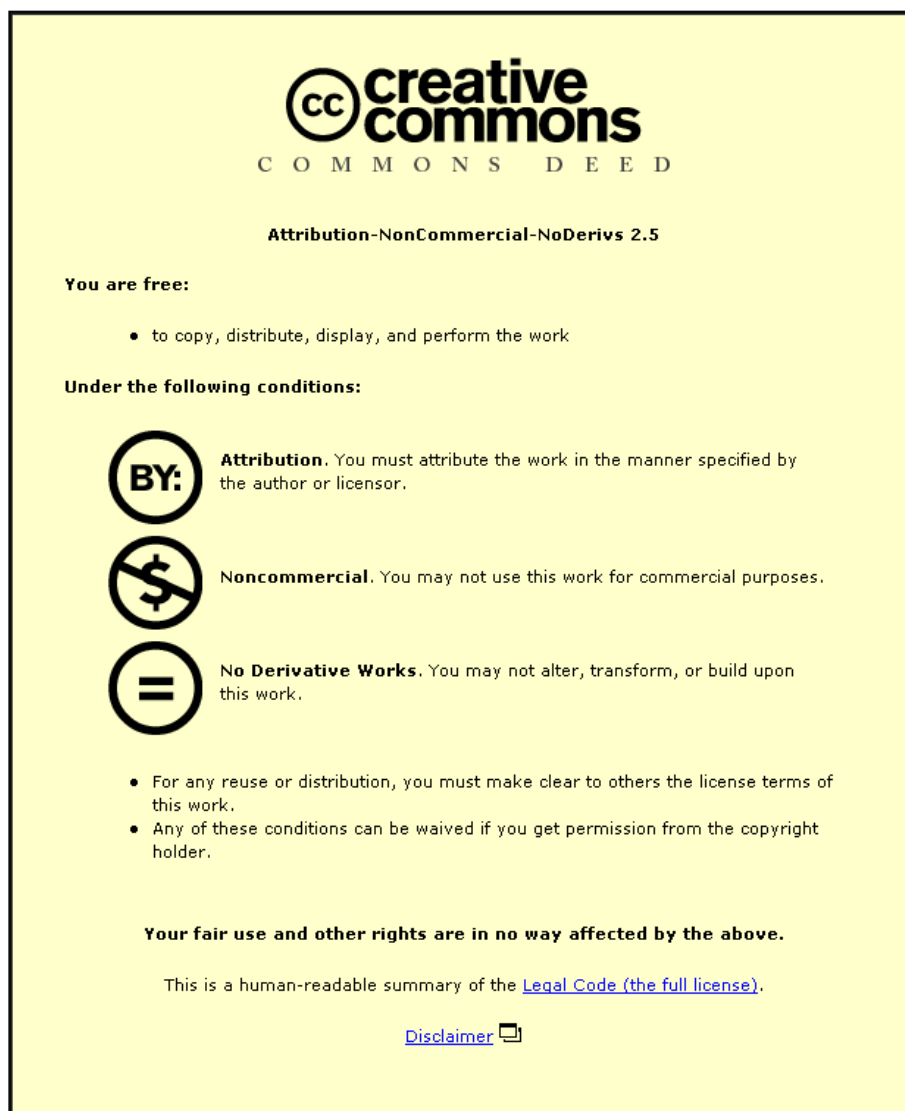


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DIGITAL COMPUTER ANALYSIS OF
VARIABLE-FREQUENCY INVERTER-INDUCTION
MOTOR DRIVE SYSTEMS

by

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A Doctoral Thesis
Submitted in Partial Fulfilment of the
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SUMMARY

Recently, the 3-phase d.c.-link inverter has found wide application in speed control schemes for induction motor drives, and several methods of analysing such systems have been presented. For an inverter with 180° conduction periods, a common approach is to assume that the motor is supplied with a precisely defined rectangular waveform. Several other papers have been based on this assumption, but have analysed the machine performance using techniques such as Fourier analysis and transition matrices, or have invoked the symmetry of the inverter motor unit to allow analysis to be performed over part of the supply cycle. While suitable for steady-state conditions, these methods may be unsuitable for transient operation, where the inverter waveforms are influenced by the motor conditions and the supply and filter impedances. Furthermore with inverters employing 120° conduction periods, one or more of the machine terminals is open circuited during each supply cycle, and during transients the machine terminal voltage waveshape will change significantly. An analysis of this type of inverter is by no means straightforward.

Several computational methods have been presented to analyse the inverter circuit; for example, the resistance method, the model subroutine method and the tensor technique based on the work of Kron, and it is found that the tensor technique has significant advantages over alternative methods of analysis. For the analysis of the machine, three methods have been studied. These are the

conventional transformed 2-axis model, the direct phase model and a new transformed 3-axis model. The last of these is recommended, due to its suitability for use with the inverter and its low computational time.

The inverter and the motor programs have been combined into one, and the performance of the whole system has been studied for various transient and fault conditions, for instance, starting, plugging, braking and terminal open and short circuit.

Practical results have been obtained from a laboratory inverter-motor system and very good agreement with theoretical results has been shown to exist.

When the variation of the inverter voltage and frequency are carefully controlled, an improvement in machine performance, for instance, a reduction in starting time, lower starting transients, has been achieved.

List of Principal Symbols

a, b, c	Suffixes denoting direct phase variables.	
d, q	Suffixes denoting transformed 2-axis variables.	
α, β, γ	Suffixes denoting transformed 3-axis variables.	
$T_1 \rightarrow T_6$	Inverter main power thyristors.	
$A_1 \rightarrow A_6$	Inverter auxiliary thyristors.	
$D_1 \rightarrow D_6$	Freewheeling or return diodes.	
D_p	Conduction period of the return diodes.	(s)
R_{t1}, R_{t2}	Commutating resistors.	(Ω)
R_f	Inverter supply and filter resistance.	(Ω)
R_{sh}	Resistance in the filter capacitor branch.	(Ω)
R_a, R_b, R_c	Phase a, b and c load resistances.	(Ω)
$R_1 \rightarrow R_6$	Thyristor/diode operational resistances.	(Ω)
R_s, R_r	Resistance per phase of the stator and rotor circuits, respectively.	(Ω)
$L_{t1} \rightarrow L_{t6}$	Commutating inductors.	(H)
L_f	Inverter supply and filter inductance.	(H)
L_a, L_b, L_c	Phase a, b and c load inductances.	(H)
L_{ss}, L_{rr}	Self inductances per phase of the stator and rotor circuits, respectively.	(H)
L_{sl}, L_{rl}	Leakage inductances per phase of the stator and rotor circuits, respectively.	(H)
L_{sm}	Mutual inductance between stator phases.	(H)
L_{rm}	Mutual inductance between rotor phases.	(H)
L_e	Exciting inductance of the induction machine equivalent circuit.	(H)
M_{sr}	Maximum mutual inductance between stator and rotor circuits.	(H)
C_{t1}, C_{t2}	Commutating capacitors.	(F)
C_{sh}	Inverter filter capacitance.	(F)
E	Inverter supply d.c. input voltage.	(V)
V_s	Inverter d.c.-link instantaneous voltage.	(V)
V_r	Inverter auxiliary d.c. supply voltage.	(V)
V_{c1}, V_{c2}	Instantaneous voltages across C_{t1} and C_{t2} respectively.	(V)
$v_1, v_2, v_3 \dots$	Instantaneous voltages across branch 1, 2, 3 ... of the primitive network.	(V)
V_{max}	Maximum phase voltage	(V)

i_{dc}	Inverter d.c. link instantaneous current.	(A)
$i_1, i_2, i_3 \dots$	Instantaneous currents through branch 1,2,3... of the primitive network.	(A)
$i_A, i_B, i_C \dots$	Instantaneous independent currents of the basic network.	(A)
$i_{x_1}, i_{x_2}, i_{x_3} \dots$	Instantaneous independent currents of the reduced network.	(A)
$x_1 \rightarrow x_4$	Instantaneous independent state variables for the model subroutine method.	
q	Instantaneous charge on C_{sh} .	(C)
θ	Relative position angle of the rotor with respect to stator.	(Elect.Rad)
w	Instantaneous motor speed	(Elect.Rad/s)
w_s	Synchronous speed.	(Elect.Rad/s)
s	Slip.	
T_e	Electromagnetic torque developed.	(N.m.)
T_m	Mechanical torque applied.	(N.m.)
J	Moment of Inertia.	(kg.m ²)
P	Number of pole pairs.	
K_f	Rotor friction coefficient.	(kg.m ² /s)
f	Instantaneous supply frequency.	(Hz)
F_{in}	Initial increase of f .	(Hz).
F_s	Steady-state operational frequency.	(Hz)
S_f	Slip frequency at which maximum torque is produced.	(Hz)
H	Time for f to reach F_s .	(s)
t	Time.	(s)
p	d/dt operator.	

$[R_p]$	Primitive network resistance tensor.
$[R_n]$	Basic network resistance tensor.
$[R_{nn}]$	Reduced network resistance tensor.
$[R_m]$	Resistance tensor of the motor system..
$[L_p]$	Primitive network inductance tensor.
$[L_n]$	Basic network inductance tensor.
$[L_{nn}]$	Reduced network inductance tensor.
$[L_m]$	Inductance tensor of the motor system.
$[G_p]$	Primitive network rotational voltage coefficient tensor of the inverter-motor system.
$[G_n]$	Basic network rotational voltage coefficient tensor of the inverter-motor system.
$[G_{nn}]$	Reduced network rotational voltage coefficient tensor of the inverter-motor system.
$[G_m]$	Rotational voltage coefficient tensor of the motor system.
$[S_p]$	Primitive network inverse-of-capacitance tensor.
$[S_n]$	Basic network inverse-of-capacitance tensor.
$[S_{nn}]$	Reduced network inverse of capacitance tensor.
$[sq_p]$	Primitive network instantaneous capacitor voltage tensor.
$[sq_n]$	Basic network instantaneous capacitor voltage tensor.
$[sq_{nn}]$	Reduced network instantaneous capacitor voltage tensor.
$[v_p]$	Primitive network instantaneous source voltage tensor.
$[v_B]$	Primitive network instantaneous branch voltage tensor.
$[v_n]$	Basic network instantaneous source voltage tensor.
$[v_{nn}]$	Reduced network instantaneous source voltage tensor.
$[v_m]$	Instantaneous voltage tensor of the motor system.
$[v_x]$	Mesh-sum thyristor/diode instantaneous voltages of the bridge inverter.
$[i_p]$	Primitive network instantaneous branch current tensor.
$[i_n]$	Basic network instantaneous independent current tensor.
$[i_{nn}]$	Reduced network instantaneous independent current tensor.
$[i_m]$	Instantaneous current tensor of the motor system.
$[\psi_m]$	Instantaneous flux tensor of the motor system.
$[q_{nn}]$	Reduced network instantaneous charge tensor.

$[C_p]$	Transformation tensor between primitive and basic networks.
$[C_h]$	Transformation tensor between basic and reduced networks.
$[C_m]$	Transformation tensor between the motor direct phase and transformed axes variables.
$[C_k]$	Row matrix link the filter capacitor current and $[i_n]$.

* All other symbols are defined as they appear.

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CHAPTER 1

INTRODUCTION

Many applications in industry require continuously adjustable speed drives. For many years d.c. commutator motors have been used in most situations where control of speed is required, because they operate at a high efficiency over a wide range of conditions, and their speed can be varied in a relatively simple manner. In many applications, however, the commutator is an embarrassment, since it requires periodic maintenance and can present considerable problems in, for instance, hazardous environments.

A.C. motors, such as the squirrel cage induction motor and the synchronous reluctance motor, have a simple rotor construction which results in a cheaper and more reliable machine. Unfortunately these machines are basically inflexible as regards speed, since this is directly related to the normally-constant supply frequency. Variable-speed a.c. drives can of course be produced, for example, by adjusting the supply voltage or by using a variable rotor resistance,¹ but the techniques used are either inherently inefficient or afford only a limited range of speed variation. For efficient wide-range speed control, the stator frequency must be varied. Variable frequency supplies from rotating machine sets have been used for many years although they, too, have many disadvantages.¹

Electronic power convertors involving thyratrons have also been used in speed control systems², but because of difficulties

encountered with these devices, power convertor technology has remained dormant for many years. However, there has been a tremendous rebirth of interest in power convertors since the development of transistors and thyristors. The thyristor, being an efficient, rugged, durable and compact device, with short switching time, is increasingly being applied in solid-state a.c. drives, while transistors are also being used in small power convertors. One very important advantage of power convertors is the production of a stable, accurate frequency, which being internally generated is independent of load and transient conditions.

There are two basic types of static frequency convertors, in the first, the cycloconverter,³⁻⁷ the a.c. supply frequency is converted directly into a.c. of variable frequency. In this case, the thyristors are used to selectively connect the load to the supply source, so that the low-frequency output voltage waveform is fabricated from segments of the supply voltage waveform. The disadvantage of this type of convertor is that the highest output frequency is limited to not more than $1/3$ of the supply frequency if too high a harmonic content is to be avoided. The second type of convertor is the d.c.-link 3-phase bridge⁷⁻¹⁵ inverter, in which the a.c. supply is first rectified to d.c. and then inverted to a.c. at a variable frequency. The main power thyristors of the inverter are triggered sequentially such that a rectangular or stepped square wave voltage is generated at the output. In contrast to the cycloconverter, the output frequency of the d.c.-link inverter can range from a few hertz up to several

hundred hertz. Mainly for this reason, d.c-link inverters have found wide application in industry and a study of the inverter-motor performance has consequently become more important. The thesis is therefore concerned with the analysis of a 3-phase bridge d.c.-link type inverter connected to an induction motor system.

One of the important points that affects the inverter as well as the motor performance is the conduction period of the main thyristors of the inverter. It can theoretically be fixed at any interval between less than 90° and 180° . However, intervals of 120° and 180° are particularly advantageous, and both are in widespread use.¹⁶⁻²²

Many papers have analysed an induction motor supplied by a d.c.-link type inverter, and a common approach has been to assume that the motor supply has a precisely defined waveform. Based on this assumption, analytic solutions have been obtained, using for example matrix methods,²³⁻²⁶ Fourier analysis,²⁷⁻²⁸ time domain analysis based on the 2-reaction theory,²⁹ time domain complex variables³⁰ and a method of multiple reference frames.³¹⁻³² While suitable for motors driven by 180° inverters, these techniques are unsuitable for systems employing 120° inverters, since in this case the inverter voltage no longer has a well-defined waveform and its shape varies considerably during transient and changing load conditions. However, analysis of the motor driven by this kind of inverter has been presented in the literature, for example, by analysing each circuit configuration separately.³³ In this study,

the d.c. voltage across the inverter was assumed constant. Another study³⁴ was to invoke the symmetry of the inverter and the motor and to analyse only 1/6 of a complete cycle. The effects of the inverter supply and filter impedances were included in this analysis. Unfortunately all the techniques mentioned have been concerned with the constant speed, normal operation of the inverter-motor system. Dynamic performance of the start-up condition has been presented using a digital computer,³⁵ but again assuming that the motor supply voltage waveform is known, and fixed. Alternatively, an analogue computer has also been used to simulate the steady-state performance and some dynamic conditions of the inverter-motor system.³⁶⁻³⁸

Although this study has included the effect of the inverter supply and filter impedances, it is limited to normal operations and to systems employing 180° inverters. It appears from the literature that a study of the dynamic behaviour of systems with 120° inverters has not been presented. Moreover, when 180° inverters are used, the output voltage is affected during abnormal and fault conditions. For an accurate analysis over a wide range of steady-state and transient conditions for both constant and variable-frequency operation, including normal and abnormal conditions and for systems with 180° and 120° inverters, it is therefore important that the inverter with its associated supply and filter impedances are included.

The analysis of the inverter-induction motor system using a digital computer is undertaken in three separate stages, and these are presented in the thesis. In the first stage, three techniques

of analysing an inductively-loaded inverter are presented and discussed, these being the model subroutine,³⁹⁻⁴² the resistance,⁴³ and the tensor technique^{44,45} methods. Among these the tensor technique is finally adopted, due to its significant advantages when compared with the other two methods. The technique, which is based on the work of Kron,^{46,47} has been successfully applied to the analysis of a line-commutated bridge convertor.⁴⁵

The second part of the analysis deals with the induction motor. Three models are presented and discussed, the direct phase,³⁵ the conventional transformed 2-axis⁴⁸ and the transformed 3-axis models.⁴⁹ Among these, the transformed 3-axis model is finally adopted since it combines the advantages of the other two models.

These techniques for analysing the inverter and the motor are then combined in the third stage of the analysis, in order to analyse the complete inverter-motor system. Several operating and fault conditions are studied, and results for start-up, loading, braking, plugging, sudden frequency changes, stator phase short circuit and stator phase open circuit conditions are presented.

Finally, the variable frequency performance of the induction motor is considered. It was shown by Lawrenson and Stephenson⁵⁰ that efficient and rapid speed changing of the motor can be achieved by linear variation of the supply frequency with time, although the study was concerned only with a sine wave supply. In the present investigation, inverter supplies are considered, and the study is extended to include a constant slip frequency

control technique. Results of the start-up performance for all variable frequency starting techniques are compared with the usual direct-on-line starting method.

Throughout the thesis, the theoretical investigation is supported by considerable experimental work, and good agreement is obtained between the computed and experimental results, thus giving a high degree of confidence in the methods of analysis developed.

The final appendix contains several publications arising from the work described in this thesis, and further publications are planned.

CHAPTER 2

THE D.C.-LINK INVERTER SYSTEM

In this chapter, the 3-phase bridge d.c.-link inverter is discussed in some detail. Various types of commutation methods can be employed in these inverters, and some possible methods are discussed. Subsequently, the individual d.c. side commutation method is explained in detail and is considered exclusively in the work described in this chapter.

Considerable attention is given to two widely used inverters (i.e. those with 120° and 180° conduction periods). All possible circuit configurations for both inverters are presented, and these form the basis of the inverter analyses presented in Chapter 3.

2.1 General

The basic circuit diagram of the 3-phase bridge d.c.-link inverter system is shown in Fig. 2.1. A rectangular or stepped voltage is generated at the output by gating thyristors ($T_1 \rightarrow T_6$) at uniform intervals. Diodes ($D_1 \rightarrow D_6$) serve as 'free wheeling' or 'return' diodes. The inverter output frequency is determined by the gating frequency of the thyristors.

The d.c. supply voltage v_s is normally derived from phase-controlled rectifiers whose control circuits operate in a closed-loop with the inverter in order to vary the alternating output voltage in proportion to the output frequency and so maintain approximately constant air gap flux in a driven motor.

Since a d.c. voltage is applied across the thyristors, there is no natural tendency for them to turn off, and means must be provided to force the current to zero (forced commutation). The most widespread application of forced commutation techniques in inverters is 'impulse commutation'⁸ which may be applied in several ways as explained in the following section.

2.2 Impulse Commutation Methods

The term impulse commutation is applied to the use of an impulse to reverse briefly the voltage on a conducting thyristor, thereby allowing it to turn off. The pulse is generally formed by means of an oscillatory inductance-capacitance network and it may be initiated, in a 3-phase bridge inverter, either

- a) by firing the thyristor which is complementary to the thyristor being turned off (complementary impulse commutation), or
- b) by firing an auxiliary thyristor (auxiliary impulse commutation).

The latter arrangement allows a wider choice of firing sequence since the turn off and firing processes are divorced from each other, moreover loss of gating signals does not result in a commutation failure.

There are many possible circuit arrangements in which auxiliary thyristors are used for turn off purposes.⁵¹ In a 3-phase bridge inverter, a thyristor in the upper half of the bridge may

be turned off by reducing momentarily the anode potential below the cathode potential, or alternatively by raising momentarily the cathode potential above the anode potential. These processes are termed d.c.-side and a.c.-side commutation respectively, since the commutation pulse is applied to the d.c. or a.c. terminals of the thyristor by the auxiliary circuit.

The auxiliary commutating circuits can be arranged either to reverse bias all inverter thyristors simultaneously (fully commutated), or half the total number (half commutated), or only one (individually commutated).

Half and individual a.c.-side commutation techniques are widely used for medium and high power inverter motor drives^{14,52} while full d.c.-side commutation is the normal process used for low-power inverter motor drives^{13,53}. However, the individual d.c.-side commutation inverter is considered in this thesis since it has the advantages explained in the following section.⁹

2.3 The Inverter Circuit

2.3.1 Description

The complete individual d.c.-side commutation inverter circuit is shown in Fig. 2.2. Each component of this type of inverter performs only one function, and there is no undesirable interaction between the various parts of the circuit. The rates-of-voltage rise are well-controlled under all operating conditions of the motor, and the circuit is particularly simple to investigate experimentally.

It also has the advantage that each commutating inductor has to withstand the commutating pulse only once per cycle, which gives a longer period for the decay of the resultant circulating current. Moreover, in this type of inverter, there is no undesirable commutation failure at very low frequencies, when the main d.c. supply voltage is also very low. This desirable feature is due to the use of an auxiliary commutation supply (shown in Fig. 2.2. as V_r).

2.3.2 Operation

The operation of the inverter is such that each load phase is alternately connected to the positive and negative supply lines, by gating the main thyristors at uniform intervals throughout the sequence 1-2-3-4-5-6. Each auxiliary thyristor is provided with a single pulse every cycle, which is timed with respect to the end of the conduction period of the corresponding main thyristor. The conducting thyristor is switched off by the action of the commutation process (as discussed in Section 2.3.3) and the diode in the corresponding opposite leg of the bridge conducts to allow the inductive current to decay to zero. A 3-phase output is obtained by preserving a phase displacement of 60° between any two successive gating pulses delivered to the main thyristors (and to the auxiliary thyristors).

The inverter output frequency depends on the switching rate of the thyristors, which is controlled in turn by the frequency of the oscillator in the control circuit (Section 2.3.4).

2.3.3 The Commutation Process

Only a brief explanation of the commutation process will be given here, since detailed analysis can be found elsewhere⁵⁴.

Consider the instant at which thyristor T_1 is given a reverse bias via the auxiliary thyristor A_1 . Fig. 2.3 shows the state of the relevant part of the circuit immediately before the commutation process begins. At this stage only thyristor T_1 is conducting, and it is therefore carrying full load current.

The commutating capacitors C_{t_1} and C_{t_2} are charged, with the polarity shown, from a previous process, and this gives thyristor A_1 a forward bias. When A_1 is triggered, point y , whose potential is below v_s by the amount v_{c_1} , is connected to the anode of thyristor T_1 . Thyristor T_1 is thus reversed biased and its current is thereby forced to zero.

Diode D_1' becomes forward biased and helps to shunt the commutated load current away from the capacitors preventing excessive charge building up on them.

2.3.4 Electronic Control Circuit

The function of the control circuit is to provide trigger signals to the main and auxiliary thyristors.

Signals to the main thyristors, properly routed for the correct sequence, must appear at 60° intervals, and each turn on signal is of either 120° or 180° duration. This type of pulse is necessary with inverters supplying inductive loads and it also produces closed-loop current via the load at starting, since the

trigger pulses for the two thyristors overlap.

The gate circuit of each thyristor is usually isolated from the associated triggering circuit by means of a pulse transformer. It is therefore necessary, and desirable, as a means of reducing gate dissipation, to present a train of short duration pulses to the gates, instead of 120° (or 180°) envelopes. These short pulses, generated by a separate oscillator, are common to all six thyristors. For each auxiliary thyristor a single turn-on pulse is used per cycle, with these pulses being sequenced and timed at the end of the conduction period of the main thyristors.

In the case of 180° conduction periods, the train of pulse signals for each main thyristor are inhibited at the beginning of the conduction period for a short time, allowing the main thyristor on the opposite leg to switch off before the other one conducts, thus preventing a short circuit fault across the d.c.-link of the bridge.

A schematic diagram of the electronic control circuit for 120° conduction periods is shown in Fig. 2.4 which is much the same as for 180° conduction periods except where indicated. Fig. 2.5 shows the formation of the main and auxiliary thyristor signals for both 120° and 180° conduction periods. Note that the frequency of the master oscillator is six times the gating frequency of the thyristors. Pictures of the practical pulses for one main thyristor (and its corresponding auxiliary thyristor) for 120° and 180° conduction periods are shown in Fig. 2.6.

2.4 Inverters of Different Conduction Periods

Regardless of the commutation method used, the inverter outputs are influenced by the period of the gating pulses for each main thyristor. In practice these periods are of either 120° or 180° duration.

2.4.1 180° Inverters

Inverters of 180° conduction periods always have three thyristors triggered simultaneously, to ensure continuous a.c. current in the motor. In general, turn-off and turn-on signals are applied almost simultaneously to corresponding top and bottom leg thyristors of the bridge.

Each main thyristor conducts for $\frac{1}{2}$ cycle for resistive loading, and less when the load is inductive.

2.4.2 120° Inverters

In inverters with 120° conduction periods, the turn-on signal for a particular thyristor is applied 60° after the complementary thyristor in the opposite half of the conducting phase has turned off. With a resistive load, each phase is therefore open circuit for a 60° period. For lower power factor loads, the period of open circuit is reduced, because of the time needed for the reactive current in the phase to decay to zero through the free-wheeling diode in the opposite half of the bridge.

As the power factor decreases the decay time becomes greater, until it eventually becomes equivalent to 60° . There is now no

open circuit for either the incoming or the outgoing phase, so that the motor is operating exactly as if the conduction period is 180° . This can be demonstrated experimentally with the inverter driving the motor under no load conditions, when the power factor is very low.

The conduction pattern of the main thyristors and the return diodes is shown in Fig. 2.7 for the two different conduction periods of 180° and 120° . Note that the period of current decay through the return diodes has been chosen arbitrarily.

2.5 Circuit Topologies

The circuit topology of the inverter feeding a 3-phase star-connected load depends obviously on the state of the conducting devices. This state is such that no two devices are conducting in the same leg of the bridge, otherwise there is a short circuit across the d.c.-link. With this proviso, each of the three phases is connected in turn to either the positive or the negative line of the supply. In the case of a 120° conduction period, a total of twelve circuit configuration possibilities exist as shown in Fig. 2.8, with six of these being due to periods of phase open-circuit. However, in the case of 180° conduction periods, there are only six possible circuit configurations.

2.6 Output Voltage Waveforms

From the derived inverter circuit topologies of Fig. 2.8, the theoretical output phase-voltage waveforms can now readily be drawn. Fig. 2.9a shows the phase voltage for the 180° inverters. This waveform is the same for any load power factor, the only difference being the period of conduction of the return diodes (D_p).

When the 120° inverter with a resistive load is employed, the phase voltage waveform is as shown in Fig. 2.9b. But for inductive load with D_p less than 60°, the phase voltage waveform becomes more complicated and is as shown in Fig. 2.9c. The lower the power factor of the load the higher the value of D_p , until 60° is reached. The phase voltage waveform is then similar to that of Fig. 2.9a.

2.7 Conclusion

This chapter has been concerned with the description and operation of d.c.-link 3-phase bridge inverters employing both 120° and 180° conduction periods.

Regardless of the commutation method used, twelve different inverter topologies were shown to exist for 120° inverters and six for 180° inverters. A 6-step square wave voltage can be obtained from these inverters, except in the case of a 120° inverter supplying a high power factor load, when the output waveform is more complicated.

To study circuits of this kind, the next chapter will present various methods for the analysis of both 120° and 180° inverters.

CHAPTER 3

ANALYSIS OF THE INVERTER CIRCUIT

In this chapter, several methods for analysing circuits containing switching devices are presented. These are used to investigate an inductively-loaded d.c.-link 3-phase bridge inverter. The advantages and disadvantages of each method are discussed and a computer program is produced. The most useful method is subsequently selected for the analysis of the inverter-motor system presented in later chapters.

Experimental and computed results of the inverter output voltage and current for the two 120° and 180° inverters are presented and compared.

3.1 General Assumptions

Before describing the different methods of analysis, the following assumptions are considered which significantly simplify the analysis.

- a) The supply to the inverter is represented by an adjustable direct voltage E with an effective supply impedance incorporated in the impedance of the filter inductance and represented by $R_f + pL_f$. The filter (smoothing) capacitors are represented by C_{sh} , as shown in Figs. 3.1, 3.2 and 3.4.
- b) Each parallel diode/thyristor in the bridge is treated as one branch, since these devices never conduct simultaneously.

- c) The thyristors and diodes can be considered as instantaneous switching devices, since at normal operating frequencies, their commutation times are very short in comparison with the inverter output period.
- d) The commutating inductors which are in series with the main thyristors are neglected (because of their small values) although they can easily be included in the analysis if desired.

3.2 Methods of Analysis

The three methods presented here are called the 'model subroutine', the 'resistance' and the 'tensor technique' methods. The aim of each is to set-up the correct system differential equations that describe a particular topology, and the differences between them lie mainly in the way the switching of the thyristors and diodes is dealt with.

Since the inverter operation involves a changing topology and, additionally, the inverter circuit contains capacitive as well as inductive elements, it is convenient to use a step-by-step numerical integration of the system differential equations. The system equations may be written in the following form suitable for a computer program solution:

$$\dot{p}x = f(x,t)$$

3.1

A numerical solution becomes even necessary when analysing the dynamic performance of the inverter-motor system, as will be shown later in chapter 4.

At the end of each integration step, conducting thyristors and diodes are tested for turn off, and non-conducting thyristors and diodes for turn on, to determine whether the inverter topology remains constant or changes, and the system equations are changed to suit the new topology.

3.2.1 Model Subroutine Method

In this method, each different operating condition of the bridge inverter, associated with a different network topology, is analysed separately. Kirchhoff's laws are used to produce the differential equations defining the circuit conditions for the particular topology, and each set of equations is assigned to a subroutine in the computer solution. Since, as mentioned in chapter 2, there are twelve different circuit topologies during normal operation of the inverter, twelve sets of differential equations (and hence twelve subroutines) are needed for a full circuit description.

Only one set of differential equations defining one particular inverter circuit topology will be given here, since the rest may be formed in a similar manner. Taking, as this example, the case when thyristors T_1 and T_6 and diode D_2 are conducting, the circuit topology is as shown in Fig. 3.1. This circuit has three nodes,

five branches and one capacitor, and four state variables $x_1 \rightarrow x_4$ are needed to define completely the network equations, where;

x_1 represents the source current

x_2 represents the capacitor voltage

x_3 represents the phase a current (towards the star point)

x_4 represents the phase b current (towards the star point).

(These variables are the same for any network topology).

The four differential equations for this configuration can be written as:

$$E = R_f x_1 + L_f p x_1 + R_{sh} C_{sh} p x_2 + \dot{x}_2$$

$$E = R_f x_1 + L_f p x_1 + R_a x_3 + L_a p x_3 - R_b x_4 - L_b p x_4 \quad 3.2$$

$$R_b x_4 + L_b p x_4 = R_c (-x_3 - x_4) + L_c (-p x_3 - p x_4)$$

$$x_1 - C_{sh} p x_2 = x_3$$

or in the form of equation 3.1

$$px_2 = (x_1 - x_3)/C_{sh}$$

$$px_1 = (E - R_f x_1 - R_{sh} C_{sh} px_2 - x_2)/L_f$$

$$px_3 = (L_b + L_c) [(E - R_f x_1 - L_f px_1 - R_a x_3 + R_b x_4 - L_b/(L_b + L_c))$$

$$(R_b x_4 + R_c x_3 + R_c x_4)]/(L_a L_b + L_a L_c + L_b L_c)$$

$$px_4 = -(R_b x_4 + R_c x_3 + R_c x_4 + L_c px_3)/(L_b + L_c) \quad 3.3$$

Once the differential equations are established for any particular topology, they can be solved numerically and the load voltage and currents can be found. The current through a conducting thyristor (or diode) for one phase is the same as the load current of the same phase.

3.2.2 Resistance Method

In this method, each parallel thyristor/diode branch is represented by an ideal resistor, as shown in Fig. 3.2, with the switching action being simulated by an appropriate choice of this resistor. Values of 0.05Ω for the on state and $300\text{ k}\Omega$ for the off state would normally be suitable. The inverter circuit of Fig. 3.2 requires seven variables (six for the currents plus one for the filter capacitor charge) to define the system equations.

These can easily be developed in the form of equation 3.1. It is obvious that under switching operation the network topology remains unchanged, but the numerical values of the system parameters have to be changed.

3.2.3 Tensor Technique Method

This method is based on the concepts associated with the tensor analysis of electrical networks.⁴⁷ It allows for the switching action of the diodes and thyristors by producing automatically the correct transformation tensor required to assemble the appropriate network equations. Williams and Smith⁴⁵ were the first to use the method for the analysis of a line commutated bridge converter and it is presented here for the analysis of the d.c.-link bridge inverter.

Before describing the steps in the analysis, it is advantageous to assume initially that each parallel diode/thyristor can be represented by one resistor branch, as shown in Fig. 3.2. This forms the network with the highest number of branches possible and is called the basic network; where all the branches are numbered as indicated. Note that in order to facilitate the analysis, the thyristor (and diode) numbers appear in a different order from their normal firing sequence.

3.2.3.1 Primitive Network

The first stage in the analysis is to build a 'primitive network' by removing all connections between circuit elements and forming, in conjunction with each of these, the simplest physically realisable circuit. Using this technique, the primitive network for the circuit of Fig. 3.2 can be developed, as shown in Fig. 3.3, from which the primitive resistance $[R_p]$, the primitive inductance $[L_p]$ and the primitive inverse of capacitance $[S_p]$ tensors can be determined.

These are:

$$[R_p] = \text{diag} [R_1, R_2, R_3, R_4, R_5, R_6, R_a, R_b, R_c, R_{sh}, R_f]$$

(The devices operational resistances $R_1 \rightarrow R_6$ can be set to the device's forward resistances).

$$[L_p] = \text{diag} [0, 0, 0, 0, 0, 0, L_a, L_b, L_c, 0, L_f]$$

and $[S_p] = \text{diag} [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/C_{sh}, 0]$ respectively.

The primitive current tensor $[i_p]$ is:

$$[i_p] = [i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9, i_{10}, i_{11}]^t$$

The primitive voltage source tensor $[v_p]$ is

$$[v_p] = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, E]^t$$

The branch voltage tensor $[v_B]$ is

$$[v_B] = [v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}]^t$$

The primitive network equations can now be written:

$$[v_B] = [R_p][i_p] + [L_p]p[i_p] + [S_p] \int [i_p] - [v_p] \quad 3.4.$$

3.2.3.2 Basic Network

The relationship between the primitive current tensor $[i_p]$ and the independent current tensor

$$[i_n] = [i_A, i_B, i_C, i_D, i_E, i_F]^t$$

of the basic network has now to be developed. With the 6-independent currents defined, as shown in Fig. 3.2, the relationship between the two sets of currents is;

$$i_1 = i_A$$

$$i_2 = i_B$$

$$i_3 = i_C$$

$$i_4 = i_D$$

$$i_5 = i_E \quad 3.5$$

$$i_6 = -i_A - i_B - i_C - i_D - i_E$$

$$i_7 = i_A + i_B$$

$$i_8 = i_C + i_D$$

$$i_9 = -i_A - i_B - i_C - i_D$$

$$i_{10} = -i_A - i_C - i_E + i_F$$

$$i_{11} = i_F$$

These equations can be written in standard form as:

$$[i_p] = [C_p][i_n] \quad 3.6$$

where the transformation tensor $[C_p]$ is

$$[C_p] = \begin{bmatrix} 1 & & & & & & & & & \\ & 1 & & & & & & & & \\ & & 1 & & & & & & & \\ & & & 1 & & & & & & \\ & & & & 1 & & & & & \\ & & & & & 1 & & & & \\ -1 & -1 & -1 & -1 & -1 & & & & & \\ & 1 & 1 & & & & & & & \\ & & & 1 & 1 & & & & & \\ -1 & -1 & -1 & -1 & & & & & & \\ -1 & & -1 & & -1 & 1 & & & & \\ & & & & & & 1 & & & \end{bmatrix} \quad 3.7$$

This tensor is then used to transform equation 3.4 to an equation which describes the basic network

$$[v_n] = [R_n][i_n] + [L_n]p[i_n] + [S_n]f[i_n] \quad 3.8$$

where $[R_n]$, $[L_n]$, $[S_n]$ and $[v_n]$ are obtained from the following standard transformations, based on invariance of power⁴⁷:

$$\begin{aligned} [R_n] &= [C_p]^t [R_p] [C_p] \\ [L_n] &= [C_p]^t [L_p] [C_p] \\ [S_n] &= [C_p]^t [S_p] [C_p] \\ \text{and } [v_n] &= [C_p]^t [v_p] \end{aligned} \quad 3.9$$

Note that the branch voltage tensor $[v_B]$ is no longer important since $[C_p]^t [v_B]$ (representing the loop voltages) is null.

3.2.3.3 Reduced Network

Although equation 3.8 is valid when all devices are conducting, as explained earlier, under operating conditions only a small number of devices are conducting at any time. Again, the tensor techniques allow the relationship between the basic network and any particular topology to be established. Thus, as an example, when only thyristors T_1 and T_6 and diode D_3 are conducting, the basic network of Fig. 3.2 becomes the reduced network of Fig. 3.4. With the 3-new independent current ($[i_{nn}] = [i_{x_1}, i_{x_2}, i_{x_3}]^t$ needed for this particular topology chosen as shown in the figure, the relationship between the old and new currents gives rise directly to the transformation tensor:

$$[C_n] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 3.10$$

This tensor is produced in a mechanical way within the computer program every time the circuit topology changes (see Appendix A).

The equation describing the reduced network, i.e.

$$[v_{nn}] = [R_{nn}][i_{nn}] + [L_{nn}]p[i_{nn}] + [S_{nn}] \int [i_{nn}] \quad 3.11$$

can now be obtained using transformations similar to equation

3.9:

$$\begin{aligned} [R_{nn}] &= [C_n]^t [R_n] [C_n] \\ [L_{nn}] &= [C_n]^t [L_n] [C_n] \\ [S_{nn}] &= [C_n]^t [S_n] [C_n] \end{aligned} \quad 3.12$$

and $[V_{nn}] = [C_n]^t [v_n]$

To rewrite equation 3.11 in the form of equation 3.1, a new charge tensor $[q_{nn}]$ may be defined as

$$p[q_{nn}] = [i_{nn}]$$

and therefore 3.13

$$p[i_{nn}] = [L_{nn}]^{-1} \{ [v_{nn}] - [R_{nn}][i_{nn}] - [S_{nn}][q_{nn}] \}$$

or in matrix form

$$\begin{bmatrix} p [q_{nn}] \\ p [i_{nn}] \end{bmatrix} = \begin{bmatrix} [0] & [1] \\ -[L_{nn}]^{-1}[S_{nn}] & -[L_{nn}]^{-1}[R_{nn}] \end{bmatrix} \begin{bmatrix} [q_{nn}] \\ [i_{nn}] \end{bmatrix} + \begin{bmatrix} [0] \\ [L_{nn}]^{-1}[v_{nn}] \end{bmatrix} \quad 3.14$$

Since there is only one capacitor in the circuit, the above system of equations can be reduced in number by an alternative treatment of the capacitor. If the instantaneous charge on the capacitor is q , the instantaneous voltage of q / C_{sh} can be used in a capacitor voltage tensor $[sq_p]$, which can be treated in the same way as $[v_p]$

$$\text{i.e. } [sq_p] = [0, 0, 0, 0, 0, 0, 0, 0, 0, q / C_{sh}, 0]^t$$

$$[sq_n] = [C_p]^t [sq_p] \quad 3.15$$

$$[sq_{nn}] = [C_n]^t [sq_n]$$

and therefore the new system equations become

$$p[i_{nn}] = [L_{nn}]^{-1} \{ [v_{nn}] - [R_{nn}][i_{nn}] - [sq_{nn}] \} \quad 3.16$$

$$pq = [C_K][C_n][i_{nn}]$$

where $[C_K]$ is row 10 of the $[C_p]$ tensor.

After solving equations 3.16 for $[i_{nn}]$ and q , the independent and primitive current tensors can be found from:

$$[i_n] = [C_n][i_{nn}] \quad 3.17$$

$$[i_p] = [C_p][i_n]$$

The thyristor and/or diode currents are given by the first six components of $[i_p]$. The current derivative tensors are calculated as above from

$$p[i_n] = [C_n]p[i_{nn}] \quad 3.18$$

$$p[i_p] = [C_p]p[i_n]$$

The branch voltage tensor $[v_B]$ can now be calculated from

$$[v_B] = [R_p][i_p] + [L_p]p[i_p] + [sq_p] - [v_p] \quad 3.19$$

It should be noted that $v_1 \rightarrow v_6$ obtained from the above equations do not represent the thyristor/diode voltages when these devices are non-conducting, because $R_1 \rightarrow R_6$ have been initially set to the device forward resistances. However the off thyristor/diode voltage can be found as follows:

By application of Kirchhoff's voltage law, an array of mesh-sum thyristor/diode voltages $[v_x]$ can be found as

$$[v_x] = [R_n][i_n] + [L_n]p[i_n] + [sq_n] - [v_n] \quad 3.20$$

where $[v_x]$ has the form

$$[v_x] = \begin{bmatrix} v_1 - v_6 \\ v_2 - v_6 \\ v_3 - v_6 \\ v_4 - v_6 \\ v_5 - v_6 \\ 0 \end{bmatrix} \quad 3.21$$

If thyristor (or diode) 6 is conducting, $v_6 = 0$ and the remaining thyristor/diode voltages may be calculated immediately. If thyristor (or diode) 6 is not conducting, v_6 can be determined from the element of $[v_x]$ which corresponds to a conducting device and $v_1 \rightarrow v_5$ can be subsequently calculated.

The inverter current i_{dc} which is the current flowing into the inverter as shown in Fig. 3.2 can be calculated as

$$i_{dc} = i_1 + i_3 + i_5 \quad 3.22$$

and the inverter voltage v_s which is the voltage across the d.c.-link (Fig. 3.2) is given by v_{10} (equation 3.19).

3.3 Programs and Results

For the three methods of inverter analysis presented in this chapter, three programs (INVERTER 1, INVERTER 2 and INVERTER 3) were developed. These are described in Appendix A, where brief program flow charts and listings are also given. The programs employ the Runge-Kutta-Merson numerical integration routine^{55,56} and the integration step length is determined by calculating the system eigenvalues, since it can be shown that for numerical stability, the step length of the Runge-Kutta-Merson method must be less than 3.5 times the reciprocal of the maximum system eigenvalue.⁵⁷

As an illustration of the validity of the programs, experimental and computed results for both 120° and 180° inverters and for the following inverter circuit parameters are presented:

$$E = 50V, R_f = 0.5\Omega, L_f = 0.02H, R_{sh} = 0.05\Omega, C_{sh} = 5000 \mu F,$$

$$R_{a,b,c} = 10\Omega \text{ and } L_{a,b,c} = 0.022H, \text{ frequency} = 50 \text{ Hz.}$$

The values of integration step length which ensures integration stability and gives results with good accuracy, are given in Table 3.1 for each method. The table also indicates the program execution times for one complete cycle of output.

Experimental tracings of the steady-state output phase voltage, phase current and inverter current, under normal operation for 180° inverter are shown in Fig. 3.5. Since the output

voltage is a 6-step, square waveform, similar to the theoretical shape derived in Section 2.6, the current waveforms consist of parts of exponential curves with the inverter current having a frequency six times the inverter frequency.

For the same system parameters, but for the 120° inverter, experimental results of the output phase voltage and current and the inverter current are shown in Fig. 3.6. The voltage waveform in this case is more complicated as a result of the open circuit periods due to the high power factor load used (as discussed also in Section 2.6). The periods of open circuit are more clearly indicated in the output phase current waveform (Fig. 3.6b). The inverter current waveform (Fig. 3.6c) is similar to that for 180° inverters but of smaller magnitude. This is because the open circuit periods mentioned above make the r.m.s. phase voltage for the 120° inverter less than the corresponding r.m.s. voltage for the 180° inverter with the same system parameters. As a result of this difference the power transmitted through the 120° inverter is less than that transmitted through the 180° inverter. This can also be explained physically by noticing that the thyristors in the 120° inverter are not used as efficiently as in the 180° inverter due to the difference in the conduction periods.

Computed results for both 120° and 180° inverters and for the same system conditions are also obtained and plotted in Figs. 3.7 and 3.8. Good agreement exists between the computed and experimental results, which give a high degree of confidence in the ability of the methods described to simulate thyristor/diode networks.

3.4 Discussion and Conclusion

The first method (the model subroutine) is suitable for the normal operation of an inductively-loaded inverter. If, however, the method is to be used in the analysis of an inverter-motor system, two important points have to be considered:

- a) An expansion of the circuit equations to allow for the motor coupling coefficients results in a new system of equations which is mathematically cumbersome and complicated.
- b) Since the system is more complicated, a study of abnormal and fault conditions will require additional subroutines. The addition of these further subroutines would lead to a lengthy and inelegant program which, being inefficient, would result in an excessive computing time.

Although in the resistance method the network topology is constant, unfortunately a very small integration step length has to be used to ensure integration stability. This is because, irrespective of the circuit conditions, the maximum system eigenvalue turns out to be very high as indicated in Table 3.1. This small step length, in addition to the number of system equations required (twice the number required by other methods) results in an unacceptably long program execution time (Table 3.1).

In contrast to these two methods, the tensor technique has several significant advantages:

- i) The approach is valid for any bridge topology.

- ii) The analysis of abnormal fault conditions presents no problem once the transformations tensors $[C_p]$ and $[C_n]$ are available.
- iii) The usefulness of the method is that the tensor $[C_n]$ is produced in a systematic and mechanical way, which is eminently suitable for computer program implementation.
- iv) The analysis can easily be extended to include the motor circuit. In this case, it merely requires additional terms in enlarged system tensors.
- v) External and additional rotor circuit components such as saturistors or capacitors can also be included in the same way.

For these reasons, the tensor method is recommended for the analysis of the complete inverter-motor system as described in later chapters.

TABLE 3.1

Comparison between different models with
regard to program execution times

Method of Analysis	Maximum negative eigenvalue (s^{-1})	Integration step length (s)	Program execution time for one complete cycle (s)
Model sub-routine method	0.43×10^3	1.0×10^{-4}	10
Resistance method	1.5×10^6	0.5×10^{-7}	15,000
Tensor technique method	0.43×10^3	1.0×10^{-4}	14

CHAPTER 4

MODELLING THE INDUCTION MOTOR

In this chapter, the application of three different models to the analysis of the induction motor is discussed. In the first of these, the machine parameters and variables are expressed in their actual phase quantities. The second is the conventional, transformed 2-axis model derived by Stanley⁴⁸, and employed subsequently by many authors⁵⁸⁻⁶¹. The final model considered is a part transformation (of the rotor circuit only), with the stator remaining unchanged. For certain systems, it thus combines the advantages of both the other models.⁴⁹

The merits of the three systems are compared, and for each a computer program is used to study the start-up performance of an induction motor when supplied from a sinusoidal voltage source. The motor is assumed to operate from a 3-phase, 3-wire system, since in an inverter-motor system, the neutral point is usually floating.

4.1 Direct 3-Phase Model

An induction motor can be approximated to the ideal cylindrical rotor machine shown in Figure 4.1, by means of the following assumptions:^{48,58}

- a) The stator and rotor windings are balanced.
- b) The coefficient of mutual inductance between any stator winding and any rotor winding is a cosine function of the

electrical angle between the axes of the two windings.

- c) The airgap is uniform.
- d) The effects of saturation, hysteresis and eddy currents are negligible.

Based on the above assumptions, the voltage balance equations for the machine can be written in matrix form as:

$$[v_m] = [R_m][i_m] + p[\psi_m] \quad 4.1$$

where $[v_m] = [v_{sa}, v_{sb}, v_{sc}, v_{ra}, v_{rb}, v_{rc}]^t$

$$[i_m] = [i_{sa}, i_{sb}, i_{sc}, i_{ra}, i_{rb}, i_{rc}]^t$$

$$[R_m] = \text{diag} [R_s, R_s, R_s, R_r, R_r, R_r]$$

and $p[\psi_m] = [p\psi_{sa}, p\psi_{sb}, p\psi_{sc}, p\psi_{ra}, p\psi_{rb}, p\psi_{rc}]^t$

The relationship between the phase flux linkages $[\psi_m]$ and the phase currents $[i_m]$ are:

$$[\psi_m] = [L_m][i_m] \quad 4.2$$

where:

$$[L_m] = \begin{bmatrix} L_{ss} & L_{sm} & L_{sm} & M_{sr} \cos \theta & M_{sr} \cos(\theta + \frac{2\pi}{3}) & M_{sr} \cos(\theta - \frac{2\pi}{3}) \\ L_{sm} & L_{ss} & L_{sm} & M_{sr} \cos(\theta - \frac{2\pi}{3}) & M_{sr} \cos \theta & M_{sr} \cos(\theta + \frac{2\pi}{3}) \\ L_{sm} & L_{sm} & L_{ss} & M_{sr} \cos(\theta + \frac{2\pi}{3}) & M_{sr} \cos(\theta - \frac{2\pi}{3}) & M_{sr} \cos \theta \\ M_{sr} \cos \theta & M_{sr} \cos(\theta - \frac{2\pi}{3}) & M_{sr} \cos(\theta + \frac{2\pi}{3}) & L_{rr} & L_{rm} & L_{rm} \\ M_{sr} \cos(\theta + \frac{2\pi}{3}) & M_{sr} \cos \theta & M_{sr} \cos(\theta - \frac{2\pi}{3}) & L_{rm} & L_{rr} & L_{rm} \\ M_{sr} \cos(\theta - \frac{2\pi}{3}) & M_{sr} \cos(\theta + \frac{2\pi}{3}) & M_{sr} \cos \theta & L_{rm} & L_{rm} & L_{rr} \end{bmatrix}$$

..... 4.3

Taking the derivatives of both sides of equation 4.2

gives:

$$p[\psi_m] = [L_m] p[i_m] + \frac{\partial}{\partial \theta} [L_m] \cdot p\theta[i_m] \quad 4.4$$

where:

$$\frac{\partial}{\partial \theta} [L_m] = -M_{sr} \begin{bmatrix} 0 & 0 & 0 & \sin\theta & \sin(\theta+\frac{2\pi}{3}) & \sin(\theta-\frac{2\pi}{3}) \\ 0 & 0 & 0 & \sin(\theta-\frac{2\pi}{3}) & \sin\theta & \sin(\theta+\frac{2\pi}{3}) \\ 0 & 0 & 0 & \sin(\theta+\frac{2\pi}{3}) & \sin(\theta-\frac{2\pi}{3}) & \sin\theta \\ \sin\theta & \sin(\theta-\frac{2\pi}{3}) & \sin(\theta+\frac{2\pi}{3}) & 0 & 0 & 0 \\ \sin(\theta+\frac{2\pi}{3}) & \sin\theta & \sin(\theta-\frac{2\pi}{3}) & 0 & 0 & 0 \\ \sin(\theta-\frac{2\pi}{3}) & \sin(\theta+\frac{2\pi}{3}) & \sin\theta & 0 & 0 & 0 \end{bmatrix} \quad 4.5$$

By substituting equation 4.4 into equation 4.1:

$$[v_m] = [R_m][i_m] + [L_m]p[i_m] + \frac{\partial}{\partial \theta} [L_m]p\theta[i_m] \quad 4.6$$

The above system of equations cannot generally be solved in closed form, because the dependence of the stator-rotor mutual inductance on the rotor electrical angle θ makes the coefficients of $[L_m]$ variable. The equations can, however, be solved by numerical

techniques using a digital computer. For this purpose, it is necessary to rearrange equation 4.6 as:

$$p[i_m] = [L_m]^{-1} \{ [v_m] - [R_m][i_m] - \frac{\partial}{\partial \theta} [L_m] p\theta[i_m] \} \quad 4.7$$

If a step-by-step numerical integration routine is used then, at any calculation step, a knowledge of θ is required to form $[L_m]$ and $\frac{\partial}{\partial \theta} [L_m]$ and thus to calculate $[L_m]^{-1}$. Additionally, a knowledge of $[v_m]$, $[i_m]$ and $p\theta$ will yield the current derivatives $p[i_m]$, from which the new values of $[i_m]$ can be found.

The total electrically developed torque T_e is:

$$T_e = \left(\frac{1}{2}\right)(P) [i_m]^t \frac{\partial}{\partial \theta} [L_m][i_m] \quad 4.8$$

The rotor electric angle θ and the speed $p\theta$ required in the next step of the calculation can be obtained by integrating the machine dynamic equation

$$\frac{J}{P} p^2\theta + K_f p\theta + T_m = T_e \quad 4.9$$

which may conveniently be rewritten

$$p\theta = w \quad 4.10$$

$$pw = \frac{P}{J} (T_e - T_m - K_f w)$$

where w is the instantaneous motor speed.

Results obtained from this analysis are presented in a later section.

4.2 Stationary 2-axis Model

If the 3-phase model of the induction machine (Fig. 4.1) is transformed to its equivalent 2-phase stationary axis (the d-q form) as shown in Fig. 4.2, the relationship between the actual 3-phase currents and the fictitious 2-axis currents are:⁶²

$$i_{sa} = i_{sd}$$

$$i_{sb} = -\frac{1}{2} i_{sd} + \frac{\sqrt{3}}{2} i_{sq}$$

$$i_{sc} = -\frac{1}{2} i_{sd} - \frac{\sqrt{3}}{2} i_{sq}$$

4.11

$$i_{ra} = i_{rd} \cos \theta + i_{rq} \sin \theta$$

$$i_{rb} = i_{rd} \cos \left(\theta + \frac{2\pi}{3} \right) + i_{rq} \sin \left(\theta + \frac{2\pi}{3} \right)$$

$$i_{rc} = i_{rd} \cos \left(\theta - \frac{2\pi}{3} \right) + i_{rq} \sin \left(\theta - \frac{2\pi}{3} \right)$$

Note that because the three actual phases are replaced by a system of 2-axis coils, the unit of the 2-axis currents is $\frac{3}{2}$ times the unit of the actual phase currents for a constant input power per phase. Note also that the zero sequence currents are

zero since there is no neutral connection.

Equation 4.11 gives rise directly to the transformation tensor

$$[C_m] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & \cos(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ 0 & 0 & \cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) \end{bmatrix}$$

4.12

which may be used to produce the 2-axis equations for the machine by applying the transformation (detailed in reference 63) to equation 4.1. The new system equations have the form

$$[v_m] = [R_m][i_m] + [G_m] p\theta [i_m] + [L_m] p [i_m] \quad 4.13$$

where $[v_m]$ is now

$$\begin{bmatrix} v_{sd} \\ v_{sq} \\ v_{rd} \\ v_{rq} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta - \frac{4\pi}{3}) \\ 0 & 0 & 0 & \sin\theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{4\pi}{3}) \end{bmatrix} \begin{bmatrix} v_{sa} \\ v_{sb} \\ v_{sc} \\ v_{ra} \\ v_{rb} \\ v_{rc} \end{bmatrix}$$

4.14

$$[i_m] = [i_{sd}, i_{sq}, i_{rd}, i_{rq}]^t$$

$$[R_m] = \text{diag} [R_s, R_s, R_r, R_r]$$

$$[G_m] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{3}{2} M_{sr} & 0 & (L_{rr} - L_{rm}) \\ -\frac{3}{2} M_{sr} & 0 & -(L_{rr} - L_{rm}) & 0 \end{bmatrix}$$

4.15

$$\text{and } [L_m] = \begin{bmatrix} L_{ss} - L_{sm} & 0 & \frac{3}{2} M_{sr} & 0 \\ 0 & L_{ss} - L_{sm} & 0 & \frac{3}{2} M_{sr} \\ \frac{3}{2} M_{sr} & 0 & L_{rr} - L_{rm} & 0 \\ 0 & \frac{3}{2} M_{sr} & 0 & L_{rr} - L_{rm} \end{bmatrix} \quad 4.16$$

The above equations are identical to those obtained by Stanley⁴⁸ and others.^{64,65} Since the equations are functions only of the rotor speed $p\theta$ they can be solved analytically when the speed is constant, although for variable-speed operation, a numerical technique will still have to be used. For this purpose equations 4.13 can be rearranged as

$$p [i_m] = [L_m]^{-1} \{ [v_m] - [R_m][i_m] - [G_m] p\theta [i_m] \} \quad 4.17$$

and the solution for the currents can proceed using a standard numerical integration routine. The actual phase currents are obtainable from equations 4.11.

The total electromagnetic torque can be calculated from

$$T_e = \left(\frac{3}{2}\right)(P) [i_m]^t [G_m][i_m] \quad 4.18$$

The rotor speed $p\theta$ which is required by equation 4.17 in the next step of the solution is obtained by integrating equations 4.10.

Using this method, a solution can be obtained in a step-by-step manner, without the need to form and invert the machine inductance matrix at each computation step. However, transformation of currents and voltages is still needed.

4.3 Stationary 3-axis Model

In this section, a stationary 3-axis model⁴⁹ is described which combines the advantages of both the direct-phase and the stationary 2-axis models. The new model can be formulated under the following two requirements:

- a) Equations 4.1 should be transformed in a similar manner to the conventional 2-axis transformation, in order to eliminate the time-dependent terms in $[L_m]$. This requirement can be fulfilled by choosing stator and rotor reference frames stationary relative to each other.
- b) The transformation should result in no change in the stator variables, in order to retain the advantage of the direct phase model. The rotor quantities must therefore be transformed to a stationary 3-axis reference frame.

The new 3-phase stationary axis model is shown in Fig. 4.3, where the fictitious stationary rotor coil currents $i_{r\alpha}$, $i_{r\beta}$ and $i_{r\gamma}$ flow through stationary windings and produce the same rotor m.m.f. as the actual rotor currents i_{ra} , i_{rb} and i_{rc} . The two

sets of currents are thus related to each other by

$$i_{ra} - \frac{1}{2} i_{rb} - \frac{1}{2} i_{rc} = \cos\theta i_{\alpha r} + \cos(\theta - \frac{2\pi}{3}) i_{\beta r} + \cos(\theta + \frac{2\pi}{3}) i_{\gamma r}$$

$$i_{rb} - \frac{1}{2} i_{ra} - \frac{1}{2} i_{rc} = \cos(\theta + \frac{2\pi}{3}) i_{\alpha r} + \cos\theta i_{\beta r} + \cos(\theta - \frac{2\pi}{3}) i_{\gamma r}$$

$$i_{rc} - \frac{1}{2} i_{ra} - \frac{1}{2} i_{rb} = \cos(\theta - \frac{2\pi}{3}) i_{\alpha r} + \cos(\theta + \frac{2\pi}{3}) i_{\beta r} + \cos\theta i_{\gamma r}$$

4.19

Since, as explained before, the stator currents are unchanged, the transformation tensor for this model can now be written

$$[C_m] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3} \cos\theta & \frac{2}{3} \cos(\theta - \frac{2\pi}{3}) & \frac{2}{3} \cos(\theta + \frac{2\pi}{3}) \\ 0 & 0 & 0 & \frac{2}{3} \cos(\theta + \frac{2\pi}{3}) & \frac{2}{3} \cos\theta & \frac{2}{3} \cos(\theta - \frac{2\pi}{3}) \\ 0 & 0 & 0 & \frac{2}{3} \cos(\theta - \frac{2\pi}{3}) & \frac{2}{3} \cos(\theta + \frac{2\pi}{3}) & \frac{2}{3} \cos\theta \end{bmatrix}$$

4.20

As with the 2-axis model, this transformation tensor is then used to produce the 3-phase stationary axes equations from equation 4.1. The new system equations are similar to equations 4.13, but with

$$[v_m] = [v_{s\alpha}, v_{s\beta}, v_{s\gamma}, v_{r\alpha}, v_{r\beta}, v_{r\gamma}]^t$$

$$[i_m] = [i_{s\alpha}, i_{s\beta}, i_{s\gamma}, i_{r\alpha}, i_{r\beta}, i_{r\gamma}]^t$$

$[R_m]$ is given by equation 4.1.

$$[L_m] = \begin{bmatrix} L_{ss} & L_{sm} & L_{sm} & M_{sr} & -\frac{1}{2} M_{sr} & -\frac{1}{2} M_{sr} \\ L_{sm} & L_{ss} & L_{sm} & -\frac{1}{2} M_{sr} & M_{sr} & -\frac{1}{2} M_{sr} \\ L_{sm} & L_{sm} & L_{ss} & -\frac{1}{2} M_{sr} & -\frac{1}{2} M_{sr} & M_{sr} \\ M_{sr} & -\frac{1}{2} M_{sr} & -\frac{1}{2} M_{sr} & L_{rr} & L_{rm} & L_{rm} \\ -\frac{1}{2} M_{sr} & M_{sr} & -\frac{1}{2} M_{sr} & L_{rm} & L_{rr} & L_{rm} \\ -\frac{1}{2} M_{sr} & -\frac{1}{2} M_{sr} & M_{sr} & L_{rm} & L_{rm} & L_{rr} \end{bmatrix}$$

4.21

$$[G_m] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2}M_{sr} & -\frac{\sqrt{3}}{2}M_{sr} & 0 & \frac{1}{\sqrt{3}}(L_{rr}-L_{rm}) & -\frac{1}{\sqrt{3}}(L_{rr}-L_{rm}) \\ -\frac{\sqrt{3}}{2}M_{sr} & 0 & \frac{\sqrt{3}}{2}M_{sr} & -\frac{1}{\sqrt{3}}(L_{rr}-L_{rm}) & 0 & \frac{1}{\sqrt{3}}(L_{rr}-L_{rm}) \\ \frac{\sqrt{3}}{2}M_{sr} & -\frac{\sqrt{3}}{2}M_{sr} & 0 & \frac{1}{\sqrt{3}}(L_{rr}-L_{rm}) & -\frac{1}{\sqrt{3}}(L_{rr}-L_{rm}) & 0 \end{bmatrix}$$

4.22

The total electric developed torque is given by

$$T_e = (P) [i_m]^t [G_m] [i_m] \quad 4.23$$

The numerical solution of the system equations can now proceed as before, except that the circuit voltage tensor does not need transformation.

4.4 Programs and Results

For each of the models described earlier, a computer program was written to study the start-up transients of an induction motor. The programs (named MOTOR1, MOTOR2 and MOTOR3) are given in Appendix B and they can be used for either sinusoidal or 6-step square wave voltage inputs.

A range of start-up transients has been considered for the machine having the parameters given in Appendix D. The results produced by the three programs are in excellent agreement with each other, but the direct phase model has involved considerably more computation time than the other models, as indicated in table 4.1.

Figures 4.4 and 4.5 show typical predicted results for motor phase current, torque and speed for the direct application of rated sine wave voltage at rated frequency, while Fig. 4.6, shows practical results of the phase current under the same conditions. The oscillatory nature of the torque at starting causes the dips in the speed curve and also results in a starting current which has a rising and falling amplitude as shown. A good agreement clearly exists between these computed and experimental results.

4.5 Discussion and Conclusion

With the machine described in terms of its transformed d-q variables, the resultant system differential equations offer computational simplicity, due to the absence of the rotor position term in the inductance matrix. These equations have been extensively applied in the study of the dynamic performance of sinusoidally-excited machines and also machines supplied by well-defined rectangular voltage waveforms.

However, in a general study of a combined inverter-motor system, it is necessary to monitor the machine phase variables and a problem thus arises in defining a set of d-q variables for the opening and closing branches of the inverter. In this case the direct phase model seems to be more suitable. This method also permits a wider range of operating conditions to be studied conveniently. Sarkar and Berg³⁵ have used this approach to study the start-up performance of an induction machine when supplied from a rectangular waveform. However when compared with the stationary 2-axis model, the direct phase method suffers from the excessive computational times required to form and invert the inductance matrix at each step.

In contrast to these methods, the stationary 3-axis model proposed by Robertson and Hebbar⁴⁹ appears to be much more powerful for use with inverter circuits. Since this model describes the machine in terms of its stator phase quantities, and at the same time transforms the rotor quantities to a stationary 3-axis reference frame, it retains the advantages of both models.

For this reason, the stationary 3-axis model will be used in conjunction with the inverter circuit model of Chapter 3 for the analysis of the whole inverter-motor system.

TABLE 4.1

Comparison between different models with
regard to program execution times

Motor Model	Program execution time for start-up range (0.37s) (s)
Direct phase model	240
Stationary 2-axis model	40
Stationary 3-axis model	45

CHAPTER 5

ANALYSIS OF INVERTER-INDUCTION MOTOR SYSTEM

From the three methods of analysis described in Chapter 3, and for the reasons given there, the tensor approach was selected for the analysis of a bridge inverter supplying an inductive load. Using this same approach, together with the stationary 3-axis induction motor model developed in Chapter 4, an analysis of a combined inverter-induction motor system was developed. Results derived from this analysis are presented in this section to illustrate starting, plugging and braking of the motor, and some other operating and fault conditions of the system.

5.1 General System Analysis

In the tensor approach adopted for the analysis of the inverter (Chapter 3), the independent equations of the basic network were established from various primitive system tensors. (The inverter is assumed to feed a star-connected inductive load, and with this regarded as the stator of an induction motor, the complete inverter-motor system can be developed by the addition of a rotor circuit.) The overall system is shown in Fig. 5.1. The new basic network contains an extra three branches and two nodes, and a further two independent equations are therefore required. The analysis of the complete system can be carried out using the same procedure as in Section 3.2.3, by enlarging the various primitive tensors to include the new rotor elements and the mutual

$$[R_p] = \text{diag} [R_1, R_2, R_3, R_4, R_5, R_6, R_s, R_s, R_s, R_{sh}, R_f, R_r, R_r, R_r]$$

5.2

- c) When the stationary 3-axis model (described in Chapter 4), is employed, the new primitive inductance tensor $[L_p]$ is no longer diagonal, due to coupling between the motor phases, and it takes the form given by equation 5.3.
- d) A new primitive tensor which is not present in the analysis of Section 3.2.3. must now appear. This is the $[G_p]$ tensor which corresponds to the $[G_m]$ tensor used in the analysis of Chapter 4 and is given by equation 5.4.
- e) The primitive source voltage tensor $[v_p]$, the capacitor voltage tensor $[sq_p]$ and the current tensor $[i_p]$ are each expanded by three more elements as given below:

$$[v_p] = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, E, 0, 0, 0]^t$$

$$[sq_p] = [0, 0, 0, 0, 0, 0, 0, 0, 0, q/C_{sh}, 0, 0, 0, 0]^t \quad 5.5$$

and $[i_p] = [i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9, i_{10}, i_{11}, i_{12}, i_{13}, i_{14}]^t$

- f) The $[C_n]$ transformation tensor (which connects the basic network to the reduced network) has to be enlarged by two rows and two columns (corresponding to the two independent equations of the rotor). For the inverter topology considered in Section 3.2.3, the new $[C_n]$ tensor becomes:

$$[C_n] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad 5.6$$

The analysis can now proceed as explained in Chapter 3 except that the new $[G_n]$ and $[G_{nn}]$ tensors have to be evaluated using the relationships:

$$[G_n] = [C_p]^t [G_p] [C_p] \quad 5.7$$

$$[G_{nn}] = [C_n]^t [G_n] [C_n]$$

Finally the system equations for any topology can be established as:

$$p[i_{nn}] = [L_{nn}]^{-1} \{ [v_{nn}] - [R_{nn}][i_{nn}] - [G_{nn}][i_{nn}]p\theta - [sq_{nn}] \} \quad 5.8$$

For the analysis of the system when the speed is varying, the two dynamic relationships of equation 4.10 are included for a full mathematical description.

5.2 D.C. Supply Representation

As explained in Chapter 2, the supply to the inverter is normally derived from a phase-controlled rectifier (i.e. a non-regenerative supply), which was represented during the analysis by a d.c. supply in series with an effective supply impedance. This representation is reasonable as long as the supply current is positive (i.e. towards the inverter). However, in a real inverter-motor system, and during some operating conditions such as plugging, the motor behaves as an induction generator which returns energy to the inverter. Since this is non-receptive, the returned energy is stored in the filter capacitor, and it is therefore necessary to modify the supply representation by the addition of the series diode shown dotted in Fig. 5.1. Within the computer program, this device is treated in the same way as any other diode (i.e. it is turned off whenever its current drops to zero).

The effect of the diode, whether conducting or non-conducting, can be included in the system equations by the presence or absence of the column which corresponds to the source current in the $[C_n]$ tensor.

5.3 Analysis of Some Operating and Fault Conditions

The usefulness of the tensor approach is obvious when it is applied to various motor operating conditions. It is shown in this section that the analysis of any operating condition requires a simple modification to the transformation tensors depending on the

particular condition being studied, while the rest of the analysis remains the same.

5.3.1 Start-up on constant voltage and frequency

This is a common system condition to which the previous analysis can be applied, and no modifications of the transformation tensors are required. The initial conditions of the independent variables of the system are as follows:

- a) All the currents are set to zero.
- b) The rotor speed $p\theta$ is set to zero.
- c) The capacitor charge q is set to $C_{sh}E$.

Experimental and computed results for this operating condition are presented in Section 5.5.1.

5.3.2 Plugging

During a plugging operation, the motor stator terminals are first disconnected from the inverter for a short period, and subsequently reconnected with two of the phases interchanged.

The circuit topology during the period of disconnection (amounting normally to only a small fraction of a second) is as shown in Fig. 5.2, where only three loops are now active. The initial values of the rotor currents immediately after disconnection are different from those immediately before, and their new values can be obtained using the constant flux linkage theorem, as explained in Appendix E. Following disconnection the currents

decay exponentially, thus inducing exponentially decreasing sinusoidal voltages in the stator windings. At the same time, the rotor speed drops slightly due to mechanical friction and to other rotational losses.

The period of disconnection can easily be represented in the analysis by the following $[C_n]$ tensor:

$$[C_n] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 5.9$$

After reconnection, the circuit configuration is similar to the original form, except that phase a is now connected to thyristors T_3 and T_4 , and phase b to thyristors T_1 and T_2 .

At the instant of reconnection, the primitive transformation tensor $[C_p]$ has to be updated to allow for the change in the circuit, as follows:

$$[C_p] = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \\ -1 & -1 & -1 & -1 & -1 \\ & & 1 & 1 & \\ 1 & 1 & & & \\ -1 & -1 & -1 & -1 & \\ -1 & & -1 & & -1 & 1 \\ & & & 1 & \\ & & & & 1 \\ & & & & & 1 \\ & & & & & & 1 \\ & & & & & & & -1 & -1 \end{bmatrix}$$

Using this new tensor, the various independent tensors are recalculated using equations 3.9, 3.12, and 3.15 and the analysis proceeds as before.

The induction machine behaves initially as an induction generator transferring energy to the filter capacitors, while the rectifier diode (in the supply) ceases to conduct. The machine speed eventually drops to zero and starts to increase in the opposite direction. The machine now operates as an ordinary induction motor and the rectifier diode is no longer reverse biased.

The plugging operation is demonstrated by experimental and computed results in Section 5.5.3.

5.3.3 Braking

Braking of an induction motor is commonly achieved either by d.c. injection⁶⁶ or by using the a.c. supply with different arrangements of specially-included diodes.^{67,68} When an induction motor is driven by a d.c. link type inverter, a very effective and quiet braking action is obtainable in the manner described below. This has the advantages that no additional equipment is needed, a high braking torque is achieved and the rotor is brought rapidly to rest and held there.

The method adopted is to switch off simultaneously the inverter d.c. supply, while keeping the filter capacitor connected to the inverter, and the control circuitry which triggers the thyristors. The devices conducting at this instant (either two or three) continue to conduct until their currents become zero. Meanwhile the capacitor, which is initially charged to a value proportional to the d.c. supply voltage, provides a d.c. voltage for injection to the motor through the conducting devices and establishes a braking action. Both the capacitor energy and the rotor kinetic energy are quickly dissipated as heat in the windings.

Assuming for example that thyristors T_2 and T_4 and diode D_5 are conducting at the instant of switching, the circuit topology of the system remains constant as shown in Fig. 5.3. The transformation tensor $[C_n]$ for this case is also constant and is simply obtained by assuming that the rectifier diode is always nonconducting, i.e.:

$$[C_n] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad 5.11$$

Using this kind of braking, experimental and computed results are obtained and presented in Section 5.5.4.

5.3.4 Stator Phase Short Circuit

One possible fault in any motor system is a short circuit of one of the stator windings.⁶⁹ By short circuiting, for example, phase c, the basic network for the system can be drawn as shown in Fig. 5.4. Since a circulating current through phase c will result from the mutual coupling between the phases, it is necessary for branch 9 of the network to be included in the analysis. The inclusion of this branch, and the short circuit, means that the number of branches in the network remains unchanged from the normal case (Fig. 5.1) while the number of nodes is less by one. A new independent current must therefore be added to the system, and this is shown in Fig. 5.4 as i_1 . The transformation tensor $[C_p]$ has now to be increased by one column to accommodate the new current, as follows:

$$[C_p] = \begin{bmatrix} 1 & & & & & & & & & & \\ & 1 & & & & & & & & & \\ & & 1 & & & & & & & & \\ & & & 1 & & & & & & & \\ & & & & 1 & & & & & & \\ & & & & & 1 & & & & & \\ -1 & -1 & -1 & -1 & -1 & & & & & & \\ 1 & 1 & & & & & & & & & \\ & & & 1 & 1 & & & & & & \\ & & & & & & & & & & 1 \\ -1 & & -1 & & -1 & 1 & & & & & \\ & & & & & & 1 & & & & \\ & & & & & & & 1 & & & \\ & & & & & & & & 1 & & \\ & & & & & & & & & -1 & -1 \end{bmatrix} \quad 5.12$$

The transformation tensor $[C_n]$ has also to be increased by one row and by one column to cope with the new current which is present for all network topologies. As an example, consider the case when thyristors T_2 and T_3 and diode D_5 are conducting, when the circuit topology is as shown in Fig. 5.5, and the $[C_n]$ tensor is:

$$[C_n] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad 5.13$$

After defining the two transformation tensors for such a fault condition, the complete analysis of the system follows the procedure described in Section 5.1.

This kind of fault condition is demonstrated by experimental and computed results in Section 5.5.6.

5.3.5 Stator Phase Open Circuit

Another possible fault condition in a motor system is the open circuiting of one of the stator phases. By open circuiting phase c, the basic network for the system becomes that shown in Fig. 5.6. There is now no current through phase c although there is still an induced e.m.f. in this phase due to mutual coupling from the remaining phases. Without taking into account the open circuit leg of the bridge (shown dotted in Fig. 5.6) the basic network can be considered as having seven branches and four nodes

(excluding the rotor circuit). Six independent currents ($i_A \rightarrow i_F$) are therefore required to define the system (including the rotor circuit) and with these chosen as shown in Fig. 5.6, the $[C_p]$ tensor can be written directly as

$$[C_p] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix} \quad 5.14$$

Since only two legs of the inverter bridge are active, there are only two devices conducting at any particular time. The inverter circuit topologies involved are therefore among the greater number involved during normal operation of the 120° inverter. For this reason no changes are necessary in the $[C_n]$ tensor and the complete analysis again follows the procedure of Section 5.1.

Results obtained experimentally and by computer for this kind of fault condition are presented in Section 5.5.7.

5.4 Computer Program

The basic computer program, INVERTER-MOTOR, written for the analysis of the complete inverter-induction motor system, is described in Appendix C, where a simplified flow chart and a program listing are given. The program employs the tensor technique for the inverter, and the stationary 3-axis model for the motor. Several versions of the program were developed for the study of the different system operating and fault conditions. In all cases the following initial values apply for the various independent variables.

- a) All the currents are zero.
- b) The rotor position angle θ is zero.
- c) The rotor speed $p\theta$ is zero for the start-up condition, otherwise it is the synchronous speed w_s .

In this case the program is run for a few cycles before starting to plot the results, which ensures that the motor has reached steady-state conditions.

5.5 Test and Computed Results

Results derived from the analyses of a number of system operational and fault conditions for both 120° and 180° inverters

are presented in the following sections, where they are supported by experimentally obtained results. All the tests were performed at the arbitrarily chosen frequency of 25 Hz, with the d.c. supply voltage set to 250V (see Appendix F), unless otherwise stated.

5.5.1 Start-up on constant voltage and frequency

This is the normal run-up of the motor when it is allowed to accelerate on no-load from standstill to full speed following direct switching of the inverter system. Experimental recordings of the motor phase voltage and current and the inverter voltage and current when the motor is fed from the 180° inverter are shown in Figs. 5.7 and 5.8 respectively. The phase voltage waveform during the first few cycles exhibits a slight drop below the steady-state values, due to the drop across the inverter input filter and supply impedances, and is more clearly indicated by the inverter voltage curve (Fig. 5.8a). The frequency of the inverter current is 6 times the inverter operational frequency, which agrees with the results obtained for the inductively-loaded inverter in Section 3.3.

Computed results of the motor phase voltage and current and the inverter voltage and current are given in Figs. 5.9 and 5.10 respectively. These all confirm the experimental results of Figs. 5.7 and 5.8 except that the inverter voltage curve (Fig. 5.10a) is smooth while the corresponding experimental tracing (Fig. 5.8a) contains a very small amount of ripple. This arises from the d.c.

supply being in practice derived from a 3-phase bridge rectifier, whereas in the computer solution the supply is assumed to be completely smooth.

Similar experimental and computed procedures described above were repeated for the motor driven from the 120° inverter, with the results given in Figs. 5.11 - 5.14. In this case the waveforms of the motor phase voltage (Figs. 5.11a and 5.13a) vary considerably during the transient period, and tend to become fixed 6-step square waves under steady-state conditions. During the transient interval, when the motor power factor tends to be high, off periods appear in the current waveforms (Figs. 5.11b and 5.13b) and under steady-state no-load conditions, when the motor power factor is very low, the voltage and current waveforms become similar to those for the 180° inverter. The inverter current waveforms (Figs. 5.12b and 5.14b) are similar to those for the 180° inverter, except that their peaks are lower during the transient interval, when the off periods mentioned above cause the r.m.s. motor phase voltage for the 120° inverter to be less than that for the 180° inverter. For the same reason the speed builds up a little slower in the case of the motor driven from the 120° inverter than from the 180° inverter. This is clear from the computed speed curves shown in Fig. 5.15, where the dips appearing in the curves are due to the oscillatory nature of the torque (Fig. 5.16). The torque also has a pulsating nature due to the presence of time harmonic m.m.f. waves in the airgap.

5.5.2 Loading

While the motor is supplied from the 180° inverter, and is running at its no load speed, an estimated torque of 5 N.m. is applied to the shaft for a short time as a sudden load. The resulting experimental graphs of the motor phase voltage and current are shown in Fig. 5.17 and the corresponding computer plots are given in Fig. 5.18. Experimental and computed results for similar tests but with the motor supplied from the 120° inverter are given in Figs. 5.19 and 5.20.

Since the motor power factor increases when load is applied it will be noticed that, in the case of the 120° inverter, off periods appear in the motor phase current waveform and that the motor phase voltage wave shape changes during the loading period. On the other hand, for the 180° inverter the motor phase voltage wave shape remains unchanged but the waveform of the motor phase current changes in such a way that its r.m.s. value increases slightly.

5.5.3 Plugging

Experimental and computed motor phase voltage and current waveforms during plugging of the induction motor fed by the 180° inverter are shown in Figs. 5.21 and 5.22 respectively. Similar waveforms for the motor fed by the 120° inverter are given in Figs. 5.23 and 5.24. As explained in Section 5.3.2, during the disconnection period (80 ms), an exponentially decaying sinusoidal voltage is induced in the stator phases and the speed drops slightly. After reconnection, with two of the motor terminals interchanged,

a peak current of up to 8 times the steady-state no load peak current results, due to the high slip of nearly 2. The speed then drops rapidly to zero and builds up towards full speed in the reverse direction, as shown in the computed curves of Fig. 5.25b.

The differences between the 120° and 180° inverters, with regard to the motor phase voltage and current waveforms and the speed curves, are due to the reasons discussed in Section 5.4.1.

5.5.4 Braking

The method of capacitor discharge braking discussed in Section 5.3.3 was applied to the induction motor fed from the 180° inverter, and experimental and computed results for the motor phase voltage and current are shown in Figs. 5.26 and 5.27 respectively. Results for the 120° inverter are not given, since they are similar to those for the 180° inverter.

The effectiveness of this kind of braking is clearly indicated by the computed motor torque and speed curves shown in Fig. 5.28. This method of braking is quite impressive since the speed drops from its steady-state no load value to rest in a fraction of a second.

5.5.5 Sudden Frequency Change

The motor speed in an inverter-motor system can be raised or lowered by making sudden increases or decreases in the inverter frequency. Test results using this kind of speed control

were obtained for the unloaded motor, when the inverter frequency was suddenly changed from 20 Hz to 30 Hz and vice versa. Since, in the experimental scheme, the output d.c. voltage from the rectifier stage of the system is manually adjusted, it is fixed during this test at 200V, which corresponds to a frequency of 20 Hz (see Appendix F). The motor phase voltage and current waveforms during the frequency increase for the 180° and 120° inverters are shown in Figs. 5.29 and 5.30 respectively. Similar waveforms for the frequency decrease are shown in Figs. 5.31 and 5.32. The corresponding computed results are shown in Figs. 5.33 - 5.36. Since the exact moment when the frequency changes is difficult to determine accurately, there is a slight difference between the experimental and computed results, but nevertheless they show good agreement.

It is important to note that immediately after the step decrease in frequency, the induction motor behaves as an induction generator and the corresponding braking action causes the speed to drop quickly to the value dictated by the new inverter frequency. For a sudden increase in frequency the motor will accelerate towards the new speed and the acceleration period is longer than the deceleration period. This difference in acceleration and deceleration times is clearly seen when the transient periods of Figs. 5.29 - 5.32 and Figs. 5.33 - 5.36 are compared.

As explained in Section 5.2, when the machine is generating power the rectifier diode stops conducting and charge builds up on the filter capacitor. This is demonstrated in Fig. 5.37, which shows computed curves of the inverter supply current and the inverter

d.c. link voltage for the frequency decrease test. Conversely, during the step frequency increase, the motor demands more power and the source current (Fig. 5.38b) increases causing the inverter voltage (Fig. 5.38a) to drop slightly.

5.5.6 Stator phase Short Circuit

In this test, the motor is switched to the 180° inverter with one of its stator windings permanently short circuited, and the inverter supply voltage set to a reduced value of 75V (at 25 Hz) to ensure that the motor line currents (where two of them represent the phase currents and the third the short circuit current, Fig. 5.4) are limited to a safe value. The steady-state waveforms of these currents as well as the inverter current are shown in Figs. 5.39 and 5.40, and the corresponding computed results are given in Figs. 5.41 and 5.42. The same procedure is repeated with the motor switched to the 120° inverter and the experimental and computed results for the same variables are given in Figs. 5.43 - 5.46. In both cases, close agreement is clearly evident between the experimental and the computed graphs. It will be noted from these figures that:

- a) For both 120° and 180° inverters, a phase shift of 60° exists between the current waveforms of the two stator phases not short circuited, (Figs. 5.41 and 5.45), instead of the 120° occurring during normal operation.
- b) As indicated in Fig. 5.4, the short circuit current at any instant is the negative sum of the currents in the two phases

not short circuited (Figs. 5.42a and 5.46a).

- c) The instantaneous inverter current for both 120° and 180° inverters consists of either one of the currents in the phases not short-circuited, or of their sum. This can be verified from the circuit topologies of Fig. 2.8, for example, by considering phase c as short circuit. If the thyristors T_1 and T_6 are conducting the inverter current is the same as the current of phase a, whereas if thyristors T_1 , T_3 and T_6 are conducting the inverter current is the sum of the phase a and phase b currents, and so on.
- d) For the 120° inverter, periods of open circuit occur in one of the phases, and the various currents are reduced from those supplied by the 180° inverter. For example, the short circuit and the inverter currents are less by 33% and 50% when compared with their corresponding values for the 180° inverter.

5.5.7 Stator Phase Open Circuit

In this test a stator terminal is disconnected permanently while the motor is running on no-load and fed from the 180° inverter, so that operation continues in a 2-phase mode. Since in this case the two remaining phases are connected in series (see Fig. 5.6), their steady-state current (and also voltage) variations are the same and these, together with the inverter current and the open-circuit induced voltage of the disconnected phase, are given in Figs. 5.47 and 5.48. The corresponding computed results are shown

in Figs. 5.49 and 5.50. Experimental and computed results for the same test, but repeated for the 120° inverter, are given in Figs. 5.51 - 5.54. As previously, good agreement exists between the experimental and computed graphs, from which it will be noted that:

- a) The phase current waveforms (Figs. 5.49a and 5.53a) are the same as those during normal operation (Fig. 5.9b) except that in this case one of the peaks disappears every half cycle due to the disconnection of one of the phases.
- b) The inverter current waveforms for both inverters have zero periods every supply cycle, as indicated in Figs. 5.49b and 5.53b. These periods occur during the time when the stator phases (excluding the open circuited one) are effectively connected in parallel. (This particular topology can be seen in Fig. 2.8 after open circuiting one of the impedances).
- c) When using the 120° inverter, the inverter current waveform (Fig. 5.53b) has extra zero periods every supply cycle, in addition to those mentioned above. These periods are due to an open circuit in one of the conducting phases, which since the stator phases are in series, results in zero current.
- d) The small curved portions appearing in the phase voltage waveforms (Figs. 5.50a and 5.54a) are due to the circulating current in the stator phases when these are effectively isolated from the supply.

- e) The open-circuit voltage present in the disconnected phase is a rotational e.m.f. induced from the rotor and it is therefore of sinusoidal form (Figs. 5.50b and 5.54b).

5.6 Conclusion

The analysis of various operating and fault conditions which may arise in an inverter-induction motor system have been dealt with efficiently and accurately using the tensor technique. This has the desirable feature that every operating condition is analysed in a similar systematic way, after adjusting the transformation tensors for the particular operating condition. The close agreement between the experimental and computed results obtained for all cases gives a high degree of confidence in the computer work described. This technique may now be applied to the study of the variable frequency performance of the system, which is continuing in the following chapter.

CHAPTER 6

MOTOR STARTING PERFORMANCE FROM A
VARIABLE-FREQUENCY (v.f.) SUPPLY

Changing the speed of an induction machine using a variable-frequency (v.f.) supply has been studied by Lawrenson and Stephenson⁵⁰ and by Berg and Sarkar.⁷⁰ Lawrenson and Stephenson have shown that the motor speed can be changed rapidly and efficiently, by varying the supply frequency uniformly with time, while maintaining a constant voltage/frequency ratio. Under these conditions, run up from standstill can be achieved more quickly than by the usual step application of voltage at rated frequency (the direct-on-line case) and the machine currents are also reduced. However, the optimum rate-of-change of frequency is obviously different for different motor systems, and depends also on the motor load. Using this v.f. starting-up procedure, a complete reversal from full speed in one direction to full speed in the reverse direction also takes a reduced time, with the motor having lower losses than in the usual plugging method.

Berg and Sarkar⁷⁰ have given approximate analytic expressions for the rates of (linear) change of frequency with time for optimum motor starting performance. Both papers^{50,70} consider only sinusoidal voltage supplies in their analysis, whereas in practice v.f. supplies, as derived from inverters, are of nonsinusoidal waveform. In this chapter, computer predictions of the v.f. start-up procedure for an experimental induction motor system, for both sine wave and inverter wave inputs, are presented and compared.

Two starting techniques are investigated, these being:

- a) linear frequency variation (open loop operation),
- b) constant slip frequency control (closed loop operation).

Since the inverter is intended to be part of a speed control drive, the study is presented for typical operating frequencies of 50, 60 and 100 Hz.

6.1 The Motor Input Voltage

Normally in v.f. a.c. drive systems, the voltage supplied to the motor is varied with the frequency in such a way as to maintain constant the airgap flux. This ensures the same normalized output torque-speed characteristic at all operating frequencies.

In this section, the derivation of the magnitude of the supply voltage to the motor for both sinusoidal and inverter waveforms, as a function of frequency for constant flux, is presented. These equations are used later when investigating the v.f. behaviour of the machine.

6.1.1 Sine Wave Input

From the steady-state motor torque equation G.2 it is clear that, except at very low frequencies, the supply voltage/frequency ratio should be kept constant to maintain approximately constant maximum output torque. However, at very low frequencies, the stator resistance drop becomes significant and results in a

reduced driving torque. This is clearly indicated in Fig. 6.1a which shows the steady-state torque-speed curves of the motor under investigation for different operating frequencies, using a constant supply voltage/frequency ratio to give rated flux density at rated voltage.

In order to obtain the same maximum output torque at all operating frequencies, it is necessary that the voltage should be boosted at low frequencies to compensate for the stator resistance drop. For this purpose the general form of the input voltage as a function of frequency can be approximated by an equation of the form:

$$V_{\max} = A + B \cdot f \quad 6.1$$

where $V_{\max}$ is the peak input phase voltage and A & B are machine-parameter-dependent constants, which may be found by calculating the voltage required to ensure the same maximum output torque at different operating frequencies using equation G.2. Values of voltage versus frequency obtained for the motor under investigation are plotted in Fig. 6.2. If an approximate straight line is drawn through these it will have the equation

$$V_{\max} = 36.2 + 5.5f \quad 6.2$$

so that A and B of equation 6.1 are determined. Using the above equation, the steady-state torque-speed curves are now as shown in Fig. 6.1b which clearly indicates that the same normalized

torque-speed characteristics are obtained at all frequencies.

6.1.2 Inverter Wave Input

When the motor is supplied from a d.c.-link inverter, the motor terminal voltage is controlled by varying the d.c. voltage E applied to the inverter. According to equation F.2, E should be $1.61 V_{\max}$ for an inverter output peak fundamental equal to $V_{\max}$. Therefore, if the motor torque caused by inverter output harmonics, is neglected, an approximate equation relating E to frequency for constant maximum output torque is obtained by multiplying equation 6.2. by the factor 1.61, i.e.

$$V_{\max} = 1.61 \times 36.2 + 1.61 \times 5.5f$$

$$\text{thus} \quad E = 58.3 + 8.85f \quad 6.3$$

This same equation will apply for both 120° and 180° inverters (see Appendix F).

6.2 Starting Performance From Variable Frequency Sine Wave Supply

6.2.1 Linear Frequency Variation

Program MOTOR3 (described in Appendix B), in which the motor is supplied by a sinusoidal voltage, is used to compute the performance of the motor under investigation from standstill to

no-load speed, with the frequency varied linearly from zero to the steady-state value F_s (50 Hz in this case) according to:

$$f = \frac{F_s}{H} t \quad (t < H)$$

6.4

$$f = F_s \quad (t > H)$$

where H is the time in seconds for the frequency to reach F_s .

With the input voltage varied according to equation 6.2, results were obtained for several values of H , and from these it is found that a value of 0.18s gives the minimum starting time. Figs. 6.3 and 6.4 show the input phase current, phase voltage, torque, speed and frequency curves for this case.

The starting time, defined as the time required for the speed to rise from zero to 95% of synchronous speed, is 0.247s, which is 15.8% less than the corresponding time taken following the sudden application of the supply (Figs. 4.4 and 4.5). The starting current is also reduced by 22.4% and the torque is less oscillatory, as indicated in Figs. 6.3b and 6.4a. The differences in the motor performance between the direct-on-line and the v.f. starting procedures can be explained as follows: when the motor is started on a fixed-frequency supply, a small torque is produced at the beginning of the starting period due to the low rotor power factor at high rotor frequencies. If, however, the v.f. starting method is used, the low frequency at the beginning of the starting period improves the rotor power factor and so increases the torque per ampere, and consequently the start-up time and

starting currents are reduced.

The curves of frequency and speed (Fig. 6.4b) with time show that the frequency is in advance of the speed by a roughly constant value, except when both are initially zero. If, however, the frequency is varied not from zero but from a certain initial value F_{in} , while still maintaining the same time H taken to reach F_s , a further reduction in the start-up time is possible. The new frequency variation is:

$$f = F_{in} + \frac{F_s - F_{in}}{H} t \quad (t < H) \quad 6.5$$

$$f = F_s \quad (t > H)$$

Several values were tried for F_{in} and a value of 11.8 Hz was found to be the optimum for this motor. As indicated in Fig. 6.1b, 11.8 Hz is in fact equal to the slip frequency S_f at which maximum torque is produced, and this is the same for any supply frequency. Using this technique, the starting time is now reduced to 0.22s, a 25% reduction from the direct-on-line case. The motor phase current, voltage, torque, speed and frequency curves are shown in Figs. 6.5 and 6.6, in which the frequency and speed curves are now displaced from each other by an almost constant value.

6.2.2 Constant Slip Frequency Control

The above methods of voltage/frequency variation are for open-loop operation. In practice, however, closed-loop operation

is more satisfactory, since the slip frequency can be controlled to ensure that operation always occurs at a predetermined value which does not exceed the rotor breakdown frequency.⁷

As shown in the last section, for optimum results, the frequency should be in advance of the speed by an almost constant value. For constant slip frequency control operation, the frequency of the supply in program MOTOR3 is now controlled according to:

$$f = S_f + \frac{p\theta}{2\pi} \quad (f \neq F_s) \quad 6.6$$

with the voltage equation remaining the same as equation 6.2.

Using $S_f = 11.8$ Hz, the start-up time is now 0.225s representing a 23% reduction from the direct-on-line case. This result is not surprising, since the frequency and speed curves (Fig. 6.8b) for the constant slip frequency technique are similar to those of Fig. 6.6b for the linear frequency variation technique, and the motor phase current and torque curves (Figs. 6.7b and 6.8a) are also similar to those of Figs. 6.5b and 6.6a. From this similarity between the two techniques, it can be concluded that the constant slip frequency technique is more favourable, since the starting results are optimum irrespective of the motor parameters and load conditions, whereas for the linear-frequency variation technique it is necessary to establish the optimum rate of change of frequency for each particular motor and load conditions.

The above study was concerned with the nominal motor frequency of 50 Hz. The constant slip frequency technique was applied again for higher operational frequencies, and for the purpose of illustration, frequencies of 60 and 100 Hz were selected. It was found that the v.f. starting technique becomes more and more useful as the operational frequencies become higher. For instance with $F_s = 100$ Hz, a considerable reduction in the start-up time (56%) is achieved, as well as a reduction of 36% in the currents, when compared with the direct-on-line case.

The results obtained so far are summarized in Table 6.1.

6.3 Starting Performance from v.f. Inverter

The program INVERTER-MOTOR used previously for the analysis of the inverter-motor system and described in Appendix C, is again employed to study the behaviour of the induction motor when supplied from an actual variable-frequency inverter.

Firstly, computed results for the direct-start-up condition at 50, 60 and 100 Hz and for both 120° and 180° inverters were obtained. The d.c. input voltage to the inverter for each frequency was calculated from equation F.3.

Secondly, a linear variation of frequency from zero to 50 Hz (equation 6.4) was considered, with the d.c. input voltage varied according to equation 6.3. It was found that for both 120° and 180° inverters, there was an optimum start-up time for a fre-

quency rate corresponding to $H = 0.18s$. This is exactly the same rate obtained in Section 6.2.1 when a sine wave supply was employed. Using this optimum rate and the same voltage equation, the program was then used to compute the performance with the frequency varied according to equation 6.5.

Finally, the constant slip-frequency technique for 50, 60 and 100 Hz operation were considered. The results of the start-up time and the peak starting current for all cases for both the 120° and the 180° inverters are given in Tables 6.2 and 6.3. Discussion of these results is presented in section 6.5.

To give an idea of the motor performance when driven from the v.f. inverter, computer graphs for four cases are presented. These are:

- a) Direct-on-line starting for a 50 Hz supply from a 120° inverter.
- b) Linear frequency variation starting technique (utilising equation 6.5) for a 50 Hz supply from a 120° inverter.
- c) Direct-on-line starting for a 60 Hz supply from a 180° inverter.
- d) Constant slip frequency starting technique for a 60 Hz supply from a 180° inverter.

The motor phase voltage and current waveforms, and the torque, speed and frequency curves for these cases are shown in

Figs. 6.9 - 6.16 respectively. The graphs of the direct-on-line cases (Figs. 6.9, 6.10, 6.13 and 6.14) are similar to those presented in Section 5.5.1 for 25 Hz frequency operation. Graphs of all other cases (Figs. 6.11, 6.12, 6.15 and 6.16) clearly demonstrate that the motor starting performance using the v.f. starting techniques is improved over the direct-on-line method. This conclusion also applies to the sine wave case.

6.4 Speed Reversal

In this section, it is shown that it is possible to reverse the motor speed far more rapidly than with the usual plugging process. The procedure is to apply the braking technique (described in Section 5.3.3) which allows the motor speed to drop to zero very quickly, the stator supply sequence is then reversed and the motor is allowed to accelerate in the reverse direction using one of the v.f. starting techniques. Reversal of the phase sequence can be achieved using the inverter control circuitry, and no switching of power leads is necessary.

Thus it has been shown by computed results that the motor speed, using the usual plugging process, changes from full speed in one direction to 95% of synchronous speed in the other direction when operated from a 180° inverter at 50 Hz frequency in a time of 1.4s, with a peak phase current of 29A. If, however, the combined braking-v.f. starting technique is used, the motor speed

drops to zero in a fraction of a second and accelerates to 95% of synchronous speed in the reverse direction in 0.242s, with a peak motor current of 12.4A (see Table 6.3). This means that the reversing time and the peak current are reduced by 80% and 57% respectively from the values when the motor is plugged.

6.5 Comparison of Results

From the results given in Tables 6.1-6.3 for the start-up times and motor peak currents of all the starting methods for sine wave and inverter inputs, it can be concluded that:

- a) The start-up times and motor peak currents for direct-on-line starting using 120° and 180° inverters are almost the same as for a sine wave supply.
- b) The start-up times for all v.f. starting techniques for the 180° inverter are slightly greater than those for a sine wave supply, and for the 120° inverter they are greater still. Two reasons can be given for these differences:
 - i) As the inverter frequency begins to increase, the electromagnetic torque (and consequently the speed) remains zero for some time (Figs. 6.12a and 6.16a), as a result of the effective d.c. voltage applied to the motor. This period is about 11% of the start-up time, when the frequency is varied according to equation 6.4, and about 5% when the frequency is varied

according to equation 6.5. In contrast to this, the torque and speed for a sine wave supply increase from the instant the supply is applied.

- ii) Since the d.c. input voltage to the inverter follows the same variation for both the 120° and 180° inverters (equation 6.3), the actual r.m.s. motor phase voltage during the starting period for the 120° inverter is less than that for the 180° inverter due to the periods of open circuit involved.
- c) Whichever starting variation method is chosen, the peak motor current of the 180° inverter is lower than the direct-on-line current, by roughly the same amount as for the sine wave supply. For the 120° inverter the current is even lower, for the reason given in (ii) above.

6.6 Conclusion

The use of an optimum rate-of-linear-frequency variation for starting an induction motor driven from a v.f. sine wave supply offers a reduction both in the start-up time and the motor current, and results in a less oscillatory torque when compared with the usual direct-on-line starting method. The optimum rate-of-change of frequency depends on both the motor parameters and the load conditions. The same improvements in the starting performance are also obtainable when constant slip frequency control is used. This has the advantage of giving optimum results irrespective of the motor parameters and the load conditions.

Computer-predicted results relating the start-up time, motor current and torque for the motor driven from a 180° inverter using the above starting techniques are roughly similar to those for a sine wave supply, while when the motor is driven from a 120° inverter, the improvement in start-up time is slight, but the motor currents are further reduced. It is also noticed that the reduction in start-up time and current become more significant when it is required to accelerate the motor to high speeds.

TABLE 6.1

Results of v.f. starting technique for the
sine wave input

Steady State Frequency, Hz (F_s)	Starting Method ($f > F_s$)	Starting Time, s	% Reduction in Starting Time	Peak Input Current, A	% Reduction in Peak Current
50	Direct-on-line	0.2935	-	16.9	-
50	$f = \frac{50}{0.18}t$	0.247	15.8	13.11	22.4
50	$f = 11.8 + \frac{37.2}{0.18}$	0.2199	25	12.4	26.6
50	$f = 11.8 + p\theta/2\pi$	0.2255	23.2	13.0	23
60	Direct-on-line	0.377	-	18.2	-
60	$f = 11.8 + p\theta/2\pi$	0.256	32	13.3	27
100	Direct-on-line	0.8796	-	21.5	-
100	$f = 11.8 + p\theta/2\pi$	0.3852	56	13.8	35.8

TABLE 6.2

Results of v.f. starting technique for the
1200 inverter signal

Steady State Frequency, Hz (F_s)	Starting Method ($f \nmid F_s$)	Starting Time, s	% Reduction in Starting Time	Peak Input Current, A	% Reduction in Peak Current
50	Direct-on-line	0.2958	-	14	-
50	$f = \frac{50}{0.18}t$	0.3174	- 7.3	10	28.57
50	$f=11.8+\frac{37.2}{0.18}$	0.29	1.96	8.12	42.
50	$f=11.8+\frac{p\theta}{2\pi}$	0.306	- 3.4	8.6	38.6
60	Direct-on-line	0.369	-	18.38	-
60	$f=11.8+\frac{p\theta}{2\pi}$	0.345	6.42	8.7	52.66
100	Direct-on-line	0.818	-	18.4	-
100	$f=11.8+\frac{p\theta}{2\pi}$	0.507	37.95	9.0	51.08

TABLE 6.3

Results of v.f. starting technique for the
1800 inverter input

Steady State Frequency, Hz (F_s)	Starting Method ($f \neq F_s$)	Starting time, s	% Reduction in Starting Time	Peak Input Current, A	% Reduction in peak current
50	Direct-on-line	0.29	-	16.7	-
50	$f = \frac{50}{0.18}t$	0.271	6.4	13.4	19.76
50	$f = 11.8 + \frac{37.2}{0.18}$	0.2325	19.77	12.26	26.58
50	$f = 11.8 + p\theta/2\pi$	0.242	16.5	12.4	25.7
60	Direct-on-line	0.369	-	18.3	-
60	$f = 11.8 + p\theta/2\pi$	0.2733	25.9	12.8	30.05
100	Direct-on-line	0.8365	-	22.8	-
100	$f = 11.8 + p\theta/2\pi$	0.404	51.7	13.2	42.1

CHAPTER 7

CONCLUSIONS

Advances in thyristor/diode technology in recent years have enabled these devices to be increasingly utilised in speed control schemes for induction machines. Their increased power-handling and reliability ensures their continued adoption for such control applications, and a technique is clearly required for the prediction of the performance of existing systems, and those which may be developed in the future.

A tensor technique has been employed in this thesis which can deal efficiently with the varying topology of the inverter system as the thyristor conduction pattern changes, and which also enables the equations of the combined inverter-motor systems to be solved in an accurate and efficient manner using a digital computer. As an example of the advantages of the tensor technique over possible alternative methods (for instance, the resistance and model subroutine methods), a typical program run required 14s to produce one cycle of output for an inductively-loaded inverter, whereas 2400s for 1/6th of a cycle was needed for the resistance method. Although only 10s was needed for the model subroutine method, the mathematically cumbersome and complicated equations involved when studying the complete inverter-motor system more than outweigh the small computer time. It has been illustrated how the tensor technique may be extended to include the motor system in an easy manner without adding complexity to the system equations. A transformed stationary 3-axis model has been employed for the motor system, which combines the advantages of the direct-phase and the conventional transformed 2-axis models. Using the direct-phase model,

it is possible to monitor the machine phase variables directly and the technique enables the combined inverter-motor system to be studied conveniently. However, the method suffers from the excessive computational times required to invert the system inductance matrix at each step of the numerical solution. The transformed 2-axis model, on the other hand, has the advantage of a low computational time (only one-sixth that of the direct-phase model) although difficulties are encountered in expressing the machine variables when the branches of the inverter open and close, and also when studying fault conditions. As the stationary 3-axis model describes the machine in terms of its stator phase quantities, and at the same time transforms the rotor quantities to a stationary reference frame, this method thus retains the advantages of both the other models, and has been used extensively in the work described.

To prove the validity of the models used in the analysis of the inverter motor systems, computer solutions for the performance of both 120° and 180° conduction period inverter systems, were obtained. The studies included system operating conditions such as start-up, braking, plugging, step frequency changes and variable frequency operations. Also included were fault conditions at the motor terminals. The computer solutions were complemented by typical experimental results taken on a small laboratory inverter-motor system and, in the majority of cases, the agreement obtained was very good.

It was concluded that the performance of the motor when fed from the 180° inverter was slightly different than when the 120°

inverter was used. The motor phase voltage waveform for the 180° inverter, for example, remained always of the 6-step square shape, whereas that for the 120° inverter, changed significantly during transient and loading conditions due to the open circuit periods involved in the motor phases during each supply cycle. The motor phase currents for the 180° and 120° inverters are also different; for example the motor peak current for the 120° inverter during plugging was 22% less than that for the 180° inverter under the same conditions. In general, the power transmitted through the 120° inverter is less than that transmitted through the 180° inverter for the same system parameters, which is due obviously to the difference in the conduction periods of the inverter thyristors.

Finally, when studying the induction motor starting performance from a variable-frequency inverter, it was concluded that if the frequency is varied linearly from a certain initial value (rather than from zero), a better starting performance is possible. The same performance is also obtainable when the frequency is controlled in such a way as to keep constant the slip frequency at which maximum torque is produced.

Suggestions for Further Work

This thesis has been concerned with the analysis of an inverter-induction motor system, and any continuing research could usefully be directed towards a number of areas, including for example:

- a) The transformed stationary 3-axis model, which has proved successful and powerful for induction machine systems may prove useful for synchronous motor applications, such as in the textile industry, where a closely-regulated speed is required.
- b) The versatility of the tensor technique may be usefully employed in studying more complicated systems such as those involving switching devices in the rotor circuit of an induction motor. An example of a system of this type, presently used in industry, is the modified Kramer drive, used to recover the bulk rotor power loss when adjusting the speed of a large induction motor by controlling the slip.
- c) The tensor approach could also be applied to a cycloconverter system and, since there are a large number of thyristors in this application, the tensor technique would prove to be a powerful tool.

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FIGURES

INSTANTANEOUS CAR SCREENWASH WATER HEATER

Features:

PROVIDES INSTANT¹ HOT² WATER TO CLEAR ANY WINDSCREEN

FOR USE WITH ANY CAR

FULLY AUTOMATIC

EASILY INSTALLED

SAFE

ELECTRICALLY POWERED

WILL NOT 'FLATTEN' THE BATTERY

1 It may take upto 10 seconds for the water temperature to reach a maximum

2 Typically the water temperature will be raised to 45°C (115°F)

3 Raymond Avenue.

842081

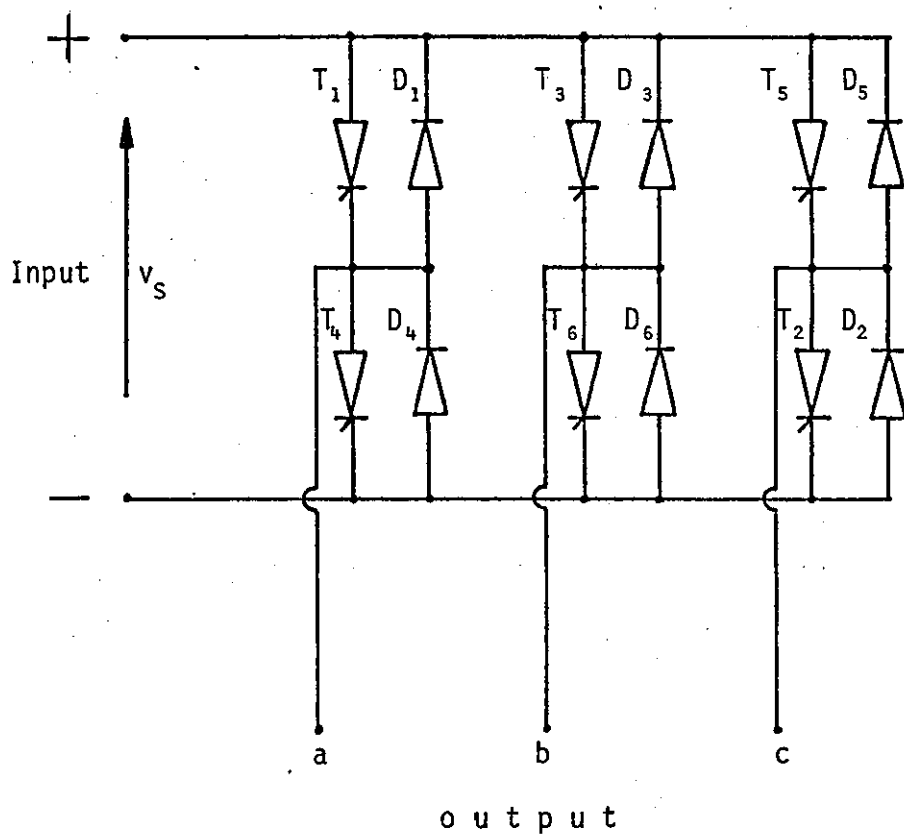


FIGURE 2.1 The general d.c.-link type inverter circuit

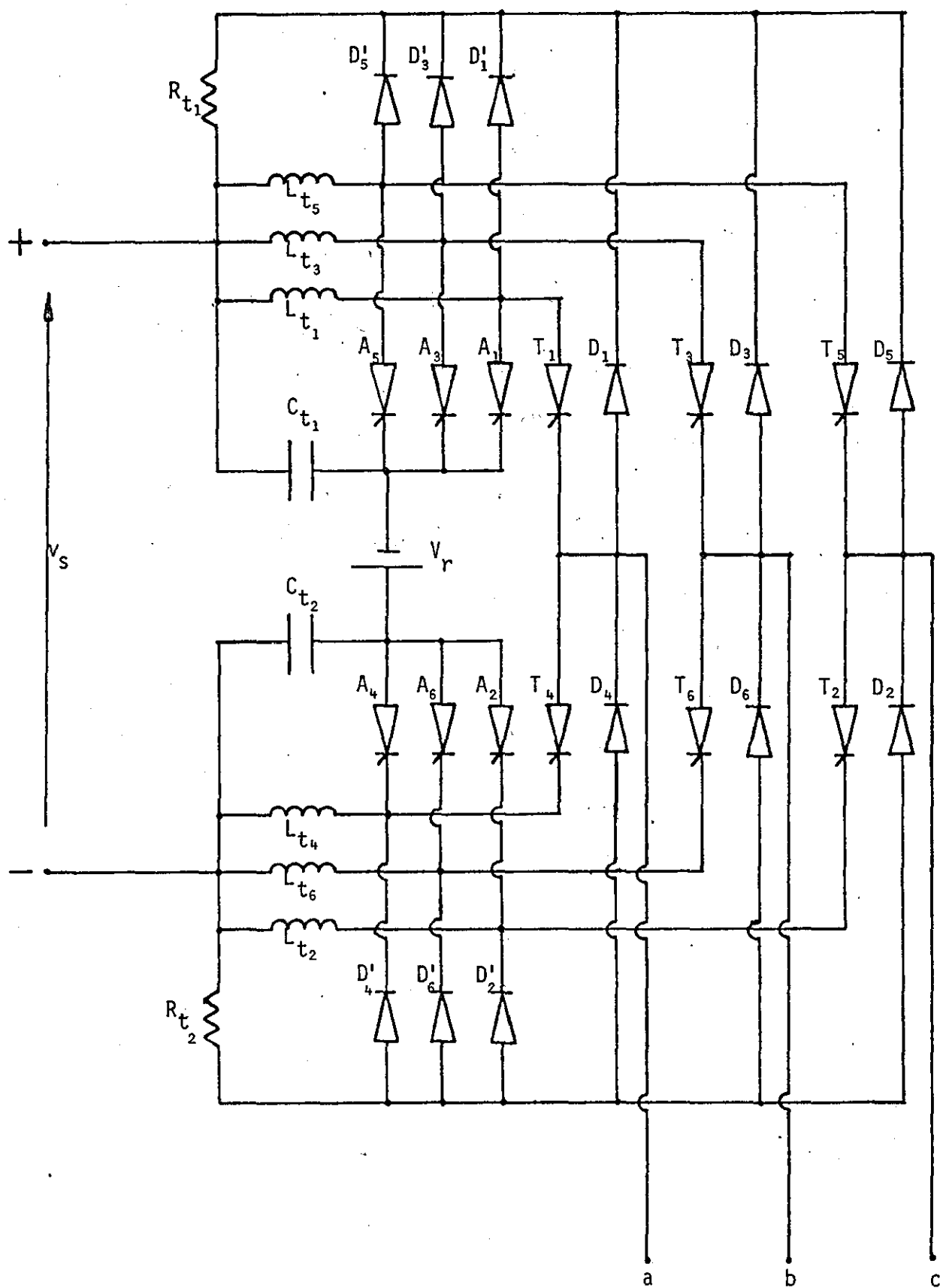


FIGURE 2.2. The individual d.c.-side commutation inverter circuit

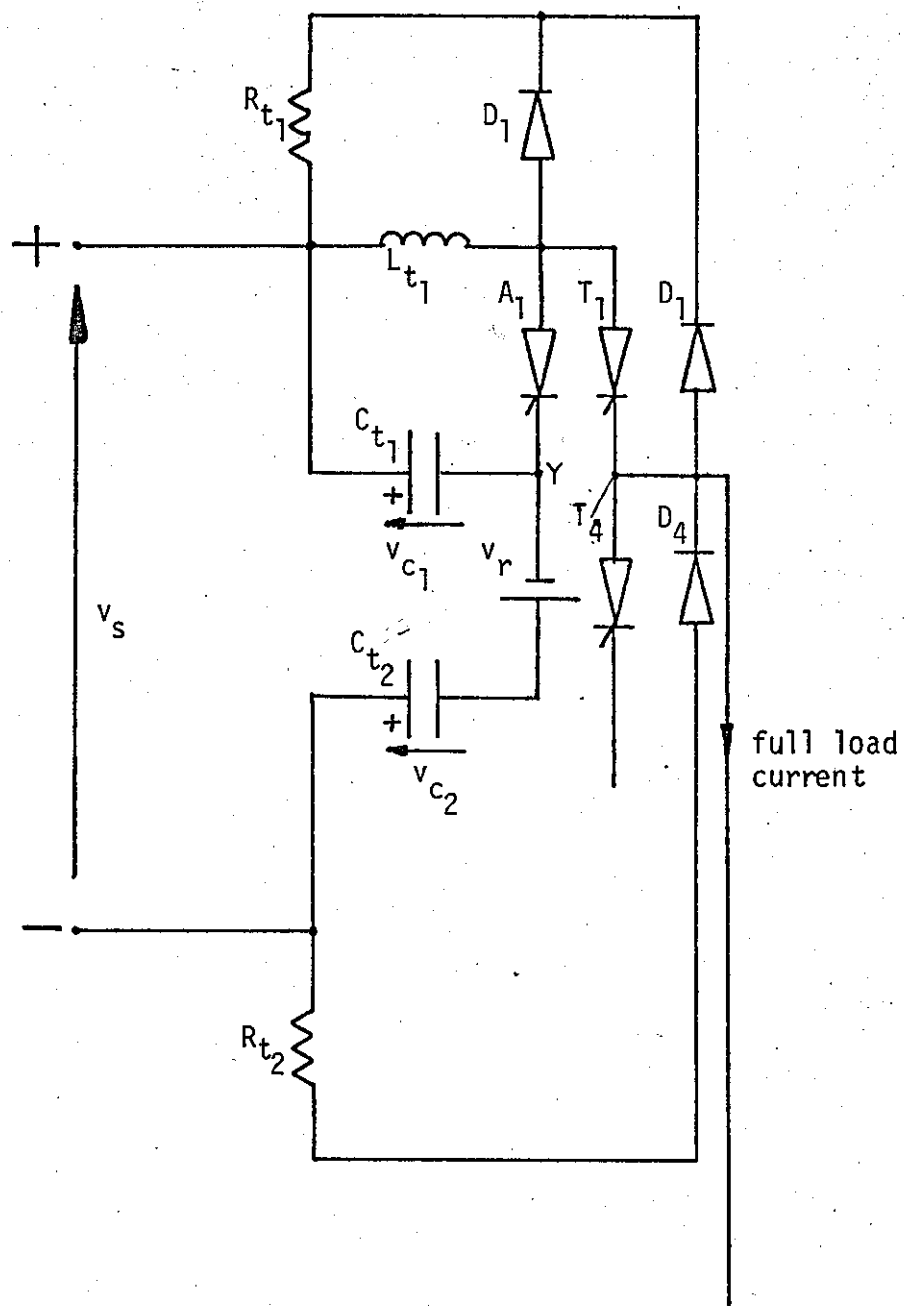


FIGURE 2.3 Illustration of the commutation process

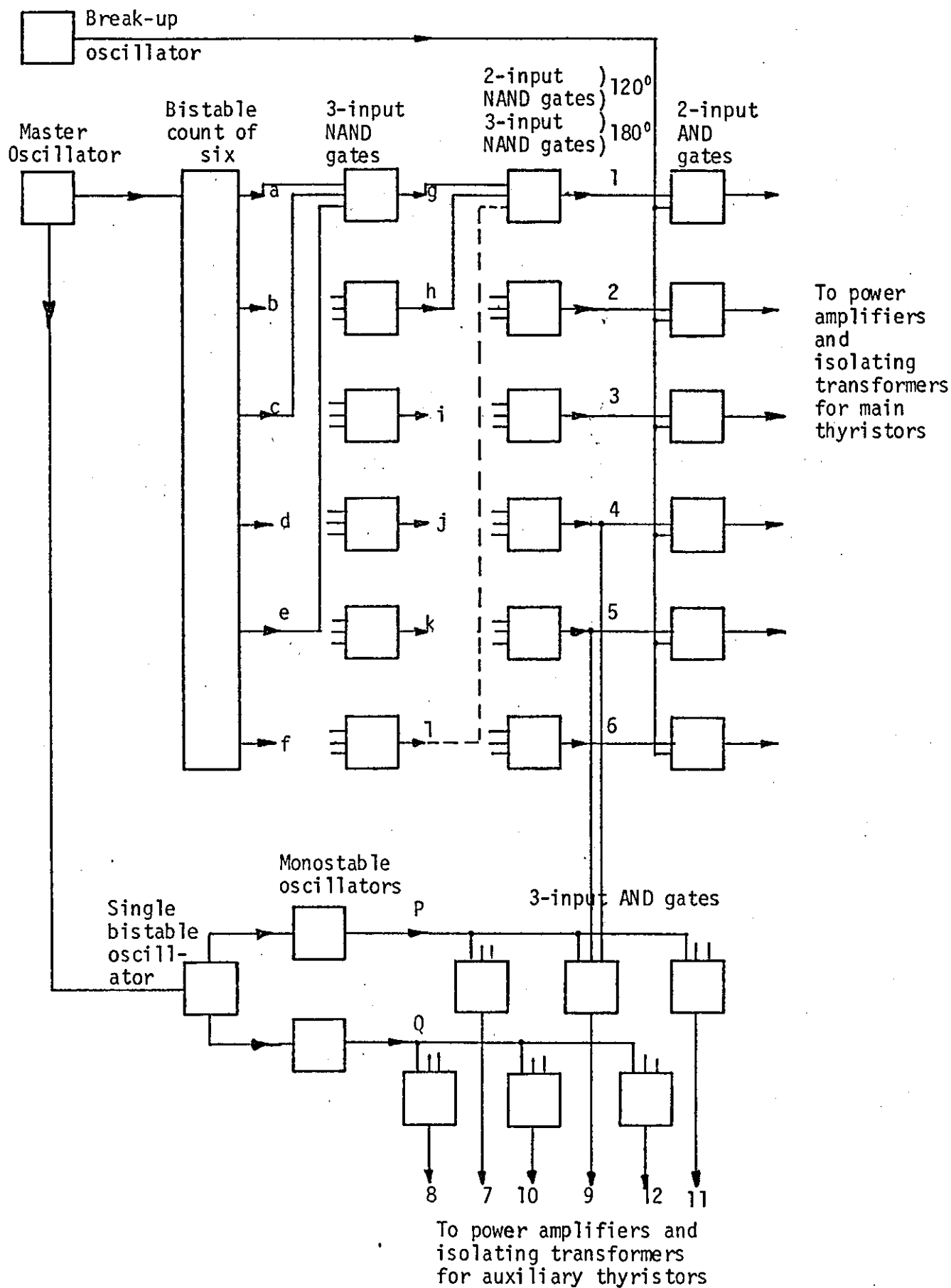


FIGURE 2.4 Block diagram of the electronic control circuit for the 120° inverter, with dotted lines for the 180° inverter. (Inhibition circuits not included)

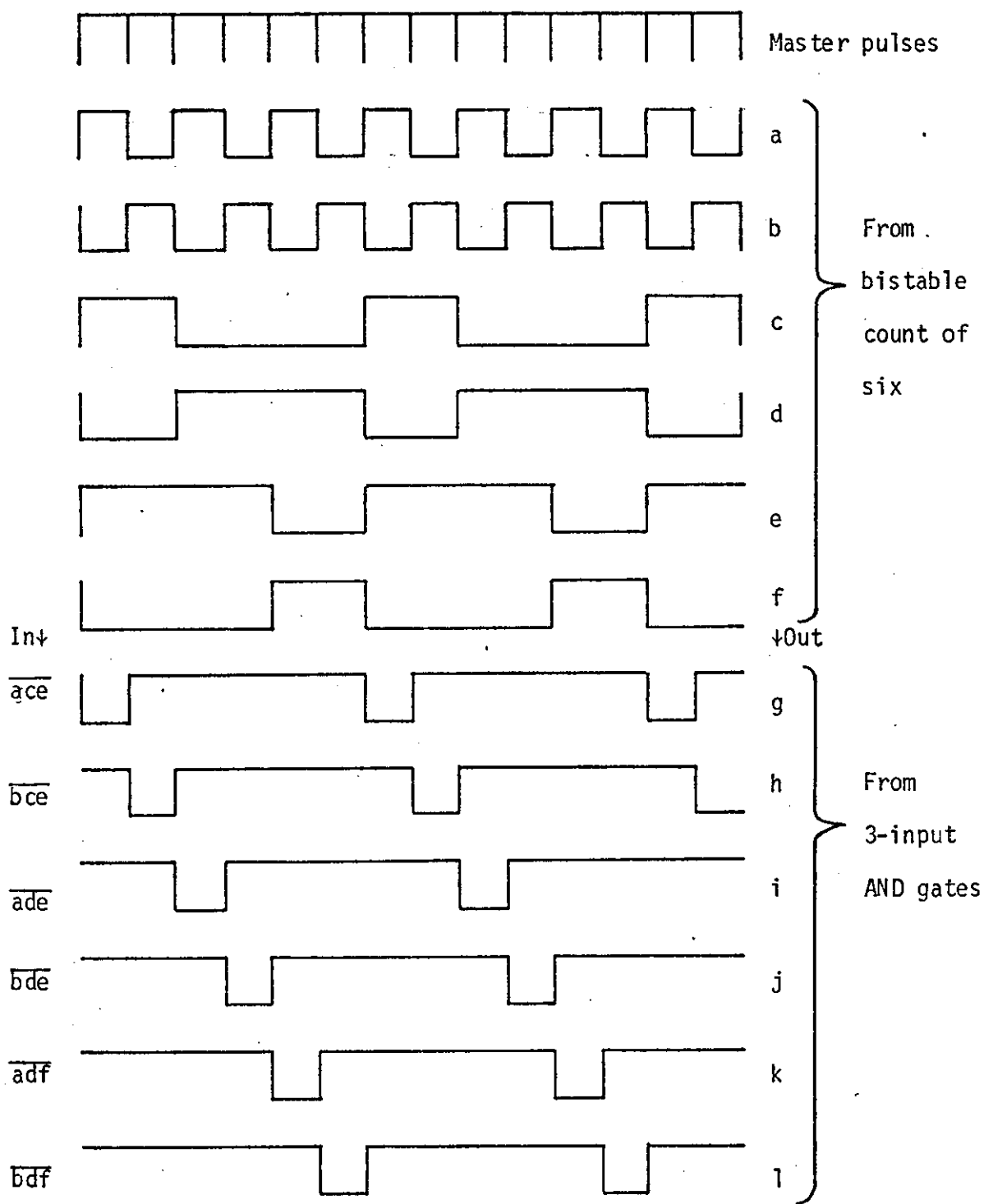


FIGURE 2.5a Formation of the main thyristor pulses (continued next page)

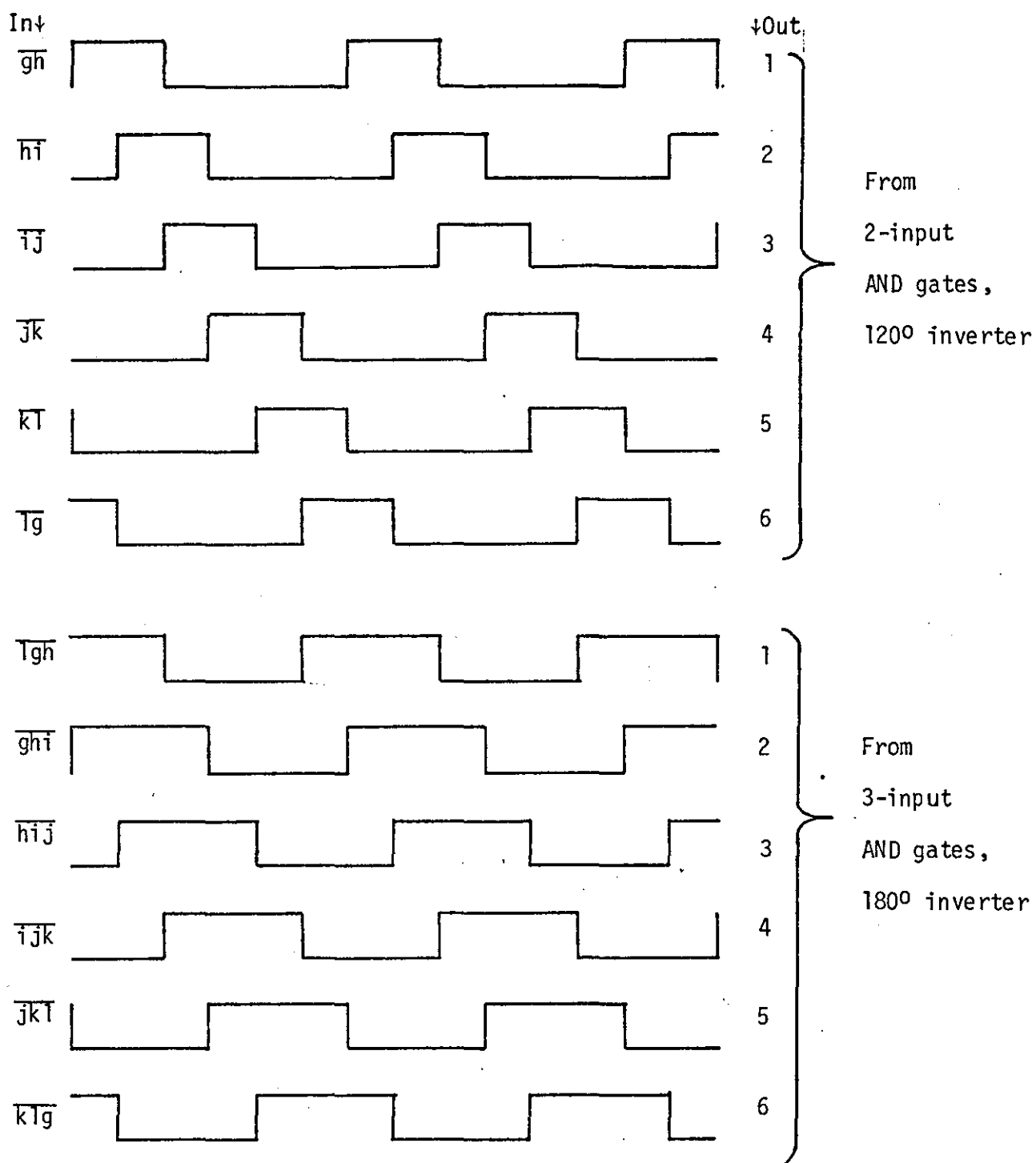


FIGURE 2.5a (continued) Formation of the main thyristor pulses

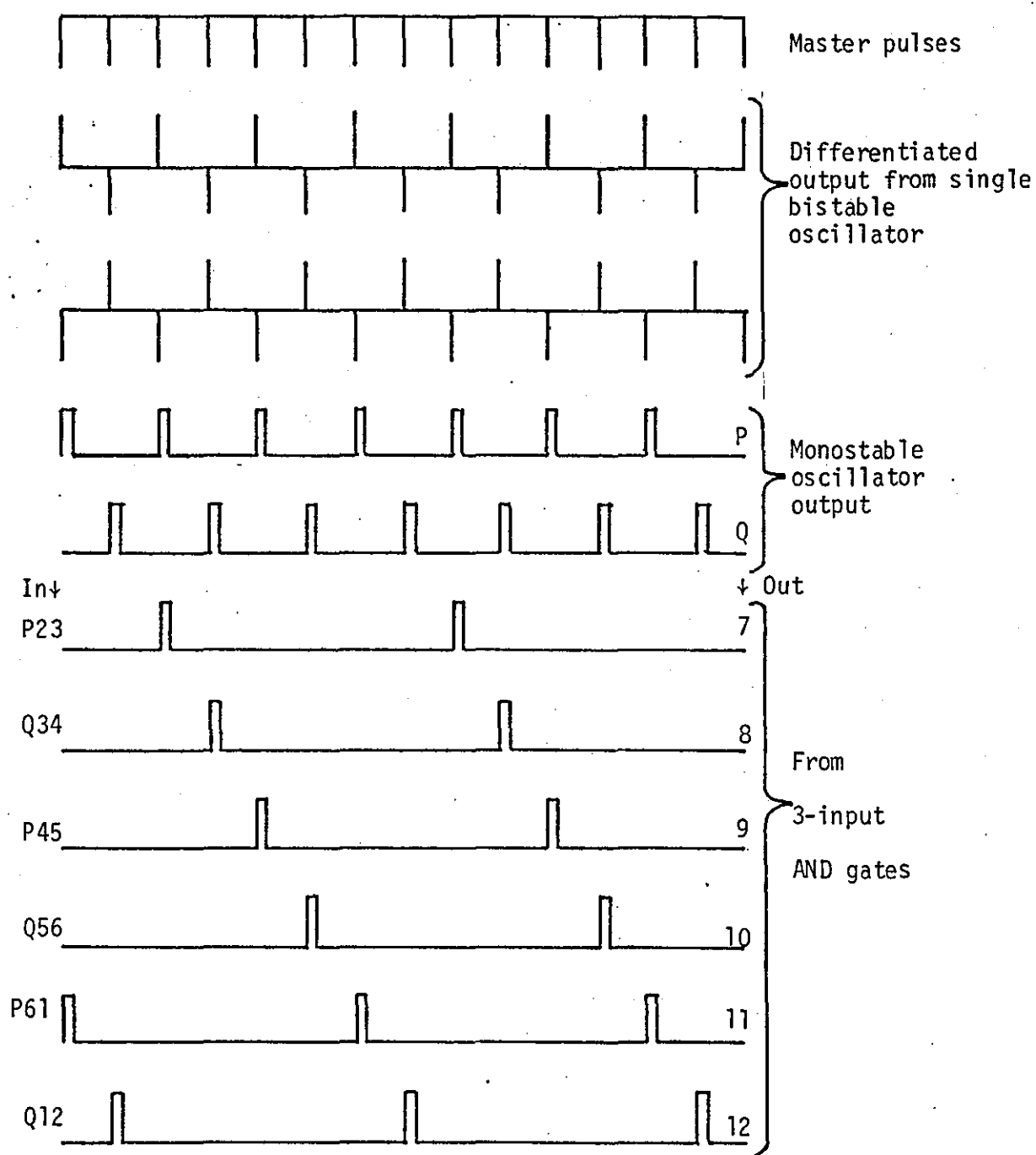


FIGURE 2.5b Formation of the auxiliary thyristor pulses

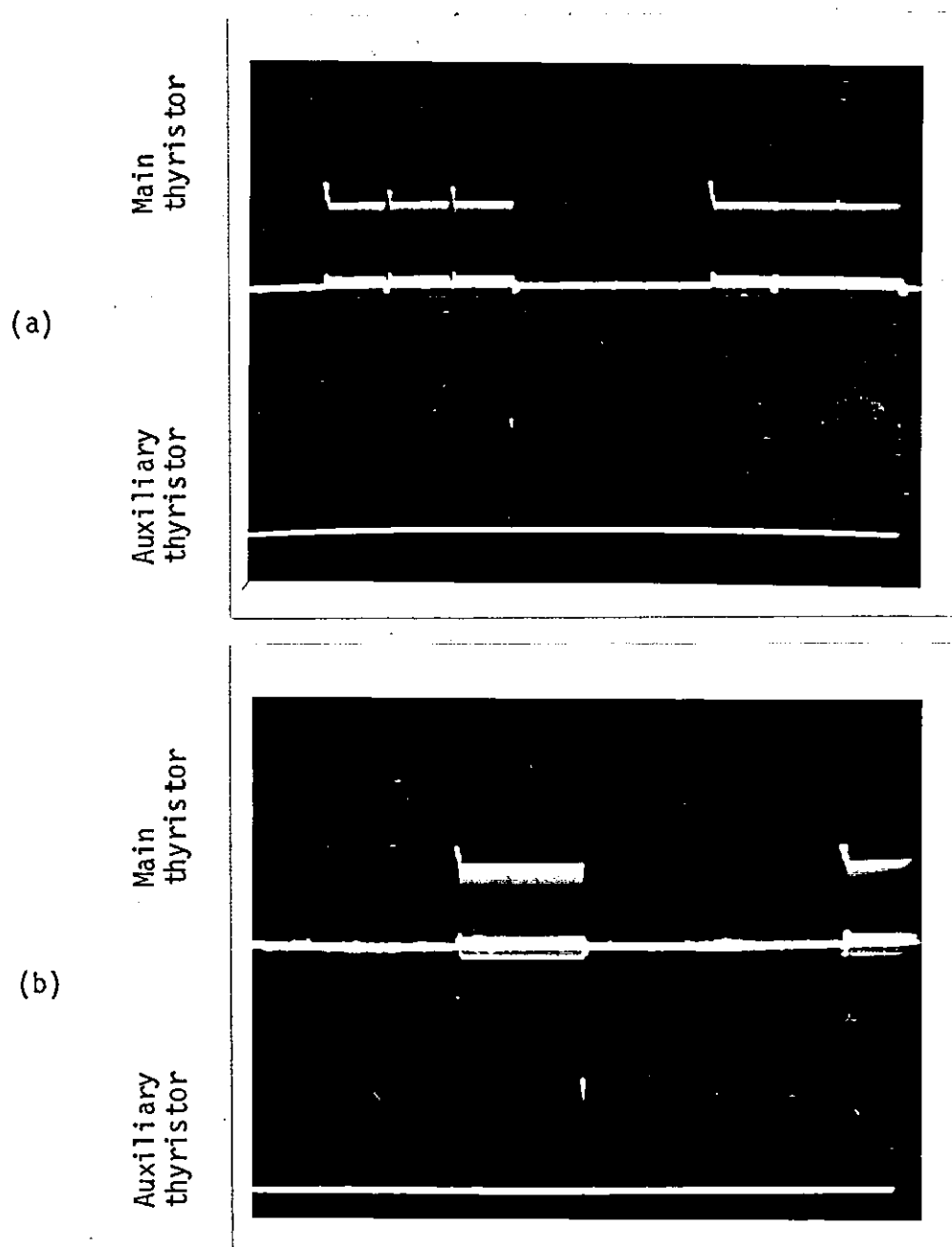
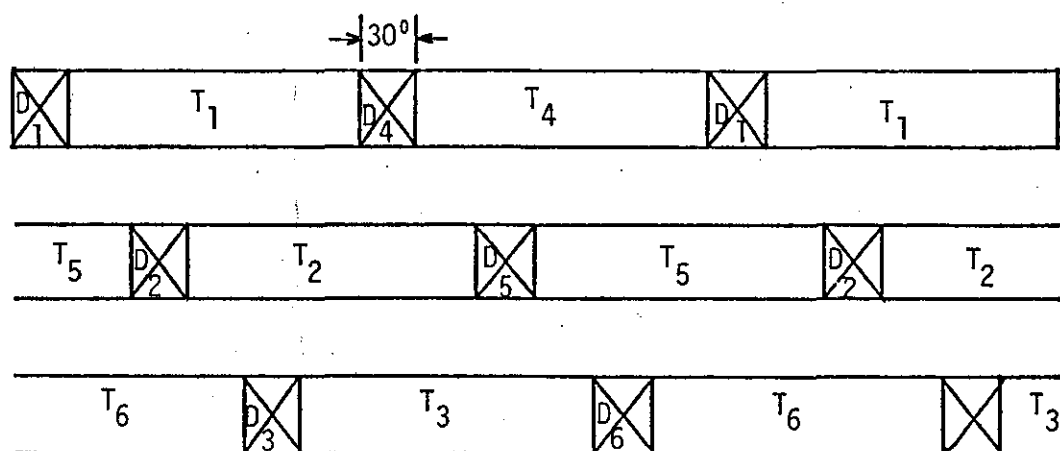
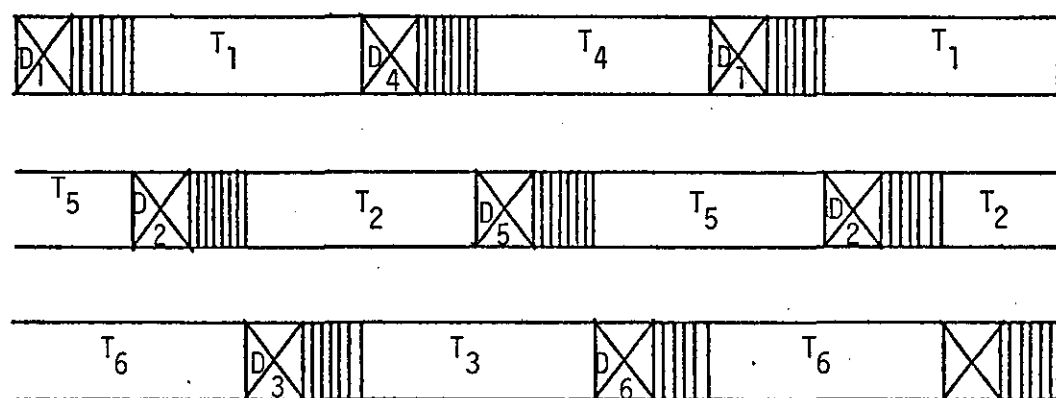


FIGURE 2.6 Practical tracings of thyristor gate signals
(a) 180° inverter
(b) 120° inverter



(a)



(b)

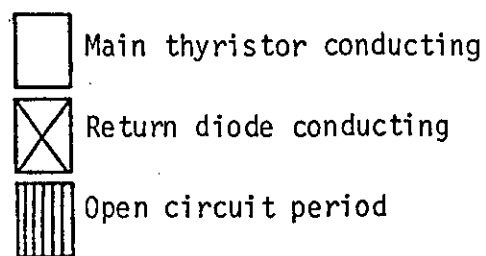


FIGURE 2.7 Thyristors and diodes conduction patterns:

- a) 180° inverter
- b) 120° inverter

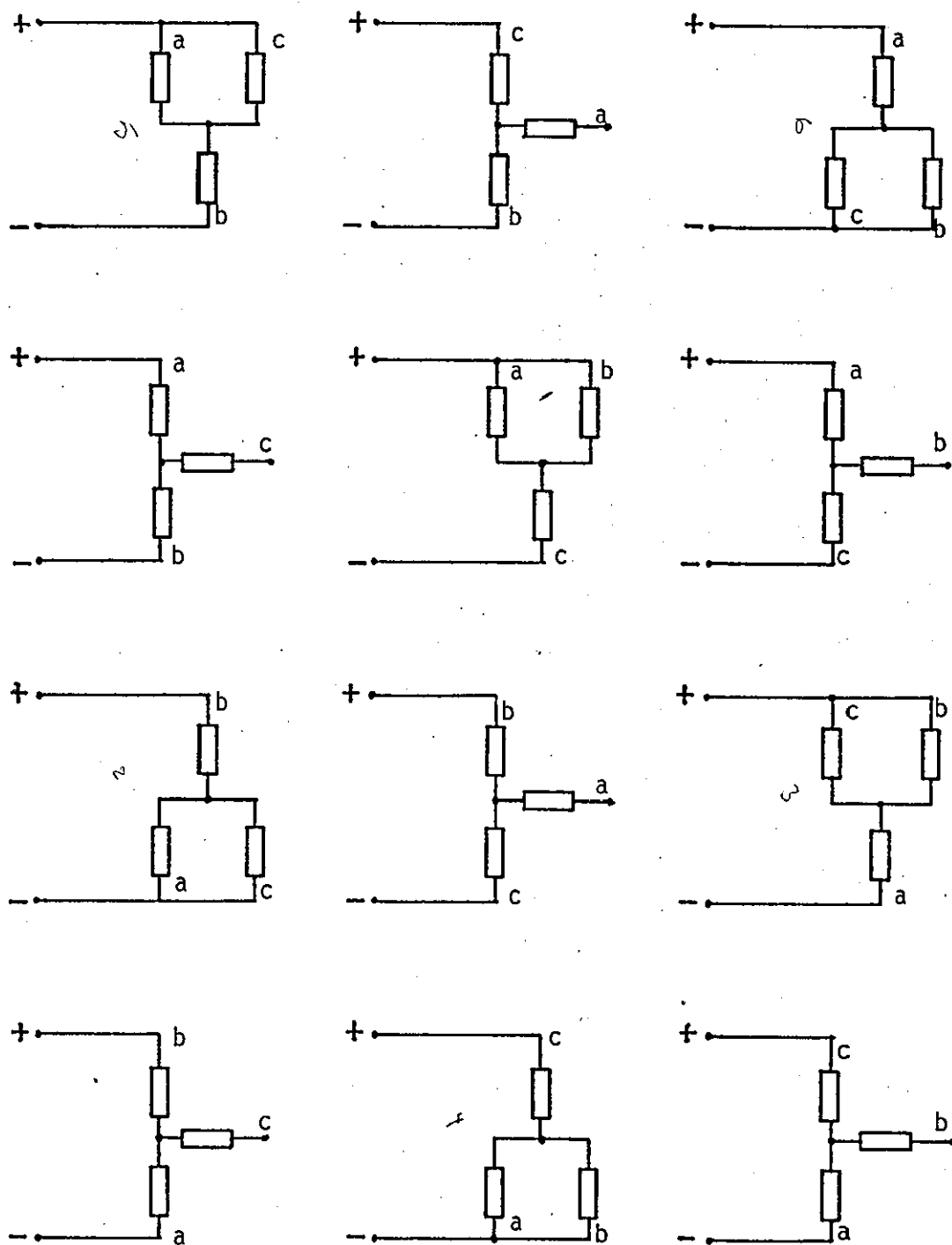


FIGURE 2.8 Inverter circuit topologies

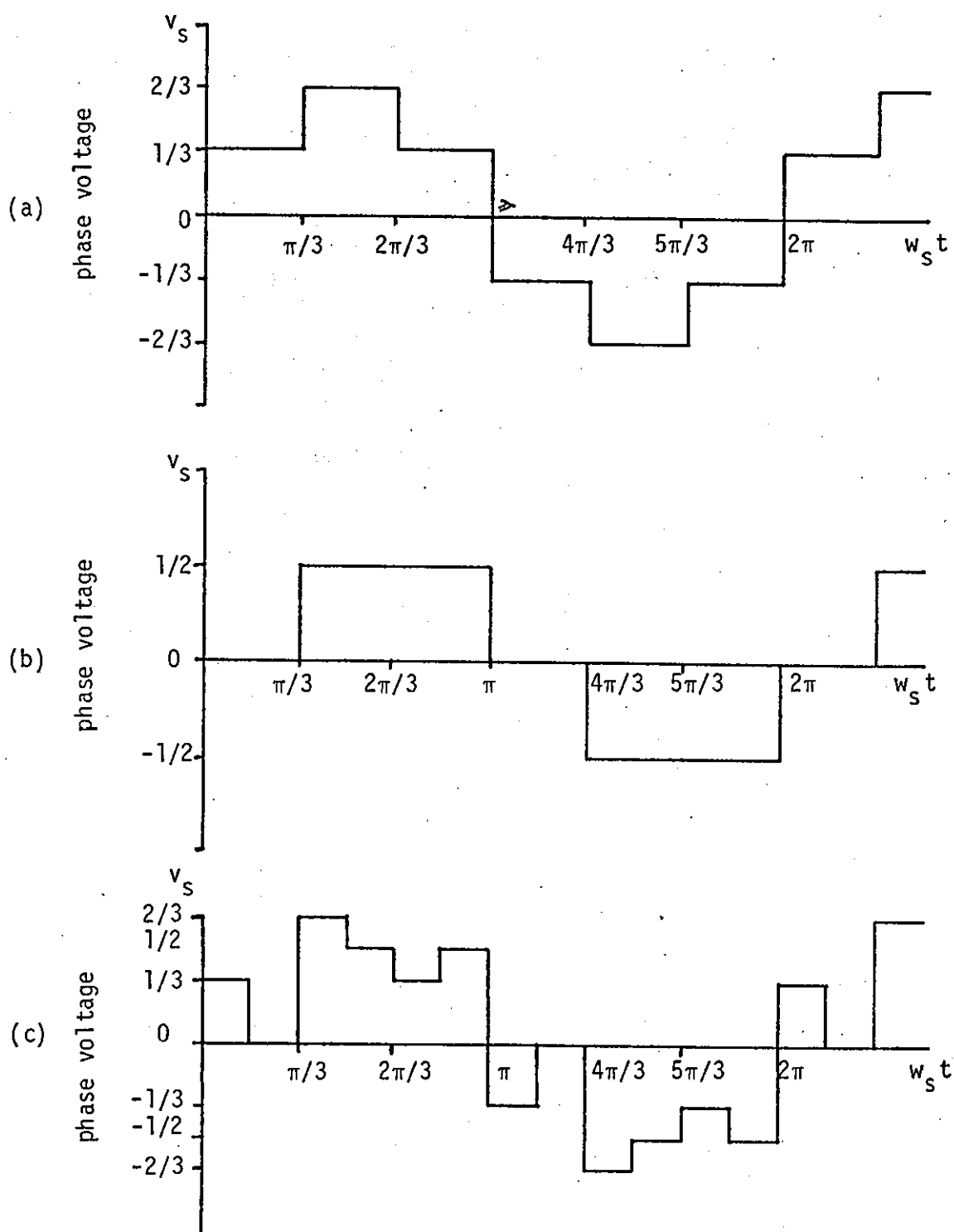


FIGURE 2.9 Theoretical output phase voltage waveforms:

- a) 180° inverter.
- b) 120° inverter with resistive load.
- c) 120° inverter with inductive load.

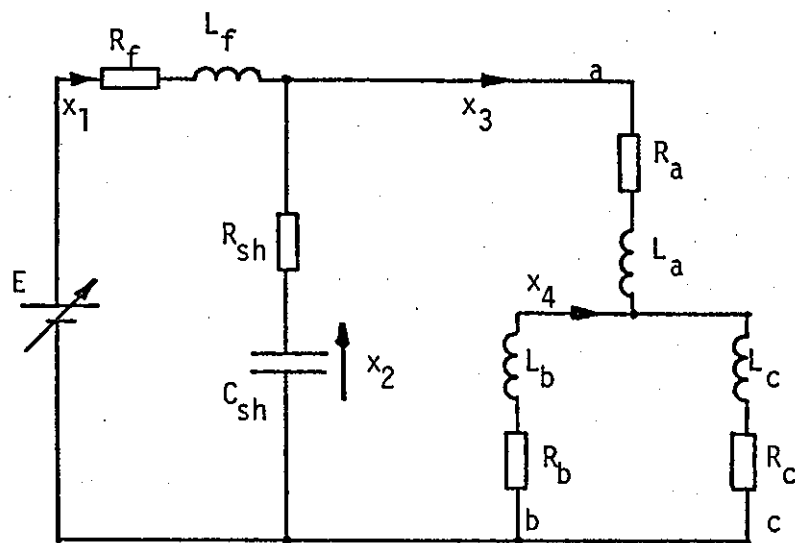


FIGURE 3.1 The circuit topology for the example given in Section 3.2.1.

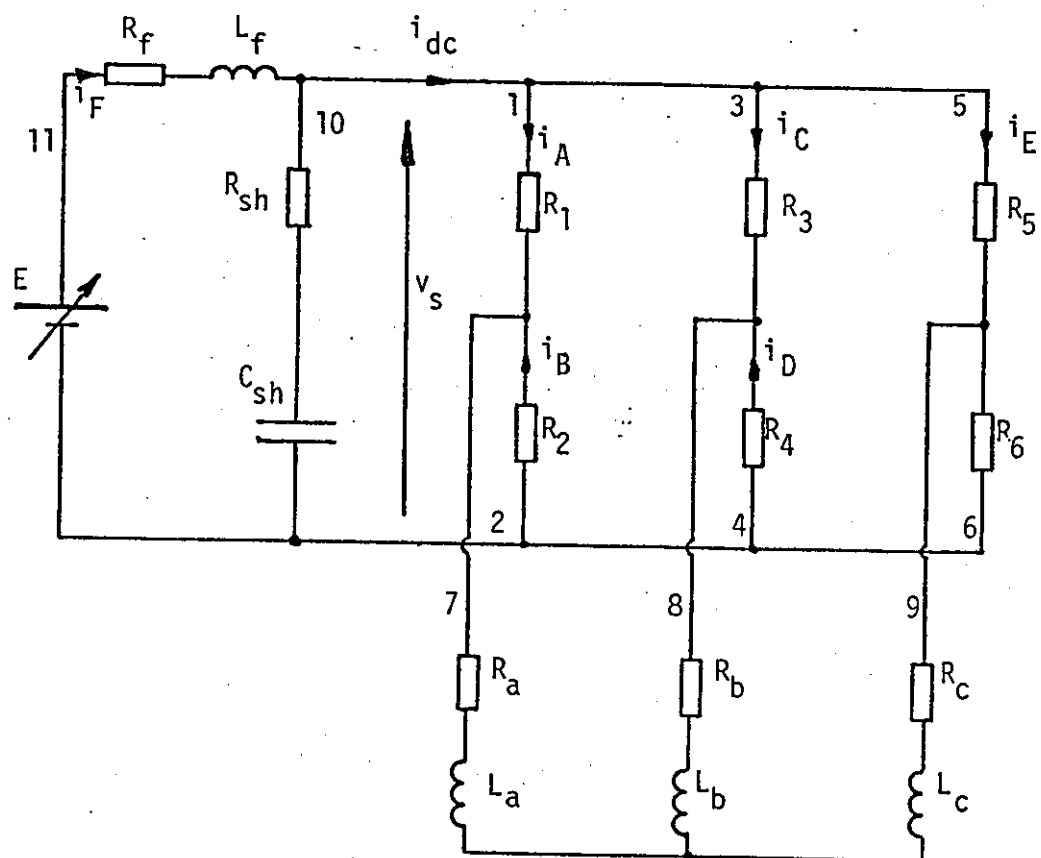


FIGURE 3.2 The basic network of the inductively-loaded inverter system.

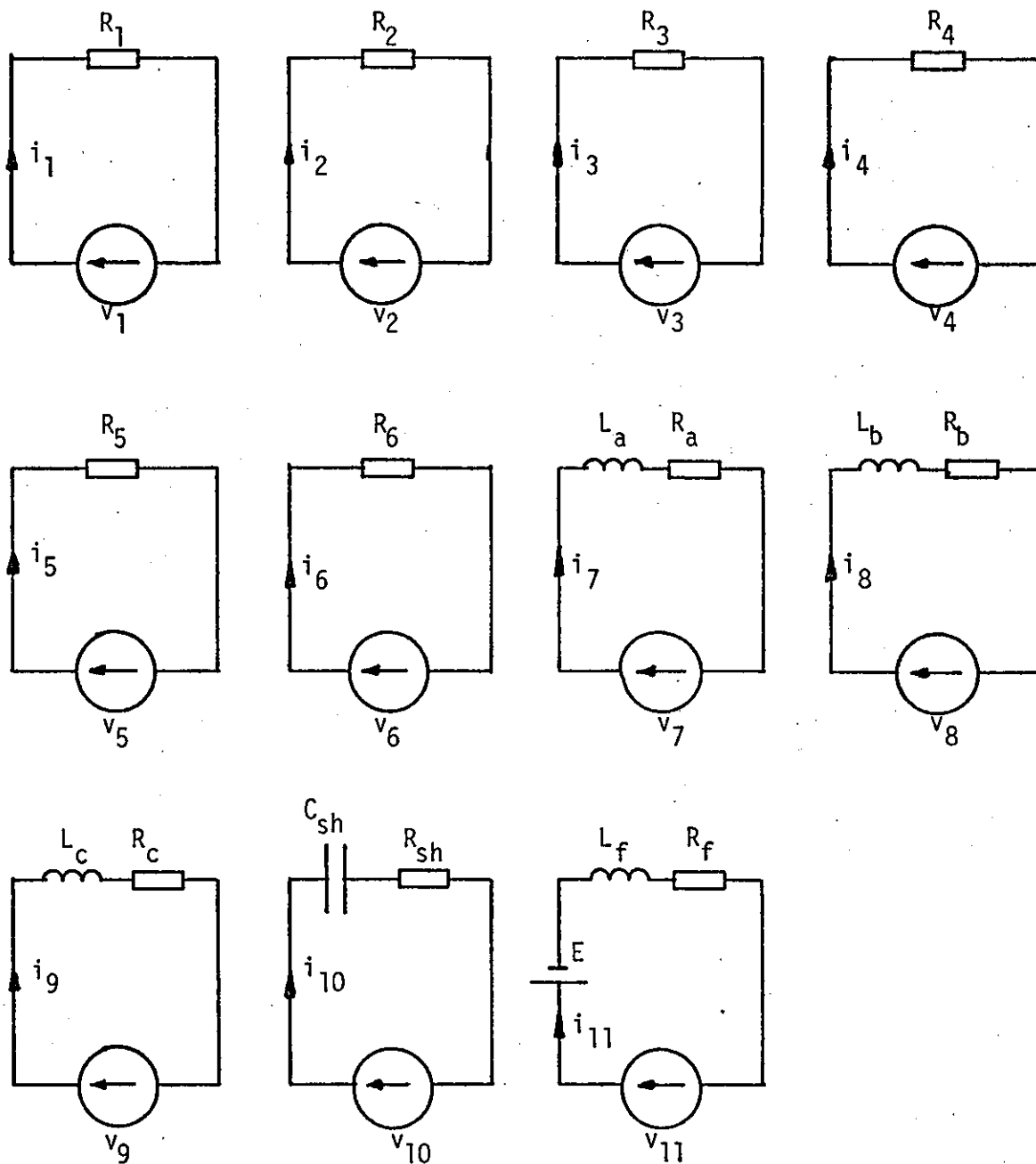


FIGURE 3.3 The primitive model of Figure 3.2.

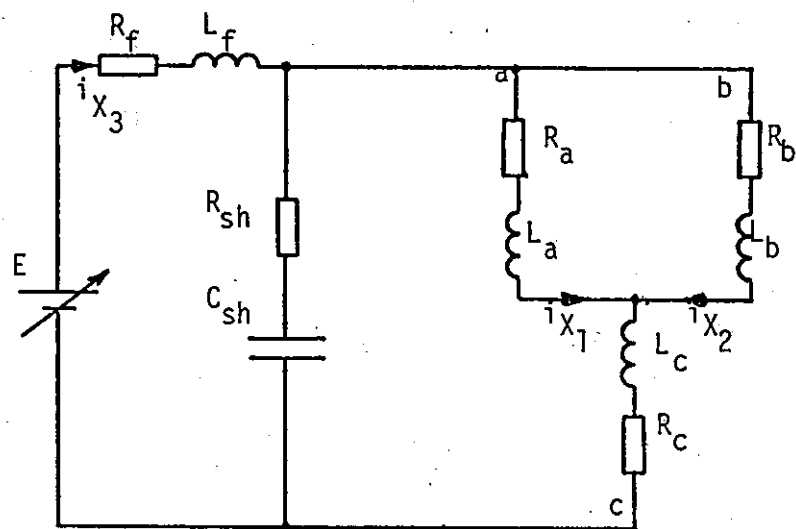
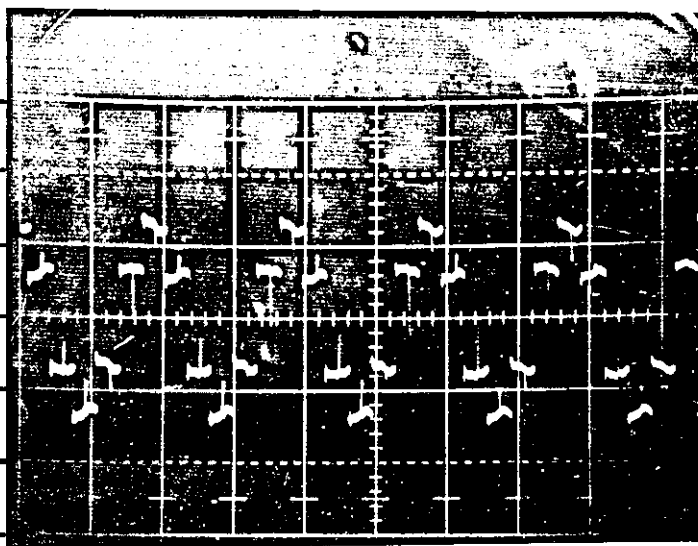


FIGURE 3.4 The reduced network for the example given in Section 3.2.3.3.

(a)

PHASE VOLTAGE (V)

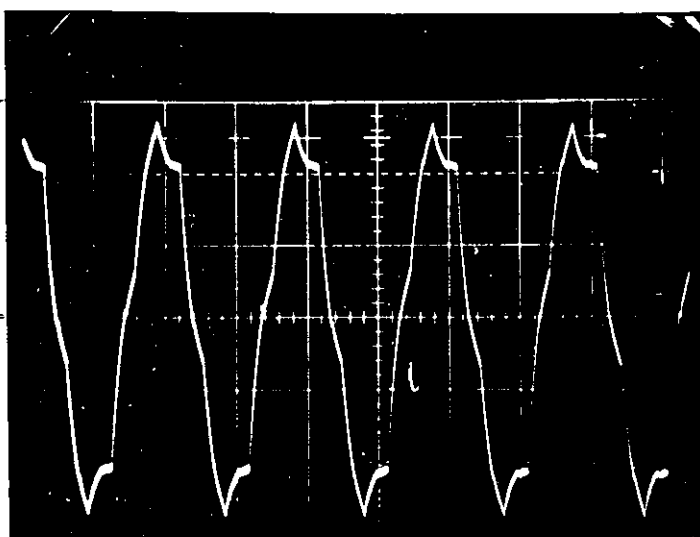
75
0
-75



(b)

PHASE CURRENT (A)

3
0
-3



(c)

INVERTER CURRENT (V)

3
0
-3

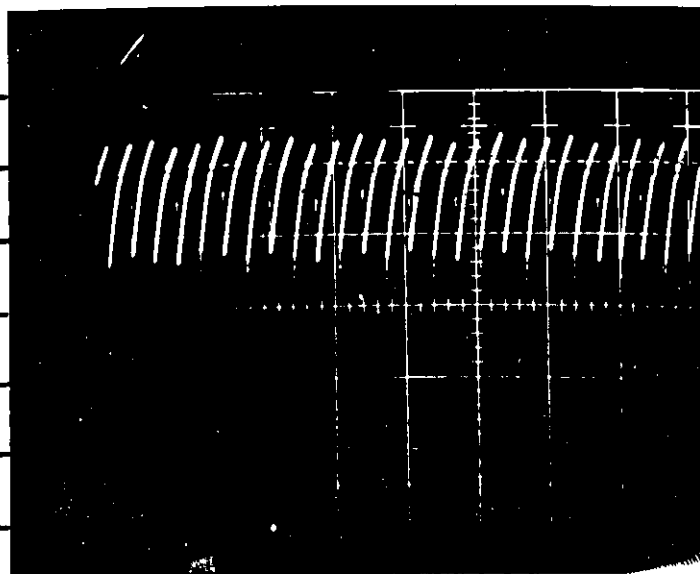
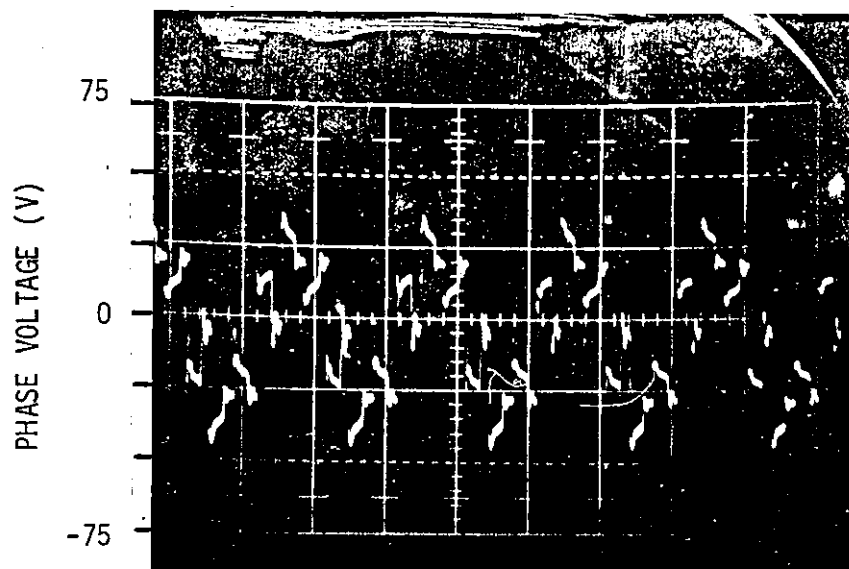


FIGURE 3.5

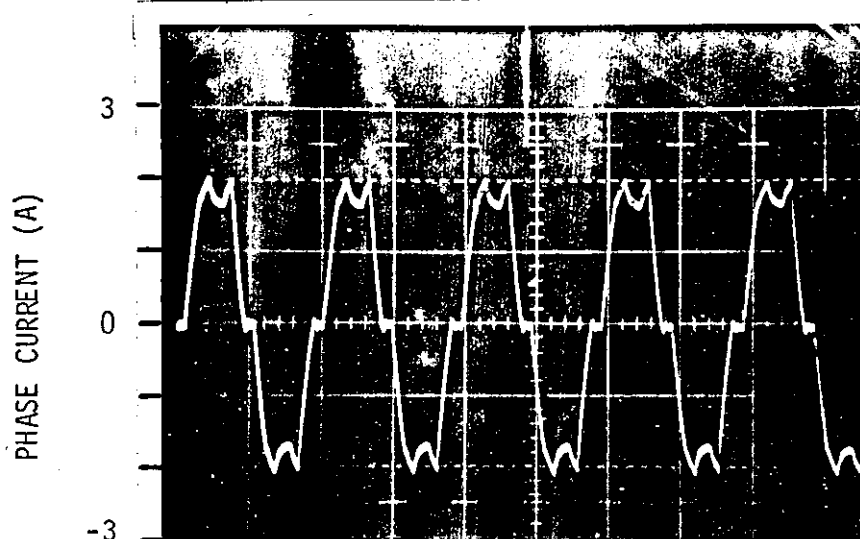
Experimental results for the inductively loaded 1800 inverter

Scale: voltage 25V/div
current 1A/div
time 10 ms/div

(a)



(b)



(c)

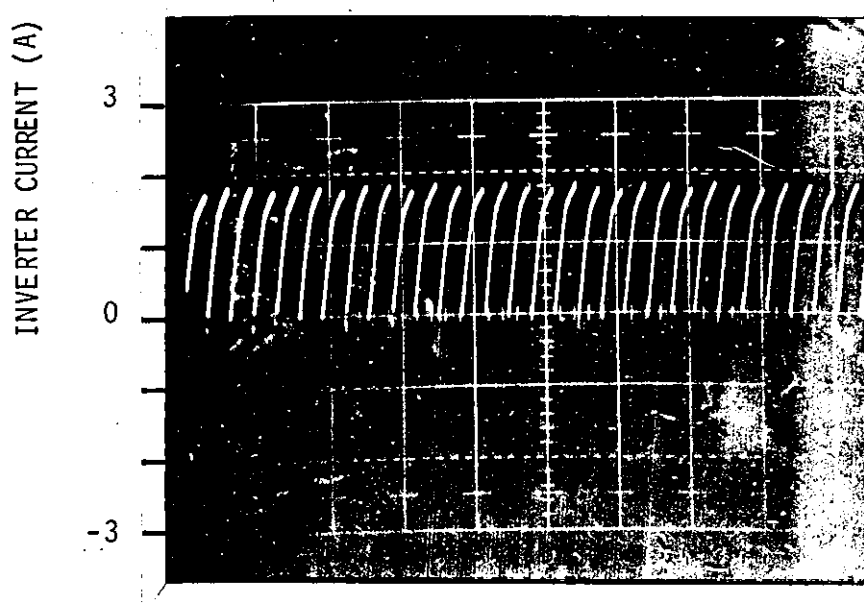


FIGURE 3.6 Experimental results for the inductively loaded 120° inverter

Scale: voltage 25V/div
current 1A/div
time 10 ms/div

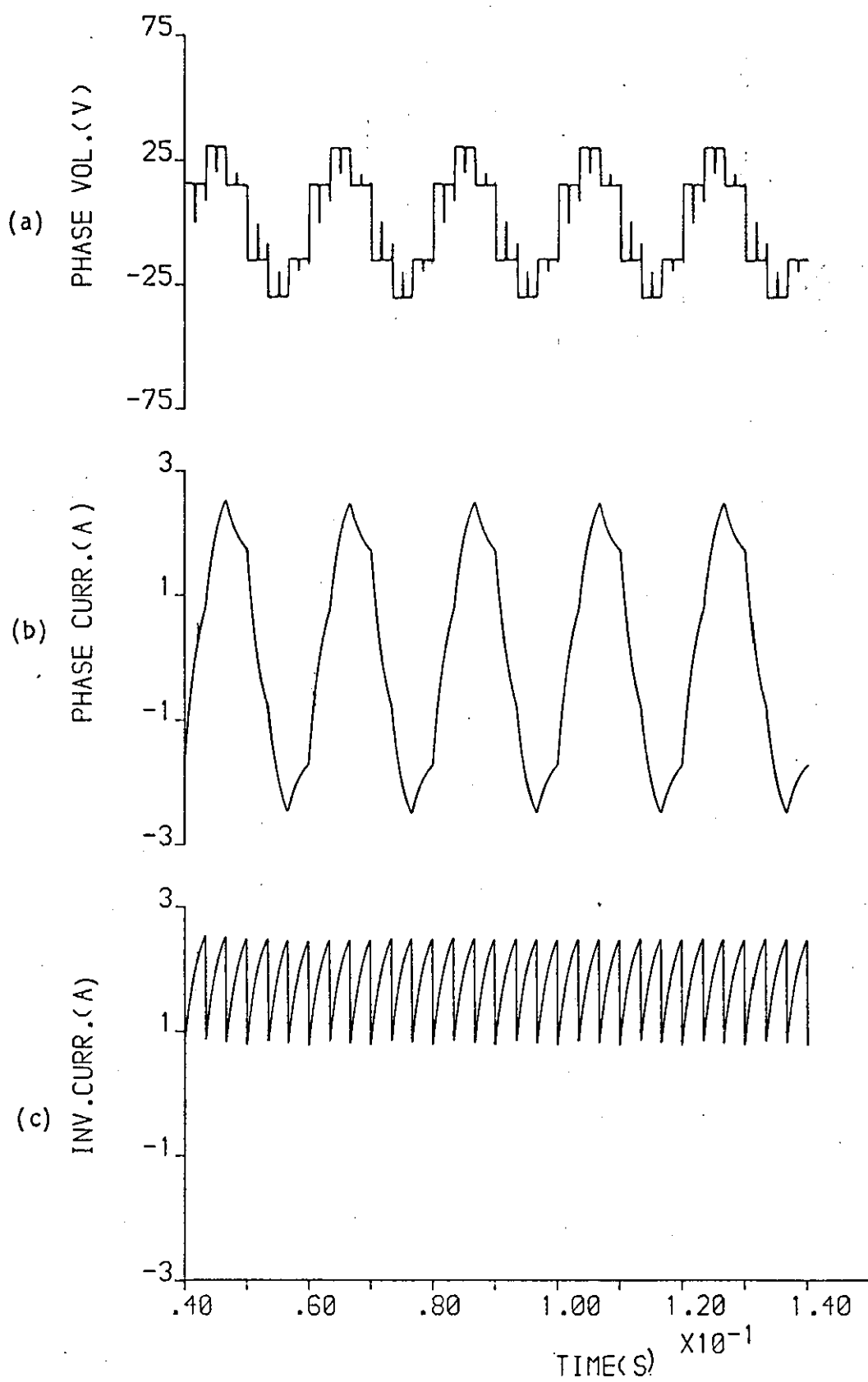


FIGURE 3.7 Computed results for the inductively loaded - 180° inverter

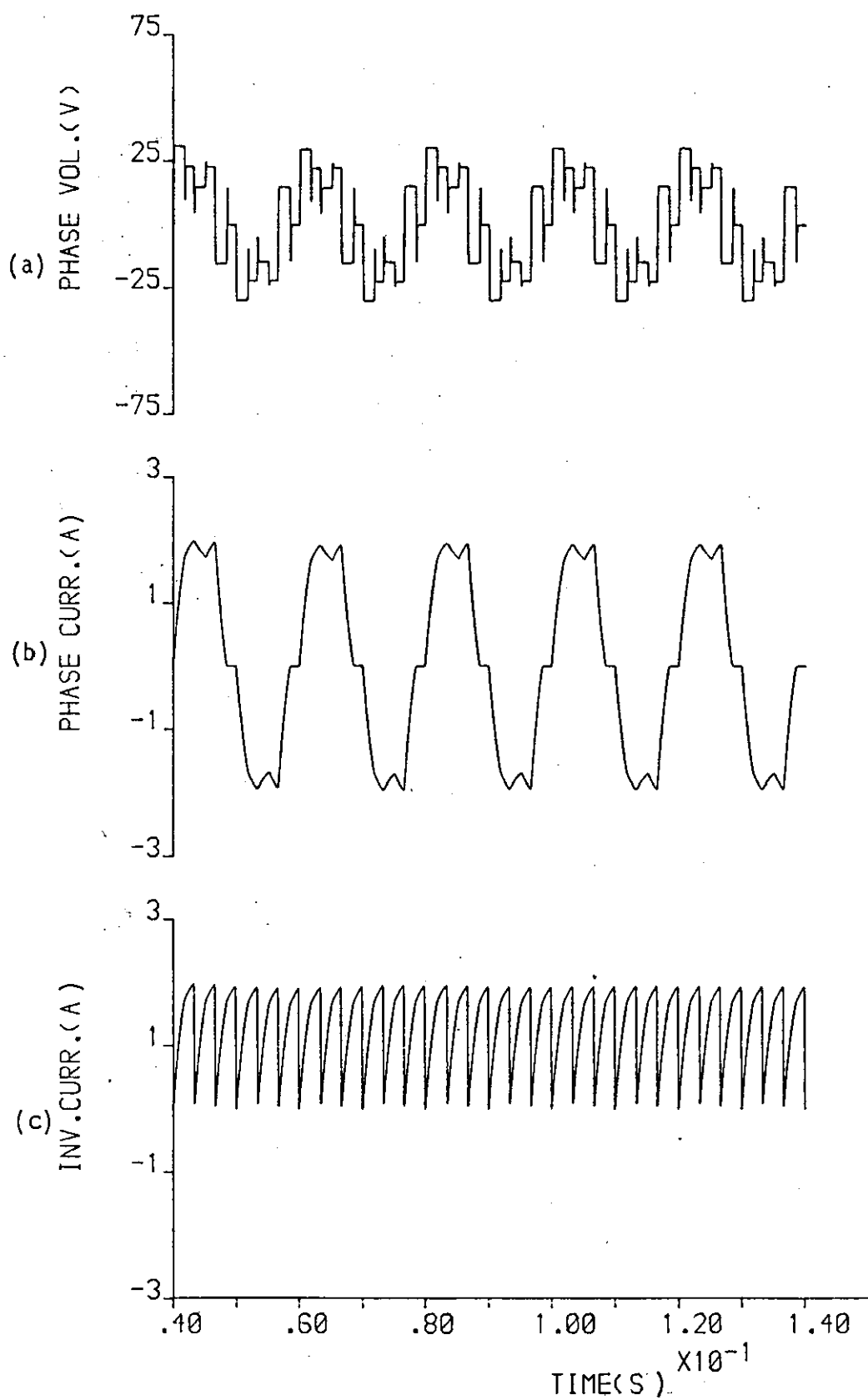


FIGURE 3.8 Computer results for the inductively loaded - 120° inverter

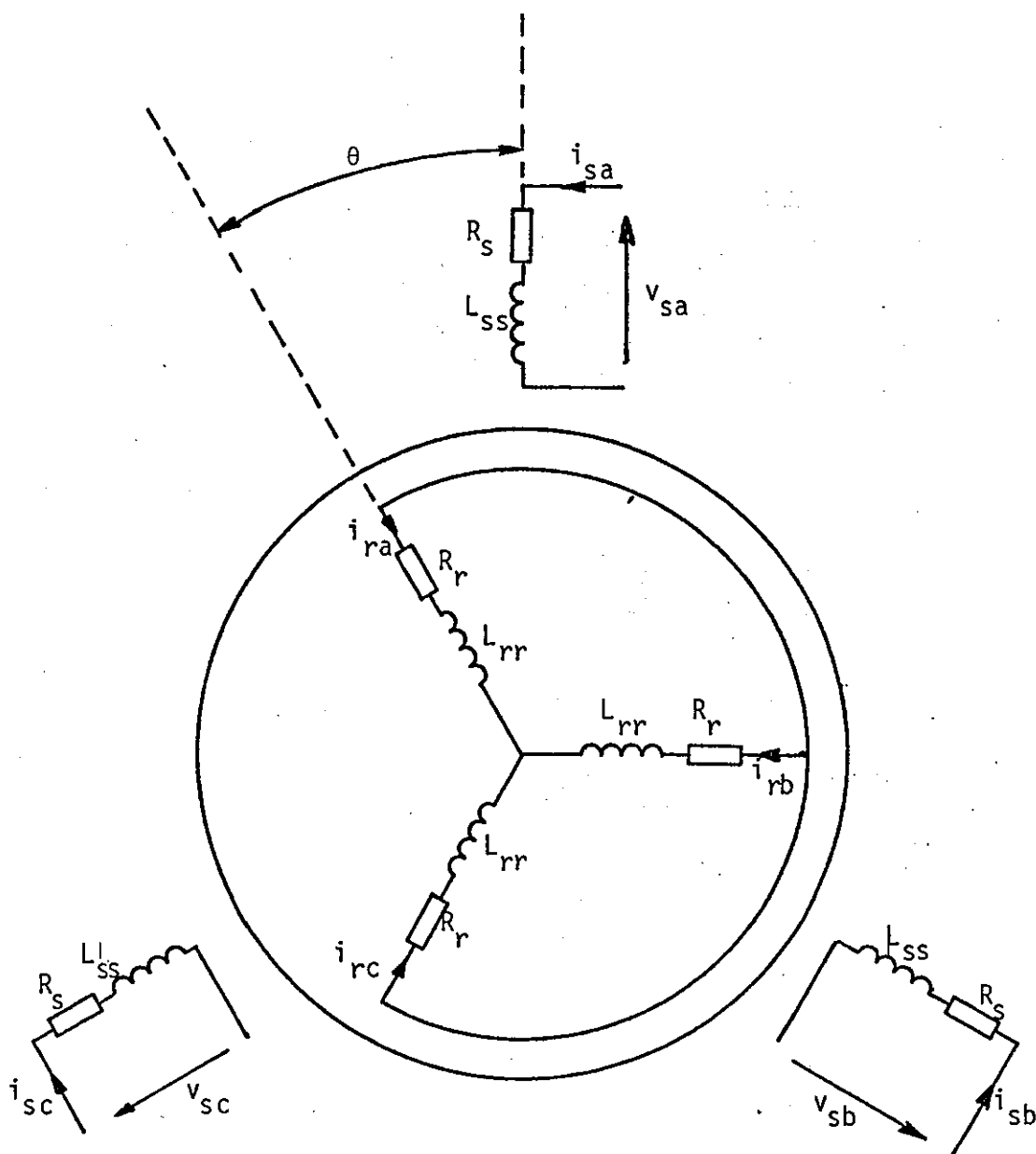


FIGURE 4.1 The direct 3-phase model for the motor system

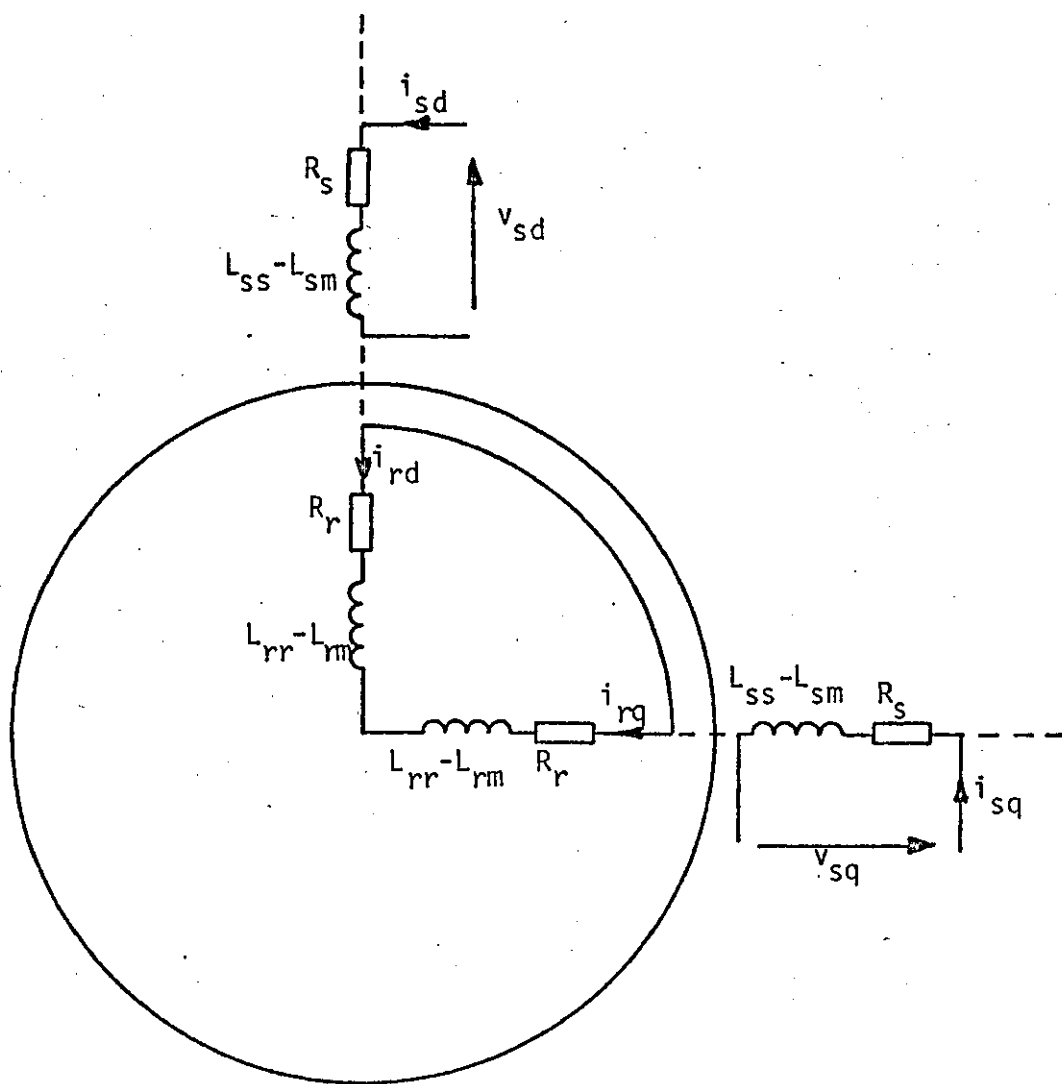


FIGURE 4.2 The stationary 2-axis model for the motor system

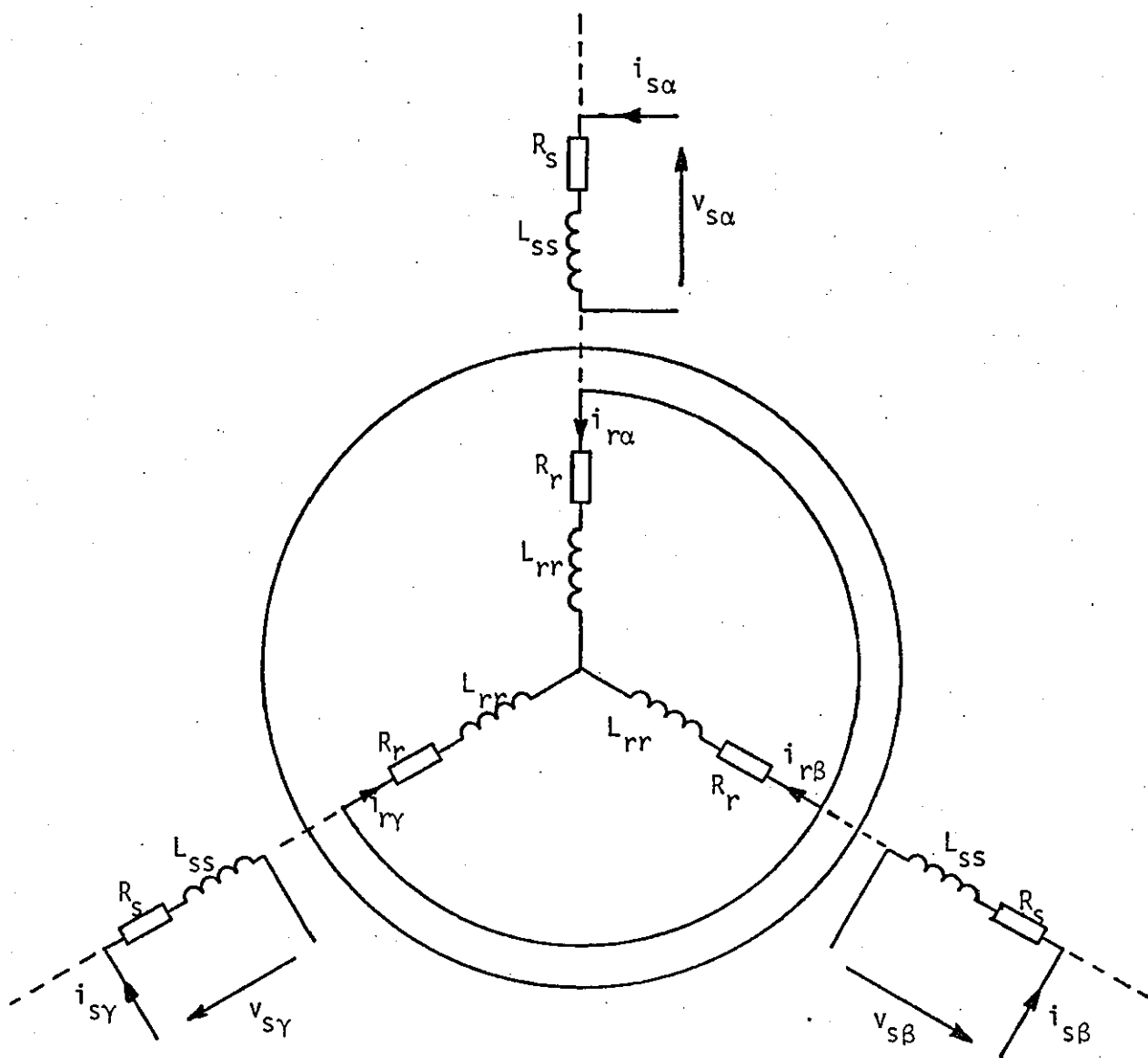


FIGURE 4.3 The stationary 3-axis model for the motor system

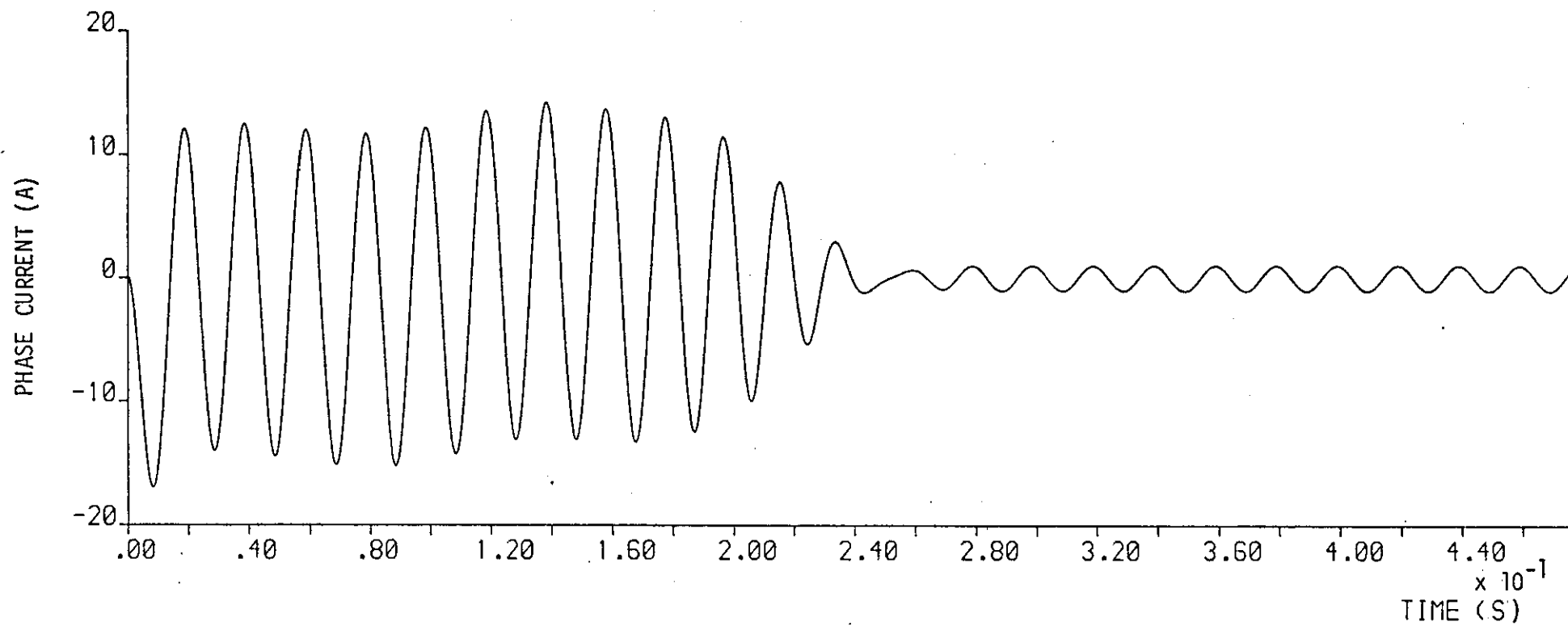


FIGURE 4.4 Computed results for start-up of the experimental motor-sine wave input

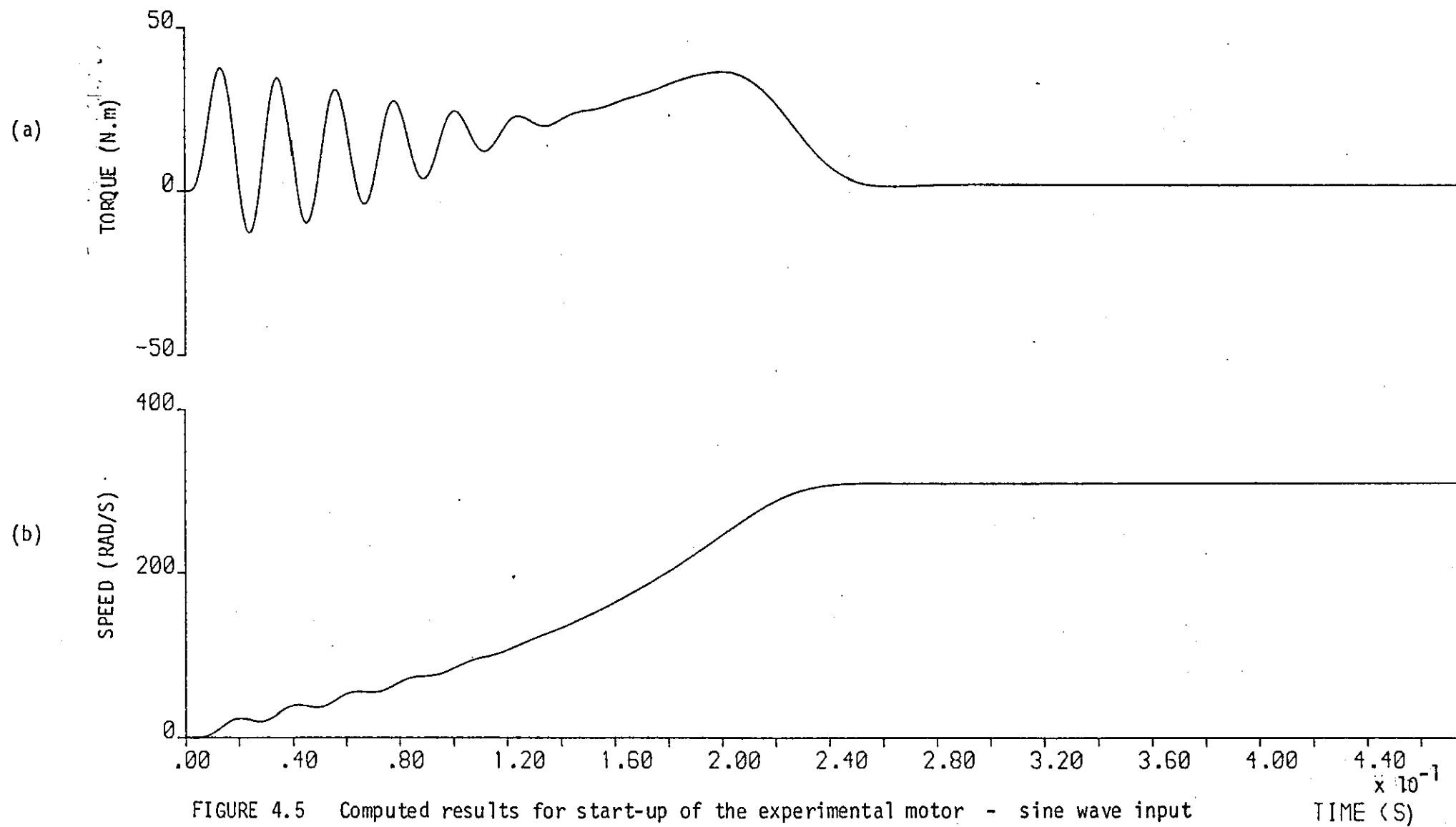


FIGURE 4.5 Computed results for start-up of the experimental motor - sine wave input

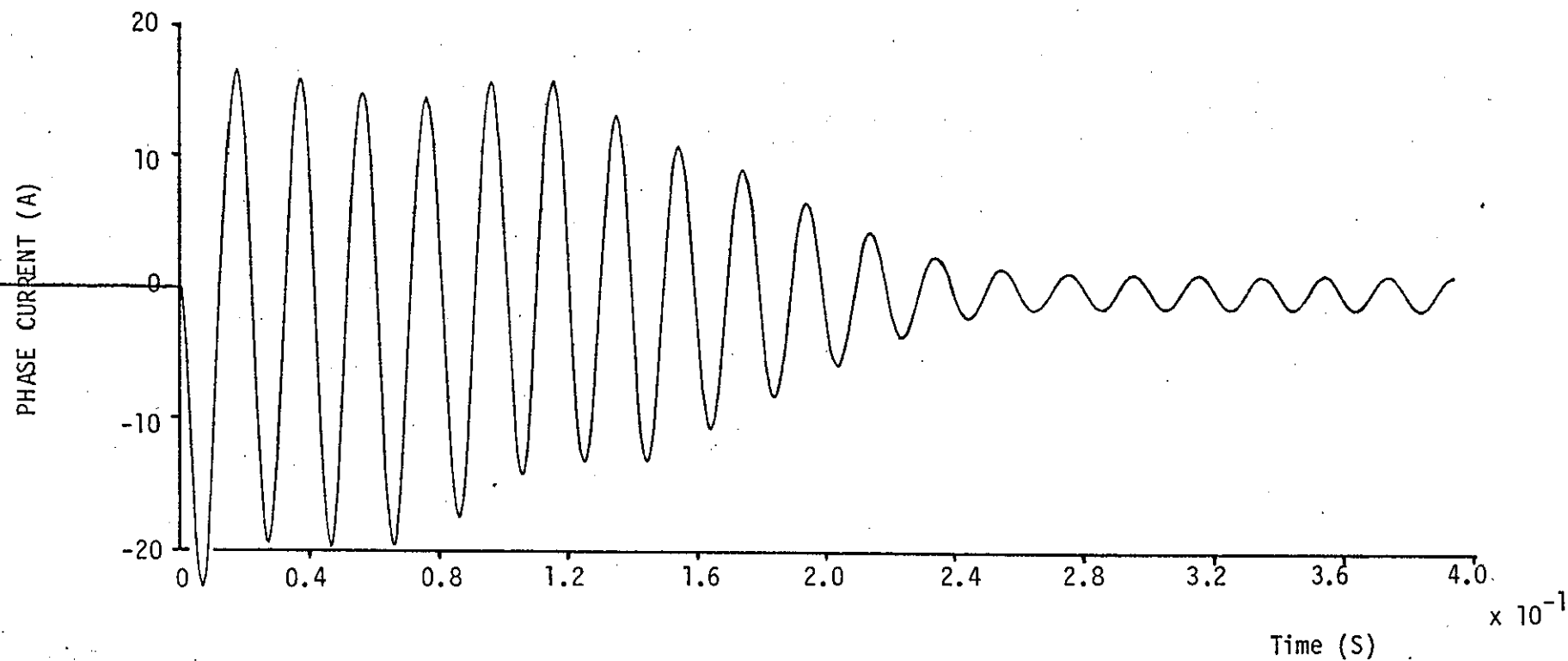


FIGURE 4.6 Experimental results for start-up of the experimental motor-sine wave input

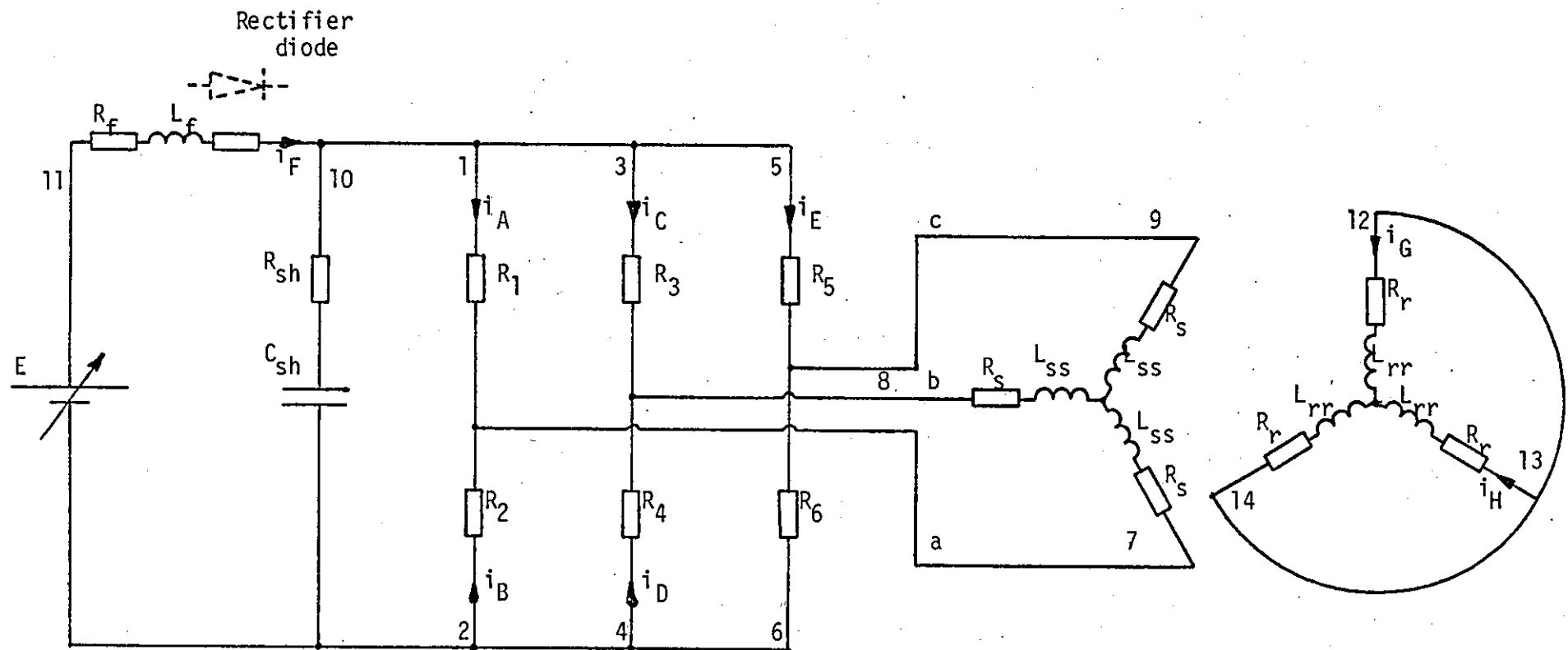


FIGURE 5.1 The basic network for the inverter-motor system during normal operation

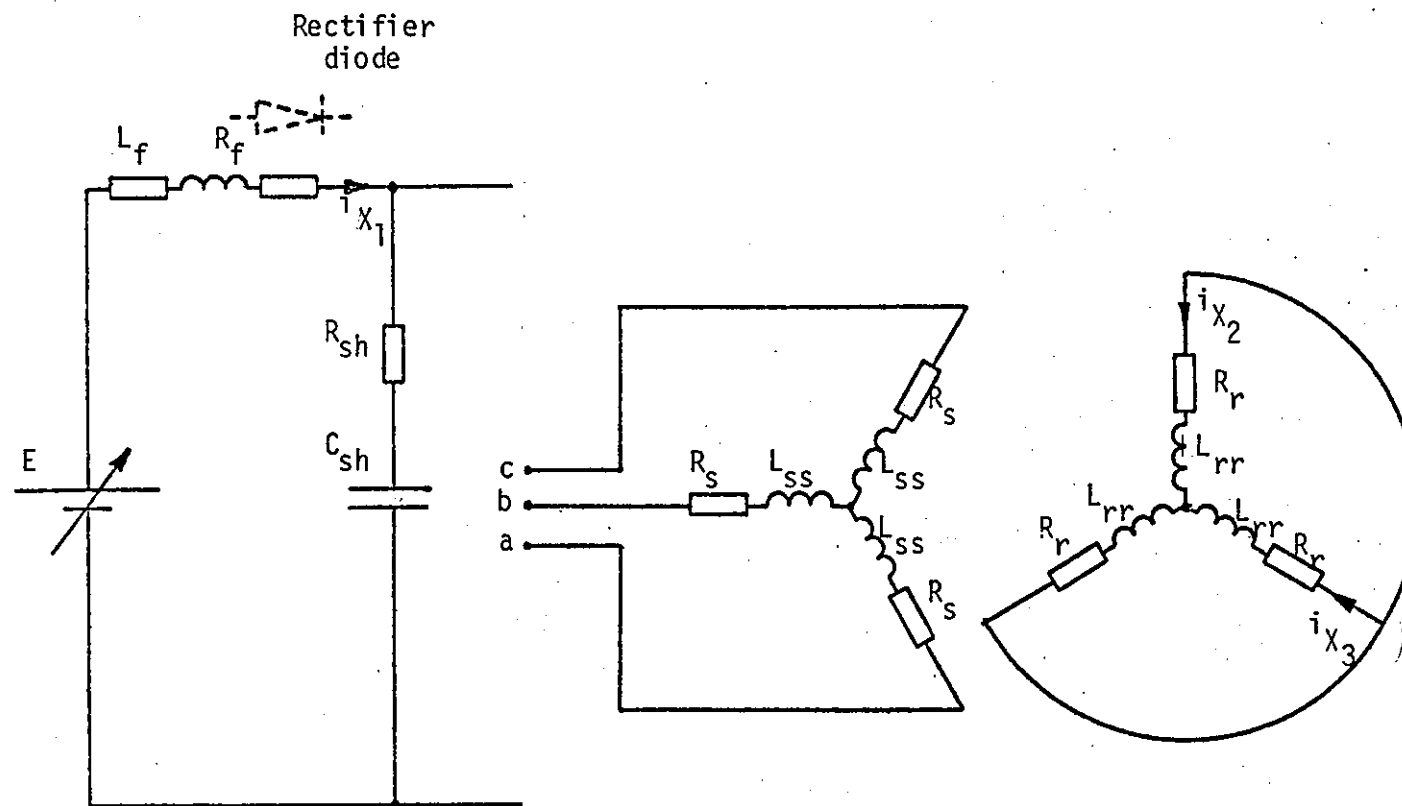


FIGURE 5.2 The circuit topology of the inverter-motor system during the disconnection period of plugging.

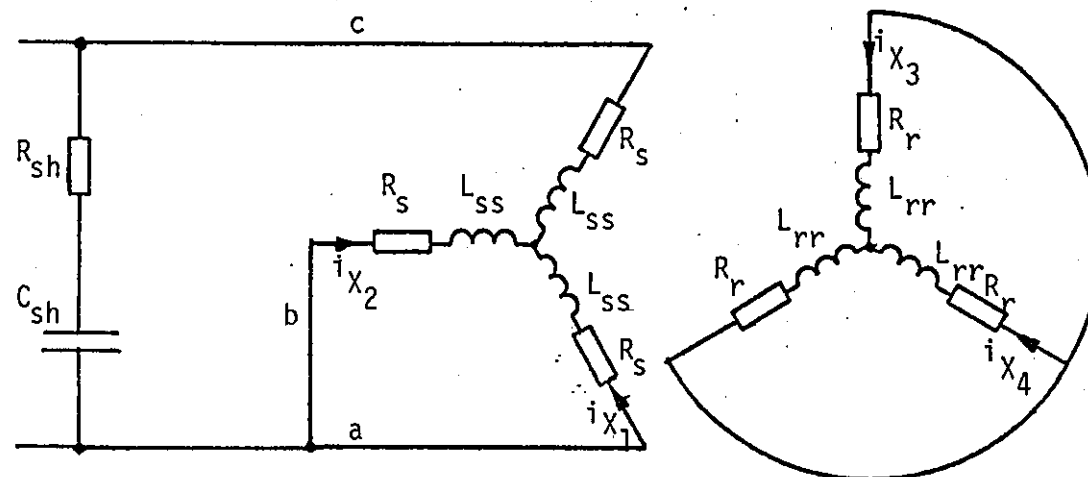


FIGURE 5.3 The circuit topology for the inverter motor system during braking

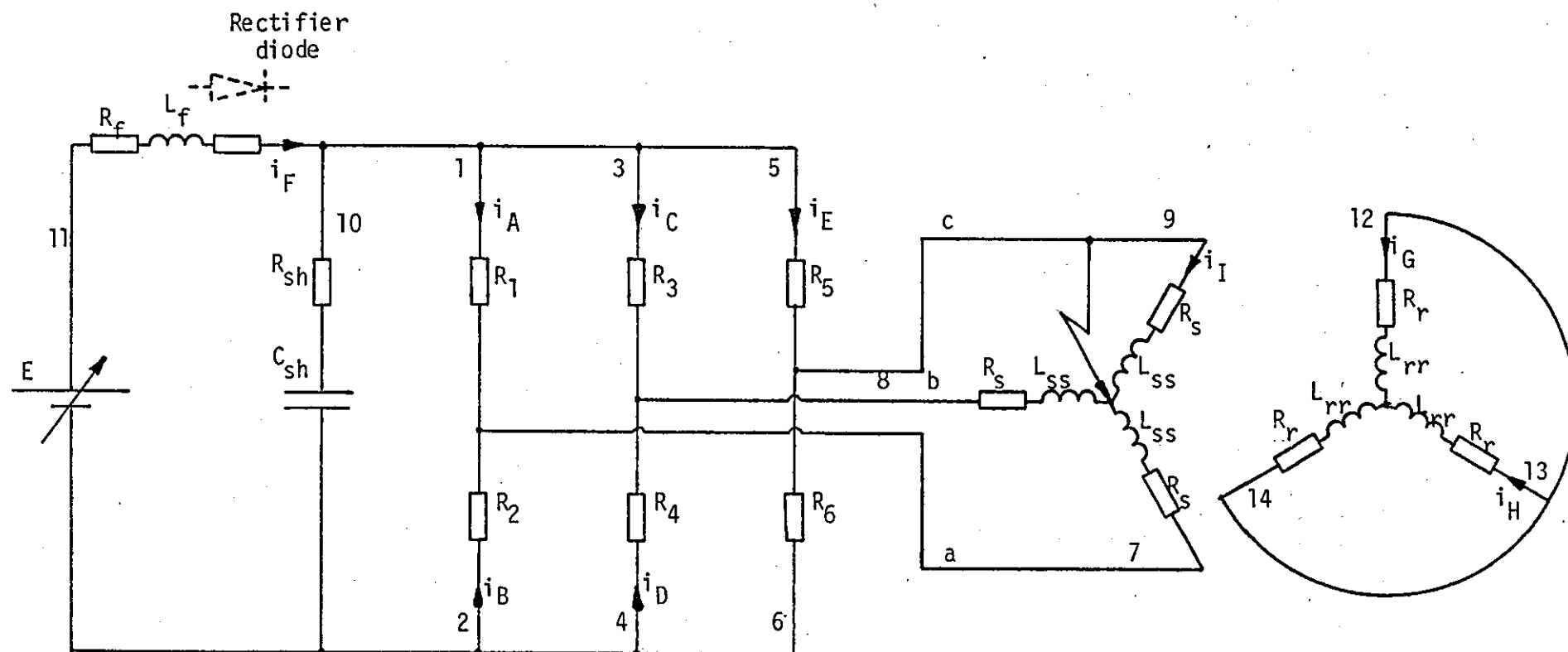


FIGURE 5.4 The basic network for the inverter-motor system during short circuit fault on stator phase c.

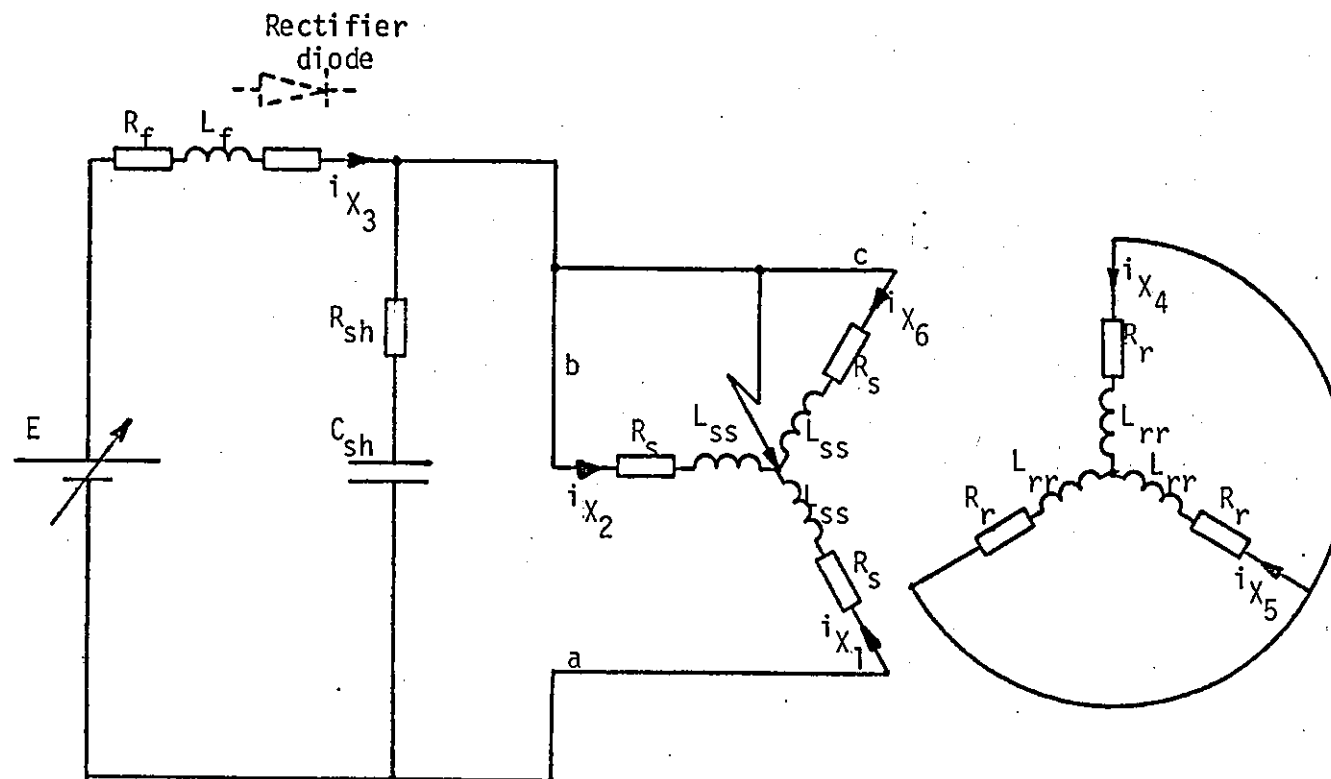


FIGURE 5.5 The circuit topology of the inverter-motor system for the example given in Section 5.3.4.

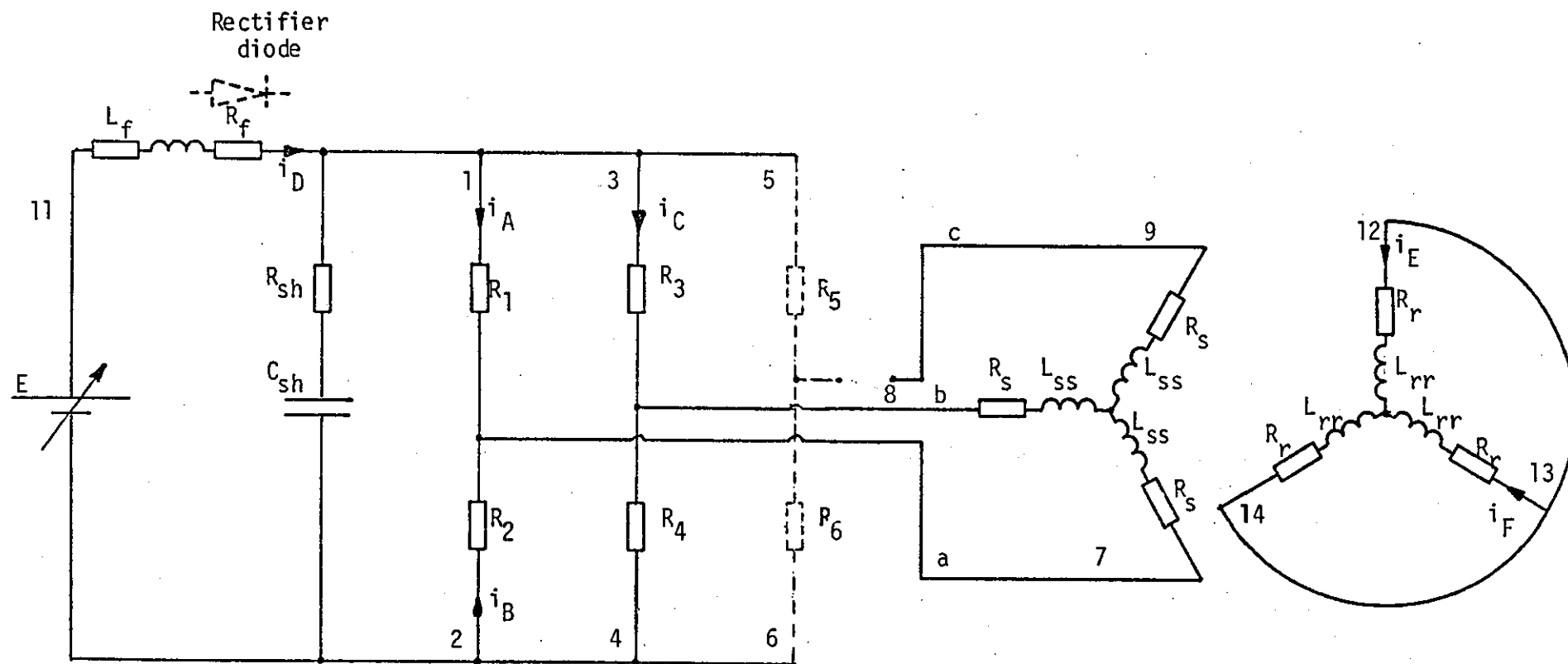


FIGURE 5.6 The basic network for the inverter-motor system during open circuit fault on stator phase c.

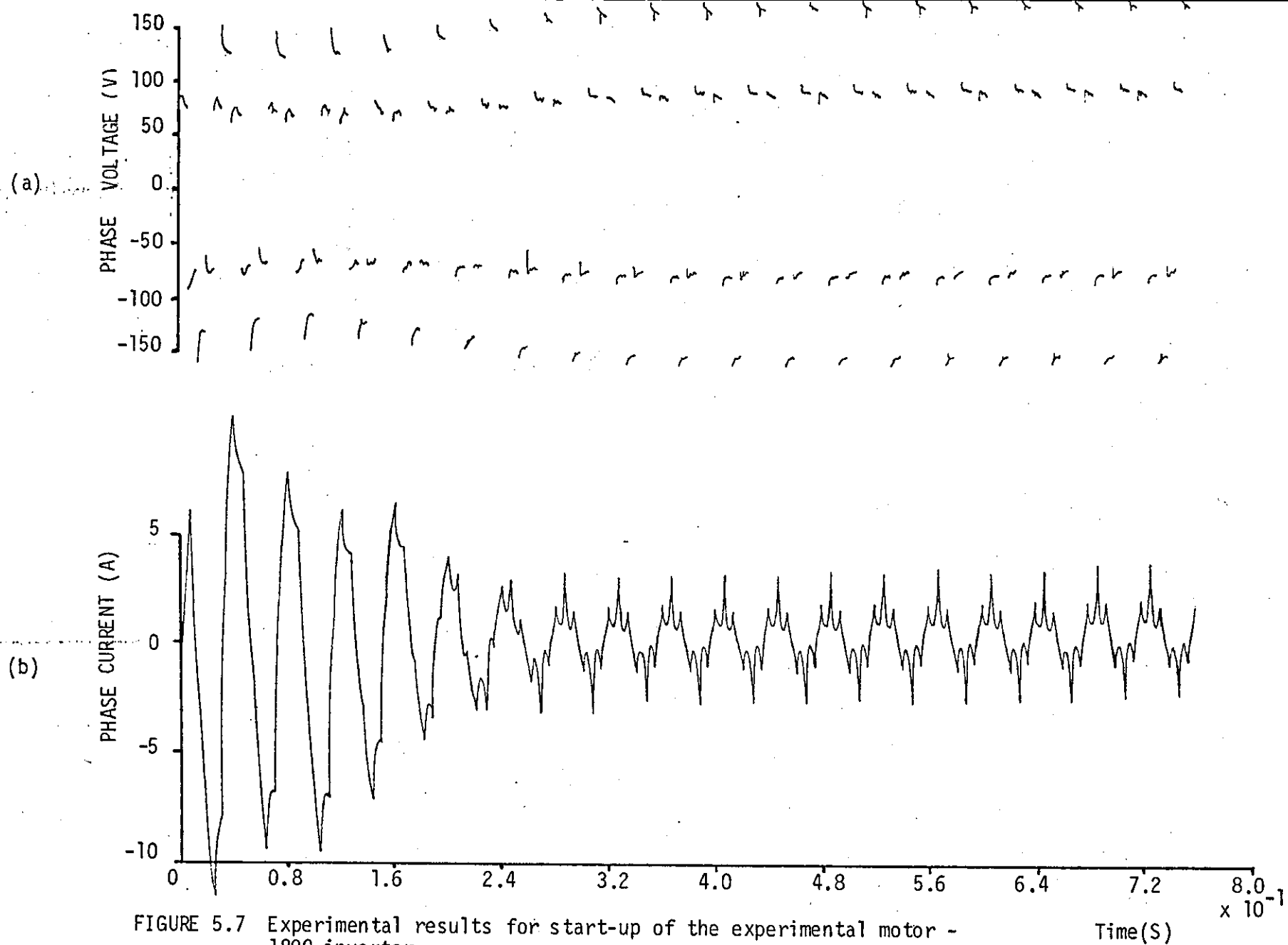


FIGURE 5.7 Experimental results for start-up of the experimental motor - 180° inverter

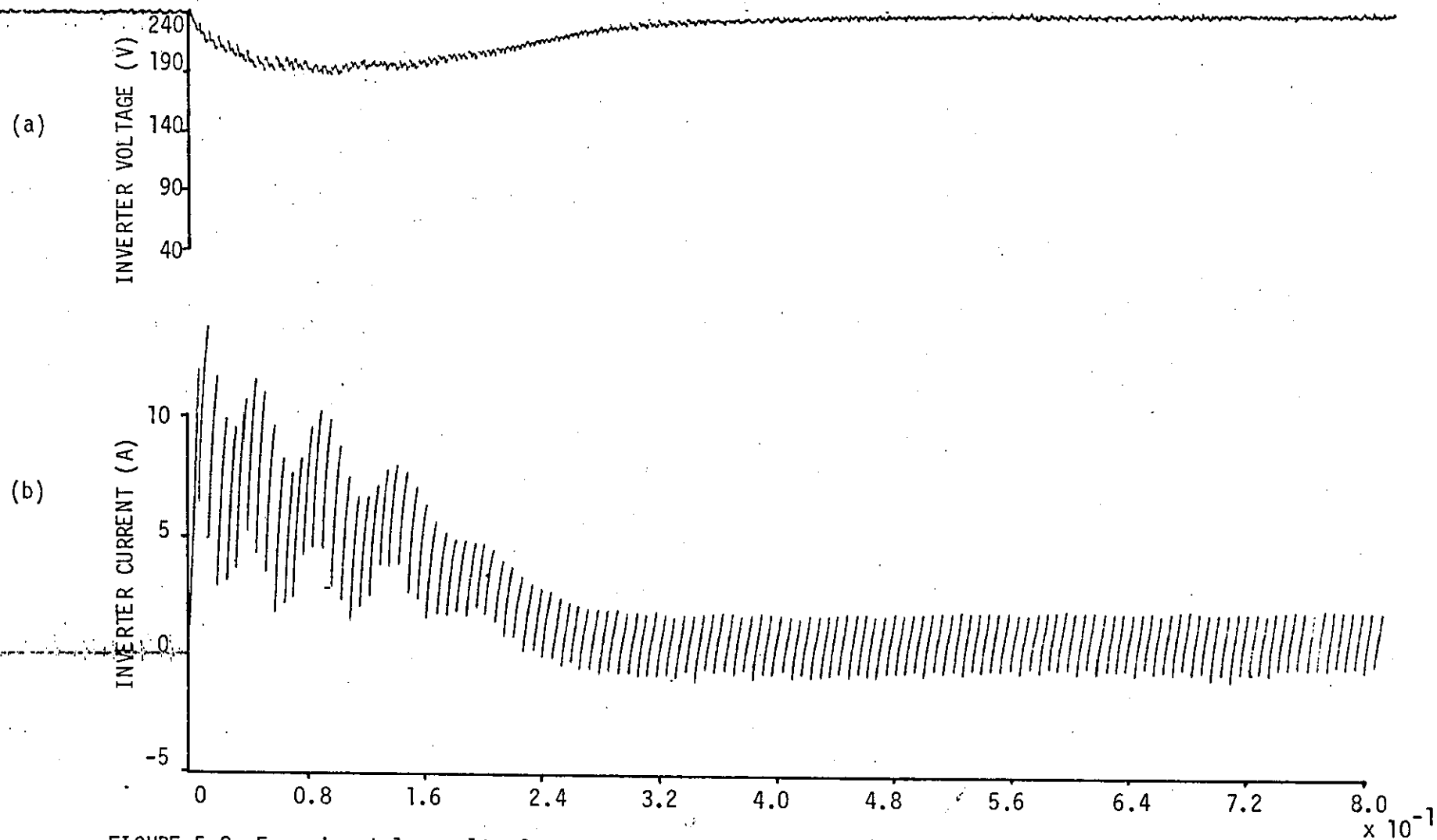


FIGURE 5.8 Experimental results for start-up of the experimental motor - 180° inverter

Time (S)

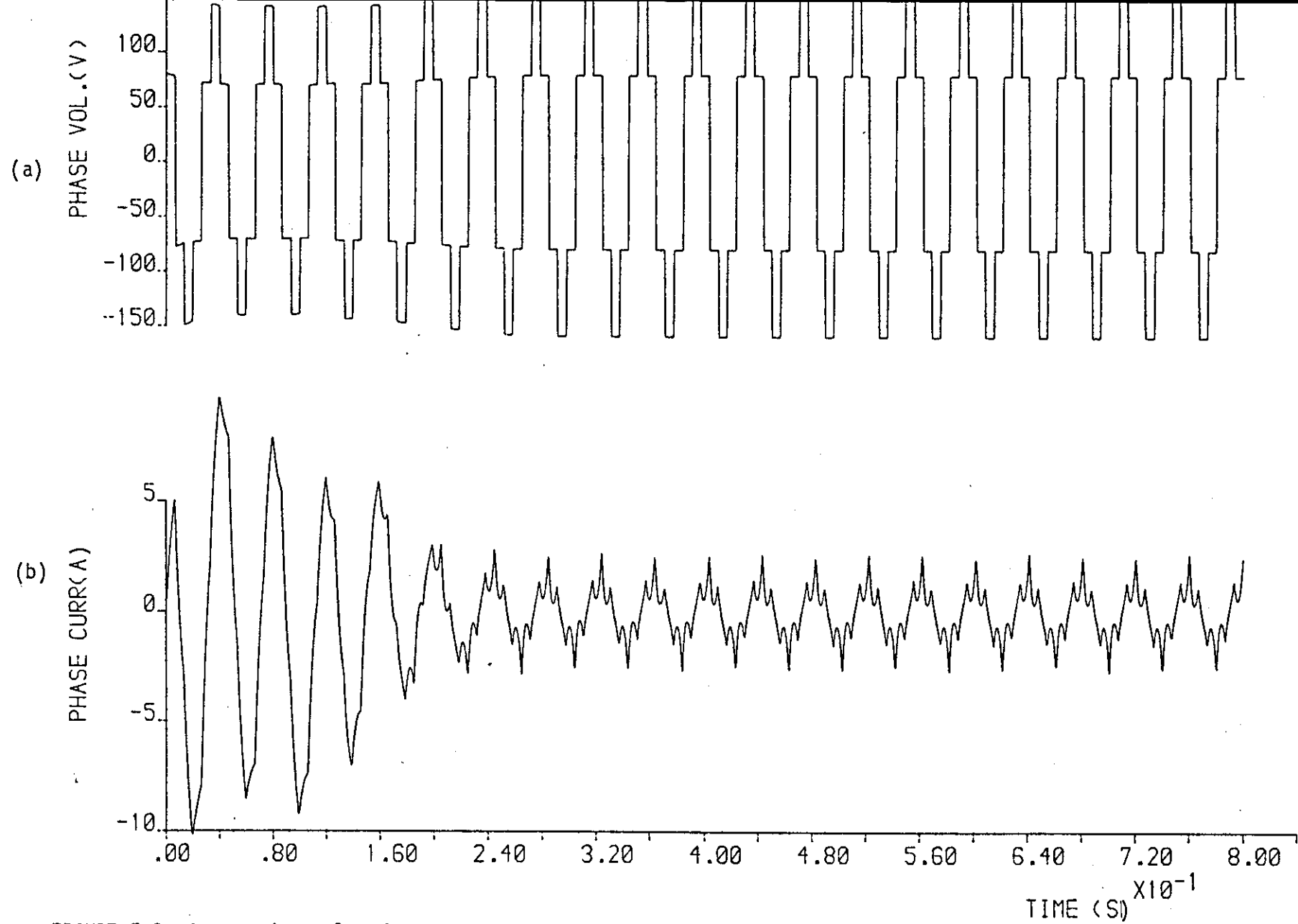


FIGURE 5.9 Computed results for start-up of the experimental motor - 180° inverter

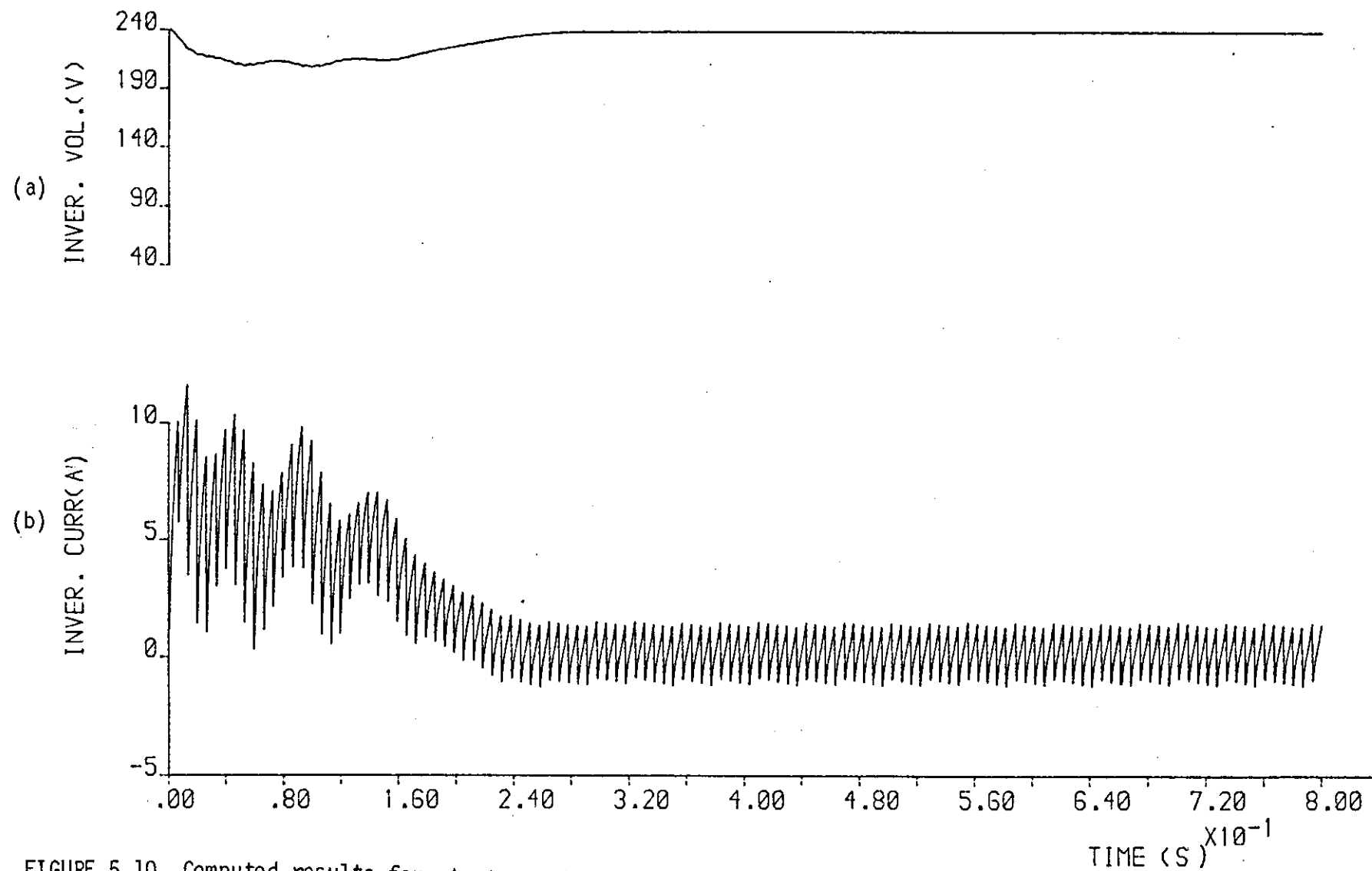


FIGURE 5.10 Computed results for start-up of the experimental motor -
180° inverter

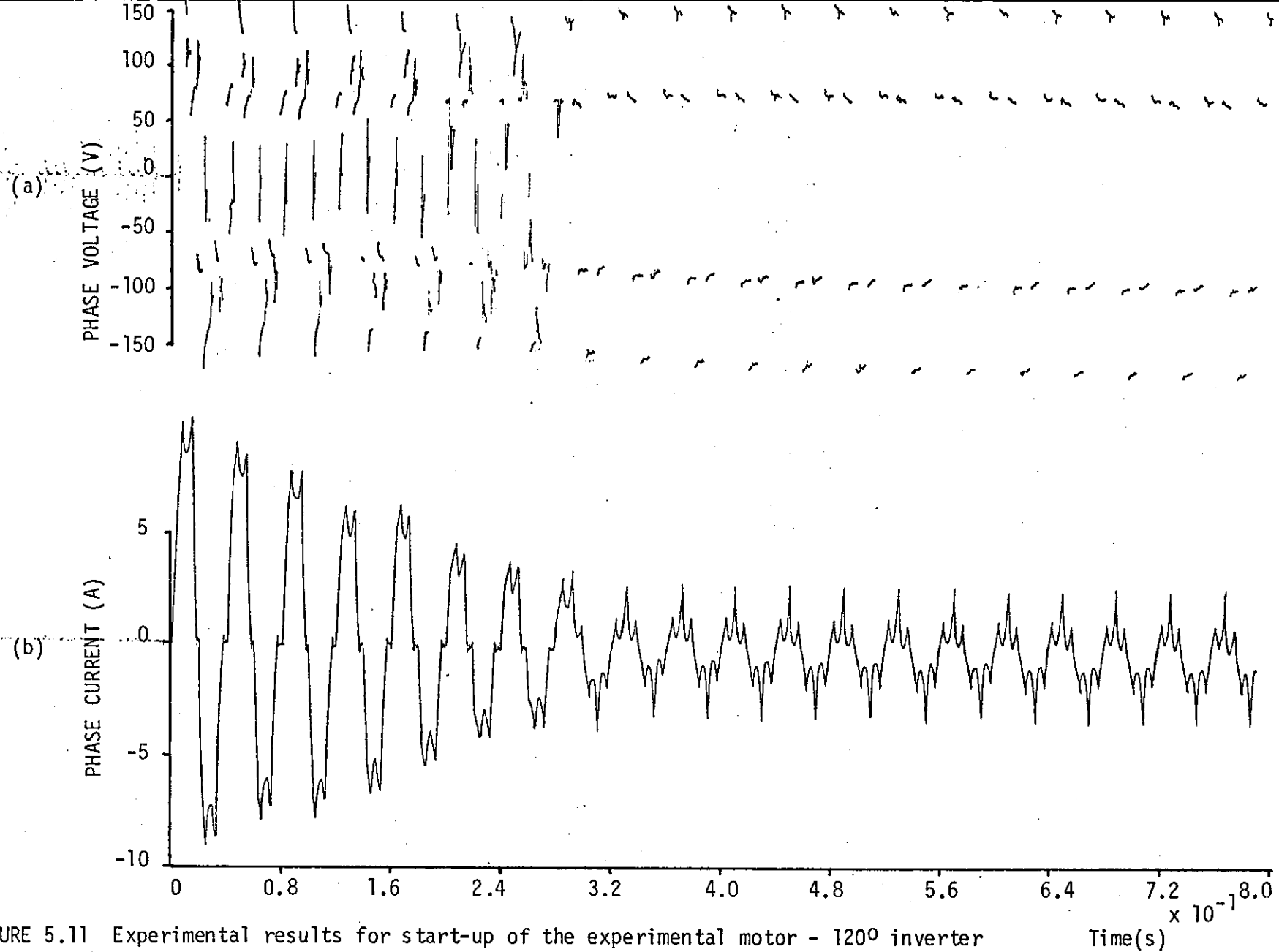


FIGURE 5.11 Experimental results for start-up of the experimental motor - 120° inverter

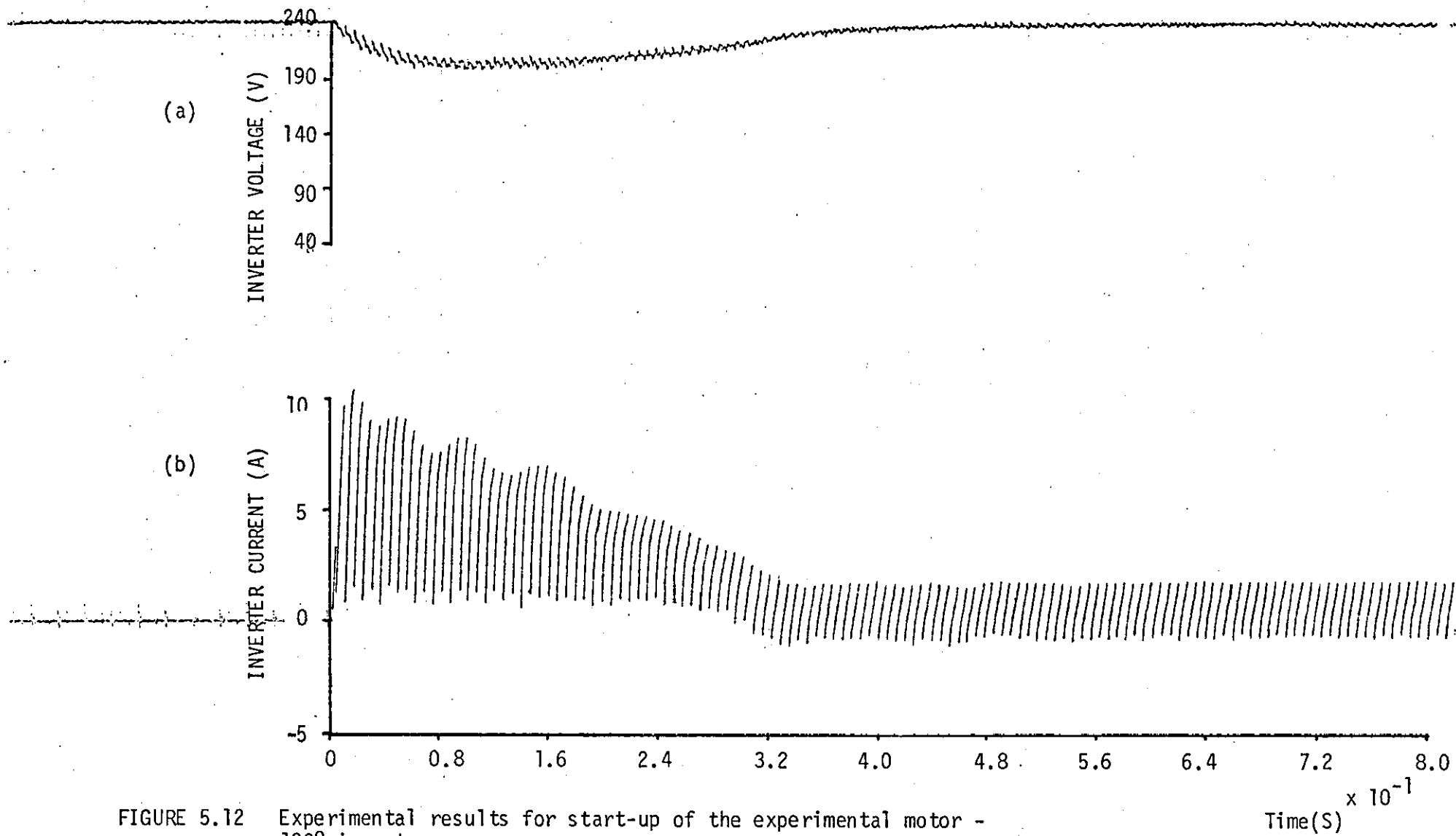


FIGURE 5.12 Experimental results for start-up of the experimental motor - 120° inverter

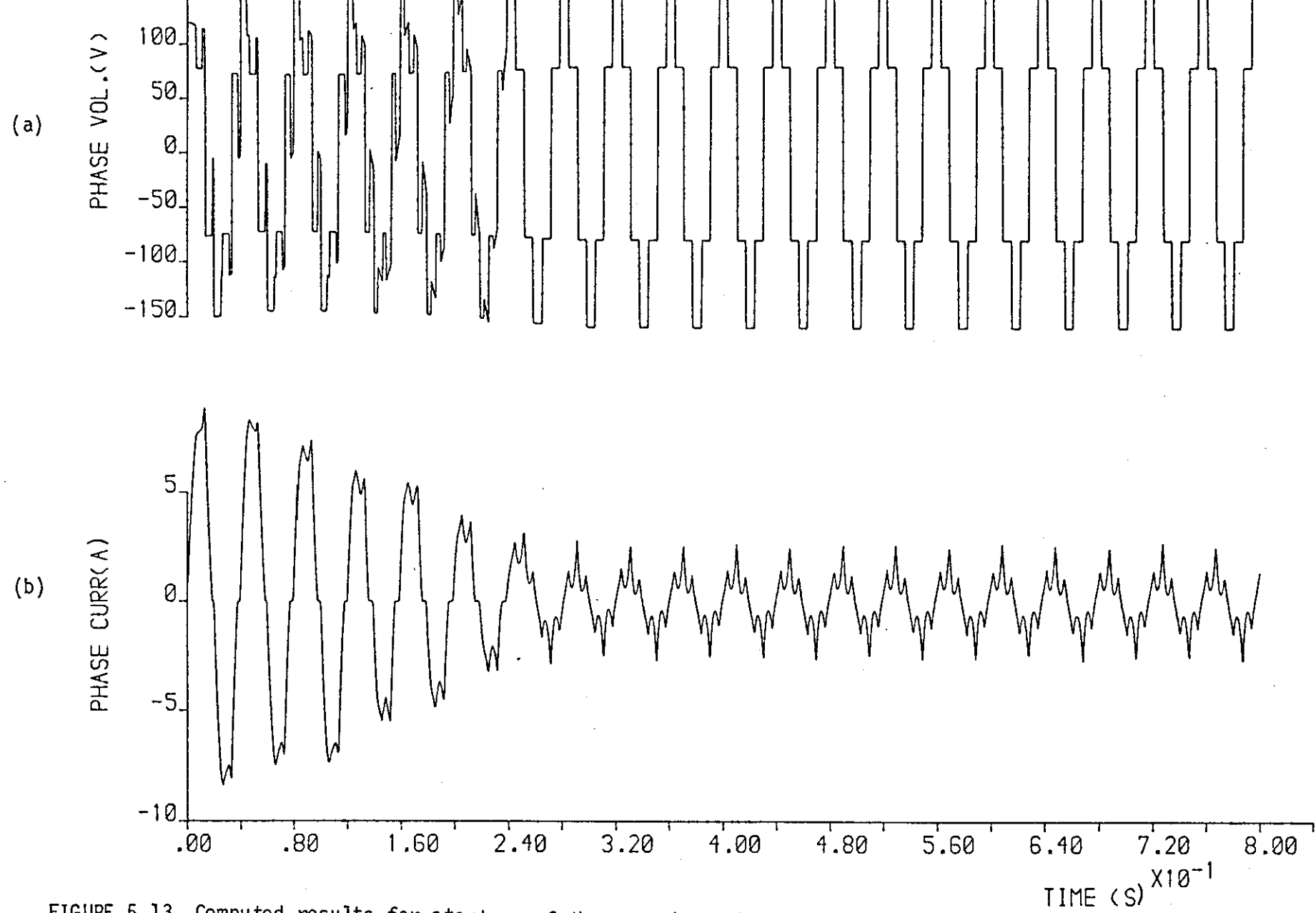


FIGURE 5.13 Computed results for start-up of the experimental motor -
120° inverter

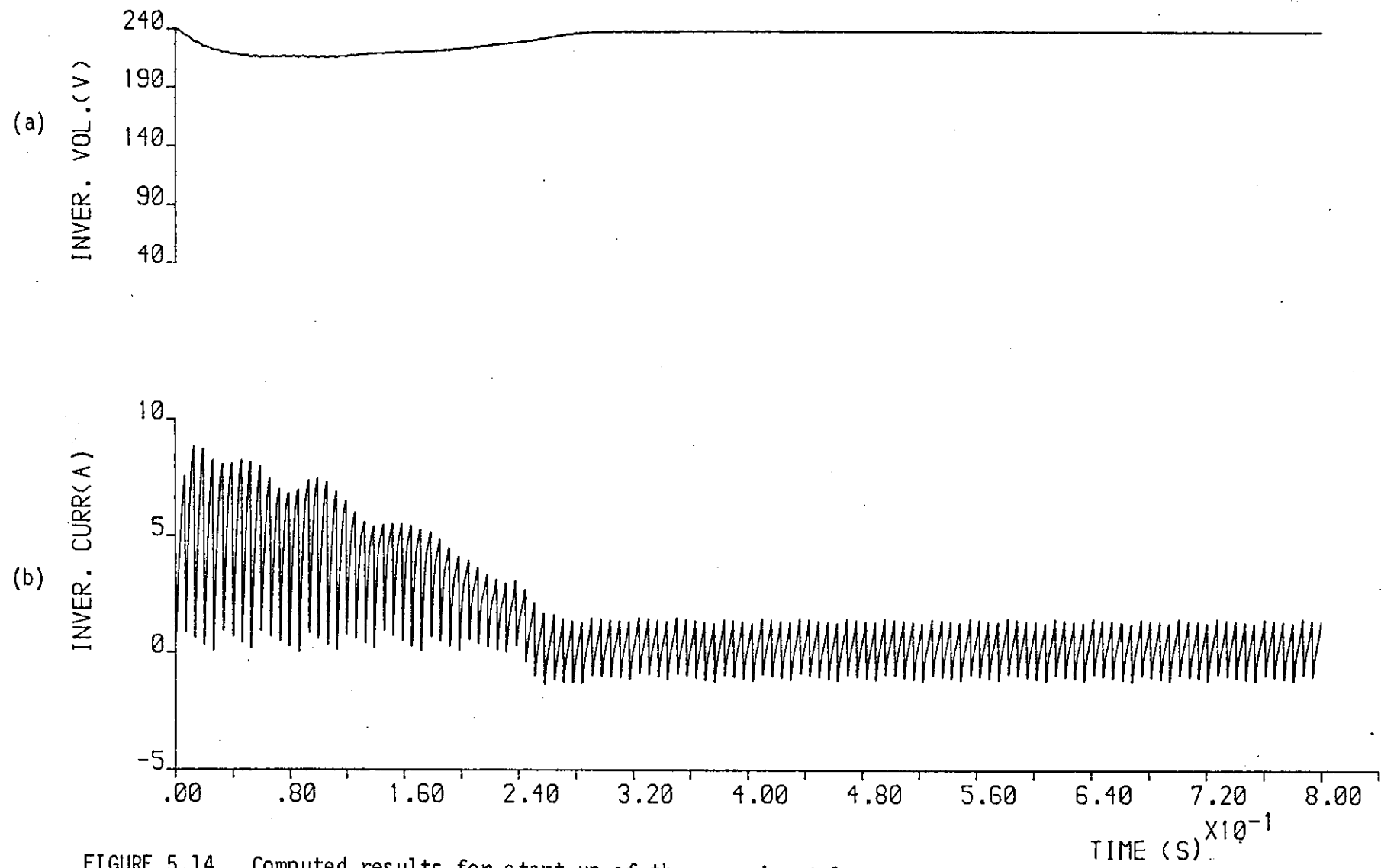


FIGURE 5.14 Computed results for start-up of the experimental motor - 1200 inverter

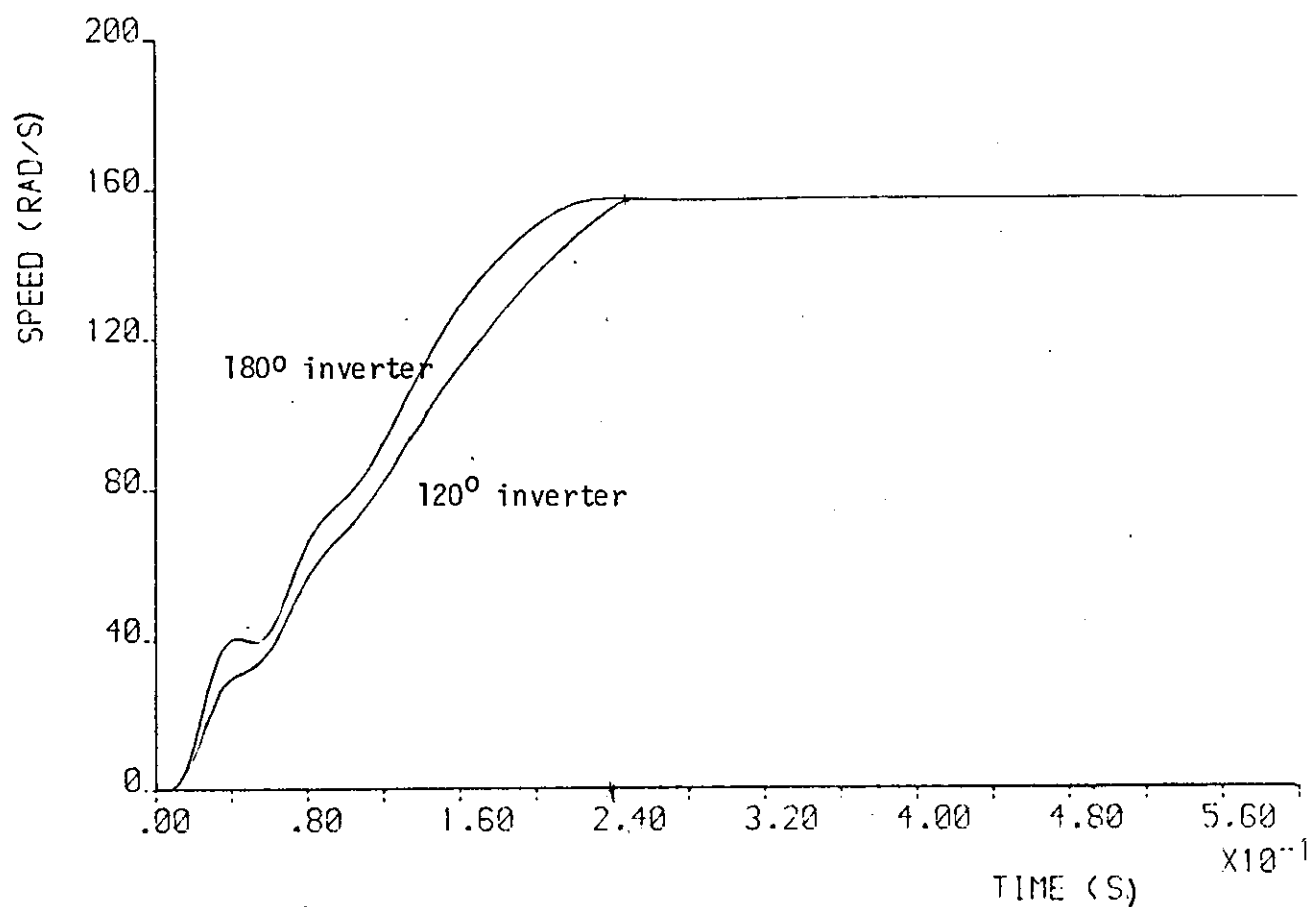


FIGURE 5.15 Computed speed curves for start-up of the experimental motor

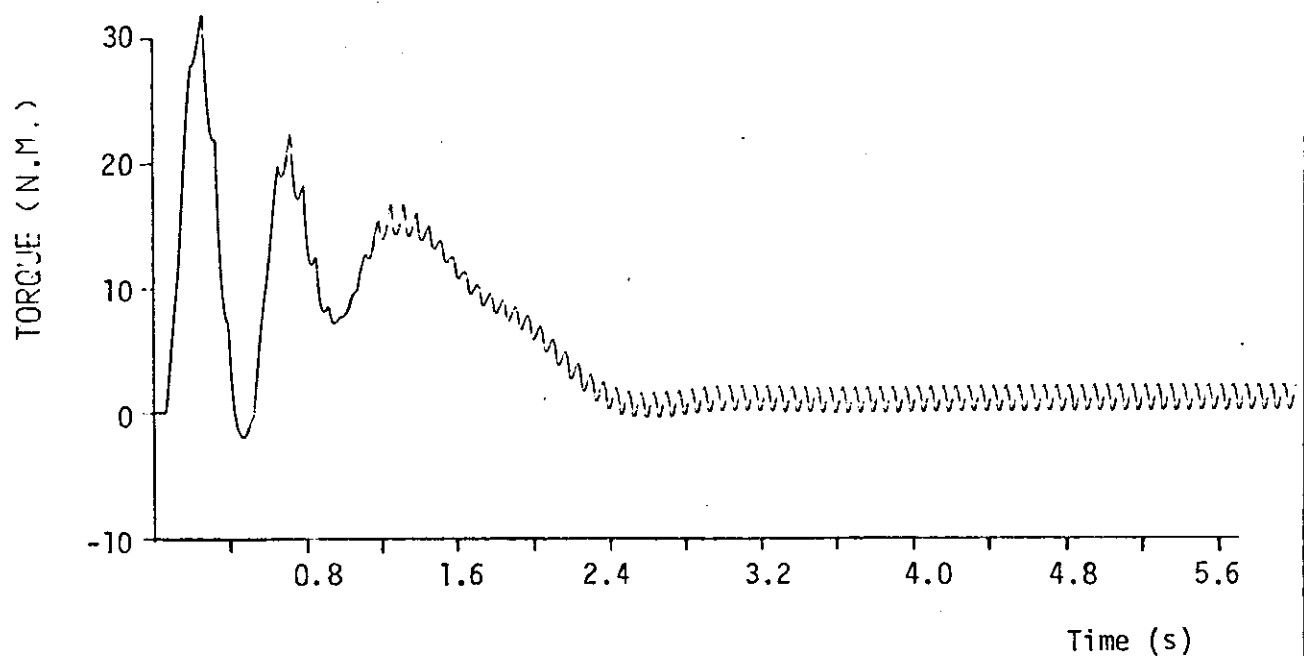


FIGURE 5.16 Computed torque curve for start-up of the experimental motor - 180° inverter

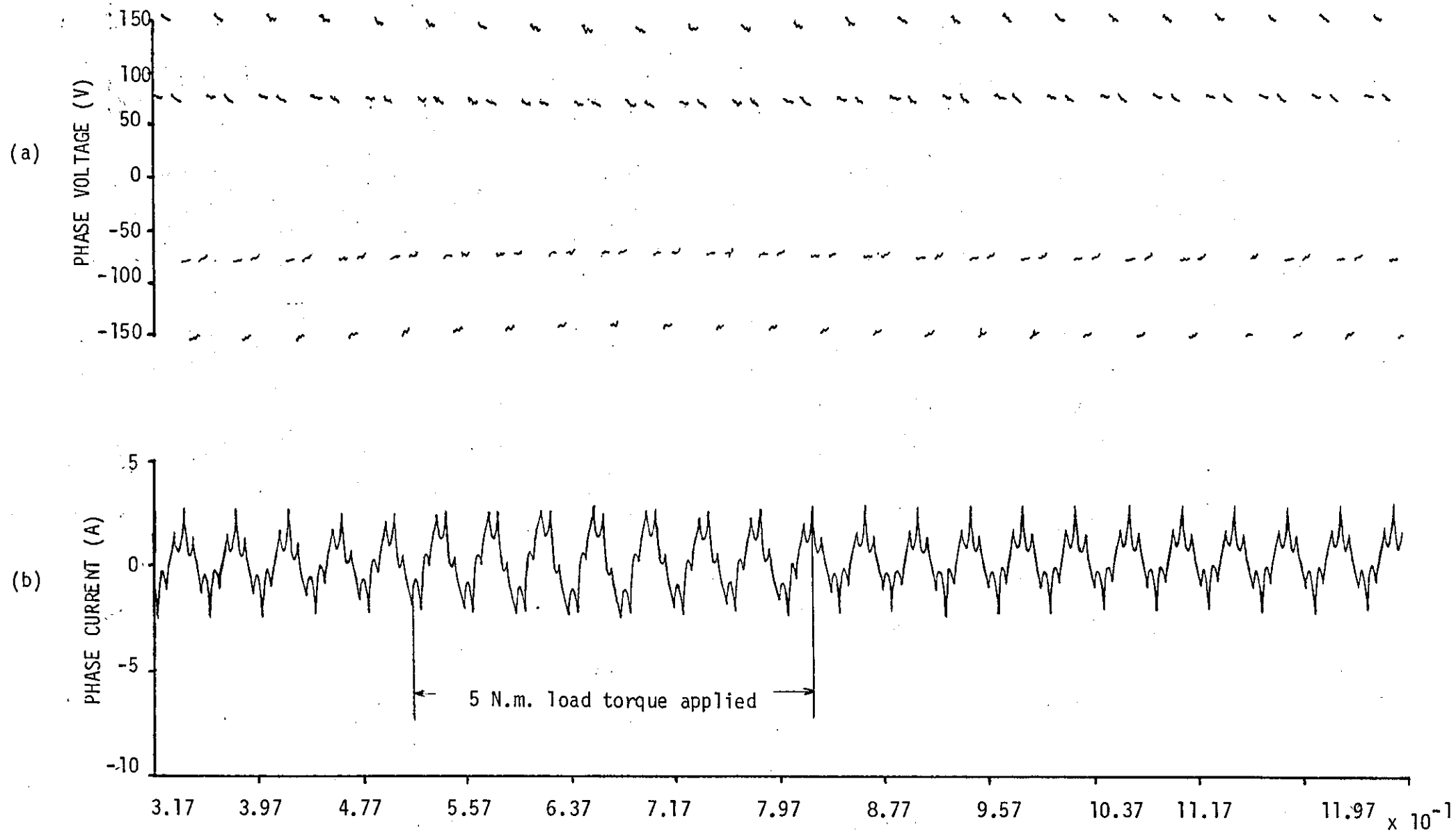


FIGURE 5.17 Experimental results for application of load to the experimental motor - 180° inverter.

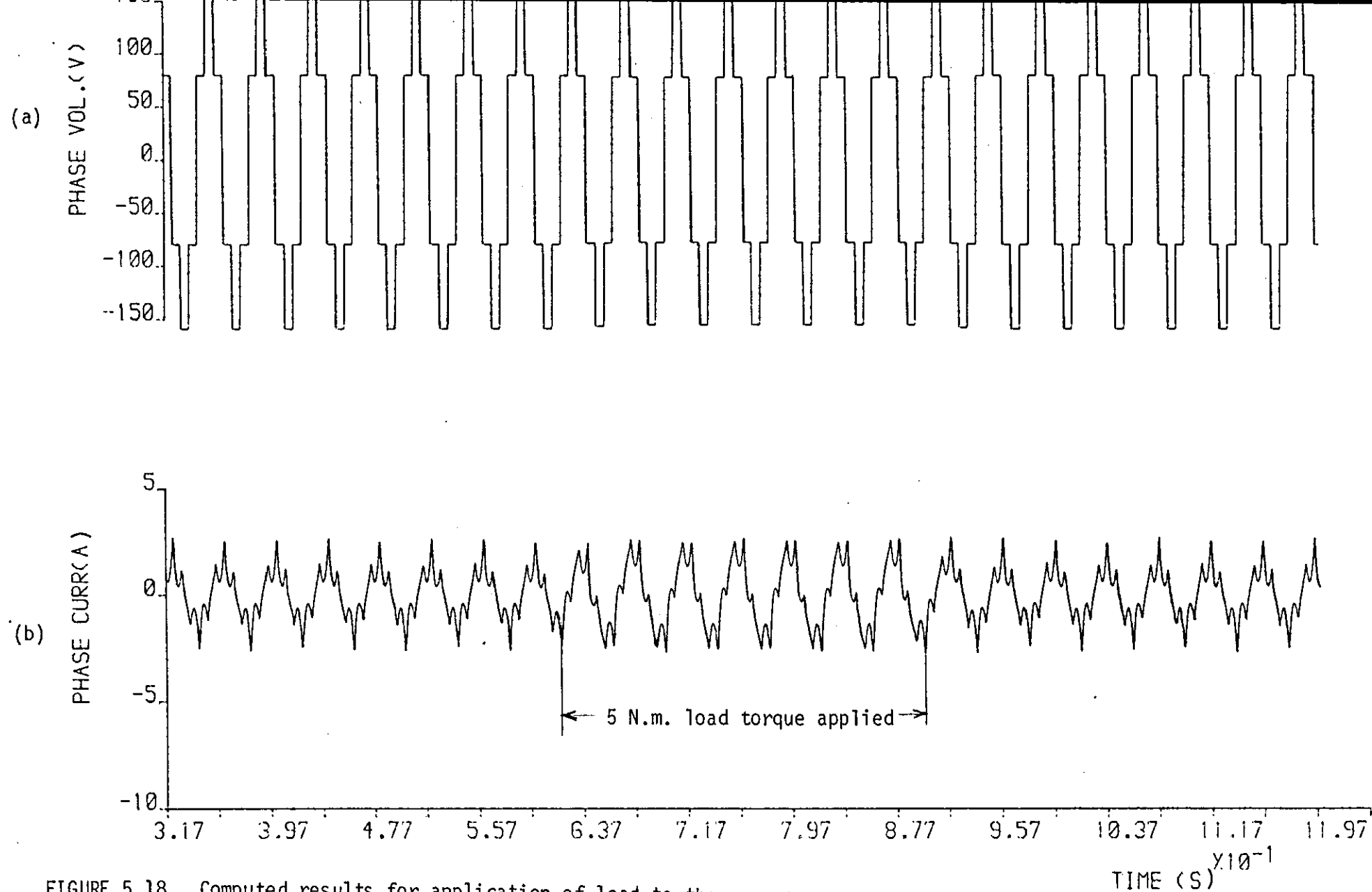


FIGURE 5.18 Computed results for application of load to the experimental motor - 180° inverter

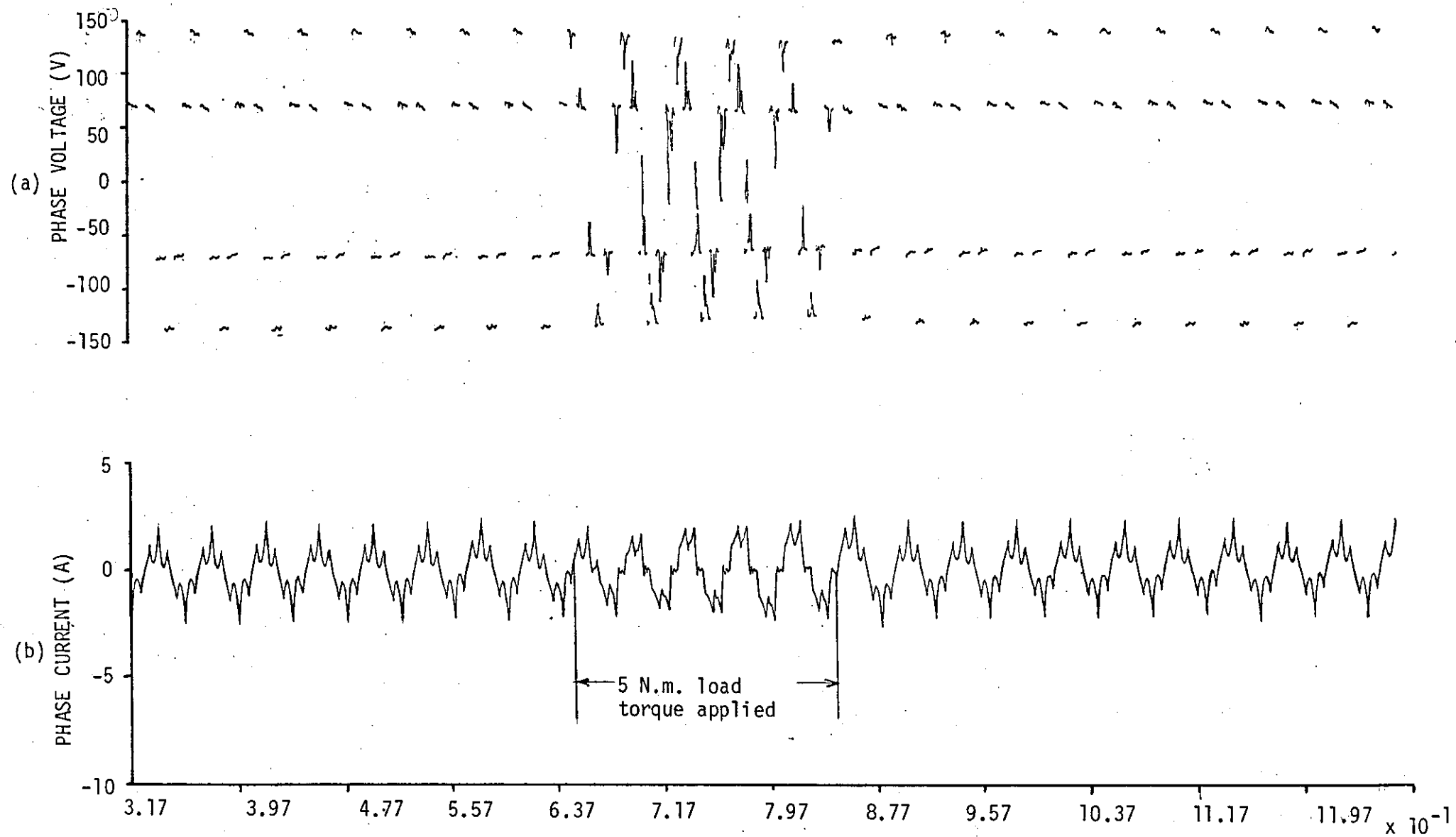


FIGURE 5.19 Experimental results for application of load to the experimental motor - 120° inverter

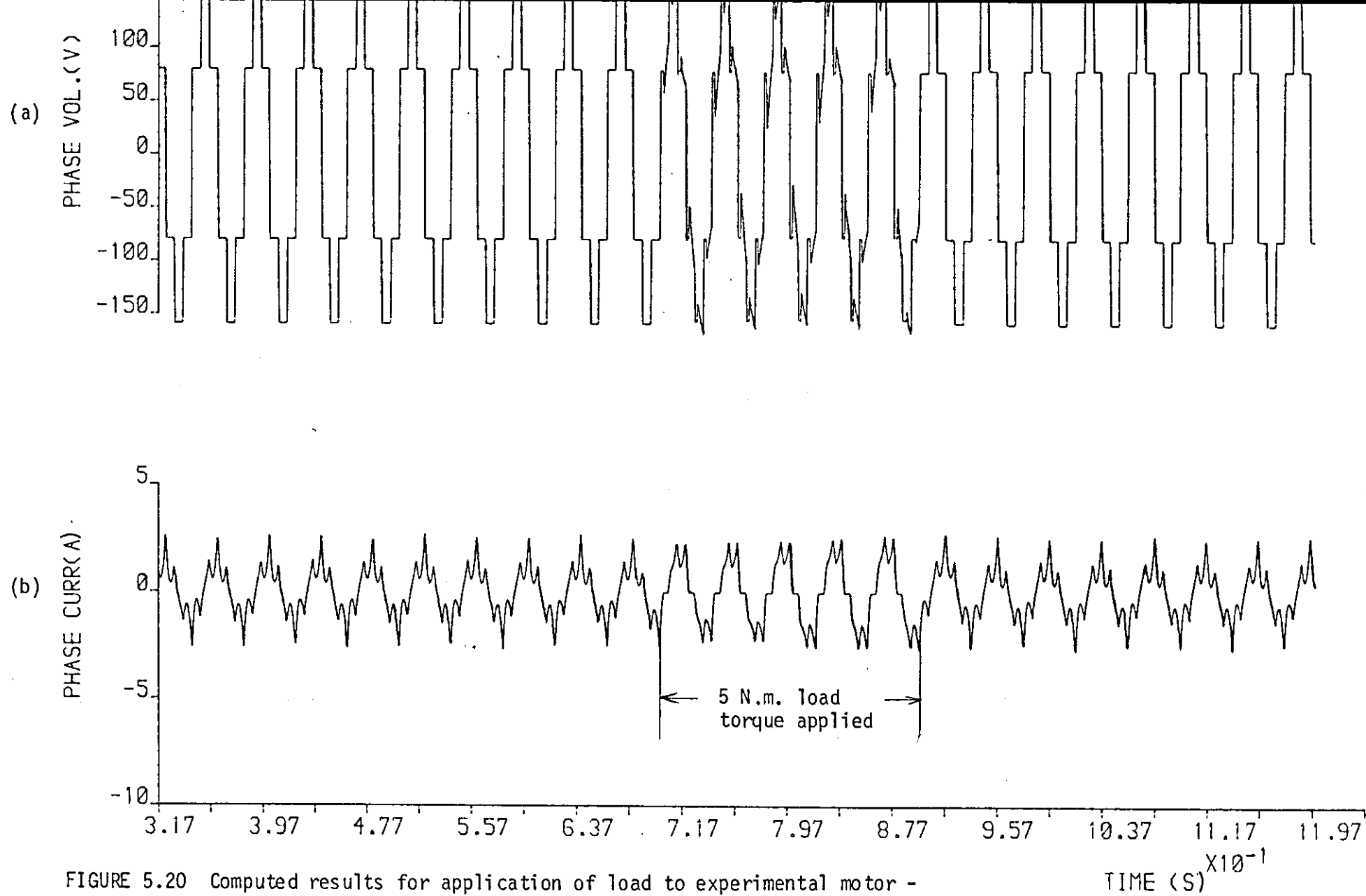


FIGURE 5.20 Computed results for application of load to experimental motor - 120° inverter

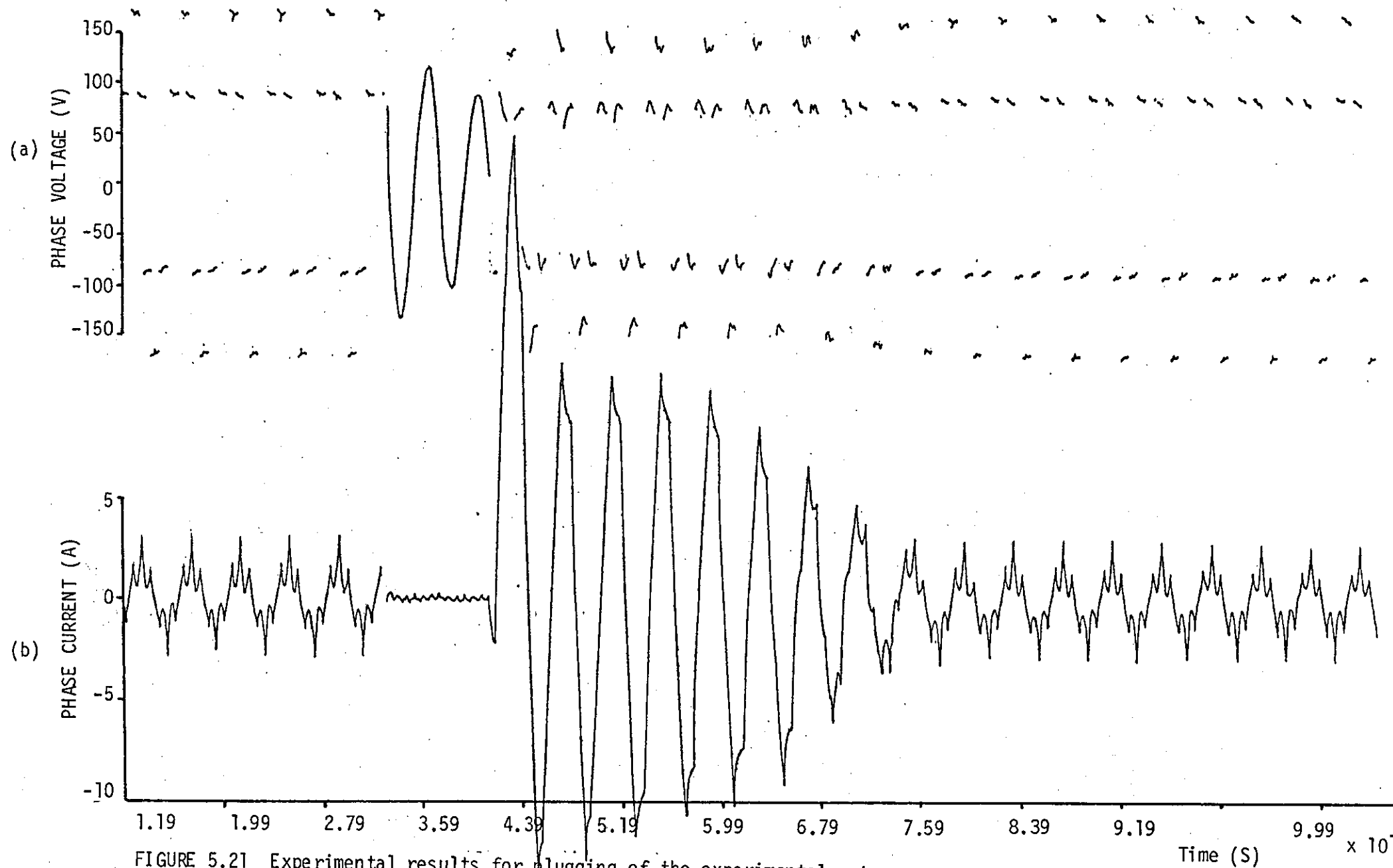


FIGURE 5.21 Experimental results for plugging of the experimental motor - 180° inverter

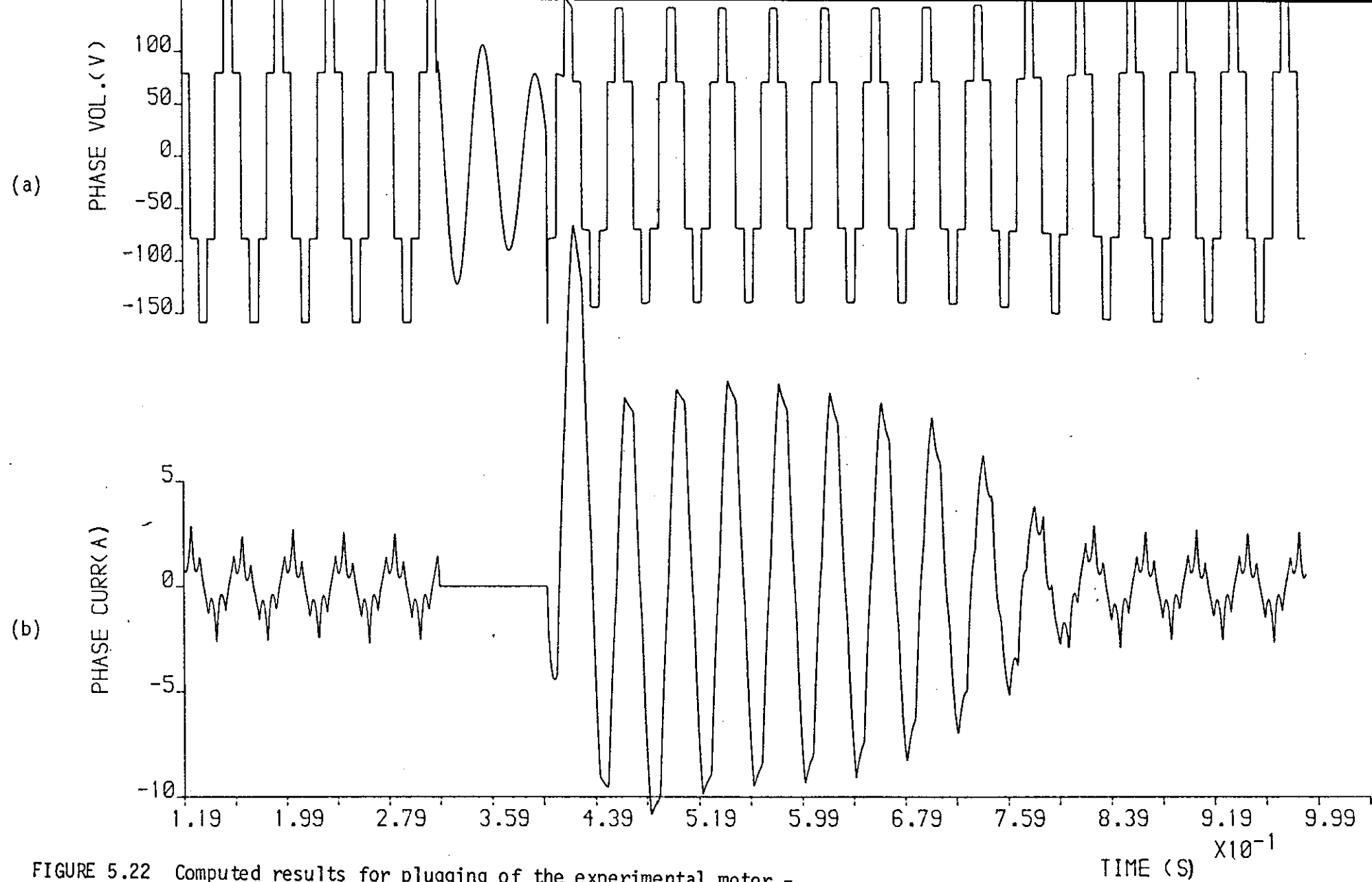


FIGURE 5.22 Computed results for plugging of the experimental motor -
180° inverter

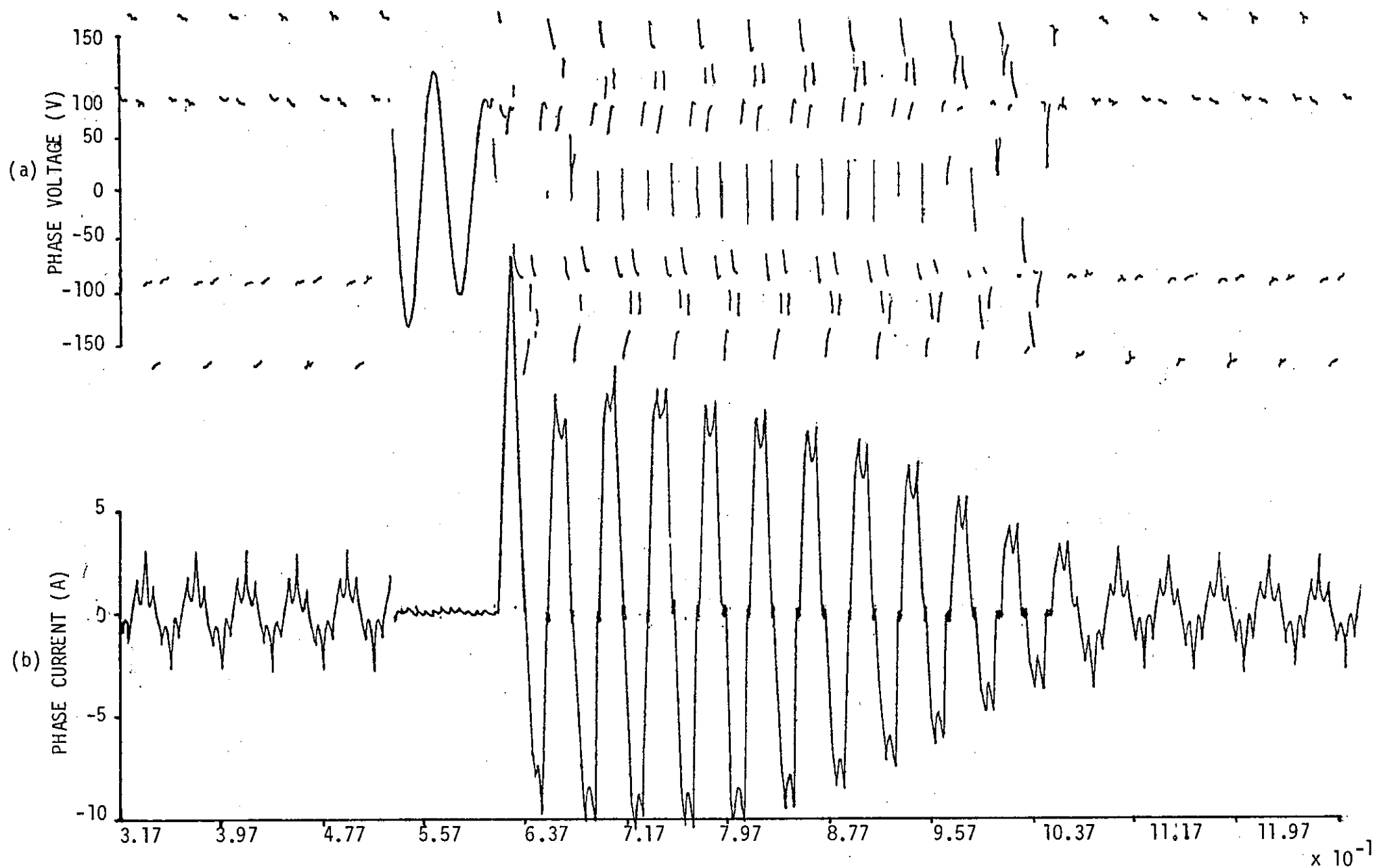


FIGURE 5.23 Experimental results for plugging of the experimental motor - 120° inverter

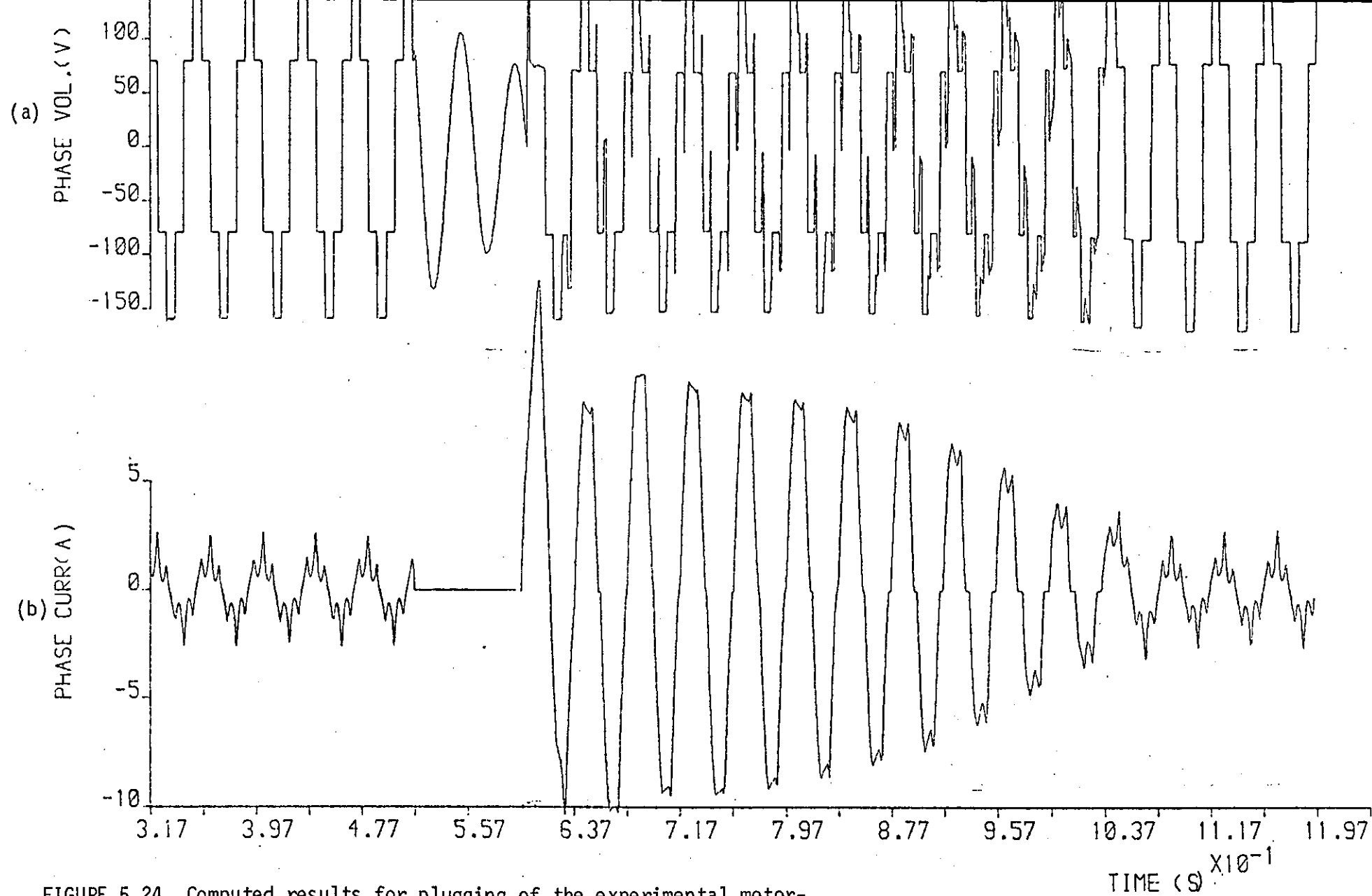


FIGURE 5.24 Computed results for plugging of the experimental motor-120° inverter

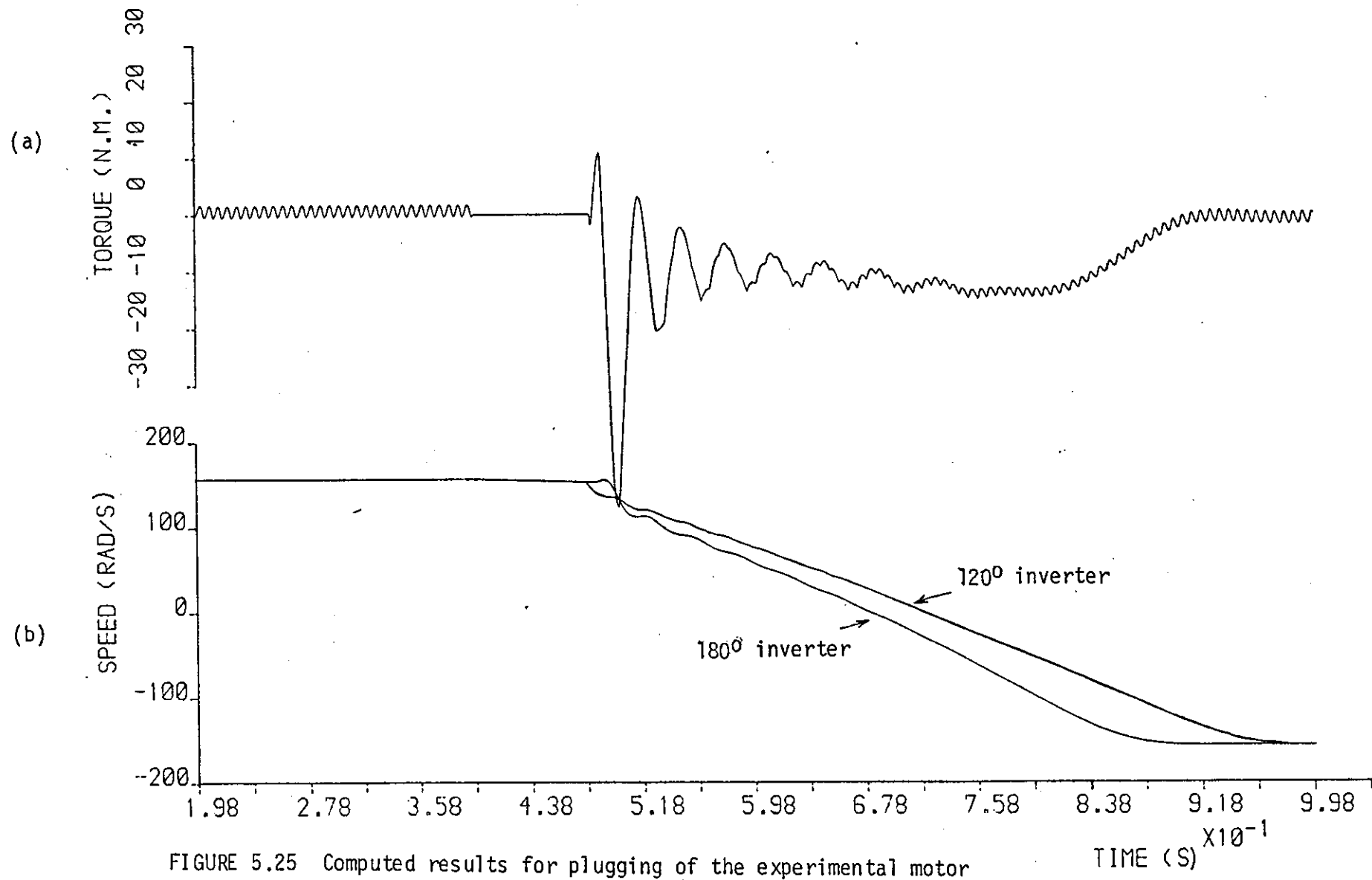


FIGURE 5.25 Computed results for plugging of the experimental motor

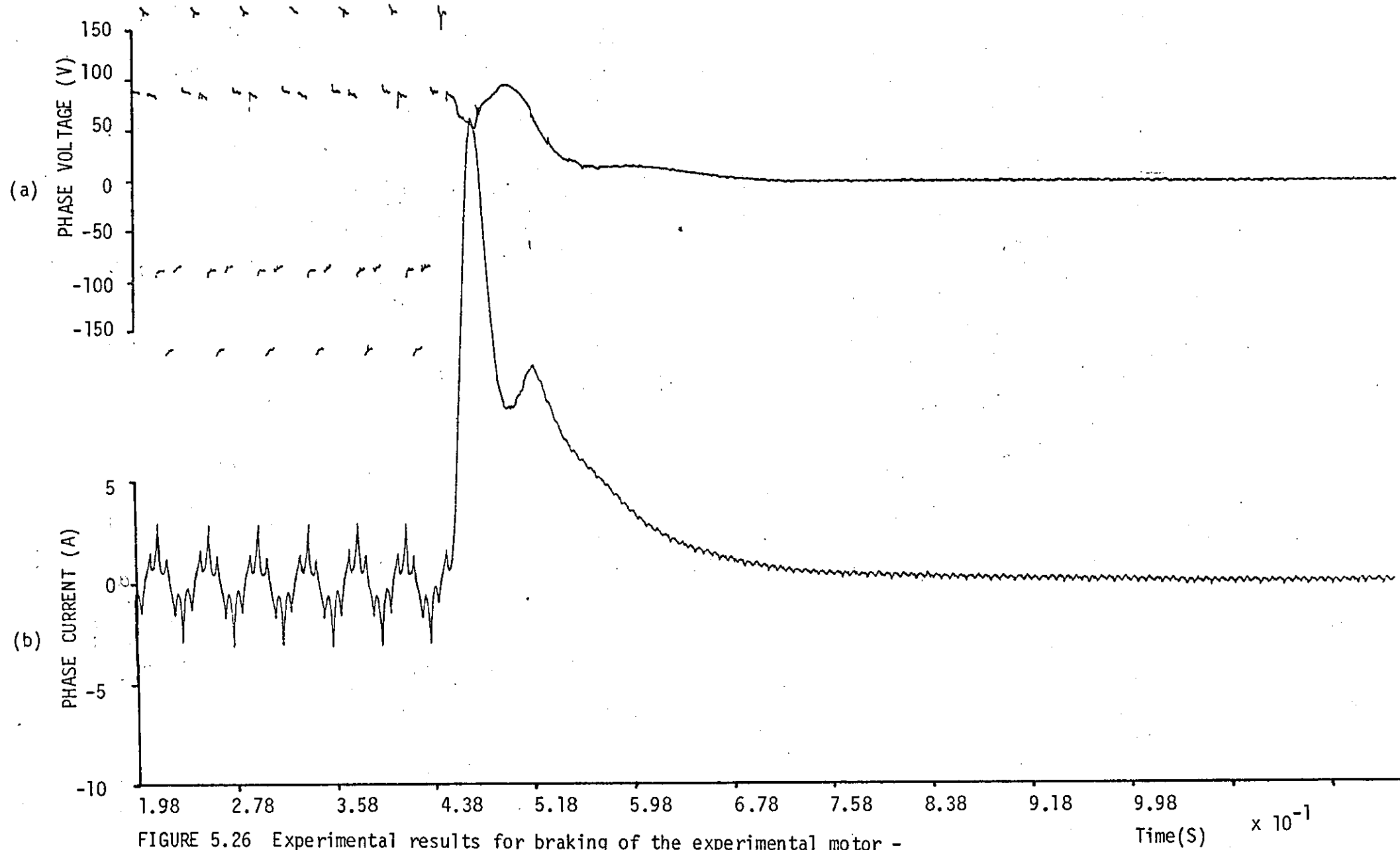


FIGURE 5.26 Experimental results for braking of the experimental motor - 180° inverter

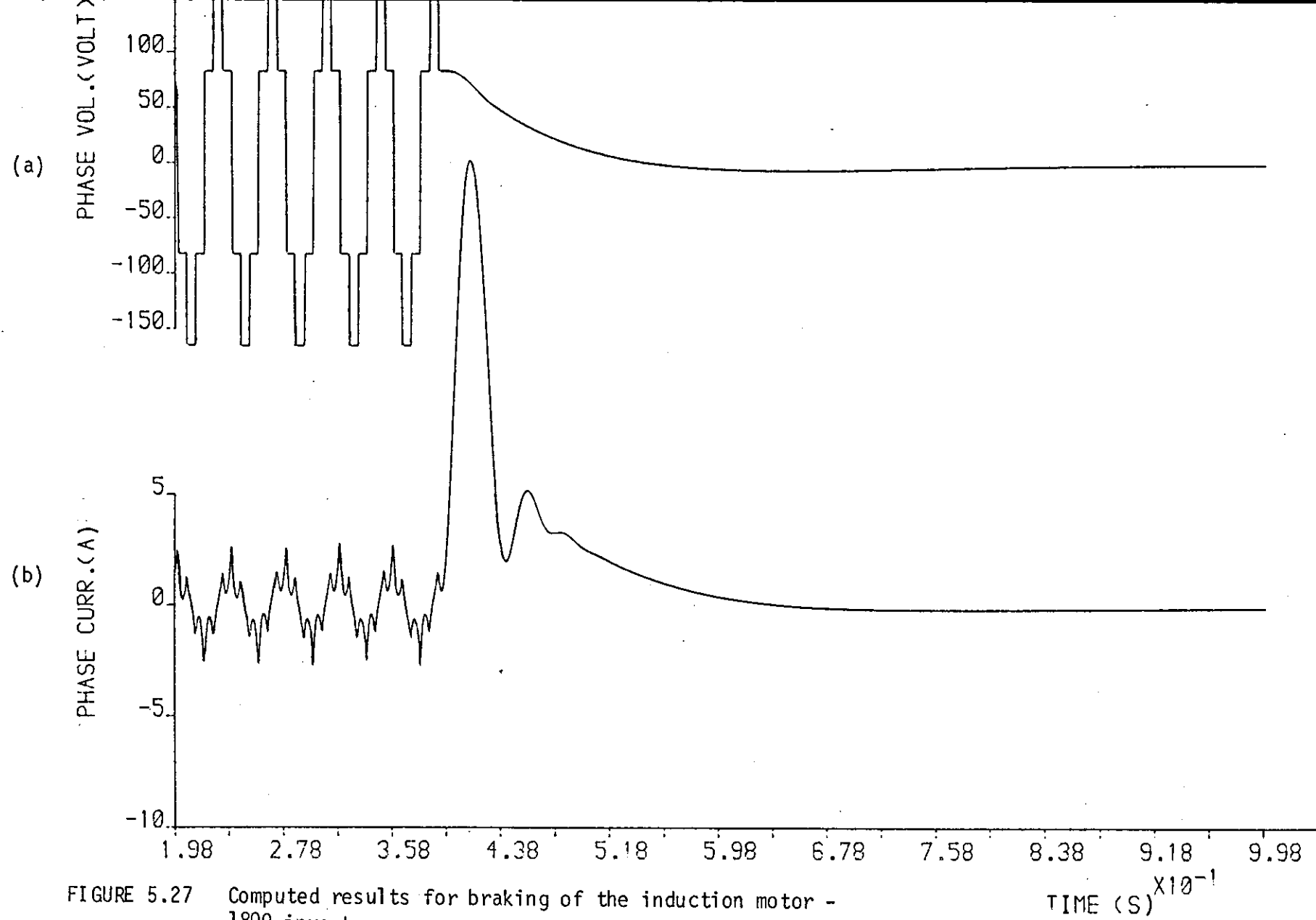


FIGURE 5.27 Computed results for braking of the induction motor - 180° inverter

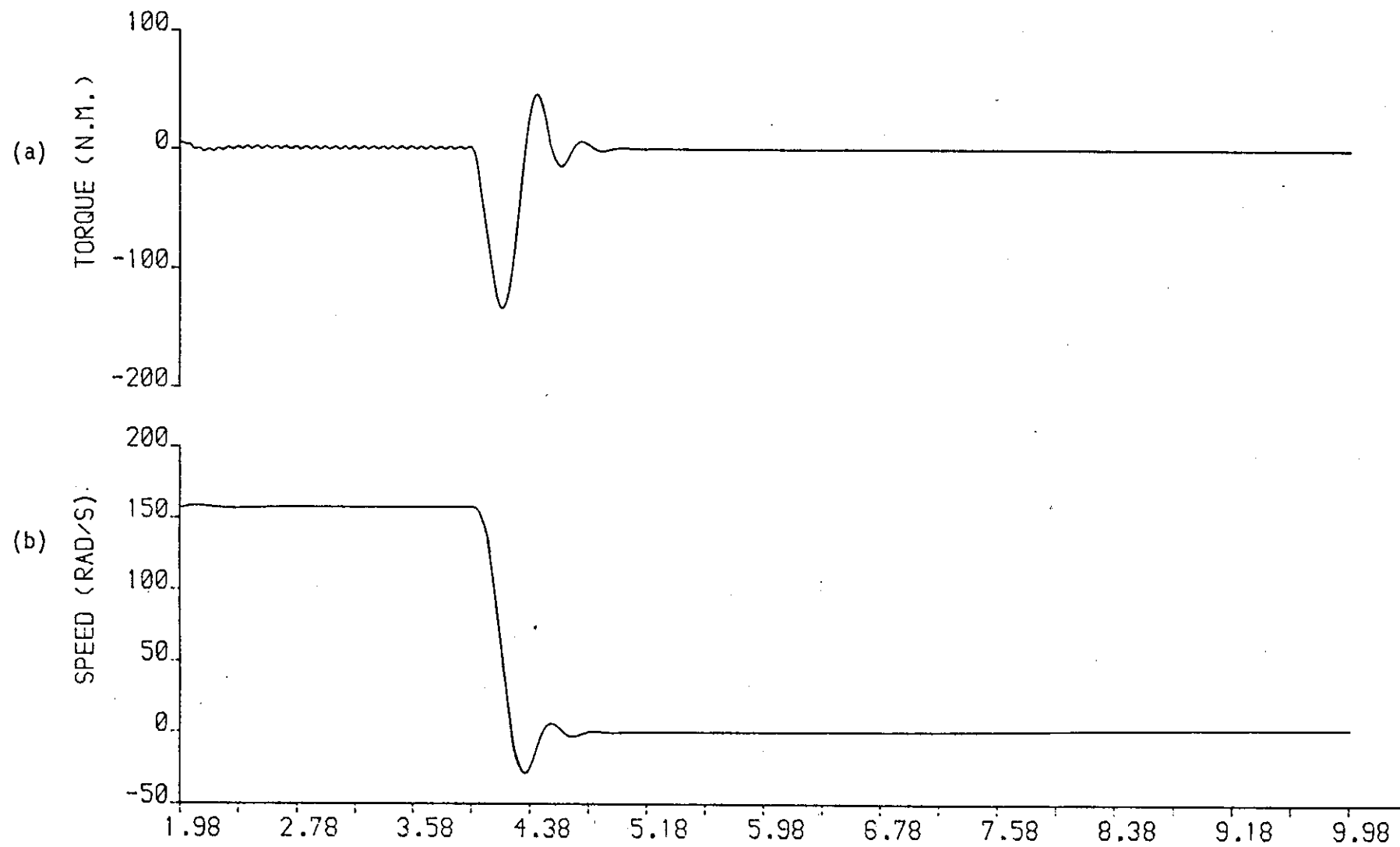


FIGURE 5.28 Computed results for braking of the experimental motor -
180° inverter

TIME (S) $\times 10^{-1}$

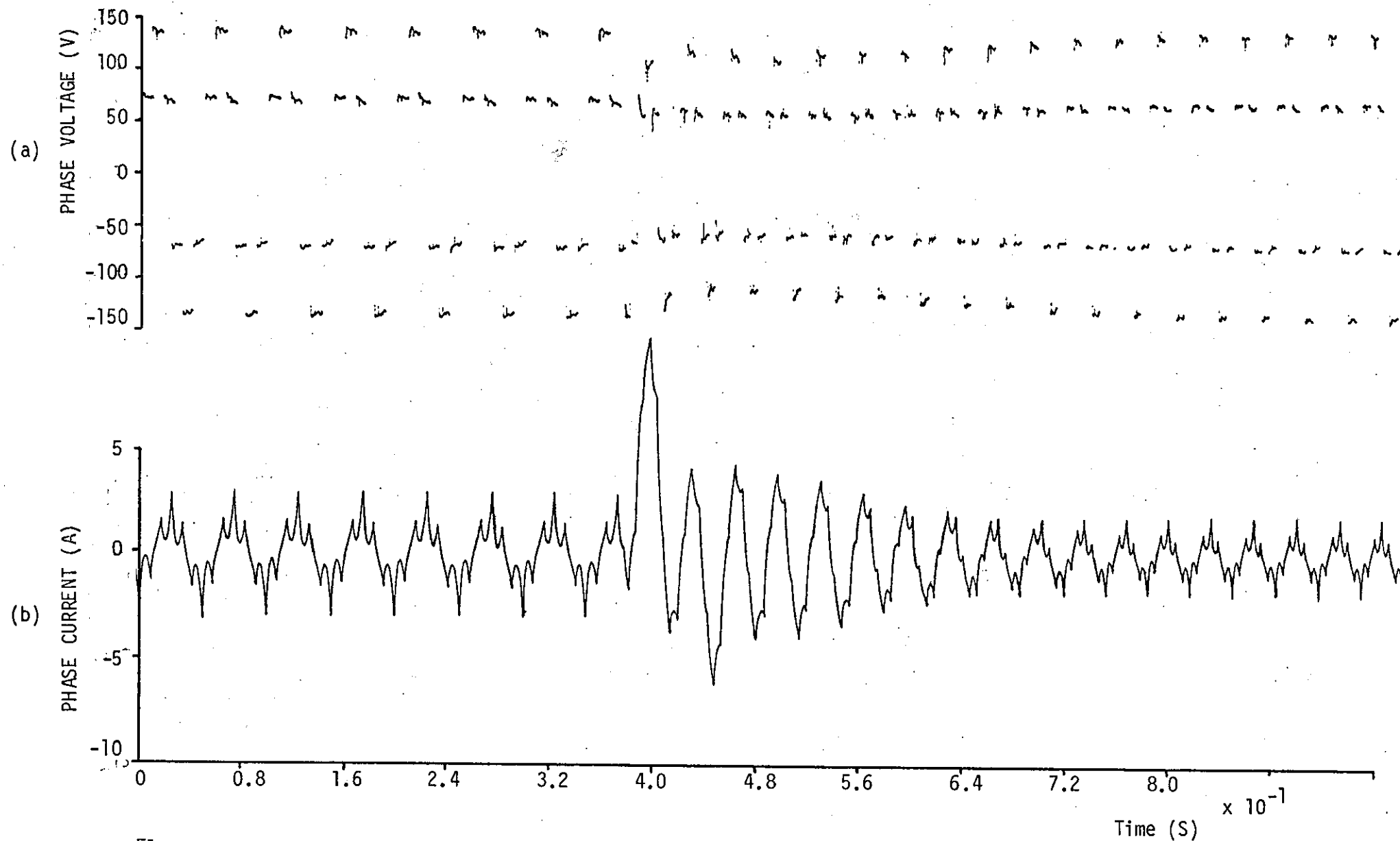
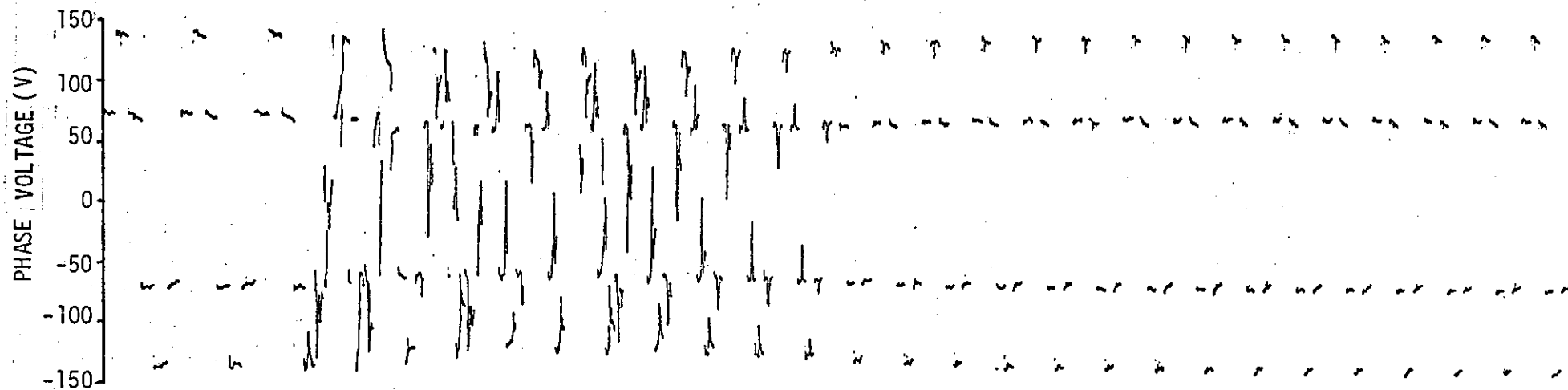


FIGURE 5.29 Experimental results for a step change from 20 to 30 Hz for the frequency supplied to the experimental motor - 180° inverter

(a)



(b)

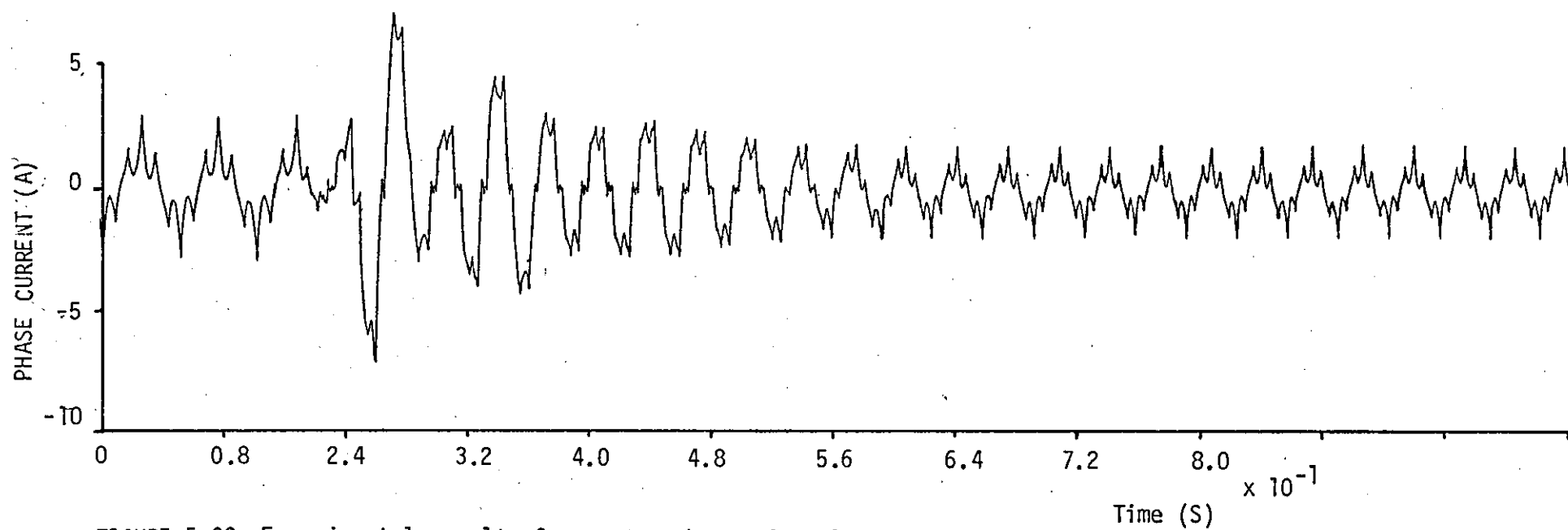


FIGURE 5.30 Experimental results for a step change from 20 to 30 Hz in the frequency supplied to the experimental motor - 120° inverter

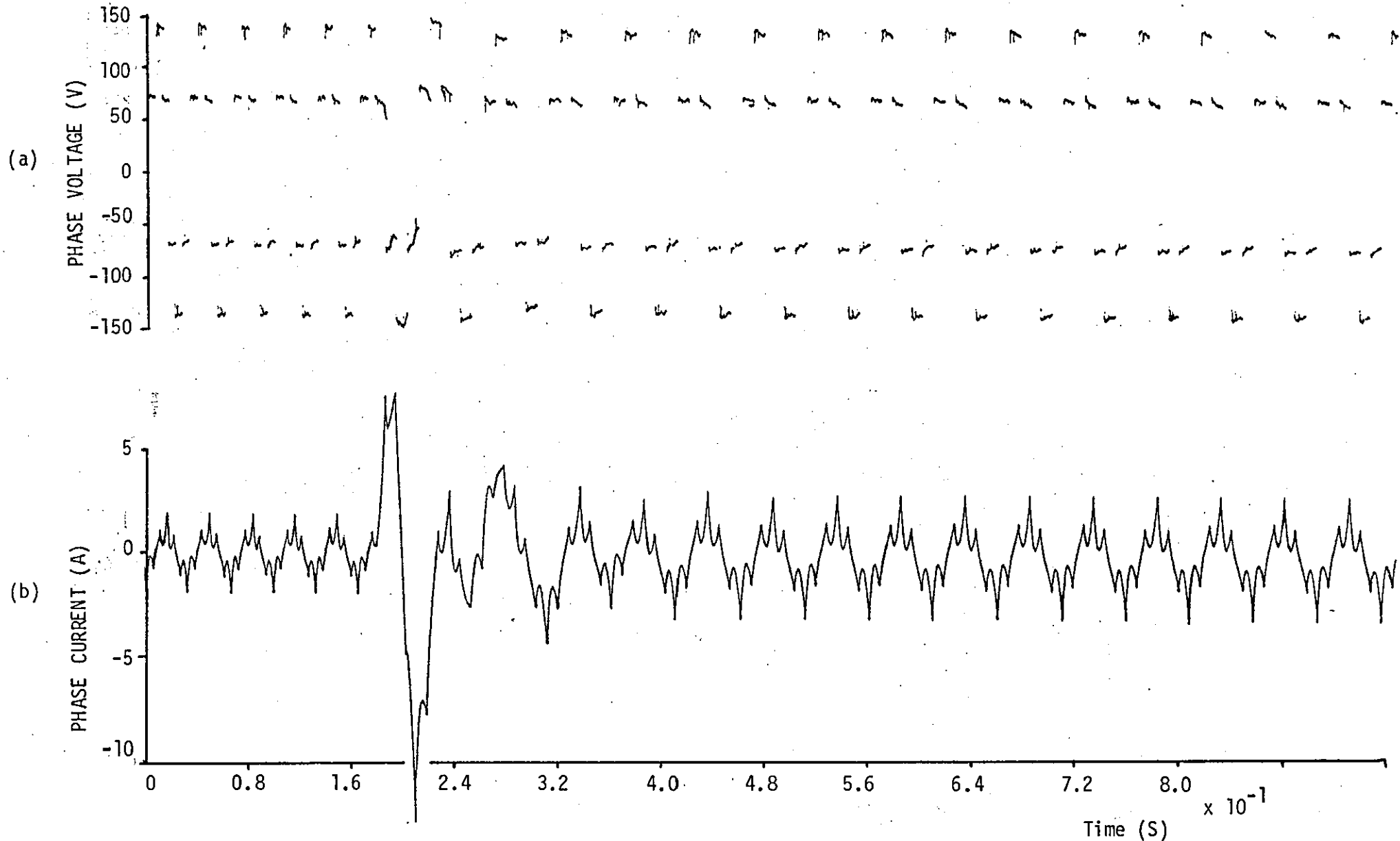


FIGURE 5.31 Experimental results for a step frequency change from 30 to 20 Hz in the frequency supplied to the experimental motor - 180° inverter

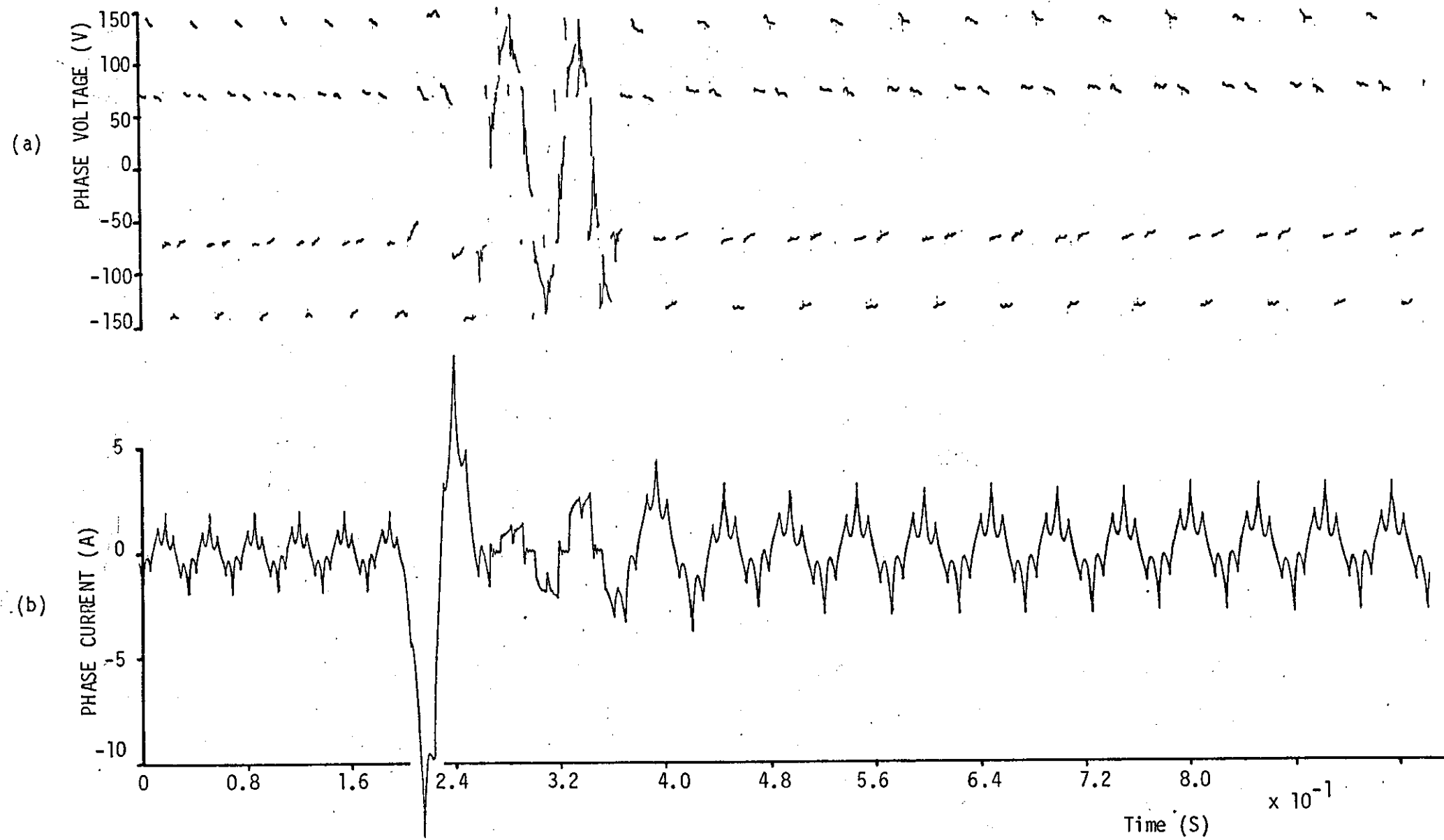


FIGURE 5.32 Experimental results for a step frequency change from 30 to 20 Hz in the frequency supplied to the experimental motor - 120° inverter

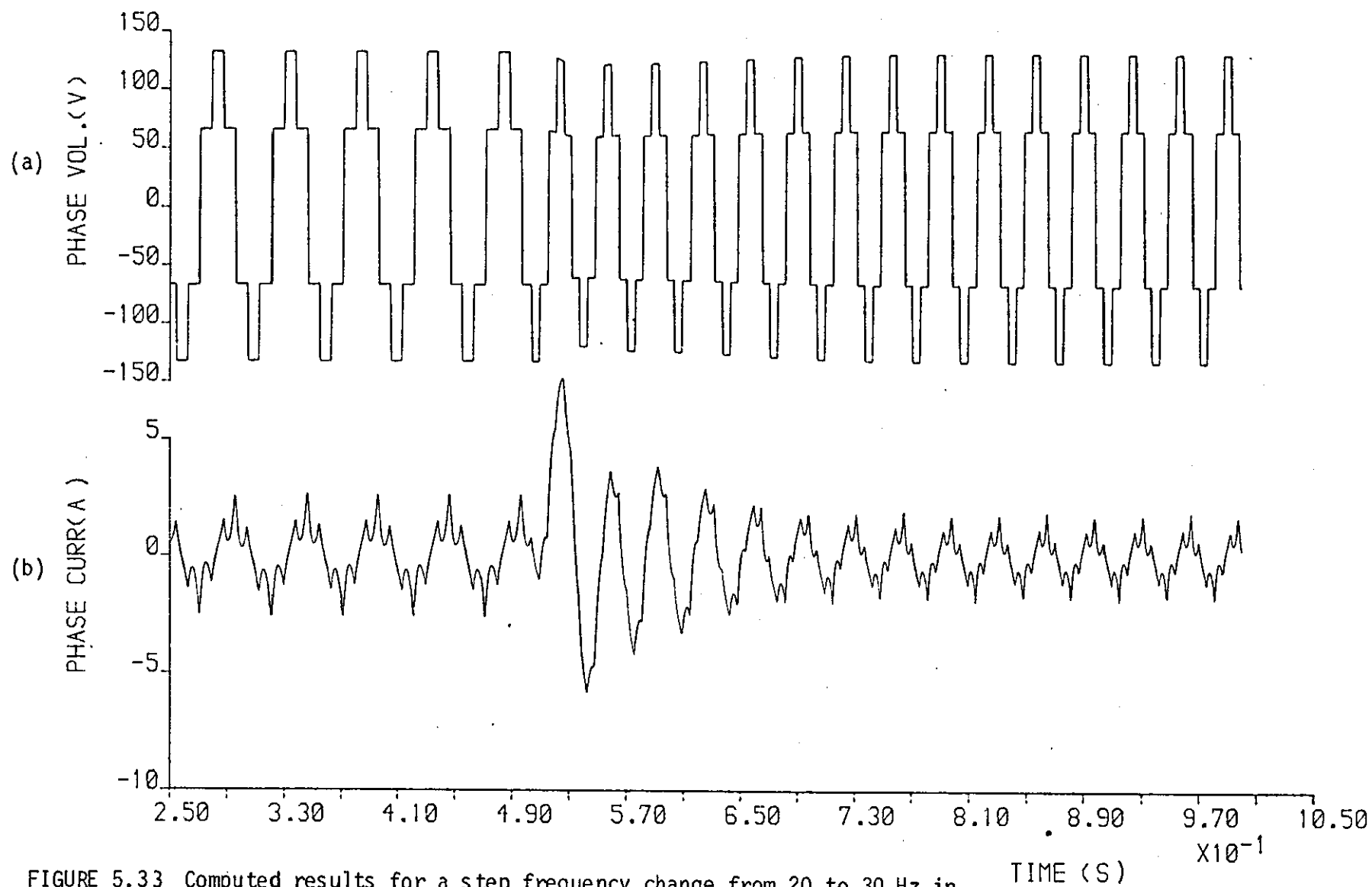


FIGURE 5.33 Computed results for a step frequency change from 20 to 30 Hz in the frequency supplied to the experimental motor - 180° inverter

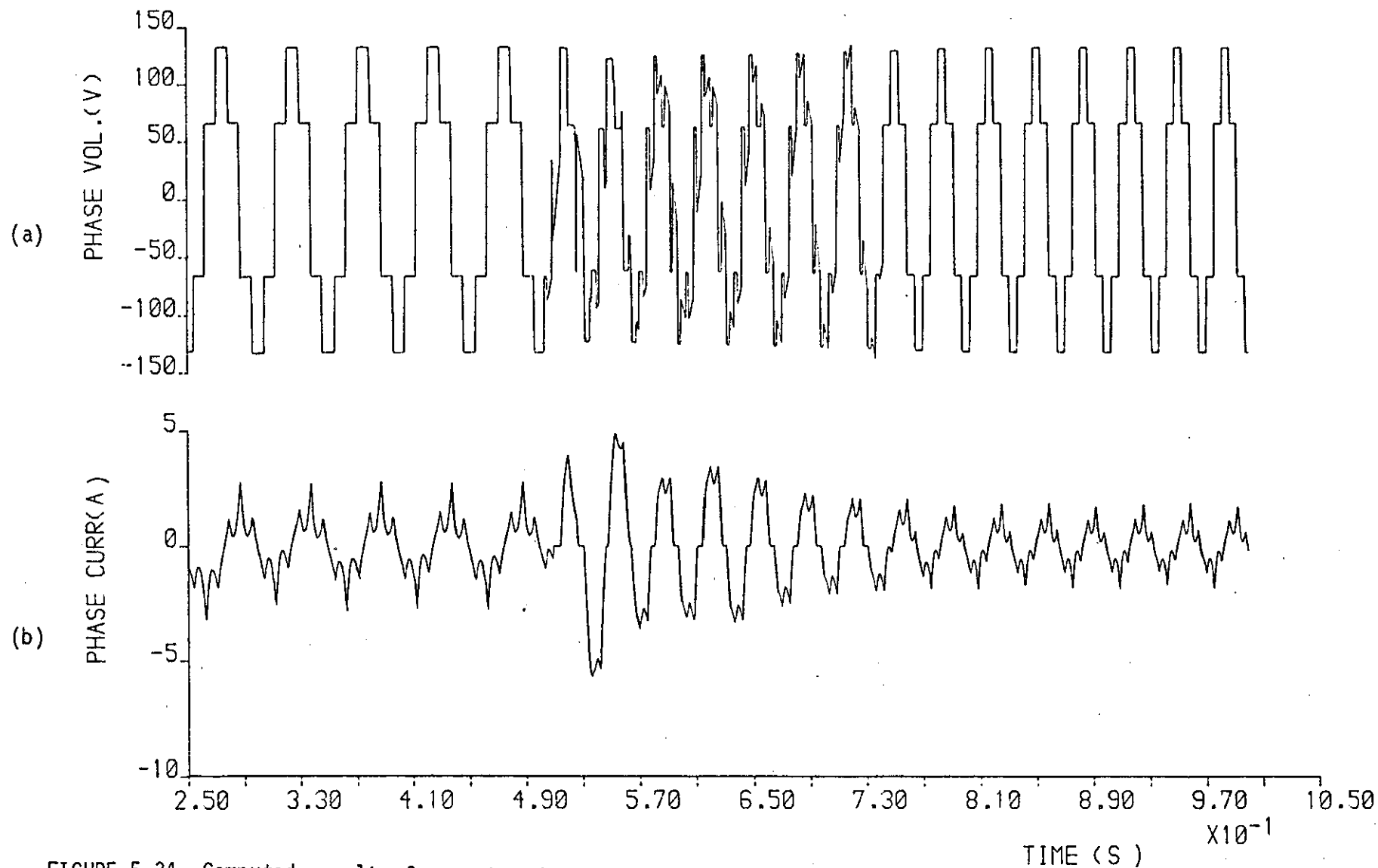


FIGURE 5.34 Computed results for a step frequency change from 20 to 30 Hz in the frequency supplied to the experimental motor - 120° inverter

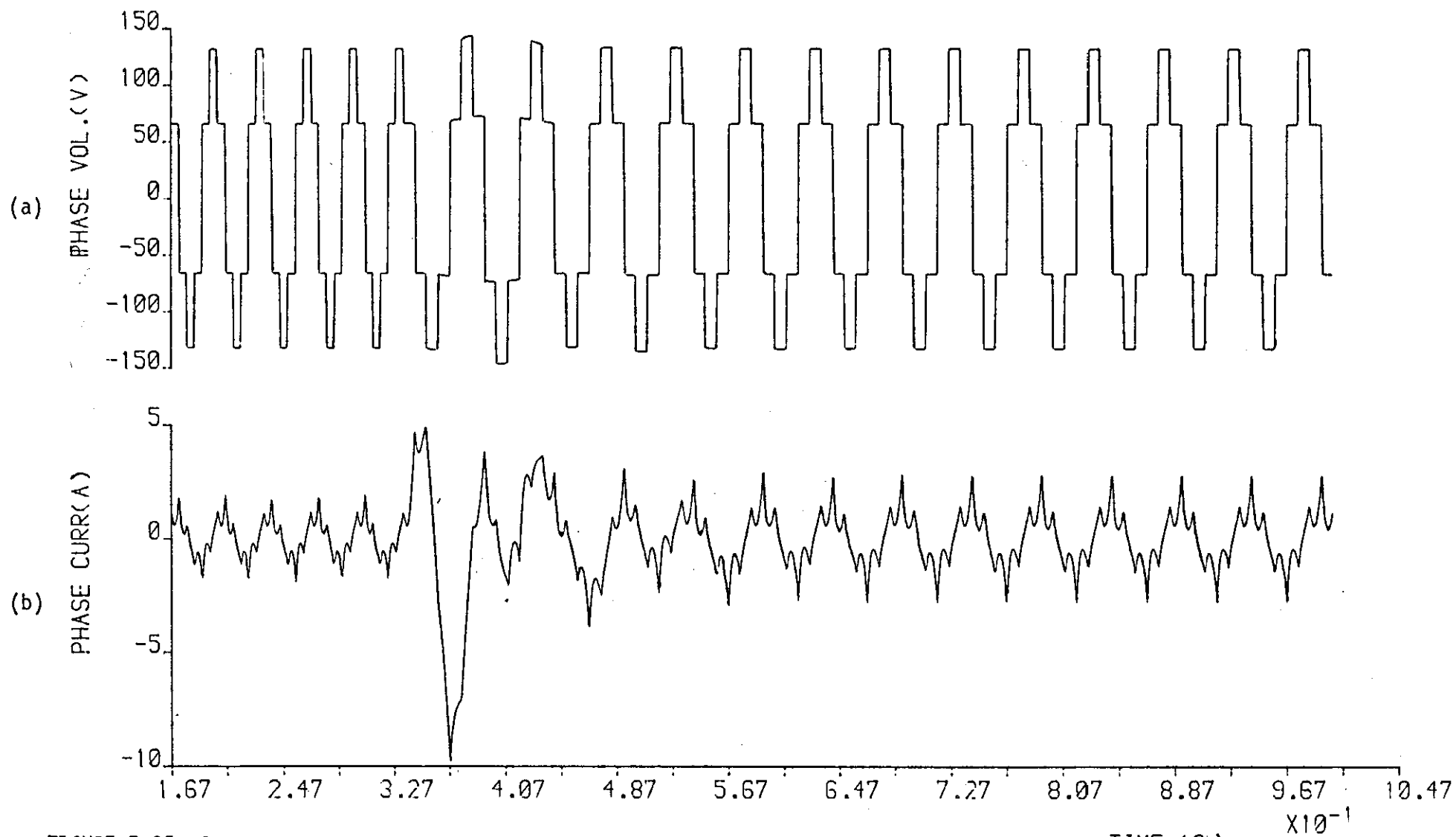


FIGURE 5.35 Computed results for a step change from 30 to 20 Hz in the frequency supplied to the experimental motor - 180° inverter

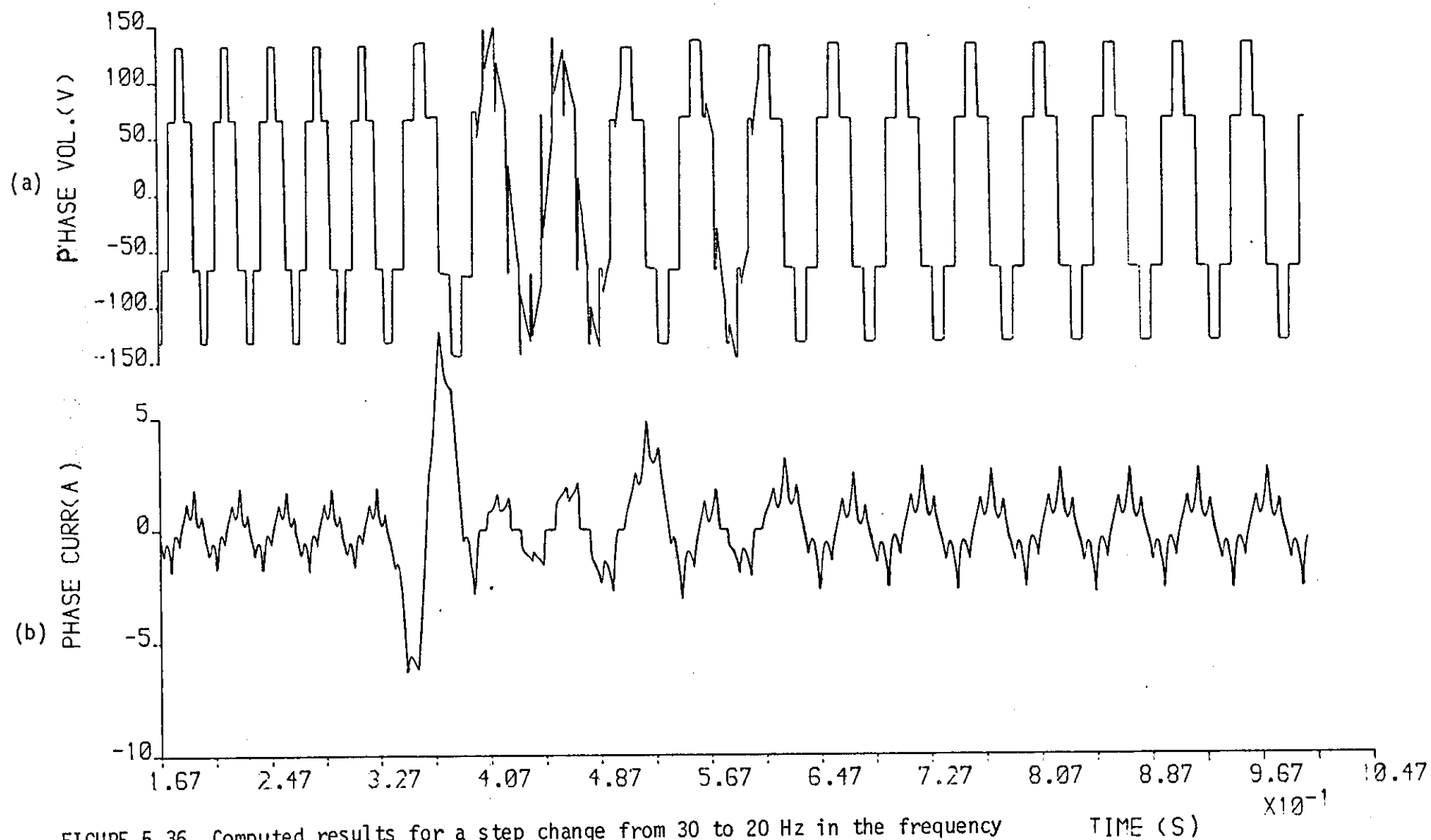


FIGURE 5.36 Computed results for a step change from 30 to 20 Hz in the frequency supplied to the experimental motor - 120° inverter

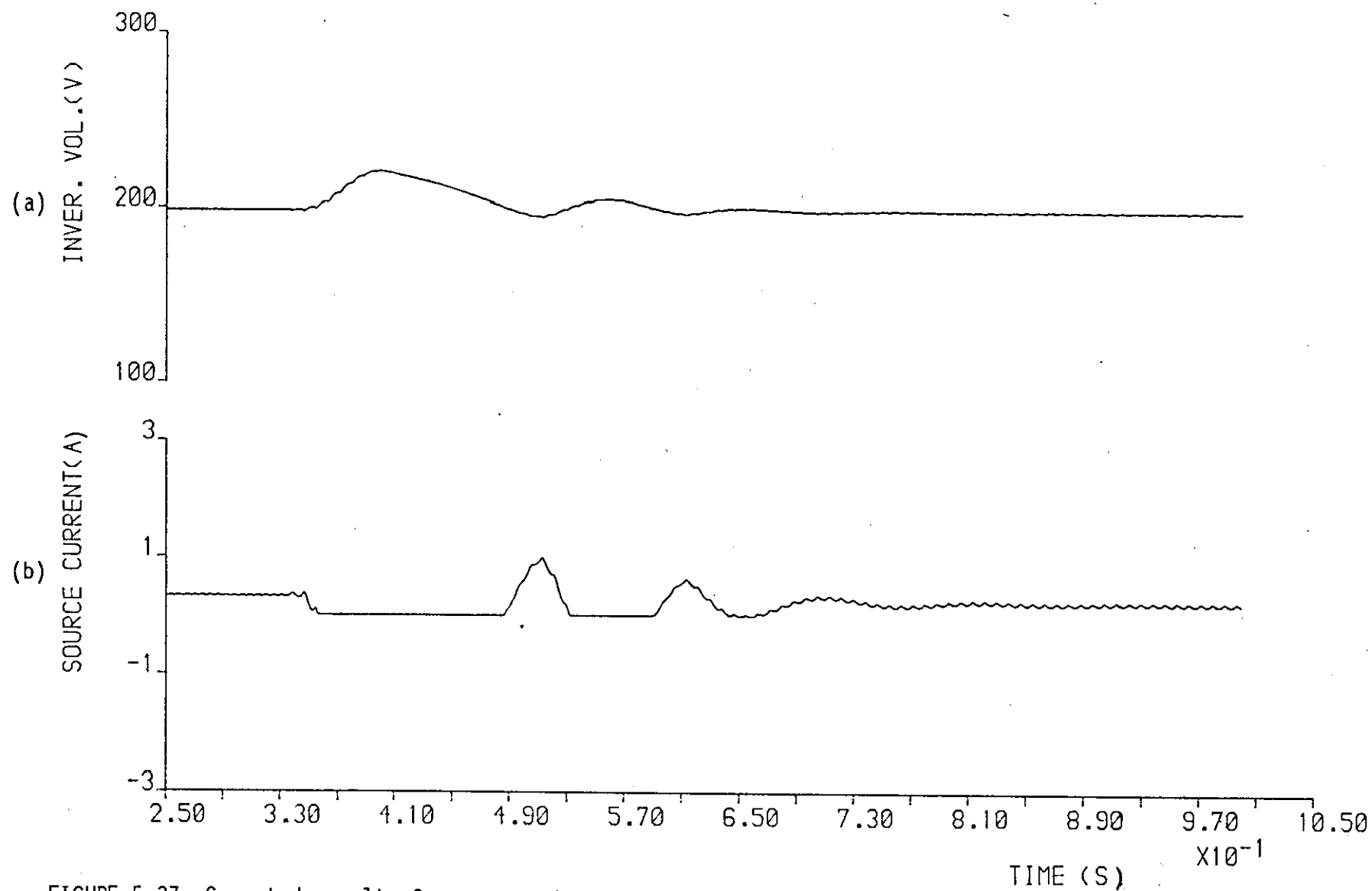


FIGURE 5.37 Computed results for a step change from 30 to 20 Hz in the frequency applied to the experimental motor - 120° inverter

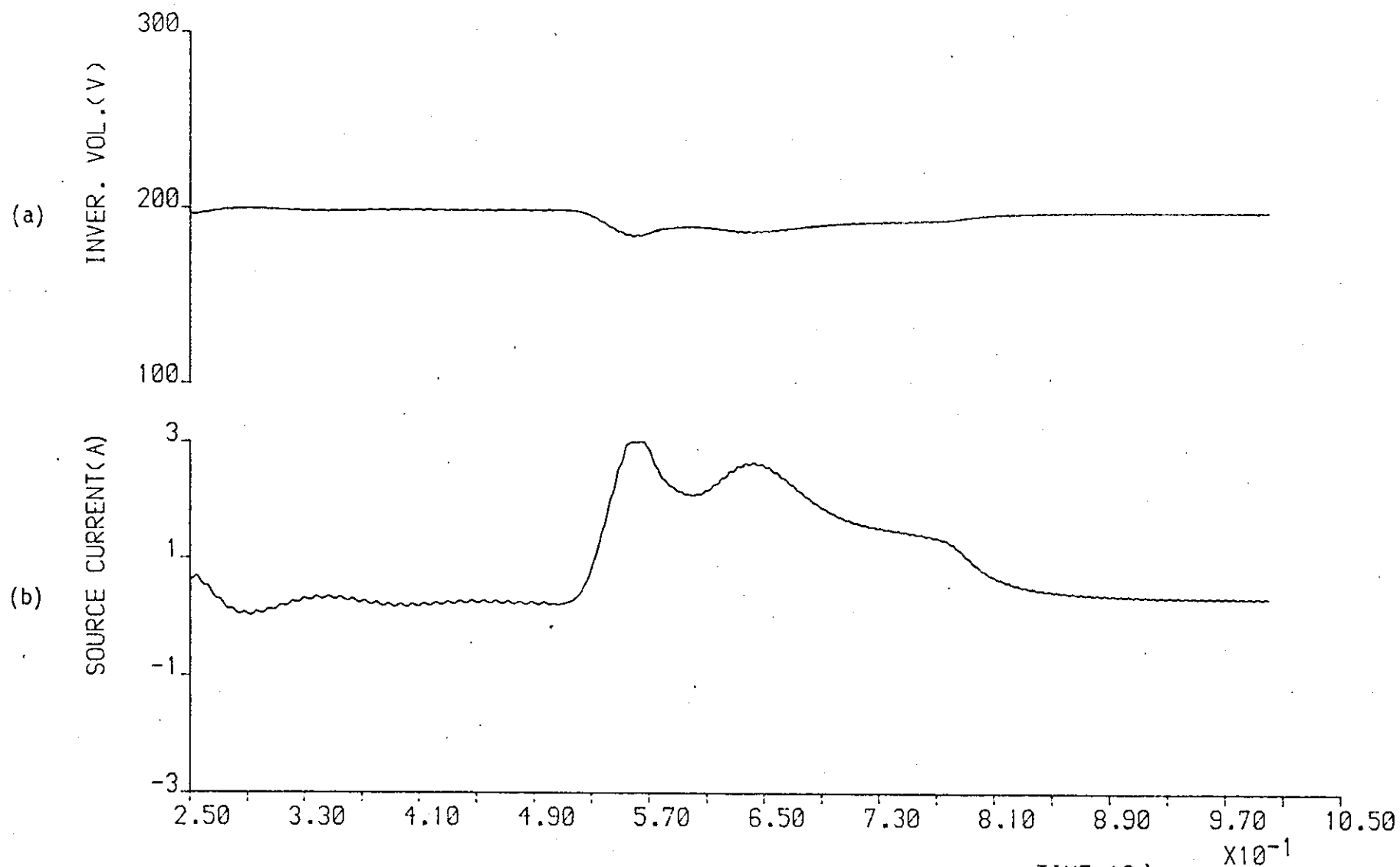


FIGURE 5.38 Computed results for a step change from 20 to 30 Hz in the frequency supplied to the experimental motor - 120° inverter

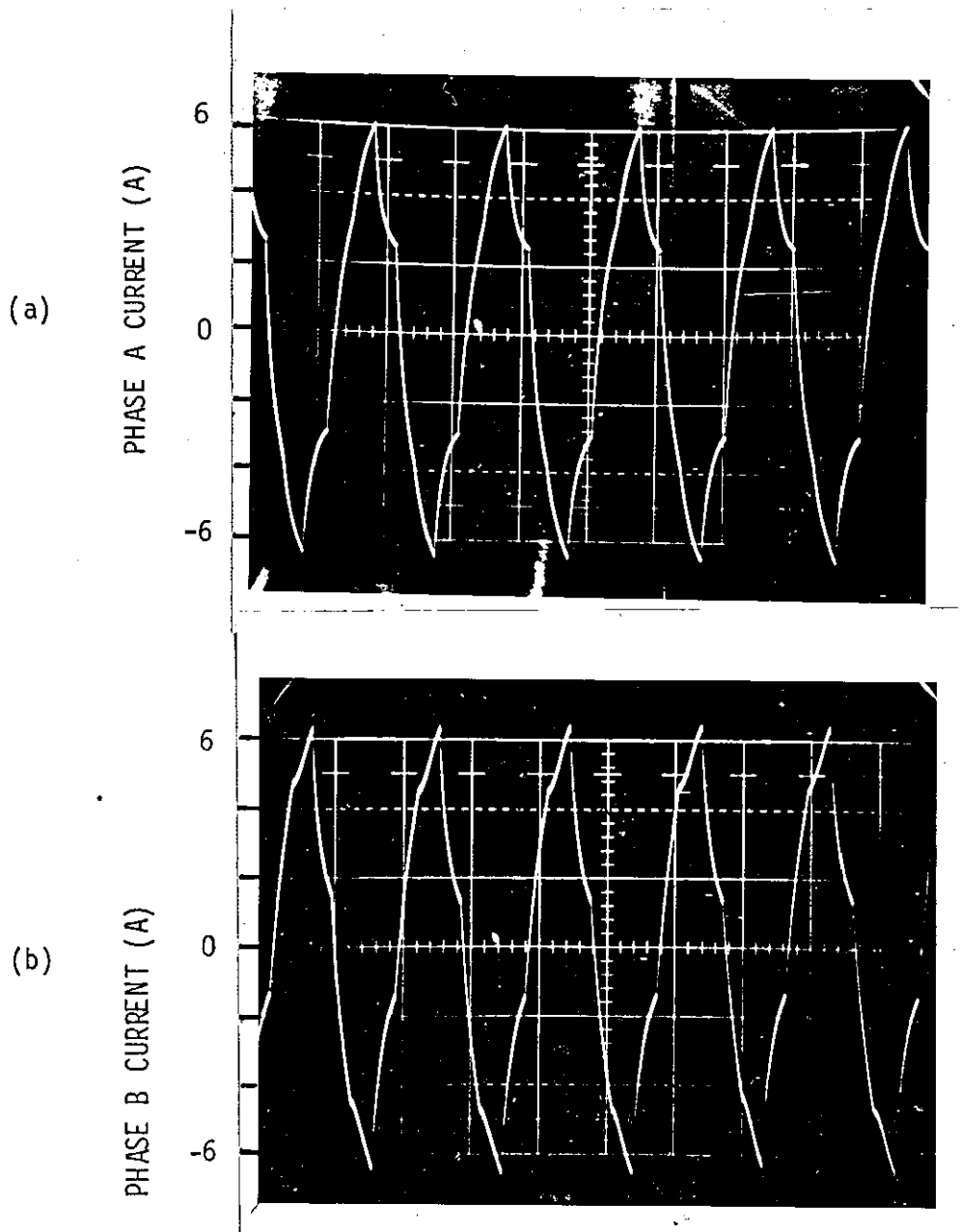


FIGURE 5.39 Experimental results for short circuit fault on phase C of the experimental motor supplied from 180° inverter

Scale: current 2A/div
time 20 ms/div

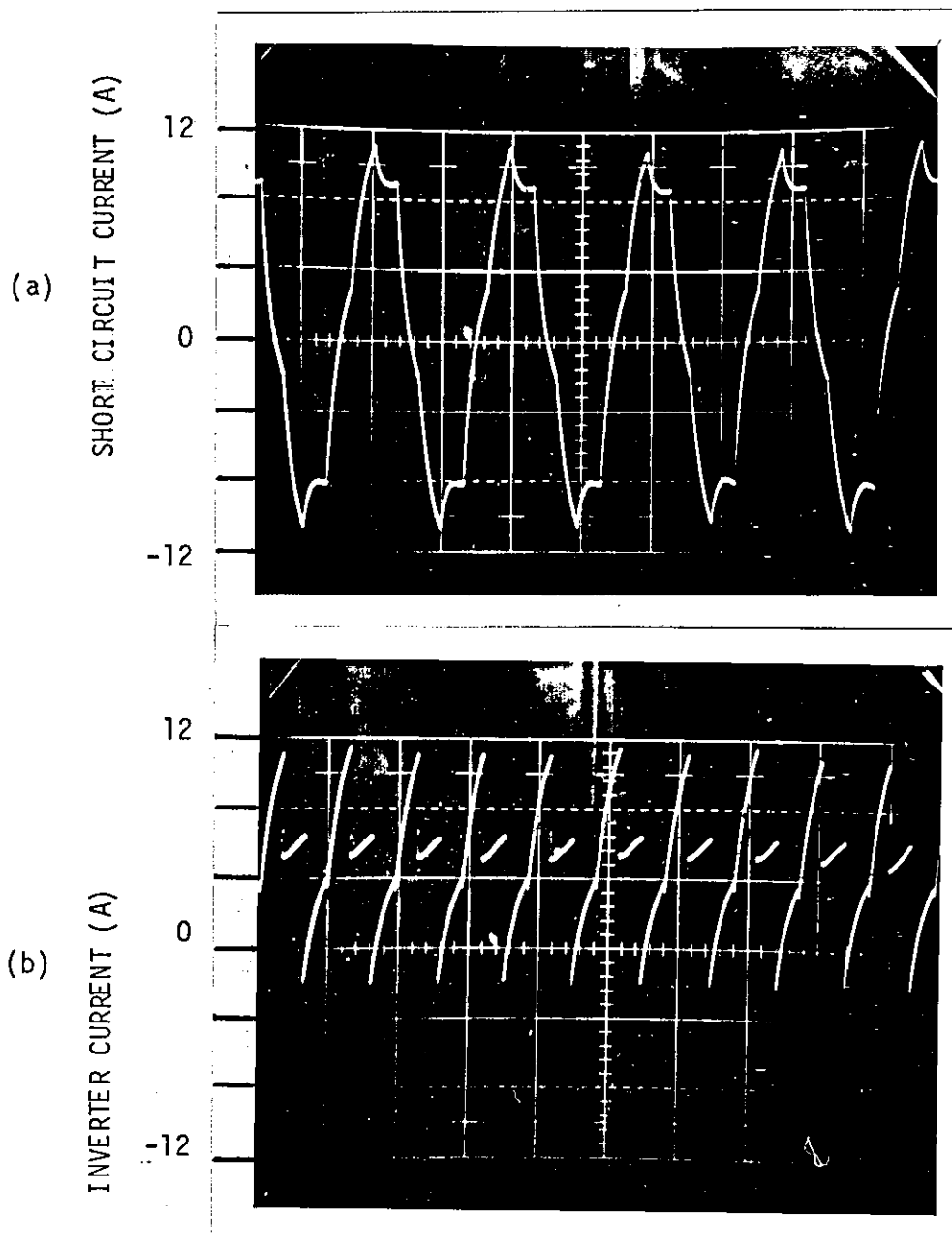


FIGURE 5.40 Experimental results for short circuit fault on phase C of the experimental motor supplied from 180° inverter

Scale: Current 4A/div
Time 20 ms/div

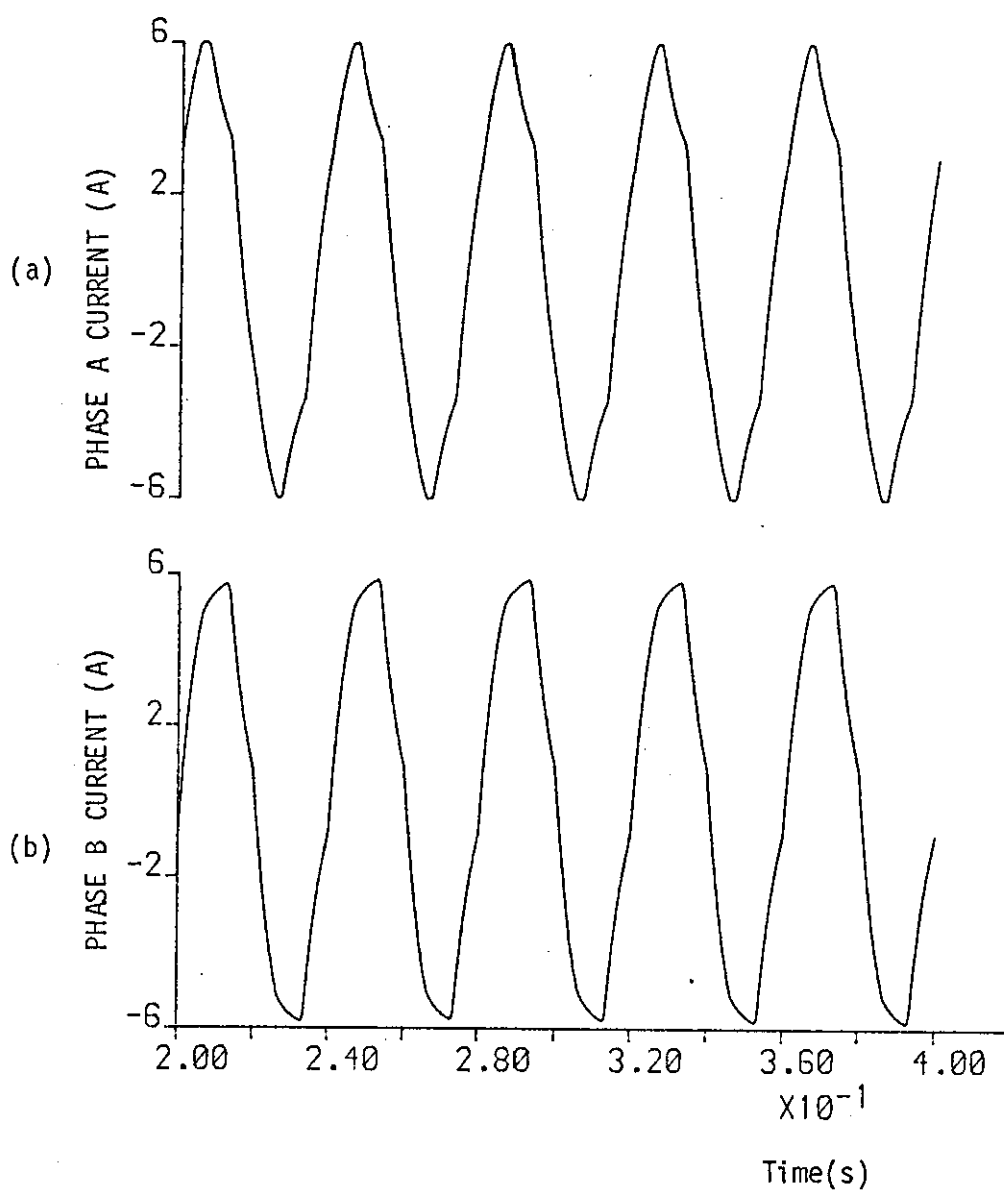


FIGURE 5.41 Computed results for short circuit fault on phase C of the experimental motor supplied from 180° inverter

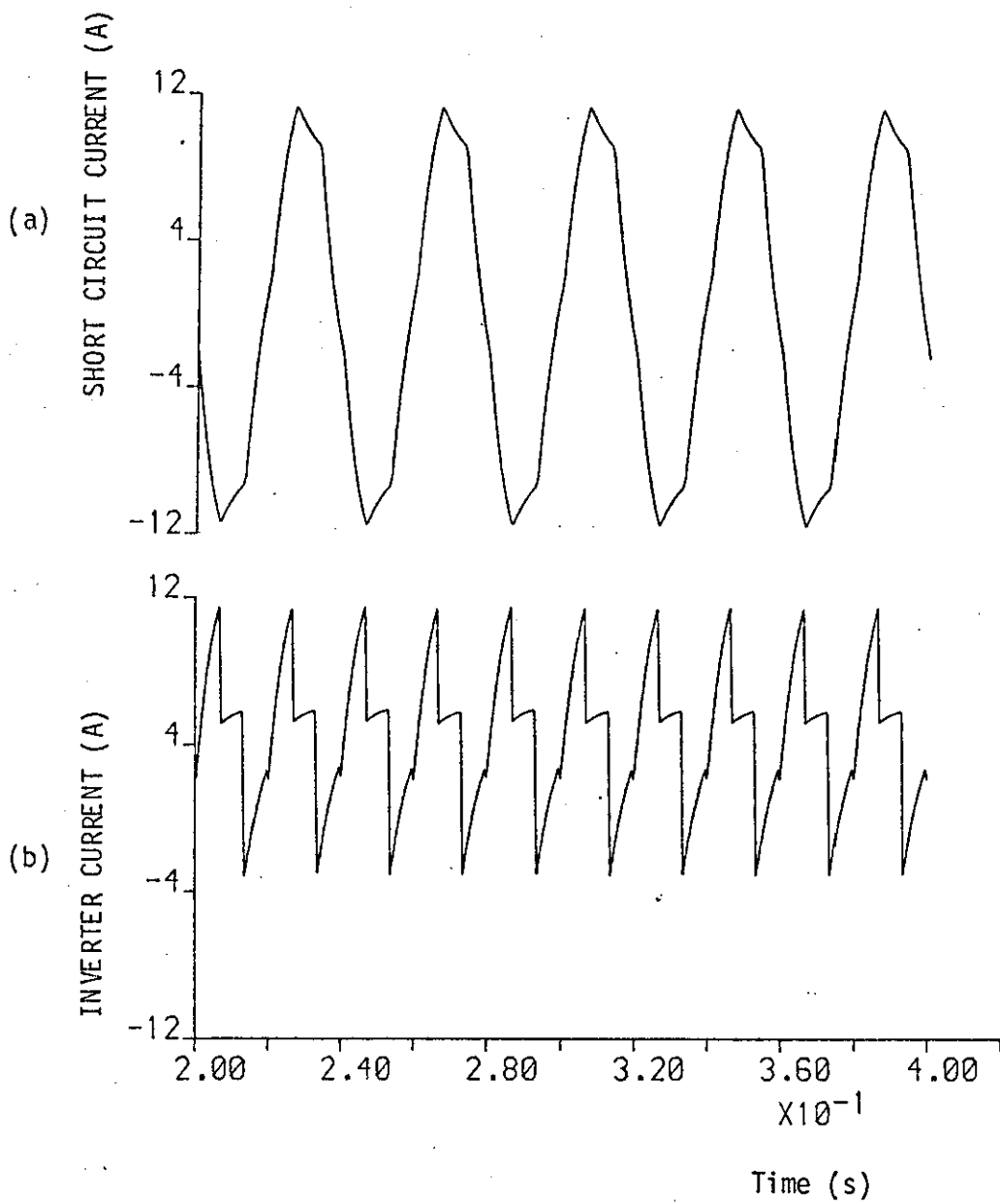
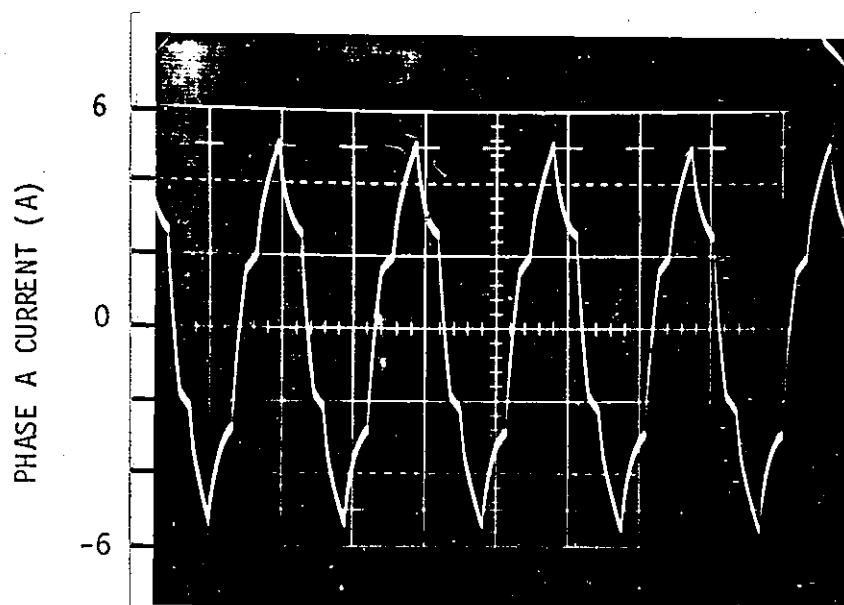


FIGURE 5.42 Computed results for short circuit faults on phase C of the experimental motor supplied from 180° inverter

(a)



(b)

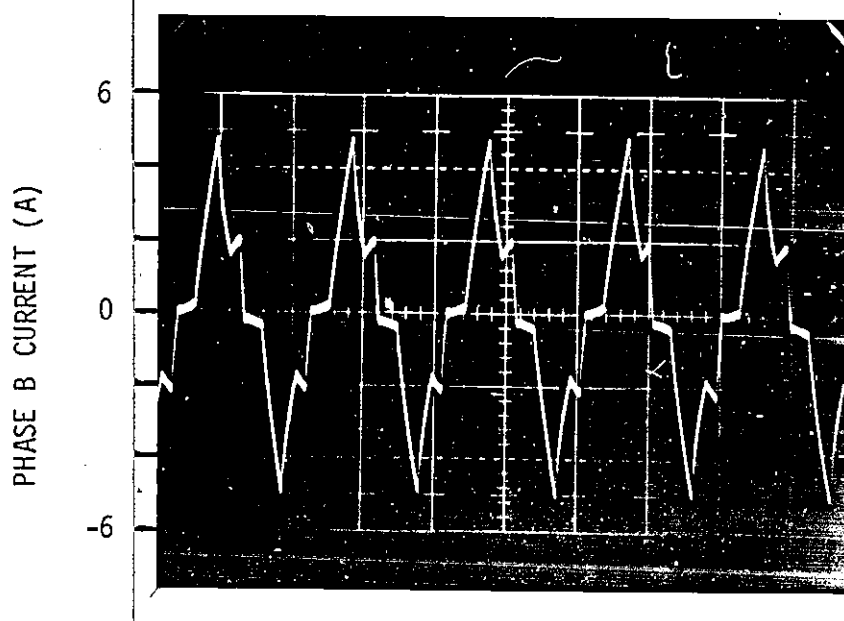


FIGURE 5.43 Experimental results for short circuit fault on phase C of the experimental motor - 120° inverter.

Scale: current 2A/div
time 20 ms/div

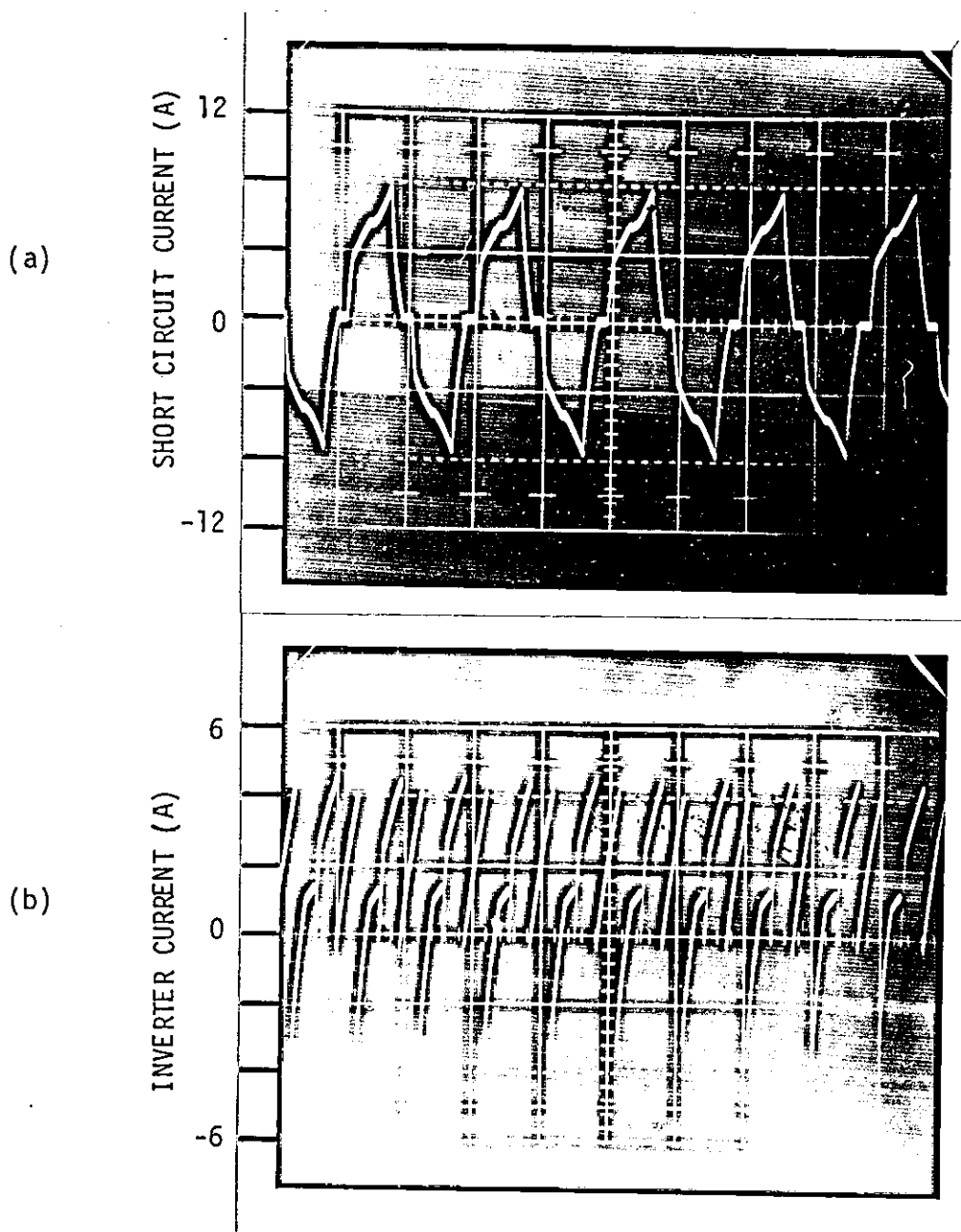


FIGURE 5.44 Experimental results for short circuit fault on phase C of the experimental motor - 120° inverter

Scale: upper trace current 4A/div
lower trace current 2A/div
time 20 ms/div

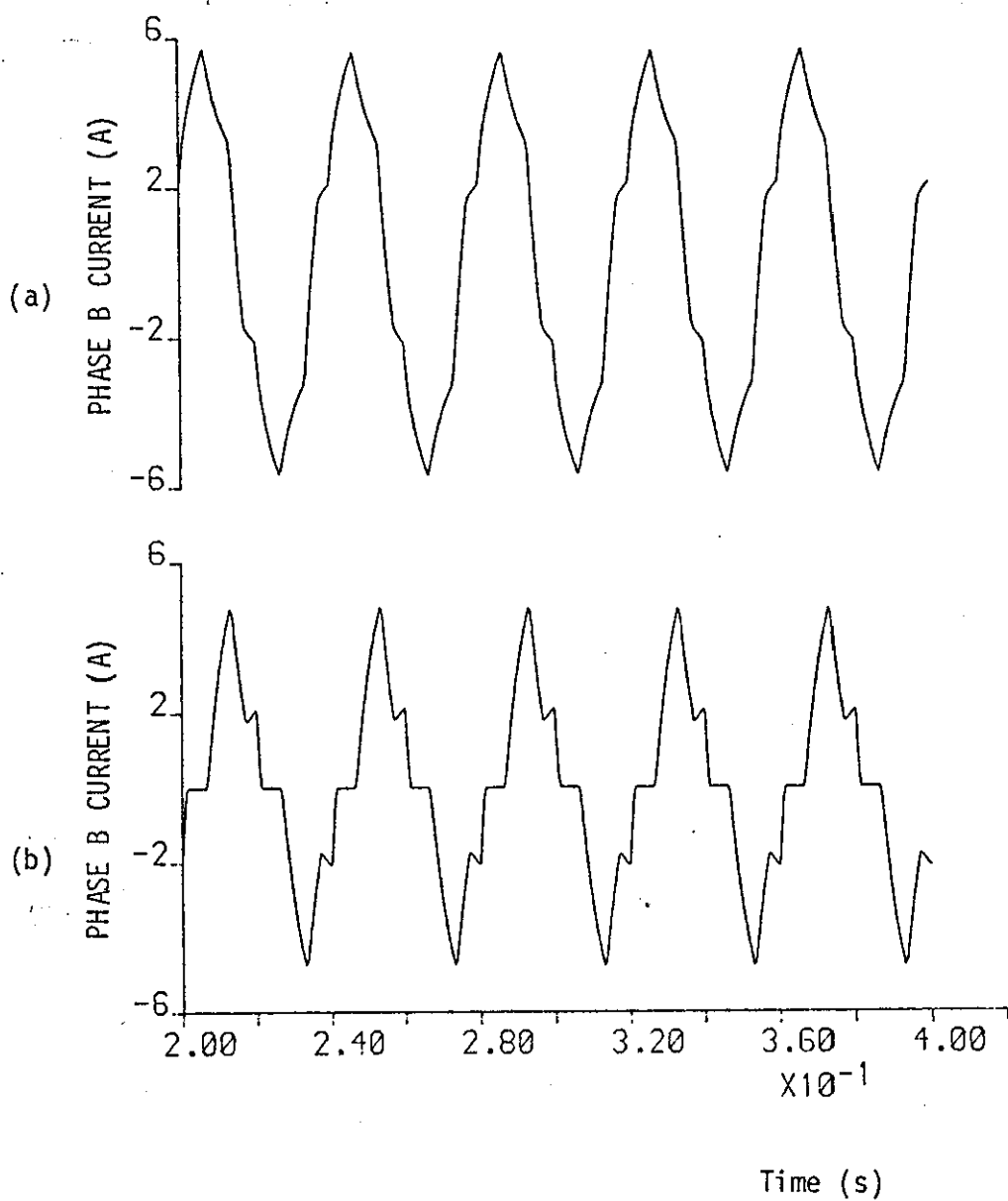


FIGURE 5.45 Computed results for short circuit faults on phase C of the experimental motor supplied from 180° inverter.

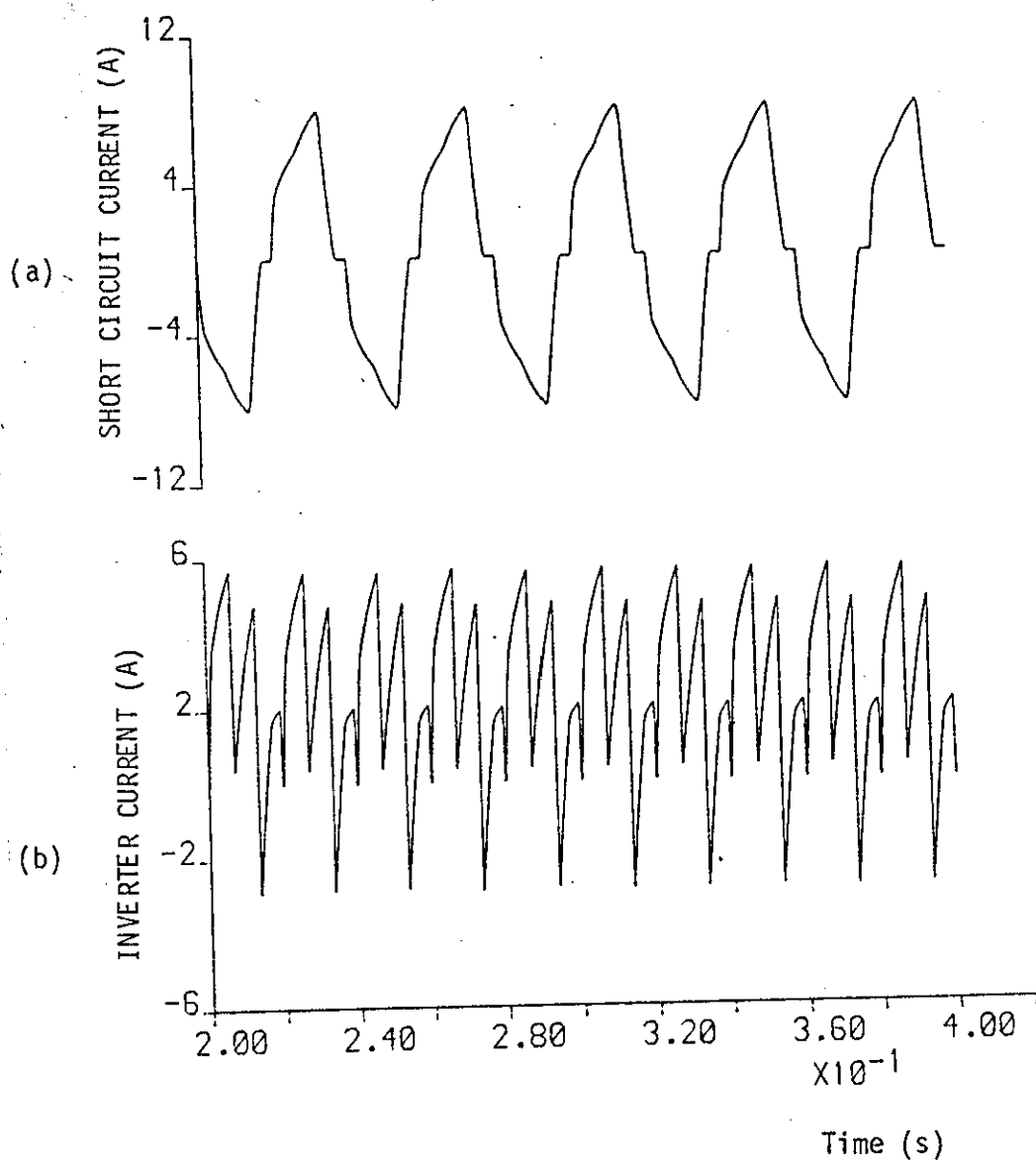


FIGURE 5.46 Computed results for short circuit faults on phase C of the experimental motor supplied from 120° inverter

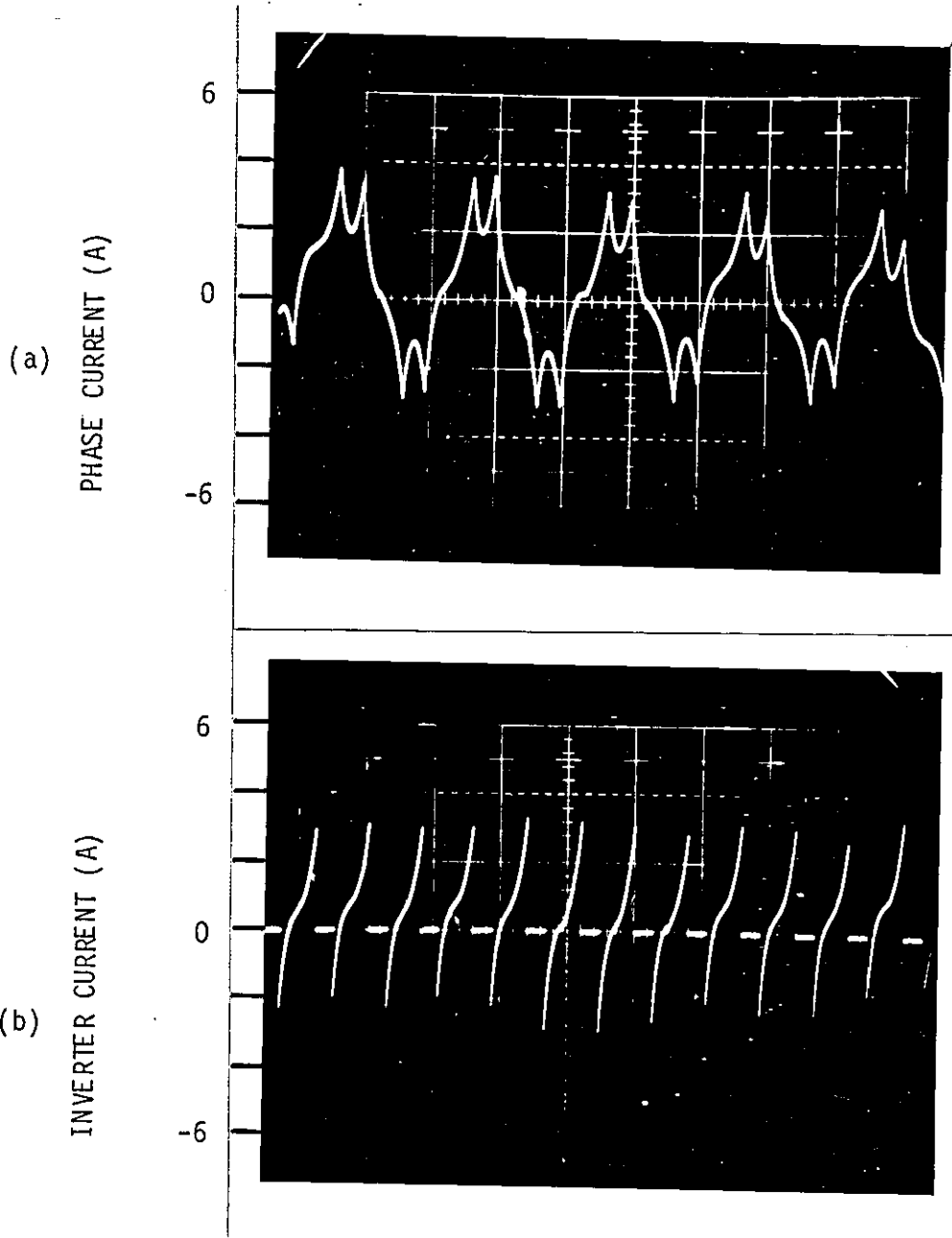


FIGURE 5.47 Experimental results for open circuit fault on phase C of the experimental motor supplied from 180° inverter

Scale: current 2A/div
time 20 ms/div

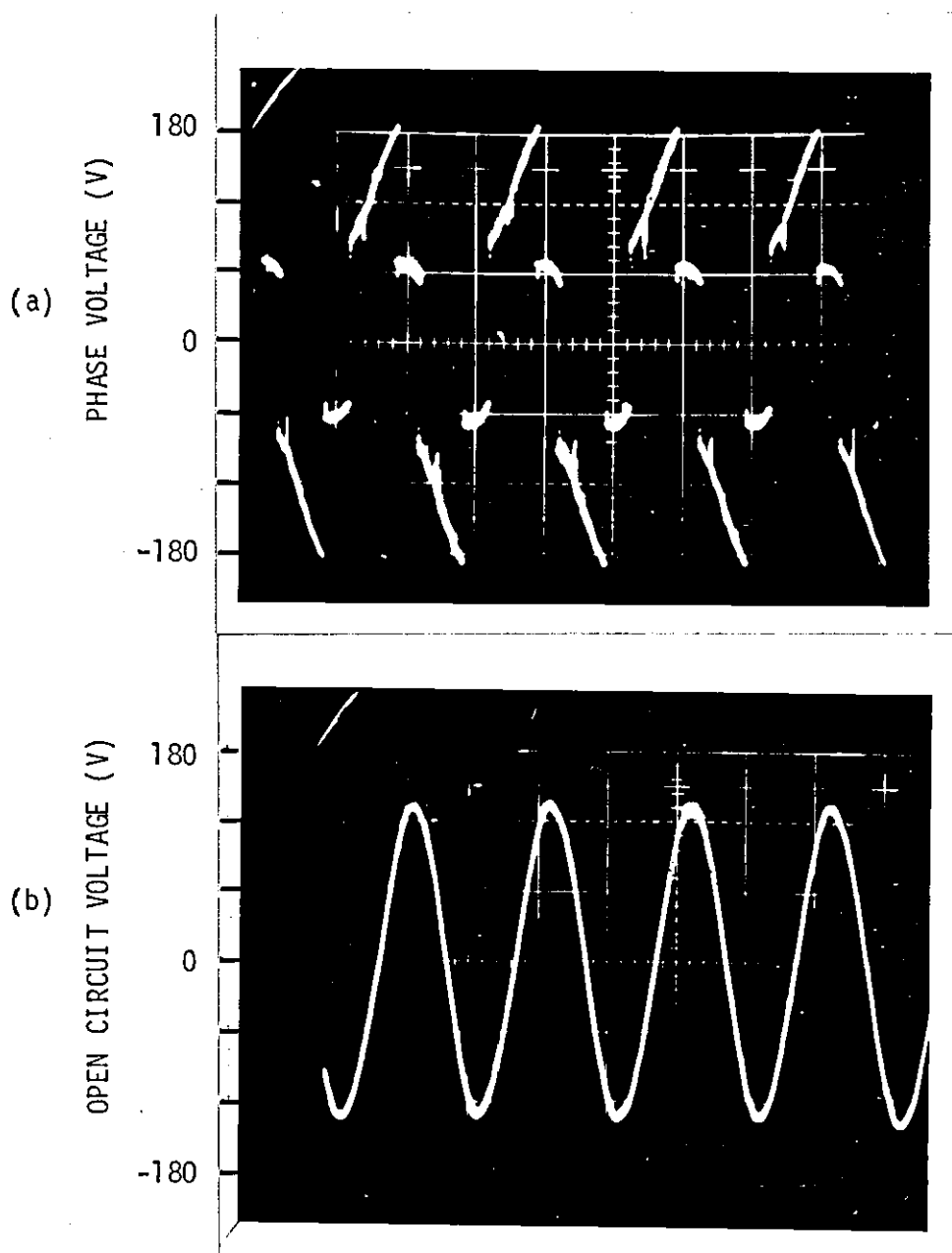


FIGURE 5.48 Experimental results for open circuit fault on phase C of the experimental motor supplied from 180° inverter

Scale: voltage 60 V/div
time 20 ms/div

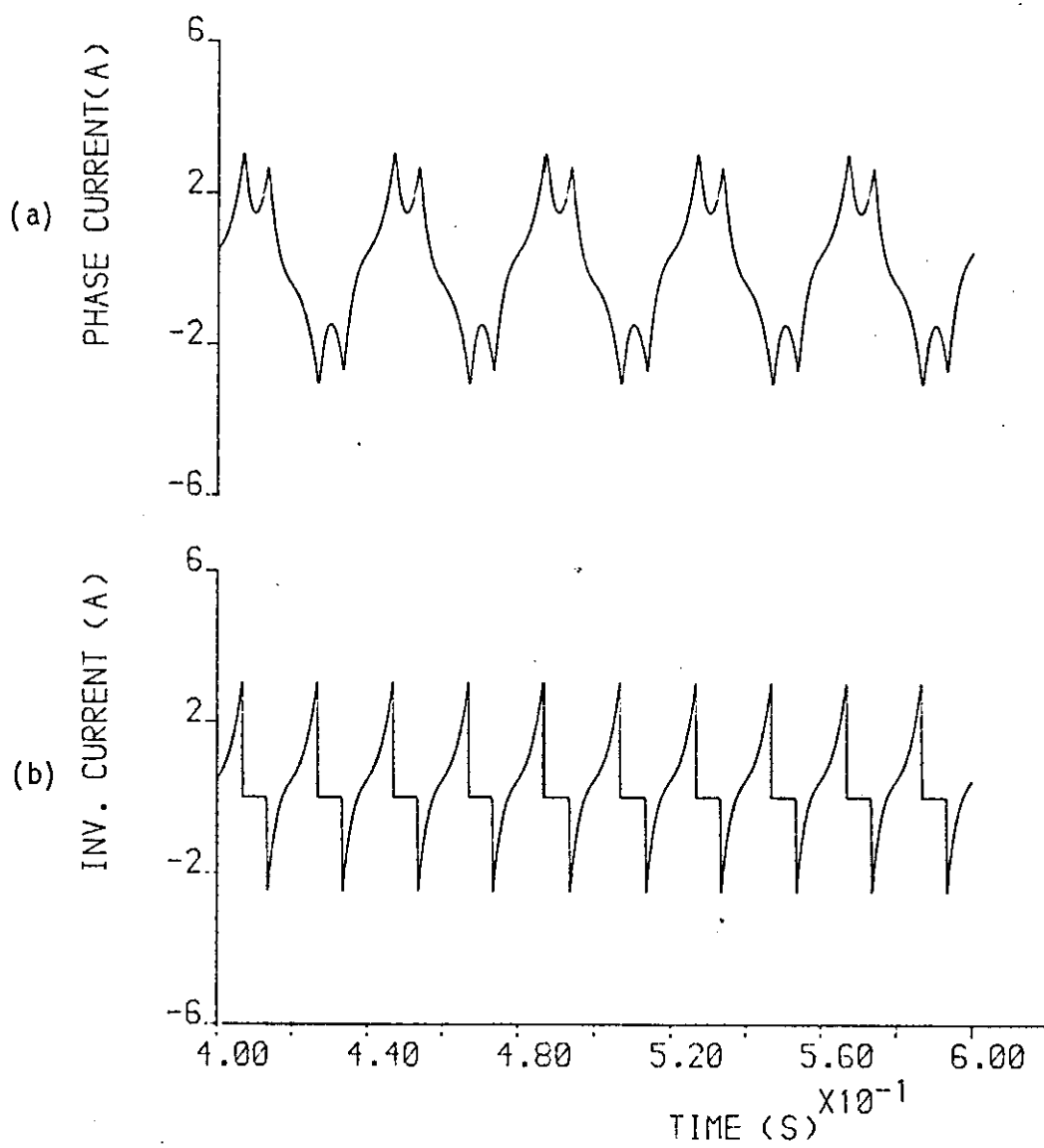


FIGURE 5.49 Computed results for open circuit fault on phase C of the experimental motor supplied from 180° inverter

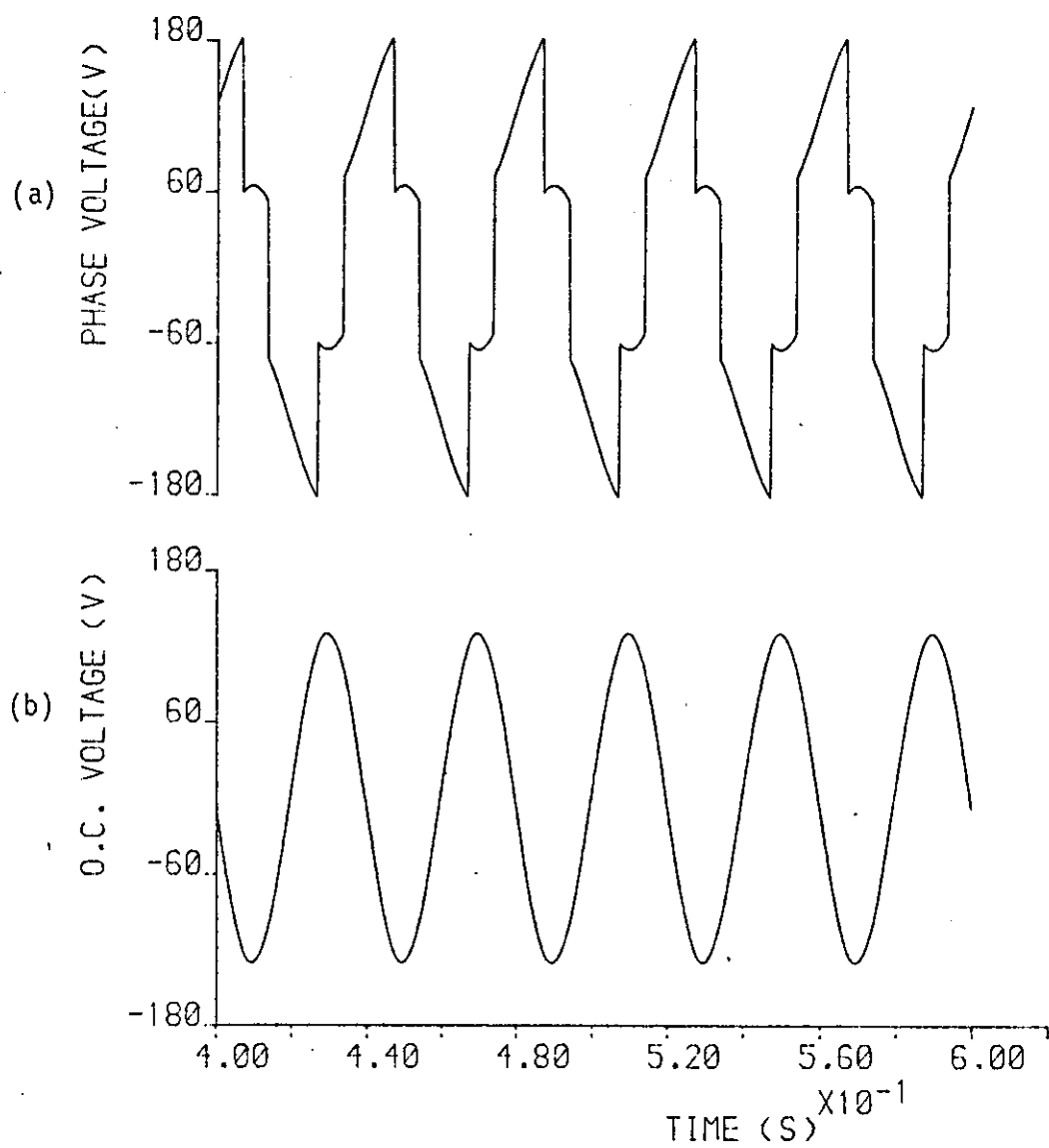


FIGURE 5.50 Computed results for open circuit fault on phase C of the experimental motor supplied from 180° inverter

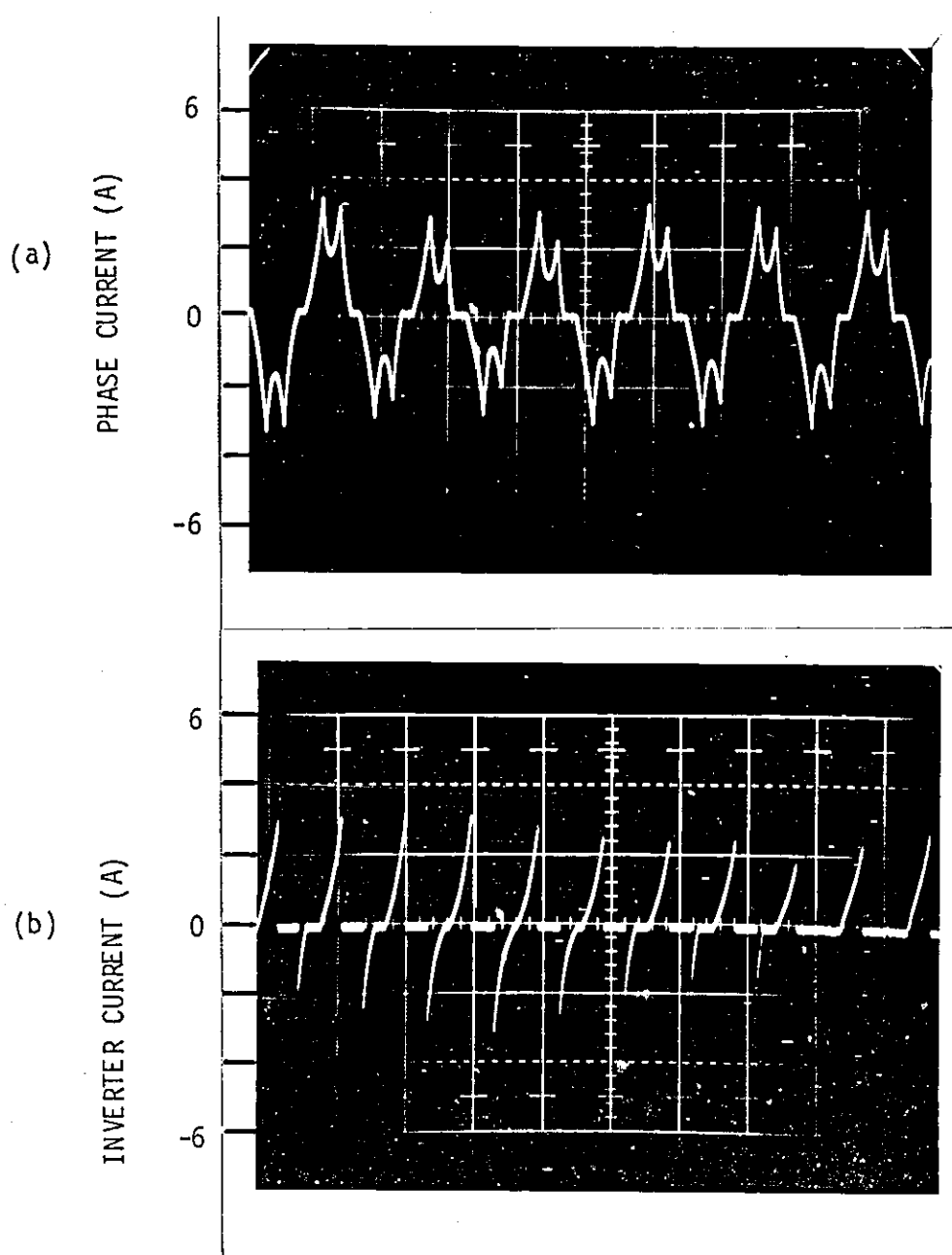


FIGURE 5.51 Experimental results for open circuit fault on phase C of the experimental motor supplied from 1200 inverter

Scale: current 2A/div
time 20 ms/div

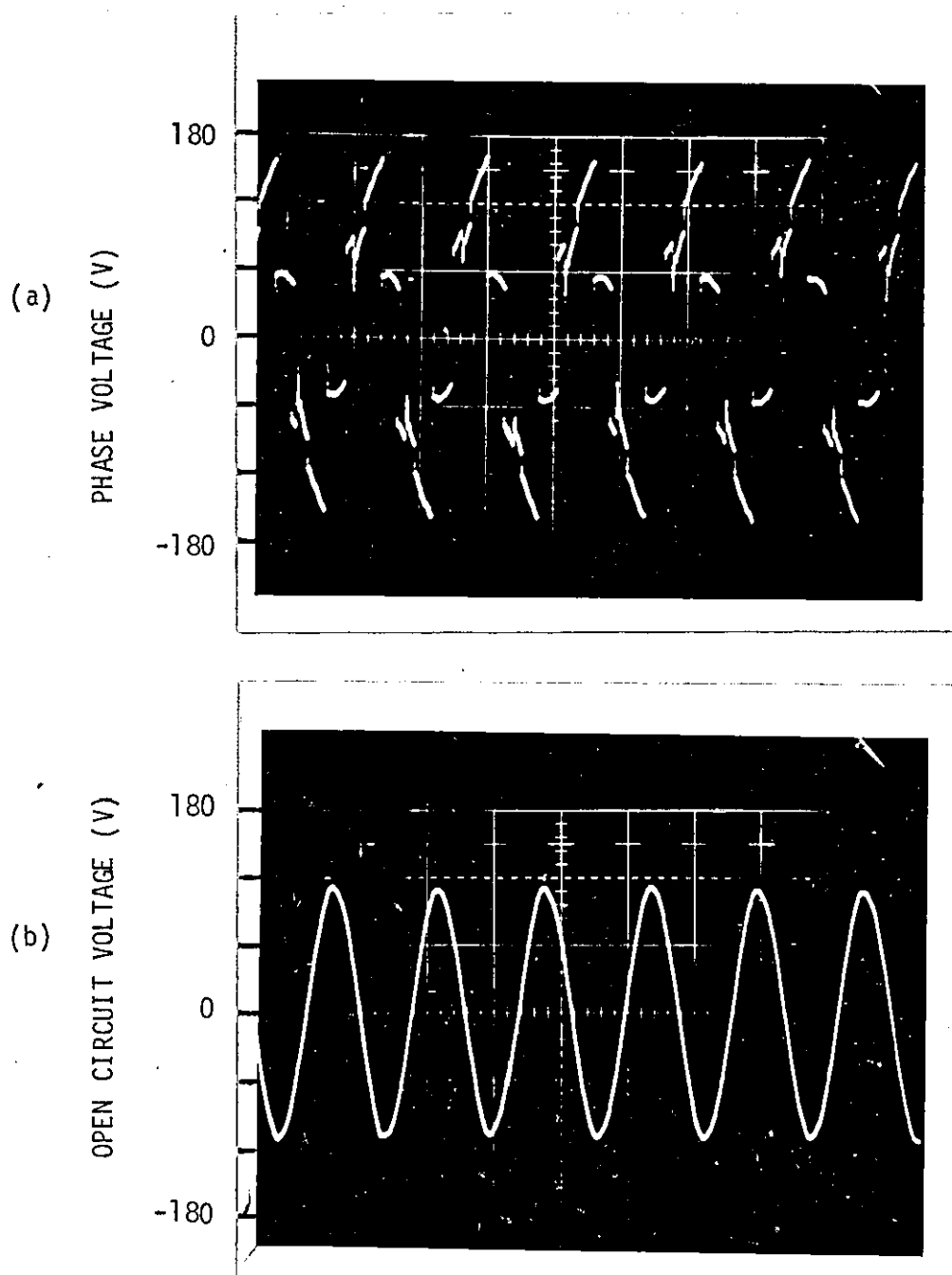


FIGURE 5.52 Experimental results for open circuit fault on phase C of the experimental motor supplied from 120° inverter

Scale: voltage 60V/div
time 20 ms/div

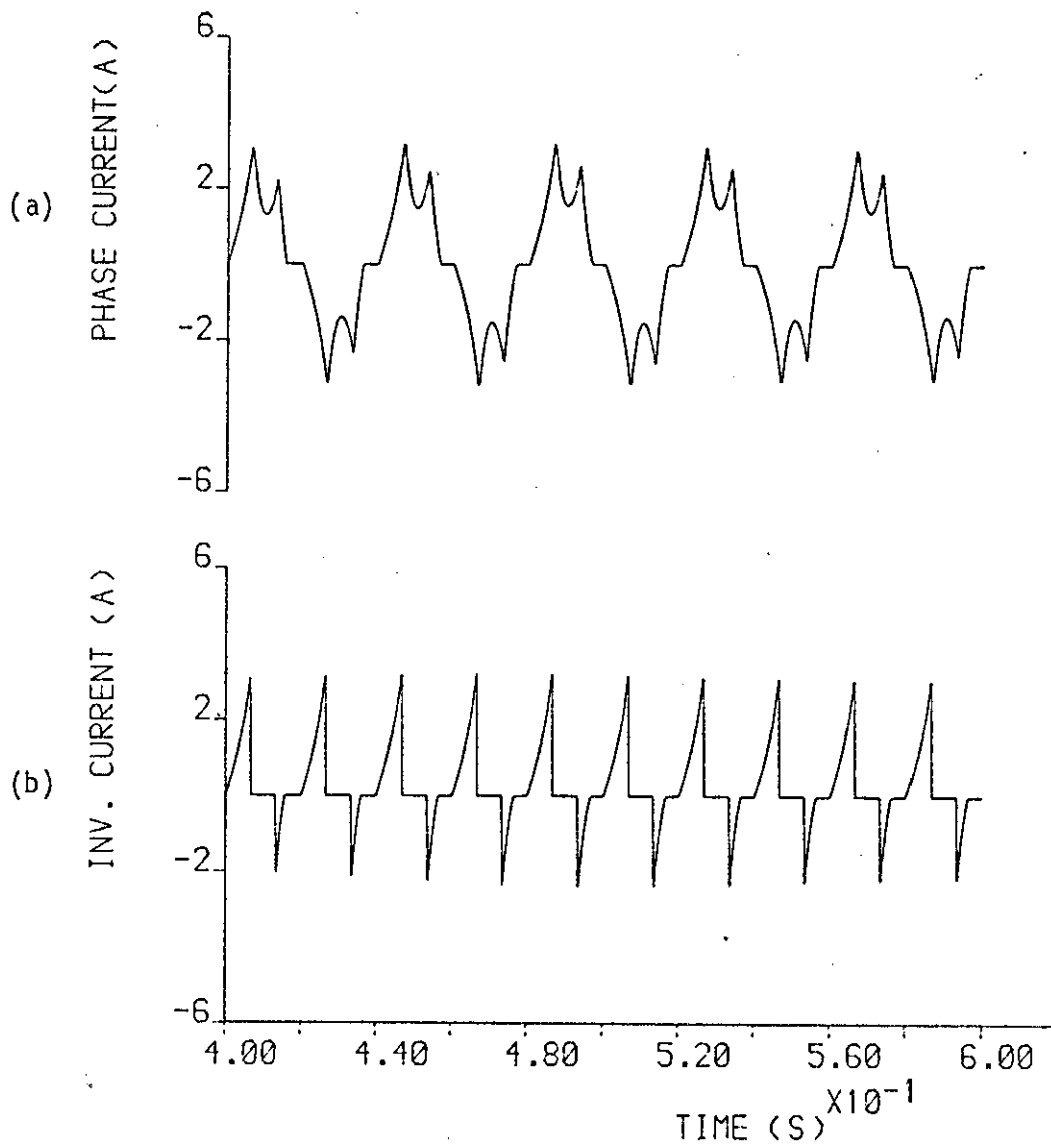


FIGURE 5.53 Computed results for open circuit fault on phase C of the experimental motor supplied from 120° inverter

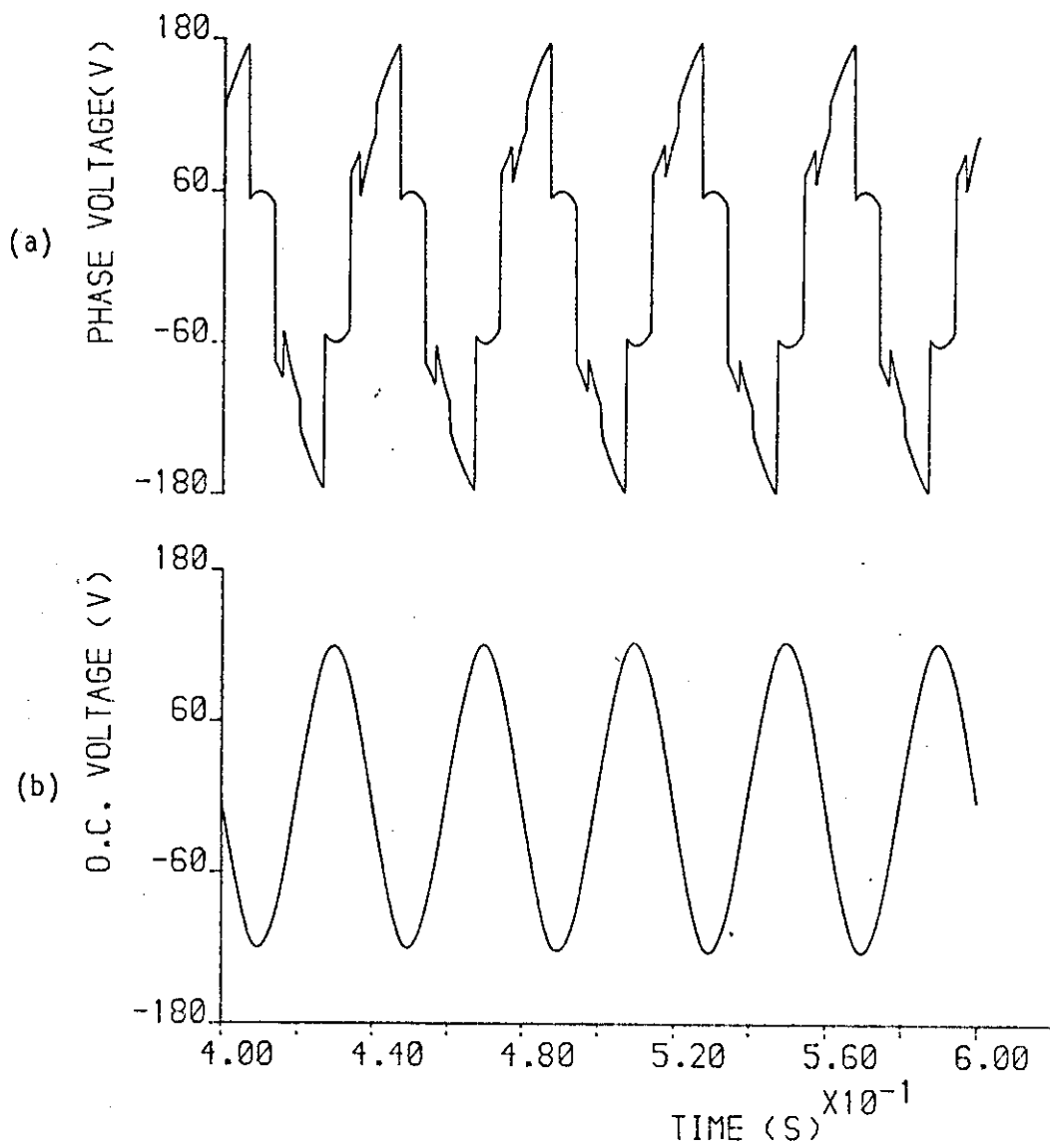


FIGURE 5.54 Computed results for open circuit fault on phase C of the experimental motor supplied from 120° inverter

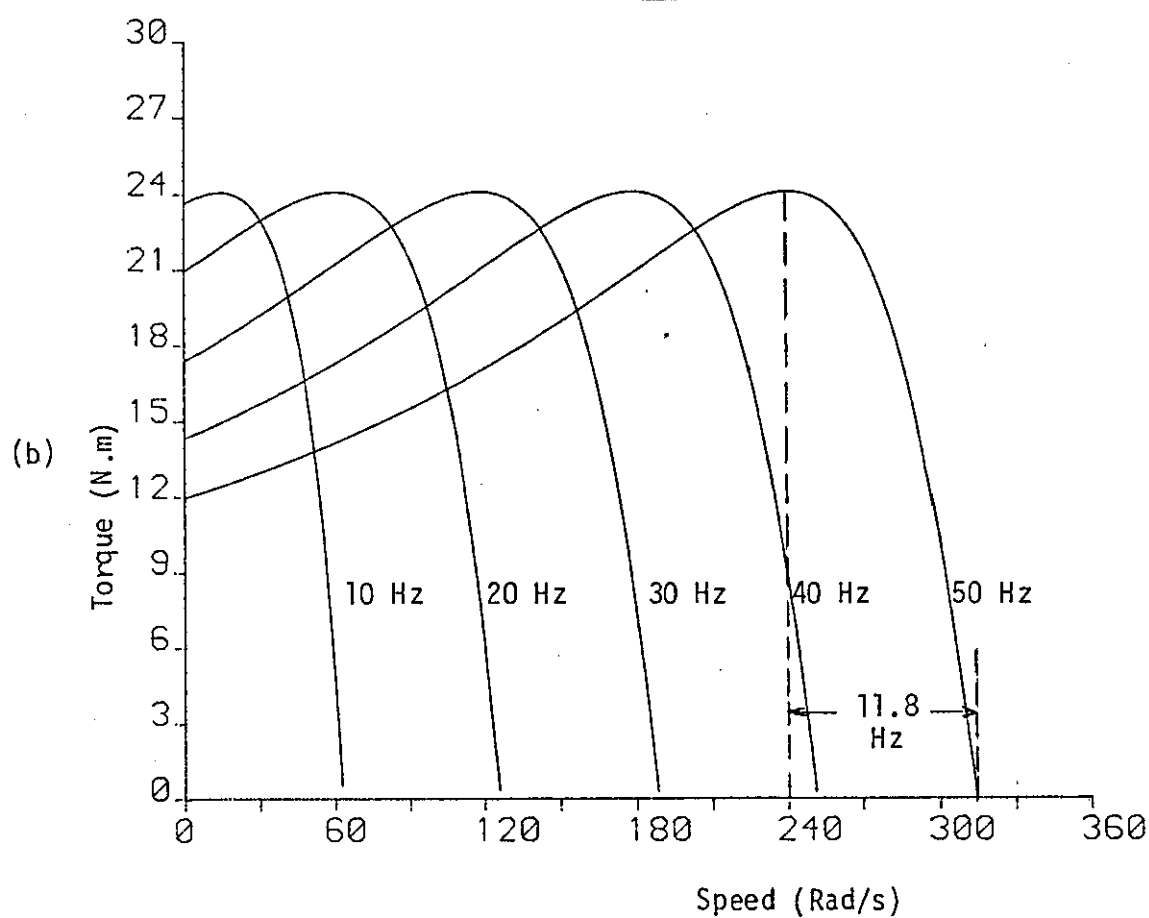
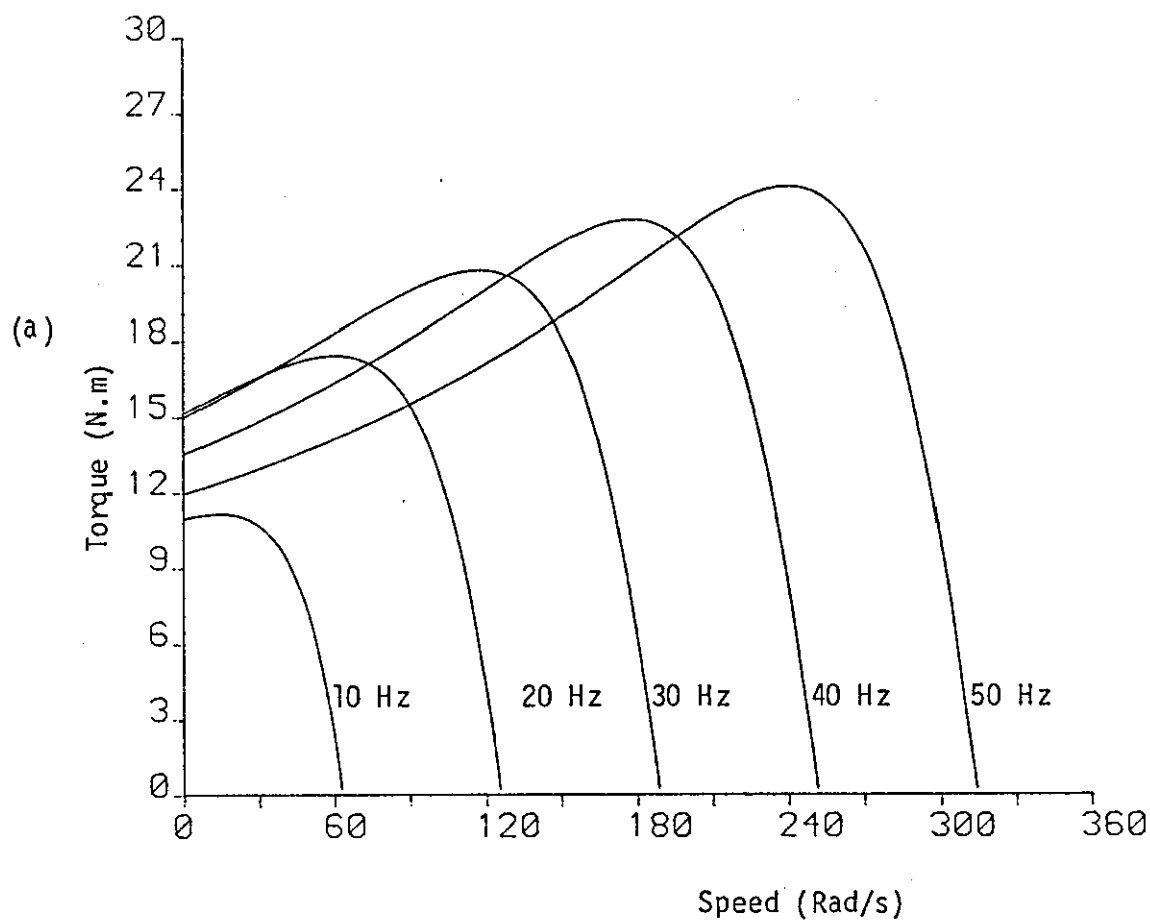


FIGURE 6.1 Steady state torque speed curves for the experimental motor,
 (a) constant supply voltage to frequency ratio
 (b) constant airgap flux

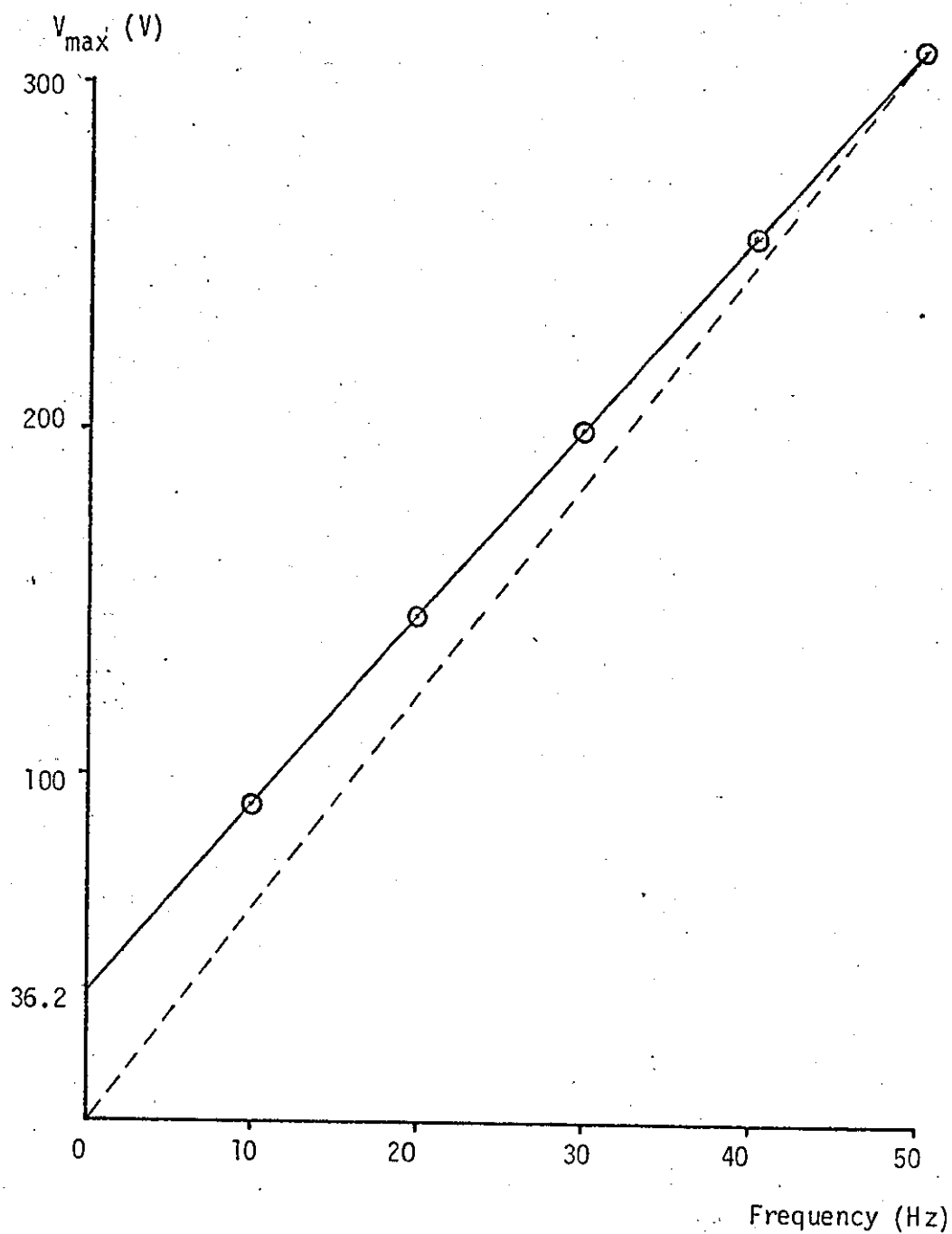


FIGURE 6.2 Graphs relating the maximum input voltage and frequency for the experimental motor

- constant voltage to frequency ratio
- constant airgap flux

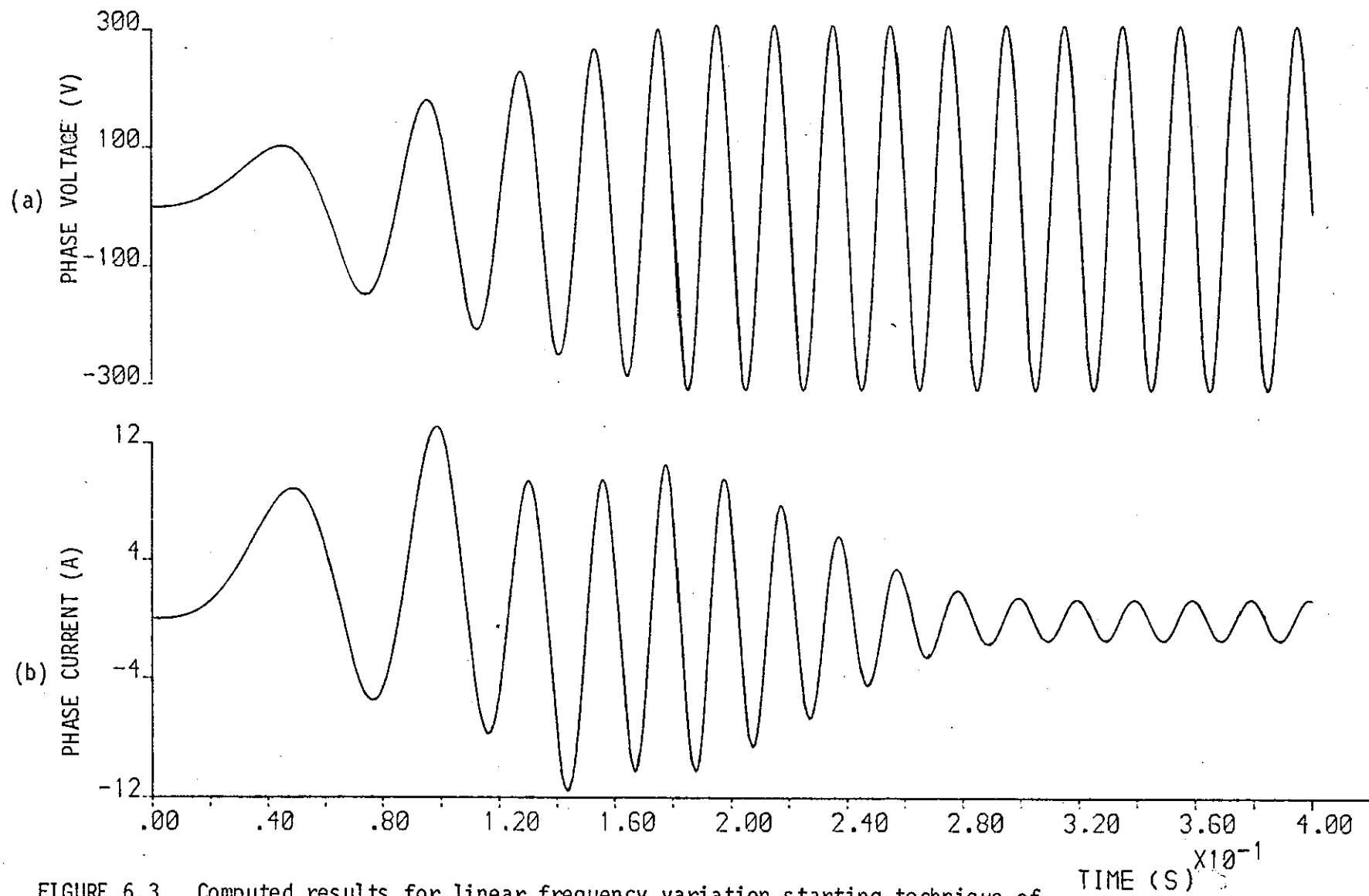


FIGURE 6.3 Computed results for linear frequency variation starting technique of the experimental motor-sine wave input-frequency increased from zero to 50 Hz

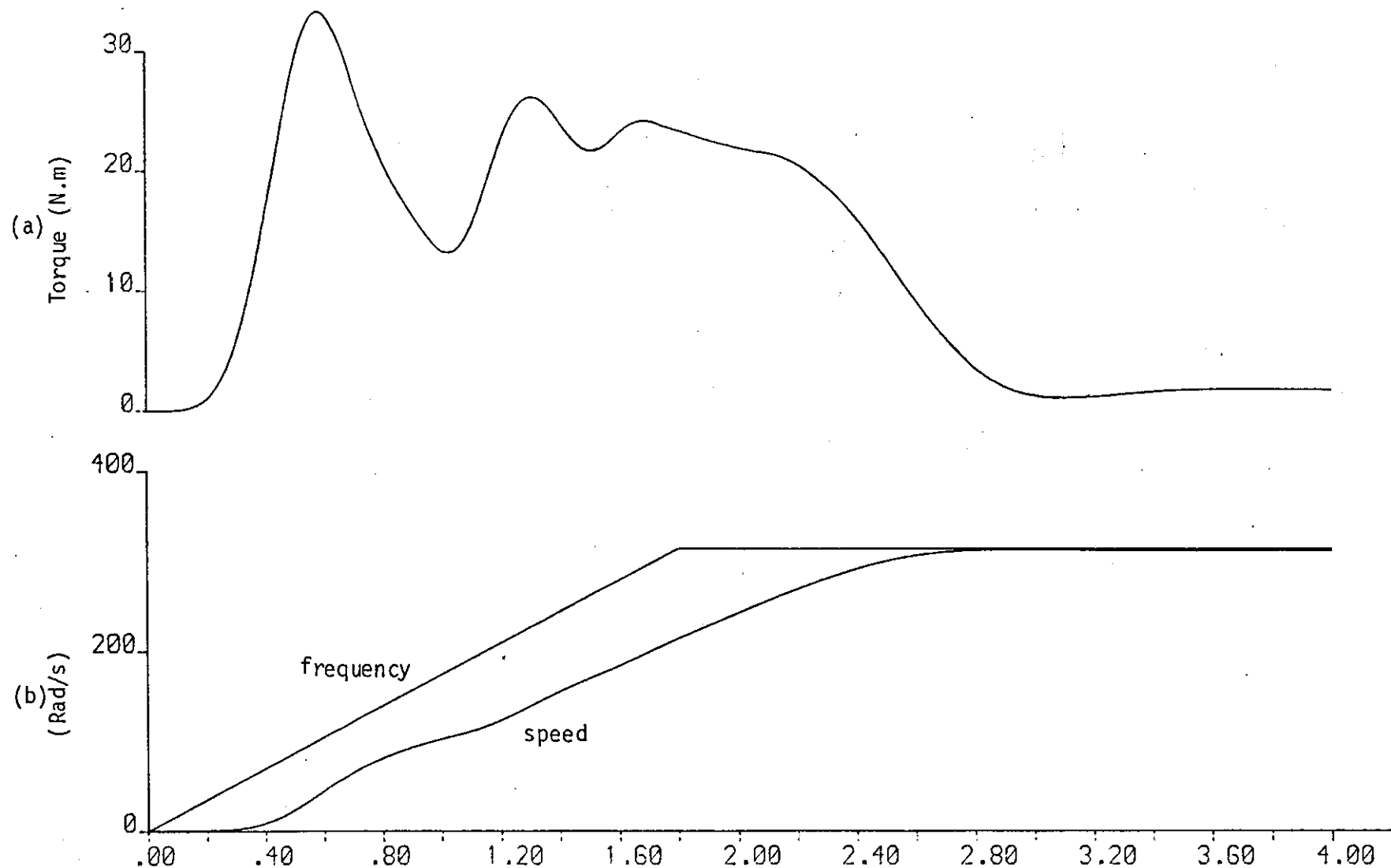


FIGURE 6.4 Computed results for linear frequency variation starting technique of the experimental motor - sine wave input frequency increased from zero to 50 Hz

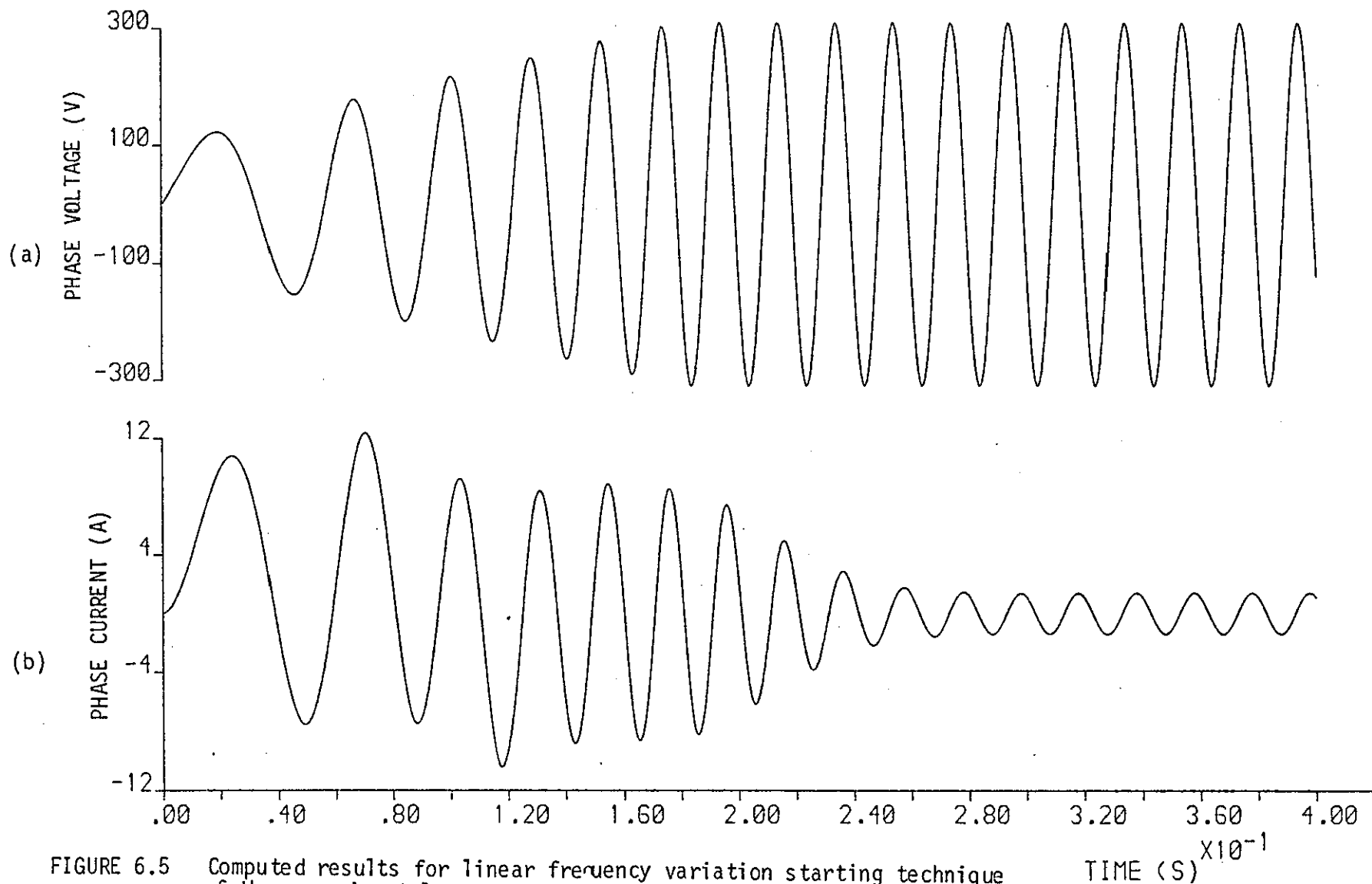


FIGURE 6.5 Computed results for linear frequency variation starting technique of the experimental motor-sine wave input-frequency increased from 11.8 to 50 Hz.

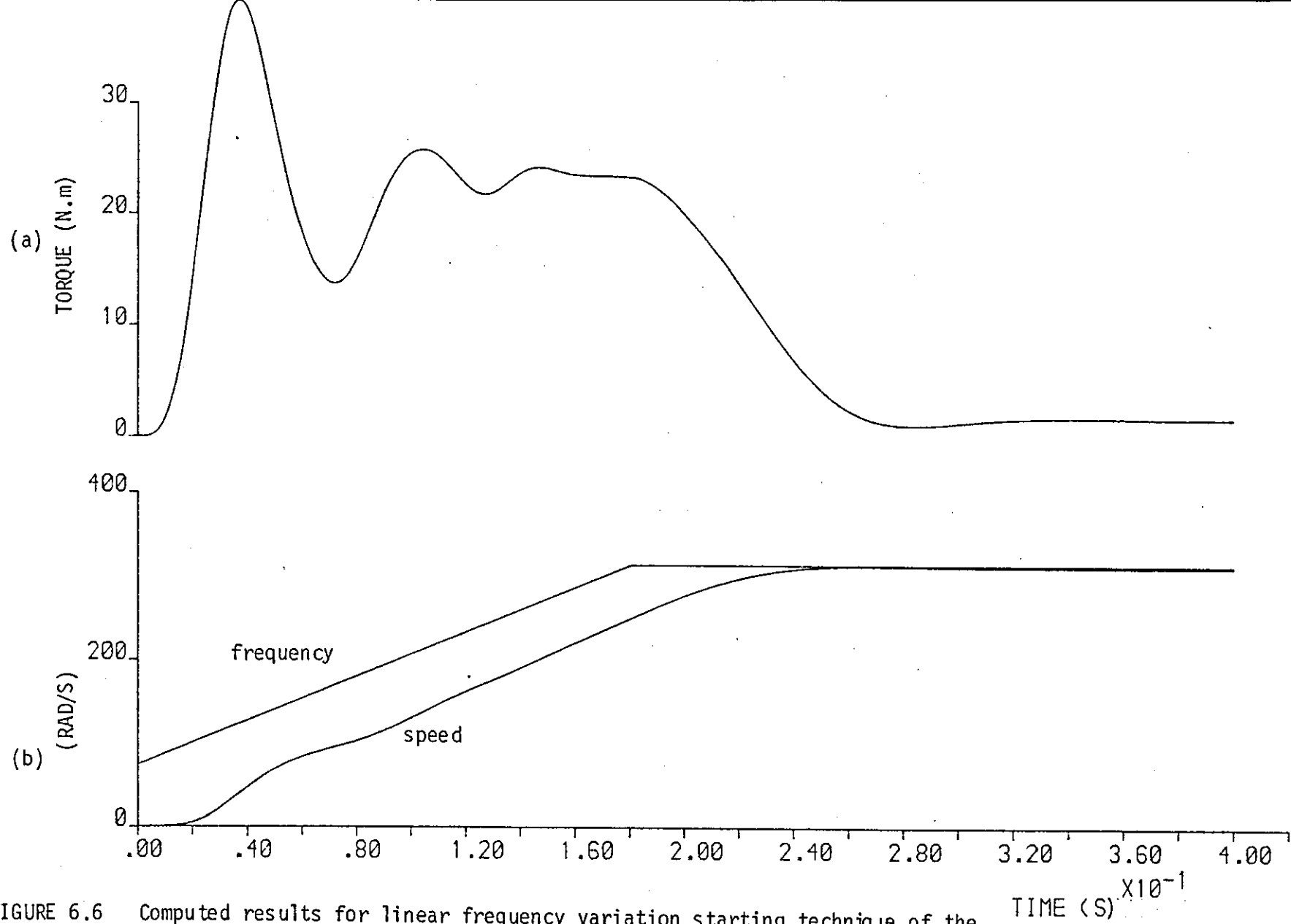


FIGURE 6.6 Computed results for linear frequency variation starting technique of the experimental motor-sine wave input-frequency increased from 11.8 to 50 Hz.

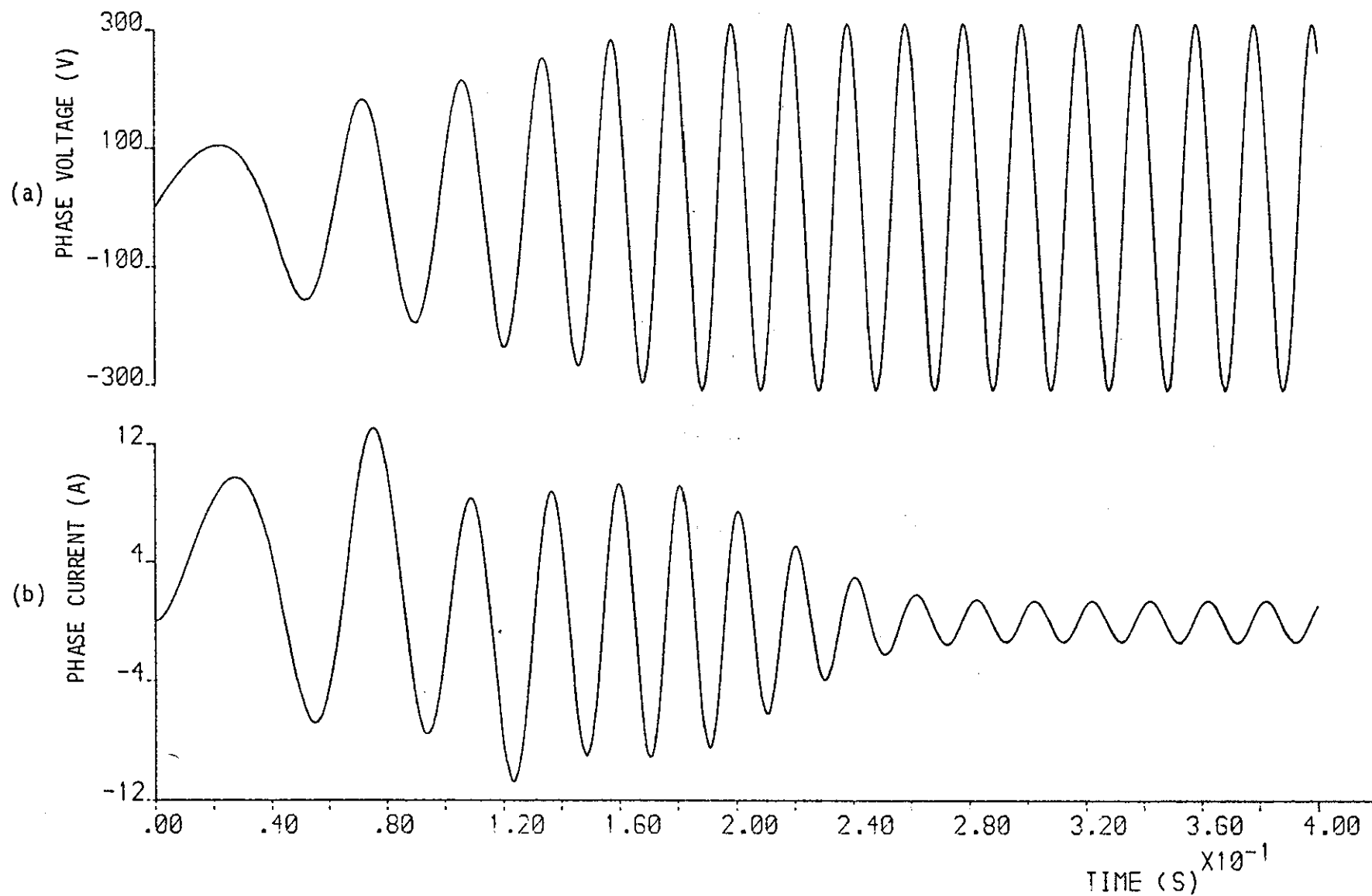


FIGURE 6.7 Computed results for constant slip frequency starting technique of the experimental motor-sine wave input-final frequency, 50 Hz

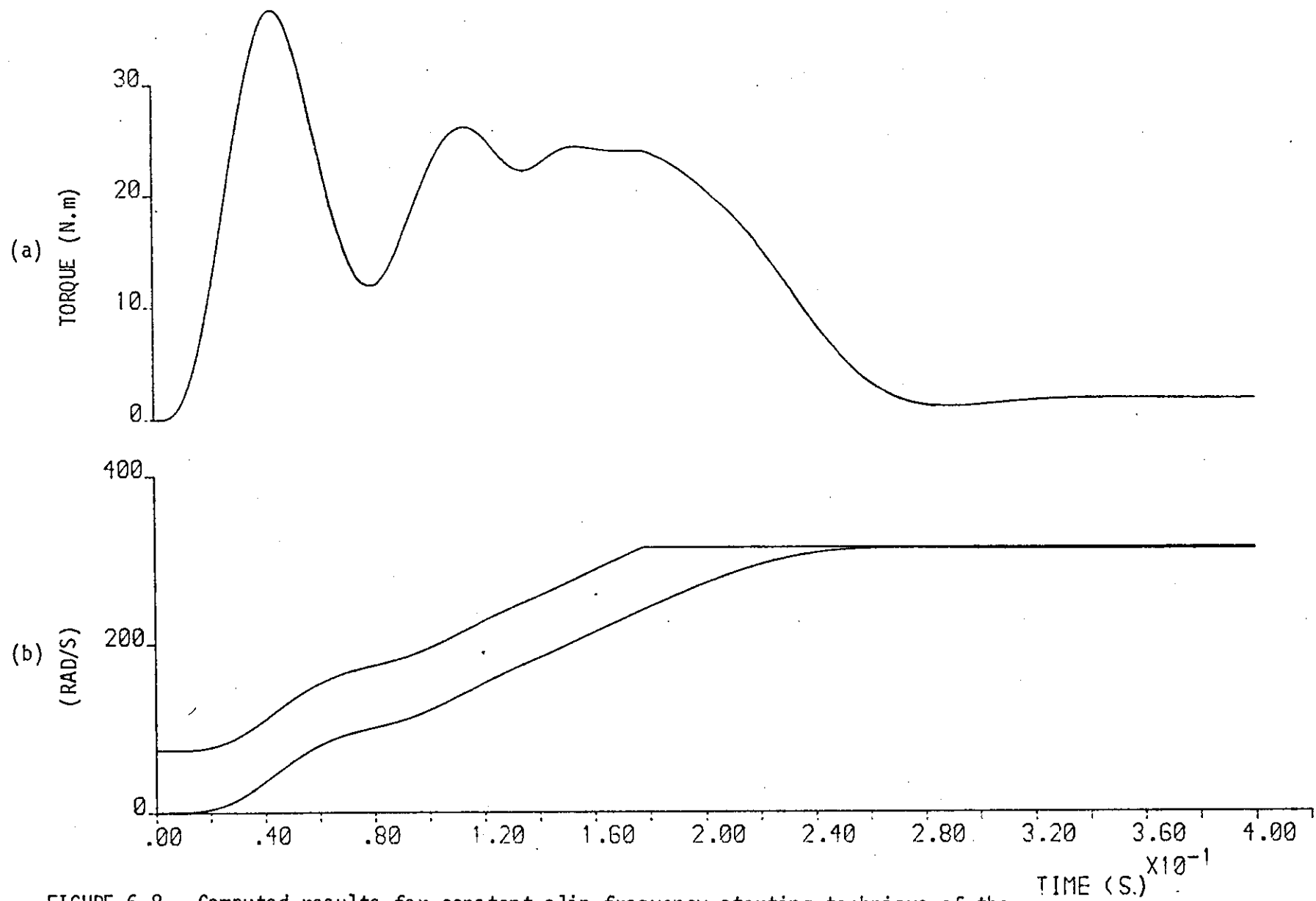


FIGURE 6.8 Computed results for constant slip frequency starting technique of the experimental motor-sine wave input-final frequency 50 Hz

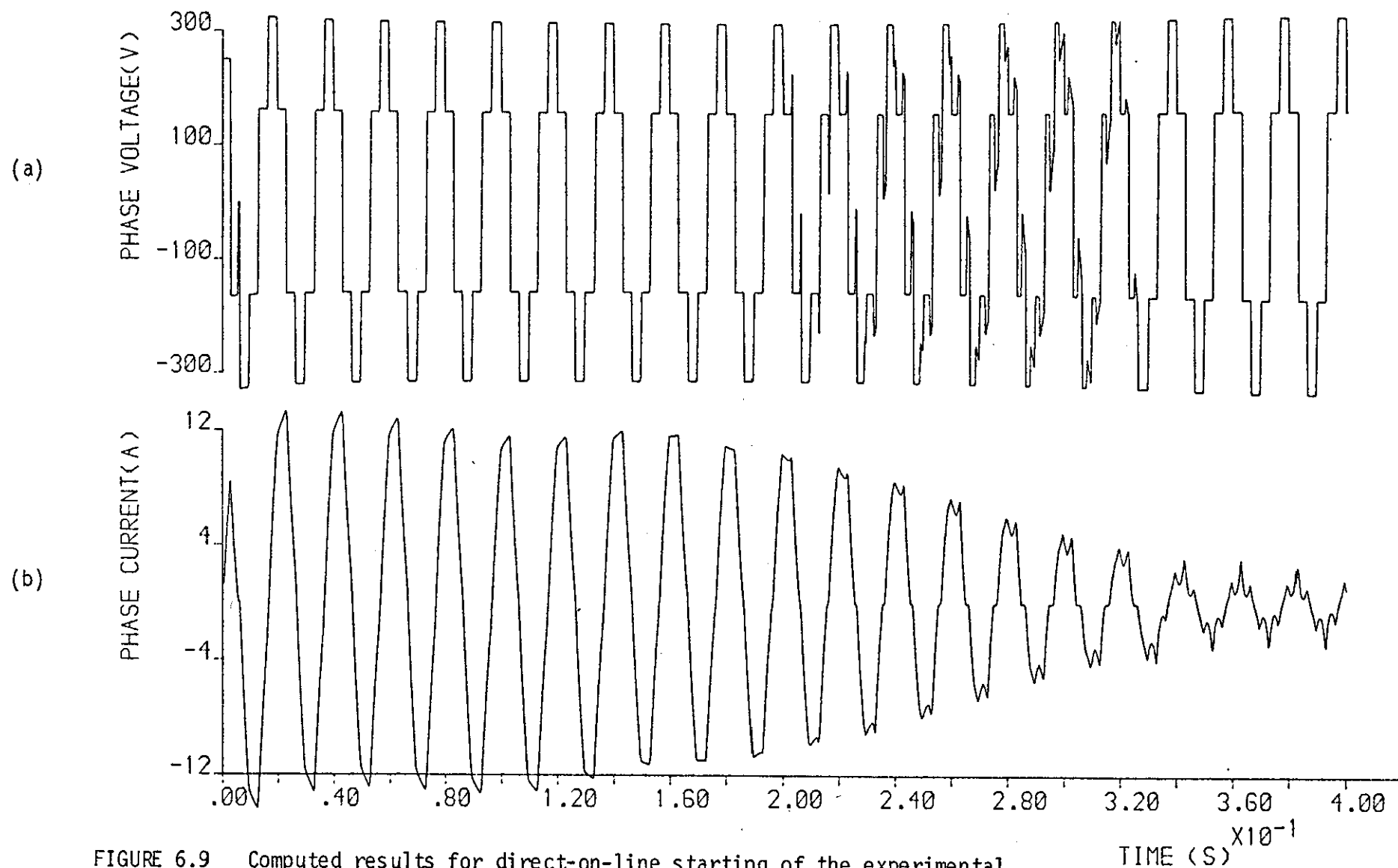


FIGURE 6.9 Computed results for direct-on-line starting of the experimental motor supplied from 120° inverter at 50 Hz

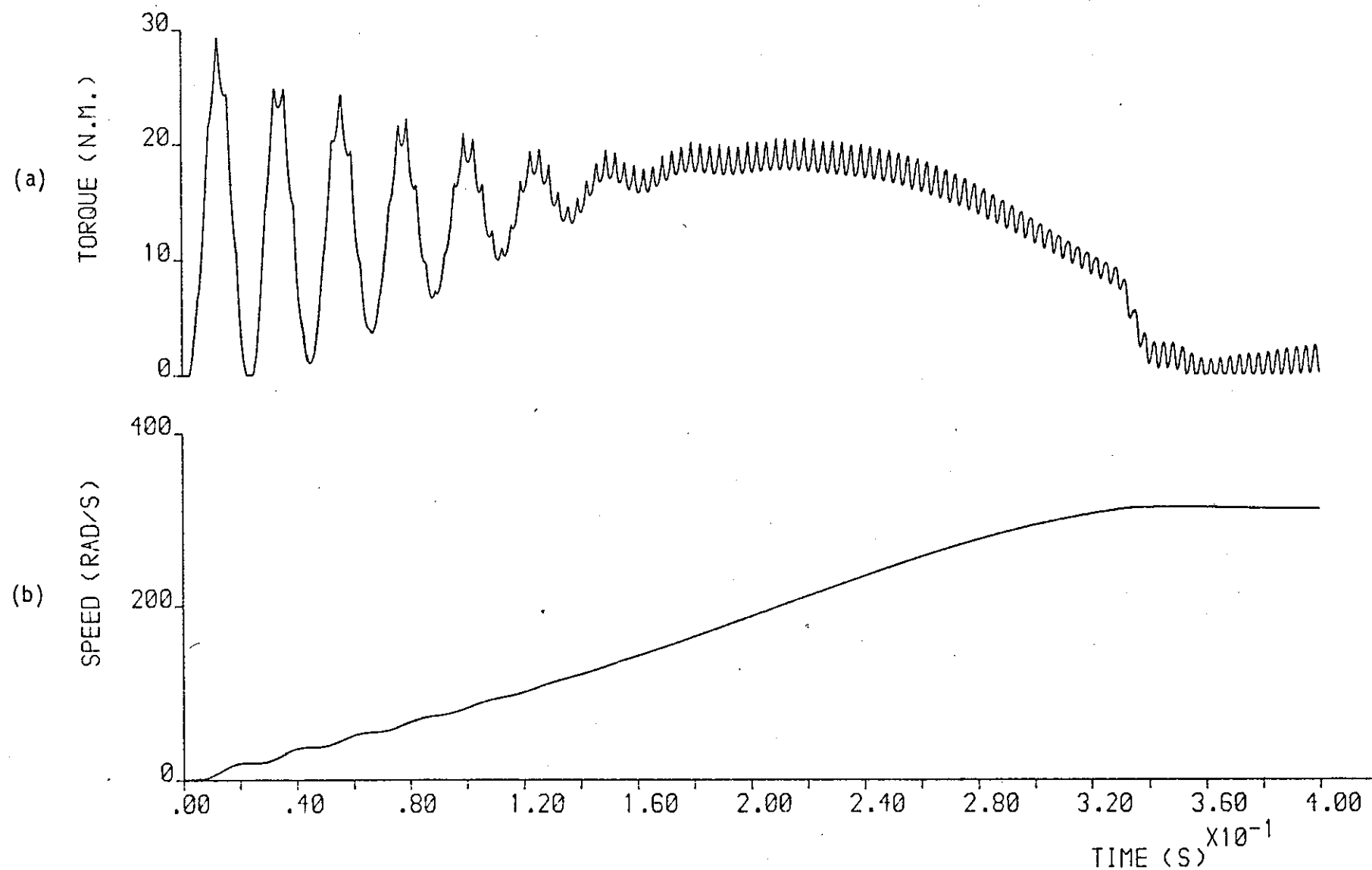


FIGURE 6.10 Computed results for direct-on-line starting of the experimental motor supplied from 120° inverter at 50 Hz

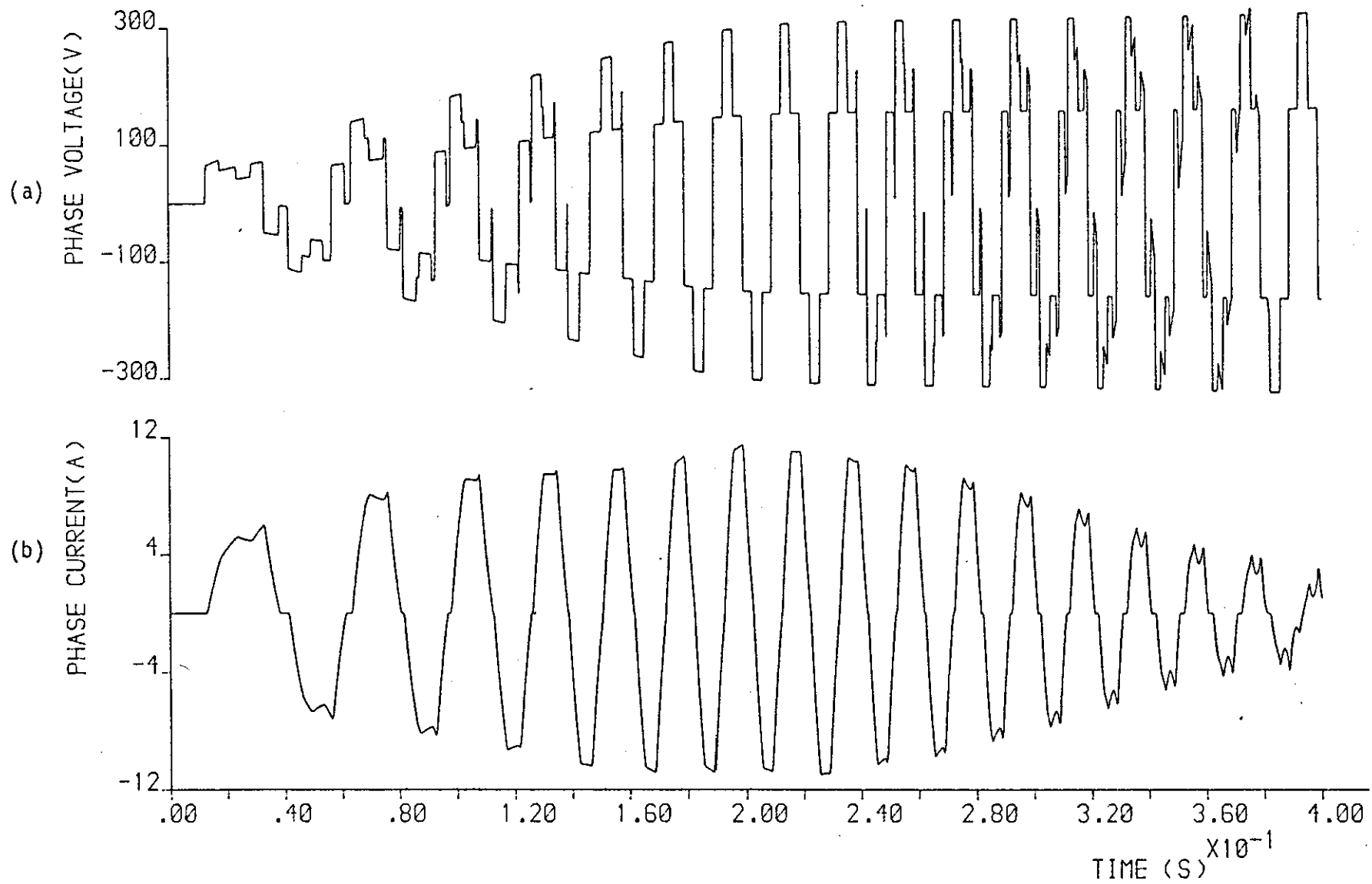


FIGURE 6.11 Computed results for linear frequency variation starting technique of the experimental motor supplied from 120° inverter—frequency increased from 11.8 to 50 Hz

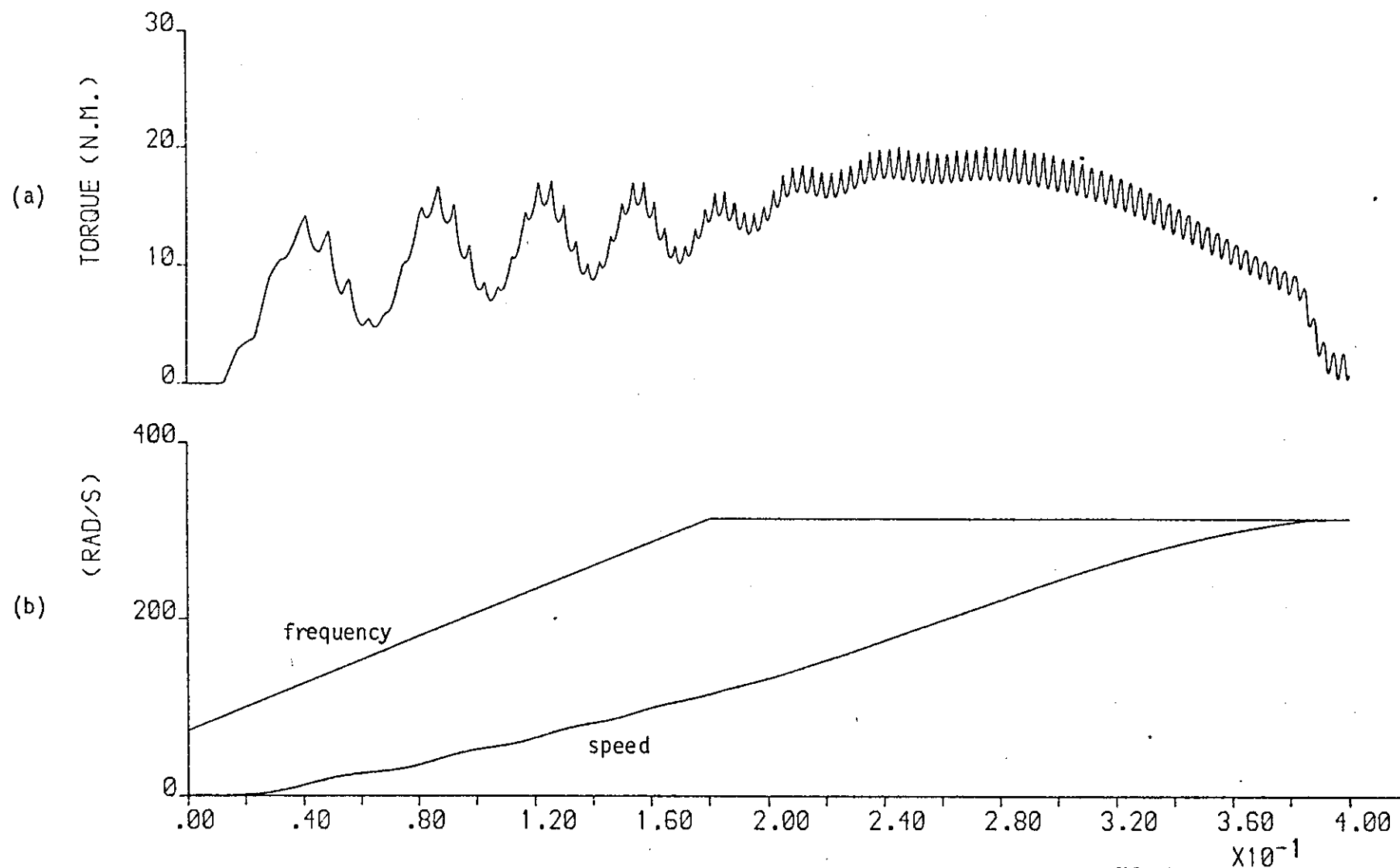


FIGURE 6.12 Computed results for linear frequency variation starting technique of the experimental motor supplied from 120° inverter-frequency increased from 11.8 to 50 Hz

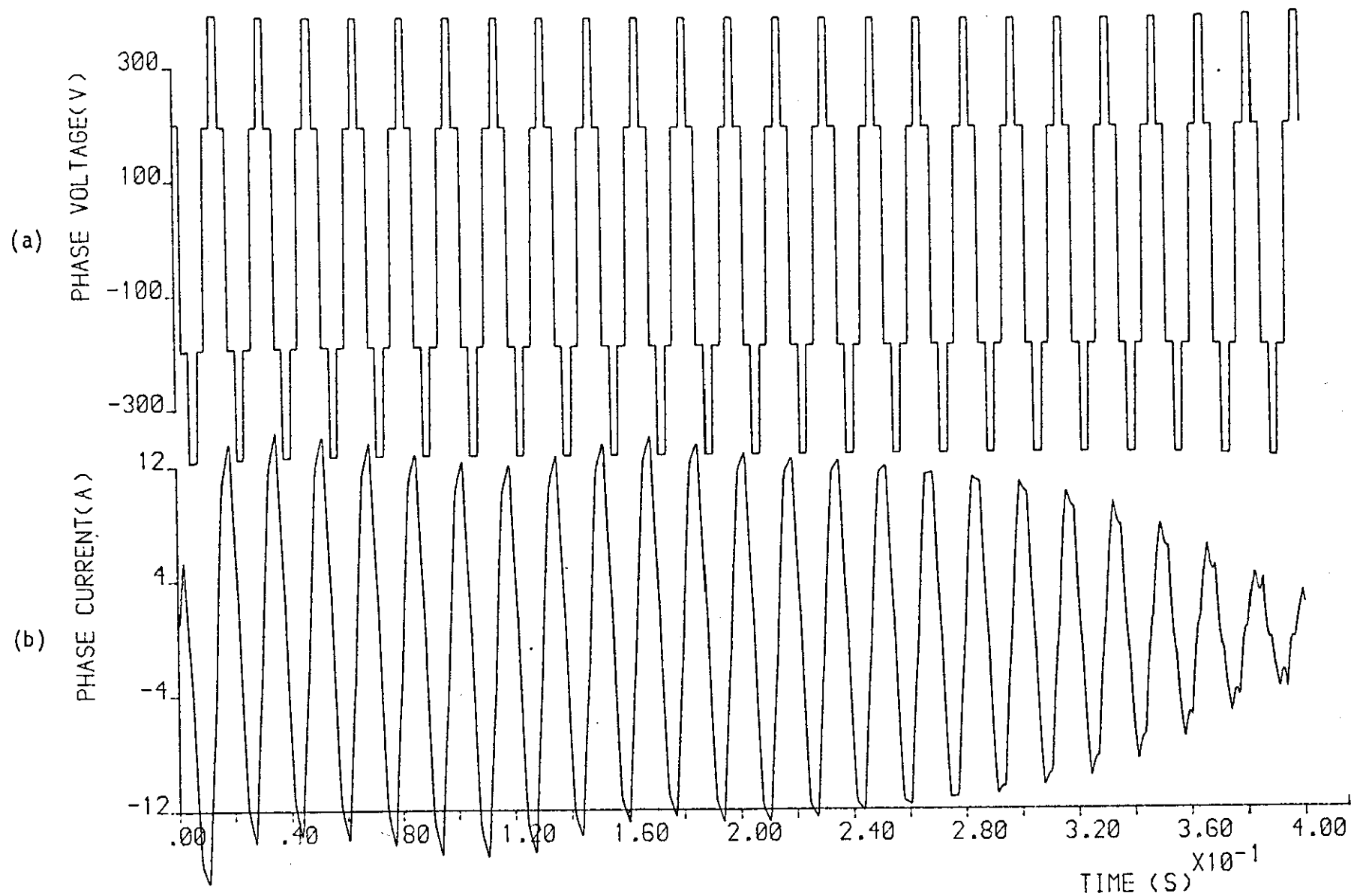


FIGURE 6.13 Computed results for direct-on-line starting of the experimental motor supplied from 1800 inverter at 60 Hz

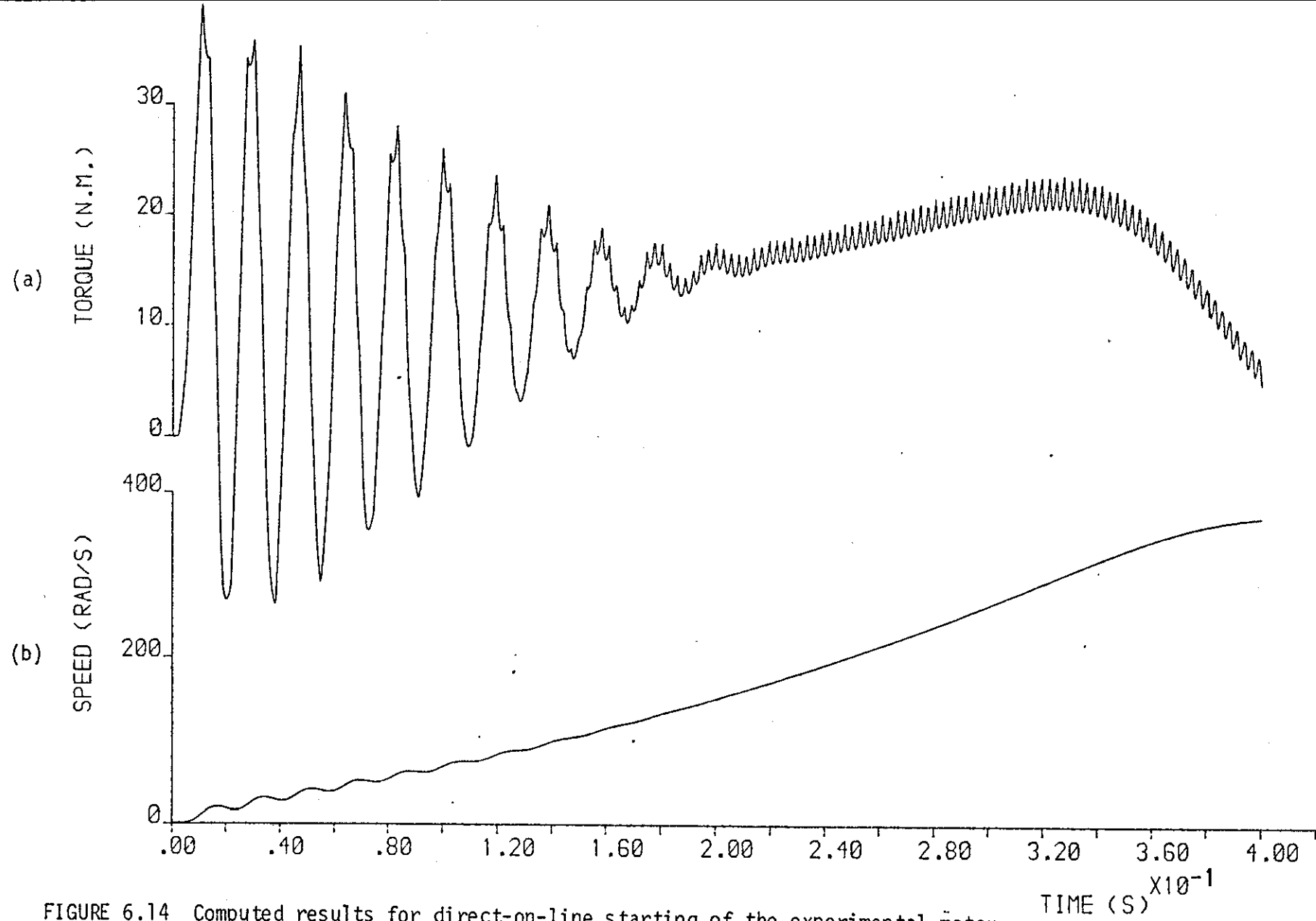


FIGURE 6.14 Computed results for direct-on-line starting of the experimental motor supplied from 180° inverter at 60 Hz

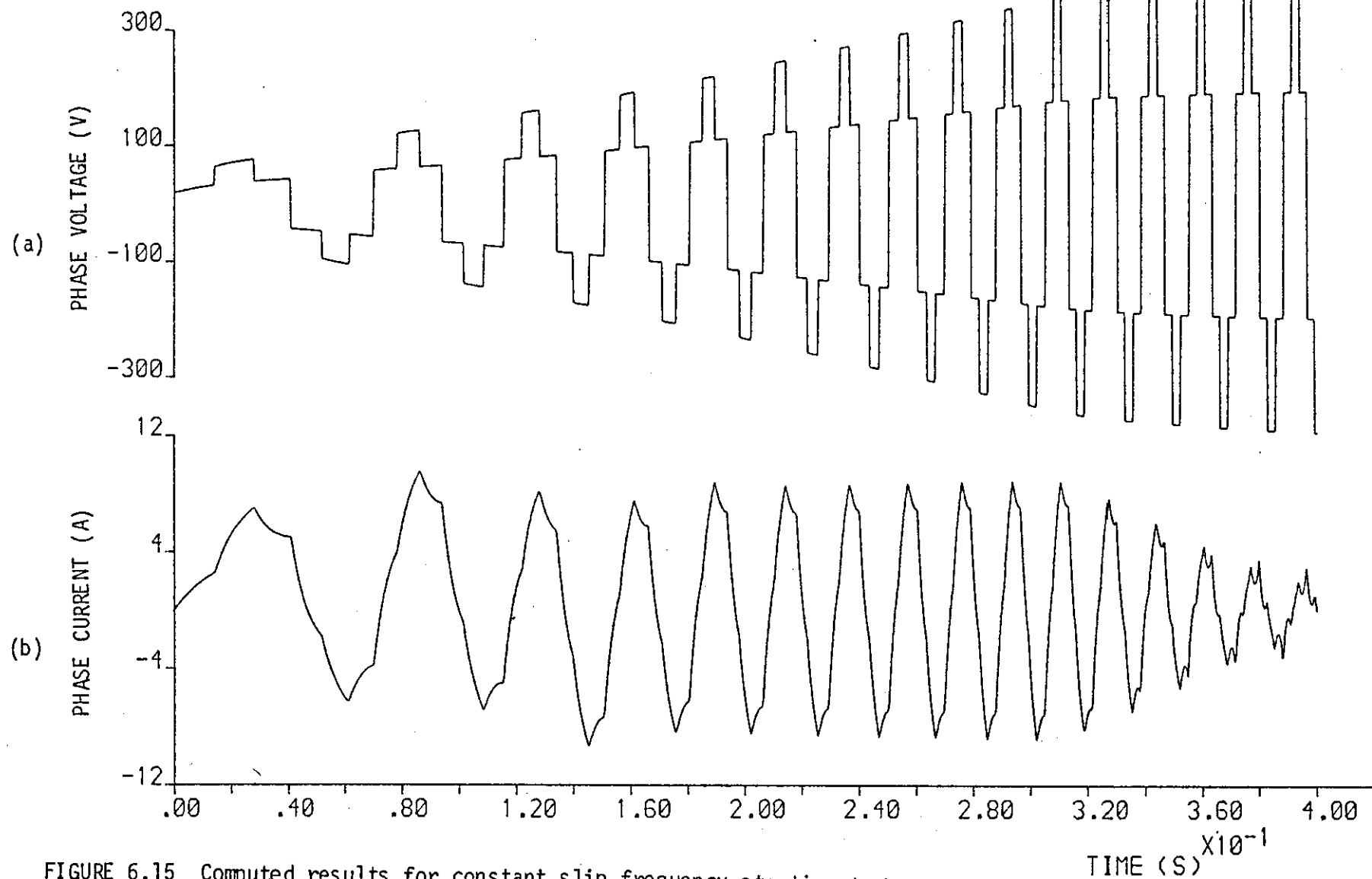


FIGURE 6.15 Computed results for constant slip frequency starting technique of the experimental motor supplied from 180° inverter-final frequency 60 Hz

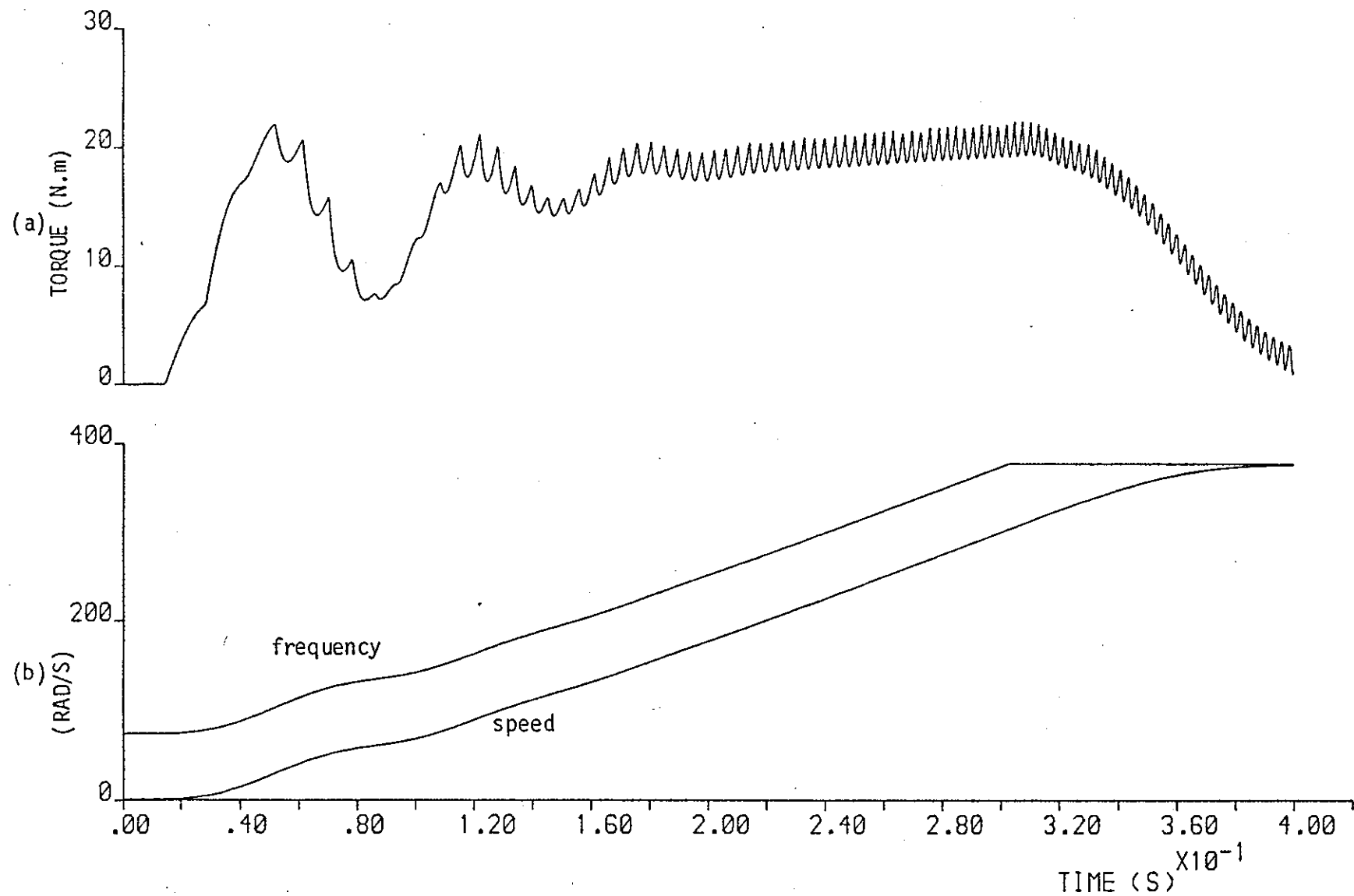


FIGURE 6.16 Computed results for constant slip frequency starting technique of the experimental motor supplied from 1800 inverter-final frequency 60 Hz

APPENDICES

APPENDIX A

COMPUTER PROGRAMS FOR THE ANALYSIS OF THE INVERTER SYSTEM

A general simplified flow chart for the computer programs INVERTER1, INVERTER2, and INVERTER3 is shown in Figure A.1, followed by the program listings. The variable names used in these programs have been chosen to be as far as possible those used throughout the analysis. The solution of the system equations is obtained from the NAGF library routine D02AAF which is based on the Runge-Kutta-Merson method of numerical integration. At each step of the solution, a special subroutine CHECK is called to test nonconducting devices for turn on and conducting devices for turn off, and if there is any change in the conduction states, the system equations are changed accordingly. When employing the tensor technique another special subroutine CONN1 is called to generate the required transformation tensor for the new system topology.

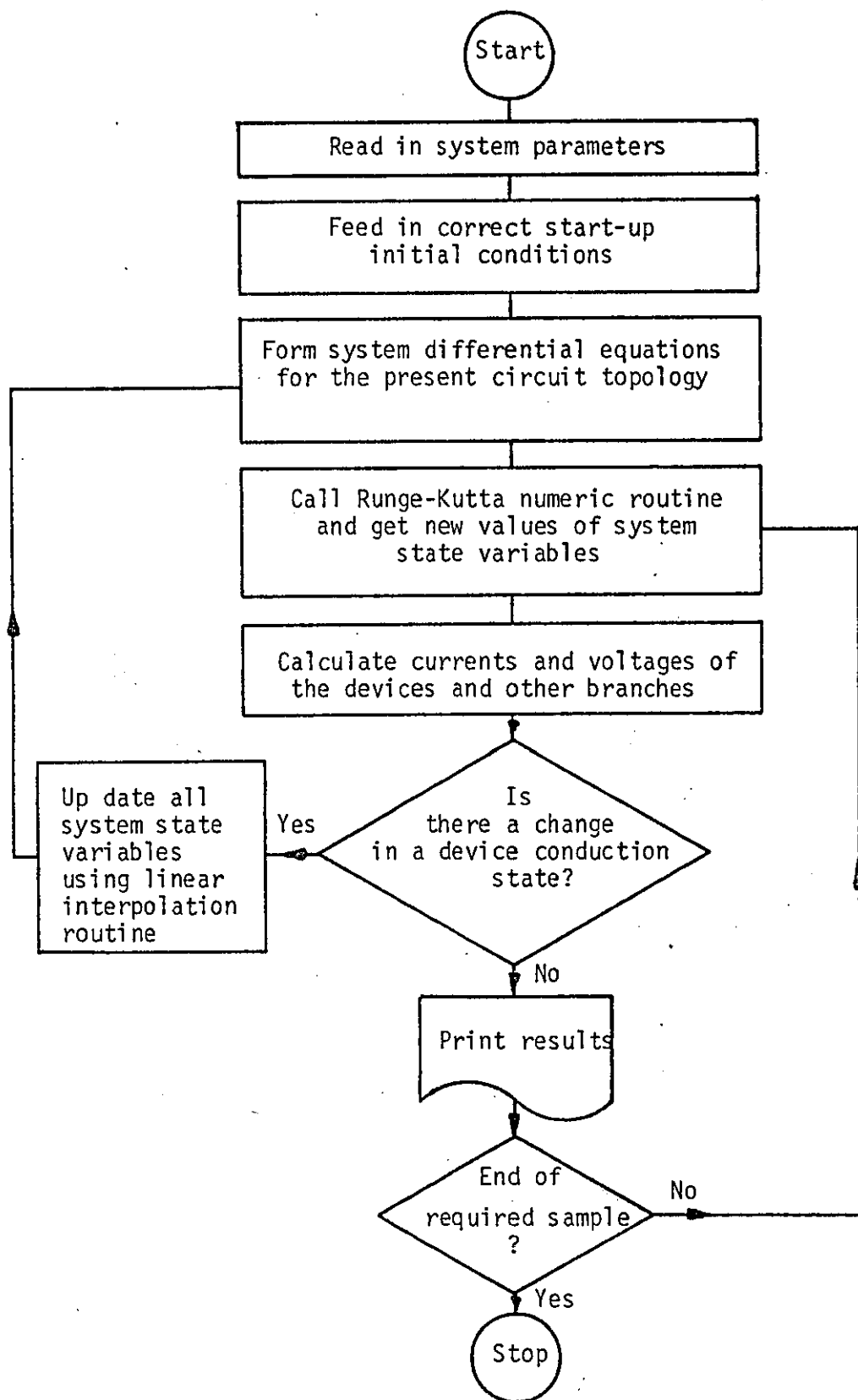


FIGURE A.1 The general flow chart for the inverter system analysis

```

C -----
C PROGRAM INVERTER 1
C ANALYSIS OF THE 3-PHASE BRIDGE INVERTER
C USING MODEL SUBROUTINE METHOD
C -----
REAL LF,LA,LB,LC,LALB,LALC,LBLC,LALBLC
DIMENSION X(5),XP(5),ERR(4),A(4),B(4),C(4),D(4)
DIMENSION ISS(6),IT(6),TT(6),ITHY(6),LABEL(18)
DIMENSION CTHY(6),CTHYP(6),VTHY(6)
COMMON /BLOCK1/RF,LF,RSH,CSH,RA,RB,RC,LA,LB,LC
& /LALB,LALC,LBLC,LALBLC
& /RARB,RARC,RBRC,SSH,RCSH
& /E,II
COMMON /BLOCK2/EPS
COMMON /BLOCK3/T,ST,ANGLE,ALP6,TT2,TT,IT,ITHY,IITHY,NEQ
EXTERNAL AUX1
DATA LABEL/37,25,22,46,38,41,36,33,24,9,18,6
& ,40,20,34,17,10,5/
EPS=.1E-10
T=0.
C READ IN CIRCUIT PARAMETERS
READ(2,1)RF,LF,RSH,CSH,RA,RB,RC,LA,LB,LC
1 FORMAT(10F0.0)
READ(2,2)E,FREQ,STEP,Tf1,INVRT
& 2 FORMAT(4F0.0,10)
ST=STEP
ALP=1./FREQ
ALP6=ALP/6.
ANGLE=ALP6*(INVRT+1)
SSH=1./CSH
RCSH=RSH*CSH
LALB=LA+LB
LALC=LA+LC
LBLC=LB+LC
LALBLC=LA*LB+LA*LC+LB*LC
RARB=RA+RB
RARC=RA+RC
RBRC=RB+RC
NEQ=4
C READ IN THE CORRECT START UP INITIAL CONDITIONS
ISS(1),ISS(4),ISS(6)=1
IT(1),IT(4)=1
TT(4)=ALP6
ITHY(1)=6
ITHY(2)=5
ITHY(3)=2
ITHY(4)=5
ITHY(5)=4
ITHY(6)=1
IITHY=1
X(3)=0.
X(4)=-E/(RF+RB+RC)
X(1)=-X(4)
X(2)=E
X(5)=-X(3)-X(4)

```

```

C      MAIN LOOP
      NEQ=5
100    CONTINUE
502    KK=ISS(6)+2*ISS(5)+4*ISS(4)+8*ISS(3)+16*ISS(2)+32*ISS(1)
      DO 504 I=1,18
      IF(KK.EQ.LABEL(I))GO TO 500
504    CONTINUE
      WRITE(3,505)ISS
505    FORMAT(1X,6I1,' INVALID TOPOLOGY')
      GO TO 1000
500    DO 12 I=1,5
12     XP(I)=X(I)
      IF(ABS(ST).LE.EPS)GO TO 501
C      CALL THE RUNGE-KUTTA INTEGRATION ROUTINE
      CALL DD2AAF(X,ERR,T,ST,4,AUX1,A,B,C,D)
      X(5)=-X(3)-X(4)
501    CONTINUE
C      CALCULATE THYRISTOR/DIODE CURRENTS AND VOLTAGES
      DO 32 I=1,6
      IF(ISS(I))33,33,0
      K=3+I/3+I/5-1/6
      CTHY(I)=X(K)
      CTHYP(I)=XP(K)
      VTHY(I)=0.
      GO TO 32
33     CTHY(I)=0.
      CTHYP(I)=0.
      VTHY(I)=X(2)
32     CONTINUE
      J=0
      CALL CHECK(X,XP,CTHY,CTHYP,VTHY,ISS,J)
      IF(J)50,50,100
50     DO 31 I=1,6
31     TT(I)=IT(I)+(TT(I)+ST)
      TT2=TT2+ST
      ST=STEP
C      CALCULATE LOAD CURRENT AND VOLTAGE
C      PRINT RESULTS
      DELT=X(3)-XP(3)
      VOLT=X(3)*RA+DELT/ST*LA
      WRITE(3,6)ISS,T,X,VOLT
6      FORMAT(1X,6I1,F7.4,9F11.5)
      IF(T-TFI)500,500,0
1000   STOP
      END

```

```

SUBROUTINE AUX1(F,Z,T)
REAL LS,LA,LB,LC,LALB,LALC,LBLC,LALBLC
DIMENSION F(4),Z(4)
DIMENSION LABEL(18)
COMMON /BLOCK1/RS,LS,RSH,CSH,RA,RB,RC,LA,LB,LC
& ,LALB,LALC,LBLC,LALBLC
& ,RARB,RARC,RBRC,SSH,RSHG
& ,E,FI
COMMON /BLOCK2/EPS
GO TO (101,102,103,104,105,106,107,108,109,110
& ,111,112,113,114,115,116,117,118),FI
101 F(2)=(Z(1)-Z(3))*SSH
F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
F(3)=LBLC*(E-RS*Z(1)-LS*F(1)-RA*Z(3)+RB*Z(4)-LB/LBLC*
+ (RB*Z(4)+RC*Z(3)+RC*Z(4)))/LALBLC
F(4)=- (RB*Z(4)+RC*Z(3)+RC*Z(4)+LC*F(3))/LBLC
RETURN
102 F(2)=(Z(1)-Z(4))*SSH
F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
F(4)=LALC*(E-RS*Z(1)-LS*F(1)-RB*Z(4)+RA*Z(3)-LA/LALC*
+ (RA*Z(3)+RC*Z(4)+RC*Z(3)))/LALBLC
F(3)=- (RA*Z(3)+RC*Z(4)+RC*Z(3)+LC*F(4))/LALC
RETURN
103 F(2)=(Z(1)+Z(3)+Z(4))*SSH
F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
F(3)=LB*(-E+RS*Z(1)+LS*F(1)-RC*(Z(3)+Z(4))-RA*Z(3)-LC/LB*
+ RA*Z(3)+LC/LB*RB*Z(4))/LALBLC
F(4)=(RA*Z(3)-RB*Z(4)+LA*F(3))/LB
RETURN
104 F(2)=(Z(1)+Z(3))*SSH
F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
F(3)=LBLC*(-E+RS*Z(1)+LS*F(1)+RB*Z(4)-RA*Z(3)-LB/LBLC*
+ (RB*Z(4)+RC*(Z(3)+Z(4))))/LALBLC
F(4)=- (RB*Z(4)+RC*(Z(3)+Z(4))+LC*F(3))/LBLC
RETURN
105 F(2)=(Z(1)+Z(4))*SSH
F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
F(4)=LALC*(-E+RS*Z(1)+LS*F(1)-RB*Z(4)+RA*Z(3)-LA/LALC*
+ (RA*Z(3)+RC*Z(4)+RC*Z(3)))/LALBLC
F(3)=- (RA*Z(3)+RC*Z(4)+RC*Z(3)+LC*F(4))/LALC
RETURN
106 F(2)=(Z(1)-Z(3)-Z(4))*SSH
F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
F(3)=LB*(E-RS*Z(1)-LS*F(1)-RC*(Z(3)+Z(4))-RA*Z(3)-LC/LB*
+ RA*Z(3)+LC/LB*RB*Z(4))/LALBLC
F(4)=(RA*Z(3)-RB*Z(4)+LA*F(3))/LB
RETURN
107 F(2)=(Z(1)-Z(3))*SSH
F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
F(3)=(E-RS*Z(1)-LS*F(1)-(RA+RB)*Z(3))/LALB
F(4)=-F(3)
RETURN

```

```

108  F(2)=(Z(1)-Z(3))*SSH
      F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
      F(3)=(E-RS*Z(1)-LS*F(1)-(RC+RA)*Z(3))/LALC
      F(4)=0.
      RETURN
109  F(2)=(Z(1)+Z(3))*SSH
      F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
      F(3)=-(E-RS*Z(1)-LS*F(1)+(RA+RB)*Z(3))/LALB
      F(4)=-F(3)
      RETURN
110  F(2)=(Z(1)-Z(4))*SSH
      F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
      F(4)=(E-RS*Z(1)-LS*F(1)-(RB+RC)*Z(4))/LBLC
      F(3)=0.
      RETURN
111  F(2)=(Z(1)+Z(3))*SSH
      F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
      F(3)=-(E-RS*Z(1)-LS*F(1)+(RC+RA)*Z(3))/LALC
      F(4)=0.
      RETURN
112  F(2)=(Z(1)+Z(4))*SSH
      F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
      F(4)=-(E-RS*Z(1)-LS*F(1)+(RC+RB)*Z(4))/LBLC
      F(3)=0.
      RETURN
113  F(2)=Z(1)*SSH
      F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
      F(4)=-RARB/LALB*Z(4)
      F(3)=-F(4)
      RETURN
114  F(2)=Z(1)*SSH
      F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
      F(4)=-RARB/LALB*Z(4)
      F(3)=-F(4)
      RETURN
115  F(2)=Z(1)*SSH
      F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
      F(3)=-RARC/LALC*Z(3)
      F(4)=0.
      RETURN
116  F(2)=Z(1)*SSH
      F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
      F(3)=-RARC/LALC*Z(3)
      F(4)=0.
      RETURN
117  F(2)=Z(1)*SSH
      F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
      F(4)=-RBRC/LBLC*Z(4)
      F(3)=0.
      RETURN
118  F(2)=Z(1)*SSH
      F(1)=(E-RS*Z(1)-RSHG*F(2)-Z(2))/LS
      F(4)=-RBRC/LBLC*Z(4)
      F(3)=0.
      RETURN
END

```

```

C -----
C PROGRAM INVERTER 2
C ANALYSIS OF THE 3-PHASE BRIDGE INVERTER
C USING THE RESISTANCE METHOD
C -----
      INTEGER INT1(6),INT2(6)
      REAL LF,LA,LB,LC
      REAL RP(11),RN(36),LP(11),LN(36),SQP(11),SQN(6),VP(11),VN(6)
      REAL IP(11),IPP(11),X(7),XP(7)
      DIMENSION CTHY(6),CTHP(6),VTHY(6)
      DIMENSION CP(66),CPT(66),R(66)
      DIMENSION ERR(7),A(7),B(7),C(7),D(7)
      DIMENSION ISS(6),IT(6),TT(6),ITHY(6)
      COMMON /BLOCK1/RN,LN,SQN,VN
      COMMON /BLOCK2/EPS
      COMMON /BLOCK3/T,ST,ANGLE,ALP6,TT2,TT,IT,ITHY,IITHY,HSMALL
      COMMON /BLOCK4/NEQ
      COMMON /BLOCK5/I1,IKS,JJ
      EXTERNAL AUX2
      DATA LP/11*.0001/
      EPS=.1E-10
C READ IN CIRCUIT PARAMETERS
      READ(2,1)RF,LF,RSH,CSH,RA,RB,RC,LA,LB,LC
      1  FORMAT(10F0.0)
      READ(2,2)E,FREQ,STEP,TFI,INVRT
      2  FORMAT(4F0.0,10)
      READ(2,3)CP
      3  FORMAT(11F0.0)
      ST=STEP
      ALP=1./FREQ
      ALP6=ALP/6.
      ANGLE=ALP6*(INVRT+1)
      OFFRES=300000.
      NEQ=7
C READ IN THE CORRECT START INITIAL CONDITIONS
      ISS(1),ISS(4),ISS(6)=1
      IT(1),IT(4)=1
      TT(4)=ALP6
      ITHY(1)=6
      ITHY(2)=5
      ITHY(3)=2
      ITHY(4)=5
      ITHY(5)=4
      ITHY(6)=1
      IITHY=1
      X(1)=E*CSH
      X(2)=EPS
      X(5)=-E/(RF+RB+RC)
C FORM RP,LP,SQP AND VP
      RP(7)=RA
      RP(8)=RB
      RP(9)=RC
      RP(10)=RSH
      RP(11)=RF
      SQP(10)=1./CSH
      VP(11)=E
C CALCULATE LN,LNINV,SQN AND VN
      CALL GHTRA(CP,CPT,11,6,66)
      CALL GHPRD(CPT,SQP,SQN,6,11,1,66,11,6)
      CALL GHPRD(CPT,VP,VN,6,11,1,66,11,6)

```

```

C      MAIN LOOP
100    DO 43 I=1,5
        RP(I)=0.
        IF(ISS(I))0,0,43
        RP(I)=OFFRES
43     CONTINUE
        LP(7)=LA
        LP(8)=LB
        LP(9)=LC
        LP(11)=LF
        CALL PROD(CPT,RP,R,6,11,66,11,66)
        CALL GHPRD(R,Cp,RN,6,11,6,66,66,36)
        CALL PROD(CPT,LP,R,6,11,66,11,66)
        CALL GHPRD(R,Cp,LN,6,11,6,66,66,36)
        CALL MINV(LN,6,DET,INT1,INT2,36,6)
C      CORRECT THE CURRENTS INITIAL CONDITIONS
        IF(JJ-1)0,0,10
        IF(IKS.NE.6)X(IKS+1)=X(J1+1)
        IF(I1.NE.6)X(I1+1)=0.
10     CONTINUE
C      CALL THE RUNGE-KUTTA INTEGRATION ROUTINE
500    DO 47 I=1,7
47     XP(I)=X(I)
        IF(ABS(ST)-EPS)501,501,0
        CALL D02AAF(X,ERR,T,ST,7,AUX2,A,B,C,D)
501    CONTINUE
C      CALCULATE THYRISTOR/DIODE CURRENTS AND VOLTAGES
        DO 44 I=1,11
            IP(I),IPP(I)=0.
            DO 44 J=1,5
                K=1*(J-1)+1
                IPP(I)=IPP(I)+CP(K)*XP(J+1)
44     IP(I)=IP(I)+CP(K)*X(J+1)
            DO 45 I=1,6
                CTHY(I)=IP(I)
                CTHYP(I)=IPP(I)
45     VTHY(I)=CTHY(I)*RP(I)
C      CALL CHECK SUBROUTINE
        J=0
        CALL CHECK(X,XP,CTHY,CTHYP,VTHY,ISS,J)
        IF(J)50,50,100
50     DO 31 I=1,6
31     TT(I)=IT(I)*(TT(I)+ST)
        TT2=TT2+ST
        ST=STEP
C      CALCULATE LOAD CURRENT AND VOLTAGE
C      PRINT RESULTS
        DELT=IP(7)-IPP(7)
        VOLT=IP(7)*RP(7)+DELT/ST*LP(7)
        WRITE(3,6)ISS,T,IP(7),IP(8),IP(9),VOLT
5     FORMAT(1X,6I1,F7.4,9F11.3)
        IF(T-TFI)500,500,0
1000  STOP
      END

```

```

SUBROUTINE AUX2(F,X,T)
DIMENSION X(7),F(7),FF(6)
REAL IN(6),RN(36),LN(36),SQN(6),VN(6),RI(6)
COMMON /BLOCK1/RN,LN,SQN,VN
F(1)=X(7)-X(2)-X(4)-X(6)
DO 10 I=1,6
10  IV(I)=X(I+1)
CALL GMPRO(RN,IN,RI,6,6,1,36,6,6)
DO 20 I=1,6
20  FF(I)=VN(I)-X(1)*SQN(I)-RI(I)
DO 30 I=1,6
  F(I+1)=0.
DO 30 J=1,6
  K=(J-1)*6+1
30  F(I+1)=F(I+1)+LN(K)*FF(J)
RETURN
END

```

```

C -----
C PROGRAM I N V E R T E R 3
C ANALYSIS OF THE 3-PHASE BRIDGE INVERTER
C USING THE TENSOR TECHNIQUE METHOD
C -----
  REAL INVCUR,INVOL
  INTEGER INT1(3),INT2(3)
  REAL LF,LA,LB,LC
  REAL RP(11),RN(36),RNN(9),LP(11),LN(36),LNN(9),RI(66)
  REAL SQP(11),SQN(6),SQNN(3),VP(11),VN(6),VNN(3)
  REAL IP(11),IPP(11),DIP(11),IN(6),INP(6),DIN(6),INN(3),DINN(3)
  REAL CTHY(6),CTHY(6),VTHY(6),VX(6),RI(6),LDI(6)
  DIMENSION CP(66),CPT(66),CN(18),CNT(18)
  DIMENSION X(4),XP(4),ERR(4),A(4),B(4),C(4),D(4)
  DIMENSION ISS(6),IT(6),TT(6),ITHY(6)
  COMMON /BLOCK1/RNN,LNN,SQNN,VNN,IN,INN,DINN,ND
  COMMON /BLOCK2/EPS
  COMMON /BLOCK3/T,ST,ANGLE,ALP6,TT2,TT,IT,ITHY,IITHY,HSMALL
  COMMON /BLOCK4/NEQ
  EXTERNAL AUX3
  EPS=.1E-10
C READ IN CIRCUIT PARAMETERS
  READ(2,1)RF,LF,RSH,CSH,RA,RB,RC,LA,LB,LC
  1  FORMAT(10F0.0)
  READ(2,2)E,FREQ,STEP,TFI,INVRT
  2  FORMAT(4F0.0,I0)
  READ(2,3)CP
  3  FORMAT(11F0.0)
  ST=STEP
  ALP=1./FREQ
  ALP6=ALP/6.
  ANGLE=ALP6*(INVRT+1)
C FEED IN THE CORRECT START UP INITIAL CONDITIONS
  ISS(1),ISS(4),ISS(6)=1
  IT(1),IT(4)=1
  TT(4)=ALP6
  ITHY(1)=6
  ITHY(2)=3
  ITHY(3)=2
  ITHY(4)=5
  ITHY(5)=4
  ITHY(6)=1
  IITHY=1
  X(1)=E*CSH
  IN(4)=-E/(RF+RB+RC)

```

```

C      FORM RP,LP,SQP AND VP
      RP(7)=RA
      RP(8)=RB
      RP(9)=RC
      RP(10)=RSH
      RP(11)=RF
      LP(7)=LA
      LP(8)=LB
      LP(9)=LC
      LP(11)=LF
      SQP(10)=1./CSH
      VP(11)=E
C      CALCULATE RN, LN, SQN AND VN
      CALL SMTRA(CP,CPT,11,6,66)
      CALL PROD(CPT,RP,R,6,11,66,11,66)
      CALL SMPRD(R,Cp,RN,6,11,6,66,66,36)
      CALL PROD(CPT,LP,R,6,11,66,11,66)
      CALL SMPRD(R,Cp,LN,6,11,6,66,66,36)
      CALL SMPRD(CPT,SQP,SQN,6,11,1,66,11,6)
      CALL SMPRD(CPT,VP,VN,6,11,1,66,11,6)
C      MAIN LOOP
100    ND=ISS(1)+ISS(2)+ISS(3)+ISS(4)+ISS(5)+ISS(6)
      NEQ=ND+1
      CALL CONN1(CN,ISS,ND,K)
      IF(K)40,40,0
      WRITE(3,4)ISS
      FORMAT(1X,6I1,' INVALID TOPOLOGY')
      GO TO 1000
C      CALCULATE RNN,LNN,LNNINV,SQNN AND VNN
40     CALL SMTRA(CN,CNT,6,ND,18)
      CALL SMPRD(CNT,RN,R,ND,6,6,18,36,66)
      CALL SMPRD(R,CN,RNN,ND,6,ND,66,18,9)
      CALL SMPRD(CNT,LN,R,ND,6,6,18,36,66)
      CALL SMPRD(R,CN,LNN,ND,6,ND,66,18,9)
      CALL SMPRD(CNT,SQN,SQNN,ND,6,1,18,6,3)
      CALL SMPRD(CNT,VN,VNN,ND,6,1,18,6,3)
      CALL MINV(LNN,ND,DET,INT1,INT2,9,3)
      IS=2*(1-ISS(1)-ISS(2))
      X(2)=IN(1+IS)+IN(2+IS)
      X(3)=IN(5)+IN(4)
      X(4)=IN(6)
C      CALL THE RUNGE-KUTTA INTEGRATION ROUTINE
500    DO 41 I=1,ND+1
41     XP(I)=X(I)
      IF(ABS(ST)-EPS)501,501,0
      CALL DD2AAF(X,ERR,T,ST,NEQ,AUX3,A,B,C,D)
501    CONTINUE

```

C CALCULATE THYRISTOR/DIODE CURRENTS AND VOLTAGES

```

DO 42 I=1,5
  INP(I)=IN(I)
  IN(I),DIN(I)=0.
  DO 42 J=1,ND
    K=6*(J-1)+1
    DIN(I)=DIN(I)+CN(K)*DINN(J)
42  IN(I)=IN(I)+CN(K)*X(J+1)
    DO 43 I=1,11
      IPP(I)=IP(I)
      IP(I),DIP(I)=0.
      DO 43 J=1,6
        K=11*(J-1)+1
        DIP(I)=DIP(I)+CP(K)*DIN(J)
43  IP(I)=IP(I)+CP(K)*IN(J)
      CALL GMPRD(RN,IN,RI,6,5,1,36,6,6)
      CALL GJPRD(LN,DIN,LDI,6,6,1,36,6,6)
      DO 44 I=1,5
        VX(I)=VN(I)-RI(I)-LDI(I)-SQN(I)*X(1)
        IF(ISS(I).EQ.1)VTHY(6)=-VX(I)
44  CONTINUE
      DO 45 I=1,5
        VTHY(I)=VX(I)+VTHY(6)
        CTHY(I)=IP(I)
45  CTHYP(1)=IPP(1)
      CTHY(5)=IP(5)
      CTHYP(6)=IPP(6)

```

C CALL CHECK

```

  J=0
  CALL CHECK(X,Xp,CTHY,CTHYP,VTHY,ISS,J)
  IF(J)50,50,0
  CALL INTERP(IN,INP,ST,HSMALL,6)
  GO TO 100
50  DO 31 I=1,6
31  TT(I)=IT(I)+(TT(I)+ST)
  TT2=TT2+ST
  ST=STEP

```

C CALCULATE LOAD CURRENT AND VOLTAGE

C PLOT RESULTS

```

  VOLT=IP(7)*RP(7)+LP(7)*DIP(7)
  INVCUR=IP(1)+IP(3)+IP(5)
  WRITE(3,6)ISS,T,IP(7),IP(8),IP(9),VOLT
6  FORMAT(1X,6I1,F7.4,9F11.3)
  NV=4
  WRITE(6)NV,T,IP(7),INVCUR,VOLT
  IF(T-TFI)500,500,0
  CALL PLOT
1000 STOP
END

```

```

SUBROUTINE AUX3(F,X,T)
DIMENSION X(NEQ),F(NEQ),FF(3)
REAL RNN(9),LNN(9),SQNN(3),VNN(3),RI(3)
REAL IN(6),INN(3),DINN(3)
COMMON /BLOCK1/RNN,LNN,SQNN,VNN,IN,INN,DINN,ND
COMMON /BLOCK4/NEQ
F(1)=IN(6)-IN(1)-IN(3)-IN(5)
DO 10 I=1,ND
10 INN(I)=X(I+1)
CALL GMPRO(RNN,INN,RI,ND,ND,1,9,3,3)
DO 20 I=1,ND
20 FF(I)=VNN(I)-RI(I)-SQNN(I)*X(1)
DO 30 I=1,ND
F(I+1)=0.
DO 30 J=1,ND
K=(J-1)*ND+I
30 F(I+1),DINN(I)=F(I+1)+LNN(K)*FF(J)
RETURN
END

```

```

SUBROUTINE CONN1(CN,ISS,ND,K)
DIMENSION CN(18),ISS(6)
K=0
DO 30 I=1,18
30 CN(I)=0.
GO TO (0,2,3),ND
K=1
RETURN
2 DO 40 I=1,4
IF(ISS(I))0,0,41
41 CN(I)=1
DO 42 II=I+1,5
42 IF(ISS(II).EQ.1)CN(II)=-1
RETURN
3 DO 32 I=1,4
J=I+1/3*6
32 IF(ISS(I).EQ.1)CN(J)=1
IF(ISS(5))33,33,0
CN(5),CN(11)=-1
33 CN(18)=1
RETURN
END

```

```

SUBROUTINE CHECK(X,XP,CTHY,CTHYP,VTHY,ISS,J)
DIMENSION X(NEQ),XP(NEQ),CTHY(6),CTHYP(6),VTHY(6)
DIMENSION ISS(6),IT(6),TT(6),ITHY(6)
COMMON /BLOCK2/EPS
COMMON /BLOCK3/T,ST,ANGLE,ALP6,TT2,TT,IT,ITHY,IITHY,HSMALL
COMMON /BLOCK4/NEQ
H1,H2,H3=ST
C CHECK FOR ANY THYRISTOR TO BE SWITCHED OFF
DO 13 I=1,6
IF(ISS(I))13,13,0
IF(TT(I)-EPS.LE.ANGLE.AND.TT(I)+ST+EPS.GT.ANGLE)GO TO 14
13 CONTINUE
GO TO 15
14 J=J+1
I1=I
H1=ANGLE-TT(I)
C THYRISTOR I1 IS EXPECTED TO SWITCH OFF AFTER H1 SEC.
C
C TEST FOR ANY THYRISTO/DIODE TURNING OFF
15 DO 16 I=1,6
IF(ISS(I))16,16,0
IF(ABS(CTHYP(I)).LE..1E-4)CTHYP(I)=0.
IF(CTHYP(I)*CTHY(I))17,16,16
16 CONTINUE
GO TO 18
17 J=J+2
H2=ABS(CTHYP(I))/ABS(CTHY(I)-CTHYP(I))*ST
I2=I
C THYRISTO/DIODE I2 IS EXPECTED TO TURN OFF AFTER H2 SEC.
C
C CHECK FOR THE GATE SIGNAL ON THE NEXT THYRISTOR IN THE SEQUENCE
18 IF(TT2-EPS.LE.ALPH6.AND.TT2+ST+EPS.GE.ALPH6)GO TO 19
GO TO 21
19 I3=ITHY(IITHY)
H3=ALP6-TT2
J=J+4
C GATE SIGNAL ON THYRISTOR I3 STARTS IN H3 SEC.
C THYRISTOR I3 IS EXPECTED TO SWITCH ON AFTER H3 SEC.
C
21 IF(J)50,50,0
C FIND HSMALL OF H1,H2 AND H3
KKK=1
HSMALL=H1
IF(HSMALL-H2)22,0,0
HSMALL=H2
KKK=2
22 IF(HSMALL-H3)23,0,0
HSMALL=H3
KKK=3

```

```

C      UPDATE THE VARIABLES AT HSMALL BY LINEAR INTERPOLATION
23     CALL INTERP(X,XP,ST,HSMALL,NEQ)
        T=T-ST+HSMALL
        DO 32 IK=1,6
32     TT(IK)=IT(IK)*(TT(IK)+HSMALL)
        TT2=TT2+HSMALL
        GO TO (24,25,26),KKK
C      SWITCH OFF THYRISTOR I1 AND TURN ON ITS COMPLEMENTARY DIODE
24     ISS(I1)=0
        IT(I1)=0
        TT(I1)=0.
        KS=1
        IF(I1/2+2-I1.EQ.0)KS=-1
        IKS=I1+KS
        ISS(IKS)=1
        WRITE(3,1)I1,IKS
1     FORMAT(1X,'THYRISTOR',I2,' IS SWITCHED OFF AND DIODE',I2,
        ' IS TURNED ON')
        GO TO 50
C
C      TURN OFF THYRISTOR/DIODE I2
25     ISS(I2)=0
        WRITE(3,2)I2
2     FORMAT(1X,'THYRISTO/DIODE',I2,' IS TURNED OFF')
        GO TO 50
C
C      SWITCH ON THYRISTOR I3
26     IT(I3)=1
        TT(I3)=0.
        TT2=0.
        I1THY=I1THY+1
        IF(I1THY.GT.6)I1THY=I1THY-6
        WRITE(3,3)I3
3     FORMAT(1X,'GATE SIGNAL ON THYRISTOR',I2,' IS STARTED')
50     DO 10I=1,6
        IF(ISS(I))0,0,10
        IF(IT(I))10,10,0
        ISS(I)=1
        WRITE(3,4)I
4     FORMAT(1X,'THYRISTOR',I2,' IS SWITCHED ON')
10     CONTINUE
        RETURN
END

```

APPENDIX B

COMPUTER PROGRAMS FOR THE ANALYSIS OF THE MOTOR SYSTEM.

A general simplified flow chart for the computer programs MOTOR1, MOTOR2, and MOTOR3 is shown in Figure B.1, followed by the program listings. Again in these programs, the variable names used are as far as possible those used in the analysis, and the solution of the system equations is also obtained from the NAGF library routine D02AAF. No special subroutines are used in these programs.

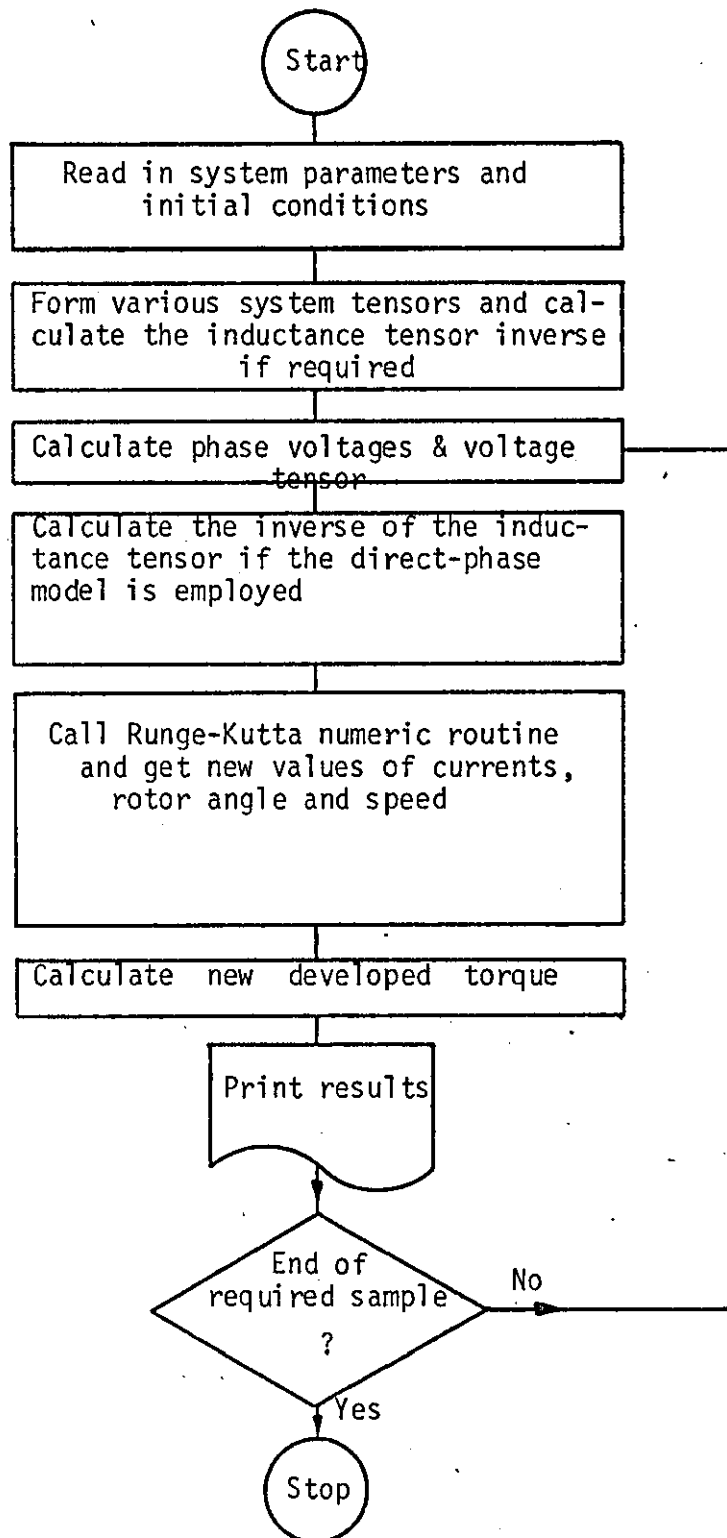


FIGURE B.1 The general flow chart for the motor system analysis

```

C-----
C   PROGRAM M Q T O R 1
C   INDUCTION MACHINE ANALYSIS
C   USING DIRECT PHASE MODEL
C-----
      REAL INER
      REAL LS,LR,LSM,LRM,MSR
      REAL LP(36),RP(6),IP(6),VP(6),DL(36)
      DIMENSION Y(8),ERR(8),A(8),B(8),C(8),D(8)
      COMMON /BLOCK1/LP,DL,RP,VP,VMAX,W,P,INER,COEFF1,TORQ,TORQM,R120
      LS,LR,LSM,LRM,MSR
      EQUIVALENCE(Y(1),THETA),(Y(2),SPEED),(Y(3),IP(1))
      EXTERNAL AUX4
      PI=4.*ATAN(1.)
      R120=27/3.*PI
      READ(2,1)RS,RR,LS,LR,LSM,LRM,MSR,VMAX,FREQ,P,INER,COEFF
1     FORMAT(15F0.0)
      W=2.*PI*FREQ
      ST=.5E-3
      TFI=2.
      RP(1),RP(2),RP(3)=RS
      RP(4),RP(5),RP(6)=RR
100    CONTINUE
      CALL DO2AAF(Y,ERR,T,ST,8,AUX4,A,B,C,D)
      TORQ=
      & -(IP(1)*IP(4)+IP(2)*IP(5)+IP(3)*IP(6))*SIN(THETA)
      & + (IP(1)*IP(5)+IP(2)*IP(6)+IP(3)*IP(4))*SIN(THETA+R120)
      & + (IP(1)*IP(6)+IP(2)*IP(4)+IP(3)*IP(5))*SIN(THETA+R120))*P*MSR
      WRITE(3,2)T,THETA,SPEED,TORQ,IP,VP(1)
2     FORMAT(1X,F6.4,11F10.3)
      IF(T.LT.TFI)GO TO 100
      STOP
      END

```

```

SUBROUTINE AUX4(F,Y,T)
REAL INER
INTEGER INT1(6),INT2(6)
REAL LS,LR,LSM,LRM,MSR
REAL LP(36),RP(6),IP(6),VP(6),DL(36)
DIMENSION F(8),Y(8),FF(6)
COMMON /BLOCK1/LP,DL,RP,VP,VMAX,W,P,INER,COEFF1,TORQ,TORQM,R120
2 ,LS,LR,LSM,LRM,MSR
THETA=Y(1)
LP(1),LP(8),LP(15)=LS
LP(22),LP(29),LP(36)=LR
LP(2),LP(3),LP(7),LP(9),LP(13),LP(14)=LSM
LP(23),LP(24),LP(28),LP(30),LP(34),LP(35)=LRM
LP(20),LP(27),LP(31),LP(6),LP(10),LP(17)=MSR*COS(THETA-R120)
LP(21),LP(25),LP(32),LP(5),LP(12),LP(16)=MSR*COS(THETA+R120)
LP(4),LP(11),LP(18),LP(19),LP(26),LP(33)=MSR*COS(THETA)
CALL MINV(LP,6,DET,INT1,INT2,36,6)
DL(4),DL(11),DL(18),DL(19),DL(26),DL(33)=SIN(THETA)*MSR
DL(5),DL(12),DL(16),DL(21),DL(25),DL(32)=SIN(THETA+R120)*MSR
DL(6),DL(10),DL(17),DL(20),DL(27),DL(31)=SIN(THETA-R120)*MSR
1 VP(1)=VMAX*SIN(W*T)
VP(2)=VMAX*SIN(W*T-R120)
VP(3)=VMAX*SIN(W*T+R120)
F(1)=Y(2)
F(2)=(TORQ-TORQM-COEFF1*Y(2))/INER*P
DO 10 I=1,6
DLI=0.
DO 30 J=1,6
K=I+(J-1)*6
30 DLI=DLI+DL(K)*Y(J+2)
10 FF(I)=VP(I)-RP(I)*Y(I+2)+DLI*Y(2)
DO 20 I=1,6
F(I+2)=0.
DO 20 J=1,6
K=I+(J-1)*6
20 F(I+2)=F(I+2)+LP(K)*FF(J)
RETURN
END

```

```

C-----
C   PROGRAM M O T O R 2
C   INDUCTION MACHINE ANALYSIS
C   USING STATIONARY 2-AXIS MODEL
C-----
      INTEGER INT1(4),INT2(4)
      REAL INER
      REAL LS,LR,LSM,LRM,MSR
      REAL LP(16),RP(4),IP(4),VP(3),G(16)
      DIMENSION Y(6),ERR(6),A(6),B(6),C(6),D(6)
      COMMON /BLOCK1/LP,RP,G,VP,VMAX,W,P,INER,COEFF1,TORQ,TORQM,R120
      EQUIVALENCE (Y(1),THETA),(Y(2),SPEED),(Y(3),IP(1))
      EXTERNAL AUX5
      PI=4.*ATAN(1.)
      R120=27*PI/3.
      STEP=.5E-3
      TFI=.5.
      READ(2,1)RS,RR,LS,LR,LSM,LRM,MSR
1    & ,VMAX,FREQ,INER,P,COEFF1,TORQM
      FORMAT(7F0.0)
      W=2.*PI*FREQ
      RP(1),RP(2)=RS
      RP(3),RP(4)=RR
      LP(1),LP(6)=LS-LSM
      LP(11),LP(16)=LR-LRM
      LP(3),LP(8),LP(9),LP(14)=1.5*MSR
      G(4)=-1.5*MSR
      G(7)=1.5*MSR
      G(12)=-(LR-LRM)
      G(15)=LR-LRM
100  CALL MINV(LP,4,DET,INT1,INT2,16,4)
      CONTINUE
      CALL DO2AAF(Y,ERR,T,STEP,6,AUX5,A,B,C,D)
      TORQ=1.5*1.5*MSR*P*(IP(2)*IP(3)-IP(1)*IP(4))
      WRITE(3,2)T,THETA,SPEED,TORQ,IP,VP(1)
2    FORMAT(1X,F6.4,11F10.3)
      IF(T.LT.TFI)GO TO 100
      STOP
      END

```

```

SUBROUTINE AUX5(F,Y,T)
REAL INER
REAL LP(16),RP(4),VP(3),VDQ(4),G(16)
DIMENSION F(6),Y(6),FF(4)
COMMON /BLOCK1/LP,RP,G,VP,VMAX,W,P,INER,COEFF1,TORQ,TORQM,R120
VP(1)=VMAX*SIN(W*T)
VP(2)=VMAX*SIN(W*T-R120)
VP(3)=VMAX*SIN(W*T+R120)
VDQ(1)=(2.*VP(1)-VP(2)-VP(3))/3.
VDQ(2)=(VP(2)-VP(3))/SQRT(3.)
F(1)=Y(2)
F(2)=(TORQ-TORQM-COEFF1*Y(2))/INER*P
DO 20 I=1,4
GI=0.
DO 10 J=1,4
K=I+(J-1)*4
10 GI=GI+G(K)*Y(J+2)
20 FF(I)=VDQ(I)-RP(I)*Y(I+2)-GI*Y(2)
DO 30 I=1,4
F(I+2)=0.
DO 30 J=1,4
K=I+(J-1)*4
30 F(I+2)=F(I+2)+LP(K)*FF(J)
RETURN
END

```

```

C-----
C   PROGRAM M U T O R 3
C   INDUCTION MACHINE ANALYSIS
C   USING STATIONARY 3-AXIS MODEL
C-----
      INTEGER INT1(6),INT2(6)
      REAL INER
      REAL LS,LR,LSM,LRM,MSR
      REAL LP(36),RP(6),IP(6),VP(6),G(36)
      DIMENSION Y(8),ERR(8),A(8),B(8),C(8),D(8)
      COMMON/BLOCK1/LP,RP,G,VP,VMAX,W,P,INER,COEFF1,TORQ,TORQM,R120
      EQUIVALENCE (Y(1),THETA),(Y(2),SPEED),(Y(3),IP(1))
      EXTERNAL AUX6
      PI=4.*ATAN(1.)
      R120=2.*PI/3.
      READ(2,1)RS,RR,LS,LR,LSM,LRM,MSR
      Q ,VMAX,FREQ,P,INER,COEFF1,TORQM
1     FORMAT(7F0.0)
      W=2.*PI*FREQ
      TORF=1.5*SQRT(3.)*P*MSR
      STEP=.5E-5
      TFI=.5
      RP(1),RP(2),RP(3)=RS
      RP(4),RP(5),RP(6)=RR
      LP(1),LP(8),LP(15)=LS
      LP(22),LP(29),LP(36)=LR
      LP(2),LP(3),LP(7),LP(9),LP(13),LP(14)=LSM
      LP(23),LP(24),LP(28),LP(30),LP(34),LP(35)=LRM
      LP(4),LP(11),LP(18),LP(19),LP(26),LP(33)=MSR
      LP(5),LP(6),LP(10),LP(12),LP(16),LP(17),LP(20),LP(21),LP(25)
      Q ,LP(27),LP(31),LP(32)=-.5*MSR
      CALL MINV(LP,6,DET,INT1,INT2,36,6)
      G(5),G(12),G(16)=-SQRT(3.)/2.*MSR
      G(6),G(10),G(17)=-G(5)
      G(23),G(30),G(34)=- (LR-LRM)/SQRT(3.)
      G(24),G(28),G(35)=-G(23)
100    CONTINUE
      CALL D02AAF(Y,ERR,T,STEP,8,AUX6,A,B,C,D)
      TORQ=TORF*(IP(1)*IP(6)-IP(3)*IP(4))
      WRITE(3,4)T,THETA,SPEED,TORQ,IP,VP(1)
2     FORMAT(1X,F6.4,11F10.3)
      IF(T.LT.TFI)GO TO 100
      STOP
      END

```

```

SUBROUTINE AUX6(F,Y,T)
REAL INER
REAL LP(36),RP(6),VP(6),G(36)
DIMENSION F(8),Y(8),FF(6)
COMMON/BLOCK1/LP,RP,G,VP,VMAX,W,P,INER,COEFF1,TORQ,TORQM,R120
VP(1)=VMAX*SIN(W*T)
VP(2)=VMAX*SIN(W*T-R120)
VP(3)=VMAX*SIN(W*T+R120)
F(1)=Y(2)
F(2)=(TORQ-TORQM-COEFF1*Y(2))/INER*P
DO 10 I=1,6
GI=0.
DO 20 J=1,6
K=I+(J-1)*6
20  GI=GI+G(K)*Y(J+2)
10  FF(I)=VP(I)-RP(I)*Y(I+2)-GI*Y(2)
DO 30 I=1,6
F(I+2)=0.
DO 30 J=1,6
K=I+(J-1)*6
30  F(I+2)=F(I+2)+LP(K)*FF(J)
RETURN
END

```

APPENDIX C

COMPUTER PROGRAM FOR THE ANALYSIS OF THE INVERTER-MOTOR SYSTEM

A general simplified flow chart for the computer program INVERTER-MOTOR is shown in Figure C.1. followed by program listings. As explained in Chapter 5 this program is a combination of the programs INVERTER3 and MOTOR3. The subroutine CONN2 is used to generate the transformation tensor for any system topology. This subroutine basically is similar to CONN1 for the inverter system analysis, but here it has been modified to cope with the additional rotor circuits and the rectifier diode.

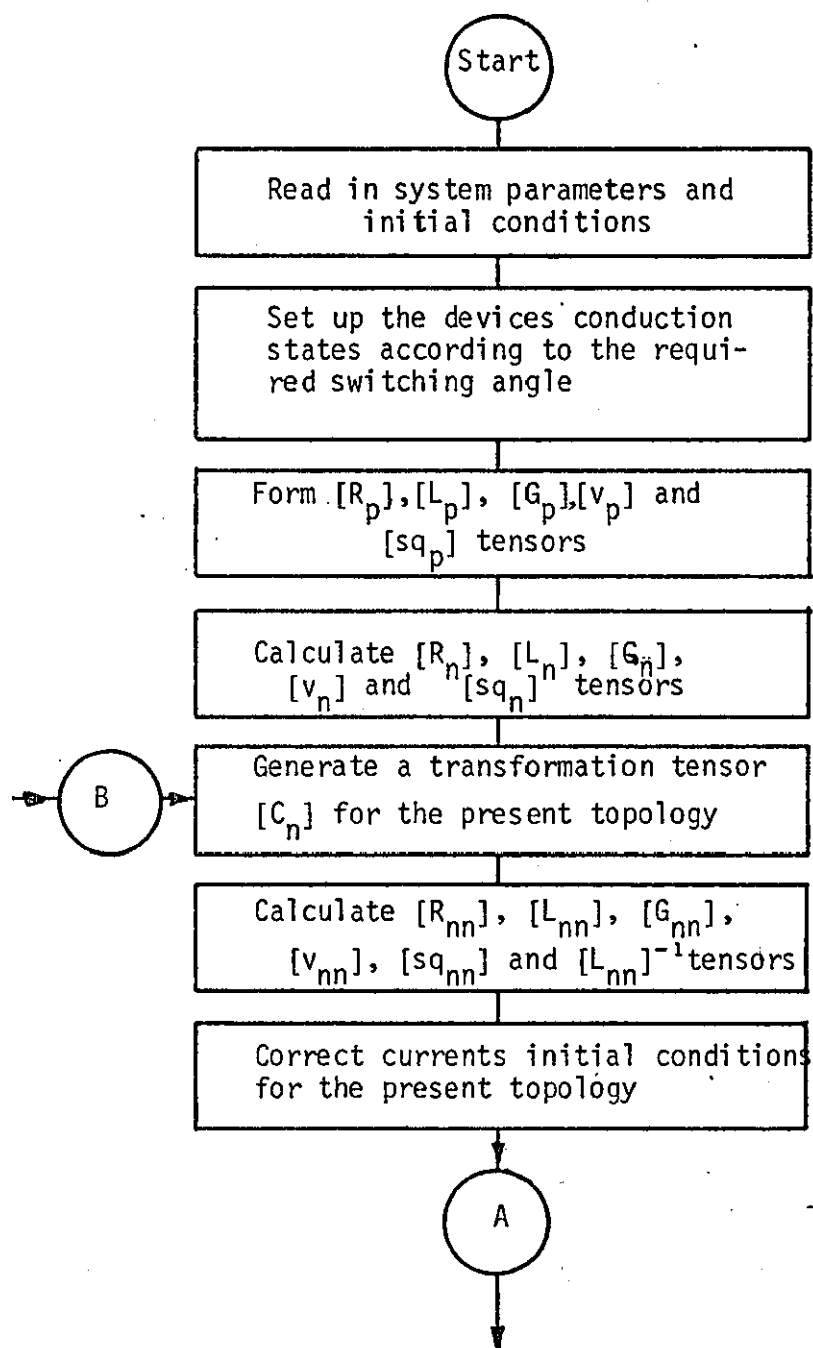


FIGURE C.1 The general flow chart for the inverter-motor system analysis (continued next page).

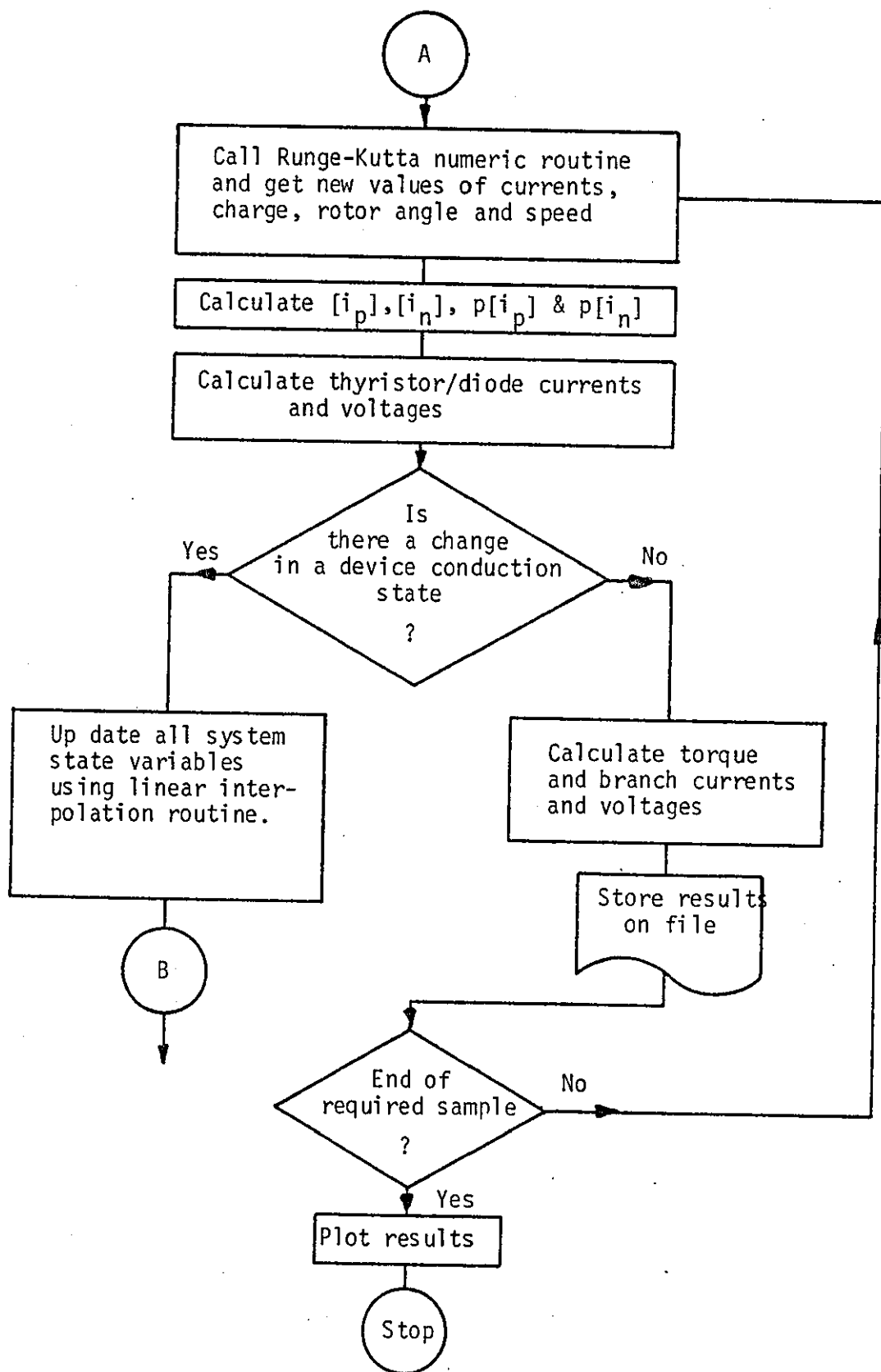


FIGURE C.1 (continued) The general flow chart for the inverter-motor system analysis

```

C -----
C PROGRAM INVERTER - MOTOR
C GENERAL ANALYSIS OF THE INVERTER-INDUCTION MOTOR SYSTEM
C -----

```

```

      INTEGER INT1(6),INT2(6)
      REAL INER,INVCUR,INVOLT
      REAL LF,LSS,LRR,LSM,LRM,MSR
      REAL CP(126),CPT(126),CN(54),CNT(54)
      REAL RP(15),RN(81),RNN(36),R(126),RI(9)
      REAL LP(126),LN(81),LNN(36),LDI(9)
      REAL GP(126),GN(81),GNN(36),GI(9)
      REAL IP(14),IPP(14),DIP(14),IN(9),DIN(9),INN(6),DINN(6)
      REAL VP(14),VN(9),VNN(6)
      REAL SQP(14),SQN(9),SQNN(6)
      REAL VB(14),VL(14),VG(14),VX(5),VTHY(6),CTHY(6),CTHYP(6)
      REAL Y(9),YP(9),F(9),ERR(9),A(9),B(9),C(9),D(9)
      DIMENSION ISS(6),ITHY(6),ITHY1(6),IT(6),TT(6),H(5),JJ(6)
      COMMON/BLOCK1/CP,CN,IP,IN,INN,DINN
      /RNN,LNN,GNN,RI,GI,R,VNN,SQNN
      /LOOP,LL,NEQ,M,M14,ML
      /TREF,TORQ,TRQM,COEFF1,INER,P
      COMMON/BLOCK4/EPS
      EQUIVALENCE (Y(1),THETA),(Y(2),SPEED),(Y(3),CHARGE)
      EXTERNAL AUX7
      DATA ITHY/4,1,6,3,2,5/
      DATA ITHY1/0,3,2,5,4,1/
      PI=4.*ATAN(1.)
      R120=2.*PI/3.
      EPS=.1E-10
      SQ3=SQRT(3.)

```

```

C
C READ IN SYSTEM PARAMETERS
C
C N=6 FOR THE STATOR PHASE OPEN CIRCUIT TEST
C N=9 FOR THE STATOR PHASE SHORT CIRCUIT TEST
C N=8 FOR ALL OTHER CASES
C INVRT =1 FOR 120-INVERTOR
C INVRT =2 FOR 180-INVERTOR
      READ(2,1)RF,LF,RSH,CSH,RS,RR,LSS,LRR,LSM,LRM,MSR,P,INER,COEFF1
1      FORMAT(20F0.0)
      READ(2,1)E,FREQ,STEP,TFI,ALPHA,TSWITCH1,TSWITCH2,SPEED
      READ(2,2)INVRT,M
2      FORMAT(10I0)
      M14=M*14
      M1=M*4
      READ(2,3)(CP(J),J=1,M14)
3      FORMAT(14F0.0)
      TREF=1.5*SQ3*P*MSR
      W=2.*PI*FREQ
      ALP=1./FREQ
      ALP6=ALP/6.
      ANGLE=(INVRT+1)*ALP6*STEP/10.
      IF(INVRT.EQ.1)I00=4
      IF(INVRT.EQ.2)I33=1
      ST=STEP

```

C DETERMINE THE DEVICE CONDUCTION STATES ACCORDING TO ALPHA

```

DO 903 I=1,6
  IF (ALPHA.LT.ALPH6*I)GO TO 904
903 CONTINUE
904 ITHON1=ITHY(I)
  ITHON2=ITHY(I+1-1/6*6)
  ISS(ITHON1),ISS(ITHON2),IT(ITHON1),IT(ITHON2)=1
  TT(ITHON1)=ALPHA-(I-2)*ALPH6
  TT(ITHON2)=ALPHA-(I-1)*ALPH6
  TT2=TT(ITHON2)
  TTT=ALPHA
  IITHY=I+2
  IF (INVRT-1) 906,906,0
  ITHON3=ITHY(I+2-1/5*6)
  ISS(ITHON3),IT(ITHON3)=1
  IT(ITHON1)=ALPHA-(I-3)*ALPH6
  TT(ITHON2)=ALPHA-(I-2)*ALPH6
  TT(ITHON3)=ALPHA-(I-1)*ALPH6
  TT2=TT(ITHON3)
  IITHY=I+3
906 CONTINUE
  ID=1
  CHARGE=E*CSH

```

C FOR I RP,LP,GP,VP AND SQP TENSORS

```

  RP(7),RP(8),RP(9)=RS
  RP(10)=RSH
  RP(11)=RF
  RP(12),RP(13),RP(14)=RR
  LP(151)=LF
  LP(91),LP(106),LP(121)=LSS
  LP(165),LP(181),LP(196)=LRR
  LP(92),LP(93),LP(105),LP(107),LP(119),LP(120)=LSM
  LP(167),LP(168),LP(180),LP(182),LP(194),LP(195)=LRM
  LP(96),LP(111),LP(126),LP(161),LP(176),LP(191)=MSR
  LP(97),LP(98),LP(110),LP(112),LP(124),LP(125),LP(162),LP(163)
  ,LP(175),LP(177),LP(189),LP(190)=-.5*MSR
  GP(97),GP(112),GP(124)=-SQ3/2.*MSR
  GP(98),GP(110),GP(125)=SQ3/2.*MSR
  GP(167),GP(182),GP(194)=-(LRR-LRM)/SQ3
  GP(168),GP(180),GP(195)=(LRR-LRM)/SQ3
  VP(11)=E
  SQP(103)=1./CSH

```

C CALCULATE RN, LN, GN, VN AND SQN TENSORS

```

  CALL GHTRA(CP,CPT,14,M,M14)
  CALL PROD(CPT,RP,R,H,14,M14,14,H14)
  CALL GHPRD(R,GP,RN,H,14,M,M14,M14,NM)
  CALL GHPRD(CPT,LP,R,H,14,14,M14,196,M14)
  CALL GHPRD(R,GP,LN,M,14,M,M14,M14,NM)
  CALL GHPRD(CPT,GP,R,H,14,14,M14,196,M14)
  CALL GHPRD(R,GP,GN,M,14,M,M14,M14,NM)
  CALL GHPRD(CPT,VP,VN,M,14,1,M14,14,M)
  CALL GHPRD(CPT,SQP,SQN,M,14,1,M14,14,M)

```

```

C      MAIN LOOP
100    DO 66 I=1,6
66      IF(IT(I).EQ.1)ISS(I)=1
        ND=ISS(1)+ISS(2)+ISS(3)+ISS(4)+ISS(5)+ISS(6)
        IF(M.EQ.6)ISS(5),ISS(6)=0
        IF(IPLUG1.EQ.2.AND.IPLUG2.EQ.1)ND=1
        LOOP=ND+ID+M/9+1
        LL=LOOP*LOOP
        NL=M*LOOP
        NEQ=LOOP*3
C      CORRECT CURRENT INITIAL CONDITIONS FOR THIS STEP
        IS=2*(1-ISS(1)-ISS(2))
        INN(1)=IP(1+IS)+IP(2+IS)
        INN(2)=IP(3)+IP(4)
        INN(ND)=IP(11)
        INN(LOOP-1)=IP(12)
        INN(LOOP)=IP(13)
        DO 74 I=1,LOOP
74      Y(I+3)=INN(I)
        CALL CONN2(CN,ISS,ND,ID,M)
C      CALCULATE RNN,LNN,LNNINV,GNN,VNN AND SQNN TENSORS
        CALL GHTRA(CN,CNT,M,LOOP,ML)
        CALL GHPRD(CNT,RN,R,LOOP,M,M,ML,MM,ML)
        CALL GHPRD(R,CN,RNN,LOOP,M,LOOP,ML,ML,LL)
        CALL GHPRD(CNT,LN,R,LOOP,M,M,ML,MM,ML)
        CALL GHPRD(R,CN,LNN,LOOP,M,LOOP,ML,ML,LL)
        CALL GHPRD(CNT,GN,R,LOOP,M,M,ML,MM,ML)
        CALL GHPRD(R,CN,GNN,LOOP,M,LOOP,ML,ML,LL)
        CALL GHPRD(CNT,VN,VNN,LOOP,M,1,ML,M,LOOP)
        CALL GHPRD(CNT,SQN,SQNN,LOOP,M,1,ML,M,LOOP)
        CALL MINV(LNN,LOOP,DET,INT1,INT2,LL,LOOP)
C      CALL THE RUNGE-KUTTA INTEGRATION ROUTINE
        IF(ABS(ST)-EPS)501,501,0
500     CONTINUE
        DO 75 I=1,14
75      IPP(I)=IP(I)
        YP(1)=Y(1)
        YP(2)=Y(2)
        YP(3)=Y(3)
        CALL DO2AAF(Y,ERR,T,ST,NEQ,AUX7,A,B,C,D)
501     CONTINUE
        DO 73 I=1,LOOP
73      INN(I)=Y(I+3)
C      CALCULATE IN,DIN,IP AND DIP TENSORS
        DO 43 I=1,M
43      IN(I),DIN(I)=0
        DO 43 J=1,LOOP
        K=M*(J-1)+1
        DIN(I)=DIN(I)+CN(K)*DINN(J)
        IN(I)=IN(I)+CN(K)*INN(J)
        DO 44 I=1,14
44      IP(I),DIP(I)=0
        DO 44 J=1,M
        K=14*(J-1)+1
        DIP(I)=DIP(I)+CP(K)*DIN(J)
        IP(I)=IP(I)+CP(K)*IN(J)

```

```

C   CALCULATE THYRISTOR/DIODE CURRENTS AND VOLTAGES
    CALL GMPRD(RN,IN,RI,M,M,1,MM,M,M)
    CALL GMPRD(GN,IN,GI,M,M,1,MM,M,M)
    CALL GMPRD(LN,DIN,LDI,M,M,1,MM,M,M)
    DO 202 I=1,5
    CTHY(I)=IP(I)
    CTHYP(I)=IPP(I)
    VX(I)=VN(I)-RI(I)-GI(I)*SPEED-LDI(I)-SQN(I)*CHARGE
    IF(ISS(I).EQ.1)VTHY(6)=-VX(I)
202  CONTINUE
    CTHY(6)=IP(6)
    CTHYP(6)=IPP(6)
    DO 207 I=1,5
207  VTHY(I)=VX(I)+VTHY(6)
C   CHECK FOR ANY CHANGE IN DEVICE CONDUCTION STATES
    J=0
    DO 34 I=1,5
34  H(I)=ST
C   CHECK FOR ANY THYRISTOR TO BE SWITCHED OFF
    DO 13 I=1,6
    IF(ISS(I))13,13,0
    IF(TT(I)-EPS.LE.ANGLE.AND.TT(I)+ST+EPS.GT.ANGLE)GO TO 14
13  CONTINUE
    GO TO 15
14  J=J+1
    I1=I
    H(1)=ANGLE-TT(I)
C   THYRISTOR I1 IS EXPECTED TO SWITCH OFF AFTER H(1) S.
C
C   TEST FOR ANY THYRISTOR/DIODE TURNING OFF
15  DO 16 I=1,6
    IF(ISS(I))16,16,0
    IF(ABS(CTHYP(I)).LE..1E-4)CTHYP(I)=0.
    IF(CTHY(I)*CTHYP(I))17,16,16
16  CONTINUE
    GO TO 18
17  J=J+1
    I2=I
    H(2)=ABS(CTHYP(I))/ABS(CTHY(I)-CTHYP(I))*ST
C   THYRISTOR/DIODE I2 IS EXPECTED TO TURN OFF AFTER H(2) S.
C
C   CHECK FOR THE GATE SIGNAL ON THE NEXT THYRISTOR IN THE SEQUENCE
18  IF(TT2-EPS.LE.ALP6.AND.TT2+ST+EPS.GE.ALP6)GO TO 19
    GO TO 21
19  J=J+1
    I3=ITHY(I1THY)
    H(3)=ALP6-TT2
C   GATE SIGNAL ON THYRISTOR I3 STARTS IN H(3) S.
C   THYRISTOR I3 IS EXPECTED TO SWITCH ON AFTER H(3) S.
C
C   CHECK IF THE RECTIFIER DIODE IS TO BE TURNED OFF
21  IF(ID)51,51,0
    IF(IP(11)*IPP(11))0,51,51
    J=J+1
    H(4)=ABS(IPP(11))/ABS(IP(11)-IPP(11))*ST
C   RECTIFIER DIODE IS EXPECTED TO TURN OFF AFTER H(4) S.

```

```

C CHECK IF THE RECTIFIER DIODE IS TO BE TURNED ON
51 IF(ID)0,0,52
   RDV=E-CHARGE/CSH
   IF(RDV)52,52,0
   J=J+1
   H(5)=0.
C RECTIFIER DIODE IS EXPECTED TO TURN ON AFTER H(5) S.
C
52 IF(J)53,53,0
C FIND HSMAL OF H ARRAY
   HSMAL=H(1)
   KKK=1
   DO 33 I=2,5
   IF(HSMAL-H(I))33,33,0
   HSMAL=H(I)
   KKK=I
33 CONTINUE
C UPDATE THE VARIABLES AT HSMAL BY LINEAR INTERPOLATION
   CALL INTERP(Y,YP,ST,HSMAL,3)
   CALL INTERP(IP,IPP,ST,HSMAL,14)
   DO 32 IK=1,6
32 IT(IK)=IT(IK)*(TT(IK)+HSMAL)
   TT2=TT2+HSMAL
   TTT=TTT+HSMAL
   ST=ST-HSMAL
   T=T-ST
   GO TO(22,23,24,25,26),KKK
C SWITCH OFF THYRISTOR I1 AND TURN ON ITS COMPLEMENTARY DIODE
22 ISS(I1)=0
   IT(I1)=0
   TT(I1)=0.
   KS=1
   IF(0.1/2+2-I1.EQ.0)KS=-1
   IKS=I1+KS
   ISS(IKS)=1
   WRITE(3,5)I1,IKS
5 FORMAT(1X,'THYRISTOR',I2,' IS SWITCHED OFF AND DIODE',I2,
  ' IS TURNED ON')
   GO TO 50
C TURN OFF THYRISTOR/DIODE I2
23 ISS(I2)=0
   WRITE(3,6)I2
5 FORMAT(1X,'THYRISTOR/DIODE',I2,' IS TURNED OFF')
   GO TO 50
C SWITCH ON THYRISTOR I3
24 IT(I3)=1
   TT(I3)=0.
   TT2=0.
   IITHY=IITHY+1
   IF(IITHY.GT.6)IITHY=IITHY-6
   WRITE(3,7)I3
7 FORMAT(1X,'GATE SIGNAL ON THYRISTOR',I2,' IS STARTED')
   GO TO 50
C TURN OFF THE RECTIFIER DIODE
25 ID=0
   WRITE(3,8)
8 FORMAT(1X,'RECTIFIER DIODE IS TURNED OFF')
   GO TO 50

```

C TURN ON THE RECTIFIER DIODE

```
26 ID=1  
WRITE (3,9)  
91 FORMAT(1X,'RECTIFIER DIODE IS TURNED ON')  
50 DO 10 I=1,6  
IF(ISS(I))0,0,10  
IF(IT(I))10,10,0  
ISS(I)=1  
WRITE(3,11)I  
11 FORMAT(1X,'THYRISTOR',I2,' IS SWITCHED ON')  
10 CONTINUE  
GO TO 100  
53 DO 31 IK=1,6  
31 TT(IK)=IT(IK)*(TT(IK)+ST)  
TT2=TT2+ST  
TTT=TTT+ST  
ST=STEP
```

C CALCULATE BRANCH VOLTAGES VB

```
CALL SMPRD(LP,DIP,VL,14,14,1,196,14,14)  
CALL SMPRD(GP,IP,VG,14,14,1,196,14,14)  
DO 12 I=1,14  
12 VB(I)=RP(I)*IP(I)+VG(I)*SPEED+VL(I)-VP(I)  
INVCUR=IP(11)-IP(10)  
TORQ=TORE*(IP(7)*IP(14)-IP(9)*IP(12))
```

C PLOT RESULTS

```
WRITE(3,4)ISS,T,THETA,SPEED,TORQ,IP(7),IP(8),IP(9),INVCUR,VB(7)  
4 FORMAT(1X,6I1, F7.4,9F11.3)  
NV=9  
WRITE(6)NV,T,THETA,SPEED,TORQ,IP(7),IP(8),IP(9),INVCUR,VB(7)  
IF(T-TFI)500,500,0  
CALL PLOT  
STOP  
END
```

```

SUBROUTINE AUX7(F,Y,T)
REAL INER
REAL IP(14),IN(9),INN(6),DINN(6)
REAL RNN(36),LNN(36),GNN(36),RI(9),GI(9),R(126)
REAL VNN(6),SQNN(6)
REAL CP(126),CN(54)
REAL F(NEQ),Y(NEQ),FF(6)
COMMON/BLOCK1/CP,CN,IP,IN,INN,DINN
& ,RNN,LNN,GNN,RI,GI,R,VNN,SQNN
& ,LOOP,LL,NEQ,M,M14,ML
& ,TORF,TORQ,TRQM,COEFF1,INER,P
DO 11 I=1,LOOP
11 INN(I)=Y(I+3)
CALL GMPRD(RNN,INN,RI,LOOP,LOOP,1,LL,LOOP,LOOP)
CALL GMPRD(GNN,INN,GI,LOOP,LOOP,1,LL,LOOP,LOOP)
CALL GMPRD(CN,INN,IN,M,LOOP,1,ML,LOOP,M)
CALL GMPRD(CP,IN,IP,14,M,1,M14,M,14)
TORQ=TORF*(IP(7)*IP(14)-IP(9)*IP(12))
F(1)=Y(2)
F(2)=(TORQ-TRQM-COEFF1*Y(2))/INER*P
F(3)=IP(11)-IP(1)-IP(3)-IP(5)
DO 10 I=1,LOOP
10 FF(I)=VNN(I)-RI(I)-GI(I)*Y(2)-SQNN(I)*Y(3)
DO 20 I=1,LOOP
F(I+3)=0.
DO 20 J=1,LOOP
K=(J-1)*LOOP+I
20 F(I+3),DINN(I)=F(I+3)+LNN(K)*FF(J)
RETURN
END

```

```

SUBROUTINE CONN2(CN,ISS,ND,ID,M)
DIMENSION CN(54),ISS(6)
L=N/2+1
DO 31 I=1,54
31  CN(I)=0.
    IF(ID)11,11,0
    CN((ND+ID-1)*M-2)=ID
11  CN((ND+ID)*M-1),CN((ND+ID+1)*M)=1
    GO TO (44,2,3),ND
2   DO 40 I=1,4
    IF(ISS(I))0,0,41
40  CONTINUE
41  CN(I)=1
    DO 42 II=I+1,L
42  IF(ISS(II).EQ.1)CN(II)=-1
44  RETURN
3   DO 32 I=1,4
    J=I+1/3*M
32  IF(ISS(I).EQ.1)CN(J)=1
    IF(ISS(5))33,33,0
    CN(5),CN(5+M)=-1
33  RETURN
END
SUBROUTINE INTERP(Z,ZP,ST,T,N)
DIMENSION Z(N),ZP(N)
COMMON/BLOCK4/EPS
IF(ABS(ST).LE.EPS)RETURN
DO 10 K=1,N
10  Z(K)=ZP(K)-(ZP(K)-Z(K))/ST*T
    RETURN
END

```

```

SUBROUTINE PLOT
DIMENSION XX(4000),YY(4000),A(15)
DIMENSION IARRAY(16,15)
READ(271)((IARRAY(I,J),I=1,16),J=1,15)
1  FORMAT(16A1)
   READ(2,3)STEP,TMIN,TFI,PS
3  FORMAT(10F0.0)
   READ(272)NCURVE
   N=TFI/STEP
   CALL C1051N
   CALL JNITS(10.)
   CALL DEVPAP(50.,29.,1)
   TAXIS=NINT(PS*(TFI-TMIN))+1.
   ITAXIS=NINT(PS*(TFI-TMIN))+1
   CALL CHASIZ(.25,.3)
   DO 20 J=1,NCURVE
   READ(272)K,Y0,YAXIS,INT,YMIN,YMAX,L,LL
2  FORMAT(10,2F0.0,10,2F0.0,2I0)
   IX=0
   DO 10 I=1,N
   READ(6)NV,T,(A(JJ),JJ=1,NV)
   IF(T-TMIN)10,10,0
   IX=IX+1
   XX(IX)=T
   YY(IX)=A(K)
10  CONTINUE
   CALL CHAANG(0.)
   CALL AXIPDS(1,3.,Y0,YAXIS,2)
   CALL AXISCA(3,INT,YMIN,YMAX,2)
   UYPCM=(YMAX-YMIN)/YAXIS
   IF(LL)0,0,11
   CALL AXIPDS(1,3.,Y0,TAXIS,1)
   CALL AXISCA(3,ITAXIS,TMIN,TMIN+ITAXIS/PS,1)
   CALL AXIDRA(1,1,1)
   XPOS=TFI-4./PS
   YPOS=YMIN-UYPCM*1.5
   CALL GRAMOV(XPOS,YPOS)
   CALL CHAARR(IARRAY(1,14),16,1)
11  CALL AXIDRA(-1,-1,2)
   XPOS=TMIN-1.5/PS
   YPOS=YMAX-4.*UYPCM
   CALL GRAMOV(XPOS,YPOS)
   CALL CHAANG(90.)
   CALL CHAARR(IARRAY(1,K),16,1)
   CALL GRAPDL(XX,YY,IX)
   IF(LL.EQ.1)CALL PICCLE
   REMIND 6
20  CONTINUE
   CALL DEVEND
   RETURN
END

```

```
SUBROUTINE GMPRD(A,B,R,N,M,L,MM,NN,LL)
DIMENSION A(MM),B(NN),R(LL)
```

```
IR=0
```

```
IK=-M
```

```
DO 10 K=1,L
```

```
IK=IK+M
```

```
DO 10 J=1,N
```

```
IR=IR+1
```

```
JI=J-N
```

```
IB=IK
```

```
R(IR)=0.
```

```
DO 10 I=1,M
```

```
JI=JI+N
```

```
IB=IB+1
```

```
10 R(IR)=R(IR)+A(JI)*B(IB)
```

```
RETURN
```

```
END
```

```
SUBROUTINE PROD(A,B,R,N,M,NN,MM,LL)
```

```
DIMENSION A(NN),B(MM),R(LL)
```

```
I=0
```

```
DO 10 J=1,M
```

```
DO 10 K=1,N
```

```
I=I+1
```

```
10 R(I)=A(I)*B(J)
```

```
RETURN
```

```
END
```

```
SUBROUTINE GMTRA(A,R,N,M,MM)
```

```
DIMENSION A(MM),R(MM)
```

```
IR=0
```

```
DO 10 I=1,N
```

```
IJ=I-M
```

```
DO 10 J=1,M
```

```
II=IJ+N
```

```
IR=IR+1
```

```
10 R(IR)=A(IJ)
```

```
RETURN
```

```
END
```

```

SUBROUTINE MINV(A,N,D,L,M,MM,NN)
DIMENSION A(MM),L(NN),M(NN)
C   SEARCH FOR LARGEST ELEMENT
D=1.
NK=-N
DO 80 K=1,N
NK=NK+N
L(K)=K
M(K)=K
KK=NK+K
BIGA=A(KK)
DO 20 J=K,N
IZ=N*(J-1)
DO 20 I=K,N
IJ=IZ+I
10 IF(ABS(BIGA)-ABS(A(IJ)))15,20,20
15 BIGA=A(IJ)
L(K)=I
M(K)=J
20 CONTINUE
C   INTERCHANGE ROWS
J=L(K)
IF(J-K)35,35,25
25 KI=K-N
DO 30 I=1,N
KI=KI+N
HOLD=-A(KI)
JI=KI-K+J
A(KI)=A(JI)
30 A(JI)=HOLD
C   INTERCHANGE COLUMNS
35 I=M(K)
IF(I-K)45,45,38
38 JP=N*(I-1)
DO 40 J=1,N
JK=NK+J
JI=JP+J
HOLD=-A(JK)
A(JK)=A(JI)
40 A(JI)=HOLD
C   DIVIDE COLUMN BY MINUS PIVOT HVALUE OF PIVOT ELEMENT IS
C   CONTAINED IN BIGA)
45 IF(ABS(BIGA)-.1E-20)46,46,48
46 D=0.
RETURN
48 DO 55 I=1,N
IF(I-K)50,55,50
50 IK=NK+I
A(IK)=A(IK)/(-BIGA)
55 CONTINUE

```

```

C      REDUCE MATRIX
DO 65 I=1,N
IK=NK+I
HOLD=A(IK)
IJ=I-V
DO 65 J=1,N
IJ=IJ+N
IF(I-K)60,65,60
60 IF(J-K)62,65,62
62 KJ=IJ-I+K
A(IJ)=HOLD+A(KJ)+A(IJ)
65 CONTINUE
C      DIVIDE ROW BY PIVOT
KJ=K-V
DO 75 J=1,N
KJ=KJ+N
IF(J-K)70,75,70
70 A(KJ)=A(KJ)/BIGA
75 CONTINUE
C      PRODUCT OF PIVOTS
D=D*BIGA
C      REPLACE PIVOT BY RECIPROCAL
A(KK)=1./BIGA
80 CONTINUE
C      FINAL ROW AND COLUMN INTERCHANGE
K=H
100 K=(K-1)
IF(K)150,150,105
105 I=L(K)
IF(I-K)120,120,108
108 JQ=N*(K-1)
JR=N*(I-1)
DO 110 J=1,N
JK=JQ+J
HOLD=A(JK)
JI=JR+J
A(JK)=-A(JI)
110 A(JI)=HOLD
120 J=H(K)
IF(J-K)100,100,125
125 KI=K-V
DO 130 I=1,N
KI=KI+N
HOLD=A(KI)
JI=KI-K+J
A(KI)=-A(JI)
130 A(JI)=HOLD
GO TO 100
150 RETURN
END

```

APPENDIX D

Inverter and Motor Details

D.1 *The inverter:*

The principal components of the inverter system are:

a) Main and auxiliary thyristors:

Westcode semiconductors, type P036QH12FG0,
average on-state current 36A,
maximum repetitive peak reverse voltage 1200V,
circuit commutation turn off time 35 μ s.

b) Return diodes:

Westcode semiconductors, type SF6GN64,
average on-state current 64A,
maximum repetitive peak reverse voltage 600V.

c) Commutating capacitors (C_{t_1} and C_{t_2})

8 μ F - 1000V each.

d) Commutating inductors (L_{t_1} , L_{t_2} , ...)

180 turns, 2.5 mm², 15A continuous rating, 1.1 mH each.

e) Smoothing capacitors (C_{sh})

made up to 5000 μ F across the inverter main and
auxiliary supplies, using 10,000 μ F, 100V electro-
lytic units.

D.2 *The motor:*

1 HP, 6-pole, 50 Hz, 220/240V (for Δ connection)

moment of inertia $J = 0.045 \text{ kg.m}^2$

$$R_s, R_r = 5.09\Omega$$

$$L_{ss}, L_{rr} = 0.499\text{H}$$

$$L_{sm}, L_{rm} = -0.233\text{H}$$

$$L_{s\ell}, L_{r\ell} = 0.034\text{H}$$

$$L_e = 0.697\text{H}$$

$$M_{sr} = 0.465\text{H}$$

all values are referred to stator.

APPENDIX E

Calculation of the rotor currents following stator supply disconnection

When the stator of the induction motor is disconnected from the supply, the stator currents will normally fall to zero in an extremely short time, dependent on the characteristics of the switch, and this time is assumed here to be zero. Since the flux in the machine cannot change instantaneously, the rotor currents adjust themselves in such a way as to maintain constant flux linkages with both the stator and the rotor windings immediately before and after the switch is opened.

In terms of the transformed stationary 3-axis variables, the flux linkage of the rotor circuit with respect to the currents immediately before disconnection are:

$$\begin{bmatrix} \psi_{r\alpha} \\ \psi_{r\beta} \\ \psi_{r\gamma} \end{bmatrix} = \begin{bmatrix} M_{sr} & -\frac{1}{2}M_{sr} & -\frac{1}{2}M_{sr} & L_{rr} & L_{rm} & L_{rm} \\ -\frac{1}{2}M_{sr} & M_{sr} & -\frac{1}{2}M_{sr} & L_{rm} & L_{rr} & L_{rm} \\ -\frac{1}{2}M_{sr} & -\frac{1}{2}M_{sr} & M_{sr} & L_{rm} & L_{rm} & L_{rr} \end{bmatrix} \begin{bmatrix} i_{s\alpha_1} \\ i_{s\beta_1} \\ i_{s\gamma_1} \\ i_{r\alpha_1} \\ i_{r\beta_1} \\ i_{r\gamma_1} \end{bmatrix}$$

E.1.

(where suffix 1 denotes the currents immediately before disconnection) or with respect to the rotor currents ($i_{r\alpha_2}$, $i_{r\beta_2}$, $i_{r\gamma_2}$)

immediately after disconnection.

$$\begin{bmatrix} \psi_{r\alpha} \\ \psi_{r\beta} \\ \psi_{r\gamma} \end{bmatrix} = \begin{bmatrix} L_{rr} & L_{rm} & L_{rm} \\ L_{rm} & L_{rr} & L_{rm} \\ L_{rm} & L_{rm} & L_{rr} \end{bmatrix} \begin{bmatrix} i_{r\alpha_2} \\ i_{r\beta_2} \\ i_{r\gamma_2} \end{bmatrix} \quad \text{E.2}$$

From the two equations E.1, and E.2, the new rotor currents are:

$$\begin{bmatrix} i_{r\alpha_2} \\ i_{r\beta_2} \\ i_{r\gamma_2} \end{bmatrix} = \begin{bmatrix} L_{rr} & L_{rm} & L_{rm} \\ L_{rm} & L_{rr} & L_{rm} \\ L_{rm} & L_{rm} & L_{rr} \end{bmatrix}^{-1} \begin{bmatrix} M_{sr} & -\frac{1}{2}M_{sr} & -\frac{1}{2}M_{sr} & L_{rr} & L_{rm} & L_{rm} \\ -\frac{1}{2}M_{sr} & M_{sr} & -\frac{1}{2}M_{sr} & L_{rm} & L_{rr} & L_{rm} \\ -\frac{1}{2}M_{sr} & -\frac{1}{2}M_{sr} & M_{sr} & L_{rm} & L_{rm} & L_{rr} \end{bmatrix} \begin{bmatrix} i_{s\alpha_1} \\ i_{s\beta_1} \\ i_{s\gamma_1} \\ i_{r\alpha_1} \\ i_{r\beta_1} \\ i_{r\gamma_1} \end{bmatrix}$$

E.3

APPENDIX F

Inverter d.c. Supply Voltage

The peak fundamental component V_{peak} of a 6-step square waveform of amplitude V_A (Fig. F.1) is:

$$V_{\text{peak}} = \frac{3}{\pi} V_A \quad \dots \quad \text{F.1}$$

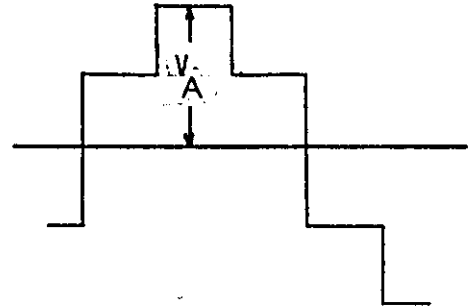


FIG. F.1

If the waveform represents the phase voltage of a motor driven from a 180° inverter, then the d.c. supply voltage to the inverter (E), for a fundamental phase voltage equal to the nominal motor phase voltage V_{max} after allowing say 2% regulation is,

$$\begin{aligned} E &= \frac{3}{2} \times \frac{\pi}{3} \times 1.02 \times V_{\text{max}} \\ &= 1.61 V_{\text{max}} \end{aligned}$$

F.2

Substituting 310.6V for V_{max} into equation F.2, for 50 Hz frequency operation:

$$E = 1.61 \times 310.6 = 500\text{V}$$

at any other operational frequency F_s the value of E to maintain approximately constant flux inside the motor is calculated from:

$$E = \frac{500}{50} \times F_s = 10F_s$$

F.3

although this is not applicable for very small values of F_s .

When the motor is supplied from a 120° inverter, the phase voltage waveform is different from that shown above, except when the motor is running on light load. To avoid saturation inside the machine, the same voltage E is therefore used for both 120° and 180° inverters.

APPENDIX G

Steady-state Motor Torque Equation

From the single-phase equivalent circuit of an induction motor (Fig. G.1), the steady-state torque equation is given by⁷¹

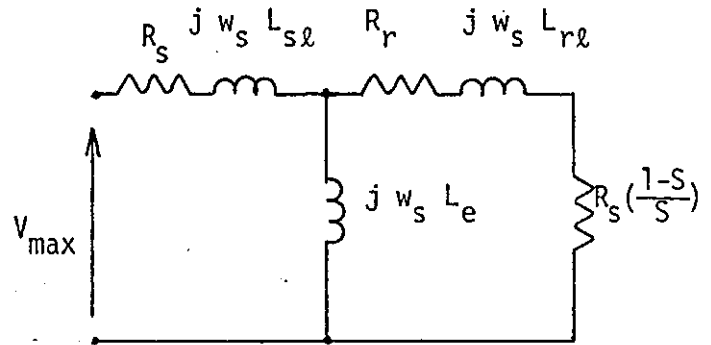


FIG. G.1

$$T_e = \frac{1.5 V_{1a}^2 (R_r/s)}{w_s [(R_{Th} + R_r/s)^2 + w_s^2 (L_{Th} + L_{r\ell})^2]} \quad \text{G.1}$$

where:

$$V_{1a} = V_{\max} \frac{j w_s L_e}{R_s + j w_s (L_{s\ell} + L_e)}$$

and $R_{Th} + j w_s L_{Th} = R_s + j w_s L_{s\ell}$ in parallel with $j w_s L_e$

The maximum torque is given by:

$$T_{\max} = \frac{0.75 V_{1a}^2}{w_s [R_{Th} + \sqrt{R_{Th}^2 + w_s^2 (L_{Th} + L_{r\ell})^2}]} \quad \text{G.2}$$

APPENDIX H
PUBLISHED WORK

1. Al-Nimma, D.A., and Williams, S.,

"Analysis of inverter-induction motor drives using a digital computer".

Proc. of the Twelfth Universities Power Engineering Conference, Brunel University, April 1977.

2. Al-Nimma, D.A., and Williams, S.,

"Computation of inverter-induction motor drives using a tensor technique".

Proc. of the International Conference on Electrical Machines, Brussels, Belgium, Sept. 1978.

3. Al-Nimma, D.A., and Williams, S.,

"Modelling a variable-frequency induction motor drive",
IEE Journal on Electric Power Applications, Vol. 2,
No. 4, Aug. 1979, pp.132-134.

ANALYSIS OF INVERTOR-INDUCTION MOTOR DRIVES USING A DIGITAL COMPUTER

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INTRODUCTION: Recently, the three phase d.c. link inverter has found a wide application in speed control schemes for induction motor drives, and several methods of analysing such systems have been presented.^{1,2} For an inverter with 180° conduction periods, a common approach¹ is to assume that the motor is supplied from a precisely defined rectangular waveform. Alternatively² the symmetry of the inverter-motor unit has been invoked to allow analysis to be performed over a part of the supply cycle. While suitable for steady state conditions, these methods may be unsuitable for transient operation where the inverter waveform will be influenced by the motor conditions and supply impedances. Additionally, for inverters with 120° conduction periods, portions of the terminal voltage waveform will depend on the rotor currents and consequently may be difficult to define precisely.

To overcome these problems, a tensor technique based on the work of Kron^{3,4} has been established both to deal efficiently with the varying topology of the inverter circuit and to allow the whole system to be analysed simultaneously. Using this technique, investigations of unusual operating and fault conditions present no problem.

TENSOR ANALYSIS OF THE INVERTOR: The most common d.c. link inverter is the three phase bridge shown in Fig. 1, where the input voltage E is frequently derived from a three-phase bridge controlled rectifier. The sequence of thyristor switching is 1 6 3 2 5 4, and a total of 6 circuit configurations are possible in the case of an inverter with 180° conduction periods. For 120° conduction periods the situation is more complicated and 12 different topologies are possible.

In order to facilitate the analysis of the circuit, the following assumptions are made:

- All the diodes and thyristors are perfect switches.
- Each diode/thyristor combination is treated as one element since they never conduct simultaneously.

The circuit of Fig. 1 (considering the stator as a load only) has 11 branches and 6 nodes; 6 independent currents $[i_n]$ (i.e. $i_a \rightarrow i_f$) are therefore necessary to completely define the system. With these chosen to be as far as possible the thyristor currents (the last being the source current) a transformation tensor linking the primitive currents of Fig. 1 and the independent currents follows as

$$[C_p] = \begin{bmatrix} 1 & & & & & & & & & & \\ & 1 & & & & & & & & & \\ & & 1 & & & & & & & & \\ & & & 1 & & & & & & & \\ & & & & 1 & & & & & & \\ & & & & & 1 & & & & & \\ -1 & -1 & -1 & -1 & -1 & & & & & & \dots 1 \\ & 1 & 1 & & & & & & & & \\ & & & 1 & 1 & & & & & & \\ -1 & -1 & -1 & -1 & & & & & & & \\ -1 & & -1 & & -1 & 1 & & & & & \\ & & & & & 1 & & & & & \\ & & & & & & 1 & & & & \end{bmatrix}$$

The primitive resistance tensor $[R_p]$ and the primitive inductance tensor $[L_p]$ are given by:

$$[R_p] = \text{diag} [0, 0, 0, 0, 0, 0, 0, r_a, r_b, r_c, r_{sh}, r_s, 0] \quad \dots 2a$$

$$[L_p] = \text{diag} [0, 0, 0, 0, 0, 0, 0, L_a, L_b, L_c, 0, L_s, 0] \quad \dots 2b$$

and the primitive voltage tensor $[V_p]$ by

$$[V_p] = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, E] \quad \dots 3$$

If the voltage across the capacitor is treated as the source voltage, the primitive capacitor voltage tensor $[SQ_p]$ is given by,

$$[SQ_p] = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, Q/C_{sh}, 0, 0] \quad \dots 4$$

where Q is the charge on the capacitor. Since this is an additional independent variable, $[SQ_p]$ must be calculated at every step.

Having determined the primitive impedance and voltage tensors, the equations for the independent system (i.e. $[V_n] = [R_n][i_n] + [L_n][i_n'] + [SQ_n]$) could be established using the familiar relationships:

$$[R_n] = [C_p]^t [R_p] [C_p], \quad [L_n] = [C_p]^t [L_p] [C_p] \quad \dots 5a$$

$$[V_n] = [C_p]^t [V_p], \quad [SQ_n] = [C_p]^t [SQ_p] \quad \dots 5b$$

Although the approach above is mathematically rigorous, since the operation of the inverter is controlled entirely by the triggering and commutation of the thyristors, in practice only two or three thyristors may conduct simultaneously. Consequently, only two or three independent currents are necessary, and this leads us to form another tensor $[C_n]$ which links the new currents $[i_{nn}]$ and the independent currents $[i_n]$. In the program this tensor is produced by a special subroutine whenever the circuit topology changes.

The new resistance, inductance and voltage tensors are therefore calculated as follows:

$$[R_{nn}] = [C_n]^t [R_n] [C_n], \quad [L_{nn}] = [C_n]^t [L_n] [C_n] \quad \dots 6a$$

$$[V_{nn}] = [C_n]^t [V_n], \quad [SQ_{nn}] = [C_n]^t [SQ_n] \quad \dots 6b$$

Solution of the network equation:

$$[i_{nn}'] = [L_{nn}]^{-1} \{ [R_{nn}][i_{nn}] + [SQ_{nn}] - [V_{nn}] \} \quad \dots 7a$$

$$Q' = [C_K][i_{nn}] \quad \dots 7b$$

(where $[C_K]$ is a row matrix linking $[i_{nn}]$ and the capacitor current) can now proceed numerically. Once these variables have been found, the currents $[i_n]$ and all branch currents $[i_p]$ can be determined from the primitive transformation tensor using the relationships:

$$[i_n] = [C_n][i_{nn}], \quad [i_p] = [C_p][i_n] \quad \dots 8$$

The stator voltages can be easily calculated using the stator impedances and the stator currents and their derivatives.

EXTENDING THE ANALYSIS TO INCLUDE THE INDUCTION MOTOR: For ease of establishing the validity of the inverter model, in the previous section it was assumed that the load was simply a star connected inductive impedance, and if a simple induction motor model is adopted, this could well form the stator of such a system. The extension to include the rotor circuit merely requires:

- additional terms in the enlarged connection and impedance tensors.
- the inclusion of the well-known dynamic equation of the motor¹.

Direct solution of the machine equations in phase quantities is used since this permits a wide class of operating conditions to be studied conveniently. In this case it is more convenient to solve for fluxes $[\psi]$ (from $[d\psi] = [R_{nn}][i_{nn}] + [SQ_{nn}] - [V_{nn}]$, the core losses, saturation effects and space harmonics being neglected) rather than for currents and to obtain the currents from:

$$[i_{nn}] = [L_{nn}]^{-1} [\psi] \quad \dots 9$$

COMPUTER PROGRAM AND RESULTS: A simplified flow chart of the computer program used in the analysis of the inverter-motor system is shown in Fig. 2. The fourth-order Runge-Kutta numerical integration procedure is used in the solution of the system equations, and an appropriate step length to ensure stability is chosen from a determination of the system eigenvalues. The program operation is controlled by a system state array (which has six elements associated with the six bridge thyristors) and this is updated whenever a change in the state of the bridge occurs. A special subroutine then uses this array to produce the correct $[C_n]$ tensor.

Two sets of computed results are presented. Fig. 3 shows motor phase current, electric torque and motor phase voltage for an inverter with 180° conduction periods. Fig. 4 shows the same information but for an inverter with 120° conduction periods. Both sets of results show the machine running up from rest under no load, until at $t = 0.3$ sec full load torque is applied.

The experimental data is taken from the results of a previous study⁵ and is as follows

Inverter	$E = 540 \text{ V}$	$\bar{r}_{sh} = 0.05 \Omega$
	$R_s = 0.5 \Omega$	$C_{sh} = 5250 \mu\text{F}$
	$L_s = 0.02 \text{ H}$	

Motor: Name plate details, 3.7 KW 230 V 4 pole 50 Hz
 Stator resistance = 1.28Ω Rotor resistance = 1.3Ω
 Self inductances of three phase stator and of rotor circuits = 0.118 H
 Mutual inductances between stator phases and between rotor phases = 0.054 H
 Maximum mutual inductance between three phase stator and rotor circuits = 0.108 H
 (All values referred to stator).

CONCLUSION: An efficient method of analysing an inverter motor system using a digital computer has been presented. The technique is able to cope with all operating conditions of the system, including transient, unbalanced and fault conditions and it may be used to study systems with both 180° and 120° inverter conduction modes.

For the computed results obtained, excellent agreement has been shown to exist with experimental results obtained previously for steady state operation⁵, and this gives a high degree of confidence in the ability of the program to solve transient and fault conditions satisfactorily.

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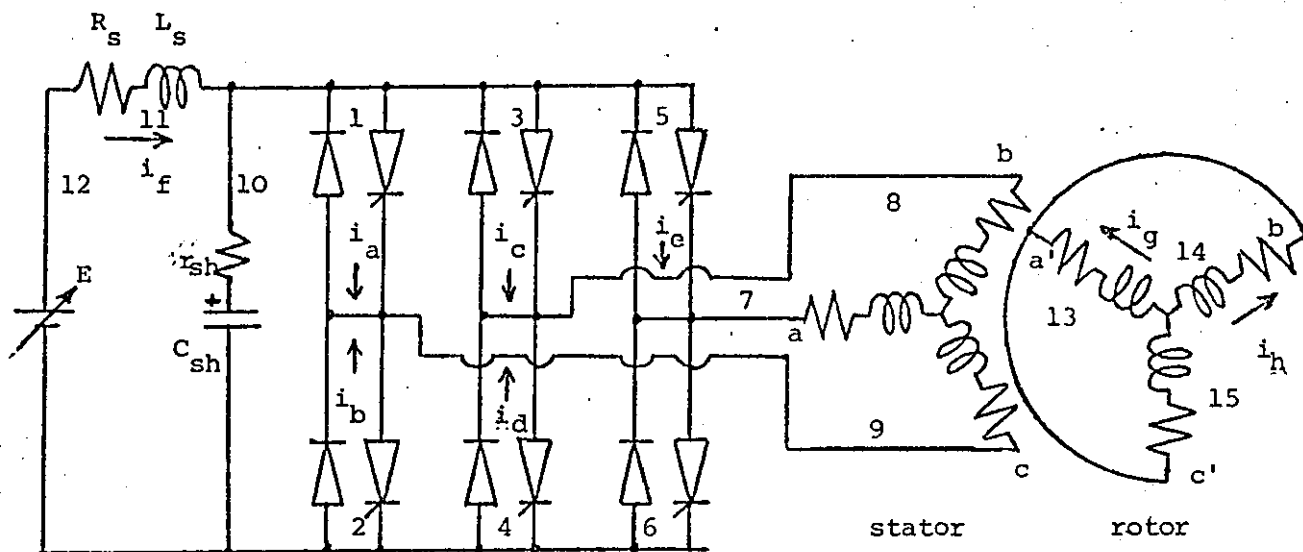


Fig. 1 The inverter motor circuit

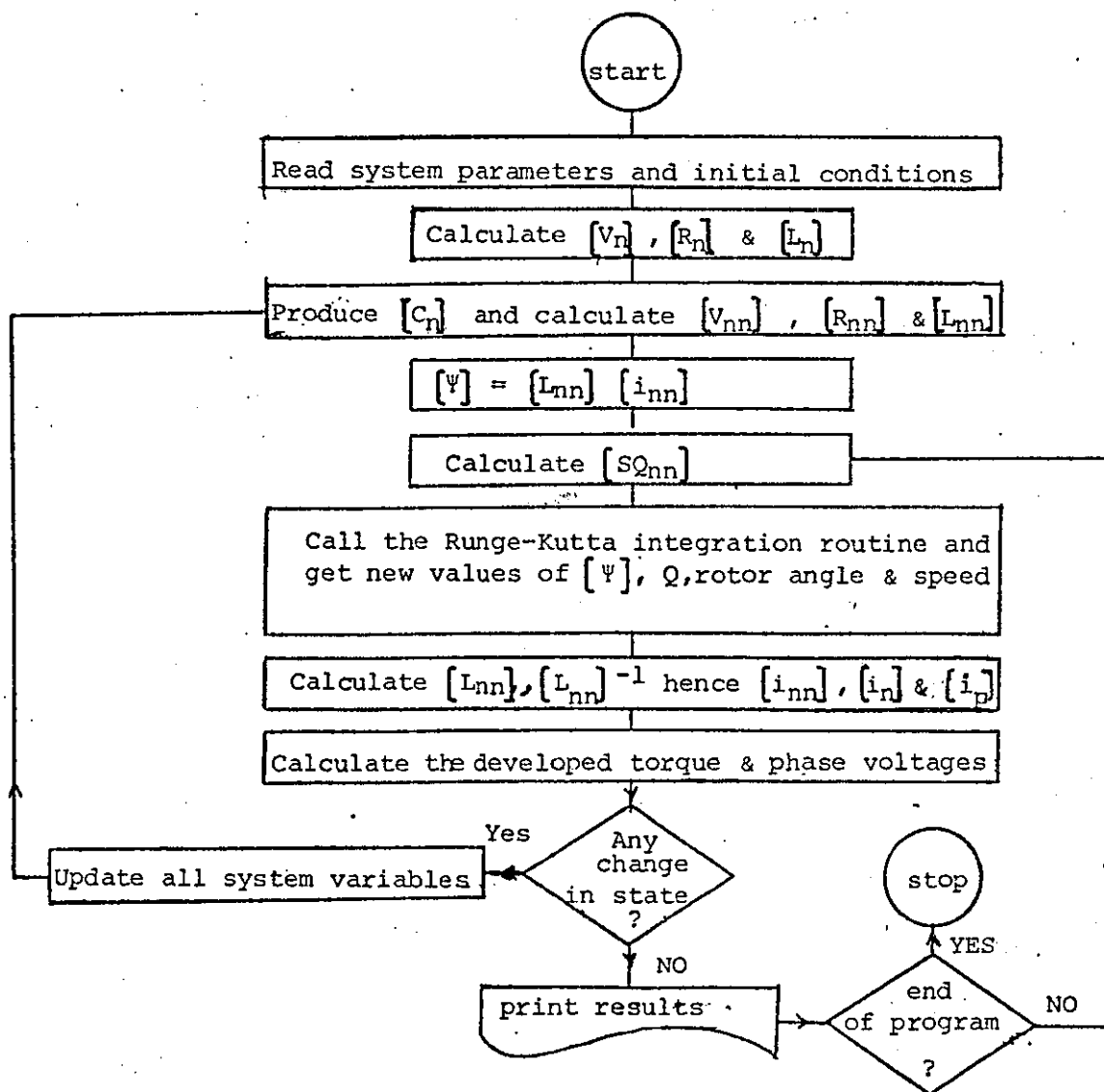
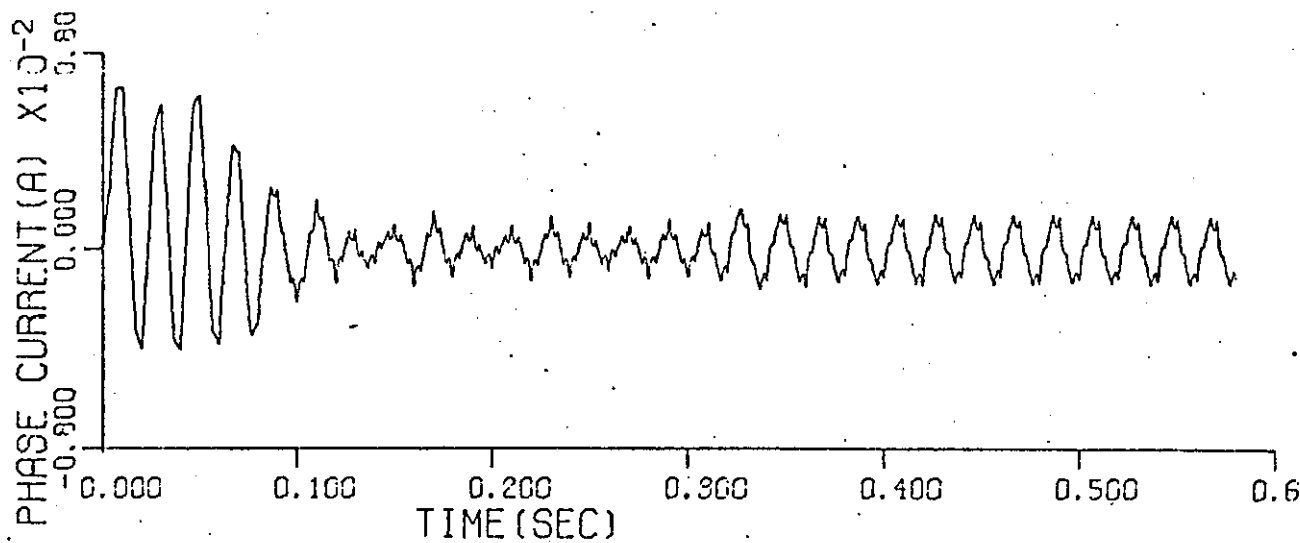
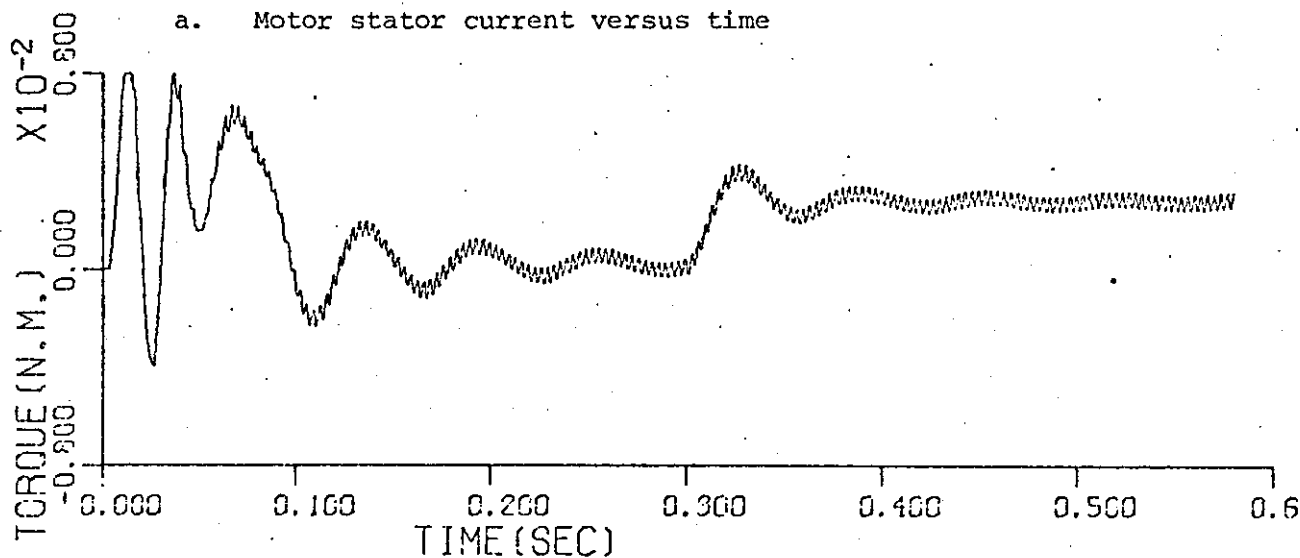


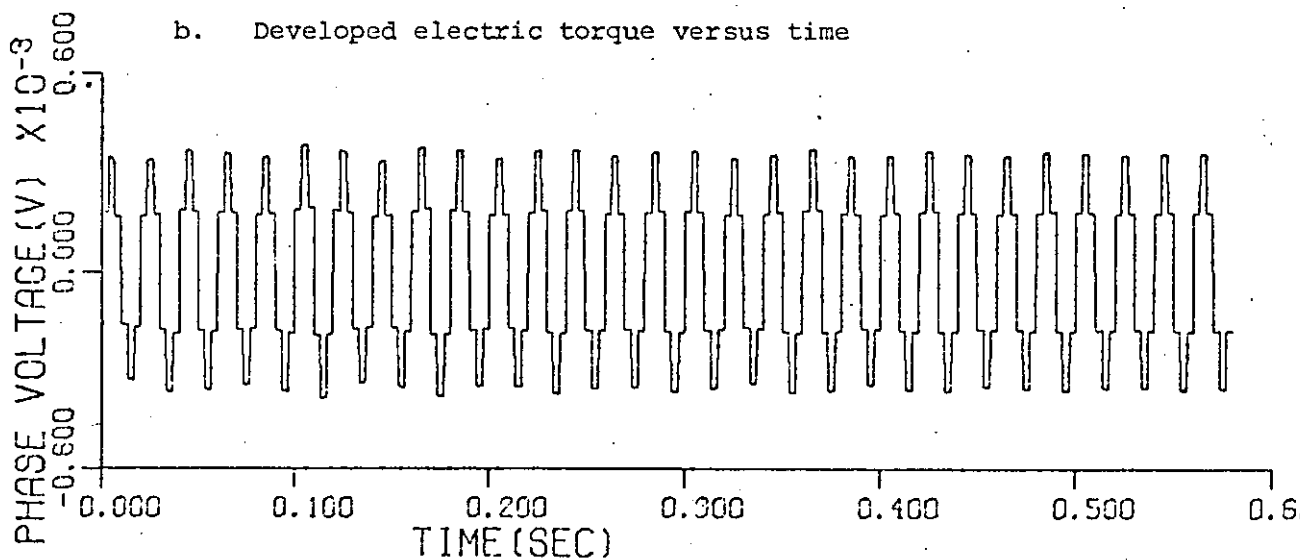
Fig. 2. Computer program flow chart



a. Motor stator current versus time

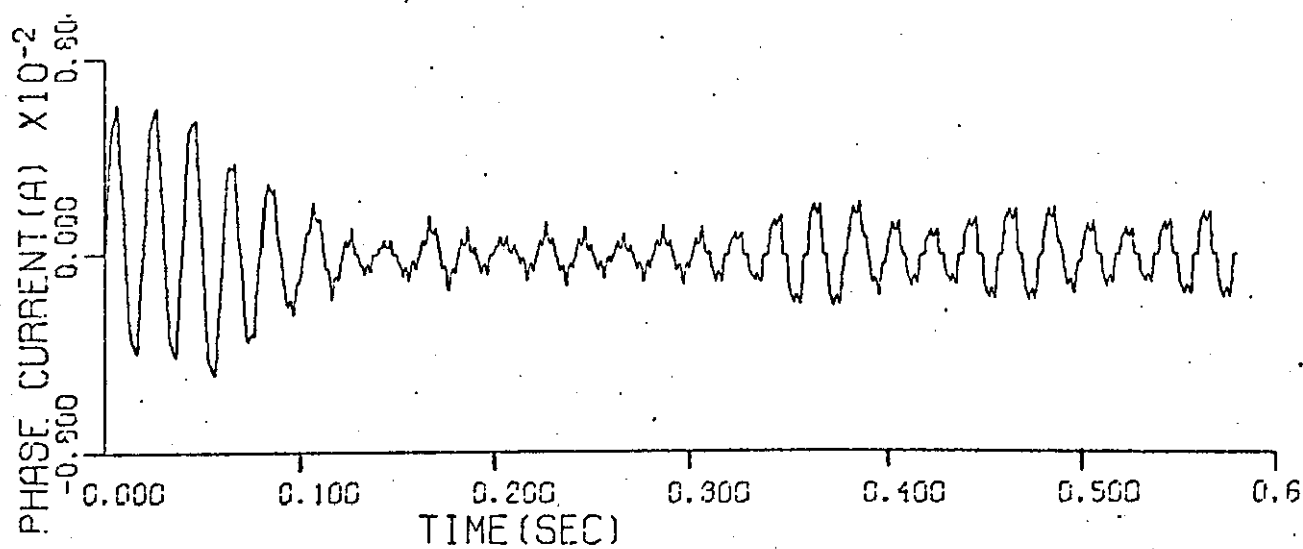


b. Developed electric torque versus time

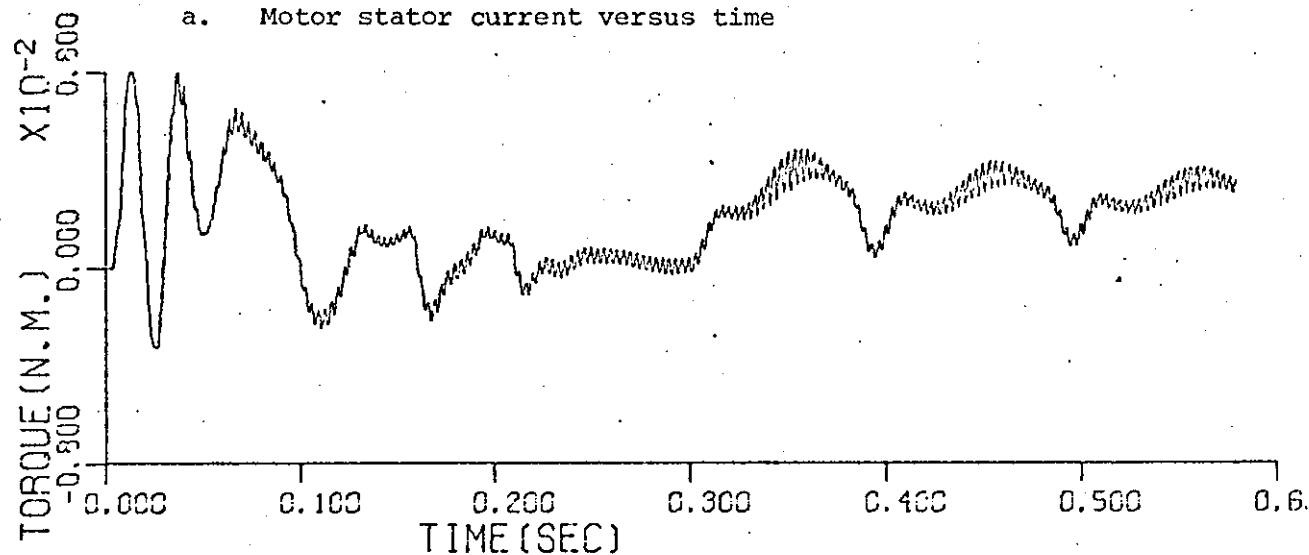


c. Motor stator voltage versus time

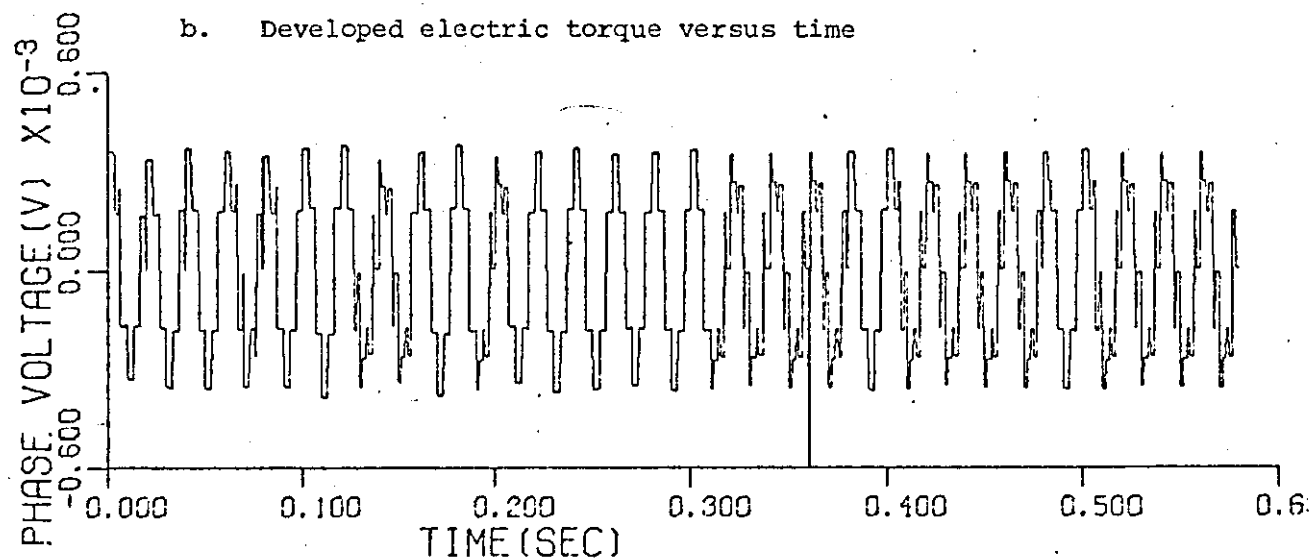
Fig. 3 Typical results for 180° operation of the invertor



a. Motor stator current versus time



b. Developed electric torque versus time



c. Motor stator voltage versus time

Fig. 4 Typical results for 120° operation of the inverter

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ABSTRACT

The paper describes a method which has been developed for the analysis of inverter-induction motor drive systems. To ensure a correct digital computer solution during all operating and fault conditions the technique is based on the work of Gabriel Kron, and can cope efficiently with the varying topology of the inverter circuit. A direct-phase model has been adopted for the induction motor and good agreement has been obtained between computed and experimental results, using both 180° and 120° conduction period inverters, over a wide range of transient and steady-state conditions.

INTRODUCTION

Recently, the three phase d.c. link inverter has found a wide application in speed control schemes for induction motor drives, and several methods of analysing such systems have been presented.^{1,2,3,4} For an inverter with 180° conduction periods, a common approach¹ is to assume that the motor is supplied from a precisely-defined rectangular waveform. Other papers have been based on this assumption, but have analysed the machine performance using techniques such as a Fourier analysis² and transition matrices³, or have invoked the symmetry of the inverter-motor unit to allow analysis to be performed over a part of the supply cycle.⁴ While suitable for steady state conditions, these methods may be unsuitable for transient operation where the inverter waveform will be influenced by the motor conditions and supply impedances. Furthermore, with inverters employing 120° conduction periods, one or more of the machine terminals will be open-circuited for various periods during each supply cycle, and during transients the machine terminal voltage wave-shape will change significantly. Thus the analysis of this type of inverter is by no means straightforward.

To overcome these problems, a tensor technique based on the work of Kron^{5,6} has been established both to deal efficiently with the varying topology of the inverter circuit and to allow the whole system to be analysed simultaneously. Using this technique, investigations of unusual operating and fault conditions present no problem.

TENSOR ANALYSIS OF THE INVERTOR

The most common d.c. link inverter is the three phase bridge shown in Fig.1, where the input voltage E is frequently derived from a three-phase bridge controlled rectifier. The sequence of thyristor switching is 1 6 3 2 5 4, and a total of 6 circuit configurations are possible in the case of an inverter with 180° conduction periods. For 120° conduction periods the situation is more complicated and 12 different topologies are possible.

In order to facilitate the analysis of the circuit, the following assumptions are made:

- a. All the diodes and thyristors are perfect switches.
- b. Each diode/thyristor combination is treated as one element since they never conduct simultaneously.

The circuit of Fig. 1 (considering the stator as a load only) has 11 branches and 6 nodes; 6 independent currents $[i_n]$ (i.e. $i_a \rightarrow i_f$) are therefore necessary to completely define the system. With these chosen to be as far as possible the thyristor currents (the last being the source current) a transformation tensor linking the primitive currents of Fig. 1 and the independent currents follows

$$[C_p] = \begin{bmatrix} 1 & & & & & & & & & & & & \\ & 1 & & & & & & & & & & & \\ & & 1 & & & & & & & & & & \\ & & & 1 & & & & & & & & & \\ & & & & 1 & & & & & & & & \\ -1 & -1 & -1 & -1 & -1 & & & & & & & & \\ 1 & 1 & & & & & & & & & & & \\ & & 1 & 1 & & & & & & & & & \\ -1 & -1 & -1 & -1 & & & & & & & & & \\ -1 & & -1 & & & -1 & & 1 & & & & & \\ & & & & & & & 1 & & & & & \\ & & & & & & & & 1 & & & & \end{bmatrix} \quad (1)$$

The primitive resistance tensor $[R_p]$, the primitive inductance tensor $[L_p]$ and the primitive inverse of capacitance tensor $[S_p]$ are given by

$$[R_p] = \text{diag} [0, 0, 0, 0, 0, 0, 0, r_a, r_b, r_c, r_{sh}, r_s, 0] \quad (2a)$$

$$[L_p] = \text{diag} [0, 0, 0, 0, 0, 0, 0, L_a, L_b, L_c, 0, L_s, 0] \quad (2b)$$

$$[S_p] = \text{diag} [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1/C_{sh}, 0, 0] \quad (2c)$$

and the primitive voltage tensor $[V_p]$ by

$$[V_p] = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, E] \quad (3)$$

Having determined the primitive impedance and voltage tensors, the equations for the independent system (i.e. $[V_n] = [R_n][i_n] + [L_n][i_n'] + [S_n][i_{10}]$) could be established using the familiar relationships:

$$[R_n] = [C_p]^t [R_p] [C_p], \quad [L_n] = [C_p]^t [L_p] [C_p], \quad [S_n] = [C_p]^t [S_p] [C_p] \quad (4a)$$

$$[V_n] = [C_p]^t [V_p] \quad (4b)$$

Although the approach above is mathematically rigorous, since the operation of the inverter is controlled entirely by the triggering and commutation of the thyristors, in practice only two or three thyristors may conduct simultaneously. Consequently, only two or three independent currents are necessary, and this leads us to form another tensor $[C_n]$ which links the new currents $[i_{nn}]$ and the independent currents $[i_n]$. In the program this tensor is produced by a special subroutine whenever the circuit topology changes.

The new resistance, inductance, inverse of capacitance and voltage tensors are therefore calculated as follows:

$$[R_{nn}] = [C_n]^t [R_n] [C_n], [L_{nn}] = [C_n]^t [L_n] [C_n], [S_{nn}] = [C_n]^t [S_n] [C_n] \quad (5a)$$

$$[V_{nn}] = [C_n]^t [V_n] \quad (5b)$$

Solution of the network equations:

$$[i_{nn}] = [L_{nn}]^{-1} \{ [R_{nn}] [i_{nn}] + [S_{nn}] Q - [V_{nn}] \} \quad (6a)$$

$$Q' = [C_K] [i_{nn}] \quad (6b)$$

(where $[C_K]$ is a row matrix linking $[i_{nn}]$ and the capacitor current and Q is the charge on the capacitor) can now proceed numerically. Once these variables have been found, the currents $[i_n]$ and all branch currents $[i_p]$ can be determined from the primitive transformation tensor using the relationships:

$$[i_n] = [C_n] [i_{nn}], [i_p] = [C_p] [i_n] \quad (7)$$

The stator voltages can be easily calculated using the stator impedances and the stator currents and their derivatives.

EXTENDING THE ANALYSIS TO INCLUDE THE INDUCTION MOTOR

For ease of establishing the validity of the inverter model, in the previous section it was assumed that the load was simply a star connected inductive impedance, and if a simple induction motor model is adopted, this could well form the stator of such a system. The extension to include the rotor circuit merely requires:

- a. additional terms in the enlarged connection and impedance tensors.
- b. the inclusion of the well-known dynamic equation of the motor.¹

Direct solution of the machine equations in phase quantities is used since this permits a wide class of operating conditions to be studied conveniently. In this case it is more convenient to solve for fluxes

$$[\psi] \text{ (from } [d\psi] = [R_{nn}] [i_{nn}] + [S_{nn}] Q - [V_{nn}], \text{ the core losses, saturation}$$

effects and space harmonics being neglected) rather than for currents and to obtain the currents from:

$$\begin{bmatrix} i_{nn} \end{bmatrix} = \begin{bmatrix} L_{nn} \end{bmatrix}^{-1} \begin{bmatrix} \psi \end{bmatrix} \quad (8)$$

COMPUTER PROGRAM AND RESULTS

A simplified flow chart of the computer program used in the analysis of the inverter-motor system is shown in Fig. 2. The fourth-order Runge-Kutta numerical integration procedure is used in the solution of the system equations, and an appropriate step length to ensure stability is chosen from a determination of the system eigenvalues. The program operation is controlled by a system state array (which has six elements associated with the six bridge thyristors) and this is updated whenever a change in the state of the bridge occurs. A special subroutine then uses this array to produce the correct $\begin{bmatrix} C_n \end{bmatrix}$ tensor. Experimental results were taken on a small laboratory induction motor which had the following parameters:

Name-plate details: 1 h.p. 220/240 V 6 pole 50 Hz

Stator resistance per phase = 5.09 Ω Rotor resistance per phase = 5.09 Ω

Self inductances of rotor and stator windings = 0.499 H

Mutual inductances between stator phases and between rotor phases = 0.232 H

Maximum mutual inductance between 3 phase stator and rotor circuits = 0.465 H
(all values referred to stator).

Very good agreement between computed and experimental results was obtained during steady-state operation and as an illustration of the agreement obtained during transient conditions the very severe test of plugging was chosen. Fig. 3 shows practical and computed results of motor phase voltage and current for both 180° and 120° invertors during the test in which,

supply frequency = 25 Hz

period of disconnection = 80 milliseconds

inverter voltage	= 250 V
source impedance	= 0.5Ω , $0.02H$
filter branch	= 0.05Ω , $5000\ \mu F$

CONCLUSION

An efficient method of analysing an inverter motor system using a digital computer has been presented. The technique is able to cope with all operating conditions of the system, including transient, unbalanced and fault conditions and it may be used to study systems with both 180° and 120° inverter conduction modes.

Excellent agreement has been shown to exist between experimental and computed results giving a high degree of confidence in the method of analysis described.

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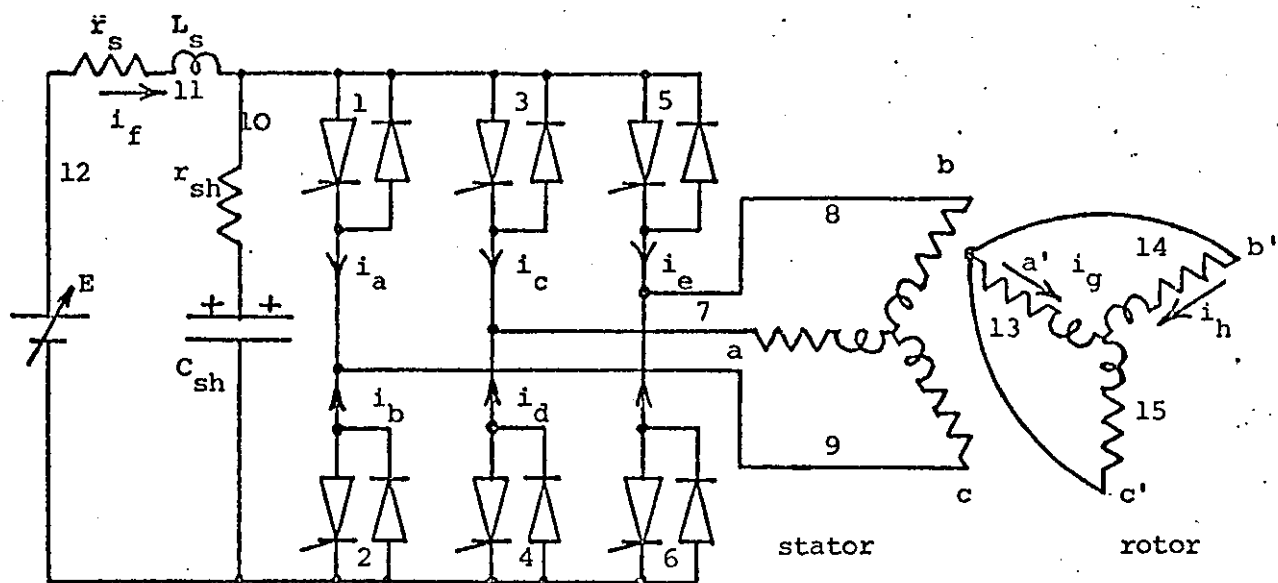


Fig. 1 The inverter motor circuit

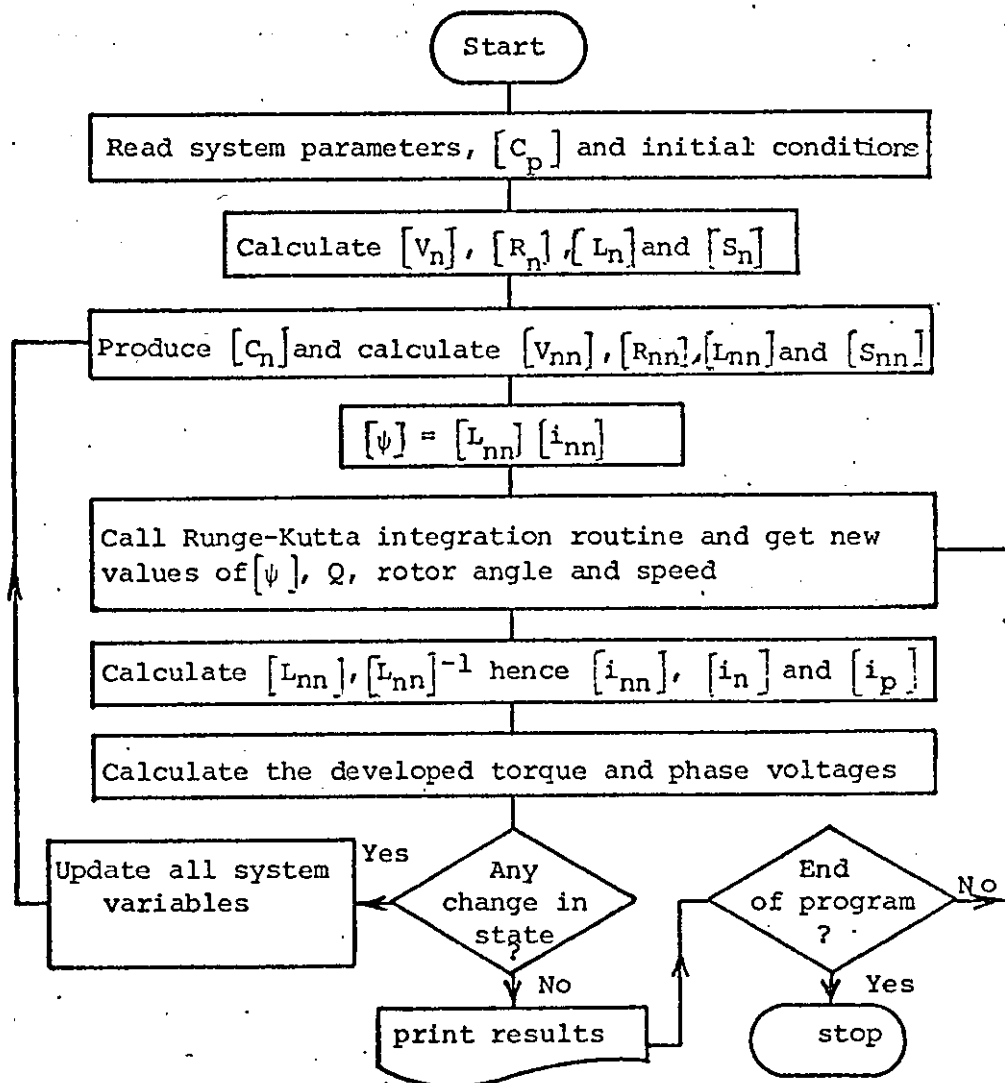


Fig. 2 Computer program flow chart

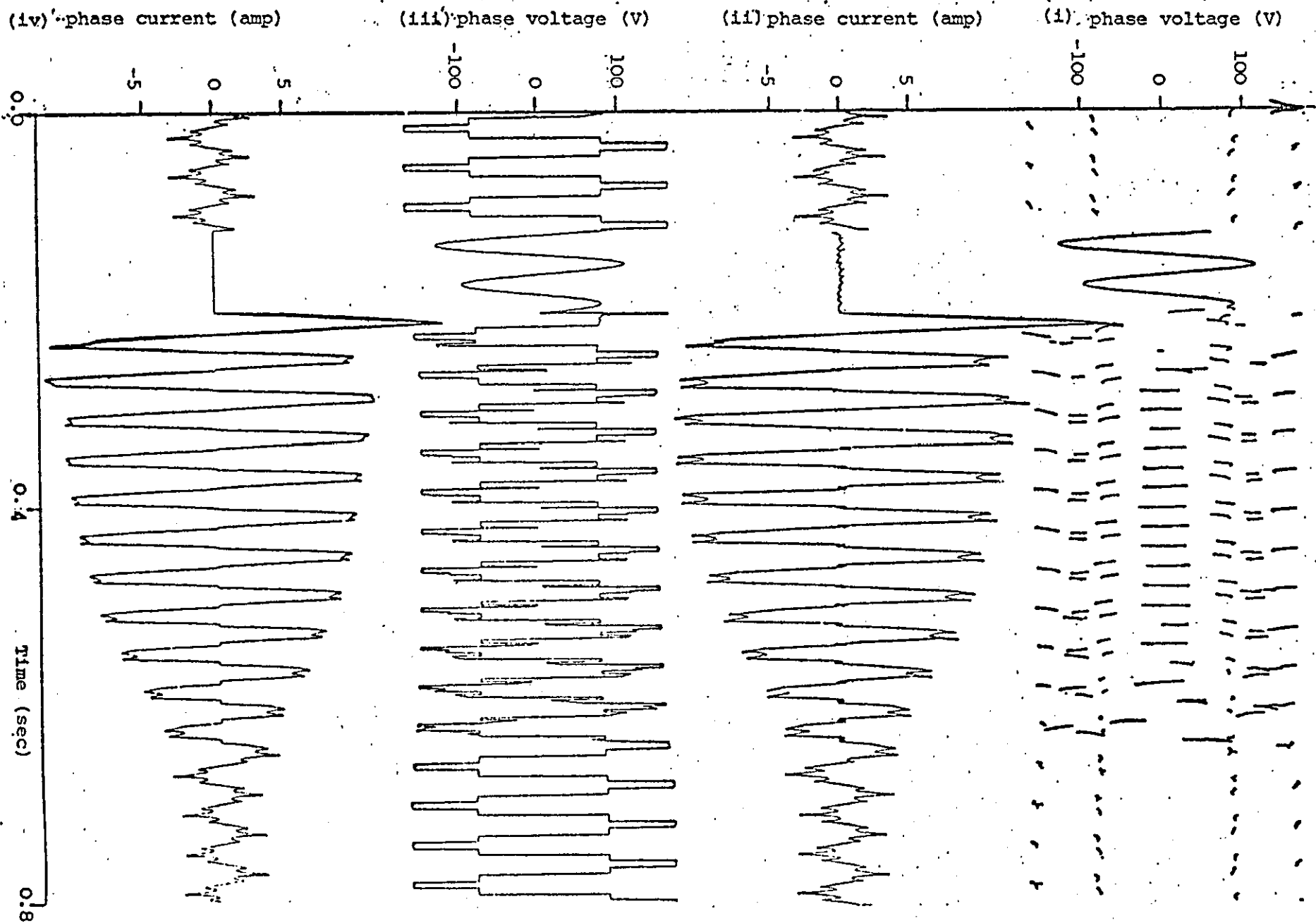


Fig. 3a: 120° operation, curves (i) and (ii) are experimental, curves (iii) and (iv) are computed.

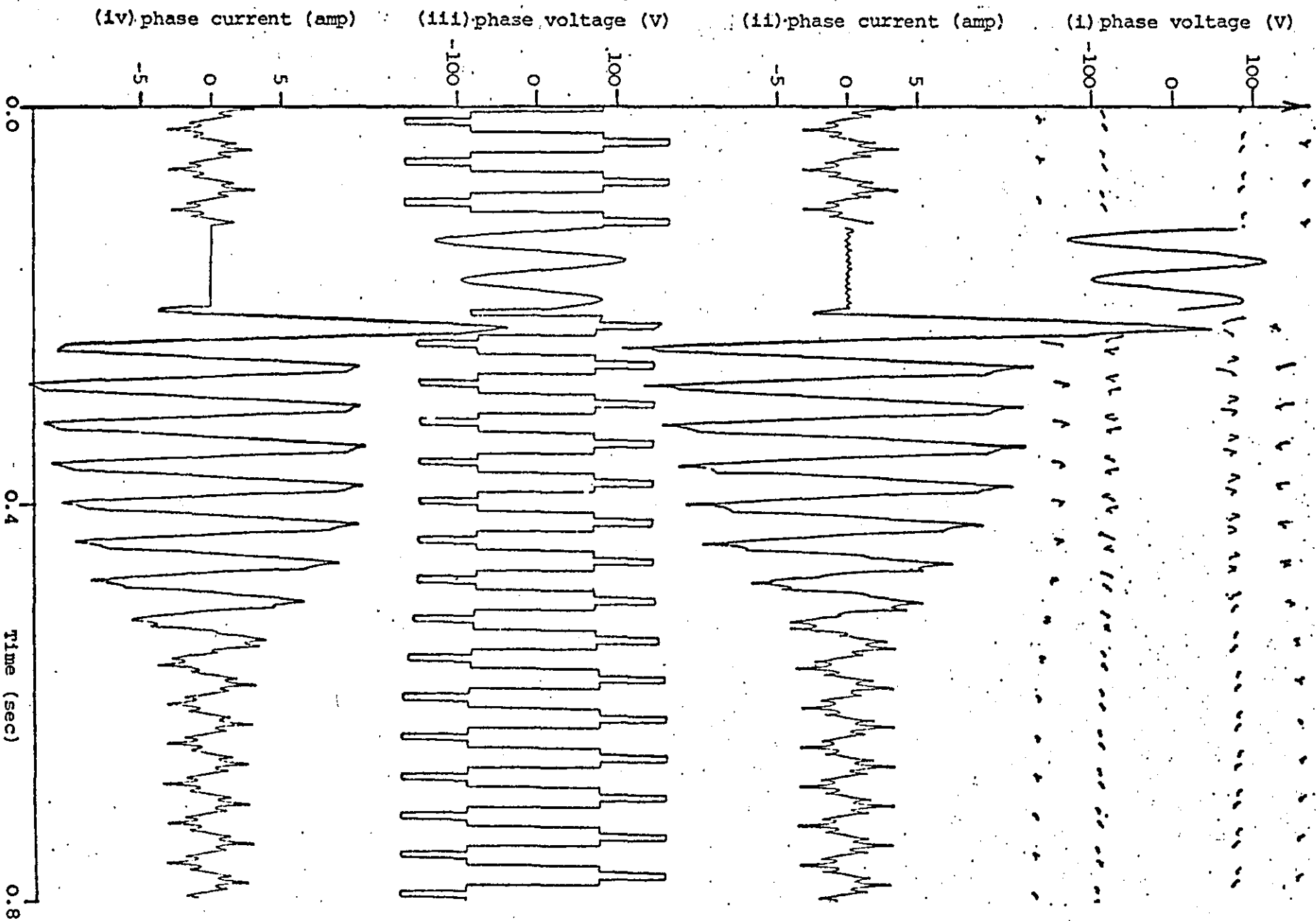


Fig. 3b: 180° operation, curves (i) and (ii) are experimental, curves (iii) and (iv) are computed.

Modelling a variable-frequency induction-motor drive

Dhiya A.B. Al-Nimma and Stephen Williams

Indexing terms: Electric drives, Induction motors, Modelling

Abstract: A tensor method has been developed for the analysis of inverter-induction-motor drives using a digital computer. Good agreement has been obtained between computed and experimental results, over a wide range of transient and steady-state conditions.

1 Introduction

The tensor techniques first developed by Gabriel Kron have already been used for the analysis of synchronous-machine-thyristor bridge systems,¹ and a recent paper has described their use for the analysis of d.c. motor drives.²

Several methods of analysing inverter-induction-motor systems have previously been presented, and a common approach has been to assume that the motor is supplied from a precisely defined waveform.^{3,4} While applicable to steady-state conditions, these methods may be unsuitable for transient operation where the inverter waveform will be influenced by the motor conditions and supply impedances. Furthermore, with inverters employing 120° conduction periods, one or more of the machine terminals will be open circuited for various periods during each supply cycle, and, during transients, the machine terminal waveshape will change significantly. Thus the analysis of this type of inverter is by no means straightforward.

To overcome these problems, the tensor method has been employed to deal efficiently with the varying topology of the inverter circuit, and investigations of unusual operating and fault conditions present no problem.

2 Tensor analysis of the inverter

The most common d.c. link inverter is the 3-phase bridge shown in Fig. 1, where the input voltage E is frequently derived from a 3-phase bridge-controlled rectifier. A total of 6 circuit configurations are possible for an inverter with 180° conduction periods but, for 120° operation, the situation is more complicated and 12 different topologies are possible during the normal operation. The following assumptions are made:

- All the diodes and thyristors are perfect switches.
- Each diode/thyristor combination is treated as one element since they never conduct simultaneously.

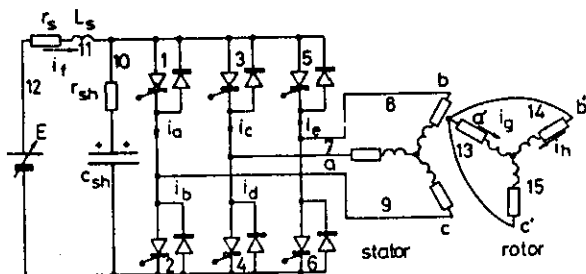


Fig. 1 Inverter-motor circuit

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The circuit of Fig. 1 (considering the stator as a load only) requires 6 independent currents $[i_n]$ (i.e. $i_a \rightarrow i_f$) to completely define the system. With these chosen to be as far as possible the thyristor currents a transformation tensor linking the primitive and independent currents of Fig. 1 follows:

$$[C_p] = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ -1 & -1 & -1 & -1 & -1 & \\ 1 & 1 & & & & \\ & & 1 & 1 & & \\ -1 & -1 & -1 & -1 & & \\ -1 & & -1 & & -1 & 1 \\ & & & & & 1 \\ & & & & & 1 \end{bmatrix} \quad (1)$$

The primitive resistance tensor $[R_p]$ and the primitive inductance tensor $[L_p]$ are given by

$$[R_p] = \text{diag} [0, 0, 0, 0, 0, 0, r_a, r_b, r_c, r_{sh}, r_s, 0] \quad (2a)$$

$$[L_p] = \text{diag} [0, 0, 0, 0, 0, 0, L_a, L_b, L_c, 0, L_s, 0] \quad (2b)$$

and the primitive voltage tensor $[V_p]$ and the primitive capacitor voltage tensor $[SQ_p]$ are given by

$$[V_p] = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, E]^t \quad (3a)$$

$$[SQ_p] = [0, 0, 0, 0, 0, 0, 0, 0, 0, q/C_{sh}, 0, 0]^t \quad (3b)$$

where q is the instantaneous charge in the filter capacitor.

Having determined the primitive impedance and voltage tensors, one could establish the equations for the independent system (i.e. $[V_n] = [R_n][i_n] + [L_n][i_n] + [SQ_n]$) using the normal tensor relationships

$$[R_n] = [C_p]^t [R_p] [C_p]$$

$$[L_n] = [C_p]^t [L_p] [C_p] \quad (4a)$$

and

$$[V_n] = [C_p]^t [V_p]$$

$$[SQ_n] = [C_p]^t [SQ_p] \quad (4b)$$

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However, since the operation of the inverter is controlled entirely by the triggering and commutation of the thyristors, in practice only two or three devices conduct simultaneously, and this leads us to form another tensor $[C_n]$ which links the new currents $[i_{nn}]$ of the 'reduced network' to those for the independent system $[i_n]$. In the program this tensor is produced by a special subroutine whenever the circuit topology changes. The new circuit tensors are therefore calculated as follows:

$$[R_{nn}] = [C_n]^t [R_n] [C_n],$$

$$[L_{nn}] = [C_n]^t [L_n] [C_n] \tag{5a}$$

$$[V_{nn}] = [C_n]^t [V_n], \quad [SQ_{nn}] = [C_n]^t [SQ_n] \tag{5b}$$

Solution of the network equations:

$$[i'_{nn}] = -[L_{nn}]^{-1} \{ [R_{nn}] [i_{nn}] + [SQ_{nn}] - [V_{nn}] \} \tag{6a}$$

$$q' = [C_K] [i_{nn}] \tag{6b}$$

(where $[C_K]$ is a row matrix linking $[i_{nn}]$ and the capacitor current) can now proceed numerically.

3 Extending the analysis to include the induction motor

In the previous Section it was assumed that the load was simply a star-connected impedance, and if a simple induction motor model is adopted, this could well form the stator of such a system. The extension to include the rotor circuit merely requires:

- (i) additional terms in the enlarged connection and impedance tensors
- (ii) the inclusion of the dynamic equation of the motor

Direct solution of the machine equations in phase quantities is used since this permits a wide class of operating conditions to be studied conveniently. In this case it is more convenient to solve for fluxes $[\psi]$ (from $-[d\psi] =$

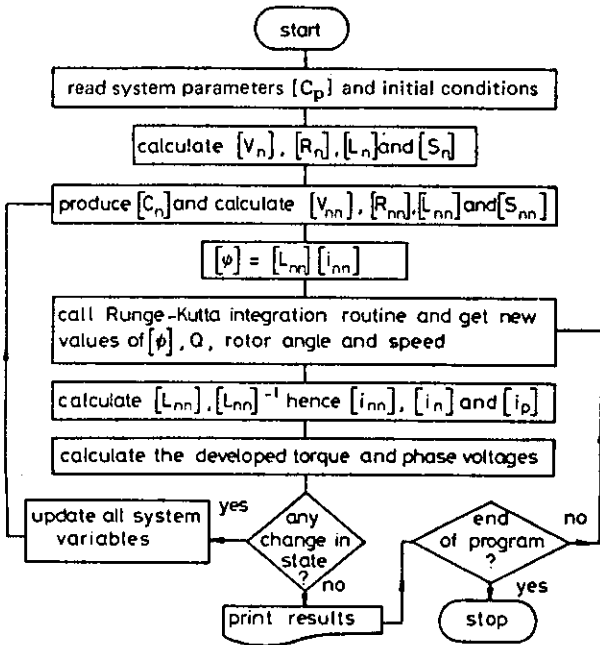


Fig. 2 Computer program flow chart

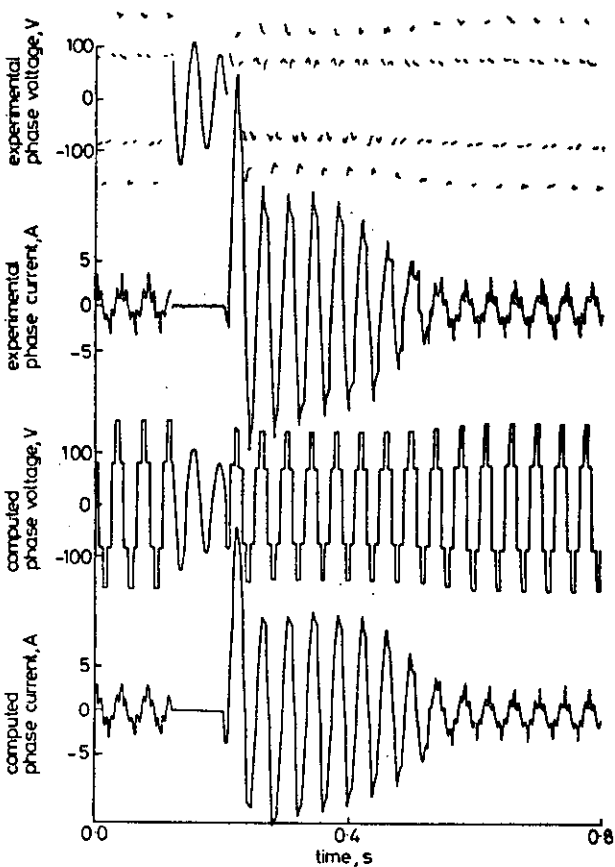


Fig. 3 Experimental and computed results for plugging with 180° inverter operation

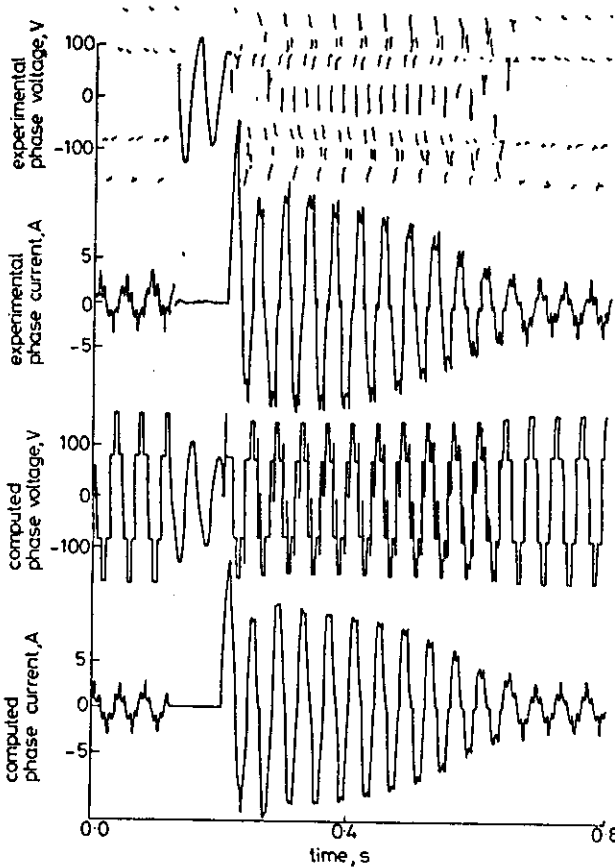


Fig. 4 Experimental and computed results for plugging with 120° inverter operation

$[R_{nn}][i_{nn}] + [SQ_{nn}] - [V_{nn}]$, the core losses, saturation effects and space harmonics being neglected) rather than for currents, and to obtain the currents from:

$$[i_{nn}] = [L_{nn}]^{-1} [\psi] \quad (7)$$

4 Computer program and results

A simplified flow chart of the computer program is shown in Fig. 2. The fourth-order Runge-Kutta numerical integration procedure is used in the solution of the system equations, and an appropriate step length is chosen from a determination of the system eigenvalues. The program operation is controlled by a system state array (which has six elements associated with the six bridge thyristors) and this is updated whenever the bridge topology changes. A special subroutine then uses this array to produce the correct $[C_n]$ tensor. Experimental results were taken on a small laboratory induction motor with the following parameters:

- (a) Name-plate details: 1 hp 220/240 V 6-pole 50 Hz
 - (b) Stator resistance per phase = 5.09Ω
 - (c) Rotor resistance per phase = 5.09Ω
 - (d) Self inductances of rotor and stator windings = 0.499 H
 - (e) Mutual inductances between stator phases and between rotor phases = 0.232 H
 - (f) Maximum mutual inductance between 3-phase stator and rotor circuits = 0.465 H
- (all values referred to stator).

Very good agreement between computed and experimental results was obtained during steady-state and transient operation and as an illustration the very severe test of plugging was chosen. Fig. 3 shows practical and computed results of motor phase voltage and current for a 180° conduction period inverter, and Fig. 4 shows the same information for an inverter with 120° conduction periods.

5 Conclusion

An efficient and accurate method of analysing an inverter motor system using a digital computer has been presented. The technique is able to cope with all operating conditions of the system, including transient, unbalanced and fault conditions and it may be used to study systems with both 180° and 120° inverter conduction modes.

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