MODELLING AND PERFORMANCE EVALUATION OF RANDOM ACCESS CDMA NETWORKS

by

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A Doctoral Thesis

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To my parents, wife and daughter

ABSTRACT

The objective of this research is to develop a Markovian model in the form of a discrete-time queueing network to assess the performance of random access code division multiple access networks (CDMA). An approximation method called equilibrium point analysis (EPA) has been used to solve the model.

The CDMA protocol is an important application of spread spectrum communications that allows simultaneous transmission of multiple users to occupy a wideband channel with small interference. This is done by assigning each user a unique pseudo noise code sequence. These codes have low cross-correlation between each pair of sequences. Both slotted direct sequence CDMA (DS) and frequency hopping CDMA (FH) are considered with an emphasis on DS-CDMA systems.

The EPA method has previously been used to evaluate the performance of other random access systems such as the ALOHA protocol, but has not previously been used in the context of a CDMA protocol. Throughput and mean packet delay of random access CDMA networks are evaluated, since these two measures are usually used in the study of the performance assessment of multiple access protocols. The analytical results of the random access model are validated against a discrete-event simulation which is run for large number of slots.

The study then proceeds by using the model to examine the effect on performance of introducing error correcting codes to the DS-CDMA systems. Optimum error correcting codes that give the best performances in terms of the throughput and the delay are determined.

The performance of random access CDMA systems applied to radio channels, as in packet radio networks, is then studied, and the effect of multipath fading on the performance is evaluated.

Finally, the performance of DS-CDMA with different user classes (non-identical users case) is investigated. An extended equilibrium point analysis (EEPA) method has been used to solve the Markovian model in this situation. This extended model is used to assess the effects on performance of the unequal powers due to varying distances of the users to an intended receiver or to a base station (near-far problem).

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ABBREVIATIONS

AWGN	Additive White Gaussian Noise
ANSI	American National Standards Institute
BCH	Bose-Chaudhuri-Hocquenghem
BPSK	Binary Phase Shift Keying
CDMA	Code Division Multiple Access
CL	Constraint length
CSMA	Carrier Sense Multiple Access
CSMA/CD	Carrier Sense Multiple Access with Collision Detection
DPSK	Differential Phase Shift Keying
DS-CDMA	Direct sequence-Code Division Multiple Access
EEPA	Extended Equilibrium Point Analysis
EIA	Electronic Industry Association
EPA	Equilibrium Point Analysis
FDMA	Frequency Division Multiple Access
FH-CDMA	Frequency Hopping-Code Division Multiple Access
FFH	Fast Frequency Hopping
FSK	Frequency Shift Keying
gen.	generator
H	Hamming
HDLC	High-level Data Link Control
IF	Intermediate Frequency
IEE	Institution of Electrical Engineers
IEEE	Institution of Electrical and Electronic Engineers
ISI	Intersymbol Interference
LAN	Local Area Network
LEO	Low Earth Orbit

LOS	Line-of-sight
LPI	Low Probability of Interception
MAI	Multiple Access Interference
MEO	Medium Earth Orbit
NLOS	Non-line-of-sight
pdf	Probability density function
PN	Pseudo Noise
PRNets	Packet Radio Networks
PSK	Phase Shift Keying
QPSK	Quadrature Phase Shift Keying
RF	Radio Frequency
RS	Reed-Solomon
S-G	Throughput-Channel traffic
seq.	sequence
S-ALOHA	Slotted-ALOHA
SIR	Signal to Interference Ratio
SFH	Slow Frequency Hopping
SNR	Signal to Noise Ratio
SS	Spread Spectrum
SSMA	Spread Spectrum Multiple Access
sync.	synchronisation
TDMA	Time Division Multiple Access
TIA	Telecommunication Industry Association

NOTATIONS

d_{free}	Free distance of a convolutional code
d_{min}	Minimum distance of a BCH code
E_b	Energy per bit
$erfc$	Error function complementary
G	Processing gain
$G_U(z)$	Generation function of random variable U
H	Number of node in a discrete-time queueing network
$I_0(.)$	Modified Bessel function of zeroth order
K_f	Rician factor
k	Number of information bits
L	Number of discrete mutipath links
L	Number of information bits in a packet
L_p	Number of bits in a packet
m	Number of simultaneous users in a CDMA system
$m(t)$	Data information sequence
N	Network population
N_0	Power spectral density of AWGN channel
n	Block length in a BCH code
P_{av}	Average received power
P_{cw}	Probability of correct code word
$P_c(m)$	Probability of packet success for m simultaneous users
$P_E(m)$	Probability of packet failure
P_{ew}	Probability of a code word error
P_{IC}	Probability of correct code word of the inner code of RS code
P_{OC}	Probability of correct code word of the outer code of RS code
$P_u(p_e)$	Union bound on the first error probability
p	Retransmission probability

p_e	Probability of bit error
p_i	Retransmission probability of user i
$p(t)$	Pseudo noise sequence
R	Propagation channel delay
R_b	Information bit rate (DS-CDMA)
R_c	Chip rate (DS-CDMA)
R_c	Code rate of an error correction code
R_H	Hopping rate (FH-CDMA)
RT	Retransmission node (S-ALOHA)
r_{ji}	Routing probability from node j to node i
$S(x_i^e)$	Mean throughput of user i
S	Throughput
T	Thinking node (S-ALOHA)
T_b	Information bit duration
T_c	Chip duration
T_H	Hopping time interval in FH-CDMA
T_m	Delay spread of a multipath channel
t	Error correction capability
W	Channel bandwidth
$W(x_i^e)$	Mean packet delay of user i
x_n^i	Number of users at node j during slot n
x_e	EPA solution
$(\Delta f)_c$	Coherent bandwidth of a fading channel
β	Path gain
γ	Path loss component
λ_i	Mean number of arrival per unit time at node i
μ_i	Serving probability for user i
σ	New packet generation probability
σ_i	New packet generation probability for user i
ψ	Variance of the Gaussian process

CONTENTS

CHAPTER 1

Introduction

1.1 Random Access CDMA-A Brief Introduction

Over the past several decades, the explosive growth in the number of computers has motivated the demand to search for efficient methods to transfer digital information almost error free among a large number of users for both large and short distances.

Data transmission requires a very low probability of bit error, thus one of the important tasks of a network protocol is to minimise the probability of bit error of the transferred data information. Both retransmission and error correction coding can be applied in order to correct errors that occur during transmission. In the case when the number of errors exceeds the capability of the error correction of the code used, the transmitted message is retransmitted. Retransmission is only possible when errors are detectable by the sender when they are due to collisions in the network. To reduce the effect that a single error can cause an entire transmitted message to be unsuccessfully received, transmitted messages are usually divided into shorter messages called packets.

The transmitted packets of different users in a network will occupy a common channel shared among these users. A multiple access protocol is an algorithm or a procedure that controls the access to the channel. Thus multiple access refers to the capability to share a communication channel bandwidth among a population of geographically distributed users.

The earliest methods for achieving the multiple access capability were the time division multiple access and the frequency division multiple access (TDMA, FDMA). These techniques allocate the channel bandwidth to the users in a static fashion, independently of their activities. These fixed assignment protocols can be very efficient if there is a constant level of transmitted information among users. However, data traffic is usually characterised as bursty, in which case these techniques are not very efficient.

Random access or contention protocols are the alternative to fixed assignment protocols, and these are more suitable for data transmission. In these protocols the entire communication channel bandwidth is available to the users to be accessed randomly. Random access CDMA is one of the important applications of spread spectrum modulation and allows multiple users to transmit simultaneously into a wideband channel. The multiple access capability of random access CDMA is achieved by assigning each user a unique pseudo noise (PN) code of rate much higher than the actual date rate. These PN codes should make good separation between the signal of a desired user and the signal of the other interfering users. The values of the cross-correlation among PN codes in a CDMA system should be as small as possible in order to mitigate the interference among users when they transmit at the same time.

1.2 Existing Work

During recent years there has been considerable interest in evaluating the performance of multiple access systems. In particular the random access CDMA has been extensively investigated due to its promising future as a strong candidate for the second and the third generations of communication systems.

Some of the researches have been directed towards finding an efficient method that can be used to assess the performance of multiple access systems including the random access CDMA systems. Abramson [Abra 70] has introduced the S-G (throughput-channel traffic) analysis in order to evaluate the performance of the pure ALOHA system. Kleinrock and Tobagi [Klei 75b] have used the S-G analysis to assess the performance of the carrier sense multiple access protocol. Tanenbaum [Tane 96] has evaluated the performance of the slotted-ALOHA protocol using the S-G analysis. The main problem associated with the S-G analysis is its deficiency to determine the stability of the modelled systems.

Kleinrock and Lam [Klei 75a] have proposed an analytical technique called Markov analysis which can solve the stability problem in slotted ALOHA with a finite number of users. Markov analysis has been used to evaluate the performance of different multiple access systems, such as slotted ALOHA [Toba 80a] and random access CDMA [Rayc 81], [Stor 86], [Stor 89] and [Pras 91]. This method can study the dynamic behaviour of the multiple access protocols. However, it cannot be applied to complex protocols that are represented as multidimensional Markov chains.

An alternative approach has been used by Fukuda and Tasaka [Fuku 83] who developed multidimensional Markovian models for the slotted-ALOHA protocol and then solved the models using an approximation technique called equilibrium point analysis (EPA). The EPA method has also been used to assess the performance of other multiple access protocols. Tasaka [Tasa 86] has used the EPA to solve a number of Markovian models of different multiple access protocols, while Rodrigo [Rodr 93] has used EPA to solve models for the slotted ring local area networks. Also, Woodward [Wood 93] has used EPA to evaluate the performance of some satellite and LAN protocols which have been modelled in the form of discrete-time queueing networks.

In this work the approach used by Woodward [Wood 93] will be used to evaluate the performance of random access CDMA protocols which will be modelled as Markov chains in the form of discrete-time queueing networks that can be solved using the EPA technique.

1.3 Organisation of the Thesis

The thesis is organised as follows. In chapter 2, a classification of multiple access protocols will be presented. Both fixed assignment (contentionless) protocols and random access (contention) protocols will be discussed, and the principle of the random access CDMA protocol as one of the most important applications of spread spectrum modulation will be discussed in some detail. Two schemes of spread spectrum modulation for CDMA will be considered, namely direct sequence spreading and frequency hopping. The multiple access capability, advantages and disadvantages of these two types will be discussed. Comparison of three analytical methods that have been used to evaluate the performance of multiple access protocols, namely the S-G, Markov, and Equilibrium point analysis (EPA) are discussed.

In chapter 3, a Markovian model in the form of a discrete-time queueing network for random access CDMA networks will be developed in order to assess the performance of these networks. The EPA analytical method will be used to solve the model. Throughput and mean packet delay characteristics will be shown. Results of the model will be compared to discrete event simulation results. Throughput performance of random access CDMA with different data modulation schemes will be presented.

In chapter 4, the model of the previous chapter will be modified to accommodate error correction codes, and the effect on performance of introducing the error correction capability will be examined. Block and convolutional codes will be considered, with the emphasis on block codes.

In chapter 5, the performance of a random access CDMA system applied to packet radio networks will be studied. The effect on the performance of multipath fading and the packet size will be examined.

In chapter 6, the assumptions made in previous chapters of identical users will be relaxed in order to evaluate the performance of random access CDMA with different user classes. Each user will be assumed to have different statistical properties to the other users, such as the new packet generation probability and the retransmission probability. The EPA method will be modified in order to solve the Markovian model for the different user classes case. The effect of the near/far problem on the performance will be also discussed.

Finally, chapter 7 will summarise the work carried out throughout this research and suggest some topics for further research.

CHAPTER 2

Random Access Code Division Multiple Access (CDMA)

2.1 Introduction

Spread spectrum modulation techniques have been used in the past mainly for military communications over several decades. That is because of their inherent antijam and low probability of intercept capabilities. More recently, spread spectrum has been used commercially in mobile communications and packet radio networks to provide a multiple access facility.

In a spread spectrum system, if different spreading codes are used, then a number of transmissions can simultaneously occupy a wide band channel, and each will suffer only slight degradation in signal quality compared to the performance of a single transmission. Such a system is referred to as spread spectrum multiple access (SSMA). A system that relies on the low cross-correlation properties of an orthogonal set of spreading codes is referred to as code division multiple access (CDMA) [Stor 86]. CDMA is one of the multiple access protocols that can be used to provide efficient communication among a number of users who share a wideband channel simultaneously.

In this chapter, the classification of multiple access protocols is discussed with an emphasis on the CDMA protocol and the areas of its applications. Finally, analytical techniques for modelling and performance evaluation of multiple access protocols are discussed in order to choose an efficient and simple method that can be applied to model and evaluate the performance of CDMA networks.

2.2 Multiple Access Protocols

Multiple access refers to the capability to share a communication resource among a population of geographically distributed users. In data transmission systems, the communication channel bandwidth is often the main resource. Whenever a common channel is accessed by more than one user, the need for multiple access arises. Thus, the algorithm or the procedure that controls the access to the shared channel is called a multiple access protocol.

A multiple access protocol should possess certain properties in order to be efficient [Pras 96]:

- The protocol should perform the allocation such that the transmission channel is used efficiently. The efficiency is usually measured in terms of channel throughput and the delay of the transmission.
- The allocation should be fair towards individual users, thus each user should receive the same allocated capacity.

- The protocol should be stable. This means that if the system is in equilibrium, an increase in load should move the system to a new equilibrium. With an unstable protocol an increase in load will make the system drift to an even higher load and lower throughput.
- The protocol should be robust with respect to changing conditions. If one user does not function correctly, this should affect the performance of the rest of the system as little as possible.

Multiple access protocols can be classified due to [Toba 80b], [Tasa 86], and [Wood 93] into five classes:

1. Fixed assignment techniques.
2. Random access (contention) techniques.
3. Demand assignment with centralised control techniques.
4. Demand assignment with distributed control techniques
5. Adaptive techniques.

Fig.(2.1) shows a classification of multiple access protocols.

In this section, protocols from the first two groups are discussed, since code division multiple access (CDMA) protocols can be classified as either fixed assignment or random access protocols. For protocols of the other three groups see [Toba 80b].

2.2.1 Fixed assignment protocols

Fixed assignment protocols consist of those techniques which allocate the channel bandwidth to the users in a static fashion, independently of their activity. These techniques take two common forms: orthogonal, such as frequency division multiple access (FDMA) or time division multiple access (TDMA), and quasi-orthogonal, such as code division multiple access (CDMA) [Toba 80b].

-Time division multiple access (TDMA)

In a TDMA protocol the time axis is divided into frames of equal duration, and each frame is divided into the same number of time slots. Each user is assigned a fixed predetermined channel time in slots. Any user has access to the entire channel bandwidth, but only during its time slots. The orthogonality in TDMA is achieved in the time domain.

The TDMA protocol has some disadvantages, such as the capacity wasting property when some users have nothing to transmit in their allocated slots. Another problem associated with TDMA is the necessity for synchronisation of all users, so that each user knows exactly when and for how long it can transmit. However, the TDMA protocol has been and is still widely used because of its relative simplicity.

-Frequency division multiple access (FDMA)

With the FDMA protocol, the bandwidth of the communication channel is divided into a number of frequency bands with guard bands between them to achieve frequency separation of adjacent bands. Thus each user is assigned a fraction of the available bandwidth. The orthogonality among users is achieved in the frequency domain.

FDMA has the same capacity-wasting property as in TDMA, because if a user has nothing to transmit, its allocated frequency band cannot be used by other users. FDMA has the advantage of simple implementation, because synchronisation of the users is not necessary.

-Code division multiple access (CDMA)

The CDMA protocol allows overlap in transmission in both frequency and time. It achieves orthogonality by the use of spreading codes, thus the users of a CDMA system can use the entire available bandwidth simultaneously.

The CDMA protocol is classified as a fixed assignment (contentionless) protocol where users are allowed to transmit simultaneously without conflict. However, if the number of users rises above a threshold, contention will occur [Pras 96]. In general, contention is assumed in a CDMA protocol, and consequently this protocol is classified as a random access protocol.

2.2.2 Random access (contention) protocols

In random access (contention) protocols, the entire bandwidth is provided to the users as a single channel to be accessed randomly. A user cannot be sure that a transmission will not collide because other users may be transmitting at the same time. Therefore, random access protocols need to resolve conflicts if they occur. To reduce the effect that a single error in a transmission can cause the whole message to be unsuccessfully received, transmitted messages are normally divided into shorter messages called packets [Stor 86]. Examples of these protocols are, ALOHA, Carrier sense multiple access (CSMA), Carrier sense multiple access with collision detection (CSMA/CD), and CDMA.

ALOHA protocol

The first data network to be based upon a random access protocol was the ALOHA network which went into operation throughout the state of Hawaii in

1970, employing packet-switching on a radio channel [Abra 94]. There are two types of ALOHA protocol, pure and slotted.

The pure ALOHA protocol permits a user to transmit any time it desires. There will of course be collisions, and the colliding packets will be destroyed. However, due to the feedback property of a radio channel, a transmitter can always know whether or not its transmitted packet was destroyed. If a packet was destroyed, the user waits a random amount of time and retransmits the same packet again. Whenever two or more packets occupy the channel simultaneously, there will be a collision. If the first bit of a new packet overlaps with the last bit of an already transmitted packet, both packets will be totally destroyed, and both have to be retransmitted again. Thus, the vulnerable period is twice as long as the packet transmission time. The maximum achievable throughput (capacity) of the pure ALOHA protocol is about 0.184. In other words, the maximum channel utilisation is 18%.

To increase the low capacity of the pure ALOHA protocol, a slotted version is obtained by dividing time into slots of duration equal to the transmission time of a single packet. Each user is required to synchronise the start of transmission of its packet to coincide with the slot boundary [Robe 75]. When two packets conflict, they will overlap completely rather than partially. Since the vulnerable period is halved to become equal to one packet transmission time, the maximum throughput of the slotted ALOHA protocol is doubled to become about 0.368.

Both pure and slotted ALOHA protocols are applicable to satellite, ground radio, and local bus environments. The slotted version has the advantage of efficiency, but in a ground radio channel it has the disadvantage that synchronisation may be hard to achieve [Toba 80b].

-Carrier sense multiple access protocol (CSMA)

With slotted ALOHA the best channel utilisation that can be achieved is about 36%. Therefore, there is a high probability of collision. In order to increase the channel utilisation and avoid collisions as much as possible, the users have to avoid transmitting when the channel is busy with another transmission. This can be achieved by listening to the channel to see if any user is transmitting. If the channel is occupied by another transmission, the user waits until the channel becomes idle. When a user detects an idle channel, it transmits a packet. If a collision occurs due to more than one user detecting an idle channel simultaneously, the user waits a random amount of time and starts the procedure again. This protocol is called 1-persistent CSMA because the user transmits with probability 1 whenever it finds the channel idle.

There are two other types of CSMA protocol called non-persistent CSMA and p-persistent CSMA. In the non-persistent protocol, if a user senses a busy channel, it does not continually sense it for the purpose of immediate transmission when detecting the idle channel (as in the 1-persistent protocol), but instead waits a random period of time and then repeats sensing the channel. In the p-persistent CSMA protocol, when a user becomes ready to transmit it senses the channel. If the channel is not busy, the user transmits its packet with probability p and does not transmit with probability $q=1-p$ and waits for the next slot. If that slot is also idle, it either transmits or defers again with probability p and q, respectively. This process is repeated until either the packet has been transmitted or some other user has begun transmission [Tane 96].

-Carrier sense multiple access with collision detection (CSMA/CD)

In CSMA/CD protocol, when two users sense an idle channel simultaneously and begin transmission they will both detect the collision immediately and stop transmission as soon as the collision is detected. CSMA/CD is widely used in Local Area Networks (LANs), and has been standardised by the IEEE in the 802.3 standard [ANSI 85].

2.3 Random Access Code Division Multiple Access (CDMA)

Random access CDMA is an important application of spread spectrum communications that allows simultaneous transmissions of multiple users to occupy a single wideband channel with small interference.

Spread Spectrum (SS) modulation was originally used for military communication systems because of the resistance against jamming signals and low probability of interception. Scholtz [Scho 82] has presented a detailed history of spread spectrum communications, and a definition of spread spectrum that adequately reflects the characteristics of this technique is given in [Pick 82] as:
"Spread spectrum is a means of transmission in which the signal occupies a bandwidth in excess of the minimum necessary to send the information; the band spread is accomplished by means of a code which is independent of the data, and a synchronised reception with the code at the receiver is used in despreading and subsequent data recovery".

Thus, the idea of spread spectrum is to transform an information data signal into a transmission signal with a much larger bandwidth. This transformation is achieved by encoding the information data signal by a code signal called a pseudo random or pseudo noise (PN) code sequence that is independent of the data signal, and has a much larger bandwidth than the data signal. The ratio of the transmitted bandwidth, BW_{RF}, to the original data bandwidth, BW_{info}, is called the processing gain, G, of the spread spectrum system

$$G = \frac{BW_{RF}}{BW_{info}} \tag{2.1}$$

The spread spectrum receiver correlates the received wideband signal with a synchronously generated replica of the PN sequence to recover the original transmitted data information. Fig.(2.2) shows a basic block diagram of a spread spectrum digital communication system.

Spread spectrum modulation has some special properties due to the PN coding and the resulting wide bandwidth signals [Pick 82], [Pras 96]. These are:

1. Anti-jamming capability.
2. Secure communication.
3. Multipath rejection.
4. Low probability of interception (LPI).
5. Interference rejection.
6. Multiple access capability.

Code division multiple access (CDMA) is an application of the multiple access property of spread spectrum modulation. This multiple access property is achieved by assigning each user a unique PN code such that there is a low cross-correlation between each pair of sequences.

If multiple users transmit a spread spectrum signal simultaneously, the intended receiver will be able to extract the signal of the desired user out of the spreading signals provided each user has its own code sequence that has a sufficiently low cross-correlation with the codes of other users. The intended receiver correlates the received wide band signal with the code signal of the desired user. The correlator will despread the desired signal, while the other spread spectrum signals will be spread more over a large bandwidth.

Within the information bandwidth the power of the desired user will be much larger than the interfering power provided there are not too many interfering users. Fig.(2.3) shows the principle of spread spectrum multiple access for three users.

CDMA systems can be classified into two main systems according to the spread spectrum modulation method used, as follows:

1. Direct Sequence (DS-CDMA).
2. Frequency Hopping (FH-CDMA).

Both DS-CDMA and FH-CDMA systems are considered in this work with an emphasis on DS-CDMA.

2.3.1 Direct sequence code division multiple access (DS-CDMA)

Fig.(2.4) shows a block diagram of the transmitter and the receiver of a DS-CDMA system. In a DS-CDMA system, the data information sequence, $m(t)$, of rate R_b bits per second is directly modulated by a PN sequence, $p(t)$, of rate R_c times per second. The reciprocal of R_c, denoted by T_c defines the duration of a rectangular pulse of a PN code, which is called a chip. If $T_b=1/R_b$ is defined as the duration of a rectangular pulse corresponding to the transmission time of an information bit, the processing gain of this system is given by

$$G = \frac{T_b}{T_c} = \frac{R_c}{R_b} \tag{2.2}$$

The resulting sequence is then modulated by a PSK modulator using an IF carrier. The PSK signal is then upconverted to the desired RF frequency in order to be transmitted through the transmitter antenna.

At the receiver, the intended user will have a synchronised despreading function to generate a replica of the PN sequence used at the corresponding transmitter. The received signal is first downconverted from RF to IF frequency, then a PSK demodulation is performed, the resulting signal is *m(t).p(t)*. This signal is then multiplied by the locally generated PN sequence which is synchronised to the PN sequence of the transmitter, thus *p(t).p(t)=1*. At the output of the multiplier the original data signal, *m(t)*, is recovered

$$m(t).\, p(t).\, p(t) = m(t).\, p(t)^2 = m(t) \tag{2.3}$$

-Multiple access capability of DS-CDMA

When N multiple users use the same wide band channel simultaneously, the intended receiver will receive the desired signal superimposed on N-1 interfering signals. Then the multiplier output signal is

$$m(t).\, p(t)^2 + \sum_{i=1}^{N-1} m_i(t).\, p_i(t).\, p(t) = m(t) + \sum_{i=1}^{N-1} m_i(t).\, p_i(t).\, p(t) \tag{2.4}$$

The desired signal, *m(t)*, is now superimposed on noise due to interference. If care has been taken to choose PN codes with low cross-correlation properties, this noise will be small. Multiplication of $\sum_{i=1}^{N-1} m_i(t).\, p_i(t)$ by *p(t)* at the receiver implies spreading the spectrum of each user's data sequence $m_i(t)$ which has been already been spread by its own $p_i(t)$ sequence. Thus, the noise spectral density of the interference term in equation (2.4) is low, and consequently, the interference noise power in the bandwidth of the desired signal is low [Mara 93].

DS-CDMA has the advantages of spread spectrum modulation, such as multipath interference rejection, security, and low probability of interception (LPI). With regard to system implementation, DS-CDMA has two advantages, namely that the generation of the coded signal is easy, since it can be done by simple multiplication, and synchronisation among users is not necessary. The two main disadvantages associated with DS-CDMA systems are the PN synchronisation, and the near/far problem.

-PN sequence synchronisation

Synchronisation of the PN sequence at the transmitter with the locally generated replica at the intended receiver is difficult. This synchronisation has to take place within a fraction of the chip time.

-Near/far problem

The power received from users close to the intended receiver or to the base station is much higher than that received from users further away. Since a user continuously transmits over the whole bandwidth, a user close to the intended receiver or to the base station will constantly create a lot of interference for the signals from distant users, making their reception impossible [Pras 96]. This near/far problem can be solved by applying what is called a power control scheme to make all users deliver the same average power to the intended receiver or to the base station. This power control has to be adaptive when the users are mobile and their distances to a receiver are not constant. An adaptive power control scheme ensures that signals received by the base station from all the mobile users remain at the same average power, independent of the movement, propagation path loss, and location of the mobile users.

2.3.2 Frequency hopping code division multiple access (FH-CDMA)

In the FH-CDMA systems, the carrier frequency of the modulated information signal is not constant but changes periodically. During a time interval T_H, the carrier frequency remains the same but after each interval the carrier hops to another (or possibly the same) frequency [Pras 96]. The selection of the carrier frequency in each interval is made pseudo randomly according to a frequency synthesiser controlled by a PN code generator.

Fig.(2.5) shows a block diagram of an FH-CDMA system. The data signal is modulated in the baseband by usually a binary or an M-ary FSK modulator (other modulation schemes can be used). The resulting FSK signal is transformed to one of the available frequencies of the frequency synthesiser driven by a PN code sequence. At the receiver, the received signal is converted down to the FSK base band signal using a locally generated PN code to control the frequency synthesiser of the receiver. The data information is then recovered after the FSK demodulation process.

FH-CDMA systems can be classified either as slow (SFH) or fast (FFH) depending on the relationships between the hopping rate and the information rate. In FFH-CDMA, the hopping rate of the RF carrier, R_H, is greater than the date rate, R_b. In this case, the carrier frequency changes a number of times during the transmission of one data bit, so that one bit is transmitted in different frequencies. In SFH-CDMA, the hopping rate is smaller than the data bit rate, so multiple data bits are transmitted at the same frequency.

-Multiple access capability of FH-CDMA

Each user in an FH-CDMA system is assigned a different PN code, and this implies that the frequency hopping pattern for each user is different. Consequently, the probability of other users transmitting in the same frequency band of the desired user is very low, thus the signal of the desired user will be received correctly most of the time. When the interfering users transmit in the same frequency band as the desired user, error correcting codes can be used to recover the data transmitted during the overlapping period.

-Advantages of FH-CDMA

In addition to the spread spectrum properties, FH-CDMA systems have some advantages over DS-CDMA systems:

1. Synchronisation is much easier than DS-CDMA. With FH-CDMA synchronisation has to be within a fraction of the hop time which is much longer than the chip time of DS-CDMA.
2. The different hopping frequencies that an FH signal can occupy do not have to be contiguous, because the frequency synthesiser can easily skip over certain parts of the spectrum, and this allows a much higher spread spectrum bandwidth to be used [Pras 96].
3. If a user far from the base station transmits, it will be received by the base station even if users close to the base station are transmitting, since those users will be transmitting at other frequencies with high probability. Thus, the near/far performance is improved over the DS-CDMA systems.

-Disadvantages of FH-CDMA [Pras 96]

1. The need for a highly sophisticated frequency synthesiser .
2. Coherent demodulation is difficult because of the problem of maintaining phase relationships during hopping.

2.4 Pseudo-Noise Codes (PN)

The key feature of a random access CDMA system is the PN codes that are assigned to the users of the system. These codes should make a good separation between the signal of a desired user and the signals of interfering users. In order to reduce the interference among users when they attempt transmission simultaneously, the value of the cross-correlation between any pair of the used codes should be as small as possible. The auto-correlation of the PN codes is also important, since it has an effect on the lock and synchronisation of the locally generated PN code to the received signal.

2.4.1 Properties of PN codes [Golo 67], [Lee 93] and [Pras 96]

1. The code must be easy to generate.
2. Long periods are required to obtain satisfactory auto-correlation.
3. The code must be difficult to reconstruct from a short segment, in order to obtain secure transmission.
4. It must have the desired randomness properties:
   - Balance property: the number of zeros and ones of a PN code are different only by one.
   - Run property: in every period of a PN code, half of the runs have length one, one-quarter of the runs have length two, one-eighth of the runs have length three, etc.
   - Low cross-correlation.
   - Low auto-correlation.

The Balance and the Run properties insure that a PN code is random as much as possible. This randomness is important in the case of using PN codes in CDMA systems, because it minimises the interference when many users transmit their packets simultaneously using the same channel.

The demands on the correlation properties are the most important demands for the development of suitable codes for random access CDMA communications. Many practical codes have been used or suggested to be used in CDMA systems, such as

Gold, Kassami, No, and Bent codes. Properties of these codes can be found in [Gold 67], [Sarw 80], [Tach 92], [Osle 82] and [Purs 77].

2.4.2 PN codes generation [Proa 95] [Fehe 95]

The most widely known binary PN code sequences are the maximum length shift register sequences (*m*-sequences). An *m*-sequence has length $n=2^m-1$ bits and can be generated by an *m*-stage shift register with linear feedback, as shown in Fig.(2.6). The generated sequence is periodic with period *n*. However, because the cross-correlation of *m*-sequences is high they are not acceptable for CDMA applications.

PN sequences with better cross-correlation properties than *m*-sequences have been suggested by Gold [Gold 67], and these are now called Gold sequences. They are derived from *m*-sequences by selecting certain pairs of *m*-sequences with low cross-correlation. These sequences are called preferred sequences. Gold sequences are generated by modulo-2 addition of two preferred *m*-sequences clocked by the same clock. Since both *m*-sequences have equal length *n*, the resulting Gold sequence is of length *n* as well. The possible number of different Gold sequences that can be generated by the same preferred pair of *m*-sequences is $L=2^{n-1}$ sequences. Fig.(2.7) shows a Gold sequences generator. Two preferred *m*-sequences of length 31 bits are used which are described by the polynomials

$$g_1(p) = p^5 + p^2 + 1$$
$$g_2(p) = p^5 + p^4 + p^2 + p + 1$$

Preferred pairs of *m*-sequences are listed in [Pete 72] and [Dixo 94].

2.5 Applications of Random Access CDMA

In recent years extensive research has been carried out into the application of the random access CDMA protocol as a multiple access method for future communication systems, such as the cellular mobile and the personal mobile satellite communications.

In the cellular environment, the CDMA protocol is a strong candidate for mobile cellular communication systems. This is because of its advantages over conventional protocols, such as TDMA and FDMA. The most important advantage which concerns the cellular application is the improved capacity that can be achieved compared to the current FDMA and TDMA. The CDMA capacity gain is of the order of 10-20 compared to FDMA, and compared to TDMA the capacity gain is of the order of 4-7 [Vite 94], [Gilh 91]. Other advantages of CDMA over FDMA and TDMA can be found in [Lee 91] and [Jung 93]. The CDMA digital cellular system was standardised in North America in 1993 as the Telecommunication Industry Association (TIA) and The Electronic Industry Association (EIA) standard IS-95 [EIA 93].

Personal mobile satellite communications uses low earth orbit (LEO) or medium earth orbit (MEO) satellite systems in order to reduce the high transmission powers and the long delays associated with geostationary satellite systems. The CDMA protocol has been suggested for use in personal mobile satellite communications by four out of five proposals to implement such a system (Odyssey, Ellipsat, Global Star, and Aries) [Rush 92], [Fehe 95].

2.6 Techniques for Performance Analysis of Multiple Access Protocols

There are several analytical techniques that can be used to evaluate the performance of packet multiple access networks. The most common ones are:

1. S-G or Poisson analysis.
2. Markov analysis.
3. Equilibrium point analysis.

2.6.1 S-G analysis

S-G analysis (Throughput-Channel traffic) was introduced by Abramson [Abra 70] to evaluate the performance of the pure ALOHA protocol in the ALOHA system at the university of Hawaii. It is also referred to as Poisson analysis.

This technique assumes [Tasa 86]:

-Steady-state conditions exist.

-The traffic source consists of an infinite number of users who collectively form an independent Poisson source.

-The sum of new packet transmissions and retransmissions in the channel (channel traffic) can be approximated as a Poisson process at a rate of G packets/slot.

The S-G technique evaluates the performance of multiple access systems by forming the relationship between the throughput S and the channel traffic G. The S-G technique can easily be applied to complex multiple access protocols. However, it cannot give any idea about the dynamic behaviour of these protocols, and consequently it cannot determine the stability of the modelled systems.

2.6.2 Markov analysis

Markov analysis formulates a Markovian model of a system and obtains the stationary probability distribution of the Markov chain from the state transition probabilities. Markov analysis was first used by Kleinrock and Lam [Klei 75a] to evaluate the performance of slotted-ALOHA systems.

This technique can study the dynamic behaviour of the multiple access protocols, so it can solve the stability problem. However, it cannot be applied to complex protocols that are modelled as multidimensional Markov chains, because the resulting models become intractable. Channel propagation delay, packet reservation, and the buffering capability of the users are some of the causes of multidimensionality of Markov chains.

2.6.3 Equilibrium point analysis (EPA) - the principle

The Equilibrium point analysis (EPA) method as proposed by Fukuda and Tasaka [Fuku 83] is a powerful analytic tool for the performance evaluation of packet broadcast networks. EPA is an approximation technique that is applied to the steady state, and it assumes that the system is always at the equilibrium point. Therefore EPA does not need to calculate the state transition probabilities needed in Markov analysis. The importance of an equilibrium point in multiple access protocols was first noticed by Kleinrock and Lam [Klei 75a], and by Carleial and Hellman [Carl 75], independently.

In order to study the performance of complicated multiple access systems represented by multidimensional Markov chains, Fukuda and Tasaka in [Fuku 83] introduced an analytical technique depending on the results of [Klei 75a] and [Carl 75], and this is called Equilibrium Point Analysis (EPA).

In essence, the equilibrium distribution of the Markov chain to be solved is approximated by a unit impulse located at a point in the state space where the system is in equilibrium. This method requires the solution of a set of non-linear equations known as the equilibrium point equations, which can often be reduced to a fixed point equation for a single variable of interest. This equation then can be solved either by iteration or bisection methods [Wood 93].

The EPA method has been used to evaluate the performance of various multiple access protocols, such as ALOHA [Fuku 83], reservation-ALOHA [Tasa 83], CSMA/CD [Tasa 86], [Wood 93], and slotted ring LAN [Rodr 93], In this work the EPA method will be applied to solve a Markovian model in the form of a discrete-time queueing network of the random access CDMA protocol. Sometimes throughout this thesis the Markovian model solved by EPA method is simply called the EPA model.

2.7 Summary

In this chapter a brief discussion of the concept of the multiple access protocols and their classification has been presented. Both fixed assignment and random access protocols have been discussed in some detail.

The principle of the random access CDMA protocol as one of the most important applications of spread spectrum modulation has been discussed. Two types of spread spectrum modulation for CDMA are considered, namely the direct sequence (DS-CDMA) and the frequency hopping (FH-CDMA). The multiple access capability, advantages and disadvantages of these systems have been discussed.

Finally, a comparison of different analytical methods used to evaluate the performance of multiple access protocols has been presented. Equilibrium point analysis (EPA) is the method that will be used throughout this work in order to solve various Markovian models of the CDMA protocol and so evaluate their performance.

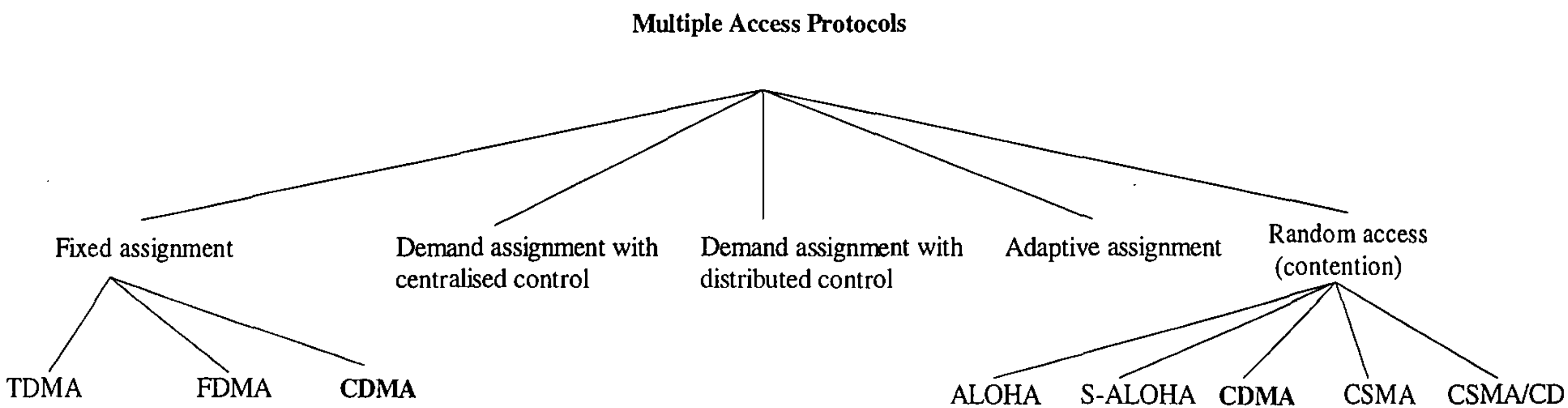


Fig.(2.1) Classification of multiple access protocols

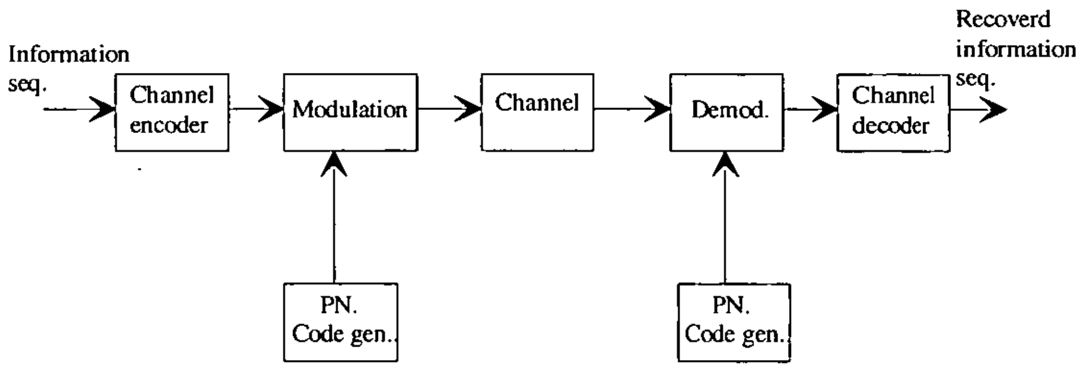


Fig.(2.2) Basic spread spectrum block diagram.

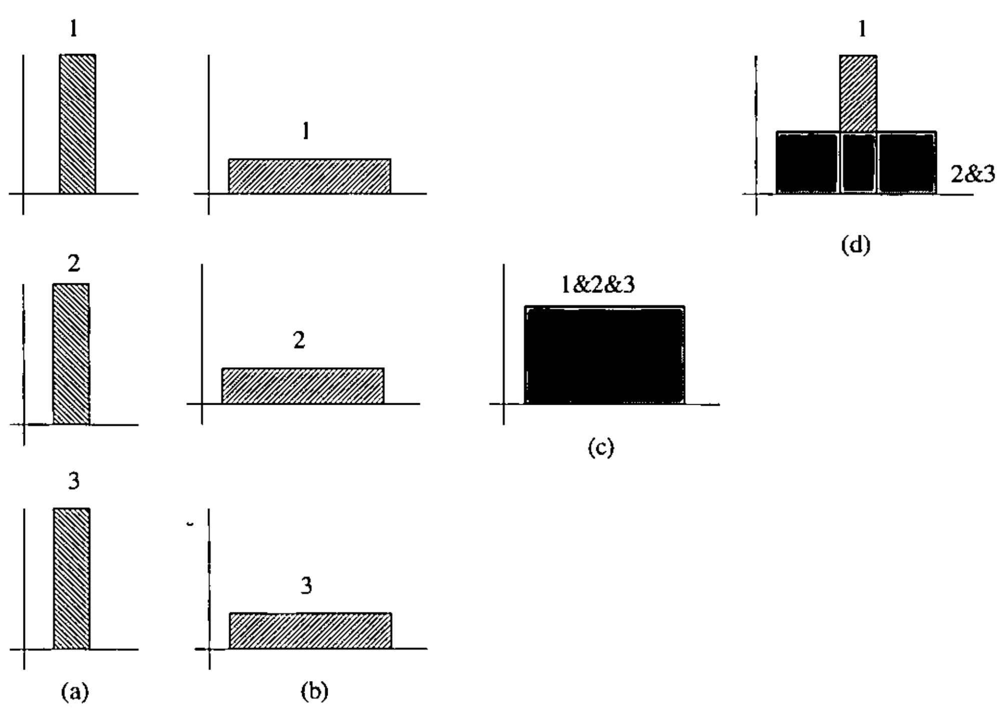


Fig.(2.3) Principle of spread spectrum multiple access.

(a) Narrowband data signals.
(b) Spread spectrum signals.
(c) Transmitted spread spectrum signals.
(d) Extracted data signal of user 1.

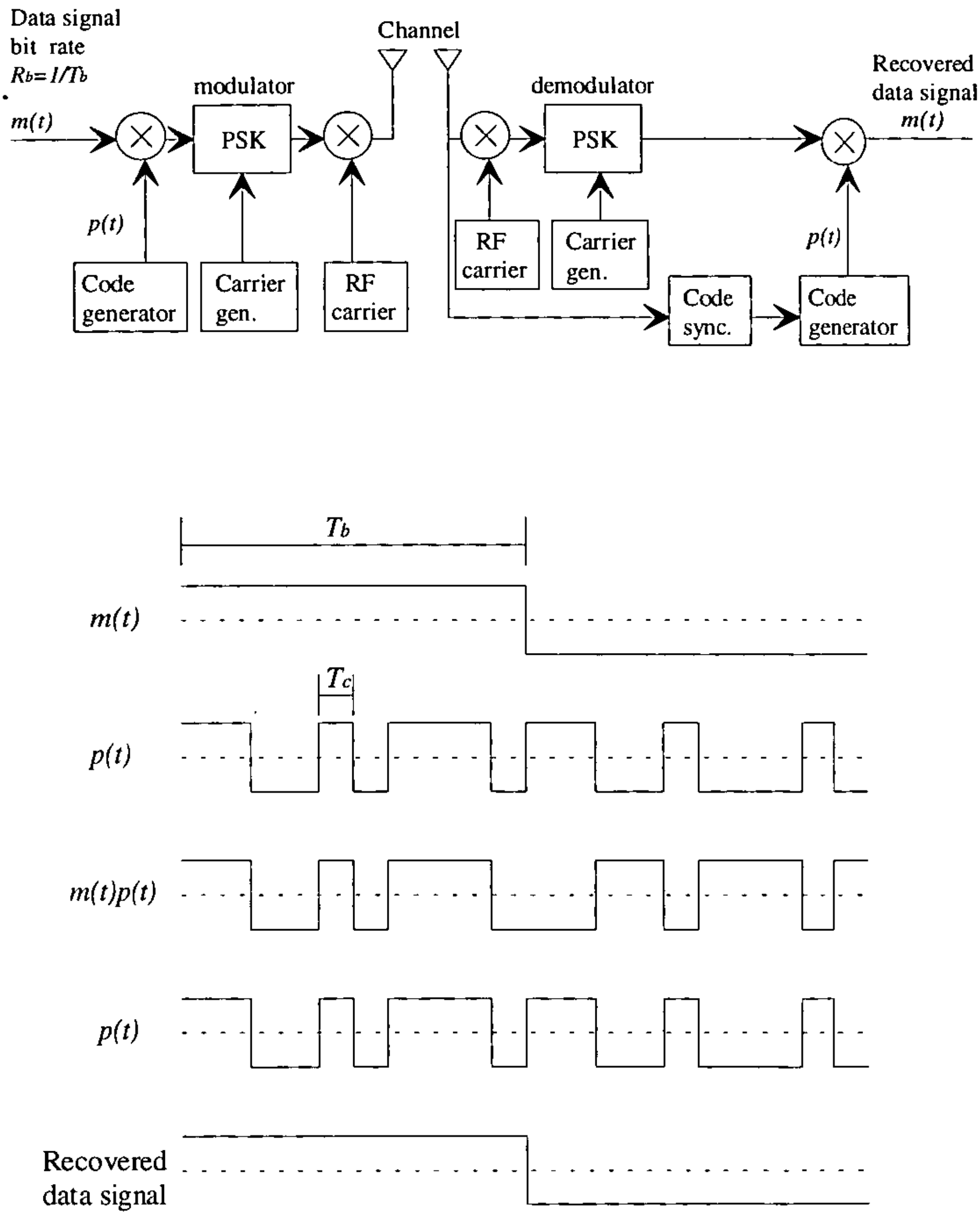


Fig.(2.4) Direct Sequence (DS-CDMA).

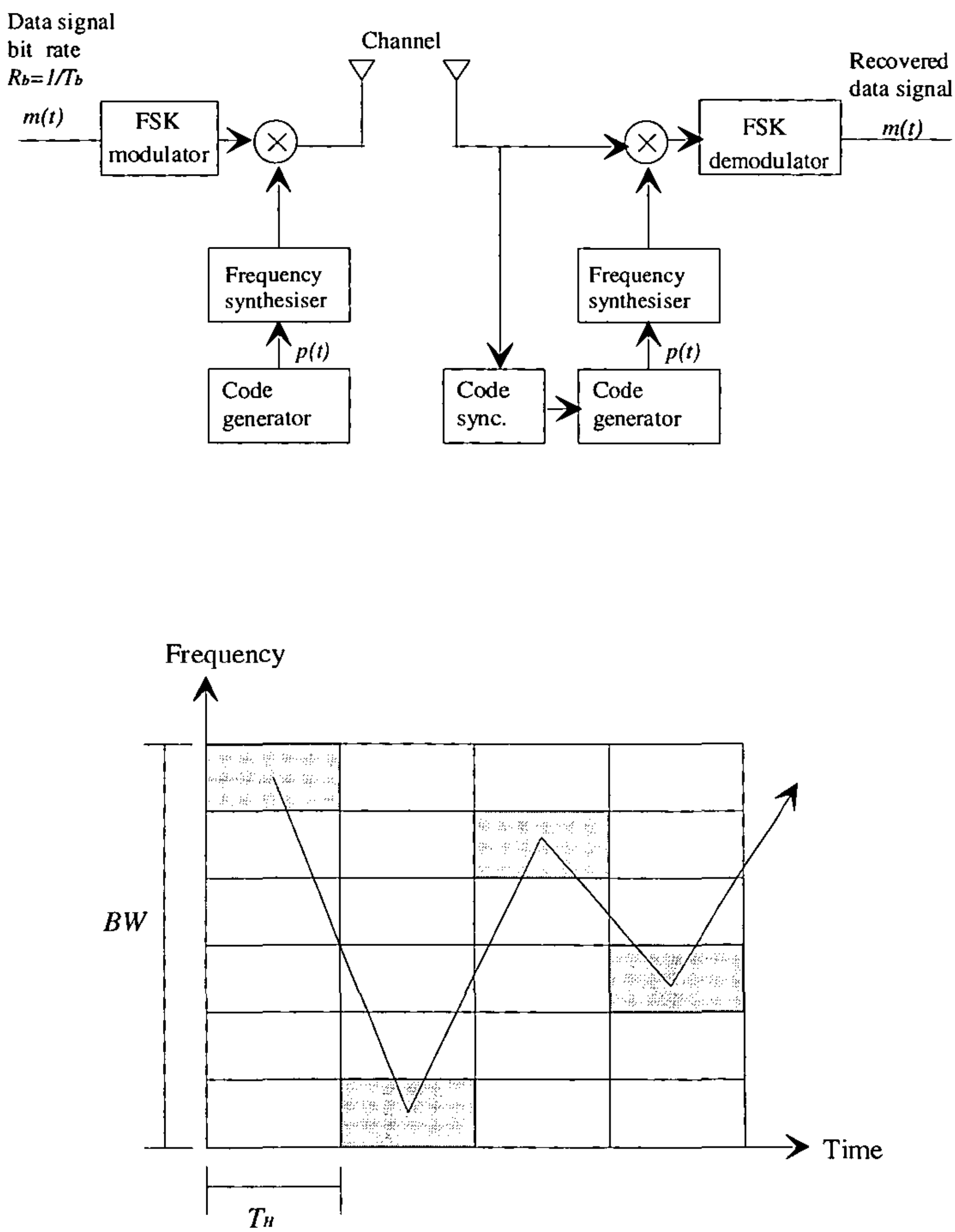


Fig.(2.5) Frequency Hopping (FH-CDMA).

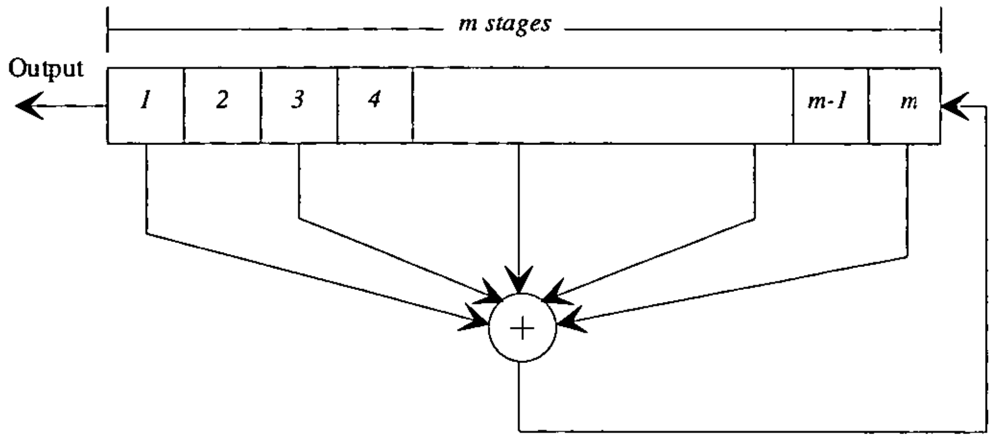


Fig.(2.6) General m-stage shift register with linear feedback.

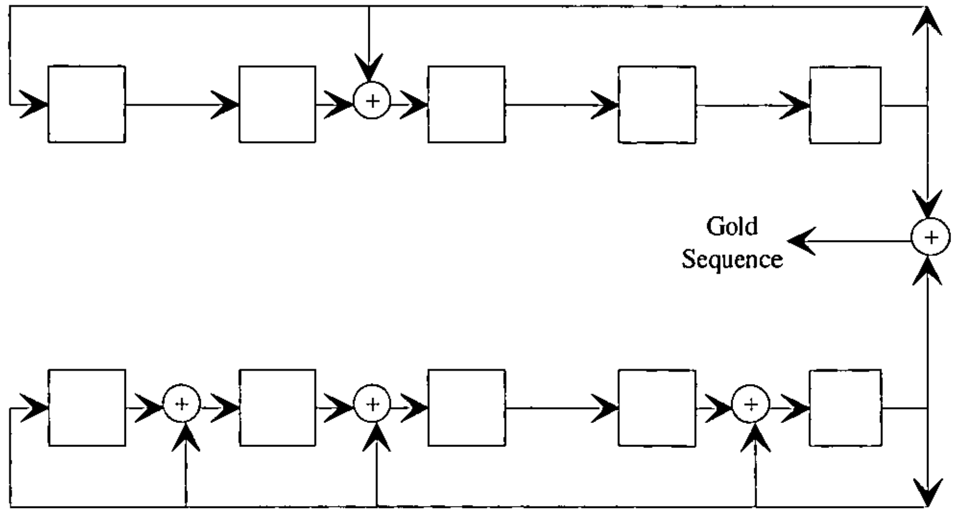


Fig.(2.7) Generation of Gold sequences of length 31 bits.

CHAPTER 3

Equilibrium Point Analysis for Random Access CDMA

3.1 Introduction

The objective of this chapter is to develop a Markovian model in the form of a discrete-time queueing network for random access CDMA systems in order to assess the performance of these systems. Throughput and mean packet delay characteristics are shown, since these two measures are usually used in the study of the performance analysis of multiple access protocols. The throughput is defined as the average number of successfully transmitted packets per transmission time of a packet. The average packet delay is defined as the average time, in unit time, from the moment a packet is generated until the instant that this packet is correctly received by its destination [Tasa 86].

The method used to solve the model and to evaluate the performance of random access CDMA systems is equilibrium point analysis (EPA). This method has previously been used to study the performance of random access Slotted-ALOHA systems in [Tasa 86] and [Fuku 83]. It has not previously been used in the context of random access CDMA.

3.2 Discrete-Time Queueing Network Model for Random Access Protocols

Many communication networks transmit information in fixed length packets and operate on the basis of time slotting. It is natural therefore to model such systems in discrete-time, where the basic time unit in a model is equal to a time slot in the physical system that is modelled [Wood 95b]. The discrete-time queueing model of a multiple access protocol [Wood 93] consists of H nodes, where H is finite. The H nodes are populated by N users circulating around these nodes. Each node in the model represents a queue of users such that all users at the same node are waiting in the same state (The queues are conceptual ones rather than a physical ones and for that reason the model may be called a virtual queueing network [Wood 95b]). A user departs from a node when an event takes place to cause a state transition. Such events are either the generation of a new packet or the transmission of a packet by a user.

If x_n^j denotes the number of users at node j during slot n, n=0, 1, 2, ..., then

$$x_n = (x_n^1, x_n^2, \dots\dots\dots, x_n^H) \tag{3.1}$$

is a state vector of the network and is a discrete-time Markov chain

$$\mathbf{X} = \{x_n = x, \qquad n = 0,1,2,\ldots\ldots.., \qquad x \in X_N\} \tag{3.2}$$

where

$$x = (x^1, x^2, \ldots\ldots\ldots.., x^H) \tag{3.3}$$

and

$$X_N = \left\{ x \in X \middle| \sum_{j=0}^{H} x^j = N \right\} \tag{3.4}$$

with

$$X = \{0, 1, 2, \ldots\ldots\ldots.\}^H \tag{3.5}$$

The flow conservation equations can be written as

$$\lambda_i = \sum_{j=1}^{H} \lambda_j r_{ji} \qquad i=1, 2, \ldots\ldots..., H \tag{3.6}$$

where λ_i is the mean number of arrivals per unit time at node i, and r_{ji} is the routing probability from node j to i.

EPA is an approximation method to solve the flow conservation equations (3.6) under the assumption that the system is always at an equilibrium point. Equations (3.6) can be written in an equivalent form by applying Little's result as

$$\mathrm{E}[x^i]\mu_i = \sum_{j=1}^{H} \mathrm{E}[x^j]\mu_j r_{ji} \qquad i = 1,2,\ldots\ldots\ldots.,H \tag{3.7}$$

where μ_i is the serving probability for node i.

The expected values in equations (3.7) are then replaced by the corresponding equilibrium point values that solve the equations

$$x^i \mu_i = \sum_{j=1}^{H} x^j \mu_j r_{ji} \qquad i = 1,2,\ldots\ldots\ldots\ldots,H \tag{3.8}$$

Equations (3.8) are the equilibrium point equations; the left hand side approximates the expected number of users leaving node i per unit time, and the right hand side approximates the expected number of users entering node i per unit time. With a network populated by N users, one of the equilibrium equations is linearly dependent on the others and can be replaced by the constraint equation

$$\sum_{i=1}^{H} x^i = N \tag{3.9}$$

Equations (3.8), with one equation removed, and (3.9) give H independent equations that are usually non-linear, and can be solved to obtain an equilibrium point

$$x_e = (x_e^1, x_e^2, \ldots\ldots\ldots\ldots, x_e^H) \tag{3.10}$$

It is clear that the equilibrium distribution of the Markov chain **X** has been approximated by a unit impulse located at x_e.

If $S(x)$ is the throughput performance of a modelled network, the expected value for this is

$$\mathrm{E}[S(x)] = \int_{X^H} S(x)\delta(x - x_e)dx = S(x_e) \tag{3.11}$$

Where the integration is over a H-dimensional Euclidean space that includes the state space as a discrete valued subset. Thus, the mean value of the throughput can be approximated by its value at an equilibrium point. The above model will be applied to Slotted-ALOHA and random access CDMA.

3.3 EPA for S-ALOHA

3.3.1 S-ALOHA with zero channel delay

The EPA method will first be described in the context of S-ALOHA before being extended to random access CDMA.

Let us consider the model in Fig.(3.1) which is an S-ALOHA system with a population of N users who generate single packet messages. Each user is assumed to be at one of two nodes, thinking *(T)* or retransmission *(RT)*. A user in node T generates a new packet in a slot with probability σ. A user whose packet has a collision enters node RT and retransmits his packet with probability p. If the system is assumed to be in equilibrium, the average number of users leaving a node per unit time is equal to the average number of users entering the same node.

When the EPA method is applied to solve a Markovian model such as the model in Fig.(3.1), a modified model is considered which makes the analysis by EPA more tractable, especially for complex systems modelled by multidimensional Markov chains with large numbers of possible nodes. The purpose of the modification is to merge the two inputs from nodes T and RT to the S-ALOHA channel into one [Tasa 86]. The modified model is shown in Fig.(3.2). The modified model is identical to the original model from a stochastic behaviour point of view. The proof of equivalency can be found in [Tasa 86] and [Wood 93]. The modified model in Fig.(3.2) is correct under the condition $\sigma \leq p$. For the case $\sigma > p$ another modified model is required. This case is discussed later in the context of modelling random access CDMA systems, since the modelling of these systems is the main objective of this research.

In the model shown in Fig.(3.2), a user who has just successfully transmitted a packet moves into nodes *T'* and *RT'* with probabilities $1-\frac{\sigma}{p}$ and $\frac{\sigma}{p}$, respectively, instead of entering node *T* with probability one. A user whose transmitted packet is involved in a collision enters node *RT'* with probability one. A user in node *T'* moves into node *RT'* at the next slot with probability σ, and a user in node *RT'* transmits a packet with probability p.

Using Fig.(3.2), let x be a random variable representing the number of users in node *RT'*, the conditional throughput can be written as

$$S(x) = x\, p\, (1-p)^{x-1} \tag{3.12}$$

So, a success can be achieved if and only if only one user at node *RT'* attempts to transmit a packet through the S-ALOHA channel.

Applying EPA to node *T'* gives

$$(N-x)\,\sigma - \left(1-\frac{\sigma}{p}\right) S(x) = 0 \tag{3.13}$$

Equation (3.12) can be solved for x by using either iterative or bisection methods. Let x_e is the solution which represents the equilibrium point, thus the throughput S can be expressed in terms of x_e as

$$S = S(x_e) = x_e\, p\, (1-p)^{x_e-1} \tag{3.14}$$

3.3.2 S-ALOHA with non-zero channel delay

Before modelling the case of non-zero channel delay, some assumptions must be made to make the analysis possible:

1. Each data packet has a fixed length of L_p bits.
2. Slot duration is equal to the packet transmission time, and each message consists of only one packet. The assumption of single-packet messages is made to simplfy the model. The model can easily be modified to accommodate multiple-packets messages, see [Wood 93].
3. The channel is error free except for collisions.
4. The network has a fixed number of users N.

An S-ALOHA model with non-zero channel delay is represented in Fig.(3.3). A user at node 0 will successfully transmit a packet in a given slot if no other users at node 0 attempt transmission in the same slot. If a transmitted packet is involved in a collision, the transmitting user will enter the node R and be routed back to node 0 from which the colliding packet will be retransmitted. However, if a packet is successfully transmitted, the user will enter the node $2R$ and be routed back to node 0 with probability $\frac{\sigma}{p}$ or to node $2R$+1 with probability 1-$\frac{\sigma}{p}$.

The state vector of this model is

$$x = \left(x^0, x^1, x^2, \ldots\ldots\ldots, x^{2R}\right) \tag{3.15}$$

and the state space is

$$X_N = \left\{ x \in X \;\middle|\; x^0 + x^{2R+1} + \sum_{i=1}^{2R} x^i = N \right\} \tag{3.16}$$

$$X = \{0, 1, 2, \ldots\ldots\ldots\}^{2R+1} \tag{3.17}$$

The EPA equations are

$$x^{2R+1}.\sigma = x^{R+1}\left(1-\frac{\sigma}{p}\right) \tag{3.18}$$

$$x^{R+1} = x^{R+2} = \ldots\ldots\ldots = x^{2R} = x^0 p(1-p)^{x^0-1} \tag{3.19}$$

$$x^1 = x^2 = \ldots\ldots\ldots = x^R = x^0 p - x^0 p(1-p)^{x^0-1} \tag{3.20}$$

The constraint equation is

$$\sum_{i=0}^{2R+1} x^i = N \tag{3.21}$$

The equation for node 0 has not been considered since it is linearly dependent on the other equations. From equations (3.18) to (3.21) a fixed point equation can be expressed in terms of the number of users in a node 0 as

$$x^0 = \frac{N - \left(1-\frac{\sigma}{p}\right)\cdot\frac{1}{\sigma}\cdot x^0 p(1-p)^{x^0-1}}{1+Rp} \tag{3.22}$$

Let x_e^0 be the solution to equation (3.22), then the expected value of throughput is

$$E[S(x)] = S(x_e^0) = S = x_e^0 p(1-p)^{x_e^0-1} \tag{3.23}$$

Fig.(3.4) shows throughput against retransmission probability ($0 \le p \le 0.12$) for σ=0.01 and σ=0.1 with N=100 users and channel delay R=12 slots. The maximum throughput achieved in the modelling is about [35% to 38%] which is very close to the maximum throughput stated in the literature. The modelling results are compared with simulation results which simulate the actual behaviour of a slotted

ALOHA system over 10^5 slots, and it can be seen that the model matches well the simulation.

3.4 EPA for Random Access CDMA

3.4.1 System model

In the previous analysis of the S-ALOHA protocol a collision (and consequently destroyed packets) occur when two or more users attempt transmission in the same slot. However, the transmitted packets can be coded in a way that makes the interference amongst them very low when they are transmitted simultaneously in the same slot. This can be achieved using CDMA, which is an important application of the spread spectrum modulation, and can be implemented in a number of different ways, such as direct sequence, frequency hopping, or time hopping [Dixo 94], [Hayk 88].

In accordance with the principle of CDMA, each user is assigned a unique code to be modulated by its data. These codes are approximately mutually orthogonal, thus each code has low cross-correlation properties with all other codes in the CDMA system.

Each user in the system is assumed to have a buffer that can hold at most one packet, and a packet must be held in the buffer until confirmation is received of its successful transmission, at that time the packet is deleted from the buffer. A user with an empty buffer is assumed to generate a new packet per slot with probability σ. No new packet is thus generated with probability $1-\sigma$ per slot. A user that generates a packet immediately attempts to transmit this packet. A packet will be successfully transmitted with probability $P_c(m)$, or unsuccessfully transmitted with probability $1-P_c(m)$, where m is the number of users simultaneously attempting transmission in the same slot. A user associated with a successful transmission immediately deletes the transmitted packet from its buffer and becomes ready to generate a new packet. A user associated with an unsuccessful transmission retains the packet in its buffer and attempts to retransmit the same packet with probability p. The retransmission procedure is repeated until the packet is successfully transmitted.

Fig.(3.5) represents a discrete-time queueing network model for a random access CDMA system with R slots channel propagation delay. The modelled system consists of N users which circulate around $2R+2$ nodes labelled 0, 1, 2,.........,$2R+1$, where each node represents a discrete-time queue of users. The nodes have binomial release probabilities, which implies that each user's sojourn time at each of the nodes is geometrically distributed and independent of the other users in the network.

Users at node $2R+1$ have no packet in their buffer and are ready to generate new packets with probability σ in a slot. Each user is thus released from node $2R+1$ in a slot with probability σ, or remains at the node with probability $1-\sigma$ depending on whether the corresponding user generates a packet or not, respectively. Users at node 0 have packets in their buffer waiting for retransmission. Each user is released from node 0 in a slot with probability p, or remains at this node with probability $1-p$. Each user released either from node 0 or node $2R+1$ in a slot either successfully transmits its packet with probability $P_c(m)$ and jumps to node $2R$, or is unsuccessful with probability $1-P_c(m)$ and jumps to node R. $P_c(m)$ is thus the probability that a user when released from either node 0 or node $2R+1$ is routed to node $2R$ when m users are released from these two nodes in a slot. Similarly, $1-P_c(m)$ is the probability that a user that is released from node 0 or node $2R+1$ is routed to node R when m users are released from these two nodes in a slot. Since m is dependent on the number of users at nodes 0 and $2R+1$, the routing probability $P_c(m)$ is dependent on the state of the queueing network. Nodes 1, 2,........, $2R$, each represents a one slot delay, and each user at these nodes has a release probability of 1. A user associated with a successful transmission returns to node $2R+1$ after an R slots delay, and a user associated with unsuccessful transmission returns to node 0 after an R slots delay. This R slots delay represents the end-to-end propagation delay over the channel.

As in S-ALOHA analysis, a modified model can be used to assess the performance of random access CDMA. This modified model is stochastically equivalent to the original model shown in Fig.(3.5). Two modified models are used, for $\sigma \leq p$ and $\sigma > p$ cases. The Stochastic transformation shown in Fig.(3.6) is performed on the CDMA channel inputs in order to merge the two channel inputs into one, thus to make the EPA analysis more tractable. Fig.(3.6b) and Fig.(3.6c) represent the stochastic transformation of the original model for $\sigma \leq p$ and $\sigma > p$, respectively. Nodes 0 and $2R+1$ which form a parallel input to the CDMA channel in Fig.(3.6a) are replaced by a single input from node $0'$. The problem is to determine the routing probability x in order to make the modified model stochastically equivalent to the original one. The proof of equivalency for the random access CDMA model is similar to that for the model of S-ALOHA given by Tasaka [Tasa 86] and Woodward [Wood 93].

For the $\sigma \leq p$ case, let U_1, W_1 and W_2 be random variables representing the number of slots for which a user remains at nodes $2R+1$, 0 and $0'$, respectively. Also, let U_2 be a random variable representing the number of slots from the instant a user enters either node $2R+1'$ or node $0'$, until the instant the user leaves node $0'$. Let the generating functions of U_1, U_2, W_1 and W_2 be denoted by $G_{U1}(z)$, $G_{U2}(z)$, $G_{W1}(z)$ and $G_{W2}(z)$, respectively. Since W_1, and W_2 are geometrically distributed with parameter p, and U_1 is geometrically distributed with parameter σ then

$$G_{W_1}(z) = G_{W_2}(z) = \frac{pz}{1-(1-p)z} \tag{3.24}$$

$$G_{U_1}(z) = \frac{\sigma z}{1-(1-\sigma)z} \tag{3.25}$$

$$G_{U_2}(z) = \frac{(1-x)\sigma z}{1-(1-\sigma)z} G_{W_2}(z) + xG_{W_2}(z) \tag{3.26}$$

For stochastic equivalence of the two models then

$$G_{U_1}(z) = G_{U_2}(z) \tag{3.27}$$

Equations (3.25), (3.26) and (3.27) can be solved to give

$$x = \frac{\sigma}{p} \tag{3.28}$$

Similarly, for the $\sigma > p$ case shown in Fig.(3.6c), the generating functions can be written as

$$G_{U_1}(z) = G_{U_2}(z) = \frac{\sigma z}{1-(1-\sigma)z} \tag{3.29}$$

$$G_{W_1}(z) = \frac{pz}{1-(1-p)z} \tag{3.30}$$

$$G_{W_2}(z) = (1-x)\frac{pz}{1-(1-p)z} G_{U_2}(z) + xG_{U_2}(z) \tag{3.31}$$

For stochastic equivalence of the two models then

$$G_{W_1}(z) = G_{W_2}(z) \tag{3.32}$$

Equations (3.30), (3.31) and (3.32) can be solved to determine the routing probability x as

$$x = \frac{p}{\sigma} \tag{3.33}$$

The resulting modified models for the $\sigma \leq p$ and $\sigma > p$ cases are shown in Fig.(3.7) and Fig.(3.8), where the nodes $0'$ and $2R+1'$ in Fig.(3.6b) and Fig.(3.6c) are relabelled as 0 and $2R+1$.

With reference to the modified model of Fig.(3.7), let M be a random variable that represents the number of attempted transmissions in a slot, and K be a random variable that represents the number of successfully received packets in a slot. If the system is at state x, the conditional distribution of K is [Wood 93]

$$P(K = k | M = m) = \binom{m}{k} P_c^k(m)\left(1 - P_c(m)\right)^{m-k} \tag{3.34}$$

The conditional mean value of throughput, given the system is at state x can be written as

$$\begin{aligned} S(x) &= E[K] \\ &= E\left[E(K|M)\right] \\ &= E\left[\sum_{k=0}^{M} k \binom{M}{k} P_c^k(M)\left(1 - P_c(M)\right)^{M-k}\right] \\ &= E\left[MP_c(M)\right] \\ &= \sum_{m=1}^{x^0} m \binom{x^0}{m} p^m (1-p)^{x^0-m} P_c(m) \end{aligned} \tag{3.35}$$

Throughput *S(x)* can be approximated by applying the EPA method to the modified model in Fig.(3.7).

The state vector of the model is

$$x = \left(x^0, x^1, \ldots\ldots\ldots, x^{2R}\right) \tag{3.36}$$

The state space is

$$X_N = \left\{ x \in X \middle| \sum_{i=0}^{2R+1} x^i = N \right\} \tag{3.37}$$

$$X = \{0, 1, 2, \ldots\ldots\ldots\}^{2R+1} \tag{3.38}$$

The EPA equations are

$$x^{2R+1} \cdot \sigma = x^{R+1}\left(1 - \frac{\sigma}{p}\right) \tag{3.39}$$

$$x^{R+1} = x^{R+2} = \ldots\ldots\ldots = x^{2R} = S(x) \tag{3.40}$$

$$x^1 = x^2 = \ldots\ldots\ldots = x^R = x^0 p - S(x) \tag{3.41}$$

The constraint equation is

$$\sum_{i=0}^{2R+1} x^i = N \tag{3.42}$$

From equations (3.39) to (3.42), x^0 the number of users in node 0 can be written as

$$x^0 = \frac{N - \left(1 - \frac{\sigma}{p}\right) \cdot \frac{1}{\sigma} \cdot S(x)}{1 + R \cdot p} \tag{3.43}$$

A full derivation of equation (3.43) can be found in Appendix A.

Equation (3.43) is a non-linear equation and can be solved numerically using either bisection or iteration methods to obtain an equilibrium value x_e^0. Note that since x_e^0 is a real valued quantity, the solution of equation (3.43) can be achieved by using nearest integer approximations, both above and below x_e^0, and then using linear interpolation.

When equation (3.43) has one solution, the modelled system is stable and the equilibrium point in this case is called a globally stable equilibrium point. Otherwise, the system is said to be unstable when the number of solutions is more than one (there is more than one equilibrium point). In this case, the equilibrium points associated with the highest and the lowest values of throughput are called locally stable equilibrium points, for that reason the unstable system may be considered as a bistable system. Thus the EPA method can give useful information about the dynamic behaviour of the modelled network. The ability of EPA to study the stability of the modelled system is one of the advantages of this method. The instability can be avoided by choosing appropriate parameters for the modelled system, such as the processing gain and the packet retransmission probability.

Equation (3.43) is a fixed point equation, and if x_e^0 is the solution of this equation, then the expected value of throughput can be approximated as

$$E[S(x)] \cong S(x_e^0) = S = \sum_{m=1}^{x_e^0} m \binom{x_e^0}{m} p^m (1-p)^{x_e^0 - m} P_c(m) \tag{3.44}$$

The mean packet delay, W, is the mean time from when a packet is generated to the successful completion of its transmission. With reference to Fig.(3.5) W is the time from when a user at node $2R+1$ generates a packet to the time when this user returns to this node. If $\bar{n}$ is the mean number of packets in the network, then W can be expressed by Little's result as

$$\begin{aligned} W &= \frac{\bar{n}}{S} \\ &= \frac{N - x^{2R+1}}{S} + \frac{x^{2R+1}\sigma}{S} \end{aligned} \tag{3.45}$$

The equilibrium equation for node $2R+1$ is

$$S = x^{2R+1} \cdot \sigma \tag{3.46}$$

From equations (3.45) and (3.46) the mean packet delay is

$$W = \frac{N}{S} - \frac{1}{\sigma} + 1 \tag{3.47}$$

Fig.(3.8) shows the modified model for the case of $\sigma > p$, where a similar analysis to that used in the case of $\sigma <= p$ can be used to analyse the $\sigma > p$ case. The conditional mean value of throughput can be written as

$$S(x) = \sum_{m=1}^{x^0} m \binom{x^0}{m} \sigma^m (1-\sigma)^{x^0 - m} P_c(m) \tag{3.48}$$

The state vector and the state space are the same as that in the $\sigma <= p$ case.

The EPA equations are

$$x^{2R+1} \cdot p = x^{R+1}\left(1-\frac{p}{\sigma}\right) \tag{3.49}$$

$$x^{R+1} = x^{R+2} = \dots\dots\dots = x^{2R} = x^0\sigma - S(x) \tag{3.50}$$

$$x^1 = x^2 = \dots\dots\dots = x^R = S(x) \tag{3.51}$$

The constraint equation is

$$\sum_{i=0}^{2R+1} x^i = N \tag{3.52}$$

From equations (3.49) to (3.52) x^0 can be expressed as

$$x^0 = \frac{N + \left(1-\frac{p}{\sigma}\right)\cdot\frac{1}{p}\cdot S(x)}{R\cdot\sigma + \frac{\sigma}{p}} \tag{3.53}$$

In a similar way to equation (3.43), if equation (3.53) has one solution the system is stable, otherwise the system is said to be unstable. A full derivation of equation (3.53) can be found in Appendix A.

If x_e^0 is the solution of this fixed point equation, then the expected value of throughput can be approximated as

$$E[S(x)] \cong S(x_e^0) = S = \sum_{m=1}^{x_e^0} m\binom{x_e^0}{m}\sigma^m (1-\sigma)^{x_e^0-m} P_c(m) \tag{3.54}$$

3.4.2 Probability of correct packet reception $P_c(m)$

In order to evaluate the performance of random access CDMA systems, the probability of correct packet reception in equations (3.44) and (3.54) has to be calculated. This $P_c(m)$ is a function of the type of spread spectrum modulation scheme and the data modulation used in the modelled random access CDMA systems. In what follows, two main types of spread spectrum modulation are used, Direct Sequence (DS) and Frequency Hopping (FH).

3.4.2.1 Direct sequence CDMA

In practical direct sequence CDMA systems, phase shift keying (PSK) as a data modulation scheme is usually used along with the spread spectrum modulation. Other types of data modulation are discussed later in this chapter.

The probability of bit error p_e can be approximated as [Proa 95]

$$p_e = \frac{1}{2} erfc\left[\sqrt{\frac{G}{m-1}}\right] \tag{3.55}$$

where G is the processing gain, or the bandwidth expansion. m is the number of users attempting transmission in the same slot, so $m-1$ represents the number of interfering users.

In equation (3.55) some assumptions are made:

1. The multiple access interference (MAI) in a DS/CDMA system is approximated by an Additive White Gaussian Noise process (AWGN), with variance equal to the MAI variance [Morr 89].
2. The synchronous case is considered, which means the interfering signals are chip and phase aligned with the desired signal. This represents the worst case of bit error probability.

3. All the received powers from the transmitting users are identical, which means there is no near/far problem.
4. The background signal to noise ratio, or the thermal noise is negligible compared to the interference from other users, $(N_0/2E_b)=0$. If the thermal noise is included, the probability of bit error becomes

$$p_e = \frac{1}{2} erfc\left[\frac{1}{\sqrt{\frac{N_0}{2E_b}+\frac{(m-1)}{G}}}\right] \tag{3.56}$$

With packets of length L_p bits, and assuming the bits are independent, the mean probability of packet error can be written as

$$P_E(m) = \sum_{n=1}^{L_p}\binom{L_p}{n} p_e^n \left(1-p_e\right)^{L_p-n} = 1-\left(1-p_e\right)^{L_p} \tag{3.57}$$

If p_e is small ($p_e < 10^{-6}$), the mean probability of packet error can be approximated as

$$P_E(m) = L_p p_e = \frac{L_p}{2} erfc\left[\sqrt{\frac{G}{m-1}}\right] \tag{3.58}$$

Then the probability of correct packet reception which will be used in the modelling equations is

$$P_c(m) = 1 - P_E(m) \tag{3.59}$$

3.4.2.2 Frequency hopping CDMA

In FH-CDMA, the partial-band interference is applied to model the multiple access interference (MAI) of the FH-CDMA systems. Partial-band interference is more appropriate for FH-CDMA than the AWGN interference model considered for DS-CDMA, that is because the interference from other users in FH-CDMA affects only part of the total available bandwidth of the intended user, depending on the overlapping of the transmitted frequencies.

Partial-band interference is modelled by Proakis [Proa 95] as a zero-mean Gaussian random process with a flat power spectral density over a fraction α, ($0 \le \alpha \le 1$), of the total available bandwidth and zero elsewhere. The desired received signal will be corrupted by the interference of the other users with probability α and will not be corrupted with probability $1-\alpha$. Then the average probability of bit error for slow FH-CDMA with binary FSK and non-coherent detection is

$$p_e(\alpha) = \frac{\alpha}{2}\exp\left(-\frac{\alpha}{2}\frac{G}{m-1}\right) \tag{3.60}$$

To obtain the value of α (α^*) which makes the worst case partial-band interference, the average probability of bit error equation (3.60) is differentiated with respect to α and solve for the extrumum with the restriction that $0 \le \alpha \le 1$

$$\alpha^* = \begin{cases} 2\dfrac{m-1}{G} & \text{if } \dfrac{G}{m-1} \ge 2 \\[2ex] 1 & \text{if } \dfrac{G}{m-1} < 2 \end{cases} \tag{3.61}$$

The corresponding error probability for the worst case of interference is

$$p_e = \begin{cases} \dfrac{m-1}{G} e^{-1} & \text{if } \dfrac{G}{m-1} \geq 2 \\ \dfrac{1}{2} \exp\left(-\dfrac{G}{2(m-1)} \right) & \text{if } \dfrac{G}{m-1} < 2 \end{cases} \tag{3.62}$$

where m is the number of simultaneous users attempting transmission, and G is the processing gain or number of the available frequency channels. The probability of packet success is the same as in DS-CDMA, equations (3.57) to (3.59).

3.5 DS-CDMA with Various Modulation Schemes

In this section, we investigate the throughput performance of random access DS-CDMA systems using various data modulation schemes. The systems are classified as synchronous or asynchronous. Synchronous systems are those in which the interfering signals are chip and phase aligned with the desired signal, which represents the worst case in terms of the probability of error. Asynchronous systems are those in which the interfering signals have random chip delays and phases with the desired signal [Morr 89].

The systems are also classified as coherent, in which the carrier phase is known perfectly at the receiver, and non-coherent, in which the received signal is demodulated with no attempt being made to estimate the carrier phase [Proa 95].

3.5.1 Probability of error for coherent DS-CDMA systems

Two types of coherent modulation are considered, binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK). The probability of error for these systems is derived in [Stor 86].

1. Synchronous BPSK

$$p_e = \frac{1}{2}\, erfc\left[\sqrt{\frac{G}{m-1}}\right] \tag{3.63}$$

2. Asynchronous BPSK

$$p_e = \frac{1}{2}\, erfc\left[\sqrt{\frac{3G}{2(m-1)}}\right] \tag{3.64}$$

3. Synchronous QPSK

$$p_e = \frac{1}{2}\, erfc\left[\sqrt{\frac{G}{2(m-1)}}\right] \tag{3.65}$$

4. Asynchronous QPSK

$$p_e = \frac{1}{2}\, erfc\left[\sqrt{\frac{3G}{4(m-1)}}\right] \tag{3.66}$$

where m is the number of simultaneous users attempting transmission, and G is the processing gain and at the same time is the period of the PN codes used. Equations (3.63) to (3.66) are used in the EPA model to evaluate the performance of these systems.

3.5.2 Probability of error for non-coherent DS-CDMA systems

Frequency shift keying (FSK), and differential phase shift keying (DPSK) are considered. The probability of error for non-coherent systems has the form [Gera 85]

$$p_e = \frac{1}{2}\exp(-\alpha) \tag{3.67}$$

where the value of α depends on the modulation type as follows:

1. Asynchronous DPSK

$$\frac{1}{\alpha} = 2\left(\frac{N_0}{2E_b} + \frac{m-1}{G}\right) \tag{3.68}$$

2. Asynchronous FSK

$$\frac{1}{\alpha} = 4\left(\frac{N_0}{2E_b} + \frac{m-1}{6G}\right) \tag{3.69}$$

3. Synchronous DPSK

$$\frac{1}{\alpha} = 2\left(\frac{N_0}{2E_b} + \frac{m-1}{2G}\right) \tag{3.70}$$

4. Synchronous FSK

$$\frac{1}{\alpha} = 4\left(\frac{N_0}{2E_b} + \frac{m-1}{4G}\right) \tag{3.71}$$

where $2E_b/N_0$ is the signal to thermal noise ratio.

3.6 Numerical Results

Fig.(3.9) shows the variations of throughput against retransmission probability p for random access CDMA using direct sequence spread spectrum modulation with

N=100, σ=0.1, R=0, L=1025 bits and different values of processing gain G. It should be noted that in these performance curves, and all subsequent ones, p=0.0 obviously gives no solution to either the equilibrium point equations or the simulation. The initial value of p taken was therefore a very small positive value (e.g. p=0.0001) although this may appear to be at p=0.0 on some of the graphs, depending on the scale used. The intervals for p is 0.05.

With processing gain equal to 30, 50 or 70 the channel is stable. However, with the processing gain increased to 80 the channel becomes locally bistable and three solutions are obtained from the model representing two locally stable equilibrium points and one unstable one. When the processing gain is increased to 500 or above, the channel becomes stable again and virtually everything is transmitted without error. In this case the throughput has a value very close to $N\sigma$, which is the maximum mean rate at which packets can be generated.

Throughput and mean packet delay results obtained from the model are shown compared with results obtained from 10^5 slots simulations in Figs. (3.10) and (3.11). Results are shown for σ=0.1 and 0.2 and processing gains of 63 and 127, respectively. In both cases the model shows a good match to the simulations for throughput and mean packet delay over the entire range of retransmission probability p.

It should be noted that the simulations are discrete event simulations [Mitr 82] based on generating random numbers which are independent and uniformly distributed on the interval (0,1). These random numbers are used to generate other random quantities, such as the new packet generation probability and the retransmission probability. The results of the simulations are obtained either by a single run for a very large number of slots (10^6), or by multiple runs for a large number of slots (10^4), since the later allows the results to be represented by the confidence interval method [Matl 88].

It should be noted as well, that the simulations were designed to follow the exact scenario of the model, so the difference between the results obtained from the model and those from the simulations should reflect the error in the equilibrium point approximations used to solve the model. A detailed flowchart of the discrete event simulations for random access CDMA networks can be found in Fig.(3.19).

Fig.(3.12) shows a comparison between the model and the simulations for the unstable case when the processing gain is 80. The modelled system shows instability for retransmission probability beyond p=0.22 and the model has three solutions. It can be seen that one of the solutions from the model follows the simulation, but the other two do not. The interpretation of this is that the channel is spending most of its time at the locally stable equilibrium point associated with the lower value of throughput, and very little time at the locally stable equilibrium point associated with the higher value of throughput. This can easily be seen by observing that the trajectory of the unstable equilibrium point (which is between the trajectories of the two locally stable ones) is very close to the trajectory of the locally stable equilibrium point associated with the higher throughput, thus making the later equilibrium point itself relatively unstable.

Fig.(3.13) and (3.14) show the throughput and mean packet delay, respectively when the channel propagation delay is introduced to the model. The model shows slight disagreement with the simulation beyond p=0.3.

Fig.(3.15) shows the throughput performance of a FH-CDMA system model using EPA. Different values of the processing gain or the available frequency channels are considered, G=500, 1000 and 1500. The maximum throughput or the capacity is nearly independent of the packet generation probability σ. For example, when G=1000, the capacity for σ=0.1 is 4.099 packets/slot, and for σ=0.2 is 4.116 packets/slot.

In the FH-CDMA model there is only one solution to the EPA non-linear equation whatever the number of available frequency channels G to each user. Consequently, this suggests that FH-CDMA systems are more stable than DS-CDMA systems.

Fig.(3.16) shows the throughput performance of DS-CDMA systems using coherent data modulation along with spread spectrum modulation. It can be seen from this figure that the BPSK systems outperform the QPSK systems, and asynchronous types outperform the synchronous ones in both BPSK and QPSK systems. In the BPSK case the capacity or the maximum throughput occurs at different values of retransmission probability p for synchronous and asynchronous systems; 0.12 for asynchronous and 0.07 for synchronous.

Fig.(3.17) shows the throughput performance against retransmission probability for the non-coherent systems discussed above. It can be seen from this figure that the asynchronous systems are better than synchronous ones in both of the considered types of modulation, DPSK and FSK. Systems with DPSK modulation have higher throughput than FSK systems.

Fig.(3.18) shows the throughput performance when the background or the thermal noise is not neglected. The maximum achieved throughput (capacity) is down from 16 packets/slot for $2E_b/N_0=\infty$ to 14.8, 12.6 and 6.4 packets/slot for $2E_b/N_0$ is 20dB, 15dB and 10dB, respectively. An appropriate value of retransmission probability should therefore be chosen according to the value of the thermal noise.

3.7 Summary

In this chapter, a Markovian model in the form of a discrete-time queueing network has been developed to evaluate the performance of random access CDMA systems and the Equilibrium Point Analysis (EPA) method has been applied to solve the model.

The EPA method is an approximation technique that is applied to the steady state, and it is assumes that the system is always at the equilibrium point. Before applying the EPA technique to the analytical model, this model has been transformed into a stochastically equivalent one that turns out to be much easier to analyse than the original model.

In terms of complexity, the EPA method needs just a single fixed point equation to be solved, irrespective of the number of users or other parameters.

Results of the model have been validated against discrete event simulation results. The model results show good match with the simulation for zero channel delay systems. There is slight disagreement between the model and the simulation in case of non-zero channel delay systems, the model is underestimating the throughput and overestimating the mean packet delay.

The EPA method determines the unstable behaviour of random access CDMA systems, and indicates how this instability can be avoided by choosing appropriate values of retransmission probability and processing gain.

Throughput performance of random access CDMA with different schemes of data modulation have been presented. It has been shown that systems with BPSK modulation have better performance than QPSK systems, and asynchronous types outperform the synchronous ones. Non-coherent modulation, systems with DPSK modulation have higher throughput than systems with FSK modulation.

The effect of the background or the thermal noise on the performance has also been presented.

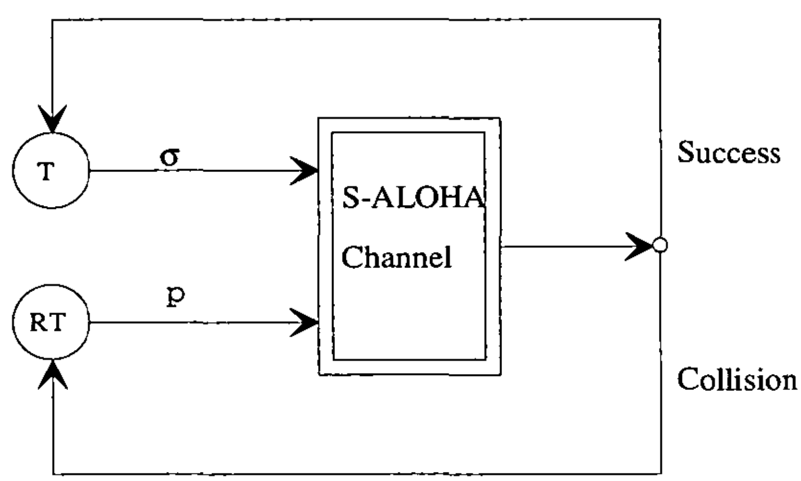


Fig.(3.1) Discrete-time queueing network model for Slotted ALOHA channel.

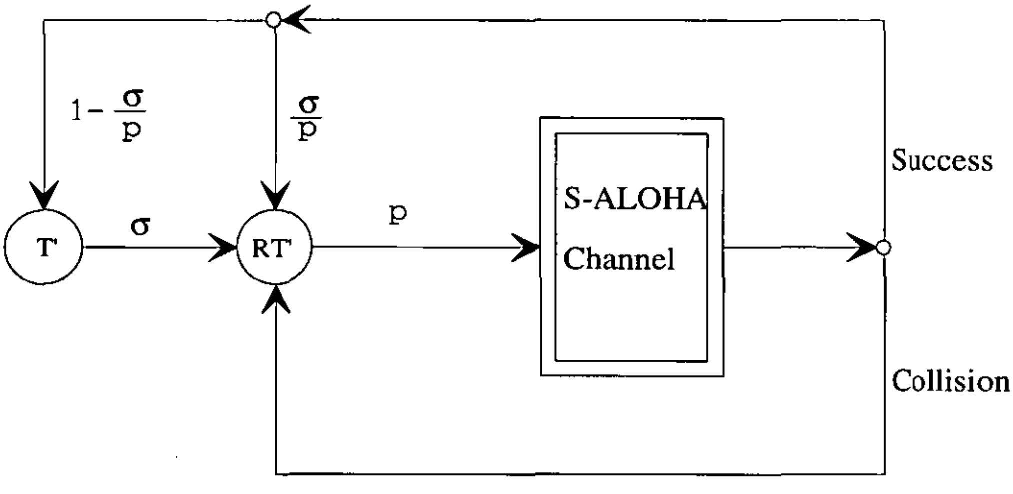


Fig.(3.2) Modified discrete-time queueing network model for S-ALOHA (σ<=p), zero channel delay.

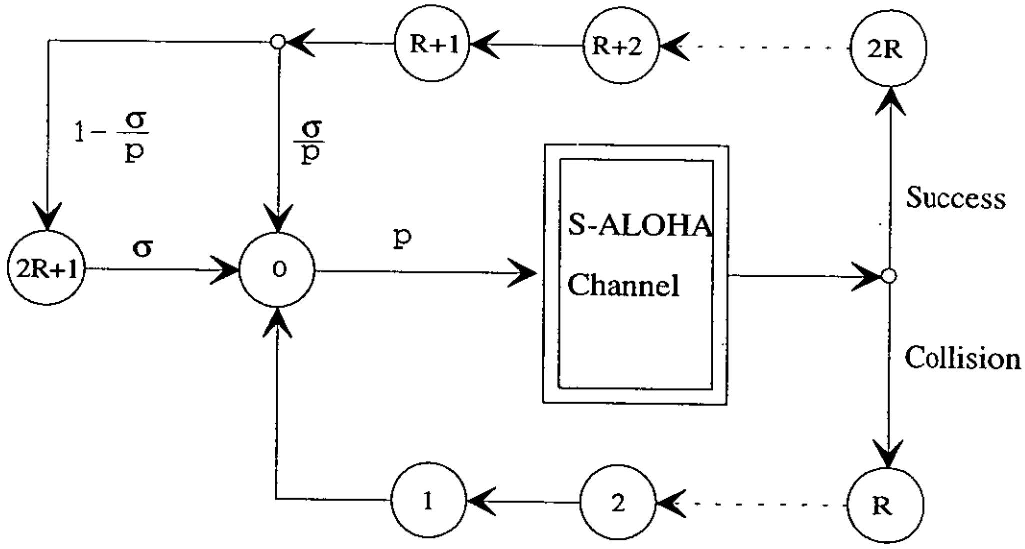


Fig.(3.3) Modified discrete-time queueing network model for S-ALOHA ($\sigma<=p$), R slots channel delay.

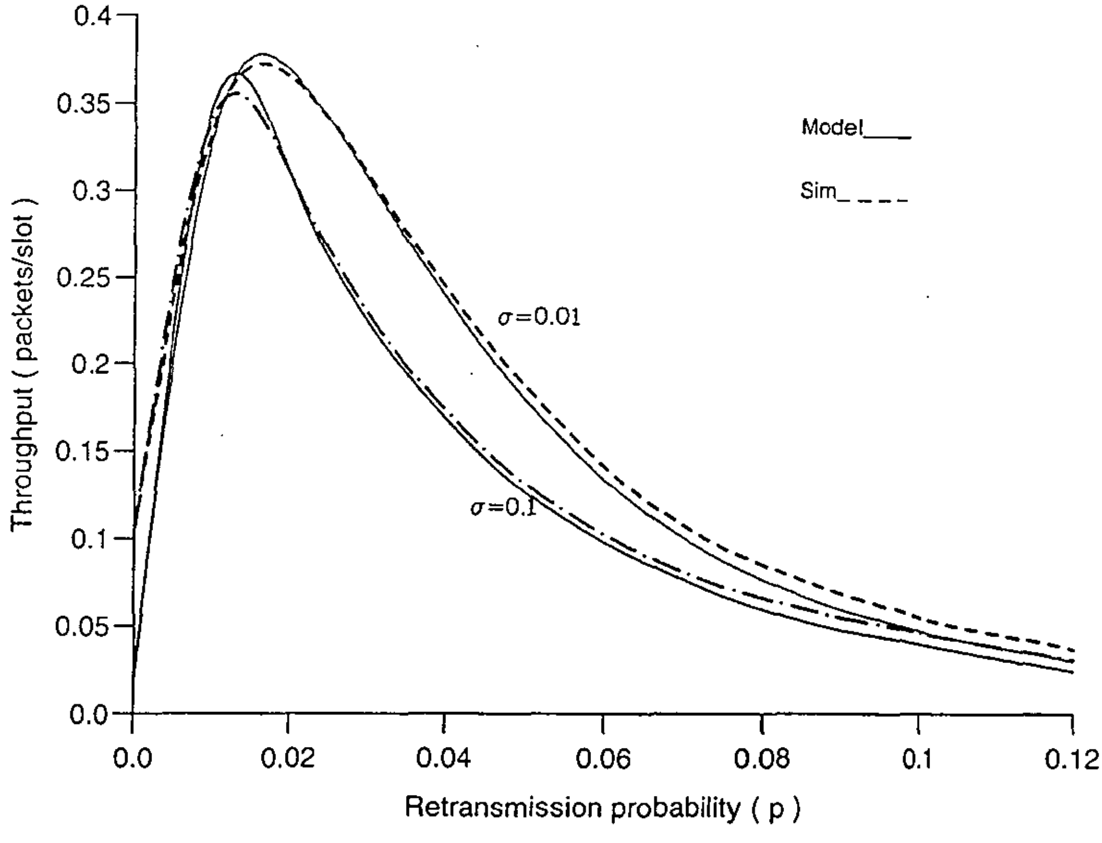


Fig.(3.4) S-ALOHA model & simulation throughput comparison, N=100, R=12 slots.

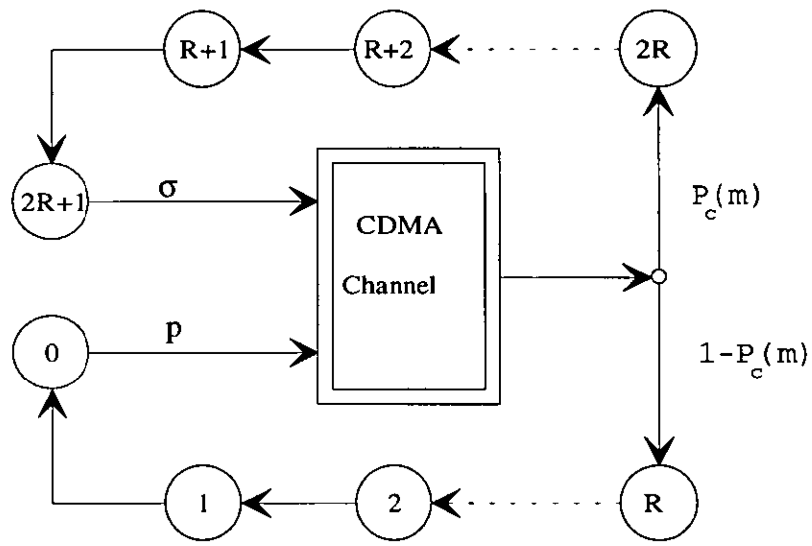


Fig.(3.5) Discrete-time queueing network model for random access CDMA.

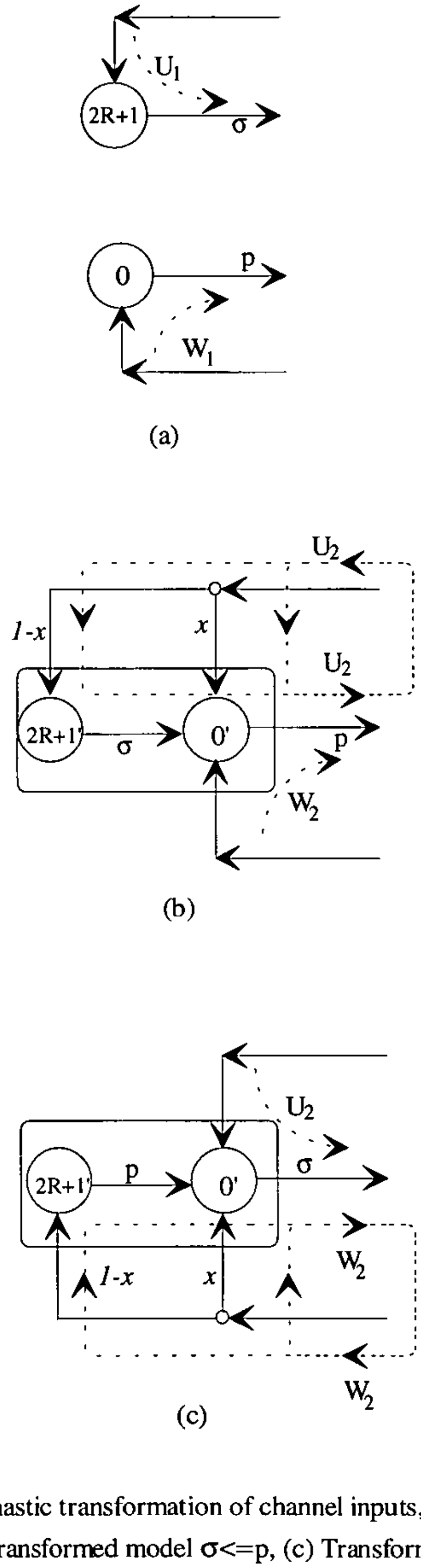


Fig.(3.6) Stochastic transformation of channel inputs, (a) Original model, (b) Transformed model $\sigma <= p$, (c) Transformed model $\sigma > p$.

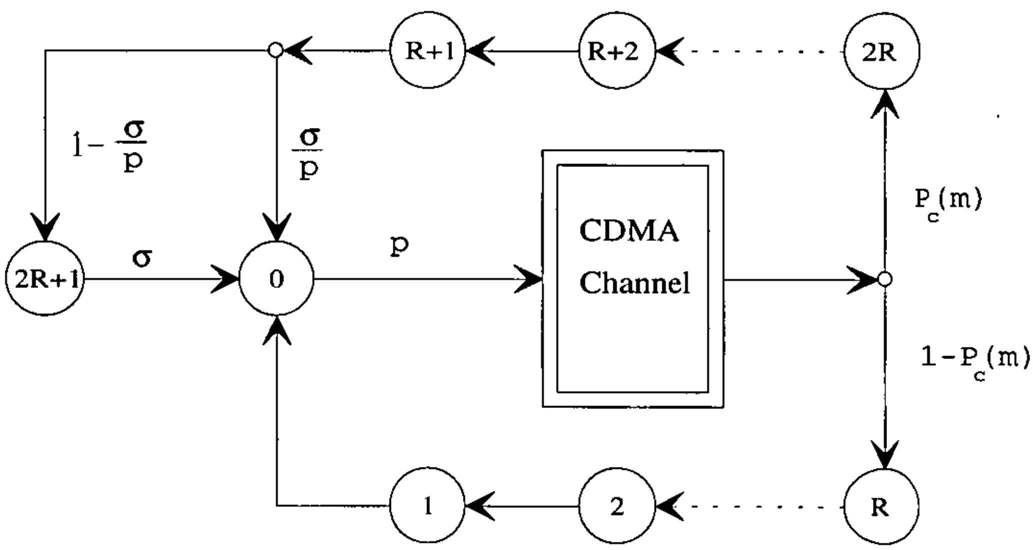


Fig.(3.7) Modified Discrete-time queueing network model for random access CDMA ($\sigma<=p$).

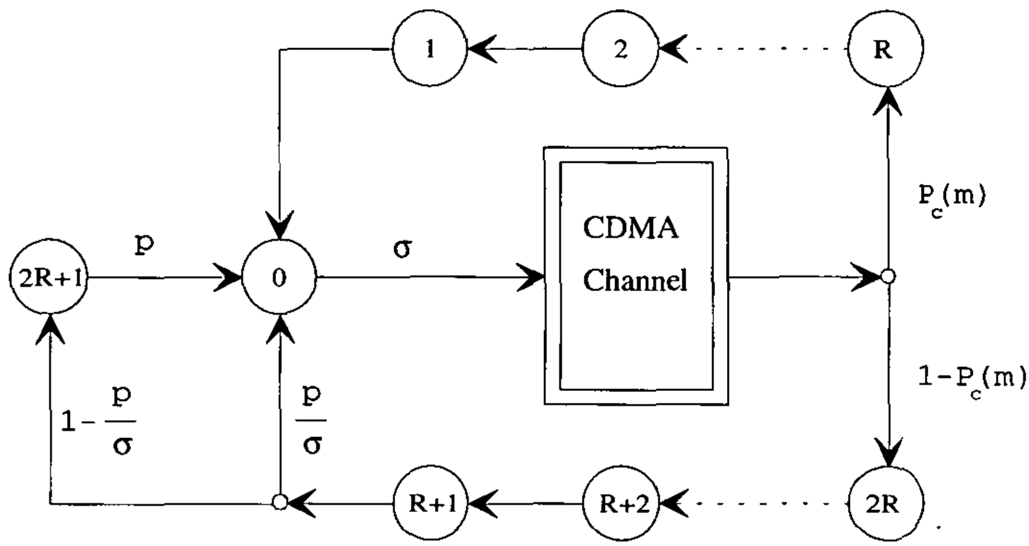
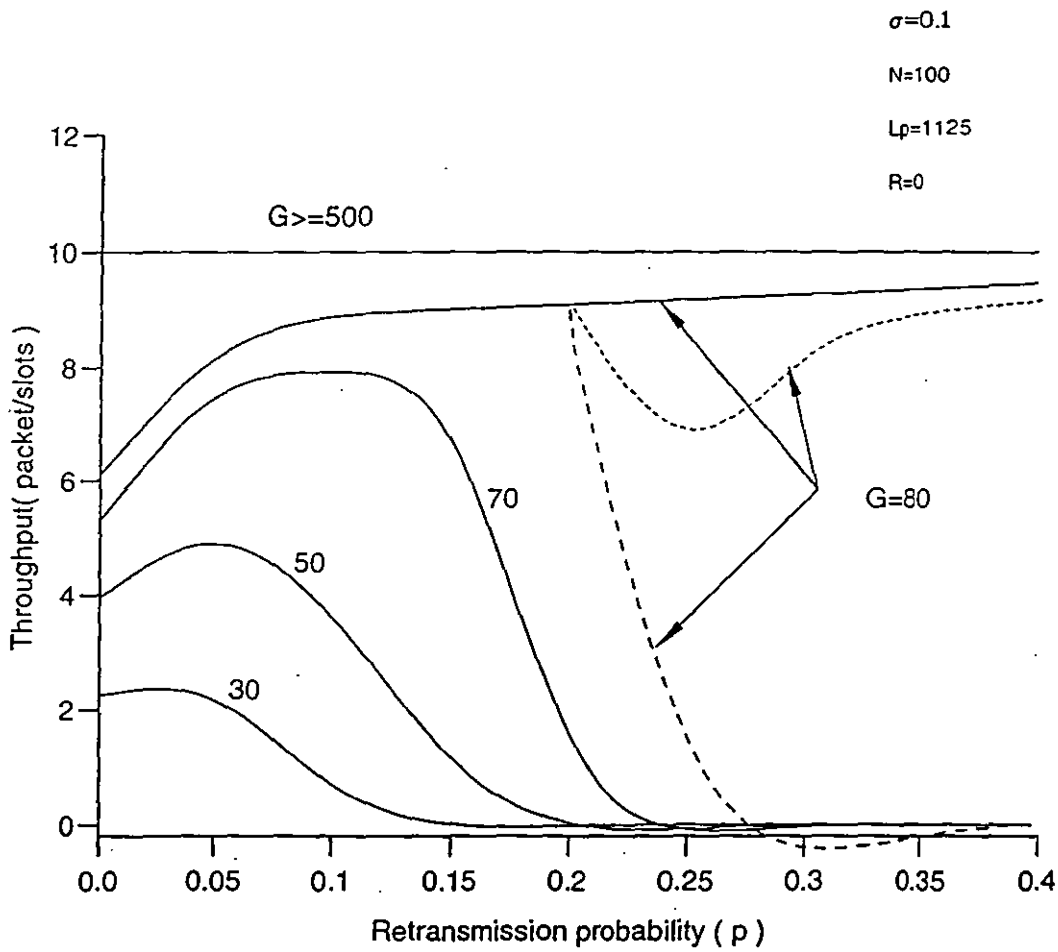


Fig.(3.8) Modified Discrete-time queueing network model for random access CDMA(σ>p).



Fig(3.9) DS-CDMA model for different values of G.

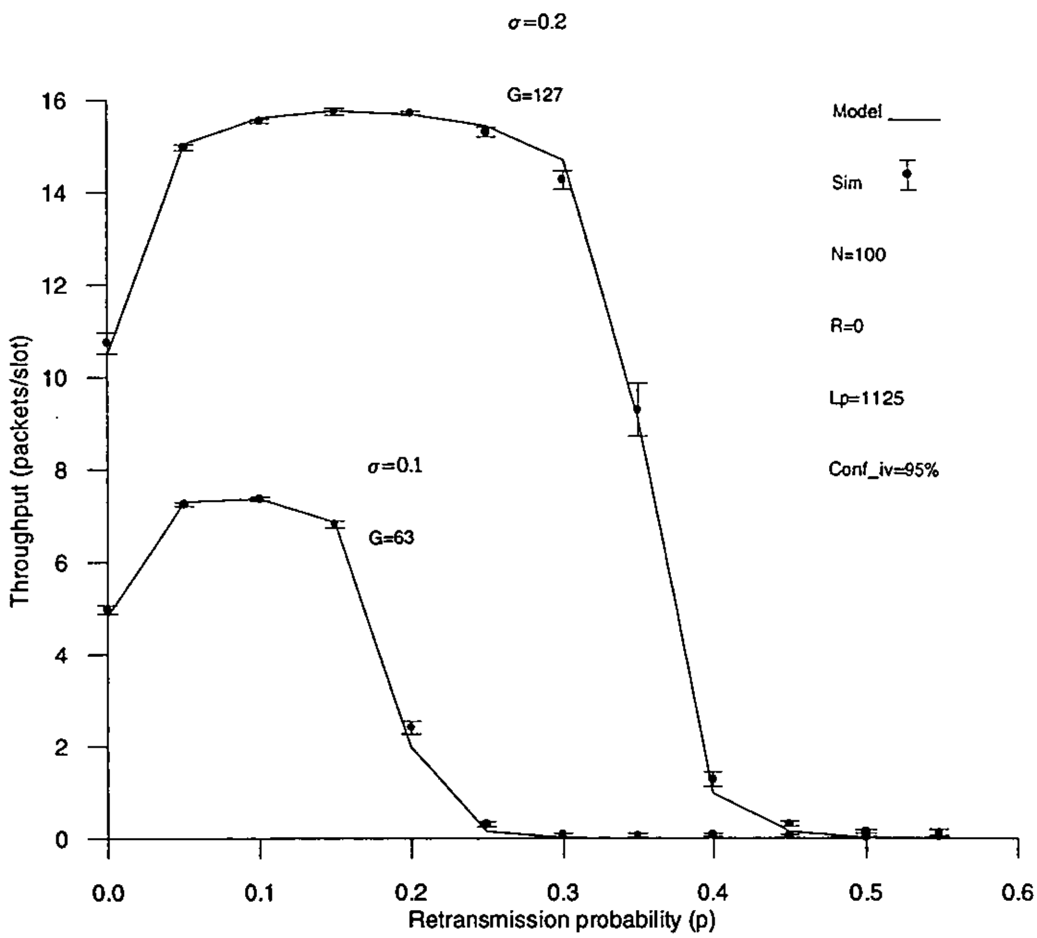


Fig.(3.10) DS-CDMA model & simulation comparison.

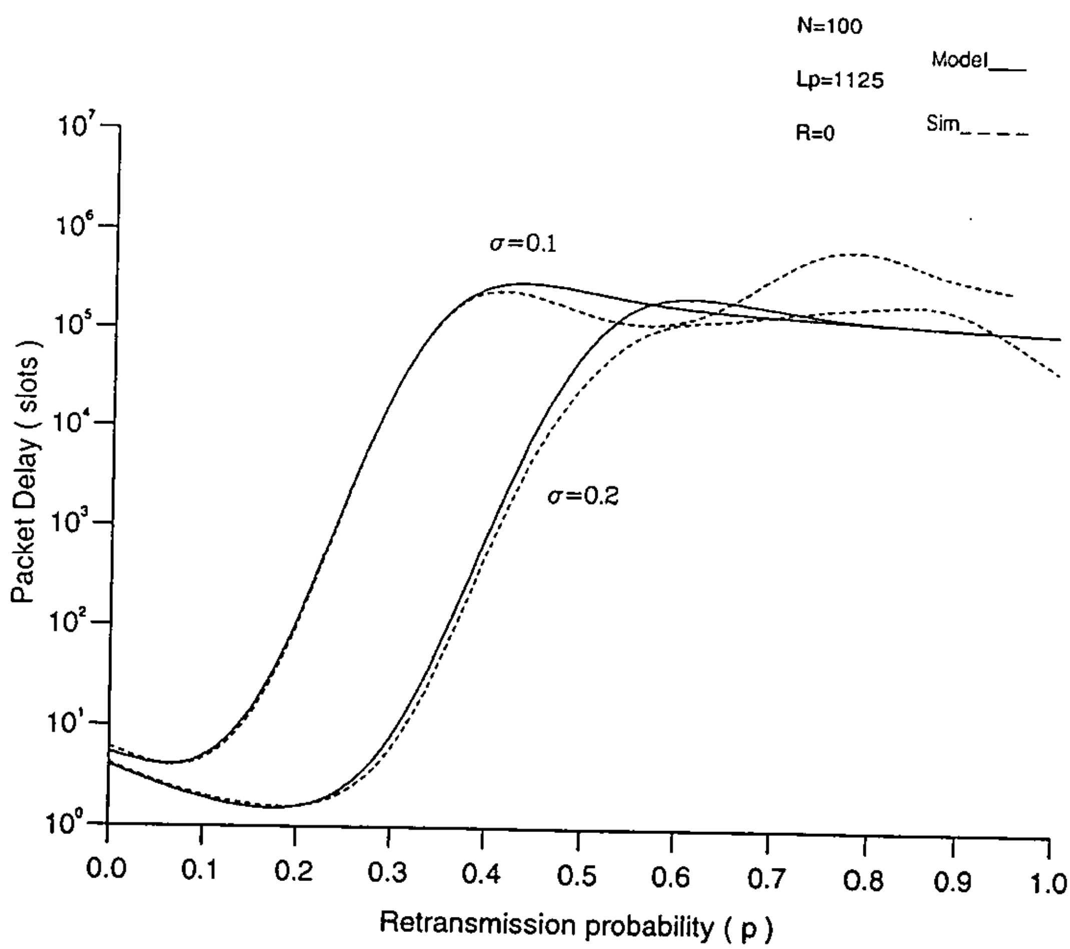


Fig.(3.11) DS-CDMA model & simulation mean packet delay comparison.

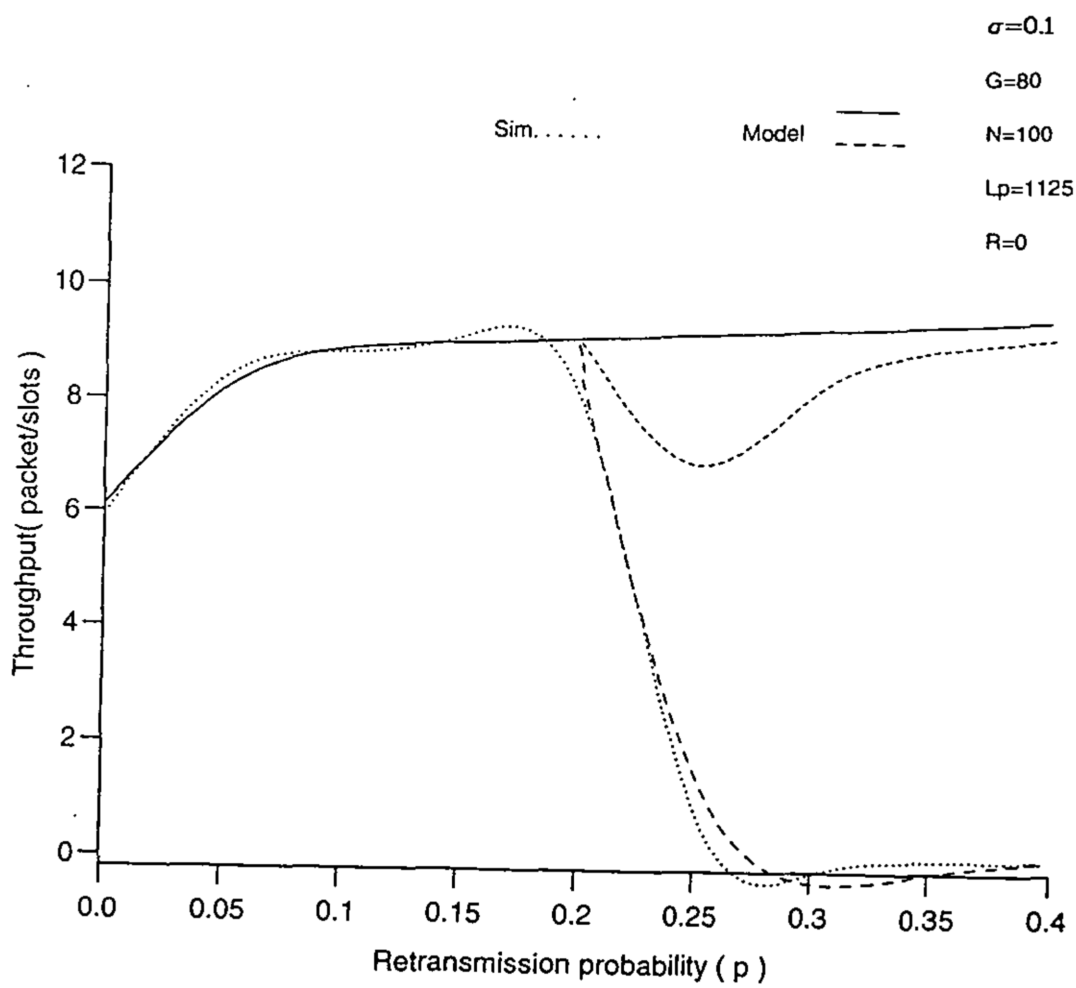


Fig.(3.12) DS-CDMA model & simulation throughput comparison (unstable case)

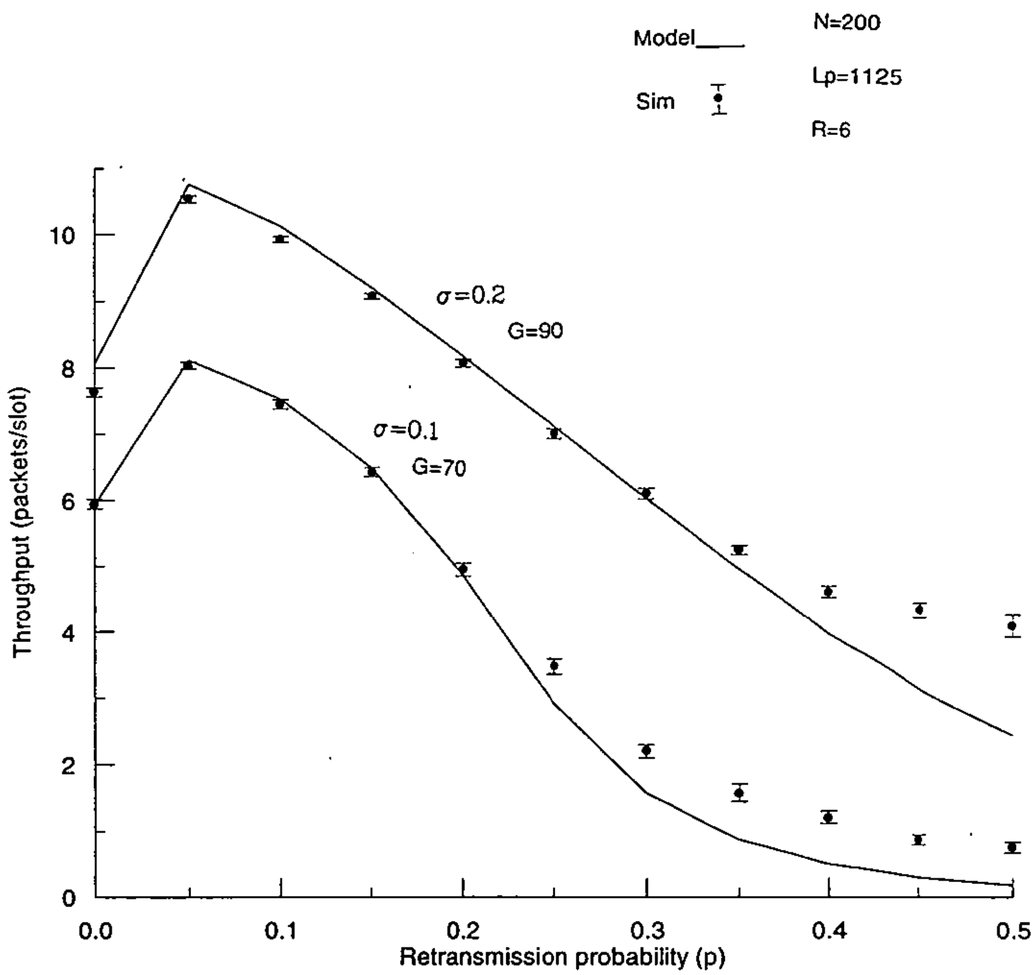


Fig.(3.13) DS-CDMA model & simulation throughput comparison, R=6 slots.

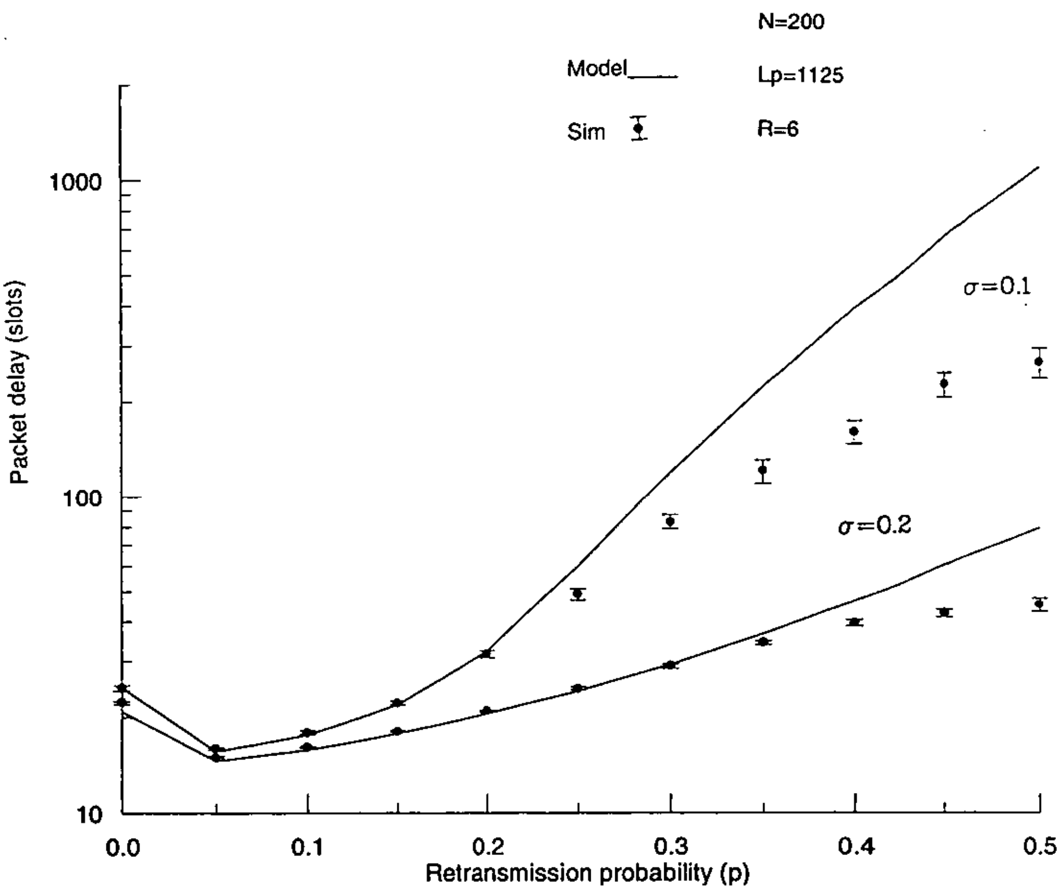
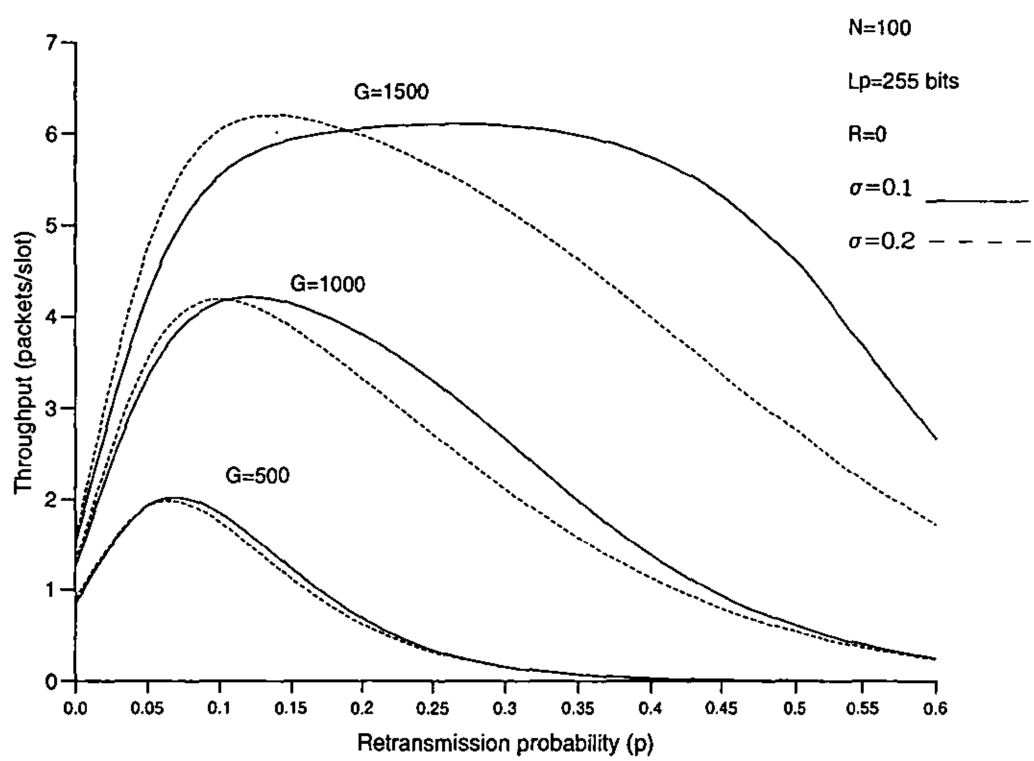
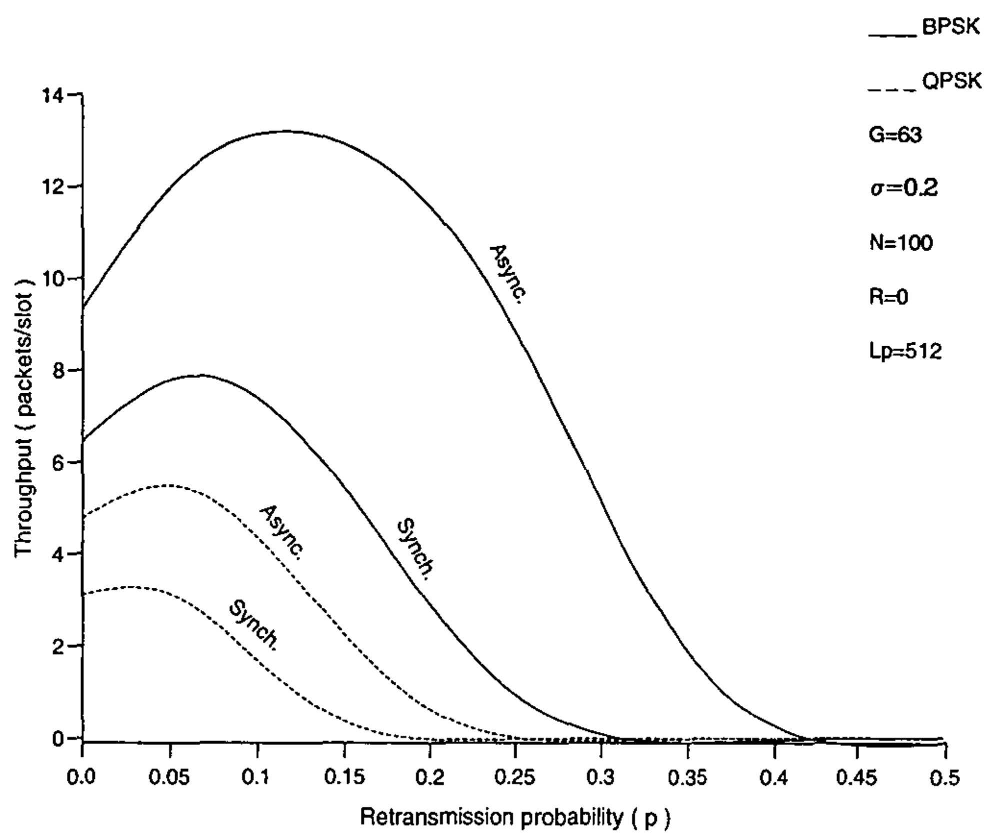


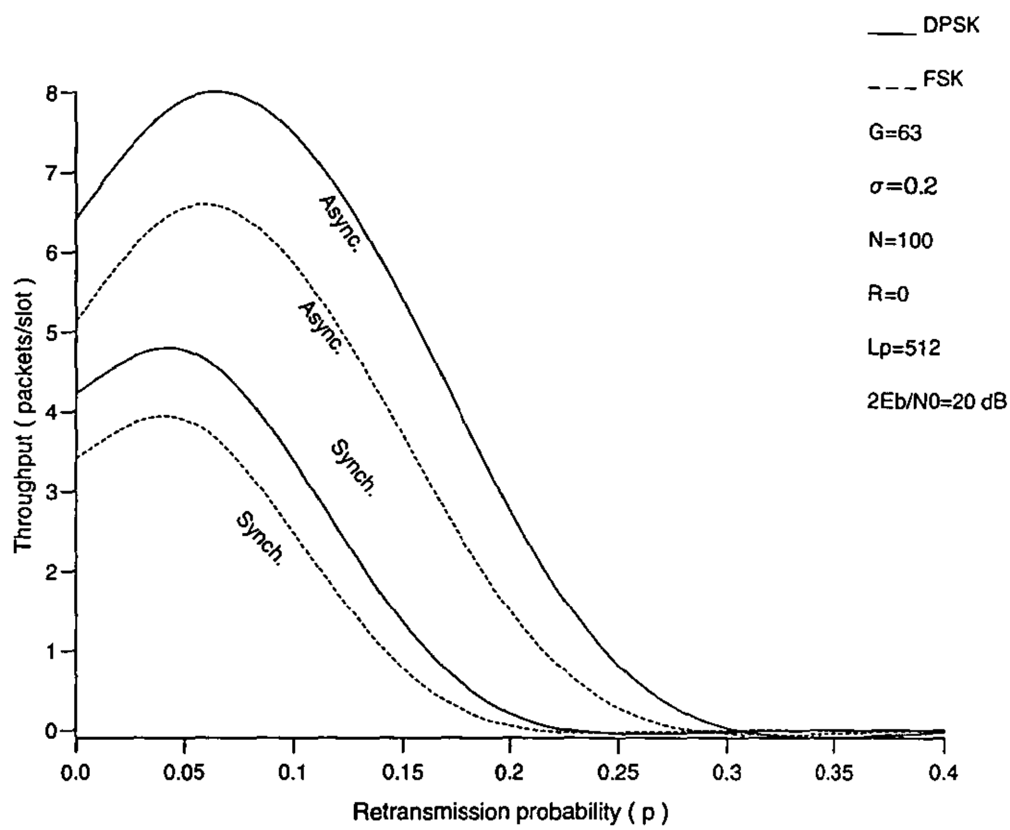
Fig.(3.14) DS-CDMA model & simulation packet delaycomparison, R=6 slots.



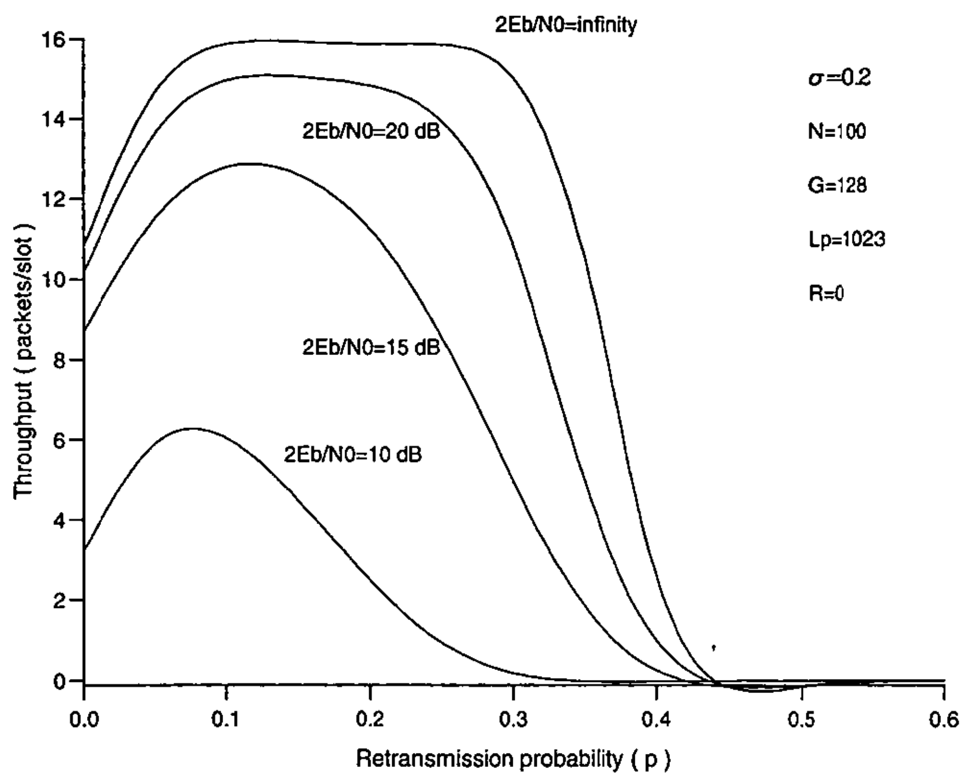
Fig(3.15) Throughput performance of SFH-CDMA.



Fig(3.16) Throughput performance of DS-CDMA with coherent modulation.



Fig(3.17) Throughput performance of DS-CDMA with non-coherent modulation.



Fig(3.18) Throughput performance of DS-CDMA with thermal noise.

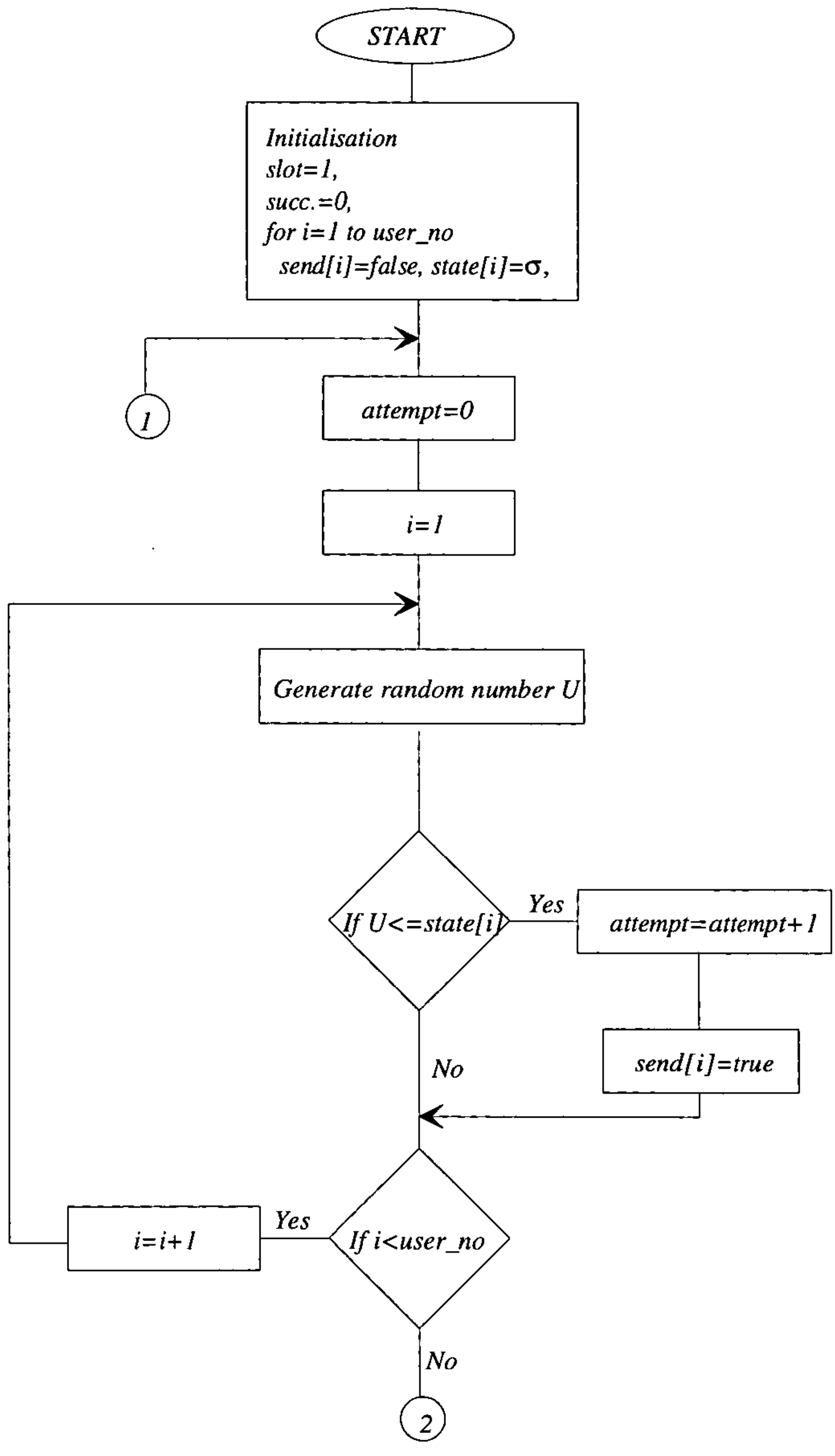


Fig.(3.19) Discrete event simulation flowchart of random access CDMA system.

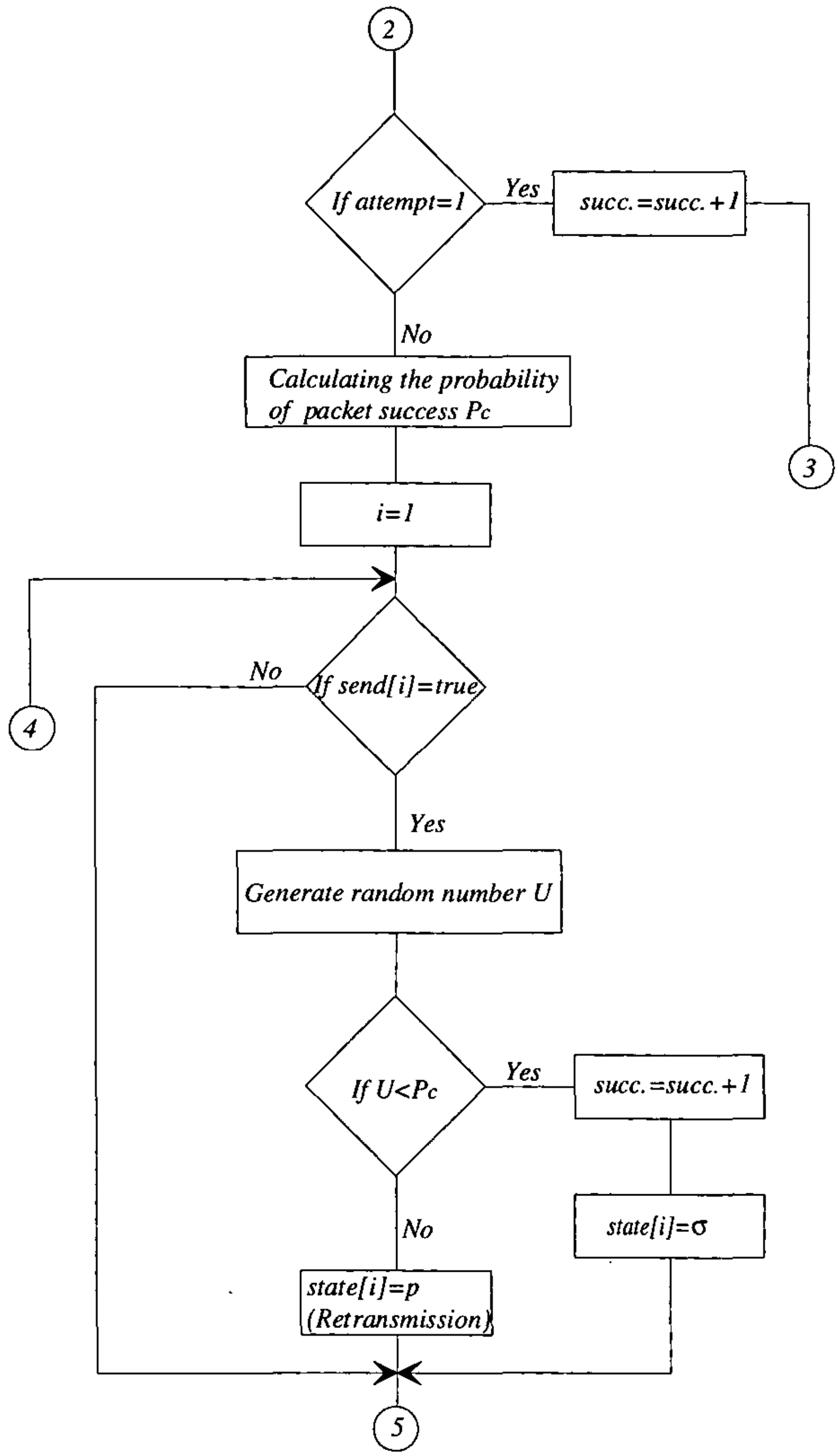


Fig.(3.19) Continued.

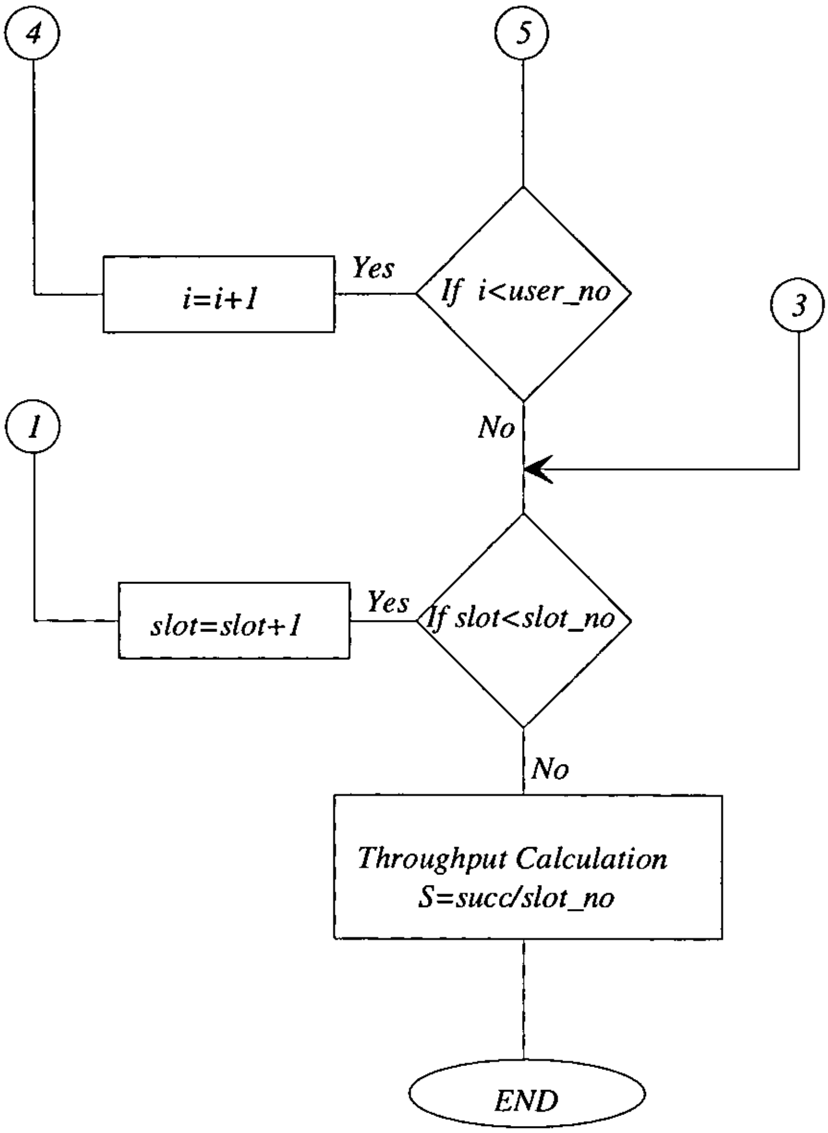


Fig.(3.19) Continued.

CHAPTER 4

Random Access DS-CDMA with Error Correction Coding

4.1 Introduction

The objective of this chapter is to modify the random access DS-CDMA model considered previously to accommodate error correction codes, then examine how error correction coding can affect the performance of the random access DS-CDMA systems in terms of the throughput and the packet delay.

Block and convolution error correction codes are considered with emphasis on block coding. Two schemes of block codes are applied, BCH codes and Reed-Solomon codes concatenated with binary Hamming codes.

The main task here is to find the probability of correct packet reception when error correction coding is applied to the random access DS-CDMA systems. This probability is then used in the model, which can be solved as before using EPA, to obtain the throughput and the mean packet delay.

4.2 DS-CDMA with Block Error Correction Coding

4.2.1 BCH codes

BCH (Bose-Chaudhuri-Hocquenghem) codes are a large class of cyclic codes for correcting multiple errors. These codes cater for a large selection of block lengths and code rates [Pete 72].

BCH codes can be constructed with parameters:

$$
\begin{aligned}
&n = 2^r - 1 \\
&n - k \leq rt \\
&d_{\min} = 2t + 1 \\
&R_c = \frac{k}{n}
\end{aligned}
\tag{4.1}
$$

where

n is the block length,

k is the number of information bits,

t is the number of correctable errors in a block of n bits,

r ($r \geq 3$) is an arbitrary positive integer,

d_{min} is the minimum distance of the (n,k) code, and

R_c is the code rate.

A table of possible BCH codes of block length of 7 to 1023 bits can be found in Appendix B.

BCH codes have been suggested as channel coding for random access CDMA systems in [Geor 90], [Owen 89]. In general, performance improvement is achieved by applying BCH coding to DS-CDMA systems, and in this chapter this improvement is investigated in detail. It has been found that the expected performance improvement due to channel coding does not always materialise, and this improvement depends on the network parameters, such as retransmission probability and the packet length, and as well as on the BCH code parameters such as the code length and the code rate.

Optimum codes which give the highest throughput (capacity), and the lowest packet delay are determined precisely.

4.2.1.1 Probability of correct packet reception for DS-CDMA with BCH codes

In order to determine the probability of correct packet reception, the probability of bit error stated in the previous chapter has to be modified to accommodate channel coding. This probability of bit error for DS-CDMA with block error correction codes and assuming independent bit errors can be expressed as [Proa 95]

$$p_e = \frac{1}{2} erfc\left[\sqrt{\frac{G}{m-1}.\frac{k}{n}}\right] \tag{4.2}$$

where

G is the processing gain or the bandwidth expansion of the DS-CDMA.

m is the number of simultaneous users attempting transmission in a given slot.

A full derivation of Equation (4.2) can be found in Appendix C.

A packet with error correction coding may consist of one or more code words, and this depends on the packet length L_p and the code length n. In general, multiple code word packets will be assumed. The probability of a code word error can be written in the form of an upper bound as [Proa 95]

$$\begin{aligned} P_{ew} &\leq \sum_{r=t+1}^{n} \binom{n}{r} p_e^r \left(1-p_e\right)^{n-r} \\ &\leq 1-\sum_{r=0}^{t} \binom{n}{r} p_e^r \left(1-p_e\right)^{n-r} \end{aligned} \tag{4.3}$$

where p_e is the probability of bit error stated in Equation (4.2), and n is the code length in bits.

For a transmitted packet to be correctly received, all code words which form the packet must have at most t errors each, which can thus be corrected in the decoder. When hard decision decoding is used, the probability of a correct code word P_{cw} can be expressed as

$$P_{cw} = 1 - P_{ew} = \sum_{r=0}^{t} \binom{n}{r} p_e^r \left(1-p_e\right)^{n-r} \tag{4.4}$$

If the packets are of length L_p bits, then assuming L_p is a multiple of n, the transmitted packets will consist of $\frac{L_p}{n}$ code words.

Finally, the probability of correct packet reception can be expressed as

$$
\begin{aligned}
P_c(m) &= \left[1 - P_{ew}\right]^{\frac{L_p}{n}} \\
&= \left[\sum_{r=0}^{t} \binom{n}{r} p_e^r \left(1 - p_e\right)^{n-r}\right]^{\frac{L_p}{n}}
\end{aligned}
\tag{4.5}
$$

Equation (4.5) is then used in the analytical model developed in chapter 3 to determine the performance of DS-CDMA systems in the presence of BCH coding, and to compare this with uncoded systems.

4.2.2 Concatenated codes

Concatenated coding is a practical technique for implementing a code with very long block length and a large error correction capability. This is accomplished by using multiple levels of coding. The most common approach uses two levels of coding as outer and inner codes. Reed-Solomon codes are commonly used as outer codes; however, many different codes have been suggested for use as the inner codes, like short block codes, orthogonal codes and convolutional codes [Clar 81].

In this section, the inner code is chosen to be a Hamming binary block code *H(n,k)* which is a short code, while the outer code is chosen to be Reed-Solomon code *RS(N,K)*.

Code words of a concatenated code are formed by subdividing a block of *kK* information bits into *K* groups, called symbols, where each symbol consist of *k* bits. The *K* *k*-bit symbols are encoded into *N* *k*-bit symbols by the outer encoder. The inner encoder takes each *k*-bit symbol and encodes it into a binary block code of length *n*. The result is a concatenated block code having block length of *Nn* bits and containing *Kk* information bits [Proa 95].

The main advantage of concatenated codes is their ability to correct burst errors, which is a very common requirement in data communications.

Concatenated codes have been suggested for use in CDMA systems by [Rahm 92] and [Cide 94].

4.2.2.1 Probability of correct packet reception with concatenated codes

Let *H(n,k)* be a Hamming binary code which is used as the inner code, and let this be concatenated with a Reed-Solomon code *RS(N,K)*, which is used as the outer code, the two codes thus forming a concatenated code. The code rate of the concatenated code, assuming independent bit errors, can be written as [Proa 95]

$$R_c = \frac{kK}{nN} \tag{4.6}$$

The probability of bit error is

$$p_e = \frac{1}{2} erfc\left[\sqrt{\frac{G}{m-1} \cdot \frac{kK}{nN}}\right] \tag{4.7}$$

The probability of a correct code word of the inner code can be written as

$$P_{IC} = \sum_{x=0}^{t} \binom{n}{x} p_e^x \left(1-p_e\right)^{n-x} \tag{4.8}$$

where t is the error correction capability of the inner code.

RS(N,K) code is used as the outer code; it has the ability to correct up to T symbols out of N symbols of the outer code, where T can be expressed as

$$T = \left[\frac{N-k}{2}\right] \tag{4.9}$$

The probability of receiving correctly N symbols of the outer code (one code word of the concatenated code) can be written as

$$P_{OC} = \sum_{r=0}^{T} \binom{N}{r} P_{IC}^{N-r} \left(1 - P_{IC}\right)^{N} \tag{4.10}$$

Finally, the probability of packet success, with packets of length, L_p bits, each of which consists of $\frac{L_p}{Nn}$ code words of the concatenated code, can be expressed as

$$P_c(m) = \left[P_{OC}\right]^{\frac{L_p}{Nn}} = \left[\sum_{r=0}^{T} \binom{N}{r} P_{IC}^{N-r} (1 - P_{IC})^{N}\right]^{\frac{L_p}{Nn}} \tag{4.11}$$

where *(N.n)* is the length of a code word of the constructed concatenated code. For example, when a Reed-Solomon $RS(15,7)$ code is used as the outer code and a Hamming $H(7,4)$ code is used as the inner code, the resulting concatenated code can correct any error pattern that affects any 4 bytes of seven bits, where each has more than a single error (up to seven errors). The Hamming inner code can correct any byte which has only a single error.

4.3 DS-CDMA with Convolutional Coding

The information sequence in block coding is divided into blocks which are encoded independently. This is not done with convolutional codes, but instead, redundant symbols are generated as a function of a span of preceding information symbols as well as the current one. The transmitted sequence can be considered as a single semi-infinite code word. Convolutional codes are widely used, mainly because of their simplicity and the relatively large coding gains that can be achieved [Clar 8].

A convolutional code is generated by passing the information sequence through a linear shift register. This shift register consists of *CL* *k*-bit stages and *n* linear algebraic function generators. The number of output bits for each *k*-bit input sequence is *n* bits. The parameter *CL* is called the constraint length of the convolutional codes [Proa 95].

When convolutional codes are used as channel coding for random access DS-CDMA, the approximation of independent bit errors assumed in the case of block coding is no longer accurate. That is because errors at the output of a Viterbi decoder are dependent even if the errors at the decoder input may be independent. Furthermore, in asynchronous DS-CDMA systems the bit errors on the channel (at the input of the decoder) are dependent due to the different delay and phases of the interfering signals [Trab 93].

In asynchronous systems the delays and phases of the interfering signals compared with the desired signal are not zero. An improved Gaussian model for the multiple access interference (MAI) is used to take into account the dependency in bit errors. In the following analysis both independent and dependent bit errors at the input of the Viterbi decoder are considered. Note that in both cases the bit errors at the output of the decoder are dependent due to the use of convolutional codes.

Convolutional codes have been suggested for use in CDMA systems by Woerner and Stark in [Woer 91] and [Woer 92], and it is interesting to note that convolutional codes have been used for channel coding in practical DS-CDMA systems in North America in the Qualcomm system [EIA 93], [Vite 95].

4.3.1 Probability of packet success in DS-CDMA with Convolutional coding

4.3.1.1 Independent bit errors

Assuming the multiple access interference (MAI) is a Gaussian process with variance ψ, the probability of bit error for asynchronous DS-CDMA systems is given by [Morr 89] as

$$p_e = \frac{1}{2} erfc\left[\frac{G}{\sqrt{2\psi}}\right] \tag{4.12}$$

In the case of independent bit errors ψ is given by

$$\psi = \frac{(m-1)G}{3} \tag{4.13}$$

then the probability of bit errors is

$$p_e = \frac{1}{2} erfc\left[\sqrt{\frac{3G}{2(m-1)}}\right] \tag{4.14}$$

where G is the processing gain and is also the PN sequence period.

In the analysis of convolutional codes, it has been assumed that bit errors at the output of a Viterbi decoder cluster within a short segment of decoded data. These clusters are knows as error events. By assuming independent error events, the probability of packet success can be approximated by [Purs 87b]

$$P_c(m) \cong \left(1 - P_u(p_e)\right)^L \tag{4.15}$$

where L is the number of information bits in a packet, and $P_u(p_e)$ is the union bound on the first error event probability which is defined in [Clar 81] as the sum of the error probabilities for all possible paths which merge with the correct path at a specific point in a Trellis diagram of the Viterbi decoder. Van de Meerberg [Van 74] has shown that $P_u(p_e)$ may be upper bounded as

$$P_u(p_e) \le \binom{2n_0 - 1}{n_0} 2^{-2n_0-1} \{[T(D) + T(-D)] + D[T(D) - T(-D)]\}_{D=2\sqrt{p_e}} \quad (4.16)$$

where $T(D)$ is the transfer function of the relevant convolutional code, and n_0 can be written as a function of the code's free distance, d_{free}, as

$$\begin{aligned} n_0 &= \frac{d_{free}}{2} && \text{if } d_{free} \text{ is even} \\ n_0 &= \frac{d_{free}+1}{2} && \text{if } d_{free} \text{ is odd} \end{aligned} \quad (4.17)$$

4.3.1.2 Dependent bit errors

To accommodate the assumption of dependent bit errors, an improved Gaussian model for the multiple access interference (MAI) derived by Morrow and Lehnert in [Morr 89] is used, where the variance of the MAI (ψ) is a function of the delays and phases of the interfering signals. ψ can thus be treated as a random variable which depends on the delays and the phases of the interfering signals. After a long derivation, Trabelsi in [Trab 93] has shown that the probability of packet success can be expressed as

$$P_c(m) = (1 - \frac{1}{\zeta^2})[1 - P_u(p_{e1})]^L + \frac{1}{2\zeta^2}[1 - P_u(p_{e2})]^L + \frac{1}{2\zeta^2}[1 - P_u(p_{e3})]^L \quad (4.18)$$

where p_{e1}, p_{e2} and p_{e3} are given by

$$p_{e1} = \frac{1}{2} erfc\left[\sqrt{\frac{3G}{2(m-1)}}\right] \tag{4.19}$$

$$p_{e2} = \frac{1}{2} erfc\left[\sqrt{\frac{G^2}{2((m-1)(G/3)+\zeta\sigma_{\psi})}}\right] \tag{4.20}$$

$$p_{e3} = \frac{1}{2} erfc\left[\sqrt{\frac{G^2}{2((m-1)(G/3)-\zeta\sigma_{\psi})}}\right] \tag{4.21}$$

where ζ is a constant, and σ_{ψ} is the variance of ψ and it is given by

$$\sigma_{\psi} = \left(\frac{m-1}{360}\left[23G^2 - (2-10m)G + (2-10m)\right]\right)^{\frac{1}{2}} \tag{4.22}$$

$P_u(p_{e1}), P_u(p_{e2})$ and $P_u(p_{e3})$ are determined by equation (4.16).

4.4 Numerical Results

Fig.(4.1) represents a comparison between the uncoded and coded DS-CDMA systems for block length n=31 bits and packet length L_p=1116 bits, so each packet consists of 36 code words each of 31 bits in length. Three codes are considered here, the (31,26), (31,16) and (31,11) which can correct up to 1, 3 and 5 errors, respectively in each block. Throughput is normalised with respect to the number of information bits in a block (i.e. excluding the coding or the redundant bits). This is to make a fair comparison between the uncoded and coded systems. This graph shows that the (31,26) coded DS-CDMA system outperforms the uncoded system for all values of retransmission probability p. For the (31,16) code the uncoded system is better than the coded one for p up to 0.07, and then the coded system is better for higher values of p. For the (31,11) code the uncoded system is better than the coded system for p up to 0.125, and vice versa beyond that.

The above results show that the normalised throughput depends on two factors: The error correction capability of the used code, and the optimum choice of retransmission probability, which is a controllable parameter [Pras 91], [Rayc 81].

Fig.(4.2) represents the normalised throughput verses the error correction capability t for a code length n=255 bits and packet length L_p=1020 bits, so each packet has 4 code words. The normalised throughput is determined for all possible BCH codes of length 255 bits, and for p=0.1. This plot shows that the normalised throughput of the coded DS-CDMA system is better than the uncoded case for codes for which the number of information bits is in the range 91 to 247 bits, and the error correction capability is less than 26 bits in a block of 255 bits. This result suggests that the best code which has the maximum normalised throughput is the (255,207) code with t=6 which will be considered in more detail precisely later on in this chapter.

Fig.(4.2) examines the normalised throughput verses error correction capability for a single value of retransmission probability, p=0.1. To make the results more general, all possible values of p must be considered. Fig.(4.3) is a 3D graph that represents the relation between the normalised throughput, error correction capability and retransmission probability. This plot shows that the normalised throughput has good values for t=5 to 10 and p around 0.1.

Fig.(4.4) represents the influence of the number of code words in the transmitted packets on the normalised throughput. The BCH (255,179) code with t=10 and R_c=0.702 is used. The improvement gained through the use of coding is about 2.6 dB over the uncoded case when L_p=1020 bits (4 code words in a packet), and about 2 dB for L_p=255 bits (one code word in a packet). On the other hand the maximum normalised throughput (capacity) is found when L_p=255 bits, so the packets of the DS-CDMA system lead to better performance when they consist of one code word rather than multiple code words.

Fig.(4.5) shows the difference between an uncoded and coded random access DS-CDMA systems when using single code word packets, L_p=1023 bits, with code length n=1023 bits. The number of information bits of the four codes used are 1013, 983, 923 and 513, with error correction capabilities t=1, 4, 10 and 57, respectively. It is found that the (1023,1013), (1023,983) and (1023,923) codes give better throughput than the uncoded case for all values of retransmission probability. The (1023,513) code with relatively large error correction capability, t=57, and relatively small code rate, R_c=0.501, compared with the previous three codes gives lower throughput than the uncoded case for retransmission probability less than 0.12.

Fig.(4.6) shows the normalised throughput verses the error correction capability for BCH codes of length n=1023 bits and packet length of only one code word, so L_p=1023 bits. Retransmission probability is set to be 0.1. Because of the large number of possible BCH codes of code length 1023 bits, a number of these codes are used. The maximum normalised throughput occurred between t=15 to t=25. To determine exactly which code is the best, the model is used to assess each possible BCH codes in the range t=15 to t=25. Fig.(4.7) shows that the best code is the (1023,858) code, with t=17 and R_c=0.839.

Fig.(4.8) is a 3D graph corresponding to Fig.(4.6) for retransmission probability p=0.05 to 0.3. This graph shows the best codes (those which have higher throughput) have error correction capability between t=16 and t=19, and the best retransmission probability p lies between 0.05 and 0.1.

To determine the optimum code for approximately fixed packet lengths chosen to be $L_p \cong 1020$ bits, (since this is a reasonable length for many applications), the model program was run for all possible BCH codes of length 1023, 511, 255 and 127 bits. The best code of each set which gives the highest normalised throughput (capacity) and the lowest packet delay is determined (the complete results can be found in Appendix D).

Table (4.1) shows the optimum codes for σ=0.2 and G=60 for 100 users. The first four columns of this table and all subsequent tables represent codes parameters. They are the code length *(n)*, the information field length *(k)*, the error correction capability *(t)* and the code rate *(R_c)*. The last four columns represent the maximum achievable throughput (capacity), the corresponding minimum packet delay, the improvement of coded DS-CDMA systems over the uncoded systems, and finally, the optimum retransmission probability at which the maximum throughput occurs.

n	*k*	*t*	R_c	*max.S*	*min.D*	*improv.(dB)*	*p*
1023	858	17	0.839	13.118	3.623	2.76	0.1
511	421	10	0.824	12.629	3.918	2.59	0.1
255	207	6	0.812	12.035	4.309	2.38	0.1
127	99	4	0.780	11.105	5.005	2.03	0.1

Table (4.1) Optimum codes for σ=0.2, G=60, N=100.

Table (4.2) shows the optimum codes for σ=0.25

n	*k*	*t*	R_c	*max.S*	*min.D*	*improv.(dB)*	*p*
1023	858	17	0.839	13.238	4.554	2.83	0.05
511	421	10	0.824	12.745	4.846	2.66	0.05
255	207	6	0.812	12.152	5.229	2.46	0.05
127	106	3	0.835	11.257	5.883	2.12	0.05

Table (4.2).Optimum codes for σ=0.25, G=60, N=100.

Table(4.3) and Table(4.4) shows the optimum codes for G=30 and 45 respectively.

n	k	t	R_c	$max.S$	$min.D$	$improv.(dB)$	p
1023	848	18	0.829	5.796	13.253	3.50	0.05
511	412	11	0.806	5.592	13.833	3.34	0.05
255	187	9	0.733	5.388	14.560	3.18	0.05
127	99	4	0.780	4.945	16.222	2.81	0.05

Table (4.3) Optimum codes for σ=0.2, G=30, N=100.

n	k	t	R_c	$max.S$	$min.D$	$improv.(dB)$	p
1023	858	17	0.839	9.560	6.460	3.01	0.05
511	421	10	0.824	9.217	6.850	2.85	0.05
255	207	6	0.812	8.801	7.362	2.65	0.05
127	99	4	0.780	8.146	8.276	2.32	0.05

Table (4.4) Optimum codes for σ=0.2, G=45, N=100.

Table (4.5) shows the optimum codes when the number of users using the DS-CDMA system is increased to 1000 users. In order to get a reasonable throughput, the processing gain is increased proportionally with increasing the number of users.

n	k	t	R_c	$max.S$	$min.D$	$improv.(dB)$	p
1023	858	17	0.839	150.02	2.666	3.18	0.1
511	421	10	0.824	142.79	3.003	2.97	0.1
255	207	6	0.812	134.03	3.461	2.70	0.05
127	106	3	0.835	123.03	4.128	2.32	0.05

Table (4.5) Optimum codes for σ=0.2, G=600, N=1000.

It should be noted that the best codes presented in Tables (4.1) through (4.5) are optimum with respect to the other parameters used in the modelled systems.

Fig.(4.9) shows the influence of the processing gain G on the normalised throughput. The BCH (1023,858) code with t=17 and R_c=0.839 is used, because it is the best code of 1023 bits code length, see Table (4.1). When the processing gain is relatively high ($G \geq 300$) this bandwidth expansion results in all packets being correctly received, so there is no need for any retransmission. The throughput of uncoded DS-CDMA reaches the highest possible average value, which is $S = N \cdot \sigma = 100 \times 0.2 = 20$ packets/slot, where N is the number of users in the DS-CDMA systems. Adding coding in this case has no improvement, since it just adds redundancy bits to each transmitted packet. The overall throughput is then normalised with respect to the code rate of the code used, thus the maximum achievable average throughput is $S = N \cdot \sigma \cdot R_c = 16.77$ packets/slot, which is less than the throughput of the uncoded case. If the processing gain is in the region of 65 to 110, the modelled systems show unstable behaviour.

Fig.(4.10) and Fig.(4.11) show good matching between the model and the simulation in terms of the normalised throughput and packet delay. Two optimum codes shown in Table (4.1) are used, the (1023,858) code with t=17, and the (255,207) code with t=6.

Fig.(4.12) shows the performance of DS-CDMA in the presence of concatenated coding. A Hamming $H(7,4)$ code is concatenated with the $RS(15,11)$, $RS(15,7)$, $RS(15,5)$ codes. These Reed-Solomon codes have error correction capabilities 2, 4 and 5, and code rates 0.419, 0.267 and 0.19, respectively. Because of these low code rates, the number of added check bits are very large, so the normalised throughput in all three codes used is lower than for the uncoded case. However, the throughput of coded DS-CDMA may have higher values than the uncoded DS-CDMA beyond certain values of retransmission probability, $p>0.1$ for $RS(15,11)$, $p>0.165$ for $RS(15,7)$, and $p>0.23$ for $RS(15,5)$.

Fig.(4.13) represents a comparison between the uncoded asynchronous DS-CDMA system and convolutional coded systems for both independent and dependent bit errors. Three codes are considered

1. $R_c = \frac{1}{2}, CL = 3, d_{free} = 5,$ and $T(D) = \frac{D^5}{1-2D},$

2. $R_c = \frac{1}{3}, CL = 3, d_{free} = 6,$ and $T(D) = \frac{D^6}{1-2D^2},$

3. $R_c = \frac{1}{2}, CL = 1, d_{free} = 3,$ and $T(D) = \frac{D^3}{1-D}$

This plot shows a good improvement for coded systems over the uncoded one, with the improvement increasing proportionally with the constraint length *CL*. On the other hand increasing the free distance of the codes has the effect of flattening the maximum throughput over a wider range of retransmission probability. The calculated achievable throughput when assuming independent bit errors overestimates the actual throughput for low to moderate values of retransmission probability p, and underestimates it for moderate to high values of p. For example, for convolutional code number 1, the throughput with independent bit errors is higher than the actual throughput for retransmission probability up to 0.17, and lower beyond this value.

4.5 Summary

In this chapter, the performance of random access DS-CDMA systems with error correction coding has been investigated. Two schemes of error correction codes have been considered, block and convolutional codes, with emphasis on BCH block codes.

In the case of BCH codes, it has been shown that there can be a considerable improvement achieved by the use of coded DS-CDMA systems over uncoded systems. Codes of length 1023, 511, 255 and 127 bits have been examined to determine the optimum codes which give the highest throughputs and the lowest packet delays. It has been found that BCH codes of code rates in the range of 0.73 to 0.84 have the best performance. These optimum code rates are relatively insensitive to the block lengths of the relevant codes, to the processing gain of DS-CDMA systems, to the number of users in the modelled networks, and to the new packet generation probability.

Due to the low code rates of the concatenated codes, these codes have not shown good results in terms of the capacity over the uncoded systems. However, these codes can give better results than the uncoded case for certain values of retransmission probability.

For convolutional codes, independent and dependent bit errors have been studied. In the case of independent bit errors, standard Gaussian noise process was used, while an improved Gaussian noise process was used for dependent bit errors. Higher throughputs were again achieved for coded systems over the uncoded systems.

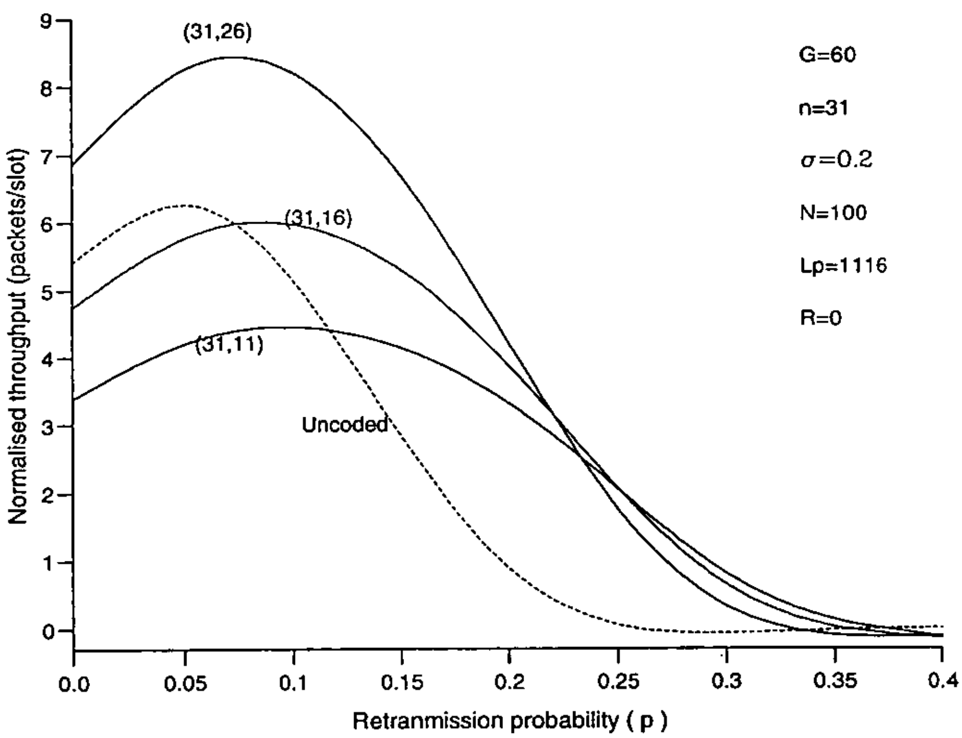


Fig.(4.1) DS-CDMA throughput performance comparison of coded & uncoded systems.

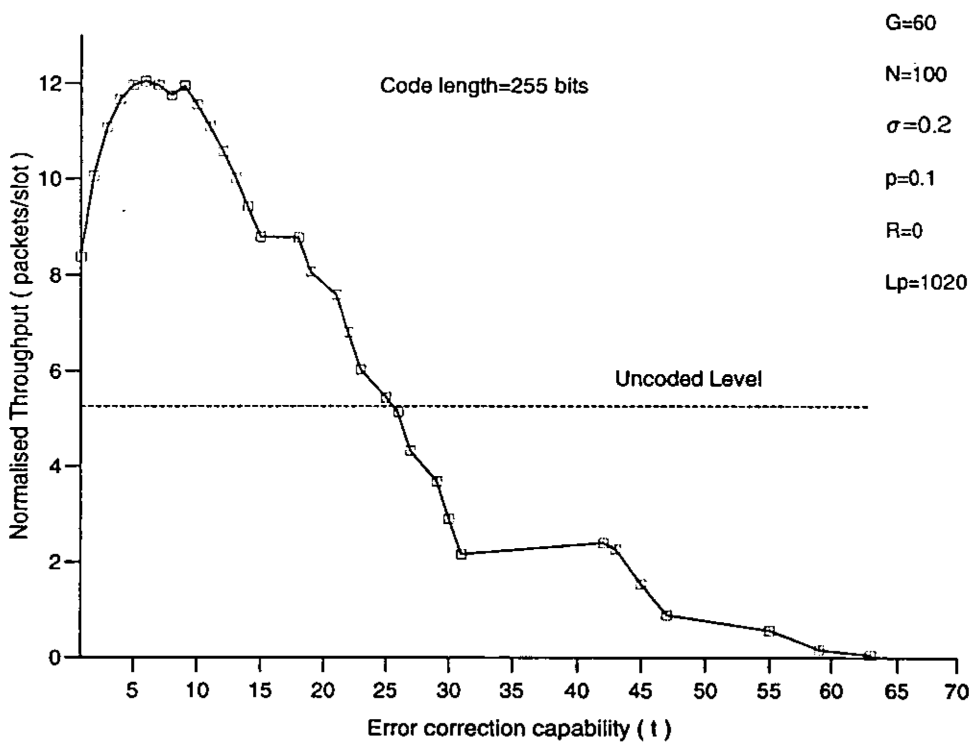
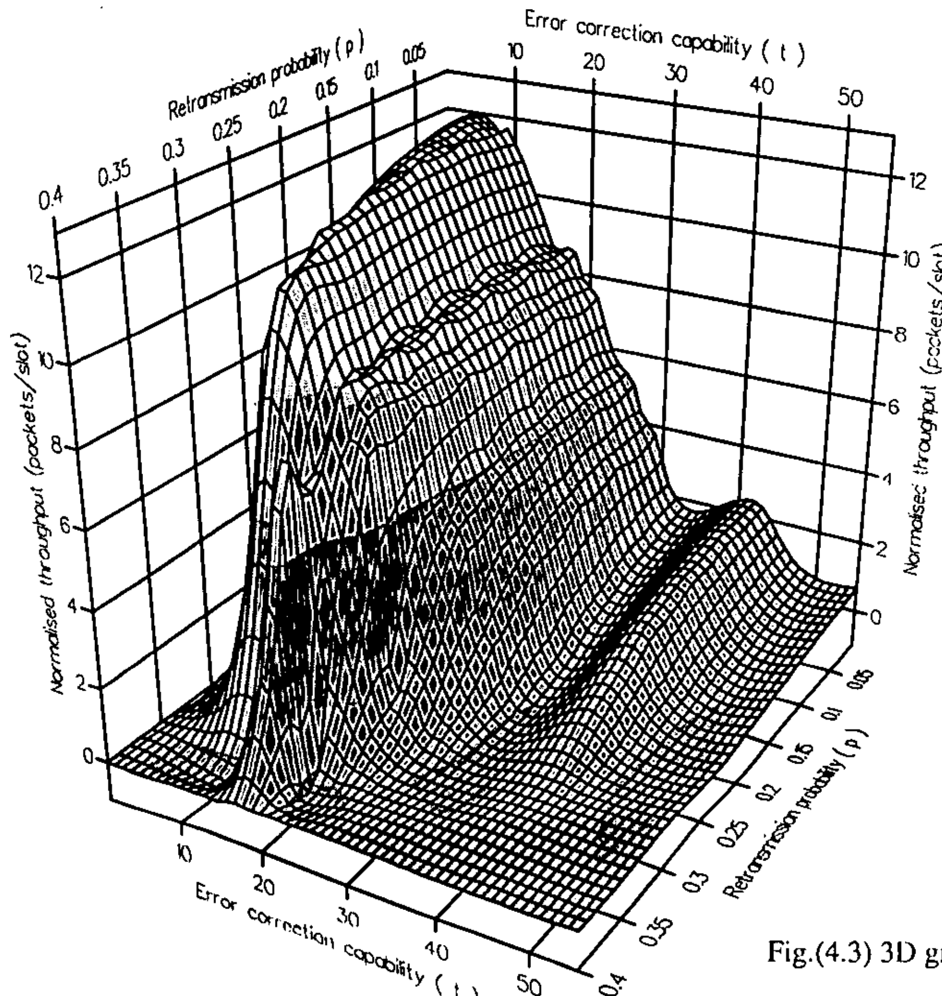


Fig.(4.2) Normalised throughput for all BCH codes of 255 bits code length.



G=60

σ=0.2

N=100

n=255

R=0

Lp=1020

Fig.(4.3) 3D graph corresponding to Fig.(4.2), (p=0.05-0.4).

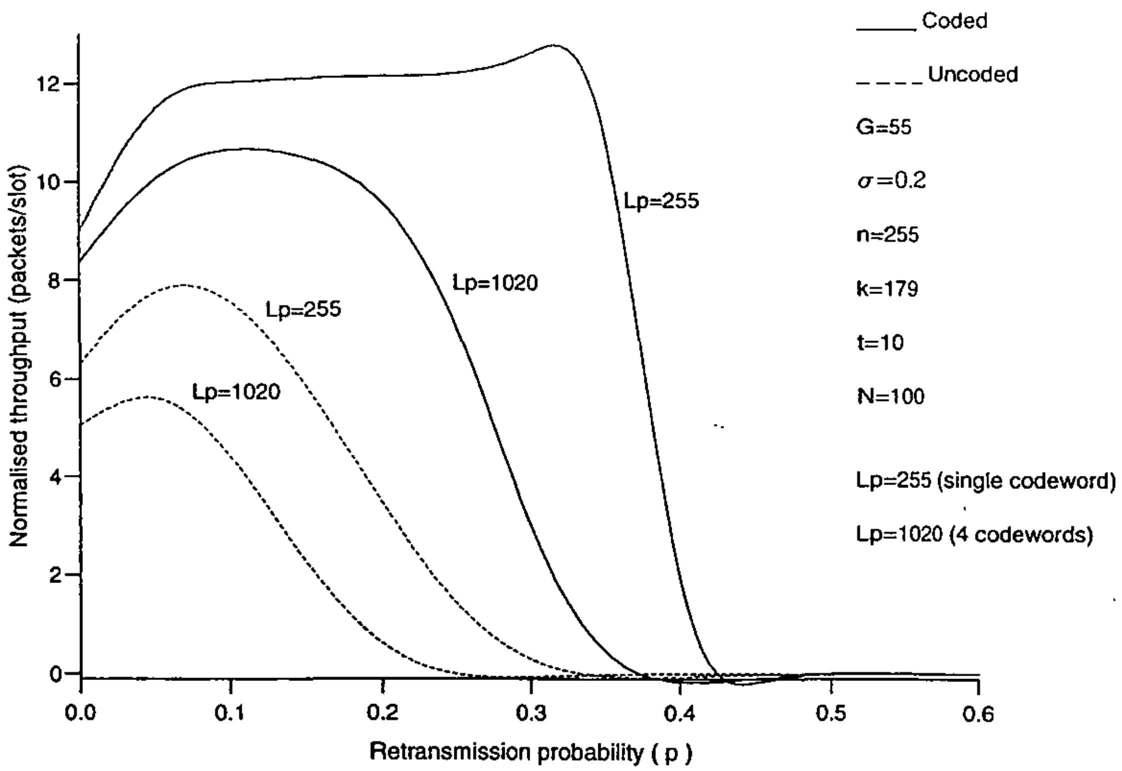


Fig.(4.4) The influence of the number of code words in a packet.

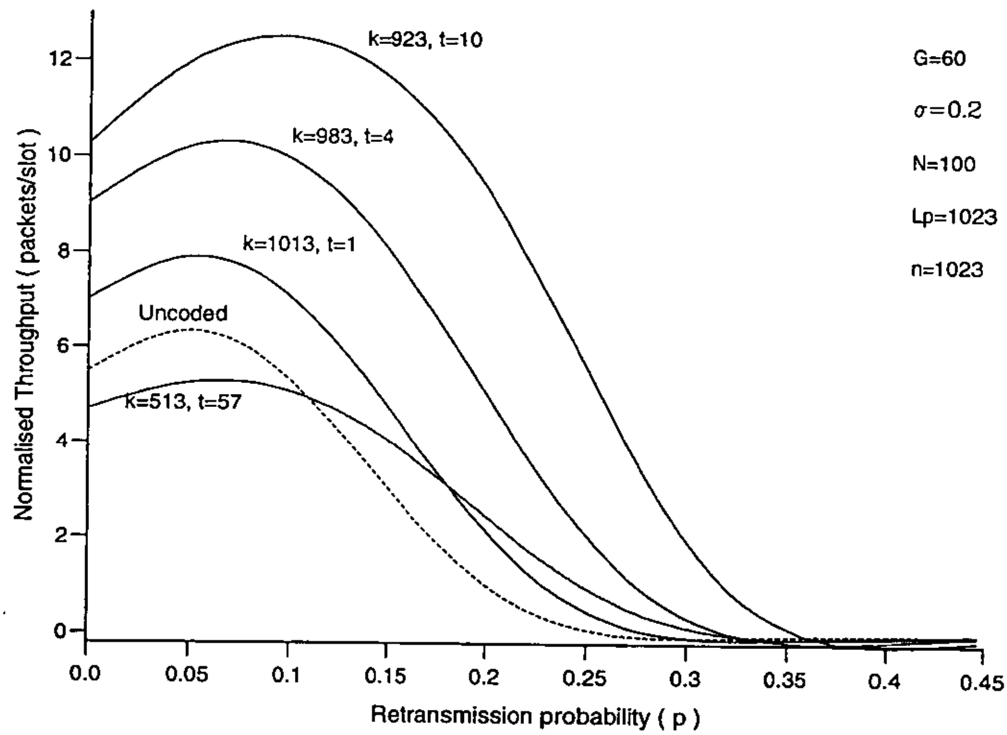


Fig.(4.5) DS-CDMA throughput performance (single code word packets).

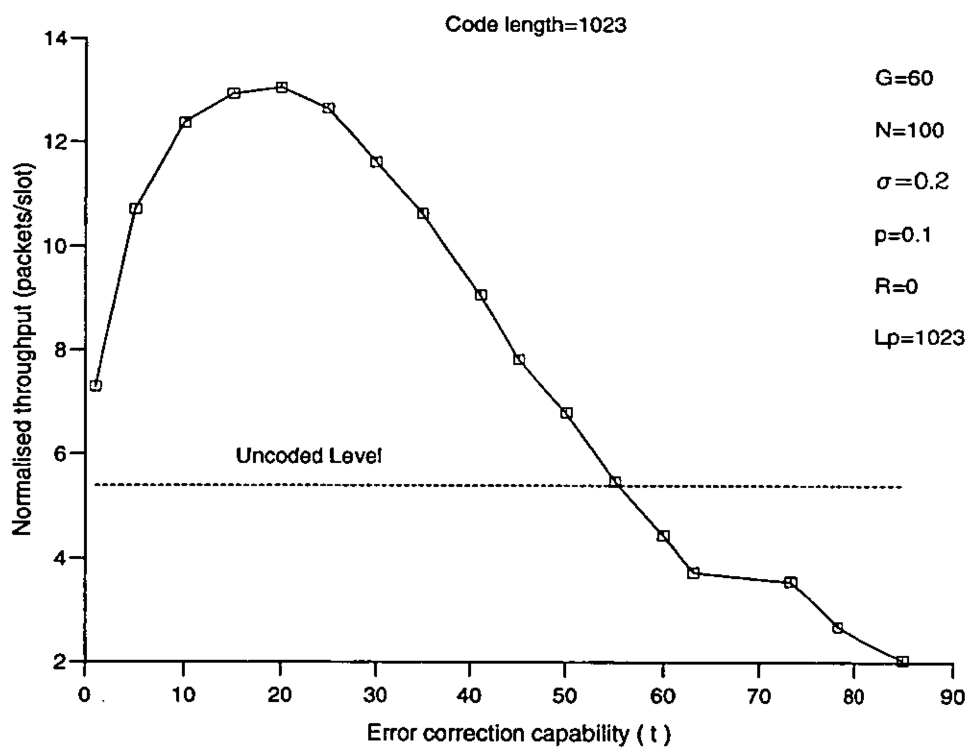


Fig.(4.6) Normalised throughput for systems with single code word packets.

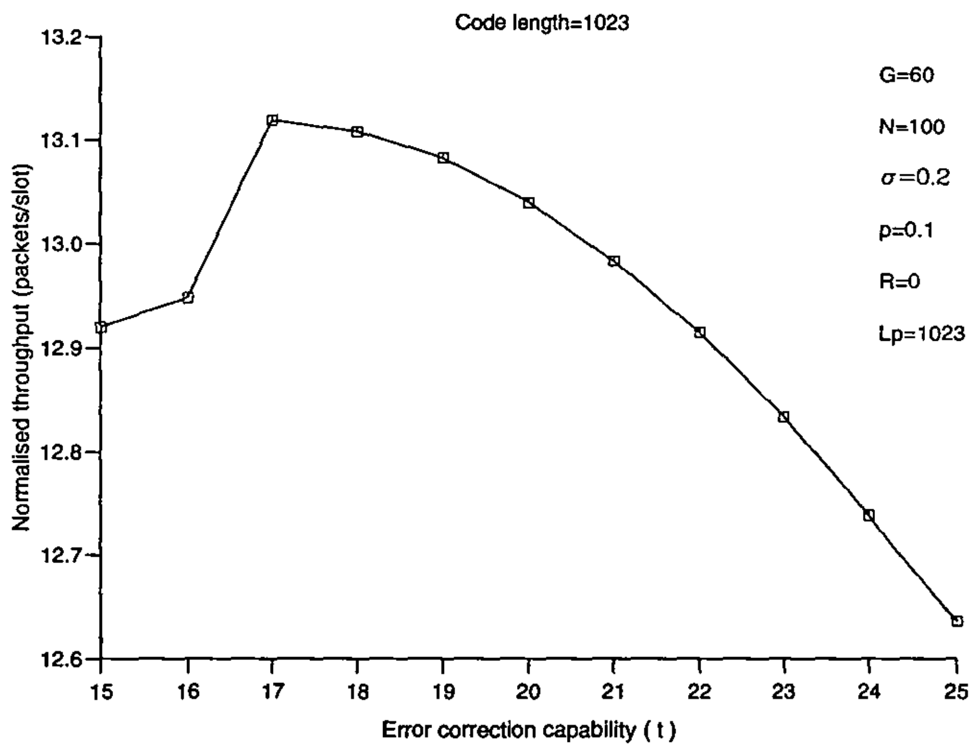
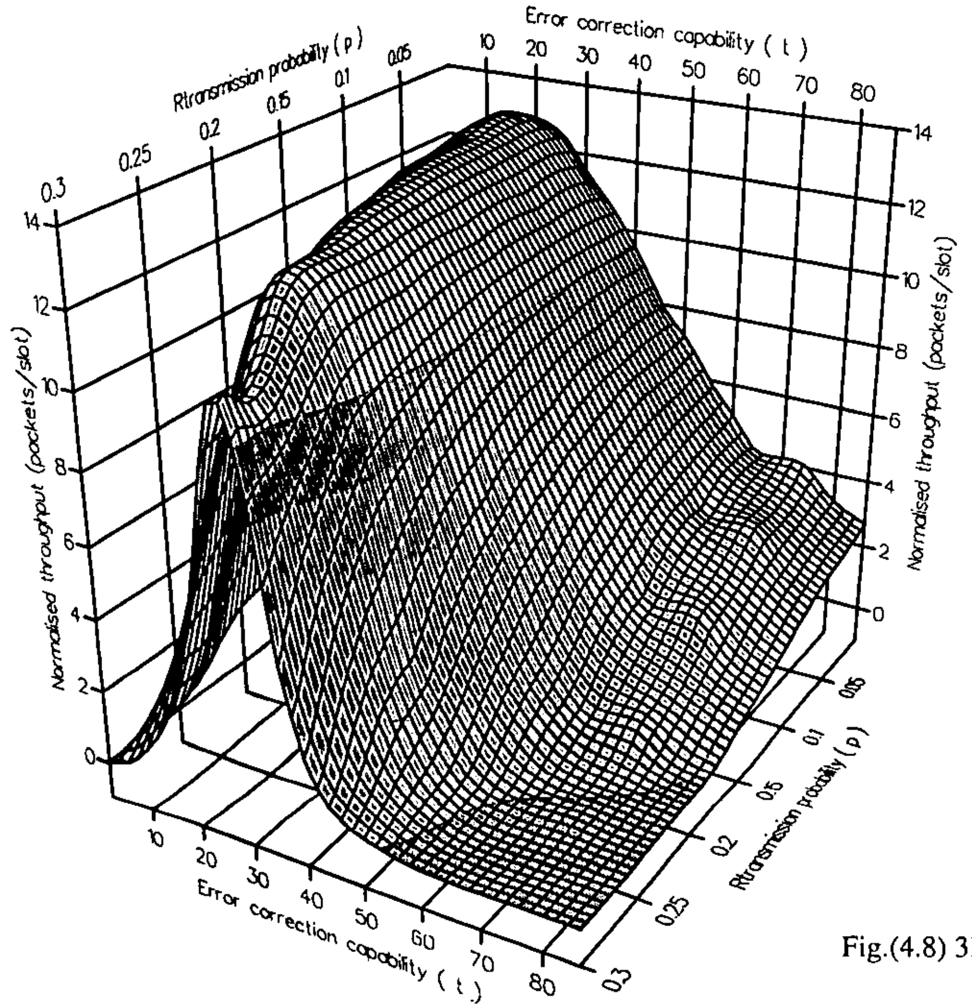


Fig.(4.7) Determination of optimum code of length 1023 bits.



G=60

σ=0.2

N=100

n=1023

R=0

Lp=1023

Fig.(4.8) 3D graph corresponding to Fig.(4.6), (p=0.05-0.3)

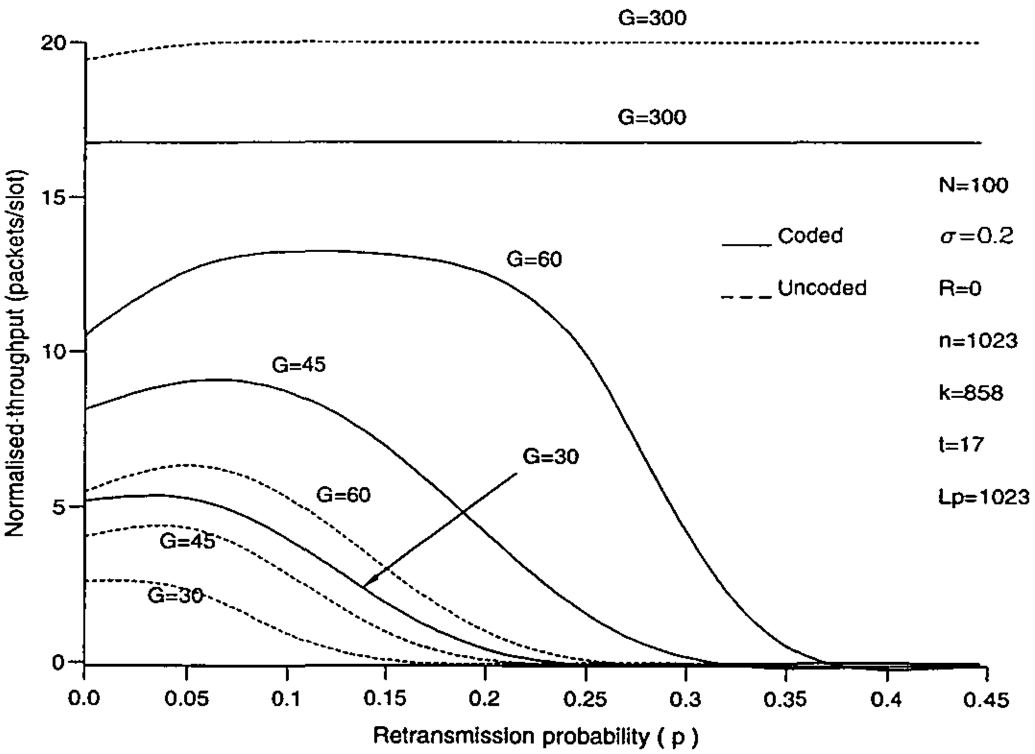


Fig.(4.9) The influence of processing gain of coded DS-CDMA systems.

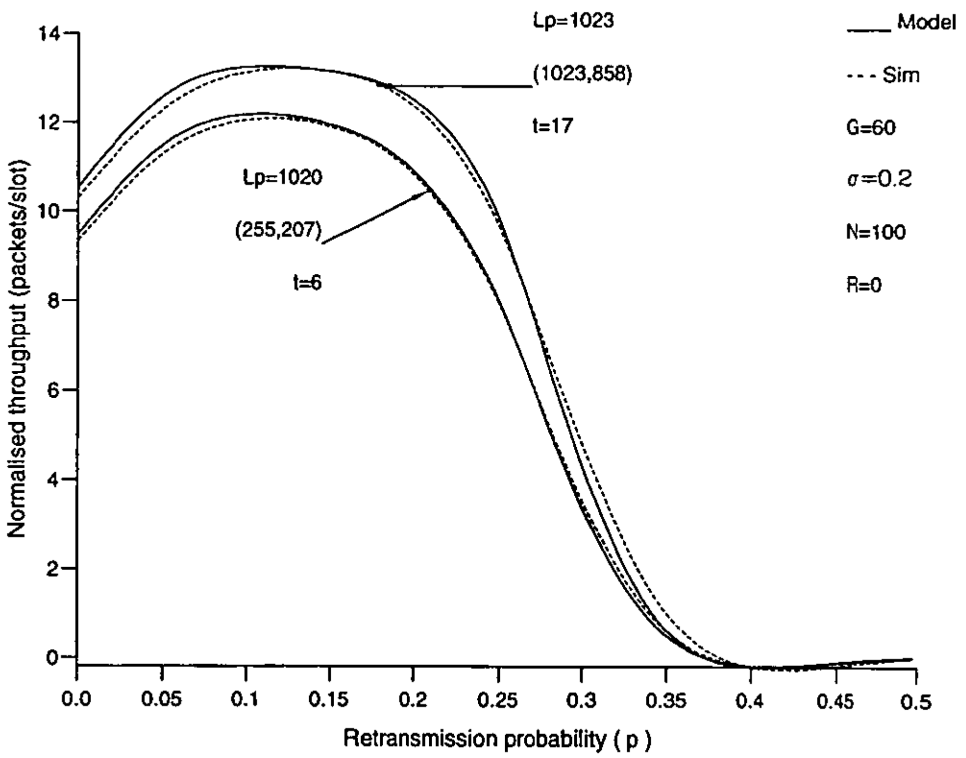


Fig.(4.10) Coded DS-CDMA model and simulation throughput comparison.

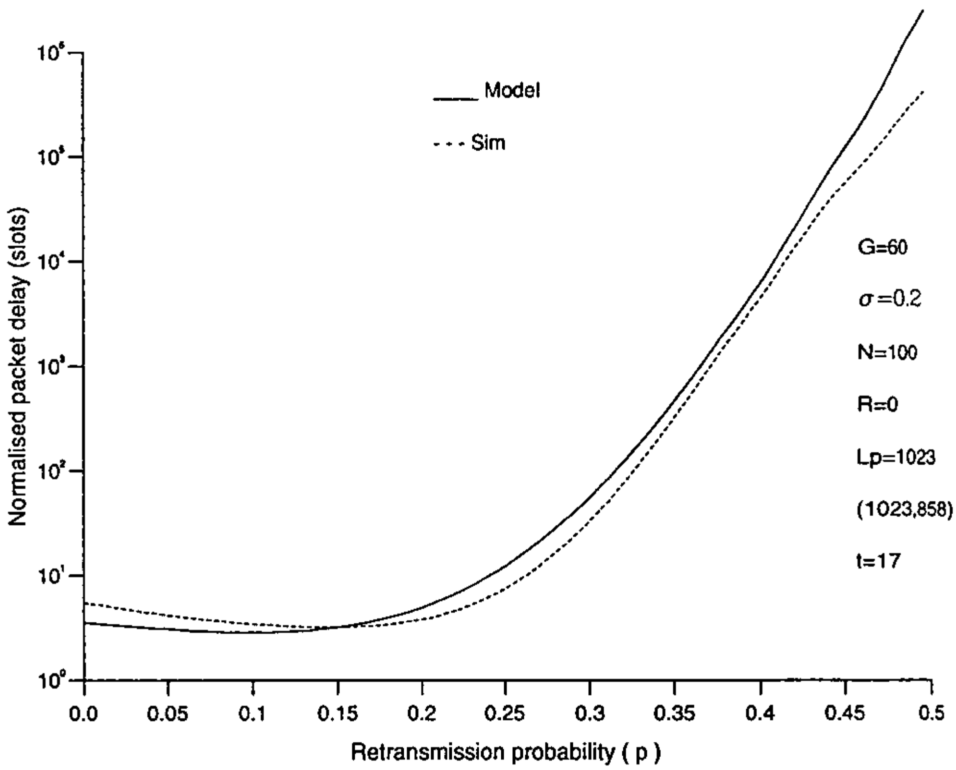


Fig.(4.11) Coded DS-CDMA model and simulation mean packet delay comparison.

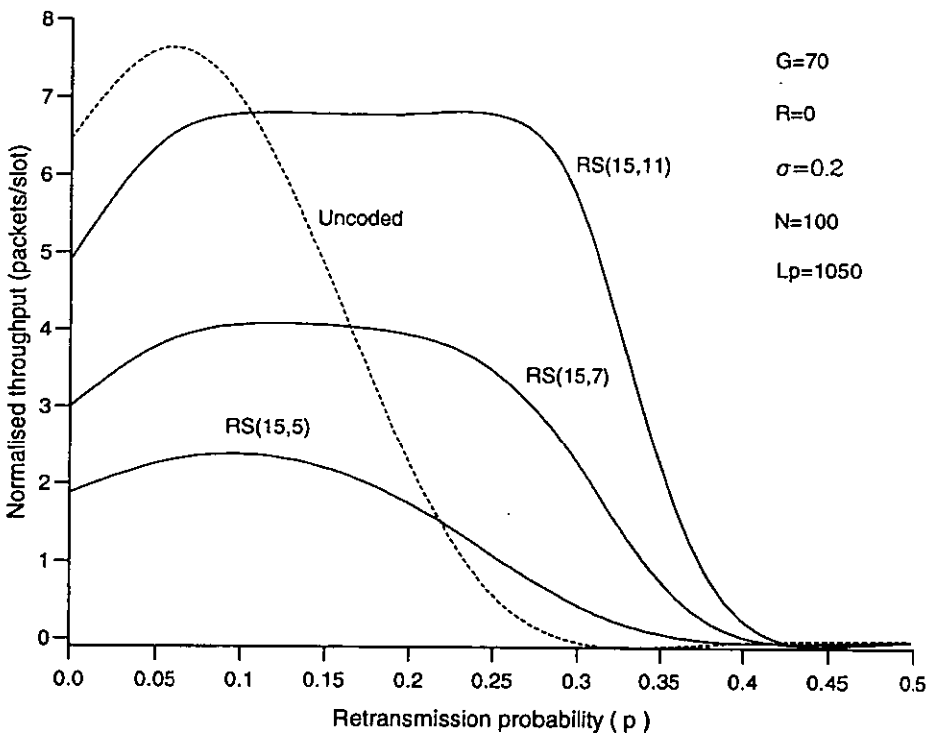


Fig.(4.12) Performance of DS-CDMA with concatenated coding.

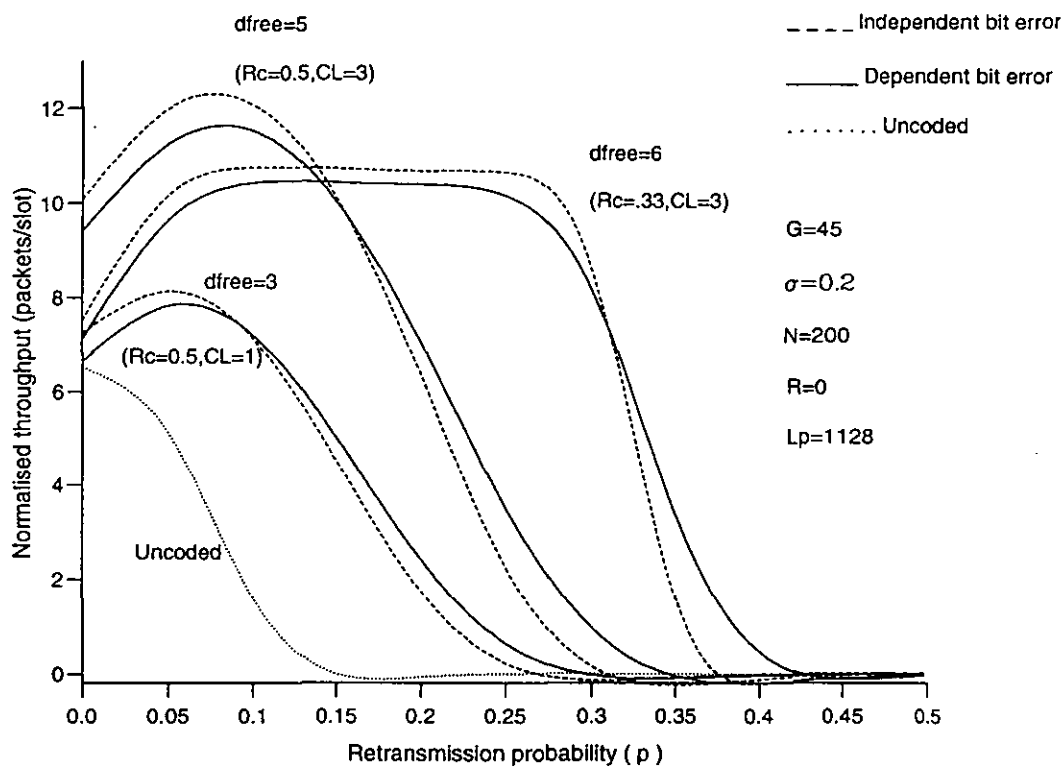


Fig.(4.13) Performance of DS-CDMA with convolutional coding.

CHAPTER 5

Random Access DS-CDMA for Packet Radio Networks

5.1 Introduction

In this chapter, the EPA model for random access DS-CDMA is applied to a radio channel with mobile users transmitting data information in the form of packets. The radio channel and the mobile users form what is called a Packet Radio Network (PRNet).

Packet Radio Networks provide data communications to users located over a broad geographic area, where wire connection among users is not practical, as in the case of mobile users. The users in PRNets communicate with each other via radio links. A central base station may exist but is not necessary. These users are mobile in general [Shei 94], [Barr 87].

The ALOHA system has been used in PRNets [Shei 90] [Arnb 87]. However, spread spectrum modulation is also considered to be a good choice for PRNets for several reasons:

- In spread spectrum, users can use the entire available bandwidth in a radio channel simultaneously, since each user has a unique PN sequence.
- Antijam and anti-interference properties of spread spectrum.
- Spread spectrum has the ability to combat multipath fading experienced in PRNets.
- Some type of security can be achieved in spread spectrum, see Chapter 2.

Spread spectrum in the form of CDMA has been given considerable attention as a potential candidate for PRNets [Purs 87a], [Stor 89], [Geor 90], and [Limi 91].

In the case of PRNets, the assumption of AWGN channel is no longer valid. This is because in a mobile radio environment, the received signals experience multipath fading, where a transmitted signal reaches its destination via paths with different delays and phases.

The probability of bit errors and consequently the probability of packet success used previously in the EPA model needs to be modified to accommodate the multipath fading effect in PRNets.

5.2 Multipath Fading Channels

In the previous chapters, the performance of random access CDMA systems is determined by assuming an AWGN channel. In packet radio networks the AWGN channel is no longer valid because of the multipath fading experienced in these networks. Multipath fading channels will therefore be considered to assess the performance of packet radio networks employing random access DS-CDMA. The characteristics of multipath fading channels for spread spectrum communications are briefly discussed in the following subsections.

5.2.1 Path loss

Transmitted signals from mobile users suffer loss in the actual delivered power which is related to a user's distance to the receivers or to the base stations. In general, the spatially averaged power P at a point a distance d from the transmitter is a decreasing function of d. Usually, this function is represented by a path-loss-power of the form [Pras 96]

$$P(d) \approx d^{-\gamma} \tag{5.1}$$

The path loss factor, γ, indicates how fast path loss increases with distance.

A lot of measurements have been carried out to obtain the value of γ in practical environments [Bult 87], [Sale 87], and [Rees 87]. Experimental results indicate that $3.5 \leq \gamma \leq 5$ for outdoor cellular mobile communications, and $2 \leq \gamma \leq 4$ for indoor channels.

In CDMA systems, the path loss is responsible for the near-far problem which is a major disadvantage of CDMA systems. This implies that the average power received by a receiver or a base station from users close to the receiver or to the base station is much higher than that for users located at a longer distance [Pras 96].

5.2.2 Multipath propagation

In fading channels, a transmitted signal arrives at the antenna of a receiver or a base station via several paths. This phenomeon is referred to as multipath propagation. In most applications, a direct line-of-sight (LOS) propagation path exists in addition to many non-line-of-sight (NLOS) reflected paths [Fehe 95].

A fading channel can be represented by multiple paths having path gain β_l, propagation delay τ_l, and phase shift γ_l, where l is the path index. These three parameters are randomly changing functions of time. The difference between the maximum and the minimum values of the propagation delay is called the maximum multipath delay spread T_m. Practical values of T_m can be as large as 15µs in urban and 3µs in sub-urban environments [Rees 87].

In DS-CDMA systems, if the data bit duration T_b is longer that the maximum delay spread, then the channel introduces a negligible amount of intersymbol interference (ISI) [Trab 95].

The reciprocal of the maximum time delay spread T_m is a measure for the coherent bandwidth of the fading channels

$$(\Delta f)_c \approx \frac{1}{T_m} \tag{5.2}$$

The coherent bandwidth $(\Delta f)_c$ is the bandwidth over which the signal propagation characteristics are correlated. If $(\Delta f)_c$ is small compared to the bandwidth of the transmitted signal W, $(W=1/T_c)$, the channel is said to be frequency selective, otherwise the channel is frequency non-selective, i.e. all frequency components of the transmitted signal are subject to the same attenuation and phase shift [Pras 96].

If L represents the number of discrete multipath links between each user and the intended receiver or base station, then L is determined by

$$L \cong \left\lfloor \frac{T_m}{T_c} \right\rfloor + 1 \tag{5.3}$$

where $\lfloor x \rfloor$ is the largest integer that is less than or equal to x, and T_c is the chip duration of spread spectrum modulation.

5.2.3 Rician channel

In a Rician channel a dominant path, which may be a line-of-sight (LOS) path, often occurs at the receiver, in addition to many scattered paths. This dominant path decreases the depth of fading of the received signals. The power density function (pdf) of the received envelope is said to be Rician [Stee 92].

An important parameter of the Rician channel called the Rician factor or parameter is defined as the ratio of the power in the dominant path to the power in the scattered paths

$$K_f = \frac{\text{Power in the dominant path}}{\text{Power in the scattered paths}} \tag{5.4}$$

when $K_f=0$, the fading channel becomes Rayleigh, while for $K_f=\infty$, the channel is Gaussian. The fades in the received signals have high probability of being deep when $K_f=0$, and to be very shallow for high values of K_f, thus approaching the Gaussian approximation.

A Rician channel can thus represent a multipath fading channel between the two extremes, Rayleigh and Gaussian, thus the Rician channel may be considered as the general case of fading channels. Usually, a Rician fading channel is used to model the communications channel in rural and indoor environments.

5.3 Probability of Bit Error for Random Access DS-CDMA over Multipath Fading Channels

Rician fading as a model of multipath radio channels as used by Trabelsi [Trab 95] is considered in the EPA model, since this is considered to be a general model which characterises most mobile radio communication channels. This model is represented by L discrete multipaths. The first path of the channel consists of a direct component (LOS) and several non-selective fading components. The differential delays for the components of the first path are much smaller than the chip duration T_c of the used PN sequence in spread spectrum modulation, Thus the fading of the first path is modelled as non-selective Rician fading. Each of the remaining L-1 paths consist of several non-selective fading channel components with small differential delays.

The probability density function (pdf) of the Rician random variable is given by [Prao 95], [Stee 92]

$$p(\beta) = \frac{\beta}{s^2}\exp\left(-\frac{\left(\beta^2 + c^2\right)}{2s^2}\right) I_0\left(\frac{\beta c}{s^2}\right) \tag{5.5}$$

where

β is random variable that represents the path gain.

$I_0(.)$ is the modified Bessel function of zeroth order.

c is a constant representing the line-of-sight component.

$2s^2$ is the power of scattered components.

The severity or the depth of fading is determined by the Rician parameter K_f as

$$K_f = \frac{c^2}{2s^2} \tag{5.6}$$

The signal to noise ratio at receiver 1 (the reference receiver) of asynchronous DS/CDMA in a fading environment can be expressed as [Trab 95]

$$snr = \frac{\beta_{1,1}^2}{\frac{m-1}{3G}\left(c^2 + 2s_1^2\right) + \frac{m(L-1)}{3G} 2s_1^2} \tag{5.7}$$

where

$\beta_{1,1}$ is a random variable that represents the path gain of the first path.

m is the number of users attempting transmission simultaneously.

G is the processing gain and is also the PN sequence length..

L is the number of discrete multipaths.

The conditional probability of bit error is given by

$$p_e(\beta_{1,1}) = \frac{1}{2} erfc\left(\sqrt{\frac{snr}{2}}\right) \tag{5.8}$$

In order to get the unconditional probability of bit error, $p_e(\beta_{1,1})$ must be averaged over the channel statistics given in equation (5.5). Thus to evaluate the integral

$$p_e = \int_0^\infty p_e(\beta_{1,1}) p(\beta_{1,1}) d(\beta_{1,1})$$

$$p_e = \int_0^\infty \frac{1}{2} erfc\left[\frac{1}{2}\frac{\beta_{1,1}^2}{\frac{m-1}{3G}\left(c^2+2s_1^2\right)+\frac{m(L-1)}{3G}2s_1^2}\right]\cdot\frac{\beta_{1,1}}{s_1^{\ 2}}\exp\left(-\frac{\left(\beta_{1,1}^{\ 2}+c^2\right)}{2s_1^2}\right)I_0\left(\frac{\beta_{1,1}c}{s_1^{\ 2}}\right)d\left(\beta_{1,1}\right)$$

(5.9)

This complicated integral was solved numerically by using the Maple-v software package [Bruc 91].

5.4 Packet Length in PRNets

The second major issue in packet radio network communications, apart from fading, is the length of transmitted packets. This packet length has a strong influence on the performance of the mobile radio links.

Because a mobile radio channel experiences fading, short packets are less likely to encounter fades than long packets [Siew 89]. However because of the header which is attached to the data bits of each packet, short packets have a data efficiency less than long packets. This means that packets ought to be sufficiently short so that most of them fall in the interfade time intervals. This leads to high transmission efficiency. However, a smaller packet size increases the overhead fraction of the packet and consequently reduces the effective throughput [Cast 92].

For the purpose of the following analysis, the structure of the transmitted packets is assumed to be the same as in High-level Data Link Control protocol (HDLC). An HDLC data packet consists of header and data bits. The header can be 6, 7 or 8 bytes long which includes address, control, check sequence, and flags [Tane 96]. This header is attached to the data bytes which can be as small as 8 bytes for short packets, or up to 120 bytes for long packets. For example, if a transmitted packet is 1024 bits, the header is 48, 56 or 64 bits. The corresponding data efficiencies are

95%, 94% or 93% which are good from the data efficiency point of view. But in terms of bit error, these long packets are more vulnerable in a fading channel, since one bit error requires the whole packet to be retransmitted. Short packets have low data efficiency 50%-60%, but they have an error probability better than long packets. Thus, a compromising choice must be made between high data efficiency (best for long packets) and high packet success probability (best for short packets).

5.5 Numerical Results

Equation (5.9) is used in the EPA model developed previously as the probability of bit error in fading environments. The probability of packet success is then calculated as before by using this probability of bit error.

Fig.(5.1) shows the probability of packet error verses the number of simultaneous users in the channel. Three different values of Rician factor K_f are considered, 10, 15 and 20 dB. When K_f is set to infinity, that is to say, the channel has no fading, it thus becomes a Gaussian channel. The number of paths is 3 for each user. In other words, the transmitted signal of any user reaches any receiver via three different paths. This figure shows that the probability of packet error increases with increasing the number of simultaneous users using the same channel. It is generally accepted that the desired bit error probability for data communications is less than 10^{-6}, thus the accepted packet error probability for packet lengths of approximately 1000 bits is 10^{-3}. As expected, the packet error probability is not acceptable when the fading channel experienced severe fading, which is represented by the Rician factor K_f. In the case of K_f=10 dB, the probability of packet error is higher than 10^{-3} even for only two users occupying the channel simultaneously. For K_f=15 dB, and 20 dB, the results show that the number of simultaneous users should not exceed 6 and 8 users, respectively. To overcome the high probability of error due to severe fading conditions, the processing gain or the length of PN sequences, G, should be increased to improve the performance by reducing the probability of packet error.

Fig.(5.2) shows the improvement of the probability of packet error when the processing gain is doubled from 63 to 127. The number of simultaneous users increases from 2 to more than 20 at a packet error rate of 10^{-3}.

Fig.(5.3) shows the improvement in the performance in terms of the throughput of DS-CDMA systems in the Rician fading channel. The maximum achievable throughput (capacity) is 3.5 packets/slot for G=63, while for G=127 the capacity becomes about 8.5 packets/slot. This improvement is achieved at the cost of a doubled bandwidth.

Fig.(5.4) shows the performance of DS-CDMA in the Rician fading channel with different levels of fading, which are represented by changes in the Rician factor K_f. The capacity, or the maximum achievable throughput is down from 11 packets/slot for a non-fading channel to 10.2, 7.9 and 3.7 for channels with K_f equal to 20, 15 and 10 dB, respectively. The optimum retransmission probability corresponding to the maximum throughput (capacity) slightly decreases as the channel fading increases.

Fig.(5.5) shows the effect of the number of paths of the Rician fading channel on the throughput performance of the DS-CDMA systems. The throughput decreases with increasing the number of multipaths. From equation (5.3), the number of paths is related to the maximum delay spread of the fading channels, T_m, thus the throughput performance degrades with increasing T_m, which is one of the statistical measures of fading channels.

Fig.(5.6) shows the normalised throughput (with respect to the actual transmitted data bits excluding the header) for different packet lengths 1024, 512, 256 and 128 bits. The attached header to any transmitted packet is set to be 48 bits. The channel has K_f =10 dB and L=3 paths. It can be seen from this plot that shorter packets have better throughput than long packets for retransmission probability p<0.1, the best packet length being 256 bits. However, beyond p=0.1 the packet length of 128 bits has the best performance. It is obvious that long packets of 1024 bits have the worst performance in these fading conditions.

Fig.(5.7) shows the throughput performance of the same packet lengths of the previous figure, but for a header of 64 bits. As the header length increases, the performance of short packets of 128 bits decreases due to a degradation in the data efficiency of this packet length.

Fig.(5.8) represents the normalised throughput with different packet lengths (same as before). The header length is 64 bits and K_f =15 dB, so the fading in this case is less than in the previous one with K_f =10 dB. It can be seen that short packets of 128 bits are no longer the best choice for retransmission probabilities up to 0.25. The performance of long packets of 1024 bits is better than short packets of 128 bits for retransmission probability less than 0.1.

In this figure, the best packet length or the optimum choice is represented by a solid line. The best packet length can be determined depending on the value of the retransmission probability. For $p \leq 0.08$ the best packet length is 512 bits (long packet); for $0.22 \geq p > 0.08$, the best packet length is 256 bits (moderate packet); for $p > 0.22$ the optimum choice is 128 bits (short packet).

These results show that the optimum choice of packet length depends on the fading conditions and the retransmission probability. The later represents the state of the DS-CDMA modelled network in terms of the load on the network. As the retransmission probability increases the network load becomes heavier.

Shorter packets are most suitable for networks with heavy load and for channels with severe fading. In contrast, longer packets become the optimum choice for networks with light load and for channels with shallow fading.

Fig.(5.9) shows the corresponding mean packet delay of Fig.(5.8). The packet delay is around 10-30 slots for all packet lengths for retransmission probability p less than 0.15. As p increases, long packets are subjected to longer delays more than short packets. For example, when p=0.3 the packet delay is 11, 24, 130 and 952 slots for packet lengths of 128, 256, 512 and 1024 bits, respectively.

5.6 Summary

In this chapter, the performance of random access DS-CDMA systems applied to packet radio networks (PRNets) has been studied. In PRNets the assumption of an AWGN channel is no longer valid because in radio environments, the received signals experience multipath fading. The probability of packet success previously used in the EPA model has been modified to accommodate the multipath effect in PRNets.

The Rician fading channel has been used as a model for the multipath radio channel since it can be considered as a general model which characterises most mobile radio communication channels.

It has been shown that the performance of DS-CDMA systems degrades with increasing the severity of fading and consequently the system can only handle a small number of simultaneous users. To increase the efficiency, the processing gain has to be increased.

The effect of packet size on the performance of random access DS-CDMA has been studied. The optimum packet length has been determined for specific fading characteristics and DS-CDMA parameters. It has been found that shorter packets are suitable for networks with heavy load and deep fading, and longer packets are suitable for lighty loaded networks and shallow fading channels.

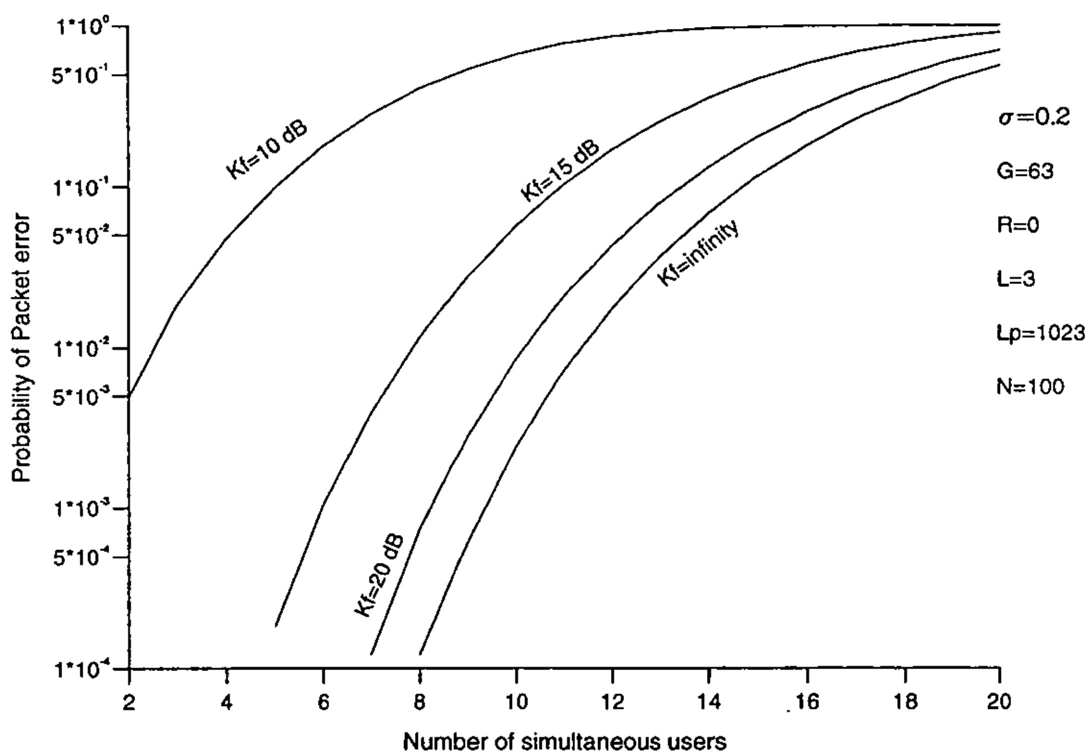


Fig.(5.1) Probability of packet error for different values of K_f.

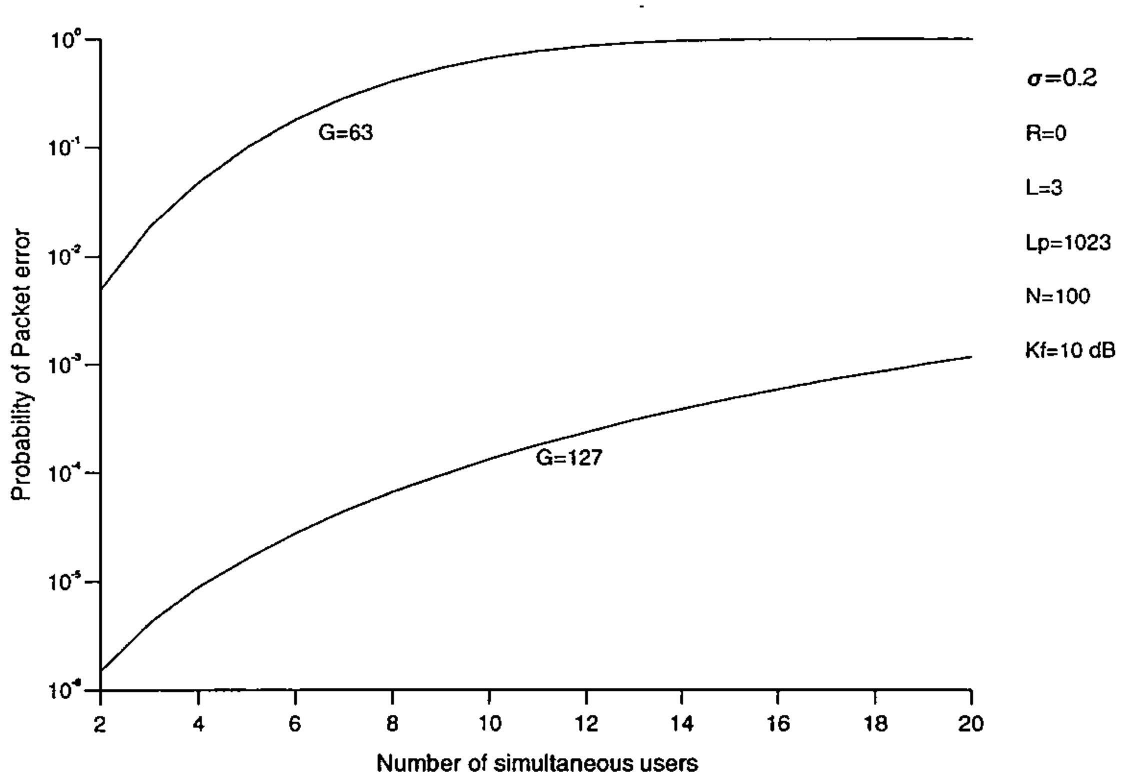


Fig.(5.2) Probability of packet error improvement when G is doubled.

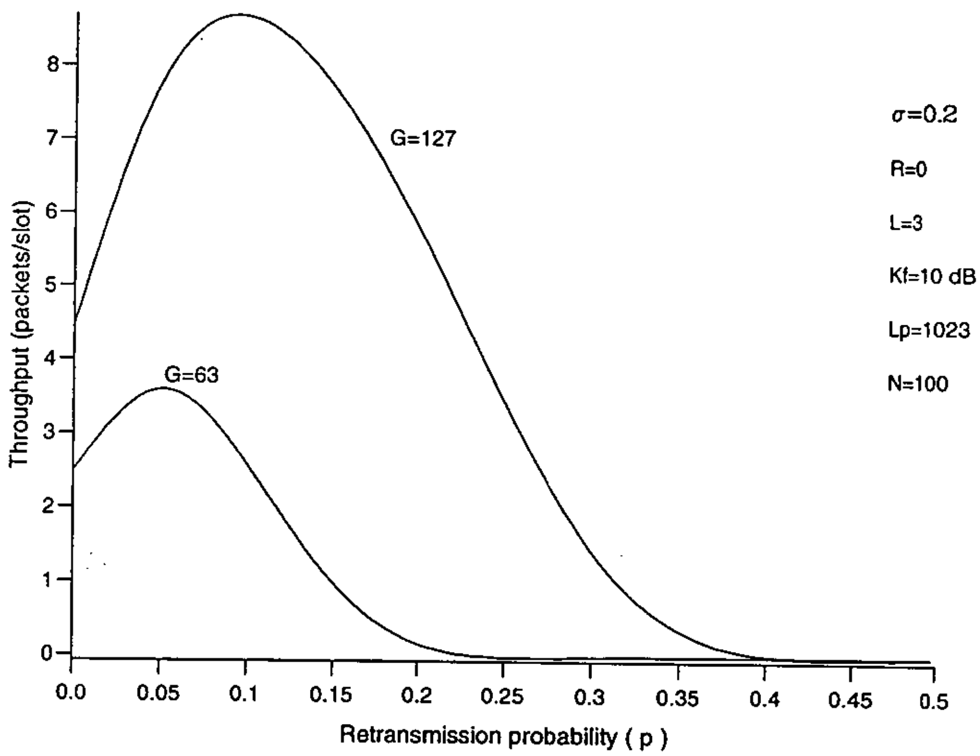


Fig.(5.3) Throughput performance improvement when G is doubled.

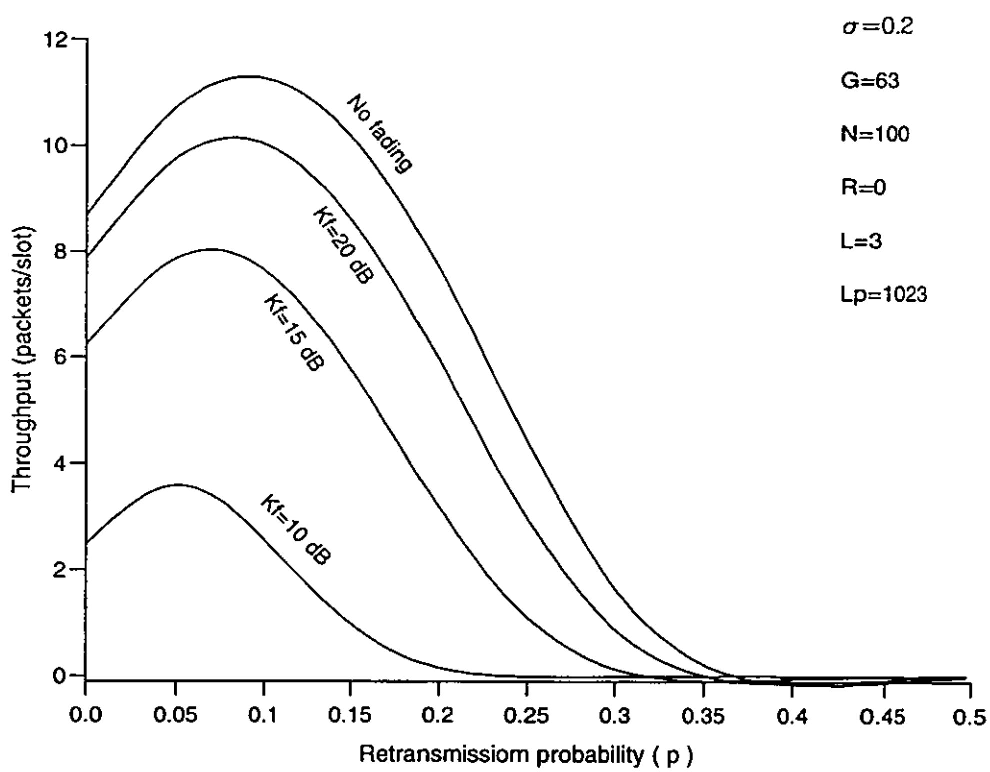


Fig.(5.4) Throughput performance for different values of Rician factor K_f.

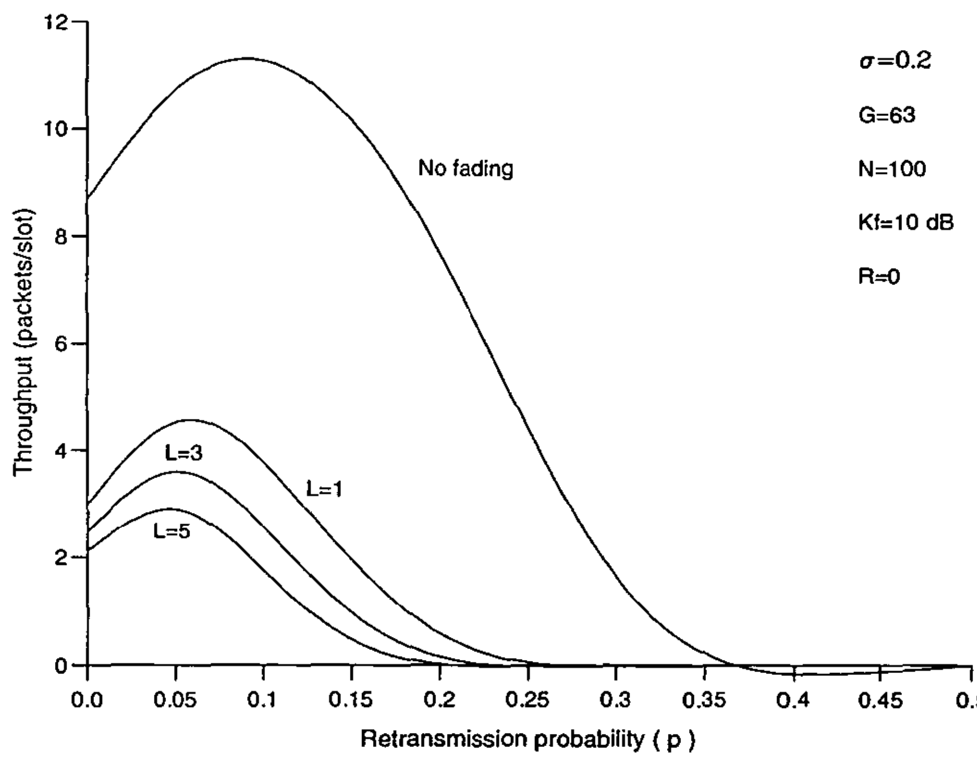


Fig.(5.5) Throughput performance for different numbers of paths.

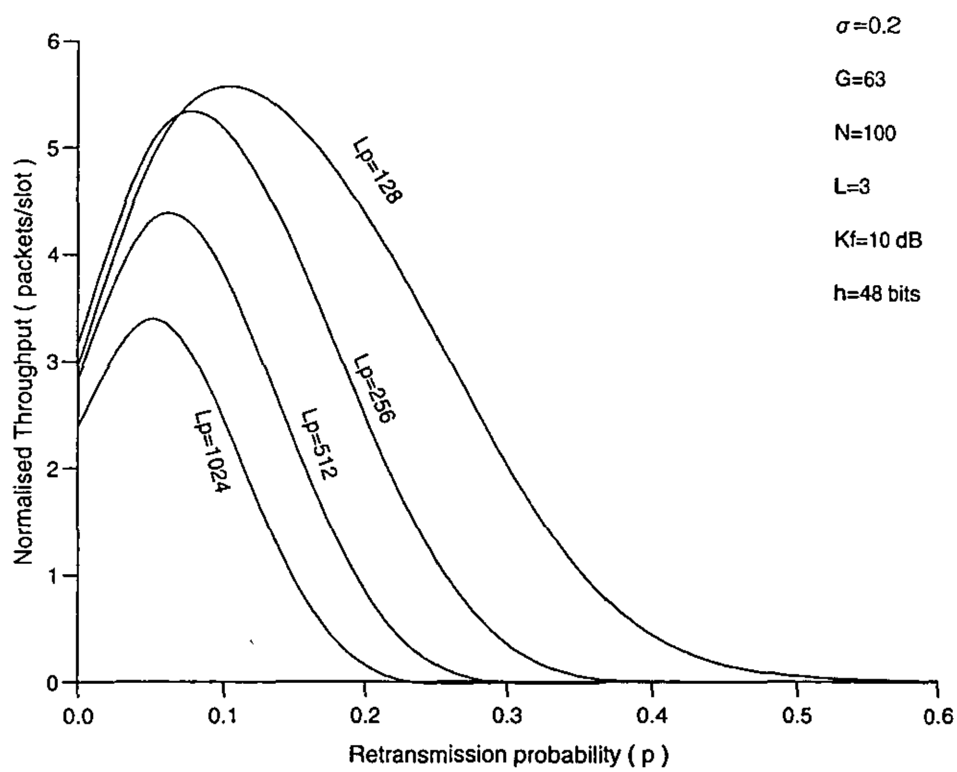


Fig.(5.6) Throughput performance of short and long packets, header=48 bits.

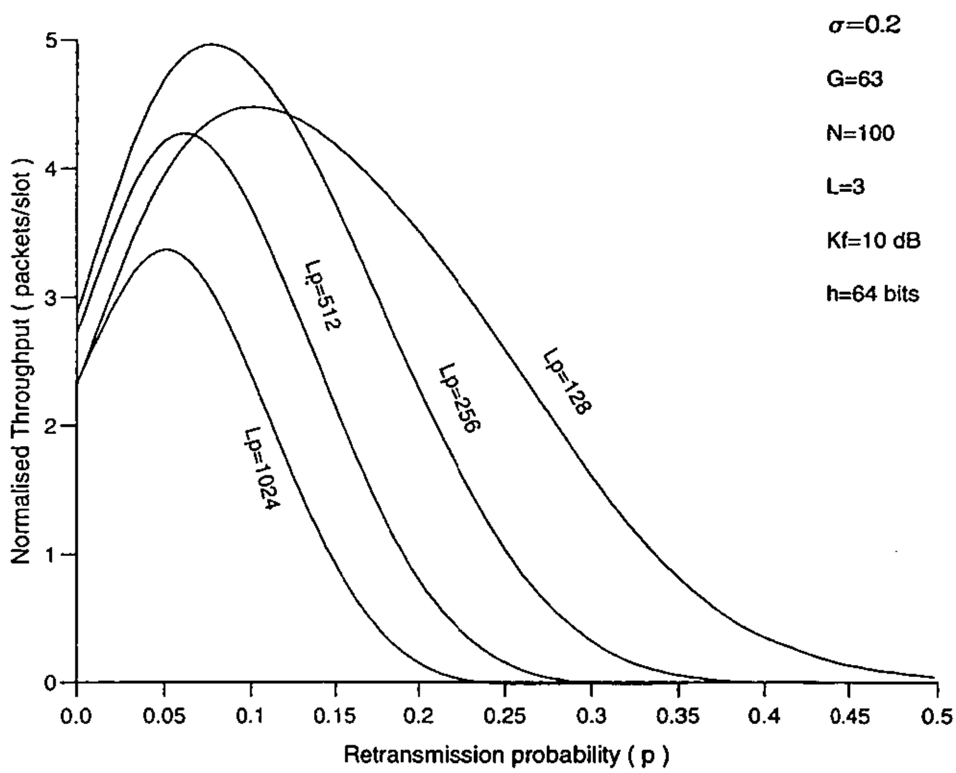


Fig.(5.7) Throughput performance of short and long packets, header=64 bits.

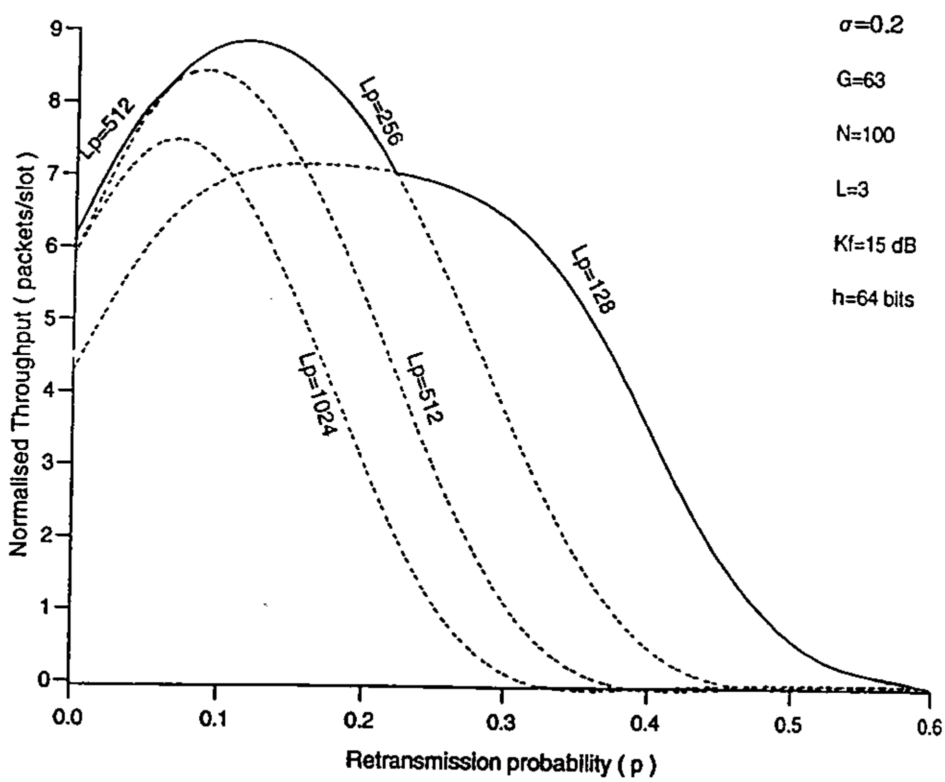


Fig.(5.8) Optimum packet length selection.

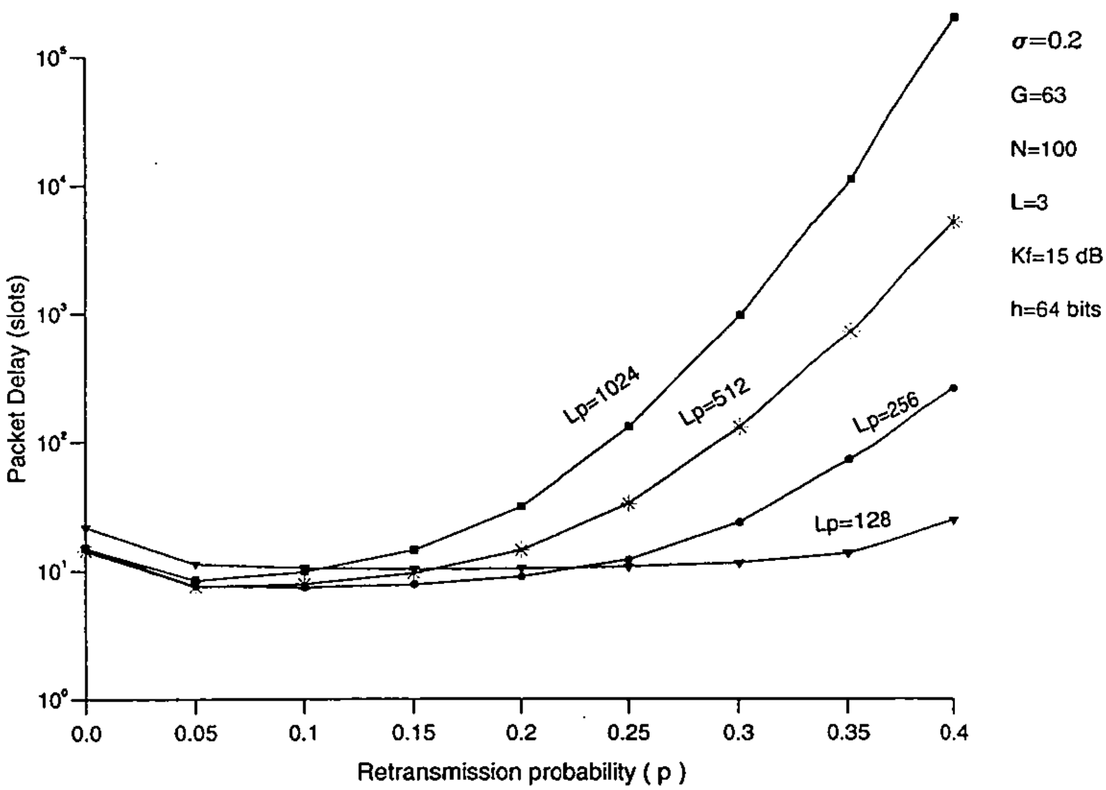


Fig.(5.9) Packet delay performance of short and long packets.

CHAPTER 6

Random Access DS-CDMA with Different User Classes

6.1 Introduction

In the previous chapters, we assume that all users using the same DS-CDMA channel have identical statistical properties (the new packet generation probability σ, and the retransmission probability p). In practice, this is not necessarily the case, and some users may have different requirements to others. All users that have statistically identical requirements are said to belong to the same class [Wood 93]. As a general case, each user is assumed to belong to a different class, and so has different σ and p to the other users in a network of N users, thus the maximum number of classes is equal to the total number of users in the modelled network.

The Equilibrium Point Analysis (EPA) method used previously has to be modified in order to solve a Markovian model in the form of a discrete-time queueing network with different user classes. A modified iterative EPA method has been used to solve the Slotted-Aloha systems in the case of different user classes, and was developed by Woodward in [Wood 93]. This method is called Extended Equilibrium Point Analysis (EEPA). EEPA will be used in this chapter to solve a Markovian model of a random access DS-CDMA network in order to evaluate the performance of these networks with different user classes.

Previously, a perfect power control is assumed in evaluating the performance of random access DS-CDMA systems; that is to say, there is no near/far problem. In this chapter the effect of the near/far problem is determined by relaxing the assumption of identical received powers of all users in the modelled systems. By using the assumption of different user classes, each user in a DS-CDMA system can deliver different power to the other users depending on its distance to the intended receiver or to the base station.

6.2 Performance of Random Access DS-CDMA with Different User Classes

In a similar way to before, the modified Markovian model in the form of a discrete-time queueing network has two versions depending on the values of σ_i and p_i ($\sigma_i \leq p_i$ and $\sigma_i > p_i$), where i is the user index.

Fig.(6.1) represents a discrete-time queueing network model for a random access DS-CDMA network with $\sigma_i \leq p_i$. Each user i at node 2 can generate a packet with probability σ_i, and packets are transmitted or retransmitted from node 1 with probability p_i, i=1, 2, ... , N. If a transmitted packet by user i is successfully received, the user i is routed to node 1 with probability (σ_i / p_i), or to node 2 with probability $1-(\sigma_i / p_i)$. In contrast, when a transmitted packet is received incorrectly, the user i is routed back to node 1 with probability 1.

Using Fig.(6.1), we define the following:

x_i is the probability that user i is at node 1.

$\phi(x_i)$ is the probability of a packet from user i being received in error.

r_{nk} is the routing probability from node n to node k.

The routing probabilities of the network shown in Fig.(6.1) are

$$r_{11} = \phi(x_i) + (1-\phi(x_i)) \cdot \frac{\sigma_i}{p_i} \tag{6.1}$$

$$r_{12} = (1-\phi(x_i)) \cdot \left(1-\frac{\sigma_i}{p_i}\right) \tag{6.2}$$

$$r_{21} = 1 \tag{6.3}$$

The equilibrium equation at node 2 for user i is

$$(1-x_i)\cdot\sigma_i = x_i p_i (1-\phi(x_i)) \cdot \left(1-\frac{\sigma_i}{p_i}\right) \tag{6.4}$$

then the probability that user i is at node 1 can be written as a function of $\phi(x_i)$ as

$$x_i = \frac{\frac{\sigma_i}{p_i}}{1+\phi(x_i)\left(1-\frac{\sigma_i}{p_i}\right)} \tag{6.5}$$

The probability of packet error $\phi(x_i)$ can be written as

$$\phi(x_i) = 1 - \left[1 - \frac{1}{2} erfc\left(\sqrt{G / \sum_{\substack{j=1 \\ j \neq i}}^{N} x_j p_j}\right)\right]^{L_p} \tag{6.6}$$

where

L_p is the packet length.

G is the processing gain.

N is the number of users in the modelled network.

Note that, the number of interfering users m-1 in the case of identical users is replaced by $\sum_{\substack{j=1 \\ j \neq i}}^{N} x_j p_j$ for the case of different user classes.

Equations (6.5) and (6.6) can be solved for x_i using the iterative algorithm specified by Woodward in [Wood 93] as follows:

Let $x_i(n)$ be the value of x_i after n iterations, and let Δ be an acceptable error tolerance.

$n := 0$

For i:=1 to N do

$\quad x_i(n) := 1/H$ {H is the total number of nodes}

Repeat

$\quad n := n+1$

$\quad$ For i:=1 to N do

$$\phi(x_i(n)) := \phi(x_2(n-1), \ldots\ldots, x_N(n-1)); \qquad i = 1$$

$$\phi(x_1(n), \ldots\ldots, x_{i-1}(n), x_{i+1}(n-1), \ldots\ldots, x_N(n-1)); \qquad 2 \le i \le N-1$$

$$\phi(x_1(n), \ldots\ldots\ldots, x_{N-1}(n)); \qquad i = N$$

$$x_i(n) := f(\phi(x_i(n)));$$

Until $\left(|x_i(n) - x_i(n-1)| \le \Delta \ for\ all\ i \in \{1, 2, \ldots\ldots, N\}\right)$;

After applying the above algorithm, let $x_i^e, i = 1,2,......., N$ be a solution. Then the mean throughput of user i can be written as

$$S(x_i^e) = \frac{(1 - x_i^e)\sigma_i}{1 - \frac{\sigma_i}{p_i}} \qquad i = 1,2,......., N. \tag{6.7}$$

The mean packet delay for user i is

$$W(x_i^e) = \frac{1}{S(x_i^e)} - \frac{1}{\sigma_i} + 1 \tag{6.8}$$

Finally, the total mean throughput of the network assuming the users of this network behave independently can be written as

$$S(x^e) = \sum_{i=1}^{N} S(x_i^e) \tag{6.9}$$

And the mean packet delay for the network is

$$W(x^e) = \frac{1}{N} \sum_{i=1}^{N} W(x_i^e) \tag{6.10}$$

Fig.(6.2) represents a discrete-time queueing network model for random access DS-CDMA in the case of $\sigma_i > p_i$. The routing probabilities r_{nk} in this case can be written as

$$r_{11} = \phi(x_i) \cdot \frac{\sigma_i}{p_i} + (1 - \phi(x_i)) \tag{6.11}$$

$$r_{12} = \phi(x_i) \cdot \left(1 - \frac{\sigma_i}{p_i}\right) \tag{6.12}$$

$$r_{21} = 1 \tag{6.13}$$

The equilibrium equation is

$$(1 - x_i) \cdot p_i = x_i \sigma_i \cdot \phi(x_i) \cdot \left(1 - \frac{\sigma_i}{p_i}\right) \tag{6.14}$$

then the probability that user i is at node 1 is

$$x_i = \frac{p_i}{p_i + \phi(x_i)\left(1 - \dfrac{\sigma_i}{p_i}\right)} \tag{6.15}$$

The probability of packet error $\phi(x_i)$ can be written as

$$\phi(x_i) = 1 - \left[1 - \frac{1}{2} erfc\left(\sqrt{G / \sum_{\substack{j=1 \\ j \neq i}}^{N} x_j \sigma_j}\right)\right]^{L_p} \tag{6.16}$$

Let $x_i^e, i = 1, 2, \ldots\ldots, N$ be a solution for equations (6.15) and (6.16). The mean throughput of user i can be written as

$$S(x_i^e) = x_i^e \sigma_i \left(1 - \phi\left(x_i^e\right)\right) \qquad i = 1, 2, \ldots\ldots, N. \tag{6.17}$$

The mean packet delay for user i, the total throughput of the network, and the mean packet delay for the network stated in equations (6.8), (6.9) and (6.10) for the $\sigma_i \leq p_i$ case are the same in this case.

6.3 Performance of Random Access DS-CDMA for Different Received Powers

In the previous chapters, the received powers from all users attempting transmission simultaneously are assumed to be identical. In other words, a perfect power control scheme is assumed that makes all the received powers at any receiver or at a base station, if one exists, to be the same. Power control is essential in any DS-CDMA system in order to mitigate the near/far problem [Simp 93]. Without power control, the closer transmitters to the intended receiver or to the base station in a DS-CDMA system will overpower the receiver, so distant transmitters have no chance to deliver their data correctly. The EEPA method enables us to assess the effect of power control on the performance of DS-CDMA systems, since in EEPA each user can be assigned different parameters, and the delivered power can be one of these parameters.

In the case of non-identical powers, each user has a different delivered power depending on its position in the network, and on how far it is from the intended receiver or from the base station. Let P_{av} be the received power from all users in the identical powers case. α_i is defined as a factor representing the change in P_{av} depending on the position of user i. α_i in terms of dB can be either negative or positive. A negative value of α_i means the user i's delivered power is less that the average power, while a positive value means the user i's delivered power is greater than the average power. Clearly, α_i reflects the relative distance of user i from the receiver, relative to other users in the network.

The signal to interference ratio for a receiver assumed to receive data from user i, assuming that m users are attempting transmission simultaneously, can be expressed as

$$SIR = \frac{\alpha_i P_{av}}{P_{av}\underset{j\neq i}{(}\alpha_1 + \alpha_2 + \ldots\ldots + \alpha_j + \ldots\ldots + \alpha_m)} \qquad (6.18)$$

With reference to the discrete-time network represented in Fig.(6.1) and to the probability of a packet received in error ($\phi(x_i)$ equation(6.6)), $\phi(x_i)$ for the case of non-identical powers can be written as

$$\phi(x_i) = 1 - \left[1 - \frac{1}{2} erfc\left(\sqrt{\alpha_i G / \sum_{\substack{j=1 \\ j \neq i}}^{N} x_j p_j \alpha_j}\right)\right]^{L_p} \tag{6.19}$$

6.4 Numerical Results

Table (6.1) shows a comparison between the EEPA model and the simulation which is run for 10^5 slots. The $\sigma_i \leq p_i$ case is considered in this table. The number of users in the system is 100 users labelled from 1 to 100. The probability of a new packet generation for user i, σ_i, is within the range 0.1 to 0.196. The retransmission probability of user i, p_i, is within the range 0.12 to 0.216. Table(6.1) and all tables in this chapter show the results for 26 users out of the 100 users in the modelled network. The last row of these tables represents the mean values for the all 100 users in the network. The results for the entire 100 users can be found in Appendix E. The EEPA model gives results that are very close to those obtained from the simulation. The mean error in the throughput is only about 1%, and the mean error in the packets delay is around 5.3%.

Table (6.2) shows a comparison between the EEPA model and a 10^5 slots simulation for the case of $\sigma_i > p_i$. The probability of new packet generation of user i, σ_i, is within the range 0.22 to 0.27. The retransmission probability of user i, p_i, is within the range 0.02 to 0.119. The mean error in the throughput is 3.39%, and the mean error in the delay is 5.74%. In Tables (6.1) and (6.2), the processing gain G is 100, and the packet length L_p is 256 bits.

Table (6.3) shows a comparison between the model and the simulation for processing gain G=80. The models match well to the simulation with very slight percentage error, 0.082% for the throughput, and 0.34% for the delay. Increasing the processing gain G has a similar effect to decreasing the load in the modelled network, and vice versa.

Table (6.4) shows a comparison between the EEPA model and 10^4 slots simulation for different values of the processing gain, 60 to 300, which means changing the load of the modelled network. It has been found, the mean error between the model and the simulation is high for network with heavy load, G=60-65. The accuracy is improved rapidly with increasing the processing gain, or in other words, when the load on the network decreases.

It can be observed that the results of Table (6.4) show that the best match between the model and the simulation is when G=80, not when the load is very light, when G=300, as was expected. This is because the mean error changes its sign, i.e. crossing the zero error approximately when G=80. Then the accuracy slightly decreases for G=90 and 100 before improving again along with increasing G from 110 to 300. In general, the mean error is small when $G \geq 70$.

Table (6.5) shows a comparison between two systems, one with power control and the other without power control. α_i is set to be in the range from -3dB for user number 1 and +3dB for user number 100. The throughput of users with negative α_i deteriorates, and consequently these users have a slim chance to get their data transmitted correctly, and their packets suffer very long delays. The packets of users with high α_i are transmitted almost without any error.

In Table (6.6) α_i is in the range from -5dB to +5dB. The first nine users in the modelled network have almost no throughput at all, and have very long packet delays (longer than 10^6 slots for the first user).

Fig.(6.3) shows the importance of power control in DS-CDMA systems. Networks without power control have very poor throughput when the received power of an intended transmitter is down 3 to 5 dB.

Fig.(6.4) shows the corresponding packet delay. This packet delay increases rapidly when the received power decreases by up to 3 to 5 dB.

Table (6.7) shows a comparison between the EEPA model and a 10^6 slots simulation. There is a disagreement when α_i is negative, and this is because the measured throughput is very small and the large percentage errors are thus more likely to occur when measuring small values.

6.5 Summary

In this chapter, the performance of DS-CDMA networks with different user classes has been determined. Each user in a modelled network is said to belong to a different user class and has different statistical properties to the other users in the network. A Markovian model in the form of a discrete-time queueing network has been used to model the DS-CDMA networks with different user classes.

The Extended Equilibrium Point Analysis (EEPA) method has been applied to solve the DS-CDMA network model in the case of non-identical users. The EEPA results show good matching with the simulation results, especially when the load of the modelled networks is light. The accuracy of the EEPA degrades with increasing network load.

The EEPA method enables us to show the effect of the near/far problem on the performance of DS-CDMA systems. That is because in EEPA each user can be assigned different delivered power depending on its distance to the receiver or to the base station. It has been shown that DS-CDMA systems are very vulnerable in the presence of the near/far problem, and these systems have very poor performance for users having their delivered powers 3 to 5 dB below the average power.

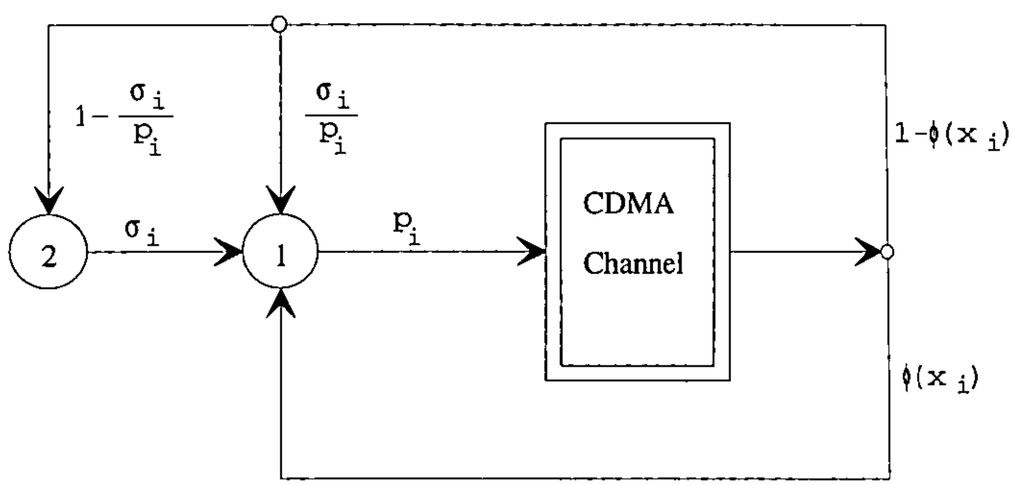


Fig.(6.1) Modified discrete-time queueing network model for random access DS-CDMA (σi<=pi).

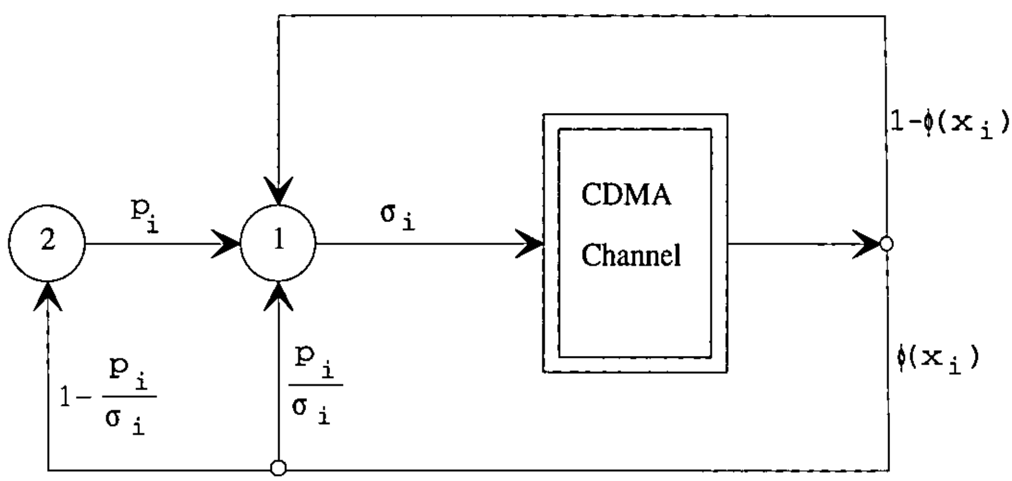


Fig.(6.2) Modified discrete-time queueing network model for random access DS-CDMA ($\sigma_i > p_i$).

Table(6.1) EEPA Model & Simulation comparison, G=100, N=100, L_p=256, ($\sigma_i \leq p_i$).

			Throughput			Delay		
user i	σ_i	p_i	*Simul.*	*EEPA*	*% Error*	*Simul.*	*EEPA*	*% Error*
1	0.100	0.120	0.094770	0.095734	1.017	1.551862	1.445644	-6.84
5	0.104	0.124	0.097320	0.099536	2.277	1.659995	1.431268	-13.78
9	0.108	0.128	0.101700	0.103337	1.610	1.573582	1.417791	-9.90
13	0.112	0.132	0.105170	0.107139	1.872	1.579843	1.405131	-11.06
17	0.116	0.136	0.109090	0.110940	1.696	1.546053	1.393215	-9.89
21	0.120	0.140	0.112990	0.114741	1.550	1.517007	1.381980	-8.90
25	0.124	0.144	0.117220	0.118541	1.127	1.466451	1.371370	-6.48
29	0.128	0.148	0.118930	0.122342	2.869	1.595807	1.361333	-14.69
33	0.132	0.152	0.124730	0.126142	1.132	1.441560	1.351824	-6.22
37	0.136	0.156	0.129670	0.129942	0.210	1.358943	1.342803	-1.19
41	0.140	0.160	0.132120	0.133742	1.228	1.426019	1.334233	-6.44
45	0.144	0.164	0.134660	0.137542	2.140	1.481665	1.326081	-10.50
49	0.148	0.168	0.140180	0.141341	0.828	1.376928	1.318317	-4.26
53	0.152	0.172	0.141720	0.145141	2.414	1.447219	1.310914	-11.26
57	0.156	0.176	0.147500	0.148940	0.976	1.369404	1.303848	-4.79
61	0.160	0.180	0.149510	0.152739	2.160	1.438516	1.297096	-5.28
65	0.164	0.184	0.155690	0.156539	0.545	1.325459	1.290637	-2.63
69	0.168	0.188	0.157030	0.160338	2.107	1.415829	1.284453	-12.10
73	0.172	0.192	0.163080	0.164137	0.648	1.318080	1.278527	-3.00
77	0.176	0.196	0.165030	0.167936	1.761	1.377686	1.272843	-7.61
81	0.180	0.200	0.169090	0.171735	1.564	1.358549	1.267386	-6.70
85	0.184	0.204	0.174400	0.175533	0.650	1.229162	1.262143	-2.85
89	0.188	0.208	0.177660	0.179332	0.941	1.309580	1.257102	-4.01
93	0.192	0.212	0.180450	0.183131	1.486	1.333368	1.252251	-6.11
97	0.196	0.216	0.186800	0.186929	0.402	1.269105	1.247580	-1.70
100	0.199	0.219	0.189280	0.189778	0.263	1.258052	1.244188	-1.10
Mean	**0.150**	**0.169**	**14.133530**	**14.276275**	**1.010**	**1.399359**	**1.325160**	**-5.30**

Table(6.2)EEPA Model & Simulation comparison, G=100, N=100, L_p=256, ($\sigma_i > p_i$).

			Throughput			**Delay**		
user i	σ_i	p_i	*Simul.*	*EEPA*	*% Error*	*Simul.*	*EEPA*	*% Error*
1	0.220	0.020	0.076750	0.078886	2.783	9.483861	9.131130	-3.714
5	0.224	0.024	0.086480	0.088966	2.875	8.099082	7.775941	-3.990
9	0.228	0.028	0.096990	0.098098	1.142	6.924376	6.807950	-1.681
13	0.232	0.032	0.106700	0.106470	-0.216	6.061726	6.081956	0.334
17	0.236	0.036	0.110590	0.114226	3.288	5.805121	5.517294	-4.958
21	0.240	0.040	0.117240	0.121474	3.611	5.362845	5.065565	-5.543
25	0.244	0.044	0.124550	0.128298	3.009	4.930543	4.695968	-4.758
29	0.248	0.048	0.128510	0.134767	4.869	4.749238	4.387971	-7.607
33	0.252	0.052	0.135830	0.140932	3.756	4.393890	4.127357	-6.066
37	0.256	0.056	0.142360	0.146838	3.146	4.118195	3.903975	-5.202
41	0.260	0.060	0.148780	0.152520	2.514	3.875180	3.710376	-4.253
45	0.264	0.064	0.152770	0.158006	3.427	3.757909	3.540978	-5.773
49	0.268	0.068	0.155430	0.163323	5.078	3.702421	3.391509	-8.398
53	0.272	0.072	0.162910	0.168489	3.425	3.461888	3.258647	-5.871
57	0.276	0.076	0.170620	0.173522	1.701	3.237789	3.139771	-3.027
61	0.280	0.080	0.174210	0.178437	2.426	3.168770	3.032782	-4.292
65	0.284	0.084	0.178370	0.183247	2.734	3.085197	2.935983	-4.836
69	0.288	0.088	0.182920	0.187963	2.757	2.994649	2.847984	-4.898
73	0.292	0.092	0.186430	0.192593	3.306	2.939286	2.767637	-5.840
77	0.296	0.096	0.191500	0.197147	2.949	2.843554	2.693985	-5.260
81	0.300	0.100	0.199930	0.201631	0.851	2.668417	2.626226	-1.581
85	0.304	0.104	0.201340	0.206052	2.340	2.677249	2.563679	-4.242
89	0.308	0.108	0.204360	0.210415	2.963	2.646572	2.505765	-5.230
93	0.312	0.112	0.208270	0.214725	3.099	2.596331	2.451987	-5.560
97	0.316	0.116	0.214950	0.240062	11.683	2.487688	2.001039	-19.56
100	0.319	0.119	0.216790	0.222154	2.474	2.477963	2.366576	-3.325
Mean	**0.270**	**0.070**	**15.657340**	**16.187905**	**3.389**	**4.167248**	**3.927968**	**-5.742**

Table(6.3) EEPA Model & Simulation comparison, G=80, N=100, L_p=256, ($\sigma_i \leq p_i$).

			Throughput			Delay		
user i	σ_i	p_i	*Simul.*	*EEPA*	*% Error*	*Simul.*	*EEPA*	*% Error*
1	0.100	0.120	0.085330	0.084390	-1.102	2.719208	2.849701	4.799
5	0.104	0.124	0.085880	0.087678	2.094	3.028770	2.790034	-8.557
9	0.108	0.128	0.090570	0.090964	0.435	2.781924	2.734095	-1.719
13	0.112	0.132	0.094160	0.094250	0.096	2.691650	2.681547	-0.375
17	0.116	0.136	0.098260	0.097535	-0.738	2.556392	2.632089	2.961
21	0.120	0.140	0.100150	0.100819	0.668	2.651689	2.585458	-2.498
25	0.124	0.144	0.103630	0.104102	0.455	2.585199	2.541418	-1.694
29	0.128	0.148	0.107490	0.107385	-0.098	2.490691	2.499758	0.364
33	0.132	0.152	0.111070	0.110668	-0.362	2.427574	2.460291	1.348
37	0.136	0.156	0.113970	0.113950	-0.018	2.421298	2.422847	0.064
41	0.140	0.160	0.116840	0.117231	0.335	2.415856	2.387276	-1.183
45	0.144	0.164	0.119660	0.120513	0.713	2.412567	2.353440	-2.451
49	0.148	0.168	0.122960	0.123793	0.677	2.375969	2.321215	-2.304
53	0.152	0.172	0.126280	0.127074	0.629	2.339963	2.290489	-2.114
57	0.156	0.176	0.129020	0.130354	1.034	2.340480	2.261160	-3.389
61	0.160	0.180	0.135000	0.133634	-1.012	2.157408	2.233134	3.510
65	0.164	0.184	0.138420	0.136913	-1.089	2.126829	2.206327	3.738
69	0.168	0.188	0.141890	0.140193	-1.196	2.095332	2.180660	4.072
73	0.172	0.192	0.143670	0.143472	-0.138	2.146442	2.156063	0.448
77	0.176	0.196	0.146950	0.146750	-0.136	2.123218	2.132470	0.436
81	0.180	0.200	0.149930	0.150029	0.066	2.114224	2.109821	-0.208
85	0.184	0.204	0.152040	0.153307	0.833	2.142434	2.088060	-2.088
89	0.188	0.208	0.156130	0.156586	0.292	2.085770	2.067135	-0.893
93	0.192	0.212	0.158820	0.159684	0.657	2.088103	2.047001	-1.968
97	0.196	0.216	0.163540	0.163141	-0.244	2.012671	2.027612	0.742
100	0.199	0.219	0.165740	0.165600	-0.084	2.008421	2.013535	0.255
Mean	**0.150**	**0.169**	**12.491140**	**12.501426**	**0.082**	**2.357637**	**2.349617**	**-0.340**

Table(6.4) EEPA Model & Simulation comparison for different G, N=100, L_p=256, Mean σ_i =0.1495, Mean p_i=0.169, ($\sigma_i \leq p_i$).

	Mean Throughput			Mean Delay		
G	*Simul.*	*EEPA*	*% Error*	*Simul.*	*EEPA*	*% Error*
60	7.93580	5.87689	25.95	7.139673	11.640706	-63.04
65	9.41150	7.68046	18.39	5.086809	7.523081	-47.89
70	10.65600	10.45833	1.855	3.808219	3.959614	-3.975
80	12.49110	12.50143	-0.082	2.357637	2.349617	0.340
90	13.54320	13.65681	-0.839	1.727503	1.652458	4.344
100	14.14650	14.27628	-0.917	1.396654	1.325160	5.119
110	14.50050	14.60080	-0.692	1.207046	1.164748	3.504
150	14.86550	14.92485	-0.339	1.035629	1.011610	2.319
300	14.91400	14.95000	-0.241	1.008359	1.000000	0.829

Table(6.5) Comparison between systems with & without power control using EEPA Model, G=100, N=100, L_p=256, ($\sigma_i \leq p_i$). α_i=-3dB to 3dB.

				Throughput		**Delay**	
user i	σ_i	p_i	α_i *(dB)*	*With pwc* $\alpha_i=0$	*Without pwc*	*With pwc* $\alpha_i=0$	*Without pwc*
1	0.100	0.120	-3.00	0.095734	0.002804	1.445644	347.6084
5	0.104	0.124	-2.52	0.099536	0.008292	1.431268	111.9815
9	0.108	0.128	-2.08	0.103337	0.017915	1.417791	47.56003
13	0.112	0.132	-1.67	0.107139	0.031081	1.405131	24.24568
17	0.116	0.136	-1.31	0.110940	0.046259	1.393215	13.99678
21	0.120	0.140	-0.97	0.114741	0.061857	1.381980	8.833004
25	0.124	0.144	-0.66	0.118541	0.076735	1.371370	5.967340
29	0.128	0.148	-0.36	0.122342	0.090290	1.361333	4.262944
33	0.132	0.152	-0.09	0.126142	0.102321	1.351824	3.197415
37	0.136	0.156	0.17	0.129942	0.112872	1.342803	2.505659
41	0.140	0.160	0.41	0.133742	0.122108	1.334233	2.046628
45	0.144	0.164	0.64	0.137542	0.130232	1.326081	1.734181
49	0.148	0.168	0.86	0.141341	0.137447	1.318317	1.518812
53	0.152	0.172	1.07	0.145141	0.143934	1.310914	1.368689
57	0.156	0.176	1.27	0.148940	0.149849	1.303848	1.236150
61	0.160	0.180	1.46	0.152739	0.155315	1.297096	1.188546
65	0.164	0.184	1.64	0.156539	0.160437	1.290637	1.135432
69	0.168	0.188	1.82	0.160338	0.165291	1.284453	1.097569
73	0.172	0.192	1.99	0.164137	0.169944	1.278527	1.070331
77	0.176	0.196	2.15	0.167936	0.174438	1.272843	1.050862
81	0.180	0.200	2.30	0.171735	0.178814	1.267386	1.036835
85	0.184	0.204	2.46	0.175533	0.183103	1.262143	1.026629
89	0.188	0.208	2.6	0.179332	0.187319	1.257102	1.019335
93	0.192	0.212	2.74	0.183131	0.191483	1.252251	1.014057
97	0.196	0.216	2.88	0.186929	0.195609	1.247580	1.010186
100	0.199	0.219	2.98	0.189778	0.198683	1.244188	1.008020

Table(6.6) Comparison between systems with & without power control using EEPA Model, G=100, N=100, L_p=256, $(\sigma_i \leq p_i)$. α_i=-5dB to 5dB.

				Throughput		Delay	
user i	σ_i	p_i	α_i (dB)	With pwc α_i=0	Without pwc	With pwc α_i=0	Without pwc
1	0.100	0.120	-5.00	0.095734	$<10^{-9}$	1.445644	$>10^{8}$
5	0.104	0.124	-3.66	0.099536	0.000001	1.431268	$>10^{6}$
9	0.108	0.128	-2.64	0.103337	0.000079	1.417791	12602.32
13	0.112	0.132	-1.82	0.107139	0.001219	1.405131	812.6956
17	0.116	0.136	-1.12	0.110940	0.006690	1.393215	141.8587
21	0.120	0.140	-0.53	0.114741	0.019605	1.381980	43.67488
25	0.124	0.144	0.00	0.118541	0.038912	1.371370	18.63449
29	0.128	0.148	0.47	0.122342	0.067131	1.361333	8.083680
33	0.132	0.152	0.89	0.126142	0.087204	1.351824	4.891566
37	0.136	0.156	1.28	0.129942	0.104013	1.342803	3.261204
41	0.140	0.160	1.63	0.133742	0.117595	1.334233	2.360939
45	0.144	0.164	1.96	0.137542	0.128945	1.326081	1.837900
49	0.148	0.168	2.26	0.141341	0.137353	1.318317	1.523763
53	0.152	0.172	2.55	0.145141	0.144724	1.310914	1.330767
57	0.156	0.176	2.82	0.148940	0.151041	1.303848	1.210446
61	0.160	0.180	3.07	0.152739	0.156626	1.297096	1.134618
65	0.164	0.184	3.30	0.156539	0.160476	1.290637	1.086440
69	0.168	0.188	3.53	0.160338	0.166444	1.284453	1.055652
73	0.172	0.192	3.74	0.164137	0.170945	1.278527	1.035890
77	0.176	0.196	3.94	0.167936	0.175283	1.272843	1.023257
81	0.180	0.200	4.14	0.171735	0.179514	1.267386	1.015054
85	0.184	0.204	4.33	0.175533	0.183671	1.262143	1.009735
89	0.188	0.208	4.50	0.179332	0.187777	1.257102	1.006314
93	0.192	0.212	4.68	0.183131	0.191849	1.252251	1.004105
97	0.196	0.216	4.84	0.186929	0.195897	1.247580	1.002686
100	0.199	0.219	4.96	0.189778	0.198923	1.244188	1.001951

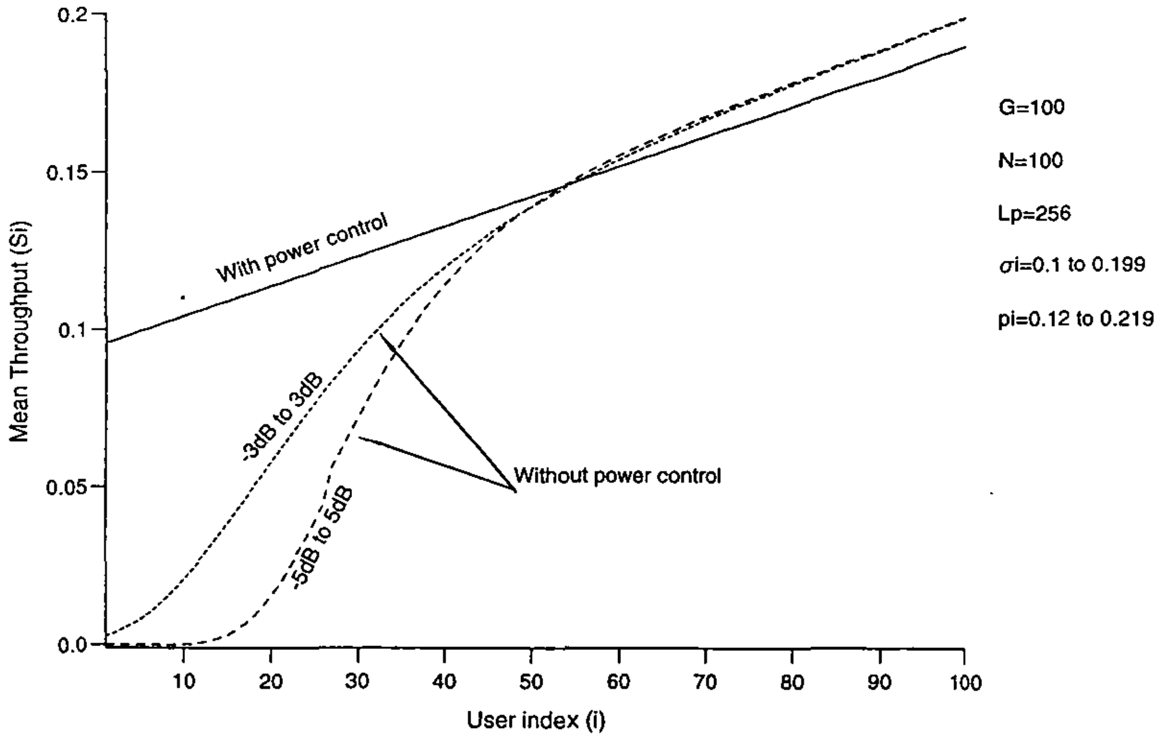


Fig.(6.3) Mean throughput comparison (with and without power control).

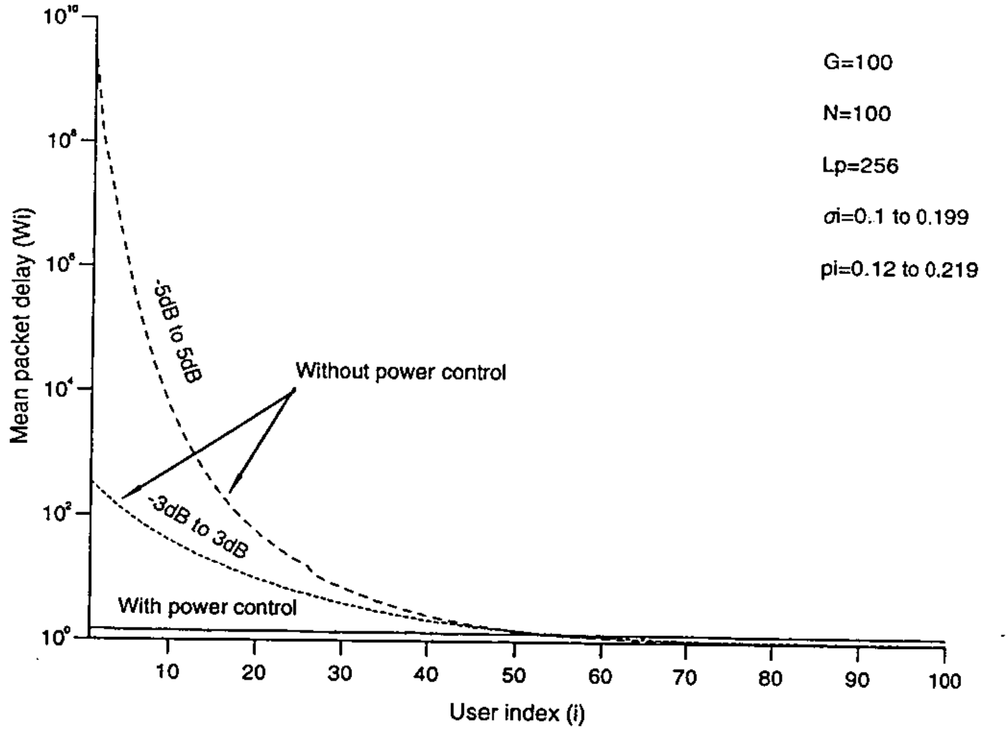


Fig.(6.4) Mean packet delay comparison (with and without power control).

Table(6.7) EEPA Model & Simulation comparison without power control, G=100, N=100, L_p=256, ($\sigma_i \leq p_i$). α_i=-3dB to 3 dB.

				Throughput			Delay		
user i	σ_i	p_i	α_i (dB)	Simul.	EEPA	% Error	Simul.	EEPA	% Error
1	0.100	0.120	-3.00	0.014899	0.002804	-81.18	58.11860	347.6084	>100
5	0.104	0.124	-2.52	0.023930	0.008292	-65.35	33.17317	111.9815	>100
9	0.108	0.128	-2.08	0.034686	0.017915	-48.35	20.57082	47.56003	>100
13	0.112	0.132	-1.67	0.046317	0.031081	-32.90	13.66177	24.24568	77.74
17	0.116	0.136	-1.31	0.058581	0.046259	-21.03	9.449692	13.99678	48.12
21	0.120	0.140	-0.97	0.070857	0.061857	-12.70	6.779599	8.833004	30.29
25	0.124	0.144	-0.66	0.082189	0.076735	-6.636	5.102562	5.967340	19.05
29	0.128	0.148	-0.36	0.092990	0.090290	-2.904	3.941345	4.262944	8.160
33	0.132	0.152	-0.09	0.103352	0.102321	0.998	3.099914	3.197415	3.145
37	0.136	0.156	0.17	0.111651	0.112872	1.094	2.603539	2.505659	-3.721
41	0.140	0.160	0.41	0.120238	0.122108	1.555	2.173981	2.046628	-5.858
45	0.144	0.164	0.64	0.127596	0.130232	2.066	1.892792	1.734181	-8.380
49	0.148	0.168	0.86	0.134921	0.137447	1.871	1.654988	1.518812	-8.228
53	0.152	0.172	1.07	0.141879	0.143934	1.448	1.469312	1.368689	-6.848
57	0.156	0.176	1.27	0.147494	0.149849	1.597	1.369681	1.236150	-7.778
61	0.160	0.180	1.46	0.153368	0.155315	1.269	1.270265	1.188546	-6.433
65	0.164	0.184	1.64	0.158176	0.160437	1.429	1.224511	1.135432	-7.275
69	0.168	0.188	1.82	0.164120	0.165291	0.714	1.140722	1.097569	-3.783
73	0.172	0.192	1.99	0.168549	0.169944	0.828	1.119039	1.070331	-4.353
77	0.176	0.196	2.15	0.172203	0.174438	1.298	1.125282	1.050862	-7.860
81	0.180	0.200	2.30	0.177796	0.178814	0.573	1.068868	1.036835	-2.997
85	0.184	0.204	2.46	0.182401	0.183103	0.385	1.047644	1.026629	-2.006
89	0.188	0.208	2.6	0.186203	0.187319	0.599	1.051334	1.019335	-3.044
93	0.192	0.212	2.74	0.191855	0.191483	-0.194	1.003937	1.014057	1.008
97	0.196	0.216	2.88	0.194501	0.195609	0.570	1.039321	1.010186	-2.803
100	0.199	0.219	2.98	0.198106	0.198683	0.291	1.022677	1.008020	-1.433

CHAPTER 7

Conclusions and Suggestions for Further Work

7.1 Conclusions

The main task of this research was to develop an analytical model for random access CDMA systems in order to assess their performance. A Markovian model in the form of a discrete-time queueing network was built where each node in the model represents a queue of users such that all users at the same node are waiting in the same state of a Markov chain.

In chapter 3, an approximation method called equilibrium point analysis (EPA) was used to solve the analytical model. The EPA method is an approximation technique that is applied to the steady state, and it assumes that the system is always at the equilibrium point. The main advantages of EPA is its simplicity compared to the conventional exact Markov analysis since it does not need to calculate the state transition probabilities needed in Markov analysis, and can be used to study the performance of the multiple access systems which are characterised by multidimensional Markov chains; such as the case of networks with non-zero channel propagation delay, and statistically different users.

The original analytical model was transformed to a stochastically equivalent one in order to make the analysis by the EPA method more tractable. Results of the analytical model were validated against discrete event simulation results in order to determine the accuracy of the EPA technique. The model results showed good matching with the simulation results for the case of zero channel propagation delay, and there was a slight disagreement between the model and the simulation when the channel delay was introduced into the model.

The EPA technique was able to determine the stability behaviour of the modelled random access CDMA networks and consequently the instability regions can be avoided by choosing the appropriate values of the network parameters, such as the processing gain and the retransmission probability.

Both direct sequence and frequency hopping CDMA performances were evaluated using the EPA technique with an emphasis on DS-CDMA. Throughput performance of random access DS-CDMA systems with coherent BPSK and QPSK data modulation schemes were assessed. Non-coherent DPSK and FSK modulation methods were also considered, and both synchronous and asynchronous systems were studied. It was found that BPSK systems have better performance than QPSK systems, and DPSK systems have higher throughput than FSK systems. In both coherent and non-coherent modulation schemes, asynchronous systems outperform synchronous ones.

In chapter 4, the performance of random access DS-CDMA was further investigated by developing the model to accommodate error correction coding. Block and convolutional codes were considered with an emphasis on BCH codes.

By applying BCH codes to the DS-CDMA systems, it was found that there is a considerable improvement in the performance over the uncoded systems. The optimum codes which led to the highest throughput and the lowest packet delay were determined. It was found that BCH codes of code rates in the range of 0.73 to 0.84 have the best performance. These optimum code rates are relatively insensitive to the block lengths, to the processing gain, to the number of users and to the new packet generation probability.

In chapter 5, the EPA model for random access DS-CDMA then was applied to a radio channel and mobile users forming a packet radio network (PRNet). The assumption of an AWGN channel was no longer valid and had to be changed to a multipath channel. The Rician fading channel was used as a model for the multipath radio channel since it can be considered as a general model which characterises most mobile radio communication channels. It was found that there is a considerable degradation in system performance due to the effect of multipath interference and fading, and that DS-CDMA systems in a fading environment can only handle a small number of simultaneous users. To enhance the systems performance in the presence of fading, the processing gain has to be increased as the fading becomes more severe.

The effect of packet size on the performance in a fading environment was studied. It was shown that the selection of the packet size depends on the fading conditions and the load on the networks. It was found that shorter packets are most suitable for networks with heavy load and for channels with severe fading. In contrast, longer packets are suitable for networks with light load and for channels with shallow fading.

Finally, in chapter 6, the assumption of identical users was relaxed, and each user in a random access DS-CDMA network was assumed to belong to a different user class and has different statistical properties to the other users in the network (new packet generation and retransmission probabilities).

An extended equilibrium point analysis (EEPA) method which is an extended version of the original EPA was used to solve the analytical model in the case of non-identical users. The results of the EEPA technique showed good matching with the simulation results when the load of the modelled network is light, then the accuracy degrades with increasing network load.

The assumption of non-identical users allowed the effect of the near/far problem associated with DS-CDMA networks to be studied. That is because in the EEPA each user can be assigned different delivered power depending on its distance to the base station. It was found that DS-CDMA networks are very vulnerable in the presence of the near/far problem, and the performance of these networks deteriorates rapidly for users having their delivered powers 3 to 5 dB below the average power.

7.2 Suggestions for Further Research

In this work slotted random access CDMA networks were considered where each user in a modelled network transmits a packet at the beginning of a time slot. An unslotted version of random access CDMA is a topic for further research, and in this case it is necessary to consider the reception of packets for which the multiple access interference varies over a packet transmission time.

The work of this thesis can also be extended to include the application of CDMA protocols to cellular systems with networks that carry voice traffic instead of the data traffic which was considered throughout this research. Furthermore combined traffic networks, such as asynchronous transfer mode (ATM) networks which can carry voice, data and video information at the same time could be assessed using the analytical model developed in this research with suitable modifications. For example, traffic in an ATM network is characterised by packets (called cells in ATM) having a correlated interarrival time, and this could be incorporated into the EPA models by the use of the various Markov modulated arrival process [Onvu 93].

Appendix A

Derivation of Equations (3.43) and (3.53)

A.1 Derivation of equation (3.43)

The EPA equations are

$$x^{2R+1} \cdot \sigma = x^{R+1}\left(1-\frac{\sigma}{p}\right) \tag{A.1}$$

$$x^{R+1} = x^{R+2} = \dots\dots\dots = x^{2R} = S(x) \tag{A.2}$$

$$x^{1} = x^{2} = \dots\dots\dots = x^{R} = x^{0}p - S(x) \tag{A.3}$$

The constraint equation is

$$\sum_{i=0}^{2R+1} x^{i} = N \tag{A.4}$$

then

$$x^{0} + x^{1} + x^{2} + \cdots\cdots + x^{R} + x^{R+1} + x^{R+2} + \cdots\cdots + x^{2R} + x^{2R+1} = N \tag{A.5}$$

Equation (A.3) gives

$$x^{1} + x^{2} + \cdots\cdots + x^{R} = R \cdot \left[x^{0}p - S(x)\right] \tag{A.6}$$

Equation (A.2) gives

$$x^{R+1} + x^{R+2} + \cdots\cdots + x^{2R} = R \cdot S(x) \tag{A.7}$$

Substituting (A.6) and (A.7) into (A.5) yields

$$x^0 + R \cdot \left[x^0 p - S(x)\right] + R \cdot S(x) + x^{2R+1} = N \tag{A.8}$$

$$x^{2R+1} = N - x^0 - R \cdot \left[x^0 p - S(x)\right] - R \cdot S(x) \tag{A.9}$$

$$x^{2R+1} = N - x^0 - R \cdot x^0 \cdot p \tag{A.10}$$

Substituting (A.10) into (A.1) yields

$$\left(N - x^0 - R \cdot x^0 \cdot p\right)\sigma = S(x) \cdot \left(1 - \frac{\sigma}{p}\right) \tag{A.11}$$

$$N\sigma - x^0\sigma - Rx^0 p\sigma = S(x) \cdot \left(1 - \frac{\sigma}{p}\right) \tag{A.12}$$

$$N\sigma - S(x)\left(1 - \frac{\sigma}{p}\right) = x^0\left(\sigma + R\sigma\, p\right) \tag{A.13}$$

$$x^0 = \frac{N\sigma - S(x)\left(1 - \frac{\sigma}{p}\right)}{\sigma + R\sigma\, p} \tag{A.14}$$

Finally, dividing by σ gives

$$x^0 = \frac{N - \left(1 - \frac{\sigma}{p}\right) \cdot \frac{1}{\sigma} \cdot S(x)}{1 + R \cdot p} \tag{A.15}$$

A.2 Derivation of equation (3.53)

The EPA equations are

$$x^{2R+1} \cdot p = x^{R+1}\left(1 - \frac{p}{\sigma}\right) \tag{A.16}$$

$$x^{R+1} = x^{R+2} = \dots\dots\dots = x^{2R} = x^{0}\sigma - S(x) \tag{A.17}$$

$$x^{1} = x^{2} = \dots\dots\dots = x^{R} = S(x) \tag{A.18}$$

The constraint equation is

$$\sum_{i=0}^{2R+1} x^{i} = N \tag{A.19}$$

then

$$x^{0} + x^{1} + x^{2} + \cdots\cdots + x^{R} + x^{R+1} + x^{R+2} + \cdots\cdots + x^{2R} + x^{2R+1} = N \tag{A.20}$$

Equation (A.18) gives

$$x^{1} + x^{2} + \cdots\cdots + x^{R} = R \cdot S(x) \tag{A.21}$$

Equation (A.17) gives

$$x^{R+1} + x^{R+2} + \cdots\cdots + x^{2R} = R \cdot \left[x^{0}\sigma - S(x)\right] \tag{A.22}$$

Substituting (A.21) and (A.22) into (A.20) yields

$$x^{0} + R \cdot S(x) + R \cdot \left[x^{0}\sigma - S(x)\right] + x^{2R+1} = N \tag{A.23}$$

$$x^{2R+1} = N - x^0 - R \cdot \left[x^0\sigma - S(x)\right] - R \cdot S(x) \tag{A.24}$$

$$x^{2R+1} = N - x^0 - R \cdot x^0 \cdot \sigma \tag{A.25}$$

Substituting (A.25) into (A.16) yields

$$\left(N - x^0 - R \cdot x^0 \cdot \sigma\right)p = \left[x^0\sigma - S(x)\right] \cdot \left(1 - \frac{p}{\sigma}\right) \tag{A.26}$$

$$N \cdot p - x^0 p - Rx^0 p\sigma = x^0 \cdot \sigma \cdot \left(1 - \frac{p}{\sigma}\right) - S(x) \cdot \left(1 - \frac{p}{\sigma}\right) \tag{A.27}$$

$$N \cdot p + S(x)\left(1 - \frac{p}{\sigma}\right) = x^0\left[p + R \cdot p \cdot \sigma + \sigma \cdot \left(1 - \frac{p}{\sigma}\right)\right] \tag{A.28}$$

$$x^0 = \frac{N \cdot p + S(x)\left(1 - \frac{p}{\sigma}\right)}{p + R \cdot p \cdot \sigma + \sigma \cdot \left(1 - \frac{p}{\sigma}\right)} \tag{A.29}$$

Finally, dividing by p gives

$$x^0 = \frac{N + S(x)\left(1 - \frac{p}{\sigma}\right) \cdot \frac{1}{p}}{1 + R \cdot \sigma + \frac{\sigma}{p} \cdot \left(1 - \frac{p}{\sigma}\right)} \tag{A.30}$$

$$x^0 = \frac{N + \left(1 - \frac{p}{\sigma}\right) \cdot \frac{1}{p} \cdot S(x)}{R \cdot \sigma + \frac{\sigma}{p}} \tag{A.31}$$

Appendix B

BCH Codes List

Table (B.1) List of all possible BCH error correction codes of block length of 7 to 1023 bits [Pete 72], where n, k and t are the block length, the number of information bits, and the number of correctable errors in a block of n bits, respectively.

n	k	t	n	k	t	n	k	t
7	4	1	127	85	6	255	131	18
15	11	1		78	7		123	19
	7	2		71	9		115	21
	5	3		64	10		107	22
				57	11		99	23
31	26	1		50	13		91	25
	21	2		43	14		87	26
	16	3		36	15		79	27
	11	5		29	21		71	29
	6	7		22	23		63	30
				15	27		55	31
63	57	1		8	31		47	42
	51	2					45	43
	45	3	255	247	1		37	45
	39	4		239	2		29	47
	36	5		231	3		21	55
	30	6		223	4		13	59
	24	7		215	5		9	63
	18	10		207	6			
	16	11		199	7	511	502	1
	10	13		191	8		493	2
	7	15		187	9		484	3
				179	10		475	4
127	120	1		171	11		466	5
	113	2		163	12		457	6
	106	3		155	13		448	7
	99	4		147	14		439	8
	92	5		139	15		430	9

n	k	t	n	k	t	n	k	t
511	421	10	511	28	111	1023	608	45
	412	11		19	119		598	46
	403	12		10	127		588	47
	394	13					578	49
	385	14	1023	1013	1		573	50
	376	15		1003	2		563	51
	367	16		993	3		553	52
	358	18		983	4		543	53
	349	19		973	5		533	54
	340	20		963	6		523	55
	331	21		953	7		513	57
	322	22		943	8		503	58
	313	23		933	9		493	59
	304	25		923	10		483	60
	295	26		913	11		473	61
	286	27		903	12		463	62
	277	28		893	13		453	63
	268	29		883	14		443	73
	259	30		873	15		433	74
	250	31		863	16		423	75
	241	36		858	17		413	77
	238	37		848	18		403	78
	229	38		838	19		393	79
	220	39		828	20		383	82
	211	41		818	21		378	83
	202	42		808	22		368	85
	193	43		798	23		358	86
	184	45		788	24		348	87
	175	46		778	25		338	89
	166	47		768	26		328	90
	157	51		758	27		318	91
	148	53		748	28		308	93
	139	54		738	29		298	94
	130	55		738	30		288	95
	121	58		718	31		278	102
	112	59		708	34		268	103
	103	61		698	35		258	106
	94	62		688	36		248	107
	85	63		678	37		238	109
	76	85		668	38		228	110
	67	87		658	39		218	111
	58	91		648	41		208	115
	49	93		638	42		203	117
	40	95		628	43		193	118
	31	109		618	44		183	119

Appendix C

Derivation of Equation (4.2)

The probability of bit error for Additive White Gaussian Noise (AWGN) channel when PSK modulation is used can be written as [Proa 95]

$$p_e = \frac{1}{2} erfc\sqrt{\gamma_b} \tag{C.1}$$

where $\gamma_b = \frac{\alpha^2 \xi_b}{N_0}$ is the received signal to noise ratio per information bit, ξ_b is the energy per information bit, N_0 is the power spectral density of AWGN, and α is the channel attenuation.

If (n,k) code is used as channel coding with code rate R_c where $R_c = \frac{k}{n}$, the energy per information bit becomes

$$\xi_b = \frac{\xi_t}{k} = \frac{n \cdot \xi}{k} = \frac{\xi}{R_c} \tag{C.2}$$

then

$$\xi = \xi_b \cdot R_c \tag{C.3}$$

where ξ is the transmitted signal energy per coded bit, and ξ_t is the transmitted signal energy per coded word

Then the probability of coded bit error becomes

$$
\begin{aligned}
p_e &= \frac{1}{2} erfc\left[\sqrt{\frac{\alpha^2 \xi}{N_0}}\right] \\
&= \frac{1}{2} erfc\left[\sqrt{\frac{\alpha^2 \xi_b}{N_0} \cdot R_c}\right] \\
&= \frac{1}{2} erfc\left[\sqrt{\gamma_b \cdot R_c}\right]
\end{aligned}
\tag{C.4}
$$

For direct sequence spread spectrum ξ_b can be written as

$$
\xi_b = P_{av} \cdot T_b = \frac{P_{av}}{R} \tag{C.5}
$$

where R is the information rate, and P_{av} is the average signal power.

The power spectral density of the jamming signal is

$$
J_0 = \frac{J_{av}}{W} \tag{C.6}
$$

where J_{av} is the average jamming power, and W is the pseudo noise (PN) rate.

The signal to jamming ratio can be written as

$$
\frac{\xi_b}{J_0} = \frac{P_{av} / R}{J_{av} / W} = \frac{W / R}{J_{av} / P_{av}} = \frac{G}{J_{av} / P_{av}} \tag{C.7}
$$

where J_{av}/P_{av} is the jamming to signal power ratio, and G is the processing gain or the bandwidth expansion.

So, the probability of bit error for uncoded spread spectrum is

$$
p_e = \frac{1}{2} erfc\left[\sqrt{\frac{G}{J_{av} / P_{av}}}\right] \tag{C.8}
$$

If a code with R_c code rate is used then the probability of coded bit error becomes

$$
\begin{aligned}
p_e &= \frac{1}{2} erfc\left[\sqrt{\frac{\xi}{J_0}}\right] \\
&= \frac{1}{2} erfc\left[\sqrt{\frac{\xi_b}{J_0} \cdot R_c}\right] \\
&= \frac{1}{2} erfc\left[\sqrt{\frac{G}{J_{av} / P_{av}} \cdot R_c}\right]
\end{aligned}
\tag{C.9}
$$

For spread spectrum DS/CDMA

$$
\frac{J_{av}}{P_{av}} = \frac{m-1}{1} \tag{C.10}
$$

where m is number of simultaneous users attempting transmission in a given slot, each transmits one packet in a given slot.

Finally, the probability of bit error for DS/CDMA can be expressed as

$$
p_e = \frac{1}{2} erfc\left[\sqrt{\frac{G}{m-1} \cdot R_c}\right] = \frac{1}{2} erfc\left[\sqrt{\frac{G}{m-1} \cdot \frac{k}{n}}\right] \tag{C.11}
$$

Appendix D

Throughput Performance of Random Access DS-CDMA Using Different BCH Error Correction Codes

Note that the throughput values which are underlined represent the maximum achievable throughput (capacity) and thus the corresponding error correction codes are optimal. The optimum codes of these results are shown in Chapter 4 in Tables(4.1) through (4.5). R_c, t and p represent the code rate, the error correction capability and the retransmission probability.

Table (D.1) n=127, L_p=1016, σ=0.2, G=60 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.945	1	9.58897	8.804846	6.975414	3.903017	0.956004	0.104283
0.89	2	10.72283	10.39796	9.290938	6.993321	2.946178	0.457684
0.835	3	11.10814	11.04628	10.41086	8.951673	5.497553	1.128179
0.78	**4**	11.02666	**11.10536**	10.74764	9.84052	7.537081	2.052019
0.724	5	10.62747	10.77291	10.56425	9.966074	8.486594	3.019183
0.669	6	10.00199	10.16809	10.02885	9.582177	8.512708	3.627826
0.614	7	9.211388	9.367488	9.247365	8.851637	7.910335	3.464702
0.559	9	8.898479	9.124455	9.153645	9.051859	8.853639	8.087323
0.504	10	7.812766	7.983976	7.958799	7.773214	7.373588	5.327585
0.449	11	6.675474	6.777342	6.67048	6.345213	5.555132	2.194964
0.394	13	5.864825	5.957935	5.869709	5.595209	4.934444	2.051239
0.339	14	4.637607	4.630102	4.40702	3.88122	2.575358	0.561379
0.283	15	3.447267	3.329407	2.958295	2.189262	0.862724	0.122937
0.228	21	3.1867	3.20607	3.093742	2.815413	2.126938	0.613916
0.173	23	2.049884	1.973896	1.740432	1.269954	0.504405	0.07898
0.118	27	1.180094	1.072452	0.832835	0.446449	0.105191	0.011799
0.063	31	0.391071	0.275216	0.117103	0.02072	0.001544	6.37E-05

Table (D.2) n=255, L_p=1020, σ=0.2, G=60 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.969	1	9.344204	8.377118	6.291263	3.088531	0.621459	0.057373
0.937	2	10.61273	10.04923	8.540382	5.64536	1.763613	0.217836
0.906	3	11.34848	11.06126	10.01971	7.789017	3.468327	0.528762
0.875	4	11.74858	11.6538	10.95499	9.351822	5.506529	1.01582
0.843	5	11.91905	11.95213	11.49711	10.37786	7.47005	1.694624
0.812	**6**	11.91522	**12.03474**	11.74616	10.97046	8.976375	2.577168
0.78	7	11.77761	11.95124	11.77584	11.23244	9.905179	3.683238
0.749	8	11.53578	11.74057	11.6378	11.24674	10.34629	5.01898
0.733	9	11.67716	11.93706	11.93769	11.7451	11.36094	9.525196
0.702	10	11.28297	11.54515	11.56797	11.4254	11.15668	10.00709
0.671	11	10.83105	11.0866	11.1191	11.00121	10.78648	9.900662
0.639	12	10.33075	10.57393	10.60513	10.49337	10.29045	9.453065
0.608	13	9.790199	10.01619	10.03692	9.91578	9.690014	8.72323
0.576	14	9.216434	9.419542	9.421963	9.276392	8.990004	7.62812
0.545	15	8.612841	8.789206	8.766042	8.57952	8.190865	5.9001
0.514	18	8.556334	8.779932	8.852315	8.845358	8.846039	8.75137
0.482	19	7.866295	8.057097	8.094537	8.034161	7.932835	7.532681
0.451	21	7.39209	7.576417	7.618826	7.576598	7.510084	7.236082
0.42	22	6.66218	6.803824	6.795221	6.669236	6.41273	5.05718
0.388	23	5.92827	6.019348	5.944282	5.699454	5.112311	2.023685
0.357	25	5.366897	5.43611	5.341229	5.066036	4.377466	1.452237
0.341	26	5.074703	5.129569	5.020626	4.720632	3.952205	1.176731
0.31	27	4.329761	4.318818	4.115259	3.63222	2.381594	0.462055
0.278	29	3.728518	3.679983	3.430695	2.866893	1.545998	0.253619
0.247	30	3.01544	2.895287	2.546223	1.8257	0.641985	0.079045
0.216	31	2.344894	2.159989	1.731346	0.972156	0.214953	0.019286
0.184	42	2.434334	2.401433	2.234233	1.859788	1.01152	0.179855
0.176	43	2.298572	2.259639	2.086604	1.705056	0.876282	0.150155
0.145	45	1.633871	1.533365	1.281602	0.809043	0.2282	0.026049
0.114	47	1.036768	0.89002	0.605339	0.231791	0.032623	0.002162
0.082	55	0.683037	0.562569	0.346739	0.109446	0.013029	0.000778
0.051	59	0.26729	0.161384	0.046894	0.004855	0.000208	4.7E-06
0.035	63	0.130749	0.057276	0.008565	0.00048	1.18E-05	2E-07

Table (D.3) n=511, L_p=1022, σ=0.2, G=60 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.982	1	8.971671	7.836883	5.545434	2.372268	0.400104	0.031918
0.93	4	11.49998	11.12148	9.92065	7.399311	2.901394	0.39244
0.877	7	12.35201	12.33432	11.78506	10.46852	7.010533	1.362684
0.824	10	12.47626	12.62934	12.41162	11.77827	10.179	3.175034
0.771	13	12.17604	12.40127	12.34072	12.02872	11.35536	6.611163
0.718	16	11.59914	11.84432	11.84857	11.67543	11.34726	9.708532
0.665	20	11.08527	11.35193	11.42458	11.38701	11.34844	11.1153
0.595	25	10.00856	10.25477	10.3342	10.32618	10.33755	10.25353
0.507	30	8.167784	8.33369	8.326762	8.188074	7.917368	6.396568
0.448	38	7.159171	7.353162	7.3894	7.326506	7.205645	6.735431
0.378	43	5.250023	5.290768	5.115183	4.675061	3.597394	1.073142
0.307	51	3.434079	3.245485	2.751851	1.825816	0.608601	0.090493
0.237	58	1.989953	1.667094	1.069936	0.372327	0.051204	0.003657

Table (D.4) n=1023, L_p=1023, σ=0.2, G=60 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.99	1	8.56885	7.294013	4.862399	1.828912	0.267356	0.018997
0.951	5	11.26517	10.713	9.224927	6.26054	1.960358	0.226235
0.902	10	12.47875	12.37209	11.66719	10.02705	5.863776	0.9624
0.853	15	12.81129	12.91958	12.61863	11.81524	9.67626	2.392133
0.809	20	12.82065	13.03931	12.95902	12.60423	11.8239	5.825145
0.761	25	12.38269	12.6359	12.64391	12.47019	12.15482	10.53761
0.712	30	11.3569	11.60824	11.60798	11.41956	11.04395	9.129479
0.682	35	10.45115	10.6197	10.50279	10.1073	9.186692	4.317619
0.633	41	9.035579	9.068613	8.743919	7.941535	5.845985	1.372518
0.594	45	7.924798	7.82253	7.295421	6.111611	3.376465	0.594636
0.56	50	7.009713	6.789925	6.088959	4.611777	1.901436	0.276038
0.511	55	5.830338	5.471565	4.576005	2.878173	0.784171	0.084064
0.472	60	4.906967	4.429896	3.395162	1.702366	0.321629	0.026216
0.443	63	4.265621	3.728019	2.64837	1.113017	0.173075	0.012499
0.433	73	4.086459	3.548838	2.47867	0.993627	0.140796	0.008897
0.394	78	3.295923	2.691722	1.619729	0.472233	0.048376	0.002341
0.36	85	2.677894	2.039243	1.029603	0.21442	0.015679	0.000551

Table (D.5) n=511, L_p=1022, σ=0.2, G=60 and N=100.

		Throughput					
R_c	t	$p=0.05$	$p=0.1$	$p=0.15$	$p=0.2$	$p=0.25$	$p=0.3$
0.912	5	11.89782	11.66919	10.73234	8.662542	4.190731	0.638674
0.894	6	12.17314	12.06166	11.34055	9.678237	5.599358	0.959791
0.877	7	12.35201	12.33432	11.78506	10.46852	7.010533	1.362684
0.859	8	12.45374	12.50843	12.09651	11.0624	8.29632	1.855297
0.841	9	12.49211	12.6026	12.29948	11.49001	9.363948	2.450489
0.824	**10**	12.47626	**12.62934**	12.41162	11.77827	10.179	3.175034
0.806	11	12.41422	12.60021	12.44778	11.95158	10.75846	4.064407
0.789	12	12.3121	12.52108	12.42067	12.02963	11.13862	5.183758
0.771	13	12.17604	12.40127	12.34072	12.02872	11.35536	6.611163
0.753	14	12.00951	12.24521	12.2134	11.96267	11.44634	8.133839

Table (D.6) n=1023, L_p=1023, σ=0.2, G=60 and N=100.

		Throughput					
R_c	t	$p=0.05$	$p=0.1$	$p=0.15$	$p=0.2$	$p=0.25$	$p=0.3$
0.853	15	12.81129	12.91958	12.61863	11.81524	9.67626	2.392133
0.844	16	12.81473	12.94784	12.70024	12.00521	10.20302	2.789178
0.839	**17**	12.94734	**13.11852**	12.93716	12.37954	11.00447	3.593853
0.829	18	12.91804	13.10795	12.96555	12.48709	11.34815	4.206108
0.819	19	12.87696	13.0822	12.9722	12.56069	11.61902	4.930516
0.809	20	12.82065	13.03931	12.95902	12.60423	11.8239	5.825145
0.8	21	12.75419	12.98263	12.92581	12.62066	11.97554	6.939039
0.79	22	12.67677	12.91434	12.87685	12.61355	12.07548	8.19708
0.78	23	12.588	12.83289	12.81266	12.58452	12.13491	9.296415
0.77	24	12.48837	12.7378	12.73289	12.5341	12.15722	10.04479
0.761	25	12.38269	12.6359	12.64391	12.47019	12.15482	10.53761

Table (D.7) n=127, L_p=1116, σ=0.25, G=60 and N=100.

		Throughput					
R_c	t	$p=0.05$	$p=0.1$	$p=0.15$	$p=0.2$	$p=0.25$	$p=0.3$
0.945	1	9.594489	8.396462	6.207567	3.257469	0.874788	0.102606
0.89	2	10.80347	10.01306	8.326199	5.640309	2.359716	0.430373
0.835	**3**	**11.25703**	10.7376	9.449614	7.230022	3.903614	0.973767
0.78	4	11.22792	10.88198	9.877464	8.064202	5.046212	1.57376
0.724	5	10.85866	10.61859	9.802765	8.288667	5.630952	2.031512
0.669	6	10.23896	10.05521	9.356792	8.041509	5.678448	2.209581

0.614	7	9.432241	9.267302	8.633424	7.436469	5.277688	2.071989
0.559	9	9.187849	9.16155	8.767113	7.963999	6.425118	3.446667
0.504	10	8.034634	7.959459	7.52865	6.6856	5.109254	2.380301
0.449	11	6.827426	6.692128	6.208278	5.301995	3.688405	1.392232
0.394	13	6.000459	5.887358	5.470274	4.687996	3.291878	1.27433
0.339	14	4.705309	4.509135	4.009555	3.135993	1.774631	0.469936
0.283	15	3.469314	3.198162	2.638536	1.752991	0.696085	0.116436
0.228	21	3.242532	3.13997	2.843527	2.314571	1.451837	0.474245
0.173	23	2.062437	1.896774	1.556576	1.027607	0.413289	0.074813
0.118	27	1.179493	1.020809	0.740418	0.374852	0.097218	0.011631
0.063	31	0.386914	0.259536	0.106209	0.018452	0.001539	6.37E-05

Table (D.8) n=255, L_p=1020, σ=0.25, G=60 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.969	1	9.330474	7.962621	5.588508	2.610372	0.583799	0.056837
0.937	2	10.65021	9.612753	7.591149	4.578993	1.519001	0.211011
0.906	3	11.43995	10.65574	8.968917	6.223444	2.693758	0.493021
0.875	4	11.89518	11.30512	9.899695	7.485258	3.893255	0.89062
0.843	5	12.11511	11.66928	10.49328	8.394167	4.965656	1.364099
0.812	**6**	**12.15231**	11.81877	10.8227	8.99691	5.828535	1.861798
0.78	7	12.04681	11.79588	10.94202	9.340653	6.451367	2.32133
0.749	8	11.8269	11.63534	10.89002	9.467531	6.834649	2.696966
0.733	9	12.02814	11.92948	11.33043	10.14809	7.879923	3.703441
0.702	10	11.6395	11.5638	11.02507	9.945781	7.860449	3.865696
0.671	11	11.17991	11.11866	10.618	9.61196	7.659596	3.848104
0.639	12	10.66348	10.60451	10.12675	9.16711	7.303911	3.664022
0.608	13	10.09945	10.03253	9.5649	8.62904	6.817893	3.339712
0.576	14	9.494086	9.413533	8.943177	8.013029	6.223949	2.910421
0.545	15	8.857498	8.754337	8.272151	7.331763	5.544226	2.41813
0.514	18	8.88053	8.890249	8.594722	7.968346	6.726577	4.010635
0.482	19	8.131569	8.099962	7.76207	7.074833	5.731734	3.01541
0.451	21	7.648655	7.629778	7.329566	6.71142	5.499456	2.996484
0.42	22	6.856297	6.78758	6.429929	5.729619	4.391976	1.991014
0.388	23	6.068421	5.946959	5.531825	4.749424	3.32205	1.217624
0.357	25	5.483694	5.354401	4.944869	4.183605	2.823711	0.964856
0.341	26	5.178261	5.041345	4.631234	3.875062	2.545301	0.826646
0.31	27	4.391301	4.199166	3.729581	2.904765	1.605127	0.390187
0.278	29	3.768808	3.557708	3.082169	2.274323	1.109932	0.227537
0.247	30	3.030247	2.772419	2.257363	1.450096	0.526061	0.075689
0.216	31	2.343888	2.051142	1.523583	0.792386	0.19561	0.019012
0.184	42	2.460641	2.323066	2.011072	1.485067	0.735837	0.160874
0.176	43	2.321383	2.182463	1.874578	1.360693	0.650069	0.136064

0.145	45	1.637079	1.462444	1.133989	0.655716	0.199974	0.025393
0.114	47	1.031461	0.840572	0.53499	0.198762	0.031558	0.002155
0.082	55	0.678282	0.530724	0.308249	0.095669	0.012777	0.000777
0.051	59	0.26368	0.151755	0.042552	0.004176	0.000208	4.7E-06
0.035	63	0.128678	0.053885	0.007662	0.000389	1.18E-05	2E-07

Table (D.9) n=511, L_p=1022, σ=0.25, G=60 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.982	1	8.942155	7.430257	4.926608	2.033454	0.383174	0.031744
0.93	4	11.57381	10.67587	8.831173	5.885593	2.319161	0.371701
0.877	7	12.53271	12.00063	10.68736	8.370801	4.674503	1.141623
0.824	10	12.74462	12.43191	11.47281	9.687949	6.480782	2.128492
0.771	13	12.50311	12.31643	11.58293	10.16589	7.482299	2.99672
0.718	16	11.9486	11.82931	11.22977	10.04491	7.747945	3.473601
0.665	20	11.4811	11.44628	11.0055	10.10109	8.300376	4.449115
0.595	25	10.38416	10.3724	10.00928	9.250497	7.733241	4.36822
0.507	30	8.407523	8.30968	7.868419	7.00388	5.330073	2.290432
0.448	38	7.40554	7.402979	7.103143	6.475542	5.241525	2.742513
0.378	43	5.345197	5.189403	4.71616	3.866223	2.465158	0.814067
0.307	51	3.445022	3.106052	2.452586	1.493537	0.525575	0.087298
0.237	58	1.978799	1.576484	0.952924	0.325037	0.049946	0.003647

Table (D.10) n=1023, L_p=1023, σ=0.25, G=60 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.99	1	8.527333	6.904296	4.326997	1.587152	0.259312	0.018933
0.951	5	11.3052	10.23406	8.159647	5.000358	1.659691	0.218776
0.902	10	12.62912	11.98121	10.49551	7.925809	4.036469	0.847107
0.853	15	13.06048	12.66332	11.5757	9.578868	6.064578	1.758299
0.809	20	13.15453	12.92558	12.11784	10.56763	7.630034	2.862445
0.761	25	12.75554	12.61782	11.9717	10.70004	8.214942	3.566088
0.712	30	11.69975	11.59775	11.00626	9.835624	7.572878	3.408658
0.682	35	10.70152	10.50468	9.803432	8.484773	6.09007	2.439724
0.633	41	9.186108	8.857725	7.98717	6.428526	3.858226	1.090209
0.594	45	8.010376	7.566647	6.565791	4.872292	2.432933	0.529879
0.56	50	7.054368	6.520743	5.423235	3.667848	1.5035	0.259804
0.511	55	5.83966	5.212696	4.037885	2.319284	0.687041	0.082062
0.472	60	4.89487	4.194966	2.983998	1.407516	0.30007	0.025975
0.443	63	4.245346	3.520131	2.331031	0.941929	0.16575	0.012437

0.433	73	4.067239	3.351264	2.183682	0.843702	0.135562	0.008864
0.394	78	3.270084	2.533246	1.434147	0.411208	0.047597	0.002339
0.36	85	2.650363	1.915139	0.917216	0.188279	0.015578	0.000551

Table (D.11) n=511, L_p=1022, σ=0.25, G=60 and N=100.

		Throughput					
R_c	t	$p=0.05$	$p=0.1$	$p=0.15$	$p=0.2$	$p=0.25$	$p=0.3$
0.912	5	12.00789	11.25252	9.607981	6.866225	3.123715	0.586537
0.894	6	12.31967	11.6842	10.21651	7.69031	3.92061	0.846294
0.877	7	12.53271	12.00063	10.68736	8.370801	4.674503	1.141623
0.859	8	12.66757	12.22102	11.04293	8.921764	5.361071	1.462459
0.841	9	12.73448	12.36067	11.30023	9.357043	5.965742	1.796502
0.824	**10**	**12.74462**	12.43191	11.47281	9.687949	6.480782	2.128492
0.806	11	12.70559	12.44455	11.5724	9.92727	6.904134	2.445674
0.789	12	12.6236	12.40316	11.60519	10.08363	7.236885	2.740073
0.771	13	12.50311	12.31643	11.58293	10.16589	7.482299	2.99672
0.753	14	12.34677	12.18981	11.50773	10.18296	7.64563	3.207497

Table (D.12) n=1023, L_p=1023, σ=0.25, G=60 and N=100.

		Throughput					
R_c	t	$p=0.05$	$p=0.1$	$p=0.15$	$p=0.2$	$p=0.25$	$p=0.3$
0.853	15	13.06048	12.66332	11.5757	9.578868	6.064578	1.758299
0.844	16	13.07727	12.71982	11.69311	9.791381	6.38487	1.950526
0.839	**17**	**13.23783**	12.93053	11.97906	10.19633	6.927106	2.288507
0.829	18	13.22594	12.9454	12.04698	10.34923	7.192946	2.486327
0.819	19	13.19531	12.94511	12.09218	10.4718	7.426964	2.679394
0.809	20	13.15453	12.92558	12.11784	10.56763	7.630034	2.862445
0.8	21	13.09624	12.89135	12.12179	10.63827	7.802717	3.033267
0.79	22	13.02895	12.84414	12.10838	10.68587	7.946553	3.190288
0.78	23	12.94949	12.78024	12.07849	10.71129	8.062112	3.331697
0.77	24	12.85532	12.70299	12.03179	10.71366	8.149745	3.456007
0.761	25	12.75554	12.61782	11.9717	10.70004	8.214942	3.566088

Table (D.13) n=127, L_p=1116, σ=0.2, G=30 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.945	1	3.942677	1.947	0.36222	0.023626	0.000638	8.8E-06
0.89	2	4.598842	2.730894	0.751744	0.070404	0.002587	4.74E-05
0.835	3	4.893427	3.198572	1.104427	0.133183	0.006054	0.000135
0.78	**4**	**4.944702**	3.416611	1.350629	0.193852	0.0102	0.00026
0.724	5	4.820543	3.437826	1.470454	0.235635	0.013613	0.000379
0.669	6	4.562273	3.30424	1.468126	0.248806	0.015069	0.000438
0.614	7	4.203674	3.050597	1.362036	0.232318	0.014133	0.000412
0.559	9	4.168225	3.19525	1.639109	0.35202	0.026469	0.000945
0.504	10	3.614916	2.704925	1.305655	0.252841	0.017302	0.000566
0.449	11	3.035213	2.185202	0.95692	0.158759	0.009464	0.000272
0.394	13	2.672029	1.930891	0.854307	0.144584	0.008829	0.000261
0.339	14	2.048152	1.363631	0.491716	0.062352	0.002953	6.86E-05
0.283	15	1.463809	0.857554	0.227076	0.020072	0.000693	1.19E-05
0.228	21	1.431708	0.988731	0.393531	0.05871	0.003341	9.56E-05
0.173	23	0.873611	0.508853	0.135115	0.012439	0.000464	8.9E-06
0.118	27	0.481124	0.231668	0.041431	0.002675	0.000074	1.1E-06
0.063	31	0.139101	0.032917	0.002012	5.85E-05	8E-07	0

Table (D.14) n=255, L_p=1020, σ=0.2, G=30 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.906	3	4.863837	2.942611	0.839216	0.079844	0.002916	5.25E-05
0.875	4	5.129947	3.318683	1.110272	0.126145	0.005332	0.00011
0.843	5	5.278692	3.578405	1.341419	0.175654	0.008341	0.000191
0.812	6	5.338141	3.741526	1.524727	0.223218	0.011626	0.000289
0.78	7	5.324679	3.823329	1.655358	0.26431	0.014795	0.000393
0.749	8	5.251157	3.835725	1.73389	0.295441	0.017458	0.000487
0.733	**9**	**5.387996**	4.054576	1.976949	0.380564	0.025027	0.00077
0.702	10	5.224604	3.960728	1.967956	0.390735	0.026376	0.000831
0.671	11	5.023113	3.821344	1.915602	0.385849	0.026352	0.000838
0.639	12	4.789675	3.643297	1.825559	0.367185	0.02502	0.000794
0.608	13	4.529757	3.432871	1.703877	0.336969	0.022597	0.000706
0.576	14	4.247044	3.19554	1.556658	0.298072	0.019407	0.00059
0.545	15	3.94747	2.936876	1.390291	0.253771	0.015825	0.000462
0.514	18	4.023962	3.143831	1.685872	0.383095	0.02922	0.001031
0.482	19	3.659401	2.804515	1.432273	0.297968	0.020962	0.000687
0.451	21	3.449355	2.657547	1.375857	0.293638	0.021191	0.000712
0.42	22	3.062376	2.291685	1.1017	0.206898	0.013306	0.000402

0.388	23	2.680256	1.933742	0.847055	0.137083	0.007731	0.000207
0.357	25	2.413158	1.717849	0.727329	0.111972	0.00606	0.000156
0.341	26	2.273021	1.601539	0.66102	0.098092	0.005149	0.000129
0.31	27	1.895888	1.255886	0.443508	0.053159	0.00232	4.89E-05
0.278	29	1.609978	1.02147	0.324247	0.034105	0.001333	2.54E-05
0.247	30	1.264339	0.723842	0.179502	0.014403	0.000445	6.8E-06
0.216	31	0.949634	0.471988	0.085537	0.005111	0.000122	1.4E-06
0.184	42	1.055384	0.670405	0.215265	0.023541	0.000982	2.03E-05
0.176	43	0.99278	0.622161	0.193511	0.020426	0.000828	1.67E-05
0.145	45	0.674974	0.358581	0.076181	0.005413	0.000155	2.3E-06
0.114	47	0.402816	0.161332	0.019663	0.00088	1.64E-05	2E-07
0.082	55	0.26012	0.092529	0.009418	0.000377	6.5E-06	1E-07
0.051	59	0.088798	0.014735	0.000598	1.26E-05	1E-07	0
0.035	63	0.0394	0.003726	9.61E-05	1.5E-06	0	0

Table (D.15) n=511, L_p=1022, σ=0.2, G=30 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.877	7	5.410885	3.600017	1.282701	0.15457	0.006719	0.00014
0.824	10	5.587362	3.965505	1.661053	0.248387	0.012873	0.000314
0.771	13	5.533049	4.070312	1.867545	0.320173	0.018564	0.000501
0.718	16	5.313994	3.982872	1.916347	0.354021	0.021834	0.000623
0.665	20	5.148952	3.964015	2.043073	0.424549	0.028994	0.000906
0.595	25	4.666508	3.620853	1.902944	0.40924	0.028776	0.000923
0.507	30	3.727514	2.778083	1.316388	0.236105	0.014105	0.000389
0.448	38	3.366442	2.581461	1.314411	0.270166	0.018476	0.000578
0.378	43	2.384676	1.662626	0.675315	0.101836	0.005573	0.000144
0.307	51	1.428879	0.778918	0.181667	0.015306	0.000561	1.12E-05
0.237	58	0.748437	0.273181	0.03023	0.001357	2.69E-05	3E-07

Table (D.16) n=1023, L_p=1023, σ=0.2, G=30 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.902	10	5.3943	3.483901	1.148661	0.124158	0.004868	9.17E-05
0.853	15	5.675375	3.950992	1.571625	0.215573	0.010232	0.000229
0.809	20	5.786192	4.224471	1.896511	0.310427	0.017023	0.000433
0.761	25	5.65122	4.224671	2.015435	0.362749	0.021499	0.000585
0.712	30	5.25606	3.945436	1.903118	0.352826	0.021756	0.000617
0.682	35	4.751001	3.453561	1.550967	0.266577	0.016442	0.000491
0.633	41	4.024873	2.757151	1.061988	0.144799	0.007215	0.000177

0.594	45	3.428533	2.179362	0.6979	0.075117	0.003048	6.11E-05
0.56	50	2.974204	1.77128	0.487913	0.04496	0.001615	2.89E-05
0.511	55	2.400414	1.277379	0.269964	0.018762	0.000518	7.2E-06
0.472	60	1.951306	0.911226	0.145945	0.007926	0.000175	0.000002
0.443	63	1.66704	0.708811	0.096093	0.00465	9.31E-05	0.000001
0.433	73	1.577837	0.643693	0.08002	0.003567	6.54E-05	6E-07
0.394	78	1.238628	0.429618	0.041091	0.001525	2.36E-05	2E-07
0.36	85	0.982958	0.28257	0.020289	0.000609	7.6E-06	1E-07

Table (D.17) n=511, L_p=1022, σ=0.2, G=30 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.877	7	5.410885	3.600017	1.282701	0.15457	0.006719	0.00014
0.859	8	5.502104	3.756382	1.427305	0.186808	0.008683	0.000192
0.841	9	5.559033	3.876904	1.553907	0.218423	0.010761	0.000251
0.824	10	5.587362	3.965505	1.661053	0.248387	0.012873	0.000314
0.806	**11**	**5.591635**	4.02535	1.748749	0.275809	0.014931	0.000379
0.789	12	5.571568	4.059572	1.81736	0.299939	0.016854	0.000443
0.771	13	5.533049	4.070312	1.867545	0.320173	0.018564	0.000501
0.753	14	5.475516	4.059831	1.900421	0.336092	0.019996	0.000552

Table (D.18) n=1023, L_p=1023, σ=0.2, G=30 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.839	17	5.790121	4.133381	1.752783	0.263349	0.013471	0.000322
0.829	**18**	**5.796277**	4.172505	1.807226	0.280133	0.014697	0.000359
0.819	19	5.794506	4.20267	1.855073	0.295864	0.015886	0.000396
0.809	20	5.786192	4.224471	1.896511	0.310427	0.017023	0.000433
0.8	21	5.771039	4.238145	1.931737	0.323711	0.018095	0.000468
0.79	22	5.74863	4.244462	1.961169	0.335633	0.019089	0.000501
0.78	23	5.720552	4.243687	1.984577	0.346109	0.019993	0.000532
0.77	24	5.686945	4.236158	2.002143	0.355086	0.020796	0.00056
0.761	25	5.65122	4.224671	2.015435	0.362749	0.021499	0.000585

Table (D.19) n=127, L_p=1116, σ=0.2, G=45 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.835	3	8.137852	7.246553	5.360462	2.494342	0.44167	0.034592
0.78	**4**	**8.145587**	7.452342	5.869008	3.17834	0.689008	0.062886
0.724	5	7.893846	7.332106	5.978417	3.532153	0.874671	0.087642
0.669	6	7.45009	6.970975	5.77771	3.559202	0.94436	0.099059
0.614	7	6.863573	6.427533	5.33955	3.30728	0.884431	0.093101
0.559	9	6.718249	6.452063	5.665582	4.058936	1.459265	0.18908
0.504	10	5.8606	5.566838	4.772753	3.211123	1.007376	0.119312
0.449	11	4.964199	4.631039	3.814447	2.312391	0.59716	0.061723
0.394	13	4.365182	4.078901	3.371616	2.065411	0.546607	0.058063
0.339	14	3.401308	3.055139	2.308724	1.127688	0.211032	0.01725
0.283	15	2.485082	2.094142	1.360227	0.468527	0.057675	0.003307
0.228	21	2.354794	2.151089	1.689879	0.91865	0.209061	0.021136
0.173	23	1.479969	1.241957	0.800618	0.277775	0.0365	0.002352
0.118	27	0.836808	0.635747	0.323003	0.072901	0.006462	0.0003
0.063	31	0.263437	0.132639	0.026538	0.001975	6.52E-05	1.2E-06

Table (D.20) n=255, L_p=1020, σ=0.2, G=45 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.906	3	8.214228	7.037845	4.755452	1.780965	0.238182	0.014496
0.875	4	8.573131	7.596729	5.559711	2.495604	0.408825	0.029192
0.843	5	8.755602	7.937338	6.129508	3.13292	0.605192	0.048877
0.812	**6**	**8.800582**	8.109659	6.500477	3.649759	0.8056	0.071557
0.78	7	8.739649	8.145536	6.703298	4.027523	0.990029	0.094383
0.749	8	8.589858	8.071095	6.764959	4.263686	1.13767	0.114275
0.733	9	8.755789	8.341009	7.209277	4.928589	1.541196	0.171796
0.702	10	8.475834	8.102502	7.05621	4.921881	1.604962	0.18317
0.671	11	8.14307	7.797307	6.814878	4.798654	1.595866	0.183982
0.639	12	7.766151	7.43591	6.498763	4.574837	1.518704	0.17457
0.608	13	7.351912	7.027445	6.119717	4.266281	1.38436	0.156634
0.576	14	6.90717	6.579306	5.688403	3.88867	1.208022	0.132841
0.545	15	6.436097	6.100356	5.214873	3.458279	1.008368	0.1063
0.514	18	6.482588	6.280158	5.632531	4.24815	1.667665	0.213468
0.482	19	5.925033	5.690626	5.010618	3.5969	1.249472	0.148111
0.451	21	5.57613	5.368693	4.750651	3.457331	1.242817	0.151159
0.42	22	4.985602	4.737245	4.072515	2.743455	0.829598	0.090423
0.388	23	4.398619	4.109083	3.40135	2.071421	0.517255	0.04951
0.357	25	3.971439	3.686242	3.006845	1.76295	0.413431	0.038031

0.341	26	3.74785	3.461693	2.79286	1.591279	0.356469	0.031814
0.31	27	3.163283	2.836094	2.135189	1.019417	0.17659	0.012944
0.278	29	2.70677	2.375447	1.700305	0.717981	0.106734	0.006934
0.247	30	2.16132	1.801368	1.137587	0.361577	0.039129	0.001934
0.216	31	1.6569	1.283954	0.675707	0.15004	0.011437	0.000411
0.184	42	1.770458	1.55329	1.112028	0.475883	0.074817	0.005327
0.176	43	1.668562	1.454156	1.024677	0.422942	0.063887	0.004416
0.145	45	1.164125	0.932769	0.535926	0.143651	0.013741	0.000636
0.114	47	0.721045	0.497253	0.196433	0.028528	0.001558	4.25E-05
0.082	55	0.471823	0.304312	0.102825	0.012488	0.000604	1.51E-05
0.051	59	0.17499	0.06945	0.008578	0.000404	8.2E-06	1E-07
0.035	63	0.082314	0.020645	0.001326	3.92E-05	5E-07	0

Table (D.21) n=511, L_p=1022, σ=0.2, G=45 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.877	7	9.026655	8.111684	6.139634	2.96255	0.515308	0.037419
0.824	10	9.217483	8.545263	6.95525	4.03724	0.911062	0.079249
0.771	13	9.062966	8.545659	7.230591	4.648727	1.249148	0.12107
0.718	16	8.671247	8.247751	7.11533	4.818228	1.429242	0.146796
0.665	20	8.350242	8.039284	7.125087	5.189744	1.811895	0.204505
0.595	25	7.555285	7.2994	6.518777	4.845633	1.775962	0.205796
0.507	30	6.094593	5.781908	4.961894	3.30837	0.943737	0.09353
0.448	38	5.465256	5.260481	4.636596	3.328683	1.143512	0.130206
0.378	43	3.868645	3.53673	2.785724	1.530713	0.352166	0.033772
0.307	51	2.468851	2.014772	1.224317	0.387985	0.049385	0.00333
0.237	58	1.363306	0.886475	0.30989	0.04082	0.002239	6.67E-05

Table (D.22) n=1023, L_p=1023, σ=0.2, G=45 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.902	10	9.063529	8.026838	5.872762	2.589671	0.393979	0.02534
0.853	15	9.414325	8.649908	6.902032	3.783387	0.764061	0.060086
0.809	20	9.51279	8.939153	7.512589	4.726225	1.194378	0.108271
0.761	25	9.242789	8.783373	7.567541	5.091696	1.459713	0.142374
0.712	30	8.455792	8.034736	6.906449	4.644662	1.382628	0.143956
0.682	35	7.788605	7.305527	6.105018	3.832754	1.043052	0.112302
0.633	41	6.621938	6.003987	4.642592	2.381622	0.465553	0.038995
0.594	45	5.73505	5.025891	3.58665	1.511486	0.227993	0.015358
0.56	50	5.118028	4.35644	2.896551	1.032984	0.129166	0.007617

0.511	55	4.119698	3.284252	1.860236	0.478141	0.042825	0.001828
0.472	60	3.480605	2.631769	1.302357	0.260619	0.018279	0.000619
0.443	63	2.967137	2.107172	0.891125	0.139805	0.007992	0.000225
0.433	73	2.811357	1.955186	0.786197	0.115129	0.006189	0.000163
0.394	78	2.253575	1.431656	0.461697	0.051124	0.002162	4.51E-05
0.36	85	1.825835	1.042175	0.258792	0.021143	0.000674	1.07E-05

Table (D.23) n=511, L_p=1022, σ=0.2, G=45 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.877	7	9.026655	8.111684	6.139634	2.96255	0.515308	0.037419
0.859	8	9.138528	8.315831	6.483478	3.368724	0.646067	0.05032
0.841	9	9.199904	8.457583	6.752917	3.728405	0.77938	0.064458
0.824	**10**	**9.217483**	8.545263	6.95525	4.03724	0.911062	0.079249
0.806	11	9.196895	8.584847	7.098817	4.293396	1.036319	0.094052
0.789	12	9.143822	8.583852	7.188329	4.496788	1.150255	0.108203
0.771	13	9.062966	8.545659	7.230591	4.648727	1.249148	0.12107
0.753	14	8.954161	8.475084	7.23009	4.751076	1.329923	0.132083

Table (D.24) n=1023, L_p=1023 σ=0.2, G=45 and N=100.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.839	**17**	**9.559829**	8.889282	7.292263	4.300294	0.97331	0.082518
0.829	18	9.555345	8.920755	7.384059	4.460445	1.050608	0.09128
0.819	19	9.538863	8.936591	7.456982	4.6024	1.124578	0.099906
0.809	20	9.51279	8.939153	7.512589	4.726225	1.194378	0.108271
0.8	21	9.475087	8.930123	7.552173	4.832446	1.259346	0.116261
0.79	22	9.42959	8.908479	7.576677	4.921446	1.31883	0.123761
0.78	23	9.37533	8.875667	7.586038	4.993564	1.372244	0.130668
0.77	24	9.308412	8.830668	7.579983	5.048104	1.418935	0.136885
0.761	25	9.242789	8.783373	7.567541	5.091696	1.459713	0.142374

Table (D.25) n=127, L_p=1116, σ=0.2, G=600 and N=1000.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.835	**3**	**123.0308**	120.5937	111.6545	87.46374	17.06456	0.297062
0.78	4	122.4689	122.0929	117.5286	104.9199	44.83827	1.024284

0.724	5	118.1794	118.7767	116.4021	109.4933	77.40041	2.067339
0.669	6	111.31	112.2476	110.8183	106.2996	88.36198	2.803122
0.614	7	102.603	103.503	102.2962	98.40646	83.31937	2.674534
0.559	9	98.73776	100.4278	100.8977	100.6608	99.71583	96.06691
0.504	10	86.90379	88.12032	88.01866	86.7982	83.22769	5.54922
0.449	11	74.43208	74.9237	73.76707	70.28951	55.40247	1.659201
0.394	13	65.32351	65.78706	64.82539	61.90899	49.94266	1.736197
0.339	14	51.72868	50.94638	47.85513	39.38715	9.271544	0.188098
0.283	15	38.36717	36.07403	30.25633	15.68935	0.829923	0.00626
0.228	21	35.26878	35.03322	33.48515	29.36449	12.74281	0.549422
0.173	23	22.58057	21.10203	17.44752	8.998751	0.71858	0.013728
0.118	27	12.82108	10.98462	7.14701	1.566722	0.048815	0.000611
0.063	31	4.020208	2.168232	0.24812	0.002325	9.5E-06	0

Table (D.26) n=255, L_p=1020, σ=0.2, G=600 and N=1000.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.906	3	126.3713	120.5113	104.6478	62.75033	4.263349	0.030058
0.875	4	131.5457	128.5575	118.6492	90.94246	12.81967	0.139888
0.843	5	133.8515	132.7833	126.9337	110.483	30.7318	0.424143
0.812	**6**	**134.0343**	134.1975	130.9711	121.7429	66.18683	0.955644
0.78	7	132.6009	133.5089	131.8969	126.8403	104.3904	1.733642
0.749	8	129.9146	131.2363	130.5695	127.7889	118.4217	2.655737
0.733	9	131.2634	133.1412	133.5695	133.0907	131.6041	6.588157
0.702	10	126.8154	128.705	129.3084	129.2151	128.6049	126.2377
0.671	11	121.7521	123.5869	124.2401	124.2958	124.0038	122.7102
0.639	12	116.1905	117.931	118.5569	118.6228	118.3748	117.2293
0.608	13	110.213	111.8285	112.3652	112.3311	111.9145	110.2564
0.576	14	103.8816	105.3335	105.7106	105.4449	104.561	4.80035
0.545	15	97.23749	98.47302	98.59383	97.89625	95.93666	3.101783
0.514	18	95.86393	97.33914	98.05176	98.47148	98.85357	99.14581
0.482	19	88.46119	89.79823	90.34982	90.54777	90.62967	90.42899
0.451	21	83.04396	84.31487	84.86936	85.11796	85.29262	85.29523
0.42	22	75.15457	76.14474	76.31139	75.92045	74.80685	3.097762
0.388	23	67.07586	67.57332	66.9656	64.95526	56.84575	0.93406
0.357	25	60.73811	61.00817	60.09217	57.36662	42.97473	0.605132
0.341	26	57.43586	57.55233	56.40268	53.08892	32.48737	0.443697
0.31	27	49.04615	48.29892	45.5592	37.69616	6.395011	0.069563
0.278	29	42.18737	40.93408	37.28208	26.79453	2.441394	0.019868
0.247	30	34.0203	31.71785	26.05254	11.52463	0.301504	0.000943
0.216	31	26.29042	22.99854	15.62483	2.663129	0.016251	1.48E-05
0.184	42	27.32176	26.45068	23.94717	17.08834	2.100623	0.035717
0.176	43	25.77112	24.82259	22.1966	15.08538	1.628148	0.026749

0.145	45	18.19602	16.36793	12.19045	3.713533	0.090134	0.000532
0.114	47	11.36191	8.851864	4.047621	0.187501	0.000705	1.1E-06
0.082	55	7.378931	5.349133	1.883449	0.056603	0.000265	8E-07
0.051	59	2.676502	0.950972	0.014404	1.47E-05	0	0
0.035	63	1.187777	0.124587	0.000122	0	0	0

Table (D.27) n=511, L_p=1022, σ=0.2, G=600 and N=1000.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.877	7	140.2367	138.3717	131.156	110.3011	19.00474	0.168179
0.824	10	142.1557	142.7856	140.8856	135.1419	102.7237	0.881071
0.771	13	138.7697	140.2794	140.3129	139.1782	136.033	2.296858
0.718	16	132.1219	133.7948	134.367	134.3667	134.0873	132.7797
0.665	20	125.7629	127.3981	128.1964	128.6773	129.1645	129.5591
0.595	25	113.3929	114.7985	115.4966	115.9325	116.3635	116.7112
0.507	30	93.39968	94.5192	94.87706	94.82319	94.55025	93.35839
0.448	38	79.56432	81.06614	81.64193	81.77414	81.62868	80.93516
0.378	43	57.83262	57.36164	54.59573	47.2997	22.83437	0.487553
0.307	51	37.89259	34.45712	26.59843	10.96367	0.897157	0.028789
0.237	58	21.19362	15.60788	6.109661	0.326768	0.004271	4.52E-05

Table (D.28) n=1023, L_p=1023, σ=0.2, G=600 and N=1000.

		Throughput					
R_c	t	p=0.05	p=0.1	p=0.15	p=0.2	p=0.25	p=0.3
0.902	10	142.9065	139.6317	129.7179	100.0855	9.119818	0.050873
0.853	15	147.5501	147.5687	144.7285	136.468	58.17573	0.361323
0.809	20	147.6881	149.0785	149.1109	148.0675	145.3084	1.386889
0.761	25	142.0685	143.8438	144.5725	144.8438	145.1029	145.0936
0.712	30	127.3808	129.0949	129.4279	128.8581	127.2077	3.570111
0.682	35	117.8851	118.7248	117.6858	114.3406	102.146	3.544908
0.633	41	102.1468	101.2012	96.82516	84.5001	24.78563	0.559957
0.594	45	89.80075	87.04949	79.19873	57.0698	6.439028	0.098557
0.56	50	79.38592	74.84851	63.7345	34.77104	1.853556	0.016286
0.511	55	65.2005	58.4865	43.32526	12.05202	0.203866	0.000698
0.472	60	54.87398	46.52947	28.70557	3.588942	0.022875	3.35E-05
0.443	63	47.38046	38.08417	19.35062	1.066631	0.002958	2.2E-06
0.433	73	44.99612	35.43375	16.54309	0.665562	0.001368	8E-07
0.394	78	36.16617	25.56883	7.509015	0.069925	4.35E-05	0
0.36	85	28.92465	17.89743	2.531529	0.005412	0.000001	0

Table (D.29) n=511, L_p=1022, σ=0.2, G=600 and N=1000.

		Throughput					
R_c	t	$p=0.05$	$p=0.1$	$p=0.15$	$p=0.2$	$p=0.25$	$p=0.3$
0.877	7	140.2367	138.3717	131.156	110.3011	19.00474	0.168179
0.859	8	141.647	140.8696	135.9741	121.9353	33.84565	0.323778
0.841	9	142.2439	142.2801	139.1172	129.9952	61.47147	0.558596
0.824	**10**	142.1557	**142.7856**	140.8856	135.1419	102.7237	0.881071
0.806	11	141.4931	142.5413	141.5376	138.035	124.3242	1.289369
0.789	12	140.338	141.6692	141.2863	139.2329	132.8186	1.769871
0.771	13	138.7697	140.2794	140.3129	139.1782	136.033	2.296858
0.753	14	136.843	138.4561	138.7606	138.1987	136.6543	2.833947

Table (D.30) n=1023, L_p=1023, σ=0.2, G=600 and N=1000.

		Throughput					
R_c	t	$p=0.05$	$p=0.1$	$p=0.15$	$p=0.2$	$p=0.25$	$p=0.3$
0.839	**17**	149.1588	**150.0161**	148.9029	145.2165	129.1556	0.744287
0.829	18	148.8415	149.9255	149.2934	146.7644	138.0234	0.936231
0.819	19	148.3433	149.6032	149.3474	147.6762	142.7588	1.151443
0.809	20	147.6881	149.0785	149.1109	148.0675	145.3084	1.386889
0.8	21	146.8831	148.3682	148.6183	148.0293	146.544	1.638811
0.79	22	145.9492	147.4983	147.9098	147.6458	146.9334	143.6588
0.78	23	144.8886	146.4812	147.0103	146.977	146.7399	145.4967
0.77	24	143.6339	145.2898	145.9164	146.0518	146.1141	145.6962
0.761	25	142.0685	143.8438	144.5725	144.8438	145.1029	145.0936

Appendix E

EEPA Results for 100 Users

Table (E.1) EEPA Model & Simulation comparison,
G=100, N=100, L_p=256, $(\sigma_i \leq p_i)$.

$user_i$	σ_i	p_i	S_i (simul.)	S_i (model)	D_i (simul.)	D_i (model)
1	0.1	0.12	0.09477	0.095734	1.551863	1.445644
2	0.101	0.121	0.0964	0.096684	1.472454	1.441961
3	0.102	0.122	0.09691	0.097635	1.514931	1.438338
4	0.103	0.123	0.09919	0.098585	1.372924	1.434774
5	0.104	0.124	0.09732	0.099536	1.659996	1.431268
6	0.105	0.125	0.10142	0.100486	1.336179	1.427818
7	0.106	0.126	0.09934	0.101436	1.632476	1.424423
8	0.107	0.127	0.10088	0.102387	1.566973	1.421081
9	0.108	0.128	0.1017	0.103337	1.573583	1.417791
10	0.109	0.129	0.10368	0.104288	1.47075	1.414552
11	0.11	0.13	0.10342	0.105238	1.578401	1.411363
12	0.111	0.131	0.10539	0.106188	1.479557	1.408223
13	0.112	0.132	0.10517	0.107139	1.579844	1.405131
14	0.113	0.133	0.10678	0.108089	1.515492	1.402084
15	0.114	0.134	0.10798	0.109039	1.489045	1.399084
16	0.115	0.135	0.10833	0.109989	1.535401	1.396128
17	0.116	0.136	0.10909	0.11094	1.546054	1.393215
18	0.117	0.137	0.11085	0.11189	1.474191	1.390345
19	0.118	0.138	0.11051	0.11284	1.574379	1.387516
20	0.119	0.139	0.11321	0.11379	1.429781	1.384728
21	0.12	0.14	0.11299	0.114741	1.517008	1.38198
22	0.121	0.141	0.11503	0.115691	1.428922	1.379271
23	0.122	0.142	0.11701	0.116641	1.349557	1.3766
24	0.123	0.143	0.11775	0.117591	1.362488	1.373967
25	0.124	0.144	0.11722	0.118541	1.466452	1.37137
26	0.125	0.145	0.11782	0.119491	1.487524	1.368809
27	0.126	0.146	0.11924	0.120441	1.44994	1.366282
28	0.127	0.147	0.11937	0.121392	1.503299	1.363791
29	0.128	0.148	0.11893	0.122342	1.595808	1.361333
30	0.129	0.149	0.1219	0.123292	1.451508	1.358908
31	0.13	0.15	0.12304	0.124242	1.435131	1.356515

32	0.131	0.151	0.12465	0.125192	1.388875	1.354154
33	0.132	0.152	0.12473	0.126142	1.44156	1.351824
34	0.133	0.153	0.12533	0.127092	1.460139	1.349524
35	0.134	0.154	0.12705	0.128042	1.408231	1.347255
36	0.135	0.155	0.12826	0.128992	1.389256	1.345014
37	0.136	0.156	0.12967	0.129942	1.358943	1.342803
38	0.137	0.157	0.13008	0.130892	1.388307	1.340619
39	0.138	0.158	0.13215	0.131842	1.320782	1.338464
40	0.139	0.159	0.13157	0.132792	1.406272	1.336335
41	0.14	0.16	0.13212	0.133742	1.42602	1.334233
42	0.141	0.161	0.13454	0.134692	1.340535	1.332157
43	0.142	0.162	0.13423	0.135642	1.407646	1.330106
44	0.143	0.163	0.13434	0.136592	1.450793	1.328081
45	0.144	0.164	0.13466	0.137542	1.481666	1.326081
46	0.145	0.165	0.13763	0.138492	1.369306	1.324104
47	0.146	0.166	0.13932	0.139441	1.328406	1.322152
48	0.147	0.167	0.13864	0.140391	1.410205	1.320223
49	0.148	0.168	0.14018	0.141341	1.376929	1.318317
50	0.149	0.169	0.14319	0.142291	1.272319	1.316433
51	0.15	0.17	0.14138	0.143241	1.40647	1.314572
52	0.151	0.171	0.14259	0.144191	1.390598	1.312732
53	0.152	0.172	0.14172	0.145141	1.47722	1.310914
54	0.153	0.173	0.14696	0.146091	1.268625	1.309117
55	0.154	0.174	0.14363	0.147041	1.468828	1.30734
56	0.155	0.175	0.14778	0.14799	1.315203	1.305584
57	0.156	0.176	0.1475	0.14894	1.369405	1.303848
58	0.157	0.177	0.145	0.14989	1.527125	1.302131
59	0.158	0.178	0.1503	0.15084	1.324246	1.300434
60	0.159	0.179	0.15203	0.15179	1.288341	1.298756
61	0.16	0.18	0.14951	0.152739	1.438516	1.297096
62	0.161	0.181	0.15116	0.153689	1.404327	1.295454
63	0.162	0.182	0.15304	0.154639	1.3614	1.293831
64	0.163	0.183	0.15485	0.155589	1.322893	1.292225
65	0.164	0.184	0.15569	0.156539	1.325459	1.290637
66	0.165	0.185	0.15694	0.157488	1.311256	1.289066
67	0.166	0.186	0.15521	0.158438	1.418788	1.287512
68	0.167	0.187	0.15751	0.159388	1.36078	1.285975
69	0.168	0.188	0.15703	0.160338	1.415829	1.284453
70	0.169	0.189	0.15954	0.161288	1.350861	1.282948
71	0.17	0.19	0.16143	0.162237	1.312283	1.281459
72	0.171	0.191	0.16185	0.163187	1.330607	1.279986
73	0.172	0.192	0.16308	0.164137	1.318007	1.278527
74	0.173	0.193	0.16245	0.165086	1.375394	1.277084
75	0.174	0.194	0.16588	0.166036	1.281328	1.275656

76	0.175	0.195	0.16314	0.166986	1.415419	1.274242
77	0.176	0.196	0.16503	0.167936	1.377686	1.272843
78	0.177	0.197	0.16749	0.168885	1.320788	1.271458
79	0.178	0.198	0.16829	0.169835	1.324146	1.270087
80	0.179	0.199	0.16803	0.170785	1.364726	1.26873
81	0.18	0.2	0.16909	0.171735	1.358455	1.267386
82	0.181	0.201	0.16825	0.172684	1.418675	1.266056
83	0.182	0.202	0.17018	0.173634	1.381626	1.264739
84	0.183	0.203	0.17427	0.174584	1.273742	1.263435
85	0.184	0.204	0.1744	0.175533	1.299163	1.262143
86	0.185	0.205	0.17456	0.176483	1.323284	1.260865
87	0.186	0.206	0.17746	0.177433	1.258729	1.259598
88	0.187	0.207	0.17761	0.178382	1.28272	1.258344
89	0.188	0.208	0.17766	0.179332	1.30958	1.257102
90	0.189	0.209	0.17705	0.180282	1.357117	1.255872
91	0.19	0.21	0.18074	0.181231	1.269652	1.254654
92	0.191	0.211	0.18106	0.182181	1.287429	1.253447
93	0.192	0.212	0.18045	0.183131	1.333368	1.252251
94	0.193	0.213	0.18383	0.18408	1.258462	1.251067
95	0.194	0.214	0.18309	0.18503	1.307156	1.249894
96	0.195	0.215	0.18289	0.18598	1.339563	1.248731
97	0.196	0.216	0.18618	0.186929	1.269106	1.24758
98	0.197	0.217	0.18707	0.187879	1.269451	1.246439
99	0.198	0.218	0.18739	0.188828	1.285959	1.245308
100	0.199	0.219	0.18928	0.189778	1.258053	1.244188

Table (E.2)EEPA Model & Simulation comparison,
G=100, N=100, L_p=256, ($\sigma_i > p_i$).

$user_i$	σ_i	p_i	S_i *(simul.)*	S_i *(model)*	D_i *(simul.)*	D_i *(model)*
1	0.22	0.02	0.07675	0.078886	9.483861	9.13113
2	0.221	0.021	0.07675	0.081507	9.504429	8.743933
3	0.222	0.022	0.08039	0.084059	8.934854	8.391936
4	0.223	0.023	0.0799	0.086544	9.03134	8.070547
5	0.224	0.024	0.08648	0.088966	8.099082	7.775941
6	0.225	0.025	0.08542	0.09133	8.262416	7.504904
7	0.226	0.026	0.0876	0.093637	7.990746	7.254715
8	0.227	0.027	0.09049	0.095892	7.645658	7.023059
9	0.228	0.028	0.09699	0.098098	6.924376	6.80795
10	0.229	0.029	0.09625	0.100256	7.022798	6.607675
11	0.23	0.03	0.09579	0.102369	7.091677	6.420753
12	0.231	0.031	0.10188	0.10444	6.486465	6.24589
13	0.232	0.032	0.1067	0.10647	6.061726	6.081956
14	0.233	0.033	0.10389	0.108462	6.33372	5.927957
15	0.234	0.034	0.10609	0.110418	6.152455	5.783017
16	0.235	0.035	0.10573	0.112338	6.202734	5.64636
17	0.236	0.036	0.11059	0.114226	5.805121	5.517294
18	0.237	0.037	0.1124	0.116082	5.677388	5.395205
19	0.238	0.038	0.11502	0.117908	5.492459	5.279542
20	0.239	0.039	0.11468	0.119704	5.535816	5.16981
21	0.24	0.04	0.11724	0.121474	5.362845	5.065565
22	0.241	0.041	0.11889	0.123217	5.261759	4.966405
23	0.242	0.042	0.12219	0.124934	5.051744	4.871966
24	0.243	0.043	0.11984	0.126628	5.229233	4.781921
25	0.244	0.044	0.12455	0.128298	4.930543	4.695968
26	0.245	0.045	0.12487	0.129947	4.926696	4.613835
27	0.246	0.046	0.12581	0.131574	4.883453	4.535274
28	0.247	0.047	0.12584	0.13318	4.898016	4.460055
29	0.248	0.048	0.12851	0.134767	4.749238	4.387971
30	0.249	0.049	0.12918	0.136335	4.725072	4.318828
31	0.25	0.05	0.13149	0.137884	4.605141	4.252452
32	0.251	0.051	0.13471	0.139417	4.43929	4.188678
33	0.252	0.052	0.13583	0.140932	4.39389	4.127357
34	0.253	0.053	0.13643	0.142431	4.377197	4.068351
35	0.254	0.054	0.14204	0.143915	4.103262	4.011529
36	0.255	0.055	0.13899	0.145384	4.273194	3.956774
37	0.256	0.056	0.14236	0.146838	4.118195	3.903975
38	0.257	0.057	0.14165	0.148278	4.168604	3.853028
39	0.258	0.058	0.14455	0.149705	4.042052	3.803838

40	0.259	0.059	0.14897	0.151119	3.851757	3.756315
41	0.26	0.06	0.14878	0.15252	3.87518	3.710376
42	0.261	0.061	0.15124	0.153909	3.78059	3.665944
43	0.262	0.062	0.1521	0.155286	3.757828	3.622945
44	0.263	0.063	0.15221	0.156652	3.767589	3.581311
45	0.264	0.064	0.15277	0.158006	3.757909	3.540978
46	0.265	0.065	0.15133	0.159351	3.83449	3.501886
47	0.266	0.066	0.15935	0.160684	3.516096	3.463979
48	0.267	0.067	0.15901	0.162008	3.543594	3.427203
49	0.268	0.068	0.15543	0.163323	3.702421	3.391509
50	0.269	0.069	0.16089	0.164627	3.497955	3.356849
51	0.27	0.07	0.16407	0.165923	3.391256	3.32318
52	0.271	0.071	0.16435	0.16721	3.394539	3.290459
53	0.272	0.072	0.16291	0.168489	3.461888	3.258647
54	0.273	0.073	0.1681	0.169759	3.285836	3.227707
55	0.274	0.074	0.16464	0.171021	3.424223	3.197603
56	0.275	0.075	0.16976	0.172275	3.254306	3.168301
57	0.276	0.076	0.17062	0.173522	3.237789	3.139771
58	0.277	0.077	0.17109	0.174761	3.234769	3.111982
59	0.278	0.078	0.17081	0.175994	3.257336	3.084905
60	0.279	0.079	0.17201	0.177219	3.229386	3.058514
61	0.28	0.08	0.17421	0.178437	3.16877	3.032782
62	0.281	0.081	0.17503	0.179649	3.154587	3.007686
63	0.282	0.082	0.17786	0.180855	3.0763	2.983202
64	0.283	0.083	0.18049	0.182054	3.006904	2.959308
65	0.284	0.084	0.17837	0.183247	3.085197	2.935983
66	0.285	0.085	0.17838	0.184435	3.097238	2.913207
67	0.286	0.086	0.18337	0.185616	2.956951	2.89096
68	0.287	0.087	0.18302	0.186792	2.979563	2.869225
69	0.288	0.088	0.18292	0.187963	2.994649	2.847984
70	0.289	0.089	0.18355	0.189128	2.987899	2.82722
71	0.29	0.09	0.188	0.190288	2.870873	2.806918
72	0.291	0.091	0.18784	0.191443	2.887254	2.787061
73	0.292	0.092	0.18643	0.192593	2.939286	2.767637
74	0.293	0.093	0.18978	0.193738	2.85629	2.74863
75	0.294	0.094	0.18862	0.194879	2.900304	2.730028
76	0.295	0.095	0.19345	0.196015	2.779464	2.711817
77	0.296	0.096	0.1915	0.197147	2.843554	2.693985
78	0.297	0.097	0.19388	0.198274	2.790826	2.676521
79	0.298	0.098	0.19857	0.199397	2.680303	2.659414
80	0.299	0.099	0.1972	0.200516	2.726512	2.642652
81	0.3	0.1	0.19993	0.201631	2.668417	2.626226
82	0.301	0.101	0.19845	0.202742	2.716794	2.610125
83	0.302	0.102	0.20221	0.203849	2.634096	2.594339

84	0.303	0.103	0.2031	0.204952	2.623353	2.57886
85	0.304	0.104	0.20134	0.206052	2.677249	2.563679
86	0.305	0.105	0.2032	0.207148	2.642571	2.548787
87	0.306	0.106	0.20392	0.20824	2.63591	2.534175
88	0.307	0.107	0.20133	0.209329	2.709641	2.519837
89	0.308	0.108	0.20436	0.210415	2.646572	2.505765
90	0.309	0.109	0.2071	0.211497	2.592339	2.49195
91	0.31	0.11	0.21039	0.212576	2.527271	2.478387
92	0.311	0.111	0.21126	0.213652	2.51807	2.465068
93	0.312	0.112	0.20827	0.214725	2.596331	2.451987
94	0.313	0.113	0.20974	0.215795	2.57292	2.439138
95	0.314	0.114	0.20961	0.237908	2.586051	2.018601
96	0.315	0.115	0.21777	0.238986	2.417398	2.009743
97	0.316	0.116	0.21495	0.240062	2.487688	2.001039
98	0.317	0.117	0.21798	0.241135	2.433003	1.992483
99	0.318	0.118	0.21797	0.242205	2.443133	1.984072
100	0.319	0.119	0.21679	0.222154	2.477963	2.366576

Table (E.3) EEPA Model & Simulation comparison,
G=80, N=100, L_p=256, ($\sigma_i \leq p_i$).

$user_i$	σ_i	p_i	S_i (simul.)	S_i (model)	D_i (simul.)	D_i (model)
1	0.1	0.12	0.08533	0.08439	2.719208	2.849701
2	0.101	0.121	0.08564	0.085212	2.775796	2.834415
3	0.102	0.122	0.08492	0.086034	2.971867	2.819378
4	0.103	0.123	0.08848	0.086856	2.593251	2.804587
5	0.104	0.124	0.08588	0.087678	3.02877	2.790034
6	0.105	0.125	0.08895	0.088499	2.718461	2.775713
7	0.106	0.126	0.09098	0.089321	2.557464	2.76162
8	0.107	0.127	0.08882	0.090143	2.912931	2.747749
9	0.108	0.128	0.09057	0.090964	2.781924	2.734095
10	0.109	0.129	0.09112	0.091786	2.800227	2.720652
11	0.11	0.13	0.09387	0.092607	2.562122	2.707417
12	0.111	0.131	0.09161	0.093428	2.90683	2.694383
13	0.112	0.132	0.09416	0.09425	2.69165	2.681547
14	0.113	0.133	0.09676	0.095071	2.485292	2.668903
15	0.114	0.134	0.09675	0.095892	2.563988	2.656449
16	0.115	0.135	0.09564	0.096713	2.760224	2.644179
17	0.116	0.136	0.09826	0.097535	2.556392	2.632089
18	0.117	0.137	0.10042	0.098356	2.411167	2.620176
19	0.118	0.138	0.09953	0.099177	2.572646	2.608436
20	0.119	0.139	0.09985	0.099998	2.611661	2.596864
21	0.12	0.14	0.10015	0.100819	2.651689	2.585458
22	0.121	0.141	0.10038	0.10164	2.697681	2.574214
23	0.122	0.142	0.10116	0.102461	2.688609	2.563128
24	0.123	0.143	0.10319	0.103281	2.56078	2.552197
25	0.124	0.144	0.10363	0.104102	2.585199	2.541418
26	0.125	0.145	0.10389	0.104923	2.625566	2.530787
27	0.126	0.146	0.1048	0.105744	2.605477	2.520302
28	0.127	0.147	0.10602	0.106565	2.558167	2.50996
29	0.128	0.148	0.10749	0.107385	2.490691	2.499758
30	0.129	0.149	0.10898	0.108206	2.424058	2.489692
31	0.13	0.15	0.10869	0.109027	2.508171	2.479761
32	0.131	0.151	0.10867	0.109847	2.568584	2.469961
33	0.132	0.152	0.11107	0.110668	2.427574	2.460291
34	0.133	0.153	0.11173	0.111488	2.431351	2.450746
35	0.134	0.154	0.11247	0.112309	2.428573	2.441326
36	0.135	0.155	0.11136	0.113129	2.572478	2.432027
37	0.136	0.156	0.11397	0.11395	2.421298	2.422847
38	0.137	0.157	0.1153	0.11477	2.373757	2.413784
39	0.138	0.158	0.11672	0.115591	2.321135	2.404836

40	0.139	0.159	0.11456	0.116411	2.534806	2.396001
41	0.14	0.16	0.11684	0.117231	2.415856	2.387276
42	0.141	0.161	0.11819	0.118052	2.368754	2.378659
43	0.142	0.162	0.11768	0.118872	2.455367	2.370149
44	0.143	0.163	0.1187	0.119692	2.431593	2.361743
45	0.144	0.164	0.11966	0.120513	2.412567	2.35344
46	0.145	0.165	0.12056	0.121333	2.398073	2.345237
47	0.146	0.166	0.12347	0.122153	2.249818	2.337133
48	0.147	0.167	0.12336	0.122973	2.303634	2.329127
49	0.148	0.168	0.12296	0.123793	2.375969	2.321215
50	0.149	0.169	0.12273	0.124614	2.436558	2.313397
51	0.15	0.17	0.12491	0.125434	2.339098	2.305672
52	0.151	0.171	0.12659	0.126254	2.277002	2.298036
53	0.152	0.172	0.12628	0.127074	2.339963	2.290489
54	0.153	0.173	0.12669	0.127894	2.357335	2.28303
55	0.154	0.174	0.1296	0.128714	2.222543	2.275656
56	0.155	0.175	0.12897	0.129534	2.302128	2.268367
57	0.156	0.176	0.12902	0.130354	2.34048	2.26116
58	0.157	0.177	0.13154	0.131174	2.232824	2.254035
59	0.158	0.178	0.13001	0.131994	2.362602	2.24699
60	0.159	0.179	0.1324	0.132814	2.263562	2.240023
61	0.16	0.18	0.135	0.133634	2.157408	2.233134
62	0.161	0.181	0.13396	0.134454	2.253735	2.226321
63	0.162	0.182	0.13778	0.135274	2.085108	2.219583
64	0.163	0.183	0.136	0.136094	2.217972	2.212919
65	0.164	0.184	0.13842	0.136913	2.126829	2.206327
66	0.165	0.185	0.13652	0.137733	2.264328	2.199806
67	0.166	0.186	0.1393	0.138553	2.154655	2.193356
68	0.167	0.187	0.14009	0.139373	2.150244	2.186974
69	0.168	0.188	0.14189	0.140193	2.095332	2.18066
70	0.169	0.189	0.13921	0.141012	2.266232	2.174414
71	0.17	0.19	0.1421	0.141832	2.154945	2.168232
72	0.171	0.191	0.14325	0.142652	2.13285	2.162116
73	0.172	0.192	0.14367	0.143472	2.146442	2.156063
74	0.173	0.193	0.14562	0.144291	2.086842	2.150073
75	0.174	0.194	0.14604	0.145111	2.100313	2.144145
76	0.175	0.195	0.14527	0.145931	2.169448	2.138278
77	0.176	0.196	0.14695	0.14675	2.123218	2.13247
78	0.177	0.197	0.14648	0.14757	2.177153	2.126722
79	0.178	0.198	0.14858	0.14839	2.112404	2.121031
80	0.179	0.199	0.14898	0.149209	2.125718	2.115398
81	0.18	0.2	0.14993	0.150029	2.114224	2.109821
82	0.181	0.201	0.14944	0.150849	2.166787	2.104299
83	0.182	0.202	0.15231	0.151668	2.071052	2.098832

84	0.183	0.203	0.15136	0.152488	2.142285	2.093419
85	0.184	0.204	0.15204	0.153307	2.142434	2.08806
86	0.185	0.205	0.15664	0.154127	1.97866	2.082752
87	0.186	0.206	0.15396	0.154947	2.11885	2.077496
88	0.187	0.207	0.15486	0.155766	2.109852	2.072291
89	0.188	0.208	0.15613	0.156586	2.08577	2.067135
90	0.189	0.209	0.15758	0.157405	2.054978	2.062029
91	0.19	0.21	0.1568	0.158225	2.114393	2.056972
92	0.191	0.211	0.15891	0.159044	2.057268	2.051963
93	0.192	0.212	0.15882	0.159864	2.088103	2.047001
94	0.193	0.213	0.15941	0.160683	2.091785	2.042085
95	0.194	0.214	0.16122	0.161502	2.048065	2.037216
96	0.195	0.215	0.16288	0.162322	2.011284	2.032391
97	0.196	0.216	0.16354	0.163141	2.012671	2.027612
98	0.197	0.217	0.1659	0.163961	1.951586	2.022876
99	0.198	0.218	0.16663	0.16478	1.950815	2.018184
100	0.199	0.219	0.16574	0.1656	2.008421	2.013535

Table (E.4) Comparison between systems with & without power control using EEPA Model, G=100, N=100, L_p=256, ($\sigma_i \leq p_i$), α_i=-3dB to 3dB.

$user_i$	σ_i	p_i	α_i	S_i with pwc	S_i without pwc	D_i with pwc	D_i without pwc
1	0.1	0.12	0.5	0.095734	0.002804	1.445644	347.6084
2	0.101	0.121	0.515	0.096684	0.003812	1.441961	253.421
3	0.102	0.122	0.53	0.097635	0.005052	1.438338	189.1336
4	0.103	0.123	0.545	0.098585	0.006541	1.434774	144.1627
5	0.104	0.124	0.56	0.099536	0.008292	1.431268	111.9815
6	0.105	0.125	0.575	0.100486	0.010309	1.427818	88.4774
7	0.106	0.126	0.59	0.101436	0.012591	1.424423	70.98678
8	0.107	0.127	0.605	0.102387	0.015131	1.421081	57.74417
9	0.108	0.128	0.62	0.103337	0.017915	1.417791	47.56003
10	0.109	0.129	0.635	0.104288	0.020925	1.414552	39.61504
11	0.11	0.13	0.65	0.105238	0.02414	1.411363	33.33452
12	0.111	0.131	0.665	0.106188	0.027534	1.408223	28.30974
13	0.112	0.132	0.68	0.107139	0.031081	1.405131	24.24568
14	0.113	0.133	0.695	0.108089	0.034754	1.402084	20.92438
15	0.114	0.134	0.71	0.109039	0.038525	1.399084	18.18502
16	0.115	0.135	0.725	0.109989	0.042369	1.396128	15.90677
17	0.116	0.136	0.74	0.11094	0.046259	1.393215	13.99678
18	0.117	0.137	0.755	0.11189	0.050173	1.390345	12.38395
19	0.118	0.138	0.77	0.11284	0.05409	1.387516	11.01317
20	0.119	0.139	0.785	0.11379	0.05799	1.384728	9.841095
21	0.12	0.14	0.8	0.114741	0.061857	1.38198	8.833004
22	0.121	0.141	0.815	0.115691	0.065676	1.379271	7.961846
23	0.122	0.142	0.83	0.116641	0.069435	1.3766	7.205272
24	0.123	0.143	0.845	0.117591	0.073125	1.373967	6.545191
25	0.124	0.144	0.86	0.118541	0.076735	1.37137	5.96734
26	0.125	0.145	0.875	0.119491	0.080261	1.368809	5.459284
27	0.126	0.146	0.89	0.120441	0.083698	1.366282	5.011237
28	0.127	0.147	0.905	0.121392	0.087041	1.363791	4.614847
29	0.128	0.148	0.92	0.122342	0.09029	1.361333	4.262944
30	0.129	0.149	0.935	0.123292	0.093441	1.358908	3.950002
31	0.13	0.15	0.95	0.124242	0.096497	1.356515	3.670734
32	0.131	0.151	0.965	0.125192	0.099456	1.354154	3.421159
33	0.132	0.152	0.98	0.126142	0.102321	1.351824	3.197415
34	0.133	0.153	0.995	0.127092	0.105093	1.349524	2.99661
35	0.134	0.154	1.01	0.128042	0.107773	1.347255	2.816074
36	0.135	0.155	1.025	0.128992	0.110364	1.345014	2.65349
37	0.136	0.156	1.04	0.129942	0.112872	1.342803	2.506659
38	0.137	0.157	1.055	0.130892	0.115298	1.340619	2.373917
39	0.138	0.158	1.07	0.131842	0.117642	1.338464	2.254002

40	0.139	0.159	1.085	0.132792	0.119911	1.336335	2.145276
41	0.14	0.16	1.1	0.133742	0.122108	1.334233	2.046628
42	0.141	0.161	1.115	0.134692	0.124235	1.332157	1.957041
43	0.142	0.162	1.13	0.135642	0.126297	1.330106	1.875579
44	0.143	0.163	1.145	0.136592	0.128294	1.328081	1.801595
45	0.144	0.164	1.16	0.137542	0.130232	1.326081	1.734181
46	0.145	0.165	1.175	0.138492	0.132113	1.324104	1.672739
47	0.146	0.166	1.19	0.139441	0.13394	1.322152	1.616692
48	0.147	0.167	1.205	0.140391	0.135718	1.320223	1.565514
49	0.148	0.168	1.22	0.141341	0.137446	1.318317	1.518812
50	0.149	0.169	1.235	0.142291	0.13913	1.316433	1.476093
51	0.15	0.17	1.25	0.143241	0.140769	1.314572	1.437153
52	0.151	0.171	1.265	0.144191	0.142371	1.312732	1.401382
53	0.152	0.172	1.28	0.145141	0.143934	1.310914	1.368689
54	0.153	0.173	1.295	0.146091	0.145461	1.309117	1.338761
55	0.154	0.174	1.31	0.147041	0.146953	1.30734	1.311408
56	0.155	0.175	1.325	0.14799	0.148415	1.305584	1.286236
57	0.156	0.176	1.34	0.14894	0.149849	1.303848	1.26315
58	0.157	0.177	1.355	0.14989	0.151252	1.302131	1.242064
59	0.158	0.178	1.37	0.15084	0.15263	1.300434	1.22266
60	0.159	0.179	1.385	0.15179	0.153984	1.298756	1.204884
61	0.16	0.18	1.4	0.152739	0.155315	1.297096	1.188546
62	0.161	0.181	1.415	0.153689	0.156623	1.295454	1.173568
63	0.162	0.182	1.43	0.154639	0.157913	1.293831	1.159748
64	0.163	0.183	1.445	0.155589	0.159183	1.292225	1.14712
65	0.164	0.184	1.46	0.156539	0.160437	1.290637	1.135432
66	0.165	0.185	1.475	0.157488	0.161673	1.289066	1.124725
67	0.166	0.186	1.49	0.158438	0.162892	1.287512	1.114933
68	0.167	0.187	1.505	0.159388	0.164098	1.285975	1.105901
69	0.168	0.188	1.52	0.160338	0.165291	1.284453	1.097569
70	0.169	0.189	1.535	0.161288	0.166471	1.282948	1.089897
71	0.17	0.19	1.55	0.162237	0.16764	1.281459	1.082811
72	0.171	0.191	1.565	0.163187	0.168796	1.279986	1.076375
73	0.172	0.192	1.58	0.164137	0.169944	1.278527	1.070331
74	0.173	0.193	1.595	0.165086	0.171081	1.277084	1.064838
75	0.174	0.194	1.61	0.166036	0.172208	1.275656	1.059807
76	0.175	0.195	1.625	0.166986	0.173327	1.274242	1.055158
77	0.176	0.196	1.64	0.167936	0.174438	1.272843	1.050862
78	0.177	0.197	1.655	0.168885	0.175542	1.271458	1.046929
79	0.178	0.198	1.67	0.169835	0.17664	1.270087	1.04327
80	0.179	0.199	1.685	0.170785	0.177731	1.26873	1.039882
81	0.18	0.2	1.7	0.171735	0.178814	1.267386	1.036835
82	0.181	0.201	1.715	0.172684	0.179894	1.266056	1.033976
83	0.182	0.202	1.73	0.173634	0.180969	1.264739	1.031297

84	0.183	0.203	1.745	0.174584	0.182038	1.263435	1.028874
85	0.184	0.204	1.76	0.175533	0.183103	1.262143	1.026629
86	0.185	0.205	1.775	0.176483	0.184161	1.260865	1.024627
87	0.186	0.206	1.79	0.177433	0.185217	1.259598	1.022723
88	0.187	0.207	1.805	0.178382	0.186269	1.258344	1.020983
89	0.188	0.208	1.82	0.179332	0.187319	1.257102	1.019335
90	0.189	0.209	1.835	0.180282	0.188365	1.255872	1.017848
91	0.19	0.21	1.85	0.181231	0.189408	1.254654	1.016451
92	0.191	0.211	1.865	0.182181	0.190447	1.253447	1.015211
93	0.192	0.212	1.88	0.183131	0.191483	1.252251	1.014057
94	0.193	0.213	1.895	0.18408	0.192517	1.251067	1.012987
95	0.194	0.214	1.91	0.18503	0.19355	1.249894	1.011996
96	0.195	0.215	1.925	0.18598	0.194579	1.248731	1.011087
97	0.196	0.216	1.94	0.186929	0.195609	1.24758	1.010186
98	0.197	0.217	1.955	0.187879	0.196635	1.246439	1.009434
99	0.198	0.218	1.97	0.188828	0.19766	1.245308	1.008688
100	0.199	0.219	1.985	0.189778	0.198683	1.244188	1.00802

Table (E.5) EEPA Model & Simulation comparison without power control, G=100, N=100, L_p=256, $(\sigma_i \leq p_i)$, α_i=-3dB to 3dB.

$user_i$	σ_i	p_i	α_i	S_i (simul.)	S_i (model)	D_i (simul.)	D_i (model)
1	0.1	0.12	0.5	0.014899	0.002804	58.1186	347.6084
2	0.101	0.121	0.515	0.016942	0.003812	50.12392	253.421
3	0.102	0.122	0.53	0.018903	0.005052	44.09773	189.1336
4	0.103	0.123	0.545	0.021554	0.006541	37.68636	144.1627
5	0.104	0.124	0.56	0.02393	0.008292	33.17317	111.9815
6	0.105	0.125	0.575	0.026153	0.010309	29.71272	88.4774
7	0.106	0.126	0.59	0.028973	0.012591	26.08093	70.98678
8	0.107	0.127	0.605	0.031845	0.015131	23.05631	57.74417
9	0.108	0.128	0.62	0.034686	0.017915	20.57082	47.56003
10	0.109	0.129	0.635	0.037522	0.020925	18.47672	39.61504
11	0.11	0.13	0.65	0.040287	0.02414	16.73099	33.33452
12	0.111	0.131	0.665	0.043439	0.027534	15.01178	28.30974
13	0.112	0.132	0.68	0.046317	0.031081	13.66177	24.24568
14	0.113	0.133	0.695	0.049484	0.034754	12.359	20.92438
15	0.114	0.134	0.71	0.051876	0.038525	11.50481	18.18502
16	0.115	0.135	0.725	0.055144	0.042369	10.43869	15.90677
17	0.116	0.136	0.74	0.058581	0.046259	9.449692	13.99678
18	0.117	0.137	0.755	0.061624	0.050173	8.680435	12.38395
19	0.118	0.138	0.77	0.064172	0.05409	8.108544	11.01317
20	0.119	0.139	0.785	0.067604	0.05799	7.388663	9.841095
21	0.12	0.14	0.8	0.070857	0.061857	6.779599	8.833004
22	0.121	0.141	0.815	0.072899	0.065676	6.453147	7.961846
23	0.122	0.142	0.83	0.076518	0.069435	5.872099	7.205272
24	0.123	0.143	0.845	0.079496	0.073125	5.449168	6.545191
25	0.124	0.144	0.86	0.082189	0.076735	5.102562	5.96734
26	0.125	0.145	0.875	0.084748	0.080261	4.799689	5.459284
27	0.126	0.146	0.89	0.087124	0.083698	4.541386	5.011237
28	0.127	0.147	0.905	0.089798	0.087041	4.26209	4.614847
29	0.128	0.148	0.92	0.09299	0.09029	3.941345	4.262944
30	0.129	0.149	0.935	0.095142	0.093441	3.758667	3.950002
31	0.13	0.15	0.95	0.097467	0.096497	3.567575	3.670734
32	0.131	0.151	0.965	0.100949	0.099456	3.272405	3.421159
33	0.132	0.152	0.98	0.103352	0.102321	3.099914	3.197415
34	0.133	0.153	0.995	0.105746	0.105093	2.937826	2.99661
35	0.134	0.154	1.01	0.107078	0.107773	2.8763	2.816074
36	0.135	0.155	1.025	0.109639	0.110364	2.713435	2.65349
37	0.136	0.156	1.04	0.111651	0.112872	2.603539	2.506659
38	0.137	0.157	1.055	0.113362	0.115298	2.522028	2.373917
39	0.138	0.158	1.07	0.116502	0.117642	2.337167	2.254002

40	0.139	0.159	1.085	0.1182	0.119911	2.265992	2.145276
41	0.14	0.16	1.1	0.120238	0.122108	2.173981	2.046628
42	0.141	0.161	1.115	0.122336	0.124235	2.08201	1.957041
43	0.142	0.162	1.13	0.12378	0.126297	2.036596	1.875579
44	0.143	0.163	1.145	0.125854	0.128294	1.952708	1.801595
45	0.144	0.164	1.16	0.127596	0.130232	1.892792	1.734181
46	0.145	0.165	1.175	0.129632	0.132113	1.817593	1.672739
47	0.146	0.166	1.19	0.131601	0.13394	1.749412	1.616692
48	0.147	0.167	1.205	0.132578	0.135718	1.740009	1.565514
49	0.148	0.168	1.22	0.134921	0.137446	1.654988	1.518812
50	0.149	0.169	1.235	0.136534	0.13913	1.612774	1.476093
51	0.15	0.17	1.25	0.138598	0.140769	1.548445	1.437153
52	0.151	0.171	1.265	0.139847	0.142371	1.528155	1.401382
53	0.152	0.172	1.28	0.141879	0.143934	1.469312	1.368689
54	0.153	0.173	1.295	0.142504	0.145461	1.481399	1.338761
55	0.154	0.174	1.31	0.14468	0.146953	1.418299	1.311408
56	0.155	0.175	1.325	0.146025	0.148415	1.39653	1.286236
57	0.156	0.176	1.34	0.147494	0.149849	1.369681	1.26315
58	0.157	0.177	1.355	0.149235	0.151252	1.331414	1.242064
59	0.158	0.178	1.37	0.150143	0.15263	1.331203	1.22266
60	0.159	0.179	1.385	0.151568	0.153984	1.308391	1.204884
61	0.16	0.18	1.4	0.153368	0.155315	1.270265	1.188546
62	0.161	0.181	1.415	0.153838	0.156623	1.289165	1.173568
63	0.162	0.182	1.43	0.155757	0.157913	1.247418	1.159748
64	0.163	0.183	1.445	0.157542	0.159183	1.212545	1.14712
65	0.164	0.184	1.46	0.158176	0.160437	1.224511	1.135432
66	0.165	0.185	1.475	0.159554	0.161673	1.206865	1.124725
67	0.166	0.186	1.49	0.160446	0.162892	1.20853	1.114933
68	0.167	0.187	1.505	0.162692	0.164098	1.15856	1.105901
69	0.168	0.188	1.52	0.16412	0.165291	1.140722	1.097569
70	0.169	0.189	1.535	0.164617	0.166471	1.157547	1.089897
71	0.17	0.19	1.55	0.166144	0.16764	1.136522	1.082811
72	0.171	0.191	1.565	0.167307	0.168796	1.129083	1.076375
73	0.172	0.192	1.58	0.168549	0.169944	1.119039	1.070331
74	0.173	0.193	1.595	0.169158	0.171081	1.131286	1.064838
75	0.174	0.194	1.61	0.170299	0.172208	1.124899	1.059807
76	0.175	0.195	1.625	0.172247	0.173327	1.091331	1.055158
77	0.176	0.196	1.64	0.172203	0.174438	1.125282	1.050862
78	0.177	0.197	1.655	0.174007	0.175542	1.097178	1.046929
79	0.178	0.198	1.67	0.175345	0.17664	1.085065	1.04327
80	0.179	0.199	1.685	0.17648	0.177731	1.079772	1.039882
81	0.18	0.2	1.7	0.177796	0.178814	1.068868	1.036835
82	0.181	0.201	1.715	0.178511	0.179894	1.077034	1.033976
83	0.182	0.202	1.73	0.179783	0.180969	1.067756	1.031297

84	0.183	0.203	1.745	0.18136	0.182038	1.049414	1.028874
85	0.184	0.204	1.76	0.182401	0.183103	1.047644	1.026629
86	0.185	0.205	1.775	0.183375	0.184161	1.047901	1.024627
87	0.186	0.206	1.79	0.183631	0.185217	1.06936	1.022723
88	0.187	0.207	1.805	0.185354	0.186269	1.047488	1.020983
89	0.188	0.208	1.82	0.186203	0.187319	1.051334	1.019335
90	0.189	0.209	1.835	0.187133	0.188365	1.052788	1.017848
91	0.19	0.21	1.85	0.188154	0.189408	1.051638	1.016451
92	0.191	0.211	1.865	0.189761	0.190447	1.034185	1.015211
93	0.192	0.212	1.88	0.191855	0.191483	1.003937	1.014057
94	0.193	0.213	1.895	0.191841	0.192517	1.031303	1.012987
95	0.194	0.214	1.91	0.192286	0.19355	1.045948	1.011996
96	0.195	0.215	1.925	0.19407	0.194579	1.024575	1.011087
97	0.196	0.216	1.94	0.194501	0.195609	1.039321	1.010186
98	0.197	0.217	1.955	0.195758	0.196635	1.032206	1.009434
99	0.198	0.218	1.97	0.197279	0.19766	1.018458	1.008688
100	0.199	0.219	1.985	0.198106	0.198683	1.022677	1.00802

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