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## Uncertainty due to misalignment in Laser Vibrometry

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# UNCERTAINTY DUE TO MISALIGNMENT IN LASER VIBROMETRY 

## BY

## MARIO TIRABASSI

## A DOCTORAL THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF

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## Nomenclature

## Roman Characters

| $a(t)$ | Target vibration displacement |
| :---: | :---: |
| $\hat{b}$ | Unit vector for the incident beam |
| $\hat{b}_{1}$ | Unit vector for the outgoing beam |
| $\hat{b}_{2}$ | Unit vector for the deflected beam |
| $\hat{b}_{3}$ | Unit vector of the deflected beam |
| $\hat{b}_{4}$ | Unit vector of the deflected beam |
| $\hat{b}_{5}$ | Unit vector of the deflected beam |
| $\hat{b}_{I}$ | Unit vector for the incident beam |
| $\hat{b}_{R}$ | Unit vector for the reflected beam |
| $\hat{b}_{T}$ | Unit vector for the refracted beam |
| $\|\overrightarrow{B D}\|$ | Dimension of the Dove prism |
| $\|\overrightarrow{D E}\|$ | Thickness of the second wedge |
| $d$ | Scan diameter for the Sever self-tracking system |
| $\vec{d}$ | Vector situated in the plane of the beams perpendicular to the beam direction |
| $d_{s}$ | Distance between the mirrors of the dual mirror SLDV system |
| $f_{D}$ | Frequency shift produced by a moving object |
| $f_{R}$ | Reference beam frequency shift |
| $F T[]$ | Fourier Transform |
| $h$ | Distance between the beams of a Laser Rotational Vibrometer (LRV) |
| $I_{M}$ | Measuring beam intensity |
| $I_{R}$ | Reference beam intensity |


| $I_{\text {res }}$ | Resultant amplitude of the Doppler signal |
| :---: | :---: |
| $\hat{n}$ | Surface normal unit vector |
| $\hat{n}_{B}$ | Surface normal unit vector at the point B |
| $\hat{n}_{B^{\prime}}$ | Surface normal unit vector at the point B' |
| $\hat{n}_{C}$ | Surface normal unit vector at the point C |
| $\hat{n}_{C^{\prime}}$ | Surface normal unit vector at the point $\mathrm{C}^{\prime}$ |
| $\hat{n}_{D}$ | Surface normal unit vector at the point D |
| $\hat{n}_{D^{\prime}}$ | Surface normal unit vector at the point D' |
| $\hat{n}_{E}$ | Surface normal unit vector at the point E |
| $\hat{n}_{E^{\prime}}$ | Surface normal unit vector at the point $\mathrm{E}^{\prime}$ |
| R | Ratio for cross-sensitivity |
| $r_{s}$ | Radius of the desired scan pattern |
| $\vec{U}$ | Vector velocity of the moving object |
| $U_{m}$ | Measured component of target velocity |
| $U_{x}$ | x - direction radial vibration measurement |
| $\tilde{U}_{x}$ | AC coupled $x$ - direction radial vibration measurement |
| $U_{y}$ | y - direction radial vibration measurement |
| $\tilde{U}_{y}$ | AC coupled y - direction radial vibration measurement |
| $\overrightarrow{V_{B}}$ | Instantaneous velocity of the point B |
| $\overrightarrow{V_{B^{\prime}}}$ | Instantaneous velocity of the point B, |
| $\overrightarrow{V_{C}}$ | Instantaneous velocity of the point C |
| $\overrightarrow{V_{C^{\prime}}}$ | Instantaneous velocity the point $\mathrm{C}^{\prime}$ |
| $\overrightarrow{V_{D^{\prime}}}$ | Instantaneous velocity of the point $\mathrm{D}^{\prime}$ |
| $\overrightarrow{V_{E^{\prime}}}$ | Instantaneous velocity of the point $\mathrm{E}^{\prime}$ |
| $\overrightarrow{V_{f}}$ | Deformation vibration velocity of the point P due to cross-flexibility |
| $\overrightarrow{V_{K}}$ | Instantaneous velocity of the point K |


| $\overrightarrow{V_{K^{\prime}}}$ | Instantaneous velocity of the point $\mathrm{K}^{\prime}$ |
| :---: | :---: |
| $\overrightarrow{V_{M}}$ | Instantaneous velocity of the mirror |
| $\overrightarrow{V_{O}}$ | Instantaneous velocity of the point O |
| $\overrightarrow{V_{P}}$ | Instantaneous velocity at the point P |
| $\overrightarrow{V_{P 1}}$ | Instantaneous velocitiy at the point $\mathrm{P}_{1}$ |
| $\overrightarrow{V_{P 2}}$ | Instantaneous velocity at the point $\mathrm{P}_{2}$ |
| $W(\omega)$ | Frequency dependent weighting function |
| $x$ | Target translational vibration displacement in x - direction |
| $\hat{x}$ | x - direction unit vector |
| $\dot{x}$ | Velocity in x - direction |
| $x_{0}$ | Known point x - coordinate |
| $\dot{X}(\omega)$ | Fourier Transform of vibration velocity in x - direction at |
| $x_{\text {A }}$ | Coordinate of the point A along the x - axis |
| $x_{\text {B }}$ | Coordinate of the point $B$ along the x - axis |
| $x_{B^{\prime}}$ | Coordinate of the point $\mathrm{B}^{\prime}$ along the x - axis |
| $x_{\text {C }}$ | Coordinate of the point C along the x - axis |
| $\dot{x}_{C^{\prime}}$ | Velocity of the point $C^{\prime}$ along the x - axis |
| $x_{\text {D }}$ | Coordinate of the point D along the x - axis |
| $x_{E}$ | Coordinate of the point E along the x - axis |
| $x_{M}$ | Coordinate of the measuring point situated at the mirror along the x - axis |
| $y$ | Target translational vibration displacement in y-direction |
| $\hat{y}$ | $y$-direction unit vector |
| $\dot{y}$ | Velocity in y- direction |
| $y_{0}$ | Known point y- coordinate |
| $\dot{Y}(\omega)$ | Fourier Transform of vibration velocity in $y$ - direction at |
| $y_{A}$ | Coordinate of the point A along the y - axis |
| $y_{B}$ | Coordinate of the point B along the y - axis |
| $y_{B^{\prime}}$ | Coordinate of the point $\mathrm{B}^{\prime}$ along the y - axis |


| $y_{D}$ | Coordinate of the point D along the y - axis |
| :--- | :--- |
| $y_{E}$ | Coordinate of the point E along the y - axis |
| $y_{M}$ | Coordinate of the measuring point situated at the mirror |
| $\hat{z}_{R}$ | Target rotation axis direction |
| $z$ | Target translational vibration displacement in z- direction |
| $\hat{z}$ | z- direction unit vector |
| $\dot{z}$ | Velocity in z- direction |
| $z_{0}$ | Known point z- coordinate |
| $z_{0}$ | Distance target-scanning head of the dual mirror SLDV system along the |
|  | z- axis (for dual mirror SLDV system) |
| $z_{A}$ | Coordinate of the point A along the z - axis |
| $z_{B}$ | Coordinate of the point E along the z - axis |
| $z_{E}$ | Velocity of the mirror along the z- axis |
| $\dot{z}_{m}$ | Unit vector for the wedge spin axis |
| $\hat{z}_{w}$ |  |

## Greek Characters

| $\alpha$ | Laser beam orientation about z-axis |
| :--- | :--- |
| $\alpha_{F}$ | Rotation angle of the fold mirror around the x - axis |
| $\alpha_{L}$ | Rotation angle of the laser source around the x - axis |
| $\alpha_{m}$ | Rotation of the mirror around x -axis |
| $\dot{\alpha}_{m}$ | Mirror angular velocity around the x - axis |
| $\alpha_{m L}$ | Rotation angle of the laser head around the x - axis (angular misalignment) |
| $\alpha_{m}(t)$ | Time varying mirror angular velocity around the x - axis |
| $\alpha_{m 0}$ | Initial mirror inclinations around the x - axis |
| $\alpha_{m 1}$ | Rotation angle of the first scanning mirror around the x - axis |
| $\alpha_{m 2}$ | Rotation angle of the second scanning mirror around the x - axis |
| $\alpha_{P}$ | Rotation angle of the Dove prism around the x - axis |


| $\beta$ | Laser beam orientation about y-axis |
| :---: | :---: |
| $\beta_{F}$ | Rotation angle of the fold mirror around the y - axis |
| $\beta_{L}$ | Rotation angle of the laser source around the y - axis |
| $\beta_{m}$ | Rotation of the mirror around x -axis |
| $\dot{\beta}_{m}$ | Mirror angular velocity around the y - axis |
| $\beta_{m L}$ | Rotation angle of the laser head around the y - axis (angular misalignment) |
| $\beta_{m}(t)$ | Time varying mirror angular velocity around the y - axis |
| $\beta_{m 0}$ | Initial mirror inclinations around the $y$ - axis |
| $\beta_{m 1}$ | Rotation angle of the first scanning mirror around the y - axis |
| $\beta_{m 2}$ | Rotation angle of the second scanning mirror around the y - axis |
| $\beta_{P}$ | Rotation angle of the Dove prism around the y - axis |
| $\delta_{x}$ | Angular offset of the x - LDV around the y - axis |
| $\delta_{y}$ | Angular offset of the y - LDV around the y - axis |
| $\gamma$ | angle |
| $\gamma_{a}$ | Amplitude of the oscillations added to the wedges |
| $\gamma_{C}$ | Rotation angle for the conical mirror |
| $\gamma_{L}$ | Rotation angle of the laser source around the z - axis |
| $\gamma_{m 1}$ | Rotation angle of the first scanning mirror around the z - axis |
| $\gamma_{m 2}$ | Rotation angle of the second scanning mirror around the z - axis |
| $\gamma_{w}$ | Whole body wedge rotation |
| $\gamma_{w 1}$ | Rotation angle for the first wedge |
| $\gamma_{w 2}$ | Rotation angle for the second wedge |
| $\gamma_{P}$ | Rotation angle for the Dove prism |
| $\gamma_{V}$ | Rotation angle for the vertex mirror |
| $\Delta U_{m}$ | Measured velocity obtained as difference between the frequency shift of two beams |
| $\Delta x_{\text {A }}$ | Translational displacement of the point A along the x - axis |
| $\Delta x_{B}$ | Translational displacement of the point B along the x - axis |


| $\Delta x_{\text {C }}$ | Translational displacement of the point C along the y - axis |
| :---: | :---: |
| $\Delta x_{D}$ | Translational displacement of the point D along the x - axis |
| $\Delta x_{E}$ | Translational displacement of the point E along the x - axis |
| $\Delta y_{\text {A }}$ | Translational displacement of the point A along the y - axis |
| $\Delta y_{B}$ | Translational displacement of the point B along the y - axis |
| $\Delta y_{C}$ | Translational displacement of the point C along the y - axis |
| $\Delta y_{D}$ | Translational displacement of the point D along the y - axis |
| $\Delta y_{E}$ | Translational displacement of the point C along the y - axis |
| $\Delta z_{\text {A }}$ | Translational displacement of the point A along the z - axis |
| $\Delta z_{B}$ | Translational displacement of the point B along the z - axis |
| $\Delta z_{\text {C }}$ | Translational displacement of the point C along the y - axis |
| $\Delta z_{D}$ | Translational displacement of the point D along the z - axis |
| $\Delta z_{E}$ | Translational displacement of the point C along the z - axis |
| $\Delta \phi$ | Relative initial angular position of the wedges |
| $\Delta \Omega_{T}$ | Total target torsional oscillation |
| $\varepsilon_{a}$ | Refractive indices of air |
| $\varepsilon_{I}$ | Index of refraction for the first material for Snell Law |
| $\varepsilon_{P}$ | Refractive index of the Dove prism |
| $\varepsilon_{T}$ | Index of refraction for the second material for Snell Law |
| $\varepsilon_{w}$ | Refractive indices of the wedge |
| $\varepsilon_{w 2}$ | Refractive index of the second wedge |
| $\varepsilon_{x}$ | Angular offset of the x -LDV around the z -axis |
| $\varepsilon_{y}$ | Angular offset of the y -LDV around the z -axis |
| $\Phi_{\text {res }}$ | Resultant phase |
| $\theta_{I}$ | Angle of incidence |
| $\theta_{\text {R }}$ | Angle of reflection |
| $\theta_{s x}$ | x - deflection mirror scan angle |
| $\theta_{\text {sy }}$ | $y$ - deflection mirror scan angle |


| $\theta_{x}$ | Target rotational vibration displacement about x - axis (pitch) |
| :---: | :---: |
| $\dot{\theta}_{x}$ | Target rotational vibration velocity about x - axis |
| $\dot{\theta}_{X}$ | Mirror oscillations around the x - axis |
| $\theta_{x 0}$ | Initial deflection mirror scan angle |
| $\theta_{x}(t)$ | Total x-deflection mirror scan angle |
| $\theta_{y}$ | Target rotational vibration displacement about x - axis (yaw) |
| $\dot{\theta}_{y}$ | Target rotational vibration velocity about y - axis |
| $\dot{\theta}_{Y}$ | Mirror oscillations around the y - axis |
| $\theta_{y 0}$ | Initial deflection mirror scan angle |
| $\theta_{y}(t)$ | Total y-deflection mirror scan angle |
| $\dot{\theta}_{z}$ | Target rotational vibration velocity about z - axis |
| $\dot{\Theta}_{x}$, | Pitch vibration measurement |
| $\dot{\Theta}_{y}$ | Yaw vibration measurement |
| k | Light wave number |
| $\lambda$ | Laser wavelength |
| $\phi_{M}$ | Measuring beam phase |
| $\phi_{R}$ | Reference beam phase |
| $\phi_{w 1}$ | Initial phase of the first wedge |
| $\phi_{w 2}$ | Initial phase of the second wedge |
| $\phi_{1}$ | Initial phase of the first scanning mirror |
| $\phi_{2}$ | Initial phase of the second scanning mirror |
| $\varphi$ | Inclination of the beams with respect to the spin axis of the target |
| $\varphi_{P}$ | Initial phase of the Dove prism |
| $\varphi_{T}$ | Initial phase of the vertex mirror |
| $\varphi_{w}$ | Wedge initial phase |
| $\omega$ | In-plane target vibration frequency |
| $\omega_{V}$ | Target vibration frequency |
| $\vec{\omega}$ | Angular velocity of the point $\mathrm{P}_{\mathrm{O}}$ about an instantaneous axis passing |

through O

| $\rho$ | Angle between the velocity vector and the bisector of the angle formed |
| :--- | :--- |
| between the incident and deflected beam directions |  |
| $\sigma$ | Scattering angle |
| $\psi_{C}$ | Characteristic conical mirror angle |
| $\psi_{P 1}$ | Characteristic Dove prism angle |
| $\psi_{P 2}$ | Characteristic Dove prism angle |
| $\psi_{w}$ | Characteristic wedge angle |
| $\psi_{w 1}$ | Characteristic wedge angle for the first wedge |
| $\psi_{w 2}$ | Characteristic wedge angle for the second wedge |
| $\Omega$ | Target rotation angular velocity |
| $\Omega_{B 1}$ | Oscillation added to the first wedge |
| $\Omega_{B 2}$ | Oscillation added to the second wedge |
| $\Omega_{P}$ | Dove prism rotational angular velocity |
| $\Omega_{T}$ | Total target rotational angular velocity |
| $\bar{\Omega}_{T}$ | Mean total target rotational angular velocity |
| $\Omega(t)$ | Torsional vibrations |
| $\Omega_{w}$ | Wedge rotational angular velocity |
| $\Omega_{w 1}$ | First wedge rotational angular velocity |
| $\Omega_{w 2}$ | Second wedge rotational angular velocity |

## Special characters

$\left[X, \alpha_{C}\right] \quad$ Rotation matrix of the conical mirror around the x - axis
$\left[X, \alpha_{F}\right] \quad$ Rotation matrix of the fold mirror around the x - axis
$\left[X, \alpha_{L}\right] \quad$ Rotation matrix of the laser source around the x - axis
$\left[X,\left(\alpha_{L}+\alpha_{m L}\right)\right]$ Rotation matrix of the laser head around the x - axis, including misalignments
$\left[X, \alpha_{m 1}\right] \quad$ Rotation matrix of the first scanning mirror around the x - axis
$\left[X, \alpha_{P}\right] \quad$ Rotation matrix of the Dove prism around the x - axis
$\left[X, \alpha_{w}\right] \quad$ Rotation matrix of the mirror around the x - axis
$\left[\mathrm{X}, \alpha_{w 1}\right] \quad$ Rotation matrix of the first wedge around the x - axis
$\left[\mathrm{X}, \alpha_{w 2}\right] \quad$ Rotation matrix of the second wedge around the x - axis
$\left\lfloor X,\left(\theta_{y}+\alpha_{m 2}\right)\right\rfloor \quad$ Rotation matrix of the second scanning mirror around the x - axis
$\left\lfloor X, \theta_{y 0}\right\rfloor \quad$ Rotation matrix of the second scanning mirror around the x - axis
$\left[X, \psi_{C}\right] \quad$ Rotation matrix of the conical mirror around the x - axis
$\left[X, \psi_{P 1}\right] \quad$ Rotation matrix of the first surface of the Dove prism around the x - axis to incorporate the Dove prism angle
$\left\lfloor X, \psi_{P_{2}}\right\rfloor \quad$ Rotation matrix of the second surface of the Dove prism around the x - axis
$\left[X, \psi_{V}\right] \quad$ Rotation matrix of the vertex mirror around the x - axis to incorporate the mirror angle
$\left[X, \psi_{w}\right] \quad$ Rotation matrix of the wedge around the x - axis
$\left[X, \psi_{w 1}\right] \quad$ Rotation matrix of the first wedge around the x - axis to incorporate the wedge angle
$\left[X, \psi_{w 2}\right] \quad$ Rotation matrix of the second wedge around the x -axis to incorporate the wedge angle
$\left[Y, \beta_{C}\right] \quad$ Rotation matrix of the conical mirror around the $y$ - axis
$\left[Y, \beta_{F}\right] \quad$ Rotation matrix of the fold mirror around the $y$ - axis
$\left[Y, \beta_{L}\right] \quad$ Rotation matrix of the laser source around the y - axis
$\left[Y,\left(\beta_{L}+\beta_{m L}\right)\right] \quad$ Rotation matrix of the laser head around the y - axis, including misalignments
$\left[Y, \beta_{m 1}\right] \quad$ Rotation matrix of the first scanning mirror around they- axis
$\left[Y, \beta_{m 2}\right] \quad$ Rotation matrix of the second scanning mirror around the y - axis
$\left[Y, \beta_{P}\right] \quad$ Rotation matrix of the Dove prism around the y - axis
$\left[Y, \beta_{w}\right] \quad$ Rotation matrix of the mirror around the y - axis
$\left[Y, \beta_{w 1}\right] \quad$ Rotation matrix of the first wedge around the y - axis
$\left[Y, \beta_{w 2}\right] \quad$ Rotation matrix of the second wedge around the y - axis
$\left[\mathrm{Z}, \gamma_{\mathrm{C}}\right] \quad$ Rotation matrix of the conical mirror around the z - axis
$\left[Z, \gamma_{L}\right] \quad$ Rotation matrix of the laser source around the z - axis
$\left[Z, \gamma_{m 2}\right] \quad$ Rotation matrix of the second scanning mirror around the z - axis
$\left[\mathrm{Z}, \gamma_{w}\right] \quad$ Rotation matrix of the wedge around the z - axis
$\left[Z, \gamma_{P}\right] \quad$ Rotation matrix of the Dove prism around the z - axis
$\left[\mathrm{Z}, \gamma_{V}\right] \quad$ Rotation matrix of the vertex mirror around the z - axis
$\left[\mathrm{Z}, \gamma_{w 1}\right] \quad$ Rotation matrix of the first wedge around the z - axis
$\left[\mathrm{Z}, \gamma_{w_{2}}\right] \quad$ Rotation matrix of the second wedge around the z - axis
$\left[\mathrm{Z}, \theta_{x 0}\right] \quad$ Rotation matrix of the first scanning mirror around the z - axis
$\left[Z,\left(\theta_{x}+\gamma_{m 1}\right)\right] \quad$ Rotation matrix of the first scanning mirror around the z - axis

## Abstract

Laser Doppler Vibrometry (LDV) is a well established technique used for non intrusive velocity measurements in fluid flows and on solid surfaces. Unlike traditional contacting vibration transducers, laser vibrometers require no physical contact with the test object. The ability to combine advanced mirror systems together with the laser source allows automated scanning LDV (SLDV) measurements, where a high number of measurement points can be measured consecutively. Non-contact vibration measurements with very high spatial resolution are possible with such a scanning system and can lead to a significantly more detailed analysis from vibration tests.

One of the main limitations of Laser Vibrometry is the difficulty to realize a perfect alignment between the investigated target and the laser beam. Frequently, for engineering applications, it is desirable to investigate different points on a target using the LDV system and, in this case, accurate knowledge of the measuring point position is required. Misalignments associated with the laser beam or the optics used to deflect the beam introduce deviation from the desired position and uncertainties in the measured velocity. All optical configurations are sensitive to misalignments, especially scanning systems able to move the laser beam around static or rotating targets.

This thesis describes advances in the application and interpretation of such measurements using Laser Vibrometry and concentrates on the analysis of the uncertainties due to the inevitable misalignments between the laser beam and the investigated target in vibration measurements on rotating components.

The work is divided into three main sections. The first part proposes a novel method to model any kind of LDV optical arrangement suitable for vibration measurements. This model has been developed with scanning LDV systems in mind but it can be used for any optical configuration. The method is based on a vector approach and integrates directly with the Velocity Sensitivity Model to determine the velocity measured by a single incident beam. The resulting mathematical models describe completely the beam path, the scan pattern and the measured velocity in the presence/absence of target vibrations and misalignments without any kind of approximation. The mathematical expressions derived are complex but easily implemented in software such as Matlab. The models are an important tool for LDV because they help the user to have a better understanding of measured data and to make the best alignment possible.

The second part of the thesis concentrates on the modelling of different optical systems using the new method. Different systems from the simplest to the most complex have been analysed using the method. For some arrangements, mathematical models have been formulated for the
first time such as for the newly proposed single and dual wedge SLDV systems and for the recently introduced Dove prism SLDV system. These systems are compared to the dual mirror SLDV system. In particular, for the single and the dual wedge SLDV systems, experimental tests have been performed to validate theoretical predictions. The results confirm the validity of the models and show the potential of these systems. Established systems such as the dual mirror and the self-tracking SLDV systems, for which generally less comprehensive models can be found in literature, have been re-analysed with the new method and theoretical predictions have been compared to the data from literature in order to confirm the validity of the new models and also to investigate for the first time some details that have previously been neglected.

The models enable identification of the main characteristics of any arrangement, in particular the sensitivity to typical misalignments and target vibration components. For tracking applications on rotating targets, the presence of misalignments causes measured velocities at DC and the first target rotation harmonic whose values depend on how the misalignments combine. The analysis of misalignment effects enables identification of the optical device(s) with the most critical alignment and supplies an initial estimation of the level of uncertainty affecting typical, practical applications. Investigation shows as the self-tracking scanning systems are much sensitive to misalignments and target vibrations than the other scanning systems.

The third part of the thesis concentrates on effects on radial and pitch/yaw vibration measurements on rotating targets of both misalignments and surface roughness of the test rotor. It is known that radial and pitch/yaw vibrations taken directly from a rotor using LDV are affected by a cross-sensitivity to the orthogonal vibration component. Resolution of the individual radial or pitch and yaw components is possible via a particular arrangement of the laser beams and using a dedicated resolution algorithm. Error sources such as instrument misalignments, rotation speed measurement error and introduce uncertainties in the resolution algorithm output. Research has quantified these uncertainties when radial vibrations with different or equal amplitude are applied to the target.

Particular attention has been given to the effects that surface roughness has on the crosssensitivity encountered in these measurements. From the tests, it is possible to identify three different ranges of surface roughness. For very smooth circular rotors, the cross-sensitivities are negligible and measurements can be made directly on the rotor without the need for a resolution algorithm. For very rough surfaces including surfaces coated in retro-reflective tape, the measurements have to be resolved to remove the cross-sensitivity. For surface roughnesses between the very smooth and the very rough, reliable measurements cannot be made because levels of the cross-sensitivity cannot be predicted making correct resolution impossible.

The significant developments in the use of Laser Vibrometry for different optical configurations and quantification of the uncertainties expected for typical applications on rotating components realised during this research project make this work a practical and important tool for the user.

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## Chapter 1

## An introduction to Laser Doppler Vibrometry

Laser Doppler Vibrometry (LDV) is a well established technique used for non intrusive velocity measurements in fluid flows and on solid surfaces. Unlike traditional contacting vibration transducers, laser vibrometers require no physical contact with the test object. Laser vibrometers offer special benefits where certain measurement constraints are imposed by the structure such as by its rotation, light weight or high temperature. Furthermore, the ability to combine advanced mirror systems together with the laser source allows automated scanning measurements, where a high number of measurement points can be measured consecutively. Non-contact vibration measurements with very high spatial resolution are possible with such a scanning system and can lead to a significantly more detailed analysis from vibration tests.

## 1.1-Definition of the problem

LDV measurements from rotating and not rotating targets can be made using stationary or moving laser beams. Since the LDVs measure velocity in the direction of the incident beam, the user has much greater freedom to orient the probe laser beam to select a desired vibration component. Both the direction of the laser beam and its incident point can be manipulated with an ease that cannot be matched by contacting transducers.

Practical applications and experiences in using LDVs for vibration measurements have highlighted the need to be able to model measured velocity in all cases where freedom
is available over laser beam orientation, from basic tripod mounting to more advanced continuous scanning and tracking applications, and where target motions are complex.

Furthermore, in recent times greater attention has fallen on the effects of instrument misalignments with the aims of minimising uncertainty and optimising data interpretation and on the effects that surface roughness has on the cross-sensitivity encountered in radial vibration measurements [1.1].

A first mathematical model to predict the measured velocity for arbitrary beam orientation and arbitrary target motion was proposed by Bell [1.2]. The velocity measured by the LDV is defined as the vector product between the direction of the incident beam and the velocity of the measured point situated on the target. The model was initially applied to perform axial and radial vibration measurements taken directly on rotors. Results confirmed that radial vibration measurements are sensitive to the radial component perpendicular to the velocity component that it is intended to measure and that this cross-sensitivity cannot be resolved by laser beam orientation or manipulation. For this reason, Halkon \& Rothberg [1.3] subsequently formulated a resolution procedure based on multiple, simultaneous measurements that is able to eliminate the cross-sensitivity terms.

The model in combination with the resolution technique was successfully used to estimate the uncertainties introduced by single instrument misalignments in radial vibration measurements [1.3]. However, practical applications are characterized by unknown combinations of misalignments and speckle noise and further investigations are necessary to define a typical level of uncertainty expected for such practical applications.

A first attempt to adapt the model to axial vibration measurements on rotors using a scanning laser beam was successful [1.4-1.5] but the new model was not readily applicable to other scanning systems such as the self-tracking LDV systems and the new recent de-rotator SLDV [1.6].

Other existing models developed for SLDV systems [1.7] have incorporated a limited set of misalignments or the modelling methods are insufficiently flexible to incorporate conveniently a full set of misalignments for all optical devices in any system. In any model, however, the contribution of the Doppler shifts occurring at the deflecting optics to the measured velocity has been largely neglected.

The specific objectives of this research work derive from the intention of investigating the LDV technique and new ways to model LDV systems in order to predict and estimate correctly the sensed velocity. These objectives can be summarised as:

1. development of a new mathematical framework for the prediction of measured velocity in diverse LDV systems and applications, especially for scanning and tracking systems and with the facility to incorporate misalignments;
2. confirmation of the universal applicability of the new framework by analysis of a range of scanning and tracking LDV systems.
3. assisted by the framework, to investigate the effect of environmental vibration on a steering mirror in an LDV system with subsequent correction of measured data.
4. assisted by the framework, to develop of a novel scanning LDV based on rotating wedges
5. using simulations, to explore the effects of LDV misalignments in measurements of radial vibration directly from rotating shafts,
6. using experimentation, to explore the effects of surface roughness in LDV measurements of radial vibration directly from rotating shafts.

The aim of this thesis is to give a comprehensive description in both mathematical and experimental points of view of various LDV systems available for different applications. Moreover, the work done within this research includes the development of various mathematical models in Matlab and the improvement of the software used for radial and pitch/yaw vibration measurements on rotors written in Labview.

## 1.2 - Fundamentals of Laser Vibrometry

The principle of operation of Laser Vibrometry relies on the detection of the Doppler frequency shift in coherent light scattered from a moving target. According to the optical scheme shown in figure 1.1, the frequency shift $f_{D}$ produced by the moving object is given by:

$$
\begin{equation*}
f_{D}=\left(\hat{b}_{2}-\hat{b}_{1}\right) \cdot \vec{U} / \lambda \tag{1.1}
\end{equation*}
$$

where $\hat{b}_{1}$ and $\hat{b}_{2}$ are the unit vectors for the incoming and scattered beam directions, $\vec{U}$ is the vector velocity of the moving object and $\lambda$ is the laser wavelength [1.7]. Equation (1.1) expresses the Doppler frequency shift using a vector formulation while Drain [1.8] defines the frequency shift as:

$$
\begin{equation*}
f_{D}=\frac{2 \mathrm{U}}{\lambda} \cos \rho \sin \left(\frac{\sigma}{2}\right) \tag{1.2}
\end{equation*}
$$

where the term $U=|\vec{U}|$ is the velocity of the moving object, $\sigma$ is the scattering angle through which the beam is deflected and $\rho$ is the angle between the velocity vector and the bisector of the angle formed between the incident and reflected beam directions. Equations (1.1) and (1.2) are equivalent and describe the same effect. Usually, in laser vibrometry applications the beam taken in consideration is that backscattered so the angle $\sigma=180^{\circ}$ and equation (1.2) becomes:
$f_{D}=\frac{2 U}{\lambda} \cos \rho$

Doppler shifts can be detected using the interferometric techniques of the heterodyne process. This method consists of a mixing between the measuring and the reference
beams, which are characterized by different frequencies. These beams are combined to produce a beat whose frequency is equal to the difference in the individual frequencies. From the analysis of the beat, the time-varying target velocity is derived.

As an example, the standard arrangement for a Polytec Laser Vibrometer is schematically shown in figure 1.2 [1.9]. The incoming laser beam is split by a beamsplitter (BS1) into two beams, the reference beam and the measuring beam, each of equal amplitude, which first travel different optical paths and are then recombined at the photodetector. After passing through a second beam splitter, BS2, the measuring beam is focused onto the object under investigation, which reflects it. The reflected beam is deviated by the beam-splitter BS2 downwards to a third beam-splitter, BS3. The reference beam passes through an acousto-optic modulator (Bragg cell) which shifts the light frequency by 40 MHz . The frequency shift from the Bragg cell is necessary to discriminate the target velocity direction. The shifted reference beam is directed towards the beam-splitter, BS3. At the beam splitter BS3, the two beams, measuring and reference, are split again and directed towards two photo-detectors. At the first, the beams are combined to produced a constructive interference while at the second photodetector are combined to produce a destructive interference. The relative change of phase at the beam splitter BS3 makes the photo-detector outputs $180^{\circ}$ out of phase. Their subtraction results in double the signal amplitude and cancels intensity noise. The output signals from both the photo-detectors are then converted to electrical signals and combined in order to derive the Doppler signal from which the velocity information is extracted.

Mathematically, the intensity on the photo-detector is the time-average of the square of the total light amplitude and is obtained as [1.10]:

$$
\begin{equation*}
I=\left(I_{R}+I_{M}\right)+2 \sqrt{I_{R} I_{M}} \cos \left[2 \pi f_{R} t-2 k a(t)+\Phi_{\text {res }}\right] \tag{1.4}
\end{equation*}
$$

where $I_{R}$ and $I_{M}$ are, respectively, the reference and the measuring beam intensities, $f_{R}$ is the reference beam frequency shift, $k$ is the light wave number and $a(t)$
represents the target vibration displacement and $\Phi_{\text {res }}$ is the resultant phase obtained as $\Phi_{\text {res }}=\left(\phi_{R}-\phi_{M}\right)$ where $\phi_{R}$ is the reference beam phase across the detector, $\phi_{M}$ is the measuring beam phase. In equation (1.4), the first term represents the mean intensity that is neglected during the subsequent signal manipulation because it is not important for the vibration measurement. The second term is an AC part alternating at the difference in frequency between the two beams whose demodulation provides the vibration velocity of the target. The frequency of this so-called 'Doppler signal' is known as the 'beat frequency', $f_{\text {beat }}$, and is given by the modulus of the time derivative of the cosine argument:

$$
\begin{equation*}
f_{\text {beat }}=\left|f_{R}-\frac{2}{\lambda} \frac{d a(t)}{d t}+\frac{1}{2 \pi} \frac{d \Phi_{\text {res }}(t)}{d t}\right|=\left|f_{R}-f_{D}\right| \tag{1.5}
\end{equation*}
$$

and equation (1.5) shows that the beat occurs at the frequency difference between the two beams.

Another method of frequency shifting is provided by a rotating diffraction grating disc positioned between the incoming beam and investigated structure [1.11\&1.12]. The diffracted orders are shifted by an amount that is proportional to the rotational speed of the grating. The maximum frequency shift obtainable is proportional to the maximum rotational speed of the grating disc but high speeds may result in problems of vibration. For this reason, for higher frequency shifts, the Bragg cell is more appropriate and in modern instruments, Bragg cells are prevalent.

## 1.3 - Laser speckle

In LDV applications, one or more laser beams are incident on the investigated structure. When a highly coherent laser light illuminates a surface whose rms surface roughness is comparable with the wavelength of the incident radiation, the resulting scattered light becomes de-phased and a speckle pattern is produced [1.13]. The intensity of the backscattered beam acquires a characteristic granular appearance characterized by a
random spatial variation of the intensity produced by the mutual interference of a set of wave fronts. This intensity distribution is known as a "speckle pattern". Each microscopic element of the scattering surface within the laser spot acts like a point source of coherent light and individual scattered waves interfere constructively or destructively producing respectively the bright and the dark regions visible on a screen, as shown in figure 1.3. In engineering applications, many surfaces can be considered rough on the scale of the optical wavelength, so the formation of a speckle pattern is important for engineering applications of laser vibrometry.

The properties of speckle patterns depend both on the coherence of the incident light and on the properties of the random scattering surface. Since the formation of speckle is considered as a random phenomenon, these properties are described statistically as first and second order probabilities. First Order statistics assert a negative exponential probability density function of intensity, with the standard deviation of the intensity equal to the mean intensity [1.14]. First Order statistics also assert that the phase is represented by a uniformly distributed probability function. The second Order statistics describe how rapidly the intensity varies from point to point in the pattern and supply information about the size of the speckles.

Analysis of the spatial distribution of intensity in the speckle patterns is important in many applications concerning the analysis of moving targets. In vibration measurements, the analysis of the speckle pattern evolution produced by a moving scattering surface assumes an important role in the accurate measurement of the target velocity. Consistent with equation (1.4), the Doppler signal can also be written as:

$$
\begin{equation*}
\Delta I=I_{\text {res }}(t) \cos \left[2 \pi f_{R} t-2 k a(t)+\Phi_{\text {res }}(t)\right] \tag{1.6}
\end{equation*}
$$

where $I_{\text {res }}$ and $\Phi_{\text {res }}$ are respectively the resultant amplitude and phase of the Doppler signal. Demodulation of $\Delta I$ provides the beat frequency from which the target velocity is derived, as explained in equation (1.5). When the laser beam illuminates a stationary target, the spatial characteristics of the speckle pattern on the photo-detector do not
change and the output from the detector is cosinusoidal with a constant beat frequency amplitude $I_{\text {res }}$ and phase $\Phi_{\text {res }}$ term. When the target moves, the detected speckle population can change and the resulting Doppler signal is modulated in amplitude and phase. When non-normal target motions are present, such as tilt, in-plane motion and rotation, which are often periodic with the same fundamental frequency as the on-axis vibration, the speckle pattern changes and the time derivative of $\Phi_{\text {res }}$ included in equation (1.5) acquires a pseudo random nature characterized by a fundamental frequency equal to that of the target vibration or rotation. The resulting spectrum of this additional signal component is characterized by approximately equal amplitude peaks at the fundamental frequency and higher Order harmonics defined as "pseudo-vibrations" [1.10] which are generally indistinguishable from the genuine vibration information. The level of these pseudo-vibrations may be sufficient to mask low level vibration information in the intended measurement.

For pseudo-vibrations, rotation is the most important non-normal target motion in this study. The test rig shown in figure 1.4 is useful to explain the presence of pseudovibrations in measurements on a rotating and vibrating structure. An electric motor is mounted on a linear slide and drives a small disc. An electro-mechanical shaker fixed to the support of the motor produces a sinusoidal vibration along the horizontal direction. A single beam vibrometer incident on the rotating target measures the total velocity of the disc along the horizontal direction while the translational movement of the motor housing is measured by an accelerometer fixed on it, as indicated in figure 1.4. The laser vibrometer and accelerometer signal outputs are connected to the same computer with a data acquisition system which records both measurements. In this thesis, this test rig will be used frequently to validate assertions.

When rotating the disc at 17 Hz in the absence of external vibrations, the accelerometer output does not show any obvious vibration, as indicated in figure 1.5 a but the laser vibrometer measures additional harmonic vibration peaks as indicated in figure 1.5 b . These harmonic peaks are characterized by similar amplitudes and are at integer multiples of the rotation frequency. They represent the pseudo-vibrations. This result is
typical when the measurement is taken under conditions where a large number of speckle pattern changes occur and the spectral shape helps in attributing characteristics to speckle pattern motion.

When an on-axis translational vibration of nominally $10 \mathrm{~mm} / \mathrm{s}$ at 30 Hz without rotation is applied to the rotor, the laser vibrometer output takes the form shown in figure 1.6 b . The genuine fundamental vibration peak and harmonic distortions closely match the measurement obtained from the accelerometer, which is shown in figure 1.6a. Since the accelerometer is not directly mounted on the rotor but on the bearing housing, some small differences can be detected in the data comparison. If the translational vibration is applied to a rotating target, the LDV output is the combination of the genuine vibration peaks plus the pseudo-vibrations due to the speckle noise, as shown in figure 1.7 b , while the output of the accelerometer is still only characterized by the fundamental vibration peak and its harmonic distortions, figure 1.7a. In the LDV measurement, it is difficult to distinguish the harmonics related to the target rotation from those due to the vibration. In this case, the vibration makes the speckle pattern repeat less pronounced so the speckle noise is more like broadband noise although a number of 20 Hz harmonics are still visible. The figures show clearly how speckle noise can mask low-level vibrations.

## 1.4-Laser Vibrometry for rotor vibration measurements

Knowledge of rotor vibrations is important from the earliest stages of design and development through to condition monitoring of installed machinery. In particular, rotating machines undergo many kinds of vibrations, which can affect the machine's effectiveness or its reliability. Although contact transducers are commonly used for machinery monitoring, they cannot be used to measure directly vibrations from rotors while the LDV systems enable these measurements.

Typical measurements taken directly from rotors can detect the axial, radial or torsional motions of the target. Since the laser vibrometer measures the velocity in the direction of the incident beam, these measurements are possible by an appropriate alignment between
the target and the LDV beam(s). For axial vibration measurements, the incident beam needs to be parallel to the target spin axis, as shown in figure 1.8a. For radial vibration measurements, the particular arrangement requires two orthogonal laser vibrometers each aligned perpendicularly to the target spin axis, as indicated in figure 1.8 b .

Torsional vibrations can be measured only using two parallel laser beams incident on the target in a plane perpendicular to the spin axis, as shown in figure 1.9 [1.5]. This optical arrangement is the basic scheme for the Laser Rotational Vibrometer (LRV) and can be also used to measure the pitch/yaw angular vibrations of a rotating target [1.16]. The beam splitter divides the outgoing beam to form two parallel and equal intensity beams directed at two different points on the structure, P1 and P2. Then, the two scattered beams pass through the beam splitter and arrive at the photo-detector where the combination of these two beams results in a measured velocity given by:

$$
\begin{equation*}
\Delta U_{m}=\hat{b} \cdot\left|\overrightarrow{V_{P 1}}-\overrightarrow{V_{P 2}}\right|=h \Omega \cos \varphi \tag{1.7}
\end{equation*}
$$

where $\overrightarrow{V_{P 1}}$ and $\overrightarrow{V_{P 2}}$ are, respectively, the instantaneous velocities at $P_{1}$ and $P_{2}$ and $\varphi$ is the inclination of the beams with respect to the spin axis of the target. Equation (1.7) establishes that the measured velocity is a function of the target rotational velocity, of the incident angle $\varphi$ and of the distance $h$.

The basis of all these kinds of vibration measurements taken directly from a rotating target can be captured in a totally general Velocity Sensitivity Model summarised in the following section.

### 1.4.1 - The "Velocity Sensitivity Model"

The comprehensive theory describing the velocity measured by a single laser beam incident on a target that is characterized by generic motion is known as the "Velocity Sensitivity Model" and was proposed for the first time by Bell [1.2]. This mathematical model considers a rigid axial element of a shaft with a length equal to the small region
illuminated by the incident beam(s) and arbitrary cross-section, rotating about its spin axis whilst undergoing arbitrary six degree of freedom vibration, as shown in figure 1.10, but it is also applicable to any non-rotating, vibrating component. The axial component rotates around an axis defined by $\hat{z}_{R}$ which differs from the z - axis of a translating reference frame xyz due to pitch/yaw vibrations. The origin of the reference frame is the point $O$ positioned on the shaft spin axis with the undeflected shaft rotation axis defining the direction and position of the z - axis. Assuming that the laser beam is incident at a point $P$, the velocity measured by the beam is defined as:

$$
\begin{equation*}
U_{m}=\hat{b} \cdot \overrightarrow{V_{P}} \tag{1.8}
\end{equation*}
$$

where the unit vector $\hat{b}$ indicates the direction of the incident laser beam and $\overrightarrow{V_{P}}$ is the total velocity of the point P. Development of equation (1.8) asserts that to predict measured velocity it is necessary to know the beam direction, $\hat{b}$, any single point situated along the beam path (which can be P but does not have to be P ) and the target vibration, $\overrightarrow{V_{P}}$ in terms of translations of O and rotations around O . All these terms have to be described in terms of measurable parameters.

Figure 1.11 shows how the unit vector $\hat{b}$, initially directed along the x - axis, is described as a combination of two rotations, by angles $\beta$ and $\alpha$ respectively around the y - and the z - axes. It is important to notice that the two rotations are finite and their order has to be maintained. In this way, the final form of the unit vector is given by:

$$
\begin{equation*}
\hat{b}=[\cos \alpha \cos \beta] \hat{x}+[\sin \alpha \cos \beta] \hat{y}-[\sin \beta] \hat{z} \tag{1.9}
\end{equation*}
$$

For an arbitrary motion of the shaft, the total velocity of the point P is a combination of three translational and three rotational velocity terms. The velocity $U_{m}$ measured by the incident laser beam is given as:

$$
\begin{align*}
& U_{m}=\cos \beta \cos \alpha\left[\dot{x}+\left(\dot{\theta}_{z}+\Omega\right) y-\left(\dot{\theta}_{y}-\Omega \theta_{x}\right) z\right]+ \\
& \cos \beta \sin \alpha\left[\dot{y}-\left(\dot{\theta}_{z}+\Omega\right) x+\left(\dot{\theta}_{x}+\Omega \theta_{y}\right) z\right]- \\
& \sin \beta\left[\dot{z}-\left(\dot{\theta}_{x}+\Omega \theta_{y}\right) y+\left(\dot{\theta}_{y}-\Omega \theta_{x}\right) x\right]- \\
& \left(y_{0} \sin \beta+z_{0} \cos \beta \sin \alpha\right)\left[\dot{\theta}_{x}+\Omega \theta_{y}\right]+  \tag{1.10}\\
& \left(z_{0} \cos \beta \cos \alpha+x_{0} \sin \beta\right)\left[\dot{\theta}_{y}-\Omega \theta_{x}\right]+ \\
& \left(x_{0} \cos \beta \sin \alpha-y_{0} \cos \beta \cos \alpha\right)\left[\dot{\theta}_{z}+\Omega\right]
\end{align*}
$$

where $\hat{x}, \hat{y}$ and $\hat{z}$ are the unit vectors for the xyz reference system, $x_{0}, y_{0}, z_{0}$ are the alignment offsets, $\dot{x}, \dot{y}, \dot{z}$ are the translational velocities of O along the $\mathrm{x}-, \mathrm{y}$ - and z axes, $\theta_{x}$ and $\theta_{y}$ are the angular vibration displacements of the shaft known also as pitch and yaw, and $\dot{\theta}_{x}, \dot{\theta}_{y}$ and $\dot{\theta}_{z}$ are the angular vibration velocities around the x-, y- and zaxes. The term $\Omega$ is the angular velocity of the shaft element combining rotational speed with any torsional oscillations around the shaft spin axis.

Equation (1.10) shows that the measured velocity is the sum of six terms, each the product of a combination of geometric parameters and a combination of motion parameters. The terms included in the square brackets, known as vibration sets, are inseparable combinations of the motion components and only these combinations can be measured directly. This means that direct measurements of pure radial, nearly axial or torsional vibration are not possible because the detected velocities are always sensitive to other motion components. In each vibration set, the quantity appearing first, for example $\dot{x}$ or $\dot{y}$, can be regarded as the intended measurement. While individual components cannot be measured directly, it is still important to be able to measure individual vibration sets and this is achieved by beam alignment. In some cases, it may be possible to assume that the effects of a particular vibration component are negligible, thereby enabling direct measurement of a specific motion component. For example, if the amplitudes of the vibration components in a vibration set are known to be similar, then the intended measurement dominates at vibration frequencies much higher than the rotation frequency. However, more reliable estimation of the motion components
requires the resolution of the outputs from two simultaneous measurements each arranged to measure a particular vibration set.

### 1.4.2 - Isolation of the radial and pitch/yaw vibration sets

The individual vibration sets shown in equation (1.10) can be isolated through an appropriate choice of geometrical parameters. Measurement of the x - radial vibration set is obtained using a laser beam accurately aligned to pass through the centre of the rotor along the x - axis with $\alpha=\beta=0^{\circ}$ and setting the offsets as $y_{0}=z_{0}=0$. The measured velocity is then equal to:

$$
\begin{equation*}
U_{m}=U_{x}=\left\lfloor\dot{x}+\left(\dot{\theta}_{z}+\Omega\right) y-\left(\dot{\theta}_{y}-\Omega \theta_{x}\right) z\right\rfloor \tag{1.11}
\end{equation*}
$$

The term $\left(\dot{\theta}_{z}+\Omega\right)$ included in equation (1.11) represents the combination between the shaft rotation and roll motion which are indistinguishable in practice so it can be replaced with the term $\Omega_{T}=\left(\dot{\theta}_{z}+\Omega\right)$ indicating the total rotational speed of the shaft element. In this way, equation (1.11) can be re-written as:

$$
\begin{equation*}
U_{m}=U_{x}=\left\lfloor\dot{x}+\Omega_{T} y-\left(\dot{\theta}_{y}-\Omega \theta_{x}\right) z\right\rfloor \tag{1.12}
\end{equation*}
$$

Similarly it is possible to isolate the radial vibration set along the $y$ - axis, setting the geometrical parameters $\alpha=90^{\circ}, \beta=0^{\circ}$ and $x_{0}=z_{0}=0$ to obtain:

$$
\begin{equation*}
U_{m}=U_{y}=\left\lfloor\dot{y}-\Omega_{T} x+\left(\dot{\theta}_{x}+\Omega \theta_{y}\right) z\right\rfloor \tag{1.13}
\end{equation*}
$$

Given the presence of three different terms in equations (1.12-1.13), the resolution of radial vibrations requires post-processing of the vibrometer outputs. The additional third terms are assumed to be an order of magnitude smaller than the first two components and thus are neglected. The x - and y - radial vibration can be written approximately as:

$$
\begin{align*}
& U_{x} \approx \dot{x}+\Omega_{T} y  \tag{1.14}\\
& U_{y} \approx \dot{y}-\Omega_{T} x \tag{1.15}
\end{align*}
$$

To isolate angular vibration components requires $\cos \alpha \cos \beta=\cos \beta \sin \beta=\sin \beta=0$ in equation (1.10) but this condition cannot be satisfied with a single incident beam. Only the arrangement comprising two parallel beams [1.17] achieves this goal. From the difference between the frequency shifts in each beam, the measured velocity is obtained as [1.3]:

$$
\begin{equation*}
\Delta U_{m}=\vec{\omega} \cdot(\vec{d} \times \hat{b}) \tag{1.16}
\end{equation*}
$$

where $\vec{\omega}$ is the vector rotational velocity around an instantaneous axis passing through $\mathrm{O}, \vec{d}$ is a vector situated in the plane of the beams perpendicular to the beam direction, $\hat{b}$, with magnitude equal to the perpendicular separation of the beams, as indicated in figure 1.12. Expanding expression (1.16), the complete expression for the measured velocity is:

$$
\begin{align*}
& \Delta U_{m}=d(\sin \gamma \sin \beta \cos \alpha-\cos \gamma \sin \alpha)\left[\dot{\theta}_{x}+\Omega \theta_{y}\right]+ \\
& d(\sin \gamma \sin \beta \sin \alpha+\cos \gamma \cos \alpha)\left[\dot{\theta}_{y}-\Omega \theta_{x}\right]+  \tag{1.17}\\
& d \sin \gamma \cos \beta\left[\dot{\theta}_{z}+\Omega\right]
\end{align*}
$$

where the angle $\gamma$ is as shown in figure 1.12. This equation contains the pitch, the yaw and the rotational vibration sets. With an appropriate alignment, the isolation of the pitch, $\dot{\Theta}_{x}$, and yaw, $\dot{\Theta}_{y}$, sets can be done. In particular, these vibration sets can be isolated by aligning the vibrometers along or at the end of the investigated shaft element. The pitch set, $d\left(\dot{\theta}_{x}+\Omega \theta_{y}\right)$ is isolated adopting $\alpha=0, \beta= \pm \pi / 2$ and $\gamma=\pi / 2$, if the measurement is realized at the end of the shaft, while $\alpha=\pi / 2$ and $\gamma=0$ is required if the measurement is made along the side of the shaft. The yaw vibration set, $d\left(\dot{\theta}_{y}-\Omega \theta_{x}\right)$, is
isolated adopting $\alpha=\pi / 2, \beta= \pm \pi / 2$ and $\gamma=\pi / 2$ at the end of the shaft and with $\alpha=0$ and $\gamma=0$ along the side of the shaft. The rotational set, $d\left(\dot{\theta}_{z}+\Omega\right)$, is only isolated along the side of the shaft using the geometrical parameters $\beta=0$ and $\gamma=\pi / 2$. The pitch $\dot{\Theta}_{x}$ and the yaw $\dot{\Theta}_{y}$ vibrational sets can be written without approximation as:

$$
\begin{align*}
& \dot{\Theta}_{x}=\dot{\theta}_{x}+\Omega \theta_{y}  \tag{1.18}\\
& \dot{\Theta}_{y}=\dot{\theta}_{y}-\Omega \theta_{x} \tag{1.19}
\end{align*}
$$

in which the cross-sensitivity terms are visible. These additional terms take the same form as in equations (1.14-1.15) and cannot be directly eliminated from the measurements. With an appropriate algorithm, however, laser vibrometer outputs can be processed, as explained in the following section.

### 1.4.3-Post processing technique

The post processing technique proposed to resolve the cross-sensitivity in laser vibrometer measurements of radial vibrations requires use of a particular optical arrangement [1.2]. Two simultaneous orthogonal laser beams are aligned with the x - and the y - axes, as was shown in figure 1.8 b . Because in practical applications it is difficult to realize a perfect alignment between the beams and the target, it is necessary to include the offsets $x_{0}$ and $y_{0}$ and equations (1.12) and (1.13) become:

$$
\begin{align*}
& U_{m}=\dot{x}+\Omega_{T}\left(y-y_{0}\right)  \tag{1.20}\\
& U_{m}=\dot{y}-\Omega_{T}\left(x-x_{0}\right) \tag{1.21}
\end{align*}
$$

The rotational angular velocity is decomposed into two components, the mean and the time dependent terms, $\Omega_{T}=\Omega$, and expressions for the AC components of the vibrometer outputs become:
$\tilde{U}_{x}=\dot{x}+\overline{\Omega_{T}} y+\Delta \Omega_{T}\left(y-y_{0}\right)$

$$
\begin{equation*}
\tilde{U}_{y}=\dot{y}-\overline{\Omega_{T}} x-\Delta \Omega_{T}\left(x-x_{0}\right) \tag{1.23}
\end{equation*}
$$

After further mathematical elaborations of equations (1.22-1.23), it is possible to obtain the following expression:

$$
\begin{array}{r}
\dot{x}+\bar{\Omega}_{T} \int_{0}^{t} x d t=\left(\widetilde{U}_{x}-\bar{\Omega}_{T} \int_{0}^{t} \widetilde{U}_{y} d t\right)-\left(\Delta \Omega_{T} y+\bar{\Omega}_{T} \int_{0}^{t} \Delta \Omega_{T} x d t\right)+ \\
\\
\left(\Delta \Omega_{T} y_{0}+\bar{\Omega}_{T} x_{0} \int_{0}^{t} \Delta \Omega_{T} d t\right)-\bar{\Omega}_{T} y(0) \\
\dot{y}+\bar{\Omega}_{T} \int_{0}^{t} y d t=\left(\widetilde{U}_{y}+\bar{\Omega}_{T} \int_{0}^{t} \widetilde{U}_{x} d t\right)+\left(\Delta \Omega_{T} x+\bar{\Omega}_{T} \int_{0}^{t} \Delta \Omega_{T} y d t\right)-  \tag{1.25}\\
\\
\left(\Delta \Omega_{T} x_{0}+\bar{\Omega}_{T} y_{0} \int_{0}^{t} \Delta \Omega_{T} d t\right)-\bar{\Omega}_{T} x(0)
\end{array}
$$

In the frequency domain, equations (1.24-1.25) become:

$$
\begin{array}{r}
\dot{X}(\omega)=W(\omega) F T\left[\left(\widetilde{U}_{x}-\bar{\Omega}_{T} \int_{0}^{t} \widetilde{U}_{y} d t\right)\right]-W(\omega) F T
\end{array} \quad\left[\left(\Delta \Omega_{T} y+\bar{\Omega}_{T} \int_{0}^{t} \Delta \Omega_{T} x d t\right)\right]+, ~ W(\omega) F T\left[\left(\Delta \Omega_{T} y_{0}+\bar{\Omega}_{T} x_{0} \int_{0}^{t} \Delta \Omega_{T} d t\right)\right]+
$$

$$
\left.\left.\begin{array}{r}
\dot{Y}(\omega)=W(\omega) F T\left[\left(\tilde{U}_{y}+\bar{\Omega}_{T} \int_{0}^{t} \tilde{U}_{x} d t\right)\right]+W(\omega) F T
\end{array}\right]\left[\Delta \Omega_{T} x-\bar{\Omega}_{T} \int_{0}^{t} \Delta \Omega_{T} y d t\right)\right]-, ~ W(\omega) F T\left[\left(\Delta \Omega_{T} x_{0}-\bar{\Omega}_{T} y_{0} \int_{0}^{t} \Delta \Omega_{T} d t\right)\right]
$$

where the component $W(\omega)$ is defined as

$$
\begin{equation*}
W(\omega)=\frac{\omega^{2}}{\omega^{2}-\bar{\Omega}_{T}^{2}} \tag{1.28}
\end{equation*}
$$

At synchronous frequency, $\omega=\Omega_{T}$, the weighting function $W(\omega)$ becomes infinite and the velocity measured by the vibrometer cannot be reconstructed to establish a solution for the synchronous vibration components. This is not a limitation of the resolution technique but a fundamental feature of the synchronous velocity component sensed by the laser vibrometer. The main consequence is that it is not possible to return from the frequency domain back to the time domain since the synchronous component will be missing. The inability to resolve vibration components at synchronous vibrations represents a serious limitation. Close to synchronous frequency, the term $W(\omega)$ becomes very large and any additional content, e.g. speckle noise, close to the synchronous frequency is amplified. To avoid an incorrect interpretation of the data, it is necessary to neglect a number of spectral components adjacent to the synchronous component.

In equations (1.26-1.27), the first bracketed terms are functions of the vibrometer outputs and the mean rotational angular velocity. The second terms are the oscillationdisplacement terms and depend on the rotational angular velocity and on the unknown vibration displacements, $x$ and $y$. The last terms are the oscillation-offset terms, which depend on the measured rotational angular velocity and on the offsets $x_{0}$ and $y_{0}$, which cannot be determined but can be minimized by a careful alignment of the laser vibrometer with respect to the shaft rotation axis. A further algorithm allows correction of initial estimates from measurements made in the presence of speed fluctuations and/or torsional vibrations.

This technique has been used to resolve the radial and pitch/yaw vibrations from simultaneous measurements on different rotating structures [1.2]. Moreover, a simulator based on these algorithms has been developed in Labview and initial simulations have shown perfect function under ideal conditions and acceptable errors apparent when
typical measurement problems such as speckle noise and errors in rotational speed measurement are introduced [1.3].

Exactly the same algorithm is used for pitch/yaw vibration measurements, $\dot{\Theta}_{x}(\omega)$ and $\dot{\Theta}_{y}(\omega)$, whose equivalent expressions are:

$$
\begin{align*}
& \dot{\Theta}_{x}(\omega)=W(\omega) F T\left[\left(\dot{\theta}_{x}-{\overline{\Omega_{T}}}_{0}^{t} \int_{0} \dot{\theta}_{y} d t\right)\right]-W(\omega) F T\left[\left(\Delta \Omega_{T} \theta_{y}+\bar{\Omega}_{T} \int_{0}^{t} \Delta \Omega_{T} \theta_{x} d t\right)\right]  \tag{1.29}\\
& \dot{\Theta}_{y}(\omega)=W(\omega) F T\left[\left(\dot{\theta}_{y}+\overline{\Omega_{T}} \int_{0}^{t} \dot{\theta}_{y} d t\right)\right]+W(\omega) F T\left[\left(\Delta \Omega_{T} \theta_{x}-\bar{\Omega}_{T} \int_{0}^{t} \Delta \Omega_{T} \theta_{y} d t\right)\right] \tag{1.30}
\end{align*}
$$

### 1.4.4 - Cross-sensitivity effects: practical applications

The measurements reported in this section have been performed to explain crosssensitivity effects on the radial measurements performed on a rotating target. The optical arrangement used in these applications is shown in figure 1.13; two single beam vibrometers have been aligned respectively along the $x$ - and the $y$ - axes. This configuration is the typical arrangement used to provide data for the post-processing technique previously cited.

Figures 1.14 a and 1.14 b show typical data in which the target was rotated nominally at 21 Hz and an on-axis translational vibration with magnitude $\approx 15 \mathrm{~mm} / \mathrm{s}$ nominally at 15 Hz was applied along the x - axis. As shown in figure 1.14a, the vibrometer aligned along the x - axis measures the genuine vibration (because the y - vibration is zero) with speckle harmonics due to the target rotation also evident. For the vibrometer aligned along the yaxis, speed harmonics are evident, as shown in figure 1.14b, but the vibration peak at 15 Hz is also present with a magnitude of $4.05 \mathrm{~mm} / \mathrm{s}$. Since the vibration is only applied along the x - direction, vibration along the y - axis is nominally zero but the target rotation causes the detected cross-sensitivity given by $U_{y}=-\Omega_{T} x$.

## 1.5 - Scanning Laser Doppler Vibrometers

One of the first applications in which the LDV was used to perform vibration measurements on rotating targets, employed a single beam vibrometer with fixed orientation incident on a passing turbine blade [1.18]. The duration of the received signal was limited because it was inversely proportional to rotational speed so conventional spectrum analysis was quite difficult, limiting the investigation only to low rotational speeds. This application demonstrated the need to develop technical solutions to maintain the beam on a chosen measurement point, in particular when vibration measurements are performed on rotating structures like disks or turbine blades. The combination of a laser head with a scanning device to modify the direction of the outgoing beam constitutes the so-called Scanning Laser Doppler Vibrometer (SLDV) [1.6, 1.19 and 1.20]. These systems provide a means to trace different scan patterns over stationary or moving targets, measuring the velocity field of the structure, obtaining spatially dense results and reducing the test time. The acquisition of data from many points requires much less time than that necessary to move the beam manually from point to point. Modal analysis [1.21] and damage identification [1.22] are typical applications in which these systems have been used.

Commercial scanning Vibrometers incorporate two orthogonally aligned mirrors to deflect the laser beam, as shown in figure 1.15 . The scanner automatically positions the laser beam at the desired measurement locations through two independent galvanometers, which rotate the mirrors with angles proportional to the input voltages after calibration. The main advantage of the dual-mirror SLDV system is the possibility to take measurements on a few or on a huge number of points situated on a fixed or moving target using a single system to perform a variety of vibration measurements. Combining different galvanometer inputs, the system can perform different scan types such as point by point, line scan, circular scan and area scan.

If the mirrors are driven by sine waves at the same frequency but $90^{\circ}$ out of phase with each other then the beam scans a circle. Then, the motion of the beam and of the rotating structure can be synchronised so that the same point on the structure is followed at all
times. The use of this so called 'tracking' technique is reported by Castellini et. al. [1.20].

The tracking of a rotating component can only be made for a limited speed range. This limitation is mainly due to the inertia of the scanning mirrors which makes these systems inadequate for high-speed rotational measurements, such those found in a jet engine, without error in the position of the measurement point. Solutions have been proposed to replace the galvanometers and the oscillating mirrors with a mechanical connection between the rotation of the measurement system and the actual measurement point. In this way, tracking of rotating components can be performed at any speed. Examples of these solutions have been reported [1.23-1.24] using two planar mirrors, one attached to the rotating target and the other positioned in order to deflect the beam towards a defined measuring point situated on the target.

When a scanning LDV system is used for tracking applications on a rotating target, the velocity measured at the target can include target vibrations and velocity components due to target flexibility. For example, when the scanning system shown in figure 1.15 is used for tracking applications on a rotating target, the velocity measured at the measuring point P can be written as:

$$
\begin{equation*}
\overrightarrow{V_{P}}=\overrightarrow{V_{O}}+\Omega_{T} \hat{z} \times\left(\overrightarrow{O P}-\overrightarrow{O O^{*}}\right)+\overrightarrow{V_{f}} \tag{1.31}
\end{equation*}
$$

where $\overrightarrow{V_{O}}$ is the translational velocity of the point O , the terms $\overrightarrow{O P}$ and $\overrightarrow{O O^{*}}$ are the vector position of the measuring point P and of the moving point $O^{*}$ with respect to the reference system $x y z, \Omega_{T}$ is the target rotational speed while $\overrightarrow{V_{f}}$ is the vibrations from target flexibility. Equation (1.31) indicates that the velocity measured at the point P is a combination of various components linked to the target vibrations and the target rotation. Equation (1.31) is fundamental to investigate the sensitivity of the various scanning systems to the target vibrations.

## 1.6 - Overview of the thesis

Sections 1.3 and 1.4 have introduced the two important areas in LDV which this thesis considers in detail. The first part of the thesis presents theoretical and experimental work to model various arrangements of particular relevance in scanning and tracking LDV applications. The second part of the thesis is concerned with uncertainty in resolution of the cross-sensitivity in radial and pitch/yaw vibration measurements on rotating components. In both cases, misalignments are of central importance to the resulting uncertainty.

Chapter 2 proposes a novel mathematical procedure to derive the direction of an incident beam for any optical configuration. The advantage of this procedure is the ease with which it readily incorporates misalignment. Integration with the Velocity Sensitivity Model then estimates the velocity measured by a single beam vibrometer in various arrangements. In this chapter, simple scanning systems based on steering mirrors and on a rotating wedge are modelled. For situations where the steering mirror itself vibrates, a procedure to correct the measurements taken by the laser vibrometer is suggested and validated.

Chapter 3 concentrates on the analysis of scanning LDV systems. All scanning LDV systems are affected by misalignments of the various optical devices. These misalignments are inevitable and introduce uncertainties that can affect the measurement. The mathematical procedure proposed in chapter 2 enables the introduction of these misalignments into the mathematical models of the analysed scanning arrangements to predict their effects. In the first part of chapter 3, the mathematical model for the dual mirror scanning system is reformulated to place particular attention on the misalignment effects. The simulated data are compared with experimental data [1.25] in order to validate the assertions. The second part of the chapter analyses two possible alternatives to the commercial dual mirror SLDV system, which incorporate either two rotating wedges [1.26] or a Dove prism [1.27]. For both the systems, the mathematical models are formulated for the first time enabled by the new procedure. Simulated data are used to identify the main characteristics for scanning
and tracking LDV applications, with a particular attention on misalignment effects. In particular, for the dual wedge scanning system simulated data are compared to experimental measurements. The comparison illustrates the potential and limitations of the system, especially in regards to tracking applications on rotating components.

Chapter 4 presents the mathematical models associated with two existing self-tracking LDV systems. These systems replace the galvanometers and the oscillating mirrors with a mechanical connection between the rotation of the measurement system and the actual measurement point. In this way, tracking of rotating components can be performed at any speed. Examples of these solutions have been reported [1.23-1.24] using two planar mirrors, one attached to the rotating target and the other positioned in order to deflect the beam towards a defined measuring point situated on the target. Chapter 4 concentrates on the main characteristics of the systems and on the misalignment effects. Simulated data are compared to experimental data from published literature in order to validate the models.

Chapter 5 considers the cross-sensitivity associated with radial and pitch/yaw vibration measurements on rotating components. The chapter comprises two parts. In the first, simulated data are used to quantify the likely error introduced by typical values for misalignment, speckle noise and errors in rotational speed measurement that occur in practical measurements. In the second part of the chapter, the relationship between cross-sensitivity and the surface roughness of rotating shafts is investigated. Recent applications [1.1-1.28] have suggested the existence of a relation between the surface roughness and the presence of cross-sensitivity terms. Initial experiments [1.29] have noticed that for very smooth, circular surfaces, the cross-sensitivities are much reduced but further investigation was needed. The relationship between cross-sensitivity and the surface roughness of rotating shafts is investigated experimentally in the hope of identification of a surface roughness range in which the cross-sensitivities can be neglected. Further recommendations are made for measurements on treated surfaces or those with roughness outside this range.

## Chapter 2

## Prediction of measured velocity in LDV systems with

## beam reflection and/or refraction

The use of the Velocity Sensitivity Model to define the measured velocity from a target requires knowledge of the final beam direction, a single known point along the line of the beam and the translational and rotational velocities of the illuminated target element. In practical applications, one of the main uncertainties in LDV results from inevitable misalignments of the various optical elements of any arrangement. To reduce the final beam misalignment, it is useful to know in advance the effects of individual misalignments through an appropriate analysis. The development of generic analysis methods is made difficult by the fact that different LDV scanning systems generally have different geometries and optical devices.

In this chapter, a versatile method to determine the velocity measured for any optical configuration and in the presence of translational and angular misalignments is proposed. This method is a useful tool for the engineer to quantify misalignment effects for any kind of LDV scheme incorporating steering or scanning optics, from the simplest to the most complex. Successively, some simple arrangements of particular interest will be analysed using this technique. Doppler shifts from deflection at optical devices within the LDV system have historically been neglected and are also included here for the first time.

## 2.1 - New approach to predict the measured velocity in LDV systems

The method proposed to predict the measured velocity is composed of the following steps:

- location of coordinate system origin on the target;
- definition of the ideal alignment (zero misalignment) for the arrangement investigated;
- incorporation of the potential misalignments associated with the various elements of the arrangement;
- step-by-step determination of the surface normals on the deflecting optics, of the beam orientations, of the points where the beam orientation changes and of the investigated point on the target;
- calculations of the measured velocity at each deflection point using the Velocity Sensitivity Model.

The method utilises a vector approach with generic mathematical expressions applicable to any LDV system and integrates with the Velocity Sensitivity Model for calculation of the sensed velocity based on assumed target vibrations. Vector description of beam orientation and surface normals is particularly important, facilitated by the use of rotation matrices to modify initial vectors and by the use of vector expressions for reflection and refraction. Vector polygons enable identification of significant locations within the optical system. The approach was developed with scanning systems in mind but is applicable to all LDV systems.

### 2.1.1 - Definition of the incident beam orientation

The first LDV arrangement analysed with the proposed method is that shown in figure 2.1 where a single beam vibrometer is positioned in front of a vibrating target. This is the simplest possible arrangement used in vibrometry but it will be a useful starting point to introduce the proposed method before incorporating deflecting optics to illuminate the structures under investigation.

The mathematical description of a system begins with the definition of the ideal alignment. A reference system xyz is fixed on the target, with origin O as shown in figure 2.1. In the ideal alignment, the incident beam is parallel to the z - axis and incident on the point O . The position of the laser head is defined by the point A whose expression is:

$$
\overrightarrow{O A}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
x_{A} & y_{A} & z_{A} \tag{2.1}
\end{array}\right]^{T}
$$

where $\hat{x}, \hat{y}$ and $\hat{z}$ are the unit vectors of the xyz reference system, $x_{A}, y_{A}$ and $z_{A}$ are the laser source coordinates with $x_{A}=y_{A}=0$. The direction of the incident beam is given by the unit vector $\hat{b}_{1}$ that can be written as:
$\hat{b}_{1}=\left[\begin{array}{lll}\hat{x} & \hat{y} & \hat{z}\end{array}\right]\left[\begin{array}{lll}0 & 0 & -1\end{array}\right]^{T}$
where $\left[\begin{array}{ccc}0 & 0 & -1\end{array}\right]^{T}$ represents the ideal laser beam orientation. The scattering direction $\hat{b}_{2}$ coincides with $\hat{b}_{1}, \hat{b}_{2}=-\hat{b}_{1}$, and, in these conditions, when the structure has a translational vibration along the z - axis, the vibrometer will measure this vibration component fully.

Inevitable angular misalignments are small rotations of the optical devices around the reference system axes. Applying two small rotations around the x - and the y - axes to the laser head defined by the angles $\alpha_{L}$, as shown in figure 2.2 , and $\beta_{L}$, the final orientation of the incident laser beam is given by:
$\hat{b}_{1}=\left[\begin{array}{lll}\hat{x} & \hat{y} & \hat{z}\end{array}\right]\left[X, \alpha_{L}\right]\left[\begin{array}{ll}Y, \beta_{L}\end{array}\right]\left[\begin{array}{lll}0 & 0 & -1\end{array}\right]^{T}$
where $\left[X, \alpha_{L}\right]$ and $\left[Y, \beta_{L}\right]$ are the matrices associated with the beam rotations defined as [2.1]:

$$
\begin{align*}
& {\left[X, \alpha_{L}\right]=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & \cos \alpha_{L} & -\sin \alpha_{L} \\
0 & \sin \alpha_{L} & \cos \alpha_{L}
\end{array}\right]}  \tag{2.4a}\\
& {\left[Y, \beta_{L}\right]=\left[\begin{array}{lll}
\cos \beta_{L} & 0 & \sin \beta_{L} \\
0 & 1 & 0 \\
-\sin \beta_{L} & 0 & \cos \beta_{L}
\end{array}\right]} \tag{2.4b}
\end{align*}
$$

Since the rotations are finite, the order of the multiplications in equation (2.3) has to be maintained except for the case of very small angles. Multiplying out equation (2.3) gives the following expression:

$$
\begin{equation*}
\hat{b}_{1}=-\left(\sin \beta_{L}\right) \hat{x}+\left(\sin \alpha_{L} \cos \beta_{L}\right) \hat{y}-\left(\cos \alpha_{L} \cos \beta_{L}\right) \hat{z} \tag{2.5}
\end{equation*}
$$

and the incident and the scattering beam directions continue to be related as $\hat{b}_{2}=-\hat{b}_{1}$. Translational misalignments of the laser source move the point A to A' whose expression can be written as:

$$
\overrightarrow{O A^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
x_{A}+\Delta x_{A} & y_{A}+\Delta y_{A} & z_{A}+\Delta z_{A} \tag{2.6}
\end{array}\right]^{T}
$$

where $\Delta x_{A}, \Delta y_{A}$ and $\Delta z_{A}$ are the translational misalignments added to the laser source position. The introduction of the misalignments also modifies the position of the measuring point which shifts from $B$ to $B^{\prime}$ as derived in the following section.

### 2.1.2 - Description of the beam path and of the "known point"

The prediction of the velocity measured by the laser vibrometer requires knowledge of any point situated along the line of the incident beam. According to figure 2.2, this might be the point A' located along the line defined by $\hat{b}_{1}$ and defined by equation (2.6). Alternatively, the laser beam crosses the xy plane at the point B', which does not
move. Considering the triangle $\mathrm{OA}^{\prime} \mathrm{B}^{\prime}$, the position of the point $\mathrm{B}^{\prime}$ is found using a vector polygon:

$$
\begin{equation*}
\overrightarrow{O B^{\prime}}=\overrightarrow{O A^{\prime}}+\left|\overrightarrow{A^{\prime} B^{\prime}}\right| \hat{b}_{1}=\overrightarrow{O A^{\prime}}+\left|\frac{\overrightarrow{O A^{\prime}} \cdot \hat{z}}{\hat{b}_{1} \cdot \hat{z}}\right| \hat{b}_{1} \tag{2.7}
\end{equation*}
$$

from which knowledge of vector $\overrightarrow{O A^{\prime}}$ and the unit vector $\hat{b}_{1}$ allows vector $\overrightarrow{O B^{\prime}}$ to be easily derived. Use of vector polygons is particularly effective when applied to the more complex arrangements where the geometries incorporate multiple changes in beam orientation. Further applications of this technique will be reported in later sections where more complex LDV systems are analysed.

### 2.1.3-Definition of the measured target velocity

With knowledge of the location of point B', the Velocity Sensitivity Model provides the velocity measured at $\mathrm{B}^{\prime}$. According to Bell [2.2], the total velocity of a single point is a combination of six velocity sets, three translational and three rotational and is written as in equation (1.10). For example, if the target undergoes a translational vibration along the z - axis, $\dot{z}$, and a rotation around the x -axis, $\dot{\theta}_{X}$, the measured velocity can be found using equation (1.8) and (2.5) and becomes:

$$
\begin{equation*}
U_{m}=\cos \alpha_{L} \cos \beta_{L}\left(\dot{z}+\dot{\theta}_{X}\left(\overrightarrow{O B^{\prime}} \cdot \hat{y}\right)\right) \tag{2.8}
\end{equation*}
$$

Equation (2.8) indicates how translational and angular misalignments affect measured velocity through the quantities $\alpha_{L}, \beta_{L}$ and $\overrightarrow{O B^{\prime}}$. When $\alpha_{L}=\beta_{L}=0^{\circ}$, equation (2.8) shows that only the translational vibration along the z - axis is measured. Although this result was readily predictable, equation (2.8) has been obtained through application of this technique to confirm its validity using a simple configuration.

### 2.1.4 - Incorporating beam deflections

In practical applications, misalignments are inevitable and difficult to identify. The introduction of additional optical devices makes the geometry and the beam path more complex and the number of sources of uncertainty increases. Systems incorporating deflecting optics and suitable for vibration measurements are the subject of the following analysis.

By knowing the incident beam direction $\hat{b}_{I}$, the direction of the deflected beams $\hat{b}_{R}$ and $\hat{b}_{T}$ are found using Snell's Law. This Law asserts that when a plane wave, such as the laser, is incident at an interface between two homogeneous media of different optical properties it is split into a transmitted and a reflected wave, figure 2.3. For the 'Law of Reflection', the angle of incidence, $\theta_{I}$, equals the angle of reflection, $\theta_{R}$. For the 'Law of Refraction', the ratio of sines of the incident and refracted beam can be formulated in terms of the absolute indices of refraction of the two media, $\varepsilon_{I}$ and $\varepsilon_{T}$, as:

$$
\begin{equation*}
\frac{\sin \theta_{I}}{\sin \theta_{T}}=\frac{\varepsilon_{T}}{\varepsilon_{I}} \tag{2.9}
\end{equation*}
$$

In vector form, the unit vector for the reflected beam can be written as [2.3]:

$$
\begin{equation*}
\hat{b}_{R}=\hat{b}_{I}-2\left(\hat{b}_{I} \cdot \hat{n}\right) \hat{n} \tag{2.10}
\end{equation*}
$$

where $\hat{b}_{I}$ and $\hat{b}_{R}$ are the unit vectors of the incident and reflected beams and $\hat{n}$ is the normal to the interface between the two surfaces. In the case of beam refraction, the unit vector of the refracted beam, $\hat{b}_{T}$, is given by [2.4]:

$$
\begin{equation*}
\hat{b}_{T}=\left(\hat{b}_{I}-\left(\hat{b}_{I} \cdot \hat{n}\right) \hat{n}\right) \frac{\varepsilon_{I}}{\varepsilon_{T}}-\left(\sqrt{1-\left(\frac{\varepsilon_{I}}{\varepsilon_{T}}\right)^{2}\left(1-\left(\hat{b}_{I} \cdot \hat{n}\right)^{2}\right)}\right) \hat{n} \tag{2.11}
\end{equation*}
$$

Equations (2.10-2.11) show that knowledge of the incident beam direction, the surface normal and each refractive index lead to outgoing beam direction for both reflection and refraction. For example, for a mirror normal initially aligned with $\hat{z}$, the final normal direction after two mirror rotations around the x - and the y - axis, with angles $\alpha_{m}$ and $\beta_{m}$ respectively, is achieved using the same vector approach employed to derive $\hat{b}_{1}$ in equation (2.3)

$$
\hat{n}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{m}\right]\left[\begin{array}{ll}
Y, \beta_{m}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & 1 \tag{2.12}
\end{array}\right]^{T}
$$

where the rotation matrices $\left[X, \alpha_{m}\right]$ and $\left[Y, \beta_{m}\right]$ are of the same form as those indicated in equations $(2.4 \mathrm{a} \& \mathrm{~b})$. The same technique can be used to derive the surface normal for an inclined wedge surface.

### 2.1.5 - Total measured velocity

The velocity measured by the laser vibrometer incident on a point P situated on the target can be written by combining equations (1.1) and (1.3):

$$
\begin{equation*}
U_{m}=\frac{1}{2}\left(\hat{b}_{2}-\hat{b}_{1}\right) \cdot \overrightarrow{V_{P}} \tag{2.13}
\end{equation*}
$$

Equation (2.13) is equivalent to equation (1.8) when $\hat{b}_{2}=-\hat{b}_{1}$. It is important to recognise that Doppler shifts can occur wherever the beam is deflected. The total Doppler shift of the beam will be the sum of the individual shifts and so it is possible to add together all the corresponding velocity components, written as in equation (2.13), from every beam deflection, not just at the target but also at beam reflections and refractions. Equation (2.13) asserts that any moving optical device can produce additional velocity terms that appear in the LDV output but are not related to target motion. These uncertainties have not been studied previously. Equation (2.13) is useful in analysis of optical arrangements characterised by various beam deflections such as the steering and scanning systems under investigation here.

## 2.2 - Application to the scanning/steering mirror optical configuration

### 2.2.1 - Modelling measured velocity

Consider the optical arrangement shown in figure 2.4 in which a plane mirror is used. The mirror might be used to perform a line scan along the z - direction on a target or simply to steer the laser beam onto an otherwise inaccessible location. For convenience, the reference system xyz has been fixed on the target while a reference system $x_{m} y_{m} z_{m}$ is fixed on the mirror. With an initial rotation around the $y$ - axis, the line scan along the $z$ - axis can be realized by moving the mirror in two distinct ways: oscillation along the $z$ - axis or around the $y$ - axis. In steering applications, vibrations of the steering mirror are also important and will also be considered in this analysis. These are undesired but occur in practical applications introducing uncertainty into measurements.

The system is ideally aligned when the outgoing beam $\hat{b}_{1}$ is parallel to the z - axis, defined by equation (2.2), and incident at the centre of the mirror at the point B. (For all the optical schemes analysed in this chapter, the direction of the outgoing beam is $\hat{b}_{1}=-\hat{z}$ when the systems are ideally aligned). The mirror is oscillating around the $\mathrm{x}-$ and the y - axes with a time varying angular velocities $\dot{\alpha}_{m}$ and $\dot{\beta}_{m}$. The mirror surface normal is determined from equation (2.12) where the angles $\alpha_{m}$ and $\beta_{m}$ are given by:

$$
\begin{align*}
& \alpha_{m}(t)=\alpha_{m 0}+\int \dot{\alpha}_{m} d t  \tag{2.14}\\
& \beta_{m}(t)=\beta_{m 0}+\int \dot{\beta}_{m} d t \tag{2.15}
\end{align*}
$$

in which the terms $\alpha_{m 0}$ and $\beta_{m 0}$ are the initial mirror inclinations around the x - and y axes. For example, for the line scan along the z-axis, $\alpha_{m 0}=0^{\circ}$ and $\beta_{m 0}=-45^{\circ}$ while $\dot{\beta}_{m}$ is the oscillation of the mirror. The terms $\dot{\alpha}_{m}$ and $\dot{\beta}_{m}$ might also contain undesirable vibrations of the mirrors whose effects determine uncertainty into measurements. The final expression for the mirror normal is:

$$
\begin{equation*}
\hat{n}_{B}=-\left(\sin \beta_{m}\right) \hat{x}+\left(\sin \alpha_{m} \cos \beta_{m}\right) \hat{y}-\left(\cos \alpha_{m} \cos \beta_{m}\right) \hat{z} \tag{2.16}
\end{equation*}
$$

With knowledge of the unit vectors for the incident beam and the mirror normal, the reflected beam orientation is readily determined. Substituting equations (2.2) and (2.16) into (2.10), the reflected beam direction is given by:

$$
\begin{equation*}
\hat{b}_{2}=\cos \alpha_{m} \sin \left(2 \beta_{m}\right) \hat{x}-\cos ^{2} \beta_{m} \sin \left(2 \alpha_{m}\right) \hat{y}+\left(2 \cos ^{2} \beta_{m} \cos ^{2} \alpha_{m}-1\right) \hat{z} \tag{2.17}
\end{equation*}
$$

In the presence of only angular oscillations of the mirror around $x$ - and $y$ - axis, the position of the reflected point $B$ is situated at the centre of the steering mirror and is known from the position of the mirror with respect to the origin O . When the mirror has also translational misalignments, the point $B$ moves to $B *$ defined as:

$$
\overrightarrow{O B}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
x_{B}+\Delta x_{B} & y_{B}+\Delta y_{B} & z_{A}+\Delta z_{B} \tag{2.18}
\end{array}\right]^{T}
$$

where the terms $\Delta x_{B}, \Delta y_{B}$ and $\Delta z_{B}$ are the translational misalignments added to the mirror. The position of the new deflection point $B^{\prime}$ is found from the vector triangles $O B^{\prime} A^{\prime}$ and $O B * B^{\prime}$ obtaining the following system:

$$
\left\{\begin{array}{l}
\overrightarrow{O A^{\prime}}+\left|\overrightarrow{A^{\prime} B^{\prime}}\right| \hat{b}_{1}=\overrightarrow{O B^{\prime}}  \tag{2.19}\\
O B *+\overrightarrow{B * B^{\prime}}=\overrightarrow{O B^{\prime}} \\
\overrightarrow{B * B^{\prime}} \cdot \hat{n}_{B}=0
\end{array}\right.
$$

where $\overrightarrow{O A^{\prime}}$ is obtained from equation (2.6) and the vector $\overrightarrow{O B^{\prime}}$ is found as:

$$
\begin{equation*}
\overrightarrow{O B^{\prime}}=\overrightarrow{O A^{\prime}}+\left|\frac{\left(\overrightarrow{O B^{*}}-\overrightarrow{O A^{\prime}}\right) \cdot \hat{n}_{B}}{\hat{b}_{1} \cdot \hat{n}_{B}}\right| \hat{b}_{1} \tag{2.20}
\end{equation*}
$$

The vector $\overrightarrow{O B^{\prime}}$ is a time dependent term through the normal $\hat{n}_{B}$. The point $C$ is the initial position of the measuring point while $C^{\prime}$ is the time-varying incident point, which is determined using a vector polygon for the triangle $O B^{\prime} C^{\prime}$ :

$$
\begin{equation*}
\overrightarrow{O C^{\prime}}=\overrightarrow{O B^{\prime}}+\left|\overrightarrow{B^{\prime} C^{\prime}}\right| \hat{b}_{2}=\overrightarrow{O B^{\prime}}+\left|\frac{\overrightarrow{O B^{\prime}} \cdot \hat{x}}{\hat{b}_{2} \cdot \hat{x}}\right| \hat{b}_{2} \tag{2.21}
\end{equation*}
$$

The point $C^{\prime}$ is then used as the known point in the Velocity Sensitivity Model to predict the velocity measured by the laser vibrometer. The complete beam path (out and back) represented in figure 2.4 includes three beam reflections, two at the mirror and one at the target. According to equation (2.13), the total velocity measured by the laser vibrometer can be written as the summation of the individual velocities detected along the beam path. This velocity can be written as:

$$
\begin{equation*}
U_{m}=\frac{1}{2} \overrightarrow{V_{B^{\prime}}} \cdot\left(\hat{b}_{2}-\hat{b}_{1}\right)+\frac{1}{2} \overrightarrow{V_{C^{\prime}}} \cdot\left(-\left(\hat{b}_{2}\right)-\hat{b}_{2}\right)+\frac{1}{2} \overrightarrow{V_{B^{\prime}}} \cdot\left(-\left(\hat{b}_{1}\right)-\left(-\hat{b}_{2}\right)\right)=\overrightarrow{V_{B^{\prime}}} \cdot\left(\hat{b}_{2}-\hat{b}_{1}\right)-\overrightarrow{V_{C^{\prime}}} \cdot \hat{b}_{2} \tag{2.22}
\end{equation*}
$$

where $\overrightarrow{V_{B^{\prime}}}$ and $\overrightarrow{V_{C^{\prime}}}$ are respectively the mirror and the target velocities at the particular points of incidence. Substituting equation (2.10) into (2.22), its final form becomes much simpler:

$$
\begin{equation*}
U_{m}=-2\left(\overrightarrow{V_{B^{\prime}}} \cdot \hat{n}_{B^{\prime}}\right)\left(\hat{b_{1}} \cdot \hat{n}_{B^{\prime}}\right)-\overrightarrow{V_{C^{\prime}}} \cdot \hat{b}_{2} \tag{2.23}
\end{equation*}
$$

Equation (2.23) shows the presence of two distinct terms, one linked to the target vibration and the second proportional to the mirror velocity component along the mirror normal. When the mirror vibrates or oscillates, the point B may no longer be a point on the mirror so the normal at the point $B^{\prime}, \hat{n}_{B^{\prime}}$, becomes time dependent.

An application in which a steering mirror is used is the measurement of an engine valve motion [2.5], see figure 2.5. Here the mirror deflects the incoming beam onto a single point on the valve or scans the complete valve during its motion in the cylinder head.

According to equation (2.23), in-plane mirror vibrations, $\dot{x}_{m}$ and $\dot{y}_{m}$, do not affect the LDV output since the term $\left(\overrightarrow{V_{B^{\prime}}} \cdot \hat{n}_{B^{\prime}}\right)$ is zero and the target velocity is measured directly. In the presence of out-of-plane mirror vibrations, the analysis shows that errors are introduced into the measurement of the target vibration. The analysis also suggests that it is possible to correct this error using an additional measurement of the mirror vibration in the direction of its normal. If the additional mirror velocity can be measured, it should be sufficient to attach accelerometers to the mirror to make this compensation, as shown in figure 2.6. If the mirror undergoes a vibration $\overrightarrow{V_{B^{\prime}}}$ directed along its normal, the output of a single accelerometer attached at the back of the mirror is sufficient to correct the LDV output and isolate the target vibration. In the presence of additional mirror oscillations around the x - and y - axes of the mirror, the velocity of the measuring point $B^{\prime}$ situated on the mirror becomes:

$$
\begin{equation*}
\overrightarrow{V_{B^{\prime}}} \cdot \hat{n}_{B^{\prime}}=\dot{z}_{m}+\dot{\alpha}_{m} y_{B^{\prime}}-\dot{\beta}_{m} x_{B^{\prime}} \tag{2.24}
\end{equation*}
$$

where $\dot{\alpha}_{m}$ and $\dot{\beta}_{m}$ are the mirror oscillations around the x - and y-axes respectively, $\dot{z}_{m}$ is the translational velocity of the mirror along the z - axis while $x_{B^{\prime}}$ and $y_{B^{\prime}}$ are the coordinates of the point $B^{\prime}$ determined from equation (2.20). In this case, the various vibration components can be determined using four accelerometers attached at the back of the mirror, as shown in figure 2.6 where the accelerometers 1 and 2 are positioned along the $y$ - axis with 3 and 4 along the x - axis. The dynamics of the mirror are determined with three combinations:

- accelerometers 1 and 2 supply the mirror rotation around the x - axis by subtraction of outputs;
- accelerometers 3 and 4 supply the mirror rotation around the $y$ - axis by subtraction of outputs;
- each pair of accelerometers supply, at the same time, the mirror vibration along the z - axis by addition of outputs.

To validate equation (2.23) and develop these ideas, some simple applications are reported in the following section.

### 2.2.2 - Practical applications: correction for steering mirror vibrations

Figure 2.7 schematically shows the test-rig used to confirm the mathematical description of measured velocity for the steering mirror arrangement. A plane mirror is excited along its normal by a shaker. The mirror-shaker structure has been mounted on a graduated plate to control the angle $\beta_{m 0}$ and maintain the excitation direction parallel to the mirror normal for any mirror inclination. The target is represented by a plane structure positioned at a fixed distance from the mirror and aligned parallel to the yz plane. A second shaker attached to the structure allows its excitation in the x -direction with velocity $\dot{x}_{C^{\prime}}$ where the point $C^{\prime}$ is the target point where the reflected beam $\hat{b}_{2}$ meets the vibrating structure. The target-shaker combination is mounted on a linear slide that ensures that the beam is incident on the target at any mirror inclination and that the original orientation of the target is maintained. For a more practical use, equation (2.23) has been expanded using measurable parameters according to figure 2.7 as:

$$
\begin{equation*}
U_{m}=V_{C^{\prime}} \sin \left(2 \beta_{m 0}\right)+2 \mathrm{~V}_{B} \cos \beta_{m 0} \tag{2.25}
\end{equation*}
$$

where $V_{C^{\prime}}=\left(\overrightarrow{V_{C^{\prime}}} \cdot \hat{x}\right)$. Maintaining the mirror angle $\alpha_{m}(t)=0$, three sets of tests have been performed to validate each velocity term in equation (2.20). For initial alignment, a vibration directed along the z- axis, $\dot{z}_{m}$, with frequency of 40 Hz was applied to the mirror inclined with $\beta_{m 0}=0^{\circ}$. An accelerometer was fixed at the back of the mirror in order to measure its velocity along the z - axis. Then, a single beam vibrometer was aligned with respect to the mirror to obtain an output similar to that of the
accelerometer. Adjustments of the position and orientation of the laser head were made using the tripod, the linear and angular stage on which the laser vibrometer is fixed and adjustments of the mirror were made using the angular stage on which the mirror was mounted. Figures 2.8 a and 2.8 b indicate respectively the output of the accelerometer and of the LDV for this case. These outputs have very close amplitudes detected at 40 Hz and the configuration was considered as the initial alignment of the test rig. From this position, the mirror is rotated around its y - axis using the graduated plate.

The first step was conducted to validate the mirror velocity term. The mirror was excited at 40 Hz and its inclination was varied from $0^{\circ}$ to $60^{\circ}$ in 13 steps of $5^{\circ}$ each. Examples of the accelerometer 1 and LDV outputs are shown in figures 2.9 a and 2.9 b where at 40 Hz the detected velocities are, respectively, 27 and $40 \mathrm{~mm} / \mathrm{s}$ for a mirror inclination of $40^{\circ}$. According to equation (2.25), with angles less than $60^{\circ}$, the velocity detected by the LDV should be bigger than that of the accelerometer and this behaviour is confirmed in figure 2.10a. The expected theoretical velocity is obtained from the second term of equation (2.25).

Figure 2.10 b reports the percentage error between the theoretical and the experimental velocities indicating an excellent correspondence and only small differences between the two sets of values, probably due to small misalignments introduced during the test by the various mirror or laser source movements. These data confirm the validity of the mirror velocity term in equation (2.25).

Analysing the experimental LDV data points, it is possible to see an inconsistent behaviour at $0^{\circ}$. According to equation (2.25), the laser vibrometer should measure a velocity of $2 V_{B}$ but the detected velocity is $V_{B}$ because, for zero mirror angle, the laser beam returns directly to the vibrometer without either scattering from the target or a second mirror reflection. This arrangement is equivalent to that shown in figure 2.1.

A second test was performed to investigate the first term in equation (2.25) giving the relationship between measured velocity and target velocity. A sinusoidal x-direction
vibration of 40 Hz was applied to the target. The mirror inclination was varied from $30^{\circ}$ to $60^{\circ}$, using 14 steps with angular increments of $2^{\circ}$. For outer values of the mirror inclination, the backscattered intensity was too weak for an acceptable signal. Examples of experimental output are shown in figure 2.11a and 2.11b, respectively, for accelerometer 2, fixed to the vibrating target, and the LDV. According to equation (2.25), the target velocity detected by accelerometer 2 should be bigger than that measured by the laser vibrometer and this outcome is shown by figure 2.12 . When the mirror is inclined at $45^{\circ}$, the accelerometer and the laser vibrometer measurements are theoretically equal but small, inevitable misalignments result in differences of approximately $1-2 \%$.

The final experiment was intended to validate the complete equation (2.25) with target and mirror vibration applied simultaneously at 60 Hz and 40 Hz respectively. For a mirror inclination of $40^{\circ}$, figure 2.13 a and 2.13 b show the outputs for the accelerometers fixed on the mirror and on the target, respectively, while figure 2.14 shows the laser vibrometer output. The vibrometer output contains both vibrations as predicted by equation (2.25). Comparison of peaks in the vibrometer spectrum originating with target and mirror vibrations with the corresponding accelerometer measurements indicate very good agreement as shown in figure 2.15a and 2.15b.

Figure 2.15a shows the errors between the experimental and the theoretical measured velocities of the vibrating mirror taken for various inclinations of the steering mirror. The data indicate errors between $2.5 \%$ and $1.5 \%$ for the investigated range of mirror inclination due to possible uncertainties in the alignment of the laser head and mirror. Figure 2.15 b shows the errors between the experimental and the theoretical velocities detected at the vibrating target. In this case, the errors assume a maximum value around $5 \%$ for a mirror inclination of $30^{\circ}$ and decrease to values around $1.5 \%$ for bigger inclinations of the steering mirror. Also in this case, the difference between the two sets of velocities, experimental and theoretical, can be associated with inevitable misalignments affecting the test rig.

These data validate the mathematical description of the measured velocity for the steering mirror optical configuration and the approach used to derive it. Theoretically and experimentally, two important results have been demonstrated: the existence of measurement uncertainties linked to the motion of deflecting optics and the possibility to correct these measurements using additional measurements.

In the next section, this approach is applied to model an optical configuration in which a rotating wedge is used for scanning measurements on rotating structures.

## 2.3 - Application to a novel scanning LDV system using a rotating wedge

### 2.3.1-Modelling measured velocity

Solutions for scanning optical arrangements can be realised using optical devices which refract the laser beam towards the investigated target. One of these novel configurations uses rotating wedges positioned between the laser vibrometer and the target to deflect the outgoing laser beam. This optical arrangement is suitable for scanning applications and a simple representation is shown in figure 2.16a where a wedge, traditionally used for beam steering, controls the deflection of the light towards the target. Rotating the wedge around its spin axis, results in a circular scan on a target. Fixing the origin O of the reference system xyz on the rotation axis of the target, the ideally aligned beam path is defined when the incident beam and the spin axis of the wedge coincide along the z axis. The positions of the laser head and of the wedge are defined by the points A and B obtained as:

$$
\begin{align*}
& \overrightarrow{O A}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
x_{A} & y_{A} & z_{A}
\end{array}\right]^{T}  \tag{2.26}\\
& \overrightarrow{O B}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}]\left[\begin{array}{lll}
x_{B} & y_{B} & z_{B}
\end{array}\right]^{T}
\end{array} .=x^{T}\right. \tag{2.27}
\end{align*}
$$

where $x_{A}=y_{A}=x_{B}=y_{B}=0$ for an ideal alignment. The position of the point C situated at the sloped wedge surface and along its spin axis is defined as:

$$
\begin{equation*}
\overrightarrow{O C}=\overrightarrow{O B}-|\overrightarrow{B C}| \vec{z} \tag{2.28}
\end{equation*}
$$

The direction of the outgoing beam, defined as in equation (2.2) is perpendicular to the first wedge surface and passes directly through it and along the wedge spin axis. The direction of the outgoing beam $\hat{b}_{1}$ coincides with that for the beam $\hat{b}_{2}$. The first refraction takes place at the point C situated on the inclined wedge surface. The wedge surface normals are described as:

$$
\begin{align*}
& \left.\hat{n}_{B}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right] \begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T}  \tag{2.29}\\
& \hat{n}_{C}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{ll}
Z, \gamma_{w}
\end{array}\right]\left[\begin{array}{ll}
X, \psi_{w}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \tag{2.30}
\end{align*}
$$

where $\psi_{w}$ is the characteristic wedge angle and the angle $\gamma_{w}$ represents the whole body wedge rotation around its spin axis given by:

$$
\begin{equation*}
\gamma_{w}(t)=\Omega_{w} t+\varphi_{w} \tag{2.31}
\end{equation*}
$$

in which $\Omega_{w}$ is the wedge rotational speed and $\varphi_{w}$ is its initial phase, making the unit vector $\hat{n}_{C}$ a function of time. The rotational matrices $\left[X, \psi_{w}\right]$ and $\left[Z, \gamma_{w}\right]$ are defined as [2.1]:

$$
\begin{align*}
& {\left[X, \psi_{w}\right]=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & \cos \psi_{w} & -\sin \psi_{w} \\
0 & \sin \psi_{w} & \cos \psi_{w}
\end{array}\right]}  \tag{2.32}\\
& {\left[Z, \gamma_{w}\right]=\left[\begin{array}{lll}
\cos \gamma_{w} & -\sin \gamma_{w} & 0 \\
\sin \gamma_{w} & \cos \gamma_{w} & 0 \\
0 & 0 & 1
\end{array}\right]} \tag{2.33}
\end{align*}
$$

The final direction of the incident beam, $\hat{b}_{3}$, is derived from equation (2.11) to give:

$$
\begin{align*}
\hat{b}_{3} & =\left[\sin \psi_{w}\left(\frac{\varepsilon_{w}}{\varepsilon_{a}} \cos \psi_{w}-m\right) \sin \gamma_{w}\right] \hat{x}-\left[\sin \psi_{w}\left(\frac{\varepsilon_{w}}{\varepsilon_{a}} \cos \psi_{w}-m\right) \cos \gamma_{w}\right] \hat{y}- \\
& {\left[\frac{\varepsilon_{w}}{\varepsilon_{a}}-\cos \psi_{w}\left(\frac{\varepsilon_{w}}{\varepsilon_{a}} \cos \psi_{w}-m\right)\right] \hat{z} } \tag{2.34}
\end{align*}
$$

where $\varepsilon_{w}$ and $\varepsilon_{a}$ are respectively the refractive indices of the wedge and the air and the term $m$ is defined as:

$$
\begin{equation*}
m=\sqrt{1-\left(\frac{\varepsilon_{w}}{\varepsilon_{a}}\right)^{2}\left(1-\left(\hat{b}_{1} \cdot \hat{n}_{C}\right)^{2}\right)} \tag{2.35}
\end{equation*}
$$

Equation (2.34) shows how quickly the complexity grows in the equation for beam direction but the equation is easily derived using the proposed technique. From knowledge of point C , the point K on the target is derived as follows from the vector polygon for triangle OCK:

$$
\begin{equation*}
\overrightarrow{O K}=\overrightarrow{O C}+\left|\frac{\overrightarrow{O C} \cdot \hat{z}}{\hat{b}_{3} \cdot \hat{z}}\right| \hat{b}_{3} \tag{2.36}
\end{equation*}
$$

Point K is used to predict the measured target velocity but the total measured velocity must also include any Doppler shift from the rotating wedge itself. For ideal alignment, point C is situated on the wedge spin axis and, therefore, no additional Doppler shift occurs. In this case, the laser vibrometer output is only dependent on target velocity. This situation changes in the presence of misalignment of the laser head. For example, translations of the laser head along the $\mathrm{x}-$, y - and $\mathrm{z}-\mathrm{axes}, \Delta x_{A}, \Delta y_{A}$ and $\Delta z_{A}$ respectively, determine a new position for the laser source, the point $A^{\prime}$, which is defined as:

$$
\overrightarrow{O A^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
x_{A}+\Delta x_{A} & y_{A}+\Delta y_{A} & z_{A}+\Delta z_{A} \tag{2.37}
\end{array}\right]^{T}
$$

while angular misalignments of the same device around the x - and the y -axes, as shown in figure 2.16 b , defined by the angles $\alpha_{L}$ and $\beta_{L}$, determine the new direction of the outgoing beam as:

$$
\hat{b}_{1}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{L}\right]\left[\begin{array}{ll}
Y, \beta_{L}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & -1 \tag{2.38}
\end{array}\right]^{T}
$$

Successively, misalignments can be added to the wedge. In this case, translational misalignments along the x - and the y - axes, $\Delta x_{B}$ and $\Delta y_{B}$, move the spin axis position on the first face from the point B to $B^{*}$ whose position becomes:
$\overrightarrow{O B *}=\left[\begin{array}{lll}\hat{x} & \hat{y} & \hat{z}\end{array}\right]\left[\begin{array}{lll}x_{B}+\Delta x_{B} & y_{B}+\Delta y_{B} & z_{B}\end{array}\right]^{T}$

Because of the translational misalignments added to the wedge and angular misalignments around the x - and y -axis, $\alpha_{w}$ and $\beta_{w}$, the point C moves to $C *$ defined as:
$\overrightarrow{O C^{*}}=\overrightarrow{O B^{*}}-\overrightarrow{B C} \hat{z}_{w}$
where $\hat{z}_{w}$ is the wedge spin axis defined as:

$$
\hat{z}_{w}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{w}\right]\left[\begin{array}{ll}
Y, \beta_{w}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & 1 \tag{2.41}
\end{array}\right]^{T}
$$

The angular misalignments also modify the wedge surface normal as:

$$
\begin{align*}
& \hat{n}_{B^{\prime}}=\hat{n}_{B^{*}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{w}\right]\left[Y, \beta_{w}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T}  \tag{2.42}\\
& \hat{n}_{C^{\prime}}=\hat{n}_{C}\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right] X, \alpha_{w}\left[I Y, \beta_{w}\right]\left[Z, \gamma_{w}\right]\left[\begin{array}{lll}
X, \psi_{w}
\end{array}\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T}\right. \tag{2.43}
\end{align*}
$$

The directions of the misaligned refracted beams $\hat{b}_{2}$ and $\hat{b}_{3}$ are again derived using equation (2.11) and their expressions become:

$$
\begin{align*}
& \hat{b}_{2}=\left(\hat{b}_{1}-\left(\hat{b}_{1} \cdot \hat{n}_{B^{\prime}}\right) \hat{n}_{B^{\prime}}\right) \frac{\varepsilon_{a}}{\varepsilon_{w}}-\left(\sqrt{1-\left(\frac{\varepsilon_{a}}{\varepsilon_{w}}\right)^{2}\left(1-\left(\hat{b}_{1} \cdot \hat{n}_{B^{\prime}}\right)^{2}\right)}\right) \hat{n}_{B^{\prime}}  \tag{2.44}\\
& \hat{b}_{3}=\left(\hat{b}_{2}-\left(\hat{b}_{2} \cdot \hat{n}_{C^{\prime}}\right) \hat{n}_{C^{\prime}}\right) \frac{\varepsilon_{w}}{\varepsilon_{a}}-\left(\sqrt{1-\left(\frac{\varepsilon_{w}}{\varepsilon_{a}}\right)^{2}\left(1-\left(\hat{b}_{2} \cdot \hat{n}_{C^{\prime}}\right)^{2}\right)}\right) \hat{n}_{C^{\prime}} \tag{2.45}
\end{align*}
$$

The modified beam path incorporates two refractions at the wedges, respectively at points $B^{\prime}$ and $C^{\prime}$ and a new measuring point situated on the target, $K^{\prime}$. Using vector polygons, the position of the point $B^{\prime}$ is found by developing the following system:

$$
\left\{\begin{array}{l}
\overrightarrow{O A^{\prime}}+\overrightarrow{A^{\prime} B^{\prime}} \mid \hat{b}_{1}=\overrightarrow{O B^{\prime}}  \tag{2.46}\\
O B *+\overrightarrow{B^{\prime} B^{\prime}}=\overrightarrow{O B^{\prime}} \\
\overrightarrow{B * B^{\prime}} \cdot \hat{n}_{B^{\prime}}=0
\end{array}\right.
$$

from which the vector $\overrightarrow{O B^{\prime}}$ is derived as:

$$
\begin{equation*}
\overrightarrow{O B^{\prime}}=\overrightarrow{O A^{\prime}}+\left|\frac{\left(\overrightarrow{O B^{*}}-\overrightarrow{O A^{\prime}}\right) \cdot \hat{n}_{B^{\prime}}}{\hat{b}_{1} \cdot \hat{n}_{B^{\prime}}}\right| \hat{b}_{1} \tag{2.47}
\end{equation*}
$$

Applying vector polygons to triangle $O B^{\prime} C^{\prime}$ the following system is derived:

$$
\left\{\begin{array}{l}
\overrightarrow{O B^{\prime}}+\left|\overrightarrow{B^{\prime} C^{\prime}}\right| \hat{b}_{2}=\overrightarrow{O C^{\prime}}  \tag{2.48}\\
O \overrightarrow{O C}^{\prime}+\overrightarrow{C * C^{\prime}}=\overrightarrow{O C^{\prime}} \\
\overrightarrow{C * C^{\prime}} \cdot \hat{n}_{C^{\prime}}=0
\end{array}\right.
$$

from which it is possible to obtain the vector $\overrightarrow{O C^{\prime}}$ as:

$$
\begin{equation*}
\overrightarrow{O C^{\prime}}=\overrightarrow{O B^{\prime}}+\left|\frac{\left(\overrightarrow{O C^{*}}-\overrightarrow{O B^{\prime}}\right) \cdot \hat{n}_{C^{\prime}}}{\hat{b}_{2} \cdot \hat{n}_{C^{\prime}}}\right| \hat{b}_{2} \tag{2.49}
\end{equation*}
$$

The measuring point $K^{\prime}$ at the target is found as:

$$
\begin{equation*}
\overrightarrow{O K^{\prime}}=\overrightarrow{O C^{\prime}}+\left|\frac{\overrightarrow{O C^{\prime}} \cdot \hat{z}}{\hat{b}_{3} \cdot \hat{z}}\right| \hat{b}_{3} \tag{2.50}
\end{equation*}
$$

The entire beam path (out and back) is comprised by four refractions (at the wedge surfaces) and one reflection (at the target) and according to equation (2.13), the total velocity measured by the LDV is defined as:

$$
\begin{equation*}
U_{m}=\overrightarrow{V_{B^{\prime}}} \cdot\left(\hat{b}_{2}-\hat{b_{1}}\right)+\overrightarrow{V_{C^{\prime}}} \cdot\left(\hat{b}_{3}-\hat{b}_{2}\right)-\overrightarrow{V_{K^{\prime}}} \cdot \hat{b_{3}} \tag{2.51}
\end{equation*}
$$

where $\overrightarrow{V_{B^{\prime}}}, \overrightarrow{V_{C^{\prime}}}$ and $\overrightarrow{V_{K^{\prime}}}$ are respectively the velocities of the two refraction points and the target. For a rotating wedge, the velocities $\overrightarrow{V_{B^{\prime}}}$ and $\overrightarrow{V_{C^{\prime}}}$ can be obtained as:

$$
\begin{align*}
& \overrightarrow{V_{B^{\prime}}}=\Omega_{w}\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{l}
X, \alpha_{w}
\end{array}\right]\left[\begin{array}{l}
Y, \beta_{w}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \times \overrightarrow{B * B^{\prime}}  \tag{2.52}\\
& \overrightarrow{V_{C^{\prime}}}=\Omega_{w}\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{w}\left[\begin{array}{l}
Y, \beta_{w}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \times \overrightarrow{C * C^{\prime}}\right. \tag{2.53}
\end{align*}
$$

while $\overrightarrow{V_{K^{\prime}}}$ for a rotating target without vibration is given by:

$$
\begin{equation*}
\overrightarrow{V_{K^{\prime}}}=\Omega_{T} \hat{z} \times \overrightarrow{O K^{\prime}} \tag{2.54}
\end{equation*}
$$

where $\Omega_{T}$ is the total velocity of the target. A more general case, the velocity measured at the point K' will also include target vibrations and velocity components due to target flexibility. According to equation (1.31), the velocity at $K^{\prime}$ can be written as:

$$
\begin{equation*}
\overrightarrow{V_{K^{\prime}}}=\overrightarrow{V_{O}}+\Omega_{T} \hat{z} \times\left(\overrightarrow{O K^{\prime}}-\overrightarrow{O O^{*}}\right)+\overrightarrow{V_{f}} \tag{2.55}
\end{equation*}
$$

where the term $\overrightarrow{O O *}$ is the vector position of the moving point $\mathrm{O}^{*}$ with respect to the reference system $x y z, \overrightarrow{V_{O}}$ is the translational velocity of the point O and $\overrightarrow{V_{f}}$ is the vibrations from target flexibility. Equation (2.51) asserts the presence of additional Doppler shifts generated by the wedge at the refraction points $B^{\prime}$ and $C^{\prime}$ as a result of wedge angular velocity in combination with incidence away from the wedge spin axis.

Equations like these are algebraically intensive when expanded but they are simple to implement sequentially, as presented, in software such as Matlab.

### 2.3.2 - Practical applications: rotating wedge optical configuration

Experiments have been performed to validate equation (2.51). The test rig shown in figure 2.17 incorporates a single rotating wedge to trace a circular scan on a rotating disk that is mounted on a translating and rotating stage. An encoder positioned behind the target drives a signal generator that controls the rotation of the wedge using a stepper motor and belt drive. Rotating the wedge in the same direction as the target and the same speed, the system enables a circular scan profile with radius of almost 6 cm .

For a real test, it is impossible to obtain a perfect alignment between the various components. It was possible to isolate the Doppler shifts produced by the rotating wedge itself by positioning a plane screen covered by reflecting tape in front of the target. For this reason, the first step was an accurate alignment between laser vibrometer and wedge in order to reduce to the smallest value possible the Doppler shift produced by the wedge.

The first test was performed to quantify the Doppler shift produced by the rotating target in the presence of translational misalignments added to the laser head. The geometrical parameters which characterized this test are: $|\overrightarrow{O A}|=0.88 \mathrm{~m},|\overrightarrow{O B}|=0.66 \mathrm{~m}$ and
$\psi_{w}=10^{\circ}$. The system traces a circular scan pattern with radius of 6 cm on the plane screen covered by reflecting tape.

When deliberately adding known translational misalignments to the laser head, which is mounted on a graduated plate and linear slide, with respect to the wedge which is maintained aligned, the velocity detected increases in magnitude at 1 x the wedge rotational speed. Examples of the measured velocities are shown in figure $2.18 \mathrm{a} \& \mathrm{~b}$. Figure 2.18a shows the LDV output obtained when rotating the wedge at 8.5 Hz the laser beam traces a circular beam path on the plane screen. The additional velocity term present at the wedge rotational frequency is very small and measures $7.13 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s}$. This indicates that the (unknown) misalignments between laser head and wedge are small and the position is considered as the best alignment possible.

Figure 2.18 b shows the LDV output obtained when the laser head is moved horizontally for 2 mm . The velocity at the wedge rotational frequency increases and measures $9.12 \mathrm{~mm} / \mathrm{s}$ as due to the misalignment introduced. The measured spectrum shows also other components that can be linked to the vibration of the system during the rotation of the wedge and speckle noise from the plane screen.
Figure $2.19 \mathrm{a} \& \mathrm{~b}$ show the rms levels of the measured harmonics when known misalignments are added to the laser head. The velocity values have been normalised in by dividing their magnitudes by the wedge rotational speed, which also equals the target rotational speed. These plots suggest a near-linear relationship between the magnitude of the additional velocity and the misalignments added to the wedge. In figure $2.19 \mathrm{a} \& \mathrm{~b}$ the experimental data are compared to the simulated uncertainty of the 1 x velocity term predicted in the presence of the same translational misalignments used in the experiments. The figures suggest a very good agreement of the two series of data. From the data a translational misalignment of 1 mm appears to introduce an rms error of almost $0.07 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$.

Figure $2.20 \mathrm{a} \& \mathrm{~b}$ shows the LDV outputs measured when the laser beam traces a circular path on the rotating target. Because the dimensions of the target are small, the
geometrical parameters of the test rig were changed and are: $|\overrightarrow{O A}|=0.64 \mathrm{~m},|\overrightarrow{O B}|=0.42 \mathrm{~m}$ and $\psi_{w}=10^{\circ}$. In this way, a circular beam path with radius of 3.5 cm was traced on the rotating target to track its rotation.

The target is fixed on a graduated plate and linear slide used to move horizontally, vertically and rotate the target around the y-direction. Figure 2.20a shows the LDV output obtained with the best alignment achieved for this arrangement. The target is rotating at 8.5 Hz and the velocity term at the wedge rotation frequency measures $5 \mathrm{e}-$ $1 \mathrm{~mm} / \mathrm{s}$. Figure 2.20 b shows the LDV output obtained when a known translational misalignment of 2 mm along the x - axis was added to the target, while maintaining the alignment of the laser head-wedge subsystem aligned. The velocity increases in magnitude at 1 x the wedge rotational speed and measures $9.38 \mathrm{~mm} / \mathrm{s}$. The velocity spectrum still shows additional components that can be linked to the vibration of the system during the rotation of the wedge and speckle noise from the rotating target.

Figures $2.21 \mathrm{a} \& \mathrm{~b}$ show the normalized rms levels of the measured 1 x component when known translational misalignments are added to the rotating target. These plots suggest a near-linear relationship between the magnitude of the additional velocity and the misalignments added to the target. In figure $2.21 \mathrm{a} \& \mathrm{~b}$ the experimental data are compared to the simulated uncertainties predicted for the 1 x velocity term considering the same misalignments. Also in this case, the comparison shows a very good agreement between the two series of data.

To analyse the effects of angular misalignments, the laser head-wedge subsystem was maintained in its ideal alignment while angular misalignments around the $y$ - axis were added to the rotating target. However, the rotation axis around the $y$ - direction is not passing through the target but is positioned 15 cm behind the target, as shown in figure 2.22. This means that angular misalignments of the target is also associated with translational misalignments along the x - direction (equal to 150 mm * $\sin$ (angular misalignment)).

Comparison between experimental and theoretical values is good only when an angular misalignment of the target of $1.3^{\circ}$ around the x -axis is included in the simulations. Figure 2.23a shows the LDV output obtained when an angular misalignment of $1^{\circ}$ (plus the additional translational misalignment of 2.16 mm along the x - direction) is added to the target. In this case, the peak at the wedge rotational frequency measures $32.16 \mathrm{~mm} / \mathrm{s}$ and it increases with the added misalignments. Figure 2.23b shows the comparison between the experimental values for different angular misalignments added to the target while the subsystem laser head-wedge is maintained at its ideal alignment. The comparison between the two series of data indicates that the experimental values are bigger than the theoretical although the trend is similar. It is possible that during the movement of the target other unknown misalignments and/or speckle noise are added that are not included in the calculation of simulated levels.

These data indicate that experimental values measured in the presence of translational misalignments are in good agreement with the predicted. Only in the presence of angular misalignments added to the target there is not a perfect accordance between the experimental and the simulated data. Nonetheless, the mathematical model closely predicts the measured velocities in the presence of misalignments.

In typical applications, the translational misalignments of the laser head and of the wedge can be controlled and minimised using linear stages on which the devices are mounted. Angular misalignments around the x - axis of the same devices are controlled and minimised using a spirit level while the alignment around the y - axis is performed by eye and can result in larger uncertainties in the measured velocity. Through the tests performed, it is apparent that alignment for the system requires two distinct steps. The first regards the alignment between the wedge and the laser head using the reflecting screen to minimise misalignment. Once aligned, this combination of laser head and wedge can be aligned to the target or, as in these tests, the target can be aligned to the combination of laser source and wedge.

In summary, a new method able to predict LDV measured velocities has been proposed and applied for the first time to some arrangements of particular interest. In the
following sections, this method will be used to derive complete mathematical descriptions of more complex scanning LDV systems including those reported by other researchers or used in commercial systems.

## Chapter 3

## Prediction of the measured velocity in scanning and tracking LDV systems

In the previous chapter, a novel mathematical procedure to predict the velocity measured by LDV systems was introduced and applied to relatively simple optical arrangements. Because of its versatility and the possibility to integrate with the Velocity Sensitivity Model, this procedure can be used to model more complicated optical schemes. In this chapter the procedure will be employed to analyze three different scanning LDV systems.

The first one is the well known dual mirror SLDV system. This system has been the subject of much published literature [3.1, 3.2 and 3.3] although the effects on the measured velocity of the mirrors in the scanning head have been largely neglected. Since the new approach provides both the target and the scanning head contribution to the velocity sensed, the model of the dual mirror SLDV system will be reformulated and the results obtained will be compared to published experimental data [3.4].

The other two arrangements investigated are the dual-wedges and the Dove prism SLDV systems, which are considered as possible alternatives to the traditional dual mirror SLDV system. These systems allow the creation of various scan patterns but the chapter concentrates on circular scans on rotating targets, for tracking applications. The analysis serves as a means to demonstrate the effectiveness of the procedure. In particular, the origin of additional measured velocity components due to the presence of
geometrical misalignments will be revealed and their influence on measured data will be discussed.

The analyses will show a full set of equations for the case of zero misalignments followed by a full set of equations for the misaligned case to demonstrate clearly how the analysis is affected. Sensitivity to in-plane and out-of-plane target vibrations will also be considered as it is an important performance parameter in these scanning systems.

## 3.1 - Dual mirror SLDV system

### 3.1.1 - Mathematical model

A schematic representation of the dual mirror SLDV system is shown in figure 3.1. A reference system xyz is fixed at the target with the $z$ - axis coincident with its spin axis. The oscillating mirrors are situated at a distance $z_{0}$ from the target and their separation is defined by the term $d_{s}$. The mirror oscillation axes are parallel to the z - and the x axes, respectively, for the first and the second mirror whose initial inclinations, $\theta_{x 0}$ and $\theta_{y 0}$, are each at $45^{\circ}$.

The zero misalignments configuration is obtained when the incoming beam is parallel to the x - axis and meets the first mirror in its middle point B and when the target spin axis passes through the middle point of the second mirror, the point C . The directions of the incoming beam and of the mirror normals are written as:
$\hat{b}_{1}=\left[\begin{array}{lll}\hat{x} & \hat{y} & \hat{z}\end{array}\right]\left[\begin{array}{lll}-1 & 0 & 0\end{array}\right]^{T}$
$\hat{n}_{B}=\left[\begin{array}{lll}\hat{x} & \hat{y} & \hat{z}\end{array}\right]\left[\begin{array}{lll}Z, \theta_{x 0}\end{array}\right]\left[\begin{array}{lll}1 & 0 & 0\end{array}\right]^{T}$
$\left.\hat{n}_{C}=\left[\begin{array}{lll}\hat{x} & \hat{y} & \hat{z}\end{array}\right]\left[\begin{array}{llll}X, \theta_{y 0}\end{array}\right] \begin{array}{lll}0 & -1 & 0\end{array}\right]^{T}$
where the rotation matrices $\left[Z, \theta_{x 0}\right]$ and $\left\lfloor X, \theta_{y 0}\right\rfloor$ introduce the initial mirror inclinations around the z - and the x - axis, respectively. The creation of a circular beam path suitable for tracking applications requires mirror scanning angles $\theta_{s x}$ and $\theta_{s y}$ defined as:

$$
\begin{align*}
& \theta_{x}(t)=\theta_{x 0}+\theta_{s x}=\theta_{x 0}-0.5 \tan ^{-1}\left(\frac{r_{s}}{z_{0}+d_{s}}\right) \cos \left(\Omega_{T} t+\phi_{l}\right)  \tag{3.4}\\
& \theta_{y}(t)=\theta_{y 0}+\theta_{s y}=\theta_{y 0}+0.5 \tan ^{-1}\left(\frac{r_{s}}{z_{0}}\right) \sin \left(\Omega_{T} t+\phi_{2}\right) \tag{3.5}
\end{align*}
$$

where $\Omega_{T}$ is the target rotational speed, $r_{s}$ is the radius of the desired scan pattern, $\phi_{1}$ and $\phi_{2}$ are the initial phases of the mirror oscillations with $\phi_{1}=\phi_{2}$ Equations (3.4\&3.5) show corrected mirror drive signals with unequal amplitudes used to describe an almost perfect circular scan pattern and indicate that the mirror scan angles are functions of the system geometry and of the desired scan radius [3.4].

The orientations of the reflected beams are found by applying Snell's law, equation (2.10), at each deflection point obtaining:

$$
\begin{align*}
& \hat{b}_{2}=\hat{b}_{1}-2\left(\hat{b}_{1} \cdot \hat{n}_{B}\right) \hat{n}_{B}  \tag{3.6}\\
& \hat{b}_{3}=\hat{b}_{2}-2\left(\hat{b}_{2} \cdot \hat{n}_{C}\right) \hat{n}_{C} \tag{3.7}
\end{align*}
$$

The final direction of the incident beam $\hat{b}_{3}$ is given by:

$$
\begin{equation*}
\hat{b}_{3}=-\left(\sin 2 \theta_{s x}\right) \hat{x}+\left(\cos 2 \theta_{s x} \sin 2 \theta_{s y}\right) \hat{y}-\left(\cos 2 \theta_{s x} \cos 2 \theta_{s y}\right) \hat{z} \tag{3.8}
\end{equation*}
$$

From knowledge of the initial geometry of the system and the reflected beam directions, the position of the deflection points can be found using vector polygons. The laser source position, the point A , is defined as:
$\overrightarrow{O A}=\left[\begin{array}{lll}\hat{x} & \hat{y} & \hat{z}\end{array}\right]\left[\begin{array}{lll}x_{A} & y_{A} & z_{A}\end{array}\right]^{T}$
while the position of the first deflection point B situated at the centre of the first oscillating mirror and on the rotation axis is given as:
$\overrightarrow{O B}=\left[\begin{array}{lll}\hat{x} & \hat{y} & \hat{z}\end{array}\right]\left[\begin{array}{lll}x_{B} & y_{B} & z_{B}\end{array}\right]^{T}$
where the component $x_{B}=0, y_{B}=y_{A}=-d_{s}$ and $z_{A}=z_{B}=z_{0}$. The point C , which is situated at the middle of the second mirror and on its rotation axis, is defined as:

$$
\overrightarrow{O C}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
x_{C} & y_{C} & z_{C} \tag{3.11}
\end{array}\right]^{T}
$$

where $x_{C}=y_{C}=0$ and $z_{C}=z_{0}$. The reflection point situated at the second mirror, the point $C^{\prime}$, is found from the development of the following system:
$\left\{\begin{array}{l}\overrightarrow{O B}+\overrightarrow{B C^{\prime}} \hat{b}_{2}=\overrightarrow{O C^{\prime}} \\ \overrightarrow{O C}+\overrightarrow{C C^{\prime}}=\overrightarrow{O C^{\prime}} \\ \overrightarrow{C C^{\prime}} \cdot \hat{n}_{C}=0\end{array}\right.$

From the system (3.12) the vector $\overrightarrow{B C^{\prime}}$ can be obtained and the final position of the reflection point $C^{\prime}$ is found as:

$$
\begin{equation*}
\overrightarrow{O C^{\prime}}=\overrightarrow{O B}+\left|\frac{(\overrightarrow{O C}-\overrightarrow{O B}) \cdot \hat{n}_{C}}{\hat{b}_{2} \cdot \hat{n}_{C}}\right| \hat{b}_{2} \tag{3.13}
\end{equation*}
$$

The description of the beam path is completed by determining the position of the measuring point K on the target:

$$
\begin{equation*}
\overrightarrow{O K}=\overrightarrow{O C^{\prime}}+\left|\frac{\overrightarrow{O C^{\prime}} \cdot \hat{z}}{\hat{b}_{3} \cdot \hat{z}}\right| \hat{b}_{3} \tag{3.14}
\end{equation*}
$$

From knowledge of the positions of the reflection points and the dynamic behaviours of the mirrors and target, the velocity measured by the laser vibrometer can be predicted. According to figure 3.1 and equation (2.13), the complete velocity measured by the system can be written as:

$$
\begin{equation*}
U_{m}=\overrightarrow{V_{B}} \cdot\left(\hat{b}_{2}-\hat{b_{1}}\right)+\overrightarrow{V_{C^{\prime}}} \cdot\left(\hat{b}_{3}-\hat{b}_{2}\right)-\overrightarrow{V_{K}} \cdot \hat{b_{3}} \tag{3.15}
\end{equation*}
$$

where $\hat{b}_{1}, \hat{b}_{2}$ and $\hat{b}_{3}$ are the beam directions defined in equations $(3.1,3.6 \& 3.8) \overrightarrow{V_{B}}$ and $\overrightarrow{V_{C^{\prime}}}$ are the velocities of the deflection points on the oscillating mirror surfaces and $\overrightarrow{V_{K}}$ is the velocity at the measuring point on the target.

For the zero misalignment configuration, the oscillations of the mirrors do not affect the measurement because the points B and $C^{\prime}$ are both positioned along the respective mirror rotation axes where surface velocities are zero. For a rotating target, the velocity of the measuring point K is given as:

$$
\begin{equation*}
\overrightarrow{V_{K}}=\Omega_{T} \hat{z} \times \overrightarrow{O K} \tag{3.16}
\end{equation*}
$$

At this point, all the terms in equation (3.15) are known and the measured velocity can be reformulated as:

$$
\begin{equation*}
U_{m}=-\left(\Omega_{T} \hat{z} \times \overrightarrow{O K}\right) \cdot \hat{b}_{3} \tag{3.17}
\end{equation*}
$$

Equation (3.17) indicates that the velocity sensed by the dual mirror SLDV system for the zero misalignment configuration is affected only by the rotating target.

### 3.1.2-Circular tracking applications on a rotating target

Figure 3.2a shows the simulated circular path with radius of 10 cm obtained by setting $z_{0}=1.2 \mathrm{~m}$ and $d_{s}=5 \mathrm{~cm}$. The scan path deviates from a perfect circle but this deviation is so small that in these applications it is considered as a perfect circular scan pattern. These results have been obtained by implementing the model described by equations (3.1-17) in Matlab. Tracking applications on a rotating but not vibrating target indicate that the system measures only a velocity term at twice the target rotational speed with rms amplitude $1.17 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, as indicated in figure 3.2 b .

Figure 3.2 b shows three different velocity spectra detected from this configuration. The first spectrum shows the velocities generated by the scanning mirrors (a combination of the Doppler shifts at each mirror). The second plot shows the velocity detected from the rotating target while the last spectrum indicates the velocity measured by the laser vibrometer and is obtained by combining the previous two spectra as indicated by equation (3.15). The magnitude of this term depends on the chosen values of $z_{0}$, $d_{s}=\left|y_{C}-y_{B}\right|$ and radius and the result agrees with the data reported by Halkon under the same conditions [3.4].

Figure 3.3a shows the scan pattern traced by the system with radius of 5 cm while figure 3.3 b shows the velocity spectrum detected by the laser vibrometer. In this case, the 2 x velocity term measures $2.94 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$. The amplitude of the 2 x velocity term is proportional $r_{s}^{2}$ so when the scan radius is reduced to half the magnitude of the 2 x velocity term decreases of 4 times. This result agrees with the theory [3.4] that indicates that the level of the 2 x velocity term is proportional to $r_{s}^{2}$.

In the presence of whole-body target vibrations, the velocity measured at the point K becomes more complex. According to equation (1.31), for a general case, the velocity measured at the target can be written as:

$$
\begin{equation*}
\overrightarrow{V_{K}}=\overrightarrow{V_{O}}+\Omega_{T} \hat{z} \times\left(\overrightarrow{O K}-\overrightarrow{O O^{*}}\right) \tag{3.18}
\end{equation*}
$$

where $\overrightarrow{V_{O}}$ is the translational velocity of the point O , the term $\overrightarrow{O O^{*}}$ is the vector position of the point on the target that coincides with O without vibrations which moves to $O^{*}$ with respect to the reference system $x y z$. In this case, the system detects a single peak for each component of z- vibration and smaller sidebands associated with any inplane ( x - or y -) vibration component.

Figure 3.4 a shows the predicted spectrum in the presence of whole-body harmonic out-of-plane and in-plane vibration both of amplitude $10 \mathrm{~mm} / \mathrm{s}$ but frequencies of $5 \Omega_{T}$ and $10 \Omega_{T}$ with amplitude $10 \mathrm{~mm} / \mathrm{s}$.

The peak at $5 \Omega_{T}$ due to the whole-body out-of-plane vibration measures 1.12 $\mathrm{mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ ( $=99.8 \mathrm{~mm} / \mathrm{s}$ ) which is almost $99.8 \%$ of the true vibration. The whole-body in-plane vibration produces a pair of sidebands at 9 x and 11 x with same amplitude of $2.01 \mathrm{e}-2$ and $2.48 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ with sensitivity around $2.21 \%$ of the genuine vibration. Moreover, the additional component at 2 x , due to the geometry of the system still measures $2.94 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$.

When the system is misaligned, the sensitivity of the system can change. If, for example, an angular misalignment around the y - axis, with $\gamma_{L}=0.2^{\circ}$, is added to the laser head, the velocity spectrum becomes that shown in figure 3.4 b . The peak at 5 x does change amplitude but various sidebands of negligible magnitude are detected around it. The whole-body in-plane vibration produces a single peak at 10 x with magnitude $3.92 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}(=3.49 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s})$ with sensitivity around $0.349 \%$ of the genuine vibration while the pair of sidebands measured at 9 x and 11 x are unchanged with respect to the system with ideal alignment. The misalignment results also in additional low harmonics, the 1x component. These data indicate that in the presence of misalignments any target vibration produces a single peak that combined to the
sidebands due to the whole-body in-plane target vibration made complicated the understanding of the vibration measurement.

### 3.1.3 - Introduction of misalignments

The new approach used to model scanning systems allows incorporation of misalignments, both translational and angular, at all stages of the system. Table 3.1 reports a typical, full set of misalignments that can affect the dual mirror arrangement. The values chosen are based on those met in practical applications after the user has taken steps to reduce these errors. In practice, translational alignment of the laser head can be controlled using the tripod and linear guides on which the vibrometer is mounted. The angular alignment of the vibrometer around the z - axis can be controlled using a spirit level while the angular alignment around the $y$ - axis is critical. For the scanning head, the alignment with respect to the target is critical, in particular the control of the angular alignment around the $y$-axis.

| Device | Misalignment | Range | $\boldsymbol{\Delta}$ step | N. steps |
| :---: | :---: | :---: | :---: | :---: |
| Laser head | $\Delta \mathrm{x}_{\mathrm{A}}$ | $\pm 5 \mathrm{~mm}$ | 5 mm | 3 |
|  | $\Delta \mathrm{y}_{\mathrm{A}}$ | $\pm 5 \mathrm{~mm}$ | 5 mm | 3 |
|  | $\Delta \mathrm{z}_{\mathrm{A}}$ | $\pm 5 \mathrm{~mm}$ | 5 mm | 3 |
|  | $\beta_{\mathrm{L}}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |
|  | $\gamma_{\mathrm{L}}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |
| $1^{\text {st }}$ oscillating | $\Delta \mathrm{x}_{\mathrm{B}}$ | $\pm 3 \mathrm{~mm}$ | 3 mm | 3 |
|  | $\Delta \mathrm{y}_{\mathrm{B}}$ | $\pm 3 \mathrm{~mm}$ | 3 mm | 3 |
|  | $\alpha_{\mathrm{m} 1}$ | $\pm 0.5^{\circ}$ | $0.5^{\circ}$ | 3 |
|  | $\beta_{\mathrm{m} 1}$ | $\pm 0.5^{\circ}$ | $0.5^{\circ}$ | 3 |
| $2^{\text {nd }}$ oscillating | $\gamma_{\mathrm{m} 1}$ | $\pm 0.5^{\circ}$ | $0.5^{\circ}$ | 3 |
| mirror | $\Delta \mathrm{y}_{\mathrm{C}}$ | $\pm 3 \mathrm{~mm}$ | 3 mm | 3 |
|  | $\Delta \mathrm{z}_{\mathrm{C}}$ | $\pm 3 \mathrm{~mm}$ | 3 mm | 3 |
|  | $\alpha_{\mathrm{m} 2}$ | $\pm 0.5^{\circ}$ | $0.5^{\circ}$ | 3 |
|  | $\beta_{\mathrm{m} 2}$ | $\pm 0.5^{\circ}$ | $0.5^{\circ}$ | 3 |
|  | $\gamma_{\mathrm{m} 2}$ | $\pm 0.5^{\circ}$ | $0.5^{\circ}$ | 3 |

Table 3.1 - Full set of misalignments used to investigate the dual mirror SLDV system

When translational misalignments along the $\mathrm{x}-, \mathrm{y}$ - and the z - axes, with displacements $\Delta x_{A}, \Delta y_{A}$ and $\Delta z_{A}$, are added to the laser head position the point A moves to $A^{\prime}$ defined as:

$$
\overrightarrow{O A^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
x_{A}+\Delta x_{A} & y_{A}+\Delta y_{A} & z_{A}+\Delta z_{A} \tag{3.19}
\end{array}\right]^{T}
$$

Angular misalignments of the laser head around the y - and the z - axes, with angles $\beta_{L}$ and $\gamma_{L}$, modify the direction of the outgoing beam as:

$$
\hat{b}_{1}=\left[\begin{array}{ll}
\hat{x} & \hat{y}
\end{array} \hat{z}\right]\left[\begin{array}{l}
{\left[, \beta_{L}\right.}
\end{array}\right]\left[Z, \gamma_{L}\right]\left[\begin{array}{lll}
-1 & 0 & 0 \tag{3.20}
\end{array}\right]^{T}
$$

where the rotation matrices $\left[Y, \beta_{L}\right]$ and $\left[Z, \gamma_{L}\right]$ accommodate angular misalignments. The same method is used to introduce angular and translational misalignments to the oscillating mirrors. For the first mirror, translational misalignments along the x - and y axes, with displacements $\Delta x_{B}$ and $\Delta y_{B}$, move the point B to $B^{*}$ whose position is found as:

$$
\overrightarrow{O B^{*}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
x_{B}+\Delta x_{B} & y_{B}+\Delta y_{B} & z_{B} \tag{3.21}
\end{array}\right]^{T}
$$

while no translational misalignments along the z - axis in order to have the point $B^{*}$ on the rotation axis of the mirror and at the centre of the mirror. Angular misalignments of the mirror around the same axes, with angles $\alpha_{m l}, \beta_{m l}$ and $\gamma_{m l}$, modify the mirror surface normal in:

$$
\hat{n}_{B^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{m 1}\right]\left[Y, \beta_{m 1}\right]\left[Z,\left(\theta_{x}+\gamma_{m l}\right)\right]\left[\begin{array}{lll}
1 & 0 & 0 \tag{3.22}
\end{array}\right]^{T}
$$

For the second oscillating mirror, translational misalignments along the y - and z - axis move the point C to $C^{*}$ defined as:

$$
\overrightarrow{O C^{*}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
x_{C} & y_{C}+\Delta y_{C} & z_{C}+\Delta z_{C} \tag{3.23}
\end{array}\right]^{T}
$$

As for the point $B^{*}$, for the point $C^{*}$ no translational misalignments along the z - axis are added to the second oscillating in order to leave the point $C^{*}$ on the mirror rotation axis and at its centre.

Angular misalignments of the second mirror around the three axes, with angles $\alpha_{m 2}$, $\beta_{m 2}$ and $\gamma_{m 2}$, determine the misaligned direction of the mirror surface normal as:

$$
\hat{n}_{C^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{l}
Y, \beta_{m 2}
\end{array}\right]\left[Z, \gamma_{m 2}\right]\left[X,\left(\theta_{y}+\alpha_{m 2}\right)\right]\left[\begin{array}{cc}
0 & -1 \tag{3.24}
\end{array} 0\right]^{T}
$$

The directions of the reflected beams are still derived from equations (3.6\&3.7) with the directions of the outgoing beam and mirror surface normals modified as in equations (3.20, 3.22\&3.24). The position of the deflection point $B^{\prime}$ is found from a vector polygon obtaining the following system:

$$
\left\{\begin{array}{l}
\overrightarrow{O A^{\prime}}+\left|\overrightarrow{A^{\prime} B^{\prime}}\right| \hat{b}_{1}=\overrightarrow{O B^{\prime}}  \tag{3.25}\\
\overrightarrow{O B^{*}}+\overrightarrow{B^{*} B^{\prime}}=\overrightarrow{O B^{\prime}} \\
\overrightarrow{B^{*} B^{\prime}} \cdot \hat{n}_{B^{\prime}}=0
\end{array}\right.
$$

from which the vector $\overrightarrow{O B^{\prime}}$ is obtained as:

$$
\begin{equation*}
\overrightarrow{O B^{\prime}}=\overrightarrow{O A^{\prime}}+\left|\frac{\left(\overrightarrow{O B^{*}}-\overrightarrow{O A^{\prime}}\right) \cdot \hat{n}_{B^{\prime}}}{\hat{b}_{1} \cdot \hat{n}_{B^{\prime}}}\right| \hat{b}_{1} \tag{3.26}
\end{equation*}
$$

At the second mirror, the reflection point $C^{\prime}$ is found by developing and resolving the following system

$$
\left\{\begin{array}{l}
\overrightarrow{O B^{\prime}}+\left|\overrightarrow{B^{\prime} C^{\prime}}\right| \hat{b}_{2}=\overrightarrow{O C^{\prime}}  \tag{3.27}\\
\overrightarrow{O C^{*}}+\overrightarrow{C^{*} C^{\prime}}=\overrightarrow{O C^{\prime}} \\
\overrightarrow{C^{*} C^{\prime}} \cdot \hat{n}_{C^{\prime}}=0
\end{array}\right.
$$

and the vector $\overrightarrow{O C^{\prime}}$ is defined as:

$$
\begin{equation*}
\overrightarrow{O C^{\prime}}=\overrightarrow{O B^{\prime}}+\left|\frac{\left(\overrightarrow{O C^{*}}-\overrightarrow{O B^{\prime}}\right) \cdot \hat{n}_{C^{\prime}}}{\hat{b}_{2} \cdot \hat{n}_{C^{\prime}}}\right| \hat{b}_{2} \tag{3.28}
\end{equation*}
$$

At the target, the position of the measuring point $K^{\prime}$ is obtained as:

$$
\begin{equation*}
\overrightarrow{O K^{\prime}}=\overrightarrow{O C^{\prime}}+\left|\frac{\overrightarrow{O C^{\prime}} \cdot \hat{z}}{\hat{b}_{3} \cdot \hat{z}}\right| \hat{b}_{3} \tag{3.29}
\end{equation*}
$$

The expression for the measured velocity is reformulated in terms of the new points of deflection:

$$
\begin{equation*}
U_{m}=\overrightarrow{V_{B^{\prime}}} \cdot\left(\hat{b_{2}}-\hat{b_{1}}\right)+\overrightarrow{V_{C^{\prime}}} \cdot\left(\hat{b}_{3}-\hat{b_{2}}\right)-\overrightarrow{V_{K^{\prime}}} \cdot \hat{b_{3}} \tag{3.30}
\end{equation*}
$$

where the velocities $\overrightarrow{V_{B^{\prime}}}$ and $\overrightarrow{V_{C^{\prime}}}$ are defined as:

$$
\begin{align*}
& \overrightarrow{V_{B^{\prime}}}=\dot{\theta}_{x}\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{m 1}\right]\left[\begin{array}{ll}
Y, \beta_{m 1}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \times \overrightarrow{B^{*} B^{\prime}}  \tag{3.31}\\
& \overrightarrow{V_{C^{\prime}}}=\dot{\theta}_{y}\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{ll}
Y, \beta_{m 2}
\end{array}\right]\left[\begin{array}{ll}
Z, \gamma_{m 2}
\end{array}\right]\left[\begin{array}{lll}
1 & 0 & 0
\end{array}\right]^{T} \times \overrightarrow{C^{*} C^{\prime}} \tag{3.32}
\end{align*}
$$

in which the vectors $\overrightarrow{B^{*} B^{\prime}}$ and $\overrightarrow{C^{*} C^{\prime}}$ are given as:

$$
\begin{align*}
& \overrightarrow{B^{*} B^{\prime}}=\overrightarrow{O B^{\prime}}-\overrightarrow{O B^{*}}  \tag{3.33}\\
& \overrightarrow{C^{*} C^{\prime}}=\overrightarrow{O C^{\prime}}-\overrightarrow{O C^{*}} \tag{3.34}
\end{align*}
$$

Equation (3.30) allows the investigation of the effects produced by the various misalignments.

### 3.1.4 - Analysis of some misaligned configurations

This section shows typical misaligned configurations that have been analysed to show the misalignment effects in the scan pattern and in the measured velocity. In the absence of any misalignment, the system measures a 2 x velocity term due to the geometry of the arrangement and traces a near circular scan path centred in the target plane. Figure 3.5a shows the scan pattern traced by the system when the laser head has a translational misalignment along the y - axis, $\Delta y_{A}=2 \mathrm{~mm}$. The circle is shifted along the x -direction as indicated in figure 3.5 a while figure 3.5 b shows additional velocity terms at 1 x and 2 x the target rotational speed with magnitudes of $1.72 \mathrm{e}-1$ and $2.94 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$. The final 1 x component measured by the vibrometer is a combination of the Doppler shifts generated at the target and at the scanning mirrors which results in an increment of the velocity terms measured by the laser vibrometer. In the same manner, the effect of translational misalignment of the laser head in the z - direction can also be predicted. For example, a translational misalignment of the laser head along the $z$ - axis, with $\Delta z_{A}=2 \mathrm{~mm}$, moves the scan pattern along the y - axis, as shown in figure 3.6 a and introduces an additional 1 x velocity of $1.74 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ as indicated in figure 3.6 b . The 2 x velocity term is unchanged.

When the laser head has an angular misalignment around the z - axis, with $\gamma_{L}=0.2^{\circ}$, the scan pattern moves along the x - direction as indicated in figure 3.7a while the vibrometer measures additional velocities at 1 x and 2 x of magnitude $6.51 \mathrm{e}-3$ and $2.94 \mathrm{e}-$ $2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, respectively, as indicated in figure 3.7b. Again, from the figure, it is evident that the 1 x velocity term results from the combination of the Doppler shifts generated at the mirrors and at the target. In the same manner, the effect of angular misalignment of the laser head around the $y$ - direction can also be predicted.

When the first scanning mirror has a translational misalignment along the $y$ - axis of $\Delta y_{B}=2 \mathrm{~mm}$, the distance between the deflection points $B^{\prime}$ and $C^{\prime}$ measured along the y - axis changes from the initial value of $d_{s}$ and this should affect the magnitude of the 2 x velocity term and on the position of the final scan pattern in the xy plane. In this
case, the 2 x term measures $2.82 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ and the additional 1 x velocity term measures $1.72 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ which is a combination of the Doppler shifts generated at the target and mirrors. The scan pattern moves along the x - axis as indicated in figure 3.8a.

When the same scanning mirror has an angular misalignment around the y - axis, $\beta_{m 1}=0.2^{\circ}$, the scan pattern moves along the y - direction, as indicated in figure 3.9a, while the laser vibrometer measures a DC, 1 x and 2 x velocity terms with magnitudes of $2.9 \mathrm{e}-4,1.08 \mathrm{e}-2$ and $2.94 \mathrm{e}-2 \mathrm{~m} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, as shown in figure 3.9 b . The same 1 x and 2 x velocity components are detected when the first mirror has an angular misalignment around the x- axis with $\alpha_{m l}=0.2^{\circ}$, as reported in figure 3.10 b , while the scan pattern is shifted along the x - axis, as reported in figure 3.10a.

For the second oscillating mirror, translational misalignments along the x - axis have no effects in the measured velocity. Translational misalignments along the $y$ - axis, with $\Delta y_{C}=2 \mathrm{~mm}$, moves the scan pattern along the y - axis, as shown in figure 3.11 a and additional 1 x and 2 x velocity terms with magnitudes of $5.65 \mathrm{e}-2$ and $3.06 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ are detected by the laser vibrometer as reported by figure 3.11 b . Translational misalignments of the second mirror along the z - axis, with $\Delta z_{C}=2 \mathrm{~mm}$, moves the scan pattern along the y - axis while the predicted additional 1 x and 2 x velocity terms measure $1.74 \mathrm{e}-1$ and $2.94 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, as indicated in figure $3.12 \mathrm{a} \& \mathrm{~b}$. An angular misalignment of the second mirror around the y - axis of $\beta_{m 2}=0.2^{\circ}$ produces a shift of the scan pattern along the x - axis and the presence of additional DC term with magnitude of $2 \mathrm{e}-4 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ as indicated in figures $3.13 \mathrm{a} \& \mathrm{~b}$. Angular misalignments of the second scanning mirror around the z - axis have no effects on the measured velocity within the scale plotted.

These results show that the effects of misalignments can be predicted, including the separate measured velocities associated with the target and the scanning mirrors.

### 3.1.5 - Quantification of the uncertainties in the presence of single and

## combined misalignments

From this point, the analysis of misalignments continues in two distinct phases. The first part concerns the estimation of uncertainty introduced by single misalignments to detect the most significant sources of error. The second part analyses some configurations in which typical, practical values of misalignments are combined. The goal is the evaluation of the uncertainty likely to affect real measurements. The geometrical parameters of the system used for this investigation are: $z_{0}=1.2 \mathrm{~m}$ and $d_{s}=5 \mathrm{~cm}$ with scanning mirrors oscillating to produce a 5 cm scan radius.

The additional DC term is detected when the second scanning mirror has angular misalignments around the $y$ - axis but these values are very small, as shown in figure 3.14a. Figure 3.14 b shows the predicted values for the 2 x velocity term in the presence of translational misalignments added to the oscillating mirrors along the $y$ - axis. As previously said, the magnitude of this additional term is directly linked to the value of $d_{s}$ and each misalignment that modifies the initial value of $d_{s}$ causes a variation of the initial 2 x velocity term. In these simulations, positive translational misalignments of the mirrors along the y - axis have been considered. Thus, translational misalignments of the first mirror (FM) along the y -axis reduce the initial value of $d_{s}$ and also the amplitude of the 2 x component while translational misalignments of the second mirror (SM) along the same axis increase the initial value of $d_{s}$, as shown in figure 3.14 b .

The additional 1 x velocity term is detected when translational or angular misalignments are added to the various optical devices. Figure 3.15a shows the values predicted for the 1 x additional velocity term when translational misalignments are added to the laser head and the mirrors. Translational misalignments of the first mirror added along the y - axis and translational misalignments added to the second mirror along the z - axis give values of additional 1x components with values similar to those reported by the trend for the laser head misalignments (LH) in figure 3.15a. Translational misalignments added to the second mirror along the y - axis give values for the additional 1 x velocity component
similar to that indicated for the translational misalignments added to the first mirror in figure 3.15a. Translational misalignments of the laser head added along the x - axis produce no additional velocities as translational misalignments along the z - axis added to the first mirror and along the x - axis added to the second mirror.

Figure 3.15 b shows the most important values of the 1 x velocity term predicted in the presence of angular misalignments of the various optical devices. The data show that the biggest uncertainty is predicted when the laser head is angularly misaligned around the $y$ - axis while the values predicted for angular misalignments of the laser head around the x - axis are slightly smaller. The data reported in figure $3.14 \mathrm{a} \& \mathrm{~b}$ and $3.15 \mathrm{a} \& \mathrm{~b}$ suggest that the main uncertainty introduced by the various misalignments is the additional 1 x velocity term affected by the optical devices.

In practical applications a combination of all the possible misalignments reported in table 3.1 is likely and simulations have been made with combined misalignments covering $531441\left(=3^{12}\right)$ different misaligned situations. The predicted additional velocities from each situation have been used to calculate RMS values, which characterize the system in terms of level of uncertainty expected in practical applications. Table 3.2 reports these RMS values. These data have been obtained considering only the different scenarios in which the scan patterns have a maximum offset on the target plane of $5 \%$ from the ideal position $(\approx 2.5 \mathrm{~mm})$. This is because in practical applications the user generally performs vibration measurements only when the scan pattern is almost centred in the target plane.

| Additional component | RMS expected value |
| :---: | :---: |
| DC | $1.32 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |
| 1 x | $1.38 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |
| 2 x | $3.16 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |

Table 3.2 - Predicted RMS values for additional velocity terms for the dual mirror SLDV system performing a circle with radius of 5 cm

The data indicate the 1 x velocity term as the biggest uncertainty detected by the system while the uncertainty expected for the 2 x velocity term is one order of magnitude smaller. The expected uncertainty for to the DC term is two orders of magnitude smaller than the 1 x term while the value predicted for the 3 x term is negligible and likely to be masked by the speckle noise level typical in real applications. The values reported in table 3.2 can be considered as the expected values in typical circular scanning applications when the system traces a circle with radius of 5 cm and the target is only rotating.

Figures $3.16 \mathrm{a} \& \mathrm{~b}$ and $3.17 \mathrm{a} \& \mathrm{~b}$ show two cases of arrangements with combined misalignments producing scan patterns centred in the target plane but also uncertainties. The scenario reported in figure $3.16 \mathrm{a} \& \mathrm{~b}$ has been obtained considering the following misalignments: $\beta_{L}=0.2^{\circ}, \gamma_{L}=0.1^{\circ}, \Delta y_{A}=3 \mathrm{~mm}, \Delta z_{A}=3 \mathrm{~mm}, \alpha_{m 1}=-0.1^{\circ}, \beta_{m 1}=0.2^{\circ}$, $\Delta x_{B}=3 \mathrm{~mm}, \Delta y_{B}=-3 \mathrm{~mm}, \gamma_{m 2}=0.2^{\circ}, \Delta y_{C}=-3 \mathrm{~mm}$ and $\Delta z_{C}=-3 \mathrm{~mm}$. In this case, the standard deviation from radial position is around $0.16 \%$ and the centre offset is around $5 \%$. The vibrometer detects the additional $\mathrm{DC}, 1 \mathrm{x}$ and 2 x velocity terms that measure $8.81 \mathrm{e}-3,2.94 \mathrm{e}-1$ and $3.01 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, respectively, as shown in figure 3.16 b while the scan pattern is almost centred in the target plane.

Figure $3.17 \mathrm{a} \& \mathrm{~b}$ have been obtained considering the following misalignments: $\gamma_{L}=0.3^{\circ}, \Delta y_{A}=3 \mathrm{~mm}, \Delta z_{A}=3 \mathrm{~mm}, \beta_{m 1}=0.1^{\circ}, \Delta x_{B}=-3 \mathrm{~mm}, \beta_{m 2}=-0.3^{\circ}, \gamma_{m 2}=-0.3^{\circ}$, $\Delta y_{C}=-3 \mathrm{~mm}, \Delta z_{C}=3 \mathrm{~mm}$. For this scenario, the standard deviation from radial position is around $2.63 \%$ while the centre offset is around $1.92 \%$. The system predicts a DC, 1 x and 2 x velocity terms with magnitudes $5.61 \mathrm{e}-3,8.75 \mathrm{e}-2$ and $2.77 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ as indicated in figure 3.17 b . These two cases indicate that in practical applications, the presence of a circular scan pattern centred in the target plane is not synonymous with perfect alignment because it can be obtained by combining the various misalignments which will result in measured velocity uncertainties despite the quality of the scan pattern.

## 3.2 - The dual-wedge SLDV system

The second scanning SLDV arrangement analysed in this chapter is the novel dualwedge SLDV system. The main advantage of the wedges is that it is easier to generate whole body rotation at high rotational speed than it is to make a device oscillate at high frequencies as in the dual mirror SLDV system. The use of a rotating wedge for scanning was proposed for the first time by Rosell [3.5]. The rotating wedge was considered as a versatile means to scan structures at very high velocities and trace various scan patterns through appropriate adjustment of the relative velocities and initial positions of two rotating wedges. Since then, similar systems have been used in various applications such as optical tracking in guidance systems [3.6], laser radar systems [3.7], inter-satellite laser communication [3.8] and in confocal reflectance microscopy [3.9]. However, until now, wedges have not been considered for LDV applications on rotating components. This work considers for the first time the dual wedge scanning system as a valid alternative to the dual mirror SLDV system and considers sensitivity to in-plane and out-of-plane vibration and to geometrical misalignments.

As cited in chapter 2, a single rotating wedge enables a circular scan but the combination of two wedges independently rotated offers greater flexibility and can enable more complicated scan patterns. In this case, the total beam deflection is dependent on the relative initial angular positions of the wedges and the beam can take any direction falling within a cone-shaped volume.

The relationship between the beam deviation and the wedge movement is quite complicated and the addition of a second wedge makes the description of the final beam path difficult. Nonetheless, this model is developed, for the first time, enabled by the new approach proposed. In this chapter, the analysis concentrates on the development of the mathematical model for the dual wedge scanning system to predict measured velocity in vibration measurements on rotating structures and on the investigation of the misalignment effects. A dual wedge scanning system was tested in the laboratory and used to confirm predictions.

### 3.2.1-Mathematical model

The development of the mathematical model for the dual wedge scanning system follows from the model for the single wedge scanning arrangement proposed in the previous chapter. Figure 3.18 shows the optical arrangement where a translating reference system $x y z$ has been fixed at the target. The z - axis has been chosen as coincident with the target spin axis with the origin $O$ as a point on the target. The two wedges rotate respectively around the axes BC and DE and are positioned at distances $z_{B}$ and $z_{E}$ from the target. The zero misalignment configuration assumes that the incident laser beam, the target and the wedge spin axes are collinear with the z - axis. The direction of the outgoing beam $\hat{b}_{1}$ and the surface normals for the first rotating wedge are still defined by equations $(2.2,2.29 \& 2.30)$ while those for the second wedge are described as:

$$
\begin{align*}
& \hat{n}_{D}=\hat{n}_{D^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[Z, \gamma_{w 2}\right]\left[X, \psi_{w 2}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T}  \tag{3.35}\\
& \hat{n}_{E}=\hat{n}_{E^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \tag{3.36}
\end{align*}
$$

where the rotation matrix $\left[X, \psi_{w 2}\right.$ ] incorporates the characteristic wedge angle $\psi_{w 2}$ while the matrix $\left[Z, \gamma_{w 2}\right]$ represents the whole body rotation of the wedge around its spin axis. The term $\gamma_{w 2}$ represents the wedge rotation angle and is defined as $\gamma_{w 2}=\Omega_{w 2} t+\phi_{w 2}$ where $\Omega_{w 2}$ and $\phi_{w 2}$ are the rotational speed and the initial angular position of the wedge, respectively. In this way, the beam deflection is introduced by the temporal variation of the angles $\gamma_{w 1}$ and $\gamma_{w 2}$ while the relative initial angular position of the wedges is defined by the term $\Delta \phi=\phi_{w 1}-\phi_{w 2}$ which controls the dimension of the scan pattern.

The orientations of the deflected beams are obtained by applying Snell's law, equation (2.11), at any deflection point. The outgoing beam $\hat{b}_{1}$ is parallel to the z - axis and perpendicular to the first surface of the first wedge. The beam passes through the first
wedge surface without deflection and the beam $\hat{b}_{2}$ coincides with $\hat{b}_{1}$. The direction of the beam refracted at the point $\mathrm{C}, \hat{b}_{3}$, is described by equation (2.34). On the second wedge, the refraction points are indicated as points $\mathrm{D}^{\prime}$ and E ' and the directions of the refracted beams $\hat{b}_{4}$ and $\hat{b}_{5}$ are obtained as:

$$
\begin{align*}
& \hat{b}_{4}=\left(\hat{b}_{3}-\left(\hat{b}_{3} \cdot \hat{n}_{D^{\prime}}\right) \hat{n}_{D^{\prime}}\right) \frac{\varepsilon_{a}}{\varepsilon_{w 2}}-\left(\sqrt{1-\left(\frac{\varepsilon_{a}}{\varepsilon_{w 2}}\right)^{2}\left(1-\left(\hat{b}_{3} \cdot \hat{n}_{D^{\prime}}\right)^{2}\right)}\right) \hat{n}_{D^{\prime}}  \tag{3.37}\\
& \hat{b}_{5}=\left(\hat{b}_{4}-\left(\hat{b}_{4} \cdot \hat{n}_{E^{\prime}}\right) \hat{n}_{E^{\prime}}\right) \frac{\varepsilon_{w 2}}{\varepsilon_{a}}-\left(\sqrt{1-\left(\frac{\varepsilon_{w_{2}}}{\varepsilon_{a}}\right)^{2}\left(1-\left(\hat{b}_{4} \cdot \hat{n}_{E^{\prime}}\right)^{2}\right)}\right) \hat{n}_{E^{\prime}} \tag{3.38}
\end{align*}
$$

In this way, the mathematical procedure allows the final beam orientation to be calculated incorporating all geometrical complexity of the configuration without any approximation. To show the complexity in the relationship between the final beam orientation and the wedge rotations, the final expression for $\hat{b}_{5}$ has been derived as follows:

$$
\begin{aligned}
\hat{b}_{5} & {\left[Q \sin \psi_{w 1} \sin \left(\Omega_{w 1} t+\phi_{w 1}\right)+\sin \psi_{w 2}\left(R-S \frac{\varepsilon_{w 2}}{\varepsilon_{a}}\right) \sin \left(\Omega_{w 2} t+\phi_{w 2}\right)\right] \hat{x} } \\
& -\left[Q \sin \psi_{w 1} \cos \left(\Omega_{w 1} t+\phi_{w 1}\right)+\sin \psi_{w 2}\left(R-S \frac{\varepsilon_{w 2}}{\varepsilon_{a}}\right) \cos \left(\Omega_{w 2} t+\phi_{w 2}\right)\right] \hat{y} \\
& -[W] \hat{z}
\end{aligned}
$$

where the terms $\mathrm{Q}, \mathrm{R}, \mathrm{S}$ and W are defined as follows:

$$
\begin{align*}
& Q=\sqrt{1-\left(\frac{\varepsilon_{w 1}}{\varepsilon_{a}}\right)^{2}\left(1-\left(\hat{b}_{2} \cdot \hat{n}_{C}\right)^{2}\right)}  \tag{3.40}\\
& R=\left(\hat{b}_{3} \cdot \hat{n}_{D^{\prime}}\right) \tag{3.41}
\end{align*}
$$

$$
\begin{align*}
& S=\sqrt{1-\left(\frac{\varepsilon_{a}}{\varepsilon_{w 2}}\right)^{2}\left(1-R^{2}\right)}  \tag{3.42}\\
& W=\sqrt{1-\left(\frac{\varepsilon_{w 2}}{\varepsilon_{a}}\right)^{2}\left(1-\left(\hat{b}_{4} \cdot \hat{n}_{E^{\prime}}\right)^{2}\right)} \tag{3.43}
\end{align*}
$$

Equation (3.39) defines the orientation of the incident beam $\hat{b}_{5}$ for a zero misalignment configuration and confirms that this expression is complex and composed by components that are periodic functions of the wedge rotational speeds. Using the technique shown, this final beam direction is easily obtainable.

From knowledge of the directions of the deflected beams and the initial geometry of the system, the deflection points can be found using vector polygons. The positions of the laser source, A , and of the points B and C situated at the first wedge surfaces can be determined as previously described in section 2.3 where the scanning system with a single rotating wedge was analysed. On the second wedge, the refraction point $D^{\prime}$ from which the beam $\hat{b}_{4}$ originates is determined from the following vector system:

$$
\left\{\begin{array}{l}
\overrightarrow{O C}+|\overrightarrow{C D}| \hat{b}_{3}=\overrightarrow{O D^{\prime}}  \tag{3.44}\\
\overrightarrow{O D}+\overrightarrow{D D^{\prime}}=\overrightarrow{O D^{\prime}} \\
\overrightarrow{D D^{\prime}} \cdot \hat{n}_{D}=0
\end{array}\right.
$$

which reveals the vector $\overrightarrow{O D^{\prime}}$ as:

$$
\begin{equation*}
\overrightarrow{O D^{\prime}}=\overrightarrow{O C}+\left|\frac{(\overrightarrow{O D}-\overrightarrow{O C}) \cdot \hat{n}_{D}}{\hat{b}_{3} \cdot \hat{n}_{D}}\right| \hat{b}_{3} \tag{3.45}
\end{equation*}
$$

In the same manner, the position of the point $E^{\prime}$, from which the beam $\hat{b}_{5}$ originates, is obtained from the following system:

$$
\left\{\begin{array}{l}
\overrightarrow{O D^{\prime}}+\mid \overrightarrow{D^{\prime} E^{\prime}} \hat{b}_{4}=\overrightarrow{O E^{\prime}}  \tag{3.46}\\
\overrightarrow{O E}+\overrightarrow{E E^{\prime}}=\overrightarrow{O E^{\prime}} \\
\overrightarrow{E E^{\prime}} \cdot \hat{n}_{E}=0
\end{array}\right.
$$

from which the vector $\overrightarrow{O E^{\prime}}$ is found as:

$$
\begin{equation*}
\overrightarrow{O E^{\prime}}=\overrightarrow{O D^{\prime}}+\left|\frac{\left(\overrightarrow{O E}-\overrightarrow{O D^{\prime}}\right) \cdot \hat{n}_{E}}{\hat{b}_{4} \cdot \hat{n}_{E}}\right| \hat{b}_{4} \tag{3.47}
\end{equation*}
$$

The geometrical description of the arrangement is completed with the determination of the point K where the beam $\hat{b}_{5}$ meets the plane of the target derived as:

$$
\begin{equation*}
\overrightarrow{O K}=\overrightarrow{O E^{\prime}}+\left|\frac{\overrightarrow{O E^{\prime}} \cdot \hat{z}}{\hat{b}_{5} \cdot \hat{z}}\right| \hat{b}_{5} \tag{3.48}
\end{equation*}
$$

By rotating the two wedges at the same rotational speed, a circular scan pattern is described. The radius of the circular scan pattern is minimum when the initial angular position of the wedges $\Delta \phi=0^{\circ}$ as shown in figure 3.18 while it is maximum when is $\Delta \phi=180^{\circ}$. By synchronising the wedge rotational speeds to the target rotational speed, the system performs the circular tracking of the target.

From knowledge of the positions of the deflection points and the dynamic behaviours of the wedges and target, the complete velocity measured by the system can be written as:

$$
\begin{equation*}
U_{m}=\overrightarrow{V_{B}} \cdot\left(\hat{b}_{2}-\hat{b_{1}}\right)+\overrightarrow{V_{C}} \cdot\left(\hat{b}_{3}-\hat{b}_{2}\right)+\overrightarrow{V_{D^{\prime}}} \cdot\left(\hat{b}_{4}-\hat{b}_{3}\right)+\overrightarrow{V_{E^{\prime}}} \cdot\left(\hat{b}_{5}-\hat{b}_{4}\right)-\overrightarrow{V_{K}} \cdot \hat{b}_{5} \tag{3.49}
\end{equation*}
$$

where $\hat{b}_{1}, \hat{b}_{2}, \hat{b}_{3}, \hat{b}_{4}$ and $\hat{b}_{5}$ are the known beam directions and $\overrightarrow{V_{B}}, \overrightarrow{V_{C}} \overrightarrow{V_{D^{\prime}}}, \overrightarrow{V_{E^{\prime}}}$ and $\overrightarrow{V_{K}}$ are the velocities of the deflection points on the rotating wedges and target. For the
zero misalignment configuration, the velocities calculated at the point B and C are zero because they lie along the wedge rotation axis while for $D^{\prime}, E^{\prime}$ and K it is possible to write:

$$
\begin{align*}
& \overrightarrow{V_{D^{\prime}}}=\Omega_{w 2} \hat{z} \times \overrightarrow{D D^{\prime}}  \tag{3.50}\\
& \overrightarrow{V_{E^{\prime}}}=\Omega_{w 2} \hat{z} \times \overrightarrow{E E^{\prime}}  \tag{3.51}\\
& \overrightarrow{V_{K}}=\Omega_{T} \hat{z} \times \overrightarrow{O K} \tag{3.52}
\end{align*}
$$

where the vectors $\overrightarrow{D D^{\prime}}$ and $\overrightarrow{E E^{\prime}}$ are given as:

$$
\begin{align*}
& \overrightarrow{D D^{\prime}}=\overrightarrow{O D^{\prime}}-\overrightarrow{O D}  \tag{3.53}\\
& \overrightarrow{E E^{\prime}}=\overrightarrow{O E^{\prime}}-\overrightarrow{O E} \tag{3.54}
\end{align*}
$$

Substituting equations (3.50-52) and the directions of the deflected beams into (3.49), the full velocity measured by the system is determined without any approximation.

### 3.2.2 - Predicted velocity for typical tracking applications

For the zero misalignment configuration, the circular scan pattern is obtained by synchronising the wedges, $\Omega_{w 1}=\Omega_{w 2}$. Figure 3.21a shows the scan pattern described for a configuration characterized by the following geometrical parameters: $z_{A}=1.4 \mathrm{~m}$, $z_{B}=1.2 \mathrm{~m}, \quad \varepsilon_{w 1}=\varepsilon_{w 2}=1.5, \quad \psi_{w 1}=\psi_{w 2}=4^{\circ}, \quad z_{E}=1.15 \mathrm{~m}, \quad \Delta \phi=\phi_{w 2}-\phi_{w 1}=75^{\circ} \quad$ and $\Omega_{w 1}=\Omega_{w 2}=\Omega_{T}$ to perform the tracking of the rotating target. These parameters have been chosen in order to trace a circular path with radius of 5 cm . In this case, no velocities are detected by the system. A deeper investigation of this result reveals that the deflection points at the wedges and the target produce equal and opposite Doppler shifts whose combination is zero as shown in figure 3.21b. As for the dual mirror SLDV system the contribution of the Doppler shifts generated by the deflection optics to the final velocity measured by the system is fundamental for the complete analysis of this scanning optical configuration.

The model also predicts measured velocity due to target vibrations. Figure 3.22a shows the predicted spectrum in the presence of whole-body harmonic out-of-plane and inplane vibration both of amplitude $10 \mathrm{~mm} / \mathrm{s}$ but frequencies of $5 \Omega_{T}$ and $10 \Omega_{T}$. The peak at $5 \Omega_{T}$ due to the whole-body out-of-plane vibration measures $1.12 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ ( $=99.8 \mathrm{~mm} / \mathrm{s}$ ) with a sensitivity around $99.8 \%$ of the true vibration. The whole-body inplane vibration produces a pair of sidebands at 9 x and 11 x with same amplitude of $2.17 \mathrm{e}-2$ and $2.63 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}(=1.91 \mathrm{e}-1$ and $2.34 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s})$ with sensitivity around $1.91 \%$ and $2.34 \%$ of the genuine in-plane vibration.

When an angular misalignment around the y - axis of $\beta_{L}=0.2^{\circ}$ is added to the laser head, a pair of sidebands with negligible magnitude are measured at $4 x$ and $6 x$, the whole-body in plane vibration produces a single peak at 10 x with magnitude $3.93 \mathrm{e}-3$ $\mathrm{mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ indicating sensitivity around $0.349 \%$ to the genuine vibration while the sidebands measured at 9 x and 11 x are the same of a system with ideal alignment. These data indicate that misalignment results in addition of a single peak for the whole-body in-plane target vibration plus sidebands around the whole-body out-of-plane target vibration.

Other scan patterns suitable for different applications are obtained using different wedge rotational speeds. A line scan is performed by rotating the wedges in opposite directions with speeds $\Omega_{w 1}=-\Omega_{w 2}$, as indicated in figure 3.19 b . The inclination of the line depends on the initial value of $\Delta \phi$, in this case $\Delta \phi=45^{\circ}$. Rotating the wedges at different speeds, $\Omega_{w 2}=n \Omega_{w 1}$ where $n \neq \pm 1$, results in beam paths that are visually appealing but generally not very useful for real applications. In figure 3.20a, a rosette has been obtained using $\Omega_{w 1}=-2 \Omega_{w 2}$ and $\Delta \phi=0^{\circ}$ while the spiral of figure 3.20 b has been obtained using $\Omega_{w 2}=0.98 \Omega_{w 1}$.

When anti-phase oscillations at frequencies $\Omega_{B 1}$ and $\Omega_{B 2}$ are applied on top of the wedge whole body rotations, as shown in figure 3.23, different scan paths are obtained. This condition allows simultaneous tracking and scanning, which has proved useful for
application on bladed disks using mirrors for beam deflection [3.10]. In this case, the mathematical expressions used for the wedges oscillations are:

$$
\begin{align*}
& \gamma_{w 1}=\phi_{w 1}+\left(\Omega_{w 1} t\right)-\gamma_{a} \sin \left(\Omega_{B 1} t\right)  \tag{3.55}\\
& \gamma_{w 2}=\phi_{w 2}+\left(\Omega_{w 2} t\right)+\gamma_{a} \sin \left(\Omega_{B 2} t\right) \tag{3.56}
\end{align*}
$$

where $\phi_{w 1}$ and $\phi_{w 2}$ are the initial angular positions of the wedges and $\gamma_{a}$ is the amplitude chosen for the oscillations. When the oscillation frequency of the wedges is higher than the rotation frequency, for example $\Omega_{B 2}=\Omega_{B 1}=4 \Omega_{T}$ and $\Delta \phi=90^{\circ}$ the result would be a fast scan up and down a blade, several times within each rotation, as described by the vector $\overrightarrow{O K}$ in figure 3.24. Similar scan paths have previously been created experimentally by using the dual mirror arrangement [3.11]. In this case, the system detects an additional velocity at DC. Velocities at $4 \Omega_{T}$ and $8 \Omega_{T}$ are linked to the oscillations added to the wedges, as indicated in figure 3.24b. An oscillation frequency smaller than the rotation frequency may be more appropriate, resulting in a slow scan up and down the blades over several rotations, as shown by the vector $\overrightarrow{O K}$ in figure 3.25 , obtained by setting $\Omega_{w 2}=\Omega_{w 1}=\Omega_{T}$ and $\Omega_{B 2}=\Omega_{B 1}=0.1 \Omega_{T}$. Again, the oscillations added to the second wedge affect the measured velocity as indicated in figure 3.25 b , where additional terms at $\mathrm{DC}, 0.1 \Omega_{T}$ and $0.2 \Omega_{T}$ are visible.

These data show how the proposed procedure is able to predict paths for complex configurations and to predict measured velocities resulting from target rotation and vibration, assisting the user in ensuring correct interpretation of measured data.

### 3.2.3 - Introduction of the misalignments

Returning to the circular tracking application and moving from the zero misalignments configuration, the analysis of the dual wedge scanning system continues with the introduction of geometrical misalignments. Translational and angular misalignments added to the laser head and wedges modify the geometry of the system, the beam path
and the measured velocity. According to the technique proposed in chapter 2 and to figure 3.26, translational misalignments added to the laser source move the point A to $A^{\prime}$ whose position is:
$\overrightarrow{O A^{\prime}}=\left[\begin{array}{lll}\hat{x} & \hat{y} & \hat{z}\end{array}\right]\left[\begin{array}{lll}x_{A}+\Delta x_{A} & y_{A}+\Delta y_{A} & z_{A}+\Delta z_{A}\end{array}\right]^{T}$
where $\Delta x_{A}, \Delta y_{A}$ and $\Delta z_{A}$ are the misalignments along the three axes. Angular misalignments of the laser head around the x - and the y - axis, with angles $\alpha_{L}$ and $\beta_{L}$, change the direction of the outgoing beam $\hat{b}_{1}$ whose equation is still defined by equation (2.38). Translational and angular misalignments added to the first rotating wedge modify the positions of the points B and C to $B^{*}$ and $C^{*}$ whose positions are defined as:

$$
\begin{align*}
& \overrightarrow{O B^{*}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}] x_{B}+\Delta x_{B} \\
y_{B}+\Delta y_{B} & z_{B}+\Delta z_{B}
\end{array}\right]^{T}  \tag{3.58}\\
& \overrightarrow{O C^{*}}=\overrightarrow{O B^{*}}-\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{ll}
X, \alpha_{w}
\end{array}\right]\left[\begin{array}{lll}
Y, \beta_{w}
\end{array}\right] \overrightarrow{B C}\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \tag{3.59}
\end{align*}
$$

while their surface normals are defined as:

$$
\begin{align*}
& \hat{n}_{B^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{w 1}\right]\left[\begin{array}{lll}
Y, \beta_{w 1}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T}  \tag{3.60}\\
& \hat{n}_{C^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{w 1}\right]\left[Y, \beta_{w 1}\right]\left[Z, \gamma_{w 1}\right]\left[X, \psi_{w 1}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \tag{3.61}
\end{align*}
$$

Applying the same technique to the second wedge, translational and angular misalignments move the points D and E to $D^{*}$ and $E^{*}$ whose positions are found as:

$$
\begin{align*}
& \overrightarrow{O E^{*}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
x_{E}+\Delta x_{E} & y_{E}+\Delta y_{E} & z_{E}+\Delta z_{E}
\end{array}\right]^{T}  \tag{3.62}\\
& \left.\overrightarrow{O D^{*}}=\overrightarrow{O E^{*}}+\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right] X, \alpha_{w 2}\right]\left[\begin{array}{l}
Y, \beta_{w 2}
\end{array}\right] \overrightarrow{D E}\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \tag{3.63}
\end{align*}
$$

where $\Delta x_{E}, \Delta y_{E}$ and $\Delta z_{E}$ are the translations added to the second wedge along the $\mathrm{x}, \mathrm{y}$ and z - axes, while the surface normals are found as:

$$
\begin{align*}
& \hat{n}_{D^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{w 2}\left[I Y, \beta_{w 2}\right]\left[Z, \gamma_{w 2} I\left[X, \psi_{w_{2}}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T}\right.\right.  \tag{3.64}\\
& \hat{n}_{E^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{l}
X, \alpha_{w 2}
\end{array}\right]\left[\begin{array}{ll}
Y, \beta_{w 2}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \tag{3.65}
\end{align*}
$$

From the wedge surface normals, the orientations of the deflected beams are found using Snell's law. The new direction of $\hat{b}_{2}$ is defined by equation (2.44), the direction of $\hat{b}_{3}$ is found using equation (2.45) while $\hat{b}_{4}$ and $\hat{b}_{5}$ are still found using equations (3.37\&38) in which the modified wedge surface normals and beam orientations are used. In the presence of misalignments, the first refraction can take place at the point B' whose position is found from the triangles $O A^{\prime} B^{\prime}$ and $O B^{*} B^{\prime}$, developing the following system of equations:

$$
\left\{\begin{array}{l}
\overrightarrow{O A^{\prime}}+\left|\overrightarrow{A^{\prime} B^{\prime}}\right| \hat{b}_{2}=\overrightarrow{O B^{\prime}}  \tag{3.66}\\
\overrightarrow{O B^{*}}+\overrightarrow{B^{*} B^{\prime}}=\overrightarrow{O B^{\prime}} \\
\overrightarrow{B^{*} B^{\prime}} \cdot \hat{n}_{B^{\prime}}=0
\end{array}\right.
$$

from which the vector $\overrightarrow{O B^{\prime}}$ is obtained as:

$$
\begin{equation*}
\overrightarrow{O B^{\prime}}=\overrightarrow{O A^{\prime}}+\left\lvert\, \frac{\left(\overrightarrow{O B^{*}}-\overrightarrow{O A^{\prime}}\right) \cdot \hat{n}_{B^{\prime}} \mid \hat{b}_{1}}{\hat{b}_{1} \cdot \hat{n}_{B^{\prime}}}\right. \tag{3.67}
\end{equation*}
$$

The other refraction points are obtained using the same operations described in section 3.2.1 to obtain the following expressions:

$$
\begin{equation*}
\overrightarrow{O C^{\prime}}=\overrightarrow{O B^{\prime}}+\left|\frac{\left(\overrightarrow{O C^{*}}-\left.\overrightarrow{O B^{\prime}} \cdot \cdot \hat{n}_{C}\right|^{\prime}\right.}{\hat{b}_{2} \cdot \hat{b}_{C^{\prime}}}\right|_{2} \tag{3.68}
\end{equation*}
$$

$$
\begin{align*}
& \overrightarrow{O D^{\prime}}=\overrightarrow{O C^{\prime}}+\left|\frac{\left(\overrightarrow{O D^{*}}-\overrightarrow{O C^{\prime}}\right) \cdot \hat{n}_{D^{\prime}}}{\hat{b}_{3} \cdot \hat{n}_{D^{\prime}}}\right|_{3}  \tag{3.69}\\
& \overrightarrow{O E^{\prime}}=\overrightarrow{O D^{\prime}}+\left|\frac{\left(\overrightarrow{O E^{*}}-\overrightarrow{O D^{\prime}}\right) \cdot \hat{n}_{E^{\prime}}}{\hat{b}_{4} \cdot \hat{n}_{E^{\prime}}}\right|_{4}  \tag{3.70}\\
& \overrightarrow{O K^{\prime}}=\overrightarrow{O E^{\prime}}+\left|\frac{\overrightarrow{O E^{\prime}} \cdot \hat{z}}{\hat{b}_{5} \cdot \hat{z}}\right| \hat{b}_{5} \tag{3.71}
\end{align*}
$$

In this way, the complete beam path is defined. The velocities of the deflection points at the wedges are determined by modifying equations (3.50\&52) to obtain:

$$
\begin{align*}
& \overrightarrow{V_{B^{\prime}}}=\Omega_{w 1}\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}]\left[X, \alpha_{w 1} \llbracket\left[\begin{array}{l}
Y, \beta_{w 1}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \times \overrightarrow{B^{*} B^{\prime}}, ~\right.
\end{array}\right.  \tag{3.72}\\
& \overrightarrow{V_{C^{\prime}}}=\Omega_{w 1}\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{w 1}\right]\left[Y, \beta_{w 1}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \times \overrightarrow{C^{*} C^{\prime}}  \tag{3.73}\\
& \overrightarrow{V_{D^{\prime}}}=\Omega_{w 2}\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{w 2}\right]\left[\begin{array}{l}
Y, \beta_{w 2}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \times \overrightarrow{D^{*} D^{\prime}}  \tag{3.74}\\
& \left.\overrightarrow{V_{E^{\prime}}}=\Omega_{w 2}\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z} \\
\hline
\end{array}\right] X, \alpha_{w 2}\right]\left[Y, \beta_{w 2}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \times \overrightarrow{E^{*} E^{\prime}} \tag{3.75}
\end{align*}
$$

where the vectors $\overrightarrow{B^{*} B^{\prime}}, \overrightarrow{C^{*} C^{\prime}}, \overrightarrow{D^{*} D^{\prime}}$ and $\overrightarrow{E^{*} E^{\prime}}$ are given as:

$$
\begin{align*}
& \overrightarrow{B^{*} B^{\prime}}=\overrightarrow{O B^{\prime}}-\overrightarrow{O B^{*}}  \tag{3.76}\\
& \overrightarrow{C^{*} C^{\prime}}=\overrightarrow{O C^{\prime}}-\overrightarrow{O C^{*}}  \tag{3.77}\\
& \overrightarrow{D^{*} D^{\prime}}=\overrightarrow{O D^{\prime}}-\overrightarrow{O D^{*}}  \tag{3.78}\\
& \overrightarrow{E^{*} E^{\prime}}=\overrightarrow{O E^{\prime}}-\overrightarrow{O E^{*}} \tag{3.79}
\end{align*}
$$

The velocity of the target is given as:

$$
\begin{equation*}
\overrightarrow{V_{K}}=\overrightarrow{V_{O}}+\Omega_{T} \hat{z} \times\left(\overrightarrow{O K}-\overrightarrow{O O^{*}}\right)+\overrightarrow{V_{f}} \tag{3.80}
\end{equation*}
$$

which combines rotation with vibrations including those due to target flexibility. Replacing equations (3.72-75) and (3.80) in (3.49), the velocity measured by the system in the presence of all the misalignments added to the system is derived.

### 3.2.4 - Analysis of some misaligned configurations

Having introduced the various misalignments, the next step is the analysis of the misalignment effects. Table 3.3 reports a full set of misalignments expected to affect the dual wedge scanning system. The values chosen are based on those met in practical applications after the user has taken steps to reduce these errors. As for the dual mirror scanning system, the alignment of the laser head can be realized using a tripod and linear guides that are able to reduce certain misalignments but not the rotation around the $y$ - axis which is critical. For the wedges, translational misalignments can be reduced using a linear guide on which the optics are mounted but, again, the angular alignment around the $y$ - axis is critical.

| Device | Misalignment | Range | $\boldsymbol{\Delta}$ step | N. steps |
| :---: | :---: | :---: | :---: | :---: |
| Laser head | $\Delta \mathrm{x}_{\mathrm{A}}$ | $\pm 5 \mathrm{~mm}$ | 5 mm | 3 |
|  | $\Delta \mathrm{y}_{\mathrm{A}}$ | $\pm 5 \mathrm{~mm}$ | 5 mm | 3 |
|  | $\alpha_{\mathrm{L}}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |
| $1^{\text {st }}$ wedge | $\beta_{\mathrm{L}}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |
|  | $\Delta \mathrm{x}_{\mathrm{B}}$ | $\pm 3 \mathrm{~mm}$ | 3 mm | 3 |
|  | $\Delta \mathrm{y}_{\mathrm{B}}$ | $\pm 3 \mathrm{~mm}$ | 3 mm | 3 |
|  | $\alpha_{\mathrm{w} 1}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |
| $2^{\text {nd }}$ wedge | $\beta_{\mathrm{w} 1}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |
|  | $\Delta \mathrm{x}_{\mathrm{E}}$ | $- \pm 3 \mathrm{~mm}$ | 3 mm | 3 |
|  | $\Delta \mathrm{y}_{\mathrm{E}}$ | $\pm 3 \mathrm{~mm}$ | 3 mm | 3 |
|  | $\alpha_{\mathrm{w} 2}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |
|  | $\beta_{\mathrm{w} 2}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |

Table 3.3- Full set of misalignments used to investigate the dual wedge SLDV system

Simulations to quantify the misalignment effects are made considering a rotating target. For the zero misalignment configuration no velocities are measured in the range of amplitudes chosen to plot.

When the laser head has an angular misalignment around the x - axis, with $\alpha_{L}=0.2^{\circ}$, the circular scan pattern shifts along the $y$ - axis as shown in figure 3.27a. Figure 3.27b shows the three velocity spectra obtained from this configuration. The first one indicates the velocity measured from the combination of the Doppler shifts generated at the wedges (considering the Doppler shifts generated by the outgoing and return beams), the second is the total velocity measured at the rotating target while the last one is the velocity measured by the laser vibrometer obtained as the combination of the previous two spectra, according to equation (3.46). The figure shows that no velocities are detected by the vibrometer. The Doppler shifts generated at the target and wedges have the same amplitudes but opposite phases so that their combination is zero. The figure shows clearly what happens at the deflecting devices, wedges and target, and how the mathematical model can predict each single contribution to the final measured velocity. A similar result is obtained when the laser head is affected by angular misalignments around the y - axis, with $\beta_{L}=0.2^{\circ}$.

When the laser head has a translational misalignment along the x - axis, with $\Delta x_{A}=2 \mathrm{~mm}$, the scan pattern moves along the same direction and the vibrometer does not detect any additional velocity in the range of amplitudes investigated, as indicated in figure $3.28 \mathrm{a} \& \mathrm{~b}$. A similar result is obtained when translational misalignments along the y - axis are added to the laser head. These data indicate that in the presence of single misalignments, angular and translational, added to the laser head the vibrometer does no detect any uncertainty in the range of amplitudes chosen to plot. When angular and translational misalignments of the laser head are combined, for example with $\alpha_{L}=0.2^{\circ}$ and $\Delta x_{A}=2 \mathrm{~mm}$, the vibrometer measures a small additional DC term with amplitude of $6.98 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ as indicated in figure 3.29 b .

When the first wedge has a translational misalignment along the x - axis of $\Delta x_{B}=2 \mathrm{~mm}$, the scan pattern slightly moves along the $y$ - axis, see figure 3.30a. The laser vibrometer measures an additional 1 x velocity term with magnitude of $4.95 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, as indicated in figure 3.30b. When the first wedge has an angular misalignment around the
x - axis of $\alpha_{w 1}=0.2^{\circ}$, the additional 1 x velocity term measures $4.31 \mathrm{e}-4 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, as shown in figure 3.31 b , and the scan pattern moves along the y - direction as indicated in figure 3.31a. A similar result is obtained misaligning the first wedge around the $y$ - axis with $\beta_{w 1}=0.2^{\circ}$. When the misalignments $\alpha_{w 1}=0.2^{\circ}$ and $\Delta x_{B}=2 \mathrm{~mm}$ are combined, the laser measures the additional 1 x velocity term with amplitude of $4.95 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, as shown in figure 3.32b while the circular pattern moves along the $y$ - axis as indicated in figure 3.32a.

When the second wedge has a translational misalignment along the x - axis of $\Delta x_{E}=2 \mathrm{~mm}$, the scan pattern moves slightly along the x - and y - axis as indicated in figure 3.33a. The vibrometer detects an additional 1 x velocity term with magnitude of $4.94 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, as shown in figure 3.33 b . When the same wedge has an angular misalignment around the x - axis of $\alpha_{w 2}=0.2^{\circ}$, the scan pattern moves in the y -direction and the laser vibrometer measures a 1 x velocity term of $1.82 \mathrm{e}-4 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, as indicated in figures $3.34 \mathrm{a} \& \mathrm{~b}$. By combining the translational and the angular misalignments of the second wedge, the laser vibrometer detects the additional 1 x velocity term with magnitude of $4.96 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, as reported in figure 3.35 b .

The data show always the presence of the 1 x velocity term for single misalignments of the wedges and of a DC term for a combined misalignment of the laser head.

### 3.2.5 - Quantification of the uncertainties in the presence of single and

 combined misalignmentsContinuing in this manner, it is possible to investigate the effects of each misalignment. The analysis, however, concentrates on the determination of the optical devices with the most critical alignment and also on the prediction of the typical level of uncertainty affecting typical vibration measurements. For this reason, as done for the dual mirror scanning system, a first investigation is performed, analysing each individual misalignment, varying the values from 0 to 1 of the range for calculation.

Figure 3.36a reports the values predicted for the 1 x component in the presence of translational misalignments along the x - axis added to the first wedge (FW). Similar values of misalignments but along the $y$ - axis and added to the same device produce similar uncertainties. Translational misalignments added to the second wedge (SW) result in similar results to those shown in figure 3.36a while no 1x uncertainties are predicted for a misaligned laser head. Figure 3.36 b shows the predicted for the 1 x velocity term in the presence of single angular misalignments around the x - axis added to the wedges. In this case, the greatest uncertainties are generated by the first rotating wedge. Angular misalignments around the $y$ - axis added to the wedges result in uncertainties similar to those indicated in figure 3.36b.

These data confirm that for single misalignments, the wedges are the devices with the most critical alignment, translational more than angular misalignments, while the 1 x velocity term is the only additional velocity term detected by the system. Similar misalignments added to the wedges produce similar uncertainties. However, the simulations presented in figure 3.29b indicate the presence of an additional DC term for combined misalignments added to the laser head. This result suggests that the level of uncertainty depends on how the misalignments are combined and in practical applications the presence of combined misalignments is inevitable. For this reason, as done for the dual mirror SLDV system, the values reported in table 3.4 have been used to run $543114\left(=3^{12}\right)$ different misaligned situations. For each situation the predicted additional velocities have been used to calculate RMS values which characterize the system in terms of level of uncertainty expected in practical applications. Table 3.4 reports the predicted RMS values for the DC, 1 x and 2 x velocity terms.

| Additional component | Predicted RMS value |
| :---: | :---: |
| DC | $6.13 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |
| 1 x | $1.11 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |
| 2 x | $1.28 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |

Table 3.4-RMS measured velocities calculated for the additional velocity terms for the various combinations of misalignments for a scan pattern with radius 5 cm

The values in table 3.4 have been obtained by considering only the different misaligned configurations, in this case 194, in which the scan centre is less than $0.5 \%$ of the radius from the position of the scan centre for the zero misalignment configuration.

Figures $3.37 \mathrm{a} \& \mathrm{~b}$ report a scenario characterized by combined misalignments whose result is a scan pattern centred in the target plane. The misalignments are: $\alpha_{L}=0.2^{\circ}$, $\beta_{L}=-0.2^{\circ}, \alpha_{w 1}=-0.5^{\circ}, \beta_{w 1}=-0.5^{\circ}, \alpha_{w 2}=-0.5^{\circ}, \beta_{w 2}=0.5^{\circ}, \Delta x_{A}=-4 \mathrm{~mm}, \Delta y_{A}=-5 \mathrm{~mm}$, $\Delta x_{B}=\Delta y_{B}=-5 \mathrm{~mm}$ and $\Delta x_{E}=\Delta y_{E}=-5 \mathrm{~mm}$. The scan pattern shows a standard deviation of the radial position of $0.02 \%$ and a centre offset of $1.72 \%$. The vibrometer measures additional DC, 1 x and 2 x velocity terms with amplitude of $3.41 \mathrm{e}-3,2.12 \mathrm{e}-1$ and $9.98 \mathrm{e}-5 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, respectively as shown in figure 3.37 b .

Figure $3.38 \mathrm{a} \& \mathrm{~b}$ show another scenario obtained in the presence of combined misalignments. In this case the misalignments used are: $\alpha_{L}=0.2^{\circ}, \beta_{L}=-0.3^{\circ}, \alpha_{w 1}=-$ $0.1^{\circ}, \beta_{w 1}=0.5^{\circ}, \alpha_{w 2}=0.5^{\circ}, \beta_{w 2}=0.5^{\circ}, \Delta x_{A}=-5 \mathrm{~mm}, \Delta y_{A}=-5 \mathrm{~mm}, \Delta x_{B}=\Delta y_{B}=5 \mathrm{~mm}$, $\Delta x_{E}=5 \mathrm{~mm}$ and $\Delta y_{E}=-5 \mathrm{~mm}$. In this case, the scan pattern indicates a standard deviation of the radial position of $0.06 \%$ and a centre offset of $4.63 \%$. The predicted additional DC, 1 x and 2 x velocity terms measure $8.51 \mathrm{e}-3,4.66 \mathrm{e}-2$ and $4.72 \mathrm{e}-5$ $\mathrm{mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ as indicated in figure.

Again the mathematical procedure has enabled investigation of what the dual-wedge scanning system measures when tracking on rotating targets, obtaining results without any kind of approximation. These examples show that a scan pattern centred in the target plane is not synonymous with no misalignment of the various devices of the scanning system but it can be obtained as a combination of different misalignments which introduce uncertainties in the vibration measurements.

### 3.2.6 - Experimental results

Experiments have been performed to confirm the mathematical model developed for the dual-wedge SLDV system. Figure 3.39 shows schematically the test rig developed for
these tests. The system is composed of an assembly which incorporates the two wedges, while a stepper motor, a system of reduction gears and a timing belt drive the wedges. An encoder is positioned behind the target and drives a signal generator which controls the rotation of the stepper motor. The choice of a stepper motor was made to synchronise the wedges and the target rotation speeds.

The wedges are housed in a particular assembly composed of two holders. The first prism holder can be rotated with respect to the other to set the initial phase difference $\Delta \phi$ and define the dimensions of the scan radius. The initial positions of the wedges have to be defined before experiments take place. An end plate is used to lock off any rotation between the wedges during the experiments.

A wedge holder cushion made of fabric is used to provide friction to stop the first wedge holder assembly from turning whilst the entire assembly rotates. Moreover, because of the shape of the wedge, counterbalances have been added in the wedge assembly.

Good measurements require a correct alignment between the laser source, the wedges and the target. During the test, the first step was a correct alignment between the laser source and the wedges to reduce to the smallest value possible the Doppler shift produced by the wedges. A panel covered by retro-reflective tape was positioned in front of the wedges while the laser head, which is mounted on a translating and rotating stage, was moved along the x - and the y - axis to reduce the additional velocity detected by the system.

Initial tests were made to quantify the Doppler shifts produced by the rotating wedges consider the following geometrical parameters: $z_{A}=0.88 \mathrm{~m}, z_{B}=0.66 \mathrm{~m}, z_{D}=0.6 \mathrm{~m}$, $\left|z_{B}-z_{D}\right|=6 \mathrm{~cm} . \psi_{w 1}=10^{\circ}$ and $\psi_{w 2}=7.4^{\circ}, \Delta \phi=90^{\circ}$. Rotating the wedge in the same direction as the target and at the same speed enables tracking with diameter of 13 cm . Figure 3.40a shows the velocity spectrum measured for the configuration with the smallest additional 1x velocity term, considered as the best alignment possible between
the wedges and the laser head. The figure shows a peak at the wedge rotation frequency, 9 Hz , with amplitude of $3.51 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s}(6.57 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s})$. When known translational misalignments along the x - and the y - axis are added to the laser head, the amplitude of this peak increases. Figure 3.40b shows the velocity spectrum measured when the laser head was moved along the x - axis with $\Delta x_{A}=1 \mathrm{~mm}$ where the peak at 9 Hz measures $1.196 \mathrm{~mm} / \mathrm{s}(2.11 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s})$.

Figure 3.41a and 3.41b show the additional 1 x velocity term detected when known translational misalignments along the x - and the y - axes are added to the laser head relative to the wedge assembly. The experimental data are compared to the theoretical data obtained with similar known misalignments. The figures show similar trends for the theoretical and the experimental data but the values are slightly different because of the presence of small, unknown misalignments in the experimental data. This is evident for the experimental configuration with zero misalignment where the measured velocity at 1 x is not zero. These tests validate the prediction of the measured velocity by the simulation.

Further tests have been performed to validate the predicted uncertainties due to misalignments for tracking on a rotating disk. The screen panel with retro-reflective tape was removed and the target was aligned to the laser head-wedges subsystem using the stage on which the target is mounted.

In these applications, the laser head-wedges subsystem was maintained in its ideal alignment while the target was aligned using the stage on which the target is fixed which controls translations along the x - and the y -axis and rotations around the y -axis. In particular, $y$ - axis angular alignment of the target is around an axis situated 15 cm behind the target surface. This means that a single rotation of the target around the $y$ direction introduces also a translation of the target along the x - axis with respect to the laser head-wedges subsystem defined as:

150 (mm) * $\sin ($ angular misalignment considered)

Figure 3.42a shows the velocity spectra measured for the set-up with best alignment. The velocity at the target rotational frequency, 5 Hz , measures $7.41 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s}$ (with a rms value $\approx 1.63 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ ). Moving the subsystem laser head-wedges along the x - axis for a displacement of 1 mm , the velocity at the target rotational speed increases to 2.60 $\mathrm{mm} / \mathrm{s}$ (with a rms value $\approx 5.81 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ ), as indicated in figure 3.42 b .

As for investigation of the Doppler shifts generated by the wedges, translations and rotations are added to the laser head-wedges subsystem in order to validate the theoretical prediction in the presence of similar misalignments. Figure 3.43a, b and c shows the comparison between theoretical and experimental data obtained by adding known translations along the x - and y - axes and rotations around the y - axis. Figure $3.43 \mathrm{a}, \mathrm{b}$ and c indicate a good agreement between theoretical and experimental data. In each case, the trends shown by experimental and theoretical data are similar but the values are different because experimental data are affected by unknown misalignments. These plots suggest a linear relationship between the magnitude of the additional velocity and the misalignments added to the target. The different values between theoretical and experimental can be associated to initial unknown misalignments and others introduced during the movement of the target. Comparison between the theoretical and the experimental values for angular misalignments was possible only by adding a fix inclination of the investigated target around the y -axis, $1.3^{\circ}$, in the simulator.

Experiments have also been made to validate the model in the presence of external target vibrations. Figure 3.44a shows an external vibration at 40 Hz with amplitude of $16.31 \mathrm{~mm} / \mathrm{s}$ applied to a target rotating at 8 Hz when the wedges are stationary. The figure shows some sidebands around 80 Hz indicating that the vibration applied to the target produces some in-plane target vibrations. Synchronising the target and the wedges speeds, the vibration measurements appear as shown in figure 3.44 b . In this case an additional 1 x velocity term at 8 Hz is measured and bigger sidebands are visible around 40 Hz . Performing the tracking of the system, it is possible to observe a target harmonic of $2.56 \mathrm{~mm} / \mathrm{s}$ due to the presence of misalignments, an increase of the peak at 41 Hz and important sidebands 32 Hz and 48 Hz the sidebands measure $4.12 \mathrm{~mm} / \mathrm{s}$ and
$5.77 \mathrm{~mm} / \mathrm{s}$. From the model sidebands are due to in-plane target vibrations and from simulations these values should be around $2 \%$ of the in-plane vibration. In this case, the measured sidebands are respectively $25 \%$ and $35 \%$ of the target vibration. These values higher than those predicted theoretically can be explained considering that the target excitation has generated high levels of in-plane vibrations so that what the system measures is a combination of in-plane and out-of-plane vibration at each frequency. The presence of sidebands with different amplitude at 40 Hz and 125 Hz is a feature of inplane target vibrations.

Although other tests have not been performed, these data shows that the mathematical model is able to predict what the system detects in the presence of target vibrations.

## 3.3 - The Dove prism tracking LDV system

The Dove prism is an optical device with the interesting property that it can rotate an image twice as fast as it is itself rotated about its longitudinal axis. Figure 3.45 shows a schematic representation of the Dove prism scanning system. The Dove prism rotates at half the speed of the target under investigation and deflects the beam towards it so as to track a fixed point. To do this, the optical unit has to be synchronised via a controller to the test object while an encoder, positioned behind the target, detects its rotational speed. When the beam enters through one of the sloped faces of the prism, it undergoes a total internal reflection from the inside of the longest face and emerges from the opposite face [3.12]. Because of this characteristic, the Dove prism has been frequently used as an image de-rotator, within optical viewfinders such as periscopes [3.13] or in interferometers to rotate one beam with respect to another [3.14, 3.15]. Recently, this device has been used to develop a novel scanning LDV system [3.16] which is the object of this investigation.

### 3.3.1-Definition of an appropriate alignment

According to figure 3.45 , the zero misalignment configuration for the Dove prism scanning system is obtained when the outgoing beam $\hat{b}_{1}$, the prism and the target spin
axes are collinear to the z - axis of a reference system $x y z$ that is fixed on the target. Rotating the prism around its spin axis with $\Omega_{P}=0.5 \Omega_{T}$, the incident beam $\hat{b}_{4}$ illuminates the point O , which is the origin of the reference system. The rotation of the prism does not modify $\hat{b}_{4}$, which remains constant such that no scan pattern results.

The creation of circular patterns with dimensions suitable for scanning LDV applications requires alignment of the system that is based on translations and/or rotations applied to the laser head or the prism. The first possible approach is to align the laser head while maintaining the prism in its zero position. If, for example, the laser head is rotated around the x - axis from its zero position, with $\alpha_{L}=2.0875^{\circ}$ and assuming $\Omega_{P}=0.5 \Omega_{T}$, the system traces the circular beam path shown in figure 3.46a. This scan path is composed of two distinct and very close circles in the xy - plane. The distance between the two circles is small enough with respect to the dimensions of the scan pattern that it is possible to consider the scan pattern as a single circle in which case the laser beam follows the same path on the target for each rotation. This arrangement delivers the circular beam path required. Similar beam paths are obtained rotating the laser head around the y - axis or translating the laser head along the x and/or y- axis. Increments of the laser head displacements, angular and translational, increase the dimensions of the circles but do not modify their position which is still centred in the target plane.

The other possible approach is to align the Dove prism while maintaining the laser head in its zero position. When the prism is rotated around the x - and/or the y - axis from its zero position, with $\alpha_{P}=1^{\circ}$, the system traces a circular scan pattern shifted in the xy plane as shown in figure 3.46b. The displacement of the scan pattern in the target plane makes this solution inconvenient for tracking applications. Similar results are obtained by translating the prism along the x - and/or y - axis. These results indicate that the Dove prism scanning arrangement can be used in circular tracking LDV measurements by appropriate alignment of the incoming beam.

### 3.3.2-Mathematical model

According to figure 3.45, the total velocity measured by the Dove prism tracking LDV system can be formulated as:

$$
\begin{equation*}
U_{m}=\overrightarrow{V_{B^{\prime}}} \cdot\left(\hat{b}_{2}-\hat{b}_{1}\right)+\overrightarrow{V_{C^{\prime}}} \cdot\left(\hat{b}_{3}-\hat{b}_{2}\right)+\overrightarrow{V_{D^{\prime}}} \cdot\left(\hat{b}_{4}-\hat{b_{3}}\right)-\overrightarrow{V_{K}} \cdot \hat{b_{4}} \tag{3.81}
\end{equation*}
$$

where the terms $\overrightarrow{V_{B^{\prime}}}, \overrightarrow{V_{C^{\prime}}}, \overrightarrow{V_{D^{\prime}}}$ and $\overrightarrow{V_{K}}$ are the velocities of the deflection points on the prism surfaces and the target while $\hat{b}_{1}, \hat{b}_{2}, \hat{b}_{3}$ and $\hat{b}_{4}$ are the beam directions. Equation (3.81) provides the complete description of the velocity measured by the system for any kind of target or prism motion including target vibrations. The terms included in equation (3.81) are found using the same technique applied to the previous scanning systems. For a configuration characterized by angular alignments around the x - and the $y$ - axis of the laser head, as shown in figure 3.47, the direction of the outgoing laser beam and of the prism surface normals can be written as:

$$
\begin{align*}
& \hat{b}_{1}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{L} \llbracket\left[Y, \beta_{L} \llbracket\left[\begin{array}{lll}
0 & 0 & -1
\end{array}\right]^{T}\right.\right.  \tag{3.82}\\
& \hat{n}_{B}=\hat{n}_{B^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[Z, \gamma_{P}\right]\left[X, \psi_{P 1} \llbracket\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T}\right.  \tag{3.83}\\
& \hat{n}_{C}=\hat{n}_{C^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[Z, \gamma_{P} \llbracket\left[\begin{array}{lll}
0 & 1 & 0
\end{array}\right]^{T}\right.  \tag{3.84}\\
& \hat{n}_{D}=\hat{n}_{D^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[Z, \gamma_{P}\right]\left[\begin{array}{l}
X, \psi_{P 2} \\
\hline 0
\end{array} 0 \quad 1\right]^{T} \tag{3.85}
\end{align*}
$$

where the rotation matrices $\left[X, \alpha_{L}\right]$ and $\left[Y, \beta_{L}\right]$ accommodate the initial angular alignment of the laser source, the rotation matrices $\left[X, \psi_{P_{1}}\right]$ and $\left[X, \psi_{P_{2}}\right]$ describe the inclination angles of the prism sloped surfaces, with $\psi_{P 1}=-45^{\circ}$ and $\psi_{P 2}=45^{\circ}$ while the matrix $\left[Z, \gamma_{P}\right]$ indicates the whole body motion of the prism around its spin axis in which the angle $\gamma_{P}$ is defined as $\gamma_{P}=\Omega_{P} t+\varphi_{P}$ where $\Omega_{P}$ and $\varphi_{P}$ are the prism rotational speed and its initial angular orientation, respectively. As $\psi_{P 1}, \psi_{P 2}$ and $\gamma_{P}$ are finite angles, the order of operations indicated in equations (3.83\&85) has to be respected.

Because of the simultaneous presence of a beam reflection and of two beam refractions at the prism surfaces, the deflected beam orientations are determined from Snell's Law, equations (2.10-2.11):

$$
\begin{align*}
& \hat{b}_{2}=\left(\hat{b}_{1}-\left(\hat{b}_{1} \cdot \hat{n}_{B}\right) \hat{n}_{B}\right) \frac{\varepsilon_{a}}{\varepsilon_{P}}-\left(\sqrt{1-\left(\frac{\varepsilon_{a}}{\varepsilon_{P}}\right)^{2}\left(1-\left(\hat{b}_{1} \cdot \hat{n}_{B}\right)^{2}\right)}\right) \hat{n}_{B}  \tag{3.86}\\
& \hat{b}_{3}=\hat{b}_{2}-2\left(\hat{b}_{2} \cdot \hat{n}_{C}\right) \hat{n}_{C}  \tag{3.87}\\
& \hat{b}_{4}=\left(\hat{b}_{3}-\left(\hat{b}_{3} \cdot \hat{n}_{D}\right) \hat{n}_{D}\right) \frac{\varepsilon_{P}}{\varepsilon_{a}}-\left(\sqrt{1-\left(\frac{\varepsilon_{P}}{\varepsilon_{a}}\right)^{2}\left(1-\left(\hat{b}_{3} \cdot \hat{n}_{D}\right)^{2}\right)}\right) \hat{n}_{D} \tag{3.88}
\end{align*}
$$

where $\varepsilon_{a}$ and $\varepsilon_{P}$ are the refractive indices of air and of the prism, respectively. From knowledge of the directions of the deflected beams, the positions of the deflection points can be found using vector polygons. The position of the laser source is defined by the point A defined by:

$$
\overrightarrow{O A}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
x_{A} & y_{A} & z_{A} \tag{3.89}
\end{array}\right]^{T}
$$

while the position of the Dove prism is determined by the points B and D situated on the prism spin axis respectively at the first and the second slope surface:

$$
\begin{align*}
& \overrightarrow{O B}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
x_{B} & y_{B} & z_{B}
\end{array}\right]^{T}  \tag{3.90}\\
& \overrightarrow{O D}=\overrightarrow{O B}-|\overrightarrow{B D}| \hat{z} \tag{3.91}
\end{align*}
$$

For this configuration the first refraction takes place at the point $B^{\prime}$ situated on the first sloped surface of the prism. The point $B^{\prime}$ can be found using the vector triangles $O B B^{\prime}$ and $O A B^{\prime}$ :

$$
\left\{\begin{array}{l}
\overrightarrow{O A}+\overrightarrow{A B^{\prime}} \mid \hat{b}_{1}=\overrightarrow{O B^{\prime}}  \tag{3.92}\\
\overrightarrow{O B}+\overrightarrow{B B^{\prime}}=\overrightarrow{O B^{\prime}} \\
\overrightarrow{B B^{\prime}} \cdot \hat{n}_{B}=0
\end{array}\right.
$$

from which the vector $\overrightarrow{O B^{\prime}}$ is derived as:

$$
\begin{equation*}
\overrightarrow{O B^{\prime}}=\overrightarrow{O A}+\left|\frac{(\overrightarrow{O B}-\overrightarrow{O A}) \cdot \hat{n}_{B}}{\hat{b}_{1} \cdot \hat{n}_{B}}\right| \hat{b}_{1} \tag{3.93}
\end{equation*}
$$

Then the beam is reflected at the point $C^{\prime}$ situated at the plane surface of the prism. To define its position, it is necessary to establish the dimensions of the prism from the points H as indicated in figure 3.45 . The point H is found as:

$$
\overrightarrow{O H}=\overrightarrow{O B}+\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}]\left[Z, \gamma_{P}\right]\left[\begin{array}{lll}
0 & -|\overrightarrow{B H}| \cos \psi_{P 1} & -|\overrightarrow{B H}| \sin \psi_{P 1}
\end{array}\right]^{T} . \tag{3.94}
\end{array}\right.
$$

where the term $|\overrightarrow{B H}|$ is the distance between the point B situated at first inclined surface of the Dove prism and the point H . The point $C^{\prime}$ is then found for the vector triangles $O B^{\prime} C^{\prime}$ and $O H C^{\prime}$ solving the following system:

$$
\left\{\begin{array}{l}
\overrightarrow{O B^{\prime}}+\left|\overrightarrow{B^{\prime} C^{\prime}}\right| \hat{b}_{2}=\overrightarrow{O C^{\prime}}  \tag{3.95}\\
\overrightarrow{O H}+\overrightarrow{H C^{\prime}}=\overrightarrow{O C^{\prime}} \\
\overrightarrow{H C^{\prime}} \cdot \hat{n}_{C}=0
\end{array}\right.
$$

The vector $\overrightarrow{O C^{\prime}}$ is derived as:

$$
\begin{equation*}
\overrightarrow{O C^{\prime}}=\overrightarrow{O B^{\prime}}+\left|\frac{\left(\overrightarrow{O H}-\overrightarrow{O B^{\prime}}\right) \cdot \hat{n}_{C}}{\hat{b}_{2} \cdot \hat{n}_{C}}\right|_{2} \tag{3.96}
\end{equation*}
$$

The second refraction point $D^{\prime}$ is determined by resolving the following system:

$$
\left\{\begin{array}{l}
\overrightarrow{O C^{\prime}}+\left|\overrightarrow{C^{\prime} D}\right| \hat{b}_{3}=\overrightarrow{O D^{\prime}}  \tag{3.97}\\
\overrightarrow{O D}+\overrightarrow{D D^{\prime}}=\overrightarrow{O D^{\prime}} \\
\overrightarrow{D D^{\prime}} \cdot \hat{n}_{D}=0
\end{array}\right.
$$

and the vector $\overrightarrow{O D^{\prime}}$ is found as:

$$
\begin{equation*}
\overrightarrow{O D^{\prime}}=\overrightarrow{O C^{\prime}}+\left|\frac{\left(\overrightarrow{O D}-\overrightarrow{O C^{\prime}}\right) \cdot \hat{n}_{D}}{\hat{b}_{3} \cdot \hat{n}_{D}}\right| \hat{b}_{3} \tag{3.98}
\end{equation*}
$$

Finally, the measuring point K situated at the target is found using the following expression:

$$
\begin{equation*}
\overrightarrow{O K}=\overrightarrow{O D^{\prime}}+\left|\frac{\overrightarrow{O D^{\prime}} \cdot \hat{z}}{\hat{b}_{4} \cdot \hat{z}}\right| \hat{b}_{4} \tag{3.99}
\end{equation*}
$$

which concludes the definition of the complete beam path traced by the system. The velocity terms included in equation (3.81) can be written as:
$\overrightarrow{V_{B^{\prime}}}=\Omega_{P} \hat{z} \times \overrightarrow{B B^{\prime}}$
$\overrightarrow{V_{C^{\prime}}}=\Omega_{P} \hat{z} \times \overrightarrow{B C^{\prime}}$
$\overrightarrow{V_{D^{\prime}}}=\Omega_{P} \hat{z} \times \overrightarrow{D D^{\prime}}$
where

$$
\begin{align*}
& \overrightarrow{B^{*} B^{\prime}}=\overrightarrow{O B^{\prime}}-\overrightarrow{O B^{*}}  \tag{3.103}\\
& \overrightarrow{C^{*} C^{\prime}}=\overrightarrow{O C^{\prime}}-\overrightarrow{O C^{*}}  \tag{3.104}\\
& \overrightarrow{D^{*} D^{\prime}}=\overrightarrow{O D^{\prime}}-\overrightarrow{O D^{*}} \tag{3.105}
\end{align*}
$$

$$
\begin{equation*}
\overrightarrow{E^{*} E^{\prime}}=\overrightarrow{O E^{\prime}}-\overrightarrow{O E^{*}} \tag{3.106}
\end{equation*}
$$

The velocity measured at the target is given as:

$$
\begin{equation*}
\overrightarrow{V_{K}}=\overrightarrow{V_{O}}+\Omega_{T} \hat{z} \times\left(\overrightarrow{O K}-\overrightarrow{O O^{*}}\right)+\overrightarrow{V_{f}} \tag{3.107}
\end{equation*}
$$

Substituting equations (3.100-106) and the directions of the deflected beams into equation (3.81), the complete velocity sensed by the system is predicted without any approximation.

### 3.3.3-Predicted velocity for circular tracking applications

Consider a system characterized by the following geometrical parameters: $z_{A}=1.4 \mathrm{~m}$, $z_{B}=1.2 \mathrm{~m}$, prism angles $\psi_{P 1}=-45^{\circ}, \psi_{P 2}=45^{\circ}$, refractive indices for air and prism $\varepsilon_{a}=1$ and $\varepsilon_{P}=1.5$, respectively, prism height and length of 0.02 m and 0.066 m respectively.

The zero configuration is not suitable for scanning applications thus it will be not investigated. Angular alignment of the laser head around the x-axis, with $\alpha_{L}=2.0875^{\circ}$, traces a circular scan pattern with radius of 5 cm as shown in figure 3.48 a while additional velocities at 0.5 x and 1 x the target rotational speed, with magnitude of $1.72 \mathrm{e}-$ 4 and $3.85 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, are detected by the vibrometer, as indicated in figure 3.48 b . Because the target is only rotating but not vibrating, these velocities have to be considered as uncertainties linked to the chosen alignment. Figure 3.48b shows that the total velocity detected by the laser vibrometer is a combination of the Doppler shifts generated at the target and the prism which have different magnitudes and phases.

When the laser head is only translated along the x - and the y - axis with $\Delta x_{A}=2 \mathrm{~mm}$, the system describes the scan path shown in figure 3.49 with radius of 2 mm . In this case, no velocity terms are detected by the vibrometer. Technically, both angular and translational alignments describe circular scan patterns suitable for tracking
applications. The translational alignments of the laser head produce Doppler shifts at each deflection point on the prism but then they cancel. Angular alignments of the laser head introduce harmonics with magnitudes proportional to the angular alignment chosen but in this way it is easier to obtain scan patterns with dimensions suitable for practical applications.

### 3.3.4-Circular tracking in the presence of target vibrations

In equation (3.107), the complete velocity measured at the measuring point K includes vibrations of the rotating target and target flexibility. When the system is aligned through single or combined rotations of the laser head around the x - and/or y - axis, the system detects both whole-body in-plane and out-of-plane target vibrations because $\hat{b}_{4}$ is not directed along the z - axis. Figure 3.50a shows the predicted spectrum in the presence of whole-body harmonic out-of-plane and in-plane vibration both of amplitude $10 \mathrm{~mm} / \mathrm{s}$ but frequencies of $5 \Omega_{T}$ and $10 \Omega_{T}$.

The whole out-of-plane vibration results in a single component with amplitude of $1.125 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}(=9.99 \mathrm{~mm} / \mathrm{s})$ with sensitivity around $99.9 \%$ of the genuine vibration. The whole-body in-plane vibration produces a pair of sidebands at 9 x and 11 x with magnitude of $1.84 \mathrm{e}-2$ and $2.25 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ with sensitivity around $1.63 \%$ and $2.01 \%$ of the genuine vibration. The low target harmonics are due to the alignment chosen for the system.

When an angular misalignment around the y - axis of $\beta_{P}=0.2^{\circ}$ is added to the Dove prism, the peak at 5 x does not change while a pair of sidebands of negligible magnitude are measured at 4 x and 6 x . The whole-body in-plane vibration, instead, produces a single peak at 10 x with magnitude $3.92 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ with sensitivity around $0.33 \%$ to the genuine vibration while the pair of sidebands at 9 x and 11 x are unchanged. These data indicate that inevitable misalignments increase the additional low harmonics and introduce a single peak linked to whole-body in-plane target vibration.

### 3.3.5-Introduction of the misalignments

Table 3.5 reports the full set of realistic misalignments that can affect practical applications made with the Dove prism scanning LDV system. During the setting of the arrangement, the user has to give particular attention to the alignment of the laser head and the prism around the y - axis for which no instruments are available.

| Device | Misalignment | Range | $\boldsymbol{\Delta s t e p}$ | N. steps |
| :---: | :---: | :---: | :---: | :---: |
| Laser head | $\Delta \mathrm{x}_{\mathrm{A}}$ | $\pm 5 \mathrm{~mm}$ | 5 mm | 3 |
|  | $\Delta \mathrm{y}_{\mathrm{A}}$ | $\pm 5 \mathrm{~mm}$ | 5 mm | 3 |
|  | $\alpha_{\mathrm{mL}}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |
|  | $\beta_{\mathrm{mL}}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |
| Dove prism | $\Delta \mathrm{x}_{\mathrm{B}}$ | $\pm 3 \mathrm{~mm}$ | 3 mm | 3 |
|  | $\Delta \mathrm{y}_{\mathrm{B}}$ | $\pm 3 \mathrm{~mm}$ | 3 mm | 3 |
|  | $\alpha_{\mathrm{P}}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |
|  | $\beta_{\mathrm{P}}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |

Table 3.5- Full set of misalignments used to investigate the Dove prism SLDV system

These misalignments are introduced in the mathematical model following the same procedure used for the previous scanning systems. Translational misalignments of the laser head move the point A to $A^{\prime}$ defined as:

$$
\overrightarrow{O A^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
x_{A}+\Delta x_{A} & y_{A}+\Delta y_{A} & z_{A}+\Delta z_{A} \tag{3.108}
\end{array}\right]^{T}
$$

while the angular misalignments of the laser head around the x-and y-axes, $\alpha_{m L}$ and $\beta_{m L}$, modify the orientation of $\hat{b}_{1}$ as:

$$
\hat{b}_{1}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X,\left(\alpha_{L}+\alpha_{m L}\right)\right]\left[Y,\left(\beta_{L}+\beta_{m L}\right)\right]\left[\begin{array}{lll}
0 & 0 & -1 \tag{3.109}
\end{array}\right]^{T}
$$

in which the angles $\alpha_{L}$ and $\beta_{L}$ are the rotations used for the intended alignment of the laser head.

For the Dove prism, translational misalignments along the $x-, y-$ and $z$ - axes, move the
point B to $B^{*}$ and D to $D^{*}$ whose expressions can be written as:

$$
\begin{align*}
& \overrightarrow{O B^{*}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z} \rrbracket x_{B}+\Delta x_{B} \\
y_{B}+\Delta y_{B} & z_{B}+\Delta z_{B}
\end{array}\right]^{T}  \tag{3.110}\\
& \left.\overrightarrow{O D^{*}}=\overrightarrow{O B^{*}}-\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}]\left[X, \alpha_{P}\right.
\end{array}\right]\left[\begin{array}{l}
Y, \beta_{P}
\end{array}\right] \overrightarrow{B D} \right\rvert\,\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \tag{3.111}
\end{align*}
$$

while angular misalignments of the prism around the x - and y - axes, with angles $\alpha_{P}$ and $\beta_{P}$, modify the prism surface normals as:

$$
\begin{align*}
& \hat{n}_{B^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{P}\right]\left[Y, \beta_{P}\right]\left[Z, \gamma_{P}\right]\left[X, \psi_{P 1}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T}  \tag{3.112}\\
& \hat{n}_{C^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{P}\right]\left[Y, \beta_{P}\right]\left[Z, \gamma_{P} \rrbracket\left[\begin{array}{ll}
0 & 1
\end{array}\right]^{T}\right.  \tag{3.113}\\
& \hat{n}_{D^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{P} \llbracket\left[Y, \beta_{P}\right]\left[Z, \gamma_{P}\right]\left[X, \psi_{P_{2}}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T}\right. \tag{3.114}
\end{align*}
$$

The directions of the deflected beams, $\hat{b}_{2}, \hat{b}_{3}$ and $\hat{b}_{4}$ are determined using equations (3.86-88) where the modified surface normals are used. All these misalignments modify the positions of the deflection points whose positions have to be determined. The first refraction takes place at the point $B^{\prime}$ whose position can be determined in line with equations (3.92\&93):

$$
\begin{equation*}
\overrightarrow{O B^{\prime}}=\overrightarrow{O A^{\prime}}+\left|\frac{\left(\overrightarrow{O B^{*}}-\overrightarrow{O A^{\prime}} \cdot \cdot \hat{n}_{B^{\prime}}\right.}{\hat{b}_{1} \cdot \hat{n}_{B^{\prime}}}\right| \hat{b}_{1} \tag{3.115}
\end{equation*}
$$

In line with equations (3.94-96), the new position of the point $H^{*}$ is defined as:

$$
\overrightarrow{O H^{*}}=\overrightarrow{O B^{*}}+\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{P}\right]\left[Y, \beta_{P}\right]\left[Z, \gamma_{P}\right]\left[\begin{array}{lll}
0 & -|\overrightarrow{B H}| \cos \psi_{P 1} & -\overrightarrow{B H} \mid \sin \psi_{P 1} \tag{3.116}
\end{array}\right]^{r}
$$

and the deflection point $C^{\prime}$ is found as:

$$
\begin{equation*}
\overrightarrow{O C^{\prime}}=\overrightarrow{O B^{\prime}}+\left|\frac{\left(\overrightarrow{O H^{*}}-\overrightarrow{O B^{\prime}}\right) \cdot \hat{n}_{C^{\prime}}}{\hat{b}_{2} \cdot \hat{n}_{C^{\prime}}}\right|_{2} \tag{3.117}
\end{equation*}
$$

At the second wedge, the deflection point $D^{\prime}$ is found in line with equations (3.97\&98) obtaining the following expression:

$$
\begin{equation*}
\overrightarrow{O D^{\prime}}=\overrightarrow{O C^{\prime}}+\left|\frac{\left(\overrightarrow{O D^{*}}-\overrightarrow{O C^{\prime}}\right) \cdot \hat{n}_{D^{\prime}}}{\hat{b}_{3} \cdot \hat{n}_{D^{\prime}}}\right| \hat{b}_{3} \tag{3.118}
\end{equation*}
$$

The measuring point $\mathrm{K}^{\prime}$ at the target is found in line with equation (3.99) obtaining:

$$
\begin{equation*}
\overrightarrow{O K^{\prime}}=\overrightarrow{O D^{\prime}}+\left|\frac{\overrightarrow{O D^{\prime}} \cdot \hat{z}}{\hat{b}_{4} \cdot \hat{z}}\right| \hat{b}_{4} \tag{3.119}
\end{equation*}
$$

In presence of misalignments, the velocities measured at the deflection points are different and in line with equations (3.100-102) can be written as:

$$
\begin{align*}
& \overrightarrow{V_{B^{\prime}}}=\Omega_{P}\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{P} \rrbracket\left[Y, \beta_{P} \rrbracket\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \times \overrightarrow{B^{*} B^{\prime}}\right.\right.  \tag{3.120}\\
& \overrightarrow{V_{C^{\prime}}}=\Omega_{P}\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{P} \llbracket\left[Y, \beta_{P}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \times \overrightarrow{B^{*} C^{\prime}}\right.  \tag{3.121}\\
& \overrightarrow{V_{D^{\prime}}}=\Omega_{P}\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z} \\
\hline
\end{array}\right], \alpha_{P} \rrbracket\left[Y, \beta_{P} \llbracket\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T} \times \overrightarrow{D^{*} D^{\prime}}\right. \tag{3.122}
\end{align*}
$$

while the velocity measured at the point $K^{\prime}$ is calculated using equation (3.107).

At this point, with knowledge of the beam directions, positions of the deflection points and their velocities, equation (3.81) can be reformulated and used to predict the velocity measured by the system in the presence of a full set of misalignments.

### 3.3.6 - Analysis of misaligned configurations

The set of equations described in section 3.3.5 were programmed in Matlab and simulations made to investigate the effects of misalignments.

If a translational misalignment along the y -axis of $\Delta y_{A}=2 \mathrm{~mm}$ is added to the laser head, the dimension of the scan pattern increases as shown in figure 3.51a while the additional velocity terms are the same as the initial aligned configuration.

Angular misalignments added to the laser head modify the dimensions of the scan pattern and the amplitude of the detected additional velocities. Figures $3.52 \mathrm{a} \& \mathrm{~b}$ show, respectively, the scan pattern and the measured velocities obtained when an angular misalignment of $\alpha_{m L}=0.2^{\circ}$ is added to the initial angle $\alpha_{L}$. The magnitudes of the 0.5 x and 1.5 x components are still negligible as in the aligned case while the 1 x velocity component increases to $4.63 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, as indicated in figure 3.52 b .

If the prism has a translational misalignment along the x - axis of $\Delta x_{B}=2 \mathrm{~mm}$, the scan pattern moves along the x -axis, as indicated in figure 3.53a. The vibrometer measures a 0.5 x with amplitudes similar to that detected for the aligned case, while the additional 1 x velocity term measures $5.53 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, as shown in figure 3.53 b . The Doppler shifts at the target and at the prism show additional DC components with equal amplitudes but opposite signs whose final combination is zero. Similar effects are detected when the Dove prism has a translational misalignment along the $y$ - axis, with $\Delta y_{B}=2 \mathrm{~mm}$.

When the prism has an angular misalignment around the x - axis of $\alpha_{P}=0.2^{\circ}$, the scan pattern moves along the y - axis as indicated in figure 3.54a. The system reveals a 1 x velocity term with amplitude $1.25 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ while the 0.5 x and 1.5 x velocity components are not changed from those obtained for the initial aligned configuration, as shown in figure 3.54b. Similar results are obtained misaligning the Dove prism around the y - axis. Misalignment around the y - axis, with $\beta_{P}=0.2^{\circ}$, added to the Dove prism introduces a DC term of $1.04 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ and additional 1 x velocity term with magnitude of $1.66 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ while the 0.5 x and 1.5 x have amplitudes similar to the initial aligned configuration, as shown in figure 3.55 b. The scan pattern moves along the x - axis as indicated in figure 3.55 a .

These data suggest that, for the chosen initial alignment, the main effects of single misalignments, angular and translational, added to the optical device are variations of the 1 x velocity term. In the following section, these uncertainties will be quantified.

### 3.3.7 - Quantification of uncertainties in the presence of single and combined

## misalignments

At this point, the device requiring the most critical alignment has to be determined by analysing the effects produced by single misalignments. The analysis is made still considering a system with the laser head initially aligned around the x - axis with $\alpha_{L}=2.0875^{\circ}$ and a scan pattern with radius of 5 cm .

Figure 3.56a indicates the values predicted for the 1 x velocity term obtained for translational misalignment added to the prism along the x - and the y - axis (DP). The variations of the 0.5 x and 1.5 x velocity terms have not been reported because the predicted amplitudes are negligible with respect to the values predicted for the 1 x term.
Figure 3.56 b reports the predicted values for the 1 x velocity term in the presence of angular misalignments added to the prism and the laser head (LH). Angular misalignments around the $y$ - axis for the prism produce the greatest uncertainty detected by the system. Figure 3.56 c shows the predicted DC terms obtained in the presence of angular misalignments around the y - axis added to the Dove prism. These values are smaller than the values of the 1 x velocity term reported in figures $3.56 \mathrm{a} \& \mathrm{~b}$. These data indicate the 1 x velocity term as the biggest uncertainty detected by the system and indicate the alignment of the Dove prism as the most critical alignment because it produces the biggest effects on the 1 x velocity term.

In practical applications, the intended configuration will be affected by a full set of inevitable misalignments. For this case, simulations have been made combining the misalignment values reported in table 3.5 by covering more than $6561\left(=3^{8}\right)$ situations. From each configuration, the predicted additional velocities have been used to calculate RMS values.

| Additional component | Predicted RMS value |
| :---: | :---: |
| DC | $1.47 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |
| 0.5 x | $2.28 \mathrm{e}-4 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |
| 1 x | $1.01 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |
| 1.5 x | $1.52 \mathrm{e}-4 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |

Table 3.6-RMS measured velocities calculated for the additional velocity terms for the various combinations of misalignments for a scan pattern with radius 5 cm

These RMS values which characterize the Dove prism scanning system in terms of expected uncertainties in practical applications have been reported in table 3.6. The data indicate the biggest uncertainty is the 1 x term followed by the DC component with an expected smaller value. The uncertainties at 0.5 x and 1.5 x have magnitudes of two order smaller than the others. These values in table 3.6 have been obtained by considering only the different misaligned configurations, 79 , in which the scan centre is less than $0.5 \%$ of the radius from the position of the scan centre for the zero misalignment configuration. Figure 3.57a shows the scan pattern traced by the system in the presence of combined misalignments: $\alpha_{m L}=0.2^{\circ}, \beta_{m L}=0.2^{\circ}, \Delta x_{A}=\Delta y_{A}=3 \mathrm{~mm}$, $\alpha_{P}=0.2^{\circ}, \Delta y_{B}=3 \mathrm{~mm}$. The scan pattern shows a standard deviation for a radial position of around $0.05 \%$ while the centre offset is around $15 \%$. The vibrometer detects a DC, 0.5 x and 1 x velocity terms with magnitude $1.10 \mathrm{e}-4,2.31 \mathrm{e}-4$ and $6.54 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ as indicated in figure 3.57 b .

Another scenario characterized by a scan pattern almost in a central position at the target plane but obtained as a combination of the various misalignments is shown in figures 3.58a\&b. The misalignments considered are the following: $\alpha_{m L}=-0.1^{\circ}$, $\beta_{m L}=0.5^{\circ}, \Delta x_{A}=\Delta y_{A}=3 \mathrm{~mm}, \alpha_{P}=-0.1^{\circ}, \beta_{P}=0.1^{\circ}$ and $\Delta x_{B}=\Delta y_{B}=3 \mathrm{~mm}$. In this case, the standard deviation for the radial position predicted for the scan pattern is around $0.05 \%$ and the centre offset is of $2.94 \%$. The additional DC and 1 x velocity term measure $6.21 \mathrm{e}-4$ and $1.24 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, respectively, while additional terms at 0.5 x and 1.5 x have magnitude smaller. As for the scanning systems analysed earlier in this chapter, a scan pattern that is almost circular and centred in the target plane can still
lead to uncertainties in practical vibration measurements. The mathematical model proposed can help the user to understand from where these uncertainties originate in order to make better measurements.

## 3.4 - Comparison of LDV scanning systems

In this chapter three different scanning LDV systems have been analysed and modelled using the mathematical procedure proposed in the previous chapter. The results obtained can be compared to provide a complete view of the advantages and limitations of the systems. Each system is able to create circular scan patterns useful for tracking vibration measurements on rotating targets.

When the systems are aligned with their zero misalignment configuration, the dual mirror system measures a consistent 2 x additional term, the Dove prism system measures additional harmonics whose amplitudes are low and likely to be lower than typical measurement noise levels while the dual-wedge system is free from any additional velocity terms.

In the presence of combined misalignments and for geometries chosen to trace circular scan patterns with radius of 5 cm , the simulations have shown that the 1 x velocity term is the biggest uncertainty affecting these systems. In particular, the values obtained for the dual mirror scanning system are slightly bigger than those detected for the other arrangements as indicated in table 3.2, 3.4 and 3.6.

Observations made on the effects of single misalignments indicate that for the dual mirror attention has to be given to the alignment of all the optical devices, in particular to the laser head. For the dual-wedge scanning system the attention has to be focused on the two rotating wedges responsible for the introduction of the 1 x velocity term while for the Dove prism system the angular alignment of the prism itself is critical.

In terms of sensitivity to target vibrations, simulations have been made considering the same whole-body target vibrations for each arrangement. Values of sensitivity to whole-
body out-of-plane target vibrations were around $99.8 \%$ for all the systems when are aligned with their zero misalignment configuration. Because for each system the final direction of the incident beam is different from $\hat{z}$, each system is unavoidably sensitive to whole-body in-plane target vibrations. This sensitivity is seen as a negative aspect because the systems have been developed having in mind to measure only out-of-plane target vibrations. Sidebands associated with in-plane target vibrations with sensitivity around $2 \%$ are predicted for all the systems. Inevitable misalignments result in a single peak produced by the whole-body in-plane target vibration with sensitivity around $0.35 \%$ for all the systems.

Definitely, these data confirm that the three SLDV systems are comparable for circular tracking applications on rotating targets in terms of sensitivity to target vibrations and level of uncertainty detected at the 1 x velocity term. Only the dual mirror scanning system shows a bigger uncertainty at the 2 x velocity term due to its geometry.

Experiments made on a rotating target using the dual-wedge scanning system confirms the theoretical prediction for the additional 1 x velocity term in the presence of single translational and angular misalignments. The presence of inevitable misalignments in the test rig introduces additional vibrations when the target undergoes in-plane and out-of-plane whole body target vibrations. Results show the complexity of the systems analysed but also confirm the goodness of the mathematical procedure used to model the systems.

## Chapter 4

## Self-tracking LDV systems

The procedure used to model scanning LDV systems is versatile enough to be applied to other arrangements suitable for vibration measurements on rotating structures. This chapter will show how to model self-tracking LDV systems using the new approach to estimate the velocity measured for circular scanning applications. Particular emphasis will be given to investigation of misalignment effects in order to help the user to perform the best alignment possible and to interpret the vibration measurements with confidence.

## 4.1 - Introduction

The scanning LDV systems analysed in the previous chapters relate the motion of the incoming beam to that of the measurement point through an electro-mechanical connection between the rotating shaft and the deflecting optics. This connection is composed of an encoder and an electronic unit. The electronic unit processes the encoder output and defines the movement of the deflecting optics to create the scan patterns suitable for the measurements.

In addition to the problem of the inertia of the scanning mirrors, the processing of the encoder signal can introduce a speed limitation due to signal lag, which affects the synchronisation between the rotating shaft and the measurement point. These limitations can result in errors, especially in applications on high speed rotating structures. In
practice, it would be very desirable to have a method of tracking that does not have such speed limitations. These speed limitations are resolved by so-called self-tracking LDV systems in which the electro-mechanical connection is replaced with a completely mechanical one. These systems can work at any rotational speed with scan patterns suitable for tracking applications. As for all the other systems, however, the selftracking arrangements are prone to misalignments that produce errors on the position of the measurement point and uncertainties in the measured velocity.

Although these systems have been analysed to some extent by their inventors, a full analysis based on the proposed technique is presented in this chapter. There are two main goals: the first is to show that the new approach is sufficiently versatile to model these systems, the second is to investigate the advantages and disadvantages in terms of sensitivity to target vibrations and misalignments for scanning applications on rotating structures.

## 4.2 - Lomenzo self-tracking LDV system

### 4.2.1 - Derivation of the mathematical model

Figure 4.1 shows the self-tracking system proposed by Lomenzo [4.1]. The mechanical connection between target and deflecting system is composed of two mirrors: the vertex mirror fixed to the rotating target and the fold mirror positioned in front of the target. The latter has a small hole for the passage of the outgoing and returning laser beam, which is reflected twice before reaching the target. The rotation of the target moves the vertex mirror while the two reflections result in a circular scan on the structure.

For the model a reference system $x y z$ is fixed at the centre of the target. In the zero misalignment configuration, the target spin axis, the vertex spin axis and the incident laser beam are collinear to the z - axis and the fold mirror is perpendicular to the z - axis, as shown in figure 4.1. The direction of the incoming laser beam and the mirror surface normals are written as follows:

$$
\begin{align*}
& \hat{b}_{1}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & -1
\end{array}\right]^{T}  \tag{4.1}\\
& \hat{n}_{B}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[Z, \gamma_{V}\right]\left[\begin{array}{lll}
X, \psi_{V}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T}  \tag{4.2}\\
& \hat{n}_{C}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & -1
\end{array}\right]^{T} \tag{4.3}
\end{align*}
$$

where the rotation matrix $\left[X, \psi_{V}\right]$ models the inclination of the vertex mirror normal and the matrix $\left[Z, \gamma_{V}\right]$ represents the rotation of the normal around the z - axis. Since the mirror is attached to the target, the rotation angle $\gamma_{V}$ is defined as $\gamma_{V}=\Omega_{T} t+\varphi_{T}$ where $\Omega_{T}$ and $\varphi_{T}$ are the target rotational speed and the initial angular position of the vertex mirror, respectively. Applying equation (2.10), the direction of the deflected beams are:

$$
\begin{align*}
& \hat{b}_{2}=\hat{b}_{1}-2\left(\hat{b}_{1} \cdot \hat{n}_{B}\right) \hat{n}_{B}  \tag{4.4}\\
& \hat{b}_{3}=\hat{b}_{2}-2\left(\hat{b}_{2} \cdot \hat{n}_{C}\right) \hat{n}_{C} \tag{4.5}
\end{align*}
$$

The final expression for the incident beam $\hat{b}_{3}$ is found as:

$$
\begin{equation*}
\hat{b}_{3}=-\left[\sin \left(2 \psi_{V}\right) \sin \left(\Omega_{T} t+\varphi_{T}\right)\right] \hat{x}+\left[\sin \left(2 \psi_{V}\right) \cos \left(\Omega_{T} t+\varphi_{T}\right)\right] \hat{y}+\left[-2 \cos ^{2} \psi_{V}+1\right] \hat{z} \tag{4.6}
\end{equation*}
$$

Equation (4.6) describes a circular scan pattern whose x - and y - components are periodic functions of the target rotational speed while the component along the z - axis is a constant close to $\approx-1$.

From knowledge of the deflected beams and the geometry of the system, vector polygons are used to find the positions of the deflection points. The initial position of the laser source is defined by the point $A$, found as:
$\overrightarrow{O A}=\left[\begin{array}{lll}\hat{x} & \hat{y} & \hat{z}\end{array}\right]\left[\begin{array}{lll}x_{A} & y_{A} & z_{A}\end{array}\right]^{T}$

For the ideal alignment, the point $A$ is situated along the z- axis with $x_{A}=y_{A}=0$. The reflection point $B$ situated at the vertex mirror is the point which intersects the line of the rotation axis and its position is defined as:

$$
\overrightarrow{O B}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & z_{B} \tag{4.8}
\end{array}\right]^{T}
$$

To find the reflection point $C$, it is necessary to know the position of the fold mirror with respect to the target. For this reason, the point of a generic point $D$ situated at the fold mirror is introduced and defined as:

$$
\overrightarrow{O D}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
x_{D} & y_{D} & z_{D} \tag{4.9}
\end{array}\right]^{T}
$$

where $x_{D}=y_{D}=0$ to have the point $D$ along the z- axis. The point $C$ is then found from the following system:

$$
\left\{\begin{array}{l}
\overrightarrow{O B}+\mid \overrightarrow{B C} \hat{b}_{2}=\overrightarrow{O C}  \tag{4.10}\\
\overrightarrow{O D}+\overrightarrow{D C}=\overrightarrow{O C} \\
\overrightarrow{D C} \cdot \hat{n}_{C}=0
\end{array}\right.
$$

from which the term $\overrightarrow{O C}$ is found to give:

$$
\begin{equation*}
\overrightarrow{O C^{\prime}}=\overrightarrow{O B}+\left|\frac{(\overrightarrow{O D}-\overrightarrow{O B}) \cdot \hat{n}_{C}}{\hat{b}_{2} \cdot \hat{n}_{C}}\right| \hat{b}_{2} \tag{4.11}
\end{equation*}
$$

Finally, the measuring point $K$ situated on the target is found as:

$$
\begin{equation*}
\overrightarrow{O K}=\overrightarrow{O C}+\left|\frac{\overrightarrow{O C} \cdot \hat{z}}{\hat{b}_{3} \cdot \hat{z}}\right| \hat{b}_{3} \tag{4.12}
\end{equation*}
$$

Equations (4.7-12) describe completely the beam path of the system without any kind of approximation.

### 4.2.2 - Definition of the measured velocity

The measured velocity follows from reformulating equation (2.13). The mirrors deflect the beam and can generate Doppler shifts whose combination provides the full velocity sensed by the system, which can be written as:

$$
\begin{equation*}
U_{m}=\overrightarrow{V_{B}} \cdot\left(\hat{b}_{2}-\hat{b}_{1}\right)+\overrightarrow{V_{C}} \cdot\left(\hat{b}_{3}-\hat{b}_{2}\right)-\overrightarrow{V_{K}} \cdot \hat{b}_{3} \tag{4.13}
\end{equation*}
$$

where the terms $\overrightarrow{V_{B}}, \overrightarrow{V_{C}}$ and $\overrightarrow{V_{K}}$ are the velocities of the deflection points on the mirror surfaces and the target. In a more general case, the velocity measured at the point $B$ will include whole-body target vibrations because the vertex mirror is attached to the target. According to equation (1.31), the velocity measured at the point $K$ will include wholebody and cross-section flexible vibrations of the target. In this way, the velocity of the measuring points $B$ and $K$ have to be written as:

$$
\begin{align*}
& \overrightarrow{V_{B}}=\overrightarrow{V_{O}}+\Omega_{T} \hat{z} \times \overrightarrow{O B}  \tag{4.14}\\
& \overrightarrow{V_{K}}=\overrightarrow{V_{O}}+\Omega_{T} \hat{z} \times\left(\overrightarrow{O K}-\overrightarrow{O O^{*}}\right)+\overrightarrow{V_{f}} \tag{4.15}
\end{align*}
$$

where $\overrightarrow{V_{O}}$ is the translational velocity of the point $O$, the term $\overrightarrow{O O *}$ is the vector position of the moving point $O *$ with respect to the reference system $x y z$ while $\overrightarrow{V_{f}}$ represents the deformation vibration velocity of the measured target point due to crosssection flexibility. The introduction of the whole-body target vibration in the velocity measured at the point $B$ makes the self-tracking system different from the other tracking systems analysed in chapter 3. Substituting equations (4.14\&4.15) in (4.13) the complete expression for the velocity measured in the presence of target vibrations and flexibility becomes:

$$
\begin{equation*}
U_{m}=\left(\overrightarrow{V_{O}}+\Omega_{T} \hat{z} \times \overrightarrow{O B}\right) \cdot\left(\hat{b}_{2}-\hat{b}_{1}\right)-\left(\overrightarrow{V_{o}}+\Omega_{T} \hat{z} \times\left(\overrightarrow{O K}-\overrightarrow{O O^{*}}\right)+\overrightarrow{V_{f}}\right) \cdot \hat{b}_{3} \tag{4.16}
\end{equation*}
$$

### 4.2.3 - Circular tracking applications on a rotating target

For a self-tracking system with zero misalignments, the vertex and the fold mirror velocities are zero because the point $B$ is situated along the z - axis while $C$ is a stationary point on the fold mirror. In these conditions, the velocity measured depends only on the target. For a rotating and not vibrating target, the system traces a circular scan and no velocities are detected by the vibrometer.

In the presence of harmonic in-plane and out-of-plane target vibrations, the velocities measured at the point $B$ and $K$ are defined by equations (4.14\&4.15). Harmonic in-plane vibrations result in sidebands while harmonic z- vibrations produce single velocity components. An example is shown in figure 4.2a where a harmonic in-plane target vibration with frequency of $10 \Omega_{T}$, is directed along the x - axis and a harmonic out-ofplane target vibration with frequency $5 \Omega_{T}$ is applied along the z- axis. For a more general description, other harmonic x - and z - vibration due to the target cross-section flexibility with frequency of $20 \Omega_{T}$ and $15 \Omega_{T}$, respectively, are added to the target rotating structure. All these vibrations have amplitude of $10 \mathrm{~mm} / \mathrm{s}$.

The z - vibration at $5 \Omega_{T}$ produces a peak of $3.34 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}(\approx 29.7 \mathrm{~mm} / \mathrm{s})$ which is obtained as the combination of the Doppler shifts generated at the vertex mirror and at the target, as indicated by equation (4.16). The peak shows a sensitivity of $297 \%$ to whole-body out-of-plane vibration. The whole-body in-plane vibration, instead, generates two sidebands at 9 x and 11 x with amplitudes of $8.79 \mathrm{e}-2$ and $1.05 \mathrm{e}-1$ $\mathrm{mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}(\approx 7.81 \mathrm{e}-1$ and $9.32 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s})$ detected at the vertex and the target whose final combination is zero. For the vibrations due to the target flexibility, the z - vibration produces a single peak at $15 \Omega_{T}$ that measures $1.11 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}(\approx 9.86 \mathrm{~mm} / \mathrm{s})$ with sensitivity around $98.6 \%$ to the flexible vibration. The in-plane flexible vibration produces two sidebands at 19 x and 21 x with equal amplitudes of $9.77 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ $(\approx 0.86 \mathrm{~mm} / \mathrm{s})$ with sensitivity around $8.68 \%$. These data show that when the system is
ideally aligned it is sensitive to the target flexibility (in-plane and out-f-plane vibrations), is insensitive to whole body in-plane target vibration while measures a very important component linked to the whole body out-of-plane vibration and this emphasises the importance of considering the frequency shift at the optical devices.

Unfortunately, misalignments are inevitable and the sensitivity to target vibrations can be different. An example is shown in figure 4.2 b where the velocity spectra are obtained considering the same target vibrations as the previous case in the presence of an angular misalignment added to the laser head, $\beta_{L}=0.2^{\circ}$, around the x - axis. The peak at 5 x due to out-of-plane vibration still measures $3.34 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}(\approx 29.7 \mathrm{~mm} / \mathrm{s})$ but the vibrometer detects two sidebands at 4 x and 6 x of smaller amplitude. The whole-body in-plane target vibration produces a single peak at 10 x of amplitude $3.92 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}(\approx 3.48 \mathrm{e}-$ $2 \mathrm{~mm} / \mathrm{s}$ ) corresponding to sensitivity of $0.34 \%$, while the combination of the associated sidebands generated at the mirror and the target cancel in the total velocity. The effect of the out-of-pane vibration due to the target flexibility is a single peak at 15 x with amplitude similar to that measured in the case of ideal alignment plus two smaller sidebands at 14 x and 16 x . The in-plane flexible vibration produces a single peak at 20 x that measures $3.91 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}(\approx 3.47 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s})$ with sensitivity around $0.34 \%$ to the in-plane flexible vibration and the same sidebands measured for the ideal alignment. This result indicates that misalignments can introduce sensitivity of the system to the whole body in-plane target vibration (a negative aspect for the system), making data interpretation more difficult. This analysis shows the importance of a good alignment for the self-tracking system.

## 4.3 - Introduction of misalignments to the Lomenzo self-tracking LDV

## system

The mathematical approach allows incorporation of misalignments at all stages of the system. Table 4.1 lists a typical, full set of misalignments that affect the optical devices included in the system. The values have been chosen considering realistic misalignments met in applications after the user has taken steps to reduce these errors.

These investigations exclude the translational misalignments of the fold mirror because these have no effects on the scan pattern and in the measured velocity and translational misalignments of the vertex mirror because the point B used to determine the position of the mirror is always situated along the z - axis.

| Device | Misalignment | Terminology | Range | Astep | N. steps |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Laser head | Translational | $\Delta x_{A}$ | $\pm 5 m m$ | $5 m m$ | 3 |
|  |  | $\Delta y_{A}$ | $\pm 5 m m$ | $5 m m$ | 3 |
|  |  | $\alpha_{L}$ | $\pm 0.5^{\circ}$ | $0.5^{\circ}$ | 3 |
|  |  | $\beta_{L}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |
| Vertex mirror | Angular | $\alpha_{V}$ | $\pm 0.3^{\circ}$ | $0.3^{\circ}$ | 3 |
|  |  | $\beta_{V}$ | $\pm 0.3^{\circ}$ | $0.3^{\circ}$ | 3 |
| Fold mirror | Angular | $\alpha_{F}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |
|  |  | $\beta_{F}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |

Table 4.1 - Values used for simulations with combined misalignments for the self-tracking system proposed by Lomenzo.

In practice, alignment of the laser head is realized using a tripod and a translational stage helps to reduce the translational misalignments. Angular alignment of the laser head around the x - axis is assisted by use of a spirit level while angular alignment around the $y$ - axis is more difficult. The alignment of the vertex mirror depends on the precision of the connection between target and mirror and small angular misalignments have been chosen. For the fold mirror the angular alignment is critical, particularly maintaining orientation parallel with the xy plane. Assuming that the angular alignment is realised by eye, misalignments of $\pm 1^{\circ}$ have been used around the $x$ - and the $y$ - axis. Figure 4.3 shows a schematic misaligned configuration. When the laser head has translational misalignments of $\Delta x_{A}, \Delta y_{A}$ and $\Delta z_{A}$, the point $A$ moves to $A^{\prime}$ whose position is defined as:

$$
\overrightarrow{O A^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
\Delta x_{A} & \Delta y_{A} & z_{A}+\Delta z_{A} \tag{4.17}
\end{array}\right]^{T}
$$

while angular misalignments of the laser beam around the x - and the y - axis, with angles $\alpha_{L}$ and $\beta_{L}$, modify the direction of the outgoing beam $\hat{b}_{1}$ :

$$
\hat{b}_{1}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{L}\right]\left[Y, \beta_{L}\right]\left[\begin{array}{lll}
0 & 0 & -1 \tag{4.18}
\end{array}\right]^{T}
$$

where the rotation matrices $\left[X, \alpha_{L}\right]$ and $\left[Y, \beta_{L}\right]$ accommodate the angular misalignments. In the same way, translational and angular misalignments can be accommodated for the vertex mirror. For the point $B$, only translational misalignments along z - axis, $\Delta z_{B}$, are considered that move the point $B$ to $B *$ which is the point where the rotation axis intersects the plane of the mirror whose position is given by:

$$
\overrightarrow{O B *}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}  \tag{4.19}\\
\hline 0 & 0 & z_{B}+\Delta z_{B}
\end{array}\right]^{T}
$$

Angular misalignments added to the vertex mirror around the x - and the y - axis, with angles $\alpha_{V}$ and $\beta_{V}$, modify the mirror surface normal:

$$
\hat{n}_{B^{\prime}}=\left[\begin{array} { l l l } 
{ \hat { x } } & { \hat { y } } & { \hat { z } }  \tag{4.20}\\
{ \hline }
\end{array} \left[X, \alpha_{V} \rrbracket\left[Y, \beta_{V}\right]\left[Z, \gamma_{V} \llbracket\left[X, \psi_{V}\right]\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]^{T}\right.\right.\right.
$$

where the rotation matrices $\left[X, \alpha_{V}\right]$ and $\left[Y, \beta_{V}\right]$ introduce the angular misalignments of the vertex mirror. For the fold mirror, translational misalignments along the $\mathrm{x}-, \mathrm{y}$ - and z axes, $\Delta x_{D}, \Delta y_{D}$ and $\Delta z_{D}$, move the point $D$ to $D *$ whose position is:

$$
\overrightarrow{O D *}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
\Delta x_{D} & \Delta y_{D} & z_{D}+\Delta z_{D} \tag{4.21}
\end{array}\right]^{T}
$$

Setting $\Delta x_{D}=\Delta y_{D}=0$ the point $D *$ remains situated along the z - axis and in the plane of the fold mirror. When $\Delta x_{D}$ and $\Delta y_{D}$ differ from 0 , the point $D *$ moves but the dimensions of the scan pattern do not change. When $\Delta z_{D}$ differs from zero, the dimensions of the scan pattern change. Angular misalignments around the x-and the y-
axis, with angles $\alpha_{F}$ and $\beta_{F}$, are accommodated using the rotation matrices $\left[X, \alpha_{F}\right]$ and [ $\left.Y, \beta_{F}\right]$ that modify the mirror surface normal as:

$$
\hat{n}_{C^{\prime}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{F}\right]\left[\begin{array}{ll}
Y, \beta_{F}
\end{array}\right]\left[\begin{array}{lll}
0 & 0 & -1 \tag{4.22}
\end{array}\right]^{T}
$$

Using vector polygons, the position of the new reflection point $B^{\prime}$ at the vertex mirror is found by resolving the following system:

$$
\left\{\begin{array}{l}
\overrightarrow{O A^{\prime}}+\left|\overrightarrow{A^{\prime} B^{\prime}}\right| \hat{b}_{1}=\overrightarrow{O B^{\prime}}  \tag{4.23}\\
O B *+\overrightarrow{B^{\prime} B^{\prime}}=\overrightarrow{O B^{\prime}} \\
\overrightarrow{B * B^{\prime}} \cdot \hat{n}_{B^{\prime}}=0
\end{array}\right.
$$

and the vector $\overrightarrow{O B^{\prime}}$ is obtained as:

$$
\begin{equation*}
\overrightarrow{O B^{\prime}}=\overrightarrow{O A^{\prime}}+\left|\frac{\left(\overrightarrow{O B^{*}}-\overrightarrow{O A^{\prime}}\right) \cdot \hat{n}_{B^{\prime}}}{\hat{b}_{1} \cdot \hat{n}_{B^{\prime}}}\right| \hat{b}_{1} \tag{4.24}
\end{equation*}
$$

At the fold mirror, the reflection point $C^{\prime}$ is given by:

$$
\left\{\begin{array}{l}
\overrightarrow{O B^{\prime}}+\left|\overrightarrow{B^{\prime} C^{\prime}}\right| \hat{b}_{2}=\overrightarrow{O C^{\prime}}  \tag{4.25}\\
O D *+\overrightarrow{D^{*} C^{\prime}}=\overrightarrow{O C^{\prime}} \\
\overrightarrow{D * C^{\prime}} \cdot \hat{n}_{C^{\prime}}=0
\end{array}\right.
$$

from which the vector $\overrightarrow{O C^{\prime}}$ is given as:

$$
\begin{equation*}
\overrightarrow{O C^{\prime}}=\overrightarrow{O B^{\prime}}+\left|\frac{\left(\overrightarrow{O D^{*}}-\overrightarrow{O B^{\prime}}\right) \cdot \hat{n}_{C^{\prime}}}{\hat{b}_{2} \cdot \hat{n}_{C^{\prime}}}\right|_{2} \tag{4.26}
\end{equation*}
$$

The position of the measuring point $K^{\prime}$ is determined as:
$\overrightarrow{O K^{\prime}}=\overrightarrow{O C^{\prime}}+\left|\frac{\overrightarrow{O C^{\prime}} \cdot \hat{z}}{\hat{b}_{3} \cdot \hat{z}}\right| \hat{b}_{3}$

In this way, the system with a full set of misalignments is described completely and the various effects can be investigated. The expression for the measured velocity is reformulated in terms of the new points of deflection:

$$
\begin{equation*}
U_{m}=\overrightarrow{V_{B^{\prime}}} \cdot\left(\hat{b}_{2}-\hat{b_{1}}\right)+\overrightarrow{V_{C^{\prime}}} \cdot\left(\hat{b}_{3}-\hat{b}_{2}\right)-\overrightarrow{V_{K^{\prime}}} \cdot \hat{b}_{3} \tag{4.28}
\end{equation*}
$$

where the velocity measured at the point $B$ can be written as:

$$
\begin{equation*}
\overrightarrow{V_{B^{\prime}}}=\overrightarrow{V_{O}}+\Omega_{T} \hat{z} \times \overrightarrow{B * B^{\prime}} \tag{4.29}
\end{equation*}
$$

the velocity at the point $C^{\prime}$ is still zero and the velocity at the measuring point $K^{\prime}$ is:

$$
\begin{equation*}
\overrightarrow{V_{K}}=\overrightarrow{V_{O}}+\Omega_{T} \hat{z} \times\left(\overrightarrow{O K}-\overrightarrow{O O^{*}}\right)+\overrightarrow{V_{f}} \tag{4.30}
\end{equation*}
$$

Equation (4.28) allows the investigation of the effects produced by the various misalignments.

### 4.3.1 - Analysis of misaligned configurations

In this section, some of the misalignments listed in table 4.1 are investigated by analysing their effects on the scan pattern and on the measured velocity for a target that rotates but does not vibrate. Investigations are made considering a system characterized by the following geometrical parameters, $\psi_{V}=5^{\circ},|\overrightarrow{O A}|=0.4 \mathrm{~m},|\overrightarrow{O B}|=0.05 \mathrm{~m}$ and $|\overrightarrow{O D}|=0.167 \mathrm{~m}$, chosen to obtain a circular scan pattern with radius of 5 cm .

Figure 4.4 a shows the scan pattern traced by the system when a translational misalignment along the x - axis, with $\Delta x_{A}=2 \mathrm{~mm}$, is added to the laser head. The scan pattern moves along the x - direction while the laser vibrometer does not detect any additional velocity (or their level is smaller than the range of amplitudes reported in these figures) because the combination of the Doppler shifts generated at the target and the vertex mirror is zero, as shown in figure 4.4 b . A similar result has been obtained when a translational misalignment along the $y$ - axis is added to the laser head. Figure $4.5 \mathrm{a} \& \mathrm{~b}$ show the effects of an angular misalignment of the laser head around the x axis, with $\alpha_{L}=0.2$. The scan pattern moves along the y - direction but again no velocities are detected by the vibrometer (in the range of the investigated amplitudes). Similar results are obtained with angular misalignment of the laser head around the $y$ axis. These data indicate that single misalignments added to the laser head move the scan pattern in the target plane but have no effects on the measured velocity. When the previous translational and angular misalignments of the laser head are combined, the vibrometer detects an additional DC term of amplitude $6.98 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, as indicated in figure 4.6 b .

Translational misalignment along the x - and $\mathrm{y}-\mathrm{axes}$ are not added to the vertex mirror because in any case the spin axis of the mirror coincides with the target spin axis and the z - axis.

Angular misalignments of the vertex mirror consider imperfections on the mounting of the mirror on the target. When an angular misalignment around the x - axis, with $\alpha_{V}=0.2^{\circ}$, is added to the vertex mirror the scan pattern moves in the target plane as shown in figure 4.7a but no additional velocities are measured by the system. When an angular misalignment around the y - axis, with $\beta_{V}=0.2^{\circ}$, is added to the vertex mirror the scan pattern moves in the target plane as indicated in figure 4.7 b but still no velocities are measured by the system

When the fold mirror is angularly misaligned around the x - axis , with $\alpha_{F}=0.2^{\circ}$, the scan pattern moves along the x - axis, as indicated in figure 4.8 a , while the laser
vibrometer detects an additional 1 x target harmonic component with magnitudes 0.101 $\mathrm{mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, as shown in figure 4.8 b . Similar results are obtained when an angular misalignment around the y - axis of $\beta_{F}=0.2^{\circ}$, is added to the fold mirror. Translational misalignments along the x - and y - axes added to the fold mirror have no effects on the scan pattern while translational misalignments along the z - axis modify the dimensions of the scan radius. However, translational misalignments of the fold mirror result in no additional velocities.

These examples for specific misaligned cases show how the model predicts the uncertainties generated. Since the zero misalignments configuration is not achievable in practice, real measurements are always affected by such errors. For this reason, the model will be now used to investigate the effects of single or combined misalignments in order to identify the major sources of error and the maximum level of uncertainty likely in typical applications. The results can be used by the user to realize the best alignment possible and to interpret the measurements correctly. Investigations are made assuming a configuration with the same geometrical parameters used in this section.

### 4.3.2 - Quantification of the uncertainties in the presence of single and

 combined misalignmentSimulations are performed by adding single misalignments with values varying from zero to 1 of the range given in table 4.1. Investigations are made considering a rotating but not vibrating target. The predicted velocities are used to identify the optical component whose alignment is critical. For a zero misalignment configuration, no velocities are predicted. Figure 4.9 shows the values of the 1 x velocity term predicted in the presence of angular misalignments of the fold mirror around the x - axis but similar values are obtained misaligning the same device around the $y$ - axis. The data suggest that in the presence of single misalignments, the alignment of the fold mirror is critical and responsible for the uncertainty at the target rotational speed. The figure indicates that each $0.1^{\circ}$ of angular misalignment for the fold mirror corresponds to an additional 1x velocity term with RMS magnitude of $0.05 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$.

However, as seen in figures 4.6 b , the presence of combined misalignments can introduce other uncertainties, as a DC or additional target harmonics, and produce levels of uncertainty different from those indicated in figure 4.9. Because in practical applications the presence of combined misalignments is inevitable, it is useful to know the level of uncertainty expected in typical applications. For this reason, the values reported in table 4.1 have been used to run $6561\left(=3^{8}\right)$ different misaligned configurations. For each misaligned scenario, the measured velocity has been predicted and used to calculate RMS values of each order that characterize the current system. Table 4.2 reports these RMS values for the uncertainty expected in a circular scanning of a rotating but not vibrating target tracing a scan pattern with a scan radius of 5 cm . The data reported in table 4.2 have been obtained considering only the different scenarios in which the scan patterns have a maximum offset on the target plane of $5 \%$ from the ideal position $(\approx 2.5 \mathrm{~mm})$. This is because practical applications are performed with scan patterns almost centred in the target plane. It was found that 142 scenarios satisfy this condition and in any case, the values in table 4.2 indicate a significant 1 x velocity term can be expected in practical applications.

| Additional component | RMS expected value |
| :---: | :---: |
| $D C$ | $3.92 e-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |
| $1 x$ | $5.93 e-1 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |
| $2 x$ | $1.26 e-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |

Table 4.2 - RMS measured velocities calculated for the additional velocity terms for the various combinations of misalignments for a scan pattern with radius 5 cm .

The data show the 1 x velocity term as the largest uncertainty, the DC term has a smaller value but with the same magnitude while the uncertainty for the 2 x term has negligible magnitudes with respect to the other terms. These results quantify the resilience to misalignment of this scanning arrangement.

Figure 4.10a shows an example of scan pattern almost centred in the target plane obtained as combination of the following misalignments: $\alpha_{L}=-0.5^{\circ}, \beta_{L}=0^{\circ}, \alpha_{v}=-0.3^{\circ}$,
$\beta_{v}=0.3^{\circ}, \alpha_{F}=0^{\circ}, \beta_{F}=1^{\circ}, \Delta x_{A}=2 \mathrm{~mm}, \Delta y_{A}=5 \mathrm{~mm}, \Delta x_{B}=\Delta y_{B}=0 \mathrm{~mm}$. The scan pattern has a radius of almost 5 cm , a standard deviation of radial position of $1.56 \%$ and a centre with an offset of $6.72 \%$ with respect to the ideal case. The associated velocity spectra are shown in figure 4.10 b where additional $\mathrm{DC}, 1 \mathrm{x}$ and 2 x velocity terms of amplitude of $1.74 \mathrm{e}-2,5.03 \mathrm{e}-1$ and $7.71 \mathrm{e}-4 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ respectively are visible. This case shows that a scan pattern centred in the target plane is not synonymous with perfect alignment or absence of uncertainties in measured velocity.

Another example is shown in figures $4.11 \mathrm{a} \& b$. Here the combined misalignments are: $\alpha_{L}=-0.5^{\circ}, \quad \beta_{L}=0^{\circ}, \quad \alpha_{v}=-0.5^{\circ}, \quad \beta_{v}=-0.2^{\circ}, \quad \alpha_{F}=-0.3^{\circ}, \quad \beta_{F}=0.5^{\circ}, \quad \Delta x_{A}=5 \mathrm{~mm}$, $\Delta y_{A}=2 \mathrm{~mm} \Delta x_{B}=\Delta y_{B}=0 \mathrm{~mm}$. In this case the scan pattern is shifted in the target plane (the centre offset is around $0.12 \%$ ) and the standard deviation of radial position is of $0.08 \%$. The detected additional DC, 1 x and 2 x velocity terms have amplitudes of $8.28 \mathrm{e}-$ $2,2.91 \mathrm{e}-1$ and $5.97 \mathrm{e}-4 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ as shown in figure 4.11 b . Compared to the previous case, this scenario shows a bigger shift of the scan pattern in the target plane and this is responsible of the increment of uncertainties at the DC and the 2 x velocity terms while the 1 x velocity term results unchanged. The mathematical model identification of the various sources of error can help the user to set a better alignment to reduce the uncertainties.

## 4.4 - Sever self-tracking LDV system

### 4.4.1 - Definition of the mathematical model

The Sever system [4.2] is a development of the Lomenzo configuration in which a conical mirror positioned on the turbine cage replaces the fold mirror, as shown in figure 4.12. When the vertex and the conical mirror are both tilted at $45^{\circ}$ and the target is rotating, the beam reflections direct the incident beam perpendicular to the investigated structure in a circular scan.

The new approach proposed can also provide a mathematical model of this system. The zero misalignments configuration is obtained by aligning the incoming beam and the axis of the conical mirror to the $z$ - axis of the reference system $x y z$ fixed at the centre of the target. The directions of the incoming beam and of the mirror normal for the vertex mirror are described by equations (4.1-4.2). To find the direction of the mirror normal surface for the conical mirror it is initially possible to imagine a plane mirror with extremities at the points $D$ and $C$. The point $D$ is situated along the z- axis at a distance $\mathrm{z}_{\mathrm{D}}$ from the target. The mirror normal direction is obtained by considering an initial orientation opposite in direction to the z - axis followed by clockwise rotation around the x - axis for an angle $\psi_{C}=-45^{\circ}$, followed by rotation around the z - axis for an angle $\gamma_{C}$. During these rotations, the point $D$ has to remain along the z - axis and in its initial position. In this way, the incident point tracks the circular base of a cone with the point $D$ and the angle $\psi_{C}$ are the apex and cone angle, respectively. The mirror normal direction at the point $C$ is given as:

$$
\hat{n}_{C}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[Z, \gamma_{C}\right]\left[X, \psi_{C}\right]\left[\begin{array}{lll}
0 & 0 & -1 \tag{4.31}
\end{array}\right]^{T}
$$

where the rotation angle $\gamma_{C}$ is defined as $\gamma_{C}=\Omega_{T} t+\varphi_{T}$ where $\Omega_{T}$ and $\varphi_{T}$ are the target rotational speed and the initial phase which defines the initial point of incidence on the conical mirror. The directions of the deflected beams, $\hat{b}_{2}$ and $\hat{b}_{3}$, are determined by applying equations (4.4-4.5) at each mirror reflection point, respectively. In the absence of misalignments, the final direction of the incident beam $\hat{b}_{3}=-\hat{z}$.

The definition of the scan pattern and the positions of the reflecting points are found using vector polygons. The position of the laser source is still defined by the point $A$ described by equation (4.7) while the centre of the vertex mirror where the beam $\hat{b}_{2}$ is generated, the point $B$, follows from equation (4.8). When the vertex mirror is inclined at $45^{\circ}$ the second reflection point, the point $C$ situated on the conical mirror, can be determined as:

$$
\begin{equation*}
\overrightarrow{O C}=\overrightarrow{O B}+\frac{d}{2} \hat{b}_{2} \tag{4.32}
\end{equation*}
$$

where $d$ is the scan diameter while the position of the point $K$ situated at the target is found as:

$$
\begin{equation*}
\overrightarrow{O K}=\overrightarrow{O C}+\left|\frac{\overrightarrow{O C} \cdot \hat{z}}{\hat{b}_{3} \cdot \hat{z}}\right| \cdot \hat{b}_{3} \tag{4.33}
\end{equation*}
$$

In this way, the zero misalignment configuration of the system is completely described without any kind of approximation. When the inclination of the vertex mirror differs from $45^{\circ}$ the position of the point $C$ changes and equation (4.32) does not work. In this case, the position of the new point $C$ will be found as explained in section 4.6.

### 4.4.2 - Velocity measured by the Sever self-tracking LDV system

As for any scanning arrangement, the velocity measured by this self-tracking system is given from the combination of all the Doppler shifts generated in the beam path and can be written as equation (4.13) while equation (4.14\&15) can be used to describe the velocities of the deflection points in absence of target vibrations or flexibility. For the zero misalignments configuration, the velocities measured at the mirrors are zero because the point $B$ is situated along the z - axis while the conical mirror is fixed. In this case, the velocity detected by the system originates solely from the target. With zero misalignments, the geometrical parameters $|\overrightarrow{O A}|=0.3 \mathrm{~m},|\overrightarrow{O B}|=0.05 \mathrm{~m},|\overrightarrow{O D}|=0.1 \mathrm{~m}$ and $\psi_{V}=\psi_{C}=-45^{\circ}$ are chosen in order to trace a circular path with radius of 5 cm , as that analysed for the Lomenzo configuration. The tracking of a rotating but not vibrating target results in no predicted velocity.

Also in this case, the mathematical model derived for the Sever arrangement can incorporate and predict the effects of target vibrations in the measured velocity. An example is shown in figure 4.13a, for an ideally aligned system. The target undergoes harmonic whole-body x- and z- vibrations respectively at $10 \Omega_{T}$ and $5 \Omega_{T}$ plus other
harmonic x - and z- vibrations at $20 \Omega_{T}$ and $15 \Omega_{T}$ due to the flexibility of the structure. For all these vibrations, the amplitude is $10 \mathrm{~mm} / \mathrm{s}$. The spectrum of the vibrometer output reveals a single peak for each out-of-plane target vibration while the in-plane target vibration results in sidebands at 9 x and 11 x . The peak at 5 x measures $2.25 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}(\approx 20 \mathrm{~mm} / \mathrm{s})$ with sensitivity around $200 \%$ to the out-of-plane target vibration directed along the z - axis obtained as combination of the Doppler shifts generated at the target and vertex mirror. The whole body in-plane target vibration produces sidebands at the vertex mirror and result in the final spectrum. These sidebands are detected at 9 x and 11 x and have magnitudes of $5.06 \mathrm{e}-1$ and $6.19 \mathrm{e}-1$ $\mathrm{mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}(\approx 4.49$ and $5.49 \mathrm{~mm} / \mathrm{s})$, respectively with sensitivity around $44.9 \%$ and $54.9 \%$ to in-plane target vibration. The peak at 15 x represents the flexible z - vibration and measures $1.125 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}(\approx 10 \mathrm{~mm} / \mathrm{s})$ with sensitivity of $100 \%$ since the direction of $\hat{b}_{3}=-\hat{z}$. The flexible x -vibration at 20 x produces no additional peaks. These data indicate that an ideally aligned system measures the complete out-of-plane target vibration, is insensitive to the flexible in-plane target vibrations, while is sensitive to a whole-body target vibrations (in-plane and out-of-plane).

As for the Lomenzo configuration, also for the Sever system the sensitivity to target vibrations can be affected by the inevitable misalignments. If the laser head has an angular misalignment of $\beta_{L}=0.2^{\circ}$, the direction of $\hat{b}_{3} \neq-\hat{z}$. In this case, considering the same target vibrations of the previous case, the relative velocity spectra are shown in figure 4.13b. The system measures a peak for each vibration plus many sidebands. The peak at 5 x due to the z -vibration still measures $2.25 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}(\approx 20 \mathrm{~mm} / \mathrm{s})$ as in the ideal case but two smaller sidebands at 4 x and 6 x with amplitude of $1.96 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ ( $\approx 1.74 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s}$ ) are measured. The whole-body in-plane target vibration produces a single peak at 10 x with amplitude of $5.89 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ (sensitivity around $0.52 \%$ ), the sidebands at 9 x and 11 x with amplitudes similar to those detected for the zero misalignment case and others smaller at 8 x and 12 x that measure $6.18 \mathrm{e}-3$ and $7.56 \mathrm{e}-$ $3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ (sensitivity around $0.54 \%$ and $0.67 \%$ to vibration due to the target flexibility). The z - vibration at 15 x due to target flexibility produces a single peak with magnitude of $1.125 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}(\approx 10 \mathrm{~mm} / \mathrm{s})$ with sensitivity of $100 \%$ to the z - vibration
due to the target flexibility. The in-plane vibration due to the target flexibility produces a single peak at 20 x and various sidebands. The peak at 20 x measures $7.85 \mathrm{e}-$ $3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}(\approx 6.98 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s})$ with sensitivity of around $0.69 \%$ while the biggest sidebands are detected at 18 x and 22 x and measure $5.89 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ (sensitivity around $0.52 \%$ ) to the in-plane flexibility of the target.

These data shows that this misalignment increases the sensitivity to the whole-body inplane vibration and introduces additional sidebands that make data interpretation more difficult. As for the Lomenzo system, also this result indicates the importance of good alignment for the scanning system.

## 4.5 - Introduction of misalignments to the Sever self-tracking LDV

## system

As in any scanning system, the Sever arrangement is affected by inevitable misalignments that cause errors in the position of the measurement point and in the measured velocity. Figure 4.14 shows a typical misaligned configuration for the Sever arrangement. Table 4.3 reports typical and realistic misalignment values for practical applications. During the alignment, the control of the angular misalignment of the laser head around the y - axis and of the conical mirror around the x - and the y - axis are critical because no devices can be used to minimize their values. For these misalignments bigger values have been chosen as indicated in table 4.3. These misalignments can be incorporated in the mathematical model following the same procedure adopted for the Lomenzo configuration.

| Device | Misalignment | Terminology | Range | पstep | N. steps |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Laser head | Translational | $x_{A}$ | $\pm 5 m m$ | $5 m m$ | 3 |
|  |  | $y_{A}$ | $\pm 5 m m$ | $5 m m$ | 3 |
|  | Angular | $\alpha_{L}$ | $0.5^{\circ}$ | $0.5^{\circ}$ | 3 |
|  |  | $\beta_{L}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |
| Vertex |  | $\alpha_{V}$ | $\pm 0.3^{\circ}$ | $0.3^{\circ}$ | 3 |
| mirror |  | $\beta_{V}$ | $\pm 0.3^{\circ}$ | $0.3^{\circ}$ | 3 |
| Conical | Translational | $x_{D}$ | $\pm 5 m m$ | $5 m m$ | 3 |
| mirror |  | $y_{D}$ | $\pm 5 m m$ | $5 m m$ | 3 |
|  | Angular | $\alpha_{C}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |
|  |  | $\beta_{C}$ | $\pm 1^{\circ}$ | $1^{\circ}$ | 3 |

Table 4.3 - Values used for the simulation of situations with combined misalignments for the selftracking system proposed by Sever.

Translational misalignments added to the laser source, $\Delta x_{A} \Delta y_{A}$ and $\Delta z_{A}$, move the point $A$ to $A^{\prime}$ as indicated in equation (4.17) while angular misalignments, $\alpha_{L}$ and $\beta_{L}$, modify the direction of the incoming beam $\hat{b}_{1}$ as indicated by equation (4.18). Translational and angular misalignments added to the vertex mirror move the point $B$ to $B *$ as indicated by equation (4.19).

The position of the reflection point $C^{\prime}$ on the conical mirror depends on the translational and angular misalignments added to the conical mirror and also on the direction of the incident beam $\hat{b}_{2}$. Translational misalignments added to the conical mirror, $\Delta x_{D}, \Delta y_{D}$ and $\Delta z_{D}$, move the point $D$ to $D *$ whose position is defined as:

$$
\overrightarrow{O D *}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{lll}
\Delta x_{D} & \Delta y_{D} & z_{D}+\Delta x_{D} \tag{4.34}
\end{array}\right]^{T}
$$

while angular misalignments added to the mirror, $\alpha_{C}$ and $\beta_{C}$, modify the direction of the conical normal surface in:

$$
\left.\hat{n}_{C^{C}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}]\left[X, \alpha_{C}\right.
\end{array}\left\|Y, \beta_{C}\right\| Z, \gamma_{C} \| X, \psi_{C}\right] \begin{array}{lll}
0 & 0 & -1 \tag{4.35}
\end{array}\right]^{T}
$$

where the rotation matrices $\left[X, \alpha_{C}\right]$ and $\left[Y, \beta_{C}\right]$ accommodate the angular misalignments of the conical mirror around the x - and the y - axes while the rotation matrix $\left[X, \psi_{C}\right]$ introduces the inclination of the conical mirror. The position of $C^{\prime}$ can be found from the following system of equations:

$$
\left\{\begin{array}{l}
\overrightarrow{O B^{\prime}}+\overrightarrow{B^{\prime} C^{\prime}} \hat{b}_{2}=\overrightarrow{O C^{\prime}}  \tag{4.36}\\
O D *+\overrightarrow{D * C^{\prime}}=\overrightarrow{O C^{\prime}}
\end{array}\right.
$$

from which the term $\overrightarrow{D * C^{\prime}}$ can be written as:

$$
\begin{equation*}
\left|\overrightarrow{B^{\prime} C^{\prime}}\right| \hat{b}_{2}-\overrightarrow{B^{\prime} D *}=\overrightarrow{D^{*} C^{\prime}} \tag{4.37}
\end{equation*}
$$

The mathematical development of equation (4.37) is complicated so the complete derivation of $\overrightarrow{O C^{\prime}}$ has been reported in appendix A . With knowledge of the point $C^{\prime}$, the position of the measuring point $K^{\prime}$ at the target is obtained in the same way as equation (4.27).

In this way, the description of the Sever system in the presence of a full set of misalignments is completed. The expression for the measured velocity is reformulated in terms of the new points of deflection as indicated in equation (4.28). In this case, the velocity measured at the point $B^{\prime}$ is described by equation (4.29), that measured at the point $C^{\prime}$ is zero because the conical mirror is stationary while the velocity at the measuring point $\mathrm{K}^{\prime}$ is indicated by equation (4.30).

### 4.5.1 - Analysis of misaligned configurations

In this section typical misaligned configurations are analysed to show the misalignment effects on the scan pattern and in the measured velocity. Simulations are made considering the geometrical parameters defined in the previous section and considering a rotating but not vibrating target.

Figure 4.15 a and 4.15 b show the effects of a translational misalignment along the x axis added to the laser source with $\Delta x_{A}=2 \mathrm{~mm}$. The scan pattern moves along the xdirection and no velocities are measured by the system. Figure 4.15 b shows that the combination of the Doppler shifts produced at the target and the vertex mirror is zero. Similar results are obtained in the presence of translational misalignments along the $y$ axis added to the laser source.

Figure 4.16a\&b show the effects produced by an angular misalignment around the x axis added to the laser head, with $\alpha_{L}=0.2^{\circ}$. In this case, the scan pattern moves along the $y$ - axis while no velocities are detected by the system. These data indicate that single misalignments added to the laser head move the scan pattern in the target plane but do not introduce uncertainties in the measured velocity. Combining the previous misalignments of the laser head, $\alpha_{L}=0.2^{\circ}$ and $\Delta x_{A}=2 \mathrm{~mm}$, the scan pattern moves in the target plane while an additional DC term with amplitude of $6.98 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ is detected, as shown in figure $4.17 \mathrm{a} \& \mathrm{~b}$.

When an angular misalignment around the x - axis, with $\alpha_{V}=0.2^{\circ}$, is added to the vertex mirror the scan pattern moves along the y-axis and its radius is reduced to 43 mm as indicated in figure 4.18a. In this case, no additional velocities are detected by the vibrometer within the scale presented. An angular misalignment around the y-axis added to the vertex mirror, $\beta_{V}=0.2^{\circ}$, moves the scan pattern along the x - direction but still no velocities are measured, see figure 418 b.

A translational misalignment of the conical mirror along the x - axis, with $\Delta x_{D}=2 \mathrm{~mm}$, results in a shift of the scan pattern along the x - axis as shown in figure 4.19a and in additional target harmonic, where the 1 x and the 2 x velocity terms measure 1.41 and $2.82 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ as indicated in figure 4.19 b . A similar result is obtained when a translational misalignment of $\Delta y_{D}=2 \mathrm{~mm}$ is added to the conical mirror.

When the conical mirror has an angular misalignment around the x - axis, with $\alpha_{C}=0.2^{\circ}$, the system detects an additional 2 x velocity terms of amplitude $1.07 \mathrm{e}-4 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ as indicated in figure 4.20b. The combination of the previous translational and angular misalignment of the conical mirror results in additional $\mathrm{DC}, 1 \mathrm{x}$ and 2 x velocity terms with amplitude of $6.98 \mathrm{e}-3,1.41$ and $2.86 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, respectively, as shown in figure 4.21 b while the scan pattern moves in the target plane.

These data assert that misalignments of the conical mirror are the most critical for the system. Moreover, these examples show how the model incorporates the various misalignments and predicts their effects.

### 4.5.2 - Quantification of the uncertainties in the presence of single and

## combined misalignment

The analysis of misalignment effects for the Sever self-tracking system continues to determine the optical devices with the most critical alignment and to predict the typical level of uncertainty affecting typical vibration measurements. This analysis follows the same procedure used for the previous self-tracking arrangement. Previous simulations have showed that single misalignments added to the laser head do not introduce uncertainties in the measured velocity while single misalignments added to the reflecting mirrors are responsible for additional target harmonics. At this point, the estimation of uncertainty introduced by single misalignments to detect the most significant source of error is necessary.

Figure 4.22a reports the values predicted for the additional 1 x velocity term in the presence of single translational misalignments along the x - axis added to the conical mirror. The figure indicates very important uncertainties characterized by a linear increment with the increment of the added misalignment. Similar values are obtained when translational misalignments along the y - axis are added to the two mirrors.

Figure 4.22 b reports the values of the 1 x velocity term predicted when single angular misalignments around the x - axis are added to the conical mirror. These values are comparable, a bit smaller than those reported in figure 4.22a but still important. Figures $4.22 \mathrm{a} \& \mathrm{~b}$ indicate that the uncertainties detected at first order have a linear behaviour with the increment of the added misalignments and identify the conical mirror as the optical device with the most critical alignment. These data emphasise the importance of good alignment of the conical mirror.

The presence of single misalignments added to the conical mirror also introduces other target harmonics whose values are smaller than those predicted for the 1 x velocity term but still important in practical applications. Figure 4.23a shows the values for the predicted 2 x velocity terms in the presence of translational misalignments added to the conical mirror along the x -axis. Similar values are obtained for similar misalignments along the $y$ - axis. Figure $4.23 b$ shows the values predicted for the 2 x velocity terms when angular misalignments around the x - axis are added to the conical mirror. Similar values are obtained when similar angular misalignments around the $y$ - axis are added to the same optical device. Figures $4.23 \mathrm{a} \& \mathrm{~b}$ indicate that the values predicted for the 2 x velocity term are one order of magnitude smaller than those predicted for the additional 1 x velocity terms. Moreover, the curves reported in figure $4.23 \mathrm{a} \& \mathrm{~b}$ show a nearparabolic relationship between measured velocity and misalignment.

Figure $4.24 \mathrm{a} \& \mathrm{~b}$ report the predicted values of the 3 x velocity term in the presence of translational and angular misalignments along/around the x - axis added to the conical mirror, respectively. The values are of two orders of magnitude smaller than the 1 x velocity terms.

These data confirm that the conical mirror is the optical device with the most critical alignment and it is also responsible for the presence of many additional target harmonics in which the 1 x velocity term is the greatest. An additional DC term, instead, is expected in the presence of combined misalignments, as shown in figures 4.17 b and 4.21 b . As for the previous scanning LDV systems, the presence of additional velocity
terms is inevitable and that the level of uncertainty depends on the combination of the various misalignments. For this reason the values reported in table 4.3 have been used to run $59049\left(=3^{10}\right)$ different misaligned situations. For each situation the predicted additional velocities have been used to calculate the RMS value which characterizes the system in term of level of uncertainty expected in typical, practical applications. Table 4.4 reports the predicted RMS values for the various additional velocity terms.

| Additional component | RMS expected values |
| :---: | :---: |
| $D C$ | $0.081 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |
| $1 x$ | $2.642 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |
| $2 x$ | $0.193 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |
| $3 x$ | $0.083 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |
| $4 x$ | $0.017 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ |

Table 4.4 - RMS measured velocities calculated for the additional velocity terms predicted for the Sever self-tracking system in the presence of various combinations of misalignments.

As for the data reported in table 4.2, the values indicated in table 4.4 have been obtained from scenarios in which the maximum offset of the scan pattern from the ideal position in the target plane is $5 \%$ of the mean of the scan radius $(\approx 2.5 \mathrm{~mm})$. This is because practical applications are performed with scan patterns centred in the target plane. In any case, 5131 scenarios satisfy this condition indicating that in many cases, the misalignments determine scan patterns not useful in practical applications. The values in table 4.4 indicate a significant level of uncertainty detected for the first four target harmonics and the biggest uncertainty is that detected at 1 x . Similar values should be expected in practical applications.

Figures $4.25 \mathrm{a} \& \mathrm{~b}$ show a simulated scenario obtained combining different misalignments whose effect is the presence of an important level of uncertainty detected by the system. The configuration is obtained considering the following misalignments:
$\alpha_{L}=-0.5^{\circ}$,
$\beta_{L}=-0.5^{\circ}$,
$\alpha_{V}=-0.3^{\circ}$,
$\beta_{V}=0.3^{\circ}$,
$\alpha_{C}=\beta_{C}=1^{\circ}$, $\Delta x_{A}=\Delta y_{A}=\Delta x_{D}=\Delta y_{D}=0 \mathrm{~mm}$. The circular scan pattern shown in figure 4.25 a has a
standard deviation of radial position of $1.17 \%$ and the centre offset is of $3.07 \%$. The velocity spectrum in figure 4.25 b shows additional $D C, 1 x, 2 x$ and $3 x$ velocity terms with important amplitudes of $1.56 \mathrm{e}-3,1.72,3.78 \mathrm{e}-2$ and $3.72 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, respectively.

Another interesting case is shown in figure $4.26 \mathrm{a} \& \mathrm{~b}$ obtained combining the following misalignments: $\alpha_{L}=0^{\circ}, \beta_{L}=1^{\circ}, \alpha_{v}=\beta_{v}=0.3^{\circ}, \alpha_{C}=0^{\circ}, \beta_{C}=-1^{\circ}, \Delta x_{A}=5 \mathrm{~mm}, \Delta y_{A}=-$ $5 \mathrm{~mm}, \Delta x_{D}=-5 \mathrm{~mm}$ and $\Delta y_{D}=5 \mathrm{~mm}$. In this case, the scan pattern differs from a circle. Its standard deviation of radial position is of $4.41 \%$ and the centre offset is of $6.71 \%$. The result is the presence of many additional DC and target harmonics, which measure $2.18 \mathrm{e}-1,4.09,5.29 \mathrm{e}-1,8.72 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, respectively, as shown by figure 4.26 b .

### 4.5.3-Further effects in self-tracking LDV

Practical applications on a rotating target [4.2] have noticed that when the laser beam moves on the test surface, the Doppler signal strength fluctuates strongly and, if the signal falls below a threshold value, the LDV demodulation circuit cannot function properly. In this way, dropouts appear in the velocity signal, limiting the accuracy of the measurement. These dropouts are periodic occurring at the same location on the structure resulting in noise at rotation frequency plus harmonics.

Another possible problem is the presence of manufacturing imperfections on the large mirror surfaces, the conical mirror in the Sever system and the fold mirror in the Lomenzo system. These imperfections can modify the normal directions defined in equations ( $4.22 \& 35$ ) and affect the final beam orientation and the shape of the scan pattern. In the proposed model, these errors can be introduced as a pseudo random periodic noise. Random because the distribution of the imperfections on the mirror surface is random while periodic because during the rotation of the target, the beam passes many times over the same path and the effects assume a periodic behaviour. Figure 4.27a shows the scan pattern and the predicted velocities produced by the conical mirror when the RMS of the variations introduced are of $0.1^{\circ}$. The beam path slightly
deviates from a perfect circle but these variations are not visible in figure 4.27a. The associated velocity spectrum is shown in figure 4.27 b and reveals the presence of additional velocities at each target order with amplitudes proportional to random noise added to model the mirror imperfections. Typical level at low orders is around $3 \mathrm{e}-$ $2 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ which would have a noticeable effect on the levels at $2 \mathrm{x}, 3 \mathrm{x}$ and 4 x in table 4.4. In this case, the scan pattern appears as not circular but slightly deformed.

## 4.6 - Comparison between the two self-tracking LDV systems

The simulations have used geometrical parameters chosen in order to produce circular scans each with radius of 5 cm . Although the two mathematical models from the selftracking configurations are similar, differences in geometry cause differences in performance. For both the systems, the optical device responsible for the major source of error is the stationary mirror used to reflect laser beam towards the target: the fold mirror for Lomenzo system and the conical mirror for Sever arrangement.

In the presence of inevitable misalignments, the level of uncertainty detected for the Sever system is higher than that shown by the Lomenzo arrangement. The main effect is a significant first order component whose amplitude is around five times bigger than that predicted for the Lomenzo system but there are also many target harmonics detected by the system.

Comparing the data for the predicted uncertainties in the presence of single misalignments, figures 4.10-11 for the Lomenzo system and figures 4.25-26 for the Sever system, it results that the Doppler shifts from the target and the vertex mirror detected for Sever are bigger than Lomenzo. Moreover, the Doppler shift generated at the target seems to dominate. This fact can explain the difference between the levels of uncertainty predicted for the two systems,

Another difference regards the sensitivity of the systems to whole-body and flexible target vibrations. The possibility to measure flexible target vibration is a fundamental request for these systems while the sensitivity to whole-body vibrations is not. When
ideally aligned, the sensitivity to whole body out-of-plane vibration is around $297 \%$ for the Lomenzo system and around $200 \%$ for the Sever system. These values are obtained as combination of the Doppler shifts generated at the vertex mirror and the target. In the presence of whole-body in-plane target vibrations, the Doppler shift generated at the vertex mirror is combined with that generated at the target. The data indicate that the Lomenzo system is completely insensitive to this vibration while the Sever system shows a high sensitivity with sidebands with amplitudes around $40-60 \%$ of the in-plane vibration.

In the presence of a z - vibration due to the target flexibility, the Sever system shows sensitivity of $100 \%$ while for the Lomenzo configuration the sensitivity is around $98.6 \%$. This difference depends on the direction of the beam $\hat{b}_{3}$ : for Sever this is parallel to the z - axis while it is inclined for the Lomenzo arrangement. The Sever system is completely insensitive to the in-plane flexible target vibration while the Lomenzo arrangement shows sensitivity with sidebands with amplitudes around $8 \%$ of the in-plane vibration.

In practical applications, the presence of inevitable misalignments modifies the sensitivity to target vibrations for both systems. Data predicted for a small angular misalignment added to the laser head, indicate that each vibration produces a single peak but also additional sidebands that make the understanding of the measurement complicated. For both the systems, the sensitivity to out-of-plane flexible target vibration decreases slightly because $\hat{b}_{3} \neq \hat{z}$. The in-plane flexible target vibration adds a peak at the vibration frequency and data indicate the Sever system more sensitive than the Lomenzo arrangement. The misalignment, however, produces sidebands around these peaks and in this case, those detected by the Lomenzo system are bigger than those measured by the Sever arrangement.

For the whole body vibrations, the misalignment slightly modifies the sensitivity to the z-vibrations measured for both systems with respect to the ideal case. The peak produced by the whole-body in-plane vibration is the same for both systems while the

Sever system continues to show high sensitivity with sidebands with amplitudes around $40-60 \%$ of the in-plane vibration plus. These results indicate the importance of good alignment for the scanning system.

## Chapter 5

## Radial vibration measurements on rotors

It is now well known that radial vibration measurements taken directly from rotors exhibit a significant sensitivity to the orthogonal vibration component. Reliable estimation of the individual motion components involves the post-processing of the outputs from several instruments configured to measure the vibration sets.

Such a technique requires a detailed treatment of the velocity sensed by an instrument and this has been made possible by the development of the comprehensive Velocity Sensitivity Model, which can be applied to any measurement configuration on any target.

As described in section 1.3.2, the x - and y - radial vibration sets are each isolated using a single beam laser vibrometer and selecting particular values for the geometric parameters, $\alpha, \beta, x_{0}, y_{0}$ and $z_{0}$ such that measurement sensitivity to other vibration sets is eliminated. In reality, however, it is not possible to align the laser beam perfectly with the respective vibration axis and it is therefore necessary to include non zero $x_{0}$ and $y_{0}$ such that the velocities sensed by the laser vibrometers are written as in equations (1.10) and (1.11). The resolution of the x - and y - vibration sets requires the application of the technique described in section 1.3.3. The same technique can be used to resolve pitch and yaw vibration measurements on a rotor, as described by equations (1.29\&30) in section1.3.3.

In this chapter, investigations are made to determine the effects of the various instrument misalignments on the accuracy of the resolved radial vibration measurements. Further investigations examine the effect of rotor surface roughness on the level of cross-sensitivity encountered in practical applications.

## 5.1 - Introduction

Figure 5.1a shows the simulated spectrum of a single frequency radial vibration at 0.5 order of a rotating target. Figure 5.1 b shows the simulated spectrum of the equivalent measurement taken by a laser vibrometer in the presence of misalignments, noise, torsional oscillations and orthogonal radial vibration at 1.5 order. The laser vibrometer output shows the velocity at 0.5 order but also an additional velocity component at 1.5 order that represents the cross-sensitivity term produced by orthogonal radial vibration. Other velocity components at integer multiple of rotation order are due to torsional vibrations, geometrical misalignments and speckle noise. Knowledge of how misalignments affect the resolution of vibration components from the laser vibrometer outputs is important to understand the real dynamic behaviour of the target and also to inform efforts to align the laser vibrometers to reduce the uncertainties in the resolved vibration components.

Resolution of orthogonal radial vibrations from simultaneous vibrometer measurements uses special algorithms [5.1]. Successful operation of these algorithms is affected by the alignment of the laser vibrometers with respect to the target rotation axis and to each other. Because in practice it is impossible to arrange a configuration without misalignments of the laser vibrometers, speckle effects and other noises that introduce uncertainties in the resolved outputs, the magnitude of likely errors has to be found through simulation incorporating values of misalignment based on practical experience.

Investigations presented in this chapter consist of simulations of typical radial vibration measurements made on rotors to quantify uncertainties due to the instrument misalignments. The results can also be implemented for pitch/yaw vibration measurements because this application uses an optical configuration similar to that used
for radial vibration, in which the single beam laser vibrometers are replaced with two parallel beam laser vibrometers. Moreover, the equations used to resolve the pitch and yaw vibrations are similar to those used for radial vibrations but without complications associated with instrument offsets. The resolution technique is identical to that used for radial vibration measurement.

## 5.2 - Analysis of misalignment effects

As indicated in section 1.3.3, the post processing technique proposed to resolve the cross-sensitivity in laser vibrometer measurements of radial vibrations uses the mathematical expressions (1.26\&27). The resulting error derived from these equations is influenced by the weighting function $W(\omega)$ which is defined by equation (1.28) and whose trend is shown in figure 5.2. The figure shows that the value of $W(\omega)$ is infinite at synchronous frequency so it is not possible to establish a solution for the synchronous vibration component. This is evident in simulations by a gap in the data at synchronous frequency by of an increase in the errors around the synchronous frequency.

This section contains a detailed examination of typical misalignment and noise effects. This analysis is divided in two steps. Simulation 1 investigates the effects of realistic, single errors in order to identify the most significant sources of uncertainty. Simulation 2 combines the various, realistic misalignments to estimate the overall level of error expected for typical applications.

### 5.2.1-Simulation 1: Errors produced by single misalignments and noises

In simulation 1 , single frequency vibrations directed along the x -, y - and z - axis are applied to a rotating rotor while the measurements are affected by the presence of misalignments and noises. Table 5.1 reports realistic vibration velocities representing medium/high severity vibrations for typical rotor systems [5.1].

| Rotational speed | $\bar{\Omega}=100 \pi r a d s^{-1}$ |
| :---: | :---: |
| Radial vibration (along $x$ - axis) | $\dot{x}=20 \cos (0.5 \bar{\Omega} t+0.5 \pi) \mathrm{mms}^{-1}$ |
| Radial vibration (along y-axis) | $\dot{y}=10 \cos (1.5 \bar{\Omega} t) \mathrm{mms}^{-1}$ |
| Axial vibration (along z-axis) | $\dot{z}=50 \cos (3 \bar{\Omega} t+\pi) \mathrm{mms}^{-1}$ |
| Torsional vibrations | $\Omega(t)=0.7 \mathrm{rads}^{-1} @ 1 / 2 \mathrm{x}$ orders |
|  | $\Omega(t)=1.4 \mathrm{rads}^{-1}, 2 \mathrm{x}$ orders |

Table 5.1 - Vibration parameters used in simulation 1

The frequencies of the vibrations have been deliberately chosen to be different to identify their effects more easily. The chosen rotational speed $\Omega_{T}$ is defined as the combination of the mean rotational velocity and the torsional oscillations, $\Omega_{T}=\overline{\Omega_{T}}+\Delta \Omega_{T}$, in order to consider a more general case of radial vibration measurements. All the data from the simulations are presented against order.

| Device | Misalignments | Terminology | Range | $\boldsymbol{\Delta}$ step | N. steps |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $x$-LDV | Translational | $y_{0}$ | $0-1 \mathrm{~mm}$ | 0.25 mm | 5 |
|  | Pseudo-random noise |  | $0-1 \mathrm{~mm} / \mathrm{s}$ | $0.25 \mathrm{~mm} / \mathrm{s}$ | 5 |
|  | Electronic noise |  | $0-1 \mathrm{~mm} / \mathrm{s}$ | $0.25 \mathrm{~mm} / \mathrm{s}$ | 5 |
| $y$ y-LDV | Translational | $x_{0}$ | $0-1 \mathrm{~mm}$ | 0.25 mm | 5 |
|  | Angular | $\varepsilon_{y}$ | $0-1^{\circ}$ | $0.25^{\circ}$ | 5 |
|  |  | $\delta_{y}$ | $0-1 \circ$ | $0.25^{\circ}$ | 5 |
|  | Pseudo-random noise |  | $0-1 \mathrm{~mm} / \mathrm{s}$ | $0.25 \mathrm{~mm} / \mathrm{s}$ | 5 |
|  | Electronic noise |  | $0-1 \mathrm{~mm} / \mathrm{s}$ | $0.25 \mathrm{~mm} / \mathrm{s}$ | 5 |

Table 5.2-Misalignments used in simulation 1

Table 5.2 lists realistic and typical ranges of misalignments and noise chosen for the analysis. In the first simulation, each misalignment is introduced singly with its value increasing from 0 to 1 of its range. For each step, the difference between the resolved and the genuine vibration amplitude is automatically determined by the simulator used to calculate an overall value combining the differences from each individual simulation.

Since the cross-sensitivities are completely eliminated by the resolution algorithm, the uncertainties detected at the genuine vibration frequencies are produced only by the added misalignments and noises.

These simulations incorporate x - and y -vibrations at different frequencies so that they can be easily distinguished. In this case, the x-direction measurement shows the correct $x$-vibration ( $20 \mathrm{~mm} / \mathrm{s}$ amplitude) but a component at the $y$-vibration order of $7 \mathrm{~mm} / \mathrm{s}$ when the true level is zero. Similarly, the y-direction measurement shows the correct yvibration ( $10 \mathrm{~mm} / \mathrm{s}$ amplitude) but a component at the $x$-vibration order of $30 \mathrm{~mm} / \mathrm{s}$ when the true level is zero. In the absence of misalignments and noises, the resolved $x$ vibration shows only a $0.5 x$ component with the correct amplitude. And the resolved yvibration shows only a 1.5 x component with the correct amplitude. Misalignments and noises result in additional uncertainties and, in the plots illustrating simulation 1, a resolved velocity error is formed based on the difference between the resolved velocity in the presence of misalignments and that without misalignments.

According to the typical arrangement used for radial vibration measurements, shown in figure 5.3a, the first misalignments analysed are the inevitable offsets of the laser vibrometers with respect to the target spin axis as shown in figure 5.3b. Mathematically, the translational offsets modify the laser vibrometer outputs as follows:

$$
\begin{align*}
& U_{x}=\dot{x}+\Omega_{T}\left(y-y_{0}\right)  \tag{5.1}\\
& U_{y}=\dot{y}-\Omega_{T}\left(x-x_{0}\right) \tag{5.2}
\end{align*}
$$

By introducing an x - offset of 0.25 mm , the measured x - and y - vibrations take the forms shown in figure $5.4 \mathrm{a} \& \mathrm{~b}$, respectively. Figure 5.4 a for the x - direction LDV measurement shows velocity components related to the genuine $x$ - and $y$ - vibrations together with small components appearing as a result of the combination of torsional vibrations and y- radial vibration. Figure 5.4 b also shows velocity components related to the genuine x - and y - vibrations, together with additional components dominated by the effect of the radial offset, $x_{0}$, in combination with torsional vibrations.

Figures $5.5 \mathrm{a} \& \mathrm{~b}$ shows how errors in the resolved x - and y - velocities vary with $x_{0}$. In the resolved x -velocity, the error apparent for the genuine vibration at 0.5 x and a 0.25 mm offset is $5.25 \mathrm{e}-3 \mathrm{~mm} / \mathrm{s}$, corresponding to $0.0263 \%$, increasing by the same amount for each additional increment in offset. In the resolved $y$-velocity, the error apparent for the genuine vibration at 1.5 x and a 0.25 mm offset is $0.307 \mathrm{~mm} / \mathrm{s}$, corresponding to $3.07 \%$, increasing by the same amount for each additional increment in offset.

In the resolved x -velocity, errors at 1.5 x order are associated with sensitivity to y vibration due to the added misalignment. For 0.25 mm offset, this error is $0.201 \mathrm{~mm} / \mathrm{s}$, corresponding to $2.01 \%$ sensitivity increasing by the same amount for each additional increment in offset. Similarly, the errors at 0.5 order in the resolved $y$-velocity are due to x -vibration. For 0.25 mm offset, this error is $6.66 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s}$, corresponding to $0.333 \%$ sensitivity, increasing by the same amount for each additional increment in offset.

Comparing figures $5.5 \mathrm{a} \& \mathrm{~b}$ the errors detected at other orders are larger in the resolved $y$ - velocity than in the resolved $x$ - velocity. For both the resolved velocities, the biggest uncertainty is at the 2 x component linked to the torsional oscillations used in the simulation. In the resolved y-velocity, this component increments by $4.67 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s}$ for each increment in offset, equivalent to $1.87 \mathrm{~mm} / \mathrm{s} / \mathrm{mm}$ of offset. Comparing to the original measured velocities, this level is higher than that seen in the larger, y - direction measurement by a factor 2 . For the resolved $x$ - vibration, the 2 x velocity term increments around by $2.32 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s}$ with each increment in offset, corresponding to $9.29 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s} / \mathrm{mm}$ of offset. This level is lower than that seen in the larger, y-direction measurement by a factor 0.5

The error measured at fourth order, again due to the torsional oscillations, is important, particularly for the resolved y - vibration, with sensitivity of $1.50 \mathrm{~mm} / \mathrm{s} / \mathrm{mm}$ of offset.

For practical applications, the laser vibrometer is generally aligned using a translational linear stage on which the device is mounted. The level of offset achievable for
translational misalignments is around 2 mm . This value can be reduced to 0.25 mm by mounting on the laser vibrometer the device shown in figure 5.6. The device is composed of two mirrors inclined at $45^{\circ}$ which reflect the outgoing beam twice, maintaining its initial orientation. The first mirror is fixed while the second is mounted on a precision translation stage which enables adjustment of the laser beam in the direction perpendicular to that of the laser beam direction. In this way a more precise translational alignment can be made. The choice of the translational misalignments was made considering the possibility to use this device in practical radial measurements.

Figures $5.7 \mathrm{a} \& \mathrm{~b}$ shows the measured (unresolved) x - and y - vibrations in the presence of offset $y_{0}=0.25 \mathrm{~mm}$. Figures $5.8 \mathrm{a} \& \mathrm{~b}$ show how errors in the resolved x - and y - velocities vary with $y_{0}$. In the resolved x -velocity, the error apparent for the genuine vibration at 0.5 x and a 0.25 mm offset is $6.66 \mathrm{e}-2 \mathrm{~mm} / \mathrm{s}$, corresponding to $0.33 \%$, increasing by the same amount for each additional increment in offset. In the resolved y-velocity, the error apparent for the genuine vibration at 1.5 x and a 0.25 mm offset is $2.01 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s}$, corresponding to $2.01 \%$, increasing by the same amount for each additional increment in offset.

In the resolved x -velocity, errors at 1.5 x order for 0.25 mm offset due to the y - radial vibration is $0.307 \mathrm{~mm} / \mathrm{s}$ that corresponds to single increments of around $3.07 \%$ sensitivity and increases by the same amount for each additional increment in offset. The errors at 0.5 order in the resolved y-velocity due to x -vibration for 0.25 mm offset is $0.126 \mathrm{~mm} / \mathrm{s}$, corresponding to $0.634 \%$ sensitivity and increases by the same amount for each additional increment in offset.

Comparing figures $5.8 \mathrm{a} \& \mathrm{~b}$ the errors detected at other orders are larger in the resolved $x$ - velocity than in resolved $y$-velocity. For both the resolved velocities, the biggest uncertainty is still at the 2 x component due to the torsional oscillations used in the simulation. In the resolved $x$ - velocity, this component increments by $0.467 \mathrm{~mm} / \mathrm{s}$ for each increment in offset, equivalent to $1.87 \mathrm{~mm} / \mathrm{s} / \mathrm{mm}$ of offset. For the resolved x-
vibration, the 2 x velocity term increments around by $2.32 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s}$ with each increment in offset, corresponding to $9.29 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s} / \mathrm{mm}$ of offset.

The error measured at fourth order is again due to the torsional oscillations and is important, particularly for the resolved $x$-vibration with sensitivity of $1.497 \mathrm{~mm} / \mathrm{s} / \mathrm{mm}$ of offset.

The angular offsets refer to inevitable inclinations of the laser vibrometer beams with respect to the x - and y - axes, making the measurement directions not perfectly orthogonal, as indicated in figure 5.3b. In this case, the measurements are sensitive to vibration sets other than that which it is intended to measure. It is possible to examine the effect that such misalignments have on the resolved outputs by considering that the y - direction vibrometer is misaligned with the y - axis by some small angles $\varepsilon_{y}$ and $\delta_{y}$ about the z - and the x - axes, respectively, as shown in figure 5.3 b . When the vibrometer has a misalignment $\varepsilon_{y}$, the measured output is derived from equation (1.9) where the angle $\beta=-\varepsilon_{y}$ and, neglecting the small cross-sensitivity terms, the modified version of the measured output becomes:
$U_{y}=\cos \varepsilon_{y}\left[\dot{y}-\Omega_{T} x\right]-\sin \varepsilon_{y}[\dot{z}]$
while the vibrometer output $U_{x}$ does not change. Equation (5.3) indicates an additional vibration term proportional to $\sin \varepsilon_{y}$ which will result in sensitivity to the axial vibration $\dot{z}$ applied to the rotor.

Figures $5.9 \mathrm{a} \& \mathrm{~b}$ show the measured (unresolved) x - and y - vibration for a $0.25^{\circ}$ offset while figures $5.10 \mathrm{a} \& \mathrm{~b}$ how errors in the resolved x - and y -velocities vary with $\varepsilon_{y}$. In the resolved x -velocity, the error apparent for the genuine vibration at 0.5 x and a $0.25^{\circ}$ offset is $2.54 \mathrm{e}-4 \mathrm{~mm} / \mathrm{s}$, corresponding to $0.00127 \%$, increasing by the same amount for each additional increment in offset. In the resolved y-velocity, the error apparent for the genuine vibration at 1.5 x and a $0.25^{\circ}$ offset is $1.72 \mathrm{e}-4 \mathrm{~mm} / \mathrm{s}$, corresponding to
$0.00172 \%$ and increases by the same amount for each additional increment in offset. These errors are very small if compared with those produced by the translational misalignments.

Also the errors detected in the resolved $x$-velocity at 1.5 order and at 0.5 order in the resolved y-velocity are negligible. For $0.25^{\circ}$ offset the error detected in the resolved $x$ velocity at 1.5 x order measures $1.15 \mathrm{e}-4 \mathrm{~mm} / \mathrm{s}$ that corresponds to single increments of around $0.00115 \%$ sensitivity while the error detected in the resolved $x$-velocity due to x -vibration for $0.25^{\circ}$ offset is $1.27 \mathrm{e}-4 \mathrm{~mm} / \mathrm{s}$, corresponding to $0.00635 \%$ sensitivity.

For both the resolved velocities, the biggest uncertainty is at the 3 x component due to the axial vibration used in the simulation. In the resolved $x$-velocity, this component increments by $0.081 \mathrm{~mm} / \mathrm{s}$ for each increment in offset, equivalent to $0.324 \mathrm{~mm} / \mathrm{s} /$ degree of offset. For the resolved x- vibration, the $3 x$ velocity term increments around by $0.245 \mathrm{~mm} / \mathrm{s}$ with each increment in offset that corresponds to $0.980 \mathrm{~mm} / \mathrm{s} /$ degree of offset.

The other angular misalignment investigated regards inevitable inclination of the vibrometer aligned to the $y$ - direction around the $z$ - axis. This misalignment, indicated as $\delta_{y}$, modifies the measured output as follows. From the reformulation of equation (1.9) in which the angle $\alpha=90^{\circ}+\delta_{y}$ and, neglecting the small cross-sensitivity terms, the measured output is modified as:

$$
\begin{equation*}
U_{y}=-\sin \delta_{y}\left[\dot{x}+\Omega_{T} y\right]+\cos \delta_{y}\left[\dot{y}-\Omega_{T} x\right] \tag{5.4}
\end{equation*}
$$

Equation (5.4) shows an additional $x$ - radial velocity term that introduces measurement errors.

Figures $511 \mathrm{a} \& \mathrm{~b}$ show the measured x - and y -vibration for a $0.25^{\circ}$ offset while figures $5.12 \mathrm{a} \& \mathrm{~b}$ show how errors in the resolved x - and y - velocities vary with $\delta_{y}$. In the
resolved x -velocity, the error apparent for the genuine vibration at 0.5 x and a $0.25^{\circ}$ offset is negligible and measures $2.19 \mathrm{e}-4 \mathrm{~mm} / \mathrm{s}$ that corresponds to $0.00109 \%$ and shows similar increments for each additional increment in offset. Also in the resolved $y$ velocity, the error apparent for the genuine vibration at 1.5 x and a $0.25^{\circ}$ offset is negligible with a value of $2.44 \mathrm{e}-4 \mathrm{~mm} / \mathrm{s}$, corresponding to $0.00244 \%$ and increases by the same amount for each additional increment in offset.

The errors detected in the resolved $x$-velocity at $1.5 x$ order due to the $y$-radial vibration is the biggest and for $0.25^{\circ}$ offset measures $0.0348 \mathrm{~mm} / \mathrm{s}$ that corresponds to single increments of around $0.348 \%$ sensitivity and increases by the same amount for each additional increment in offset. The error detected in the resolved $y$-velocity due to $x$ vibration for $0.25^{\circ}$ offset is $0.0290 \mathrm{~mm} / \mathrm{s}$ that corresponds to $0.145 \%$ sensitivity and is the biggest error detected by the x - vibrometer. For other orders, the level of error detected is negligible.

For practical applications, the angular alignment of the laser vibrometer is realized using the angular stage on which the device is mounted. The level of angular misalignment achievable depends on the kind of angular misalignment considered. For example, for the laser vibrometer aligned along the x - axis the angular alignment around the x -direction can be controlled using a spirit level and the expected uncertainty can vary between $\pm 0.1^{\circ}$. The alignment around the $y$ - axis, however, is critical because no devices can be used and it is typically performed by eye. In this case, the expected misalignment varies between $\pm 0.5^{\circ}$. The same consideration can be made for the laser vibrometer aligned to the $y$-axis. The alignment around the x - axis is controlled with the spirit level while that around the z - axis is critical.

Table 5.3a lists the percent errors at the genuine vibration frequencies found from this simulation in the presence of geometrical misalignments.

| Misalignment |  | Error (\%) |  |
| :---: | :---: | :---: | :---: |
| Type | Value | x-resolved | y-resolved |
| $x_{0}$ | 0.25 mm | 0.0263 | 3.07 |
| $y_{0}$ | 0.25 mm | 0.337 | 2.01 |
| $\varepsilon_{y}$ | $0.25^{\circ}$ | 0.0012 | 0.002 |
| $\delta_{y}$ | $0.25^{\circ}$ | 0.0014 | 0.002 |

Table 5.3a-Single percent increments for errors at the resolved $x$ - and $y$-vibrations

Table 5.3b lists the percent errors at the orthogonal vibration frequencies found from this simulation in the presence of geometrical misalignments.

| Misalignment |  | Error (\%) |  |
| :---: | :---: | :---: | :---: |
| Type | Value | $\boldsymbol{x}$-resolved | $\boldsymbol{y}$-resolved |
| $x_{0}$ | 0.25 mm | 2.01 | 0.333 |
| $y_{0}$ | 0.25 mm | 0.634 | 2.01 |
| $\varepsilon_{y}$ | $0.25^{\circ}$ | 0.0011 | 0.0006 |
| $\delta_{y}$ | $0.25^{\circ}$ | 0.348 | 0.145 |

Table 5.3b-Single percent increments of the residual sensitivity to the orthogonal vibration in the resolved $x$ - and $y$-vibrations

These investigations of offsets demonstrate that instrument alignment is an important factor worthy of attention when making such measurements. In addition to these offsets, additional uncertainties are introduced by the various noises that affect the laser vibrometer outputs. An initial distinction can be made between a random electronic noise and a pseudo-random noise associated with the laser speckle effect in measurements on a rotating target.

The random electronic noise generates an underlying broadband noise floor for the entire frequency range investigated while the pseudo-random noise added to the two laser vibrometers causes errors at the target integer orders. For a target rotating at
constant velocity, measurement experience indicates that the combination of these noise sources typically results in amplitude errors of approximately $6 \%(\approx 0.06 \mathrm{~mm} / \mathrm{s})$ at integer multiples of rotation frequency with an underlying broadband noise floor of approximately $1 \%$ of $x$ - resolved measurements. In the $y$ - direction the uncertainties detected at the integer multiples are around $9 \%$ of the genuine $y$-vibration while the error due to the underlying broadband noise floor is approximately $1.5 \%$ of the same radial vibration.

## 5.3 - Simulation 2: Errors produced by combined misalignments and noises in presence of single frequency radial vibrations

In practical applications, the arrangement used to resolve radial vibrations suffers from the simultaneous presence of the various unknown misalignments and their combined effects can differ from the individual effects reported in simulation 1. For this reason, simulation 2 considers typical, combined misalignments to estimate the maximum level of uncertainty affecting typical radial measurements. In these simulations, radial vibrations along the x - and the y - axis plus the torsional vibrations are applied to the rotor, as reported in table 5.4.

| Rotational speed | $100 \pi 4 \mathrm{rads}^{-1}$ |
| :---: | :---: |
| Radial vibration (along $x$ - axis) | $\dot{x}=A_{x} \cos \left(B_{x} \bar{\Omega} t+C_{x}\right) \mathrm{mms}^{-1}$ |
| Radial vibration (along $y$ - axis) | $\dot{y}=A_{v} \cos \left(B_{v} \bar{\Omega} t+C_{v}\right) \mathrm{mms}^{-1}$ |
| Torsional vibrations | $\Omega(t)=0.7 \mathrm{rads}^{-1} @ 1 / 2 x$ orders |
|  | $\Omega(t)=1.4 \mathrm{rads}^{-1,2 x \text { orders }}$ |

## Table 5.4 - Vibration parameters used in simulation

The terms $A_{x}$ and $A_{y}$ reported in table 5.3 are the amplitudes chosen for the x - and y radial vibrations applied to the rotor, $B_{x}$ and $B_{y}$ are the respective orders while $C_{x}$ and $C_{y}$ are the initial phases. Table 5.5 reports the realistic ranges of misalignments chosen for this simulation.

The amplitude and phase errors are automatically calculated by the simulator, which has been modified to run a huge number of scenarios. These errors are still RMS values and are defined as:

- Amplitude Error (\%) $=[$ RMS (Resolved - Genuine) $/$ RMS (Genuine) $] \times 100$
- Phase Error (\%) $=[$ RMS (Resolved - Genuine)/ $2 \pi] \times 100$

| Device | Misalignments | Terminology | Range | पstep | N. steps |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -LDV | Translational | $y_{0}$ | $\pm 1 \mathrm{~mm}$ | 0.5 mm | 5 |
|  | Angular | $\varepsilon_{x}$ | $\pm 0.5^{\circ}$ | $0.5^{\circ}$ | 3 |
|  |  | $\delta_{x}$ | $\pm 0.5^{\circ}$ | $0.5^{\circ}$ | 3 |
|  | Pseudo-random noise |  | $0.2 \mathrm{~mm} / \mathrm{s}$ | $0.1 \mathrm{~mm} / \mathrm{s}$ | 3 |
|  | Electronic noise |  | $0.2 \mathrm{rad} / \mathrm{s}$ | $0.1 \mathrm{rad} / \mathrm{s}$ | 3 |
|  | Translational | $x_{0}$ | $\pm 1 \mathrm{~mm}$ | 0.5 mm | 5 |
|  | Angular | $\varepsilon_{y}$ | $\pm 0.5^{\circ}$ | $0.5^{\circ}$ | 3 |
|  |  | $\delta_{y}$ | $\pm 0.5^{\circ}$ | $0.5^{\circ}$ | 3 |
|  | Pseudo-random noise |  | $0.2 \mathrm{~mm} / \mathrm{s}$ | $0.1 \mathrm{~mm} / \mathrm{s}$ | 3 |
|  | Electronic noise |  | $0.2 \mathrm{rad} / \mathrm{s}$ | $0.1 \mathrm{rad} / \mathrm{s}$ | 3 |
| LTV | Speed error (\%) |  | $\pm 1 \%$ | $1 \%$ | 3 |

Table 5.5 - Misalignments used in simulation 2

Simulation 2 considers a rotor subject to single frequency radial vibrations of amplitudes, $A_{x}$ and $A_{y}, 3 \mathrm{~mm} / \mathrm{s}$ or $30 \mathrm{~mm} / \mathrm{s}$. The vibrations are combined to create scenarios characterized by amplitude ratios, $A_{x} / A_{y}$, equal to $0.1,1$ and 10 . For each amplitude ratio, the initial phase of the y - vibration, $C_{y}$, steps through values of $0^{\circ}, 45^{\circ}$, $90^{\circ}, 135^{\circ}, 180^{\circ}, 225^{\circ}, 270^{\circ}$, and $315^{\circ}$ while that of the x - vibration is always set to $0^{\circ}$. In this way, 24 different vibrational cases are analysed. Each simulation incorporates harmonic radial vibrations of the same frequency, $B_{x}=B_{y}$ in the range from $0.1 \overline{\Omega_{T}}$ to $10 \overline{\Omega_{T}}$. This frequency range from $0.1 \overline{\Omega_{T}}$ to $1.5 \overline{\Omega_{T}}$ has been divided in steps of $0.1 \overline{\Omega_{T}}$ while the range from $1.5 \overline{\Omega_{T}}$ to $10 \overline{\Omega_{T}}$ has been divided in steps of $0.5 \overline{\Omega_{T}}$. For each frequency, the errors are calculated considering over 16000 different misaligned situations.

### 5.3.1 -Vibrations characterized by equal amplitudes

The first set of simulations considers radial vibrations with the same amplitudes, frequencies and phases. Figure 5.13a shows a typical x- velocity spectrum of the radial vibrations. The corresponding y- radial vibration spectrum has the same form. The peak at $0.1 \overline{\Omega_{T}}$ is the first point of these simulations. Figure 5.13 b shows a typical laser vibrometer output directed along the x - axis obtained in the presence of x - and y - radial vibrations, torsional vibrations, misalignments and noises. The sum and difference sidebands seen are due to the combination of y-radial vibration and torsional vibrations as indicated by equations (5.4\&5.5).

The uncertainties found are shown in figures 5.14 a and 5.14 b. From figure 5.14 a , it is evident that the amplitude errors determined for the resolved x - radial are very similar to those for the y - radial vibrations in the entire frequency range analysed. This result could be predicted having used the same range of misalignments for both the vibrometers and equal amplitudes for radial vibrations. Larger errors are visible at the $2^{\text {nd }}, 3^{\text {rd }}$ and $4^{\text {th }}$ orders with total values of $8.5 \%, 6 \%$ and $5 \%$ while the uncertainties affecting other components are smaller than $2 \%$. The important uncertainties detected at the $2^{\text {nd }}$ and $4^{\text {th }}$ orders are associated with the torsional oscillations and speckle noise used in this simulation. In the absence of torsional oscillations these uncertainties should be smaller. This figure indicates that, for this application, measurements can be affected by high levels of uncertainty, bigger than $5 \%$, in the frequency range around synchronous frequency, from 0.7 x to 1.2 x , and for the first 4 order components and negligible for the rest of the frequency range investigated.

From the data, it is possible to associate the $x$-and $y$ - resolved vibrations an average ratio of errors of $3.02 \%$ from 0.1 x to 0.7 x , an average ratio of errors of $3.75 \%$ for the range between 1.2 x to 4 x and an average ratio of $1.05 \%$ for the orders bigger than 4 x . In this case, it is possible to associate.

Figure 5.14 a shows the errors in resolved outputs but also the way that the weighting function $W(\omega)$ influences these errors around the synchronous frequency. Errors increase significantly around the condition of synchronous vibration with an infinite
value at 1 x due to $W(\omega)$. For this reason, the points at 0.9 x and 1.1 x have been eliminated in figure 5.14a.

For the phase errors, figure 5.14 b indicates errors for resolved x - radial and y - radial vibrations with levels close to or less than $1 \%$ across the frequency range investigated except close to 1 x where the error is around $1.5 \%$. The phase errors in each resolved output are different while the amplitude errors are almost the same. Larger errors are still at integer multiples due to torsional vibrations and speckle noise. In this case the average error for the x - and y - resolved phase is around $0.5 \%$ for the entire order range.

### 5.3.2-Vibrations characterized by amplitudes with a ratio of 0.1

The second set of simulations considers an $x$ - radial vibration with amplitude of $3 \mathrm{~mm} / \mathrm{s}$ and a $y$ - radial vibration with amplitude of $30 \mathrm{~mm} / \mathrm{s}$, both with the same frequency. Figure 5.15 a and 5.15 b show errors in resolved outputs along the x - and y - directions. In the $x$ - resolved output from 0.1 x to 0.7 x the uncertainty are between $4 \%$ and $11 \%$. For frequencies between 1.2 x and 4 x the maximum uncertainties are detected at the $2^{\text {nd }}$ and the $3^{\text {rd }}$ velocity components with values bigger than $6 \%$. After the $4^{\text {th }}$ order component the levels of uncertainty assume values smaller than $5 \%$.

In the $y$ - radial resolved output the uncertainty shows similar behaviour to that found for the x -direction but, in this case, the values are smaller. From 0.1 x to 0.7 x , the errors go from $2 \%$ to $7 \%$. From $1.2 x$ to $4 x$ the uncertainties decrease from $3 \%$ to $1 \%$ while after the $4^{\text {th }}$ order component the values remain smaller than $1 \%$ for the entire remaining frequency range. In this case, average ratio of errors for orders from 0.1 x to 0.7 x is around $4.37 \%$, of $8 \%$ for the range between 1.2 x to 4 x and $4 \%$ for orders bigger than $4 x$.

The phase errors are shown in figure 5.15 b. For the errors in the x -direction it is possible to associate an average ratio error of $0.72 \%$ from 0.1 x to 0.7 x , of $2.33 \%$ from 1.2 x to 4 x and of $0.75 \%$ for order bigger than 4 x . For the errors in the y -resolved
vibration, instead, the average ratio is around $0.5 \%$ from the entire range of orders investigated.

### 5.3.3 -Vibrations characterized by amplitudes with a ratio of 10

The third set of simulations performed for simulation 2 considers the same rotor and x and $y$ - radial vibrations with amplitudes of $30 \mathrm{~mm} / \mathrm{s}$ and $3 \mathrm{~mm} / \mathrm{s}$. The errors in resolved outputs are shown in figures 5.16a and 5.16b and indicate that the trend for the x resolved output is similar to that detected for the $y$ - resolved output in the simulations in the previous sub-section. This result was predictable because similar misalignments and noises were chosen for both the vibrometers and similar vibrations were applied along the x - and the y -axis.

Figures 5.14-16 are the first attempt to estimate the level of uncertainties expected in resolved radial vibration measurements. The study indicates that the percentage error in the resolved outputs depends on the ratio of vibration amplitudes that estimates of vibration amplitudes can be combined with typical misalignments to estimate likely errors, and that likely errors are generally of an acceptable level outside a range of $1 \pm 0.5$ order. The level of error detected at the $2^{\text {nd }}$ and $4^{\text {th }}$ order is linked to the torsional vibrations used in these simulations.

### 5.3.4 - Further simulations with varying initial phase

Further simulations have been performed considering different initial phases for the three amplitude ratios and the full range of misalignments. The overall RMS errors for the 8 initial phase differences differ little from those presented in figure 5.14-16. For this reason, figures 5.17 and 5.18 show the overall errors quantified for the resolved x and $y$ - resolved output for the ratios 0.1 and 1 . These errors represent the averages of the various amplitude and phase uncertainties obtained for the 8 different series of simulations defined in section 5.3 mathematically obtained as:

- $\sum_{i=1}^{8}\left[(\text { Amplitude } \operatorname{Error}(\%))_{i}\right] / 8$
- $\sum_{i=1}^{8}\left[(\text { Phase Error }(\%))_{i}\right] / 8$

The values of error affecting the resolved amplitudes and phases shown in figure $5.17 \mathrm{a} \& \mathrm{~b}$ and figures $5.18 \mathrm{a} \& \mathrm{~b}$ obtained combined the 8 different series are very close to that reported for single ratio in the previous sections.

## 5.4 - The effects of surface roughness on cross-sensitivity in radial

## vibration measurements

Research has shown how sensitivity appears in radial vibration measurements on rotors with rough surfaces. Recent research has shown how cross-sensitivity does not appear in measurements on rotors with smooth surfaces [5.3]. This part of the research is focused on how surface roughness can affect the magnitude of the cross-sensitivity detected in radial vibration measurements on rotating targets using laser vibrometers. The laser vibrometer supplies a beam with a Gaussian intensity profile. When the beam is normally incident on a flat, smooth surface, the reflected light collected through the aperture of the laser vibrometer has an effective centre that coincides with the centre of the beam intensity distribution (the peak of the Gaussian curve). As surface roughness increases, speckle begins to form until roughness reaches a level comparable with wavelength and a fully developed speckle pattern is formed. In all these cases, the effective centre of the beam (based on light collected) is the same as the geometric centre of the beam.

The situation is not so simple when the target surface has a curvature, like a rotor. In this case, the effective centre of the beam, based on collected light, depends on the curvature and roughness of the target, on target motions and on the direction of the incident beam. How the effective centre of the beam changes in relation to all these parameters is hard to quantify.

A simple representation of how the effective centre of the beam can differ from the actual centre of the beam on a smooth, circular rotor is shown in figure 5.19. When the laser beam is aligned to a smooth rotor, there is the tendency to collect light only from the portion of the incident beam close to and aligned with the line passing through the centre of the rotor, as shown in figure 5.19a. In this case the effective centre of the beam is the same as actual centre of the beam. When the laser beam is not aligned with the centre of the rotor, however, the actual centre of the beam moves away from the line passing through the centre of the rotor but wavelets of the incident beam always from the centre line have the tendency to be reflected away from the collecting aperture. This draws the effective centre of the beam away from its actual centre and back towards the rotor centreline. In the extreme, only the ray passing along the rotor centreline is collected by the aperture regardless of radial target vibration orthogonal to the incident beam and cross-sensitivity is eliminated.

For a rough rotor (or retro-reflective tape), the effective centre of the beam is always the same as the actual centre of the beam because light is scattered from all regions of the illuminating beam and collected by the aperture. This results in cross-sensitivity to radial vibration perpendicular to the laser beam direction as explained in section 1.

Thus, for a laser beam incident on a circular rotor, the effective centre of the beam moves from the line passing through the centre of the rotor to the geometric centre of the beam as surface roughness increases from smooth to rough. To predict mathematically how the effective centre of the beam moves is not currently possible and so the degree of cross-sensitivity encountered cannot be predicted either. The only way to investigate the relationship between cross-sensitivity and surface roughness in radial vibration measurements on rotors is experimentally.

### 5.4.1 - Experimental study

Experimental measurements have been conducted using the arrangement shown in figure 5.2. A rotor with a defined surface roughness is rotated by an electric motor using a flexible belt. The rotor system is mounted on a linear guideway and excited by
an electromagnetic shaker along a single radial direction only. This direction is defined to be the x - axis while the rotor spin axis is defined as the z - axis. The rotor is mounted in bearings that can be opened in order to change the test rotor while maintaining alignment of the laser vibrometers.

The laser vibrometers are positioned along the x - and the y - axes, respectively, reproducing the optical arrangement used to resolve the radial vibrations. With $x$ - radial vibration only, the vibrometer aligned along the $x$ - axis registers the correct measurement under the rotating condition. The vibration in the $y$ - direction is zero but cross-sensitivity causes a velocity term (related to $\Omega x$ see equation (1.12)) to be detected at the vibration frequency. The laser vibrometers used are Polytec OFV400 Rotational Vibrometers ( 2 parallel beams) with one of the beams capped to configure the instruments for translational vibration measurements. The beam diameter is around $700 \mu \mathrm{~m}$ at the stand-off distance of 0.7 m used in these experiments.

On the y- laser vibrometer was mounted the optical device shown in figure 5.6 which enables better alignment with the rotating target. The $y$ - laser vibrometer is aligned such that translation of the second mirror in the device moves the beam in the x - direction, as shown in figure 5.20. In this way, measurements can be made at different points located on the rotor circumference.

The test rotors are steel cylinders of diameter 3 cm , length 10 cm and surfaces with roughness ( Ra ) in a range between very smooth $(9 \mathrm{~nm})$ and rough $(\approx 1 \mu \mathrm{~m})$. In addition to the surface roughness, the roundness values of the test rotors are also known although a control of the roundness in the area where the measurements were taken was not possible. Table 5.6 reports the values of surface roughness and roundness for the test rotors.

| Sample | Roughness (nm) | Roundness ( $\boldsymbol{\mu m}$ ) |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Long end ( $\boldsymbol{\mu m}$ ) | Short end ( $\boldsymbol{\mu m}$ ) | Average ( $\boldsymbol{\mu m}$ ) |
| 1 | 9 | 7.14 | 6.48 | 6.81 |
| 2 | 12 | 5.07 | 4.84 | 4.93 |
| 3 | 65 | 27.35 | 22.25 | 24.8 |
| 4 | 87 | 11.22 | 14.34 | 12.7 |
| 5 | 263 | 1.67 | 1.02 | 1.34 |
| 6 | 431 | 3.69 | 1.96 | 2.82 |
| 7 | 903 | 5.25 | 8.16 | 6.70 |
| 8 | Reflective tape |  |  |  |

Table 5.6 - Roughness and roundness of the test cylinders

The procedure used in these tests is in two steps. The first regards identification of the point illuminated for alignment of the laser beam through the centre of the rotor. During the initial measurements, it was noticed that, when moving the $y$ - laser vibrometer along the x - direction, the magnitudes of the cross-sensitivity increased for illuminated points away from a line through the centre of the rotor with a minimum for the illuminated point closest to a line through the rotor centre.

To investigate how the cross-sensitivity changes in relation to the surface roughness, the alignment from which the lowest cross-sensitivity was detected has been assumed to be that which is closest to a line passing through the centre of the rotor and, from this point, 6 other measurements were taken moving the $y$ - laser vibrometer left and right along the x - direction, in steps of 0.25 mm .

Each rotor was excited at three different frequencies, $15 \mathrm{~Hz}, 30 \mathrm{~Hz}$ and 50 Hz , and rotated at 20 Hz . These frequencies were chosen deliberately to avoid speckle noise. For each excitation frequency, three measurements were taken at each location for each test rotor. In each case, the ratio of the measured $y$ - vibration to the value $\Omega x$ associated with full cross-sensitivity was calculated as:
$R=\frac{\left(\tilde{U}_{y}\right)_{f=f_{V}}}{\frac{\Omega}{w_{V}}\left(U_{x}\right)_{f=f_{V}}}$

This ratio $R$ assumes values between zero, for no cross-sensitivity, and 1 for full crosssensitivity.

### 5.4.2 - Consideration of the form of the intensity patterns

Figure 5.21 shows typical backscattered intensity patterns for different surface roughness/treatment. The intensity profile produced by a very smooth rotor comprises specular reflection forming a single narrow line. If the reflection is not centred on the vibrometer aperture and the extent of the reflection is greater than the aperture, light is collected only from a limited part of the incident beam. In this case, the effective centre of the beam is not coincident with the geometric centre of the beam. During target rotation, the intensity pattern moves noticeably and rapidly up and down on the second reflecting mirror of the vibrometer alignment optics. This motion is due principally to the vibration of the rotor. This means that during the measurement the light collected by the laser vibrometer originates from different areas within this specular reflection, at different times in the vibration cycle. If the reflected light moves too far from the centre of the aperture, poor measurements can result. Out-of roundness can also modify the direction of the reflected light and result in poor measurements.

Figure 5.22a shows the output from the laser vibrometer directed along the $y$ - axis taken on a point away from the best measurement point on a smooth rotor (bad alignment). The figure shows many signal drop-outs.

With increasing surface roughness, a partially developed speckle pattern appears in combination with a less distinct specular reflection. A fully developed speckle pattern appears in the presence of optically rough rotors. Collecting light from a fully developed speckle pattern is equivalent to collecting light from all points of the incident beam. During target rotation, it was noticed that the intensity envelope of the speckle
pattern produced by a rough surface maintained its position centred on the collecting aperture which means that the effective centre of the beam is close to the geometric centre of the beam. Moreover, the resulting signal amplitude was typically higher than in measurements on smooth surfaces because adequate light intensity was collected at all times preventing drop-outs. Figure 5.22b shows the y- laser vibrometer signal output taken on a rough rotor at a measurement point different from that of the best alignment and shows no drop-outs, in contrast to figure 5.22a for the smooth surface. For a good alignment, it was always possible to have good signal for a rough surface. For smooth rotors, however, the alignment was often critical and complicated by the vibration added to the rotor which causes the reflected beam to move away from the collecting aperture.

### 5.4.3 - Experimental data

The perpendicular distance between the effective centre of the beam and the line passing through the centre of the rotor affects the magnitude of the detected crosssensitivity. Figures 5.23 a and 5.23 b show the time and the frequency domain for the $y$ measurement taken at the point with the lowest cross-sensitivity on a rotor with surface roughness $\mathrm{Ra}=9 \mathrm{~nm}$ which rotates at 20 Hz and vibrates at 15 Hz along the x - axis. Figures 5.24 a and 5.24 b show the same measurement taken at a point situated at a distance of 0.25 mm from the previous point under the same rotor conditions. The figures indicate a difference in amplitude and shape between the two output signals. For the incident point considered as that of best alignment, the signal range is around $\pm$ $2 \mathrm{~mm} / \mathrm{s}$ while for the other point, drop-outs are prominent. The magnitudes of the crosssensitivities expressed as a ratio and detected at 15 Hz are respectively 0.06 and 0.56 . Figures 5.25 a and 5.25 b show the time and frequency domain for the measurement taken along the x - axis where the genuine vibration measures $10.8 \mathrm{~mm} / \mathrm{s}$.

Figure 5.26a shows the ratios calculated for the cylinders of varying roughness with vibration at 15 Hz . The figure shows how magnitudes of the cross-sensitivity progress with changing roughness. When the roughness is $\mathrm{Ra}=9 \mathrm{~nm}$, the magnitudes of the cross-
sensitivity are smaller than 0.01 around the position of best alignment but then increase rapidly moving from this point.

The ratios estimated for the rotor with roughness of $\mathrm{Ra}=12 \mathrm{~nm}$ show a behaviour similar to that of the smoothest rotor. The same can be said for the rotor with roughness of $\mathrm{Ra}=65 \mathrm{~nm}$ although the curve seems less flat. As surface roughness increases further, the measured cross-sensitivity magnitudes become bigger. For Ra between $0.1 \mu \mathrm{~m}$ and $0.5 \mu \mathrm{~m}$, the measurements indicate ratios between 0.1 and 0.5 with more variability away from optimum alignment than for the very smooth cylinders. The ratios seem smaller for alignment through the centre of the rotor because of the influence of specular reflection. Away from this alignment, scatter is more diffuse and ratios increase but in both cases it is difficult to predict the values of the cross-sensitivity and no mathematical models can help to define these uncertainties.

The final test was conducted on a rotor coated with retro-reflective tape. In this case, the ratios are close to 1 and the different positions tested on the rotor circumference gave similar values. In this case, it is not essential to perform a perfect alignment because the various locations will give more or less similar results for the crosssensitivities.

When the rotors are excited with frequency of 30 Hz and then of 50 Hz , the ratios found are very similar to those just analysed. Figure 5.26 b reports the data for vibration at 30 Hz , while figure 5.26 c shows data for vibration at 50 Hz . All measurements were made for vibrations which similar amplitudes, around $12 \mathrm{~mm} / \mathrm{s}$.

The data presented demonstrate the merit in making Laser vibrometer measurements of radial vibration from polished circular rotor surfaces because the cross-sensitivity exhibited is negligible for roughness up to 65 nm . Recommendations are:

- when measurements are taken on a very circular, polished rotor ( $\mathrm{Ra} \leq 65 \mathrm{~nm}$ ), the cross-sensitivities are negligible (ratio close to zero) and measurements can be made
without the need for resolution. In this case, however, it is necessary to assure careful alignment between laser beams and rotor to maintain signal quality;
- when measurements are taken on surfaces coated in retro-reflective tape, the ratio is 1 and reliable. Post-processing of the measurements with the mathematical algorithms cited in chapter 1 . Alignment through the centre of the rotor is necessary to minimise errors in resolved outputs as indicated in section 5.3;
- in other cases, including surfaces with roughness around $1 \mu \mathrm{~m}$ which should generate fully diffuse scatter, an unpredictable but significant level of crosssensitivity is encountered and reliable measurements cannot be made.


## Chapter 6

## Discussion and conclusions

## 6.1 - Summary

This thesis has investigated two main areas of LDV applications. The first one has presented theoretical and experimental work to model optical arrangements of relevance in scanning, steering and tracking LDV applications. The second has investigated the uncertainties affecting radial vibration measurements on rotating components. In both areas, misalignments are of central importance and the resulting effects have been analysed and quantified.

## 6.2 - A new mathematical procedure to model LDV performance

This project has presented a novel mathematical procedure to model LDV systems. The method has been formulated with scanning LDV systems in mind but can be used to model any optical arrangement. The technique uses a vector expression of Snell's Law and rotation matrices to determine the orientation of a deflected laser beam, uses vector positions and surface normals modified by rotation matrices to incorporate the inevitable misalignments affecting optical devices, traces beam paths using vector polygons and integrates directly with the Velocity Sensitivity Model to predict the measured velocity. The velocity measured results from the combination of Doppler shifts at deflecting optics as well as at the target. The technique enables, for the first time, quantification of the effect of the scanning head itself on the measured velocity.

The analysis of misalignment effects enables identification of the devices with the most critical alignment. Sensitivity to different target vibration components, both desired and undesired, can also be made. Such information helps to optimise alignment efforts and interpret measured data correctly.

The first system analysed with the new approach was an arrangement incorporating a steering mirror. The equation for measured velocity in the presence of vibrations of the mirror and target vibration was formulated. The equation expresses the measured velocity as a combination of the dynamic behaviours of the vibrating devices, the target and the mirror, with misalignments between the devices seen to introduce uncertainties in the measured velocity. Experiments confirmed the theoretical predictions and showed that measurements can be corrected for vibration of a steering mirror

The method was next applied to a novel scanning arrangement based on a rotating wedge. Rotating the wedge in the same direction and at the same rotation frequency as the target the scan pattern results in a circular beam path on the target suitable for tracking applications. The derived model shows the algebraic complexity of a mathematical description of the system but describes completely and without any kind of approximation the scan pattern and the velocity measured in presence/absence of misalignments and target vibrations. The model is easily implemented in Matlab to run simulations.

In the absence of misalignments, vibration measurements made on a rotating but not vibrating target indicate no measured velocities. Misalignments result in Doppler shifts at the wedge surfaces as well as the target due to its rotation and can introduce additional harmonics and a DC term in the measured velocity. The level of uncertainty introduced depends on how the misalignments are combined. Experiments made on a test rig developed for this project have confirmed the theoretical predictions in the presence of known misalignments added to the laser head and scanning wedge.

### 6.2.1 - Results from the SLDV systems

Chapter 3 analysed three different scanning LDV systems suitable for scanning and tracking applications on rotating targets. For each system, mathematical models were formulated with the new procedure. The main effects of misalignments are to introduce additional low order harmonics and a DC term in the measured velocity. The level of uncertainty affecting typical applications on a rotating but not vibrating target has been determined from different scenarios characterized by realistic combined misalignments.

Initially, the well-known dual mirror SLDV system was modelled. The analysis showed the presence of the 2 x velocity term always detected in circular tracking applications. Simulations made of circular tracking applications on a rotating target have supplied similar results to those found in literature and confirmed the validity of the new model. The results indicate the 1 x velocity component as the greatest uncertainty affecting the system. Simulations made considering different scenarios in the presence of combined misalignments indicate for the 1 x velocity term a magnitude around $1.38 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$ for applications with a circular scan pattern of radius 5 cm . Smaller values were detected for the DC and the 2 x velocity terms. An analysis of single misalignments indicates that all the optical devices have critical alignments and have effects on the 1 x velocity term.

Although the sensitivity of the system to different target vibrations depends on configuration, investigations showed a value over $99.8 \%$ to whole-body out-of-plane vibrations and values around $2 \%$ for whole-body in-plane target vibrations. In the frequency spectrum, any harmonic out-of-plane vibration provides a single peak at vibration frequency while in-plane vibrations appear as sidebands at ${ }^{ \pm} \square_{T}$ around the genuine vibration frequency with similar amplitudes.

The second system analysed in chapter 3 was a novel dual-wedge SLDV system. The system incorporates two rotating wedges to deflect a laser beam towards the target. The system can be used for vibration measurements on a rotating target because of its flexibility to describe different scan patterns but it can make other scan patterns suitable for rotating and not-rotating targets. The system was analysed for the first time as an
alternative solution to the dual mirror SLDV. The formulated mathematical model indicates the complexity of the system but described completely the scan pattern, the measured velocity and the misalignment effects without any approximation.

Investigations were concentrated on circular tracking applications on a rotating target in presence/absence of vibrations and misalignments. The simulated data showed that, for a zero misalignment configuration and in the absence of target vibrations, no velocities are measured. Misalignments are responsible for Doppler shifts at the wedge surfaces and target introducing a DC term and additional harmonics in the measured velocity. Simulation of various scenarios characterized by realistic misalignments showed the 1 x velocity term as the greatest uncertainty expected in measurements with both wedges as the devices with the most critical alignment. The sensitivity to whole-body out-of-plane target vibration is around $99.8 \%$ while the sensitivity is around $2 \%$ for whole-body inplane vibrations for the investigated case. The sensitivity depends on the geometry of the system and, in particular, on the initial value of the wedge angular position $\Delta \varphi$ which defines the dimension of the scan radius. Harmonic in-plane vibrations are characterized by a pair of sidebands with almost similar amplitudes while any harmonic out-of-plane vibration appears as a single peak at the vibration frequency in the measured velocity spectrum.

Experimental tests made to quantify the uncertainties in the presence of known misalignments added to the optical devices have indicated a good correspondence with theoretical results. During the tests, uncertainties due to known misalignments added to the laser head-wedges were quantified for measurements made on a stationary target and a rotating disk with and without vibrations. The tests confirmed the 1 x velocity term as the largest uncertainty affecting the system in the presence of misalignments and confirmed the presence of sidebands in the presence of in-plane target vibrations. The test rig for the single/dual wedge SLDV realized for this project is a prototype and works only for circular tracking applications. Further development needs to reach high rotational speeds and/or to trace scan patterns different from the circular scan. The
flexibility of the system makes this arrangement one of the best alternative to the dual mirror SLDV system.

The third system analysed in chapter 3 was the Dove prism scanning system. The arrangement is based on a rotating Dove prism to deflect the beam towards the target. The analysis suggests circular tracking applications can be made with the system only when the prism is rotated at half the target rotational speed. A circular scan pattern composed of two distinct circles is traced on the target but because the distance between the two circles is negligible compared to the scan pattern dimension, it is possible to consider the scan pattern as a single circle. The initial alignment of the laser beam controls the scan radius. Suitable alignments are translations or rotations of the laser head along or around the x - and y - axes such that the initial beam path deviates from the optical axis of the system. The choice of the alignment affects not only the dimensions of the scan pattern but also the measured velocity and the sensitivity to target vibrations.

When the system is aligned by translating the laser head along the $x$ - and $y$-axes, there is sensitivity to out-of-plane target vibrations but not to in-plane vibrations. Configurations aligned by rotating the laser head around the x - and y - axes shows a single peak at the vibration frequency associated to the out-of-plane vibration with a sensitivity around $99.9 \%$ and sidebands linked to the in-plane vibrations with sensitivity around $2 \%$.

The model formulated for the system describes completely the optical arrangement in terms of scan pattern, measured velocity and beam path including misalignment effects. Simulations made on different scenarios characterized by combined misalignments indicated a DC and 1 x velocity term as the main uncertainties for the system. The 1 x velocity term was the largest uncertainty found in typical applications, with values around $1.01 \mathrm{e}-1 \mathrm{~mm} / \mathrm{s} / \mathrm{rad} / \mathrm{s}$, while the translational and angular alignment of the prism is critical for the system. An optical scanning system based on the Dove prism configuration is already commercially available and used for circular tracking applications on high speed rotating components.

Comparison of the levels of uncertainty due to misalignments detected for the three scanning LDV systems suggested that, in terms of 1 x velocity term, the systems can be considered as almost similar although for typical, realistic misalignments the values predicted for the dual mirror SLDV are the biggest. While for the dual mirror SLDV all the optical devices are critical for the alignment, for the dual wedge and the Dove prism the alignment is critical for the deflecting optics.

The sensitivity to the out-of-plane target vibrations are almost similar and depend on the geometry chosen for the various systems. Anyway, all the systems indicate a very small sensitivity to in-plane target vibration and this is a negative aspect because the arrangements have been developed to measure out-of-plane target vibrations.

### 6.2.2 - Results from the self-tracking LDV systems

In chapter 4, the self-tracking LDV systems of Lomenzo and Sever were analysed using the new approach. Self-tracking LDV prototypes were developed by their inventors but no commercial systems were realised. The self-tracking systems are very useful in applications with target of big radial dimensions, such as a turbine. Simulations were made to quantify the effects of misalignments in circular tracking applications on rotating targets in the presence/absence of target vibrations.

In the absence of misalignments, neither system measures any velocity on a rotating target without vibration. Additional terms appear in the measured velocity spectrum in the presence of misalignments. For both systems, the 1 x velocity term is the main uncertainty introduced by misalignments which also causes displacements of the scan pattern in the target plane. The data show that for the configuration proposed by Lomenzo the alignment of the fold mirror is the most critical while for the arrangement proposed by Sever the vertex mirror is the critical component.

The models derived for the arrangements are similar but the performances show different results. In the presence of harmonic in-plane and out-of plane target vibrations and a z - vibration due to the target flexibility, the Lomenzo system perfectly aligned
shows sensitivity around $98.6 \%$ to the out-of-plane vibration due to the target flexibility, is sensitive to the in-plane target vibrations due to the target flexibility, shows a sensitivity around $297 \%$ to the whole-body out-of-plane target vibration and is insensitive to the whole-body in-plane target vibrations. This high value found for the target vibration can be explained considering that the target and the vertex mirror are attached and that the relative Doppler shifts produced at the vertex mirror and the target are combined. The system, moreover, results insensitive to in-plane vibrations.

In the presence of inevitable misalignments, the direction of the incident beam changes. For an angular misalignment added to the laser head, the system indicates that sensitivity to the out-of-plane target vibrations does not change but many smaller sidebands are introduced. The harmonic in-plane flexible vibration, instead, produces a single peak of magnitude $0.34 \%$ of the genuine vibration.

Adding the same target vibrations to a Sever system perfectly aligned, it results that the whole-body out-of-plane vibration produces a single peak of magnitude around $200 \%$ of the genuine vibration due to the fact that the target and the vertex mirror are attached. The whole-body in-plane vibration results in two sidebands with sensitivity around $50 \%$ of the genuine vibration. The out-of-plane vibration due to the target flexibility results in a single peak of amplitude equal to the genuine vibration. The system results insensitive to the in-plane target vibrations due to the target flexibility.

The presence of an angular misalignment added to the laser head determines the presence of many additional terms. In particular, the sensitivity to whole-body out-ofplane vibration does not change but sidebands of amplitude around $1.75 \%$ of the genuine z -vibration are detected. The sensitivity to the out-of-plane vibration due to the target flexibility does not change. The in-plane vibration, instead, produces a single peak with amplitude around $0.52 \%$ of the genuine vibration plus many sidebands. The biggest sidebands have amplitude around $50 \%$ of the genuine whole-body in-plane vibration. The misalignment determines the presence of a single peak at the frequency
of the flexible in-plane target vibration with sensitivity around $0.7 \%$ of the genuine vibration.

Comparing the level of uncertainty detected for the self-tracking systems to those determined for the other tracking LDV systems, the data indicated that self-tracking systems are more sensitive to misalignments and the presence of many sidebands makes hard the understanding of the vibration measurements.

## 6.3-Investigations of cross-sensitivity in radial and pitch/yaw vibration

## measurements

The second part of the thesis considers the cross-sensitivity associated with radial and pitch/yaw vibration measurements on rotating components. As known from literature, the resolution of individual radial or pitch/yaw vibration components, based on measurements taken directly from a rotor, requires post-processing of two orthogonal measurements combined with a rotation speed measurement.

The chapter 5 concentrated on the effects of the misalignments in the individual radial and pitch/yaw vibration measurements on the final resolved output. An existing simulator developed in Labview to reproduce typical radial vibration measurements using two orthogonal laser beams was modified. The simulator enabled investigation of different scenarios characterized by single or combined sources of error affecting the measurements.

Initial investigations were made to quantify the uncertainties produced by single sources of misalignment in the resolved vibration amplitudes but, in practical applications, it is more realistic to assume combined misalignments. For this reason, simulations combining misalignments and measurement noise in radial vibration measurements on a rotor subject to orthogonal sinusoidal radial excitations were made.

In the presence of radial vibrations with the same phase, amplitude and frequency, typical uncertainties around the $1^{\text {st }}$ order assumed very high values. This behaviour was expected and indicates that synchronous vibration measurements cannot be made.

Uncertainties between $8.5 \%$ and $5 \%$ were found in the resolved amplitudes at the $2^{\text {nd }}$, the $3^{\text {rd }}$ and the $4^{\text {th }}$ order. The $2^{\text {nd }}$ and the $4^{\text {th }}$ orders are associated with the torsional oscillations and speckle noise used in the simulations. Beyond $3{ }^{\text {rd }}$ order, uncertainties are less than $2 \%$. The data an average ratio of errors of $3.02 \%$ from 0.1 x to 0.7 x , an average ratio of errors of $3.75 \%$ for the range between $1.2 x$ to $4 x$ and an average ratio of $1.05 \%$ for the orders bigger than 4 x . In this case, it is possible to associate.

For the resolved phases the error was around $1 \%$ for the entire frequency range tested (up to $10^{\text {th }}$ order) with the exception of the uncertainties found around the $1^{\text {st }}$ order component that assumed high values. The data suggest that for this scenario no data from the resolved measurements have to be neglected, with the exception of resolved components around the $1^{\text {st }}$ order.

In the presence of radial vibrations with the same frequency but different amplitudes, amplitude errors above $7 \%$ appear across the first 3 orders in the resolved radial vibration with the smaller amplitude. A similar behaviour was found in the resolved phases with errors above $1.5 \%$ for the first 3 orders.

The second part of the chapter concentrated on the relationship between surface roughness and the cross-sensitivity encountered in radial vibration measurements on rotors. Experiments showed that the magnitude of the cross-sensitivity in radial vibration measurements is affected by the surface roughness of the tested rotor. When radial vibration measurements are taken directly from a rotor, the effective centre of the beam, based on collected light, depends on the shape and roughness of the target, on target motions and on the direction of the incident beam. Unfortunately, how the effective centre of the beam changes in relation to all these parameters is hard to quantify. Theoretically, for an aligned laser beam the effective centre of the beam
moves from a line passing through the centre of the rotor for a smooth, circular rotor to the geometric centre of the beam for a rough rotor with arbitrary cross-section.

For this reason, experimental tests were made on rotors with roughness ( Ra ) in a range between very smooth ( 9 nm ) and very rough $(1 \mu \mathrm{~m})$ and covered by a retro-reflective tape excited along a radial direction (the x - axis) and measured along the x - and the y axes. Measurements have been used to determine the ratio between the measured yvibrations to the value of $\Omega \mathrm{x}$ associated with full cross-sensitivity.

Tests have considered three different vibration frequencies and the effect of offset along the x - direction of the laser beam for the y -vibration measurement. The data showed that the cross-sensitivity is very small for smooth rotor surfaces but increases with surface roughness or when retro-reflective tape is attached at the target. Frequency of vibration made no difference to cross-sensitivities.

The tests suggested that for a very circular, polished rotor the cross-sensitivity was negligible and measurements could be quantified without the need for additional processing of the LDV outputs. For surface coated in retro-reflective tape or rough rotors, the level of cross-sensitivity was close to 1 . Measurements can be made but postprocessing of the LDV outputs is required. When the rotor surface roughness assume values in between the two extremities, the level of the detected cross-sensitivities depends on many factors such as the rotor roughness, the roundness of the target, the presence of unknown offsets or misalignments, the amplitude of the radial excitation (which can result in drop-outs) and it is unpredictable. In these cases, reliable measurements cannot be made.

## 6.4 - Conclusions

The first main objective of this study was to develop a novel way to model any optical LDV systems suitable for vibration measurements on rotating and not rotating targets which can also incorporate a full list of misalignments. This task was achieved by proposing a new procedure that introduces geometrical misalignments as position
vectors and rotational matrices to determine the direction of the incident laser beam. The models derived integrate completely with the Velocity Sensitivity Model to predict the velocity measured by the single beam. The novel framework was successfully applied to various LDV optical schemes and investigations led to the following conclusions:

- For the steering mirror scanning system, the velocity measured by the LDV system is a combination of the vibrating devices, target and deflecting mirror. Experimental results confirm the theoretical predictions and show that the LDV vibration measurement can be corrected measuring directly the vibrations of the steering mirror.
- New models were developed for the dual mirror, the Dove prism and the selftracking SLDV systems. The models describe completely the scan patterns and the measured velocity spectrum in the presence of single /combined misalignments and target vibrations without any kind of approximation. For circular tracking applications on rotating targets, investigations show additional velocity terms at the target rotational order and harmonics in the measured velocity spectrum as main effects of misalignments. These velocity terms can be reduced with a careful alignment of the optical devices. Misalignments introduce also sidebands at $\pm \Omega_{\mathrm{T}}$ around the genuine in-plane target vibration frequency and reduce the sensitivity of the system to out-of-plane target vibrations. All these information can be used by the user to realize better alignments and interpret the measured data. The successful application of the procedure to various LDV systems and investigation of sensitivity to misalignments and target vibrations fulfil one of the main objectives of the project and demonstrate the value of the work.
- Optical systems incorporating a single or two rotating wedges have been investigated for the first time as possible scanning LDV systems suitable for tracking applications on rotating targets. Mathematical models able to describe completely the systems without any approximation were developed. Investigations indicate the misalignment of the wedges as the major source of uncertainty an additional velocity term at 1 x is the biggest uncertainty in the measured velocity spectrum. This velocity term can be reduced through a careful alignment. Prototypes
of the systems were expressly developed for this work and used to validate theoretical predictions of misalignment effects. Experimental and theoretical results show a good correspondence for the estimated Doppler shifts generated by the rotating wedges and for the velocity measurements made on a rotating target in the presence of known misalignments. The complete analysis indicates the dual wedge SLDV as a valid alternative to the dual mirror SLDV system for vibration measurements on rotating targets.

The second main objective of this study was to investigate the effects of combined realistic misalignments and surface roughness on the accuracy of the radial vibration measurements directly from rotors using LDV. Simulations showed as the level of uncertainty increases around the condition of synchronous frequency reducing the estimation of the measured velocity around this range. Investigations led to the following conclusions:

- For radial vibrations with different frequencies and amplitudes, single translational misalignments result in uncertainties at the genuine vibrations and in an increment of the sensitivity to orthogonal vibrations.
- For radial vibrations with different frequencies and amplitudes angular misalignments can generate an increment of the sensitivity to the axial vibration which results in additional velocity terms.
- For radial vibrations with similar frequencies and amplitudes, combined misalignments generate similar uncertainties along the measuring directions. Investigations indicate an average uncertainty around $3-3.75 \%$ in the resolved amplitudes from order components between 0.1 x to 4 x and uncertainties less than $1.05 \%$ for higher orders. Average uncertainties around $1 \%$ are detected for the resolved phase for the entire frequency range tested (up to 10th order).
- In the presence of radial vibrations with the same frequencies but different amplitudes, combined misalignments generate different uncertainties. The ratio between the uncertainties is proportional to the ratio between the amplitudes of the chosen radial vibration. Investigations show an average uncertainty for the amplitude and the phase above $7 \%$ and $1.5 \%$ across the first three orders in the
resolved radial vibration components with the smaller amplitude. For the vibration component with bigger amplitude, the amplitude and the phase errors are smaller than $1 \%$ for the entire frequency range tested.
- Experimental investigations made to investigate surface roughness effects on the cross-sensitivity show that direct measurements can be made without additional processing of the LDV outputs only on very circular, polished ( $\mathrm{Ra}<65 \mathrm{~nm}$ ) rotors because the cross-sensitivity encountered is negligible. For rough ( $\mathrm{Ra}>900 \mathrm{~nm}$ ) rotors or those coated in retro-reflective tape, however, post-processing of LDV outputs is necessary. For rotors with surface roughness with values between the two extremes, the level of the cross-sensitivity depends on many factors and is unpredictable such that reliable measurements cannot be made.


## 6.5-Recommendations for future work

The overall project was very extensive but both main objectives were achieved. Nevertheless, some aspects can be studied further and improved.

In chapter 2, the mathematical model formulated for the steering mirror was validated by experimental tests made on a steering mirror that is vibrated along a single direction. In practice the arrangement can also be used to realize a line scan rotating the mirror around one axis, as for the dual mirror SLDV system. In this case, further theoretical predictions and experiments should be made to validate the model and the procedure proposed to correct the vibration measurements in the presence of angular oscillations. In this way, the analysis of this optical arrangement which can be useful for many applications can be completed.

For the single and dual wedge SLDV system, further work is necessary to develop the test rig. The investigations reported in chapter 3 show that the wedge SLDV system is an alternative to the dual mirror SLDV system in applications with very high speed. The potential of the dual wedge system for applications with high speed requires demonstration in a test rig which can reach rotational frequency in excess of 50 Hz . At the moment, however the test rig used for the tests was good enough to validate
theoretical predictions but only runs at very low speeds and does not incorporate a precise alignment procedure to minimise the misalignment effects. Now that the components whose alignment is critical can be identified, the new solution should be designed to minimise critical misalignments to include a practical alignment procedure. This might position the laser head and the wedges in one sub-system to enable a good alignment between these devices and reduce the Doppler shifts at the wedges. A possible alignment procedure could be realized in two steps. The first requires alignment between the laser head and the wedges using a stationary retro-reflective panel in front of the target. When the panel is removed the laser head-wedge sub-system can be aligned to the target. In this way, the mathematical description which defines the measured velocity will have only four misalignments: two translational along the x - and $y$ - axes and two angular around the $x$ - and $y$ - axes. Running real experiments and the use of the mathematical model the misalignments can be found and the alignment corrected.

Since the goal of the new design is to achieve high rotational speeds to perform vibration measurements, the new test rig will require a powerful motor. The choice to have the wedges, the motor and also the set of gears to rotate the wedges in the same test rig could result in the presence of noise and additional velocities in the measured velocity. These additional terms can be easily distinguished from the target vibrations or the effects of misalignments but the development of the test rig requires a careful design. Moreover, the realization of scan patterns different from circular, such as the line tracking suitable to measure the dynamic behaviour of a blade turbine, requires that the two wedges are independently rotated and this solution requires a further development on the design and the electronic of the test rig. These goals could be achieved by a next project.

For the Dove prism scanning system, the mathematical model predicts uncertainties due to misalignments. Recently, a commercial scanning system based on a similar arrangement was commercialized. Theoretical prediction of measured velocity circular tracking applications from the mathematical model might be compared to experimental
tests performed using the real system. Knowledge of the most critical device and also the development of an adequate alignment procedure can be could help to realize a good alignment for more accurate applications.

For the self-tracking LDV systems, the proposed models need further developments in order to realize more complex scan patterns suitable for different applications such as the line scanning. Theoretical predictions of the measured velocities could be compared to experimental data found in literature and used to define an alignment procedure to correct the alignment and obtain more accurate measurements.

The mathematical models developed for the various scanning systems can be developed into a software. In particular, the software should operate as a simulator rather than as a data acquisition for scanning vibration measurements. Target velocities and misalignment effects can be predicted while real measurements are acquired. The analysis of the real measurements and the comparison to predicted data can provide useful information regarding the alignment of the system. Moreover, by developing an algorithm that determines the main misalignments from the measured velocity, useful indications towards a better alignment for more accurate measurements can be obtained. If realised, this software could become a very useful tool for scanning and tracking vibration measurements.

In chapter 5, the analysed showed that it is possible combine knowledge of expected misalignments and noise with approximate values of vibration amplitudes and phases to calculate expected errors in the amplitude and phases of resolved vibration components. This could be developed into a software comparison for LDV users. For radial measurements on rotors, further tests are necessary to consider different surface treatments (like spray) and to investigate accurately the effects of the roundness on the measurements. However, the work made on surface roughness is ready to be presented as a practical guide for LDV users.

## Appendix

As defined in section 4.6, the position of the reflection point $C^{\prime}$ situated at the conical mirror requires a more complex development in the presence of misalignments. The position of $C$ ' can be written in two different ways as indicated in the following system:

$$
\left\{\begin{array}{l}
\overrightarrow{O B}+\left|\overrightarrow{B^{C} C}\right| \hat{b}_{2}=\overrightarrow{O C}  \tag{al}\\
\overrightarrow{O D^{*}}+\overrightarrow{D^{*} C}=\overrightarrow{O C}
\end{array}\right.
$$

where the term $\overrightarrow{D^{*} C^{\prime}}$ is given by:

$$
\begin{equation*}
\left|\overrightarrow{B^{\prime} C}\right| \hat{b}_{2}-\overrightarrow{B^{\prime} D^{*}}=\overrightarrow{D^{*} C^{\prime}} \tag{a2}
\end{equation*}
$$

Using a coordinate system fixed in the cone of which the conical mirror is a part, see figure 4.12 , the vector $\overrightarrow{D^{*} C^{\prime}}$ can also be written as:

$$
\begin{equation*}
\overrightarrow{D^{*} C^{\prime}}=-a \tan \psi_{c} \sin \gamma_{c} \hat{x}_{c}+a \tan \psi_{C} \cos \gamma_{c} \hat{y}_{c}-a \hat{z}_{c} \tag{a3}
\end{equation*}
$$

where the terms $\hat{x}_{c}, \hat{y}_{c}$ and $\hat{z}_{c}$ are unit vectors defining the axes of the conical mirror reference system while the angles $\psi_{C}$ and $\gamma_{c}$ are the cone angle and the angular position of $C^{\prime}$ in the cone. Because all the other main points of the Sever system are expressed in terms of the reference system fixed at the rotating target, the vector $\overrightarrow{D^{*} C^{\prime}}$
must also be written in the same way. The relationship between the conical and the target reference system is given by:

$$
\left[\begin{array}{lll}
\hat{x}_{c} & \hat{y}_{c} & \hat{z}_{c}
\end{array}\right]=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[X, \alpha_{c}\right]\left[Y, \beta_{c}\left[\begin{array}{lll}
1 & 0 & 0  \tag{a4}\\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right]\right.
$$

where the rotation matrices $\left[X, \alpha_{c}\right]$ and $\left[Y, \beta_{c}\right]$ accommodate the angular misalignments of the conical mirror with respect to the target reference system and their combination supplies the following rotational matrices:

$$
\left[X, \alpha_{c}\right]\left[Y, \beta_{c}\right]=\left[\begin{array}{ccc}
\cos \beta_{c} & 0 & \sin \beta_{c}  \tag{a5}\\
\sin \alpha_{c} \sin \beta_{c} & \cos \alpha_{c} & -\sin \alpha_{c} \cos \beta_{c} \\
-\cos \alpha_{c} \sin \beta_{c} & \sin \alpha_{c} & \cos \beta_{c} \cos \beta_{c}
\end{array}\right]
$$

Because the angular misalignments of the conical mirror are considered small, the terms in (a5) can be approximated and equation (a4) becomes:
$\left[\begin{array}{lll}\hat{x}_{c} & \hat{y}_{c} & \hat{z}_{c}\end{array}\right] \approx\left[\begin{array}{lll}\hat{x} & \hat{y} & \hat{z}\end{array}\right]\left[\begin{array}{ccc}1 & 0 & \beta_{c} \\ 0 & 1 & -\alpha_{c} \\ -\beta_{c} & \alpha_{c} & 1\end{array}\right]$

Combining equation (a2), (a3) and (a6):

$$
\left|\overrightarrow{B^{\prime} C}\right| \hat{b}_{2}-\overrightarrow{B^{\prime} D^{*}}=\left[\begin{array}{lll}
\hat{x} & \hat{y} & \hat{z}
\end{array}\right]\left[\begin{array}{c}
-a\left(\tan \psi_{C} \sin \gamma_{c}+\beta_{c}\right)  \tag{a7}\\
a\left(\tan \psi_{C} \cos \gamma_{c}+\alpha_{c}\right) \\
-a\left(1-\alpha_{c} \tan \psi_{C} \cos \gamma_{c}-\beta_{c} \tan \psi_{C} \sin \gamma_{c}\right)
\end{array}\right]
$$

At this point, it is convenient to decompose the vector $\overrightarrow{D^{*} C^{\prime}}$ in term of its $\mathrm{x}, \mathrm{y}$ and z components obtaining the following three equations:

$$
\begin{align*}
& \left|\overrightarrow{B^{\prime} C}\right| \hat{b}_{2 x}-\left(\overrightarrow{B^{\prime} D^{*}}\right)_{x}=-a\left(\tan \psi_{C} \sin \gamma_{c}+\beta_{c}\right)  \tag{a8}\\
& \left|\overrightarrow{B^{\prime} C}\right| \hat{b}_{2 y}-\left(\overrightarrow{B^{\prime} D^{*}}\right)_{y}=a\left(\tan \psi_{C} \cos \gamma_{c}+\alpha_{c}\right)  \tag{a9}\\
& \left|\overrightarrow{B^{\prime} C}\right| \hat{b}_{2 z}-\left(\overrightarrow{B^{\prime} D^{*}}\right)_{z}=-a\left(1-\alpha_{c} \tan \psi_{C} \cos \gamma_{c}-\beta_{c} \tan \psi_{C} \sin \gamma_{c}\right) \tag{a10}
\end{align*}
$$

Substituting equations (a8) and (a9) into (a10), the terms $\sin \gamma_{c}$ and $\cos \gamma_{c}$ can be eliminated and equation (a10) becomes:

$$
\begin{equation*}
a\left(1+\alpha_{c}^{2}+\beta_{c}^{2}\right)=\left|\overrightarrow{B^{\prime} C^{\prime}}\right|\left(-\hat{b}_{2 z}+\alpha_{c} \hat{b}_{2 y}-\beta_{c} \hat{b}_{2 x}\right)-\left(-\left(\overrightarrow{B^{\prime} D^{*}}\right)_{z}+\alpha_{c}\left(\overrightarrow{B^{\prime} D^{*}}\right)_{y}-\beta_{c}\left(\overrightarrow{B^{\prime} D^{*}}\right)_{x}\right) \tag{al1}
\end{equation*}
$$

that can be rearranged in the form:

$$
\begin{equation*}
a=\left|\overrightarrow{B^{\prime} C}\right| s+t \tag{a12}
\end{equation*}
$$

where the terms $s$ and $t$ are:

$$
\begin{align*}
& s=\left(-\hat{b}_{2 z}+\alpha_{c} \hat{b}_{2 y}-\beta_{c} \hat{b}_{2 x}\right) /\left(1+\alpha_{c}^{2}+\beta_{c}^{2}\right)  \tag{a13}\\
& t=\left(-\left(\overrightarrow{B^{\prime} D^{*}}\right)_{z}+\alpha_{c}\left(\overrightarrow{B^{\prime} D^{*}}\right)_{y}-\beta_{c}\left(\overrightarrow{B^{\prime} D^{*}}\right)_{x}\right) /\left(1+\alpha_{c}^{2}+\beta_{c}^{2}\right) \tag{a14}
\end{align*}
$$

At this point, equations (a8\&9) are combined using $\sin \gamma_{c}^{2}+\cos \gamma_{c}^{2}=1$ while equation (a11) can be rearranged as a quadratic expression:

$$
\begin{align*}
& a^{2}\left(\tan ^{2} \psi_{C}-\alpha_{c}^{2}-\beta_{c}^{2}\right)-2 a\left(\beta_{c}\left(\overrightarrow{B^{\prime} C} \mid \hat{b}_{2 x}-\left(\overrightarrow{B^{\prime} D^{*}}\right)_{x}\right)-\alpha_{c}\left(\overrightarrow{B^{\prime} C^{\prime}} \hat{b}_{2 y}-\left(\overrightarrow{B^{\prime} D^{*}}\right)_{y}\right)\right)+ \\
& \left(\left(\overrightarrow{B^{\prime} C} \mid \hat{b}_{2 x}-\left(\overrightarrow{\vec{B}^{\prime} D^{*}}\right)_{x}\right)^{2}-\left(\left|\overrightarrow{B^{\prime} C}\right| \hat{b}_{2 y}-\left(\overrightarrow{B^{\prime} D^{*}}\right)_{y}\right)\right)^{2}=0 \tag{a15}
\end{align*}
$$

Substituting equation (a12) into (a15), the complete development of the quadratic form becomes:

$$
\begin{align*}
& \left|\overrightarrow{B^{\prime} C^{\prime}}\right|^{2}\left[s^{2}\left(\tan ^{2} \psi_{c}-\alpha_{c}^{2}-\beta_{c}^{2}\right)-2 s\left(\beta_{c} \hat{b}_{2 x}-\alpha_{c} \hat{b}_{2 y}\right)-\hat{b}_{2 x}^{2}-\hat{b}_{2 y}^{2}\right] \\
& +2\left|\overrightarrow{B^{\prime} C^{\prime}}\right|\left[s t\left(\tan ^{2} \psi_{c}-\alpha_{c}^{2}-\beta_{c}^{2}\right)+s\left(\beta_{c}\left(\overrightarrow{B^{\prime} D^{*}}\right)_{x}-\alpha_{c}\left(\overrightarrow{B^{\prime} D^{*}}\right)_{y}\right)-t\left(\beta_{c} \hat{b}_{2 x}-\alpha_{c} \hat{b}_{2 y}\right)\right.  \tag{a.16}\\
& \left.+\hat{b}_{2 x}\left(\overrightarrow{B^{\prime} D^{*}}\right)_{x}+\hat{b}_{2 y}\left(\overrightarrow{B^{\prime} D^{*}}\right)_{y}\right] \\
& +\left[t^{2}\left(\tan ^{2} \psi_{C}-\alpha_{c}^{2}-\beta_{c}^{2}\right)+2 t\left(\beta_{c}\left(\overrightarrow{B^{\prime} D^{*}}\right)_{x}-\alpha_{c}\left(\overrightarrow{B^{\prime} D^{*}}\right)_{y}\right)-\left(\overrightarrow{B^{\prime} D^{*}}\right)_{x}^{2}-\left(\overrightarrow{B^{\prime} D^{*}}\right)_{y}^{2}\right]=0
\end{align*}
$$

From the two possible solutions, the negative root gives positive $\left|\overrightarrow{B^{\prime} C}\right|$. This which is then used to calculate $\gamma_{c}$ by rearranging equation (a8\&9) to derive the following equation:
$\tan \gamma_{c}=\frac{-\left(\overrightarrow{B^{\prime} C^{\prime}} \mid \hat{b}_{2 x}-\left(\overrightarrow{B^{\prime} D^{*}}\right)_{x}\right)-\left(\overrightarrow{B^{\prime} C} \mid s+t\right) \beta_{c}}{\left(\left|\overrightarrow{B^{\prime} C^{\prime}}\right| \hat{b}_{2 y}-\left(\overrightarrow{B^{\prime} D^{*}}\right)_{y}\right)-\left(\left|\overrightarrow{B^{\prime} C^{\prime}}\right| s+t\right) \alpha_{c}}$

Known the angle $\gamma_{c}$, the term $\overrightarrow{O C^{\prime}}$ is found from equation (4.36) and the mirror normal $\hat{n}_{C^{\prime}}$ is found from equation (4.35).

## Publications resulting from this research

1.1 M. Tirabassi and S.J. Rothberg, "Scanning LDV using wedge prisms", Optics and Lasers in Engineering, Volume 47, Issues 3-4, pp. 454-460, March-April (2009).
1.2 M. Tirabassi and S.J. Rothberg, "Advanced modelling of tracking LDV systems incorporating rotating optical wedges", Eighth International Conference on Vibration Measurements by Laser Techniques: Advances and Applications , Proc. SPIE, Vol. 7098, 709806 (2008).

## References

## Chapter 1

1.1 M. Rantatalo, K. Tatar, P. Norman, "Laser Doppler vibrometry measurements of a rotating milling machine spindle", Proceedings of the Eighth International Conference on Vibrations in Rotating Machinery, University of Wales, Swansea, UK, pp. 231-240, (2004).
1.2 J. R. Bell and S.J. Rothberg, "Laser vibrometers and contacting transducers, target rotation and 6 degree-of-freedom vibration: what do we really measure?" Journal of Sound and Vibration, Vol. 237, 2, pp. 245-261(17), October (2000).
1.3 B.J. Halkon and S. J. Rothberg, "Rotor Vibration Measurements Using Laser Doppler Vibrometry: Essential Post-Processing for Resolution of Radial and Pitch/Yaw Vibrations", Transactions of the ASME, Journal of Vibrations \&Acoustics, 8, Vol. 128, pp. 8-20, (2006).
1.4 B.J. Halkon and S. J. Rothberg, "Vibration measurements using continuous scanning Laser Doppler Vibrometry: theoretical velocity sensitivity analysis", Measurements Science and Technology, Vol. 14, pp. 382-393, (2003).
1.5 B.J Halkon., S.R Frizzel and S.J Rothberg, "Vibration Measurements Using Continuous Scanning Laser Vibrometry: Velocity Sensitivity Model Experimental Validation", Measurement Science and Technology, Vol. 146, 1st, 773-783, June (2003).
1.6 X. Zeng, L.D. Mitchell and B.L. Agee, "A laser position determination algorithm for an automated mechanical mobility measurement system", Proceedings of the 11th International Modal Analysis Conference, Vol. 1, pp. 122-129, Kissimmee, Florida (February 1993).
1.7 B.M. Watrasiewicz and M.J. Rudd, "Laser Doppler measurements", London, Boston, Butterworths, (1976).
1.8 L.E. Drain, "The Laser Doppler Technique", J. Wiley \& Sons, (1980).
1.9 www.polytec.com/eur/_files/LM_AN_INFO_0103_E_Vibrometry_Basics.
1.10 S.J. Rothberg, J.R. Baker and N.A. Halliwell, "Laser vibrometry: pseudovibrations", Journal of Sound and Vibration 135, pp. 516-522, (1989).
1.11 J. Oldengarm, A.H. van Krieken, H.J. Raterink, "Laser Doppler velocimeter with optical frequency shifting", Optics and Laser Technology, pp. 249-252, (December 1973).
1.12 W.H. Stevenson, "Optical frequency shifting by means of a rotating diffraction grating", Applied Optics, 9 (3), pp. 649-652, (1970).
1.13 J.C. Dainty, "Laser Speckle and related phenomena", Springer-Verlag, (Berlin, 1975).
1.14 J.W. Goodman, "Some fundamental properties of speckle", J. Opt. Soc. Am., Vol. 66 (11), pp. 1145-1150, (November 1976).
1.15 N.A. Halliwell, "The Laser Torsional Vibrometer: a step forward in rotating machinery diagnostics", Journal of Sound and Vibration, 190, 3, pp. 399418, (1996).
1.16 T.J. Miles, M. Lucas, N.A. Halliwell and S.J. Rothberg, "Torsional and Bending Vibration Measurement on Rotors Using Laser Technology," Journal of Sound and Vibration, 2263, pp. 441-467, (1999).
1.17 J.R. Bell and S.J. Rothberg, "On the application of laser vibrometry to translational and rotational vibration measurements on rotating shafts", Measurement 35, pp. 201-210, (2004).
1.18 Q.V. Davis and W.K. Kulczyk, "Vibrations of turbine blades measured by means of a laser", Nature 222, pp. 475-476, (1969).
1.19 X. Zeng, A. L. Wicks and L. D. Mitchell, "Geometrical Method for the Determination of the Position and Orientation of a Scanning Laser Doppler Vibrometer", Optics and Lasers in Engineering 25, pp. 247-264, (1996).
1.20 P. Castellini and N. Paone, "Development of the Tracking Laser Vibrometer: Performance and Uncertainty Analysis," Review of Scientific Instrumentation, 71(12), pp. 4639-4647, (2000).
1.21 A.B. Stanbridge, D.J. Ewins, "Modal testing of rotating disc using a scanning LDV" Proceedings of ASME Design Engineering Technical Conference 3(B), Boston, USA, pp. 1207-1213, (1985).
1.22 A.Z. Khan, A.B. Stanbridge and D.J. Ewins, "Detecting damage in vibrating structure with a scanning LDV", Optics and Laser in Engineering, 32, pp. 583-592, (2000).
1.23 R.A. Lomenzo, A.J. Barker, A.L. Wicks and P.S. King, "A Laser Vibrometry System for Measuring Vibrations on Rotating Disks," presented at the 4th National Turbine Engine High Cycle Fatigue (HCF) Conference, Monterey, CA, pp. 277-282, (February 1999).
1.24 I.A. Sever, "Turbomachinery Blade Vibration Measurements with Tracking LDV under rotation", Seventh International Conference on Vibration Measurements by Laser Techniques: Advances and Applications, Proc. of SPIE Vol. 6345, Ancona, Italy, (2006).
1.25 B.J Halkon., S.R Frizzel and S.J Rothberg, "Vibration Measurements Using Continuous Scanning Laser Vibrometry: Velocity Sensitivity Model Experimental Validation", Measurement Science and Technology, 146, 1st, 773-783, June (2003).
1.26 M. Tirabassi and S.J. Rothberg, "Scanning LDV using wedge prisms", Optics and Lasers in Engineering, Volume 47, Issues 3-4, pp. 454-460, March-April 2009.
1.27 S. Boedecker, A. Dräbenstedt, L. Heller, A. Kraft, A. Leonhardt, C. Pape, S. Ristau, E. Reithmeier, C. Rembe, "Optical Derotator for Scanning Vibrometer Measurements on Rotating Objects" Seventh International Conference on Vibration Measurements by Laser Techniques: Advances and Applications, Proc. of SPIE Vol. 6345, Ancona, Italy, (2006).
1.28 K. Tatar, M. Rantatalo, P. Gren, "Laser vibrometry measurements of an optically smooth rotating spindle", Mechanical Systems and Signal Processing 21, pp. 1739-1745, (2007).
1.29 S. J. Rothberg, "Radial Vibration Measurements Directly from Rotors using Laser Vibrometry: Uncertainty due to Surface Roughness", Proceedings of the IMAC-XXV:A Conference \& Exposition on Structural Dynamics, Orlando, Florida USA February 2007, Paper \#371.

## Chapter 2

2.1 H.R. Harrison \& T. Nettleton, "Advanced Engineering Dynamics", Arnold Edition, (1997).
2.2 J. R. Bell and S.J. Rothberg, "Laser vibrometers and contacting transducers, target rotation and 6 degree-of-freedom vibration: what do we really measure?", Journal of Sound and Vibration, Volume 237, 2, pp. 245-261(17), October (2000).
2.3 B.J. Halkon and S.J. Rothberg, "Vibration measurements using continuous scanning laser vibrometry: theoretical velocity sensitivity analysis with applications", Meas. Sci. Technol. 14 pp. 382-389, (2003).
2.4 C.T. Amirault and C.A. DiMarzio, "Precision pointing using a dual-wedge scanner", Appl. Opt. 24 (9) pp. 1302-1308, (1985).
2.5 Polytec GmbH Application notes, "Measuring Valve Train Dynamics Using the HSV-2002 High speed Laser Vibrometer", Polytec, http://www.polytec.com/eur/_files/LM_AN_VIB-C-03_2005_11.

## Chapter 3

3.1 X. Zeng, L.D. Mitchell and B.L. Agee, "A laser position determination algorithm for an automated mechanical mobility measurement system", Proceedings of the 11th International Modal Analysis Conference, Vol. 1, pp. 122-129, Kissimmee, Florida (February 1993).
3.2 X. Zeng, A. L. Wicks and L. D. Mitchell, "Geometrical Method for the Determination of the Position and Orientation of a Scanning Laser Doppler Vibrometer", Optics and Lasers in Engineering 25, pp. 247-264, (1996).
3.3 P. Castellini and N. Paone, "Development of the Tracking Laser Vibrometer: Performance and Uncertainty Analysis," Review of Scientific Instrumentation, 71(12), pp. 4639-4647, (2000).
3.4 B.J. Halkon and S.J. Rothberg, "Synchronised-Scanning Laser Vibrometry", Proceedings of Sixth International Conference on Vibration Measurements by Laser Techniques, 5503, Ancona, Italy, 2004: 260-271
3.5 F.A. Rosell, "Prism Scanner," Journal Optical Society America, 50(6), 521 526 (1960).
3.6 W.L. Wolfe, G.J. Zissis, "The Infrared Handbook", Office of Naval Research, Department of the Navy, Arlington, 1978.
3.7 C.T. Amirault and C.A. DiMarzio, "Precision pointing using a dual-wedge scanner," Appl. Opt. 24(9), 1302 - 1308, (1985).
3.8 A. Li, L. Liu, J. Sun, X. Zhong, D. Xu, Q. Shen, Y. Zhou Z. Luan, L. Wang, "Double-prism Scanner for Testing Tracking Performance of Inter-satellite Laser Communication Terminals," Proc. SPIE 6304, 63041R-1 - 63041R-10 (2006).
3.9 J. Wu, M. Conry, C. Gu, F. Wang, Z. Yaqoob, C. Yang, "Paired-anglerotation scanning optical coherence tomography forward-imaging probe," Opt. Lett., 31(9), 1265 - 1267 (2006).
3.10 W. C. Warger II, S. A. Guerrera, C. A. DiMarzio, "Toward a Compact DualWedge Point-Scanning Confocal Reflectance Microscope", Proc. of SPIE Vol. 6443, 644311-7.
3.11 W. L. Wolfe, "Nondispersive prisms," in Handbook of Optics, 2nd ed., M. Bass, E. W. Van Stryland, D. R. Williams, and W. L.Wolfe, pp. 4.1-4.29, eds. McGraw-Hill, New York, (1995).
3.12 M. Born, and Wolf, "Principles of Optics", Sixth edition (Oxford: Pergamon), section 6.3, E., (1980).
3.13 J. D Armitage and Lohmann, A., 1965, Optica Acta, 12, 185.
3.14 D. Coutts, Conference on Lasers and Electro-Optics Europe, paper CWI4, (1998).
3.15 S. Boedecker, A. Dräbenstedt, L. Heller, A. Kraft, A. Leonhardt, C. Rembe, "Optical Derotator for Scanning Vibrometer Measurements on Rotating Objects", Seventh International Conference on Vibration Measurements by Laser Techniques: Advances and Applications, Proc. of SPIE Vol. 6345, Ancona, Italy, (2006).

## Chapter 4

4.1 R.A. Lomenzo, A.J. Barker, A.L. Wicks and P.S. King, "A Laser Vibrometry System for Measuring Vibrations on Rotating Disks," presented at the 4th National Turbine Engine High Cycle Fatigue (HCF) Conference, Monterey, CA, 277-282, (February 1999).
4.2 I.A. Sever, "Turbomachinery Blade Vibration Measurements with Tracking LDV under rotation", Seventh International Conference on Vibration Measurements by Laser Techniques: Advances and Applications, Proc. of SPIE Vol. 6345, Ancona, Italy, (2006).

## Chapter 5

5.1 B.J. Halkon \& S. J. Rothberg, "Rotor Vibration Measurements Using Laser Doppler Vibrometry: Essential Post-Processing for Resolution of Radial and Pitch/Yaw Vibrations", Transactions of the ASME, Journal of Vibrations \& Acoustics, 8, Vol. 128, pp. 8-20, (2006).
5.2 J.R. Bell \& S. J. Rothberg "Rotational vibration measurements using laser Doppler Vibrometry: Comprehensive theory and practical application", Journal of Sound and Vibration, Vol. 238, Issue 4, pp. 673-690, (2000).
5.3 K. Tatar, M. Rantatalo, P. Gren, "Laser vibrometry measurements of an optically smooth rotating spindle", Mechanical Systems and Signal Processing 21, pp. 1739-1745, (2007).


Figure 1.1 - Scattering condition [1.2]


Figure 1.2 - Polytec LDV arrangement [1.3]


Figure 1.3 - Speckle pattern from a rough surface


Figure 1.4 - Test rig used for laser vibrometer vibration measurement on rotor


Figure 1.5 - a) Accelerometer and b) laser vibrometer output from rotor rotating at 20 Hz


Figure 1.6 - a) Accelerometer and b) laser vibrometer output from the rotor (non-rotating) undergoing vibration at 30 Hz


Figure 1.7 - a) Accelerometer and b) laser vibrometer output from rotor rotating at 20 Hz and undergoing an on-axis vibration at 30 Hz

a)

b)

Figure 1.8 - Optical arrangement for: a) axial vibration measurement,
b) radial vibration measurement


Figure 1.9 - Scheme of the Laser Torsional Vibrometer (LTV)


Figure 1.10 - Definition of the point P on a rotating component undergoing arbitrary vibration


Figure 1.11 - Orientation of beam direction $\hat{b}$


Figure 1.12 - Orientation of parallel beam separation defining angle $\gamma$


Figure 1.13 - Optical scheme proposed for the simultaneous measurements of radial vibrations on a rotating target


Figure 1.14 - Laser vibrometer output: a) along the x - axis, b) along the $y$ - axis from a rotor undergoing vibration at 15 Hz and rotating at 20 Hz


Figure 1.15 - Typical scheme for the dual mirror SLDV system


Figure 2.1 - Simplest scheme used in vibrometry with ideal alignment


Figure 2.2 - Simplest scheme used in vibrometry including laser head (a) translational and (b) angular misalignments


Figure 2.3 - Refraction and reflection of an incident laser beam


Figure 2.4 - Schematic representation of the steering mirror optical system


Figure 2.5 - Optical scheme used by Polytec GmbH to measure valve train dynamics [2.3]


- Accelerometers

Figure 2.6 - Schematic set-up of four accelerometers positioned on a steering mirror to determine its dynamics

## Plane xz



Figure 2.7 - Experimental test rig for the steering mirror system validation


Figure 2.8 -a) Accelerometer 1 and b) laser vibrometer output from a mirror vibrating at 40 Hz and inclined at $\beta_{m 0}=0^{\circ}$ (initial alignment)


Figure 2.9 - a) Accelerometer 1 and b) laser vibrometer output from a mirror vibrating at 40 Hz and inclined at $\beta_{m 0}=40^{\circ}$


Figure 2.10 - a) Experimental accelerometer and laser vibrometer outputs from a mirror vibrating at 40 Hz inclined at different values of $\beta_{m 0}$, b) error between theoretical and experimental data


Figure 2.11 - a) Accelerometer 2 and b) laser vibrometer output from a target vibrating at 40 Hz using a reflecting stationary mirror inclined at $\beta_{m 0}=40^{\circ}$


Figure 2.12 - a) Ratio between theoretical and experimental laser vibrometer output from a target vibrating at 40 Hz obtained for different inclinations of the stationary mirror, b) error between theoretical and experimental data


Figure 2.13 -a) Accelerometer 1 output from the mirror vibrating at 40 Hz and inclined $\beta_{m 0}=40^{\circ} ; \mathrm{b}$ ) accelerometer 2 output from the target vibrating at 60 Hz


Figure 2.14 - Laser vibrometer output from the target vibrating at 60 Hz and deflected by the mirror vibrating at 40 Hz and inclined at

$$
\beta_{m 0}=40^{\circ}
$$



Figure 2.15 - Error between experimental and theoretical laser vibrometer (according to equation (2.25) from: a) the vibrating mirror;
b) the vibrating target obtained for various steering mirror inclinations
a) Zero misafignments configuration


## 6) Misaligned configuration



Figure 2.16 - Schematic representation of a scanning system based on a rotating wedge


Figure 2.17 - Test rig for the single wedge SLDV


Figure 2.18 - Laser vibrometer output corresponding to a circular path on the retro-reflective screen: a) best alignment, b) translational misalignment of the laser head along the x - axis, $\Delta x_{A}=2 \mathrm{~mm}$


Figure 2.19 - Comparison between experimental and simulated data for the 1 x velocity term for: a) horizontal target misalignment, b) vertical target misalignment on a reflective screen


Figure 2.20 - Laser vibrometer output corresponding to a circular path on the rotating target: a) best alignment, b) translational misalignment of the laser head along the x- axis, $\Delta x_{A}=2 \mathrm{~mm}$


Figure 2.21 - Comparison between experimental and simulated data for the 1 x velocity term for: a) horizontal target misalignment, b) vertical target misalignment on the rotating target


Figure 2.22 - Particular of the investigated target used in tests made with the single wedge SLDV system


Figure 2.23 - a) Laser vibrometer output corresponding to a circular path on the rotating target in the presence of angular and translational misalignments of the target around the y - axis and along the x - axis; b ) comparison between experimental and simulated data for the 1 x velocity term for angular misalignments of the target around $y$ - axis and translational misalignments of the target along the x - axis


Figure 3.1 - Schematic representation of the dual mirror scanning system


Figure 3.2 - a) Simulated beam path and b) velocity spectra for the zero misalignments configuration of the dual mirror scanning system


Figure 3.3 - a) Simulated beam path and b) velocity spectra for the zero misalignments configuration of the dual mirror scanning system


Figure 3.4 - Predicted velocity spectra in the presence of whole body target vibrations, $\dot{z}_{\text {WB }}(t)=10 \cos \left(5 \Omega_{T}\right)$ and $\dot{x}_{\text {WB }}(t)=10 \cos \left(10 \Omega_{T}\right)$ : a) zero misalignment configuration, b) with an angular misalignment added to the laser head, with $\gamma_{L}=0.2^{\circ}$


Figure 3.5 - a) Scan pattern and b) velocity spectra of the measured velocity in the presence of a translational misalignment of the laser source along the y - axis with $\Delta y_{A}=2 \mathrm{~mm}$


Figure 3.6 - a) Scan pattern and b) velocity spectra of the measured velocity in the presence of a translational misalignment of the laser source along the z - axis with $\Delta z_{A}=2 \mathrm{~mm}$


Figure 3.7 - a) Scan pattern and b) velocity spectra of the measured velocity in the presence of angular misalignment of the laser source around the z-axis with $\gamma_{L}=0.2^{\circ}$


Figure 3.8 - a) Scan pattern and b) velocity spectra of the measured velocity in the presence of a translational misalignment added to the first oscillating mirror along the y-axis with $\Delta y_{B}=2 \mathrm{~mm}$


Figure 3.9 - a) Scan pattern and b) velocity spectra of the measured velocity in the presence of an angular misalignment with $\beta_{m 1}=0.2^{\circ}$ added to the first oscillating mirror around the $y$ - axis


Figure 3.10 - a) Scan pattern and b) velocity spectra of the measured velocity in the presence of an angular misalignment with $\alpha_{m 1}=0.2^{\circ}$ added to the first oscillating mirror around the x - axis


Figure 3.11 - a) Scan pattern and b) velocity spectra of the measured velocity in the presence of a translational misalignment with $\Delta y_{C}=2 \mathrm{~mm}$ added to the second oscillating mirror along the y - axis


Figure 3.12 - a) Scan pattern and b) velocity spectra of the measured velocity in the presence of a translational misalignment with $\Delta z_{\mathrm{C}}=$ 2 mm added to the second oscillating mirror along the z - axis


Figure 3.13 - a) Scan pattern and b) velocity spectra of the measured velocity in the presence of an angular misalignment with $\beta_{m 2}=0.2^{\circ}$ added to the second oscillating mirror around the $y$ - axis


Figure 3.14 - a) DC component estimated for angular misalignments of second oscillating mirror; b) 2 x component estimated for translational misalignments of the oscillating mirrors


Figure $3.15-1 \mathrm{x}$ component predicted for: a) translational, b) angular misalignments of the optical devices


Figure 3.16 - a) Predicted beam path and b) velocity spectra for a scenario with: $\beta_{L}=0.2^{\circ}, \gamma_{L}=0.1^{\circ}, \Delta y_{A}=3 \mathrm{~mm}, \Delta z_{A}=3 \mathrm{~mm}, \alpha_{m 1}=-0.1^{\circ}$, $\beta_{m 1}=0.2^{\circ}, \Delta x_{B}=3 \mathrm{~mm}, \Delta y_{B}=-3 \mathrm{~mm}, \gamma_{m 2}=0.2^{\circ}, \Delta y_{C}=-3 \mathrm{~mm}$ and $\Delta z_{C}=-$ 3 mm


Figure 3.17 - a) Predicted beam path and b) velocity spectra for a scenario with combined misalignments: $\gamma_{L}=0.3^{\circ}, \Delta y_{A}=3 \mathrm{~mm}$, $\Delta z_{A}=3 \mathrm{~mm}, \quad \beta_{m 1}=0.1^{\circ}, \quad \Delta x_{B}=-3 \mathrm{~mm}, \quad \beta_{m 2}=-0.3^{\circ}, \quad \gamma_{m 2}=-0.3^{\circ}, \quad \Delta y_{C}=-$ $3 \mathrm{~mm}, \Delta z_{\mathrm{C}}=3 \mathrm{~mm}$


Figure 3.18 - Optical scheme of the dual wedge SLDV system


Figure 3.19 - Scan patterns described by the dual-wedge SLDV system: a) circular scan, b) line scan

a)

b)

Figure 3.20 - Scan patterns described by the dual-wedge SLDV system: a) Rosette, b) spiral


Figure 3.21 - a) Scan pattern and b) velocity spectrum of the measured for the zero misalignment configuration


Figure 3.22 - Predicted velocity spectra in the presence of whole body target vibrations, $\dot{z}_{\text {WB }}(t)=10 \cos \left(5 \Omega_{T}\right)$ and $\dot{x}_{\text {WB }}(t)=10 \cos \left(10 \Omega_{T}\right)$ : a) zero misalignment configuration, b) with an angular misalignment added to the laser head, with $\gamma_{L}=0.2^{\circ}$

$\Omega_{B 1}=$ Oscillation frequency on the top of the first body wedge
$\Omega_{B 2}=$ Oscillation frequency on the top of the second body wedge
$\Omega_{1}=$ Rotational speed of the first wedge
$\Omega_{2}=$ Rotational speed of the second wedge
$\Omega_{T}=$ Rotational speed of the target

Figure 3.23 - Schematic representation of all the rotations and oscillations considered for the double wedge scanning arrangement


Figure 3.24 - a) Scan patterns and b) predicted velocity spectrum by the dual-wedge SLDV system in absence of misalignments but in the presence of oscillations applied to the wedges

a)

b)

Figure 3.25 - a) Scan patterns and b) predicted velocity spectrum by the dual-wedge SLDV system in absence of misalignments but in the presence of oscillations applied to the wedges


Figure 3.26 - Optical scheme of the dual wedge SLDV system with translational misalignments of the laser head along the $y$-axis.


Figure 3.27 - a) Simulated scan pattern and b) velocity spectra for the dual wedge scanning system with an angular misalignment around the x - axis added to the laser head, $\alpha_{L}=0.2^{\circ}$


Figure 3.28 - a) Simulated scan pattern and b) velocity spectra for the dual wedge scanning system with laser head translated along the x axis with $\Delta x_{A}=2 \mathrm{~mm}$


Figure 3.29 - a) Simulated scan pattern and b) velocity spectra for the dual wedge scanning system when the laser has angular and translational misalignments with respect the x- axis, $\alpha_{L}=0.2^{\circ}$, $\Delta x_{A}=2 \mathrm{~mm}$


Figure 3.30 - a) Simulated scan pattern and b) velocity spectra for the dual wedge scanning system when the first wedge has a translational misalignment along the x - axis $\Delta x_{B}=2 \mathrm{~mm}$


Figure 3.31 - a) Simulated scan pattern and b) velocity spectra for the dual wedge scanning system when the first wedge has an angular misalignment around the x- axis, $\alpha_{w 1}=0.2^{\circ}$


Figure 3.32 - a) Simulated scan pattern and b) velocity spectra for the dual wedge scanning system when the first wedge has angular and translational misalignments with respect the x- axis, $\alpha_{w 1}=0.2^{\circ}, \Delta x_{B}=$ 2 mm


Figure 3.33 - a) Simulated scan pattern and b) velocity spectra for the dual wedge scanning system when the second wedge has a translational misalignment along the x - axis with $\Delta x_{E}=2 \mathrm{~mm}$


Figure 3.34 - a) Simulated scan pattern and b) velocity spectra for the dual wedge scanning system when the second wedge has an angular misalignment around the x- axis, $\alpha_{w 2}=0.2^{\circ}$


Figure 3.35 - a) Simulated scan pattern and b) velocity spectra for the dual wedge scanning system when the second wedge has an angular misalignment around the x - axis $\alpha_{w 2}=1^{\circ}$ and a translational misalignment along the x - axis with $\Delta x_{E}=2 \mathrm{~mm}$


Figure 3.36 - Values predicted for the 1 x additional term in presence of a) translational misalignments, b) angular misalignments


Figure 3.37 - a) Simulated scan pattern and b) velocity spectra obtained combining different misalignments: $\alpha_{L}=0.2^{\circ}, \beta_{L}=-0.2^{\circ}$, $\alpha_{w 1}=-0.5^{\circ}, \quad \beta_{w 1}=-0.5^{\circ}, \quad \alpha_{w 2}=-0.5^{\circ}, \quad \beta_{w 2}=0.5^{\circ}, \quad \Delta x_{A}=-4 \mathrm{~mm}, \quad \Delta y_{A}=-$ $5 \mathrm{~mm}, \Delta x_{B}=\Delta y_{B}=-5 \mathrm{~mm}$ and $\Delta x_{E}=\Delta y_{E}=-5 \mathrm{~mm}$.


Figure 3.38 - a) Simulated scan pattern and b) velocity spectra obtained combining different misalignments: $\alpha_{L}=0.2^{\circ}, \beta_{L}=-0.3^{\circ}$, $\alpha_{w 1}=-0.1^{\circ}, \beta_{w 1}=0.5^{\circ}, \alpha_{w 2}=0.5^{\circ}, \beta_{w 2}=0.5^{\circ}, \Delta x_{A}=-5 \mathrm{~mm}, \Delta y_{A}=-5 \mathrm{~mm}$, $\Delta x_{B}=\Delta y_{B}=5 \mathrm{~mm}, \Delta x_{E}=5 \mathrm{~mm}$ and $\Delta y_{E}=-5 \mathrm{~mm}$.


Figure 3.39 - Test rig for the dual wedges SLDV


Figure 3.40 - a) Laser vibrometer output corresponding to a circular path on the retro-reflective screen panel for a) the best alignment possible, b) in the presence of translational misalignment of the laser head along the x-axis, $\Delta x_{A}=2 \mathrm{~mm}$


Figure 3.41 - Comparison between experimental and simulated of the 1x velocity term obtained on the retro-reflective screen panel for: a) horizontal laser head displacements, b) vertical laser head displacements


Figure 3.42 - Laser vibrometer output corresponding to a circular path on the rotating target $a$ ) the best alignment possible, $b$ ) in the presence of translational misalignment of the target of 2 mm along the x - axis.


Figure 3.43 - Comparison between experimental and simulated of the 1 x velocity term obtained on the rotating target for: a) horizontal target displacements, b) vertical target displacements


Figure 3.43 - c) Comparison between experimental and simulated of the 1 x velocity term obtained on the rotating target for angular and translational misalignments of the target around the y - axis and along the x - axis


Figure 3.44 - Laser vibrometer output corresponding to: a) vibrating target in absence of wedge rotations, b) vibrating and rotating target in the presence of rotating wedges at 6 Hz .


Figure 3.45 - Schematic representation of the Dove prism SLDV system (zero misalignment configuration)


Figure 3.46 - Scan pattern described when a) the laser beam is aligned around the x-axis, with $\alpha_{L}=2.0875^{\circ}, \mathrm{b}$ ) the Dove prism is aligned around the x-axis, with $\alpha_{P}=1^{\circ}$


Figure 3.47 - Schematic representation of the Dove prism SLDV system initially aligned rotating the laser head around the x - and the y axis with $\alpha_{L}=\beta_{L}=2^{\circ}$


Figure 3.48 - a) Scan pattern and b) velocity spectra of the measured velocity with the laser beam aligned around the x - axis with $\alpha_{L}=2.0875^{\circ}$


Figure 3.49 - Scan pattern predicted when the laser beam is aligned along the x - axis with $\Delta x_{A}=2 \mathrm{~mm}$


Figure 3.50 - Predicted velocity spectra in the presence of whole body target vibrations, $\dot{z}_{\text {WB }}(t)=10 \cos \left(5 \Omega_{T}\right)$ and $\dot{x}_{\text {WB }}(t)=10 \cos \left(10 \Omega_{T}\right)$ : a) zero misalignment configuration, b) with an angular misalignment added to the Dove prism, with $\beta_{p}=0.2^{\circ}$


Figure 3.51 - a) Scan pattern and b) velocity spectra of the measured velocity with the laser beam aligned around the x - axis with $\alpha_{L}=2.0875^{\circ}$ and translational misalignment of the laser head along the $y$ - axis $\Delta y_{A}=2 \mathrm{~mm}$


Figure 3.52 - a) Scan pattern and b) velocity spectra of the measured velocity with the laser beam aligned around the x-axis with $\alpha_{L}=$ $2.0875^{\circ}$ plus an angular misalignment of the laser head around the $x-$ axis with $\alpha_{m L}=0.2^{\circ}$


Figure 3.53 - a) Scan pattern and b) velocity spectra of the measured velocity with the laser beam aligned around the x - axis with $\alpha_{L}=$ $2.0875^{\circ}$ and a translational misalignment of the prism along the x - axis of $\Delta x_{B}=2 \mathrm{~mm}$


Figure 3.54 - a) Scan pattern and b) velocity spectra of the measured velocity with the laser beam aligned around the x - axis with $\alpha_{L}=$ $2.0875^{\circ}$ and the prism rotated around the x- axis with $\alpha_{P}=0.2^{\circ}$


Figure 3.55 - a) Scan pattern and b) velocity spectra of the measured velocity with the laser beam aligned around the x - axis with $\alpha_{L}=$ $2.0875^{\circ}$ and the prism rotated around the y - axis with $\beta_{p}=0.2^{\circ}$


Figure 3.56 - a) Predicted amplitudes for the 1 x additional velocity term in the presence of: a) translational misalignments, b) angular misalignments

c)

Figure 3.56 - c) Predicted amplitudes for the DC additional velocity term in the presence of angular misalignments


Figure 3.57 - a) Predicted scan pattern and b) velocity spectra obtained by combining different misalignments: $\alpha_{m L}=0.2^{\circ}, \beta_{m L}=0.2^{\circ}$, $\Delta x_{A}=\Delta y_{A}=3 \mathrm{~mm}, \alpha_{P}=0.2^{\circ}, \Delta y_{B}=3 \mathrm{~mm}$.


Figure 3.58 - Predicted a) scan pattern and b) velocity spectra obtained by combining different misalignments: $\alpha_{m L}=-0.1^{\circ}, \beta_{m L}=0.5^{\circ}$, $\Delta x_{A}=\Delta y_{A}=3 \mathrm{~mm}, \alpha_{P}=-0.1^{\circ}, \beta_{P}=0.1^{\circ}$ and $\Delta x_{B}=\Delta y_{B}=3 \mathrm{~mm}$


Figure 4.1 - Schematic representation of the self-tracking system proposed by Lomenzo (zero misalignment configuration)


Figure 4.2 - Velocity spectrum predicted in the presence of target vibrations: a) zero misalignments configuration, b) with angular misalignment of the laser head around the y - axis, $\beta_{L}=0.2^{\circ}$


Figure 4.3 - Misaligned configuration of the self-tracking system proposed by Lomenzo (with laser head misalignments)


Figure 4.4 - a) Predicted scan pattern and b) velocity spectra for a circular scan in the presence of a translational misalignment of the laser head along the x-axis, with $\Delta x_{A}=2 \mathrm{~mm}$


Figure 4.5 - a) Predicted scan pattern and b) velocity spectra for a circular scan in the presence of an angular misalignment of the laser head around the x- axis, with $\alpha_{L}=0.2^{\circ}$


Figure 4.6 - a) Predicted scan pattern and b) velocity spectra for a circular scan in the presence of an angular and translational misalignment of the laser head along and around the x - axis, with $\alpha_{L}=0.2^{\circ}$ and $\Delta x_{A}=2 \mathrm{~mm}$


Figure 4.7 - a) Predicted scan pattern in the presence an angular misalignment of the vertex mirror around: a) the $x$ - axis with $\left.\alpha_{V}=0.2^{\circ} ; \mathrm{b}\right)$ the $\mathrm{y}-\mathrm{axis}$ with $\beta_{V}=0.2^{\circ}$


Figure 4.8 - a) Predicted scan pattern and b) velocity spectra for a circular scan in the presence of an angular misalignment of the fold mirror along and around the x-axis, with $\alpha_{F}=0.2^{\circ}$


Figure 4.9 - Values predicted for the 1x additional velocity term in the presence of angular misalignments of the fold mirror around the $x$ axis.


Figure 4.10 - a) Predicted scan pattern and b) velocity spectra for a circular scan in the presence of the following combined misalignments: $\alpha_{L}=-0.5^{\circ}, \beta_{L}=0^{\circ}, \alpha_{v}=-0.3^{\circ} \beta_{v}=0.3^{\circ}, \alpha_{F}=0^{\circ}, \beta_{F}=1^{\circ}$, $\Delta x_{A}=2 \mathrm{~mm}, \Delta y_{A}=5 \mathrm{~mm}, \Delta x_{B}=\Delta y_{B}=0 \mathrm{~mm}$


Figure 4.11 - a) Predicted scan pattern and b) velocity spectra for a circular scan in the presence of the following combined misalignments: $\alpha_{L}=-0.5^{\circ}, \beta_{L}=0^{\circ}, \alpha_{v}=-0.5^{\circ}, \beta_{v}=-0.2^{\circ}, \alpha_{F}=-0.3^{\circ}$, $\beta_{F}=0.5^{\circ}, \Delta x_{A}=5 \mathrm{~mm}, \Delta y_{A}=2 \mathrm{~mm} \Delta x_{B}=\Delta y_{B}=0 \mathrm{~mm}$


Figure 4.12- Schematic representation of the self-tracking system proposed by Sever (zero misalignment configuration) [4.2]


Figure 4.13 - Velocity spectrum predicted in the presence of target vibrations: a) zero misalignments configuration, b) with angular misalignment of the laser head around the $y$ - axis, $\beta_{L}=0.2^{\circ}$


Figure 4.14 - Possible misalignments for the self-tracking system proposed by Sever


Figure 4.15 - a) Predicted scan pattern and b) velocity spectra for a circular scan in the presence of a translational misalignment of the laser head along the x - axis, with $\Delta x_{A}=2 \mathrm{~mm}$


Figure 4.16 - a) Predicted scan pattern and b) velocity spectra for a circular scan in the presence of an angular misalignment of the laser head around the x- axis, with $\alpha_{L}=0.2^{\circ}$


Figure 4.17 - a) Predicted scan pattern and b) velocity spectra for a circular scan in the presence of an angular misalignment of the laser head around the x - axis and a translational misalignment along the x axis, with $\alpha_{L}=0.2^{\circ}$ and $\Delta x_{A}=2 \mathrm{~mm}$


Figure 4.18 - a) Predicted scan pattern in the presence of an angular misalignment of the vertex mirror: a) around the $x$ - axis, with $\left.\alpha_{V}=0.2^{\circ}, \mathrm{b}\right)$ around the y - axis, with $\beta_{V}=0.2^{\circ}$


Figure 4.19 - a) Predicted scan pattern and b) velocity spectra for a circular scan in presence of a translational misalignment of the conical mirror along the x - axis with $\Delta x_{D}=2 \mathrm{~mm}$


Figure 4.20 - a) Predicted scan pattern and b) velocity spectra for a circular scan in presence of an angular misalignment of the conical mirror around the x- axis with $\alpha_{C}=0.2^{\circ}$


Figure 4.21 - a) Predicted scan pattern and b) velocity spectra for a circular scan in the presence of an angular misalignment of the conical mirror around the x - axis and a translational misalignment along the x axis, with $\alpha_{C}=0.2^{\circ}$ and $\Delta x_{D}=2 \mathrm{~mm}$


Figure 4.22 - Values predicted for the 1 x additional velocity term in the presence of a) translational misalignments of the vertex and conical mirror, $b$ ) angular misalignments of the conical mirror.


Figure 4.23 - Values predicted for the 2 x additional velocity term in the presence of a) translational misalignments of the conical mirror, b) angular misalignments of the conical mirror.


Figure 4.24 - Values predicted for the 3 x additional velocity term in the presence of a) translational misalignments of the conical mirror, b) angular misalignments of the conical mirror.


Figure 4.25 - a) Predicted scan pattern and b) velocity spectra for a circular scan in the presence of the following combined misalignments: $\quad \alpha_{L}=-0.5^{\circ}, \quad \beta_{L}=-0.5^{\circ}, \quad \alpha_{V}=-0.3^{\circ}, \quad \beta_{V}=0.3^{\circ}$, $\alpha_{C}=\beta_{C}=1^{\circ}, \Delta x_{A}=\Delta y_{A}=\Delta x_{D}=\Delta y_{D}=0 \mathrm{~mm}$.


Figure 4.26 - a) Predicted scan pattern and b) velocity spectra for a circular scan in the presence of the following combined misalignments: $\alpha_{L}=0^{\circ}, \quad \beta_{L}=1^{\circ}, \quad \alpha_{v}=\beta_{v}=0.3^{\circ}, \quad \alpha_{C}=0^{\circ}, \quad \beta_{C}=-1^{\circ}$, $\Delta x_{A}=5 \mathrm{~mm}, \Delta y_{A}=-5 \mathrm{~mm}, \Delta x_{D}=-5 \mathrm{~mm}$ and $\Delta y_{D}=5 \mathrm{~mm}$


Figure 4.27 - a) Scan pattern and b) predicted velocity obtained in the presence of manufacturing imperfections on the conical mirror


Figure 5.1 - a) Spectrum for the genuine radial vibration of the rotating target; b) spectrum of the equivalent LDV output in the presence of geometrical misalignments, torsional vibrations, noises.


Figure 5.2 - Behaviour of the function $W(\omega)$


Figure 5.3 - a) Typical arrangements used to isolate radial vibration sets; b) typical misalignments of the laser vibrometers

a)

b)

Figure 5.4 - Spectrum of the: a) unresolved x- radial vibration, b) unresolved $y$ - radial vibration in the presence of a translational misalignment $x_{0}=0.25 \mathrm{~mm}$


Figure 5.5 - a) Resolved velocity errors for the a) resolved x-radial vibration, b) resolved $y$ - radial vibration in the presence of translational misalignment $x_{0}$


Figure 5.6 - Optical device mounted on the laser vibrometer to perform a more accurate alignment


Figure 5.7 - Spectrum of the: a) unresolved x- radial vibration, b) unresolved $y$ - radial vibration in the presence of a translational misalignment $y_{0}=0.25 \mathrm{~mm}$


Figure 5.8 - a) Resolved velocity errors for the a) resolved x-radial vibration, b) resolved $y$ - radial vibration in the presence of translational misalignment $y_{0}$

a)

b)

Figure 5.9 - Spectrum of the: a) unresolved x- radial vibration, b) unresolved $y$ - radial vibration in the presence of angular misalignment $\varepsilon_{y}=0.25 \mathrm{~mm}$


Figure 5.10 - a) Resolved velocity errors for the a) resolved x-radial vibration, b) resolved $y$ - radial vibration in the presence of angular misalignment $\varepsilon_{y}$


Figure 5.11 - Spectrum of the: a) unresolved $x$ - radial vibration, b) unresolved y - radial vibration in the presence of angular misalignment $\delta_{y}=0.25 \mathrm{~mm}$


Figure 5.12 - a) Resolved velocity errors for the a) resolved x-radial vibration, b) resolved $y$ - radial vibration in the presence of angular misalignment $\delta_{y}$


Figure 5.13 - a) Spectrum of $x$ - radial vibration used for simulation with ratio 1 ; b) Spectrum of the x-radial vibration measured by the laser vibrometer in the presence of misalignments, noises, torsional vibrations and $y$ - radial vibration


Fig. 5.14 - a) Simulated velocity errors and b) simulated phase errors for single frequency radial vibrations characterized by an amplitude ratio of 1 and initial phase difference of $0^{\circ}$


Fig. 5.15 - a) Simulated velocity errors and b) simulated phase errors for single frequency radial vibrations characterized by an amplitude ratio of 0.1 and initial phase difference of $0^{\circ}$


Fig. 5.16 - a) Simulated velocity errors and b) simulated phase errors for single frequency radial vibrations characterized by an amplitude ratio of 10 and initial phase difference of $0^{\circ}$


Fig. 5.17 - a) Overall RMS amplitude error and b) phase error calculated along x - and the y - axis for the case with ratio 0.1


Fig. 5.18 - a) Overall RMS amplitude error and b) phase error calculated along x - axis for the case with ratio 1

b) With misalignments or as result of target vibration perpendicular to laser beam


Figure 5.19 - Case of aligned and misaligned incident laser beam on a smooth rotating target with circular cross-section


## Tested rotor



Figure 5.20 - Test rig used for the surface roughness investigations

$\mathbf{R a}=\mathbf{0 . 0 1 2 \mu m}$

$\mathbf{R a}=\mathbf{2 5 0} \boldsymbol{\mu \mathrm { m }}$
$\mathbf{R a}=1000 \mu \mathrm{~m}$

$\mathbf{R a}=$ Reflective tape

Figure 5.21 - Scattered light patterns from some of the tested cylinders


Figure 5.22 - Time domain for the $y$ - laser vibrometer signal output taken: a) on a smooth rotor in a measurement point different from the best aligned; b) on a rough rotor in a measurement point different from the best aligned


Figure 5.23 - a) Time domain and b) Frequency spectrum for the yLaser vibrometer signal output taken at the best measurement point on smooth rotor.


Figure 5.24 - a) Time domain and b) frequency spectrum for the $y$ Laser vibrometer signal output taken at a distance of 0.25 from the best measurement point on smooth rotor.


Figure 5.25 - a) Time domain and b) frequency spectrum for the $x$ laser vibrometer signal output.


Figure 5.26 - Ratios calculated for the various tested rotors in presence of vibration directed along the $x-a x i s$ at a) 15 Hz ; b) 30 Hz


Figure 5.26 - Ratios calculated for the various tested rotors in presence of vibration directed along the x -axis at c) 50 Hz

