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Synchronous code-division multiplex systems

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SYNCHRONOUS CODE-DIVISION MULTIPLEX SYSTEMS

by

ROGER B. HANES   B.Tech.

A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy of the University of Technology, Loughborough.

September, 1975

Supervisor: Dr A. P. Clark

Department of Electronic and Electrical Engineering

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ABSTRACT

The investigation is concerned with various synchronous multiplexing and demultiplexing processes suitable for use with serial baseband data-transmission systems. The multiplexed signals are transmitted in orthogonal groups over a channel which introduces additive white Gaussian noise but no signal distortion.

Techniques are considered for increasing both the capacity and tolerance to additive noise, when the number of multiplexed signals may vary with time, and may exceed the maximum number of orthogonal multiplexed signals.

Several different multiplexing schemes have been proposed together with a variety of demultiplexing and detection processes. The optimum detection process is of limited practical value because of the very large number of sequential operations required when there are more than a few signals in a group. The more effective of the suboptimum detection processes achieve a tolerance to additive white Gaussian noise approaching that of the optimum detector but require far fewer sequential operations and can be implemented quite simply.

The tolerances to noise of the various multiplexing and demultiplexing schemes have been assessed by computer simulation for different numbers of multiplexed signals.

A particular scheme utilising a ternary transmitted signal has been proposed, which shows a significant advantage over a conventional time-division multiplex system. A trade off exists between the number of signals multiplexed and the tolerance to additive noise. The number of signals multiplexed may exceed the maximum number of orthogonal multiplexed signals with a slowly deteriorating tolerance to noise.

A hardware model has been constructed using this scheme. It is capable of multiplexing and demultiplexing up to eight signals. The performance of the model agrees well with the results of the corresponding computer simulation tests.

The theoretical aspect of the optimum multiplexing arrangement has been considered briefly. The different transmitted signals are here represented as points in n-dimensional Euclidean signal space, and are positioned in such a way as to maximise the minimum distance between these points.

GLOSSARY OF SYMBOLS AND TERMS

m	number of active channels multiplexed.
n	number of sample values corresponding to a group of transmitted or received signal elements.
S	n-component row vector whose components carry the transmitted element values of the transmitted group of signal elements.
R	n-component row vector whose components are the sample values of the received group of signal elements.
W	n-component row vector whose components are sample values of a Gaussian random variable with zero mean and variance σ^2.
σ^2	two-sided power spectral density of zero mean additive white Gaussian noise at the input to the receiver filter.
$\|x\|$	magnitude (absolute value) of x, if x is a scalar.
$\|X\|$	length (Euclidean norm) of X, if X is a vector.
$\{x_i\}$	the components of X, if X is a row vector.
$\{X_i\}$	the rows of X, if X is a matrix.
a_{ij}	the component of matrix A, located in the ith row and jth column.
A_i	the ith row of the matrix A.
A^{-1}	the inverse of matrix A.
A^T	the transpose of matrix A.

signs (A) the operator "signs" replaces each component of the vector A by ±1, the selected sign being the same as the component of A.

A signal element is a unit component of a digitally-coded signal.

Vectors are treated as matrices having one row or column.

SYNCHRONOUS CODE-DIVISION MULTIPLEX SYSTEMS

CHAPTER 1

INTRODUCTION

1.1 Subject of Thesis

This thesis is concerned with improving conventional multiplexing techniques with respect to their capacity and tolerance to additive white Gaussian noise. The multiplexed signals are transmitted over a common transmission path, from a single transmitter to a single receiver, and the demultiplexing of the signals is achieved in the detection process at the receiver.

1.2 Conventional methods of multiplexing signals

Multiplexers enable signals from several data sources to be transmitted simultaneously, but independently over a common transmission path from a single transmitter to a single receiver. Figure 1.1-1 shows a general multiplex system which consists of a multiplexer, the transmission path and a demultiplexer. The techniques of multiplexing involve coding the input data signals at the transmitter, corresponding to the independent channels in a manner which ensures non-interference, and allows the original data signals to be identified correctly at the receiver. In the demultiplexer an inverse operation is performed to separate the multiplexed data signals. Multiplexing is possible and of economic value because the data signals that are multiplexed require a much narrower bandwidth than that of the common transmission path.

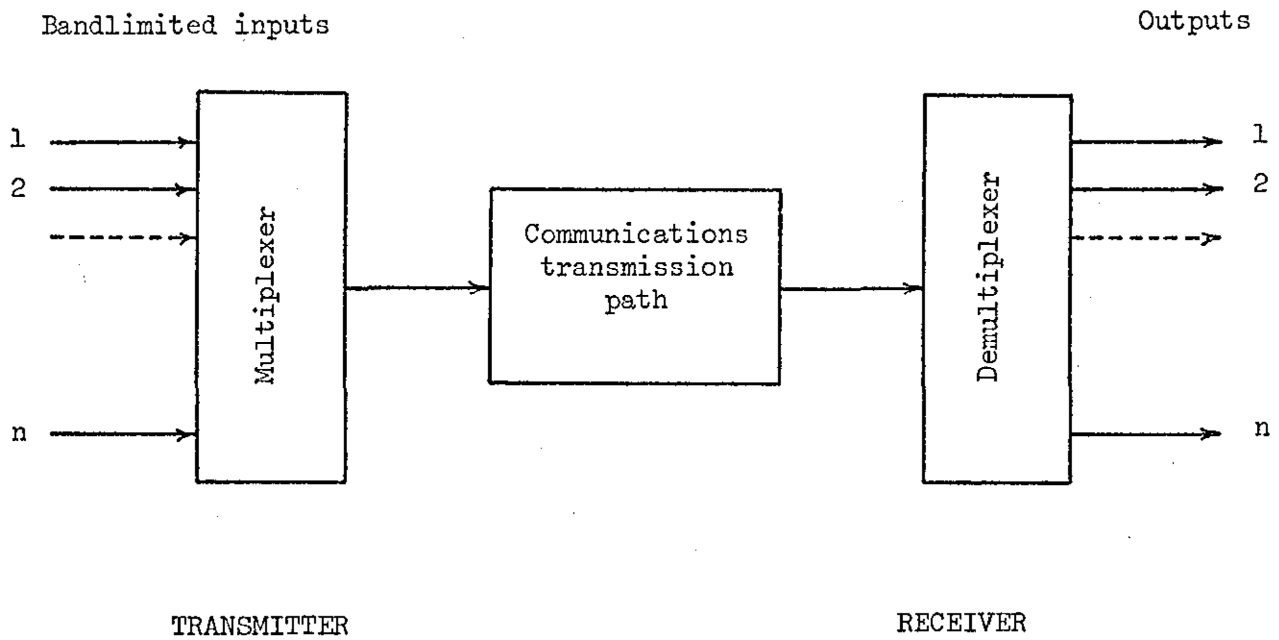


Figure 1.1-1 General multiplex system

The most widely used methods of multiplexing signals are frequency-division multiplex (FDM), and time-division multiplex (TDM).[1,16-25] Whereas only FDM may be applied to analogue signals, both FDM and TDM may be applied to digital signals. The important property of these multiplex methods is that the different signals are orthogonal.

The two functions $f(j,x)$ and $f(k,x)$ are orthogonal in the interval $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$ if the integral

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} f(j,x) \cdot f(k,x)\, dx \begin{array}{ll} = 0 & \text{for } j \neq k \\ = \text{finite, non zero} & \text{for } j = k \end{array} \qquad (1.1\text{-}1)$$

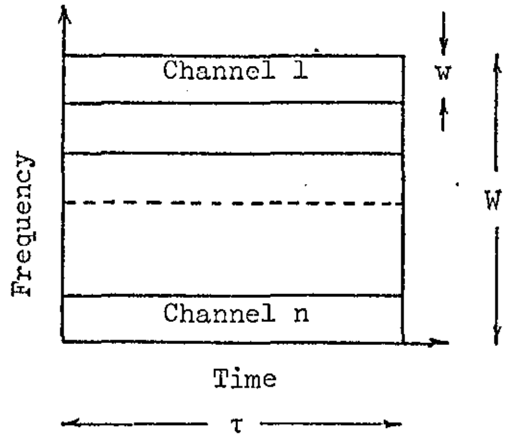
They are orthogonal and normal, or orthonormal if the integral is equal to 1 for $j = k$.

In an FDM system the total available frequency bandwidth W is divided into narrow bands of bandwidth w, each being used by a separate channel corresponding to the data sources. Individual input data signals have exclusive use of a frequency band. No inference is made as to the type of signals transmitted or to the methods of modulation and detection used.

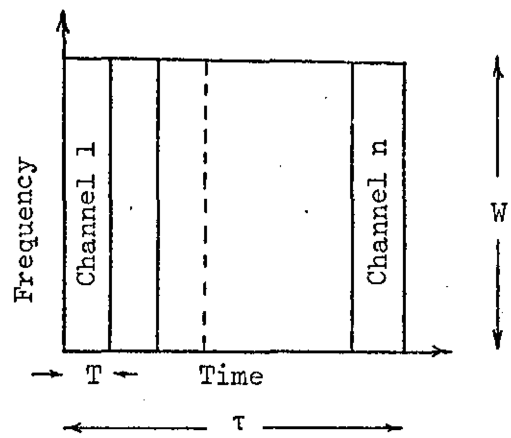
In a TDM system each interval of τ seconds called an element period, is divided into n discrete time slots of T seconds. Each input data signal is assigned a specific time slot, but has the total bandwidth W available.

The frequency bands and time slots for these signals are shown in Figure 1.1-2. There are W/w frequency bands and τ/T separate time slots in the frequency-time space allocated to n signals corresponding to n different channels or data sources.

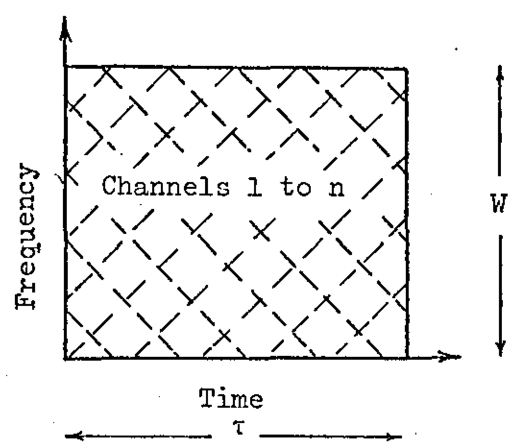
$$n = \frac{W}{w} = \frac{\tau}{T} \qquad (1.1\text{-}2)$$



a) Frequency division multiplex



b) Time division multiplex



c) Code division multiplex

Figure 1.1-2 Three basic multiplex methods

In a code-division multiplex system (CDM),[31-38] the frequency-time space is not divided (in a well defined way) amongst the input data signals (Figure 1.1-2). A system which separates the signals by coding is called a CDM system. Each independent channel has continuous, but not exclusive usage of the available bandwidth, by employing a coded signal waveform which is repeated in each element period of τ seconds.

The sampling theorem states that a signal waveform which is strictly band-limited to the frequency range 0 to W Hz and which therefore has an infinite duration, can be completely specified by a knowledge of its values at sampling points regularly spaced at intervals of $\frac{1}{2W}$ seconds over the whole of its infinite duration.

In a practical situation when the waveform is non-zero over the period τ seconds ($\tau \gg \frac{1}{2W}$) and zero at all points outside this interval, the waveform can be completely specified by a knowledge of its values at the $2W\tau$ sampling points which are spaced at $\frac{1}{2W}$ seconds over the period τ. The information conveyed in a period of τ seconds is given by the value of $n = 2W\tau$ sample values for that element period, and thus the detection of the received signals can be carried out entirely by operating upon the $2W\tau$ sample values per element period.

These $2W\tau$ samples give the maximum number of orthogonal waveforms that may be represented in the time τ. An infinite number of sets of orthogonal functions exist, the simplest being a TDM system where the signals are rendered orthogonal by employing independent sample values corresponding to the different multiplexed signals.

In a CDM system, the individual data signals are first coded into a unique combination of n pulses, given by the particular set of orthogonal functions used, these functions being the discrete codes. The coincident pulses belonging to the different data signals are in synchronism and all have the same width. They are combined using various techniques to give the resultant transmitted signal.[39-71]

Multiplex communications systems may be divided into two basic categories, linear systems and non-linear systems.[21] A multiplex system is linear or non-linear according to whether the transmitted signal is a linear function or not, of the individual signal codes. Conventional FDM and TDM systems fall into the category of linear multiplexing, whereas CDM systems may be either linear (Chapters 3 and 4) or non-linear (Chapters 5 to 8).

1.3 Limitations of existing systems

Systems of a conventional nature appear to exhibit two limitations.[32]

Firstly, they are designed to multiplex up to a given maximum number of channels with a specified performance, which is by their nature independent of the number of channels in operation (active channels). In practice however, the maximum number of channels simultaneously used will rarely exceed 50% of the total number of channels, and on average the number of active channels is typically between 10% and 35%.[61] Thus a high percentage of the available capacity remains totally unused, and at present with conventional FDM and TDM techniques, the system performance, in terms of tolerance to noise, does not improve with a reduced number of channels. By contrast a better multiplex system would provide a specified performance for an average number of active channels, improved performance for few active channels, and a degraded performance for more active channels. Thus the total available bandwidth would be usefully employed at all times.

The second limitation evident with multiplex systems using sinewaves and pulses for the channel carriers as in FDM and TDM, is that they are prone to disruption by interference, since unpredictable impulsive noise normally occurs in forms similar to sinewaves and pulses.[32] Therefore systems are designed whose susceptibility to such interference is rendered tolerable by employing high signal/noise ratios and by including special additional subsystems such as error correcting units.[17] These make use of redundant information encoded into the transmitted signal to correct errors caused by additive noise. A better approach would be to use carriers which were not readily simulated by interference, these using specially coded waveforms where the precautions outlined with conventional systems would be either automatically inbuilt or unnecessary.

To summarise, a system is required to be inherently flexible as regards the maximum number of multiplexed channels, in which a trade off should exist between the number of active channels and the tolerance to noise. The transmitted signal should be less sensitive to interference than existing conventional techniques.

CHAPTER 2

MULTIPLEXING SIGNALS FOR A BASEBAND CHANNEL

2.1 Model of the data transmission system

The model of the data transmission system is shown in Figure 2.1-1. It is a synchronous serial baseband system, where the coder and multiplexer transmit a group of n binary or multilevel signal elements over an element period of nT seconds. This corresponds to the information presented synchronously to the coder and multiplexer of m active channels. The multiplexed signals are transmitted over a common channel, from a single transmitter to a single receiver and the demultiplexing is achieved in the detection process at the receiver.

At the transmitter an element timing waveform having an element period of nT seconds, is fed from the coder and multiplexer to the data sources whose signals consist of binary element values and take the value ±1. They have fixed values over the element duration of nT seconds, and reach the coder and multiplexer in element synchronism. Only the m sources and destinations of data, which are actually in operation are shown in Figure 2.1-1. Over any given element period, the m received binary element values at the coder and multiplexer are stored. The coder converts each of the m binary element values to a sequence of n impulses. These n impulses form a code, which gives the codewords for the m channels present. Each codeword is used by a single channel only and they are thus referred to as channel carriers. The multiplexer then combines these codewords, using linear and non-linear techniques, and transmits the resulting signal

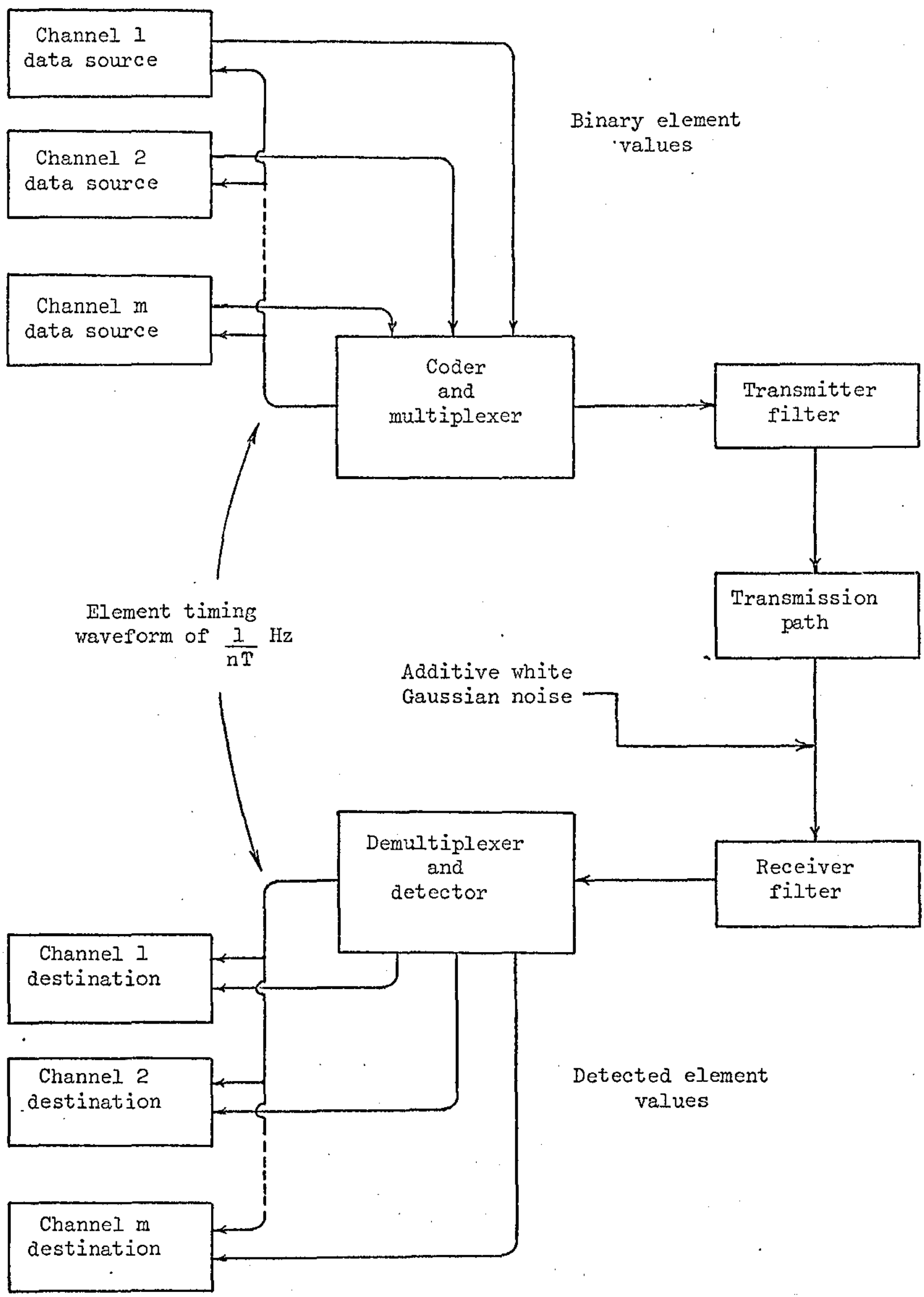


Figure 2.1-1 Model of the multiplexer, transmission system and demultiplexer.

over the transmission path. The transmitted signal consists of n signal elements or impulses forming a signal group, which is transmitted over the element period of nT seconds, the duration of the input binary element values.

At irregular intervals, new channels will start operation and channels already in operation will cease transmission, since the channels are completely independent of each other. During a transmission, each channel has a unique codeword associated with it, and that codeword remains unchanged and is used by that channel only. It is assumed that when a new channel starts operation its codeword is either selected at random from those not already in use, or that the channel has a codeword uniquely associated with it. Thus not only does the number of channels in operation m, vary over the full range from 0 to n, where n is the maximum number of orthogonal codewords for a transmitted group length of n digits over the element period of nT seconds, but for any given number of channels in operation at two widely separate times, two different sets of codewords will in general be in use.

The transmission path is assumed to be a linear baseband channel, which could include a modulator, bandpass channel and demodulator, and which introduces no signal delay, attenuation or distortion. The transmitter and receiver filters in Figure 2.1-1 are equivalent to all transmitter and receiver filters respectively, including any involved in modulation or demodulation. Thus the data signal at the output of the transmission path is an identical copy of that at the input.

Over some practical channels such as voice frequency channels using HF radio links, the most important type of noise introduced by the channel is additive noise, which can for practical purposes be taken to be additive white Gaussian noise. The difference between the two is sufficiently small not to introduce any serious discrepancies in the performance, in terms of tolerance to noise, when the noise actually present is taken to be white Gaussian noise.

Over telephone circuits, however, the most important source of noise is impulsive noise which sometimes resembles short bursts of Gaussian noise.[17] It has been shown that, if one data transmission system has a better tolerance to additive white Gaussian noise than another, it will also in general, have a better tolerance to the additive noise over telephone circuits.[28] It follows therefore, that the relative tolerance of two systems to additive white Gaussian noise is a good measure of their relative tolerance to the additive noise over telephone circuits.[17,28] Furthermore, whereas Gaussian noise is easily produced in the laboratory and analysed theoretically, the impulsive noise over telephone circuits is not

easily simulated accurately in the laboratory. Nor is it easy to achieve more than a probabalistic theoretical analysis to this noise.[17] For these reasons, in the model of the data transmission system, it is assumed that additive white Gaussian noise is introduced at the output of the transmission path. The noise has zero mean, and a two sided power spectral density of σ^2.

The receiver filter removes the noise components outside a frequency band approximately corresponding to the bandwidth of the received signal.

The impulse response h(t) of the transmitter and receiver filters in cascade, and hence the impulse response of the baseband channel, is assumed to be such that $h(0) = 1$ and $h(jT) = 0$ for all non-zero integers of j, the delay introduced by the filters and transmission path being neglected, so that these are in fact non-physical. This impulse response is achieved in a conventional manner by using the same transfer function $H^{\frac{1}{2}}(f)$ for the transmitter and receiver filters, where

$$H(f) = \begin{cases} \frac{1}{2}\, T\, (1 + \cos \pi f T) & \text{for } -\frac{1}{T} < f < \frac{1}{T} \\ 0 & \text{elsewhere} \end{cases} \qquad (2.1\text{-}1)$$

The use of the same transfer function for the transmitter and receiver filters is conventional[7,29] and enables an easy comparison to be made with other systems. Alternative transfer functions for the filters are available, and some of these make more efficient use of bandwidth.[29]

If C(f) is the transfer function of the transmission path, then the channel transfer function expressed in terms of the transfer functions of the transmission path and filters is,

$$Y(f) = H(f)\ C(f) \qquad (2.1\text{-}2)$$

and the impulse response of the channel y(t) is given by the inverse Fourier transform of Y(f), that is,

$$y(t) = F^{-1}\{Y(f)\} = \int_{-\infty}^{\infty} C(f)\, H(f)\, e^{j2\pi ft}\, df \qquad (2.1\text{-}3)$$

When no signal distortion is introduced by the transmission path, that is when $C(f) = 1$,

$$y(t) = \int_{-\infty}^{\infty} H(f)\, e^{j2\pi ft}\, df \qquad (2.1\text{-}4)$$

From Eqn. (2.1-1),

$$y(t) = \tfrac{1}{2}T \int_{-1/T}^{1/T} (1 + \cos \pi fT)\, e^{j2\pi ft}\, df$$

$$= \tfrac{1}{2}T \int_{-1/T}^{1/T} (1 + \tfrac{1}{2}e^{j\pi ft} + \tfrac{1}{2}e^{-j\pi ft})\, e^{j2\pi ft}\, df$$

$$= \tfrac{1}{2}T \int_{-1/T}^{1/T} \{e^{j\pi f2t} + \tfrac{1}{2}e^{j\pi f(2t+T)} + \tfrac{1}{2}e^{j\pi f(2t-T)}\}\, df$$

$$= \tfrac{1}{2}T \left| \frac{e^{j\pi f2t}}{j\pi 2t} + \tfrac{1}{2}\,\frac{e^{j\pi f(2t+T)}}{j\pi(2t+T)} + \tfrac{1}{2}\,\frac{e^{j\pi f(2t-T)}}{j\pi(2t-T)} \right|_{-1/T}^{+1/T}$$

$$= \frac{e^{j\pi\frac{2t}{T}} - e^{-j\pi\frac{2t}{T}}}{2j\pi\frac{2t}{T}} + \tfrac{1}{2}\,\frac{e^{j\pi(\frac{2t}{T}+1)} - e^{-j\pi(\frac{2t}{T}+1)}}{2j\pi(\frac{2t}{T}+1)} + \tfrac{1}{2}\,\frac{e^{j\pi(\frac{2t}{T}-1)} - e^{-j\pi(\frac{2t}{T}-1)}}{2j\pi(\frac{2t}{T}-1)}$$

$$= \frac{\sin \pi\frac{2t}{T}}{\pi\frac{2t}{T}} + \tfrac{1}{2}\,\frac{\sin \pi(\frac{2t}{T}+1)}{\pi(\frac{2t}{T}+1)} + \tfrac{1}{2}\,\frac{\sin \pi(\frac{2t}{T}-1)}{\pi(\frac{2t}{T}-1)} \qquad (2.1\text{-}5)$$

Figures 2.1-2 and 2.1-3 show the transfer Function H(f) and impulse response y(t) respectively.

Clearly, when C(f) = 1,

$$y(0) = 1 \qquad y(\pm \frac{T}{2}) = \tfrac{1}{2} \qquad (2.1\text{-}6)$$

and

$$y(\pm \tfrac{1}{2}\, iT) = 0 \qquad \text{for } i \neq 0 \text{ or } \pm 1 \qquad (2.1\text{-}7)$$

The received signal r(t) at the output of the receiver filter is sampled at time instants t = iT, for all integers i. This assumes that the receiver has prior knowledge of the time of arrival of each signal element, that is, the receiver is in element synchronism with the received signal. Techniques for achieving correct element synchronism have been widely studied and will not be considered further.[28]

The ith received element is sampled at time t = iT to give the sample value,

$$r(iT) = s_i y(0) + w(iT) \qquad (2.1\text{-}8)$$

or

$$r_i = s_i + w_i \qquad (2.1\text{-}9)$$

where $r_i = r(iT)$ and $w_i = w(iT)$, and it is assumed that C(f) = 1.

With additive white Gaussian noise having a two sided power spectral density of σ^2 at the input to the receiver filter, the noise power spectral density at the output of the receiver filter is,

$$\sigma^2 \; |H^{\frac{1}{2}}(f)|^2 = \sigma^2 H(f) \qquad (2.1\text{-}10)$$

so that the mean noise power is

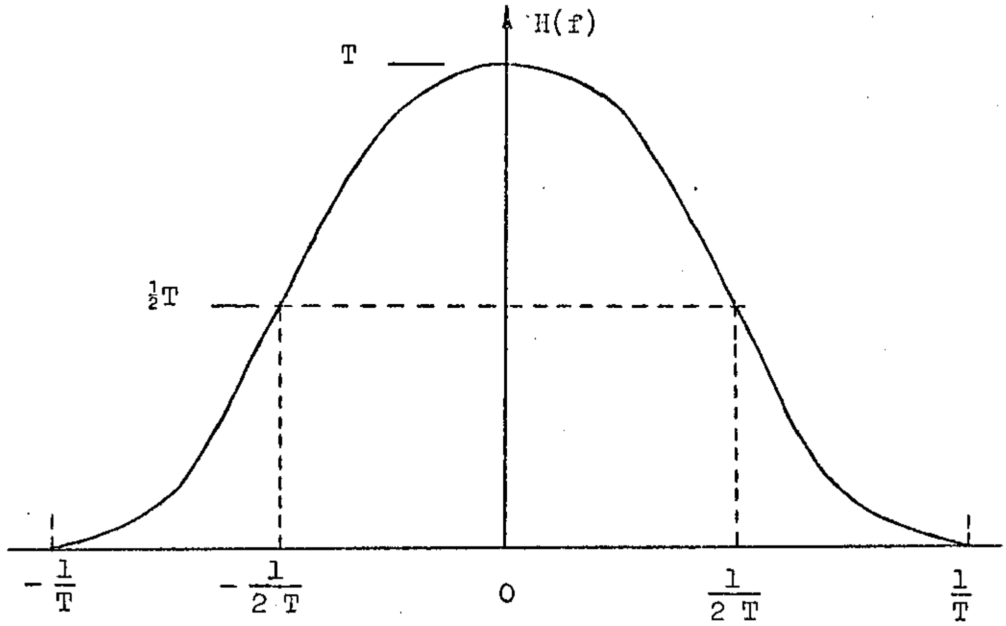


Figure 2.1-2 Transfer function H(f) of the transmitter and receiver filters.

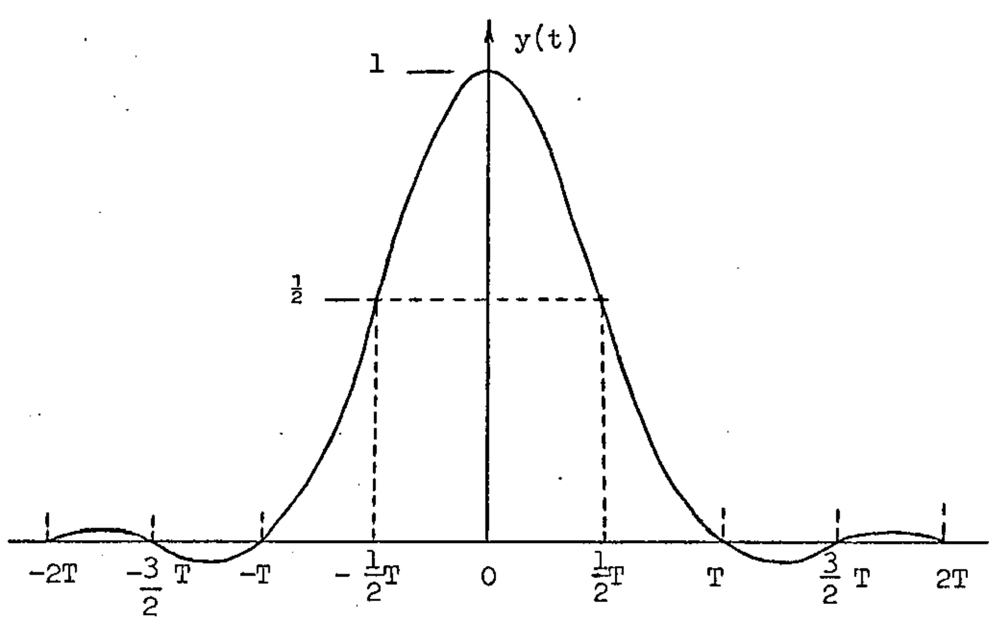


Figure 2.1-3 Impulse response y(t) of the baseband channel when no signal distortion is introduced by the transmission path.

$$\sigma^2 \int_{-\infty}^{\infty} | H(f) | \, df = \sigma^2 \qquad (2.1\text{-}11)$$

Thus w_i in Eqn. 2.1-9 is a sample value of a Gaussian random variable with zero mean and variance σ^2.

From the Wiener-Kinchine Theorem,[3] the autocorrelation function of the noise signal w(t) at the output of the receiver filter is,

$$d(\tau) = \int_{-\infty}^{\infty} \sigma^2 \, H(f) \, e^{j2\pi f\tau} \, df \qquad (2.1\text{-}12)$$

$$= \sigma^2 \left| \frac{\sin \pi \frac{2\tau}{T}}{\pi \frac{2\tau}{T}} + \tfrac{1}{2} \frac{\sin \pi(\frac{2\tau}{T} + 1)}{\pi(\frac{2\tau}{T} + 1)} + \tfrac{1}{2} \frac{\sin \pi(\frac{2\tau}{T} - 1)}{\pi(\frac{2\tau}{T} - 1)} \right.$$

and from Eqn. 2.1-4, clearly

$$d(0) = \sigma^2$$

and $$d(iT) = 0$$

for any non zero integer i. Since the mean value of w(iT) is zero, it follows that the noise component w(iT) is uncorrelated with the noise component w(hT), where the integer $h \neq i$, so that the $\{w_i\}$ are sample values of statistically independent Gaussian random variables with zero mean and variance σ^2.

The detector samples the received signal n times per element period at regular intervals of T seconds, the sampling instants being suitably phased with respect to the received data signal, such that, in the absence of noise, a transmitted impulse of value x gives a value x for the corresponding received sample, and a value zero for all other received samples.

While one store holds the n sample values for a detection process, another store is receiving the next n sample values, so that nT seconds are available for a detection process. In the detection process, the m element values corresponding to the m channels multiplexed are detected simultaneously by operating on the corresponding n sample values.

It is assumed that the detector has prior knowledge of the number of channels in use, m, and the codewords corresponding to these channels. This information must be fed to the receiver, possibly via a separate channel and updated immediately a channel ceases transmission, or transmission commences on a new channel. The techniques involved are not considered here, but are briefly considered elsewhere.[29,38]

2.2 Outline of investigation

The investigation is concerned with improving conventional multiplexing techniques with respect to their capacity and tolerances to additive white Gaussian noise. The multiplexed signals are transmitted over a common channel, from a single transmitter to a single receiver, and the demultiplexing is achieved in the detection process at the receiver. The primary aim of the investigation has been to obtain a better understanding of these systems and hence to develop the most cost-effective arrangement. Since the various systems studied are all arrangements for processing sets of numerical values, these are computer like systems which are best simulated on a computer rather than tested on a practical model. The latter would simply be a special purpose digital computer with the appropriate analogue/digital converter interfaces.

Chapter 3 starts with a survey of linear multiplexing systems in which the transmitted signal is a linear function of the individual channel codewords. The resultant transmitted signal is therefore multilevel. In particular, a system is described which employs a combination of time- and code-division multiplexing, in which the TDM signal elements are orthogonal as are the CDM signal elements, but simultaneously transmitted TDM and CDM signal elements are not orthogonal. This arrangement uses a non-linear combination of the linear sums of the TDM and CDM orthogonal set codewords to form the resultant transmitted signal. It is particularly well suited to applications where the number of multiplexed channels is typically a little greater than the maximum number that may be orthogonally multiplexed using TDM alone.

Chapter 4 develops the previous multiplexing arrangement and describes a system which combines non-linearly a TDM and two CDM sets of orthogonal signals. Up to three times as many channels may be multiplexed than is possible using TDM alone.

Chapter 5 surveys non-linear multiplexing systems using Walsh functions[34] for the channel codewords. These systems use a non-linear majority logic multiplexing operation,[56-60] and generate a resultant binary transmitted signal.

Chapter 6 describes two non-linear multiplexing arrangements which overcome the disadvantages of proposed Walsh majority logic multiplexing systems. The first arrangement generates a multilevel transmitted signal, whereas the second user majority logic multiplexing resulting in a ternary transmitted signal. The available bandwidth and power is well utilised for any number of active channels.

Its performance compares favourably with conventional TDM systems operating under the same conditions of transmitted signal energy and signalling rate. The number of channels multiplexed may exceed the maximum number of orthogonal channels with a slowly deteriorating tolerance to noise as the number of channels increases.

Chapters 7 and 8 present various demultiplexing arrangements for use with the two multiplexing arrangements of the previous chapter. The optimum detection process, which minimises the probability of error in the detection of the m element values of a group, is of limited practical value because of the very large number of sequential operations required when there are more than a few signals in a group. The more effective of the suboptimum detection processes achieve a tolerance to additive white Gaussian noise approaching that of the optimum detector, but requires far fewer sequential operations and can be implemented quite easily.

Chapter 9 describes a hardware model of the most attractive system, capable of multiplexing and demultiplexing up to eight channels. It was designed and constructed in order to focus attention on the practical realisation and economic aspects of a multiplex system that has hitherto been tested by computer simulation only.

Chapter 10 considers briefly the theoretical aspect of the optimum multiplexing arrangement. This gives the lowest probability of error of any arrangement in the detection of the m element values, when used in conjunction with the optimum detection process. The different transmitted signals are here represented as points in n-dimensional Euclidean signal space, and are positioned in such a way as to maximise the distance of these points. The overall complexity is demonstrated by a series of relatively simple examples.

CHAPTER 3

LINEAR CODE-DIVISION MULTIPLEXING

3.1 Introduction

In recent years, the availability of inexpensive digital circuitry has focused considerable attention on the possibility of applying easily generated and manipulated binary functions to tasks exclusive to sinusoidal functions. Emphasis has been given to the use of Walsh functions as a basis for multiplexing various data sources for transmission over a common channel. They were first described by Walsh[39] in 1922, and simultaneously but independently, Rademacher presented a system of functions which were later shown to be a subset of Walsh functions. Little attention was devoted to Walsh functions from a engineering standpoint until in 1969 when Harmuth published an article in the I.E.E.E. Spectrum[40] which aroused much interest in the area. The possibility of replacing many tasks previously the domain of sinusoidal functions with an easily generated binary function, and the increasing availability of digital integrated circuits, was a contributing factor to the emergence recently of nine international conferences on Walsh functions and their applications, in Washington and at the Hatfield Polytechnic.

Walsh functions[41-46] are a complete set of binary functions that are periodic and orthogonal. Figure 3.1-1 shows the first sixteen functions. A mathematical definition has been given by Davidson,[46] although many variations exist. The functions are defined over the interval $0 \leqslant \theta < 1$ where θ is the normalised time variable. A popular symbol for a function is,

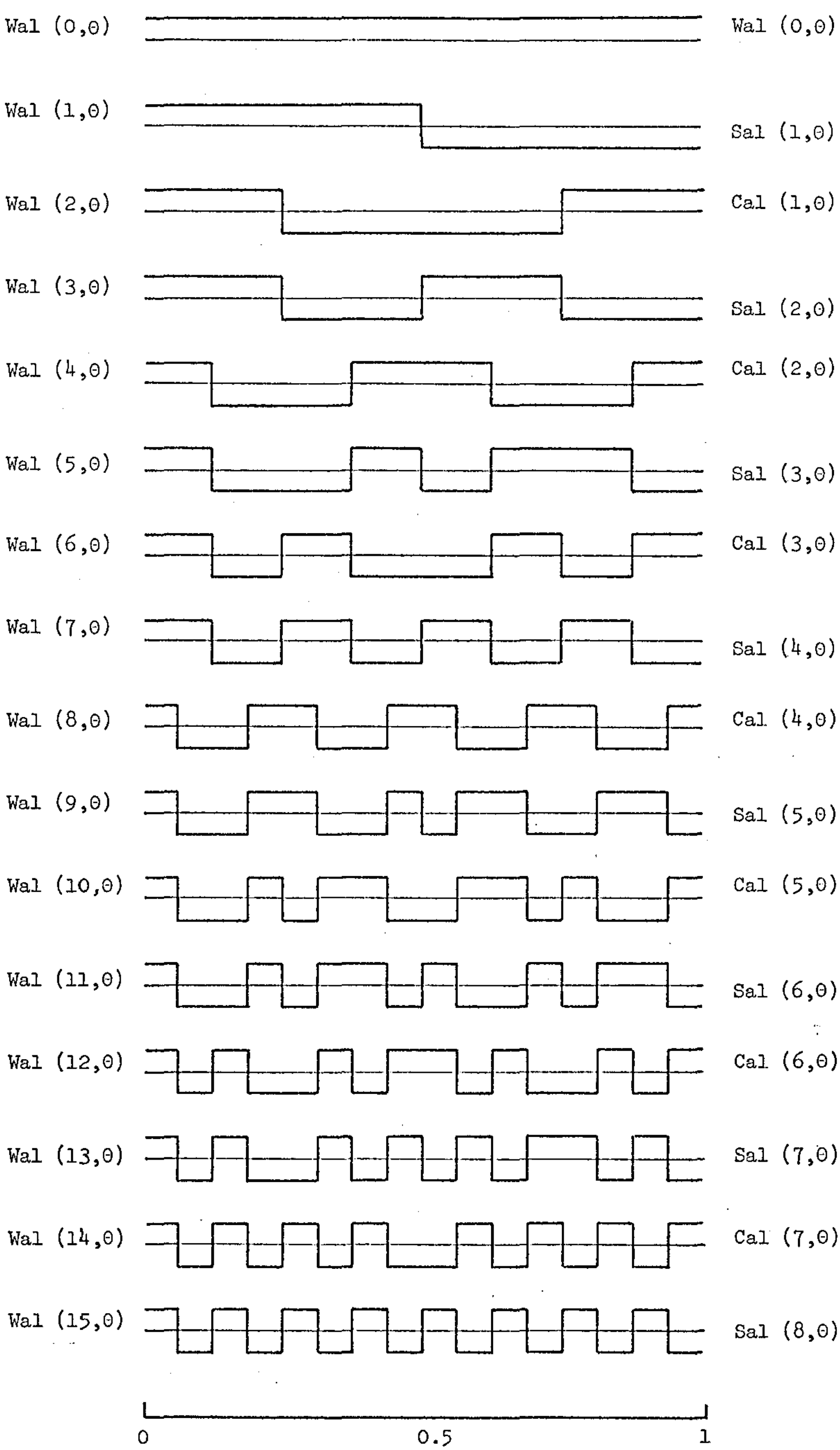


Figure 3.1-1 The first sixteen Walsh functions

Wal (k,θ)

The parameter k is the order of the function, and is equal to one half the number of zero crossings in the interval $0 \leqslant \theta < 1$. k is referred to as the sequency, and is analagous to the frequency of circular functions, although the sign changes are not equidistant. Walsh functions are divided into two groups called Sal and Cal functions,[45,46] which correspond to the circular functions Sine and Cosine. However, the complete set can be described by a single function that includes the Sal and Cal functions.

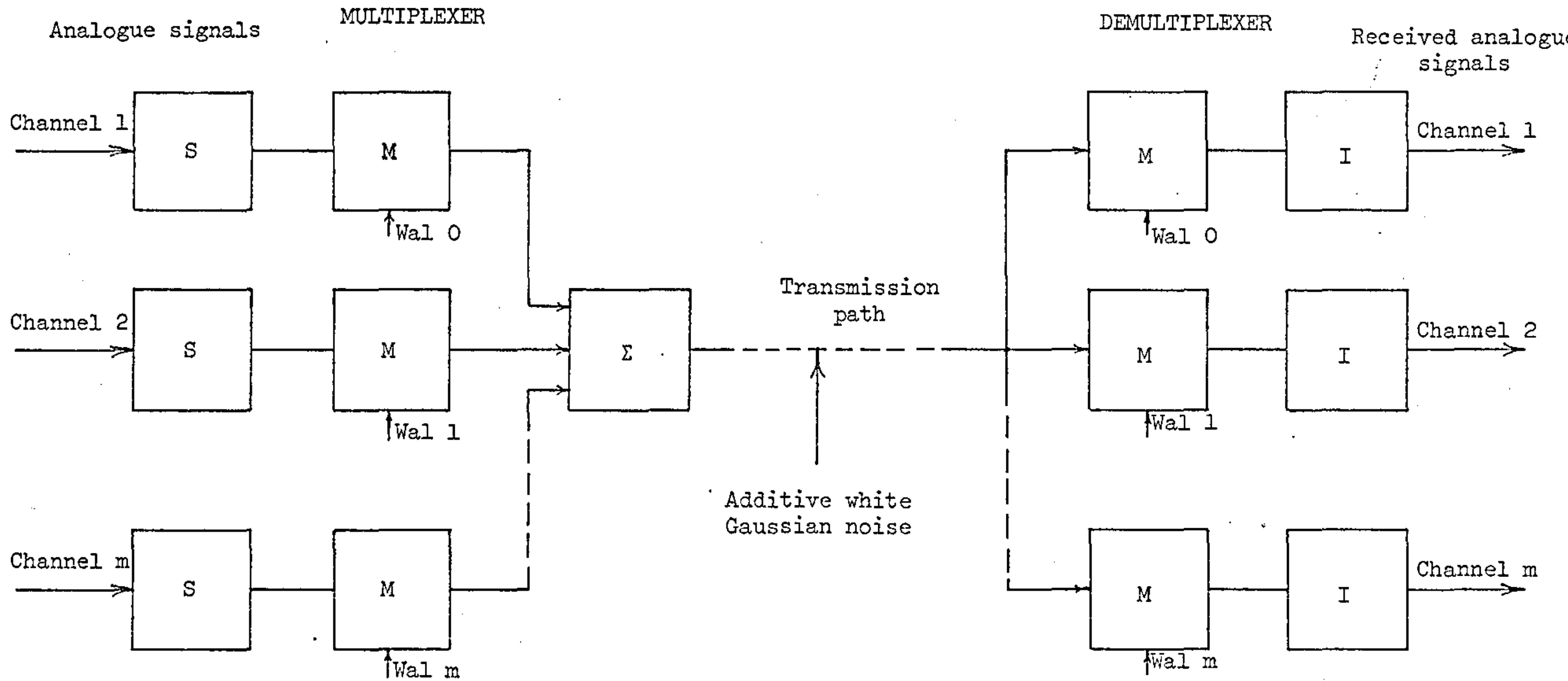
Figure 3.1-1 shows a set of sixteen Walsh functions ordered in terms of their sequency. Expressed as components ±1 they are by definition of length 2^n, $n = 1,2 \ldots$. Other ordering exist, and in particular, a set of Walsh functions set down in appropriate order as lines of a matrix constitute the best-known form of Hadamard matrix, namely one of order $m = 2^n$.[34]

Because Walsh functions are orthogonal it is possible to use them as signal carriers for multiplexing systems like the circular functions.[42,46-48] However, being two values, they are very easily generated,[43,45,70] and have considerable computational and equipment advantages. Analogous to the amplitude, the frequency and the phase modulation associated with the conventional trigonometric functions, the information is equally contained in the amplitude, sequency, and time position of the Walsh carriers. However, the orthogonality is only preserved if the Walsh functions are synchronised and are in phase.[54] Similar to conventional demultiplexing techniques, information that is amplitude modulated on to a given Walsh function, may be recovered by a process of correlation or matched filter detection.[10,27] This minimises the probability of error in the detection of the individual channel element values by maximising the ratio of the energy level of the wanted signal, to the average energy level of the noise components.

3.2 Linear multiplexing using Walsh functions

Figure 3.2-1 shows a block diagram of the multiplex system using Walsh functions as the channel carriers. The m active input analogue signals are passed through sample and hold circuits S, and multiplied by the corresponding Walsh functions using analogue multipliers over an element period of nT seconds. The summation circuit Σ adds linearly the modulated codewords to form the resultant transmitted signal. The transmission path introduces additive white Gaussian noise, having a noise power spectral density σ^2 and zero mean. At the receiver, the demultiplexer consists of a process of correlation detection in which the received signal is multiplied by Walsh functions identical to those used in the multiplexer. Because the signal carriers are orthogonal, the received signals are extracted by integrating the resulting analogue signals. Interference from other channels having high frequency components is thus suppressed.

This method was first described by Judge[49] in 1962, and later by Bagdasarjanz and Loretan,[50] where they consider the cross talk generated by various parts of the system. A working system designed for 1024 channels has been described by Hübner of the West German Post Office.[51] The main problem with these systems is that of cross talk caused by inaccurate synchronisation,[52,54] and realising sufficiently linear analogue multipliers. Synchronisation problems may be eased if Rademacher functions (square waves) are used, the orthogonality of which is invariant with a time shift.[55] Another problem is the widespread development of conventional FDM systems causing a justifiable reluctance to change for even quite considerable technical gains. The advantage of this system over FDM is



S - sample and hold circuit

M - analogue multiplier

Σ - summation circuit

I - integrator

Figure 3.2-1 A linear multiplex system using Walsh functions the absence of a set of single sideband filters. The Walsh function multipliers replace these and because the functions are two valued the process is very easily implemented especially using integrated circuit technology. Also, Walsh function generation is much simpler than frequency synthesis.

The position is different, however, for the Walsh multiplexing of binary signals.[46,51,68] The transmission of binary data for communication and computer purposes is beginning to impose its own requirements for which the equipment in service is as yet limited in quality.[16,20] The input binary signals are assigned to individual channels in the multiplex system. For n channels in operation, because of the linear summation of the Walsh codewords, the resultant transmitted signal has n+1 amplitude levels. Such a system, therefore, would require the provision of extensive regenerative repeaters to deal with multi-amplitude signals. These systems give very low probability of error in the received data signals, for they are not susceptible to a pulse type disturbance, because the individual channel signal energies are spread over the entire element period.

The next section discusses in detail, an interesting system for the combination of a TDM and a digital CDM system.

3.3 System A1

This arrangement, proposed by Clark,[35] is capable of extending a conventional binary TDM system with additional channels using CDM codewords, such that the overall tolerance to additive white Gaussian noise of the system is only degraded slightly by the addition of a few extra channels.

The arrangement uses a combination of TDM and CDM, in which the TDM signal elements are orthogonal as are the CDM elements, but simultaneously transmitted TDM and CDM elements are not orthogonal. With this arrangement, up to twice as many channels may be multiplexed, for a given transmission path and signal element rate per channel, than is possible with orthogonal multiplexing using either TDM or CDM alone.

The transmitted signal elements are arranged in separate groups, which are transmitted sequentially, and there is no intersymbol interference between elements in different groups. At any particular time, the total number of channels may have any value from 0 to 2n, where n is the maximum number of orthogonal TDM or CDM channels. If a group of m elements contain u elements from different TDM channels and v elements from different CDM channels, then clearly $u \leqslant n$, $v \leqslant n$ and $u + v = m$.

The TDM codewords which are used as the signal carriers for the n TDM channels are given by the rows $\{A_i\}$ of an $n \times n$ identity matrix. The complete set of n TDM codewords will be referred to as the orthogonal set A. If the ith codeword, from the set of n codewords corresponding to the ith TDM channel is given by $\sum_{j=i}^{n} a_{ij}\, \delta\,(t - jT)$, it may be represented by the n-component row vector,

$$A_i = 0 \;.\; .\; 0 \;\; a_{ii} \;\; 0 \;.\; .\; 0 \qquad (3.3\text{-}1)$$

whose ith component is $a_{ii} = 1$.

The CDM codewords which are used as the signal carriers for the different channels are given by the rows $\{B_i\}$ of an $n \times n$ Hadamard matrix. For the particular case where a codeword or signal element contains 16 components, that is $n = 16$, the matrix B is shown in Figure 3.3-1. The complete set of CDM codewords will be referred to as the orthogonal set B. If the ith codeword from the set of n CDM codewords is given by

$$
B = \frac{1}{4}\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1
\end{bmatrix}
$$

Figure 3.3-1 The matrix of set B codewords

$\sum_{j=1}^{n} b_{ij}\, \delta\, (t - jT)$, it can be represented by the n-component row vector,

$$B_i \;=\; b_{i1}\;\; b_{i2}\;\; .\;\; .\;\; .\;\; b_{in} \tag{3.3-2}$$

The element value of the signal element in the ith of the n channels whose codewords belong to the orthogonal set A, is $x_i = \pm 1$ when a signal is present in this channel, or $x_i = 0$ when no signal is present. Similarly, the element value of the signal element in the ith of the n channels whose codewords belong to the orthogonal set B, is $y_i = \pm 1$ when a signal is present in this channel, or $y_i = 0$ when no signal is present. Let X and Y be the n-component row vectors with ith components x_i and y_i respectively.

It is assumed that the $u\{x_i\}$, $v\{y_i\}$ for the m active channels are statistically independent and equally likely to have either binary value. The $u\{x_i\}$, $v\{y_i\}$ are not necessarily the first u and v of the $n\{x_i\}$ $n\{y_i\}$, but may be any of the $n\{x_i\}$, $n\{y_i\}$.

The coder and multiplexer combine the m codewords for the two orthogonal sets over the period 0 to nT seconds.

The orthogonal set A and set B codewords, $\{A_i\}$ and $\{B_i\}$, are multiplied by the corresponding binary element values $\{x_i\}$ and $\{y_i\}$, $i = 1 - - n$, so that each codeword given by (3.3-1) and (3.3-2) is binary antipodal. The orthogonal set A codewords are added linearly to give,

$$XA \tag{3.3-3}$$

and the orthogonal set B added to give,

$$YB \tag{3.3-4}$$

The n components of the signal vector YB for the orthogonal set B are mutliplied by a scalar, whose value is positive and equal to c, and determines the level of the vector YB to give,

$$cYB \tag{3.3-5}$$

The n components of the vector XA are now combined non-linearly with the n components of the vector cYB as follows. For each j, if the jth component of XA is negative, then the sign of the jth component of cYB is reversed. The jth components are now added linearly to give the jth component of the transmitted signal S.

$$S = XA + \text{signs}(XA)\ (cYB) \tag{3.3-6}$$

where the operator "signs" replaces each term of the vector XA by ±1 corresponding to the sign of the components of XA. For components of XA equal to zero, then the operator "signs" on those components gives a value +1.

The non-linear combination described may be regarded as a process of amplitude modulation, the components of XA being modulated or systematically altered respectively by the coincident components of cYB. The reason for using a non-linear combination rather than a linear one, lies in the detection process, which is now capable of detecting the orthogonal set B element values without prior knowledge of the orthogonal set A element values. Error extension effects are thus minimised and the probability of error is reduced relative to linear coding.

In the model of the system (Figure 2.1-1), white Gaussian noise with a two sided power spectral density of σ^2 is added to the data signal at the output of the transmission path, giving the Gaussian waveform w(t) added to the data signal at the output of the receiver filter, as described in

Section 2.1.

The signal at the output of the receiver filter over the duration of a single group of coincident signal elements is sampled at regular time intervals of T seconds to give the n components of the received data signal vector R.

$$R = S + W \tag{3.3-7}$$

where S and W are n-component vectors, and for the particular receiver filter in use, the $\{w_i\}$ are sample values of statistically independent Gaussian random variables with zero mean and variance σ^2.
From Eqns. (3.3-6) and (3.3-7)

$$R = XA + \text{signs}(XA)\,(cYB) + W \tag{3.3-8}$$

The detection process uses two separate sets of correlation detectors matched to the orthogonal sets A and B. These minimise the probability of error in the detection of the individual channel element values by maximising the ratio of the energy level of the wanted signal, to the average energy level of the noise components.

In general, the ith element value p_i, corresponding to the orthogonal set Z (any orthogonal set) is detected by feeding Q (the set of n sample values of the input signal to the correlation detectors) to the correlation detector matched to Z_i. The correlation detector multiplies the jth component of Q by the jth component of Z_i, for $j = 1 - - n$, and adds the products to give the output signal. The ith element value is detected from the sign of the output signal.

$$p_i = \text{sign} \sum_{j=1}^{n} q_j z_{ij} \qquad (3.3\text{-}9)$$

where q_j and z_{iz} are the jth components of the row vectors Q and Z_i.
Using matrix rotation,

$$P_i = \text{signs}\ (QZ^T) \qquad (3.3\text{-}10)$$

where the operator "signs" replaces each term of the vector QZ^T by ±1 corresponding to the sign of the components of QZ^T.

Figure 3.3-2 shows a block diagram of the demultiplexing and detection process which operates in an iterative fashion, thus saving hardware and reducing complexity. For convenience the process is divided into the first and subsequent cycles, as the first detection cycle differs slightly from the following cycles which are identical. In the first cycle of the iterative process, the detector determines the binary element values $\{x_i'\}$ for the orthogonal set A from the signs of the components of R. This is because the matrix A is an identity matrix. Let X' be the n-component row vector with components $\{x_i'\}$

$$\begin{aligned} X' &= \text{signs}\ (RA^T) \\ &= \text{signs}\ (R) \end{aligned} \qquad (3.3\text{-}11)$$

and for those channels not in operations, the corresponding element values $\{x_i'\}$ are set to zero.
From Eqn. (3.3-8),

$$X' = \text{signs}\ (XA + \text{signs}(XA)\ (cYB) + W) \qquad (3.3\text{-}12)$$

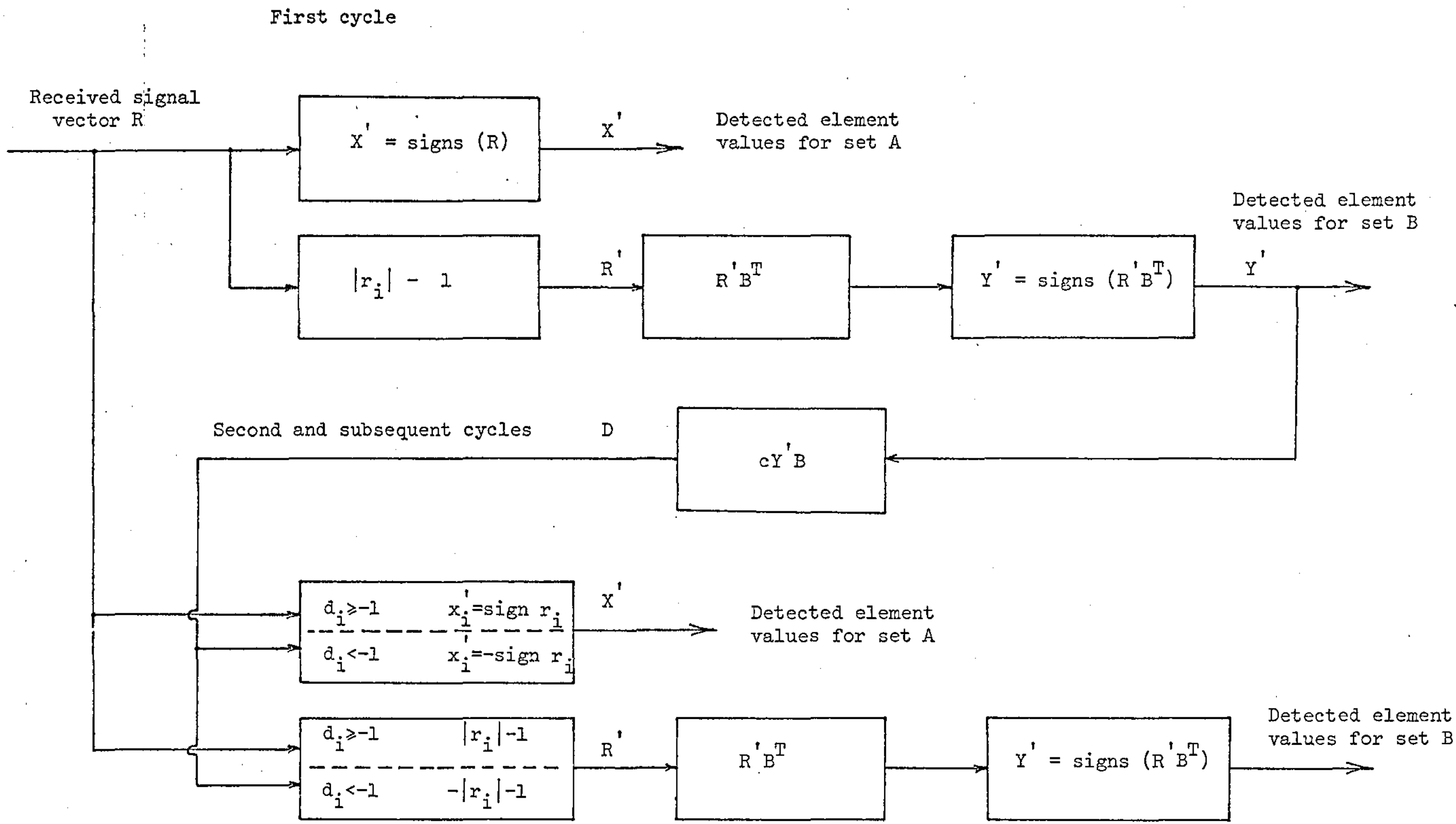


Figure 3.3-2 System A1. Block diagram of the iterative demultiplexing and detection process.

If the ith component of the second term of Eqn. (3.3-12) due to orthogonal set B is of greater magnitude and of opposite sign to the first term a temporary error will be made in the ith component of X'. This may be corrected in the second cycle of the iterative process, when the value of this second term (Eqn. 3.3-12) is estimated from the detected element values for the orthogonal set B, determined in the first cycle. If the ith component of the noise term W is of greater magnitude and of opposite sign to the first term, then a permanent error will occur which cannot be corrected. This is because the magnitude and sign of the noise components $\{w_i\}$ of the vector W are unknown. In general, both second and third terms of Eqn. (3.3-12) contribute interference in the detection of the ith component of X'.

The signs of all $\{r_i\}$ which contain received elements of the orthogonal set A are now made positive, so that each of these becomes the corresponding $|r_i|$. The value of 1 is then subtracted from each of these $\{|r_i|\}$. The remaining $\{r_i\}$ contain no elements of set A and are left unchanged. The resulting n-component vector R' is fed to the correlation detectors matched to the codewords B_i of the received elements of the orthogonal set B. The element values in this set $\{y_i'\}$ are detected from the signs of the corresponding correlation detector output signals to give the n components $\{y_i'\}$ of the vector Y'.

$$Y' = \text{signs}\ (R'B^T) \qquad (3.3\text{-}13)$$

and for those channels not in operation, the corresponding element values $\{y_i'\}$ are set to zero.

In the second cycle of the iterative detection process, the detected binary element values $\{y'\}$ for the orthogonal set B are used to generate the corresponding codewords, which are then added together to give the detected value of the sum of the received elements in set B. This is identical to that performed in the multiplexing process (Eqn. 3.3-5), only now the detected element values $\{y_i'\}$ are used. Let this be the n-component vector D, with components $\{d_i\}$ where,

$$D = cY'B \tag{3.3-14}$$

Referring to Eqn. (3.3-6) in which XA is an n-component vector with components equal to ±1, it is clear that if the ith component of cYB is more negative than -1, then the ith component of S will be of opposite sign to that of the ith component of XA. An incorrect detection in the ith component of X' will have occurred in the first cycle of the iterative process.

The orthogonal set A element values $\{x_i'\}$ are redetected from the sign of the corresponding components $\{r_i\}$, except when d_i is more negative than -1 when the component x_i' is detected as -sign (r_i).

The sign of each r_i that contains a received element in set A is now made positive, except for the $\{r_i\}$ whose corresponding $\{d_i\}$ are more negative than -1. The signs of these $\{r_i\}$ are made negative. The remaining $\{r_i\}$ are left unchanged as before. The value of +1 is then subtracted from each of the resultant components containing an element of set A. The n-component vector R' obtained from the operation is fed to the correlation detectors matched to the set B codewords, to give the n-component vector Y', the detected binary element values for the orthogonal set B.

$$Y' = \text{signs}\ (R'B^T) \tag{3.3-15}$$

and for those channels not in operation, y_i' is set to zero.

The cycle may be repeated as often as required. The most frequent cause of non-unique detectability of the detected element values occurs when c takes certain values. If the ith component of the n-component vector cYB at the transmitter is -1, exact cancellation between the ith component of the sum of the two vectors XA and cYB, will occur, whether the value of the ith component of XA is +1 or -1.

Exact cancellation between the ith digits of the sums of the set elements may occur if $\sqrt{n}/c$ (where $c \leqslant \sqrt{n}$) is an integer value. If $\sqrt{n}/c = k$ is even, then k, k + 2, k + 4 _ _ _ _ n channels in operation may cause exact cancellation and non-unique detectability. If $\sqrt{n}/c = k$ is odd then k, k + 2, k + 4 _ _ _ n-1 channels in operation may cause cancellation and non-unique detectability.

3.4 Computer simulation tests

The relative performances, in the prescence of additive white Gaussian noise, of the various systems discussed, have been compared by computer simulation. All the programs have been written in FORTRAN IV and run on the ICL 1904A computer at Loughborough University of Technology. Appendix A2 shows a selection of programs for the more important systems.

Figure 3.4-1 shows a flow diagram of the computer simulation model for a single test of m active channels. The total number of signal groups transmitted ℓ, and the number of active channels are first selected. For every signal group transmitted, a random selection is made of the m codewords from the total number of codewords available, which are stored permanently for easy access in the multiplexer and demultiplexer.

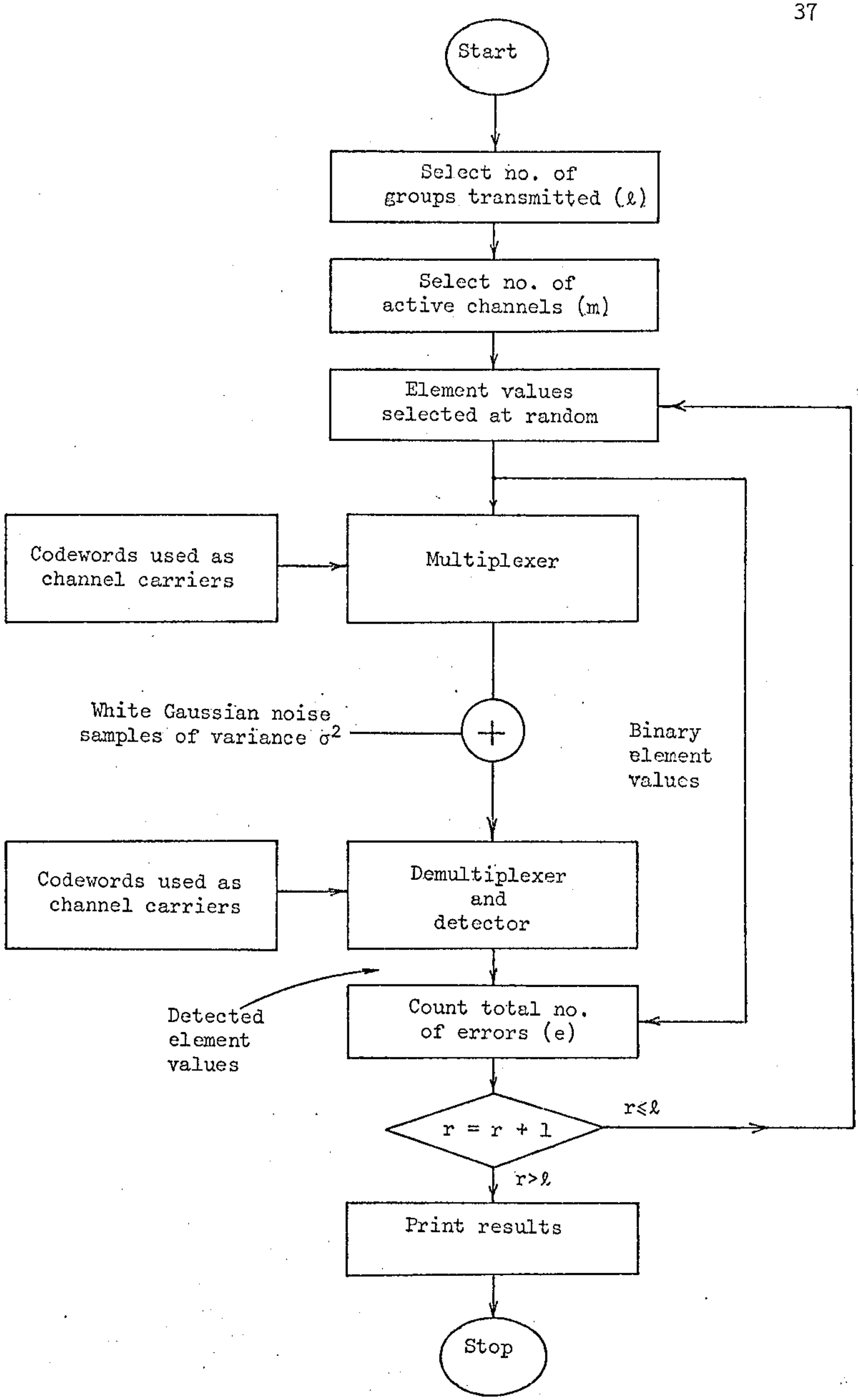


Figure 3.4-1 Flow diagram of the computer simulation model for a single test of m active channels.

The detector is assumed to have prior knowledge of the m codewords selected The data element values are ±1 and are statistically independent and equally likely to take either value.

The variance σ^2 of the additive white Gaussian noise samples introduced into the transmission path, is adjusted to obtain a given error probability per channel. The demultiplexer operates on the received n sample values given by the vector R, to give the m detected element values. These are compared with the corresponding multiplexed element values and the number of errors counted (differences in sign). The test proceeds for ℓ transmitted signal groups, after which the error probability per channel p (for active channels) is calculated from,

$$p = \frac{e}{m\,\ell} \qquad (3.4\text{-}1)$$

where e is the total number of errors counted for the test of m active channels. Finally, a print-out is obtained of the relevant test details. Another test commences with a different number of active channels, and the computer simulation program finishes when all the different number of channel tests are completed.

In a practical system, error probabilities of 1 in 10^5 or less may be expected. It is not possible to test systems with such low error probabilities because, for a reasonable estimate of the error rate, some 20 to 30 errors must be obtained in a computer simulation test. This implies a very large number of trials. A compromise is therefore necessary between the error probability per channel, for 20 to 30 errors, and the computer time necessary. For all system arrangements tested, an error probability per channel of 0.003 has been chosen which for a total of 30 errors was found to give an acceptable computer run time even for detection processes that require a vast number of sequential operations. Tests with different numbers of active channels naturally require a different number of transmitted signal groups for the given error probability per channel and for a total of about 30 errors.

For each system tested, the performance, in terms of tolerance to noise has been compared with a conventional binary TDM system (with components of amplitude ±1), with the same transmission rate and error probability per channel of 0.003. The average energy per component of the transmitted signal for each system has been normalised to unity so that it has the same average energy as a component in the TDM system. In this way a true comparison can be made.

A measure of the tolerance of a system to additive white Gaussian noise, is given by the ratio of the noise variance σ^2 for the system under test, to the noise variance σ^2_{TDM} for a conventional binary TDM system under the prescribed conditions. Expressed in decibels, the noise level relative to a binary TDM system is given by

$$10 \log_{10} \left(\frac{\sigma^2}{\sigma^2_{TDM}} \right) \qquad (3.4\text{-}2)$$

For the multiplexing of more channels than may be multiplexed orthogonally, an interesting comparison is made with a conventional quaternary TDM system, having the same error probability per channel and average energy per component of the transmitted signal. It should be pointed out that the error probability per channel now corresponds to the worst case error probability per channel, where two bits of information are conveyed by one component of the transmitted signal. If the four possible amplitude levels of the quaternary TDM signal are 3a,a, -a, -3a, then the average energy per component is $5a^2$.

If the average energy per component is now set to unity, this represents a reduction in the average energy per bit of information of 1/5, or expressed in dB, a reduction of almost 7 dB. Thus the tolerance to noise of a quaternary TDM system with the same average energy per component of the transmitted signal as a binary TDM system, is 7 dB lower than the corresponding binary TDM system.

3.5 Confidence limits

Because a compromise has by necessity been accepted between the available computer time and the number of errors obtained for a given error probability, the question naturally arises as to the confidence level of the results.

For a given value of the average element error probability per channel, p, the number of errors e obtained in a simulation test is given by

$$e = \ell p m \tag{3.5-1}$$

where ℓ is the total number of signal groups transmitted in a test with m active channels.

It has been shown that if the errors are statistically independent, $e > 30$, $p \ll 1$, and if an accuracy of no better than 20% is required for the confidence limits, then it can be assumed that e has a Gaussian probability density with a mean $\mu = e$ and a standard deviation $\eta = \sqrt{e}$.
For a given value of $p > 0$, the 95% confidence limits for the value of p are approximately,[38]

$$\pm \frac{2\eta}{\mu} p \qquad (3.5\text{-}2)$$

$$\pm \frac{2}{\sqrt{e}} p \qquad (3.5\text{-}3)$$

where the limits are expressed as deviation from the given value of p. Where the number of errors is less than 30, the 95% confidence limits are estimated from the results of reference (30)

In any test with orthogonal groups of signals, there may be a degree of dependence between the individual element errors of a group in a detection process. The result of this dependence is to reduce the number of independent errors obtained in a test and so to widen the confidence limits. Thus e in Eqn. (3.5-1) does not represent the effective number of errors and therefore is only an indication as to the confidence limits. When this occurs, a series of tests may be performed for a given number of active channels. Let the total number of errors counted for each test be e_1, e_2 - - - e_r, for r successive tests. The mean μ and standard deviation η of the total number of errors counted are given by,[80]

$$\mu = \frac{1}{r} \sum_{i=1}^{r} e_i \qquad (3.5\text{-}4)$$

$$\eta = \left(\frac{1}{r-1} \sum_{i=1}^{r} (e_i - \mu)^2 \right)^{\frac{1}{2}} \qquad (3.5\text{-}5)$$

(Bessel's formulae)

The 95% confidence limits in the value of error probability is now given by Eqn. (3.5-2).

3.6 Results of computer simulation tests

The tests simulate the multiplexing and demultiplexing of 16 channels from the orthogonal set A, and between 0 and 16 channels from the orthogonal set B. For each test the value of c, the level of the set B signal elements was adjusted to give the same error probability per channel for sets A and B, at the end of the second detection cycle. The variance of the additive white Gaussian noise samples was simultaneously adjusted to give an error probability of 0.003 for both sets at the end of the second cycle, subsequent cycles being found to give no significant improvement in the total number of errors counted. For each test 1000 signal groups were transmitted.

Figure 3.6-1 gives the noise level for an error probability per channel of 0.003 expressed in decibels relative to a binary TDM system, for 0 to 16 active channels in set B. Also shown is the relative noise level of the corresponding quaternary TDM system. Both binary and quaternary TDM systems have the same average energy per component of the transmitted signal, the same transmission rate, and the same error probability per channel as the system under test, as explained in Section 3.4.

This system is more attractive in terms of tolerance to additive white Gaussian noise than conventional quaternary TDM, when the orthogonal set A is at maximum capacity, and when there are up to 7 active channels from the orthogonal set B. For a greater number of active channels in set B the tolerance to noise decreases rapidly.

The confidence limits of Eqn. (3.5-3) may be applied to each test where approximately 50 errors were counted for the orthogonal set A. Rather fewer errors were counted for the set B, the number depending on the number of active channels in set B. However, because the additive white Gaussian noise affects the number of errors counted for each set equally, it is

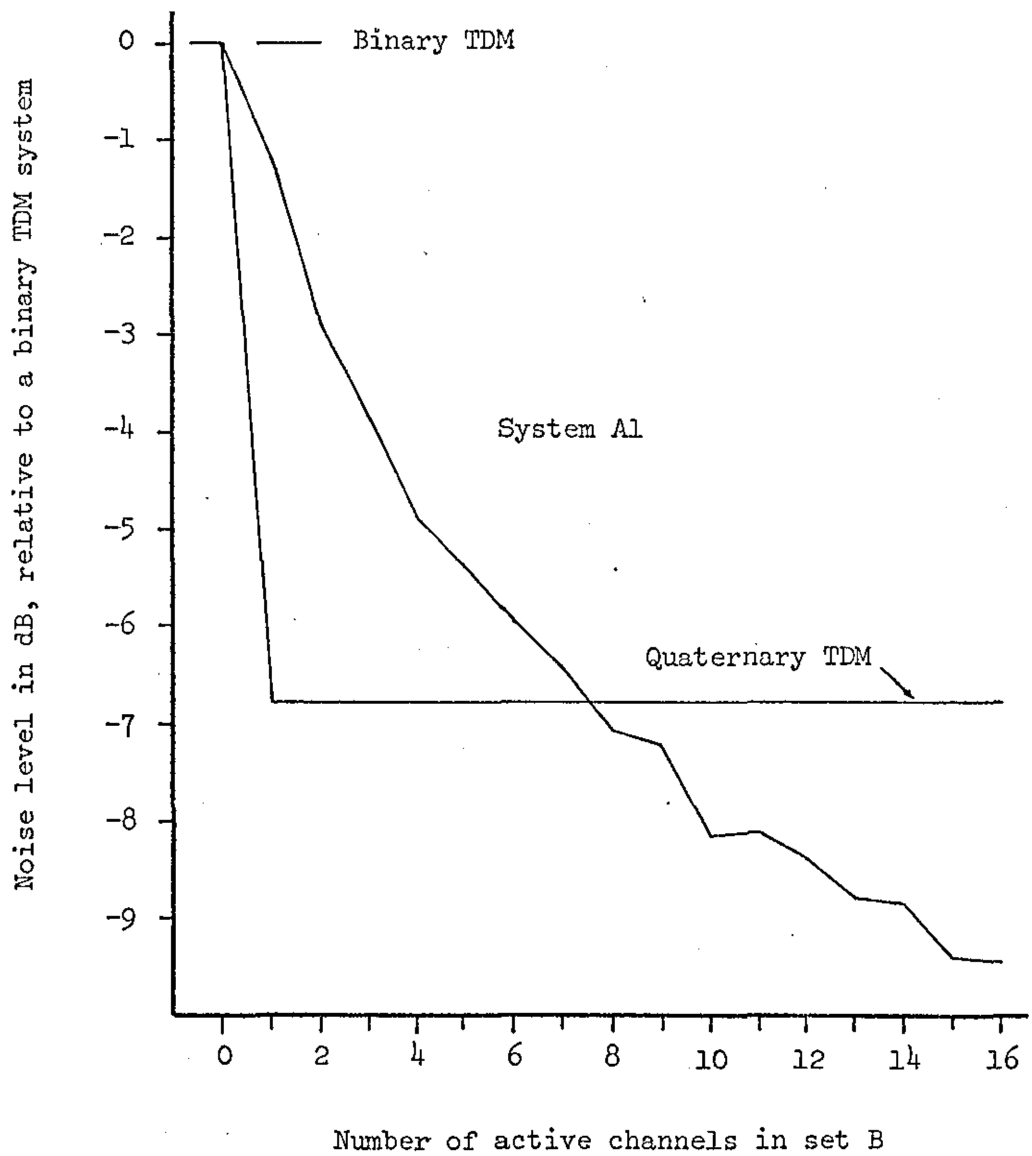


Figure 3.6-1 System A1. Noise level for an error probability per channel of 0.003, expressed in dB relative to a binary TDM system, for a varying number of active channels in set B.

reasonable to accept 50 errors as an indication to the confidence limits for an error probability per channel of 0.003. The 95% confidence limits are therefore 0.003 ± 0.00085, or expressed in decibels, +0.30 and -0.35 on the measured value of relative noise level of Figure 3.6-1.

Non-unique detectability caused by cancellation of coincident components of the orthogonal sets A and B is examined by an alternative approach. The level of the set B signal elements c, is maintained at a constant level, as is the noise variance σ^2 of the additive white Gaussian noise samples, for a varying number of active channels in set B.

Figures 3.6-2 and 3.6-3 show the error probability per channel, at the end of the first and second cycles of the iterative detection process. The parameters associated with the different graphs are as below.

GRAPH		c	σ
1A	1B	0.4	0.125
2A	2B	0.4	0.0
3A	3B	0.3636	0.125
4A	4B	0.3636	0.0

The letters A and B against a graph indicate the orthogonal set to which the value of p apply. The error probability for the set B channels remain approximately constant, regardless of the number of channels in set B. Thus, to simplify Figure 3.6-2 and 3.6-3, the graphs plotted for 1B to 4B show in each case a constant value of p, which is its average value determined over the range 1 to 16 channels in set B. Under noiseless conditions, the first detection cycle produces a considerable error probability, but for certain conditions is reduced to zero in the second detection cycle. When c = 0.4 a good tolerance to noise is obtained for

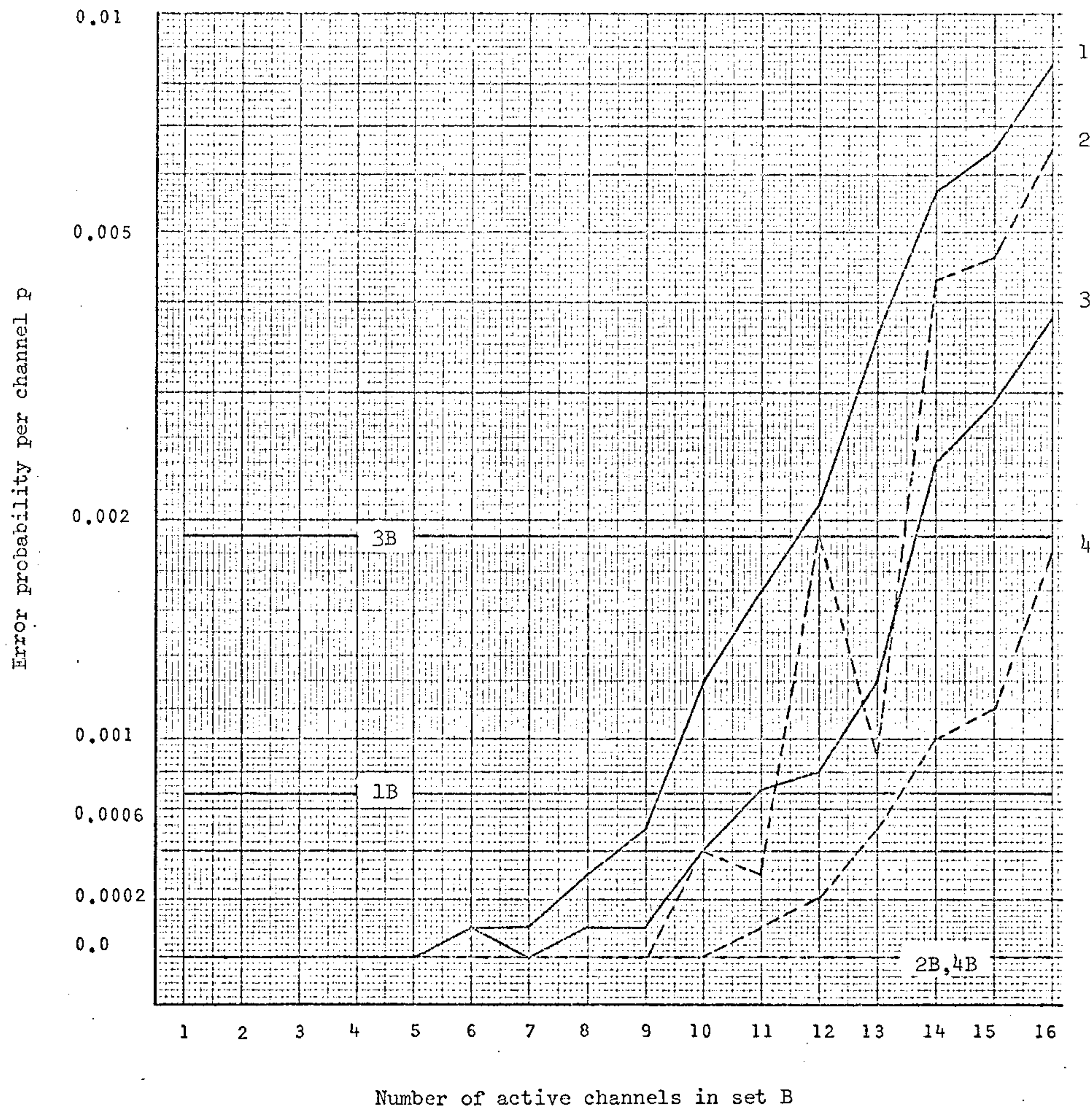


Figure 3.6-2 System A1. First detection cycle. Error probability per channel, for a varying number of active channels in set B.

---- noiseless conditions

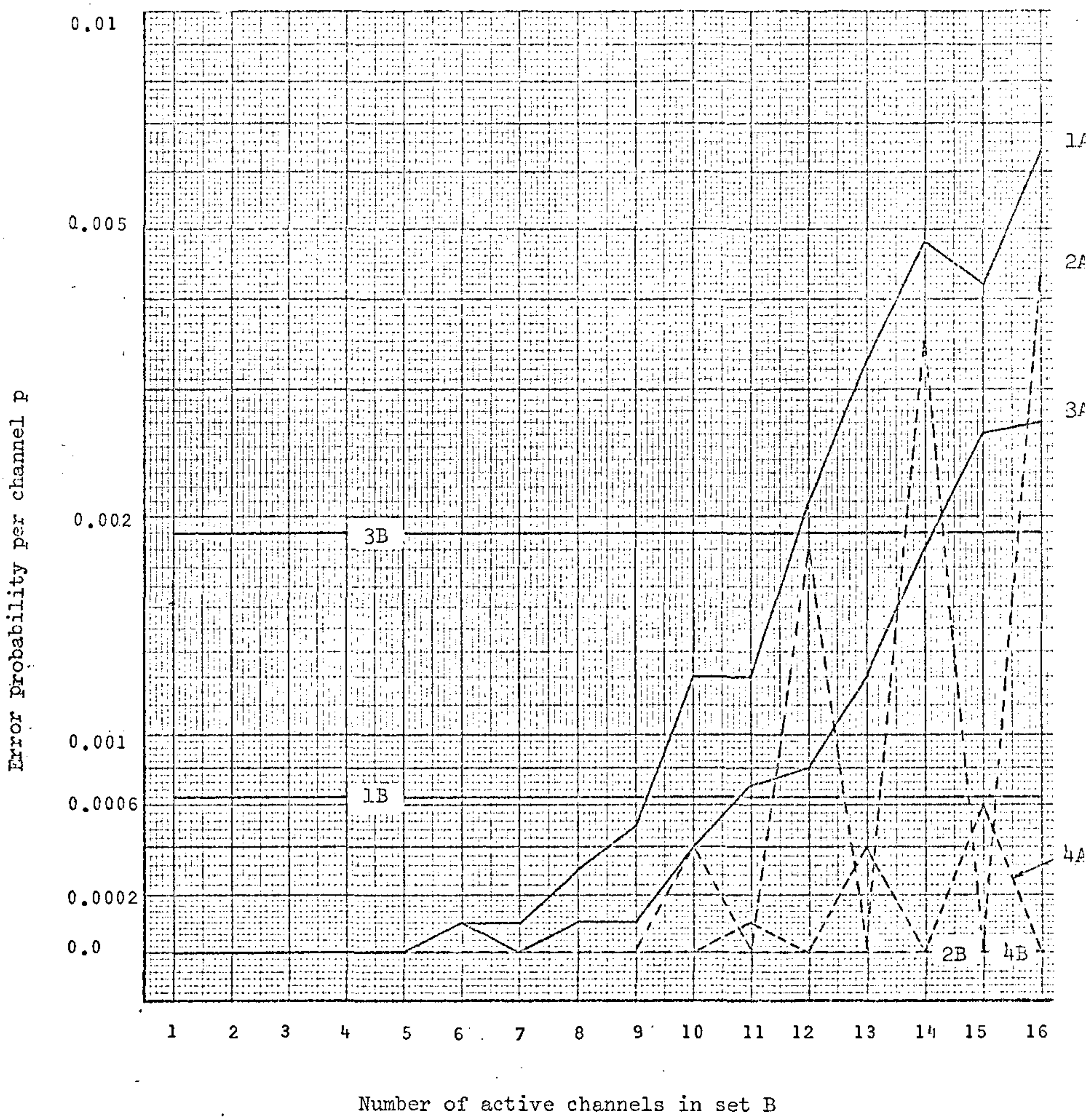


Figure 3.6-3 System A1. Second detection cycle. Error probability per channel, for a varying number of active channels in set B.

---- noiseless conditions

an odd number of channels in set B. However, exact cancellation described causing non-unique detectability occurs in the set A channels, even under noiseless conditions for m even and $\geqslant 10$. For $c = 0.3636$ a good tolerance to noise is obtained for an even number of channels in set B, but causes non-unique detectability for m odd and $\geqslant 11$. Graph 2A and 4A of Figure 3.6-3 shows this clearly.

More detailed tests[35] show that the error probability per channel of set B is unchanged if the number of channels in set A is reduced.

3.7 Assessment of System A1

The non-linear multiplexing arrangement of System A1 described, is most suitable when the number of multiplexed channels is a little over the maximum number that may be multiplexed orthogonally. In particular, with a suitable choice of signal level for the set B, unique detectability can be ensured for up to twice as many channels as may be multiplexed orthogonally. The arrangement gains an advantage in tolerance to additive white Gaussian noise over the corresponding conventional quaternary TDM system where the transmitted signal has the same average energy per component, and the same transmission rate as the System A1.
The significance of the system is that the tolerance to additive white Gaussian noise gradually decreases as the number of channels in set B increases. A trade-off exists between the number of additional multiplexed channels and the tolerance to noise.

CHAPTER 4

DEVELOPMENTS OF SYSTEM A1

4.1 System A2

This is a modification of System A1, in which the most frequent cause of non-unique detectability of the detected element values occurs when the value of c, determining the level of the signal elements of set B, takes certain values. System A2 introduces a simple and effective non-linearity into the multiplexing method to overcome this.

From Eqn. (3.3-6) the n-component transmitted signal vector S is given by,

$$S = XA + \text{signs}(XA)\,(cYB) \qquad (4.1\text{-}1)$$

The channel element values $\{x_i\}$ and $\{y_i\}$ are given by the n components of the vectors X and Y whose ith components are ±1, or 0 for those channels not in operation. The matrix A is an identity matrix whose rows are the codewords of the orthogonal set A. The components of XA are therefore ±1 or 0.

If the value of c is such that the ith component of cYB is -1, then if the ith component of XA = ±1, exact cancellation will occur, and the information conveyed by the ith channel of the orthogonal set A will be completely obliterated.

When this occurs, the ith component of S, given by Eqn. (4.1-1) is now modified to take the value ±k, the selected sign being the same as that of the ith component of XA. Let the n-component vector S' be the modified transmitted signal.

$$\left.\begin{array}{l} \text{If } s_i = 0 \\ \text{and } (XA)_{ith} = \pm 1 \end{array}\right\} s_i' = k\,(XA)_{ith} \qquad (4.1\text{-}2)$$

$$\text{otherwise} \qquad s_i' = s_i$$

where the ith component of XA is denoted by $(XA)_{ith}$

Errors in the detected element values are also caused by components of the transmitted signal taking small values, whose signs are easily corrupted by the additive white Gaussian noise introduced into the transmission path. If the modulus of the ith component of s_i is less than a value g, then this component is now modified to take the value ±g, the selected sign being the same as that of the ith component of s_i.

$$\begin{array}{llll} \text{if} & |s_i| \geq g & s_i' = s_i & \\ & |s_i| < g & s_i' = g \text{ sign } (s_i) & \end{array} \qquad (4.1\text{-}3)$$

For simplification the value g is assumed equal to k and the two non-linearities are combined.

$$\begin{array}{ll} \left.\begin{array}{l} \text{if } s_i = 0 \\ \text{and } (XA)_{ith} = \pm 1 \end{array}\right\} & s_i' = k\,(XA)_{ith} \\ \text{if } |s_i| < k & s_i' = k \text{ sign } (s_i) \\ \text{otherwise} & s_i' = s_i \end{array} \qquad (4.1\text{-}4)$$

Increasing the value of k should increase the tolerance to noise of the orthogonal set A element values, but because the orthogonal set B element values are detected from all n components of the received data signal, the orthogonal set B error probability should remain reasonably constant for small values of k. For k larger, the error probability for the orthogonal set B is expected to increase, due to excessive interference from residual components introduced by the non-linearities, and not removed with the detection of the orthogonal set A element values.

4.2 Results of computer simulation tests

The tests applied here to System A2 are identical to those for System A1. Figure 4.2-1 shows the error probability per channel, at the end of the second detection cycle, as the number of channels in the orthogonal set B is increased from 1 to 16. The parameters associated with the different graphs are as below.

Graph		c	σ	k
1A	1B	0.4	0.125	Unmodified system A1
2A	2B	0.4	0.125	0.2
3A	3B	0.4	0.125	0.3
4A	4B	0.4	0.125	0.4
5A	5B	0.4	0.0	Unmodified system A1
6A	6B	0.4	0.0	0.3

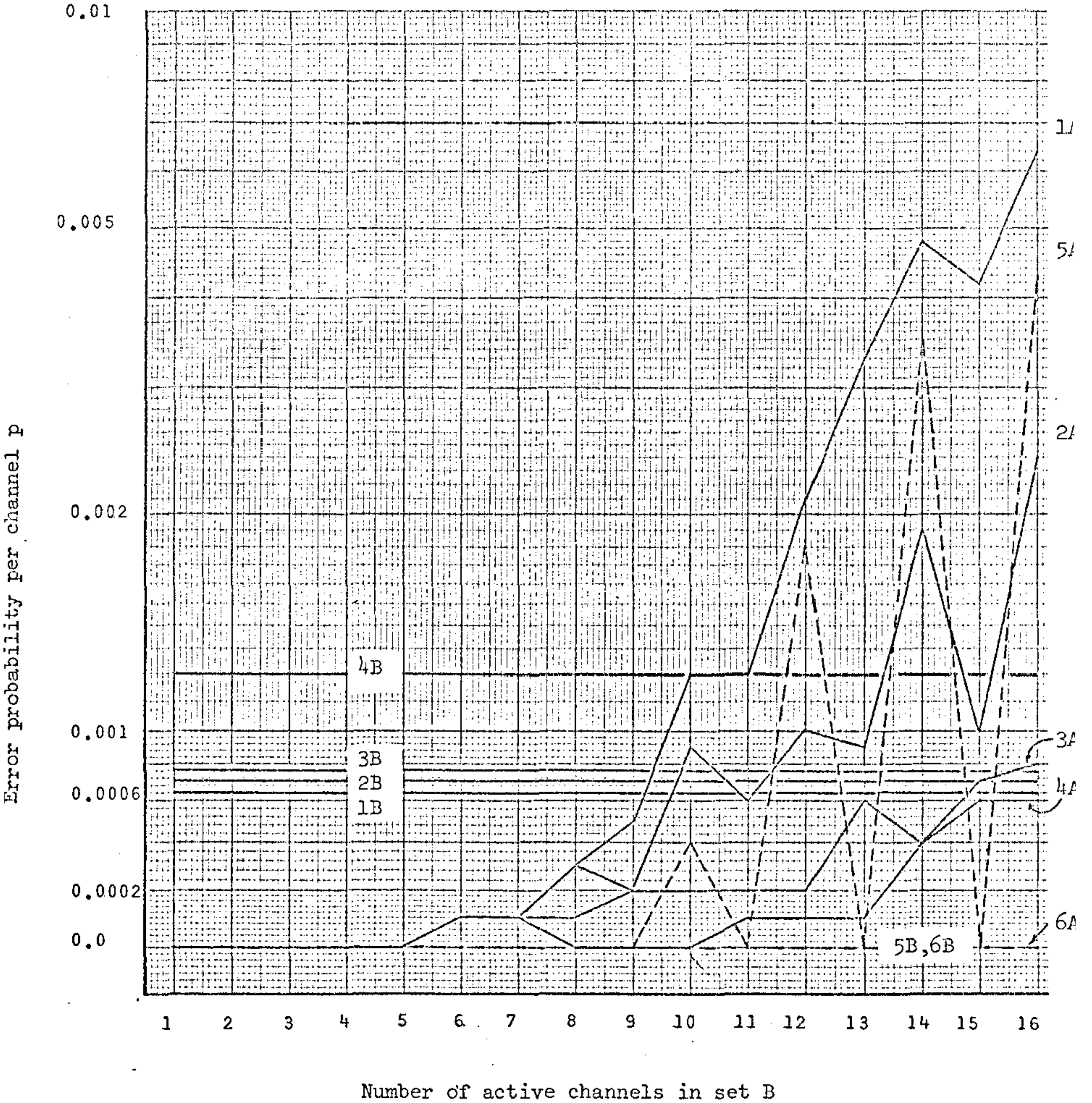


Figure 4.2-1 System A2. Second detection cycle. Error probability per channel, for a varying number of active channels in set B.

---- noiseless conditions

Computer simulation results show that the error probability per channel for the orthogonal set B, remains approximately constant regardless of the number of channels in set B. Thus for simplification, the curves 1B to 6B are shown as a constant value, the average value of error probability taken for all 1 to 16 channels in set B. The curves 1A and 5A refer to system A1 and are shown here for comparison.

Increasing the value of k decreases significantly the error probability in set A, whilst the error probability in set B only increases marginally, because these element values are detected from all n components of the received data signal. A value of k = 0.3 results in approximately equal error probabilities for the sets A and B. The error probability per channel in set A is then reduced to about one tenth of its previous value.

Figure 4.2-2 gives the noise level for an error probability per channel of 0.003 at the end of the second detection cycle, expressed in decibels relative to a binary TDM system, for 0 to 16 active channels in set B, and for k having a value 0.3. Also shown is the relative noise level of the corresponding quaternary TDM system, with the same average energy per component of the transmitted signal, the same transmission rate, and the same error probability per channel as the system under test, as explained in Section 3.4. The relative noise level curve for system A1 is shown for comparison. For more than five active channels from the orthogonal set B, the non-linearities of System A2 introduced give an advantage of about 0.6 dB over system A1. For up to 9 active channels in set B the system is more attractive in terms of tolerance to additive white Gaussian noise than conventional quaternary TDM.

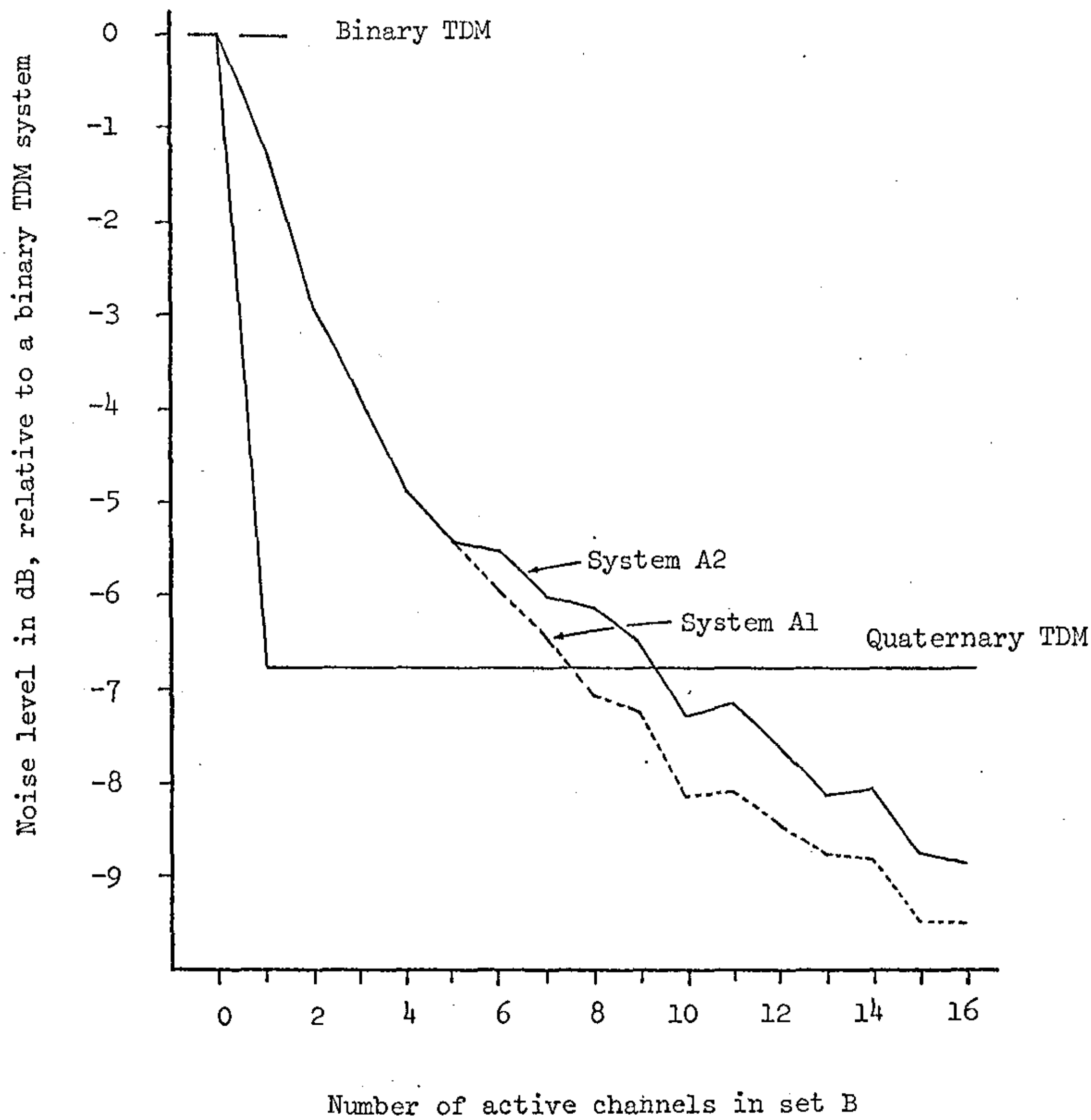


Figure 4.2-2 System A2. Noise level for an error probability of 0.003, expressed in decibels relative to a binary TDM system, for a varying number of active channels in set B. k = 0.3

4.3 Assessment of System A2

By modifying the multiplexing arrangement of System A1 slightly, the error probability is reduced by a significant amount, with only a relatively trivial increase in equipment complexity. Additionally, under noiseless conditions, unique detectability is ensured for any number of channels in the orthogonal set B, irrespective of the value of c determining the level of the set B signal elements. When there are up to 50% more channels than may be multiplexed orthogonally, the System A2 has a greater tolerance to noise over the corresponding conventional quaternary TDM system.

4.4 System A3

Systems A1 and A2 are arrangements involving a non-linear combination of two sets of orthogonal signals. The TDM elements are orthogonal as are the CDM elements, but simultaneously transmitted TDM and CDM elements are not orthogonal. With this system, up to twice as many channels may be multiplexed, for a given transmission path and signal element rate per channel, than is possible with orthogonal multiplexing using TDM or CDM alone.

The techniques used in System A1 are now extended to System A3, involving three sets of orthogonal signals, a TDM set and two CDM sets. It has been considered, to investigate whether the advantages of System A1 apply to the multiplexing of an additional CDM orthogonal set.
System A3 is capable of multiplexing up to three times the maximum number of channels than is possible with orthogonal multiplexing using TDM or CDM alone, for a given transmission path and signal element rate per channel. At any time, the total number of channels m, may take any value from 0 to 3n, where n is the maximum number of orthogonal TDM or CDM elements.

The TDM codewords which are used as the signal carriers for the n TDM channels are given by the rows $\{A_i\}$ of an $n \times n$ identity matrix. The complete set of n TDM codewords will be referred to as the orthogonal set A. If the ith codeword, from the set of n codewords, corresponding to the ith TDM channel is given by $\sum_{j=1}^{n} a_{ij}\, \delta(t-jT)$, it may be represented by the n-component row vector,

$$A_i = 0 \;.\; .\; 0 \;\; a_{ii} \;\; 0 \;.\; .\; 0 \tag{4.4-1}$$

where the ith component $a_{ii} = 1$.

The two sets of CDM codewords which are used as the remaining channel carriers are given by the rows $\{B_i\}$ and $\{C_i\}$ of $n \times n$ matrices B and C, and are referred to as the orthogonal sets B and C. The set C was originally chosen as a 16×16 Hadamard matrix (as in System A1), and the set B was therefore chosen as a Hadamard matrix with four non-zero components. In this way it was thought that by suitable choice of signal levels for the elements of sets B and C, the interference between the sets would be minimised. However, because of the particular multiplexing method used, the set C codewords are modified, such that the individual elements of the orthogonal set C are contained in all 16 components of the transmitted signal. The matrices B and C are shown in Figure 4.1-1. As explained in Section 3.1, Walsh functions could be used, but these are only an alternative ordering of the rows of a Hadamard matrix.

If the ith codeword from the orthogonal set B of n CDM codewords is given by $\sum_{j=1}^{n} b_{ij}\, \delta(t-jT)$ it may be represented by the n-component row vector,

$$B_i = b_{i1} \;\; b_{i2} \;.\; .\; b_{in} \tag{4.4-2}$$

$$B = \frac{1}{2}\begin{bmatrix}
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1
\end{bmatrix}
\qquad
C = \frac{1}{2}\begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1
\end{bmatrix}$$

Figure 4.4-1 The orthogonal matrices B and C

Similarly, the ith codeword from the orthogonal set C may be represented by the n-component row vector,

$$C_i = c_{i1} \; c_{i2} \; \cdot \; \cdot \; c_{in} \tag{4.4-3}$$

The element value of the signal element in the ith of the n channels whose codewords belong to the orthogonal set A, is $x_i = \pm 1$ when a signal is present in this channel, or $x_i = 0$ when no signal is present. Similarly, the element value of the signal element in the ith of the n channels whose codewords belong to the orthogonal set B, is $y_i = \pm 1$ when a signal is present in this channel, or $y_i = 0$ when no signal is present. Similarly, for the orthogonal set C, $z_i = \pm 1$ when the ith signal is present, or $z_i = 0$ when no signal is present. Let X, Y and Z be the n-component row vectors with ith components x_i, y_i and z_i respectively.

It is assumed that the $\{x_i\}$, $\{y_i\}$ and $\{z_i\}$ for the active channels are statistically independent and equally likely to have either binary value. The $\{x_i\}$, $\{y_i\}$ and $\{z_i\}$ for the active channels are not necessarily the first of the $n\{x_i\}$, $n\{y_i\}$ and $n\{z_i\}$, but may be any of the $n\{x_i\}$, $n\{y_i\}$ and $n\{z_i\}$.

The operations involved in the multiplexing and demultiplexing processes are basically identical to those of System A1. The signal elements for different orthogonal sets are combined non-linearly so that the element values of a particular set may be detected without prior knowledge of element values previously detected. Thus if an element value is detected in error, due to the additive white Gaussian noise introduced, the cancellation of the signal elements from that set from the received data signal, does not affect the detection of other element values, as would occur with a linear coding scheme. The probability of correct detection of the element values is thus increased.

The coder and multiplexer combine the m channel codewords from the three orthogonal sets over the period 0 to nT to give the resultant transmitted signal which may be represented as an n-component row vector. The orthogonal set codewords $\{C_i\}$ are multiplied by the corresponding binary element values $\{z_i\}$ so that each codeword given by Eqn. (4.4-3) is binary antipodal. The codewords are added linearly to give the n-component vector ZC, which is multiplied by a scaling factor, whose value is positive and equal to h, and determines the level of the vector ZC to give,

$$hZC \qquad (4.4\text{-}4)$$

The n components of the vector hZC are now combined non-linearly with the element values $\{y_i\}$ of set B as follows. For each j, j = 1 -- n, if the jth component of Y is negative, then the sign of the jth component of hZC is reversed. The jth components are now added linearly to give,

$$Y + \text{signs}(Y)\ (hZC) \qquad (4.4\text{-}5)$$

where the operator "signs" replaces each term of the vector Y by ±1 corresponding to the sign of the components of Y.

The orthogonal set B codewords $\{B_i\}$ are multiplied by the components given by (4.4-5), and the elements are then added linearly . The resulting n-component vector is multiplied by a scaling factor, whose value is positive and equal to f, and determines the level of the signal elements of set B to give,

$$f(Y + \text{signs}(Y)\ (hZC)\ )\ B \qquad (4.4\text{-}6)$$

The n components of the vector given by (4.4-6) are now combined non-linearly with the element values $\{x_i\}$ as follows. For each j, j = 1 _ _ n, if the jth component of X is negative, then the sign of the jth component of (4.4-6) is reversed. The jth components are added linearly to give,

$$X + \text{signs}(X)\ f(Y + \text{signs}(Y)\ (hZC)\ )\ B \tag{4.4-7}$$

The orthogonal set A codewords $\{A_i\}$ would now logically be multiplied by the n-components given by (4.4-7), but as the matrix A is an identity matrix, this operation becomes unnecessary, and (4.4-7) gives the n-component transmitted signal vector S.

$$S = X + \text{signs}(X)\ f(Y + \text{signs}(Y)\ (hZC)\ )\ B \tag{4.4-8}$$

In the model of the system (Figure 2.1-1), white Gaussian noise with a two sided power spectral density of σ^2 is added to the data signal at the output of the transmission path, giving the Gaussian waveform w(t) added to the data signal at the output of the receiver filter. This has been described in detail in Section 2.1.

The signal at the output of the receiver filter over the duration of a single group of coincident signal elements is sampled at regular time intervals of T seconds to give the n components of the received data signal.

$$R = S + W \tag{4.4-9}$$

where S and W are n-component vectors, and the $\{w_i\}$ are sample values of statistically independent Gaussian random variables with zero mean and variance σ^2 as before. From Eqn. (4.4-8) and Eqn. (4.4-9),

$$R = X + \text{signs}(X)\ f(Y + \text{signs}(Y)\ (hZC)\ )\ B + W \qquad (4.4\text{-}10)$$

The detection process uses three separate sets of correlation detectors matched to the orthogonal sets A, B and C, and operates in a similar fashion to System A1. Figure 4.4-2 shows a block diagram of the iterative detection process which is divided for convenience into the first and subsequent cycles, the first cycle differing slightly from the following cycles.

In the first cycle of the iterative process, the detector determines the binary element values $\{x_i'\}$ of the vector X' for the orthogonal set A from the signs of the corresponding components $\{r_i\}$ of R,

$$X' = \text{signs}\ (R) \qquad (4.4\text{-}11)$$

and for those channels not in operation, the corresponding element values $\{x_i'\}$ are set to zero.

The signs of all $\{r_i\}$ which contain received elements of the orthogonal set A are now made positive, so that each of these becomes the corresponding $|r_i|$. The value of 1 is then subtracted from each of these $\{|r_i|\}$. These operations remove components due to the orthogonal set A from the received data signal, and require no prior knowledge of the actual element values of set A. Thus, it remains valid even for incorrectly detected element values. The remaining $\{r_i\}$ contain no elements of the set A and are left unchanged. The resulting n-component vector R' is fed to the correlation detectors matched to the codewords B_i, of the received elements of the orthogonal set B. The element values in this set $\{y_i'\}$, are detected from the signs of the corresponding correlation detector output signals to give the n components $\{y_i'\}$ of the vector Y'.

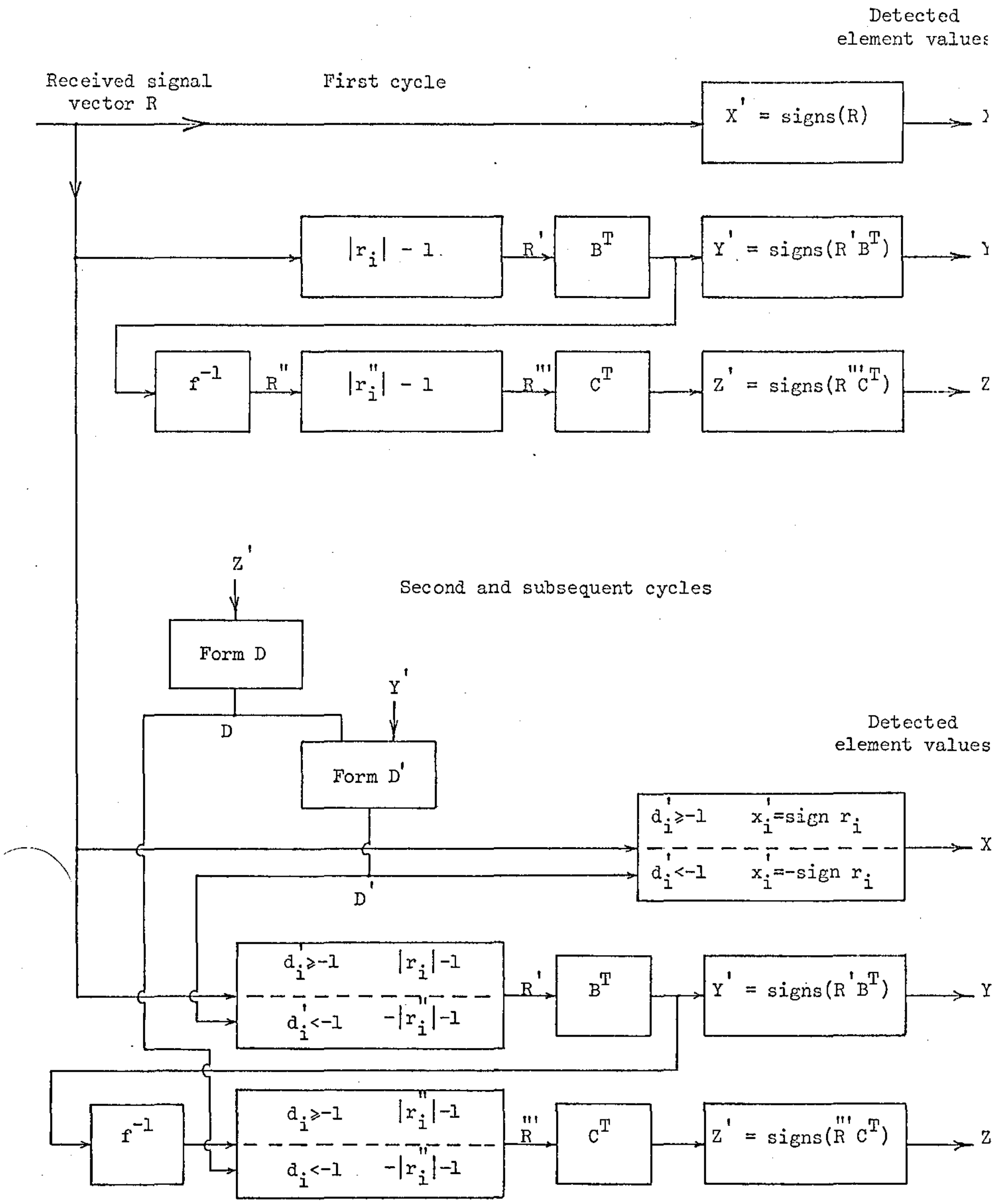


Figure 4.4-2 System A3. Block diagram of the iterative demultiplexing and detection process.

$$Y' = \text{signs } (R'B^T) \qquad (4.4\text{-}12)$$

and for those channels not in operation, the corresponding element values $\{y_i'\}$ are set to zero.

The correlation detector output signals are multiplied by the scalar f^{-1} thereby offsetting the amplitude scaling in the multiplexer. Let the resultant n-component vector be R''.

$$R'' = f^{-1}R'B^T \qquad (4.4\text{-}13)$$

The signs of all component $\{r_i''\}$ of the vector R'' which contain received elements of the orthogonal set B are now made positive, so that each of these becomes the corresponding $|r_i''|$. The value of 1 is subtracted from each of these $\{|r_i''|\}$. The components due to the orthogonal set B are thus removed from the received signal and without prior knowledge of the element values of the set B. The remaining $\{r_i''\}$ contain no elements of the set B and are left unchanged. The resulting n-component vector R''' is fed to the correlation detectors matched to the codewords C_i of the received elements of the orthogonal set C. The element values in the set $\{z_i'\}$ are detected from the signs of the corresponding correlation detector output signals to give the n components $\{z_i'\}$ of the vector Z'.

$$Z' = \text{signs } (R'''C^T) \qquad (4.4\text{-}14)$$

and for those channels not in operation, the corresponding element values $\{z_i'\}$ are set to zero.

In the second cycle of the iterative detection process, new estimates are made of the detected element values, and those incorrectly detected initially may now, to some extent, be corrected. Under noiseless conditions all incorrectly detected element values are corrected in this second cycle.

The reason for incorrectly detected element values in the first cycle can be seen by referring to Eqn. (4.4-8), that is,

$$S = X + \text{signs}(X)\ f\ (Y + \text{signs}(Y)\ (hZC)\ )\ B$$

For those channels in operation for the orthogonal sets A and B, the corresponding components of the vectors X and Y have the value ±1. If then, the ith component of the term $f(Y + \text{signs}(Y)\ (hZC)\ )\ B$ has a value more negative than -1, irrespective of whether the ith component of X is ±1, the sign of the ith component of S will be of opposite sign to the ith component of X (provided the ith channel of the orthogonal set A is in operation). Because the element values of the orthogonal set A are detected from the signs of the received data signal R, where $R = S + W$ from Eqn. (4.4-9), an incorrect detection in the ith component of X' will occur.

Similarly, if the ith component of the term (hZC) is more negative than -1, then, irrespective of whether the ith component of Y is ±1, the sign of the ith component of $Y + \text{signs}(Y)\ (hZC)$ will be of opposite sign to ith component of Y. In the detection of the ith element value y_i' and incorrect detection will occur.

In the second cycle of the detection process, the element values $\{x_i'\}$, $\{y_i'\}$ and $\{z_i'\}$ are again detected, but the vectors $f(Y + \text{signs}(Y)\ (hZC))\ B$ and (hZC) are examined for components more negative than -1, by reconstituting these vectors from the element values previously detected.

The detected binary element values $\{z_i'\}$ for the orthogonal set C are first used to generate the n-component vector D with components $\{d_i\}$.

$$D = hZ'C \tag{4.4-15}$$

If the ith component d_i is more negative than -1, then the component y_i' will have been incorrectly detected and its sign is now changed.

The detected binary element values $\{z_i'\}$ and $\{y_i'\}$ for the orthogonal sets C and B are now used to generate the n-component vector D' with components $\{d_i'\}$.

$$D' = f(Y' + \text{signs}(Y')\ (hZ'C)\ )\ B \tag{4.4-16}$$

If the ith component d_i' is more negative than -1, then the component x_i' will have been incorrectly detected in the first cycle.

The second cycle continues as follows, using the same principle as the first cycle of the iterative detection process. The orthogonal set A element values $\{x_i'\}$ are redetected from the signs of the corresponding components of R, except when d_i' is more negative than -1, when the component x_i' is detected as -sign (r_i).

$$X' = \text{signs}(R) \tag{4.4-17}$$

If $d_i' < -1$ then x_i' is set to -sign (r_i), and for those channels not in operation the corresponding element values $\{x_i'\}$ are set to zero.

The sign of each r_i that contains a received element in set A is now made positive, except for the $\{r_i\}$ whose corresponding $\{d_i'\}$ are more negative than -1. The signs of these $\{r_i\}$ are made negative. The remaining $\{r_i\}$ are left unchanged as before. The value of -1 is then subtracted from each of the resultant components containing an element in set A. The n-component vector R' obtained from this operation is fed to the correlation detectors matched to the set B codewords. The element values in this set $\{y_i'\}$ of the vector Y' are detected from the signs of the corresponding correlation detector output signals, except when d_i is more negative than -1, when the component y_i' is detected of opposite sign to the detector output.

$$Y' = \text{signs } (R'B^T) \tag{4.4-18}$$

If $d_i < -1$ set $y_i' = -\text{sign } (R'B^T)_{ith}$
where the ith component of $R'B^T$ is denoted by $(R'B^T)_{ith}$. For those channels not in operation, the corresponding element values $\{y_i'\}$ are set to zero.

The correlation detector output signals are multiplied by the scalar f^{-1} thereby offsetting the amplitude scaling in the multiplexer. Let the resultant n-component vector be R'',

$$R'' = f^{-1}R'B^T \tag{4.4-19}$$

The signs of all $\{r_i''\}$ which contain received elements of the orthogonal set B are now made positive, so that each of these becomes the corresponding $|r_i''|$. The value of 1 is subtracted from each of these $\{|r_i''|\}$.
The remaining $\{r_i''\}$ contain no elements of the set B and are left unchanged. The resulting n-component vector R''' is fed to the correlation detectors matched to the codewords $\{C_i\}$, of the received elements of the orthogonal set C. The element values in this set $\{z_i'\}$ are detected from the signs of the corresponding correlation detector output signals to give the n components

$\{z_i'\}$ of the vector Z',

$$Z' = \text{signs}\ (R''' C^T) \tag{4.4-20}$$

and for those channels not in operation, the corresponding element values $\{z_i'\}$ are set to zero.

This second cycle of the iterative detection process may now be repeated as often as required using the most recently detected binary element values to obtain new estimates of these binary element values. In practice, little or no advantage is gained with more than two cycles, especially at high signal/noise ratios (low probability of error).

4.5 Results of computer simulation tests.

The tests simulate the multiplexing and demultiplexing of 16 channels from the orthogonal set A, and between 0 and 16 channels from each of the orthogonal sets B and C. Set C channels are not used until all channels from the orthogonal set B are in operation. The codewords used in the sets B and C are chosen at random from the available codewords for every group transmitted, and the detector has prior knowledge of those chosen. After each cycle of the iterative detection process, the number of element values in error are counted separately for the sets A, B and C, and at the end of the test, the error probability per channel for each set is calculated. For all tests 500 groups were transmitted.

For every test the value of f and h, the levels of the set B and set C signal elements respectively, were adjusted to give the same error probability per channel for each of the three orthogonal sets. The variance of the additive white Gaussian noise samples was simultaneously adjusted to give an error probability per channel of 0.003 for each set at the end of the third cycle of the iterative detection process. Subsequent cycles were found to give no improvement.

Figure 4.5-1 gives the noise level for an error probability per channel of 0.003 expressed in decibels relative to a binary TDM system with the same average energy per component of the transmitted signal, the same transmission rate, and the same error probability per channel as the system under test, for 0 to 16 active channels in the sets B and C. Also shown are the relative noise levels of the corresponding quaternary and 8-level bipolar TDM systems for the same conditions as a binary TDM system, and as explained in Section 3.4 .

With the orthogonal set A at maximum capacity, for up to 10 channels in set B, the system is more attractive in terms of tolerance to additive white Gaussian noise than conventional quaternary TDM. The system is also more attractive than 8-level TDM, when the sets A and B are fully occupied and there are up to about 7 channels in the orthogonal set C. Figure 4.5-1 shows that a trade-off exists between the number of active channels, and the tolerance to additive white Gaussian noise.

The confidence limits of Eqn. (3.5-3) may be applied to each test where approximately 25 errors were counted for the orthogonal set A. Rather fewer errors were counted for the sets B and C, the number depending on the number of active channels in the sets. However, because the additive white Gaussian noise affects the number of errors counted for each set equally, it is reasonable to accept 25 errors as an indication to the confidence limits for an average error probability per channel of 0.003. The 95% confidence limits are therefore 0.003 ± 0.0012, or expressed in decibels +0.37 and -0.49 on the measured value of relative noise level of Figure 4.5-1.

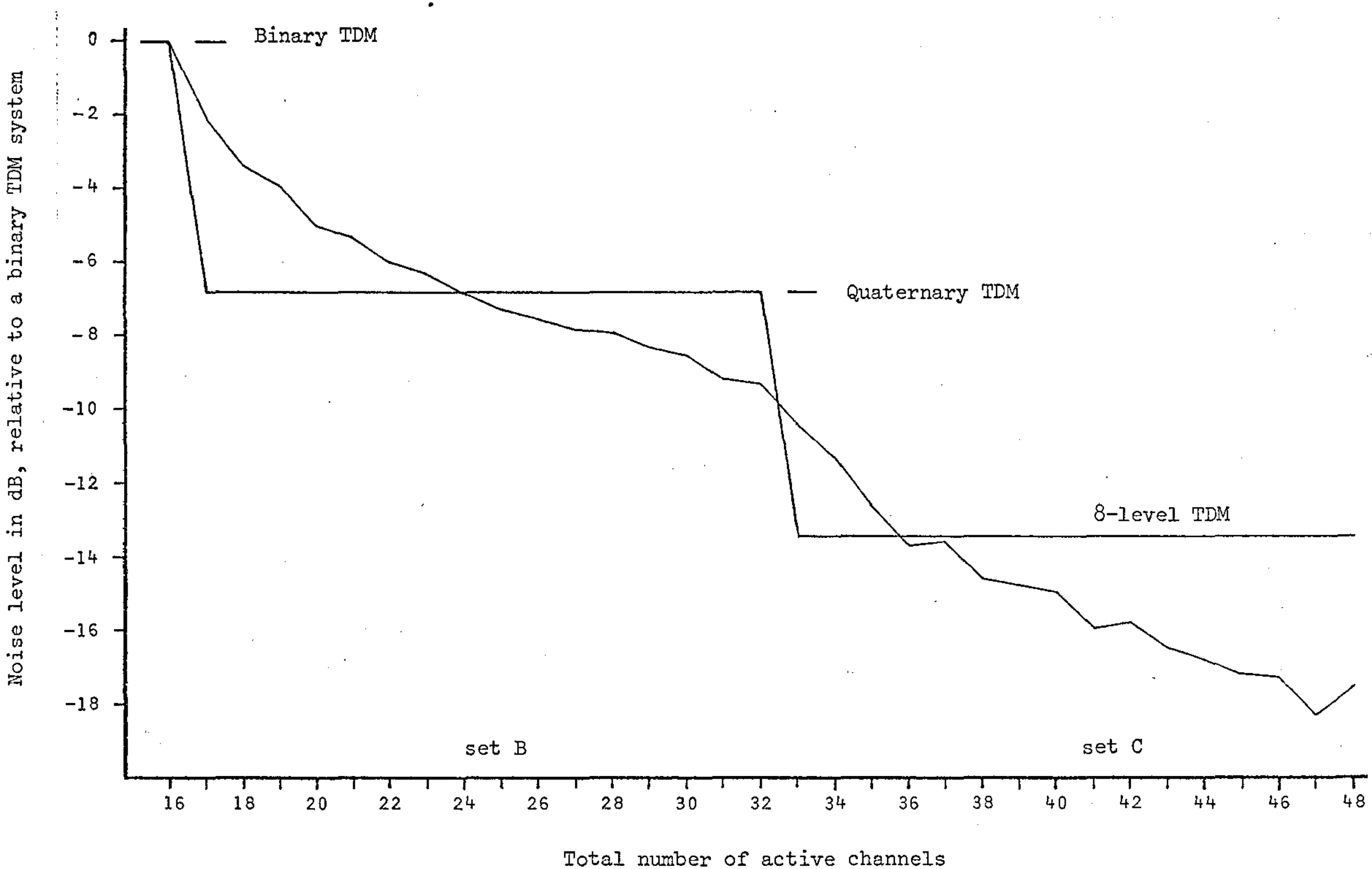


Figure 4.5-1 System A3. Noise level for an error probability per channel of 0.003, expressed in dB relative to a binary TDM system, for a varying number of channels in the orthogonal sets B and C .

4.6 Assessment of System A3

The non-linear arrangement of System A3 described, capable of multiplexing up to three times the maximum number of orthogonal channels, is most suitable when there are up to about 50% more channels than may be multiplexed by conventional means, whether it be binary or quaternary TDM. A trade-off exists between the number of active channels and the tolerance to additive white Gaussian noise.

4.7 System A4

Systems A1 and A3 are arrangements involving a non-linear combination of two and three sets of orthogonal signals respectively. The TDM elements are orthogonal as are the TDM elements, but simultaneously transmitted TDM and CDM elements are not orthogonal. As explained previously, a trade-off exists between the number of channels in operation and the tolerance to additive white Gaussian noise. System A4 is an extension using the same techniques whereby up to five times as many channels may be multiplexed, for a given transmission path and signal element rate per channel, than is possible with orthogonal multiplexing using TDM or CDM alone.

The transmitted signal elements are arranged in separate groups comprising a single TDM orthogonal set and four CDM orthogonal sets. At any time the total number of channels m may take any value from 0 to 5n, where n is the maximum number of orthogonal TDM or CDM elements.

The TDM codewords which are used as the signal carriers for the n TDM channels are given by the rows $\{A_i\}$ of an $n \times n$ identity matrix as before. The complete set of n TDM codewords will be referred to as the orthogonal set A.

The four sets of CDM codewords are given by the rows $\{B_i\}$, $\{C_i\}$, $\{D_i\}$ and $\{E_i\}$ of $n \times n$ matrices, B,C,D and E, which are shown in Figure 4.7-1 for the particular case where a codeword contains 16 components. These particular sets of codewords are used, so that after multiplexing, the components corresponding to each element value of the various sets present in the resultant transmitted signal, interfere minimally. In the resultant transmitted signal, the orthogonal set A element values correspond to 1 non-zero component, the orthogonal set B element values correspond to 2 non-zero components, the orthogonal set C to 4 non-zero components, the orthogonal set D to 8 non-zero components and the orthogonal set E to 16 non-zero components.

To avoid any unnecessary confusion, it is sufficient to say that the multiplexing and demultiplexing arrangements are identical to System A3, where only three orthogonal sets are combined. The detected element values for each set are detected sequentially for each cycle of the iterative detection process and error correction is performed by reconstituting various signals from the detected element values of the previous cycle.

This system has not been computer simulated due to the immense complexity involved with five orthogonal sets. However, the performance of this arrangement may be extrapolated from the results of System A3 shown in Figure 4.5-1. It is clear that although a trade-off exists between the number of channels in operation and the tolerance to additive white Gaussian noise, the incorporation of additional orthogonal sets only reduces the relative advantage over conventional multilevel TDM systems, having the same average energy per component of the transmitted signal. When there are more than 3 times the maximum number of channels that may be multiplexed orthogonally, there is little or no advantage in using this system. System A4 therefore remains only an interesting extension using

$$B = \frac{1}{\sqrt{2}}\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{bmatrix}$$

$$C = \frac{1}{\sqrt{2}}\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1
\end{bmatrix}$$

$$D = \frac{1}{\sqrt{2}}\begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1
\end{bmatrix}$$

$$E = \frac{1}{\sqrt{2}}\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{bmatrix}$$

Figure 4.7-1 The orthogonal matrices B, C, D and E.

the basic techniques of Systems A1 and A3.

4.8 Correlative coding scheme suitable for hardware implementation

The multiplexing arrangement of System A3 previously described is obviously complicated. This technique produces the identical transmitted signal vector but in a different manner, and could be easily implemented with digital integrated circuits.

The TDM codewords which are used as the signal carriers for the n TDM channels are given by the rows $\{A_i\}$ of an $n \times n$ identity matrix as before. They may be represented by the n-component row vector,

$$A_i = 0 \;.\;.\; 0 \;\; a_{ii} \;\; 0 \;.\;.\; 0 \tag{4.8-1}$$

whose ith component is $a_{ii} = 1$.

The two sets of CDM codewords which are used as the remaining channel carriers are given by the rows $\{B_i\}$ and $\{D_i\}$ of $n \times n$ matrices B and D. The matrix D differs from the matrix C used in System A3, because the individual set codewords are multiplied by the element values of that set only, and then added linearly. In System A3 the codewords of the orthogonal set B are multiplied by a non-linear combination of the element values from the orthogonal set B, and the signal elements from the orthogonal set C. It should be noted, however, that D = BC. The ith codewords from the orthogonal sets B and D are given by the vectors,

$$B_i = b_{i1} \;\; b_{i2} \;.\;.\; b_{in} \tag{4.8-2}$$

$$C_i = c_{i1} \;\; c_{i2} \;.\;.\; c_{in} \tag{4.8-3}$$

and are shown in Figure 4.8-1 for the particular case where $n = 16$.

The element value of the signal element in the ith of the n channels whose codewords belong to the orthogonal set A, is $x_i = \pm 1$ when a signal is present, or $x_i = 0$ when no signal is present. Similarly, the element value of the signal element in the ith of the n channels whose codewords belong to the orthogonal set B, is $y_i = \pm 1$ when a signal is present, or $y_i = 0$ when no signal is present. Similarly, for the orthogonal set D, $z_i = \pm 1$ when the ith signal is present, or $z_i = 0$ when no signal is present. Let X,Y and Z be the n-component row vectors with ith components x_i, y_i and z_i respectively.

At the transmitter, the coder and multiplexer combine the codewords from the three orthogonal sets over the period 0 to nT, to give the resultant transmitted signal. Each complete set of codewords forming an $n \times n$ matrix is modified, by sign changes of the components, according to the element values of lower order sets, where set A is of lowest order. The signal elements so formed are multiplied by the coincident element values of the corresponding sets, and added linearly to form the transmitted signal.

The orthogonal set A codewords $\{A_i\}$ are multiplied by the corresponding binary element values $\{x_i\}$, so that each codeword given by Eqn. (4.8-1) is binary antipodal. The codewords are added linearly to give the n-component vector,

$$XA \tag{4.8-4}$$

$$B = \tfrac{1}{2}\begin{bmatrix}
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1
\end{bmatrix}
\qquad
D = \tfrac{1}{4}\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1
\end{bmatrix}$$

Figure 4.8-1. The orthogonal matrices B and D for the correlative coding scheme of System A3

$$\begin{bmatrix}
1 & 1 & 1 & 1 & 5 & 5 & 5 & 5 & 9 & 9 & 9 & 9 & 13 & 13 & 13 & 13 \\
2 & 2 & 2 & 2 & 6 & 6 & 6 & 6 & 10 & 10 & 10 & 10 & 14 & 14 & 14 & 14 \\
3 & 3 & 3 & 3 & 7 & 7 & 7 & 7 & 11 & 11 & 11 & 11 & 15 & 15 & 15 & 15 \\
4 & 4 & 4 & 4 & 8 & 8 & 8 & 8 & 12 & 12 & 12 & 12 & 16 & 16 & 16 & 16 \\
1 & 1 & 1 & 1 & 5 & 5 & 5 & 5 & 9 & 9 & 9 & 9 & 13 & 13 & 13 & 13 \\
2 & 2 & 2 & 2 & 6 & 6 & 6 & 6 & 10 & 10 & 10 & 10 & 14 & 14 & 14 & 14 \\
3 & 3 & 3 & 3 & 7 & 7 & 7 & 7 & 11 & 11 & 11 & 11 & 15 & 15 & 15 & 15 \\
4 & 4 & 4 & 4 & 8 & 8 & 8 & 8 & 12 & 12 & 12 & 12 & 16 & 16 & 16 & 16 \\
1 & 1 & 1 & 1 & 5 & 5 & 5 & 5 & 9 & 9 & 9 & 9 & 13 & 13 & 13 & 13 \\
2 & 2 & 2 & 2 & 6 & 6 & 6 & 6 & 10 & 10 & 10 & 10 & 14 & 14 & 14 & 14 \\
3 & 3 & 3 & 3 & 7 & 7 & 7 & 7 & 11 & 11 & 11 & 11 & 15 & 15 & 15 & 15 \\
4 & 4 & 4 & 4 & 8 & 8 & 8 & 8 & 12 & 12 & 12 & 12 & 16 & 16 & 16 & 16 \\
1 & 1 & 1 & 1 & 5 & 5 & 5 & 5 & 9 & 9 & 9 & 9 & 13 & 13 & 13 & 13 \\
2 & 2 & 2 & 2 & 6 & 6 & 6 & 6 & 10 & 10 & 10 & 10 & 14 & 14 & 14 & 14 \\
3 & 3 & 3 & 3 & 7 & 7 & 7 & 7 & 11 & 11 & 11 & 11 & 15 & 15 & 15 & 15 \\
4 & 4 & 4 & 4 & 8 & 8 & 8 & 8 & 12 & 12 & 12 & 12 & 16 & 16 & 16 & 16
\end{bmatrix}$$

Figure 4.8-2 The matrix M

The components of the orthogonal set B codewords are modified by the orthogonal set A element values as follows. If the jth element value of the orthogonal set A, x_j is negative, then the components of the jth column of matrix B are changed in sign. Let this give the modified n × n matrix B' of the set B elements. These elements $\{B_i'\}$ are multiplied by the corresponding binary element values $\{y_i\}$, which are then added linearly to give the n-component vector,

$$YB' \tag{4.8-5}$$

The components of the orthogonal set D codewords are modified by both the orthogonal sets A and B element values as follows. If the jth element value of the orthogonal set A, x_j is negative, then the components of the jth column of matrix D are changed in sign. Let this give the modified n × n matrix D'. If the jth component of the orthogonal set B, y_j is negative, then the components of matrix D' are changed in sign, where the components are identified by the number j in the matrix M shown in Figure 4.8-2. For example, if $y_3 = -1$, then the components d_{31}', d_{32}', d_{33}', d_{34}', d_{71}', d_{72}' etc. are changed in sign. Let this give the modified n × n matrix D'' of the set D elements. These elements $\{D_i''\}$ are multiplied by the corresponding binary element values $\{z_i\}$, and are added linearly to give the n-component vector,

$$ZD'' \tag{4.8-6}$$

The vector YB' (4.8-5) is multiplied by a scalar f to determine the level of the signal elements of the orthogonal set B. Similarly, ZD'' (4.8-6) is multiplied by the scalar g.

The n components of each of the three signal elements for the three orthogonal sets are added linearly, to give the n-component resultant transmitted signal vector,

$$S = XA + fYB^{'} + gZD^{''} \qquad (4.8\text{-}7)$$

The multiplexing of System A4 involving five orthogonal sets of signals may be similarly simplified using this technique. Different orthogonal matrices are used, whose rows are codewords used as the channel carriers. The matrix modifications depend on the lower order set element values, and involve changes in sign only.

CHAPTER 5

NON-LINEAR CODE-DIVISION MULTIPLEXING

5.1 Introduction

The technique of multiplexing is usually based on the orthogonality of the channel carriers. The waveforms of the carriers assigned to each of the channels are such that, if $f_1(t)$ and $f_2(t)$ are the carriers assigned to channels i and j respectively, if $i \neq j$, then over the period 0 to T,

$$\int_0^T f_1(t).f_2(t)\,dt = 0 \tag{5.1-1}$$

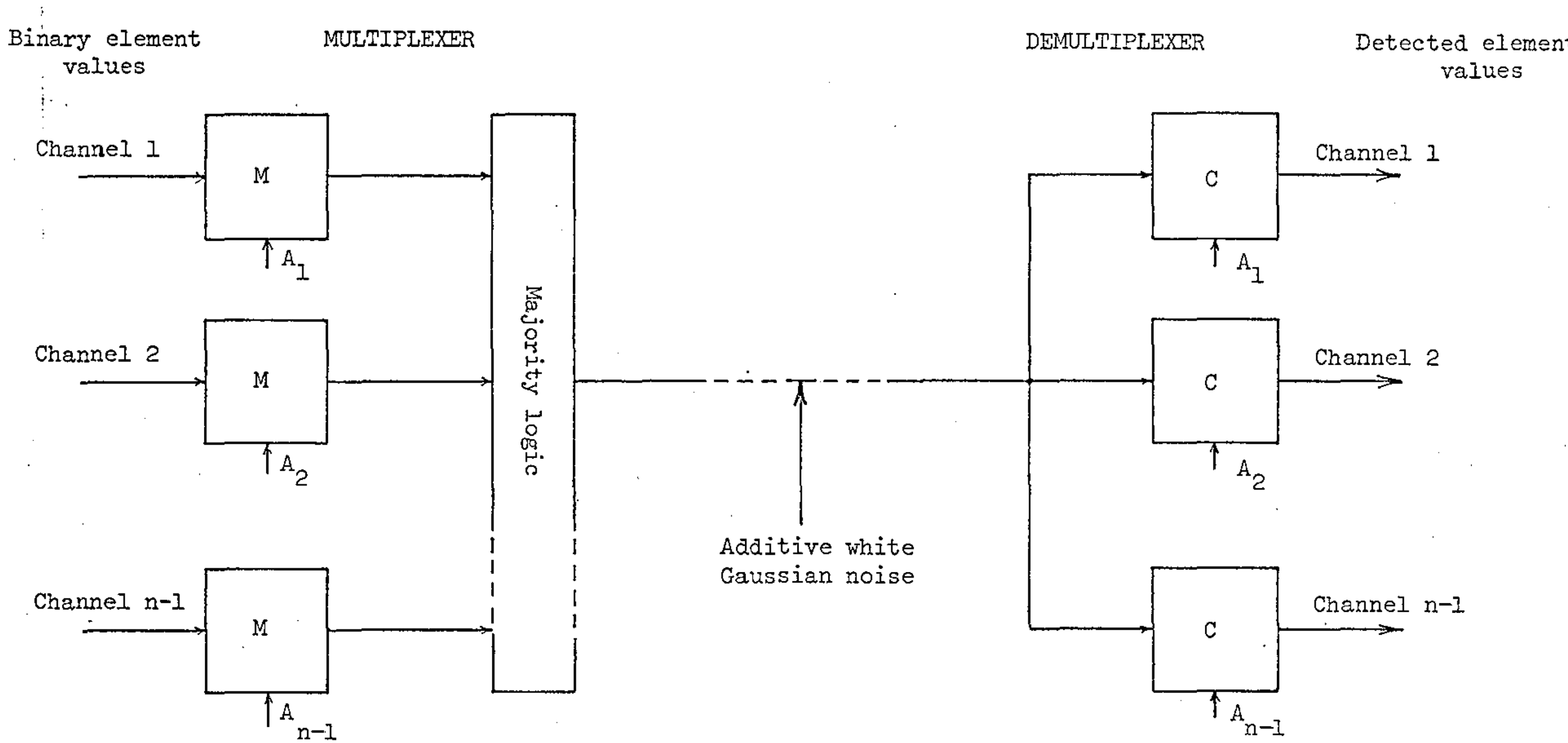
This condition is met in FDM and TDM systems by the use of non-overlapping bands in the frequency and time domains respectively. Section 3.1 introduces linear code-division multiplexing systems in which different channels are assigned orthogonal codes which are multiplied by the corresponding analogue signals. The demultiplexer recovers the data for each channel by correlating the received signal with locally generated codewords, the correlation coefficients being proportional to the multiplexed analogue signals.

For binary data signals, such a process is wasteful, which enables the data to be recovered correctly in both sign and magnitude, at least in the absence of noise. Only the sign, for binary data is in reality necessary, and the presence of other channels may be allowed to corrupt the magnitude of the received data signal. A non-linear code-division multiplex system is one in which the transmitted signal is not a linear function of the individual signal codes.

Titsworth[56] in 1962 proposed a system in which the codewords, multiplied by the corresponding element values are added linearly as before, only now, the components of the transmitted signal are binary bipolar equal to ±1, the sign corresponding to that of the components of the vector previously obtained by the linear addition of the codewords. Barrett and Karran[57] have developed a similar system which employs pseudo-random noise carriers as the codewords, and correlation detection at the receiver. Such systems are now known as majority multiplex systems, from the prominent work in this field by Gordon and Barrett of the Hatfield Polytechnic.

5.2 Gordon and Barrett

A system has been proposed which exhibits a trade-off between the number of active channels and the tolerance to additive white Gaussian noise.[58-60] Two prototypes have been built using different methods of channel simulation, but the multiplexing and demultiplexing techniques are identical. Figure 5.2-1 shows a block diagram of the majority multiplex system. The Walsh functions used as the signal carriers for the different channels are given by the $n-1$ component row vector $\{A_i\}$, forming an $(n-1)\times(n-1)$ matrix A, which consists of the first n Walsh functions with the first row and column omitted. For the particular case where a codeword contains 7 components, the matrix A is as below, and is the truncated form of an 8×8 matrix of the first 8 Walsh functions.



M - binary multipliers (EX OR)

C - correlation detectors

A_1 to A_{n-1} - truncated Walsh functions

Figure 5.2-1 A non-linear multiplex system using majority logic of Walsh functions.

$$A = \begin{bmatrix} 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}$$

If the component values of the carriers of an individual signal element in the ith channel are given by $\bigcup_{j=1}^{n-1} a_{ij}$ they can be represented by the $n-1$ component row vector.

$$A_i = a_{i1} \; a_{i2} \; \cdot \; \cdot \; a_{i(n-1)} \tag{5.2-1}$$

The element value of the signal element in the ith of the $n-1$ channels x_i, of the $n-1$ component row vector X, is $x_i = \pm 1$, when a signal is present in this channel, or $x_i = 0$ when no signal is present.

The multiplexer multiplies the codewords $\{A_i\}$ by the binary element values $\{x_i\}$ to give the corresponding signal elements, which are added linearly to give the $n-1$ component vector XA. The ith component s_i of the resultant transmitted signal vector S is given by ± 1, the selected sign being the same as that of the ith component of XA, so that the transmitted signal vector corresponding to the m coincident signal elements is the $n-1$ component vector,

$$S = \text{signs } (XA) \tag{5.2-2}$$

where the operator "signs" replaces each term of the vector XA by ±1 corresponding to the sign of the components of XA. For an even number of active channels, the ith component of XA may equal zero, for which the operator signs cannot be applied. The multiplexing scheme is therefore only valid for an odd number of active channels.

Two prototypes have been constructed and tested with different channel simulators. In the first,[58] white Gaussian noise is added to the transmitted signal, which is passed through an active 4th order Butterworth low-pass filter, with a 3 dB point of 2KHz. The transmission rate is 2.4 kilobits/sec. The input of the demultiplexer slices the received signal, recovering the binary waveform.

The second prototype channel simulator[59] uses a digital random-error generator which introduces digital binary errors into the transmitted signal stream with a given probability of error. The generator incorporates a set of random number generators, with bases of 10, 5 and 2. Each generator produces a random number for each component of the data signal. If the set of random numbers fits a prescribed set of conditions, an error is introduced into the data stream. By varying the set of conditions it is possible to introduce digital binary errors into the data signal with any probability which may be expressed in terms of the numbers 10, 5 and 2. For instance, error probabilities of 1 in 2, or 1 in 2×10^7, or 1 in 5×10^4 may be introduced.

The demultiplexer correlates the received binary n-1 component signal vector R with the identical truncated Walsh functions used in the multiplexing process. The correlation detector multiplies the jth component of R by the jth component of A_i for $j = 1 - - n-1$ and adds the product to give the output signal (correlation coefficient) for each of the m channels. The detected element values $\{x_i'\}$ of the vector X' are given by the signs of these output signals.

$$X' = \text{signs}\ (RA^T) \tag{5.2-3}$$

A block diagram of the experimental arrangement is shown in Figure 5.2-2.

The results for the first prototype using the additive white Gaussian noise channel simulator are shown in Figure 5.2-3. Also shown are the theoretical curves for the error probability per channel against a variation of signal/noise ratio for different numbers of active channels. The signal/noise ratio was measured at the output of the channel simulator, the signal and noise energies being measured separately using a thermocouple arrangement. An attenuator feeding the thermocouple was used to measure the quantities, the attenuator being adjusted until the thermocouple gave a standard reading. In this way the relative energies of the signal and noise may be measured accurately. The error rates were measured by counting a large number of errors, typically between 500 and 20,000, to obtain statistically significant results. For seven active channels, the arrangement gives a performance approaching that of a conventional binary TDM system, having the same average energy per component of the transmitted signal, and the same transmission rate. At high signal/noise ratios, with one active channel only, the tolerance to noise increases by about $7\frac{1}{2}$ dB relative to 1 active channel.

The results using the digital random error-generator channel simulator of the second prototype are shown in Figure 5.2-4. Experimental and theoretical curves show the detected element value error probability per channel against the transmission error probability for different numbers of active channels. For less than the maximum number of channels in operation, a significant reduction in the element value error probability per channel

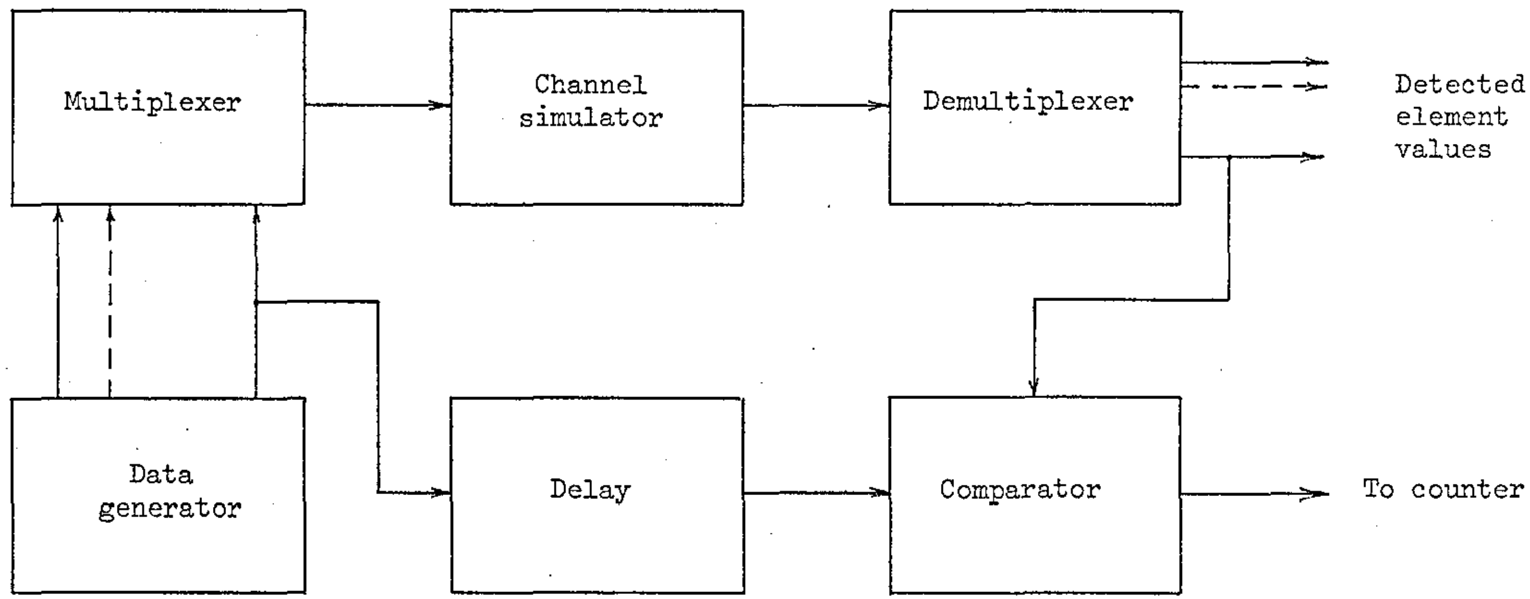


Figure 5.2-2 The experimental arrangement

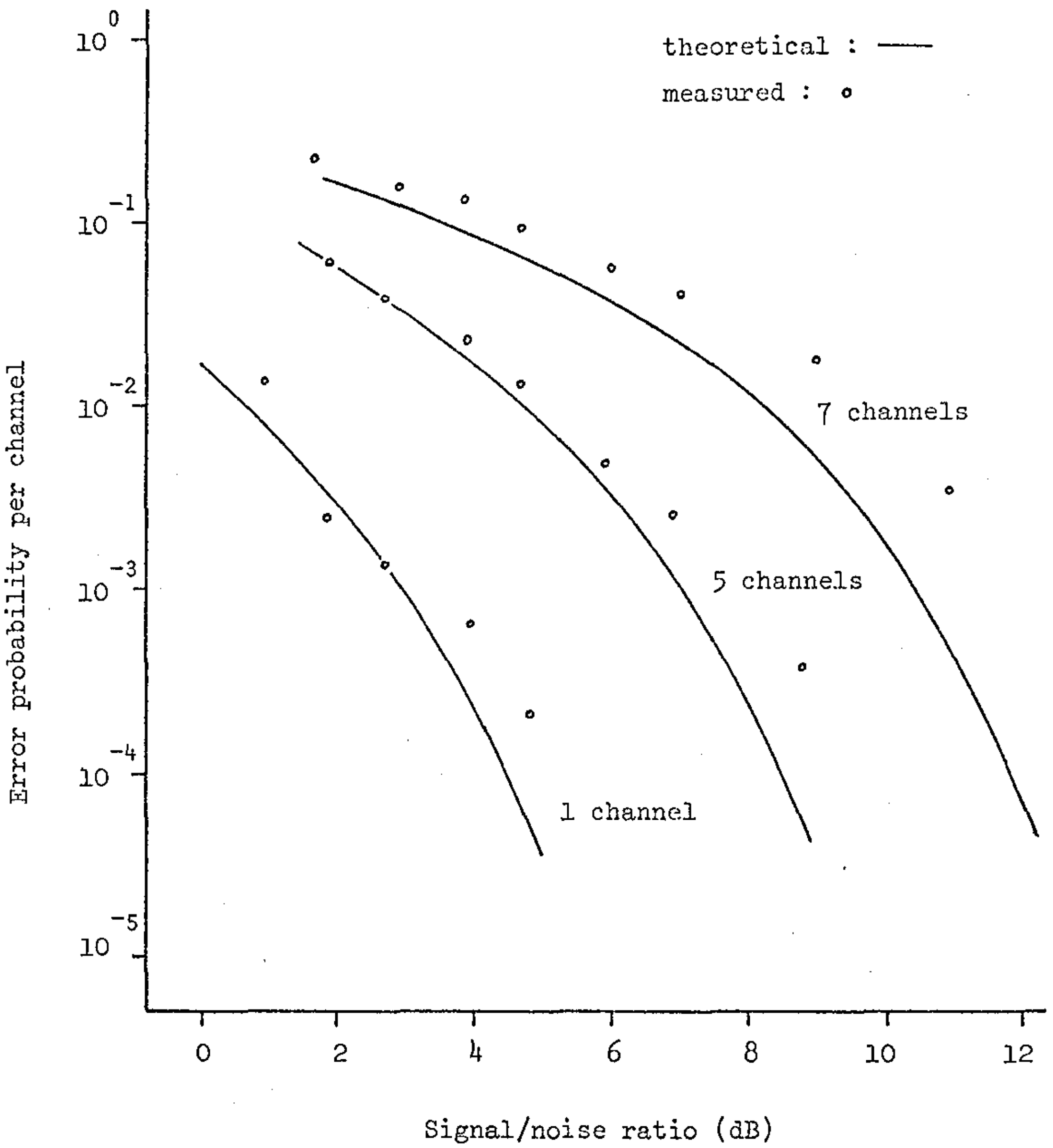


Figure 5.2-3 Additive white Gaussian noise channel simulator. Variation of the error probability per channel with signal/noise ratio in decibels, for different numbers of active channels.

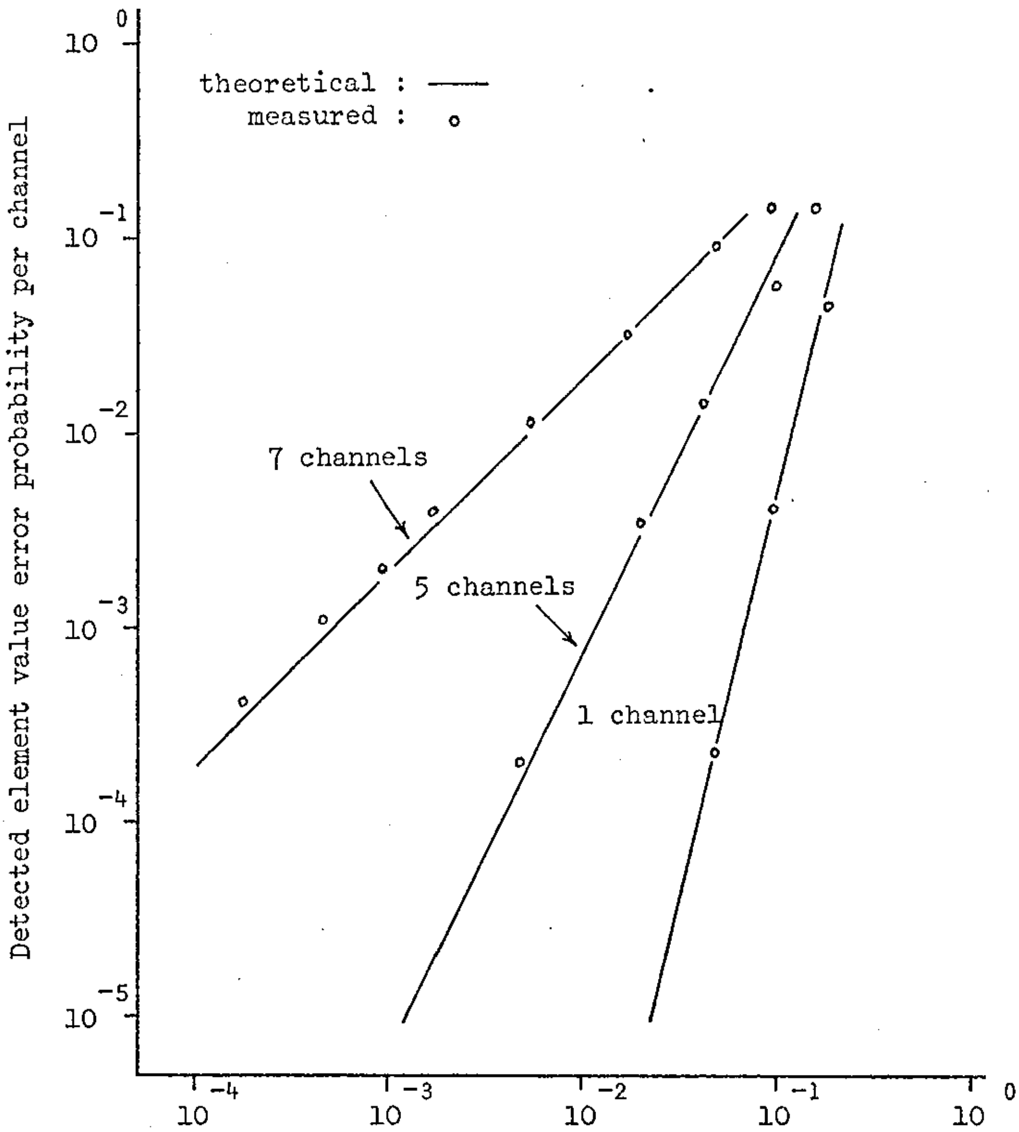


Figure 5.2-4 Digital random error generator channel simulator. Variations of the detected element value error probability per channel, with the error probability of the digital binary errors introduced into the data signal, for different numbers of active channels.

is obtained compared to the introduced digital binary errors in the data signal. For example, for one active channel, the element value error probability per channel is about 10^{-6} for a digital binary error probability in the data signal of 10^{-2}.

The advantages of this system are twofold. When the system is not operated at maximum capacity, the data is redundantly encoded, and a considerable measure of error correction takes place without additional circuitry, this correction taking place quite automatically as a result of the encoding and correlation detection in the demultiplexing process. Thus a trade-off exists between the number of channels in operation at any time and the tolerance to additive white Gaussian noise of the data signal.

The second advantage is that the transmitted signal is binary, which simplifies the design of any repeater equipment that may exist.

However, there are several disadvantages. The coding scheme is only valid for a codeword length n-1 of 7 or 3 components, accommodating a maximum capacity of 7 and 3 channels only. It has been shown[62] that there are no matrices which provide any improvement over this, and it is merely fortuitous that the Walsh matrix majority multiplexing scheme works at all. As explained, only an odd number of active channels may be multiplexed. For an even number of active channels, a dummy signal representing an additional channel must be introduced.

An extension to Gordon and Barrett's majority multiplexing scheme has been proposed by Hashim[63] to enable more than 7 channels to be simultaneously multiplexed. The total number of channels must be a multiple of 3 or 7. Different groups of codewords are interleaved, each group using majority multiplexing independently of the others. For a few channels in operation only, each channel may use several codewords, one from each independent group, the element values being detected from the sum of the correlation detector outputs for each group.

Gordon and Barrett have more recently proposed a group multiplexing system by concatenation, in which the outputs of several independent multiplexers form the input to another multiplexer. In this way, larger, more powerful error correcting groups are formed. The results are given in reference (61).

CHAPTER 6

A CODE-DIVISION MULTIPLEX SYSTEM USING AN ADAPTIVE WALSH FUNCTION CODING SCHEME

6.1 Introduction

From the previously discussed proposed systems, three factors appear significant in an arrangement that uses the available power and bandwidth optimally to give the best possible tolerance to noise.

a) A conventional binary TDM system whose individual channels occupy one component of the transmitted signal group only, and whose element values are statistically independent, is considered optimum when all channels are in operation and the system is used at maximum capacity. For this condition no alternative arrangement will give a superior tolerance to additive white Gaussian noise.

b) A CDM system whose individual channels are assigned reference carriers with components spread over the entire element period, is optimum for the particular case when one channel only is in operation. The transmitted signal is binary antipodal, and the selected channel has exclusive use of the entire bandwidth.

c) For a good tolerance to noise performance, equality is necessary between the peak component energy and the average transmitted energy. This is because the transmitted energy per component increases with the square of the component amplitude, whereas the tolerance to noise increases linearly. Thus large peak energies are not beneficial. Also, for transmitted signal components of equal amplitude, additional orthogonal sets of signals may be added in a similar fashion to multilevel TDM, where the components of an additional orthogonal set are added to the previous orthogonal set at half the amplitude.

An optimum multiplexing arrangement would generate a transmitted signal similar to a CDM codeword and TDM, for minimum and maximum capacities respectively, and changing gradually from one arrangement to the other as the number of channels increases. The utilisation of available power could then be optimum (best possible arrangement) at all times.

A necessary requirement is also a demultiplexing arrangement whose operation is uncomplicated, fast and whose performance matches up to the optimum detection process. This detector minimises the probability of error (that is, the probability of one or more element errors) in the detection of the element values of a group.

The following two closely related multiplexing arrangements C and D, fulfil the conditions previously outlined. These arrangements use an adaptive coding scheme, such that the transmitted signal automatically adjusts itself to the number of multiplexed channels. In so doing, the tolerance to additive white Gaussian noise of the transmitted signal over the communication link is improved, relative to the corresponding TDM system, for any number of channels. The technique is capable of multiplexing any number of signals, quasi-orthogonally, up to the maximum number of signal elements over an element period of nT seconds. The number of signals multiplexed may exceed the maximum number of orthogonal multiplexed signals using a multilevel transmitted signal, in which the tolerance to noise deteriorates slowly as the number of channels increases. Thus, a trade-off exists between the number of channels transmitted and the tolerance to additive noise.

6.2 The multiplexing arrangement C

The multiplexing procedure is based on a non-linear combination of Walsh functions which are used as the signal carriers for the different channels. The Walsh functions are given by the n-component row vectors $\{A_i\}$, forming the rows of the $n \times n$ matrix A. For the particular case where a codeword or signal element contains 8 components, the matrix A is as below.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}$$

If the component values of the carriers of an individual signal element in the ith channel are given by $\bigcup_{j=1}^{n} a_{ij}$, they can be represented by the n-component vector,

$$A_i = a_{i1} \quad a_{i2} \quad . \quad . \quad a_{in} \tag{6.2-1}$$

The element value of the signal element in the ith of the n channels x_i, of the n-component vector X, is $x_i = \pm 1$ when a signal is present in this channel, or $x_i = 0$ when no signal is present.

It is assumed that the $m\{x_i\}$ for the m channels in use are statistically independent and equally likely to have either binary value. These $\{x_i\}$ are not necessarily the first m of the $n\{x_i\}$, but may be any of the $n\{x_i\}$.

The coder and multiplexer (Figure 2.1-1) combine the codewords of the different channels as follows. For each integer j in the range 1 to n, if $x_j = \pm 1$, set a_{ij} to zero for each $i \neq j$, and leave a_{jj} unchanged. For $x_j = 0$, leave a_{ij} unchanged for each i. The modified matrix of codewords is given by A'.

The modified codewords $\{A_i'\}$ are now multiplied by the binary element values $\{x_i\}$ to give the corresponding signal elements, which are added linearly to give the n-component vector XA'. This is the transmitted signal vector corresponding to the m coincident signal elements, and is given by the n components s_i of the vector S.

$$S = XA' \tag{6.2-2}$$

The transmitted signal components are multilevel, whose amplitude may take any integer value up to $\pm m$. The vector S may be considered to contain two types of components, "independent" and "grey" components. An independent component s_i is one which depends only on the element value of the signal in the ith channel, so that $s_i = x_i a'_{ii}$. s_i has no component from any other channels. A grey component is dependent on the element values of the signals in all the channels in use. For two channels in operation, when $n = 8$, there are two independent components and six grey components, whereas for 8 channels in use, there are no grey components at all.

In the model of the system, white Gaussian noise with two sided power spectral density σ^2 is added to the transmitted signal at the output of the transmission path, giving the Gaussian waveform w(t) added to the transmitted signal at the output of the receiver filter.

The signal at the output of the receiver filter over the duration of a single group of coincident signal elements, is sampled at regular time intervals of T seconds to give the n-component receiver vector,

$$R = S + W \tag{6.2-3}$$

where S and W are n-component vectors, and the $\{w_i\}$ are sample values of statistically independent Gaussian random variables with zero mean and variance σ^2.

From Eqns. (6.2-2) and (6.2-3),

$$R = XA' + W \tag{6.2-4}$$

6.3 The multiplexing arrangement D

This arrangement is similar to the multiplexing arrangement C, only now the multiplexing includes a non-linear majority logic multiplexing operation.

The channel carriers are given by the rows of the $n \times n$ Walsh function matrix A, and the channel element values by the components $\{x_i\}$ of the n-component vector X, as in Section 6.2 .

The coder and multiplexer combine the codewords of the different channels as before. For each integer j in the range 1 to n, if $x_j = \pm 1$, set a_{ij} to zero for each $i \neq j$, and leave a_{jj} unchanged. For $x_j = 0$, leave a_{ij} unchanged for each i. The modified matrix of codewords is given by A'.

The modified codewords $\{A_i'\}$ are now multiplied by the binary element values $\{x_i\}$ to give the corresponding signal elements, which are added linearly to give the n-component vector XA'. The ith component s_i of the resultant transmitted signal vector S is given by ± 1, the selected sign being the same as that of the ith component of XA', so that the transmitted signal vector corresponding to the m coincident signal elements is the n-component vector,

$$S = \text{signs}\ (XA') \qquad (6.3\text{-}1)$$

where the operator "signs" replaces each term of the vector XA' by ± 1 corresponding to the sign of the components of XA'. However, if the ith component of XA' is zero, then the ith component of S, s_i is set to zero. The transmitted signal so formed is ternary. From Eqns. (6.3-1) and (6.2-3),

$$R = \text{signs}(XA') + W \qquad (6.3\text{-}2)$$

6.4 The multiplexing of more channels than may be multiplexed orthogonally

The multiplexing arrangements C and D may be extended to the multiplexing of more than the maximum number of channels that may be multiplexed orthogonally, by dividing the total number of channels into distinct sets. The following coding scheme applies equally to the arrangements C and D, although arrangement C requires a rather more complicated demultiplexing process.

Let the set A contain n channels, where n is the maximum number of orthogonal channels (equal to the number of signal elements), over an element period of nT seconds, and the set B an additional m channels, where $m \leq n$. Each set of channel element values $\{x_i\}$ and $\{y_i\}$ for the sets A and B respectively, are multiplexed completely separately using the same set of Walsh function codewords, to form two n-component vectors S_A and S_B, for the sets A and B. The multiplexing of a single set has been described in sections 6.2 and 6.3. The vector S_B is multiplied by a scalar whose value is positive and equal to c, and determines the level of the signal elements of S_B.

The n components of the vector S_A are now combined non-linearly with the n components of the vector cS_B as follows. For each integer j, $j = 1 \_\_ n$, if the jth component of S_A is negative, then the sign of the jth component of cS_B is reversed. The jth components are now added linearly to give the jth component of the transmitted signal S.

$$S = S_A + \text{signs}(S_A)\ (cS_B) \qquad (6.4\text{-}1)$$

The reason for using a non-linear operation, rather than a linear one, lies in the detection process, in which the set B element values may now be detected without prior knowledge of the detected element values of set A. Thus, errors in the detection of the element values of set A do not affect the detection of the set B element values as would occur with a linear coding scheme. The probability of correct detection of the element values is thus increased.

As in Section 6.2 the received n-component vector is,

$$R = S + W \qquad (6.4\text{-}2)$$

At the receiver the two sets of signal elements are separated, and demultiplexed independently as a set of 1 to n channels only. The demultiplexing processes vary in complexity and performance and are described in Chapters 7 and 8.

Because the individual components of the vectors S_A and S_B are ±1 or 0 for arrangement D, and multilevel for arrangement C, two different techniques are used for the separation of the set A and set B signals.

The demultiplexing of the received signals for arrangement C is performed using an iterative process of two cycles shown in Figure 6.4-1. In the first cycle the set A signals are demultiplexed and detected from the n components $\{r_i\}$ of the vector R. The set B signals are demultiplexed and detected from the n-component vector R', whose ith component is,

$$r_i' = |r_i| - 1 \qquad i = 1 \text{ -- } n \qquad (6.4\text{-}3)$$

This operation effectively nullifies the non-linear "signs" operation in the multiplexing of the two vectors S_A and S_B (Eqn. 6.4-1), where each component of S_A is ±1. The element values $\{x_i'\}$ and $\{y_i'\}$ are detected for the sets A and B respectively.

In the second cycle of the iterative detection process, new estimates are made of the detected element values. Under noiseless conditions, element values detected incorrectly in the first cycle are now corrected. The reason for incorrectly detected element values in the first cycle can be seen by referring to Eqn. (6.4-1), that is,

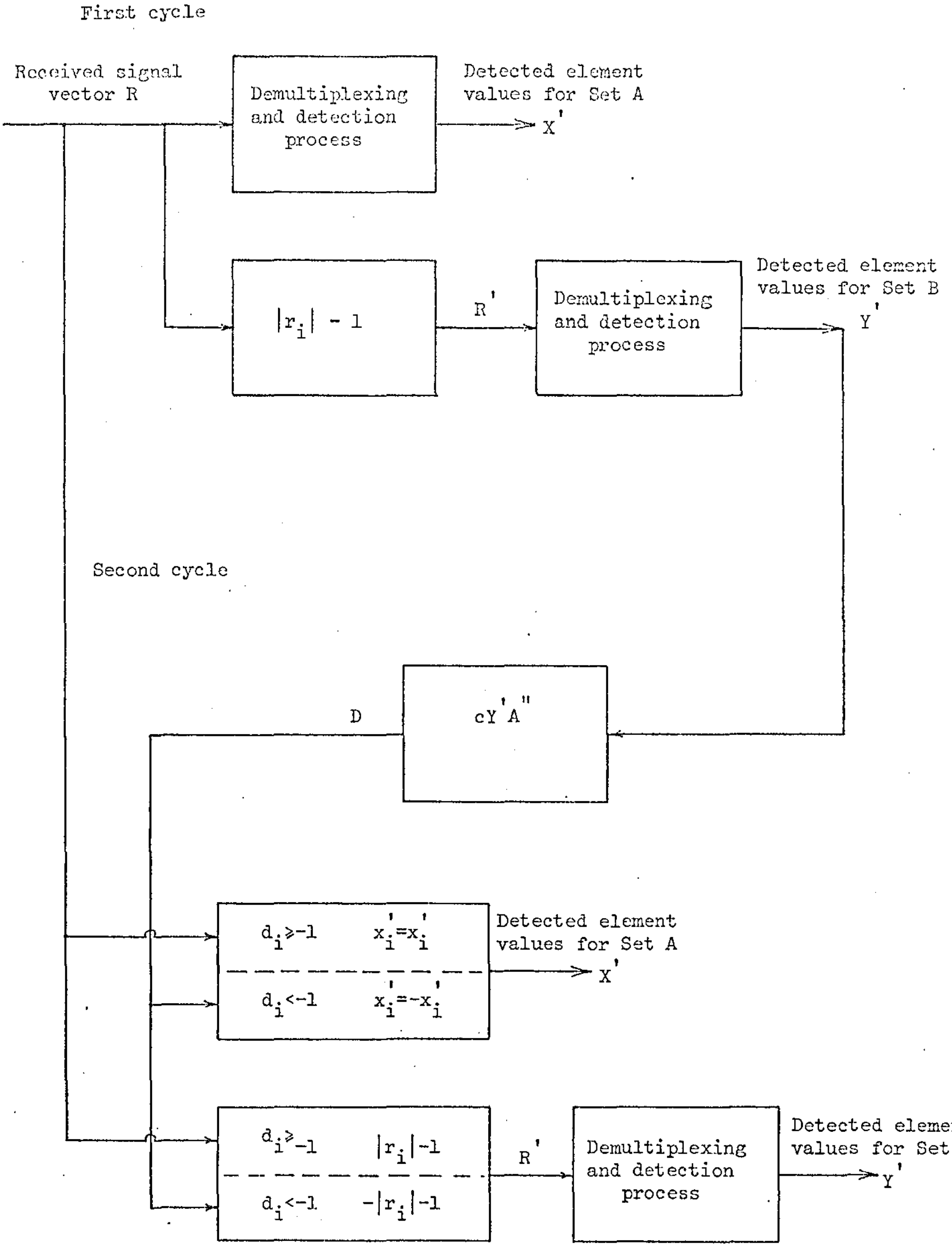


Figure 6.4-1 Arrangement C. Block diagram for the separation of two sets of signals.

$$S = S_A + \text{signs}(S_A)\ (cS_B)$$

For n channels in set A, the components of S_A are independent, and the information corresponding to the ith channel is contained in the ith component of S_A alone. If the ith component of the term cS_B has a value more negative than -1, then, irrespective of whether the ith component of S_A is ±1, the sign of the ith component of S will be of opposite sign to the ith component of S_A.

In the second cycle, the element values are again detected, but the vector cS_B is examined for components more negative than -1, by reconstituting the vector cS_B from the element values $\{y_i'\}$ of the vector Y' previously obtained for set B. Let D be the n-component vector equal to cS_B with components $\{d_i\}$.

$$\begin{aligned} D &= cS_B \\ &= cY'A'' \end{aligned} \qquad (6.4\text{-}4)$$

where the matrix A'' corresponds to the modified matrix of codewords for the element values $\{y_i'\}$.

The set A element values $\{x_i'\}$ remain unchanged, except when d_i is more negative than -1, when the component x_i' is detected as $-x_i'$.

The sign of each r_i is now made positive, except for the $\{r_i\}$ whose corresponding $\{d_i\}$ are more negative than -1. The signs of these $\{r_i\}$ are made negative. The value of 1 is subtracted from each of the components to give the new n-component vector R', which is used for the detection of the set B element values.

Under noisy conditions some error correction takes place depending on the noise components of the vector W.

The separation of the two sets of signals for the multiplexing arrangement D is performed in a single operation, shown in Figure 6.4-2. The set A element values are demultiplexed and detected from the n components $\{r_i\}$ of the vector R. The set B element values are demultiplexed and detected from the n-component vector R', whose ith component is,

$$r_i' = |r_i| - 1 \qquad i = 1 \ldots n \qquad (6.4\text{-}5)$$

The actual demultiplexing processes vary in complexity and performance and are described in Chapter 8. Not only does majority multiplexing at the transmitter (arrangement D) produce a relatively simple transmitted signal, that is ternary, but the separation of the signal sets is comparatively trivial.

For arrangements C and D, when two sets both containing n channels are in operation, an additional third set of signals may be incorporated using the same principle as for two sets. The separations of more than two sets also follows similar lines, where each individual set is demultiplexed completely separately.

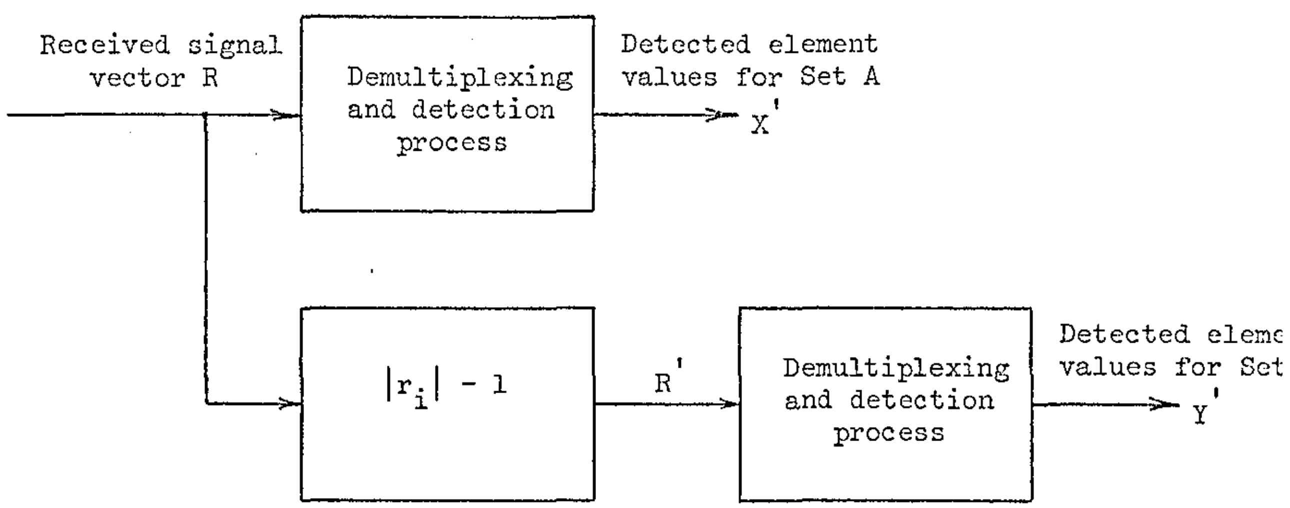


Figure 6.4-2 Arrangement D. Block diagram for the separation of two sets of signals.

CHAPTER 7

DETECTION PROCESSES FOR THE MULTIPLEXING ARRANGEMENT C

7.1 Detection process 1

A useful upper performance bound to a system is provided by the optimum detection process. It has been shown that when the transmitted signal elements are statistically independent and equally likely to take either binary value, the detector which minimises the probability of error (that is the probability of one or more element errors) in the detection of m elements of a group, is the detector that determines which of the 2^m possible transmitted signal vectors is at the minimum distance from the received vector R, in the n-dimensional Euclidean vector space containing R.[11,26,38] The detector knows the exact positions of each possible transmitted signal in the vector space. At high signal/noise ratios, this detection process also minimises the probability of error in the detection of any one of the m elements in a group corresponding to the m channels in operation.

The detection process cannot be implemented by a linear network followed by the appropriate decision thresholds, but is best performed by an iterative process as follows. The receiver generates in turn the vectors {S} where $S = XA'$ from Eqn. (6.2-2), corresponding to the different combinations of the element values $\{x_i\}$ of the m signal elements in a group. The receiver has prior knowledge of which m out of the n channels are in operation. The components of R are stored throughout the detection process for a group of n signal elements. The first vector S from the 2^m possible vectors is subtracted from the received vector R. The components of the difference vector are squared and added, to give the square of the distance between the vector R and the generated vector S.

$$d^2 = \sum_{j=1}^{n} (r_j - s_j)^2 \qquad (7.1\text{-}1)$$

where d is the distance between the vectors R and S. In the first subtraction process, the distance measure d^2, together with the associated vector X are stored. In subsequent subtraction processes no action is taken, unless the value of d^2 is smaller than that stored. When this occurs, the new d^2 together with the associated vector X, replace those stored. Thus, at the end of the detection process, the receiver has the vector X which minimises the distance between S and R and takes this vector X to give the detected element values $\{x_i'\}$ of the m multiplexed channels in the received group. Since the separate operations in the detection process are carried out sequentially, they can be performed by a simple piece of equipment. However, because of the very large number of sequential operations required when there are more than a few multiplexed channels in a group, this process is of limited value.

The n-dimensional vector space may be divided into 2^m decision regions separated by decision boundaries. These decision boundaries are hyperplanes which perpendicularly bisect the lines joining the different signal vectors {S} in the n-dimensional vector space containing the received vector R. The distance of any signal point to a decision boundary is half the distance between the two signal points separated by the decision boundary. Figure 7.1-1 shows the particular case where the two dimensional vector space is divided into four decision regions corresponding to four possible transmitted signals S_1 to S_4.

From Eqn. (6.2-3) $\qquad R = S + W$

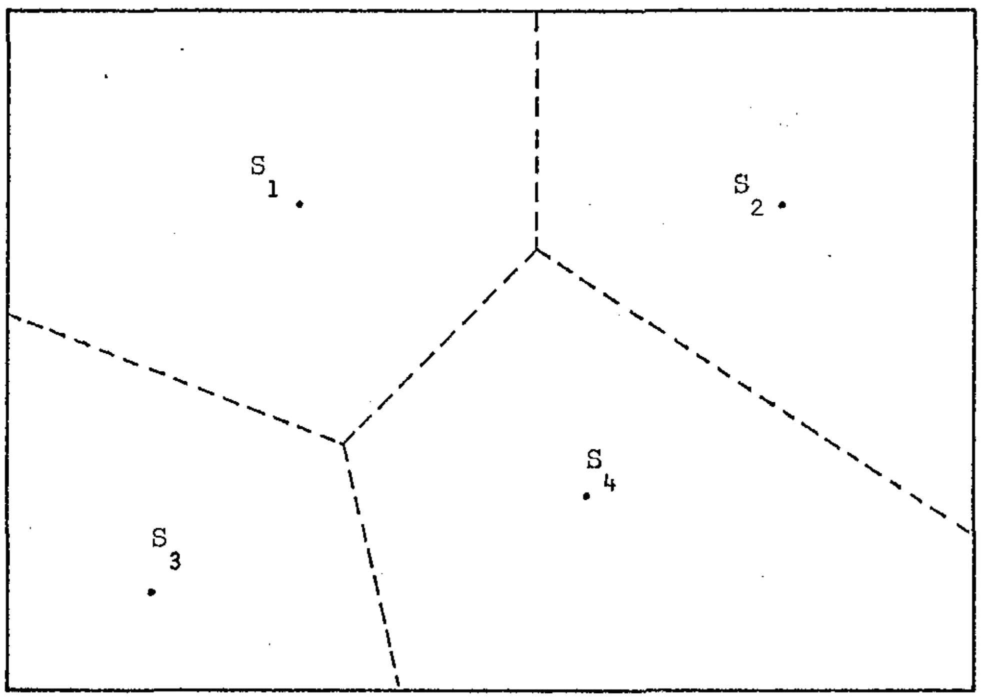
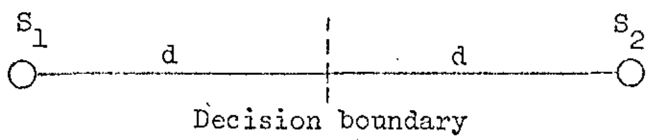


Figure 7.1-1 Decision regions and decision boundaries for the optimum detection process, for four possible transmitted signals in two dimensional vector space.

where W is the n-component noise vector whose projection on to any direction of the n-dimensional vector space is a sample value of a Gaussian random variable with zero mean and variance σ^2. The probability of error in the detection of S_i from R may be deduced with the aid of the following diagram.



When S_i (= S_1 or S_2) is received in the presence of the noise vector W, it is wrongly detected if R is on the opposite side of the decision boundary to S_i. The probability of this occurring is that the orthogonal projection of W on to a line joining S_1 and S_2 has a value greater than d in the direction from S_i to the decision boundary. Noise components in directions parallel to the decision boundary cannot produce errors, nor do they affect the error probability. Thus, the probability of an error in the detection of S_i, whether i = 1 or 2 is,

$$p = \int_{d}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-w^2}{2\sigma^2}\right) dw \tag{7.1-2}$$

$$= \int_{d/\sigma}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-w^2}{2}\right) dw$$

$$= Q\left(\frac{d}{\sigma}\right) \tag{7.1-3}$$

where $$Q(u) = \int_{u}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-x^2}{2}\right) dx$$

d is the distance from S_1 and S_2 to the decision boundary. The Q function, or variaton of the element error probability with d/σ has been widely tabulated and is shown in Appendix A1. The result applies for any value of n, so long as the Gaussian noise sample of zero mean and variance σ^2 are statistically independent.

In the general case where k decision boundaries exist, the total probability of error is determined by the sum of the k individual probabilities of error, due to the various distances to the decision boundary.

$$p \leqslant \sum_{i=1}^{k} Q\left(\frac{d_i}{\sigma}\right) \tag{7.1-4}$$

At high signal/noise ratios with additive white Gaussian noise, even a very small increase in the distance to a decision boundary produces a considerable reduction in the corresponding probability of error (Appendix A1). Thus, the probability of error is effectively determined by the nearest decision boundary, the remaining boundaries having in comparison a very small effect on the probability of error.

An approximate upper bound is given by the value of p_i for the smallest d_i. Let the minimum value of d_i be d, and the corresponding value of p_i be p. Then at high signal/noise ratios, the average element error probability is equal to the probability of error and is approximately equal to,

$$p = Q\left(\frac{d}{\sigma}\right) \tag{7.1-5}$$

7.2 Computer simulation tests

A general description of the computer tests performed and the confidence limits relating to the results obtained have been given in Sections 3.4 and 3.5. In particular, these tests, for systems C1, C2 and D1 to D4 simulate the multiplexing and demultiplexing of between 1 and 8 channels. Between

10000 and 1500 signal groups are transmitted for 1 to 8 active channels, such that about 30 errors are obtained for an error probability of 0.003 per channel. For every test the variance of the additive white Gaussian noise samples σ^2, introduced into the transmission path was adjusted to give an error probability per channel of 0.003.

From (3.5-3), the 95% confidence limits in the value of p are given by,

$$\pm \frac{2}{\sqrt{e}} \; p \tag{7.2-1}$$

where the limits are expressed as deviation from the given value of element value error probability per channel p, and e is the total number of errors counted. Table 7.2-1 summarises the test details and shows the 95% confidence limits, expressed as deviation from the value of p, and also expressed in decibels as deviation from the given value of noise level obtained. These results apply to all systems C1 to D4.

For each system, the noise level expressed in decibels for the same given error probability of 0.003 is compared relative to a conventional binary TDM system (with components of amplitude ±1), having the same transmission rate. For systems C1 and C2, the transmitted signal is multilevel. The average energy per component of the transmitted signal has been normalised to unity, so that it has the same average energy as a non zero component in the TDM system. For systems D1 to D4, the transmitted signal components are given by ±1 or 0.

7.3 Results of computer simulation tests for System C1

The results of computer simulation tests (outlined in Section 7.2) for System C1 are shown in Figure 7.3-1. Although as discussed in Section 6.1,

Number of active channels m	Total no. of groups transmitted	Total no. of errors counted e	Error probability per channel p	95% confidence limits	
				expressed as deviation from the given value of p	of σ expressed in dB as deviation from the given value of σ
1	10000	30	0.003	± 0.0011	
2	5000	30	0.003	± 0.0011	
3	3000	27	0.003	± 0.0012	
4	2500	30	0.003	± 0.0011	All approximately + 0.35 − 0.45
5	2000	30	0.003	± 0.0011	
6	1500	27	0.003	± 0.0012	
7	1500	31	0.003	± 0.0011	
8	1500	36	0.003	± 0.0010	

Table 7.2-1 Summary of test details and 95% confidence limits

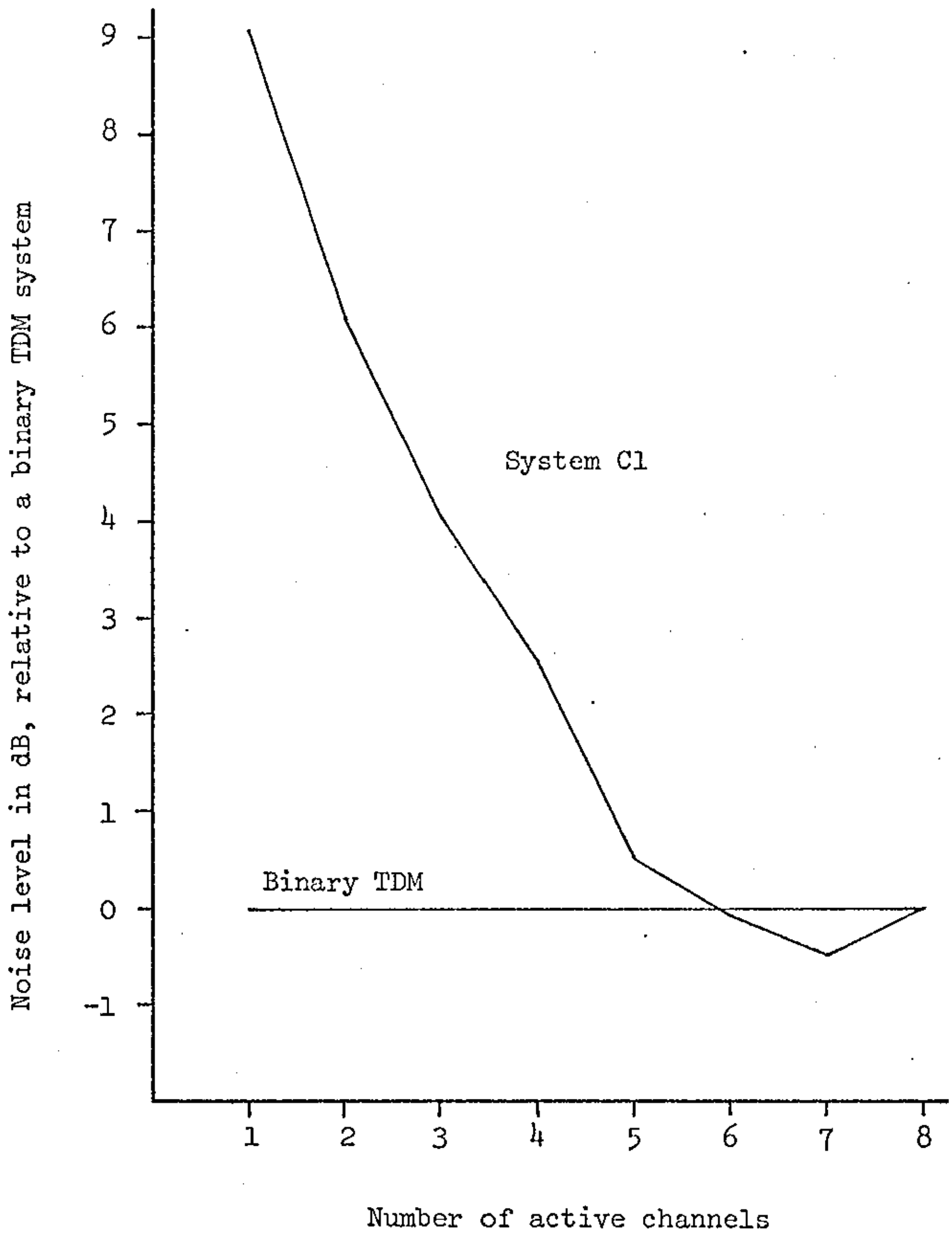


Figure 7.3-1 System C1. Noise level for an error probability per channel of 0.003, expressed in dB relative to a binary TDM system for a varying number of active channels.

the multiplexing arrangement is optimum for 1 and 8 channels in operation, its performance is inferior to that of a conventional binary TDM system for 6 or 7 active channels.

The theoretical performance has been evaluated using an entirely separate computer program. This program calculates the distance in n-dimensional Euclidean vector space between each possible transmitted signal vector, and all other possible 2^m transmitted signal vectors for a given number of active channels m. The average number of signals at various distances is calculated. Using Eqn. (7.1-4), the variance of the additive white Gaussian noise samples σ^2 is calculated for an error probability per channel of 0.003, and from this the relative noise level compared to a conventional binary TDM system with the same average energy per component of the transmitted signal.

Table 7.3-1 shows the theoretical and computer simulation results, together with the 95% confidence limits expressed in decibels as deviation from the relative noise level corresponding to the given value of σ.
The results lying outside the confidence limits are explained by the degree of dependence between the individual element errors in a group in the detection process. The effect of this dependence is to reduce the number of independent errors obtained in a test, and hence the confidence limits should be rather wider than the simplified theory gives (Section 3.5).

The multiplexing and demultiplexing arrangements for more channels than may be multiplexed orthogonally has been described in Section 6.4. Computer simulation results for such a system using the optimum detection process are shown in Figure 7.3-2. Also shown is the relative noise level of the corresponding quaternary TDM system. Both binary and quaternary TDM systems have the same average energy per component of the transmitted signal, the

Number of active channels	Noise level for an error probability per channel of 0.003, expressed in dB relative to a binary TDM system.		95% confidence limits of σ, expressed in dB as deviation from the given value of σ
	Theoretical	Computer simulation	
1	9.05	9.05	All approximately + 0.35 − 0.45
2	6.08	6.07	
3	4.04	3.99	
4	2.53	2.58	
5	0.55	0.49	
6	-0.08	-0.49	
7	-0.46	-2.37	
8	0.00	0.00	

Table 7.3-1 System Cl. Theoretical and computer simulation results

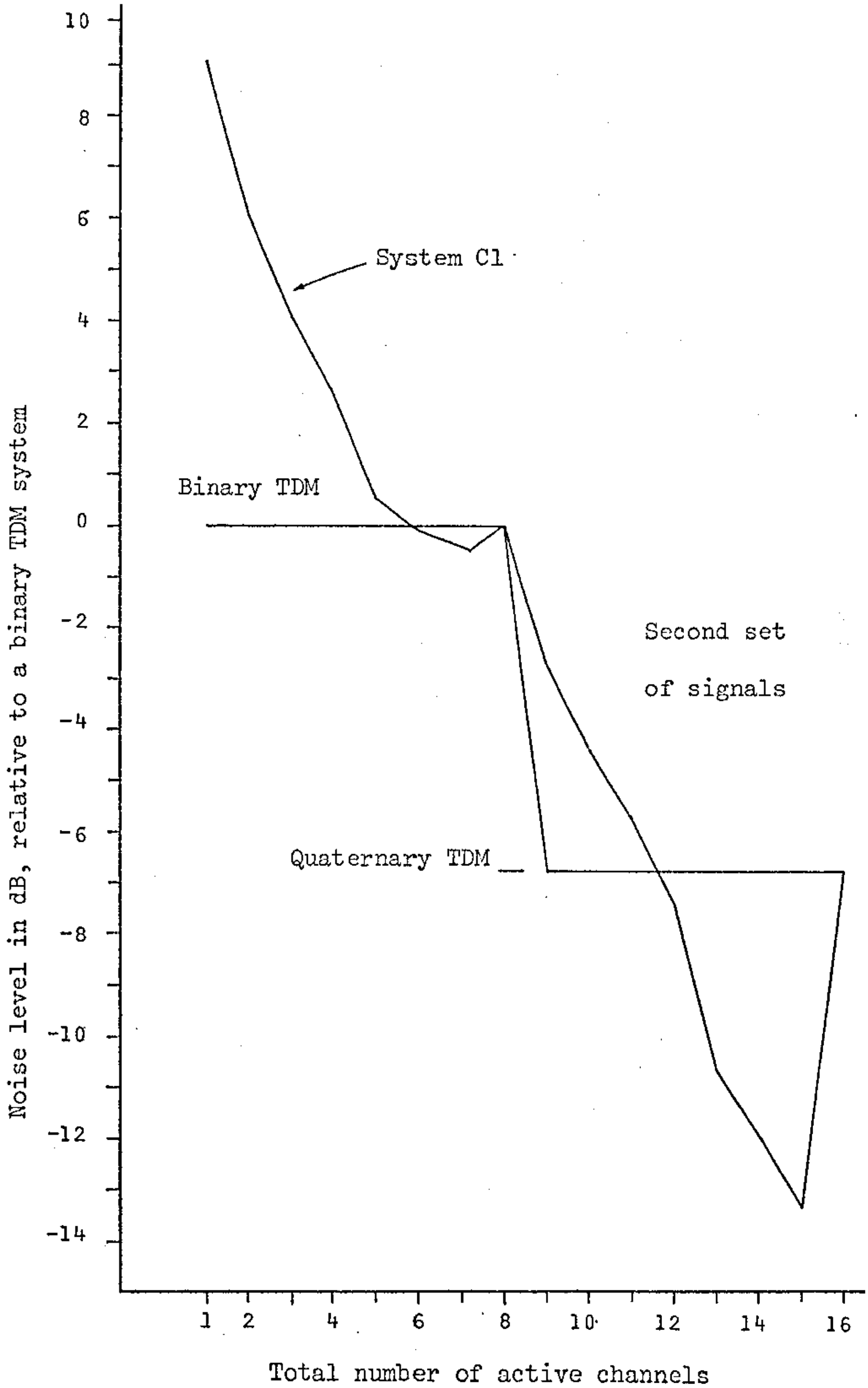


Figure 7.3-2 System C1. The multiplexing of two signal sets. Noise level for an error probability per channel of 0.003 expressed in decibels relative to a binary TDM system for a varying number of active channels.

same transmission rate and the same error probability per channel as explained in Section 3.4. Channels in the second set are not used until the first set channels are all in operation.

7.4 Detection process 2

The process of correlation or matched filter detection is well established in the field of orthogonal signal elements, and like the optimum detection process, at high signal/noise ratios, minimises the error probability per channel of the individual recieved elements. The detection maximises the ratio of the energy level of the wanted signal, to the average energy level of the noise components. The received signal vector R is fed to a set of correlation detectors matched to the orthogonal codewords, and the correlation coefficients obtained give the sign of the received data in each channel. A modification of this standard technique using an iterative process enables quasi-orthogonal signal elements of the multiplexing arrangement C to be detected.

The receiver has prior knowledge of which m out of the n possible channels are in operation. The n-component codewords $\{A_i\}$ corresponding to those m channels are modified appropriately, as was performed in the multiplexing process, as follows. For each codeword A_i, for each integer j in the range 1 to n, if $x_j = \pm 1$, set a_{ij} to zero for each $i \neq j$, and leave a_{jj} unchanged. For $x_j = 0$, leave a_{ij} unchanged for each i. This results in the modified matrix of codewords given by A'.

In the first cycle of the iterative process, the n-component received signal vector R is fed to the m correlation detectors matched to the codewords $\{A_i'\}$ for those m channels in operation. A single element value x_i' is detected from the sign of the correlation coefficient having the largest modulus. The other m-1 element values $\{x_i'\}$ remain undetected. The n components of the codeword A_i' corresponding to the detected element value x_i' are multiplied by x_i', and subtracted from the n components of the received signal vector R, to give the modified n-component received signal vector R'.

In the second cycle of the iterative detection process, the modified received signal vector R is fed to the m-1 correlation detectors matched to the m-1 codewords $\{A_i'\}$, for the m-1 undetected element values. A second element value x_i' is detected from the sign of the correlation coefficient having the largest modulus. The other m-2 element values $\{x_i'\}$ remain undetected. The n components of the codeword A_i' corresponding to the previously detected element value x_i' are multiplied by x_i', and subtracted from the n components of the modified received signal vector R', to give a new modified received signal vector R''.

Subsequent cycles follow using one fewer correlation detectors in each cycle until all m element values are detected. In this way, those signal elements of the transmitted signal which receive the least interchannel interference are detected first, and when cancelled from the received signal, enable other element values to be detected with a lower probability of error.

7.5 Results of computer simulation tests for System C2

This system, using the previously described demultiplexing arrangement, has been tested by computer simulation under identical conditions to System C1 (outlined in Section 7.2). The results for both Systems C1 and C2 are shown in Figure 7.5-1. There is negligible difference between the performance for the correlation and cancellation technique and the optimum detection process. The confidence limits are given in Table 7.2-1

7.6 Assessment of Systems C1 and C2

Detection process 2, using a correlation and cancellation technique, shows that a performance equal to the optimum may be achieved using very simple iterative equipment. Only m sequential operations are required compared to 2^m for the optimum detector. However, even the optimum performance is below that of the corresponding conventional binary TDM system for the same transmission rate, and with the same average energy per component of the transmitted signal, for 6 or 7 active channels. The number of channels may exceed the maximum number of orthogonal multiplexed channels using a second set of signals. Even with the optimum detection process, the performance with 8 channels in the first set and between 4 and 7 channels in the second set, yields a very inferior performance compared to the corresponding quaternary TDM system, with the same average energy per component of the transmitted signal. The separation of the two signal sets described in Section 6.4 is obviously fairly complicated, and for these reasons the multiplexing arrangement C is not further investigated.

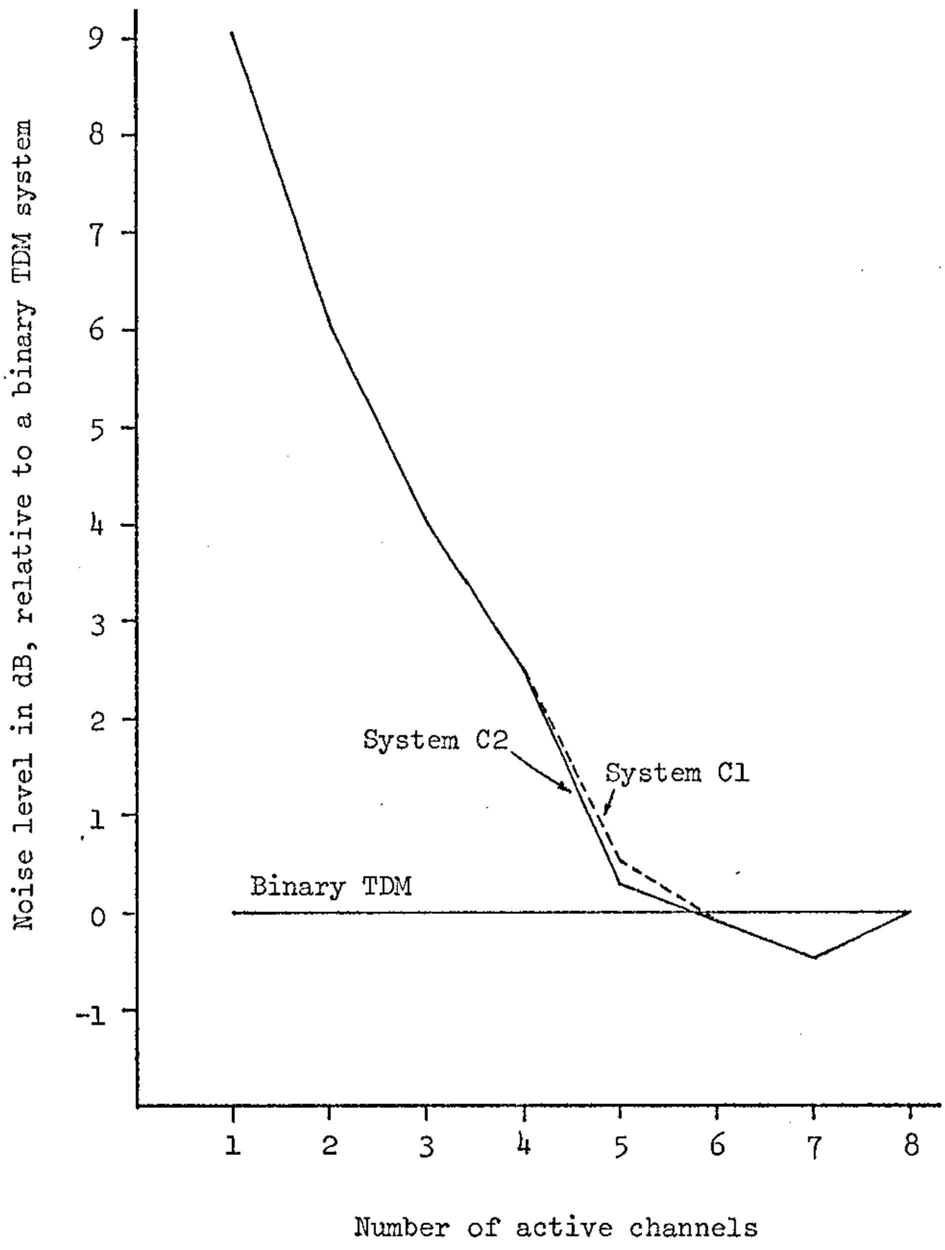


Figure 7.5-1 System C2. Noise level for an error probability per channel of 0.003, expressed in dB relative to a binary TDM system, for a varying number of active channels.

CHAPTER 8

DETECTION PROCESSES FOR THE MULTIPLEXING ARRANGEMENT D

8.1 Detection process 1

The optimum detection process described in Section 7.1, gives an upper performance bound to a system irrespective of the multiplexing arrangement. This detector minimises the probability of error (that is the probability of one or more element errors) in the detection of the m elements in a group, by determining which of the 2^m possible transmitted signal vectors {S} is at the minimum distance from the received vector R, in the n-dimensional Euclidean vector space containing R. System D1 uses this optimum detection process for the multiplexing arrangement D.

8.2 Results of computer simulation tests for System D1

The results of computer simulation tests (outlined in Section 7.2) are shown in Figure 8.2-1. The transmitted signal amplitude is given by ±1 or 0, so that it has the same maximum energy per component of the transmitted signal as a conventional binary TDM system with components equal to ±1. For 2,4 or 6 channels in operation, zero components in the transmitted signal cause a degradation of the system performance as shown in Figure 8.2-1, due to the reduced average energy per component of the transmitted signal.

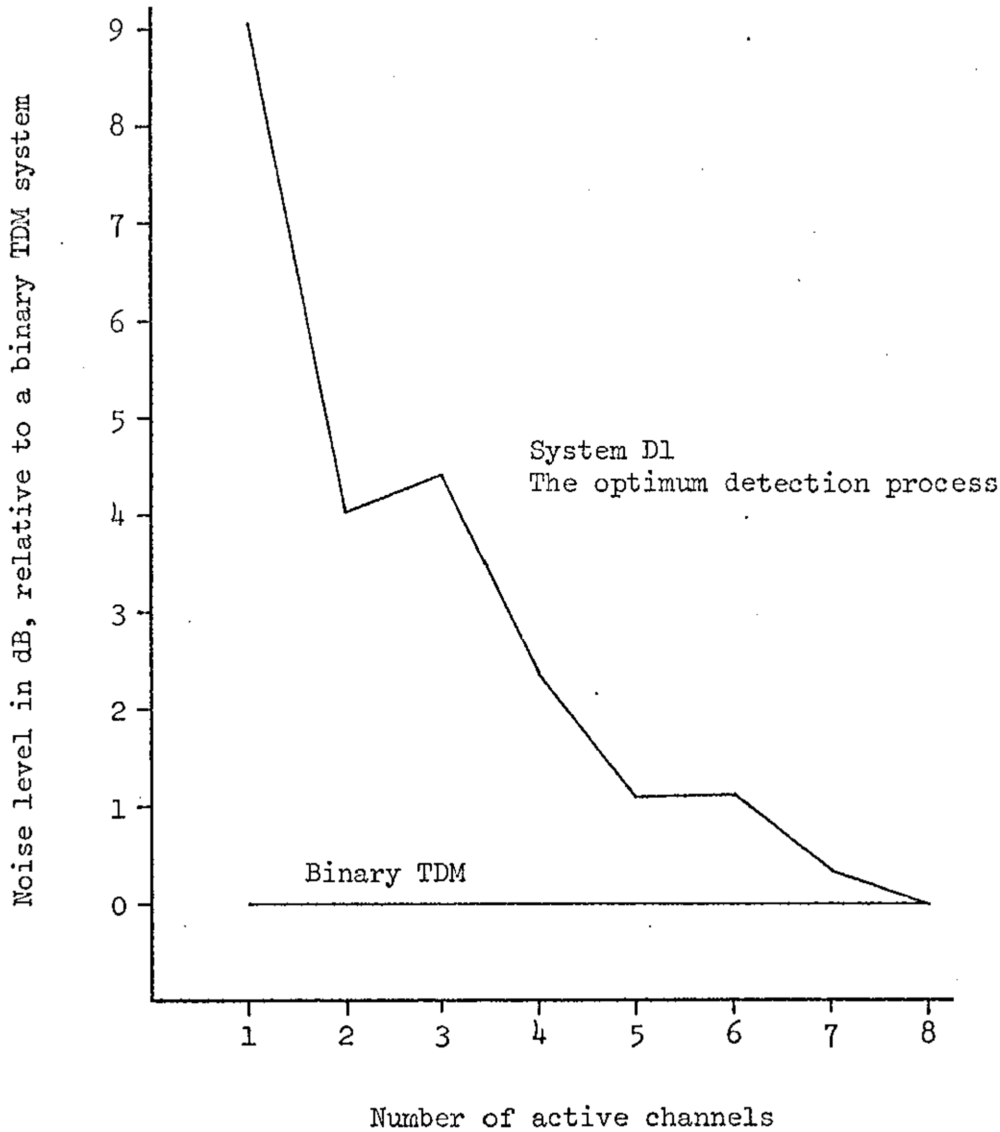


Figure 8.2-1 System D1. Noise level for an error probability per channel of 0.003, expressed in dB relative to a binary TDM system, for a varying number of active channels.

The theoretical optimum performance has been calculated by considering the distances in n-dimensional Euclidean vector space, between the 2^m signal vectors for m active channels, as described in Section 7.2.

Assuming independent errors, the 95% confidence limits in the value of p are given by,

$$\pm \frac{2}{\sqrt{e}} p \tag{8.2-1}$$

where the limits are expressed as deviation from the given value of element value error probability per channel p, and e is the total number of errors counted. In a test with orthogonal or quasi-orthogonal groups of signals there may be a degree of dependence between the individual element errors of a group in the detection process. The result of this dependence is to reduce the number of independent errors obtained in a test and so to widen the confidence limits. Thus e of (8.2-1) does not represent the effective number of errors, and therefore gives only an indication as to the confidence limits. To assess the degree of dependence between errors of the same received signal group, additional tests have been performed. Each individual test with m active channels was repeated several times using different random noise sequences. If the total number of errors counted for each test is $e_1, e_2 -- e_r$, for r successive tests, then the mean μ and standard deviation η of the total number of errors counted are given by,[80]

$$\mu = \frac{1}{r} \sum_{i=1}^{r} e_i \tag{8.2-2}$$

$$\eta = \left(\frac{1}{r-1} \sum_{i=1}^{r} (e_i - \mu)^2 \right)^{\frac{1}{2}} \tag{8.2-3}$$

From (3.5-2) the 95% confidence limits in the value of p are given by,

$$\pm \frac{2\eta}{\mu} \; p \tag{8.2-4}$$

where the limits are expressed as deviation from the given value of element value error probability per channel p. For one and eight active channels the signals are independent and the confidence limits given by (8.2-1) and (8.2-4) should agree. For several active channels some divergence is expected.

Table 8.2-1 summarises the theoretical and computer simulation results together with the confidence limits assuming independent and dependent errors. The confidence limits are expressed in decibels, as deviation from the value of σ, expressed in decibels relative to a conventional binary TDM system, with components ±1, and having the same transmission rate and error probability per channel.

Figure 8.2-2 shows theoretical curves of error probability per channel against signal/noise ratio expressed in decibels, for different numbers of multiplexed signals. For eight channels in operation the individual signal components are independent and equally likely to take either binary value, giving the familiar Q function curve (Appendix A1). Fewer channels in operation correspond to an appropriate sideways shift of this curve. Additional computer simulation tests, besides those for an error probability per channel of 0.003 give good agreement with the corresponding theoretical curves.

The number of signals multiplexed may exceed the maximum number of orthogonal multiplexed signals as described in Section 6.4. Computer simulation results for the multiplexing of two signal sets and using the optimum detection process, are shown in Figure 8.2-3. Also shown are the

Number of active channels	Noise level for an error probability per channel of 0.003, expressed in dB relative to a binary TDM system		95% confidence limits of σ expressed in dB as deviation from the given value of σ	
	Theoretical	Computer simulation	Independent errors $\frac{2}{\sqrt{e}}\,p$	Dependent errors $\frac{2\eta}{\mu}\,p$
1	9.03	9.05		+ 0.39 − 0.51
2	3.97	4.05		+ 0.52 − 0.78
3	5.03	4.40		+ 0.62 − 1.08
4	2.29	2.38	All approximately + 0.35 − 0.45	+ 0.37 − 0.50
5	1.34	1.10		+ 0.47 − 0.68
6	0.92	1.12		+ 0.42 − 0.55
7	0.27	0.37		+ 0.50 − 0.81
8	0.00	0.00		+ 0.32 − 0.41

Table 8.2-1 System D1. Theoretical and computer simulation results

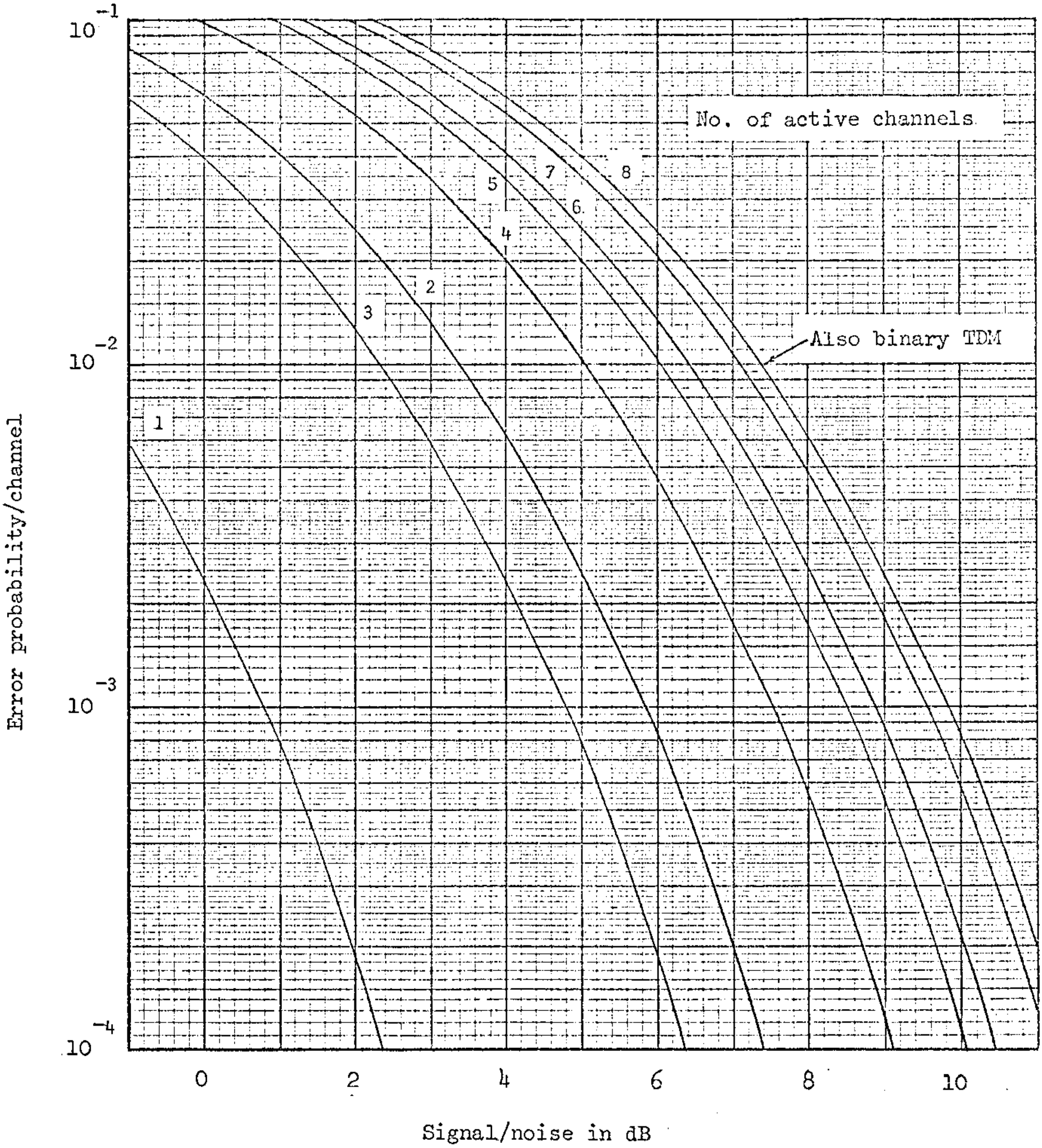


Figure 8.2-2 System D1. Theoretical error probability per channel against signal/noise ratio in decibels, for different numbers of multiplexed channels.

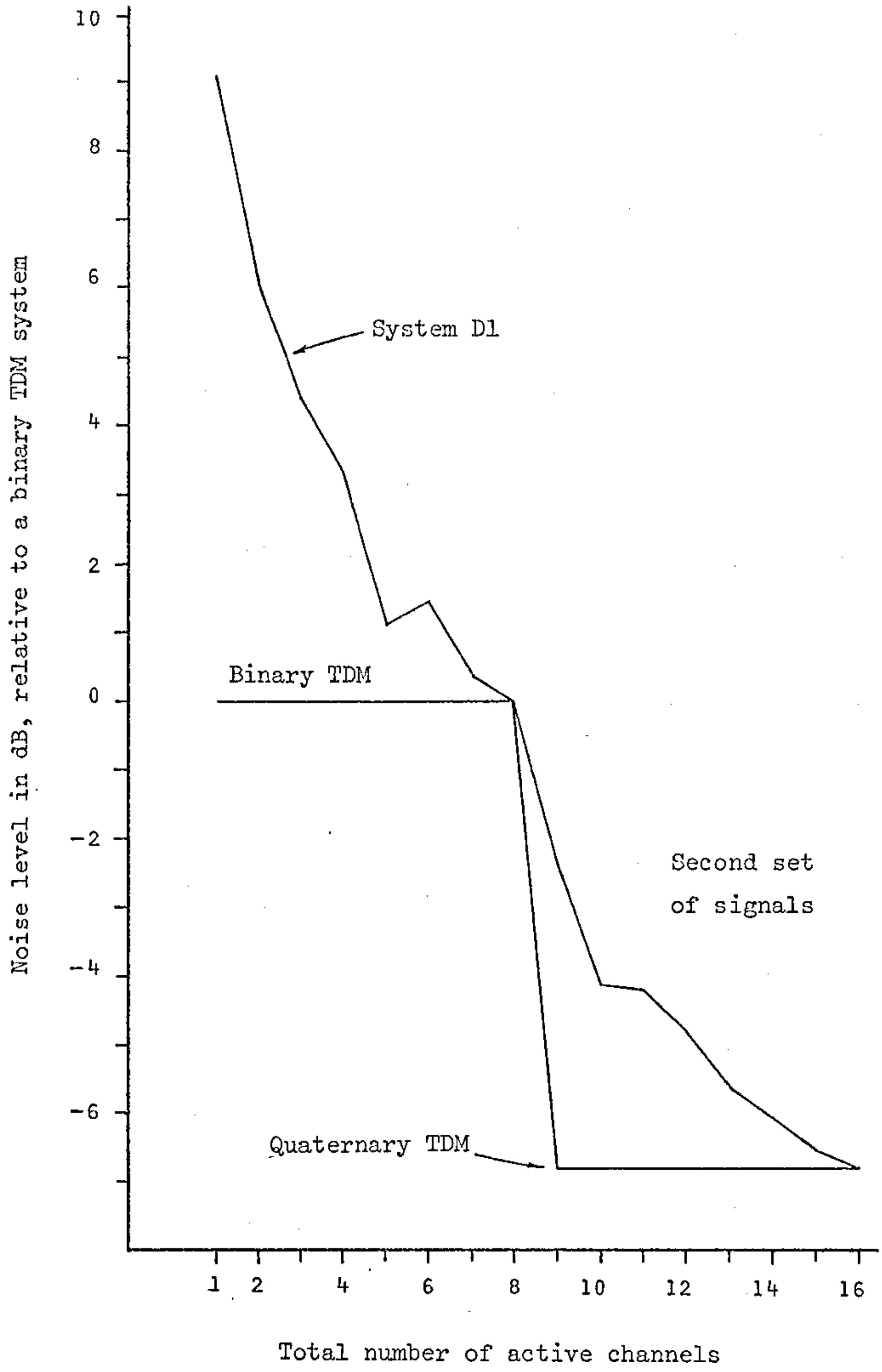


Figure 8.2-3 System D1. The multiplexing of two signal sets. Noise level for an error probability per channel of 0.003, expressed in dB relative to a binary TDM system, for a varying number of channels. The average energy per component of the transmitted signal is here normalised to unity.

corresponding TDM systems. Both binary and quaternary TDM systems here have the same average energy per component (equal to unity) of the transmitted signal, the same transmission rate and the same error probability per channel of 0.003 as explained in Section 3.4. The average energy per component of the transmitted signal of the system under test has been normalised to unity (Figure 8.2-3 only), so that it has the same average energy as a component in the TDM systems. Channels in the second set are not used until the first set channels are all in operation.

8.3 Detection process 2

The optimum detection process (Section 7.1) uses all possible transmitted signal vectors {S} in the detection of the m multiplexed element values in a group. The iterative detection process involves a vast number of sequential operations (2^m) which becomes impractical for values of m greater than about 8 to 10.

This null-zone detection process, System D2, requires substantially fewer sequential operations by applying the optimum detection process to a carefully selected subset of the total number of possible transmitted signal vectors.

The n-component transmitted signal vector S is composed of m binary independent components, and n-m ternary grey components, for m active channels. An independent component s_i is one which depends only on the element value of the signal in the ith channel so that $s_i = x_i a_{ii}$, s_i has no component from any other channels. However, in the detection of x_i, all the grey components are a contributing factor. The ith detected element value x_i' may be detected from the ith component of the received signal vector R alone, and is given by,

$$x_i' = a_{ii} \text{ sign } (r_i) \tag{8.3-1}$$

and provided the received component r_i is not corrupted in sign, correct detection results.

Consider a single received component r_i of the vector R where $r_i = s_i + w_i$, and w_i is a Gaussian noise sample of zero mean and variance σ^2. Figure 8.3-1 (a) shows the signal plus noise probability density functions, for a single component s_i which may take either binary value ±1 and are equiprobable. The probability of receiving r_i given s_i^+ is derivable from the curve $p(r_i/s_i^+)$. With a single decision boundary equidistant from s_i^+ and s_i^-, s_i is detected from the sign of r_i with a probability of error given by,

$$a = b = \int_1^\infty \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{r^2}{2\sigma^2}\right) dr$$

$$= Q\left(\frac{1}{\sigma}\right) \tag{8.3-2}$$

and will yield the best estimate of s_i provided no additional information from other components is available.

Figure 8.3-1 (b) shows the same probability density functions, but with two symmetrical decision boundaries or threshold levels separated by a null-zone of width 2d. If the ith received signal component of the vector R, r_i, is greater than +d, then s_i is detected as s_i^+, with as small an error probability as is required, depending on the value of threshold level d.

$$\alpha = \beta = \int_{1+d}^\infty \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-r^2}{2\sigma^2}\right) dr$$

$$= Q\left(\frac{1+d}{\sigma}\right) \tag{8.3-3}$$

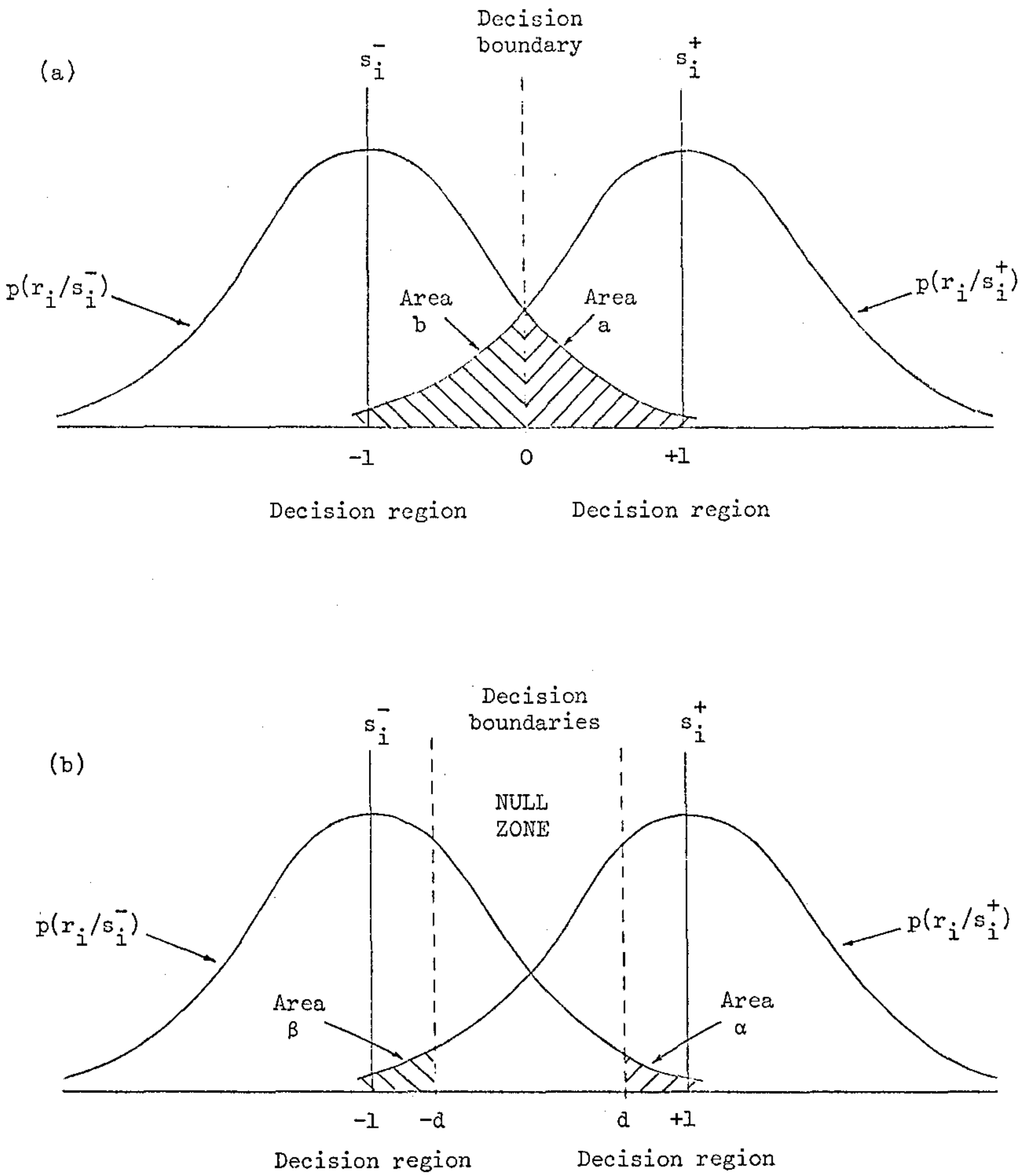


Figure 8.3-1 Signal plus noise probability density function for a single bipolar binary component. (a) A single decision boundary (b) Null zone detection involving two symmetrical decision boundaries.

In this detection process, if $|r_i| \geq d$ then x_i' is detected from the ith component of R,

$$x_i' = a_{ii} \text{ sign } (r_i) \qquad (8.3\text{-}4)$$

For those components $|r_i| < d$, an uncertainty exists as to whether s_i^- or s_i^+ was transmitted and the optimum detection process is applied to both combinations.

The receiver generates in turn each of the vectors {S} where S = signs (XA') from Eqn. (6.3-1), corresponding to the different combinations of element values, of the undetected signal elements $(|r_i| < d)$ in a group of received signals, the detected element values $(|r_i| \geq d)$ remaining unchanged. The distances between the vectors {S} and the received signal vector R are calculated iteratively as for the optimum detection process, and that having the minimum distance gives the detected element values $\{x_i'\}$ of the m signal elements in the received group.

The value of threshold level d is selected such that the probability of incorrectly detecting x_i', when $|r_i| \geq d$, is very small. This probability is $\alpha = \beta$ (Eqn. 8.3-3). As d increases so this probability decreases but the number of sequential operations required in the optimum detection process increases.

Thus the null-zone detector judiciously selects a small volume of the n-dimensional Euclidean vector space, having a very high probability of containing the vector R. The optimum detection process is then applied to these vectors {S} contained in this volume only.

Referring to Figure 8.3-1(b), the probability of detecting s_i as s_i^- when s_i^+ was transmitted is α, where,

$$\alpha = Q\left(\frac{1+d}{\sigma}\right) \tag{8.3-5}$$

The probability of r_i falling in the null-zone given s_i^+ or s_i^- is

$$|d| \leq 1 \quad p' = Q\left(\frac{1-d}{\sigma}\right) - \alpha \qquad\qquad |d| \geq 1 \quad p' = 1 - Q\left(\frac{d-1}{\sigma}\right) - \alpha$$

$$\simeq Q\left(\frac{1-d}{\sigma}\right) \qquad\qquad \simeq 1 - Q\left(\frac{d-1}{\sigma}\right) \tag{8.3-6}$$

The number of sequential operations required in the optimum detection process is,

$$2m(p')(1-p')^{m-1} + 4\frac{m(m-1)}{1.2}(p')^2(1-p')^{m-2} + 8\frac{m(m-1)(m-2)}{1.2.3}(p')^3(1-p')^{m-3} + \ldots \tag{8.3-7}$$

where each term of the binomial expansion corresponds to the probability of one, two .... components of the received signal falling in the null-zone.

For example, suppose the probability of s_i being detected incorrectly α by the null-zone is to be 1×10^{-6}. From Eqn. (8.3-5)

$$1 \times 10^{-6} = Q\left(\frac{1+d}{\sigma}\right)$$

$$\frac{1+d}{\sigma} = 4.72$$

Suppose now that the additive white Gaussian noise samples have variance 0.09 ($\alpha = 0.3$), then, the threshold level is,

$$d = 4.72\,\sigma - 1 = 0.41$$

From Eqn. (8.3-6), the probability of a received component r_i falling in the null-zone, $|r_i| < 0.41$ is,

$$p' = Q\left(\frac{1-d}{\sigma}\right)$$

$$= Q\left(\frac{0.59}{0.3}\right) \simeq 0.025$$

If there are six channels in operation say, instead of performing $2^6 = 64$ sequential operations using the optimum detection process, this null-zone detector would require on average, from Eqn. (8.3-7), 0.28 sequential operations. A phenomenal saving in time for no significant degradation in the detector performance.

At maximum capacity with all n channels in operation, there are no grey components in the transmitted signal vector, and no advantage is gained by using the optimum detector. The element values are given by the signs of the received components of R from Eqn. (8.3-4).

8.4 Results of computer simulation tests for System D2

The results of computer simulation tests (outlined in Section 7.2) are shown in Figure 8.4-1, for different values of α, which determines the threshold level d, and the subsequent number of sequential operations required in the detection of the m element values of a signal group. For $\alpha = 10^{-6}$, the majority of the received components $\{r_i\}$ of the received signal vector R fall within the null-zone. Those independent components of R falling outside the null-zone are detected with a very low probability of error, whilst those within cause the detector to consider both the +ve and the -ve possibilities of element values in the ensuing optimum detection process. Consequently, a comparatively large number of sequential operations are required. For $\alpha = 10^{-4}$, more components $\{r_i\}$ lie outside the null-zone, producing a reduction in the number of sequential operations required. Figure 8.4-2 shows the average number of sequential operations required for the various values of α, for different numbers of active channels, the results being obtained from computer simulation tests. Figure 8.4-3 shows theoretical curves for the number of sequential operations required for

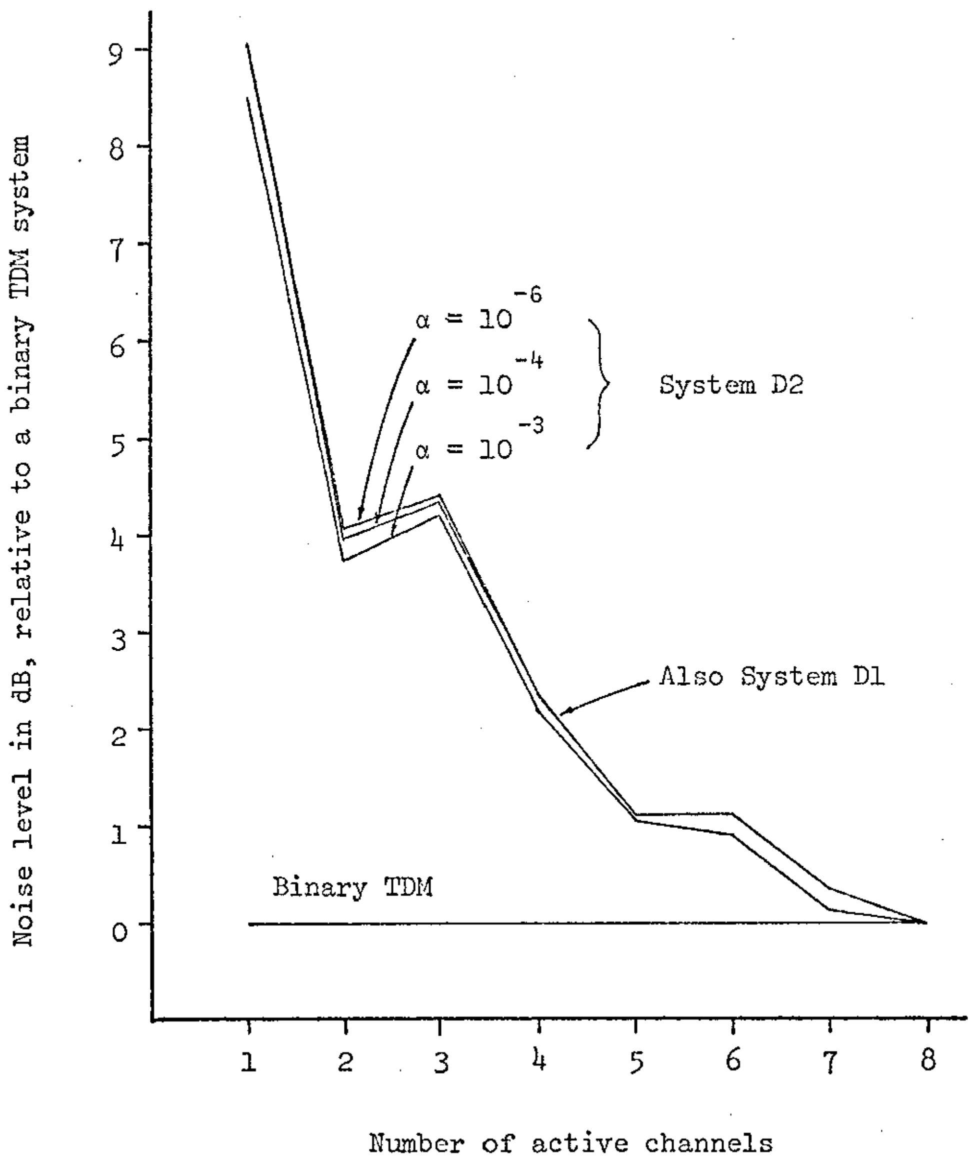


Figure 8.4-1 System D2. Noise level for an error probability per channel of 0.003, expressed in decibels relative to a binary TDM, for various values of α.

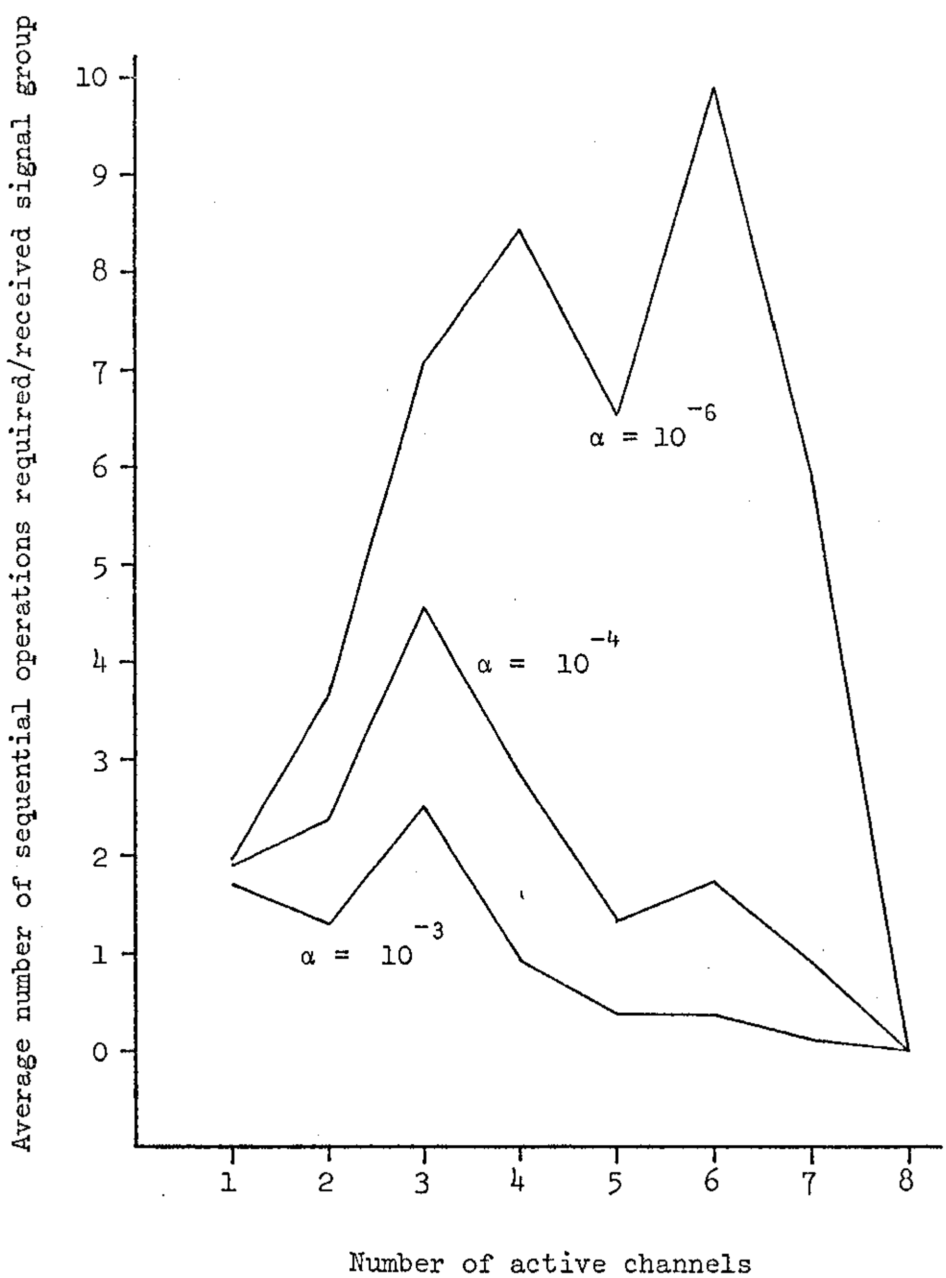


Figure 8.4-2 System D2. Average number of sequential operations required per signal group for various values of α, for an error probability per channel of 0.003.

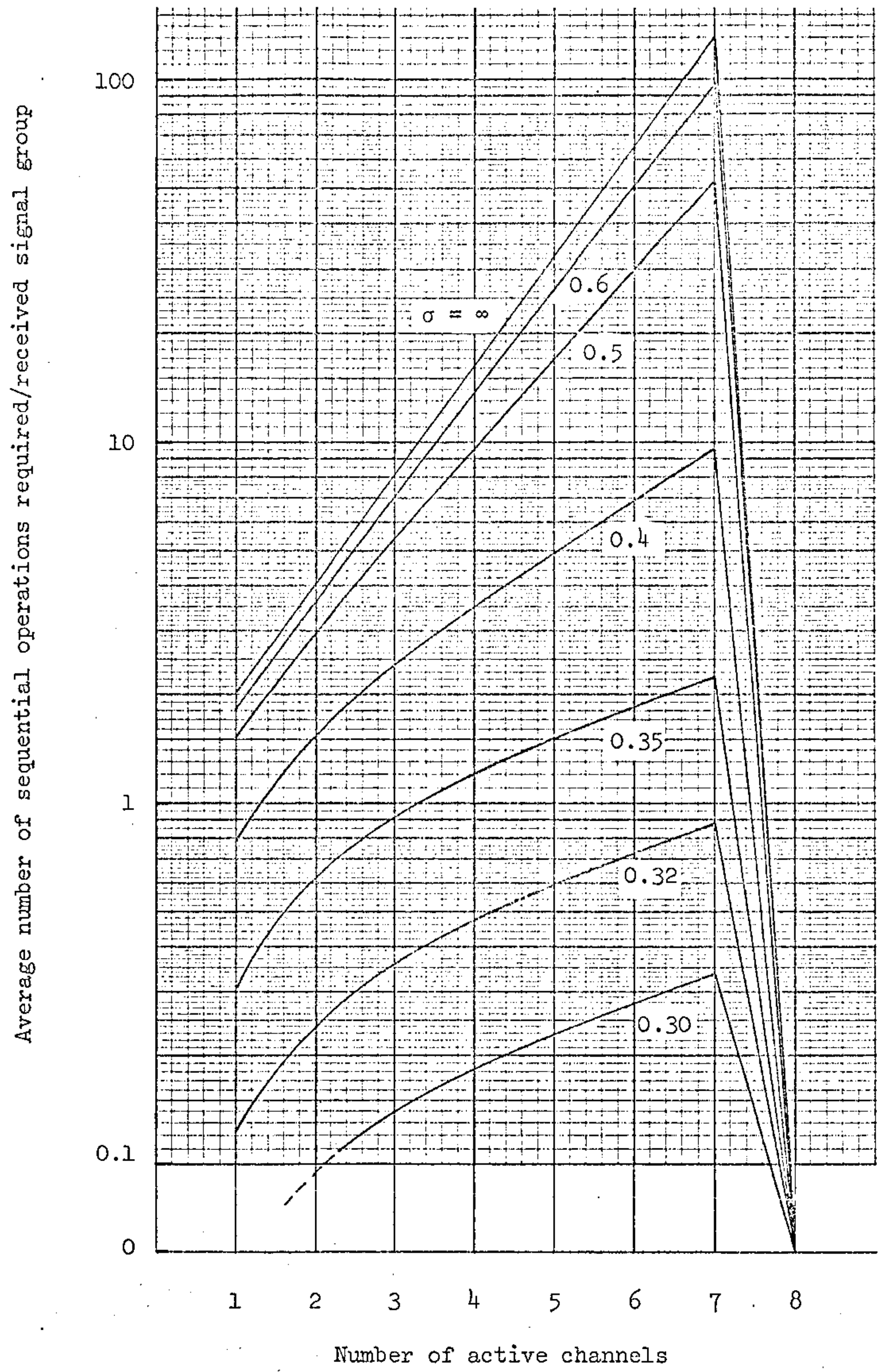


Figure 8.4-3 System D2. Average number of sequential operations required per received signal group, for various noise levels for $\alpha = 10^{-6}$.

various noise levels using Eqn. (8.3-7) and for $\alpha = 10^{-6}$. The corresponding computer simulation tests show good agreement. For 8 active channels the element values are given by the signs of the received components of R. For $\sigma = \infty$, all received components $\{r_i\}$ of the received signal vector R lie within an infinitely wide null-zone, and the optimum detection process is applied to all independent components. The number of sequential operations is therefore 2^m.

8.5 Detection process 3

This detection process examines the n-component received vector R from the aspect of minimisation of the channel noise vector W, where

$$R = S + W$$

Assuming that the n-component transmitted vector S has components $\{s_i\}$ given by ±1, an estimate S' of S which minimises the length (Euclidean norm) of the noise vector W is given by,

$$S' = \text{signs } (R) \qquad (8.5\text{-}1)$$

where the operator "signs" replaces each term of the vector R by ±1, the selected sign being the same as that of the components of R.
Provided S' contains a valid combination of components $\{s_i\}$ corresponding to the m channels in operation, the m element values $\{x_i\}$ associated with this estimate are accepted as the detected element values. A valid combination check is made as follows. The m element values $\{x_i'\}$ of the vector X' are given by the independent component of R,

$$\begin{aligned} x_i' &= a_{ii} \text{ sign } (r_i) \\ &= a_{ii}\, s_i' \end{aligned} \qquad (8.5\text{-}2)$$

and from these m element values, the detector generates an estimate of the transmitted signal vector. Let this be the n-component vector S''.
From Eqn. (6.3-1),

$$S'' = \text{signs}\ (X'A') \tag{8.5-3}$$

where A' are the modified codewords corresponding to the m active channels. If the signs of the n components of S'' agree with the corresponding n components of S', then S' is a valid combination and the $m\{x_i\}$ are accepted. If however, any component signs disagree, the components of the estimated vector S' do not form a valid combination. A new estimate is made from R by changing the sign of the component s_i' of S' corresponding to the smallest value (modulus) r_i of R. The length of the noise vector W is thus increased by a minimum amount. From this new estimate S' a new set of m element values are given by Eqn. (8.5-2) and the detector proceeds as previously.
If a valid combination of the components $\{s_i'\}$ is not found, the estimate S' is changed again, such that the noise vector length is increased by a minimum amount, by changing the sign of the component s_i' corresponding to the second smallest component value r_i of R. This process continues until a valid combination of the $\{s_i'\}$ is found.

Provided the noise vector length increases incrementally by a minimum amount, the detector minimises the probability of error in the detection of the m element values of a group. However, modifying S' such that the noise vector increases by a minimum amount involves time and equipment complexity approaching that of the optimum dection process. As a compromise, the components of the received signal vector $\{r_i\}$ are assigned an order according to their absolute value. The components of S' are changed in sign, in a binary sequence, the smallest weight corresponding to the smallest absolute value of r_i. For example, if the assigned absolute value order of the components of R is, a,b,c,d _ _ n, then the component s_i' corresponding to a is first changed, then b, followed by a and b, followed by d, etc. This does not increase the noise vector by a minimum amount always, but is a fairly close approximation.

Because components of the transmitted signal vector R may equal zero for an even number of active channels, Eqn. (8.5-1) is clearly only valid for an odd number of active channels. For an even number, a component s_i'' may equal zero, in which case a comparison between S'' and S' is only made for those components $\{s_i''\}$ for which $s_i'' = \pm 1$.

8.6 Results of computer simulation tests for System D3

The results of computer simulation tests (outlined in Section 7.2) are shown in Figure 8.6-1. For $m = 3,5,7$ or 8, the detector gives optimum performance with very few sequential operations, on average between 1 and 2. For $m = 2,4$ or 6, zero components of the internally generated transmitted signal S'' are omitted in the comparison process and well below the optimum performance results.

For $m = 1$, an error probability of 0.003 requires a large value of additive white noise. An average of 5 sequential operations are necessary, giving below optimum performance, because of the approximate incrementation process of the minimum noise vector. For lower noise levels (lower error probability per channel) $m = 1$ gives optimum performance.

8.7 Detection process 4

Like the null-zone detection process, this detector applies the optimum detection process to a carefully selected subset of the total number of possible transmitted signal vectors, thereby reducing the number of sequential operations required. The subset selection uses different criteria,

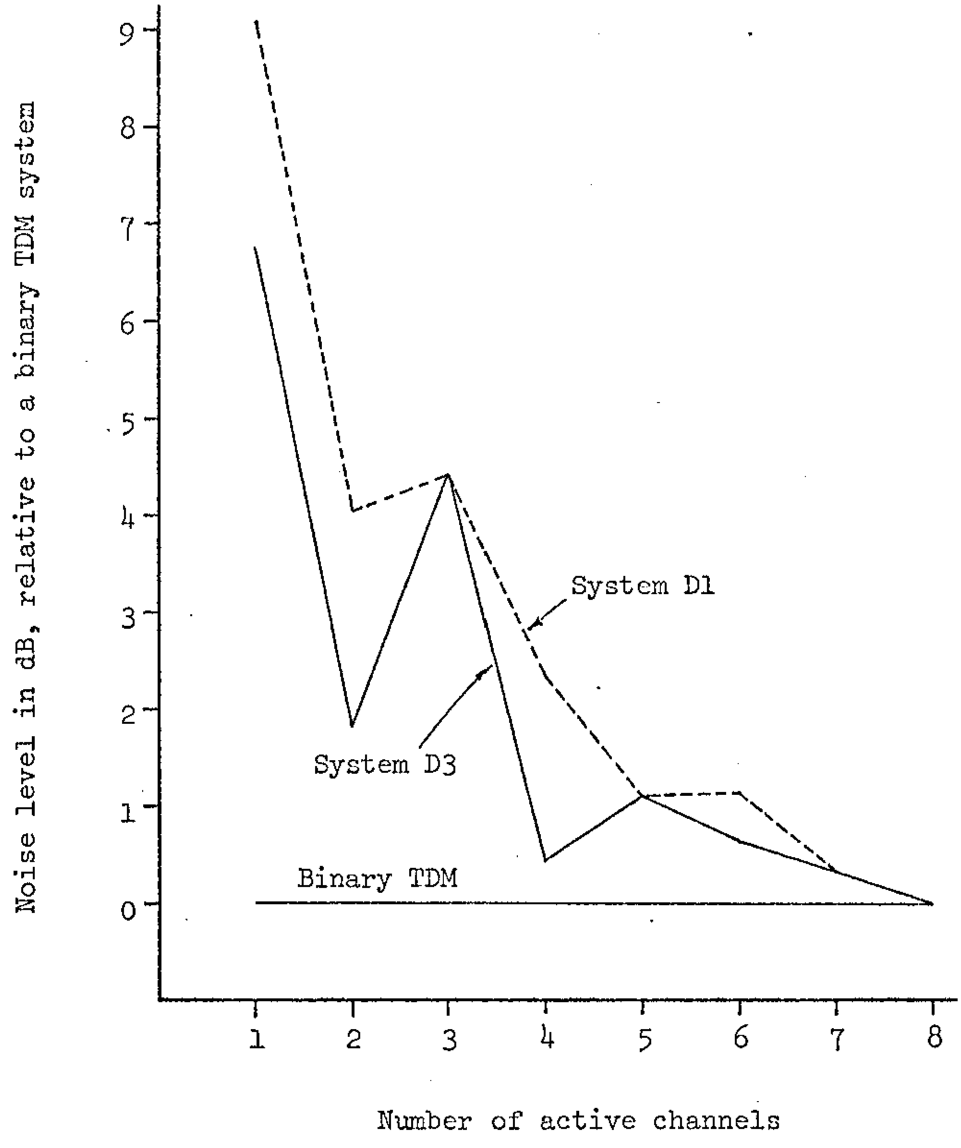


Figure 8.6-1 System D3. Noise level for an error probability per channel of 0.003, expressed in dB relative to a binary TDM system, for a varying number of active channels.

and relies on no more than two independent components of the n-component received vector R being corrupted in sign.

The equipment complexity is simplified by replacing the optimum 'distance' detection process by the optimum 'correlation' detection process. Again, this minimises the probability of error (that is the probability of one or more channel errors) in the detection of the m elements in a group. The detector generates sequentially the most likely transmitted signal vectors $\{S\}$, correlating each one with the received vector R, and determining that vector S corresponds with the largest correlation coefficient. The vector X' of element values associated with this S gives the m detected element values $\{x_i'\}$.

The detection process starts by making an initial estimate of the m element values. Let this estimate be the n-component vector X', whose ith component is x_i', as is determined as follows. For each i, corresponding to a channel in operation,

$$x_i' = a_{ii} \text{ sign } (r_i) \tag{8.7-1}$$

so that x_i' is determined from the sign of r_i to give the initial detected values of $\{x_i'\}$.

The detected element values $\{x_i'\}$ are fed to a multiplexer, identical to the one used at the transmitter, to generate an n-component vector S', which is an estimate of the original transmitted signal.

The inner product of the vectors R and S' is now formed by means of a correlator which multiplies the jth component of R by the jth component of S' for $j = 1 - - n$, and adds the product to give the output signal c_o,

$$c_o = \sum_{j=1}^{n} r_j s_j' \tag{8.7-2}$$

which is stored.

The first non-zero component of X' is now changed in sign, and the vector used as above to generate a new n-component vector S'. The inner product of this vector with R has the value c_1. Its value is compared with c_o and if larger, the value c_1 is stored with the corresponding modified vector X'.

The first non-zero component of X' is now changed back in sign, and the second non-zero component of X' is now changed in sign, and the inner product of S' and R is formed as before to give c_2. This is compared with the previously stored value of c_i, and if it is larger, it replaces c_i and the new vector X' replaces that stored.

This continues until all m non-zero components of the n-component vector X' have been changed successively. The resultant stored vector X' gives the estimate of the element values obtained in the first part of the detection process.

The stored vector X' is now processed as was the first estimate X', each component being changed in turn. At the end of this process, the resultant stored vector X' gives the final detected values of the $m\{x_i'\}$

No improvement in tolerance to noise, at high signal/noise ratios has been found by repeating this process.

The correlation process given by Eqn. (8.7-2) has been simplified for clarity, and is only valid when the components $\{s_j'\}$ are $\pm$ 1. However, for an even number of channels in operation, the grey components of S may contain zeros causing a decrease in the value of c_i.

From Eqn. (7.1-1), the optimum detection process yields the square of the distance between the n-component signal vector R, and the n-component generated vector S'.

$$d^2 = \sum_{j=1}^{n} (r_j - s_j')^2 \qquad (8.7\text{-}3)$$

$$= \sum_{j=1}^{n} r_j^2 + s_j'^2 - 2r_j s_j'$$

For a given R

$$d^2 = 2\,(k + \tfrac{1}{2} \sum_{j=1}^{n} s_j'^2 - \sum_{j=1}^{n} r_j s_j') \qquad (8.7\text{-}4)$$

where $k = \sum_{j=1}^{n} r_j^2$

From Eqn. (8.7-2) the optimum correlation detection process yields the correlation coefficient,

$$c_i = \sum_{j=1}^{n} r_j s_j' \qquad (8.7\text{-}5)$$

Because both detection processes give optimum performance in terms of tolerance to additive white Gaussian noise, the vector S' corresponding to the minimum distance, is the same vector that corresponds to the maximum correlation coefficient. However, if now the jth component of the n-component vector S' is zero, c_i will decrease by $r_j s_j'$. The distance measure increases by $r_j s_j'$, but also decreases by $\frac{1}{2}$ has there are now only n-1 components of S' equal to ±1. Therefore, for d^2 and c_i to remain in proportion, the value of $\frac{1}{2}$ must be added to the value of c_i obtained, for every zero component in the generated vector S'. For the components of the vector S' equal to ±1 or 0, Eqn. (8.7-2) becomes,

$$c_i = \sum_{j=1}^{n} r_j s_j' + \tfrac{1}{2}\,(\text{No. of zero components in } S') \qquad (8.7\text{-}6)$$

and the correlation process gives optimum performance despite the presence of zero components of the transmitted signal vector S.

8.8 Results of computer simulation tests for System D4

The results of computer simulation tests (outlined in Section 7.2) are shown in Figure 8.8-1. For an error probability per channel of 0.003, optimum performance is obtained, except when $m = 3$. Occasionally all three independent components of the n-component received signal vector R are corrupted in sign, but a maximum of two components may be corrected only. For $m = 1$ or 2, then even if both independent components are corrupted in sign, both may be corrected and optimum performance results.
For $m = 4,5 -- 8$, the reduced variance of the additive white Gaussian noise samples does not cause more than two independent components of R to be changed in sign, and again optimum performance results. At higher signal/noise ratios $m = 3$ also gives optimum performance. $2m + 1$ sequential operations are required irrespective of the additive white Gaussian noise level, which corresponds to the two sign changing processes of m independent components, plus another operation at the outset, when no components of the received vector R are changed in sign. Figure 8.8-2 shows the performance of an identical system for a group length $n = 16$. The optimum detection process requires 2^{16} sequential operations which is prohibitive both in practice and computer simulation tests. The results are shown relative to a conventional binary TDM system, with the same transmission rate and error probability per channel, and has the same maximum energy per component of the transmitted signal.

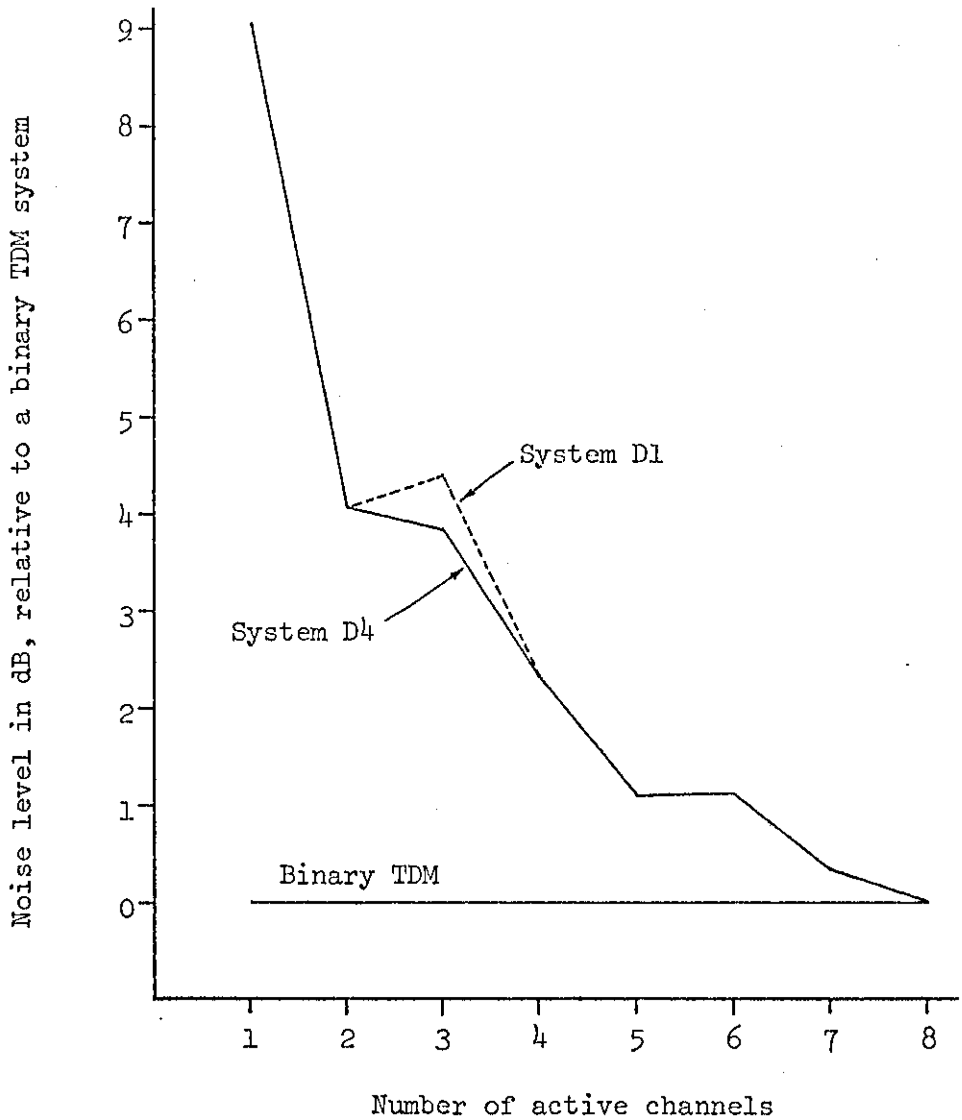


Figure 8.8-1 System D4. Noise level for an error probability per channel of 0.003, expressed in dB relative to a binary TDM system, for a varying number of active channels.

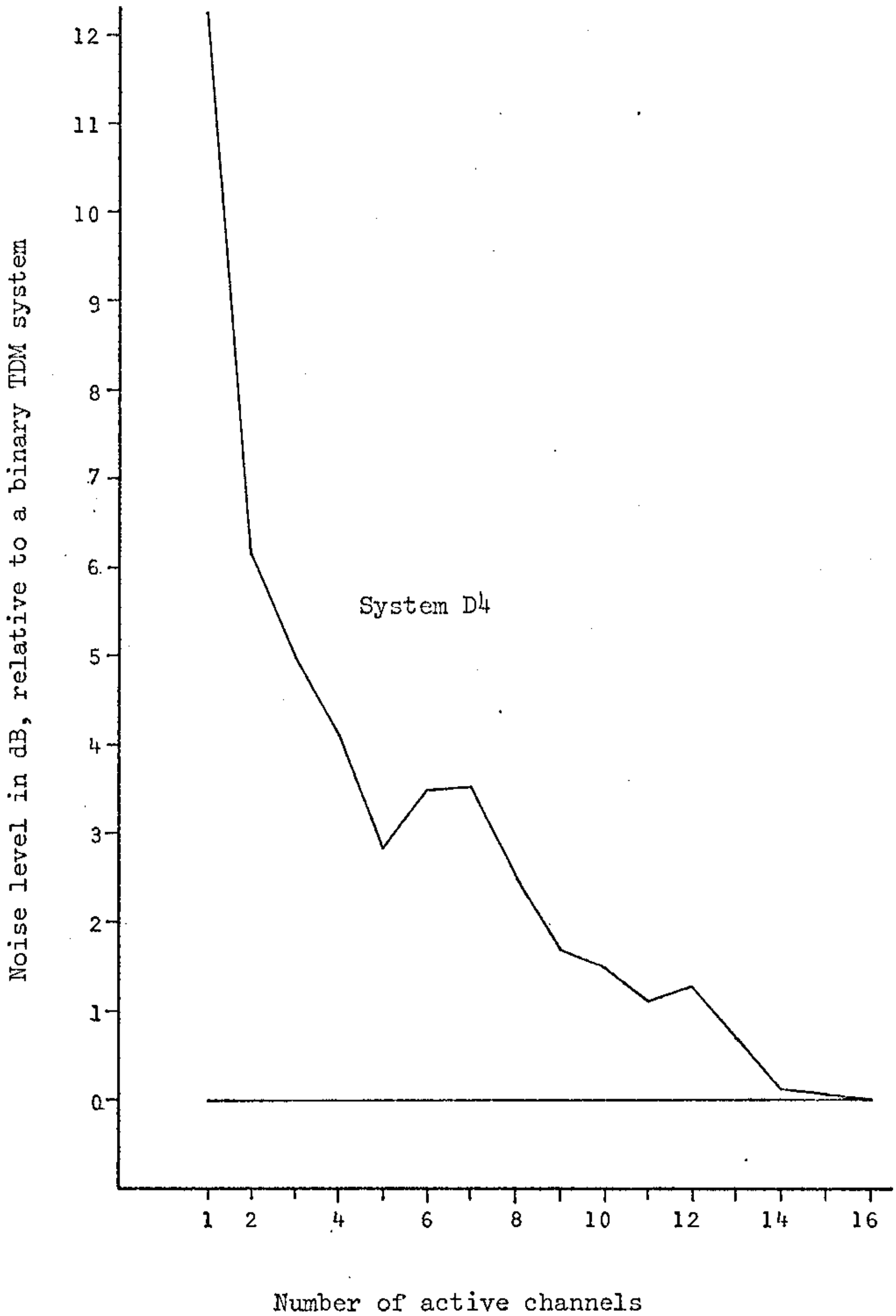


Figure 8.8-2 System D4, n = 16. Noise level for an error probability per channel of 0.003, expressed in dB relative to binary TDM system, for a varying number of active channels.

8.9 Assessment of the Systems D1 to D4

System D1 using the optimum detection process, indicates clearly the advantage of the multiplexing arrangement D in terms of additive white Gaussian noise performance, in so much that irrespective of the number of channels in operation, the performance is always equal or superior to that of the corresponding conventional TDM system, having the same maximum energy per component of the transmitted signal. Regarding demultiplexing, the optimum detection process of System D1 requires 2^m sequential operations which is prohibitive for m greater than about 8 to 10. System D3 is unsuitable due to non-optimum performance for 2,4 and 6 active channels. The demultiplexing arrangements of System D2 and D4 require far fewer sequential operations, but slightly greater equipment complexity for the subset selection of the possible transmitted signal vectors. System D2 requires a measure of the average noise vector length to determine the null-zone detector threshold levels, whereas System D4 functions optimally at high signal/noise ratios, irrespective of the additive noise level. System D4 is therefore the best overall arrangement.

CHAPTER 9

HARDWARE MODEL FOR SYSTEM D4

9.1 Introduction

A hardware model of System D4 has been designed and constructed in order to focus attention on the practical realisation and economic aspects of a multiplex system that has hitherto been tested by computer simulation only. Also, from a personal viewpoint it was considered a valuable experience.

Unlike computer simulation tests where computer time is severely limited, the hardware model enables a large number of errors to be counted, and measurements to be taken at high signal/noise ratios (low probability of error).

Figure 9.1-1 shows a simplified block diagram of the hardware model. The data signals consisting of binary element values corresponding to the m active channels, arrive in element synchronism at the multiplexer where the first n Walsh functions are generated. These are combined using the non-linear multiplexing arrangement D to form the resultant data signal which is transmitted over the duration of the following element period. Bandlimited white Gaussian noise is introduced into the transmitted path, and at the receiver input, the signal/noise ratio is measured, the signal and noise energies being measured separately with the other removed. The received signal is sampled n times per element period, and whilst one store holds the n samples for a detection process, another store is receiving the next n samples. The transmitted element values are

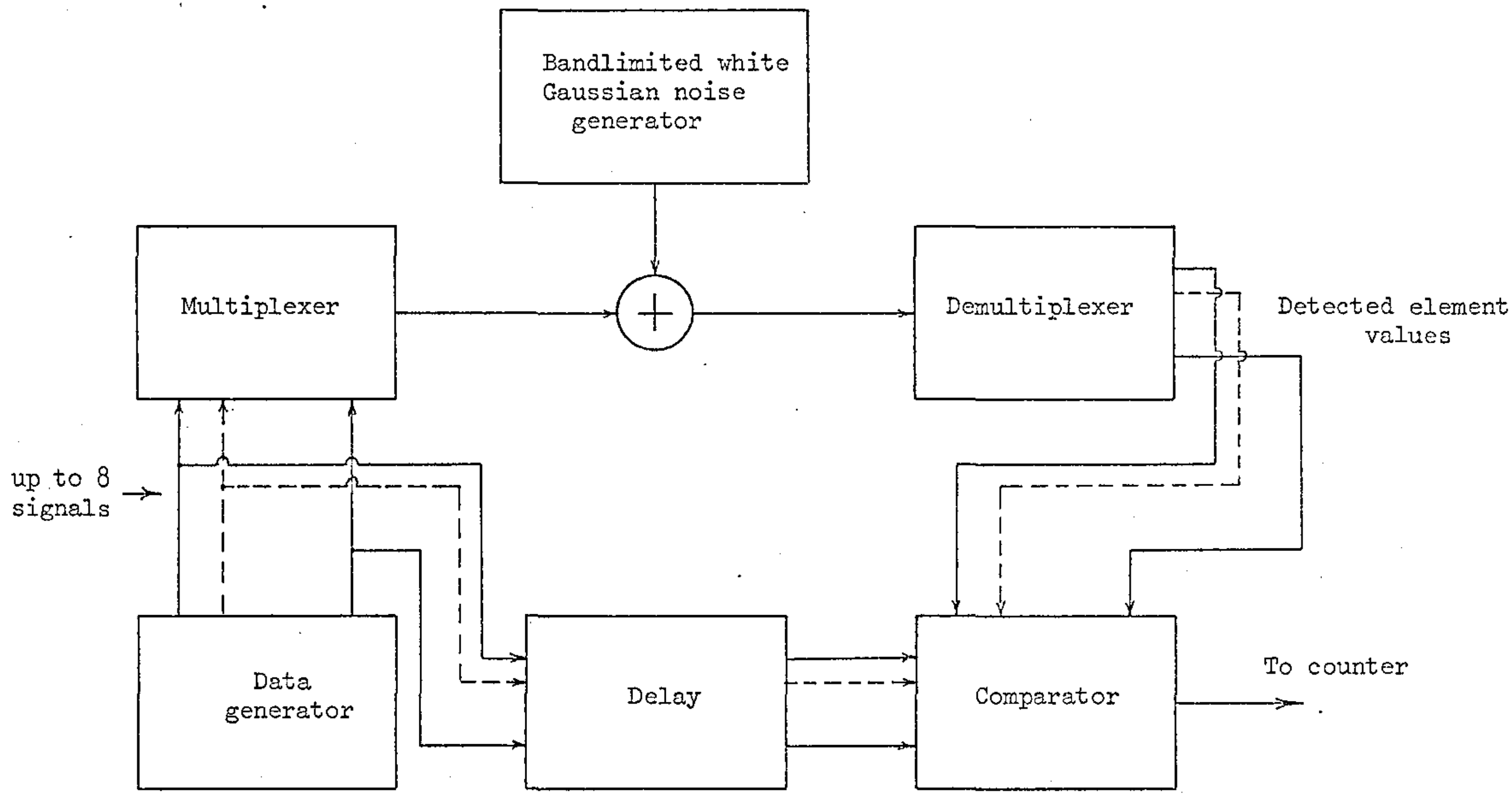


Figure 9.1-1 Simplified block diagram of the hardware model

therefore delayed by two element periods before a comparison with the detected element values. Discrepancies in sign are counted for a given number of transmitted signal groups.

9.2 Description of equipment

A transmission rate of 2400 bauds was chosen for compatability with existing equipment using the local subsciber network,[16] which gives an element rate of 300 bauds for each of the eight individual data sources. Readily available 74 TTL series integrated circuits were chosen, there being no special circuit requirements in terms of speed or power consumption. Before the detailed design, several arrangements varying in complexity and cost were investigated, the total estimated cost of the final arrangement being about £150.

The hardware block diagram is shown in Figure 9.2-1, and detailed circuits are given in Appendix A3. Due to the lack of time, the hardware model transmission system (Figure 2.1-1) has by necessity, been simplified. The transmitter and receiver filters are omitted and the transmitted signal consists of square pulses to which bandlimited Gaussian noise is added. This is sampled at regular intervals of T seconds. In a practical system the received signal would be integrated over the interval of T seconds before sampling. The hardware model performance is therefore compared relative to the corresponding TDM system using square pulses.

The element values corresponding to the eight multiplexed channels are generated in a pseudo random fashion, using a nine-stage shift register with modulo-2 addition feedback. The element values are thus changed each element period. Alternatively, the element values may be selected manually, a particularly useful facility during the initial testing phase. The channels in operation are also selected manually.

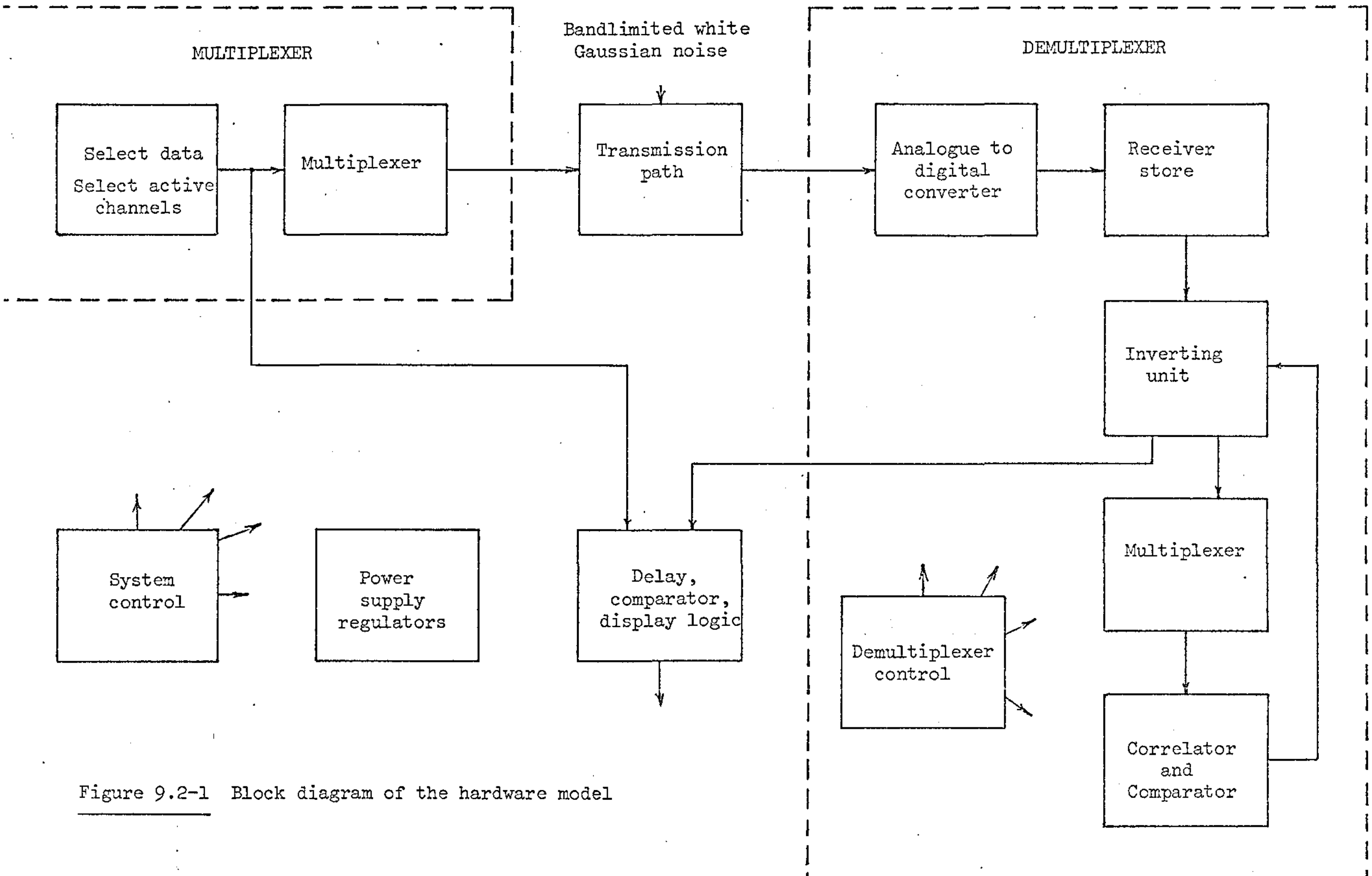


Figure 9.2-1 Block diagram of the hardware model

The first eight Walsh functions, generated in the multiplexer by a diode matrix, are modulated by the channel element values using exclusive OR gates. The majority logic function is conveniently implemented with tri-state logic devices, whose summed outputs form an analogue signal. A pair of voltage comparators determine whether the transmitted signal is ±1 or 0.

White Gaussian wideband noise from a commercial instrument is bandlimited to 15 KHz, and added via a precision potentiometer to the resultant transmitted signal. Because the analogue to digital converter used does for simplicity not contain a sample and hold circuit, any wider bandwidth causes malfunctioning of the converter. With the potentiometer set at maximum, the noise level is adjusted, such that the same reading is obtained as the transmitted signal, when connected to a thermocouple type electronic voltmeter. The ratio of the signal and noise energies is thus 0 dB, which may be accurately increased to any desired value with the precision potentiometer.

The received signal passes to a ramp type analogue to digital converter giving a 4 bit output (3 bits + sign), a total of 15 distinct levels. Computer simulation model tests using 2,3,4 and 5-bit converters on the received signal, show that at high signal/noise ratios, no significant advantage results with more than 4 bits. The received analogue signal is quantised at the mid point of each received digit. Whilst one store holds eight 4-bit words for a detection process, another store is receiving the next eight words.

Referring to Figure 9.2-1, the inverting unit estimates the m multiplexed element values from the signs of the eight word samples. This first estimate is fed to a multiplexer, identical to the one used at the transmitter, to generate an eight component ternary signal which is an estimate of the original transmitted signal.

The correlator forms the inner product of the estimated transmitted signal and that actually received, multiplying the corresponding components of each signal and adding the products to give the output signal c_o. As explained in Section 8.7, a zero component of the estimated transmitted signal requires that the value $\frac{1}{2}$ be added in place of the product term.

The inverting unit now changes the sign of the first estimated element value, and the multiplexing and correlation procedure described generates another output signal c_1. This is compared with the previous value c_o, and if it is larger, replaces it, and the corresponding estimated element values are recorded in the inverting unit.

The process is repeated until all estimated element values have been changed in sign. The largest value of c_i stored, corresponds to the m element values obtained in the first part of the detection process.

These estimated element values are now processed as were the first estimates, each component being changed in turn. At the end of this process, the resultant stored element values give the final detected element values.

The transmitted element values, delayed by two element periods, are compared in sign with the detected element values, and the number of discrepancies counted using a commercial instrument. This proceeds for a given number of transmitted signal groups.

9.3 Tests performed

Prior to each test the Gaussian noise level was adjusted as described previously, to compare its energy with that of the transmitted signal using a thermocouple type instrument. With additive white Gaussian noise even a very small increase in the noise level produces a considerable increase in

the corresponding probability of error (Appendix A1), and because of this, the level was checked regularly for possible drift. A second source of error, the analogue to digital converter at the receiver input was also checked at intervals. For a given number of active channels, the noise level was adjusted in one dB steps to obtain an error probability/channel of between 0.0001 and 0.01. The corresponding total number of errors counted was between 30 and 10000 for 100000 groups transmitted.
The tests were repeated several times using different channel selections, and the total number of errors counted averaged. Figure 9.3-1 shows the results obtained, of error probability/channel against signal/noise ratio in dB, for a varying number of multiplexed channels. Also shown is the performance of the corresponding binary TDM system whose maximum component energy is the same as that of the system under test.

The overall system complexity has made an exhaustive testing of the equipment almost impossible. The digital circuitry was tested with given data selections and channels in operation, but without noise. The analogue transmission path and analogue to digital converter were checked regularly to prevent drifting. The confidence limits of the results are therefore uncertain, but ignoring inaccuracies due to the equipment, the 95% confidence limits are given by,

$$\pm\, p \, \frac{2}{\sqrt{e}} \qquad (9.4\text{-}1)$$

where the limits are expressed as deviation from the given value of error probability p. The total number of errors counted in a test is e.

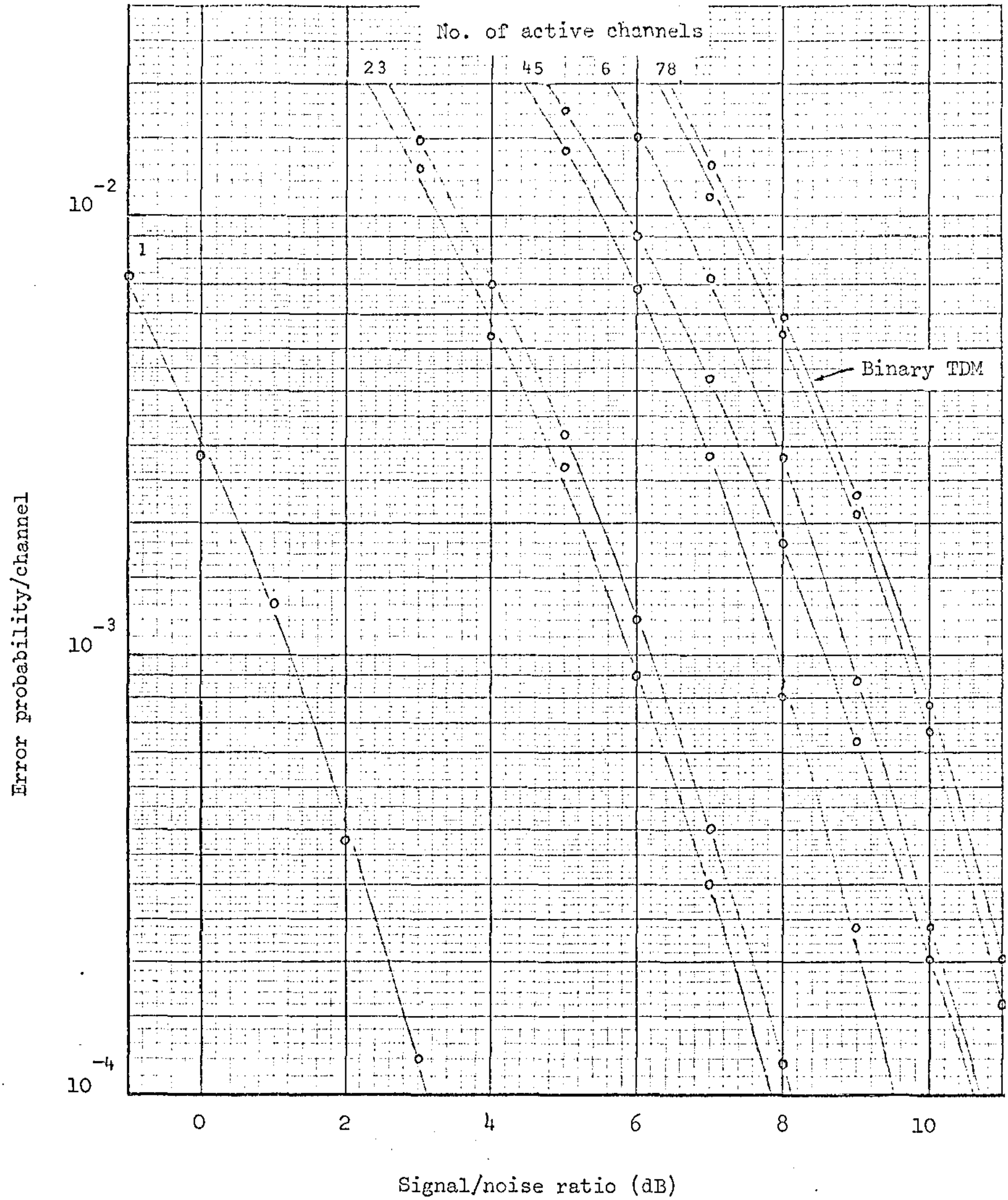


Figure 9.3-1 Error probability per channel against signal/noise ratio in decibels, for different numbers of multiplexed channels.

The confidence limits may be summarised as follows,

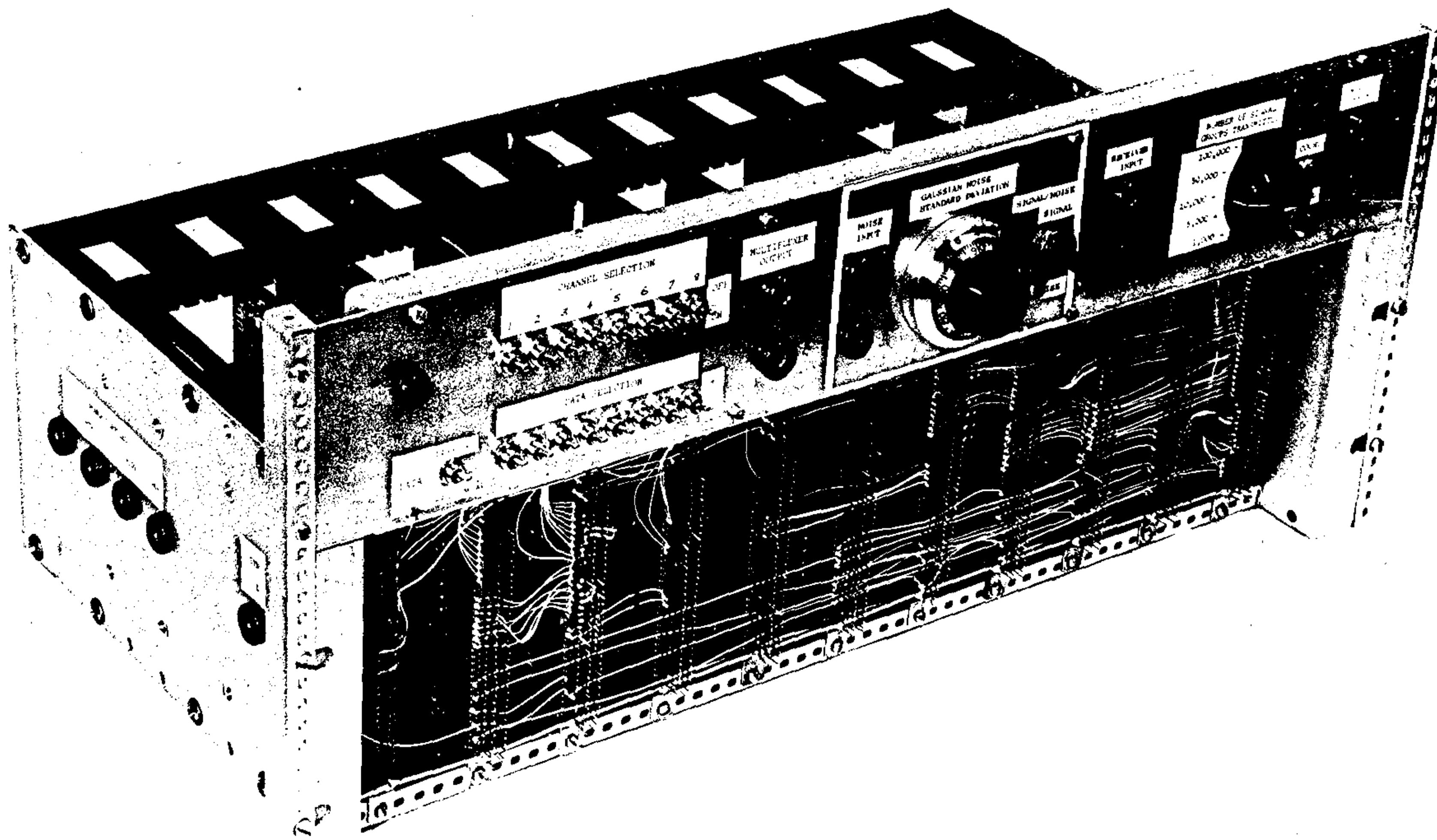
No. of active channels	95% confidence limits expressed as deviation from the value of error probability.		
	Probability of error/channel		
	1×10^{-2}	1×10^{-3}	1×10^{-4}
1	± .00063	± .00020	± .000063
2	± .00045	± .00014	± .000045
3	± .00037	± .00012	± .000037
4	± .00032	± .00010	± .000032
5	± .00028	± .000090	± .000028
6	± .00026	± .000082	± .000026
7	± .00024	± .000076	± .000024
8	± .00022	± .000071	± .000022

9.4 Hardware model assessment

Table 9.4-1 compares the system performance of :- the theoretical optimum detection process, the computer simulation optimum detection process, detection process D4 and the detection process D4 with a 4-bit a/d converter, the hardware model using detection process D4 with a 4-bit a/d converter. Good agreement is found between the performance of the hardware model and the corresponding computer simulation tests.

Number of active channels	Noise level for an error probability per channel of 0.003, expressed in dB relative to a binary TDM system				
	Theoretical	Computer simulation			Hardware model
	Optimum detection process D1	Optimum detection process D1	Detection process D4	Detection process D4 4-bit A/D	Detection process D4 4-bit A/D
1	9.03	9.05	9.05	8.93	8.70
2	3.97	4.05	4.05	3.94	3.96
3	5.03	4.40	3.82	3.72	3.71
4	2.29	2.38	2.35	2.25	1.88
5	1.34	1.10	1.10	1.10	1.32
6	0.92	1.12	1.12	1.04	0.80
7	0.27	0.37	0.37	0.39	0.12
8	0.00	0.00	0.00	0.00	0.00

Table 9.4-1 Comparison of results, theoretical, computer simulation and hardware model.



The Hardware Model

CHAPTER 10

THE OPTIMUM MULTIPLEXING ARRANGEMENT

In the previous chapters, various multiplexing arrangements have been proposed. Besides their simplicity as an attractive feature, demultiplexing processes have been developed which give a relatively good performance compared to the optimum detection process, which requires a vast number of sequential operations.

It is therefore pertinent to ask whether an optimum multiplexing arrangement exists, which working in conjunction with the optimum detection process would yield the overall optimum multiplexing system giving the lowest possible probability of error in the detected element values. Operational complexity would, by necessity, be unimportant at this stage. A simplified economically feasible practical system derived from the optimum system would naturally entail a compromise between the reduced complexity and an inevitable, slightly inferior performance.

The optimum detection process, applicable to any multiplexing arrangement, minimises the probability of error in the detection of the m element values $\{x_i'\}$ of the received signal elements in a group, selecting the vector X', such that the corresponding transmitted vector S' is at the minimum distance from the received vector R, in the n-dimensional Euclidean vector space containing these vectors. The detection process requires 2^m sequential operations.

As stated in Section 7.1, the n-dimensional vector space may be divided into 2^m decision regions separated by decision boundaries, where these decision boundaries are hyperplanes which perpendicuarly bisect the lines joining the different signal vectors {S}. In the general case where k decision boundaries exist, the total probability of error p, is given by the sum of the k individual probabilities of error, due to the various distances $\{d_i\}$ to the decision boundaries.

$$p = \sum_{i=1}^{k} Q\left(\frac{d_i}{\sigma}\right) \qquad (10.1\text{-}1)$$

At high signal/noise ratios with additive white Gaussian noise, even a very small increase in the distance to a decision boundary produces a considerable reduction in the corresponding probability of error. (Appendix A1). Thus the probability of error is effectively determined by the nearest decision boundary, the remaining boundaries having in comparison a very small effect on the probability of error.

The multiplexing problem is therefore concerned with positioning the 2^m possible transmitted vectors {S} in n-dimensional Euclidean vector space such that their proximity is maximised, and in particular, of utmost importance is the maximising of the minimum distance between the 2^m vectors in the vector space, as this effectively determines the probability of error.

It is assumed that the

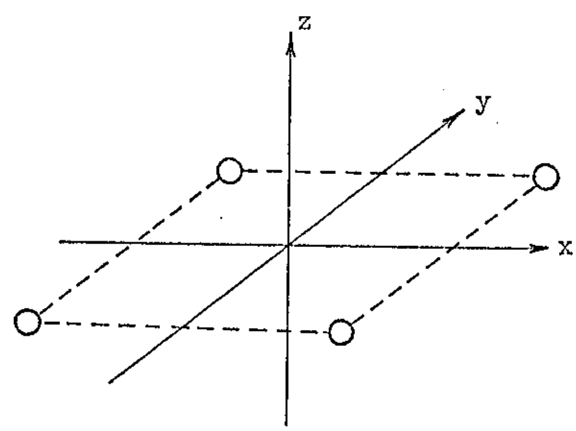
vector length, does not exceed $\sqrt{n}$, that is, the signals lie on or within a hypersphere of radius $\sqrt{n}$. The problem may be visualised as the packing of 2^m hyperspheres into a hypersphere of radius $\sqrt{n}$, such that the packing density is maximised. The 2^m hypersphere centres may lie on the circumference of the hypersphere of radius $\sqrt{n}$ and indeed probably will, for n large (high dimensionality) when a large proportion of the volume of a hypersphere lies near the circumference. The radius of the 2^m hyperspheres gives the smallest distance d, which effectively determines the probability of error.

No directly relevant references have been found to this particular problem although related topics concerned with the packing density of spheres in n-dimensional space are of interest.[72-75]

An appreciation of the problem complexity in n-dimensional vector space is conveniently illustrated by considering several simple examples where visual inspection offers an alternative approach to a mathematical analysis.

Two data sources, producing four possible transmitted signal vectors may be positioned in three dimensional vector space as follows.
The vectors are,

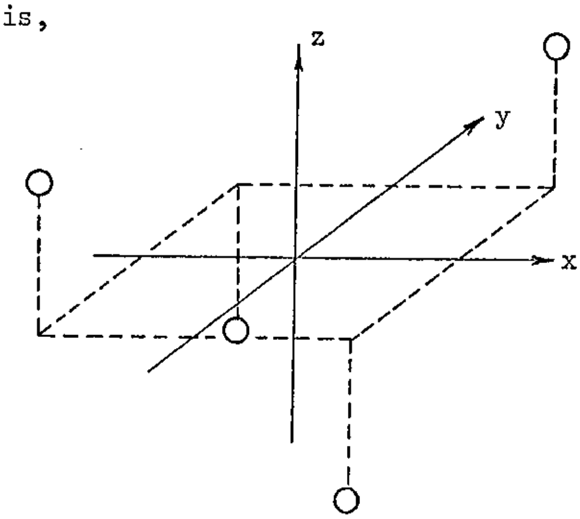
$$\left(\sqrt{\tfrac{3}{2}}\quad \sqrt{\tfrac{3}{2}}\quad 0\right)$$
$$\left(\sqrt{\tfrac{3}{2}}\quad -\sqrt{\tfrac{3}{2}}\quad 0\right)$$
$$\left(-\sqrt{\tfrac{3}{2}}\quad \sqrt{\tfrac{3}{2}}\quad 0\right)$$
$$\left(-\sqrt{\tfrac{3}{2}}\quad -\sqrt{\tfrac{3}{2}}\quad 0\right)$$



The distance to the nearest decision boundary separating the nearest vectors is $\sqrt{\frac{3}{2}}$. Solving Eqn. (10.1-1) for a total probability of error $p = 1 \times 10^{-4}$, gives $\sigma = 0.329$, or expressed as signal/noise ratio in decibels, 9.66 dB.

An alternative vector arrangement is,

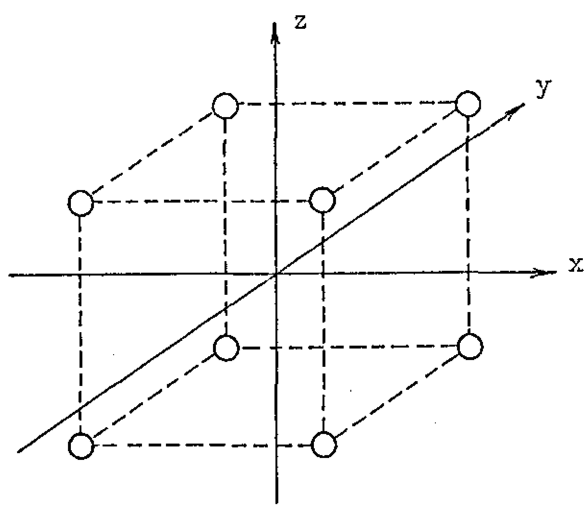
$$(\ 1\quad 1\quad 1\ )$$
$$(\ 1\quad -1\quad -1\ )$$
$$(\ -1\quad 1\quad -1\ )$$
$$(\ -1\quad -1\quad 1\ )$$



where the minimum distance to a decision boundary is $\sqrt{2}$. The four equidistant vectors now form the vertices of a tetrahedron, which, offering the closest possible packing density, represents the optimum arrangement of four signal vectors in 3 dimensional vector space. The signal/noise ratio is 8.78 dB, for $p = 1 \times 10^{-4}$, an advantage of almost 1 dB over the previous example.

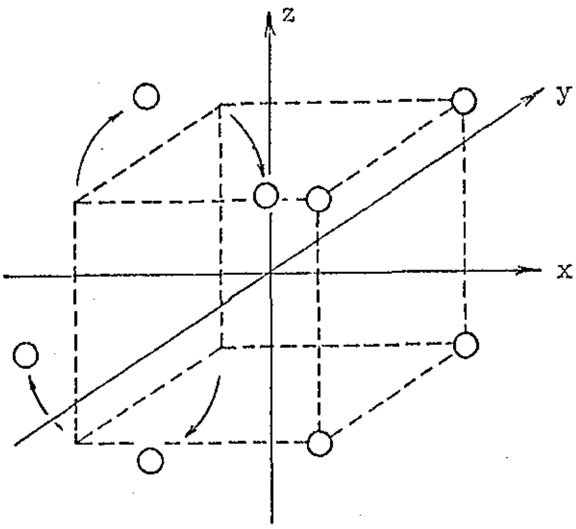
The tetrahedron structure provides the basis for an additional four vectors placed symmetrically, perpendicular to the face centres of the tetrahedron, forming a cube.

$$
\begin{array}{c}
(\;1 \;\; 1 \;\; 1\;) \\
(\;1 \;\; 1 \;\; -1\;) \\
(\;1 \;\; -1 \;\; 1\;) \\
(\;1 \;\; -1 \;\; -1\;) \\
(\;-1 \;\; 1 \;\; 1\;) \\
(\;-1 \;\; 1 \;\; -1\;) \\
(\;-1 \;\; -1 \;\; 1\;) \\
(\;-1 \;\; -1 \;\; -1\;)
\end{array}
$$



Each vector is surrounded by three other vectors having distances to the decision boundaries of 1. The signal/noise ratio is 11.41 dB, for $p = 1 \times 10^{-4}$. This arrangement represents a three channel TDM system with all three channels active. Although symmetrical and an obvious extension of the optimum tetrahedron arrangement, a higher packing density is obtained by rotating the last four vectors through 45° about the x axis as follows,

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$
$$\begin{pmatrix} -1 & 0 & \sqrt{2} \end{pmatrix}$$
$$\begin{pmatrix} -1 & 0 & -\sqrt{2} \end{pmatrix}$$
$$\begin{pmatrix} -1 & \sqrt{2} & 0 \end{pmatrix}$$
$$\begin{pmatrix} -1 & -\sqrt{2} & 0 \end{pmatrix}$$



Each vector row has two adjacent nearest vectors at distances to the decision boundaries of 1. The signal/noise ratio for $p = 1 \times 10^{-4}$ now decreases to 11.28 dB.

A further subtle refinement fractionally adjusts the vector positions such that the minimum distance between the vectors is increased slightly. More remote vectors, however, approach each other slightly, but with no significant effect on the probability of error. The adjusted vectors given below, have been calculated by considering the highest packing density of eight spheres where four are rotated by 45° about one axis from the cubic structure. The vector positions are virtually identical to the model of the previous example.

$$\begin{pmatrix} a & b & b \end{pmatrix}$$
$$\begin{pmatrix} a & b & -b \end{pmatrix}$$
$$\begin{pmatrix} a & -b & b \end{pmatrix}$$
$$\begin{pmatrix} a & -b & -b \end{pmatrix}$$
$$\begin{pmatrix} -a & 0 & c \end{pmatrix}$$
$$\begin{pmatrix} -a & 0 & -c \end{pmatrix}$$
$$\begin{pmatrix} -a & c & 0 \end{pmatrix}$$
$$\begin{pmatrix} -a & -c & 0 \end{pmatrix}$$

where $b = \sqrt{\dfrac{6}{4 + \sqrt{2}}}$

$$a = \sqrt[4]{2}\; b$$

$$c = \sqrt{2}\; b$$

Each vector now has four adjacent nearest vectors at distances to the decision boundaries of 1.05, giving a signal/noise ratio of 11.27 dB, for $p = 1 \times 10^{-4}$. Although not conclusive, it appears that no alternative arrangement will give a better signal/noise ratio. A most important result is therefore, that a conventional TDM system, with all channels in operation, is not necessarily the optimum arrangement as intuition would have us believe. In particular, for three active channels in a three channel system, a multilevel arrangement gives an improved performance over a binary bipolar TDM system with the same average energy per component of the transmitted signal.

To illustrate the dilemma further, 8 vectors, corresponding to three active channels are distributed in 8 dimensional vector space. A possible arrangement is the orthogonal rows that form a Hadamard matrix,

$$
\begin{array}{c}
(\ 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1\ ) \\
(\ 1 \ -1 \quad 1 \ -1 \quad 1 \ -1 \quad 1 \ -1\ ) \\
(\ 1 \quad 1 \ -1 \ -1 \quad 1 \quad 1 \ -1 \ -1\ ) \\
(\ 1 \ -1 \ -1 \quad 1 \quad 1 \ -1 \ -1 \quad 1\ ) \\
(\ 1 \quad 1 \quad 1 \quad 1 \ -1 \ -1 \ -1 \ -1\ ) \\
(\ 1 \ -1 \quad 1 \ -1 \ -1 \quad 1 \ -1 \quad 1\ ) \\
(\ 1 \quad 1 \ -1 \ -1 \ -1 \ -1 \quad 1 \quad 1\ ) \\
(\ 1 \ -1 \ -1 \quad 1 \ -1 \quad 1 \quad 1 \ -1\ )
\end{array}
$$

Each of the eight equidistant vectors is separated from the nearest decision boundary by a distance of 2. However, the first dimension components are all positive, indicating an uneven vector distribution in the 8 dimensional hypersphere.

The following vectors form an alternative arrangement,

$$
\begin{array}{cccccccc}
(& 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 &) \\
(& 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 &) \\
(& 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 &) \\
(& 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 &) \\
(& -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 &) \\
(& -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 &) \\
(& -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 &) \\
(& -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 &)
\end{array}
$$

Each vector has six adjacent vectors at a distance of 2, and another vector at a distance of $2\sqrt{2}$ to the decision boundary. Despite the marginal advantage of the arrangement, it clearly suggests that a re-arrangement exists, whereby the minimum distance of 2 is increased slightly, at the expense of the seventh vector at a distance of $2\sqrt{2}$.

The following vectors illustrate this.

$$
\begin{array}{cccccccccc}
(& a & a & a & 0 & 0 & 0 & \frac{1}{2}a & \frac{1}{2}a &) \\
(& a & -a & -a & 0 & 0 & 0 & \frac{1}{2}a & \frac{1}{2}a &) \\
(& -a & a & -a & 0 & 0 & 0 & \frac{1}{2}a & \frac{1}{2}a &) \\
(& -a & -a & a & 0 & 0 & 0 & \frac{1}{2}a & \frac{1}{2}a &) \\
(& 0 & 0 & 0 & a & a & a & -\frac{1}{2}a & -\frac{1}{2}a &) \\
(& 0 & 0 & 0 & a & -a & -a & -\frac{1}{2}a & -\frac{1}{2}a &) \\
(& 0 & 0 & 0 & -a & a & -a & -\frac{1}{2}a & -\frac{1}{2}a &) \\
(& 0 & 0 & 0 & -a & -a & a & -\frac{1}{2}a & -\frac{1}{2}a &)
\end{array}
\qquad \text{where } a = \sqrt{\frac{8}{3\frac{1}{2}}} \simeq 2.14
$$

All eight vectors are equidistant with a distance to the decision boundaries of 2.14, which is a significant improvement. The first three dimensions have a tetrahedral structure, as do the fourth to sixth dimensions. The seventh and eighth dimensions increase the distance between the first and second groups of four vectors. Equidistant vectors are surely a guideline to the optimum arrangement, but misleading, as the uneven distribution problem has revealed in the first example of 8 dimensions.

From the foregoing it is evident that a non-mathematical intuitive approach may well be the only method for determining the optimum arrangement of vectors in n-dimensional Euclidean vector space. It remains to be seen whether mathematical analysis will yield conclusive results. An important result obtained from the positioning of 3 signals (8 vectors) in 3-dimensional vector space, is that a multilevel transmitted signal arrangement can give an improved performance over a binary bipolar TDM system with the same average energy per component of the transmitted signal when used with all channels in operation. This may apply to higher dimensional arrangements. To conclude, it appears that the optimum multiplexing arrangement is undefined and may be approached only through specific examples.

As a final note, the optimum theoretical method, in the sense of minimising the error probability, for transmitting data through a Gaussian channel, consists of waiting until all data has been accumulated at the transmitted, and then sending a single waveform to represent the entire message.[25] The optimum receiver in the presence of white noise consists of filters matched to each message waveform. The disadvantage of this form of communication lies in the fact that transmitter and receiver complexity grows exponentially with message length. Thus, system designers usually restrict system complexity by not waiting for the entire message before transmission. Short portions of the message are encoded systematically, and transmitted sequentially as they arrive, using relatively simple terminal equipment.

CHAPTER 11

COMMENTS ON THE RESEARCH PROJECT

11.1 Originality

To the best of the Author's knowledge, the following chapters of this thesis are believed to be original. Developments of a multiplex system using a combination of time- and code-division multiplexing (Chapter 4). A code-division multiplex system using adaptive coding of Walsh functions (Chapter 6), and all detection processes other than the optimum detection process relating to the multiplexing arrangements C and D (Chapters 7 and 8). The hardware model circuitry and tests performed (Chapter 9). Discussion on the design of an optimum multiplexing arrangement (Chapter 10). All computer simulation tests and computer programs.

11.2 Suggestions for further investigations

The research project has been concerned with various multiplexing and demultiplexing processes suitable for use in a synchronous serial baseband data-transmission system, where the signals are transmitted in orthogonal groups over a channel which introduces additive white Gaussian noise only.

From the foregoing theoretical work, further investigations appear promising in the following areas:-

a) The multiplexing arrangement D (Section 6.3) achieves a useful performance over the corresponding TDM system when used with the optimum detector. Various demultiplexing arrangements have been proposed with performances approaching that of the optimum detector, but an even further reduced operational complexity would be desirable.

b) Considerable scope exists for developing multiplexing arrangements which need not necessarily be confined to a binary or ternary transmitted signal. Indeed, the optimum multiplexing arrangement for a varying number of active channels would probably employ a multilevel signal.

c) In Chapter 10, the optimum multiplexing arrangement was briefly considered from the aspect of maximising the minimum distance in n-dimensional Euclidean vector space, between the possible transmitted signals represented as vectors in the vector space. This introduction indicates the problem complexity, and clearly forms the basis for a detailed theoretical investigation.

d) The data-transmission system considered has for simplicity, introduced additive white Gaussian noise only into the transmission path. Over practical systems, distortion or intersymbol interference may be a significant factor, and although this has received wide attention for serial data-transmission, its effect on multiplexing arrangements, together with additive Gaussian noise, has yet to be investigated.

CHAPTER 12

CONCLUSIONS

From the foregoing it is evident that considerable scope exists to investigate multiplex systems other than those based on conventional FDM and TDM techniques.

The systems discussed provide advantages in keeping with the improvements suggested in Section 1.3. That is, they are inherently flexible, they have no well defined overload characteristics, and are inherently less sensitive to interference than existing conventional techniques.

System A1 is particularly well suited to applications where the number of multiplexed channels is typically a little greater than the maximum number orthogonally multiplexed using TDM. For up to 50% more channels, the system gains an advantage in tolerance to additive white Gaussian noise over the corresponding quaternary TDM system, where this has the same average energy per component of the transmitted signal, the same transmission rate, and the same error probability per channel as System A1.
This advantage decreases slowly as the number of channels increases.
System A2 is identical to System A1 except for a simple modification at the transmitter. This not only ensures unique detectability of the received element values, but has an advantage of up to 1 dB over System A1.
System A3 extends the non-linear multiplexing technique of System A1, for multiplexing three orthogonal signal sets. The results are a natural

extension of those obtained for System A1.

The majority multiplex arrangement of Gordon and Barrett, although ingenious, has several disadvantages. The coding scheme is only valid for a codeword length of 7 or 3 components, accommodating a maximum capacity of 7 and 3 channels only. It has been shown that there are no matrices which provide any improvement over this, and it is merely fortuitous that the Walsh matrix majority multiplexing scheme works at all. Also, and odd number of active channels only, may be multiplexed.

An interesting scheme is the multiplexing arrangement C, which generates a transmitted signal similar to a CDM codeword and TDM, for minimum and maximum capacities respectively, and gradually changes from one arrangement to the other as the number of channels increases. However, no more than the maximum number of orthogonal signals may be multiplexed satisfactorily. The detection process of System C2 achieves a performance equal to that of the optimum detector, System C1, but with far fewer sequential operations.

The multiplexing arrangement D, a majority multiplexed form of arrangement C, again generates a transmitted signal similar to CDM and TDM, for minimum and maximum capacities respectively, only now, the transmitted signal is binary, or ternary for an even number of active channels.
The number of channels multiplexed may exceed the maximum number of orthogonal channels, with a slowly deteriorating tolerance to noise.
System D3 only functions for an odd number of active channels.
System D2 achieves a performance approaching System D1, using the optimum detection process, but requires far fewer sequential operations.
The detector, however, must determine the threshold detector levels from the average magnitude of the noise vector. At high signal/noise ratios,

System D4 also achieves the optimum performance with a further reduced complexity, and is the best overall CDM arrangement considered.

It is evident that the optimum multiplexing arrangement does not lend itself to mathematical analysis. Its performance appears undefined and may only be approached through specific examples. An important result obtained from the positioning of 3 signals (8 vectors) in 3-dimensional vector space, is that a multilevel transmitted signal arrangement can give an improved performance over a binary bipolar TDM system with the same average energy per component of the transmitted signal when used with all channels in operation.

APPENDIX A1

ERROR PROBABILITY AND SIGNAL/NOISE RATIO

When the signal element values in a group are statistically independent and are equally likely to have the two possible values ±1, the probability of error in the detection of the ith element value of a group from Section 7.1 is,

$$p_i = Q\left(\frac{d_i}{\sigma}\right) \qquad \text{(A1-1)}$$

where σ^2 is the power spectral density of the additive white Gaussian noise at the input to the receiver filter, and d_i is the distance to the single decision boundary in the detection of the ith element of the group of m. Let p_i be equal to p, and d_i equal to d, so that,

$$p = Q\left(\frac{d}{\sigma}\right) \qquad \text{(A1-2)}$$

The variation of the element error probability p with d/σ is obtained from probability distribution tables and is shown in Figure A1-1.

At high signal/noise ratios, that is when p has a value around 1×10^{-5} it can be seen from Figure A1-1 that for a given change in the error probability, the corresponding change in the signal/noise ratio is relatively small. For $p = 3 \times 10^{-5}$, the corresponding value of d/σ is 4.05 For $p = 6 \times 10^{-5}$, a doubling of the error probability, the corresponding value of d/σ is 3.85, a change in tolerance to noise of 0.34 dB.

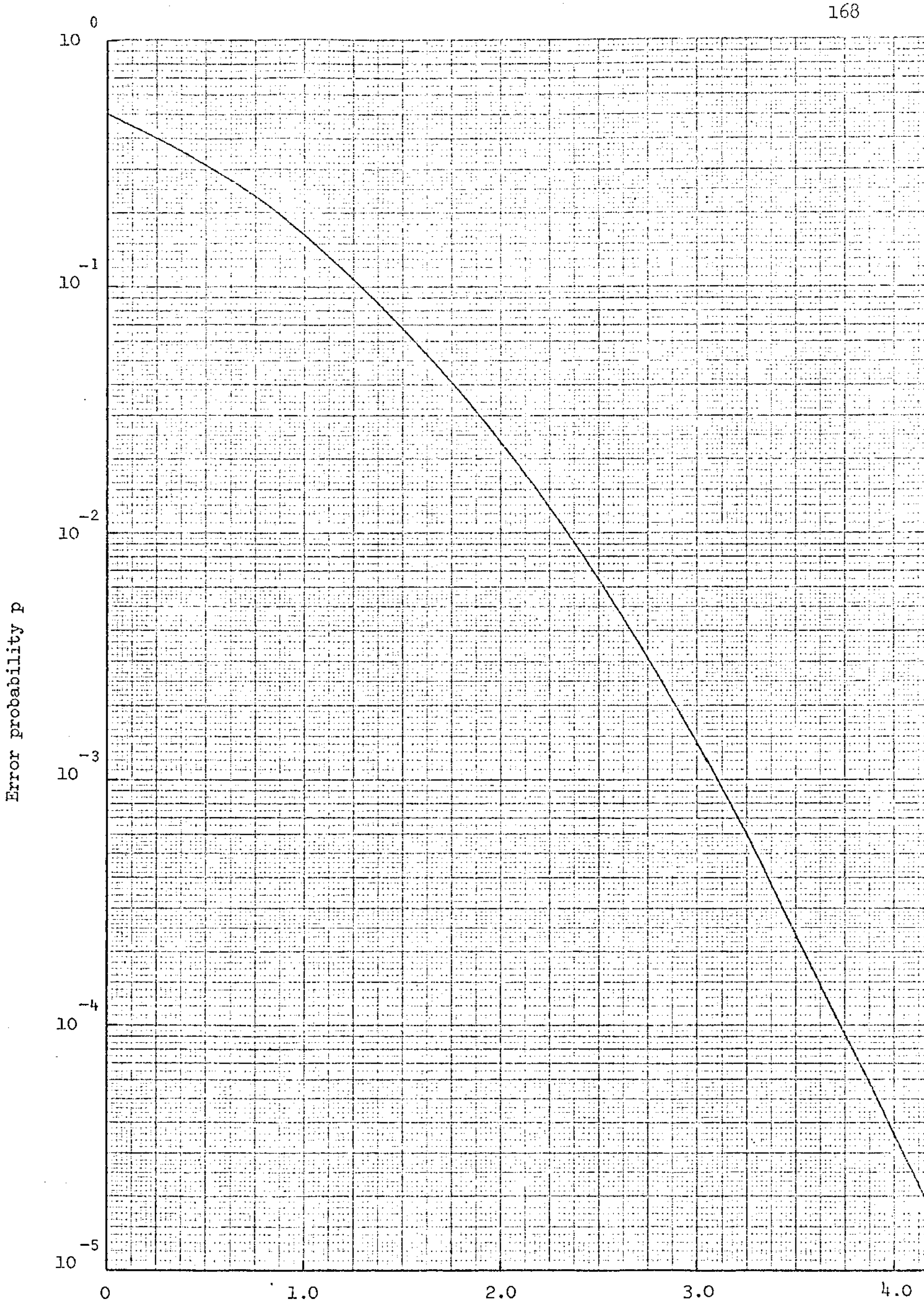


Figure A1-1 Variation of the element error probability with d/σ

At high signal/noise ratios even the doubling of the error probability produces a negligible change in the signal/noise ratio. On the other hand, a small change in signal/noise ratio produces a relatively large change in the element error probability. At high signal/noise ratio a change of 1 dB approximately alters the element error probability by 10 times.

Consider that there are two binary element values in a group having possible values ±1. From Eqn. (A1-1),

$$p_1 = Q\left(\frac{d_1}{\sigma}\right) \qquad \text{and} \qquad p_2 = Q\left(\frac{d_2}{\sigma}\right)$$

Assume now that, the signal/noise ratio is high and furthermore, $d_1/\sigma = 3.0$ and $d_2/\sigma = 4.0$ (say). From Figure A1-1, p_1 corresponding to $d_1/\sigma = 3.0$ is 1.4×10^{-3}, and p_2 corresponding to $d_2/\sigma = 4.0$ is 3.5×10^{-5}. Clearly, $p_1 >> p_2$. It therefore follows, that the average element value error probability in the detection of the two element values of the group is effectively given by p_1 which corresponds to the smaller of the two distances d_1 and d_2, providing that the signal/noise ratio is high. If there are m element values in a group, the average element value error probability, is approximately given by the p_i of Eqn. (A1-1), which corresponds to the smallest value of d_i.

APPENDIX A2

COMPUTER SIMULATION PROGRAMS

The following computer programs are shown as typical examples of multiplexing and demultiplexing arrangements. For completeness, they are shown in their entirety, including control cards, document data and results.

System A1

System D1 (two orthogonal sets multiplexed)

System D2 (" " " ")

Computer simulation program for System A1

```
JOB THA1,E,RBH1522
EDSFILES 1
LUFORTRAN ,,W
JOBCORE 32K
DOWN 22
RUN ,,2000
****
DOCUMENT SOURCE
      LIBRARY (ED,SUBGROUPNAGF)
      WORK(ED,POSTFILEUSER)
      PROGRAM(THA1)
      ABNORMAL FUNCTIONS
      COMPACT
      INPUT 1 = CR0
      OUTPUT 2 = LP0
      COMPRESS INTEGER AND LOGICAL
      TRACE 0
      END

      MASTER SYSTEM A1

C   COMPUTER PROGRAM TO SIMULATE SYSTEM A1. TWO ORTHOGONAL SETS ,
C   A TDM AND A CDM OF 16 CHANNELS EACH , ARE COMBINED NON-
C   LINEARLY AND DETECTED USING CORRELATION DETECTION .

C   SAA,SBB = MATRIX OF CHANNEL CODEWORDS - SET A, SET B
C   NA,NB = NO OF CHANNELS - SET A, SET B
C   AB = LEVEL OF THE SET B CODEWORDS
C   SD = STANDARD DEVIATION OF THE CHANNEL NOISE SAMPLE VALUES
C   L = TOTAL NO. OF GROUPS TRANSMITTED
C   ZA,ZB = CHANNEL ELEMENT VALUES TO BE MULTIPLEXED - SET A,SET B
C   SA,SB = LINEAR SUM OF SET CODEWORDS - SET A,SET B
C   R = TRANSMITTED SIGNAL VECTOR
C   A,B = DETECTED ELEMENT VALUES - SET A, SET B
C.  RSB = RECONSTRUCTED LINEAR SUM OF SET B CODEWORDS
C   RNL = RELATIVE NOISE LEVEL IN DB
C   RSE = RELATIVE SIGNAL ENERGY IN DB
C   EA1 = TOTAL NO. OF ERRORS - SET A,FIRST CYCLE
C   PA1 = AVERAGE ERROR PROBABILITY / CHANNEL - SET A,FIRST CYCLE
C   G05BAF(X) INITIALISES RANDOM NUMBER GENERATOR
C   G05AEF(A,B) R.N.G. WITH GAUSSIAN DIST.  MEAN A  ST.DEV. B
C   G05AAF(Y) R.N.G. WITH UNIFORM DIST. BETWEEN 0 AND 1
C   G05ABF(A,B) R.N.G. WITH UNIFORM DIST. BETWEEN A AND B

      INTEGER SAA(16,16),SBB(16,16),ZA(16),ZB(16),SA(16),SB(16)
      INTEGER A(16),B(16),EA1,EB1,EA2,EB2
      DIMENSION R(16),RR(16),OP(16),RSB(16)

C   READ MATRIX OF CODEWORDS - SET A,SET B
      READ (1,10) ((SAA(I,J),J=1,16),I=1,16)
      READ (1,10) ((SBB(I,J),J=1,16),I=1,16)
   10 FORMAT (16I0)
```

```
C   WRITE OUTPUT TITLES
       WRITE (2,11)
    11 FORMAT (1H1////45X,'SYSTEM A1'/45X,'----------'//'  NO  OF'
      1,'     AMP   GAUSS.  NO   OF   RELATIVE   NO OF ERRS   ERROR '
      2,'PROBABILITY  / CHANNEL  RELATIVE'/' CHANNELS  SET B  NOI'
      3,'SE    GROUPS  NOISE L.  1 CYC  2CYC   1ST CYCLE            '
      4,'2ND CYCLE  SG.ENERGY'/' (NA)(NB)  (AB)     (SD)      (L)   '
      5,' (RNL)DB    A  B    A  B    SET A    SET B  SET A   SET B    '
      6,'(RSE)DB'//)

C   READ SET OF DATA
       DO 200 NN=1,17
       READ (1,12) NA,NB,AB,SD,L
    12 FORMAT (2I0,2F0.0,I0)
       EA1,EB1,EA2,EB2=0
       TSE=0.0

C   THE PROGRAM NOW RUNS FOR L TRANSMITTED SIGNAL GROUPS
       DO 100 LL=1,L
       CALL RANDOM (NA,ZA)
       CALL RANDOM (NB,ZB)

C   FORMATION OF THE TRANSMITTED SIGNAL  R(J)
       DO 17 J=1,16
       SB(J)=0
       DO 15 I=1,16
       IF (ZB(I)) 13,15,14
    13 SB(J)=SB(J)-SBB(I,J)
       GO TO 15
    14 SB(J)=SB(J)+SBB(I,J)
    15 CONTINUE
       IF (ZA(J)) 16,17,17
    16 SB(J)=-SB(J)
    17 R(J)=ZA(J)+SB(J)*0.25*AB

C   CAL. TOTAL SIGNAL ENERGY
       DO 18 J=1,16
    18 TSE=TSE+R(J)*R(J)

C   ADD GAUSSIAN NOISE OF STD. DEV. SD
       DO 19 J=1,16
    19 R(J)=R(J)+G05AEF(0.0,SD)
```

```
C   DETECTION OF THE RECEIVED SIGNAL  R(J)+NOISE
C   FIRST CYCLE
C   DETECT SET A
        DO 24 J=1,16
        IF (R(J)-0.0001) 20,23,23
    20  IF (R(J)+0.0001) 22,22,21
    21  IF (G05AAF(Z)-0.5) 22,23,23
    22  A(J)=-1
        GO TO 24
    23  A(J)=+1
    24  RR(J)=ABS(R(J))-1
C   DETECT SET B
        DO 30 J=1,16
        OP(J)=0.0
        IF (ZB(J)) 25,28,25
    25  DO 26 I=1,16
    26  OP(J)=OP(J)+RR(I)*SBB(I,J)
        IF (OP(J)) 27,28,29
    27  B(J)=-1
        GO TO 30
    28  B(J)=0
        GO TO 30
    29  B(J)=+1
    30  CONTINUE
C   COUNT TOTAL NO OF ERRORS EA1,EB1
        DO 34 J=1,16
        IF (A(J)-ZA(J)) 31,32,31
    31  EA1=EA1+1
    32  IF (B(J)-ZB(J)) 33,34,33
    33  EB1=EB1+1
    34  CONTINUE
C   SECOND CYCLE
C   RECONSTRUCT RSB(J) FROM SET B AND REDETECT SET A
        IS=0
        DO 39 J=1,16
        RSB(J)=0.0
        DO 37 I=1,16
        IF (B(I)) 35,37,36
    35  RSB(J)=RSB(J)-SBB(I,J)
        GO TO 37
    36  RSB(J)=RSB(J)+SBB(I,J)
    37  CONTINUE
        IF (RSB(J)*0.25*AB+1) 38,39,39
    38  A(J)=-A(J)
        RR(J)=(-ABS(R(J)))-1
        IS=1
    39  CONTINUE
        IF (IS) 47,47,40
```

```
C  DETECT SET B
   40 DO 46 J=1,16
      OP(J)=0,0
      IF (ZB(J)) 41,44,41
   41 DO 42 I=1,16
   42 OP(J)=OP(J)+RR(I)*SBB(I,J)
      IF (OP(J)) 43,44,45
   43 B(J)=-1
      GO TO 46
   44 B(J)=0
      GO TO 46
   45 B(J)=+1
   46 CONTINUE

C  COUNT TOTAL NO OF ERRORS EA2,EB2
   47 DO 51 J=1,16
      IF (A(J)-ZA(J)) 48,49,48
   48 EA2=EA2+1
   49 IF (B(J)-ZB(J)) 50,51,50
   50 EB2=EB2+1
   51 CONTINUE

  100 CONTINUE

C  CAL. AVERAGE ERROR PROBABILITIES  PA1,PB1,PA2,PB2
      PA1=EA1/FLOAT(L*NA)
      PA2=EA2/FLOAT(L*NA)
      IF (NB.EQ.0) GO TO 52
      PB1=EB1/FLOAT(L*NB)
      PB2=EB2/FLOAT(L*NB)

C  CAL. RELATIVE NOISE LEVEL IN DB
   52 RNL=20*ALOG10(SD/0.364)

C  CAL. AVERAGE SIGNAL ELEMENT ENERGY / TRANSMITTED COMPONENT
      ASE=TSE/(L*16)

C  CAL. RELATIVE SIGNAL ENERGY PER COMPONENT IN DB
      RSE=10*ALOG10(ASE/1.0)

      WRITE (2,53) NA,NB,AB,SD,L,RNL,EA1,EB1,EA2,EB2,PA1,PB1,PA2,
     1PB2,RSE
   53 FORMAT (I4,I4,F8.3,F7.3,I8,F9.2,I5,I5,I4,I3,F9.4,F7.4,F8.4,
     1F7.4,'    +',F4.2/)

  200 CONTINUE

      WRITE (2,54)
   54 FORMAT (' ***RBH***')
      STOP
      END
```

```
      SUBROUTINE RANDOM (NA,ZA)
C   SUBROUTINE RANDOM GENERATES A 16 COMPONENT VECTOR ZA WHICH
C   HAS NA ELEMENTS SET AT RANDOM TO +1 OR -1 , AND POSITIONED
C   RANDOMLY THROUGHOUT THE VECTOR.
      INTEGER ZA(16)
      IF (NA-8) 10,10,18
   10 DO 11 J=1,16
   11 ZA(J)=0
      IF (NA) 12,25,12
   12 DO 17 I=1,NA
   13 M=G05ABF(1,0,16 999)
      IF (ZA(M)) 13,14,13
   14 IF (G05AAF(Y)-0.5) 16,15,15
   15 ZA(M)=+1
      GO TO 17
   16 ZA(M)=-1
   17 CONTINUE
      GO TO 25
   18 DO 21 J=1,16
      IF (G05AAF(Y)-0.5) 20,19,19
   19 ZA(J)=+1
      GO TO 21
   20 ZA(J)=-1
   21 CONTINUE
      IF (NA-16) 22,25,25
   22 DO 24 I=1,16-NA
   23 M=G05ABF(1,0,16.999)
      IF (ZA(M)) 24,23,24
   24 ZA(M)=0
   25 RETURN
      END
      FINISH
****
```

```
DOCUMENT DATA
   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0
   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0
   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0
   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0
   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0
   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0
   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
   1  -1   1  -1   1  -1   1  -1   1  -1   1  -1   1  -1   1  -1
   1   1  -1  -1   1   1  -1  -1   1   1  -1  -1   1   1  -1  -1
   1  -1  -1   1   1  -1  -1   1   1  -1  -1   1   1  -1  -1   1
   1   1   1   1  -1  -1  -1  -1   1   1   1   1  -1  -1  -1  -1
   1  -1   1  -1  -1   1  -1   1   1  -1   1  -1  -1   1  -1   1
   1   1  -1  -1  -1  -1   1   1   1   1  -1  -1  -1  -1   1   1
   1  -1  -1   1  -1   1   1  -1   1  -1  -1   1  -1   1   1  -1
   1   1   1   1   1   1   1   1  -1  -1  -1  -1  -1  -1  -1  -1
   1  -1   1  -1   1  -1   1  -1  -1   1  -1   1  -1   1  -1   1
   1   1  -1  -1   1   1  -1  -1  -1  -1   1   1  -1  -1   1   1
   1  -1  -1   1   1  -1  -1   1  -1   1   1  -1  -1   1   1  -1
   1   1   1   1  -1  -1  -1  -1  -1  -1  -1  -1   1   1   1   1
   1  -1   1  -1  -1   1  -1   1  -1   1  -1   1   1  -1   1  -1
   1   1  -1  -1  -1  -1   1   1  -1  -1   1   1   1   1  -1  -1
   1  -1  -1   1  -1   1   1  -1  -1   1   1  -1   1  -1  -1   1
16  0 0.000 0,364 1000
16  1 0.780 0,320 1000
16  2 0.770 0,270 1000
16  3 0.678 0,244 1000
16  4 0.604 0,217 1000
16  5 0.564 0,204 1000
16  6 0.525 0,192 1000
16  7 0.500 0,182 1000
16  8 0.479 0,172 1000
16  9 0.458 0,168 1000
16 10 0.428 0,150 1000
16 11 0.426 0,152 1000
16 12 0.404 0,146 1000
16 13 0.390 0,141 1000
16 14 0.376 0,140 1000
16 15 0.364 0,130 1000
16 16 0.361 0,130 1000
****
```

SYSTEM A1

NO OF CHANNELS (NA)	(NB)	AMP SET B (AB)	GAUSS. NOISE (SD)	NO OF GROUPS (L)	RELATIVE NOISE L. (RNL)DB	NO OF ERRS 1 CYC A	1 CYC B	2CYC A	2CYC B	ERROR PROBABILITY / CHANNEL 1ST CYCLE SET A	1ST CYCLE SET B	2ND CYCLE SET A	2ND CYCLE SET B	RELATIVE SG.ENERGY (RSE)DB
16	0	0.000	0.364	1000	0.00	46	0	46	0	0.0049	0.0000	0.0029	0.0000	+0.00
16	1	0.780	0.320	1000	-1.12	47	3	47	3	0.0049	0.0030	0.0029	0.0030	+0.15
16	2	0.770	0.27[illegible]	1000	-2.59	47	6	47	6	0.0049	0.0030	0.0029	0.0030	+0.32
16	3	0.678	0.244	1000	-3.47	46	9	46	9	0.0049	0.0030	0.0029	0.0030	+0.37
16	4	0.604	0.217	1000	-4.49	47	12	47	12	0.0049	0.0030	0.0029	0.0030	+0.38
16	5	0.564	0.204	1000	-5.03	49	15	49	15	0.0031	0.0030	0.0031	0.0030	+0.38
16	6	0.525	0.192	1000	-5.56	49	18	49	18	0.0031	0.0030	0.0031	0.0030	+0.41
16	7	0.500	0.182	1000	-6.02	49	20	49	20	0.0031	0.0029	0.0031	0.0029	+0.44
16	8	0.479	0.172	1000	-6.51	46	24	46	24	0.0049	0.0030	0.0029	0.0030	+0.52
16	9	0.458	0.168	1000	-6.72	52	27	49	27	0.0033	0.0030	0.0031	0.0030	+0.50
16	10	0.428	0.150	1000	-7.70	54	31	47	31	0.0033	0.0031	0.0029	0.0031	+0.45
16	11	0.426	0.152	1000	-7.59	52	34	48	34	0.0033	0.0031	0.0030	0.0031	+0.49
16	12	0.404	0.146	1000	-7.93	46	36	48	35	0.0049	0.0030	0.0030	0.0029	+0.48
16	13	0.390	0.141	1000	-8.24	55	38	50	37	0.0034	0.0029	0.0031	0.0028	+0.54
16	14	0.376	0.14[illegible]	1000	-8.30	57	40	48	40	0.0036	0.0029	0.0030	0.0029	+0.50
16	15	0.364	0.13[illegible]	1000	-8.94	49	50	46	47	0.0031	0.0033	0.0029	0.0031	+0.50
16	16	0.361	0.13[illegible]	1000	-8.94	62	49	47	48	0.0039	0.0031	0.0029	0.0030	+0.54

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Computer simulation program for System D1

```
JOB THD1,E,RBH1522
EDSFILES 1
LUFORTRAN ,,W
JOBCORE 32K
DOWN 22
RUN ,,2500
****
DOCUMENT SOURCE
      LIBRARY (ED,SUBGROUPNAGF)
      WORK(ED,USEREDSFILE1)
      PROGRAM(THD1)
      ABNORMAL FUNCTIONS
      COMPACT
      INPUT 1 = CR0
      OUTPUT 2 = LP0
      COMPRESS INTEGER AND LOGICAL
      TRACE 0
      END


      MASTER SYSTEM D1

C   COMPUTER PROGRAM TO SIMULATE SYSTEM D1. TWO ORTHOGONAL COM
C   SETS , OF 8 CHANNELS EACH ARE MULTIPLEXED , AND DETECTED
C   USING THE OPTIMUM DETECTION PROCESS .

C   SAA,SBB = MATRIX OF CHANNEL CODEWORDS - SET A, SET B
C   NA,NB = NO OF CHANNELS - SET A, SET B
C   AB = LEVEL OF THE SET B CODEWORDS
C   SD = STANDARD DEVIATION OF THE CHANNEL NOISE SAMPLE VALUES
C   L = TOTAL NO. OF GROUPS TRANSMITTED
C   ZA,ZB = CHANNEL ELEMENT VALUES TO BE MULTIPLEXED - SET A,SET B
C   SA,SB = LINEAR SUM OF SET CODEWORDS - SET A,SET B
C   R = TRANSMITTED SIGNAL VECTOR
C   A,B = DETECTED ELEMENT VALUES - SET A, SET B
C   RNL = RELATIVE NOISE LEVEL IN DB
C   RSE = RELATIVE SIGNAL ENERGY IN DB
C   EA = TOTAL NO. OF ERRORS - SET A
C   PA = AVERAGE ERROR PROBABILITY / CHANNEL - SET A
C   G05BAF(X) INITIALISES RANDOM NUMBER GENERATOR
C   G05AEF(A,B) R.N.G. WITH GAUSSIAN DIST.  MEAN A  ST.DEV. B
C   G05AAF(Y) R.N.G. WITH UNIFORM DIST. BETWEEN 0 AND 1

      INTEGER SAA(8,8),ZA(8),ZB(8),SA(8),SB(8),A(8),B(8),X(8)
      INTEGER XA(256,8),XB(256,8),EA,EB
      DIMENSION R(8),RR(8),XRA(256,8),XRB(256,8)

      CALL G05BAF(1.0)
      READ (1,10) ((SAA(I,J),J=1,8),I=1,8)
   10 FORMAT (8I5)
```

```
      WRIYE(2,11)
   11 FORMAT (1H1////45X,'SYSTEM D1'/45X,'----------'//'  NO  OF'
     1,'    AMP   GAUSS,  NO  OF  RELATIVE   NO OF ERSS   ERROR '
     2,'PROBABILITY  / CHANNEL  RELATIVE'/' CHANNELS  SET B  NOI'
     3,'SE   GROUPS  NOISE L.   SETA  SETB        SET A       '
     4,'SET B       SG.ENERGY'/' (NA)(NB)  (AB)    (SD)     (L)    '
     5,' (RNL)DB     (EA)  (EB)          (PA)           (PB)      '
     6,'(RSE)DB'//)

      DO 300 NNN=1,16
      READ (1,12) NA,NB,AB,SD,L
   12 FORMAT (2I0,2F0.0,I0)
      EA,EB=0
      TSE=0.0

C  FORMATION OF ALL POSSIBLE TRANS. VECTORS XRA(NNA,K),XRB(NNB,K)
      NNA=2**NA
      NNB=2**NB

      DO 31 M=1,2
      IF (M.EQ.2.AND.NB.EQ.0) GO TO 31
      IF (M.EQ.1.AND.SY.EQ.1) GO TO 31
      IF (M.EQ.1) NNO=NNA
      IF (M.EQ.2) NNO=NNB
      IF (M.EQ.1) NO=NA
      IF (M.EQ.2) NO=NB
      DO 13 J=1,8
   13 X(J)=0

      DO 30 K=1,NNO

      DO 15 J=1,NO
      X(J)=X(J)+2
      IF (X(J)-1) 16,16,14
   14 X(J)=-1
   15 CONTINUE

   16 DO 30 J=1,8
      R(J)=0.0
      IF (X(J)) 17,19,18
   17 R(J)=-SAA(J,J)
      GO TO 23
   18 R(J)=+SAA(J,J)
      GO TO 23
   19 DO 22 I=1,8
      IF (X(I)) 20,22,21
   20 R(J)=R(J)-SAA(I,J)
      GO TO 22
   21 R(J)=R(J)+SAA(I,J)
   22 CONTINUE

   23 IF (R(J)) 24,25,26
   24 R(J)=-1
      GO TO 27
   25 R(J)=0
      GO TO 27
   26 R(J)=+1
```

```
   27 IF (M-1) 28,28,29
   28 XA(K,J)=X(J)
      XRA(K,J)=R(J)
      GO TO 30
   29 XB(K,J)=X(J)
      XRB(K,J)=R(J)
   30 CONTINUE
   31 CONTINUE

      IF (NA.EQ.8) IY=1
C  THE PROGRAM NOW RUNS FOR L TRANSMITTED SIGNAL GROUPS
      DO 200 NN=1,L
      CALL RANDOM (NA,NB,ZA,ZB)
C  FORMATION OF THE TRANSMITTED SIGNAL  R(J)
      DO 56 J=1,8
      SA(J),SB(J)=0
      IF (ZA(J)) 32,34,33
   32 SA(J)=-SAA(J,J)
      GO TO 38
   33 SA(J)=+SAA(J,J)
      GO TO 38
   34 DO 37 I=1,8
      IF (ZA(I)) 35,37,36
   35 SA(J)=SA(J)-SAA(I,J)
      GO TO 37
   36 SA(J)=SA(J)+SAA(I,J)
   37 CONTINUE
   38 IF (SA(J)) 39,40,41
   39 SA(J)=-1
      GO TO 42
   40 SA(J)=0
      GO TO 42
   41 SA(J)=+1

   42 IF (NB) 56,56,43
   43 IF (ZB(J)) 44,46,45
   44 SB(J)=-SAA(J,J)
      GO TO 50
   45 SB(J)=+SAA(J,J)
      GO TO 50
   46 DO 49 I=1,8
      IF (ZB(I)) 47,49,48
   47 SB(J)=SB(J)-SAA(I,J)
      GO TO 49
   48 SB(J)=SB(J)+SAA(I,J)
   49 CONTINUE
   50 IF (SB(J)) 51,52,53
   51 SB(J)=-1
      GO TO 54
   52 SB(J)=0
      GO TO 54
   53 SB(J)=+1

   54 IF (SA(J)) 55,56,56
   55 SB(J)=-SB(J)
   56 R(J)=SA(J)+SB(J)*AB
```

```
C   CAL. TOTAL SIGNAL ENERGY
       DO 57 J=1,8
    57 TSE=TSE+R(J)*R(J)

C   ADD GAUSSIAN NOISE OF STD. DEV. SD
       DO 58 J=1,8
    58 R(J)=R(J)+G05AEF(0.0,SD)

C   DETECTION OF THE RECEIVED SIGNAL  R(J)+NOISE
       IF (NA-8) 63,59,59
    59 DO 62 J=1,8
       IF (R(J)) 60,61,61
    60 A(J)=-SAA(J,J)
       GO TO 62
    61 A(J)=+SAA(J,J)
    62 CONTINUE
       GO TO 68

    63 OPA=10000
       DO 66 K=1,NNA
       OP=0.0
       DO 64 J=1,8
       XX=XRA(K,J)-R(J)
    64 OP=OP+XX*XX
       IF (OP-OPA) 65,66,66
    65 OPA=OP
       MA=K
    66 CONTINUE

       DO 67 J=1,NA
    67 A(J)=XA(MA,J)

    68 IF (NB) 80,80,69

    69 AAB=1/AB
       DO 70 J=1,8
    70 RR(J)=(ABS(R(J))-1)*AAB

       IF (NB-8) 75,71,71
    71 DO 74 J=1,8
       IF (RR(J)) 72,73,73
    72 B(J)=-SAA(J,J)
       GO TO 74
    73 B(J)=+SAA(J,J)
    74 CONTINUE
       GO TO 80

    75 OPA=10000
       DO 78 K=1,NNB
       OP=0.0
       DO 76 J=1,8
       XX=XRB(K,J)-RR(J)
    76 OP=OP+XX*XX
       IF (OP-OPA) 77,78,78
    77 OPA=OP
       MB=K
    78 CONTINUE

       DO 79 J=1,NB
    79 B(J)=XB(MB,J)
```

```
C   COUNT TOTAL NO OF ERRORS  EA,EB
    80 DO 84 J=1,8
       IF (A(J)-ZA(J)) 81,82,81
    81 EA=EA+1
    82 IF (B(J)-ZB(J)) 83,84,83
    83 EB=EB+1
    84 CONTINUE

   200 CONTINUE

C   CAL. AVERAGE ERROR PROBABILITY  PA,PB
       PA=EA/FLOAT(L*NA)
       IF (NB.EQ.0) GO TO 85
       PB=EB/FLOAT(L*NB)

C   CAL. RELATIVE NOISE LEVEL IN DB
    85 RNL=20*ALOG10(SD/0.364)

C   CAL. AVERAGE SIGNAL ELEMENT ENERGY / TRANSMITTED COMPONENT
       ASE=TSE/(L*8)

C   CAL. RELATIVE SIGNAL ENERGY PER COMPONENT IN DB
       RSE=10*ALOG10(ASE/1.0)

       IF (NB) 88,86,88
    86 WRITE (2,87) NA,NB,AB,SD,L,RNL,EA,PA,RSE
    87 FORMAT (I4,I4,F8.5,F7.5,I8,F9.2,I8,F20.4,21X,F5.2/)
       GO TO 300
    88 IF (NB.EQ.1) WRITE (2,89)
    89 FORMAT (/)
       WRITE (2,90) NA,NB,AB,SD,L,RNL,EA,EB,PA,PB,RSE
    90 FORMAT (I4,I4,F8.5,F7.5,I8,F9.2,I8,I6,F14.4,F14.4,7X,
     1'*',F4.2/)

   300 CONTINUE

       WRITE (2,91)
    91 FORMAT (' ***RB****')

       STOP
       END
```

```
      SUBROUTINE RANDOM (NA,NB,ZA,ZB)
C   SUBROUTINE RANDOM GENERATES TWO 8 COMPONENT VECTORS ZA
C   AND ZB WITH NA AND NB COMPONENTS SET RANDOMLY TO + OR -1

      INTEGER ZA(8),ZB(8)

      DO 10 J=1,8
   10 ZA(J),ZB(J)=0

      DO 13 I=1,NA
      IF (G05AAF(Y)-0.5) 11,11,12
   11 ZA(I)=-1
      GO TO 13
   12 ZA(I)=+1
   13 CONTINUE

      IF (NB) 14,18,14
   14 DO 17 I=1,NB
      IF (G05AAF(Y)-0.5) 15,15,16
   15 ZB(I)=-1
      GO TO 17
   16 ZB(I)=+1
   17 CONTINUE

   18 RETURN
      END

      FINISH
****


DOCUMENT DATA
    1     1     1     1     1     1     1     1
    1    -1     1    -1     1    -1     1    -1
    1     1    -1    -1     1     1    -1    -1
    1    -1    -1     1     1    -1    -1     1
    1     1     1     1    -1    -1    -1    -1
    1    -1     1    -1    -1     1    -1     1
    1     1    -1    -1    -1    -1     1     1
    1    -1    -1     1    -1     1     1    -1
1 0 0.000 1.032 10000
2 0 0.000 0.580 5000
3 0 0.000 0.604 3000
4 0 0.000 0.479 2500
5 0 0.000 0.413 2000
6 0 0.000 0.414 1500
7 0 0.000 0.380 1500
8 0 0.000 0.364 1500
8 1 0.279 0.287 10000
8 2 0.432 0.238 5000
8 3 0.400 0.242 3000
8 4 0.462 0.227 2500
8 5 0.470 0.210 2000
8 6 0.500 0.201 1500
8 7 0.519 0.193 1500
8 8 0.520 0.188 1500
****
```

SYSTEM D1

NO OF CHANNELS (NA)	(NB)	AMP SET B (AB)	GAUSS. NOISE (SD)	NO OF GROUPS (L)	RELATIVE NOISE L. (RNL) DB	NO OF ERRS SETA (EA)	SETB (EB)	ERROR PROBABILITY / CHANNEL SET A (PA)	SET B (PB)	RELATIVE SG.ENERGY (RSE) DB
1	0	0,000	1,03[illegible]	1000[illegible]	9.05	30		0.0030		0.00
2	0	0,000	0,58[illegible]	500[illegible]	4,05	30		0,0030		-2.04
3	0	0,000	0,604	300[illegible]	4,40	28		0.0031		0.00
4	0	0,000	0,479	250[illegible]	2,38	31		0,0031		-0.95
5	0	0,000	0,413	200[illegible]	1,10	30		0.0030		0.00
6	0	0,000	0,414	150[illegible]	1,12	27		0.0030		-0.35
7	0	0,000	0,38[illegible]	150[illegible]	0.37	31		0.0030		0.00
8	0	0,000	0,364	150[illegible]	0,00	36		0.0030		0.00
8	1	0.279	0,287	1000[illegible]	-2,06	246	30	0.0031	0,0030	+0.31
8	2	0.432	0,236	500[illegible]	-3,69	117	30	0.0029	0,0030	+0.47
8	3	0.400	0,242	300[illegible]	-3.55	74	27	0,0031	0.0030	+0.67
8	4	0,462	0,227	250[illegible]	-4.10	61	31	0.0031	0.0031	+0.70
8	5	0.470	0,21[illegible]	200[illegible]	-4,78	48	30	0,0030	0.0030	+0.89
8	6	0.500	0.20[illegible]	150[illegible]	-5.16	36	27	0.0030	0.0030	+0.89
8	7	0.519	0.193	150[illegible]	-5,51	37	32	0,0031	0.0030	+1.01
8	8	0.520	0.186	150[illegible]	-5,74	36	36	0.0030	0.0030	+1.04

Computer simulation program for System D4

```
JOB THD4,E,RBH1522
EDSFILES 1
LUFORTRAN ,,W
JOBCORE 32K
DOWN 22
RUN ,,1700
****
DOCUMENT SOURCE
      LIBRARY (ED,SUBGROUPNAGF)
      WORK(ED,USEREDSFILE1)
      PROGRAM(THD4)
      ABNORMAL FUNCTIONS
      COMPACT
      INPUT 1 = CRO
      OUTPUT 2 = LPO
      COMPRESS INTEGER AND LOGICAL
      TRACE 0
      END

      MASTER SYSTEM D4

C   COMPUTER PROGRAM TO SIMULATE SYSTEM D4. TWO ORTHOGONAL COM
C   SETS , OF 8 CHANNELS EACH ARE MULTIPLEXED , AND DETECTED
C   USING A SUB-OPTIMUM DETECTION PROCESS .

C   SAA,SBB = MATRIX OF CHANNEL CODEWORDS - SET A, SET B
C   NA,NB = NO OF CHANNELS - SET A, SET B
C   AB = LEVEL OF THE SET B CODEWORDS
C   SD = STANDARD DEVIATION OF THE CHANNEL NOISE SAMPLE VALUES
C   L = TOTAL NO. OF GROUPS TRANSMITTED
C   ZA,ZB = CHANNEL ELEMENT VALUES TO BE MULTIPLEXED - SET A,SET B
C.  SA,SB = LINEAR SUM OF SET CODEWORDS - SET A,SET B
C   R = TRANSMITTED SIGNAL VECTOR
C   A,B = DETECTED ELEMENT VALUES - SET A, SET B
C   RNL = RELATIVE NOISE LEVEL IN DB
C   RSE = RELATIVE SIGNAL ENERGY IN DB
C   EA = TOTAL NO. OF ERRORS - SET A
C   PA = AVERAGE ERROR PROBABILITY / CHANNEL - SET A
C   G05BAF(X) INITIALISES RANDOM NUMBER GENERATOR
C   G05AEF(A,B) R.N.G. WITH GAUSSIAN DIST.  MEAN A  ST.DEV. B
C   G05AAF(Y) R,N.G. WITH UNIFORM DIST. BETWEEN 0 AND 1

      INTEGER SAA(8,8),ZA(8),ZB(8),SA(8),SB(8),A(8),B(8),X(8)
      INTEGER XA(256,8),XB(256,8),EA,EB,JN(8)
      DIMENSION R(8),RR(8),XRA(256,8),XRB(256,8)

      CALL G05BAF (1,0)
      READ (1,10) ((SAA(I,J),J=1,8),I=1,8)
   10 FORMAT (8I5)
```

```
      WRITE(2,11)
   11 FORMAT (1H1////45X,'SYSTEM D4'/45X,'---------'//'  NO  OF'
     1,'      AMP    GAUSS.  NO   OF   RELATIVE    NO OF ERRS   ERROR '
     2,'PROBABILITY  / CHANNEL  RELATIVE'/' CHANNELS  SET  B  NO.'
     3,'SE    GROUPS   NOISE L.    SETA   SETB          SET A          '
     4,'SET B        SG ENERGY'/' (NA)(NB)  (AB)      (SD)      (L)    '
     5,'  (RNL)DB     (EA)   (EB)            (PA)               (PB)   '
     6,'(RSE)DB'//)

      DO 300 NNN=1,16
      READ (1,12) NA,NB,AB,SD,L
   12 FORMAT (2I0,2F0.0,I0)
      EA,EB=0
      TSE=0.0

C     FORMATION OF ALL POSSIBLE TRANS. VECTORS XRA(NNA,K),XRB(NNB,K)
      NNA=2**NA
      NNB=2**NB

      DO 31 M=1,2
      IF (M.EQ.2.AND.NB.EQ.0) GO TO 31
      IF (M.EQ.1.AND.IY.EQ.1) GO TO 31
      IF (M.EQ.1) NNO=NNA
      IF (M.EQ.2) NNO=NNB
      IF (M.EQ.1) NO=NA
      IF (M.EQ.2) NO=NB
      DO 13 J=1,8
   13 X(J)=0

      DO 30 K=1,NNO

      DO 15 J=1,NO
      X(J)=X(J)+2
      IF (X(J)-1) 16,16,14
   14 X(J)=-1
   15 CONTINUE

   16 DO 30 J=1,8
      R(J)=0.0
      IF (X(J)) 17,19,18
   17 R(J)=-SAA(J,J)
      GO TO 23
   18 R(J)=+SAA(J,J)
      GO TO 23
   19 DO 22 I=1,8
      IF (X(I)) 20,22,21
   20 R(J)=R(J)-SAA(I,J)
      GO TO 22
   21 R(J)=R(J)+SAA(I,J)
   22 CONTINUE

   23 IF (R(J)) 24,25,26
   24 R(J)=-1
      GO TO 27
   25 R(J)=0
      GO TO 27
   26 R(J)=+1
```

```
   27 IF (M-1) 28,28,29
   28 XA(K,J)=X(J)
      XRA(K,J)=R(J)
      GO TO 30
   29 XB(K,J)=X(J)
      XRB(K,J)=R(J)
   30 CONTINUE
   31 CONTINUE

      IF (NA.EQ.8) IY=1

C.  THE PROGRAM NOW RUNS FOR L TRANSMITTED SIGNAL GROUPS
      DO 200 NN=1,L
      CALL RANDOM (NA,NB,ZA,ZB)

C   FORMATION OF THE TRANSMITTED SIGNAL R(J)
      DO 56 J=1,8
      SA(J),SB(J)=0
      IF (ZA(J)) 32,34,33
   32 SA(J)=-SAA(J,J)
      GO TO 38
   33 SA(J)=+SAA(J,J)
      GO TO 38
   34 DO 37 I=1,8
      IF (ZA(I)) 35,37,36
   35 SA(J)=SA(J)-SAA(I,J)
      GO TO 37
   36 SA(J)=SA(J)+SAA(I,J)
   37 CONTINUE
   38 IF (SA(J)) 39,40,41
   39 SA(J)=-1
      GO TO 42
   40 SA(J)=0
      GO TO 42
   41 SA(J)=+1

   42 IF (NB) 56,56,43
   43 IF (ZB(J)) 44,46,45
   44 SB(J)=-SAA(J,J)
      GO TO 50
   45 SB(J)=+SAA(J,J)
      GO TO 50
   46 DO 49 I=1,8
      IF (ZB(I)) 47,49,48
   47 SB(J)=SB(J)-SAA(I,J)
      GO TO 49
   48 SB(J)=SB(J)+SAA(I,J)
   49 CONTINUE
   50 IF (SB(J)) 51,52,53
   51 SB(J)=-1
      GO TO 54
   52 SB(J)=0
      GO TO 54
   53 SB(J)=+1

   54 IF (SA(J)) 55,56,56
   55 SB(J)=-SB(J)
   56 R(J)=SA(J)+SB(J)*AB
```

```
C  CAL. TOTAL SIGNAL ENERGY
      DO 57 J=1,8
   57 TSE=TSE+R(J)*R(J)

C  ADD GAUSSIAN NOISE OF STD. DEV. SD
      DO 58 J=1,8
   58 R(J)=R(J)+G05AEF(0.0,SD)

C  DETECTION OF THE RECEIVED SIGNAL  R(J)+NOISE
      DO 61 J=1,NA
      IF (R(J)) 59,60,60
   59 A(J)=-SAA(J,J)
      GO TO 61
   60 A(J)=+SAA(J,J)
   61 CONTINUE

      IF (NA-8) 62,74,74

   62 JN(1)=1
      JN(2)=2
      JN(3)=4
      JN(4)=8
      JN(5)=16
      JN(6)=32
      JN(7)=64
      JN(8)=128

      DO 73 NX=1,5

      OPA=10000
      DO 71 I=1,NA+1
      K=1
      DO 64 J=1,NA
      IF (A(J)) 64,63,63
   63 K=K+JN(J)
   64 CONTINUE

      OP=0.0
      DO 65 J=1,8
      XX=XRA(K,J)-R(J)
   65 OP=OP+XX*XX
      IF (OP-OPA) 66,67,67
   66 OPA=OP
      M=I-1

   67 IF (I-1) 69,69,68
   68 A(I-1)=-A(I-1)
   69 IF (I-9) 70,71,71
   70 A(I)=-A(I)
   71 CONTINUE

      IF (M) 74,74,72
   72 A(M)=-A(M)
   73 CONTINUE
```

```
74 IF (NB) 92,92,75

75 AAB=1/AB
   DO 76 J=1,8
76 RR(J)=(ABS(R(J))-1)*AAB

   DO 79 J=1,NB
   IF (RR(J)) 77,78,78
77 B(J)=-SAA(J,J)
   GO TO 79
78 B(J)=+SAA(J,J)
79 CONTINUE

   IF (NB-8) 80,92,92

80 DO 91 NX=1,5

   OPA=10000
   DO 89 I=1,NB+1
   K=1
   DO 82 J=1,NB
   IF (B(J)) 82,81,81
81 K=K+JN(J)
82 CONTINUE

   OP=0,0
   DO 83 J=1,8
   XX=XRB(K,J)-RR(J)
83 OP=OP+XX*XX
   IF (OP-OPA) 84,85,85
84 OPA=OP
   M=I-1

85 IF (I-1) 87,87,86
86 B(I-1)=-B(I-1)
87 IF (I-9) 88,89,89
88 B(I)=-B(I)
89 CONTINUE

   IF (M) 92,92,90
90 B(M)=-B(M)
91 CONTINUE
```

```
C   COUNT TOTAL NO OF ERRORS  EA,EB
    92 DO 96 J=1,8
       IF (A(J)-ZA(J)) 93,94,93
    93 EA=EA+1
    94 IF (B(J)-ZB(J)) 95,96,95
    95 EB=EB+1
    96 CONTINUE

   200 CONTINUE

C   CAL. AVERAGE ERROR PROBABILITY  PA,PB
       PA=EA/FLOAT(L*NA)
       IF (NB.EQ.0) GO TO 97
       PB=EB/FLOAT(L*NB)

C   CAL. RELATIVE NOISE LEVEL IN DB
    97 RNL=20*ALOG10(SD/0.364)

C   CAL. AVERAGE SIGNAL ELEMENT ENERGY / TRANSMITTED COMPONENT
       ASE=TSE/(L*8)

C   CAL. RELATIVE SIGNAL ENERGY PER COMPONENT IN DB
       RSE=10*ALOG10(ASE/1.0)

       IF (NB) 100,98,100
    98 WRITE (2,99) NA,NB,AB,SD,L,RNL,EA,PA,RSE
    99 FORMAT (I4,I4,F8.5,F7.3,I8,F9.2,I8,F20.4,21X,F5.2/)
       GO TO 300
   100 IF (NB.EQ.1) WRITE (2,101)
   101 FORMAT (/)
       WRITE (2,102) NA,NB,AB,SD,L,RNL,EA,EB,PA,PB,RSE
   102 FORMAT (I4,I4,F8.3,F7.3,I8,F9.2,I8,I6,F14.4,F14.4,7X,
      1'+',F4.2/)

   300 CONTINUE

       WRITE (2,103)
   103 FORMAT (' ***RBH***')

       STOP
       END
```

```
      SUBROUTINE RANDOM (NA,NB,ZA,ZB)
C   SUBROUTINE RANDOM GENERATES TWO 8 COMPONENT VECTORS ZA
C   AND ZB WITH NA AND NB COMPONENTS SET RANDOMLY TO + OR -1

      INTEGER ZA(8),ZB(8)

      DO 10 J=1,8
   10 ZA(J),ZB(J)=0

      DO 13 I=1,NA
      IF (G05AAF(Y)-0.5) 11,11,12
   11 ZA(I)=-1
      GO TO 13
   12 ZA(I)=+1
   13 CONTINUE

      IF (NB) 14,18,14
   14 DO 17 I=1,NB
      IF (G05AAF(Y)-0.5) 15,15,16
   15 ZB(I)=-1
      GO TO 17
   16 ZB(I)=+1
   17 CONTINUE

   18 RETURN
      END

      FINISH
****
```

```
DOCUMENT DATA
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     1    -1     1    -1     1    -1     1    -1
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1 0 0.000 1.032 10000
2 0 0.000 0.580 5000
3 0 0.000 0.565 3000
4 0 0.000 0.477 2500
5 0 0.000 0.413 2000
6 0 0.000 0.414 1500
7 0 0.000 0.380 1500
8 0 0.000 0.364 1500
8 1 0.279 0.287 10000
8 2 0.432 0.238 5000
8 3 0.424 0.232 3000
8 4 0.464 0.226 2500
8 5 0.473 0.207 2000
8 6 0.500 0.201 1500
8 7 0.519 0.193 1500
8 8 0.520 0.188 1500
****
```

SYSTEM D4

NO OF CHANNELS (NA)	(NB)	AMP SET B (AB)	GAUSS. NOISE (SD)	NO OF GROUPS (L)	RELATIVE NOISE L. (RNL)DB	NO OF ERRS SETA (EA)	SETB (EB)	ERROR PROBABILITY / CHANNEL SET A (PA)	SET B (PB)	RELATIVE SG.ENERGY (RSE)DB
1	0	0.000	1,032	1000	9,05	30		0.0030		0.00
2	0	0.000	0.58[illegible]	500	4.05	30		0.0030		-2.04
3	0	0.000	0.565	300	3,82	27		0.0030		0.00
4	0	0.000	0,477	250	2,55	31		0.0031		-0.95
5	0	0.000	0,413	200	1,10	30		0.0030		0.00
6	0	0.000	0,414	150	1,12	27		0.0030		-0.35
7	0	0.000	0.38[illegible]	150	0.37	31		0.0030		0.00
8	0	0.000	0,364	150	0.00	36		0.0030		0.00
8	1	0.279	0,287	1000	-2,06	246	30	0.0031	0.0030	+0.31
8	2	0.432	0,236	500	-3,69	117	30	0.0029	0.0030	+0.47
8	3	0.424	0,232	300	-3,91	72	28	0.0030	0.0031	+0.75
8	4	0.464	0.226	250	-4,14	59	30	0.0029	0.0030	+0.71
8	5	0.473	0.207	200	-4.90	46	30	0.0029	0.0030	+0.90
8	6	0.500	0,201	150	-5,16	36	27	0.0030	0.0030	+0.89
8	7	0,519	0,193	150	-5,51	37	32	0.0031	0.0030	+1.01
8	8	0.520	0,186	150	-5,74	36	36	0.0030	0.0030	+1.04

APPENDIX A3

CIRCUIT DIAGRAM FOR THE HARDWARE MODEL OF SYSTEM D4

The following detailed circuit diagrams are shown for the hardware model described in Chapter 9.

Description of integrated circuits

7400	Quad 2 input NAND
7402	Quad 2 input NOR
7404	Hex inverter
7408	Quad 2 input AND
7413	Dual 4 input NAND schmitt
7430	Eight input NAND
7432	Quad 2 input OR
7472	Gated MS flip flop
7473	Dual JK MS flip flop
7474	Dual D flip flop
7483	4 bit binary full adder
7485	4 bit comparator
7486	Quad 2 input EX OR
7490	Decade counter
7493	4 bit binary counter
7495	4 bit shift register P1/PO
74126	Tristate bus driver
710	Differential comparator
741	Operational amplifier

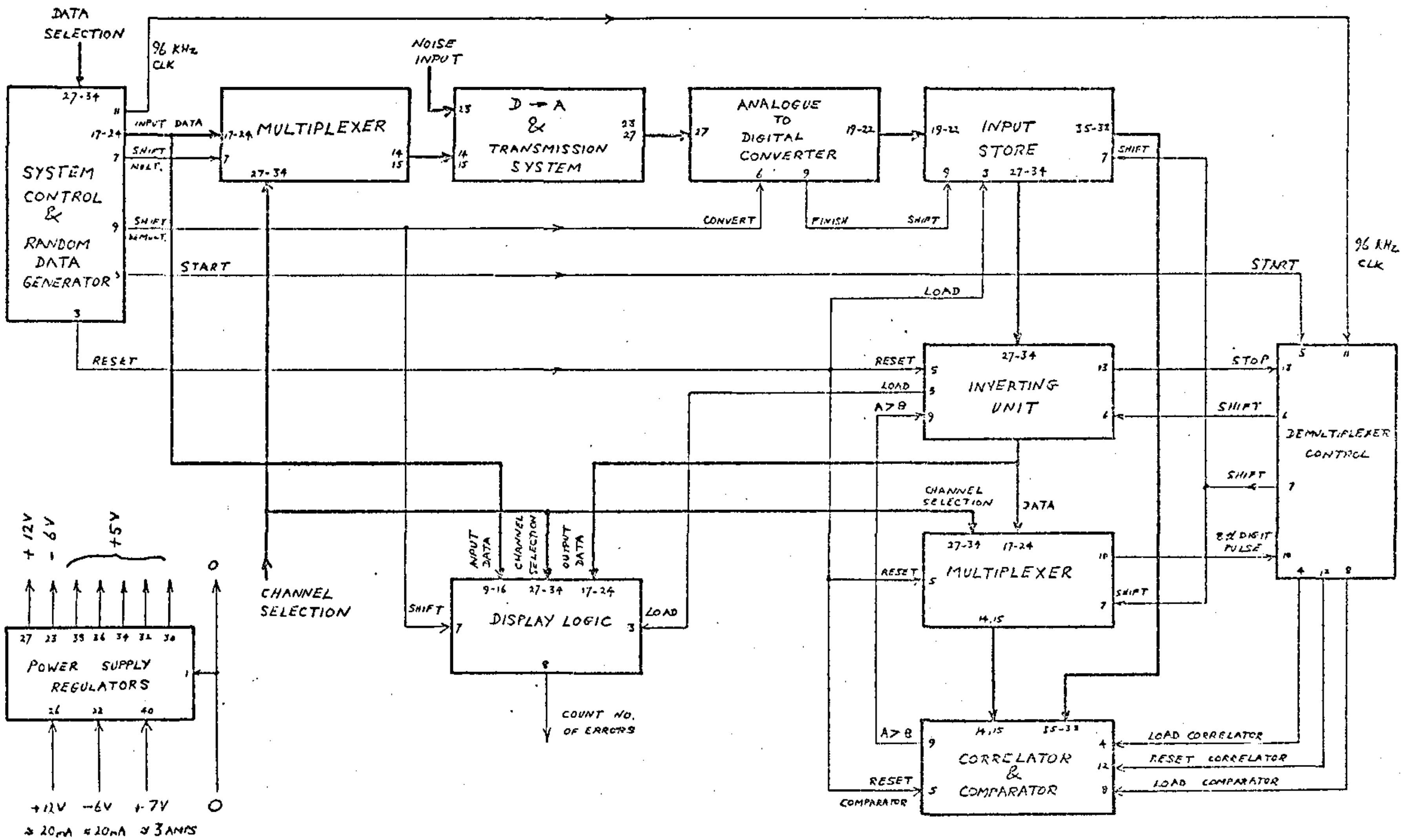

Figure A3-1 Flow diagram for the complete system

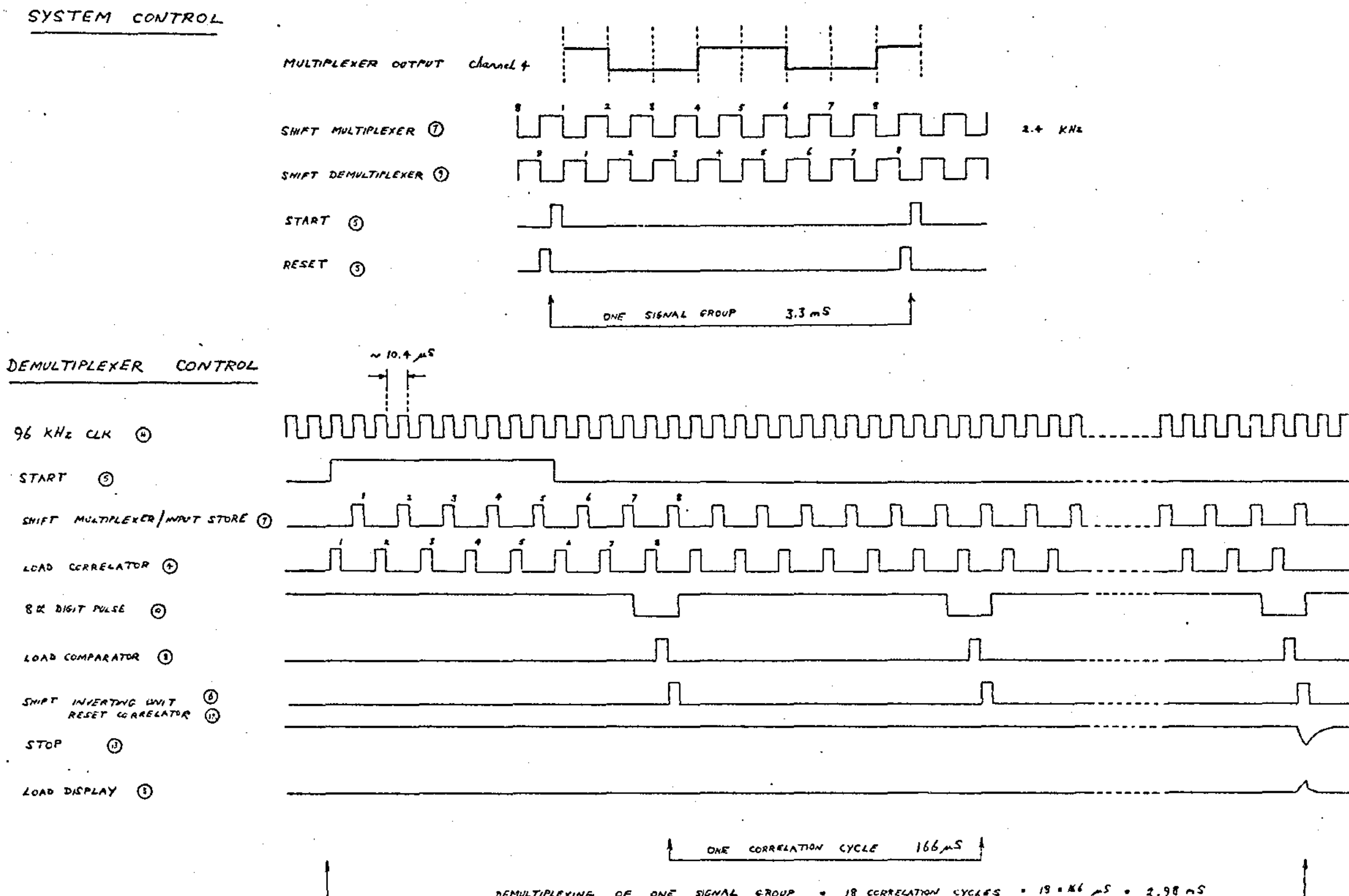

Figure A3-2 Control logic waveforms

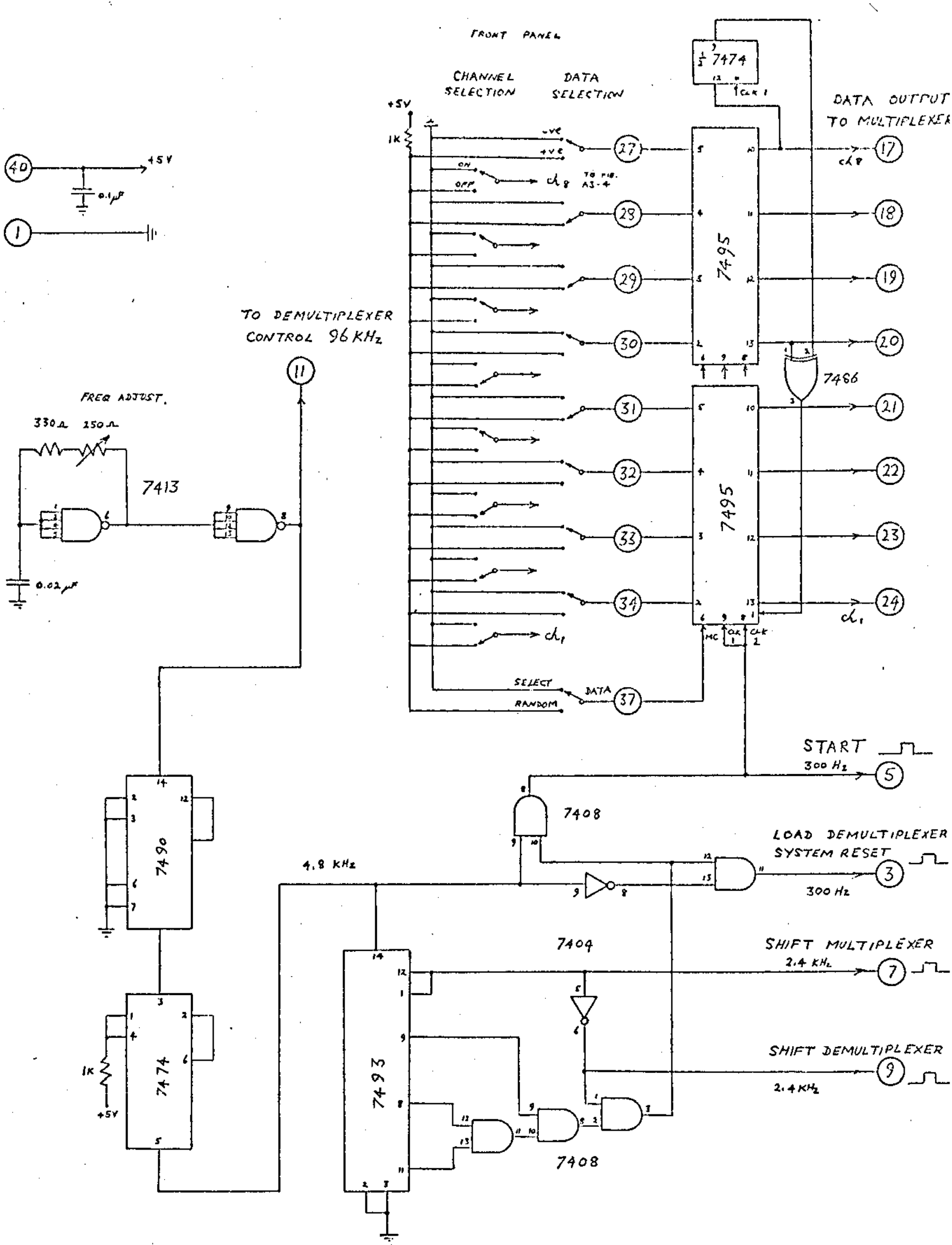

Figure A3-3 System control and random data generator

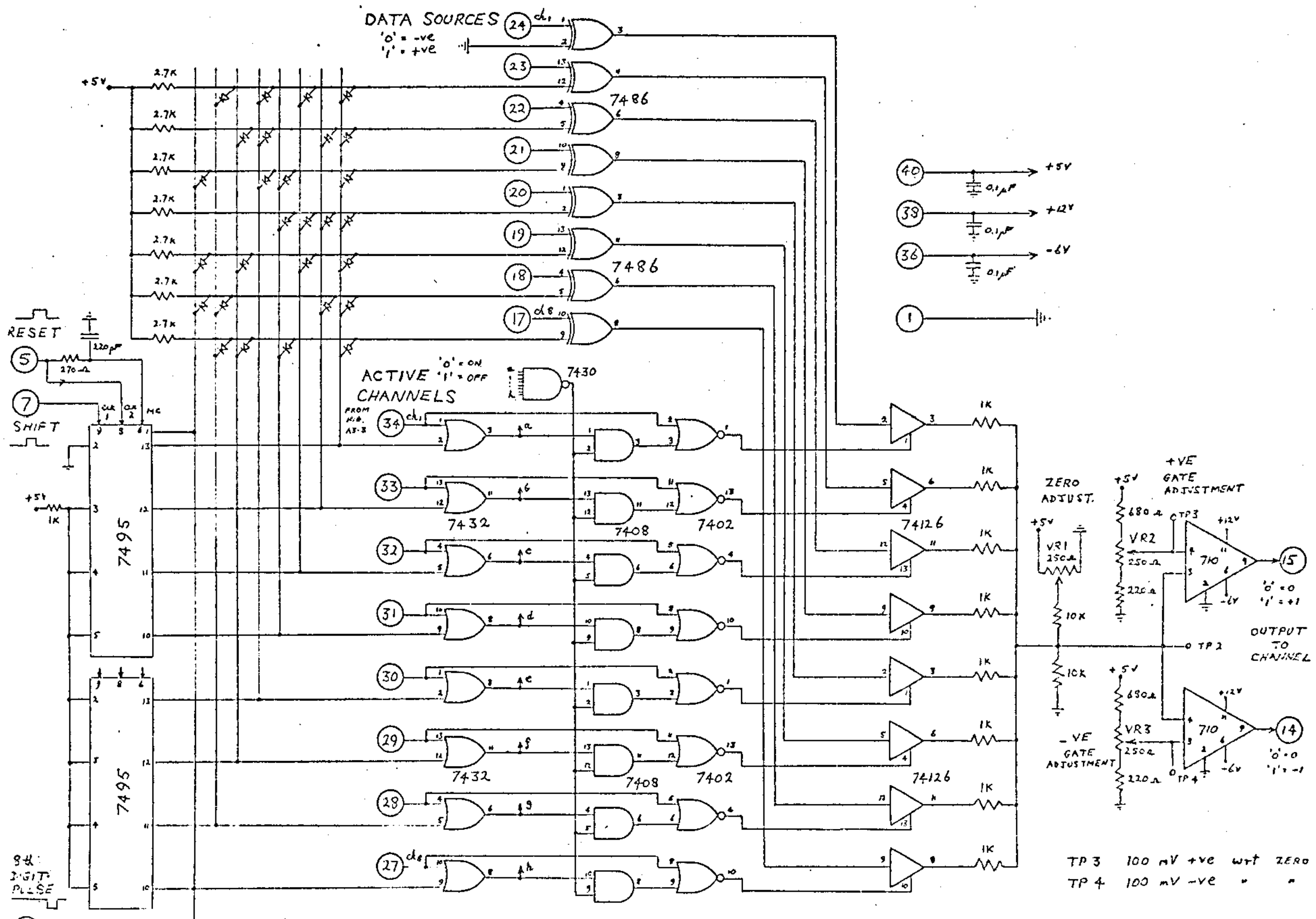

Figure A3-4 Multiplexer

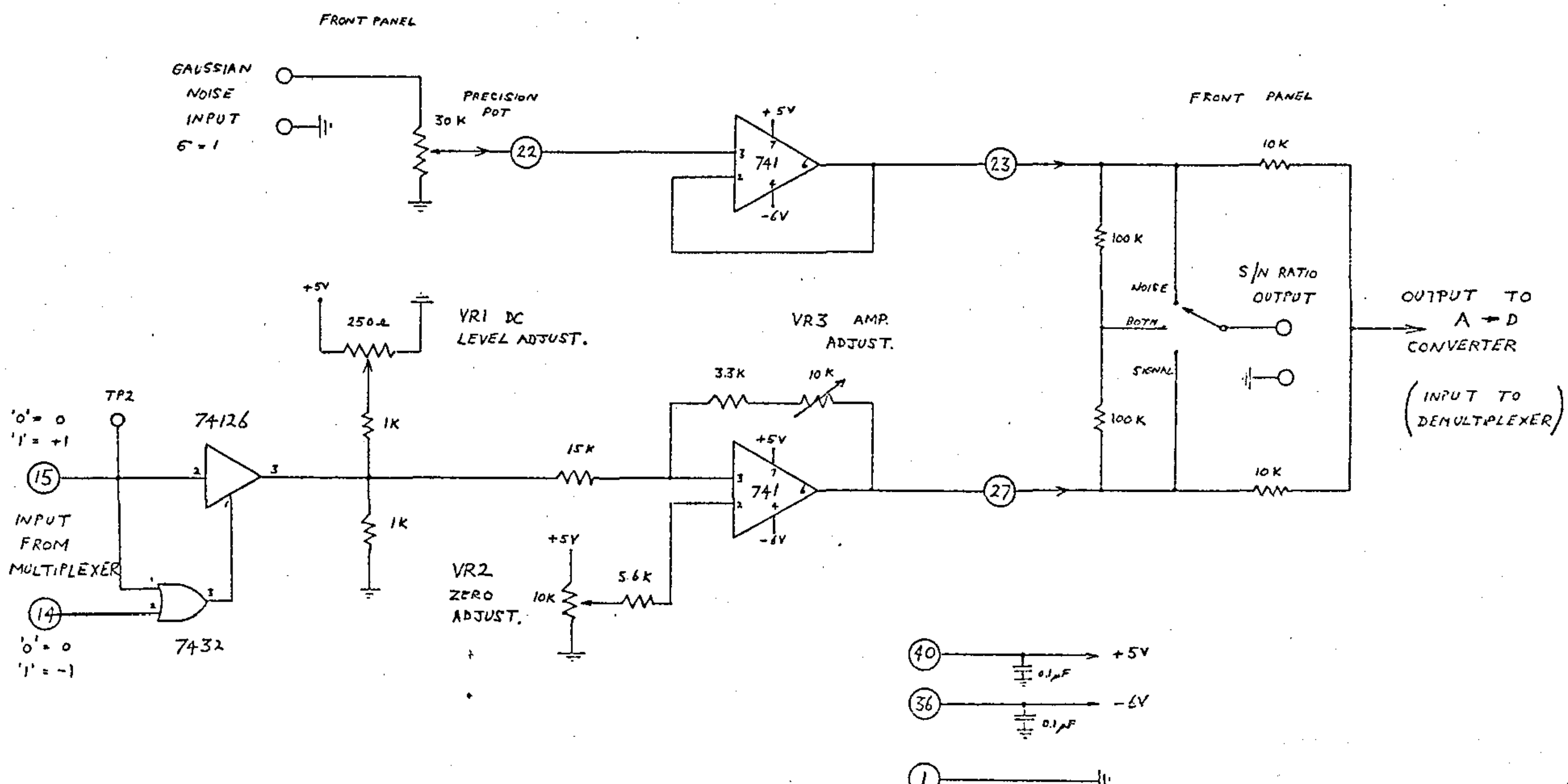

Figure A3-5 Transmission channel

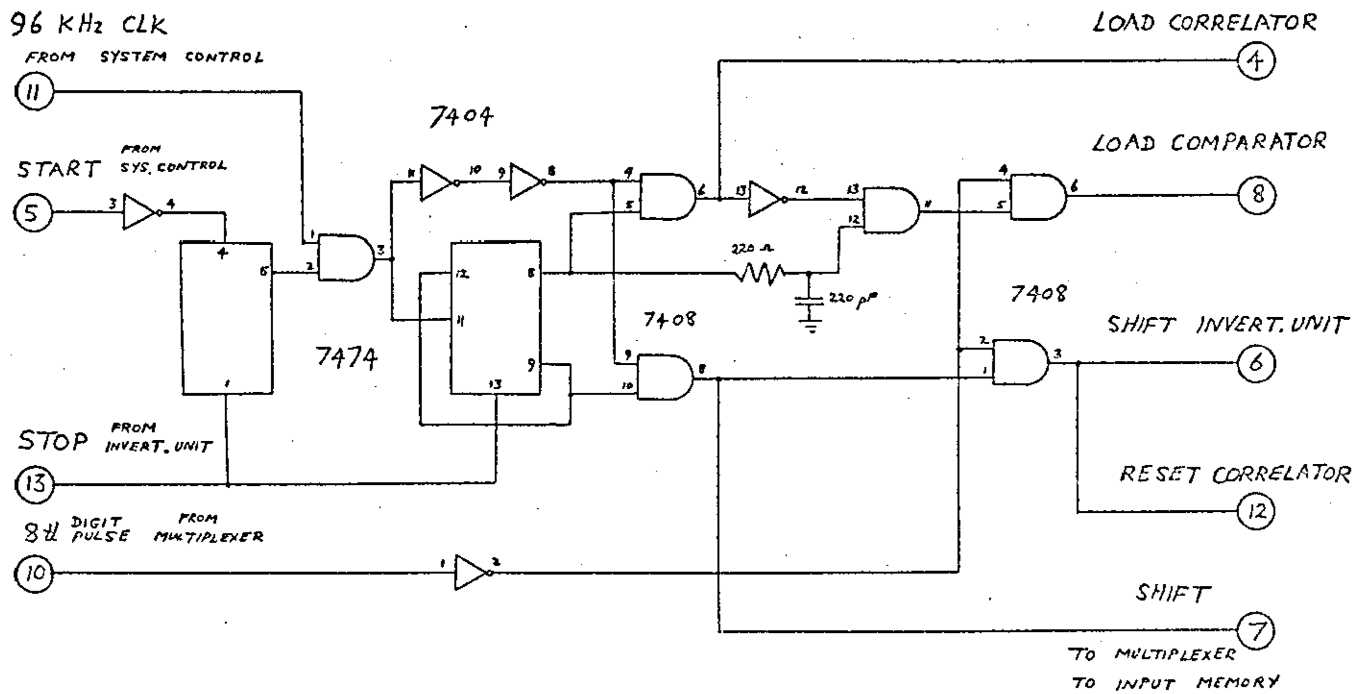

Figure A3-6 Demultiplexer control

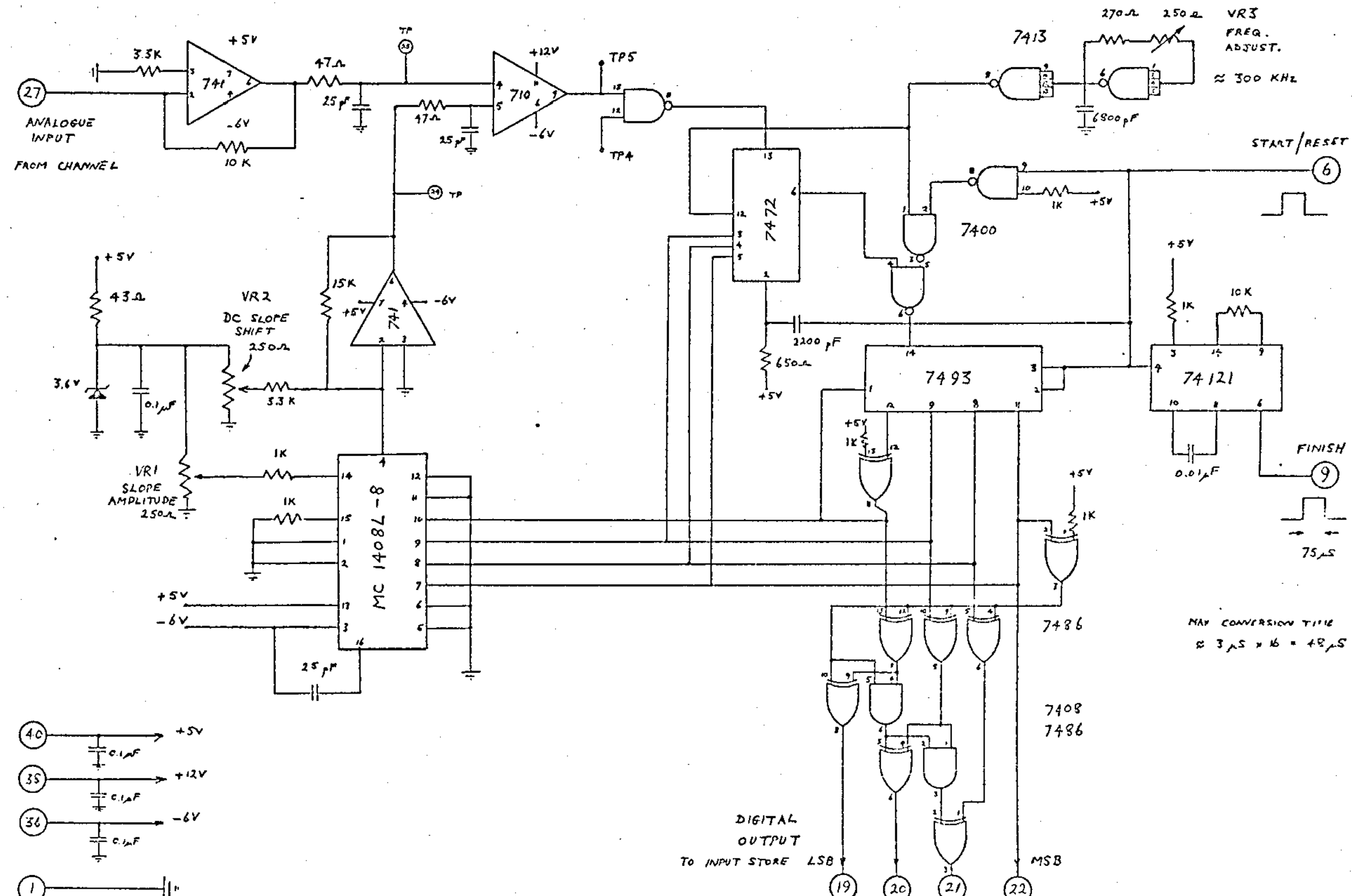

Figure A3-7 Analogue to digital converter

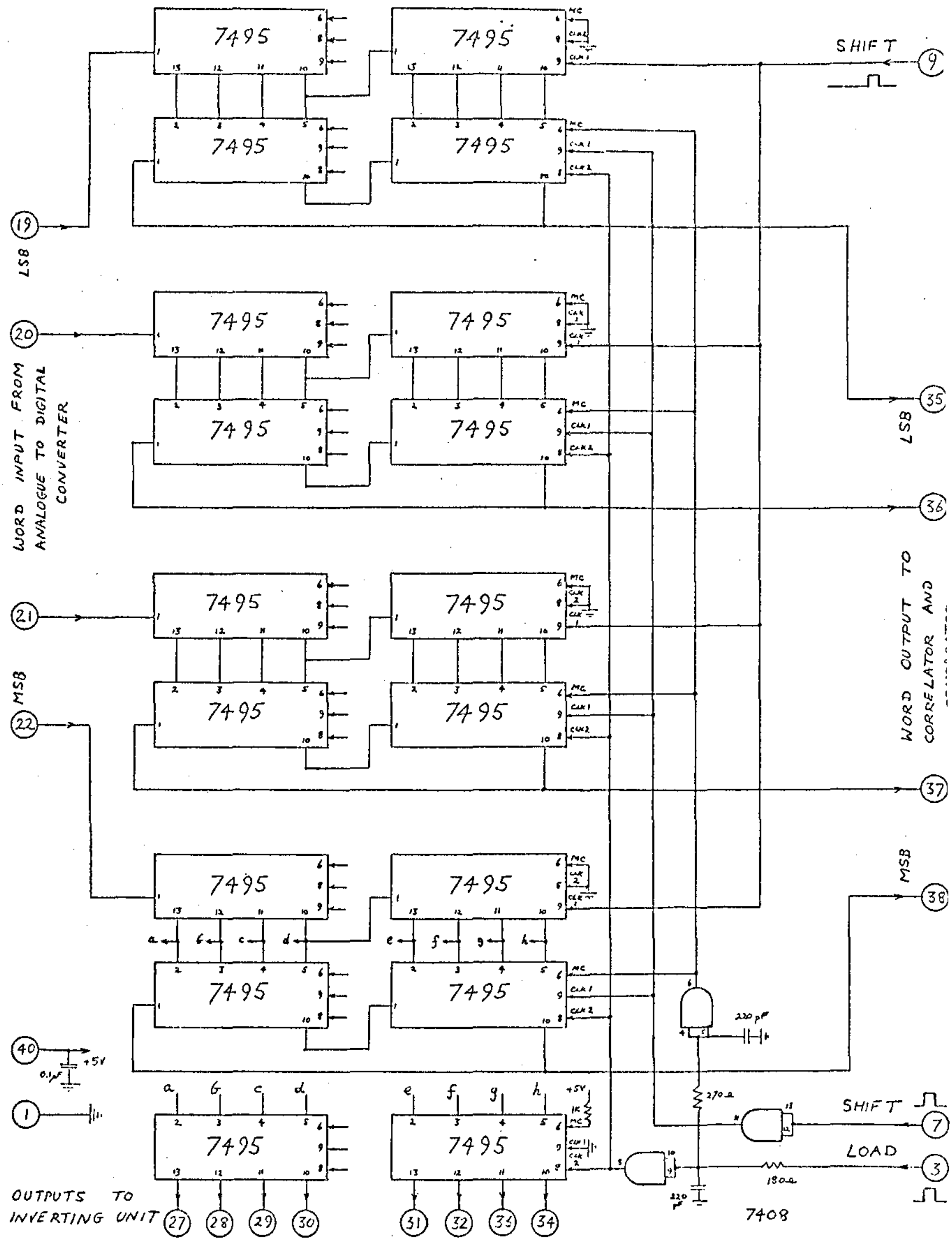

Figure A3-8 Input store

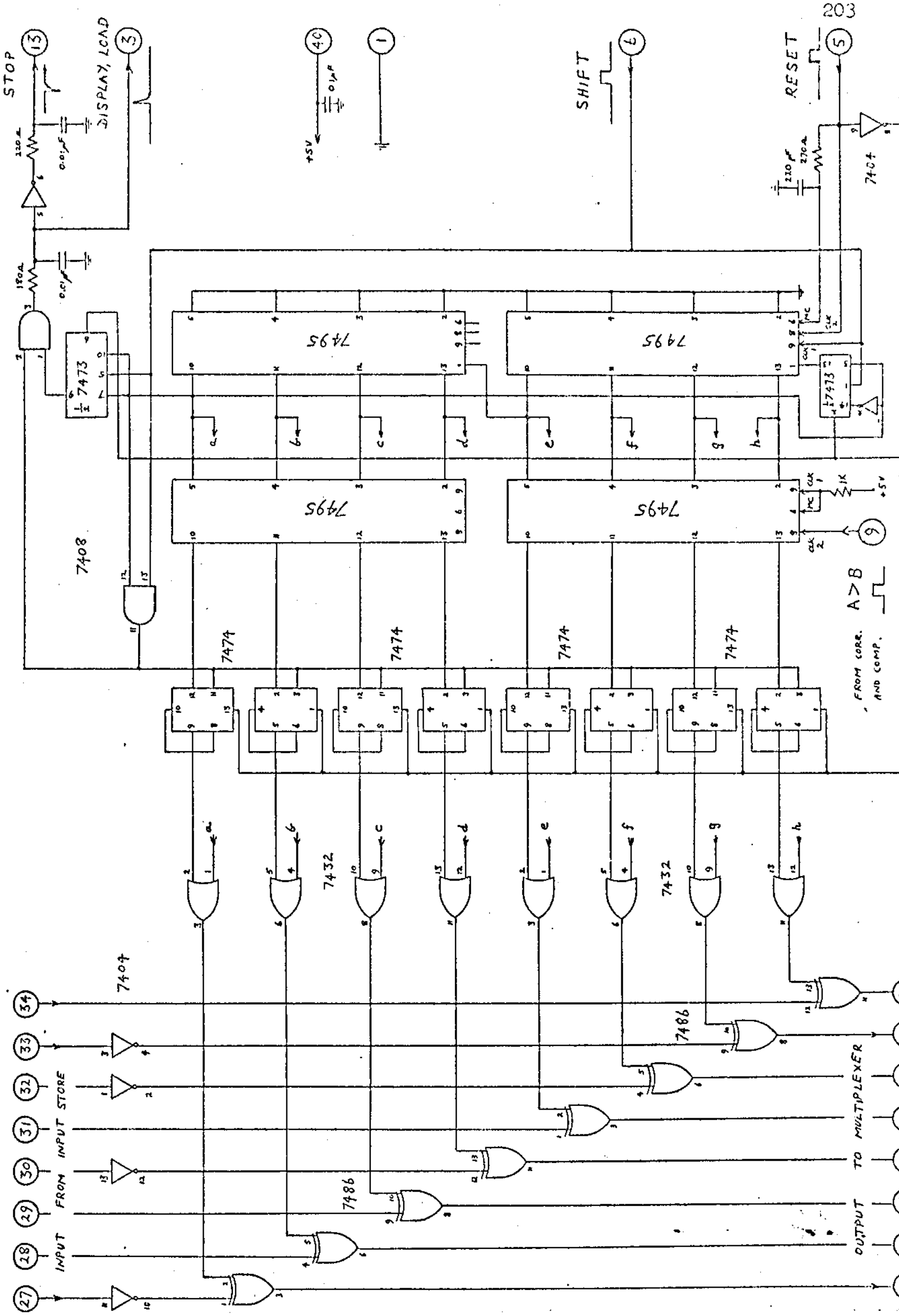

Figure A3-9 Inverting unit

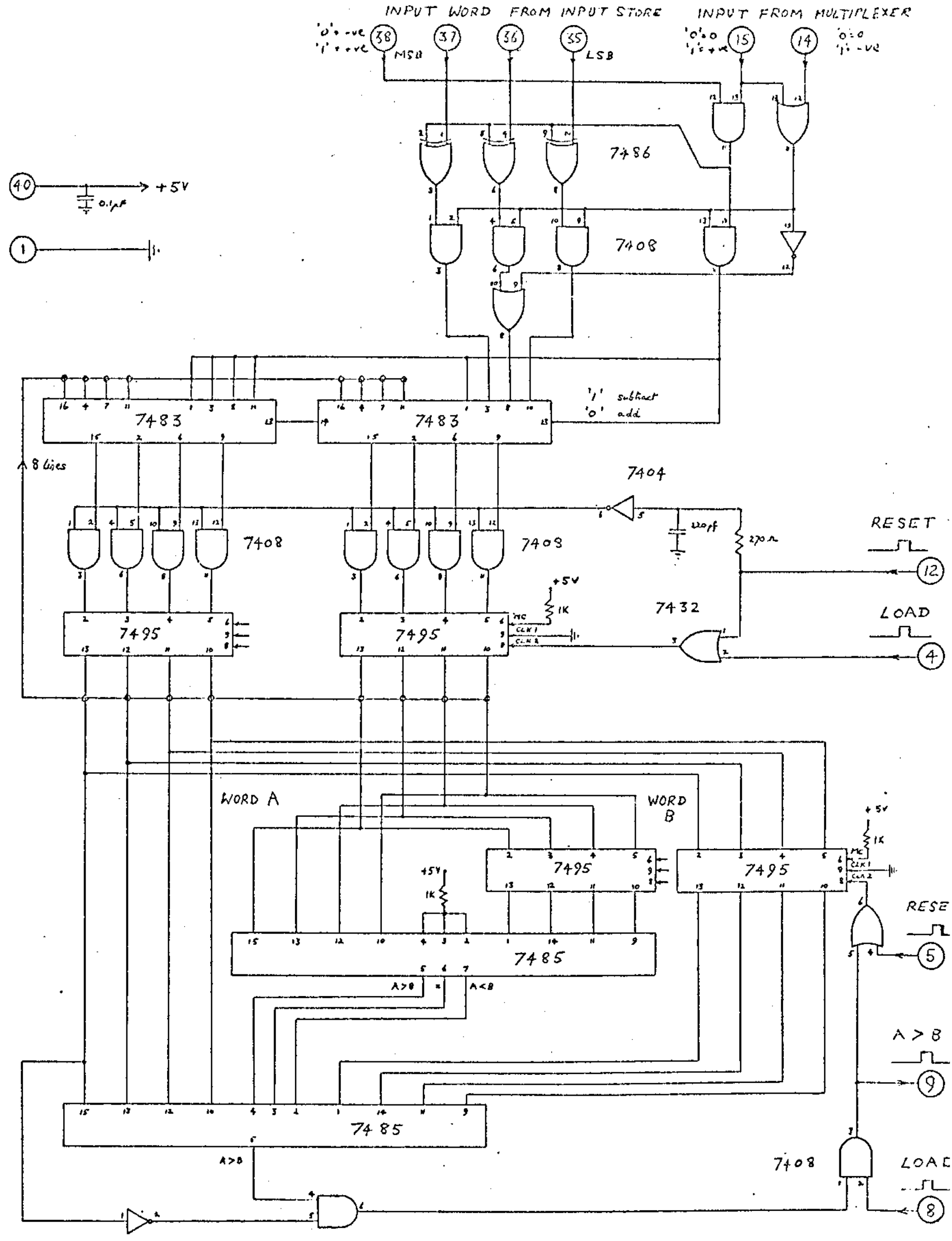

Figure A3.10 Correlator and comparator

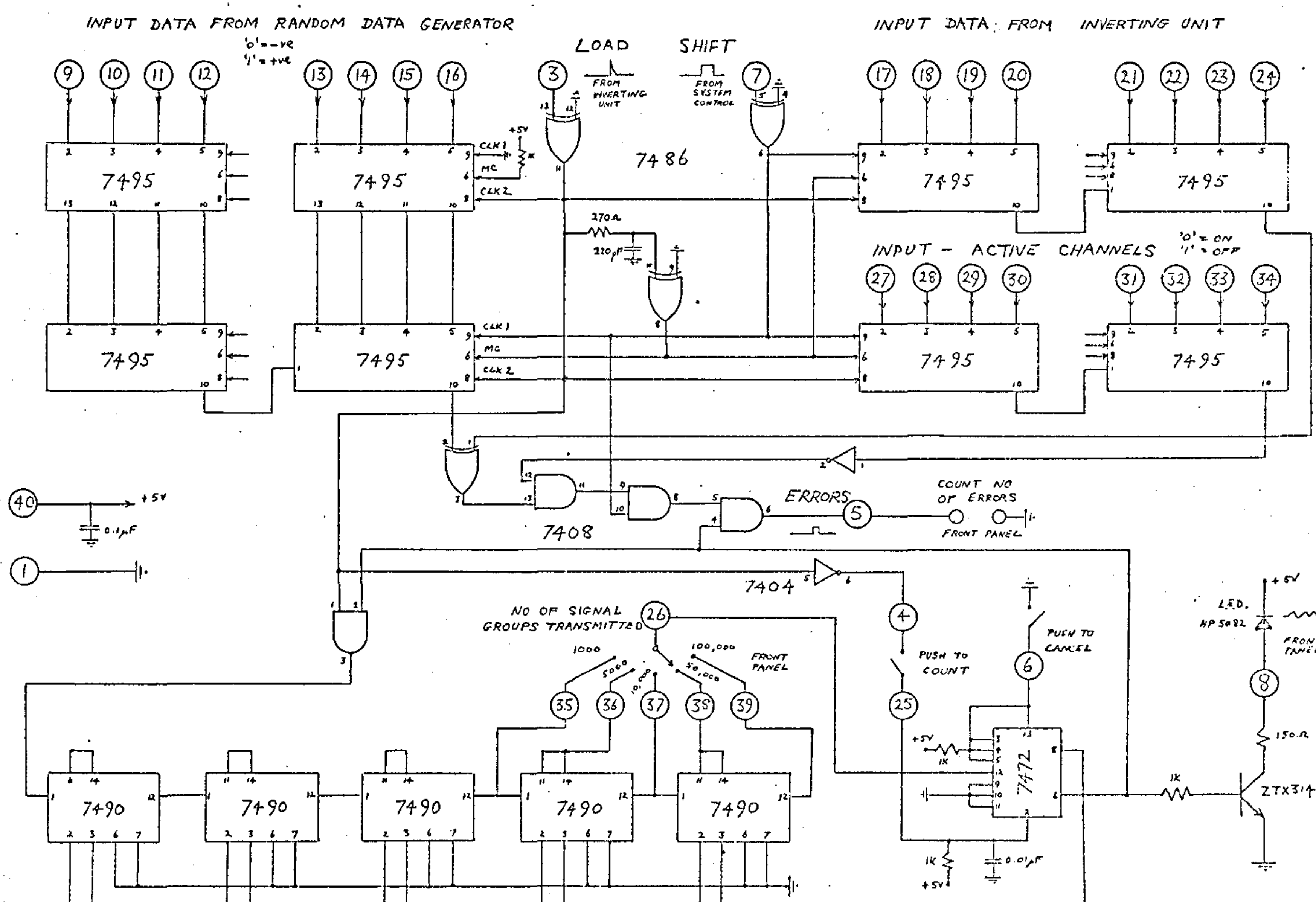

Figure A3-11 Display logic

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