case where the sector at the top of the diagram has a charge of 1 Coulomb, and no charge is applied to any of the other conductors. The field is seen to be greatest at the sector corners and also approximately constant along the parallel edges between pairs of adjacent sectors. This demonstrates that approximating the sector shapes as being circular and as having a uniform charge density does not give an accurate model of the electric field for closely spaced sector-shape conductors. The corresponding sequence admittance results, calculated using (73), and then (67), (68) and (69), are shown in Table 4-6.

Cable size, mm <sup>2</sup>	95	185	300
Positive sequence admittance, $y_{11}$ , $\mu$ S/km	133	135	151
Zero sequence admittance, $y_{00}$ , $\mu$ S/km	48	53	59

#### Table 4-6 – Shunt admittance at 50 Hz for 3-core Waveform cable from simulation method

The simulations were repeated with an alternative configuration in which the ground was added as an additional conductor. The model assumes a perfect multi-grounded neutral, such that the ground could be treated as an additional neutral conductor in the same manner as with the concentric neutral strands. These simulations provided the same results as shown in Table 4-6, confirming that the electric field outside of the concentric neutral can be considered to be zero.

The admittance results from the simulation model are significantly higher than those predicted by the analytical approach. However, even the highest of these values, a positive sequence admittance of 151  $\mu$ S/km for the 300 mm<sup>2</sup> cable, corresponds to a current of only 35 mA/km at 230 V. The capacitance effects of Waveform cables might therefore reasonably be neglected for LV network simulations.



Figure 4-7 – FEMM electric field solution of Waveform 3-core 95 mm<sup>2</sup> cable

# 4.4 BS 5467 cable series impedance

## 4.4.1 Cable description

Chapters 5 and 6 describe measurements of the currents and voltages on an LV feeder on the Loughborough campus. In preparation for these measurements, the series impedance of the LV cable has been estimated using the finite element modelling technique described in Section 4.2.3 and in Appendix A.

This test feeder is a 4-core cable with 95 mm<sup>2</sup> copper stranded conductors, constructed according to standard BS 5467 (BSI 2008). In addition to the sector conductors, there is an armour conductor consisting of concentric strands of galvanised wire. These have a cable lay rotation relative to the sector conductors, although this has a continuous increase in rotation angle, rather than the sinusoidal pattern of the Waveform cable.

Ideally a sample of the actual cable type used in the feeder being tested would have been obtained but this was not possible. Instead, a sample of a similar cable, designed to standard BS 7846 (BSI 2009a) was provided by Prysmian Cables and used as a reference for the dimensions. This cable type has similar construction to BS 5467 but with additional layers added for fire retardant material, measured to have approximately 0.2 mm thickness.

Although the internal dimensions of the test cable could not be measured, the outer diameter was found to be 40 mm, a little less than the standard value of 41.7 mm from BS 5467.

The sector conductors for the test cable have stranded copper construction, such that the total area of the sector shape includes gaps between the strands. Although the sectors nominally have a 95 mm<sup>2</sup> conductor area, the area occupied by the sector shape is therefore a little larger than this. The standard BS 5467 defines the overall resistance of the conductors, but does not specify their exact shape. The width and depth of the sector shape were therefore assumed to be equivalent to the BS 7846 cable sample.



Figure 4-8 – BS 7846 4-core 95 mm<sup>2</sup> cable sample

Cable size, mm <sup>2</sup>	95
Sector area, $a_s$ , mm <sup>2</sup> (based on sector dimensions)	106.8
Sector radius, b, mm, from measured BS 7846 sample	12.5
Corner radius, <i>c</i> , mm, from measured BS 7846 sample	2.0
Sector width, w, mm, from measured BS 7846 sample	15.0
Sector angle, $\phi$ , degrees, assumed the same as for Waveform cable, BS 3988	89
Sector depth, s, mm, from measured BS 7846 sample	11.0
Sector resistance at 20 °C, Ω/km, BS 5467	0.193
Sector resistivity temperature coefficient, 20 °C, IEC 60287-1-1 (BSI 2006)	0.00393
Insulation thickness, t, mm, from measured BS 7846 sample	1.3
Concentric earth area, mm <sup>2</sup> , BS 5467	147
Earth wire strand radius, $R_{\rm S}$ , mm, BS 5467	1.0
Number of earth wires, $N_{\rm S}$ , based on concentric earth area and radius of each wire	47
Sheath thickness, mm, BS 5467	2.2
Earth wire concentric radius, $R_{\rm N}$ , mm	16.8
Earth wire resistance at 20 °C, Ω/km, BS 5467	1.1
Outer radius, $R_0$ , mm, from measurement of test feeder	40
Earth wire relative permeability, $\mu_{\rm r}$ , (Stølan 2009)	300

Table 4-7 -	- Parameters	for 4-core	95 mm <sup>2</sup> BS	5467 cable
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Based on the AEI manufacturer's data, the positive sequence impedance of the 4core 95 mm<sup>2</sup> cable for 50 Hz at 20 °C is  $0.193 + j0.0723 \Omega/km$  (AEI Cables 2015). The zero sequence impedance is not provided.

## 4.4.2 Finite element model

The analytical techniques described in Section 4.2.2 could not be used for this cable type (at least, not without modification) since the steel wire armour is ferromagnetic. The impedance was therefore modelled using the FE approach, where the appropriate relative permeability could easily be entered into the design model. Although the number of wires is not specified in the standard, this can be determined from the required overall resistance of the armour wires and their diameter.

The cable conductors are shown in Figure 4-9. The feeder is installed with TN-S grounding and so the neutral and earth remain separate at the customer connections. The earth connections therefore differ from those for the 3-core Waveform cable described in Section 4.2.3 where the neutral and ground were assumed to be

shorted together. In this case, the concentric earth (labelled with voltage  $V_e$  in Figure 4-9) is connected to the physical ground at the substation, but the grounding further along the cable is uncertain although it is likely that the earth wire connects to grounded metalwork in the building served by the feeder. Since this grounding is uncertain, two extreme cases can be considered: 1) where the grounding is perfect at both ends of the cable, such that there is a loop created around the concentric earth and the ground; and 2) where the concentric earth and physical ground are isolated at the customer end of the cable, such that the loop circuit is not closed.

For each solution frequency, the FE simulation method generates results for six conductors (the three phases, neutral, concentric earth and the ground), although the FE solution is actually generated individually for each neutral strand in order to allow for the rotation of the concentric earth due to the cable lay, as described in Appendix A.





The individual FE solutions allow for eddy currents within each of the conductors, but not circulating between them. Several post -processing steps are therefore required to model the connectivity between the conductors. These steps are as follows:

1. At any node where the cable is connected, all of the concentric earth strands are at the same potential and circulating currents can flow from one strand to

another. This is modelled by applying a Kron reduction, as described in Appendix D (although with different numbers of conductors). All the strands are represented as a single concentric earth conductor, reducing the matrix to a 6x6 form.

2. If the node at the customer end of the cable is assumed to have the concentric earth connected to ground, then the same process as outlined in Appendix D is applied again, making the assumption that there is zero potential difference between the concentric earth and the physical ground. This allows circulating currents to flow around the loop formed by the concentric earth and the ground. The concentric earth and the ground are represented as a single ground conductor, reducing the conductor impedance matrix to a 5x5 form.

The phase conductors are then used to form circuits with the neutral as the return path, following (9) and (10). This gives a 3x3 circuit impedance matrix, and a corresponding 3x3 sequence impedance matrix. The combined conductor representing the concentric earth and ground in parallel is neglected at this stage as the load current is not normally connected to the earth (other than with a fault). Although the ground is not included in the circuit impedances, currents in the phase conductors could induce circulating currents to flow in the concentric earth and ground conductors. Since the losses associated with these currents dissipate real power, and since there is a zero voltage between the concentric earth and ground conductors, the dissipated power must cause an increased voltage drop in the phase conductors, effectively increasing their resistance.

The modelling provided the results as in Table 4-8, shown for the case where the concentric earth is grounded at both ends of the cable. Since the neutral is isolated from this combined ground, the phase impedance is given by the 3x3 sub-matrix for the phase conductor within the circuit impedances matrix. The FE simulation predicts an 8% higher positive sequence reactance than the manufacturer (0.078  $\Omega$ /km, compared to 0.0723  $\Omega$ /km). As indicated in Section 3.6, the zero sequence impedance is 4 times the positive sequence impedance.

In the alternative case where the concentric earth and ground are open circuit at one end of the cable, the impedance results were found to have negligible differences from those in Table 4-8. This is consistent with the mutual impedances between each conductor in the cable and the ground conductor being equal, such that the total voltage induced in the ground is zero if the sum of currents in the cable conductors is zero.

The value assumed for the relative permeability of the steel wire armour could potentially have a significant impact on these results, and the adopted value  $\mu_r = 300$  (Stølan 2009) is not definite. The impedance was therefore also modelled for values of  $\mu_r = 50$  and  $\mu_r = 1000$ , for which the sequence impedances are shown in Table 4-8. There are minimal differences relative to the results for  $\mu_r = 300$ .

Results for a third option are shown where the concentric earth is excluded from the model. This approximation might be thought reasonable since none of the unbalance current connects to the earth. However, this gives a lower reactance for both the zero and positive sequence impedances, indicating that the concentric earth needs to be included in the model. The high relative permeability for these wires affects the magnetic field, changing the inductance of the other conductors.
Finite element modelling, default case with $\mu_{\rm r}=300$					
Circuit impedance with return path in	$\hat{\mathbf{z}} = \begin{bmatrix} 0.392 + j0.140 & 0.197 + j0.095 & 0.194 + j0.046 & 0.196 + j0.070 \\ 0.197 + j0.095 & 0.395 + j0.189 & 0.197 + j0.095 & 0.197 + j0.095 \\ 0.194 + j0.046 & 0.197 + j0.095 & 0.392 + j0.140 & 0.196 + j0.070 \\ 0.196 + j0.070 & 0.197 + j0.095 & 0.196 + j0.070 & 0.499 + j0.561 \end{bmatrix}$				
the neutral, $\Omega/km$	The fourth row and column here provide the circuit impedance of the				
	combined conductor with the armour and ground in parallel				
Phase impedance, Ω/km	$\hat{\boldsymbol{z}}_{abc} = \begin{bmatrix} 0.392 + j0.140 & 0.197 + j0.095 & 0.194 + j0.046 \\ 0.197 + j0.095 & 0.395 + j0.189 & 0.197 + j0.095 \\ 0.194 + j0.046 & 0.197 + j0.095 & 0.392 + j0.140 \end{bmatrix}$				
Sequence impedance, Ω/km	$\hat{\boldsymbol{z}}_{012} = \begin{bmatrix} 0.785 + j0.314 & 0.027 - j0.018 & -0.029 - j0.014 \\ -0.029 - j0.014 & 0.196 + j0.078 & -0.014 + j0.009 \\ 0.027 - j0.018 & 0.015 + j0.007 & 0.196 + j0.078 \end{bmatrix}$				
Finite el	ement modelling, reduced steel wire relative permeability $\mu_{\rm r}=50$				
Sequence impedance, Ω/km	$\hat{\boldsymbol{z}}_{012} = \begin{bmatrix} 0.785 + j0.312 & 0.027 - j0.018 & -0.029 - j0.014 \\ -0.029 - j0.014 & 0.196 + j0.078 & -0.013 + j0.009 \\ 0.027 - j0.018 & 0.015 + j0.007 & 0.196 + j0.078 \end{bmatrix}$				
Finite eler	nent modelling, increased steel wire relative permeability $\mu_{ m r}=1000$				
Sequence impedance, Ω/km	$\hat{\boldsymbol{z}}_{012} = \begin{bmatrix} 0.785 + j0.314 & 0.027 - j0.018 & -0.029 - j0.014 \\ -0.029 - j0.014 & 0.196 + j0.078 & -0.014 + j0.009 \\ 0.027 - j0.018 & 0.015 + j0.007 & 0.196 + j0.078 \end{bmatrix}$				
Finite element modelling, no steel wire armour					
Sequence impedance, Ω/km	$\hat{\boldsymbol{z}}_{012} = \begin{bmatrix} 0.782 + j0.282 & 0.024 - j0.016 & -0.026 - j0.013 \\ -0.026 - j0.013 & 0.196 + j0.071 & -0.012 + j0.008 \\ 0.024 - j0.016 & 0.013 + j0.007 & 0.196 + j0.071 \end{bmatrix}$				
Finite element modelling, with gaps to represent space between sector conductor strands					
Sequence impedance, Ω/km	$\hat{\boldsymbol{z}}_{012} = \begin{bmatrix} 0.785 + j0.311 & 0.027 - j0.018 & -0.029 - j0.014 \\ -0.029 - j0.014 & 0.196 + j0.078 & -0.014 + j0.009 \\ 0.027 - j0.018 & 0.015 + j0.007 & 0.196 + j0.078 \end{bmatrix}$				

# Table 4-8 – Impedances for the 4-core 95 mm<sup>2</sup> BS 5467 cable at 50 Hz, with concentric earth and ground connected at both ends

As an example of the impact of neglecting the concentric earth, the voltage drop is considered for a balanced current of 50 A over a cable of length 100 m. If the concentric earth is included, the RMS voltage drop is

$$|\Delta V_{\rm no \, earth}| = \begin{pmatrix} 1.20\\ 1.07\\ 0.85 \end{pmatrix}$$
(78)

If the concentric earth is included with  $\mu_r = 300$  the voltage drop for the same current and cable length is:

$$|\Delta V_{\text{with earth}}| = \begin{pmatrix} 1.24\\ 1.10\\ 0.85 \end{pmatrix}$$
(79)

Neglecting the steel wire armour therefore introduces an error of around 3% into the voltage calculation. Although this error is not large, it is avoidable, unlike other uncertainties relating to temperature and variations in the cross-sectional profile of the cable.

These voltage calculations also illustrate the impact of the asymmetry between the three phases due to the four-core structure of the the cable

The impedance data for the case with  $\mu_r = 300$  has been adopted for use in Chapter 5 where the measured impedance is compared against values predicted by these FE simulations. Impedance data has been generated for each of the harmonic frequencies.

The FE model has also been used to investigate the approximation made in modelling the sector conductors with a uniform conductivity. From Figure 4-8 it can be seen that there are gaps between the circular strands when they are compacted into the sector shapes. Based on the measured dimensions of the BS 7846 sample, the total area of a sector increases to 106 mm<sup>2</sup>, compared to the nominal 95 mm<sup>2</sup> for a solid conductor, an increase of 12%. The impact of this has been modelled by inserting non-conductive gaps within the sector shape, as shown in . The inclusion of the strand gaps has a slight impact on the zero sequence reactance, but the approximation using uniform sector resistivity appears to be reliable.



Figure 4-10 – BS 5467 FE model with non-conductive gaps to represent stranded conductor

The effects described here are unfortunately difficult to investigate using the conventional analysis based on the GMR and GMD of the conductors, highlighting the benefits of the FE approach. However, the result that the stranded cable can be approximated as a having conductors with a uniform sector area of equivalent conductivity is useful as it means confirms that the simpler analytical method can then be used to provide a close approximation. Unfortunately this is not the case for the model of the sheath where the presence of the steel wire armour affects the impedances, even though this is not included in the circuits with the phase conductor and neutral, and needs to be included in order to obtain an accurate model.

## 4.5 Conclusions

The impedances of 3-core Waveform cables have been modelled using several analytical techniques and by FE simulation.

In the simplest model the sector shapes were approximated as being circular with uniform current distribution. An approximation is also made that the concentric earth is isolated from the ground. The second method introduces a more detailed model of the sector-shape geometry, such that the close spacing between the sides of the sectors is represented, although the current is still assumed to have a uniform density. This gives a 14% decrease in the zero sequence reactance and a 7%

increase in the positive sequence reactance compared to the approximation with circular conductors. The third model includes corrections to the AC resistance calculated according to IEC 60287. At 50 Hz, these corrections have negligible impact for the 95 mm<sup>2</sup> cable but there is a 6% increase in the AC resistance for the 300 mm<sup>2</sup> cable size. Finally, the ground path is added to the model and assumed to be connected to the neutral. This decreases the zero sequence resistance but increases the zero sequence reactance by a factor of 4.

The results from the analytical methods were then compared with impedances from an FE simulation. At 50 Hz, the analytical methods give a good approximation to the FE results provided that the sector geometry is taken into account and using corrections to allow for the AC resistance. At harmonic frequencies, analytical methods diverge from the FE results, which are assumed to be more accurate.

At 50 Hz, the FE simulations matched the analytical results with the AC resistance corrections and ground path to within 1%. However, the results diverge at harmonic frequencies with the FE model predicting differences at 450 Hz of 16% for the 95 mm<sup>2</sup> cable and 33% for the 300 mm<sup>2</sup> cable. These results suggest that the significantly increased complexity of using an FE model is not required for impedances at 50 Hz but is needed in order to obtain accurate impedance data at harmonic frequencies.

A key observation from the FE simulations, also supported by Carson's equations, is that the assumption of a multi-grounded neutral implies a current distribution in the ground with a very wide cross-sectional area and a radius of up over 1 km needs to be considered in the model. This dimension is much longer than the typical length between nodes for an LV cable and so the 'end effects' where the current enters and leaves the ground at earth electrodes cannot be neglected. This is considered further in Chapter 9.

The FE modelling method has been also used to generate impedance data for a BS 5467 4-core cable that will be used in Chapters 5 and 6 to compare against impedances estimated from measurements of a feeder on the Loughborough campus. In this case, the neutral of the cable is not grounded, so the impedance of this cable is not subject to the ground current concerns noted above. However, it is possible for the outer armour layer to be grounded and so this and the ground conductor have been included in the model. Since there is the mutual impedances between each of the sector conductors (in which the current sums to zero) and the ground are equal, the mean of the induced eddy currents in the armour and ground is zero. However, the armour is ferromagnetic and so this must be retained in the model in order to model the reactance accurately.

The impacts of several possible manufacturing tolerances have been investigated for the Waveform cable type. The permitted tolerances for the conductor resistance allow up to 3% variation in the magnitude of the impedance and also affect the zero sequence resistance. Other variations in the geometry have less impact, but a 50% variations in the insulation thickness was found to cause variations of +9% in the positive sequence reactance and up to -11% in the zero sequence reactance for an ungrounded neutral.

Perhaps more importantly, the resistance is sensitive to the conductor temperature, and therefore sensitive to seasonal variations and to temperature change due to losses varying with the demand. The positive resistances of the cable increase according to the temperature coefficient of the conductor material. For a cable with grounded neutral, the zero sequence reactance also increases with temperature since more of the unbalance current flows through the ground.

Finally, the shunt admittance has also been considered. This is addressed using a simple analytical model and also with an FE simulation approach. The FE model allows for the non-uniform electric field between the closely spaced sector conductors. However, the currents due to the shunt admittance are small at LV for the cable type considered and could be omitted from a network simulation.

# 5 MEASUREMENTS WITH RMS DATA

# 5.1 Introduction

There are few studies in which simulations of voltages on LV distribution networks are validated against measurements. Whilst there is no concern that the voltage differences can be predicted by Ohm's law, a great many practical uncertainties remain. These uncertainties include approximations in determining the cable impedance, the errors associated with the measurement equipment, and the methods used by the instrumentation to summarise the time varying waveforms so that the current and voltage can be captured and recorded. In order to investigate these uncertainties, new measurements have been made using the LV distribution network on the Loughborough University campus

These measurements had the following objectives.

- 1. Verify the series impedance matrix used in the calculation of voltage drops and losses.
- 2. Provide a high resolution reference so that the impacts of using lower resolution monitoring data can be assessed.

New measurement data was required for this work due to the difficulty of obtaining access to detailed information to describe the test configurations for the data that was already available. Typically it was unclear exactly where the instrumentation had been installed and there was limited information available to describe the network topology. In the data measured on live networks, the current was typically measured at the substation, but not at each of the customer connections. Without time-synchronised current measurements for each customer, the current in each network branch was not known and so it was not possible to predict the voltages differences in the feeder.

It was therefore decided to make new measurements on an LV cable on the Loughborough University campus. Two power quality analysers were available for this work and so the measurement setup required a single section of cable with the voltage difference calculated between the two end nodes. There were no intermediate junctions along the length of this cable and so no currents entering or leaving the cable between the two end points. The currents were fully therefore characterised by the measurements at either of the power quality analysers.

Since the same current was measured by both power quality analysers, this provided a means of time-aligning the recorded data so that the voltages measured by one analyser could be compared with voltages measured at the corresponding measurement interval from the other analyser.

The selected feeder cable section provides the connection between a secondary substation and a distribution cabinet at a nearby building. Conveniently, there was space available at both the substation and the distribution cabinet the power quality analysers to be installed, as well as terminals that could be used to attach the voltage probes without needing to switch off the supply. A 230 V mains socket was installed at both the substation and cabinet so that the power quality analysers could be used for long periods without needing to re-charge their internal batteries.

A detailed description of the underground cable route and of the type and size of the cable was also available from the Loughborough University Facilities Management team. It was also necessary for a number of visits to the test site to be made so that the protection equipment and the exact connectivity of the neutral and ground conductors could be examined and represented in the models. This level of detail is not typically included in other published studies.

The building supplied by the cable was in active use and so the demand varied as appliances were switched on or off by the occupants. The building contained office space with computing equipment and kitchen appliances and also a laboratory area used by the Geography Department. Additional appliances were connected for the purposes of the tests so that the demand could be varied. This allowed measurements to be made with different power factors and varying balance between the currents in the feeder.

The laboratory also housed a riverbed flume installation which incorporated a 9 kW three-phase variable speed water pump. The flume could be switched on with power output gradually increased over a period of a few minutes, in order to vary the demand on the feeder and to introduce additional current distortion.

By using these different demand conditions, the tests described here take a different approach to methods used in recent network modelling projects where the aim is to consider a demand that is statistically representative of a wide range of feeders (Electricity North West Ltd 2015a). This is not necessary for the test described here where the aim is to compare results predicted using simulations with results that are actually measured. This comparison needs to consider a wide range of demand variations, but it is not necessary to know the probability that these variations would occur on a real network. Consequently, although the measurements here required a high time and frequency resolution and detailed characterisation of the network, it was not necessary to capture the data for a long time duration.

The following sections describe the test configuration in detail, and then provide some examples of the demand and the measured distortion. The results of the measurements are then described and two different approaches are taken with the analysis. In the first method, the impedance of the cable is estimated from the measured voltages and currents. The estimated impedance is then compared with the impedance predicted by the cable impedance simulations presented in Section 4.4. This provides a useful means of quantifying the difference between the measured and simulated cases and the estimated impedance matrix can also then be used to model the voltages and losses on the cable for other demand conditions.

The second method of analysis approaches this comparison from the opposite direction, with the predicted cable impedance being used to calculate the expected voltage drop along the cable. This voltage drop is then compared to the measured voltage drop. This second approach demonstrates whether differences between the estimated and predicted impedance matrices are sufficient to cause significant differences in the power quality metrics applied to the network.

# 5.2 Test configuration

#### 5.2.1 Feeder cable

Power quality analysers were installed at the two end locations on a point-to-point cable between an LV substation and a distribution cabinet at the 'Ann Packer' building on the university campus, as shown in Figure 5-1.



Figure 5-1 – Test cable layout

The cable was 4-core 95 mm<sup>2</sup> copper-stranded according to BS 5467 (BSI 2008) throughout with no intermediate branch junctions. This structure consists of four sector-shaped cores with concentric steel wire armour.

During campus maintenance works, the route of the cable was verified using a cable avoidance tool ('CAT scanner'), with markers on the ground at approximately 5 m intervals. The accuracy of these marking positions was claimed by the scanner operator to be approximately  $\pm 0.1$  m. The length of the underground cable route was then measured using a surveyor's wheel and found to be 119.7 m, with a variation of  $\pm 0.2$  m between repeated measurements.

At each end of the feeder section, the cable rises up into the equipment cabinets. The length of these sections was estimated as 5.5 m at the substation and 1.5 m at the distribution cabinet. The cable route within the substation was not easily visible and so this length was difficult to measure. An error of  $\pm 0.5$  m has been assumed for this length. Combining these measurements gives a total length of 126.7 m with an RMS error of  $\pm 0.55$  m or  $\pm 0.4\%$  of the total length. These errors might have been avoided if the measurements had used a section of cable in the laboratory, but this would not have been representative of a cable installed in a real network.

Although there are no branches or service cable connections between the two end points of the measured section, there is an underground repair to the cable approximately mid-way between the two end points, such that a short section of the feeder may have different characteristics to the rest of the cable.

It is assumed that the cable installed has a similar cross-sectional profile to cables defined by the BS 5467 standard, but this cannot be verified while the feeder is in service.

The connectivity of the test cable is shown in Figure 5-2. This shows the two measurement locations with voltage  $V_{A,iht}$  at the substation and voltage  $V_{B,iht}$  at the distribution cabinet. The voltage is recorded for each conductor *i*, frequency *h* h and measurement interval *t*. The current is measured at both ends of the cable, giving two measurements  $I_{A,iht}$  from the substation and  $I_{B,iht}$  from the cabinet, both of which should ideally provide the same data.



Figure 5-2 – Test cable circuit diagram

At the substation, both the neutral conductor and the steel wire armour were connected to the substation metalwork. The substation metalwork is also connected to the physical ground via a buried grounding electrode. At the distribution cabinet, the armour was again connected to the cabinet metalwork but there was no connection between the neutral and the armour. The cable installation therefore conforms to a TN-S grounding scheme (Cronshaw 2005) where the neutral is isolated from the ground (other than in fault conditions), as shown in Figure 5-2. This would potentially allow eddy currents to circulate either within the multiple wires of the armour conductor, or in loop between the armour and ground.

The impedances of the grounding electrodes  $Z_{gr}$  and  $Z_{gr}'$  shown in Figure 5-2 are unknown, as is the resistivity of the ground path between the two cable ends. However, based on the modelling work described in Section 4.4.2, the mean current in both the armour and the ground is expected to be zero. The circuit impedances of the phase conductors are then expected to be the same with the armour and ground being either isolated or connected at the distribution cabinet end of the cable. Despite this, it is important that the armour wires are included in this model as eddy currents can flow around the individual wires, and because the high relative permeability of the steel affects the magnetic fields due to current in the other conductors.

## 5.2.2 Power quality analysers

The measurements used two Fluke 435-II power quality analysers, as shown in Figure 5-3. These have accuracy specifications as shown in Table 5-1 for recording RMS voltage and current harmonics (Fluke 2012; Fluke 2006b). These are taken as indicative of the accuracy in the waveform capture mode although this is not explicitly stated in the documentation. A key observation is that the accuracy of the Rogowski coil is dependent on the positioning of the coil around the conductor, with a  $\pm 2\%$  error if the conductor is not aligned with the centre axis.

Power quality analyser				
Current resolution	1 A (prior to scaling factor of 0.38 for waveform capture,			
	giving net resolution of 0.38 A)			
Current accuracy	±0.5%			
Voltage resolution	0.1 V			
Voltage accuracy for harmonic number $h$	$\pm 0.1\% \pm h \times 0.4\%$ relative to nominal voltage, 230 V			
Phase angle resolution	1°			
Phase angle accuracy	$\pm h \times 1^{\circ}$			
i430flex current transformer				
Basic accuracy	±1% of reading at 25 °C			
Linearity	±0.2% of reading at 10% to 100% of range			
Temperature dependence	0.08% of reading /° C for 0 °C to 70 °C			
Coil positioning	±2% of reading for non-axial CT installation			
Phase shift	±1° for 45 Hz to 65 Hz			

#### Table 5-1 – Fluke 435-II accuracy specifications

Both analysers were equipped with a GPS reference. This provides time synchronisation to a tolerance of 20 ms at each analyser and therefore a combined tolerance 40 ms for both analysers (Fluke 2012). The recorded data can therefore be considered to be synchronised in terms of the RMS measurement intervals of 250 ms, but not such that individual 50 Hz cycles are aligned. In practice, it was later found that the start of the measurement periods often differed by multiples of 10 cycles (equivalent to a data block size within the event data file format), suggesting that there is some variation in the start time of the sampling. This is discussed further in Section 6.2.

For these measurements the analysers were configured in a logging mode, allowing longer data capture periods of several hours, but retaining only the amplitude and phase of each harmonic, rather than the full time domain waveforms. The analysers were set to record at the maximum supported rate, giving a set of samples for each harmonic frequency every 250 ms. Within each 250 ms interval, the analysers recorded the RMS amplitude and mean phase angle, calculated over 10 waveform cycles at 50 Hz.

For the measurements described in this chapter, the analysers were configured to record the system frequency, plus the following parameters for both current and voltage:

- Amplitude and phase of fundamental frequency and up to the 21<sup>st</sup> harmonic
- RMS of the total waveform
- THD

The data was found to be subject to errors due to two measurement effects. Firstly, the logged harmonic amplitudes were smoothed by a filter within the Fluke analyser software (Fluke 2012). This filtering is defined by IEC 61000-4-7 (BSI 2002) as a digital low pass filter with a 1.5 s time constant. The effect of this filter can be observed with a step change in current, such as when an appliance is switched off for which the recorded RMS values show the step change but the recorded harmonics are seen to decay in accordance with the time constant. This filtering applies to all of the frequency spectral data recorded by the analyser (i.e. the fundamental frequency and all of the harmonics) and so limits the effective time resolution of the analyser to 1.5 s, rather than the recorded measurement interval of 250 ms.

A second complication is that although IEC 61000-4-7 defines the filtering to be applied to the amplitude of the harmonics, it does not define how the corresponding phase of the smoothed output should be recorded. Similarly the algorithm used to compute the RMS amplitude over the 250 ms period is defined, but the computation of the mean phase angle is less clear.

From examination of the data, it appears that the software within the Fluke analyser calculates an arithmetic mean of the phase of individual cycles within the 250 ms period. This gives rise to occasional measurements with unexpected results where the phase angle is close to the wrap-around value of 360°. The phase angle for such measurements would in some cases be recorded as 180°, being the arithmetic mean of 0° or 360°. This effectively inverts the sinusoidal waveform for these samples.

Owing to the effects of the filtering and the phase angle averaging, the data contained a number of measurement intervals for which the sum of currents in all conductors significantly deviated from zero. In order to confirm that these unexpected results were due to the processing of the data, a measurement was made with the Rogowski coil placed around the complete cable assembly, including the phase, the neutral and the armour, and so measuring the residual current as the sum of currents in each conductor. This test is described in detail in Section 6.6 of the following chapter. For the analysis of the data described here, any measurement intervals for which the magnitude of the residual current was greater than 0.5 A were excluded from the analysis.



Figure 5-3 – Fluke 435-II power quality analyser installed at the substation

#### 5.2.3 Protection equipment impedances

The measured voltage difference between the substation and cabinet includes a voltage drop across the LV circuit protection equipment, as shown in Figure 5-2. At the substation, the downstream circuits are connected in parallel to the LV bus-bar, with each circuit protected by fuses in the phase conductors. For the test measurements, the voltage probes were attached to the fuse terminals of a spare circuit. Assuming that the voltage drop across the bus-bar can be neglected, the voltage on the fuses of the spare circuit (with no current flowing) is assumed to be equal to the voltage on the bus-bar side of the fuses in the Ann Packer circuit.

The fuses for the Ann Packer building circuit were Lawson CTFP parts, rated at 200 A. The power dissipation in this type of fuse has been measured by Lawson

Fuses as 14.9 W at the rated 200 A current. This corresponds to a resistance at the rated current of 0.37 m $\Omega$ . This value is typically expected to be 35% greater than the resistance of the fuse when cold, suggesting a lower bound to the resistance of 0.28 m $\Omega$ . The power dissipation in the fuse therefore varies with the square of the current, and in proportion to a temperature dependent resistance that lies between these bounds.

The impact of the fuses on the reactance has not been included here. This impact may be small since the fuses represent a very short length in relation to that of the cable. However, the conductor dimensions are likely to be narrower than those of the cable conductors, and the separation between them is much greater than that in the cable bundle, so a much higher inductance per unit length would be expected. This could potentially be modelled using the FE simulation technique although a 2-D planar model may not be valid since the length of the fuses is less than the spacing between them.

At the distribution cabinet, shown in Figure 5-4, the incoming cable connects to a Merlin-Gerin NS240 NA switch disconnector, which is then connected to an NS160 N circuit breaker. The voltage probes are attached on the downstream side of this circuit breaker, such that the measured voltage differences between the substation and the cabinet includes the voltage drop through both the incoming and outgoing protection devices.

These products are now supported by Schneider Electric (Schneider-Electric 2015). The circuit breaker operation is based on thermal heating of a bi-metallic strip which has a power dissipation of 14 W at the 160 A rated current, giving an equivalent resistance of 0.55 m $\Omega$ . Data for the corresponding resistance at lower currents was not available.

The disconnector is a manual switch with no automatic protection function. Since this consists only of metallic contacts, no additional resistance has been included for this part.

As with the fuses, the impact of this protection equipment on the reactance has not been included.



Figure 5-4 – Protection equipment in cabinet, showing incoming feeder cable from substation (red, yellow blue colouring), connected to switch disconnector, circuit breaker, and outgoing feeder (brown, black, grey colouring)

The fuses and protection equipment are in series with the phase conductors, but not included on the neutral. The resistances noted above are included in the impedance matrix by their addition of a total of  $0.83 \text{ m}\Omega$  to the resistive components of the self-impedance terms. At 20 °C, the cable has a nominal conductor resistance of  $0.193 \Omega/\text{km}$ , or 24.5 m $\Omega$  for the 126.7 m length so neglecting the resistance through the protection equipment would introduce a 3.4% error to the calculation.

#### 5.2.4 Cable termination impedance

Figure 5-5 shows the cable routing within the switch frame at the substation. The cable from the Ann Packer circuit connects to the floor of the switch frame, with the armour connected to the frame metalwork. The four sector conductors continue vertically, initially maintaining their square configuration, but becoming further separated as the separate conductors are routed towards the fuses. The conductors enter the fuse compartment in a side-by-side configuration, entering this part of the frame through four vertically spaced holes at approximately 50 mm centre-to-centre spacing. The cable runs for approximately 1 m length in this configuration, with the

geometry variations account for only 1% of the total cable length from the substation to the cabinet, significant differences in the reactance might be expected.

In order to estimate the impact of this conductor routing on the impedance, three variations to the standard cable geometry have been modelled using the finite element simulation method. These 2-D planar simulations provide an approximate model of the actual configuration, which would ideally require a 3-D model.



Figure 5-5 – Cable routing within the substation frame

The three configurations considered were:

- 1. The cable in conventional quad-sector configuration but with the sectors offset an additional 10 mm from the central axis, as in Figure 5-6. This models the cable where routed from the switch frame floor.
- The cable with conductors side-by-side, ordered as shown in Figure 5-5. A centre-to-centre spacing of 50 mm was assumed. This models the cable where the conductors are positioned so that they can be routed in to the fuse enclosure.
- 3. As above, but with the addition of a steel box, 1 mm thickness at 50 mm spacing from the row of conductors, and with outer dimensions 200 mm x 180

mm, as in Figure 5-7. This models the cables within the fuse enclosure. The steel was defined to have relative permeability of 1000, and electrical conductivity of 7 MS/m.

The circuit impedance matrices for these geometry variations are shown in Table 5-2. Although the lengths of the sections with these different geometries are short, the results show that there can be a significantly higher reactance.



Figure 5-6 – Cable model for variation 1 with increased radial offset of 10 mm



Figure 5-7 – Cable model for variations 2 and 3 with side-by-side conductors Steel frame to model fuse enclosure included for variation 3 only.

Configuration	Circuit impedance matrix		
Cable according to BS 5467, $\Omega/km$	$\hat{\boldsymbol{z}}_{50,\text{cable}} = \begin{bmatrix} 0.392 + 0.140j & 0.197 + 0.095j & 0.194 + 0.046j \\ 0.197 + 0.095j & 0.395 + 0.189j & 0.197 + 0.095j \\ 0.194 + 0.046j & 0.197 + 0.095j & 0.392 + 0.140j \end{bmatrix}$		
Variation 1 with radial offset 10 mm, $\Omega/km$	$\hat{\mathbf{z}}_{V1} = \begin{bmatrix} 0.388 + 0.219 j & 0.194 + 0.131 j & 0.194 + 0.088 j \\ 0.194 + 0.131 j & 0.388 + 0.262 j & 0.194 + 0.131 j \\ 0.194 + 0.088 j & 0.194 + 0.131 j & 0.388 + 0.218 j \end{bmatrix}$ Increased reactance of self- and mutual impedances		
Variation 2 with conductors adjacent, $\Omega/km$	$\hat{\mathbf{z}}_{V2} = \begin{bmatrix} 0.387 + 0.295j & 0.194 + 0.190j & 0.194 + 0.172j \\ 0.194 + 0.190j & 0.387 + 0.381j & 0.194 + 0258j \\ 0.194 + 0.172j & 0.194 + 0.258j & 0.387 + 0.432j \end{bmatrix}$ Reactance of self- and mutual impedances approximately doubled		
Variation 3 with conductors adjacent and steel frame, $\Omega$ /km	$\hat{\mathbf{z}}_{V3} = \begin{bmatrix} 0.3970 + 0.378j & 0.209 + 0.287j & 0.211 + 0.267j \\ 0.209 + 0.287j & 0.416 + 0.557j & 0.230 + 0446j \\ 0.211 + 0.267j & 0.230 + 0.446j & 0.441 + 0.714j \end{bmatrix}$ Reactance of self- and mutual impedances more than doubled, matrix more asymmetrical, and increased resistance due to eddy current effects		

#### 5.2.5 Combined impedance matrix

The impedances of the protection equipment and the impacts of the cable geometry at the substation have been combined with the cable impedance to give the total predicted impedance matrix for the feeder between the two measurement locations.

For each harmonic frequency, this is given by:

$$\widehat{\boldsymbol{Z}}_{\boldsymbol{h}, \mathbf{feeder}} = l_{\mathrm{C}}\widehat{\boldsymbol{z}}_{\boldsymbol{h}, \mathbf{cable}} + \sum_{i=1}^{3} l_{Ti} \left[ \mathrm{re}(\widehat{\boldsymbol{z}}_{\mathbf{V}i}) + \mathrm{j}h \operatorname{im}(\widehat{\boldsymbol{z}}_{\mathbf{V}i}) \right] + R_{\mathrm{protection}} \times U$$
(80)

where:

 $\hat{z}_{h,\text{cable}}$  is the frequency-dependent impedance matrix of the cable, from Table 4-8,  $l_c$  is the total length of the cable with standard geometry (125.2 m),

 $\hat{z}_{Vi}$  is the impedance of one if the three sections with alternative geometry from Table 5-2, (since these have only a small contribution to the total, the impedance calculated for 50 Hz is simply scaled for other harmonics rather than being modelled again for each frequency)

 $l_{Ti}$  is the corresponding length (each 0.5 m)

 $R_{\text{protection}}$  is the sum of the resistance contributions of the fuses and protection

#### equipment from Section 5.2.3, and

U is the 3  $\times$  3 identity matrix.

The combined impedance predicted for the feeder section as shown in Table 5-3:

Impedance for cable only for 126.7 m length, 50 Hz				
Circuit impedance, $\Omega$	$\widehat{\boldsymbol{Z}}_{cable} = \begin{bmatrix} 0.0496 + 0.0178j & 0.0250 + 0.0120j & 0.0246 + 0.0058j \\ 0.0250 + 0.0120j & 0.0500 + 0.0240j & 0.0250 + 00120j \\ 0.0246 + 0.0058j & 0.0250 + 0.0120j & 0.0496 + 0.0178j \end{bmatrix}$			
Sequence impedance, $\Omega$	$\boldsymbol{Z_{012,cable}} = \begin{bmatrix} 0.0995 + 0.0397j & 0.0034 - 0.0023j & -0.0037 - 0.0018j \\ -0.0037 - 0.0018j & 0.0249 + 0.0099j & -0.0017 + 0.0012j \\ 0.0034 - 0.0023j & 0.0019 + 0.0009j & 0.0249 + 0.0099j \end{bmatrix}$			
Impedance for combined feeder with geometry variations, 50 Hz				
Circuit impedance, $\Omega$	$\widehat{\mathbf{Z}}_{\mathbf{feeder}} = \begin{bmatrix} 0.0504 + 0.0180j & 0.0250 + 0.0122j & 0.0246 + 0.0060j \\ 0.0250 + 0.0122j & 0.0509 + 0.0243j & 0.0250 + 00123j \\ 0.0246 + 0.0060j & 0.0250 + 0.0123j & 0.0505 + 0.0183j \end{bmatrix}$			
Sequence impedance, $\Omega$	$\boldsymbol{Z}_{012, \text{feeder}} = \begin{bmatrix} 0.1004 + 0.0405j & 0.0034 - 0.0024j & -0.0037 - 0.0019j \\ -0.0037 - 0.0019j & 0.0257 + 0.0101j & -0.0017 + 0.0012j \\ 0.0034 - 0.0024j & 0.0019 + 0.0009j & 0.0257 + 0.0101j \end{bmatrix}$			

Table	5-3 –	Cable	impedance	and	combined	impedance	matrix	for	feeder	length

Collectively, including the additional allowances for the protection equipment and for the geometry variations has increased the positive sequence resistance by around 3% and the positive sequence reactance by about 2%. The zero sequence resistance is increased by 1% and the zero sequence reactance by 2%.

Clearly these are not large differences, and the relative impact of these factors would be less for a feeder of several hundred metres. Despite this, these additional factors can have some impact on the impedance and so they are included here where the aim is to quantify the effect these contributions.

The circuit impedances for harmonic frequencies up to h = 13 are given in Table 5-4.

Frequency, Hz	Circuit impedance, Ω			
50	$\widehat{\boldsymbol{Z}} = \begin{bmatrix} 0.050 + 0.018j & 0.025 + 0.012j & 0.025 + 0.006j \\ 0.025 + 0.012j & 0.051 + 0.024j & 0.025 + 0.012j \\ 0.025 + 0.006j & 0.025 + 0.012j & 0.051 + 0.018j \end{bmatrix}$			
150	$\widehat{\boldsymbol{Z}} = \begin{bmatrix} 0.055 + 0.052j & 0.029 + 0.035j & 0.026 + 0.018j \\ 0.029 + 0.035j & 0.059 + 0.070j & 0.029 + 0.035j \\ 0.025 + 0.016j & 0.028 + 0.032j & 0.054 + 0.047j \end{bmatrix}$			
250	$\widehat{\boldsymbol{Z}} = \begin{bmatrix} 0.063 + 0.084j & 0.034 + 0.055j & 0.028 + 0.029j \\ 0.034 + 0.055j & 0.070 + 0.110j & 0.034 + 0.056j \\ 0.028 + 0.029j & 0.034 + 0.056j & 0.063 + 0.085j \end{bmatrix}$			
350	$\widehat{\boldsymbol{Z}} = \begin{bmatrix} 0.071 + 0.112 j & 0.040 + 0.073 j & 0.030 + 0.040 j \\ 0.040 + 0.073 j & 0.081 + 0.146 j & 0.040 + 0.074 j \\ 0.030 + 0.040 j & 0.040 + 0.074 j & 0.071 + 0.113 j \end{bmatrix}$			
450	$\widehat{\boldsymbol{Z}} = \begin{bmatrix} 0.079 + 0.138j & 0.045 + 0.089j & 0.032 + 0.050j \\ 0.045 + 0.089j & 0.092 + 0.179j & 0.045 + 0.090j \\ 0.032 + 0.050j & 0.045 + 0.090j & 0.079 + 0.140j \end{bmatrix}$			
550	$\widehat{\boldsymbol{Z}} = \begin{bmatrix} 0.086 + 0.162j & 0.050 + 0.105j & 0.035 + 0.059j \\ 0.050 + 0.105j & 0.101 + 0.210j & 0.050 + 0.106j \\ 0.035 + 0.059j & 0.050 + 0.106j & 0.086 + 0.165j \end{bmatrix}$			
650	$\widehat{\boldsymbol{Z}} = \begin{bmatrix} 0.093 + 0.186j & 0.055 + 0.120j & 0.038 + 0.068j \\ 0.055 + 0.120j & 0.110 + 0.240j & 0.055 + 0.121j \\ 0.038 + 0.068j & 0.055 + 0.121j & 0.093 + 0.189j \end{bmatrix}$			

Table 5-4 – Combined circuit matrix vs. frequency

## 5.3 Measured demand characteristics

#### 5.3.1 Demand variation

Data was recorded for a period of approximately 21.5 hours, beginning at 4 pm on the 19<sup>th</sup> January 2015, and continuing until 1:30 pm on the 20<sup>th</sup>. The three-phase active power demand for this period is shown in Figure 5-8.

The flume equipment was switched on at 9 am on the 20<sup>th</sup> with the pump speed gradually increased over a period of 10 minutes, then held at full speed for a further 10 minutes. With the flume at full speed, a maximum current of 55 A was reached on the L2 phase. Figure 5-9 shows the current variation for the 20 minute period as the flume was switched on.

After the flume had been switched off, the demand increased to higher levels due to the appliance use by occupants of the building. The demand was significantly unbalanced throughout the test period with much lower current on phase L1 and L3, and therefore high currents in the neutral.







Figure 5-9 – RMS current, demand variation, showing period with flume switched on

## 5.3.2 Current and voltage distortion

Due to the characteristics of the variable speed pump, the current was highly distorted with up to 75% THD when the flume was at full speed, as shown in Figure 5-10. The THD figures used in this plot have been calculated from the recorded harmonic data from the analyser at the substation using (102) (and substituting voltage values for current). These results were found to differ from the THD values recorded directly by the power quality analyser, for which the maximum THD was around 60%. The same comparison was made for the current data recorded at the cabinet, where both methods provided THD data that is consistent with Figure 5-10. It was therefore assumed that the calculated THD provides more reliable results than the recorded THD values.

The mean current distortion over the full measurement period of 17 hour was 20%, and so considerable distortion was present even when the variable speed pump was not in use

The RMS voltage on all three phases is relatively constant, maintained by a PowerPerfector unit at the substation to a target voltage of around 220 V. This has been installed with the aims of reducing demand through conservation voltage reduction, and of managing power quality by reducing harmonics and improving the power factor at the substation.

The voltage distortion is shown in Figure 5-12. This plot shows the voltage THD calculated using the measured frequency amplitude data from the substation and the cabinet.

The voltage distortion is greater at the cabinet, particularly when the flume is at full speed and when the current distortion is greatest. The voltage distortion at the substation also increases for this period, such that it cannot necessarily be treated as a perfect voltage source when determining the harmonic voltages on the LV feeder. This is considered further in Section 6.9.2.



Figure 5-10 – Current THD



Figure 5-11 – Current harmonic distortion at substation with flume at full speed



Figure 5-12 – Voltage THD

#### 5.4 Estimation of cable impedance from measurements

#### 5.4.1 Estimation method

This section describes the estimation of the cable impedance from the measured voltages and current and compares this to the impedance matrix predicted from the modelling.

For each measurement interval *t*, and for each harmonic *h*, the feeder impedance is the 3 × 3 matrix  $\hat{Z}_h$  such that the line to neutral voltage vector at the cabinet  $V_{B,ht}$  is related to the measured phase current vector  $I_{ht}$  by and the voltage vector at the substation  $V_{A,ht}$  by:

$$V_{\mathbf{B}_{h}ht} = V_{\mathbf{A}_{h}ht} - \hat{\mathbf{Z}}_{h} \times I_{ht}$$
(81)

Although this equation cannot be resolved to find  $\hat{Z}_h$  for a single measurement interval, it would be possible to assemble a 3 × 3 current matrix  $I_{h,t_1:t_3}$  from three independent current measurements, and similarly, the corresponding 3 × 3 voltage matrix  $\Delta V_{h,t_1:t_3}$ . This would then allow an estimate of the impedance matrix  $\hat{Z}_{h,t_1:t_3}$  as:

$$\widehat{Z}_{h,t_1:t_3} = \left( V_{\mathrm{A},h,t_1:t_3} - V_{\mathrm{B},h,t_1:t_3} \right) \times I_{h,t_1:t_3}^{-1}$$
(82)

As there are many possible solutions to (82) using the many samples from the measurement data, a curve-fitting approach has been adopted so as to find an estimate of the impedance with the best fit to the set of time samples within the measurement period using the Matlab function 'Isqcurvefit'. For each frequency, this was configured to find a matrix  $\hat{Z}_h$  to minimise the mean square error between the  $3 \times N_t$  matrix  $V_{B,measured,ht}$  of voltages measured at the cabinet, and a predicted  $3 \times N_t$  matrix of voltages  $V_{B,predicted,ht} = V_{A,ht} - \hat{Z}_h \times I_{ht}$ .

Since the analysers have no common phase reference, they can only record the relative angles between the voltages and currents. The convention adopted by the analysers is that these are normalised relative to the phase of the L1 voltage. This phase normalisation needs to be implemented in the estimation function in order to compare the predicted and measured voltages. The Matlab curve fitting function therefore finds a matrix  $\hat{Z}_h$  that gives the minimum square error *E* for all  $N_t$  measurement intervals and for the 3 conductors, given by:

$$E_{h} = \min_{Z} \sum_{t=1}^{N_{t}} \sum_{i=1}^{3} \left| V_{\text{B,measured},iht} - V_{\text{B,predicted},iht} \cdot e^{-j \angle V_{\text{B,predicted},1ht}} \right|^{2}$$
(83)

The impedance  $\hat{Z}_h$  was constrained to be a symmetrical matrix such that the curve fitting process identified 12 scalar variables, giving the resistance and reactance for the six independent terms. This allows the self-impedances of all three line conductors to be different and so does not impose the constraint that the geometry is symmetrical. Conversely, in the FE simulation, a symmetrical geometry was assumed. This gave identical impedances for phases L1 and L3 (adjacent to the neutral) and only the impedances for phase L2 (opposite the neutral) were different.

The impedance estimation process has been found to give the best fit to the measured voltage drops if the input data is based only on the time period for which the flume was switched on. Outside of this time period, the current harmonic distortion is much lower and so the estimation of the harmonic impedances was less reliable. A subset of the data was therefore selected, using the 20 minute time interval plotted in Figure 5-9.

Impedances have been estimated for up to the 13<sup>th</sup> harmonic. The amplitude of current voltage components for higher frequencies was too low for the estimation algorithm to give reliable results.

A key assumption made in this curve fitting method is that the impedance matrix is constant for the duration of this time period. This is not necessarily the case if the resistance changes due to heat dissipation within the cable. However, the analysis in Section 4.2.6 suggests that the resistance will not vary significantly over this time period.

The results show obtained show a good fit between the voltage amplitude difference as measured and as calculated from the estimated impedance. The agreement between the measured and estimated data is shown in for the fundamental frequency and in for the 7<sup>th</sup> harmonic. This demonstrates that the curve fitting has found an impedance matrix that is consistent with the measured data.



Figure 5-13 – Voltage amplitude difference between substation and cabinet for 50 Hz from measured data and calculated using best fit impedance with voltage amplitude data



Figure 5-14 – Voltage amplitude difference between substation and cabinet for 350 Hz frequency from measured data and calculated using best fit impedance with voltage amplitude data

Since both analysers measure the same current, an alternative phase normalisation process could be defined so that the angles of the voltage and current vectors are expressed relative to a one of the current vectors, this preserving the phase offset due to the vector voltage difference along the cable. This process was applied using the phase angle of the current in conductor L2 as the reference. However, the curve-fitting algorithm did not find a good fit to the measured data, shown in Figure 5-15.

The poor convergence of the curve-fitting algorithm is thought to be due to errors with the phase normalisation. Since the current is the same at both analysers, it would ideally be possible to use any of the conductors as the reference. However, different normalisation angles could be obtained for each conductor. This may be partly due to the concerns relating to the phase angle data described in Section 5.2.2 but the tolerances of the current sensors are likely to be the dominant factor. These tolerances are large in comparison to the change in phase angle from one end of the cable to the other. The impact of these errors is investigated further in Chapter 6.

The results that follow therefore use the method from (83).



Figure 5-15 – Voltage amplitude difference between substation and cabinet for 50 Hz from measured data and calculated using best fit impedance data with voltage vector data

#### 5.4.2 Impedance results

The curve fitting algorithm provided the circuit impedance data shown in Table 5-5. The corresponding sequence impedances are shown in Table 5-6 and plotted in Figure 5-16 and Figure 5-17. The figures show the comparison between the estimated impedance and the predicted impedances from the FE simulations.

At 50 Hz, the resistances of the sequence impedance agree to within 1% of the values predicted by the FE simulations. However, the reactance values of the sequence impedances are higher than the predicted values, by 11% for the positive sequence and 15% for the zero sequence.

As the frequency increases the estimated and predicted impedances follow similar trends with differences of 10%-20% at some frequencies. The estimated reactances of the sequence impedances are consistently higher than the predicted values.

These results provide an experimental verification of the AC resistance effect due to eddy currents in the conductors. The eddy currents cause the current density in the conductors to be less uniform as the frequency increases, with resistance therefore increasing. The current density becomes higher in parts of a conductor that are nearer to conductors carrying current in the opposite direction (the proximity effect), and so the inductance of a circuit around these two conductors is reduced.

The figures also show some significant deviations at 650 Hz and results for higher frequencies and for the even harmonics (with very low current distortion) were similar. A less accurate estimate of the resistance might be expected at high frequencies since the magnitude of the impedance is increasingly dominated by the reactance. The curve fitting algorithm then finds an acceptable best fit solution which is insensitive to the parameters corresponding to the estimated resistance. At 650 Hz, the curve fitting algorithm returns the initialising value of zero for the resistance for some of the impedance terms. The diverging results at 650 Hz therefore appear to be due to limitations of the impedance estimation method, rather than indicating significantly different characteristics of the feeder cable.

	Circuit impedance, Ω			
Fraguanay	$\hat{Z}_{11}$	$\hat{Z}_{12}$	$\hat{Z}_{13}$	
	$\hat{Z}_{21}$	$\hat{Z}_{22}$	$\hat{Z}_{23}$	
112	$\hat{Z}_{31}$	$\hat{Z}_{32}$	$\hat{Z}_{33}$	
	0.050+j0.018	0.024+j0.014	0.027+j0.007	
50	0.024+j0.014	0.051+j0.028	0.025+j0.014	
	0.027+j0.007	0.025+j0.014	0.052+j0.023	
	0.044+j0.062	0.028+j0.046	0.020+j0.024	
150	0.028+j0.046	0.059+j0.084	0.027+j0.041	
	0.020+j0.024	0.027+j0.041	0.053+j0.068	
	0.055+j0.092	0.034+j0.060	0.023+j0.038	
250	0.034+j0.060	0.065+j0.132	0.020+j0.064	
	0.023+j0.038	0.020+j0.064	0.060+j0.085	
	0.066+j0.117	0.038+j0.080	0.032+j0.040	
350	0.038+j0.080	0.082+j0.160	0.037+j0.078	
	0.032+j0.040	0.037+j0.078	0.069+j0.132	
	0.080+j0.150	0.048+j0.095	0.036+j0.054	
450	0.048+j0.095	0.093+j0.192	0.047+j0.099	
	0.036+j0.054	0.047+j0.099	0.081+j0.155	
	0.087+j0.177	0.060+j0.119	0.035+j0.066	
550	0.060+j0.119	0.101+j0.243	0.054+j0.119	
	0.035+j0.066	0.054+j0.119	0.088+j0.189	
	0.000+j0.217	0.000+j0.034	0.000+j0.000	
650	0.000+j0.034	0.000+j0.415	0.000+j0.028	
	0.000+j0.000	0.000+j0.028	0.000+j0.371	

Table 5-5 – Estimated circuit impedance, best fit over measured data

#### Table 5-6 – Estimated and predicted sequence impedance

	Sequence impedance at 50 Hz, $\Omega$			
	Z <sub>00</sub>	$Z_{01}$	Z <sub>02</sub>	
	$Z_{10}$	$Z_{11}$	Z <sub>12</sub>	
	$Z_{20}$	Z <sub>21</sub>	Z <sub>22</sub>	
	0.100+j0.041	0.003-j0.002	-0.004-j0.002	
Predicted	-0.004-j0.002	0.026+j0.010	-0.002+j0.001	
	0.003-j0.002	0.002+j0.001	0.026+j0.010	
	0.101+j0.047	0.003-j0.003	-0.004-j0.005	
Estimated	-0.004-j0.005	0.026+j0.011	-0.004-j0.001	
	0.003-j0.003	0.002+j0.001	0.026+j0.011	







Figure 5-17 – Estimated and predicted sequence impedance, inductance terms

## 5.5 Measured and predicted power quality metrics

This section compares the voltage drop and losses that can be calculated from the measured data with those that are calculated using the predicted impedance data. This analysis uses the same measured data as above in Section 5.4, but makes an alternative comparison. Whereas the previous section used the measured voltages and currents to calculate the impedance, then compared this with the prediction, this section calculates the voltages that would be expected with the predicted impedances and compares them with the measurement. This aim of this second comparison is to show whether differences between the predicted impedance matrix and the actual impedance of the cable have an impact on the estimated voltages and losses.

## 5.5.1 Voltage drop

The previous section has demonstrated that the zero sequence impedance of the cable appears higher than the values predicted in the FE cable modelling simulations. This section now considers whether these differences affect the voltage drop that would be calculated along the cable, in comparison with the measured voltage drops. The comparison uses the same measurement data as in the previous section and so provides and so is providing an alternative means of viewing the agreement between the estimated and predicted impedance data.

Using the measured current and simulated impedance, the predicted RMS voltage drops are calculated using the sum of the voltages for each frequency. The calculation of the voltage at the cabinet end of the cable uses the measured currents for each frequency and also a frequency-dependent impedance matrix. This is a considerably more detailed calculation than is normally employed, and is included here so that the impact of harmonics on the voltage drop is fully represented.

The voltage drops for the measured and simulated cases are given by:

$$V_{\rm d, predicted, it} = \sqrt{\sum_{h=1}^{N_{\rm h}} |V_{\rm A, iht}|^2} - \sqrt{\sum_{h=1}^{N_{\rm h}} |V_{\rm A, iht} - \sum_{k=1}^{3} \hat{Z}_{h, ik} I_{\rm A, kht}|^2}$$
(84)

(85)

predicted RMS voltage drop.

$$V_{d,measured,it} = \sqrt{\sum_{h=1}^{N_{h}} |V_{A,iht}|^{2}} - \sqrt{\sum_{h=1}^{N_{h}} |V_{B,iht}|^{2}}$$

The measured and predicted voltage drops are compared in Figure 5-18. This shows that the voltage drop calculated using the predicted impedances agrees closely with the measured voltage drop with the worst-case voltage drop on phase L2 matching to within 0.4%. However, there are slightly greater differences between the measured and predicted voltage drops for phases L2 and L3 with differences up to 2% for the period shown in Figure 5-18. Although the results of Section 5.4.2 suggest that the cable has a higher zero sequence reactance at 50 Hz than was expected from the FE modelling, these impedance differences do not significantly affect the





The voltage drop with the estimated impedance is now also compared with results based on simpler models of the cable impedance. This contrasts with the predicted impedance matrix used above which has been developed through detailed FE modelling and careful consideration of the impacts of protection equipment. This level of detail is not normally available so the following section compares the

measurements with calculated results that would be obtained if more approximate cable impedance data had been used.

The first approximation models the case where the positive and zero sequence impedance of the cable are known accurately, but the mutual sequence impedances are unknown. As discussed in Section 3.6, this provides the circuit impedances of the cable with the conductors transposed, and therefore does not represent any unbalance introduced due to the asymmetry of the cable. Effectively, the cable is approximated as having balanced impedances.

Two further approximations are considered where the impedance matrix is derived from the positive sequence impedance quoted by a cable manufacturer (AEI Cables 2015). The zero sequence impedance is not stated and so impedance matrices have been calculated based on the assumption that  $z_{00} = 4 \times z_{11}$  and that  $z_{00} = 3 \times z_{11}$ . The scaling factor of 4 is indicated by the FE simulation, and also consistent with the symmetry of the cable, as discussed in Section 3.6. However, the positive sequence reactance of 0.0723  $\Omega$ /km is lower than predicted by the FE model. The use of a scaling factor of 3 follows general guidance for a scaling factor between 2 and 3.5 (Nagrath 2008). Although this guidance is more applicable to overhead lines and to higher voltage distribution where unbalanced load currents return through the ground, the use of the same factors for LV networks is not contra-indicated.

The RMS voltage drop for these approximated impedances are shown in Figure 5-19, Figure 5-20 and Figure 5-21. The voltage calculation here uses only the fundamental as the approximated impedance matrices are not defined for harmonic frequencies.

For phase L2, the voltage drop with the transposed impedance is within 1% of the results with the full impedance matrix. However, the results with the transposed impedance are around 30% lower for phase L1 and 40% higher for phase L3. This demonstrates the same effect illustrated in Section 3.6.

The voltage drop results with the impedances based on the manufacturer's data differ more from the measured voltage drop. For the case where the zero sequence impedance  $z_{00} = 4 \times z_{11}$ , the worst case voltage drop on phase L2 is underestimated by 5%, and for the approximation that  $z_{00} = 3 \times z_{11}$ , the worst case voltage drop is under-estimated by 13%.







Figure 5-20 – Measured and predicted RMS voltage drop for 50 Hz, phase L2


Figure 5-21 – Measured and predicted RMS voltage drop for 50 Hz, phase L3

#### 5.5.2 Losses in the LV cable

The losses calculated from using the predicted impedances are now compared with losses derived from the measured power difference between the two ends of the cable. This evaluates whether the differences between the predicted and estimated impedances cause significant errors in the calculated losses.

The losses calculated based on the power difference are calculated for each measurement interval by:

$$P_{\text{loss,power difference},t} = \text{re}\left(\sum_{i=1}^{3}\sum_{h=1}^{N_{\text{h}}} \left(V_{\text{A},iht} \times I_{\text{A},iht}^{*} - V_{\text{B},iht} \times I_{\text{B},iht}^{*}\right)\right)$$
(86)

This method does not require any phase synchronisation between the two analysers.

Alternatively, the losses can be calculated using an  $I^2R$  metric where the resistance is derived from the impedance matrix.

$$P_{\text{loss},I^2R,t} = \text{re}\left(\sum_{i=1}^{3}\sum_{h=1}^{N_h} \left( \left(\sum_{k=1}^{3} \hat{Z}_{h,ik} I_{A,kht}\right) \times I_{A,iht}^* \right) \right)$$
(87)

In this calculation,  $\hat{Z}$  may be either the estimated impedance matrix or the predicted impedances from the FE simulations.

The losses for these three cases are compared in Figure 5-22. The mean losses calculated by the  $I^2R$  method with the predicted impedances are within 0.4% of those using the estimated impedances. The differences between these two impedance matrices therefore cause minimal impact to the loss calculations.



Figure 5-22 – Losses based on power difference, or I<sup>2</sup>R method using estimated and predicted impedance matrices

The losses calculated using the power difference method are not consistent with either of the  $I^2R$  results and appear highly noisy with no obvious relationship to the trends expected based on the demand profile. Examination of the measurement data shows that currents measured at the cabinet are generally slightly higher than the currents measured at the substation, with the relative difference being similar to the proportion of delivered power that is dissipated. Errors in the current amplitude measurements affect the calculation of the delivered power (rather than the loss power). The difference of these two power calculations is then subject to the combined errors of both sensors. Where the losses are small relative to the sensor tolerances, the power difference results are largely determined by the measurement accuracy. This can cause the losses to appear negative, as in Figure 5-22. This issue is explored in detail in Chapter 6, where both a constant offset and a noise-like variation in the amplitude measurements are observed. However, the results here show that the power difference method is not usable where the percentage errors in current amplitude estimates are of a comparable scale to the losses.

As in the previous section, the results with the estimated impedances can also be compared with results generated using approximated impedance matrices. The same examples are assumed as in Section 5.5.1, giving the results shown in Figure 5-23. The loss calculation here includes only the fundamental component as the approximations are not defined for harmonic frequencies. As in Figure 5-22, the mean losses using the estimated impedance matrix are very close to those using the impedances predicted from the FE model, with only a 0.2% difference. There is negligible difference to the losses if the impedances from the FE model are transposed (not plotted). However the losses calculated using the positive sequence impedance from the manufacturer and with approximated zero sequence impedances are lower, under-estimating the mean losses by 4% for  $z_{00} = 4 \times z_{11}$  and by 12% for  $z_{00} = 3 \times z_{11}$ .





# 5.6 Conclusions

This chapter has described measurements of the voltages and currents on a section of LV feeder cable. The selected cable was a single branch with no intermediate junctions. The same currents were therefore measured at both ends of the cable and a differential measurement of the voltage could be made. The feeder was located on the university campus where it was possible to gain access to the cabinet rooms, allowing the test configuration to be characterised in detail. This allowed for detailed consideration of the neutral and ground connectivity, the locations of current sensors and voltage probes in relation to protection equipment, and of the underground route of the cable.

The measurements used a time resolution of 250 ms and recorded the amplitude and phase of frequency components up to the 21<sup>st</sup> harmonic. This high resolution data is explored further in Chapter 8 where the effect of impact of using lower resolution monitoring is discussed.

The feeder cable was of BS 5467 type with 4-core 95 mm<sup>2</sup> conductors. This cable type has been modelled, as described in Section 4.4.2, so that the circuit impedance for the fundamental and harmonic frequencies is available for comparison with the measurements. Further impedance contributions have been added to this matrix to allow for the fuses at the substation and the disconnectors at the distribution cabinet.

The impedance of the test cable was then estimated from the measured voltages and currents using a curve fitting algorithm. This algorithm was configured to find an impedance matrix that would give a predicted voltage amplitude drop with the best fit to the measured voltage amplitude drop. The resistance terms of the estimated and predicted sequence impedances agreed to within 1% at 50Hz, but the estimated reactance terms were consistently higher by a ratio of 10%-20%.

For harmonics up to the 550 Hz, the estimated sequence impedances follow the same trends as the predicted values, providing a verification of the eddy current effects predicted by the FE modelling.

The measured voltage drop along the cable has been compared with the voltage drop that would be calculated using the predicted impedance matrix for the measured currents. This makes an alternative comparison using the same measurement data as used to estimate the impedances, and more clearly demonstrates the practical impact of the differences in the impedance matrices. This showed that the worst-case voltage drop using the predicted impedances agreed to within 0.4% of the measured voltage drop, although typically there were differences of up to 2% of the voltage drop in the most heavily loaded conductor. Despite the estimated impedance having a higher zero sequence reactance, this shows that network voltage calculations using this data differ only slightly from the measured voltage drop is compared against the voltage drop calculating using impedances based on the manufacturer's datasheet, with differences of between 5% and 13% depending on the approximation used to provide the zero sequence impedance. The results also demonstrate significant errors in estimating the voltage drop if the cable if the sequence impedance matrix is defined using only the positive and zero sequence impedance terms.

The losses within the cable were also calculated, firstly by finding the difference between the power supplied to and delivered from the cable, then by an  ${}^{\prime}I^{2}R'$  method using the impedance data. The mean losses calculated using the estimated and predicted impedances agreed to within 0.4%. Again, there are significant differences if the estimated losses are compared with calculations using impedances derived from the manufacturer's datasheet, with the mean losses under-estimated by between 5% and 13% depending on the approximation used to provide the zero sequence impedance.

The loss calculation based on power differences provided noise-like results, as a result of amplitude response differences in the Rogowski coil current transformers. The power difference method does not give usable results where the percentage tolerances on the CTs are of a similar scale to the percentage of delivered power that is dissipated as losses.

Although the measurement of these impedances might seem straightforward, the work has highlighted a number of practical difficulties that affect the results, many of which would also affect LV network measurements in general. Uncertainties relating to the feeder cable itself are:

<u>Cable length</u>: This was measured here to  $\pm 0.4\%$ , but this tolerance is based on an accurate marking of the route with a CAT scanner. Wider tolerances are likely to apply for underground cables where the routed is less clearly defined from network records.

<u>Fuses and protection equipment</u>: The additional resistance of this equipment added 3% to the circuit resistances. This increase would be less in relative terms for a longer feeder cable.

<u>Cable routing within the cabinet</u>: The cable conductors are widely spaced within equipment cabinets, in this case increasing the reactance by an estimated 1-2%.

<u>Underground joints</u>: The reactance may also be increased in cable repair fittings or service cable cut-outs where the conductors are spread out so as to facilitate the joints. This has not been quantified here, but the uncertainty has been highlighted by considering the cable routing within the cabinet.

Further uncertainty is introduced in relation to the power quality analysers:

<u>Phase averaging</u>: The method used to determine a mean phase over a measurement interval is unclear. In this particular case further issues were noted with the arithmetic averaging of phase angles at 360°.

<u>Harmonic filtering</u>: The smoothing filter defined in IEC 61000-4-7 distorts the measurement of step changes in the current. Over these transition periods, the sum of currents calculated from the measured data is not zero. The smoothing filter has the effect of reducing the time resolution of the recorded fundamental and harmonic frequency amplitudes.

<u>THD calculation</u>: The THD calculated from the recorded harmonic amplitudes was not always equal to the THD recorded by the power quality analyser.

<u>Sensor and measurement tolerances</u>: It was noted that the current from one analyser were higher in amplitude than those from the second analyser. The impact of these tolerances is explored further in Chapter 6.

Ideally, the impedance estimation algorithm used for this work would have incorporated both the amplitude and the phase data from the measurements. This would be expected to give a more accurate estimate of the reactance than the method using only the voltage amplitudes, since the reactive contribution to the voltage difference causes a rotation of the voltage phase angle rather than a significant change in amplitude. However, the impedance estimation method did not give usable results when using both phase and amplitude and the uncertainties noted above regarding the phase angle data are a possible cause of this problem. Further information was also required to characterise the impact of the tolerances on the current and voltage sensors. In order to address this, a second set of measurements was made using captured waveform data, as described in the next chapter.

For DNOs making future measurements, this work shows that the reliability of the network diagram is a key concern, as expected, but also that other aspects such as the protection equipment, can also affect the results. Monitoring instrumentation needs to be characterised before installation in order to determine the response to harmonics, to ensure that the desired measurement resolution is achieved in practice and to calibrate the measurements from sensors relative to a common reference.

# 6 MEASUREMENTS WITH WAVEFORM DATA

# 6.1 Introduction

The previous chapter provided estimates of the LV feeder impedance, with the positive sequence impedances showing close agreement with the predicted values. However, the reactance of the zero sequence impedance was higher than expected. As noted in Section 5.6, there was uncertainty over the reliability of the phase data recorded by the power quality analysers which provided a mean phase angle for each frequency, averaged over the measurement interval.

The analysis of the measurements with recorded RMS data also demonstrated a case in which the losses could not be calculated as the difference of the power into the cable and the power supplied from it as described in Section 5.5.2. Due to the amplitude tolerances of the Rogowski coil CTs, there were differences in the current measured by the two analysers, and these tolerances were of a similar proportion to the losses in the cable.

To investigate the amplitude and phase angles of the measured data, a second set of measurements has been made with the current and voltage waveforms sampled in the time domain with multiple samples per cycle. These measurements are described in this chapter.

In these measurements, the power quality analysers were configured to record waveform data using the 'PowerWave' operating mode. This mode records samples at 3.75 kHz rate, providing 75 samples per period of a 50 Hz cycle. The maximum duration of a single measurement is 5 minutes and so multiple measurements are needed to capture the demand over longer periods. The feeder cable and test setup was the same as used for the RMS data measurements described in Section 5.2. As before, the tests relied partly on the demand that occurred on the network over the period when the data was captured, but the power factor was also varied by connecting a capacitive load to a three-phase socket supplied by the feeder. Due to the practicalities of obtaining access, it was not possible to make use of the flume equipment with variable speed pump (see Section 5.2) for these measurements.

The data captured in this mode was saved by the power quality analysers into 'event' waveform files. The software supplied with the analysers does not provide a means of exporting the waveform samples as numeric data and so a software function was written to convert this from the binary-coded data files. This required the event waveform to be interpreted as a series of 16-bit integer values and then multiplied by 32-bit floating point scaling factors from a configuration file. This method provided numeric data outputs that corresponded to the values displayed graphically by the power quality analyser data analysis software.

# 6.2 Timing synchronisation

The tolerances of the timing alignment provided by the GPS reference allow for a timing alignment error between the two power quality analysers of one or more cycles. In practice, it was found that the triggered start of the waveform capture could differ by a greater error. The first step in analysing the data was therefore to re-align the two captured waveforms.

Where the analysers were placed at each end of the cable for measurement of the voltage difference, the synchronisation was based on aligning the current waveforms. Where the analysers were both placed at the substation for verification tests, the same voltage would be measured by both analysers, the synchronisation was based on aligning voltage waveforms which are less subject to error than the current waveform measurements.

In both cases, the following process has been followed so that the synchronisation error between the two waveforms is minimised.

To align the two recorded waveforms the RMS amplitude of each line phase was calculated for each cycle period. The waveforms were then aligned based on the waveform with the maximum RMS amplitude, assuming that this would have the minimum errors due to measurement noise. The timing error between the two analysers is then calculated by finding a number of offset cycles for which there is a minimum RMS difference between the RMS amplitude measured at the two analysers. An example of this timing offset calculation is shown in Figure 6-1 where it was found that the data capture from the analyser at the cabinet was triggered 50 frequency cycles later than in the analyser at the substation.



Figure 6-1 – Cycle synchronisation results

Having identified the synchronisation error to the nearest 50 Hz cycle, the timing offset was then refined to find the number of offset samples within one cycle. This offset was given by the number of samples with the minimum RMS difference between the waveforms from the two analysers, over a range of plus or minus half of a cycle. As before, the line phase with the greatest maximum RMS amplitude is taken as the reference. An example of this sample synchronisation is shown in Figure 6-2. This shows a search window of  $\pm 37$  samples either side of the cycle offset (50 x 75 samples) from Figure 6-1. In this example, the optimum alignment was achieved with the waveform from the analyser at the cabinet advanced by 3761 samples. (This method cannot resolve beyond  $\pm 0.5$  samples and it can be seen that an offset of 3760 samples would also give a close alignment.)

The waveforms were then time aligned to the nearest sample, applying the same offset to all voltage and current channels from each analyser. However, with a time resolution of 75 samples per cycle, this alignment still allows a variation of  $\pm 2.4^{\circ}$  and a further phase alignment was therefore needed. The phase alignment method was developed based on observations from the verification tests and so this is described later in Section 6.4.4.



Figure 6-2 – Sample synchronisation results

# 6.3 Spectral analysis

Following the synchronisation process described above, the data was then transformed into frequency domain samples by calculating a Fast Fourier Transform (FFT) of the samples in each cycle. It is assumed here that the frequency is sufficiently close to 50 Hz that one waveform cycle is exactly equivalent to 75 samples.

The 75 samples used to calculate each FFT are scalar values and so the FFT output provides 37 complex-valued phasors, plus a scalar-value DC offset. Each phasor in the resulting spectrum represents an integer harmonic, and there are no inter-harmonic components. It would therefore be possible to reconstruct the original sampled time domain waveform exactly by applying an inverse FFT.

The amplitudes of the frequency components can be combined to calculate the true RMS  $U_{\text{RMS},it}$  (with *U* representing either current or voltage) of the waveform on conductor *i* for measurement interval *t*, given by:

$$U_{\rm RMS, it} = \sqrt{\sum_{i=1}^{N_{\rm h}} |U_{iht}|^2}$$
(88)

where  $U_{iht}$  is the RMS amplitude of a single frequency component. Since the harmonic data from the FFT has been configured such that it fully represents the sampled waveform, this true RMS value is that equal to the value that would be given by calculating the RMS over the corresponding set of samples the same cycle in the time domain. In practice, the analysers also record a time averaged true RMS data while capturing the waveform data but these values are averaged over measurement interval and so differ slightly from the true RMS calculated per cycle from the harmonics.

This frequency analysis described here is subtly different to that used by the software when logging RMS data, as used for the measurements described in Chapter 5. In this logging mode, the power quality analyser software calculates the frequency domain data with a higher resolution and so gives information about the spectral components at inter-harmonic frequencies. To reduce the volume of data that results from this spectral analysis, the current or voltage amplitudes from blocks of frequencies are grouped, as defined in IEC 6100-4-7 (BSI 2002) so that the recorded data provides a single amplitude and phase to represent the frequency range defined for each group. In this case, the original time domain waveform cannot be re-constructed exactly from this grouped frequency domain data, unless the inter-harmonic content is zero.

Since the operation of the current transformer is based on integration of the voltage induced in the Rogowski coil, the sensor would ideally not respond to any DC component of the current, and there would be no DC component in the recorded data. However, observation of the recorded data shows that a DC offset exists at the start of the recorded current waveforms, typically decaying to zero as the measurement progresses. It is assumed that this DC offset arises in the integrator circuit used to represent the voltage induced in the Rogowski coil as a current waveform (Power Electronic Measurements 2013). The DC component in the FFT output is therefore ignored.

# 6.4 Verification tests

This section describes a series of calibration measurements that have investigated the amplitude and phase tolerances of the power quality analysers. Initially, the consistency of measurements from multiple sensor channels on the same analyser is considered, followed by comparisons of the results recorded by the two analysers (referred to as analysers A and B). These tests were conducted with the analysers installed at the substation so that interactions between the CTs and magnetic fields from other nearby conductors might be detected, and so that the test setup would be representative of a typical installation where there is limited space available within the cabinets and the CTs are in close proximity to each other.

A further test was needed to quantify the phase angle alignment between the current and voltage measurements. This test used a heater as a reference load with unity power factor so that the phases of the measured currents and voltages could be compared.

### 6.4.1 Comparison between measurement channels

The Fluke 435 power quality analyser has four Rogowski coil CTs, four line voltage measurement probes and a ground probe, enabling four relative voltages to be measured. The first verification test was therefore to consider differences between these measurement channels.

Each analyser was installed at the substation with all four CTs and all line voltage probes placed on the L2 phase conductor, such that they should record the same current and line to neutral voltage waveforms. Due to the limited space available, only one analyser could be tested in each measurement.

The amplitudes and phases of the currents measured at the fundamental frequency are shown in Figure 6-3 and Figure 6-4. The plots show the RMS amplitude and the relative phase angle of the line currents in each waveform cycle, taking the current measured in the neutral channel as a phase reference. There is an approximately constant offset of approximately  $\pm 0.9\%$  in amplitude and  $\pm 1^{\circ}$  in angle, plus a noise component from cycle to cycle of approximately  $\pm 0.7\%$  in amplitude and  $\pm 0.3^{\circ}$  in angle.

Similar plots for the voltage are shown in Figure 6-5 and Figure 6-6 where the line to neutral voltage measured in the L1 and L2 channels is compared with that from phase L3. This shows that the line to neutral voltage measurements are consistent to within 0.01 % in amplitude and 0.01° in angle.

The amplitude and phase differences are summarised in Table 6-1 for both analyser A and analyser B and for harmonics up to h = 9 and averaged over the 5 minute measurement duration. The table shows that the differences in current data between channels remain similar over frequency, although the phase errors appear lower for h = 3 and h = 9.

The voltage measurements are very consistent between channels for frequencies up to h = 9. This repeatability between channels is very much better than the calibration tolerance quoted by the manufacturer and listed in Table 5-1, although the verification test described here does not compare the measured voltages to an absolute standard so all of the channels may be subject to a common error.



Figure 6-3 – Current at analyser A, fundamental frequency component, amplitude ratio of measurement of line phase channels relative to measurement on neutral channel



Figure 6-4 – Current at analyser A, fundamental frequency component, phase angle of measurement of line phase channels relative to measurement on neutral channel



Figure 6-5 – Line to neutral voltage at analyser A, fundamental frequency component, amplitude of measurement on L1 and L2 channels relative to measurement on L3 channel



Figure 6-6 – Line to neutral voltage at analyser A, fundamental frequency component, phase angle of measurement on L1 and L2 channels relative to measurement on L3 channel

Harmonic	1	3	5	7	9		
Analyser A							
RMS amplitude of current measured in CTs on	+0 89	±1.74	±1.23	±0.45	±0.94		
L1, L2 and L3 relative to CT on neutral, %	_0.00						
Mean phase angle of current measured in line	±0.84	±0.19	±0.62	±1.24	±0.08		
CTs relative to neutral CT, degrees							
RMS amplitude of line to neutral voltage	±0.004	±0.038	+0 005	+0 037	+0.071		
measured in L1 and L2 relative to L3, %	s, %		_0.000	_0.001			
Mean phase angle of line to neutral voltage		±0.155	±0.026	±0.031	±0.047		
measured in L1 and L2 relative to L3, degrees		_000					
Ana	Analyser B						
RMS amplitude of current measured in line CTs	±0.97	±1.24	±0.78	±0.90	±1.67		
relative to neutral CT, %							
Mean phase angle of current measured in line	±0.23	±0.21	±0.48	±0.75	±0.21		
CTs relative to neutral CT, degrees							
RMS amplitude of line to neutral voltage	±0.002	±0.070	±0.026	±0.022	±0.111		
measured in L1 and L2 relative to L3, %							
Mean phase angle of line to neutral voltage	±0.001	.001 ±0.146	±0.033	±0.018	±0.048		
neasured in L1 and L2 relative to L3, degrees							

Table 6-1 – Comparison of results between measurement channels

## 6.4.2 Sensor positioning

The causes of the variation in current amplitude and phase have been investigated by repeating the test described in Section 6.4.1 but with one CT moved during the measurement. This aims to test how contribution of the errors due to non-axial positioning of the CT.

For this test, each of the four current sensors was fitted to the same conductor. The sensor on the L2 measurement channel was moved to a new position approximately every 30 seconds, allowing for movements of the CT up, down, left or right, and with orientation perpendicular or slanted relative to the conductor.

The current measured in the neutral channel has been taken as a reference, giving the amplitude and phase differences for the L1, L2 and L3 channels as shown in Figure 6-7 and Figure 6-8. The plots include graphics to indicate the approximate positioning of the L2 channel CT. Due to the limited space available, the CTs on the other channels were not ideally centralised or axial.

The CT positioning clearly has an impact on the amplitude and phase response, with deviations occurring both when the conductor is to one side of the sensor loop and when the orientation is not axial. Both the amplitude and phase response return approximately to the original condition when the CT is moved back to the initial position. The errors were found to affect both amplitude and phase, with approximately  $\pm 2\%$  in amplitude and  $\pm 1^{\circ}$  in phase angle. This is consistent with the tolerance indicated by the manufacturer data (Fluke 2006b), and forms a significant contribution to the potential errors. The observed variation in the phase angle differs from previous wok where the CT positioning was found to affect the measured current amplitude but to have no significant impact on the phase angle (Cataliotti et al. 2011).

As a consequence, it is not possible to use this test as a calibration since the CTs would then need to be moved when installed on their respective cables, changing the amplitude and phase offsets. Unfortunately, there is limited space available in the cabinets where the CTs are installed, particularly at the distribution cabinet, and so it is not possible to achieve an optimum positioning of the Rogowski coils.



Figure 6-7 – Current at analyser B with movement of CT on L2 channel, fundamental frequency component, amplitude ratio of measurement of line phase channels relative to measurement on neutral channel





The phase errors relating to the CT movement are shown for harmonic frequencies in Figure 6-9. Although the cycle-to-cycle variation increases for higher order harmonics, the mean phase error still remain centred around zero. This demonstrates that the sensitivity to the sensor positioning does not increase with frequency.





#### 6.4.3 Comparison between power quality analysers

The verification test described above indicates that the multiple channels on each analyser are consistent with each other, but does not indicate whether results from one analyser differ from those on another. A similar test was therefore configured in which both analysers were installed at the substation, with CTs and voltage probes measuring the same currents and voltages. There was insufficient space to connect all six voltage probes to the same line phase and so the CTs and probes were attached to the three phases in the conventional manner.

The results were initially analysed with the waveform data from the two analysers synchronised to the nearest sample, following the methods outlined in Section 7.16.2.

The mean phase angle between current waveforms from analyser A and analyser B was then calculated, giving results as shown in Figure 6-10, where the mean phase error is seen to increase with frequency. As noted above, this trend did not occur when phase errors were considered for multiple channels on the same analyser. However, this increase in error with frequency is consistent with a timing misalignment, where the phase angle impact of this timing error increases proportionally with frequency.



Figure 6-10 – Mean phase of current at analyser A relative to current at analyser B following sample-level synchronisation

Since the two analysers were co-located, it can be assumed that the voltage measured by each analyser is equal. The waveforms were therefore finely synchronised by rotating the phasors from analyser B, normalising these so that the voltage phase angles of the voltage in phase L2 (having the greatest mean RMS) were the same in the measurements from both analysers. For the example case shown in Figure 6-10, this gave a correction of 0.224° at 50 Hz. This phase rotation was then applied to measurements of all currents and voltages from analyser B.

A similar adjustment is applied to the harmonics, with the phase angle multiplied by the harmonic number such that the impact at each frequency is equivalent to a constant time advance or delay. With this phase adjustment applied, the mean phase angles between currents from analysers A and B were as shown in Figure 6-11. The errors are now seen to be flat with frequency.



Figure 6-11 – Mean phase of current at analyser A relative to current at analyser B following phasor adjustment based on voltage synchronisation

The amplitude and phase differences after this phase rotation had been applied are summarised in Table 6-2. This shows that there is minimal difference in the RMS amplitude or mean phase of voltages between the two analysers. The differences between current measurements at the two analysers are similar to those presented in Section 6.4.1 comparing different channels of the same analyser.

Harmonic	1	3	5	7	9
RMS amplitude of current, %	±1.05	±1.42	±1.12	±1.05	±1.29
Mean phase angle of current, degrees	±0.56	±0.10	±0.46	±0.45	±0.10
RMS amplitude of line to neutral voltage, %	±0.010	±0.086	±0.037	±0.062	±0.059
Mean phase angle of line to neutral voltage, degrees	±0.003	±0.072	±0.069	±0.029	±0.039

### 6.4.4 Phasor synchronisation

The cable impedance estimation method described later in Section 6.8 requires as an input the measured voltage difference between the two ends of the cable. Since this is a vector quantity, the phase angles of voltages recorded by the analyser at the cabinet need to be aligned to the same phase reference as the voltages recorded at the substation. This phase alignment is partly provided by the timing synchronisation described earlier in Section 7.16.2 where the two sets of data are aligned to the nearest sample. However, with 75 samples per cycle, this still allows for a phase offset between the two sets of waveform data of  $\pm 2.4^{\circ}$ . This is a significant phase error relative to the small variations in the phase angle of the voltage due to the reactance of the cable. The error particularly affects estimates of the loss within the cable where the angle of the small voltage difference vector  $\Delta V = V_i - V'_i$  is sensitive to errors in the angles of vectors  $V_i$  and  $V'_i$ .

Since the same current is measured by both analysers, the two sets of data can be aligned by rotating the current phasors measured at the cabinet so that they are normalised to the current phasors measured at the substation. The same normalisation phase offset is then also applied to the voltage phasors. However, there is a phase offset between samples from each current measurement channel due to the positioning of the CT on the conductor, as described in Section 6.4.2, and this applies to the currents measured at both analysers, doubling the possible error range. The error varies independently for each CT and so the normalisation phase offset calculated using phase L2, for example, will not necessarily align the currents in phases L1 and L3. The phase offset used to normalise the voltage phasors is also subject to these errors.

An improved method is suggested by the linear trend shown in Figure 6-10. A timing misalignment produces a mean phase error between the current measurements that is proportional to frequency. Since the mean errors due to the CT positioning do not significantly increase with frequency, as shown in Figure 6-9, the total phase error at harmonic frequencies is dominated by the contribution due to the timing misalignment. A better estimate of the phase offset at 50 Hz can therefore be made by taking the phase error at a harmonic frequency and dividing by the harmonic number.

It can be seen from Figure 6-10 that the phase errors for the neutral current follow a linear relationship more closely than for the line currents. A phase offset can therefore be determined such that the slope of this relationship is reduced to zero. The 9<sup>th</sup> harmonic is taken as a reference, since triplen harmonics combine in the neutral, giving a greater amplitude and therefore a more reliable phase estimate. Use of a higher harmonic (e.g. the 15<sup>th</sup>) would further reduce the impact of the CT positioning errors, but with increased noise since the magnitude of the 15<sup>th</sup> harmonic is much lower. From Figure 6-10 the mean phase difference for the 9<sup>th</sup> harmonic is

2.03°, corresponding to a difference of 0.226° at 50 Hz, and equivalent to the offset determined by aligning the voltages.

The timing offset between samples measured at the two analysers varies over the 5 minute measurement period, suggesting that there is a drift in the sampling frequency. This effect is distinct from the slight variations in the AC frequency about the nominal 50 Hz, for which the phase variations are common to both analysers. An example of the timing offset is shown in Figure 6-12, represented as a phase offset at 50 Hz. This shows the phase difference for each cycle between the neutral current 9<sup>th</sup> harmonic from analysers A and B, averaged over a time window of ± 1 second. This averaging time period was selected as an optimum value after experimentation showed that longer periods reducing the dynamic response to the phase variations, and shorter periods increased the noise in the estimate of the mean phase difference.

This phase synchronisation method has been applied for the measurement analysis described below in Section 6.8.



Figure 6-12 – Phase difference between current waveforms at 50 Hz following synchronisation to nearest sample, estimated based on neutral current 9<sup>th</sup> harmonic

#### 6.4.5 Voltage and current phase relationship

The tests described above consider phase offsets between each of the current and voltage sensors on multiple measurement channels, but do not demonstrate the relationship between voltage and current measurements.

Measurements of current and voltage have therefore been made using a resistive load, for which the power factor is assumed to be unity. This used a 4.5 kW heater, connected to a single-phase supply. The heater was connected via a test assembly in which the live and neutral could be separated such that the CTs could be fitted. All four CTs were fitted to the line conductor.

An example of the phase angle between the recorded voltage and current is shown in Table 6-3 for the 50 Hz component. The phase offsets shown are within the tolerance range noted previously due to the CT positioning on the conductor, and so do not indicate that there is any significant additional phase offset between the measured voltages and currents.

Analyser channel	Voltage to current phase angle
L1	1.5°
L2	1.1°
L3	0.7°
Neutral	1.1°

Table 6-3 – Phase angle at 50 Hz between voltage and current for 4.5 kW heater

# 6.5 Capacitor assembly for reactive power tests

Initial measurements of the currents on the selected LV network indicated that the aggregated demand due to connected appliances had a power factor close to unity. A capacitive 'test load' was therefore assembled so that the measurements could also include some configurations with currents at different phase relationships to the voltage.

The test load consisted of nine Electronicron 50  $\mu$ F, 300 V rms AC capacitors, each drawing a current of 3.6 A when connected at 230 V at 50 Hz. The capacitors could be configured as a balanced load of 10.8 A on each phase, or as a single load of 32.4 A with all nine capacitors connected to the same phase.

The capacitors were fitted inside a custom-built test assembly, protected by a 63 A fuse and 30 mA residual current circuit breaker module, and connected to a three-phase supply. An internal discharge resistor was included within each AC capacitor casing, such that the voltage would decay to below 50 V within 60 seconds of

disconnection from the supply. As an additional safety feature, a light bulb was fitted between each line phase and neutral to provide a voltage indicator, with the bulb visibly glowing for voltages above 30 V.

The capacitor assembly is shown in Figure 6-13 and Figure 6-14.



Figure 6-13 – Capacitor test load, box with capacitors and three-phase protection equipment



Figure 6-14 – Capacitor test load, internal assembly

# 6.6 Measured sum of currents in the cable

A key assumption throughout this work, both in the derivation of the cable impedances and in the network modelling, is that the sum of currents in all of the conductors is zero. For the LV cable measured here with separate neutral and earth conductors, the sum of currents is expected to be zero within the cable and there would only be current flowing from the phase or neutral conductors to the ground in the case of a fault condition. Based on the FE modelling of the cable (see Section 4.4.2, it is also assumed that the mutual impedance between each of the conductors within the cable is equal. Since the sum of currents in the cable conductors is zero there will therefore be no eddy currents induced in the ground. This implies that there the current in the loop formed between the cable armour and the ground (as shown in Figure 5-2) will also be zero, although there may be eddy currents circulating within the separate wires of the armour.

These assumptions were tested by placing a CT around the entire cable, such that any net current would be recorded. The CTs on the other measurements channels were connected but not placed around any conductors, so as to indicate the 'zero' current reading.

This test provided RMS current readings over the 5 minute measurement period of 0.18 A for the CT placed around the cable, and 0.08 to 0.09 A for the CTs not attached to conductors.

These results support the general assumption that the total vector sum of the current within the cable is zero, although it is unclear whether the very slight increase in current measured with the CT around the cable is due to a net current or due to each conductor having a different flux linkage with the Rogowski coil.

# 6.7 Measured demand characteristics

### 6.7.1 Test cases

Results are presented for eight tests, each recorded for a 5 minute duration, as described in Table 6-4.

Test case	Demand	Date
1	Demand due to appliances in building	5 February 2015
2	Demand due to appliances in building	5 February 2015
3	Demand due to appliances in building	11 March 2015
4	Demand due to appliances in building	25 June 2015
5	Demand plus heaters on L1 and 100 $\mu$ F capacitance on all phases	25 June 2015
6	Demand plus heaters on L1 and 300 µF capacitance on L3	25 June 2015
7	Demand plus heaters on L1 and 300 µF capacitance on L2	25 June 2015
8	Demand plus 300 µF capacitance on L1	25 June 2015

#### Table 6-4 – Waveform network measurement data sets

The measurement data for each case as prepared for analysis following the process outlined in Section 6.2 whereby the data records from both analysers were synchronised and then converted to phasors for the fundamental and harmonic frequencies. The current and voltage phasors from analyser B were then rotated, aligning the phase angle of the 9<sup>th</sup> harmonic current in the neutral, as described in Section 6.4.4.

### 6.7.2 Load current variation

The load current and voltage variation for one of the 5 minute measurements (test case 2) are shown in Figure 6-15. The values shown here are based on the true RMS, calculated from (88). The demand is highly unbalanced, with the L2 current being approximately 10 times that in L1, and therefore with a high neutral current. There are also significant spikes in the demand on both L2 and L3 phases. These spikes are not apparent on the recorded L1 current, indicating that the CTs are not vulnerable to coupling between the phases.



Figure 6-15 – Current amplitude

The set of test cases encompasses a range of current unbalance conditions, as shown in Table 6-5. Unfortunately it has not been possible to configure the demand on the network such that the tests also would include a case with the phases closely balanced.

Test case	L1 current, A	L2 current, A	L3 current, A	Neutral current, A
1	2.9	40.8	14.6	34.4
2	5.3	43.4	18.9	30.9
3	9.2	17.9	10.7	12.5
4	64.2	23.6	6.7	54.3
5	62.3	26.7	13.4	51.2
6	62.1	23.1	32.5	41.6
7	62.0	40.8	6.1	84.0
8	32.6	10.2	9.3	26.9

Table 6-5 – True RMS	current cale	culated over	5 minute	measurement	period
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An example of the voltage variation over the 5 minute measurement period is shown in Figure 6-16. As noted in Section 5.3.2 the voltage at the substation is held towards the lower end of the permitted range. However, the plot shows that there is considerable short term variation in the substation voltage, and this is observed therefore also observed in all of the figures in this chapter where voltages are plotted.



Figure 6-16 – RMS line to neutral voltage at the substation

#### 6.7.3 Current distortion

An example of the current waveform at the start of the 5 minute measurement is shown in Figure 6-17, showing that the current waveform includes considerable harmonic distortion. This includes significant contributions from up to the 13<sup>th</sup> harmonics, as shown in Figure 6-18. In this example, phase L1 has relatively high distortion (14% of the fundamental), but this makes a low contribution to the neutral current. However, the distortion in the neutral is higher than in the line conductors due to the combination of triplen harmonics and the cancellation of the balanced portion of the line currents.

The harmonics vary over the 5 minute measurement. If the THD is calculated for individual cycles then up to 25% THD could be found on the L1 conductor. However, this high THD occurs when the fundamental component is relatively low so the actual RMS amplitude of the harmonic currents is also low. Figure 6-19 shows the THD for the current measured at the substation, calculated individually for each cycle over the 5 minute measurement. In this case, the distortion forms a relatively constant fraction of the fundamental current, although there are transient increases and decreases as the loads with high power peaks (shown in Figure 6-15) switch on and off.

The current THD, calculated as in EN 50160 (BSI 2011b), is summarised over the 5 minute measurement period for each of the test measurements in Table 6-6.

Throughout all of these tests, the current distortion was much lower than that shown in Figure 5-11 for the measurement recorded using RMS data. This is likely to represent the current distortion for a more typical office electricity demand than in Figure 5-11 where the variable speed pump was in operation. A higher level of distortion would have been desirable for the purposes of estimating the cable impedance since that would reduce the sensitivity of the results to measurement noise. However, this exemplifies the difficulties that are inherent in measuring network characteristics with the live demand rather than with test loads.



Figure 6-17 – Example of current waveform, substation analyser, test case 2



Figure 6-18 – Current harmonic magnitude as percentage of fundamental, test case 2



Figure 6-19 – Current total harmonic distortion, test case 2

Test case	L1 THD	L2 THD	L3 THD	Neutral THD
1	24.4 %	10.1 %	15.3 %	16.1 %
2	13.9 %	9.5 %	14.4 %	19.8 %
3	19.5 %	23.4 %	16.9 %	49.3 %
4	5.3 %	10.7 %	24.6 %	10.3 %
5	6.4 %	11.6 %	15.9 %	13.0 %
6	5.6 %	12.3 %	8.5 %	15.3 %
7	5.5 %	8.2 %	22.5 %	7.1 %
8	17.0 %	22.3 %	14.8 %	27.9 %

Table 6-6 - Current THD calculated over 5 minute measurement period

#### 6.7.4 Consistency tests

Two tests can be made to check the consistency of the current measurements. These are applied in the frequency domain for the fundamental and for each harmonic frequency, such that the waveforms are synchronised, as described above, and so that compensation is applied for calibration errors.

Since there is no connection between neutral and ground, the mean sum of measured current phasors in each cycle and at each analyser should equal zero. In

practice, the sum of currents adds to a non-zero residual, characterised as an RMS value given by

$$I_{\text{sum\_error},h} = \sqrt{\frac{1}{N_{\text{t}}} \sum_{t=1}^{N_{\text{t}}} \left| \sum_{i=1}^{4} I_{iht} \right|^2}$$
(89)

where  $I_{iht}$  represents the current phasor in conductor *i* at harmonic frequency *h* for cycle *t* of the total set of  $N_t$  phasors.

Example results considering the sum of currents measured at the substation are shown in Figure 6-20. This plot shows the mean and maximum residual current error for each harmonic frequency h over the 5-minute duration, and indicate a mean error at 50 Hz of 0.25 A for analyser A and 0.3 A for analyser B.

In this example, and in general, the errors at analyser A (at the substation) were slightly lower and so current samples from this analyser have been used in predicting the voltage drop and losses. The poorer consistency of current measurements at analyser B may arise from the placement of the CTs being less optimal due to the limited space available in the cabinet, increasing the errors as discussed in Section 6.4.2.



Figure 6-20 – Mean and maximum amplitude of vector sum of currents, test case 2

Secondly, the difference between the synchronised current waveforms recorded by both analysers should equal zero, so that

(90)

$$I_{\text{amplitude\_difference\_error},ih} = \sqrt{\frac{1}{N_{\text{t}}} \sum_{t=1}^{N_{\text{t}}} \left| I_{\text{B},iht} - I_{\text{A},iht} \right|^2}$$

The RMS amplitude of the difference between currents is shown in Figure 6-21, with a mean error at the fundamental frequency of around 0.4 A.



Figure 6-21 – Mean amplitude of current differences between analysers, test case 2

Similarly, the mean phase error between currents recorded from the two analysers is given by

$$I_{\text{phase\_difference\_error},ih} = \frac{1}{N_{\text{t}}} \sum_{t=1}^{N_{\text{t}}} (\angle I_{\text{B},iht} - \angle I_{\text{A},iht})$$
(91)

Figure 6-22 shows the mean phase difference between currents at the two analysers, following synchronisation of the current waveforms, as described in 6.4.4. Only the odd-numbered harmonics are plotted as the phase estimation for even harmonics is subject to a high level of measurement noise due to their low amplitude. The remaining phase errors after synchronisation are between  $\pm 3^{\circ}$ , and much lower for phases L2 and L3 and for the neutral, for which the current magnitude is greater.

Figure 6-23 shows the same data as Figure 6-22 but plotting only the triplen harmonics. The phase errors are much reduced for this set of frequencies, with the dominant remaining error being that of the L1 phase at 50 Hz.



Figure 6-22 – Mean phase of current differences between analysers, test case 2



Figure 6-23 – Mean phase of current differences between analysers, showing only triplen harmonics, test case 2

For test cases 4 to 8, it was found that the sum of currents was significantly above zero for some portion of the measurement period, but then similar to the example described above for the remainder of the measurement. This unexpected behaviour occurs simultaneously at both analysers and coincides with the presence of a lower frequency oscillation superimposed on the 50 Hz waveforms. Since the operation of the Rogowski coil CTs depends on an inducted voltage, proportional to the rate of change of the magnetic field due to the current, the response of the sensors at DC is undefined, and is assumed to be un-calibrated at low frequencies. The observed low

frequency alternating current is therefore assumed to arise within the measurement equipment rather than being present in the actual cables.

In the post-processing of the measured data, the low frequency alternating current appears in the Fourier analysis as a DC component for cycles where this behaviour occurs. Further investigation is required in order to diagnose the cause of these disturbances. However, in the analysis here, the DC component has been excluded from the results of the Fourier analysis. The calculations of the line impedance in Section 6.8 are also based only on the portions of the measurement period for which the sum of currents appears consistent.

These consistency tests show that it has been possible (within some tolerance limits) to demonstrate that the sum of currents in the conductors is zero over the required range of harmonic frequencies, and that the two analysers measuring the same current give consistent readings. The assumption that the sum of currents equals zero is a key step in the development of the cable impedance theory and so this demonstration is an essential requirement in preparing for the next step of estimating the impedances.

# 6.8 Estimation of cable impedance from measurements

### 6.8.1 Impedance results at 50 Hz

The circuit impedance matrix has been estimated using a curve fitting process, following the same approach as outlined previously in Section 5.4. The method here retains the phase difference between the two voltage measurements, rather than normalising the phase angles to a reference local to each analyser as in the analysis of the RMS measurement data in Section 5.4.1.

Initially, the impedances have been estimated separately for each of the test cases listed in Table 6-4. The resulting sequence impedance matrices for each of these tests are shown in Table 6-7. The sequence impedance predicted by FE modelling, as described in Section 5.2.5, is shown here for comparison.

Taking the positive and zero sequence impedances as the key metrics, the range of variation over the individual test cases is shown in Figure 6-24 and Figure 6-25. It
should be noted that, in this case, the positive and zero sequence impedances alone do not completely represent the impedance matrix (since the phase impedances are not transposed and the mutual sequence impedances are non-zero).

Table 6-7 also shows results a further 'combined' case in which the measurement data from all of the individual tests are concatenated to create a single dataset with 40 minutes duration. Although the actual measurements were not contiguous in practice, this provides one single dataset that includes all of the unbalance and reactive power conditions that were configured. The impedance estimation then finds an impedance matrix that gives the best fit to all of this measured data. This makes the assumption the impedance matrix is constant over all of the measurements. In practice, this is likely to be a less secure assumption than to assume that the impedance is constant over individual measurement periods as the multiple tests were recorded on different days, with possible differences in the ground temperature and in the preceding history of the electrical demand.

Test case	Estimated sequence impedance at 50 Hz, $\Omega$					
	Z <sub>00</sub>	Z <sub>01</sub>	Z <sub>02</sub>			
	$Z_{10}$	$Z_{11}$	Z <sub>12</sub>			
	$Z_{20}$	Z <sub>21</sub>	Z <sub>22</sub>			
	0.095+j0.044	-0.001-j0.005	-0.010-j0.005			
1	-0.010-j0.005	0.023+j0.007	-0.004-j0.001			
	-0.001-j0.005	-0.002-j0.003	0.023+j0.007			
	0.102+j0.048	0.002-j0.002	-0.007-j0.004			
2	-0.007-j0.004	0.027+j0.011	-0.002+j0.005			
	0.002-j0.002	0.002+j0.000	0.027+j0.011			
	0.101+j0.046	0.002-j0.000	-0.005+j0.005			
3	-0.005+j0.005	0.029+j0.015	0.001+j0.004			
	0.002-j0.000	0.001-j0.000	0.029+j0.015			
	0.059+j0.043	0.015-j0.016	0.026+j0.006			
4	0.026+j0.006	0.014+j0.018	-0.011-j0.002			
	0.015-j0.016	0.007+j0.009	0.014+j0.018			
	0.105+j0.043	0.005-j0.000	-0.004-j0.003			
5	-0.004-j0.003	0.026+j0.014	0.003+j0.001			
	0.005-j0.000	0.002-j0.000	0.026+j0.014			
	0.074+j0.043	0.004-j0.026	-0.006+j0.013			
6	-0.006+j0.013	0.020+j0.012	0.000+j0.000			
	0.004-j0.026	0.016+j0.002	0.020+j0.012			
	0.086+j0.042	0.015-j0.013	-0.001+j0.021			
7	-0.001+j0.021	0.010+j0.006	0.015-j0.012			
	0.015-j0.013	0.005+j0.014	0.010+j0.006			
	0.087+j0.046	0.015-j0.012	0.001+j0.017			
8	0.001+j0.017	0.016+j0.002	0.020-j0.010			
	0.015-j0.012	-0.003+j0.014	0.016+j0.002			
	0.103+j0.047	0.004-j0.002	-0.004-j0.003			
All tests	-0.004-j0.003	0.028+j0.012	-0.002+j0.005			
concalenaled	0.004-j0.002	0.002+j0.001	0.028+j0.012			
	Predicted	I impedance from FE model a	t 50 Hz, Ω			
	0.100+j0.041	0.003-j0.002	-0.004-j0.002			
	-0.004-j0.002	0.026+j0.010	-0.002+j0.001			
concatenated	0.003-j0.002	0.002+j0.001	0.026+j0.010			



Figure 6-24 – Variation of sequence impedance resistance at 50 Hz over set of test cases



Figure 6-25 – Variation of sequence impedance reactance at 50 Hz over set of test cases

Over the individual test cases, there is considerable variation in both the resistance and reactance terms of the estimated impedance. For tests 4 to 8 there was less variation in the demand than for tests 1 to 3. Since multiple independent demand conditions are needed to estimate the  $3 \times 3$  impedance matrix from the  $3 \times 1$  current and voltage vectors, the impedances estimated from tests 4 to 8 are therefore more vulnerable to measurement noise. The demand for these test cases was also dominated by phase L1, such that the best fit process used to find the impedance could converge to a good solution with little impact on the errors if the impedances for phases L2 and L3 were incorrect.

The estimated positive sequence impedance from this combined data has 8% higher resistance and 19% higher reactance than the predicted values. The estimated zero sequence impedance has 2% higher resistance and 17% higher reactance than the predicted values.

### 6.8.2 Impedance results for harmonics

The sequence impedance is now considered for harmonic frequencies. The magnitude of the harmonic voltages is very low for some of the test cases and so only the combined data set has been used for the impedance estimates at harmonic frequencies. By using this combined data, the curve fitting algorithm is provided with a greater variety of load current vectors with different balance and power factors and the impedance estimation is based on a longer time period of data such that the impact of measurement noise is reduced. The impedance has been estimated for the odd harmonic frequencies, up to the 13<sup>th</sup> harmonic.

The sequence impedances are shown in Table 6-8 for results estimated from the measurements data, and can be compared to the predicted impedances from Table 5-4 for results derived from the FE simulation model.

As the frequency increases, the impedance becomes dominated by the reactance and so the iterative curve fitting process used to derive the impedance depends less on the resistive term. At the 13<sup>th</sup> harmonic, the algorithm converges to a best fit without making any change to the estimated resistance from that provided as an initialisation.

Frequency, Hz	Circuit impedance, Ω
50	$\widehat{\boldsymbol{Z}} = \begin{bmatrix} 0.053 + j0.022 & 0.026 + j0.014 & 0.024 + j0.006 \\ 0.026 + j0.014 & 0.051 + j0.028 & 0.025 + j0.015 \\ 0.024 + j0.006 & 0.025 + j0.015 & 0.054 + j0.021 \end{bmatrix}$
150	$\widehat{\boldsymbol{Z}} = \begin{bmatrix} 0.053 + j0.066 & 0.024 + j0.044 & 0.020 + j0.021 \\ 0.024 + j0.044 & 0.051 + j0.088 & 0.023 + j0.044 \\ 0.020 + j0.021 & 0.023 + j0.044 & 0.050 + j0.070 \end{bmatrix}$
250	$\widehat{\boldsymbol{Z}} = \begin{bmatrix} 0.050 + j0.105 & 0.025 + j0.066 & 0.024 + j0.037 \\ 0.025 + j0.066 & 0.047 + j0.135 & 0.025 + j0.068 \\ 0.024 + j0.037 & 0.025 + j0.068 & 0.049 + j0.109 \end{bmatrix}$
350	$\widehat{\boldsymbol{Z}} = \begin{bmatrix} 0.042 + j0.142 & 0.024 + j0.091 & 0.018 + j0.054 \\ 0.024 + j0.091 & 0.046 + j0.177 & 0.022 + j0.093 \\ 0.018 + j0.054 & 0.022 + j0.093 & 0.045 + j0.149 \end{bmatrix}$
450	$\widehat{\boldsymbol{Z}} = \begin{bmatrix} 0.029 + j0.177 & 0.013 + j0.113 & 0.013 + j0.065 \\ 0.013 + j0.113 & 0.026 + j0.224 & 0.011 + j0.112 \\ 0.013 + j0.065 & 0.011 + j0.112 & 0.028 + j0.177 \end{bmatrix}$
550	$\widehat{\boldsymbol{Z}} = \begin{bmatrix} 0.015 + j0.205 & 0.003 + j0.128 & 0.011 + j0.075 \\ 0.003 + j0.128 & 0.004 + j0.255 & 0.006 + j0.133 \\ 0.011 + j0.075 & 0.006 + j0.133 & 0.011 + j0.211 \end{bmatrix}$
650	$\widehat{\boldsymbol{Z}} = \begin{bmatrix} 0.000 + j0.205 & 0.000 + j0.128 & 0.000 + j0.075 \\ 0.000 + j0.128 & 0.000 + j0.255 & 0.000 + j0.133 \\ 0.000 + j0.075 & 0.000 + j0.133 & 0.000 + j0.211 \end{bmatrix}$

Table 6-8 – Measure	d circuit impedance,	best fit over	aggregated d	data from all tests
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The measurement results are compared with the simulated impedances in Figure 6-26 and Figure 6-27. Both plots show similar trends with the inductance estimated from measurements being higher than that from the FE simulation.

In the FE simulation, the results for  $Z_{aa}$  and  $Z_{cc}$  are equal, due to the symmetry of the cable, as are the results for  $Z_{ab}$  and  $Z_{bc}$  and this is also observed in the estimated impedance data. The relative difference between the inductances for  $Z_{aa}$  and  $Z_{bb}$  is also similar in both sets of results, and this ratio remains similar across the frequency range.



Figure 6-26 – Estimated and simulated phase self-impedance, reactance represented as inductance





The resistive and inductive components of the sequence impedance are plotted in Figure 6-28 and Figure 6-29. The estimated resistance agrees well with the predicted values at 50 Hz but deviates significantly at higher frequencies. This is due to limitations with the curve fitting algorithm, as noted above. The estimated sequence impedance inductances again follow a similar trend to the predicted values, but both the estimated values are higher for both the positive and zero sequence. Differences of approximately 20% are maintained over the frequency range.

These results appear to confirm the conclusions of Chapter 5, again finding that that the estimated sequence inductance values are higher than the values predicted from the FE simulation.

Section 6.4 highlighted the impact of amplitude and phase tolerances on the measured values. These are now explored further to determine whether the differences between the estimated and predicted impedances are significant when a range of likely measurement tolerances is considered.



Figure 6-28 – Estimated and simulated sequence impedance, resistance terms



Figure 6-29 – Estimated and simulated sequence impedance, reactance represented as inductance

### 6.8.3 Error analysis

The analysis described above relies on making the assumption that the currents measured at both analysers are the same, such that the phase of readings from analyser B can be rotated to align with the phase of readings from analyser A. The phase rotation to correct the timing alignment is applied equally to the current and voltage samples from each channel of analyser B.

However, as shown in Section 6.4.2, the amplitude and phase readings of the current are subject to errors, largely due to the positioning of the CT around the conductors. There are also errors that apply to the voltage readings. The consistency between voltage channels on the power quality analyser has been found to be much better than the calibration tolerance stated by the manufacturer.

The effect of these errors has been investigated by repeating the impedance estimation process with random variations applied to the amplitude and phase of each current measurement, and with amplitude variations applied to the voltage measurements. The errors are selected from a uniform distribution centred on the nominal value from the measured data from test case 2. Based on the trends shown in the plots in Section 6.4.1, the variations are applied independently for each frequency component and for each conductor, but they are held constant for all of the samples within the measurement duration. A set of 100 Monte-Carlo trials has been calculated with the error ranges, based on the results in Section 6.4, as follows:

- Current amplitude: ±1%
- Current phase: ±1°
- Voltage amplitude: ±0.1%

The results for the positive sequence inductance are shown in Figure 6-30, with the predicted results being within the distribution range of the estimated values for all but the third harmonic. Although the estimated positive sequence inductances reported in Section 6.8.1 and 6.8.2 are above the predicted value, this difference is within the range of variation expected due to the sensor tolerances.

For the zero sequence inductance, the distribution of values resulting from the error analysis is shown in Figure 6-31. In this case, the distribution range of the estimated inductance is clearly higher than the predicted values. The difference between the estimated and predicted zero sequence inductance does not appear to be due to the sensor tolerances.



Figure 6-30 – Error analysis for positive sequence inductance



Figure 6-31 – Error analysis for zero sequence inductance

The sequence impedances estimated from the waveform measurements therefore broadly agree with those from the RMS data described in Chapter 5, both of which indicate that the sequence impedances reactances are 10-20% higher than predicted from the FE model. These increases are consistent over the frequency range for which the curve-fitting method worked successfully. The error analysis described above shows that the observed difference in the positive sequence reactance is within the range of values that would be expected given a likely range of current sensor tolerances. However, the difference in the zero sequence reactance is outside of the range of variation indicated by the error analysis, suggesting that the estimated value reflects a real difference rather than being a consequence of measurement tolerances.

#### 6.9 Measured and simulated power quality metrics

This section takes the same approach as previously in Section 5.5 and compares measured power quality metrics (voltage difference, distortion and losses) with those that would be predicted by simulations using the predicted impedance matrix. As

before, this uses the same measurement data as used to derive the impedances but makes an alternative comparison.

### 6.9.1 Voltage drop

The discussion above demonstrates that there appear to be differences between the measured cable impedance and that predicted by the FE simulations. The following sections now compare the impact of these differences on several key performance metrics in order to determine how results obtained with the impedances from the FE simulation differ from the measured data.

The measured and predicted voltage drops are calculated for each conductor using (84) and (85) but with the variable t now representing a single cycle rather than a measurement interval of 250 ms as before.

An example of the RMS voltage drop for both measurement and simulation is shown in Figure 6-32. On phase L2, where the demand is greatest and so where errors are of greatest concern, the simulation matches to better than 2% of the measured RMS voltage difference. The agreement is a little less close for phases L1 and L3 where the voltage differences are much lower.



Figure 6-32 – RMS voltage drop

The agreement between the voltage drops for the full set of tests is shown in Table 6-9, summarised as a mean voltage drop across the 5 minute test duration. Here the RMS voltage is averaged over all the cycles in the measurement, including all harmonic components from h = 1 to  $N_h$  as:

$$V_{d,simulated,i} = \sqrt{\frac{1}{N_{t}} \sum_{t=1}^{N_{t}} \sum_{h=1}^{N_{h}} |V_{A,iht}|^{2}} - \sqrt{\frac{1}{N_{t}} \sum_{t=1}^{N_{t}} \sum_{h=1}^{N_{h}} |V_{A,iht} - \sum_{k=1}^{3} \hat{Z}_{h,ik} I_{A,kht}|^{2}}$$
(92)  
$$V_{d,measured,i} = \sqrt{\frac{1}{N_{t}} \sum_{t=1}^{N_{t}} \sum_{h=1}^{N_{h}} |V_{A,iht}|^{2}} - \sqrt{\frac{1}{N_{t}} \sum_{t=1}^{N_{t}} \sum_{h=1}^{N_{h}} |V_{B,iht}|^{2}}$$
(93)

This metric is similar to that in EN 50160 (BSI 2011b) although calculated over the available measurement period rather than the standard duration of 10 minutes.

This shows agreement between the simulated and measured voltage drops to within 0.3 V. The percentage errors are shown, taking the maximum voltage drop over phases L1 to L3 as a reference. The simulated and measured voltage drops agree to

within 0-10% in most cases and to within 5% for the conductors with the greatest voltage drop.

Test	Mean measured voltage		Mean simulated voltage		Percentage error of				
case	drop, V			drop, V		simulation relative to			
							maximum measured voltage		
							drop		
	L1	L2	L3	L1	L2	L3	L1	L2	L3
1	-0.11	2.01	-0.31	-0.18	1.97	-0.25	-3.2%	-2.0%	3.0%
2	-0.02	1.97	-0.11	-0.08	1.93	-0.06	-3.3%	-1.7%	2.4%
3	0.33	0.71	0.02	0.30	0.67	-0.07	-4.3%	-5.6%	-11.7%
4	3.36	-0.34	-0.81	3.17	-0.24	-0.83	-5.5%	2.8%	-0.5%
5	3.13	-0.38	-0.90	2.97	-0.30	-0.87	-4.9%	2.4%	0.8%
6	2.66	0.65	-1.45	2.52	0.61	-1.33	-5.3%	-1.6%	4.3%
7	4.20	-1.21	-1.33	3.97	-1.00	-1.33	-5.5%	5.0%	-0.1%
8	0.28	-0.38	0.68	0.32	-0.41	0.64	5.6%	-4.5%	-5.4%

 
 Table 6-9 – Simulated and measured RMS voltage drop, conductors with the highest voltage drop highlighted in red

#### 6.9.2 Voltage distortion

The THD is calculated individually for each measurement cycle as:

$$V_{\text{THD}i} = 100 \times \sqrt{\sum_{h=2}^{N_{\text{h}}} \left(\frac{|V_{hi}|}{|V_{1i}|}\right)^2}$$

An example of the comparison between the measured and simulated voltage THD is shown for phase L2 in Figure 6-33. This shows that the increase in distortion at the cabinet end of the cable is greater in the measured data than in the simulation.

This trend appears consistently over the set of measurements, as shown in Table 6-10, comparing the mean voltage over the measurement periods. In all cases, the simulation under-estimates the voltage THD at the cabinet, compared to the measured data.

This difference is expected since the estimated impedances have higher reactance than in the predicted cable impedance. At harmonic frequencies, the impedance

(94)

magnitude is dominated by the reactance, and so the difference in increase in voltage THD is similar to the difference in the reactance.



Figure 6-33 – Voltage THD at substation and cabinet phase L2

Test case	Measured analyser A, %			Measured analyser B, %			Simulated analyser B, %		
	L1	L2	L3	L1	L2	L3	L1	L2	L3
1	1.53	1.51	1.14	1.58	1.74	1.23	1.57	1.69	1.20
2	1.48	1.43	1.17	1.55	1.66	1.31	1.54	1.60	1.26
3	1.62	1.44	1.12	1.73	1.68	1.19	1.70	1.64	1.17
4	0.78	0.63	0.60	0.94	0.77	0.73	0.88	0.72	0.69
5	0.82	0.65	0.58	1.01	0.84	0.75	0.94	0.79	0.70
6	0.80	0.71	0.70	0.95	0.86	0.85	0.90	0.82	0.80
7	0.78	0.66	0.66	0.94	0.83	0.75	0.89	0.77	0.72
8	0.80	0.63	0.67	1.04	0.75	0.82	0.96	0.71	0.78

Table 6-10 – Voltage THD calculated over 5 minute measurement period

## 6.9.3 Losses in the LV cable

The losses in the cable can be calculated using three different methods.

As in Section 5.5.2, the losses can be calculated as the difference of power supplied into the cable and the power delivered from it, as in (86). However, as demonstrated

before, this calculation is vulnerable to amplitude tolerances in the measured current amplitude. Losses can also be calculated using an  $I^2R$  metric with (87) where the resistance is based on the predicted impedance matrix.

A third method is also possible for the waveform measurements described here. Using the phase synchronisation method of Section 6.4.4, there is a common phase reference for voltages measured at both ends of the cable. The losses can then be calculated using the vector voltage difference between the two analysers. The voltage measurements have been shown to be much more consistent from one analyser to another (Section 6.4.3) than the current measurements, and so the sensitivity to amplitude measurement errors is reduced. The loss calculation is then given by:

$$P_{\rm loss} = \rm re\left(\sum_{i=1}^{3} \sum_{h=1}^{N_{\rm h}} (V_{A,hi} - V_{B,hi}) \times I_{A,hi}^{*}\right)$$
(95)

A comparison of the measured and simulated losses is shown in Figure 6-34. This shows losses calculated with the power difference method of (86). As demonstrated previously in Section 5.5.2 for the RMS measurement data, this method does not give usable results. The method based on voltage differences using (95) gives results that are very close to the losses calculated using the predicted impedance and the measured currents with (87).

For the test case shown in Figure 6-34, the mean losses are around 0.57% of the input power from the substation and there is negligible difference between the losses calculated with the simulated cable impedances or from the measured voltage differences using (95). The agreement is less close for some of the other test cases, as summarised in Table 6-11. Taking the combined data set from all of the test cases, the losses calculated using the predicted impedance matrix are 4.5% below those calculated using the vector voltage difference method. This is consistent with the estimated resistances being higher than those in the predicted impedance matrix.

The differences in the mean losses were greater for the test measurements made later in the year. For the measurements in June, the mean demand was also greater. This suggests that some of the differences between the measured and predicted losses in the later tests may be due to an increase in the cable conductor temperature, with a 4% difference being consistent with a 10 °C rise (Section 4.2.6).





Test case	Measured loss, % of	Simulated	Difference	Date
	delivered real power	loss, % of	between	
		delivered real	simulation and	
		power	measurement	
1				5 February
	0.62	0.62	-0.3%	2015
2				5 February
	0.57	0.57	-0.2%	2015
3	0.24	0.23	-6.5%	11 March 2015
4	0.98	0.94	-4.1%	25 June 2015
5	0.97	0.92	-4.4%	25 June 2015
6	0.96	0.91	-4.8%	25 June 2015
7	1.67	1.60	-4.3%	25 June 2015
8	0.69	0.66	-4.0%	25 June 2015
All tests concatenated	0.69	0.66	-4.5%	

## 6.10 Conclusions

The measurements described in this chapter been made using time domain waveform samples from two power quality analysers at opposite ends of an LV feeder cable. These measurements have provided a more detailed understanding of the voltage difference along the cable than the previous measurements which recorded RMS amplitude and phase data averaged over a 250 ms period. By capturing the time domain waveform samples, it was possible to obtain a more accurate measurement of the phase angles of each frequency component of the currents and voltages, avoiding the uncertainties relating to the harmonic smoothing and phase angle averaging noted in Chapter 5. The use of waveform measurements described here enabled both the amplitude and phase differences between the voltages to be compared.

These measurements have also provided a demonstration that the sum of currents in the cable equals zero. This is a key assumption but it is rarely demonstrated in practice. It is also notable that considerable effort has been required in order to provide this demonstration of a basic assumption in the theory. It was not possible to show this using the measurements described in Chapter 5 due to the phase averaging errors and the smoothing filter applied to the RMS current amplitudes, although this is due to the standards implemented in the power quality analyser than the underlying physics. However, even using industry standard power quality analysers, it has been necessary to resolve samples to a single-cycle resolution in order to obtain this result.

Some error still remains, which is assumed to be due to the tolerances of the current sensors. The current measurements were found to be dependent on the positioning of the current transformer (CT) around the conductors with tolerances of approximately  $\pm 2\%$  in amplitude and  $\pm 1^\circ$  in phase angle. It was observed that the magnitude of the phase angle tolerances is similar over a range of frequencies.

In order to calculate the vector voltage differences, the measurements from both analysers were phase aligned using the current measurement as a reference. A technique was developed to minimise the impact of the phase errors by normalising the currents based on a harmonic as a reference. This use of a harmonic current reduces the timing error approximately in proportion to the harmonic number. The timing alignment was optimised by normalising to the neutral conductor current as this has greater magnitude (for a triplen harmonic) than in the phase conductors.

The cable impedances estimated from the measured data were found to have slightly higher resistance (8% for positive sequence at 50 Hz) than the predicted values from the FE simulation. The reactances of the sequence impedances were around 20% higher than the predicted values. An error analysis has shown that the increased positive sequence reactance may be explained by the measurement tolerances but not the zero sequence reactance. The higher reactances are consistent with those in Chapter 5 where the estimation process did not attempt any phase synchronisation between the measurements from the two analysers. The measurements here have also demonstrated in practice the differences in reactance between sectors that are adjacent or opposite each other in the cable bundle.

Further work would be needed to identify the reasons for the differences in reactance more clearly. The estimation process here has assumed that the impedance is constant over the duration of the measurement activity but the resistance is sensitive to temperature variation, as discussed in Section 4.2.6. It has not been possible to

obtain a sample of the actual cable type in use and so the cross-sectional profile may not be as assumed in the FE model. The reactances are sensitive to variations in the conductor insulation thickness, as noted in Section 4.2.5. Whilst the predicted impedance has been developed to account for the resistance impact of protection equipment and variations in the reactance due to visible differences in the conductor spacing, the reactance impact of the protection equipment is unknown, as is the effect of a buried repair junction in the feeder cable. The power quality analysers also have a tolerance relative to temperature and it would be useful to characterise this in a lab trial.

The RMS voltage drop, distortion and losses have been calculated from the voltage measurements at both analysers and also by modelling the voltage drop that would occur based on the impedances from the FE simulation. If the network were to be modelled assuming that the impedances from the FE simulation, the worst case RMS voltage drop would be under-estimated by up to approximately 5%. Similarly, the increase in voltage distortion along the feeder cable is under-represented by approximately 20%, highlighting that accurate characterisation of the voltage distortion requires accuracy in the reactance terms of the cable impedance matrix.

A loss calculation technique has been proposed using the measured vector voltage difference and using the measured current data to provide a phase synchronisation. The calculated losses match the losses calculated using impedances from the FE simulation to within approximately 5% (with the difference being consistent with the differences in the estimated and predicted resistance). This avoids the problems noted in Chapter 5 where it was not possible to calculate the losses based on the power difference since the losses were of similar magnitude to the error tolerances in the CTs. Using the waveform data enables the loss to be calculated based on the voltage vector difference, for which the measurement tolerances are much reduced.

This technique would therefore be recommended if similar power quality analysers are used for future loss measurements on LV cables. Whilst not being applicable for estimating losses on a network-scale, it allows for an improvement in accuracy where detailed measurements are needed of a specific feeder branch.

# 7 HARMONIC DISTORTION

## 7.1 Introduction

Chapters 5 and 6 have described measurements taken to determine the impedance of an LV feeder cable where the amplitude and phase of frequency harmonics has been recorded. The impedance of the cable has been predicted in Chapter 4 with careful consideration of the accuracy of the impedance at harmonic frequencies. It is therefore possible to calculate the voltage drop and losses in the cable that would occur with and without the harmonic currents. This addresses the question of whether these metrics are affected by the presence of harmonics.

The effect on the voltage drop is first addressed in Section 7.2 via a theoretical approach, where the relationship between the permitted THD and the worst case voltage drop is outlined. Section 7.3 utilises the measurements of a cable between a substation and a distribution cabinet, as presented in Chapter 5, to show the impact on the voltage drop and losses due to harmonics.

The following section 7.4 then considers a subtly different question of whether simulation results will be in error if harmonics are present in reality but omitted from the model. Different approximations can be made using measured or synthesized data for models that use only the fundamental frequency.

Finally, Section 7.5 describes a technique that could be used to improve estimates of losses from a model at the fundamental frequency where the harmonic currents are not defined but where empirical results can be used to define the relationship between the distortion and the RMS current.

# 7.2 Worst case impact on voltage drop

The RMS voltage metric defined in EN 50160 is interpreted in IEC 61000-4-30 as the RMS of the time domain waveform (BSI 2009b). The measurement therefore includes the harmonic components.

The RMS value  $V_{\text{rms},h}$  of a harmonic frequency component is given by:

$$V_{\mathrm{rms},h}^{2} = \frac{\omega_{1}}{2\pi} \int_{t=0}^{\frac{2\pi}{\omega_{1}}} \left( \hat{V}_{h} \cos(\omega_{h}t + \phi_{h}) \right)^{2} \cdot dt$$
(96)

where  $\hat{V}_h$  is the peak amplitude of the harmonic, with angular frequency  $\omega$  and phase angle  $\phi$ . The RMS is integrated with respect to time *t* over the period  $2\pi/\omega_1$ , corresponding to a single cycle of the fundamental component.

Considering a case in which the time varying voltage can be represented by just two frequency components, the RMS value  $V_{\rm rms}$  of the combined voltage is given by

$$V_{\rm rms}^{2} = \frac{\omega_{1}}{2\pi} \int_{t=0}^{\frac{2\pi}{\omega_{1}}} (\hat{V}_{1} \cos(\omega_{1}t + \phi_{1}) + \hat{V}_{h} \cos(\omega_{h}t + \phi_{h}))^{2} \cdot dt$$
(97)

where  $\hat{V}_1$  and  $\hat{V}_h$  are the peak amplitude values of the fundamental and harmonic components, and where  $\omega_h = k\omega_1$  with *k* being an integer.

This can be expanded to give:

$$V_{\rm rms}^{2} = \frac{\omega_{1}}{2\pi} \int_{t=0}^{\frac{2\pi}{\omega_{1}}} \hat{V}_{1}^{2} \cos^{2}(\omega_{1}t + \phi_{1}) \cdot dt$$

$$+ \frac{\omega_{1}}{2\pi} \int_{t=0}^{\frac{2\pi}{\omega_{1}}} 2\hat{V}_{1}V_{h} \cos(\omega_{1}t + \phi_{1}) \cos(\omega_{h}t + \phi_{h}) \cdot dt$$

$$+ \frac{\omega_{1}}{2\pi} \int_{t=0}^{\frac{2\pi}{\omega_{1}}} \hat{V}_{h}^{2} \cos^{2}(\omega_{h}t + \phi_{h}) \cdot dt$$

$$V_{\rm rms}^{2} = V_{\rm rms,1}^{2} + V_{\rm rms,h}^{2} + \frac{\omega_{1}}{2\pi} \int_{t=0}^{\frac{2\pi}{\omega_{1}}} 2\hat{V}_{1}\hat{V}_{h} \cos(\omega_{1}t + \phi_{1}) \cos(\omega_{h}t + \phi_{h}) \cdot dt$$
(98)
(98)
(98)

The third term in (99) can be re-written as

$$\cos(\omega_1 t + \phi_1) \cos(\omega_h t + \phi_h)$$
(100)  
=  $\frac{1}{2} [\cos((1-k)\omega_1 t + \phi_1 - \phi_2) + \cos((1+k)\omega_1 t + \phi_1 + \phi_2)]$ 

The integration of (100) over the period of the fundamental frequency is zero provided that k is an integer (i.e. corresponding to a harmonic, rather than an inter-harmonic).

This can be extended with the addition of further frequency components such that the RMS of the combined waveform is given by:

$$V_{\rm rms} = \sqrt{\sum_{h=1}^{N_{\rm h}} V_{\rm rms,h}^2}$$
(101)

The voltage THD is calculated according to (BSI 2011b) as:

$$\text{THD}_{V} = 100 \times \frac{1}{V_{\text{rms},1}} \sqrt{\sum_{h=2}^{N_{h}} V_{\text{rms},h}^{2}}$$
(102)

The error in the calculation of the RMS voltage due to omitting harmonics is then given by  $V_{\rm rms} - V_{\rm rms,1}$ , expressed as:

Error = 
$$V_{\rm rms} - V_{\rm rms,1} = V_{\rm rms,1} \left( \sqrt{1 + \left(\frac{\rm THD_V}{100}\right)^2} - 1 \right)$$
 (103)

ENA G5-4 specifies that the voltage THD at a customer connection should be no greater than 5% (Energy Networks Association 2005). From (103), the impact of including harmonics in the RMS voltage calculation is therefore limited to 0.12%, or a voltage difference of approximately 0.3 V for a 230 V fundamental component. Provided that the power supply complies with the distortion metrics, this gives an upper bound to the impact of harmonics on the RMS voltage.

Typically the impact of including harmonics in calculating the RMS voltage drop along an LV cable will be less than this maximum figure, even with the maximum permitted distortion, since the voltage at the substation will also be distorted. The error in the RMS voltage drop is then  $(V_{A,rms} - V_{B,rms}) - (V_{A,rms,1} - V_{B,rms,1})$ :

Error = 
$$V_{A,rms,1} \left( \sqrt{1 + \left(\frac{THD_{V,A}}{100}\right)^2} - 1 \right) - V_{B,rms,1} \left( \sqrt{1 + \left(\frac{THD_{V,B}}{100}\right)^2} - 1 \right)$$
 (104)

The error due to not including harmonics in the RMS calculation at the customer node is therefore offset by the error of not including harmonics at the substation.

## 7.3 Impact of harmonics in measured data

The section uses measured data to compare the voltage drop and losses in the LV cable when harmonics are included the in calculations and when harmonics are omitted. This can also be seen as a comparison of the impact on these metrics for a load that creates distortion compared to a load that has a completely linear behaviour. The measured data described in Section 5.2.5 is used as a practical example. These measurements included a period for with highly distorted load currents caused by a variable speed three-phase pump. For this period, the current THD reached 75% on the phase conductor with the least demand, and 45% on the most heavily loaded phase conductor.

### 7.3.1 Voltage drop

For the case with the harmonics included, the vector voltage drop is calculated for each frequency using a frequency-dependent impedance matrix, and the voltage drop is calculated as the difference of the RMS voltages at each end of the cable. The RMS voltage is referred to as the true RMS, as all of the frequency components are included (with the exception of any DC voltage difference which is not represented in the impedance matrix). The true RMS voltage drop for conductor iand sample t is then calculated as:

$$V_{\rm d,it} = \sqrt{\sum_{h=1}^{N_{\rm h}} \left| V_{\rm A,iht} \right|^2} - \sqrt{\sum_{h=1}^{N_{\rm h}} \left| V_{\rm A,iht} - \sum_{k=1}^3 \hat{Z}_{h,ik} I_{\rm A,kht} \right|^2}$$
(105)

where  $\hat{Z}_{h,ik}$  is the circuit impedance term  $\hat{Z}_{ik}$  for harmonic *h*, and  $I_{A,kht}$  is the current on conductor *k* for harmonic *h* and sample *t* measured at analyser A.

For the case without the harmonics, only the fundamental component of the current is included when calculating the voltage at node B. It is assumed that the voltage distortion at the substation (node A) is largely due to harmonic distortion elsewhere on the network (rather than just this particular cable) and so the harmonic voltage components at node A are retained. With the assumption that there are no harmonic currents in the cable, the same harmonic voltages exist at the downstream end (node B). The voltage drop calculation is then:

$$V_{d,it} = \sqrt{\sum_{h=1}^{N_{h}} |V_{A,iht}|^{2}} - \sqrt{\left|V_{A,i1t} - \sum_{k=1}^{3} \hat{Z}_{1,ik} I_{A,k1t}\right|^{2} + \sum_{h=2}^{N_{h}} |V_{A,iht}|^{2}}$$

(106)

If new low carbon technologies are added to the network, this suggests that there will be negligible impact on the RMS customer voltages.



Figure 7-1 – RMS voltage drop with harmonics included or omitted

### 7.3.2 Losses in the LV cable

A similar comparison now follows for the losses in the cable. For the case with the harmonics included, the loss for measurement interval t, calculated over all three conductors and for all frequencies, is given by:

$$P_{\text{loss},t} = \text{re}\left(\sum_{i=1}^{3}\sum_{h=1}^{N_{h}} \left[ \left(\sum_{k=1}^{3} \hat{Z}_{h,ik} I_{\text{A},kht}\right) \times I_{\text{A},iht}^{*} \right] \right)$$
(107)

where  $\hat{Z}_{h,ik}$  is the circuit impedance term  $\hat{Z}_{ik}$  for harmonic *h*, and  $I_{A,ikht}$  is the current on conductor *k* measured at analyser A.

For the case with only the fundamental current included, the losses are given by:

$$P_{\text{loss},t} = \text{re}\left(\sum_{i=1}^{3} \left[ \left( \sum_{k=1}^{3} \hat{Z}_{1,ik} I_{\text{A},k1t} \right) \times I_{\text{A},i1t}^{*} \right] \right)$$
(108)

The two cases are compared in Figure 7-2, showing that the harmonics cause a significant increase of 38% in the maximum losses. The mean losses over the full measurement period of 17 hours increased by 4.4% with harmonics included, although for much of this period the distortion was much lower, with an average of 20% THD, compared to the maximum of 75% THD for the period when the pump was in use.



Figure 7-2 – Losses with harmonics included or omitted

Simulation model	Mean loss, W	Increase in mean	Maximum loss	Increase in maximum
		loss with harmonics	W	loss with harmonics
With harmonics	22.54	-+4.4%	182.3	+38.5%
No harmonics	21.58	-	131.6	-

Table 7-1 – Losses with harmonics included or omitted

As expected, these results show that current distortion does increase the losses in the LV cable. If LCTs cause significant current distortion, then the additional losses need to be taken into account when assessing the capacity of cables.

## 7.4 Simulation models omitting harmonics

The preceding section considered the impacts on the voltage drop and losses if harmonic currents were either present or absent. This section assumes that the current does have distortion and considers how errors arise in simulations if the models only include the fundamental frequency.

## 7.4.1 Voltage drop

Simulation models typically calculate line currents such that they are consistent with loads specified by their active or complex power. This load data might be derived from measurements or, as with the CREST demand model, synthesized using the rated power of each appliance (Richardson & Thomson 2011). The power is usually specified at the fundamental frequency and the harmonics are not defined. The power factor is then entirely represented as an angular displacement of the current from the voltage. No allowance is then included for the contribution to the power factor due to distortion. A more complex demand model could add harmonic currents in a defined proportion to the fundamental component (Collin et al. 2010). However, in most cases, the active power is assumed to apply to a waveform at the fundamental frequency and the model omits any current distortion that might occur in practice

Conversely, network measurements without sophisticated spectral analysis are likely to record the true RMS current and voltage, including any distortion in the time domain waveform. Rather than being omitted from the measurements, the harmonics are included but not distinguished from the fundamental component. In a more advanced measurement setup where the power quality analyser includes harmonic analysis, the waveform is decomposed such that the constituent harmonics can be identified (such as in Chapter 5). This requires higher specification monitoring equipment than is often used so a more typical case is that the harmonics form part of the measured RMS values.

Measured true RMS data could be used to specify either the current or the voltage data in a network model. The RMS voltage could be used to provide a voltage profile for the substation in a network simulation so that, for example, a model of voltages at the customer connection could allow for variations due to the primary substation tap changer. The measured true RMS voltage is:

$$V_{\text{Arms},it} = \sqrt{\sum_{h=1}^{N_{\text{h}}} \left| V_{\text{A},iht} \right|^2}$$
(109)

An example calculation is shown below with the amplitude of the substation voltage equal to the measured true RMS value and used to specify a sinusoid at the fundamental frequency. The phase angle for the substation voltage is taken to be that of the measured fundamental component, although a power quality analyser may have other means (such as by calculations of the zero crossing points) to define this if it does not have a spectral analysis capability. In this example the harmonic current data is retained and so the vector voltage difference along the cable is known at each frequency. The substation voltage is set to zero at all frequencies other than the fundamental. The voltage drop for each sample t is then calculated as:

$$V_{d,it} = V_{Arms,it} - \sqrt{\left| V_{Arms,it} e^{j \angle V_{A,i1t}} - \sum_{k=1}^{3} \hat{Z}_{1,ik} I_{k1t} \right|^{2} + \left( \sum_{h=2}^{N_{h}} \left| \sum_{k=1}^{3} \hat{Z}_{h,ik} I_{A,kht} \right|^{2} \right)}$$
(110)

This calculation gives negligible change in the voltage, with the worst-case voltage drop on phase L2 increasing by 0.014 V. This is a change of only 0.8% relative to the worst-case voltage drop of 1.7 V shown in Figure 7-1. There is therefore little error introduced by approximating the substation voltage as a single sinusoid at the fundamental frequency, with the same amplitude as the measured true RMS voltage.

Similarly, the load current data may be based on measurements with the true RMS of the distorted current being used to specify the amplitude of a waveform at the fundamental frequency. This approximation is less secure than the corresponding

approximation used to derive the substation voltage fundamental component from the measured true RMS since the current harmonics are usually much greater in proportion to the fundamental component. By representing harmonic currents as a contribution to the fundamental frequency, the combination of these currents in the neutral is changed. For example, if the load currents were perfectly balanced but distorted, this approximation would model zero current in the neutral, whereas the balanced triplen harmonics would actually combine coherently in the neutral.

In the following example the current vector for the voltage drop calculation is formed by combining the amplitude based on the measured true RMS with the phase angle of the measured fundamental component. The true RMS current is:

$$I_{\text{Arms},it} = \sqrt{\sum_{h=1}^{N_{\text{h}}} \left| I_{\text{A},iht} \right|^2}$$
(111)

The same approximation is made as above with the substation voltage based on the measured RMS, and so the voltage drop is given by:

$$V_{d,it} = V_{Arms,it} - \left| V_{Arms,it} e^{j \angle V_{A,i1t}} - \left( \sum_{k=1}^{3} \hat{Z}_{1,ik} I_{Arms,kt} e^{j \angle I_{A,k1t}} \right) \right|$$
(112)

The difference in RMS voltage drop introduced by this approximation is shown in , calculated using the estimated impedances from Section 5.4.2. The greatest impact in on phase L1 for which the worst-case voltage drop increases by 35% from 0.8 V to 1.1 V. However, the maximum voltage drop of 1.7 V is on conductor L2, for which the worst case increase in RMS voltage is 0.1 V, or 6% relative to the maximum voltage drop.



Figure 7-3 – RMS voltage drop with harmonics included or with measured RMS at fundamental frequency

Finally, a third approximation is considered where the load current is based on the active power requirements of a demand model, without including any harmonic currents. The substation voltage is again based on the measured RMS and the voltage drop is then calculated as:

$$V_{d,it} = V_{Arms,it} - \left| V_{Arms,it} e^{j \ge V_{A,i1t}} - \sum_{k=1}^{3} \hat{Z}_{1,ik} I_{A,k1t} \right|$$
(113)

Relative to the results with the harmonics fully included, the worst-case voltage drop with this approximation increases by about 0.02 V, or 1.2%. This shows that a more accurate approximation is obtained if the harmonics are omitted completely than if they are included as a contribution to the amplitude of the sinusoid at the fundamental frequency.

### 7.4.2 Losses in the LV cable

If the two power quality analysers used for the measurements were configured to record the instantaneous active power, the losses could be calculated as the difference of the two sets of readings. The active power calculation within the analysers is a time domain summation of the product of voltage and current and so includes harmonics (Fluke 2012). As noted in Chapter 5, the accuracy of the results from this method is critically dependent on the tolerances of the current and voltage sensors being smaller relative to the proportion of power that is dissipated as losses.

The same approximations as discussed in Section 7.4.1 where the amplitude of the substation voltage and load currents are specified by measured RMS data are considered now with regard to losses. However, only the approximation for the load currents is relevant here as the losses depend on the differential voltage along the cable but not the voltage at any one node.

With the load currents specified such that the measured RMS current represents a sinusoid at the fundamental frequency, the losses are given by:

$$P_{\text{loss},t} = \text{re}\left(\sum_{i=1}^{3} \left[ \left( \sum_{k=1}^{3} \hat{Z}_{1,ik} I_{\text{Arms},kt} e^{j \angle I_{\text{A},k1t}} \right) \times \left( I_{\text{Arms},it} e^{j \angle I_{\text{A},i1t}} \right)^* \right] \right)$$
(114)

If the load currents are specified using the fundamental component alone, such as with demand models based on the load powers, then the loss calculation is the same as shown previously in (108) for the case when harmonics are absent.

The losses for each of these cases are compared in Figure 7-4 for the time period with the variable speed pump switched on. For this period, the peak losses were 182 W with the harmonics included, and 131 W if the harmonics are omitted. The peak loss power is therefore under-estimated by approximately 28% where harmonics are omitted from the load currents.

If the current at the fundamental frequency is defined by the measured RMS amplitude, then the peak losses are 167 W and the error is reduced to 8%.



Figure 7-4 – Losses with harmonics included, with measured RMS at fundamental frequency, or harmonics omitted

Although there is a significant impact of neglecting the harmonics for the measurement period shown in the figures for the period with the variable speed pump operating, there is less impact when the mean loss is considered over the full length of the measurement. For most of this period, the demand was due to conventional office appliances with a lower level of distortion than with the pump in operation. The impact of including harmonics in the simulation is summarised in Table 7-2, showing a more modest difference in the calculated losses of approximately 4.3% when the harmonics are neglected. If the current is defined according to the measured true RMS then the estimated losses are closer to those predicted when the harmonics are taken into account, with the error reduced to 2.4%.

Simulation model	Mean loss,	Mean loss	Maximum loss	Maximum loss
	W	relative to	with pump	relative to
		case 1	operating W	case 1
With harmonics	22.54	-	182.3	-
Load current amplitude from				
measured RMS	22.00	-2.4%	167.0	-8.4%
No harmonics	21.58	-4.3%	131.6	-27.8%

 
 Table 7-2 – Losses with harmonics included, with measured RMS at fundamental frequency, or harmonics omitted

These results demonstrate that the calculations are affected if harmonics are neglected. There was a significant impact (under-estimating by close to 1/3 of the losses) when the current was highly distorted, but a more modest impact over full measurement period with the electrical demand from office working. A single-frequency calculation at the fundamental frequency gives a better approximation where the amplitude is based on the RMS of the distorted current. This result differs from that for the voltage drop where omitting the harmonics provided a better approximation than using the measured RMS to specify the current amplitude.

## 7.5 Loss approximation method to allow for harmonics

For most simulation studies using synthesised demand models, where the currents are specified according to the load power, there is no data to specify the RMS current that would include the harmonic distortion. If this information were to be available, the results from Section 7.4.2 show that a better estimate of the losses could be made. If the current distortion were known, then the true RMS could be predicted from (103) (substituting the voltage notation for current). However, in order to replicate the true RMS exactly, the THD would need to be known for each current phasor sample and for each phase conductor.

It is more likely that a general estimate of the level of distortion may be available from previous characterisations of similar feeders, and so a new method can be proposed where this is used to approximate the RMS current. This would be then be used in a similar manner to the loss load factor calculations where measurement data from a small number of feeders is used to approximate the losses on other feeders for which no measurement data is available. To model the outcome from this approximation, the mean current THD has been estimated from the measured data, as follows:

$$\text{THD}_{I,\text{all}} = 100 \times \sqrt{\frac{\sum_{i=1}^{3} \sum_{t=1}^{N_{t}} \sum_{h=2}^{N_{h}} |I_{A,iht}|^{2}}{\sum_{i=1}^{3} \sum_{t=1}^{N_{t}} |I_{A,i1t}|^{2}}}$$
(115)

This provides a mean current distortion of 20.4%, which can be applied to estimate the true RMS current from the current at the fundamental frequency, giving:

$$I_{\text{A rms},it} = \left| I_{\text{A},i1t} \right| \times \sqrt{1 + \left(\frac{\text{THD}_{\text{I,all}}}{100}\right)^2}$$
(116)

The loss can then be calculated as before using (114).

A second and more refined method takes account of the variation of distortion with the load current. Typically higher power loads are likely to be more resistive, and therefore introduce less distortion, whereas appliances with power electronic converters are typically lower powered. A single figure for the distortion as a percentage of the fundamental current may therefore not always be applicable. This relationship is illustrated in Figure 7-5, showing points separately for each line conductor, and also a best fit curve. The graph includes measurements for the period with the flume in operation for which the distortion increased approximately proportionally with the current, giving the three distinct lines that deviate from the curve. These lines are different for each line conductor since the current for the flume was superimposed on different background loads.



Figure 7-5 – Current THD variation with amplitude of the fundamental frequency component

From observation of the data, it was found that the root mean square of the current distortion (in amperes) increases logarithmically with the amplitude of the current at the fundamental frequency, so that:

$$\sum_{h=2}^{N_{\rm h}} \left| I_{{\rm A},iht} \right|^2 = k_{\rm THD} \ln \left| I_{{\rm A},iht} \right|$$
(117)

Substituting from (102) (and replacing  $|I_{A,iht}|$  for  $V_{rms,h}$ ) gives

$$\text{THD}_{I,it} = 100 \times \frac{k_{\text{THD}} \ln \left| I_{A,iht} \right|}{\left| I_{A,iht} \right|}$$
(118)

For the measured data in this test case,  $k_{\text{THD}} = 1.13$ . The true RMS current can then be approximated using this THD value as before.

Using these estimation approaches, the losses were calculated for the measured data, as shown in Table 7-3.

Both methods give a better approximation to the losses than if the calculation is based on the fundamental frequency component alone. The curve-fitting approach, in which the distortion varies according to the current, gives a slightly less accurate prediction of the mean losses but a better prediction of the losses for the peak load currents within the measurement. This method is more general as it allows for variations in the mean load and distortion, while assuming that these two parameters follows the same empirical logarithmic relationship.

 Table 7-3 – Losses predicted using approximated distortion data, calculated over full measurement length

Simulation model	Mean loss,	Mean loss	Maximum loss	Maximum loss
	W	relative to	over full	relative to
		case 1	measurement W	case 1
Case 1	22.54	-	197.8	-
RMS currents estimated using				
mean THD	22.48	-0.3%	204.8	+3.5%
RMS currents estimated using				
curve fit to distortion	22.17	-1.6%	197.2	-0.3%

The application of this method clearly relies on the assumption that either the mean levels of current distortion, or the relationship between the distortion and the demand, have similar characteristics on multiple feeders. This has not been demonstrated here as the measurements provide data for only one feeder. These characteristics are likely to vary significantly between industrial and commercial feeders and domestic feeders. However, it seems likely that the characteristics would be similar for feeders with similar types of customer, particularly for domestic feeders where the demand is based on many aggregated loads and where there is a similar probability that the same types of appliances are used. Further measurements would be required to demonstrate this in practice.

## 7.6 Conclusions

This chapter investigates whether the accuracy of network models is significantly degraded if the harmonics are omitted from calculations of the RMS voltage drop and losses.

Using measured data with highly distorted current waveforms, it has been demonstrated that there is minimal impact on the RMS voltage drop due to the

Harmonic distortion

presence of harmonics. Only a 1% difference was found in the voltage drop for current waveforms with up to 75% THD. This suggests that any increase in current distortion due to the introduction of new low carbon technologies will not have an adverse impact on the RMS customer voltage.

However, the losses in the LV cable were found to increase due to the current distortion, with mean losses for the current including distortion being around 4.4% higher than without the distortion. Over a period with high current distortion due to a variable speed pump, the maximum losses were 38% higher with the distortion included than without. If new LCTs cause current distortion, this will need to be taken into account when considering the thermal capacity ratings of cables.

Although networks already have significant current distortion, simulations typically only model the fundamental frequency. Where the currents are specified using a demand model based on the active and reactive power, the harmonic currents are typically omitted. The errors with this approach for the measurement period with highly distorted currents are the same as described above with only 1% difference for the voltage drop and with the mean losses under-estimated by 4.3%.

Alternatively, where the currents are specified using measured RMS data, the harmonics are typically included in the measurement. If the measured RMS current was used to define the amplitude of a sinusoid at the fundamental frequency then then the worst-case voltage drop for the measured data was over-estimated by 6%, and by up to 35% on the conductor with the lowest voltage drop. Conversely, this approximation gives a better approximation of the losses, under-estimating by 2.4% rather than by 4.3% for the model with the harmonics omitted.

Therefore, although a better approximation was given for the voltage drop by considering only the fundamental component rather than by approximating this with the measured RMS amplitude, the reverse is true when calculating losses.

Measured data to define the true RMS is not typically available for network simulations and so a method is proposed whereby this can be approximated based on past experience of the distortion. This assumes that the relationship between current distortion and the current amplitude can be measured for some feeders, giving empirical factors that can then be applied more generally. Simplistically, a
scaling factor based on the average current distortion could be applied to allow for the additional losses caused by harmonics. Alternatively, an empirical relationship between the distortion and the fundamental component of the current could be used to estimate the true RMS of the load currents. The measurements have demonstrated that there is an approximately logarithmic empirical relationship between the RMS distortion current and the fundamental with the distortion reducing as a proportion of the fundamental as the load increases. Losses calculated using both of these approaches have been shown to give improved estimates of the mean losses but the use of a single scaling factor based on the average distortion overestimates the peak losses. This is resolved using the second of these approaches where the true RMS is estimated as a function of the fundamental.

A simulation may also use measured data to define the substation voltage, again with the RMS value being used to define the amplitude of a sinusoid at the fundamental frequency. However this makes negligible difference to the results.

# 8 DEMAND MODEL TIME RESOLUTION

## 8.1 Introduction

The measurements described in previous chapters have captured voltage and current data at a much higher resolution than is used in most simulations studies which typically use demand data averaged over periods of between 1 minute and 1 hour. This chapter therefore considers how calculations of metrics such as voltage drop and losses are affected by the use of longer demand data averaging periods.

A key concern here is the demand data is typically derived from an arithmetic mean measurement of the customer demand. The demand varies approximately in proportion to the current but the copper losses vary in proportion to the square of the current. If the arithmetic mean demand does not represent the 'spiky' nature of the real customer demand then the copper losses will be under-estimated. This issue is well known and covered by standard texts (Lakervi & Holmes 1995) and has also been investigated for LV networks (Brandauer et al. 2013). Although this demonstrates the problem, it does not provide guidance on how to select an adequate resolution so as to avoid significant under-estimation of the losses.

This issue is considered first in Section 8.2 in the context of single-phase loads. This allows the underlying concepts to be established, without the additional complexity of considering the variation of the demand across the three phases. This section describes the loss ratio, as the ratio of the estimated mean losses to the actual mean losses, and proposes a method by which the actual losses can be estimated from measured data.

In Section 8.3, the analysis is applied to the measurements recorded from the LV cable on the Loughborough campus. This allows the impacts to be demonstrated for a three-phase case. In this case the loss ratio depends on the unbalance of currents between the three phases and also on their power factor.

The impacts of the time resolution are also considered with respect to the RMS voltage drop and the unbalance factor.

# 8.2 Time resolution and losses in single-phase cables

The discussion in this section is a summary of the published work (Urquhart & Thomson 2015a) included here as Appendix B.

## 8.2.1 Analysis of simple switched loads

In a simple example, the losses to supply a 1 kW load for 10 seconds, are one tenth of the losses caused by supplying a 10 kW load for 1 second. Both cases will have the same arithmetic mean demand if the 10 kW load is switched on for 1 second and then switched off for the remainder of 10 second averaging period but the losses will be under-represented by a factor of 10. However, even the measurement of demand over a 1 second interval may hide further variation of the demand current such that this is also an under-estimate. In order to know the extent by which the calculated losses are in error, it is necessary to know the 'true' losses that would occur if the demand data was not averaged.

This was investigated using several idealised test cases in which a simplified current variation was modelled. It is assumed here that the line to neutral voltage is approximately constant and so the demand current varies in proportion to the arithmetic mean power over the averaging period defined by the demand data time resolution.

The analysis first considers a case where the current has a step change between two states  $I_1$  and  $I_2$ , as shown in Figure 8-1. The current amplitude variation is characterised by a ratio  $x = I_2/I_1$ , and the switch timing is defined such that  $\beta$  is the proportion of the averaging period for which current state  $I_1$  occurs. The radio of the estimated loss to the actual loss can then be calculated over the averaging time period of length  $t_M$ . This ratio of estimated losses to the actual losses is referred to as the 'loss ratio' and is given by:

$$r_{\rm A} = \frac{\left(\beta + x(1 - \beta)\right)^2}{\beta + x^2(1 - \beta)}$$
(119)



Figure 8-1 – Step change in current

If the current is constant then  $r_A = 1$  and the losses are accurately represented. Where the demand is highly aggregated, and the current variation is small relative to the magnitude, then the error in the estimated losses will be less than where the demand is highly variable. The under-estimation of losses is therefore a greater risk for network branches that are closer to the individual customers. Fortunately, the magnitude of the losses is generally less for these less aggregated ends of the network.

Conversely, if the ratio between the currents is close to zero, such as with a load that switches on and then completely off for part of the averaging period, then  $r_A = \beta$ . In this case, as the proportion of the time for which the load is switched on reduces, the proportion of the actual losses that are estimated also reduces, and the error in the estimated losses increases.

The loss ratio is the same regardless of whether the load is on for many short bursts, adding up to a total proportion  $\beta$ , or switched on for a single longer burst. Equation (119) is therefore equivalent to the loss ratio for the case where the averaging period is longer than the duty cycle of the load with a cyclic switching behaviour, with either one or many switching cycles within the period  $t_{\rm M}$ .

A second situation is then considered where the averaging period is shorter than the dwell times for the two current states, as shown in Figure 8-2. It is shown in Appendix B that the loss ratio for this case is given by:

$$r_{\rm A} = 1 - \left(\frac{(1-x)^2}{3(\beta + (1-\beta)x^2)}\right) \cdot \left(\frac{n_{\rm t}}{t_{\rm M}}\right) \cdot t_{\rm A}$$
(120)

where  $n_t$  is the number of transitions from state  $I_1$  to state  $I_2$  (over a long period, the expected number of transitions is the same from state  $I_2$  to state  $I_1$ ) and  $t_A$  is the averaging period (now much shorter than the total measurement time  $t_M$ ).

The loss ratio in (120) contains a term subtracted from 1 that is given by the product of three factors. The first factor depends on parameters x and  $\beta$  which relate to the 'spikiness' of the demand and the second factor  $n_t/t_M$  describes the rate at which the current states change relative to the total length of the measurement. These factors are determined by characteristics of the demand, rather than the averaging used to measure it. The third factor indicates that the loss ratio is also proportional to the averaging period length.



Figure 8-2 – Step change in current averaging periods shorter than the duty cycle

Having established these relationships, Appendix B then provides simulation results using current states generated by randomised sequences, and demonstrates that the loss ratio from the simulation agrees with the analysis. Where the averaging period is much longer than the switching duty cycle, the loss ratio tends towards the ratio given in (119) and is independent of the averaging period. Similarly, where the averaging period is much shorter than the duty cycle, then the loss ratio tends towards the ratio from (120). Where the averaging period is similar to the duty cycle, the simulation demonstrates a smooth transition between the two conditions covered by the analysis.

The simulations are then extended to consider a more complicated case where there are multiple appliances, and so multiple possible current states. As the number of appliances increases, the total aggregated current becomes less spiky and so the errors in the loss estimates are reduced.

## 8.2.2 Mean loss estimates with measured data

The loss ratio has also been considered for measured data. Initially, this was investigated for the demand from a single dwelling. For a single-phase grid connection, the loss ratio here represents the errors in the phase and neutral conductors of the service cable between the feeder and the customer connection point.

The analysis used data measured at 1 second resolution over a period of 7 days (Taylor 2010). Over much of the 7 day period, the demand was relatively constant. However, for short periods, such as shown in Figure 8-3, the demand switched frequently, and estimates made with 1 minute or 30 minute resolution would not fully represent the variation in the currents.



Figure 8-3 – Example 30-min interval of demand for one dwelling

The loss ratio was calculated for this data treating the 1 second data as a reference, and then calculating the losses with the demand averaged over different intervals, from 2 seconds up to hourly, giving the curve shown in Figure 8-4. For this measurement data, the estimated losses with 1 minute averaging would be 89% of the losses with 1 second data. For 30 minute data, the estimated losses are 62% of the losses with 1 second data, giving an error of 38%.



Figure 8-4 – Loss ratio for one dwelling, based on 1 second data

These errors represent the extent to which losses in a specific cable or line serving one customer would be under-estimated. If losses were to be calculated for such a cable, then the ratios from Figure 8-4 could be used to scale-up the estimated losses and to allow for the effects of time averaging.

However, the curve in Figure 8-4 is based on one customer and is not necessarily representative. The following section presents a method that can be used more generally to improve loss estimates.

## 8.2.3 Extrapolation method to estimate the actual losses

From Figure 8-4, it can be seen that the gradient of the loss ratio curve increases as the averaging period reduces towards zero. This is perhaps an unexpected result, since it might be expected that the curve would converge towards a line with the loss ratio equal to unity as the averaging period reduces towards zero. This intuitive trend can be demonstrated if the averaging period axis is plotted using a logarithmic scale as in Figure 8-5. However, the results in Figure 8-4 suggest that there is a

convergence to a linear relationship for the gradient of the loss ratio with respect to the averaging period, rather than for the loss ratio itself.

This can be explained from (119) and (120) if the loss ratio is considered to have contributions from multiple loads with different length duty cycles, at least for the idealised state where loads switch between two states. Where the averaging period is longer than the duty cycle, the loss ratio is given by (119) and is independent of the averaging period length. If the averaging period is shorter than the duty cycle, then (120) applies and the loss ratio is proportional to the averaging period length. For a mixed set of loads, an averaging period that is shorter than any of the duty cycles will follow (120) and have a gradient that is proportional to the averaging period. As the averaging period increases, it will become longer than the duty cycles of some of the loads, with their contribution to the loss ratio being flat in relation to the averaging period. When the averaging period is longer than any of the duty cycles, the loss ratio curve will decrease no further.



Figure 8-5 – Loss ratio for one dwelling, based on 1 second data, plotted on logarithmic scale

Using this reasoning, it is possible to predict a scaling factor to allow for the actual losses without the effects of demand data averaging by extrapolating the loss ratio curve towards an averaging period of zero. The method takes the losses predicted

with the demand data resolution available, and then finds the losses for an alternative solution where adjacent demand samples are averaged together. The loss ratio for an averaging period of zero can then be determined by linear extrapolation. The estimate of the actual losses  $e_{\rm R}$ ' is then then given by:

$$e_{\rm R}' = 2e_{\rm A,1} - e_{\rm A,2} \tag{121}$$

where  $e_{A,1}$  is the loss estimate with the original demand data and  $e_{A,2}$  is the loss estimate for the case where successive pairs of demand samples are averaged together.

Based on the discussion above, if the best available demand data resolution is shorter than the fastest duty cycles of the loads then the extrapolation will provide the actual losses. However, the duty cycles of the load appliances are not generally known when considering demand data. Based on the reasoning outlined above, the extrapolated estimate will be a better estimate of the actual losses and will be an under-estimate rather than an over-estimate.

A second set of measurements has been considered in which the demand has been recorded with 1 minute averaging periods from a group of 22 residential dwellings in Loughborough (Richardson & Thomson 2010). Although this data has poorer resolution than the 1 second data discussed above, the availability of demand data for multiple dwellings allows the impact of demand aggregation on the loss ratio to be investigated. For the purposes of this study, it has been assumed that all of the dwellings are supplied with a single-phase connection (such as might occur in practice for some rural customers).

The loss ratio curve for this measurement data is shown in Figure 8-6. For each curve, an estimate of the actual losses has been made using (121) and the loss ratios are then normalised to this value. These results exhibit the same characteristic as noted above, whereby the gradient of the loss ratio curve (literally, the magnitude of the gradient, since the gradient is negative) increases as the averaging period reduces, and also demonstrate the reduction in errors where the demand aggregation is greater. This is highlighted by the lines showing the extrapolation towards an averaging period zero. As the averaging period reduces, the loss ratio curves tend towards these linear projections.



Figure 8-6 – Loss ratio for groupings of dwellings, based on 1 minute data

This interpolation method can be used by DNOs to improve estimates of losses based on measured data. Ideally measurements would have a high resolution such that the under-estimation of losses is minimised, but whatever resolution is available, the results here show that interpolation method will provide an improved estimate.

### 8.2.4 Short term heating effects

The above discussion considers the losses as a long term average and relates to the energy lost from the power system. Models considering the heating within the cable need to consider the losses over shorter time periods.

This has been addressed using the data sets described above taking a time period of 30 minutes as an example of an interval that may be relevant for thermal considerations within the cable. An example is shown in Figure 8-7 for the aggregated demand of a group of 22 dwellings with demand data measured at 1 minute resolution. Figure 8-7 shows the ratio of the loss calculated from the 30 minute mean demand to the mean loss over 30 minute calculated from 1 minute demand. This loss ratio is plotted against the mean demand over the 30 minute period. Typically the greatest errors in the loss estimates occur when the demand is lower, and the loss ratio is closer to unity for the peak demand. However, at the peak demand of 25 kW in this example, losses calculated with 30 minute data would be under-estimated by 4% compared to the loss estimated with 1 minute data. The errors in peak loss estimates for less aggregated demand can be considerably greater (although the magnitude of the losses is typically lower).



Figure 8-7 – Energy loss for group of 22 dwellings using 30 minute data, relative to mean loss over 30 minutes, based on 1 minute data

The worst case error condition can be defined by a peak loss ratio  $r_{A,pk,k}$ , calculated as the peak loss using the averaged demand, relative to the peak loss over the same duration using un-averaged data. The peaks in the averaged and un-averaged data do not necessarily occur in the same time period. The peak loss ratio is given by:

$$r_{A,pk,k} = \frac{k \cdot \max\{(I_{A,k,i}^{2}) : i = 1, \cdots, n_{A}/k\}}{\max\{(\sum_{t=(i-1)k+1}^{t=ik} I_{t}^{2}) : i = 1, \cdots, n_{A}/k\}}$$
(122)

where  $I_{A,k,i}$  is the current for the arithmetic mean demand for averaged sample *i* with averaging block size *k*.

This highlights a disparity that may occur between network simulations and measurements of actual currents if the maximum currents are calculated using the arithmetic mean demand, but the measured currents are recorded by a maximum demand indicator. These indicators typically operate by measuring the heating effect of the current in a bi-metallic strip and so the recorded value gives an RMS measurement over the time lag period, typically 15 to 30 minutes (Kamaraju 2009). The sensitivity of the indicator to spikes in the demand is further increased since the influence of the heating effect on temperature rise is not uniformly weighted over the nominal time lag period (unlike a digitally calculated average), such that short spikes of high demand have a greater influence on the maximum temperature. There is also a risk that simulations and measurements will give different results if the time lag of the maximum demand indicator is different to that used for the demand data time resolution. If, for example, a maximum demand indicator with a 15 minute time lag is used to calibrate the maximum current of a demand profile with hourly data, then the currents will be over-estimated.

## 8.2.5 Loss load factor

In many cases detailed demand data is not available and so losses might be estimated as a function of the recorded maximum demand, scaling the losses according to the loss load factor. This is defined as the ratio of the mean losses to the losses for the peak demand (Lakervi & Holmes 1995). This raises the question of the time resolution used to define the peak demand.

The loss load factor has been calculated for the group of 22 dwellings described above, as shown in Figure 8-8. Results are shown for two methods of calculation, as described more fully in Appendix B, using either RMS or arithmetic mean averaging for the demand. The figure shows that the loss load factor is dependent on the level of demand aggregation, with much lower values for a single dwelling than for the group of 22. The loss load factor also increases as the averaging period increases.

The main concern in estimating the mean loss through the of the loss load factor is that the averaging period used to define the peak demand must be consistent with that used in computing the loss load factor figure. In an example DNO calculation with hourly data, there is no inaccuracy provided that the peak demand is also measured for a period of an hour (E.ON Central Networks 2006). However, if the

loss load factor is applied using peak demand from instrumentation with a 15 or 30 minute integration period, such as from a maximum demand indicator, then errors could be introduced into the loss calculation.



Figure 8-8 – Loss load factor for groupings of dwellings, based on 1 minute data

From a practical perspective, the recommendation from this study for DNOs is therefore to recognise that the loss load factor metric depends on the time resolution used to calculate the demand profile, and so to ensure that the losses for the maximum demand are calculated using the same time resolution. This avoids further inaccuracy due to the use of the loss load factor metric. It should be noted that even with a consistent use of the loss load factor calculation, the errors discussed in Section 8.2.2 due to the use of time-averaged demand would still apply.

## 8.3 Time resolution impacts on three-phase measured data

This section considers the impacts of the demand data time resolution using the three-phase measurements described in Chapter 5. The measured RMS data has been used here as these tests have a long duration and so represent a much greater

variation in demand conditions than the 5 minute waveform captures. This also enables the use of a 10 minute average period, as specified in the EN 50160 (BSI 2011b).

The measured LV feeder serves a building used partly for office work and also to house laboratory equipment. The demand profile could be considered to model the demand of a feeder with industrial or commercial customers. This data therefore differs from the data considered in Section 8.2.2 which was representative of domestic demand. The aim here is to investigate whether the same trends apply as in Section 8.2.2 when the losses are dependent on the balance between the phases in addition to the time variation of the demand, and also to consider the impacts on the RMS voltage drop and the unbalance factor.

### 8.3.1 Data averaging

The measured data with resolution of 250 ms is used to synthesize demand data with longer averaging periods of blocks of  $N_{\rm m}$  measurement intervals, as follows. :

$$S_{\text{mean},im} = \sqrt{\frac{1}{N_{\text{m}}} \sum_{t=(m-1)N_{\text{m}}+1}^{t=mN_{\text{m}}} \left( \left( V_{\text{B,rms},it} \cdot e^{j \angle V_{\text{B},i1t}} \right) \left( I_{\text{A,rms},it} \cdot e^{j \angle I_{\text{A},i1t}} \right)^{*} \right)}$$
(123)

where  $S_{\text{mean},im}$  is the mean complex power in block *m*. The true RMS measurements recorded directly by the power quality analysers have been used for this investigation as this data is not affected by the smoothing filter applied to the amplitude and phase data for the individual frequency components. The harmonics are therefore included here, and the total RMS amplitude is assumed to represent a waveform at the fundamental frequency.

A power-flow simulation method is coded in Matlab in order to calculate the current and voltages for the averaged demand which is assumed to model a constant power load. Since only one cable branch is being modelled, this is a simpler algorithm compared to network power-flow simulations. The iterative process finds the branch currents and the voltage at the cabinet that supplies the required average demand power. This process also requires an averaged voltage to be specified at the substation, which is given by taking the RMS of the recorded true RMS voltages over the block of measurement intervals for the corresponding averaged demand. A perfect three-phase angle configuration of 0°, -120° and -240° is assumed, so that the averaged substation voltage vector is given by:

$$V_{A,\text{mean},im} = \left( \sqrt{\frac{1}{N_{\text{m}}} \sum_{t=(m-1)N_{\text{m}}+1}^{t=mN_{\text{m}}} V_{A,\text{rms},it}^{2}} \right) e^{j\frac{2\pi i}{3}}$$
(124)

For each average period *m* and for each conductor *i*, the power-flow process is initialised by setting  $V_{B,mean,im} = V_{A,mean,im}$ . The current is then estimated using  $I_{mean,im} = (S_{mean,im}/V_{B,mean,im})^*$ . A new estimate of the voltage at analyser B can then be found using:

$$V_{\rm B,mean,im}' = V_{\rm A,mean,im} - \sum_{k=1}^{3} \hat{Z}_{1,ik} I_{\rm mean,im}$$
 (125)

This process continues, setting  $V_{B,mean,im} = V_{B,mean,im}'$  and repeating until the maximum difference of the magnitudes of  $V_{B,mean,im}$  is less than a defined threshold, in this case 10<sup>-8</sup> V.

This averaging, followed by the power-flow solution, then gives a set of voltages and currents for different averaging period lengths. Results have been generated for with the raw recorded data resolution of 250 ms, and with averaging block sizes corresponding to 1 second, 30 seconds, 1 minute, 2 minutes, 5 minutes, 10 minutes, 30 minutes, and 1 hour. These are now compared so see the impacts on the voltage drop along the cable, the losses, and the unbalance factor.

### 8.3.2 Losses in the LV cable

The variation of the loss power for different averaging periods is shown in Figure 8-9. The plot shows results for a time interval of 2 hours duration, including the period with the flume switched on. Results for a longer portion of the measured data have been plotted here so as to show the impact of the data averaged for periods of up to 1 hour.



#### Figure 8-9 – Loss on all conductors, comparison of averaging periods

The variation of the mean losses over the full measurement period relative to the averaging period is shown in Figure 8-10. The losses are under-estimated by a very small proportion if the data is averaged over 1 second, but there is a 1% error for 1 minute data, a 4% error for 10 minute data, a 6% error for half-hourly data, and a 7% error for hourly data.

The loss ratio figures are lower here than those presented in Section 8.2.2 for individual dwellings. This relates to the level of demand aggregation being higher as this particular cable section serves the aggregated demand from multiple occupants and appliances in the building. The overall impact of time resolution on estimates of losses needs to take account of the few sections of cable (such as this) with higher losses and lower estimation errors, plus the many cables serving individual customers where the losses are lower but the estimation errors are higher. This is addressed in Section 10.6 where the losses are considered throughout the feeder.

The results in Figure 8-10 also demonstrate the same trend as highlighted in Figure 8-4 such that the gradient of the loss ratio vs. the averaging period increases (increasingly negative) as the averaging period reduces towards zero, and that this tends towards a linear relationship. This demonstrates that the extrapolation method

to estimate the actual losses with no averaging, as proposed in Section 8.2.3, can also be applied to three-phase data.



Figure 8-10 – Mean loss over measurement period, comparison of averaging periods, normalised to loss for 250 ms measurement interval

These results demonstrate that the under-estimation of losses due to demand data averaging, previously demonstrated for single-phase cables in Section 8.2, applies similarly to three-phase cables.

It should be noted here that the results of the analysis in this chapter are concerned with the impacts on particular sections of cable. The errors due to demand data averaging are more significant in single-phase service cables serving a single customer (typically with low losses) and less in three-phase cables serving an aggregated customer demand (where the losses are higher). Chapter 10 considers the errors in the mean losses due to demand data averaging when all of the cables in an LV feeder are included.

## 8.3.3 Voltage drop

The impact of demand time averaging on the RMS voltage drop is now considered.

For the averaged data, this is given by  $|V_{A,\text{mean},im}| - |V_{B,\text{mean},im}|$ .

The RMS voltage drop for conductor L2 (which has the greatest current) is shown in Figure 8-11.



#### Figure 8-11 – RMS voltage drop on L2 conductor, comparison of averaging periods

If the simulation were to use the full recorded resolution, there would be a maximum voltage drop of 1.8 V over the time interval shown. However, if the simulation were to be based on demand data with 10 minute data, then the highest voltage observed over this period would be 1.4 V, significantly under-estimating the maximum voltage drop.

The maximum voltage drops over the full duration of the measurement are shown in Figure 8-12, with the maximum of 2.6 V on conductor L2 for the 250 ms data being approximately 50% higher than the maximum of 1.7 V for 10 minute data. As expected, the maximum voltage drop is much less using the averaged data, partly because the demand currents are smoothed, but also due to the reduction in unbalance caused by this smoothing.

These differences may need to be taken into account for UK networks since the use of 10 minute RMS data is not specified in the Electricity Safety, Quality and Continuity Regulations that dictate the tolerance bands for LV customer connections (UK Government 2002).



Figure 8-12 – Maximum RMS voltage drop, comparison of averaging periods

For European countries where BS EN50160 is used to define power quality, the tolerance bands specified for LV customer voltages are based on 10 minute RMS data. However, a higher resolution may be used in simulations if this is needed for other purposes (such as loss estimation), or if this is dictated by the resolution of the demand model.

This has been tested using the voltage data based on the averaged demand by applying further averaging to find the 10 minute RMS voltages. This gives results that are almost exactly equal to the curve in Figure 8-11 where the original averaging period is 10 minutes. In effect, the 10 minute mean voltage is the same, regardless of whether the demand data is averaged to 10 minute periods, or whether the demand has a higher resolution and the resulting voltage data is averaged to 10 minute periods. This outcome is to be expected since the voltage difference varies linearly with the current and so approximately in proportion to the arithmetic mean demand.

Clearly, if demand data averaging periods longer than 10 minutes are used then the voltage results will not accurately reflect the 10 minute averages. Using half-hourly data gives a park voltage drop of 1.6 V over the duration of the measurement, rather than the 1.7 V for the standard metric of 10 minute data.

# 8.3.4 Unbalance

There are several different definitions of unbalance in use in standards (Beharrysingh 2014). In the UK, Engineering Recommendation P29 requires that the unbalance for LV supplies is less than 2% based on an average over a 1 minute period (Electricity Council 1990). Voltage unbalance is defined in EN 50160 as the ratio of the 10 minute RMS of the negative sequence component to 10 minute RMS of the positive sequence component (BSI 2011b). The unbalance is required to be less than 2% for 95% of the time within 1 week. This metric is used in the analysis here.

EN 50160 defines that the sequence transform is performed first, followed by the RMS averaging of the sequence components. Starting with the voltages  $V_{B,mean,im}$  generated by the power-flow method of Section 8.3.1, the sequence voltages are calculated as:

$$\begin{pmatrix} V_{B,zero,m} \\ V_{B,pos,m} \\ V_{B,neg,m} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{4\pi j/3} & e^{2\pi j/3} \\ 1 & e^{2\pi j/3} & e^{4\pi j/3} \end{pmatrix}^{-1} \times \begin{pmatrix} V_{B,mean,1m} \\ V_{B,mean,2m} \\ V_{B,mean,3m} \end{pmatrix}$$
(126)

Where the demand was averaged for a block size of  $N_{\rm m}$  measurement intervals, the RMS of the sequence voltages is then calculated over a block size of  $N_{\rm n}$  blocks, where  $N_{\rm n} = 60 \times 60 \times 10 \times 4/N_{\rm m}$  for an original resolution of 250 ms. The 10-minute unbalance factor is therefore given by:

$$F_{\text{B,unbalance},n} = \sqrt{\frac{\sum_{m=(n-1)N_{\text{n}}+1}^{m=nN_{\text{n}}} |V_{\text{B,neg},m}|^2}{\sum_{m=(n-1)N_{\text{n}}+1}^{m=nN_{\text{n}}} |V_{\text{B,pos},m}|^2}}$$
(127)

The maximum 10-minute voltage unbalance factor based on the measured demand is shown in Figure 8-13. For demand averaging periods of up to 10 minutes, this

demonstrates that the unbalance factor is not significantly affected by the demand data averaging period.



Figure 8-13 – Maximum voltage unbalance factor, comparison of averaging periods

The unbalance factor can also be calculated directly on the sequence components without applying any further averaging. The maximum unbalance factor is then dependent on the averaging period, as shown in Figure 8-13. For all but the longest averaging periods, it can be seen that the maximum unbalance factor follows an approximately linear relationship with the logarithm of the averaging period.

Although the UK P29 standard and EN 50160 both define a 2% limit, these two standards require different time averaging periods. The results here highlight that the calculated unbalance factor varies with the averaging period. A simulation using 10 minute demand data will therefore not provide accurate results for comparison with the UK standard based on 1 minute data.

# 8.4 Conclusions

The chapter investigates the impact of the demand data time resolution on calculations of losses, RMS voltage drop and voltage unbalance factor.

When arithmetic mean averaged demand data is used to specify the current in network models, the spiky characteristics of real customer demands are not represented and network losses could be significantly under-estimated. Using measured data for the demand at a single dwelling, it was found that losses were under-estimated by 40% if half-hourly demand data was used, and by between 4% and 11% if one minute data was used. As the level of demand aggregation increases, the time variation is smoother and so there is less error caused by the use of arithmetic mean demand data.

These errors should be taken into account in measurements or studies of the effect of low carbon technologies on losses in the LV network. Where DNOs are considering the change in losses if LCTs are introduced to a network, then accurate 'before' and 'after' loss estimates are required since the losses in both cases may be under-estimated. If the demand has a more spikey characteristic than that of new embedded generation, such as from photovoltaics, then the reduction in losses due to introduction of this embedded generation will be under-estimated.

It has been demonstrated that, as the averaging period reduces to become shorter than the switching state dwell times of the demand, the error in estimating the losses converges to a linear relationship with the averaging period. This has been illustrated by considering an analytical model of a single load with a cyclic demand, in simulation models with multiple loads, and also from measurements on the Loughborough campus of a three-phase LV cable with real demand.

Using this linear relationship, a method has been proposed which can be used to estimate the actual losses without errors due to the demand data averaging. This estimate will still be an under-estimate of the actual losses, but will be a better estimate than is obtained directly by using the measured averaged demand data. This relationship can be used by DNOs to improve estimates of losses from future measurements.

Errors in the short term losses have also been considered, with the greatest errors fortunately occurring when the demand is lower, but errors of a few percent were still noted in losses calculated with half-hourly data, relative to losses over a half-hour calculated from 1 minute demand data.

The loss load factor, used to calculate losses where demand data is not available, also varies significantly with the averaging period and the level of demand aggregation. Loss load factors for hourly demand profiles are therefore different to those for 30 minute profiles. When applying the loss load factor to calculate losses, it is also necessary to ensure that the averaging period used to define the peak demand is the same as that used to calculate the loss load factor. This may not be the case where peak demands are based on maximum demand indicators.

The measured demand data has also been used to study the impacts of varying time resolution on the customer voltages levels. Where EN 50160 is applied, there is no significant impact to the voltage drop or to the voltage unbalance factor if these are calculated using 10 minute resolution data or calculated using with data at a higher resolution and with the averaging applied to the voltage results.

However, in the absence of this RMS averaging (as in the UK regulations), the worst-case voltage drop was 50% higher at the original 250 ms resolution than would be seen if the demand data was averaged over 10 minute periods. This difference is due to the smoothing of the spikes in the demand and also to the reduction in unbalance between the three-phase currents. The voltage unbalance also varied from 0.27 for 250 ms data to 0.19 for 10 minute data, with the longer averaging periods smoothing out the short-term unbalance due to individual customer loads

This is an expected outcome, given the linear dependency of the voltage drop on the arithmetic mean demand and highlights the significant differences between results with the RMS metrics of EN 50160 and results without an averaging period as in the UK regulations. The voltage drop (or rise) is a key metric for determining the impact of new low carbon technologies connections on a distribution network and the hosting capacity for these technologies therefore depends on the voltage averaging methods.

Results of simulation studies using one standard cannot be directly applied to systems where the averaging methods are different. Future research studies should therefore clearly state which averaging methods have been employed. The results also suggest that that a review of the UK voltage standards to adopt EN 50160 would formalise the use of 10 minute RMS averaging. This would prevent concerns over short-term voltage deviations that are accepted in other countries where the same domestic appliances are used.

# 9 MODELLING NEUTRAL AND GROUND CURRENTS

# 9.1 Introduction

The literature review identified that different approaches can be taken to represent the impact of the ground path. For a feeder with separate neutral and earth, the neutral is isolated from the ground, as with the LV cable used for the measurements described in Chapters 5 and 6, and so the ground path need not be considered.

However, for a cable with combined neutral and earth, the neutral is connected to the ground at a number of locations in order to meet the protective multiple earthing requirements (E.ON Central Networks 2006). The unbalance current from the loads can therefore return to the substation via both the neutral and the ground. Carson's equations can be used to include the ground path in the circuit impedance matrix but, in calculating the voltage difference along a network, the individual currents within the neutral and ground must be known. This is commonly resolved by applying the Kron reduction which assumes a perfect connection between the neutral and ground such that they can be treated as a single combined conductor. For radial distribution networks, as assumed here, the forwards/backward sweep method is commonly adopted (Kersting 2012).

However, more detailed methods aim to allow for the fact that the connection between the neutral and ground conductors is not zero resistance, and that there is no connection at some nodes. This requires the forward/backward sweep method to be modified, and also requires values to be defined for the grounding resistances. Methods published in the literature tend to focus on the first of these requirements, but give less consideration to the interpretation of the grounding connections and how to specify their locations and appropriate values.

This chapter considers several different methods that could be used to calculate the neutral voltages and currents. These are summarised in Table 9-1 and their advantages and limitations are reviewed in the following discussion. The chapter then continues by proposing new methods that aim to resolve these limitations.

Method	Description
Previous methods	
Ciric	Neutral and ground circuit power divider approximation (Ciric et al. 2003; Ciric et
	al. 2004; Sunderland & Conlon 2012)
Beharrysingh	Neutral currents calculated as function of neutral currents in adjoining branches
	(Beharrysingh 2014)
Demirok	Neutral currents calculated relative to upstream neutral to ground voltage and
	downstream neutral currents (Demirok et al. 2012)
Methods proposed here	
M1	Neutral to ground current calculated according to the neutral to ground voltage at
	the same node
M2	Neutral currents calculated as matrix operation after each forward/backward
	sweep
МЗ	Neutral to ground voltages calculated as matrix operation after each
	forward/backward sweep
M4	Neutral to ground voltages calculated as matrix operation after each
	forward/backward sweep, including interactions between grounding electrodes

#### Table 9-1 – Summary of simulation methods

Ciric's method incorporated grounding resistances into the network by including voltage corrections at each grounded node and by defining a power divider equation to calculate the proportions of the unbalance current from the loads that flows in the neutral or ground conductors (Ciric et al. 2003). The circuit model does not take account of the mutual impedance effects between the neutral or phase conductors and the ground (Beharrysingh 2014). It also omits the voltage difference across the grounding resistance at the upstream node.

If the equivalent circuit is defined without this power divider approximation, the neutral to ground currents can in some cases be solved by treating the resistance between the neutral and ground conductors in the same manner as for a constant impedance load between the phase and neutral conductors (method M1). The current through the grounding resistance at each node is calculated on the backward sweep using the neutral to ground voltages at the node defined from the forward sweep. While this seems an intuitive approach, it has been found to be subject to difficulties as the power-flow simulation does not always converge.

Similar difficulties with convergence of the power-flow solution for Ciric's method have been noted by Beharrysingh where a revised method was proposed in which the neutral current could be calculated as a function of the neutral currents in the adjoining branches. Beharrysingh's method allowed for the grounding resistance to be zero, but did not accommodate an infinite grounding resistance (such that the neutral and ground are not connected). A further concern arises since the calculation requires knowledge of the upstream neutral currents in advance of them being updated as the backward sweep progresses.

Demirok's method uses a similar approach to Beharrysingh and applies Kirchoff's law to the same neutral and ground circuit model (Demirok et al. 2012). The equations are arranged such that the neutral current in a branch is calculated as a function of the downstream neutral currents and the upstream neutral to ground voltage. This differs from Beharrysingh's method as the current calculation in the backward sweep only requires knowledge of branch currents that have already been calculated. A second benefit of this method is that it accommodates nodes with no connection between neutral and ground. However, where the grounding resistance is high compared to the magnitude of the line impedances, this method has been found to suffer from the same convergence problems as method M1.

To address this, a new approach has been proposed here which extends Beharrysingh's method such that the iterative solution of the load currents in the phase conductors is separated from the linear solution of the neutral and ground currents (method M2). The neutral and ground circuit currents can then be solved by a single matrix operation after the backward sweep has been completed. For each iteration of the power-flow solution, there is therefore a three-stage process consisting of i) forward sweep voltage calculation for each branch, ii) backward sweep load current and phase conductor current calculation for each branch, iii) neutral and ground current calculation as a single operation for all branches.

Method M2 described above does not address the constraint that an infinite grounding resistance can be specified and so a further evolution of this approach has been developed (method M3). This has been achieved by re-formulating the circuit equations used to solve the neutral and ground conductor network in terms of the neutral to ground at each node, rather than the neutral current in branch. This

allows for the neutral and ground to be isolated at some nodes, but with the consequence that the grounding resistance cannot be specified as zero.

The discussion then reviews the basic theory employed in defining the grounding resistance in which the interface between the grounding electrode and the earth is modelled such that the current density reduces as an inverse square law. This conceptual model differs significantly from the approach taken in Carson's equations where the ground current is assumed to flow longitudinally along the cable in parallel with the phase and neutral conductors. A fourth power-flow solution method is then presented (method M4) in which the radial current flow of the ground currents.

## Terminology

The radial network consists of a set of 'branches' representing the cable sections and 'nodes' which are the junctions between them. From any location in the network, the nodes and branches that are 'downstream' are those nearer to the customers. The 'upstream' nodes and branches are those that are nearer to the substation. Some nodes represent customer connections with either loads or generators attached and others represent junctions in the network with no loads. Generators and loads are all referred to here as 'loads', so that a generator is modelled as a load with negative real power.

The power-flow method considered here is described as a four-wire model, in which the phase and neutral currents are represented explicitly. The sum of currents in each branch is assumed to be zero and so the ground current can also be determined.

## 9.2 Forward/backward sweep including ground current calculation

## 9.2.1 Network solution method

This section considers method M1 from Table 9-1, where the conventional forward/backward sweep power-flow simulation is modified in order to calculate the current flow through the grounding resistance at by applying Ohm's law to the neutral to ground voltage at the same node (Kersting 2012).

An iterative method for the network solution is required since the loads and generators are typically specified as having a constant active and reactive power, such that the current does not vary linearly with the voltage. This method operates equally well with constant impedance and constant current loads and so can be adopted as a general method regardless of the load model. The resistance between the neutral and ground conductors is treated here in the same manner as constant impedance loads between the phase and neutral conductors. The network is then solved using the forward/backward sweep process, calculating the voltage differences along the branches on the forward sweep and the currents between conductors at each node on the backward sweep.

The network branches are configured as shown in Figure 9-1 and Figure 9-2. In Figure 9-1, the substation is shown, consisting of the transformer and the connection to node 1, together with a set of  $N_1$  branches that are downstream of node 1. The transformer is defined as a perfect voltage source with no impedance between this and node 1. The model allows for node 1 to have a load connected, although typical networks would consist of several feeder cable branches before the first downstream customer connection. Figure 9-2 shows a branch k between node k - 1 and node k, again with a set of  $N_k$  branches that are downstream of node k. It is assumed that the sum of currents in all conductors equals zero and the branch is represented by a  $4 \times 4$  circuit impedance  $\hat{Z}_k$ , as defined in Section 3.3. In this case, the circuit consists of an outward conductor in the cable (either a phase conductor or the neutral) and a return path via the ground (where the current is defined as positive in the downstream direction). The circuit impedance  $\hat{Z}_k$  in ohms of a particular branch is calculated from the circuit impedance per unit length  $\hat{x}_k$  and the length  $l_k$  of section ksuch that:

$$\widehat{\boldsymbol{Z}}_{\boldsymbol{k}} = l_k \widehat{\boldsymbol{z}}_{\boldsymbol{k}} \tag{128}$$

At each node, the voltage relative to ground is defined by a 4-element vector  $V_{abcn,k}$ . In this four-wire equivalent circuit model, as described in Section 3.3, the impact of the ground conductor impedances are incorporated into the circuit impedances and so  $V_{abcn,k}$  represents the voltage relative to the local ground at each node.

At node 1 the phase voltages  $V_{abc,1}$  are determined by the transformer.



Figure 9-1 – Radial network for forward/backward power-flow solution, substation model consisting of the transformer and the first node



Figure 9-2 - Radial network for forward/backward power-flow solution, branch between two nodes

Following the conventional power-flow method, the network is initialised with all of the currents set to zero (Kersting 2012). The first forward sweep, starting from the substation, therefore propagates the three-phase transformer voltage throughout the network. The neutral to ground voltage is initially zero at all nodes but later forward sweeps can result in a voltage difference through this grounding resistance such that current flows between the neutral and the ground. The process is described in detail below. On the backward sweep, the phase current in each branch is calculated as follows:

$$I_{abc,k} = I_{load,abc,k} + \sum_{j \in N_k} I_{abc,j}$$
(129)

where  $I_{load,abc,k}$  is the phase current vector due to loads at node k, and  $I_{abc,j}$  is the phase current vector from each branch within the set of  $N_k$  branches that are downstream of node k. The backward sweep starts with branches that are at the downstream ends of the network, for which the downstream current is zero.

At each node, the vector sum of the three-phase currents flows into the neutral. This is referred to as the unbalance current at node k, defined as:

$$I_{u,k} = I_{load,a,k} + I_{load,b,k} + I_{load,c,k}$$
(130)

At all nodes except the substation node, the neutral to ground current is calculated according to the grounding impedance  $Z_{\text{gr},k}$ . The value of  $Z_{\text{gr},k}$  is assumed to be purely resistive, but the methods through this chapter allow for a complex impedance if this were to be required.

If the neutral is not grounded at this node then  $I_{gr,k} = 0$ . Otherwise:

$$I_{\mathrm{gr},k} = V_{\mathrm{ng},k} / Z_{\mathrm{gr},k}$$
(131)

This requires a constraint that  $Z_{\text{gr},k} > 0$  and so a perfect ground connection cannot be defined here.

The neutral current is then given by:

$$I_{n,k} = I_{gr,k} - I_{u,k} + \sum_{j \in N_k} I_{n,j}$$
(132)

where  $I_{n,j}$  is the neutral current in one of the  $N_k$  branches that are downstream of node k. The ground current is given by:

$$I_{\mathrm{g},k} = -I_{\mathrm{gr},k} + \sum_{j \in N_k} I_{\mathrm{g},j}$$
(133)

At the substation node, it would be possible to calculate  $I_{gr,1} = V_{ng,1}/Z_{gr,1}$  but, since there is no ground current flowing upstream of node 1, all of the unbalance current must return to the transformer in the neutral, as shown in Figure 9-1. Therefore  $I_{g,1} = 0$  and:

$$I_{a,1} + I_{b,1} + I_{c,1} + I_{n,1} = 0$$
(134)

$$I_{\text{gr},1} = \sum_{j \in N_k} I_{\text{g},j} \tag{135}$$

The grounding resistance at the substation node therefore has no impact on the branch currents as part of the backward sweep as the neutral to ground current is already determined by the downstream currents in the ground.

The following forward sweep then re-calculates the voltages for the next iteration. At the substation, the phase to neutral voltages  $V_{abc,k}$  at node 1 are set by the transformer and the neutral to ground voltage is now calculated with reference to the grounding resistance, giving:

$$V_{\rm ng,1} = Z_{\rm gr,1} I_{\rm gr,1} \tag{136}$$

For the substation node, it is therefore possible to define  $Z_{gr,1} = 0$ , but not for the grounding resistance to be an open-circuit.

The voltage at successive downstream nodes is then re-calculated. For a downstream node k + 1 the voltage vector  $V_{abcn,k+1}$  is given by the upstream node k with voltage  $V_{abcn,k}$  and the phase and neutral currents in the branch between the two nodes  $I_{abcn,k}$  so that:

$$V_{\text{abcn},k+1} = V_{\text{abcn},k} - \widehat{Z}_k \times I_{\text{abcn},k}$$
(137)

where  $I_{abcn,k}$  is the phase current in the branch to node k in the direction from the substation,

The forward and backward sweeps continue until a convergence criteria is reached, given by:

$$\max_{i=1\cdots N} |V_i - V_{i,\text{previous iteration}}| < V_{\text{convergence threshold}}$$
(138)

where  $V_{\text{convergence}}$  is a pre-defined threshold. This was set to  $10^{-6}$  V for the simulations described in Chapter 10 in order that errors in the software coding would not be masked by residual differences due to non-convergence, although it is recognised that this resolution is not needed for practical applications.

## 9.2.2 Verification of the converged solution

Once the convergence has been reached, the solution can be tested to ensure that it satisfies Ohm's law and Kirchoff's current law, and also that the required power is delivered to the loads. These tests need to be applied in terms of an acceptance threshold since the solution convergence is not perfect, and to allow for the limits of numerical accuracy in the simulation method.

In each branch, the sum of currents should equal zero. The solution as considered valid where the error was below a threshold  $I_{T1} = 10^{-6}$  A:

$$\left|I_{a,i} + I_{b,i} + I_{c,i} + I_{n,i} + I_{g,i}\right| < I_{T1}$$
(139)

At each node, the sum of phase currents from all connected branches should equal zero, and be below a node current error threshold  $I_{T2} = 10^{-6}$  A:

$$\left| I_{\text{load},\text{abc},k} - I_{\text{abc},k} + \sum_{j \in N_k} I_{\text{abc},j} \right| < I_{\text{T2}}$$
(140)

The current supplied by the transformer should equal the sum of all of the load currents, and be within an error threshold  $I_{T3} = 10^{-6}$  A:

$$\left| \boldsymbol{I}_{abc,1} - \sum_{i=1}^{N_i} \boldsymbol{I}_{load,abc,i} \right| < \boldsymbol{I}_{T3}$$
 (141)

For each constant power load, the power delivered should be within an error threshold  $S_{T4} = 10^{-6}$  VA of the specified rated power, so that:

$$\left|V_{\text{load},i}I_{\text{load},i}^* - S_{\text{load},i,\text{rated}}\right| < S_{\text{T4}}$$
(142)

Finally, the power delivered by the transformer should also equal the total power received by the loads, plus the total of power losses within the branch cables and in the grounding resistances, within an error threshold  $P_{\rm T5} = 10^{-6}$  VA:

$$V_{abc,1}I_{abc,1}^* - \sum_{i=1}^{N_i} V_{load,i}I_{load,i}^* - \sum_{i=1}^{N_i} (\widehat{Z}_k I_{abcn,k}) I_{abcn,k}^* - \sum_{i=1}^{N_i} V_{ng,k}I_{gr,k}^* < P_{T5}$$
(143)

## 9.2.3 Convergence with iterative method

The method described above has been coded into Java in order to set up the powerflow simulation, as described in detail in Section 10.2 of the following chapter. For some network configurations, this solution method converges successfully. This has been demonstrated for the test network shown later in Figure 10-1. As an initial test of the method, all of the grounding resistances were set to 50  $\Omega$ , except at the substation node where the grounding resistance was set to zero. The simulation was configured to run for1440 time steps. The number of forward/backward sweep iterations depended on the loads configured for each customer node, but up to 30 iterations were required on some time steps. This is a much less rapid convergence than for simpler cases (such as with the neutrals at each node isolated from the ground) where only 7 iterations were needed.

If the grounding resistances at each node were increased from 50  $\Omega$  to 100  $\Omega$ , then no more than 14 iterations were needed. Conversely, if the grounding resistances were reduced from 50  $\Omega$  to 40  $\Omega$ , then up to 48 iterations were required. If the grounding resistances were reduced further to 30  $\Omega$  then the convergence threshold was not reached in 100 iterations.

The convergence was also sensitive to the grounding resistance defined at the substation node. Initially this was set to zero but if the resistance was increased to 0.1  $\Omega$  then up to 69 iterations were needed. With a further increase to 0.2  $\Omega$ , convergence was not reached within 100 iterations and the maximum voltage difference between iterations was not reducing.

These tests indicate that the simulation convergence with this method is highly dependent on the values selected for the grounding resistances. This is a considerable limitation if only particular combinations of grounding resistances can be selected while ensuring that the simulation will converge to a solution.

In order to gain more insight into the mechanism by which the iterative solution develops, the convergence behaviour has been investigated in detail for a much simpler resistive circuit topology. The network model for this test consisted of only 5 nodes, arranged along a single-phase linear route (i.e. with no branches), as shown in Figure 9-3. The nodes were each configured to have a constant power load with the reactive power set to zero. The cable impedances were also defined to be resistive and all of the mutual conductor impedances were set to zero. The cables are represented in Figure 9-3 by their conductor impedances  $\overline{Z}_k$  and are the same for each branch. The corresponding circuit impedances used in the power-flow simulation were calculated from (9) and (10).

For verification, this network was also modelled using the Falstad circuit simulator (Falstad 2015). This simulator does not support constant power loads and so the loads were configured as resistors, with the resulting power dissipation then being used in order to specify the constant power loads in the power-flow simulation.



Figure 9-3 – Test network to investigate convergence mechanism

The results from the forward/backward sweep process are given in Table 9-2. This shows the voltages at nodes 2 and 5 after each forward/backward sweep, and also the currents in the branch leading to these nodes. The power-flow solution proceeded as follows:

<u>First forward sweep</u>: All currents are zero and so the substation voltage is applied throughout the network. The ground conductor is then at the same potential as the substation neutral and so at zero voltage. The line to neutral voltage is applied to the phase conductors at all nodes.

<u>First backward sweep</u>: The load currents are calculated according to the line to neutral voltages at each node. Since the neutral and ground node voltages are zero, there is no current through the grounding resistance. All of the unbalance current therefore flows through the neutral.

<u>Second forward sweep</u>: The line voltage reduces along the cable. Similarly, the neutral voltage rises along the cable due to the current flowing through the neutral self-impedance.

<u>Second backward sweep</u>: The neutral to ground voltages are now non-zero so there is a positive potential difference through the grounding resistances at each node. The backward sweep therefore calculates a ground current which accumulates as the sweep progresses towards the substation. This current flows in the same direction as the neutral current, towards the substation, such that the total unbalance current is shared between the neutral and the ground.

<u>Third forward sweep</u>: Although the current in the ground conductor is much smaller than in the neutral, the grounding resistance at node 1 (100  $\Omega$ ) is much greater than the line resistance in the first neutral branch (0.25  $\Omega$ ). The neutral to ground voltage at node 2 is therefore negative. The line resistances along the cable are relatively small compared to the first grounding resistance and so all nodes along the cable have a similarly negative neutral to ground voltage. Given that this is a completely resistive network, this negative voltage already indicates that the iterative process is not converging towards the required solution.

<u>Third backward sweep</u>: Since the neutral to ground voltage is negative at each node, the ground current is now calculated to be positive, i.e. flowing towards the end of the feeder. Again, this indicates that the method is not iterating towards convergence. Since the sum of all currents is zero, the neutral current is then increased relative to the previous sweep, as it must provide a return path for the outward flow of current in both the line conductor and the ground.

<u>Fourth forward sweep</u>: As before, the neutral to ground voltage at the first node is set by the 100  $\Omega$  grounding resistance. The ground current direction is now positive, and so the neutral to ground voltage is also positive. It also has a greater magnitude than after the second forward sweep, when it was previously positive.

This process continues, with the neutral to ground voltage alternating between positive and negative after each sweep, and progressively increasing in magnitude such that the solution for the overall network does not converge.
		Sweep	1	2	3	4	5	6	7	8	9	10
Node 1	Voltage	Line to neutral	100	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
		Neutral to ground	0	0	-0.468	1.450	-6.405	25.754	-105.903	433.096	-1773.540	7260.310
	Current	Line	0.5215	0.522	0.522	0.522	0.522	0.522	0.521	0.527	0.503	0.650
		Neutral	-0.5215	-0.522	-0.522	-0.522	-0.522	-0.522	-0.521	-0.527	-0.503	-0.650
		Ground	0	0	0	0	0	0	0	0	0	0
Node 2	Voltage	Line to neutral	100	99.850	99.851	99.846	99.866	99.786	100.115	98.768	-1691.351	81.709
		Neutral to ground	0	0.075	-0.399	1.544	-6.410	26.151	-107.152	438.585	1795.633	7351.134
	Current	Line	0.2993	0.300	0.300	0.300	0.300	0.300	0.299	0.305	0.281	0.428
		Neutral	-0.2993	-0.295	-0.314	-0.236	-0.557	0.759	-4.630	17.430	-72.884	296.810
		Ground	0	-0.005	0.015	-0.064	0.258	-1.059	4.331	-17.735	72.603	-297.238
Node 3	Voltage	Line to neutral	100	99.764	99.766	99.757	99.792	99.651	100.227	97.867	107.530	67.968
		Neutral to ground	0	0.118	-0.361	1.600	-6.427	26.436	-108.106	442.702	-1812.275	7419.482
	Current	Line	0.1728	0.173	0.173	0.173	0.173	0.173	0.172	0.177	0.159	0.273
		Neutral	-0.1728	-0.169	-0.184	-0.125	-0.367	0.624	-3.432	13.173	-54.806	223.454
		Ground	0	-0.004	0.011	-0.049	0.193	-0.798	3.259	-13.350	54.647	-223.727
Node 4	Voltage	Line to neutral	100	99.730	99.732	99.721	99.765	99.584	100.326	97.288	109.725	58.805
		Neutral to ground	0	0.135	-0.347	1.626	-6.451	26.615	-108.755	445.442	-1823.411	7465.153
	Current	Line	0.0688	0.069	0.069	0.069	0.069	0.069	0.069	0.071	0.063	0.120
		Neutral	-0.0688	-0.066	-0.076	-0.036	-0.198	0.464	-2.247	8.852	-36.587	149.412
		Ground	0	-0.003	0.007	-0.033	0.129	-0.533	2.178	-8.923	36.524	-149.532
Node 5	Voltage	Line to neutral	100	99.720	99.722	99.710	99.759	99.558	100.383	97.005	110.832	54.224
		Neutral to ground	0	0.140	-0.344	1.635	-6.466	26.701	-109.084	446.811	-1828.991	7488.020
	Current	Line	0.0198	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.018	0.037
		Neutral	-0.0198	-0.018	-0.023	-0.004	-0.085	0.247	-1.111	4.448	-18.308	74.844
		Ground	0	-0.001	0.003	-0.016	0.065	-0.267	1.091	-4.468	18.290	-74.880

# Table 9-2 – Test network with 100 $\Omega$ grounding resistances, voltages and currents after each forward/backward sweeps

One means of dealing with this convergence problem would be to set the grounding resistance at the first node to zero. This avoids the instability described above and the solution converges after 7 iterations. However, modelling the first grounding resistance as zero is inconsistent with the practical reality that the electrode has a finite size (as discussed later in Section 9.5) and so the grounding resistance is non-zero.

Furthermore, if the grounding resistances at the other nodes are reduced from 100  $\Omega$  to 10  $\Omega$ , then the solution no longer converges, even with the grounding resistance at node 1 set to zero.

In each of these examples, the line to neutral part of the network reaches a near steady state after several iterations but the convergence of the neutral to ground part of the network is highly sensitive to the resistance values used. This raises questions as to why the forward/backward sweep process appears reliable for the line to neutral part of the circuit, but not for the neutral to ground part. To investigate this further, the network has been simplified further so that only the neutral to ground part of the circuit is retained. In this reduced network, the resistances for the line conductor were replaced with those previously in the ground path and the loads replaced by the grounding resistances. These new loads are defined to have a constant impedance characteristic. This gives the circuit shown in Figure 9-4, where the voltage is applied in place of the grounding resistance at the first node. The forward/backward sweep process implemented here did not converge for this network. This shows that the forward/backward sweep is not appropriate for ladder networks with the values selected here, and that an alternative method is required.



Figure 9-4 – Reduced test network to investigate convergence mechanism

The following sections of this chapter describe modified versions of the forward/backward sweep process that addresses this problem. In these new approaches, the line to neutral part of the circuit is solved using a conventional forward/backward sweep method, but the neutral to ground part of the circuit is considered separately.

#### 9.3 Matrix method for neutral currents

This section describes a proposed new method (method M2 in Table 9-1) in which the currents flowing through the loads and generators are calculated as part of the backward sweep but the neutral and ground currents are calculated separately. The key principle here is that the non-linear (constant power) loads are connected only between the phase and neutral conductors (excluding fault conditions) and so, once the unbalance current due to the loads at each node has been determined, the remaining network comprising the neutral and ground conductors can be treated as linear. For any particular set of currents through the loads and generators, the currents through the neutral and ground paths can then be calculated without needing an iterative method.

The analysis shown below in Section 9.3 extends the analysis presented in Beharrysingh's method by allowing for a network with multiple branches at both the upstream and downstream nodes of each branch. In Beharrysingh's method, the currents in each of the conductors are calculated for each branch in succession as part of the backward sweep. Since the resulting equations require knowledge of currents from the upstream branches, this requires current information from the previous sweep to be utilised.

The methods presented here differ from these previous approaches as all of the neutral currents are calculated in a single matrix operation once all of the phase currents have been determined in the forward/backward sweep process, thereby avoiding the use of phase current information from the previous iterative sweep.

## 9.3.1 Network solution method

As in Beharrysingh's method, the neutral and ground conductor network is solved here by finding an expression for the neutral current in a branch as a function of the neutral currents in the adjoining branches. The circuit model is shown below in Figure 9-5. This shows a single branch, indicated by subscript k, with nodes k1 and k2 at either end. The flow of current is designated as positive in the direction from node k1 to node k2. Nodes k1 and k2 have  $N_{k1}$  and  $N_{k2}$  branches, each with current flowing away from branch k.

This calculation of neutral and ground currents does not require a radial structure for the network, although this is needed for the forward/backward sweep used to calculate the line to neutral voltages and phase currents. The direction of current flow from the branches attached to nodes k1 and k2 is determined by parameters  $\alpha_{k1,i}$  and  $\alpha_{k2,j}$ . For a configuration that is radial in terms of the phase conductors,  $\alpha_{k1,i} = 1$  and  $\alpha_{k2,j} = 1$  for all branches except for the one upstream branch connected to node k1 where  $\alpha_{k1,i} = -1$ .

The line impedance for the cable is represented by circuit impedances  $\hat{Z}_k$ , which includes terms  $\hat{Z}_{nn,k}$  representing the self-impedance of the neutral (a single complex value here, if only one neutral is considered), and a matrix  $\hat{Z}_{n:abc,k}$  representing the mutual impedance of the phase conductors to the neutral.

At nodes k1 and k2, the neutral and ground conductors have an input current due to the load unbalance of  $I_{u,k1}$  and  $I_{u,k2}$ .



Figure 9-5 – Network branch representation for branch current method

Applying Kirchoff's current and voltage laws:

$$V_{\mathrm{ng},k1} = V_{\mathrm{ng},k2} + \widehat{\boldsymbol{Z}}_{\mathbf{n}:\mathbf{abc},k} \boldsymbol{I}_{\mathbf{abc},k} + \widehat{\boldsymbol{Z}}_{\mathrm{nn},k} \boldsymbol{I}_{\mathrm{n},k}$$
(144)

$$V_{\rm ng,k1} = Z_{\rm gr,k1} I_{\rm gr,k1}$$
 (145)

$$V_{\mathrm{ng},k2} = Z_{\mathrm{gr},k2} I_{\mathrm{gr},k2} \tag{146}$$

$$I_{\text{gr},k1} = I_{\text{u},k1} - I_{\text{n},k} - \sum_{i=1}^{N_{k1}} \alpha_{k1,i} I_{\text{n},k1,i}$$
(147)

$$I_{\text{gr},k2} = I_{\text{u},k2} + I_{\text{n},k} - \sum_{j=1}^{N_{k2}} \alpha_{k2,j} I_{\text{n},k2,j}$$
(148)

Re-arranging and substituting from the equations above:

$$Z_{\text{gr},k1}\left(I_{\text{u},k1} - I_{\text{n},k} - \sum_{i=1}^{N_{k1}} \alpha_{k1,i} I_{\text{n},k2,i}\right)$$

$$= Z_{\text{gr},k2}\left(I_{\text{u},k2} + I_{\text{n},k} - \sum_{j=1}^{N_{k2}} \alpha_{k2,j} I_{\text{n},k2,j}\right) + \widehat{\mathbf{Z}}_{\mathbf{n}:\mathbf{abc},k} \mathbf{I}_{\mathbf{abc},k} + \widehat{Z}_{\text{nn},k} I_{\text{n},k}$$
(149)

This can be re-arranged to give:

$$I_{n,k} = \frac{1}{Z_{gr,k2} + Z_{gr,k1} + \hat{Z}_{nn,k}} \left( -Z_{gr,k1} \sum_{i=1}^{N_{k1}} \alpha_{k1,i} I_{n,k1,i} + Z_{gr,k2} \sum_{j=1}^{N_{k2}} \alpha_{k2,j} I_{n,k2,j} + Z_{gr,k1} I_{u,k1} - Z_{gr,k2} I_{u,k2} - \hat{Z}_{n:abc,k} I_{abc,k} \right)$$
(150)

This can be written as:

$$I_{n,k} = A_k \sum_{i=1}^{N_{k1}} \alpha_{k1,i} I_{n,k1,i} + B_k \sum_{j=1}^{N_{k2}} \alpha_{k2,j} I_{n,k2,j} + C_k$$
(151)

where:

$$A_{k} = \frac{-Z_{\text{gr},k1}}{Z_{\text{gr},k2} + Z_{\text{gr},k1} + \hat{Z}_{\text{nn},k}}$$
(152)

$$B_{k} = \frac{Z_{\text{gr},k2}}{Z_{\text{gr},k2} + Z_{\text{gr},k1} + \hat{Z}_{\text{nn},k}}$$
(153)

$$C_{k} = \frac{1}{Z_{\text{gr},k2} + Z_{\text{gr},k1} + \hat{Z}_{\text{nn},k}} \left( Z_{\text{gr},k1} I_{u,k1} - Z_{\text{gr},k2} I_{u,k2} - \widehat{Z}_{n:\text{abc},k} I_{\text{abc},k} \right)$$
(154)

This neutral current in branch *k* is now defined in terms of the neutral currents in the other connecting branches, plus a constant which depends on the forward sweep calculation of currents in each of the loads (defining  $I_{u1,k}$ ,  $I_{u2,k}$ , and  $I_{abc,k}$ ).

The constant  $C_k$  can be further defined as

$$C_k = C_{\mathbf{u},k1}I_{\mathbf{u},k1} + C_{\mathbf{u},k2}I_{\mathbf{u},k2} + C_{\mathbf{abc},k}I_{\mathbf{abc},k}$$
(155)

where:

$$C_{u,k1} = \frac{Z_{gr,k1}}{Z_{gr,k2} + Z_{gr,k1} + \hat{Z}_{nn,k}} = -A_k$$
(156)

$$C_{\rm u,k2} = -\frac{Z_{\rm gr,k2}}{Z_{\rm gr,k2} + Z_{\rm gr,k1} + \hat{Z}_{\rm nn,k}} = -B_k$$
(157)

$$\boldsymbol{C}_{\mathbf{abc},\boldsymbol{k}} = -\frac{\widehat{\boldsymbol{Z}}_{\mathbf{n}:\mathbf{abc},\boldsymbol{k}}}{Z_{\mathrm{gr},k2} + Z_{\mathrm{gr},k1} + \widehat{Z}_{\mathrm{nn},k}}$$
(158)

It is then possible to assemble the terms above for each branch into a matrix structure such that:

$$I_n = M \times I_n + C \tag{159}$$

in which the parameters for each branch  $A_k$  and  $B_k$  are populated into matrix **M** and the parameters  $C_k$  formed into vector **C**.

For a network with *N* nodes, there are N - 1 branches. However, a further condition is required for the substation node, where the application of Kirchoff's law requires the neutral current from the transformer. This is provided by the backward sweep calculation of phase currents and given by  $I_{n,1} = -(I_{a,1} + I_{b,1} + I_{c,1})$ . In the matrix equation, an additional branch is defined for which  $C_1 = I_{n,1}$  and with  $A_1 = B_1 = 0$ .

The matrix equation can then be solved to provide the neutral currents as:

$$\boldsymbol{I}_n = (\boldsymbol{U} - \boldsymbol{M})^{-1} \times \boldsymbol{C} \tag{160}$$

where:  $\boldsymbol{U}$  is the  $N \times N$  identity matrix.

Conveniently, matrix M depends only on the impedance data and so the matrix inversion can be performed once at the initialisation of the power-flow simulation, rather than repeatedly for each time step or iteration.

An example of this method is provided for the radial network shown in Figure 9-6. The diagram indicates that each has a grounding electrode (although this does not imply a zero resistance connection). The matrix equation (159) for this example then becomes:

$$\begin{pmatrix} I_{n,1} \\ I_{n,2} \\ I_{n,3} \\ I_{n,4} \\ I_{n,5} \\ I_{n,6} \\ I_{n,7} \\ I_{n,8} \end{pmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -A_2 & 0 & B_2 & 0 & 0 & A_2 & B_2 & 0 \\ 0 & -A_3 & 0 & B_3 & 0 & 0 & A_3 & 0 \\ 0 & 0 & -A_4 & 0 & B_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & -A_5 & 0 & 0 & 0 & 0 \\ -A_6 & A_6 & 0 & 0 & 0 & 0 & 0 & B_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & -A_8 & 0 \end{bmatrix} \times \begin{pmatrix} I_{n,1} \\ I_{n,2} \\ I_{n,3} \\ I_{n,4} \\ I_{n,5} \\ I_{n,6} \\ I_{n,7} \\ I_{n,8} \end{pmatrix} + \begin{pmatrix} I_{n,1} \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{pmatrix}$$

$$(161)$$

Figure 9-6 – Example radial network

#### 9.3.2 Nodes with zero resistance neutral to ground connections

If it is assumed that node 1 is perfectly grounded then  $z_{gr,k1} = 0$  and:

$$I_{n,k,SC1} = \frac{1}{Z_{gr,k2} + \hat{Z}_{nn,k}} \left( Z_{gr,k2} \sum_{j=1}^{N_{k2}} \alpha_{k2,j} I_{n,k2,j} - Z_{gr,k2} I_{u,k2} - \widehat{Z}_{n:abc,k} I_{abc,k} \right)$$
(162)

Similarly with node 2 perfectly grounded  $Z_{gr,k2} = 0$  and:

$$I_{n,k,SC2} = \frac{1}{Z_{gr,k1} + \hat{Z}_{nn,k}} \left( -Z_{gr,k1} \sum_{i=1}^{N_{k1}} \alpha_{k1,i} I_{n,k1,i} + Z_{gr,k1} I_{u,k1} - \hat{Z}_{n:abc,k} I_{abc,k} \right)$$
(163)

If nodes at both ends of the branch are grounded, the equation reduces to:

$$I_{n,k,SC} = \frac{-\widehat{Z}_{n:abc,k}}{\widehat{Z}_{nn,k}} (I_{abc,k})$$
(164)

This is equivalent to the Kron reduction calculation (Kersting 2012). In this case, the neutral current in each branch is seen to be independent of currents in any other branch. In the conventional forward/backward sweep method the neutral and ground currents can be calculated together with the phase currents for each branch. However, in the more generic case where the grounding resistances are non-zero,

the unbalance current from every node affects the neutral currents in each of the branches.

Method M2 outlined above can therefore be applied in a network where some or all of the nodes have non-zero grounding impedances defined, but it is also valid for a perfect multi-grounded network.

## 9.3.3 Nodes with neutral and ground isolated

If node 1 has no connection between neutral and ground then  $Z_{\text{gr},k1} = \infty$  and the neutral current calculation reduces to:

$$I_{n,k,OC1} = -\sum_{i=1}^{N_{k1}} \alpha_{k1,i} I_{n,k1,i} + I_{u,k1}$$
(165)

Similarly if  $Z_{\text{gr},k2} = \infty$  and:

$$I_{n,k,OC2} = \sum_{j=1}^{N_{k2}} \alpha_{k2,j} I_{n,k2,j} - I_{u,k2}$$
(166)

In both cases the neutral current is determined by the sum of currents at the ungrounded node.

Considering the example of two branches connected by an un-grounded node, with no side branches, the calculation method defined above therefore provides only a single equation to resolve the two unknown neutral currents either side of the ungrounded node, neither of which take account of the mutual impedances between this neutral current and the phase currents.

Equally, if the network has a grounded node from which all of the branches are otherwise un-grounded, then the calculation above will make no reference to the grounding impedance at the grounded node.

This causes a problem if this method is to be applied to real networks since there is typically no connection between the neutral and ground at most nodes. Ground electrodes are only provided where required for protective multiple earthing and where the earth bonding at customer connections gives a route between the neutral and the ground. Method M3 is presented in Section 9.4 to address this.

## 9.3.4 Comparison to Demirok's method

Before leaving the discussion of Method M2, the similarities to Demirok's method are considered. Demirok's method has also been implemented in the Java power-flow simulation model described below in Section 10.2, but was found to have similar convergence problems to method M1 outlined in Section 9.2.3. Demirok's method applies Kirchoff's laws as described here in (144) to (148), but without making the substitution of voltage  $V_{ng1,k}$  from (145). This gives:

$$I_{n,k} = \frac{V_{ng,k1} - \widehat{Z}_{n:abc,k} I_{abc,k} + Z_{gr,k2} \left( \sum_{j=1}^{N_{k2}} \alpha_{k2,j} I_{n,k2,j} - I_{u,k2} \right)}{\widehat{Z}_{nn,k} + Z_{gr,k2}}$$
(167)

This is subtly different from (150) and uses the upstream voltage  $V_{\text{ng1},k}$  from the previous forward sweep. It can be applied within the backward sweep process since the neutral current in each branch can be determined without knowledge of the upstream currents. However, since in practical cases  $Z_{\text{gr},k2}$  is very much larger than the cable impedance terms  $\hat{z}_{\text{nn},k}$  and  $\hat{z}_{n:abc,k}$ , the neutral current from (167) is approximately given by:

$$I_{n,k} \simeq \frac{V_{ng,k1}}{Z_{gr,k2}} + \sum_{j=1}^{N_{k2}} \alpha_{k2,j} I_{n,k2,j} - I_{u,k2}$$
(168)

Substituting from (148), this then gives:

$$I_{\mathrm{gr},k2} \cong \frac{V_{\mathrm{ng},k1}}{Z_{\mathrm{gr},k2}} \tag{169}$$

This is subject to the same convergence issues as when the neutral to ground current was calculated as part of the backward sweep using (131).

## 9.4 Matrix method for neutral to ground voltages

#### 9.4.1 Network solution equations

The network analysis is now revised so as to address the difficulty noted above with un-grounded nodes in method M2. In this new method M3, the currents within the neutral and ground paths are calculated by a similar matrix approach as above, but with the equations expressed in terms of the neutral to ground voltage at each node, rather than the neutral current in each branch. The network diagram is re-drawn as shown in Figure 9-7. The diagram shows a single node, indicated by subscript k, at which  $N_k$  branches are connected. The diagram shows one of these, branch l. The flow of current is designated as positive in the direction away from the node.

As above, the current direction of the *i*<sup>th</sup> branch from node *k* is referenced according to the orientation of each branch in the network diagram by a parameter  $\alpha_{k,i}$ . For a configuration that is radial in terms of the phase conductors,  $\alpha_{k,i} = -1$  for the upstream branch and  $\alpha_{k,i} = 1$  for all branches in the downstream direction.

The line impedance for the *i*<sup>th</sup> branch from node *k* is represented by circuit impedances  $\hat{z}_{k,i}$ , with the ground defined as the circuit return path. At node *k*, there is an unbalance current from the loads of  $I_{u,k}$ .



Figure 9-7 – Network node representation for node voltage method

Kirchoff's current and voltage laws are applied as above, giving:

$$V_{\mathrm{ng},k} = V_{\mathrm{ng},i} + \alpha_{k,i} \widehat{\boldsymbol{Z}}_{\mathbf{n}:\mathbf{abc},k,i} \boldsymbol{I}_{\mathbf{abc},k,i} + \alpha_{k,i} \widehat{\boldsymbol{Z}}_{\mathrm{nn},k,i} \boldsymbol{I}_{\mathrm{n},k,i}$$
(170)

$$V_{\mathrm{ng},k} = Z_{\mathrm{gr},k} I_{\mathrm{gr},k} \tag{171}$$

$$I_{\text{gr},k} = I_{\text{u},k} - \sum_{i=1}^{N_k} \alpha_{k,i} I_{\text{n},k,i}$$
(172)

Re-arranging and substituting from the equations above:

$$\alpha_{k,i}I_{\mathbf{n},k,i} = \frac{1}{\widehat{Z}_{\mathbf{n}\mathbf{n},k,i}} \left( V_{\mathbf{n}\mathbf{g},k} - V_{\mathbf{n}\mathbf{g},i} - \alpha_{k,i}\widehat{Z}_{\mathbf{n}:\mathbf{abc},k,i}I_{\mathbf{abc},k,i} \right)$$
(173)

$$V_{\mathrm{ng},k} = Z_{\mathrm{gr},k} \left( I_{\mathrm{u},k} - \sum_{i=1}^{N_k} \left( \frac{1}{\hat{Z}_{\mathrm{nn},k,i}} \left( V_{\mathrm{ng},k} - V_{\mathrm{ng},i} - \alpha_{k,i} \widehat{Z}_{\mathbf{n}:\mathrm{abc},k,i} I_{\mathrm{abc},k,i} \right) \right) \right)$$
(174)

This can be re-arranged to give:

$$V_{\mathrm{ng},k} = \frac{Z_{\mathrm{gr},k}}{1 + Z_{\mathrm{gr},k} \sum_{i=1}^{N_k} \frac{1}{\hat{Z}_{\mathrm{nn},k,i}}} \left( \sum_{i=1}^{N_k} \frac{1}{\hat{Z}_{\mathrm{nn},k,i}} V_{\mathrm{ng},i} + I_{\mathrm{u},k} + \sum_{i=1}^{N_k} \frac{\alpha_{k,i} \widehat{Z}_{\mathrm{n:abc},k,i}}{\hat{Z}_{\mathrm{nn},k,i}} I_{\mathrm{abc},k,i} \right)$$
(175)

This can be written as:

$$V_{\rm ng,k} = \sum_{i=1}^{N_k} A_{k,i} V_{\rm ng,i} + C_k$$
(176)

where:

$$A_{k,i} = \frac{Z_{\text{gr},k}}{\left(1 + Z_{\text{gr},k} \sum_{i=1}^{N_k} \frac{1}{\hat{Z}_{\text{nn},k,i}}\right) \hat{Z}_{\text{nn},k,i}}$$
(177)

$$C_{k} = \frac{Z_{\text{gr},k}}{1 + Z_{\text{gr},k} \sum_{i=1}^{N_{k}} \frac{1}{\hat{Z}_{\text{nn},k,i}}} \left( I_{\text{u},k} + \sum_{i=1}^{N_{k}} \frac{\alpha_{k,i} \widehat{Z}_{\text{n:abc},k,i}}{\hat{Z}_{\text{nn},k,i}} I_{\text{abc},k,i} \right)$$
(178)

At the substation node, the sum of currents is defined with the neutral current from the transformer  $I_{n,t} = -(I_{a,t} + I_{b,t} + I_{c,t})$  included as a separate term since this is provided from the phase current calculation rather than being dependent on the voltage at another node. This gives:

$$I_{\rm gr,1} = I_{\rm n,t} + I_{\rm u,1} - \sum_{i=1}^{N_1} \alpha_{1,i} I_{{\rm n},1,i}$$
(179)

Following the same process as above then gives:

$$V_{\rm ng,1} = \frac{Z_{\rm gr,1}}{1 + Z_{\rm gr,1} \sum_{i=1}^{N_1} \frac{1}{\hat{Z}_{\rm nn,1,i}}} \left( \sum_{i=1}^{N_1} \frac{1}{\hat{Z}_{\rm nn,1,i}} V_{\rm ng,i} + I_{\rm n,t} + I_{\rm u,1} + \sum_{i=1}^{N_1} \frac{\alpha_{1,i} \hat{Z}_{\rm n:abc,1,i}}{\hat{Z}_{\rm nn,1,i}} I_{\rm abc,1,i} \right)$$
(180)

In this case,  $R_1$  and  $A_1$  are defined as above but:

$$C_{1} = R_{1} \left( I_{n,t} + I_{u,1} + \sum_{i=1}^{N_{1}} \frac{\alpha_{1,i} \widehat{Z}_{n:abc,1,i}}{\widehat{Z}_{nn,1,i}} I_{abc,1,i} \right)$$
(181)

The neutral to ground voltages can then be determined by assembling an  $N \times N$  matrix from parameters  $A_{k,i}$  and  $C_k$  such that:

$$V_{\rm ng} = M \times V_{\rm ng} + C \tag{182}$$

This can be solved to give:

$$V_{\rm ng} = (U - M)^{-1} \times C \tag{183}$$

As with the first method based on branch currents, the matrix inversion needs only to be performed once at the start of the power-flow simulation.

The neutral currents can then be determined using (173).

#### 9.4.2 Nodes with neutral and ground isolated

If node *k* has no connection between neutral and ground then  $Z_{\text{gr},k} = \infty$  and the neutral to ground voltage calculation reduces to:

$$V_{\rm ng,k,OC} = \frac{1}{\sum_{i=1}^{N_k} \frac{1}{\hat{Z}_{{\rm nn},k,i}}} \left( \sum_{i=1}^{N_k} \frac{1}{\hat{Z}_{{\rm nn},k,i}} V_{{\rm ng},i} + I_{{\rm u},k} + \sum_{i=1}^{N_k} \frac{\alpha_{k,i} \hat{Z}_{{\rm n:abc},k,i}}{\hat{Z}_{{\rm nn},k,i}} I_{{\rm abc},k,i} \right)$$
(184)

#### 9.4.3 Nodes with zero resistance neutral to ground connections

If node k has a perfect connection between neutral and ground then  $Z_{\text{gr},k} = 0$  and the neutral to ground voltage calculation reduces to  $V_{\text{ng},k} = 0$ .

This is correct where there is perfect grounding but does not allow the neutral currents to the node to be determined since the terms  $A_{k,i}$  and  $C_k$  in matrix M are zero.

There is therefore a constraint in method M3 that  $Z_{\text{gr},k} \neq 0$ . However, as shown in Section 9.5, it is not possible for the condition where  $Z_{\text{gr},k} = 0$  to arise in practice so the constraint that  $Z_{\text{gr},k} \neq 0$  does not limit the use of this method.

## 9.5 Ground electrode resistance theory

## 9.5.1 Grounding resistance for individual electrodes

The methods outlined above require a means of defining the impedance between the neutral conductor and the ground. This includes a contribution due to the resistance of the electrode itself and the wire used to bond this to the neutral conductor, but the local resistance of the ground is a much larger factor.

The impedance of connections between an electrode and the ground has been studied extensively for the purpose of defining the potential rise caused by fault currents entering the ground, often using an analytical method defined by Tagg (Tagg 1964). This approach provides the theoretical background to the 'fall of potential' method used for ground resistivity measurements using test instruments (Fluke 2006a).

Following Tagg's method, an idealised electrode is defined with a hemispherical shape, as shown in Figure 9-8. The ground reference is also assumed to be hemispherical (US Military 1987).



Figure 9-8 – Idealised ground electrode model, cross-section through ground

A current *I* flows through the electrode into the ground and spreads out according to the inverse square law, such that the current density at a radius x from the electrode centre is:

$$i = \frac{I}{2\pi x^2} \tag{185}$$

The field strength is given by  $e = \rho i$ , where  $\rho$  is the ground resistivity, and so the total potential  $V_{\text{GE}}$  between the ground reference at radius  $x_{\text{G}}$  and the ground electrode with radius  $x_{\text{E}}$  is:

$$V_{\rm EG} = -\int_{x=x_{\rm G}}^{x_{\rm E}} e \cdot dx = -\frac{\rho I}{2\pi} \int_{x=x_{\rm G}}^{x_{\rm E}} \frac{1}{x^2} \cdot dx = \frac{\rho I}{2\pi} \left(\frac{1}{x_{\rm E}} - \frac{1}{x_{\rm G}}\right)$$
(186)

If radius  $x_G$  is large compared to  $x_E$ , then the potential can be approximated as

$$V_{\rm EG} = \frac{\rho I}{2\pi x_{\rm E}} \tag{187}$$

The resistance of the electrode, considering the potential between the electrode and the reference distance, is then  $R_{EG} = V_{EG}/I$  and so:

$$R_{\rm EG} = \frac{\rho}{2\pi x_{\rm E}} \tag{188}$$

In practice, the electrode is unlikely to be hemispherical, and so radius  $x_E$  is then considered to be the radius of an equivalent hemispherical electrode having the same ground resistance as the actual non-hemispherical electrode.

As an example of practical dimensions, the grounding resistance at an LV substation is required to be less than 20  $\Omega$  (East Midlands Electricity 2001). For earth resistivity of 100  $\Omega$ m, this gives an equivalent hemispherical electrode of 0.8 m, a plausible dimension for the equivalent hemispherical radius of the substation ground electrode. Similarly, PME earths installed on the feeder have a nominal resistance of 100  $\Omega$ and have an equivalent hemispherical radius of 0.16 m.

A notable conclusion from (188) is that  $R_{EG}$  can only be zero if  $x_E = \infty$ , such that a grounding resistance of zero cannot be realised in practice. The constraint noted in Section 9.4 whereby the power-flow current calculations could not accommodate  $z_{gr,k} = 0$  is therefore not an obstacle to the use of this method, since this condition cannot occur.

The 3-probe 'fall of potential' method can be used to determine the electrode grounding resistance where the dimensions of the electrode are unknown (Tagg 1964). A current is passed into the ground at the electrode (E) under test, returning to a second 'current electrode' (C) some distance away. A third 'voltage probe electrode' (P) is inserted between these two, as shown in Figure 9-9.



Figure 9-9 – Electrode layout for 3-probe fall of potential method, plan view

Current entering the ground at the test electrode E raises the potential at E by  $\rho I/(2\pi x_{\rm E})$  relative to the ground reference hemisphere, as shown above. This current also raises the potential at P relative to the ground reference hemisphere, by a voltage given by

$$V_{\rm PG} = -\int_{x=x_{\rm G}}^{d_{\rm P}} e \cdot dx = \frac{\rho I}{2\pi} \left( \frac{1}{d_{\rm P}} - \frac{1}{x_{\rm G}} \right)$$
(189)

For a large  $x_{\rm G}$ , the potential at P relative to the ground reference hemisphere is  $\rho I/(2\pi d_{\rm P})$ .

Similarly, the potential due to current leaving the ground at C, for a measurement at the test electrode E is  $-\rho I/(2\pi d_c)$  and for a measurement at P is:

$$-\rho I / \left(2\pi \sqrt{d_{\rm C}^2 + d_{\rm P}^2 - 2d_{\rm C}d_{\rm P}\cos\theta}\right).$$

The resistance between the test electrode E and the probe P is then given by:

$$R_{\rm EP} = \frac{V_{\rm EP}}{I} = \frac{V_{\rm EG} - V_{\rm PG}}{I} = \frac{\rho}{2\pi} \left( \frac{1}{x_{\rm E}} - \frac{1}{d_{\rm C}} - \frac{1}{d_{\rm P}} + \frac{1}{\sqrt{d_{\rm C}^2 + d_{\rm P}^2 - 2d_{\rm C}d_{\rm P}\cos\theta}} \right)$$
(190)

If probe P lies on the line between E and C ( $\theta = 0$ ) then the measured resistance  $R_{\rm EP}$  is equal to the test electrode ground resistance  $R_{\rm EG}$  in the special case that  $1/d_{\rm C} + 1/d_{\rm P} - 1/(d_{\rm C} - d_{\rm P}) = 0$ . This occurs where  $d_{\rm P} = d_{\rm C} (\sqrt{5} - 1)/2$ , a ratio known as the 'golden ratio',  $d_{\rm P} = 0.618 d_{\rm C}$ .

This theory is now developed in the following section for application to LV networks.

## 9.5.2 Grounding resistance between two electrodes

An LV network simulation could be configured using electrode grounding impedances based on (188), assuming that the ground resistivity and the dimensions of an equivalent hemispherical electrode are known. The grounding impedance would then be defined for each node as  $Z_{gr,k} = R_{E,k}$ .

The conductive path through the ground is therefore modelled by a lumped impedance to represent the resistance as the current spreads out to a large hemispherical radius, followed by the ground impedance per unit length of the line, and then a further lumped resistance to represent the connection to the second ground electrode. This appears preferable to a model based only on the line impedances (i.e. assuming a perfect multi-grounded neutral) as it allows for the interface between the current being concentrated at the electrode and being widely dispersed, as implied by the line impedance model.

However, if the current enters and leaves the ground at two electrodes that are closely spaced, then it would be expected to flow directly from one to the other, rather than flowing out to an infinite reference hemisphere and then back again to the other electrode. The theory shown above for the fall of potential measurement method is now extended here to model this scenario.

A set of *N* nodes with ground electrodes is defined, as shown in Figure 9-10. The equivalent hemispherical radius of node *i* is  $x_{ii}$ , and the distance from node *i* to node *j* is  $x_{ij}$ . A current  $I_i$  enters the ground at each node. One node is defined to be the substation, node *s*.



Figure 9-10 – Electrode layout for LV network model, plan view

The effective resistance between node *i* and node *j* is given by considering the difference in potential between each of the nodes and the infinite reference hemisphere. The potential at *i* due to current entering the ground at node *i* is  $\rho I_i/(2\pi x_{ii})$ , and the potential at *i* due to current entering the ground at node *j* is  $\rho I_j/(2\pi d_{ij})$ , so that:

$$V_{iG} = \frac{\rho}{2\pi} \left( \frac{I_i}{x_{ii}} + \frac{I_j}{d_{ij}} \right) \tag{191}$$

By symmetry:

$$V_{jG} = \frac{\rho}{2\pi} \left( \frac{I_j}{x_{jj}} + \frac{I_i}{d_{ji}} \right)$$
(192)

Assuming that  $I_j = -I_i$  and  $V_{ij} = V_{iG} - V_{jG}$  gives:

$$R_{ij} = \frac{V_{ij}}{I_i} = \frac{\rho}{2\pi} \left( \frac{1}{x_{ii}} - \frac{1}{d_{ij}} - \frac{1}{d_{ji}} + \frac{1}{x_{jj}} \right) = \frac{\rho}{2\pi} \left( \frac{1}{x_{ii}} - \frac{2}{d_{ij}} + \frac{1}{x_{jj}} \right)$$
(193)

If the electrodes were considered independently, the combined resistance would be:

$$R_{ij,\text{independent}} = \frac{\rho}{2\pi} \left( \frac{1}{x_{ii}} + \frac{1}{x_{jj}} \right)$$
(194)

The model proposed here therefore includes a term  $2/d_{ij}$  that allows for the current taking a direct path between the two electrodes, rather than travelling to and from the conceptual reference hemisphere.

The variation of the total resistance between two nodes is shown in Figure 9-11, where one node has an individual grounding resistance of 20  $\Omega$  and the second node has an individual grounding resistance of 100  $\Omega$ . This suggests that there is negligible interaction between the ground electrodes for separation distances over 20 m, and that the total ground resistance can be considered as the sum of the grounding resistances.



Figure 9-11 – Ground resistance between substation and node with PME earth electrode

Measured ground electrodes at substations have been found to have resistances between 2  $\Omega$  and 10  $\Omega$  (Mather 1958). The resistances of earth electrodes at domestic premises via metal pipework was between 2.5  $\Omega$  and 16.5  $\Omega$ , although these values were measured at rural farm locations and so may not be representative of more urban housing. Compared to the figures in a DNO network manual (East Midlands Electricity 2001) these lower ground resistance values imply larger equivalent hemispherical electrodes and greater separations would be required in order that interactions between electrodes could be neglected.

With many closely spaced ground connections on the network, it is possible that the overall resistance of the interface between the cable and ground is significantly reduced. The analysis above is therefore extended in the following section to consider an LV network with multiple ground electrodes, as shown in Figure 9-10.

#### 9.5.3 Grounding resistance between multiple electrodes

At node *i*, the potential between the electrode and the infinite reference hemisphere includes contributions from current entering the ground at that node, and also due to the currents from each of the other nodes, so that:

$$V_{iG} = \frac{\rho}{2\pi} \sum_{j=1}^{N} \frac{I_j}{x_{ij}}$$
(195)

Similarly, the potential between the substation node s and the infinite reference hemisphere is:

$$V_{sG} = \frac{\rho}{2\pi} \sum_{j=1}^{N} \frac{I_j}{x_{sj}}$$
(196)

Taking the ground at the substation as a reference, the potential difference between node i and node s is then:

$$V_{is} = V_{iG} - V_{sG} = \frac{\rho}{2\pi} \left( \sum_{j=1}^{N} \frac{I_j}{x_{ij}} - \sum_{j=1}^{N} \frac{I_j}{x_{sj}} \right)$$
(197)

The total current entering and leaving the ground is zero so:

$$I_s = -\sum_{j=1, j \neq s}^N I_j \tag{198}$$

Re-arranging (197) and (198) gives:

$$V_{is} = \frac{\rho}{2\pi} \left( \left( \sum_{j=1, j \neq s}^{N} \frac{I_j}{x_{ij}} \right) + \frac{I_s}{x_{is}} - \left( \sum_{j=1, j \neq s}^{N} \frac{I_j}{x_{sj}} \right) - \frac{I_s}{x_{ss}} \right)$$
(199)

$$V_{is} = \frac{\rho}{2\pi} \left( \left( \sum_{j=1, j \neq s}^{N} \frac{I_j}{x_{ij}} \right) - \left( \frac{1}{x_{is}} \sum_{j=1, j \neq s}^{N} I_j \right) - \left( \sum_{j=1, j \neq s}^{N} \frac{I_j}{x_{sj}} \right) + \left( \frac{1}{x_{ss}} \sum_{j=1, j \neq s}^{N} I_j \right) \right)$$
(200)  
$$V_{is} = \frac{\rho}{2\pi} \sum_{j=1, j \neq s}^{N} \left( \frac{1}{x_{ij}} - \frac{1}{x_{is}} - \frac{1}{x_{sj}} + \frac{1}{x_{ss}} \right) I_j$$
(201)

This can be written as:

$$V_{is} = \sum_{j=1, j \neq s}^{N} \hat{z}_{gr,ij} I_j$$
(202)

where:

$$\hat{Z}_{\text{gr},ij} = \frac{\rho}{2\pi} \left( \frac{1}{x_{ij}} - \frac{1}{x_{is}} - \frac{1}{x_{sj}} + \frac{1}{x_{ss}} \right)$$
(203)

The impedances  $\hat{Z}_{\text{gr},ij}$  are analogous to the circuit impedances  $\hat{Z}_{ij}$  developed for the cables, with the circuit here consisting of current entering the ground at a network node and returning at the substation node. The neutral to ground voltage at each node now depends on the current entering the ground at that node, multiplied by a self-impedance term, and on the current entering the ground at all the others nodes,

multiplied by their respective mutual impedance terms. As with the ground conductor in the circuit impedances for cables, the grounding resistance at the substation has been incorporated into the grounding circuit impedances for the other nodes.

According to this model, current flowing from one node to another via the ground is no longer assumed to travel via a reference hemisphere at infinite distance. The circuit impedance  $\hat{Z}_{\text{gr},ij}$  includes terms that allow for the potential at one node to be raised due to current from another node, such that the overall voltage drop between them is reduced. This effectively models the current taking a more direct path between the two electrodes.

Based on this approach to the grounding resistance theory alone, (203) could be taken to fully define the impedance between one ground electrode and another, with no further need for any contribution due to the ground path in the line impedance calculation. For a DC system, this would be valid since the contribution of the ground path to the resistive component of the circuit impedance would be zero (this is given by the term  $\mu_0 \omega/8$  in (28)). For a finite ground resistivity, that would imply an infinite cross-sectional area for the ground path, which is again consistent with the concepts in the ground electrode resistance theory where the electrodes are widely separated.

However, for an AC system, the theory described here for a resistance between one grounding electrode and another does not take into account the inductance that would occur when this current path forms part of a circuit. This is accommodated within the line impedance calculations according to Carson's equations. An analysis including the lumped grounding impedances from (203) and also the line impedances based on Carson's equations therefore provides a method that allows for both the resistive and the inductive effects. However, the model remains imperfect, as (203) implies a reduced voltage difference as current flows from one electrode to another without dispersing to a large radius, whereas the line impedance at 50 Hz calculated from Carson's equations implies that the current path still has a wide cross-sectional area.

## 9.6 Matrix method with interactions between ground electrodes

## 9.6.1 Network solution equations

The network analysis method M3 described in Section 9.4.1 can now be revised to take account of the interaction between the grounding resistances. This method M4 takes account of the fact that current entering the ground at one electrode will affect the local ground potential of each of the other electrodes.

The analysis presented here maintains the constraint that the currents in each cable, and in the ground that runs parallel, sum to zero. The current in the ground must therefore follow the route of the cable rather than taking a more direct path.

In this analysis, node 1 is taken to be the substation node, for which  $Z_{gr,11} \neq 0$ , but the circuit grounding impedance  $\hat{Z}_{gr,1i} = \hat{Z}_{gr,11} = 0$ , as described in Section 9.5.3.

In (202), the voltage at each grounding electrode is shown to depend on the currents from each of the other grounding electrodes. However, in the context of an LV network, the set of nodes with grounding electrodes is a subset of the total set of nodes. The analysis here is therefore developed separately for nodes with grounding electrodes, and for those without.

Referring to Figure 9-7, (202) is re-written to define the neutral voltage based on grounding currents at the set of nodes  $N_{\rm gr}$  (excluding the substation node) at which a grounding electrode is present, giving:

$$V_{\mathrm{n},k1,\hat{z}_{\mathrm{gr},ii}\neq\infty} = \sum_{i\in N_{\mathrm{gr}}} \hat{Z}_{\mathrm{gr},ki} I_{\mathrm{gr},i}$$
(204)

where the voltage of the neutral conductor  $V_{n,k,1}$  is now expressed relative to the neutral conductor at node 1, and  $V_{n,k1} = V_{ng,k} - V_{ng,1}$ .

Equations (172) and (173) are re-written to define the neutral current at node i in terms of each of the set of  $N_i$  branches from node i this to other nodes:

$$I_{\text{gr},i} = I_{\text{u},i} - \sum_{j \in N_i} \alpha_{i,j} I_{\text{n},i,j}$$
(205)

$$\alpha_{i,j}I_{\mathbf{n},i,j} = \frac{1}{\hat{Z}_{\mathbf{n}\mathbf{n},i,j}} \left( V_{\mathbf{n},i1} - V_{\mathbf{n},j1} - \alpha_{i,j} \widehat{\boldsymbol{Z}}_{\mathbf{n}:\mathbf{abc},i,j} \boldsymbol{I}_{\mathbf{abc},i,j} \right)$$
(206)

The set  $N_i$  can include the substation node, although  $V_{n,11}$  is defined to be zero.

Substituting these into (204) then gives:

$$V_{\mathbf{n},k\mathbf{1},\hat{z}_{\mathrm{gr},ii}\neq\infty} = \sum_{i\in N_{\mathrm{gr}}} \left( \hat{Z}_{\mathrm{gr},ki} \sum_{j\in N_{i}} \left( \frac{1}{\hat{Z}_{\mathrm{nn},i,j}} V_{\mathrm{n},j\mathbf{1}} \right) - \hat{z}_{\mathrm{gr},ki} \sum_{j\in N_{i}} \left( \frac{1}{\hat{Z}_{\mathrm{nn},i,j}} V_{\mathrm{n},i\mathbf{1}} \right) + \hat{Z}_{\mathrm{gr},ki} I_{\mathrm{u},i} + \hat{Z}_{\mathrm{gr},ki} \sum_{j\in N_{i}} \left( \frac{1}{\hat{Z}_{\mathrm{nn},i,j}} \alpha_{i,j} \hat{\mathbf{Z}}_{\mathrm{n:abc},i,j} I_{\mathrm{abc},i,j} \right) \right)$$

$$(207)$$

For a node k without a grounding electrode, (205) becomes:

$$0 = I_{\mathbf{u},k} - \sum_{j \in N_k} \alpha_{k,j} I_{\mathbf{n},k,j}$$
(208)

Substituting (206) then gives:

$$V_{\mathbf{n},k\mathbf{1},\hat{z}_{\mathrm{gr},ii}=\infty} = \frac{1}{\sum_{j\in N_k} \frac{1}{\hat{Z}_{\mathbf{nn},k,j}}} \left( \sum_{j\in N_k} \frac{1}{\hat{Z}_{\mathbf{nn},k,j}} V_{\mathbf{n},j\mathbf{1}} + I_{\mathbf{u},k} + \sum_{j\in N_k} \frac{1}{\hat{Z}_{\mathbf{nn},k,j}} \alpha_{k,j} \widehat{\mathbf{Z}}_{\mathbf{n}:\mathbf{abc},k,j} I_{\mathbf{abc},k,j} \right)$$

$$(209)$$

With subscript *i* now referring to the full set of nodes, both (207) and (209) can be written as:

$$V_{n,k1} = \sum_{i=2}^{N} A_{k,i} V_{n,i1} + C_k$$
(210)

The neutral voltages are defined here for nodes 2 to *N*, relative to the neutral voltage at node 1. The network can therefore be solved as an  $(N - 1) \times (N - 1)$  matrix, using equation (183). The neutral currents in the branches to node 1 can then be determined from (206) with  $V_{n,11} = 0$ .

Although this method allows for  $\hat{Z}_{\text{gr},ii} = \infty$ , it does not accommodate  $\hat{Z}_{\text{gr},ii} = 0$  since this would imply that  $x_{ii} = \infty$  from (203), effectively requiring an infinite size of grounding electrode.

#### 9.6.2 Limitations of this method

In order to make use of the line impedance  $\hat{z}_k$  based on Carson's equations, it is necessary to maintain the constraint that the sum of currents in each branch equals zero. However, this assumption is now more questionable, since the ground resistances calculated from (203) depend on the physical spacing between the electrodes, rather than on their position in the electrical network topology. This concern is highlighted in the example layout shown in Figure 9-12 where the grounding electrodes are installed only at the substation (node 1) and at the end of the feeder (node 5). The end of the feeder is physically close to the transformer. According to the analysis defined here, any unbalance current at the end of the feeder would travel around the feeder route, rather than flowing directly between the ground electrodes at node 5 and the substation. In practice, some fraction of the ground current would be expected to take a more direct route.



Figure 9-12 – Network layout showing possibility of loop currents in the ground

A second example is shown in Figure 9-13 where the feeder has two parallel sections. Grounding electrodes are installed at the end of each section and at the substation. The cable impedances for each branch calculated from Carson's equations are based on the assumption that each cable is installed in an entirely separate area of ground, such that each cable has an independent ground conductor. However, in this example, the spacing between the feeders may be small relative to the cross-sectional profile of the ground current distribution implied by Carson's equations at 50 Hz, so the two feeder sections are effectively sharing the same ground conductor. If the two cables are precisely parallel, the impedance calculation with Carson's equations could be extended to allow for this by representing the two feeders as a single set of conductors, with six phase conductors and two neutrals. This would provide an  $8 \times 8$  circuit impedance matrix including the self-impedance of each feeder cable, and also the mutual impedance between the conductors in opposite feeders.

Although this would resolve the inconsistency regarding the shared ground conductor, it is difficult to extend this approach to a more general network layout, for which it is more practical to assume that there are no mutual interactions between the feeders, and that each cable is associated with a separate ground conductor. By including the interactions between the grounding electrodes, as defined in the analysis method presented in Section 9.6.1, it is possible to include some representation of the shared ground conductor within the model, albeit with known limitations.



Figure 9-13 – Network layout showing parallel paths for two feeder mains

## 9.7 Conclusions

This chapter has developed several approaches by which the conventional forward/backward sweep method can be extended to allow for the grounding resistance between the neutral conductor and the physical ground. Initially, a simple method M1 was considered in which the resistances between the neutral and ground are treated in the same manner as constant impedance loads between the line and neutral conductors. This method only converges successfully if certain constraints are applied to the grounding resistance values in the network. In an example simulation, it was found that the convergence failed if the grounding resistance at the substation node was raised too high, or if the grounding resistances at the other nodes were set too low. These constraints prevent the modelling of realistic networks and so several alternative approaches have been considered.

A second method M2 was then developed, similar to Beharrysingh's method but with the difference that the solution does not require the use of current data from the previous iteration in the backward sweep calculation. The new method also differs in that the neutral and ground current calculations are not included in the backward sweep process. Instead, only the load currents and phase conductor currents are determined during the backward sweep, with the neutral and ground currents determined afterwards in a single matrix-based calculation. The matrix inversion needed for this analysis depends only on the circuit model and so need only be calculated once for each network configuration.

A third method M3 has been developed to overcome the constraint of method M2 (and also Beharrysingh's method) that nodes without ground electrodes could not be represented. Method M3 uses Kirchoff's laws as before but in a different arrangement so that the neutral to ground voltages at each node are calculated, rather than the neutral currents in each branch. Rearranging the equations in this manner replaces the constraint that the neutral to ground resistance cannot be infinite with the constraint that it cannot be zero. Since a perfect short circuit between the neutral and ground cannot be realised (as this would imply an infinitely sized grounding electrode), this is not a constraint in practice. The key advantage is that the model allows for the realistic case in which the neutral to ground resistance is infinite at nodes without a grounding electrode.

The theory used in defining the resistance of a ground electrode has been reviewed as a means of providing numeric values for use in the simulations. This theory has then been developed to find the resistance of the ground path between a pair of electrodes, which is less than the combined resistance of each individual electrode. For practical grounding resistance values and feeder lengths, the interactions between electrodes installed by the DNO at the ends of the feeders will be minimal. However, where the model also includes the many possible grounding connections at customer premises and street lights, it is less clear that the grounding resistances can be considered to be independent. In this case, current entering the ground at one electrode may raise the local ground potential for other nearby ground connections.

Finally, method M4 has been presented in which the ground electrode resistance theory is combined with the network analysis of method M3 above. This development models the interactions between electrodes whereby current flowing into the ground at one node increases the ground potential at all of the other nodes. The grounding model then consists of circuit impedances, analogous to the circuit impedances developed for the cables, in which each electrode has both selfimpedance and mutual impedance components.

The discussion highlights a conceptual difference between the model assumed in Carson's equations where the current flows in the ground parallel to the cable, and the model used to derive the grounding resistance where the current disperses radially from the grounding electrode. Although these models can be combined to represent both the interface between the electrode and the ground, and the impedance of the path between the electrodes, this is still an imperfect model. A number of limitations have been identified, in particular that the model assumes that the sum of currents in each branch is zero. This enforces that the unbalance current in the ground must follow the topology of the cable network, rather than follow the path of least resistance to the substation.

One way of addressing these concerns would be to develop a 3-D model of the ground current distribution. However, it is also shown here that the interaction between one grounding electrode and another is relatively low when these are spaced at typical distances to be expected in an LV network. While acknowledging the inconsistencies implied by models, it is hoped that this chapter presents a useful contribution to the understanding of the assumptions typically applied when modelling ground currents in a power-flow simulation, and also in proposing several new approaches that improve the analytical method.

# **10 POWER-FLOW SIMULATIONS**

## **10.1 Introduction**

Previous chapters have discussed several 'building blocks' for network simulations, considering the cable impedance, the time resolution of the demand model, and the connectivity of the neutral and ground circuits. This chapter assembles these into a network simulation such that the importance of accuracy in these different aspects can be evaluated in the context of an LV feeder model.

The network design selected for the model is based on a realistic layout for a feeder in Loughborough (Richardson 2011) serving a residential urban area with 100 domestic customers. The voltage drops and losses on this network are then compared for a number of test cases with different assumptions being made about the grounding, the cable impedances, the time resolution of the data, and the connection of the loads to the feeder.

The aim here is to provide an indication of the sensitivity to these different assumptions. This differs from other studies where the objective is to quantify the proportion of low carbon technologies that can be connected. Instead, the aim here is to understand which changes to the model are likely to affect the outcome of such simulation studies. Although the quantitative differences between results for different assumptions are not necessarily valid for networks in general, the intention is to highlight assumptions that have the potential to affect the results. The extent to which different networks are affected will, of course, vary depending on the demand and the network design.

The power-flow simulations have used a new software tool written for this work in Java. Commercially available tools such as PowerFactory DIgSILENT were initially reviewed but were not appropriate for this work as the software code is proprietary and so could not be viewed or modified. Open source software such as OpenDSS was also considered as this could potentially have provided a useful framework in which the incorporate the new LV solution methods described in Chapter 9 (Dugan 2015). However, the calculations required for these new methods would require changes to the structure of the network solution process, and in the case of method M4, the calculations for each branch depend on each of the other nodes in the

network. These calculations do not readily fit into the existing OpenDSS software architecture. Moreover, OpenDSS is a significantly more complex software package than was required for this work as it allows the LV components to be integrated into a comprehensive distribution network model. Including the new code within the OpenDSS software package would therefore have placed constraints on the structure of the code as it would have needed to be compatible with the existing higher level modules, which were not required in this case. It was therefore decided to write a new simulation tool in order to test and compare the methods described in Chapter 9.

The Java language was selected for this work as this would allow the code to be run on a variety of computing platforms and for it to be usable with minimal requirements to install software or to manage licensing. This facilitates use of the code either by the project sponsors or for future publication as open source software. The strongly enforced type checking and object-orientated architecture of the Java software development environment also assisted in validating that the code provided a correct implementation of the required algorithms.

Based on the results of Section 7.3, the model operates at 50 Hz and so does not include harmonics. However, it is recognised that the losses will be under-represented by modelling only the fundamental, as shown in Section 7.3.2, and the development of a demand profile model including harmonics would be an interesting area for further work.

The software is described in the following section.

## **10.2 Simulation method**

#### 10.2.1 Network model

The feeder network is shown in Figure 10-1 and Figure 10-2, for which the node locations and branch topology follow a representative layout based on a feeder in Loughborough, a town in the East Midlands, UK (Richardson 2011). The network serves 100 domestic customers, mostly with single-phase connections

The figures show the feeder connecting at the distribution substation with two main branch feeders, giving a radial layout that is suited to a forward/backward sweep power-flow simulation. There are 100 customer loads connected at approximately equal spacing along the feeders, most of which are connected via single-phase service cables. In general, the service cables are relatively short with distances of only a few metres from the feeder in the street to the meter point in the houses. However, several connections at the ends of the feeder have longer service cables of over 20 m.

As a default configuration, the loads have been assigned in equal proportion to the three phases. These are allocated in a randomised manner along the feeder length. One spur of the network has a group of customers connected via a single-phase cable.

Figure 10-2 shows the same network but with the colouring indicating the cable types. The specification for the cables used in the real network was not known and so the cable sizes have been assigned based on the numbers of customers connected. As a default case, the feeder is modelled with 3-core 95 mm<sup>2</sup> cable, apart from a short section near to the substation which is modelled with 3-core 185 mm<sup>2</sup> cable, although the cable parameters are varied as part of the modelling investigation. These cable sizes were selected so as to create a scenario with relatively large voltage drops such that the effects of different assumptions were more distinct. The results therefore do not indicate the voltage ranges on the actual network that was originally described (Richardson 2011).









The network cables are assumed to have a concentric combined neutral and earth conductor, and the customers therefore have TN-C-S connections (Cronshaw 2005). Based on the design guidance for LV networks with PME in the East Midlands, the combined neutral and earth conductor should be connected to a ground electrode at the ends of each main and at junctions (East Midlands Electricity 2001). A grounding electrode is also installed at the substation, as shown in Figure 10-3.

It is also possible that the connections at customer meter points will provide a neutral to ground connection due to the earth bonding between the neutral cable and gas and water pipes. However, the design guidance states that the PME design cannot rely on these customer earth bonding connections to provide a guaranteed ground electrode. These grounding connections are uncertain as the contact between the metal pipes and the ground is unknown, and also since a conductive path is not guaranteed if sections of the metal pipes are replaced with plastic.



Figure 10-3 – Grounding connections

The simulation model uses the following set of cable types, defined according to BS 7870 (E.ON Central Networks 2006).

Cable type	Configuration
Single-phase 10 mm2, 25 mm <sup>2</sup> , 35 mm <sup>2</sup>	BS 7870, single core with solid aluminium and concentric neutral (BSI 1996)
Three-phase, 70 mm <sup>2</sup> , 95 mm <sup>2</sup> , 185 mm <sup>2</sup>	BS 7870, 3-core with solid aluminium sector-shape conductors and concentric neutral (BSI 2001)

#### Table 10-1 – Network model cable types

The conductor impedance data for these cables has been generated using the finite element modelling method described in Chapter 3, giving the circuit impedance data shown in Table 10-2. For single-phase cables, the matrices provide impedances for the phase and the combined concentric earth and neutral conductors. For the three-phase cable, the matrices provide impedances for phases L1, L2 and L3 and the combined concentric earth and neutral. The ground path is included as the circuit return path.

	-						
Cable type	Configuration						
Single-phase	[3.127 + j0.829 0.047 + j0.773]						
$10 \text{ mm}^2$	$\mathbf{z} = \begin{bmatrix} 0.047 + j0.773 & 3.251 + j0.772 \end{bmatrix}$						
Cinala abasa							
Single-phase	$\hat{z} = \begin{bmatrix} 1.247 + j0.801 & 0.047 + j0.753 \end{bmatrix}$						
25 mm <sup>2</sup>	2 - [0.047 + j0.753  1.348 + j0.752]						
Single-phase	_ [0.915 + j0.789 0.047 + j0.745]						
35 mm <sup>2</sup>	$\mathbf{z} = \begin{bmatrix} 0.047 + j0.745 & 0.958 + j0.743 \end{bmatrix}$						
	[0.491 + j0.767  0.047 + j0.705  0.047 + j0.705  0.047 + j0.690]						
Three-phase	0.047 + j0.705 $0.491 + j0.767$ $0.047 + j0.705$ $0.047 + j0.690$						
$70 \text{ mm}^2$	$z = \begin{bmatrix} 0.047 + j0.705 & 0.047 + j0.705 & 0.491 + j0.767 & 0.047 + j0.690 \end{bmatrix}$						
70 11111	0.047 + j0.690 $0.047 + j0.690$ $0.047 + j0.690$ $0.490 + j0.690$						
	[0.368 + i0.757  0.047 + i0.697  0.047 + i0.697  0.047 + i0.684]						
Three-phase	0.047 + j0.697 $0.368 + j0.757$ $0.047 + j0.697$ $0.047 + j0.684$						
$95 \text{ mm}^2$	$z = \begin{bmatrix} 0.047 + j0.697 & 0.047 + j0.697 & 0.368 + j0.757 & 0.047 + j0.684 \end{bmatrix}$						
	0.047 + j0.684 $0.047 + j0.684$ $0.047 + j0.684$ $0.367 + j0.683$						
	[0.214 + j0.733  0.047 + j0.673  0.047 + j0.673  0.047 + j0.661]						
Three-phase	0.047 + j0.673 $0.214 + j0.733$ $0.047 + j0.673$ $0.047 + j0.661$						
$185 \text{ mm}^2$	$z = \begin{bmatrix} 0.047 + j0.673 & 0.047 + j0.673 & 0.214 + j0.733 & 0.047 + j0.661 \end{bmatrix}$						
	0.047 + j0.661 $0.047 + j0.661$ $0.047 + j0.661$ $0.211 + j0.661$						

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#### 10.2.2 Demand model

The customer demand is defined using a set of time stepped profiles with 1 minute resolution, created using the CREST demand model (Richardson & Thomson 2011). This was modified so that both the active and reactive power would be included, making use of the power factor data already populated in the model with values varying between 0.8 and 1. An example of the daily demand profile for one dwelling is shown in Figure 10-4. For each customer, the demand consists of loads due to lighting and from the various appliances that may exist at each dwelling. The modified model was configured to generate the demand for an extended period of 10 days, and for the 100 different customers. In configuring the model, the appliances at

each property were assumed to remain constant for each day but were different for each customer.



Figure 10-4 – Example of active (kW) and reactive (kvar) demand for one customer, first day of synthesized profile

An example of the aggregated demand profile for all100 customers is shown in Figure 10-5. This shows the phase unbalance that can occur, as individual loads switch on and off according to their stochastic behaviour, even with a balanced allocation of customers to the three phases. The aggregated demand has a similar profile over each day of the 10 day period, as in Figure 10-6. This plot shows the mean active and reactive power over hourly periods. A slight dip in the demand can be seen after midnight as the data for each day was synthesised from a separate run of the demand model which initialised with the appliances switched off. As a default, the loads are assumed to have a constant power relationship between current and voltage, although the impact of varying this model is also investigated.

Although the model can provide data that combines the demand with generation from photovoltaic arrays, the modelling here has used only the demand data. The time variation of the demand is greater than that of the photovoltaic generation and so exercises a greater range of unbalance conditions within the length of the synthesized profile. Consequently, the key voltage metric considered here is the worst-case RMS voltage drop due to the demand, although the results also demonstrate the voltage rise that can occur due to the unbalance of the demand.



Figure 10-5 – Aggregated demand for first day of synthesized profile



Figure 10-6 – Aggregated active (kW) and reactive (kvar) power for 10 day profile, hourly mean

#### **10.2.3 Simulation process**

The structure of the Java software tool is described by the flow chart in Figure 10-7. The simulation process begins with the definition of the network, based on a set of nodes and branches, with one node being defined as the substation. Nodes are defined with X and Y geographic co-ordinates, and then branches are defined in terms of their beginning and end nodes. The physical route taken between these two nodes is then defined by a set of vectors, such that the branches do not necessarily follow a straight line path from one node to another.



Figure 10-7 – Simulation software flow chart for method M3 and M4, steps modified for this method shown by shading

The customer loads are then defined in terms of their connection node, the allocated service cable phase, their voltage/current characteristics (constant power, constant current or constant impedance). Each load is also assigned a synthesized demand

profile consisting of active and reactive power, sampled at 1 minute intervals, with 14400 samples to model a period of 10 days.

The software then interprets the topology of the network and defines a sequence in which the branches must be solved, as required by the forward/backward sweep process. This network is then checked to ensure that:

- There are no nodes that are not connected by branches
- The topology contains no loops
- Multiple loads connected to a single-phase cable are assigned the same phase
- There are no three-phase cables connected downstream of a single-phase cable

The currents and voltages in the network are then solved for each time step, assuming a constant voltage at the substation. Although this voltage could vary in time, it has been held constant at 240 V for this modelling so that differences in the voltage drops within the LV feeder can be more easily identified without the additional variation of the reference voltage.

The different solution methods utilised for the results presented here, allowing for different assumptions regarding the neutral to ground connectivity of the network, are as follows:

- <u>Ground isolated</u>: There is an open circuit between the neutral and ground and so all of the unbalance current from the phase conductor currents returns via the neutral with no current in the ground.
- <u>Multi-grounded neutral</u>: The neutral and ground are connected by a shortcircuit so that there is a zero voltage between them. For a given unbalance current from the phase conductors, the current in the neutral and in the ground can be calculated using the Kron reduction.
- <u>Matrix method M3, as described in Section 9.4</u>: Some or all nodes have a defined resistance between the neutral conductor and the ground. Nodes without a grounding resistance can have an open circuit between the neutral and ground, as is the case for service cable connections on to a main feeder.
The neutral and ground currents are then determined at the end of each backward sweep.

 <u>Matrix method M4, as described in Section 9.6</u>: This mode develops the case above and allows for the increase in ground potential at each grounded node due to the currents entering the ground at all of the other grounded nodes. This models the interactions that occur when the ground electrodes are in close proximity.

Methods M3 and M4 require a network matrix to be computed at the start of the power-flow method. This is then applied on each subsequent time step.

The power-flow calculation then continues the forward/backward sweep process until a convergence threshold has been reached. This is defined such that the magnitude of the voltage difference between successive sweeps must be less than  $10^{-6}$  V. (This threshold is clearly beyond practical relevance, but is useful in verification of the software.)

Once the voltages have reached convergence, the load demands are then advanced to the next time step in their synthesized demand profiles. The power-flow process is repeated, making use of the previous state as the initialisation for the new time step.

#### **10.2.4 Simulation outputs**

Once all the required time steps have completed, statistics are generated based on the history of voltages at each node, the power provided to the network at the substation, and the power delivered to the loads at each node. A sequence of 10 minute RMS voltage values is also calculated, as specified in EN 50160.

A reference node, identified on Figure 10-1, is used for the voltage measurements. This node has been found to have the worst case voltage profile in the majority of simulation runs and so acts as the 'end of feeder' test point.

This data is used to record the following voltage metrics:

- Minimum and maximum voltage, over all nodes, for the three phases and for all time steps
- Minimum and maximum 10 minute RMS voltage, over all nodes, for the three phases and for all 10 minute intervals

Minimum 95% confidence level, calculated as the minimum of the 95% confidence level for all nodes and for the three phases, where this confidence level for each node and phase is the voltage that 95% of 10 minute RMS values are above. There is no calculation of 95% maximum confidence level as this tolerance is only specified on the lower side of the voltage range (see Section 2.1.10).

The following metrics are recorded for evaluation of the losses:

- Mean losses over simulation duration
- Mean input power on each phase at the substation over the simulation duration

The voltage variation is also plotted for the reference node as a cumulative distribution function, showing the probability that the voltage will be below a given value. An example plot is shown in Figure 10-8.

A second plot is generated to show the maximum and minimum voltage profile along the feeder, as shown in Figure 10-9. For each node, this plots the maximum and minimum 10 minute RMS voltage calculated for over all three phases and all 10 minute intervals. The horizontal axis shows the distance along the route of the feeder from the substation, from zero up to the longest distance of 417 m.







Figure 10-9 – Example of voltage profile from power-flow simulations

## 10.3 Neutral and ground connectivity

The impact of variations in the neutral to ground connectivity has been explored by comparing 5 test cases.

A simple approximation can be made by modelling the network with the ground isolated from the cable. This represents the cable as if it were constructed with a separate neutral and earth. In this case, all the unbalance current returns through the combined neutral and earth conductor and there is no current in the ground.

An opposite case is then provided by the assumption of a multi-grounded neutral, in which there is a perfect connection between the neutral and ground and the Kron reduction can be applied to treat both the combined neutral and earth and the ground as a single conductor.

The cases with non-zero grounding resistances represent approximations that are between these two extremes. The grounding resistances are introduced into the network in four steps:

- <u>Grounding resistance configuration A</u>: Grounding resistances are defined at the substation, and at PME earths that are required at specific locations along the feeders, as shown on Figure 10-1. The resistances have been specified according to worst case values (E.ON Central Networks 2006; East Midlands Electricity 2001) with the grounding resistance at the substation of 20 Ω and the resistance at PME earths of 100 Ω. This configuration represents the case that could be required in network planning where only the guaranteed ground connections (i.e. those under the direct control of the network operator) can be taken into account.
- <u>Grounding resistance configuration B</u>: The grounding resistances are subject to measurement difficulties and also dependent on local ground conditions. It therefore seems likely that the many installations will have resistances that are better than the specified maximum values, in order to ensure a good safety margin. In this test case, the substation and PME earth resistances were reduced by a factor of 10, giving resistances in a similar range to those assumed by Sunderland (Sunderland & Conlon 2012). The grounding resistance at the substation was therefore 2 Ω and the resistance at PME earths was 10 Ω.

- <u>Grounding resistance configuration C</u>: Grounding connections at customer loads were included, in addition to those at the substation and PME earths. The grounding resistances were set to 100 Ω at each of the nodes with loads connected.
- <u>Grounding resistance configuration D</u>: The above case was repeated with the grounding resistances reduced to 10 Ω at nodes with loads. This is consistent with the previous measured values for premises with good grounding (Mather 1958). Similar values have been used in other studies (Alam et al. 2015; Werda et al. 2008; Sunderland & Conlon 2012).

Results from these different models are compared in Table 10-3. These simulations assume that the loads have a constant power characteristic. The cases with finite grounding resistances are solved with method M4 from Section 9.6 where the interactions between grounding electrodes are taken into account, and also with the simulation method M3 described in Section 9.4 where the grounding electrodes are each considered as being independent. Results from method M3 are shown in parentheses.

Compared to the model with the multi-grounded neutral, the maximum voltage drop is 2.7 V (10%) higher for 1 minute samples and 1.2 V (5%) higher for 10 minute RMS voltages. At the 95% confidence level, the maximum voltage drop is only 0.2 V (1.5%) higher. There is also an impact on the maximum voltage rise which is 1.2 V greater for 1 minute samples and 1.7 V higher for the 10 minute RMS voltages.

The differences in the grounding assumptions have the greatest impact on the samples for which the demand is highly unbalanced, such that there is a greater total current flowing in the neutral and ground. Where the voltage metrics are less sensitive to the extreme load conditions, then the results are less sensitive to the assumptions made within the grounding model.

	Maximum voltage			Maximum voltage		
	drop, V			rise, V		
	1 min	10 min	95%	1 min	10 min	Mean loss
Neutral and ground connections		RMS			RMS	power, W
Ground isolated	29.3	24.5	13.8	8.3	6.2	822.9
Configuration A: Substation 20 $\Omega$ ,	29.2	24.5	13.8	83	6.2	822.7
PME earths 100 $\Omega$ ,	(20.2)	(24.5)	(13.8)	(8.3)	(6.2)	(822.7)
other nodes open circuit	(23.2)	(24.3)	(13.0)	(0.5)	(0.2)	(022.7)
Configuration B: Substation 2 $\Omega$ ,	29.0	24.3	13.7	8.1	6.1	821.0
PME earths 10 $\Omega$ ,	(20.0)	(24.3)	(13.7)	(8.1)	(6.1)	(820.8)
other nodes open circuit	(29.0)	(24.3)	(13.7)	(0.1)	(0.1)	(020.0)
Configuration C: Substation 2 $\Omega$ ,	28.0	24.2	13.7	8.0	6.0	820.1
PME earths 10 $\Omega$ , loads 100 $\Omega$ ,	(20.0)	(24.4)	(12.7)	(9.0)	(5.0)	(810.4)
other nodes open circuit	(20.0)	(24.1)	(13.7)	(8.0)	(5.9)	(019.4)
Configuration D: Substation 2 $\Omega$ ,	28.6	-23.9	13.6	78	5.8	818.0
PME earths 10 $\Omega$ , loads 10 $\Omega$ ,	(27.7)	(22.2)	(12.4)	(7.0)	(5.1)	(812.2)
other nodes open circuit	(27.7)	(23.3)	(13.4)	(7.0)	(5.1)	(012.2)
Multi-grounded neutral,	26.6	23.4	13.6	7 1	4.5	702 1
all nodes 0 Ω	20.0	20.4	10.0	1.1	5	152.1

# Table 10-3 – Simulation results investigating neutral and ground connectivity, main feeder 3-core 95 mm<sup>2</sup> cable (figures in parentheses for solutions with ground electrodes modelled as being independent)

The mean loss power is 4% higher for the case with the neutral and ground isolated than with the multi-grounded neutral. Although reactive currents can then flow in the loop created around the neutral and ground, the overall losses with the multigrounded neutral are reduced due to the additional conductive path of the ground.

The two different solution methods give slightly different results, with the method including the ground interactions predicting slightly greater voltage drops and losses. Where the ground connections only exist at the substation and at the PME earths the results are the same from both methods, as expected since the electrodes are far apart in this case with negligible interaction between them.

The modelling was repeated with the main feeder replaced with 3-core 185 mm<sup>2</sup> cables, rather than the 95 mm<sup>2</sup> cables assumed as a default, giving results as shown in Table 10-4. This shows between 1% and 3% variation in the voltage metrics and around 3% variation in the calculated losses. The voltage deviations and losses are

lower than with the 95 mm<sup>2</sup> cable since the resistances are lower and so a greater proportion of the load unbalance current returns through the neutral than in the ground path (where the resistance contribution is unchanged in relation to the cable size).

	Maximum voltage			Maximum voltage		
	drop, V			rise, V		
	1 min	10 min	95%	1 min	10 min	Mean loss
Neutral and ground connections		RMS			RMS	power, W
Ground isolated	15.3	13.1	7.6	4.4	3.1	451.8
Configuration A: Substation 20 $\Omega$ , PME earths 100 $\Omega$ ,	15.3 (15.3)	13.1 (13.1)	7.6 (7.6)	4.4 (4.4)	3.1 (3.1)	451.8 (451.8)
other nodes open circuit	· · · ·	, , , , , , , , , , , , , , , , , , ,			~ /	、 ,
Configuration B: Substation 2 $\Omega$ , PME earths 10 $\Omega$	15.2	13.1	7.5	4.4	3.1	451.3
other nodes open circuit	(15.2)	(13.1)	(7.5)	(4.4)	(3.1)	(451.3)
Configuration C: Substation 2 $\Omega$ , PME earths 10 $\Omega$ , loads 100 $\Omega$ .	15.2	13.1	7.5	4.4	3.1	451.1
other nodes open circuit	(15.2)	(13.0)	(7.5)	(4.4)	(3.0)	(450.9)
Configuration D: Substation 2 $\Omega$ , PME earths 10 $\Omega$ loads 10 $\Omega$	15.1	13.0	7.5	4.3	3.0	450.5
other nodes open circuit	(14.9)	(12.8)	(7.4)	(4.2)	(2.9)	(449.0)
Multi-grounded neutral, all nodes 0 $\Omega$	14.8	13.0	7.5	3.5	2.4	438.8

 Table 10-4 – Simulation results investigating neutral and ground connectivity,

 main feeder 3-core 185 mm<sup>2</sup> cable (figures in parentheses for solutions with ground electrodes modelled as being independent)

Overall, the differences in voltage drops are not large, other than if the worst-case is considered, suggesting that the results are not highly sensitive to the neutral to grounding connections. This is fortunate since there is usually little practical information on which to base an accurate model of these connections.

If a network is planned assuming an isolated ground, then these results suggest that the actual voltage drops will be less than predicted (all other aspects being equal). Conversely, the assumption of a multi-grounded neutral model (using Carson's equations and applying the Kron reduction) will under-estimate the voltage deviations since not all nodes are grounded and sincethose that are have finite grounding resistances.

Since there is significant uncertainty regarding the grounding resistances which are unlikely to be known in practice, DNOs could make an approximation of the actual voltage drop and losses by taking the average of results for the case with the isolated ground and the multi-grounded neutral. Alternatively, if most of the customer nodes are thought to be ungrounded, then the assumption of an isolated ground gives results that are closer to those for a network model that included PME earths, than the assumption of a multi-grounded neutral.

#### 10.4 Demand unbalance

The scenarios described in Section 10.3 have been repeated with the phase allocations of the loads changed so that one phase (L1) has twice the number of customers as the other two phases.

The voltage metrics and mean losses for this case are shown in Table 10-5, where the power-flow method from Section 9.6. Compared to the results in Table 10-3 where the customers are assigned in equal proportion to the three phases, the voltage drops and losses are higher, as expected due to the increased load on demand L1.

The variation between the two cases with isolated ground or multi-grounded neutral is similar to the previous results where the minimum voltage is considered, but the different grounding models have a much greater impact on the 95% confidence level voltage metric. The different grounding models here give approximately 5% variation in the RMS voltage drop metrics and also 5% variation in the losses.

For these results, with a different random allocation of loads, the demand unbalance that causes the occurrence of the worst-case RMS voltage drop is not necessarily any greater than for the results in Table 10-3. However, since the mean demand on each phase is unbalanced, the likelihood of the demand being unbalanced in each time step is much greater and so a greater difference is noted when the balanced and unbalanced cases are compared with the 95% confidence level metric.

	Minimum voltage			Maximum voltage		
	drop, V		rise, V			
	1 min	10 min	95%	1 min	10 min	Mean loss
Neutral and ground connections		RMS			RMS	power, W
Isolated	38.1	31.6	20.5	10.1	7.1	1056.2
Configuration A: Substation 20 $\Omega$ ,						
PME earths 100 $\Omega$ ,	38.1	31.6	20.5	10.0	7.1	1055.7
other nodes open circuit						
Configuration B: Substation 2 $\Omega$ ,						
PME earths 10 $\Omega$ ,	37.8	31.3	20.3	9.8	6.9	1051.5
other nodes open circuit						
Configuration C: Substation 2 $\Omega$ ,						
PME earths 10 $\Omega$ , loads 100 $\Omega$ ,	37.6	31.2	20.2	9.7	6.8	1049.6
other nodes open circuit						
Configuration D: Substation 2 $\Omega$ ,						
PME earths 10 $\Omega$ , loads 10 $\Omega$ ,	37.2	31.0	20.1	9.4	6.6	1045.1
other nodes open circuit						
Multi-grounded neutral,	35.8	30.4	19.5	11.1	6.3	999.4
all nodes 0 Ω						

Table 10-5 – Simulation results investigating neutral and ground connectivity,
main feeder 3-core 95 mm <sup>2</sup> cable, with unbalanced assignment of customers to phase

As with the results above for a mean balance of the demand, the models with ground resistances included give an RMS voltage drop that is approximately the mean of the results for an isolated ground and for a multi-grounded neutral.

## 10.5 Demand variation with voltage

The simulations described above have assumed that the loads can be modelled with a constant power relationship, as often assumed in power-flow studies, although recent work has suggested that other relationships may be more appropriate.

A general load model can be defined so that the demand power varies exponentially in accordance with the line to neutral voltage, giving:

$$S = P_0 \left(\frac{|V|}{|V_0|}\right)^{K_{\rm P}} + jQ_0 \left(\frac{|V|}{|V_0|}\right)^{K_{\rm Q}}$$
(211)

where  $P_0$  and  $Q_0$  are the rated active and reactive powers and  $V_0$  is the rated voltage. The exponents  $K_P$  and  $K_Q$  determine the load model characteristic. These exponents can take any real value but specific cases are given for constant power loads with  $K_P = K_Q = 0$ ; constant current loads with  $K_P = K_Q = 1$ ; and constant impedance loads with  $K_P = K_Q = 2$ .

A model could potentially be configured with the demand current model assigned individually for each load, or even for individual appliances at each load connection. However, to show the overall range of sensitivity here, all loads are configured together according to one of these three current models, with results as shown in Table 10-6. The default network model with 3-core 95 mm<sup>2</sup> cables is assumed, together with the grounding resistances given by configuration D from Section 10.3. The solution method of Section 9.6 has been used.

		Minimum voltage			Maximum voltage		
		drop, V			rise, V		
Rated	Demand model		10 min			10 min	Mean loss
voltage, V	exponents	1 min	RMS	95%	1 min	RMS	power, W
	$K_{\rm P}=K_{\rm Q}=0$	28.6	23.9	13.6	7.8	5.8	818.0
	$K_{\rm P} = K_{\rm Q} = 1$	25.1	21.6	12.7	6.8	5.1	763.0
240	$K_{\rm P} = K_{\rm Q} = 1.3$	24.3	21.0	12.5	6.6	4.9	748.5
	$K_{\rm P} = 1.3, K_{\rm Q} = 6.0$	24.0	20.7	12.5	5.7	4.4	735.1
	$K_{\rm P} = K_{\rm Q} = 2$	22.7	19.9	12.0	6.1	4.6	717.4
230	$K_{\rm P} = 1.3, K_{\rm Q} = 6.0$	25.5	22.0	13.3	6.1	4.8	847.0

Table 10-6 – Simulation results investigating variations with demand current model

For the results with  $V_0 = 240$  V, and so equal to the substation voltage, the minimum 10 minute RMS voltage varies by approximately 20% between the constant power and constant impedance cases, and the 95% confidence level varies by around 13%. The losses also reduce by around 13% between these two cases. These results are consistent with the trends shown by Ciric where constant power and constant admittance loads were compared (Ciric et al. 2003).

Table 10-6 also provides results for a case with  $K_{\rm P} = 1.3$ , and  $K_{\rm Q} = 6.0$  as proposed from the Electricity North West CLASS project (Electricity North West Ltd 2015a),

showing significant reductions in the voltage drop and losses compared to the case with constant power loads. Results are also shown for a similar case in which the separate exponential factor for the reactive power is neglected and  $K_P = K_Q = 1.3$ . This simplification changes the results by around 2%, indicating that slight errors would be introduced if the simulation software were only to allow for one exponential factor for both active and reactive power.

Although there may be uncertainty over the exact load model parameters to use, there are clearly significant differences in results to those assuming a constant power characteristic. The reductions in losses and voltage drops observed in these results are to be expected since loads with lower voltages towards the ends of the feeder consume less power and so the currents provided to the feeder are lower.

However, changing the load model from the constant power case, as typically assumed, to a model with higher exponential factors does not necessarily lead to lower voltage drops and losses. This is demonstrated by the results with  $V_0 = 230$  V where the rated powers  $P_0 + jQ_0$  are specified for a lower voltage than the voltage at the substation on the LV feeder. This situation may apply where demand data is based on the rated power data for domestic appliances with the assumption that this is defined at 230 V. Typically, distribution substation voltages are set higher than this and so loads that are near to the substation will have greater demand than their rated power. This is demonstrated in Table 10-6 with parameters  $K_P = 1.3$ , and  $K_Q = 6.0$ , a rated voltage for the loads of 230 V, and a voltage of 240 V set at the substation. The increase in demand at nodes with voltages above 230 V more than offsets the reductions from nodes at the ends of the feeder where the voltages are lower. The overall losses are also increased.

A similar concern may apply if demand data recorded by smart meters is used to create demand data profiles for use in power-flow studies. Demand profiles based on such measured data could be used, for example, to model the demand for nodes at other locations in the network, or for nodes with distributed generation connected. When recording demand data for such models from smart meters, it is therefore recommended that DNOs should also record the voltage at which the demand occurred, such that the corresponding currents can be scaled correctly when the demand samples are used for model customers at different voltages.

## 10.6 Demand averaging

The simulation model allows the impacts using time averaged demand data to be investigated at the scale of a full LV feeder. This builds on the discussion in Section 8.2 which considers the effect on loss estimates for single-phase loads and Section 8.3 where the effects were considered for a section of three-phase cable on which the demand was highly aggregated.

The comparisons are now repeated using the load simulation model so that the effects can be investigated for all of the branches from the three-phase cables at the substation to the single-phase cables at customer connections. In exchange for this increased coverage of the network, it is accepted that the simulation must use synthesized data rather than real measurements, and that the results are based on a demand model with a native resolution of 1 minute. This data has been used to create data sets with the active and reactive power averaged over blocks of 2, 10, 30 and 60 minutes, representing the demand data that would have been generated if the model had used these measurement intervals.

The simulations again use the method M4 of Section 9.6 together with the grounding resistances given by configuration D from Section 10.3. Results have been generated for cases with i) constant power loads, and ii) exponential loads with  $K_{\rm P} = 1.3$ ,  $K_{\rm Q} = 6.0$ , and  $V_{\rm Q} = 240$  V. The results are shown in Table 10-7.

As expected from Section 8.3, the minimum RMS 10 minute voltage is unchanged provided the demand data resolution is 10 minutes or better and the same applies to the 95% confidence level metric. At 1 minute resolution, there is a significant difference of 4 V to 5 V in the minimum RMS voltage.

As before, the variation of losses is plotted in Figure 10-10 as a loss ratio defined as the ratio of estimated losses to the 'true' losses without averaging. An estimate of the 'actual' losses without averaging has been made using (121). Since the demand here is determined by a simulation method, it could be argued that the 1 minute data already allows the actual losses to be calculated exactly. However, the model is constructed based on the mean demand for individual appliances over the 1 minute period. Although shorter term variations are not represented, the results from Section 8.2.2 demonstrate that they occur in practice.

	Averaging	Minimum voltage			Maximum voltage		
	period	drop, V		rise, V			
			10 min			10 min	Mean loss
Load model		1 min	RMS	95%	1 min	RMS	power, W
	1 minute	28.6	23.9	13.6	7.8	5.8	818.0
	2 minutes	28.4	23.9	13.6	7.3	5.8	808.6
Constant power	5 minutes	25.1	23.9	13.6	6.6	5.8	789.3
Constant power	10 minutes	23.9	23.9	13.6	5.8	5.8	771.0
	30 minutes	17.2	17.2	12.7	4.7	4.7	730.2
	60 minutes	15.2	15.2	11.9	3.0	3.0	697.1
	1 minute	24.0	20.7	12.5	5.7	4.4	735.1
$K_{\rm r} = 1.3$	2 minutes	23.9	20.7	12.5	5.4	4.4	728.0
$K_{\rm p} = 1.3$ $K_{\rm Q} = 6.0$	5 minutes	21.6	20.7	12.5	5.1	4.4	713.0
	10 minutes	20.7	20.7	12.5	4.4	4.4	698.6
$v_0 = 240 v$	30 minutes	15.5	15.5	11.8	3.5	3.5	666.2
	60 minutes	13.9	13.9	11.0	2.2	2.2	640.0

Table 10-7 – Simulation results investigating variations with demand data averaging





The results show that the 10 minute data would under-estimate the losses by around 7%, 30 minute data under-estimates the losses by 10% and the use of hourly data would under-estimate this by around 15%. However, the error reduces to around 1% with 1 minute data.

The loss ratio depends slightly on the load model that has been assumed. With the exponential model, and with the rated voltage set equal to the voltage at the substation, a load near the end of the feeder will require a lower current than a load of the same power near the substation. The errors in the loss estimates are greatest at the feeder ends since the demand is less aggregated and so more 'spiky'. With the exponential model, there is therefore a lower contribution to the total error from these end nodes.

These simulations suggest that DNOs should aim to use the a time resolution of the order of 1 minute in order to minimise the errors in representing losses. The methods outlined in Chapter 8 could then be applied to approximate the mean loss power that would be observed without the impact of demand averaging in the measurement sample periods.

## **10.7 Service cable connections**

The network database may have limited description of the service cables. One approach to resolving this would be to model the customers as being connected directly to the feeder, omitting the service cable from the network topology. The impact of this is explored here, comparing the results both with and without the service cables.

The model that excludes the service cables was created by re-assigning the loads to the nearest node on the main feeder, retaining the same phase assignments as when on the single-phase service cable. This raises a difficulty in modelling the ground resistances since, if the service cables are not represented, it is not possible to model grounding resistances directly at the customers' dwellings. The results in Section 10.3 have also demonstrated that the inclusion of grounding connections only at the substation and at PME earths has little impact on the network voltages. The case without service cables is therefore modelled with the neutral and ground isolated, and also with a perfect multi-grounded neutral, these being the two modelling approximations most likely to be employed.

These cases are compared in Table 10-8. If the service cables are omitted, the worst-case voltage drop is under-estimated by between 0.3 V to 0.7 V, depending on

the modelling case and the chosen metrics. Based on the 95% confidence level, omitting the service cables under-estimates the voltage drop by 0.1 V to 0.2 V.

		Minimum voltage			Maximum voltage		
Neutral and		drop, V		rise, V			
ground		1 min	10 min	95%	1 min	10 min	Mean loss
connections	Service cables		RMS			RMS	power, W
Isolated	Included	29.3	24.5	13.8	8.3	6.2	822.9
15010100	Omitted	28.6	23.9	13.7	8.0	5.9	798.4
Multi-grounded	Included	26.6	23.4	13.6	7.1	4.5	792.1
neutral	Omitted	26.0	23.1	13.4	7.0	4.4	776.5

 Table 10-8 – Simulation results investigating impact of omitting service cables from the model

These differences might be estimated simply by adding an allowance for the voltage drop through a service cable onto the metrics obtained from a model with loads connected directly at the feeders. However, the worst-case voltage drop in the service cable for the end of feeder reference node was between 1.5 V and 2 V depending on the grounding model and between 0.7 V and 0.9 V for the 10 minute RMS voltage. Adding this value to the worst-case voltage at the service cable connection would give an unduly pessimistic result with an error between 1% and 4% as it does not allow for the low probability that the worst-case load at a particular node coincides with the worst-case voltage at the feeder.

## **10.8 Conclusions**

This chapter has presented results from power-flow simulations of an LV feeder with a combined neutral and earth conductor and connections between this conductor and the ground at PME earth electrodes. These simulations implement the matrixbased methods used to calculate the neutral and ground currents described in Chapter 9 which have been coded into a new software tool written in Java. This allows different assumptions regarding the neutral and ground connectivity to be compared in terms of the impact on RMS voltage drop and the losses. These results can also be compared with simpler approximations in which the neutral and ground are either regarded as isolated or as being perfectly connected with a multi-grounded neutral. If the modelling only includes ground electrodes where these are under the direct control of the network operator, then the results are closely approximated by assuming that the neutral is isolated from the ground. However, if each of the customer nodes also has a 10  $\Omega$  connection between neutral and ground, then assuming an isolated ground will over-estimate the voltages drops and losses. Conversely, assuming a multi-grounded neutral will under-estimate the voltage drops and losses. The errors in the voltage drop can be up to 10% and errors in the estimated losses up to 4% depending on the size of the cable and on the averaging period selected in making the comparison of voltages.

Although more accurate results can be achieved by modelling the grounding connections in detail, network operators do not normally possess the data to define the grounding resistances at each node. One of the two approximations is therefore needed. If the customer nodes provide PME ground electrodes (assumed here to have 10  $\Omega$  grounding resistance), then a better estimate of the maximum voltage drop and mean losses can be obtained by taking the arithmetic mean of the results for the isolated ground and multi-grounded neutral cases. If the customer nodes are not expected to provide a PME ground electrode then more accurate results are obtained by assuming that the neutral is isolated from the ground. This allows for the use of a simpler calculation than would be needed if the ground conductor was included.

The power-flow simulations have also been used to explore the impacts of assuming different load models, comparing constant power and constant impedance characteristics as well as other intermediate exponential models. The choice of load model has a greater impact than the use of different grounding assumptions, with up to 20% greater maximum voltage drop and 13% greater mean losses for a constant power model than for a constant impedance model. The voltage drop and losses also depend on the reference voltage assumed when defining load models with non-zero exponents (i.e. not a constant power model). It is therefore important that monitoring data used to create demand models also includes voltage at which the demand was measured. Without this voltage information, there is a risk that simulation models will misrepresent customer demands if demand data (such as from smart meters) is not correctly normalised.

The impacts of demand data averaging have been considered using the simulation model so that the effect on losses can be investigated for a full LV feeder, extending the work presented earlier in Section 8.2 for single-phase demands and in Section 8.3 for a single section of three-phase cable. Over the full LV feeder, the results indicate errors in the loss estimates of around 15% if hourly demand data were to be used, and 7% with 10 minute data.

A comparison has also been made to show the impact of omitting the service cables from the model. The results show a relatively low error of 1-2% in the worst-case voltage drop, but a 3% error in the losses. However, the worst-case voltage drop along the service cable does not coincide with the worst-case voltage at the nearest connection point, and so there is an error of 1%-4% introduced by adding the two worst-case conditions.

Although the variations in some of these voltage comparisons are not large, it is notable that the ranges of worst-case voltage, 10 minute RMS, and the 95% confidence level voltage (as required by EN 50160) are significantly different. A change in the UK regulations to permit the use of RMS averaging (as discussed in Chapter 8) and to allow either an extended lower voltage tolerance or a probabilistic confidence limit would significantly increase the hosting capacity of UK networks.

# **11 CONCLUSIONS**

This work has investigated the accuracy of simulations of LV networks and has proposed new methods that can improve the results. There is increased interest in modelling LV networks as new low carbon technologies (LCTs) are becoming connected, with electrical demand that is greater and more variable than has previously been assumed. Upgrading network cables is costly and so there is a need to operate networks closer to design limits for power quality limits and capacity. This requires accurate network models.

A wide range of LV network studies and simulations from published literature has been reviewed, identifying that there are many assumptions and approximations in the models (Chapter 2). There are significant uncertainties in predicting the future demand and generation that will be connected to the network, and also in obtaining an accurate description of the feeder network. Data describing the cable types, and the location and phase allocation of service cable connections is not always available or up-to-date. However, in addition to these forecasting and operational uncertainties, there are also a number of approximations introduced by the modelling methods. The impacts of the following five key aspects have been investigated in detail:

- The models used to determine the line impedance of the cables, including the conductive path through the ground.
- The time resolution of the demand data.
- The impact of harmonics, and of neglecting them in the model.
- The methods used to include the ground in power-flow models, allowing for the resistance of the interface between the earth electrodes and the ground.
- The practical difficulties associated with making current and voltage measurements to provide the input data for simulation models.

The conclusions from these investigations are described below.

## 11.1 Cable impedances

A detailed finite element (FE) method for estimating the impedance matrix of underground cables has been developed, extending an initial version from previous work. This new FE model takes account of the rotation of conductor cores relative to a concentric neutral due to the cable lay and also models circulating currents in the sheath, armour and the ground. The revised model has been used to provide accurate impedance matrices for use in network simulations and for comparison with impedances derived from measurements. This modelling method could be adopted by DNOs to review or re-confirm the accuracy of impedance data in use for network planning (Chapter 4).

For impedances at 50 Hz, it has been demonstrated that simpler analytical techniques can give a very close approximation to the FE results, provided that the geometry of the sector conductors is modelled in detail and that AC resistance corrections from IEC 60287 are included. At harmonic frequencies, the FE model provides improved accuracy as the impact of eddy currents is included (Chapter 3).

The asymmetry in four-core cables, significant errors in the voltage drops can be introduced if the cable is approximated as being transposed, as when the impedance matrix is specified by only the positive and zero sequence terms. For an example of BS 5467 cable, the worst-case voltage drop was under-estimated by 17% (Chapter 3).

The theory used in modelling cable impedances has been examined in detail, identifying a number of assumptions that are applied in the literature but not always stated or recognised. Although the theory relies on the assumption that the sum of currents equals zero, several practical scenarios are described where this may not be the case (Chapter 3).

A clear distinction in the theory is drawn between conductor impedances and circuit impedances. Whilst Carson's equations provide the impedance of a circuit with a ground return path, approaches in the literature partition these impedances to provide individual conductor impedances such that the voltage difference along the ground can be calculated. However, this method is shown here to rely on arbitrary assumptions for the geometry of the equivalent conductors and so is unreliable (Chapter 3).

The currents due to capacitance between the cable conductors were simulated using the FE model and found to be negligible (Chapter 4).

There was also negligible difference in the impedance for stranded conductors (with gaps) compared to the impedance for a uniform solid material with the same average conductivity (Chapter 4).

Cables installed in the ground are also subject to manufacturing tolerances. A range of likely variations has been assessed, indicating that the spacing between conductors due to the insulation thickness has the greatest impact on the reactance. The resistance varies with temperature, increasing by 4% for a 10 °C change, but the zero sequence reactance is also increased by 6% if the neutral is grounded.

Manufacturers generally provide limited impedance data and further assumptions are needed to provide the complete impedance matrix. A section of LV cable has been measured and the voltage drop was found to be under-estimated by between 5% and 13% from simulations based on the manufacturer's data, depending on the approximation used to provide the zero sequence impedance. Similar errors were found with the estimated losses (Chapter 5).

#### **11.2 Time resolution effects**

Losses can be significantly under-estimated if the network currents are specified by an arithmetic mean demand, and if the demand varies over the averaging period. Using measured data for the demand at a single dwelling, it was found that mean losses over 7 days were under-estimated by 40% if half-hourly demand data was used, and by between 4% and 11% if one minute data was used (Chapter 8). The error reduces as the level of demand aggregation increases and models of an LV feeder showed that losses were under-estimated by 10% using half-hourly demand data and by 1% for one minute data. A time resolution of around one minute is therefore recommended for studies of losses on LV feeders. A resolution of the order of a few seconds is needed to study the losses for individual domestic customers.

These errors in loss estimates apply to feeders with conventional demand and also to feeders with new low carbon technologies (LCTs) connected. If the time variation of demand or generation due to LCTs is smoother than that of the existing demand, then the increase in losses due to additional loads will be over-estimated and the decrease in losses due to additional generation will be under-estimated.

The under-estimation of losses from measured data can partly be offset using an approximation technique. This utilises the ratio of the estimated mean losses to the actual losses which, for individual switched loads, has been shown to vary linearly with the averaging period if this is longer than the dwell times of the load. With multiple loads, the gradient of this ratio increases as the averaging period reduces. Using high resolution measurements of an LV feeder cable on campus, the same relationship between the estimated and actual losses has been shown to apply for three-phase demand measurements. A better estimate of the actual losses can therefore be obtained from measured data by extrapolating the loss ratio curve towards a zero averaging period. Any remaining error between this improved estimate and the actual losses will still result in the losses being under-estimated, rather than being over-estimated. This technique can be used by DNOs to improve estimates of losses in future measurement work.

The maximum losses (e.g. half-hourly) and calculations of the loss load factor are also dependent on the time resolution of the data. DNOs should also ensure that the averaging period used to calculate the loss load factor is consistent with that used when calculating the peak demand.

The use of arithmetic mean demand data does not affect the customer voltage ranges, provided that the voltage is calculating as an RMS average over a period that is at least as long as the demand data averaging period.

The UK ESQCR regulations do not define an RMS averaging period although the use of a 10 minute RMS voltage metric appears to be common UK industry practice. The interpretation of this standard therefore has a significant impact on the hosting capacity for low carbon technologies. Examples have been shown where the worst-case voltage rise at the highest resolution can be 50% above calculations with a 10 minute RMS average (Chapters 8 and 10).

Studies of network hosting capacities for low carbon technologies assuming EN 50160 are therefore not necessarily applicable to the UK, and vice versa. A change in the UK ESQCR to adopt the probabilistic limits of EN 50160, together with the wider tolerances, would increase the hosting capacity for low carbon technologies on UK networks.

Voltage unbalance is defined in EN 50160 based on 10 minute RMS values and is unaffected by the demand data resolution provided this is no longer than 10 minutes (Chapter 8).

#### 11.3 Harmonics

The impacts of distortion were tested by measuring the harmonic voltage differences along a cable caused by the operation of a load with highly distorted current. Omitting the current harmonics from the voltage drop calculation caused a 1% increase in the worst case voltage drop. This relatively minor difference suggests that an increase in harmonics due to LCTs will not have a significant adverse impact on the customer voltages.

There was a greater impact on the mean losses which were 4.4% higher with the harmonics included than for current at the fundamental frequency. If new LCTs cause current distortion, this will need to be taken into account when considering the thermal capacity ratings of cables.

Simulation studies typically do not allow for harmonic distortion and model only the fundamental frequency. In models where the demand is specified in terms of the active and reactive power of the load, the current harmonics are typically omitted. This gives the minimal error to estimates of the voltage drop, as if the harmonics were absent. However, load currents could be specified using measured RMS data which includes contributions due to the distortion. With this data used to specify the amplitude of a waveform at the fundamental frequency, the worst-case voltage drop was over-estimated by 6%.

Conversely, simulations at the fundamental frequency give a better approximation to the actual losses if the load currents are specified using this RMS data, than if the harmonics are simply omitted.

This suggests that a loss estimates from a simulation at the fundamental frequency could be improved if the RMS current, including distortion, were to be known. The feeder measurements described here have shown that the distortion has a logarithmic relationship with the current at the fundamental frequency, allowing an

approximation to the RMS of the distorted current waveform to be calculated. Measurements of representative feeders could therefore be used to define an empirical relationship that would allow the true RMS to be approximated without direct measurement. This could then allow for improved estimates of the current distortion and of the losses in network simulations where only the fundamental component is known.

#### 11.4 Network models

There is considerable variation between networks with some using PME and others having separate earth conductors. The locations and resistances of ground electrodes are also difficult to determine. Models of LV networks therefore make a number of assumptions and approximations relating to the equivalent circuit of the neutral and ground conductors. In some cases, the ground path is included and the impedance calculated using Carson's equations, but other models assume that the neutral and ground are isolated (Chapter 2). More sophisticated four-wire network models in the literature have been found to either simplify the equivalent circuit or calculate currents in such a way that convergence is uncertain. Previous methods to resolve this have not allowed for nodes with no connection between neutral and ground. In practice, this is the case for most network nodes (Chapter 9).

The conceptual models of the ground currents implied by Carson's equations are different to those used when considering grounding resistances for calculations of earth potential rise. The two conceptual models have been combined to give a new four-wire simulation method that calculates the increase in ground potential at each network node due to current entering the ground at all the other nodes. This revised method models the end effects in the ground current distribution for LV networks with short branches. However, this is still an approximation as it does not model the transition between the current distributions implied by the grounding resistance theory and by Carson's equations. This highlights an inherent limitation arising from the use of multiple 2-dimensional models to represent a 3-dimensional phenomenon.

The new four-wire method extends the conventional forward/backward sweep method such that only the phase conductor currents are calculated in the backward sweep. The neutral and ground currents for all network branches are then calculated in a single matrix operation after each backward sweep. The matrix inversion for this only needs to be solved once at the beginning of the simulation. This new method could be adopted by DNOs to provide an accurate network model allowing for the impacts of neutral to ground connections and of the ground electrodes (Chapter 9).

Using this new method, simulations of an LV feeder with the ground resistances represented in detail are compared to simpler approximations where the neutral is isolated from the ground, or where there is a perfect multi-grounded neutral, giving differences in the voltage drop of up to 10% depending on the voltage metrics used. This suggests that assuming a multi-grounded neutral would under-estimate the voltage drop by 10% or similarly under-estimate the voltage rise by 10% for a generaton-led network (Chapter 10).

For a model that includes PME ground electrodes (but not any other grounding provided at the customer connections), the voltage drop and loss calculations were closely approximated by a model in which the neutral and ground were assumed to be isolated. Unless additional ground electrodes need to be taken into account, or details of the actual grounding configuration are available, DNOs are therefore recommended to use this approximation. Conveniently, this avoids the need to use the Carson's equations.

Alternatively, for a model with multiple ground electrodes at the customer connections, the voltage drop and loss calculations were close to the mean values of a model with the neutral and ground isolated and a model with the neutral assumed to be perfectly grounded. DNOs could use this approximation since detailed ground resistance data is not usually available.

Differences in the assumption used to model the variation of load current with voltage have a greater impact on the voltage drops and losses than the neutral and ground model, giving differences of up to 20%. The results are also dependent on the reference voltage assumed in defining the nominal demand, but this is rarely stated in published studies. This highlights the need for demand data, such as from smart meters, to be recorded together with the voltage.

Some studies omit service cables from the model and assume that the load is directly connected to the LV feeder. The test case studied represents a feeder in an urban area with relatively short service cables. With these cables omitted the model

under-estimated the worst-case voltage drop by 1-2% and the losses by 3%. The worst-case voltage drop in the service cables does not typically occur simultaneously with the worst-case voltage at the nearest cut-out and there would be an over-estimate of up to 4% if the two worst-case voltage drops were simply added.

#### 11.5 Measurement accuracy

Few studies of LV networks compare simulations with real measurement data but this work has included measurements of the voltage differences and impedances on a section of feeder cable in the university campus. Tests with RMS current and voltage logging showed agreement between the resistance of the measured and predicted sequence impedances within 1% at 50 Hz. The reactances of the measured sequence impedances were 10%-20% higher than predicted. The increase in resistance with frequency aligned with predictions from the FE impedance model, validating the simulated effects of eddy currents on the AC resistance (Chapter 5).

To investigate the difference in the reactance further measurements were made in which the time domain current and voltage waveforms were recorded. This was intended to overcome uncertainties in the algorithm used to provide average phase angles together with RMS amplitude data. These measurements again indicated higher reactances than predicted by the FE impedance model with the difference increased to 20% (Chapter 6).

In order to estimate impedances from the waveform data, a method was developed to align the waveforms measured by the two analysers to a common phase reference, enabling the voltage vector difference along the cable to be calculated. This was required as the measurements were made using industry standard power quality analysers with no phase synchronisation. To minimise the impact of phase errors related to the positioning of the Rogowski coil current sensors, the waveforms were aligned using the 9<sup>th</sup> harmonic of the neutral current as a reference.

Although these measurements initially appeared to be a straightforward demonstration of Ohm's law, the work identified a number of practical constraints that apply in general to the use of network measurements in modelling:

- Most significantly, the amplitude and phase angle tolerances of the Rogowski coil sensors limit the accuracy of current measurements. It is not possible to estimate losses from the difference of power input and power output from the cable where these tolerances are similar to the losses as a proportion of the delivered power.
- Vagaries in the measurement equipment can affect the results. In selecting
  power logging equipment, case needs to be taken so that the averaging
  methods applied to the phase angles can be understood and the time
  resolution is not compromised by smoothing filter such as defined by IEC
  61000-4-7. This function distorted the recorded current amplitudes.
- There can be considerable uncertainty over the length of the cables. If accurate models are required, it is recommended that the actual route of cables is traced rather than relying on database records.
- The resistance of protection equipment including fuses and circuit breakers should be considered. This added 3% to the predicted resistance of the measured cable.
- The routing of cables in equipment cabinets increases the reactance, although the affected sections are typically short. The higher reactance is due to the wide spacing between conductors and the proximity of steel frames. For the cable measured here, this added 1%-2% to the predicted reactance.

## 11.6 Summary

This work has proposed new methods that improve the accuracy of low voltage network simulations.

Accurate cable impedance data is a key requirement. This work has demonstrated the improvements can be achieved through finite element modelling of the cable geometry, but also that good approximations can be achieved using simpler analytical methods for results at the fundamental frequency.

The time resolution of the model is also critical to accurate models of the losses. The potential for losses to be under-estimated has been demonstrated and an extrapolation technique to reduce the errors has been proposed. This can be used by DNOs to improve the estimates of losses in LV cables from measured demand data.

Models assuming a multi-grounded neutral do not reflect the reality that only some nodes in the network are grounded, and that there is a non-zero resistance for this connection. A new extension to the forward/backward sweep power-flow technique has been presented such that a network with realistic neutral to ground connectivity can be modelled. This new method brings together the theoretical approaches used for cable impedances with the theory used to analyse the resistances of ground electrodes.

Finally, despite the difficulties noted with practical measurements, it has been demonstrated that the simulated voltage differences, which are higher than expected from published impedance data, agree closely with measured data, validating the impedance modelling techniques.

Although these developments in the modelling techniques cannot overcome the difficulties associated with predicting the demand or in obtaining an accurate database description of the cable network in the ground, this work provides a clearer understanding of the modelling methods such that further uncertainties can be avoided.

## 11.7 Further work

The following areas for further study have been highlighted by this work. The first two areas have been specifically identified here as current research is limited by the availability of measurement data.

<u>Development of the LV modelling tool:</u> The software developed for the studies described in this thesis provides a sophisticated model of the neutral and ground currents, coupled with a high resolution demand model. This tool can be developed for use by DNOs in network design and planning, allowing for detailed characterisation of the expected customer voltage ranges and losses.

<u>Cable impedance modelling</u>: The LV modelling software utilises a highly detailed model of the cable conductor profile in order to estimate the impedance matrix. The FE simulation tool developed for this work can be employed to review the accuracy of cable impedance data used by DNOs. Some cable impedance data has already been published but the work could be extended to include a wider variety of cable design standards.

Addition of LCTs to the model: The work here has intentionally concentrated on the accuracy of the modelling methods, rather than on applying the model to determine a hosting capacity for new LCTs. With a more accurate model now defined, the work can now be extended to evaluate the impact of adding LCTs to LV feeders.

<u>Measurement of grounding resistances</u>: This work has shown that modelling results depend on the grounding assumptions but limited measurement data is available. New measurements in collaboration with DNOs are needed to confirm the grounding resistances at domestic dwellings and at grounded installations such as street lights.

<u>Mesh network configurations</u>: LV networks are typically operated in a radial configuration but neutrals of adjacent feeders are typically inter-connected at link boxes and so form a mesh configuration. Measurements of the current at link boxes are needed to show whether the circulating currents exist, which would invalidate the assumptions in the impedance data used by DNOs for network planning.

<u>Impacts of harmonics</u>: The work here demonstrates that losses in the LV cable can be affected by harmonics. A method has been proposed by which the impact of harmonics on losses can be estimated based on an empirical relationship between the distortion and the fundamental current. DNO measurements are needed so demonstrate whether this relationship holds for multiple feeders, such that measurements on one feeder can be used more generally to predict the impact on losses

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## **APPENDIX A**

### Series impedance of distribution cables with sector-shaped conductors

Andrew J Urquhart, Murray Thomson

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### Andrew J. Urquhart <sup>™</sup>, Murray Thomson

Centre for Renewable Energy Systems Technology (CREST), School of Electronic, Electrical and Systems Engineering, Loughborough University, Loughborough LE11 3TU, UK © E-mail: a.j.urquhart@lboro.ac.uk

**Abstract**: Models of power distribution networks require accurate cable impedance data. For unbalanced networks, both the self-impedances and the mutual impedances are needed. However, published studies use differing approaches to determine cable impedances, leaving uncertainty over the level of detail required. This study compares impedances provided by the manufacturer with those from several analytical methods, showing the impact of modelling the noncircular geometry and of including corrections allowing for the AC resistance. The analysis is compared to results from a freely available finite element (FE) solver where the current distribution is modelled in detail, taking account of eddy currents and the rotation of the cores relative to the neutral due to the cable lay. At 50 Hz, the analytical methods provide a good approximation, but the FE results show that eddy currents affect the impedance at harmonic frequencies. The results also show the impact of including the ground path in the impedance calculation. The current distribution in the ground has a wide cross-sectional area, suggesting that the assumption of a perfect multi-grounded neutral is inappropriate for low voltage networks with short cable lengths.

### 1 Introduction

Low-carbon technologies such as electric vehicles, heat pumps and solar photovoltaic panels are increasingly being connected to the low voltage (LV) distribution network. This may lead to voltages at customer connections being outside of their permitted ranges. Accurate modelling is therefore needed to ensure that cables are correctly sized and to determine whether connections of new technologies can be permitted. This requires accurate series impedance data for the cables.

The uneven allocation of customers to the three phases and the stochastic nature of their demands causes the currents to be highly unbalanced [1]. This unbalance is likely to be worsened by the addition of large single-phase low-carbon technologies. Models of these unbalanced networks require data for both the self-impedance and mutual impedance of the cables. Since the neutral conductor may be grounded at multiple locations along the feeder, the ground path is frequently also included [2].

Increasing levels of harmonics also cause concern due to the need to meet voltage distortion metrics and due to the increased losses and heating effects, particularly in the neutral conductor [3]. Impedance data is therefore also required for harmonic frequencies.

Data from cable manufacturers is not always sufficient to derive the full frequency-dependent matrix with self- and mutual impedance terms [4]. The impact of the ground path is normally excluded as this varies with local ground resistivity and depends on the location of earth electrodes. In response to this lack of data, a number of techniques have previously been used to estimate the impedances.

In some studies, the impedance is specified as a single complex value without defining the mutual impedances [5, 6]. In other cases, the impedance is defined to be the positive sequence term and the zero sequence term is then approximated by applying a scaling factor of between 3 and 5 to the positive sequence [7, 8]. However, voltage calculations for unbalanced demands are sensitive to this uncertain scaling factor [8]. This approximation also provides a symmetrical phase impedance matrix and is equivalent to assuming that the phases are fully transposed,

potentially introducing further errors in voltage and loss calculations [9].

Other studies have utilised software tools such as OpenDSS [10] or DIgSILENT [11]. These tools allow for cable impedances to be determined using Carson's equations [12] where a ground path can be included, typically assuming a perfect multi-grounded neutral with zero voltage between the neutral and the ground. Carson's equations were intended for widely spaced overhead lines, but are also employed for underground cables (with some uncertainty, as in [2]). A modified form of Carson's equations is commonly used to reduce the computational complexity [13] and the errors introduced by the approximation have been found to be negligible for underground cables [14].

These approaches typically assume a uniform current distribution within the cable conductors, neglecting induced eddy currents and so not allowing for the skin effect or the proximity effect with closely spaced conductors. Analytical expressions for the skin effect are available for circular conductors [15] and an analysis has also been developed for the proximity effect in a cable with four sectors [16]. Studies of harmonics have used correction factors for the AC resistance from IEC 60287 [17, 18] although these do not allow for the many variations in the structures of the sector-shaped conductors and cable lay.

Finite element (FE) methods have been developed to provide a more accurate model of the current distribution within the conductors, and modern computing allows these models to include the ground surrounding the cable [19]. A hybrid approach was taken in [20] where the current distribution within the cable was solved using a numerical method, combined with corrections from [15] for the ground path. These techniques may provide a high degree of accuracy, but tend to be complex to apply and published models for specific cable types cannot easily be adapted for new applications.

The use of this wide range of different approaches suggests that there is some uncertainty over the level of detail needed so that impedances are adequately represented. This paper, therefore reviews the underlying theory (Section 2), and evaluates the differences between modelling approaches for the example case of

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waveform cable (Section 3). Four analytical calculation methods are compared, progressively adding more detail to the model (Section 4). The use of a freely available FE solver is introduced (Section 5) and results are compared with those from the analysis (Section 6). The impedance data is available for download [21].

This paper does not include calculation of shunt admittance due to capacitance, because its effect is small relative to that of series impedance, in the context of LV distribution networks at 50 Hz. The phase-to-neutral admittance is calculated in [13] for a single core cable with similar concentric neutral dimensions to the waveform cable considered here as  $60 \,\mu$ S/km, giving currents at 230 V of just 14 mA/km. The phase-to-phase capacitance in sector cables has also been measured at around 75 nF/km, or 24  $\mu$ S/km at 50 Hz [22]. At higher voltages and harmonic frequencies, however, capacitance does become significant. It may be estimated assuming circular conductors and uniform charge density or, for greater accuracy, FE models using similar concepts to those presented in this paper could be developed.

### 2 Impedance definitions

### 2.1 Conductor impedances

The cable can be modelled as a set of conductors with associated self- and mutual impedances, as shown in Fig. 1. This shows two conductors i and j and a ground conductor g. The equivalent circuit is shorted together at one end to represent only the voltage drops due to the cable (excluding those due to the loads).

By assuming a uniform current distribution (neglecting eddy currents), the conductor resistances can be calculated based on their cross-sectional area and resistivity. The inductances include contributions due to the flux linkage that is internal to the conductor, and also due to the external flux linkage. The external flux linkage can be obtained by integrating the magnetic field from the conductor surface at radius R to a point P. In the case of the FE model described below in Section 5, this represents the distance to the boundary of the finite solution area.

Following the established approach as outlined by Glover *et al.* [23], if the magnetic field is considered to a finite distance P, the total flux linkage for conductor *i* with a total of  $N_{\text{cond}}$  conductors is

$$\lambda_{iP} = \frac{\mu_0}{2\pi} \sum_{j=1}^{N_{\text{cond}}} I_j \ln \frac{D_{Pj}}{D_{ij}}$$
(1)

where  $D_{Pj}$  is the distance from conductor *j* to point *P* and  $D_{ij}$  is the geometric mean distance (GMD) between conductors *i* and *j*. For *i* = *j*, the distance  $D_{ii}$  is the geometric mean radius (GMR) of conductor *i*. The GMR of a circular conductor is given by  $D_{ii} = e^{-1/4}R$  where *R* is the physical radius.

The GMD can be determined by considering each conductor to be formed from a set of sub-conductors, each having uniform current density and carrying an equal share of the total current. The GMD can then be calculated as

$$D_{ij} = e^{\left(\sum_{m=1}^{N_{\text{sub},i}} \sum_{k=1}^{N_{\text{sub},i}} \ln d_{mk}\right) / \left(N_{\text{sub},i} N_{\text{sub},i}\right)}$$
(2)

where  $N_{\text{sub},i}$  is the number of sub-conductors in conductor *i*. When



Fig. 1 Conductor impedance model

k=m, distance  $d_{km}$  is the GMR of one sub-conductor. Otherwise when  $k \neq m$ , distance  $d_{mk}$  is strictly defined as the GMD between the sub-conductors [24]. However, this gives a recursive definition and the centre-to-centre distance between the sub-conductors is used instead. This gives a negligible error provided that the sub-conductors are small compared to the distances between them. For a high density of sub-conductors, the formulation of (2) using the arithmetic mean of logarithms is less subject to numerical rounding errors than the direct calculation of the geometrical mean as in [23].

### 2.2 Circuit impedances

It is generally assumed that the conductors belong to circuits where the sum of currents in the cable and the ground is zero. As the distance to point P tends to infinity, the total magnetic field then tends to zero [23]. The terms in (1) relating to distance  $D_{Pj}$  then cancel and the total flux linkage is

$$\lambda_{iP} = \mu_0 / (2\pi) \sum_{m=1}^{M} I_m \ln(1/D_{im})$$
(3)

Taking the self- or mutual inductance of a conductor as the component of (3) proportional to the corresponding current, the conductor impedances can be expressed as

$$\bar{z}_{ii} = r_i + j(\omega\mu_0/(2\pi)) \cdot \ln(1/D_{ii})$$
(4)

$$\bar{z}_{ij} = \mathbf{j}(\omega\mu_0/(2\pi)) \cdot \ln\left(1/D_{ij}\right)$$
(5)

where  $r_i$  is the resistance of conductor *i* in  $\Omega/m$ , and  $\omega$  is the angular frequency. Although (4) and (5) appear to be properties of individual conductors, they are more correctly contributions from each conductor to the total impedance of a circuit.

The circuit impedance is commonly represented as a single parameter, following the method of [13]. Referring to Fig. 1, Kirchoff's voltage law for  $V_{ig}$  gives

$$V_{ig} = \bar{z}_{ii}I_i + \bar{z}_{ij}I_j + \bar{z}_{ig}I_g - \bar{z}_{gi}I_i - \bar{z}_{gj}I_j - \bar{z}_{gg}I_g$$
(6)

With the sum of currents equal to zero such that  $I_g = -(I_i + I_j)$ , this can be written

$$V_{ig} = \hat{z}_{ii}I_i + \hat{z}_{ij}I_j \tag{7}$$

in which the circuit impedances are defined as

$$\hat{z}_{ii} = \bar{z}_{ii} - \bar{z}_{gi} - \bar{z}_{ig} + \bar{z}_{gg} \tag{8}$$

$$\hat{z}_{ij} = \bar{z}_{ij} - \bar{z}_{gj} - \bar{z}_{ig} + \bar{z}_{gg}$$
 (9)

For circuits with a ground return path where the distance to the equivalent ground conductor is unknown, the circuit impedances in  $\Omega/m$  can be calculated using Carson's equations. Using the modified equations from [13] in SI units, these are

$$\hat{z}_{ii} = r_i + \mu_0 \omega/8 + j\mu_0 \omega/(2\pi) \cdot \ln\left(658.9/\left(D_{ii}\sqrt{f/\rho}\right)\right)$$
(10)

$$\hat{z}_{ij} = \mu_0 \omega/8 + j\mu_0 \omega/(2\pi) \cdot \ln\left(658.9/\left(D_{ij}\sqrt{f/\rho}\right)\right)$$
(11)

where  $\rho$  is the ground resistivity in  $\Omega$ m.

These circuit impedances are dependent on the assumption noted above that the total current sums to zero, but this may not be strictly accurate in meshed networks where feeders are looped. A mesh configuration can arise in LV networks where the neutrals remain permanently connected at link boxes, even if the network is considered radial with regard to the phase conductors.

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Table 1 Parameters for 3-core  $95\,\text{mm}^2$  cable [25, 26], with DC resistances are quoted at  $20^\circ\text{C}$ 

sector area, a	92.14 mm <sup>2</sup>	insulation thickness, t	1.1 mm
sector radius, D	10.24 11111		
corner radius, c	1.02 mm	neutral strand radius, R <sub>S</sub>	0.79 mm
sector width, w	15.76 mm	neutral radius, <i>R</i> <sub>N</sub>	14.36 mm
sector angle, $\phi$	119°	neutral resistance	0.32 Ω/km
sector depth, s	9.14 mm	outer radius, <i>R</i> o	17.25 mm
sector lay length	800 mm	neutral lay length	>250 mm
sector resistance	0.32 Ω/km		

#### 2.3 Phase and sequence impedances

Where the ground path is included in the impedance matrix, the ground currents depend on the earthing method. The neutral and ground may be isolated (for networks with independent earths), or may be connected at a number of earth electrodes (as with protective multiple earthing). Typically the impedance of these grounding connections is high compared to that of the cable [2].

A multi-grounded neutral can be modelled by assuming a short circuit between the neutral and the ground at each node [13]. The Kron reduction can then be applied to the circuit impedance matrix, to give a  $3 \times 3$  phase impedance matrix  $z_{abc}$  [13]. This can be transformed to give a  $3 \times 3$  sequence impedance matrix  $z_{012}$ . For a cable with rotational symmetry between phases, the impedances are then fully represented by the zero and positive sequence impedances.

### 3 Waveform cable

Impedance calculation methods are compared here for waveform cables, designed for underground use in LV networks. The cable consists of either 3 or 4 aluminium sector conductors surrounded by copper concentric neutral/earth conductor, as described in Table 1 and Fig. 2.

The nominal cable design is standardised [25–27], but dimensions such as the insulation thickness may be greater than the specified minimum to allow for manufacturing tolerances. The standards have evolved over the years and newer editions require fewer copper strands, but with increased diameter to maintain the overall resistance. Installed cables (possibly several decades in age) may therefore differ from those in current product datasheets.

The sector conductors have a cable lay, rotating about the central axis of [27]. The lay length is long in comparison to the width and radial offset of the sector and so the total conductor length is approximately equal to that of the cable. The neutral strands have a shorter lay length with an approximately sinusoidal waveform (rather than a continuously advancing rotation). Along the length of the cable, the sector cores therefore rotate relative to the neutral strands.

Impedance data for the cable is shown in Table 2, where the manufacturer's data is interpreted as a contribution from each conductor to the circuit impedance. In this example, the impedance of the neutral conductor is assumed to equal that of the phase conductor (typically the case for the resistance, although not necessarily so for the reactance). The table shows the implied impedances from (8) and (9) for circuits with the neutral as the



**Fig. 2** Sample cross-section of 3-core 95 mm<sup>2</sup> cable and dimensions

Table 2 Impedances based on manufacturer's data for 3-core 95  $\mbox{mm}^2$  cable

Manufacturer provided data: DC resistance at 20°C $r = 0.32 \Omega/k$ $x = 0.0735$ kmapproximate reactance at 50 Hz $x = 0.0735$ km					
Impl	ied conductor in	npedance matrix	k, Ω/km		
	∑0.32 + j0.0735	0	0	0 7	
_	0	0.32 + j0.0735	0	0	
<b>z</b> =	0	Ó	0.32 + j0.0735	0	
	0	0	0	0.32 + j0.0735	
Circi grou	uit impedance m ınd path, Ω/km	atrix with neutra	al as return path	, no	
	∇0.64 + j0.147	0 + j0.0735	0 + i0.0735		

0 + j0.0735

0.64 + j0.147

Phase impedance matrix with no ground path  $\boldsymbol{z}_{abc} = \hat{\boldsymbol{z}}$ 

0.64 + j0.147

0 + j0.0735

0 + i0.0735

0 + i0.0735

Sequence impedance matrix with neutral as return path, no ground path,  $\Omega/km$ 

	☐ 1.28 + j0.294	0	0
$z_{012} =$	0	0.32 + j0.0735	0
	Lo	0	0.32 + j0.0735

return path (the cable is isolated from the ground). The terms here are double the individual conductor impedances, allowing for the circuit loop through the sector core and neutral return. In the corresponding sequence impedance matrix, the positive sequence term is then equal to the individual conductor impedance. The zero sequence term is exactly four times the positive sequence.

### 4 Analytical methods

### 4.1 Approximating sector shapes as circular

The impedances derived from the manufacturer's data are now compared with impedances from several analytical techniques. A simple estimate of the impedance can be made with the conductors approximated as being circular [28]. The cable lay is neglected in this approach.

The GMR of the sector is assumed to be that of a circle with the same area a

$$D_{ii} = e^{-1/4} \sqrt{a/\pi}$$
 (12)

The distances between sectors are calculated relative to a nominal centre at distance m from the cable axis, as shown in Fig. 2 given by

$$m = b - s/2 + \delta \tag{13}$$

The centre of rotation of the sector arcs may be displaced slightly from the centre axis of the cable assembly (as discussed in [21]). This increases the gaps between sectors to allow for the thickness of their insulation sleeves. The offset  $\delta$  is given by

$$\delta \cong t / \sin(\theta_{12}/2) - b + s + c(1/\sin(\phi/2) - 1)$$
(14)

where  $\theta_{12}$  is the angular separation between two adjacent sectors (120° for a 3-core cable).

The GMD between sectors is approximated by the distance between their nominal centres, given by

$$D_{ij} = m\sqrt{(1 - \cos\theta_{ij})^2 + \sin^2\theta_{ij}}$$
(15)

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The GMD between a sector and the neutral can be found from (2), where the strands comprise the sub-conductors in the neutral and with the sector represented as a single conductor at distance m from the cable centre.

The GMR of the neutral could be also found from (2), but for a ring of circular conductors is obtained more easily from

$$D_{nn} = \sqrt[N_{\rm S}]{R'_{\rm S} \cdot N_{\rm S} \cdot R_{\rm N}^{N_{\rm S}-1}}$$
(16)

where  $R'_{\rm S} = e^{-1/4} R_{\rm S}$  is the GMR of one strand [13].

The impedances for circuits with a neutral return path can then be determined by applying (4) and (5), giving the results as shown in Table 3.

### 4.2 Modelling sectors using multiple sub-conductors

The results above are now compared with those for a more detailed model of the sector geometry, in which the sector shapes are represented by a set of sub-conductors in parallel. This provides an improved estimate of the GMR and GMD parameters without needing the approximations for a nominal centre point of the sector shape. The current distribution is still assumed to be uniform, both within each sub-conductor and across the sector shape.

The method used to define the outline of the sector shape is described in detail in [21]. A rectangular grid of sub-conductors is defined within this outline, as shown in Fig. 3. Each sub-conductor is assumed to be a square with a GMR of 0.447 times the side length [24]. The neutral strands do not require further sub-division since they are circular and their GMR is already known.

The GMR and GMD parameters of the combined sector and neutral conductors can then be determined using (2). This provides a  $4 \times 4$  matrix  $D_{ij}$ , equivalent to that derived in Section 4.1. Compared to the approximation using circular conductors, the sub-conductor method gives a slight increase in the GMR of a sector (from 4.2 to 4.4 mm) and also an increase in the GMD between sectors (from 10.4 to 11.6 mm).

The circuit equations are applied as above, to derive the sequence impedances included in Table 3. Compared to the simpler method of Section 4.1, there is a 14% decrease in the zero sequence reactance and a 7% increase in the positive sequence reactance. The resistances are unaffected since the conductor areas are equal in both cases and the current distribution is still assumed to be uniform. The simple method of approximating the sectors as being circular gives a useful estimate, but the more detailed representation using sub-conductors is assumed to be more accurate. Both analytical

**Table 3** Comparison of impedances from analysis with manufacturer's data for circuits with neutral as return path

Calculation technique	Positive sequence z <sub>11</sub> , Ω/km	Zero sequence <i>z</i> <sub>00</sub> , Ω/km	
From manufacturer's data	0.32 + j0.0735	1.28 + j0.294	
Analysis with sectors approximated as circles (Section 4.1)	0.32 + j0.057	1.28 + j0.116	
Sectors modelled using sub-conductors (Section 4.2)	0.32 + j0.061	1.28 + j0.100	
Adding corrections for AC resistance (Section 4.3)	0.323 + j0.061	1.283 + j0.100	
Adding ground path in parallel with neutral (Section 4.4)	0.323 + j0.061	1.101 + j0.445	
FE model for circuits with neutral and ground in parallel (Section 6.1)	0.322 + j0.060	1.096 + j0.450	



**Fig. 3** Sub-conductor model of 3-core 95 mm<sup>2</sup> cable

methods suggest lower reactances than in the manufacturer's data, with the zero sequence reactance approximately halved.

#### 4.3 Analytical corrections for AC resistance

Standard IEC 60287 provides a means to determine the current ratings of cables and so includes methods to calculate the AC resistance of cables [29]. Although the equations are developed for circular conductors, compensation factors are included to allow for asymmetry in sector shapes. This method does not define the current distribution associated with the correction factors and so the reactance calculation here is still based on a uniform current distribution as above.

The standard defines the total AC resistance as

$$r_{i,\text{total}} = r_{i,\text{DC}}(1 + y_{\text{S}} + y_{\text{P}})(1 + \lambda_1)$$
(17)

where  $y_{\rm S}$  allows for the skin effect,  $y_{\rm P}$  allows for the proximity effect and  $\lambda_1$  allows for the resistive effect of losses due to eddy currents in the sheath.

The results in Table 3 for the 3-core  $95 \text{ mm}^2$  cable show that the corrections make a minor difference to the impedance at 50 Hz.

The comparison has been repeated for the 3-core 300 mm<sup>2</sup> cable where the conductor dimensions are larger compared to the skin depth. Including the IEC 60287 corrections increase the positive sequence resistance by 6% (from 0.1 to 0.106  $\Omega$ /km). The corresponding parameters are then  $y_{\rm S} = 0.008$ ,  $y_{\rm P} = 0.014$  and  $\lambda_1 =$ 0.04 so the sheath losses give the greatest contribution.

### 4.4 Including the ground path

The impedances are now calculated using Carson's equations (10) and (11) to include the ground in parallel with the neutral. The resulting sequence impedances are shown in Table 3.

Adding the ground in parallel with the neutral does not affect the positive sequence impedance (with no unbalance current), but has reduced the zero sequence resistance by 14% and increased the zero sequence reactance by a factor of 4.

These results were also repeated for comparison using the 'full' Carson's equations [14], giving differences in the circuit impedances of <0.25% at 50 Hz, and <2% at 3 kHz. The differences are similar for all of the self-impedances and all of the mutual impedances, such that the resulting zero sequence impedances are almost identical with the full and modified Carson's equations.

### 5 FE model

The impedances obtained using the analytical techniques are now compared with results from FE analysis obtained using the FE method magnetics (FEMM) software [30]. This is freely available, such that it is possible for the results here to be replicated or extended in other work. The current distribution and magnetic field are solved for a planar cross-section of the conductor geometry, giving impedance results per unit length, and assuming an infinite longitudinal projection.

The geometry for the waveform cable was entered into FEMM using the sector outlines as described in Section 4.2, and drawing



**Fig. 4** *FEMM model showing of 3-core 95 mm<sup>2</sup> waveform cable a* Conductors with mesh

b Ground conductor and boundary conditions

the neutral strands as circles. This defines a set of conducting regions placed within a non-conductive outer circle defined by the radius  $R_0$  of the cable, as shown in Fig. 4*a*. The voltage is constant across each conductor since the software only models longitudinal currents (a valid approximation for power frequencies).

Two further semi-circular regions were defined to represent the ground conductor surrounding the cable and the air above the ground surface, as shown in Fig. 4b. The ground resistivity is therefore constant over the planar cross-section. A boundary condition with magnetic vector potential A = 0 is applied at the edge of the solution space, effectively truncating the magnetic field at the boundary radius. A magnetic permeability of 1 was assumed throughout.

A separate simulation run was configured for each solution frequency and for each conductor. There are therefore 34 simulations for the 3-core 95 mm<sup>2</sup> cable (3 sectors, 30 neutral strands and the ground). In each run, a mean current of 1 A was applied to one 'active' conductor with the other conductors having a mean current of zero. This allows for eddy currents, but prevents currents from circulating between conductors (maintaining an open circuit at one end of the circuit, as shown in Fig. 1). Using default mesh parameters, the set of 34 simulation runs required less than 10 min.

For each run, the solver provides the self-impedance of the 'active' conductor and also the mutual inductance with each of the others. The mutual impedance has a complex value in which the imaginary term is negative and represents a resistive component in the mutual impedance, allowing for losses due to induced eddy currents.

It is again assumed that the dimensions of the cable lays are long relative to the conductor widths and spacings so that it is valid to model the conductors as being longitudinal when calculating the flux linkage. It has also been found that the mutual impedance between conductors and the ground is independent of their orientation.

The impact of the cable lay on the eddy currents needs further consideration since the sectors and neutral rotate relative to each other. At one position along the cable length, a strand will be adjacent to a particular sector conductor, but further along the cable it will be on the opposite side of the circle. Over a length of a few metres (and provided that the lay lengths are not exact multiples of each other), each strand has an approximately equal probability of being at any angle relative to the sector cores.

This transposing effect can be modelled by averaging the strand conductor impedances over the set of  $N_{\rm S}$  strand positions around the cable. The mean self- or mutual conductor impedance for a strand is then

$$\overline{\overline{z}}_{ij} = \frac{1}{N_{\rm S}} \sum_{k=0}^{N_{\rm S}-1} \overline{z}_{i+k,j+k \; (\mathrm{mod} \; N_{\rm S})}$$
(18)

and the mutual impedance between sector i and strand j is

$$\bar{\bar{z}}_{ij} = \frac{1}{N_{\rm S}} \sum_{k=0}^{N_{\rm S}-1} \bar{z}_{i,j+k \; ({\rm mod} \; N_{\rm S})}.$$
(19)



**Fig. 5** Positive  $(z_{11})$  and zero  $(z_{00})$  sequence impedances for 3-core 95 and 300 mm<sup>2</sup> cables

These averaged conductor impedances are then used to calculate the impedances of circuits with a conductor and ground return according to (8) and (9). As in Section 2.2, the impact of the magnetic field truncation at the simulation boundary is then cancelled out in the resulting circuit impedances. This  $4 \times 4$  matrix is then reduced to a  $3 \times 3$  sequence impedance matrix, assuming a multi-grounded neutral as in Section 2.3.

### 6 Simulation results

### 6.1 Waveform cable impedances

The model was configured with the cable located 1 m below the ground surface, using ground resistivity of 100  $\Omega$ m, and with a 3 km simulation boundary radius. Using the DC resistance and cross-sectional area from Table 1, the conductor conductivity was defined as 33.9 MS/m for the aluminium sector and 53.1 MS/m for the copper neutral.

The FE simulations are compared with the analysis of Section 4.4, showing the impact of including a detailed representation of the current distribution in the cable. The ground is also modelled as a physical conductor rather than being included through analysis of the fields (as in Carson's equations). At 50 Hz, the sequence impedances from the FE simulations for the 95 mm<sup>2</sup> cable are within 1% of those obtained from the analysis, as shown in Table 3. At higher frequencies, the FE simulation results diverge from the analytical results, as shown in Fig. 5. At 450 Hz, the positive sequence impedance from the FE simulation has 16% lower resistance and 10% lower for resistance and 6% lower for reactance.

Fig. 5 also shows the same comparison for the  $300 \text{ mm}^2$  cable size where eddy currents would be expected to have greater impact. In this case, the results agree to within 2% at 50 Hz, but the positive sequence at 450 Hz has 33% lower resistance and reactance. The corresponding zero sequence results are 6% lower for resistance and 28% lower for reactance.

The results from the FE simulations support the conclusion that Carson's equations are valid for use with underground cables. Although differences arise between the FE results and the analysis results at higher frequencies, these are attributed to the limitations

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Fig. 6 Relative current density from FEMM results, 350 Hz, 3-core 95 mm<sup>2</sup> cable

a Positive sequence

b Zero sequence

of the AC resistance correction method which does not model the non-uniform current distribution within the cable conductors.

### 6.2 Three-phase current distributions

The FE model was re-configured with currents applied to each phase conductor so that the current distributions of the sequence modes could be observed. The neutral and ground conductors were configured in parallel to model a multi-grounded neutral. At 50 Hz, the current distribution was close to uniform and so the plots are shown for 350 Hz where the impact of eddy currents is more clearly visible.

Fig. 6a shows a FEMM plot of the current density for the positive sequence, where balanced three-phase currents of 1 A were applied to the sector conductors. The current density in the sectors is greatest on the edges orientated towards the sector leading in phase and the lowest on the opposite edge. This gives a higher resistance than for a uniform distribution.

The plot also shows eddy currents in the concentric neutral strands. These have a similar magnitude to those in the sectors and a phase angle that varies around the circle. However, the plot from a single cross-section represents the eddy currents that would occur if the strands remained at the same angle relative to the sectors for the full length of the cable. In practice, the currents in the strands are transposed as the cores rotate relative to the neutral due to the cable lay. This highlights a risk with FE models where this transposing effect is not taken into account.

A similar plot for the zero sequence at 350 Hz is shown in Fig. 6b. For this case, the model was configured with  $1 \angle 0^\circ$  A in each sector and -3 A in the neutral and the ground. The current density in the sectors is slightly higher towards the outer edges, as expected due to the proximity effect. At 350 Hz, the current within the ground is negligible and almost all of the current returns through the neutral.

### 6.3 Ground conductor current distribution

For the simulations described above, the boundary radius was selected so that the ground conductor truncation did not significantly affect the results. As shown in Fig. 4b, the boundary limits the cross-sectional area of the ground, changing its DC resistance. This is a different concern with that noted in Section 2 where the truncation of the magnetic field is cancelled out when the currents inside the boundary sum to zero.

Fig. 7 shows sequence impedances for varying boundary radius. The positive sequence impedance is unaffected since this has no unbalance current, but a radius of at least 1 km is required for the zero sequence impedance at 50 Hz to converge. At higher frequencies, a lower radius can be used, as the increased proximity effect causes currents flowing in opposite directions (outwards via the cable and returning via the ground) to have a higher current density at closer separations.

A l km boundary radius may seem large, but is consistent with the dimensions implied by Carson's equations, in which the self-impedance term  $\mu_0 \omega/8$  represents the additional resistance of a

circuit with a ground return. At 50 Hz, with ground resistivity of 100  $\Omega$ m, an equivalent resistance would be provided by a semi-circular conductor with a radius of 1136 m. This suggests that the ground current for short LV cables would be subject to significant end effects. Where unbalance current enters the ground at earth electrodes, the current density is much greater, and the ground path resistance much higher than in the infinite longitudinal projection, giving the high grounding resistances noted in [2]. In the absence of a three-dimensional network model, this suggests that it would better to approximate the neutral as being isolated from the ground, rather than to apply Carson's equations and then a zero impedance connection between the neutral and the ground.

### 6.4 Impact of ground resistivity and cable depth

To determine the sensitivity of the FE model to ground resistivity, the simulations have been repeated with resistivity increased or decreased by a factor of 10, as shown in Table 4. A higher boundary radius of 30 km was used here to allow for the higher ground resistivity. The zero sequence impedance varies from 3 to 4% for each order of magnitude change, and so is relatively insensitive to variations in the ground resistivity.

The simulations were repeated with the cable position varied between 2 m below ground and 2 m above, with the results remaining within 0.1% of the values shown in Table 4. Since the current distribution within the ground reduces gradually over hundreds of metres, a relatively small difference in the cable position has minimal impact.

### 7 Conclusions

Accurate cable impedance data is needed to evaluate the impacts of connecting new low carbon technologies to LV networks. Published studies have adopted a range of impedance models with differing approaches used to represent the geometry of the conductors and the current distribution within them. This paper compares the manufacturer's data for 3-core waveform cable with impedances calculated using analytical techniques, showing the differences that arise as the complexity of the model is increased.

In the simplest analytical model, the sector shapes were approximated as being circular with uniform current distribution and no connection to the ground. Representing the sector shapes more accurately by using a grid of sub-conductors gave a difference in the reactance at 50 Hz of 14%, suggesting that the more detailed geometry representation is needed for more accurate results. Including the AC resistance corrections from IEC 60287 had minimal impact at 50 Hz for the 95 mm<sup>2</sup> cable, although a 6% difference was noted for the 300 mm<sup>2</sup> cable size. The reactance results from all of the analytical techniques were lower than those indicated by the manufacturer. The addition of the ground path in parallel with the neutral affected the zero sequence impedance



**Fig. 7** Positive  $(z_{11})$  and zero  $(z_{00})$  sequence impedances versus boundary radius, 3-core 95 mm<sup>2</sup> cable, ground resistivity of 100  $\Omega$ m, cable located 1 m below the surface

 Table 4
 Zero sequence impedance at 50 Hz for 3-core 95 mm<sup>2</sup> cable

Ground resistivity, $\Omega m$	Zero sequence impedance, $\Omega/km$
10	1.062 + 0.469j
1000	1.122 + 0.430j

with a 14% reduction in the resistance and a 4x increase in the reactance.

The use of a freely-available FE solver (FEMM) is then described as an accurate means of allowing for the sector-shape geometry and for the non-uniform current distribution due to induced eddy currents. The results at 50 Hz matched those from the analytical methods to within 1% so there was little to be gained from the more complex FE approach. However, at harmonic frequencies, the FE results diverge from the results with the IEC 60287 corrections, with differences at 450 Hz of 16% for the 95 mm<sup>2</sup> cable and 33% for the 300 mm<sup>2</sup> cable. The use of the FE model would therefore be recommended where accurate impedance data is required at harmonic frequencies. The results are available for download from [21].

At 50 Hz, the FE results are consistent with those obtained from the modified form of Carson's equations, providing confidence in their use for underground cables. The FE results were relatively insensitive to variations in ground resistivity and were unaffected by likely variations in the depth of the cable within the ground.

However, both the FE model and Carson's equations assume an infinite longitudinal projection of the current distribution. Examination of the implied current distribution suggests that the ground path will be subject to much higher resistance where unbalance current enters the ground at earthing electrodes. For short LV cables, the grounding resistance would be significantly higher than that of the neutral conductor. This suggests that modelling the neutral as isolated from the ground would be a better approximation than using Carson's equations and then assuming a perfect short circuit between the neutral and the ground when applying the Kron reduction.

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### **APPENDIX B**

Impacts of demand data time resolution on estimates of distribution system energy losses

Andrew J Urquhart, Murray Thomson

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# Impacts of Demand Data Time Resolution on Estimates of Distribution System Energy Losses

Andrew J. Urquhart and Murray Thomson

Abstract—Copper losses in low voltage distribution circuits are a significant proportion of total energy losses and contribute to higher customer costs and carbon emissions. These losses can be evaluated using network models with customer demand data. This paper considers the under-estimation of copper losses when the spiky characteristics of real customer demands are smoothed by arithmetic mean averaging. This is investigated through simulation and by analysis of measured data. The mean losses in cables and equipment supplying a single dwelling estimated from half-hourly data were found to have significant errors of 40%, compared to calculations using high resolution data. Similar errors were found in estimates of peak thermal loading over a half-hour period, with significant variation between results for each customer. The errors reduce as the demand is aggregated, with mean losses for a group of 22 dwellings under-estimated by 7% using half-hourly data. This paper investigates the relationship between the demand data time resolution and errors in the estimated losses. Recommendations are then provided for the time resolution to be used in future measurements and simulation studies. A linear extrapolation technique is also presented whereby errors due to the use of averaged demand data can be reduced.

*Index Terms*—Average demand, distribution system, losses, low-voltage networks, time resolution.

### I. INTRODUCTION

**E** NERGY losses in the distribution network contribute to the greenhouse gas emissions associated with the electricity supply and also to the costs paid by the customer. In the U.K., distribution losses are between 5% to 6% of the energy delivered [1], [2], adding around 7% to domestic customer bills [3]. Distribution network operators (DNOs) are under pressure to minimize losses and receive financial rewards or penalties via the Losses Incentive Mechanism according to their performance [4].

These losses are due to both technical and non-technical factors, with the technical losses including a fixed "iron loss" and a "copper loss" component that varies with the load. The copper losses from the low voltage (LV) network are approximately one quarter of the distribution losses [1], [2].

The authors are with the Centre for Renewable Energy Systems Technology (CREST), School of Electronic, Electrical and Systems Engineering, Loughborough University, Loughborough, LE11 3TU, U.K. (e-mail: A.J.Urquhart@lboro.ac.uk; M.Thomson@lboro.ac.uk).

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With new low carbon technologies (LCTs) for the electrification of heat and transport, there is an increasing likelihood that LV network infrastructure will need to be reinforced and new cables installed [5]. Distributed generation may in some cases reduce losses, but could also cause them to increase [6]. To minimize losses, the use of oversized cables is planned for new installations in areas where high penetrations of new LCTs are predicted [3]. The use of larger cables is also considered in [7] where the increased capital costs are compared to the lower cost of losses over the cable lifetime.

An accurate evaluation of losses is needed to understand the relative proportion of the different loss mechanisms. However, network models can be subject to errors if they use using averaged demand data that does not accurately represent the spiky nature of real customer demand profiles. This source of error arises since copper losses vary with the square of the current and so will be under-estimated if calculated based on the arithmetic mean current.

Taking a simple example, the losses to supply a 1 kW load for 10 s, are one tenth of the losses caused by supplying a 10 kW load for 1 s. It is important to recognize that the long thermal time constant associated with a cable or transformer will help to smooth the associated temperature rise but this does not remove the risk of under-estimating the total amount of energy lost. This concern has been noted previously where errors of over 20% were noted for the losses in the outer branches of LV feeders, if estimated using demand data averaged over 15 min [8]. Further investigation is required into the dependency of this error on the time resolution and the degree of demand aggregation.

In the absence of detailed demand data for individual networks, average losses can be estimated by multiplying the losses for the peak demand by a loss load factor [9], [10]. This factor may be calculated from an average demand curve, typically with an hourly resolution. The loss load factor may also be estimated from an empirical relationship with the load factor [9], [11]. Such approaches using standardized loss metrics have been categorized as "top-down" methods [12].

In the alternative "bottom-up" approach, losses are calculated for the currents and impedances in the network. This requires a full network model and inputs to describe the demand and generation for individual customers. The demand might be defined by half-hourly "average" profiles [13], possibly in combination with randomization techniques to create different profiles for each customer [14], [15]. Other studies have used shorter time steps, as in [16] where a 1 minute resolution was selected so that the impact of losses could be more accurately represented. A resolution of 5 min was recommended in [17] for representation of mean customer voltages and utilization of the main

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segment of the feeder cable. This study, among others, uses a domestic demand with a 1-min resolution, based on statistics for the occupancy of each house and for the use of individual appliances [18]. Demand data from smart meters typically has a resolution between 15 min and 1 h [19].

High resolution data is increasingly available from network monitoring projects and modern equipment can record demand and RMS current at better than 1 second resolution [20]. Losses calculated from RMS current data avoid the under-estimation problem noted above. However, this only provides the losses for cables carrying the measured currents, not for those throughout the network, and so network loss calculations remain dependent on measured or estimated demand data.

This paper investigates the relationship between the demand data time resolution (or averaging period) and the extent to which copper losses are under-estimated. This provides a rationale for selecting the time resolution in future energy loss studies. A terminology has been adopted so that, for example, "30-min data" refers to the arithmetic mean of the demand over 30 min. The "theoretical loss" is the loss that would be calculated given perfect knowledge of the variations in the demand.

We define the "loss ratio" as the ratio of the losses estimated from mean averaged data to the theoretical loss. This ratio may be applied as a correction factor to improve the estimation of copper losses when using mean-averaged demand data. Considering an individual single-phase dwelling, the loss ratio would apply to losses in the line and neutral conductors of service cables and to any power management equipment such as might be installed for voltage optimization. The loss ratio also applies to groupings of consumers on a common single-phase circuit, including single-phase distribution transformers and the line and neutral conductors of any single-phase mains.

On a three-phase LV network, the loss ratio presented below may also be applied to losses calculated for the phase conductors, but a different ratio would apply to the neutral. The currents in the neutral are dependent not only on the time varying demand in each phase, but also on the coincidence of these variations with the currents in the other phases. To represent this accurately requires high resolution data for both the angle and amplitude of the phase currents. Whilst acknowledging the importance of the three-phase case, this paper is confined to the single phase case in order to present the underlying concepts as clearly as possible. A future paper will consider the combined impacts on line and neutral conductors of three-phase systems.

### II. ANALYTICAL MODEL

### A. Loss Ratio for Step Changes in Demand

Assuming an approximately constant voltage, the demand data can be scaled to provide an estimate of the mean current. Fig. 1 shows a measurement period  $t_{\rm M}$  in which the current experiences a step change from  $I_1$  to  $I_2$ . The change occurs at a time interval  $t_1$  after the start. With a nominal resistance R, the theoretical losses (without any impact due to averaging) are given by the sum of the losses for each current level:

$$e_{\rm R} = I_1^2 R \cdot t_1 + I_2^2 R \cdot (t_{\rm M} - t_1). \tag{1}$$



Fig. 1. Step change in current within an averaging period.

If the demand is represented by an arithmetic mean average for period  $t_M$ , (again with constant voltage), the arithmetic mean current will be as indicated by the shading on Fig. 1. The time-weighted mean current is

$$I_{\rm A} = \frac{(I_1 t_1 + I_2 (t_{\rm M} - t_1))}{t_{\rm M}}.$$
 (2)

The loss from the mean averaged data is therefore

$$e_{\rm A} = I_{\rm A}^2 R \cdot t_{\rm M} = \frac{(I_1 t_1 + I_2 (t_{\rm M} - t_1))^2 R}{t_{\rm M}}.$$
 (3)

The step change can be described in terms of a time ratio  $\beta = t_1/t_M$  such that current  $I_1$  occurs as a proportion  $\beta$  of the measurement time, and also by the relative magnitude of the two current states  $x = I_2/I_1$ . Re-writing (1) and (3)

$$e_{\rm R} = I_1^2 R \left(\beta + x^2 (1 - \beta)\right) t_{\rm M} \tag{4}$$

$$e_{\rm A} = I_1^2 R \left(\beta + x(1-\beta)\right)^2 t_{\rm M}.$$
 (5)

The loss based on the mean averaged demand can be expressed relative to the theoretical loss  $r_A = e_A/e_R$ :

$$r_{\rm A} = \frac{(\beta + x(1 - \beta))^2}{(\beta + x^2(1 - \beta))}.$$
(6)

It can be seen that, if  $x \approx 1$ , then  $r_A \approx 1$ . This represents the case with a smooth profile, such as for the aggregated demand of many customers. In this case, changes in the current due to one appliance are small compared to the total demand, and so the averaging effects introduce a low error.

Conversely, if x = 0 then  $r_A$  is proportional to  $\beta$ . This could represent a single appliance with an on/off activity pattern, for which the loss ratio is determined by the duty cycle. For the example given in Section I, the losses for an appliance with a 10% duty cycle would be found to be only 10% of their true value if the demand is taken as the arithmetic mean average over the duty cycle.

Since (6) depends only on the proportion of time for which the current is in each state, regardless of the sequence in which it occurs, the same equation would apply if the demand were repeatedly switching between the two states. This therefore also represents the case of a cyclic, and where the averaging period is long compared to the duty period.

# *B.* Loss Ratio for Averaging Periods Shorter Than the Duty Cycle

The analysis is now extended to describe a second scenario in which the averaging period is much shorter than the duty cycle of the demand variation, as in Fig. 2.



Fig. 2. Step change in current averaging periods shorter than the duty cycle.

The cyclic demand profile requires a current varying between states  $I_1$  and  $I_2$ , over a measurement time  $t_M$ . There are  $n_A$  averaging periods, each of length  $t_A$ . Within this total, there are  $n_1$  periods with constant current  $I_1$  and  $n_2$  periods with constant current  $I_2$ . There is no error in the loss estimation for these periods since the current is constant. There are also averaging periods during which there is a step change in the current, and it is assumed that these periods are sufficiently short that there is only one transition per period. Over a long measurement time, an equal number of transitions  $n_t$  is expected from  $I_1$  to  $I_2$  and similarly  $I_2$  to  $I_1$ .

The current variations do not necessarily conform to a regular duty cycle, with the length of each current state being different each time. Each transition from  $I_1$  to  $I_2$  occurs at time  $t_{E1i}$  after the start of the averaging periods, and similarly each transition from  $I_2$  to  $I_1$  occurs at  $t_{E2i}$ . Over a long measurement period, the values  $t_{E1i}$  and  $t_{E2i}$  have a uniform distribution between 0 and  $t_A$ .

The theoretical energy loss  $e_{\rm R}$  is determined as above as the sum of the losses in each averaging period:

e

$$R = n_1 I_1^2 t_A R + n_2 I_2^2 t_A R + \sum_{i=1}^{n_t} \left( I_1^2 t_{E1i} + I_2^2 (t_A - t_{E1i}) R \right) + \sum_{i=1}^{n_t} \left( I_2^2 t_{E2i} + I_1^2 (t_A - t_{E2i}) R \right)$$
(7)

A similar approach can be taken for the energy losses estimated from the arithmetic mean averaged samples  $e_A$ :

$$e_{A} = n_{1}I_{1}^{2}t_{A}R + n_{2}I_{2}^{2}t_{A}R + \sum_{i=1}^{n_{t}} \frac{(I_{1}t_{E1i} + I_{2}(t_{A} - t_{E1i}))^{2}}{t_{A}}R + \sum_{i=1}^{n_{t}} \frac{(I_{2}t_{E2i} + I_{1}(t_{A} - t_{E2i}))^{2}}{t_{A}}R.$$
(8)

Setting  $x = I_2/I_1$  and  $t_{E1i} = \alpha_{1i}t_A$  and  $t_{E2i} = \alpha_{2i}t_A$ , where  $\alpha_{1i}$  and  $\alpha_{2i}$  have a uniform distribution between 0 and 1 gives

$$e_{\rm R} = I_1^2 R t_{\rm A} \left( n_1 + x^2 n_2 + n_{\rm t} (1 + x^2) + (1 - x^2) \right) \\ \times \sum_{i=1}^{n_{\rm t}} \alpha_{1i} - (1 - x^2) \sum_{i=1}^{n_{\rm t}} \alpha_{2i} \right)$$
(9)

$$e_{A} = I_{1}^{2} R t_{A} \left( n_{1} + x^{2} n_{2} + (1 + x^{2}) n_{t} + (1 - x)^{2} \right)$$
$$\times \sum_{i=1}^{n_{t}} \left( \alpha_{1i}^{2} + \alpha_{2i}^{2} \right) + 2(1 - x) \sum_{i=1}^{n_{t}} (x \alpha_{1i} - \alpha_{2i}) \right)$$
(10)

For values  $\alpha_i$  taken from a uniform distribution between 0 and 1, the mean over a long period is 0.5, such that

$$\sum_{i=1}^{n} \alpha_i = \frac{n}{2}.$$
(11)

The sum of squares term in (10) is evaluated by considering that the probability density function of  $\alpha$  is the same as that of a single cycle of a saw-tooth waveform between 0 and 1 (by re-arranging the values of  $\alpha$  in ascending order). For large *n*, the mean sum of squares of discrete samples tends towards the mean square of the waveform found by integration, so that

$$\frac{1}{n}\sum_{i=1}^{n}\alpha_{i}^{2} \to \int_{0}^{1}t^{2} \cdot dt = \frac{1}{3}.$$
(12)

The loss estimates therefore reduce to

$$e_{\rm R} = I_1^2 R \left( n_1 + x^2 n_2 + n_{\rm t} (1 + x^2) \right) t_{\rm A} \tag{13}$$

$$e_{\rm A} = e_{\rm R} - I_1^2 R \left( (1-x)^2 \frac{n_{\rm t}}{3} \right) t_{\rm A}.$$
 (14)

The loss based on arithmetic mean averaged data is therefore an under-estimate, by an extent that is determined by the demand profile and the averaging period. As above, the loss ratio is defined as  $r_{\rm A} = e_{\rm A}/e_{\rm R}$  so that

$$r_{\rm A} = 1 - \frac{(1-x)^2 n_{\rm t}}{3\left(n_1 + x^2 n_2 + n_{\rm t}(1+x^2)\right)}.$$
 (15)

The duty cycle of the irregular cyclic demand pattern is defined such that the current is at level  $I_1$  on average for a fraction  $\beta$  of the total measurement time  $t_M$ , such that

$$\beta t_{\rm M} = n_1 t_{\rm A} + \sum_{i=1}^{n_{\rm t}} t_{{\rm E}1i} + \sum_{i=1}^{n_{\rm t}} (t_{\rm A} - t_{{\rm E}2i}).$$
 (16)

Making the substitutions as above, and using (11) gives

$$n_1 = \beta t_{\rm M} / t_{\rm A} - n_{\rm t}. \tag{17}$$

Since  $n_1 + n_2 + 2n_t = t_M/t_A$ , the number of periods  $n_2$  is

$$n_2 = (1 - \beta) t_{\rm M} / t_{\rm A} - n_{\rm t}.$$
 (18)

The loss ratio can then be expressed as

$$r_{\rm A} = 1 - \frac{(1-x)^2}{3\left(\beta + (1-\beta)x^2\right)} \cdot \frac{n_{\rm t}}{t_{\rm M}} \cdot t_{\rm A}.$$
 (19)

In (19), the extent to which the loss is under-estimated depends on three terms. The first term is determined by current ratio x and time ratio  $\beta$ , and the second term is a factor  $n_t/t_M$  which determines how frequently the current switches from one level to another, relative to the measurement period. Both of these factors depend only on the demand variation itself and are

independent of the sampling method. Finally, the ratio is also proportional to the averaging period.

For a specific cyclic demand profile, the loss ratio therefore varies linearly with the averaging period. When this period approaches zero, the loss estimate tends to the theoretical loss.

This result can be anticipated if the total loss is considered to be the sum of 1) correctly estimated losses from periods with constant demand, plus 2) under-estimated losses from periods containing a step change. As the averaging period reduces, the expected error in the  $I^2R$  power of the loss remains constant (when considered over many periods). The number of periods with step changes also remains constant. However, the contribution of this power loss to the total energy loss scales in proportion with the averaging period.

### **III. SIMULATION MODEL**

### A. Validation of Analysis for a Single Switched Appliance

A Matlab simulation has been developed to model the example demand profile described above. This provides a validation of the above analysis and also allows the errors to be investigated where the averaging period has a similar magnitude to the periodicity of the demand profile.

The model has been configured to simulate the demand from a 2.3-kW electric hob at a low power setting, thermostatically controlled and switching on and off with a cyclic pattern. This has been found to represent a particularly "spikey" domestic load, having both high peak power and short duration cycles of around 20-s periods [21].

The demand profile is generated as a sequence of switching events between two states of 11 A and 1 A (representing a constant background current), so that x = 1/11. The appliance duty cycle has a mean period of 8 s at the 11 A state, and a mean period of 12 s at the 1 A state, giving  $\beta = 8/20$ . The switching times are randomized by adding a uniformly varying offset between  $\pm 1$  second onto the length of each switching state period.

The simulation calculates the theoretical losses (without any impact due to time averaging) by combining the losses for each steady state period in the switching sequence. A second data set is then generated for which the losses are calculated based on the arithmetic mean average current over a series of sampling periods. The loss ratio is calculated as the ratio of the loss from averaged data to the theoretical loss.

The results in Fig. 3 show the loss ratio simulation results. The figure also shows linear approximations from (19) for which the averaging period is shorter than the duty cycle, and from (6) where the averaging period is longer than the duty cycle. For this specific scenario representing the electric hob, there is a worst case loss ratio of approximately 50%.

The loss ratio results from the simulation are shown to be a good fit to the linear approximations. The relationship from (19) remains valid until the averaging period increases so that there is more than one step change within a single period.

### B. Energy Losses for Multiple Switching Appliances

So far the illustrations and modelling presented have been restricted to the simple case of an individual appliance with cyclic



Fig. 3. Loss ratio, for 1 appliance, simulation versus analysis.

switching. This serves to illustrate the concepts but now we consider a more practical case with multiple appliances.

Two further simulation configurations were defined with the demand from either 2 or 5 appliances combined. As before, a series of switching event times was generated for each appliance. These were then sorted chronologically to create a single sequence of switching events for the combined demand. The theoretical losses were calculated from this combined sequence. The switching sequence was then sampled as before to represent the data that would be generated by an averaged demand profile.

The appliances were each configured with the same current states of 11 A and 1 A, but with slightly different mean duty cycles. With a long measurement period, this ensured that the phase alignment of the duty cycles would slide relative to each other, such that the results were not dependent on the starting times for the sequences of the multiple appliances.

Fig. 4 shows the loss ratio for the simulation with multiple appliances, plotted now for averaging periods up to 20 s so that the trends for short averaging periods can be seen more clearly. The linear approximation for a single appliance from (19) is included as before. Although the analytical approach only allows for a single appliance, it can be seen that the loss ratio for multiple appliances also follows a linear trend, deviating from this approximation where then averaging period becomes longer than the mean time periods of the current states. The point at which the curves deviate from a linear relationship is shown to depend on the characteristics of the appliance cycling, rather than on the number of appliances that are aggregated. However, the extent to which the losses are under-estimated decreases as the degree of demand aggregation increases.

For appliances with a relatively fast switching pattern (such as a thermostatically controlled hob), these simulations show that averaging periods for data recording should be in the range of seconds, rather than minutes, if energy losses are to be accurately determined.

### IV. MEASURED DATA ANALYSIS METHOD

The simulation results presented above are illustrative of problematic scenarios involving small numbers of high-power switching appliances. We may anticipate, however, that, for a typical dwelling, the overall demand profile will, in practice,



Fig. 4. Loss ratio, for multiple appliances, simulation results.

have less frequent transitions between high and low current states and more frequent occurrence of intermediate currents, and therefore that the effect of mean averaging on loss estimation will be less than that shown above. The following analysis uses high-resolution measured data to quantify the magnitude of errors that may be expected in realistic situations within a distribution network, particularly in all single-phase equipment leading to final customer meter points.

By calculating the arithmetic mean over successive blocks of the measured data, it is possible to show how the losses would have appeared with longer averaging periods. The *i*-th averaged current  $I_{A,k,i}$  is calculated from the original time series  $I_t$  for a block period of k samples as

$$I_{\mathbf{A},k,i} = \frac{1}{k} \sum_{t=(i-1)k+1}^{t=ik} I_t.$$
 (20)

The total energy losses can then be determined by combining the losses for each block average current:

$$e_{A,k} = \sum_{i=1}^{\frac{n_A}{k}} I_{A,k,i}^2 R \cdot k \cdot t_A.$$
 (21)

The loss ratio for block size k is the ratio of  $e_{A,k}$  to the loss calculated from RMS average currents:

$$r_{\mathrm{A},k} = e_{\mathrm{A},k}/e_{\mathrm{R}} = k \cdot \sum_{i=1}^{\frac{n_{\mathrm{A}}}{k}} I_{\mathrm{A},k,i}^2 / \sum_{t=1}^{n_{\mathrm{A}}} I_t^2.$$
 (22)

A similar ratio can be defined based on the worst case loss estimates over individual periods of block size k. This shows the impact of averaging on the peak thermal loading, for example over a half-hourly period:

$$r_{A,pk,k} = \frac{k \cdot \max\left\{ \left( I_{A,k,i}^2 \right) : i = 1, \cdots, n_{A/k} \right\}}{\max\left\{ \left( \sum_{t=(i-1)k+1}^{t=ik} I_t^2 \right) : i = 1, \cdots, n_{A/k} \right\}}.$$
(23)

If the measured data is based on RMS current data, then the denominator in the above ratios represents the theoretical losses exactly. Otherwise, the theoretical losses are approximated by



Fig. 5. Example 30-min interval of demand for one dwelling.

the losses calculated from the original data, which may already have some error due to mean averaging.

### V. MEASURED DATA ANALYSIS RESULTS

### A. Measurements From One Dwelling at 1 Second Resolution

The demand at a single dwelling has been recorded with 1-s resolution for a period of 7 days in August 2010 [21]. The load current was estimated from the measured real and reactive power, assuming a constant 230-V supply.

The demand profile was found to consist of short periods with frequent switching events, and also long intervals when the demand was relatively constant. Fig. 5 shows a 30-min period that was selected as an example of a time when the demand is switching frequently. The plot shows the original 1-s data, together with averaged 1-min data and 30-min data. The 30-min data shows the mean demand, but omits all of the short-term variation. The 1-min data captures some the switching pattern but mostly does not represent the extremes of the variation.

The loss ratios for the full 7 day monitoring period are shown in Fig. 6. For 1-min data, the estimated losses are 89% of the losses with 1-s data. For 30-min data, the estimated losses are 62% of the losses with 1-s data, thereby under-estimated the losses by 38%.

The common practice use of 30-min average demand data is therefore expected to introduce significant errors in estimates of losses within cables or equipment that serve individual dwellings. Fig. 6 illustrates that an averaging period of the order of seconds is required in order to ensure that the loss ratio is close to unity and so to provide good accuracy of losses calculated directly from such data.

# *B.* Method to Estimate Full Losses When Using Mean Averaged Data

Inspection of the 1-s demand data shows that there are a number of periods for which the demand is similar to that shown in Fig. 5. Based on the analysis presented above, the loss ratio would be expected to vary linearly with the averaging period for periods up to about 1 min, as confirmed in Fig. 6.

This allows for the use of a longer averaging period than noted above, by exploiting the linear relationship illustrated in



Fig. 6. Loss ratio for one dwelling, based on 1-s data.

Fig. 6. Using the original 1-s resolution as a reference, the loss ratio can be determined for block sizes of 2 and 3 s, etc.

In this region of the curve, the losses  $e_A$  can be approximated by a linear relationship with gradient m, relative to varying integer multiples k of the averaging period, giving

$$e_{\rm A} = e'_{\rm R} - mkt_{\rm A}.$$
 (24)

where  $e'_{\rm R}$  is an extrapolated estimate of the theoretical losses. Combining results for the original resolution  $e_{\rm A,1}$  and a second result  $e_{\rm A,2}$  for blocks of two consecutive samples (k = 2) the estimated full losses according to the linear approximation are

$$e'_{\rm R} = 2e_{\rm A,1} - e_{\rm A,2}.$$
 (25)

For the results in Fig. 6, an original measurement resolution of 30 s would have allowed for pairs of samples remaining within the linear region of the graph, and would therefore allow a good estimate of the full losses to be made. A measurement resolution of 20 s would allow for a third averaging point within the linear region up to 60 s, thereby giving greater confidence in the linear relationship.

A key observation from Fig. 6 is that the gradient increases in magnitude as the averaging period reduces. If the extrapolation method were to be used with an original measurement resolution greater than 30 s, the estimated losses would be closer to the actual full losses, but some under-estimation error would still remain.

The results in Fig. 6 have been re-normalized to an extrapolated estimate of the full energy losses, as above. The re-normalized loss ratio for the 1 second data is effectively unity, and so the losses estimated from this can be considered to be equivalent to the theoretical losses.

# *C. Measurements From Multiple Dwellings at 1-Min Resolution*

The impact on estimates of energy losses due to using averaged demands has been considered for a second set of measured data, recorded at multiple dwellings. This allows the errors in energy loss estimates to be considered for different degrees of demand aggregation.



Fig. 7. Loss ratio for groupings of dwellings, based on 1-min data.

This data recorded the demand with 1 minute resolution at 22 dwellings in Loughborough, for a period of 42 days in May and June 2008 [22]. The active power recorded by the meters is assumed to be proportional to the load current (effectively assuming a constant voltage and power factor).

By adding the demands, the losses for a single dwelling have been compared with losses for groups of 5, 11, and 22 dwellings, and calculated for different averaging periods as in (20). The mean losses were averaged over all 22 individual houses and for multiple groups of dwellings (e.g. for 11 groups of 2 dwellings or 2 groups of 11 dwellings, etc.). This meant that all of the data points were represented in each result, such that the difference in losses depended on the method by which the data was combined, rather than on which particular dwellings were selected. (For groups of 5, the first 20 dwellings were included). All of the dwellings are considered to be on a common single-phase supply.

Fig. 7 shows the loss ratio for the varying aggregation group sizes. The curves are close to linear for short averaging periods, as indicated by the dotted lines on the plot and the extrapolation method of (25) has been used to provide an improved estimate of the theoretical losses. The loss ratio derived from averaged demand data is then re-normalized to this estimate of the theoretical losses.

From Fig. 7, the mean losses calculated from 30 minute data for single dwellings show only 60% of the estimated theoretical losses, meaning that losses are under-represented by 40%. Similarly, the mean losses calculated from 1 minute data were 96% of the estimated theoretical losses. The averaging has less impact if more customer demands are aggregated together. With the combined demand from 22 houses, the losses estimated from 30-min data are 93% of the estimated theoretical losses.

Loss ratio results were also generated with the 22 dwellings considered individually. In this case the mean losses for 30-min data are found to vary from 39% to 74% of the losses with 1-min data.

Similarly, the losses from 1-min data were between 93% and 98% of the estimated theoretical losses. The corresponding ratio of 89% from the 1-s data (Fig. 6) is outside of this range. This suggests again that although linear extrapolation from 1 min



Fig. 8. Loss ratio for one dwelling, using 30-min data, relative to mean loss over 30 min, based on 1-s data.

gives a better estimate of the full theoretical losses, a higher resolution is ideally required.

### D. Effect of Averaging on Short-Term Heating Effects

The discussion so far has considered losses from the perspective of energy efficiency, and the results have shown the impact of averaging the demand data on the losses over the full length of the available data sets. To examine the impact of the loss estimation errors on peak thermal loading, it is necessary to consider the losses over a shorter period.

Fig. 8 shows the relative energy losses for a single dwelling calculated over a 30-min period. Each point shows the ratio between the losses based on 30-min data, and the mean of the losses based on 1-s data over the same 30-min period. The results show that the 30-min data can significantly under-estimate the losses, indicating down to 10% of the loss that would be calculated from 1-s data. However, these more extreme ratios occur rarely and are found at times when the demand is relatively low. For the 30-min period with greatest mean demand (3.9 kW), the losses estimated with 30-min data were 74% of the losses calculated as the mean of 1-s data. Peak losses would therefore be under-represented by 26% if using 30-min data for this particular time period.

A number of points in Fig. 8 follow a linear pattern. These relate to the use of a 3.3-kW appliance, together with a low background demand. The loss ratio for these points is then described by (6) with  $x \cong 0$ , and is approximately proportional to the fraction of time  $\beta$  for which the appliance was switched on within the 30-min period. The mean demand for these points is also proportional to  $\beta$ , giving the linear pattern in Fig. 8. The two points to the right of the graph are due to the rare concurrent use of a second high power appliance.

As above, results were also generated for the 22 dwellings using 1-min data. In general, the peak losses from 30-min data did not occur in the same 30-min period as the peak losses from 1-min data. The ratio of these peaks varied between 43% and 96%.



Fig. 9. Energy loss for group of 22 dwellings using 30-min data, relative to mean loss over 30 min, based on 1-min data.

A similar plot in Fig. 9 shows the relative losses for the demand aggregated over the 22 dwellings. The loss ratio is expressed here relative to the losses for 1-min data, since the theoretical losses over the 30-min periods are not known.

With a greater level of demand aggregation, there is less impact of the averaging. The minimum ratio between the loss based on the 30-min average demand and the losses over a 30-min period is 65%. For the maximum demand within the monitoring period, the loss based on the 30-min demand is 96% of the loss with 1 minute data.

These error ratios would need to be considered where simulation results are to be compared with measured maximum demand data. Maximum demand indicators can operate by measuring the heating effect of the load current (e.g., in a bi-metallic strip) and the recorded value therefore gives an RMS measurement over the specified time lag period, typically 15 to 30 min [23]. Simulation models based on average demand data would provide a lower maximum demand than seen in indicator equipment, with a ratio given by the square root of the loss ratios presented above.

These results show that comparisons between simulation results and measurements from meters installed at feeder junctions (with less demand aggregation than at substations) could have significant errors.

### E. Loss Load Factor

The sections above have considered the impacts of time resolution on loss calculations where individual customer demand data is available. We now consider the impacts on the loss load factor, i.e., the ratio of the mean losses to the losses with the peak load [11]. This is typically derived from a sample of demand data use as a general loss estimation metric [24].

Assuming that the "peak demand" is defined over a time period of k samples, the loss load factor  $F_{LL,rms}$  based on RMS averages would be

$$F_{\rm LL,rms} = \frac{\sum_{t=1}^{n_{\rm A}} I_t^2}{\frac{n_{\rm A}}{k} \cdot \max\left\{\left(\sum_{t=(i-1)k+1}^{t=ik} I_t^2\right) : i = 1, \cdots, n_{\rm A}/k\right\}}$$
(26)

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Fig. 10. Loss load factor for groupings of dwellings, based on 1-min data.

However, in the absence of high resolution RMS current data, the loss load factor can be determined from an average demand profile, as in [24], so that

$$F_{\rm LL,av} = \frac{\sum_{i=1}^{\frac{n_A}{k}} I_{\rm A,k,i}^2}{\frac{n_A}{k} \max\left\{ \left( I_{\rm A,k,i}^2 \right) : i = 1, \cdots, n_A/k \right\}}.$$
 (27)

The difference in these two calculations then becomes

$$\frac{F_{\rm LL,av}}{F_{\rm LL,rms}} = \frac{r_{\rm A,k}}{r_{\rm A,pk,k}}$$
(28)

where  $r_{A,k}$  and  $r_{A,pk,k}$  are given by (22) and (23) above.

These factors have been calculated for varying averaging periods from the 1 minute data described above, as shown in Fig. 10. Results are presented for single dwellings and for groups of 5, 11, or 22. Results are also shown for both RMS averaging (26) and arithmetic mean averaging (27).

The loss load factor (and also the load factor) increases as the degree of load aggregation increases. The results are clearly also dependent on the averaging period used when defining the losses associated with the peak load.

There are only slight differences between calculations based on RMS average losses and those based on arithmetic mean demand data. This indicates that the use of mean averaged data has similar impact on the long term energy losses  $r_{A,k}$  and the peak energy losses  $r_{A,pk,k}$ .

The results presented here emphasize the need for consistency in the definition of the time period for which the peak demand is measured. Errors could potentially arise if one time period is used in the calculation of the loss load factor, with another averaging period being used by the maximum demand indicators. For example, considering the loss load factor for the group of 22 dwellings derived from hourly data, losses would be over-estimated if this factor were combined with peak losses calculated from readings of a maximum demand indicator with 30-min time lag.

Clearly the recorded peak current is likely to be higher if the demand is measured over a longer period. The loss load factor results in Fig. 10 are based on the peak losses occurring over the 42 day measurement and in any of the groups of dwellings.

Alternatively, the loss load factors might be calculated based on an average daily load curve, such as with an hourly averaging period [24]. In this case, the average peak-hour losses would be lower than those in the worst-case hour, giving a higher loss load factor. Applying this approach to the data for the group of 22 dwellings gives a loss load factor of 0.4, compared to the value of 0.18 in Fig. 10. As expected from the discussion above, losses calculated from an average load curve are subject to a further slight under-estimation if the averaging between days uses an arithmetic mean. However, a much more significant error could arise if losses were calculated using a loss load factor derived from a daily load curve (using the averaged peak hour losses), but combined with one-off peak losses based from a maximum demand reading.

### VI. CONCLUSIONS

When arithmetic mean averaged demand data is used in modelling low-voltage distribution networks, the spiky characteristics of real customer demands are removed and network losses may be significantly under-estimated.

For the demand at a single dwelling, the estimated losses from half-hourly mean-averaged data were found to be approximately 60% of the losses without the impact of averaging, therefore under-estimating the losses by 40%. Even with 1 minute data, the estimated losses were found to be between 89% and 96% of the losses without the impacts of averaging. These results are directly relevant to the operation of assets associated with a single dwelling (or similar), including service cables, small pole-mounted distribution transformers or voltage-optimization equipment that might be introduced. Using half-hourly data in these cases could significantly under-estimate the losses.

Moving back through the network, the total aggregated demand becomes much smoother, greatly reducing the error in the estimated losses due to mean demand averaging. With demand aggregated from a group of 22 dwellings, the losses estimated from half-hourly data were 97% of those from 1 minute data. Half-hourly data therefore appears adequate for the quantification of losses at this level of aggregation.

In seeking to provide guidance on the time resolution needed to avoid significant errors, this paper has illustrated that calculated losses contain an error that varies linearly with the averaging period, once this is reduced below the shortest switching state periods of high-power appliances. This effect has been illustrated analytically for single loads with a cyclic profile, expanded through simulation for multiple loads and further demonstrated through the analysis of measured demand data. Two sets of high-resolution measured demand data were used to quantify the effects of reduced resolutions, indicating a need for very high-resolution data if errors in calculated losses are to be avoided. For the demand at a single dwelling, an averaging period in the order of a few seconds is suggested.

Alternatively, an improved estimate of the actual losses may be obtained through extrapolation of losses estimated from lower resolution data. If the averaging period is sufficiently short that the linear relationship noted above can be demonstrated, then this provides a good estimate of the full losses. Where the switching of high-power appliances has periods of 30–60 s (as in the measured 1 second data), this allows for an averaging period of around 30 s. Note however that a higher resolution may still be required for other purposes such as power quality studies.

The use of mean averaged demand data also affects estimates of peak thermal loading, and even small percentage errors in losses may be significant if they coincide with the peak demand. For the demand of a single dwelling, the worst-case losses from half-hourly data were found to vary between 43% and 96% of the losses over the same 30-min period using 1-min data. Again, a smaller error occurs when the demand is more highly aggregated.

The loss load factor decreases as the averaging period used to determine the peak losses is reduced. This highlights a potential error in calculations if, for example, an hourly averaging period used in deriving the loss load factor and the peak losses are determined based on current readings from a maximum demand meters with 30-min time lag.

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Andrew J. Urquhart received the B.Sc. degree in engineering science from Durham University, Durham, U.K., in 1985, and the M.Sc. degree in renewable energy systems technology from Loughborough University, Loughborough, U.K. He is now pursuing the Ph.D. degree, working on the integration of low carbon technologies on LV networks, with interests in improving the verification of simulations methods against real world data.

He worked in the telecoms industry as a radio systems engineer. In 2009, he began studying with CREST, Loughborough University.



**Murray Thomson** received the B.Sc. degree in electrical and electronic engineering from Leeds University, Leeds, U.K., in 1985 and the Ph.D. degree from Loughborough University, Loughborough, U.K., in 2004.

He worked in the automation industry as a software and electrical systems engineer. He joined CREST, Loughborough University in 1997 to study the integration of renewable-energy powered generators into electricity networks. He is now a Senior Lecturer in Electrical Networks and Systems, with research in-

terests including demand response, grid balancing, wind power, solar photovoltaics and high-resolution LV network analysis.

### **APPENDIX C**

# Assumptions and approximations typically applied in modelling LV networks with high penetrations of low carbon technologies

Andrew J Urquhart, Murray Thomson

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# Assumptions and Approximations Typically Applied in Modelling LV Networks with High Penetrations of Low Carbon Technologies

Andrew J. Urquhart and Murray Thomson Centre for Renewable Energy Systems Technology Loughborough University Loughborough, LE11 3TU, UK <u>A.J.Urquhart@lboro.ac.uk; M.Thomson@lboro.ac.uk</u>

*Abstract*— Uncertainties in the assessment of LV network capacity to accommodate PV and other low-carbon technologies can lead to installation constraints or costly network reinforcements that may not be entirely necessary. This paper reviews the numerous assumptions often used in such assessments and highlights those relating to time resolution of demand models, harmonics, network grounding and impedance modelling as being particularly questionable. In many cases, the individual assumptions may be low risk, but there is greater uncertainty when assumptions are applied in combination.

Keywords-component; network simulation, harmonics, photovoltaics, reinforcement planning

### I. INTRODUCTION

This paper considers the assumptions often used in low voltage (LV) network modelling, particularly power flow analysis or 'load flow' where the objective is to calculate voltages and currents within the network in response to a specified set of connected loads and generators. Other operational parameters determined can include unbalance, harmonic distortion, losses and thermal impacts.

Such modelling is often used in the assessment of networks to accommodate proposed PV and other lowcarbon technologies and plays an important role in ensuring that network operational parameters are maintained within suitable limits. On the other hand, over-cautious assessment can lead to constraints being applied to the installation or operation of low-carbon technologies or to costly network reinforcement that may not be entirely necessary. The accuracy of such models is therefore an important matter.

This paper focuses on networks in which power is distributed around a local area at LV, as is commonplace in Europe. In a typical network, as described in [1], a primary substation supplies several medium voltage (MV) feeders, each routed to a number of LV distribution transformers. Power from these transformers is routed via underground mains cables or on overhead lines. Customer service connections are attached where required along the distribution route. The details of LV network design practices can vary significantly between different countries and even local regions with different operating companies and these variations present additional challenges in the modelling.

Some of the assumptions typically used in LV network modelling are adopted from experience of modelling higher voltage networks. Others may stem from limitations in data available to describe the actual network configurations, and the varying characteristics of the loads and generators that are connected. This paper aims to provide a general review of all assumptions and uses the following categories:

*Loads, generators and substation nodes*— The characteristics of power import and export at the nodes on the LV network.

*Network topology*— The connectivity between the nodes of the LV network. This covers the cables and overhead lines and the connections between them.

*Conductor impedances*— The impedances of the cables or overhead lines between the junction nodes.

In each section, a summary table provides a list of assumptions, including references to examples where they have been applied. Brief comments are included to review the impacts of the assumptions on modelling results. The text then describes some of the more questionable assumptions which are reviewed in further detail. The paper concludes by identifying the modelling assumptions that appear to be the most critical for further evaluation.

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### TABLE I. MODELLING ASSUMPTIONS FOR LOADS GENERATORS AND SUBSTATION NODES

Assumption	Review of risks and impacts
Customer demand is represented by national	Assuming a common demand profile for all customers may neglect regional variations, and differences
mean profiles [2].	between individual customers (e.g. for shift workers) or attitudes to energy use. See section II.A.
Individual customer demand generated from statistical distributions [2].	Impacts of phase unbalance depend on adequate representation of the deviations of individual customer demands from the mean . <i>See section II.A</i>
All PV installations on the LV network are subject to the same irradiance variation	Simulations of PV on an LV network showed significantly different voltage rise if the effect of cloud movement was applied to the irradiance data, rather than all customers having the same irradiance [21]. The peak irradiance data were noted as occurring on cloudy days.
Commercial demand is not modelled [3]	Phase balance and loads may be inaccurate, due to differing demand and use of three phase supplies.
Loads on each phase are balanced [22].	If unbalanced networks are simulated as being balanced, neutral currents and losses will be under- estimated. Voltage extremes are under-represented if currents are averaged between phases [23]. <i>See section II.B.</i>
Demand is unbalanced due to load variations	Effects of unbalance on voltage range and losses are under-estimated if simulations only model
but mean demand on each phase is balanced [1], [24].	scenarios with equal mean demand and generation on each phase, for example with customers allocated sequentially to each phase and having the same mean demand profile. <i>See section II.B</i>
Loads and generation can be represented by time averaged samples [1], [2], [8].	If currents are averaged over too long a period, short term voltage deviations will not be represented and losses will be under-estimated. The proportion of exported power from generators will be over-estimated if demand from loads is time averaged [10]. <i>See section II.C.</i>
Loads have constant power vs. voltage [1][8], or a constant current model [22] [9].	Risks in accuracies in customer node voltages, unbalance and neutral currents [13]. See section II.D.
Generators have a constant power output vs. voltage variation.	Generators driven by renewable energy are commonly assumed to provide output power dependent on the renewable resource available, and that this is independent of the network voltage [22].
The network can be simulated as sinusoidal 50/60 Hz with no harmonics [24].	This neglects the voltage drops due to increased reactance at higher frequency, and assumes a greater cancellation of three phase currents in the neutral than will occur in practice. <i>See section II.E.</i>
A constant power factor is assumed for loads, for example 0.9 as in [22].	Variation with customer loads and throughout the day would be omitted. All phases are balanced in terms of phase angle relative to the three phase voltage. The approach in [3] addresses this by assigning a power factor for each appliance. <i>See section II.E.</i>
A constant power factor is assumed for generators, typically unity [22].	A low risk assumption as this may be defined by grid connection regulations.
Non-metered demand such as due to street lighting is neglected [25].	Estimated losses and voltages would be inaccurate if the full demand is not modelled, for example if based on data from customer smart meters [26].
The distribution transformer (or primary	The On Load Tap Changer (OLTC) at the primary substation maintains the voltage within a specified
substation, if the MV feeder is modelled) are	bandwidth, adjusted in fixed steps. Tap changes only occur if the bandwidth is exceeded for a defined
a constant voltage source [9], [1].	time period. A voltage uncertainty of 2% has been allowed for the OLTC accuracy [27].

### II. LOADS, GENERATORS AND SUBSTATION NODES

Table I lists assumptions relating to loads, generators and the substation.

### A. Demand profiles

The variation of demand in time can be characterized according to standard load profiles. These profiles are based on many aggregated customers so do not reflect the stochastic variation of individual customer loads.

One approach to creating individual customer demand samples is to assume that a particular statistical distribution applies. In [2], a normal distribution was used with assumptions made for the standard deviation.

An alternative approach is to build up the customer demands based on individual appliance use, and then scale the total power to match known demand profiles [3]. The loads are based on statistics relating to active occupancy of homes and data describing the appliances. An assumption is needed to determine the correlation between daily occupancy patterns. Assuming the same profile each day ignores potential variations, but using an independent profile each day models every customer according to the same statistical distribution. In [3] the model provided good agreement with measured data but underestimated the low and high extremes of the average demand per customer.

### B. Phase balance

If unbalanced networks are simulated as being balanced, neutral currents and losses will be under-estimated. Voltage extremes may be under-represented if currents are averaged. With single phase service connections, the time varying characteristics of demand and generation cause currents in the three phase mains to be unbalanced. Where three phase connections are provided, as in Germany, high power heating loads will be balanced across all three phases, but lower power appliances are still on single phase circuits so some unbalance remains.

In addition to short term unbalance due to appliance activity, the mean demand on each phase may be unbalanced. This could be due to the fact that the network serves a mixture of residential and commercial customers with different demand profiles, or just due to the differences between customers. Single phase connections may not be equally shared between phases. A possible cause of this where 4-core main cables are used is that it is less easy to separate the bundle to select the core opposite the neutral.

Where customers have single phase connections, distributed generation is likely to be unbalanced, as installations build up in a randomized pattern on each phase. Where customers have three phase connections, smaller PV inverters still have single phase operation. Since all three phases are available, the phase allocation is selected by the installer. The balance of the aggregated generation depends on the evenness of these phase selection decisions.

Phase unbalance has been highlighted by results from a program of LV substation monitoring [4]. A distribution substation (on a feeder with a high penetration of PV) was monitored over two months, with mean currents on the three phases of 69 A, 99 A and 126 A. Similar results were found

in [5] where the mean current unbalance was 27%, (ratio of peak phase to average phase current).

This suggests that modelling should include scenarios in which the mean demand and generation are unbalanced.

### C. Variation with time

Detailed simulations often use a time step approach in which each sample represents the demand or generation over a fixed time interval. A high resolution is needed to represent 'spiky' demand characteristics. If currents are averaged over too long a period, short term voltage deviations will not be represented. Since power dissipated is proportional to the square of the current, losses are underestimated if calculated using an average current.

The required time resolution could be determined by standards, for example where the power delivered to customers is required to conform to EN 50160. This defines that voltage magnitude, unbalance and harmonic distortion should be averaged over 10 minutes [6]. The resolution can also be considered in relation to the typical activity periods of appliances. Thermostatically controlled heating has a significant impact on residential demand models due to the high power required and short time periods. An example was given in [7] of a cooker hob on a low setting, modelled as a 2 kW load switched on for 30 s then off for 120 s.

Simulation models have used a wide range of time resolutions, including 1 minute [1], 15 minutes [8], and 30 minutes [2]. The impact of selecting different time intervals has been reviewed for periods of 1 to 30 minutes [9]. For a single customer, the maximum demand with 30 minute averaging was 16% below that for 1 minute samples. When the demand from 16 customers was aggregated (and therefore more balanced), there was only a 6% difference.

The proportion of energy imported for a house with a hypothetical constant power generator was reviewed in [10]. If the demand is smoothed by 30 minute averaging then it appears that on-site generation meets a greater proportion of the demand than if 1 minute samples were used.

### D. Load power variation with voltage

Typical domestic appliances have been reviewed in [11] in which an aggregated residential load model is proposed. During most of the day, the demand is approximately 20% constant impedance, and 80% constant power. In the evening, the proportion with constant impedance increases to 40% due to resistive heating loads. At night, when the resistive power peaks are absent, the characteristic reverts to a constant power model. Work on conservation voltage reduction also suggests that a constant power model is not fully representative, with one study suggesting that demand reduces by 0.5% to 1% for a 1% reduction in voltage [12].

Simulations of loads with constant power and constant impedance were compared in [13]. The neutral currents and voltage unbalance for the constant power model were doubled compared to the constant impedance model. The end node customer voltage also varied by up to 7%.

A constant power model is probably not representative of real domestic appliances, and making this approximation does have impacts to the overall simulation accuracy.

### E. Harmonic distortion and power factor

Analysis often uses voltage and current phasors, assuming sinusoidal operation with no harmonics. In practice, significant harmonic distortion appears to occur.

Harmonics from domestic appliances have been reviewed in [11], where a combined model of the appliances in a house includes  $3^{rd}$  and  $5^{th}$  harmonics at 20% and 8% relative to the fundamental. Tests of PV inverters indicate a lower current distortion, with Total Harmonic Distortion (THD) between 2.1% and 4.8% [14].

Monitoring at a UK distribution substation on a network with a high penetration of PV connected has shown voltage THD between 2% and 3.5% [4]. This also showed that current harmonics vary between the three phases, with up to 25%, 13%, 9% and 8% for the  $3^{rd}$ ,  $5^{th}$ ,  $7^{th}$  and  $9^{th}$  harmonics.

Currents harmonics propagate through the network, causing voltage distortion. Loads and generators may simply be treated as a source of harmonic currents, with no dependency on the voltage. Alternatively, loads could also accept active power at harmonic frequencies and act as a sink for the distortion.

The power factor allows for both distortion (since the average power delivered is zero if voltage and current have different frequencies) and reactance. Simulations at the fundamental frequency represent the power factor entirely as a phase displacement between current and voltage.

### III. NETWORK CONNECTIVITY

Table II lists assumptions related to the connectivity. The following sections provide further discussion on two assumptions that affect the network simulation method.

### A. Connectivity of Neutrals and Ground

Where the currents in the three phases are unbalanced or include harmonics, currents will flow in the neutral

TABLE II. MODELLING ASSUMPTIONS FOR NETWORK CONNECTIVITY

Assumption	Review of risks and impacts		
Cable types, routes and connectivity are as described in the network database.	There is relatively high confidence in the database for HV networks, but the accuracy of data describing the LV network is less certain. In [24] service cables were approximated by straight line routes from the house centre to the nearest point on the LV main.		
Phase allocations for customers with single	Where customers are provided with single phase service connections, the records of phase allocations		
phase phase supplies are known.	may be missing or incorrect, so that the current balance between phases may not be correctly modelled.		
Service cables are omitted from the model,	The impact of neglecting the voltage drop in the service cable may be minimal for customers in urban		
as in [2], [9].	areas, but could be greater for older installations in rural areas.		
Neutral conductors, concentric neutral or	Risk of inaccuracies in voltage calculations, proportion of current in neutral and ground calculations, and		
sheath are grounded at each node.	therefore for losses in neutral conductors, if ground connections are not as modelled. See section III.A.		
Neutral and earth connections at link boxes	Simulations would not include circulating currents that may exist bewteen separate LV network		
can be ignored, as in [1], [28].	branches, even though the branches are radial for phase conductors. See section III.B.		

conductor. The accuracy of simulations of network voltages therefore depends on the modelling of the routes available for the neutral current to flow back to the sub-station.

As described in section IV.D, it is common practice to simplify the cable impedance matrix to a  $3 \times 3$  form on the basis that the voltage between neutrals and the earth at each end of a line is zero, i.e. that the neutrals are grounded.

In European networks, it is common for the LV side of the distribution transformer to have a wye configuration with a grounded neutral. The connection is provided by electrodes designed to ensure a low earth potential rise in the presence of fault currents. Metallic sheaths and concentric neutrals of underground cables are also connected to ground.

At the customer premises, the regulations governing the earthing system define several different connection configurations [15]. For TT earthing, there is no protective earth provided by the network and so the customer must install an earth electrode. For TN configurations, the earth conductor provided by the network is connected to pipes at the customer meter point. This creates an equipotential zone within the customer's premises, but may also provide a ground connection if the pipes are metallic. For TN-S earthing, there is no connection between neutral and ground.

LV mains include branched joints and junctions between different cable types. Junction boxes connect the cable cores and concentric neutral or sheath, but have no connection between the neutral and the sheath, and neutrals are not grounded. Similarly, neutral cores may not be grounded at link boxes or where service cables are attached to mains.

However, where a combined neutral/earth conductor is provided, it is important for safety that the earth does not become broken. Additional earth electrodes are added at nodes within the LV network such as cable joints and at the ends of feeder mains. It is also possible for LV networks to include a combination of sections with separate neutral and earth and sections with combined neutral and earth.

In summary, there is a wide variation in earth

configurations, each with different ground connections for the neutral. This issue is usually considered carefully in regard to safety and fault conditions but treated less rigorously in simulation models. There is no concern if the network is balanced and has negligible harmonics, but the consequences for real networks require investigation.

### B. Links Between Radial Branches

Link boxes between radial branches allow the supply to be re-routed in the case of faults. Where the link box is used as a normally open point, the phase conductors links are removed but the neutral and sheath conductors may remain connected through. This creates loops within the neutrals or sheaths and allows circulating currents to flow.

The forward/backward sweep algorithm provides an efficient means of solving the network power flow, but is most easily implemented if the network has radial branches. Assuming that neutrals and sheaths are disconnected at the link boxes (in addition to the phase conductors) allows the network to be simplified to a radial structure. However, the impact of this assumption is not clear.

### IV. CONDUCTOR IMPEDANCES

Table III lists assumptions related to the conductor impedances, with further discussion below.

### A. Full conductor model

A full model of these conductors would include the series impedance and shunt admittance, plus the lumped impedances of neutral or sheath connections to ground. Typical underground LV mains cables include the three phase conductors, a conductive sheath or concentric neutral, and in some cases an additional neutral core. If connections to ground exist, the earth provides a further conductor.

Carson's equations are frequently used to provide a matrix  $\hat{z}$ , containing the self-impedance and mutual impedances for each conductor in a circuit with a ground return path [16]. For a 3-core cable with concentric neutral, this would be a  $4 \times 4$  matrix.

 TABLE III.
 MODELLING ASSUMPTIONS FOR CONDUCTOR IMPEDANCES

Assumption	Review of risks and impacts		
Shunt admittance is neglected [13]	Calculations in [17] conclude that neglecting capacitance has minimal impact, at least in an example for overhead line. Shunt admittatance is also assumed to be negligible.		
The simplified Carson's equations are valid for cable and overhead line impedances.	Comparison between impedance matrices obtained with the full Carson's equations and with the simplified equations suggests that little inaccuracy is caused by this approximation, at least for the fundamental mains frequency [29]. <i>See section IV.A.</i>		
Ground resistivity is a constant, e.g. $100 \Omega m$ [17].	Although negligible impact on voltage was found for resistivity varying from 10 to 1000 $\Omega$ m in [29], it would be useful to review this for a case with significant unbalance and for underground cables.		
End effects are negelected in calculating cable impedances for LV cables	Carson's equations assume that end effects are negligible so that the current distribution in the ground and cable is the same all along the cable [16]. This appears questionable, as discussed in <i>section IV.C</i> .		
Carson's equations define separate conductor and earth voltages.	Calculated earth voltage drops are dependent on arbitrary assumptions made when separating Carson's equatons into terms for the conductor and for the earth. See <i>section IV.B</i> .		
Sum of currents equals zero in each line segment.	Where there are loops in neutral conductors due to link boxes, circulating currents may exist. Circulating currents can flow in the ground if branches with unequal earth potentials are co-located.		
Conductor impedances can be reduced to $3 \times 3$ form using the Kron reduction.	Assuming a multi-grounded network, if this is not the case in practice, introduces an approximation to neutral voltages if impedances are reduced to $3 \times 3$ phase impedance matrices. <i>See section IV.D.</i>		
Impedances defined by positive and zero sequence impedance values [1].	This approach neglects any coupling between sequence modes, and makes an approximation that the cables are fully balanced. Any asymmetry due to the cable is not modelled. <i>See section IV.E.</i>		
Zero sequence impedance can be estimated to be a multiple of the positive sequence impedance [1], [2].	Results in [1] where found to be insensitive to the scaling factor used, but results in [2] with more unbalanced currents showed that the zero sequence impedance significantly affected the proportion of voltage range and unbalance constraint violations.		
Conductors represented by phase and neutral conductor impedances [9].	The model does not include mutual coupling between the conductors or currents in the earth. <i>See section IV.E.</i>		
Neutral to ground impedance is zero.	The grounding impedance at LV substations may be up to 20 $\Omega$ [27]. Results in [30] compared a model with different grounding resistances and noted significant difference in the neutral voltage.		

The full equations include an infinite summation term and so a simplification is generally made in which only one resistive term and two reactive terms are retained [17].

### B. Ground impedance derived from Carson's equations

Carson's equations define the impedance of a conductor together with an earth return path. In order to model the conductor and earth voltage drops separately, terms within Carson's equations have been partitioned in order to provide separate ground and conductor impedance equations.

The ground resistance can easily be isolated from the combined circuit resistance since the conductor resistance is known. However, different approaches have been developed to separate the reactance given by Carson's equations into terms due to the ground and conductor self-inductances and the mutual inductance between them.

The approach by Ciric [13] is described for overhead lines and follows the physical concept from Carson [16]. The earth return path is modelled as an equivalent conductor that is the image of the conductor above ground.

Anderson [18] uses a re-arranged form of Carson's equations in which the earth return is represented as a wire with a specified geometric mean radius (GMR) and depth within the ground. The conceptual earth return wire is arbitrarily selected to have a GMR of unity and the depth in the ground then calculated accordingly.

The two approaches give different equations for the selfreactance of the earth return path, and for the mutual reactance between this and the wire conductor. This seems to be an area in which there is some uncertainty as both approaches require an arbitrary definition of the reactance.

### C. Axial current flow assumed in cable direction

Based on the modified Carson's equations from [17], the resistance of the earth return path is  $r_d = 0.0592 \,\Omega/\text{km}$  and is independent of the ground resistivity. For  $\rho = 100 \,\Omega\text{m}$ , this suggests an effective ground conductor with area  $1.67 \times 10^6 \,\text{m}^2$ , equivalent to a semi-circular profile with radius 1037 m. This appears surprising, and is a dimension much greater than the typical node to node distance in an LV network. A key assumption in Carson's equations is that currents only flow in the axial direction [16], but this might be questioned where the implied current distribution is of a scale greater than the actual length of the cable.

A different assumption is made when calculating the earth potential rise due to fault currents, for which the potential reduces approximately in inverse proportion to the radius from the fault point, rather than linearly along a cable.

### D. Kron reduction

Simulation methods may utilize the Kron reduction in order to reduce cable impedance data from an  $n \times n$  matrix  $\hat{z}$ , to a 3 × 3 phase impedance matrix  $z_{abc}$  as in [17]. This allows the cable models to be integrated into a network simulation with components such as transformers that are also modelled by a 3 × 3 matrix. The Kron reduction applies a constraint that the multiple neutral or earth paths are connected together at each end of a line. However, the technique is questionable if the neutral to ground connections absent at some junction nodes in the network.

Using the notation from [17], the Kron reduction provides the phase impedance matrix:

$$\mathbf{z}_{abc} = \hat{\mathbf{z}}_{ij} - \hat{\mathbf{z}}_{in} \cdot \hat{\mathbf{z}}_{nn}^{-1} \cdot \hat{\mathbf{z}}_{nj}$$
(1)

The neutral and ground currents can also be determined:

$$\mathbf{I_n} = -\hat{\mathbf{z}_{nn}}^{-1} \cdot \hat{\mathbf{z}}_{nj} \cdot \mathbf{I_{abc}}$$
(2)

$$I_{g} = -(I_{n} + I_{a} + I_{b} + I_{c})$$
(3)

For example, if two cable segments of different types are connected in series, and if the neutrals are grounded between the segments, then the combined impedance can be represented by the sum of the  $3 \times 3$  phase impedance matrices for each line section. Alternatively, if there is no ground between the two sections, the  $n \times n$  conductor impedances must be combined first, and the Kron reduction applied to the result. This gives a different result to that with the Kron reduction performed first and the results combined.

In both cases, the sum of currents in each line is assumed to be zero. Phase currents are the same in both segments. If the neutral is grounded between the two line segments, then the proportion of current in the ground varies depending on the self- and mutual impedances of the conductors. If the neutral is not grounded between the two segments, the currents in the neutral and ground are constant throughout.

Assuming additional neutral to ground connections makes an approximation that each section can be treated independently. A comparative study would be needed to determine the significance of this approximation.

### E. Approximated cable impedances

In the absence of data to provide the full  $n \times n$  conductor impedance matrix, an approximate model may be defined based on only the self-impedance data.

One approach is to use the impedances for the conductors from manufacturers' datasheets. These define the conductor alone, without an earth return path (as would be given by Carson's equations). Mutual coupling data is not normally available so is assumed to be zero. Since no earth currents are included, it is implied that the circuits are isolated from the ground. If all the phase conductor impedances are equal, these are also equal to the positive sequence impedances, the positive and zero sequence modes could be simulated separately.

The impact of approximating the impedances using only the positive sequence value was reviewed in [19] and shown to introduce considerable error into voltage calculations.

Alternatively, the impedances might be defined by positive and zero sequence values. These are used to populate the leading diagonal of a  $3 \times 3$  sequence impedance matrix  $z_{012}$  which can be transformed to give a phase impedance matrix  $z_{abc}$ . As the sequence matrix contained no coupling, the corresponding phase impedance matrix is fully balanced. If the cable being modelled is asymmetrical, this is equivalent to making an approximation that the phases are transposed. Since there is no information available to expand the  $3 \times 3$  matrix into a  $4 \times 4$  matrix, if  $z_{012}$  represents a cable with an earth path, it is implied that the neutral to ground voltage is zero.

The impact of this approximation was shown to have minimal impact on voltage magnitudes [19]. However, there is a greater error on estimates of voltage unbalance and losses [20].

### V. CONCLUSIONS

This paper has presented a broad review of assumptions made when modelling power flow in LV networks, particularly where the impact of distributed generation is to be assessed. Many of these are more questionable for LV networks than for higher voltages as the demand is more subject to the stochastic variations of customer loads and as the networks are less well characterized.

A formal comparative study would be needed to fully assess the impact of all of these assumptions. However, the following appear to present some level of risk as they clearly affect the numeric results:

- The use of time averaged demand samples for periods much longer than the typical on-time of appliances
- Assuming a constant power vs. voltage model for loads.
- Assuming mean demand is balanced across each phase.
- Modelling the network as sinusoidal without harmonics.
- Assuming one earthing scheme throughout, when many configurations and combinations may occur in practice.
- Applying the Kron reduction technique when ground connections may not exist, or have non-zero impedance.
- The use of separate terms from Carson's equations to provide an impedance model for the earth currents.

Assumptions regarding time resolution, current balance and harmonics, all have impacts on the models of neutral currents, losses and voltage unbalance. These assumptions are particularly questionable when combined.

Further work is planned to evaluate the impact of these simulation assumptions, initially addressing the questions of harmonics and grounding assumptions.

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### APPENDIX D

### Kron reduction of neutral strand conductors

The finite element method used for the impedance data requires a separate simulation for each conductor. In order that the conductor impedances can be averaged to allow for the cable lay rotation, a separate simulation is needed for each concentric earth strand. For the example of the 3-core 95 mm<sup>2</sup> cable, this results in a 34x34 matrix of conductor impedances (3 sectors, 30 concentric earth strands, and the ground).

However, the voltage difference between the strands is zero, and so these can be considered as one combined conductor, as in the analytical methods, with the impedances for the above example reduced to a 5x5 matrix. This uses a Kron reduction method, as in (Beharrysingh 2014). It should be noted that this is a separate process to the Kron reduction that is used to combine the concentric earth and ground conductors as a multi-grounded neutral.

The method proceeds as shown in the following example, described for a simpler case of a cable with three sectors, two concentric earth strands k and m and the ground conductor g. This gives a 6x6 conductor impedance matrix, re-ordered here so that the concentric earth strands are at the lower right of the impedance matrix.

$$\begin{bmatrix} V_{a} \\ V_{b} \\ V_{c} \\ V_{g} \\ V_{k} \\ V_{m} \end{bmatrix} - \begin{bmatrix} V_{a}' \\ V_{b}' \\ V_{c}' \\ V_{g}' \\ V_{k}' \\ V_{m}' \end{bmatrix} = \begin{bmatrix} \Delta V_{a} \\ \Delta V_{b} \\ \Delta V_{c} \\ \Delta V_{c} \\ \Delta V_{c} \\ \Delta V_{k} \\ \Delta V_{m} \end{bmatrix} = \begin{bmatrix} \bar{z}_{aa} & \bar{z}_{ba} & \bar{z}_{ca} & \bar{z}_{ga} & \bar{z}_{ka} & \bar{z}_{ma} \\ \bar{z}_{ab} & \bar{z}_{bb} & \bar{z}_{cb} & \bar{z}_{gb} & \bar{z}_{kb} & \bar{z}_{mb} \\ \bar{z}_{ac} & \bar{z}_{bc} & \bar{z}_{cc} & \bar{z}_{gc} & \bar{z}_{kc} & \bar{z}_{mc} \\ \bar{z}_{ag} & \bar{z}_{bg} & \bar{z}_{cg} & \bar{z}_{gg} & \bar{z}_{kg} & \bar{z}_{mg} \\ \bar{z}_{ak} & \bar{z}_{bk} & \bar{z}_{ck} & \bar{z}_{gk} & \bar{z}_{mk} & \bar{z}_{mk} \\ \bar{z}_{am} & \bar{z}_{bm} & \bar{z}_{cm} & \bar{z}_{gm} & \bar{z}_{km} & \bar{z}_{mm} \end{bmatrix} \begin{bmatrix} I_{a} \\ I_{b} \\ I_{c} \\ I_{g} \\ I_{k} \\ I_{m} \end{bmatrix}$$

Row k is now subtracted from row m. All the strands voltages are equal so  $V_k = V_m$ and  $V_k' = V_m'$ , giving:

$$\begin{bmatrix} \Delta V_{a} \\ \Delta V_{b} \\ \Delta V_{c} \\ \Delta V_{g} \\ \Delta V_{k} \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{z}_{aa} & \bar{z}_{ba} & \bar{z}_{ca} & \bar{z}_{ga} & \bar{z}_{ka} & \bar{z}_{ma} \\ \bar{z}_{ab} & \bar{z}_{bb} & \bar{z}_{cb} & \bar{z}_{gb} & \bar{z}_{kb} & \bar{z}_{mb} \\ \bar{z}_{ac} & \bar{z}_{bc} & \bar{z}_{cc} & \bar{z}_{gc} & \bar{z}_{kc} & \bar{z}_{mc} \\ \bar{z}_{ag} & \bar{z}_{bg} & \bar{z}_{cg} & \bar{z}_{gg} & \bar{z}_{kg} & \bar{z}_{mg} \\ \bar{z}_{ak} & \bar{z}_{bk} & \bar{z}_{ck} & \bar{z}_{gk} & \bar{z}_{kk} & \bar{z}_{mk} \\ \bar{z}_{am} - \bar{z}_{ak} & \bar{z}_{bm} - \bar{z}_{bk} & \bar{z}_{cm} - \bar{z}_{ck} & \bar{z}_{gm} - \bar{z}_{gk} & \bar{z}_{km} - \bar{z}_{kk} & \bar{z}_{mm} - \bar{z}_{mk} \end{bmatrix} \begin{bmatrix} I_{a} \\ I_{b} \\ I_{c} \\ I_{g} \\ I_{k} \\ I_{m} \end{bmatrix}$$

The current from strand m is now added to that from strand k, giving:

$\begin{bmatrix} \Delta \\ \Delta \\ \Delta \\ \Delta \end{bmatrix}$	$ \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_g \\ V_k \end{bmatrix} $						
	- <i>Ī</i> aa	$\bar{z}_{ba}$	$\bar{z}_{ca}$	$\bar{z}_{ m ga}$	$\bar{z}_{\mathrm{ka}}$	$\bar{z}_{\rm ma} - \bar{z}_{\rm ka}$	г <i>I</i> а т
	$\bar{z}_{ab}$	$ar{z}_{ m bb}$	$\bar{z}_{ m cb}$	$ar{z}_{ m gb}$	$ar{z}_{ m kb}$	$\bar{z}_{ m mb} - \bar{z}_{ m kb}$	I <sub>b</sub>
	$\bar{z}_{ac}$	$ar{z}_{ m bc}$	$\bar{z}_{cc}$	$ar{z}_{ m gc}$	$ar{z}_{ m kc}$	$\bar{z}_{ m mc} - \bar{z}_{ m kc}$	I <sub>c</sub>
_	$\bar{z}_{ag}$	$ar{z_{ ext{bg}}}$	$ar{z}_{ m cg}$	$ar{z}_{ m gg}$	$ar{z}_{ m kg}$	$ar{z}_{ m mg} - ar{z}_{ m kg}$	Ιg
	$\bar{z}_{ak}$	$ar{z}_{ m bk}$	$\bar{z}_{ck}$	$\bar{z}_{ m gk}$	$\bar{z}_{\mathrm{kk}}$	$\bar{z}_{mk} - \bar{z}_{kk}$	$I_{\rm k} + I_{\rm m}$
	$\bar{z}_{am} - \bar{z}_{ak}$	$\bar{z}_{\rm bm} - \bar{z}_{\rm bk}$	$\bar{z}_{\rm cm} - \bar{z}_{\rm ck}$	$\bar{z}_{\rm gm} - \bar{z}_{\rm gk}$	$\bar{z}_{\rm km} - \bar{z}_{\rm kk}$	$\bar{z}_{\rm mm} - \bar{z}_{\rm mk} - \bar{z}_{\rm km} + \bar{z}_{\rm kk}$	L I <sub>m</sub> ]

The total current in the concentric earth is given by:

$$I_{\rm e} = I_{\rm k} + I_{\rm m}$$

And the voltage of all concentric earth strands is equal to that on strand k so  $V_e = V_k$ and  $V_e' = V_k'$ , giving:

$$\begin{bmatrix} \Delta V_{a} \\ \Delta V_{b} \\ \Delta V_{c} \\ \Delta V_{g} \\ \Delta V_{e} \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{z}_{aa} & \bar{z}_{ba} & \bar{z}_{ca} & \bar{z}_{ga} & \bar{z}_{ka} & \bar{z}_{ma} - \bar{z}_{ka} \\ \bar{z}_{ab} & \bar{z}_{bb} & \bar{z}_{cb} & \bar{z}_{gb} & \bar{z}_{kb} & \bar{z}_{mb} - \bar{z}_{kb} \\ \bar{z}_{ac} & \bar{z}_{bc} & \bar{z}_{cc} & \bar{z}_{gc} & \bar{z}_{kc} & \bar{z}_{mc} - \bar{z}_{kc} \\ \bar{z}_{ag} & \bar{z}_{bg} & \bar{z}_{cg} & \bar{z}_{gg} & \bar{z}_{kg} & \bar{z}_{mg} - \bar{z}_{kg} \\ \bar{z}_{ak} & \bar{z}_{bk} & \bar{z}_{ck} & \bar{z}_{gk} & \bar{z}_{kk} & \bar{z}_{mk} - \bar{z}_{kk} \\ \bar{z}_{am} - \bar{z}_{ak} & \bar{z}_{bm} - \bar{z}_{bk} & \bar{z}_{cm} - \bar{z}_{ck} & \bar{z}_{gm} - \bar{z}_{gk} & \bar{z}_{km} - \bar{z}_{kk} & \bar{z}_{mm} - \bar{z}_{mk} - \bar{z}_{km} + \bar{z}_{kk} \end{bmatrix} \begin{bmatrix} I_{a} \\ I_{b} \\ I_{c} \\ I_{g} \\ I_{e} \\ I_{m} \end{bmatrix}$$

Partitioning the impedance matrix between rows e and m, and between columns e and m gives:

$$\begin{bmatrix} \Delta V_{abcge} \\ 0 \end{bmatrix} = \begin{bmatrix} \overline{z}_{abcge:abcge} & \overline{z}_{abcge:m} \\ \overline{z}_{m:abcge} & \overline{z}_{mm} \end{bmatrix} \begin{bmatrix} I_{abcge} \\ I_m \end{bmatrix}$$
$$0 = \overline{z}_{m:abcge} I_{abcge} + \overline{z}_{mm} I_m$$
$$I_m = -\overline{z}_{mm}^{-1} \overline{z}_{m:abcge} I_{abcge}$$

## $\Delta V_{abcge} = \overline{z}_{abcge:abcgn} I_{abcge} + \overline{z}_{abcge:m} I_m$

$$\Delta V_{abcge} = \overline{z}_{abcge:abcge} I_{abcge} - \overline{z}_{abcge:m} \overline{z}_{mm}^{-1} \overline{z}_{m:abcge} I_{abcge}$$

$$\Delta V_{\text{abcge}} = \left(\overline{z}_{\text{abcge:abcge}}I_{\text{abcge}} - \overline{z}_{\text{abcge:m}}\overline{z}_{\text{mm}}^{-1}\overline{z}_{\text{m:abcge}}I_{\text{abcge}}\right)I_{\text{abcge}}$$

This provides a 5x5 matrix  $\bar{\bar{z}}_{abcge}$  where:

$$\bar{\bar{z}}_{abcge} = \bar{z}_{abcge:abcge} I_{abcge} - \bar{z}_{abcge:m} \bar{z}_{mm}^{-1} \bar{z}_{m:abcge} I_{abcge}$$