Mathematical Optimization and Game Theoretic Techniques for Multicell Beamforming

by

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CERTIFICATE OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this thesis, that the original work is my own except as specified in acknowledgements or in footnotes, and that neither the thesis nor the original work contained therein has been submitted to this or any other institution for a degree.

.... (Signed)

.... (Candidate)

I dedicate this thesis to my dear parents Yongping Wu and Ling Ma.

Abstract

The main challenge in mobile wireless communications is the incompatibility between limited wireless resources and increasing demand on wireless services. The employment of frequency reuse technique has effectively increased the capacity of the network and improved the efficiency of frequency utilization. However, with the emergence of smart phones and even more data hungry applications such as interactive multimedia, higher data rate is demanded by mobile users. On the other hand, the interference induced by spectrum sharing arrangement has severely degraded the quality of service for users and restricted further reduction of cell size and enhancement of frequency reuse factor.

Beamforming technique has great potential to improve the network performance. With the employment of multiple antennas, a base station is capable of directionally transmitting signals to desired users through narrow beams rather than omnidirectional waves. This will result users suffer less interference from the signals transmitted to other co-channel users. In addition, with the combination of beamforming technique and appropriate power control schemes, the resources of the wireless networks can be used more efficiently.

In this thesis, mathematical optimization and game theoretic techniques have been exploited for beamforming designs within the context of multicell wireless networks. Both the coordinated beamforming and the coalitional game theoretic based beamforming techniques have been proposed. Initially, coordinated multicell beamforming algorithms for mixed design criteria have been developed, in which some users are allowed to achieve target signal-to-interference-plus-noise ratios (SINRs) while the SINRs of rest of the users in all cells will be balanced to a maximum achievable SINR. An SINR balancing based coordinated multicell beamforming algorithm has then been proposed which is capable of balancing users in different cells to different SINR levels. Finally, a coalitional game based multicell beamforming has been considered, in which the proposed coalition formation algorithm can reach to stable coalition structures. The performances of all the proposed algorithms have been demonstrated using MATLAB based simulations.

Contents

Statement of Originality

The contributions of this thesis are mainly on the development of various beamforming algorithms for multicell wireless networks. The novelty of the contributions is supported by the following international journal and conference papers.

Journal Papers

1. Y. Wu and S. Lambotharan, “Coalitional Games for Downlink Multicell Beamforming”, has been submitted to IEEE Transactions on Vehicular Technology.

Conference Papers

2. Y. Wu and S. Lambotharan, “A Coordinated Multicell Beamforming Technique with Multiple Interference Constraints”, IET International Conference on Intelligent Signal Processing (ISP), London, UK, Dec. 2013.

3. Y. Wu, G. Bournaka and S. Lambotharan, “Coordinated Beamforming with Mixed SINR-Balancing and SINR-Target-Constraints for Multicell Wireless Networks”, IEEE Wireless Communications and Networking Conference (WCNC), Istanbul, Turkey, Apr. 2014.

4. Y. Wu and S. Lambotharan, “An Interference Constraint and SINR Balancing Based Coordinated Multicell Beamforming”, IEEE International Conference on Digital Signal Processing (DSP), Singapore, Jul. 2015.

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List of Acronyms

1G	First Generation
2G	Second Generation
3G	Third Generation
4G	Fourth Generation
5G	Fifth Generation
AMPS	Advance Mobile Phone Service
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BSs	Base Stations
CCI	Co-Channel Interference
CDMA	Code Division Multiple Access
CSI	Channel State Information
DOA	Direction of Arrival
EDGE	Enhanced Data Rates for GSM Evolution
EGC	Equal Gain Combining
ESPRIT	Estimating Signal Parameters via Rotational Invariance

	Techniques
FDMA	Frequency Division Multiple Access
FM	Frequency Modulation
GMSK	Gaussian Mask Shift Keying
GP	Geometric Programming
GPRS	General Packet Radio Service
GSM	Global Systems for Mobile Communications
ICI	Intercell Interference
IFC	Interference Channel
IMT-2000	International Mobile Telecommunications 2000
ISI	Inter-Symbol Interference
LMI	Least Mean Inequality
LMS	Linear Matrix Square
LOS	Line of Sight
LP	Linear Programming
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
MMSE	Minimum Mean Square Error
MRC	Maximal Ratio Combining
MRT	Maximum Ratio Transmission
MUSIC	Multiple Signal Classification

MVDR	Minimum-Variance Distortionless Response
NE	Nash Equilibrium
NRTUs	Non-Real-Time Users
NTU	Non-Transferable Utility
OFDM	Orthogonal Frequency Division Multiplexing
PDF	Probability Distribution Function
QCQP	Quadratically Constrained Quadratic Programming
QoS	Quality of Service
RTUs	Real-Time Users
QP	Quadratic Programming
SC	Selection Combining
SDP	Semidefinite Programming
SIMO	Single-Input Multiple-Output
SINR	Signal to Interference plus Noise Ratio
SNR	Signal to Noise Ratio
SNSG	Strategic Non-cooperative Sub Game
SOCP	Second-Order Cone Programming
SVD	Singular Value Decomposition
TACS	Total Access Communication System
TC	Threshold Combining
TDMA	Time Division Multiple Access

TD-SCDMA	Time Division-Synchronous Code Division Multiple Access
TU	Transferable Utility
WCDMA	Wideband Code Division Multiple Access
WIM	Walfisch-Ikegami Model

List of Symbols

Scalar variables are denoted by plain lower-case letters, (i.e., x), vectors by bold-face lower-case letters, (i.e., $\mathbf{x}$), and matrices by upper-case bold-face letters, (i.e., $\mathbf{X}$). Some frequently used notations are as follows:

$\mathrm{E}\{\cdot\}$	Statistical expectation
$(\cdot)^T$	Transpose
$(\cdot)^H$	Hermitian transpose
$\lvert . \rvert$	Absolute value or cardinality depending on the context
$\lVert . \rVert_2$	Euclidean norm
$\lVert . \rVert_p$	l_p norm
$\lVert . \rVert_{\mathcal{F}}$	Frobenius norm
$\mathbf{I}$	Identity matrix
$\mathbf{1}_M$	$M \times 1$ vector of ones
$\mathbf{0}_M$	$M \times 1$ vector of zeros
$(.)^{-1}$	Matrix inverse
$\mathrm{tr}\{\cdot\}$	Trace operator
$\mathbf{diag}(\mathbf{x})$	Diagonal matrix with vector $\mathbf{x}$
$\mathbf{dom}$	Domain

$x!$	Factorial of x
$\exp(\cdot)$	Exponential function
$\emptyset$	Empty set
$\cup$	Union of sets
$\cap$	Intersection of sets
$\mathbb{S}^M$	Set of symmetric $M \times M$ matrices
$\max(\cdot)$	Maximum value
$\min(\cdot)$	Minimum value

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Chapter 1

INTRODUCTION

1.1 Development of Celluar Wireless Communications

With the acceleration of informatization, the demand on communications has dramatically increased. Communication systems have become the foundation to maintain the normal running of the human society. Wireless communication has been playing an increasingly significant role in the whole communication systems due to its advantages of universality of use and convenience of access, which can effectively break the fetter on communications. The explosive increase on wireless communication services has brought new challenges on frequency management and resource allocation. These challenges will further influence the development of wireless communication techniques.

The cellular mobile communication system emerged in the 1980s and has been developing for generations with the evolution of techniques as shown in Figure 1.1. The first-generation (1G) cellular system only provided speech services with analog transmission based on Frequency Modulation (FM) technique. In addition, Frequency Division Multiple Access (FDMA) is employed as the spectral sharing scheme, where the whole bandwidth is divided into several disjoint bands. For example, in the United States, the Advance Mobile Phone Service (AMPS) was developed in which a total allocated bandwidth of 50MHz was divided into two 25MHz bands for uplink and downlink channels respectively [3]. Similar to the AMPS in US, other sys-

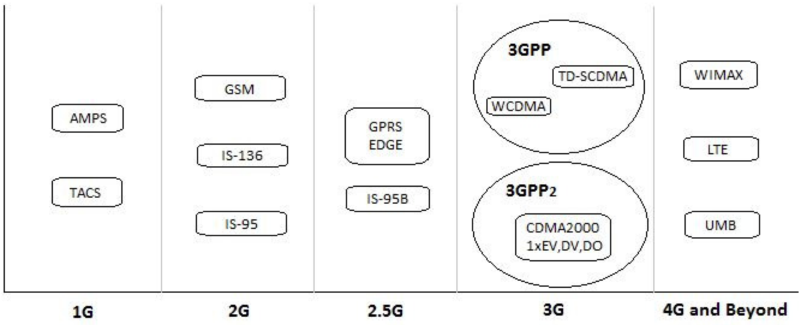

Figure 1.1. Evolution of cellular communication systems.

tems such as Total Access Communication System (TACS) has also been developed in the same period.

With the development of digital devices and the increasing demand on high quality mobile communications, the 1G cellular system quickly evolved to the second-generation (2G) digital cellular system in the early 1990s. One of the most successful 2G systems is the Global Systems for Mobile Communications (GSM) which integrated all 1G standards in Europe into a uniform standard. In addition to FDMA, GSM uses Time Division Multiple Access (TDMA). The IS-95 is another 2G standards based on the Code Division Multiple Access (CDMA) technique which allows a maximum of 64 users transmitting signals on a 1.25MHz frequency channel simultaneously. 2G was then evolved to 2.5G, where the feature of packet data service was added and the voice service was improved. For example, the General Packet Radio Service (GPRS) was developed as the enhancement of the GSM which was further evolved to the Enhanced Data rates for GSM Evolution (EDGE) with the highest data rate of 384 kbps [4]. In 2G, a range of techniques was developed to improve the system performance. For example, due to promising anti-interference capability, the Gaussian Minimum Shift Keying (GMSK) and QPSK were employed as modulation techniques for GSM and CDMA, respectively. To overcome the slow fading and near-far effect, the power control technique was introduced into the CDMA system. The

Rake receiver and adaptive equalization had been applied to overcome the frequency-selective fading caused by multipath effect [5].

The third-generation (3G) mobile system emerged with the development of mobile Internet and the increasing demand on data service. Three 3G standards, collectively known as International Mobile Telecommunications 2000 (IMT-2000), have been accepted as the 3G worldwide standards. The three standards are Wideband Code Division Multiple Access (WCDMA), Time Division-Synchronous Code Division Multiple Access (TD-SCDMA) and CDMA2000. Compared to 2G, the most prominent contribution of 3G is that it can provide higher data rates to afford various wireless services such as mobile Internet, video call and mobile TV. However, as shown in Figure 1.2, 3G could not satisfy the increasing demand on data and other mobile multimedia services, which gives rise of the fourth-generation (4G). In 4G, a data rate of 100Mbps is required to meet the need of most users, which means that the quality of service (QoS) needs to be improved dramatically. To achieve this target, some key techniques have been introduced such as Orthogonal Frequency Division Multiplexing (OFDM), Multiple-input Multiple-output (MIMO), and smart antenna [6]. In recent years, the fifth-generation (5G) mobile system has been proposed. By employing new techniques such as massive MIMO and multicell cooperative processing, 5G aims to provide the data rate which ten times higher than that in the 4G era [7]. The main focus of this thesis is the spatial diversity technique used in the 4G system and beyond within the context of multicell cooperation.

1.2 Antenna Array Processing in Wireless Communications

One of the main targets in designing cellular wireless communication systems is to improve the capacity of the system with allocated radio frequency. One common way to achieve this is to decrease the size of cells to improve the

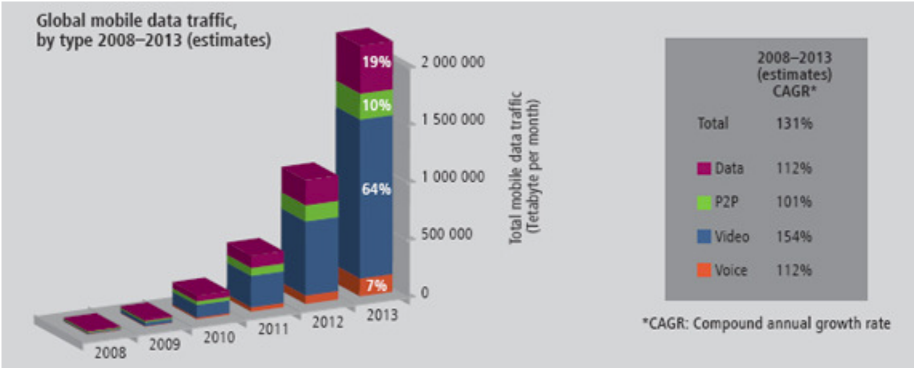

Figure 1.2. Explosion in the demand on data and video services [1].

frequency utilization efficiency, which then results in higher system capacity. However, such method could lead to severe inter-cell interference, especially to those users at cell edges. In addition, the configuration of smaller cells will increase the cost of network construction due to the fact that more base stations (BSs) are needed. To overcome these bottlenecks, an alternative way is to equip BSs and/or mobile users with multiple antennas and develop antenna array processing on the respective terminals. The antenna array was originally developed for military applications such as radar and sonar system design. With the development of digital signal processing technique, such technique was gradually applied in the civil communications as a potential technique to improve the system performance. By employing antenna array, the system performance can be improved in the following aspects:

- *Overcome channel fading*

In terrestrial wireless communications, both the complexity of radio propagation environment and the mobility of user terminals will lead to severe fading problems. By deploying antenna arrays, a signal can be transmitted through several statistically independent channels, which can effectively suppress the negative effect caused by channel fading.

- *Interference cancellation*

In cellular communications, due to the implementation of frequency reuse, the signal-to-noise ratio (SNR) obtained by a user is impacted by the co-channel interference (CCI). By deploying antenna array to the BS, it has the ability to form desired beam patterns by steering the main lobe towards the user of interest and nulls towards the rest of the users, which can effectively reduce the CCI and, improve the SNR.

• *Improvement of frequency utilization efficiency and system capacity*

Reduction of interference will increase the frequency reuse factor and improve the frequency utilization efficiency. In addition, for interference limited systems such as the CDMA system, the implementation of antenna array processing will allow more users to be served due to the reduction of transmit power for both BSs and user terminals.

Beamforming plays a key role in antenna array processing and it can be traced back to the concept of 'adaptive antenna' first proposed by Van Atta in the 1940s [8]. The research on beamforming started from the design of receivers. A least mean square (LMS) algorithm was first proposed in [9], which allows the antenna array be trained so that it can form a pattern comprising a main lobe in the specified direction. In such LMS based array processing, by automatically adjusting the variable weights, the signal and noise occurring outside the main lobe can be rejected. The constrained LMS based beamformer design was then proposed in [10], in which only the frequency band and the direction of arrival (DOA) are needed as a priori information. In [11], the Capon spatial filter was proposed to estimate the DOA. Following the work of [11], the minimum-variance distortionless response (MVDR) algorithm was developed. Based on estimated DOA, such method can maintain the desired signal be distortionless while minimizing the total power of noise and interference at the output [12].

Many DOA estimation techniques have been developed. The most famous DOA algorithm is the multiple signal classification (MUSIC) developed in the 1980s. The principle of the MUSIC algorithm is that if the number of array units is greater than the number of signal sources, the covariance matrix of the signal received by the antenna array can be decomposed into signal subspace and noise subspace, which are orthogonal to each other. Then, the DOA can be found by using the singular decomposition method [13]. If either the sample size or the SNR is large enough, MUSIC can return DOAs with high accuracy. Some other methods such as the estimating signal parameters via rotational invariance techniques (ESPRIT) has also been proposed for DOA estimation at the receiver [14].

In addition to the beamformer designs at the receiver, beamforming can also be applied to the BS if it is equipped with multiple antennas. An advantage of using beamforming at the BS is that BSs can provide higher processing power than that in the receiver. Different to the receiver beamforming, the channel state information (CSI), acquired through a feedback channel from the receiver, is needed for the design of the transmit beamformers. A Variety of transmit beamforming techniques have been proposed since 1990. In [15], a full feedback adaptive transmit beamforming was considered. By sending probing signals to the mobile users and obtaining the feedback, the unknown propagation environment could be identified. A blind adaptive transmit beamforming algorithm has been proposed in [16], which can design beamformers without receiving any feedback information. In this case, the covariance matrices of the downlink and uplink can be connected through a rotation matrix. Then, the transmit beamformers can be obtained by calculating the maximum eigenvalue and corresponding eigenvector of the covariance matrix in the uplink. In [17], a joint power control and transmit beamforming technique was proposed, which converts the downlink beamforming problem into the beamformer design and power control problem in the uplink. Other downlink beamforming designs such as the zero forcing beamforming and the semidefinite programming (SDP) based beamforming have also been proposed [18–20]. The details for both receiver and transmitter beamforming designs will be reviewed in Chapter 2.

1.3 Multicell Cooperation Networks

In conventional cellular networks, a specific radio frequency is reused by a cluster of cells. The frequency reuse efficiency is dependent on the value of frequency reuse factor. In practice, this value is always much less than 1, which means that the frequency reuse efficiency is low and the CCI can be well controlled. However, with the increasing demand on wireless services, the frequency reuse efficiency needs to be further improved. By employing the CDMA technique, the universal frequency reuse is allowed. However, the disadvantage is that the CCI becomes severe. This may significantly deteriorate the quality of communications, especially for those users at the cell edge. Hence, interference mitigation technique is necessary for each BS to fairly sustain the QoS for all its users. Multicell cooperation is considered as an effective way to solve the problem. Instead of performing coding and decoding separately, BSs can operate in a coordinated way by exchanging information and parameter through the backhaul channels among several cells and optimize the resource allocation over the whole network.

From the angle of network structure, multicell cooperation can be classified as centralized cooperation and distributed cooperation. In centralized multicell cooperation, information and data for all BSs are sent to a central control unit and all computation only takes place in the central control unit. The disadvantage of this type of cooperation is the need of enormous signaling overhead [2]. Another way of multicell cooperation is that all BSs cooperate with each other in a distributed way where each BS optimizes the resource allocation for its users based on the information exchange with all other BSs and a central control unit is no longer necessary. The comparison of centralized cooperation and distributed cooperation is shown in Figure

1.3.

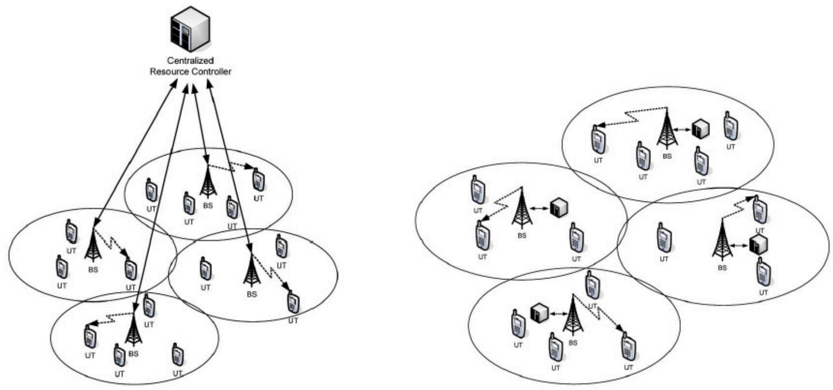

Figure 1.3. Comparison of centralized cooperation and distributed cooperation [2].

From the angle of information exchange, multicell cooperation can be classified as interference coordination and signal-level coordination [21]. Interference coordination is achieved by sharing CSI of inter-cell channels among neighboring BSs through backhaul channels. Based on the CSI of the whole network, BSs can coordinately optimize their resource allocation strategies such as power and beam directions, if the beamforming technique is employed. Different from the interference coordination, BSs can also coordinate by sharing not only CSI, but also signal data, which can achieve better performance improvement. The challenge of this type of coordination is the requirement of perfect signal-level synchronization and increased traffic in the backhaul channels. A common way of signal-level coordination is to construct a cooperative MIMO by jointly processing the antennas of several BSs.

1.4 Thesis Outline

In wireless communications, the reliability of communications depends on the QoS that the system can provide to each user. To improve the frequency utilization efficiency, the frequency reuse technique has been deployed, which will introduce CCI and degrade the QoS for users. In addition, services such as mobile multimedia urge wireless networks to provide higher data rate, which means that the QoS of users must be further improved. Beamforming is a promising technique to improve the QoS by suppressing the CCI and introducing array gain. However, in conventional wireless networks, the single BS-based beamforming may not suppress CCI satisfactorily. Hence, this thesis proposes various coordination based beamforming techniques for multicell multi-user wireless networks.

Chapter 2 provides a survey on MIMO and power control techniques used in wireless networks. To begin with, wireless communication channels, especially the fading channels are first discussed. Several models for characterizing fading channels are introduced. Following on this, the MIMO technique compromising spatial diversity and spatial multiplexing is introduced. The beamforming techniques are then discussed for both the uplink and the downlink. Finally, the power control and power allocation techniques are introduced for the multi-user wireless networks.

In Chapter 3, the convex optimization technique and game theoretic methods are briefly reviewed. For convex optimization, fundamental concepts and typical optimization problems are covered while for game theory, both the strategic game and coalitional game are discussed.

The novel contributions of this thesis are provided in Chapters 4, 5 and 6.

In chapter 4, a coordinated downlink beamforming technique for multicell wireless networks with mixed QoS will be presented. Instead of attaining an overall balanced signal-to-interference-plus-noise ratio (SINR) to all users in all cells, the proposed algorithm allows a specific subset of users in each cell to achieve certain target SINRs while the SINRs of the remaining users in all cells are balanced subject to the total transmission power. Two scenarios are considered in the chapter. In the first scenario, all BSs jointly design beamformers for all users in all cells while in the second scenario, a subset of BSs coordinates to beamforming for their users while ensuring the interference leakage to users in the neighboring cells is below a threshold. The novelty of this work is supported by [22, 23].

Chapter 5 considers a coordinated multicell downlink beamforming based on the SINR balancing technique within the context of users in different BSs achieve maximized balanced SINRs with different levels. Instead of attaining an overall balance of SINRs to all users in all cells, a multiple steps optimization algorithm is proposed that allows users in various cell to achieve different maximum possible balanced SINRs subject to the individual BS transmission power. Different to the existing work using SINR constraints, the optimization algorithm introduces interference constraints at each optimization stage to guarantee the balanced SINRs for users served by those BSs that have used their full transmission power are not degraded. An interference modified rebalancing technique is proposed to improve the flexibility of the proposed algorithm. This work has been published in [24].

A coalitional game based multicell downlink beamforming is proposed in Chapter 6, in which BSs are allowed to partially cooperate by forming coalitions. The target of a BS is to minimize its power consumption while allowing its users to achieve a set of SINR targets. Different to existing coordinated multicell beamforming that all BSs form full cooperation to improve the overall performance, by introducing cooperation cost, not all BSs have incentive to join the cooperation. Hence, a coalition formation algorithm is proposed which allows BSs to minimize their resource consumption by forming appropriate coalitions. Both the Pareto order and the Majority order are considered as the decision rule in the coalition formation process. To improve the performance of the coalition formation algorithm, an $\boldsymbol{\alpha}$-Modification algorithm is proposed.

Finally, conclusions are drawn in Chapter 7 with a discussion on potential future works.

Chapter 2

MULTIPLE ANTENNA AND POWER CONTROL TECHNIQUES IN WIRELESS COMMUNICATION NETWORKS

In this chapter, the background techniques for the contribution chapters are reviewed. The main focus is on the beamforming and power control techniques in wireless networks. To comprehensively understand the motivation of these background techniques, this chapter starts from the introduction of wireless communication channels. The interference channel caused by co-channel interference is particularly emphasized. An overall review of MIMO technique is then provided which includes spatial diversity, spatial multiplexing and beamforming. Finally, power allocation methods for both downlink and uplink communications are introduced.

2.1 Wireless Communication Channels

The radio channel is a fundamental factor for analyzing wireless communication systems. Different to the wired channels, the radio channels are not stationary and may not be predictable since the transmission path between the transmitter and the receiver can vary due to the change of transmission environment [3]. The communication quality of a wireless system is highly dependent on these channel states. Hence, modeling radio channels is one the most significant tasks in designing a wireless system.

2.1.1 Radio Propagation and Fading

In wireless communications, the radio channel is a physical media for the propagation of signals between the transmitter and the receiver [4]. The basic characteristic of radio propagation is that the strength of the signal transmitted through a radio channel will decrease along with the propagation. Such phenomenon is called signal attenuation. For radio channels without any specific characteristics, the signal attenuation depends on the propagation distance. The fundamental model to describe the distance dependent attenuation is the ideal free space model, i.e., line-of-sight (LOS) scenario, where there is no obstruction between the transmitter and the receiver. In free space propagation, the power of the received signal is inversely proportional to the square of the distance d, as [25]:

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}, \tag{2.1.1}$$

where P_t is the transmitted power, G_t is the gain of the transmitter, G_r is the gain of the receiver, λ is the wavelength, and L is the system loss factor. Then, the signal attenuation is the difference between the transmitted power and the received power, which is called *path loss* can be expressed as

$$PL(dB) = 10log_{10}\frac{P_t}{P_r} = -10log_{10}\left[\frac{G_tG_r\lambda^2}{(4\pi)^2d^2}\right]. \tag{2.1.2}$$

In practice, the complexity of radio propagation environment implies that the idealized model is inadequate to characterize all channel behaviors. For example, in terrestrial wireless systems, in addition to the path loss, the signal can be attenuated by buildings and hills located between the transmitter and the receiver. Due to the nature of electromagnetic wave, there are three mechanisms that impact the radio signal propagation: reflection, diffraction and scattering [26]. The reflection of radio waves generates multiple propagation paths for the receiveres, which is called multipath propagation. In addition, if the propagation of radio waves is obstructed by a dense body with large dimension (compare to λ), secondary waves can be formed behind the obstruction body through diffraction and the transmitted signal can still arrive at the receiver. Such phenomenon is called shadowing. Hence, in wireless communications, the received signal is the superposition of multiple copies of the transmitted signal with different delay, phase shift and attenuation. If such additivity is destructive to the signal, fading incurs.

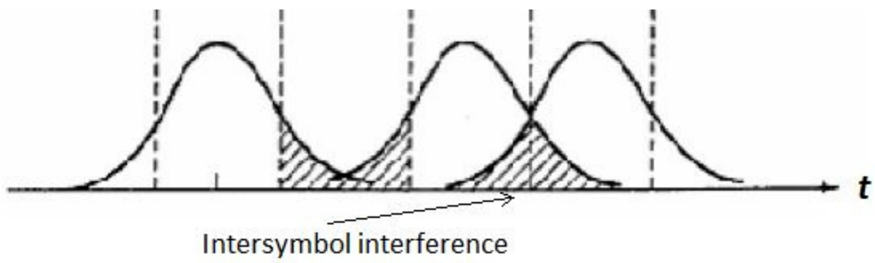

Figure 2.1. Channel induced intersymbol interference.

Fading (also called small-scale fading in some articles) is both frequency dependent and time variant. In practice, it is mainly dominated by multipath propagation and Doppler effects, which refer to the two mechanisms for the manifestation of fading: time dispersion (time-delay spreading) and time variant [25]. In multipath dominant fading, a channel is said to be frequency selective if $T_s < T_m$, where T_s and T_m represent the symbol duration and the maximum delay spread of the channel, respectively. For a single transmitted impulse, the maximum delay spread is the time duration of the received multipath signal where the power between the first and the last components is over some threshold value. Hence, for a frequency selective channel, the length of the received components for a symbol is longer than the symbol's duration. For this case, there is significant overlap between neighboring symbols. Such phenomenon is called inter-symbol interference (ISI) as shown in Figure 2.1. From the view of frequency domain, a frequency selective channel also means that the spectral components of the signal are not equally influenced by the channel. Hence, the coherence bandwidth is smaller than the bandwidth of the signal, where the coherence bandwidth is a range of frequencies over which the channel has equal gain and linear phase [26]. In comparison to the frequency selective channel, a channel is said to be frequency non-selective or flat fading if $T_s > T_m$. Illustrated in a reciprocal way, all spectral components of the signal are affected by a flat fading channel in the same manner. Hence, for a flat fading channel, ISI will not occur. However, the performance degradation still exists due to the destructive additivity of phase components [26].

In mobile wireless communications, the degradation of signal not only depends on the time dispersion but also depends on the time-variant nature of the channel due to relative motion between the transmitter and the receiver. Fading caused by the time-variant is called Doppler effect and can be classified into two categories: fast fading and slow fading. In fast fading, the time that channel behaves in a correlated manner is shorter than the symbol's duration T_s and the distortion manifests as the baseband pulse distortion rather than the ISI. In slow fading, such time is longer than the symbol's duration, which means that the baseband pulse distortion will not occur.

However, similar to the flat fading, in slow fading, the degradation on SNR will still take place [26].

2.1.2 Channel Models

In practice, due to the complexity and variability of wireless communication environment, wireless communication channels are characterized by various empirical based channel models. One of the widely used models is the Okumura-Hata model, which is applicable for the frequency range from 150MHz to 1500MHz. By extending the application frequency to 2000MHz, the Hata model was adopted as the reference model for the PCS system [27]. Another famous model is the Walfisch-Ikegami model (WIM), which has been employed by the GSM system. Compared to the Hata model, more factors are introduced into the WIM model to measure the loss caused by diffraction and scattering and the fading caused by different obstructions.

In addition to empirical channel models, various statistical models have been developed to specifically characterize channel fading. For the propagation environment with many objects scattering radio signals such as the central areas of cities, radio channels can be characterized by Rayleigh fading. Rayleigh fading is used for the environment that there is no dominant stationary signal component, or roughly speaking, the LOS between the transmitter and the receiver does not exist. Then, the signal envelope r follows the Rayleigh distribution with the following probability distribution function (PDF) [28]:

$$p(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right),\ r \geq 0 \tag{2.1.3}$$

where σ^2 is the average power of the received signal. If the LOS exists, then the Rician fading is used and the signal envelope has the following PDF [28]:

$$p(r) = \frac{r}{\sigma^2}\exp\left(-\frac{(r^2+A^2)}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right),\ r \geq 0, A \geq 0, \tag{2.1.4}$$

where A is the peak amplitude of the dominant signal and $I_0(\cdot)$ is the modified Bessel function of the first kind and zero order. It should be noticed that if $A = 0$, the dominant signal component is absent, then the Rician distribution is reduced to the Rayleigh distribution. In this thesis, all simulations have been drawn based on the Rayleigh and flat fading channels.

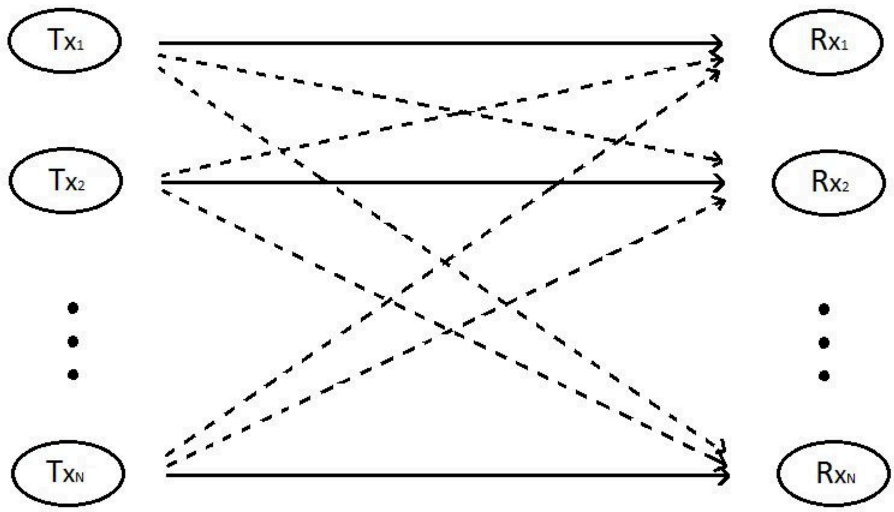

Figure 2.2. Interference channel model.

2.2 Interference Channel

When the performance of a wireless communication systems is analyzed, a starting point is to consider the radio channel only with noise. A primarily considered noise is the thermal noise generated at the receiver, which has zero mean Gaussian PDF. Hence, the received signal is simply modeled by assuming the channel is an additive white Gaussian noise (AWGN) channel. However, in practice, the external interference has more significant effect than the thermal noise, such as the filter-induced ISI and the channel-induced ISI. With the increasing number of users in wireless networks, the frequency reuse scheme is employed to accommodate the frequency insufficiency. The share of radio frequency within several communication channels will cause co-channel interference, which becomes the dominant interference in wireless communications. A channel with co-channel interference is called interference channel (IFC) [25]. Figure 2.2 shows a typical IFC with N pairs of transmitter-receivers sharing the same frequency band. Then, the received signal at the kth receiver can be written as

$$y_k(n) = \sum_{i=1}^{N} h_{i,k} x_i(n) + \eta_k(n), \tag{2.2.1}$$

where $x_i(n)$ is the signal transmitted by the ith transmitter, $\eta_k(n)$ is the noise at the kth receiver, and $h_{j,k}$ represents the channel gain from the jth transmitter to the kth receiver.

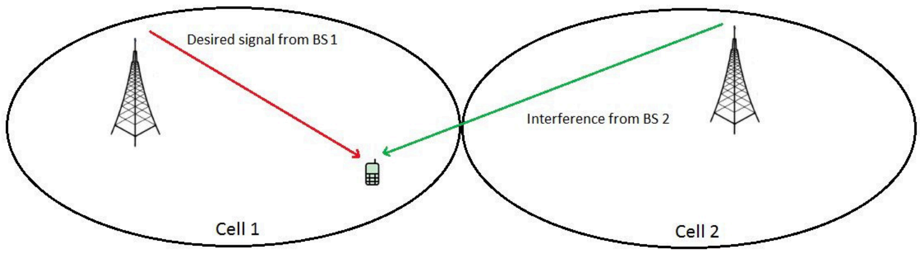

Figure 2.3. Inter-cell interference.

In mobile wireless communication systems with universal frequency reuse such as the CDMA system, all mobile users in all cells share the same frequency band. Hence, users in a specific cell suffer from CCI caused by the communications of adjacent cells, which is called inter-cell interference (ICI) as shown in Figure 2.3. In traditional mobile networks, ICI is generally considered as background noise and the quality of the received signal is evaluated by the SNR at the receiver. Such consideration is effective only if the ICI is weak. In practice, if some mobile users are at the cell edge, they will suffer severe ICI and the capacity of the network will decrease dramatically. Hence, instead of using SNR, the SINR is more appropriate when designing a mobile wireless network. In chapter 4, 5 and 6 of this thesis, the aim of the proposed techniques is to suppress ICI to enhance SINR using multiple antenna techniques.

2.3 Multiple Antenna Technique

The multiple-input multiple-output technique has emerged for improving the data rates and reliability in wireless communications. In traditional wireless communications, the deployment of single antenna at both the transmitter and the receiver restricts the data processing to only within the time and frequency domain. By deploying multiple antennas, the spatial dimension is extended and the system performance such as coverage and error rate can be improved. A typical point-to-point MIMO system is shown in Figure 2.4.

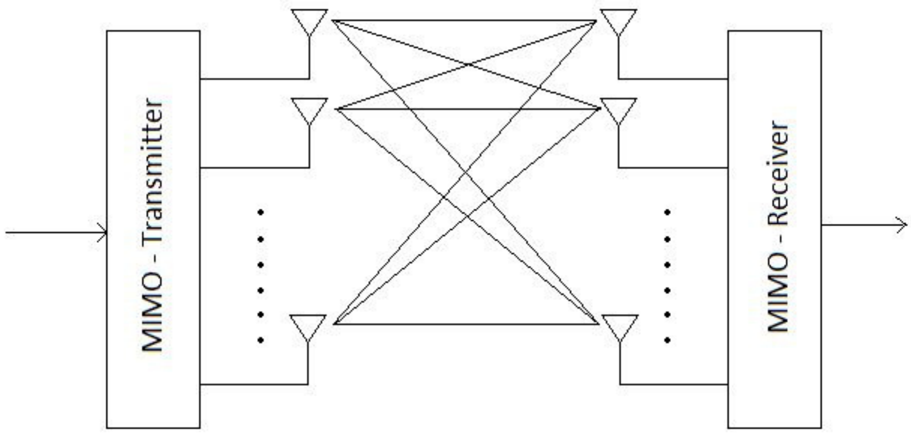

Figure 2.4. A point-to-point MIMO communication system.

Depending on the deployment of multiple antennas at the transmitter or the receiver, MIMO communications can be classified as single-input multiple-output (SIMO) systems, multiple-input single-output (MISO) systems, and MIMO systems. A further classification is point-to-point MIMO and point to multiple points MIMO, where the latter is also known as multiuser multiplexing. A wireless communication system can benefit from using MIMO communications by obtaining array gain, spatial diversity gain and spatial multiplexing gain.

Spatial Diversity Gain

As discussed in Section 2.1, channel fading caused by the random fluctuation of space and frequency could severely impact signal propagation and degrade the performance of wireless systems. Diversity is a technique used to combat the effect of channel fading. The principle of diversity is to provide the receiver multiple copies of the same signal by transmitting the signal over multiple independent fading paths. The philosophy behind this technique arises from the fact that independent paths are unlikely to suffer a fade at the same time [29]. Hence, with the increasing number of independent copies, the probability that all copies of the signal suffering a deep fade becomes low. The spatial diversity gain refers to the link reliability and can be characterized by diversity order. A MIMO system with M_t transmit antennas and M_r receive antennas can achieve a maximum spatial diversity order of M_tM_r.

Arrary Gain

Array gain is the improvement of SNR at the receiver, which can be obtained by combining signals at the receiver. Different to the spatial diversity gain, either the multiple paths are independent or correlated, the array gain can be obtained. The SNR at the receiver will increase linearly with the number of receiver antennas.

Spatial Multiplexing Gain

With the deployment of multiple antenna at both sides of the link, the data rates can be linearly increased, which is called spatial multiplexing gain. In spatial multiplexing, each transmitter antenna sends an independent data stream. At the receiver, the multiple data streams are separated and the data rate is improved.

In addition to the benefits discussed above, MIMO can also be used for interference suppression with the employment of beamforming techniques. In the following of this section, both the spatial diversity and the spatial multiplexing will be reviewed. Particular attention will be paid on beamforming techniques.

2.3.1 Spatial Diversity

Diversity technique can be accomplished in different domains. Time diversity is achieved by transmitting the same signal at different time slots with a time interval greater than the channel coherence time. It is often used to suppress the error bursts caused by time-varying channels [30]. In practice, time diversity can be achieved through coding or interleaving [4]. In frequency diversity, the same signal is transmitted at different carrier frequencies. This technique is used to overcome the frequency selective fading since there is low probability that signals suffer severe attenuation at different frequencies simultaneously. The use of frequency diversity requires higher transmission power. With the deployment of multiple antennas at the transmitter or receiver, the spatial diversity (also called space diversity) can be achieved. Instead of sacrificing time or frequency, in spatial diversity schemes, the spatial dimensions are exploited by using multiple antennas at one or both sides of the transmission link. Spatial diversity can be classified into two types: receiver diversity and transmitter diversity.

Receiver Diversity

Spatial diversity can be obtained by using multiple antennas at the receiver. If these antennas are spaced sufficiently apart (the space is greater than half of the wavelength), the signal paths corresponding to different receiver antennas can be thought of as independently faded. Here, each of these independent fading path is called a branch. Then, by combining these branches, a combined signal can be obtained at the receiver and passed to a demodulator. It is important to choose appropriate combining scheme to accommodate the complexity and overall performance.

The simplest combining strategy is to choose the branch with the highest SNR, which is called selection combining (SC). Instead of using SNR, other metrics such as absolute power and bit error rate (BER) can also be adopted [29]. Though by using SC, the highest SNR among all branches can be obtained at the output, such method needs continuous monitor on SNRs of all branches if the source signal is transmitted continuously. Hence, the implementation complexity is increased. To avoid this problem, another method called threshold combining (TC) is used. Here, a branch with the SNR higher than a threshold value will be chosen. Since there are generally more than one SNRs greater than the threshold, a preset decision rule should be applied that sequentially detect the SNR of each branch and choose the first qualified branch.

Instead of choosing a single branch, the output signal can also be obtained by linearly combining all the branches with the assignment of a weight a_i to each branch, which is called gain combining. When the weights are set as $a_i = e^{-j\omega_i}$, the combiner is called equal gain combiner (EGC). Here, ω_i is the phase of the incoming signal on the ith branch and the multiplication of $e^{-j\omega_i}$ means that signals received by different antennas are co-phased. Hence, to use EGC, knowledge of signal phases is required at the combiner [29]. In EGC, each branch has the same contribution to the array gain at the combiner output. Another linear combiner is the the maximal ratio combiner (MRC). Different to EGC, the goal of MRC is to maximize the output SNR of the combiner. Hence, branches with high SNRs should be assigned to more weights than branches with low SNRs. By letting the weight of each branch be propotional to its corresponding SNR, the output SNR is maximized. In MRC, in addition to signal phases, the knowledge of channel gains for all signal paths is also required.

Transmitter Diversity

When the transmitter is equipped with multiple antennas, diversity can also be obtained. The motivation of using transmit diversity is that in some wireless communication systems, the transmitter has more capability in terms of processing power and space to accommodate multiple antennas. For example, in cellular wireless communications, the BSs can provide more power and have stronger computational capability than mobile users. In receiver diversity, the CSI is always known by the combiner due to the fact that it can be easily estimated by the receiver. However, this is not the case at the transmitter, where the channel knowledge needs to be fed back from the receiver.

When the channel information is known at the transmitter, transmit diversity can be obtained similar to the receiver diversity. The signal is multiplied by a combining weight before it is transmitted by an antenna and the signal at the receiver is the summation of the weighted signals transmitted by all antennas. Transmit diversity can also be obtained if the CSI is not known at the transmitter. For example, in [31], a space-time coding based diversity was proposed for a digital communication system with a two antenna diversity, which is called Alamouti space-time block coding.

2.3.2 Spatial Multiplexing

In MIMO communication systems, by equipping multiple antennas at both the transmitter and the receiver, additional spatial dimension is provided for communications [32, 33]. Several data streams can be spatially multiplexed onto the MIMO channel by decomposing the MIMO channel matrix into several independent spatial sub-channels. Then, data rate can be increased through a factor that is not greater than the rank of the MIMO matrix [4,34]. Consider the point-to-point MIMO system shown in Figure 2.4. With M_t antennas at the transmitter and M_r antennas at the receiver, the received signal can be expressed as

$$\mathbf{y}(n) = \mathbf{H}\mathbf{x}(n) + \boldsymbol{\eta}(n), \tag{2.3.1}$$

where $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ is the channel matrix with the entries $h_{j,k}$ representing the complex channel gain between the jth transmitter and kth receiver; $\mathbf{x}(n) = [x_1(n), \cdots, x_{M_t}(n)]^T$ and $\mathbf{y}(n) = [y_1(n), \cdots, y_{M_r}(n)]^T$ are the transmitted signal vector and received signal vector, respectively, where $x_i(n)$ is the symbol transmitted by the ith antenna and $y_i(n)$ is the signal received by the ith antenna; $\boldsymbol{\eta}(n) \in \mathbb{C}^{M_r \times 1}$ is the noise vector at the receiver, where each $\eta_i(n)$ is AWGN. It is assumed that the channel matrix $\mathbf{H}$ is known to both the transmitter and the receiver. Then, $\mathbf{H}$ can be decomposed through the singular value decomposition (SVD) as [35]

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H, \tag{2.3.2}$$

where $\mathbf{U} \in \mathbb{C}^{M_r \times M_r}$ and $\mathbf{V} \in \mathbb{C}^{M_t \times M_t}$ are unitary singular matrices; $\boldsymbol{\Sigma} \in \mathbb{C}^{M_r \times M_t}$ is a diagonal matrix obtained from the singular values $\{v_i\}$ of $\mathbf{H}$, where $v_i = \sqrt{\lambda_i}$ and λ_i is the ith eigenvalue of matrix $\mathbf{H}\mathbf{H}^H$. By performing precoding at the transmitter, the modulated symbol stream $\tilde{\mathbf{x}}$ can be coded from $\mathbf{x}$ as

$$\mathbf{x} = \mathbf{V}\tilde{\mathbf{x}} \tag{2.3.3}$$

In addition, the received signal can be shaped through

$$\tilde{\mathbf{y}} = \mathbf{U}^H\mathbf{y}. \tag{2.3.4}$$

Such transmitter precoding and receiver shaping can decompose the MIMO channel into R_H number of independent SISO channels as shown in Figure 2.5, where R_H is the rank of matrix $\mathbf{H}$. Then, the shaped receiving signal can be written as

$$\tilde{\mathbf{y}} = \mathbf{U}^H(\mathbf{H}\mathbf{x}(n) + \boldsymbol{\eta}(n)) = \boldsymbol{\Sigma}\tilde{\mathbf{x}} + \tilde{\boldsymbol{\eta}}, \tag{2.3.5}$$

where $\tilde{\boldsymbol{\eta}} = \mathbf{U}^H\boldsymbol{\eta}$ and $\tilde{\mathbf{y}} = [\tilde{y}_1, \cdots, \tilde{y}_{R_H}]^T$. By the above channel decomposition method, the data rate can be increased up to R_H times compared to the SISO counterpart.

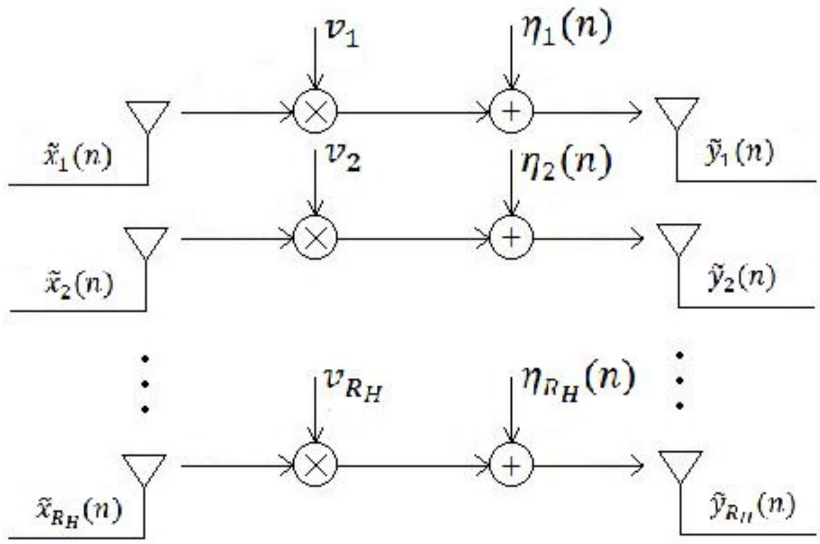

Figure 2.5. Parallel decomposition of the MIMO channel.

2.3.3 Beamforming Technique

Beamforming is a classical technique in array processing that is based on the combination of multiple antenna technique and digital signal processing. As introduced in Chapter 1, it has been widely used in both military affairs and civil communications. The principle of beamforming is to multiply a complex weight to the signal at each antenna branch before the signal is transmitted or after the signal is received. Different to the spatial diversity that aims to overcome deep fades, for beamforming, the antenna array is used to adjust the beam pattern of the transmitter or receiver to improve the channel gain towards the desired directions while minimizing the interference power at the receiver. Such characteristic is especially appropriate for the processing of multiple spatially separated signals. Hence, beamforming can be used to improve the capacity of cellular networks by supporting multiple co-channel users in each cell, where signal for each user manifests as interference to other users. In this subsection, both receiver and transmitter beamforming are discussed.

Receiver Beamforming

Receiver beamforming is employed when the receiver is equipped with multiple antennas while the transmitter has only one antenna. The objective of receiver beamforming is to extract the desired signal from the received signal which is corrupted by both noise and interfering signals. A typical receiver beamformer with M antennas is shown in Figure 2.6. By linearly combining signals from all antenna branches, the output signal of the beamformer $y(n)$ can be written as

$$y(n) = \mathbf{w}^H \mathbf{r}(n), \tag{2.3.6}$$

where n is the time index, $\mathbf{r}(n) = [r_1(n), \cdots, r_M(n)]^T$ represents the signal received by the antenna array, and $\mathbf{w} = [w_1, \cdots, w_M]^T$ is the beamformer vector. Consider a wireless communication system with J transmitting sources. The signal received by the antenna array $\mathbf{r}(n)$ is the superposition of signals from all sources and the noise at the receiver that can be expressed as

$$\mathbf{r}(n) = \mathbf{s}_d(n) + \mathbf{s}_{-d}(n) + \boldsymbol{\eta}(n), \tag{2.3.7}$$

where $\mathbf{s}_d(n) \in \mathbb{C}^{M\times 1}$ is the desired signal, $\mathbf{s}_{-d}(n) \in \mathbb{C}^{M\times 1}$ is the superposition of all signals except the desired signal, and $\boldsymbol{\eta}(n)$ represents the noise

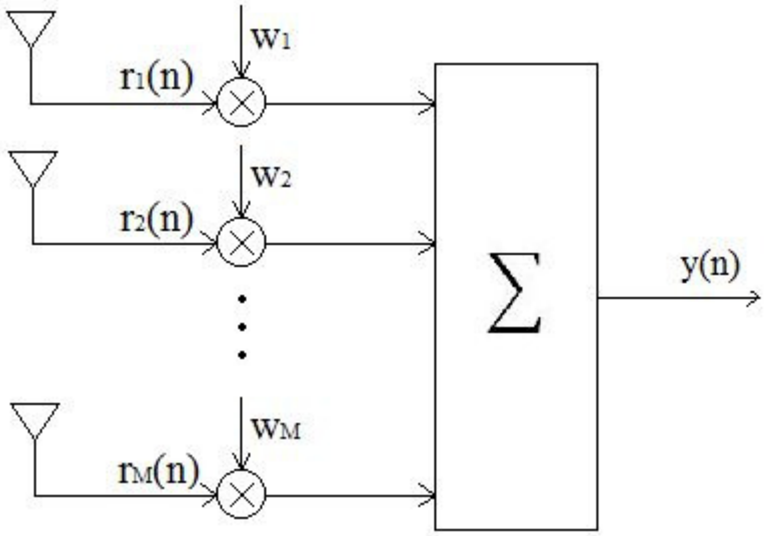

Figure 2.6. Mathematical structure of a receiver beamformer.

vector. Hence, to obtain the desired signal, the weight vector $\mathbf{w}$ needs to be designed to suppress the interference caused by other signals.

Different methods have been proposed to design beamfomers at the receiver depending on various design criteria. One of the well known algorithms is the MVDR proposed in the early stage of the development of beamforming technique [12]. The philosophy of this method is to keep the gain of the desired signal as one at the beamformer output while minimizing the interference induced by signals from all other transmitting sources. By assuming arrays at the receiver are all linearly placed and transmit sources are far away from the receiver, the desired signal $\mathbf{s}_d(n)$ can be written as

$$\mathbf{s}_d(n) = s_d(n)\mathbf{s}(\theta_d), \tag{2.3.8}$$

where θ_d is the direction of arrival of the desired signal and $\mathbf{s}(\theta_d) \in \mathbb{C}^{M\times 1}$ is the steering vector that defined as

$$\mathbf{s}(\theta_d) = [1, e^{-j\theta_d}, \cdots, e^{-j(M-1)\theta_d}]^T. \tag{2.3.9}$$

Then, for the MVDR algorithm, the receiver beamformer is given by [36]

$$\mathbf{w}_d = \frac{\mathbf{R}^{-1}\mathbf{s}(\theta_d)}{\mathbf{s}^H(\theta_d)\mathbf{R}^{-1}\mathbf{s}(\theta_d)}, \tag{2.3.10}$$

where $\mathbf{R} = E\{\mathbf{r}(n)\mathbf{r}^H(n)\}$ is the covariance matrix of signals received at the antenna array. It is obvious that the MVDR algorithm depends on the knowledge of DOA θ_d. Several methods have been developed to estimate the DOA such as MUSIC in [13], ESPRIT in [14], and the Capon method.

Another widely used beamformer design method is the minimum mean square error (MMSE) receiver arises from the concept of Wiener filter [36]. The principle of Wiener filter is to find the optimal filtering coefficients that minimize the estimation error. In receiver beamforming, the error is the difference between the desired signal and the output signal of the beamformer that can be written as

$$e(n) = d(n) - y(n) = s_d(n) - \mathbf{w}^H\mathbf{r}(n). \tag{2.3.11}$$

The purpose is to find a set of beamformer weights that minimizes the MSE. Hence, the MMSE beamformer is given by [36]

$$\mathbf{w}_{opt} = \mathbf{R}^{-1}\mathbf{P}, \tag{2.3.12}$$

where $\mathbf{R} = E\{\mathbf{r}(n)\mathbf{r}^H(n)\}$ and $\mathbf{P} = E\{\mathbf{r}(n)s_d(n)\}$ is the cross correlation vector between the desired signal and the signal received at the antenna array. Since the optimal solution only depends on the covariance matrix $\mathbf{R}$ and the cross correlation vector $\mathbf{P}$, in practice, by transmitting training signals to the receiver, the MMSE beamformer can be determined. Beamformer at the receiver can also be designed by maximizing the SINR of the desired signal. For the desired signal $s_d(n)$, the SINR at the output of the beamformer is given by

$$\mathrm{SINR}_d = \frac{\mathbf{w}^H \mathbf{R}_d \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{-d} \mathbf{w}}, \tag{2.3.13}$$

where $\mathbf{R}_d = E\{\mathbf{s}_d(n)\mathbf{s}_d^H(n)\}$ and $\mathbf{R}_{-d} = E\{[\mathbf{s}_{-d}(n)+\boldsymbol{\eta}(n)][\mathbf{s}_{-d}(n)+\boldsymbol{\eta}(n)]^H\}$. Then, the optimal beamformer is equivalent to the generalized eigenvector of $[\mathbf{R}_d,\ \mathbf{R}_{-d}]$ that satisfies [37]

$$\mathbf{R}_{-d}^{-1}\mathbf{R}_d\mathbf{w} = \lambda_{max}\mathbf{w}, \tag{2.3.14}$$

where λ_{max} is the largest eigenvalue of the matrix $\mathbf{R}_{-d}^{-1}\mathbf{R}_d$.

Transmitter Beamforming

Contrary to the receiver beamforming, in transmitter beamforming, multiple antennas are equipped at the transmitter and each receiver is only equipped with one antenna. In cellular wireless systems, the use of transmitter beamforming can bring special benefits. On one hand, since the throughput in the downlink is typically much higher than that in the uplink, transmitter beamforming at the BS can optimize the downlink transmission and significantly improve the system performance; on the other hand, due to the higher capability on data processing, it is easier to realize beamforming at the BS than at the user terminals. There are several differences between designing transmitter beamforming and receiver beamforming. In the latter, beamformer for a specific user can be indepedently designed since it will not affect the performance of the other users. In transmitter beamforming, to avoid the interference to the unintended users covered in the same area, beamformers for all users must be jointly designed. In addition, for receiver beamforming, by using training signals, channel coefficients can be directly estimated by the receiver. In transmitter beamforming, however, such knowledge need to be obtained by receiving the CSI from the receiver through a feedback channel [38–41].

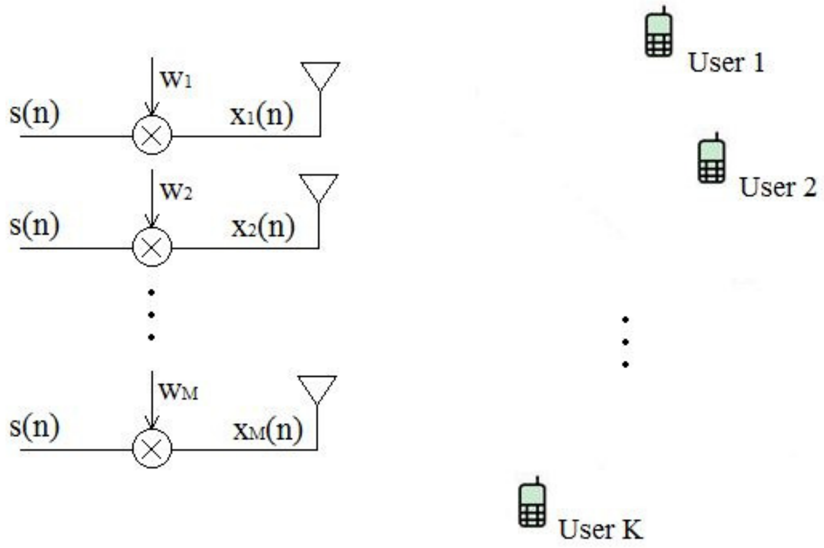

Figure 2.7. Transmitter beamformer with multiple users.

Consider a cellular wireless network with one BS serving K users. It is assumed that the BS is equipped with M antennas and each users only has one antenna. Then, the transmitter beamforming at the BS is shown in Figure 2.7, and the received signal at the kth user is given by

$$y_k(n) = \mathbf{h}_k^H \mathbf{x}(n) + \eta_k(n), \tag{2.3.15}$$

where $\eta_k(n)$ is a AWGN received at user k, $\mathbf{h}_k \in \mathbb{C}^{M\times 1}$ is the channel coefficient vector between the BS and the kth user, and $\mathbf{x}(n)$ is the signal transmitted by the BS with the following expression

$$\mathbf{x}(n) = \mathbf{W}\mathbf{s}(n), \tag{2.3.16}$$

where $\mathbf{s}(n) = [s_1(n), \cdots, s_k(n)]^T$ and $s_k(n)$ is the signal symbol for the kth user for all $k \in \{1, \ldots, K\}$, $\mathbf{W} = [\mathbf{w}_1, \cdots, \mathbf{w}_K]$ and $\mathbf{w}_k \in \mathbb{C}^{M\times 1}$ is the transmitter beamformer vector for the kth user. To design beamformers at the transmitter, various methods with different design criteria have been proposed. The simplest method is the zero forcing beamforming, in which the beamformer for a specific user manifests as null to the channels of all other users [18, 19, 42]. To perform zero forcing, the received signals at all users are stacked in one vector as

$$\mathbf{y} = \mathbf{H}\mathbf{x}(n) + \boldsymbol{\eta}(n), \tag{2.3.17}$$

where $\mathbf{H} = [\mathbf{h}_1, \cdots, \mathbf{h}_K]^T$ is the channel matrix for all users and $\boldsymbol{\eta}(n)$ is the noise vector. Then, the SINR at the kth user can be written as

$$\mathrm{SINR}_k = \frac{[\mathbf{HW}]^2_{k,k}}{\sum_{j\neq k}[\mathbf{HW}]^2_{k,j} + \sigma_k^2}, \tag{2.3.18}$$

where σ_k^2 is the noise variance for the kth user.

The objective of the zero forcing beamforming is to design beamformers for all users to force the interference between users as zero, i,e, $[\mathbf{HW}]^2_{k,j} = 0$ for all $j \neq k$. By assuming $E\{\mathbf{s}(n)\mathbf{s}^H(n)\} = \mathbf{I}$ and $[\mathbf{HW}]^2_{k,k} \geq 0$, for all $k \in \{1, \ldots, K\}$, if the zero forcing is achievable, then the following condition should hold:

$$\mathbf{HW} = \mathrm{diag}\{\boldsymbol{\tau}\}, \tag{2.3.19}$$

where $\boldsymbol{\tau} = [\sqrt{\tau_1}, \cdots, \sqrt{\tau_K}]^T$. Then, the MISO channels can be decoupled into K independent sub-channels as

$$y_k(n) = \sqrt{\tau_k} s_k(n) + \eta_k(n), \; k = 1, \ldots, K. \tag{2.3.20}$$

In zero forcing, the vector $\boldsymbol{\tau}$ is the diagonal elements of the matrix $\mathbf{HW}$. Hence, it is also known as block diagonalization [43, 44]. Though zero forcing can completely mitigate the interference between all users, a significant drawback of this method is that it requires the number of antennas at the transmitter greater than the number of users [45]. Once the number of antennas is less than the number of users, the antenna array does not have sufficient degree of freedom to steer beams and mitigate interference among all users.

Instead of using zero forcing, another commonly used method in transmitter beamforming is to minimize the total transmission power at the BS while allowing users to achieve a set of target SINRs, which can be expressed as [20, 46]

$$\begin{aligned} &\min_{\mathbf{w}_l} \quad \sum_{l=1}^{K} \|\mathbf{w}_l\|_2^2 \\ &\text{subject to} \quad \frac{\mathbf{w}_k^H \mathbf{R}_k \mathbf{w}_k}{\sum_{l \neq k}^{K} \mathbf{w}_l^H \mathbf{R}_k \mathbf{w}_l + \sigma_k^2} \geq \gamma_k, \ k = 1, \ldots, K, \end{aligned} \tag{2.3.21}$$

where γ_k is the SINR target for the kth user and $\mathbf{R}_k = \mathbf{h}_k \mathbf{h}_k^H$. Since problem (2.3.21) is a quadratically constrained nonconvex problem, it can not be directly solved. Hence, (2.3.21) is always converted into a SDP problem by introducing Lagrangian relaxation, and then can be directly solved using existing convex optimization tool boxes [47–49]. A problem induced by using this method is that if the total allowable transmission power at the BS is constrained by some value, the optimization problem may be infeasible.

To overcome this drawback, another framework has been proposed to achieve the fairness for all users by maximizing the SINR of worst-case user under a certain total power constraint at the BS [37, 50, 51]. This method is also known as SINR balancing technique that can be expressed as

$$\begin{aligned} &\max_{\mathbf{U},\mathbf{p}} \min_{k} \quad \frac{\mathrm{SINR}_k(\mathbf{U}, \mathbf{p})}{\delta_k}, \ k = 1, \cdots, K, \\ &\qquad\qquad \mathbf{1}^T \mathbf{p} \leq \mathrm{P}_{max}, \end{aligned} \tag{2.3.22}$$

where δ_k and P_{max} are the weighting factor for the kth user and the total allowable transmission power, respectively. In (2.3.22), $\mathbf{U} = [\mathbf{u}_1, \cdots, \mathbf{u}_k]$ where $\mathbf{u}_k$ is the beam patter vector for the kth user with $\|\mathbf{u}_k\|_2 = 1$, $\mathbf{p} = [p_1, \cdots, p_K]^T$ is the power allocation vector for all users and p_k is the allocated power for the kth user with $\mathbf{w}_k = \sqrt{p_k}\mathbf{u}_k$. Since problem (2.3.22) is a quasiconvex problem that difficult to be directly solved, an iterative based method has been proposed [52], in which the problem is converted into iteratively updating power allocation and beamformers in the uplink.

2.4 Power Control in Wireless Communication Networks

Transmission power is a valuable resource in wireless communications. How to assign the power hence becomes a significant issue in designing wireless communication systems. In cellular wireless networks, power control comprises of managing and adjusting transmission power for both BSs and user terminals. In wireless communications, specific SINR should be achieved at the output of the receiver to guarantee the QoS for communication. To improve the SINR for a specific link, a simple way is to increase the transmission power for this link. However, due to the employment of frequency reuse scheme, the increase of power for a link will induce more CCI to other links and degrade their QoS. Hence, the objective of power control is to guarantee the communication quality for a certain link while reducing the CCI to other links.

Various of power control schemes have been proposed. A simple classification on the power control schemes is based on how to measure the communication quality., i.e., power strength-based, SINR-based and BER-based power control, where the QoS is measured based on signal strength, SINR and BER, respectively. Then, the control of transmission power depends on whether the obtained measurement is higher than a threshold value.

In cellular wireless communications, based on directions of communication, power control can be classified as uplink power control and downlink power control. Power control in the uplink is one of the most important requirements for mobile communication systems. On one hand, the uplink transmit power cannot be arbitrarily large since it is limited by the battery used at the mobile terminals. On the other hand, for those systems suffering near-far effect, without uplink power control, the link quality of the user far away from the BS will be severely interfered by the user near to the BS [53]. The main challenge of uplink power control is the low computational capability of mobile terminals. Different to the uplink case, in the downlink, the main concern is to reduce the interference to other cells. The advantage of downlink power control is the strong computational capability of BSs. However, the complex optimization process will cause extra communication overhead. In the following, algorithms for both uplink and downlink power control schemes will be discussed.

2.4.1 Power Control in the Downlink

In cellular wireless networks, if there is a centralized controller that has the knowledge of all channels in the system, a centralized scheme can be used for the power control in the downlink. Consider a multicell network with J cells, in which each BS serves only one mobile user. It is assumed that all BSs and mobile users are equipped with only one antenna and all links are on the same frequency channel. Then, the SINR at the user in the ith cell is given by

$$\Gamma_i = \frac{h_{i,i}p_i}{\sum_{j\neq i} h_{j,i}p_j + \sigma_i^2}, \tag{2.4.1}$$

where $h_{j,i}$ is the channel gain between the jth BS and the user in the ith cell, p_i is the power transmitted by the ith BS, and σ_i^2 represents the noise variance at the user served by the ith BS. The simplest standard for designing such a network is to let all users achieve the same target SINR γ_0. In this case, the objective of power control is to keep SINRs for all users over γ_0 by adjusting transmission power of all BSs. This SINR balancing based power control has been well studied in [54], which can be solved through the maximal eigenvalue based method [55]. Following the problem proposed in [54], a more general case is considered in which each user is allowed to achieve a specific target SINR while the total transmission power over all BSs is minimized. This problem can be stated as

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{J} p_i \\ \text{subject to} \quad & \Gamma_i \geq \gamma_i, \ i = 1, \ldots, J, \end{aligned} \tag{2.4.2}$$

where γ_i is the target SINR for the user in the ith cell. This problem can be solved by writing all SINR constraints into a matrix form as

$$(\mathbf{I} - \mathbf{DF})\mathbf{p} \geq \mathbf{d}, \tag{2.4.3}$$

where $\mathbf{p} = [p_1, \cdots, p_J]^T$ is the power allocation vector for all users, $\mathbf{D} = \text{diag}\{\gamma_1, \cdots, \gamma_J\}$, $\mathbf{F}$ is a matrix with the following structure

$$\mathbf{F} = \begin{cases} \frac{h_{j,i}}{h_{i,i}} > 0, & \text{if } j \neq i \\ 0, & \text{if } j = i, \end{cases}$$

and $\mathbf{d} \in \mathbb{R}^{J \times 1}$ is a vector whose elements are defined as

$$d_i = \frac{\gamma_i \sigma_i^2}{h_{i,i}}. \tag{2.4.4}$$

Then, problem (2.4.2) is converted to the following optimization problem

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{J} p_i \\ \text{subject to} \quad & (\mathbf{I} - \mathbf{DF})\mathbf{p} \geq \mathbf{d} \end{aligned} \tag{2.4.5}$$

It has been proved that if the spectral radius of $\mathbf{DF}$ is less than one, $(\mathbf{I}-\mathbf{DF})$ is an invertible matrix [55]. This means that problem (2.4.5) is feasible and the optimal solution can be obtained when equality holds for (2.4.3). Then, the optimal power allocation is given by

$$\mathbf{p}^* = (\mathbf{I} - \mathbf{DF})^{-1}\mathbf{d}. \tag{2.4.6}$$

Here, the optimal power allocation means that for any other feasible power allocation $\mathbf{p}$, it satisfies

$$\mathbf{p}^* \leq \mathbf{p}. \tag{2.4.7}$$

2.4.2 Power Control in the Uplink

Different to the downlink power control, in the uplink, each user only knows its local information and updates its own power. Hence, distributed based algorithms should be used [53]. Consider the same multicell network discussed in the downlink case. Denote q_i as the transmit power of the user in the ith cell. The uplink SINR for the user in the ith cell is given by

$$\Lambda_i = \frac{h_{i,i}q_i}{\sum_{j\neq i} h_{i,j}q_j + \sigma^2}. \tag{2.4.8}$$

The objective of power control in the uplink is to meet the SINR target of each user by letting each user control its own power. This problem can be solved by employing the algorithm proposed in [56], in which the SINR constraints are converted into corresponding constraints as

$$\mathbf{q} \geq \mathbf{b}(\mathbf{q}), \tag{2.4.9}$$

where $\mathbf{q} = [q_1, \cdots, q_J]^T$ is the transmitted power vector in the uplink and $\mathbf{b}(\mathbf{q}) = [b_1(\mathbf{q}), \cdots, b_J(\mathbf{q})]^T$, where $b_i(\mathbf{q})$ is the effective interference from other users to user i, which can be defined as

$$b_i(\mathbf{q}) = \frac{\gamma_i(\sum_{j\neq i} h_{i,j}q_j + \sigma^2)}{h_{i,i}}. \tag{2.4.10}$$

Hence, the feasibility of the system is equivalent to whether the inequality (2.4.9) can hold.

To analyze the convergence of the algorithm, a significant concept named *standard function* has been proposed in [56], which can be defined as:

Definition 2.4.1. (Standard function) [56]: *The interference function* $\mathbf{b}(\mathbf{q})$ *is said to be standard if for all* $\mathbf{q} \geq 0$*, it possesses the following properties:*

- *Positivity:* $\mathbf{b}(\mathbf{q}) > 0$.
- *Monotonicity: If* $\mathbf{q}_1 \geq \mathbf{q}_2$*, then* $\mathbf{b}(\mathbf{q}_1) \geq \mathbf{b}(\mathbf{q}_2)$.
- *Scalability: For any* $\alpha > 1$, $\alpha\mathbf{b}(\mathbf{q}) > \mathbf{b}(\alpha\mathbf{q})$.

If $\mathbf{b}(\mathbf{q})$ is a standard function, the optimal uplink power vector can be obtained by iteratively updating the following equation

$$\mathbf{q}(t+1) = \mathbf{b}(\mathbf{q}(t)). \tag{2.4.11}$$

In addition, according to [56], the iterative based uplink power control algorithm has the following properties:

- If a fixed convergence point exists, the fixed point is unique.
- For both synchronous and asynchronous cases, the algorithm can converge.

2.5 Summary

This chapter introduced various spatial diversity techniques and power control methods for wireless networks. Particular focus has been on the beamforming techniques and power control methods using convex optimization techniques. These methods will be used in later chapters for coordinated multicell processing and beamforming.

Chapter 3

CONVEX OPTIMIZATION TECHNIQUE AND GAME THEORY

In this chapter, the convex optimization technique and game theory are reviewed. The basic concepts of these techniques are briefly discussed. For convex optimization, the formulation of canonical convex problems and quasi-convex problems are introduced. For game theory, both the non-cooperative games and cooperative games are studied. The combination of these techniques gives the formulation and solution for the beamforming problems proposed in this thesis.

3.1 Convex Optimization

Mathematical optimization has been developing for several decades and being everywhere from engineering to daily life [49,57]. When studying communications and signal processing, mathematical optimization is an undoubtedly indispensable technique since many problems in these areas can be formulated into optimization problems with appropriate constraints [49,58,59]. One of the widely used optimization methods is the convex optimization. The main advantage of a convex problem is that it can be efficiently solved using interior point method [49,60]. In addition, the emerge of software tool boxes for solving convex optimization problems has further reinforced its attraction [47,48,61]. Hence, when solving a generic optimization problem, an efficient way is to reformulate it into a convex form with appropriate mathematical manipulations [58,59].

3.1.1 Basic Concepts of Convex Optimization

In this subsection, the basic concepts in convex optimization are briefly introduced.

Convex Sets

Definition[section]

Definition 3.1.1. (Convex set) [59]: *A set $\mathcal{K} \in \mathbb{R}^n$ is said to be convex if the following condition is satisfied:*

$$\theta \mathbf{x}_1 + (1-\theta)\mathbf{x}_2 \in \mathcal{K}, \tag{3.1.1}$$

where $\mathbf{x}_1, \mathbf{x}_2 \in \mathcal{K}$ and $\theta \in [0,1]$.

As shown in Figure 3.1, a convex set means that for any two points in the set, the connection line of the two points still lies in the set. A typical convex set is the Euclidean ball, which is defined as

$$\mathcal{B}(\mathbf{x}_c, r) = \{\mathbf{x} \mid \|\mathbf{x} - \mathbf{x}_c\|_2 \leq r\}, \tag{3.1.2}$$

where vector $\mathbf{x}_c$ is the center point of the ball and r is the radius of the ball where $r > 0$. It is important to differentiate whether a set is convex or not when solving an optimization problem.

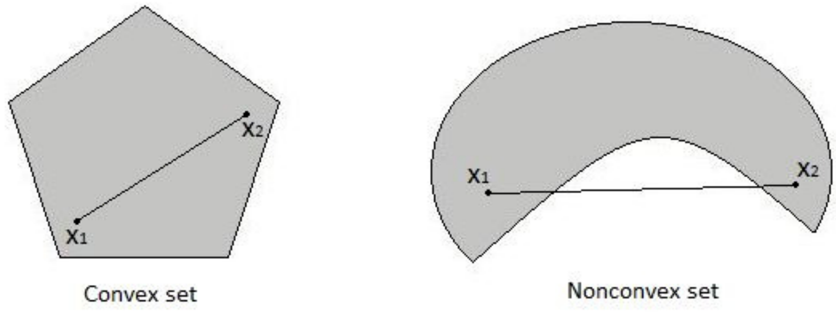

Figure 3.1. Comparison of convex set and nonconvex set.

Convex Functions

Definition 3.1.2. (Convex function) [49]: *A function* $f(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ *is said to be a convex function if for any two points* $\mathbf{x}_1, \mathbf{x}_2 \in \textbf{\textit{dom}}\ f(\mathbf{x})$*, the following condition is satisfied:*

$$f(\theta \mathbf{x}_1 + (1-\theta)\mathbf{x}_2) \leq \theta f(\mathbf{x}_1) + (1-\theta) f(\mathbf{x}_2), \tag{3.1.3}$$

where $\textbf{\textit{dom}}\ f(\mathbf{x})$ *is a convex set and* $\theta \in [0, 1]$*.*

Inequality (3.1.3) means that for a convex function, the connection line between any $(\mathbf{x}_1, f(\mathbf{x}_1))$ and $(\mathbf{x}_2, f(\mathbf{x}_2))$ should lie above the $f(\mathbf{x})$. Based on the definition of convex function, a function $f(\mathbf{x})$ is said to be concave if $-f(\mathbf{x})$ is convex.

An equivalent way to define a convex function is based on the differentiability of function $f(\mathbf{x})$. If $f(\mathbf{x})$ is differentiable, then $f(\mathbf{x})$ is convex if and only if the following inequality is satisfied

$$f(\mathbf{x}_1) \geq f(\mathbf{x}_2) + \nabla f(\mathbf{x}_2)^T (\mathbf{x}_1 - \mathbf{x}_2), \tag{3.1.4}$$

where $\mathbf{x}_1, \mathbf{x}_2 \in \mathbf{dom}\ f(\mathbf{x})$ and $\mathbf{dom}\ f(\mathbf{x})$ is a convex set. Inequality (3.1.4) is also called the first-order condition, which means that the first-order Taylor approximation of $f(\mathbf{x})$ is a *global underestimator* [49].

In addition, if function $f(\mathbf{x})$ is twice differentiable, the convexity of function

$f(\mathbf{x})$ is further equivalent to satisfy [49]

$$\nabla^2 f(\mathbf{x}) \succeq 0, \tag{3.1.5}$$

where $\mathbf{dom}\ f(\mathbf{x})$ is a convex set and $\mathbf{x} \in \mathbf{dom}\ f(\mathbf{x})$. The above inequality means that a function $f(\mathbf{x})$ is convex if and only if $\mathbf{dom}\ f(\mathbf{x})$ is a convex set and the second derivative of $f(\mathbf{x})$ is positive semidefinite [49].

Sublevel Sets

Definition 3.1.3. (Sublevel set) [62]: *For a function $f(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$, the β-sublevel set of $f(\mathbf{x})$ is defined as*

$$\mathcal{K}_\beta = \{\mathbf{x} \in \boldsymbol{dom}\ f(\mathbf{x}) \mid f(\mathbf{x}) \leq \beta\}. \tag{3.1.6}$$

If $f(\mathbf{x})$ is a convex function, all possible β-sublevel sets of $f(\mathbf{x})$ are convex. However, if all possible β-sublevel sets of $f(\mathbf{x})$ are convex, it can not guarantee that $f(\mathbf{x})$ is a convex function [49]. For example, $f(x) = -a^x$, $x \in \mathbb{R}$ with $a > 0$, is a concave function though any of its β-sublevel sets are convex.

Quasiconvex functions

Definition 3.1.4. (Quasiconvex function) [63]: *A function $f(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ is said to be quasiconvex if $\boldsymbol{dom}\ f(\mathbf{x})$ is a convex set and its β-sublevel sets are convex for all $\beta \in \mathbb{R}$.*

For a convex function, since all its sublevel sets are convex, it is also quasiconvex. A typical example of quasiconvex function is the $log(x)$ on $\mathbb{R}_+$. The comparison of convexity and quasiconvexity is shown in Figure 3.2.

Quasiconvexity can also be defined through Jenson's inequality similar to the convexity. Then, a function $f(\mathbf{x})$ is quasiconvex if $\mathbf{dom}\ f(\mathbf{x})$ is a convex set and

$$f(\theta\mathbf{x}_1 + (1-\theta)\mathbf{x}_2) \leq max\{f(\mathbf{x}_1), f(\mathbf{x}_2)\}, \tag{3.1.7}$$

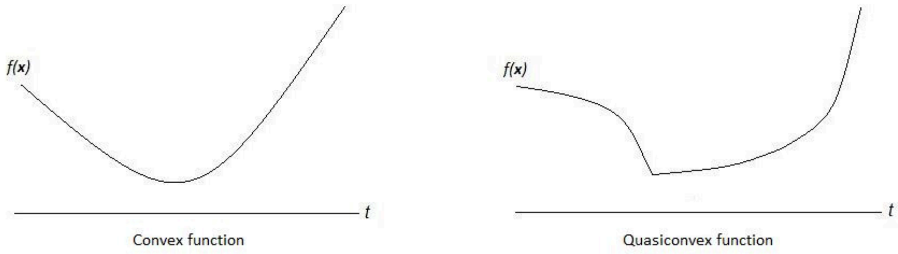

Figure 3.2. Comparison of convex function and quasiconvex function.

where $\mathbf{x}_1, \mathbf{x}_2 \in \textbf{dom } f(\mathbf{x})$ and $\theta \in [0, 1]$ [49]. The above definition means that for a quasiconvex function, the value of a segment between any two points is always smaller than the maximum value of the two endpoints.

3.1.2 Convex Optimization Problems

The standard form of optimization problems is expressed as [49]

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f_0(\mathbf{x}) \\ \text{subject to} \quad & f_i(\mathbf{x}) \leq 0, \qquad i = 1, \ldots, m, \\ & h_i(\mathbf{x}) = 0, \qquad i = 1, \ldots, p, \end{aligned} \tag{3.1.8}$$

where $f_0(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ is the objective function, $\mathbf{x} \in \mathbb{R}^n$ is the optimization variable, $f_i(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ are inequality constraint functions, and $h_i(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ are equality constraint functions. The domain of the optimization problem (3.1.8) is defined as

$$\mathcal{D}_c = \bigcap_{i=0}^{m} \textbf{dom } f_i \cap \bigcap_{i=0}^{p} \textbf{dom } h_i. \tag{3.1.9}$$

A point $\mathbf{x} \in \mathcal{D}_c$ is said to be feasible if it can satisfy all constraints in (3.1.8). If a feasible point exists, then problem (3.1.8) is feasible. If problem (3.1.8) is feasible, the optimal value of (3.1.8) is given by

$$f_0^\star = \inf\{f_0(\mathbf{x}) \mid f_i(\mathbf{x}) \le 0,\ i = 1, \cdots, m,\ h_i(\mathbf{x}) = 0,\ i = 1, \cdots, p\}. \quad (3.1.10)$$

The optimal point $\mathbf{x}^\star$ of (3.1.8) is a point in $\mathcal{D}_c$, which is feasible and satisfies $f_0(\mathbf{x}^\star) = f_0^\star$. In the following, some standard form convex optimization problems are reviewed. The advantage of the standard form convex optimization is that it can be easily solved using software packages [48].

Linear Programming

The simplest form of convex optimization is the linear programming (LP), which is generally expressed as

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \mathbf{c}^T\mathbf{x} + d \\ \text{subject to} \quad & \mathbf{G}\mathbf{x} \preceq \mathbf{h}, \\ & \mathbf{A}\mathbf{x} = \mathbf{b}, \end{aligned} \quad (3.1.11)$$

where $\mathbf{G} \in \mathbb{R}^{m\times n}$ and $\mathbf{A} \in \mathbb{R}^{p\times n}$. In LP problem, both the objective function and constraint functions are affine.

Quadratic Programming

A quadratic programming (QP) optimization has the following form

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & (1/2)\mathbf{x}^T\mathbf{P}\mathbf{x} + \mathbf{q}^T\mathbf{x} + r \\ \text{subject to} \quad & \mathbf{G}\mathbf{x} \preceq \mathbf{h}, \\ & \mathbf{A}\mathbf{x} = \mathbf{b}, \end{aligned} \quad (3.1.12)$$

where $\mathbf{P} \in \mathbb{S}_+^n$, $\mathbf{G} \in \mathbb{R}^{m\times n}$, and $\mathbf{A} \in \mathbb{R}^{p\times n}$. In QP, the objective function is quadratic and all constraint functions are affine. It can be found that LP is a special case of QP if matrix $\mathbf{P}$ is set to $\mathbf{0}$. By replacing the inequality constraint of (3.1.12) with a set of quadratic inequality constraints, the QP problem can be transferred into a quadratically constrained quadratic programming (QCQP) problem, which has the following standard form [49]

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & (1/2)\mathbf{x}^T\mathbf{P}_0\mathbf{x} + \mathbf{q}_0^T\mathbf{x} + r_0 \\ \text{subject to} \quad & (1/2)\mathbf{x}^T\mathbf{P}_i\mathbf{x} + \mathbf{q}_i^T\mathbf{x} + r_i \leq 0, \qquad i = 1, \ldots, m, \\ & \mathbf{A}\mathbf{x} = \mathbf{b}, \end{aligned} \tag{3.1.13}$$

where $\mathbf{P}_i \in \mathbb{S}_+^n, \quad i = 0, \cdots, m$. Different to the QP that minimizes a quadratic function over a polyhedron, in QCQP, a quadratic function is minimized over a feasible region which is obtained by intersecting a set of ellipsoids.

Second-Order Cone Programming

A second-order cone programming (SOCP) has the following standard form

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \mathbf{f}^T\mathbf{x} \\ \text{subject to} \quad & \|\mathbf{A}_i\mathbf{x} + \mathbf{b}_i\|_2 \leq \mathbf{c}_i^T\mathbf{x} + \mathbf{d}_i, \qquad i = 1, 2, \ldots, m, \\ & \mathbf{F}\mathbf{x} = \mathbf{g}, \end{aligned} \tag{3.1.14}$$

where $\mathbf{A}_i \in \mathbb{R}^{n_i \times n}$ and $\mathbf{F} \in \mathbb{R}^{p \times n}$. The inequality constraints in (3.1.14) are called second-order cone constraints. SOCP is a more general form of optimization than QCQP and LP since (3.1.14) can be reduced to a QCQP by setting $\mathbf{c}_i = \mathbf{0}, \quad i = 1, \ldots, m$, or transferred into a LP by setting $\mathbf{A}_i = \mathbf{0}, \quad i = 1, \ldots, m$.

Geometric Programming

Geometric programming (GP) problems are not convex in their natural form. However, by changing variables and transforming both objective function and constraint functions, GP can be transferred into convex form optimization problems. To begin with, the concepts of monomial function and posynomial function are first introduced.

Definition 3.1.5. (Monomial function) [49]: *A monomial function* $f(\mathbf{x}) : \mathbb{R}^n_{++} \to \mathbb{R}$ *is defined as*

$$f(\mathbf{x}) = cx_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}, \tag{3.1.15}$$

where $c > 0$ *and* $\textbf{\textit{dom}}\ f = \mathbb{R}^n_{++}$.

The exponents $a_i,\ i = 1, \cdots, n$, could be any real numbers.

Definition 3.1.6. (Posynomial function) [49]: *A posynomial function is the summation of a set of monomials with the following expression*

$$f(\mathbf{x}) = \sum_{k=1}^{K} c_k x_1^{a_{1k}} x_2^{a_{2k}} \cdots x_n^{a_{nk}}, \tag{3.1.16}$$

where $c_k > 0$.

Then, a standard form GP problem is given as

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f_0(\mathbf{x}) \\ \text{subject to} \quad & f_i(\mathbf{x}) \leq 1, \quad i = 1, \cdots, m, \\ & h_i(\mathbf{x}) = 1, \quad i = 1, \cdots, p, \end{aligned} \tag{3.1.17}$$

where all inequality constraint functions $f_i(\mathbf{x})$ are posynomials and all equality constraint functions $h_i(\mathbf{x})$ are monomials.

Semidefinite Programming

Semidefinite programming is a generalization of the standard form convex optimization problem, which has the following form:

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{x}_1\mathbf{F}_1 + \mathbf{x}_2\mathbf{F}_2 + \ldots + \mathbf{x}_n\mathbf{F}_n + \mathbf{G} \preceq \mathbf{0}, \\ & \mathbf{A}\mathbf{x} = \mathbf{b}, \end{aligned} \tag{3.1.18}$$

where $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{A} \in \mathbb{R}^{p\times n}$. The inequality constraint in (3.1.18) is called linear matrix inequality (LMI) with $\mathbf{G}, \mathbf{F}_1, \ldots, \mathbf{F}_n \in \mathbb{S}^k$. Compared to other canonical optimization problems, SDP is a more general form. For example, if $\mathbf{G}, \mathbf{F}_1, \ldots, \mathbf{F}_n$ are all diagonal matrices, the SDP problem (3.1.18) reduces to a LP.

By introducing symmetric matrices $\mathbf{C}, \mathbf{A}_1, \ldots, \mathbf{A}_p \in \mathbb{S}^n$, the standard form SDP is expressed as

$$\begin{aligned} \underset{\mathbf{X}}{\text{minimize}} \quad & \boldsymbol{tr}(\mathbf{C}\mathbf{X}) \\ \text{subject to} \quad & \boldsymbol{tr}(\mathbf{A}_i\mathbf{X}) = b_i, \quad i = 1, \ldots, p \\ & \mathbf{X} \succeq \mathbf{0}, \end{aligned} \tag{3.1.19}$$

where $\mathbf{X} \in \mathbb{S}^n$ is the optimization variable.

3.1.3 Quasiconvex Optimization Problem

The standard form quasiconvex optimization problem is expressed as

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f_0(\mathbf{x}) \\ \text{subject to} \quad & f_i(\mathbf{x}) \le 0, \qquad i = 1, \ldots, m, \\ & \mathbf{A}\mathbf{x} = b, \end{aligned} \tag{3.1.20}$$

where the objective function $f_0(\mathbf{x})$ is quasiconvex and the inequality constraint functions are convex. The convexity of the inequality constraint functions is based on the fact that a quasiconvex function can be replaced by the corresponding convex functions [49].

The main difference between convex optimization and quasiconvex optimization is that in quasiconvex optimization, local optimal solutions are not globally optimal [49]. If the objective function $f_0(\mathbf{x})$ of (3.1.20) is differentiable, a solution $\mathbf{x} \in \mathcal{X}_q$ is said to be optimal if for all $\mathbf{y} \in \mathcal{X}_q$, the following conditions are satisfied [49]

$$\nabla f_0(\mathbf{x})^T(\mathbf{y} - \mathbf{x}) > 0, \tag{3.1.21}$$

where $\mathcal{X}_q$ is the feasible set of the quasiconvex optimization problem. It should be noticed that (3.1.21) is a sufficient condition rather than a necessary and sufficient condition due to the local optimality in quasiconvex optimization. In addition, the condition (3.1.21) holds only if $\nabla f(\mathbf{x}_0)$ is nonzero.

Bisection Method for Quasiconvex Optimization

A quasiconvex optimization problem can be solved by solving a sequence of convex optimization problems. This relies on the fact that the sublevel sets of a quasiconvex function can be represented by a family of convex inequalities. Define $\phi_t(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}, t \in \mathbb{R}$, as a family of convex functions with the following property:

$$f_0(\mathbf{x}) \leq t \iff \phi_t(\mathbf{x}) \leq 0, \tag{3.1.22}$$

where $\phi_t(\mathbf{x})$ is a non-increasing function of t for a given $\mathbf{x}$. Then the quasiconvex optimization problem (3.1.20) can be transferred to consider the following feasibility problem

$$\begin{aligned} \text{find} \quad & x \\ \text{subject to} \quad & \phi_t(\mathbf{x}) \leq 0 \\ & f_i(\mathbf{x}) \leq 0, \qquad i = 1, \ldots, m \\ & \mathbf{A}\mathbf{x} = b. \end{aligned} \tag{3.1.23}$$

Since all inequality constraint functions in (3.1.23) are convex and the equality constraints are linear, (3.1.23) is a convex feasibility problem. Let $p^\star$ be the optimal value of the quasiconvex optimization problem (3.1.20). If (3.1.23) is feasible, $p^\star \leq t$; if (3.1.23) is infeasible, $p^\star \geq t$. For a given t, $p^\star \leq t$ means that any feasible point $\mathbf{x}$ is feasible for the quasiconvex optimization problem (3.1.20).

By solving the feasibility problem (3.1.23) at each step, the quasiconvex optimization problem (3.1.20) can be solved using the bisection method [49,64]. This method is based on the assumption that problem (3.1.23) is feasible and $p^\star$ lies in an initial interval $[l, u]$. Let $t = (l + u)/2$ and solve the feasibility problem (3.1.23). If $p^\star \leq t$, the optimal value is in the lower half of the interval, then update the interval $[l, u]$ by reducing u. If $p^\star \geq t$, the optimal value is in the upper half of the interval, then update the interval $[l, u]$ by increasing l. The optimal $p^\star$ value can be obtained once $u - l \leq \epsilon$, where $\epsilon > 0$ is a tolerance value [64].

3.1.4 Lagrangian Duality

By introducing a set of weights, the objective function and all constraints in (3.1.8) can be integrated into one function, which is called Lagrangian duality. The Lagrangian $L : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^p \to \mathbb{R}$ for problem (3.1.8) is defined as

$$L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^{p} \nu_i h_i(\mathbf{x}), \tag{3.1.24}$$

where the domain of (3.1.24) is $\mathcal{D}_c \times \mathbb{R}^m \times \mathbb{R}^p$ and $\mathcal{D}_c$ is the domain of problem (3.1.8). $\boldsymbol{\lambda}$ and $\boldsymbol{\nu}$ are dual variables which are obtained by writing all λ_i and ν_i into the vector form, respectively. λ_i and ν_i are Lagrange multipliers associated with the ith inequality constraints and ith equality constraints of (3.1.8), respectively. Based on L, the Lagrangian dual function $g : \mathbb{R}^m \times \mathbb{R}^p \to \mathbb{R}$ is defined as

$$g(\boldsymbol{\lambda}, \boldsymbol{\nu}) = \inf_{\mathbf{x} \in \mathcal{D}_c} \left(f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^{p} \nu_i h_i(\mathbf{x}) \right). \tag{3.1.25}$$

The Lagrangian dual function is always concave due to the fact that (3.1.25) is the pointwise infimum of a family of affine functions of $(\boldsymbol{\lambda}, \boldsymbol{\nu})$. For any $\boldsymbol{\lambda} \succeq 0$ and $\boldsymbol{\nu}$, there exists lower bounds for the dual function (3.1.25) on the optimal value $p^\star$. This can be derived in the following way. Let $\bar{\mathbf{x}}$ be a feasible point for problem (3.1.8). Hence, $f_i(\bar{\mathbf{x}}) \leq 0$ and $h_i(\bar{\mathbf{x}}) = 0$ hold. By multiplying each constraint with associated Lagrangian multiplier, the following inequality can be obtained

$$\begin{aligned} f_0(\mathbf{x}) &\geq f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^{p} \nu_i h_i(\mathbf{x}) \\ &\geq \inf_{\mathbf{z} \in \mathcal{D}_c} \left(f_0(\mathbf{z}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{z}) + \sum_{i=1}^{p} \nu_i h_i(\mathbf{z}) \right) \\ &= g(\boldsymbol{\lambda}, \boldsymbol{\nu}). \end{aligned} \tag{3.1.26}$$

Since $\bar{\mathbf{x}}$ is any feasible point that satisfies the inequality (3.1.26), the following inequality holds

$$g(\boldsymbol{\lambda}, \boldsymbol{\nu}) \leq f_0(\mathbf{x}^\star). \tag{3.1.27}$$

The difference between $g(\boldsymbol{\lambda}, \boldsymbol{\nu})$ and $f_0(\mathbf{x}^\star)$ is called the duality gap. If $g(\boldsymbol{\lambda}, \boldsymbol{\nu}) < f_0(\mathbf{x}^\star)$, the weak duality holds for the original problem and its dual problem. If $g(\boldsymbol{\lambda}, \boldsymbol{\nu}) = f_0(\mathbf{x}^\star)$, the strong duality holds [49]. Hence, solving problem (3.1.8) is equivalent to find the best lower bound obtained from the Lagrange dual function through solving the following problem

$$\begin{aligned} \underset{\boldsymbol{\lambda}, \boldsymbol{\nu}}{\text{maximize}} \quad & g(\boldsymbol{\lambda}, \boldsymbol{\nu}) \\ \text{subject to} \quad & \boldsymbol{\lambda} \geq 0. \end{aligned} \tag{3.1.28}$$

In this context, the original problem (3.1.8) is called the primal problem and problem (3.1.28) is called the Lagrange dual problem associated with the primal problem. The optimal solution of the Lagrange dual problem is called the dual optimal denoted as $(\boldsymbol{\lambda}^\star, \boldsymbol{\nu}^\star)$. By observing problem (3.1.28), it can be found that whether the primal problem is convex or not, the Lagrange dual problem is a convex optimization problem.

3.2 Game Theory

Game theory is a powerful tool to analyze the interactions among several rational decision makers [65]. It has been playing an increasingly important role in a wide range of disciplines such as economics, engineering and political science [66]. Game theory is especially attractive in engineering in recent years due to its advantages of flexibility and low complexity. The modern game theory was first introduced in [67], in which the concept of game was systematically described through giving the rules of the game, the moves of decision makers and the outcome of each decision maker.

3.2.1 Fundamentals of Game Theory

There are three main elements in formulating a game: *players*, *strategies* and *payoffs* [68].

• *Players*: all decision makers involved in the game are called players.

• *Strategies*: decisions of action are called strategies of the players.

• *Payoffs*: the payoffs, also known as *the utility*, are used to quantitatively measure the satisfaction of each player for a chosen strategy.

A well-known game in the field of game theory is the prisoner's dilemma, which was first presented in [69]. This game analyzes a scenario in which the conflict of interest exists due to independent decision-making. The hypothetical setting of this game is described in the following. Two persons are arrested by the police as suspects of a crime and placed in two separate rooms, hence they cannot communicate each other. It is assumed that the police cannot convict either of the two suspects due to the lack of evidence. Thus, the police decide to offer a deal to the suspects by allowing them confess to reduce sentence. In this game, the players are the two suspects and there are two available strategies for each player: confess and deny. With the deal provided by the police, players know the payoffs obtained based on their strategies, which are concluded as follows:

• If one of the suspects confesses the crime and the other denies, the denier will be sent to jail for five years and the confessor will be set free.

• If both suspects confess, both of them will be sent to jail for three years.

• If both suspects deny, both of them will be sent to jail for one year.

The prisoner's dilemma problem is depicted in Figure 3.3, in which suspect 1 and suspect 2 act as the column player and the row player, respectively. The payoff of the game is represented by a pair (p_1, p_2), where p_1 represents the payoff for the column player and p_2 represents the payoff for the row player. There are four possible pairs of strategies: (*Confess*, *Confess*), (*Confess*, *Deny*), (*Deny*, *Confess*) and (*Deny*, *Deny*). Since the two suspects are sep-

		Suspect 2	
		Deny	**Confess**
Suspect 1	**Deny**	(1, 1)	(5, 0)
	Confess	(0, 5)	(3, 3)

Figure 3.3. Prisoners' dilemma.

arately interrogated, there is no cooperation between them and one suspect is unaware of the decision made by the other suspect. Hence, a player has to independently choose the most preferable strategy by evaluating the outcomes of all four possible pairs of strategies. From Figure 3.3, it can be found that a player in this game has an incentive to always confess regardless of the strategy chosen by the other player since by choosing confess, a player can always have a better payoff. Then, due to the greediness of players, $(Confess, Confess)$ will be achieved as the equilibrium point though more efficient outcomes can be obtained if both players choose $(Deny, Deny)$. The prisoner's dilemma has profoundly revealed the insight of game theory in the conflicting situation.

Games can be classified in several ways. Depending upon the total payoffs of all the players, a game can be classified as *zero-sum game* and *non-zero sum game*. Depending upon the information to all players, games can be classified as *games with complete inforamtion* and *games with incomplete inforamtion*. By observing the competitiveness between players, games can be generally classified as non-cooperative games and cooperative games [70]. In cooperative games, the groups of players are allowed to cooperate with each other to improve their individual payoffs or group payoffs while in non-cooperative games, all players independently make decisions and compete with each other. In the following, the non-cooperative games and cooperative games will be reviewed in details, respectively.

3.2.2 Non-cooperative Game Theory

A non-cooperative game focuses on analyzing the interactions between several competitive decision-makers with conflicting interests. As one of the most important branches, it has been widely used in economics, political science and other disciplines. There are two major classes of games in non-cooperative game theory: simultaneous games and sequential games. A sequential game is always represented in a extensive form in which a player knows the actions chosen by those players that acted before them and can act more than once with a predefined order. In contrast, simultaneous games are always in a strategic form that players takes their actions simultaneously without any knowledge about other player's actions. In the following, the strategic form non-cooperative game will be emphatically discussed.

Non-cooperative Games in Strategic Form

A basic type of strategic form games is the game in matrix form as shown in Figure 3.4, which is used to characterize non-cooperative games with two players . A *matrix game* has the following characteristics [25]:

• Rows and columns refer to player 1 and player 2 respectively.

• Each row represents a strategy of the row player and each column represents a strategy of the column player.

• Each entry of the matrix represents a pair of outcome of the game, where the elements are the payoffs of the row player and the column player, respectively.

		Player 2	
		Strategy A	***Strategy B***
Player 1	***Strategy A***	(P_{1AA}, P_{2AA})	(P_{1AB}, P_{2AB})
	Strategy B	(P_{1BA}, P_{2BA})	(P_{1BB}, P_{2BB})

Figure 3.4. Strategic game in matrix form.

The prisoner's dilemma discussed above is a typical matrix game. The limitation of the matrix form games is that it can only depict games with two players. In the following, a general form of strategic game is discussed.

Definition 3.2.1. (Strategic game): *A non-cooperative strategic game can be expressed using a triplet as*

$$\mathcal{G} = (\mathcal{N}, \{\mathcal{S}_i\}_{i\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}}) \tag{3.2.1}$$

where

- $\mathcal{N} = \{1, \ldots, N\}$ *is the set of players.*
- $\mathcal{S}_i$ *is the strategy set for player* i*.*
- u_i *is the utility function for player* i*.*

With above definition, the *strategy space* of the strategic game (3.2.1) is defined as $\mathcal{S} := \mathcal{S}_1 \times \cdots \times \mathcal{S}_N$. For any player i, $s_i \in \mathcal{S}_i$ represents the strategy of i and $\mathbf{s}_{-i} = \{s_j\}_{j\in\mathcal{N}, j\neq i}$ denotes the vector of strategies of all players except player i. A joint choice of strategies of all players is called a *strategy profile* and denoted by $(s_i, \mathbf{s}_{-i}) \in \mathcal{S}$. Then, the payoff function of player i can be rewritten as a function of the strategies of all players as $u_i(s_i, \mathbf{s}_{-i})$. If for any player i, the strategy set $\mathcal{S}_i$ is finite, game (3.2.1) is called a finite game.

For a strategic form game, a problem is that whether a player knows the common knowledge of the game such as the identities of all the other players and their strategies and payoffs. If all elements of a game is known by all players in the game, the game is said to be a game with complete information. Otherwise, it is said to be a game with incomplete information [71].

Solutions of Non-cooperative Games

Given the model of a non-cooperative strategic game, it is important to find the solution of the game. A commonly used concept in analyzing the solutions of non-cooperative games is *Nash equilibrium*, which was first proposed by John Nash in [72]. For a non-cooperative game with rational players, the Nash equilibrium (NE) is an outcome led by a strategy profile that no player has incentive to move to another strategy. Before giving the definition of Nash equilibrium, some concepts are first stated in the following. To solve non-cooperative strategic games, the concept of *Dominant strategy* is first introduced.

Definition 3.2.2. (Dominant strategy) [73]: *The dominating strategy for player i is a strategy $s_i \in \mathcal{S}_i$ satisfying*

$$u_i(s_i, \mathbf{s}_{-i}) \geq u_i(s_i^{'}, \mathbf{s}_{-i}), \quad s_i^{'} \in \mathcal{S}_i \textit{ and } \forall \mathbf{s}_{-i} \in \mathcal{S}_{-i}, \tag{3.2.2}$$

where $\mathcal{S}_{-i}$ is the set of all possible strategy profiles for all players except player i. Hence, by choosing a dominating strategy, a player can always obtain the highest payoff regardless of the strategies chosen by other players. If dominant strategies exist for all players, then all players will choose their dominant strategies and the set of dominant strategies for all players is naturally the solution of the game. Such solution is called the *Dominant-strategy equilibrium* and defined as

Definition 3.2.3. (dominant-strategy equilibrium) [25]: *The dominant-strategy equilibrium is a strategy profile $\mathbf{s}^\star \in \mathcal{S}$ in which every strategy $s_i^\star$ belongs to $\mathbf{s}^\star$ is a dominant strategy for the corresponding player.*

A typical game with dominant strategy equilibrium is the prisoner's dilemma presented in Figure 3.3, in which *confess* is the dominant strategy for both suspects. Hence, (*confess*, *confess*) is the dominant-strategy equilibrium and (3, 3) is the outcome payoffs of the game. However, such dominant-strategy equilibrium is inefficient since the payoffs obtained are not the best for both players. It should also be noticed that the existence of the dominant-strategy equilibrium is not guaranteed since for some games, dominant strategies may not exist for all players. Another concept for solving the non-cooperative games is the *best-response function*, which is defined as

Definition 3.2.4. (Best response) [25]: *The best response function of player i to the profile of strategies $\mathbf{s}_{-i}$ is the set of strategies with the following expression*

$$\mathcal{B}r_i(\mathbf{s}_{-i}) = \{s_i \in \mathcal{S}_i \mid u_i(s_i, \mathbf{s}_{-i}) \geq u_i(s_i^{'}, \mathbf{s}_{-i}),\ \forall s_i^{'} \in \mathcal{S}_i\}. \tag{3.2.3}$$

Hence, every strategy s_i in the set $\mathcal{B}r_i(\mathbf{s}_{-i})$ is the best response to a fixed set of strategies of all the other players except player i. For a given strategy profile $\mathbf{s}_{-i}$, any strategy in $\mathcal{B}r_i(\mathbf{s}_{-i})$ should perform no worse than every other available strategy in $\mathcal{S}_i$.
As mentioned above, not all non-cooperative games have dominant strategy equilibrium. Hence, the concept of Nash equilibrium is introduced as an alternative solution for non-cooperative games. For non-cooperative games with pure strategies, the Nash equilibrium is defined as

Definition 3.2.5. (Nash equilibrium) [73] : *The Nash equilibrium of a non-cooperative game $\mathcal{G}$ with pure strategies is a strategy profile $\mathbf{s}^\star \in \mathcal{S}$ satisfying*

$$u_i(s_i^\star, \mathbf{s}_{-i}^\star) \geq u_i(s_i, \mathbf{s}_{-i}^\star),\ \forall s_i \in \mathcal{S}_i, \forall i \in \mathcal{N}. \tag{3.2.4}$$

If (3.2.4) only holds for inequality, the Nash equilibrium is said to be strict. For example, by observing the game of prisoner's dilemma, it can be found that (*confess*, *confess*) is a strict Nash equilibrium. A Nash equilibrium can be alternatively defined using the best response function as:

Proposition 3.2.1. : *The Nash equilibrium of a non-cooperative game $\mathcal{G}$ is a strategy profile $\mathbf{s}^\star \in \mathcal{S}$ satisfying*

$$s_i^\star \in \mathcal{B}r_i(\mathbf{s}_{-i}^\star), \forall i \in \mathcal{N}. \tag{3.2.5}$$

This means that when a Nash equilibrium is achieved, every player's strategy is a best response to the other players' strategies. Thus, the Nash equilibrium of a non-cooperative game can be obtained by finding a strategy profile in which the strategy of each player can be expressed using equation (3.2.5). When solving a non-cooperative game by applying Nash equilibrium, the key interest mainly involves the following aspects [25]:

- The existence of Nash equilibrium.
- The number of Nash equilibrium points: unique or multiple?
- The efficiency of Nash equilibrium: whether the outcome is optimal?

For generic non-cooperative games, the existence of Nash equilibrium is not generally guaranteed and it is difficult to prove the existence of Nash equilibrium [74, 75]. However, for a non-cooperative game, if its best response functions can be expressed in closed form, the pure strategy Nash equilibrium can be obtained by finding the intersection point of all best response functions [25]. By using the concept of *standard function* given in section 2.4, the following theorem is given:

Theorem 3.2.1. [56]: *For a non-cooperative game $\mathcal{G}$, if the best response functions exist and are standard for all players, there exists a unique pure-strategy Nash equilibrium for $\mathcal{G}$.*

Beyond using best response functions, other theorems have also been proposed. In [76–78], it has shown that if the strategy sets and utility functions posses certain properties, the pure-strategy Nash equilibrium exists. In this thesis, only the best response based method is applied for analyzing the existence and uniqueness of Nash equilibrium.

3.2.3 Cooperative Game Theory

Different to the non-cooperative game theory, in cooperative games, rational players are allowed to cooperate with each other, which can affect the strategies chosen by players and further impact their utilities [79]. One of the main branches in cooperative game theory is the coalitional game, in which players can improve their utilities by forming cooperating groups. In the following, two types of coalitional games: canonical coalitional games and coalition formation games are reviewed, respectively.

Canonical Coalition Games

In contrast to the strategic form games, in coalitional games, a set of players $\mathcal{N} = \{1, \ldots, N\}$ intend to form cooperative groups to strengthen their benefits rather than fully competing with each other. A cooperative group, denoted by C, is called a *coalition*, in which all players agree to act as a single entity. To analyze the benefits to form coalitions, in coalitional games, the concept *coalition value*, denoted by v, is used to quantify the worth of a coalition. Then, based on $\mathcal{N}$ and v, a coalitional game is defined as:

Definition 3.2.6. (Coalitional game) [25]: *A coalitional game can be expressed as a pair $(\mathcal{N}, v)$, where $\mathcal{N}$ is the set of players and v is a mapping function that assigns payoffs for players in the game.*

The most commonly studied coalitional games are the games in characteristic form, in which the value of a coalition C depends only on the members in C. This type of coalitional games was first introduced in [67] with the concept of *transferable utility* (TU). TU means that the total utility of a coalition can be divided and distributed to members in the coalition in any manner. For coalitional games in characteristic form with TU, the value is also called the *characteristic function* and defined as: $v : 2^{\mathcal{N}} \to \mathbb{R}$ with $v(\emptyset) = 0$, where $\emptyset$ is an empty set. For each coalition $C \subseteq \mathcal{N}$, such characteristic function generates a real value $v(C)$ to quantify the gain of the coalition. Let x_i be the payoff received by player $i \in C$, then the *payoff allocation* vector of coalition C is denoted as $\mathbf{x} \in \mathbb{R}^{|C| \times 1}$.

Different to coalitional games in characteristic form with TU, in some games,

the value assigned to each coalition is not a specific real value. These games are called coalitional games with *non-transferable utility* (NTU), in which the payoff of a player in a coalition C depends on the joint strategies chosen by all players in the coalition [80]. Hence, in NTU games, the value of a coalition $v(C)$ is a set of payoff allocation vectors rather than a real value.
Another significant concept for coalitional games in characteristic form is *superadditivity.* For NTU games, superadditivity is defined as

Definition 3.2.7. (Superadditivity) [25]: *An NTU coalitional game $(\mathcal{N}, v)$ is said to be superadditive if the following the condition is satisfied*

$$\begin{aligned} v(C_1 \cup C_2) \supset \quad & \{\mathbf{x} \in \mathbb{R}^{|C_1 \cup C_2| \times 1} \mid (x_i)_{i \in C_1} \in v(C_1), \\ & (x_j)_{j \in C_2} \in v(C_2), \forall C_1 \subset \mathcal{N}, C_2 \subset \mathcal{N}, \\ & \textit{and } C_1 \cap C_2 = \emptyset\}, \end{aligned} \tag{3.2.6}$$

where $\mathbf{x}$ is the payoff allocation of the coalition $C_1 \cup C_2$. Since a TU game can be seen as a special case of the NTU game, for TU games, condition (3.2.6) reduces to the following inequality [79]:

$$v(C_1 \cup C_2) \geq v(C_1) + v(C_2), \forall C_1 \subset \mathcal{N}, C_2 \subset \mathcal{N}, C_1 \cap C_2 = \emptyset. \tag{3.2.7}$$

From (3.2.6), supperadditivity means that players can do no worse by forming disjoint small coalitions into a larger coalition since for any two disjoint coalitions, if they forms, players in these two coalitions can receive the same payoffs as they received before the formation.
Following the definitions above, coalitional games can be classified as canonical coalitional games if the following requirements are met [79]:

- The coalitional game is in characteristic form.
- Players in the game will never receive detriment by forming larger coalitions, that the superadditivity property always holds.

The superadditivity property also gives the fact that in canonical games, players can always obtain best joint benefits by forming the *grand coalition*, where a grand coalition $\mathcal{N}$ is a coalition with all players in the game. This means that to solve a canonical coalitional game, the main task is to find the payoff allocation that can guarantee the stability of the grand coalition. In the following, several concepts for solving canonical coalitional games are reviewed.

Solutions for Canonical Coalitional Games

A well known solution for canonical coalitional games is the *core*. Before giving the definition of the core, some concepts are first introduced. In a canonical coalitional game $(\mathcal{N}, v)$, a payoff vector $\mathbf{x} \in \mathbb{R}^{|\mathcal{N}|\times 1}$ of the grand coalition $\mathcal{N}$ is said to be *group rational* if $\sum_{i\in\mathcal{N}} x_i = v(\mathcal{N})$. In addition, if $x_i \geq v(\{i\})$ for every player $i \in \mathcal{N}$, the payoff vector $\mathbf{x}$ is said to be *individually rational*. If a payoff vector is both group rational and individually rational, it is called an imputation. Then, the following definition is given:

Definition 3.2.8. (Core) [25]: *The core of a TU canonical coalitional game $(\mathcal{N}, v)$ is defined as a set of imputations expressed as follows:*

$$\mathcal{C}r_{TU} = \big\{\mathbf{x} \mid \sum_{i\in\mathcal{N}} x_i = v(\mathcal{N}) \ \textit{and} \ \sum_{i\in C} x_i \geq v(C), \forall C \subseteq \mathcal{N}\big\}. \tag{3.2.8}$$

Hence, for TU canonical coalitional games, the core can guarantee that no players have incentive to deviate from the grand coalition to form a smaller coalition $C \subset \mathcal{N}$. The grand coalition is stable if there exists a payoff allocation $\mathbf{x} \in \mathbb{R}^{|\mathcal{N}|\times 1}$ lies in the core. For NTU games, the core can be applied as the solution only if the value v satisfies the following conditions [66]:

- The value $v(C)$ for any coalition $C \subseteq \mathcal{N}$ must be a closed and convex subset of $\mathbb{R}^{|C|\times 1}$.
- If $\mathbf{x}_1 \in v(C)$ and $\mathbf{x}_2 \in \mathbb{R}^{|C|\times 1}$ satisfies $\mathbf{x}_2 \leq \mathbf{x}_1$, then $\mathbf{x}_2 \in v(C)$.

• The set $\{\mathbf{x}|\mathbf{x} \in v(C)\ and\ x_i \geq z_i, \forall i \in C\}$ with $z_i = max\{y_i|\mathbf{y} \in v(\{i\})\}, \forall i \in \mathcal{N}$ must be a bounded subset of $\mathbb{R}^{|C|\times 1}$.

Then, the core for NTU canonical coalitional games is defined as

$$\mathcal{C}r_{NTU} = \{\mathbf{x} \in v(\mathcal{N}) \mid \forall C, \nexists \mathbf{y} \in v(C),\ such\ that\ y_i > x_i, \forall i \in C\}. \quad (3.2.9)$$

Different to the TU case, the stability of NTU games depends on all elements of the payoff vector rather than the sum of the payoff vector.

Although the core is a powerful tool for solving canonical coalitional games, it suffers from some drawbacks [79]. Firstly, the core is not always nonempty; secondly, it is difficult to find a payoff allocation lies in the core; thirdly, the fairness among players cannot be guaranteed with the payoff allocation lies in the core. Hence, an alternative concept called *Shapley value* has also been proposed for solving TU coalitional games $(\mathcal{N}, v)$, in which a unique payoff vector in $\mathbb{R}^{|\mathcal{N}|\times 1}$ can be obtained as the value of a game. The Shapley value is denoted by $\phi(v)$ and possesses following properties [25]:

• *Efficiency*: $\sum_{i\in\mathcal{N}} \phi_i(v) = v(\mathcal{N})$.

• *Symmetry*: If for any coalition $C \subseteq \mathcal{N}$ and players $i, j \notin C$, $v(C \cup \{i\}) = v(C \cup \{j\})$ holds, then $\phi_i(v) = \phi_j(v)$.

• *Dummy*: If for any coalition $C \subseteq \mathcal{N}$ and players $i \notin C$, $v(C) = v(C \cup \{i\})$ holds, then $\phi_i(v) = 0$.

• *Additivity*: For any two characteristic functions u and v, there is $\phi(u+v) = \phi(v + u) = \phi(u) + \phi(v)$.

In above properties, ϕ_i represents the payoff assigned to player i by the payoff mapping ϕ. These properties have also shown the nature that Shapley value is a payoff vector with group rationality and for any two players, if they have the same contribution to a coalition, their payoffs assigned by the Shapley value ϕ are equal. Given any TU coalitional games, the payoff assigned to player $i \in \mathcal{N}$ by the Shapley value is given as:

$$\phi_i(v) = \sum_{C \subseteq \mathcal{N} \setminus \{i\}} \frac{|C|!(|\mathcal{N}| - |C| - 1)!}{|N|!}[v(C \cup \{i\}) - v(C)]. \tag{3.2.10}$$

Equation (3.2.10) gives another interpretation of Shapley value by considering the order of players join the grand coalition, in which $v(C \cup \{i\}) - v(C)$ is the marginal contribution of player i to coalition C.

Although both the core and the Shapley value are solutions for canonical coalitional games, they are generally not related to each other. However, in some special cases, the Shapley value may also lie in the core. An typical example is the *convex game*.

Definition 3.2.9. (Convex game): *A TU game is said to be a convex game if*

$$v(C_1) + v(C_2) \leq v(C_1 \cup C_2) + v(C_1 \cap C_2), \forall C_1, C_2 \subseteq \mathcal{N}. \tag{3.2.11}$$

Given a coalitional game, if the core of the game exists and the Shapley value lies in the core, it means that the solution of the Shapley value possesses both stability and fairness. It should be noticed that the Shapley value discussed above is developed for TU games. However, it is also applicable for NTU games [66].

Coalition Formation Games

Another branch of coalitional games is the coalition formation game. In canonical coalitional games, the main task is to analyze the stability of the grand coalition. However, one significant assumption for such games is that there is no cost for cooperation, which means that players always benefit from forming coalitions. However, in practice, the cooperation among players always incurs cost which could weaken the gain from forming coalitions.

Hence, the main task for coalition formation games is to analyze the coalition formation process and the stability of specific coalition structures.

Different to the canonical coalitional games, in a coalition formation game, the value of the game v is not always a characteristic function, which means that the value of a coalition C is dependent not only on the coalition itself, but also on the structure formed outside coalition C. Such type of games is called coalitional games in *partition form*, which is first introduced in [81].

Definition 3.2.10. (Partition): *A partition of $\mathcal{N}$ is a coalition structure S comprising of a collection of coalitions satisfying:*

$$S = \{C_1, \ldots, C_L\}, \; where \; \bigcup_{l=1}^{L} = \mathcal{N} \; and \; C_i \cap C_j = \emptyset, \forall i, j \in \mathcal{N} \; and \; i \neq j. \tag{3.2.12}$$

The value of a coalition $C \in S$ in a partition form coalitional game is written as $v(C, S)$.

The main challenge in coalition formation games is how to form a suitable coalition structure for a specific game. Many coalition formation algorithms have been proposed in the literatures. A general method for coalition formation is to form a coalition structure in a distributed way. To develop a coalition formation algorithm for a given coalition formation game, there are three aspects of rules need to be given [82]:

• *Order for comparison*: A pre-defined order for comparing different collections of coalitions.

• *Forming and breaking*: Rules for forming or breaking a coalition.

• *Evaluation of stability*: The rule for evaluating the stability of a coalition structure.

For a coalitional game with a set of players $\mathcal{N}$, a collection of coalitions is defined as [25]:

Definition 3.2.11. (Collection of coalitions)*: A collection of coalitions is* *defined as a coalition set $\mathcal{C} = \{C_1, \ldots, C_l\}$ comprising of any group of mutually disjoint coalitions of $\mathcal{N}$, where $C_i \subseteq \mathcal{N}$ and $C_i \cap C_j = \emptyset\ \forall i, j \in \{1, \ldots, l\}$ and $i \neq j$. If $\bigcup_{i=1}^{l} C_i = \mathcal{N}$, then the collection $\mathcal{C}$ becomes a partition of $\mathcal{N}$.*

By denoting $\rhd$ as the comparison relation, the order of comparison is defined as:

Definition 3.2.12. (Comparison order)*: Define two collections $\mathcal{C} = \{C_1, \ldots, C_l\}$ and $\mathcal{D} = \{D_1, \ldots, D_p\}$ as two partitions of the same subset $\mathcal{A} \subseteq \mathcal{N}$ that $\bigcup_{i=1}^{l} C_i = \bigcup_{j=1}^{p} D_j = \mathcal{A}$. Then, $\mathcal{C} \rhd \mathcal{D}$ means that $\mathcal{C}$ is a preferred partition of $\mathcal{A}$ to $\mathcal{D}$.*

For coalition formation games with TU, the two generally used relations are *utilitarian order* and *Nash order*, where two coalition structures are compared in the collective way. Let $a = \{a_1, \ldots, a_n\}$ and $b = \{b_1, \ldots, b_n\}$ be two sequences of elements, then the utilitarian order and the Nash order are defined as follows respectively:

• the utilitarian order:

$a \rhd_{ut} b$ iff $\sum_{i=1}^{n} a_i > \sum_{j=1}^{n} b_i$,

• the Nash order:

$a \rhd_{Nash} b$ iff $\prod_{i=1}^{n} a_i > \prod_{j=1}^{n} b_i$.

For coalition formation games with NTU, individually based comparison should be applied such as *majority order* and *Pareto order*, which can be defined as:

• the majority order:

$a \rhd_m b$ iff $|\{i|a_i > b_i\}| > |\{i|b_i > a_i\}|$,

• the Pareto order:

$a \rhd_p b$ iff $a_i \geq b_i\ \forall i$ and $\exists i \in \{1, \ldots, n\}$ such that $a_i > b_i$.

In coalition formation games, a coalition structure can be reached by breaking from the grand coalition $\mathcal{N}$. The rule of breaking a large coalition into several disjoint small coalitions is called *split*, which can be defined as [82]:

Definition 3.2.13. (Split Rule): *A coalition* $\bigcup_{i=1}^{l} C_i$ *can be split into a set of coalitions* $\{C_1, \ldots, C_l\}$ *if and only if* $\{C_1, \ldots, C_l\} \triangleright \{\bigcup_{i=1}^{l} C_i\}$*. Such process is expressed as* $\{\bigcup_{i=1}^{l} C_i\} \rightarrow \{C_1, \ldots, C_l\}$*.*

Instead of splitting from the grand coalition, a coalition structure can also be reached by forming from the coalition structure that all coalitions are singletons. The rule of forming several disjoint coalitions into a single coalition is called *merge*, which can be defined as [82]:

Definition 3.2.14. (Merge Rule)*: A set of coalitions* $\{C_1, \ldots, C_l\}$ *can be merged into a single coalition* $\bigcup_{i=1}^{l} C_i$ *if and only if* $\{\bigcup_{i=1}^{l} C_i\} \triangleright \{C_1, \ldots, C_l\}$*. Such process is expressed as* $\{C_1, \ldots, C_l\} \rightarrow \{\bigcup_{i=1}^{l} C_i\}$*.*

Given above rules, coalition formation algorithms can be established for both TU and NTU coalition formation games. Once a coalition structure is reached, it is important to evaluate whether it is stable. The concepts of $\mathbb{D}_{hp}$-stable partition and $\mathbb{D}_c$-stable partition have been proposed in [82] for analyzing the stability of partitions obtained with the merge-and-split formation algorithm. A partition is called $\mathbb{D}_{hp}$-stable if no groups of players have incentive to either split from the partition or merger into new coalitions. Compared to $\mathbb{D}_{hp}$-stable, $\mathbb{D}_c$-stable is more strict and possesses the following properties:

- The $\mathbb{D}_c$-stable partition is $\mathbb{D}_{hp}$-stable.
- The $\mathbb{D}_c$-stable partition is the unique outcome of the merge-and-split algorithm with any iteration.
- For a given comparison order, the $\mathbb{D}_c$-stable partition is preferred to all other partitions.

Another concept named *coalition structure stable set* has also been proposed in [83] for analyzing the stability of sequential based coalition formation games.

3.3 Summary

This chapter summarized various convex optimization techniques and game theoretic methods. Some of the reviewed techniques will be used in the subsequent sections. In Chapter 4, the SDP method will be used for proving the validity of the proposed multicell downlink beamforming algorithm. In Chapter 5, the GP method will be employed for solving the power allocation problems. In Chapter 6, the coalition formation game in partition form will be considered for the cooperative downlink multicell beamforming; in particular, both the Pareto and the majority comparison methods will be used and their performance will be analyzed.

Chapter 4

COORDINATED BEAMFORMING FOR MULTICELL WIRELESS NETWORKS WITH MIXED QUALITY OF SERVICE

In this chapter, coordinated multicell beamforming based on the SINR balancing technique within the context of mixed QoS is proposed. Instead of attaining an overall balance of SINRs to all users in all cells, the proposed algorithm allows a specific subset of users in each cell to achieve certain target SINRs while the SINRs of the remaining users in all cells are balanced subject to the total transmission power. Two scenarios are considered in the chapter. In the first scenario, all BSs design beamformers cooperatively for all users in all cells while in the second scenario, a subset of BSs coordinates the beamformer design for users in their cells, while ensuring interference leakage to users in other neighboring cells is below a threshold.

4.1 Introduction

Depending on the requirements on SINR, various downlink beamforming techniques have been proposed in recent years. One of the most widely studied beamformer designs for wireless networks is based on allowing users achieving a set of target SINRs by minimizing total transmission power of BSs [84]. Another strategy in designing beamformers is to balance the SINRs of all users by maximizing the SINR of the worst case user subject to a constraint on total transmission power [37, 85–88]. The advantage of this approach is that it ensures the fairness among users while maximizing their SINRs. It also ensures the optimization problem is always feasible.

To improve efficiency further, coordinated multicell beamforming have also been considered in recent years [86, 87, 89, 90]. In [89], where the Lagrangian duality, a special uplink-downlink duality approach, is employed to formulate an iterative algorithm. The uplink-downlink duality technique has been widely used for solving beamformer design problems with the capability of enabling complicated downlink problems be replaced by simpler uplink problems [17, 37, 52, 91–93].

In [94], a beamformer design algorithm was proposed to provide both delay-intolerant real-time services and delay-tolerant packet data services for a single cell wireless network. For delay-intolerant real-time services such as the voice service, due to the 'real-time' nature, specific SINR targets need to be achieved all the time to guarantee the QoS for corresponding users. For delay-tolerant packet data services such as mail services, since users are not urgent for such services, certain delay is allowed. Hence, instantaneous SINR targets are not required. The aim of the algorithm proposed in [94] is to fairly balance non-real-time users (NRTUs) to the same SINR level while satisfying target SINRs for real-time users (RTUs) under a total power constraint.

In this chapter, the work of [94] is extended to a multicell scenario in which mobile users are affected not only by the intra-cell interference, but also by the intercell interference. In addition, a more practical wireless network scenario that comprises of both cooperative cells and independent cells, is also considered.

4.2 Coordinated Multicell Beamforming with Mixed Quality of Service

4.2.1 System Model and Problem Statement

Signal Model

A multicell multi-user wireless network with J cells and K users in each cell is shown in Figure 4.1. Each BS consists of M antennas, while each of users is equipped with a single antenna. The downlink transmit beamforming technique is employed to perform spatial multiplexing. Let $s_{j,k}$ be the information symbol for the kth user in the jth cell and $\mathbf{u}_{j,k} \in \mathbb{C}^{M\times 1}$ be the corresponding beamformer vector, where $\|\mathbf{u}_{j,k}\|_2 = 1, \forall j, \forall k$. The received signal at the kth user in the jth cell can be written as

$$y_{j,k} = \sum_{l=1}^{K} \mathbf{h}_{j,j,k}^{H} \sqrt{p_{j,l}}\mathbf{u}_{j,l} s_{j,l} + \sum_{\substack{i=1\\ i\neq j}}^{J} \sum_{m=1}^{K} \mathbf{h}_{i,j,k}^{H} \sqrt{p_{i,m}}\mathbf{u}_{i,m} s_{i,m} + \eta_{j,k}, \quad (4.2.1)$$

where $\mathbf{h}_{i,j,k} \in \mathbb{C}^{M\times 1}$ is the channel vector from the BS of the ith cell to the kth user in the jth cell, $p_{j,k}$ denotes the power allocated to the kth user in the jth cell, and $\eta_{j,k}$ is assumed to be complex AWGN with zero mean and total variance of $\sigma_{j,k}^2$.

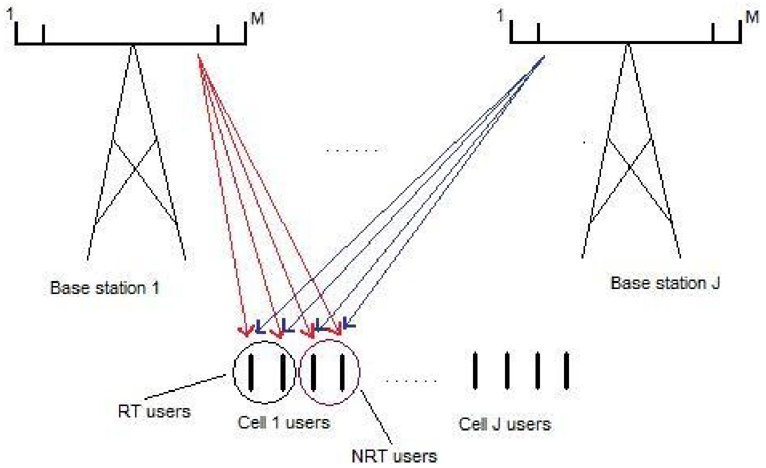

Figure 4.1. Mixed QoS-based multicell wireless network.

Downlink Transmit Beamforming

By defining $\mathbf{R}_{i,j,k} \triangleq \mathbf{h}_{i,j,k}\mathbf{h}_{i,j,k}^H$, the SINR of the kth user in the jth cell in the downlink can be written as

$$\Gamma_{j,k} = \frac{p_{j,k}\mathbf{u}_{j,k}^H \mathbf{R}_{j,j,k}\mathbf{u}_{j,k}}{\sum\limits_{\substack{l=1\\l\neq k}}^{K} p_{j,l}\mathbf{u}_{j,l}^H\mathbf{R}_{j,j,k}\mathbf{u}_{j,l} + \sum\limits_{\substack{i=1\\i\neq j}}^{J}\sum\limits_{m=1}^{K} p_{i,m}\mathbf{u}_{i,m}^H\mathbf{R}_{i,j,k}\mathbf{u}_{i,m} + \sigma_{j,k}^2}. \qquad (4.2.2)$$

With the aim of designing the downlink beamformers on the basis of mixed SINR balancing and SINR target constraints in the multicell wireless network, without loss of generality, it is assumed the first K_j users out of the K users in the jth cell are RTUs employing delay-intolerant real-time services, whereas the rest of the users are NRTUs employing delay-tolerant services. Therefore, target SINRs need to be achieved all the time for those RTUs.

The mixed QoS based multicell beamforming design can be formulated as:

$$\max_{\mathbf{U},\mathbf{p}} \min_{j,k} \quad \frac{\Gamma_{j,k}(\mathbf{U},\mathbf{p})}{\delta_{j,k}}, \qquad k = K_j+1,\cdots,K, \forall j \tag{4.2.3a}$$

$$\text{s.t.} \quad \Gamma_{j,k}(\mathbf{U},\mathbf{p}) \geq \gamma_{j,k}, \qquad k = 1,\cdots,K_j, \forall j \tag{4.2.3b}$$

$$\mathbf{1}^T\mathbf{p} \leq P_{max}, \tag{4.2.3c}$$

where $\mathbf{p} = [\mathbf{p}_1^T,\cdots,\mathbf{p}_J^T]^T$ is the power allocation vector for all users and $\mathbf{U} = [\mathbf{U}_1,\cdots,\mathbf{U}_J]$ denotes the muticell beamformer matrix. In (4.2.3), $\mathbf{U}_j = [\mathbf{u}_{j,1},\cdots,\mathbf{u}_{j,K}]$ and $\mathbf{p}_j = [p_{j,1},\cdots,p_{j,K}]^T$ are the beamformers of the users in the jth cell and their associated downlink power allocation, respectively. $\gamma_{j,k}$ denotes the target SINR for the kth user in the jth cell when the user (j,k) is a RTU, whereas $\delta_{j,k}$ is a given priority weighting factor for the SINR of the corresponding NRTUs. It is assumed that all BSs share a total power of P_{max}. However, to account for flexibility at BS in terms of its transmission power, in the subsequent chapters, individual power constraints will also be used.

4.2.2 Coordinated Multicell Beamforming Algorithm

The multicell mixed QoS beamformer design problem can be related to the ordinary SINR balancing problem since the SINR balancing problem has been very well established through using the uplink-downlink duality [37]. However, as stated in [94], beamformers for RTUs and NRTUs need to be jointly designed, which can result problem (4.2.3) to be nontrivial. Hence, to make the design problem straightforward, the uplink power vector of RTUs is first written as a function of the uplink power vector of NRTUs. Here, vectors $\boldsymbol{\sigma}_j = [\sigma_{j,1}^2 \cdots \sigma_{j,K}^2]^T$ and $\mathbf{q}_j = [q_{j,1} \cdots q_{j,K}]^T$ are first introduced, which represent the noise variance vector and virtual uplink power allocation vector of the jth cell, respectively. Then, problem (4.2.3) can be reformulated as the following dual uplink problem on the basis of uplink-downlink duality [95]:

$$\max_{\mathbf{U},\mathbf{q}} \min_{j,k} \quad \frac{\Lambda_{j,k}(\mathbf{u}_{j,k},\mathbf{q})}{\delta_{j,k}}, \qquad k = K_j+1,\cdots,K, \forall j \tag{4.2.4a}$$

$$\text{s.t.} \quad \Lambda_{j,k}(\mathbf{u}_{j,k},\mathbf{q}) \geq \gamma_{j,k}, \qquad k = 1,\cdots,K_j, \forall j \tag{4.2.4b}$$

$$\boldsymbol{\sigma}^T\mathbf{q} \leq P_{max}, \tag{4.2.4c}$$

where $\boldsymbol{\sigma} = [\boldsymbol{\sigma}_1^T \cdots \boldsymbol{\sigma}_J^T]^T$, and $\mathbf{q} = [\mathbf{q}_1^T \cdots \mathbf{q}_J^T]^T$. The uplink SINR of the kth user in the jth cell $\Lambda_{j,k}(\mathbf{u}_{j,k},\mathbf{q})$ can be written as

$$\Lambda_{j,k} = \frac{q_{j,k}\mathbf{u}_{j,k}^H\mathbf{R}_{j,j,k}\mathbf{u}_{j,k}}{\mathbf{u}_{j,k}^H(\sum_{(i,l)\neq(j,k)} q_{i,l}\mathbf{R}_{j,i,l} + \mathbf{I})\mathbf{u}_{j,k}}, \qquad \forall k, \forall j. \tag{4.2.5}$$

So far, the downlink transmitter beamforming problem has been converted to its dual uplink problem.

Uplink Power Allocation for a Given Set of Beamformers

For a given set of beamformers in the uplink, the constraints in (4.2.4) should satisfy with equality when the optimal value is achieved. Hence, (4.2.4a) and (4.2.4b) are modified to equality to satisfy the optimality, and the problem (4.2.4) can be further rewritten as

$$\frac{\Lambda_{j,k}(\widetilde{\mathbf{u}}_{j,k},\widetilde{\mathbf{q}})}{\delta_{j,k}} = \frac{1}{\lambda}, \qquad k = K_j+1,\cdots,K, \forall j \tag{4.2.6a}$$

$$\Lambda_{j,k}(\widetilde{\mathbf{u}}_{j,k},\widetilde{\mathbf{q}}) = \gamma_{j,k}, \qquad k = 1,\cdots,K_j, \forall j \tag{4.2.6b}$$

$$\boldsymbol{\sigma}^T\widetilde{\mathbf{q}} = P_{max}, \tag{4.2.6c}$$

where $1/\lambda$ is a balanced SINR of the NRTUs, and $\widetilde{\mathbf{q}}$ is the optimal virtual uplink power vector obtained by employing a given set of beamformers $\widetilde{\mathbf{U}}$. By substituting (4.2.5) into (4.2.6b), the optimal virtual uplink power for an individual RTU can be written as

$$\widetilde{q}_{j,k} = \gamma_{j,k} \frac{\widetilde{\mathbf{u}}_{j,k}^H (\sum_{(i,l)\neq(j,k)} \widetilde{q}_{i,l} \mathbf{R}_{j,i,l} + \mathbf{I}) \widetilde{\mathbf{u}}_{j,k}}{\widetilde{\mathbf{u}}_{j,k}^H \mathbf{R}_{j,j,k} \widetilde{\mathbf{u}}_{j,k}}, \qquad k = 1, \cdots, K_j, \forall j. \quad (4.2.7)$$

By rearranging (4.2.7) for all RTUs, the optimal uplink power allocation vectors for RTUs $\widetilde{\mathbf{q}}_R = [\widetilde{q}_{1,1} \cdots \widetilde{q}_{1,K_1}, \cdots, \widetilde{q}_{J,1} \cdots \widetilde{q}_{J,K_J}]^T$ can be expressed as

$$\widetilde{\mathbf{q}}_R = \mathbf{D}_R \mathbf{\Psi}_A \widetilde{\mathbf{q}}_R + \mathbf{D}_R \mathbf{1} + \mathbf{D}_R \mathbf{\Psi}_B \widetilde{\mathbf{q}}_N, \quad (4.2.8)$$

where $\widetilde{\mathbf{q}}_N = [\widetilde{q}_{1,K_1+1} \cdots \widetilde{q}_{1,K}, \cdots, \widetilde{q}_{J,K_J+1} \cdots \widetilde{q}_{J,K}]^T$ is the optimum power allocation vector for the NRTUs, and

$$\begin{aligned}\mathbf{D}_R = \mathrm{diag}[&(\gamma_{1,1}/\widetilde{\mathbf{u}}_{1,1}^H \mathbf{R}_{1,1,1} \widetilde{\mathbf{u}}_{1,1}) \cdots (\gamma_{1,K_1}/\widetilde{\mathbf{u}}_{1,K_1}^H \mathbf{R}_{1,1,K_1} \widetilde{\mathbf{u}}_{1,K_1}) \cdots \\ &(\gamma_{J,1}/\widetilde{\mathbf{u}}_{J,1}^H \mathbf{R}_{J,J,1} \widetilde{\mathbf{u}}_{J,1}) \cdots (\gamma_{J,K_J}/\widetilde{\mathbf{u}}_{J,K_J}^H \mathbf{R}_{J,J,K_J} \widetilde{\mathbf{u}}_{J,K_J})],\end{aligned}$$

$$\mathbf{\Psi}_A = \begin{cases} \widetilde{\mathbf{u}}_{j,k}^H \mathbf{R}_{j,i,s} \widetilde{\mathbf{u}}_{j,k}, & (i,s) \neq (j,k) \\ & k = 1, \cdots, K_j, \forall j \\ & s = 1, \cdots, K_i, \forall i \\ 0, & (i,s) = (j,k), \end{cases}$$

$$\mathbf{\Psi}_B = \begin{cases} \widetilde{\mathbf{u}}_{j,k}^H \mathbf{R}_{j,i,l} \widetilde{\mathbf{u}}_{j,k}, & k = 1, \cdots, K_j, \forall j \\ & l = K_i + 1, \cdots, K, \forall i. \end{cases}$$

Similar to (4.2.7), by substituting (4.2.5) into (4.2.6a) with appropriate rearrangements, an equation about $\widetilde{\mathbf{q}}_N$ can be obtained as follow

$$\lambda \widetilde{\mathbf{q}}_N = \mathbf{D}_N \mathbf{\Psi}_D \widetilde{\mathbf{q}}_N + \mathbf{D}_N \mathbf{1} + \mathbf{D}_N \mathbf{\Psi}_C \widetilde{\mathbf{q}}_R, \quad (4.2.9)$$

where

$$\begin{aligned}\mathbf{D}_N =&\text{diag}[(\delta_{1,K_1+1}/\widetilde{\mathbf{u}}^H_{1,K_1+1}\mathbf{R}_{1,1,K_1+1}\widetilde{\mathbf{u}}_{1,K_1+1})\cdots(\delta_{1,K}/\widetilde{\mathbf{u}}^H_{1,K}\mathbf{R}_{1,1,K}\widetilde{\mathbf{u}}_{1,K})\\ &\cdots(\delta_{J,K_J+1}/\widetilde{\mathbf{u}}^H_{J,K_J+1}\mathbf{R}_{J,J,K_J+1}\widetilde{\mathbf{u}}_{J,K_J+1})\cdots(\delta_{J,K}/\widetilde{\mathbf{u}}^H_{J,K}\mathbf{R}_{J,J,K}\widetilde{\mathbf{u}}_{J,K})],\end{aligned}$$

$$\mathbf{\Psi}_C = \begin{cases} \widetilde{\mathbf{u}}^H_{j,k}\mathbf{R}_{j,i,s}\widetilde{\mathbf{u}}_{j,k}, & k = K_j + 1, \cdots, K, \forall j \\ & s = 1, \cdots, K_i, \forall i, \end{cases}$$

$$\mathbf{\Psi}_D = \begin{cases} \widetilde{\mathbf{u}}^H_{j,k}\mathbf{R}_{j,i,l}\widetilde{\mathbf{u}}_{j,k}, & (i,l) \neq (j,k) \\ & k = K_j + 1, \cdots, K, \forall j \\ & l = K_i + 1, \cdots, K, \forall i \\ 0, & (i,l) = (j,k). \end{cases}$$

Then the optimal uplink power allocation vector can be rewritten by composing $\widetilde{\mathbf{q}}_R$ and $\widetilde{\mathbf{q}}_N$ as $\widetilde{\mathbf{q}} = [\widetilde{\mathbf{q}}_R^T\ \widetilde{\mathbf{q}}_N^T]^T$, and finally (4.2.6c) is reformulated as

$$\boldsymbol{\sigma}_R^T\widetilde{\mathbf{q}}_R + \boldsymbol{\sigma}_N^T\widetilde{\mathbf{q}}_N = P_{max}, \tag{4.2.10}$$

where $\boldsymbol{\sigma}_R = [\sigma_{1,1}\cdots\sigma_{1,K_1}, \cdots, \sigma_{J,1}\cdots\sigma_{J,K_J}]^T$ and $\boldsymbol{\sigma}_R = [\sigma_{1,K_1+1}\cdots\sigma_{1,K}, \cdots, \sigma_{J,K_J+1}\cdots\sigma_{J,K}]^T$. So far, all constraints in the uplink beamforming problem have been reformulated into associated matrix forms. By rearranging (4.2.8), the optimal uplink power allocation for RTUs $\widetilde{\mathbf{q}}_R$ can be written in terms of $\widetilde{\mathbf{q}}_N$ as

$$\widetilde{\mathbf{q}}_R = (\mathbf{I} - \mathbf{D}_R\mathbf{\Psi}_A)^{-1}\mathbf{D}_R\mathbf{1} + (\mathbf{I} - \mathbf{D}_R\mathbf{\Psi}_A)^{-1}\mathbf{D}_R\mathbf{\Psi}_B\widetilde{\mathbf{q}}_N. \tag{4.2.11}$$

Here, (4.2.11) can hold only when $(\mathbf{I}-\mathbf{D}_R\mathbf{\Psi}_A)$ is invertible and $(\mathbf{I}-\mathbf{D}_R\mathbf{\Psi}_A)^{-1}$ is a nonnegative matrix. The sufficient condition to enable $(\mathbf{I}-\mathbf{D}_R\mathbf{\Psi}_A)$ nonsingular and $(\mathbf{I} - \mathbf{D}_R\mathbf{\Psi}_A)^{-1}$ nonnegative is $\rho(\mathbf{D}_R\mathbf{\Psi}_A) \leq 1$, where $\rho(\mathbf{D}_R\mathbf{\Psi}_A)$

is the spectral radius of $\mathbf{D}_R\mathbf{\Psi}_A$ [94]. Substituting (4.2.11) into (4.2.9), the following equation can be obtained

$$\lambda\widetilde{\mathbf{q}}_N = \mathbf{D}\widetilde{\mathbf{q}}_N + \mathbf{d}, \tag{4.2.12}$$

where

$$\mathbf{D} = \mathbf{D}_N\mathbf{\Psi}_D + \mathbf{D}_N\mathbf{\Psi}_C(\mathbf{I} - \mathbf{D}_R\mathbf{\Psi}_A)^{-1}\mathbf{D}_R\mathbf{\Psi}_B \tag{4.2.13}$$

$$\mathbf{d} = \mathbf{D}_N\mathbf{1} + \mathbf{D}_N\mathbf{\Psi}_C(\mathbf{I} - \mathbf{D}_R\mathbf{\Psi}_A)^{-1}\mathbf{D}_R\mathbf{1}. \tag{4.2.14}$$

By substituting (4.2.11) into (4.2.10), the power constraint can also be expressed in terms of $\widetilde{\mathbf{q}}_N$ as

$$\mathbf{c}^T\widetilde{\mathbf{q}}_N = P_{max} - c, \tag{4.2.15}$$

where

$$\mathbf{c}^T = \boldsymbol{\sigma}_R^T(\mathbf{I} - \mathbf{D}_R\mathbf{\Psi}_A)^{-1}\mathbf{D}_R\mathbf{\Psi}_B + \boldsymbol{\sigma}_N^T \tag{4.2.16}$$

$$c = \boldsymbol{\sigma}_R^T(\mathbf{I} - \mathbf{D}_R\mathbf{\Psi}_A)^{-1}\mathbf{D}_R\mathbf{1}. \tag{4.2.17}$$

Multiplying $\mathbf{c}^T$ to both sides of (4.2.12) and combining with (4.2.15), a new equation can be obtained as

$$\lambda = \frac{1}{P_{max} - c}\mathbf{c}^T\mathbf{D}\widetilde{\mathbf{q}}_N + \frac{1}{P_{max} - c}\mathbf{c}^T\mathbf{d}. \tag{4.2.18}$$

Finally, the conjunction of (4.2.12) and (4.2.18) provides an eigensystem for solving the SINR balancing problem as shown below:

$$\lambda\widetilde{\mathbf{q}}_{\text{ext}} = \mathbf{\Upsilon}(\widetilde{\mathbf{U}})\widetilde{\mathbf{q}}_{\text{ext}} \tag{4.2.19}$$

where

$$\boldsymbol{\Upsilon}(\widetilde{\mathbf{U}}) = \begin{bmatrix} \mathbf{D} & \mathbf{d} \\ \frac{1}{P_{max}-c}\mathbf{c}^T\mathbf{D} & \frac{1}{P_{max}-c}\mathbf{c}^T\mathbf{d} \end{bmatrix}, \tag{4.2.20}$$

and $\widetilde{\mathbf{q}}_{\text{ext}} = [\widetilde{\mathbf{q}}_N^T \ 1]^T$. The optimal uplink power allocation for NRTUs $\widetilde{\mathbf{q}}_N$ can then be obtained by determining the eigenvector of the matrix (4.2.20) based on the Perron-Frobenious theory [37], [94] and [96].

Beamformer Design for a Given Power Allocation

To find the optimal solution of the uplink SINR balancing problem formulated above, the optimal beamformers for a given power allocation need to be determined first. The optimal beamformers of the virtual uplink can be determined by independently maximizing the SINR of each user in each cell as [37]

$$\widetilde{\mathbf{u}}_{j,k} = \max_{\mathbf{u}_{j,k}} \frac{\mathbf{u}_{j,k}^H \mathbf{R}_{j,j,k} \mathbf{u}_{j,k}}{\mathbf{u}_{j,k}^H \mathbf{Q}_{j,k} \mathbf{u}_{j,k}}, \ \text{s.t.} \ ||\mathbf{u}_{j,k}||_2 = 1, \tag{4.2.21}$$

where

$$\mathbf{Q}_{j,k} = \sum_{\substack{l=1 \\ l \neq k}}^{K} q_{j,l} \mathbf{R}_{j,j,l} + \sum_{\substack{i=1 \\ i \neq j}}^{J} \sum_{m=1}^{K} q_{i,m} \mathbf{R}_{j,i,m} + \boldsymbol{\Omega}_j \quad \forall j, \forall k.$$

Virtual Uplink Power Initialization

To ensure the validity of the algorithm to be proposed, an appropriate initial uplink power allocation need to be determined. Here, the uplink power is initialed by considering a special case that only RTUs exist, and the corresponding uplink SINR balancing problem can be formulated as

$$\begin{aligned} \max_{\mathbf{U}_R, \mathbf{q}_R} \min_{j,k} \quad & \frac{\text{SINR}_{j,k}^{UL}(\mathbf{u}_{j,k}, \mathbf{q}_R)}{\gamma_{j,k}}, \quad k = 1, \cdots, K_j, \forall j \\ \text{s.t.} \quad & \boldsymbol{\sigma}_R^T \mathbf{q}_R \leq P_{max}. \end{aligned} \tag{4.2.22}$$

If (4.2.22) is feasible, then the original problem (4.2.3) is also feasible. Hence, the uplink power initialed through (4.2.22) can guarantee the feasibility of the original problem. Similar to (4.2.19) and (4.2.20), for a given set of beamformers $\widetilde{\mathbf{U}}_R = [\widetilde{u}_{1,1} \cdots \widetilde{u}_{1,K_1}, \cdots, \widetilde{u}_{J,1} \cdots \widetilde{u}_{J,K_J}]^T$, the problem formulated above can be tackled by solving the following eigensystem:

$$\lambda_R \widetilde{\mathbf{q}}_{R_{\text{ext}}} = \mathbf{\Upsilon}_R \widetilde{\mathbf{q}}_{R_{\text{ext}}}, \tag{4.2.23}$$

where

$$\mathbf{\Upsilon}_R = \begin{bmatrix} \mathbf{D}_R \mathbf{\Psi}_A & \mathbf{D}_R \mathbf{1} \\ \frac{1}{P_{max}} \boldsymbol{\sigma}_R^T \mathbf{D}_R \mathbf{\Psi}_A & \frac{1}{P_{max}} \boldsymbol{\sigma}_R^T \mathbf{D}_R \mathbf{1} \end{bmatrix}, \tag{4.2.24}$$

and $\widetilde{\mathbf{q}}_{R_{\text{ext}}} = [\widetilde{\mathbf{q}}_R^T \ 1]^T$ is the extended RTUs power allocation vector. The power allocation vector $\mathbf{q}_R^{opt}$ can be obtained when the optimal λ_R^{opt} is achieved. Finally, the initial power allocation for the proposed multicell beamforming algorithm can be written as $\mathbf{q}^{(0)} = [\mathbf{q}_R^{optT} \ \mathbf{0}^T]^T$, where $\mathbf{q}^{(0)} \in \mathbb{R}^{JK \times 1}$.

Iterative Solution

The coordinated multicell beamformer allocation algorithm is presented in Algorithm 4.1, in which the optimal uplink beamformer allocation and associated virtual uplink power vector can be determined. From the uplink-downlink duality, though the same set of the beamformers can be used for achieving the same SINR values in both the uplink and downlink, the associated power allocations for uplink and downlink are different. Hence, the obtained optimal uplink beamformers $\widetilde{\mathbf{U}}^*$ need to be further used to determine the downlink power allocation. Similar to the uplink case, the downlink power allocation can be composed by the downlink power allocation vectors for RTUs and NRTUs as $\mathbf{p} = [\mathbf{p}_R^T \ \mathbf{p}_N^T]^T$. By rearranging (4.2.3) and (4.2.3b), the downlink power allocation vectors for RTUs and NRTUs can be written as

$$\mathbf{p}_N^* = (\lambda^* \mathbf{I} - \mathbf{D}_D^*)^{-1} \mathbf{d}_D^* \tag{4.2.25}$$

$$\begin{aligned} \mathbf{p}_R^* &= (\mathbf{I} - \mathbf{D}_R^* \mathbf{\Psi}_A^{*T})^{-1} \mathbf{D}_R^* \mathbf{\Psi}_C^{*T} \mathbf{p}_N^* \\ &\quad + (\mathbf{I} - \mathbf{D}_R^* \mathbf{\Psi}_A^{*T})^{-1} \mathbf{D}_R^* \boldsymbol{\sigma}_R, \end{aligned} \tag{4.2.26}$$

where

$$\mathbf{D}_D^* = \mathbf{D}_N^* \mathbf{\Psi}_D^{*T} + \mathbf{D}_N^* \mathbf{\Psi}_B^{*T} (\mathbf{I} - \mathbf{D}_R^* \mathbf{\Psi}_A^{*T})^{-1} \mathbf{D}_R^* \mathbf{\Psi}_C^{*T} \tag{4.2.27}$$

$$\mathbf{d}_D^* = \mathbf{D}_N^* \boldsymbol{\sigma}_N + \mathbf{D}_N^* \mathbf{\Psi}_B^{*T} (\mathbf{I} - \mathbf{D}_R^* \mathbf{\Psi}_A^{*T})^{-1} \mathbf{D}_R^* \boldsymbol{\sigma}_R, \tag{4.2.28}$$

and $\mathbf{D}_R^*$, $\mathbf{D}_N^*$, $\mathbf{\Psi}_A^*$, $\mathbf{\Psi}_B^*$, $\mathbf{\Psi}_C^*$ and $\mathbf{\Psi}_D^*$ are matrices obtained using $\widetilde{\mathbf{U}}^*$.

Algorithm 4.1 Coordinated Multicell Beamformer Design

1. **Initialize** $\mathbf{q}^{(0)}$ by solving (4.2.22)
2. $n = 0$
3. **Repeat**
4. $n \Leftarrow n + 1$
5. Solve (4.2.21) using $\mathbf{q}^{(n-1)}$ to obtain $\widetilde{\mathbf{U}}^{(n-1)}$
6. Compute $\mathbf{D}_R^{(n-1)}$, $\mathbf{D}_N^{(n-1)}$, $\mathbf{\Psi}_A^{(n-1)}$ $\mathbf{\Psi}_B^{(n-1)}$, $\mathbf{\Psi}_C^{(n-1)}$ and $\mathbf{\Psi}_D^{(n-1)}$ using $\widetilde{\mathbf{U}}^{(n-1)}$
7. Solve (4.2.19) and obtain $\lambda^{(n)}$ and $\widetilde{\mathbf{q}}_N^{(n)}$
8. Obtain $\widetilde{\mathbf{q}}_R^{(n)}$ from $\widetilde{\mathbf{q}}_N^{(n)}$ and (4.2.11)
9. Define $\mathbf{q}^{(n)} = [\widetilde{\mathbf{q}}_R^{(n)T} \; \widetilde{\mathbf{q}}_N^{(n)T}]^T$
10. **Until** $\lambda^{(n-1)} - \lambda^{(n)} \leq \epsilon$
11. $\widetilde{\mathbf{U}}^* = \widetilde{\mathbf{U}}^{(n-1)}$ and $\lambda^* = \lambda^{(n)}$

4.2.3 Simulation Results

A two-cell coordinated beamforming with two users in each cell is investigated. The first user of each cell is considered as RTU that need to achieve a specific target SINR, whereas all the remaining users in all cells are NRTUs whose SINRs need to be balanced. Some key parameters of the cellular network for simulation are set as follow: the BS consist of four antennas; each user is equipped with only one antenna; the total transmission power of the multicell network is set to 3W. Random channels are generated to describe the channels between each user and all BSs using complex Gaussian variables with zero mean and unity variance. To simplify the simulation, the noise variance at all users are set to 0.01. The step size t is set to 0.01, and the stopping criterion ϵ is set to 0.0005. The target SINRs for RTUs in cell 1 and cell 2 are set to 40 and 70 respectively, whereas the priority weighting factor for balanced SINR for the NRTUs is set to 1. Simulation results for the proposed algorithm are shown in Table 4.1. Here, five different set of random channels are adopted, and both power allocation and SINRs are listed. The obtained SINRs of RTUs achieved the preset target values, whereas SINRs of the NRTUs in both cells have been balanced.

In order to check the validity of the proposed algorithm, the downlink power allocation determined by employing the proposed algorithm is compared with those obtained using the SDP approach [97]. The simulation configuration for the SDP method is the same as those settings in the proposed algorithm. In addition, SINR targets for all users need to be set when using the SDP approach. Here, SINR targets of all users are set the same as those obtained through using the proposed algorithm. The simulation results obtained using the SDP method are shown in Table 4.2. Comparing Table 4.1 and Table 4.2, it can be found that the power allocation obtained using both the methods are completely the same. Therefore, the proposed algorithm is valid for determining the optimal beamformers since the obtained results match that of the optimal solution provided by the SDP method.

Table 4.1. Power allocations and the achieved SINRs using the proposed method

	Power Allocation				Total Power	Achieved SINR			
	BS1		BS2			BS1		BS2	
Channels	User 1	User 2	User 1	User 2		User 1	User 2	User 1	User 2
Channel 1	1.2900	0.3758	0.9170	0.4172	3	40	4.1133	70	4.1133
Channel 2	0.9059	1.3075	0.4102	0.3764	3	40	49.1845	70	49.1845
Channel 3	0.3322	1.0226	0.8127	0.8324	3	40	156.7103	70	156.7103
Channel 4	0.7938	0.7915	0.6844	0.7303	3	40	39.5429	70	39.5429
Channel 5	0.2235	0.9708	1.1195	0.6863	3	40	171.4480	70	171.4480

Table 4.2. Target SINRs and the user power consumption using the SDP-based method

	Target SINR				Total Power	Power Allocation			
	BS1		BS2			BS1		BS2	
Channels	User 1	User 2	User 1	User 2		User 1	User 2	User 1	User 2
Channel 1	40	4.1133	70	4.1133	3	1.2900	0.3758	0.9170	0.4172
Channel 2	40	49.1845	70	49.1845	3	0.9059	1.3075	0.4102	0.3764
Channel 3	40	156.7103	70	156.7103	3	0.3322	1.0226	0.8127	0.8324
Channel 4	40	39.5429	70	39.5429	3	0.7938	0.7915	0.6844	0.7303
Channel 5	40	171.4480	70	171.4480	3	0.2235	0.9708	1.1195	0.6863

4.3 Coordinated Multicell Beamforming with Multiple Interference Constraints

4.3.1 System Model and Problem Formulation

System Model

In section 4.2, a multicell multi-user wireless network was considered in which all J BSs involve in the coordination. However, it is impractical for all BSs to coordinately design beamformers. Therefore, in this section, in addition to considering J coordinating cells, it is assumed that there are V adjacent independent cells in the network where each independent cell has L users. In this case, the J BSs belonging to coordinating cells coordinate the beamformer design for their users, while ensuring interference leakage to users in other neighboring independent cells is below a threshold. In this section, it is assumed that the interference thresholds are known to all coordinating cells. However, in practice, the value of the threshold needs to be determined by considering system level performance. The system model is illustrated in Figure 4.2.

Downlink Transmit Beamforming

By considering adjacent independent cells, the mixed QoS-based beamforming for coordinating cells is constrained not only by the total transmission power and target SINRs for RTUs, but also by the interference leakage to users in those independent cells. Interference leakage to the lth user in the vth independent cell from all coordinating BSs can be defined as $\mathbf{g}_{v,l} = [\|\tilde{h}^H_{1,v,l}\mathbf{u}_{1,1}\|^2_2, \cdots, \|\tilde{h}^H_{1,v,l}\mathbf{u}_{1,K}\|^2_2]^T, \cdots, \|\tilde{h}^H_{J,v,l}\mathbf{u}_{J,1}\|^2_2,$
$\cdots, \|\tilde{h}^H_{J,v,l}\mathbf{u}_{J,K}\|^2_2]^T$, where $\tilde{h}_{j,v,l} \in \mathbb{C}^{M\times 1}$ is the channel vector between the jth coordinating BS and the lth user in the vth independent cell.

Then the mixed QoS-based multicell beamformer design with interference

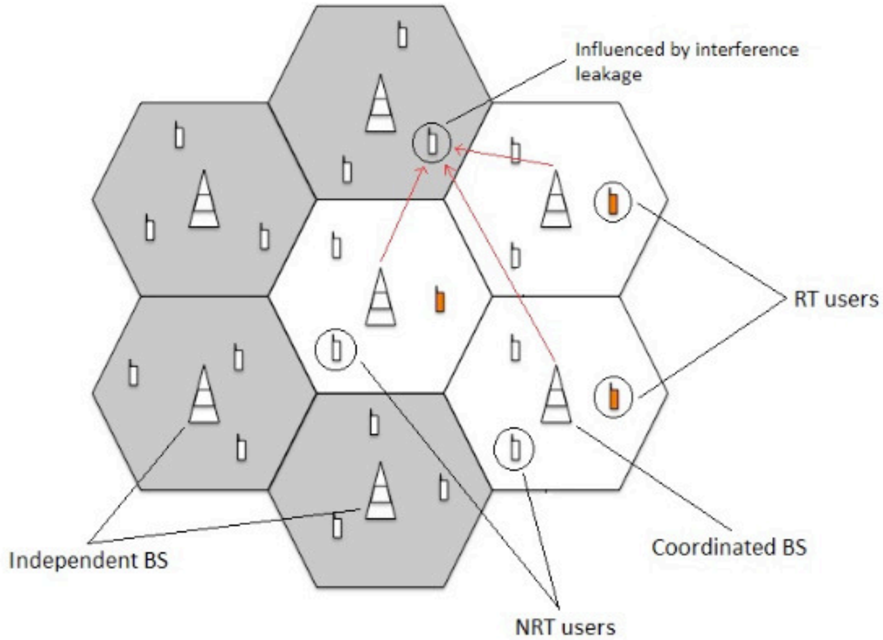

Figure 4.2. Mixed QoS-based multicell beamforming with local coordination.

constraints is formulated as follows:

$$\max_{\mathbf{U},\mathbf{p}} \min_{j,k} \quad \frac{\Gamma_{j,k}(\mathbf{U},\mathbf{p})}{\delta_{j,k}}, \qquad k = K_j + 1, \cdots, K, \forall j \tag{4.3.1a}$$

$$\text{s.t.} \quad \Gamma_{j,k}(\mathbf{U},\mathbf{p}) \geq \gamma_{j,k}, \quad k = 1, \cdots, K_j, \forall j \tag{4.3.1b}$$

$$\mathbf{g}_{v,l}^T \mathbf{p} \leq P_{v,l}, \qquad \forall v, \forall l, \tag{4.3.1c}$$

$$\mathbf{1}^T \mathbf{p} \leq P_{max}, \tag{4.3.1d}$$

where $P_{v,l}$ is the interference threshold for the lth user in the vth independent cell. It is assumed all J coordinated BSs share a sum power constraint of P_{max}.

4.3.2 Interference Constraints-based Coordinated Multicell Beamforming Algorithm

Different to problem (4.2.3) with only a single power constraint, in (4.3.1), both power constraint and interference constraints exist. Hence, to solve problem (4.3.1) by using the same method applied to (4.2.3), a set of auxiliary variables are introduced to integrate the multiple constraints in (4.3.1) into a single constraint as follows:

$$\sum_{v=1}^{V}\sum_{l=1}^{L} a_{v,l}(\mathbf{g}_{v,l}^T\mathbf{p} - P_{v,l}) + a_p(\mathbf{1}_{JK}^T\mathbf{p} - P_{max}) \leq 0, \tag{4.3.2}$$

where $a_{v,l} \in \mathbb{R}_+$ is the auxiliary variable associated with the interference to the lth user in the vth independent cell while $a_p \in \mathbb{R}_+$ is the auxiliary variable associated with the total power constraint. By writing all auxiliary variables as a vector $\mathbf{a} = [a_{1,1}, \cdots, a_{1,L}, \cdots, a_{V,1}, \cdots, a_{V,L}, a_p]^T$, a vector $\mathbf{b} = [\mathbf{g}_{1,1}, \cdots, \mathbf{g}_{1,L}, \cdots, \mathbf{g}_{V,1}, \cdots, \mathbf{g}_{V,L}, \mathbf{1}_{JK}]\mathbf{a}$ and $P = \sum_{v=1}^{V}\sum_{l=1}^{L} a_{v,l}P_{v,l} + a_pP_{max}$ is then constructed. Using these definitions, the constraints (4.3.1c) and (4.3.1d) can be replaced by a new constraint $\mathbf{b}^T\mathbf{p} \leq P$. Then, (4.3.1) can be rewritten as the following optimization problem

$$\max_{\mathbf{U},\mathbf{p}} \min_{j,k} \quad \frac{\Gamma_{j,k}(\mathbf{u}_{j,k}, \mathbf{p})}{\delta_{j,k}}, \qquad k = K_j + 1, \cdots, K, \forall j \tag{4.3.3a}$$

$$\text{s.t.} \quad \Gamma_{j,k}(\mathbf{u}_{j,k}, \mathbf{p}) \geq \gamma_{j,k}, \quad k = 1, \cdots, K_j, \forall j \tag{4.3.3b}$$

$$\mathbf{b}^T\mathbf{p} \leq P. \tag{4.3.3c}$$

It should be noticed that the optimization problem (4.3.3) is not equivalent to the problem (4.3.1). However, the optimal solution of the problem (4.3.3) is an upper bound of that of the problem (4.3.1). This can be proved as follows:

Proof. If $(\mathbf{U}, \mathbf{p})$ is feasible for the problem (4.3.1), then it is also feasible for

the problem (4.3.3). Therefore, the feasible set of the problem (4.3.1) is a subset of that of the problem (4.3.3). Since the optimal $(\mathbf{U}^{*}, \mathbf{p}^{*})$ associated with the optimal solution of the problem (4.3.1) is also feasible for the problem (4.3.3), the optimal solution of the problem (4.3.3) could not be lower than the optimal solution of the problem (4.3.1).
In addition, by choosing an appropriate set of auxiliary variables, the optimal solution of the problem (4.3.1) can achieve the upper bound; hence, the upper bound is tight. Thus, though (4.3.1) and (4.3.3) are not equivalent, the optimal solution of the problem (4.3.1) can be obtained by solving (4.3.3) with an appropriate set of positive auxiliary variables. In the following, the algorithm of solving the problem (4.3.3) for a given set of auxiliary variables is first considered. The method of finding the optimal set of auxiliary variables will be explained later in this chapter. □

According to [95], for a given set of auxiliary variables, the above downlink beamforming problem can be reformulated as a dual uplink problem on the basis of uplink-downlink SINR duality:

$$\max_{\mathbf{U},\mathbf{q}} \min_{j,k} \quad \frac{\Lambda_{j,k}(\mathbf{u}_{j,k}, \mathbf{q})}{\delta_{j,k}}, \qquad k = K_j + 1, \cdots, K, \forall j \tag{4.3.4a}$$

$$\text{s.t.} \quad \Lambda_{j,k}(\mathbf{u}_{j,k}, \mathbf{q}) \geq \gamma_{j,k}, \quad k = 1, \cdots, K_j, \forall j \tag{4.3.4b}$$

$$\boldsymbol{\sigma}^T \mathbf{q} \leq P. \tag{4.3.4c}$$

In this case, the uplink SINR of the kth user in the jth coordinating cell $\Lambda'_{j,k}(\mathbf{u}_{j,k}, \mathbf{q})$ can be written as

$$\Lambda'_{j,k} = \frac{q_{j,k}\mathbf{u}_{j,k}^H \mathbf{R}_{j,j,k}\mathbf{u}_{j,k}}{\mathbf{u}_{j,k}^H (\sum\limits_{(i,l)\neq(j,k)} q_{i,l}\mathbf{R}_{j,i,l} + \Omega_j)\mathbf{u}_{j,k}}, \quad \forall k, \forall j \tag{4.3.5}$$

where Ω_j is the interference-plus-noise-variance matrix for users in the jth coordinating cell defined as $\Omega_j = \sum\limits_{v=1}^{V}\sum\limits_{l=1}^{L} a_{v,l}\tilde{\mathbf{h}}_{j,v,l}\tilde{\mathbf{h}}_{j,v,l}^H + a_p\mathbf{I}$. Solution of the downlink transmit beamforming with interference constraints can also be obtained by solving its dual uplink problem.

4.3.3 Power Allocation and Beamformers Design in the Uplink

Similar to (4.2.6a), by setting constraints (4.3.4b) and (4.3.4c) to equality, problem (4.3.4) can be rewritten as

$$\frac{\Lambda_{j,k}(\widetilde{\mathbf{u}}_{j,k}, \widetilde{\mathbf{q}})}{\delta_{j,k}} = \frac{1}{\lambda}, \qquad k = K_j + 1, \cdots, K, \forall j \tag{4.3.6}$$

$$\Lambda_{j,k}(\widetilde{\mathbf{u}}_{j,k}, \widetilde{\mathbf{q}}) = \gamma_{j,k}, \qquad k = 1, \cdots, K_j, \forall j \tag{4.3.7}$$

$$\boldsymbol{\sigma}^T \widetilde{\mathbf{q}} = P. \tag{4.3.8}$$

Following the mathematical manipulation stated in section 4.2, the optimal uplink power allocation vector for the RTUs $\widetilde{\mathbf{q}}_R$ can be expressed as

$$\widetilde{\mathbf{q}}_R = \mathbf{D}_R \Psi_A \widetilde{\mathbf{q}}_R + \mathbf{D}_R \mathbf{b}_R + \mathbf{D}_R \Psi_B \widetilde{\mathbf{q}}_N \tag{4.3.9}$$

where $\widetilde{\mathbf{q}}_N$ is the optimal power allocation vector for the NRTUs in all coordinating cells, and

$$\begin{aligned} \mathbf{b}_R = [&\widetilde{\mathbf{u}}_{1,1}^H \Omega_1 \widetilde{\mathbf{u}}_{1,1} \cdots \widetilde{\mathbf{u}}_{1,K_1}^H \Omega_1 \widetilde{\mathbf{u}}_{1,K_1}, \cdots, \\ &\widetilde{\mathbf{u}}_{J,1}^H \Omega_J \widetilde{\mathbf{u}}_{J,1} \cdots \widetilde{\mathbf{u}}_{J,K_J}^H \Omega_J \widetilde{\mathbf{u}}_{J,K_J}]. \end{aligned}$$

Using the same approach stated in section 4.2, substituting (4.3.5) into (4.3.6) and with appropriate rearrangements, the optimum uplink power allocation vector for the NRTUs $\widetilde{\mathbf{q}}_N$ can be expressed as follows

$$\lambda \widetilde{\mathbf{q}}_N = \mathbf{D}_N \Psi_D \widetilde{\mathbf{q}}_N + \mathbf{D}_N \mathbf{b}_N + \mathbf{D}_N \Psi_C \widetilde{\mathbf{q}}_R, \tag{4.3.10}$$

where

$$\mathbf{b}_N = [\widetilde{\mathbf{u}}_{1,K_1+1}^H \Omega_1 \widetilde{\mathbf{u}}_{1,K_1+1} \cdots \widetilde{\mathbf{u}}_{1,K}^H \Omega_1 \widetilde{\mathbf{u}}_{1,K}, \cdots ,$$
$$\widetilde{\mathbf{u}}_{J,K_J+1}^H \Omega_J \widetilde{\mathbf{u}}_{J,K_J+1} \cdots \widetilde{\mathbf{u}}_{J,K}^H \Omega_J \widetilde{\mathbf{u}}_{J,K}].$$

In equations (4.3.9) and (4.3.10), matrices $\mathbf{D}_R$, $\mathbf{D}_N$, $\boldsymbol{\Psi}_A$, $\boldsymbol{\Psi}_B$, $\boldsymbol{\Psi}_C$ and $\boldsymbol{\Psi}_D$ are the same as those defined in section 4.2. Then, equation (4.3.8) can be reformulated as

$$\boldsymbol{\sigma}_R^T \widetilde{\mathbf{q}}_R + \boldsymbol{\sigma}_N^T \widetilde{\mathbf{q}}_N = P. \tag{4.3.11}$$

By rearranging (4.3.9), the optimal uplink power allocation for RTUs $\widetilde{\mathbf{q}}_R$ can finally be written as a function of the optimal uplink power allocation for NRTUs $\widetilde{\mathbf{q}}_N$ as

$$\widetilde{\mathbf{q}}_R = (\mathbf{I} - \mathbf{D}_R \Psi_A)^{-1} \mathbf{D}_R \mathbf{b}_R + (\mathbf{I} - \mathbf{D}_R \Psi_A)^{-1} \mathbf{D}_R \Psi_B \widetilde{\mathbf{q}}_N. \tag{4.3.12}$$

Similar to the algorithm formulation process stated in section 4.2, the optimal uplink power allocation for NRTUs $\widetilde{\mathbf{q}}_N$ can be determined by solving the following eigen problem:

$$\lambda' \widetilde{\mathbf{q}}_{\text{ex}} = \Upsilon'(\widetilde{\mathbf{U}}) \widetilde{\mathbf{q}}_{\text{ex}}, \tag{4.3.13}$$

where

$$\Upsilon'(\widetilde{\mathbf{U}}) = \begin{bmatrix} \mathbf{D}_1 & \mathbf{d}_1 \\ \frac{1}{P-c_1} \mathbf{c}_1^T \mathbf{D}_1 & \frac{1}{P-c_1} \mathbf{c}_1^T \mathbf{d}_1 \end{bmatrix}, \tag{4.3.14}$$

and $\widetilde{\mathbf{q}}_{\text{ex}} = [\widetilde{\mathbf{q}}_N^T \; 1]^T$.

The elements in matrix $\Upsilon'(\widetilde{\mathbf{U}})$ are defined as follows:

$$\mathbf{D}_1 = \mathbf{D}_N \Psi_D + \mathbf{D}_N \Psi_C (\mathbf{I} - \mathbf{D}_R \Psi_A)^{-1} \mathbf{D}_R \Psi_B, \tag{4.3.15}$$

$$\mathbf{d}_1 = \mathbf{D}_N \mathbf{b}_N + \mathbf{D}_N \Psi_C (\mathbf{I} - \mathbf{D}_R \Psi_A)^{-1} \mathbf{D}_R \mathbf{b}_R, \tag{4.3.16}$$

$$\mathbf{c}_1^T = \boldsymbol{\sigma}_R^T(\mathbf{I} - \mathbf{D}_R\Psi_A)^{-1}\mathbf{D}_R\Psi_B + \boldsymbol{\sigma}_N^T, \tag{4.3.17}$$

$$c_1 = \boldsymbol{\sigma}_R^T(\mathbf{I} - \mathbf{D}_R\Psi_A)^{-1}\mathbf{D}_R\mathbf{b}_R. \tag{4.3.18}$$

Then, using the same method stated in section 4.2, the optimal beamformers in the uplink can be determined by independently maximizing the SINR of each user in each coordinating cell based on the dominant generalized eigenvector method and the initialed uplink power can be obtained by solving the case only RTUs exist.

Iterative Solution

Based on the description above, the overall algorithm for solving the proposed coordinated multicell beamforming problem is presented in Algorithm 4.2. The algorithm is composed of two iteration processes. The outer iteration is employed to update auxiliary variables while the inner iteration is applied to determine a set of optimum beamformers and power allocation for a given set of auxiliary variables. For the inner iteration, the uplink beamformer and associated uplink power allocation are first determined. The obtained uplink beamformers $\widetilde{\mathbf{U}}^*$ can then be used in the downlink since the same set of the beamformers in both the uplink and downlink can be used for achieving the same SINR values but with different power allocations due to the uplink-downlink duality. The expected downlink power allocation vector for all users in all coordinating cells can be obtained by determining power vectors for RTUs and NRTUs respectively as shown in section 4.2. Once the optimal downlink power allocation is obtained, the auxiliary variables can be updated by using the equations below

$$a_{v,l}^{(m+1)} = a_{v,l}^{(m)} + t(\mathbf{g}_{v,l}^T\mathbf{p}^{(m)} - P_{v,l}), \quad \forall v, \forall l \tag{4.3.19}$$

$$a_p^{(m+1)} = a_p^{(m)} + t(\mathbf{1}^T\mathbf{p}^{(m)} - P_{max}), \tag{4.3.20}$$

where t is a small positive step size. The optimal auxiliary variables are obtained at the point where the change in their values is below a threshold. The stoping criteria for this subgradient algorithm is given below

$$|a_{v,l}^{(m+1)}(\mathbf{g}_{v,l}^T\mathbf{p}^{(m)} - P_{v,l})| \leq \epsilon, \qquad \forall v, \forall l \tag{4.3.21}$$

$$|a_p^{(m+1)}(\mathbf{1}^T\mathbf{p}^{(m)} - P_{max})| \leq \epsilon, \tag{4.3.22}$$

where ϵ is a small threshold value.

4.3.4 Simulation Results

A simple three-cell wireless network with two coordinated BSs and one independent BS is investigated. It is assumed each cell has two users that share the same frequency. For the coordinating cells, the first user in both cells is considered as a RTUs that needs to achieve a specific target SINR, whereas the remaining two users in both cells are NRTUs whose SINRs need to be balanced. Other parameters used for the simulation are as follow: the BS consists of four antennas; each user is equipped with only one antenna; the total transmission power for all coordinating cells is set to 3W. Random channels are generated to describe the channels between all BSs and users using complex Gaussian variables. The noise variance at all users are set to 0.01. The target SINRs for RTUs in the coordinating cell 1 and cell 2 are set to 40 and 50 respectively. The interference leakage threshold is assumed to be 0.1 for all users in the adjacent cell. All auxiliary variables are initialized to 0.1. The step size t has been set to 0.01, and the stopping criterion ϵ has been set to 0.001.

Simulation results are shown in Table 4.3. Here, three different set of random channels have been adopted. Both the power allocation and the achieved SINRs are listed. The obtained SINRs of RTUs are the same as the preset target values, whereas SINRs of the NRTUs in all coordinated cells have

Algorithm 4.2 Coordinated Multicell Beamformer Design with Interference Constraints

1. **Initialize** t, ϵ, $a_p^{(0)}$ and $a_{v,l}^{(0)}$, $\forall v$, $\forall l$
2. $m = 0$
3. **Repeat**
4. $m \Leftarrow m + 1$
5. Initialize $\mathbf{q}^{(0)}$
6. $n = 0$
7. Repeat
8. $n \Leftarrow n + 1$
9. Obtain uplink beamformers $\widetilde{\mathbf{U}}^{(n-1)}$ using $\mathbf{q}^{(n-1)}$
10. Compute $\mathbf{D}_R^{(n-1)}$, $\mathbf{D}_N^{(n-1)}$, $\Psi_A^{(n-1)}$, $\Psi_B^{(n-1)}$,$\Psi_C^{(n-1)}$ and $\Psi_D^{(n-1)}$ using $\widetilde{\mathbf{U}}^{(n-1)}$
11. Solve (4.3.13) to obtain $\lambda^{(n)}$ and $\widetilde{\mathbf{q}}_N^{(n)}$
12. Determine $\widetilde{\mathbf{q}}_R^{(n)}$ using $\widetilde{\mathbf{q}}_N^{(n)}$ and (4.3.12) and
13. obtain $\mathbf{q}^{(n)} = [\widetilde{\mathbf{q}}_R^{(n)^T} \ \widetilde{\mathbf{q}}_N^{(n)^T}]^T$
14. Until $\lambda^{(n-1)} - \lambda^{(n)} \leq \epsilon$
15. $\widetilde{\mathbf{U}}^* = \widetilde{\mathbf{U}}^{(n-1)}$ and $\lambda^* = \lambda^{(n)}$
16. Obtain the optimal $\mathbf{p}^{(m)} = [\widetilde{\mathbf{p}}_R^T \ \ \widetilde{\mathbf{p}}_N^T]^T$ using (4.2.25) and (4.2.26)
17. Update auxiliary variables using (4.3.19) and (4.3.20)
18. **Until** (4.3.21) and (4.3.22) are satisfied.

been balanced.

To check the optimality of the design, the SDP method is again adopted. By comparing Table 4.3 and 4.4, it can be observed that same power allocation is obtained using these two approaches. It can also be observed that both methods provided the same set of beamformers. Hence, algorithm 4.2 can provide optimum results.

4.4 Summary

A mixed QoS-based coordinated multicell beamforming technique has been considered. The constraints of SINR-balancing and target-SINR are jointly considered with a total power constraint in designing the beamformers. Two scenarios have been considered. In the first scenario, each BS designs beamformers by coordinating with all other BSs and in the second scenario, only a subset of BSs join the coordination while rest of the BSs design their beamformers independently. The proposed iterative algorithms are capable of finding the optimal solutions. Hence, the proposed technique has the ability to satisfy the real time users with guaranteed SINRs while ensuring fairness to non-real time users by balancing and maximizing their SINRs. The employment of SDP technique has proved the optimality of the simulation results for both scenarios.

Table 4.3. Power allocations and the achieved SINRs using the proposed method

	Power Allocation				Total	Achieved SINR				Inter. Leakage	
	BS1		BS2		Power	BS1		BS2		Int 1	Int 2
Channels	User 1	User 2	User 1	User 2		User 1	User 2	User 1	User 2		
Channel 1	0.234	1.201	0.770	0.795	3	40	196.94	50	196.94	0.1	0.1
Channel 2	0.969	1.893	0.737	0.401	3	40	3.502	50	3.502	0.1	0.1
Channel 3	0.322	1.122	0.631	0.925	3	40	176.70	50	176.70	0.1	0.1

Table 4.4. Target SINRs and the user power consumption using the SDP-based method

	Target SINR				Total	Power Allocation				Inter. Leakage	
	BS1		BS2		Power	BS1		BS2		Int 1	Int 2
Channels	User 1	User 2	User 1	User 2		User 1	User 2	User 1	User 2		
Channel 1	40	196.94	50	196.94	3	0.234	1.201	0.770	0.795	0.1	0.1
Channel 2	40	3.502	50	3.502	3	0.969	1.893	0.737	0.401	0.1	0.1
Channel 3	40	176.70	50	176.70	3	0.322	1.122	0.631	0.925	0.1	0.1

Chapter 5

COORDINATED MULTICELL BEAMFORMING WITH MULTIPLE SINR BALANCING CRITERIA

In this chapter, a coordinated multicell beamformer design based on the SINR balancing technique within the context of users in different BSs achieve maximized SINRs with different levels is considered. Instead of attaining an overall balance of SINRs to all users in all cells, a multiple-step optimization algorithm is proposed which allows users in various cells to achieve different maximum possible balanced SINRs subject to the individual BS transmission power constraint. The uplink-downlink duality is used for converting the downlink problem into the uplink beamformer design problem in all optimization stages. An interference modified rebalancing technique is also proposed to provide the flexibility to the proposed algorithm.

5.1 Introduction

As discussed in Chapter 4, SINR balancing based beamforming has been studied widely in the literature [37, 86, 87, 98]. However, most of these existing works considered balancing SINR of all users in all cells to the same level. In [99], a new design was proposed, in which SINR of users in various cells are balanced and maximized to different levels rather than balancing SINR of all users in all cells to the identical value. The aim of the algorithm developed in [99] was to fairly balance users in a specific cell to the same SINR level while guaranteeing that the achieved SINR is the best that the users could achieve in the corresponding cell with its maximum transmission power. This problem is solved by balancing SINR of users in various cells sequentially while constraining SINR of users in those cells that have already achieved the maximum achievable balanced SINR to specific target SINR. Instead of employing target SINR constraints as proposed in [99], in this chapter, a set of interference constraints is introduced at each round of the optimization to find the optimum beamformers and power allocation. Compared to [99], the advantage of the proposed method is that for a specific round of optimization, only users served by those BSs that have excess transmission power need to be optimized. However, in [99], all users in all cells have to be optimized in every optimization round, which is computationally unattractive.

5.2 System Model and Problem Statement

A multicell multi-user wireless network consisting of J cells is considered as shown in Figure 5.1. It is assumed that there are K users in each cell. The MISO technique is employed in the downlink where each BS is equipped with M antennas and each user terminal has a single antenna. Let $\mathbf{h}_{i,j,k} \in \mathbb{C}^{M\times 1}$ represent the channel vector from the ith BS to the kth user in the jth cell and $p_{j,k}$ be the allocated power to the kth user in the jth cell. By denoting $\mathbf{u}_{j,k} \in \mathbb{C}^{M\times 1}$ as the beamformer vector for the kth user in the jth cell, where

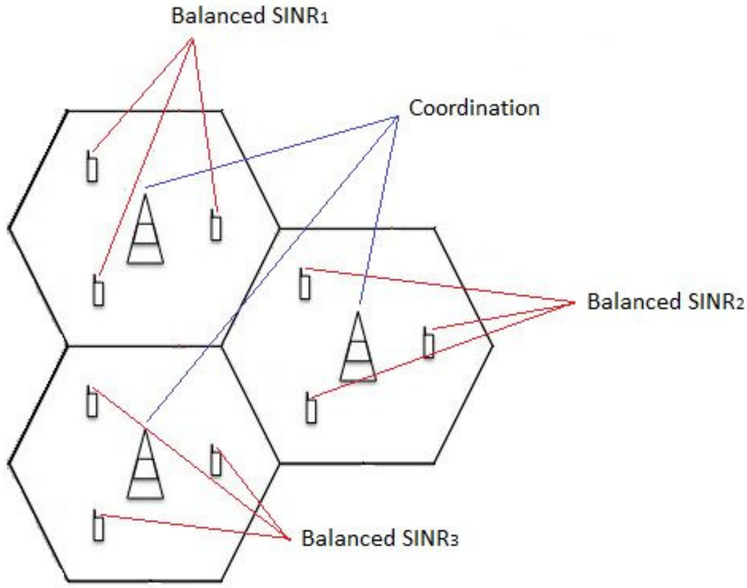

Figure 5.1. Coordinated multicell beamforming with multi-level SINR balancing criterion.

$\|\mathbf{u}_{j,k}\|_2 = 1,\ \forall j\ \forall k$, the SINR of the kth user in the jth cell in the downlink can be written as

$$\Gamma_{j,k} = \frac{p_{j,k}\mathbf{u}_{j,k}^H\mathbf{R}_{j,j,k}\mathbf{u}_{j,k}}{\sum\limits_{\substack{l=1\\l\neq k}}^{K} p_{j,l}\mathbf{u}_{j,l}^H\mathbf{R}_{j,j,k}\mathbf{u}_{j,l} + \sum\limits_{\substack{i=1\\i\neq j}}^{J}\sum\limits_{m=1}^{K} p_{i,m}\mathbf{u}_{i,m}^H\mathbf{R}_{i,j,k}\mathbf{u}_{i,m} + \sigma_{j,k}^2}, \tag{5.2.1}$$

where $\mathbf{R}_{i,j,k} \triangleq \mathbf{h}_{i,j,k}\mathbf{h}_{i,j,k}^H$, and $\sigma_{j,k}^2$ is the noise variance at the kth user in the jth cell. To balance and maximize SINRs of users in each cell, beamformers for all users in all cells need to be jointly designed. The multicell beamformer design with multi-level SINR balancing criteria can be formulated as:

$$\begin{aligned}
&\text{maximize} && r_j\ ,\forall j \in \{1, 2, \cdots, J\} && (5.2.2a)\\
&\text{s.t.} && \min_{1\leq k\leq K} \frac{\Gamma_{j,k}(\mathbf{U},\mathbf{p})}{\rho_{j,k}} \geq r_j, \forall j && (5.2.2b)\\
& && \mathbf{1}^T\mathbf{p}_j \leq P_j^{max},\ \forall j, && (5.2.2c)
\end{aligned}$$

where $\mathbf{p} = [\mathbf{p}_1^T, \cdots, \mathbf{p}_J^T]^T$ is the power allocation vector and $\mathbf{U} = [\mathbf{U}_1, \cdots, \mathbf{U}_J]$ denotes the beamformer matrix for all users. In (5.2.2), $\mathbf{U}_j = [\mathbf{u}_{j,1}, \cdots, \mathbf{u}_{j,K}]$ and $\mathbf{p}_j = [p_{j,1}, \cdots, p_{j,K}]^T$ are the beamformers of the users in the jth cell and their associated downlink power allocation, respectively. r_j is the *maxmin* SINR for the jth cell, whereas $\rho_{j,k}$ represents a priority weighting factor for the kth user in the jth cell. It is assumed the maximum consumable power for the jth BS is P_j^{max}.

5.3 Multi-Stage Coordinated Multicell Beamforming

To balance SINRs of users in different cells to different levels, a two-stage optimization scheme is adopted. In the first stage, all users in all cells are initially balanced to the same SINR level through the method proposed in [99]. In the second stage, the SINR levels for those BSs that have excess transmission power will be further improved.

5.3.1 First Stage: Balance All Users to the Same SINR Level

In the first stage, a *maxmin* problem is formulated by maximizing the minimum SINR of users in all cells with individual transmission power constraint P_j^{max} for each base station as follows

$$\max_{\mathbf{p}\succeq 0, \mathbf{U}} \min_{j,k} \quad \frac{\Gamma_{j,k}(\mathbf{U}, \mathbf{p})}{\rho_{j,k}}, \ \forall j, \forall k \tag{5.3.1a}$$

$$\text{s.t.} \quad \mathbf{1}^T \mathbf{p}_j \leq P_j^{max}, \ \forall j. \tag{5.3.1b}$$

Such SINR balancing problem has been well studied in the literature for both single and multi-cell wireless networks [37], [86], [95]. To solve (5.3.1), multiple linear per-BS power constraints in (5.3.1b) are put into a single linear constraint by introducing auxiliary variables as follows

$$\sum_{j=1}^{J} a_j(\mathbf{1}^T\mathbf{p}_j - P_j^{max}) \leq 0, \tag{5.3.2}$$

where $a_j \in \mathbb{R}_+$ are auxiliary variables that can be updated to find the optimal solution [95]. By defining the auxiliary variables vector $\mathbf{a} = [a_1\mathbf{1}_K^T \cdots a_J\mathbf{1}_K^T]^T$ and the integrated power constraint $P_{max} \triangleq \sum_{j=1}^{J} a_j P_j^{max}$, problem (5.3.1) can be reformulated as

$$\max_{\mathbf{p}\succeq 0, \mathbf{U}} \min_{j,k} \quad \frac{\Gamma_{j,k}(\mathbf{U}, \mathbf{p})}{\rho_{j,k}} \tag{5.3.3a}$$

$$\text{s.t.} \quad \mathbf{a}^T\mathbf{p} \leq P_{max}. \tag{5.3.3b}$$

Downlink Power Allocation for a Given Set of Beamformers

To solve problem (5.3.3), the power allocation for a given set of beamformers $\widetilde{\mathbf{U}}$ is first considered, where beamformers $\widetilde{\mathbf{U}}$ are fixed. Then, (5.3.3) can be rewritten as

$$\max_{\mathbf{p}\succeq 0} \min_{j,k} \quad \frac{\Gamma_{j,k}(\widetilde{\mathbf{U}}, \mathbf{p})}{\rho_{j,k}} \tag{5.3.4a}$$

$$\text{s.t.} \quad \mathbf{a}^T\mathbf{p} \leq P_{max}. \tag{5.3.4b}$$

The above *maxmin* problem has been solved in [37] for the single cell case by assuming SINR of all users can achieve an identical value. However, for multicell scenario, as the transmission power from various cells cannot be traded off, such identical balanced SINR value might not be possible. Hence, instead of using the Perron-Frobenious method proposed in [37], the downlink power allocation is determined by solving the following geometric programming problem:

$$
\begin{aligned}
\max_{\mathbf{p}} \quad & \gamma \\
\text{s.t.} \quad & \frac{\Gamma_{j,k}(\widetilde{\mathbf{U}}, \mathbf{p})}{\rho_{j,k}} \geq \gamma, \qquad \forall j, \forall k \qquad (5.3.5a) \\
& \mathbf{1}^T \mathbf{p}_j \leq P_j^{max}, \qquad \forall j, \qquad (5.3.5b)
\end{aligned}
$$

where γ is the expected *maxmin* SINR in the downlink for all users in all cells and $\widetilde{\mathbf{p}}$ is the optimal downlink power allocation vector.

Uplink Power Allocation for a given set of Beamformers

Since it is difficult to directly find the optimal beamformers $\widetilde{\mathbf{U}}_{opt}$ in the downlink, due to the uplink-downlink duality, the optimal beamformers in the downlink can be obtained by solving the corresponding uplink SINR balancing problem with the same power constraint P_{max}. By defining $\mathbf{q}_j = [q_{j,1}, \cdots, q_{j,K}]^T$ as the virtual uplink power allocation vector of the jth cell, the virtual uplink power allocation vector for all users in all cells can be written as $\mathbf{q} = [\mathbf{q}_1^T, \cdots, \mathbf{q}_J^T]^T$. Then, for a given set of beamformers $\widetilde{\mathbf{u}}_{j,k}$, the uplink SINR balancing problem can be expressed as

$$
\begin{aligned}
& \max_{\mathbf{q} \succeq 0} \min_{j,k} \quad \frac{\Lambda_{j,k}(\widetilde{\mathbf{u}}_{j,k}, \mathbf{q})}{\rho_{j,k}} \qquad (5.3.6a) \\
& \quad \text{s.t.} \quad \boldsymbol{\sigma}^T \mathbf{q} \leq P_{max}, \qquad (5.3.6b)
\end{aligned}
$$

where

$$
\Lambda_{j,k} = \frac{q_{j,k} \widetilde{\mathbf{u}}_{j,k}^H \mathbf{R}_{j,j,k} \widetilde{\mathbf{u}}_{j,k}}{\widetilde{\mathbf{u}}_{j,k}^H (\sum\limits_{\substack{l=1 \\ l \neq k}}^{K} q_{j,l} \mathbf{R}_{j,j,l} + \sum\limits_{\substack{m=1 \\ m \neq j}}^{J} \sum\limits_{l=1}^{K} q_{m,l} \mathbf{R}_{j,m,l} + \mathbf{\Omega}_j) \widetilde{\mathbf{u}}_{j,k}}
$$

and $\mathbf{\Omega_j} = a_j \mathbf{I}$ represents the corresponding noise covariance matrix in the uplink. As proved in [99], for a given set of $\widetilde{\mathbf{U}}$ and P_{max}, the optimal solution of the primal problem (5.3.4) and the dual uplink problem (5.3.6) are the same. This means the optimal beamformers $\widetilde{\mathbf{U}}_{opt}$ for (5.3.6) should also be the optimal beamformers obtained by solving (5.3.4).

Similar as in the downlink, (5.3.6) can also be solved through the GP optimization as

$$\begin{aligned} \max_{\mathbf{q}} \quad & \beta \\ \text{s.t.} \quad & \frac{\Lambda_{j,k}(\widetilde{\mathbf{u}}_{j,k}, \mathbf{q})}{\rho_{j,k}} \geq \beta, \qquad \forall j, \forall k \qquad (5.3.7a) \\ & \boldsymbol{\sigma}^T \mathbf{q} \leq P_{max}, \qquad (5.3.7b) \end{aligned}$$

where β is the expected $maxmin$ SINR in the uplink.

Beamformer Design for a given Power Allocation

By independently maximizing the SINR of each user in each cell in the uplink, the optimal beamformers for all users in the uplink can be determined by solving the following generalized eigenvalue problem

$$\widetilde{\mathbf{u}}_{j,k} = \max_{\mathbf{u}_{j,k}} \frac{\mathbf{u}_{j,k}^H \mathbf{R}_{j,j,k} \mathbf{u}_{j,k}}{\mathbf{u}_{j,k}^H \mathbf{Q}_{j,k} \mathbf{u}_{j,k}}, \text{ s.t. } ||\mathbf{u}_{j,k}||_2 = 1, \qquad (5.3.8)$$

where

$$\mathbf{Q}_{j,k} = \sum_{\substack{l=1 \\ l \neq j}}^{K} q_{j,l} \mathbf{R}_{j,j,l} + \sum_{\substack{i=1 \\ i \neq j}}^{J} \sum_{m=1}^{K} q_{i,m} \mathbf{R}_{j,i,m} + \mathbf{\Omega_j}, \forall j, \forall k. \qquad (5.3.9)$$

Iterative Solution for First Stage Optimization

According to [94] and [99], for a given set of auxiliary variables, the optimal uplink beamformers for problem (5.3.6) can be determined by iteratively solving (5.3.7) and (5.3.8). The obtained optimal uplink beamformers is then used in the downlink to determine the downlink power allocation and update auxiliary variables. Once no auxiliary variables can be further updated, the optimal downlink beamformers and power allocation for problem (5.3.3) are obtained.

5.3.2 Second Stage: SINR Improvement for Certain Cells

After the first stage of the optimization, all users in all cells should have achieved a maxmin SINR. There is at least one BS that has used its full transmission power and achieved the optimal balanced SINR. However, the users served by those BSs that have not used the full transmission power may not have achieved the optimal balanced SINR since their balanced SINR can be improved further by allocating more power to these users. Hence, in the following, the SINR improvement for users belonging to those BSs that have excess transmission power is considered.

Nth Round Balancing Problem

It is assumed that after $(N-1)$th round of optimization (including the first stage optimization), the first J_1 cells out of J cells have used their full transmission power. According to [99], the balanced SINR of users in the remaining $J - J_1$ cells can be further improved by solving a problem similar to (5.3.3), in which the SINRs of users in the first J_1 cells are constrained to a set of target SINRs obtained after the $(N-1)$th round optimization. Here, instead of employing target SINR constraints, a set of interference constraints is introduced to guarantee at the Nth round optimization, interference from the remaining $J - J_1$ BSs to users in the first J_1 cells is not greater than the same set of interference generated after the $(N-1)$th round optimization. By defining two sets of $\Theta_1 = \{1, \cdots, J_1\}$ and $\Theta_2 = \{J_1 + 1, \cdots, J\}$, the

SINR optimization for the remaining $J-J_1$ cells can be formulated as

$$\max_{\mathbf{U}^*,\mathbf{p}^*}\min_{j,k} \quad \frac{\Gamma_{j,k}(\mathbf{U}^*,\mathbf{p}^*)}{\rho_{j,k}}, \quad \forall j\in\Theta_2,\forall k \tag{5.3.10a}$$

$$\text{s.t.} \quad \mathbf{g}_{i,k}^T\mathbf{p}^*\leq t_{i,k}, \quad \forall i\in\Theta_1,\forall k \tag{5.3.10b}$$

$$\mathbf{1}^T\mathbf{p}_j\leq P_j^{max}, \quad \forall j\in\Theta_2, \tag{5.3.10c}$$

where $\mathbf{g}_{i,k}^T\mathbf{p}^*$ is the inter-cell interference to the kth user in the ith cell from the remaining $J-J_1$ cells when $i\in\Theta_1$, and $\mathbf{g}_{i,k}$ can be defined as

$$\begin{aligned}\mathbf{g}_{i,k} = [\,&\|h_{J_1+1,i,k}^H\mathbf{u}_{J_1+1,1}\|_2^2,\cdots,\|h_{J_1+1,i,k}^H\mathbf{u}_{J_1+1,K}\|_2^2,\cdots,\\ &\|h_{J,i,k}^H\mathbf{u}_{J,1}\|_2^2,\cdots,\|h_{J,i,k}^H\mathbf{u}_{J,K}\|_2^2]^T.\end{aligned}$$

$\mathbf{p}^*=[\mathbf{p}_{J_1+1}^T,\cdots,\mathbf{p}_J^T]^T$ and $\mathbf{U}^*=[\mathbf{U}_{J_1+1},\cdots,\mathbf{U}_J]$ are the power allocation vector and beamformer matrix for users in the remaining $J-J_1$ cells, respectively, and $t_{i,k}$ is the interference threshold obtained after the $(N-1)$th optimization round that can be calculated using corresponding beamformers and power allocation obtained in the $(N-1)$th optimization round as $t_{i,k}=\mathbf{g}_{i,k}^{(N-1)T}\mathbf{p}^{*(N-1)}$.

Similar to the method used in the first stage optimization, the multiple constraints developed above can be integrated into a single constraint by introducing two sets of auxiliary variables as follows

$$\sum_{i=1}^{J_1}\sum_{k=1}^{K}b_{i,k}(\mathbf{g}_{i,k}^T\mathbf{p}^*-t_{i,k})+\sum_{j=J_1+1}^{J}b_j(\mathbf{1}^T\mathbf{p}_j-P_j^{max})\leq 0,$$

where b_j and $b_{i,k}$ are auxiliary variables associated with the power constraints and inter-cell interference constraints, respectively. By writing all auxiliary variables as a vector of $\mathbf{b}=[b_{1,1},\cdots,b_{1,K},\cdots,b_{J_1,1},\cdots,b_{J_1,K},b_{J_1+1},\cdots,b_J]^T$, the above constraint can be rewritten in a simple way by setting $\mathbf{c}=[\mathbf{g}_{1,1},\cdots,\mathbf{g}_{1,K},\cdots,\mathbf{g}_{J_1,1},\cdots,\mathbf{g}_{J_1,K},\mathbf{r}_1,\cdots,\mathbf{r}_{J-J_1}]\mathbf{b}$ and $P_{max}^*=$

$\sum_{i=1}^{J_1}\sum_{k=1}^{K} b_{i,k}t_{i,k} + \sum_{j=J_1+1}^{J} b_j P_j^{max}$, where $\mathbf{r}_n$ is defined as a length $(J - J_1)K$ column vector, in which the nth length-K-block vector is set to $\mathbf{1}_K$ and all other length-K-block vectors are set to $\mathbf{0}_K$. Finally, problem (5.3.10) is reformulated with a single constraint as

$$\max_{\mathbf{U}^*,\mathbf{p}^*} \min_{j,k} \quad \frac{\Gamma_{j,k}(\mathbf{U}^*, \mathbf{p}^*)}{\rho_{j,k}}, \qquad j \in \Theta_2, \forall k \tag{5.3.11a}$$

$$\text{s.t.} \quad \mathbf{c}^T\mathbf{p}^* \leq P_{max}^*. \tag{5.3.11b}$$

Downlink Power Allocation for a Given Set of Beamformers at the Nth Round

Similar to the solution of problem (5.3.3), for a given set of beamformers $\widetilde{\mathbf{U}}^*$, the optimal power allocation in the downlink at the Nth round optimization can be obtained by solving the following GP problem

$$\max_{\mathbf{p}^*} \quad \gamma^*$$

$$\text{s.t.} \quad \frac{\Gamma_{j,k}(\widetilde{\mathbf{U}}^*, \mathbf{p}^*)}{\rho_{j,k}} \geq \gamma^*, \qquad \forall j \in \Theta_2, \forall k \tag{5.3.12a}$$

$$\mathbf{g}_{i,k}^T\mathbf{p}^* \leq t_{i,k}, \qquad \forall i \in \Theta_1, \forall k \tag{5.3.12b}$$

$$\mathbf{1}^T\mathbf{p}_j \leq P_j^{max}, \qquad \forall j \in \Theta_2. \tag{5.3.12c}$$

It should be noticed the downlink SINR of the kth user in jth cell for $j \in \Theta_2$ should be modified as follows

$$\Gamma_{j,k}(\mathbf{U}^*, \mathbf{p}^*) = \frac{p_{j,k}\mathbf{u}_{j,k}^H\mathbf{R}_{j,j,k}\mathbf{u}_{j,k}}{\sum\limits_{\substack{l=1\\l\neq k}}^{K} p_{j,l}\mathbf{u}_{j,l}^H\mathbf{R}_{j,j,k}\mathbf{u}_{j,l} + \sum\limits_{\substack{i=J_1+1\\i\neq j}}^{J}\sum\limits_{m=1}^{K} p_{i,m}\mathbf{u}_{i,m}^H\mathbf{R}_{i,j,k}\mathbf{u}_{i,m} + \xi_{j,k}}$$

where $\xi_{j,k} = \sum_{i=1}^{J_1}\sum_{m=1}^{K}\hat{p}_{i,m}\hat{\mathbf{u}}_{i,m}^H\mathbf{R}_{i,j,k}\hat{\mathbf{u}}_{i,m} + \sigma_{j,k}^2$ consists of noise variance and the inter-cell interference to the kth user in the jth cell from those BSs that have used their full transmission power after the $(N-1)$th round of optimization.

Beamformer Design and Power Allocation in the Uplink at the Nth Round of Optimization

Similar to the SINR balancing process in the first stage, at the Nth round of optimization, the optimal beamformers $\widetilde{\mathbf{U}}^*_{opt}$ should also be determined in the uplink. By writing the virtual uplink power allocation vector for users in all $j \in \Theta_2$ as $\mathbf{q}^* = [\mathbf{q}^T_{J_1+1}, \cdots, \mathbf{q}^T_J]^T$ and defining $\boldsymbol{\xi_n} = [\xi_{n,1}, \cdots, \xi_{n,K}]^T$, for a given set of beamformers, the corresponding virtual uplink SINR balancing problem at the Nth round of optimization can be written as

$$\max_{\mathbf{q}^* \succeq 0} \min_{j,k} \quad \frac{\Lambda_{j,k}(\widetilde{\mathbf{u}}_{j,k}, \mathbf{q}^*)}{\rho_{j,k}}, \qquad \forall j \in \Theta_2, \forall k \tag{5.3.13a}$$

$$\text{s.t.} \quad \boldsymbol{\xi}^T \mathbf{q}^* \leq P^*_{max}, \tag{5.3.13b}$$

where $\boldsymbol{\xi} = [\boldsymbol{\xi}^T_{J_1+1}, \cdots, \boldsymbol{\xi}^T_J]^T$, and the uplink SINR of the kth user in the jth cell for $j \in \Theta_2$ is given by

$$\Lambda_{j,k} = \frac{q_{j,k}\widetilde{\mathbf{u}}^H_{j,k}\mathbf{R}_{j,j,k}\widetilde{\mathbf{u}}_{j,k}}{\widetilde{\mathbf{u}}^H_{j,k}(\sum\limits_{\substack{l=1\\l\neq j}}^{K} q_{j,l}\mathbf{R}_{j,j,l} + \sum\limits_{\substack{i=J_1+1\\i\neq j}}^{J}\sum\limits_{m=1}^{K} q_{i,m}\mathbf{R}_{j,i,m} + \boldsymbol{\Omega}^*_j)\widetilde{\mathbf{u}}_{j,k}},$$

where the noise covariance matrix of the jth cell at the Nth round of optimization should be defined as $\boldsymbol{\Omega}^*_j = \sum_{i=1}^{J_1}\sum_{k=1}^{K} b_{i,k}h_{j,i,k}h^T_{j,i,k} + b_j\mathbf{I}$.
The optimal virtual uplink power vector $\widetilde{\mathbf{q}}^*$ and the uplink balanced SINR are obtained by solving the following GP problem

$$\max_{\mathbf{q}^*} \quad \beta^*$$

$$\text{s.t.} \quad \frac{\Lambda_{j,k}(\widetilde{\mathbf{u}}_{j,k}, \mathbf{q}^*)}{\rho_{j,k}} \geq \beta^*, \quad \forall j \in \Theta_2, \forall k \tag{5.3.14a}$$

$$\boldsymbol{\xi}^T \mathbf{q}^* \leq P^*_{max}. \tag{5.3.14b}$$

In addition, for a given uplink power allocation, the uplink beamformers at the Nth round of optimization can also be obtained by solving (5.3.8). However, the matrix $\mathbf{Q}_{j,k}$ defined in (5.3.9) should be redefined as

$$\mathbf{Q}^*_{j,k} = \sum_{\substack{l=1 \\ l\neq j}}^{K} q_{j,l}\mathbf{R}_{j,j,l} + \sum_{\substack{i=J_1+1 \\ i\neq j}}^{J} \sum_{m=1}^{K} q_{i,m}\mathbf{R}_{j,i,m} + \mathbf{\Omega}^*_j \qquad \forall j \in \Theta_2, \forall k. \tag{5.3.15}$$

5.3.3 Algorithm Formulation of the Overall Optimization

So far, the procedure of designing beamformers for a given set of auxiliary variables at the Nth round of optimization has been given. However, since the obtained optimal solution is the upper bound of the optimal solution obtained through solving the problem (5.3.10), to find the optimal solution of (5.3.10), an appropriate set of auxiliary variables is required to be found. As stated in [95], the optimal auxiliary variables can be obtained through the subgradient-based updating method. For the problem (5.3.10), auxiliary variables can be updated through the following updating equations:

$$b_j^{(m+1)} = b_j^{(m)} + t(\mathbf{1}^T\mathbf{p}_j^{(m)} - P_j^{max}),\ \forall j \in \Theta_2 \tag{5.3.16}$$

$$b_{i,k}^{(m+1)} = b_{i,k}^{(m)} + t(\mathbf{g}_{i,k}^T\mathbf{p}^{*(m)} - t_{i,k}),\ \forall i \in \Theta_1, \forall k \tag{5.3.17}$$

$$|b_j^{(m+1)}(\mathbf{1}^T\mathbf{p}_j^{(m)} - P^*_{max})| \leq \epsilon,\ \forall j \in \Theta_2 \tag{5.3.18}$$

$$|b_{i,k}^{(m+1)}(\mathbf{g}_{i,k}^T\mathbf{p}^{*(m)} - t_{i,k})| \leq \epsilon,\ \forall i \in \Theta_1, \forall k. \tag{5.3.19}$$

Then, the algorithm for solving problem (5.3.10) at the Nth round of optimization can be formulated following the similar steps stated in section 5.3. Finally, the algorithm for the optimization problem (5.2.2) is obtained and shown in Algorithm 5.1.

Algorithm 5.1
Solution of the Multiple Criteria SINR Balancing Problem

First Stage Balancing:

Balance all users in all cells to the same SINR level

Second Stage Balancing:

    Initialize $m \Leftarrow 0$, $n \Leftarrow 0$, $\mathbf{q}^{*(0)}$, $\mathbf{b}^{(0)}$, t, ϵ

    Repeat

        $m \Leftarrow m + 1$

        Repeat

            $n \Leftarrow n + 1$

            Solve (5.3.8) using $\mathbf{q}^{*(n-1)}$ to obtain $\widetilde{\mathbf{U}}^{*(n-1)}$

            Using $\widetilde{\mathbf{U}}^{*(n-1)}$ to solve (5.3.14) and obtain $\beta^{*(n)}$ and $\widetilde{\mathbf{q}}^{*(n)}$

        Until $\beta^{*(n-1)} - \beta^{*(n)} \leq \epsilon$

        $\beta^{*}_{opt} = \beta^{*(n)}$ and $\widetilde{\mathbf{U}}^{*}_{opt} = \widetilde{\mathbf{U}}^{*(n-1)}$

        Obtain $\widetilde{\mathbf{p}}^{*(m)}$ and $\gamma^{*(m)}$ by solving (5.3.12)

        Update auxiliary variables using (5.3.16) and (5.3.17)

    Until (5.3.18) and (5.3.19) are satisfied.

5.4 Simulation Results

A wireless network with two cells is considered, in which each cell serves two users and all users in the network share the same frequency. The BS consists of four antennas while each user is equipped with one antenna. Random channels were generated to describe the channels between all BSs and users using complex additive white Gaussian variables with zero mean. The noise variance at all users were set to 0.01. The step size t was set to 0.0001 and the stopping criterion ϵ has been set to 0.00001. The maximum total transmission power for the first BS and the second BS were initialed to 0.4W and 5W, respectively. All priority factors $\rho_{j,k}$ were set to 1. In the first stage of optimization, SINRs of users in both cells have been balanced to 14.74dB. At this stage, the first BS has used its full transmission power of 0.4W while the second BS used only 4.5656W. This means that the second BS is not able to use all its transmission power at this stage. Hence, Algorithm 5.1 was employed to improve the balanced SINR of users served by the second BS. As shown in Figure 5.2, at the convergence, users in the second cell achieved a higher balanced SINR of 15.13dB and the second BS used its full transmission power of 5W.

To further verify the effectiveness of the proposed algorithm, the results are compared by varying transmission power of the second BS while keeping the first BS power at 0.4W. Figure 5.2 shows the balanced SINRs for users in both cells against the transmission power of the second BS. It is clear that the SINR achieved by users in the first cell remains the same while the balanced SINR of users in the second cell increases.

Hence, using the proposed algorithm, both the cells could use their full transmission power and different levels of balanced SINRs could be achieved by users in different cells. However, it should be noticed that the simulation results were based on the setting that each BS is equipped with four antennas

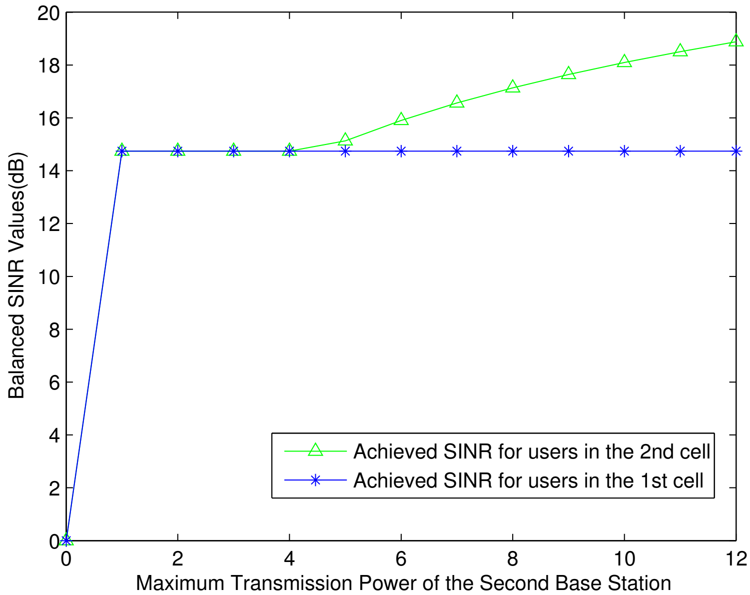

Figure 5.2. Balanced SINRs versus the maximum transmission power of the 2nd BS

and there are only four users in the network. In practice, if the total number of users is greater than the number of antennas equipped by each BS, once the first BS has used its full transmission power, the balanced SINR of the second BS may not be improved dramatically. This is because the second BS can not use its full transmission power due to inadequate degrees of freedom to suppress interference to users in other cells with increasing transmission power. To show this, a new set of channels with $M = 3$ was generated to repeat the simulation illustrated above. The maximum transmission power for the two BSs were set to 1W and 10W respectively. As shown in Table 5.1, for users in both cells, the SINRs have been balanced to 21.89; however, though the first BS has used all its transmission power of 1W, the second BS used only 2.1245W. This means that the second BS could not consume power beyond to improve the SINRs of its users. At this point, if the balanced SINR of users served by the second BS still needs to be improved, an effective way is to reduce the interference from the first BS to users in the second cell. With coordinated beamforming scheme, the second BS could request the first BS to modify its beamformers and power allocation to mitigate the interference to its users. However, this may reduce the SINRs of users served by the first BS. The simulation results with modified interference process are shown in Table 5.1. It is assumed that after the SINR improvement process of the second stage of optimization, if the interference modified scheme is not employed, the interference from the first BS to users in the second cell are denoted as int_1 and int_2, respectively. At this point, the second BS is unable to use its full transmission power and the balanced SINR of users served by the second BS cannot go beyond 21.89. To further improve the SINRs of the users in the second cell, both int_1 and int_2 are proportionally decreased. By introducing a modification factor θ, the modified interference from the first BS to users in the second cell can be set to $\theta \cdot int_1$ and $\theta \cdot int_2$. Here, the value of θ is between 0 and 1, and $\theta = 1$ means there is no interference modification. As shown in Table 5.1, once θ is set to 0.9, the balanced SINR of users in the second cell is improved from 21.89 to 23.58, which is 7.7% improvement. At the same time, the balanced SINR for users in the first cell has only dropped by 1.8%. This means that the balanced SINR of users in the second cell can be improved dramatically by sacrificing the SINR for users in the first cell by a small amount.

Table 5.1. Balanced SINRs using the proposed method with modified interference

	Achieved SINR			
	BS1		BS2	
θ	User 1	User 2	User 1	User 2
1	21.89	21.89	21.89	21.89
0.9	21.49	21.49	23.58	23.58
	Power of BSs			
	Total Power		Used Power	
θ	BS 1	BS 2	BS 1	BS 2
1	1	10	1	2.1245
0.9	1	10	0.9986	2.1954

5.5 Summary

A coordinated multicell beamforming technique with multiple SINR balancing criteria has been considered. Users in different cells can achieve a different level of maximum possible SINR. The proposed algorithm consists of two stages. In the first stage of optimization, all users in all cells are balanced to the same SINR level while in the second stage, SINR for users served by those BSs that have excess transmission power will be further optimized sequentially by introducing interference constraints. The geometric programming method is employed to solve the *maxmin* problem. An interference modified rebalancing method has also been proposed to introduce flexibility to the proposed technique.

Chapter 6

COALITIONAL GAMES FOR DOWNLINK MULTICELL BEAMFORMING

The coordinated multicell beamforming discussed in chapter 4 and chapter 5 can effectively suppress the intercell interference and improve the SINRs for users in all cells. However, cooperation between BSs will increase the burden of the network due to the information exchange between BSs. In this chapter, a coalitional game based multicell downlink beamforming is proposed. Each BS intends to minimize its transmission power while aiming to attain a set of target SINRs for its users. To improve the performance of power consumption and avoid the disadvantages arised from full coordination, BSs are allowed to form partial cooperation. For a given coalition, BSs in the coalition greedily minimize the total weighted transmission power without considering interference to users in other coalitions. By introducing a cost for cooperation, the coalition formation game is considered for the power minimization based downlink beamforming. A merge-regret based sequential coalition formation algorithm has been developed that proved to be capable of reaching a unique stable coalition structure.

6.1 Introduction

Coordinated multicell beamforming is an effective method to suppress intercell interference and to improve the overall performance of the network. However, in multicell networks with coordination design, messages have to be exchanged between BSs through backhaul channels which will lead to considerable cost and burden to the network. To overcome this disadvantage, game theoretic beamforming has been proposed in recent years. Different to the coordination based design, in non-cooperative multicell beamforming, each BS is considered as a player that greedily maximizes its own utility. In [100], a game theory based MISO interference channel was considered, in which each BS equipped with multiple antennas served only one user equipped with a single antenna. A two BSs based MISO IFC beamforming has been studied in [101] in both competitive and cooperative manner. In [101], it has found that the Nash equilibrium point of the MISO IFC game is equivalent to the point obtained through the maximum ratio transmission (MRT) and it is generally inefficient. The competition in a multicell multiuser network has been studied using game theory in [102] where each BS employs downlink beamforming to greedily minimize its own transmission power.

The inefficiency of the Nash equilibrium point obtained in the game theory based multicell beamforming urges BSs to coordinate with other BSs. However, when conditions required in the coordinated beamforming such as perfect channel reciprocal and strict synchronization are considered as extra cost, not all BSs are guaranteed to benefit from full coordination. For this reason, a cooperation mechanism should be considered in which each BS is allowed to selectively cooperate with some of the BSs to maximize its own benefit. Coalitional game aims to balance the competitiveness and coordination by allowing players to partially cooperate with each other. The coalitional game has been applied to the MISO IFC beamforming in which an operating point can be found through the strategic bargaining [103]. In [104], a MISO IFC beamforming based on coalition formation was studied. Coalition structures obtained through the algorithm proposed in [104] have been proved always in a coalition structure stable set. However, in both [103] and [104], only the scenario that each BS serves only one user has been considered.

In this chapter, a multicell beamforming is considered where the aim of each BS is to minimize its transmission power while satisfying a set of target SINRs for its users. In [89], this problem has been solved by jointly designing beamformers over all BSs in the network, in which the cost arised from the cooperation was not taken into consideration and each BS had the incentive to cooperate with all other BSs to reduce its transmission power as much as possible. However, in practice, BSs may not benefit from such full joint processing since the transmission power reduction for each BS through the coordination design may not provide any gain as compared to the cost introduced to each BS for forming coordination. Hence, in this chapter, it is assumed cost always exists in the cooperation and its value is linearly proportional to the number of BSs involved in the cooperation. Instead of cooperating over all BSs, a set of BSs are allowed to locally cooperate with each other by forming a coalition to maximize their benefits.

Various scenarios for multicell downlink beamforming are considered. In the non-cooperative multicell beamforming, each BS competes with all other BSs to minimize its power consumption while in the fully coordinated downlink multicell beamforming, all BSs cooperate with each other to reduce their overall power consumption. For a coalitional form multicell beamforming, different groups of BSs form coalitions and each coalition competes with all other coalitions to minimize the power consumption of its members. The main contribution of this chapter is the development of a merge-regret based coalition formation algorithm, which could effectively merge singleton BSs into large coalitions and reduce the resource consumption of each BS when the cooperation cost is taken into consideration.

6.2 Mathematical Background

Some definitions and statements in mathematics are first given, which will be used in the following sections.

In this section, $\mathbf{A} \geq \mathbf{0}$ denotes each element of matrix $\mathbf{A}$ is nonnegative while $\mathbf{A} > \mathbf{0}$ means that $\mathbf{A} \geq \mathbf{0}$ and at least one element of matrix $\mathbf{A}$ is positive. Then, to distinguish with the normal matrix inequality, the following inequalities are defined:

$$\mathbf{A} \geq \mathbf{B} \ \text{ if } \ a_{i,j} \geq b_{i,j}, \ \forall i, j,$$
$$\mathbf{A} > \mathbf{B} \ \text{ if } \ \mathbf{A} \geq \mathbf{B} \ \text{ and } \ \mathbf{A} \neq \mathbf{B},$$

where $\mathbf{A}$ and $\mathbf{B}$ are square matrices with the same dimension; $a_{i,j}$ and $b_{i,j}$ are the $(i, j)th$ elements of matrices $\mathbf{A}$ and $\mathbf{B}$, respectively.

Three types of square matrices are also defined as follows:

Definition 6.2.1. (Z-matrix) [105]: *A square matrix* $\mathbf{A}$ *is a Z-matrix if all of its off-diagonal elements are nonpositive.*

Definition 6.2.2. (P-matrix) [105]: *A square matrix* $\mathbf{A}$ *is a P-matrix if all of its principle minors are positive.*

Definition 6.2.3. (M-matrix) [106], [107]: *A square matrix* $\mathbf{A}$ *is M-matrix if* $\mathbf{Ay} \geq \mathbf{0}$ *implies* $\mathbf{y} \geq \mathbf{0}$ *for all* $\mathbf{y}$.

Definition 6.2.4. (Inverse-positive matrix) [105]: *A square matrix* $\mathbf{A}$ *is an inverse-positive matrix if* $\mathbf{A}^{-1} > \mathbf{0}$.

6.3 System Model

A multicell multi-user wireless network consisting of J cells is considered. Let $\Omega = \{1, \cdots, J\}$ be the set of all cells. It is assumed there are K users in each cell. The MISO technique is employed where each BS is equipped with M antennas and each user terminal has a single antenna. $\mathbf{h}_{i,j,k} \in \mathbb{C}^{M\times 1}$ represents the channel vector from the ith BS to the kth user in the jth cell and $s_{j,k}$ is the information symbol to the kth user in the jth cell, where $\mathrm{E}\{|\mathrm{s_{j,k}}|\} = 1$. By denoting $\mathbf{w}_{j,k} \in \mathbb{C}^{M\times 1}$ as the downlink transmitter beamformer vector for the kth user in the jth cell, the received signal at the kth user in the jth cell can be written as

$$y_{j,k} = \sum_{l=1}^{K} \mathbf{h}_{j,j,k}^{H}\mathbf{w}_{j,l}s_{j,l} + \sum_{\substack{i=1\\i\neq j}}^{J}\sum_{m=1}^{K} \mathbf{h}_{i,j,k}^{H}\mathbf{w}_{i,m}s_{i,m} + \eta_{j,k}, \tag{6.3.1}$$

where $\eta_{j,k}$ in (6.3.1) is assumed to be complex additive white Gaussian noise with zero mean and variance σ^2. The downlink SINR of the kth user in the jth cell can be written as

$$\Gamma_{j,k} = \frac{|\mathbf{w}_{j,k}^{H}\mathbf{h}_{j,j,k}|^2}{\sum\limits_{l\neq k}^{K}|\mathbf{w}_{j,l}^{H}\mathbf{h}_{j,j,k}|^2 + \sum\limits_{i\neq j}^{J}\sum\limits_{m=1}^{K}|\mathbf{w}_{i,m}^{H}\mathbf{h}_{i,j,k}|^2 + \sigma^2}. \tag{6.3.2}$$

6.4 Downlink Coalitional Beamforming

The transmitter beamformers of users in the jth cell can be denoted in a matrix form as $\mathbf{W}_j = [\mathbf{w}_{j,1}, \cdots, \mathbf{w}_{j,K}]$, where $\mathbf{W}_j \in \mathcal{B}_j$ and $\mathcal{B}_j$ is the strategy space of BS j, defined as

$$\mathcal{B}_j := \{\mathbf{W}_j \in \mathbb{C}^{M\times K}\}. \tag{6.4.1}$$

Then, the strategy profile of all BSs is the joint of all possible strategies, defined as

$$(\mathbf{W}_1, \cdots, \mathbf{W}_J) \in \mathcal{X} := \mathcal{B}_1 \times \cdots \times \mathcal{B}_J. \tag{6.4.2}$$

The aim of each BS is to minimize the transmission power while ensuring that the downlink SINRs for its users are greater than a set of threshold values, i.e, $\Gamma_{j,k} \geq \gamma_{j,k}$. Hence, the utility function of the jth BS is defined as the transmission power at BS j,

$$p_j = \sum_{k=1}^{K} \|\mathbf{w}_{j,k}\|_2^2 = \|\mathbf{W}_j\|_F^2, \quad \forall j \in \Omega. \tag{6.4.3}$$

Let $S = \{C_1, \ldots, C_{N_s}\}$ be a partition of Ω with the following characteristics: $\bigcup_{q=1}^{N_s} C_q = \Omega$ and $C_x \cap C_y = \emptyset$ for any $C_x, C_x \in S$. Then, S is a coalition structure of Ω. Based on all the definitions above, the partition form game for the downlink multicell beamforming can be expressed as [104]

$$\langle \Omega,\ \mathcal{X},\ \mathcal{F},\ (p_j)_{j\in\Omega} \rangle, \tag{6.4.4}$$

where $\mathcal{F}$ is the partition function that assigns all possible partitions to the game. In the following, the coalitional beamforming problem will be discussed in terms of different coalition structures.

6.4.1 Non-cooperative Multicell Beamforming

In coalitional games, a special coalition structure is that all coalitions are singletons. For this special coalition structure, each player is competing with all other players without any cooperation, which falls into a traditional strategic non-cooperative game (SNG). For the transmission power minimization problem, players are BSs and each BS will greedily minimize its own transmission power without constraining interference to users in other cells, where the transmission power is the utility function of each player.

Hence, for the non-cooperative game, the beamforming strategy set of the jth BS is defined as

$$\mathcal{B}_j^{'} := \{\mathbf{W}_j \in \mathbb{C}^{M\times K} : \Gamma_{j,k}(\mathbf{W}_j, \mathbf{W}_{-j}) \geq \gamma_{j,k}, \forall k\}, \tag{6.4.5}$$

where $\mathbf{W}_{-j}$ is the strategy of all BSs except BS j. Then, the coalitional beamforming game with non-cooperative coalition structure can be expressed as

$$\langle \Omega,\ \{\mathcal{B}_j^{'}\}_{j\in\Omega},\ (p_j)_{j\in\Omega}\rangle. \tag{6.4.6}$$

This game has been well studied in [102], in which the best response strategy of the jth BS is the solution of the following optimization problem:

$$\begin{aligned} &\text{minimize} \quad \|\mathbf{W}_j\|_F^2 && \text{(6.4.7a)}\\ &\text{subject to} \quad \frac{|\mathbf{w}_{j,k}^H \mathbf{h}_{j,j,k}|^2}{\sum_{l\neq k}^{K} |\mathbf{w}_{j,l}^H \mathbf{h}_{j,j,k}|^2 + z_{j,k}} \geq \gamma_{j,k},\ \forall k && \text{(6.4.7b)}\end{aligned}$$

where $z_{j,k} = \sum_{i\neq j}^{J} \sum_{m=1}^{K} |\mathbf{w}_{i,m}^H \mathbf{h}_{i,j,k}|^2 + \sigma^2$ is the noise power plus the intercell interference from all the rest of BSs to the kth user in BS j. It has been proven in [102] that if the Nash equilibrium of the game (6.4.6) exists, it is unique. Then, the best strategies $\{\mathbf{W}_1^*, \cdots, \mathbf{W}_J^*\}$ are a set of beamformers satisfying

$$p_j(\mathbf{W}_j^*) \leq p_j(\mathbf{W}_j),\quad \forall \mathbf{W}_j \in \mathcal{B}_j^{'},\ \forall j \in \Omega, \tag{6.4.8}$$

where $p_j(\mathbf{W}_j^*)$ is the transmission power of the jth BS with the best strategy. It has also been found in [102] that for each BS, beam patterns of its users are independent to the value of the intercell interference. This means that for a given set of target SINRs, each BS can design a set of fixed beam patterns for its users regardless of interference value $z_{j,k}$. Therefore, the strategies of the non-cooperative beamforming game can be reduced to a set of power allocation as follows:

$$\mathcal{B}_j'(\mathbf{p}) := \{\mathbf{p}_j \in \mathbb{R}_+^K : \Gamma_{j,k}(\mathbf{p}_j, \mathbf{p}_{-j}) \geq \gamma_{j,k}, \forall k\}, \tag{6.4.9}$$

where $\mathbf{p}_j$ and $\mathbf{p}_{-j}$ are power allocation of the jth BS and all other BSs except the jth BS respectively. By submitting the obtained beam patterns into (6.4.7b) and setting equality for (6.4.7b), the best response strategy of the jth BS can be obtained through

$$\mathbf{p}_j^* = \mathbf{G}_j^{-1}\mathbf{z}_j, \tag{6.4.10}$$

where $\mathbf{z}_j = [z_{j,1}, \cdots, z_{j,K}]^T$, and $\mathbf{G}_j \in \mathbb{R}^{K\times K}$, defined as $[\mathbf{G}_j]_{k,k} = (\frac{1}{\gamma_{j,k}})$ $|\mathbf{u}_{j,k}^H \mathbf{h}_{j,j,k}|^2$ and $[\mathbf{G}_j]_{k,l} = -|\mathbf{u}_{j,l}^H \mathbf{h}_{j,j,k}|^2$, if $l \neq k$. $\mathbf{u}_{j,k}$ is the beam pattern of the kth user in the jth cell that can be obtained through $\mathbf{u}_{j,k} = \mathbf{w}_{j,k}/\|\mathbf{w}_{j,k}\|_2$. Hence, once the intersection point of (6.4.10) is obtained, the Nash equilibrium is achieved. It has also been discussed in [102] that the Nash equilibrium of (6.4.6) exists if and only if matrix $\mathbf{G}$ is a $\boldsymbol{M}$-matrix. Matrix $\mathbf{G}$ is defined as

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_1 & -\mathbf{G}_{21} & \dots & -\mathbf{G}_{J1} \\ -\mathbf{G}_{12} & \mathbf{G}_2 & \dots & -\mathbf{G}_{J2} \\ \vdots & \vdots & \ddots & \vdots \\ -\mathbf{G}_{1J} & -\mathbf{G}_{2J} & \dots & \mathbf{G}_J \end{bmatrix}, \tag{6.4.11}$$

where $\mathbf{G}_{ij}$ is the intercell interference matrix from ith cell to the jth cell. By summarizing all above, the best response strategies and utilities at the Nash equilibrium can be determined through Algorithm 6.1.

6.4.2 Coordinated Downlink Multicell Beamforming

Another special coalition structure in coalitional multicell beamforming is that all BSs in the network join to form a grand coalition and coordinately design beamformers for their users, which is known as fully coordinated

Algorithm 6.1
Downlink Beamforming with Strategic Non-cooperative Game

1. Determine the downlink beamformer $\mathbf{w}_{j,k}$ for all users using the method in [37] with a given set of interference;
2. Find the beam pattern $\mathbf{u}_{j,k}$ for all users;
3. Submit $\mathbf{u}_{j,k}$ into (6.4.7b) and set (6.4.7b) as equality to obtain (6.4.10);
4. For a given $\mathbf{z}_j$, determine $\mathbf{p}_j$ using (6.4.10);
5. Update $\mathbf{z}_j$ with $\mathbf{p}_{-j}$ and repeat step 4 until the optimal $\mathbf{p}_j^*$ is obtained;
6. Obtain the downlink beamformer using $\mathbf{w}_{j,k} = \sqrt{p_{j,k}^*}\mathbf{u}_{j,k}$.

multicell beamforming. By jointly designing beamformers for all users in all cells, intercell interference can be effectively mitigated while the transmission power of each BS is reduced. Here, the coordinated multicell beamforming proposed in [89] is considered, in which when a grand coalition is formed, beamformers are designed by minimizing the weighted total transmission power over all BSs. This multicell power minimization problem can be stated as follows:

$$\text{minimize} \quad \sum_{j=1}^{J} \hat{\alpha}_j \|\mathbf{W}_j\|_F^2 \tag{6.4.12a}$$

$$\text{subject to} \quad \frac{|\mathbf{w}_{j,k}^H \mathbf{h}_{j,j,k}|^2}{\sum_{l \neq k}^{K} |\mathbf{w}_{j,l}^H \mathbf{h}_{j,j,k}|^2 + \sum_{i \neq j}^{J} \sum_{m=1}^{K} |\mathbf{w}_{i,m}^H \mathbf{h}_{i,j,k}|^2 + \sigma^2} \geq \gamma_{j,k}, \ \forall j,k, \tag{6.4.12b}$$

where $\hat{\alpha}_j$ is the weighting factor assigned to the jth BS in the grand coalition Ω. As stated in [89], the optimal solution of (6.4.12) can be obtained by equivalently solving a dual uplink problem for the same set of SINRs. By introducing the Lagrangian technique, problem (6.4.12) can be transferred into the following uplink problem:

$$\text{minimize} \quad \sum_{j=1}^{J} \sum_{k=1}^{K} \hat{\lambda}_{j,k} \sigma^2 \tag{6.4.13a}$$

$$\text{subject to} \quad \frac{\hat{\lambda}_{j,k} |\hat{\mathbf{w}}_{j,k}^H \mathbf{h}_{j,j,k}|^2}{\sum_{(i,m) \neq (j,k)} \hat{\lambda}_{i,m} |\hat{\mathbf{w}}_{j,k}^H \mathbf{h}_{j,i,m}|^2 + \hat{\alpha}_j \hat{\mathbf{w}}_{j,k}^H \hat{\mathbf{w}}_{j,k}} \geq \gamma_{j,k}, \ \forall j,k, \tag{6.4.13b}$$

where $\hat{\lambda}_{j,k}$ and $\hat{\mathbf{w}}_{j,k}$ are the uplink power and receiver beamformer of the kth user in the jth cell in the grand coalition Ω, respectively. The optimal uplink power $\hat{\lambda}_{j,k}^*$ can be iteratively obtained through the method proposed in [56], and the receiver beamformer $\hat{\mathbf{w}}_{j,k}$ can then be calculated through the following equation

$$\hat{\mathbf{w}}_{j,k} = (\sum_{i=1}^{J}\sum_{m=1}^{K}\hat{\lambda}_{i,m}\mathbf{h}_{j,i,m}^{H}\mathbf{h}_{j,i,m} + \hat{\alpha}_j\mathbf{I})^{-1}\mathbf{h}_{j,j,k}. \tag{6.4.14}$$

According to [89], the downlink beamformers should be a scaled version of the uplink beamformers as $\mathbf{w}_{j,k} = \sqrt{\hat{\delta}_{j,k}}\hat{\mathbf{w}}_{j,k}$, where the scaling factors can be obtained using equation (18) in [89]. Based on this, the fully coordinated beamforming proposed in [89] is summarized in Algorithm 6.2.

Algorithm 6.2
Fully Coordinated Downlink Beamforming

1. Iteratively find the optimal uplink power $\hat{\lambda}_{j,k}^{*}$;
2. Determine the receiver beamformers using (6.4.14) based on the set of optimal uplink power;
3. Obtain the scaling factor $\hat{\delta}_{j,k}$ using equation (18) in [89];
4. Calculate downlink beamformers using $\mathbf{w}_{j,k} = \sqrt{\hat{\delta}_{j,k}}\hat{\mathbf{w}}_{j,k}$.

6.4.3 Beamformers Design for a Given Coalition Structure

After considering the strategic non-cooperative and fully coordinated cases, the multicell downlink beamforming for a given coalition structure is formulated. In the coalitional beamforming, disjoint cells merge to several coalitions and BSs in each coalition jointly design beamformers for their users. A typical coalitional game based beamforming is shown in Figure 6.1, in which cell 1 and cell 2 has formed as a coalition while cell 3 and cell 4 are singleton coalitions. In this case, the cooperation only exists within each coalition, which means that beamformers are designed with partial coordination and each coalition is still competing with other coalitions. Hence, to find the optimal beamformers for all users in all coalitions, the coalitional game is transferred to a strategic non-cooperative sub game (SNSG), in which players are all coalitions. For a singleton coalition, the utility function is still the transmission power of the corresponding BS while for a coalition with multiple cells, utility function is the weighted total transmission power of all users in the coalition. In SNSG, each coalition competes with other coalitions by greedily minimizing its utility.

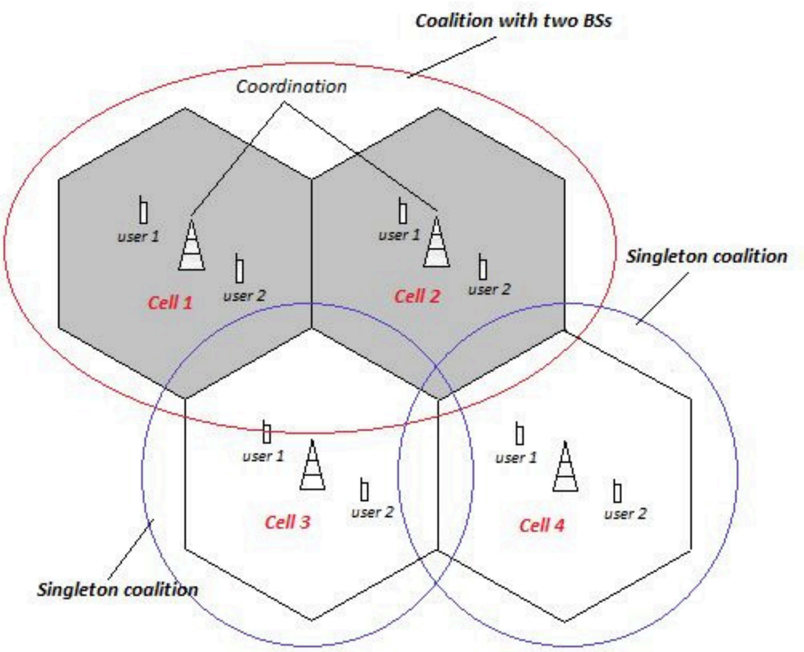

Figure 6.1. Multicell beamforming for a give coalition structure.

A coalition structure S with N_s coalitions $\{C_1, \cdots, C_{N_s}\}$ is considered, where $\bigcup_{q=1}^{N_s} C_q = \Omega$. Let $\Omega_s = \{1, \cdots, N_s\}$ be the set of players for the SNSG with coalition structure S and coalition C_q be the qth player of the SNSG, $\forall q \in \Omega_s$. The utility function of the qth player for the SNSG is defined as

$$T_q = \sum_{j \in C_q} \sum_{k=1}^{K} \breve{\alpha}_j \|\mathbf{w}_{j,k}\|_2^2 = \sum_{j \in C_q} \breve{\alpha}_j \|\mathbf{W}_j\|_F^2 \qquad (6.4.15)$$

where $\breve{\alpha}_j$ is the weighting factor of the jth BS with $\sum_{j \in C_q} \breve{\alpha}_j = 1$. It should be noticed that for a single coalition, $\breve{\alpha}_j = 1$ and (6.4.15) reduces to (6.4.3). Define the beamformer matrix of the coalition C_q in the coalition structure S as $\mathbf{W}_q\langle S \rangle$. This is the strategy of the qth player for the SNSG. Then, the beamforming strategy of all coalitions except coalition C_q is defined as $\mathbf{W}_{-q}\langle S\rangle$. By introducing downlink SINRs, the admissible strategy set for coalition C_q is defined as

$$\mathcal{B}_q\langle S\rangle = \{\mathbf{W}_q \in \mathbb{C}^{M\times K|C_q|} : \Gamma_{j,k}(\mathbf{W}_q\langle S\rangle, \mathbf{W}_{-q}\langle S\rangle) \geq \gamma_{j,k}, \forall j \in C_q, \forall k\}, \tag{6.4.16}$$

where $\gamma_{j,k}$ is the target SINR at the kth user in the jth cell for all $j \in C_q$. The interference induced by all BSs outside coalition C_q to the kth user in the jth cell can be written as $\sum_{i\notin C_q}\sum_{m=1}^{K}|\mathbf{w}_{i,m}^H\mathbf{h}_{i,j,k}|^2$. Then, the SNSG for a given coalition structure S can be written as

$$\langle \Omega_s,\ \{\mathcal{B}_q\langle S\rangle\}_{q\in\Omega_s},\ \{T_q\}_{q\in\Omega_s}\rangle. \tag{6.4.17}$$

The optimal strategy of the qth coalition for this game can be obtained by solving the following optimization problem

$$\text{minimize} \quad \sum_{j\in C_q} \breve{\alpha}_j \|\mathbf{W}_j\|_F^2 \tag{6.4.18a}$$

$$\text{subject to} \quad \frac{|\mathbf{w}_{j,k}^H\mathbf{h}_{j,j,k}|^2}{\sum\limits_{l\neq k}^{K}|\mathbf{w}_{j,l}^H\mathbf{h}_{j,j,k}|^2 + \sum\limits_{\substack{i\in C_q\\ i\neq j}}\sum\limits_{m=1}^{K}|\mathbf{w}_{i,m}^H\mathbf{h}_{i,j,k}|^2 + \check{z}_{j,k}} \geq \gamma_{j,k},\ \forall j \in C_q, \forall k, \tag{6.4.18b}$$

where $\check{z}_{j,k}$ is the inter-coalition interference from BSs outside coalition C_q to the kth user in the jth cell plus noise power for all BSs $j \in C_q$, can be written as

$$\check{z}_{j,k} = \sum_{\substack{x\in\Omega_s\\ x\neq q}}\sum_{i\in C_x}\sum_{m=1}^{K}|\mathbf{w}_{i,m}^H\mathbf{h}_{i,j,k}|^2 + \sigma^2. \tag{6.4.19}$$

For the qth coalition C_q with multiple BSs and a given set of $\check{z}_{j,k}$, problem (6.4.18) can be solved using the method proposed in [89]. The optimal transmitter beamformers can be obtained via solving the corresponding dual uplink problem. Similar to the fully coordination case, by introducing the Lagrangian duality, problem (6.4.18) can be transformed to the following optimization problem

$$\text{maximize} \quad \sum_{j \in C_q} \sum_{k=1}^{K} \check{\lambda}_{j,k} \check{z}_{j,k} \tag{6.4.20a}$$

$$\text{subject to} \quad \check{\mathbf{\Sigma}}_j \succeq (1 + \frac{1}{\gamma_{j,k}}) \check{\lambda}_{j,k} \mathbf{h}_{j,j,k} \mathbf{h}_{j,j,k}^H \tag{6.4.20b}$$

where

$$\check{\mathbf{\Sigma}}_j = \sum_{i \in C_q} \sum_{m=1}^{K} \check{\lambda}_{i,m} \mathbf{h}_{j,i,m}^H \mathbf{h}_{j,i,m} + \check{\alpha}_j \mathbf{I} \tag{6.4.21}$$

and $\check{\lambda}_{j,k}$ is the uplink power of the kth user in the jth cell with the coalition structure S. According to [89], problem (6.4.20) is equivalent to the following optimization problem

$$\text{minimize} \quad \sum_{j \in C_q} \sum_{k=1}^{K} \check{\lambda}_{j,k} \check{z}_{j,k} \tag{6.4.22a}$$

$$\text{subject to} \quad \frac{\check{\lambda}_{j,k} |\check{\mathbf{w}}_{j,k}^H \mathbf{h}_{j,j,k}|^2}{\sum\limits_{i \in C_q} \sum\limits_{m=1}^{K} \check{\lambda}_{i,m} |\check{\mathbf{w}}_{j,k}^H \mathbf{h}_{j,i,m}|^2 + \check{\alpha}_j \check{\mathbf{w}}_{j,k}^H \check{\mathbf{w}}_{j,k}}$$

$$\geq \frac{\gamma_{j,k}}{1 + \gamma_{j,k}}, \tag{6.4.22b}$$

where $\check{\mathbf{w}}_{j,k}$ is the uplink beamformer vector for the kth user in the jth cell with the coalition structure S. According to [89] and [102], the optimal uplink power $\check{\lambda}_{j,k}^*$, $\forall j \in C_q$, $\forall k$, can be determined through the following iterative fixed point process

$$\check{\lambda}_{j,k} = \frac{\gamma_{j,k}}{1 + \gamma_{j,k}} \cdot \frac{1}{\mathbf{h}_{j,j,k}^H \check{\mathbf{\Sigma}}_j^{-1} \mathbf{h}_{j,j,k}}. \tag{6.4.23}$$

For a given set of uplink power, the beamformer in the uplink $\check{\mathbf{w}}_{j,k}$ is the

MMSE receiver, can be calculated through

$$\check{\mathbf{w}}_{j,k} = (\sum_{i \in C_q} \sum_{m=1}^{K} \check{\lambda}_{i,m} \mathbf{h}_{j,i,m}^H \mathbf{h}_{j,i,m} + \check{\alpha}_j \mathbf{I})^{-1} \mathbf{h}_{j,j,k}. \tag{6.4.24}$$

It is clear that the solution of uplink power $\check{\lambda}_{j,k}$ and receiver beamformer $\check{\mathbf{w}}_{j,k}$ depends only on the intra-coalition channels and weighting factors in coalition C_q, and it is independent of the interference induced by BSs outside C_q. It has been proved in [89] that the optimal transmitter beamformer $\mathbf{w}_{j,k}$ is the scaled version of the optimal receiver beamformer $\check{\mathbf{w}}_{j,k}$. Hence, the transmitter beamformer $\mathbf{w}_{j,k}$ should also be a scaled version of the beam pattern $\mathbf{u}_{j,k}$, which can be obtained through the following equation

$$\mathbf{u}_{j,k} = \frac{\check{\mathbf{w}}_{j,k}}{\|\check{\mathbf{w}}_{j,k}\|_2}. \tag{6.4.25}$$

Hence, for a coalition $C_q \in S$, a fixed set of beam patterns for users inside C_q can be designed without considering any inter-coalition interference. By writing the transmitter beamformer as $\mathbf{w}_{j,k} = \sqrt{p_{j,k}} \mathbf{u}_{j,k}$, where $p_{j,k}$ is the downlink power allocated to the kth user in the jth cell, the weighted total power minimization problem (6.4.18) can be restated as

$$\text{minimize} \quad \sum_{j \in C_q} \sum_{k=1}^{K} \check{\alpha}_j p_{j,k} \tag{6.4.26a}$$

s.t

$$\frac{p_{j,k} |\mathbf{u}_{j,k}^H \mathbf{h}_{j,j,k}|^2}{\sum\limits_{\substack{l=1 \\ l \neq k}}^{K} p_{j,l} |\mathbf{u}_{j,l}^H \mathbf{h}_{j,j,k}|^2 + \sum\limits_{\substack{i \in C_q \\ i \neq j}} \sum\limits_{m=1}^{K} p_{i,m} |\mathbf{u}_{i,m}^H \mathbf{h}_{i,j,k}|^2 + \check{z}_{j,k}}$$
$$\geq \gamma_{j,k}, \ \forall j \in C_q, \forall k. \tag{6.4.26b}$$

Since all SINR constraints should achieve equality when the optimal beamformers $\mathbf{w}_{j,k}^* = \sqrt{p_{j,k}^*} \mathbf{u}_{j,k}$ is obtained $\forall j \in C_q$,$\forall k$, constraints (6.4.26b) can be rewritten as

$$p_{j,k}\frac{|\mathbf{u}_{j,k}^{H}\mathbf{h}_{j,j,k}|^2}{\gamma_{j,k}} - \sum_{\substack{l=1\\ l\neq k}}^{K} p_{j,l}|\mathbf{u}_{j,l}^{H}\mathbf{h}_{j,j,k}|^2$$
$$- \sum_{\substack{i\in C_q\\ i\neq j}}\sum_{m=1}^{K} p_{i,m}|\mathbf{u}_{i,m}^{H}\mathbf{h}_{i,j,k}|^2 = \check{z}_{j,k},\ \forall j\in C_q, \forall k. \tag{6.4.27}$$

To simplify the expression, it is assumed all the BSs in coalition C_q are re-numbered in the ascending order such that BSs with ascending indexes are renumbered from 1 to $|C_q|$. Then, parameters can be re-denoted in the following way: $\mathbf{h}_{(i)_x,(j,k)_q} \in \mathbb{C}^{M\times 1}$ is the channel vector from the ith BS in coalition C_x to the kth user of the jth cell in coalition C_q and $\mathbf{w}_{(j,k)_q}$ is the downlink transmit beamformer vector for the kth user in the jth cell in coalition C_q. $p_{(j,k)_q}$ represents the allocated power to the kth user in the jth cell in coalition C_q. The power allocation vector of coalition C_q in coalition structure S is denoted as $\check{\mathbf{p}}_q = [\mathbf{p}_{(1)_q}^T, \cdots, \mathbf{p}_{(|C_q|)_q}^T]^T$, $\forall C_q \in S$, where $\mathbf{p}_{(j)_q}$ is the power allocation vector of the jth cell in coalition C_q. By setting all SINR constraints in coalition C_q to equality, the following equation can be obtained

$$\mathbf{F}_q\check{\mathbf{p}}_q = \check{\mathbf{z}}_q, \tag{6.4.28}$$

where $\check{\mathbf{z}}_q = [\check{\mathbf{z}}_{(1)_q}^T, \cdots, \check{\mathbf{z}}_{(|C_q|)_q}^T]^T$ and $\check{\mathbf{z}}_{(v)_q}$ is the inter-coalition interference vector of the vth BS in coalition C_q. $\mathbf{F}_q$ is a $K|C_q| \times K|C_q|$ matrix with the following structure

$$\mathbf{F}_q = \begin{bmatrix} \mathbf{F}_q^{(1,1)} & \mathbf{F}_q^{(1,2)} & \dots & \mathbf{F}_q^{(1,|C_q|)} \\ \mathbf{F}_q^{(2,1)} & \mathbf{F}_q^{(2,2)} & \dots & \mathbf{F}_q^{(2,|C_q|)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{F}_q^{(|C_q|,1)} & \mathbf{F}_q^{(|C_q|,2)} & \dots & \mathbf{F}_q^{(|C_q|,|C_q|)} \end{bmatrix} \tag{6.4.29}$$

where $\mathbf{F}_q^{(j,i)}$ is a $K \times K$ sub-matrix with the following entries

$$[\mathbf{F}_q^{(j,i)}]_{k,m} = \begin{cases} \frac{|\mathbf{u}^H_{(j,k)_q}\mathbf{h}_{(j)_q,(j,k)_q}|^2}{\gamma_{(j,k)_q}} & i = j,\ m = k, \\ -|\mathbf{u}^H_{(j,m)_q}\mathbf{h}_{(j)_q,(j,k)_q}|^2 & i = j,\ m \neq k, \\ -|\mathbf{u}^H_{(i,m)_q}\mathbf{h}_{(i)_q,(j,k)_q}|^2 & i \neq j \end{cases} \tag{6.4.30}$$

for $j, i = 1, \cdots, |C_q|$ and $k, m = 1, \cdots, K$. It is clear that the best response strategy of the SNSG can be expressed as

$$\check{\mathbf{p}}_q = R_q(\check{\mathbf{p}}_{-\mathbf{q}}) = \mathbf{F}_q^{-1}\check{\mathbf{z}}_q. \tag{6.4.31}$$

It should be noticed that the best response strategy exists only if $\mathbf{F}_q$ is invertible. By rewriting the inter-coalition interference in a matrix form, the best response strategy can be expressed as

$$\check{\mathbf{p}}_q = \mathbf{F}_q^{-1}\Big(\sum_{\substack{x\in\Omega_s \\ x\neq q}} \mathbf{F}_{xq}\check{\mathbf{p}}_x + \mathbf{1}\sigma^2\Big) \tag{6.4.32}$$

where $\mathbf{F}_{xq}$ is the inter-coalition interference matrix of size $K|C_q| \times K|C_x|$ from the xth coalition to the qth coalition with the following structure

$$\mathbf{F}_{xq} = \begin{bmatrix} \mathbf{F}_{xq}^{(1,1)} & \mathbf{F}_{xq}^{(1,2)} & \cdots & \mathbf{F}_{xq}^{(1,|C_x|)} \\ \mathbf{F}_{xq}^{(2,1)} & \mathbf{F}_{xq}^{(2,2)} & \cdots & \mathbf{F}_{xq}^{(2,|C_x|)} \\ \vdots & \vdots & & \vdots \\ \mathbf{F}_{xq}^{(|C_q|,1)} & \mathbf{F}_{xq}^{(|C_q|,2)} & \cdots & \mathbf{F}_{xq}^{(|C_q|,|C_x|)} \end{bmatrix} \tag{6.4.33}$$

in which $\mathbf{F}_{xq}^{(j,i)}$ is a $K \times K$ sub-matrix with the following entries

$$[\mathbf{F}_{xq}^{(j,i)}]_{k,m} = |\mathbf{u}_{(i,m)_x}^H \mathbf{h}_{(i)_x,(j,k)_q}|^2, \quad \begin{aligned} & j = 1, \cdots, |C_q|, \\ & i = 1, \cdots, |C_x|, \\ & k = 1, \cdots, K, \\ & m = 1, \cdots, K. \end{aligned} \tag{6.4.34}$$

It should be noticed that (6.4.32) is the best response obtained based on a coalition C_q with multiple BSs in which the downlink beam pattern $\mathbf{u}_{j,k}$ is obtained using (6.4.24) and (6.4.25). However, for a singleton coalition, the downlink beam pattern should be determined using the method presented in Algorithm 6.1. Based on all the discussion above, the downlink beamforming for a given coalition structure can be summarized in Algorithm 6.3.

Algorithm 6.3
Coalitional beamforming algorithm

1. Find downlink beam pattern $\mathbf{u}_{j,k}$ via Algorithm 6.1 and 6.2 for both singleton coalitions and coalitions with multiple BSs;
2. Submit $\mathbf{u}_{j,k}$ into (6.4.26b) to obtain (6.4.31);
3. For a given $\check{\mathbf{z}}_q$, determine $\check{\mathbf{p}}_q$ using (6.4.31);
4. Update $\check{\mathbf{z}}_q$ and repeat step 3 until the optimal $\check{\mathbf{p}}_q^*$ is obtained;
5. Calculate the optimal downlink beamformer using $\mathbf{w}_{j,k}^* = \sqrt{p_{j,k}^*}\mathbf{u}_{j,k}$.

Similar to the strategic non-cooperative game presented in [102], for a given coalition structure S, the best response function (6.4.32) of the qth coalition is a standard function. According to the fixed point theorem stated in [56], a standard function means that if the Nash Equilibrium of the SNSG for a given coalition structure exists, then the NE point is unique. Hence, another issue of the SNSG game for a given coalition structure S is that whether the

NE exists. For a given coalition structure S, by rewriting the downlink SINRs for all users in all cells into the matrix form, the following equation can be obtained

$$\mathbf{G}^{'}\check{\mathbf{p}}^{*} = \mathbf{1}\sigma^{2} \tag{6.4.35}$$

where $\mathbf{G}^{'}$ is a matrix has the same structure of $\mathbf{G}$ as defined in (6.4.11). The difference between $\mathbf{G}^{'}$ and $\mathbf{G}$ is that they are obtained through different sets of beam patterns. In [102], the sufficient and necessary conditions for the existence and uniqueness of the NE for the non-cooperative game (6.4.7) has been given. Here, it is assumed that the NE for the strategic non-cooperative game always exists. Then, in the following, it will be shown that if some conditions are satisfied, the NE of the SNSG game exists.

Proposition 6.4.1. *If the NE of the non-cooperative game* (6.4.7) *exists, the NE of the SNSG game for a given coalition structure S, $S \neq \Omega$ exists if there exists an inverse-positive matrix $\mathbf{A}_1$ satisfies $\mathbf{A}_1 \leq \mathbf{G}^{'}$.*

Proof. First, since matrix $\boldsymbol{G}^{'}$ is a square matrix with the same structure as $\boldsymbol{G}$, $\boldsymbol{G}^{'}$ is a Z-matrix. As proved in [102], if the NE of the non-cooperative game (6.4.7) exists, $\boldsymbol{G}$ is a M-matrix.

Let $\boldsymbol{A}_2$ be a Z-matrix satisfies $\boldsymbol{A}_2 \geq \boldsymbol{G}$ and $\boldsymbol{A}_2 \geq \boldsymbol{G}^{'}$. Since $\boldsymbol{G}$ is a M-matrix, $\boldsymbol{G}^{-1}$ exists and $\boldsymbol{G}^{-1} > \mathbf{0}$. In addition, since $\boldsymbol{A}_2$ is a Z-matrix satisfies $\boldsymbol{A}_2 \geq \boldsymbol{G}$, according to [106], the inverse matrix $\boldsymbol{A}_2^{-1}$ exists and $\boldsymbol{G}^{-1} \geq \boldsymbol{A}_2^{-1} \geq \mathbf{0}$. Hence, $\boldsymbol{A}_2$ is an inverse-positive matrix.

If there exists an inverse-positive matrix $\boldsymbol{A}_1$ satisfies $\boldsymbol{A}_1 \leq \boldsymbol{G}^{'}$, then both $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are inverse-positive and according to [107], they are also monotone. Since $\boldsymbol{A}_1 \leq \boldsymbol{G}^{'} \leq \boldsymbol{A}_2$, then $\boldsymbol{G}^{'}$ is also monotone [36, Corollary 3.5], which means $\boldsymbol{G}^{'-1}$ exists and $\boldsymbol{G}^{'-1} > \mathbf{0}$. Hence, equation (6.4.35) can be written as $\check{\boldsymbol{p}}^{*} = \boldsymbol{G}^{'-1}\mathbf{1}\sigma^{2} > \mathbf{0}$, which means that there exists positive solutions $\check{\boldsymbol{p}}^{*}$ for (6.4.35). Thus, the NE of the SNSG game exists. □

It should be noticed that if coalition structure S is a partition of Ω with all singleton coalitions, Algorithm 6.3 will reduce to Algorithm 6.1. If S is the grand coalition Ω, Algorithm 6.3 is equivalent to Algorithm 6.2. In addition, for a given coalition structure, the best response strategy of the SNSG is obtained based on fixed sets of weighting factors for all coalitions in S with multiple BSs. Hence, by changing the weighting factors assignment for each coalition, the NE point of the SNSG will change.

6.5 Coalition Formation Process

Based on the discussion above, the coalition formation process for the downlink multicell beamforming can be formulated. To develop the coalition formation algorithm, some definitions are first given. The concept of *q-Deviation* has been proposed in [104] as a deviation rule descirbing how a coalition structure transits to another coalition structure in the coalition formation process. Here, by modifying the *q-Deviation* to accommodate the proposed downlink multicell beamforming problem, the concept of $\boldsymbol{\alpha}$-*Deviation* is introduced.

Definition 6.5.1. ($\boldsymbol{\alpha}$ – *Deviation*) : $\mathrm{S}_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_{n+1}$ represents the process of transiting the coalition structure S_n to the coalition structure S_{n+1} by merging coalitions in Θ to a new coalition $C_M = \bigcup \Theta$ with a given $\boldsymbol{\alpha}$, where $\Theta \subset S_n$; $C_M \in S_{n+1}$ and $S_{n+1} = S_n \backslash \Theta \cup C_M$; $\boldsymbol{\alpha}$ comprises of the weighting factors for all BSs in C_M with $\boldsymbol{\alpha} \in \mathbb{R}_+^{|C_M|}$ and $\|\boldsymbol{\alpha}\|_1 = 1$.

With above definition, a coalition structure S_n can transit to coalition structure S_{n+1} by merging all coalitions in the set Θ into one coalition. However, the new coalition C_M can be successfully formed only if all BSs in $\bigcup \Theta$ agree such transition for a given decision rule. In the proposed coalition formation problem, the individual utility based decision rule is applied by comparing utilities obtained in S_{n+1} and S_n for all BSs in $\bigcup \Theta$. In the following, both *Pareto Order* and *Majority Order* comparisons rules will be applied in the coalition formation process.

In existing works of coordinated multicell beamforming, beamformers for users in different BSs are jointly designed without considering the cost of cooperation. However, in practice, such cooperation cost cannot be ignored. Hence, in the proposed coalitional beamforming problem, it is assumed that once a coalition is formed, the associated cooperation cost will be introduced to all BSs in the coalition. The cooperation cost of a BS is assumed to be linearly proportional to the size of the coalition it stays in. Then, the cooperation cost of BS j in coalition structure S can be defined as

$$\epsilon_j(S) = (|C| - 1)\epsilon^* \tag{6.5.1}$$

where ϵ^* is the cost factor and C is the coalition of S that BS j stays in. It is clear that for BSs without cooperating with other BSs, $|C| = 1$ and there is no cost for cooperation.

In the downlink coalitional beamforming, each BS intends to reduce its transmission power by cooperating with other BSs; hence, once a new coalition C_M is formed, the benefit obtained by BSs in C_M is the reduced transmission power. However, in practice, it cannot guarantee that all BSs will benefit from the deviation especially when the cooperation cost is taken into consideration. Hence, the concept of *deviation gain* is introduced as the total benefit obtained through the deviation $S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_{n+1}$ by each BS. By defining the *resource consumption* of the jth BS in coalition structure S_n as

$$r_j(S_n) = p_j(S_n) + \epsilon_j(S_n),\ \forall j, \tag{6.5.2}$$

the *deviation gain* of BS j obtained by $S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_{n+1}$ is given as

$$\nu_j(S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_{n+1}) = r_j(S_n) - r_j(S_{n+1}),\ \forall j. \tag{6.5.3}$$

In addition, for deviation $S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_{n+1}$, it is assumed only BSs in $\bigcup\Theta$ can decide whether to form coalition C_M and all the rest of BSs are not allowed to make decisions. Based on these definitions and rules, both strongly independent comparison and weakly independent comparison can be given as follows:

Strong independence comparison: Pareto Order

$$\begin{aligned} & S_{n+1} \succ_P S_n \quad iif \\ & \nu_j(S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_{n+1}) \geq 0,\ \forall j \in \bigcup\Theta, and \\ & \exists j \in \bigcup\Theta \ satisfy\ \nu_j(S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_{n+1}) > 0. \end{aligned} \tag{6.5.4}$$

In Pareto order comparison, S_n can transit to S_{n+1} only if the deviation gains of all BSs in $\bigcup\Theta$ are nonnegative and at least one BS $j \in \bigcup\Theta$ has positive deviation gain. For any BSs in $\bigcup\Theta$ have obtained negative deviation gains, they will refuse to stay in coalition C_M and then the coalition structure S_{n+1} can not be formulated.

Weak independence comparison: Majority Order

$$\begin{aligned} & S_{n+1} \succ_M S_n \quad iif \\ & |\{j|\nu_j(S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_{n+1}) > 0, \forall j \in \bigcup\Theta\}| > \\ & |\{j|\nu_j(S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_{n+1}) < 0, \forall j \in \bigcup\Theta\}|. \end{aligned} \tag{6.5.5}$$

Different to the Pareto order comparison, in majority order, if majority number of BSs in $\bigcup\Theta$ have positive deviation gains of $S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_{n+1}$, S_n is allowed to transit to S_{n+1}. Hence, in this way of comparison, no BSs can independently reject a coalition formation. In the following, $\rhd = \{\succ_P, \succ_M\}$ denotes as the comparison strategy set that includes both the two comparison rules. Then, it can be concluded that $S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_{n+1}$ is reacheable if and only if $S_{n+1} \rhd S_n$ holds.

6.5.1 Coalition Formation Algorithm

As discussed above, for a given coalition structure, the beamforming design is a SNSG, which means that for the deviation $S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_{n+1}$, BSs in $\bigcup \Theta$ can only decide whether to stay in the new formed coalition C_M after the coalition structure S_{n+1} forms and the Nash equilibrium point of the SNSG game with S_{n+1} is achieved. Hence, a merge-regret formation strategy is adopted to formulate the coalition formation algorithm. The coalition formation algorithm for the multicell downlink beamforming is shown in Algorithm 6.4. It is assumed the coalition formation process always starts from the non-cooperative game in which all coalitions are singletons. Each BS has a preset index and knows indexs of other BSs. It is also assumed that BSs can communicate with each other and share coalition information and there is no extra cost for this. For a coalition structure S_n, all coalitions are numbered in some way. Then, each coalition in S_n first generates a set of l-combinations with lexicographical order, where $l = min\{b, |S_N|\}$, b is the maximum allowable size of merging. By using the same sequence of l-combinations, coalitions in the first l-combination merge into a temporary coalition C_t with a given $\boldsymbol{\alpha} = \frac{1}{|C_t|}\mathbf{1}$, and the coalition structure S_n is temporarily transited to S_t. Then, BSs in C_t will decide whether C_t is valid. Based on algorithm 3, each BS in C_t could find the best response beamforming strategy when the NE is reached; then its transition gain can be calculated and sent to all other BSs in C_t. A decision can be made by all BSs in C_t. If $S_t \triangleright S_n$ holds, C_t is valid and $S_{n+1} = S_t$; else, BSs in C_t will split from the temporary coalition C_t and reform the original coalitions S_n. Then, another temporary coalition with the next l-combination will be formed. The above process is repeated until a valid coalition structure is found. If for all l-combinations, no valid coalition structure S_{n+1} is found; this means l is too large for coalitions in S_n to transit to a new coalition structure via $\boldsymbol{\alpha}$-Deviation. Then each coalition will generate a new set of l-combinations with lexicographical order by reducing l to $l-1$ and repeat the process of temporary coalition formation until $l < 2$. If no valid coalition structure S_t can be found, algorithm stops and coalition structure S_n will be sustained; else, S_n is successfully transited to S_{n+1} and the coalition formation goes on.

6.5.2 α-Modification Algorithm

As discussed in section 6.3, for a given coalition structure, beam patterns for users in a coalition only depend on the intra-coalition channels of the coalition and the weighting factors assigned to all BSs within the coalition. This means that by modifying weighting factors assignment, beam patterns of all users in the coalition will be redesigned and the transmission power of the BS will change. In Algorithm 6.4, once a temporary coalition C_t is formed with a given $\boldsymbol{\alpha} = \frac{1}{|C_t|}\mathbf{1}$, the coalition structure S_n is temporarily transit to S_t. If $S_t \triangleright S_n$ holds, the formation of C_t is valid, then S_n will formally transit to S_{n+1}, where $S_{n+1} = S_t$; else C_t will split and the next temporary coalition will be formed. However, once a temporary coalition C_t is found invalid, by modifying the weighting factors vector $\boldsymbol{\alpha}$ of C_t, the transmission power of BSs in C_t will be changed, which might lead $S_t \triangleright S_n$ holds and C_t becomes valid.

The effect of this modification process can be explained in Figure 6.2, in which a network with two cells and two users in each cell is considered. It can be found that if both BSs individually design beamformers for their users in a competitive way, a Nash eqiulibrium can be obtained with the consumed power of 0.45W and 0.8W for BS1 and BS2 respectively. When the two BSs coordinately design beamformers with $\boldsymbol{\alpha} = [0.4 \;\; 0.6]$, the consumed power of BS 2 has been reduced to 0.57W as compared to the competitive design; however, the power consumed by BS has increased to 0.46W. This means that if the Pareto comparison is applied, even the coordination cost is not taken into consideration, the two BSs will still decide not to join as a

Algorithm 6.4
Merge-regret based coalition formation algorithm

1. **Input**: Ω, b, ϵ^*
2. **Initialize**: $n = 0$, S_n, $l = min\{b, |S_n|\}$
3. **while** $l \geq 2$
4. Each coalition generates b^* l-combinations of S_n in the lexicographical order;
5. **Initialize**: $k = 1$
6. S_n temporarily transits to S_t by merging coalitions in the kth combination Θ_k into a single temporary coalition C_t;
7. Compute utility $p_j(S_t)$ for all BSs $j \in C_t$ with $\boldsymbol{\alpha} = \frac{1}{|C_t|}\mathbf{1}$ using Algorithm 6.3;
8. Each BS $j \in C_t$ computes $\nu_j(S_n \xrightarrow{\boldsymbol{\alpha},\Theta_k} S_t)$ by (6.5.2)and (6.5.3);
9. **if** $S_t \rhd S_n$ holds
10. $n = n + 1$;
11. Update $S_n = S_t$ and $l = min\{b, |S_n|\}$;
12. Numbering all coalitions in S_n;
13. Go to step 4;
14. **elseif** $k < b^*$
15. BSs in C_t split from C_t and re-form S_n;
16. $k = k + 1$;
17. Go to step 6;
18. **else**
19. BSs in C_t split from C_t and re-form S_n;
20. $l = l - 1$;
21. Go to step 4;
22. **Output**: S_n, $\boldsymbol{\alpha}$

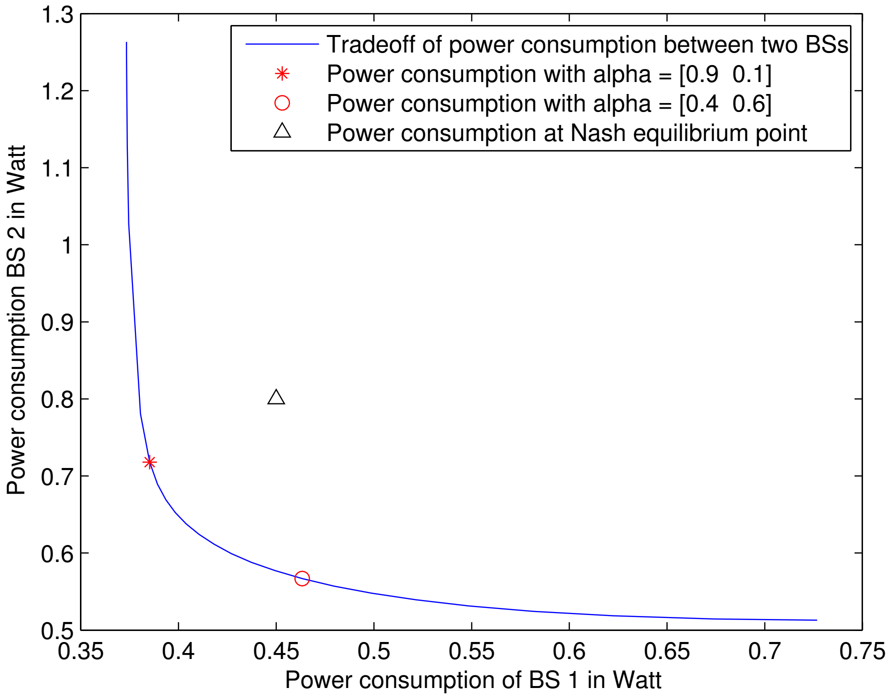

Figure 6.2. Power consumption of individual BSs for different beamforming methods.

coalition. However, once $\boldsymbol{\alpha}$ is modified to $[0.9 \;\; 0.1]$, the power consumption for both two BSs has been successfully reduced as compared to the competitive design. This will lead to a positive decision for the coalition formation. Hence, for the proposed coalition formation algorithm, by employing the $\boldsymbol{\alpha}$-Modification scheme, there is more chance to successfully deviate to a new coalition structure. To formulate the $\boldsymbol{\alpha}$-Modification algorithm, the concept of *deviation gain ratio* for the $\boldsymbol{\alpha}$-Deviation is first introduced as

$$\beta_j = \frac{\nu_j(S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_{n+1})}{r_j(S_n)}, \;\forall j. \tag{6.5.6}$$

Then, if $S_t \rhd S_n$ does not hold, $\boldsymbol{\alpha}$ can be modified through the following equations:

$$\alpha_j = \alpha_j + \zeta, \quad \forall j \in \Omega_1, \; for\ Pareto\ order; \qquad (6.5.7)$$
$$\forall j \in \Omega_3, \; for\ Majority\ order,$$
$$\alpha_j = \alpha_j - \zeta, \quad \forall j \in \Omega_2, \; for\ Pareto\ order; \qquad (6.5.8)$$
$$\forall i \in \Omega_4, \; for\ Majority\ order,$$

where ζ is the step size for updating.

For Pareto comparison, $\Omega_1 = \{j|\nu_j(S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_t) < 0, \forall j \in C_t\}$ is the set of BSs with $\beta_j < 0$ while $\Omega_2 \subset C_t$ is a set of $|\Omega_1|$ BSs with $|\Omega_1|$ largest β values. It is obvious that $\boldsymbol{\alpha}$ can be modified only if Ω_1 satisfies $|\Omega_1| < |\{j|\nu_j(S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_t) > 0, \forall j \in C_t\}|$. However, in practice, even the above condition is satisfied but $|\Omega_1|$ is very large, it is still difficult to achieve $S_t \succ_P S_n$ with the modified $\boldsymbol{\alpha}$ due to the limitation of flexibility for those BSs in Ω_2. Hence, an upper bound μ is introduced so that the Pareto comparison based $\boldsymbol{\alpha}$-Modification is applicable only if $|\Omega_1| \leq \mu < |\{j|\nu_j(S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_t) > 0, \forall j \in C_t\}|$.

Different to the Pareto case, for majority comparison, α_j of BSs in Ω_3 and Ω_4 will be modified, where Ω_3 is the set of N_m BSs with smallest $|\beta_j|$ values and $\beta_j < 0$; while $\Omega_4 \subset C_t$ is a set of $|\Omega_2|$ BSs with largest β values. N_m is the minimum number of BSs that need to improve β_j values and can be obtained by

$$N_m = \lceil \frac{N_d + 1}{2} \rceil,$$

where $\lceil \cdot \rceil$ is the ceiling function defined as

$$\lceil x \rceil = min\{y \in \mathbb{Z} \mid x \leq y\} \qquad (6.5.9)$$

and

$$\begin{aligned} N_d = & |\{j|\nu_j(S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_t) < 0, \forall j \in C_t\}| \\ & - |\{j|\nu_j(S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_t) > 0, \forall j \in C_t\}|. \end{aligned} \tag{6.5.10}$$

Similar to the Pareto case, the upper bound μ is introduced so that the majority comparison based $\boldsymbol{\alpha}$-Modification is applicable only if $N_m \leq \mu < |\{j|\nu_j(S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_t) > 0, \forall j \in C_t\}|$. Then, by integrating the Pareto case and majority case into one algorithm, the $\boldsymbol{\alpha}$-Modification algorithm for Algorithm 6.4 is summarized in Algorithm 6.5.

6.5.3 Stable Coalition Structures

In the coalition formation game, a main concern is that whether the output coalition structure is stable. To analyze the stability of the coalition structures obtained by the proposed algorithm, the following definition is given.

Definition 6.5.2. ($\boldsymbol{\alpha}-b$ *dominance*) :S_{n+1} $\boldsymbol{\alpha}$-b *dominates* S_n, if there exists a coalition $\Theta \subset S_n$, $|\Theta| \leq b$, with a given $\boldsymbol{\alpha} \in \mathbb{R}_+^{|\bigcup \Theta|}$ such that $S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_{n+1}$, and $S_{n+1} \rhd S_n$. The $\boldsymbol{\alpha}$-b dominance can be written as $S_{n+1} \gg^{\boldsymbol{\alpha}-b} S_n$.

Then, the solution of the coalition formation game can be described by introducing the concept of *Coalition Structure Stable Set* proposed in [83]. By defining the coalition formation game proposed in Algorithm 6.4 as $(\mathcal{P}, \gg)$, the Coalition Structure Stable Set can be defined as follows:

Definition 6.5.3. Coalition Structure Stable Set: *The set of coalition structures $\mathcal{R} \subset \mathcal{P}$ is a coalition structure stable set of $(\mathcal{P}, \gg)$ only if the following conditions are satisfied [83]:*

** $\mathcal{R}$ is internally stable for $(\mathcal{P}, \gg)$ if there do not exist $S, S' \in \mathcal{R}$ such that $S \gg^{\boldsymbol{\alpha}-b} S'$;*

Algorithm 6.5 $\boldsymbol{\alpha}$-Modification algorithm

1. **Input**: S_t, C_t, ζ, μ, N_M, $\{r_j(S_n), \forall j \in C_t\}$
2. **Initialize** $\boldsymbol{\alpha}^{(m)} = \frac{1}{|C_t|}\mathbf{1}$, $m = 0$
3. Compute utility $p_j^{(m)}(S_t)$ for all $j \in C_t$ with $\boldsymbol{\alpha}^{(m)}$ using Algorithm 6.3;
4. Each BS $j \in C_t$ computes $\nu_j^{(m)}(S_n \xrightarrow{\boldsymbol{\alpha},\Theta} S_t)$ and $\beta_j^{(m)}$ using (6.5.2), (6.5.3), (6.5.6) and $r_j(S_n)$;
5. **if** $S_t \triangleright S_n$ holds
6. $n = n + 1$;
7. $S_n = S_t$;
8. Go to step 17;
9. **elseif** $|\Omega_1| > \mu$ (or $|\Omega_3| > \mu$)
10. Go to step 17;
11. **elseif** $m < N_M$
12. Update $\boldsymbol{\alpha}^{(m)}$ using (6.5.7) and (6.5.8);
13. m = m + 1;
14. Go to Step 3;
15. **else**
16. Go to step 17;
17. **Output**: S_n, $\boldsymbol{\alpha}^{(m)}$

* *$\mathcal{R}$ is externally stable for $(\mathcal{P}, \gg)$ if for all $S \in \mathcal{P}/\mathcal{R}$, there exists $S^{'} \in \mathcal{R}$ such that $S^{'} \gg^{\boldsymbol{\alpha}-b} S$;*
* *$\mathcal{R}$ is a coalition structure stable set for $(\mathcal{P}, \gg)$ if it is both internally and externally stable.*

For the proposed coalition formation algorithm with a certain comparison rule (Pareto or majority), it is a sequential process in which coalitions can only merge into larger size coalitions. Such characteristic could guarantee the formation process always reach some points. Thus, the main concern of the coalition formation game is whether the output points are stable. In other words, by considering the concept of coalition structure stable set $\mathcal{R}$, whether output coalition structures of the coalition formation game $(\mathcal{P}, \gg)$ are in $\mathcal{R}$. This can be directly proved by considering an output coalition structure S_o of Algorithm 6.4. It is assumed S_o is in the coalition structure stable set $\mathcal{R}$ and a coalition structure $S_o^{'}$ can be found in $\mathcal{R}$ that has $S_o^{'} \gg^{\boldsymbol{\alpha}-b} S_o$. This means that S_o can further transit to $S_o^{'}$ via Algorithm 6.4 and S_o is not the output of $(\mathcal{P}, \gg)$, which contradicts to the assumption. In addition, by assuming S_o is outside the coalition structure stable set $\mathcal{R}$ that $S_o \in \mathcal{P}/\mathcal{R}$. According to external stability, there should be a coalition structure $S_o^{'}$ in $\mathcal{R}$ that satisfy $S_o^{'} \gg^{\boldsymbol{\alpha}-b} S_o$, which also contradicts to the assumption. Hence, output coalition structures from Algorithm 6.4 must be in the coalition structure stable set.

Proposition 6.5.1. *The coalition formation game $(\mathcal{P}, \gg)$ can reach a unique coalition structure, if and only if the following conditions are satisfied:*

- *A numbering strategy is adopted at all coalition formations;*
- *If $\boldsymbol{\alpha}$ is allowed to be modified, a given $\boldsymbol{\alpha}$-Modification algorithm must be applied to all coalition formations.*

Proof: This is a direct result from Definition 6.4.1 and Algorithm 6.4. If the $\boldsymbol{\alpha}$-Modification algorithm is not employed, only if there is a fixed numbering strategy, the sequence of the coalition formation process is unique and then, an unique output is guaranteed.

Once the $\boldsymbol{\alpha}$-Modification algorithm is employed, if for different coalition formation processes, different modification schemes are allowed, the sequence of the coalition formation process may be alterable. Hence, to guarantee the uniqueness of the output, it is necessary to ensure that the same $\boldsymbol{\alpha}$-Modification algorithm is employed to all coalition formations.

6.6 Numerical Results

Some numerical results for the proposed coalition formation algorithms are presented. A seven-cell network in which each BS serves two users is considered. It is assumed that each BS employs six antennas while each user is equipped with a single antenna. To simplify the simulation setting, the target SINRs for all the users in all cells are set to identical. The noise variance σ^2 at the receiver of all users is set to 0.01. The channel coefficients have been generated using zero mean complex Gaussian random variables. To practically model the simulation scenario, the distance dependent path loss model with path loss exponent 3 is introduced to calculate channel gains for both inter-cell and intra-cell channels. The distance between a BS and its users is set to 0.9 kilometers for all BSs while the distance between any two neighboring BSs is set to six kilometers. It is assumed that the coalition formation game always starts from the non-cooperative game. Once a new coalition structure is reached, a postpositional numbering strategy is used so that the newly formed coalition is always numbered as the last coalition while all other coalitions are numbered in the ordinary way. It is assumed that the cooperation cost can be characterized by the same dimension as power hence the cooperation cost factor ϵ has a unit of Watt.

Before demonstrating the benefits of the proposed formation algorithm, the

Table 6.1. Some possible coalition strutures for a multicell network with seven cells.

	Coalition structure
S_1	$\{\{1,2\},\{3,5\},\{4,6\},\{7\}\}$
S_2	$\{\{1,2,7\},\{3,5\},\{4,6\}\}$
S_3	$\{\{1,2,5\},\{3,7\},\{4,6\}\}$
S_4	$\{\{1,4,5\},\{2,6\},\{3\},\{7\}\}$
S_5	$\{\{1,4,5\},\{2,6,7\},\{3\}\}$

cases presented in Table 6.1 is considered to show merging coalitions has the potential to reduce the transmission power most of the time. As an example, for the coalition structure transit $S_1 \rightarrow S_2$, the coalition $\{7\}$ is forced to merge with the coalition $\{1,2\}$. Out of 10000 various channel realizations, it is observed that 98.21% of the time, this random merge has reduced the total power consumption of all seven BSs as shown in Table 6.2. The table also indicated the percentage of power reduction for a different transit $S_4 \rightarrow S_5$.

Table 6.2. Probability of performance improvement for different coalition transit process and target SINRs.

Coalition deviation	Probability of power reduction	
	Target SINR = 17dB	Target SINR = 18dB
$S_1 \xrightarrow{\boldsymbol{\alpha},\Theta} S_2$	98.21%	97.02%
$S_4 \xrightarrow{\boldsymbol{\alpha},\Theta} S_5$	98.27%	97.22%

Figure 6.3 compares the performance of the proposed coalition formation algorithm with that of the fully competitive beamforming discussed in [102] and the fully coordinated beamformer design algorithm developed in [89]. In this case, target SINRs for all users are set to 18.5dB while the cooperation cost factor ϵ is set to 0.005 Watt. It is observed that the fully coordinated design has the advantage of reducing the power consumption for individual BSs most of the time. For both the Pareto comparison and the majority comparison, the proposed algorithm resulted into lower power consumption for individual BSs as compared to a fully competitive design. However, once

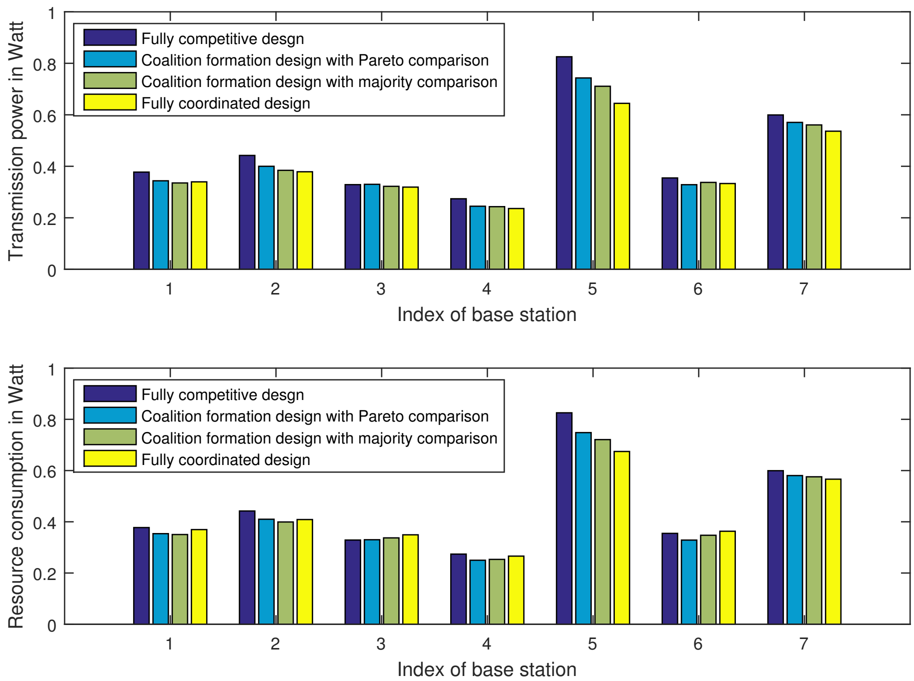

Figure 6.3. Transmission power and resource consumption of individual BSs for various beamforming design methods.

the cooperation cost is taken into consideration, BSs may not always benefit from cooperation. Hence, as shown in Figure 6.3, the proposed algorithm has a better performance in terms of resource consumption than the fully competitive design and fully coordinated design most of the time. Hence, when there exists a cooperation cost, the proposed algorithm improves the performance of the network.

The effect of the α-Modification algorithm on the proposed coalition formation algorithm is then investigated. As seen in Figure 6.4, α-Modification is very sensitive to the Pareto comparison, however for most cases, α-Modification does not change the number coalitions significantly for majority comparison mode. Lower the number of coalitions is likely to provide higher the saving in transmission power. Therefore, it is noticed that the majority comparison performs better than the Pareto comparison for most SINR targets. The effect of these four schemes on the total resource consumption

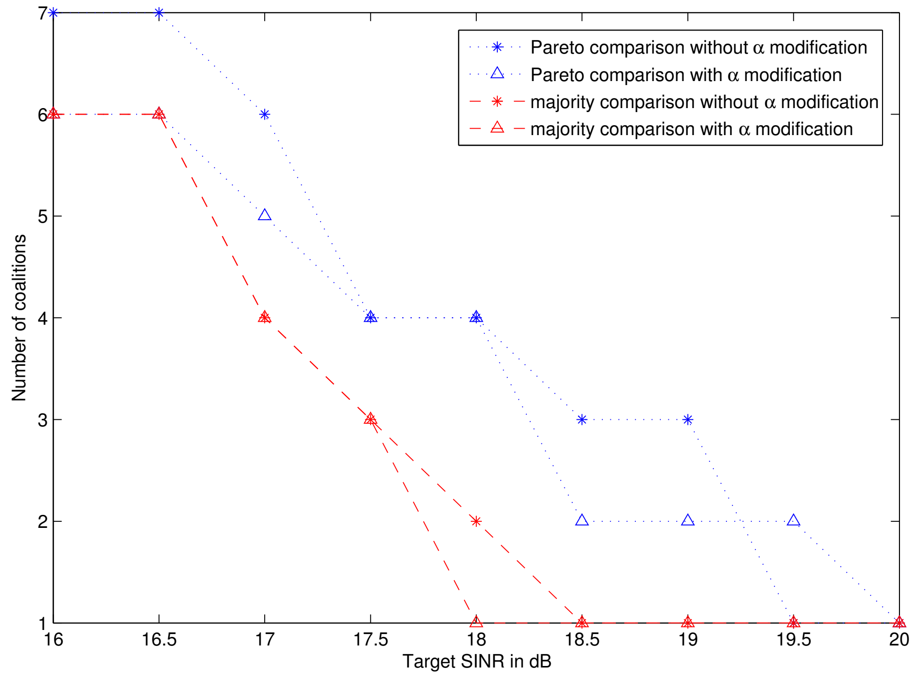

Figure 6.4. The effect of $\boldsymbol{\alpha}$-Modification algorithm on the number of coalitions.

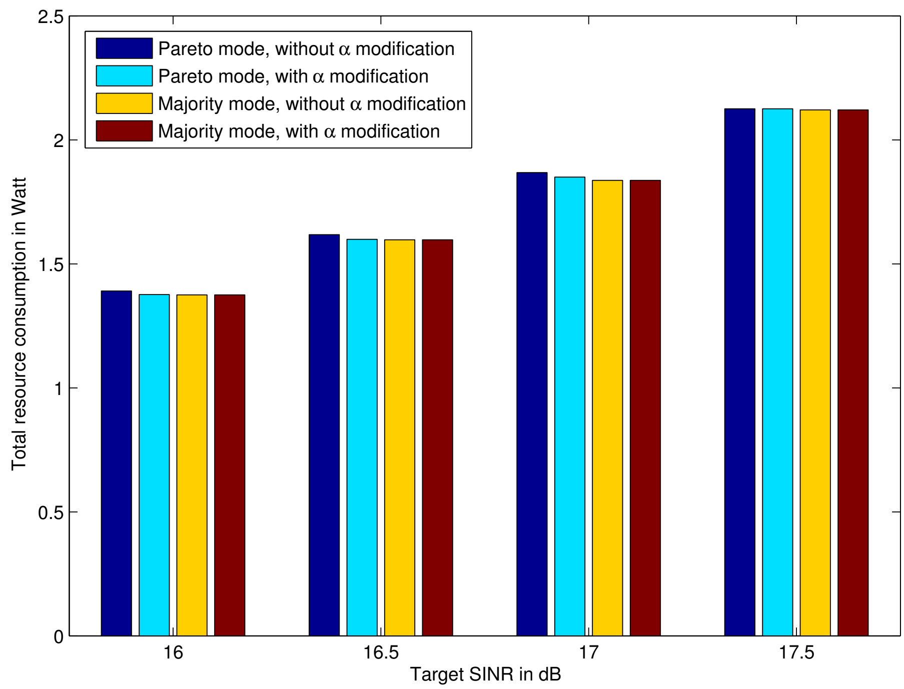

Figure 6.5. The effect of $\boldsymbol{\alpha}$-Modification algorithm on the total transmission power.

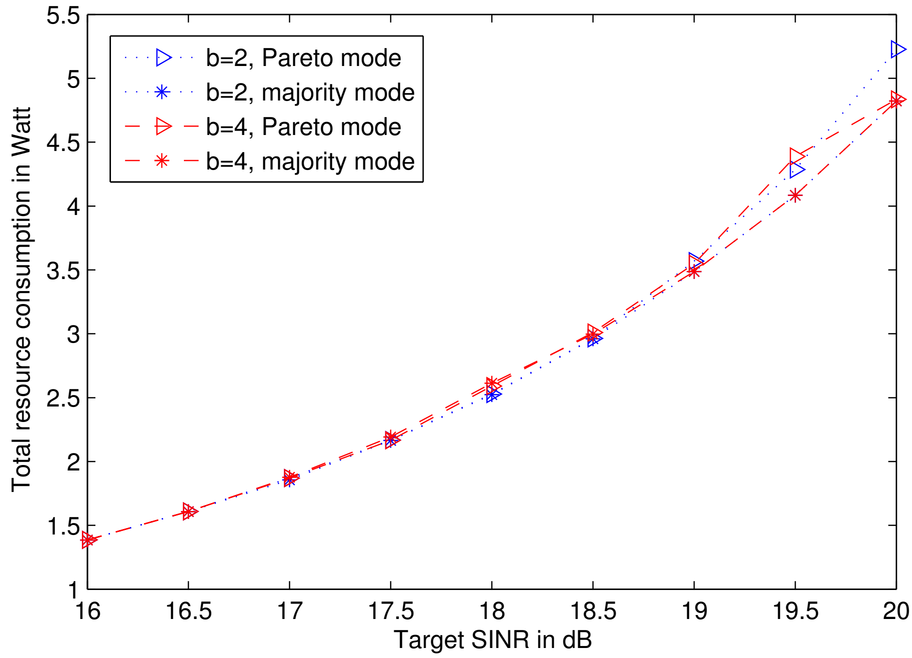

Figure 6.6. Total resource consumption versus various SINR targets.

is shown in Figure 6.5. As seen, all four schemes incur into almost same resource consumption, however, a closer look reveals, the majority comparison based algorithm provides more saving in resource.

Figure 6.6 compares the total resource consumption of the output obtained with different b values. As seen, both cases of $b = 2$ and $b = 4$ have resulted almost the same total resource consumption. Hence, value of b has limited effect on the performance of the coalition formation process in terms of resource consumption. However, in practice, in addition to resource consumption, other factors such as the formation speed should be considered. Figure 6.7 shows the performance in terms of the number of temporary coalitions formed, for different b values. Larger the number of temporary coalitions means more the time it takes for final coalition formation. As the SINR target increases, the number of temporary coalitions decreases and converges to almost the same value for both $b = 2$ and $b = 4$. However, when target SINR is small, the number of temporary coalitions is significantly low for

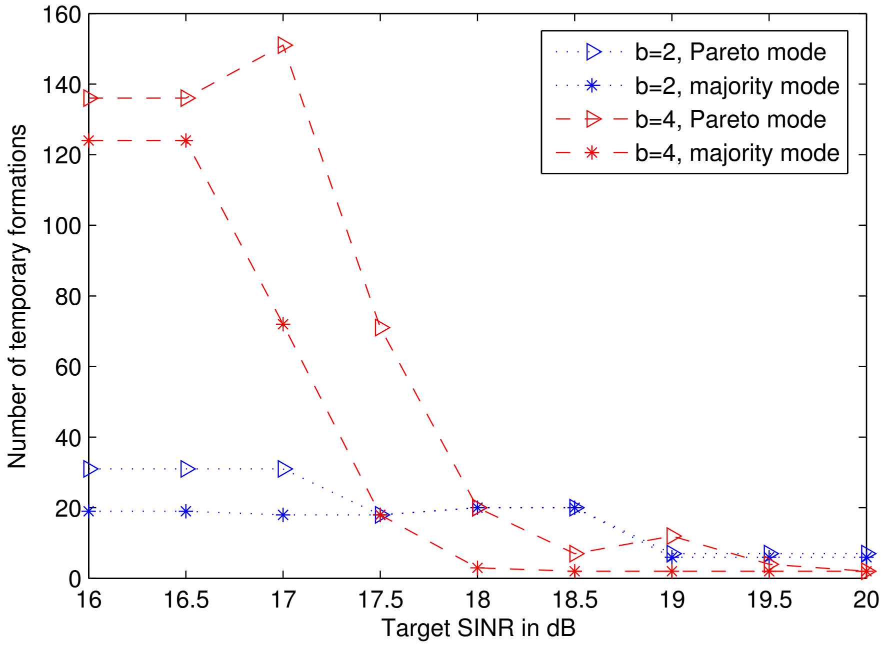

Figure 6.7. Number of temporary formations versus various SINR targets.

$b = 2$. Hence, it is normally a good practice to choose a lower b value.

So far, the key parameters of the proposed algorithm and their effect on the coalition formation process have been analyzed. However, all these simulations are based on the assumption that the cooperation cost factor is set to 0.005W. Now, the effect of the cooperation cost on the coalition formation process is investigated. As seen in Figure 6.8, once again a lower b value provides a better performance almost for all values of cooperation cost.

6.7 Summary

A multicell multiuser downlink beamforming technique using coalitional games was proposed. Due to the benefits of coordination, each BS has incentive to cooperate with other BSs via forming coalitions. However, by considering the associated cost, cooperation is not always preferred by all BSs since any benefits in terms of reduction of transmission power may be overwhelmed by the cost for cooperation. The beamformer design for a given

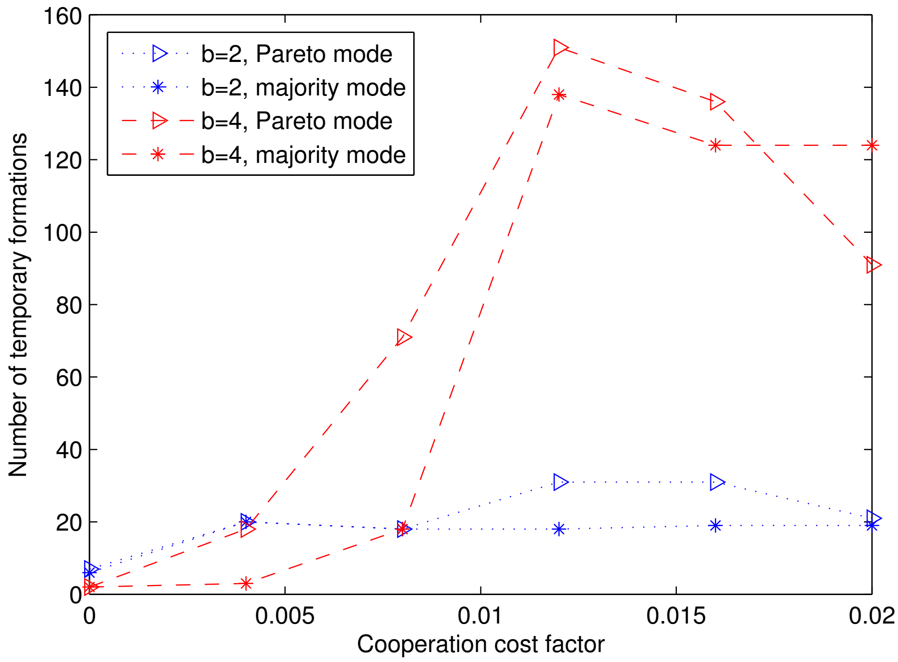

Figure 6.8. Number of temporary formations versus various cooperation cost factors.

coalition structure was first considered and the process of finding downlink beamformers for all users was illustrated. Then, the coalition formation game and a merge-regret based sequential coalition formation algorithm were proposed. It has shown that the output of the proposed algorithm is in a coalition structure stable set. With certain constraints, the proposed algorithm can produce a unique stable coalition structure. The simulation results have shown that the majority mode can always provide a better performance. Generally, it is better to choose a smaller b value to accelerate the coalition formation process. As a part of the proposed coalition formation algorithm, the $\boldsymbol{\alpha}$-Modification algorithm has been developed for a range of comparison rules. The simulation proved that when the Pareto mode is used, the employment of $\boldsymbol{\alpha}$-Modification algorithm can help to reduce the number of coalitions at the output, and decrease the total power consumption.

Chapter 7

SUMMARY, CONCLUSION AND FUTURE WORK

The focus of this thesis has been on the beamforming techniques for multicell wireless networks. The coordinated mulitcell beamforming algorithms for mixed QoS were developed in Chapter 4. Two scenarios were considered. In the first scenario, all BSs jointly designed beamformers for their users. Each BS served both real-time users and non-real-time users. The philosophy of this design is that with coordinated beamforming, each BS assigns its real-time users the minimum allowable power to guarantee their SINR targets while allocating the rest of the power to the non-real-time users to achieve a maximized balanced SINR. This beamforming design was then extended to a wireless network comprising of both cooperative cells and independent cells. The cooperative cells jointly designed beamformers with the same criterion as in the first scenario while ensuring the interference to users in the independent cells is below a threshold value. The proposed algorithms were capable of allowing real time users to achieve a set of SINR targets while ensuring non-real time users in all cells obtain a maximized balanced SINR. By comparing with the SDP method, it was shown that results obtained through the proposed algorithm were optimal.

Chapter 5 investigated coordinated beamforming however with a design criterion that SINRs of users in different cells are balanced to different levels.

An interference constraints based algorithm was proposed that has a number of optimization stages. In the first round of optimization, users in all cells were balanced to the same SINR level. At this stage, at least one of the BSs has used its full transmission power. In subsequent rounds of the optimization, BSs which have excess transmission power will sequentially improve the balanced SINRs by allocating more power to their users. To avoid the SINR degradation to BSs that have already used their full transmission power, interference constraints were introduced at each round of the optimization rather than using SINR constraints. The advantage of the proposed algorithm is that in each round of optimization, only beamformers of users served by those BSs that have excess power are required to be designed. Hence, the complexity has been effectively reduced. For the case that the number of antennas equipped by each BS is smaller than the total number of users, the balanced SINR may not be further improved by allocating more power to corresponding users. Hence, an interference modification method was proposed to rebalance the SINRs of different cells.

In Chapter 6, a coaltional game based multicell beamforming was proposed. The aim of each BS is to minimize its power consumption while ensuring its users could achieve a set of SINR targets. Different to the traditional multicell beamforming with full cooperation, with the introduction of cooperation cost, BSs may prefer local cooperations by forming coalitions. A merge-regret based coalition formation algorithm was developed, in which the coalition structure with all singletons could be sequentially transit to a coalition structure with less coalitions. It has been proved that the proposed algorithm could lead to a stable coalition structure at the output. Both the Pareto order and majority order have been used as the comparison rules of the proposed algorithm. To further improve the effectiveness of the proposed algorithm, an $\boldsymbol{\alpha}$-modification algorithm was proposed, which can further reduce the number of coalitions of the obtained coalition structure. The simulation has shown that for different target SINR regions, the time consumption for the coalition formation can be reduced by choosing appropriate b values.

7.1 Future Work

Several extensions can be made based on the works presented in this thesis. In Chapter 6, a coalition formation algorithm was proposed, in which only one new coalition is allowed to be formed at each coalition formation stage. Hence, to improve the efficiency of the algorithm further, it is possible to allow two or multiple coalitions to be formed at each stage. In addition, in the coalitional game considered in Chapter 6, for a given coalition structure, all coalitions are disjoint, which means that each BS can only stay in one of the coalitions. However, if some BSs are allowed to stay in more than one coalitions to enlarge the cooperation, their performance may be improved further. This is called coalitional game with overlapping coalitions.

Other possible direction is to apply non-cooperative strategic game to the SINR balancing based multicell beamforming design. Since the SINR balancing problem is quasiconvex, the challenge in exploiting this problem is the way of finding the conditions for the existence and uniqueness of Nash equilibrium. To relax the assumption of perfect channel state information, robust optimization technique can be developed to all methods proposed in this thesis.

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