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# COHERENT DETECTION OF QAM SIGNALS <br> IN LAND MOBILE RADIO 

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A Doctoral Thesis submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of the Loughborough University of Technology
$28^{\text {th }}$ November 1988

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## Abstract

This thesis forms part of a joint universities project in which it is required to design and build a digital modem for the transmission of speech or data over a 900 MHz land mobile radio channel. The main objectives being to try to maximize the bandwidth efficiency and attain near-optimum system performance.

The theoretical modem design is presented here. The other parts of the system, that is the error control coding, speech coding, RF design and the actual hardware implementation are described elsewhere. All the systems described here have been designed to satisfy the overall system requirements. In particular, it must be possible to build this modem with existing technology without undue equipment complexity.

All aspects of the digital modem design are addressed, namely the choices of modulation scheme, pulse shaping filtering, packet structure and timing and synchronization methods suitable for the transmission of a digital signal over the fading radio channel. The important problems of channel estimation and data detection are examined in more detail.

The first system described is one in which only one digital signal is transmitted in a narrowband channel. In the second system a novel technique of transmitting two signals in the same frequency band from two different mobiles to a single base station is described, which makes use of the fact that these two transmission paths are fading independently. The third system describes a method for transmitting back to these mobiles from the base station, again in the same frequency band.

Although these systems have been designed specifically for use over 900 MHz cellular land mobile radio channels, the techniques described are directly applicable to digital signal transmission over any flat fading channel.

## Acknowledgements

I would like to express my thanks to Prof. A.P.Clark for his guidance and encouragement throughout this project, and to my supervisor Dr. S.D.Smith for his invaluable help and advice, particularly during the writing-up stage. Also I am grateful to the SERC for their financial support which made this work possible.

Special thanks go to the staff at the Computer Centre for their friendly nature and great understanding in the allocation of computer resources.

On a personal level I would like to thank my family for their support. Most important of all Dee, who has shown wonderful patience and understanding throughout the course of this work.

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## List of Principal Symbols

```
b = small positive constant in Gradient Algorithm estimator
c}\mp@subsup{i}{i}{}=\mathrm{ cost associated with a stored vector Q Q 
di}\mp@subsup{i}{}{2}=\mathrm{ maximum likelihood cost of a possible received data point from
    ri
e}\mp@subsup{i}{i}{}=\mp@subsup{r}{i}{\prime}\mp@subsup{|}{i}{\prime}=\mathrm{ error signal in the Gradient Algorithm estimator at
    time t=iT
```



```
E}\mp@subsup{b}{b}{}=\mathrm{ average transmitted energy per bit
E
j}=\sqrt{}{-1
\frac{1}{2}}\mp@subsup{N}{0}{}=\mp@subsup{\sigma}{}{2}=\mathrm{ two-sided power spectral density of real-valued additive
    white Gaussian noise at input to the receiver
q}\mp@subsup{i}{i}{=}\mathrm{ detected value of s}\mp@subsup{s}{i}{}\mathrm{ in the stored vector }\mp@subsup{Q}{i}{
rim}(,\mp@subsup{r}{a.i}{\prime},\mp@subsup{r}{b.i}{\prime}
    = sample value of the received demodulated waveform (at receiving
    antenna a,b) at time t=iT
r' }\mp@subsup{i}{i}{\prime}=\mathrm{ estimate of ri
    (similarly for r'a.i' r'b.i)
si
    = data symbol value of signal (from mobile 1,2) at time t=iT
s'i
    (similarly for s'1.i' s!2.i)
T}==\mathrm{ symbol duration in seconds
wi
        = additive white Gaussian noise component in rri (, ra.i, rb.i)
Yi
        = For Systems 1 and 3: sample of the baseband fading channel (at
    receiving antenna a,b) at time t=iT
```



```
        = For System 2: sample of the baseband fading channel at time
    t=iT. (Transmission paths 1,2,3,4 as defined in Chap. 4)
Y'}\mp@subsup{}{i}{\prime}=\mathrm{ estimate of }\mp@subsup{Y}{i}{
    (similarly for y'a.i' Y' b.i' Y' 1.i' Y' 2.i, Y' 3.i', Y' 4.i ).
Y' i,i-1 =one-step prediction of }\mp@subsup{y}{i}{}\mathrm{ made at time t=(i-1)T
    (similarly for y'a.i,i-1' Y' b.i,i-1' Y'1.i,i-1' Y' 2.i,i>1'
    y'3.i,i-1, Y'4.i,i-1)
```

```
Y' }\mp@subsup{i}{i,i-1}{\prime}=\mathrm{ one-step prediction of the slope of }\mp@subsup{Y}{i}{}\mathrm{ made at time t=(i-1)T
    (similarly for y'a.i,i-1' Y' b.i,i-1' ' '' 1.i,i-1' y' 2.i,i-1'
    y'3.i,i-1' y'4.i,i-1)
\alpha i,1 的,2
    = two binary digits transmitted at time t=iT, for QPSK modulation
```



```
\mp@subsup{\beta}{i,1}{1}}\mp@subsup{\boldsymbol{\beta}}{i,2}{
    = two differentially encoded binary digits transmitted at time
        t=iT
    0 = small, real-valued constant in fading memory predictor
\lambdae}=\mathrm{ measured mean-square error in estimation
    = 10 年 }\mp@subsup{}{10}{}(\frac{1}{N}\mp@subsup{\sum}{i=1}{N}|\mp@subsup{Y}{i}{}-\mp@subsup{Y}{}{\prime}\mp@subsup{}{i}{\prime}\mp@subsup{|}{}{2})d
\lambda
    = lolog}10(\frac{1}{N}\mp@subsup{\sum}{i=1}{N}|\mp@subsup{y}{i}{\prime}-\mp@subsup{Y}{}{\prime}\mp@subsup{}{i,i-1}{}\mp@subsup{|}{}{2})d
\pi}=\mathrm{ constant 3.141592654
\sigma}=\frac{1}{2}\mp@subsup{N}{0}{}=\mathrm{ variance of both real and imaginary part of w w, wa.i
    w
\psi = E E }/\mp@subsup{N}{0}{}=\mathrm{ signal-to-noise ratio
\psidB}=10\mp@subsup{\operatorname{log}}{10}{(E}(\mp@subsup{E}{b}{}/\mp@subsup{N}{0}{})d
{\cdot} = set of .
Re[\cdot] = real part of complex-valued quantity -
Im[\cdot] = imaginary part of complex-valued quantity .
|\cdot| = \sqrt{}{Re[\cdot\mp@subsup{]}{}{2}+\operatorname{Im[\cdot]}}\mp@subsup{}{2}{=}=\mathrm{ amplitude of complex valued quantity .}
L}==\mp@subsup{\operatorname{tan}}{}{-1}(\operatorname{Im}[\cdot]/\operatorname{Re}[\cdot])=\mathrm{ phase angle of complex-valued quantity .
```


## Notes:

All samples r,s,y,w and their estimates are complex-valued. All other symbols represent real-valued quantities.

The * superscript denotes the complex conjugate of the given sample. The ' superscript represents either a channel estimate $y$ ' or a detected symbol s'.

Subscripts $1,2, a, b$ appearing before the . refer to the transmitted
signals 1 and 2, and the receiving antennas $A$ and B. Subscript ${ }_{i}$ after the represents a sample of the continuous waveform taken at time $t=i T$. Subscript ${ }_{i, i-1}$ in $y_{i, i-1}$ is used to show that this sample is a prediction of $y_{i}$, the prediction being made at time $t=(i-1) T$.

## Introduction

The ultimate objective of mobile communications is to allow anyone who is travelling in a vehicle to talk to anyone else by radiotelephone in exactly the same way as would be achieved using an ordinary fixed telephone connected to the Public Switched Telephone Network (PSTN). That is, via a private, full-duplex (two-way) channel, with the quality of reception normally expected from the PSTN. In the United Kingdom, up until 1986 there were only very limited mobile communications facilities available to the general public. The most commonly used were push-to-talk systems such as Citizens Band, where any call is free of charge but there is no privacy and the range is limited to only a few miles, after which the quality of reception becomes very poor. Radiophone System 4 was the only mobile radiotelephone system in operation, but the charge for calls was always kept artificially high to keep demand low because the system capacity was very limited. There was no way this system could ever cope with the estimated nationwide demand of many millions of users.

The concept of a cellular radio telephone network developed in 1979 [1-6] at Bell Labs was seen as the solution to this problem. With cellular radio, the whole country is divided into small areas called cells, with a small number of the available full-duplex channels allocated to each cell. The channels used in any cell are also used in other cells which are far enough apart to ensure that serious co-channel interference is avoided. Thus the demand for the whole country can be met with as few as sevèral hundred separate channels in all, occupying a bandwidth of less than 100 MHz . The size of any cell is carefully chosen so that the N channels in that cell are enough to serve the demand in that area. Theoretically, the increasing demand for channels can always be met by splitting one cell into two smaller cells, still with $N$ channels in each cell, so halving the area that is covered by $N$ cells. In 1986, the first United Kingdom cellular system TACS [7] came into operation. This is essentially a narrowband system, with a separate 25 kHz full-duplex channel allocated to each mobile in the 900 MHz region of the spectrum. An analogue frequency modulation scheme is used to transmit the speech. It
is planned to phase out the existing TACS system in the mid 1990's, to be replaced by an all digital system.

The work reported in this thesis forms part of a project in which it is required to design, build and test a digital modem for use in such a system. The overall functions of this modem have been broken down as shown in Fig.1.1. This thesis describes the theoretical design of the modulator / demodulator sections shown. At the transmitter, the modulator must convert the bit stream of information that is to be transmitted into a suitable baseband digital signal ready to be mixed up to RF (radio frequency) and transmitted. In the receiver demodulator, the bit stream is recovered from the received baseband waveform. The standard 25 kHz channel spacing is used to make it compatible with the existing analogue system [7]. The emphasis in the design of the different systems is on:

1) Increased bandwidth efficiency, in terms of the number of bits per second transmitted for each Hertz of bandwidth occupied (bit/s/Hz). This is the most important consideration.
2) Optimizing the system performance for a given average transmitted energy per bit.
3) Cost. The equipment at the mobile is kept relatively cheap at the expense of the base station equipment cost.
4) Complexity. The modem must be simple enough to build with existing hardware.

Novel techniques of combined detection and estimation with regular retraining of the channel estimator are used in the digital modem. With this method, coherent detection of narrowband quadrature amplitude modulated (QAM) signals is achieved, when these signals are transmitted over the 900 MHz mobile radio channel. Probably the most important result from this work is the demonstration of the fact that it is possible to transmit simultaneously two 4-level QAM signals in the same frequency band where the two signals originate from two different sources and fade independently at the receiver. The independent fading of the signals itself performs a process of collaborative coding that enables the signals to be distinguished from each other at the receiver. Thus a doubling of the bandwidth efficiency is achieved over the conventional single 4-level QAM transmission system. This is a completely new multiplexing method for fading channels.

All aspects of the digital modem design are addressed in this thesis. Namely the choices of; modulation schemes, pulse shaping filtering, packet


Fig.1.1 Modem functions
structure and timing and synchronization methods suitable for the transmission of a digital signal over the 900 MHz land mobile radio channel. The important problems of channel estimation and data detection are examined in more detail.

However, before any digital speech/data transmission system can be designed, the effect that the transmission path has on the signal must be known in detail. These channel properties are described in Chapter 2, the most important of which is known as flat Rayleigh fading. Also discussed here are; the lognormal fading component $[8,9,37-44]$ of the transmission path, the additive noise $[8,23,31-36]$ and co-channel and adjacent channel interference [4-9,45-48]. The quadrature amplitude modulated (QAM) signals used throughout this thesis $[52,53]$ have been chosen to give a good performance under these conditions, with coherent detection at the receiver. As a consequence of the Rayleigh fading some form of frequency modulation (FM) is usually proposed as a suitable signalling scheme because the frequency of the signal is effected less by Rayleigh fading than are its amplitude and phase [8,9]. Tamed FM [85-87] and Gaussian minimum shift keying (GMSK) [88,89] are generally said to be particularly suitable for a bandwidth efficient system [43,90]. But in practice with these schemes, particularly for Tamed $F M$, it has proved difficult to extract a timing waveform. Also as a result of the Rayleigh fading it is almost invariably stated that coherent demodulation cannot be used because it would be practically impossible to achieve an accurate phase reference [8]. The only exception to this in the literature seems to be a method known as SSB with TTIB and FFSR [78-83]. That is, single sideband with transparent tone-in-band and feedforward signal regeneration. This narrowband SSB scheme achieves coherent demodulation by multiplying the received data signal by the received pilot tone to remove the effects of the Rayleigh fading. The results suggest [81, 82] that this works with a bandwidth efficiency approaching $1 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$, but the signal processing techniques required to extract the pilot tone and generate the received signal are very complex. Also the transmitted signal does not have a constant envelope and power is wasted in transmitting the pilot tone. However, the two bandlimited quadrature amplitude modulated (QAM) signals described in Chapter 2 give a more bandwidth efficient modulation scheme than Tamed FM and GMSK. Coherent demodulation of these QAM signals can be achieved giving a good tolerance to additive noise without using a pilot tone. Methods are described by which it should be possible to achieve
carrier frequency synchronization and symbol timing simply and inexpensively with this system.

It soon becomes clear in this work that the key to successful system performance lies in the quality of the channel estimation process. It is often quoted in the literature $[63,114]$ that a complicated Kalman type estimation process must be used in a fast fading channel because of its fast converging properties. Other estimation processes such as the Gradient (steepest descent) algorithm are deemed to be too slow to react to changing channel conditions. But in contrast, a wealth of successful research has been carried out in recent years at Loughborough University applying simple Gradient algorithm estimators to fading channels [64,65,69-71/[ ${ }^{124-125}$. These two types of estimation processes and a few novel ideas are tested in this thesis.

It is also important to note the improvements in system performance that can be achieved by suitably combining the signals received from two spatially separated antennas, when there is uncorrelated Rayleigh fading at each antenna. It is unlikely that a bigger improvement could be obtained wịth an error correcting code [43].

In Chapter 3, one 4-level QAM signal is transmitted in the narrowband channel, System 1. The data is detected coherently by maximum likelihood detection or by Viterbi-type detection. Several different channel estimation processes are tested. In the rest of this thesis, the aim is to extend the successful coherent methods of System 1 to different modulation schemes that would double the spectral efficiency from about. 1 bit/s/Hz to $2 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$.

In Chapter 4, a novel technique of transmitting two of these 4-level QAM signals in the same frequency band from two different mobiles to a base station is described, which makes use of the fact that the two transmission paths are fading independently when separating the two signals at the receiver. This is called System 2.

System 3 is described in Chapter 5. Here, a single 16-level QAM signal is transmitted from the base station back to these two mobiles, again in the same 25 kHz frequency band.

The logical sequence followed in this thesis is shown in Fig.1.2. Chapter 2 is seen as a pre-requisite for all the chapters that follow. It contains all the information required to model the systems and then analyse their performances. The reasons for choosing the particular coherent demodulation / detection techniques used throughout this


Fig.1.2 Logical sequence of main chapters
investigation can only be appreciated after carefully studying this theory. In a similar way, System 1 is seen as a pre-requisite for Systems 2 and 3, in that the methods of Viterbi detection, channel estimation and retraining used in Systems 2 and 3 are all basically extensions of the successful methods used in System 1. So whenever possible in Chapters 4 and 5 reference is made to descriptions already given in Chapter 3 to avoid needless repetition. Only descriptions of the various techniques tested and the main conclusions are described in Chapters 3 to 5. The computer simulation results are all given in Chapter 6.

## BASIC THEORY

### 2.1 Introduction

Before a digital speech/data transmission system can be designed and tested by computer simulation, a suitable mathematical model of this transmission system must be defined. The aim of this chapter is to define such a model and analyse it to show the way the data signal is affected during transmission. Then, a general method is proposed by which coherent demodulation can be acheived at the receiver. This method forms the basis for all systems tested in this thesis.

Firstly, the important channel properties are. discussed in detail in Sec.2.2. Then the bandwidth efficient modulation schemes used in this thesis are described in Sec.2.3, and their baseband equivalent models are defined from which all computer simulation results are obtained. Then in Secs.2.4-2.6 the general descriptions of the complete data transmission systems tested in the following chapters are given. Finally, the timing and synchronization methods used are described in Sec.2.7.

### 2.2 Channel Properties

In a typical 900 MHz land mobile radio link, full duplex radio signals are passed between mobile and base station. The mobile unit is surrounded by tall buildings and other mobiles whereas the base station is mounted upon a nearby rooftop or any other convenient high point. Generally there is no direct line-of-sight path between mobile and base station, so the mode of radio wave propagation from transmitter to receiver is largely by way of scattering either by reflection from, or diffraction around other mobiles, buildings or terrain features [8-11], as in Fig.2.2.1.

Each path in Fig.2.2.1 represents a radio wave travelling at the speed of light. Different path lengths mean that there will be a corresponding time difference in the arrival of the wave along each path. These propagation paths change with time as the mobile moves among the scatterers causing the received signal to undergo rapid fading known as Rayleigh fading.


Fig.2.2.1 Scattering model of signal propagation

### 2.2.1 Rayleigh Fading

The well known mathematical model of the Rayleigh fading that characterizes this land mobile radio system was first derived by R.H.Clarke [10], and is often referred to as the Clarke model. This original theory was further developed in the references [ $8,9,11$ ], and many field trials have served to confirm the accuracy of the model [12-17]. The basic theoretical development has been repeated in Appendix $A$ and the important properties of the Rayleigh fading that follow from this model are summarized here.

Consider first the case where the modulation is removed and only a constant amplitude carrier waveform is transmitted from the base station. The transmitted signal is

$$
\begin{equation*}
s(t)=v \cos \omega_{c} t \tag{2.2.1}
\end{equation*}
$$

and the signal phasor at the input to the mobile receiving antenna is

$$
\begin{equation*}
V Y(t)=V A(t) \cos \left(\omega_{c} t+\theta(t)\right) \tag{2.2.2}
\end{equation*}
$$

Where, $\omega_{C}=2 \pi f{ }_{c}$ is the angular frequency of the carrier in radians/sec. $f_{c} \approx 900 \mathrm{MHz}$ is the carrier frequency. $A(t), \theta(t)$ are respectively the time varying amplitude and phase waveforms of the received carrier.
$A(t)$ and $\theta(t)$ are independent lowpass random variables and are assumed to vary slowly in time relative to the rapid variations exhibited by the cosine function of the carrier (Eq.(2.2.1)).

By expanding the cosine function in Eq.(2.2.2) a second representation of $Y(t)$ is obtained, namely

$$
\begin{align*}
V Y(t) & =V A(t) \cos \theta(t) \cos \omega_{c} t-V A(t) \sin \theta(t) \sin \omega_{C} t  \tag{2.2.3}\\
& =V_{Y}(t) \cos \omega_{C} t-V_{Q}(t) \sin \omega_{C} t \tag{2.2.4}
\end{align*}
$$

where,

$$
\begin{equation*}
Y_{I}(t)=A(t) \cos \theta(t), \quad Y_{Q}(t)=A(t) \sin \theta(t) \tag{2.2.5}
\end{equation*}
$$

and conversely

$$
\begin{equation*}
A(t)=\sqrt{y_{I}^{2}(t)+y_{Q}^{2}(t)}, \quad \theta(t)=\tan ^{-1}\left(y_{Q}(t) / y_{I}(t)\right) \tag{2.2.6}
\end{equation*}
$$

Finally, a third representation for $Y(t)$ is obtained from Eq. (2.2.2) by defining the complex envelope $y(t)$ as

$$
\begin{equation*}
y(t)=A(t) \exp (j \theta(t))=y_{I}(t)+j y_{Q}(t) \tag{2.2.7}
\end{equation*}
$$

so that

$$
\begin{equation*}
V Y(t)=V \cdot \operatorname{Re}\left[y(t) \exp \left(j \omega_{C} t\right)\right] \tag{2.2.8}
\end{equation*}
$$

where $\operatorname{Re[]}$ denotes the real part of the complex-valued quantity in the brackets. Thus the received fading carrier is completely described by any one of the three equivalent forms given in Eqs.(2.2.2),(2.2.4) or (2.2.8).

The signals $y_{I}(t)$ and $y_{Q}(t)$, termed the in-phase and quadrature
components of $\mathrm{Y}(\mathrm{t})$ respectively are $[8-11,18-23]$; identically distributed, zero mean, statistically independent Gaussian random variables, that are varying slowly in time relative to the rapid variations of the carrier waveform $S(t)(E q .(2.2 .1))$. It now follows [24.25] that the amplitude $A(t)$ has a Rayleigh probability density function (hence Rayleigh fading) and the phase is uniformly distributed from $-\pi$ to $+\pi$.

It is assumed throughout this thesis that the variances of both $y_{I}(t)$ and $y_{Q}(t)$ are always equal to $\frac{1}{2}$. And since $y_{I}(t)$ and $y_{Q}(t)$ are statistically independent, the mean-square value of $A(t)$ is 1 . So there is no change in average signal level during transmission. This avoids unnecessary complications in determining the signal-to-noise ratio in the computer simulation tests.

The statistical properties of these random variables $y_{I}(t), y_{Q}(t)$, $A(t), \theta(t)$ are summarized in Table 2.2.1 and Fig.2.2.2. Where, for any random variable $x$ with probability density function $f(x)$, its mean, mean-square and variance are defined by

$$
\begin{gather*}
\text { mean }=\bar{x}=\int_{-\infty}^{\infty} f(x) x d x  \tag{2.2.9}\\
\text { mean square }=\overline{x^{2}}=\int_{-\infty}^{\infty} f(x) x^{2} d x  \tag{2,2.10}\\
\text { variance }=\overline{x^{2}}-(\bar{x})^{2}=\text { mean'square }-(\text { mean })^{2} \tag{2.2.11}
\end{gather*}
$$

Now consider the frequency characteristics of $Y(t)$ in Eq. (2.2.2). The individual radio waves (Fig.2.2.1) are assumed to arrive at the mobile from all (horizontal) directions with equal probability. Both transmitting and receiving antennas are omnidirectional, vertical monopole antennas. The transmitted signal is assumed to be vertically polarized and the polarization unchanged during transmission $[9,10]$ so the antenna receives the electric field component of the signal, which due to the motion of the mobile has the power/spectrum given by [8-11]

$$
|Y(f)|^{2}= \begin{cases}\frac{1}{\pi f_{m} \sqrt{1-\left(\left(f-f_{c}\right) / f_{m}\right)^{2}}} & \text { for } f_{c}-f_{m} \leqslant f_{c} \leqslant f_{m}+f_{m}  \tag{2.2.12}\\ 0, & \text { elsewhere }\end{cases}
$$

where,
mean received fading power $=1$
$\mathrm{f}_{\mathrm{C}}=$ carrier frequency $=900 \mathrm{MHz}$
$f_{m}=$ maximum Doppler frequency shift $=v / \lambda=80 \mathrm{~Hz}$
$v=$ vehicle speed $\approx 60$ miles/hour $\approx 26.8$ metres/second

Table 2.2.1 Statistical properties of $y_{1}(t), y_{Q}(t), A(t), \theta(t)$

| Random <br> Variable | Probability <br> Density | Mean | Mean- <br> square | Variance |
| :---: | :---: | :---: | :---: | :---: |
| $y_{1}$ or $y_{Q}$ | $\frac{1}{\pi} \exp \left(-y_{1}^{2}\right)$, for $-\infty<y<\infty$ | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $A$ | A.exp $\left(-A^{2}\right), \quad$ for $A>0$ <br> 0, | $\sqrt{\pi}$ | 1 | $1-\pi$ |
| $\theta$ | $\frac{1}{2 \pi}, \quad$ for $-\pi<\theta<\pi$ <br> 0, <br> elsewhere | 0 | $\frac{\pi^{2}}{3}$ | $\frac{\pi^{2}}{3}$ |



(a) Gaussian $y_{1}$ or $y_{Q}$
$\lambda=$ carrier wavelength $\approx 0.33$ metres
This is shown in Fig.2.2.3. It is usually referred to as the Doppler power spectrum. The spectral shape shown in Fig.2.2.3 has been observed in field trials [17].
. The vehicle speed of 60 miles/hour was chosen to represent urban worst case fading conditions. Some tests are also carried out for a simulated vehicle speed of $30 \mathrm{miles} /$ hour ( $\mathrm{f}_{\mathrm{m}}=40 \mathrm{~Hz}$ ) for comparison purposes.

This Doppler power spectrum has been suitably scaled so that the mean-square received fading power is still unity. That is,

$$
\begin{equation*}
\int_{f_{m}+f_{m}}^{f_{c}+f_{m}}|Y(f)|^{2} d f=1 \tag{2.2.13}
\end{equation*}
$$

The autocorrelation function with time of the Rayleigh fading channel $Y(t)$ is given by the inverse Fourier transform of $|Y(f)|^{2}$. For the vertical monopole antenna which senses the electric field, this is (see Appendix A).

$$
\begin{align*}
R_{Y Y}(\tau) & =\int_{f_{c}}^{f_{c}+f_{m}}|Y(f)|^{2} \exp (j 2 \pi f \tau) d f  \tag{2.2.14}\\
& =J_{0}\left(2 \pi f_{m} \tau\right) \tag{2.2.15}
\end{align*}
$$

where $J_{0}$ () is the zero-order Bessel function of the first kind, as shown in Fig.2.2.3. This can be used to indicate how the fading is likely to change in short periods of time $\tau$.

The discussion thus far describes the received fading waveform at the mobile where the transmitted signal is an unmodulated carrier. Now consider the received fading signal when a modulated carrier waveform is transmitted.

Digital quadrature amplitude modulated signals are considered in this thesis (Sec.2.3), which occupy a bandwidth of 24 kHz centered around the carrier frequency of 900 MHz . The duration of each individual transmitted signal element, $T$, is about $83 \mu \mathrm{~s}$. In an urban environment, the time dispersion or spread in time delay of the various received signals arriving at the mobile along the different propagation paths (Fig.2.2.1) are small in comparison with the nominal duration $T$ of a received signal element. This small time delay spread of generally less than $3 \mu \mathrm{~s}$ (for this carrier frequency) $[8,9,12,13]$ does not cause any intersymbol interference. Under these conditions, all frequency components of the received data signal can be considered to fade in unison. Such fading is described as "flat" fading or "frequency non-selective" fading.

So, the transmitted quadrature amplitude modulated signal can be


Flg.2.2.3 (a) Theoretical power spectrum and (b) autocorrelation function with time, of received carrier
represented by

$$
\begin{equation*}
S(t)=M(t) \cos \left(\omega_{C} t+\phi(t)\right) \tag{2.2.16}
\end{equation*}
$$

where $M(t), \phi(t)$ are the amplitude and phase components respectively of the digital modulation.

Now, ignoring any delay in transmission, the signal phasor at the output of the Rayleigh fading transmission path at time $t$ is assumed to be accurately represented by

$$
\begin{equation*}
R(t)=A(t) M(t) \cos \left(\omega_{C} t+\phi(t)+\theta(t)\right) \tag{2.2.17}
\end{equation*}
$$

This is simply the transmitted signal phasor $S(t)$ shifted in amplitude (by $A(t)$ ) and phase (by $\theta(t))$ by the Rayleigh fading, with no time dispersion in the received signal. This signal representation is in polar coordinates. An equivalent representation of this fading process in Cartesian form can be derived as follows, that is easier to simulate on the computer.

Since the band of frequencies occupied by $S(t)(24 \mathrm{kHz})$ is small
relative to $f_{c}(900 \mathrm{MHz})$ the signal is a narrowband bandpass signal. Now, the cosine function in Eq. (2.2.16) can be expanded to give

$$
\begin{align*}
S(t) & =M(t) \cos \phi(t) \cos \omega_{C}(t)-M(t) \sin \phi(t) \sin \omega_{C} t  \tag{2.2.18}\\
& =s_{I}(t) \cos \omega_{C} t-s_{Q}(t) \sin \omega_{C} t
\end{align*}
$$

with complex envelope

$$
\begin{equation*}
s(t)=s_{I}(t)+j s_{Q}(t) \tag{2.2.19}
\end{equation*}
$$

where $s_{I}(t)=M(t) \cos \phi(t)$ and $s_{Q}(t)=M(t) \sin \phi(t)$ are the in-phase and quadrature components respectively of $s(t)$.

Similarly, Eq.(2.2.17) can be expressed in Cartesian form

$$
\begin{align*}
R(t)= & A(t) M(t)(\cos \theta(t) \cos \phi(t)-\sin \theta(t) \sin \phi(t)) \cos \omega_{C} t- \\
& A(t) M(t)(\cos \theta(t) \sin \phi(t)+\sin \theta(t) \cos \phi(t)) \sin \omega_{C} t \\
= & \left(s_{I}(t) Y_{I}(t)-s_{Q}(t) Y_{Q}(t)\right) \cos \omega_{C} t- \\
& \left(s_{I}(t) Y_{Q}(t)+s_{Q}(t) Y_{I}(t)\right) \sin \omega_{C} t \tag{2.2.20}
\end{align*}
$$

where $y_{I}(t), y_{Q}(t)$ were defined in Eqs.(2.2.5) and (2.2.7)
The complex envelope of $R(t)$ is

$$
\begin{equation*}
r(t)=r_{I}(t)+j r_{Q}(t) \tag{2.2.21}
\end{equation*}
$$

where, from Eq. (2.2.20)

$$
\begin{align*}
& r_{I}(t)=s_{I}(t) Y_{I}(t)-s_{Q}(t) Y_{Q}(t) \\
& r_{Q}(t)=s_{I}(t) Y_{Q}(t)+s_{Q}(t) Y_{I}(t) \tag{2.2.22}
\end{align*}
$$

So,

$$
\begin{equation*}
r(t)=s(t) y(t) \tag{2.2.23}
\end{equation*}
$$

This important restult forms the basis for the computer simulation model used in all tests. In the baseband equivalent model of this
microwave radio transmission system (described in Appendix B), only the complex envelopes of the signals need be simulated [26,27]. All linear frequency translations involved in modulation and demodulation are assumed to be ideal so can be ignored in the simulation. So first of all, the complex envelopes of the data and channel, $s(t)$ and $y(t)$, must be generated and the signal at the output of the Rayleigh fading transmission path is given by $r(t)$, which is formed from Eqs.(2.2.22)-(2.2.23).

The above discussion gives all that needs to be known to model the fading on a computer. However, a good understanding of the fading is essential in order to develop the best possible system. A detailed analysis of this flat Rayleigh fading for land mobile radio is given elsewhere [8-11]. Some of the important results of this analysis are listed below.

It is well known that errors in signal detection usually occur during deep fades. The deep fades encountered in Rayleigh fading are a property of the Rayleigh distribution. For example, the amplitude distribution in Table 2.2.1 predicts that for $99 \%$ of the time $A \geqslant 0.1$. ( 0.1 is 20 dB below the root-mean-square value of the amplitude level). This result is independent of the Doppler power spectrum of the fading [11-28].

However, to evaluate receiver performance it is of interest to know the rate of fading and the average duration of deep fades. This can only be determined by considering the additional information contained in the Doppler power spectrum of the fading signal. With the omnidirectional vertical monopole antenna, the rate at which the fading amplitude falls below a level $A$ is given by [8]

$$
\begin{equation*}
N(A)=\sqrt{2 \pi} f_{m} A \exp \left(-A^{2}\right) \tag{2.2.24}
\end{equation*}
$$

$N(A)$ is plotted in Fig.2.2.4. The average duration of this fade is given by

$$
\begin{equation*}
\tau(A)=\frac{\exp \left(A^{2}\right)-1}{\sqrt{2 \pi f_{m}}{ }^{A}} \tag{2.2.25}
\end{equation*}
$$

$\tau(\mathrm{A})$ is plotted in Fig.2.2.5
For example, at $60 \mathrm{miles} /$ hour and 900 MHz , 20 dB fades occur at a rate of about 22 fades per second with an average duration of about $500 \mu \mathrm{~s}$. $500 \mu \mathrm{~s}$ is about 6 symbols duration, which accounts for the fact that errors tend to occur in short bursts.

These probability density functions in Table 2.2.1 and Eqs.(2.2.24)(2.2.25) say nothing about the way the fading changes over short intervals of time. This a function of the Doppler power spectrum (Eq.(2.2.10)). It


Fig.2.2.4 Normalized level-crossing rate


Fig.2.2.5 Normalized average duration of fade
is worth repeating here that at any time $t$, the baseband equivalent Rayleigh fading can be expressed either in terms of its amplitude (A(t)) and phase $(\theta(t))$ components, or in terms of its in-phase $\left(y_{I}(t)\right)$ and quadrature $\left(y_{Q}(t)\right)$ components. The theoretical power spectral densities $S_{Y}(f), S_{Y}(f), S_{A}(f), S_{\dot{\theta}}(f)$ of the baseband quantities $Y_{I}, Y_{Q}, A, \dot{\theta}$ can be derived from the Doppler power spectrum of Eq. (2.2.12) $[8,9,11,18]$ and are shown in Fig. 2.2.6. Here, $\dot{\theta}$ is the rate of change of $\theta$, commonly termed random FM.

The in-phase and quadrature components $y_{I}, Y_{Q}$ have no frequency components greater than $\mathrm{f}_{\mathrm{m}}$, whereas the amplitude A is seen to occupy twice the bandwidth with frequencies up to $2 \mathrm{f}_{\mathrm{m}}$. It is important to note that the random $F M$ theoretically. exists at all frequencies. The meansquare value of the random $F M$ is infinite which means that instantaneous phase changes occur which would be impossible to predict. Clearly it follows that any channel estimation process which tracks the components $Y_{I}, Y_{Q}$ would perform much better than one which attempted to track the corresponding components $\mathrm{A}, \Theta$ instead.

So far the discussion in this Sec.2.2.1.has described the Rayleigh fading in the transmission path from the base station to the mobile unit. In fact, as a result of the reciprocity theorem [8] this equally well describes the Rayleigh fading in the reverse direction from the mobile unit to the base station. That is, the statistical properties of the fading signal and the autocorrelation versus time (or Doppler power spectrum) are identical for signals received at the mobile unit and at the base station. However, the correlation versus distance is different at mobile and base station which is important when considering space diversity reception.

This difference in spatial correlation arises because the mobile unit is surrounded by nearby scatterers (Fig.2.2.1) so generally receives its component waves equally from all directions. In contrast, the base station is mounted up and away from the scatterers so the waves arriving at the base station from the mobile are generally restricted to a narrow angular spread. The effect this has on the correlation versus distance at both mobile and base station has been considered theoretically in references [8,9,i1]. The important results from this are that two receiving antennas about six inches apart ( $\frac{1}{2}$ wavelength) in the same horizontal plane at the mobile can be assumed to receive practically


Fig.2.2.6 Power spectra of baseband fading components;
(a) $y_{1}$ or $y_{0}$
(b) A
(c) $\theta$
uncorrelated fading of their signals. Field trials suggest that this distance is nearer 0.8 of a wavelength due to a departure from pure Rayleigh statistics in the real world [9]. At the base station, the two receiving antennas might typically have to be 20 to 30 wavelengths apart to give the same decorrelation [11,16]. Also the actual placing of these antennas has an important effect on the decorrelation. In fact, if two antennas are separated along the line between base and mobile, there is practically no decorrelation in their fading signals. However, Parsons and Feeney at Liverpool University have shown that [16] a vertical separation of receiving antennas at the base station can be used to give a decorrelation in the fading which is practically independent of the direction of the mobile from the base station at any time. This vertical separation of space diversity receiving antennas at the base station is assumed throughout this thesis.

Throughout this thesis wherever two antennas are used at the receiver, whether at the mobile or the base station, they are assumed to be spaced sufficiently far apart to reduce to zero any correlation between their two fading signals. In practice there is likely to be some correlation between these signals with a corresponding degradation in performance $[8,9,29,30]$. But without making specific assumptions about the locations where these antennas are mounted, it is difficult to model the correlation reliably.

### 2.2.2 Additive Noise

In 900 MHz land mobile radio, a level of background noise $W(t)$ is added to the information signal. This is almost equally composed of man-made noise from vehicle ignition systems, and receiver front-end noise [8]. The man-made noise is impulsive in nature [31] whereas the receiver noise is generally described as additive white Gaussian noise [23]. Throughout this thesis, the additive noise $W(t)$ is modelled as stationary additive white Gaussian noise that is added to the data signal at the output of the Rayleigh fading transmission path. It is generally considered in digital communications that the system which has the best tolerance to additive white Gaussian noise will usually perform best in practice [31-33]. The baseband equivalent model of this noise is quite simple to simulate on the computer (Appendix B) whereas there is no such widely accepted model for the impulsive noise.

This additive noise waveform $W(t)$ has a Gaussian probability density


FIg.2.2.7 Probability density function of white Gaussian noise


Flg.2.2.8 Power spectral density of white Gaussian noise
function with zero mean and variance $\sigma^{2}=\frac{1}{2} N_{0}$, as shown in Fig.2.2.7. W(t) has a two-sided power spectral denstity of $\frac{1}{2} N_{0}$ over all positive and negative frequencies as shown in Fig.2.2.8 [31-36]. In other words, the behaviour of this Gaussian distributed noise waveform $W(t)$ over any length of time, however short, is completely unpredictable. Its mean-square value or average power level is $\sigma^{2}=\frac{1}{2} N_{0}$. It is unnecessary to consider here any power units (in watts) since the noise power is always related to the signal power through the signal-to-noise ratio.

### 2.2.3 Average Received Signal Strength

It has been shown $[8,9,37,38]$ that although the short term statistics are Rayleigh, in the long term the average received signal strength is not constant but varies slowly with time as the vehicle moves. This is caused by two effects known as shadowing and path loss.
(1) Shadowing:- Shadowing of the radio signal by buildings and hills leads to differences in the mean received signal level for different locations of the mobile. As the moving vehicle changes its location, this local mean signal strength fluctuates due to shadowing by typically 6 to 12 dB . This local mean expressed in decibels is normally distributed about its average value. This lognormal distribution has been confirmed in a number of propagation surveys $[8,16,38]$.

The effect of this lognormal component on the fading signal is shown in Fig.2.2.9 [39-41]. A complete cycle of the lognormal component typically lasts for several seconds. The local mean therefore changes so slowly that it would be tracked along with the fast Rayleigh fading component in the channel. estimation process. But it is unavoidable that it would cause a further degradation in tolerance to noise in any system compared with pure Rayleigh fading $[37,39,42,43]$. When designing a mobile radio system, the transmitter power is increased above that which would be required in pure Rayleigh fading to allow for this.
(2) Path Loss:- The received signal power decreases as the mobile unit moves away from the base station. A commonly used approximation [9,38,44] is that the received power is inversely proportional to the fourth power of this distance separation. The mobile transmitters have the ability to switch between different output power levels on request from the base station to keep the received power level roughly constant at all times [7].



FIg.2.2.9 (a) Typical envelope of signal (b) Pure Rayleigh envelope

No extra signal processing is carried out at the receiver to combat either of these long term fading effects. So, throughout this thesis pure Rayleigh fading is assumed with a constant mean-square value of unity.

### 2.2.4 Co-channel and Adjacent channel Interference

The discussion so far in this chapter has considered one isolated mobile radio link between a mobile unit and a base station. But within a cellular radio network there is interference from adjacent channels in the frequency spectrum. Also there is co-channel interference from signals transmitting on the same carrier frequency that has been re-used in other cells. Now, a good overall system capacity is probably the most important factor to be taken into account when planning a cellular network. So each individual radio link must have good adjacent channel interference properties so that more channels can be squeezed into a given frequency bandwidth. Also, each radio link must have a good tolerance to co-channel interference so that more channels can be fitted into a smaller coverage area.

In this thesis co-channel and adjacent channel interference are not actually simulated, but all modems tested must be designed to cope well with both of these problems. These two types of interference are now considered separately.
(1) Co-channel interference:- In a practical mobile radio network, the co-channel re-use distance would be calculated to give the required bit-error-rate performance. The Rayleigh fading, lognormal shadowing and path loss of the wanted signal and of the co-channel interferers must be taken into account in these calculations [4-6].

Some steps can be taken in the modem design to combat co-channel interference. Firstly, coherent detection of the digital signal is considered to have a higher immunity to co-channel interference than any non-coherent methods $[8,9,45]$ as long as the coherent reference of the wanted signal can be maintained in the presence of the Rayleigh fading. Coherent detection is used in the receiver of all systems tested in this thesis (Secs.2.3-2.5).

Secondly, the tolerance of a system to co-channel interference can be greatly improved by coherent maximal ratio combining of the signals received from spatially separated antennas [43,46,47]. This and other well known combining methods are discussed in Sec.3.2.

Error control coding can also be effective in reducing the number of
bit errors caused by co-channel interference [48]. Work is being carried out at Manchester University to develop error control codes for use with the modems developed in this thesis $[49,50]$. No error control codes are described here.
(2) Adjacent channel interference:- The channel spacing has been fixed at 25 kHz . The system performance in relation to the adjacent channel interference is then determined by; the frequency, tolerances of the transmitter and receiver local oscillators, and the effective bandwidth of the digital data signal together with the method of spectral shaping employed [6].

Economic considerations make it difficult to achieve oscillator stabilities better than about 2 parts per million at the mobile [6,7] (that is, $\pm 8 \mathrm{kHz}$ at 900 MHz ). In contrast, the base station oscillator may be assumed to maintain almost perfect stability (that is, $\pm 100 \mathrm{~Hz}$ at 900 MHz ). However, in Sec. 2.8 a cost effective method is proposed by which the mobile units can transmit with a carrier frequency within about 50 Hz of the base station reference.

The quadrature amplitude modulated signals used in this thesis always have a fully raised-cosine spectral shaping. The out-of-band radiation is about $60 d B$ down on the average power in the centre of the passband.

With this signalling arrangement, all data signals are passed between mobile and base station with a total bandwidth of 24 kHz , leaving a 1 kHz guard band between each frequency division multiplexed channel in the spectrum. Thus, in the model assumed here, significant levels of adjacent channel interference are avoided.

### 2.3 Quadrature Amplitude Modulation Schemes (QAM)

In the cellular land mobile radio system, frequency division multiplexing is assumed with a channel spacing of 25 kHz in the 900 MHz region of the spectrum [43,51]. Two different narrowband quadrature amplitude modulation (QAM) schemes have been proposed in this investigation for use in Systems 1,2 and 3 [52,53]. In every case, signal transmission along any path between one transmitting antenna and one receiving antenna is either by 4-level QAM or 16 -level QAM. The same basic modulator / demodulator structure is used in both cases, as shown in Fig.2.3.1 [32,54,55]. The mathematical development of this model has been described in detail in Appendix $B$. The main points are summarized in this section.


Fig.2.3.1 QAM data transmission system

### 2.3.1 4-level QAM (or bandlimited QPSK)

The data signal $S(t)$ transmitted over the flat fading channel is a $24 \mathrm{kbit} / \mathrm{s}$, 4-level QAM signal that has a carrier frequency of 900 MHz and an element rate of 12 kbaud . The bandwidth occupied by this signal is 24 kHz , thus resulting in a 1 kHz frequency guard band between adjacent channels to handle Doppler shifts, variations in filter characteristics and local oscillator innaccuracies / drift.

The information to be transmitted is carried by the complex-valued data symbol values $\left\{s_{i}\right\}$. They take the form of impulses $\left\{s_{i} \delta(t-i T)\right\}$ that only exist at the input to the transmitter filter (Fig.2.3.1) at the discrete instants in time $t=i T$, for all integers $\{i\}$. T here is the element duration. The $\left\{s_{i}\right\}$ are statistically independent and equally likely to be any one of their four possible values $\pm 1 \pm j$. Therefore at time t=iT,

$$
\begin{equation*}
s_{i}=s_{I_{. i}}+j s_{Q . i} \tag{2.3.1}
\end{equation*}
$$

where, $j=\sqrt{-1}$ and $s_{I . i}, s_{Q . i}= \pm 1$. The real and imaginary data streams $\left\{s_{I_{. i}}\right\},\left\{s_{Q . i}\right\}$, are separately filtered through identical root-raisedcosine lowpass filters and the resulting baseband signal waveforms at their outputs are used to modulate their respective carrier waveforms $\sqrt{2} \cos 2 \pi f_{c} t$ and $-\sqrt{2} \sin 2 \pi f_{c} t$, where $f_{c}=900 \mathrm{MHz}$. So the resulting QAM signal is the sum of the in-phase and quadrature components that carry the data symbols $\left\{s_{I_{. i}}\right\},\left\{s_{Q . i}\right\}$ respectively and whose carriers are in phase quadrature, since

$$
\begin{equation*}
\cos \left(2 \pi f_{c} t+90^{\circ}\right)=-\sin 2 \pi f_{c} t \tag{2.3.2}
\end{equation*}
$$

This 4-level QAM signal can also be considered as a QPSK signal having a considerable envelope ripple caused by bandlimiting (Fig.2.3.2). Techniques are now available that enable a high-power amplifier in the transmitter to handle the large envelope ripple in this QAM signal [56-59].

In any practical system a bandpass filter (Fig.2.3.1) is required in the transmitter to remove spurious frequency components generated in the modulator. In the receiver demodulator, the noisy, fading QAM signal is filtered with a wideband filter (several MHz ) centered about $f_{c}$ to limit the noise power without changing the QAM signal characteristics. The in-phase and quadrature components of the received baseband signal waveform $r(t)$ are formed by multiplying this filtered QAM signal with the waveforms $\sqrt{2} \cos \left(2 \pi f_{c} t+\gamma\right)$ and $-\sqrt{2} \sin \left(2 \pi f_{c} t+\gamma\right)$ respectively, and filtering as shown in Fig.2.3.1: Where $\gamma$ is a constant phase difference


Flg.2.3.2 Baseband four-level QAM signal. (a) In-phase component (b)quadrature component (c) envelope
between the local oscillators in the modulator and demodulator sections.
All root-raised-cosine lowpass filters in Fig.2.3.1 have the same transfer function $H^{\frac{1}{2}}(f)$ such that the resultant transfer function of the transmitter and receiver lowpass filters in cascade is

$$
H(f)=\left\{\begin{array}{lc}
\frac{1}{2} T(1+\cos \pi f T), & -\frac{1}{T} \leqslant f \leqslant \frac{1}{T}  \tag{2,3.3}\\
0, & \text { elsewhere }
\end{array}\right.
$$

This gives the conventional fully raised-cosine spectral shaping to the QAM signal [31]. The filtering is equally divided between transmitter and receiver, so the receiver filter is said to be matched to the transmitter filter. As such, it is well known that [31,35,36] the transmitter and receiver filters in cascade can be assumed to introduce no intersymbol interference. And ignoring any propagation delay, the optimum detection of the data symbol value $s_{i}$ from the sample $r_{i}$ ( $r(i t)$ ) is also the optimum detection of $s_{i}$ from the received baseband waveform $r(t)$. No other samples of $r(t)$ can be used to improve the detection of $s_{i}$.

These are still accurate assumptions to make when the flat Rayleigh fading is in between the two filters, even though the receiver filter is not now matched to the fading signal at its input. The results in Appendix B suggest that this is because the simulated Doppler frequency spread of $2 f_{m}=160 \mathrm{~Hz}$ is such a small fraction of the total QAM signal bandwidth of 24 kHz . As a rule of thumb, it seems that the effect of the Rayleigh fading on these matched filtering assumptions is negligible with fully raised-cosine filtering as long as $2 f_{m} \leqslant 1 \%$ of the total occupied bandwidth of the QAM signal.

With this arrangement described of matched filtering (Eq. (2.3.3)) and with the $\sqrt{2}$ scaling factors in all carrier modulating / demodulating waveforms in Fig.2.3.1, the received baseband sample at time $t=i T$ (ignoring any delay in transmission) is given by (see Appendix B.1)

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{2.3.4}
\end{equation*}
$$

where $r_{i}, s_{i}, y_{i}, w_{i}$ are all samples of the complex envelopes of their corresponding waveforms at time $t=i T$.

From the statistics of the $\left\{s_{i}\right\}$ it has been shown (Appendix B) that the average transmitted energy per bit in the $\left\{s_{i}\right\}$ is

$$
\begin{equation*}
E_{b}=1 \tag{2,3.5}
\end{equation*}
$$

With this model of the modulator (Fig.2.3.1), $E_{b}$ is also the average transmitted energy per bit of the real-valued QAM signal at the output of the modulator.

Thus, since only these received samples $\left\{r_{i}\right\}$ are used in the detector,


Fig.2.3.3 Baseband model of data transmission system as simulated


Fig.2.3.4 Coding of binary digits
the data transmission system of Fig.2.3.1 can be simulated at baseband as shown in Fig.2.3.3 (according to Eq. (2.3.4)) and so saving a considerable amount of computer time. The computer simulation method is described in Appendix B. All computer simulation tests in this thesis with this modulation scheme have assumed this model. The assumptions have been made here of perfect carrier frequency synchronization and perfect symbol timing recovery at the receiver with ideal sampling of $r(t)$ every $T$ seconds (that is, once per symbol). If any of these conditions were not satisfied, then the raised-cosine filtering would also have to be simulated.

The timing and synchronization methods built into the prototype modem are outlined in Sec.2.8, though no test results showing their effectiveness are given in this thesis. Their operation is always assumed to be ideal.

So far in this discussion, the transmitted information has been assumed to be contained in the $\left\{s_{i}\right\}$. In fact, in the transmitter the $\left\{s_{i}\right\}$ must be encoded from the binary digits $\left\{\alpha_{i}\right\}$ which represent the coded speech/data, and in the receiver the detected values of these binary digits $\left\{\alpha_{i}{ }_{i}\right\}$ must be found by decoding the detected symbol values $\left\{s^{\prime}{ }_{i}\right\}$ in a reverse process. This is depicted in Fig.2.3.4.

Two different methods of coding the binary digits of this 4-level QAM (or bandlimited QPSK) signal are considered in this thesis. The two different resulting waveforms are known from now on as QPSK and DQPSK. The signal constellation, which is the same in each case is shown in Fig.2.3.5. The binary digits that represent the coded speech/data are statistically independent and equally likely to be either value or 1 . Two binary digits $\alpha_{i, 1}, \alpha_{i, 2}$ are associated with the data symbol $s_{i}$. The mapping between bits and symbols (Fig.2.3.5) described below, uses an arrangement of Gray coding [36,60]. Thus, adjacent points in the signal constellation differ by only one binary digit.
(1) QPSK - Coherent coding of binary digits

In the coherent system, the data symbol $s_{i}$ is determined from Fig:2.3.5 where the binary coded number shown against any signal point is $\alpha_{i, 1} \alpha_{i, 2}$. In the receiver, the detected data symbol $s^{\prime}{ }_{i}$ determines the corresponding signal point in Fig.2.3.5 and the associated binary coded number gives the detected values of $\alpha_{i, 1} \alpha_{i, 2}$.


Fig.2.3.5 Signal constellation of both QPSK and DQPSK

Table 2.3.1 Differential coding of binary digits

| $\alpha_{i, 1} \alpha_{i, 2}$ | $\beta_{i-1,1} \beta_{i-1,2}$ | $\beta_{i, 1} \beta_{i, 2}$ |
| :---: | :---: | :---: |
| 00 | 00 | 00 |
| 01 | 00 | 01 |
| 11 | 00 | 11 |
| 10 | 00 | 10 |
| 00 | 01 | 01 |
| 01 | 01 | 11 |
| 11 | 01 | 10 |
| 10 | 01 | 00 |
| 00 | 11 | 11 |
| 01 | 11 | 10 |
| 11 | 11 | 00 |
| 10 | 11 | 01 |
| 00 | 10 | 10 |
| 01 | 10 | 00 |
| 11 | 10 | 01 |
| 10 | 10 | 11 |

(2) DQPSK - Differential coding of binary digits

The binary digits $\alpha_{i, 1}, \alpha_{i, 2}$ are differentially encoded into the binary digits $\beta_{i, 1}, \beta_{i, 2}$ according to Table $2.3: 1$ [61], using also the given values $\beta_{i-1,1}, \beta_{i-1,2}$ of the previous two differentially encoded bits. (At the start of every data packet, $\beta_{i-1,1}, \beta_{i-1,2}$ are arbitrarily set to zero). The corresponding data symbol $s_{i}$ is then determined from Fig.2.3.5 where the binary coded number shown against any signal point is now assumed to be $\beta_{i, 1} \beta_{i, 2}$. In the receiver, the detected data symbol $s^{\prime}{ }_{i}$ determines the corresponding signal point in Fig.2.3.5 and the associated binary-coded number gives the detected values of $\beta_{i, 1}, \beta_{i, 2}$. Finally, the detected values of $\alpha_{i, 1}, \alpha_{i, 2}$ are determined from Table 2.3.1 using the detected values of $\beta_{i-1,1}, \beta_{i-1,2} \cdot \beta_{i, 1}, \beta_{i, 2}$

### 2.3.2 16-level QAM

This data signal is a $48 \mathrm{kbit} / \mathrm{s}, 16$-level QAM signal that has a carrier frequency of 900 MHz and an element rate of 12 kbaud . Again this signal occupies a bandwidth of 24 kHz with fully raised-cosine spectral shaping, thus resulting in a 1 kHz frequency guard band between adjacent channels. The only difference with this 16 -level QAM signal lies in the data symbol values $\left\{s_{i}\right\}$. The $\left\{s_{i}\right\}$ are now statistically independent and equally likely to be either one of their 16 possible values $( \pm 1$ or $\pm 3)+j( \pm 1$ or +3). Therefore, at time $t=i T$

$$
\begin{equation*}
s_{i}=s_{I_{. i}}+j s_{Q . i} \tag{2.3.6}
\end{equation*}
$$

where $j=\sqrt{-1}$ and $s_{I . i}, s_{Q . i}= \pm 1$ or $\pm 3$. In fact, the modulation and demodulation methods (Fig.2.3.1) are exactly the same as for the bandlimited QPSK scheme just discussed. A similar analysis can be carried out here to show that the same baseband simulation model of Fig.2.3.3 can be used, under the same assumptions of ideal timing and synchronization. So the sample of the received baseband waveform at time $t=i T$ is again given by

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{2.3.7}
\end{equation*}
$$

where $r_{i}, s_{i}, y_{i}, w_{i}$ are all complex-valued samples.
It has been shown in Appendix $B$ that

$$
\begin{equation*}
E_{b}=2.5 \tag{2.3.8}
\end{equation*}
$$

which is the average transmitted energy per bit in the $\left\{s_{i}\right\}$, and is also equal to the average transmitted energy per bit of the real-valued QAM signal at the output of the modulator in Fig.2.3.1.

Again the coding between the binary digits $\left\{\alpha_{i}\right\}$ representing the coded


| $\begin{array}{cc} \text { Imaginary } \\ \text { Part } \\ 10^{\circ} 00 & 10^{\circ} 10^{+3} \end{array}$ |  |  |  |
| :---: | :---: | :---: | :---: |
|  |  | $11^{\circ} 01$ | 1100 |
| 1001 | $10^{\circ} 11^{+1}$ | $1{ }^{\bullet} 11$ | 1110 |
| - 3 | -1 | $+1$ | $+3$ |
| $00^{\circ} 10$ | $00^{\circ} 11^{-1}$ | 0111 | 0101 |
| 0000 | $0001^{-3}$ | $01^{\circ} 10$ | 0100 |

(b)

Fig.2.3.6 Signal constellation of (a) 16-QAM (b) 16-DQAM
speech/data, and the $\left\{s_{i}\right\}$ of the $16-1$ evel QAM signal, is achieved either coherently or with differential coding. From now on in this thesis, these two different methods will be known as 16 -QAM and 16 -DQAM respectively. The signal constellations which are different for each case are shown in Fig.2.3.6. Gray coding cannot be achieved with differential coding across the whole 16 -point constellation but is only satisfied in each individual quadrant. Thus, adjacent points in the signal constellation that are in different quadrants may differ in their binary codes by more than one digit.

The binary digits that represent the coded speech/data are statistically independent and equally likely to be either value 0 or 1 . Four binary digits $\alpha_{i, 1}, \alpha_{i, 2}, \alpha_{i, 3}, \alpha_{i, 4}$ are associated with the data symbol $s_{i}$. The mapping between binary digits and data symbol values for 16-QAM and 16-DQAM (Fig.2.3.6) is now described.
(1) 16-QAM - Coherent coding of binary digits

In the coherent system, the data symbol $s_{i}$ is determined from Fig.2.3.6(a) where the binary coded number against any signal point is
$\alpha_{i, 1} \alpha_{i, 2} \alpha_{i, 3} \alpha_{i, 4}$. In the receiver, the detected data symbol value $s^{\prime}{ }_{i}$
determines the corresponding signal point in Fig.2.3.6(a) and the
associated binary coded number gives the detected values of
$\alpha_{i, 1} \alpha_{i, 2} \alpha_{i, 3} \alpha_{i, 4}$.
(2) 16-DQAM - Differential coding of binary digits

With 16-DQAM, the binary digits $\alpha_{i, 1}, \alpha_{i, 2}$ are differentially encoded into the binary digits $\beta_{i, 1}, \beta_{i, 2}$ according to Table 2.3.1 [61], but the digits $\alpha_{i, 3}, \alpha_{i, 4}$ are left unchanged. The corresponding data symbol $s_{i}$ is then determined from Fig.2.3.6(b) where the binary coded number shown against any signal point is $\beta_{i, 1} \beta_{i, 2} \alpha_{i, 3} \alpha_{i, 4}$. In the receiver the detected data symbol $s^{\prime}{ }_{i}$ determines the corresponding signal point in Fig.2.3.6(b) and the associated binary-coded number gives the detected values of $\beta_{i, 1}$, $\beta_{i, 2}, \alpha_{i, 3}, \alpha_{i, 4}$ Finally the detected values of $\alpha_{i, 1} \alpha_{i, 2}$ are determined from Table 2.3 .1 using the detected values of $\beta_{i-1,1}, \beta_{i-1,2}, \beta_{i, 1}, \beta_{i, 2}$ As for the DQPSK signal, $\beta_{i-1,1}=\beta_{i-1,2}=0$ at the start of every data packet.

### 2.3.3 Differential Coding

It was shown in Sec.2.2.1 that although the in-phase and quadrature components of the Rayleigh fading waveform $y(t)$ are changing quite slowly and predictably with time, the channel phase is likely to exhibit
instantaneous and unpredictable phase jumps during deep fades, often termed random $\mathrm{FM}[8,9,11,62]$. An example of this is shown in Fig.2.3.7. Thus, in practice it would be impossible to track the phase effectively, which is disasterous for coherent detection. This is why the in-phase and quadrature components of the fading are estimated instead.

However, this does mean that there is likely to be a constant phase ambiguity of $\pm 90^{\circ}$ or $180^{\circ}$ in the estimated phase of the channel over a period of time, accompanied by a corresponding error in the phase of the detected data symbols. The differential coding operation described in Secs.2.3.1-2.3.2 would give correct detection of the binary digits in this situation. The way this works is best described by example:- Ignoring the effects of noise for now, then at time $t=i T$

$$
\begin{equation*}
r_{i}=s_{i} y_{i} \tag{2.3.9}
\end{equation*}
$$

Assume that the channel estimate from $t=T$ to $t=10 \mathrm{~T}$ is $+90^{\circ}$ out of phase. That is,

$$
\begin{equation*}
y_{i}^{\prime}=j y_{i}, \quad \text { for } i=1,2, \ldots, 10 \tag{2.3.10}
\end{equation*}
$$

where $j=\sqrt{-1}$. Or

$$
\begin{equation*}
y_{I . i}^{i}=-y_{Q . i} \text { AND } y_{Q . i}^{\prime}=y_{I . i}, \text { for } i=1,2, \ldots, 10 \tag{2.3.11}
\end{equation*}
$$

Then, the detected symbols over the same period of time will be $-90^{\circ}$ out of phase. That is,

$$
\begin{equation*}
s_{i}^{\prime}=j s_{i}, \quad \text { for } i=1,2, \ldots, 10 \tag{2.3.12}
\end{equation*}
$$

or

$$
\begin{equation*}
s^{\prime} I_{. i}=-s_{Q . i} \text { AND } s_{Q . i}^{\prime}={ }^{s_{I . i}}, \quad \text { for } i=1,2, \ldots ., 10 \tag{2.3.13}
\end{equation*}
$$

since,

$$
\begin{equation*}
\left(-j s_{i}\right) \times\left(j y_{i}\right)=s_{i} y_{i}=r_{i} \tag{2.3.14}
\end{equation*}
$$

So if the binary digits $\alpha_{i, 1}, \alpha_{i, 2},\left(\alpha_{i, 3}, \alpha_{i, 4}\right)$ are detected coherently from the value of $s^{\prime}{ }_{i}$, then they will also be in error for $i=1,2, \ldots . .10$. But, if the differential encoding and decoding operations are carried out according to Table 2.3.1, then the information in the two binary digits $\alpha_{i, 1}, \alpha_{i, 2}$ is contained in the phase change between $s_{i-1}$ and $s_{i}$. Now, a constant phase error in the $s^{\prime}{ }_{i}$ for $i=1,2, \ldots, 10$ means that there is no error in the detected phase changes for $i=2,3, \ldots, 10$ and therefore no error in the detected binary digits $\left\{\alpha_{i, 1}\right\},\left\{\alpha_{i, 2}\right\}$ for $i=2,3, \ldots . .10$.

However, there is a penalty to be paid for this in tolerance to noise. Consider an isolated symbol error. For example, if

$$
s_{5}^{\prime}=j s_{5}
$$

and




Flg.2.3.7 Typical step change in channel phase in going through a deep fade

$$
\begin{equation*}
s_{i}^{\prime}=s_{i}, \quad \text { for all } i \neq 5 \tag{2.3.15}
\end{equation*}
$$

The phase changes between $s_{4}$ and $s_{5}$ and between $s_{5}$ and $s_{6}$ are both detected in error. By studying Table 2.3 .1 it can be shown that generally, with isolated symbol errors, differential coding always doubles the number of errors in the detected binary digits.

Also, differential coding generally worsens the bit error rate when there is a burst of several consecutive random errors in the $\left\{s_{i}\right\}$, though on average, the error rate in this case will be less than doubled. Such a burst of errors is common during a deep fade in the channel.

### 2.4. System 1

Up to this point, the characteristics of the QAM signal, Rayleigh fading and noise that comprise the bandpass channel have been described separately. Now, the first of the complete data transmission systems proposed in this thesis can be described in terms of its equivalent baseband model (developed in Appendix B). A coherent demodulation receiver is used in this system. It is the development of this receiver from its basic form described here [63-68], that is the aim of Chapter 3 .

### 2.4.1 Model of System

A four-level QAM signal is transmitted from one mobile to a base station as shown in Fig.2.4.1. By the theorem of reciprocity, this is equally applicable to transmission in the reverse direction [8,11]. The system is tested with both one and two receiving antennas (Systems 1A and 1B respectively) and with both QPSK and DQPSK modulation.

The baseband equivalent models of Systems 1A and 1B are given in Fig.2.4.2. In the computer simulations the baseband received samples at time $t=i T$, for all \{i\}, are given by:

For one receiving antenna, System 1A

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{3.2.1}
\end{equation*}
$$

For two receiving antennas, System 1B

$$
\begin{align*}
& r_{a, i}=s_{i} y_{a, i}+w_{a, i} \\
& r_{b, i}=s_{i} y_{b, i}+w_{b, i} \tag{3.2.2}
\end{align*}
$$

Where the letters $s, y, w$ refer to the complex-valued data, channel and noise waveforms respectively. The subscript $i$ means that the corresponding waveforms have been sampled at time $t=i T$. The subscripts a and $b$ before the dot refer to receiving antennas $a$ and $b$. The methods of


Fig.2.4.1 Block diagram of System 1 with (a) one receiving antenna (System 1A) and (b) two receiving antennas (System 1B)


Fig.2.4.2 Baseband model of data transmission system used in computer simulation tests for (a) System 1A and (b) System 1B
generating the $\left\{s_{i}\right\},\left\{y_{i}\right\},\left\{w_{i}\right\}$ and so forming the $\left\{r_{i}\right\}$ in the computer simulations are described in Appendix B.

The important assumptions from which these equations have been derived in Appendix $B$ were described in detail in Sec. 2.3 and are summarized here as follows:

1) Perfect linear modulation and demodulation are assumed, with perfect nominal carrier frequency synchronization between transmitter and receiver. No assumption need be made about carrier phase synchronization.
2) The baseband received signal is sampled every $T$ seconds (once per symbol) with perfect symbol timing recovery assumed at the receiver. 3) Matched filtering using root-raised-cosine frequency response filters.
3) Frequency non-selective fading with the frequency spread of the fading very small compared with the signal bandwidth (less than 1\%).
4) With two receiving antennas, separate ideal matched filtering and sampling is carried out at each antenna. Uncorrelated fading is assumed in the signals arriving at these two antennas.

It has been shown in Appendix $B$ that under these assumptions, the relevant statistical properties of the samples $\left\{r_{i}\right\},\left\{s_{i}\right\},\left\{y_{i}\right\},\left\{w_{i}\right\}$ can be summarized as follows:

Data: $\left\{s_{i}\right\}$
With the five assumptions just described, the samples $\left\{s_{i}\right\}$ on the computer are simply the data symbol values, $\left\{s_{i}\right\}=\{ \pm 1 \pm j\}$. That is, $s_{i}$ is any one of the four points in the constellation shown in Fig.2.4.3 with equal probability. The individual $\left\{s_{i}\right\}$ being statistically independent. The bit-to-symbol mapping always follows the arrangement of Gray coding shown in Fig.2.4.3, where each symbol $s_{i}$ is formed by mapping the two random dibits $\alpha_{i, 1} \alpha_{i, 2}$ as shown. These $\left\{\alpha_{i}\right\}$ are statistically independent and equally likely to be either value 0 or 1 . In the coherent "QPSK" system, the pseudo-random bit stream is directly encoded as shown. But in the differentially coded "DQPSK" system, the random dibits are first differentially encoded as in Sec.2.3.1, then these differentially encoded bits are mapped into symbols as shown in Fig.2.4.3.

The transmitted data signal is assumed to be a 12 kbaud four-level QAM signal, so $12000\left\{s_{i}\right\}$ are transmitted every second in the baseband simulation model. The transmitted energy $E_{s}$ in every data symbol in the baseband model is the same.


Fig.2.4.3 QPSK signal constellation



Fig.2.4.4 (a) Theoretical and (b) simulated power spectral densties of $\operatorname{Re}\left[y_{i}\right]$ and $\operatorname{Im}\left[y_{i}\right]$



Fig.2.4.5 (a) Constellation of $\mathrm{s}_{i}$
(b) Example constellation of $\mathrm{s}_{\mathrm{i}} \mathrm{y}_{\mathrm{i}}$, with an example of $r_{i}$

$$
E_{S}=( \pm 1)^{2}+( \pm \dot{j})^{2}=2
$$

Since two bits are encoded to give each symbol.

$$
\begin{equation*}
E_{b}=\frac{1}{2} E_{s}=1 \tag{2.4.3}
\end{equation*}
$$

This is the same average transmitted energy per bit as in the real-valued QAM waveform, when modulation, demodulation and filtering are as described in Sec.2.3. It can be shown that both baseband and modulated carrier signals have the same tolerance to additive white Gaussian noise [31,55].

## Channel: $\left\{y_{i}\right\}$

The channel $\left\{y_{i}\right\}$ is represented on the computer as a complex-valued Gaussian random process. The real and imaginary parts of $y_{i}$ are samples of statistically independent baseband Gaussian waveforms, zero mean, variance $\frac{1}{2}$, each with the power spectral density [8-11]

$$
|Y(f)|^{2}= \begin{cases}\frac{1}{2 \pi f_{m} \sqrt{1-\left(f / f_{m}\right)^{2}}}, & \text { for }-f_{m} \leqslant f \leqslant+f_{m}  \tag{2.4.4}\\ 0, & \text { elsewhere }\end{cases}
$$

Where $f_{m}$ is the maximum Doppler frequency shift. In the computer simulations $f_{m}=80 \mathrm{~Hz}$ or 40 Hz corresponding to a vehicle speeds of 60miles/hour or 30miles/hour respectively. The theoretical and simulated spectral densities are shown in Fig.2.4.4. The channel simulation method is described in Appendix B.

It now follows $[24,25]$ that the channel amplitude given by

$$
\begin{equation*}
\left|y_{i}\right|=\sqrt{\operatorname{Re}\left[y_{i}\right]^{2}+\operatorname{Im}\left[y_{i}\right]^{2}} \tag{2,4.5}
\end{equation*}
$$

is Rayleigh distributed. This amplitude has a mean-square value of 1.
That is, the average channel power is unity since

$$
\begin{equation*}
\lim _{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^{N}\left|y_{i}\right|^{2}=1 \tag{2.4.6}
\end{equation*}
$$

It also follows $[24,25]$ that the channel phase given by

$$
\begin{equation*}
\angle y_{i}=\tan ^{-1} \frac{\operatorname{Im}\left[y_{i}\right]}{\operatorname{Re}\left[y_{i}\right]} \tag{2.4.7}
\end{equation*}
$$

is uniformly distributed over all phase angles.
In all tests with two antennas, both complex-valued transmission paths $Y_{a . i}$. Yb.i in Fig.2.4.2 are modelled exactly as just described. These two fading channels are statistically independent with the same mean-square value and the same value of $f_{m}$.

Noise: $\left\{w_{i}\right\}$
The noise samples $\left\{w_{i}\right\}$ in the computer simulations are complex-valued
samples of a white Gaussian noise process. The real and imaginary components of $w_{i}$ are both zero mean, variance $\sigma^{2}$. $\sigma^{2}$ is fixed for any given signal-to-noise ratio $\psi$. But, the real-valued white noise input to the receiver filter has a two-sided power spectral density of $\frac{1}{2} \mathrm{~N}_{0}$ over all positive and negative frequencies. With the choice of linear modulation and demodulation as shown in Sec.2.4 it was shown in Appendix $B$ that

$$
\begin{equation*}
\sigma^{2}=\frac{1}{2} N_{0} \tag{2.4.8}
\end{equation*}
$$

Since the real and imaginary parts of $w_{i}$ are independent, the average power of the complex-valued noise samples $\left\{w_{i}\right\}$ is

$$
E\left[\left|w_{i}\right|^{2}\right]=2 \sigma^{2}=N_{0}
$$

In all tests with two receiving antennas, the $\left\{w_{a . i}\right\},\left\{w_{b . i}\right\}$ are independent of each other and both have the same value of $\sigma^{2}$.

## Received Samples: $\left\{r_{i}\right\}$

The received signal constellation of $s_{i} y_{i}$ in $r_{i}$ is the four-point constellation of $s_{i}$, simply shifted in amplitude and phase by the Rayleigh fading, as in Fig.2.4.5. This is true for all \{i\}. It is important to emphasize that there are no intersymbol interference components in $r_{i}$, only $s_{i} Y_{i}$ plus white Gaussian noise (see Eq.(2.4.1) and Fig.2.4.5). With two receiving antennas, the two received signals are separately - but identically - demodulated, filtered and sampled to give the $\{r a . i\}$ and $\left\{r_{b . i}\right\}$. The signal constellations are generally shifted in amplitude and phase by different amounts at each antenna due to the uncorrelated fading samples $y_{a . i}, y_{b . i}$.

## Signal-to-noise ratio

The signal-to-noise power ratio per transmitted bit in the received samples $\left\{r_{i}\right\}$ at the input to the detector is equal to (Appendix B)

$$
\begin{equation*}
\psi=\frac{E_{b}}{N_{0}} \tag{2.4.10}
\end{equation*}
$$

Therefore, in decibels

$$
\begin{equation*}
\psi=10 \log _{10} \frac{\mathrm{E}_{\mathrm{b}}}{\mathrm{~N}_{\mathrm{O}}} d \mathrm{~d} \tag{2.4.11}
\end{equation*}
$$

In all tests with two antennas, the signal-to-noise ratio at each antenna is the same and is given by Eq. (2.4.11). This is also taken to bethe system signal-to-noise ratio in every case, no matter how the signals are combined.

This completes the discussion of the statistical properties of the $\left\{r_{i}\right\},\left\{s_{i}\right\},\left\{y_{i}\right\},\left\{w_{i}\right\}$. The general operation of the receivers for Systems 1A and 1B are now described.

### 2.4.2 Coherent Demodulation Receiver for System 1A

For ideal coherent detection of a QAM signal, the received carrier phase must always be known exactly at the receiver, which usually means that a phase-coherent local carrier must be generated at the receiver. However, this cannot be achieved here without a certain amount of signal processing, even if very stable and costly oscillators are used in both transmitter and receiver. This is because the Rayleigh fading introduces a time varying random phase rotation into the transmitted signal that cannot be controlled (Sec.2.2). This usually changes considerably faster than any phase drift caused by the local oscillators. It is for this reason that the use of coherent demodulation is usually ruled out for land mobile radio [8].

It is assumed throughout this thesis that the transmitter and receiver local oscillators are synchronized in freguency around 900 MHz (Sec.2.8), no attempt is made to adaptively control the phase of the reference carriers in the demodulation process. The Rayleigh fading samples $\left\{y_{i}\right\}$ in
Eq. (2.4.1) can now be assumed to contain any constant phase errors between these two local oscillators ( $\gamma$ radians) and any relatively slow drifts in carrier frequencies/phases, without significantly affecting the statistical properties of the $\left\{y_{i}\right\}$ described in Sec.2.4.1.

In recent years some quite simple channel estimation processes have been developed at Loughborough University $[64,65,69-71]$ that successfully track the slower Rayleigh fading encountered in HF radio links. It remains to be seen whether the faster Rayleigh fading encountered in land mobile radio could be successfully tracked using similar methods. With an accurate estimation of the $\left\{y_{i}\right\}$ in the receiver, coherent detection of the QAM signal could be achieved. It is important to note that in this thesis the $\left\{y_{i}\right\}$ are always estimated in terms of their in-phase and quadrature components rather than their corresponding amplitude and phase because the former components are easier to track (Sec.2.2.1)

This coherent demodulation receiver for System 1A is shown in Fig.2.4.2(a) as a combined detection and estimation process. A more detailed model is shown in Fig.2.4.6, where the channel estimation process is shown to perform the two separate functions of estimation and


Fig.2.4.6 Combined detector and estimator for System 1A
prediction. The general principle of operation for this receiver is now described.

It is required here to recover the information in the transmitted speech/data by detecting the data symbols $\left\{s_{i}\right\}$ from the received samples $\left\{r_{i}\right\}$. These samples $\left\{r_{i}\right\}$ are given by Eq. (2.4.1), for all $\{i\}$, as

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{2.4.12}
\end{equation*}
$$

Ignoring the effects of the noise for now, Eq.(2.4.12) is essentially one equation in two unknowns $s_{i}, y_{i}$. So there is not enough information in Eq. (2.4.3) alone to detect $s_{i}$ from $r_{i}$. However, in Sec. 2.2 it was shown that $y(t)$ is a lowpass random waveform whose quadrature components have a maximum frequency of $f_{m}=80 \mathrm{~Hz}$. So the samples $\left\{y_{i}\right\}$, which are samples of $y(t)$ taken every $T$ seconds (or every $1 / 12000$ seconds), represent gross oversampling at 75 times the Nyquist rate [35]. This means that the value of $y_{i}$ does not change appreciably from one sample to the next so it would be quite simple to form an accurate prediction of the value of $y_{i}$ from the previous set of samples $\left\{y_{n}\right\}$, for $n \leqslant i-1$. This knowledge that the samples $\left\{y_{i}\right\}$ are slowly changing, used together with Eq. (2.4.12) means that there is enough information in the $\left\{r_{i}\right\}$ to detect the $\left\{s_{i}\right\}$ with the arrangement shown in Fig.2.4.6.

First of all, at time $t=i T$ the detected value $s^{\prime}{ }_{i}$ of the data symbol $s_{i}$ is formed in a process of coherent detection described in Chap. 3. In detecting $s_{i}$ the detector assumes that the prediction $y^{\prime}{ }_{i, i-1}$ made at time $t=(i-1) T$ is an exact estimate of $y_{i}$. (In fact the $\left\{y_{i, i-1}^{\prime}\right\}$ are generally noisy estimates of the $\left\{y_{i}\right\}$ ). Then the detected value $s^{\prime}{ }_{i}$ is fed back to the channel estimator together with $r_{i}$, and the channel estimate $y^{\prime}{ }_{i}$ is formed (Chap.3). The channel estimator relies very heavily on the correct detection of the data symbols. The channel predictor then predicts along the samples $\left\{y^{\prime}{ }_{n}\right\}$, for $n \leqslant i$, to give $y_{i+1, i}$ which is held in store ready for the detection of the next data symbol $s_{i+1}$.

It is important to understand this general principle of operation for the combined detection and estimation process because every System tested in this thesis uses an adaption of the same basic receiver structure. However, this basic system (used successfully in HF radio links [64,65]) is developed in a novel way in this thesis to be more suited to the fast Rayleigh fading environment encountered in land mobile radio. This is described in detail in Chapters 3 to 5.


Fig.2.4.7 Combined detector and estimator for System 1.B

### 2.4.3 Coherent Demodulation Receiver for System 1B

Now, with two receiving antennas, the combined detection and estimation process is adapted as shown in Fig.2.4.7. The channel estimation process corresponding to the two antennas $A$ and $B$ both work in exactly the same way as that described in System 1A. It is important to note that these two estimation processes work in parallel and both use the same set of values $\left\{s^{\prime}{ }_{i}\right\}$ at their input. However, in this case, $s^{\prime}{ }_{i}$ is coherently detected from a combination of the signals from the two antennas (Chap.3). A big improvement in the performance is expected over System 1A $[8,9,22,23,29]$ because the fading sequences at each antenna, $\left\{y_{a, i}\right\}$, $\left\{y_{b . i}\right\}$ are uncorrelated. It was noted in Sec. 2.2 that errors tend to occur during deep fades. With uncorrelated fading sequences it is much more unlikely that a deep fade will occur at both antennas at the same time than it is for a similar deep fade to occur at either one antenna. It therefore follows that detection errors caused by these fades are much less likely with two antennas than with one.

## 2. 5 System 2

After the successful development of System 1 in Chapter 3, the aim of the rest of the thesis is to devolop a mobile radio link with twice the bandwidth efficiency of System 1. System 2 described in Chapter 4 is such a system that would be used to transmit in the direction from two mobiles to the base station. It is the development of the receiver from its basic form introduced here that is the aim of Chapter 4.

### 2.5.1 Model of System

A $24 \mathrm{kbit} / \mathrm{s}$ four-level QAM signal is transmitted from each of two mobile units to the same base station. The two signals transmit on the same carrier frequency, so the sum of the two signals is received at the base. station within a 25 kHz portion of the spectrum around 900 MHz . The system is tested with both one and two receiving antennas (Systems 2A and 2B respectively) and with both QPSK and DQPSK modulations (Sec.2.3). The baseband equivalent models of Systems 2 A and 2 B are given in Fig.2.5.2. If either of the two mobiles were to stop transmitting, then the model of the system (Figs.2.5.1-2.5.2) would reduce to that of System 1
(Figs.2.4.1-2.4.2)
In the computer simulations, the basebend received samples at time


Fig.2.5.1 Model of System 2 with (a) one receiving antenna (System 2A) and (b) two receiving antennas (System 2B)

(a)


Fig.2.5.2 Data transmission system used in computer
simulation tests
(a) System 2A
(b) System 2B
$t=i T$, for all \{i\}, are given by:
For System 2A

$$
\begin{equation*}
r_{i}=s_{1, i} y_{1, i}+s_{2, i} y_{2, i}+w_{i} \tag{2.5.1}
\end{equation*}
$$

For System 2B

$$
\begin{align*}
& r_{a, i}=s_{1 . i} y_{1 . i}+s_{2, i} y_{2 . i}+w_{a, i} \\
& r_{b, i}=s_{1, i} y_{3 . i}+s_{2, i} y_{4 . i}+w_{b, i} \tag{2.5.2}
\end{align*}
$$

The signal, channel and noise samples have been represented by the letters s, $y$, w respectively as for System 1. The subscript $i$ after the dot shows that samples of the baseband waveforms have been taken at time $t=i T$. The subscripts $a$ and $b$ of $r$ and $w$ refer to receiving antennas $A$ and $B$. The subscripts 1 and 2 of $s$ refer to the data signals from mobiles 1 and 2. To be consistent with this notation the four channels in Fig.2.5.1(b) would have been labelled $y_{1 a}, y_{2 a}, y_{1 b}, y_{2 b}$ denoting the transmission paths between signals 1 and 2 and antennas $A$ and $B$, but the double subscript proved to be too confusing. Hence they have been labelled here as $y_{1}, y_{2}$, $\mathrm{y}_{3}, \mathrm{y}_{4}$ respectively.

Of course, for these Eqs.(2.5.1)-(2.5.2) to be valid the same five ideallistic assumptions as for System 1 must be made (see Sec.2.4.1). But now, two additional important assumptions must be made:
6) The signals originating from the two mobiles must arrive perfectly synchronized in time at the receiving antennas, so that the ideal sampling instant at the output of the matched filter is exactly the same for both signals $s_{1}$ and $s_{2}$.
7) The nominal carrier frequency ( 900 MHz ) must be the same for both local oscillators.

Neither of these conditions can be met exactly in practice. It is assumed in this thesis that the simple timing and synchronization methods outlined in Sec. 2.8 result in a close approximation to these conditions. Now, none of the results given in Chap. 6 would be adversely affected by such timing and synchronization errors.

Now consider the statistical properties of these data, channel and noise samples. The two data streams $\left\{s_{1 . i}\right\},\left\{s_{2 . i}\right\}$ each have exactly the same properties is described for the $\left\{s_{i}\right\}$ in System 1 . The information transmitted from both mobiles are independent of each other, so the two data streams are assumed to be uncorrelated. All four sets of channel samples $\left\{y_{1 . i}\right\},\left\{y_{2 . i}\right\},\left\{y_{3 . i}\right\},\left\{y_{4 . i}\right\}$ have exactly the same properties as described for the channels in System 1, and are all statistically independent of each other. Uncorrelated Rayleigh fading from the two
mobiles is assured because the distance separating them need only be greater than about 9 inches [8,9,44]. In all simulation tests, both mobiles are assumed to be travelling at the same speed, (60miles/hour). This is the worst case since the two fading data signals are now most likely to be confused with each other (Sec.4.4).

The received signal constellation of $s_{1 . i} y_{1 . i}{ }^{+} s_{2 . i} Y_{2 . i}$ in $r_{i}$ is a l6-point constellation. Each four point constellation of $s_{1 . i}$ and $s_{2 \text {.i }}$ is shifted in amplitude and phase by their uncorrelated Rayleigh fading samples. The sum of these two four-point fading constellations gives the required 16 -point constellation. This is true for all \{i\}. The shape of this 16-point constellation changes with time as the two constituent four-point constellations fade independently of each other. An example of this is shown in Fig.2.5.3. It is important to emphasize that there are no intersymbol interference components in $r_{i}$.

In Systems 2A and 2B the signal-to-noise ratio is again given by

$$
\begin{equation*}
\psi=E_{b} / N_{0} \tag{2.5.3}
\end{equation*}
$$

or, in decibels

$$
\begin{equation*}
\psi=10 \log _{10}\left(E_{b} / N_{0}\right)=10 \log _{10}\left(1 / 2 \sigma^{2}\right) d B \tag{2.5.4}
\end{equation*}
$$

Where $E_{b}=1$ is the average transmitted energy per bit of each signal $\left\{s_{1, i}\right\},\left\{s_{2 . i}\right\}$. $E_{b}=1$ is also the average transmitted energy per bit of the sum of the two independent transmitted signals $\left\{s_{1 . i}+s_{2 . i}\right\}$, so there can be no confusion. Thus the signal-to-noise ratio in the channel is unaffected by transmitting a second signal, that is, in going from System 1 to System 2.

The general operation of the receivers for Systems $2 A$ and $2 B$ can now be described.

### 2.5.2 Coherent demodulation receiver for System 2

The receiver used here achieves coherent detection of the data symbols with a similar combined detection and estimation process to that used for System 1. The block diagram of this basic system is shown in Fig. 2.5.4. Again, the real and imaginary parts of the complex-valued channel components $\left\{y_{1 . i}\right\},\left\{y_{2 . i}\right\},\left\{y_{3 . i}\right\}$, $\left\{y_{4 . i}\right\}$ are estimated. These estimates can be used to achieve coherent detection of the data symbols $\left\{s_{1 . i}\right\}$, $\left\{s_{2 . i}\right\}$ without the need to generate a phase-coherent local carrier in the demodulator.

The receiver for System 2A (Fig.2.5.4(a)) basically works as follows. First of all at time $t=i T$, the detected values $s^{\prime} 1 . i^{\prime} s^{\prime}{ }_{2 . i}$ of the


Fig.2.5.3 (a) Constellation of $s_{1 . i}$ and $s_{2 . i}$.
(b) Example constellation of $s_{1} y_{1}$ and $s_{2} y_{2}$.
(c) 16-point constellation of $\mathrm{s}_{1} \mathrm{y}_{1}+\mathrm{s}_{2} \mathrm{y}_{2}$


Fig.2.5.4(a) Combined detector and estimator for System 2A


Fig.2.5.4(b) Combined detector and estimator for System 2B
corresponding data symbols $s_{1 . i}, s_{2 . i}$ are both formed in a process of coherent detection described in Chap.4. In detecting the symbols $s_{1 . i}$, $s_{2 . i}$ from $r_{i}$ the detector assumes that the predictions $Y^{\prime}{ }_{1 . i}, i-1$, $y_{2 . i, i-1}^{\prime}$ made at time $t=(i-1) T$ are exact estimates of $y_{1 . i}, Y_{2 . i}$ respectively. (In fact, the $\left\{y_{1 . i, i-1}^{\prime}\right\},\left\{y_{2 . i, i-1}^{\prime}\right\}$ are generally noisy estimates of the $\left\{y_{1 . i}\right\},\left\{y_{2 . i}\right\}$ ). Then the detected values $s^{\prime}{ }_{1 . i}{ }^{\prime} s^{\prime}{ }_{2 . i}$ are fed back to the channel estimator together with $r_{i}$, and the channel estimates $Y^{\prime}{ }_{1.1}, Y^{\prime}{ }_{2 . i}$ are formed (see Chap.4). The channel estimator relies very heavily on the correct detection of the data symbols. The channel predictors then predict along the samples $\left\{y^{\prime}{ }_{1 . n}\right\},\left\{y^{\prime}{ }_{2 . n}\right\}$ for $n \leqslant i$, to give $y^{\prime}{ }_{1 . i+1, i}, y^{\prime}{ }_{2 . i+1, i}$ which are held in store ready for the detection of the next data symbols $s_{1 . i+1}, s_{2 . i+1}$.

For System 2B with two receiving antennas, the combined detection and estimation process is shown in Fig.2.5.4(b). The channel estimation process corresponding to the two antennas A and B both work in exactly the same way as just described for System 2A. It is important to note that these two estimation processes work in parallel and both use the same sets of samples $\left\{s^{\prime}{ }_{1 . i}\right\},\left\{s^{\prime}{ }_{2 . i}\right\}$ at their inputs. However, in this case $s^{\prime}{ }_{1 . i}$. $s^{\prime} 2 . i$ are coherently detected from a combination of the signals from the two antennas (Chap.4). Again, a big improvement in performance is expected over System 2A $[8,9,22,23,29]$ because of the uncorrelated fading at the two receiving antennas.

Of course, this is only a description of how the information flows through the functional blocks of the basic Systems shown in Fig.2.5.4. The actual signal processing carried out at each stage, and the development of the model from its basic form to the final System, is described in Chap. 4 .

This System 2 uses a completely new way of multiplexing two digital radio signals in the same frequency space. The two data streams \{s' $\left.{ }_{1.1}\right\}$, \{s'2.i\} can be separated at the receiver without the use of error control coding because these two independent data streams fade independently. So this independent fading is itself a form of coding. This is probably the most important result in this thesis.

### 2.6 System 3

In Chapter 5, System 3 is described in which a single signal is transmitted back from the base station to the two mobiles that are in turn
channels
(a)

mobile 2


base station
channels

base station

Fig.2.6.1 Model of System 3 with (a) one receiving antenna (System 3A) (b) two receiving antennas (System 3B)
transmitting according to System 2. This system also has twice the bandwidth efficiency of System 1. It is the development of the receiver from its basic form introduced here that is the aim of Chapter 5.

### 2.6.1 Model of System

A single $48 \mathrm{kbit} / \mathrm{s} 16$-level QAM signal transmitted from a base station is received by two different mobiles in the cell. Half of the bits are allocated to each mobile. Again the carrier frequency is about 900 MHz with a channel spacing of 25 kHz . The system is tested with both one and two receiving antennas (Systems $3 A$ and $3 B$ respectively) and with both 16-QAM and 16-DQAM modulations.

Only the signal received at one of these mobiles need be simulated, since it is assumed that the performance of the receiver at the other mobile travelling at the same speed would be exactly the same. The simulation model is not complicated here by considering how the information in the 16 -level signal would be allocated to each mobile. Instead the bit error rate in the received 16 -level QAM signal is measured in the simulation tests. This allows more useful comparisons to be made. with the other modulation schemes in Systems 1 and 2.

As for System 1, the same 5 assumptions of ideal timing, synchronization, linear modulation / demodulation and raised-cosine filtering are made (Sec.2.4.1). Under these assumptions, the baseband received samples at the output of the matched filter at time $t=i T$ are

For System 3A

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{2.6.1}
\end{equation*}
$$

For System 3B

$$
\begin{align*}
& r_{a, i}=s_{i} y_{a . i}+w_{a . i} \\
& r_{b . i}=s_{i} y_{b . i}+w_{b . i} \tag{2.6.2}
\end{align*}
$$

These equations are identical to those used to represent System 1 , and so the basic simulation model (Fig.2.4.2) is used. The only difference between Systems 1 and 3 lies in the set of data symbol values $\left\{s_{i}\right\}$ that is now used to represent a 16 -level QAM signal. Under the five idealistic assumptions outlined in Sec.2.4.1, the samples $\left\{s_{i}\right\}$ in Eqs.(2.6.1)-(2.6.2) are the data symbol values

$$
\left\{s_{i}\right\}=\{( \pm 1 \text { or } \pm 3)+( \pm j \text { or } \pm 3 j)\}
$$

That is, any one of the sixteen points in the signal constellation of Fig.2.6.2(a) with equal probability. The individual $s_{i}$ being statistically independent.



Fig.2.6.2 (a) Constellation of 16-level QAM signal, $s_{i}$ (b) Example constellation of $s_{i} y_{i}$

In Systems $3 A$ and $3 B$, the signal-to-noise ratio is always given by

$$
\psi=10 \log _{10}\left(E_{b} / N_{0}\right)=\operatorname{lol}_{10}\left(2.5 /\left(2 \sigma^{2}\right)\right) d B
$$

where $E_{b}=2.5$ is the average transmitted energy per bit of the 16-level QAM signal (Sec.2.3).

Again, when two receiving antennas are used at the mobile receiver, they are assumed to be spaced sufficiently far apart that the fading sequences $\left\{y_{a . i}\right\},\left\{y_{b . i}\right\}$ are uncorrelated. The sets of fading samples $\left\{y_{i}\right\},\left\{y_{a . i}\right\},\left\{y_{b . i}\right\}$ all have a mean-square value of 1 . The received signal constellation of $s_{i} y_{i}$ in $r_{i}$ is the 16 -point constellation of $s_{i}$ that is simply shifted in amplitude and phase by the Rayleigh fading as in Fig.2.6.2, with no intersymbol interference. The relative distances between the different points of the received signal constellation does not change with time, as it did in System 2.

The coherent demodulation receiver for System 3 has the same basic structure of combined detection and estimation as that used in System 1 (Figs.2.4.4-2.4.5). So the general description of how it works is also exactly the same as for System 1. However, the actual signal processing carried out in the detector and estimator is different for Systems 1 and 3 because of the different constellations of the data signal $s_{i}$. It is the detailed development of this combined detection and estimation process in a way that is suitable for System 3 that is the aim of Chapter 5 .

### 2.7 Reasons for using coherent detection of narrowband QAM signals

It is mentioned in Chapter 1 that nowadays probably the most important requirement of a mobile radio link design is that the digital modulation scheme used must provide a high bandwidth efficiency in terms of $\mathrm{bit} / \mathrm{s} / \mathrm{Hz}$ for a given quality of reception. With this in mind, a narrowband QAM scheme with coherent detection in the receiver was chosen (Secs.2.3-2.6). This technique is used in all systems tested in this thesis and is expected to perform favourably compared with other digital transmission techniques that have been reported to be applicable to mobile radio reception $[43,72,73]$.

Coherent detection is performed at the receiver without the need to transmit a pilot carrier, by using a channel estimation process to track the Rayleigh fading in the received signal. The in-phase and quadrature components of the fading are tracked rather than the corresponiding amplitude and phase since the latter representation is more rapidly time
varying (Sec.2.2). The irreducible error rate normally associated with the random $\operatorname{FM}[8,9,62,74-76]$ should now be largely avoided. The signal processing involved in such an estimation process [63-71] is generally much less complicated than that used in the pilot tone, single sideband (SSB) schemes using coherent detection [77-83] that have been publicised so much in recent years. It is well known that in the presence of additive white Gaussian noise, coherent detection results in a lower probability of error in the detection of the $\left\{s_{i}\right\}$ than any other data recovery scheme. It should also give the best performance in the presence of interference from any other signals (Sec.2.2.4). Although differential coding of the binary digits (Sec.2.3.3) increases the bit error rate compared with ideal coherent detection, it does give correct recovery of the binary digits when there is a constant phase ambiguity of $\pm 90^{\circ}$ or $180^{\circ}$ in the phase of the channel estimate. Such a phase error is likely to be caused by the random $F M$ in the Rayleigh fading. This method of differential coding does not use differential ( / differentially-coherent) detection at the receiver $[31,36,84]$. Modems using differential detection in fading mobile radio channels often exhibit serious irreducible error rates caused by the random FM. This error rate gets worse as the speed of the mobile increases [62,74-76]. The differential coding method used here should not suffer this limitation, if the channel estimate performs well.

The narrowband QAM signals used in this thesis all have a fully raised-cosine spectral shape [31] in an arrangement of matched filtering. So no intersymbol interference is introduced by this filtering and it is well known that the tolerance to noise of these bandimited QAM signals is identical to that for the corresponding signals that have not been bandlimited (if ideal coherent detection is assumed at the receiver [31]). The out-of-band radiation is expected to be at least 60dB below the average signal power at the centre of the passband (Appendix B). So with this filtering, adjacent channel interference can be kept to a very low level. A further very useful consequence of this filtering is that the element-timing waveform can be simply extracted at the receiver ( $\sec .2 .8$ ) .

It is interesting to note that the use of coherent detection is usually ruled out in land mobile radio links because of the difficulty in generating a coherent phase reference at the receiver in the presence of such fast Rayleigh fading [8]. This rapid amplitude fading is also assumed to rule out the use of amplitude modulation (AM) schemes in favour
of frequency modulation (FM) techniques which, of course, do not use the amplitude information. In recent years, the narrowband digital FM transmission techniques of tamed FM (TFM) [85-87] and Gaussian baseband filtered minimum shift keying (GMSK) [ 88,89$]$ have been reported to be applicable to mobile radio transmission $[43,90]$. Both of these digital FM techniques have a constant envelope which means they suffer a minimum of distortion in the equipment's non-linear amplifiers. They are both considered to be contained in a sufficiently compact power spectrum for a cellular land mobile radio service. However, the QAM signals used here that are not constant envelope signals occupy an even more compact spectrum $[32,72,72]$. Techniques are now available that could enable a high-power amplifier at a mobile to handle the large envelope ripple in the QAM signal [56-59]. A further important problem with these constant envelope techniques TFM and GMSK is that a very much more complicated process is required to extract the element timing waveform [85,87,91] than is used with the QAM signals in this thesis (see Appendix C).

### 2.8 Timing and Synchronization

It is assumed in all computer simulation tests carried out in this thesis for Systems 1,2 and 3 that ideal timing and synchronization has been achieved. Under these ideal conditions the following assumptions are made:
(i) Perfect linear modulation and demodulation is assumed, with perfect nominal carrier frequency synchronization between (both) transmitter(s) and the receiver. No assumptions need be made about carrier phase synchronization.
(ii) The ideal element and frame-timing waveforms are assumed to have been recovered perfectly at the receiver. When the signals from two mobiles share the same channel (System 2), the two signals are assumed to arrive at the base station exactly synchronized in time. Under this condition, the ideal timing waveforms corresponding to the two received signals are coincident in time.

Of course, in practice there will be a small departure from these ideal conditions, but this should not greatly affect the results of any tests.

The base station transmits, in addition to the digital signals to the mobiles, a single uninterrupted modulated-carrier signal in which the same
sequence of data symbols is continually repeated. The carrier frequency of this synchronizing signal is used to control the carrier frequency transmitted by every mobile in that cell. The signal is also used to achieve both element-timing and frame-timing synchronization of the signals transmitted by the mobiles in the cell, such that the corresponding elements and frames in these signals are coincident in time (at least approximately.)..when they reach the base station.

In the prototype modem being built at Loughborough and Liverpool Universities [92,93], a simple $12 k b a u d$ binary FSK (frequency shift keying [31]) synchronizing signal was used with a carrier frequency about 10 MHz apart from that of the mobile transmitter. Initial hardware tests indicate very promising synchronization results. The basic methods of achieving carrier frequency synchronization and element/frame-timing synchronization are now described.

### 2.8.1 Carrier Frequency Synchronization

Economic considerations make it difficult to achieve oscillator stabilities better than $2 \frac{1}{2}$ parts per million $[6,7]$ (i.e. $\pm 2.25 \mathrm{kHz}$ at 900 MHz ) at the mobile unit. It is feasible to control the base station to at least an order better than this. For the purpose of analysis, the base station is assumed to maintain perfect stability.

The synchronizing signal is received by the mobile with a Doppler frequency shift, which is determined by the arrival angles of the component radio waves relative to the direction of motion of the mobile (Appendix A). This received carrier frequency is shifted by several MHz in the mobile to give the carrier frequency of the transmitted digital QAM signal. The errors in the carrier frequencies received at the base station are now confined (at least approximately) to within twice the maximum Doppler shift. That is, within about 160 Hz of the base station reference of 900 MHz , for a vehicle speed of $60 \mathrm{miles} / \mathrm{hour}$.

Had the 900 MHz carrier frequency been generated directly in the mobile, the frequency error of $\pm 2.25 \mathrm{kHz}$ would have caused some serious problems. Firstly, adjacent channel interference would be introduced since the frequency guard band is only 1 kHz . Also, with two signals transmitted in the channel (System 2), the best that could be done in the base station receiver would be to set the demodulating carrier frequency half-way between these two carrier frequencies. Significant distortion would almost certainly be introduced into the two received baseband signals by
the matched filtering process. It would be extremely difficult to track the baseband channel waveforms with such high frequency components.

### 2.8.2 Element and Frame Synchronization

The binary FSK synchronizing signal transmitted by the base station is shown in Fig.2.8.1. This signal is continuously repeated. The mobile unit locates the unique word (which indicates the start of each frame) and extracts the symbol timing from the bit reversals. The mobile then transmits its QAM data signal with the same element and frame timing waveforms.

The unique word

$$
\left\{\begin{array}{lllllllllllll}
1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1
\end{array}\right\}
$$

is the Barker code of length 13 [94,95]. Its autocorrelation function is easily distinguished from the autocorrelation function of the bit reversals

```
{.... 1 -1 1 1 -1 1 -1 1 1 -1 1 1 -1 .....}
```

as shown in Fig.2.8.2. Hence, the unique word is easily located. The symbol timing extraction from the bit reversals is described in Appendix C.

In practice; a synchronizing packet of the form shown in Fig.2.8.3 is inserted into the QAM data signal at regular intervals to aid in the accurate recovery of element and frame timing at the receiver [92-93]. Also it has been shown theoretically (Appendix C) that the timing waveform can be simply extracted from the received baseband waveform in a novel technique, even when two QAM signals are received in the same frequency space: So fine adjustments to this timing waveform are made continuously from the received data signal.

System 2 has an extra important requirement for element-timing synchronization: The two digital QAM signals transmitted independently from the two different mobiles must arrive at the base station almost perfectly synchronized in time. This could be simply arranged by taking advantage of the sophisticated vehicle location system that is in operation at the base station (the details of which are beyond the scope of this thesis). The base station could instruct each mobile to delay its transmission (after receiving the synchronizing signal) so that the signals from both mobiles arrive back at the base station with say, a 20, s total loop delay. As long as the distance between mobile and base station is known to within about 0.25 km , then the signals from the mobiles can be

| unique <br> word | bit reversals |
| :---: | :---: |

Fig.2.8.1 Format of FSK synchronizing signal


Fig.2.8.2 Autocorrelation functions of (b). bit reversals

| carrier <br> burst | bit reversals | unique <br> word |
| :---: | :---: | :---: |

Fig.2.8.3 Format of synchronizing packet transmitted within the QAM data signal
expected to arrive at the base station with no more than about $2 \mu s$ error in time. (i.e. $\approx 1 / 40^{\text {th }}$ of a symbol duration).

If the two mobiles simply transmitted a signal element immediately on receiving the corresponding synchronizing signal element, then the delays in transmission would impose an upper limit on the allowable cell radius. For example, consider that a timing error in each received signal of $\pm 0.1 \mathrm{~T} \approx \pm 8 \mu$ s can be tolerated. (The arrangement used here of matched raised-cosine filtering is very robust to symbol timing errors). Radio waves travelling at the speed of light $\left(3 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)$ cover a distance of 1.5 km in about $5 \mu \mathrm{~s}$. However, there is generally no line-of-sight path between a mobile and base station 1.5 km apart. So it can be expected to take longer, typically up to about $8 \mu \mathrm{~s}$ for the signal to pass between this mobile and base station. This gives a total loop delay of about $16 \mu \mathrm{~s}$. Assume the worst case, with the second mobile very close to the base station, with practically no delay. Then a loop delay of $8 \mu s$ is assumed at the receiver which gives a timing error of $8 \mu$ s in both received signals and 1.5 km becomes the maximum cell radius.

### 2.9 Summary

The mathematical models of the complete data transmission systems have been described. There is enough information here to allow the baseband model to be simulated and any subsequent analysis of a system's performance to be carried out.

A narrowband digital signal with a total bandwidth of 24 kHz and a carrier frequency 900 MHz as used throughout this thesis undergoes many changes during transmission in the cellular land mobile radio environment.

In the following chapters, the performances of the various systems are measured in the presence of flat Rayleigh fading (no intersymbol interference) and additive white Gaussian noise only. Any of the systems tested that give a good performance under these conditions can be expected to work well in practice. Although interference from other signals is not simulated in this thesis, all systems tested have been designed to have a good tolerance to these effects.

The rest of this thesis is now devoted to the development and testing by computer simulation of Systems 1 to 3 . The basic receiver structure used in all these systems is a combined detection and estimation process. The key to the successful operation of this system lies in the channel
estimation. A thorough study of the fading has shown that the best estimation process would track the in-phase and quadrature components rather than the amplitude and phase of the fading. With state-of-the-art estimation processes of the type used in recent years at Loughborough University in tests over HF radio links [64-71], it should be possible to track the fast Rayleigh fading and achieve coherent demodulation at the receiver. Any sudden phase changes that are likely to occur during deep fades should not appreciably affect the bit error rate if differential encoding and decoding of the binary digits is carried out as described in Sec.2.3.

## SYSTEM 1

### 3.1 Introduction

The aim of this chapter is to design System 1; that is, a digital data transmission system to transmit one QPSK signal in a 24 kHz signal bandwidth over the 900 MHz land mobile radio channel. By the reciprocity theorem [8] this system applies equally well to base station-to-mobile $(\mathrm{B} \rightarrow \mathrm{M}$ ) transmission and mobile-to-base station ( $\mathrm{M} \cdot \mathrm{B}$ ) transmission. A total of $24 \mathrm{kbit} / \mathrm{s}$ is transmitted in this 24 kHz channel giving a channel bandwidth efficiency of $1 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$. With $20 \%$ of the bits set aside for retraining and synchronization purposes this still leaves $19.2 \mathrm{kbit} / \mathrm{s}$, which is enough for a coded speech signal with some extra redundancy for error control coding. At this time, typically $16 \mathrm{kbit} / \mathrm{s}$ is used for efficiently coded speech $[102,103]$.

The key to the successful operation of System 1 lies in the coherent demodulation receiver. This receiver has been introduced in Chapter 2, and is now developed to its final form in this chapter. In Sec.3.2 the model of the system as simulated on the computer is given. Then in Sec.3.3 the channel estimation process is assumed to be ideal and the optimum detection process is described for both one and two receiving antennas. This defines the theoretically optimum system performance.

However, the key to successful data detection lies in achieving a good channel estimate. So in Sec.3.4 perfect detection is assumed and many different estimation processes are investigated in detail. In Sec.3.5 methods of regularly retraining the chosen channel estimator are developed which should safeguard against a likely total system collapse [68].

Now all the component parts of System 1 have been defined. In Sec.3.6 the performance of the final combined detection and estimation process is carefully analysed. Particular note is taken here of the error extension effects caused by feeding back incorrectly detected symbols into the estimation process.

### 3.2 Model of System

In the computer simulations, at time $t=i T$ the baseband received sample at


Fig.3.2.1 Baseband model of data transmission system used in computer simulation tests with: (a) One receiving antenna (System 1A). (b) Two receiving antennas (System 1B)
the output of the receiver filter is given by:
For System 1A

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{3.2.1}
\end{equation*}
$$

For System 1B

$$
\begin{align*}
& r_{a . i}=s_{i} y_{a . i}+w_{a . i} \\
& r_{b . i}=s_{i} y_{b . i}+w_{b . i} \tag{3.2,2}
\end{align*}
$$

The letter's $s, y, w$ refer to the data, channel and noise waveforms respectively. The subscript $i$ shows that these waveforms have been sampled at time $t=i T$. The subscripts $a$ and $b$ before the dot refer to receiving antennas $A$ and $B$. The computer simulation model is shown in Fig.3.2.1 and the detailed simulation method is described in Appendix B. The important assumptions from which these equations are derived have been summarized in Sec.2.4.1. Also the relevant properties of the data, channel, noise and received samples have been summarized in this section.

The general operation of the coherent demodulation receiver has been described in Secs.2.4.2-2.4.3. The detailed operation of the data detection and channel estimation processes that comprise this receiver are investigated in the rest of this chapter.

### 3.3 Detection

The aim of this section is to investigate different methods of detecting the data symbols $\left\{s_{i}\right\}$ in Systems $1 A$ and $1 B$, assuming perfect channel estimation at the receiver. The best method tested here should still be the best detection process when used with the actual channel estimates.

### 3.3.1 Model of Detection Process

The baseband received received sample(s) at the input to the detector at time $t=i T$ are;

For System 1A

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{3.3.1}
\end{equation*}
$$

For System 1B

$$
\begin{align*}
& r_{a . i}=s_{i} y_{a . i}+w_{a . i} \\
& r_{b . i}=s_{i} y_{b . i}+w_{b . i} \tag{3.3.2}
\end{align*}
$$

It is assumed that the estimates of the channels used in the detector are exact, as shown in Fig.3.3.1. So the detector must minimize the probability of error in the detection of $s_{i}$. The detector has full knowledge of the four possible values of $s_{i}( \pm 1 \pm j)$ as shown in Fig.3.3.2.


Fig.3.3.1 Block diagram of detector for
(a) System 1A (b) System

1B


Fig.3.3.2 Set of all possible values of $s_{i}$


Fig.3.3.3 Example received signal constellation of $s_{i} y_{i}$ when $y=0.606+j 0.35^{\prime \prime}$,

### 3.3.2 Maximum Likelihood Detection

The samples $\left\{r_{i}\right\},\left\{r_{a . i}\right\},\left\{r_{b, i}\right\}$ are the samples of the received signals at the output of the matched filters, with perfect timing and synchronization. Since the fading data signal is received in the presence of white Gaussian noise, it is well-known that the sample $r_{i}$ (or samples $r_{a . i} r_{b . i}$ ) forms a sufficient statistic for the optimum detection of $s_{i}$ [31.104,105] even in the presence of the fast. Rayleigh fading considered here (Appendix B.1). That is, the detection process for the detection of $s_{i}$ cannot be improved by double-sampling or indeed by including any other samples of the received waveform in the detection process. So the optimum detection of $s_{i}$ from $r_{i}$ (or $r_{a . i}, r_{b . i}$ ) is also the optimum detection of $s_{i}$ from $r(t)$ (or $r_{a}(t), r_{b}(t)$ ). Maximum likelihood detection is the optimum detection process when the channel, and all possible values of $s_{i}$ are known exactly at the receiver. No other detection process gives a lower probability of error $[31,104,105]$.

For System 1A, this optimum maximum likelihood detector that has exact prior knowledge of $y_{i}$, takes as the detected value of $s_{i}$, the possible value $s^{\prime}{ }_{i}$ for which

$$
\begin{equation*}
d_{i}^{2}=\left|r_{i}-s_{i}^{\prime} y_{i}\right|^{2} \tag{3.3.3}
\end{equation*}
$$

is minimum over all four combinations of the possible values of $s^{\prime}{ }_{i}$ $( \pm 1 \pm j)$. Where $|x|$ is the absolute value of the complex-valued quantity x .

For System 1B, with exact prior knowledge of $y_{a . i}, Y_{b, i}$ and statistically independent $w_{a . i}, w_{b . i}$. Eq.(3.3.3) for optimum maximum likelihood detection becomes

$$
\begin{equation*}
d_{i}^{2}=\left|r_{a . i}-s_{i}^{\prime} y_{a . i}\right|^{2}+\left|r_{b . i}-s_{i}^{\prime} y_{b . i}\right|^{2} \tag{3.3.4}
\end{equation*}
$$

Note that $d_{i}{ }^{2}$ is the maximum likelihood distance of the two-component vector [s'i, $\left.y_{a . i} s^{\prime}{ }_{i} Y_{b . i}\right]$ from [ra.i $\left.r_{b . i}\right]$. In practice, the estimator must use estimates of the channel samples in place of the $\left\{y_{i}\right\},\left\{y_{a . i}\right\}$, $\left\{y_{b . i}\right\}$ themselves. This inevitably degrades the detection process which is therefore no longer optimum.

It is easier to see the mechanism involved here by considering an example for System 1A: Suppose that at time $t=i T$ it is known at the receiver that $y_{i}=0.7 / 30^{\circ}=0.606+j 0.35$. The received system constellation can be constructed as shown in Fig.3.3.3. Imagine drawing four straight lines connecting the received point $r_{i}$ to all four possible points $s^{\prime}{ }_{i} y_{i}$. The lengths of these lines gives the distances $\left|r_{i}{ }^{-s}{ }_{i} y_{i}\right|$ in the complex-number plane between $r_{i}$ and the corresponding signal points $s^{\prime}{ }_{i} y_{i}$.

The maximum likelihood detector selects as the detected value of $s_{i}$ the possible value $s^{\prime}{ }_{i}$ for which $s^{\prime}{ }_{i} Y_{i}$ is the shortest distance from $r_{i}$. Eq.(3.3.3) actually calculates the squares of these distances, $\left|r_{i} s^{\prime}{ }_{i} y_{i}\right|^{2}$. Squaring in no way affects the order of these four distances $\left\{d_{i}\right\}$ from shortest to longest, since whenever $\left|x_{1}\right|<\left|x_{2}\right|$ then $\left|x_{1}\right|^{2}<\left|x_{2}\right|^{2}$. With System 1B, the corresponding uncorrelated distance variables at both antennas are summed to give the optimum decision rule [104]:

## Theoretical Probabilities of Error

The tolerance to noise of this optimum detection process using Eqs.(3.3.3) or (3.3.4) is well-known $[8,9,22,23]$. This has been derived theoretically in Appendix $D$ for several cases of interest. It is shown in Appendix $D$ for QPSK modulation (that is, with no differential coding) that:
(i) Assuming one receiving antenna and detection according to Eq(3.3.3). For the special case with no fading (that is, where $y_{i}=1$ and $r_{i}=s_{i}+w_{i}$ for all \{i\}). The bit error rate in the detection of the $\left\{s_{i}\right\}$ for any given signal-to-noise ratio $\psi$ is given by

$$
\begin{equation*}
P_{b}=\int_{\sqrt{2 \psi}}^{\infty} \frac{1}{\sqrt{2 \pi}} e^{-\frac{1}{2} z^{2}} d u=Q(\sqrt{2 \psi}) \tag{3.3.5}
\end{equation*}
$$

Where the $Q$-function $Q()$ is tabulated in the references [106]. And where $\psi=E_{b} / N_{0}$, as defined in Eq. (2.4.10).
(ii) But with flat Rayleigh fading (that is, where $r_{i}=s_{i} y_{i}+w_{i}$ and the $\left\{y_{i}\right\}$ are as described in sec.2.4.2, with a mean-square value of unity).

$$
\begin{equation*}
P_{b}=\frac{1}{2}\left[1-\sqrt{\frac{\psi}{1+\psi}}\right] \tag{3.3.6}
\end{equation*}
$$

(iii) Now assuming two receiving antennas and detection according to Eq. (3.3.4). For the special case with no fading (that is, where $y_{a . i}=y_{b, i}=1, r_{a . i}=s_{i}+w_{a . i}$ and $r_{b . i}=s_{i}+w_{b, i}$ for all \{i\}). The bit error rate is given by

$$
\begin{equation*}
P_{b}=\int_{2 \sqrt{\psi}}^{\infty} \frac{1}{\sqrt{2 \pi}} e^{-\frac{1}{2} 2^{2}} d u=Q(2 \sqrt{\psi}) \tag{3.3.7}
\end{equation*}
$$

(iv) But for the general case of System 1B with flat Rayleigh fading (that is, where $r_{a . i}=s_{i} y_{a . i}+w_{a . i}, r_{b . i}=s_{i} y_{b, i}+w_{b . i}$ and the uncorrelated $\left\{y_{a . i}\right\},\left\{y_{b . i}\right\}$ each have a mean-square value of 1 and are as described in Sec.2.4.2). The bit error rate is given by

$$
\begin{equation*}
P_{b}=\frac{1}{2}\left[1-\sqrt{\frac{\psi}{1+\psi}}-\frac{1}{2 \psi}\left(\sqrt{\frac{\psi}{1+\psi}}\right)^{3}\right] \tag{3.3.8}
\end{equation*}
$$

These four bit error rate curves are shown in Fig.3.3.4.


Flg.3.3.4 Theoretical probabilities of error for QPSK modulation (no differential coding)

This theory in Appendix D gives a good insight into the mechanisms that cause these errors to occur. Most importantly it is shown that the deeper the fade at any time, the more likely there is to be an error in detection. The big difference in tolerance to noise between the fading and corresponding non-fading cases is caused entirely by the Rayleigh probability density function of the fading. The fading rate has no effect on the bit error rate curve. This is because when the fading rate is slow, deep fades tend to last a long time resulting in long bursts of errors. For fast fading rates, deep fades occur more frequently but last a shorter length of time resulting in shorter, but more frequent error bursts, with the same average error rate.

For the non-fading case, there is a constant 3dB improvement in tolerance to noise in going from one to two receiving antennas, which is hardly worth the extra equipment complexity. However in the fading case, typically as much as 12 dB can be gained by using a second receiving antenna. This big improvement is because with two antennas, most of the errors now occur when both antennas are in a deep fade at the same time. For uncorrelated fading, the probability of this event happening is very much less than the probability of any one channel being in such a deep fade. Hence the marked decrease in the error rate at any given $\psi$.

The bit error rate curves with differential coding are best obtained by computer simulation. Generally, with isolated errors in the detected data symbols $\left\{s^{\prime}{ }_{i}\right\}$, the bit error rate at any fixed signal-to-noise ratio is doubled with differential coding. So the loss in tolerance to noise caused by the differential coding, measured in decibels, depends on the slope of the bit error rate curve. For the non-fading channel, this loss would be about $\frac{1}{2} \mathrm{~dB}$ at $10^{-4}$. For the fading channel, this loss would be about 3 dB with one receiving antenna, and about 1.5 dB with two receiving antennas. However, with a fading channel there is always a tendency for the errors to arrive in bursts, so the losses here would be a little less than this (see Sec.2.3.3).

### 3.3.3 Threshold Level Detection

For System 1A, the optimum maximum likelihood detection of $s_{i}$ can also be achieved by a threshold level detection. This method is computationally more efficient than executing Eq. (3.3.3) in full, but gives the identical result. The decision rule is given in Table 3.3 .1 , where

$$
\begin{equation*}
\underline{r}_{i}=\underline{r}_{I_{. i}}+\underset{\underline{r}_{Q . i}}{ }=r_{i} Y^{\star}{ }_{i} \tag{3.3.9}
\end{equation*}
$$

Table 3.3.1 Threshold level detection for System 1A

| Condition |  |  |  |
| :---: | :---: | :---: | :---: |
| If | $\left(\underline{r}_{1 . i} \geqslant 0\right)$ | AND | $\left(\underline{r}_{Q . i}>0\right)$ |
| If | $\left(\underline{r}_{1 . i}>0\right)$ | AND | $\left(\underline{r}_{Q . i} \leqslant 0\right)$ |
| If | $\left(\underline{r}_{1 . i}<0\right)$ | AND | $\left(\underline{r}_{Q . i} \geqslant 0\right)$ |
| If | $\left(\underline{r}_{1 . i} \leqslant 0\right)$ | AND | $\left(\underline{r}_{Q . i}<0\right)$ |
| $s_{i}^{\prime}=+1-j$ |  |  |  |

$$
\begin{align*}
& =s_{i} y_{i} Y^{*}{ }_{i}+w_{i} Y^{\star}{ }_{i} \\
& =s_{i}\left|Y_{i}\right|^{2}+w_{i} y^{\star}{ }_{i} \tag{3.3.10}
\end{align*}
$$

and $y_{i}{ }_{i}$ is the complex conjugate of $y_{i}$.
The received sample $r_{i}$ is multiplied by $y^{*}{ }_{i}$ to remove the phase rotation in the received signal constellation caused by the fading. So now the real and imaginary axes become the decision boundaries, and the value of $s^{\prime}{ }_{i}$ is determined by the quadrant that $\underline{r}_{i}$ lies in, according to Table 3.3.1.

There is no equivalent threshold level detection for signals from two receiving antennas. So for System 1B, Eq.(3.3.5) must be executed in full which requires far more complex equipment. However, if the signals from. these two antennas were combined into one signal before detection, then simple threshold level detection would be possible.

### 3.3.4 Combining Techniques

Combining is usually done at IF (an intermediate frequency) rather than at baseband. But with exact prior knowledge of the channel values, ideal combining can be achieved quite simply at baseband, allowing simple threshold level detection.

There are three main pre-detection diversity combining techniques used to counteract the fading in mobile radio communications; maximal ratio combining, equal gain combining and selection diversity [8,9,23]. Their implementations at baseband are now described.

## Maximal Ratio Combining

At time $t=i T$, the received samples at the outputs of the matched filters at antennas A and B are respectively

$$
\begin{align*}
& r_{a, i}=s_{i} y_{a, i}+w_{a . i} \\
& r_{b, i}=s_{i} y_{b, i}+w_{b, i} \tag{3.3.11}
\end{align*}
$$

These received baseband samples are weighted by their instantaneous channel amplitudes $\left|y_{a . i}\right|,\left|y_{b . i}\right|$ and these two weighted signals are co-phased and summed to give the combined signal $r_{\text {MR.i }}$. When the two signals are combined in this way, the signal-to-noise ratio in the combined sample $r_{\text {MR.i }}$ has been maximized - hence maximal ratio combining [8,107]. The signal-to-noise ratio in the combined signal equals the sum of the branch signal-to-noise ratios. That is,

$$
\begin{equation*}
\psi_{M R}=2 \psi=2 \mathrm{E}_{\mathrm{b}} / \mathrm{N}_{\mathrm{O}} \tag{3.3.12}
\end{equation*}
$$

It has also been assumed in these weighting factors $[8,107$ ] that the noise
power is the same in each branch.
The two operations of multiplying by scaling factors and co-phasing are equivalent to the single operation of multiplying the samples ra.i* $r_{b . i}$ by the complex conjugates of their channel components, $Y^{*}{ }_{a . i}{ }^{\prime} Y^{*}{ }_{b . i}$ respectively. So,

$$
\begin{equation*}
r_{M R . i}=s_{i}\left(\left.\left|y_{a . i}\right|^{2+\mid y_{b . i}}\right|^{2}\right)+y_{a . i}^{*} w_{a . i}+y_{b . i}^{*} w_{b . i} \tag{3.3.13}
\end{equation*}
$$

This operation is depicted in Fig.3.3.5. Now $s_{i}$ can be detected from $r_{M R . i}$ by threshold level detection, exactly as for System 1A.

The formal maximum likelihood detection calculations can be performed on $r_{\text {MR.i. }}$. In this case, the Euclidean distance between $r_{\text {MR.i }}$ and the correct data point $s_{i}$ is (from Eqs.(3.3.3) and (3.3.13))

$$
\begin{align*}
d_{i}^{2} & =\left|r_{\text {MR.i }}-s_{i}\left(\left|y_{a, i}\right|^{2}+\left|y_{b, i}\right|^{2}\right)\right|^{2} \\
& =\left|y^{\star}{ }_{\text {a.i }} w_{a . i}+y_{b, i}^{\star}{ }_{b, i}\right|^{2} \tag{3.3.14}
\end{align*}
$$

Whereas in the optimum two component maximum likelihood detection (from. Eqs.(3.3.4) and (3.3.11))

$$
\begin{align*}
d_{i}^{2} & =\left|r_{a . i}-s_{i} y_{a . i}\right|^{2}+\left|r_{b_{. i}}-s_{i} y_{b, i}\right|^{2} . \\
& =\left|w_{a . i}\right|^{2}+\left|w_{b . i}\right|^{2} \tag{3.3.15}
\end{align*}
$$

Eqs.(3.3.14) and (3.3.15) are slightly different which means that a different mechanism is involved with detection from the combined signal compared with the optimum detection process. However, it is shown theoretically in Appendix $D$ that the bit error rate expressions are identical in both cases. Also computer simulation tests (Chapter 6) have shown that identical errors occured in exactly the same place every time, whichever of these two methods is used. So with perfect channel estimation these two methods are equivalent.

## Equal Gain Combining

If the baseband received samples $r_{a . i}, r_{b . i}$ from the two antennas are co-phased and added together as before, but with both branch gains set equal to a constant value of unity, equal gain combining results. This combining method is only very slightly inferior to maximal ratio, so is useful when it is inconvenient to provide a variable weighting capability. However, in this case, where all the mathematical calculations are done on the complex-valued baseband samples $r_{a . i}, r_{b, i}, y_{a . i}, y_{b, i}$, there is no simpler way of co-phasing the two received samples than by multiplying them by the complex-conjugates of their channel components - which is maximal ratio.

To combine the signals with an equal gain would actually require


Fig.3.3.5 Maximal ratio combining of baseband received samples


Fig.3.3.6 Selection diversity from baseband received samples
further computation which is certainly not worthwhile just to give a worse performance. Equal gain combining is really more applicable when co-phasing is done at IF/RF with phase comparator circuits [108,109]. Therefore it is not considered further for System 1B.

## Selection Diversity

One of the two received baseband samples $r_{a . i}, r_{b . i}$ is chosen for which the instantaneous channel amplitude/power is the greater. Then maximum likelihood detection is carried out on that sample only, preferably by threshold level detection. The selection diversity process is depicted in Fig.3.3.6 where the selector operates according to the decision rule in Table 3.3.2. An example of the reduction in fading experienced with this combiner is shown in Fig.3.3.7. The signal whose power is reduced most by the fading at any time is rejected. It is clear that the deep fades that cause most of the detection errors are largely removed.

So in fact, the two signals are not actually combined here. Instead, at any time the strongest signal is selected and only that signal is used in the detector. The computational complexity required to calculate the channel power for each diversity branch and select between them is not much less than is required for maximal ratio combining, when the combining is done at baseband. So, since the received carrier phase in each diversity branch is known, it would be pointless to discard half of the available information in this selection, with the accompanying degradation in performace. For this reason selection diversity is not considered further in this thesis.

### 3.3.4 Detection Conclusions

This completes the discussion of the different detection methods considered for System 1. Computer simulation tests have been carried out on Systems 1A and 1B to show the performance of the optimum maximum likelihood detection process operating with perfect channel estimation. The results of these tests are shown in Chapter 6. Simulation tests have also been carried out on System 1B with maximal ratio combining. The accuracy of these simulation results has been confirmed by the theoretical results in Appendix D.

It is interesting to note that the bit error rate in detection does not depend on vehicle speed (fading rate), or on the shape of the power spectrum of the fading or on the depth or duration of fades. It depends

Table 3.3.2 Selection diversity

| Condition | Selection |
| :---: | :---: |
| $\mid f\left(\left\|y_{\text {a.i }}\right\|^{2} \geqslant\left\|y_{\text {b.i }}\right\|^{2}\right)$ | $r_{i}=r_{\text {a.i }}$ |
| $\mid f\left(\left\|y_{\text {a.i }}\right\|^{2}<\left\|y_{\text {b.i }}\right\|^{2}\right)$ | $r_{i}=r_{\text {b.i }}$ |



Fig.3.3.7 Example selection diversity output $r(t)$ from two antenna input $r_{a}(t), r_{b}(t)$
only on the Rayleigh probability density function of the fading. In the presence of Rayleigh fading, the times at which errors in detection occur mainly correspond to times of deep fades. Because these deep fades typically last for several symbols, errors tend to occur in short bursts. The tolerance to noise of the optimum maximum likelihood detector is greatly improved with space diversity reception. With two receiving antennas, most of the errors seem to occur when both signals are in a deep fade at he same time. With uncorrelated fading in the two branches, this event is very much less likely than is a deep fade at either one antenna - hence a marked reduction in the error rate. For System 1B, threshold level detection with pre-detection maximal ratio. combining gives identical results to the optimum two-component maximum likelihood detection, when perfect channel estimation at the receiver is assumed.

### 3.4 Channel Estimation

The aim of this section is to find an estimation process that will result in near optimum data detection, with a reasonable level of equipment complexity. It is assumed that all detected symbols that are fed back to the estimator are correct. The best estimation process found in this way will almost certainly be the best in the actual system where the estimator relies on the input of detected data symbols, some of which are inevitably wrong.

### 3.4.1 Model of Estimation Process

For convenience, the estimation processes considered in this Sec.3.4 will be confined to estimating $\left\{y_{i}\right\}$ from the $\left\{r_{i}\right\}$ (and $\left\{s_{i}\right\}$ ), bearing in mind that when two receiving antennas are used, an exactly similar estimation process would be applied to both sets of received samples $\left\{r_{\text {a.i }}\right\}$ and $\left\{r_{b . i}\right\}$ when estimating $\left\{y_{a_{. i}}\right\},\left\{y_{b_{. i}}\right\}$ respectively.

The estimation process is in fact a combination of estimation and prediction (Fig.3.4.1) and operates as follows:

At time $t=i T$, the detected data symbol value $s_{i}$ is fed back to the estimator, along with the received baseband sample $r_{i}$, which is given by (see Sec.3.2)

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{3.4.1}
\end{equation*}
$$

The $\left\{s_{i}\right\}$ are comprised entirely of the random data symbol values $( \pm 1 \pm j)$


Fig.3.4.1 Model of estimation process as simulated
since retraining and synchronization are assumed ideal here. The channel estimator forms the estimate $y^{\prime}{ }_{i}$. But the detector requires an estimate of $\dot{y}_{i+1}$ ready for the next symbol. This is formed in the predictor as $y_{i+1, i}^{\prime}$ and held in store until time $t=(i+1) T$, when it is fed into the detector. This store is represented in Fig.3.4.1 as introducing a delay of $T$ seconds, since if its input signal at time $t=i T$ is $y^{\prime}{ }_{i+1, i}$, its output signal is $y^{\prime}{ }_{i, i-1}$.

It is in fact the quality of this one-step prediction that is important in reducing the probability of error in detection. So this section is concerned with finding the estimation process that gives the "best" one-step prediction $\left\{y^{\prime}{ }_{i+1, i}\right\}$. Where the "best" prediction is defined as that which results in the lowest average bit error rate for the optimum detector given in Sec.3.3. However, in evaluating the performance of any estimation process it is impractical to generate bit error rate curves every time as it would take up far too much computer time. Instead, the minimum mean-square error in $y_{i, i-1}^{\prime}$ is the main criterion used here. for comparing the performance of different estimation processes. As seen in the previous section on detection, the optimum detector chooses as the detected symbol the one from all possible values of $s^{\prime}{ }_{i}$ for which $s^{\prime}{ }_{i} Y^{\prime}{ }_{i, i-1}$ is the smallest Euclidean distance from the received data point. That is, the point which indicates the smallest possible squared-error in $s^{\prime}{ }_{i} Y^{\prime}{ }_{i, i-1}$. So it follows that if the average or mean-square error in $Y^{\prime}{ }_{i, i-1}$ for a given estimation process is smaller than for every other estimation process, then this will almost certainly be the best estimation process out of those tested. This mean-square error is usually calculated for a typical, fixed fading sequence.

The baseband channel samples $\left\{y_{i}\right\}$ are complex-valued quantities. In all the following methods the $\left\{y_{i}\right\}$ are estimated in terms of their real and imaginary components rather than their amplitude and phase, since they vary a lot more predictably from one sample to the next (see Sec.2.2.1).

### 3.4.2 Unbiased Estimator

This is the simplest possible estimation process. The channel estimate at time $\mathrm{t}=\mathrm{iT}$ is given by

$$
\begin{equation*}
y_{i}^{\prime}=s_{i}^{\prime} r_{i}^{-1}=s_{i}^{\prime}{ }_{i}^{-1} s_{i} y_{i}+s_{i}^{\prime}{ }^{-1} w_{i} \tag{3.4.2}
\end{equation*}
$$

Assuming correct detection, $s^{\prime}{ }_{i}=s_{i}$, so

$$
\begin{equation*}
y_{i}^{\prime}=s_{i}^{-1} r_{i}^{1}=y_{i}+s_{i}^{-1} w_{i} \tag{3.4.3}
\end{equation*}
$$

As shown in Fig. 3.4 .2 , the reciprocal of $s^{\prime}{ }_{i}$ is formed by taking the


Fig.3.4.2 Model of unbiased channel estimator


Fig.3.4.3 Block diagram of unbiased estimator and polynomial predictor
complex conjugate of $s^{\prime}{ }_{i}$ and dividing by the amplitude squared. For the QPSK signal, $\left|s_{i}\right|^{2}=2$ for all $\{i\}$, therefore

$$
\begin{equation*}
s_{i}^{\prime}{ }^{-1}=\frac{s_{i}^{\prime}{ }_{i}^{*}}{\left|s_{i}\right|^{2}}=\frac{1}{2} s_{i}^{\prime}{ }^{*} \tag{3.4.4}
\end{equation*}
$$

The mean-square error of this estimate can now be determined theoretically as follows. Assuming correct data symbols are fed back from the detector (Eq.(3.4.3)), then in the absence of noise

$$
\begin{equation*}
y_{i}^{\prime}=s_{i}^{-1} r_{i}=y_{i} \tag{3.4.5}
\end{equation*}
$$

In other words, $s_{i}{ }^{-1} r_{i}$ gives an exact estimate of the channel for all $\{i\}$. The mean-square in estimation is

$$
\begin{equation*}
\lambda_{e}=0 \tag{3.4.6}
\end{equation*}
$$

This is of course the optimum estimate of $y_{i}$ in the absence of noise.
Now, assuming correct data symbols fed back (Eq.(3.4.3)). In the presence of additive white Gaussian noise

$$
\begin{equation*}
y_{i}^{\prime}=s_{i}^{-1} r_{i}=y_{i}+s_{i}^{-1} w_{i} \tag{3.4.7}
\end{equation*}
$$

Where $s_{i}{ }^{-1} r_{i}$ is an unbiased estimate of $Y_{i}$. This is the optimum raw estimate of $y_{i}$. This means that the only way to improve on this estimate is by smoothing the noise sequence $\left\{s_{i}{ }^{-1} w_{i}\right\}$, to reduce its average power.

The mean-square error in the estimate of $y_{i}$, is given'by the mean-square value of the noise component $s_{i}^{-1} w_{-1}$. That is

$$
\begin{align*}
\lambda_{e} & =\lim _{N \rightarrow \infty} \frac{1}{N} \sum_{j=1}^{N}\left|s_{i}^{-1} w_{i}\right|^{2}=E\left[\left|s_{i}-1_{w_{i}}\right|^{2}\right]  \tag{3.4.8}\\
& =E\left[\left|s_{i}^{-1}\right|^{2}\right] . E\left[\left|w_{i}\right|^{2}\right]
\end{align*}
$$

(for independent $s, w$ and constant $\left|s_{i}^{-1}\right| 2$ for all \{i\}). Therefore

$$
\begin{aligned}
\lambda_{e} & =\frac{1}{2} \cdot 2 \sigma^{2} \\
& =\sigma^{2}
\end{aligned}
$$

Thus, the real and imaginary parts of $s_{i}{ }^{-1} w_{i}$ are statistically independent Gaussian random variables with zero mean and variance $\frac{1}{2} \sigma^{2}$. But, since the signal-to-noise ratio is $\psi=\frac{1}{2} \sigma^{2}$, then

$$
\begin{equation*}
\lambda_{e}=\frac{1}{2} \psi^{-1} \tag{3.4.10}
\end{equation*}
$$

or in decibels

$$
\begin{equation*}
\lambda_{e}=\operatorname{lol}_{10}\left(\frac{1}{2} \psi^{-1}\right)=-(\psi+3) \mathrm{dB} \tag{3.4.11}
\end{equation*}
$$

Since there is no prediction in this estimation process, the best one-step prediction of $y_{i+1}$ for use in the detector is

$$
\begin{equation*}
y_{i+1, i}^{\prime}=y_{i}^{\prime}=y_{i}+s_{i}^{-1} w_{i} \tag{3.4.12}
\end{equation*}
$$

The mean-square error in this prediction can now be determined theoretically. Assuming correct detection and in the absence of noise (Eqs.(3.4.5) and (3.4.12))

$$
\begin{equation*}
y_{i+1, i}^{\prime}=y_{i} \tag{3.4.13}
\end{equation*}
$$

Therefore, the mean-square error in this prediction over $N\left\{y_{i, i-1}\right\}$ is given by

$$
\begin{align*}
\lambda_{p}=\frac{1}{N} \sum_{i=1}^{N}\left|y_{i}-y_{i, i-1}^{\prime}\right|^{2} & =\frac{1}{N} \sum_{i=1}^{N}\left|y_{i}-y_{i-1}\right|^{2} \\
& =E\left[\left|y_{i}-y_{i-1}\right|^{2}\right] \tag{3.4.14}
\end{align*}
$$

Thus for a given symbol rate, the mean-square error in prediction with no noise depends on the fading rate, since $E\left[\left|y_{i}-y_{i-1}\right|^{2}\right]$ is different for each different fading rate. Consequently the prediction error will level off to an irreducible error given by Eq.(3.4.14) as the signal-to-noise ratio is increased. Clearly, the faster the fading, the worse this irreducible error would be.

In the presence of noise, the prediction error would be

$$
\begin{aligned}
\lambda_{p} & =\frac{1}{N} \sum_{i=1}^{N}\left|y_{i}-y_{i-1}-s_{i-1}^{-1} w_{i-1}\right|^{2} \\
& \left.=E\left[\left|y_{i}-y_{i-1}\right|^{2}\right]+E\left[\left|s_{i}^{-1} w_{i}\right|^{2}\right], \quad \text { (for uncorrelated } y, s, w\right)
\end{aligned}
$$

As the signal-to-noise ratio is decreased to the point where $E\left[\left|y_{i}-y_{i-1}\right|^{2]}\right.$ is negligible compared with E[ $\left|s_{i}{ }^{-1} w_{i}\right|^{2]}$, then

$$
\begin{equation*}
\lambda_{p} \approx E\left[\left|s_{i}^{-1} w_{i}\right|^{2}\right]^{i}=\lambda_{e}^{i}=-(\psi+3) d B \tag{3.4.17}
\end{equation*}
$$

### 3.4.3 Unbiased Estimator with Least-Squares Fading-Memory Polynomial

## Prediction

It should be possible to improve the performance of the unbiased estimator by incorporating some form of prediction as shown in Fig.3.4.3.
Least-squares fading memory prediction is tested here using polynomial filters. These prediction filters have been used successfully at Loughborough University in tests over HF radio links [64,65,68-71].

From Eq. (3.4.2), the unbiased estimate of $y_{i}$ is
$y^{\prime}{ }_{i}=s^{\prime}{ }_{i}{ }^{-1} r_{i}=s^{\prime}{ }_{i}{ }^{-1} s_{i} y_{i}+s^{\prime}{ }_{i}{ }^{-1} w_{i}$
and assuming correct detection

$$
y_{i}^{\prime}=s_{i}{ }^{-1} r_{i}=y_{i}+s_{i}^{-1} w_{i}
$$

The predictor now forms the one-step prediction $y_{i+1, i}$ of $y_{i+1}$. First of all, the error signal $E_{i}$ is formed as

$$
\begin{equation*}
E_{i}=y_{i}^{\prime}-y_{i, i-1}^{\prime} \tag{3.4.20}
\end{equation*}
$$

Then, for any given degree of the polynomial predictor, the equations in Table 3.4.1 are evaluated in the order shown resulting in the one-step prediction $Y^{\prime}{ }_{i+1, i}$. The terms $\dot{Y}^{\prime}{ }_{i+1, i}, \dddot{z}^{\prime}{ }_{i+1, i}, \dddot{z}^{\prime}{ }_{i+1, i}$ defined in. Eq.(3.4.21) are functions of the first, second and third derivatives

Table 3.4.1 Least-squares fading memory prediction using a polynomial filter

Degree of
one-step prediction polynomial, $p$ at time $t=i T$
$0 \quad y_{i+1, i}^{\prime}=y_{i, i-1}^{\prime}+(1-\theta) E_{i}$

1

$$
\begin{aligned}
& \dot{y}_{i+1, i}^{\prime}=\dot{y}_{i, i-1}^{\prime}+(1-\theta)^{2} E_{i} \\
& y_{i+1, i}^{\prime}=y_{i, i-1}^{\prime}+\dot{y}_{i+1, i}^{\prime}+\left(1-\theta^{2}\right) E_{i}
\end{aligned}
$$

2

$$
\begin{aligned}
& \ddot{z}_{i+1, i}^{\prime}=\dddot{z}_{i-1, i}^{\prime}+\frac{1}{2}(1-\theta)^{3} E_{i} \\
& \dot{y}_{i+1, i}^{\prime}=\dot{y}_{i, i-1}^{\prime}+2 \ddot{z}_{i+1, i}^{\prime}+\frac{3}{2}(1-\theta)^{2}(1+\theta) E_{i} \\
& y_{i+1, i}^{\prime}=y_{i, i-1}^{\prime}+\dot{y}_{i+1, i}^{\prime}-\dddot{z}_{i+1, i}^{\prime}+\left(1-\theta^{3}\right) E_{i}
\end{aligned}
$$

3

$$
\begin{aligned}
\dddot{z}_{i+1, i}^{\prime}= & \dddot{z}_{i-1, i}^{\prime}+\frac{1}{6}(1-\theta)^{4} E_{i} \\
\dddot{z}_{i+1, i}^{\prime}= & \dddot{z}_{i-1, i}^{\prime}+3 \dddot{z}_{i+1, i}^{\prime}+(1-\theta)^{3}(1+\theta) E_{i} \\
\dot{y}_{i+1, i}^{\prime}= & \dot{y}_{i, i-1}^{\prime}+2 \dddot{z}_{i+1, i}^{\prime}-3 \dddot{z}_{i+1, i}^{\prime} \\
& +\frac{1}{6}(1-\theta)^{2}\left(11+14 \theta+11 \theta^{2}\right) E_{i} \\
y_{i+1, i}^{\prime}= & y_{i, i-1}^{\prime}+\dot{y}_{i+1, i}^{\prime}-\dddot{z}_{i+1, i}^{\prime}+\dddot{z}_{i+1, i}^{\prime}+\left(1-\theta^{4}\right) E_{i}
\end{aligned}
$$

respectively of $y^{\prime}{ }_{i+1}$, $i$ with respect to time. They are considered in detail elsewhere [110].

$$
\begin{align*}
& \dot{y}_{i+1, i}=T \cdot \frac{d y^{\prime}}{d t}{ }_{i+1, i} \\
& \ddot{z}_{i+1, i}^{\prime}=T^{T^{2} \cdot d^{2} y^{\prime}} \frac{d+1, i}{d t^{2}} \\
& \dddot{z}_{i+1, i}=\frac{T^{3} \cdot \frac{d^{3} y^{\prime}}{3 t^{3}} i+1, i}{} \tag{3.4.21}
\end{align*}
$$

Under the following assumptions the predictor of degree-p minimizes the mean-square error in the prediction $y_{i+1, i}$ of $y_{i+1}$ by fitting the samples $y^{\prime}{ }_{i}, Y^{\prime}{ }_{i-1}, Y^{\prime}{ }_{i-2}, \ldots$ to a polynomial of degree-p;
Assumption 1:- The sequence of samples $y_{i}, y_{i-1}, Y_{i-2}, \ldots$ satisfies a polynomial equation in $t$ (time) of degree-p.
Assumption 2:- These samples $\left\{y_{i}\right\}$ satisfy $y_{i}{ }_{i}=y_{i}$ +additive white noise, for all \{i\}.
Assumption 3:- $\theta$ is matched to the channel conditions.
Assumption 2 is satisfied by the unbiased estimator. Assumption 1 is only satisfied for a Rayleigh fading channel over very short periods of time. As the degree of the polynomial is increased, Assumption 1 is satisfied over longer periods of time.

Of course, if the channel $\left\{y_{i}\right\}$ is assumed to fit a polynomial of degree- $p$, then all derivatives of $y_{i}$ greater than the $p^{\text {th }}$ are by definition equal to zero. Hence, as shown in Table 3.4.1, the polynomial predictor of degree-p has p+1 equations to evaluate.

For all the predictors $\theta$ is a real constant in the range -1 to +1 (but nearly always positive), and is in effect an exponential weighting factor. All previous estimates $y_{i}^{\prime}, y^{\prime}{ }_{i-1}, \ldots$ are involved in the evaluation of the prediction $Y^{\prime}{ }_{i+1}, i$ in these recursive prediction filters. But in practice, only the latter estimates are fitted to a polynomial of degree-p. There is a gradual departure from this polynomial as the age of the estimate increases. The predictor places exponentially decaying weighting factors on all previous estimates. The value of $\theta$ determines the rate of decay. Increasing the value of $\theta$ towards +1 increases the number of estimates that are effectively involved in the prediction. This gives a better smoothing of the additive noise in the unbiased estimates but results in a slowing down of the ability of the predictor to track changes in the channel. So generally, as the signal-to-noise ratio decreases, $\theta$ tends to increase towards +1 . The optimum value of $\theta$ for any
given fading rate and signal-to-noise ratio is best evaluated by computer simulation.

### 3.4.4 Unbiased Estimator with Modified Least-Squares Fading Memory

## Polynomial Filters

These polynomial filters are not necessarily optimum for this fading channel because, as pointed out in Sec.3.4.3, these filters have been defined as optimum where the channel samples $\left\{y_{i}\right\}$ can be assumed to fit exactly to a polynomial of degree-p. This is not true here. The degree-1 polynomial predictor is now slightly modified to see if it can be improved for this fading channel.

The set of equations for this modified degree-1 predictor now become

$$
\begin{align*}
E_{i} & =\theta_{a}\left(y_{i}^{\prime}-y_{i, i-1}^{\prime}\right)  \tag{3.4.22}\\
\dot{y}_{i+1, i}^{\prime} & =\theta_{b} \dot{y}_{i, i-1}^{\prime}+(1-\theta)^{2} E_{i}  \tag{3.4.23}\\
y_{i+1, i}^{\prime} & =\theta_{c} y_{i, i-1}^{\prime}+\dot{y}_{i+1, i}^{\prime}+\left(1-\theta^{2}\right) E_{i} \tag{3.4.24}
\end{align*}
$$

$\theta_{a}, \theta_{b}, \theta_{c}$ are all real-valued constants. which can be set to minimize the mean-square error in prediction, $\lambda_{p}$, in an exactly similar way as was done for $\theta$ in the previous section. The values of $\theta_{a}, \theta_{b}, \theta_{c}$ are found experimentally by computer simulation. If $\theta_{a}=\theta_{b}=\theta_{c}=1$, then this modified predictor reduces to its standard form given in Table 3.4.1.
$\theta_{a}$ in the range 0 to +1 has the effect of smoothing the output of the predictor. The predictor is now only updated in the direction of the error $E_{i}$ rather than by the whole error.

The effect of $\theta_{b}$ on the degree-1 predictor can be seen by expanding out Eq. (3.4.23) into its non-recurșive form. Assuming the predictor has been running from $i=1,2, \ldots i-1, i$, then

$$
\begin{equation*}
y_{i+1, i}^{\prime}=(1-\theta)^{2}\left(E_{i}+\theta_{b} E_{i-1}+\theta_{b}^{2} E_{i-2}+\ldots .+\theta_{b}^{i-1} E_{1}\right) \tag{3.4.25}
\end{equation*}
$$

With $\theta_{b}=1$ in its standard form, the degree-1 predictor takes equal notice of all $\left\{E_{i}\right\}$ in the updating of $y^{\prime}$. But with $0<\theta_{b}<1$, this prediction of the slope of the channel now has a fading memory, taking less notice of older error measurements.

Similarly it can be shown that $\theta_{C}$ (where $0<\theta_{C}<1$ ) has a further fading memory effect on the degree-1 predictor. Expanding Eq.(3.4.24) into its non-recursive form gives

$$
\begin{align*}
y_{i+1, i}^{\prime}= & \dot{y}_{i+1, i}+\theta_{c} \dot{y}_{i, i-1}+\theta_{c}^{2} \dot{y}_{i-1, i-2}^{\prime}+\ldots+\theta_{c}^{i} \dot{y}^{\prime} 1,0 \\
& +\left(1-\theta^{2}\right)\left(E_{i}+\theta_{c} E_{i-1}+\theta_{c}^{2} E_{i-2}+\ldots+\theta_{c}^{i-1} E_{1}\right) \tag{3.4.26}
\end{align*}
$$

The prediction of the channel value now takes less notice of older error measurements and older predictions of slope.


Fig.3.4.4 Block diagram of unbiased estimator with Taylor expansion predictor

Table 3.4.2 p-component Taylor expansion predictors

| $p$ | $c_{0}$ | $c_{1}$ | $c_{2}$ | $c_{3}$ | $c_{4}$ | $c_{5}$ |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 2 | 2 | -1 | 0 | 0 | 0 | 0 |
| 3 | 3 | -3 | 1 | 0 | 0 | 0 |
| 4 | 4 | -6 | 4 | -1 | 0 | 0 |
| 5 | 5 | -10 | 10 | -5 | 1 | 0 |
| 6 | 6 | -15 | 20 | -15 | 6 | -1 |

### 3.4.5 Unbiased Estimator with Taylor's Expansion Predictor

For such a fast fading channel it may be better to predict along only the most recent samples of $\left\{y_{i}^{\prime}\right\}$ rather than to take account of all the previous estimates. After all, the channel is practically uncorrelated with itself over distances greater than about 100 samples.

The predictor shown in Fig.3.4.4 is a p-component linear feedforward transversal filter. It forms the one-step prediction $y^{\prime}{ }_{i+1}$, from the $p$ samples $Y^{\prime}{ }_{i}, Y^{\prime}{ }_{i-1}, Y^{\prime}{ }_{i-2}, \ldots, y^{\prime}{ }_{i-p+1}$ as

$$
\begin{equation*}
y_{i+1, i}^{\prime}=c_{0} y_{i}^{\prime}+c_{1} y_{i-1}^{\prime}+c_{2} y_{i-2}^{\prime}+\ldots+c_{p-1} y_{i-p+1}^{\prime} \tag{3.4.27}
\end{equation*}
$$

It has been shown $\{111,112]$ that the coefficients $\left\{c_{k}\right\}$ for one-step prediction may be determined from the Taylor series expansion of the continuous transfer function $H(s)=1$, given by

$$
\left[\begin{array}{l}
c_{0}  \tag{3.4.28}\\
c_{1} \\
\vdots \\
c_{p-1}
\end{array}\right]=\left[T_{p}{ }^{-1}\right]\left[\begin{array}{l}
1 \\
1 \\
\vdots \\
1
\end{array}\right]
$$

where $T_{p}^{-1}$, the inverse Taylor matrix, is a pxp square matrix defined in Appendix $E[111,112]$. The p-component predictor coefficients so obtained for $p=2,3,4,5,6$ are given in Table 3.4.2. (It is interesting to note that the Lagrange predictor [110] uses exactly the same set of predictor coefficients, but has been derived in a different way).

### 3.4.6 Unbiased Estimator with Sinewave Scheme of Channel Prediction

It was seen in Sec.2.2.1 that the fading channel has a dominant high frequency component at the maximum Doppler frequency shift, $f_{m}$. If this high frequency component could be removed, then it should be possible to track the fading that remains more accurately.

The sinewave component and the residual fading component of the fading channel $y_{i}$ are now described. At time $t=i T$, the real part of $y_{i}$ can be represented as

$$
\begin{equation*}
y_{I_{, i}}=x_{i}+v_{i} \tag{3.4.29}
\end{equation*}
$$

where $x_{i}$ is the sinewave component and $v_{i}$ is the residual fading component. An example of this is shown in Fig.3.4.5. For convenience only the real part of the channel component is considered here, bearing in mind that an exactly similar description can be given for the imaginary part.

The sinewave component is generally represented by

$$
\begin{equation*}
x_{i}=\left(a_{k}+b_{k}\right) \sin \left(\Theta_{i}+c_{k}\right) \tag{3.4.30}
\end{equation*}
$$



Fig.3.4.5 Typical fading channel with its constituent sinewave and residual fading components


Fig.3.4.6 Block diagram of estimation process incorporating a sinewave removal scheme


Fig.3.4.8 (a) Sinewave component $x_{i}$ in $y_{1 . i}$ (b) Prediction of $x_{i}$
as shown in Fig．3．4．8（⿳亠丷⿵冂⿱八口刂）．$\Theta_{i}+c_{k}$ is the phase angle of $x_{i}$ in radians． $0 \leqslant \Theta_{i}<2 \pi . \quad c_{k}$ is assumed to be constant over any half－cycle of the sinewave between a peak and a trough．$\left(\Delta_{k}+C_{k}\right) T$ is the duration of time between the $k$ ．h． and $k+1^{\text {th }}$ peak／trough，where $T$ is the symbol period．$\Delta_{k}+\mathbb{U}_{k}$ is generally NOT a whole number of symbols．It is assumed that $\Delta_{k} \approx \Delta_{k+1}$ and $c_{k} \approx c_{k+1}$ for all $\{k\}$ ，so these small changes in instantaneous frequency with time can be tracked．The position of each successive peak／trough is given by $\tau_{k}+C_{k}, \tau_{k+1}+C_{k+1} \ldots \ldots$ and generally lies between two adjacent samples of $\left\{x_{i}\right\}$ ． In the absence of any phase errors（that is，when $c_{k}=0$ for all $\{k\}$ ），the positions of the successive zero crossings are given by
$\tau_{k+\frac{1}{2}}, \tau_{k+1 \frac{1}{2}}, \tau_{k+2 \frac{1}{2}} \ldots$ where $\tau_{k+\frac{1}{2}}=\frac{1}{2}\left(\tau_{k}+\tau_{k+1}\right)$ ．The absolute value of the $k^{t h^{2}}$ peak／trough is given by $a_{k}+b_{k}$ ，so $a_{k}+b_{k} \geqslant 0$ ．It is assumed that $a_{k}+b_{k} \approx a_{k+1}+b_{k+1}$ for all $\{k\}$ ，so these small changes in instantaneous amplitude can be tracked．The residual fading component $v_{i}$ in Eq．（3．4．29） is now defined such that Eqs．（3．4．29）－（3．4．30）are satisfied，for all \｛i\}.

The sinewave scheme of channel prediction shown in Fig．3．4．6 consists of two separate operations：sinewave prediction followed by residual fading prediction．The channel prediction $y^{\prime}{ }_{i+1, i}$ used in the detector is given by the sum of these two separate predictions．The method tested in Chapter 6 is now described．

At time $t=i T$, assuming $s^{\prime}{ }_{i}=s_{i}$ ，the real part of the unbiased channel estimate $Y^{\prime}{ }_{i}$ is（from Eq．（3．4．3））
$y^{\prime}{ }_{I_{i} i}=y_{I . i}+\operatorname{Re}\left[s_{i}{ }^{-1} w_{i}\right]$
Substituting $W_{i}=\operatorname{Re}\left[s_{i}{ }_{w_{i}}\right]$ and Eq．（3．4．29）into Eq．（3．4．31）

$$
\begin{equation*}
y_{I, i}^{\prime}=x_{i}+v_{i}+W_{i} \tag{3.4.32}
\end{equation*}
$$

where the $\left\{W_{i}\right\}$ are statistically independent Gaussian random variables with zero mean and variance $\frac{1}{2} \sigma^{2}$（given in Sec．2．4．1）．

It is assumed that at any time $t=i T$

$$
\begin{equation*}
x_{i}=\left(a_{k}+b_{k}\right) \sin \left(\theta_{i}+c_{k}\right) \tag{3.3.33}
\end{equation*}
$$

and the prediction of $x_{i}$ in this sinewave scheme is given by

$$
\begin{equation*}
x_{i}^{\prime}=a_{k} \sin \Theta_{i} \tag{3.3.34}
\end{equation*}
$$

Hence there is an error of $-b_{k}$ in the prediction of the peak value and an error of $-c_{k}$ radians in the prediction of the phase angle of the sinewave component．This is shown in Fig．3．4．8（b）．In order to predict this sinewave component，the $\left\{b_{k}\right\}$ and $\left\{c_{k}\right\}$ must be tracked as they change with time．A further assumption must be made that the residual fading $v_{i}$ remains at a constant value over any half－cycle of the fading．So the
sinewave estimate, $x^{\prime}{ }_{i}$ ' in Eq(3.4.34) is defined as follows:
When $0 \leqslant \Theta_{i}<\pi / 2$ and $\tau_{k-\frac{1}{2}} \leqslant i<\tau_{k}$

$$
\begin{equation*}
x_{i}^{\prime}=a_{k} \sin \Theta_{i}, \quad \text { where } \Theta_{i}=\frac{\pi}{\Delta_{k-1}}\left(i-\tau_{k-\frac{1}{2}}\right) \tag{3.4.35}
\end{equation*}
$$

When $\pi / 2 \leqslant \Theta_{i}<\pi$ and $\tau_{k} \leqslant i<\tau_{k+\frac{1}{2}}$

$$
\begin{equation*}
x_{i}^{\prime}=a_{k} \sin \theta_{i}, \quad \text { where } \theta_{i}=\frac{\pi}{\Delta_{k}}\left(i-\tau_{k}+\frac{\downarrow \Delta_{k}}{}\right) \tag{3.4.36}
\end{equation*}
$$

When $\pi \leqslant \Theta_{i}<3 \pi / 2$ and $\tau_{k+\frac{1}{2}} \leqslant i \leqslant \tau_{k+1}$

$$
\begin{equation*}
x_{i}^{\prime}=a_{k+1} \sin \theta_{i}, \quad \text { where } \theta_{i}=\frac{\pi}{\Delta_{k}}\left(i-\tau_{k}+\frac{\xi \Delta_{k}}{}\right) \tag{3.4.37}
\end{equation*}
$$

When $3 \pi / 2 \leqslant \Theta_{i}<2 \pi$ and $\tau_{k+1} \leqslant i<\tau_{k+1 \frac{1}{2}}$

$$
\begin{equation*}
x_{i}^{\prime}=a_{k+1} \sin \Theta_{i}, \quad \text { where } \theta_{i}=\frac{\pi}{\Delta_{k+1}}\left(i-\tau_{k+1}+1 \cdot 5 \Delta_{k+1}\right) \tag{3.4.38}
\end{equation*}
$$

Referring to Fig.3.4.7, the best estimate of the peak value $a_{k}+b_{k}$ (at $\Theta_{i}=\pi / 2$ ) from $n$ measurements $\left\{y^{\prime}{ }_{I . i}\right\}$, regularly spaced over the interval $0 \leqslant \theta_{i}<\pi$, is

$$
\begin{align*}
& \frac{\sum_{i=1}^{n} Y^{\prime}{ }_{I . i} \sin \theta_{i}-\frac{2}{\pi} \sum_{i=1}^{n} Y_{I . i}^{\prime}}{\left(1-8 / \pi^{2}\right) \sum_{i=1}^{n} \sin ^{2} \theta_{i}} \\
& \quad \approx \frac{10.55796}{n}\left(\sum_{i=1}^{n} Y^{\prime}{ }_{I . i} \sin \Theta_{i}-0.63662 \sum_{i=1}^{n} y^{\prime} I_{i . i}\right)
\end{align*}
$$

This has been derived in Appendix $F$. So let

$$
\begin{equation*}
a_{k}+b_{k}^{\prime}=\frac{10.55796}{n}\left(\sum_{i=1}^{n} Y^{\prime} I_{. i} \sin \theta_{i}-0.63662 \sum_{i=1}^{n} Y^{\prime} I_{. i}\right) \tag{3.4.40}
\end{equation*}
$$

${ }^{b^{\prime}}{ }_{k}$ can now be taken as the raw measurement of $b_{k}$. Clearly $b_{k}$ is the error or discrepancy between the measured value of $a_{k}+b_{k}$ and its predicted value $a_{k}$. The predicted value of the next peak level is $a_{k+1}$ and this is taken to be

$$
\begin{equation*}
a_{k+1}=a_{k}+\left(1-\theta_{b}\right) b_{k}^{\prime} \tag{3.4.41}
\end{equation*}
$$

where $\theta_{b}$ is an appropriate positive constant a little less than +1 , whose optimum value needs to be determined experimentally. The above equation gives a degree-0 least squares fading memory prediction and can be extended to degree-1 or degree-2 as in Sec.3.4.3

Similarly, the best estimate of the peak value $a_{k+1}+b_{k+1}$ (at $\Theta_{i}=3 \pi / 2$ ) from $n$ measurements $\left\{y_{I_{. i}}\right\}$ regularly spaced over the interval $\pi \leqslant \rho_{i}<2 \pi$ is

$$
\begin{equation*}
a_{k+1}+b_{k+1}^{\prime}=\frac{10.55796}{n}\left(\sum_{i=1}^{n} y^{\prime}{ }_{I . i} \sin \theta_{i}+0.63662 \sum_{i=1}^{n} y^{\prime}{ }_{I . i}\right) \tag{3.4.42}
\end{equation*}
$$

So the absolute value of all peaks of the sinewave is given by Eq. (3.4.40) and all troughs by Eq.(3.4.42). The prediction of the absolute value of the next peak or trough is given by Eq. (3.4.40)

Referring to Fig.3.4.7, the best estimate of $c_{k-1}$ from $n$ measurements $\left\{Y^{\prime} I_{. i}\right\}$ regularly spaced over the interval $-\pi / 2 \leqslant \Theta_{i}<\pi / 2$ is (see Appendix F)

$$
\begin{gather*}
\tan ^{-1}\left[\frac{\sum_{i=1}^{n} Y^{\prime} I_{. i} \cos _{i}-\frac{2}{\pi} \sum_{i=1}^{n} Y^{\prime} I_{. i}}{\left(1-8 / \pi^{2}\right) \sum_{i=1}^{n} Y^{\prime} I_{. i} \sin \Theta_{i}}\right] \\
\approx \tan ^{-1}\left[\frac{5.27898 \sum_{i=1}^{n} Y^{\prime} I_{. i} \cos \Theta_{i}-0.63662 \sum_{i=1}^{n} Y^{\prime} I_{. i}}{\sum_{i=1}^{n} Y^{\prime} I_{. i} \sin \Theta_{i}}\right] \tag{3.4.43}
\end{gather*}
$$

So, let

$$
\begin{equation*}
c_{k-1}^{\prime}=\frac{5.27898 \sum_{i=1}^{n} y^{\prime} I_{. i}{ }^{\cos \Theta_{i}-0.63662 \sum_{i=1}^{n} y^{\prime} I . i}}{\sum_{i=1}^{n} y^{\prime} I_{. i} \sin \Theta_{i}} \tag{3.4.44}
\end{equation*}
$$

Similarly, the best estimate of $c_{k}$ from $n$ measurements $\left\{y^{\prime} I_{i}{ }^{i}\right\}$ regularly spaced over the interval $\pi / 2 \leqslant \Theta_{i}<3 \pi / 2$ is

$$
\begin{equation*}
c_{k}^{\prime}=\frac{5.27898 \sum_{i=1}^{n} Y^{\prime} I_{. i}{ }^{\cos \theta_{i}}+0.63662 \sum_{i=1}^{n} Y^{\prime} I_{. i}}{\sum_{i=1}^{n} Y^{\prime} I_{. i} \sin \theta_{i}} \tag{3.4.45}
\end{equation*}
$$

A small phase error is assumed here, such that $\tan (c) \approx c . \quad c^{\prime}{ }_{k-1}$ can now be taken as the raw measurement of $c_{k-1}$. Now

$$
\begin{equation*}
c_{k-1}^{\prime}=-c_{k-1}^{\prime} \frac{\Delta_{k-1}}{\pi} \tag{3.4.46}
\end{equation*}
$$

Clearly, $C^{\prime}{ }_{k-1}$ is the error or discrepancy (as a number of symbols) between the measured time $\tau_{k-\frac{1}{2}}+C_{k-1}$ of the $(k-1)^{\text {th }}$ zero crossing of the sinewave and its predicted time $\tau_{k-\frac{1}{2}}$. The minus sign in Eq. (3.4.46) is necessary because a positive phase value of $c_{k-1}$ radians results in a negative value of $C_{k-1}$ samples.

The measured time of the $k^{\text {th }}$ peak/trough is $\tau_{k}+C_{k-1}^{\prime}$, since this phase error is assumed constant over the half-cycle of the sinewave in the range $-\pi / 2 \leqslant \Theta_{i}<\pi / 2$. So, the measured value of the number of samples between
$\Theta_{i}=-\pi / 2$ and $\Theta_{i}=\pi / 2$ is $\Delta_{k-1}+C '{ }_{k-1}$, whereas the predicted number was $\Delta_{k-1}$. The predicted time interval between the $(k-1)^{\text {th }}$ and $k^{\text {th }}$ zero crossings as a number of symbol periods is now given by

$$
\begin{equation*}
\Delta_{k}=\Delta_{k-1}+\left(1-\theta_{c}\right) C_{k-1}^{\prime} \tag{3.4.47}
\end{equation*}
$$

where $\theta_{C}$ is an appropriate positive constant just less than +1 , whose optimum value needs to be determined experimentally. Eq(3.4.47) gives a degree-O least squares fading memory prediction of the zero crossing interval. It can be extended to degree-1 or degree-2 as before.

It is important to observe that the correction is applied to the predicted time interval of the next half-cycle rather than to the time instants of any of previous predicted zero crossings. This is a correction to the frequency of the signal rather than its phase.

The prediction $X^{\prime}{ }_{i}$ is now subtracted from the raw measurement $y^{\prime} I_{i}$ to give the residual fading estimate $v^{\prime}{ }_{i}$, where

$$
v_{i}^{\prime}=y_{I_{i}}^{\prime}-x_{i}^{\prime} \approx v_{i}+w_{i}
$$

The simple polynomial filter (degree-1 or degree-2) now operates on the $\left\{v_{i}{ }_{i}\right\}$, to track the residual fading. The prediction $v^{\prime}{ }_{i+1, i}$ of $v_{i+1}$ is then added to $x^{\prime}{ }_{i+1}$ to give the resultant prediction $x^{\prime}{ }_{i+1}+v^{\prime}{ }_{i+1, i}$ of $Y^{\prime}{ }_{I}{ }^{i+1, i}$. This prediction together with the corresponding prediction $Y_{Q . i+1, i}^{\prime}$ is used in the appropriate detection process.

### 3.4.7 Equalizer

It was shown in Sec.3.4.2 that the unbiased estimator gives the optimum "raw measurement" of $y_{i}$ from $s_{i}$ and $r_{i}$. But it does not use any information gained from previous channel estimates $y_{i-1}{ }^{\prime} y_{i-2}^{\prime}, \ldots$. However, the channel samples $\left\{y_{i}\right\}$ actually change quite slowly from one sample to the next. So by using this knowledge it should be possible to reduce the mean-square error in the estimate of the channel. This should in turn lead to a lower probability of error in the detector. The linear feedforward equalizer described here is such an estimation process. At $\mathrm{t}=\mathrm{i} \mathrm{T}$ it updates the estimate of the inverse of the channel by an amount depending on the samples $r_{i}, s_{i}$.

Equalizer equations:-
After the receipt of $r_{i}$

$$
\begin{equation*}
x_{i}=c^{\prime}{ }_{i-1} r_{i} \tag{3.4.48}
\end{equation*}
$$

Then, after the maximum likelihood detection of $s_{i}$ from $x_{i}$


Fig.3.4.9 Equalizer

$$
\begin{align*}
e_{i} & =s^{\prime}{ }_{i}-x_{i}  \tag{3.4.49}\\
c_{i}^{\prime} & =c_{i-1}^{\prime}+a e_{i} r^{*}{ }_{i} \tag{3.4.50}
\end{align*}
$$

Where "a" is a small, positive real-valued quantity whose optimum value is determined experimentally. All other quantities are complex-valued. This operation is depicted in Fig.3.4.9. The square containing the symbol $\sum$ is an accumulator that adds the error signal $a e_{i} r^{*}{ }_{i}$ to the stored value $c^{\prime}{ }_{i-1}$ to give the value $c^{\prime}{ }_{i}$, which is stored ready for the next sample.

This quantity $c^{\prime}{ }_{i}$ is an estimate of $c_{i}$, where $c_{i}$ satisfies the equation

$$
\begin{equation*}
c_{i} y_{i}=1 \tag{3.4.51}
\end{equation*}
$$

for all \{i\}. (Because $y_{i}$ is time varying, $c_{i}$ must also be time varying.) In other words, the equalizer acts as the inverse of the channel. It corrects for the multiplicative fading introduced by the channel and restores the received signal into a copy of the transmitted signal, (neglecting for the moment the effects of noise).

The optimum value of "a" can be determined experimentally as follows. Put Eq. (3.4.49) into Eq.(3.4.50)

$$
\begin{equation*}
c_{i}^{\prime}=c_{i-1}^{\prime}+a\left(s_{i}^{\prime}-x_{i}\right) r_{i}^{*} \tag{3.4.52}
\end{equation*}
$$

Put Eq. (3.4.48) into Eq.(3.4.52)

$$
\begin{align*}
c^{\prime} & =c^{\prime}{ }_{i-1}+a s^{\prime}{ }_{i} r^{\star}{ }_{i}-a c_{i-1}{ }_{i} r_{i} r^{*}{ }_{i} \\
& =\left(1-a r_{i} r^{\star}{ }_{i}\right) c^{\prime}{ }_{i-1}+a s^{\prime}{ }_{i} r^{*}{ }_{i} \tag{3.4.53}
\end{align*}
$$

Put $a=1 /\left(r_{i} r_{i}\right)=1 /\left|r_{i}\right|^{2}$ into Eq.(3.4.53)

$$
\begin{equation*}
c_{i}^{\prime}=\frac{s_{i}^{\prime}}{r_{i}} \tag{3.4.54}
\end{equation*}
$$

where $r_{i}$ is given in Eq.(3.4.1) as

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{3.4.55}
\end{equation*}
$$

So, for perfect detection ( $s_{i}{ }_{i}=s_{i}$ ), no noise ( $r_{i}=s_{i} y_{i}$ ) and $a=1 /\left|r_{i}\right|^{2}$

$$
\begin{equation*}
c_{i}^{\prime}=\frac{1}{y_{i}} \tag{3.4.56}
\end{equation*}
$$

But, in the presence of noise

$$
\begin{equation*}
c_{i}^{\prime}=\frac{s_{i}}{s_{i} y_{i}+w_{i}} \tag{3.4.57}
\end{equation*}
$$

Clearly, this is the optimum value of "a" in the absence of noise, and $c^{\prime}{ }_{i}$ is independent of all previous values of $c^{\prime}{ }_{i}$. But it may well be better to use a small constant value for "a" under noisy conditions. The value of $c^{\prime}{ }_{i}$ would then only change gradually from one sample to the next. The equalizer would then change more predictably, but would rely very heavily on the previous estimates $\left\{c^{\prime}{ }_{i}\right\}$ being accurate.

A major problem with this method is that the equalizer is very likely to go unstable during deep fades in the channel. Since, as $y_{i}$ tends to zero, $c_{i}$ tends to infinity. The equalizer would have to be restarted as soon as the signal strength returned.

### 3.4.8 Gradient Algorithm Estimator [64,65,67-71.,124,125]

This linear feedforward estimator forms the estimate $y^{\prime}{ }_{i}$ by updating the previous estimate $y_{i-1}^{\prime}$ by an amount depending on the values of $r_{i}, s_{i}^{\prime}$. This is a very similar method to the equalizer of the previous section. However, this Gradient algorithm should perform better because it tracks the channel rather than its inverse.

Gradient estimator equations:-

$$
\begin{align*}
r_{i}^{\prime} & =s^{\prime}{ }_{i} y^{\prime}{ }_{i-1}  \tag{3.4.57}\\
e_{i} & =r_{i}-r_{i}^{\prime}  \tag{3.4.58}\\
y_{i}^{\prime} & =y_{i-1}^{\prime}+b e_{i} s^{\prime *}{ }_{i} \tag{3.4.59}
\end{align*}
$$

where $b$ is a small, real-valued positive constant whose value is determined experimentally. All other quantities are complex-valued. $s^{\prime *}{ }_{i}$ is the complex conjugate of $s^{\prime}{ }_{i}$.

After the receipt of $r_{i}$ and the detection of $s_{i}$, the estimator works by executing Eqs.(3.4.57)-(3.4.59) in numerical order, for all \{i\}. This operation is depicted in Fig.3.4.10(a). The sample $y^{\prime}{ }_{i}$ output from this estimator (see Fig.3.4.10(b)) is fed into a fading memory polynomial filter as described in Sec.3.4.4. The subsequent predicted sample $y^{\prime}{ }_{i+1, i}$ is fed into the detector ready for the detection of the next symbol $s_{i+1}$. The square containing $\sum$ is an accumulator that adds the error signal be ${ }_{i} s^{\prime *}{ }_{i}$ to the stored value $y^{\prime}{ }_{i-1}$ to give the value $y^{\prime}{ }_{i}$, which is stored ready for the next sample. This accumulator can alternatively be represented as shown in Fig.3.4.10(b). The square marked $T$ is a store that holds the channel estimate $Y^{\prime}{ }_{i}$ and each time the store is triggered on the receipt of a sample $r_{i}$, the stored value is shifted one place to the right.

This estimator is in fact a recursive digital filter, and the output $y_{i}^{\prime}$ depends on all previous estimates $y^{\prime}{ }_{i-1}, y_{i-2}^{\prime}{ }_{i} \ldots$ by an amount depending on the value of the constant $b$. The smaller the value of $b$, the smaller the effect of the additive noise on $y^{\prime}{ }_{i}$, but the slower the rate of response of $Y^{\prime}{ }_{i}$ to changes in $Y_{i}$.

The performance of this estimation process can be analysed theoretically.

(a)

Fig.3.4.10 (a) Gradient algorithm estimator with
(b) an alternative representation of $\Sigma$

Put Eq. (3.4.58) into Eq.(3.4.59)

$$
\begin{equation*}
y_{i}^{\prime}=y_{i-1}^{\prime}+b\left(r_{i}-r_{i}^{\prime}\right) s^{\prime *}{ }_{i} \tag{3.4.60}
\end{equation*}
$$

But Eq. (3.4.1) shows that

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{3.4.61}
\end{equation*}
$$

So putting Eqs.(3.4.57) and (3.4.61) into Eq.(3.4.60)

$$
\begin{align*}
y_{i}^{\prime} & =y_{i-1}^{\prime}+b\left(s_{i} y_{i}+w_{i}-s_{i}^{\prime} y^{\prime}{ }_{i-1}\right) s^{\prime *}{ }_{i} \\
& =b s_{i} s^{\prime *}{ }_{i} y_{i}+\left(1-b s_{i}^{\prime} s^{\prime *}{ }_{i}\right) y^{\prime}{ }_{i-1}+b s^{\prime *}{ }_{i} w_{i} \tag{3.4.62}
\end{align*}
$$

Now put $b=1 /\left(s^{\prime}{ }_{i} s^{\prime *}{ }_{i}\right)=1 /\left|s_{i}{ }_{i}\right|^{2}$ into Eq.(3.4.62)

$$
\begin{equation*}
y_{i}^{\prime}=s_{i} s_{i}^{\prime}{ }^{-1} y_{i}+s_{i}^{\prime}{ }^{-1} w_{i} \tag{3.4.63}
\end{equation*}
$$

This is identical to Eq.(3.4.2). So the Gradient algorithm with $b=1 /\left|s_{i}^{\prime}\right|^{2}$ is exactly equivalent to the unbiased estimator (Sec.3.4.2), even with incorrect detection. For the QPSK signal used in System $1, b=\frac{1}{2}$ for all \{i\}.

So now, assuming correct detection

$$
\begin{equation*}
y_{i}^{\prime}=y_{i}+s_{i}^{-1} w_{i} \tag{3.4.64}
\end{equation*}
$$

and with no noise

$$
\begin{equation*}
y_{i}^{\prime}=y_{i} \tag{3.4.65}
\end{equation*}
$$

This estimate is independent of all previous channel estimates and is optimum in the absence of noise. However, this may not necessarily be the optimum estimate of $y_{i}$ from $s_{i}, r_{i}$ and $Y_{i-1}$ in all practical levels on noise. It is possible that other values of $b$ in this Gradient algorithm may give an estimate of $y_{i}$ with a smaller mean-square error. Since, with correct detection and $b \neq 1 /\left|s_{i}\right|^{2}$, from Eq. (3.4.62)

$$
\begin{equation*}
y_{i}^{\prime}=b\left|s_{i}\right|^{2} y_{i}+\left(1-b\left|s_{i}\right|^{2}\right) y_{i-1}^{\prime}+b s_{i}^{*} w_{i} \tag{3.4.66}
\end{equation*}
$$

So for example, consider $b=0.8 /\left|s_{i}\right|^{2}$

$$
y_{i}^{\prime}=0.8 y_{i}+0.2 y_{i-1}^{\prime}+0.8 s_{i}^{-1} w_{i}
$$

The mean-square error in this estimate over $N\left\{y^{\prime}{ }_{i}\right\}$ is

$$
\lambda_{e}=\frac{1}{N} \sum_{i=1}^{n}\left|y_{i}-y_{i}^{\prime}\right|^{2}=\left|y_{i}-y_{i}^{\prime}\right|^{2}
$$

(where $\bar{x}$ is the average or expected value of $x$ ). So, for independent $y_{i}$, $s_{i}, w_{i} \quad \lambda_{e} \approx 0.04 \overline{\left|y_{i}-y^{\prime}{ }_{i}\right|^{2}}+0.64 \overline{\left|s_{i}{ }^{-1} w_{i}\right|^{2}}$
If the estimate is assumed to be good enough and the fading rate slow enough so that generally $y_{i-1} \approx y_{i}$, then

$$
\begin{equation*}
\lambda_{e} \approx 0.64\left|\mathrm{~s}_{\mathrm{i}}^{-1}{ }_{w_{i}}\right|^{2} \tag{3.4.68}
\end{equation*}
$$

This represents a 2 dB improvement in this channel estimate over the case when $b=1 /\left|s_{i}\right|^{2}$-or the unbiased estimator.

The smaller the value of $b$ here, the more this mean-square er ror
caused by the noise component $\mathrm{bs}_{\mathrm{i}}{ }^{-1} \mathrm{w}_{\mathrm{i}}$ will be reduced. But, as b gets smaller, so the estimator is slower to respond to the fast fading. This in turn degrades the assumption that $Y^{\prime}{ }_{i-1} \approx Y_{i}$ which tends to increase the mean-square error. The optimum value of $b$ for any given signal-to-noise ratio and fading rate is best evaluated by computer simulation.

### 3.4.9 Gradient algorithm estimator incorporating feedback from a fading memory polynomial predictor

This is the standard arrangement of the Gradient algorithm estimation process that has been used successfully at Loughborough University over HF radio links, where the fading rate is slower [64,65].

Gradient estimator equations:-

$$
\begin{align*}
r_{i}^{\prime} & =s^{\prime}{ }_{i} y^{\prime}{ }_{i, i-1}  \tag{3.4.69}\\
e_{i} & =r_{i}-r_{i}^{\prime}  \tag{3.4.70}\\
y_{i}^{\prime} & =y_{i, i-1}^{\prime}+b e_{i} s^{\prime *}{ }_{i} \tag{3.4.71}
\end{align*}
$$

Where again, b is a small, real-valued positive constant whose optimum value is determined experimentally. All other quantities are complexvalued. $s^{\prime \prime}{ }_{i}$ is the complex conjugate of $s^{\prime}{ }_{i}$.

This estimation process operates in an exactly similar way to that of the previous Sec.3.4.8, but with the predictor now incorporated into the estimator. The only difference being that the estimate $y^{\prime}{ }_{i-1}$ stored in the estimator of Sec.3.4.8 and used in Eqs.(3.4.57)-(3.4.59) is replaced by the prediction $y^{\prime}{ }_{i, i-1}$. This prediction from the degree-p fading memory predictor is determined according to the corresponding equations in Table 3.4.1.

The input to the degree-p predictor is

$$
\begin{equation*}
E_{i}=y_{i, i-1}^{\prime}-y_{i}^{\prime} \tag{3.4.72}
\end{equation*}
$$

which, from Eq.(3.4.71) is equivalent to

$$
\begin{equation*}
E_{i}=b e_{i} s^{\prime *}{ }_{i} \tag{3.4.73}
\end{equation*}
$$

So, Eq.(3.4.71) need not be executed since the $\left\{y^{\prime}{ }_{i}\right\}$ are not required. Only the $\left\{y^{\prime}{ }_{i, i-1}\right\}$ are used in the detector. Hence the lower diagram of Fig.3.4.11(b).

The optimum values of $b$ (in Eq.(3.4.71)) and $\theta$ (in Table 3.4.1) are found experimentally by computer simulation for any given fading and noise conditions.

The theoretical analysis of this estimator's performance is exactly similar to that for the previous estimator, Eqs.(3.4.60)-(3.4.68), with $y^{\prime}{ }_{i-1}$ replaced by $y^{\prime}{ }_{i, i-1}$. Hence, from Eq. (3.4.62)

(b)
(a)

Fig.3.4.11 (a) Gradient algorithm with prediction
(b) Alternative representations of $\Sigma$

$$
\begin{equation*}
y^{\prime}{ }_{i}=b s_{i} s^{\prime *}{ }_{i} y_{i}+\left(1-b s s_{i} s^{\prime *}{ }_{i}\right) y^{\prime}{ }_{i, i-1}+b s{ }^{\prime *}{ }_{i} w_{i} \tag{3.4.74}
\end{equation*}
$$

and with $b=1 /\left(s^{\prime}{ }_{i} s^{\prime *}{ }_{i}\right.$ ) $=1 /\left|s^{\prime}{ }_{i}\right|^{2}=\frac{1}{2}$, and correct detection ( $s^{\prime}{ }_{i}=s_{i}$ ), Eq. (3.4.74) becomes

$$
\begin{equation*}
y_{i}^{\prime}=y_{i}+s_{i}^{-1} w_{i} \tag{3.4.75.}
\end{equation*}
$$

Again, with $b=1 /\left|s^{\prime}{ }_{i}\right|^{2}$, this estimation process for $y_{i}{ }_{i}$ is identical to the unbiased estimator (Sec.3.4.2) even with incorrect detection. This estimate $y^{\prime}{ }_{i}$ is independent of all previous channel estimates and predictions and is optimum in the absence of noise, since when $w_{i}=0$

$$
\begin{equation*}
y_{i}^{\prime}=y_{i} \tag{3.4.76}
\end{equation*}
$$

This is not necessarily the optimum value of $b$ in the presence of noise. With correct detection and $b \neq 1 /\left|s_{i}\right|^{2}$, Eq. (3.4.74) becomes

$$
\begin{equation*}
y_{i}^{\prime}=b\left|s_{i}\right|^{2} y_{i}+\left(1-b\left|s_{i}\right|^{2}\right) y_{i, i-1}+b s_{i}{ }_{i} w_{i} \tag{3.4.77}
\end{equation*}
$$

So, for the example where $b=0.8 /\left|s_{i}\right|^{2}$

$$
y_{i}^{\prime}=0.8 y_{i}+0.2 y_{i, i-1}^{\prime}+0.8 s_{i}^{*} w_{i}
$$

The mean-square estimate in this estimate is

$$
\lambda_{e}=\frac{1}{N} \sum_{i=1}^{N}\left|y_{i}-y_{i}^{\prime}\right|^{2}=\overline{\left|y_{i}-y_{i}^{\prime}\right|^{2}}
$$

So for independent $s_{i}, Y_{i}, w_{i}$

$$
\begin{equation*}
\lambda_{e} \approx 0.04 \overline{\left|y_{i}-y_{i, i-1}^{\prime}\right|^{2}}+0.64 \overline{\left|s_{i}{ }^{-1} w_{i}\right|^{2}} \tag{3.4.78}
\end{equation*}
$$

Comparing Eq.(3.4.78) with Eq.(3.4.67), one would intuitively expect this estimation process to be an improvement over that of Sec.3.4.8 since generally, $y^{\prime}{ }_{i, i-1}$ is likely to be much closer to $y_{i}$ than is $y^{\prime}{ }_{i-1}$.

BUT the analysis must be carried one stage further. How does this process of feeding-back the prediction $y_{i, i-1}{ }_{i n t o}$ the Gradient estimator affect the subsequent performance of the predictor? The input to the predictor is now highly dependent on previous outputs from the predictor, and so does not conform to the assumptions made by Morrison [110] (see Sec.3.4.3).

The input to any of the least-squares fading memory polynomial predictors is

$$
\begin{equation*}
E_{i}=y_{i}^{\prime}-y_{i, i-1}^{\prime} \tag{3.4.79}
\end{equation*}
$$

But, using the unbiased estimator (or Gradient algorithm with $b=1 /\left|s_{i}\right|^{2}$ ), assuming correct detection

$$
\begin{equation*}
y_{i}^{\prime}=y_{i}+s_{i}^{-1} w_{i} \tag{3.4.80}
\end{equation*}
$$

Put Eq.(3.4.80) into Eq.(3.4.79). The prediction error is

$$
\begin{equation*}
E_{i}=y_{i}-y_{i, i-1}^{\prime}+s_{i}^{-1} w_{i} \tag{3.4.81}
\end{equation*}
$$

But for this estimator, assuming $b \neq 1 /\left|s_{i}\right|^{2}$ and correct detection. From Eq.(3.4.74)

$$
\begin{equation*}
y_{i}^{\prime}=b\left|s_{i}\right|^{2} y_{i}+\left(1-b\left|s_{i}\right|^{2}\right) y_{i, i-1}+b s_{i}{ }_{i} w_{i} \tag{3.4.82}
\end{equation*}
$$

Put Eq. (3.4.82) into Eq.(3.4.79). The prediction error now is

$$
\begin{equation*}
E_{i}=b\left|s_{i}\right|^{2}\left(y_{i}-y_{i, i-1}^{\prime}+s_{i}^{-1} w_{i}\right) \tag{3.4.83}
\end{equation*}
$$

which equals $b\left|s_{i}\right|^{2} \times\left(E_{i}\right.$ for the unbiased estimator).
Therefore, it can be seen that the prediction obtained from this arrangement of the Gradient algorithm (even with incorrect detection) is EXACTLY EQUIVALENT to that obtained by:
(i) Forming the unbiased estimate (Sec.3.4.2), then
(ii) Forming the prediction from these unbiased estimates with a slight modification to the prediction algorithm that

$$
\begin{equation*}
E_{i}=b\left|s_{i}\right|^{2}\left(y_{i}-y_{i, i-1}^{\prime}\right) \tag{3.4.84}
\end{equation*}
$$

In fact, this has already been tested in $\operatorname{Sec} .3 .4 .4$, with $\theta_{a}=b\left|s_{i}\right|^{2}$ and $\theta_{b}=\theta_{c}=1$.

### 3.4.10 Double sampling in the Gradient estimator

In Sec.3.3 it was stated that the $\left\{r_{i}\right\}$ form a sufficient statistic for the optimum detection of the $\left\{s_{i}\right\}$ given the $\left\{y_{i}\right\}$. So nothing is gained in the detection process by increasing the sampling rate. But it does not follow that the same $\left\{r_{i}\right\}$ and $\left\{s_{i}\right\}$ form a sufficient statistic for the optimum estimation of the $\left\{y_{i}\right\}$. In fact, sampling the channel $y(t)$ more frequently reduces the fading rate in the channel samples. It was shown in the previous two sections that the accuracy of the Gradient algorithm estimation process improves as the fading rate in these samples is reduced.

The Gradient estimator described in Sec.3.4.9 is modified here to allow for double sampling. The sequence of operations are listed as follows. They must be performed in the numerical order shown. $t=\left(i-\frac{1}{2}\right) T$
(i) From $s^{\prime}{ }_{i-1},\left[s^{\prime}{ }_{i}\right]_{4}$
(ii) From $r_{i-\frac{1}{2}},\left[s^{\prime}{ }_{i-\frac{1}{2}}\right]^{\prime}, Y^{\prime}{ }_{i-\frac{1}{2}, i-1}$
calculate $\left[\mathrm{s}^{\prime}{ }_{\mathrm{i}-\frac{1}{2}}\right]_{4}$
$t=i T$

denoted here by
$\left[s_{i}^{\prime}\right]_{4}=\left[s^{\prime}{ }_{i, 1} \quad s^{\prime}{ }_{i, 2} s^{\prime}{ }_{i, 3} \quad s^{\prime}{ }_{i, 4}\right]=[-1-j \quad-1+j+1-j+1+j]$ (3.4.85) The corresponding four possible values of $s^{\prime}{ }_{i-\frac{1}{2}}$ are denoted by $\left[s^{\prime}{ }_{i-\frac{1}{2}}\right]_{4}{ }^{\prime}$ where

$$
\begin{equation*}
s_{i-\frac{1}{2}, m}^{\prime}=\frac{1}{2}\left(s_{i-1}^{\prime}+s_{i, m}^{\prime}\right) \text { for } m=1,2,3,4 \tag{3.4.86}
\end{equation*}
$$

This is correct because the sampled impulse response of the transmitter and receiver lowpass filters in cascade is (Appendix B)

$$
\begin{equation*}
h_{i+\frac{1}{2}}=\ldots 0,0,0, \frac{1}{2}, 1, \frac{1}{2}, 0,0,0, \ldots \tag{3.4.87}
\end{equation*}
$$

Where ideal timing is assumed with sampling every $T / 2$ seconds. (ii) After the arrival of $r_{i-\frac{1}{2}}$, the vector of the four preditions $\left[y^{\prime}{ }_{i, i-\frac{1}{2}}\right]_{4}$ can be formed - one for each corresponding $\left[s^{\prime}{ }_{i-\frac{1}{2}}\right]_{4}$. The one-. step prediction is performed exactly as described in Sec.3.4.9. The best value of the constant $b$ is found experimentally. The channel behaviour is now predicted over the time interval $T / 2$ rather than $T$, which should give greater accuracy.
(iii) At time $t=i T$ after the arrival of $r_{i}$, the value of $s^{\prime}{ }_{i}$ is detected (see Sec.3.3). Note that each possible value of $s^{\prime}{ }_{i}$ now has a different value of $y_{i, i-\frac{1}{2}}$ associated with it in the detection process.
(iv) Now that $s^{\prime}{ }_{i}$ is known, $s^{\prime}{ }_{i-\frac{1}{2}}$ is simply given by

$$
\begin{equation*}
s_{i-\frac{1}{2}}^{\prime}=\frac{1}{2}\left(s_{i-1}^{\prime}+s_{i}^{\prime}\right) \tag{3.4.87}
\end{equation*}
$$

(v) and (vi) The one-step predictions $y_{i, i-\frac{1}{2}}^{\prime}$ and $y_{i+\frac{1}{2}, i}$ can now be found as for step (iii). The value of $y^{\prime}{ }_{i+\frac{1}{2}, i}$ is held in store until the arrival of $r_{i+\frac{1}{2}}$.

Only the predictions $\left\{y^{\prime}{ }_{i, i-\frac{1}{2}}\right\}$, for all integers $\{i\}$, are used in the detector. The mean-square error in this prediction for a run of $N$ transmitted $\left\{s_{i}\right\}$ is

$$
\begin{equation*}
\lambda_{p}=\frac{1}{N} \sum_{i=1}^{N}\left|y_{i}-y_{i, i-\frac{1}{2}}^{\prime}\right|^{2} \tag{3.4.88}
\end{equation*}
$$

The values of $b, \theta$ that minimize $\lambda_{p}$ for any given fading rate and signal-to-noise ratio are best found experimentally. Since these predictions have been made over a time interval $T / 2$ they should be more accurate than the corresponding $\left\{y^{\prime}{ }_{i, i-1}\right\}$ taken with single sampling, as in Sec.3.4.9.

### 3.4.11 Kalman estimator

A Kalman estimator $[63,71,113-116]$ is often used in preference to a Gradient algorithm estimator for fast fading channels, because it is quicker to respond to changes in the channel conditions so generally gives
a more accurate estimate of a fast fading channel. This improved performance is gained at the cost of an increase in the equipment complexity.

Kalman estimator equations:-

$$
\begin{align*}
r^{\prime}{ }_{i} & =s^{\prime}{ }_{i} y^{\prime}{ }_{i-1}  \tag{3.4.89}\\
e_{i} & =r_{i}-r^{\prime}{ }_{i}  \tag{3.4.90}\\
K_{i} & =\frac{P_{i-1} s^{\prime}{ }_{i}^{*}}{\omega+s^{\prime}{ }_{i} P_{i-1} s^{\prime}{ }_{i}^{*}}  \tag{3.4.91}\\
P_{i} & =\frac{1}{w}\left[P_{i-1}-K_{i} s^{\prime}{ }_{i} P_{i-1}\right]  \tag{3.4.92}\\
y_{i}^{\prime} & =y_{i-1}^{\prime}+K_{i} e_{i} . \tag{3.4.93}
\end{align*}
$$

Where $r_{i}$ is given by (Eq.(3.4.1))

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{3.4.94}
\end{equation*}
$$

$\boldsymbol{\omega}$ is a real-valued constant in the range 0 to +1 . All the other variables are complex-valued quantities. $s^{\prime}{ }_{i}{ }^{*}$ is the complex conjugate of $s^{\prime}{ }_{i}$.

Note that the Kalman estimator equations have been slightly simplified from their standard form [63,114-116] because there is no intersymbol interference in Eq. (3.4.94). Here, all variables $K_{i}, P_{i}, s_{i}^{\prime} y^{\prime} y_{i}$ are single component vectors.

Again, the one-step prediction can be incorporated into these equations by replacing $y^{\prime}{ }_{i-1}$ in Eqs.(3.4.89) and (3.4.93) by $y_{i, i-1}{ }_{i}$

### 3.4.12 Conclusions of channel estimation

A number of different estimation processes that perform coherent demodulation have been tested under the assumption that all detected symbols $\left\{s^{\prime}{ }_{i}\right\}$ fed back from the detector into the estimator are correct. That is, $s^{\prime}{ }_{i}=s_{i}$ for all \{i\}. Results obtained from these tests in Chapter 6 give a useful measure of the capabilities of these estimation processes.

The best estimation process was seen to be the Gradient algorithm with feedback from the fading memory predictor (Sec.3.4.9). This is, in fact, equivalent to the unbiased estimator with modified fading memory
prediction (Sec.3.4.4). The fading memory filters significantly improved the performance of the unbiased and Gradient estimators with a modest increase in equipment complexity. The degree-l predictor being the most cost-effective of those tested. When this estimator is combined with the detector in Sec.3.3, only about 1 dB is lost in tolerance to noise compared with perfect channel estimation.

Double sampling (Sec.3.4.10) would improve the performance of this
estimation process. However, only a small improvement of less than 1 dB is possible, which would not justify the extra complexity. So double sampling is not considered further.

The sinewave scheme (Sec.3.4.6) did not work because the dominant sinewave component in the fading channel has powerful low frequency components superimposed on it. This caused almost instantaneous changes in the amplitude and frequency of this sinewave component that could not be tracked.

The Taylor predictor is also not considered further because its performance is generally worse than any of the fading memory predictors and is no simpler to implement.

The equalizer tracks the inverse of the channel and since the inverse of a deep fade is a peak approaching infinity, the equalizer output tends to overflow during deep fades. It is quite useless for this fading channel. Also the Kalman estimator gave no improvement over the successful estimators mentioned above. Its increased complexity and well known instability problems $[114,115]$ rule out its use here.

### 3.5 Retraining of the Channel Estimator

Tests have shown that, with detection and estimation processes of the general type used here, there is likely to be a catastrophic failure in the system over the transmission of a message of typical duration [68]. The failure usually occurs after a deep fade or a prolongued loss of signal power such as when the mobile passes under a bridge. This causes a long burst of errors in the detected data symbols, which in turn reduces the accuracy of the channel estimate, which further increases the probability of error and so on. Thus, regular retraining of the channel estimate must be used.

### 3.5.1 Model of the retraining process

The packet structure assumed in the computer simulation tests is shown in Fig.3.5.1. Each packet of information consists of $N$ transmitted symbols $\left\{s_{i}\right\}$ (numbered $i=1,2, \ldots, N$ ), of which the first $R$ are known retraining symbols (numbered $i=1,2, \ldots, R$ ) and the last $N-R$ are data symbols (numbered $i=R+1, R+2, \ldots, N)$. $10 \%$ retraining is employed so $R=0.1 N$. Different packet lengths are tested up to a maximum of $N=120$, so $1<R \leqslant 12$. This upper limit has been imposed to keep the hardware complexity in the prototype modem to


Fig.3.5.1 Packet structure used in computer simulation tests

Table 3.5.1 Training signal

| $i$ | $s_{i}$ |
| :---: | :---: |
| 1 | -1 |
|  | $-j$ |
| 3 | +1 |

a manageable level [92]. In practice, every tenth packet is used for synchronization purposes to give a total redundancy of about $20 \%$ in the transmitted signal. However, since we are not here concerned with synchronizing methods, the latter redundant packets are omitted and the system operates with only $10 \%$ redundancy.

Retraining methods are only described here for the channel $\left\{y_{i}\right\}$. When two receiving antennas are used (System 1B), it is understood that an exactly similar operation is carried out to retrain the estimators for the two channels $\left\{y_{a_{. i}}\right\},\left\{y_{b . i}\right\}$. So, the baseband received samples during the retraining period are $\left\{r_{i}\right\}$ for $i=1,2, \ldots$, . From Eq. (3.2.1)

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{3.5.1}
\end{equation*}
$$

and the $\left\{s_{i}\right\}$ for $i=1,2, \ldots, R$ are known at the receiver. The particular sequence used for the training signal is not critical. The training signal in Table 3.5 .1 has therefore been chosen to optimize the performance of the retraining algorithm for System 2 (see Chapter 4), while at the same time enabling effective symbol timing to be achieved over the duration of the training signal (see Appendix C).

It is assumed during retraining (for $i=1,2, \ldots, R$ ) that

$$
\begin{equation*}
y_{i+1}-y_{i}=y_{i}-y_{i-1} \approx \dot{y}_{i, i-1}^{\prime} \tag{3.5.2}
\end{equation*}
$$

so that $y_{i}$ varies linearly with $i$. The quantity $\dot{y}^{\prime}{ }_{i, i-1}$, previously defined in Eq. (3.4.21), is the one-step prediction of the rate of change of $y_{i}$ with i.

It is very important that the retraining method is reliable because if the channel estimators are badly retrained, it is quite probable that the whole of the following data packet would be lost. The "best" retraining method is defined here as that which results in the lowest bit error rate in detection for System 1. This best method should also give the minimum mean-square error in the estimate of the channel and its slope at the end of the retraining burst. That is, at the start of data transmission.

The methods tested for retraining the channel estimator of Sec . 3.4.9 are now described. Only the degree-1 and degree-2 fading memory predictors are tested. The results of these tests are shown in Chapter 6.

### 3.5.2 Ideal Retraining

With ideal retraining and the packet structure shown in Fig.3.5.1

$$
\begin{aligned}
y_{R}^{\prime} & =y_{R} \\
y_{R+1, R}^{\prime} & =y_{R+1}
\end{aligned}
$$

$$
\begin{equation*}
\dot{y}_{R+1, R}=y_{R+1}-y_{R} \tag{3.5.3}
\end{equation*}
$$

The Gradient estimator of sec.3.4.9 is restarted with $Y^{\prime}{ }_{R}{ }^{\prime} y^{\prime}{ }_{R+1, R}$, $Y^{\prime}{ }_{R+1, R}$ given by their ideal values in Eq. (3.5.3). This is used as a benchmark by which actual retraining methods are compared.

No attempt is made here to estimate the second derivative of the channel during retraining. The high level of equipment complexity required for a relatively small improvement in system performance make such attempts impractical. It is important to note that a degree-1 predictor is completely initialised by these estimates of channel and slope whereas a degree-2 predictor would also require an estimate of the second derivative (see Table 3.4.1). Thus the degree-2 predictor is bound to be more seriously degraded by the retraining process than is the degree-1 predictor. This is of course, another reason in favour of using a degree-1 predictor rather than degree-2 in the estimation process.

### 3.5.3 Reset estimator/predictor to zero at the start of retraining

The simplest possible retraining method is to reset the estimatior and predictor to zero just before the arrival of the training signal. So, just before the arrival of $r_{1}$, set

$$
\begin{equation*}
y_{0}^{\prime}=y_{1,0}^{\prime}=\dot{y}_{1.0}^{\prime}=\ddot{z}_{1,0}=0 \tag{3.5.4}
\end{equation*}
$$

Then the estimation/prediction process described in Sec.3.4.9 is restarted from this point, where the data symbols $s_{1}, s_{2}, \ldots, s_{R}$ are known at the receiver. After the arrival of $r_{R}$ ' the predictions $Y^{\prime}{ }_{R+1}, R, \dot{y}^{\prime}{ }_{R+1}, R$ ' $\ddot{z}^{\prime}{ }_{R+1, R}$ are calculaţed ready for the detection of data symbol $s_{R+1}$.

Clearly, if the predictor has reached steady state by the time $i=R$, then this is the best retraining method. The problem here is that the maximum number of training symbols is $R=12$, and the transient response for the fading-memory predictors typically lasts several times longer than this. So in this case the channel predictions at the end of the retraining burst would generally be unsatisfactory, restulting in a serious degradation in tolerance to noise of System 1.

### 3.5.4 No Retraining if System 1 has not failed

During normal operation of System 1 when it has not failed, the channel estimator would have reached steady state by the end of the previous packet. In this case it would be best to let the estimator continue with

$$
\begin{aligned}
y_{0}^{\prime} & =y^{\prime} N \quad \text { of the previous packet } \\
Y^{\prime}{ }_{1,0} & =Y_{N+1, N}^{\prime \prime}
\end{aligned}
$$

$$
\begin{align*}
& \dot{y}_{1,0}^{\prime}=\dot{y}^{\prime}{ }_{N+1, N} \quad \text { of the previous packet } \\
& \ddot{z}^{\prime}{ }_{1,0}=\ddot{z}_{N+1, N}^{\prime \prime} \tag{3.5.5}
\end{align*}
$$

on the arrival of $r_{1}$, rather than to reset their values to zero as in Eq.(3.5.4).

The problem here is that it is not a trivial operation to decide whether the combined detector and estimator has failed. Even during correct operation, the channel estimator is likely to have switched through $\pm 90^{\circ}$ or $180^{\circ}$ after passing through a deep fade, with a corresponding shift in the detected data symbol values (see Sec.3.6). If the estimator is allowed to run on unchanged from the previous packet, then during the retraining period any such phase change would have to be determined and corrected. A retraining method that does this would either be too unreliable or too complicated to be used in the prototype modem [92]. So this method is not considered further.

### 3.5.5 Least-Squares methods

From Secs.3.5.3 and 3.5.4 it is clear that a method must be found which accurately retrains the channel estimator from scratch during every retraining period, regardless of whether or not the system has failed during the previous data packet. The method considered here is a three stage process, and operates as follows: •

Stage 1: Firstly, for each retraining symbol, the unbiased channel
estimate is formed (see Sec.3.4.3). Since

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{3.5.6}
\end{equation*}
$$

and the complete training signal ( $\left\{s_{i}\right\}$, for $i=1,2, \ldots, R$ ) is known at the receiver. The estimator now obtains $R$ estimates $\left\{x_{i}\right\}$, for $i=1,2, \ldots, R$. Where

$$
\begin{align*}
x_{i} & =s_{i}{ }^{-1} r_{i} \\
& =y_{i}+s_{i}{ }^{-1} w_{i} \tag{3.5.7}
\end{align*}
$$

is an unbiased estimate of $Y_{i}$.

Stage 2: Secondly, from these unbiased estimates, the receiver forms a least-squares estimate of both the channel and its slope in the centre of the retraining packet.

Two different least-squares estimates of slope are tested. The least-squares estimate of the rate of change of $y_{i}$ with $i$, as $i$ increases from $D-1$ to $D$ is:

EITHER Method 1

$$
\begin{equation*}
\dot{y}_{D}^{\prime}=\frac{\frac{1}{R} \sum_{i=1}^{R} i x_{i}-\left(\frac{1}{R} \sum_{i=1}^{R} i\right) \times\left(\frac{1}{R} \sum_{i=1}^{R} x_{i}\right)}{\frac{1}{R} \sum_{i=1}^{R} i^{2}-\left(\frac{1}{R} \sum_{i=1}^{R} i\right)^{2}} \tag{3.5.8}
\end{equation*}
$$

OR Method 2,

$$
\begin{equation*}
\dot{y}_{D}^{\prime}=\frac{x_{R}-x_{1}}{R-1} \tag{3.5.9}
\end{equation*}
$$

Where, in Eqs.(3.5.8) and (3.5.9) $R \geqslant 2$ and $D$ is an integer corresponding to the centre of the retraining packet, given by

$$
D= \begin{cases}\frac{1}{2} R+1, & \text { if } R \text { is even }  \tag{3.5.10}\\ \frac{1}{2} R+\frac{1}{2}, & \text { if } R \text { is odd }\end{cases}
$$

Eqs.(3.5.8) and (3.5.9) for Methods 1 and 2 are derived from first principles in Appendix $G$.

It is well known $[117,118]$ that the least-squares straight line through a set of $R$ data points $\left\{x_{i}\right\}$ passes through the centroid. The centroid is positioned at the arithmetic mean of the \{i\}, and its value is given by the arithmetic mean of the $\left\{x_{i}\right\}$. So the least-squares estimate of the channel at $i=D$ derived from the complete training signal is

$$
y_{D}^{\prime}= \begin{cases}\frac{1}{R} \sum_{i=1}^{R} x_{i}+\frac{1}{2} \dot{Y}^{\prime} D^{\prime} & \text { if } R \text { is even }  \tag{3.5.11}\\ \frac{1}{R} \sum_{i=1}^{R} x_{i}, & \text { if } R \text { is odd }\end{cases}
$$

Where again, D is defined by Eq.(3.5.10)
In practice, the number of training symbols $R$ is known beforehand, and therefore the terms $\sum i$ and $\sum i^{2}$ in Eq.(3.5.8) would also be known. So the general equation of slope for Method 1 can be greatly simplified with no need for any divisions. For example, if $\mathrm{R}=12$

$$
\begin{equation*}
y_{7}^{\prime}=0.006993 \sum_{i=1}^{R} i x_{i}-0.45454 \sum_{i=1}^{R} x_{i} \tag{3.5.12}
\end{equation*}
$$

Similarly, Eq.(3.5.9) for Method 2, if $R=12$

$$
\begin{equation*}
y_{7}^{\prime}=0.090909\left(x_{12}-x_{1}\right) \tag{3.5.13}
\end{equation*}
$$

Stage 3: Finally, these estimates of the channel and slope are used to initialise the Gradient estimator with degree-1 (or degree-2) fading memory predictor (Sec.3.4.9), which is restarted at this point, $i=D$.

The estimator/predictor is initialised with

$$
\begin{aligned}
& Y_{D}^{\prime}{ }_{D}-1=y^{\prime}{ }_{D} \\
& \dot{Y}_{D, D}^{\prime}{ }_{D}=\dot{Y}^{\prime}{ }_{D}
\end{aligned}
$$

$$
\begin{equation*}
\ddot{z}_{D, D-1}^{\prime}=0 \tag{3.5.14}
\end{equation*}
$$

The received samples $r_{D}, r_{D+1}, \ldots, r_{R}$ have been stored so the estimator / predictor is run for $i=D, D+1, \ldots, R$ according to Eqs.(3.4.69)-(3.4.71) with degree-1 or -2 prediction (Table 3.4.1). Now, the predictions $y^{\prime} R+1, R^{\prime}$ $\dot{Y}^{\prime}{ }_{R+1}, R^{\prime} \ddot{z}_{R+1, R}$ are stored ready for the arrival of the first data symbol $\mathrm{S}_{\mathrm{R}+1}$.

This three stage retraining process is tested by computer simulation for R=2,3,....12, with different signal-to-noise ratios and a fixed simulated vehicle speed of $60 \mathrm{miles} / \mathrm{hour}$. The results of these tests are given in Chapter 6, and are used to determine the best number of training symbols $(R)$ to use in the prototype modem.

Eqs.(3.5.8)-(3.5.11) used here to estimate the channel and its slope, have been derived in Appendix $G$ for a general scenario of fitting a straight line to a set of $n$ sample points by the method of least-squares. To directly apply these equations to this system, the following observations are made:

The real and imaginary parts of the complex-valued noise components $\left\{s_{i}^{-1} w_{i}\right\}$ in Eq.(3.5.7) are statistically independent Gaussian random variables with zero mean and a fixed variance $\frac{1}{2} \sigma^{2}$ (Eq. (3.4.9)). The $R$ $\left\{y_{i}\right\}$ for $i=1,2, \ldots, R$ are assumed to lie on a straight line (Eq. (3.5.2)), as shown in Fig.3.5.2(a). Under these conditions, Method 1 specifies that the least-squares straight line $\left\{y_{i}{ }_{i}\right\}$ is fitted to the $R$ unbiased estimates $\left\{x_{i}\right\}$, for $i=1,2, \ldots, R$, such that

$$
\begin{equation*}
\sum_{i=1}^{R_{n}}\left|y_{i}^{\prime}-x_{i}\right|^{2} \text { is a minimum } \tag{3.5.15}
\end{equation*}
$$

The estimates of the channel and its slope at $i=D$ that satisfy this equation, are given by Eqs.(3.5.11) and (3.5.8) respectively, where $D$ is defined in Eq. (3.5.10).

Method 1 gives a straight line $\left\{y^{\prime}{ }_{i}\right\}$ that minimizes the mean-square error between this line and the $R$ unbiased estimates $\left\{x_{i}\right\}$. But a better measure of the slope may be given by Method 2. Here the straight line $\left\{y^{\prime}{ }_{i}\right\}$ is found that minimizes the mean-square error between the slope of the straight line and the slope of the unbiased estimates $\left\{x_{i}\right\}$. This least-squares straight line $\left\{y^{\prime}{ }_{i}\right\}$, for $i=1,2, \ldots, R$ satisfies the condition

$$
\begin{equation*}
\sum_{i=1}^{R}\left|\left(y_{i}^{\prime}-y_{i-1}^{\prime}\right)-\left(x_{i}-x_{i-1}\right)\right|^{2} \text { is a minimum } \tag{3.5.16}
\end{equation*}
$$

It is shown in Appendix $G$ that the estimate of the slope of $y_{i}$ that satisfies Eq.(3.5.16) is given by Eq.(3.5.9).

Thus, the mathematical model of the least-squares methods used here has been defined. It is now shown that Method 1 generally gives a better estimate of the slope of $y_{i}$ than Method 2. For Method 1 , substitute Eq.(3.5.7) into Eq.(3.5.8)

$$
\begin{align*}
& \dot{y}_{D}=\frac{\frac{1}{R} \sum_{i=1}^{R} i\left(y_{i}+s_{i}{ }^{-1} w_{i}\right)-\left(\frac{1}{R} \sum_{i=1}^{R} i\right) \times\left(\frac{1}{R} \sum_{i=1}^{R}\left(y_{i}+s_{i}{ }^{-1} w_{i}\right)\right.}{\frac{1}{R} \sum_{i=1}^{R} i^{2}-\left(\frac{1}{R} \sum_{i=1}^{R} i\right)^{2}}  \tag{3.5.17}\\
& =\frac{\frac{1}{R} \sum_{i}^{i y_{i}}-\left(\frac{1}{R} \sum_{i} i\right) \times\left(\frac{1}{R} \sum_{i} y_{i}\right)}{\left.\frac{1}{R} \sum_{i}^{i^{2}-\left(\frac{1}{R} \sum_{i} i\right.}\right)^{2}}+\frac{\frac{1}{R} \sum_{i} i s_{i}{ }^{-1} w_{i}-\left(\frac{1}{R} \sum_{i}^{i}\right) \times\left(\frac{1}{R} \sum_{i} s_{i}{ }^{-1} w_{i}\right)}{\underbrace{}_{\text {noise-error }}(1)}
\end{align*}
$$

Where, the first term in Eq. (3.5.18) is an exact estimate of the slope of the $\left\{y_{i}\right\}$, for $i=1,2, \ldots, R$. This is only true if the $R\left\{y_{i}\right\}$ lie on a straight line as assumed in Eq.(3.5.2) (see Fig.3.5.2(a))

Similarly for Method 2, substitute Eq.(3.5.7) into Eq.(3.5.9) to give

$$
\begin{align*}
\dot{y}_{D}^{\prime} & =\frac{\left(y_{R}+s_{R}^{-1} w_{R}\right)-\left(y_{1}+s_{1}^{-1} w_{1}\right)}{R-1}  \tag{3.5.19}\\
& =\underbrace{\frac{y_{R}-y_{1}}{R-1}+\underbrace{s_{R}^{-1} w_{R}-s_{1}^{-1} w_{1}}_{\text {noise-error }(2)}}_{\dot{y}_{D}} \tag{3.5.20}
\end{align*}
$$

Where again, the first term in Eq. (3.5.20) is an exact estimate of the slope of the channel, as long as the $R\left\{y_{i}\right\}$ lie on a straight line.

Clearly noise-errors (1) and (2) in Eqs.(3.5.18) and (3.5.20) respectively, are the errors in the estimates of the slope of $y_{i}$ for the two different methods. In Appendix $G$ it is shown that the noise-errors (1) and (2) are complex-valued Gaussian random processes, whose real and imaginary parts both have zero mean, and variances $\nabla_{1}$ and $\nabla_{2}$ respectively. Now,

$$
\begin{equation*}
\nabla_{1}=\frac{12}{(R-1) R(R+1)} \cdot \frac{\sigma^{2}}{2} \quad \text { for } R \geqslant 2 \tag{3.5.21}
\end{equation*}
$$

and

$$
\begin{equation*}
\nabla_{2}=\frac{2}{(R-1)^{2}} \cdot \frac{\sigma^{2}}{2} \quad \text { for } R \geqslant 2 \tag{3.5.22}
\end{equation*}
$$

where $\sigma^{2 / 2}$ is the variance of both the real and imaginary parts of $s_{i}{ }^{-1}{ }^{w}{ }_{i}$


Fig.3.5.2 Example of real or imaginary part of $\left\{y_{i}\right\}$ and its estimate during retraining. (a) Assumed by Eq.(3.5.2) (b) Actual

| $x$ | $=$ real or imaginary part of channel |
| ---: | :--- |
|  | $=$ unbiased estimate |
|  | $=$ least-squares straight line |

Table 3.5.2 $\quad \nabla_{1}$ and $\nabla_{2}$ for different values of $R$

| R | $\nabla_{1 \times \frac{2}{\sigma^{2}}}$ | $\nabla_{2 \times} \times \frac{2}{\sigma^{2}}$ | 10log $_{10} \nabla_{1}-10 \log _{10} \nabla_{2} \mathrm{~dB}$ |
| :--- | :--- | :--- | :--- |
| 2 | 2 | 2 | 0 |
| 3 | 0.5 | 0.5 | 0 |
| 4 | 0.2 | 0.2222 | -0.46 |
| 5 | 0.1 | 0.125 | -0.97 |
| 6 | 0.05714 | 0.08 | -1.46 |
| 7 | 0.03571 | 0.05555 | -1.92 |
| 8 | 0.02381 | 0.04082 | -2.34 |
| 9 | 0.01667 | 0.03125 | -2.73 |
| 10 | 0.01212 | 0.02469 | -3.09 |
| 11 | 0.009091 | 0.02 | -3.42 |
| 12 | 0.006993 | 0.01653 | -3.74 |

and $R$ is the number of retraining symbols $2 \leqslant R \leqslant 12$.
So the mean-square error in the estimate of the slope caused by noise-errors (1) and (2) depends only on the values of $R$ and $\sigma^{2} / 2$. The effect on $\nabla_{1}$ and $\nabla_{2}$ of using different values of $R$ is shown in Table 3.5.1. Clearly for $\mathrm{R}=2$ or 3 , the error in the estimate of slope is the same for Methods (1) and (2). As $R$ increases, the measure of slope becomes more accurate with both Methods. Also as $R$ increases, Method (1) steadily improves over Method (2). The reduction in the .mean-square error of Method (1) compared with Method (2) is expressed in decibels in the last column of Table 3.5.1. In fact

$$
\log _{10} \nabla_{1}-\log _{10} \nabla_{2}=\log _{10} \frac{\nabla_{1}}{\nabla_{2}}=\operatorname{lol}_{10}\left(\frac{12 /[(\mathrm{R}-1) \mathrm{R}(\mathrm{R}+1)]}{2 /(\mathrm{R}-1)^{2}}\right)
$$

$$
\begin{equation*}
=10 \log _{10}\left(\frac{6(\mathrm{R}-1)}{\mathrm{R}(\mathrm{R}+1)}\right) \approx \operatorname{lol}_{10 \frac{6}{R}} \text {, for large } \mathrm{R} \tag{3.5.23}
\end{equation*}
$$

Method (1) gives a better measure of slope because all $R$ samples of the retraining signal are used in the calculation. The effects of noise are averaged out more than in Method (2) where only two samples are used whatever the value of $R$.

Clearly, the larger the value of $R$, the better will be the estimate of the slope of the channel, if the $R\left\{y_{i}\right\}$ for $i=1,2, \ldots, R$ all lie on a straight line as assumed in Eq.(3.5.2) (see Fig.3.5.2(a)). Of course, if the $R\left\{y_{i}\right\}$ did lie on a straight line, the estimate of slope would be equally accurate over the entire training packet. But the actual channel samples lie on a smooth curve as shown in Fig.3.5.2(b). Here, the least squares straight line through the $R\left\{x_{i}\right\}$ runs almost parallel to the tangent of the curve $\left\{y_{i}\right\}$ in the middle of the retraining packet. So the estimate of the slope given by Eq. (3.5.8) or (3.5.9) is most accurate at $i=D$. For both Methods 1 and 2, the estimate of slope at $i=D$ can now be expressed by

$$
\begin{equation*}
\dot{y}_{D}^{\prime}=\dot{y}_{D}+\text { curvature-error }+ \text { noise-error } \tag{3.5.24}
\end{equation*}
$$

where the curvature-error and the noise-error are independent of each other. As $R$ increases the noise-error decreases and the curvature-error increases. So, for any given fading rate and signal-to-noise ratio, there is an optimum length $R$ for the estimate of $\dot{Y}_{D}$. This optimum length $R$ is best found experimentally.

The mean-square error in the least-squares estimate of $Y_{D}$ is now found theoretically. Assume that $R$ is an odd number, and substitute Eq. (3.5.7)
into (3.5.11)

$$
\begin{align*}
y_{D}^{\prime} & =\frac{1}{R} \sum_{i=1}^{R}\left(y_{i}+s_{i}^{-1} w_{i}\right)  \tag{3.5.24}\\
& =\underbrace{\frac{1}{R} \sum_{i=1}^{R} y_{i}}_{y_{D}}+\underbrace{\frac{1}{R} \sum_{i=1}^{R} s_{i}^{-1} w_{i}}_{\text {noise-error }} \tag{3.5.25}
\end{align*}
$$

Where the first term in Eq.(3.5.25) is an exact estimate of $y_{D^{\prime}}$ as long as the $R\left\{Y_{i}\right\}$ lie on a straight line. Clearly, the noise-error in Eq.(3.5.25) is the error in the estimate of $Y_{D}$. This noise-error is a complex-valued Gaussian random process, whose real and imaginary parts have zero mean and variance [119]

$$
\begin{equation*}
=R \cdot \frac{1}{R^{2}} \cdot \frac{\sigma^{2}}{2}=\frac{1}{R} \cdot \frac{\sigma}{2}^{2} \tag{3.5.26}
\end{equation*}
$$

where $\sigma^{2 / 2}$ is the variance of both the real and imaginary parts of $s_{i}{ }^{-1} w_{i}$, and $R$ is the number of retraining symbols.

Clearly, the larger the value of $R$ the better is the estimate of $Y_{D}$, if the $R\left\{y_{i}\right\}$ for $i=1,2, \ldots, R$ all lie on a straight line as assumed in Eq.(3.5.2) (see Fig.3.5.2(a)). But, the actual channel samples lie on a smooth curve as shown in Fig 3.5.2(b). The first term in Eq.(3.5.25) is no longer an exact estimate of $y_{D}$, and as the curvature in the $R\left\{y_{i}\right\}$ increases, so this estimate quickly deteriorates. The estimate of $y_{D}$ can now be expressed as

$$
\begin{equation*}
y^{\prime}=y+\text { curvature-error }+ \text { noise-error } \tag{3.5.27}
\end{equation*}
$$

where the curvature-error and noise-error are independent of each other. As $R$ increases, the noise-error decreases and the curvature-error increases. So for any given fading rate and signal-to-noise ratio, there is an optimum length $R$ for the estimate of $Y_{D}$. This optimum length $R$ is best found experimentally. This is likely to be smaller than the optimum length $R$ for the estimate of $\dot{Y}_{D}$, discussed earlier.

### 3.5.6 Conclusions for Retraining

Regular retraining of the channel estimators can be successfully achieved using the least-squares methods of Sec.3.5.5. In the final system, 12 retraining symbols are used, least-squares Method 1 is used to estimate the slope of the channel and a degree-1 predictor is used in the restarted channel estimation process. Tesst results in Chapter 6 indicate that with all the correct data symbols fed back, the tolerance to noise of the estimator with this retraining method is only about $\ddagger \mathrm{dB}$ worse than with
ideal retraining. Whereas if the estimators are just reset to zero at the start of retraining, (Sec.3.5.3), then about 4 dB would be lost in tolerance to noise at high signal-to-noise ratios.

### 3.6 Combined Detection and Estimation

The maximum likelihood detection described in Sec.3.3.2 is now combined with the Gradient estimator described in Sec.3.4.9. The estimator uses a degree-1 fading memory predictor (Table 3.4.1) and is regularly retrained using the method described in Sec.3.5.5. This is the combined detector and estimator for System 1, that performs coherent detection at the receiver. It is shown in Fig.3.6.1.

Computer simulation tests in Sec.6.3 clearly show that differential coding of the binary digits must be used with this combined detector and estimator for both Systems 1 and 2. This is because a deep fade often causes a phase change of $\pm 90^{\circ}$ or $180^{\circ}$ to be introduced into the prediction of the channel. The following stream of detected data symbols are now all rotated in phase by the appropriate multiple of $90^{\circ}$, as described in Sec.2.3.3. Correct detection is achieved with differential coding of the binary digits (DQPSK), whereas $50 \%$ errors are received without it (QPSK). The phase error in the estimator is corrected after the next retraining burst (or by chance after another deep fade).

Clearly, any error extension effects caused by feeding incorrectly detected data symbols back into the estimator are negligible, since the tolerance to noise of the combined detector and estimator is the same when the detected data symbols are fed back into the estimator as when the actual $\left\{s_{i}\right\}$ are fed back. In fact, it was noted that the combined detector and estimator with differential coding worked as a very stable system even without retraining. That is, deep fades in the channel only caused short bursts of errors of several symbols duration. The combined detector and estimator never. completely collapsed so retraining was not necessary. This is no doubt helped by the fact that the $\left\{s_{i}\right\}$ all have the same amplitude. Thus an error in $s^{\prime}{ }_{i}$ is always a phase error of $\pm 90^{\circ}$ or $180^{\circ}$, which causes a compensating phase error in the channel estimate. The differentially coded binary digits can be correctly detected in the presence of a constant phase error of $\pm 90^{\circ}$ or $180^{\circ}$.

(a)


Fig.3.6.1 Simple combined detector and estimator for (a) System 1A (b) System 1B


Fig.3.6.2 Final combined detector and estimator for
(a) System 1A
(b) System
1B

### 3.6.1 Viterbi-type Detection

A weakness of the estimation process is that it relies on the correct detection of the data symbols. A technique is described in this section which overcomes this weakness by permitting the estimator to consider simultaneously several different possible values of each detected data symbol. This technique uses a Viterbi-type detection algorithm [120] in a novel way to improve the tolerance to noise of the combined detection and estimation process.

The block diagram of the combined detector and estimator is shown in Fig.3.6.2. The received sample at the output of the receiver lowpass filter of System 1A is

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{3.6.1}
\end{equation*}
$$

Similarly for System 1B

$$
\begin{align*}
& r_{a, i}=s_{i} y_{a . i}+w_{a . i} \\
& r_{b, i}=s_{i} y_{b . i}+w_{b . i} \tag{3.6.2}
\end{align*}
$$

This final combined detector and estimator for System 1 is now described in detail. The Viterbi-type detector holds in store the most likely vectors (or sequences) of detected data symbol values, where $m=1,2$ or 4 . (Generally, for an M-level signal, $m=1,2, \ldots$ or $M$. For the QPSK signal of System 1, M=4). Each vector has its own channel prediction associated with it. Detection and estimation is done simultaneously for each of the $m$ vectors, hence the thick lines in the feedback loops in Fig.3.6.2. There are assumed to be $N$ data symbols in every packet, for $i=1,2, \ldots, N$. Each packet begins with $R$ retraining symbols, $i=1,2, \ldots, R$, followed by $L=N-R$ data symbols, $i=R+1, R+2, \ldots, N .10 \%$ redundancy for retraining is assumed (Sec.3.5.1), so $R=0.1 \mathrm{~N}$ and is an even number. In practice every tenth packet is used for synchronization purposes, to give a total redundancy of about 20\% in the transmitted signal. However, since we are not concerned here with synchronization methods, the latter redundant packets are omitted. The combined detector and estimator operates as follows.

Just prior to the receipt of $r_{i}$, (or $r_{a . i}, r_{b . i}$ ) for $R<i \leqslant N$, the detector holds in store $m$ different ( $i-R-1$ )-component vectors, $Q_{i-1}$ where

$$
Q_{i-1}=\left[\begin{array}{llll}
q_{R+1} & q_{R+2} & \cdots & q_{i-1} \tag{3.6.3}
\end{array}\right]
$$

Each vector $Q_{i-1}$ represents a different possible sequence

$$
\left[s_{R+1}^{\prime} s_{R+2}^{\prime} \cdots \cdots s_{i-1}^{\prime}\right]
$$

Clearly, every $q_{i}$ has taken up one of the $M$ different possible detected values of $s_{i}$. For the QPSK signal of System $1, M=4$ and the possible values of $q_{i}$ are $\pm 1 \pm j$. Associated with each vector $Q_{i-1}$ is stored its
cost $c_{i-1}$ (to be defined presently) which is a measure of the likelihood that the vector is correct; the lower the cost, the higher being the likelihood.

On receipt of the signal $r_{i}$, each vector $Q_{i-1}$ is expanded into $m$ ( $i-R$ )-component vectors $Q_{i}$, where

$$
\begin{equation*}
Q_{i}=\left[q_{R+1} q_{R+2} \cdots \cdots q_{i}\right] \tag{3.6.4}
\end{equation*}
$$

In each group of $m$ vectors $\left\{Q_{i}\right\}$ derived from any one vector $Q_{i-1}$; the first $i-R-1$ components are as in the original $Q_{i-1}$ and the last component $q_{i}$ takes on $m$ different values. Each of the resulting mectors $\left\{Q_{i}\right\}$ has the cost given by either

$$
\begin{equation*}
c_{i}=c_{i-1}+\left|r_{i}-q_{i} y_{i, i-1}^{\prime}\right|^{2} \tag{3.6.5}
\end{equation*}
$$

for System 1A, or

$$
\begin{equation*}
c_{i}=c_{i-1}+\left|r_{a, i}-q_{i} y_{a, i, i-1}^{\prime}\right|^{2}+\left|r_{b, i}-q_{i} y_{b, i, i-1}^{\prime}\right|^{2} \tag{3.6.6}
\end{equation*}
$$

for System 1B. The quantities $y^{\prime}{ }_{i, i-1}{ }^{\prime} y^{\prime}{ }_{a . i, i-1} y^{\prime} y_{b, i, i-1}$ are one-step predictions of their corresponding $y_{i}, y_{a_{.}}, y_{b, i}$ and are considered later. The quantity $c_{i-1}$ is the cost of the vector $Q_{i-1}$ from which $Q_{i}$ was derived, such that either; For System 1A

$$
\begin{equation*}
c_{i}=\sum_{n=R+1}^{i-1}\left|r_{n}-q_{n} Y_{n, n-1}^{\prime}\right|^{2} \tag{3.6.7}
\end{equation*}
$$

or, for System 1B

$$
\begin{equation*}
c_{i-1}=\sum_{n=R+1}^{i-1}\left(\left|r_{a, n}-q_{n} y_{a, n, n-1}^{\prime}\right|^{2}+\left|r_{b, n}-q_{n} y_{b, n, n-1}^{\prime}\right|^{2}\right) \tag{3.6.8}
\end{equation*}
$$

and $C_{R}=0$. The $m\left\{Q_{i}\right\}$ derived from any one $Q_{i-1}$ are the $m$ of the $M(=4)$ possible $\left\{Q_{i}\right\}$ with the smallest costs. There are now altogether $m^{2}$ selected vectors $\left\{Q_{i}\right\}$ together with their costs ready for the receipt of $r_{i+1}$. The remaining $m^{2}-m\left\{Q_{i}\right\}$ and their costs are discarded. The process continues like this until the receipt of $r_{N}$ (or $r_{a . N}, r_{b, N}$ ) at the end of the packet. The vector $Q_{N}$ with the smallest cost now gives the values of all detected symbols in that packet.

This new system uses m different estimation and prediction processes to derive the $\left\{y^{\prime}{ }_{i, i-1}\right\},\left\{y^{\prime}{ }_{\text {a.i,i-1 }}\right\}$, $\left\{y^{\prime}{ }_{b, i, i-1}\right\}$ in Eqs.(3.6.5)-(3.6.8). Each one is associated with a different one of the $m$ stored vectors $\left\{Q_{i-1}\right\}$, where $m=1,2$ or 4 . Thus together with each $Q_{i-1}$ and its cost $c_{i-1}$ are stored also the one-step channel prediction(s) $y^{\prime}{ }_{i, i-1}{ }^{\prime}$ (or $y^{\prime}$ a.i,i-1' $Y^{\prime}{ }_{b . i, i-1}$ ) and also the one-step prediction(s) of their rates of change with $i, \dot{y}_{i, i-1}^{\prime}\left(o r \dot{y}^{\prime}{ }_{\text {a.i,i-1 }}, \dot{y}^{\prime}{ }_{b . i, i-1}\right.$ ). Of course, in evaluating the cost $c_{i}$ of any vector $Q_{i}$, the channel predictions(s) $y_{i, i-1}$ (or $y^{\prime}{ }_{a . i, i-1}, y^{\prime}{ }_{b . i, i-1}$ ) used in Eq. (3.6.5) (or Eq.(3.6.6)) are those
determined for the vector $Q_{i-1}$ from which $Q_{i}$ was derived. Finally, once the $m\left\{Q_{i}\right\}$ have been selected, the one-step prediction(s) $y^{\prime}{ }_{i+1, i}$ (or $y^{\prime}{ }_{a . i+1, i} y^{\prime}{ }_{b . i+1, i}$ ) must be formed for each $Q_{i}$. This is simply a process of updating the Gradient estimator (Sec.3.4.9) associated with the $Q_{i-1}$ from which the $Q_{i}$ was derived, as follows. For System 1A

$$
\begin{align*}
r^{\prime}{ }_{i} & =q_{i} y^{\prime}{ }_{i, i}{ }_{i-1}  \tag{3.6.9}\\
e_{i} & =r_{i}-r_{i}^{\prime}  \tag{3.6.10}\\
y_{i}^{\prime} & =y^{\prime}{ }_{i, i-1}+b e_{i} q^{*}{ }_{i}  \tag{3.6.11}\\
E_{i} & =y^{\prime}{ }_{i}-y^{\prime}{ }_{i, i-1}  \tag{3.6.12}\\
\dot{y}^{\prime}{ }_{i+1, i} & =\dot{y}^{\prime}{ }_{i, i-1}+(1-\theta)^{2} E_{i}  \tag{3.6.13}\\
y_{i+1, i}^{\prime} & =y_{i, i-1}^{\prime}+\dot{y}^{\prime}{ }_{i+1, i}+\left(1-\theta^{2}\right) E_{i} \tag{3.6.14}
\end{align*}
$$

The same values of $b, \theta$ are used in forming all $m$ predictions $\left\{y^{\prime}{ }_{i+1, i}\right\}$.
This is the standard form of the Gradient algorithm estimator. However, since $y^{\prime}{ }_{i}$ is not required in the detection process, a simplification to the algorithm can be made as follows. Replace Eqs.(3.6.11)-(3.6.12) by

$$
\begin{equation*}
E_{i}=b e_{i} q^{*}{ }_{i} \tag{3.6.15}
\end{equation*}
$$

Also, this algorithm is equivalent to the unbiased estimator with a modified degree-1 fading memory predictor (see Sec.3.4.5). Therefore, the standard algorithm can be simplified as follows. Replace Eqs.(3.6.9)(3.6.12) by

$$
\begin{align*}
y_{i}^{\prime} & =q_{i}{ }^{-1} r_{i}  \tag{3.6.16}\\
E_{i} & =b\left(y_{i}^{\prime}-y_{i, i-1}^{\prime}\right) \tag{3.6.17}
\end{align*}
$$

For System 1B, the same algorithm is repeated in exactly the same way for the signals at the two receiving antennas. That is, the samples $r_{i}$, $y^{\prime}{ }_{i, i-1}, \dot{y}^{\prime}{ }_{i, i-1}$ input to the algorithm are replaced by their corresponding samples $r_{a . i} y^{\prime}{ }_{a . i, i-1}, \dot{Y}^{\prime}{ }_{a . i, i-1}$ for antenna $A$ and $r_{b, i}$, $y^{\prime}{ }_{b . i, i-1}{ }^{\prime} \dot{Y}^{\prime}{ }_{b, i, i-1}$ for antenna $\cdot$.

So now there are m stored vectors $\left\{Q_{i}\right\}$. Associated with each $Q_{i}$ are stored its cost $c_{i}$ and its predictions $y^{\prime}{ }_{i+1, i}{ }^{\prime} \dot{y}^{\prime}{ }_{i+1, i}$ (or $y^{\prime}{ }_{a, i+1, i}$, $\dot{y}^{\prime}{ }_{a . i+1, i} y^{\prime}{ }_{b . i+1, i} \dot{y}^{\prime}{ }_{b . i+1, i}$ ) ready for the next process of detection and estimation on the receipt of $r_{i}$ (or $r_{a . i}, r_{b . i}$ ). At the end of the packet when $i=N$, the vector $Q_{N}$ with the smallest cost now gives the values of all detected symbols in that packet. That is

$$
Q_{i}=\left[\begin{array}{llll}
q_{R+1} & q_{R+2} & \cdots & q_{N} \tag{3.6.18}
\end{array}\right]=\left[s_{R+1}^{\prime} s_{R+2}^{\prime} \ldots . s_{N}^{\prime}\right]^{\prime}
$$

The detection and estimation processes are now terminated. This completes the description of the combined detection and estimation process that is
in operation during any packet of.information where $R<i \leqslant N$. All..that remains is to describe the process of retraining the $m$ channel estimators at the start of each packet. The operation procedes when $1 \leqslant i \leqslant R$, as follows.

The receiver has prior knowledge of $s_{1}, s_{2}, \ldots, s_{R}$. So, after the receipt of $r_{i}$, (for $1 \leqslant i \leqslant R$ ) the unbiased estimate of $y_{i}$ is calculated as

$$
\begin{equation*}
x_{i}=s_{i}{ }^{-1} r_{i} \tag{3.6.19}
\end{equation*}
$$

The $R$ estimates $\left\{x_{i}\right\}$ for $i=1,2, \ldots \ldots, R$ are obtained in this way. From these $R$ estimates a single Gradient estimator with degree-l predictor is restarted. The channel estimator retraining routine of Sec.3.5.5 is used. Hence, after the receipt of $r_{R}$ the estimate of the rate of change with i of $y_{i}$, as $i$ increases from $\frac{1}{2} R$ to $\frac{1}{2} R+1$ (Eq.(3.5.8)) becomes

$$
\begin{equation*}
\dot{y}^{\prime}{ }_{\frac{1}{2} R+1, \frac{1}{2} R}=\frac{\frac{1}{R} \sum_{i=1}^{R} i x_{i}-\left(\frac{1}{R} \sum_{i=1}^{R} i\right) \times\left(\frac{1}{R} \sum_{i=1}^{R} x_{i}\right)}{\frac{1}{R} \sum_{i=1}^{R} i^{2}-\left(\frac{1}{R} \sum_{i=1}^{R} i\right)^{2}} \tag{3.6.20}
\end{equation*}
$$

Similarly, the estimate of $y_{\frac{1}{2} R+1}$ derived from the complete training signal becomes

$$
\begin{equation*}
y_{\frac{1}{2} R+1, \frac{1}{2} R}=\frac{1}{R} \sum_{i=1}^{R} x_{i}+\frac{1}{2} \dot{y}^{\prime}{ }_{\frac{1}{2} R+1}, \frac{1}{2} R \tag{3.6.21}
\end{equation*}
$$

(An even number of training symbols must be used so that $\frac{1}{2} R$ is an integer). The estimator then operates according to Eqs.(3.6.9)-(3.6.14) for $i=\frac{1}{2} R+1, \frac{1}{2} R+2, \ldots \ldots, R$, setting $q_{i}=s_{i}$ for each $i$. At the end of this process the receiver for System 1A has formed $y^{\prime}{ }_{R+1, R^{\prime}} \dot{y}^{\prime}{ }_{R+1, R}$. Similarly the reciver for System 1B performs the identical process on both the $\left\{r_{a, i}\right\},\left\{r_{b, i}\right\}$ to give $Y^{\prime}{ }_{a . R+1, R} \dot{Y}^{\prime}{ }_{a, R+1, R^{\prime}} y^{\prime}{ }_{b, R+1, R^{\prime}} \dot{Y}^{\prime}{ }_{b, R+1, R}$ The detector now uses the $Y^{\prime}{ }_{R+1, R}$ (or $Y^{\prime} a^{\prime} R+1, R^{\prime} y^{\prime}{ }_{b, R+1, R}$ ) to select the $m$ possible values of $q_{R+1}$ having the smallest costs $\left\{c_{R+1}\right\}$ (Eq.(3.6.5) or (3.6.6)), where it is assumed that $c_{R}=0$. The $m\left\{q_{R+1}\right\}$ are then stored as the corresponding one-component vectors $\left\{Q_{R+1}\right\}$, together with their costs. The resulting $\left\{\mathrm{Q}_{\mathrm{R}+1}\right\}$ are next employed in Eqs.(3.6.9)-(3.6.14) with $i=R+1$ to give for every $Q_{R+1}$, a corresponding set of $m$ predictions of channel and slope $\left\{Y^{\prime}{ }_{R+2, R+1}\right\},\left\{\dot{Y}^{\prime}{ }_{R+2, R+1}\right\}$ (or $\left\{Y^{\prime}{ }_{a, R+2, R+1}\right\}$, $\left.\left\{\dot{Y}^{\prime}{ }_{a, R+2, R+1}\right\},\left\{y^{\prime}{ }_{b, R+2, R+1}\right\},\left\{\dot{Y}^{\prime}{ }_{b, R+2, R+1}\right\}\right)$. The process then continues as described.

Clearly, the additional costs added to $c_{i-1}$ in Eqs.(3.6.5) and (3.6.6) are an implementation of the maximum likelihood detection process described in Sec.3.3.2. Here, $s_{i}$ is detected in the $m^{\text {th }} Q_{i}$ as $q_{i}$. The
detector is degraded from its optimum performance because it now has to use the channel predictions $\left\{y^{\prime}{ }_{i, i-1}\right\},\left\{y^{\prime}{ }_{a . i, i-1}\right\},\left\{y_{b, i, i-1}^{\prime}\right\}$ in place of the actual channel values $\left\{y_{i}\right\},\left\{y_{a_{i}}\right\},\left\{y_{b . i}\right\}$ in Eqs.(3.6.5) and (3.6.6). By summing these costs over the whole packet in these equations, the Viterbi-type detector is expected to find the maximum likelihood sequence of symbols for this packet. Each vector $Q_{i}$ contains a different sequence of detected data symbol values $\left\{q_{i}\right\}$ and hence the channel prediction associated with each $Q_{i}$ is generally different.

Of course, when $m=1$ there is only one combined detector and estimator in operation. The bit-error-rate performance is now identical to that of the simple detector of Eqs.(3.4.5) and (3.4.6) -using $y_{i, i-1}^{\prime} y^{\prime}{ }_{a . i, i-1}{ }^{\prime}$ $y_{\text {b.i, } i-1}$ in place of $y_{i}, y_{a . i}, y_{b . i}$. But since there is no intersymbol interference in any of the channels of Fig.3.6.2, the Viterbi algorithm with $\mathrm{m}>1$ is generally considered to give no advantage over the simple detector [106]. However, the performance of the combined detector and estimator is improved here because of the m different channel predictions. A vector $Q_{i}$ containing a burst of errors would tend to have a correspondingly degraded prediction and a large cost $c_{i}$. This $Q_{i}$ is likely to be discarded when forming the $m\left\{Q_{i+1}\right\}$.

The process of combined detection and estimation with retraining described in this section is repeated in exactly the same way for every packet of information. The Systems 1A and 1B are tested by computer simulation for $m=1,2$ and 4. The values of the parameters $b, \theta, N, R$ have been determined as follows: The packet length is set as $N=120$. This was the longest packet that could be used in the prototype modem [92]. So with $10 \%$ retraining the number of retraining symbols is $R=0.1 \mathrm{~N}=12$. The values of $\mathrm{b}, \theta$ for any given signal-to-noise ratio are given in Table 6.3.1. These have been found to roughly minimize the mean-square error in prediction when correctly detected data symbols are assumed.

### 3.6.1 Conclusions for combined detection and estimation

Computer simualation results in Sec.6.3 clearly show that differential coding of the binary digits must be used with this combined detector and estimator. This is because a deep fade often causes a phase change of $\pm 90^{\circ}$ or $180^{\circ}$ to be introduced into the prediction of the channel. Thus the following stream of detected data symbols are now all. rotated in phase by the appropriate multiple of $90^{\circ}$. Without differential coding this would lead to a long burst of errors.

The Viterbi-type detector with $m>2$ and differential coding was shown to give a better tolerance to noise than the single detector and estimator with correct data symbols fed back into the estimator. This is because this Viterbi algorithm chooses the sequence of detected data symbols that gives the best tracking of the channel through a fade. It seems to follow that the tolerance to noise is improved because the predictor has the freedom to track the channel $\pm 90^{\circ}$ or $180^{\circ}$ out of phase rather than in spite of this fact.

With two receiving antennas in System 1B, computer simulation results have shown that maximal ratio combining in the detector gives a worse tolerance to noise than the method used here, which is two-component maximum likelihood detection. This is probably because of the non-ideal co-phasing when the channel predictions $\left\{y^{\prime} a_{, i, i-1}\right\},\left\{y^{\prime}{ }_{b, i, i-1}\right\}$ are used in place of their actual values $\left\{y_{a_{. i}}\right\}$, $\left\{y_{b_{. i}}\right\}$. With perfect prediction, these two methods are equivalent and optimum (Sec.3.3).

A great improvement in tolerance to noise can be achieved by using a second receiving antenna, as shown by the improvement of System 1B over 1A. This is due to the better Rayleigh statistics for the sum of two independently fading sequences, than for either one sequence.

### 3.7 Summary for System 1

A digital modem employing novel techniques of detection and estimation has been developed and tested by computer simulation. Test results suggest that this coherent demodulation receiver with differential coding of the binary digits (DQPSK) can achieve a good tolerance to additive white Gaussian noise. It does not require undue equipment complexity at the transmitter or receiver so is suitable for transmission in either direction between a mobile and a base station. This system would achieve a bandwidth efficiency of just under $1 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$ in. the mobile radio network. Two receiving antennas should be used wherever possible - the improved performance should justify the extra equipment complexity.

The key to the successful development of System 1 is a novel technique of combined detection and estimation, with regular retraining of the channel estimator. The channel estimation process is completely restarted every $1 / 100^{\text {th }}$ of a second so should be quick to recover from any prolongued loss of signal power. This also avoids problems in estimation during hand-off as the mobile unit crosses a boundary between one cell to
another. The combined detection and estimation process is particularly robust to error extension effects caused by feeding incorrectly detected data symbols back into the channel estimator. Even with the simple maximum likelihood detector ( $\mathrm{m}=1$ ), the system showed no sign of giving an avalanche of errors.

## SYSTEM 2

### 4.1 Introduction

It was shown in Chapter 3 that one bandimited QPSK signal could be successfully transmitted in a narrowband, 900 MHz mobile radio channel. Near optimum coherent demodulation is achieved at the receiver with a reasonable level of equipment complexity. The simplicity of both transmitter and receiver equipment make it a cost effective method for both mobile-to-base station and base station-to-mobile transmission. It is particularly interesting to note the stability of the system, which works well even without retraining (under the assumed conditions).

The aim of the rest of this thesis is to try to develop a narrowband system with coherent demodulation at the receiver, that acheives double the bandwidth efficiency of System 1. System 2 described in this chapter is designed to transmit simultaneously two bandlimited QPSK signals from two mobiles to the same base station within the same 24 kHz signal bandwidth at 900 MHz . A total of $48 \mathrm{kbit} / \mathrm{s}$ is transmitted from the two mobiles giving a total bandwidth efficiency of about $2 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$.

A novel multiplexing method is proposed here, where a different bandimited QPSK signal is transmitted from each mobile simultaneously. The signals originating from the two mobiles are fading independently, and the sum of these two signals is received at the base station. A method is proposed in this chapter by which the two signals can be separated at the receiver by simultaneously performing coherent demodulation of both signals.

System 2 is developed in this chapter following an exactly similar procedure as that taken for developing System 1. That is, first of all in Sec.4.2 the system model is described. Then the possible detection methods (Sec.4.3) and channel estimation methods (Sec.4.4) are considered separately, for both one and two receiving antennas. The methods for retraining the channel estimator are described in Sec.4.5 before finally testing the combined detection and estimation process in Sec.4.6

Each transmitted signal considered separately has exactly the same properties as the signal described for System 1 and all of the methods
tested are extensions of the successful methods used in System 1. So inevitably, there are many points of similarity between these two systems and wherever, possible in this chapter, reference is made to descriptions already given in Chapter 3 to avoid needless repetition. The important points to note in this chapter are the differences between Systems 1 and 2.

## 4. 2 Model of System

In the computer simulations at time $t=i T$, the baseband received sample at the output of the receiver matched filter is given by:

For System 2A

$$
\begin{equation*}
r_{i}=s_{1 . i} y_{1 . i}+s_{2, i} y_{2 . i}+w_{i} \tag{4.2.1}
\end{equation*}
$$

For System 2B

$$
\begin{align*}
& r_{a . i}=s_{1 . i} y_{1 . i}+s_{2 . i} y_{2 . i}+w_{a . i}  \tag{4.2.2}\\
& r_{b . i}=s_{1 . i} y_{3 . i}+s_{2 . i} y_{4 . i}+w_{b . i} \tag{4.2.3}
\end{align*}
$$

The letters s,y,w represent the data, channel and noise waveforms respectively. The subscript $i$ shows that these waveforms have been sampled at time $t=i T$. The data symbols ( $s$ ) are also given the subscripts 1 and 2 denoting signals transmitted from mobiles 1 and 2. The received samples ( $r$ ) and noise samples ( $w$ ) have been given the subscripts $a$ and $b$ corresponding to the samples taken at antennas A and B. To be consistent with this notation, the channel samples (y) should have been given the subscripts $1 a, 2 a, 1 b, 2 b$ denoting the transmission paths between mobiles 1 and 2 and receiving antennas $A$ and $B$. This double subscript proved to be too confusing. So the channels have been numbered from 1 to 4 as shown in Eq.(4.2.2) and Fig.4.2.1. The important assumptions from which these equations are derived have been summarized in Sec.2.5.1. Also, the relevant properties of the data, channel, noise and received samples have been summarized in that section.

The general operation of the coherent demodulation receiver has been described in Sec.2.5.2. The detailed operation of the data detection and channel estimation processes that comprise this receiver are investigated in the rest of this chapter.

### 4.3 Detection

The aim of this section is to investigate different methods of detecting the data symbols $\left\{s_{1 . i}\right\},\left\{s_{2 . i}\right\}$ in Systems $2 A$ and $2 B$, assuming perfect

(a)


Fig.4.2.1 Baseband model of data transmission system used in computer simulation tests with (a) one receiving antenna (System 2A) and (b) two receiving antennas (System 2B).
channel estimation at the receiver. The best of the methods tested here should still be the best detection process when used with the actual channel estimates.

### 4.3.1 Model of Detection Process

The baseband received sample(s) at the input to the detector at time t=iT are;

For System 2A

$$
\begin{equation*}
r_{i}=s_{1 . i} y_{1 . i}+s_{2 . i} y_{2 . i}+w_{i} \tag{4.3.1}
\end{equation*}
$$

For System 2B

$$
\begin{align*}
& r_{a . i}=s_{1 . i} y_{1 . i}+s_{2 . i} y_{2 . i}+w_{a . i} \\
& r_{b . i}=s_{1 . i} y_{3 . i}+s_{2 . i} y_{4 . i}+w_{b . i} \tag{4.3.2}
\end{align*}
$$

It is assumed that the estimates of the channels used in the detector are exact, as shown in Fig.4.3.1. So the detector must minimize the probability of error in the detection of both $s_{1 . i}$ and $s_{2 . i}$. The detector has full knowledge of the 16 possible combinations $( \pm 1 \pm j),( \pm 1 \pm j)$ of the data symbols $s_{1 . i}, s_{2 . i}$, as shown in Table 4.3.1.

### 4.3.2 Maximum Likelihood Detection

The optimum detection process of $s_{1 . i}, s_{2 . i}$ from $r_{i}$ (or from $r_{a . i}, r_{b . i}$ ), when the channel samples $Y_{1 . i}, Y_{2, i},\left(y_{3 . i}, \dot{y}_{4, i}\right)$, and all the possible values of $s_{1 . i} s_{2 . i}$ are known at the receiver is again "maximum likelihood detection". That is, no other detection process gives a lower average probability of error in the detection of both $s_{1 . i}$ and $s_{2 . i}$ [52,68]. So the detector used here is simply an adaption of that used for System 1.

For System 2A the optimum detector that has exact prior knowledge of $y_{1 . i}, Y_{2 i}$, takes as the detected values of $s_{1 . i}, s_{2 . i}$ the possible values $s^{\prime}{ }_{1 . i} s^{\prime}{ }_{2 . i}$ for which

$$
\begin{equation*}
d_{i}^{2}=\left|r_{i}-s_{1 . i}^{\prime} Y_{1 . i}-s_{2 . i}^{\prime} Y_{2 . i}\right|^{2} \tag{4.3.3}
\end{equation*}
$$

is minimum over all 16 possible combinations $( \pm 1 \pm j),( \pm 1 \pm j)$ of the values $s^{\prime}{ }_{1.1} \mathbf{' s}^{\prime}{ }_{2 . i}$. Where $|x|$ is the absolute value of the complex-valued quantity $x$.

For System 2B with exact prior knowledge of $Y_{1 . i}, Y_{2 . i}, Y_{3 . i}, Y_{4 . i}$ and statistically independent $w_{a . i}, w_{b . i}$. Eq.(4.3.3) for optimum detection becomes

$$
\begin{equation*}
d_{i}^{2}=\left|\dot{r}_{a . i}{ }^{-s_{1 . i}} y_{1 . i^{-s}} s_{2 . i} \dot{y}_{2 . i}\right|^{2}+\left|r_{b . i} s_{1 . i} y_{3 . i}^{-s_{2 . i}} Y_{4 . i}\right|^{2} \tag{4.3.4}
\end{equation*}
$$

In practice, the detector must use estimates of the channel samples in


Fig.4.3.1 Block diagram of detection process for (a) System 2A (b) System 2B

Table 4.3.1 The 16 possible combinations of $s_{1 . i}, s_{2 . i}$

| $m$ | $s_{1, i}$, | $s_{2, i}$ |
| :---: | :---: | :---: |
| 0 | $-1-j$, | $-1-j$ |
| 1 | $-1-j$, | $-1+j$ |
| 2 | $-1-j$, | $+1-j$ |
| 3 | $-1-j$, | $+1+j$ |
| 4 | $-1+j$, | $-1-j$ |
| 5 | $-1+j$, | $-1+j$ |
| 6 | $-1+j$, | $+1-j$ |
| 7 | $-1+j$, | $+1+j$ |
| 8 | $+1-j$, | $-1-j$ |
| 9 | $+1-j$, | $-1+j$ |
| 10 | $+1-j$, | $+1-j$ |
| 11 | $+1-j$, | $+1+j$ |
| 12 | $+1+j$, | $-1-j$ |
| 13 | $+1+j$, | $-1+j$ |
| 14 | $+1+j$, | $+1-j$ |
| 15 | $+1+j$, | $+1+j$ |

place of the $\left\{y_{1, i}\right\},\left\{y_{2, i}\right\},\left\{y_{3 . i}\right\},\left\{y_{4, i}\right\}$ themselves. This inevitably degrades the detection process which is therefore no longer optimum.

It is easier to see the mechanism involved here by considering an example for System 2A. Suppose that at time $t=i T$, the channel values $y_{1 . i}, Y_{2 . i}$ are known at the receiver. The received signal constellation can be constructed by calculating $s^{\prime}{ }_{i .1} Y_{1 . i}{ }^{+}{ }^{\prime}{ }_{2 . i} Y_{2 . i}$ for all possible combinations of $s^{\prime} 1 . \mathrm{i}^{\prime} \mathrm{s}^{\prime}$.i as shown in Fig.4.3.2.

Imagine drawing 16 straight lines correcting the received point $r_{i}$ to all 16 possible points $s^{\prime}{ }_{1 . i^{\prime}} Y_{1 . i}{ }^{\prime \prime}{ }_{2 . i^{\prime}} \mathrm{Y}_{2 . i}$. The lengths of these lines
 between $r_{i}$ and the corresponding signal points $s^{\prime}{ }_{1 . i} Y_{1 . i}{ }^{\prime} \prime_{2 . i} Y_{2 . i}$. The maximum likelihood detector selects as the detected value of $s_{1 . i}, s_{2 . i}$ the possible value $s^{\prime}{ }_{1 . i} \mathbf{s}^{\prime}{ }_{2 . i}$ for which $s^{\prime}{ }_{1 . i} y_{1 . i}{ }^{+s^{\prime}}{ }_{2 . i} y_{2 . i}$ is the shortest distance from $r_{i}$. Eq. (4.3.4) actually calculates the squares of these distances, $\left|r_{i} s^{\prime}{ }_{1 . i^{\prime}} Y_{1 . i^{\prime}}{ }^{\prime}{ }_{2 . i^{\prime}} Y_{2 . i}\right|^{2}$. Squaring in no way affects the order of these 16 distances $\left\{d_{i}\right\}$ from shortest to longest, since whenever $\left|x_{1}\right|<\left|x_{2}\right|$ then $\left|x_{1}\right|^{2}<\left|x_{2}\right|^{2}$.

With System 2B, the corresponding independent distance variables at both antennas are summed to give the optimum decision rule [104].

## Theoretical Probability of Error.

Clearly, the shape of the 16 point constellation of $s^{\prime}{ }_{1 . i} \mathrm{Y}_{1 . i} \mathrm{~s}^{\prime}{ }_{2 . i} \mathrm{Y}_{2 . i}$ is not fixed, but is changing with time as the fading channels change independently in both amplitude and phase (see Fig.4.3.2). As a result, the theoretical calculation of the bit error rate curves are much more complicated than for System 1. The calculation now depends on both the Rayleigh amplitude AND uniform phase distributions of the fading channels $Y_{1 . i} . Y_{2 . i}$. This theoretical calculation of the bit error rate curves is beyond the scope of this thesis. Since this System 2 uses a completely new multiplexing method, this theoretical derivation is at this time unknown.

Several example 16 point constellations are now considered to study the mechanisms which are likely to cause most of the errors. The bit-error-rate curves for each of these examples are found by computer simulation and shown later in Chapter 6.

Example (1): Assume one receiving antenna and no fading. That is, where $y_{1 . i}=y_{2 . i}=1$ for all \{i\}. So at time $t=i T$ the baseband received sample at the input to the detector is


Fig.4.3.2 Example constellations of received signal component
$s_{1 . i} y_{1 . i}+s_{2 . i} y_{2 . i}$ when
(a) $y_{1 . i}=0.61+j 0.35, y_{2 . i}=0.15+j 0.13$
(b) $y_{1 . i}=0.45, y_{2 . \mathrm{i}}=0.4$

* $=$ received sample $r_{i}$
- = possible received signal point s. ${ }_{1.1}{ }_{1 . i}{ }^{+5}{ }_{2 i}{ }^{y} 2 . i$


Fig.4.3.3 Received signal constellation when $y_{1 . i}=y_{2 . i}=1$
$\bigcirc=2$ coincident points

Table 4.3.2 The received signal constellation $s_{1 . \mathrm{i}} \mathrm{y}_{1 . \mathrm{i}}+\mathrm{s}_{2 . \mathrm{i}} \mathrm{y}_{2 . \mathrm{i}}$ when $y_{1 . i}=y_{2 . i}=1$

| $m$ | $s_{1 . i}$ | $s_{2 . i}$ | $s_{1 . i} y_{1 . i}+s_{2 . i} y_{2 . i}$ |
| :---: | :---: | :---: | :---: |
| 0 | $-1-j$ | $-1-j$ | $-2-2 j$ |
| 1 | $-1-j$ | $-1+j$ | -2 |
| 2 | $-1-j$ | $+1-j$ | $-2 j$ |
| 3 | $-1-j$ | $+1+j$ | 0 |
| 4 | $-1+j$ | $-1-j$ | -2 |
| 5 | $-1+j$ | $-1+j$ | $-2+2 j$ |
| 6 | $-1+j$ | $+1-j$ | 0 |
| 7 | $-1+j$ | $+1+j$ | $+2 j$ |
| 8 | $+1-j$ | $-1-j$ | $-2 j$ |
| 9 | $+1-j$ | $-1+j$ | 0 |
| 10 | $+1-j$ | $+1-j$ | $+2-2 j$ |
| 11 | $+1-j$ | $+1+j$ | +2 |
| 12 | $+1+j$ | $-1-j$ | 0 |
| 13 | $+1+j$ | $-1+j$ | $+2 j$ |
| 14 | $+1+j$ | $+1-j$ | +2 |
| 15 | $+1+j$ | $+1+j$ | $+2+2 j$ |

$$
\begin{equation*}
r_{i}=s_{1 . i}+s_{2 . i}+w_{i} \tag{4.3.5}
\end{equation*}
$$

The received 16 point constellation is the vector addition of the two individual 4-point constellations of $s_{1 . i}$ and $s_{2 . i}$ as shown in Fig.4.3.3. There are in fact only 9 distinct points in this constellation, because some of the points coincide with each other in the signal space. For example, from Table 4.3.2 it is shown that the four recieved signal points for $\left[s_{1, i}=-1-j, s_{2 . i}=+1+j\right],\left[s_{1, i}=-1+j, s_{2, i}=+1-j\right],\left[s_{1, i}=+1-j, s_{2 . i}=-1+j\right]$ $\left[s_{1, i}=+1+j, s_{2, i}=-1-j\right]$ all coincide with each other at the origin. The bit error rate here, even in the absence of noise, would be little better than $\frac{1}{2}$.

Example (2): Assume one receiving antenna, with constant but different amplitude and phase components in each of the two channels $\left\{y_{1, i}\right\},\left\{y_{2, i}\right\}$.

It turns out that the poor performance of Example (1) is not a great cause for concern. In the case of interest in this thesis, both channels $\left\{y_{1 . i}\right\},\left\{y_{2 . i}\right\}$ would be time varying with independent Rayleigh fading so the probability of the event $y_{1 . i}=y_{2 . i}=1$ occuring is practically nil. An infinite variety of received signal constellations are possible, most of which do not have overlapping/coincident points and yield a much better probability of error. A few examples are shown in Fig.4.3.4 to illustrate this point.

In Fig. $4.3 .4(\mathrm{a})$ it is interesting to note that this signal constellation is exactly the same as that for the 16 -point QAM signal of System 3 which is the optimum 16-point signal constellation. That is, the one' with the best tolerance to additive white Gaussian noise for a given $E_{b}$.

In Fig.4.3.4(b) one channel is much smaller in amplitude than the other. The bit error rate for this signal would be worse than for the larger amplitude signal.

In Fig.4.3.4(c) and (d), both channels have the same amplitude (1.0), but $\mathrm{Y}_{2 . i}$ is shifted in phase by $+45^{\circ}$ (Fig.c) or $+22.5^{\circ}$ (Fig.d), giving 16 distinct points in both cases. This should give an acceptable bit error rate performance in both $\left\{s_{1 . i}\right\}$ and $\left\{s_{2 . i}\right\}$, though not as good as in Fig.4.3.4(a).

So for time invariant channel values $Y_{1, i}, Y_{2, i}$, the probability of error for any given signal-to-noise ratio depends on the particular received signal constellation of $s_{1 . i} Y_{1 . i}{ }^{+s_{2}} ._{i} Y_{2 . i}$, which in turn depends on the relative amplitudes and phases of $y_{1}$ and $y_{2}$.


Fig.4.3.4 Constellation of received signal $s_{1 . i} y_{1 . i}+s_{2 . i} y_{2 . i}$ when

$$
\begin{aligned}
& y_{1 . i}=1 \text {, and (a) } y_{2 . i}=2 \quad \text { (b) } y_{2 . i}=0.2 \\
& \begin{array}{ll}
\text { (d) } y_{2 . i}=1 \angle 22.5^{\circ}
\end{array}
\end{aligned}
$$

Example 3 : Now assume one receiving antenna and independent flat Rayleigh fading in the two channels $\left\{y_{1, i}\right\},\left\{y_{2, i}\right\}$.

To evaluate theoretically the probability of bit error for maximum likelihood detection in any one QPSK signal as a function of its signal-to-noise ratio, would require averaging the bit-error-rates over all possible signal constellations given by all possible combinations of $\left|y_{1 . i}\right|,\left|y_{2 . i}\right|, / y_{1 . i}, / y_{2 . i}$, Where $|x|$ and $\angle x$ are respectively the amplitude and phase of the complex-valued quantity $x$. It is not clear how to achieve this theoretically, but it can be readily obtained by computer simulation, as indeed is done in Chapter 6.

It is important at this stage to note where the errors are most likely to occur. With only one QPSK signal in the channel (System 1) it was noted in Sec.3.3 that the detection errors occured mainly during deep fades. But, from the previous examples in this section it is clear that with two QPSK signals in the channel there are now two types of errors;
i) Errors corresponding to deep fades.
ii)Errors due to different signal points overlapping (see Fig.4.3.3) or nearly overlapping.

Example 4: Assume two receiving antennas and independent flat Rayleigh fading in the four channels $\left\{y_{1 . i}\right\},\left\{y_{2 . i}\right\},\left\{y_{3 . i}\right\},\left\{y_{4 . i}\right\}$.

The second receiving antenna turns out to be vital to the successful operation of System 2. The improvement over System 2A is even more marked than the improvement of System 1B over System 1A. The reason for this is best understood by considering the example in Fig.4.3.5. In this case, if the signal from either antenna was used on its own in Eq. (4.3.4) to detect $s_{i}$. then errors in detection would by very likely to occur. At both antennas the problem is caused by different signal points nearly overlapping. However, when the corresponding maximum likelihood distances from.both antennas are added together in Eq. (4.3.5) the detector should have no difficulty in correctly detecting $s_{1 . i}, s_{2 . i}$. In fact it is generally true for System $2 B$ that errors caused by signal points (nearly) overlapping only occur when the same two signal points come "close" together (relative to the noise power) at both antennas at the same time. That is, when

$$
\left|y_{1 . i}\right| \approx A\left|Y_{3 . i}\right| \quad A N D \quad\left|y_{2 . i}\right| \approx A\left|Y_{4 . i}\right| \quad \text { (for constant } A \text { ) }
$$

AND
So clearly, errors due to signal points overlapping occur much less frequently with two receiving antennas than with one. This effectively


Fig.4.3.5 Signal points nearly overlapping in the constellations at both antennas
means that the data symbols $\left\{s_{1 . i}\right\},\left\{s_{2 . i}\right\}$ can be simultaneously detected with very little interference from each other.

### 4.3.3 Threshold Level Detection

 fixed but is changing with time as the fading channels change independently in both amplitude and phase (see Fig.4.3.2). As a result there is no simple threshold level detection method for System 2A equivalent to the optimum detection of Eq.(4.3.3)

Additionally for System 2B, the 16 point constellations of
 shapes. They are not simply shifted in amplitude and phase relative to each other. It is therefore impossible to coherently combine these two signals. Thus maximal ratio combining cannot be used with System 2B.

### 4.3.4 Probability of error with differential coding

The differential encoding and decoding operations are carried out separately for the two signals $\left\{s_{1 . i}\right\}$ and $\left\{s_{2 . i}\right\}$, in exactly the same way as for System 1. So with perfect channel estimation a similar degradation in performance would be expected in going from QPSK to DQPSK as was experienced in System 1.

It is interesting to note here that the differentially-coherent DQPSK method often discussed in the literature $[8,9,23,36,106]$ that uses differential detection cannot be used here. This method involves multiplying the received, modulated signal by a delayed version of itself to remove the random phase of the fading channel. But since the sum of two 4-level QAM signals is received in System 2, unwanted cross-products in the two signals would be formed by this multiplication. These would swamp the wanted signal rendering this DQPSK system useless.

### 4.3.5 Conclusions of Detection

Computer simulation tests have been carried out on Systems 2 A and 2 B to show the performance of the optimum maximum likelihood detection process operating with perfect channel estimation. The results of these tests are shown in Chapter 6.

The tolerance to noise of this optimum detector for System 2 is now compared with that for System 1. System 2A loses about 5dB in tolerance to noise compared with System 1A, whereas System 2B only loses about 1.75dB
in tolerance to noise compared with System 1B. This indicates the reduction in the errors caused signal points overlapping obtained with two receiving antennas. Of course, if these errors had been completely removed in System 2B, there would have been no interference between the two data signals and the tolerance to noise of System $2 B$ would have been exactly the same as for System 1B. In view of this great improvement in performance through the use of two antennas at the receiver of System 2 rather than one, two antennas should be used if at all possible.

### 4.4 Channel Estimation

The aim of this section is to find an estimation process that will result in near-optimum data detection, when used in the maximum likelihood detector just described. This must be achieved with a reasonable level of equipment complexity. The methods tested here are derived from those which gave the most success with System 1. It is assumed in this section that all detected symbols that are fed back to the estimator are correct.

### 4.4.1 Model of Estimation Process

For convenience, the estimation process considered in sec.4.4 will be confined to estimating the $\left\{y_{1 . i}\right\},\left\{y_{2 . i}\right\}$ from the $\left\{r_{i}\right\}$ (and $\left\{s_{1 . i}\right\}$, $\left\{s_{2 . i}\right\}$ ) for System 2A, bearing in mind that an exactly similar process would be applied to both sets of received samples $\left\{r_{a_{. i}}\right\}$ and $\left\{r_{b_{. i}}\right\}$ when estimating $\left\{y_{1, i}\right\},\left\{y_{2 . i}\right\}$ and $\left\{y_{3 . i}\right\}$, $\left\{y_{4 . i}\right\}$ respectively for System 2B. Of course, since all corresponding signal properties at both antennas are the same but independent, which ever estimation process gives the best performance with one receiving antenna is also likely to be the best with two receiving antennas.

The $\left\{s_{1 . i}\right\},\left\{s_{2 . i}\right\}$ are comprised entirely of random data symbols with both sequences independent of each other. Timing and synchronization are assumed ideal here. So, at time $t=i T$,

$$
\begin{equation*}
r_{i}=s_{1, i} y_{1, i}+s_{2 . i} y_{2 . i}+w_{i} \tag{4.4.1}
\end{equation*}
$$

The properties of the data, channel and noise samples have been described in Sec. 2.5.

The estimation process depicted in Fig.4.4.1 is again a combination of estimation and prediction as for System 1. The important difference here is that the channel estimator must simultaneously estimate the two fading channels. At time $t=i T, y_{1 . i}$ and $Y_{2 . i}$ are estimated from $r_{i}$ and the


Fig.4.4.1 Model of estimation process as simulated
detected data symbols $s^{\prime} 1 . i^{\prime} s^{\prime} 2 . i$ fed back from the detector. In this Sec.4.4, these detected symbols are all assumed to be correct. So, at the input to the estimator

$$
\begin{equation*}
s_{1 . i}^{\prime}=s_{1 . i} \text { AND } s_{2 . i}^{\prime}=s_{2 . i}, \quad \text { for all }\{i\} \tag{4.4.2}
\end{equation*}
$$

Now, the predictions of the two channels are each calculated separately from their estimates $\left\{y^{\prime}{ }_{1 . i}\right\},\left\{y^{\prime}{ }_{2 . i}\right\}$ in exactly the same way as described for System 1.

There are some important points to note when analysing the estimation methods. In all tests, the estimator has been allowed enough time to start-up so that only the steady-state performance of the estimators is measured. The criterion for judging the performance of an estimation process is the mean-square error in prediction $\lambda_{p}$ (defined later), since the best (smallest) $\lambda_{p}$ generally results in the best bit error rate performance. All channel estimators work by estimating the real and imaginary parts of the channel (which represent the in-phase and quadrature components respectively) rather than their amplitude and phase. Coherent detection is achieved with these estimates.

The estimation processes considered here are the unbiased and Gradient estimators with fading memory polynomial prediction. These were seen to give the best results for System 1, so they are adapted (where possible) to work with 2 QPSK signals in the channel. All test results are shown in. Chapter 6.

### 4.4.2 Unbiased Estimator

The unbiased estimator for System 1 was the simplest possible channel esimator. To extend the idea to System 2, the unbiased estimates of $y_{1 . i}$, $y_{2 . i}$ would be given by $s^{\prime}{ }_{1 . i}{ }^{-1} r_{i}, s^{\prime}{ }_{2 . i}{ }^{-1} r_{i}$ respectively, where $r_{i}$ is given by Eq(4.4.1). So,

$$
\begin{align*}
& s^{\prime}{ }^{-1} r_{i}=y_{1 . i}+s^{\prime}{ }_{1 . i^{-1}} s_{2 . i} y_{2 . i}+s^{\prime}{ }_{1 . i^{i}}^{-1} w_{i}  \tag{4.4.3}\\
& s^{\prime}{ }_{2 . i} r_{i}=y_{2 . i}+s^{\prime}{ }_{2 . i} s_{1 . i} y_{1 . i}+s^{\prime}{ }_{2 . i} w_{i}
\end{align*}
$$

It is immediately seen that it is impossible to accurately estimate either quantity $y_{1 . i}, Y_{2 . i}$ from Eqs.(4.4.3) and (4.4.4) or from any combination of these equations. Always in the estimation of $y_{1 . i}$ (or $y_{2 . i}$ ) there is an unwanted term in $Y_{2 . i}$ (or $Y_{1 . i}$ ) that renders the estimate useless.

Therefore, the unbiased estimator does not exist for two signals in the same frequency space because basically, Eq.(4.4.1) is one equation with two unknowns $y_{1 . i}, y_{2 . i}$ (assuming $s_{1 . i}, s_{2 . i}$ are both correctly detected and ignoring the effects of noise). Therefore a second linearly
independent equation in $Y_{1 . i}, Y_{2 . i}$ is needed.

### 4.4.3 Gradient Algorithm Estimator

The Gradient algorithm was used with System 1 to try to improve on the unbiased estimator, by using the additional information that the channel. samples are slowly time varying. This is an assumption that can be made about both channels $\left\{y_{1, i}\right\},\left\{y_{2 . i}\right\}$ here, and this assumption in fact constitutes the second linearly independent equation that now makes it possible to estimate both channels $\left\{y_{1 . i}\right\},\left\{y_{2 . i}\right\}$ from the $\left\{r_{i}\right\}$.

Gradient estimator equations:

$$
\begin{align*}
r_{i}^{\prime} & =s^{\prime} 1_{1 . i} y^{\prime}{ }_{1 . i-1}+s^{\prime}{ }_{2 . i} y^{\prime}{ }_{2 . i-1}  \tag{4.4.5}\\
e_{i} & =r_{i}-r^{\prime}{ }_{i}  \tag{4.4.6}\\
y^{\prime}{ }_{1 . i} & =y_{1 . i-1}^{\prime}+b e_{i} s^{\prime}{ }_{1 . i}{ }^{*}  \tag{4.4.7}\\
y_{2 . i}^{\prime} & =y_{2 . i-1}^{\prime}+b e_{i} s^{\prime}{ }_{2 . i}^{*} \tag{4.4.8}
\end{align*}
$$

where $b$ is a small, positive real-valued constant whose optimum value is determined experimentally. $s^{\prime} 1 . i^{*}, s^{\prime} 2 . i^{*}$ are the complex conjugates of $s^{\prime} 1_{1 . i}{ }^{\prime \prime}{ }^{\prime}{ }_{2 . i}$ respectively.

This estimator works in exactly the same way as the Gradient estimator in Sec.3.4.8 for System 1, except that Eqs.(3.4.57)-(3.4.59) are replaced by Eqs.(4.4.5)-(4.4.8), and Fig.3.4.10 is replaced by Fig.4.4.2. The predictions of $y_{1 . i+1}, Y_{2 . i+1}$ are formed by passing $y^{\prime}{ }_{1 . i}, Y^{\prime}{ }_{2 . i}$ through separate but identical degree-p fading memory polynomial filters, as described in Sec.3.4. The optimum values of the constants $b$ and $\theta$ for any given signal-to-noise ratio are again found experimentally, and are generally different to the optimum value for System 1.

The performance of this Gradient estimator can be analysed theoretically as follows. For the estimate of $Y_{1 . i}$. Put Eq. (4.4.6) into Eq.(4.4.7)

$$
\begin{equation*}
y_{1 . i}^{\prime}=y_{1 . i-1}^{\prime}+b\left(r_{i}-r_{i}^{\prime}\right) s_{1 . i}^{\prime}{ }_{1} \tag{4.4.9}
\end{equation*}
$$

But Eq. (4.4.1) shows that

$$
\begin{equation*}
r_{i}=s_{1 . i} Y_{1 . i}+s_{2 . i} y_{2 . i}+w_{i} \tag{4.4.10}
\end{equation*}
$$

So, putting Eqs.(4.4.5) and (4.4.10) into Eq.(4.4.9)

$$
\begin{aligned}
& y_{1 . i}^{\prime}=Y_{1 . i-1}{ }^{+}
\end{aligned}
$$

$$
\begin{align*}
& b s^{\prime}{ }_{1 . i}{ }^{*}\left(s_{2 . i} Y_{2 . i}-s^{\prime}{ }_{2 . i} y^{\prime}{ }_{2 . i-1}\right)+b s_{1 . i}{ }^{*} W_{i} \tag{4.4.11}
\end{align*}
$$



Fig.4.4.2 (a) Gradient algorithm estimator with
(b) an alternative representation of top

Now put $b=1 /\left(s^{\prime} 1 . i^{s^{\prime}} 1 . i^{*}\right)=1 /\left|s^{\prime} 1 . i\right|^{2}$ into Eq. (4.4.11).

$$
\begin{equation*}
y_{1 . i}^{\prime}=y_{1 . i}+s_{1 . i}^{\prime \prime}\left(s_{2 . i^{\prime}}^{y_{2 . i}}-s_{2 . i}^{\prime} y_{2 . i-1}^{\prime}\right)+s_{1 . i}^{\prime-1} w_{i} \tag{4.4.12}
\end{equation*}
$$



$$
\begin{equation*}
y_{1 . i}^{\prime}=y_{1 . i}+s_{1 . i}{ }^{-1} s_{2 . i}\left(y_{2 . i}^{1.1}-y_{2 . i-1}^{\prime}\right)+s_{1 . i}{ }^{1} \underline{1}_{i}^{1} \tag{4.4.13}
\end{equation*}
$$

Of course, an exactly similar analysis can be carried out for the estimate of $Y_{2 . i}$, to give

$$
\begin{equation*}
y_{2 . i}^{\prime}=y_{2 . i}+s_{2 . i}{ }^{-1} s_{1 . i}\left(y_{1 . i}-y_{1, i-1}^{\prime}\right)+s_{1 . i}{ }^{-1} w_{i} \tag{4.4.14}
\end{equation*}
$$

Now, the error in the estimate of $y_{1 . i}$ is

$$
y_{1 . i}-y_{1 . i}^{\prime}=-s_{1 . i} i_{2 . i}\left(y_{2 . i}-y_{2 . i-1}^{\prime}\right)-s_{1 . i}{ }^{-1} w_{i}
$$

and the mean-square error in this estimate, measured over $N\left\{y_{i}{ }_{i}\right\}$ is

$$
\begin{align*}
\lambda_{e} & =\frac{1}{N} \sum_{i=1}^{N}\left|y_{1 . i}-y_{1 . i}^{\prime}\right|^{2}=\overline{\left|y_{1 . i}-y_{1 . i}^{\prime}\right|^{2}}  \tag{4.4.15}\\
& =\overline{\left|s_{1 . i}{ }^{-1} s_{2 . i}\right|^{2} \cdot} \overline{\left|y_{2 . i}-y_{2 . i-1}\right|^{2}}+\overline{\left|s_{1 . i}{ }^{-1} w_{i}\right|^{2}}
\end{align*}
$$

(for independent $s_{1 . i}, s_{2 . i}, y_{2 . i}, w_{i}$ ). Therefore, since $\left|s_{1 . i}{ }^{-1} s_{2 . i}\right|^{2=1}$ for all $\{i\}$,

$$
\begin{equation*}
\lambda_{e}=\overline{\left|y_{2 . i}-y_{2 . i-1}^{\prime}\right|^{2}}+\overline{\left|s_{i}^{-1} w_{i}\right|^{2}} \tag{4.4.16}
\end{equation*}
$$

So even with no noise (that is, $w_{i}=0$ for all \{i\}), there is an irreducible error in the estimate of $y_{1 . i}$, because the channel $\left\{y_{2 . i}\right\}$ is time varying.

In Chapter 3, it was shown that the optimum value of $b$ in the Gradient estimator for System 1 in the absence of noise was $b=1 /\left|s^{\prime}{ }_{i}\right|^{2}\left(=\frac{1}{2}\right.$, for all \{i\}). However, it is now shown that for System $2, b=0.5 /\left|s^{\prime}{ }_{1 . i}\right|^{2}\left(=\frac{1}{4}\right.$, for all \{i\}) generally gives the best estimate of $\left\{y_{1 . i}\right\}$ in the absence of noise. Since, with correct detection in Eq. (4.4.11)

$$
\begin{align*}
y_{1 . i}^{\prime}=b\left|s_{1 . i}\right|^{2} y_{1 . i} & +\left(1-b\left|s_{1 . i}\right|^{2}\right) y_{1 . i-1}^{\prime} \\
& +b s_{1 . i}{ }^{*} s_{2 . i}\left(y_{2 . i}-y_{2 . i-1}^{\prime}\right)+b s_{1 . i}{ }^{\star} w_{i} \tag{4.4.17}
\end{align*}
$$

Now, for the general case $b=B /\left|s_{1 . i}\right|^{2}$

$$
\begin{align*}
& y_{1 . i}^{\prime}=B y_{1 . i}+(1-B) y_{1 . i-1}^{\prime} \\
&+B s_{1 . i}^{-1} s_{2 . i}\left(y_{2 . i}-y_{2 . i-1}^{\prime}\right)+B s_{1 . i}{ }^{-1} w_{i} \tag{4.4.18}
\end{align*}
$$

The error in this estimate is

$$
\begin{align*}
& y_{1 . i}-y_{1 . i}^{\prime}=(1-B)\left(y_{1 . i}-y^{\prime}{ }_{1 . i-1}\right) \\
&-B s_{1 . i}{ }^{s_{2 . i}\left(y_{2 . i}-y^{\prime}{ }_{2 . i-1}\right)-B s_{1 . i}{ }^{-1} w_{i}} \tag{4.4.19}
\end{align*}
$$

with a mean-square error of, (from Eq.(4.4.15)

$$
\lambda_{e}=(1-B)^{2} \overline{\left|y_{1 . i}-y_{1, i-1}^{\prime}\right|^{2}} \begin{align*}
& +B^{2}\left|y_{2 . i}-y_{2 . i-1}^{\prime}\right|^{2}
\end{align*}+B^{2} \overline{\left|s_{1 . i}{ }^{-1} w_{i}\right|^{2}}
$$

So, when $b=0.5 /\left|s_{1 . i}\right|^{2}$

$$
\begin{align*}
& \lambda _ { e } = 0 . 2 5 \longdiv { | y _ { 1 . i } - y ^ { \prime } } \begin{array} { r l } 
{ 1 . i - 1 }
\end{array} | ^ { 2 } \\
&+0.25 \overline{\left|y_{2 . i}-y_{2 . i-1}^{\prime}\right|^{2}}+0.25 \overline{\left.s_{1 . i}{ }^{-1} w_{i}\right|^{2}} \tag{4.4.21}
\end{align*}
$$

Clearly, the smaller the value of $b$, the more the mean-square error caused by the noise component $b s_{1 . i}{ }^{-1} w_{i}$ is reduced. In going from $b=1 /\left|s_{1 . i}\right|^{2}$ down to $b=0.5 /\left|s_{1 . i}\right|^{2}$, the mean-square error due to $b s_{1 . i}{ }^{-1} w_{i}$ is reduced by about 6 dB , (since $\operatorname{lol}_{10} 0.25 \approx-6$ ). But also, as b gets smaller, so the error caused by the fading component $y_{2 . i} y^{\prime}{ }_{2 . i-1}$ is reduced at the expense of an increased error caused by $y_{1 . i^{\prime}} y^{\prime} 1 . i-1^{*}$. Now because of the symmetry of this estimator

$$
\begin{equation*}
\overline{\left|Y_{1 . i}-y_{1 . i-1}^{\prime}\right|^{2}} \approx \overline{\left|Y_{2 . i}-y_{2 . i-1}^{\prime}\right|^{2}} \tag{4.4.22}
\end{equation*}
$$

and so the minimum possible value of $(1-B)^{2} \overline{\left|Y_{1 . i} Y_{1 . i-1}\right|^{2}}+$ $B^{2} \overline{\mid Y_{2 . i}-y^{\prime}} 2 .\left.i_{-1}\right|^{2}$ can similarly be shown to occur when $B=0.5$, or $b=0.5 /\left|s_{2 . i}\right|^{2}$.

This is clearly the optimum value of $b$ for both channel estimates in the absence of noise, though at low signal-to-noise ratios smaller values of $b$ may be used. The optimum value of $b$ for any given signal-to-noise ratio and fading rate is best evaluated by computer simulation.

### 4.4.4 Gradient algorithm estimator incorporating feedback from the fading memory polynomial filter

The estimator of Sec.3.4.9, which was the best estimator tested with System 1, is now adapted for use here.

Gradient-estimator equations:

$$
\begin{align*}
& r^{\prime}{ }_{i}=s^{\prime}{ }_{1 . i} y^{\prime}{ }_{1 . i, i-1}+s^{\prime}{ }_{2 . i} Y^{\prime}{ }_{2 . i, i-1}  \tag{4.4.23}\\
& e_{i}=r_{i}-r_{i}  \tag{4.4.24}\\
& y^{\prime}{ }_{1 . i}=y_{1 . i, i-1}^{\prime}+b e_{i} s_{1 . i}^{\prime *}  \tag{4.4.25}\\
& y^{\prime}{ }_{2 . i}=y^{\prime}{ }_{2 . i, i-1}+b e_{i} s^{\prime}{ }_{2 . i}{ }^{\star} \tag{4.4.26}
\end{align*}
$$

where again, b is a small, positive, real-valued constant whose optimum value is determined experimentally. $s^{\prime} 1 . i^{*}, s^{\prime} 2 . i^{*}$ are the complex conjugates of $s^{\prime}{ }_{1 . i}{ }^{\prime \prime} s^{\prime} \dot{2} . i$ respectively.

This estimation process operates in an exactly similar way to that of the previous Sec.4.4.3, but with the predictor now incorporated into the estimator. The only difference being that the estimates $y^{\prime}{ }_{1 . i-1}{ }^{\prime} y^{\prime}{ }_{2, i-1}$ stored in the estimator of Sec.4.4.3 and used in Eqs.(4.4.5)-(4.4.8) are replaced by the predictions $y^{\prime}{ }_{1 . i, i-1} y^{\prime} y_{2 . i, i-1}$ respectively. These predictions from the degree-p fading memory predictors are determined according to the corresponding equations in Table 3.4.1.

The inputs to the degree-p predictors at time $t=i T$ are

$$
\begin{align*}
& E_{1 . i}=y^{\prime} 1_{1 . i, i-1}-y^{\prime}{ }_{1 . i}  \tag{4.4.27}\\
& E_{2 . i}=y_{2, i, i-1}^{\prime}-y_{2 . i}^{\prime} \tag{4.4.28}
\end{align*}
$$



Fig.4.4.3
(a) Gradient algorithm with prediction
(b) Alternative representations of
which, from Eqs.(4.4.25) and (4.4.26) are equivalent to

$$
\begin{align*}
& E_{1 . i}=b e_{i} s^{\prime}{ }_{1 . i^{*}}  \tag{4.4.29}\\
& E_{2 . i}=b e_{i} s^{\prime}{ }_{2 . i}{ }^{\star} \tag{4.4.30}
\end{align*}
$$

So Eqs.(4.4.25) and (4.4.26) need not be executed since the $\left\{y^{\prime}{ }_{1.1}\right\}$, $\left\{y^{\prime}, 2, i\right\}$ are not required. Only the $\left\{y^{\prime}{ }_{1, i+1, i}\right\},\left\{y^{\prime}{ }_{2, i+1, i}\right\}$ are used in the detector. Hence the lower diagram of Fig.4.4.3(b). The optimum values of $b$ (in Eqs.(4.4.25) and (4.4.26)) and $\theta$ (in Table 3.4.1): are found experimentally by computer simulation for any given fading and noise conditions.

The theoretical analysis of this estimator's performance is exactly similar to that for the previous estimator in Sec.4.4.3. The only difference being that $Y^{\prime} 1_{1 . i-1}{ }^{\prime} Y^{\prime}{ }_{2 . i-1}$ in Eqs.(4.4.9)-(4.4.22) are replaced by $y^{\prime}{ }_{1, i, i-1}, y^{\prime}{ }_{2 . i, i-1}$ respectively. The mean-square errors in the estimates of $Y_{1 . i}, Y_{2, i}$ shows an improvement over those of Sec.4.4.3 since generally, $y^{\prime}{ }_{1, i, i-1} y^{\prime} y_{2, i, i-1}$ are likely to be much closer to $y_{1 . i}, Y_{2 . i}$ than are $y^{\prime}{ }_{1 . i-1}, y^{\prime}$ 2.i-1 $^{\circ}$

But the analysis must be carried one stage further. How does this process of feeding back the predictions $y_{1 . i, i-1}^{\prime} Y^{\prime}{ }_{2 . i, i-1}$ into the Gradient algorithm estimator affect the subsequent performance of the predictors? The input to each predictor is now highly dependent on previous outputs from that predictor, and so does not conform to the assumptions made by Morrison [110] given in Sec.3.4.2. A similar analysis was carried out for System 1. It was shown in Sec.3.4.9 that the Gradient algorithm incorporating feedback from the predictor is not a new Gradient algorithm at all. It is exactly equivalent to the Gradient algorithm without this feedback, with $b=1 /\left|s^{\prime}{ }_{i}\right|^{2}$, and with a modified predictor where $E_{i}=y^{\prime}{ }_{i}-y^{\prime}{ }_{i, i-1}$ is replaced by $E_{i}=b\left|s_{i}^{\prime}\right|^{2}\left(y^{\prime}{ }_{i}-y^{\prime}{ }_{i, i}, 1\right)$. The analysis for System 2 continues as follows.

The input to any of the least-squares fading memory polynomial
predictors, for the prediction of $Y_{1 . i+1}$ is

$$
\begin{equation*}
E_{1 . i}=y_{1 . i}^{\prime}-y_{1 . i, i-1}^{\prime} \tag{4.4.31}
\end{equation*}
$$

Where, for this estimator, assuming correct detection (from Eq. (4.4.17)
with $Y^{\prime}{ }_{1 . i-1}, Y^{\prime}{ }_{2 . i-1}$ replaced by $y^{\prime}{ }_{1 . i, i-1}, Y^{\prime}{ }_{2 . i, i-1}$ )
$y^{\prime}{ }_{1 . i}=b\left|s_{1 . i}\right|^{2} y_{1 . i}+\left(1-b\left|s_{1 . i}\right|^{2}\right) y_{1, i, i-1}^{\prime}$
$+b s_{1 . i}{ }^{*} s_{2 . i}\left(y_{2 . i}-y_{2 . i, i-1}\right)+b s_{1 . i}{ }^{*} w_{i}$
Put Eq.(4.4.32) into Eq.(4.4.31). The prediction error here is

$$
\begin{align*}
E_{1 . i}=b\left|s_{1 . i}\right|^{2}\left[\left(y_{1 . i}-y^{\prime}\right.\right. & \left.1_{1 . i, i-1}\right)  \tag{4.4.33}\\
& \left.+s_{1 . i}{ }^{-1} s_{2 . i}\left(y_{2 . i}-y_{2 . i, i-1}^{\prime}\right)+s_{1 . i}{ }^{-1} w_{i}\right]
\end{align*}
$$

But, for the Gradient estimator of the previous Sec.4.3.3 that did not incorporate feedback from the predictor. With $b=1 /\left|s_{1 . i}\right|^{2}$ and correct detection (from Eq.(4.4.17))

$$
\begin{equation*}
y_{1, i}^{\prime}=y_{1, i}+s_{1, i}{ }_{s_{2, i}}\left(y_{2, i}-y_{2, i, i-1}^{\prime}\right)+s_{1, i}{ }^{-1} w_{i} \tag{4.4.34}
\end{equation*}
$$

So in this case, the prediction error is (put Eq. (4.4.34) into Eq.(4.4.31))

It is noted that

$$
\begin{equation*}
\mathrm{E}_{1 . i} \text { of Eq. (4.4.33) } \approx \mathrm{b}\left|\mathrm{~s}_{1 . i}\right|^{2} \times \mathrm{E}_{1 . i} \text { of Eq.(4.4.35) } \tag{4.4.36}
\end{equation*}
$$

The resulting prediction $y^{\prime}{ }_{1 . i+1, i}$ with this estimation process will be better than that for the estimator of the previous Sec.4.4.3, because $y^{\prime}{ }_{2 . i, i-1}$ in Eq. (4.4.33) is generally closer to $y_{2 . i}$ than is $y_{2 . i-1}$ in Eq.(4.4.35). So the estimator of this Sec.4.4.4 is a new and better Gradient estimator for System 2 than the one in Sec.4.4.3.

### 4.4.5 Estimator conclusions

The Gradient estimators of Secs.4.4.3-4.4.4 have been tested by computer simulation under the assumption that all detected symbols \{s'1.i\}, \{s'2.i\} fed back from the detector to the estimator are correct. Results of these tests shown in Chapter 6 confirm that the estimator of Sec.4.4.4 gives the best performance. This estimator is now used exclusively in the rest of this chapter.

A new method has been proposed here by which it should be possible to simultaneously estimate two independently fading channels when only the sum of the two fading signals is known at the receiver. It has yet to be proved whether an adequate retraining method can be found, and also whether this estimator will work effectively with the detected data symbols at its input.

### 4.5 Retraining. of the Channel Estimator

It is shown later in Sec. 4.6 that a catastrophic failure often occurs in the combined detector and estimator for System 2 after a deep fade, from which it does not recover. Thus, regular retraining of the channel estimate must be used. The retraining method used for System 1 is adapted for use here.

### 4.5.1 Model of Retraining Process

The packet structure of both signals $\left\{s_{1 . i}\right\}$, $\left\{s_{2 . i}\right\}$ in the computer simulation tests is shown in Fig.4.5.1. This is exactly the same as that for System 1 in Sec.3.5.1. That is, each frame of $N\left\{s_{1 . i}\right\}$ or $\left\{s_{2 . i}\right\}$ consists of $N-R$ random data symbols preceded by $R$ known training symbols. 10\% retraining is assumed so $R=0.1 N$. Also $R$ is an even number and $R \leqslant 12$. Additionally here, the corresponding packets from the two mobiles arrive perfectly synchronized in time at the receiver.

Retraining methods are only described here for the channels $\left\{y_{1 . i}\right\}$, $\left\{y_{2 . i}\right\}$. When two receiving antennas are used (System 2B), it is understood that an exactly similar operation is carried out to retrain the estimator for $\left\{y_{3 . i}\right\}$, $\left\{y_{4, i}\right\}$. So, the baseband received samples during the retraining period are $\left\{r_{i}\right\}$ for $i=1,2, \ldots, R$. From Eq. (4.2.1)

$$
\begin{equation*}
r_{i}=s_{1 . i} y_{1, i}+s_{2 . i} y_{2 . i}+w_{i} \tag{4.5.1}
\end{equation*}
$$

and the $\left\{s_{1, i}\right\},\left\{s_{2, i}\right\}$ for $i=1,2, \ldots, R$ are known at the receiver. The particular sequences used for the two training signals are different. They have been chosen to satisfy certain conditions, so are described later.

It is very important that the retraining method is reliable because if the channel estimators are badly retrained, it is quite probable that the whole of the following data packet would be lost. The "best" retraining method is defined here as that which results in the lowest bit error rate in detection for System 2. This best method should also give the minimum mean-square error in the estimates of both channels and their slopes at the end of the retraining burst. That is, at the start of data transmission.

The methods tested for retraining the channel estimator of Sec.4.4.4 are now described. Only the degree-1 fading memory predictor is tested. (The other predictors in Table 3.4 .1 were discarded in Chapter 3.) The results of these tests are shown in Chapter 6 .

### 4.5.2 Ideal Retraining

With ideal retraining and the packet structure shown in Fig.4.5.1

$$
\begin{align*}
& Y^{\prime}{ }_{1 . R}=Y_{1 . R}, \quad Y^{\prime}{ }_{2 . R}=Y_{2 . R} \\
& Y^{\prime}{ }_{1, R+1, R}=Y_{1 . R+1} \text {. } \\
& \dot{Y}_{1, R+1, R}=Y_{1, R+1}-Y_{1 . R}, \quad \quad \dot{Y}_{2, R+1, R}=Y_{2 . R+1}-y_{2 . R} \tag{4.5.2}
\end{align*}
$$

The Gradient estimator of Sec.4.4.4 is restarted with the ideal values shown in Eq.(4.5.2). This is used as a benchmark by which the actual retraining method is compared.


Fig.4.5.1 Packet structure used in computer simulation tests

(b)

Fig.4.5.2 Example of real or imaginary part of $\left\{y_{1 . j}\right\}$ or $\left\{y_{2 . i}\right\}$ and its estimate during retraining

| $x$ | $=$ real or imaginary part of channel |
| ---: | :--- |
|  | $=$ raw estimate |
|  | $=$ least-squares straight line |

### 4.5.3 Least-Squares Retraining

The method considered here uses the same basic three-stage process as for System 1 in Sec.3.5.5. Stages 2 and 3 are very nearly the same as for System 1. Again, the channel samples over the duration of the training signal lie on a smooth curve as shown in Fig.4.5.2. Here, the least-squares straight line through the $R$ raw estimates of the channel, (calculated in Stage 2), runs almost parallel to the tangent of the curve $\left\{y_{i}\right\}$ in the middle of the retraining packet. The main problem here is in Stage 1, because there is no simple unbiased estimator for System 2 that will give the initial raw estimates of the channels. Four different methods of obtaining these raw measurements are investigated. The first. method involves simply switching one transmitter off at a time so the received samples reduce to the same form as for System 1. The last three methods were originally proposed by A.P.Clark [123] for tracking fast fading channels. These four methods are now considered in detail.

Method (1): Switch off one transmitter to estimate the channel of the other signal.

The training sequences used here are shown in Table 4.5.1. There are an even number of retraining symbols, $R$, where $R \leqslant 12$. For all odd numbered training symbols $i=1,3, \ldots, R-1, s_{2 . i}=0$. From Eq. (4.5.1)

$$
\begin{equation*}
r_{i}=s_{1 . i} y_{1 . i}+w_{i} \tag{4.5.3}
\end{equation*}
$$

$s_{1 . i}$ is known at the receiver, so

$$
\begin{equation*}
x_{1, i}=s_{1, i}{ }^{-1} r_{i}=y_{1, i}+s_{1, i}{ }^{-1} w_{i} \tag{4.5.4}
\end{equation*}
$$

$x_{1 . i}$ is the unbiased estimate of $y_{1 . i}$ and is taken to be the raw measurement of $y_{1 . i}$.

Similarly, $s_{1, i}=0$ for the even numbered training symbols $i=2,4, \ldots, R$. So from Eq.(4.5.1)

$$
\begin{equation*}
r_{i}=s_{2 . i} Y_{2 . i}+w_{i} \tag{4.5.5}
\end{equation*}
$$

$s_{2 . i}$ is known at the receiver, so

$$
\begin{equation*}
x_{2, i}=s_{2, i}{ }^{-1} r_{i}=y_{2 . i}+s_{2, i}{ }^{-1} w_{i} \tag{4.5.6}
\end{equation*}
$$

$x_{2 . i}$ is the unbiased estimate of $y_{2 . i}$ and is taken to be the raw measurement of $Y_{2 . i}$.

It was shown in Sec.3.4 that the mean-square error in $N$ unbiased
estimates $\left\{x_{i}\right\}$ is

$$
\begin{align*}
\lambda_{e} & =\frac{1}{N} \sum_{i=1}^{N}\left|y_{1 . i}-x_{1 . i}\right|^{2}=\frac{1}{N} \sum_{i=1}^{N}\left|s_{1 . i}{ }^{-1} w_{i}\right|^{2} \\
& =\sigma^{2}=\frac{1}{2 \psi} \tag{4,5,7}
\end{align*}
$$

Table 4.5.1 Training signal used in least-squares Method (1)

| $i$ | $s_{1, i}$ | $s_{2 . i}$ |
| :---: | :---: | :---: |
| 1 | $-1-j$ | 0 |
| 2 | 0 | $-1+j$ |
| 3 | $+1+j$ | 0 |
| 4 | 0 | $+1-j$ |
| 5 | $-1-j$ | 0 |
| 6 | 0 | $-1+j$ |
| 7 | $+1+j$ | 0 |
| 8 | 0 | $+1-j$ |
| 9 | $-1-j$ | 0 |
| 10 | 0 | $-1+j$ |
| 11 | $+1+j$ | 0 |
| 12 | 0 | $+1-j$ |

Table 4.5.2 Training signal used in least-squares Methods (2) and (3)

| $i$ | $s_{1 . i}$ | $s_{2 \cdot i}$ |
| :---: | :---: | :---: |
| 1 | $-1-j$ | $-1+j$ |
| 2 | $+1-j$ | $+1+j$ |
| 3 | $+1+j$ | $+1-j$ |
| 4 | $-1+j$ | $-1-j$ |
| 5 | $-1-j$ | $-1+j$ |
| 6 | $+1-j$ | $+1+j$ |
| 7 | $+1+j$ | $+1-j$ |
| 8 | $-1+j$ | $-1-j$ |
| 9 | $-1-j$ | $-1+j$ |
| 10 | $+1-j$ | $+1+j$ |
| 11 | $+1+j$ | $+1-j$ |
| 12 | $-1+j$ | $-1-j$ |

Where $\psi=E_{b} / N_{0}$ as defined in Eq. (2.5.3). When fitting a least-squares straight line to these points in Stage 2 the mean-square errors in the estimates of the quadrature components of the channel and slope are approximately given by Eq. (3.5.26) and Table 3.5 .2 and respectively. Now of course, only $R / 2$ raw measurements of each channel can be formed from the $R$ retraining samples. This causes a degradation in these least-squares estimates of channel and slope compared with System 1 - where $R$ raw estimates are obtained from $R$ retraining symbols.

## Method (2): "Slow Fading" Assumption

The estimator assumes that

$$
\begin{equation*}
y_{1 . i}=y_{1 . i-1} \quad \text { AND } \quad y_{2 . i}=y_{2 . i-1} \tag{4.5.8}
\end{equation*}
$$

To estimate $Y_{1 . i}$ it is necessary to remove $Y_{2 . i}$ from the received sample $r_{i}$. This can be achieved by operating on the two received samples $r_{i}$, $r_{i-1}$. It is shown in Appendix $G[123]$ that when Eqs. (4.5.1) and (4.5.8) both hold and $s_{1 . i}, s_{1, i-1}, s_{2 . i}, s_{2 . i-1}$ are known at the receiver, a good estimate of $y_{1 . i}$ from $r_{i}, r_{i-1}$ is given by

$$
\begin{equation*}
x_{1, i}=a_{1 . i}{ }^{-1} p_{1 . i} \tag{4.5.9}
\end{equation*}
$$

where

$$
\begin{equation*}
a_{1, i}=s_{2 . i}{ }^{-1} s_{1, i}-s_{2, i-1}{ }^{-1} s_{1, i-1} \tag{4.5.10}
\end{equation*}
$$

and

$$
\begin{equation*}
p_{1 . i}=s_{2 . i}{ }^{-1} r_{i}-s_{2 . i-1}{ }^{-1} r_{i-1} \tag{4.5.11}
\end{equation*}
$$

Similarly, a good estimate of $y_{2 . i}$ is given by

$$
\begin{equation*}
x_{2 . i}=a_{2 . i}^{-1} p_{2 . i} \tag{4.5.12}
\end{equation*}
$$

where

$$
\begin{equation*}
a_{2 . i}=s_{1 . i}{ }^{-1} s_{2 . i}-s_{1 . i-1}{ }^{-1} s_{2 . i-1} \tag{4.5.13}
\end{equation*}
$$

and

$$
\begin{equation*}
p_{2 . i}=s_{1 . i}{ }^{-1} r_{i}-s_{1, i-1}{ }^{-1} r_{i-1} \tag{4.5.14}
\end{equation*}
$$

This retraining estimator obtains $R-1\left\{x_{1, i}\right\}$ and $\left\{x_{2, i}\right\}$ (for $i=2,3, \ldots, R-1)$. Estimates at $i=1$ cannot be obtained because $s_{1.0}$ and $s_{2.0}$ are not known.

Also in Appendix $G$, it is shown that

$$
\begin{equation*}
x_{1, i}=y_{1, i}+a_{1, i}^{-1} u_{1, i}+c_{1, i} \tag{4.5.15}
\end{equation*}
$$

Where $C_{1 . i}$ is the error in $x_{1 . i}$ caused by the curvature in the channel $\left\{y_{1 . i}\right\}$ (see Fig.4.5.2): Clearly, $C_{1 . i}=0$ if Eq. (4.5.8) holds true. $a_{1 . i}{ }^{-1} u_{1 . i}$ is the error in $x_{1, i}$ caused by the additive noise, where

$$
\begin{equation*}
u_{1, i}=s_{2 . i}{ }_{1_{w_{i}}^{i}}^{i}-s_{2, i-1}^{-1} w_{i-1} \tag{4.5.16}
\end{equation*}
$$

Now, these channel estimates are examined theoretically to see which
particular training sequences $\left\{s_{1 . i}\right\},\left\{s_{2 . i}\right\}$ minimize the mean-square errors in the estimates. The complex-valued noise components $w_{i}, w_{i-1}$ are statistically independent with zero mean and a fixed variance. Thus, the mean-square value of $u_{1 . i}$ is independent of the values ( $\pm 1 \pm j$ ) of the data symbols $s_{1 . i}, s_{1 . i-1}$. So for the minimum mean-square error in $x_{1 . i}$, $\left|a_{1 . i}\right|$ must be maximized which means that

$$
\begin{equation*}
s_{2, i-1}{ }^{-1} s_{1, i-1}=-s_{2 . i}{ }^{-1} s_{1 . i} \tag{4.5.17}
\end{equation*}
$$

The mean-square value of $a_{1 . i} u_{1 . i}$ is shown in Appendix $G$ to be $\sigma^{2 / 2=1 / 4 \psi}$, where $\psi=E_{b} / N_{0}$ (or $\sigma^{2} / 2=-(\psi+6) d B$, where $\psi=\log _{10}\left(E_{b} / N_{0}\right.$ )dB). In fact, with the training signals chosen to satisfy Eq. (4.5.17), the mean-square values of both $a_{1 . i}{ }^{-1} u_{1 . i}$ in $x_{1 . i}$ and $a_{2 . i}{ }^{-1} u_{2 . i}$ in $x_{2 . i}$ are minimized. The chosen training signals in Table 4.5 .2 satisfy Eq.(4.5.17), and at the same time enable effective symbol timing to be achieved over the duration of the fading signal. Details of the symbol timing are discussed in Appendix $C$.

So, the mean-square errors in the $\left\{x_{1 . i}\right\},\left\{x_{2 . i}\right\}$ are seen to be 3 dB better than for Method (1), if the assumption of slow fading (Eq.(4.5.8)) is an accurate one. However, the noise components $a_{1 . i}{ }^{-1} u_{1 . i}$ and $a_{1 . i-1}{ }^{-1}{ }_{u_{1 . i-1}}$ in adjacent samples $x_{1 . i}$ and $x_{1 . i-1}$ are correlated since two received samples are used to estimate each channel value. So when the least-squares straight lines are fitted to the $R-1\left\{x_{1 . i}\right\},\left\{x_{2 . i}\right\}$ in Stage 2 , the resulting estimates of the channels are not as accurate as if the R-1 raw measurements were uncorrelated.

## Method (3): "Fast Fading" Assumption -

The estimator assumes that

$$
\begin{align*}
& y_{1 . i+1}-y_{1 . i}
\end{align*}=y_{1 . i}-y_{1 . i-1}, ~\left(y_{2 . i+1}-y_{2 . i}=y_{2 . i}-y_{2 . i-1} .\right.
$$

To estimate $y_{1 . i}$ it is necessary to remove $y_{2 . i}$ from the received sample $r_{i}$. This can be achieved by operating on the three received samples $r_{i-1}$, $r_{i}, r_{i+1}$. It is shown in Appendix $G[68,123]$ that when Eqs.(4.5.1) and (4.5.18) both hold and $s_{1 . i-1}, s_{1 . i}, s_{1 . i+1}, s_{2 . i-1}, s_{2 . i}, s_{2 . i+1}$ are all known at the receiver, a good estimate of $y_{1 . i}$ from $r_{i-1}, r_{i}, r_{i+1}$ is given by

$$
\begin{equation*}
x_{1 . i}=a_{1 . i}{ }^{-1} p_{1 . i} \tag{4.5.19}
\end{equation*}
$$

where

$$
\begin{equation*}
p_{1 . i}=2 s_{2 . i}{ }^{-1} r_{i}-s_{2 . i-1}{ }^{-1} r_{i-1}-s_{2 . i+1}{ }^{-1} r_{i+1} \tag{4.5.20}
\end{equation*}
$$

and

$$
\begin{equation*}
a_{1 . i}=4 s_{2 . i}{ }^{-1} s_{1 . i} \tag{4.5.21}
\end{equation*}
$$

and

$$
\begin{equation*}
s_{2, i-1}{ }^{-1} s_{i, i-1}=s_{2, i+1}{ }^{-1} s_{1, i+1}=-s_{2, i}{ }^{-1} s_{1, i} \tag{4.5.22}
\end{equation*}
$$

Similarly, a good estimate of $y_{2}$. $i$ is given by

$$
\begin{equation*}
x_{2, i}=a_{2 . i}^{-1} p_{2, i} \tag{4.5.23}
\end{equation*}
$$

where

$$
\begin{equation*}
p_{2 . i}=2 s_{1 . i}{ }^{-1} r_{i}-s_{1 . i-1}{ }^{-1} r_{i-1}-s_{1 . i+1}{ }^{-1} r_{i+1} \tag{4.5.24}
\end{equation*}
$$

and

$$
\begin{equation*}
a_{2 . i}=4 s_{1 . i}{ }^{-1} s_{2 . i} \tag{4.5.25}
\end{equation*}
$$

and

$$
\begin{equation*}
s_{1, i-1}-^{1} s_{2, i-1}=s_{1, i+1}{ }^{-1} s_{2, i+1}=-s_{1, i}{ }^{-1} s_{2, i} \tag{4.5.26}
\end{equation*}
$$

Of course, Eq. (4.5.26) must be true if Eq. (4.5.22) is true. This
retraining estimator obtains $R-2$ estimates $\left\{x_{1, i}\right\}$, $\left\{x_{2, i}\right\}$ for $i=2,3, \ldots, R-1$. Estimates at $i=1$ and $R$ cannot be obtained because $s_{1.0}$, $s_{2.0} s_{1 . R+1}, s_{2 . R+1}$ are not known.

Also in Appendix $G$ it is shown that

$$
\begin{equation*}
x_{1, i}=y_{1, i}+a_{1, i}{ }^{-1} u_{1, i}+c_{1, i} \tag{4.5.27}
\end{equation*}
$$

Where $C_{1 . i}$ is the error caused by the curvature in the channel $\left\{y_{1 . i}\right\}$ (see Fig.4.5.2) Clearly, $c_{1 . i}=0$ if Eq. (4.5.18) holds true. $a_{1 . i}{ }^{-1} u_{1 . i}$ is the error in $x_{1}, i$ caused by the additive noise, where

$$
\begin{equation*}
u_{1 . i}^{1 . i}=2 s_{2 . i}{ }^{-1} w_{i}-s_{2 . i+1}{ }^{-1} w_{i+1}-s_{2 . i-1}{ }^{-1} w_{i-1} \tag{4.5.28}
\end{equation*}
$$

Now, the channel estimates are examined theoretically to see what their mean-square errors are likely to be. The mean-square values of both $a_{1, i}{ }^{-1} u_{1, i}$ in $x_{1, i}$ and $a_{2 . i}{ }^{-1} u_{2, i}$ in $x_{2 . i}$ are shown in Appendix $G$ to be $3 \sigma^{2} / 8=3 / 16 \psi$, where $\psi=\mathrm{E}_{\mathrm{b}} / \mathrm{N}_{0}=1 / 2 \sigma^{2}$ (or $3 \sigma^{2} / 8=-(\psi+7.3$ ) dB, where $\left.\psi=10 \log _{10}\left(E_{b} / N_{0}\right) d B\right)$. The chosen training signals in. Table 4.5 .2 satisfy Eq. (4.5.22), and at the same time enable effective symbol timing to be achieved over the duration of the fading signal. Details of the symbol timing are discussed in Appendix C.

So, the mean-square error in $x_{1 . i}$ and $x_{2 \text {. }}$ is seen to be about 4.3 dB better than in Method (1) and about 1.3dB better than in Method. (2). This assumes of course that the fast fading assumption of Eq. (4.5.18) is an accurate one. It is clearly more accurate than the slow fading assumption (Eq.(4.5.8)) of Method (2). However, the noise components $a_{1 . i-1}{ }^{-1} u_{1 . i-1}$, $a_{1 . i}{ }^{-1} u_{1, i}, a_{1, i+1}{ }^{-1} u_{1, i+1}$ in adjacent estimates $x_{i-1}, x_{i}, x_{i+1}$ are all correlated, since three received samples are used to estimate each channel value. So when the least-squares straight lines are fitted to the $\mathrm{R}-2$ $\left\{x_{1, i}\right\},\left\{x_{2, i}\right\}$ in Stage 2 , the resulting estimates of the channels are not
as accurate as if these $\mathrm{R}-2$ raw estimates were uncorrelated.

## Method (4): "Very Fast Fading" Assumption

The estimator assumes that

$\left(y_{2 . i+1}-y_{2 . i}\right)-\left(y_{2 . i}-y_{2 . i-1}\right)=\left(y_{2 . i}-y_{2 . i-1}\right)-\left(y_{2 . i-1}-y_{2 . i-2}\right)$
It is shown in Appendix $G[68,123]$ that when Eqs.(4.5.1) and (4.5.29) both
 are all known at the receiver, a good estimate of $y_{1 . i}$ from $r_{i-2}, r_{i-1}$, $r_{i}, r_{i+1}$ is given by

$$
\begin{equation*}
x_{1, i}=a_{1, i}{ }^{-1} p_{1, i} \tag{4.5.30}
\end{equation*}
$$

where

$$
\begin{equation*}
{ }^{p_{1 . i}}=3 s_{2 . i}{ }^{-1} r_{i}-s_{2 . i+1}{ }^{-1} r_{i+1}-3 s_{2 . i-1}{ }^{-1} r_{i-1}+s_{2 . i-2}{ }^{-1} r_{i-2} \tag{4.5.31}
\end{equation*}
$$

$$
\begin{equation*}
a_{1 . i}=8 s_{1 . i}{ }^{-1} s_{2 . i} \tag{4.5.32}
\end{equation*}
$$

and

$$
\begin{align*}
s_{2 . i}{ }^{-1} s_{1 . i}=-s_{2 . i-2}{ }^{-1} s_{1 . i-2}=-s_{2 . i-1}{ }^{-1} s_{1 . i-1} & \\
& =s_{2 . i+1}{ }^{-1} s_{1 . i+1} \tag{4.5.33}
\end{align*}
$$

The estimate of $y_{2.1}$ is determined in a similar manner. Also in Appendix $G$ it is shown that

$$
\begin{equation*}
x_{1 . i}=y_{1 . i}+a_{1 . i}{ }^{-1} u_{1 . i}+c_{1 . i} \tag{4.5.34}
\end{equation*}
$$

Where $C_{1 . i}$ is the error caused by the curvature in the channel $\left\{y_{1 . i}\right\}$ (see Fig.4.5.2). Clearly, $C_{1 . i}=0$ if Eq. $(4.5 .19)$ holds true. $a_{1 . i}{ }^{-1} u_{1 . i}$ is the error in $x_{1 . i}$ caused by the additive noise and

$$
\begin{align*}
& u_{1 . i}=3 s_{2 . i}{ }^{-1} w_{i}-s_{2 . i-2 \ldots}{ }^{-1} w_{i-2}-3 s_{2 . i-1}{ }^{-1} w_{i-1} \\
&-s_{2 . i+1}{ }^{-1} w_{i+1} \tag{4.5.35}
\end{align*}
$$

Now, the channel estimates are examined theoretically to see what their mean-square errors are likely to be. The mean-square values of both $a_{1 . i}{ }^{-1} u_{1 . i}{ }^{\text {in }} x_{1 . i}$ and $a_{2 . i}{ }^{-1} u_{2 . i}$ in $x_{2 . i}$ are shown in Appendix $G$ to be $5 \sigma^{2} / 16=5 / 32 \psi$, where $\psi=E_{b} / N_{0}=1 / 2 \sigma^{2}$ (or $-(\psi+8.1) \mathrm{dB}$, where $\psi=10 \log _{10}\left(E_{b} / N_{0}\right)$ $\mathrm{dB})$. This is better than for Method (3) by only about 0.7dB.

So, although the very fast fading assumption (Eq.(4.5.29)) is more accurate than the "fast fading" assumption (Eq.(4.5.18)), this method is very unlikely to give a great improvement in the estimates $x_{1 . i} x_{2 . i}$ over Method (3) that would justify the extra complexity. Another problem with this method is that the restriction on the values of $\left\{s_{1 . i}\right\},\left\{s_{2 . i}\right\}$ (Eq.(4.5.33)) make it impossible to choose a training sequence that
satisfies these conditions for two adjacent symbols. Also, by using four received samples in every estimate, this complicated estimation process now gives highly correlated estimates which would degrade the least-square straight line fitting of Stage 2. Only estimates $\left\{x_{1 . i}\right\}$ more than four symbols apart are uncorrelated. In view of these above reasons, this method is most unlikely to give any improvement over Method (3) so is not tested in this investigation.

The question that still remains is, which of Methods (1), (2) or (3) should be used in the retraining process of System 2? Clearly, Method (3) will give more accurate raw measurements $\left\{x_{1, i}\right\},\left\{x_{2, i}\right\}$ than Method (2) for two reasons. Firstly, the error caused by the additive noise component $\left\{w_{1}\right\}$ is about 1.3dB lower for Method (3) than for Method (2). Secondly, the error caused by the curvature in the channel samples will be lower, with the fast fading assumption of Method (3), than with the slow fading assumption of Method (2). In fact, experience with the unbiased estimator of System 1 has shown that a slow fading assumption of Eq. (4.5.8) is likely to result in an unacceptable irreducible error rate at high signal-to-noise ratios, with the fast fading rates experienced here. So no tests are carried out on Method (2).

Also, Method (1) is discarded in preference to Method (3), for the following reasons. Firstly, the error caused by the additive noise components $\left\{w_{i}\right\}$ is about 4.3dB lower for Method (3) than for Method (1). Although Method (3) relies on thet accuracy of the fast fading assumption (Eq.(4.5.18)), the mean-square error in the raw measurements $x_{1 . i} \cdot x_{2 . i}$ is still at least 4 dB better for Method (3). Secondly, $\mathrm{R}-2$ raw measurements $\left\{x_{1, i}\right\},\left\{x_{2, i}\right\}$ can be found using Method (3) against only $R / 2$ for Method (1). So, although there is some correlation in the raw measurements of Method (3), the least-squares straight line fitted to these measurements should generally give a much better result than for Method (1). Thirdly, the training signals used with Method (3) shown in Table 4.5.2 enable effective symbol timing to be achieved over the duration of the fading signal, (see Appendix G). Whereas, for Method (I), the insertion of alternate zeros in the training signals would certainly cause errors in the symbol timing recovery at the receiver. So no tests are carried out on Method (1).

The three-stage retraining process for System 2 now procedes as follows.

Stage 1: Firstly, the raw measurements of the channels are formed for the entire retraining signal.

That is, Eqs.(4.5.19)-(4.5.26) are executed for $i=2,3, \ldots, R-1$ to give the $R-2\left\{x_{1, i}\right\},\left\{x_{2 . i}\right\}$.

Stage 2: Secondly, from these raw measurements the receiver forms the least-squares estimates of both channels and their slopes in the centre of the retraining packet.

The least squares estimate of the rate of change of $y_{1 . i}$ with $i$, as $i$ increases from $\frac{1}{2} R$ to $\frac{1}{2} R+1$ is

$$
\begin{equation*}
\dot{y}_{1 . D}^{\prime}=\frac{\frac{1}{R-2} \sum_{i=2}^{R-1} i x_{1 . i}-\frac{1}{R-2} \sum_{i=2}^{R-1} i x^{R-2} \sum_{i=2}^{R-1} x_{1 . i}}{\frac{1}{R-2} \sum_{i=2}^{R-1} i^{2}-\left(\frac{1}{R-2} \sum_{i=2}^{R-1}\right)^{2}} \tag{4.5.36}
\end{equation*}
$$

where

$$
\begin{equation*}
D=\frac{1}{2} R+1 \tag{4.5.37}
\end{equation*}
$$

and $R$ is an even number.
The least-squares estimate of the channel at $i=D=\frac{1}{2} R+1$ is

$$
\begin{equation*}
y_{1 . D}^{\prime}=\frac{1}{R-2} \sum_{i=2}^{R-1} x_{1 . i}+\frac{1}{2} \dot{y}_{1 . D} \tag{4.5.38}
\end{equation*}
$$

In practice, the number of retraining symbols $R$ is known beforehand, and therefore the terms $\sum_{i}$ and $\sum_{i}{ }^{2}$ in the denominator of Eq.(4.5.36) would also be known. So the general equation of slope in Eq.(4.5.36) can be greatly simplified with no need for divisions. For example, if $\mathrm{R}=12$,

$$
\begin{equation*}
\dot{y}_{1.7}^{\prime}=0.01212 \sum_{i=2}^{R-1} i x_{1 . i}-0.07879 \sum_{i=2}^{R-1} x_{1 . i} \tag{4.5.39}
\end{equation*}
$$

Estimate of $\dot{Y}^{\prime}{ }_{2 . \frac{1}{2} R+1}, Y^{\prime}{ }_{2 . \frac{1}{2} R+1}$ are determined in a similar manner.
Stage 3: Finally, these estimates of the two channels and their slopes are used to initialize the Gradient estimator with degree-1 fading memory predictor (Sec.4.4.4), which is restarted at this point $i=D=\frac{1}{2} R+1$.

The estimator/predictor is initialized with

$$
\begin{array}{ll}
\dot{y}_{1 . D, D-1}=\dot{Y}_{1}^{\prime}{ }_{1 . D}, & \dot{y}_{2 . D, D-1}^{\prime}=\dot{y}^{\prime}{ }_{2 . D} \\
Y_{1 . D, D-1}^{\prime}=Y_{1 . D}^{\prime}, & y_{2 . D, D-1}^{\prime}=Y_{2 . D}^{\prime} \tag{4.5.40}
\end{array}
$$

The received samples $r_{D}, r_{D+1}, \ldots, r_{R}$ have been stored, so the Gradient estimator is run for $i=D, D+1, \ldots, R$ according to Eqs.(4.4.23)-(4.4.28), with degree-1 polynomial prediction (Table 3.4.1). Now the predictions $Y^{\prime}{ }_{1 . R+1, ~} R^{\prime} \dot{Y}^{\prime}{ }_{1 . R+1, R}{ }^{\prime} Y^{\prime}{ }_{2 . R+1, ~}{ }^{\prime} \dot{Y}^{\prime}{ }_{2 . R+1, R}$ are stored ready for the arrival of the first data symbol $s_{R+1}$.

Eqs.(4.5.36)-(4.5.39) used here to estimate the channel and its slope have been derived in Appendix G. Their performance has been analysed theoretically in Sec.3.5.5. The mean-square errors in these channel and slope estimates will not be quite as good here as for System 1. This is because there is some correlation in the additive noise components of the $\left\{x_{1, i}\right\},\left\{x_{2, i}\right\}$.

This three-stage retraining process is tested by computer simulation for $R \leqslant 12$ with different signal-to-noise ratios and a fixed simulated vehicle speed of $60 \mathrm{miles} /$ hour. The results of these tests are given in Chapter 6. They indicate the expected degradation in performance when using less than 12 retraining symbols. $R=12$ retraining symbols was seen. to give the best results with System 1.

### 4.5.4 Conclusions for Retraining

Regular retraining of the channel estimators can be successfully achieved with $\mathrm{R}=12$ retraining symbols, using the least-squares methods of Sec.4.5.3. Test results are given in Chapter 6 with all correct data symbols fed back into the estimator. These results indicate that the estimator with this retraining method loses about 1 dB in tolerance to noise compared to the estimator with ideal retraining. If more retraining symbols could be used, then it should be possible to improve on this performance. Unfortunately, the maximum of $R=12$ was imposed by hardware restrictions in the prototype modem [92].

### 4.6 Combined Detection and Estimation

The maximum likelihood detector described in Sec.4.3.2 in now combined with the Gradient estimator described in Sec.4.4.4. The estimator uses a degree-1 fading memory predictor (Table 3.4.1) and is regularly retrained using the method described in Sec.4.5.3, with $R=12$ retraining symbols. This is the combined detector and estimator for System 2 that performs coherent demodulation at the receiver. It is shown in Fig.4.6.1.

Computer simulation tests in Chapter 6 have shown that this combined detector and estimator is inherently unstable. This is in marked contrast to System 1 which was seen to perform well even without retraining. To explain this, consider an eroneous detection of the symbol $s_{1 . i}$ in System 2 A caused by the channel $\mathrm{y}_{1 . i}$ being in a deep fade. This error causes a large error in the estimate of the channel $y_{1, i}$, and a corresponding large


Fig.4.6.1 Simple combined detector and estimator for
(a) System 2A
(b) System 2B


Fig.4.6.2 Final combined detector and estimator for (a) System 2A (b) System 2B
error in the prediction $y^{\prime}{ }_{1 . i+1, i}$. This further increases the probability of error in the detection of $s_{1 . i+1}$, and so on. A fading channel in a. deep fade could cause a burst of errors several symbols long, which would result in predicted channel values far removed from the channel's actual in-phase and quadrature components. Consequently correct detection is now highly improbable, causing an avalanche of erroneous predictions and detected symbols. Since the signals from the two mobiles have been added in the channel and must therefore be considered as one single 16 -point signal at the receiver, collapse of one of the signals soon ensures the collapse of the second. Random data is output and the channel predictors follow completely random paths. Correct operation resumes only after the next retraining burst when the channel predictions are correctly reset. However, during correct operation of the combined detector and estimator of System 2A, shifts in the channel estimators for $y_{1 . i}$ or $y_{2 . i}$ of $\pm 90^{\circ}$ or $180^{\circ}$ occur quite often (especially just after a deep fade), with the corresponding shift in the subsequent detected symbol values $\left\{s^{\prime}{ }_{1 . j}\right\}$ or \{s'2.i\}. This is corrected for automatically by the differential coding. So correct operation is only possible here with DQPSK modulation. The QPSK modulation (without differential coding) cannot be used - this was also found to be the case for System 1.

Computer simulation tests indicate that on average, without retraining, total system collapse can be expected to occur in this System 2 A , only after about 400 symbols duration. Clearly, retraining is essential for acceptable performance of this system. Effective retraining every 120 symbols ensures that very long bursts of errors are avoided here.

However, simulation tests on System 2B indicate that a similar total system collapse is expected after about 12000 symbols duration. This marked improvement in the stability of the..system is due to the two facts discussed in Sec.4.3.2. That is, firstly, with two receiving antennas errors caused by signal points overlapping are very much reduced. Secondly, errors caused by deep fades are very much less likely with uncorrelated fading at the two receiving antennas than they are with fading at one antenna. So, with effective retraining every 120 symbols, very long bursts of errors are avoided in this System 2B.

Clearly, any method which can reduce the error extension effects caused by feeding back incorrectly detected symbols into the channel estimator is bound to give a noticeable improvement in performance,
especially for System 2A. Hence, the Viterbi-type detector for System 2 is now investigated.

### 4.6.1 Viterbi-type Detection

The weakness of the estimation process used here is that it relies very heavily on the correct detection of the data symbols. The Viterbi-type detector described in Sec.3.6.1 overcomes this weakness by permitting the estimator to consider simultaneously several different possible values of each detected data symbol: This technique uses the Viterbi-type detection algorithm [120] in a novel way to improve the tolerance to noise of this combined detection and estimation process. It is interesting to note that this Viterbi algorithm does not improve the performance of the detector or the estimator when they are tested separately as in Secs.4.3 and 4.4.

The block diagram of this final combined Viterbi-type detector and estimator is shown in Fig.4.6.2. The way it works for System 1 has been described in Sec.3.6.1. Rather than repeat this lengthy description here, the reader is referred to Sec.3.6.1 - but the following changes must be made to the equations for the description in Sec.3.6.1 to apply to System 2:
(i) Eqs.(3.6.1)-(3.6.2) are replaced by Eqs.(4.6.1)-(4.6.2). Thus, the received sample at the output of the receiver lowpass filter of system 2 A is given by

$$
\begin{equation*}
r_{i}=s_{1 . i} y_{1 . i}+s_{2 . i} y_{2 . i}+w_{i} \tag{4.6.1}
\end{equation*}
$$

Similarly, for System 2B

$$
\begin{align*}
& r_{a . i}=s_{1 . i} y_{1 . i}+s_{2 . i} y_{2 . i}+w_{a . i} \\
& r_{b_{. i}}=s_{1 . i} y_{3 . i}+s_{2 . i} y_{4 . i}+w_{b . i} \tag{4.6.2}
\end{align*}
$$

(ii) $q_{i}$ in Eq. (3.6.3) is now assumed to be an ( $M=16$ ) -level composite data symbol, given by the two-component vector

$$
q_{i}=\left[\begin{array}{ll}
q_{1, i} & q_{2, i} \tag{4.6.3}
\end{array}\right]
$$

where $q_{1 . i}$ and $q_{2 . i}$ take on possible values of $s_{1 . i}$ and $s_{2 . i}$ respectively. Thus, $q_{i}$ has $M=16$ different possible values corresponding uniquely to the 16 different possible combinations of $s_{1 . i}$ and $s_{2 . i}$ (see Table 4.3.1). So now, the vector $Q_{i-1}=\left[q_{R+1} q_{R+2} \ldots q_{i-1}\right]$ in Eq.(3.6.3) represents a pair of sequences.

$$
\left[s^{\prime} 1_{1 . R+1} s^{\prime}{ }_{1 . R+2} \quad \cdots, s_{1 . i-1}^{\prime}\right]
$$

and

$$
\left[\begin{array}{llll}
s^{\prime} & s^{\prime} R+1 & s^{\prime}, R+2 & \cdots \tag{4.6.4}
\end{array} s^{\prime}{ }_{2, i-1}\right]^{\prime}
$$

(iii) The costs $\left\{c_{i}\right\}$ are determined from their maximum likelihood
distances $\left\{\mathrm{d}_{i}{ }^{2}\right\}$. So, for System 2A, Eq. (3.6.5) is replaced by

$$
\begin{equation*}
c_{i}=c_{i-1}+\left|r_{i}-q_{1, i} y_{1, i, i-1}^{\prime}-q_{2, i} y_{2, i, i-1}^{\prime}\right|^{2} \tag{4.6.5}
\end{equation*}
$$

For System 2B, Eq.(3.6.6) is replaced by

$$
\begin{align*}
c_{i}=c_{i-1}+\mid r_{a . i} & -q_{1 . i} y_{1 . i, i-1}^{\prime}-\left.q_{2 . i} y_{2 . i, i-1}^{\prime}\right|^{2} \\
& +\left|r_{b . i}-q_{1 . i} y_{3 . i, i-1}^{\prime}-q_{2 . i} Y_{4 . i, i-1}^{\prime}\right|^{2} \tag{4.6.6}
\end{align*}
$$

Thus, for System 2A, Eq.(3.6.7) is replaced by

$$
\begin{equation*}
c_{i-1}=\sum_{n=R+1}^{i-1} \mid r_{n}-q_{1, n^{\prime}} y_{1, n, n-1}^{\prime}-q_{2, n^{\prime}}^{\left.y_{2, n, n-1}^{\prime}\right|^{2}} \tag{4.6.7}
\end{equation*}
$$

and for System 2B, Eq. (3.6.8) is replaced by
(iv) The Gradient estimator for System 2 is as described in Sec.4.4.4. So Eqs.(3.6.9)-(3.6.14) are replaced by Eqs.(4.6.9)-(4.6.18) below.

$$
\begin{align*}
r_{i}^{\prime} & =q_{1, i} y^{\prime}{ }_{1 . i, i-1}+q_{2, i} y^{\prime} 2 . i, i-1  \tag{4.6.9}\\
e_{i} & =r_{i}-r_{i}^{\prime} \tag{4.6.10}
\end{align*}
$$

Now,

$$
\begin{align*}
& y_{1 . i}^{\prime}=y_{1 . i, i-1}^{\prime}+b e_{i} q_{1 . i}{ }^{*}  \tag{4.6.11}\\
& \mathrm{E}_{1 . i}=y^{\prime}{ }_{1 . i}{ }^{-y^{\prime}}{ }_{1 . i, i-1}  \tag{4.6.12}\\
& \dot{y}^{\prime}{ }_{1, i+1, i}=\dot{y}^{\prime}{ }_{1 . i, i-1}+(1-\theta)^{2} E_{1 . i}  \tag{4.6.13}\\
& y_{1, i+1, i}^{\prime}=y_{1, i, i-1}^{\prime}+\dot{y}_{1, i+1, i}+\left(1-\theta^{2}\right) E_{1 . i} \tag{4.6.14}
\end{align*}
$$

And

$$
\begin{align*}
y_{2 . i}^{\prime} & =y^{\prime} 2 . i, i-1+b e_{i} q_{2 . i}{ }^{*}  \tag{4.6.15}\\
E_{2 . i} & =y^{\prime}{ }_{2 . i}-y_{2 . i, i-1}^{\prime}  \tag{4.6.16}\\
\dot{y}_{2, i+1, i}^{\prime} & =\dot{y}_{2 . i, i-1}^{\prime}+(1-\theta)^{2} E_{2 . i}  \tag{4.6.17}\\
y_{2, i+1, i}^{\prime} & =y_{2 . i, i-1}^{\prime}+\dot{y}_{2, i+1, i}^{\prime}+\left(1-\theta^{2}\right) E_{2 . i} \tag{4.6.18}
\end{align*}
$$

As for System 1, a simplification can be made to this algorithm since $Y^{\prime}{ }_{1 . i}, Y^{\prime}{ }_{2 . i}$ are not required in the detector. Replace Eqs.(4.6.11)(4.6.12) by

$$
\begin{equation*}
E_{1 . i}=\text { be }_{i} q_{1 . i}^{*} \tag{4.6.19}
\end{equation*}
$$

and replace Eqs.(4.6.15)-(4.6.16) by

$$
\begin{equation*}
\mathrm{E}_{2 . i}=\mathrm{be}_{i} \mathrm{q}_{2 . i}^{*} \tag{4.6.20}
\end{equation*}
$$

However, unlike System 1, a simple unbiased estimator for System 2 does not exist so no further simplifications to this algorithm can be made. The estimates $y^{\prime}{ }_{1 . i+1, i}{ }^{\prime} \dot{y}^{\prime}{ }_{1 . i+1, i}, y^{\prime}{ }_{2 . i+1, i}, \dot{y}^{\prime}{ }_{2 . i+1, i}$ associated, with each vector $Q_{i}$ are stored ready for the detection of $q_{i+1}$. For System 2B, $y^{\prime} 3 . i+1, i \prime \dot{y}_{3 . i+1, i}^{\prime}, y_{4 . i+i, i}^{\prime} \dot{y}_{4 . i+1, i}^{\prime}$ for each vector must also be stored.
(v) The least-squares retraining process is as described in Sec.4.5.3. So

Eq. (3.6.19) is replaced by Eqs.(4.6.21)-(4.6.23) below for the estimate of $Y_{1 . i}$.

$$
\begin{equation*}
x_{1, i}=a_{1, i}{ }^{-1} p_{1, i} \tag{4.6.21}
\end{equation*}
$$

where

$$
\begin{equation*}
p_{1 . i}=2 s_{2 . i}{ }^{-1} r_{i}-s_{2 . i-1}{ }^{-1} r_{i-1}-s_{2 . i+1}{ }^{-1} r_{i+1} \tag{4.6.22}
\end{equation*}
$$

and

$$
\begin{align*}
a_{1 . i} & =4 s_{2 . i}{ }^{-1} s_{1, i} \\
& =2 s_{2 . i}{ }^{-1} s_{1 . i}-s_{2 . i-1}{ }^{-1} s_{1, i-1}-s_{2, i+1}{ }^{-1} s_{1, i+1} \tag{4.6.23}
\end{align*}
$$

Also. for the estimate of $Y_{2 . i}$

$$
\begin{equation*}
x_{2, i}=a_{2 . i}{ }^{-1} p_{2 . i} \tag{4.6.24}
\end{equation*}
$$

where

$$
\begin{equation*}
p_{2 . i}=2 s_{1 . i}{ }^{-1} r_{i}-s_{1 . i-1}{ }^{-1} r_{i-1}-s_{1 . i+1}{ }^{-1} r_{i+1} \tag{4.6.25}
\end{equation*}
$$

and

$$
\begin{align*}
a_{2 . i} & =4 s_{1 . i}{ }^{-1} s_{2 . i} \\
& =2 s_{1 . i}{ }^{-1} s_{2 . i}-s_{1 . i-1}{ }^{-1} s_{2 . i-1}-s_{1 . i+1}{ }^{-1} s_{2 . i+1} \tag{4.6.26}
\end{align*}
$$

The $R-2$ estimates $\left\{x_{1, i}\right\}$ and $\left\{x_{2, i}\right\}$ for $i=2,3, \ldots, R-1$ are obtained in this way.

Now, for the channel $Y_{1}$, Eqs. (3.6.20) and (3.6.21) are replaced by Eqs.(4.6.27) and (4.6.28) below respectively.

$$
\begin{equation*}
\dot{Y}_{1 . \frac{1}{2} R+1, \frac{1}{2} R}=\frac{\frac{1}{R-2} \sum_{i=2}^{R-1} i x_{1 . i}-\frac{1}{R-2}\left(\sum_{i=2}^{R-1} i\right) \times \frac{1}{R-2}\left(\sum_{i=2}^{R-1} x_{1} 1 . i\right)}{\frac{1}{R-2} \sum_{i=2}^{R-1} i^{2}-\left(\frac{1}{R-2} \sum_{i=2}^{R-1} i\right)^{2}} \tag{4.6.27}
\end{equation*}
$$

and

$$
\begin{equation*}
Y_{1 . \frac{1}{2} R+1, \frac{1}{2} R}^{\prime}=\frac{1}{R-2} \sum_{i=2}^{R-\frac{1}{2}} x_{1 . i}+\frac{1}{2} \dot{Y}^{\prime}{ }_{1 . \frac{1}{2} R+1, \frac{1}{2} R} \tag{4.6.28}
\end{equation*}
$$

A similar procedure is carried out on the $\left\{x_{2, i}\right\},\left\{x_{3 . i}\right\},\left\{x_{4 . i}\right\}$ for channels $Y_{2}, Y_{3}, Y_{4}$.

The Systems 2A and 2 B are tested by computer simulation for $m=1,2$ and 4. That is, with either 1,2 or 4 vectors in the Viterbi-type detector. The packet length is $N=120$, with $R=12$ retraining symbols followed by $N-R=108$ random data symbols. The values of $b, \theta$ used in the estimator for any given signal-to-noise ratio are given in Table 6.4.1. These have been found to roughly minimize the mean-square error in prediction when correctly detected data symbols are fed back into the estimator.

### 4.6.2 Conclusions for Combined Detection and Estimation

Computer simulation results in Sec. 6.4 clearly show that with only one vector in the Viterbi-type detector ( $m=1$ ), System 2 performs exactly as described in Sec.4.6.1 with the single detector and estimator. That is, differential coding must be used, and total system collapse occurs on average, once every three or four packets for System 2A and about once every 100 packets for system 2B. This collapse is now accompanied by a very sharp rise in the cost $c_{i}$ of the stored vector $Q_{i}$.

With more than one vector in the Viterbi detector ( $m \geqslant 2$ ) differential coding must still be used, but such a system collapse was not observed once in any of the computer simulation tests for Systems 2A or 2B. Thus, the Viterbi-type detector has completely stabilized System 2. In fact, the tolerance to noise of this system is even better than for the single detector and estimator with correct data symbols fed back into the estimator. This is because this Viterbi algorithm chooses the sequence of detected data symbols that gives the best tracking of the channel through a fade. It seems to follow that the tolerance to noise is improved because the predictor has the freedom to track the channel $\pm 90^{\circ}$ or $180^{\circ}$ out of phase rather than in spite of this fact. A similar observation was made for System 1.

However, when one receiving antenna is used (System 2A) with differential coding, long bursts of errors in detection sometimes occur in both sets of data symbols $\left\{s_{1 . i}\right\},\left\{s_{2 . i}\right\}$ at the same time, without a sharp increase in the cost $c_{i}$. This is most often the result of an interchange in the channel predictors part-way through a packet. Thus, the channel estimator is still tracking the fading signals, but is associating the data symbols with the wrong channels. Differential coding in no way reduces the risk of this happening, nor does it correct these errors. It seems that the only practical way to correct the errors in these bursts is to apply different codes to the two data streams so that this interchange can be identified when it happens. Work is proceding on this at Manchester University. In the real world, both mobiles would generally be travelling at different speeds so the fading rates in the two channels $\left\{y_{1 . i}\right\},\left\{y_{2 . i}\right\}$ would be different. Under this condition, $y_{1 . i, i-1}$ and $Y^{\prime}$ 2.i,i-1 are actually less likely to swap over and follow the wrong channels. Also, this type of error does not seem to occur when there are two receiving antennas. So in the real world, this type of error probably has very little effect on the performance of System 2B.

### 4.7 Summary for System 2

System 2, a digital modem employing a completely new multiplexing method, has been developed and tested by computer simulation. It uses novel techniques of detection and estimation, similar to those used for System 1. It is possible with this method to transmit simultanously two four-level QAM signals in the same frequency band, where the two signals originate from different mobiles and fade independently at the base station receiver. The independent fading of the signals itself performs a process of collaborative coding that enables the signals to be detected and separated at the receiver without seriously interfering with each other.

Test results indicate that this coherent demodulation receiver with differential coding of the binary digits (DQPSK) can achieve a good tolerance to additive white. Gaussian noise. Differentially-coherent DQPSK which employs differential detection $[8,9,23,36,106]$ could not be used with this multiplexing method. This System 2 would achieve a bandwidth efficiency of just under $2 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$ in the mobile radio network - which is twice that for System 1. Two antennas must be used at the receiver for satisfactory operation to be achieved. This is largely because there is a dramatic improvement in the Rayleigh fading statistics with two antennas, as was the case for System 1. But also because errors caused by sigrial points overlapping and errors caused by an interchange in the estimators for channels $y_{1 . i}$ and $y_{2 . i}$ are greatly reduced in System 2 when two receiving antennas are used.

As for System 1, the key to the successful development of System 2 is a novel technique of combined detection and estimation with regular retraining of the estimator. As few as two stored vectors in the Viterbi-type detector are enough to ensure a stable System. In fact, the receiver with two or more stored vectors has a better performance than the receiver that has all correct data symbols fed back to the estimator. Since the channel estimation process is completely restarted every 1/100 second, it should be quick to recover from any prolongued loss of signal power. This regular retraining also avoids problems in estimation during hand-off as the mobile moves from one cell to another.

A particular virtue of the basic system studied here is that the most complex of the processes are involved at the base station, allowing the simpler processes to be implemented in the mobiles. Thus, a mobile transmits a four-level QAM signal, which is a bandlimited QPSK signal and
has only a limited ripple in the envelope. Consequently, a relatively simple high power amplifier can be used in the mobile. In contrast, the base station must perform all the complex processes in the combined detector and estimator.

## SYSTEM 3

### 5.1 Introduction

In Chapter 3 a combined detection and estimation process was used successfully at the receiver to achieve near-optimum coherent demodulation of a four-level QAM signal in fast Rayleigh fading. The bandwidth efficiency was about $1 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$. It was then shown in Chapter 4 that this basic system could be extended to simultaneously receive two four-level QAM signals in the same frequency band. These two signals originate from two different mobiles and fade independently at the receiver. This is a completely new multiplexing method, giving a bandwidth efficiency of about 2 bit/s/Hz.

The aim of this chapter is to develop System 3, which would allow the transmission back from the base station to these two mobiles in the same frequency band, with a bandwidth efficiency of about $2 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$. The coherent modulation scheme is different to that of the previous two chapters. Here, a 16-level QAM signal is transmitted with fully raised cosine spectral shaping and differential coding of the binary digits (see Sec.2.3). A $48 \mathrm{kbit} / \mathrm{s}$ (12kbaud), $16-1$ evel QAM signal is transmitted with a carrier frequency of about 900 MHz and with a total signal bandwidth of 24 kHz . The same signal is received by both mobiles, with half of the binary digits allocated to each. That is, the same information rate in each mobile-base station link as in Systems 1 and 2.

Four-level QAM modulation was chosen for the mobile transmitters in Systems 1 and 2 because it is relatively simple to generate with cheap equipment. However, it is generally considered that the base station equipment will be more expensive. So it is quite feasible to transmit a 16-level QAM signal from the base station in System 3, even though an expensive linear high-power amplifier isorequired to generate it.

System 3 is developed in this chapter following an exactly similar procedure as was taken for Systems 1 and 2. That is, first of all in Sec.5.2 the system model is described. Then in Sec.5.3 the best possible system performance is evaluated by testing the optimum maximum likelihood detector with perfect channel estimation. In Sec.5.4 correct detection is
assumed and the various estimation processes are tested. The method of retraining the channel estimator is described in Sec.5.5 before finally testing the combined detector and estimator in Sec.5.6.

Many similarities exist between System 3 and System 1. So to avoid needless repetition and confusion, only the important differences that exist between Systems 3 and 1 are highlighted. Wherever possible, references are given to descriptions already given for System 1.

Throughout this chapter the performance of System 3 is compared with that of System 1 from the point of view of the penalty paid in equipment complexity and tolerance to noise in going from System 3 to System 1 when doubling the spectral efficiency. System 3 is compared with System 2 to show the difference in performance between these two 16-point constellations which have the same spectral efficiency. The 16 -level QAM signal used in System 3 has the optimum tolerance to noise of any fixed 16 -point constellation $[31,36,104]$.

### 5.2 Model of System

In the computer simulations at time $t=i T$, the baseband received sample at the output of the receiver matched filter is given by:

For System 3A

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{5.2.1}
\end{equation*}
$$

For System 3B

$$
\begin{align*}
& r_{a . i}=s_{i} y_{a, i}+w_{a, i} \\
& r_{b . i}=s_{i} y_{b, i}+w_{b . i} \tag{5.2.2}
\end{align*}
$$

The letters s,y,w refer to the data channel and noise waveforms respectively. The subscript i shows that these waveforms have been sampled at time $t=i T$. The subscripts $a$ and $b$ before the dot refer to receiving antennas $A$ and $B$. The computer simulation model is shown in Fig.5.2.1 and the detailed simulation method is described in Appendix B. The important assumptions from which these equations have been derived are summarized in Sec.2.4.1. Also the relevant properties of the channel and noise samples have been summarized in that section.

The computer simulation model of System 3 given by Eqs.(5.2.1)(5.2.2) and Fig.5.2.1 is, in fact, the same as the model for System 1 (Eqs.(3.2.1)-(3.2.2) and Fig.3.2.1). The only differences between these two models are due to the different properties of the data symbols $\left\{\mathrm{s}_{\mathrm{i}}\right\}$. These differences are discussed in Sec.2.6.1.


Fig.5.2.1 Baseband model of data transmission system used in computer simulation tests with: (a) One receiving antenna (System 3A). (b) Two receiving antennas (System 3B)

The general operation of the coherent demodulation receiver has been described in Sec.2.6.2. The detailed operation of the data detection and channel estimation processes that comprise this receiver are investigated in the rest of this chapter.

### 5.3 Detection

The aim of this section is to investigate different methods of detecting the data symbols $\left\{s_{i}\right\}$ in Systems $3 A$ and $3 B$, assuming perfect channel estimation at the receiver. The best method tested here should still be the best detection process when used with the actual channel estimates.

### 5.3.1 Model of the Detection Process

The baseband received samples at the input to the detector at time $t=i T$, are:

For System 3A

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{5.3.1}
\end{equation*}
$$

For System 3B

$$
\begin{align*}
& r_{a . i}=s_{i} y_{a . i}+w_{a . i} \\
& r_{b . i}=s_{i} Y_{b . i}+w_{b . i} \tag{5.3.2}
\end{align*}
$$

It is assumed that the estimates of the channels used in the detector are exact, as shown in Fig.5.3.1. So the detector must minimize the probability of error in the detection of $s_{i}$. The detector has full prior knowledge of the sixteen possible values of $s_{i}( \pm 1$ or $\pm 3)+( \pm j$ or $\pm 3 j)$ as shown in Fig.5.3.2.

### 5.3.2 Maximum Likelihood Detection

For System 3A, the optimum maximum likelihood detector that has exact prior knowledge of $Y_{i}$, takes as the detected value of $s_{i}$, the possible value $s^{\prime}{ }_{i}$ for which

$$
\begin{equation*}
d_{i}^{2}=\left|r_{i}-s_{i}^{1} y_{i}\right|^{2} \tag{5.3.3}
\end{equation*}
$$

is minimum over all sixteen combinations of the possible values of $s^{\prime} i^{\prime}$ $( \pm 1$ or $\pm 3)+( \pm j$ or $\pm 3 j)$. Where $|x|$ is the absolute value of the complex valued quantity $x$.

For System 3B, with exact prior knowledge of $y_{a . i}, Y_{b . i}$ and statistically independent $w_{a . i} w_{b . i}$. Eq. (5.3.3) for optimum maximum likelihood detection becomes

$$
\begin{equation*}
d_{i}^{2}=\left|r_{a . i}-s^{\prime}{ }_{i} Y_{a . i}\right|^{2}+\left|r_{b . i}-s_{i}^{\prime} Y_{b . i}\right|^{2} \tag{5.3.4}
\end{equation*}
$$



$$
\left\{y_{i}^{\prime}\right\}=\left\{y_{i}\right\}
$$


$\left\{y_{\text {a. }}{ }^{\prime}\right\}=\left\{y_{\text {a.i }}\right\}$

Fig.5.3.1 Block diagram of detector for
(a) System 3A
(b) System 3B


Fig.5.3.2 Set of all possible values of $\mathrm{s}_{\mathrm{i}}$


Fig.5.3.3 Example received signal constellation of $s_{i} y_{i}$ when $y=0.6+j 0.12$

In practice, the detector must use estimates of the channel samples in place of the $\left\{y_{i}\right\},\left\{y_{a . i}\right\},\left\{y_{b . i}\right\}$ themselves. This inevitably degrades the detection process which is therefore no longer optimum.

The optimum detector for System 3 requires much more complex equipment than for System 1, because $d_{i}{ }^{2}$ must be calculated for sixteen rather than four possible values of $s^{\prime}{ }_{i}$. In contrast, it is much more straightforward than for System 2 because the 16 -point constellation transmitted from the base station is received at the mobile with the same basic shape. That is, all sixteen points fade together so at any instant in time, the received constellation is simply shifted in amplitude and phase relative to the transmitted constellation (see Figs.5.3.2-5.3.3). Thus, System 3 gives the multiplexing of two signals in the same 24 kHz frequency band, with only one fading channel to estimate at the receiver.

## Theoretical Probabilities of Error

The tolerance to additive white Gaussian noise of this optimum detection process using Eqs.(5.3.3)-(5.3.4) is well-known [8,9,22,23]. This has been derived theoretically in Appendix $D$, for the following four cases: (16-QAM signalling is assumed - that is, no differential coding).
(i) Assuming one receiving antenna and detection according to Eq.(5.3.3). For the special case with no fading (that is, where $y_{i}=1$ and $r_{i}=s_{i}+w_{i}$ for all \{i\}). The bit error rate in the detection of the $\left\{s_{i}\right\}$ for any given signal-to-noise ratio $\psi$ is given by

$$
\begin{equation*}
P_{b}=\frac{3}{4} Q(\sqrt{0.8 \psi})+\frac{1}{2} Q(3 \sqrt{0.8 \psi})-4 Q(5 \sqrt{0.8 \psi}) \tag{5,3.5}
\end{equation*}
$$

where $\psi=E_{b} / N_{0}$, as defined in Eq. (2.6.3). The $Q$-function
$Q(u)=\int_{u}^{\infty} \frac{1}{\sqrt{2 \pi}} \exp \left(-\frac{1}{2} z^{2}\right) d z$ is tabulated in the references [106].
(ii) But with flat Rayleigh fading (that is, where $r_{i}=s_{i} y_{i}+w_{i}$ and the $\left\{y_{i}\right\}$ are as described in Sec.2.4.2)

$$
\begin{equation*}
P_{b}=\frac{1}{2}-\frac{3}{8} \sqrt{\frac{2 \psi}{5+2 \psi}}-\frac{3}{4} \sqrt{\frac{2 \psi}{5+18 \psi}}+\frac{5}{8} \sqrt{\frac{2 \psi}{5+50 \psi}} \tag{5.3.6}
\end{equation*}
$$

(iii) Now, assuming two receiving antennas and detection according to Eq.(5.3.4). For the special case with no fading (that is, where $y_{a . i}=y_{b . i}=1, r_{a . i}=s_{i}+w_{a . i}$ and $r_{b . i}=s_{i}+w_{b . i}$, for all \{i\}). The bit error rate is given by

$$
\begin{equation*}
P_{b}=\frac{3}{4} Q(\sqrt{1.6 \psi})+\frac{1}{2} Q(3 \sqrt{1.6 \psi})-\frac{1}{4} Q(5 \sqrt{1.6 \psi}) \tag{5.3.7}
\end{equation*}
$$

(iv) But, for the general case for System $3 B$ with flat Rayleigh fading (that is, where $r_{a . i}=s_{i} y_{a . i}+w_{a . i}, r_{b . i}=s_{i} y_{b, i}+w_{b . i}$ and the uncorrelated $\left\{y_{a . i}\right\},\left\{y_{b . i}\right\}$ are exactly as described in Sec.2.4.2). The bit error rate


Flg.5.3.4 Theoretical probabilities of error for 16-QAM modulation (no differential coding)
is given by

$$
\begin{align*}
P_{b}=\frac{1}{2} & -\frac{3}{8} \sqrt{\frac{2 \psi}{5+2 \psi}}-\frac{3}{4} \sqrt{\frac{2 \psi}{5+18 \psi}}+\frac{5}{8} \sqrt{\frac{2 \psi}{1+10 \psi}} \\
& -\frac{15}{32 \psi}\left(\sqrt{\frac{2 \psi}{5+2 \psi}}\right)^{3}-\frac{15}{16 \psi}\left(\sqrt{\frac{2 \psi}{5+18 \psi}}\right)^{3}+\frac{25}{32 \psi}\left(\sqrt{\frac{2 \psi}{5+50 \psi}}\right)^{3} \tag{5.3.8}
\end{align*}
$$

These four bit error rate curves are shown in Fig.5.3.4.
The mechanisms that cause the errors in these four cases are exactly as described for System 1 (in Sec.3.3.1). That is; For the non-fading case, there is a 3dB improvement in tolerance to additive white Gaussian noise in going from one to two receiving antennas. This is caused by the $3 d B$ improvement in the signal-to-noise ratio in the received samples by using the second antenna. The large degradation in performance in going from a non-fading to a fading channel is caused entirely by the Rayleigh distribution of the amplitude of the fading. The rate of fading and hence the duration of fades has no effect on the tolerance to additive white Gaussian noise of the optimum detector. For the fading case, the big improvement in going from one to two receiving antennas is caused by the improvement in the Rayleigh statistics gained by coherently adding two independently fading channels. The corresponding curves with differential coding are best obtained by computer simulation. The degradation caused by this differential coding process is not as bad as for Systems 1 and 2 because here, only the first two binary digits in the $\left\{s_{i}\right\}$ are differentially coded. So the only errors in detection affected by the differential coding are where the detected symbol is in the wrong quadrant (Fig.5.3.2)

When theoretically deriving the bit error rate curves for this optimum detector for System 3, a simplifying approximation is usually made [31,104]. It is usually assumed that when an error in detection occurs at high signal-to-noise ratios, the detected data symbol value is adjacent to the correct element value. The probability of all other types of error is assumed negligible. This is a valid assumption to make in the non-fading case. In fact, for signal-to-noise ratios greater than about $\psi=5 \mathrm{~dB}$, Eq.(5.3.5) approximates closely to

$$
\begin{equation*}
P_{b}=3 Q(\sqrt{0.8 \psi}) \tag{5,3.9}
\end{equation*}
$$

It is important to note that in the presence of Rayleigh fading, errors normally occur during deep fades in the channel. As such, they are likely to be random symbol errors across the whole constellation. So in fading,


Flg.5.3.5 Performances of detectors for Systems 2 and 3 in Rayleigh fading with coherent coding of the binary digits
it is no longer a good approximation to assume that all errors are adjacent symbol errors.

An extremely important result is obtained by comparing the bit error rate curves with Rayleigh fading for Systems 2 and 3, as shown in Fig.5.3.5. (The curves for System 2 have been obtained by computer simulation). Although it is a fact that the 16-level QAM signal of System 3 has the best tolerance to noise of any 16 -point signal in a non-fading channel $[31,36,104]$, it is shown here that in the presence of Rayleigh fading and with two receiving antennas, System 2B has a better tolerance to noise than System 3B. The independent fading of the two bandlimited QPSK signals of System 2, itself performs a process of collaborative coding. This enables the two signals to be detected simultaneously without seriously interfering with each other when there are two receiving antennas. Though with one receiving antenna, System 3 A has a better tolerance to noise than System 2A.

### 5.3.3 Threshold Level Detection and Combining Techniques.

A threshold level detection method similar to that described for System 1A (Sec.3.3.3) can be applied to System 3A. This is equivalent to the optimum detector (Eq.(5.3.3)) but is computationally more efficient. Also, all the combining techniques discussed for System 1B are equally applicable for use with System 3B - again allowing a threshold level detector to be used. However, none of these methods are tested in this thesis. The reason being that the Viterbi detector that will be used in the final system, requires that all $16\left\{\mathrm{~d}_{\mathrm{i}}{ }^{2}\right\}$ in Eq. (5.3.3) be calculated in determining the costs of the stored vectors. The threshold level detector can only find the possible value $s^{\prime}{ }_{i}$ with the smallest $d_{i}{ }^{2}$. It does not actually calculate the individual $\left\{d_{i}{ }^{2}\right\}$, so cannot be used here.

### 5.3.4 Conclusions for Detection

Computer simulation tests have been carried out on Systems 3A and 3B to show the performance of the optimum maximum likelihood detection process operating with perfect channel estimation. The results of these tests are shown in Chapter 6. The accuracy of these simulation results is confirmed to be correct by the theoretical results of Appendix $D$. The mechanisms which cause the errors in detection have been described.

A most important result is that the optimum detector for System 3 B has a worse tolerance to noise than that for System 2B, when there is
independent Rayleigh fading at the two receiving antennas. This means that the 16 -point constellation used in System 3 in not always the optimum 16-point constellation, but is only optimum in a non-fading channel. With independently fading channels at two receiving antennas, the tolerance to noise of the 16 -point constellation formed in System 2 is better.

### 5.4 Channel Estimation

The aim of this section is to find an estimation process that will result in near-optimum data detection, when used in the maximum likelihood detector just described. This must be achieved with a reasonable level of equipment complexity. The methods tested here are simple adaptions of the unbiased and Gradient estimators used with System 1. It is assumed in this section that all detected symbols that are fed back to the estimator are correct.

### 5.4.1 Model of Estimation Process

At time $t=i T$, the received baseband sample is given by (from Eq.(5.2.1))

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{5.4.1}
\end{equation*}
$$

which is the same as for System 1. The model of the estimation process for System 3 used in the computer simulation tests is shown in Fig.5.4.1. This is also exactly the same as for System 1 (see Fig.3.4.1). Thus, the model of the estimation process for System 3 is exactly the same as for System 1 and is therefore as described in Sec.3.4.1. So, the unbiased and Gradient estimators that gave the most success with System 1 can be used with System 3 in exactly the same way. However, the performance of an estimation process will generally be worse when used in System 3 compared with its performance in System 1. This is caused by the different constellations of $s_{i}$ in the two Systems. This effect is examined more closely in the rest of sec .5 .4 .

### 5.4.2 Unbiased Estimator

As for System 1, the channel estimate at time $t=i T$ is given by

$$
\begin{equation*}
y_{i}^{\prime}=s_{i}^{\prime}{ }^{-1} r_{i}=s_{i}^{\prime}{ }^{-1} s_{i} y_{i}+s_{i}^{\prime}{ }_{w_{i}} \tag{5.4.2}
\end{equation*}
$$

Assuming correct detection,

$$
\begin{equation*}
y_{i}^{\prime}=s_{i}^{-1} r_{i}=y_{i}+s_{i}^{-1} w_{i} \tag{5.4.3}
\end{equation*}
$$

As shown in Fig.5.4.2, the reciprocal of $s^{\prime}{ }_{i}$ is formed by taking the complex conjugate of $s^{\prime}{ }_{i}$ and dividing by the amplitude-squared. For the


Fig.5.4.1 Model of estimation process as simulated


Fig.5.4.2 Model of unbiased channel estimator

Table 5.4.1 Possible values of $\left|s_{i}^{\prime}\right|^{2}$ with their corresponding $s_{i}^{\prime}$

| $s_{i}^{\prime}$ | $\left\|s_{i}^{\prime}\right\|^{2}$ |
| :---: | :---: |
| $-1-j$ | 2 |
| $-1+j$ | 2 |
| $+1-j$ | 2 |
| $+1+j$ | 2 |
| $-3-j$ | 10 |
| $-3+j$ | 10 |
| $-1-3 j$ | 10 |
| $-1+3 j$ | 10 |
| $+1-3 j$ | 10 |
| $+1+3 j$ | 10 |
| $+3-j$ | 10 |
| $+3+j$ | 10 |
| $-3-3 j$ | 18 |
| $-3+3 j$ | 18 |
| $+3-3 j$ | 18 |
| $+3+3 j$ | 18 |

16-level QAM signal, this amplitude-squared, $\left|s^{\prime}{ }_{i}\right|^{2}$, is one of three possible values 2,10 or 18 , as shown in Table 5.4 .1 , since

$$
s_{i}^{\prime}=s^{\prime} I_{. i}+j s_{Q . i}^{\prime}=( \pm 1 \text { or } \pm 3)+( \pm j \text { or } \pm 3 j)
$$

and

$$
\begin{equation*}
\left|s_{i}^{\prime}\right|^{2}=s_{I . i}^{\prime}{ }_{I} s_{Q . i^{2}}^{\prime} \tag{5.4.4}
\end{equation*}
$$

Thus, the unbiased estimator for System 3 is more complicated than for System 1, where $\left|s_{i}\right|^{2=2}$, for all $\{i\}$.

The mean-square error ( $\lambda_{e}$ ) of this estimate $y_{i}{ }_{i}$ can now be determined theoretically. This analysis is very similar to that for System 1 (Eqs.(3.4.5)-(3.4.17)). The only difference being that, for System 3

$$
\begin{gather*}
\left|s_{i}\right|^{2}=2, \quad \text { for } \frac{1}{} \text { of the }\left\{s_{i}\right\} \\
\left|s_{i}\right|^{2}=10, \text { for } \frac{1}{2} \text { of the }\left\{s_{i}\right\} \\
\left|s_{i}\right|^{2}=18, \text { for } \frac{1}{} \text { of the }\left\{s_{i}\right\} \\
E_{b}=2.5 \\
\psi=\frac{E_{b}}{N_{0}}=\frac{2.5}{2 \sigma^{2}} \tag{5.4.5}
\end{gather*}
$$

whereas, for System 1

$$
\begin{gather*}
\left|s_{i}\right|^{2}=2, \text { for all }\{i\} \\
E_{b}=1 \\
\psi=\frac{E_{b}}{N_{0}}=\frac{1}{2 \sigma^{2}} \tag{5.4.6}
\end{gather*}
$$

So, that for System 3, Eq.(3.4.9) becomes

$$
\begin{align*}
\lambda_{e} & =\left(\frac{1}{4} \cdot \frac{1}{2}+\frac{1}{2} \cdot \frac{1}{10}+\frac{1}{4} \cdot \frac{1}{18}\right) 2 \sigma^{2} \\
& =0.18889 \times 2.5 \psi^{-1} \\
& \approx-(\psi+3.26) \mathrm{dB} \tag{5.4.7}
\end{align*}
$$

The effect this has on the mean-square error curves is shown in Fig.5.4.3.

### 5.4.3 Unbiased Estimator with least-squares fading memory polynomial prediction

The performance of the unbiased estimator can be improved by incorporating a least-squares fading memory polynomial predictor - as was the case for System 1. This predictor operates on the $\left\{y^{\prime}{ }_{i}\right\}$ output from the unbiased estimator in exactly the same way as for System 1. This is described in detail in Sec.3.4.3, so is not repeated here. The predictor can also be modified as described in Sec.3.4.4.

The value of $\theta$ that minimizes the mean-square error in the prediction


Fig.5.4.3 Variation of $\lambda_{\theta}$ and $\lambda_{p}$ for Systems $1 A$ and $3 A$ '(unbiased estimator, no predictor)
$Y^{\prime}{ }_{i, i-1}$, for any given signal-to-noise ratio and fading rate, is found by computer simulation. This value of $\theta$ is generally different for Systems 1 and 3 because of the different properties of the $\left\{s_{i}{ }^{-1} w_{i}\right\}$ in Eqs.(3.4.3) and (5.4.3).
5.4.4 Gradient algorithm estimator incorporating feedback from a degree-1 fading memory polynomial predictor
The Gradient estimator for System 3 is identical to that for System 1, though its performance will be different because of the different constellations of $s_{i}$ in the two Systems. This effect is examined in this section.

The equations of this Gradient estimator with degree-1 prediction are as follows

$$
\begin{align*}
r_{i}^{\prime} & =s^{\prime}{ }_{i} y^{\prime}{ }_{i, i}-1  \tag{5.4.8}\\
e_{i} & =r_{i}-r^{\prime}{ }_{i}  \tag{5.4.9}\\
y^{\prime}{ }_{i} & =y^{\prime}{ }_{i, i-1}+b e_{i} s^{\prime}{ }_{i}{ }^{*}  \tag{5.4.10}\\
E_{i} & =y^{\prime}{ }_{i, i-1}-y^{\prime}{ }_{i}  \tag{5.4.11}\\
\dot{y}^{\prime}{ }_{i+i, i} & =\dot{y}^{\prime}{ }_{i, i-1}+(1-\theta)^{2} E_{i}  \tag{5.4.12}\\
y^{\prime}{ }_{i+1, i} & =y^{\prime}{ }_{i, i-1}+\dot{y}^{\prime}{ }_{i+1, i}+\left(1-\theta^{2}\right) E_{i} \tag{5.4.13}
\end{align*}
$$

This is depicted in Fig. 5.4.4. A simplification to this algorithm can be made because the $\left\{y^{\prime}{ }_{i}\right\}$ are not required, only the $\left\{y^{\prime}{ }_{i+1, i}\right\}$ are used in the detector. So Eqs.(5.4.10)-(5.4.11) can be replaced by

$$
\begin{equation*}
E_{i}=b e_{i} s_{i}^{\prime}{ }^{*} \tag{5.4.14}
\end{equation*}
$$

Hence, the lower diagram of Fig.5.4.4(b). The optimum values of the real valued constants $b, \theta$ are found experimentally by computer simulation for any given fading and noise conditions. These optimum values of b, $\theta$ minimize the mean-square error in the prediction $y^{\prime}{ }_{i, i-1}$, and should therefore give the best possible bit error rate.

The estimator is exactly the same as the one described in sec.3.4.9. In Chapter 3 this was shown to be the best arrangement of the Gradient algorithm estimator with prediction, for a single fading QAM signal.

The performance of this estimator is now analysed theoretically. For the moment, consider that

$$
\begin{equation*}
b=c /\left(s^{\prime}{ }_{i} s^{\prime}{ }_{i}^{*}\right)=c /\left|s^{\prime}{ }_{i}\right|^{2} \tag{5.4.15}
\end{equation*}
$$

for all $\{i\}$, where $c$ is a small real-valued positive constant. The theoretical mean-square errors in $Y^{\prime}{ }_{i}$ and $Y_{i, i-1}$ can now be derived in exactly the same way as for System 1 (in Sec.3.4.9). Of course $\left.\left|s^{\prime}\right|^{\prime}\right|^{2}$ is not constant for all $\{i\}$ because the 16 -points in the constellation of $s_{i}$

(b)
(a)

Fig.5.4.4 (a) Gradient algorithm with prediction
(b) Alternative representations of $\Sigma$
do not have a constant amplitude. The values of $\left.\left|s^{\prime}\right|^{\prime}\right|^{2}$ for every possible $s_{i}^{\prime}$ are shown in Table 5.4.1. However, b in Eq. (5.4.10) has been defined as a constant, so Eq.(5.4.15) cannot apply here. (This problem does not arise for System 1, where $\left|s_{i}\right|^{2=2}$, for all $\{i\}$ ). For this reason it seems likely that a better estimate $y_{i}{ }_{i}$ may be obtained if Eq. (5.4.10) is replaced by

$$
\begin{equation*}
y_{i}^{\prime}=y_{i, i-1}^{\prime}+\frac{c}{\left|s_{i}^{\prime}\right|^{2}} e_{i} s_{i}^{\prime}{ }^{*}=y_{i, i-1}^{\prime}+c e_{i} s_{i}^{\prime}{ }^{-1} \tag{5.4.16}
\end{equation*}
$$

Both versions of this estimation process are tested for System 3.
Clearly, in the absence of noise, the alternative estimator using Eq.(5.4.16) is the optimum estimation process because it gives an exact estimate of $y_{i}$ for all \{i\} (see Sec.3.4.9). But in typical levels of additive white Gaussian noise, the standard arrangement of the Gradient estimator using Eq.(5.4.10), is expected to give a more accurate channel estimate for the following reason. If $\left|s_{i}\right|^{2=18}$, the signal-to-noise ratio in the received sample $r_{i}$ is about 2.55 dB higher than if $\left|s_{i}\right|^{2}=10$, which is in turn about 7 dB higher than it would be if $\left|s_{i}\right|^{2}=2$. Hence, it follows that the higher the value of $\left|s_{i}\right|^{2}$, the more accurate is $e_{i}$ in Eq.(5.4.9) as a measure of the error in $Y_{i}{ }_{i}$. But, the alternative estimator using Eq. (5.4.16) scales the error signal $\left\{e_{i}\right\}$ in such a way that it takes least notice of the best error measurements.

For completeness, this principle can be taken one stage further and a second alternative estimator can be proposed in which Eq.(5.4.10) is replaced by

$$
\begin{equation*}
y_{i}^{\prime}=y_{i, i-1}^{\prime}+d\left|s_{i}^{\prime}\right|^{2} e_{i} s_{i}^{\prime}{ }^{*} \tag{5.4.17}
\end{equation*}
$$

This version of the estimation process is also tested for System 3.

### 5.4.5 Conclusions of Channel Estimation

The estimation processes described in Secs.5.4.2-5.4.4 have been tested under the assumption that all detected symbols $\left\{s^{\prime}{ }_{i}\right\}$ fed back from the detector into the estimator are correct. That is, $s^{\prime}{ }_{i}=s_{i}$ for all \{i\}. Results obtained from these tests in Chapter 6 indicate which is the best estimator to use in the complete System.

The best estimation process was seen to be the standard form of the Gradient algorithm in Sec.5.4.4. The mean-square error of $y_{i, i-1}$ was worse with the two modified Gradient algorithms considered in Eqs.(5.4.16) and (5.4.17). The improvement of the Gradient estimator over the unbiased estimator should be enough to justify the modest increase in equipment
complexity.
The Gradient estimator for System 3 tracks the fading channel with a very similar accuracy to the Gradient estimator for System 1. But the degradation in tolerance to noise caused by innaccuracies in the channel estimate, relative to the case with perfect channel estimation, is typically about 3.5dB. Whereas for System 1, this degradation was only about ldB. Clearly, the $16-1$ evel QAM signal is much more seriously affected by fading than the 4 -level QAM signal because the 16 points in the constellation of $s_{i}$ do not have constant amplitude. So for correct detection, a good estimate of the amplitude of the fading channel is needed as well as a good estimate of its phase.

This degradation in performance is marginally better than was seen for System 2, so should give an acceptable performance in the combined detector and estimator. This is an encouraging result because it is widely assumed that constant envelope signals must be used for mobile radio [8,43], because of the considerable problems in tracking the fading amplitude. The simple Gradient estimator gets round this problem by tracking the in-phase and quadrature components of the fading channel rather than the more unpredictable amplitude and phase components.

### 5.5 Retraining of the Channel Estimator

It is shown later in Sec.5.6 that a catastrophic failure often occurs in the combined detector and estimator for System 3 from which it does not recover. Thus, regular retraining of the channel estimator must be used.

It has already been pointed out in Sec.5.4.1 that, even though a 16 -level QAM signal is used in System 3 instead of the 4 -level QAM signal in System 1, the channel estimators of these two Systems are exactly the same. Now, the training signal that has been chosen here ensures that all retraining methods used for System 1 apply equally well to System 3. So no further tests on retraining methods need be made. Of course, the best is described in detail in Sec.3.5.6.

The training signal is shown in Table 5.5.1. It is the same as that used for System 1 (Table 3.5.1) except that all $\pm 1$ 's and $\pm j$ 's have been replaced by $\pm 3^{\prime \prime} s$ and $\pm 3 j^{\prime \prime} s$ respectively. Thus, the signal-to-noise ratio in the received samples has been maximized.

Table 5.5.1 Training signal

| $i$ | $s_{i}$ |
| :---: | :---: |
| 1 | $-3-3 j$ |
| 2 | $+3-3 j$ |
| 3 | $+3+3 j$ |
| 4 | $-3-3 j$ |
| 5 | $-3-3 j$ |
| 6 | $+3-3 j$ |
| 7 | $-3+3 j$ |
| 8 | $-3-3 j$ |
| 9 | $+3-3 j$ |
| 10 | $+3+3 j$ |
| 12 | $-3+3 j$ |

### 5.6 Combined Detection and Estimation

The maximum likelihood detector described in Sec.5.3.2 is now combined with the Gradient estimator described in Sec.5.4.4. The estimator is regularly retrained using the method described in Sec.3.5.5, with the 12-symbol training signal given in Table 5.5.1. This is the simple combined detector and estimator for System 3. It is shown in Fig.5.6.1. Computer simulation tests in Chapter 6 have shown that correct operation is only possible with 16-DQAM modulation. The 16-QAM modulation (without differential coding) gives $50 \%$ errors here, so cannot be used. This was also found to be the case for Systems 1 and 2, for the same reasons. That is, shifts in the channel estimator of $\pm 90^{\circ}$ or $180^{\circ}$ occur quite often just after a deep fade, with the corresponding shift in the subsequent detected symbol values $\left\{s^{\prime}{ }_{i}\right\}$. This phase shift in the $\left\{s^{\prime}{ }_{i}\right\}$ is corrected by the differential coding (see Sec.2.3.3).

However, this combined detector and estimator is inherently unstable. To explain this, consider an erroneous detection of the symbol $s_{i}$ in System 3A caused by the channel $Y_{i}$ being in a deep fade. This error in detection causes a large error in the estimate of the channel $y_{i}$ and a corresponding large error in the prediction $y^{\prime}{ }_{i+1, i}$. This further increases the probability of error in the detection of $s_{i+1}$, and so on. $A$ fading channel in a deep fade could cause a burst of errors several symbols long, which would result in a channel prediction far removed from the channel's correct in-phase and quadrature components. Consequently, correct detection is now highly improbable causing an avalanche of erroneous predictions and detected symbols. Random data is output and the channel predictor follows a completely random (noise like) path. Correct operation resumes only after the next retraining burst when the channel predictor is correctly reset.

Computer simulation tests indicate that typically, with no retraining, total system collapse can be expected to occur in this System 3A after only about 800 symbols duration. Clearly, retraining is essential for acceptable performance of this system. Effective retraining every 120 symbols ensures that very long bursts of errors are avoided here. However, simulation tests on System 3B indicate that a total system collapse is expected after about 12000 symbols duration. This marked improvement in the stability of the system is due to the following reason. Errors caused by deep fades are much less likely with uncorrelated fading at two receiving antennas than they are with fading at one receiving antenna. So,

(a)


Fig.5.6.1 Simple combined detector and estimator for (a) System 3A (b) System 3B


Fig.5.6.2 Final combined detector and estimator for (a) System 3A (b) System 3B
with effective retraining every 120 symbols, System $3 B$ appears to give a quite stable performance.

This instability problem of System 3 is in marked contrast to the simple combined detector and estimator for System 1, which was seen to perform well even without retraining. The error extension effects in System 1 caused by feeding back incorrectly detected symbols into the estimator were negligible. This was because any error in detection was simply an error of $\pm 90^{\circ}$ of $180^{\circ}$ in the detected symbol value $s^{\prime}{ }_{i}$, which caused a corresponding phase shift in the estimators for $y_{i}\left(y_{a . i}\right.$ and $\mathrm{y}_{\mathrm{b} . \mathrm{i}}$ ). Any prolongued, constant phase shift was corrected by the differential coding, with no loss in performance. However, tests with System 3 have shown that as much as 1.5 dB in tolerance to noise may be lost by error extension effects. This is because an incorrectly detected symbol fed back to the estimator is generally in error in both amplitude and phase, where the phase error is not a multiple of $90^{\circ}$. This causes a corresponding error in the channel estimate. Differential coding is no help here. An avalanche of errors as described above is always likely. A very similar unstable performance was observed with System 2. Though in that case it was caused by the two fading QAM signals interfering with each other. A bad estimate of one of these fading signals caused the collapse of the estimate of the other fading signal.

Clearly, any method which can reduce the error extension effects caused by feeding incorrectly detected symbols into the channel estimator is bound to give a naticeable improvement in the performance of System 3. Hence the Viterbi-type detector is now investigated.

### 5.6.1 Viterbi-Type Detection

The combined Viterbi-type detector shown in Fig.5.6.2 has been described for System 1 in Sec.3.6.1. Rather than repeat this lengthy description here, the reader is referred to Sec.3.6.1. Only three changes have to be made for this description to apply to System 3.
(i) It has been stated that every $q_{i}$ in Eq.(3.6.3) has taken on one of the M different possible detected values of $s_{i}$. So, for the 16 -level QAM signal of System 3, $M=16$ and the possible values of $q_{i}$ are $( \pm 1$ or $\pm 3)+$ ( $\pm \mathrm{j}$ or $\pm 3 \mathrm{j}$ ).
(ii) The Gradient estimator of System 3 is not equivalent to the unbiased estimator. So, the simplification to the standard algorithm given by Eqs.(3.6.16)-(3.6.17) does not apply here.
(iii) The values of $b, \theta$ for any given signal-to-noise ratio are different to those used for System 1. They are shown in Table 6.5.1.

### 5.6.2 Conclusions for Combined Detection and Estimation

Computer simulation results in Sec. 6.5 clearly show that with only one vector in the Viterbi-Type detector ( $m=1$ ), the receiver performs exactly as described in Sec.5.6.1, with the single detector and estimator. That is, differential coding must be used, and a total system collapse occurs typically, about once every five or six packets for System 3A, and about once every 100 packets for System 3B (where there are 108 data symbols in every packet). This collapse is now accompanied by a very sharp rise in. the cost $c_{i}$ of the stored vector $Q_{i}$.

With more than one vector in the Viterbi detector (m32), differential coding must still be used. But, a system collapse was not observed once in any of the computer simulation tests for Systems 3A or 3B. Thus, the Viterbi-type detector has completely stabilized System 3. In fact, the tolerance to noise of this system is even better than for the single detector and estimator with correct data symbols fed back into the estimator. This is because the Viterbi algorithm chooses the sequence of detected data symbols that gives the best tracking of the channel through a fade. It seems to follow that the tolerance to noise is improved because the predictor has the freedom to track the channel $\pm 90^{\circ}$ or $180^{\circ}$ out of phase rather than in spite of this fact. A similar observation was made for Systems 1 and 2.

### 5.7 Summary for System 3

A digital modem, System 3, has been developed in this chapter and tested by computer simulation. It is possible with this method to transmit a l6-level QAM signal from a base station to two mobiles. This System would achieve a bandwidth efficiency of just under $2 b i t / s / H z$ in the mobile radio network - which is the same as for System 2 and twice that for System 1. Two receiving antennas (System 3B) should be used wherever possible with four stored vectors ( $m=4$ ) in the combined detector and estimator. The improved performance should justify the extra equipment complexity.

The receiver uses novel techniques of detection and estimation very similar to that of System 1 , though its performance is quite different. Test results indicate that this coherent demodulation receiver with
differential coding of the binary digits (16-DQAM) can achieve a good tolerance to additive white Gaussian noise. The degradation in performance caused by inaccuracies in the channel estimate (with $m=4$ ), relative to the case with perfect channel estimation is about 3.5 dB for System 3A and about 1.5dB for System 3B. As might be expected, this degradation in performance when a 16-level QAM signal (System 3) is received, is much greater than that for one 4-level QAM signal (System 1), but is slightly less than that when two 4-level QAM signals (System 2) are received. A most important result is that with two antennas at the receiver, System 2 B gives a better performance than System 3 B . But, the 16-level QAM signal used in System 3 gives the best possible performance of any fixed, 16 -point constellation. So it must follow that the time varying 16 -point constellation in System 2 formed from the sum of two independently fading 4-level QAM signals, has a better tolerance to noise, with two receiving antennas.

As for Systems 1 and 2, the key to the successful development of System 3 is a novel technique of combined detection and estimation with regular retraining of the channel estimator. As few as two stored vectors in the Viterbi detector are enough to ensure a system stable against error extension effects. In fact, the receiver with $m \geqslant 2$ has a better performance than the receiver that has all correct data symbols fed back to the estimator. Since the channel estimation process is completely restarted every $1 / 100$ second, it is quick to recover from any prolongued loss of signal power. This regular retraining also avoids problems in estimation during hand-off as the mobiles move from one cell to another.

The base station transmitter requires a truly linear high-power amplifier to generate the 16 -level QAM signal. A particular virtue of System 3 is that this costly equipment is needed in the base station transmitter rather than the mobile.

## Results

### 6.1 Introduction

In the last three chapters, the combined detector and estimator that performs coherent detection at the receiver has been described in detail for Systems 1, 2 and 3. The results of all computer simulation tests carried out on these Systems are given in this chapter. The computer programs used to generate these results are given in Appendix $H$.

In Sec.6.2, the different ways the results are presented in this chapter are described. Then, the results for Systems 1, 2 and 3 are given in Secs.6.3, 6.4 and 6.5 respectively. Finally, the relative performances of these different Systems are assessed in Sec.6.6.

## 6. 2 Presentation of computer simulation results

All the results given in this chapter are presented in one of three ways. That is, they are either in the form of bit error rate curves, estimation error curves or estimator/detector output curves. The bit error rate curves convey the most important results. The performances of the final Systems 1, 2 and 3 are assessed from these curves. The estimation error curves are the most important tool for assessing the relative performances of different estimation processes. These curves indicate which estimator will give the best performance in the final System and take a great deal less computer time to generate than the bit error rate curves. The estimator/detector output curves are simply plots of channel estimates or errors in detection. These curves can give a useful insight into the mechanisms which are causing the errors to occur.

A three character alphanumeric code is used to label every curve in this chapter. The first character is the numeral 1, 2, or 3, whose value indicates respectively, one 4-level QAM signal, two 4-level QAM signals or one 16 -level QAM signal transmitted in the given frequency band. The second character in a label is the letter $A$ or $B$ which indicates that there are either one or two antennas at the receiver respectively. Thus the first two characters in the code are the System names that have been
used throughout this thesis. The third character is a numeral which gives the value of $m$ and therefore the number of stored vectors $\left\{Q_{i}\right\}$ used by the combined detector and estimator. So for example, the curve labelled 1B4 assumes one transmitted 4-level QAM signal, with two antennas at the receiver and $m=4$. When the third character is $P$ it indicates perfect channel estimation. When it is $C$ it indicates that all the detected data symbol values fed back from the detector to the estimator are correct. When the third character is $P$ or $C$, only a single stored vector $Q_{i}$ is employed, no advantage now being gained through the use of more stored vectors.

The methods of generating the three different types of curves from the computer simulation results are now described in detail.

### 6.2.1 Bit error rate curves

For each system tested, the error rate in the detected binary digits is plotted along the vertical axis, against the signal-to-noise ratio $\psi \mathrm{dB}$. Where

$$
\begin{equation*}
\text { bit error rate }=\frac{\text { number of incorrectly detected binary digits }}{\text { total number of binary digits transmitted }} \tag{6.2.1}
\end{equation*}
$$

and

$$
\begin{equation*}
\psi=10 \log _{10}\left(\frac{E_{b}}{N_{0}}\right) d B \tag{6.2.2}
\end{equation*}
$$

Every bit error rate curve is drawn as either a solid line or a dashed line. A solid line indicates that the binary digits have been
differentially coded (DQPSK or $16-D Q A M$ ) whereas a dashed line indicates that they have been coherently coded (QPSK or 16-QAM) as described in Sec. 2.3 .

Each run of the computer simulation calculates the bit error rate from a total of about 20000 transmitted symbols \{s $\left.{ }_{i}\right\}$ at a fixed signal-to-noise ratio. The bit error rate is calculated in this run with both coherent and differential coding and with $m=1,2,4, P$ and $C$, to avoid changes in the performance caused by changes in the fading sequences. To give the bit error rate result for any system at any signal to noise ratio, the bit error rate results calculated for several runs of about 20000, transmitted $\left\{s_{i}\right\}$ are all averaged. Different sequences of $\left\{s_{i}\right\},\left\{y_{i}\right\},\left\{w_{i}\right\}$ are used for each run, being called from random seed integers. When it appears that further averaging will not appreciably affect the result, this final average is plotted on the graph. The curves shown in this chapter have
been obtained by drawing the "best" smooth curve between these points. The curve does not generally pass through the points, but will most likely pass almost straight through all points where the bit error rate is greater than $10^{-2}$. The results become less accurate as the signal-tonoise ratio increases, particularly for error rates below $10^{-3}$. This must be taken into account when drawing the smooth curve from the results. So a certain amount of judgement is required in producing the best curves.

Where tests were carried out. with no Rayleigh fading, results for each curve were obtained by running the System for signal-to-noise ratios $\psi=-4,-2-0,2, \ldots, 14 \mathrm{~dB}$. In plotting any one point on a curve for high bit error rates above $10^{-2}$, a total of typically about $80000\left\{s_{i}\right\}$ were transmitted, and for low bit error rates below $10^{-2}$, typically about $200000\left\{s_{i}\right\}$ were transmitted. In plotting any one curve, a total of about $1.25 \times 10^{6}\left\{s_{i}\right\}$ were transmitted.

Where tests were carried out with Rayleigh fading and one receiving antenna, results for each curve were obtained by running the System for signal-to-noise ratios $\psi=0,5,10, \ldots, 60 \mathrm{~dB}$. With two receiving antennas, the System was run for signal-to-noise ratios $\psi=-2.5,0,2.5, \ldots, 25 \mathrm{~dB}$. In plotting any one point on a curve where the bit error rate is greater than $10^{-2}$, a total of typically $120000\left\{s_{i}\right\}$ were transmitted, and for bit error rates less than $10^{-2}$, typically $300000\left\{s_{i}\right\}$ were transmitted. In plotting any one curve, a total of about $2 \times 10^{6}\left\{s_{i}\right\}$ were transmitted. A vehicle speed of about $60 \mathrm{miles} /$ hour is assumed unless stated otherwise, corresponding to a maximum Doppler frequency shift of $f_{m}=80 \mathrm{~Hz}$.

With these arrangements, the $95 \%$ confidence limits for all the bit error rate curves are no greater than about $\pm 0.5 \mathrm{~dB}$. More $\left\{s_{i}\right\}$ must be transmitted over fading channels to give results of this accuracy than over the non-fading channel. There are two main reasons for this:

1) In a fading channel, the detection errors largely correspond to deep fades. So the results are highly dependent on the fading sequence used, especially for low bit error rates below $10^{-3}$. Many different, long fading sequences must be tested before the results start to average out.
2) A small discrepancy in the error rate measurements at a given signal-to-noise ratio $\psi$ is likely to cause a much greater error when plotting the smooth bit error rate curve in a fading channel than it would in a non-fading channel. This is because the slope in these curves for a fading channel are much shallower. For example see Fig.6.2.1. The same vertical error at a given point causes a greater horizontal (dB) error in


Fig.6.2.1 Effect of a given error in plotting the smooth curve for (b) fading channel
the curves for a fading channel.
The results for all these curves are generated using the FORTRAN program MREC.FORTRAN, with all the relevant subroutines (see Appendix H ).

### 6.2.2 Estimation error curves

For each channel estimation process tested, the quantity plotted along the vertical axis is $\lambda_{e}$ or $\lambda_{p}$, which are respectively the mean-square error in the estimate or prediction of the channel, expressed in decibels relative to unity. Thus, for Systems 1 A and 3A,

$$
\begin{equation*}
\lambda_{e}=10 \log _{10}\left(\frac{1}{6000} \sum_{i=1201}^{7200}\left|y_{i}-y_{i}^{\prime}\right|^{2}\right) d B \tag{6.2.3}
\end{equation*}
$$

for Systems $1 B$ and $3 B$

$$
\begin{equation*}
\lambda_{e}=\operatorname{lolog}_{10}\left(\frac{1}{6000} \sum_{i=1201}^{7200}\left|y_{a}-y_{a . i}^{\prime}\right|^{2}+\left|y_{b . i}-y_{b . i}^{\prime}\right|^{2}\right) d B \tag{6.2.4}
\end{equation*}
$$

for System 2A
$\lambda_{e}=\log _{10}\left(\frac{1}{2} \cdot \frac{1}{6000}\left(\sum_{i=1201}^{7200}\left|y_{1 . i^{-1}} y_{1 . i}\right|^{2}+\sum_{i=1201}^{7200}\left|y_{2 . i^{-y^{\prime}}}^{2 . i^{\prime}}\right|^{2}\right)\right) d B$
and for System 2B

$$
\begin{align*}
\lambda_{e}=\log _{10}\left(\frac{1}{2} \cdot \frac{1}{6000}\right. & \left(\sum_{i=1201}^{7200}\left|y_{1 . i^{-y^{\prime}} 1, i}\right|^{2}+\sum_{i=1201}^{7200} \mid y_{2 .\left.i^{-y_{2 . i}^{\prime}}\right|^{2}}\right. \\
& \left.\left.+\sum_{i=1201}^{7200}\left|y_{3 . i^{\prime}-\left.y^{\prime}{ }_{3, i}\right|^{2}}+\sum_{i=1201}^{7200}\right| y_{4 . i^{\prime}}-\left.y_{4 . i}^{\prime}\right|^{2}\right)\right) d B \tag{6.2.6}
\end{align*}
$$

where $|x|$ is the absolute value (modulus) of the complex-valued quantity $x$. The corresponding $\lambda_{p}$ 's are obtained by substituting $y^{\prime}{ }_{i}$ and $y^{\prime}{ }_{x . i}$ in Eqs.(6.2.3)-(6.2.6) with $y_{i, i-1}^{\prime}$ and $y_{x, i, i-1}^{\prime}$ (where $x=a, b, 1,2,3$ or 4). The first $1200\left\{y^{\prime}{ }_{i}\right\}$ and $\left\{y^{\prime}{ }_{i, i-1}\right\}$ are ignored here in order to eliminate the transient effects that sometimes occur at the start of operation. Thus, $\lambda_{e}$ and $\lambda_{p}$ give a measure of the steady state performance of the channel estimation process. The estimator is always assumed to operate with all correctly detected data symbols and with no retraining.

Since these curves are only used to compare different estimation processes to see which is the best, tests are only carried out with one receiving antenna, in the presence of Rayleigh fading and with the idealized condition that all the detected data symbols are correct. That is, all these tests are carried out on Systems $1 \mathrm{AC}, 2 \mathrm{AC}$ and 3 AC only. Whichever estimation process gives the lowest $\lambda_{p}$ under these conditions should perform best generally.

Each computer simulation test calculates $\lambda_{e}$ and $\lambda_{p}$ (Eqs.(6.2.3)-
(6.2.4)) at a fixed signal-to-noise ratio $\psi \mathrm{dB}$ (Eq. (6.3.2)). When the Gradient estimator is tested, a fixed value of $b$ must also be used. When a fading memory polynomial predictor is used, the degree of the predictor and the value of $\theta$ must also be fixed. (see Table 3.4.1). Several different tests are carried out to calculate $\lambda_{e}$ and $\lambda_{p}$ in this way for different values of either $\psi$ b or $\theta$. Smooth curves showing the variations of $\lambda_{e}$ and $\lambda_{p}$ with either $\psi, b$ or $\theta$ are then drawn. This. is done by plotting $\lambda_{e}$ (or $\lambda_{p}$ ) against the different values of either $\psi$, b or $\theta$ and drawing a smooth curve through all these points. (Only one of these variables $\psi, b, \theta$ can be changed with each test. The remaining two variables must remain constant for all tests). In all these tests, the same seed integers are used to generate the $\left\{s_{i}\right\},\left\{s_{1, i}\right\},\left\{s_{2 . i}\right\},\left\{y_{i}\right\}$, $\left\{y_{1, i}\right\},\left\{Y_{2 . i}\right\},\left\{w_{i}\right\}$ to avoid changes in the performances of the estimators caused by changes in the channel conditions (see subroutines SFADE. FORTRAN and SBBCHAN.FORTRAN in Appendix $H$ ). That is, the same sequences of $\left\{s_{i}\right\},\left\{s_{1 . i}\right\},\left\{s_{2 . i}\right\},\left\{y_{i}\right\},\left\{y_{1, i}\right\},\left\{y_{2 . i}\right\}$ are used in every test, and the same sequence $\left\{w_{i}\right\}$ is used but is suitably scaled depending on the signal-to-noise ratio, as described in Appendix $B$.

The results for these curves are generated using the FORTRAN program MESTIMPRED.FORTRAN, with all the relevant subroutines, except for the Kalman estimator where the program KALMEST.FORTRAN is used instead (see Appendix H).

### 6.2.3 Estimator / detector output curves

The estimator output curve is simply a plot of $y^{\prime}{ }_{i}$ on the vertical axis against $i$ on the horizontal axis. The $\left\{y_{i}\right\}$ are always plotted on the same axis for comparison purposes. The detector output graph is simply a plot of the errors in the detected symbols. Thus for signal $s_{i}$, "Errors in $\left\{s^{\prime}{ }_{i}\right\} "$ is plotted on the vertical axis against $i$ on the horizontal axis where

$$
\text { Error in } s_{i}^{\prime}= \begin{cases}0, & \text { if } s^{\prime}{ }_{i}=s_{i}  \tag{6.2.5}\\ 1, & \text { if } s_{i}^{\prime}=s_{i}\end{cases}
$$

All these curves are output from the FORTRAN programs MREC.FORTRAN or MESTIMPRED.FORTRAN (with all the relevant subroutines).

### 6.3 System 1

## Detection

Fig.6.3.1 shows results for the maximum likelihood detector described in

Sec.3.3.2. Clearly, there is a close agreement between the theoretical and simulated curves for QPSK (no differential coding). At high error rates (above $10^{-2}$ ) all the theoretical and simulated curves are practically indistinguishable from each other. As expected (Sec.6.2.1) the biggest error in the simulation results is at low error rates in the presence of Rayleigh fading.

The degradation in tolerance to additive white Gaussian noise caused by the differential coding is just less than $\frac{1}{2} d B$ at $10^{-4}$, over a non-fading channel. This loss decreases as the error rate decreases because the slope of the curve is increasing. At any fixed signal-tonoise ratio at low error rates, the error rate with DQPSK (differential coding) is twice that for QPSK, as expected (Sec.3.3.2).

Now consider the performance at low error rates (below $10^{-2}$ ) in the presence of Rayleigh fading. At a fixed signal-to-noise ratio, the bit error rate with one receiving antenna for DQPSK is about 1.7 times that for QPSK, whereas with two receiving antennas the bit error rate for DQPSK is about 1.8 times that for QPSK. The greater the tendency for error bursts rather than isolated errors, the more this ratio decreases from 2 towards 1. The slopes of these error rate curves are constant at these low error rates, so the degradation in tolerance to noise is about 2.4 dB for System 1A and about 1.3dB for System 1B.

With maximal ratio combining, the simulation results for System 1B are exactly the same as described above. Not only are the error rates the same, but all the same errors occur at the same times. So these two methods are equivalent when the detector operates with perfect channel estimation.

Fig.6.3.2 shows that the detection errors in System lAP described above tend to occur during deep fades in the baseband channel $\left\{y_{i}\right\}$. Similarly, in System 1BP, the errors in detection tend to occur when both channels $Y_{a . i} Y_{b . i}$ are in a deep fade at the same time. At the slower vehicle speed (smaller $f_{m}$ ), deep fades occur less frequently and tend to last longer. Hence, the error bursts are also less frequent and last longer such that the bit error rate is not affected

## Estimation

Fig.6.3.3 shows results for the unbiased estimator described in Sec.3.4.2, which operates with all correctly detected symbols $\left\{s^{\prime}{ }_{i}\right\}$. As expected (Sec.3.4.2), $\lambda_{e} \approx-(\psi+3) d B$. There is no separate prediction here so
$y_{i+1, i}{ }^{\prime} y^{\prime}{ }_{i}$. The curves show that at low signal-to-noise ratios $\lambda_{p} \approx-(\psi+3) d B$, whereas at high signal-to-noise ratios $\lambda_{p}$ levels off to an error floor which depends on the fading rate. This confirms the theoretical analysis in Sec.3.4.2. The fading rate at a vehicle speed of $60 \mathrm{miles} /$ hour $i s$ effectively twice that for a speed of $30 \mathrm{miles} / \mathrm{hour}$ and consequently the error floor in $\lambda_{p}$ is higher.

Figs.6.3.4 and 6.4 .5 show the improvement in $\lambda_{p}$ that can be achieved when least-squares fading memory polynomial prediction (described in Sec.3.4.3) is used with the unbiased estimator. The estimator operates with all correctly detected symbols $\left\{s^{\prime}{ }_{i}\right\}$. Fig.6.3.4 shows that the fading memory prediction filters typically improve $\lambda_{p}$ by between 6 and 10dB. Generally, the higher the degree of the predictor, the better is the prediction, particularly at high signal-to-noise ratios $\psi$. Clearly, the performances of the degree-1,2 and 3 predictors are quite similar. Their optimum values of $\lambda_{p}$ are usually within about $2 d B$ of each other, whereas the degree-0 predictor is typically 3 to 8 dB worse. For this reason the degree-0 predictor is rejected here. The degree-3 predictor never gains more than about $\frac{1}{2} d B$ in $\lambda_{p}$ over the degree-2 predictor. This small improvement in performance does not justify the extra complexity in the filter, so the degree-3 predictor is also rejected here.

The value of the constant $\theta$ that minimizes $\lambda_{p}$ decreases as the signal-to-noise ratio $\psi$ increases. Fig.6.3.5 clearly shows that $\theta$ should be optimised at each different value of $\psi$ if the degree-1 predictor is to give a good performance over a wide range of $\psi$. A change in the value of $\theta$ every $10 d B$ should be quite adequate. This would safeguard against an error floor at high signal-to-noise ratios.

The modified degree-1 predictors described in Sec.3.4.4 give very little improvement in $\lambda_{p}$ (less than $0.1 d B$ ) when $\theta_{b}$ and $\theta_{c}$ are optimized. Significant improvements are possible with $\theta_{\text {a }}$ optimized, though this was shown to be equivalent to the Gradient estimator of Sec.3.4.9. These results are discussed later.

Fig.6.3.6 shows that none of the Taylor expansion predictors described in Sec.3.4.5 are as good as the degree-1 predictor with $\theta=0.58$. They are not considered further.

Figs.6.3.7 and 6.3 .8 show the performance of the sinewave scheme of channel prediction described in Sec.3.4.6. Fig.6.3.7 shows that the sinewave is successfully extracted from a noisy signal which also contains; a constant term, a ramp or a low frequency sinewave. Fig.6.3.8
shows that the sinewave scheme can recover from a small error in estimated amplitude or phase, within several cycles of the sinewave. However, the sinewave cannot be reliably extracted from a typical fading sequence. Clearly the amplitude and phase of the sinewave component in the fading vary too unpredictably, probably due to the fact that there is too much power in the lower frequency components.

Fig.6.3.9 shows a typical result of tests on the equalizer described in Sec.3.4.7. In all tests, the equalizer was seen to go unstable within about 3000 symbols. This was the case both when $a_{i}=1 /\left|r_{i}\right|^{2}$ and when "a" was a small constant. The equalizer tracks the inverse of the channel, so when the channel goes into a deep fade, the equalizer output tends towards $\pm \infty$ 。

Fig. 6.3 .10 confirms that the Gradient estimator described in Sec.3.4.8 gives an exact estimate of the channel in the absence of noise, when $b=0.5$. Though, as the signal-to-noise ratio decreases, the optimum value of $b$ decreases, down to about $b=0.12$ at $\psi=5 \mathrm{~dB}$. The improvement in $\lambda_{e}$ at the optimum value of $b$ compared with $\lambda_{e}$ at $b=0.5$ is, of course, also the improvement of the Gradient estimator over the unbiased estimator (see Sec.3.4.8). This improvement is about 6 dB at $\psi=5 \mathrm{~dB}$, but only about 0.75 dB at $\psi=35 \mathrm{~dB}$. The optimum value of $b$ also decreases by about 0.05 as the vehicle speed decreases from 60 to 30 miles/hour. A small decrease was expected from the theoretical analysis.

Figs.6.3.11-6.3.13 show the performance of the Gradient estimator with degree-1 prediction described in Sec.3.4.9. Now, the values of both b and $\theta$ need to be optimized together to minimize $\lambda_{p}$. Generally for both Systems 1A (Fig.6.3.11) and 1B (Fig.6.3.12), as the signal-to-noise ratio increases, the optimum value of $b$ increases and the optimum value of $\theta$ decreases. $b$ and $\theta$ are roughly optimized every 10 dB with the values shown in Table 6.3.1. Fig.6.3.13 shows that if constant values of $b, \theta$ were used over a wide range of $\psi$, then there would be an error floor in $\lambda_{p}$ at high signal-to-noise ratios. By comparing Fig.6.3.11 with Fig.6.3.4. we can see that $\lambda_{p}$ is about $\frac{1}{2}$ to $1 d B$ lower for the Gradient estimator than for the unbiased estimator, when $b$ and $\theta$ are optimized. In a similar way it can also be shown that the Gradient estimator in Sec.3.4.8 which does not include a feedback loop from the degree-1 predictor would lose between and $\frac{1}{2} d B$ in $\lambda_{p}$ compared with the curves in Fig.6.3.11. Thus, the arrangement of the Gradient estimator described in Sec.3.4.9 is the best estimation process tested so far.

Fig.6.3.14 shows the performance of the Gradient estimator with double sampling described in Sec.3.4.10. By comparing these results with Fig. 6.3.10 we can see that $\lambda_{p}$ for can be improved by about 5.5 to 6 dB by introducing double sampling into the Gradient estimator. Unfortunately, this very significant improvement in $\lambda_{p}$ has been achieved at the expense of greatly increased equipment complexity. This technique would be well worth pursuing in systems where the bit error rate performance is unacceptable due to problems in tracking a very fast fading signal. However, it is shown later that the bit error rate performance of System 1 without double sampling in the Gradient estimator is only about ldB worse than the best possible performance with perfect channel estimation. Such a small improvement in the tolerance to additive white Gaussian noise of less than $1 d B$ would not justify the extra equipment complexity. For this reason the double sampling method is not considered further.

Fig.6.3.15 shows the performance of the Kalman estimator described in Sec.3.4.11. These results can be compared with Fig.6.3.10 to show that $\lambda_{e}$ is generally about $\frac{1}{2}$ to $\frac{1}{2} d B$ worse here than for the Gradient estimator. A particular advantage of Kalman estimators is that they converge much more rapidly to a good channel estimate after start-up than. does the Gradient estimator. However, a good steady-state performance is all that is needed here because accurate retraining of the channel estimator is assumed to be achieved at regular intervals. Since the Kalman estimator is also more complex than the Gradient estimator it is not considered further.

## Retraining

Fig.6.3.16 shows the tolerance to additive white Gaussian noise of System 1A with an unbiased estimator and degree-1 and 2 predictors, as described in Sec.3.4.3. Differential coding of the binary digits is assumed. The estimator operates with all correctly detected data symbols \{s' $\left.{ }_{i}\right\}$. With no retraining (Sec.3.5.4), the degree-2 predictor gives a better performance than the degree-1 predictor at the high signal-to-noise ratios, being about 1.5 dB better at around $\psi=40 \mathrm{~dB}$. However, with "ideal" retraining (Sec.3.5.2), this advantage is lost because the term $\ddot{z}_{i, i-1}$, which represents the estimate of the second derivative of $y_{i}$ with $i$, is simply reset to zero. (Since in practice, no attempt is made to estimate this quantity). Thus, information is lost in the retraining process when the degree-2 predictor is used. For this reason, the degree-2 predictor
is now rejected in favour of the simpler degree-1 predictor:
In Fig.6.3.17, the same two estimation processes are tested with the reset-to-zero retraining described in Sec.3.5.3. The bit error rate performance above about $\psi=20 d B$ rapidly deteriorates compared with ideal retraining (see Fig.6.3.18). System 1AC with the degree-1 predictor loses about 2 dB at an error rate of $10^{-4}$. The degree-2 predictor would lose a further 3 dB .

In Fig.6.3.18, the degree-1 predictor is tested with the least-squares retraining Methods 1 and 2 (Sec.3.5.5). It is seen that with $R=12$ retraining symbols, there is no difference in the performances with these two methods. Only about $\frac{1}{4}$ to $\frac{1}{2} d B$ is lost in tolerance to noise compared with ideal retraining. However, if only $R=6$ retraining symbols were used then the performance with Method 1 at low signal-to-noise ratios would be about $l d B$ worse than for $R=12$ and Method 2 would be a further $\ddagger d B$ worse. There is no noticable difference between the performances of any of the retraining methods above $\psi=20 \mathrm{~dB}$. Clearly, $R=12$ retraining symbols should be used in the prototype modem.

## Combined. Detection and Estimation

In Fig.6.3.19 System 1A is tested with both an unbiased and Gradient estimator with no retraining. The estimator now operates with the detected data symbol values $\left\{s^{\prime}{ }_{i}\right\}$. Only the results with differential coding are shown, since the bit error rate is $\frac{1}{2}$ for both estimators without differential coding. Clearly, without prediction in the unbiased estimator there is an irreducible bit error rate that depends on the fading rate. Whereas with the degree-1 predictor, System 1 loses only about 1.5 dB in tolerance to noise compared with perfect estimation, without an irreducible error rate. However, the value of $\theta$ has been roughly optimized for different signal-to-noise ratios (see Fig.6.3.5). That is; $\theta=0.88$ (for $\psi \leqslant 10 \mathrm{~dB}$ ), $\theta=0.82$ (for $10<\psi \leqslant 20 \mathrm{~dB}$ ), $\theta=0.72$ (for $20<\psi \leqslant 30 \mathrm{~dB}$ ) and $\theta=0.58$ (for $\psi>30$ ). The Gradient estimator (Sec.3.4.9) uses the values of $b, \theta$ shown in Table 6.3.1, and gains a further $\frac{1}{4}$ to $\frac{1}{2}$ dB over the previous estimator.

Fig.6.3.20 shows a typical example of the channel prediction going through a phase change of $+90^{\circ}$ after a deep fade at around $i=1200$. From this point onwards, $y^{\prime}{ }_{i, i-1} \approx j y_{i}$ ( or $y^{\prime} I_{. i, i-1} \approx^{-y_{Q . i}}$ and $y_{Q . i-1, i}^{\prime}{ }^{\prime} y_{I . i}$ ) and similarly $s^{\prime}{ }_{i}=-j s_{i}$. Thus, a bit error rate of $\frac{1}{2}$ results if differential coding is not employed (see Sec.2.3.3). In fact, this kind
of phase change is common in all Systems tested in this thesis where the estimator operates with the detected data symbol values $\left\{s_{i}{ }_{i}\right\}$. It never happens when the estimator operates with the actual $\left\{s_{i}\right\}$.

Fig.6.3.21 shows the performance of the final Systems $1 A$ and $1 B$ described in Sec.3.6.2. Perhaps the most significant of the results obtained is that when four stored vectors are used in the combined detector and estimator ( $m=4$ ), the degradation in performance caused by inaccuracies in the channel estimate relative to the case with perfect channel estimation is about 0.5dB. This applies for both one and two antennas at the receiver. Bearing in mind that the fading rate is up to about 160 fades per second, giving perhaps as few as 75 data symbols per fade, very accurate tracking of the fading signal is achieved here by the estimator. It is interesting to note that Systems 1 A4 and 1B4 perform better than Systems 1 AC and 1 BC respectively. This seems to be because the Viterbi detector with 4 stored vectors has the freedom to switch through any multiple of $90^{\circ}$ and so find the channel estimates that best fit the sequence of samples through a deep fade.

Fig.6.3.22 shows the performance of System 1B with pre-detection maximal ratio combining. Earlier simulation tests have shown that with perfect channel estimation, maximal ratio combining is optimum. So it is interesting to see that about $\frac{1}{2} d B$ is lost in the performance of System 1B4 with maximal ratio combining, when the detector operates with the actual channel estimates. Clearly, the ability of the Viterbi detector to choose the best sequence of data symbol values has been impaired. This is probably due mainly to the non-ideal co-phasing of the signals from the two antennas.

### 6.4 System 2

## Detection

Figs.6.4.1-6.4.2 show the performance of the maximum likelihood detector described in Sec.4.3.2, when operating with perfect channel estimation. The bit error rate curves for several different non-fading channels are shown in Fig.6.4.1(a) and the corresponding received signal constellations are shown in Fig.6.4.1(b). These curves highlight the fact that the tolerance to additive white Gaussian noise of System 2A is greatly affected by the relative phases of the channels $y_{1}$ and $y_{2}$. In all the examples tested, $y_{1 . i}=1$ for all $\{i\}$, and the amplitude level of
$Y_{2 . i}$ is always equal to 1 . Only the phase of $y_{2 . i}$ is altered. The best bit error rate curve in Fig.6.4.1(a) is the one where $/ y_{2.1}=/ 33.75^{\circ}$, for all \{i\}. This curve is about 6 dB worse than the curve for System 1 AP with no fading, shown in Fig.6.3.1. So clearly, in the absence of fading, System 2A would have a very poor performance. However, in Fig.6.4.2 the bit error rate curves for Systems $2 A P$ and $2 B P$ are shown in the presence of Rayleigh fading, (where all the channels $Y_{1 . i}, Y_{2 . i}, Y_{3 . i}, Y_{4 . i}$ are fading independntly). Now, comparing these results with Fig.6.3.1, we can see that System 2AP only loses about 5 dB compared with System 1AP and System 2BP only loses about 2.5 dB compared with System 1BP. The degradation in tolerance to noise caused by the differential coding in Systems 2AP and $2 B P$ is the same as for systems $1 A P$ and $1 B P$ respectively.

Fig.6.4.3 shows that when errors in detection in System 2AP occur in only one of the detected data streams, this is usually caused by the corresponding channel being in a deep fade. When errors occur in both detected data streams at the same time, this is usually seen to be when both channels have similar amplitudes. This was expected from the theoretical analysis in Sec.4.3.2. Two examples of this are shown in Fig.6.4.4(a) and (b). In Fig.6.4.4(a) the two channel samples $y_{1 . i}$ and $y_{2 . i}$ have similar amplitudes and their relative phases are such that several points in the constellation are almost overlapping. This causes an error in the detection of $s_{i}$. In Fig. 6.4.4(b), both channels $y_{1 . i}$ and $Y_{2 . i}$ are in a deep fade, causing an error in detection. A particularly interesting example for System 2AP is shown in Figs.6.4.4(c) and (d). Both channel samples $Y_{1 . i}$ and $Y_{2 . i}$ at antenna $A$ are in deep fade. Both channel samples $Y_{3 . i}$ and $Y_{4 . i}$ at antenna $B$ have similar amplitudes and their relative phases are such that several points in the constellation are almost coincident. The data symbol values would not be correctly detected here from $r_{a . i}$ or $r_{b . i}$ only. However, Fig.6.4.3(b) shows that $s_{1.1127}$ and $s_{2.1127}$ are correctly detected by System 2BP (though there are errors at $i=1126$ and 1131).

## Estimation

Fig.6.4.5 shows results of tests on the arrangement of the Gradient estimator described in Sec.4.4.3. There is no feedback from the degree-1 predictor to the Gradient estimator so the constants $b$ and $\theta$ can be optimized independently. It is not possible to achieve an exact estimate
of the channels $Y_{1 . i}$ and $Y_{2 . i}$. here in the absence of noise. The minimum value of $\lambda_{e}$ in the absence of noise is about -27 dB .

Fig.6.4.6 shows results for the Gradient estimator described in Sec.4.4.4. Now, the values of $b$ and $\theta$ need to be optimized together to minimize $\lambda_{p}$. These results can be compared with Fig.6.4.5 to show a quite remarkable improvement in this estimator over the previous one. With b and $\theta$ optimized, $\lambda_{p}$ can be improved by about 10 dB at $\psi=40 \mathrm{~dB}$. This is clearly the best estimator of the two tested. As the signal-to-noise ratio increases, the optimum value of $b$ increases and the optimum value of $\theta$ decreases, (as for System 1A). So again, to achieve a good tolerance to noise over a wide range of $\psi$, the values of $b$ and $\theta$ must be changed as $\psi$ changes. The values of $b$ and $\theta$ have been roughly optimized with the values shown in Table 6.4.1.

## Retraining

Fig.6.4.9 shows the tolerance to additive white Gaussian noise of System 2A described in Sec.4.6.1. Differential coding of the binary digits is assumed and the estimator operates with all correctly detected data symbols $\left\{s^{\prime}{ }_{i}\right\}$. Clearly $R=12$ retraining symbols should be used in the prototype modem since only about $\frac{1}{2} \mathrm{~dB}$ in tolerance to noise is lost compared with ideal retraining. If only $R=6$ retraining symbols were used, a further 1 dB would be lost.

## Combined Detection and Estimation

When the combined detector and estimator described in Sec.4.6 is tested without retraining, there is likely to be a catastrophic failure in the system over the transmission of a message of typical duration. This failure typically occurs after about 400 symbols in System 2 A and after about 12000 symbols in System 2B. The failure usually occurs after a deep fade that causes a large burst of errors in the detected data symbols. These errors in turn reduce the accuracy of the channel estimates, which further increases the probability of error, and so on. This system collapse is characterised by a sharp rise in the cost $c_{i}$ of the lowest cost vector, as the channel estimates go unstable and follow a completely random path. The use of differential coding does nothing to prevent the failure of the system here. Thus, regular retraining of the channel estimate must be used with System 2.

Fig.6.4.10 illustrates another problem in the combined detector and
estimator that has been observed in the computer simulation tests. At about $i=630$, the estimator has ceased to operate correctly, leading to an extended burst of errors in the detected data symbols. This is in fact the result of an interchange of $y^{\prime} 1_{1 . i, i-1}$ and $y^{\prime}{ }_{2 . i, i-1}$ in the estimator, such that the $\left\{y^{\prime}{ }_{1 . i, i-1}\right\}$ are associated with the corresponding $\left\{s^{\prime}{ }_{2 . i}\right\}$ and the $\left\{y^{\prime}{ }_{2 . i, i-1}\right\}$ are associated with the corresponding $\left\{s^{\prime}{ }_{1 . i}\right\}$. Thus the channel estimator is in fact still tracking the fading signals and the cost $c_{i}$ of the lowest cost vector is still at its typical low level, but the detector is associating the data symbols with the wrong channels. Clearly, if the occurrence of this effect can be identified when it occurs, the large majority of the errors in the burst can be corrected. Work is proceding on this at Manchester University using collaborative
 for about $630 \leqslant i<825$. At about $i=825$ the estimator corrects itself (by chance) so that $y^{\prime} 1 . i, i-1^{\sim_{-j}-j y_{1 . i}}$ and $y^{\prime}{ }_{2 . i, i-1} \approx j y_{2 . i}$ for about $825 \leqslant i<1560$. Differential coding corrects the constant phase error in the detected data symbols over this period. But then, at about $i=1560$, there is another interchange in the channel estimates, and a corresponding extended burst of errors in the detected data symbols up to $i=2000$.

Fig.6.4.11 shows the performance of the final Systems $2 A$ and $2 B$ described in Sec.4.6.2. When four stored vectors ( $m=4$ ) are used in the combined detector and estimator for System 2A, the degradation in performance caused by inaccuracies in the channel estimates, relative to the case with perfect channel estimation, is about 4 dB at the high bit error rates. Though the curve reaches a floor in the bit error rate of about $3 \times 10^{-4}$. With $\mathrm{m}=4$ in System 2 B , the degradation in the performance caused by inaccuracies in the channel estimate, increases from about 1.5 dB to 3.5 dB , as the bit error rate increases. Systems 2 A 4 and 2 B 4 perform better than Systems $2 A C$ and $2 B C$ respectively at bit error rates below about $10^{-3}$. Again this seems to be because the Viterbi detector with 4 stored vectors can switch through any multiple of $90^{\circ}$ and find the channel estimates that best fit the sequence of samples through a deep fade. At error rates above $10^{-3}$, there must be significant error extension effects in the Systems 2A4 and 2B4, because the Systems $2 A C$ and $2 B C$ respectively, perform better.

### 6.5 System 3

## Detection

Fig.6.5.1 shows results for the maximum likelihood detector described in Sec.5.3.2. Clearly there is a close agreement between the theoretical and simulated curves for $16-Q A M$ (no differential coding). Over a non-fading channel, the degradation in tolerance to additive white Gaussian noise caused by the differential coding is about $\frac{1}{2} d B$ at the bit error rate of $10^{-4}$, and increases steadily to just under 2 dB at $10^{-1}$. Over a fading channel, this degradation in performance caused by the differential coding is about 1.6dB for System 3 A and about 1.1 dB for System 3 B .

## Estimation

Fig.6.5.2 shows results for the unbiased estimator described in Sec.5.4.2, which operates with all correctly detected symbols $\left\{s^{\prime}{ }_{i}\right\}$. As expected (Sec.5.4.2) $\lambda_{e} \approx-(\psi+3.26) \mathrm{dB}$. There is no separate prediction here so $y_{i+1, i}^{\prime}{ }^{\prime} y_{i}^{\prime}$. At high signal to noise ratios $\lambda_{p}$ levels off to an error floor which depends on the fading rate, as expected.

Fig.6.5.3 shows that an improvement can be achieved when a degree-1 predictor is used with the unbiased estimator as described in Sec.5.4.3, particularly at high signal-to-noise ratios. About 20 dB can be gained in $\lambda_{p}$ at about $\psi=40 \mathrm{~dB}$, if $\theta$ is optimised.

Figs.6.5.4-6.5.6 show that the arrangement of the Gradient estimator described is Sec.5.4.4 can improve $\lambda_{p}$ still further. If $b$ and $\theta$ are carefully optimized together, this estimator can gain about 2 to 3 dB in $\lambda_{p}$, over the curves in Fig.6.5.3. The values of $b$ and $\theta$ have been roughly optimized as shown in Table 6.5.1.

Fig.6.5.7 shows the results of the two alternative arrangements of the Gradient estimator discussed in Sec.5.4.4. Both alternatives are shown to have a worse value of $\lambda_{p}$ at $\psi=20 \mathrm{~dB}$ than the standard arrangement of the Gradient estimator. This is in fact true over all signal-to-noise ratios of interest, so these alternatives are not considered further.

## Combined Detection and Estimation

Systems $3 A$ and $3 B$, did not give a stable performance without retraining. A total failure in the system typically occurs after about 800 symbols in System 3A and about 12000 symbols in System 3B. Thus regular retraining of the channel estimate must be used. With the arrangement as described in Sec.5.6.2, System 3 with $m \geqslant 2$ was seen to be completely stable.

Fig.6.5.8 shows the performance of the final Systems $3 A$ and $3 B$ described in Sec.5.6.2. The degradation in the performance of Systems 3A4 and $3 B 4$, relative to the case with perfect channel estimation is about 3.5 dB and 1.5 dB respectively.

### 6.6 Comparison of Systems

Fig.6.6.1 shows the performances of the Systems 1,2 and 3 with four stored vectors in the Viterbi detector ( $m=4$ ) and differential coding of the binary digits. As might be expected, the degradation in performance of System 3 caused by inaccuracies in the channel estimate is much greater than for System 1 but is usually less than for System 2. However, it might also be expected that the performance of System 3 would always be better than System 2, because it is well known that in a non-fading channel, no 16 -point constellation has a better tolerance to additive white Gaussian noise than that used in System 3. But it is shown in Fig.6.6.1 that System 2BP gains about ldB in tolerance to noise over System 3BP (although, System 2AP loses about 2.75 dB compared with System 3A). Clearly two antennas should be used at the receiver of System 2 rather than one, if at all possible. So with one receiving antenna, the tolerance to noise of System 1A4 is about 4.5 dB better than for System 3A4, which is in turn about 4 dB better than for System 2A (at low error rates). With two receiving antennas, the tolerance to noise of System 1B4 is about 2.5 to 4 dB better than for System 2B4, which is in turn about 0.5 to' 1 dB better than for System 3B4.


FIg.6.3.1 Systems 1AP and 1BP, maximum likelihood detector (Sec.3.3.2):
Performance of detector operating with perfect channel estimates


Flg.6.3.2 (a) System 1AP, $f_{m}=80 H z$, $(\psi=20 d B)$. (b) System 1BP, $f_{m}=80 H z,(\psi=20 d B)$.
(c) System 1AP, $\mathrm{f}_{\mathrm{m}}=40 \mathrm{~Hz}$, $(\psi=20 \mathrm{~dB})$. (d) System $1 \mathrm{BP}, \mathrm{f}_{\mathrm{m}}=40 \mathrm{~Hz},(\psi=10 \mathrm{~dB})$.

Maximum likelihood detector (Sec.3.3.2): Typical examples of errors in detection
shown against the corresponding channel amplitude


Flg.6.3.3 System 1AC, unbiased estimator, no prediction (Sec.3.4.2): Variation of $\lambda_{e}$ or $\lambda_{p}$ with $\psi$, for different vehicle speeds, $v$

degree-1 predictor

degree-2 predictor


Flg.6.3.5 System 1AC, unbiased estimator with least-squares fading memory polynomial prediction (Sec.3.4.3): Variation of $\lambda_{p}$ with $\psi$ for different values of $\theta$, with (a) degree-1 predictor (b) degree-2 predictor


Flg.6.3.6 System 1AC unbiased estimator with Taylor's expansion predictor (Sec.3.4.5): Variation of $\lambda_{p}$ with $\psi$ for the different $p$-tap predictors



Flg.6.3.8 System 1AC, sinewave estimation scheme (Sec.3.4.6):
Estimate of the the sinewave component $\left\{x_{1}\right\}$ in $\left\{y_{1.1}\right\}$, where $\left\{y_{1.1}^{\prime}\right\}$ is
(a) Sinewave with phase error (b) Sinewave with amplitude error
(c),(d) Typical fading components

$\left\{\begin{array}{l}1,1 \\ \}\end{array}\right.$
fro.z.

$\qquad$
$\left\{c_{0.1}\right.$

Flg.6.3.9 System 1AC, equalizer (Sec.3.4.7): Typical example of equalizer output $\left\{c_{1}\right\}$ shown against the corresponding $\left\{y_{1}\right\}$

$$
v=60 \text { miles/hour }
$$



$$
\mathrm{v}=30 \mathrm{miles} / \text { hour }
$$



Flg.6.3.10 System 1AC, Gradient estimator (Sec.3.4.8): Variation of $\lambda_{0}$ with $b$ for different values of $\psi$, with vehicle speed (a) $v=60 \mathrm{miles} / \mathrm{hour}$ (b) $v=30 \mathrm{miles} /$ hour


Fig.6.3.11 System 1AC, Gradient estimator incorporating feedback from a degree-1 fading memory polynomial predictor (Sec.3.4.9): Variation of $\lambda_{b}$ with $\theta$, for different values of b , with (a) $\psi=5 \mathrm{~dB}$. (b) $\psi=15 \mathrm{~dB}$ (c) $\psi=25 \mathrm{~dB}$ (d) $\psi=35 \mathrm{~dB}$


Fig.6.3.12 System 1BC, Gradient estimator incorporating feedback from a degree-1 fading memory polynomial predictor (Sec.3.4.9): Variation of $\lambda_{p}$ with $\theta$, for different values of b , with (a) $\psi=5 \mathrm{~dB}$ (b) $\psi=15 \mathrm{~dB}$

Table 6.3.1 Values of $b, \theta$ used in the tests

| System | signal-to-noise <br> ratio $(\psi \mathrm{dB})$ | b | $\boldsymbol{\theta}$ |
| :---: | :---: | :---: | :--- |
| 1 A | $\psi \leqslant 10$ | 0.15 | 0.72 |
|  | $10<\psi \leqslant 20$ | 0.16 | 0.64 |
|  | $20<\psi \leqslant 30$ | 0.16 | 0.525 |
|  | $\psi>30$ | 0.17 | 0.45 |
| 1 B | $\psi \leqslant 10$ | 0.14 | 0.72 |
|  | $\psi>10$ | 0.15 | 0.625 |

## System 1AC



## System 1BC



Flg.6.3.13 (a)System 1AC and (b)System 1BC, Gradient estimator with feedback from a degree-1 predictor (Sec.3.4.9): Variation of $\lambda_{p}$ with $\psi$, with $b, \theta$ as in Table 6.3.1

$v=60$ miles/hour
$v=30$ miles/hour

Flg.6.3.14 System 1AC, gradient estimator with double sampling (Sec.3.4.10): Variation of $\lambda_{e}$ with $b$ for different values of $\psi$


Flg.6.3.15 System 1AC, Kalman estimator (Sec.3.4.11): Variation of $\lambda_{e}$ with $\omega$ for different values of $\psi$


Flg.6.3.16 System 1AC, unbiased estimator with degree-1 and 2 fading memory polynomial predictors (Sec.3.4.3): Performance with no retraining (Sec.3.5.4) and ideal retraining (Sec.3.5.2)


Fig.6.3.17 System 1AC, unbiased estimator with degree-1 and 2 fading memory polynomial predictors (Sec.3.4.3): Performance with reset-to-zero retraining (Sec.3.5.j)


Flg.6.3.18 System 1AC, unbiased estimator with degree-1 fading memory polynomial prediction (Sec.3.4.3): Performance with least-squares retraining methods (Sec.3.5.5)


Fig.6.3.19 System 1A1, simple combined detector and estimator (Sec.3.6.1): Performance with no retraining, using different estimation processes


Flg.6.3.20 System 1A1, simple combined detector and estimator (Sec.3.6.1): Typical example of channel prediction $\left\{y^{\prime}{ }_{1.1, i-1}\right\}$, with $+90^{\circ}$ phase shift introduced during a deep fade


Flg.6.3.21 Performance of final Systems 1Am and 1Bm (Sec.3.6.2)


Flg.6.3.22 Performance of System 1Bm with maximal ratio combining (Sec.3.3.4)


Flg.6.4.1(a) System 2AP, maximum likelihood detector (Sec.4.3.2):
Performance of detector with different fixed channel values


Fig.6.4.1(b) System 2AP. Signal constellations used in tests in Fig.6.4.1(a)


Flg.6.4.2 System 2AP, maximum likelihood detector (Sec.4.3.2): Performance of detector in Rayleigh fading, operating with perfect channel estimates

## System 2A




## System 2B




Flg.6.4.3 (a) System 2AP, $\psi=20 \mathrm{~dB}$, (b) System 2BP, $\psi=15 \mathrm{~dB}$. Maximum likelihood detector (Sec.4.3.2): Typical examples of errors in detection shown against the corresponding channel amplitudes


Fig.6.4.4 Systems 2AP and 2BP. Example constellations from tests in Fig.6.4.3
estimation errors

prediction errors


Flg.6.4.5 System 2AC, Gradient estimator without feedback to the degree-1
polynomial predictor (Sec.4.4.3): (a) Variation of $\lambda_{\theta}$ with $b$ for different values of $\psi$.
(b) Variation of $\lambda_{p}$ with $\theta$ for different values of $\psi$, with $b$ roughly optimized in each case.


Fig.6.4.6 System 2AC, Gradient estimator incorporating feedback from a degree-1 fading memory polynomial predictor (Sec.4.4.4): Variation of $\lambda_{8}$ with $\theta$, for different values of $b$, with (a) $\psi=10 \mathrm{~dB}$ (b) $\psi=20 \mathrm{~dB}$ (c) $\psi=30 \mathrm{~dB}$ (d) $\psi=48 \mathrm{~dB}$


Fig.6.4.7 System 2BC, Gradient estimator incorporating feedback from a degree-1 fading memory polynomial predictor (Sec.4.4.4): Variation of $\lambda_{\mathrm{p}}$ with $\theta$, for different values of b , with (a) $\psi=5 \mathrm{~dB}$ (b) $\psi=15 \mathrm{~dB}$ (c) $\psi=25 \mathrm{~dB}$

Table 6.4.1 Values of $b, \theta$ used in the tests

| System | signal-to-noise <br> ratio $(\psi \mathrm{dB})$ | b | $\theta$ |
| :---: | :---: | :--- | :--- |
| 2 A | $\Psi \leqslant 35$ | 0.155 | 0.6 |
|  | $\psi>35$ | 0.21 | 0.4 |
| 2 Z | $\psi \leqslant 10$ | 0.15 | 0.75 |
|  | $10<\psi \leqslant 20$ | 0.15 | 0.7 |
|  | $\psi>20$ | 0.16 | 0.6 |

## System 2AC



System 2BC


Flg.6.4.8 (a)System 2AC and (b)System 2BC, Gradient estimator with feedback from a degree-1 predictor (Sec.4.4.4): Variation of $\lambda_{p}$ with $\psi$, with b, $\theta$ as in Table 6.4.1


FIg.6.4.9 System 2AC (Sec.4.6.1): Performance of estimator with least-squares retraining (Sec.4.5.3), compared with ideal retraining (Sec.4.5.2)
signal 1



Flg.6.4.10 System 2A2 with no retraining and $\psi=20 \mathrm{~dB}$, (Sec.4.6.2): Typical example of estimator output showing an interchange in the estimates of the two channels


Flg.6.4.11 Performance of final Systems 2Am and 2Bm (Sec.4.6.2)


Fig.6.5.1 Systems 3AP and 3BP, maximum likelihood detector (Sec.5.3.2): Performance of detector operating with perfect channel estimates


Flg.6.5.2 System 3AC, unbiased estimator, no prediction (Sec.5.4.2): Variation of $\lambda_{e}$ or $\lambda_{p}$ with $\psi$, for different vehicle speeds, $v$


Flg.6.5.3 System 3AC, unbiased estimator with degree-1 polynomial prediction (Sec.5.4.3): Variation of $\lambda_{p}$ with $\theta$, at different values of $\psi$


Fig.6.5.4 System 3AC, Gradient estimator incorporating feedback from a degree -1 fading memory polynomial predictor (Sec.5.4.4): Variation of $\lambda_{\mathrm{p}}$ with $\theta$, for different values of b , with (a) $\psi=15 \mathrm{~dB}$ (b) $\psi=25 \mathrm{~dB}$ (c) $\psi=35 \mathrm{~dB}$ (d) $\psi=45 \mathrm{~dB}$

$\psi=25 \mathrm{~dB}$

(c)

Fig.6.5.5 System 3BC, Gradient estimator incorporating feedback from a degree- 1 fading memory polynomial predictor (Sec.5.4.4): Variation of $\lambda_{\mathrm{p}}$ with $\theta$, for different values of $b$, with (a) $\psi=5 \mathrm{~dB}$ (b) $\psi=15 \mathrm{~dB}$ (c) $\psi=25 \mathrm{~dB}$

Table 6.5.1 Values of $b, \theta$ used in the tests

| System | signal-to-noise <br> ratio $(\psi \mathrm{dB})$ | b | $\theta$ |
| :---: | :---: | :--- | :--- |
| 3 A | $\psi \leqslant 20$ | 0.025 | 0.6 |
|  | $20<\psi \leqslant 30$ | 0.0325 | 0.45 |
|  | $30<\psi \leqslant 40$ | 0.04 | 0.275 |
|  | $\psi>40$ | 0.045 | 0.1 |
| $3 B$ | $\psi \leqslant 10$ | 0.0325 | 0.7 |
|  | $10<\psi<20$ | 0.0325 | 0.58 |
|  | $\psi \geqslant 20$ | 0.0325 | 0.4 |

System 3A


System 3B


Fig.6.5.6 Systems 3A and 3B, Gradient estimator incorporating feedback from a degree -1 fading memory polynomial predictor (Sec.5.4.4): Variation of $\lambda_{p}$ with $\psi$, for the different values of $b, \theta$ shown in Table 6.5.1


Fig.6.5.7 System 3A, Gradient estimator incorporating feedback from a degree -1 fading memory polynomial predictor (Sec.5.4.4): Variation of $\lambda_{p}$ with $\theta$, at $\psi=20 \mathrm{~dB}$, with variable and constant b


Fig.6.5.8 Performance of final Systems 3Am and 3Bm (Sec.5.6.2)


Fig.6.6.1 Performances of final Systems 1A4, 1B4, 2A4, 2B4, 3A4, $3 B 4$ with differential coding

## Conclusions

Three different transmission systems for narrowband digital modems have been developed in this thesis for use in 900MHz cellular land mobile radio. Though in fact, these systems can be directly applied to any flat Rayleigh fading channel. In each case, near optimum coherent demodulation of a rapidly fading QAM signal(s) has been achieved at the receiver using novel techniques of combined detection and estimation, with regular retraining of the channel estimator.

Computer simulation results show that a bandwidth efficiency of about 2 bit/s/Hz could be achieved together with a good tolerance to additive white Gaussian noise, as long as the binary digits are differentially encoded and decoded as described in Sec.2.3.3. This differential coding is essential to all three systems, to correct for phase ambiguities of $\pm 90^{\circ}$ or $180^{\circ}$ that often occur in the channel estimator after a deep fade. No other error correcting coding is necessary, though about $20 \%$ of the data symbols should be set aside for retraining and synchronization purposes. A theoretical analysis has been carried out wherever possible to confirm the accuracy of the simulation results. Since field trials are due shortly to be carried out with a practical model of the modem, the computer simulation tests have been confined to the idealised conditions described in Chapter 2 and Appendix B. In Sec. 2.8 and Appendix C some quite simple methods of achieving symbol timing recovery and carrier frequency synchronization at the receiver have been described to show that in practice it should be possible to achieve results quite close to those given in Chapters 3 to 6 , where timing and synchronization have been assumed to be ideal. The signal processing involved in Systems 1, 2 and 3 was simple enough that the digital modems could be built with existing hardware. A particular virtue of all three systems is that any expensive or complex equipment needed is usually situated at the base station rather than the mobile..

In Chapter 3, that is, System 1, one 12 kbaud 4 -level QAM signal was transmitted in this fast fading channel. (This signal may alternatively be described as a bandlimited QPSK signal with a considerable envelope
ripple caused by raised cosine spectral shaping). It proved quite easy to achieve coherent demodulation at the receiver and a stable system was achieved without retraining, even with the simplest possible unbiased estimation process. The best system tested (Sec.3.6) used a Gradient (or equivalent unbiased) estimator with the appropriate arrangement of degree-1 least squares fading-memory polynomial prediction. Regular retraining of the channel and slope estimates are carried out using a "least squares straight line" method. Four stored vectors are used in a Viterbi type detector. The tolerance of this system to additive white Gaussian noise was only about $\ddagger \mathrm{dB}$ worse than the theoretical optimum. Viterbi detection is used here to give added stability against er ror extension effects. It is important to note that a large improvement in tolerance to noise can be obtained by using two spatially separated receiving antennas (System 1B) compared with just one receiving antenna (System 1A). This improvement is about 9 dB at a bit error rate of $10^{-2}$ and about 19 dB at $10^{-4}$. This is due to the improvement in the Rayleigh statistics achieved by coherently combining two uncorrelated Rayleigh fading signals. An optimum maximum likelihood combination of these two signals is used in the detector. However, if maximal ratio combining is carried out on the signals at the two receiving antennas before Viterbi detection, then a further loss in performance of almost $\frac{1}{2} \mathrm{~dB}$ would be expected. This seems to be due to the accumulation of errors in the co-phasing operation, as discussed in Sec.3.6.

Probably the most important result obtained from this investigation is the demonstration that it is possible to transmit simultaneously two such 4-level QAM signals in the same frequency space, where the two signals originate from different sources and fade independently at the receiver. Thus, System 2 described in Chapter 4 is a new way of multiplexing two 4-level QAM signals that uses the fact that the fading is independent in the two transmission paths to distinguish between these two signals in the receiver. In fact, this multiplexing method would not work at all well in the absence of fading. It appears that no non-coherent or pilot tone scheme could successfully be used with two fading QAM signals in the channel, which makes this System 2 so interesting. The best version of System 2 tested (Sec.4.6) can be seen as an extension of System 1 described above. That is, the receiver uses a Gradient algorithm estimator with fading memory prediction, a Viterbi type detector and a least-squares retraining method. However, because of the way the two
independently fading signals add together in the channel, it was shown that two antennas must be used at the receiver (System 2B) for satisfactory operation to be achieved. In this system, regular retraining and Viterbi detection are now essential for a stable operation. Differential coding does not achieve stability on its own. The tolerance to noise of this System 2B with four vectors in the Viterbi detector is about 1.5 to 2 dB worse than for perfect channel estimation. With one receiving antenna (System 2A) this loss is about 4 dB at low signal-tonoise ratios, with an error floor of about $3 \times 10^{-4}$ at high signal-to-noise ratios.

In Chapter 5, System 3 is described in which one 12 kb aud 16 -level QAM signal is transmitted over this same flat fading channel. Again, the best System 3 tested (Sec.5.7) is very similar to that of System 1. That is, the receiver uses a Gradient estimator with fading memory prediction, a Viterbi type detector and an exactly similar retraining method to that of System 1. The Viterbi detector and regular retraining are necessary here for stable system operation, as was the case for System 2. This suggests that the instability in System 2 was caused by the received 16 -point constellation as much as by the fact that there are two fading channels to track. The tolerance to noise of this System 3 with four stored vectors in the Viterbi detector is about 3.5 (1.5) dB worse than for perfect channel estimation with one (two) receiving antennas.

A comparison of the results of Systems 1, 2 and 3 brings out a most important point. With two receiving antennas and perfect channel estimation, the loss in tolerance to noise of System 3 compared with System 1, with the same average transmitted energy per bit is about 3.5dB. This is actually worse than for System 2 by about 1dB. In contrast, with one receiving antenna, System 3 gains about 2.75 dB in tolerance to noise over System 2. This shows that with two receiving antennas, two 4-level QAM signals fading independently can be received simultaneously in the same frequency band with such little interference between each other, that System 2 is more power efficient than System 3. This is surprising since the 16 -point constellation of System 3 is the most power efficient fixed 16-point constellation there is. This advantage of System 2 over System 3 is even more marked when considering peak-power limited transmitters, which is generally the case for the mobile transmitters.

Any further work to be done in continuing this investigation should really concentrate on System 2, since this is the most novel and potentially most useful, bandwidth efficient system of the future. (As well as probably having the most scope for improvement).

One way to improve the system performance would be to improve the estimation process. Two possible improvements might be to, adjust the estimator parameters $b, \theta$ according to a measured signal-to-noise ratio, or to use double sampling to effectively halve the fading rate. Two obvious improvements worth testing would be to increase the number of receiving antennas and the number of vectors in the Viterbi detector, if this ever became cost effective. It would also be useful to include in the computer simulations the error correcting codes that have been specifically designed for this system, particularly if they can prevent the two messages from interchanging in the receiver.

System 2 could be simulated under less idealistic conditions, since it is important when designing a cellular radio system to know how the performance would be degraded by correlated fading at the two receiving antennas, or by co-channel or adjacent channel interference. There may also be problems when the average received energy of one signal is at a much higher level than that of the other.

It would also be worth exploring the possibilities of adapting System 2 for different applications. In particular, whether it is possible to achieve acceptable performance in the presence of intersymbol
interference, or indeed whether it is possible to transmit 3 or more QAM signals in the same channel.

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## Rayleigh fading in an unmodulated carrier signal

In a typical full-duplex mobile radio link, radio signals are passed between mobile (M) and base station (B). The mobile unit is generally surrounded by hills, tall buildings and other mobiles whereas the base station is mounted upon a nearby rooftop, or any other convenient high point. Generally, there is no direct line-of-sight path between mobile and base station so the mode of radio wave propagation from transmitter to receiver is largely by way of scattering from hills, buildings and other mobiles as in Fig.A.1. Since each path in Fig.A.l represents a radio wave travelling at the speed of light, different path lengths mean that there will be a corresponding time difference in the arrival of the wave along each path.

Consider the case where the signal modulation is removed and only a continuous tone (unmodulated carrier) $P(t)$ is transmitted from the base station to the mobile $(B \rightarrow M)$, where

$$
\begin{equation*}
P(t)=V \cos \omega_{C} t \tag{A.1}
\end{equation*}
$$

The signal $P(t)$ is propagated along $N$ different paths (Fig.A.1). The signals that are transmitted simultaneously along each path will actually arrive at the receiver at different times. But, all are continuous sine waves and only the result of their superposition is actually seen. The only effect of the differences in path delays will now be to introduce relative phase shifts on the component tones. The N different signals may add either constructively or destructively according to the values of the relative phase shifts to give a single resultant phasor at the mobile receiving antenna. When the mobile moves through the scattering medium, changes in different path lengths occur continually and randomly and the observed resultant carrier will correspondingly change randomly in envelope and RF (radio frequency) phase relative to a fixed phase reference.

If the $N$ individual received phasors of random phase are resolved into quadrature components, then it can be readily shown that these quadrature components are uncorrelated [10]. And, since all N paths are independent, the quadrature components of the resultant carrier independently approach


Fig.A. 1 Scattering model of signal propagation

Gaussian variates as the number of phasors increases. This latter result follows from the Central Limit Theorem [36]. Actually it has been noted that as few as $N=6$ contributing phasors gives a good approximation to Gaussian bahaviour [23].

Now, the resultant received carrier phasor at time $t$ can be represented mathematically as

$$
\begin{equation*}
V Y(t)=V A(t) \cos \left(\omega_{c} t+\theta(t)\right) \tag{A.2}
\end{equation*}
$$

Where $\omega_{C}=2 \pi f_{C}$ is the angular frequency of the carrier in radians/second. The carrier frequency is $f_{c}=900 \mathrm{MHz} . A(t), \theta(t)$ are the continuously, randomly changing amplitude and phase respectively of the received carrier.

It is shown later that $Y(t)$ is completely contained within a narrow band of frequencies around $f_{c}$. So this narrowband process $Y(t)$ expressed by Eq. (A.2) can be expanded from polar to Cartesian coordinates as

$$
\begin{align*}
Y(t) & =A(t) \cos \theta(t) \cos \omega_{C} t-A(t) \sin \theta(t) \sin \omega_{C} t \\
& =Y_{I}(t) \cos \omega_{C} t-Y_{Q}(t) \sin \omega_{C} t \tag{A.3}
\end{align*}
$$

where,

$$
\begin{equation*}
y_{I}(t)=A(t) \cos \theta(t), \quad Y_{Q}(t)=A(t) \sin \theta(t) \tag{A.4}
\end{equation*}
$$

and,

$$
\begin{equation*}
A(t)=\sqrt{Y_{I}^{2}(t)+Y_{Q}{ }^{2}(t)}, \quad \theta(t)=\tan ^{-1}\left(y_{Q}(t) / Y_{I}(t)\right) \tag{A.5}
\end{equation*}
$$

So the quadrature components $y_{I}(t), y_{Q}(t)$ are identically distributed, statistically independent lowpass Gaussian random variables, each with zero mean and variance $\sigma^{2}$ [18-21]. The problem here is to determine the statistics of the random envelope $A(t)$ and of the random phase $\theta(t)$. This is done by firstly finding the joint statistics of $A(t)$ and $\theta(t)$. Then integrating this over all possible values of A gives the probability density function of $\theta$, and vice versa.

From the independence and Gaussian statistics of $Y_{I}$ and $Y_{Q}$ (discarding "(t)" here for clarity), the joint probability density function of $y_{I}$ and $Y_{Q}$ is

$$
\begin{align*}
f_{Y_{I}} Y_{Q}\left(y_{I}, Y_{Q}\right) & =f_{Y_{I}\left(y_{I}\right) \cdot f_{Y_{Q}}\left(y_{Q}\right)} \\
= & \frac{\exp \left(-y_{I}{ }^{\left.2 /\left(2 \sigma^{2}\right)\right)}\right.}{\sqrt{2 \pi \sigma^{2}}} \times \frac{\exp \left(-y_{Q}{ }^{\left.2 /\left(2 \sigma^{2}\right)\right)}\right.}{\sqrt{2 \pi \sigma^{2}}} \\
& =\frac{\exp \left(-\left(y_{I}{ }^{2}+y_{Q}{ }^{2}\right) /\left(2 \sigma^{2}\right)\right)}{2 \pi \sigma^{2 *}} \tag{A.6}
\end{align*}
$$

Where $\sigma^{2}$ is the variance of both $y_{I}$ and $Y_{Q}$. Now, substituting Eq. (A.4)
into Eq.(A.6),

$$
\begin{equation*}
f_{I} Y_{Q}\left(y_{I} Y_{Q}\right)=\frac{\exp \left(-A^{2} / 2 \sigma^{2}\right)}{2 \pi \sigma^{2}} \tag{A.7}
\end{equation*}
$$

The next step is to find the joint probability density function of $A$ and $\theta$. A well-known formula for transforming differential areas [24,25] is given by

$$
\begin{equation*}
d y_{I} \cdot d y_{Q}=A \cdot d A \cdot d \theta \tag{A.8}
\end{equation*}
$$

Now the probability density function for the polar coordinates $A, \theta$ is given from

$$
\begin{equation*}
f_{A \theta}(A, \theta) \cdot d A \cdot d \theta=f y_{I} y_{\dot{Q}}\left(y_{I}, y_{Q}\right) \cdot d y_{I} \cdot d y_{Q} \tag{A.9}
\end{equation*}
$$

Substituting Eqs.(A.7) and (A.8) in Eq. (A.9)

$$
f_{A \theta}(A, \theta) \cdot d A \cdot d \theta=\frac{A \cdot \exp \left(-A^{2} /\left(2 \sigma^{2}\right)\right)}{2 \pi \sigma^{2}} \cdot d A \cdot d \theta
$$

Therefore,

$$
\begin{equation*}
f_{A \theta}(A, \theta)=\frac{A \cdot \exp \left(-A^{2} /\left(2 \sigma^{2}\right)\right)}{2 \pi \sigma^{2}} \tag{A.10}
\end{equation*}
$$

To find the density function for the phase alone, $f_{\theta}(\theta)$, simply average Eq.(A.10) over all possible. amplitudes. So,

$$
\begin{align*}
f_{\theta}(\theta) & =\int_{0}^{\infty} f_{A \theta^{\prime}}(A, \theta) d A  \tag{A.11}\\
& =\frac{1}{2 \pi} \int_{0}^{\infty} \frac{A \cdot \exp \left(-A^{2} /\left(2 \sigma^{2}\right)\right)}{\sigma^{2}} \cdot d A \\
& =\frac{1}{2 \pi}\left[-\exp \left(-A^{2} /\left(2 \sigma^{2}\right)\right)\right]_{0}^{\infty}
\end{align*}
$$

(And since $\frac{d}{d A}\left(\exp \left(-A^{2} /\left(2 \sigma^{2}\right)\right)=-\frac{A}{\sigma^{2}} \exp \left(-A^{2} /\left(2 \sigma^{2}\right)\right)\right.$ )

$$
\begin{equation*}
\mathrm{f}_{\theta}(\theta)=\frac{1}{2 \pi} \tag{A.12}
\end{equation*}
$$

This is the uniform phase distribution and is depicted in Fig.A.2.
Similarly, to find the probability density function $f_{A}(A)$ for the amplitude alone, simply average Eq. (A.10) over all possible phases. So,

$$
\begin{aligned}
f_{A}(A) & =\int_{-\pi}^{\pi} f_{A \theta}(A, \theta) d \theta \\
& =A \cdot \frac{\exp \left(-A^{2} /\left(2 \sigma^{2}\right)\right)}{2 \pi \sigma^{2}} \int_{-\pi}^{\pi} d \theta
\end{aligned}
$$



Fig.A. 2 Two alternative representations (a),(b) of the uniform phase distribution


Fig.A. 3 Rayleigh amplitude distribution

$$
=A \cdot \frac{\exp \left(-A^{2} /\left(2 \sigma^{2}\right)\right)}{\sigma^{2}}
$$

This is the Rayleigh distribution -hence the term "Rayleigh fading" - and is shown in Fig.A.3.

This completes the discussion of the statistical properties of the Rayleigh fading. It has been assumed that $Y(t)$ is a narrowband random process, though the actual frequency content of $Y(t)$ has not yet been defined. This frequency information is now determined by examining the mobile radio scattering model more closely, according to the well-known Clarke model [10].

We are still considering the transmission of an unmodulated carrier from base station to mobile. Remember, the unmodulated carrier $S(t)$.. (Eq.(A.l)) is received at the mobile as a time varying random phasor VY(t) (Eq.A.2)). The received carrier component $Y(t)$ can be represented either in polar form (Eq.A.2)) with amplitude $A(t)$ and phase $\theta(t)$ or equivalently in Cartesian form (Eq. (A.3)) with in-phase and quadrature components $Y_{I}(t), Y_{Q}(t)$ respectively. The signal received by the mobile at any point would consist of a large number ( $N$ ) of generally horizontally travelling uniform plane waves, (Fig.A.1) that are all independent of each other. The amplitudes, phases and angles of arrival of these waves relative to the direction of vehicle motion are random. It is shown in Fig.A. 4 that the vehicle motion in this horizontal plane introduces a Doppler frequency shift $f_{d}$ in every wave, where for the $n{ }^{\text {th }}$ wave

$$
\begin{equation*}
f_{d}(n)=\frac{v}{\lambda} \cos \gamma_{n}=f_{m} \cos \gamma_{n} \tag{A.14}
\end{equation*}
$$

$f_{m}=v / \lambda$ is the maximum Doppler frequency shift ( Hz ) at vehicle speed $v$ (metres/second) and carrier wavelength $\lambda$ (metres).

The total field at any received location is given by the superposition of the $N$ component waves. The $n^{\text {th }}$ wave of amplitude $V C_{n}$ arrives at angle $\gamma_{n}$ to the direction of motion, with Doppler frequency shift $f_{d}(n)$ and random phase $\theta_{n}$. The transmitted signal is vertically polarized and the polarization is assumed to remain unchanged during transmission [9,10]. The electric field component seen at the mobile can be written

$$
\begin{equation*}
E_{z}(t)=v \sum_{n=1}^{N} c_{n} \cos \left(w_{c} t+w_{d}(n) t+\theta_{n}\right) \tag{A.15}
\end{equation*}
$$

where

$$
\begin{equation*}
W_{d}(n)=2 \pi f_{d}(n) \tag{A.16}
\end{equation*}
$$

$\omega_{C}$ is the angular frequency of the carrier in radians per second. The $\left\{C_{n}\right\}$ are normalized so that $\sum_{n=1}^{N} C_{n}{ }^{2}=1$. Therefore the received signal


Fig.A. 4 Geometry of mobile unit communication
power $V^{2}$ is assumed constant.
A vehicle speed $v=26.8$ metres/second $=60 \mathrm{miles} / \mathrm{hour}$, and a carrier frequency $f_{c}=900 \mathrm{MHz}$ ( $\lambda \approx 0.33$ metres) are assumed throughout this thesis. So the maximum Doppler frequency shift is $f_{m}=v / \lambda \approx 80 H z$. Since $f_{m} \ll f_{c}$ the electric field component at the receiver is a narrowband random process.

The signal at the terminals of the receiving antenna on the mobile can now be considered. The spectrum of the electric field component of this signal will consist of a set of spectral lines that occur at random in the range $\pm f_{m}$ about the carrier frequency $f_{c}$. The probability that one of these spectral lines will occur in the range from $f_{c}+f$ to $f_{c}+f+d f$ is given by the probability density function $f(f)$ which may be obtained $[10,20]$ from the probability density function $f(\gamma)$ by equating the differential probabilities

$$
\begin{equation*}
f(f)|d f|=V(f(+\gamma)+f(-\gamma))|d \gamma| \tag{A.17}
\end{equation*}
$$

since $+\gamma$ and $-\gamma$ give the same Doppler shift. Where $f(\gamma)$ is the probability distribution of angular wave arrival. So $f(\gamma) d \gamma$ is proportional to the total power of the plane waves arriving within dr of angle $\gamma$. But from Eq. (A.14),

$$
\begin{equation*}
f(\gamma)=f_{c}+f_{m} \cos \gamma \tag{A.18}
\end{equation*}
$$

Therefore, taking derivatives of both sides with respect to $\gamma$

$$
\frac{d f(\gamma)}{d \gamma}=-f{ }_{m} \sin \gamma
$$

So,

$$
\begin{equation*}
d \gamma=\frac{-\mathrm{df}(\gamma)}{\mathrm{f}_{\mathrm{m}} \sin \gamma} \tag{A.19}
\end{equation*}
$$

But from Eq. (A.18)

$$
\cos \gamma=\frac{f(\gamma)-f_{c}}{f_{m}}
$$

Therefore

$$
\begin{equation*}
\sin \gamma=\sqrt{1-\cos ^{2} \gamma}=\sqrt{1-\left(\left(f(\gamma)-f_{c}\right) / f_{m}\right)^{2}} \tag{A.20}
\end{equation*}
$$

Substituting Eq.(A.20) into Eq.(A.19)

$$
d \gamma=-\frac{1}{f_{m} \sqrt{1-\left(\left(f(\gamma)-f_{c}\right) / f_{m}\right)^{2}}}
$$

Therefore substituting this into Eq. (A.17)

$$
\begin{equation*}
f(f)=\frac{V \cdot(f(+\gamma)+f(-\gamma))}{f_{m} \sqrt{1-\left(\left(f(\gamma)-f_{c}\right) / f_{m}\right)^{2}}} \tag{A.21}
\end{equation*}
$$

In this general mobile radio propagation model, no assumptions are made about the actual positions of any local scatterers. The mobile is
assumed to be surrounded by a uniform ring of scatterers and so the distribution of power with arrival angle $\gamma$ is a uniform distribution. That is,

$$
\begin{equation*}
f(\gamma)=\frac{1}{2 \pi}, \quad \text { for } \quad-\pi<\gamma \leqslant \pi \tag{A.22}
\end{equation*}
$$

So now, Eq(A.21) becomes

$$
\begin{equation*}
f(f)=V \cdot \frac{1}{\pi f_{m} \sqrt{1-\left(\left(f-f_{C}\right) / f_{m}\right)^{2}}} \tag{A.23}
\end{equation*}
$$

The power spectrum of the received electric field, $\left|E_{z}(f)\right|^{2}$, is generally the average energy of the electric field in the frequency range $f$ to $f+d f$, and is given by

$$
\begin{equation*}
\left|E_{z}(f)\right|^{2}=f(f) \cdot g(\gamma) \tag{A.24}
\end{equation*}
$$

Where $g(\gamma)$ is the gain of the antenna in the angular direction $\gamma$.
The practical case of most frequent interest is that for a vertical monopole antenna, which receives this vertically polarized electric field component. All antennas are assumed to be omnidirectional, vertical monopole antennas in this thesis with gain pattern

$$
g(\gamma)=1, \quad \text { for }-\pi<\gamma \leqslant \pi
$$

Thus,

$$
\left|E_{z}(f)\right|^{2}=V \cdot \frac{1}{\pi f_{m} \sqrt{1-\left(\left(f-f_{c}\right) / f_{m}\right)^{2}}}
$$

So when a unit amplitude ( $\mathrm{V}=1$ ) unmodulated carrier wave is transmitted, the signal spectrum at the receiver antenna terminals is

$$
|Y(f)|^{2}= \begin{cases}\frac{1}{\pi f_{m} \sqrt{1-\left(\left(f-f_{c}\right) / f_{m}\right)^{2}}} & \text { for } f_{c}-f_{m} \leqslant f \leqslant f_{c}+f_{m}  \tag{A.25}\\ 0, & \text { elsewhere }\end{cases}
$$

Since the gain, $g(\gamma)=1$, the mean received power is

$$
\begin{equation*}
\int_{f_{c}-f_{m}}^{f_{m}}|Y(f)|^{2} d f=1 \tag{A.26}
\end{equation*}
$$

Now, since the in-phase and quadrature components $y_{I}(t), y_{Q}(t)$ at any time $t$, are independent narrowband Gaussian random variables, their power spectra are

$$
\left|Y_{I}(f)\right|^{2}=\left|Y_{Q}(f)\right|^{2}= \begin{cases}\frac{1}{2 \pi f_{m} \sqrt{1-\left(f / f_{m}\right)^{2}}} & \text { for }-f_{m} \leqslant f \leqslant+f_{m}  \tag{A.27}\\ 0, & \text { elsewhere }\end{cases}
$$

This power spectrum, shown in Fig.A. 5 is $|Y(f)|^{2}$ (Eq.(A.25)) shifted down to baseband, with the mean received power in each quadrature path equal to


Flg.A. 5 Power spectra of $y_{l}(t), y_{Q}(t)$


Fig.A. 6 Autocorrelation function with time of $y_{1}(t), y_{0}(t)$

$$
\begin{equation*}
\int_{-f_{m}}^{f_{m}}\left|y_{I}(f)\right|^{2} d f=\frac{1}{2} \tag{A.28}
\end{equation*}
$$

The autocorrelation function $R_{Y Y}(\tau)$ of these quadrature components gives an indication of how the fading is likely to change in short periods of time, $\tau . R_{y y}(\tau)$ is given by the inverse Fourier transform of $\left|y_{I}(f)\right|^{2}$. That is,

$$
\begin{align*}
& R_{Y Y}(\tau)=\int_{-\infty}^{\infty}\left|Y_{I}(f)\right|^{2} \exp (j 2 \pi f \tau) d f  \tag{A.29}\\
& =\frac{1}{2 \pi f_{m}} \int_{-f}^{f_{m}} \frac{\exp (j 2 \pi f \tau)}{\sqrt{1-\left(f / f_{m}\right)^{2}}} d f \\
& =\frac{1}{2 \pi f_{m}} \int_{-f_{m}}^{f_{m}} \underbrace{\frac{\cos 2 \pi f \tau}{\sqrt{1-\left(f / f_{m}\right)^{2}}}}_{\text {EVEN }} d f+\underbrace{j \frac{1}{2 \pi f_{m}}}_{\text {function }} \int_{\text {ODD }}^{f_{m}} \underbrace{\underbrace{\frac{\sin 2 \pi f \tau}{\sqrt{1-(f / f})^{2}}}}_{\text {function }} d f \\
& =\frac{1}{\pi f} \int_{m}^{f} m \frac{\cos 2 \pi f \tau}{\sqrt{1-\left(f / f_{m}\right)^{2}}} d f \tag{A.30}
\end{align*}
$$

To solve this integral,
Substitute

$$
f=f_{m} \sin \zeta
$$

therefore,

$$
\begin{aligned}
\left(f / f_{m}\right)^{2} & =\sin ^{2} \zeta \\
\sqrt{1-\left(f / f_{m}\right)^{2}} & =\sqrt{1-\sin ^{2} \zeta}=\cos \zeta
\end{aligned}
$$

and,

$$
d f=f_{m} \cos \zeta d \zeta
$$

The limits of the integral (Eq.(A.30)) in terms of $\zeta$ are given by when $f=f_{m}, \quad f_{m}=f m \sin \zeta$, therefore $\sin \zeta=1$, so $\zeta=\pi / 2$ when $\mathrm{f}=0, \quad 0=\mathrm{f}_{\mathrm{m}} \sin \zeta$, therefore $\sin \zeta=0$, so $\zeta=0$

The integral Eq.(A.30) now becomes

$$
\begin{align*}
R_{Y Y}(\tau) & =\frac{1}{\pi f_{m}} \int_{0}^{\pi / 2} \frac{\cos \left(2 \pi f_{m} \tau \cdot \sin \zeta\right) f_{m} \cos \zeta d \zeta}{\cos \zeta} \\
& =\frac{1}{2} \cdot \frac{2}{\pi} \int_{0}^{\pi / 2} \cos \left(2 \pi f_{m} \tau \cdot \sin \zeta\right) d \zeta \\
& =\frac{1}{2} J_{0}\left(2 \pi f_{m} \tau\right) \tag{A.31}
\end{align*}
$$

where $J_{0}()$ is the zero-order Bessel function of the first kind. $R_{Y Y}(\tau)$ is shown in Fig.A.6. In a similar way, it may be shown that the autocorrelation function of $Y(t)$ in Eq. (A.3) is $J_{0}\left(2 \pi f f_{m}\right)$.

# Baseband equivalent model: Mathematical derivation and computer simulation 

The general model assumed in this thesis for the digital speech/data communication system between one transmitting antenna and one receiving antenna is shown in Fig.B.1. This is a synchronous serial system which transmits the information carried in the sequence of binary digits $\left\{\alpha_{i}\right\}$, where $\alpha_{i}=0$ or 1 . The coded signal is a sequence of multilevel complexvalued symbols $\left\{s_{i}\right\}$ which are uniquely determined by the $\left\{\boldsymbol{\alpha}_{i}\right\}$ (see Sec.2.3). So, if the transmitted modulated carrier signal $S(t)$ is a 4-level QAM signal, then $s_{i}$ is a complex-valued symbol with four possible values $\pm l \pm j$. Each $s_{i}$ is determined from a corresponding adjacent pair of binary digits $\alpha_{i, 1}, \alpha_{i, 2}$ as described in Sec.2.3. Alternatively if a 16-level QAM signal $S(t)$ is transmitted, then $s_{i}$ has one of sixteen possible values $( \pm 1$ or $\pm 3)+( \pm j$ or $\pm 3 j)$, determined by a corresponding set of four binary digits $\alpha_{i, 1}, \alpha_{i, 2}, \alpha_{i, 3}, \alpha_{i, 4}$ (see Sec.2.3). However, the linear mobile radio channel shown in Fig. B. 1 between the $\left\{s_{i}\right\}$ and the $\left\{r_{i}\right\}$, is exactly the same for both of these QAM signals.

First of all in this Appendix, the detailed mathematical model of the linear bandpass channel is given. Then the equivalent linear baseband channel is derived. It is shown that a useful simplification to this baseband model can be made, but only after carefully analysing the effect of the Rayleigh fading on the matched filtering. Finally, the method of simulating the linear baseband channel on a digital computer is described.

## B. 1 Bandpass and baseband equivalent models

The linear bandpass channel is shown in Fig.B.2. The important details of this bandpass channel have been described in Sec.2.2-2.3. The mathematical representation and statistical properties of the signals at each point in Fig.B. 2 are now described.

The signals at the outputs of the two lowpass filters in the transmitter are $\sum_{i} S_{I . i} a(t-i T)$ and $\sum_{i} S_{Q . i} a(t-i T)$. Where, for a 4-level QAM signal


Fig.B. 1 Digital communication system


Fig.B. 2 Detailed bandpass model of digital communication system

$$
\begin{equation*}
s_{i}=s_{I_{. i}}+j s_{Q . i}= \pm 1 \pm j \tag{B.1}
\end{equation*}
$$

or for a 16-level QAM signal

$$
\begin{equation*}
s_{i}=s_{I_{. i}}+j s_{Q . i}=( \pm 1 \text { or } \pm 3)+( \pm j \text { or } \pm 3 j) \tag{B.2}
\end{equation*}
$$

And $a(t)$ is the real-valued impulse response of the lowpass filter.
For all integer values $\{i\}, s_{I_{. i}}$ and $s_{Q . i}$ are statistically independent and equally likely to be any one of their possible values. So it follows that for a 4-level QAM signal, the mean-square value of the complex-valued quantity $s_{i}$ is

$$
\begin{equation*}
E_{s}=\overline{\left|s_{i}\right|^{2}}=2 \tag{B.3}
\end{equation*}
$$

and in fact $E_{s . i}=2$ for all \{i\}. The mean-square value of the $\left\{s_{i}\right\}$ per transmitted bit of information is

$$
\begin{equation*}
E_{b}=\frac{1}{2} E_{S}=1 \tag{B.4}
\end{equation*}
$$

For a 16 -level QAM signal, the corresponding mean-square values are

$$
\begin{equation*}
E_{S}=\overline{\left|s_{i}\right|^{2}}=\frac{1}{}\left(1^{2}+1^{2}\right)+\frac{1}{2}\left(1^{2}+3^{2}\right)+\frac{1}{2}\left(3^{2}+3^{2}\right)=10 \tag{B.5}
\end{equation*}
$$

and

$$
\begin{equation*}
E_{b}=4 E_{s}=2.5 \tag{B.6}
\end{equation*}
$$

The lowpass filter with impulse response $a(t)$ has a root-raised-cosine frequency response given by

$$
A(f)=H^{\frac{1}{2}}(f)= \begin{cases}\left.\sqrt{\frac{1}{2} T(1+\cos \pi f T}\right) & \text { for } \frac{-1}{\bar{T}} \leqslant f \leqslant \frac{1}{T}  \tag{B.7}\\ 0, & \text { elsewhere }\end{cases}
$$

and is shown in Fig.B.3(a). $H(f)=A^{2}(f)$ is the transfer function of the transmitter and receiver filters in cascade. In this filter it is assumed that $T=1 / 12000 \mathrm{sec}$, so the transmitted QAM signal $\sqrt{2} \mathrm{~S}(\mathrm{t})$ occupies a bandwidth of 24 kHz . The impulse response $a(t)$ is the inverse Fourier transform of $A(f)$. That is,

$$
\begin{align*}
a(t) & =\int_{-\infty}^{\infty} A(f) e^{j 2 \pi f t} d f  \tag{B.8}\\
& =\sqrt{\frac{T}{2}} \int_{-1 / T}^{1 / T} \sqrt{1+\cos \pi f T} \cdot \exp (j 2 \pi f t) d f \\
& =\sqrt{\frac{T}{2}} \int_{-1 / T}^{1 / T} \sqrt{1+2 \cos ^{2} \frac{\pi f T}{2}-1} \cdot \exp (j 2 \pi f t) d f \\
& =\sqrt{T} \int_{-1 / T}^{1 / T} \cos \frac{\pi f T}{2} \cdot \exp (j 2 \pi f t) d f \\
& =\sqrt{T} \frac{1 / T}{2} \int_{-1 / T}^{1 / \exp \left(\frac{j \pi f T}{2}\right.}+\exp \left(-j \frac{\pi f T}{2}\right] \exp (j 2 \pi f t) d f
\end{align*}
$$



Flg.B. 3 (a) Frequency response and (b) impulse response of ideal root-raised cosine lowpass filter


Flg.B. 4 (a) Frequency response and (b) impulse response of a practical root-raised cosine lowpass filter. (With $1 / \tau=12000 \mathrm{~Hz}, \mathrm{~d}=12.5 \mathrm{~T}$ seconds)
(see [31] for the solution to a similar problem). Therefore,

$$
\begin{equation*}
a(t)=\frac{1}{\sqrt{T}} \frac{\sin \pi\left(2 t / T+\frac{1}{2}\right)}{\pi\left(2 t / T+\frac{1}{2}\right)}+\frac{1}{\sqrt{T}} \frac{\sin \pi\left(2 t / T-\frac{1}{2}\right)}{\pi\left(2 t / T-\frac{1}{2}\right)} \tag{B.9}
\end{equation*}
$$

This is shown in Fig.B.3(b). Theoretically $a(t)$ is of infinite duration, so in practice it is limited to $d$ seconds duration. This impulse response must then be delayed in time by $\mathrm{d} / 2$ seconds to make it physically realisable (causal). This in fact changes the frequency response from zero phase to linear phase, as shown in Fig.B.4, without affecting the amplitude response.

Now in every case a single transmitted signal element at the input to the multipliers has the waveform $s_{i} a(t-i T)$, with $s_{I_{.}} a(t-i T)$ in the in-phase channel and $s_{Q . i}(t-i T)$ in the quadrature channel. The Fourier transform (frequency spectrum) of the signal element is $s_{i} \exp (-j 2 \pi f i T) A(f)$. Thus, its energy density spectrum is

$$
\begin{equation*}
\left|s_{i} \exp (-j 2 \pi f i T) A(f)\right|^{2}=\left|s_{i}\right|^{2}|A(f)|^{2} \tag{B.10}
\end{equation*}
$$

and its energy is

$$
\begin{equation*}
E_{s . i}=\left|s_{i}\right|^{2} \int_{-\infty}^{\infty}|A(f)|^{2} d f \tag{B.11}
\end{equation*}
$$

Since the signal elements are statistically independent and have zero means, making them statistically orthogonal ( $E\left[s_{i} \cdot s_{j}\right]=0$, $i \neq j$ ), the average transmitted energy per signal element at the output of the lowpass filter in the transmitter (Fig.B.2) is the average or expected value of $E_{s . i}$, and so is

$$
\begin{equation*}
E_{s}=\left|s_{i}\right|^{2} \int_{-\infty}^{\infty}|A(f)|^{2} d f \tag{B.12}
\end{equation*}
$$

where $\bar{x}$ is the average or expected value of $x$. But [31],

$$
\int_{-\infty}^{\infty}|\mathrm{A}(\mathrm{f})|^{2} \mathrm{df}=\int_{-1 / T}^{1 / T} T(1+\cos \pi f T) \mathrm{df}=1
$$

so

$$
\begin{equation*}
E_{s}=\overline{\left|s_{i}\right|^{2}} \tag{B.13}
\end{equation*}
$$

Thus the lowpass filtering introduces no change in signal level. Therefore, the average energy per bit in the signal at the input to the multipliers in the transmitter (Fig.B.2) is

$$
\begin{array}{ll}
E_{b}=\frac{1}{2} E_{s}=\frac{1}{2} \overline{\left.s_{i}\right|^{2}}=1, & \text { for 4-level QAM } \\
E_{b}=\frac{1}{4} E_{s}=\frac{1}{\left|s_{i}\right|^{2}}=2.5, & \text { for 16-level QAM } \tag{B.14}
\end{array}
$$

Now, after multiplying by the quadrature carrier components $\sqrt{2} \cos 2 \pi f{ }_{c} t$ and $\sqrt{2} \sin 2 \pi f_{c} t$, the 4 (or 16 )-level QAM signal at the input to the transmitter bandpass filter (Fig.B.2) is

$$
\begin{equation*}
\sqrt{2} S(t)=\sqrt{2} \sum_{i} S_{I \cdot i} a(t-i T) \cos 2 \pi f_{C} t-\sqrt{2} \sum_{i} S_{Q \cdot i} a(t-i T) \sin 2 \pi f_{C} t \tag{B.15}
\end{equation*}
$$

where

$$
\begin{equation*}
S(t)=s_{I}(t) \cos 2 \pi f_{C} t-s_{Q}(t) \sin 2 \pi f_{c} t \tag{B.16}
\end{equation*}
$$

with complex envelope

$$
\begin{equation*}
s(t)=s_{I}(t)+j s_{Q}(t) \tag{B.17}
\end{equation*}
$$

 quadrature components respectively of $S(t)$.

The factor $\sqrt{2}$ in $\sqrt{2} \cos 2 \pi f_{c} t$ and $-\sqrt{2} \sin 2 \pi f_{c} t$ gives each of these signals a mean-square value (average power level) of unity $[31,36,55]$. Thus the modulation (/ mixing / multiplying) process introduces no change in signal level, since $\mathrm{f}_{\mathrm{C}} \gg 1 / \mathrm{T}[31,36,55]$.

The bandpass filter in the transmitter has no effect on the signal $S(t)$, which passes through unchanged. However, this bandpass filter is required in a practical modem to remove the spurious frequency components that would be generated in the modulator.

So, the average transmitted energy per element of the real-valued modulated-carrier signal $\sqrt{2} \mathrm{~S}(\mathrm{t})$ is

$$
\begin{align*}
E_{S}=\overline{\left|s_{i}\right|^{2}} & =2, & & \text { for 4-level QAM } \\
& =10, & & \text { for } 16 \text {-level QAM } \tag{B.18}
\end{align*}
$$

and the average transmitted energy per bit is

$$
\begin{align*}
E_{b} & =\frac{1}{2} \overline{\left|s_{i}\right|^{2}}=1, \quad & & \text { for 4-level QAM } \\
& =\frac{1}{4} \overline{\left|s_{i}\right|^{2}}=2.5, & & \text { for } 16 \text {-level QAM } \tag{B.19}
\end{align*}
$$

Now the transmitted QAM signal expressed by Eqs.(B.15)-(B.17)
undergoes flat Rayleigh fading: The* Rayleigh fading is represented by its complex envelope

$$
\begin{equation*}
y(t)=y_{I}(t)+j y_{Q}(t) \tag{B.20}
\end{equation*}
$$

where $Y_{I}(t)$ and $Y_{Q}(t)$ are statistically independent lowpass Gaussian random processes whose properties have been discussed in detail in Sec.2.2. Since the mean-square value of $y(t)$ is unity, there is no change in average signal energy level caused by the fading.

The fading QAM signal is given by (see Sec.2.2)

$$
\begin{align*}
\sqrt{2} Q(t)= & \sqrt{2}\left(s_{I}(t) Y_{I}(t)-s_{Q}(t) y_{Q}(t)\right) \cos 2 \pi f{ }_{c} t \\
& -\sqrt{2}\left(s_{I}(t) Y_{Q}(t)+s_{Q}(t) Y_{I}(t)\right) \sin 2 \pi f{ }_{c} t \tag{B.21}
\end{align*}
$$

So

$$
\begin{equation*}
Q(t)=q_{I}(t) \cos 2 \pi f_{c} t-q_{Q}(t) \sin 2 \pi f c_{c} t \tag{B.22}
\end{equation*}
$$

with complex envelope

$$
\begin{align*}
q(t) & =q_{I}(t)+j q_{Q}(t) \\
& =s(t) \cdot y(t) \tag{B.23}
\end{align*}
$$

In this bandpass model of the channel (Fig.B.2), stationary white Gaussian noise is now added to the fading QAM signal. This random noise waveform has a two-sided power spectral density of $\frac{1}{2} N_{0}$ over all positive
and negative frequencies, as shown in Fig.B.5(a). It has a Gaussian probability density function, with zero mean and variance

$$
\begin{equation*}
\sigma^{2}=\lim _{T \rightarrow \infty} \frac{1}{2 T} \int_{-T}^{T} \frac{1}{2} N_{0} d f=\frac{1}{2} N_{0} \tag{B.24}
\end{equation*}
$$

The mean-square value or average power level of this real-valued noise waveform is also $\sigma^{2}=\frac{1}{2} N_{0}[31,35,55]$.

This noisy and fading modulated-carrier signal now passes through the receiver bandpass filter (Fig.B.2) which limits the noise power going into the modulator. This filter has a passband $f_{C}-B \leqslant f \leqslant f_{c}+B$ that includes the data signal but is much wider, so the fading data signal is assumed to pass through unchanged. The bandlimited noise waveform $V(t)$ has the power spectral density $|V(f)|^{2}$ shown in Fig.B.5(b). The bandwidth of the bandpass noise, 2 B , is now assumed to be small compared with its centre frequency $f_{c}$. It is narrowband bandpass noise and can be represented as

$$
\begin{equation*}
V(t)=v_{I}(t) \cos 2 \pi f f_{c} t-v_{Q}(t) \sin 2 \pi f_{c} t \tag{B.25}
\end{equation*}
$$

with complex envelope

$$
\begin{equation*}
v(t)=v_{I}(t)+j v_{Q}(t) \tag{B.26}
\end{equation*}
$$

where $V(t), v_{I}(t), v_{Q}(t)$ are all real-valued Gaussian random processes with zero mean and variance $2 N_{0} B . v_{I}(t), v_{Q}(t)$ are independent lowpass waveforms with a power spectral density of $N_{0}$ over their bandwidth as shown in Fig.B.5(c). So $v(t)$ is complex-valued with zero mean and a mean-square value of $4 N_{0} B$.

The output from the multiplier (linear demodulator) in the in-phase channel of Fig.B. 2 is $(\sqrt{2} Q(t)+V(t)) \sqrt{2} \cos \left(2 \pi f_{c} t+\gamma\right)$. This unknown constant phase offset $\gamma$ only complicates the mathematics here while having no important effect on the signal properties. It can be ignored in the subsequent analysis [31]. So

$$
\begin{align*}
(\sqrt{2 Q}(t) & +V(t)) \cdot \sqrt{2} \cos 2 \pi f_{C} t \\
= & \left(\sqrt{2} q_{I}(t) \cos 2 \pi f_{C} t-\sqrt{2} q_{Q}(t) \sin 2 \pi f_{C} t+v_{I}(t) \cos 2 \pi f_{C} t\right. \\
& \left.-v_{Q}(t) \sin 2 \pi f_{C} t\right) \cdot \sqrt{2} \cos 2 \pi f_{C} t  \tag{B.27}\\
= & \left(2 q_{I}(t)+\sqrt{2} v_{I}(t)\right) \cos ^{2} 2 \pi f_{C} t-\left(2 q_{Q}(t)+\sqrt{2} v_{Q}(t)\right) \sin 2 \pi f_{C} t \cdot \cos 2 \pi f_{C} t \\
= & \left(q_{I}(t)+\frac{1}{\sqrt{2}} v_{I}(t)\right) \cos 4 \pi f_{C} t+\left(q_{I}(t)+\frac{1}{\sqrt{2}} V_{I}(t)\right) \\
- & \left(q_{Q}(t)+\frac{1}{\sqrt{2}} v_{Q}(t)\right) \sin 4 \pi f_{C} t \tag{B.28}
\end{align*}
$$

The high frequency components of Eq. (B. 28) are subsequently removed in the lowpass filter. So the received demodulated waveform in the in-phase


Fig.B. 5 Power spectral densities of (a) white Gaussian noise (b) bandpass white noise $\cdot V(t)$ (c) quadrature noise components $v_{1}(t), v_{Q}(t)$
channel is

$$
\begin{equation*}
r_{I}(t)=\left(q_{I}(t)+\frac{1}{\sqrt{2}} v_{I}(t)\right) * a(t) \tag{B.29}
\end{equation*}
$$

where * indicates convolution and $a(t)$ is the real-valued impulse response of the lowpass filter (Eq.(B.5)). Similarly it can be shown that the received demodulated waveform in the quadrature channel is

$$
\begin{equation*}
r_{Q}(t)=\left(q_{Q}(t)+\frac{1}{\sqrt{2}} v_{Q}(t)\right) * a(t) \tag{B.30}
\end{equation*}
$$

So the complex-valued received demodulated waveform is

$$
\begin{align*}
r(t) & =r_{I}(t)+j r_{Q}(t) \\
& =q_{I}(t) *_{a}(t)+j q_{Q}(t) * a(t)+\frac{1}{\sqrt{2}} v_{I}(t) * a(t)+j \frac{1}{\sqrt{2}} v_{Q}(t) *_{a}(t) \\
& =q(t) *_{a}(t)+\frac{1}{\sqrt{2}} v(t) * a(t) \\
& =\left(\sum_{i s_{i}} a(t-i T) \cdot y(t)\right) *_{a}(t)+\frac{1}{\sqrt{2}} v(t) *_{a}(t) \tag{B.31}
\end{align*}
$$

Now, let the complex-valued noise component in $r(t)$ be

$$
\begin{equation*}
w(t)=\frac{1}{\sqrt{2}} v(t) * a(t) \tag{B.32}
\end{equation*}
$$

So,

$$
w(t)=w_{I}(t)+j w_{Q}(t)
$$

where

$$
\begin{equation*}
w_{I}(t)=\frac{1}{\sqrt{2}} v_{I}(t) * a(t), \quad w_{Q}(t)=\frac{1}{\sqrt{2}} v_{Q}(t) * a(t) \tag{B.33}
\end{equation*}
$$

The noise waveforms $w_{I}(t), w_{Q}(t)$ are independent, real-valued Gaussian random processes. Each has zero mean and power spectral density

$$
\begin{equation*}
\left|w_{I}(f)\right|^{2}=\left|w_{Q}(f)\right|^{2}=\left.\left.\left(\frac{1}{\sqrt{2}}\right)^{2}\right|_{v_{I}}(f)\right|^{2} \cdot|A(f)|^{2}=\frac{1}{2} N_{0}|A(f)|^{2} \tag{B.34}
\end{equation*}
$$

as shown in Fig.B.6. $A(f)$ is the transfer function of the receiver lowpass filter (Eq.(B.3)). The factor $\frac{1}{2}$ arises in Eq. (B.34) because half of the noise power is lost in the high frequency components during demodulation. Thus, the variance of both noise waveforms $w_{I}(t), w_{Q}(t)$ is

$$
\begin{equation*}
\sigma^{2}=\frac{1}{2} N_{0} \int_{-1 / T}^{1 \cdot / T}|A(f)|^{2} d f=\frac{1}{2} N_{0} \tag{B.35}
\end{equation*}
$$

Since $w_{I}(t), w_{Q}(t)$ are statistically independent processes, the average power or mean-square value of the complex-valued noise waveform $w(t)$ is $2 \sigma^{2}=N_{0}$ 。
A.further important property of the noise waveform $w(t)$ can be obtained from the wiener-Kinchine theorem [31,35]. It can be shown [31] that with the matched root-raised-cosine filtering at the transmitter and receiver, any two samples of $w(t)$ separated by integer multiples of $T$ seconds, are uncorrelated and therefore statistically independent Gaussian random variables.

It is well known that the signal-to-noise ratio in $r(t)$ has been

(b)


Fig.B. 6 Characteristics of $w_{1}(\dagger), w_{a}(\dagger)$. Power spectral density, $\left|w_{1}(f)\right|^{2}$, $\left|w_{Q}(f)\right|^{2}$ - (b) Probability density function $f\left(w_{1}\right), f\left(w_{a}^{\prime \cdot}\right)$
maximized by this arrangement of matched filtering [31,35]. The signal-to- noise power ratio in this thesis is always taken to be

$$
\begin{equation*}
\psi=10 \log _{10}\left(E_{b} / N_{0}\right) d B \tag{B.36}
\end{equation*}
$$

where

$$
\begin{array}{rlrl}
E_{b} & =\frac{1}{2} \overline{\left.s_{i}\right|^{2}}=1, & & \text { for } 4 \text {-level QAM } \\
& =\frac{1}{k}\left|s_{i}\right|^{2} & =2.5, &  \tag{B.37}\\
\text { for } 16 \text {-level QAM }
\end{array}
$$

and where

$$
\begin{equation*}
N_{0}=2 \sigma^{2} \tag{B.38}
\end{equation*}
$$

With this passband model, $E_{b}$ is also the average transmitted energy per bit in the real-valued modulated carrier signal $\sqrt{2} S(t)$. $E_{b}$ is also the average received signal energy per bit in $r(t)$ (and in the $\left\{r_{i}\right\}$ ), averaged over the fading.

With this passband model, $\frac{1}{2} N_{0}$ is the two-sided power spectral density of the real-valued white Gaussian noise at the input to the receiver. $N_{0}=2 \sigma^{2}$ is also the mean-square value of the complex-valued noise component $w(t)$ in $r(t)$, (and of the $\left\{w_{i}\right\}$ in $\left\{r_{i}\right\}$ ).

Now, perfect linear modulation and demodulation is assumed throughout this thesis, so all simulation tests can be carried out at baseband. The baseband equivalent model is shown in Fig.B.7. The signal $s(t)$ shown in Fig.B. 7 at the output of the transmitter lowpass filter is exactly the same as the corresponding signal in the passband model of Fig.B. 2 (Eqs.(B.17) and (B.18)). The real-valued white Gaussian noise in this baseband model has a two-sided power spectral density of $\frac{1}{2} N_{0}$, as in the passband model. Thus, the signal $r(t)$ at the output of the receiver lowpass filter is given by Eq. (B. 31), being exactly the same as $r(t)$ in Fig.B.2. The mobile radio transmission system (Fig.B.2) is described in terms of its baseband equivalent model (Fig.B.7) from now on.

The baseband equivalent model of the channel (Fig.B.7) is simulated on a digital computer according to Fig.B.8. It is shown later (Sec.B.2) that Fig.B. 8 is the sampled equivalent model of the corresponding continuous model Fig.B.7. Of course, the $\left\{r_{i}\right\}$ in Fig.B. 8 are now exactly the same as the $\left\{r_{i}\right\}$ in Fig.B.7. Perfect symbol timing is assumed in these samples. The subscript $k$ in $s_{k}, y_{k}, w_{k}, r_{k}$ denotes samples of the signal waveforms taken at time $t=k T_{s}$ (for all integers $\{k\}$ ), where $T_{s}=1 / 48000$ is the sampling period in seconds.

Strictly speaking, with a time-varying transmission path, the baseband


Fig.B.7 Baseband equivalent model of channel


Fig.B. 8 Sampled equivalent of Fig.B. 7 used in computer simulation tests


Fig.B.9 Simplified baseband equivalent model of channel used in computer simulation tests
channel should be modelled as in Fig.B.8. Thus, the Rayleigh fading is applied to the signal at the output of the transmitter filter, and the resulting fading signal is then fed through the receiver filter. However, it will now be shown that, to a very close approximation, the baseband channel in this thesis can alternatively be modelled by combining the transmitter and receiver lowpass filters into a single filter with the transfer function $H(f)$ in Eq. (B.3), and applying the fading to the signal at the output of the combined filter. This leads to the very much simpler computer simulation model shown in Fig.B.9. This second model (Fig.B.9) requires only a small fraction of the computer time needed by the first (Fig.B.8), and so makes it possible to carry out many more tests. The second model has therefore been used in all tests. The accuracy of this second model is seen to depend on two factors:
i) The fading rate and thus the frequency spread ( $2 f_{m}$ ) of the Rayleigh fading, which depends on the vehicle speed.
ii) The number of taps $(g+1)$ in the root-raised cosine digital lowpass filters $A\left(n / T_{s}\right)$.

The received samples $\left\{r_{i}\right\}$ in Fig.B. 8 are now carefully analysed. Consider for a moment that $y(t)=1$ for $-\infty<t<\infty$, so that the Rayleigh fading channel in the mobile radio transmission path (Fig.B.7) is replaced by this ideal channel. The received signal component in $r(t)$ is (from Eq.(B.31))

$$
\begin{align*}
\left(\sum_{i} S_{i} a(t-i T) \cdot y(t)\right) * a(t) & =\sum_{S_{i}} a(t-i T) * a(t-i T) \\
& =\sum_{i} S_{i} h(t-i T)
\end{align*}
$$

where $h(t)=a(t) \star a(t)$ is the real-valued impulse response of the transmitter and receiver lowpass filters in cascade. With this ideal non-fading channel, the receiver filter is said to be "matched" to the signal at its input [31,35]. These two filters in cascade do not cause any intersymbol interference [31,36]. Perfect symbol timing recovery is assumed at the receiver throughout this thesis. Thus, with ideal filtering

$$
\begin{equation*}
r_{i}=s_{i}+w_{i} \tag{B.37}
\end{equation*}
$$

for all integers \{i\}, where $r_{i}=r(i T), s_{i}=s(i T)$ and $w_{i}=w(i T)$. The delay in transmission has been ignored for clarity. The $\left\{s_{i}\right\}$ are the data symbol values given by Eq. (B.1) or (B.2). This is exactly the same as would be observed if the transmitter and receiver lowpass filters were combined into one composite filter. Since the sampled impulse response of this composite filter is [31]

$$
h(i T)= \begin{cases}1, & \text { for } i=g / 2  \tag{B.38}\\ 0, & \text { for } i=\ldots,-2,-1,1,2, \ldots\end{cases}
$$

for all integers $\{i\}$. As a consequence of the Wiener-Kinchine theorem $[31,35]$, the noise samples $\left\{w_{i}\right\}$ are uncorrelated samples of a complex-valued Gaussian random variable, whose real and imaginary parts both have zero mean and variance $\sigma^{2}=\frac{1}{2} N_{0}$.

Since the Rayleigh fading does not introduce any intersymbol interference, it would seem to follow that Eqs.(B.36)-(B.37) can be extended to the general fading case to give

$$
\left(\sum_{i} s_{i} a(t-i T) \cdot y(t)\right) * a(t)=\sum_{i} s_{i} h(t-i T) \cdot y(t)
$$

and

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{B.39}
\end{equation*}
$$

This is in fact correct, to a very close approximation, for the fading conditions assumed in this thesis combined with this particular choice of matched filtering. But this cannot be assumed to be true generally. It is extremely important in the general case to realise that although the fading alone or the matched filtering alone do not cause any intersymbol interference, when put together there is generally intersymbol interference introduced. This is because the receiver filter is no longer matched to the fading signal at its input. The faster the fading, the more the samples $\left\{r_{i}\right\}$ will differ from Eq. (B.39). Obviously, as the fading rate is reduced to the limit where there is no fading, $y_{i}=1$ for all \{i\}, and Eq. (B.37) applies. In fact, it can be shown [96] that if the $\left\{y_{k}\right\}$ vary linearly over the duration of one transmitted pulse $\left(\left\{a_{k}\right\}\right.$, for $k=0,1, \ldots.)^{\prime}$, then all the lowpass filtering can be performed at the transmitter, with exactly the same results as are observed when the channel is simulated strictly correctly according to Fig.B.8. So to show that this condition holds in this thesis, computer simulation tests are performed in a way now described. (The detailed computer simulation techniques are discussed later in Sec.B.2).

The baseband equivalent model is simulated "correctly" in the absence of noise according to Fig.B.8. Perfect symbol timing is assumed in the samples $\left\{r_{i}\right\}$. Then, the system is simulated according to the simplified baseband model shown in Fig.B.9, where Eq.(B.39) is formed directly without filtering. Again no noise is added, that is $w_{i}=0$ for all \{i\}. The same fading sequence $\left\{y_{i}\right\}$ is used in both cases. The squared-error in the "simplified" $r_{i}$ is taken to be

$$
\begin{equation*}
e_{i}=\mid \text { correct } r_{i}-\text { simple }\left.r_{i}\right|^{2} \tag{B.39}
\end{equation*}
$$

and the mean-square error for a run of 6000 transmitted $\left\{s_{i}\right\}$ is calculated as

$$
\begin{equation*}
e=\log _{10}\left(\frac{1}{6000} \sum_{i=1}^{6000} e_{i}\right) d B \tag{B.40}
\end{equation*}
$$

Under these conditions, the $95 \%$ confidence interval for these results is $\pm 1.5 \mathrm{~dB}$.

This test was repeated for the different fading frequency spreads $2 f_{m}=0$ (no fading), $160,640,1280 \mathrm{~Hz}$ with $\mathrm{T}=1 / 12000$ seconds throughout. The effect on e of using different numbers of taps $\mathrm{g}+1=51,101,201$ in the root-raised-cosine filters of Fig.B. 7 was noted. The results of these tests are shown in Tables B.1-B.3.

In the real world, and in the "correct" simulation model of Fig.B.8, intersymbol interference can be introduced into the $\left\{r_{i}\right\}$ in two ways:
i) Multiplication in the time domain of the transmitted QAM signal by the fading $y(t)$, results in convolution in the frequency domain (see Figs.B.3, B. 4 and B.10). Thus, the fading causes a frequency spread in the transmitted QAM signal of $2 \mathrm{f}_{\mathrm{m}} \mathrm{Hz}$ (see Table B.l). For example, a vehicle speed of $\mathrm{v}=60 \mathrm{miles} /$ hour $=26.8$ metres $/$ second and carrier wavelength $\lambda=0.33$ metres (for $f_{c}=900 \mathrm{MHz}$ ).

$$
\begin{equation*}
2 \mathrm{f}_{\mathrm{m}}=2 \mathrm{v} / \lambda=160 \mathrm{~Hz} \tag{B.42}
\end{equation*}
$$

This represents a frequency spreading of only $0.66 \%$ in the transmitted QAM signal bandwidth, since

$$
\begin{equation*}
\frac{160}{24000} \times 100=0.66 \tag{B.43}
\end{equation*}
$$

ii) Truncating the sampled impulse response of the root-raised cosine filters causes a widening by $2 \Delta_{f} \mathrm{~Hz}$ of the QAM signal bandwidth [97] (Table B.2). For example, with $g+1=51$ taps in the filter, $2 \Delta_{f} \approx 520 \mathrm{~Hz}$, as shown in Fig.B.10. This represents a frequency widening in the QAM signal of about $2.16 \%$.

Generally, the bigger the total frequency spread $2 f_{m}+2 \Delta_{f}$, the more intersymbol interference will be introduced, and hence, the bigger the value of e in Eq.(B.40) will become, (as shown in Table B.3). Whether or not this presents a serious problem depends on the effect it has on the performances of Systems 1 to 3. Tests have shown that with $2 \mathrm{f}_{\mathrm{m}}=160 \mathrm{~Hz}$ and 51 taps used in the root-raised cosine filters, there is no significant difference in the bit error rate curves obtained with either simulation model, Fig.B. 8 or B.9. In fact, simulation results suggest that Fig.B. 9 is probably a valid simulation model as long as $2 \mathrm{f}_{\mathrm{m}} \leqslant 1.58$ of the QAM signal

Table B. 1 Frequency spread in the transmitted QAM signal caused by the fading

| Simulated vehicle <br> speed <br> (miles/hour) |  | Frequency spread <br> $2 f^{m}(\mathrm{~Hz})$ |
| :---: | :---: | :---: | | Percentage of |
| :---: |
| signal |

Table B. 2 Frequency. spread in the transmitted QAM signal caused by truncating the impulse response of the root-raised cosine filter to ( $\mathrm{g}+1$ )-taps

| Number of <br> taps <br> $(\mathrm{g}+1)$ | Frequency spread <br> $2 \Delta_{\mathrm{f}}(\mathrm{Hz})$ | Percentage of <br> signal <br> bandwidth |
| :---: | :---: | :---: |
| 31 | 880 | 3.66 |
| 51 | 520 | 2.16 |
| 101 | 260 | 1.08 |
| 201 | 130 | 0.54 |

Table B. 3 Variation of edB with ( $\mathrm{g}+1$ )-taps and $2 \mathrm{f}{ }_{\mathrm{m}} \mathrm{Hz}$ frequency spread

| Number of <br> taps $(\mathrm{g}+1)$ | Fading frequency <br> spread $2 f\left(\mathrm{~m}_{\mathrm{m}}(\mathrm{Hz})\right.$ | e dB |
| :---: | :---: | :---: |
| 31 | 0 | -47.38 |
| 51 | 0 | -58.54 |
| 101 | 0 | -68.81 |
| 201 | 0 | -81.57 |
| 31 | 160 | -46.95 |
| 51 | 160 | -58.17 |
| 101 | 160 | -68.41 |
| 201 | 160 | -79.46 |
| 31 | 640 | -47.10 |
| 51 | 640 | -57.41 |
| 101 | 640 | -61.28 |
| 201 | 640 | -61.90 |
| 31 |  |  |
| 51 | 1280 | -46.16 |
| 101 | 1280 | -50.32 |
| 201 | 1280 | -50.70 |




Flg.B. 10 Amplitude response of 51-łap root-raised cosine filter ( $T=1 / 12000$ seconds, $T_{s}=1 / 48000$ seconds)
bandwidth and at least 51 taps are used in the root-raised-cosine filters. (.Other filter shapes would generally give quite different restrictions).

## B. 2 Computer simulation of the linear baseband channel

The linear baseband channel can be simulated on the digital computer according to the models in Fig.B. 8 or B.9. Throughout this thesis all computer simulation tests are carried out in standard FORTRAN 77, using the NAG GO5 routines [98] for random number generation. Basically, the simulation method involves generating the random sequences $\left\{s_{i}\right\},\left\{y_{i}\right\}$, $\left\{w_{i}\right\}$ such that they conform to the statistical and spectral properties outlined in the previous section, and then calculating the $\left\{r_{i}\right\}$. The simulation method used in this thesis is now described. (The FORTRAN programs are given in Appendix F).

## 1) Call random number generator routine.

This routine is called once only, before any random numbers are generated. The FORTRAN command CALL GO5CBF(I) sets the basic generator routine to a repeatable initial state, with seed integer I. Or, the command CALL G05CCF sets the basic generator routine to a non-repeatable initial state.

## 2) Generate pseudorandom binary digits: $\left\{\alpha_{i}\right\}$.

The FORTRAN command $W=\operatorname{GO} \operatorname{DDAF}(-1.0,+1.0)$ returns a pseudorandom real number taken from a uniform distribution between -1 and +1 . If this number $w$ is negative then $\alpha_{i}=0$, otherwise $\alpha_{i}=1$.

## 3) Form data symbol values: $\left\{s_{i}\right\}$

The $\left\{s_{i}\right\}$ are encoded from the $\left\{\alpha_{i}\right\}$ in one of four different ways depending on whether the modulation scheme is QPSK, DQPSK, 16-QAM, 16-DQAM. This is described in detail in Sec.2.3.

## 4) Lowpass filter the data symbols

The more complicated baseband model of Fig.B. 7 requires that the baseband modulating waveform $s(t)$ be generated by lowpass filtering the $\left\{s_{i}\right\}$. The root-raised-cosine lowpass filter must be simulated as a digital filter on the digital computer. The tap gains of this digital filter are obtained from the ideal continuous impulse response $a(t)$ given in Eq. (B.5). a(t)
is delayed in time by $\frac{1}{2} g T_{s}$ seconds and sampled every $k T_{s}$ seconds to give the sampled impulse response

$$
a_{k}= \begin{cases}c\left[\frac{1}{\sqrt{T}} \cdot \frac{\sin \pi\left(2 k T_{S} / T+\frac{1}{2}\right)}{\pi\left(2 k T_{S} / T+\frac{1}{2}\right)}+\frac{1}{\sqrt{T}} \cdot \frac{\sin \pi\left(2 k T_{S} / T-\frac{1}{2}\right)}{\pi\left(2 k T_{S} / T-\frac{1}{2}\right)}\right], & \text { for } k=0,1, \ldots g \\ 0, & \text { elsewhere (B. 43) }\end{cases}
$$

where, $T=1 / 12000$ seconds. The taps are all scaled by the constant $c$ so that

$$
\begin{equation*}
\sum_{i=0}^{g} a_{i}^{2}=1 \tag{B.44}
\end{equation*}
$$

In fact, $c \approx \sqrt{T_{s}}$. In the limit when $g=\infty, c=\sqrt{T_{s}}$ exactly.
The number of taps $\mathrm{g}+1$, and the sampling frequency $1 / \mathrm{T}_{\mathrm{s}}$ in Eq. (B.43) are carefully chosen to give a good representation of the frequency response with negligible aliasing and an acceptable level of equipment complexity. A good choice was seen to be

$$
\begin{equation*}
\mathrm{g}+1=51, \quad \mathrm{~T}_{\mathrm{s}}=\downarrow \mathrm{T}=1 / 48000 \text { seconds } \tag{B.45}
\end{equation*}
$$

for the reasons discussed previously (see Fig.B.10).
The transmitted samples input to this filter every Ts seconds are,

$$
\begin{align*}
\left\{S_{I N . k}\right\} & =\ldots, 0,0, s(i T), 0,0,0, s((i+1) T), 0,0,0, s((i+2) T), 0,0, \ldots  \tag{B.46}\\
& =\ldots, 0,0, s_{i}, 0,0,0, s_{i+1}, 0,0,0, s_{i+2}, 0,0, \ldots
\end{align*}
$$

and the output samples are given by

$$
\begin{equation*}
\left\{s_{k}\right\}=\left\{s_{I N_{.} k}\right\}^{*}\left\{a_{k}\right\} \tag{B.47}
\end{equation*}
$$

Where * means convolution. So

$$
\begin{equation*}
s_{k}=\sum_{m=0}^{g} s_{I N, k-m} \cdot a_{m} \tag{B.48}
\end{equation*}
$$

## 5) Generate the fading: $\left\{y_{i}\right\}$

It is well known $[11,26,64]$ that the samples of the Rayleigh fading, $\left\{y_{i}\right\}$, can be simulated by passing white Gaussian noise through a digital filter with a suitable spectral shape. Two independent white Gaussian noise sequences, each with zero mean and variance $\frac{3}{2}$, must be separately filtered to give the real and imaginary sequences of $\left\{y_{i}\right\}=\left\{y_{I_{. i}}+j y_{Q . i}\right\}$. The two noise shaping filters ideally have the frequency response

$$
C(f)= \begin{cases}\frac{1}{\sqrt{\pi f_{m}}} \cdot \frac{1}{\sqrt[4]{1-\left(f / f_{m}\right)^{2}}}, & \text { for }-f_{m} \leqslant f \leqslant+f_{m}  \tag{B.49}\\ 0, & \text { elsewhere }\end{cases}
$$

as shown in Fig.B.4(a).
The first problem is to find the corresponding impulse response of this filter, $c(t)$. The impulse response of a filter is given by the
inverse Fourier transform of its frequency response. So,

$$
\begin{align*}
c(t) & =\int_{-\infty}^{\infty} C(f) \exp (j 2 \pi f t) d f \\
& =\int_{-\infty}^{\infty} C(f) \cos 2 \pi f t d f+j \int_{-\infty}^{\infty} C(f) \sin 2 \pi f t d f \tag{B.50}
\end{align*}
$$

and since $C(f)$ and $\cos 2 \pi f t$ are even functions and $\sin 2 \pi f t$ is an odd function, Eq.(B.50) reduces to

$$
\begin{equation*}
c(t)=2 \int_{0}^{\infty} C(f) \cos 2 \pi f t d f \tag{B.51}
\end{equation*}
$$

Therefore,

$$
\begin{equation*}
c(t)=\frac{2}{\sqrt{\pi f_{m}}} \int_{0}^{f_{m}} \frac{\cos 2 \pi f t}{\sqrt[4]{1-\left(f / f_{m}\right)^{2}}} d f \tag{B.52}
\end{equation*}
$$

Now, substitute $f=f_{m} \sin \gamma$ into Eq.(B.52). Therefore,

$$
\begin{equation*}
\sqrt[4]{1-(f / f})_{m}^{2}=\sqrt[4]{1-\sin ^{2} \gamma}=\sqrt[4]{\cos ^{2} \gamma}=\sqrt{\cos \gamma} \tag{B.53}
\end{equation*}
$$

and,

$$
\begin{equation*}
d f=f_{m} \cos \gamma d \gamma \tag{B.54}
\end{equation*}
$$

With limits,

$$
\begin{array}{lll}
\text { when } f=f \\
\text { when } f=0, & f_{m}=f_{m} \sin \gamma, & \sin \gamma=1,  \tag{B.55}\\
0=f_{m} \sin \gamma, & \sin \gamma=0, & \text { therefore } \gamma=\pi / 2 \\
\text { whe } \gamma=0
\end{array}
$$

Substituting Eqs.(B.53)-(B.55) into Eq.(B.52)

$$
\begin{align*}
c(t) & =\frac{2}{\sqrt{\pi f}} \int_{m}^{\pi / 2} \frac{\cos \left(2 \pi f_{m} t \cdot \sin \gamma\right) \cdot f m \cos \gamma}{\sqrt{\cos \gamma}} d \gamma \\
& =2 \sqrt{\frac{f_{m}^{m}}{\pi}} \int_{0}^{\pi / 2} \cos \left(2 \pi f_{m} t \cdot \sin \gamma\right) \cdot \sqrt{\cos \gamma} d \gamma \tag{B.56}
\end{align*}
$$

Eq. (B.52) cannot be integrated in the normal way by substitution with elementary functions. Also Eq. (B.52) cannot be solved using Simpsons rule numerical.. integration because the function $\left(\sqrt[4]{1-\left(f / f_{m}\right.}\right)^{-1}=\infty$ at $f=f_{m}$. But the integral must exist because the area under the curve is finite (Fig.B.11).

Eq. (B.56) however, is finite at its boundaries and it can be shown that numerical integration by Simpsons rule gives a quick convergence to the required result. (That is, about six decimal places accuracy with 5000 strips). So, the sampled impulse response of the noise shaping filter $\left\{c_{n}\right\}$ can be found by applying Simpsons rule [99] to Eq. (B.56) for each value of $t$ required.

Throughout this thesis, the noise shaping filter used is a 401-tap linear feedforward transversal filter, where the taps $\left\{c_{n}\right\}$ are samples of the impulse response taken every $1 / 600$ seconds. The impulse response and


Fig.B. 11 Amplitude response of noise shaping filter given by Fig.B. 12


Flg.B. 12 Tap gains of a 401-tap transversal noise shaping filter
frequency response of this filter are shown in Fig.B. 12 and Fig.B. 11 respectively. The tap gains of this filter are obtained by applying Simpsons rule to Eq. (B.56) with $f_{m}=80$ for every tap and $t=(n-200) / 600$ for tap $n$ (where $n=200,201, \ldots, 400$ ). The impulse response has been sampled every $1 / 600$ seconds and has been delayed by $200 / 600$ seconds to make it physically realiseable (causal). It is now symmetrical about tap $\mathrm{n}=200$, so $c_{n}=c_{400-n}$ for $n=0,1, \ldots, 199$. The taps are scaled so that $\sum_{n=0}^{400} c_{n}^{2}=1$. The sampled impulse response $\left\{c_{n}\right\}$ can alternatively be obtained from Eq. (B.52) via the conventional inverse-DFT filter design method [97,100,101]. It has been shown that the Simpsons rule method described here gives a better stopband attenuation by about 3dB (Fig.B.13). This degradation in the inverse-DFT method may be caused by unsatisfactory sampling near the discontinuities at $f= \pm f_{m}$ in Eq.(B.52).

Finally, the sampling period in the fading samples must be matched to that of the data signal at the input to the Rayleigh fading channel in Fig.B. 8 or Fig.B.9. This is done by interpolating the fading sequences at the outputs of the noise shaping filters. For example, if the baseband simulation model of Fig.B. 8 is used with $T_{S}=1 / 48000$ seconds, then the fading must be interpolated to give 80 equally spaced samples for each sample output from the noise shaping filters (since $80 \times 600=48000$ ). However, if the simplified simulation model of Fig.B. 9 is used, $T=1 / 12000$ seconds, so the interpolation rate is only 20 times per sample (since $(20 \times 600=12000)$. A cosine roll-off interpolating filter proposed by Wesolowski [27] has been used in this thesis. A roll-off factor of 0.6 was used in the interpolating filter so that the frequency response is flat only over the passband of the fading filter, as shown in Fig.B.14.

## 6) Generate the additive white Gaussian noise: $\left\{w_{i}\right\}$

The FORTRAN Command $\omega=\operatorname{GOSDDF}(0, \sigma)$ returns a pseudorandom real number $w$ taken from a Gaussian distribution with zero mean and standard deviation $\sigma$. Successive samples from this routine are used to represent the real and imaginary parts of the complex-valued white Gaussian noise in Figs.B.8. and B.9. The two-sided power spectral density of this white noise is $\frac{1}{2} N_{0}=\sigma^{2}$.
7) Form the received samples: $\left\{r_{i}\right\}$

If the simplified baseband simulation model Fig.B. 9 is used, then the


Fig.B. 13 Comparison of noise shaping filters obtained by two different methods


FIg.B. 14 Amplitude responses of fading filter and interpolating filter
received samples are simply

$$
\begin{equation*}
r_{i}=s_{i} y_{i}+w_{i} \tag{B.57}
\end{equation*}
$$

for all $\{i\}$. These samples $\left\{r_{i}\right\}$ represent samples of the baseband received signal $r(t)$ taken every $T$ seconds (once per symbol) with perfect symbol timing. The $\left\{s_{i}\right\}$ are the data symbol values defined in Eqs.(B.1)(B.2). The $\left\{y_{i}\right\}$ and $\left\{w_{i}\right\}$ are samples of the fading and white Gaussian noise respectively taken every $T=1 / 12000$ second.

If the computer simulation model includes the matched filtering (Fig.B.8) then the received samples $\left\{r_{k}\right\}$ are given by

$$
\begin{equation*}
r_{k}=\left(s_{k} y_{k}\right) * a_{k}+v_{k}^{*} a_{k} \tag{B.58}
\end{equation*}
$$

for all $\{k\}$, where $r_{k}, s_{k}, y_{k}, v_{k}$ are all complex-valued and $a_{k}$ is realvalued. These samples $\left\{r_{k}\right\}$ represent samples of the baseband received signal $r(t)$ taken every $T_{s}=1 / 48000$ seconds. The $\left\{s_{k}\right\}$ have been defined in Eqs.(B.46)-(B.48). The $\left\{y_{k}\right\}$ are samples of the fading taken every $T_{s}$ seconds. The $\left\{a_{k}\right\}$, for $k=0,1, \ldots, g$ is the sampled impulse response of the receiver matched filter (Eq. (B.5)). The $\left\{\mathrm{v}_{\mathrm{k}}\right\}$ are samples of the white Gaussian noise taken every $T_{s}$ seconds, where the real and imaginary parts of the $\left\{v_{k}\right\}$ have zero mean and variance $\sigma^{2}$.

Perfect symbol timing can now be assumed and the samples $\left\{r_{i}\right\}$ taken every $\mathrm{T}=4 \mathrm{~T}$ s seconds. Now, to a very close approximation

$$
\begin{equation*}
r\left(i T+g T_{s}\right) \approx s(i T) Y\left(i T+\frac{1}{2} g T_{s}\right)+w(i T) \tag{B.59}
\end{equation*}
$$

The delays of $\frac{1}{2} g T_{s}$ seconds through the transmitter and receiver lowpass filters can be corrected for in the receiver so that

$$
\begin{equation*}
r_{i} \approx s_{i} y_{i}+w_{i} \tag{B.60}
\end{equation*}
$$

Finally in this Appendix, the signal and noise components in the $\left\{r_{i}\right\}$ must be shown to be the same in both the computer simulation model (Fig.B.8) and the continuous model that it represents (Fig.B.7)

In the computer simulation model, the sampled impulse response of the transmitter and receiver filters in cascade is

$$
\begin{equation*}
\left\{h_{k}\right\}=\left\{a_{k}\right\}^{*}\left\{a_{k}\right\} \tag{B.61}
\end{equation*}
$$

where,

$$
\begin{equation*}
h_{k}=\sum_{m=0}^{g} a_{m} a_{k-m} \tag{B.62}
\end{equation*}
$$

The $\left\{a_{k}\right\}$ are defined in Eqs.(B.43)-(B.44). The sampling period here is $T_{S}=1 / 48000$ seconds. So, if the $\left\{h_{k}\right\}$ are themselves sampled to give the $\left\{h_{i}\right\}$ with a sampling period of $T=1 / 12000$ seconds and ideal sampling once per symbol, then

$$
h\left(g T_{S}\right)=1
$$

and

$$
\begin{equation*}
h(i T) \approx 0, \quad \text { for all } i T \neq g T_{S} \tag{B.63}
\end{equation*}
$$

So in a slow fading channel (which is the case here) and in the absence of noise

$$
\begin{equation*}
r(i T+g T s) \approx s_{i} y\left(i T+\frac{1}{2} g T_{s}\right) \tag{B.64}
\end{equation*}
$$

as would be the case in the continuous system (see Eq. (B.39))
Since $\sum_{m=0}^{g} a_{m}^{2}=1$, the mean-square value of the complex-valued noise samples is the same at the input and output of this filter, $2 \sigma^{2}$. It is assumed that $2 \sigma^{2}=N_{0}$ in this simulation model, as in the continuous model.

Symbol Timing Recovery

## C. 1 Introduction

The procedure for symbol timing recovery at the receiver falls into two categories. Firstly, the symbol timing algorithm must be started up during the synchronizing packet (see Fig.C.1). A sequence of phase reversals is transmitted (Fig.C.2) which is used in a fast acquisition procedure. Secondly, symbol timing must be maintained throughout a frame of transmitted data. To achieve this, fine adjustments are made to the timing waveform every symbol, from measurements taken of the received demodulated waveform $r(t)$. No knowledge is required of the actual data symbol values.

It is shown in this appendix that the same simple procedure for achieving symbol timing recovery can be used in all Systems 1,2 and 3. That is, the equations for symbol timing recovery are independent of whether one 4-level QAM signal, two 4-level QAM signals or one 16-level QAM is received. With two receiving antennas, the same procedure would be carried out on both received signals $\left\{r_{a . i}\right\},\left\{r_{b . i}\right\}$ and the measurements from the two antennas would simply be averaged. No computer simulation tests have been carried out on this symbol timing method, though initial hardware tests have shown promising results [92].

## C. 2 Basic Assumptions

The chief strategy for both types of symbol timing recovery is based on the use of $100 \%$ raised cosine filtering in Systems 1, 2 and 3 . In Appendix $B$ it was shown that the signal component in the baseband received waveform $r(t)$ is given by (Eq.(B.31))

$$
\begin{equation*}
\left(\sum_{i}{ }_{i} a(t-i T) \cdot y(t)\right) * a(t) \tag{C.1}
\end{equation*}
$$

Which, because of the narrow fading frequency spread $2 \mathrm{f}_{\mathrm{m}}=160 \mathrm{~Hz}$, can alternatively be represented by (Eq. (B.39))

$$
\begin{equation*}
\sum_{i} S_{i} h(t-i T) \cdot y(t) \tag{C.2}
\end{equation*}
$$

Where $h(t)=a(t) * a(t)$ is the impulse response of the transmitter and receiver lowpass filters in cascade. $h(t)$ has a $100 \%$ raised cosine frequency response $H(f)$ (Eqs.(B.7) and (2.3.3)). This impulse response


Fig.C. 1 Frame of transmitted data

| Carrier |
| :---: | :---: | :---: |
| Burst |$\quad$ Phase Reversals $\quad$| Unique |
| :---: |
| Word |

Fig.C. 2 Format of synchronizing packet

| Training <br> Signal | Coded Speech / Data |
| :---: | :---: |

Fig.C. 3 Format of data packet


Fig.C. 4 Impulse response of filter which has a $100 \%$ raised cosine frequency response
$h(t)$ is shown in Fig.C.4. The important point to note here is that

$$
\begin{align*}
h(0) & =1 \\
h\left(-\frac{1}{2} T\right) & =h\left(+\frac{1}{2} T\right)=\frac{1}{2} \\
h\left(\frac{1}{2} i T\right) & =0, \quad \text { for all integers } i \neq 0 \text { or } \pm 1 \tag{C.3}
\end{align*}
$$

So that the sample value of $\sum_{i} S_{i} h(t-i T)$ at time $t=\left(i+\frac{1}{2}\right) T$ is the arithmetic mean of the sample values at times $t=i T$ and $t=(i+1) T$. There are no
components from any $\left\{s_{i}\right\}$ other than $s_{i}$ and $s_{i+1}$.
Thus, after the receiver has demodulated the received signal into two quadrature components $r_{I}(t)$ and $r_{Q}(t)$, it uses filtered samples of each component twice per symbol for symbol timing recovery. That is, it uses the samples

$$
\begin{align*}
& {\left[\ldots, r_{I_{. i-1}}, r_{I_{. i-\frac{1}{2}}}, r_{I_{. i}}, r_{r_{. i+\frac{1}{2}}}, r_{I_{, i+1}}, \ldots\right]} \\
& {\left[\ldots, r_{Q . i-1}, r_{Q . i-\frac{1}{2}}, r_{Q . i}, r_{Q . i+\frac{1}{2}}, r_{Q, i+1}, \ldots\right]} \tag{C.4}
\end{align*}
$$

where $r_{I_{. i}}=r_{I}(i T), r_{I_{. i-\frac{1}{2}}=r_{I}}\left(\left(i-\frac{1}{2}\right) T\right)$ and so on. From now on in this Appendix, the samples $\left\{r_{i}\right\}$, for all integers $\{i\}$, are called the "integer-samples", whereas the remaining samples $\left\{\dot{r}_{i+\frac{1}{2}}\right\}$, for all integers $\{i\}$, are called the " $\frac{1}{2}$-samples" (where $r_{i}=r_{I . i}+j r_{Q . i}$ ). Now, the receiver assumes that the integer-samples coincide with the optimum sampling points for no intersymbol interference. So, 100\% raised cosine filtering should give, for every $\frac{1}{2}$-sample

$$
\begin{equation*}
r_{I_{. i+\frac{1}{2}}}=\frac{1}{2}\left(r_{I_{. i}}+r_{I_{. i+1}}\right) \quad \text { AND } \quad r_{Q . i+\frac{1}{2}}=\frac{1}{2}\left(r_{Q . i}+r_{Q . i+1}\right) \tag{C.5}
\end{equation*}
$$

if no noise is present. If symbol timing is slightly early or late, then the value of the $\frac{1}{2}$ sample will be biased towards the value of the prior or subsequent integer-sample respectively. Thus, a value may be accumulated over a large number of symbols, which averages to zero for correct timing and which will have a near linear relationship with timing offset for small timing errors. So for fast start up, a block averaging is carried out over the packet of phase reversals, whereas during data transmission the value is used to continuously adjust the sampling instances by small increments.

Unfortunately, as shown by the Wiener-Kinchine theorem [31.35], there is some correlation between the adjacent noise samples $w_{i-\frac{1}{2}}, w_{i}, w_{i+\frac{1}{2}}$ which will degrade the performance of the timing recovery algorithm.

To simplify the subsequent analysis, the complex-valued baseband received waveform is represented by

$$
\begin{equation*}
r(t)=p(t) y(t)+w(t) \tag{C.6}
\end{equation*}
$$

where

$$
\begin{equation*}
p(t)=\sum_{i} s_{i} h(t-i T) \tag{C.7}
\end{equation*}
$$

So that

$$
\begin{gather*}
r_{i}=p_{i} y_{i}+w_{i}  \tag{C.8}\\
r_{i+\frac{1}{2}}=p_{i+\frac{1}{2}} y_{i+\frac{1}{2}}+w_{i+\frac{1}{2}} \tag{C.9}
\end{gather*}
$$

And indeed, with a timing error of 0.1 T ,

$$
\begin{equation*}
r_{i+0.1}=p_{i+0.1} Y_{i+0.1}+w_{i+0.1} \tag{C.10}
\end{equation*}
$$

Assuming noise is absent. From Eq. (C.6)

$$
\begin{equation*}
r(t)=p(t) Y(t) \tag{C.11}
\end{equation*}
$$

That is

$$
\begin{align*}
& r_{I}(t)=p_{I}(t) y_{I}(t)-p_{Q}(t) y_{Q}(t) \\
& r_{Q}(t)=p_{I}(t) y_{Q}(t)+p_{Q}(t) \dot{y}_{I}(t) \tag{C.12}
\end{align*}
$$

where

$$
\begin{align*}
& r(t)=r_{I}(t)+j r_{Q}(t)  \tag{C.13}\\
& p(t)=p_{I}(t)+j p_{Q}(t)  \tag{C.14}\\
& y(t)=y_{I}(t)+j y_{Q}(t) \tag{C.15}
\end{align*}
$$

and $j=\sqrt{-1}$.
Some specific examples are now studied to see how the symbol timing recovery scheme works.
C. 3 One QAM signal received in the channel. Fast acquisition of symbol timing from a packet of phase reversals

N phase reversals are transmitted so that

$$
\begin{array}{ll}
s_{i}=+1+j, & \text { for } i=1,3,5, \ldots, N-1 \\
s_{i}=-1-j, & \text { for } i=2,4,6, \ldots, N \tag{C.16}
\end{array}
$$

The in-phase and quadrature components of $p(t)$ resulting from these $\left\{s_{i}\right\}$ are, to a very close approximation, pure sinewaves as shown in Fig.C.5. (They would be exactly sinewaves if $h(t)$ was a fully raised cosine impulse response). Clearly, with ideal timing and no noise (from Eqs.(C.3), (C.7) and (C.16))

So that (from (Eqs.(C.17) and (C.12))

$$
\begin{array}{rlllllll}
\left\{r_{I . k}\right\} & =Y_{I .1} Y_{Q .1} & 0 & -Y_{I .2^{+Y_{Q .2}}} & 0 & Y_{I .3^{-y_{Q .3}}} & 0 & \ldots \cdot \\
\left\{r_{Q . k}\right\} & =Y_{I .1}+Y_{Q .1} & 0 & -Y_{I .2^{-Y_{Q .2}}} & 0 & Y_{I .3}+Y_{Q .3} & 0 & \ldots \tag{C.18}
\end{array}
$$

The timing error at the receiver is generally $+\alpha$ radians (or $+\alpha T / \pi$ seconds) as shown in Fig.C.5(b). That is, the receiver uses the samples $r_{i+\alpha / \pi^{\prime}} r_{1 \frac{1}{2}+\alpha / \pi^{\prime}} r_{2+\alpha / \pi^{\prime}} \ldots . . r_{N+\alpha / \pi}$ for symbol timing recovery. So in


Fig.C. 5 (a) $p(t)$ with phase reversals and (b) $r(t)$ in the absence of noise
the absence of noise (see Fig.C.5).

$$
\begin{array}{rlrlrl}
\left\{\mathrm{p}_{\mathrm{I}, \mathrm{k}}\right\}=\left\{\mathrm{p}_{Q . k}\right\} & =\cos \alpha & -\sin \alpha & -\cos \alpha & \sin \alpha & \cos \alpha \\
\text { for } k & =1+\alpha / \pi & 1 \frac{1}{2}+\alpha / \pi & 2+\alpha / \pi & 2 \frac{1}{2}+\alpha / \pi & 3+\alpha / \pi \\
& 3 \frac{1}{2}+\alpha / \pi & \ldots & \text { (c.19) }
\end{array}
$$

So that (from (Eqs.(C.19) and (C.12))

$$
\begin{aligned}
& \left\{r_{Q . k}\right\}=\cos \alpha\left(Y_{I, k}+Y_{Q . k}\right)-\sin \alpha\left(y_{I . k}+y_{Q . k}\right)-\cos \alpha\left(y_{I . k}+y_{Q . k}\right) \quad \ldots \\
& \text { for } k=1+\alpha / \pi \quad 1 \frac{1}{2}+\alpha / \pi \quad 2+\alpha / \pi \quad \ldots \text { (C.20) }
\end{aligned}
$$

Clearly, with perfect symbol timing, $\alpha=0$ and Eqs.(C.19)-(C.20) reduce to Eqs.(C.17)-(C.18). Thus,

The fading is assumed to remain constant over any length of time $T / 2$ seconds, such that $y_{k}=y_{k-\frac{1}{2}}$. So the best estimate of $\alpha$ from $r_{1+\alpha / \pi}$ and $r_{1 \frac{1}{2}+\alpha / \pi}$ is

$$
\begin{align*}
& \alpha^{\prime}=-\frac{1}{2}\left[\tan ^{-1}\left(\frac{r_{I .1 \frac{1}{2}+\alpha / \pi}+r_{Q .1 \frac{1}{2}+\alpha / \pi}}{r_{I .1+\alpha / \pi}+r_{Q .1+\alpha / \pi}}\right)\right. \\
& \left.+\tan ^{-1}\left(\frac{r_{Q .1 \frac{1}{2}+\alpha / \pi}-r_{I .1 \frac{1}{2}+\alpha / \pi}}{r_{Q .1+\alpha / \pi}-r_{I .1+\alpha / \pi}}\right)\right] \tag{C.22}
\end{align*}
$$

In fact generally, the best estimate of $\alpha$ from $r_{k+\alpha / \pi}$ and $r_{k+\frac{1}{2}+\alpha / \pi}$ is $\alpha_{k^{\prime}}{ }^{\prime}$ for $k=1,2,3,4 ; \ldots, N$, where
$\alpha_{k}^{\prime}=\frac{-1}{2} \cdot\left[\tan ^{-1}\left(\frac{r_{I . k+\frac{1}{2}+\alpha / \pi}+r_{Q . k+\frac{1}{2}+\alpha / \pi}}{r_{I . k+\alpha / \pi}+r_{Q . k+\alpha / \pi}}\right)\right.$

$$
\begin{equation*}
\left.+\tan ^{-1}\left(\frac{r_{Q \cdot k+\frac{1}{2}+\alpha / \pi^{-}} r^{r_{1 . k+\frac{1}{2}}+\alpha / \pi}}{r_{Q \cdot k+\alpha / \pi}}\right)\right] \tag{C.23}
\end{equation*}
$$

For $N$ transmitted phase reversed symbols $\left\{s_{i}\right\}$, the $\left\{\alpha_{k}^{\prime}\right\}$ in Eq. (C.23) are averaged for $k=1,2,3, \ldots, N$ to give the best estimate of $\alpha$, which is assumed to be constant over this time interval $0 \leqslant t \leqslant N T$. The timing waveform is now adjusted. Clearly, a positive value of $\alpha$ radians indicates that the timing waveform is late by $\alpha T / \pi$ seconds, whereas a negative value of $\alpha$ radians indicates that the timing waveform is early by $\alpha T / \pi$ seconds.

## C. 4 One QAM signal received in the channel. Maintaining symbol timing throughout a frame of transmitted data

Assume that a 4-level QAM signal is transmitted. There are four possible values of each $s_{i},( \pm 1 \pm j)$. Therefore there are sixteen possible values of
$\left[s_{i-1} s_{i}\right],[( \pm 1 \pm j)( \pm 1 \pm j)]$ ．Now，assuming perfect symbol timing recovery at the receiver when a 4 or 16 －level QAM signal is transmitted （from Eq．（C．5））

$$
\begin{align*}
& r_{I_{. i-\frac{1}{2}}}-\frac{1}{2}\left(r_{I_{. i-1}}+r_{I . i}\right)=0 \\
& r_{Q . i-\frac{1}{2}}-\frac{1}{2}\left(r_{Q . i-1}+r_{Q . i}\right)=0 \tag{C.24}
\end{align*}
$$

for all possible combinations of $\left[s_{i-1} s_{i}\right]$ ．This is shown in Table C．l， for the case where

$$
\begin{equation*}
y_{i-1}=y_{i-\frac{1}{2}}=y_{i} \quad\left(=y=y_{I}+j y_{Q}\right) \tag{C.25}
\end{equation*}
$$

Generally，the fading needs to change linearly over the duration of one symbol（（i－1）T太tsiT），for Eq．（C．24）to give an exact result．

However，it is assumed here that there is a small timing error of $\alpha$ radians（or $\alpha T / \pi$ seconds）during data transmission．（The fast acquisition procedure（Sec．C．3）is assumed to have obtained a good initial estimate of the timing waveform during the synchronization packet）．So for example， if the sequence

$$
\left[\begin{array}{lll}
s_{1} & s_{2} & s_{3}
\end{array}\right]=[+1+j \quad-1-j \quad+1+j]
$$

was transmitted，then in the absence of noise（from Eq．（C．19））

$$
\begin{align*}
\left\{p_{I, k}\right\}=\left\{p_{Q . k}\right\} & \approx 1  \tag{C.26}\\
\text { for } k & \approx 1
\end{align*} \begin{array}{rlrll}
1 \frac{1}{2} & -1 & \alpha & 1 & \ldots
\end{array}
$$

So that（from Eq．（C．20））

$$
\begin{aligned}
& \left\{r_{I . k}\right\} \approx\left(y_{I . k}-y_{Q . k}\right)-\alpha\left(y_{I . k}-y_{Q . k}\right)-\left(y_{I . k}-y_{Q . k}\right) \quad \alpha\left(y_{I . k}-y_{Q . k}\right) \quad \ldots \\
& \left\{r_{Q . k}\right\} \approx\left(y_{I . k}+Y_{Q . k}\right) \quad-\alpha\left(Y_{I . k}{ }^{+y_{Q . k}}\right) \quad-\left(y_{I . k}{ }^{+Y_{Q . k}}\right) \quad \alpha\left(y_{I . k}+Y_{Q . k}\right) \quad \ldots \\
& \text { for } \mathrm{k} \approx 1 \text { 1立 } 2 \text { 2 } \quad 1 \text {... (C.27) }
\end{aligned}
$$

Since，for $\alpha \approx 0$ ，

$$
\begin{gather*}
\cos \alpha \approx 1 \\
\sin \alpha \approx \alpha \\
k+\alpha / \pi \approx k \tag{C.28}
\end{gather*}
$$

This example has highlighted the important principle here．With fully raised cosine filtering，a small timing error generally causes a negligible error in the integer－samples $\ldots, p_{i}, p_{i+1}, p_{i+2}, \ldots$ ，whereas it causes an error proportional to $\alpha$ in the $\frac{1}{2}$－samples $\ldots, p_{i+\frac{1}{2}}, p_{i+1 \frac{1}{2}, \ldots}$ ． Thus，the timing error at time $t=i T$（or strictly speaking $t=(i+\alpha / \pi) T$ ）can be estimated by（see Eq．（C．24）

$$
\begin{aligned}
& \alpha_{I_{\text {.i }}}=\operatorname{sgn}\left(r_{I_{. i}}-r_{I_{. i-1}}\right) .\left(r_{I_{. i-\frac{1}{2}}}-\frac{1}{2}\left(r_{I_{. i-1}}+r_{I_{. i}}\right)\right) \text { radians } \\
& \alpha_{Q . i}^{\prime}=\operatorname{sgn}\left(r_{Q . i}-r_{Q . i-1}\right) .\left(r_{Q . i-\frac{1}{2}}-\frac{1}{2}\left(r_{Q . i-1}+r_{Q . i}\right)\right) \text { radians (C.29) }
\end{aligned}
$$

where

$$
\operatorname{sgn}(x)= \begin{cases}+1, & \text { if } x \geqslant 0  \tag{C.30}\\ -1, & \text { if } x<0\end{cases}
$$

Table C. 1 All 16 possible combinations of $\left[s_{i-1} s_{i}\right]$ with the corresponding received samples $\left[r_{i-1} r_{i-\frac{1}{2}} r_{i}\right]$. Assuming perfect timing and $y_{i-1}=y_{i-\frac{1}{2}}=y_{i}=X+j Y$

| s |  |  |  |  |  |  |  |  |  | r |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\mathrm{S}_{\text {Q.i }}$ |  |  |  |  |  |  | ${ }^{\text {li.i-1 }}$ | ${ }^{\text {li. }}$ - $\frac{1}{2}$ | ${ }^{1.1}$ | ${ }^{\text {Q }, \text {-1-1 }}$ | ${ }^{r}{ }_{\text {a.i- }}$ | ${ }^{\text {a }}$. $i$ |
| -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | $-X+Y$ | $-X+Y$ | $-X+Y$ | -X-Y | -X-Y | -X-Y |
|  |  | -1 | +1 |  |  |  | -1 | 0 | +1 | $-X+Y$ | -X | -X-Y | -X-Y | -Y | +X-Y |
|  |  | +1 | -1 |  |  |  | +1 | 0 | -1 | -X-Y | -X | $-X+Y$ | +X-Y | -Y | -X-Y |
|  |  | +1 | +1 |  |  |  | +1 | +1 | +1 | -X-Y | -X-Y | -X-Y | +X-Y | +X-Y | +X-Y |
| -1 | +1 | -1 | -1 | -1 | 0 | +1 | -1 | -1 | -1 | $-X+Y$ | +Y | +X+Y | -X-Y | -X | $-X+Y$ |
|  |  | -1 | +1 |  |  |  | -1 | 0 | +1 | $-X+Y$ | 0 | +X-Y | -X-Y | 0 | +X+Y |
|  |  | +1 | -1 |  |  |  | +1 | 0 | -1 | -X-Y | 0 | +X+Y | +X-Y | 0 | $-X+Y$ |
|  |  | +1 | +1 |  |  |  | +1 | +1 | +1 | -X-Y | -Y | +X-Y | +X-Y | +X | $+X+Y$ |
| +1 | -1 | -1 | -1 | +1 | 0 | -1 | -1 | -1 | -1 | +X+Y | +Y | $-X+Y$ | $-X+Y$ | -X | -X-Y |
|  |  | -1 | +1 |  |  |  | -1 | 0 | +1 | $+X+Y$ | 0 | -X-Y | $-X+Y$ | 0 | +X-Y |
|  |  | +1 | -1 |  |  |  | +1 | 0 | -1 | +X-Y | 0 | $-X+Y$ | $+X+Y$ | 0 | -X-Y |
|  |  | +1 | +1 |  |  |  | +1 | +1 | +1 | +X-Y | -Y | -X-Y | $+X+Y$ | +X | +X-Y |
| +1 | +1 | -1 | -1 | +1 | +1 | +1 | -1 | -1 | -1 | $+X+Y$ | +X+Y | +X+Y | $-X+Y$ | $-X+Y$ | $-X+Y$ |
|  |  | -1 | +1 |  |  |  | -1 | 0 | +1 | +X+Y | +X | +X-Y | $-X+Y$ | +Y | +X+Y |
|  |  |  | -1 |  |  |  | +1 | 0 | -1 | +X-Y | +X | +X+Y | $+X+Y$ | +Y | $-X+Y$ |
|  |  |  | +1 |  |  |  | +1 | +1 | +1 | +X-Y | +X-Y | +X-Y | $+X+Y$ | +X+Y | +X+Y |

$\begin{aligned} \text { Table C. } 2 \quad \text { All } 16 \text { possible combinations of }\left[s_{i-1}\right. & \left.s_{i}\right] \text { with the corresponding received samples }\left[r_{i-1} r_{i-\frac{1}{2}} r_{i}\right] \\ & \text { and with } \alpha_{i}^{\prime} \text {. Assuming symbol timing error of } \alpha \text { radians and } y_{i-1}=y_{i-1}=y_{i}=X+j Y\end{aligned}$ and with $\alpha_{i}^{\prime}$. Assuming symbol timing error of $\alpha$ radians and $y_{i-1}=y_{i-\frac{1}{2}}=y_{i}=X+j Y$

| S |  |  |  |  |  |  |  |  | r |  |  |  |  |  | $\alpha^{\prime}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\mathrm{S}_{\text {Q.i }}$ |  |  |  |  |  | $r_{\text {li.-1 }}$ | $r_{\text {l.i. }} \frac{1}{2}$ | ${ }^{1} .1$. | ${ }^{\text {Q }, \text { i-1 }}$ ( | $r_{\text {Qi. }-\frac{1}{2}}$ | ${ }^{\text {a }}$, | $\alpha_{1.1}$ | $\alpha^{\prime}{ }_{\text {Q }, ~}$ |
| -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | $\begin{array}{ll}-1 & -1\end{array}$ | $-X+Y$ | $-X+Y$ | $-X+Y$ | -X-Y | -X-Y -X | -X-Y | 0 | 0 |
|  |  | -1 | +1 |  |  |  | -1 | $+\alpha+1$ | $-X+Y$ | -X- $\alpha$ d | -X-Y | -X-Y | $+\alpha X-Y+$ | +X-Y | - $\alpha^{\text {Y }}$ | $+\infty$ |
|  |  |  | -1 |  |  |  | +1 | $\begin{array}{cc}-\alpha & -1\end{array}$ | -X-Y | $-\mathrm{X}+\alpha \mathrm{Y}$ | $-X+Y$ | +X-Y | - $\alpha$ X-Y -X | -X-Y | + $\alpha$ Y | - $\alpha x$ |
|  |  | +1 | +1 |  |  |  | +1 | +1 +1 | -X-Y | -X-Y | -X-Y | +X-Y | +X-Y + | +X-Y | 0 | 0 |
| -1 | +1 | -1 | -1 | -1 | $+\alpha$ | +1 | -1 | $\begin{array}{ll}-1 & -1\end{array}$ | $-X+Y$ | $+\alpha X+Y$ | +X+Y | -X-Y | $-X+\alpha Y-X$ | -X+Y | $+\alpha \mathrm{X}$ | ${ }^{+} \times$ |
|  |  | -1 | +1 |  |  |  | -1 | $+\alpha+1$ | $-X+Y$ | $+\alpha X-\alpha Y$ | +X-Y | -X-Y | $+\alpha X+\alpha Y+X$ | $+X+Y$ | ${ }_{+}+\mathrm{X}-\alpha \mathrm{Y}$ | ${ }_{+\alpha X}{ }^{\text {P }}+\alpha$ Y |
|  |  | +1 | -1 |  |  |  | +1 | $\begin{array}{ll}-\alpha & -1\end{array}$ | -X-Y | $+\alpha X+\alpha Y$ | $+X+Y$ | +X-Y | $-\alpha X+\alpha Y-X+$ | - $\mathrm{X}+\mathrm{Y}$ | $+\alpha X+\alpha Y$ | $-\alpha X+\alpha Y$ |
|  |  | +1 | +1 |  |  |  | +1 | +1 +1 | -X-Y | $+\alpha X-Y$ | +X-Y | +X-Y | $+X+\alpha Y+$ | $+X+Y$ | $+\alpha X$ | + $\alpha$ Y |
| +1 | -1 | -1 | -1 | +1 | - $\alpha$ | -1 | -1 | $\begin{array}{ll}-1 & -1\end{array}$ | +X+Y | + $\alpha X+Y$ | $-X+Y$ | $-X+Y$ | -X- $\alpha Y$ - | - -Y | - $\alpha \mathrm{X}$ | - OY |
|  |  | -1 | +1 |  |  |  | -1 | $+\alpha+1$ | +X+Y | - $\alpha X-\alpha Y$ | -X-Y | $-X+Y$ | $+\alpha \mathrm{X}-\alpha \mathrm{Y}+$ | +X-Y | $-\alpha X-\alpha Y$ | $+\alpha X-\alpha Y$ |
|  |  |  | -1 |  |  |  | +1 | $\begin{array}{ll}-\alpha & -1\end{array}$ | +X-Y | - $\alpha X+\alpha Y$ | $-X+Y$ | $+X+Y$ | $-\alpha X-\alpha Y$ | X-Y | $-\alpha X+\alpha Y$ | - $\alpha X-\alpha Y$ |
|  |  | +1 | +1 |  |  |  | +1 | +1 +1 | +X-Y | - $\alpha X-Y$ | -X-Y | $+X+Y$ | $+X-\alpha Y+$ | +X-Y | - $\alpha$ X | - $\alpha$ Y |
| +1 | +1 | -1 | -1 | +1 | +1 | +1 | -1 | $\begin{array}{ll}-1 & -1\end{array}$ | +X+Y | +X+Y | +X+Y | $-X+Y$ | $-X+Y \quad-X$ | X+Y | 0 | 0 |
|  |  | -1 | +1 |  |  |  | -1 | $+\alpha+1$ | +X+Y | $+X-\alpha Y$ | +X-Y | $-X+Y$ | $+\alpha X+Y+X+$ | +X+Y | - $\alpha$ Y | $+\infty \times$ |
|  |  |  | -1 |  |  |  | +1 | - $\alpha$-1 | +X-Y | $+\mathrm{X}+\alpha \mathrm{Y}$ | $+X+Y$ | $+X+Y$ | $-\alpha X+Y-X+$ | $-X+Y$ | $+\alpha Y$ | - $\alpha$ X |
|  |  |  | +1 |  |  |  | +1 | +1 +1 | +X-Y | +X-Y | +X-Y | +X+Y | +X+Y + | +X+Y | 0 | 0 |

Again it is assumed that $y(t)$ varies linearly over the time interval (i-1)T太t $i T$. It is shown in Table C. 2 that
$\alpha_{I . i}^{\prime}$ or $\alpha_{Q . i}^{\prime}= \begin{cases}0, & \text { if } s_{i}=s_{i-1} \\ \alpha \cdot\left(\left|y_{I}\right| \text { or }\left|y_{Q}\right| \text { or }\left| \pm y_{I} \pm y_{Q}\right|\right) \text { rads, } & \text { elsewhere }\end{cases}$
(Clearly, no timing information can be extracted if $s_{i}=s_{i-1}$ ). Now, the measured timing error at time $t=i T$ is

$$
\begin{equation*}
\alpha_{i}^{\prime}=K\left(\frac{\alpha_{I . i}^{\prime}+\alpha_{Q . i}^{\prime}}{2 \pi}\right)^{T} \text { seconds } \tag{C.32}
\end{equation*}
$$

Where $K$ is a constant. The optimum value of $K$ is best found by computer simulation. In fact, some form of (fading memory) averaging should be applied to the $\left\{\alpha_{i}^{\prime}\right\}$ to smooth out the scaling components $\left|y_{I}\right|,\left|y_{Q}\right|$, $\left| \pm y_{I} \pm y_{Q}\right|, 0$. The best averaging process needs to be found by computer simulation.

## C. 5 Two 4-level QAM signals received in the channel

It will be shown here that the methods described in Secs.C.3-C. 4 for fast acquisition and maintaining symbol timing recovery with one QAM signal, give the required result with two $4-1$ evel QAM signals in the channel.

It is assumed here that the symbol timing waveform at the receiver is lagging by $\alpha$ radians for signal $s_{1}(t)$ and by $\beta$ radians for signal $s_{2}(t)$. $\alpha$ and $\beta$ are assumed to differ by no more than $\pi / 5$ radians, which is $T / 5=16 \mu s$ (see Sec.2.8.2). The timing algorithms should adjust the timing waveform to its correct position half-way between the two. That is, with an error of $+\frac{1}{2}(\alpha+\beta)$ radians for $s_{1}(t)$ and $-\frac{1}{2}(\alpha+\beta)$ for $s_{2}(t)$.

The same sequence of $N$ phase reversals is transmitted in both $\left\{s_{1 . i}\right\}$ and $\left\{s_{2 . i}\right\}$ for fast acquisition, such that

$$
s_{1 . i}=s_{2 . i}= \begin{cases}+1+j, & \text { for } i=1,3, \ldots, N-1  \tag{C.33}\\ -1-j, & \text { for } i=2,4, \ldots, N\end{cases}
$$

So, to a very close approximation, the two waveforms $p_{1}(t) y_{1}(t)$ and $p_{2}(t) y_{2}(t)$ from the two mobiles are pure sinewaves of the same frequency, with different amplitude and phase components. Thus, in the absence of noise, the received signal waveform $r(t)$ is a pure sinewave with the same frequency but different amplitude and phase to the two constituent waveforms. The fast acquisition, symbol timing recovery algorithm will lock onto this third sinewave. The phase of this sinewave is now investigated.

Assuming noise is absent, from Eq.(C.6)

$$
\begin{equation*}
r(t)=p_{1}(t) y_{1}(t)+p_{2}(t) y_{2}(t) \tag{C.34}
\end{equation*}
$$

That is,

$$
\begin{align*}
& r_{I}(t)=p_{1 . I}(t) y_{1 . I}(t)-p_{1 . Q}(t) y_{1 . Q}(t) \\
& +p_{2 . I}(t) Y_{2 . I}(t)-p_{2 . Q}(t) y_{2 . Q}(t) \\
& r_{Q}(t)=p_{1 . I}(t) y_{1 . Q}(t)+p_{1 . Q}(t) Y_{1 . I}(t) \\
& +p_{2 . I}(t) Y_{2 . Q}(t)+p_{2 . Q}{ }^{(t) Y_{2 . I}}(t) \tag{C.35}
\end{align*}
$$

Where

$$
\begin{align*}
r(t) & =r_{I}(t)+j r_{Q}(t) \\
p_{1}(t) & =p_{1 . I}(t)+j p_{1 . Q}(t) \\
p_{2}(t) & =p_{2 \cdot I}(t)+j p_{2 . Q}(t) \\
y_{1}(t) & =Y_{1 . I}(t)+j Y_{1 . Q}(t) \\
Y_{2}(t)= & Y_{2 . I}(t)+j Y_{2 . Q}(t)  \tag{C.36}\\
\ldots & j=\sqrt{-1}
\end{align*}
$$

The waveforms from the two signals arrive at the receiver such that $p_{1}(t+\alpha T / \pi)$ coincides with $p_{2}(t+\beta T / \pi)$, for all $t$. So, the samples of $r(t)$ are late by $\alpha_{T} / \pi$ seconds for $p_{1}(t)$ and by $\beta T / \pi$ seconds for $p_{2}(t)$. From Fig.C.6.

$$
\begin{align*}
& p_{1 . I}(t)=p_{1 \cdot Q}(t)=\cos \left(\frac{\pi t}{T}+\alpha\right) \\
& p_{2 \cdot I}(t)=p_{2 \cdot Q}(t)=\cos \left(\frac{\pi t}{T}+\beta\right) \tag{C.37}
\end{align*}
$$

So, from Eqs.(C.35) and (C.37)

$$
\begin{align*}
& r_{I}(t)=\left(y_{1 . I}(t)-Y_{1 . Q}(t)\right) \cos \left(\frac{\pi t}{T}+\alpha\right) \\
& +\left(y_{2 . I}(t)-y_{2 . Q}(t)\right) \cos \left(\frac{\pi t}{T}+\beta\right) \\
& r_{Q}(t)=\left(y_{1 . I}(t)+y_{1 . Q}(t)\right) \cos \left(\frac{\pi t}{T}+\alpha\right) \\
& +\left(y_{2 . I}(t)+Y_{2 . Q}(t)\right) \cos \left(\frac{\pi t}{T}+\beta\right) \tag{C.38}
\end{align*}
$$

Expanding the cos terms in $r_{I}(t)$

$$
r_{I}(t)=\left(y_{1 . I}(t)-y_{1 . Q}(t)\right)\left(\cos \alpha \cos \frac{\pi t}{T}-\sin \alpha \sin \frac{\pi t}{T}\right)
$$

$$
\begin{equation*}
+\left(y_{2 . I}(t)-y_{2 . Q}(t)\right)\left(\cos \beta \cos \frac{\pi t}{T}-\sin \beta \sin \frac{\pi t}{T}\right) \tag{C.39}
\end{equation*}
$$

Now, let

$$
\begin{equation*}
r_{I}(t)=R \cos \left(\frac{\pi t}{T}+\gamma\right)=R \cos \gamma \cos \frac{\pi t}{T}-R \sin \gamma \sin \frac{\pi t}{T} \tag{C.40}
\end{equation*}
$$

Equating coefficients in Eqs.(C.39) and (C.40)

$$
\begin{align*}
& R \cos \gamma=\left(y_{1 . I}(t)-y_{1 . Q}(t)\right) \cos \alpha+\left(y_{2 . I}(t)-y_{2 . Q}(t)\right) \cos \beta  \tag{C.41}\\
& R \sin \gamma=\left(y_{1 . I}(t)-y_{1 . Q}(t)\right) \sin \alpha+\left(y_{2 . I}(t)-y_{2 . Q}(t)\right) \sin \beta \tag{C.42}
\end{align*}
$$

Eq.(C.42)/Eq.(C.41) gives

$$
\begin{equation*}
\tan \gamma=\frac{\left(y_{1 . I}(t)-y_{1 . Q}(t)\right) \sin \alpha+\left(y_{2 . I}(t)-y_{2 . Q}(t)\right) \sin \beta}{\left(y_{1 . I}(t)-y_{1 . Q}(t)\right) \cos \alpha+\left(y_{2 . I}(t)-y_{2 . Q}(t)\right) \cos \beta} \tag{C.43}
\end{equation*}
$$



Fig.C. 6 (a) $p_{1}(t)$ and (b) $p_{2}(t)$ with phase reversals

Now, consider the case where

$$
y_{1 . I}(t)-y_{1 . Q}(t)=y_{2 . I}(t)-y_{2 . Q}(t)
$$

The $Y_{1 . I}(t), Y_{1 . Q}(t), Y_{2, I}(t), Y_{2 . Q}(t)$ all have identical statistical properties, so in fact, this would be the average or expected case. Here

$$
\begin{equation*}
\tan \gamma=\frac{\sin \alpha+\sin \beta}{\cos \alpha+\cos \beta} \tag{C.44}
\end{equation*}
$$

But, [99]

$$
\begin{aligned}
& \sin \alpha+\sin \beta=2 \sin \left(\frac{1}{2}(\alpha+\beta)\right) \cdot \cos \left(\frac{1}{2}(\alpha-\beta)\right) \\
& \cos \alpha+\cos \beta=2 \cos \left(\frac{1}{2}(\alpha+\beta)\right) \cdot \cos \left(\frac{1}{2}(\alpha-\beta)\right)
\end{aligned}
$$

Giving

$$
\tan \gamma=\frac{\sin \left(\frac{1}{2}(\alpha+\beta)\right)}{\cos \left(\frac{1}{2}(\alpha+\beta)\right)}=\tan \left(\frac{1}{2}(\alpha+\beta)\right)
$$

and

$$
\begin{equation*}
\gamma=\frac{1}{2}(\alpha+\beta) \tag{C.45}
\end{equation*}
$$

So on average the resultant signal waveform $p_{1}(t) y_{1}(t)+p_{2}(t) y_{2}(t)$ with phase reversals transmitted, is a sinewave with phase $\frac{1}{2}(\alpha+\beta)$. The fast acquisition procedure in Eq. (C.23) would give an estimate of this angle.

However, when $y_{1}(t)$ is in a deep fade

$$
\begin{equation*}
y_{1 . I}(t) \approx y_{1 \cdot Q}(t) \approx 0 \tag{C.46}
\end{equation*}
$$

and

$$
\begin{equation*}
\gamma \approx \beta \tag{C.47}
\end{equation*}
$$

The recovered symbol timing waveform would tend to drift towards the ideal timing waveform for $p_{2}(t)$, and tend to ignore $p_{1}(t)$. Similarly, if $y_{2}(t)$ was in a deep fade, the recovered symbol timing waveform would tend to drift towards the ideal timing waveform for $p_{1}(t)$ and tend to ignore $p_{2}(t)$. Clearly, if a good estimate of $\frac{1}{2}(\alpha+\beta)$ is to be made during the fast acquisition procedure, that is not biased towards $a$ or $\beta$ by the fading, the individual estimates $\gamma_{k}$ (see Eqs. (C.23) and (C.45)) must be averaged over many symbols.

A similar analysis would show that during random data transmission, symbol timing recovery can be maintained half way between the two constituent waveforms using Eq. (C.29), where $\alpha^{\prime}{ }_{i}$ is now replaced by $\gamma_{i}=\frac{1}{2}\left(\alpha_{i}+\beta_{i}\right)^{\prime}$. Again, the phase of the recovered timing waveform would average out to $\frac{1}{2}(\alpha+\beta)$, though it would tend to drift between $\alpha$ and $\beta$ depending on the fading, if the individual estimates of the timing error were not averaged over enough symbols. The length of time over which this averaging should be performed is best evaluated by computer simulation or by hardware tests.

## Theoretical probabilities of errors in detection

The bit error rate versus signal-to-noise ratio curves $P_{b}(\psi)$ are derived theoretically in this appendix, for several cases of interest. Coherent coding of the binary digits is assumed throughout.

## D. 1 Mathematical Background

Some useful mathematical formulae are defined in this section.
The $Q$-function of $x$ is given by $[31,104,106]$

$$
\begin{gather*}
Q(x)=\int_{x}^{\infty} \frac{1}{\sqrt{2 \pi}} \exp \left(-\frac{z^{2}}{2}\right) d z=\int_{-\infty}^{-x} \frac{1}{\sqrt{2 \pi}} \exp \left(\frac{-z^{2}}{2}\right) d z  \tag{D.1}\\
Q(0)=\frac{1}{2}, \quad Q(-\infty)=1, \quad Q(\infty)=0  \tag{D.2}\\
Q(-x)=1-Q(x) \tag{D.3}
\end{gather*}
$$

$Q(x)$ is tabulated in [106] for different values of $x$.
The Gaussian probability density function of $x$ is given by [34]

$$
\begin{equation*}
f(x)=\frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left(-\frac{(x-\bar{x})^{2}}{2 \sigma^{2}}\right) \tag{D.4}
\end{equation*}
$$

where $x$ has mean $\vec{x}$ and variance $\sigma^{2}$

$$
\begin{align*}
\int_{a}^{\infty} f(x) d x & =\int_{a}^{\infty} \frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left(-\frac{(x-\bar{x})^{2}}{2 \sigma^{2}}\right) d x=\int_{\frac{a-\bar{x}}{\sigma}}^{\infty} \frac{1}{\sqrt{2 \pi}} \exp \left(-\frac{z^{2}}{2}\right) d z \\
& =Q\left(\frac{a-\bar{x}}{\sigma}\right) \tag{D.5}
\end{align*}
$$

Proof of Eq.(D.5):
Substituting $(x-\bar{x}) / \sigma=z$ in $f(x)$

$$
\mathrm{dx}=\sigma \mathrm{d} \mathrm{z}
$$

Limits: when $x=a, z=(a-\bar{x}) / \sigma$

$$
\text { . } \mathrm{x}=\infty, \mathrm{z}=\infty
$$

Therefore,

$$
\begin{aligned}
& \int_{a}^{\infty} \frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left(-{\frac{(x-\bar{x})^{2}}{2 \sigma^{2}}}^{2}\right) d x=\int_{\frac{a-\bar{x}}{\sigma}}^{\infty} \frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left(\frac{-}{2}^{2}\right) \sigma d z \\
& =\int_{\frac{a-\bar{x}}{\sigma}}^{\infty} \frac{l}{\sqrt{2 \pi}} \exp \left(-\frac{z}{2}^{2}\right) d z \quad \text { Q.E.D }
\end{aligned}
$$

The integral of a $Q$-function of $x$ times a chi-square distribution of $x$ with two degrees of freedom is given by

$$
\begin{equation*}
\int_{0}^{\infty} Q(\sqrt{a x}) \cdot \frac{1}{\bar{x}} \exp \left(-\frac{x}{\bar{x}}\right) d x=\frac{1}{2}\left[1-\frac{a \bar{x}}{2+a \bar{x}}\right] \tag{D.6}
\end{equation*}
$$

where $\vec{x}$ is the average or expected value of the random variable x , ( $\bar{x}=E[x]$ ). "a" is a constant.

Proof of Eq. (D.6):


Now, integrate by parts

$$
\begin{equation*}
\int \frac{u d v}{d x} \cdot d x=[u v]-\int \frac{v d u}{d x} \cdot d x \tag{D.8}
\end{equation*}
$$

Thus,

$$
\begin{aligned}
\frac{d u}{d x} & =\frac{d}{d x}\left[\frac{1}{2}-\int_{0}^{\sqrt{a x}} \frac{1}{\sqrt{2 \pi}} \exp \left(-\frac{z}{2}^{2}\right) d z\right] \\
& =\frac{-1}{\sqrt{2 \pi}} \cdot \frac{d}{d x} \int_{F(x)}^{\sqrt{a x}} \underbrace{\exp \left(-\frac{z}{2}^{2}\right) d z}_{0}
\end{aligned}
$$

Now, use the identity [122]

$$
\cdot \frac{d F}{d x}(x)=\int_{b}^{c} \frac{\partial f(z, x) d x-f(z=b, x) \frac{d b}{d x}+f(z=c, x) \frac{d c}{d x}, d e r}{d x}
$$

where

$$
F(x)=\int_{b(x)}^{c(x)} f(z, x) d z
$$

So that

$$
\begin{align*}
\frac{d u}{d x} & =-\frac{1}{\sqrt{2 \pi}} \cdot\left\{\int_{0}^{\sqrt{a x}} 0-\frac{1}{d x}-\exp (0) \cdot 0+\exp \left(-\frac{1}{2} a x\right) \cdot \frac{1}{2} \sqrt{a x^{-\frac{1}{2}}}\right\} \\
& =-\sqrt{\frac{a}{8 \pi}} x^{-\frac{1}{2}} \exp \left(-\frac{1}{2} a x\right) \tag{D.9}
\end{align*}
$$

And

$$
\begin{equation*}
v=\frac{1}{\bar{x}} \int \exp \left(\frac{-x}{\bar{x}}\right) d x=-\exp \left(\frac{-x}{\bar{x}}\right) \tag{D.10}
\end{equation*}
$$

Substitute Eqs(D.9) and (D.10) into Eq. (D.8)

$$
\begin{align*}
\int_{0}^{\infty} Q(\sqrt{a x}) \cdot \frac{1}{\bar{x}} \exp \left(\frac{x}{\bar{x}}\right) d x & =\left[-Q(\sqrt{a x}) \cdot \exp \left(-\frac{x}{\bar{x}}\right)\right]_{0}^{\infty}-\sqrt{\frac{a}{8 \pi}} \int_{0}^{\infty} x^{-\frac{1}{2}} \exp \left(-\frac{1}{2} a x\right) \cdot \exp \left(-\frac{x}{\bar{x}}\right) d x \\
& =[-Q(\infty) \exp (\infty)+Q(0) \exp (0)]-\sqrt{\frac{a}{8 \pi}} \int_{0}^{\infty} x^{-\frac{1}{2}} \exp \left(-\left(\frac{a}{2}+\frac{1}{\bar{x}}\right)^{x}\right) d x \\
& =\frac{1}{2}-\sqrt{\frac{a}{8 \pi}} \int_{0}^{\pi} x^{-\frac{1}{2}} \exp \left(-\left(\frac{a \bar{x}+2}{2 \bar{x}}\right)^{x}\right) d x \tag{D.11}
\end{align*}
$$

But if

$$
I=\int_{0}^{\infty} x^{-\frac{1}{2}} \exp (-b x) d x
$$

Substituting $\mathrm{x}=\mathrm{y}^{2}$ in I

$$
d x=2 y d y
$$

Limits: when $x=0, y=0$

$$
\text { " } x=\infty, y=\infty
$$

$$
I=\int_{0}^{\infty} \frac{1}{y} \exp \left(-b y^{2}\right) 2 y d y=2 \int_{0}^{\infty} \exp \left(-b y^{2}\right) d y
$$

Now, substituting by ${ }^{2}=\frac{1}{2} z^{2}$

$$
d_{y}=\frac{1}{\sqrt{2 b}} \mathrm{dz}
$$

Limits: when $y=0, z=0$
" $\mathrm{y}=\infty, \mathrm{z}=\infty$

$$
\begin{align*}
I & =2 \int_{0}^{\infty} \exp \left(-\frac{z}{2}^{2}\right) \cdot \frac{1}{\sqrt{2 b}} d z \\
& =\sqrt{\frac{2}{b}} \cdot \sqrt{2 \pi} \int_{0}^{\infty} \frac{1}{\sqrt{2 \pi}} \exp \left(-\frac{z^{2}}{2}\right) d z \\
& =2 \sqrt{\frac{\pi}{b}} \cdot Q(0) \quad \text { (from Eq. (D.1)) } \\
& =\sqrt{\frac{\pi}{b}} \tag{D.12}
\end{align*}
$$

Substituting Eq.(D.12) into Eq.(D.11)

$$
\begin{aligned}
\int_{0}^{\infty} Q(\sqrt{a x}) \cdot \frac{1}{\bar{x}} \exp \left(\frac{x}{\bar{x}}\right) d x & =\frac{1}{2}-\sqrt{\frac{a}{8 \pi}} \cdot \sqrt{\frac{\pi \cdot 2 \bar{x}}{a \bar{x}+2}} \\
& =\frac{1}{2}\left[1-\sqrt{\frac{a \vec{x}}{2+a \bar{x}}}\right] \quad \text { Q.E.D }
\end{aligned}
$$

Similarly, it can be shown that the integral of a $Q$-function times a chi-square distribution with four degrees of freedom is given by

$$
\begin{equation*}
\int_{0}^{\infty} Q(\sqrt{a x}) \cdot \frac{2 x}{\bar{x}^{2}} \exp \left(-\frac{2 x}{\bar{x}}\right) d x=\frac{1}{2}\left[1-\sqrt{\frac{a \bar{x}}{4+a \bar{x}}}-\frac{2}{a \bar{x}}\left(\sqrt{\frac{a \bar{x}}{4+a \bar{x}}}\right)^{3}\right] \tag{D.13}
\end{equation*}
$$

## D. 2 Bit error rate curves for System 1, with no differential coding and perfect channel estimation

One 4-level QAM signal is transmitted in the channel. Assume one receiving antenna and maximum likelihood detection according to Eq.(3.3.3). Consider the special case with no fading, that is where $y_{i}=1$ and $r_{i}=s_{i}+w_{i}$ for all $\{i\}$.

The transmitted data symbol value at time $t=i T$ is $s_{i}= \pm 1 \pm j$. It is useful to consider here that $s_{i}= \pm k \pm j k$ (where $k=1$ ), as shown in Table D. 1 and Fig.D.1. Thus (see Appendix B)

$$
\begin{equation*}
E_{b}=k^{2} \tag{D.14}
\end{equation*}
$$

At time $t=i T$, the baseband received sample at the input to the detector is

$$
\begin{equation*}
r_{i}=s_{i}+w_{i} \tag{D.15}
\end{equation*}
$$

where $r_{i}, s_{i}, w_{i}$ are all complex-valued

$$
\begin{align*}
& r_{i}=r_{I . i}+j r_{Q . i} \\
& s_{i}=s_{I_{. i}}+j s_{Q . i} \\
& w_{i}=w_{I_{. i}}+j w_{Q . i} \tag{D.16}
\end{align*}
$$

${ }^{w_{I . i}}$ and $W_{Q . i}$ are independent Gaussian random variables with zero mean and variance (see Appendix B)

$$
\begin{equation*}
\sigma^{2}=\frac{1}{2} N_{0} \tag{D.17}
\end{equation*}
$$

The probability density function of each quadrature noise component is (from Eq. (D.4))

$$
\begin{equation*}
\mathrm{f}(\mathrm{w})=\frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left(-\frac{\mathrm{w}^{2}}{2 \sigma^{2}}\right) \tag{D.18}
\end{equation*}
$$

If the symbol $s_{i}=+k+j k$ is transmitted, then the quadrature components of $r_{i}$ are independent Gaussian random variables, mean +k , variance $\sigma^{2}$. That is,

$$
f\left(r_{Q}\right)=f\left(r_{I}\right)=\frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left(\frac{-\left(r_{I}-k\right)^{2}}{2 \sigma^{2}}\right)
$$

Let $P_{M-N}$ denote the probability that $s_{i}$ with point number $M$ is transmitted and is detected as $s^{\prime}{ }_{i}$ with point number $N$ (Table D.1). Then the probability of correct detection of $s_{i}=+k+j k$ is


Fig.D. 1 Constellation of $s_{i}$ for System 1

Table D. 1 Constellation of $s_{i}$ for System 1

| point number | $s_{i}$ | binary digits |
| :---: | :---: | :---: |
| 0 | $-k-j k$ | 00 |
| 1 | $+k-j k$ | 01 |
| 2 | $-k+j k$ | 10 |

$$
P_{3-3}=\int_{0}^{\infty} \frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left(\frac{-\left(r_{I}-k\right)^{2}}{2 \sigma^{2}}\right) d r_{I} \cdot \int_{0}^{\infty} \frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left(\frac{-\left(r_{Q}-k\right)^{2}}{2 \sigma^{2}}\right) d r_{Q}
$$

which is the probability that both $r_{I}$ and $r_{Q}$ are positive. Thus, from Eq. (D.5)

$$
\begin{align*}
P_{3-3} & =[Q(-k / \sigma)]^{2} \\
& =[1-Q(k / \sigma)]^{2} \quad \text { (from Eq.(D.3)) } \\
& =1-2 Q(k / \sigma)+[Q(k / \sigma)]^{2} \tag{D.19}
\end{align*}
$$

The probability that the symbol $s_{i}{ }_{i}=-k-j k$ is detected with $s_{i}=+k+j k$
transmitted is

$$
P_{3-0}=\int_{-\infty}^{0} \frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left(\frac{-\left(r_{I}-k\right)^{2}}{2 \sigma^{2}}\right)^{d r} I \cdot \int_{-\infty}^{0} \frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left(\frac{-\left(r_{Q}-k\right)^{2}}{2 \sigma^{2}}\right) d r_{Q}
$$

which is the probability that both $r_{I}$ and $r_{Q}$ are negative. Thus, from Eq. (D.5)

$$
\begin{equation*}
P_{3-0}=[Q(k / \sigma)]^{2} \tag{D.20}
\end{equation*}
$$

Now, by symmetry $P_{3-1}=P_{3-2}$. So

$$
P_{3-0}+P_{3-1}+P_{3-2}+P_{3-3}=1
$$

Therefore

$$
\begin{align*}
P_{3-1}=P_{3-2} & =\frac{1}{2}\left(1-P_{3-0}-P_{3-3}\right) \\
& =Q(k / \sigma)-[Q(k / \sigma)]^{2} \tag{D.21}
\end{align*}
$$

Therefore, the bit error rate with $s_{i}=+k+j k$ transmitted is

$$
P_{b}=\frac{2}{2} P_{3-0}+\frac{1}{2} P_{3-1}+\frac{1}{2} P_{3-2}+\frac{O}{2} P_{3-3}
$$

(since $P_{3-0}$ gives 2 bit errors, $P_{3-1}$ and $P_{3-2}$ give 1 bit error and $P_{3-3}$ gives no bit errors, with 2 bits per symbol).

$$
\begin{align*}
P_{b} & =P_{3-0}+P_{3-1} \\
& =Q(k / \sigma) \tag{D.22}
\end{align*}
$$

However, from Eqs.(D.14) and (D.17)

$$
\begin{equation*}
\frac{k}{\sigma}=\sqrt{\frac{E_{b}}{\frac{1}{2} N_{0}}}=\sqrt{2 \psi} \tag{D.23}
\end{equation*}
$$

where

$$
\begin{equation*}
\psi=\frac{E_{b}}{N_{0}} \tag{D.24}
\end{equation*}
$$

Therefore

$$
\begin{equation*}
P_{b}(\psi)=Q(\sqrt{2 \psi}) \tag{D.25}
\end{equation*}
$$

From the symmetry of the constellation, and since the $\left\{s_{i}\right\}$ are statistically independent and equally likely to have any value ( $\pm k \pm j k$ ), this is also the overall bit error rate for System 1A, with perfect estimation and no differential coding.

Now, consider the general case for flat Rayleigh fading, with $r_{i}=s_{i} y_{i}+w_{i}$, for all \{i\}. Eq. (D.25) can be considered to be the bit error rate for a time-invarient channel with fixed attenuation $\left|y_{i}\right|^{2}=1$, where

$$
\begin{equation*}
\psi_{i}=\left|Y_{i}\right|^{2} \frac{E_{b}}{N_{0}} \tag{D.26}
\end{equation*}
$$

Eq. (D.26) is now taken to be the equation defining instantaneous signal-to-noise ratio. If $\left|y_{i}\right|^{2=1}$ for all $\{i\}$ as defined previously, then $\psi_{i}$ reduces to the original form in Eq. (D.24). So Eq. (D.25) is the conditional bit error rate, where the condition is that $\left|y_{i}\right|^{2}$ is constant. To obtain the bit error rate when $\left|y_{i}\right|^{2}$ is random, $P_{b}(\psi)$ must be averaged over the probability density function of $\psi(f(\psi))$. That is

$$
\begin{equation*}
P_{b}(\bar{\psi})=\int_{0}^{\infty} P_{b}(\psi) f(\psi) d \psi \tag{D.27}
\end{equation*}
$$

Where $\bar{\psi}$ is the average signal-to-noise ratio (averaged over the fading) defined mathematically as

$$
\begin{equation*}
\bar{\psi}=\frac{E_{b}}{N_{0}} \cdot E\left[\left|y_{i}\right|^{2}\right] \tag{D.28}
\end{equation*}
$$

The term $E\left[\left|y_{i}\right|^{2}\right]$ is simply the average value of $\left|y_{i}\right|^{2}$, and has been set to 1 throughout this thesis. So

$$
\begin{equation*}
\bar{\psi}=\frac{E_{b}}{N_{0}} \tag{D.29}
\end{equation*}
$$

It is important to note that in a fading channel, the bit error rate can only be defined against the average signal-to-noise ratio $\bar{\psi}$. Also, it is interesting to note that the bit error rate $P_{b}(\bar{\psi})$ does not depend on; vehicle speed, the rate of fading, the average depth / duration of fades or the shape of the power spectrum of $y(t),|Y(f)|^{2}$. It only depends on the Rayleigh probability density function of the fading.

It can be shown $[8,22,23]$ that $f(\psi)$ is a chi-square distribution with two degrees of freedom such that

$$
\begin{equation*}
f(\psi)=\frac{1}{\bar{\psi}} \exp \left(\frac{\psi}{\bar{\psi}}\right), \quad \text { for } \psi \geqslant 0 \tag{D.30}
\end{equation*}
$$

Therefore, in flat Rayleigh fading, the bit error rate for System 1 A with no differential coding and perfect channel estimation is (substitute Eqs.(D.25) and (D.30) into Eq.(D.27))

$$
\begin{equation*}
P_{b}(\bar{\psi})=\int_{0}^{\infty} Q(\sqrt{2 \psi}) \cdot \frac{1}{\bar{\psi}} \exp \left(-\frac{\psi}{\bar{\psi}}\right) d \psi \tag{D.31}
\end{equation*}
$$

Which is, from Eq.(D.6)

$$
\begin{align*}
P_{b}(\vec{\psi}) & =\frac{1}{2}\left[1-\sqrt{\frac{2 \bar{\psi}}{2+2 \bar{\psi}}}\right] \\
& =\frac{1}{2}\left[1-\sqrt{\frac{\bar{\psi}}{1+\bar{\psi}}}\right] \tag{D.32}
\end{align*}
$$

Now consider the corresponding bit error rate curves with two receiving antennas. The bit error rate curve in Eq. (D.25), for no fading, has been derived by comparing $r_{I}$ and $r_{Q}$ with simple threshold levels. There is no simple threshold level detection that can be used with two receiving antennas. Alternatively, Eq. (D.25) could be derived by considering the distance $d$ between $s_{i}$ and the nearest decision boundary [104]. The decision boundary always lies half way between two possible data points in the received signal constellation. Clearly (see Fig.D.1),

$$
\begin{equation*}
d=k, \quad \text { for all } s_{i}= \pm k \pm j k \tag{D.33}
\end{equation*}
$$

since $y_{i}=1$, for all \{i\} here. This is shown mathematically as
$d=\frac{1}{2}\left|s_{0}-s_{1}\right|=\frac{1}{2}|(-k-j k)-(+k-j k)|=\frac{1}{2}|-2 k|=k$
for the decision boundary between the two possible points $s_{0}=-k-j k$ and $s_{1}=+k-j k$ (Table D.1). Where $|x|$ is the absolute value of the complex valued quantity $x$. Thus, substituting Eq.(D.34) into Eq.(D.22), the bit error rate in the absence of fading with one receiving antenna is

$$
\begin{equation*}
P_{b}=Q(d / \sigma) \tag{D.35}
\end{equation*}
$$

It is shown in [104] that Eq. (D.35) is in fact true generally for optimum maximum likelihood detection with N receiving antennas. However, the value of $d$ increases as the number of receiving antennas increases. So with two receiving, antennas and optimum detection according to Eq. (3.3.4)

$$
\begin{equation*}
d=\frac{1}{2}\left|S_{0}-s_{1}\right| \tag{D.36}
\end{equation*}
$$

where

$$
\begin{align*}
& s_{0}=\left[\begin{array}{ll}
s_{a .0} & s_{b .0}
\end{array}\right]=[-k-j k \quad-k-j k] \\
& s_{1}=\left[\begin{array}{ll}
s_{a .1} & s_{b .1}
\end{array}\right]=[+k-j k \quad+k-j k] \\
&\left(s_{a .0} \text { is point } s_{0} \text { at antenna A, and so on for } s_{a .1}, s_{b .0^{\prime}} s_{b .1}\right) \text {. So } \\
& d=\frac{1}{2} \sqrt{\left|s_{a .0^{-s} a .1}\right|^{2}+\left|s_{b .0^{-s}} s_{b .1}\right|^{2}} \\
&=\frac{1}{2} \sqrt{4 k^{2}+4 k^{2}}  \tag{D.37}\\
&=\sqrt{2} k
\end{align*}
$$

Thus, the bit error rate for System 1B with perfect estimation and no differential coding is (from Eqs.(D.35) and (D.37))

$$
\begin{equation*}
P_{b}=Q(\sqrt{2} k / \sigma) \tag{D.38}
\end{equation*}
$$

So,

$$
\begin{equation*}
P_{b}(\psi)=Q(2 \sqrt{\psi}) \tag{D.39}
\end{equation*}
$$

Where

$$
\psi=\frac{E_{b}}{N_{0}}=\frac{k^{2}}{2 \sigma^{2}}
$$

is the signal-to-noise ratio in the signal at each antenna,
The same bit error rate results (Eq. (D.39) if the received signals from the two antennas are coherently combined (as in maximal ratio combining) and a maximum likelihood detection is carried out on the single combined signal according to Eq.(3.3.3). This is because the signal-to-noise ratio out of the combiner, $\psi_{M R}$, equals the sum of the branch signal-to-noise ratios [8,107]. That is

$$
\begin{equation*}
\psi_{M R}=2 \psi=2 \mathrm{E}_{\mathrm{b}} / \mathrm{N}_{0} \tag{D.40}
\end{equation*}
$$

Clearly, in this case (from Eq. (D.25))

$$
\begin{equation*}
P_{b}(\psi)=Q\left(\sqrt{2 \psi_{M R}}\right)=Q(2 \sqrt{\psi}) \tag{D.41}
\end{equation*}
$$

which is the same as Eq. (D.39) for the optimum detection process.
Now, consider the case where there is independent flat Rayleigh fading of the signals at the two receiving antennas. It can be shown that with two receiving antennas $[8,9,22,23] f(\psi)$ is a chi-square distribution with four degrees of freedom, such that

$$
\begin{equation*}
f(\psi)=\frac{2 \psi}{\bar{\psi}^{2}} \exp \left(-\frac{2 \psi}{\bar{\psi}}\right) \tag{D.42}
\end{equation*}
$$

Where again, $\psi$ and $\bar{\psi}$ are defined by Eqs.(D.24) and (D.29) respectively. (The same distribution is true for maximal ratio combining, so it must have the same bit error rate).

So with independent, flat Rayleigh fading at antennas $A$ and $B$, the bit error rate for System 1B with no differential coding and perfect channel estimation is (substitute Eqs.(D.41) and (D.42) into Eq.(D.27))

$$
\begin{equation*}
P_{b}(\bar{\psi})=\int_{0}^{\infty} Q(2 \sqrt{\psi}) \cdot \frac{2 \psi \exp }{\bar{\psi}^{2}}\left(-\frac{2 \psi}{\bar{\psi}}\right) d \psi \tag{D.43}
\end{equation*}
$$

which is, from Eq.(D.13)

$$
\begin{align*}
P_{b}(\bar{\psi}) & =\frac{1}{2}\left[1-\sqrt{\frac{4 \bar{\psi}}{4+4 \bar{\psi}}}-\frac{2}{4 \bar{\psi}}\left(\sqrt{\frac{4 \bar{\psi}}{4+4 \bar{\psi}}}\right)^{3}\right] \\
= & \frac{1}{2}\left[1-\sqrt{\left.\frac{\bar{\psi}}{1+\bar{\psi}}-\frac{1}{2 \bar{\psi}}\left(\sqrt{\frac{\bar{\psi}}{1+\bar{\psi}}}\right)^{3}\right]}\right. \tag{D.44}
\end{align*}
$$

## D. 3 Bit error rate curves for System 3 with no differential coding and

## perfect channel estimation

One 16 -level QAM signal is transmitted in the channel. Assume one receiving antenna and maximum likelihood detection according to

Eq.(5.3.3). Consider first the special case with no fading, that is where $y_{i}=1$ and $r_{i}=s_{i}+w_{i}$ for all $\{i\}$.

The transmitted data symbol value is $s_{i}=( \pm 1$ or $\pm 3)+( \pm j$ or $\pm 3 j)$. It is useful to consider that $s_{i}=( \pm k$ or $\pm 3 k)+( \pm j$ or $\pm j 3 k)$, (where $k=1$ ), as shown in Table D. 2 and Fig.D.2. Thus, (see Appendix B)

$$
\begin{equation*}
E_{b}=2.5 k^{2} \tag{D.45}
\end{equation*}
$$

At time $t=i T$, the baseband received sample at the input to the detector is

$$
\begin{equation*}
r_{i}=s_{i}+w_{i} \tag{D.46}
\end{equation*}
$$

where $r_{i}, s_{i}, w_{i}$ are all complex-valued

$$
\begin{align*}
& r_{i}=r_{I_{. i}}+j r_{Q . i} \\
& s_{i}=s_{I_{. i}}+j s_{Q . i} \\
& w_{i}=w_{I_{. i}}+j w_{Q . i} \tag{D.47}
\end{align*}
$$

$w_{\text {I.i }}$ and $w_{\text {Q.i }}$ are independent Gaussian random variables with zero mean and variance (see Appendix B)

$$
\begin{equation*}
\sigma^{2}=\frac{1}{2} N_{0} \tag{D.48}
\end{equation*}
$$

The probability density function of each quadrature noise component is (from Eq. (D.4))

$$
\begin{equation*}
f(w)=\frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left(\frac{-w^{2}}{2 \sigma^{2}}\right) \tag{D.49}
\end{equation*}
$$

If the symbol $s_{i}=+k+j k$ is transmitted, then the quadrature components of $r_{i}$ are independent Gaussian random variables, mean $+k$, variance $\sigma^{2}$. That is,

$$
\begin{equation*}
f\left(r_{Q}\right)=f\left(r_{I}\right)=\frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left(\frac{-\left(r_{I}-k\right)^{2}}{2 \sigma^{2}}\right) \tag{D.50}
\end{equation*}
$$

Let $P_{M-N}$ denote the probability that $s_{i}$ with point number $M$ is transmitted and is detected as $s^{\prime}{ }_{i}$ with point number $N$ (Table D.1). Then the probability of correct detection of $s_{i}=+k+j k$ is

$$
\begin{equation*}
P_{15-15}=\int_{0}^{2 k} \frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left(\frac{-\left(r_{I}-k\right)^{2}}{2 \sigma^{2}}\right) d r_{I} \cdot \int_{0}^{2 k} \frac{I}{\sqrt{2 \pi \sigma^{2}}} \exp \left(\frac{-\left(r_{Q}-k\right)^{2}}{2 \sigma^{2}}\right) d r_{Q} \tag{D.51}
\end{equation*}
$$

which is the probability that both $r_{I}$ and $r_{Q}$ are in the range 0 to $+2 k$. Thus, from Eq.(D.5)

$$
\begin{align*}
P_{15-15} & =[Q(-k / \sigma)-Q(k / \sigma)]^{2} \\
& =[1-2 Q(k / \sigma)]^{2} \quad \text { (from Eq. (D.3)) } \\
& =1-4 Q(k / \sigma)+4[Q(k / \sigma)]^{2} \tag{D.52}
\end{align*}
$$

An exactly similar procedure can be carried out to give the probability of all 15 possible incorrectly detected symbols to give


Fig.D. 2 Constellation of $s_{i}$ for System 3

Table D. 2 Constellation of $s_{i}$ for System 3

| point number | $s_{i}$ | binary digits |
| :---: | :---: | :---: |
| 0 | $-3 k-j 3 k$ | 0000 |
| 1 | $-k-j 3 k$ | 0001 |
| 2 | $-3 k-j k$ | 0010 |
| 3 | $-k-j k$ | 0011 |
| 4 | $3 k-j 3 k$ | 0100 |
| 5 | $k-j 3 k$ | 0101 |
| 6 | $3 k-j k$ | 0110 |
| 7 | $k-j k$ | 0111 |
| 8 | $-3 k+j 3 k$ | 1000 |
| 9 | $-k+j 3 k$ | 1001 |
| 10 | $-3 k+j k$ | 1010 |
| 11 | $-k+j k$ | 1011 |
| 12 | $3 k+j 3 k$ | 1100 |
| 13 | $k+j 3 k$ | 1101 |
| 14 | $3 k+j k$ | 1110 |
| 15 | $-k+j k$ | 1111 |

$$
\begin{align*}
P_{15-0} & =[Q(3 k / \sigma)]^{2} \\
P_{15-1} & =Q(k / \sigma) Q(3 k / \sigma)-[Q(3 k / \sigma)]^{2} \\
P_{15-2} & =P_{15-1} \\
P_{15-3} & =[Q(k / \sigma)]^{2}-2 Q(k / \sigma) Q(3 k / \sigma)+[Q(3 k / \sigma)]^{2} \\
P_{15-4} & =Q(k / \sigma) Q(3 k / \sigma) \\
P_{15-5} & =-2 Q(k / \sigma) Q(3 k / \sigma)+Q(3 k / \sigma) \\
P_{15-6} & =[Q(k / \sigma)]^{2}-Q(k / \sigma) Q(3 k / \sigma) \\
P_{15-7} & =Q(k / \sigma)-2[Q(k / \sigma)]^{2}+2 Q(k / \sigma) Q(3 k / \sigma)-Q(3 k / \sigma) \\
P_{15-8} & =P_{15-4} \\
P_{15-9} & =P_{15-6} \\
P_{15-10} & =P_{15-5} \\
P_{15-11} & =P_{15-7} \\
P_{15-12} & =[Q(k / \sigma)]^{2} \\
P_{15-13} & =Q(k / \sigma)-2[Q(k / \sigma)]^{2} \\
P_{15-14} & =P_{15-13} \tag{D.53}
\end{align*}
$$

A useful check here is that

$$
\begin{equation*}
\mathrm{P}_{15-0}+\mathrm{P}_{15-1}+\ldots+\mathrm{P}_{15-15}=1 \tag{D.54}
\end{equation*}
$$

Therefore, the bit error rate when $s_{i}=+k+j k$ is transmitted is

$$
\begin{align*}
P_{b}\left(\text { with } s_{i}=+k+j k\right) & =\frac{4}{4} P_{15-0}+\frac{3}{4} P_{15-1}+\frac{3}{4} P_{15-2}+\frac{2}{4} P_{15-3}+\frac{3}{4} P_{15-4} \\
& +\frac{2}{4} P_{15-5}+\frac{2}{4} P_{15-6}+{ }^{\frac{1}{4} P_{15-7}+\frac{3}{4} P_{15-8}+\frac{2}{4} P_{15-9}} \\
& +\frac{2}{4} P_{15-10}+\frac{1 P_{15-11}}{}+\frac{2}{4} P_{15-12}+\frac{1}{4} P_{15-13}+\frac{1 P_{15}}{} 15-14 \\
& +\frac{O P_{15-15}}{15} \tag{D.55}
\end{align*}
$$

Where, the fraction multiplying each $\mathrm{P}_{15-\mathrm{N}}$ is the ratio of the number of binary digits detected in error if $s^{\prime}{ }_{i}$ is detected as point number $N$ (Table D.2), divided by 4 (the total number of binary digits in $s_{i}$ ). So (substituting Eq. (D.53) into Eq.(D.55))

$$
\begin{equation*}
P_{b}\left(w i t h s_{i}=+k+j k\right)=Q(k / \sigma)+\frac{1}{2} Q(3 k / \sigma) \tag{D.56}
\end{equation*}
$$

In an exactly similar way it can be shown that

$$
\begin{align*}
P_{b}\left(\text { with } s_{i}=+k+j 3 k\right) & =\frac{3}{4} Q(k / \sigma)+\frac{1}{2} Q(3 k / \sigma)-\frac{1}{4} Q(5 k / \sigma)  \tag{D.57}\\
P_{b}\left(\text { with } s_{i}=+3 k+j k\right) & =P_{b}\left(\text { with } s_{i}=+k+j 3 k\right)  \tag{D.58}\\
P_{b}\left(\text { with } s_{i}=+3 k+j 3 k\right) & =\frac{1}{2} Q(k / \sigma)+\frac{1}{2} Q(3 k / \sigma)-\frac{1}{2} Q(5 k / \sigma) \tag{D.59}
\end{align*}
$$

Now, because of the symmetry of the constellation and the Gray coded bit mapping, the overall bit error rate for System 3A in the absence of fading with no differential coding and perfect channel estimation is

$$
\begin{aligned}
P_{b}=\frac{4}{16} P_{b}\left(\text { with } s_{i}=+k+j k\right) & +\frac{8}{16} P_{b}\left(\text { with } s_{i}=+k+j 3 k\right) \\
& +\frac{4}{16} P_{b}\left(\text { with } s_{i}=+3 k+j 3 k\right)
\end{aligned}
$$

Where the fraction multiplying each $P_{b}$ is the ratio of the number of points in the constellation with this bit error rate divided by 16 (the total number of points in the constellation). So, substituting Eqis.(D.56)-(D.59) into Eq.(D.60) gives

$$
\begin{equation*}
P_{b}=\frac{3}{4} Q(k / \sigma)+\frac{1}{2} Q(3 k / \sigma)-\frac{1}{4} Q(5 k / \sigma) \tag{D.61}
\end{equation*}
$$

But, from Eqs.(D.45) and (D.48),

$$
\begin{equation*}
\frac{k}{\sigma}=\sqrt{\frac{E_{b} / 2.5}{\frac{1}{2} N_{0}}}=\sqrt{0.8 \%} \tag{D.62}
\end{equation*}
$$

where

$$
\begin{equation*}
\psi=E_{b} / N_{0} \tag{D.63}
\end{equation*}
$$

Therefore,

$$
\begin{equation*}
P_{b}(\psi)=\frac{3}{4} Q(\sqrt{0.8 \psi})+\frac{1}{2} Q(3 \sqrt{0.8 \psi})-4 Q(5 \sqrt{0.8 \psi}) \tag{D.64}
\end{equation*}
$$

So, for System 3A in the presence of flat Rayleigh fading (see Eq.(D.27))

$$
\begin{equation*}
P_{b}(\bar{\psi})=\int_{0}^{\infty} P_{b}(\psi) \cdot f(\psi) d \psi \tag{D.65}
\end{equation*}
$$

where $P_{b}(\psi)$ and $f(\psi)$ are defined by Eqs.(D.64) and (D.30) respectively, and

$$
\begin{equation*}
\bar{\psi}=\frac{E_{b}}{N_{0}}\left[\left|Y_{i}\right|^{2}\right]=\frac{E_{b}}{N_{0}} \tag{D.66}
\end{equation*}
$$

So, using Eq.(D.6),

$$
\begin{align*}
\mathrm{P}_{b}(\bar{\psi}) & =\frac{3}{8}\left[1-\sqrt{\frac{0.8 \bar{\psi}}{2+0.8 \bar{\psi}}}\right]+\frac{1}{4}\left[1-\sqrt{\frac{7.2 \bar{\psi}}{2+7.2 \bar{\psi}}}\right]-\frac{1}{8}\left[1-\sqrt{\frac{20 \vec{\psi}}{2+20 \bar{\psi}}}\right] \\
& =\frac{1}{2}-\frac{3}{8} \sqrt{\frac{2 \bar{\psi}}{5+2 \bar{\psi}}}-\frac{3}{4} \sqrt{\frac{2 \vec{\psi}}{5+18 \bar{\psi}}}+\frac{5}{8} \sqrt{\frac{2 \bar{\psi}}{5+50 \bar{\psi}}} \tag{D.67}
\end{align*}
$$

It was shown in Sec.D. 2 that when going from one to two receiving antennas, the distance between each point and the nearest decision boundary is effectively doubled. So, with two receiving antennas (System 3B), with optimum detection according to Eq.(5.3.4) in the absence of fading and no differential coding

$$
\begin{equation*}
P_{b}(\psi)=\frac{3}{2} Q(\sqrt{1.6 \psi})+\frac{1}{2} Q(3 \sqrt{1.6 \psi})-\frac{1}{4} Q(5 \sqrt{1.6 \psi}) \tag{D.68}
\end{equation*}
$$

That is, $\sqrt{W}$ is replaced by $\sqrt{2 \psi}$ in Eq. (D.64).
So, for System 3B in the presence of Rayleigh fading (see Eq. (D.27))

$$
\begin{equation*}
P_{b}(\bar{\psi})=\int_{0}^{\infty} P_{b}(\psi) \cdot f(\bar{\psi}) d \psi \tag{D.69}
\end{equation*}
$$

Where $P_{b}(\psi)$ and $f(\psi)$ are defined by Eqs. (D.68) and (D.42) respectively and

$$
\begin{equation*}
\vec{\psi}=\frac{E_{b}}{N_{0}} \cdot E\left[\left|y_{a \cdot i}\right|^{2}\right]=\frac{E_{b}}{N_{0}} \cdot E\left[\left|Y_{b \cdot i}\right|^{2}\right]=\frac{E_{b}}{N_{0}} \tag{D.70}
\end{equation*}
$$

So, using Eq.(D.13)

$$
\begin{align*}
& \mathrm{P}_{\mathrm{b}}(\bar{\psi})= \frac{3}{8}\left[1-\sqrt{\frac{1.6 \bar{\psi}}{4+1.6 \bar{\psi}}}-\frac{2}{1.6 \bar{\psi}}\left(\sqrt{\frac{1.6 \bar{\psi}}{4+1.6 \bar{\psi}}}\right)^{3}\right] \\
& \quad+\frac{1}{4}\left[1-\sqrt{\frac{14.4 \bar{\psi}}{4+14.4 \bar{\psi}}}-\frac{2}{14.4 \bar{\psi}}\left(\sqrt{\frac{14.4 \bar{\psi}}{4+14.4 \bar{\psi}}}\right)^{3}\right] \\
&-\frac{1}{8}\left[1-\sqrt{\frac{40 \bar{\psi}}{4+40 \bar{\psi}}}-\frac{2}{40 \bar{\psi}}\left(\sqrt{\frac{40 \bar{\psi}}{4+40 \bar{\psi}}}\right)^{3}\right] \\
&= \frac{1}{2}-\frac{3}{8} \sqrt{\frac{2 \bar{\psi}}{5+2 \bar{\psi}}}-\frac{3}{4} \sqrt{\frac{2 \bar{\psi}}{5+18 \bar{\psi}}}+\frac{5}{8} \sqrt{\frac{2 \bar{\psi}}{5+50 \bar{\psi}}} \\
&-\frac{15}{32 \bar{\psi}}\left(\sqrt{\frac{2 \bar{\psi}}{5+2 \bar{\psi}}}\right)^{3}-\frac{15}{16 \vec{\psi}}\left(\sqrt{\frac{2 \bar{\psi}}{5+18 \bar{\psi}}}\right)^{3}+\frac{25}{32 \bar{\psi}}\left(\sqrt{\frac{2 \vec{\psi}}{5+50 \bar{\psi}}}\right)^{3} \tag{D.71}
\end{align*}
$$

The theoretical bit error rate curves are shown in Fig.D. 3.


FIg.D. 3 Theoretical bit error rate curves for Systems 1 and 3 with perfect channel estimation and no differential coding

## Inverse Taylor Matrices

$$
\mathrm{T}_{6}^{-1}=\frac{1}{5!}\left[\begin{array}{rrrrrr}
120 & 274 & 225 & 85 & 15 & 1 \\
0 & -600 & -770 & -355 & -70 & -5 \\
0 & 600 & 1070 & 590 & 130 & 10 \\
0 & -400 & -780 & -490 & -120 & -10 \\
0 & 150 & 305 & 205 & 55 & 5 \\
0 & -24 & -50 & -35 & -10 & -1
\end{array}\right]
$$

$$
\begin{aligned}
& \mathrm{T}_{2}^{-1}=\left[\begin{array}{rr}
1 & 1 \\
0 & -1
\end{array}\right] \\
& T_{3}{ }^{-1}=\frac{1}{2!}\left[\begin{array}{rrr}
2 & 3 & 1 \\
0 & -4 & -2 \\
0 & 1 & 1
\end{array}\right] \\
& \mathrm{T}_{4}{ }^{-1}=\frac{1}{3!}\left[\begin{array}{rrrr}
6 & 11 & 6 & 1 \\
0 & -18 & -15 & -3 \\
0 & 9 & 12 & 3 \\
0 & -2 & -3 & -1
\end{array}\right] \\
& \mathrm{T}_{5}{ }^{-1}=\frac{1}{4!}\left[\begin{array}{rrrrr}
24 & 50 & 35 & 10 & 1 \\
0 & -96 & -104 & -36 & -4 \\
0 & 72 & 114 & 48 & 6 \\
0 & -32 & -56 & -28 & -4 \\
0 & 6 & 11 & 6 & 1
\end{array}\right]
\end{aligned}
$$

APPENDIX F

## Sinewave Scheme: Measuring amplitude and phase of a sinewave in noise

In this appendix, Eqs.(3.4.40), (3.4.42), (3.4.44) and (3.4.45) are derived, which are used to estimate the amplitude and phase of a sinewave in additive white Gaussian noise. The equations automatically correct for any dc bias superimposed on this sinewave.

## F. 1 Basic Assumptions

The unbiased channel estimate at time $t=i T$, with $s^{\prime}{ }_{i}=s_{i}$, is given by (Eq.(3.4.3))

$$
\begin{equation*}
y_{i}^{\prime}=s_{i}^{-1} r_{i}=y_{i}+s_{i}^{-1} w_{i} \tag{F.1}
\end{equation*}
$$

The real part of this estimate is (from Sec.3.4.2)

$$
\begin{equation*}
Y_{I_{. i}}^{\prime}=Y_{I_{. i}}+W_{i} \tag{F.2}
\end{equation*}
$$

where, the $\left\{W_{i}\right\}$ are statistically independent Gaussian random variables with zero mean and variance $\frac{1}{2} \sigma^{2}$.

Let the general expression for $Y_{I_{. i}}$ be

$$
\begin{equation*}
y_{I_{. i}}=x_{i}+v_{i} \tag{F.3}
\end{equation*}
$$

Where

$$
\begin{equation*}
x_{i}=(a+b) \sin \left(\theta_{i}+c\right) \tag{F.4}
\end{equation*}
$$

is the sinewave component of $y_{I_{\text {. }}}$ and $v_{i}$ is the residual fading component of $Y_{I_{. i}}$. In the subsequent analysis, $v_{i}$ is assumed to remain constant over any half cycle of the sinewave $\left\{x_{i}\right\}$.

The estimate of $x_{i}$ is given by

$$
\begin{equation*}
x_{i}^{\prime}=a \sin \theta_{i} \tag{F.5}
\end{equation*}
$$

Hence, there is an error of $-b$ in the estimate of the peak value and an error of $-c$ radians in the estimate of the phase angle of the sinewave conponent $x_{i}$, as shown in Fig.F.1. $a+b$ is assumed to remain constant over any half cycle of the sinewave for $0 \leqslant \theta_{i}<\pi$ or $\pi \leqslant \theta_{i}<2 \pi$. c is assumed to remain constant over any half cycle of the sinewave for $-\pi / 2 \leqslant \theta_{i}<\pi / 2$ or $\pi / 2 \leqslant \theta_{i}<3 \pi / 2 . \quad c$ is small such that $\cos c \approx 1$ and $\sin c \approx \tan c \approx c$.

The chief strategy for estimating the amplitude ( $a+b$ ) and phase (c) of $\left\{x_{i}\right\}$ from the $\left\{y^{\prime} I_{i}\right\}$ depends on a block sequential summation (integration) of the sinewave over different half-cycles. The error in an


Fig.F. 1 Sinewave component $\left\{x_{i}\right\}$ in $\left\{y^{\prime}{ }_{1 . j}\right\}$ and its estimate $\left\{x^{\prime}\right\}$
individual $x_{i}^{\prime}$ contains components of both $a+b$ and $c$. When averaged over a half cycle of the sinewave, $a l l$ errors caused by $a+b$ cancel out leaving an estimate of $c$, or vice versa.

$$
\begin{align*}
& \text { The following integrals are well-known. } \\
& \int_{0}^{\pi} \sin \theta d \theta=\int_{-\pi / 2}^{\pi / 2} \cos \theta d \theta=2  \tag{F.6}\\
& \int_{\pi}^{2 \pi} \sin \theta d \theta=\int_{\pi / 2}^{3 \pi / 2} \cos \theta d \theta=-2  \tag{F.7}\\
& \int_{-\pi / 2}^{\pi / 2} \sin \theta d \theta=\int_{\pi / 2}^{3 \pi / 2} \sin \theta d \theta=\int_{0}^{\pi} \cos \theta d \theta=\int_{x}^{2 \pi} \cos \theta d \theta=0 \quad \text { (F.8) }  \tag{F.8}\\
& \int_{0}^{\pi} \sin ^{2} \theta d \theta=\int_{\pi}^{2 \pi} \sin ^{2} \theta d \theta=\int_{-\pi / 2}^{\pi / 2} \sin ^{2} \theta d \theta=\int_{\pi / 2}^{3 \pi / 2} \sin ^{2} \theta d \theta \\
& =\int_{-\pi / 2}^{\pi / 2} \cos ^{2} \theta d \theta=\int_{\pi / 2}^{3 \pi / 2} \cos ^{2} \theta d \theta=\frac{\pi}{2} \text { (F.9) } \\
& \int_{0}^{\pi} \sin \theta \cos \theta d \theta=\int_{\pi}^{2 \pi} \sin \theta \cos \theta d \theta \\
& =\int_{-\pi / 2}^{\pi / 2} \sin \theta \cos \theta d \theta=\int_{\pi / 2}^{3 \pi / 2} \sin \theta \cos \theta d \theta=0  \tag{F.10}\\
& \int_{0}^{\pi} v d \theta=\int_{\pi}^{2 \pi} v d \theta=\int_{-\pi / 2}^{\pi / 2} v d \theta=\int_{-\pi / 2}^{3 \pi / 2} v d \theta=v \pi \tag{F.11}
\end{align*}
$$

The sampled equivalents of Eqs.(F.6)-(F.11) are useful in the subsequent analysis. Here, the integrals are replaced by a summation of $n$ samples, where the $n\left\{\Theta_{i}\right\}$ are equally spaced over the corresponding intervals of $\pi$ radians. Now, the result of the integral (Eq. (F.6)-(F.11)) is multiplied by $n / \pi$ to give the result of the corresponding summation of samples. For example

$$
\begin{equation*}
\sum_{i=1}^{n} \sin \theta_{i}, \quad \text { for } 0 \leqslant \theta_{i}<\pi \approx \frac{2 n}{\pi} \tag{F.12}
\end{equation*}
$$

and

$$
\begin{equation*}
\sum_{i=1}^{n} \sin ^{2} \theta_{i}, \quad \text { for } 0 \leqslant \theta_{i}<\pi \approx \frac{n}{2} \tag{F.13}
\end{equation*}
$$

For large values of $n$, these are very close approximations.

## F. 2 Estimate of $a+b$

Consider $n$ measurements $\left\{y^{\prime} I_{i}\right\}$ regularly spaced over the interval $0 \leqslant \theta_{i}<\pi$. From Eqs.(F.2)-(F.4)

$$
y_{I_{. i}}^{\prime}=x_{i}+v_{i}+w_{i}
$$

$$
\begin{align*}
& =(a+b) \sin \left(\theta_{i}+c\right)+v_{i}+W_{i} \\
& =(a+b) \sin (c) \cos \theta_{i}+(a+b) \cos (c) \sin \theta_{i}+v_{i}+W_{i} \tag{F.14}
\end{align*}
$$

Therefore,

$$
y_{I_{. i}}^{\prime} \sin \theta_{i}=(a+b) \sin (c) \cos \theta_{i}+(a+b) \cos (c) \sin ^{2} \theta_{i}+v_{i} \sin \theta_{i}+w_{i} \sin \theta_{i}
$$

So, from Eqs.(F.6). (F.9) and (F.10), with $\mathrm{v}_{\mathrm{i}} \approx \mathrm{v}$ (constant) over this interval and $\sum_{W_{i}} \approx 0$

$$
\begin{equation*}
\sum_{i=1}^{n} y^{\prime} I_{\cdot i} \sin \theta_{i} \approx(a+b) \cos (c) \cdot \frac{n}{2}+v \cdot \frac{2 n}{\pi} \tag{F.15}
\end{equation*}
$$

(In fact, $v_{i}$ only needs to vary linearly over this interval, with an average value of $v$, for this equation to hold true). Also, from Eq. (F.14) with Eqs.(F.6), (F.8) and (F.11)

$$
\begin{equation*}
\sum_{i=1}^{n} y^{\prime} I_{\cdot i} \approx(a+b) \cos (c) \cdot \frac{2 n}{\pi}+v n \tag{F.16}
\end{equation*}
$$

Therefore, from Eqs.(F.9), (F.15) and (F.16)

$$
\begin{align*}
\frac{\sum_{i=1}^{n} y^{\prime} I_{. i} \sin \theta_{i}-\frac{2}{\pi} \sum_{i=1}^{n} y^{\prime} I . j_{i}}{\left(1-8 / \pi^{2}\right) \sum_{i=1}^{n} \sin ^{2} \theta_{i}} & \approx \frac{(a+b) \cos (c)\left(\frac{n}{2}-\frac{4 n}{\pi^{2}}\right)}{\left(1-8 / \pi^{2}\right) \cdot \underline{n}} \\
& =(a+b) \cos (c) \\
& \approx(a+b) \tag{F.17}
\end{align*}
$$

for small $c$. Thus, the estimate of $(a+b)$ is given by

$$
\begin{equation*}
(a+b)^{\prime}=\frac{10.55796}{n}\left(\sum_{i=1}^{n} y^{\prime} I_{. i} \sin \theta_{i}-0.63662 \sum_{i=1}^{n} y^{\prime} I_{. i}\right) \tag{F.18}
\end{equation*}
$$

where

$$
1 / \frac{1}{2}\left(1-8 / \pi^{2}\right) \approx 10.55796
$$

and

$$
2 / \pi \approx 0.63662
$$

Similarly, it can be shown that if the $n\left\{y^{\prime}{ }_{I . i}\right\}$ are equally spaced over the interval $\pi \leqslant 0_{i}<2 \pi$, then

$$
\begin{equation*}
(a+b)^{\prime}=\frac{10.55796}{n}\left(\sum_{i=1}^{n} y^{\prime} I_{. i} \sin \theta_{i}+0.63662 \sum_{i=1}^{n} y_{I . i}^{\prime}\right) \tag{F.19}
\end{equation*}
$$

The different sign in Eq.(F.19) compared with Eq. (F.18) is due to the fact that

$$
\int_{0}^{\pi} \sin \theta d \theta=+2 \quad \text { AND } \quad \int_{\pi}^{2 \pi} \sin \theta d \theta=-2
$$

as shown in Eqs.(F.6) and (F.7)

## F. 3 Estimate of c radians

Consider $n$ measurements $\left\{y^{\prime} I_{i}\right\}$ regularly spaced over the interval $-\pi / 2 \leqslant \Theta_{i}<\pi / 2$, where $y^{\prime} I_{\text {. }}$ is given by Eq. (F.14). Then

$$
\begin{aligned}
y^{\prime}{ }_{I . i} \cos \theta_{i}=(a+b) \sin (c) \cos ^{2} \theta_{i} & +(a+b) \cos (c) \sin \theta_{i} \cos \theta_{i} \\
& +v_{i} \cos \theta_{i}+w_{i} \cos \theta_{i}
\end{aligned}
$$

So, from Eqs.(F.6), (F.9) and (F.10), with $v_{i} \approx v$ over this interval, and $\sum W_{i}=0$

$$
\begin{equation*}
\sum_{i=1}^{n} y^{\prime}{ }_{I \cdot i} \cos \theta_{i} \approx(a+b) \sin (c) \cdot \frac{n}{2}+v \cdot \frac{2 n}{\pi} \tag{F.20}
\end{equation*}
$$

Similarly,

$$
\begin{gathered}
Y_{I . i}^{\prime} \sin \theta_{i}=(a+b) \sin (c) \cos \theta_{i} \sin \theta_{i}+(a+b) \cos (c) \sin 2 \theta_{i} \\
\because
\end{gathered}
$$

So, from Eqs.(F.8)-(F.10)

Also, from Eq.(F.14) with Eqs.(F.6), (F.18) and (F.11)

$$
\begin{equation*}
\sum_{i=1}^{n} y^{\prime} I_{\cdot i}=(a+b) \sin (c) \cdot \frac{2 n}{\pi}+v n \tag{F.22}
\end{equation*}
$$

Therefore, from Eqs.(F.20)-(F.22), for small c

$$
\begin{equation*}
\frac{\sum_{i=1}^{n} Y^{\prime} I \cdot i \cos \theta_{i}-\frac{2}{\pi} \cdot \sum_{i=1}^{n} y^{\prime} I \cdot i}{\left(1-8 / \pi^{2}\right) \sum_{i=1}^{n} Y^{\prime} I_{\cdot i} \sin \theta_{i}} \approx \frac{(a+b) \sin (c)\left(\frac{n}{2}-\frac{4 n}{\pi^{2}}\right)}{\left(1-8 / \pi^{2}\right)(a+b) \cos (c) \cdot \frac{n}{2}}=\tan (c) \approx c \tag{F.23}
\end{equation*}
$$

Thus, the estimate of $c$ is given by

$$
c^{\prime}=\frac{5.27898 \sum_{i=1}^{n} Y^{\prime} I . i \cos \theta_{i}-0.63662 \sum_{i=1}^{n} Y^{\prime} I . i}{\sum_{i=1}^{n} Y^{\prime} \cdot I_{. i} \sin \theta_{i}}
$$

where

$$
1 /\left(1-8 / \pi^{2}\right) \approx 5.27898
$$

and

$$
2 / \pi \approx 0.63662
$$

Similarly it can be shown that if the $\dot{n}\left\{y^{\prime} I_{i}\right\}$ are regularly spaced over the interval $\pi / 2 \leqslant \theta_{i}<3 \pi / 2$, then

$$
\begin{equation*}
c^{\prime}=\frac{5.27898 \sum_{i=1}^{n} Y^{\prime} I_{. i} \cos \theta_{i}+0.63662 \sum_{i=1}^{n} Y^{\prime} I_{. i}}{\sum_{i=1}^{n} Y^{\prime} I_{. i} \sin \theta_{i}} \tag{F.25}
\end{equation*}
$$

The different sign in Eq. (F.24) compared with Eq. (F.25) is due to the fact that

$$
\int_{-\pi / 2}^{\pi / 2} \cos \theta d \theta=2 \quad \text { AND } \quad \int_{\pi / 2}^{3 \pi / 2} \cos \theta d \theta=-2
$$

as shown in Eqs.(F.6) and (F.7)

## Retraining Algorithms

## G. 1 Least-squares fitting of a straight line to a set of $n$ data points

It is required to fit a straight line to a set of $n$ sampled data points to give the "best" estimate of the slope of the line at any point. The best estimate here is the one with the lowest mean-square error. Two methods are considered theoretically in this Sec.G.1. The first method minimizes the mean-square error between the straight line and the data at the $n$ sample points by the method of least squares. The second method attempts to minimize the mean-square error between the slope of the straight line and the slope of the data at the $n$ sample points by the method of least squares. In both methods, the general expression for the slope of the straight line can be simplified if the $n$ points are equally spaced in time.

## Method 1

The $n$ sampled data points can be represented using Cartesian coordinates
as

$$
\begin{equation*}
\left(x_{1}, Y_{1}\right),\left(x_{2}, Y_{2}\right), \ldots,\left(x_{n}, Y_{n}\right) \tag{G.1}
\end{equation*}
$$

Where $x_{i}$ is the independent variable (which is time in the retraining algorithms). $Y_{i}$, for $i=1,2, \ldots, n$ is the measurement of $Y_{i}$. All the $n$ $\left\{y_{i}\right\}$ are assumed to lie on a straight line. $y_{i}$ and $y_{i}$ are real-valued.

The principle of least-squares states that the best straight line approximation $\left\{y^{\prime}{ }_{i}\right\}$ to the $n\left\{y_{i}\right\}$ is that for which the sum of the squares of the differences between the $\left\{Y_{i}\right\}$ and the $\left\{Y^{\prime}{ }_{i}\right\}$ of the approximating function is a minimum. That is, it is necessary to fit the straight line

$$
\begin{equation*}
y^{\prime}(x)=a_{0}+a_{1} x \tag{G.2}
\end{equation*}
$$

to the data such that

$$
\begin{equation*}
\sum_{i=1}^{n}\left(y_{i}^{\prime}-Y_{i}\right)^{2} \text { is a minimum } \tag{G.3}
\end{equation*}
$$

The slope of the line $y^{\prime}(x)$ is $a_{1}$. $a_{1}$ now defines the estimate of the slope of $y(x)$. Now let

$$
\begin{equation*}
R_{i}=y_{i}^{\prime}-Y_{i}=a_{0}+a_{1} x_{i}-Y_{i}, \quad \text { for } i=1,2, \ldots, n \tag{G.4}
\end{equation*}
$$

Therefore,

$$
\begin{equation*}
\sum_{i=1}^{n}\left(y_{i}^{1}-Y_{i}\right)^{2}=R_{1}^{2}+R_{2}^{2}+\ldots+R_{n}^{2}=\sum_{i=1}^{n} R_{i}^{2} \tag{G.5}
\end{equation*}
$$

The $a_{0}, a_{1}$ must be chosen to make this equation $\sum_{R_{i}}{ }^{2}$ a minimum. From the methods of calculus, this is a mimimum (or maximum) when all the partial derivatives of $\sum_{R_{i}}{ }^{2}$ with respect to both of the $a_{j}$ 's are zero. That is, when

$$
\begin{align*}
\frac{\partial}{\partial a_{j}} \sum_{i=1}^{n} R_{i}^{2} & =\frac{\partial}{\partial a_{j}}\left[R_{1}^{2}+R_{2}^{2}+\ldots+R_{n}^{2}\right]=0, \quad \text { for } j=0,1 \\
& =2\left[R_{1} \frac{\partial R_{1}}{\partial a_{j}}+R_{2} \frac{\partial R_{2}}{\partial a_{j}}+\ldots+R_{n} \frac{\partial R_{n}}{\partial a_{j}}\right]=0, \quad \text { for } j=0,1 \tag{G.6}
\end{align*}
$$

But, from Eq. (G.4)

$$
\begin{equation*}
\frac{\partial R_{i}}{\partial a_{0}}=1, \quad \frac{\partial R_{i}}{\partial a_{1}}=x_{i}, \quad \text { for } i=1,2, \ldots, n \tag{G.7}
\end{equation*}
$$

So there are $j=2$ equations of partial derivatives (from Eqs.(G.6) and (G.7))

$$
\begin{gather*}
R_{1}+R_{2}+\ldots+R_{n}=0 \\
R_{1} x_{1}+R_{2} x_{2}+\ldots+R_{n} x_{n}=0 \tag{G.8}
\end{gather*}
$$

Replacing $R_{i}$ by $a_{0}+a_{1} X_{i}{ }^{-Y}$

$$
\begin{align*}
& n a_{0}+a_{1} \sum_{i=1}^{n} x_{i}=\sum_{i=1}^{n} x_{i} y_{i} \\
& a_{0} \sum_{i=1}^{n} x_{i}+a_{1} \sum_{i=1}^{n} x_{i}^{2}=\sum_{i=1}^{n} x_{i} Y_{i} \tag{G.9}
\end{align*}
$$

So, Eq. (G.9) reduces to two simultaneous equations in the two unknowns $a_{0}$, $a_{1}$. But it is only $a_{1}$, the slope of the straight line that minimizes $\sum\left(y_{i}{ }_{i}-Y_{i}\right)^{2}$, that is required here. Therefore, solving Eq. (G.9) for $a_{l}$ gives

$$
\begin{equation*}
a_{1}=\frac{\frac{1}{n} \sum_{i=1}^{n} x_{i} y_{i}-\left(\frac{1}{n} \sum_{i=1}^{n} y_{i}\right) \cdot\left(\frac{1}{n} \sum_{i=1}^{n} x_{i}\right)}{\frac{1}{n} \sum_{i=1}^{n} x_{i}^{2}-\left(\frac{1}{n} \sum_{i=1}^{n} x_{i}\right)^{2}} \tag{G.10}
\end{equation*}
$$

In practice, the $\left\{x_{i}\right\}$ would be known beforehand, so the terms $\sum x_{i}, \sum x_{i}{ }^{2}$ would not have to be calculated. For example, for the most likely case of equally spaced samples, the $n$ values $\left\{x_{i}\right\}$ (for $i=1,2, \ldots, n$ ) can be arbitrarily be set to $1,2, \ldots, n$ giving

$$
\begin{equation*}
a_{1}=\frac{\frac{1}{n} \sum_{i=1}^{n} i Y_{i}-\left(\frac{1}{n} \sum_{i=1}^{n} Y_{i}\right) \cdot\left(\frac{1}{n} \sum_{i=1}^{n} i\right)}{\frac{1}{n} \sum_{i=1}^{n} i^{2}-\left(\frac{1}{n} \sum_{i=1}^{n} i\right)^{2}} \tag{G.11}
\end{equation*}
$$

So for $n=10$

$$
a_{1}=0.0121212 \sum_{i=1}^{10} i Y_{i}-0.06667 \sum_{i=1}^{10} Y_{i}
$$

Eq. (G.10) applies equally well when the $n$ samples $\left\{X_{i}, Y_{i}\right\}$ are not equally spaced apart in $\mathbf{x}_{i}$.

## Method 2

Method 1 has given a straight line that minimizes the mean-square er ror between that line and the $n$ sampled points. But a better measure of slope may be found from the straight line that miminizes the mean-square error between the slope of the line and the slope of the $n$ sample points. Applying the principle of least squares, the straight line that satisfies this condition is the one for which the sum of the squares of the differences between the $\left\{Y_{i}-Y_{i-1}\right\}$ and the $\left\{Y_{i}^{\prime}{ }_{i} Y^{\prime}{ }_{i-1}\right\}$ is a minimum. That is, it is necessary to fit the straight line

$$
\begin{equation*}
y_{i}^{\prime}=a_{0}+a_{1} x_{i} \tag{G.12}
\end{equation*}
$$

to the data such that

$$
\begin{equation*}
\sum_{i=2}^{n}\left[\left(y_{i}^{\prime}-y^{\prime}{ }_{i-1}\right)-\left(Y_{i}-Y_{i-1}\right)\right]^{2} \text { is a minimum } \tag{G.13}
\end{equation*}
$$

The slope of this line $y^{\prime}{ }_{i}$ now defines the estimate of the slope of $y_{i}$. Note that $i$ goes from 2 to $n$ in the summation because $Y_{0}$ does not exist. Now, let

$$
\begin{align*}
R_{i} & =\left(y_{i}^{\prime}-y_{i-1}^{\prime}\right)-\left(Y_{i}-Y_{i-1}\right) \\
& =a_{1}\left(x_{i}-x_{i-1}\right)-\left(Y_{i}-Y_{i-1}\right), \quad \text { for } i=2,3, \ldots, n \tag{G.14}
\end{align*}
$$

Therefore,

$$
\begin{equation*}
\sum_{i=2}^{n}\left[\left(y_{i}^{\prime}-y_{i-1}^{\prime}\right)-\left(Y_{i}-Y_{i-1}\right)\right]^{2}=R_{1}^{2}+R_{2}^{2}+\ldots+R_{n}^{2}=\sum_{i=2}^{n} R_{i}^{2} \tag{G.15}
\end{equation*}
$$

This is a minimum (or maximum) when all the partial derivatives of $\sum_{R_{i}}{ }^{2}$ with respect to each of the $a_{j}$ 's is zero. That is, when

$$
\begin{align*}
\frac{\partial}{\partial a} \sum_{j=2}^{n} R_{i}^{2} & =\frac{\partial}{\partial a}\left[R_{j}^{2}+R_{3}^{2}+\ldots+R_{n}^{2}\right]=0, \quad \text { for } j=0,1 \\
& =2\left[R_{2} \frac{\partial R_{2}}{\partial a_{j}}+R_{3} \frac{\partial R_{3}}{\partial a_{j}}+\ldots+R_{n} \frac{\partial R_{n}}{\partial a_{j}}\right]=0, \quad \text { for } j=0,1 \tag{G.16}
\end{align*}
$$

But, from Eq.(G.14)

$$
\begin{equation*}
\frac{\partial R_{i}}{\partial a_{0}}=0, \quad \frac{\partial R_{i}}{\frac{\partial a_{1}}{}}=x_{i}-x_{i-1}, \quad \text { for } i=2,3, \ldots, n \tag{G.17}
\end{equation*}
$$

So now there is only one equation of partial derivatives (from (Eqs.(G.16) and (G.17))

$$
\begin{equation*}
R_{2}\left(x_{2}-x_{1}\right)+R_{3}\left(x_{3}-x_{2}\right)+\ldots+R_{n}\left(x_{n}-x_{n-1}\right)=0 \tag{G.18}
\end{equation*}
$$

$a_{0}$ is undefined because only the slope is optimized. Now, replacing $R_{i}$ by $a_{1}\left(x_{i}{ }^{-x_{i-1}}\right)-\left(Y_{i}-Y_{i-1}\right)$ (Eq. (G.14)) gives

$$
\begin{equation*}
a_{1} \sum_{i=2}^{n}\left(x_{i}-x_{i-1}\right)=\sum_{i=2}^{n}\left(x_{i}-x_{i-1}\right)\left(Y_{i}-Y_{i-1}\right) \tag{G.19}
\end{equation*}
$$

Thus, the slope of the straight line that minimizes Eq. (G.13) is given by

$$
\begin{equation*}
a_{1}=\frac{\sum_{i=2}^{n}\left(x_{i}-x_{i-1}\right)\left(Y_{i}-Y_{i-1}\right)}{\sum_{i=1}^{n}\left(x_{i}-x_{i-1}\right)} \tag{G.20}
\end{equation*}
$$

This is the general equation for any set of $n$ sampled data points $\left\{X_{i}, Y_{i}\right\}$. However, for the most common case of samples equally spaced in $x_{i}$, the difference function

$$
\begin{equation*}
x_{i}-x_{i-1}=d \tag{G.21}
\end{equation*}
$$

is constant for all $i=2,3, \ldots, n$. In this case

$$
\begin{aligned}
a_{1} & =\frac{d \sum_{i=2}^{n}\left(Y_{i}-Y_{i-1}\right)}{(n-1) d} \\
& =\frac{\left(Y_{2}-Y_{1}\right)+\left(Y_{3}-Y_{2}\right)+\ldots+\left(Y_{n}-Y_{n-1}\right)}{n-1}
\end{aligned}
$$

Therefore

$$
\begin{equation*}
a_{1}=\frac{Y_{n}-Y_{1}}{n-1} \tag{D.22}
\end{equation*}
$$

So, given a set of $n$ sampled points $\left\{x_{i}, Y_{i}\right\}$ equally spaced in the $x$ variable, Methods 1 and 2 (Eqs. (G.11) and (G.22) respectively) minimize the mean-square errors in the estimates of $\left\{Y_{i}\right\}$ and slope of $\left\{Y_{i}\right\}$ respectively. Where $Y_{i}$ is a measure of $y_{i}$. . But the important question that still remains to be answered is:- Which method gives the lowest mean-square error in the estimate of the slope of $\left\{y_{i}\right\}$ ?

In the retraining algorithms in this thesis, the $n$ samples are complex-valued and are generally assumed to be given by

$$
\begin{equation*}
Y_{i}=y_{i}+w_{i}, \quad \text { for } i=1,2, \ldots, n \tag{G.23}
\end{equation*}
$$

Where $y_{i}$ here is the complex-valued sample of the channel at time $t=i T$. The real and imaginary parts of the $\left\{y_{i}\right\}$ are independent of each other. The real parts of the $n$ samples $\left\{y_{i}\right\}$ are assumed to vary linearly with $i$, as are the $n$ imaginary parts. $w_{i}$ is a complex-valued sample of additive white Gaussian noise at time $t=i T$. The real and imaginary parts of the $\left\{w_{i}\right\}$ are independent Gaussian random variables, each with zero mean and variance $\sigma^{2}$ (see Appendix B). $a_{1}$ in Eqs.(G.11) and (G.22) is now complex valued.

In the absence of noise (that is, when $Y_{i}=y_{i}$ for all \{i\}), both measurements of slope would be the same and exact, when the $n\left\{y_{i}\right\}$ vary linearly with i. However, in the presence of noise, Method 1 (Eq. (G.11)) should surely give a better measure of the slope of $\left\{y_{i}\right\}$ than Method 2 (Eq. (G.22)) because all $n$ samples $\left\{Y_{i}\right\}$ are used in the calculation. Thus, the effects of noise are averaged out more than in Method 2 where only the first and last samples are used, whatever the value of $n$. This is shown to be true below.

For Method 1, the measure of slope is (from Eqs.(G.11) and (G.23))

$$
\begin{equation*}
a_{1}=\frac{\frac{1}{n} \sum_{i=1}^{n} i\left(y_{i}+w_{i}\right)-\left(\frac{1}{n} \sum_{i=1}^{n}\left(y_{i}+w_{i}\right)\right)\left(\frac{1}{n} \sum_{i=1}^{n} i\right.}{\frac{1}{n} \sum_{i=1}^{n} i^{2}-\left(\frac{1}{n} \sum_{i=1}^{n} i\right)^{2}} \tag{G.24}
\end{equation*}
$$

So,

$$
a_{1}=\frac{\frac{1}{n} \sum_{i=1}^{n} i y_{i}-\left(\frac{1}{n} \sum_{i=1}^{n} y_{i}\right)\left(\frac{1}{n} \sum_{i=1}^{n} i\right)}{\underbrace{\frac{1}{n} \sum_{i=1}^{n} i^{2}-\left(\frac{1}{n} \sum_{i=1}^{n} i\right)^{2}}_{\text {Exact slope of }\left\{y_{i}\right\}}}+\frac{\frac{1}{n} \sum_{i=1}^{n} i Y_{i}-\left(\frac{1}{n} \sum_{i=1}^{n} Y_{i}\right)\left(\frac{1}{n} \sum_{i=1}^{n} i\right)}{\underbrace{\frac{1}{n} \sum_{i=1}^{n} i^{2}-\left(\frac{1}{n} \sum_{i=1}^{n} i\right)^{2}}_{\text {noise-error }(1)}}
$$

For Method 2, the measure of slope is (from Eqs.(G.22) and (G.23))

$$
\begin{equation*}
a_{1}=\frac{\left(y_{n}+w_{n}\right)-\left(y_{1}+w_{1}\right)}{n-1} \tag{G.26}
\end{equation*}
$$

$=$

$$
\begin{equation*}
\underbrace{\frac{y_{n}-y_{1}}{n-1}}_{\text {Exact }}+\underbrace{\frac{w_{n}-w_{1}}{n-1}}_{\text {noise-error }} \tag{G.27}
\end{equation*}
$$

The variances of the noise-errors (1) and (2) will show which method gives the best estimate of slope. The denominator of noise-error (2) is

$$
\frac{1}{n} \sum_{i=1}^{n} i^{2}-\left(\frac{1}{n} \sum_{i=1}^{n} i\right)^{2}=\frac{(n+1)(2 n+1)}{6}-\left(\frac{(n+1)}{2}\right)^{2}=\frac{(n+1)(n-1)}{12}
$$

Since [99]

$$
\begin{gather*}
\frac{1}{n} \sum_{i=1}^{n} i=\frac{n+1}{2} \quad \text { (arithmetic progression) } \\
\frac{1}{n} \sum_{i=1}^{n} i^{2}=\frac{(n+1)(2 n+1)}{6} \quad \text { (sum of a power series) } \tag{G.29}
\end{gather*}
$$

Therefore, (from Eqs.(G.25) and (G.28))

$$
\text { noise-error }(1)=\frac{\frac{1}{n} \sum_{i=1}^{n} i w_{i}-\left(\frac{1}{n} \sum_{i=1}^{n} w_{i}\right) \cdot\left(\frac{n+1}{2}\right)}{\frac{(n+1)(n-1)}{12}}
$$



$$
\begin{equation*}
=\frac{6 \sum_{i=1}^{n}(2 i-(n+1)) w_{i}}{n(n+1)(n-1)} \tag{G.30}
\end{equation*}
$$

Thus, noise-error (1) is a complex-valued Gaussian random variable, whose real and imaginary parts have zero mean and variance [119]

$$
\begin{equation*}
\nabla_{1}=\left(\frac{6}{n(n+1)(n-1)}\right)^{2} \sum_{i=1}^{n}(2 i-(n+1))^{2} \sigma^{2} \tag{G.31}
\end{equation*}
$$

But,

$$
\begin{align*}
\sum_{i=1}^{n}(2 i-(n+1))^{2} & =\sum_{i=1}^{n}\left(4 i^{2}-4 i(n+1)+(n+1)^{2}\right) \\
& =4 \sum_{i=1}^{n} i^{2}-4(n+1) \sum_{i=1}^{n} i+(n+1)^{2} \sum_{i=1}^{n} 1 \\
& =\frac{4 n(n+1)(2 n+1)}{6}-4(n+1) \frac{n(n+1)}{2}+n(n+1)^{2} \text { (from Eq. (G. 29) } \\
& =\frac{n(n+1)(n-1)}{3} \tag{G.32}
\end{align*}
$$

$$
\begin{align*}
\nabla_{1} & =\left(\frac{6}{n(n+1)(n-1)}\right)^{2} \frac{n(n+1)(n-1)}{3} \cdot \sigma^{2} \\
& =\frac{12}{n(n+1)(n-1)} \cdot \sigma^{2} \tag{G.33}
\end{align*}
$$

Noise-error (2) is a complex-valued Gaussian random process, whose real and imaginary parts have zero mean and variance [119]

$$
\begin{align*}
\nabla_{2} & =\frac{\left(1^{2}+1^{2}\right)}{(n-1)^{2}} \cdot \sigma^{2} \\
& =\frac{2}{(n-1)^{2}} \cdot \sigma^{2} \tag{G.34}
\end{align*}
$$

Thus, the mean-square error in the estimate of the slope of both the real and imaginary parts of $\left\{y_{i}\right\}$, is $\nabla_{1}$ for Method 1 and $\nabla_{2}$ for Method 2. It is shown in Table $G .1$ that the greater the value of $n$, the greater the accuracy of both Methods and the greater the advantage of Method 1 over Method 2. (Though for $\mathrm{n}=2$ or 3, the accuracy of both methods is the same). The reduction of the mean-square error of Method 1 compared with Method 2 is expressed in decibels in the last column of Table G.1. In fact

$$
\begin{align*}
10 \log \nabla_{1}-\log _{10} \nabla_{2} & =\operatorname{lolog}_{10} \frac{\nabla_{1}}{\nabla_{2}} \\
& =\operatorname{lolog}_{10} \frac{12 /[n(n-1)(n+1)]}{2 /(n-1)^{2}} \\
& =\operatorname{lolog}_{10} \frac{6(n-1)}{n(n+1)} \\
& \approx \operatorname{lolog}_{10} \frac{6}{n} \quad \text { for large } n \tag{G.35}
\end{align*}
$$

For the fading rate assumed in this thesis (Sec.2.2), the channel samples over the duration of the 12-symbol training signal lie on a smooth curve as shown in Fig.G.1(b). In this analysis they have been assumed to lie on a straight line as shown in Fig.G.l(a). Of course, if all the $\left\{y_{i}\right\}$ did lie on a straight line, the estimate of slope would be equally accurate over the entire training signal. However, with the curved channel, the least squares straight line through the $n=12$ raw estimates $\left\{Y_{i}\right\}$, runs almost parallel to the tangent to the curve in the middle of the retraining packet. So the estimate of the slope given by Eq. (G.11) or (G.22) is most accurate at the central point of the training sequence.

Table G. 1 Variation of $\nabla_{1}$ and $\nabla_{2}$ with $n$

| n | $\nabla_{1} \times \frac{1}{\sigma^{2}}$ | $\nabla_{2 \times \frac{1}{\sigma^{2}}}$ | $10 \log _{10} \nabla_{1}-10 \log _{10} \nabla_{2} \mathrm{~dB}$ |
| :---: | :---: | :---: | :---: |
| 2 | 2 | 2 | 0 |
| 3 | 0.5 | 0.5 | 0 |
| 4 | 0.2 | 0.2222 | -0.46 |
| 5 | 0.1 | 0.125 | -0.97 |
| 6 | 0.05714 | 0.08 | -1.46 |
| 7 | 0.03571 | 0.05555 | -1.92 |
| 8 | 0.02381 | 0.04082 | -2.34 |
| 9 | 0.01667 | 0.03125 | -2.73 |
| 10 | 0.01212 | 0.02469 | -3.09 |
| 11 | 0.009091 | 0.02 | -3.42 |
| 12 | 0.006993 | 0.01653 | -3.74 |
| 15 | 0.003571 | 0.01020 | -4.56 |
| 25 | $7.692 \times 10^{-4}$ | 0.003472 | -6.55 |
| 50 | $9.604 \times 10^{-5}$ | $8.33 \times 10^{-4}$ | -9.38 |
| 100 | $1.2 \times 10^{-5}$ | $2.04 \times 10^{-4}$ | -12.30 |
| 500 | $9.6 \times 10^{-8}$ | $8.032 \times 10^{-6}$ | -19.23 |
| 1000 | $1.2 \times 10^{-8}$ | $2.004 \times 10^{-6}$ | -22.23 |



Fig.G. 1 Example of real or imaginary part of $\left\{y_{i}\right\}$ and its estimate during retraining; (a) Assuming $\left\{y_{i}\right\}$ varies linearly with $i$ (b) Actual

| $x$ | $=$ real or imaginary part of channel |
| ---: | :--- |
|  | $=$ raw estimate |
|  | $=$ least-squares straight line |

## G. 2 Retraining Algorithms for System 2

Methods (2), (3), and (4) described in Sec.4.5.3 for calculating the raw estimates of the two channels are considered here. They are based on methods first proposed by A.P.Clark for tracking fast fading channels [68,123]. For each Method, the equations are derived from first principles and the mean-square er ror in these raw estimates is derived.

The baseband received sample at the input to the estimator during retraining is

$$
\begin{equation*}
r_{i}=s_{1, i} Y_{1, i}+s_{2, i} Y_{2 . i}+w_{i}, \quad \text { for } i=1,2, \ldots, R \tag{G.35}
\end{equation*}
$$

Where the $R$ training symbols $\left\{s_{1, i}\right\},\left\{s_{2, i}\right\}$ are known at the receiver. Here, the raw estimates of the complex-valued channel samples $y_{1 . i}, Y_{2 . i}$ are denoted $x_{1 . i}, x_{2 . i}$ respectively. Where, the samples $\left\{y_{1 . i}\right\},\left\{y_{2 . i}\right\}$ are equally spaced in time, every $T$ seconds.

## Method (2): "Slow Fading" Assumption

The estimator assumes that

$$
\begin{equation*}
y_{1, i}=y_{1, i-1} \quad \text { AND } \quad y_{2, i}=y_{2, i-1} \tag{G.36}
\end{equation*}
$$

To estimate $y_{1 . i}$ it is necessary to remove $y_{2 . i}$ from the received sample $r_{i}$. This can be achieved by operating on the two received samples $r_{i}$, $r_{i-1}$. From Eq. (G.35)

$$
\begin{align*}
s_{2 . i}{ }^{-1} r_{i} & =s_{2 . i}{ }^{-1} s_{1 . i} y_{1 . i}+y_{2 . i}+s_{2 . i}{ }^{-1} w_{i} \\
s_{2 . i-1}{ }^{-1} r_{i-1} & =s_{2 . i-1}{ }^{-1} s_{1 . i-1} y_{1 . i-1}+y_{2 . i-1}+s_{2 . i-1}{ }^{-1} w_{i-1} \tag{G.37}
\end{align*}
$$

From which it follows that

$$
\begin{align*}
s_{2 . i}{ }^{-1} r_{i .}-s_{2 . i-1}{ }^{-1} r_{i-1} & =s_{2 . i}{ }^{-1} s_{1 . i} y_{1 . i}-s_{2 . i-1}{ }^{-1} s_{1 . i-1} y_{1 . i-1} \\
& +y_{2 . i}-y_{2 . i-1}+s_{2 . i}{ }^{-1} w_{i}-s_{2 . i-1}{ }^{-1} w_{i-1} \tag{G.38}
\end{align*}
$$

Now, substituting Eq. (G.36) into (G.38)

$$
\begin{align*}
s_{2 . i}{ }^{-1} r_{i}-s_{2 . i-1} r_{i-1}=\left(s_{2 . i}{ }^{-1} s_{1 . i}-\right. & \left.s_{2 . i-1}{ }^{-1} s_{1 . i-1}\right) y_{1 . i-1} \\
& +s_{2 . i}{ }^{-1} w_{i}-s_{2 . i-1}{ }^{-1} w_{i-1} \tag{G.39}
\end{align*}
$$

An unbiased estimate of $Y_{1 . i}$ is now given by

$$
\begin{equation*}
x_{1, i}=a_{1, i}{ }^{-1} p_{1, i} \tag{G.40}
\end{equation*}
$$

where

$$
\begin{equation*}
a_{1, i}=s_{2 . i}{ }^{-1} s_{1, i}-s_{2 . i-1}{ }^{-1} s_{1, i-1} \tag{G.41}
\end{equation*}
$$

and

$$
\begin{equation*}
p_{1 . i}=s_{2 . i}^{-1} r_{i} \rightarrow s_{2 . i-1}^{-1} r_{i-1} \tag{G.42}
\end{equation*}
$$

From Eqs.(G.39)-(G.42)

$$
\begin{equation*}
x_{1, i}=y_{1, i}+a_{1, i}{ }^{-1} u_{1, i} \tag{G.43}
\end{equation*}
$$

$a_{1 . i}{ }^{-1} u_{1 . i}$ is the error in $x_{i}$ caused by the additive noise, where

$$
u_{1 . i}=s_{2 . i} \stackrel{w}{w}^{-1}-s_{2 . i-1}{ }^{-1} w_{i-1}
$$

For the minimum mean-square error in $x_{1 . i}\left|a_{1 . i}\right|$ must be maximized, which means that

$$
s_{2 . i-1}{ }^{-1} s_{1 . i-1}=-s_{2 . i}{ }^{-1} s_{1 . i}
$$

so that
Since $s_{1 . i}{ }^{-1}$ and $s_{2 . i}{ }^{-1}{ }^{a_{1 . i}}$ are ${ }^{\frac{1}{2}( \pm 1 \pm j)}{ }^{2} s_{2 . i^{-1}} s_{1 . i}$, the maximum value of $\left|a_{1 . i}\right|$ occurs when

$$
\begin{equation*}
a_{1 . i}= \pm 2 \text { or } \pm 2 j \tag{G.45}
\end{equation*}
$$

The real and imaginary parts of the complex-valued noise components $\left\{w_{i}\right\}=\left\{w_{I_{. i}}{ }^{+j w_{Q . i}}\right\}$ are statistically independent Gaussian random variables with zero mean and variance $\sigma^{2}$ (Eq. (2.4.8)). So

$$
\begin{equation*}
s_{2 . i}{ }^{-1} w_{i}=\frac{1}{2}\left( \pm w_{I . i} w_{Q . i}\right)+j \frac{1}{2}\left( \pm w_{w_{1 . i}} w_{Q . i}\right) \tag{G.46}
\end{equation*}
$$

The real and imaginary parts of the $\left\{s_{2 . i}{ }^{-1} w_{i}\right\}$ are statistically independent Gaussian random variables with zero mean and variance [119],

$$
\begin{equation*}
\left(\frac{1}{2}{ }^{2}+\frac{1}{2} 2\right) \sigma^{2}=\frac{1}{2} \sigma^{2} \tag{G.47}
\end{equation*}
$$

(independent of the values $( \pm 1 \pm j)$ of the data symbols $s_{2 . i-1}, s_{2 . i}$ ). So, the mean-square value of $a_{1 . i}{ }^{-1} u_{1 . i}$ is [119]

$$
\begin{equation*}
2\left(\frac{1}{2} 2+\frac{1}{2} 2\right)\left(\frac{1}{2} \sigma^{2}\right)=\frac{1}{2} \sigma^{2} \tag{G.48}
\end{equation*}
$$

which is twice the variance of the real or imaginary parts of $a_{1 . i}{ }^{-1} u_{1 . i}$.
Now, because Eq. (G.36) is only approximately true, the general
expression for $X_{i}$ is

$$
x_{1 . i}=y_{1 . i}+a_{1 . i}^{-1} u_{1 . i}+c_{1 . i}
$$

Where $C_{1 . i}$ is the error in $x_{1 . i}$ caused by the curvature in the channel $Y_{1 . i}\left(\right.$ see Fig.G.l(b)). Clearly, $C_{1 . i}=0$ if Eq. (G.36) holds true.

The estimate of $y_{2 . i}$ is determined in a similar manner.

## Method (3):"Fast Fading" Assumption

The estimator assumes that
$Y_{1 . i+1}-y_{1 . i}=y_{1 . i}-y_{1 . i-1}$ AND $Y_{2 . i+1}-y_{2 . i}=y_{2 . i}-y_{2 . i-1} \quad(G .49)$
so that

$$
\begin{equation*}
2 y_{1 . i}=y_{1 . i-1}+y_{1 . i+1} \quad \text { AND } \quad 2 y_{2 . i}=y_{2 . i-1}+y_{2 . i+1} \tag{G.50}
\end{equation*}
$$

To estimate $Y_{1 . i}$ it is necessary to remove $Y_{2 . i}$ from the received sample $r_{i}$. This can be achieved by operating on the three received samples $r_{i-1}$, $r_{i}, r_{i+1}$. From Eq. (G.35)

$$
\begin{align*}
& 2 s_{2 . i}{ }^{-1} r_{i}=2 s_{2 . i}{ }^{-1} s_{1 . i} y_{1 . i}+2 y_{2 . i}+2 s_{2 . i}{ }^{-1} w_{i} \\
& s_{2 . i-1}{ }^{-1} r_{i-1}=2 s_{2 . i-1}{ }^{-1} s_{1, i-1} y_{1 . i-1}+2 y_{2 . i,-1}+2 s_{2 . i-1}{ }^{-1} w_{i-1} \\
& s_{2 . i+1}{ }^{-1} r_{i+1}=2 s_{2, i+1}{ }^{-1} s_{1, i+1} y_{1, i+1}+2 y_{2, i+1}+2 s_{2, i+1}{ }^{-1} w_{i+1} \tag{G.51}
\end{align*}
$$

from which it follows that

$$
\begin{align*}
2 s_{2 . i} & { }^{-1} r_{i}-s_{2} 1_{i-1}{ }^{-1} r_{i-1}-s_{2 . i+1}{ }^{-1} r_{i+1} \\
& =2 s_{2 . i} s_{1 . i} y_{1 . i}-s_{2 . i-1} s_{1 . i-1} y_{1 . i-1}-s_{2 . i+1}{ }^{-1} s_{1 . i+1} y_{1 . i+1}  \tag{G.52}\\
& +2 y_{2 . i}-y_{2 . i-1}-y_{2, i+1} \\
& +2 s_{2 . i}{ }_{w_{i}}-s_{2 . i-1}{ }_{w_{i-1}}-s_{2 . i+1}{ }^{-1} w_{i+1}
\end{align*}
$$

Now, substituting Eq.(G.50) into Eq.(G.52), under the additional condition that

$$
\begin{equation*}
s_{2, i-1}{ }^{-1} s_{1, i-1}=s_{2, i+1}{ }^{-1} s_{1, i+1}=s_{2, i}{ }^{-1} s_{1, i} \tag{G.53}
\end{equation*}
$$

gives

$$
\begin{align*}
& 2 s_{2 . i}{ }^{-1} r_{i}-s_{2 . i-1}{ }^{-1} r_{i-1}-s_{2 . i+1}{ }^{-1} r_{i+1} \\
& \quad=4 s_{2 . i}{ }^{-1} s_{1 . i} y_{1 . i}+2 s_{2 . i}{ }^{-1} w_{i}-s_{2 . i-1}{ }^{-1} w_{i-1}-s_{2 . i+1}{ }^{-1} w_{i+1} \tag{G.54}
\end{align*}
$$

An unbiased estimate of $y_{1 . i}$ is now given by

$$
\begin{equation*}
x_{1 . i}^{1.1}=a_{1 . i}{ }^{-1} p_{1 . i} \tag{G.55}
\end{equation*}
$$

where

$$
\begin{align*}
a_{1 . i} & =2 s_{2 . i}{ }^{-1} s_{1 . i}-s_{2 . i-1}{ }^{-1} s_{1 . i-1}-s_{2 . i+1}{ }^{-1} s_{1 . i+1} \\
& =4 s_{2 . i}{ }^{-1} s_{1 . i} \tag{G.56}
\end{align*}
$$

and

$$
\begin{equation*}
p_{1 . i}=2 s_{2 . i}{ }^{-1} r_{i}-s_{2 . i-1}-1 . s_{2 . i+1}{ }^{-1} r_{i+1} \tag{G.57}
\end{equation*}
$$

From Eqs.(G.54)-(G.57)

$$
\begin{equation*}
x_{1 . i}=y_{1 . i}+a_{1 . i}{ }^{-1} u_{1 . i} \tag{G.58}
\end{equation*}
$$

$a_{1 . i}{ }^{-1} u_{1 . i}$ is the error in $x_{1 . i}$ caused by the additive noise, where

$$
\begin{equation*}
u_{1, i}=2 s_{2 . i}{ }^{-1} w_{i}-s_{2, i-1}{ }^{-1} w_{i-1}-s_{2, i+1}{ }^{-1} w_{i+1} \tag{G.59}
\end{equation*}
$$

Now, because Eq.(G.49) is only approximately true, the general expression for $x_{1 . i}$ is

$$
\begin{equation*}
x_{1, i}=y_{1 . i}+a_{1 . i}{ }^{-1} u_{1, i}+c_{1, i} \tag{G.60}
\end{equation*}
$$

Where $C_{1 . i}$ is the error in $x_{1 . i}$ caused by the curvature in the channel $y_{1 . i}\left(\right.$ see Fig.G.1(b)). Clearly, $C_{1 . i}=0$ if Eq.(G.36) holds true. Now,

$$
\begin{equation*}
a_{1 . i}= \pm 4 \text { or } \pm 4 j \tag{G.61}
\end{equation*}
$$

and (from Eq.(G.47)), the mean-square value of $a_{1 . i^{-1}} u_{1 . i}$ is [119]

$$
\begin{equation*}
2\left(\frac{1}{2} 2+\frac{1}{2}+\frac{1}{2}\right)\left(\frac{1}{2} \sigma^{2}\right)=\frac{3}{8} \sigma^{2} \tag{G.62}
\end{equation*}
$$

which is twice the variance of the real and imaginary parts of

```
a 1.i -1 un_i.
    The estimate of }\mp@subsup{y}{2.i}{}\mathrm{ is determined in a similar manner.
```


## Method (4): "Very Fast Fading" Assumption

The estimator assumes that

$$
\left(y_{1 . i+1}-y_{1 . i}\right)-\left(y_{1 . i}-y_{1 . i-1}\right)=\left(y_{1 . i}-y_{1 . i-1}\right)-\left(y_{1 . i-1}-y_{1 . i-2}\right)
$$

AND

$$
\begin{equation*}
\left(y_{2 . i+1}-y_{2 . i}\right)-\left(y_{2 . i^{-1}}^{-y_{2 . i-1}}\right)=\left(y_{2 . i^{-}} y_{2 . i-1}\right)-\left(y_{2 . i-1}-y_{2 . i-2}\right) \tag{G.63}
\end{equation*}
$$

so that

$$
\begin{align*}
3 y_{1 . i} & =y_{1 . i+1}+3 y_{1 . i-1}-y_{1 . i-2} \\
\text { AND } \quad 3 y_{2 . i} & =y_{2 . i+1}+3 y_{2 . i-1}-y_{2 . i-2} \tag{G.64}
\end{align*}
$$

The estimator now operates with four received samples. From Eq.(G.35)

$$
\begin{align*}
3 s_{2 . i}{ }^{-1} r_{i} & =3 s_{2 . i}{ }^{-1} s_{s_{1 . i}} y_{1 . i}+3 y_{2 . i}+3 s_{2 . i}{ }^{-1} w_{i} \\
s_{2 . i+1}{ }^{-1} r_{i+1} & =s_{2 . i+1}{ }^{-1} s_{1 . i+1} y_{1 . i+1}+y_{2 . i+1}+s_{2 . i+1}{ }^{-1} w_{i+1} \\
3 s_{2 . i-1}{ }^{-1} r_{i-1} & =3 s_{2 . i-1}{ }^{-1} s_{1 . i-1} y_{1 . i-1}+3 y_{2 . i-1}+3 s_{2 . i-1}{ }^{-1}{ }_{w_{i-1}} \\
s_{2 . i-2}{ }^{-1} r_{i-2} & =s_{2 . i-2}{ }^{-1} s_{1 . i-2} y_{1 . i-2}+y_{2 . i-2}+s_{2 . i-2}{ }^{-1} w_{i-2} \tag{G.65}
\end{align*}
$$

So that, from Eqs.(G.64), (G.65) and the additional assumption that

$$
\begin{equation*}
s_{2, i-2}{ }^{-1} s_{1, i-2}=s_{2, i-1}{ }^{-1} s_{1, i-1}=-s_{2 . i+1}{ }^{-1} s_{1, i+1}=-s_{2, i}{ }^{-1} s_{1, i} \tag{G.66}
\end{equation*}
$$

It follows that

$$
\begin{align*}
& 3 s_{2 . i}{ }^{-1} r_{i}-s_{2 . i+1}{ }^{-1} r_{i+1}-3 s_{2 . i-1}{ }^{-1} r_{i-1}+s_{2 . i-2}{ }^{-1} r_{i-2} \\
& \quad=8 s_{2 . i}{ }^{-1} s_{1 . i} y_{1 . i}+3 s_{2 . i}{ }^{-1}{ }_{w_{i}}-s_{2 . i+1}{ }^{-1}{ }_{w_{i+1}}  \tag{G.67}\\
& -3 s_{2 . i-1}{ }_{w_{i-1}}-s_{2 . i-2}{ }^{-1}{ }_{w_{i-2}}
\end{align*}
$$

An unbiased estimate of $y_{1 . i}$ is now given by

$$
\begin{equation*}
x_{1 . i}=a_{1 . i}{ }^{-1} p_{1 . i} \tag{G.68}
\end{equation*}
$$

where

$$
\begin{align*}
a_{1 . i} & =3 s_{2 . i}{ }^{-1} s_{1 . i}-s_{2 . i+1}{ }^{-1} s_{1 . i+1}-{ }^{3} s_{2 . i-1}{ }^{-1} s_{1 . i-1}+s_{2 . i-2}{ }^{-1} s_{1 . i-2} \\
& =8 s_{2 . i}{ }^{-1} s_{1 . i} \tag{G.69}
\end{align*}
$$

and

$$
\begin{equation*}
p_{1 . i}=3 s_{2 . i}{ }^{-1} r_{i}-s_{2 . i+1}{ }^{-1} r_{i+1}-3 s_{2 . i-1}{ }^{-1} r_{i-1}+s_{2 . i-2}{ }^{-1} s_{1 . i-2} \tag{G.70}
\end{equation*}
$$

Now, the general expression for $x_{1} . i_{-1}$ is

$$
\begin{equation*}
x_{1, i}=y_{1, i}+a_{1, i}^{-1} u_{u_{1, i}}+c_{1, i} \tag{G.71}
\end{equation*}
$$

Where $C_{1 . i}$ is the error in $x_{1 . i}$ caused by the curvature in $\left\{y_{-1}\right.$.i $\}$ (see Fig.G.1(b)). Clearly, $C_{1 . i}=0$ if Eq. (G.63) holds true. $a_{1 . i}{ }^{-1} u_{1 . i}$ is the error in $x_{1}$.i caused by the additive noise, where

$$
\begin{equation*}
u_{1 . i}=3 s_{2 . i}{ }^{-1} w_{i}-s_{2 . i+1}{ }^{-1} w_{i+1}-3 s_{2 . i-1}{ }_{w_{i-1}}+s_{2 . i-2}{ }^{-1} w_{i-2} \tag{G.72}
\end{equation*}
$$

Now,

$$
\begin{equation*}
a_{1 . i}= \pm 8 \text { or } \pm 8 j \tag{G.73}
\end{equation*}
$$

and (from Eq. (G.47)), the mean-square value of $a_{1 . i}{ }^{-1} u_{1 . i}$ is [119]

$$
\begin{equation*}
2\left(\left(\frac{3}{8}\right)^{2}+\left(\frac{1}{8}\right)^{2}+\left(\frac{3}{8}\right)^{2}+\left(\frac{1}{8}\right)^{2}\right)\left(\frac{1}{2} \sigma^{2}\right)=\frac{5}{16} \sigma^{2} \tag{G.74}
\end{equation*}
$$

which is' twice the variance of the real and imaginary parts of $a_{1 . i}{ }^{-1} u_{1 . i}$.

The estimate of $Y_{2 . i}$ is determined in a similar manner.

However, if the data values $s_{i-2}, s_{i-1}, s_{i}, s_{i+1}$ are chosen to satisfy Eq. (G.61) for the estimate $\mathrm{x}_{1.1}$, then Eq. (G.61) cannot be satisfied by $s_{i-1}, s_{i}, s_{i+1}, s_{i+2}$ for the next estimate $x_{1, i+1}$. Therefore, it is impossible to get a good estimate of both $Y_{1 . i}$ and $Y_{1 . i+1}$ with this method. This was not the case for Methods (2) and (3).

## Computer Programs

MREC.FORTRAN: - Program to set up the main parameters for the subroutine SREC.FORTRAN. (The other relevant subroutines are SFADE.FORTRAN, SBBCHAN.FORTRAN, SBEQP.FORTRAN and SBEQA.FORTRAN.
SREC.FORTRAN: - Subroutine to perform all the receiver functions SFADE.FORTRAN: - Subroutine to generate the flat Rayleigh fading, $\left\{y_{i}\right\}$. SBBCHAN.FORTRAN: - Subroutine to generate the $\left\{s_{i}\right\},\left\{w_{i}\right\}$, interpolate the $\left\{y_{i}\right\}$ and calculate the $\left\{r_{i}\right\}$.
SBEQP.FORTRAN:- Subroutine to calculate the bit error rate in the QPSK signal.

SBEQA.FORTRAN: - Subroutine to calculate the bit error rate in the 16-point QAM signal.

RC_SAMP2.FORTRAN:- Program to generate the tap gains of the rootraised cosine filters and output them to the files rc_s4/rc_s8. These tap gains are used in the subroutine SBBCHAN.FORTRAN.

INTERPOLATE_RC.FORTRAN:- Program to generate the interpolating matrix "a", which is output to the files interal0/intera20/intera40. This matrix is used in the subroutine SBBCHAN.FORTRAN.

SIM2.FORTRAN:- Program to generate the noise shaping filter and output it to the file omvermon80. This filter is used in the subroutine SFADE. FORTRAN.

MESTIMPRED.FORTRAN:- Program to generate all the estimation / prediction error curves, except for the Kalman estimator. (The relevant subroutines are SFADE.FORTRAN and SBBCHAN.FORTRAN).

KALMEST.FORTRAN: - Program to generate the estimation error curves for the Kalman estimator. (The relevant subroutines are SFADE.FORTRAN and SBBCHAN.FORTRAN). SINEWAVE.FORTRAN:- Program to test the sinewave estimation scheme. EQUALIZER2.FORTRAN:- Program to test the equalizer.

**************
SREC.FORTRAN

MREC.FORTRAN
. . . . . 40 .

## inftialise

nteger isinm isampl,ipacl imetes,imetb,iordp
double precision snrdb, btemp, theta
*mmodulation scheme 1 s modsch=OPSK or 16-pt QAM
**no. of signalis) is isinum=1 or 2
**packet length is ipacl=??? symbol
**no. of samples (symbols) is isampl=? ??+2*ivien
**SNR is snrdb=???? de
**ESTIMATION :- if jmetes=0 unbiased estimator
if imetes=2 Gradient Alg. With feedback
*** if imetb=i varlable b in Gradient Ale. btemp/[si**2]
*** else imetb=0 constant $b$ in Gradient Alg. b=btemp
**in Gradient estimator, btemp=???
else degree of predictor $=$ iordp
***n fad. mem. poly. predictor, theta=???
isinum=1
pacl=120
sampl $=24000+2 * 1$ pac
imetes:2
1 metb=0
btemp $=0.2 \mathrm{do}$
lordp=1

Print*, modsch=, modsch, Print*,'isampl=',isampl, ipacl=',ipacl
Print*, imetes=',imetes,' imetb=',jmetb,' btemp=',btemp Print*,'iordp=', iordp:'
call srec (modsch,isinum, ianum,isampl,ipacl,snrdb Print*.'RETURNED srec
en


* imetes, imetb,btemp,iordp, thetal
double precision double precision double precision integer imetrt integer iimin double precision double precision double precision double precision double precision double precision double precision integer ishift
integer mm, mddmin
double precision rxpos(0:15,2), rypos(0:15,2) double precision dd(0:15), ddmin
double preciston sxpos(0:15,2), sypos(0:15,2)
integer imetco
double precision
double precision double precision integer Iperfe double precision $q x(4,-120:-1,2)$, qy(4,-120:-1,2) double precision qxold(4,-120:-2,2), qyold(4,-120:-2,2) integer istor, iqfrom(4,-120:-1)
double precision $p x(4,4,2)$, py $(4,4,2)$
integer isampl, ispsym, kddmin, ibite(0:2), ibitc (0:2)
integer $i, j, k, 1, m$
integer if, jj, ijcmín, jcmin, jJmin(4), jmin(4)
integer ivnum, ivien, istart
integer ipacl, irtsym
double precision rxest, ryest
double precision ex, ey
double precision $b$, btemp
double precision xest(4,2,2), yest(4,2,2)
double precision xetem(4,-120:-1,2,2), yetem(4,-120:-1,2,2)
double precision xeold(4,-120:-1,2,2), yeold(4,-120:-1,2,2)
double precision xeplo(-120:60000,2,2), yeplo(-120:60000,2,2
integer iperfd, imetes,imetb
double precision theta, th1, th2, th3, th4, ths
double precision
double precision
double precision
double precision
double precision
double precision
double precision
double precision integer iordp
integer jef
Integer imetde
$r x(0: 60000,2), r y(0: 60000,2)$
sx(0:60000,2), sy(0:60000.2)
$x(0: 60000,2,2), y(0: 60000,2,2)$
ssy(2), srx(2), sry(2)
xrto(0:11,2,2), yrto(0:11,2,2)
absave(2), acorab(2)
xrtave(2,2), yrtave(2,2)
xrtcor $(2,2)$, yrtcor $(2,2)$
$x \operatorname{rtm}(2,2)$, yrtmo $(2,2)$
mldo. mld9, mld27, mldi8, mlmin(2)
mrx, mry, ymaz, sdymag, isd sxdet(0:60000,2), sydet $0: 60000,2$ cq(4),cqitot, cp(4,4), cpmin
e2x. ${ }^{2}$
22x(4,2,2), z2y(4,2,2)
1x(4,2,2), ziy(4,2,2)
21x(4, $4,2,2), 21$ yol $4,2,2$
2xol(4,2,2),22yol(4,2,2)
xpred(4,2,2), ypred(4,2,2)
xpold(4,-1:9:0,2,2), ypold(4,-1:9:0,2,2)
xpplo(-119:60000,2,2), ypp10(-119:60000,2,2)
double precision snrdb
integer inter
integer iant, ianum
nteger iobbc
character\#9 modsch
double preciston svar
c ***print parameters read in from main program
Print*.'modsch=',modsch,' isinum=',isinum,' lanum=',ianum
Print*, isampl=., isampl, ipacl=' ipacl
Print*,'isampl=', isampl
Print*,'snrdb='.snrdb
Print*,'snrdb=', snrdb
Print*,' imetes $=$, imetes
Print*, imetes=, imetes, imetb=',imetb, btemp=',btemp
Print*,'iordp=',iordp,' theta=', theta
c ***svar is the average transmitted energy per bit.
*** needed in calcn. of noise variance
if(modsch.eq.'QPSK') then
svar=1.0d0
elseif (modsch.eq.'16-pt QAM') then
svar=2.5d0
else , MODULATION SCHEME NOT AVAILABLE
stop
endif
**set $b=b t e m p$. (though if imet $b=1$ then $b=b t e m p /(s i) * * 2)$
$b=\mathrm{btemp}$
vlen=1*1pac
(lvaen.eq. 1 ) then
istart=120
Print*,' Give predictors ',istart,' symbols to start-up
lse
istartifulen
Pint..' Give predictors one packet to start-up'
endif
**if iobbco then read in $s, r, y$ arrays from stored files
**else run subroutine sbbchan
obbc = 1
*"for no retraining set imetrt=0
**Cor ideal retraining set imetrt=
**for slope retralning:- subtracting one from other get imetrt=2
imetrt=3
**Assume $10 \%$ retraining, unless no retraining
if (imetrit.eq.0) then
irtsym=0
else
Irtsym = ipacl/10
endif
** bit error measurements
if (mod (isampl, lpacl).ne. 0) then
PRINT*,'isampl is NOT a whole number of packets
stop
**assume rading is generated at 600 samples per sec.
** this must be interpolated to give inter*600 samples per sec.
inter=20
*if fixed delay in detection then imetder
** else if detect each packet, as a block l.e. start from
** one vector after retraining then imetde=2
**if imetsv=1 then store/swap q vectors around according to
** Adrian's method.
** elseif imetsv=2 then trace back along vectors to detect each
*** packet in one go during retraining.
**(If no retraining then imetsv MUST=1, and imetde MUST=1)
if (imetrt.eq. o) then
imetsu=1
Print*,'store/swap vectors around according to general method
imetde $=1$
Print*, f ixed delay in detection
else
1 metsv=2
Print*,'trace back along vectors during retraining for detection imetde $=2$
Print*.'detect each packet as a block, starting from one vector
Print*, after retraining'
endif
**if outputing error files then fef:1 lelse ief=0
ef $=0$
**if plotifing channel est \& pred waveforms then ifplot=ivlen
*** else set iiplot=2
ifplot=2
***PRINT system constants for thls run
PRINT*, fanum, ANTENNAE
PRINT*,tsinum,' •,modsch,' SIGNAL(S)
if (imetes.eq. 0 ) then
PRINT*, 'Unbiased estimator'
if (isinum.gi. il then
PRINT*, CANNOT do unbiased estimation for 2 signals in channel' stop
endif
elseif(imetes.eq. 1 ) then
PRINT*,'NO feedback from predictor to estimator
elseif (imetes.eq.2) then
PRINT*.'USE feedback from predictor to estimator'
endif
if (imetes.ge. i) then
PRINT*.' Gradient Estimator ... variable b= ',b,'/[si**2]
PRINT*. Gradient Estimator ... constant $b=$, $b$
endif
end if
if (iordp.eq. -1) then
PRINT*.'No Prediction'
else
PRINT*, LSFM Predictor .... order = ',iordp,' theta= ',theta endif
PRINT*,'vector length a ', ivlen
PRINT*.' Packet Leneth $=$, ipacl
if (imetrt.eq.0) then
PRINT*, NO retraining
elseif (imetri.eq.1) then
PRINT*,' IDEAL retraining
PRINT*, $10 \%$ retraining therefore ', Irtsym, retraining symbols
else
PRINT*,' Retraining method ',imetrt
RINT*,' $10 \%$ retraining therefore ',irtsym,' retraining symbols endif

PRINT*,'isampl= ',isampl
PRINT*,'inter='.inter
Print*, ispsym,' sample(s) per symbol
PRINT*,'SNR= ',snrdb,' dB
if (ifplot.eq.ivlen) then
Print*.'channel est and pred files are output
else
Print*,'channel est and pred files are not output' endif
If (ief.eq. 1) Print*, 'error files are output'
if (imetde.eq.1) then
Print*, imetdex ', imetde, $\quad$.e. fixed delay in detection'
elseif(imetde.eq.2) then
Print*,' imetde= ',imetde.' $\quad$.e. black detection' endif
if (imetsv.eq. 1) then
Print*,' Update vectors with each symbol
elseif(imetsv.eq. 2) then
Print*, Detect symbols by tracine back along full vector'
else
PRINT*, imetsv ..error
stop
endif
***read in files of signal, channel. received signal If (lobbc .eq. 0) then
Print*, READ in files
open(unit=7, file='sxlo', form='formatted')
open(unit=B, file='syio', form='formatted')
open(unita, file='xio', form='formatted')
open(unit $=10$, file='yio', form='formatted')

read( $\theta, *)((s y(i, i s i g), i=0, i s a m p l, 1), i s i g=1, i s i n u m, 1)$
read(9,*) ((xxi,isig,iant), $i=0,19 a m p i, i), i s i e=1, i s i n u m, 1)$. iant =1, ianum,1)
read(10,") ((iy(i,isie,iant), i=0,isampl,i), isig=1,isinum,l),

* iant=1, ianum,1)
close (7)
close(8)
close (9)
close (10)
open(unit=11, file='rxio', form='formatted')
opencunit=12, file='ryio', form='formatted')
read(11,*) (rx(i,iant), $i=0, i s a m p l, 1)$, iant=1, ianum, 1 )
read (12,*) (ry(i,iant), i=0,isampl,i), iant=1, lanum, 1 )
close(11)
close(12)
else
Print*: SUBROUTINE sbbchan
call sbbchancisampl,ipacl,irtsym,inter,snrdb.
modsch,svar, isinum, ianum, $r x, r y, s x, s y, x, y)$
Print*,'RETURNED sbbchan
endif
do 9991 iant=1, ianum. 1
Print
if (iant.eq. 1 ) then
Print:.'ANTENNA a
endif
if(iant.eq.2) then

Print*.'ANTENNA b'
endif
Print*, $\quad$ rx ry sx sy x y
do 9993 i=1, isampl.i
(f (i.eq. 3) i=isampl-i
do 9995 isig=1,isinum. 1
Print*.'SIGNAL •isige
Print 9001, i,sx(i,isig),sy(i,isig), x(i,isig,iant),y(i,isig,iant
9001 format (16, 12x, 2f9.1, 2f12.4)
continue
Print 9002, rx(i,iant), ry(i,iant)
9002 format(10x, 2f9.4)
9993 continue
9991 continue
***do for feeding "Detected" data symbols and "correct" data *** symbois into estimator
do 4 iperfd=0,0,
Print

PRINT*, DETECTED data symbols fed back into estimator'
else PRINT, CORRECT data symbols fed back into egtimator...'
endif
c ***do for actual estimation and perfect est.
do 1 Iperfe=0,1,1
Print

if (iperfe.eq. 0) then
PRINT*,' Actual estimation'
else
PRINT*., assume perfect estimation'
endif
do 2 for 2 and 3 vectors
Print

if (ivnum.eq. 3) ivnum=4
if (iperfd.eq. 11 ivnum=1
c \#\#\#i.e. cannot have more than one vector if all detected
*** symbols are assumed correct
if (iperfe.eq. 1 ) ivnum=1
***i.e. no advantage of Viterbi detector over Maxm. Like
\#** for perfect estimation.
PRINT*, ivnum,' vectors in the Viterbi detector.
***do for actual combining(1) or maximal ratio combining(2)
*** or selection diversity combining(3)
do 3 imetco=1,1,1
Print
Print*,'---------------------------1
if (imetco.ge. 2).and.(isinum.ne. 1)) then
PRINT*,'CANNOT do this combining with', isinum.' signals. stop
endif
if (imetco.eq. 1) PRINT*. ACTUAL combining method , imetco
If (imetco.eq. 2) PRINT*, Maximal Ratio combining
if limetco.eq. 3) PRINT*, Selection diversity combining'
if (iperfe.eq. 1) imetrt=3
if lianum.eq. i) then
If (fanrdb.le. $35.0 d 0$ ).and. (iordp.eq. 1)) then btemp $=0.155 \mathrm{do}$
theta $=0.6 \mathrm{~d} 0$
elself((snrdb.gt. 10.0d0).and.(snrdb.le. 20.0d0)

- $c$ and.i(iordp.eq. 1 ) then
btemp $=0.16 \mathrm{~d} 0$
elseif(isnrdb.gt. 20.0d0).and.(3nrdb.1e. 30.0d0)
*c $c$.and.(iordp.eq. 1) then
btemp $=0.16 \mathrm{do}$
btemp $=0.16 \mathrm{do}$
theta $=0.525 \mathrm{do}$
elseif((snrdb.gt. 35.0d0).and.(fordp.eq. 1)) then btemp $=0.21 \mathrm{do}$
theta=0.4do
elseif((snrdb.le. 10.0d0).and.(fordp.eq. 2)) then
theta=0.92do
elseif( (snrdb.et. 10.0d0).and. (snrdb.le. 20.0d0)
* . and.(iordp.eq. 21) then
theta $=0.88 \mathrm{~d} 0$

$*$
.and.(iordp.eq. 2)) then
elseifi(snrdb.gt. 30.0d0).and.liordp.eq. 2)) then
elseta $=0.80 \mathrm{do}$
else
PRINT*,'EH:!:'
stop
endif
if (snrdb.le. 10.0 dO ) then
btemp $=0.14 \mathrm{do}$
theta=0.72d0
else
btemp $=0.15 \mathrm{~d} 0$
theta $=0.625 \mathrm{~d} 0$
endif


## endif

***ianum endif:! b=btemp
PRINT.,'tordp='.iordp,' theta='.theta,' babtemp=',b
PRINT*,'imetrt=', imetrt
if (imetrt.eq. i) then
PRINT*,'ideal retrainine'
elseif(imetrt.eq. 3) then
PRINT*'slope retrainine, least-squ. st. line fit,
endir
***SHOULD NOT NEED TO ALTER ANY PARAMETERS PAST THIS POINT

***set up all the posiible signal points
*** depending on modulation scheme AND no. of signals
if ((modsch.eq.'GPSK').and. (isinum.eq. 1)) then
$k$ max $=3$
$2021=-1,+1,2$
do $202 \mathrm{~J}=-1,+1,2$
$\operatorname{sxpos}(m, 1)=d b l e(i)$
sypos $(m, 1)=d b l e(j)$
$m=m+1$
202 continue
elseif((modsch.eq.'OPSK').and.(isinum.eq.2)) then $k$ max $=15$
$m=0$
do $203 i=-1,+1,2$
do $203 \quad j=-1,+1,2$
do $203 \mathrm{k}=-1,+1,2$
do $2031=-1,+1,2$
sxpos $(m, 1)=d b l e(i)$
sypos(m,l) = dble(j)
sxpos(m,2) $=d b l e(k)$
sypos(m,2) = dble(l)

## $m=m+$

203 continue
elseif((modsch.eq.'16-pt QAM').and.(isinum.eq.1)) then $k_{\text {max }}=15$
$\mathrm{m}=0$
do $206 i=-3,+3,2$
do $206 \mathrm{j}=-3,+3,2$
sxpos(m,l)=dblefi)
sypos (m, 1 )=dble( $f$ )
$m=m+1$
206 continue
PRINT*,' NOT POSSIBLE
stop
endif
do channel est/pr
do 199 isig=i, isinum,
do 199 ii=1, ivnum,
xest(if,isig,iant) $=0.0 \mathrm{do}$
yest(1l.isig.iant) $=0.0 \mathrm{do}$
z1x(11,isie,iant) $=0.0 \mathrm{~d} 0$
z)y(il,isig, (ant) $=0.000$

22x(ii,isig,tant) $=0.0 \mathrm{~d} 0$
z2y(il,isig,iant) $=0.000$
xpred(il, isic, iant) $=0.0 d 0$
ypred(if,isig,iant) $=0.0 \mathrm{dO}$
***alculate theta constants once and for alt
th1 $=(1.0 \mathrm{~d} 0-\mathrm{theta}) *(1.0 \mathrm{~d} 0-\mathrm{theta})$
th2 $=1.0 \mathrm{~d} 0$ - theta*theta
th3 $=0.5 \mathrm{dow}(1.0 \mathrm{~d} 0-\mathrm{theta}) * * 3$

th5 = 1.0d0 - theta*: 3

Print*, DETECTOR - ESTIMATOR - PREDICTOR - (RETRAINING)' do 111 i=1,isampl. 1

If (1.1t. istart) then
..DETECTOR
**assume perfect detection at beginning of packet
do 298 isie=1, isinum,
sxdet(i,igig) = sx(i,isig)
sydet(i,isig) $=s y(i, i s i g$
continue
*iset up a vector
do 293 isig=1,isinum,
do 293 ii $=1$,ivnum,
ax(ii, i-ivien,isig) $=$ sxdetif,isie
ay(ii,i-ivien.isig) $=$ sydetii.isig
iqfrom(ii,i-ivien) = ii
continue - endif
***set up vector components q(ii,-i,isig)
*** for use in the estimator
**Since sdet(i) used in the estimator is q(il,-1,isig)
do 215 isigei, isinum.
do 215 ii=i, ivnum,
qx(ii, -1,isig) $=$ sxdeti,isig
qy(ii,-1,isig) $=$ sydet(i,isig)
***set up costs of $q$ vectors
if (i .eq. (istart-1)) then
$\mathrm{cq}(1)=0.0 \mathrm{do}$
do 216 il:=2.ivnum.
$c q(i i)=1.0 d 5$
continue
endif
if (ivien.ne. 1) then
...ESTIMATOR and PREDICTOR vector inltializations
do 217 iant=1, ianurn. 1
do 217 isiga, iginum,
do 217 ii=1,ivnum,
xeold(if, (-ivienti),isig,iant) = xest(il,isig.iant)
yeold(if,i-(ivien+1),isig,iant) = yest(ii,isig,iant)
xeold(ii, i-ifven+1), isie,iant) $=x e s t(i f, i s i e . i a n t ~$
yeold(if,i-ivienti),isig,iant) = yestifi,isig,lant
xpold(ii,i+1-(ivien+i),isie,iant) $=x p r e d i l i s i g . i a n t)$
ypold(i,i+1-(ivien+i),isig,iant) =ypred(ii,isig,iant
xpoldil,i+i-(ivienti),isig,iant) =xpredilisie,iant ypold(ii,i+i-(ivlen+1),isig,iant) = ypred(ii,isig,iant) continue

## endif

elself( (modifipaci).eq. 0$)$ and.
(irtsym.ne. 0) and. (ifirtsym-1).le.isampl)) then ...RETRAINING
**FOR EACH RETRAINING SYMBOL
***Do detection of symbols at time i-ivien.
*** (or do block detection of previous packet).
**Also shift $p$ vectors along to make $q$ vectors
*** ready for the next gample. (Shift the corresponding
*** est and pred vectors along).
**ALSO TO RETRAIN CHANNEL EST AND PRED
***Form raw measurements of :- the channels yilit) and y2(it
*** and their first derivatives
*** from r(i), r(i+1), r(i+2)......., r(i+irtsym-1
** knowing sifi, s(i+1), s(i+2),......s(i+irtsym-1)
***************************************************
**must store and reset after loop 252
*** for use in est and pred retraining
do 249 iant $=1$, 1anum, 1
do 249 isig=1,isinum,
xetem(1,-1.isig,iant) $=x e s t(1, i s i e, i a n t)$
yetem(1,-1,isig,iant) =yest(1,isig,iant)
xptem(1,0,isig,iant) $=x p r e d(1, i s i q, i a n t)$
yptem(l,o,isig,iant $=$ ypredifisig,iant) continue
**The training bits are known therefore perfect detection do 251 isig=i,isinum,
do $251 k=i, i+j r t s y m-1$.
sxdet (k,isig)=sx(k,isig)
sydet(k,isig) $=$ sy(k,isig
continue
if (imetsv.eq. 2) then
***if method of storing(swapping) vectors $=2$, then now must
*** do block detection of previous packet
ilmin=1
do $240 \mathrm{j}=\mathrm{i}$,iven,
do 2401 isig=1,isinum, 1
sxdet(i-j,isig) $=q x(i i m i n,-f, i s i e$
sydet(i-j,isig) = qy(ilimin,-j,isig)
ontinue
do $252 k=i, t+m$
do $252 \mathrm{k}=\mathrm{i}, 1+\mathrm{irtsym}-1,1$
if (imetsv.eq. 1) then
***do detection i.e.
***sdet(i-ivlen) is taken to be q(i-ivientof the lowest cost ** vector $p$
do 2521 isig=1, isinum, 1
sxdet(k-ivien,isig) $=q x(1,-$ lvien,isig)
sydet(k-ivien,isig) = qy(1,-ivlen,isig)
continue

## endif

***shift all $p$ vectors along to make $q$ vectors ready for ** the next sample.
***Also shift all corresponding estimate and prediction
*** vectors along ready for the next sample.
***The data point q(1,-ivlen, isig) has now been detected so is
*** shifted out, and the points q(if,-1,isig) have yet to *** be calculated so are set to zero for now
do 253 ii=1,ivnum,i
if (imetsv.eq. 1 ) then
do 254 isigaidisinum.
do $254 \mathrm{j}=$-ivien, $-2,1$
$q \times(i i, j, i s i g)=q \times(i i, j+1, i s i g)$
qy(ii,j,isig) = qy(ii, j+1,isig)

> endif
> do 2541 iant=1, ianum,
> do 2541 isig=1, isinum,
do $2542 \mathrm{j}=-1$ 1plot, $-2,1$
xeold(if,j,isig,iant $=x e o l d(i l, j+1$, isig.iant $)$
yeold(if.j,isie,iant) = yeoldifi,j+1,isig,iant)
xpold(if,j+1,isig,iant) = xpold(ii, j+2,isie,iant)
ypold(ii,j+1,isig,iant) $=y p o l d(i f, j+2, i g i e, i a n t)$
2542 continue
qx(il, $-1, i g i f)=0.0 d 0$
qy(if,-1,isig)=0.0d0
xeold(if,-1,isig,iant) = xestifi,isig,iant)
yeold(ii,-1,isig,iant) $=$ yest(if.isie,iant) xpold(ii,o,isig,iant) $=x p r e d(i i, i s i g, i a n t)$
xest(il,isig,iant) $=0.0 \mathrm{dO}$
yest(idiab.
xpred(il, isig,iant) $=0.000$
ypred(if lsig (ant) $=0.0 \mathrm{do}$
253 continue

## continue

do 2532 iant=1, ianum, 1
xeplo(k-iiplot,isig,iant $)=x e o l d(1,-i f(p l o t, i s i g, i a n t)$
yeplo(k-ifplot,isig,iant) $=y e o l d(1,-i$ iplot,isig iant)
xpplo(k-ifplot+1,isig,iant) $=x p o l d(1,-i j p l o t+1, i s i g, i a n t)$
ypplo(k-ifplot+1,isig.iant) $=y p o l d(1,-i f p l o t+1$,isig,iant
$\qquad$
***set up
do 256 isig=1, isinum, 1
do 256 ii $=1$, ivnum,
qx(ii,-1,isig) $=\operatorname{sxdet}(k, i s i g)$
qy(il, -1,isig) $=$ sydet (k,isig)
if. (imetsu.eq. 21 then
***if imetsv=2 then restart build-up of $q$ vectors
*** Cold iNcorrect info in vectors is overuritten with each
*** symbol)
istor $=\bmod (k, i v i e n)-i v l e n$
do $2540 \quad i f=1$, ivnum, 1
iqfrom(ii.istor) $=1 i$
do 2540 isig=1.isinum, 1
$\mathrm{qx}(\mathrm{ii}$, istor, isig$)=\mathrm{qx}(\mathrm{if},-1, \mathrm{isig})$
$q y(11, i s t o r, 1 s i g)=q y(i i,-1, i s i g)$
2540 continue
endif.
do 261 isigel.isinum, 1
do 261 iji=1, ivnum, 1
***reset est and pred as before loop 252
do 2491 iant=1, ianum. 1
xest(1, isig, iant) $=x \operatorname{ctem}(1,-1, i s i g, i a n t)$
yest(t,isig,iant $=\operatorname{yetem}(1,-1$, isie,iant $)$
xpred (i,isig.iant) $=$ xptem(i,o.iste,iant)
ypred(i,isig,iant) $=y p t e m(1,0, i s i g, i a n t$
continue
...TO RETRAIN CHANNEL EST AND PRED
if (imetrt.eq.1) then
***ideal retralning
do 2501 iant=1, ianum, 1
do $2501 \mathrm{isig}=1$, isinum, 1
do $250 \quad 11=1$, ivnum, 1
xest(ili,isig,iant) $=x(i+i r t s y m-2, i s i g, i a n t)$
yest(ii, isig,iant) $=y(i+i r t s y m-2, i s i g, i a n t)$
zix(if,isig, fant) $=x(i+i r t s y m-1, i s i g, i a n t)-x(i+i r t s y m-2, i s i g, i a n t)$ $z 1 y(i i, i s i g, i a n t)=y(i+i r t s y m-1$, isig,iant)-y(i+irtsym-2,isig,iant) $z 2 x(i f, i s i g, i a n t)=0.0 \mathrm{~d} 0$
z2y(ii,isig,iant) $=0.0 \mathrm{~d} 0$
xpred (if.igig,fant) $=x(i+i r t s y m-1, i s i g, i a n t)$
ypred(ij,isicg,iant) $=y(i+i r t s y m-1$, isig,iant $)$
continue
elseif(imetrt.ge. 2) then
*** of least squares
do 2601 iant $=1$, ianum.
absave(lant) $=0.0 \mathrm{~d} 0$
acorab(iant) $=0.0 \mathrm{~d} 0$
do 2601 isig=1, isinum,
xrtave(isig, iant) $=0.0 \mathrm{do}$
yrtave (igig, iant) $=0.0 d 0$
$\operatorname{xrtcor}(i s i g, \operatorname{tant})=0.0 d 0$
yrtcor(isig,iant) $=0.0 \mathrm{~d} 0$

## if (isinum.eq. 1 then

do $267 \mathrm{k}=\mathrm{i}$, i+irtsym-1,
do 267 iant=1, ianum,1,
$\operatorname{xrto}(k-i, 1, i a n t)=(s x(k, 1) * r x(k, i a n t)+s y(k, 1) * r y(k, i a n t)) /$
*


* (sx(k,i)wsx(k,i) +sy(k,1)wsy(k,1))
absave(iant) = absave(iant) + dble(k-i)
acorab(iant) = acorab(iant) + dble(k-i)*dble(k-i)
xrtave (1, iant) $=x$ xtave (1,iant) $+x r_{\text {to }}(k-i, 1$, iant $)$
yrtave(i,iant) $=y r t a v e(1, i a n t)+y r t o(k-i, 1, i a n t)$
$\operatorname{xrtcor}(1$, iant $)=\operatorname{xrtcor}(1$, iant $)+d b l e(k-i) \# x \operatorname{to}(k-i, 1, i a n t)$ yrtcor(1,iant) $=\operatorname{yrtcor}(1, i a n t)+d b l e(k-i) * y r t o(k-i, 1, i a n t)$ continue
do 260 tant=1, ianum, 1
absave(iant) = absave(iant)/dble(1rtsym)
acorab(iant) = acorab(iant)/dble(irtsym)
xrtave(1,iant) $=x r t a v e(1, i a n t) / d b l e(i r t s y m)$
yrtave(1,iant) $=y r t a v e(1, i a n t) / d b l e(i r t s y m$
$x r t c o r(1, i a n t)=x r t c o r(1, i a n t) / d b l e(i r t s y m)$
yrtcor(1,iant) $=y r t c o r(1, i a n t) / d b l e f i r t s y m)$
if (imetrt.eq. 2) then
$\operatorname{xrtmO}(1, i a n t)=\left(x r_{0}(i r t s y m-1,1, i a n t)-x r t 0(0,1, i a n t)\right) /$
*yrtmo(t) dble(irtsym-1)
yrtmo(1,fant) $=(y r t o(i r t s y m-1,1, i a n t)-y r t o(0,1, i a n t)) /$ dble(irtsym-1)
elseif(imetrt.eq. 3) then
xrtmo(1,iant) $=(x r t c o r(1,1 a n t)-a b s a v e(i a n t) * x$ (ave(1,iant)) * (acorab(iant) - absave(iant)*absave(iant))
yrtmo(1,iant) $=(y r t c o r(1, i a n t)-a b s a v e(i a n t) * y r t a v e(1, i a n t))$ (acorab(iant) - absave(iant)*absave(iant)) endif
if (modifrtsym,2).eq. 0) then
c ***i.e. if number of retraining symbols is EVEN
xrto(irtsym/2,1,iant) $=x$ xtave(1,iant $)+0.5 d 0 * x r t m 0(1, i a n t)$
$y r t 0(1 r t s y m / 2,1, i a n t)=y r t a v e(1, i a n t)+0.5 d 0 * y r t m 0(1, i a n t)$
xrto(irtsym/2-1,1,iant) $=x r t a v e(1, i a n t)-0.5 d 0 * x r t m 0(1, i a n t)$ yrto(irtsym/2-1,1,iant) $=y r t a v e(1, i a n t)-0.5 d o * y r t m 0(1, i a n t)$ else
***i.e. if number of retrainine symbois is ODD
xrto(irtsym/2,1,iant) $=x r t a v e(1, i a n t)$
yrto(irtsym/2,1,iant) $=y r t a v e(1$, iant $)$
xrto(irtsym/2-1, i, iant) = xrtave(1,iant) -xrtmo(i,iant)
yrto(irtsym/2-1,1,iant) $=$ yrtave(1,iant) $-y r t m 0(1, i a n t)$
***i.e iant loop
elseif ( (modsch.eq.'QPSK') .and. (isinum.eq. 2)) then ***Retraining est and pred for 2 QPSK signals
do $257 \mathrm{k}=\mathrm{i}+2$, i+irtsym-1,1
do 257 lant:1, ianum, 1
ssy(1) $=\operatorname{sx}(k-1,1) * s y(k-1,2)-\operatorname{sy}(k-1,1) * s x(k-1,2)$ $\operatorname{srx}(1)=2.0 \mathrm{~d} 0 *(\mathrm{sx}(\mathrm{k}-1,2) * r x(k-1$, iant $)+\operatorname{sy}(k-1,2) * r y(k-1$, iant $))-$ * $\quad s x(k-2,2) * r x(k-2$, iant $)-s y(k-2,2) * r y(k-2, i a n t)-$

sry(1) $=2.0 d 0 *(s x(k-1,2) * r y(k-1$, iant $)-s y(k-1,2) * r x(k-1$, iant $))$ sx(k-2,2)*ry(k-2,iant) + sy(k-2,2)*rx(k-2,iant) sx(k,2)*ry(k,1ant) +sy(k,2)*rx(k,iant)
ssy (2) $=-s x(k-1,1)$ msy $(k-1,2)+s y(k-1,1) * s x(k-1,2)$
$\operatorname{srx}(2)=2.0 \mathrm{~d} 0 *(\mathrm{ax}(\mathrm{k}-1,1) * r \times(k-1, \operatorname{iant})+\operatorname{sy}(k-1,1) * r y(k-1$, iant $))-$ sx(k-2,1)*rx(k-2,iant) - sy(k-2,1)nry(k-2,iant) sx(k,1)*rx(k,iant) - sy(k,1)*ry(k,iant)
sry(2) $=2.0 \mathrm{don}(s x(k-1,1) * r y(k-1$, iant $)-\operatorname{sy}(k-1,1) \mathrm{mrx}(k-1$, iant $))$ sx(k-2,1)*ry(k-2,iant) +sy(k-2,1)*rx(k-2,iant) sx(k,l)*ry(k,iant) $\quad$ sy(k,1)*rx(k,iant)
xrto(k-i-1,1,iant) $=-0.0625 d 0 * s s y(1) * s r y(1)$
yrto(k-i-1,1,iant) $=0.0625 d 0$ nssy(1)*srx(1)
xrto(k-i-1,2,iant) $=-0.0625 d 0 * s s y(2) * s r y(2)$
yrto(k-1-1,2,iant) $=0.0625 d 0$ nssy(2)*srx(2)
absave(iant) $=$ absave(iant) + dble(k-i)
acorab(iant) * acorab(iant) + dble(k-i)*dble(k-i)
xrtave(l,iant) $=x r t a v e(1, i a n t)+x r t 0(k-i, f, i a n t)$
yrtave(l,iant) artave(1,iant) +yrto(k-i,1,iant)
$x r \cos (1, i a n t)=x r t c o r(1, i a n t)+d b l e(k-i) * x r t 0(k-i, 1, i a n t)$ yrtcor(1,iant) $=y r t c o r(1,1 a n t)+d b l e(k-i) \# y r t o(k-i, 1, i a n t)$ xrtave(2,iant) $=x r t a v e(2,1 a n t)+x r t o(k-1,2, i a n t)$
yrtave(2,iant) $=y r t a v e(2, i a n t)+y r t o(k-i, 2, i a n t)$
xrtcor(2,iant) $=x r t c o r(2, i a n t)+d b l e(k-i) * x r t o(k-1,2, i a n t)$ $y r t c o r(2, i a n t)=y r t c o r(2, l a n t)+d b l e(k-i) \neq y r t o(k-i, 2, i a n t)$ continue
do 257 fant=1, lanum,
do 2571 isig=1, fsinum,
absave(iant) = absave(iant)/dble(irtsym-2)
acorab(iant) = acorab(iant)/able(irtsym-2)
xrtave(isig,iant) axtave(isie.iant)/dble(irtsym-2)
yrtave(isig, iant) $=$ yrtave(isig, iant)/dble(irtsym-2) xrtcor(isig,iant) $=x r t c o r(i s i g, i a n t) / d b l e(l r t s y m-2)$

if (imetri.eq. 2) then
 - /dble(irtsym-3)
 /dble(irtsym-3)
elseif (imetrt.eq. 3) then
***slope
xrtmolisic,iant $)=(x r t c o r(i s i g, i a n t)$
absave(iant) *xrtave(isig, lant))/
(acorab(iant) - absave(iant)wabsave(iant)
* 

yrtmotisie.lant $)=(y r t c o r(i s i g, i a n t)$
${ }^{*}$
absave(lanti*yrtave(isig,iant.)) /
(acorab(iant) - absave(lant)*absave(iant)
if (modirtsym,2).eq. 0) then
***i.e. if number of retraining symbols is EVEN
xrto(irtsym/2,isig,iant) $=x$ tave(isig,iant) ${ }^{( }$ 0.5 dO *xtmo(lsie,iant)
yrto(irtsym/2,isig,iant) = yrtave(isig,iant)
0.5d0*yrtmolisig,iant
xrto(irtsym/2-1,isie.fant) - xrtave(isig,iant) 0.500 xrtmolisig,iant = yrtave(isig,iant
$0.5 d o n y t m o(i s i g, i a n t ~$
yrto(irtsym/2-i,isig.iant)
*
**i.e. if number of retraining symbols is ODD
xrto(irtsym/2,isig,iant) $=x$ rtave(isig,iant
yrto(irtsym/2,isig,iant) = yrtave(isig,iant)
xrto(irtsym/2-1.isig,iant) $=x r t a v e(i s i g, i a n t)-x r t m o(i s i g, i a n t)$ yrto(irtsym/2-1,isig,iant) =yrtave(isig,iant) - yrtmo(isig,iant) endif
\#**initialise estimator/predictor
do 2579 lant=1, ianum, 1
do 2579 isif=1.isinum, 1
do 2579 i $i=1$, ivnum, 1
xest(ii,isig,iant) $=x$ to (irtsym/2-1,isig,iant)
yest(if,isig, (ant) $=y r t 0(1 r t s y m / 2-1, i s i f . i a n t)$
z2x(ii,isig.iant) $=0.0 \mathrm{~d} 0$
z2y(ifisig.iant) $=0.0 \mathrm{~d} 0$
z1x(il.isig,iant) $=x r t m 0(i s i g, i a n t)$
ziy(ili,igig,iant) $=y r t m o(i s i g . i a n t)$
xpred(il,isig.iant) $=x r t o(i r t s y m / 2, i s i g, i a n t)$

***Run 1 sfm estimator/predictor from $i=i r t s y m / 2+1$ to irtaym
** to give predictions of channel atad alnpe for the
** first symbol arter retraining.
do $2572 \mathrm{k}=\mathrm{i}+\mathrm{irtsym} / 2, i+i r t s y m-2,1$
do 2320 iant $=1$, ianum. 1
do 2321 i $i=1$, ivnum, 1
if cimetes.eq. 0 ) then
c ***unbiased estimator for 1 signal
if (modsch.eq.' $16-\mathrm{pt}$ QAM') then
$b=1.0 d 0 /(s x(k, 1) w s x(k, 1)+s y(k, 1) * s y(k, 1)$
else
$b=0.5 \mathrm{~d} 0$
endif
 yest(il,i,iant) $=b *(s x(k, 1) * r y(k, i a n t)-s y(k, i) * r x(k, i a n t))$ else
**gradient estimator
rxest =0.0d0
ryest $=0.0 \mathrm{dO}$
do 2322 isig=1, isinum, 1
If (imetes.eq.1) then
rxest $=$ rxest +

* sx(k,isig)*xest(ii,isig,iant) sy(k,isigi*yest(ii,isig,lant)
ryest $=$ ryest +
* sx(k,istg)*yest(ii,isie.iant)

```
            elself(imetes.eq.2) then
    rxest = rxest.
        sx(k,isig)*xpred(if,isig,iant) -
    * sx(k,isie)*xpred(il.isig,iant)
    ryest = ryest
    * sx(k,isig)*ypred(il,isig,iant) +
        sx(k,isig)*ypred(il,isig,iant) +
            endif
232 continue
    ex = rx(k,iant) - rxest
    ex = rx(k,iant) - rxest
    ey = ry(k,lant) - ryest
        if (imetb.eq. i) then
        b btemp/(sx(k,isig)**2 + sy(k,isig)**2)
            endif
            if (imetes.eq.1) then
        xest(il,isig,iant)= xest(il,isie,iant) +
        b*( ex*sx(k,isig) + ey*sy(k,isig))
    * b*(-ex*sy(k isig) + ey*sx(k isig))
        b*(-ex*sy(k,isig) + ey*sx(k,isig))
    xest(ii,isig,iant) = xpred(ij,isig,iant) +
    b*( ex*sx(k,isig) + ey*sy(k,isig))
    yest(ii,isig,iant)= ypred(ii,isie,iant) +
    * . b*(-ex*sy(k,igig) + ey*sx(k,isig))
223 continue
    conif
    endif (
    **imetes endlr::!
    do 2324 {glg=i,isinum,i
    ezx = xest(if,isig,iant) - xpred(ii,isig,iant)
                    if (iordp.eq. -1) then
    ...no prediction
    xpred(if,isig,iant) = xest(if,isig,iant)
    ypred(il,isig,iant) = yest(ii,isig,iant)
                elseif(iordp.eq.i) then
c ...degree !
    zlx(if.isig.iant) = zix(if,igie,iant) + thl*ezx
    zly(if,isig,ianz) = zly(if,isig,iant) + thi*ezy
    xpred(ii,isig,iant) = xpred(if,isig,iant) + zlx(ii,isig,iant) +
    th2*ezx
    |* ypred(ifi,isig,iant)=
    ,* elseir(iordp.eq.2) then
c ...degree 2
            elseif(iordp.eq.2) then
g,iant) = z2x(1i,isig,iant) + th3*ezx
    z2y(if,isig,iant) = z2y(ii,isig,iant) + th3*ezy
    z1x(ii,isig,iant) = 21x(ii,isig,iant) + 2.0d0*z2x(ii,isig,iant) +
```



```
    z1y(il.igie,iant)= 21y(if.is
    xpred(if,isif,iant) = xpred(ifi,isig,iant) + zix(ij,isig,iant) -
    *ypred(if,isig,iant) = ypred(ii,isig,iant) + zly(ii,isig,iant) -
    ypred(il,isig,iant) = ypred(ii,isig,iant) + zly(ii,isig,iant) -
        z2y(ili,isig,iant) + th5*ezy
        else
endif
```

572 continue
do 2325 isig: 1 isinum, 1
xeplo(k-ifplot,isig,iant) $=x e o l d(1,-i f(o t, i s i g, i a n t)$
xeplo(k-ipplot,isig,iant $=$
yeplo(k-ilplot,isig,iant) $=$ yeold(i, iifiot,isig,iant)
xpplo(k-iiplot+i,isig,iant) =xpold(1,-ifplot+1,isie,iant)
ypplo(k-ifiplot+1,isig,iant) $=y p o l d(1,-i f p l o t+1, i s i g, i a n t)$ continue

572 continue
continue
endif
**i.e. retraining method endif do 2551 iant $=1$, ianum, 1
do 2551 isig=1,isinum,
do 255 1i=1.ivnum, 1
xeold(if,-i,isig,iant) = xest(ifi,isig,fant) yeold(if, - i, isig,iant) $=$ yest(if.isig.iant) xpold(if,o,isig,iant) $=\operatorname{xpred}(1 i, i s i g, i a n t)$
ypold(if,o, fsig,iant) $=y p r e d(i f, i s i g, i a n t)$

## =itirtsym-1 <br> <br> continue

 <br> <br> continue}tsym-1
imetde=1 then fixed delay in detection
***else if imetde=2 then do block detection (equivalent:!)
do 280 il: 2 , ivnum 1
cq(ij) $=c q(i i)+1000.0 d 0$
continue
endif
***set cq(1) total for packet $=0$
cq1 tot $=0.040$

## \section*{else} <br> VITERBI TYPE DETECTION

** Maximum likelyhood detection.
***expand each of the 49 vectors 4 ways
**i.e. the 4 lowest cost vectors are expanded and
*** their 4 louest cost vectors are stored.
***i.e. 16 p vectors in all.
do 221 il: 1 , 1 vnum, 1
***expand each $q$ vector in all 16 possible directions *** and calculate additional costs
do $2212 \mathrm{k}=0$, kmax, 1
dd $(k)=0.0 d 0$
2212 continue
if (imetco.eq. 1) then
***i.e. actual combining method
do $222 \mathrm{k}=0, \mathrm{kmax}, \mathrm{t}$
do 2221 iant =1, ianum, 1
rxpos(k,iant) $=0.0 \mathrm{~d} 0$
rypos(k,iant) $=0.0 \mathrm{~d} 0$
if 2221 isig=1,isinum,
ifiperfe.eq. 1$\}$ then
**"i.e. assume perfect estimation
rxpos(k,iant) $=$ rxpos(k,iant) +

* sxpos(k,isig)*x(i,isig,iant) - sypos(k,isig)*y(i,isig,iant)
* sxpos(k,isig)*y(i,isig,iant) + sypos(k,isig)*x(i,isig,iant)
elseif ifperfe.eq. 0) then

```
c
    rxpos(k,iant) = rxpos(k,iant) +
    * sxpos(k,isig)#xpred(li,isig,iant)
    rypos(k,iant)= rypos(k,iant) +
    *
2221 endif
continue (ant=1, {anum,
    dd(k)=dd(k)}
222
222 continue
        continue
                            elseif(imetco.eq. 2) then
                    ***i.e. Maximal Ratio Combinine
    ***Multiply by ampltude, co-phase and add.
    *** i.e. multiply by complex conjugate and add.
    mrx=0.0d0
    mry=0.0d0
    do 2213 iant=1,ianum,
            If (iperf'e.eq. 1' then estimation
    mrx = mrx + x(i,l,iant)mrx(i,iant) + y(i,l,iant)#ry(i,iant)
    mry = mry + x(i,l,lant)*ry(i,iant) - y(i,1,iant)*rx(i,iant)
    ymag = ymag + x(i,1,iant)**2 + y(i,1,iant)**2
            elseif(iperfe.eq. 0) then
            ***i.e. Actual estimation
        mrx=mrx +xpred(if,l,iant)#rx(i,iant) +ypred(il,l,iant)*ry(i,iant)
        mry=mry +xpred(il,l,jant)*ry(i,iant) -ypred(ii,i,iant)*rx(i,iant 
        ymag = ymag + xpred(if,1,lant)**2 + ypred(1f,1,iant)**2
            endif
2213 continue
    ***Calculate Maxm. Likelyhood costs
    do 2214 k=0,kmax,l
    dd(k) = (mrx - ymag*sxpos(k,1))**2 + (mry - ymag*sypos(k,i))**2
2214 continue
                                    elseif(smetco.eq. 3) then
                    ***.i.e. Selection diversity combining
c ***Use the antenna signai with the least fading at time t=iT
    ymag=0.0d0
    do 2216 iant=1,fanum,
            If (lperfe.eq. 1) then
                **i.e. assume perfect estimation
            sdymag = x(i,f.iant)**2 + y{i,l,iant)**2
            elseif(iperfe.eq. 0) then
            ***l.e. Actual estimation
        sdymag = xpred(if,1,iant)**2 + ypred(ii,1,iant)**2
                endif
    if (sdymag.ge. ymag) then
        ymag=sdymag
        isd = iant
    endif
c ***caiculate maximum likelihood costs
    do 2217 k=0,kmax,l
    **l.e. assume perfect.estimation
```

c
sypos(k,i)*ypred(ii, 1, isd)
* $\quad$ ypos(k, $=\operatorname{sxpos}(k, 1)$ mypred(ii, $1, i s d)$
$d d(k)=(r x(i, i s d)-r x p o s(k, i s d)) * * 2+(r y(i, i s d)-r y p o s(k, i s d)) * * 2$
2217 continue
$c \quad * * i . e . i m e t c o$ endif
c ***store 4 lowest cost $p$ vectors for each q vector
do $223 j=1$. ivnum,
cp(ii, $j)=1.0 \mathrm{~dB}$
do $224 \mathrm{k}=0, \mathrm{kmax}, 1$
if (dd(k).1t. cp(iif.j)) then
cp(ili,j) $=d d(k)$
kpmin $=k$
endif
continue
px(ifif,isig) $=\operatorname{sxpos}(k p m i n, i s i g)$
py(ii, J,isig) $=$ sypos(kpmin,isig)
2231 continue
dd(kpmin) $=1$. Odio
223 continue
221 continue
$c \quad$ ***add $q$ and $p$ costs to give 16 total costs
do 225 i $i=1$, ivnum,
do $226 \mathrm{j}=1$, ivnum, 1
cp(ii,j) $=c q(i i)+c p(i f, j)$
c $\quad$ encind smallest cost vector of the 16 p vectors
**rind smalle
cpmin $=1.0 \mathrm{~d} B$
do 227 ili=1, ivnum,
do $228 \mathrm{j}=1$, ivnum, 1
if (cp(ji,j). it. cpmin) then
cpmin $=\operatorname{cp(1i}, j)$
ijcmin a ii
jemin $=\mathrm{J}$
else
endif
228 continu
227 continue
**sdet(l-ivlen) is taken to be x(i-ivien) of the lowest cost ** vector $p$
do 2281 isig=1, isinum, 1
sxdet(i-ivien,isie) = qx(iicmin,-ivlen,isie)
sydet(i-ivlen,isig) $=q y(i f c m i n,-i v l e n, i s i g)$
continue
endif
if timetav.eq. 11 then
***(almost) discard all p vectors for which
*** x(i-ivien) ne. sdet(i-ivlen)
do 229 isigei, isinum, 1
do 229 ifi=1,ivnum,
if ( $q$ (ifi,-ivien,isif) . ne. qx(itcmin,-ivien,isif) or.

```
        (qy(if,-ivien,isig).ne. qxiticmin,-ivlen,isig))) then
        do 2291 j=1, ivnum,i
        cp(ii,j) = cp(ii,j) + 1.OdS
        continue
        endif
        continue
            endif
        **shift all p vectors alone to make 4 possible q vectors
        ** to choose from ready for the next sample
        ** AND STORE WITH THEM THEIR CORRESPONDING ESTIMATOR AND
    *** PREDICTOR VALUES.
    ***the data polnts q(if,-32) have now been detected so are
    *** shifted out, and the points q(if,-1) have yet to
    *** be decided so are set to zero for now
    do 241 i i=1,ivnum,1
                                    if (imetsv.eq. 1) then
    do 242 isig=1,isinum,
    do 242 j=-ivlen,-2.1
    qxold(il,j,igig)=qx(if,j+1,isig)}\mp@subsup{}{}{\circ
    qyold(ij,j,isig)=qy(ii,j+1,isig)
**if plotting estimator output set last ivlen values
** else only need to store last 2
**This varlable is ilplot
                    do 2421 iant=1,ianum,1
                    do 2421 isig=1,isinum,
    do 2422 j=-ilplot,-2,1
    xetem(ii,j,isig,iant) = xeold(ij,j+l,isig,iant 
    yetem(ii,j,isie,iant) = yeoldifi,j+1,isie,iant)
    xptem(if,j+1,isig,iant)= xpold(ii,j+2,isig,iant,
    yptem(ii,j+1,isig,iant)= ypold(if,j+2,isie,iant 
lom
    xetem(if,-i,isig,iant) = xest(if,isie,iant)
    yetem(if,-1,isig,lant) = yest(li,isig,iant)
    xptem(ii,o,isig,iant)=xpred(ii,isig,iant)
    yptem(if,o,isig,iant)= ypred(li,isig.iant)
    zixol(ii,,isig,iant) = zlx(ii,isig,iant)
    ziyol(ifi,isig,iant) = zly(if,isig,iant
    z2xol(if,isig,iant)=z2x(ij,isig,iant 
    z2yol(ii,isig.lant) = z2y(1i,isic,iant)
242!
continue
    **store the 4 lovest cost p vectors as the 4 next q vectorg
    **i.e. set q(if,-1) in order of costs. (ili=l corr.to lowest)
    ** and store correspondine costs
    do 230 li=1,ivnum,
    cq(ji) = 1.0d8
    do 23, jj=1,ivnum,
    do 232 j=1,ivnum,i
    ft (cp(jj,j) .lt.cq(il)) then
    cq(ij) = cp(jj,j)
    jjmin(ii) = j
    jmin(ii) = j
    endif
232 continue
231 continue
do 2311 tsig=1.1sinum,1
```

C
c
***This variable is ijplot
do 2351 iant=1, ianum, 1
do 2351 isie=1, isinum, 1
do $2352 \mathrm{j}=-\mathrm{ifplot},-1,1$
xeold(ii,j,isig,iant) = xetem(jjmin(if),j,isig,iant)

- yeold(il, j, isie,iant) $=\operatorname{yetem}(j \operatorname{jain}(i f), j, i s i e, i a n t)$
xpold(if,j+1,isie,iant) $=x p t e m(j j m i n(j i), j+1$, isig.iant)
ypold(il, $j+1$, isig,iant $=y p t e m(j j m i n(i i), j+1$, isig,iant)
2352 continue
xest(ii, isig,iant) $=x \operatorname{tem}(j J m i n(1 i),-1, i s i g, i a n t)$
yest(if,isig,iant) $=\operatorname{yetem}(j j m i n(i f),-1, i s i g, i a n t)$
xpredii,isig,iant $=x p t e m(j J m i n(i i), 0, i s i g, i a n t)$
ypredifisig,iant) $=y p t e m(j \min (i f), 0, i s i g, i a n t)$

z1y(il, isig, iant) = 21yol(jjmintiil,isigejant)
z2x(ii,isig,iant) $=z 2 x o l(j j m i n(i i), j s i g, i a n t)$
z2y(if.isie.iant) $=22 y o l(j j m i n(i f), i s i g, i a n t)$
*** 4 lowest costors and their costs have now been
** stored ready for the next sample.
***subtract the smallest cost irom all costs so that the lowest
*** cost is always zero.
cqttot $=c q 1 t o t+c q(1)$
do 233 ifi=2, ivnum,i
$c q(i i)=c q(i i)-c q(1)$
continue
$\mathrm{cq}(1)=0.0 \mathrm{~d} 0$
endif
$q \times(1 i,-1, i s t g)=p x(j j m i n(i f), j m i n(i i), i s i g$
qy(ii,-f,isie) $=p y(j j m i n(i f), j m i n(i i), i s i e)$
cp(jjmin(ij).jmin(ii)) $=1.0 d 10$
continue
do 2302 ii=1,ivnum,
***match the rest of the $q$ vectors with these q(ii,-i) from
** the 4 possible qoldifi,j) vectors
* AND MATCH THE CORRESPONDING ESTIMATORS AND PREDICTORS
do 234 ili= , ivnum,

$$
\text { if (imetsv.eq. } 1 \text { ) then }
$$

do 235 isigel,isinum,
quifi,j,isig) $=q \times o l d(j) m i n(i i), j, i s i g)$
qy(if,j,isig) $=$ qyold(jjmin(if),j,isig
continue
elseif imetsv.eq. 21 then
lstor $=$ modi.ivien) - ivien
aromili,istorj = JJinitil
ax(ii)
qx(if,istor,isig) $=q x(1,-1, i s i g)$
stor,isig) $=$ qy(ili,-I.isig
continue
endi
. . Channel estimator do 320 lant $=1$, ianum, 1
do 321 if=1, ivnum, 1
it limetes.eq. 0) then
***unbiased estimator for 1 signa
(modsch.eq.'16-pt QAM') then
if (iperfd.eq. 0) then
(1) ax(ii,-1,1)+qy(ii,-1,1)*qy(ii, -1,1) elself(iperfd.eq. 1$)$ then
$\mathrm{b}=1.0 \mathrm{~d} 0 /(\mathrm{sx}(\mathrm{i}, 1) * \sin (\mathrm{i}, \mathrm{l})+\operatorname{sy}(\mathrm{i}, 1) * \mathrm{sy}(1,1))$ endif
else
$=0.5 \mathrm{~d} 0$
if
if (iperfd.eq. 0) then
xest(ii, $1, i a n t)=b(q x(i f,-1,1) \neq r x(i, j a n t)+q y(i f,-1,1) \# r y(i, i a n t))$
yest(ii, $1, \operatorname{iant})=b *\left(q x\left(i i_{1}-1,1\right) * r y(1, i a n t)-q y(i i,-1, i) * r x(i, i a n t)\right)$ elself(iperfd.eq. 11 then

yest(if,i,iant) $=b *(\operatorname{sx}(i, 1) * r y(i, i a n t)-s y(i, i) * r x(i, i a n t))$ endif
else
***gradient estimator
$r$ xest $=0.0 \mathrm{~d} 0$
-
do 3211 isigel.isinum,
if liperfd.eq. O) then
if (imetes.eq.1) then
rxest $=r x e s t+$
*
qx(ij,-1,igig)*xest.(ii,isig,iant) -
ryest $=$ ryest
*
qx(if, -1,isig)*yest(if,isig,iant)
qy(iif, -1,isig)wxest(ii, isig,iant)
elseif(imetes.eq. 2 ) then
rxest $=$ rxest
*
qx(ii, - $1, i s i g) * x p r e d(i f, i s i g, f a n t)-$
qy(il,-1,isig)*ypred(il,isig,iant)
*
qxifi, -1, isig)\#ypred(li,isig,iant) + qy(ii, $-1, i s i g)=x p r e d(i i, i s i g, i a n t)$ endif
elself(iperfd.eq. 1) then
rxest $=$ rxest

* sx(i,isig)*xest(ifisig.iant) -
sy(i,isig)\#yest(ii,isig,iant)
ryest $=$ ryest
sxif,isig)*yest(ti,isig,iant) + sy(i, isig)*xest(ii,isig,iant)
rxest $=$ rxest +
$*$
y(i,isig)*ypred (ii isig lant)
ryest $=$ ryest sx(i,fsig)"ypred(ii,isig,iant) + sy(i,isig)*xpred(ii,isig,iant) endif
continue
ex $=r x(f, f a n t)-r x e s t$
ey $=$ ry(i,iant) -ryest
do 3212 isig=1,isinum, 1 if (iperfd.eq. o) then
$b=b t e m p /(q \times(i f,-1, i g i g) * * 2+q y(i f,-1, i s i g) * k ?$ endit
if (imetes.eq. 1 ) then
xest(if.isig,iant) $=$ xest(ii,isig,iant)
* beg(ex*qx(ii,-1,isig) +ey*qy(ii, -1,isig)
yest(if,isig,iant) =yest(if,isig,iant) +
b*(-ex*qy(if, $-1, i s i g)+$ ey*qx(ii, -1 ,isig))
elseifimetes.eq. 2) then

yest(ii,jsie,iant) = ypred(ii,isig,iant) +
$b *(-e x \neq q y(i f,-1, i s i g)+e y * q x(i),-1, i s i g))$
endit
elseif(iperfd.eq. 11 then
if (imetb.eq. 1) then
 endif
if (imetes.eq. 1 ) then
xest(il,isig.iant) =xestifi,isig,iant)
b*( ex*sx(i,isig) + ey*sy(i,isig)
yest(ii,isig,iant) $=y e s t(i f i, i s i g, i a n t) ~$
$b *(-e x * s y(i, i s i g) ~+~ e y * s x(i, i s i g)) ~$
(-ex*sy(i,isig) *ey*sx(i;isig)
xest(il,isig,iant) = xpred(il,isig,iant) +
b*(exisx(i,isig) + ey*sy(i,isig))
yest(if,isig,iant) $=$ ypred(ii,isig,iant) +
b*(-ex*sy(i,isig) +ey*sx(i,isig)


## endif

3212 continue
endif
$c \quad$ **imetes endif::
c ...PREDICTOR
do 3213 isigat.isinum,
ezx = xest(if,isig,iant) - xpred(if,isif,iant) ezy = yest(il,isie,iant) - ypred(ii,isig,iant) if liordp.eq. -il then
c ...no prediction
xpred(if,isig,iant) $=x e s t(i f, i s i g, i a n t)$
ypred(il.isig,iant) =yest(ii,isig,iant)
c ...degree 1
elseif(iordp.eq.i) then
ix(iq,isig,iant) $=21 x(i 1, i s i g$, thant + thiezx zly(if,isig,iant) $=z 1 y(1 i, j s i g, i a n t)+$ thl*ezy xpred(ii,isig,iant) $=x p r e d(i i, i s i g, i a n t)+z i x(i i, i s i g, i a n t)+$ th2*ezx
ypred(if,isig,iant) = ypred(if,isig,iant) + ziy(if,isig,iant) +
th2*ezy

22x(if,isig.fant) = 22x(if,isig.iant) thy*ezx
z2y(ifi,isig,iant) $=22 y(i f, i s i g, i a n t)+$ th3eezy
z1x(if,isig,iant) = zix(ii,isie,iant) + 2.0do*z2x(ii,isig,iant) +
*2ycil thy*ezx
*

```
                                    z2x(ifidsig,lant) + th5*ezx
    ypred(if,isig,iant) = ypred(ifi,isig.iant) t ziy(ifi,isig,iant) -
                                    z2y(il,isig,iant) + th5*ezy
                                    else
                                    endif
3213 continue
321 continue
do 323 isig=1,isinum,1
    xeplo(i-ifplot,isig,iant) = xeold(l,-ifplot,isig,iant)
    yeplo(i-ifplot,isig,iant) = yeold(1,-if(iot,isig,iant 
    xpplo(i-iiplot+1,isig,iant) = xpold(1,-iiplot+1,isig,iant,
    ypplo(i-ifplot+1,isig,iant)=ypold(i,-iiplot+1,isig,iant)
    323
    320
c
```




```
        signal(s) with Gray coding only
        AND --" -- Differential and Gray codine.
        (And output error files --(if ief=1)
        and output soft decision info files ..;
                --if (ief=1).and.ifiplot=ivlen)
    cocococococococococococcococococococecocococococcococococ
    **Set yplot()=y, if iperfe=1 and ief=1 and ifplot=ivlen
            if (fiperfe.eq., ),and.(ief.eq. ), and.(iliplot.eq.ivien)) then
    PRINT*,'Set ypplo(i)=y(i). ror perfect estimation'
    do 4989 iant=1,ianum,1
    do 4989 la|,i,laim,
    do 4989 i=0 igampl i
    xpplo(i,isizianti
    =x(i,isig,iant)
    ypplo(i,isig,iant) = y(i,isie,iant)
    989 continue
            endif info gubroutin
    do 4991 isig=1,isinum,1
    Print
    Print
    Print*.: SIGNAL ',isie sudet sydet xpplo ypplo.
    Print*, i sx sy
    if (i.eq. 3) i=isampl-1
    Print 9491,i,sx(i,isig),sy(i,isig), sxdet(i,isig), gydet(i,isif)
    9491 formatci6,12x,2f9.t 4x, 2f9, (1)
    do }4995\mathrm{ iant=1, ianum,1
    if(tant.eq.l) then
    Print.:ANTENNA
    Print*,
    endif
    if(iant.eq.2) then
    Print*.'ANTENNA
    endif
    Print 9002, xpplo(i,isig,iant), ypplo(i,isig,iant)
        continue
    4993 continue
4991 continue
    if (modsch.eq.'OPSK') then
    Printmodscheq.
    Print*,' SUBROUTINE sbeqP'
    call sbeqpisinum,ianum,isampl,
    Print.a, RFTIIRNFD ghean'
```


## SFADE.FORTRAN <br> ****


subroutine sfade (isampl,jspsym, inter, isinum, ianum,
subroutine afade(i
$\times f a d, y f a d)$
c
Onitiallse
ble precision
g05ddf, wwx, $\quad$ x(-400:7000), dcealn
нну, wy (-400:7000)
double precision why, wy
double precision yfadi-400:6050,2 2
ouble preciaion t10:400) fm
double precision $t(0: 400)$, $f$
integer isampl,jgpgym,inter,iterms, lanum,lant, iginum,isie
integer iosbli, itaps
** terms of 600 corresponds to 1 sec
iterms=(isampl/inter) +50
Print., iterms.' samples BEFORE interpolation'
Print*, Isinum, slenals in the same channel with'
Print., fanum,' recelving antennae.
degain=dsqrt (0.5do)
**if fosbli=1 then output fading files, else iosbli=0 iosbl1 $=0$
**Random no. generator cbf-gauss
PRINT",' random seed integer for fading.
call gosccf
RINT*' seed integer 63 for fading'
all g05cbf(63)
-- use seed 63 for estimation curves
.. FORM IN-PHASE AND QUADRATURE COMPONENTS OF THE FADINC
.. BEFORE INTERPOLATION
...1.E. ONE FADING CHANNEL. BETWEEN EACH MOBILE AND EACH ANTENNA. (i.e, ianum*isinum fading channels)
do 1 isieli,isinum, 1
do 2 iant $=1$, ianum,
** read in though THIS FIR-FILTER..
if (isig.eq. 1).and. (iant.eq. 1)) then
open(unit=7, file='omvermon $80^{\circ}$. (rorm='formatted')
readi7,*) fm, itaps
read(7,") (i,ii), i=0,itaps-1,1)
close (7)
PRINT*,'cut-off freq. = ',fm,' Hz .
endif
do $99 \mathrm{i}=-$ (itaps-1), iterms,
$w \times(1)=0.0 \mathrm{do}$
$x f a d(i, i s i g$, iant) $=0.0 \mathrm{~d} 0$
vy(i) $=0.0 \mathrm{do}$
wy(l)=0.0d
yfad(i.isig,iant) $=0.0 d 0$
..PASS NOISE THROUGH FILTERS
do 110 i=-(1taps-1), iterms.
$w_{w x}=g 05 d d f(0.0 d 0,+1.0 d 0)$
wwy $=$ g05ddf $(0.0 \mathrm{~d} 0,+1.0 \mathrm{do})$
$w x(i)=w u x * d c g a i n$
wy(i) =wwydcgain

## *SBUCHAN FORTRAN

****************
 subroutine sbbchantigampl,ipacl, irtsym,inter,snrdb

* modsch,suar,isinum,ianum,rxi,ryi,sxi,syi,xi,yi)


## INITIAL.1SE

double preciaion sxi(0:60000,2), syito:60000,2)
double precision xi $0: 60000,2,21$, yi $(0: 60000,2,2)$
double precision rxi(0:60000,2), ryi( $0: 60000,2$ )
integer ibpg,ibpsym, sa(4), sb(2), sbold(2), sbost (2,2)
double precision qai6x(0:1,0:1,0:1,0:1), qai6y(0:1,0:1, 0:1, 0:1)
double precision sx(-402:50900,2), sy(-402:50900,2)
double precision bsx(-402:50900,2), bsy(-402:50900,2)
double precision $\mathrm{H}, \mathrm{g} 05 \mathrm{daf}$, g05ddf
double precision t(0;200)
double precision blsx(-402:50900,2), blsyi-402:50900,2)
double precision xfad(-400:6050,2,2), yfad(-400:6050,2,2
integer ifadst(2,2), iyst(2,2)
double precision $x(-1160: 50900,2,2), y(-1160: 50900,2,2)$ double precision $a(6,320)$
integer num, kmax, inter, konst, isubs
double precision fblsx(-402:50900,2), fblsy(-402:50900,2)
double precision wx(-402:50900,2), wy(-402:50900,2)
double precision rpowx(2), rроwy(2), wpowx, wpowy
double precision spowx(2),spowy(2), ypowx(2),ypowy(2)
double precision snrdb, wsd, snrsir
double precision nftolsx(-402:50900.2), nfblsy(-402:50900,2)
double precision rx(-402:50900,2), ry(-402:50900,2)
integer $i, j, k, n, i s a m p l, i s p s y m, ~ i t a p s$
integer jsampl,jspsym
integer ipacl, irtsym, irt
integer iant,ianum, isig.tsinum
character*9 modsch
double precision suar
Integer iobli
Integer idenc
double precision checkx(0:60000,2), checky(0:60000,2)
** raust set fbpsym=no. of bits per symbo
( $m o d s c h . e q .{ }^{\prime}$ QPSK') then
ibpsym = 2
elseif(modsch.eq.'16-pt QAM') then
else
PRINT*,' CANNOT GENERATE DATA FOR THIS MOD. SCH.'
stop
endif
**if
if tidenc.eq. 0 ) then
PRINT*.' COHERENT ENCODING
elseif(idenc.eq. () then
PRINT*,'DIFFERENTIAL ENCODING'
else
PRINT*,' idenc = ERROR
stop
1spsym=

1start=0
itaps = 1
**if jpgfil=1 then do pulseshaping filtering (root-raised-cos).
jpsfil=1
If (jpsfil.eq. 1) then
jspsym=4
else
jspsym=1
endif
irt $=0$
***if lobli=0 then read in rading arrays from stored files
*** elself iobli=1 then generate fading in subroutine sfade
*** elseif iobli=2 then set up fading files manually (constant)
iobli=2
if (ioblif.eq. O) then
PRINT*, Read in fading arrays from gtored files'
elseif(iobli.eq. 1) then
PRINT*,'Generate fading in subroutine sfade'
elseif(lobli.eq. 2) then
PRINT*, Set up fading files manually (constant).
elseif((iobli.1t. 0).or. (iobli.gt. 2)) then
PRINT*,'INVALID iobli
stop
**store output files if iosbbc:1
10sbbc =0
***only works for no retraining or 4 to 12 retraining symbols
if ((iftsym.ge.1).and.(irtsym.le.3)).or. (irtsym.ge.l3)) then
PRINT*,' NOT ENOUGH RETRAINING SYMBOLS (or too many)'
stop
endif
If (jpsfil.eq. 1 ) then
**read in Tx/Rx filter tap gains
*"****FIR root-raised-cos fifter
*FAlso must allow for delay through these filters.
*** i.e. centre of first symbol must arrive after receiver filter *** at time t=0 (istart = istart - total delay)
if (jspsym.eq. 2) then
opencunit=7, file='rc_s2', form='formatted'
elseif(jspsym.eq. 4) then
pentunit=7, file='rc_s4', form='formatted'
elseif(Jspsym.eq. B) then
opencundt=7, file='rc_s8', form='formatted')
elseif'(jopsym.eq.16) then
openiunit=7, flle='rc_s16', form='formatted')
else
PRINT*, Root-Raised-Cos filter coeffs not available
stop
endif
read (7,*) itaps
read (7,*) (t(i), $i=0,1$ taps-1,1)
close (7)
**total Tx Rx filtering delay is itaps-1
start=istart-(itaps-1)
Print*,itaps,' taps in Tx (and Rx) filter
endif
i.e. read in $T x / R x$ filter taps endif
**initiallse all differential coding bits to zero

## do 73 isigei,isinum,

sbost $(1, i s i g)=0$
sbost(2
continue
***set up bit mapping for coherent/differentially and
** Gray coded 16-pt QAM signal
qa $16 \times(0,0,0,0)=-3.0 \mathrm{~d} 0$
qa $16 y(0,0,0,0)=-3.0 \mathrm{~d} 0$
qa $16 \times(0,0,0,1)=-1.0 \mathrm{~d} 0$
qai $6 x(0,0,0,1)=-1.0 d 0$
qa $16 y(0,0,0,1)=-3.0 d 0$ qa $6 \times(0,0,1,0)=-3.0 d 0$ qai $6 x(0,0,1,0)=-3.0 d 0$
qai $6 y(0,0,1,0)=-1.0 d 0$ qa $16 \mathrm{y}(0,0,1,0)=-1.0 \mathrm{~d} 0$
qa $16 \times(0,0,1,1)=-1.0 \mathrm{~d} 0$ qa $16 x(0,0,1,1)=-1.0 \mathrm{do}$
qa $16 y(0,0,1,1)=-1.0 \mathrm{~d} 0$ qa $6 y(0,0,1,1)=-1.0 \mathrm{~d} 0$
qa $16 \times(0,1,0,0)=3.0 \mathrm{~d} 0$ qa $16 \times(0,1,0,0)=3.0 \mathrm{~d} 0$
qa $16 \mathrm{y}(0,1,0,0)=-3.0 \mathrm{~d} 0$ if (idenc.eq. 0 ) then ***coherent.
qa $16 \times(0,1,0,1)=1.0 \mathrm{~d} 0$
qai $6 x(0,1,0,1)=-1.0 d 0$
qa $16 y(0,1,0,1)=-3.0 d 0$
qa $16 \times(0,1,1,0)=3.0 \mathrm{~d} 0$
qa $16 \mathrm{y}(0,1,1,0)=-1.0 \mathrm{~d} 0$
else
***differentlal
qai $6 \times(0,1,0,1)=3.0 \mathrm{~d} 0$ qa $16 y(0,1,0,1)=-1.0 d 0$ qa $16 \times(0,1,1,0)=1.0 \mathrm{~d} 0$ qa $16 \mathrm{y}(0,1,1,0)=-3.0 \mathrm{~d} 0$ endif
qai $6 \times(0,1,1,1)=1.0 \mathrm{~d} 0$ qa $16 y(0,1,1,1)=-1.0 \mathrm{~d} 0$ qa $16 \times(1,0 ; 0,0)=-3.0 \mathrm{~d} 0$ qa16y(1,0,0,0) $=3.0 \mathrm{~d} 0$

## If lidenc.e

qa $16 \times(1,0,0,1)=-1.0 \mathrm{do}$ qa16y(1,0,0,1) $=3.0 \mathrm{~d} 0$ qa $16 \times(1,0,1,0)=-3.0 \mathrm{do}$ qaity $(1,0,1,0)=1.0 \mathrm{~d} 0$ elge
***differential
qa $16 \times(1,0,0,1)=-3.0 \mathrm{~d} 0$ qa16y(1,0,0,1) $=1.0 \mathrm{do}$ qa16x(1,0,1,0) $=-1.0 \mathrm{do}$ qal6y(1,0,1,0) $=3.0 \mathrm{do}$ endif
qa16x(1,0,1,1) $=-1.0 \mathrm{do}$ qa16y(1,0.1.1) $=1.0 \mathrm{do}$ qa $16 \times(1,1,0,0)=3.0 \mathrm{do}$ qa $16 y(1,1,0,0)=3.0 \mathrm{~d} 0$ qa $16 \times(1,1,0,1)=1.0 \mathrm{do}$ qaify(1.1,0,1) $=3.0 \mathrm{do}$ qa $16 \times(1,1,1,0)=3.0 \mathrm{do}$ qaity (1,1,1,0) $=1.0 \mathrm{do}$ qa $16 \times(1,1,1,1)=1.0 \mathrm{do}$ qa16y(1.1.1.1) $=1.0 \mathrm{dO}$ do $198 \quad 1=0,1,1$
do $198 \quad \mathrm{j}=0,1$.
do $198 \mathrm{k}=0,1$,
do $19 \mathrm{H} \quad 1=0,1$.
anitialise all data symbols to zero
do 199 isig=i, isinum,
sx(1) $=-(2 *(1$ taps-1)), isampl,
sx(i,isig) $=0.0 \mathrm{~d} 0$
sy(i.isig) $=0.0 \mathrm{dO}$
***calculate gtand. dev. for noise to give required snrdb wsd = dsqrt(svar*0.5d0*10**(-0.1d0*snrdb)
***read in Rayleigh fading components
sampled every $1 / 600$ secs ****
do 99, READ in radi
do 99 isig=i isinum
if ((iant.eq.1), and.(isig.eq. 1)) then
open(unit=7, file='xfadal', form='formatted')
open(unit=B, file='yfadal', form='formatted')
elseif(iiant.eq. 1).and.(isig.eq. 2)) then
opencunit = \%, file='xfada2', form='formatted')
opencunit=t, file='yfadaz', form='formatted')
elself(ifant.eq. 2).and. (isig.eq. 1)) then
opencunit=7, flle='xfadbi', form='formatted',
open(unit=8, file='yfadbl', form='formatted')

- elself(ifant.eq. 2).and.(1sig.eq. 2)) then
open(unit=7, file='xf'adb2', form='formatted')
open(unit=8, file='yfadb2', form='formatted')
else
PRINT, 'fading arrays NOT Avallable
stop
endif
read (\%,*) (xfad(i,isig,iant), $i=1$, isampl/inter $+20,1)$
read(8,*) (yfad(i,isig,iant), $i=1$, isampl/inter+20,1)
close(7)
close(8)
continue
elseif(iobli.eq. 1) then
Print.,'SUBROUTINE sfade
call sfade (isampl,jspsym,inter, isinum,ianum,
xfad,yfad)
Print*,'RETURNED sfade
do 8日81 lant=1, ianum,
if (iant.eq. 1) Print*.'Antenna a'
if Ciant.eq. 2) Print*,'Antenna b'

do 8883 $1=1$, (isampl/inter) $+50,1$
if (i.eq. 3) $i=(i s a m p l / f n t e r)+50-1$
do 88B5 isig=1, isinum,
Print*,'sicinal $\cdot$,isig
Print 8001, i, xfadifisie,iant),yfadifisig,iant)
continue
B881 cantinue
elseiftiobli.eq. 2) then
Print*,'set up fading files manually tconstant:! ’
do 991 lant=1,ianum,1
do $991 \mathrm{lstg}=1$, isinum,
do 991 i=1, (isampl/inter +20),
if (isig.eq. i) then
$x f$ ad (f,isip.iant) $=1.0$ OHO

c**must set ifadst=0, ifadst is the number of the rad array
andif
** that is read in next into the chonnel arrayg
do 98 iant=1, ianum, 1
o 98 isig=1,isinum,
ifadstisie,iant) $=0$
continue
**read in array a for interpolation
** INTERPOLATION
**Data signal 12000 symbols per sec
** therefore, if sampling once per symbol
** must interpolate each sample 20 times
*** i.e. $20 * 600=12000$
nter*jspsym
PRINT* $\cdot 600$ FADING SAMPLES PER SEC, INTERPOLATED •, kmax, $\cdot$ TIMES
PRINT*, i.e. ', 600*inter, symbolg per gec with'
PRINT*,' $\quad$ i.e. ', $600 *$ inter,' symbolg per sec
PRINT*,'
c
if (kmax matrix a
opencunlta7, file='interalo', form='formatted'
elseif(kmax .eq. 20) then
open(unlt=7, file='intera20', form='formatted')
elseif(kmax.eq. 40) then
opentunit=7, file='Intera40', form='formatted'
elself(kmax.eq. 80) then
opencunit=7, file='interabo', form='formatted'
elseif(kmax.eq. 160) then
opencunit =7, file='interal60', form='formatted'
elseif (kmax.eq. 320) then
opentunit=7, file='Intera320', form='formatted'
PRINT*. 'INTERPOLATING matrix not available'
stop
endif
ead(7.") num
read (7,*) ( $(a(n, k), n=1, n u m, 1), k=1, k m a x, 1$
close(7) integer for data and noise is $B$, (for fading 9)
PRINT*,' random seed integer for data and noise.
call goscer
PRINT*.' seed integer 89 for data and noise'
call go5cbr(89)
-- use seed 89 for estimation curves
. RECEIVED SAMPLES ARE CALCULATED IN (blnum=jspsym/ispsym
do 21 iblock $=1$, iblnum.
c ...generate the random, bandlimited signal for each user do ll isig=1,isinum, 1
***reset sbold's for each signal in each block sbold(1) $=$ sbost(t,isig)
sbold(2) $=$ sbost(2,isig

```
c ...GENERATE RANDOM SIGNAL
Printe,'generate Random ', modsch, sicnal , isig
if (idenceq 1) then
Print., with differential. ENCODING and GRAY CODED BIT MAPPING' else
Print*.' with COHERENT ENCODING and GRAY CODED BIT MAPPING.
endif
do 100 i=istart, isampl, jspsym
if (iblock.ne. 1 ).and.(1.le. 0)) goto 100
c Either set up fixed sequences for retraining
c or generate a random palr of symbols
or generate a random palr
iconst =isampl*(iblock-1)
if ( ( \(\bmod (i \operatorname{const}+1+(i t a p s-1), i p a c l \# j s p s y m) . e q\). 0\()\).and
* (irtsym .ne. 0)) .or. (irt.ne. 0)) then
**"retraining symbol no. irt

*** -3-j3 for \(16-\mathrm{pt}\) QAM
***s2 rotates clockwise starting from \(-1+j\) ror QPSK
***
if (isig.eq. 1\()\) then
if (irt.eq.o).or.(irt.eq.4).or.(irt.eq. 8 )) then
sx(i,isig) \(=-1\).Odo
sx(i,isig) \(=-1\). odo
sy(i,isig) \(=-1\). odo
elseif(irt.eq.i).or.(irt.eq.5).or.(irt.eq.9)) then \(\mathbf{s x}(\mathrm{i}, \mathrm{isig})=+1\). Odo
sy(i,isig) \(=-1.0 \mathrm{do}\)
elseif((irt.eq.2).or.(irt.eq.6).or.(irt.eq.10)) then \(\operatorname{sx}(1, \mathrm{isig})=+1.0 d 0\)
sy(i,isig) \(=+1.0 d 0\)
elseff((irt.eq. 3 ).or.(irt.eq.7).or.(irt.eq. 11)) then
\(\mathbf{s x}(\mathrm{i}, \mathrm{lsig})=-1.0 \mathrm{~d} 0\)
sy(i.isig) \(=+1.0 \mathrm{dO}\)
endif
elseififigieq. 2) then
if (irt.eq.0).or.(irt.eq.4).or.(irt.eq.8)) then sx(i,isig) \(=-1.0 \mathrm{~d} 0\)
sy(ifisig) = +1.0d0 sx(i,isig) \(=+1\).0do
sy(i,isig) \(=+1.0 d 0\)
elseif(irt.eq.2).or.(irt.eq.6).or.(irt.eq. 10)) then ex(1,isig) \(=+1.000\)
sy(i.isig) \(=-1.0 \mathrm{do}\)
elseff(irt.eq.3).or.(irt.eq.7).or.(irt.eq.11)) then
sx(i.isig) \(=-1.0 \mathrm{~d} 0\)
sy(i,isie) \(=-1.0 \mathrm{~d} 0\)
endif
If (modsch.eq.'16-pt QAM') then
\(\operatorname{sx}(\mathrm{i}, \mathrm{isig})=\mathrm{sx}(\mathrm{i}, \mathrm{isig})(3.0 \mathrm{dO}\)
sy(i,isig) \(=s y(i, i s i g) * 3.0 d 0\)
endif
if (irt .eq.(irtsym-1)) then
irt \(=0\)
sbold \((1)=0\)
sbold(2) \(=0\)
else
endif
c \(\quad\) **Random symbols, NOT retraining
do 112 ibps \(=1\), ibpsym,
\(\omega=0.0 \mathrm{dO}\)
\(w=0.05 \mathrm{dar}(-1.0 \mathrm{do}+1.0 \mathrm{do}\)
if (y it 0.0do) then
sa(ibps) \(=0\)
else
sa(ibes) =
endif
***DIFFERENTIALLY ENCODE RANDOM DATA BITS
if ( i bold(1).eq. 0).and. (sbold (2).eq. 0)) then
\(\operatorname{sb}(1)=s a(1)\)
elself((sbold (1).eq. 0). and. (sbold(2).eq. 1)) then
if (isati).eq. o).and. (sa(2).eq. 0)) then
sb(1) = 0
sb(1) \(=0\)
\(s b(2)=1\)
elseif(isa(1).eq. 0).and.(sa(2).eq. 1)) then sb(1) \(=1\)
sb(2) = \(\quad\) (f).eq. 1).and.(sa(2).eq. 1)) then
elselficsa \(\operatorname{sb}(1)=1\)
\(\operatorname{sb}(2)=0\)
else
sb(1) \(=0\)
endif
elseif((sbold(1).eq. 1).and.(sbold(2).eq. 1)) then
if (lsa(1).eq. 0).and. (sa(2).eq. 0)) then
sb(1) = 1
elseif(isa(1).eq. 0). and.(sa(2).eq. 1)) then
gbit) \(=1\)
iseif(isa(1).eq. 1).and.tsa(2).eq. 1)) then
elselflisa
sbil) \(=0\)
\(s b(1)=0\)
\(s b(2)=0\)
else
sb(1) \(=0\)
sbif
else
if (sa(t).eq. 0).and.tsa(2).eq. 0)) then \(\mathrm{sb}(1)=1\)
\(\mathrm{sb}(2)=0\)
elseif((sa(1).eq. 0).and.(sa(2).eq. 1)) then
sb(1) \(=0\)
\(\operatorname{sb}(2)=0\)
elseif(isa(1).eq. 1 ).and. (sa(2).eq. il) then
\(s b(1)=0\)
sb(2) =
else
\(\operatorname{sb}(1)=1\)
endif
end if
do 181 i=istart,isampl.
blsx(i,isig) \(=s \times(i, i s i g)\)
blsy(i.isige z sy(i,isig)
bsx(i,isig) \(=s \times(1, i s i g)\)
bsy(i,isig) \(=s y(i, i s i g)\)

\section*{else}
.. CONVOLUTION OF DATA SIGNAL HITH TX FILTER Print", 'CONVOLUTION OF ', modsch.' SIGNAL WITH Tx Filter do \(2991=-(2 *(\) (itaps-1)),isampl,
blsx(1,isig) \(=0.0 \mathrm{dO}\)
blsy(i.isig) \(=0.0 \mathrm{do}\)
do 211 i=istart,isampl.
do \(212 \mathrm{~m}=0\), itaps-1,1
blsx(i,isig) = blsx(i,isig)
blsy(i,isig) \(=\) blsy(i.isig)
continue
**COHERENT ENCODING OF RANDOM DATA BITS
sb(1) = sa(1)
\#**i.e. idenc endif
***do bit mapping DIFFERENTIAL AND GRAY CODED
if (modsch.eq.' QPSK ') then
if (sb(2).eq. 0) then
sx(i.isig) \(=-1.0 \mathrm{do}\)
else
sx(i,isig) = 1.0d0
endif
if (sb(1).eq. 0) then
sy(1.isig) \(=-1.0 \mathrm{do}\)
else
endif
elseif (modsch.eq.' 16 -pt QAM') then
sx(i,isig) \(=q a 16 x(s b(1), s b(2), s a(3), s a(4))\)
sy(f,isig) \(=q a 16 y(s b(1), s b(2), s a(3), s a(1)\)
else
PRINT*, ' CANNOT BIT MAP this mod. gch.'
stop
***set up sbold's for next sample
sbold(1) \(=s b(1)\)
endif
***i.e. retraining or random endif
***allow for jspsym samples per symbol
do \(120 \mathrm{j}=1\), (jepsym-1), 1
\(\left.\begin{array}{ll}s \times(i+j, i s i g\end{array}\right)=0.0 d 0\)
continue
continue
***save sbold of this signal for next block
sbost(1,isig) \(=\) sbold(1
sbost(2,isig) \(=\) sbold(2
```

continue
continue generate rancomi fandlimited s(i) for each isig
c ...REPEAT FOR 2 SIGNAIS THEN FOR 2 ANTENNAE
do 1 lant=1, lanum,1
do 2 isie=1.isinum,
Print B010
Print*'', antenna ',iant.' signal ',isie
c

```

```

c
c signal twice. i.e. Once for each antenna channel
cce
Do Rayleigh rading, nolse and receiver filtering on this SAME

```

```

    ***important to initialise fading arrays to zero
    konst=(num/2)*kmax
    do 391 =istart-konst,isampl+konst,
        x(i,isig,iant) = 0.0do
        y(i,isie,iant) = 0.0d0
        do }392\textrm{i}=1\mathrm{ istart-konst,1sampl+konst,kmax
        ifadst(isig,iant)=ifadst(isie,iant)+1
    ifadst(isig,iant
    yst(isig,iant)=
        x(i,isie,iant) = xfad(ifadst(isig,iant),isig,iant)
        y(i,isig,iant) = yfad(ifadst(isig,iant),isie,iant)
        continue
    ***set up ifadstt.....) ready for next block
    ifadst(isig,lant)=ifadst(isig,iant)-(num+1)
    *:*INTERPOLATE each noise sample (1/600 gec) kmax times
    #*:INTERPOLATE each nolse sample (1/600 gec) kmax 
    do 312 k=2,kmax,
    isubs = i-(num/2)*kmax +k-1
    do 313 n=1,num,1
    x(isubs,isig,iant) = x(igubs,isig,iant) *
    * x(i-(n-1)*kmax,isig,tant)*a(n,k)
    y(isubs,isig,iant) = y(isubs,isig,iant) +
    * y(i-(n-1)*kmax,isig,iant)*a(n,k)
    *
    continue
        if (iobli.eq. 2) then (isubs,isig,iant) = x(i-(num/2)#kmax,isig,iant)
        y(isubs,isig,iant ) = x(i-(num/2)*kmax,isig,iant )
        endif
    continue
    ***Torm quadrature compts. of faded bandlimited Data signal
    do 321 l=istart,isampl,1
    fblsx(i,isig)= blsx(i,isig)*x(l,isig,iant)
    * blsy(i,isle)*y(i,isig,iant)
    fblsy(i,isig)= blsx(i,isig)*y(j,isig,iant) t
    1' continue
    c}2\mathrm{ continue
c 'i.e. do for all signals in this antenna channel
do t19! f=igtart,isampl.l
do 49{ I=igtart,1samp
wx(i,iant) = 0.0dO
wy(i,iant) = 0.0d0
REMEMERE: : calculate average signal power per Hit
***total noise power
**total no
wpowx $=0.0 \mathrm{do}$
wpowx $=0$. 0 do
do $421 \quad i=1+i$ start/2,isampl+istart/2,1
$\begin{aligned} & \text { do } \\ & \text { whowx }=\text { wpowx }+ \text { wx (i,iant)wwx(i,iant }\end{aligned}$
wpowx = wowx + wx (i,iant)ww(i,iant)
wpowy $=$ wpowy $+w y(i, i a n t) * w y(i, i a n t)$
**total signal power, channel power and received signal
*** power for EACH signal
*** power for EACH sig
do 422 isig=i, isinum,l
do 422 isig=1, isinum
spowx(isig) $=0.0 \mathrm{~d} 0$
spowy(isie) $=0.0 \mathrm{~d} 0$
ypowx(isig) $=0.0 \mathrm{do}$
ypowy(isig)
ypowy(isig) $=0.0 \mathrm{~d} 0$
rpowx (isig) $=0.0 \mathrm{~d} 0$
do $422 i=1+i$ start/2, $i s a m p l+i s t a r t / 2,1$
spowx(isie) $=$ spowx(isig) + blsx(i,isig)*blsx(i,isig)
spowx (isig) $=$ spowx(isig) + blsx(i,isig)*blsx(i,isig
spowy (isig) $=$ spowy(isig) + blsy(i,isig)*blsy(i,isig)
ypowx(isig) $=y p o w x(i s i g)+x(i, i s i g, i a n t) * x(i, i s i g, i a n t)$
ypowy(isig) =ypowy(isig) +y(i,isig,iant)ky(i,isig,iant
rpoux(isig) $=$ rpoux(isig) +fbisx(i,isig)\#rblsx(i,isig)
rpown(isi )
rpowy(isig) $=$ rpony(isig) + fblsy(i,isig)*fblsy(i,isig)
continue
... additive white gaussian noise
Print*, AHGN ... SNR = •, snrdb, dB'
***generate isampi random noise compts wx /wy
do 111 i =igtart, isampl, 1
$u=$ rosddf (0.0do, wsd)
wx(i,iant) =
wxilostio

EMEMBER:! calculate average signal power per BlT
(
***total noise power
wpowy=0.0d0
do $421 i=1+i s t a r t / 2, i s a m p l+i s t a r t / 2,1$
wpowx $=$ wpowx + wx (i,iant)*wx(i,iant
wpowy $=$ wpowy + wy (i,iant)*wy (i,iant
wpory $=$
continue
**total signal power, channel power and received signal
do 422 isig=i, isinum, 1
spowx(isig) $=0.0 \mathrm{~d} 0$
ypowx(isig) $=0.0 \mathrm{do}$
$\begin{array}{ll}\text { ypowy (isig) } & =0.0 \mathrm{~d} 0 \\ \text { rpoux }\end{array}$
rpowy (isig) $=0.0 \mathrm{~d} 0$
do 422 i=1+1start/2,isampl+istart/2,
spowx(isig) $=\operatorname{spowx}(i s i g)+b l s x(i, i s i g) * b l s x(i, i s l g)$
spowy(isig) $=s p o w y(i s i g)+b l s y(i, i s i g) * b l s y(i, i s i g)$
ypowy (isig) = ypowyisig) +y(i,isig,iant)*y(i,isie,iant)
rpowy(isig) $=$ rpowy(isig) + fblsy(i,isig)*fblsy(i,isig)
continue
\#\#\#average transmitted signal energy per bit for each user, Eb
*** mean-square value of channel amplitude
*** average received signal energy per bit for each user *** Measured SNR as lologio(Eb/NO)
do 421 isigel, isinum, 1
spowx(lsig) = (spowx(isig)*dble(jspaym))/dble(isampl*ibpsym) spowy (isig) $=(s p o w y(i s i g) * d b l e(j s p s y m)) / d b l e(i s a m p l i f p s y m)$ $\begin{aligned} & \text { spowy (isig) } \\ & \text { ypowx (isig) }\end{aligned}=$ ypowx(isige)/dble(isampl)
ypowx (isig) $=$ ypowx(isig)/dble(isampl)
ypowy(isig) $=$ ypouy(isig)/dble(isampl)
rpowx (isig) $=($ rpowx(isig)/dble(jspsym))/dble(isampl*ibpsym) rpowy(isig) $=($ rpowy (isig)/dble(jspsym))/dble(isampl*ibpsym)
rpouy(isig $=(r p o w y(i s i g) / d b l e(j s p s y m) ~$
Print*, 'Antenna= , iant,' Signal=', isig
Print, modsch,' Bits per gymbol:', ibpsym

Print*, Average Tx energy per bit .... real =., spowx(isig)

Print*, $E b=$, spowx(isig) +spowy $i$ isig)

Print*, average channel power .... real =',ypowx(isig)
Printw,' average channel power .... imag =',ypowy(isig)
Print*, average channel power =',ypowx(isig)+ypowy(isig)
Print*,' average kx energy per bit .... real =t, rpowx(isig)
Print*, average Rx energy per bit .... imag =., rpowy(isig)
Print". average Rx eneryy per bit, imag =, rpowy (isig)
continue average Rx energy per bit=', rpowx(isig)+rpowy(lsig)
wpowx $=$ wpowx/dble(isampl)
wpowy = wpowy/dble(lsampl)
Print*, 'average noise power
Print*.
Print*, average nolse power measured $=$ ', wpouxtupowy
do 425 isige1,isinum, 1
smrsir = 10.0dO*diogio((spowx(isif) tspowy(isig))/(wpowx thpowy) Print*, MEASURED SNR: 10logio(Eb/NO)'
Print*,'signal ',isig,' antenna ', iant,' SNR= ',snrsnr.' dE' continue endif
.. ADD TOGETHER: RECEIVED FADING SIGNALS AND AWGN
Print*." ADD TOGETHER SIGNALS + NOISE IN CHANNEL'
do $46 i^{\prime} 1=-(2 *(i t a p s-1))$, isampl, 1
nfblsx(i, iant) $=0.0 \mathrm{do}$
nf blsy (i, iant) $=0.0 \mathrm{~d} 0$
nfblsx(i,iant) $=n f$ blsx(i,iant) + fblsx(i,isie)
nfblsy(i,iant) $=n f b l s y(i, i a n t)$ folsy(i.isie)
62 continue
nfblsx(i,iant) $=n f b l s x(i, i a n t)+w(i, i a n t)$
nfblsy(i, iant) a nfblsy(i,iant) + wy(i, lant)
do 581 imistart,isampl,
rx(i,iant) $=n f b l s x(i, i a n t)$
$r y(i, i a n t)=n f b l g y(i, i a n t)$
contínue
.. CONVOLUTION OF NOISY FADING SICNAL UITH Rx FILTER
Print*, 'Rx FILTER'
do $599 \mathrm{i}=-(2$ (jtaps-i) ), isampl, 1
rx(i,iant $)=0.0 d 0$
$r y(i, i a n t)=0.0 d o$
do 511 i=istart, isampl,
do $512 \mathrm{~m}=0$, itaps-1,
rx(i.iant) $=r \times(1, i a n t)+t(m) * n f b l s x(i-m, i a n t)$ ry(i,iant) $=r y(i, i a n t)+t(m) * n f b l s y(i-m, i a n t)$
12 continue
continue

## endif

**i.e. for antennae 1 \& 2
. OUTPUT RESULTS FOR SAMPLING RATE ispgym

REMEMBER: Only samples [rij are actually avaliable, but the corresponding [sil,[yi] are required for comparison purposes. Samples [si] are delayed through 2 filters i.e. delay of 50 Samples [yi] are delayed through 1 fllter i.e. delay of 25.
So, the samples must be picked out at the rate jspsym/ispsym with their delays normalised to correspond with [ri].
ji=jspaym/ispsym
ilast = /sampl/ji
iconst = ilast* (iblock-1)
if((ji*ilast),ne.isampl) then
PRINT*,'INVALID isampl-- isampl/(jspsym/ispsym) MUST BE INTEGER'
stop
endif
do 611 iant=1, ianum,
do 611 l=0.ilast.
rxi(iconstionant) $=r x(i * j i, i a n t)$
ryi(iconst+i,iant) $=r y(i * j i, i a n t)$
11 continue
do 612 isig=1,isinum,
do $612 \mathrm{i}=0$,ilast, 1
sxi(iconst+i,isig) $=s x(i * j i-(i \operatorname{taps-1}), i s i g$
syi(iconst+i,isig) $=s y(i * j i-(i t a p s-1), 1 s i g$
do 613 iant=1, ianum,
do 613 isiget, isinum.
do 613 1s0.11ast,
$x i(i c o n s t+i, 1 s i g, l a n t)=x(i * j i-0.5 *(i t a p s-1), i s i g, i a n t)$
$y i(i c o n s t+i, i s i g, i a n t)=y(i * j i-0.5 *(i t a p s-1), i s i g, i a n t)$
do 651 lant=1, ianum, 1
do 651 i=iconst,iconst+llast.l
checkx(1,iant) $=0.0 \mathrm{dO}$
checky(i,tant) $=0.0 \mathrm{~d} 0$
do 651 isig=1, isinum,
checkx(i,iant $=\operatorname{checkx}(i, i a n t)+\operatorname{sxi}(f, i s i g) * x i(i, i s i g, i a n t)-$

* c c c syifi,isig)*yi(i,isig,iant

51 continue
c if ((iosbbc .eq. U), and. (iblock.eq. iblnum)) then
PRINT*, OUTPUT CHECK FILES•
opentunit=7, rile='checkxo')
opentunit $=8$, file='checkyo')
write (7,*) ( (checkx(i,iant), $i=1$, isampl,i), iant=1,ianum, $)$
write ( $8, *$ ) ( (checky(i,iant), $i=1$, isampl,i), iant $=1$, ianum, 1 )
close (7)
close (8)
c endif
 if (iblock.ne. iblnum) then
...RE-INITIALISE ARRAYS FOK NEXT HLOCK OF RECEIVED SAMPLES
***Only need to store $s, y, w$ arrays. All other arrays can be
*** recovered from these values
do $7: 2$ isig=1, isinum,
do $71201=-(2 *(1$ taps -1$)), 0,1$
$s \times(i, i s f g)=s \times($ lsamplti,isig)
sy(i,isig) = sy(isampl+i,isig)
do 712 ix $1, i \operatorname{sampl}, 1$
$s \times(i, i s i g)=0.0 \mathrm{do}$
sy(i,isig) $=0.0 \mathrm{do}$
712 continue
do 714 iant=1, fanum. 1
do $7140 \quad 1=-12 *(1 t a p s-1) 1), 0$,
$w x(i, i a n t)=w x(i s a m p l+i, i a n t)$
wy(i,iant) $=$ wy (isampl+i,iant)
7140 continue
do 714 i=1, isampl,
$w x(1, i$ ant $)=0.0 d 0$
wy(i,iant) $=0.0 \mathrm{~d} 0$
711 continue


## endif

**i.e. re-initialise ready for next block endif
c**i e. do next block
**i.e. do next block
if (ispsym.eq. 2 ) then
***DOUBLE SAMPLING, SE
do 821 isigai, isinum, 1
do $821 \mathrm{i}=1$, $1 \mathrm{sampl}, 1$
if(sxi(i,iglg).eq. 0.0 d 0$)$

* $\quad$ sxi(i,isig) $=0.5 d 0 *(s x i(i-1, i s i g)+s x i(t+1, i s i g))$

If(syi(i,igie).eq. 0.0d0)
21 continue
endif
endif
*** ispaym endif
if (iosbbc.eq. 11 then
.OUTPUT RESULTS
open(unit=", file='sxio')
pentunit=o, rife= syio
urite(\%,*) (rsxikig), $i=1$,isampl, 1 , isig=1, isinum, 1
write(8,*) (tsyi(i,isig), i=1,isampl,1), isig=1, isinum,
close( 7 )
open(unit=7, rile='xio')
open(unit=B, rile='yio')
write(7.*) (i(xi(i,isig,iant), i=1,jsampl,i), isig=1,isinum,i),
*urite( $8, ~((y)(i, i s i g(a n t), i a n t=1, i a n u m, 1)$
*
sig=1,isinum,1)
close (7)
close(8)
open(unit=7, rile='rxio')
pen(unlt = B, flle ='ryio.)
write(7,*) ( (rxi(i,iant), $i=1$, isampl, 1$),$ iant $=1$, ianum, $)$
urite( $8, n)((r y i(i, i a n t), i=1, i s a m p l, 1)$, iant=1, lanum, 1$)$
close(7)
close (8)
open(unit=7, file='rxo', form='formatted')
open(unitto o $^{\prime}$, rile='ryo', form='formatted')
urite( $7, *)((r x(i, i a n t), i=1, i s a m p l, 1), i a n t=1$, ianum, 1$)$
write ( $8,{ }^{\prime \prime}$ ) ( (ry(i,ianti, $\left.\left.i=1, i s a m p i, i\right), i a n t=1, i a n u m, i\right)$
close(7)
close (8)
closela
endif
do 9991 fant=1, ianum,
if(iant.eq.1) Print*,'ANTENNA a'
if(iant.eq.2) Print*, 'ANTENNA b.
Print*i rxi ryi sxi syi
do $i=1$
(i.eq. 3) $i=1$ sampl-1
do 999 isiey, isia
Print*,'
Print 9001 sxi(i, laig), ayi
9001 format (12x, 2f9.1,2f12.4)
9995
inue
Print 9002 , rxi(i,iant), ryi(i,iant)
9002 Format (10x, 2f9.4)
9993 continue
9991 continue
c
return
end

## SBEQP, FDRTRAN

SBEQP.fORTRAN
c.... 7........... . 20 . . . . . . . 30 . . . . . . . 40 . . . . . . . 50 . . . . . . . 60 . . . . . . . . 70
subroutine sbeqpifisinum, ianum, isampl, ivien, ipaci, irtsym

* ief,ifplot, sx,sy,sxdet,sydet,xpplo,ypplo)
$c$
$c$


## initialise

double precision $s \times(0: 60000,2)$, sy(0:60000,2)
double precision sxdet(0:60000,2), sydet $0: 60000,2)$
double precision xpplo(-119:60000,2,2), ypplo(-119:60000,2,2)
integer isinum,ianum, isampl, ivien, istart, ief,ifiplot
nteger ibitcio:2), ibitel0:2)
nteger i, isic, ibps, ibpsym
nteger sa(2), sb(2), sbold(2)
integer sadel 120000
integer jeido:120000)
ouble precision sdi(0:60000,2)
nteger iefs, m, max
double precision acfs (0:20)
**if iefs=1 then form autocorrelation fn. of symbol errors lefs $=0$
c ...CHECK TRANSFER OF INFO FROM MAIN PROG
Print*,' isinum= 'isinum,' ianum= ',ianum,' isampl= ,isampl
Print*, fulen=, ivien,' ipacl= ,ipacl.' irtsym=, irtsym
Print*, lef = , ief. ifplot =, ifiplot

do 4991 isiét,isinum,
Print*, SIGNAL , isig
Print*, i sx sy sxdet sydet xpplo ypplo.

if (i.eq. 3) $1=1$ isampl-1
Print 949:, i, sx(i,isig),sy(i,isig), sxdetif.isig),sydet(i,isig) 491 'rormat(i6, 12x, 2f9.1. 4x, 2f9.1)
do 4995 iant=1, ianum, 1
if(iant.eq. 1 ) then.
Print*. ANTENNA a.
endif
if (iant.eq. 2 ) then
Print.'ANTENNA $b$.
endif
Print 9002, xpplo(i,isig,iant), ypplo(i,isig,iant)
002 format (10x, 2f9.4)
4993, continue
4991 continue

Count the number of bit errors for :-
gignal(s) with Gray coding only
And $\quad$ Differential and Gray codine.
(And output error files - (if ief=1)

1bpsym=2
***i.e. QPSK has 2 bits per symbol
if Ifven.eq. 1 ) then
istart $=120$
Print*.' Give predictors ', istart, ' symbols to start-up'
else
istart=ivlen
Print*, Give predictors one packet to start-up.
end! f
Print*, BERg FOR QPSK SIGNAL(S)
Print., ' COHERENT AND GRAY CODING'
do 11 isig=1, isinum,
Print*,'COUNT NUMBER OF BIT ERRORS ... signal ',isig
**initialise for each gienal
(bitc(isig) $=0$
biterisie) = -- Coherent and Gray Coding
do $21 i=i s t a r t,(i s a m p l-i v i e n-1)$,
**Do decoding on actual s(i) and on detected s(i)
** By LOOK-UP TABLES
***actual
f (sy(i,isig).jt. 0.0do) then
sa(i) $=0$
else
sa(1) $=1$
endif
$f(s x(i, f s i g) .1 t, 0.0 d o)$ then
sa(2) $=0$
else
sa(2) $=1$
endif
if (sydet(i isig).1t. 0.0d0) then
sadet(1) $=0$
else
sadet(1) $=1$
$f(s x d e t(f, i s i g) .1 t .0 .0 d 0)$ then
sadet(2) $=0$
else
sadet(2) = 1
endit
*Keep running total of bit errors ror this
** skip past them.
o 22 ibps=1, ibpsym,
if (salibps), eq.sadet(ibps)) then
bitc (isie) $=1 b i t c(i s i g)+1$
else
bite (isig) $=$ ibite(isig) + 1
endif
continue
endil
**i.e. i loop
**Bit error total of this signal has now been calculated
***Now print results
PRINT*. 'No. of BIT errors is , ibite(isie


ber = dble(ibite(igig))/(dble(ibite(isig)) + dbie(ibitc(isig))) Print*.' HIT ERROR RATE $=$, ber
continue
c ***i.e. isig loop
ibitc $(0)=0$
iblte(0) $=0$
do 24 isig=1 isinum
ibitc(0) $=$ ibitc (0) + ibitc(isig)
ibite(0) $=$ ibite(0) + Ibite(isig)
21 continue
Print 8010
RINT* "TOTAL No. of BIT errors is , fbitelo
Print*, TOTAL No of BiTs correct is, ibitc (0)

Per $=$ dble(ibite(0))/(dble(fbite(0)) + dble(ibitc(0))
Print*.'日IT ERROR RATE FOR ALL ', isinum.' SICNALS = •, ber endif

C ...count bit errors with DIFFERENTIAL DECODINC

c Do decoding of actual s(i) and detected a(i) by Look-UP TABLES
o dirferential decoding on actual s(l) and on detected sli)
oction errors as eing alone
form error file array as eine alone
form sort deciston array as going along
ive BER results
 Print*, 'DIFFERENTIAL AND GRAY CODING'
do 51 isig=1, isinum, 1.
Print*.' COUNT NUMBER OF DIFFERENTIALLY CODED BIT ERRORS'
Print*, ... signal , isig
c **initialige for each signal
jbitc(lsig)=0
bite(isle) $=0$
sbold (1) $=0$
sbold (2) $=0$
abdold(1) $=0$
bdold (2) $=0$
*E*QPSK sifenal -- Differential and Gray Codine
do 61 i=istart, (isampl-ivien-1).
*: Do decoding on actual s(i) and on detected s(i)
*** By LOOK-UP TABLES
***actual
if (syif,isige.1t. 0.odo) then
$f$ syli,
sbisil
else
sb(1) $=1$
endif
(ft (sx(ifisig).1t. 0.0 O 0 ) then
$s b(2)=0$
else
sb(2) $=1$
endif
*ndetected
if (sydet(i,isig).lt. 0.0do) then sbdet(1) = 0
else
sbdet(1)=1
if (sxdet(i,isig).1t. 0.0d0) then sbdet(2) $=0$
** * Do difí
***actual
If ((sb(1).eq.sbold(1)) .and. (sb(2).eq.sbold(2))) then sa(1) $=0$
sa(2) $=0$ (sb(1).ne gbold(1)) and (sb(2) ne sbold(2))) then
elself
sa(1)
a
sa $(1)=1$
$\operatorname{sa}(2)=1$
elseif (lsbold(1).eq. 0). and. (sbold(2).eq. 0).and.
(sb(1) .eq. 0).and.(sb(2) .eq. 1)) the
9a(1) $=0$
elseif ((sbold(1).eq. 0).and. (sbold(2).eq. 1), and.
(sbil
eq. ().and.(sb(2) .eq. 1))
then
sa(1) $=0$
elseif ((sbold(1).eq. 1). and. (sbold(2).eq. 1).and.
(sb(1) .eq. 1).and.(sb(2) .eq. 0))
sa(1) $=0$
elseif ( (sbold(1).eq. 1). and. (sbold(2).eq. 0). and.

* (sb() $)$.eq. 1).and. (sbold(2).eq. 0).and.
sa(1) $=0$
sa $(2)=1$
else
sa(1) $=1$
sa(2) $=0$
endif
***detected
if(isbdet(1),eq.sbdold(1)), and.(sbdet(2).eq.sbdold(2)))then sadet(1)=0
sadet (2) $=0$
elseif((sbdet(1).ne.sbdold(1)). and.
* (sbdet(2).ne.sbdold(2))) then
sadet (1) =
sadif (sbdold(1).eq. 0).and.(sbdold(2).eq. 0).and.
* (sbdet(1) .eq. 0).and.(sbdet(2).eq. 1)) then sadet(1) $=0$
sadet (2)=1
* (sbdeti) eq. 1).and. (sbdet(2).eq. I) then sadet (1) =0
sadet(2)=1
elseif ( $(3$ bdold(1).eq. 1).and. $\{3$ bdold(2).eq. 1).and.
* (sbdet(1) .eq. 1).and.(sbdet(2).eq. 0)) then sadet (1) $=0$
elseif ( (sbdold(1).eq. 1).and. (sbdold(2).eq. 0).and.
* (sbdet(1).eq. 0).and.(sbdet(2).eq. 0)) then
sadet(l)=0
else
sadet(1)=1
sadet(2) $=0$
end!f
solds for next symbol
if (mod(l,ipacl).eq.(irtsym-1)) then
sbold(1) $=0$
sbold (2) $=0$
sbdold(1) $=0$
sbdold(2) $=0$
else
sbold(1) $=s b(1)$
sbold(2) =sb(2)
sbdold(1) = sbdet(1
sbdold $(2)=\operatorname{sbdet}(2)$
endif
***Keep running total of bit errors for this
* ${ }^{\prime \prime}$ * differentiat and Cray coded opsk sienal
***Do not count bit errors/correct for retraining symbols,
*** skip past them
if (modif,ipaci).ge.irtsym) then
do 62 ibpg=1,ibpsym,
if (sa(ibps).eq.sadet(fbps)) then
ibitce(isig) $=$ ibitc(isie) +
else
(bite(isig) $=$ ibite(igig) +
endif
continue
endif
if (ief.eq. 1) then
$\therefore$. FORM SOFT DECISION INFO (IF IEF=1 AND IIPLOT=IVIEN)
. FFORM SOFT DECISION INFO (IF IEF=i AND IIPLOT=IVLEN)
If (iiplot.eq. ivien) then
sdi(l.isig) $=0.0 \mathrm{dO}$
do 66 iant=1, ianum, 1
sdi(i,isig) = sdi(i,igig) + xpplo(i,isig,iant)**2 +


## 6

continue
**stop log of zero
if (sdi(i,isig). It. 1.0d-9) then
sdi(tivig) $=-99.0 d 0$
else
sdi(i,isig) $=10.0 \mathrm{~d} 0 * \mathrm{diog} 10(\mathrm{sdi}(\mathrm{i}, \mathrm{isig})$ )
endif

> endif endif
**ief endif:
61 continue
***i.e. loop for 1
..OUTPUT ERROR FILES (IF IEF=1)
if (ief.eq. 11 then
lo(1, isig, iant)**2
do 65 ibps $=1$, ibpsym,
if (sadet(ibps) .eq. sa(ibps)) then
iefd(i*ibpsym-ibpsym+ibps) $=0$
else
iefd(i*ibpgym-ibpsym+ibps)=1
endif
if $(\bmod (i, i p a c l) .1 t . i r t s y m) ~ l e f d(i * i b p s y m-i b p s y m+i b p s)=9$ continue
***output error files riles'
Print*,' output error f
open(unit=7, filez'efdio', form='formatted')
elseifisigig.eq. 2) then
openlunit=7, file='efd2o', form='formatted') plap

PRINT*, 'Output Error file not available.
stop
endif
writel?
write (7,*) isampl,ibpsym
write (7,*) (iefd(i), $i=(i s t a r t * 2-1),(i s a m p l-i v i e n-1) * 2,1)$
..OUTPUT SOFT DECISION INFO (IF IEF=1 AND IIPLOT=JVI.EN) if (iiplot eq. ivlen) then
Print*, output soft decision info files.
if (isig.eq. i) then
opentunit=7, ifle='sdidbio', form='formatted'
elgelf(isle .eq. 2) then
open(unit=7, file='sdidb2o', form='formatted'
PRINT*, Output soft decision file not avallable
gtop
write(7,*) (sdi(i,fsig), i=1,isampl-ivien-i,i)

83
**Calculate autocorrelation functi
*** to show burstiness of symbol errors
Print*, 'form ACF of symbol errors up to lag ',mmax
do 80 i=istart, (isampl-ivien-1),
if ((sxdet(i,isig).eq. sx(i,isig)).and.
(sydet(j.isig).eq. sy(i,isig))) the
iefr
elge
iefd(i)=1
endif
continue
***Calculate autocorrelation fn. of iefd
do $81 \mathrm{~m}=0$,mmax, 1
acfs(m) $=0.0 \mathrm{~d} 0$
do 82 i=istart, (isampl-ivlen-1-m),
acfs(m) = acfs(m) + iefdii)*iefd(i+m)
continue
acfs(m) = acfs(m)/dble(isampl-ivlen-m-istart)
continue
**Normalise ACF
do $83 \mathrm{~m}=1$, mmax, 1
acfs(m) = acfs(m)/acfs(0)
format(10r7.3)
close(7)

endif
**if.e. ifplot endif:
endif
**Bit error total of this signal has now been calculated P*NOH print results
PRINT*, 'No. of DIFFERENTIALLY CODED BIT errors is , ibitefisie **the only bits detected are in samples 33 to isampl-32
**since the rirst 32 symbols assume perfect detection
the last detected symbol is isampl-32
Pint, No. of DIFFERENTIALLY CODED BITS correct is , ibitc (isie)

(dale) Printa,'BIT ERROK RATE with DIFFERENTIAL CODING = ',ber

$$
\text { if (iefs.eq. } 1 \text { ) then }
$$

continue
$\operatorname{acfs}(0)=1.0 \mathrm{Od} 0$
PRINT*, 'Normalised ACF of sdet() errors, lag 0 to , mmax
PRINT 9083 (acfs(m) $m=0, m m a x, 1$
PRINT 9083,
و083 Tormatistion
51 continue
c **i.e. loop for isig .ne. 1) then
ibitc $(0)=0$
ibite(0) $=0$
do 74 isig=i, isinum, 1
bitc $(0)=$ ibitc $(0)+i b i t c(i s i e)$
(bite(0) $=$ ibite(0) +ibite(isig
continue
PRINT*, TOTAL No. of BIT errors with DIFF. CODING is ', ibite (0) Print*, TOTAL No. of BITS correct with DIFF. CODING is , ibitc 10


Print*, BER with differential CODINC ...
Print", $\quad$.. FOR ALL $\cdot$, Isinum,' SIGNALS $=$, ber
endif

PRINT

SBEQA.FORTRAN *
****************
 subroutine sbeqalisinum, ianum, isampl, ivien, ipacj, irtsym

* lef,ifplot, sx,sy,sxdet,sydet,xpplo,ypplo)
initialise
\#\#\#\#\#\#\#\#*
**********
double precision sx(0:60000,2), sy(0:60000.2
double precision sxdet(0:60000,2), sydet(0:60000.2)
double precision xpplo(-119:60000,2,2), ypplo(-119:60000,2,2)
integer isinum,ianum, isampl,ivien,istart,ief,ifplot
integer ibitc(0:2),ibite (0:2)
integer $i$, isig, ibps, ibpsym
integer sa(4), sb(2), sbold(2)
integer sadet(4), sbdet(2), sbdold(2)
integer lefd(0:240000)
double precision sdi(0:60000.2)

Print*, ' isinum= , isinum, ianum= •, lanum.' isampl= ',isampl Print*,' ivien= , ívien,' ipacl= , ipacl,' irtsym= .irtsym
Print*, ief =, ief. ifplot= , iliplot
do $499 i$ isig=i,isinum, 1
Printe, SIGNAL., isig
Print*, i sx sy sxdet sydet xpplo ypplo.
do $4993 \quad 1=1$, isampl, 1
ir (i.eq. 3; i=isampl-1
Print $9491, i, s x(i, i s i g), s y(i, i s i g), s x d e t(i, f s i e), s y d e t(f, i s i g)$
9491 formatif6. 12x, 2f9.1, 4x, 2f9.1)
do 4995 iant=1,ianum,
irtiant.eq. 11 then
Print*, ANTENNA a*
elseif(iant.eq.2) then
Print*,'ANTENNA b
endif
Print 9002, xpplo(i,isie,iant), ypplo(i,isig,iant)
9002 format(10x, 2f9.4)
4995 continue
4993 continue

$c \quad$ Count the number of bit errors for :-
sienal(s) with Gray codine only
AND --"-- Ditferential and Gray coding.
(And output error files --lif ief=1)
and output soft decision info files ..

ibpsym=14
***i.e. 16-pt QAM has 4 bits per symbol
if ivien.eq. ') then
istart=120
Print*,' Give predictors ',igtart,' symbols to start-up
elge
istart=ivien
Print*,' Give predictors one packet to start-up
endif
Print*,' BERs for 16-pt QAM SICNAL(S)'
Print*,'COHERENT AND GRAY CODING'
do 11 isig=1, isinum, 1
Print", COUNT NUMEER OF BIT ERRORS ... sienal , isie
c **initialise for each signal
ibitc(isig) $=0$
ibite(isig) $=0$
c**16-pt QAM signal -aherent and Gray Coding
do 21 i=istart, ifsampl-ivien-1), 1
**Do decoding on actual $s(i)$ and on detected s(i)
*** By LOOK-UP TABLES
***actus)
if (sy(i,isig).1t. 0.0d0) then
sa(1) $=0$
else
sa(1) $=1$
endif
if (sx(i,isig).1t. 0.0dO) then
clsa $\operatorname{siz}=0$
else
sa(2) $=1$
endif
if ((dabs(sx(i,isig)).1t. 2.0d0).and.
(dabsisy(i,isig)).lt. 2.0d0)) then
sa(3) = ?
$\operatorname{sa}(4)=1$
elseif((dabs(sx(i,isig)).ee. 2.0d0).and.

* (dabs(sy(i,isig)).ge. 2.0d0)) then
- $\operatorname{sa}(3)=0$
$\mathrm{sa}(4)=0$
elseif(tsx(1,isig).ge. 2.0do).and.
$4 \quad(s y(i, i s i g) . g e . ~ o .0 d 0) . a n d .(s y(i, i s i g) .1 t .2 .0 d 0))$ then sa(3) $=$ ?
sa(4) $=0$
elseif( $(s x(1, i s i g) .1 t,-2.0 d 0)$, and.
* (sy(i,isig).ge. 0.0d0).and.(sy(i,isig).lt. 2.OdO)) then sa(3) $=1$
sa(4) $=0$
elseif( $s x(1, i s i g) .1 t .-2.0 d 0)$.and.
* (syil.isig).ge.-2.0d0).and.(sy(i.isig).it. 0.odo)) then sa(3) $=1$ sa(4) $=0$
elseif(sx(i,isig).ge. 2.0d0). and.
(sy(i,isig).ge.-2.0do).and.(sy(i,isig).lt. 0.0d0)) then sa(3) $=1$ sal
else
sa $(3)=$
sa $(4)=$
sa(4) $=1$
endif
if (sydet(i,isig).lt. o.odo) then sadet(1) = 0
else
sadet(1) $=1$
endir
if (sxdet(i,isig).1t. o.OdO) then
sadet(2) $=0$
else
sadet (2) =
endif
* (dabs(sydet(i,isig)).lt. 2.Od0)) then sadet(3) $=1$
sadet(4) $=1$
elseif((dabs(sxdet(i,isig)).ge. 2.0do).and.
* (dabs(sydet(1,isle)).ge. 2.0d0)) then gadet(3) $=0$
sadet(4) $=0$
elseif(sxdet(i,isig).ge. 2.0do).and.
* (sydet(i,isig).ge. 0.OdO).and.(sydetitisig).it. 2.0doi) then sadet(3) =1
sadet(4) $=0$
elseif( $\operatorname{sxdet(1,1sig).1t.-2.0d0).and.~}$
* (sydet(i,isig).ge. 0.0dO).and.(sydet(i.isig).lt. 2.0d0)) then sadet(3) =
sadet(4) $=0$
elseif(lsxdet(i,isig).1t.-2.0d0). and.
* (sydet(f.isig).ge.-2.OdO).and.(sydeti,isig).lt. 0.0do)) then sadet(3) $=1$
sadet(4) =
elseif (sxdet(i,isig).ge. 2.0dO).and.
* (sydet (i, islg).ge.-2. OdO).and.(sydet(i,isig).it. o.odol) then sadet(3) $=1$
sadet(4)
else
sadet (3) $=0$
sadet(4) $=1$
endif
**Keep runjine total of bit errors for this
** coherent and Gray coded 16-pt QAM signal.
**Do not count bit errors/correct for retraining symbols.
** skip past them.
if (mod(i,ipaci).ge.irtsym) then
do 22 ibps=1.ibpsym,i
if (sa(ibps).eq.sadet(ibps)) then
(bitc(isig) $=$ ibitc(isig) +1
else
bite(isig) $=i b i t e(i s i g)+1$
endif
continue
endif
continue
**i.e. 1 loop
**Bit error total of this signal has now been calculated
arint. No. result
rors is , ibitelisig
rint*,' No. of BITS correct is ', ibitc (isig)
Prinl.
er = dble(fbite(isig))/(dble(fbite(isig)) + dble(ibitc(isig)))
Print*.'BIT ERROR RATE = , ber
continue
***i.e. isig loop
(isinum .ne. 1) then
bitc $(0)=0$
a 24 (sig=1 isinum,
o 24 isig=1,isinum, 1
bitc(0) $=$ (bitc (0) +ibitc(isig)
ibite(0) $=$ ibite(0) + ibite(isig)
continue
PRINT*, 'TOTAL No. of BIT errors is ', 1bite(0)
Print*. -TOTAL No of BITs correct $1 s^{\circ}$. ibitc $(0)$

'ber = dble(ibite (0) )/(dble(fbite(0)) + dble(fbltc(0)))
'Print*,' BIT ERHOR RATE FOR ALI. ', isinum,' SIGNALS = ', ber endif
accumulate bit errors as going along.
(form error file array as golng along).
(fiorm soft decision array as going along)
give ber results

'Print*.' DIfferential and cray coding'
तo 51 isig=1, isinum, 1
Print*, COUNT NUMBER OF DIfferentially CODED BIt ERRORS'
Print*, $\cdot$.
c ***initialise for each signal
ibitc(isig) $=0$
ibite(isig) $=0$
sbold (1) $=0$
sbold (2) $=0$
sbdold (1) $=0$
sbdold(2) =0
***16-pt QAM sienal - DDifferential and Gray Codine
do 61 l=istart. (isampl-Ivien-1), 1
***Do decoding on actual s(i) and on detected s(i)
*** By LOOK-UP TABLES
***actual
if (sy(1,isie).1t. 0.OdO) then
sb(1) $=0$
else
sb(1) $=1$
endif
if (sx(i,isig).lt. 0.0dO) then
sb(2) $=0$
else
sb(2) =
endif
it ( (dabs(sx(f,19ig)).1t. 2.0d0).and.
* (dabs(sy(i,isig)).it. 2.0do)i then sa(3) = ?
sa(4) $=1$
elseif(ldabs(sx(i,isig)).ge. 2.0d0). and.
* (dabs(sy(i,isig)).ge. 2.0d0)) then sa $(3)=0$
sa(4) $=0$
elseif(tsxif.isig).ee. 2.0d0). and.
* (sy(i,isig).ee. O.OdO).and.(sy(i,isie).lt. 2.0d0)) then sa(3) $=1$
sa(4) $=0$
elseif((sxif,isig).ge. - 2.0才0). and. (sx(1,isig).1t. 0.0do).and.
* isy(i,isig).ee. 2.0 d 0 ) t then sa(3) $=1$
sa(4) $=0$
elseif(sx(i,isig).1t.-2.OdO). and.
(sy(i,isig).ee.-2.odo).and.(syif.isig).lt, o.odo)) then sa(3) $=1$
elseif( $(s \times(i, j s i g) . g e .0 .0 d 0)$, and. (sx(i,isig).1t. 2.0d0). and
* (sy(l.isig).1t.-2.0dO)) then
sa(3) $=1$
sa(4) $=0$
else
sa(3) $=0$
sa(4) $=1$
endif
if (sydet(i,igie).1t. 0.OdO) then sbdet(1) $=0$
else
sbdet(1) =
endif
if (sxdet(1,fsig). 1 t .0 .0 d 0 ) then sbdet(2) $=0$
else sbdet(2) $=$
endif
if ((dabs(sxdet(1,isig)).1t. 2.0do).and.
(dabs(sydet(1,isig)).1t. 2.OdO) then sadet(3) $=1$
sadet (4) $=1$
elseif((dabs(sxdet(i,isig)).ge. 2.0do).and.
*. (dabs(gydet(i.isig)).ge. 2.0dO)) then sadet(3) $=0$
sadet (4) $=0$
elseif(csxdet(i,isig).ge. 2.0dot.and.
* (sydet(i, isig).ge. 0.odo).and.(sydet(ifisig).lt. 2.odo)) then sadet(3) $=1$
sadet(4) $=0$
elseif((sydet(i,isig).ge. 2.0do).and.
* (sxdetif.isig).ee.-2.0d0), and.(gxdet(i.isig).lt, o.odo)) then sadet(3) $=1$ sadet(4) $=0$
elseif( $s x d e t(f, 1 s i g) .1 t,-2.0 d 0)$. and.
* (sydet(i,isig).ge.-2.0do).and.(sydet(i,isig).it. o.odo)) then sadet(3) $=$
sadet(4) $=0$
lseif(sydet(i,isig).1t. - 2.0 d 0 ). and.
 sadet(3) $=1$ sadet $(4)=0$
else
sadet(3) $=0$
sadet $(4)=1$
ndif
**Do differential decoding on actual $s(1)$ and on detected s(ij
**actual
if (tsb(1).eq.sbold(1)) and. (sb(2).eq.sbold(2)) then sa(1) $=0$
lseif ((sb(1).ne.sbold(1)) .and. (sb(2).ne.sbold(2))) then sal sa(2)=1
elseif (sbold(i).eq. 0).and. (sbold(2).eq. 0). and
* (sb(1) .eq. 0).and.(sb(2) .eq. 1)) then sa(1) =0
sa(2) $=1$
alsatf (chold(1) an 0 ) and (sholdg) an il and
- (sb(1) .eq. 11.and.(sb(2) .eq. l)) then sa(1) $=0$
seif (fsbold(1).eq. 1).and. (sbold(2).eq. 1). and
* (sb(1) .eq. il.and. (sb(2) .eq. 0)) then sa(1) $=0$
a (2) $=1$
elseif (isbold(1).eq. 1).and. (sbold(2).eq. 0), and
$\operatorname{sa}(1)=0$
$\operatorname{sa}(2)=1$
else
sa(1) $=1$
nalf
endif
if((sbdet(1).eq.sbdold(1)).and.(sbdet(2).eq.sbdold(2)))then sadet(1) $=0$
sadet $(2)=0$
elseif ((sbdet (1).ne.sbdold(1)). and.
* : (sbdet(2).ne.sbdold(2))) then
sadet(1)=1
sadet (2) $=1$
elseif ((sbdold(1).eq. 0).and. (sbdold(2).eq. 0).and.
* (sbdet (1) .eq. 0).and.(sbdet (2).eq. i) then
sadet(1) $=0$
sadet(2)=1
elseif ((sbdold(i).eq. 0).and. (sbdold(2).eq. 1).and.
* (sbdet(1) .eq. 1).and.(sbdet(2).eq. 1)) then sadet (1) $=0$
sadet(2)=1
elseif (isbdold(1).eq. 1).and.(sbdold(2).eq. 1).and.
* (sbdet(1).eq. 1).and. (sbdet(2) .eq. 0)) then sadet(1)=0 sadet(2)=1
elseif ((sbdold(1).eq. 1).and. (sbdold(2).eq. 0).and.
*. (sbdet(1).eq. 0).and.(sbdet(2).eq. 0)) then sadet(1)=0
sadet(2)=1
else
sadet(1)=1
sadet(2)=0
endif
**set up sbold's for next symbol
if (modilipacl).eq.(irtsym-1)) then
gbold(1) $=0$
sbold(2) $=0$
sbdold(1)=0
else
sbold(1) $=\operatorname{sb}(1)$
sbold(2) $=s b(2)$
sbdold(1) = sbdet(1)
sbdold(2) = sbdet(2)
endif
**Keep running total of bit errors for this
*** differential and Gray coded 16-pt QAM signal
***Do not count bit errors/correct for retraining symbols.
** skip" past them

do 62 lops=1, ibpsym, 1
if (sa(ibps).eq.sadet(ibps)) then
iblic(isig) $=$ ibitc(isig) +
elge
ibite(isig) $=$ ibite(isig) +1
endif
continue
endit
***output error files
Print*, output error files
if lisig .eq. 1) then
pen(unit=7, filex'efdio', form='formatted')
elseifisig.eq. 2 then
open(unit=7, files'efd2o', form='formatted')
else
PRINT*.'Output Error file not available
stop
endif
write (7,9521) (iefd(i), i=(istart*2-1), (isampl-ivien-1)*2,1) format(20i2)
close (7)
***output soft decision info

Print*, output soft decision info files
if (isie.eq. 1) then
opentunit=?, file='sdidblo', rorm='rormatted',
elseif(igig .eq. 2) then
open(unlt $=7$, file='sdidb20', form='rormatted')
else
PRINT, 'Output soft decision file not available' stop
(ite (7,9522) (sdi(i,isig), i=1start,isampl-ivlen-1, 1)
9522 format(ior7.3)
close (7)

```
endif
***i.e. iiplot endif:
endif
***ief endif
```

**Bit error total of this signal has now been calculated ***Now print results

PRINT*,' No. of DIFFERENTIALLY CODED BIT errors is , ibite(isig)
Print., No. of DIFFERENTIALLY CODED BITS correct is , ibitctisig)

ber = dblelibite(isig) /(ddble(ibite(isig)) + dble(ibitc(isig)))
Print*,'日IT ERFOR RATE Hith DIFFERENTIAL CODING = •, ber
continue
**i.e. loop for isig
bltc(0) $=0$
bite(0) $=0$
do 74 isigei.isinum,
ibitc(0) $=$ ibitc(0) + ibite(isig)
ibite(0) $=$ ibite( 0 ) + ibite(ligig)
ibiteto
Print 8010
PRINT*, 'TOTAL No. of BIT errors is , ibite (0)
Print*, 'Total No. of Bits correct is , ibitc 10

ber = dble(ibite(0))/(dble(ibite(0)) + dble(ibitc(0))
Print*, BER with DIfFERENTIAL CODING ..'
Print*,' .. FOK ALL ', isinum,' SIGNALS = •,ber endif


## return

end
*******************
RC SAMP2.FORTRAN
*RC_SAMP2.FORTRAN *
. $30 . . .$. . . . $40 . .$. . . . . 5
Initialise
program rc_samp2.
***********
double precision $h(-100: 100)$, fe, fs, ts, pi double precision ah(-200:200)
integer $Q$

$r e=12000$
s=4.0d0*re
$\mathrm{ts}_{\mathrm{s}}=1.0 \mathrm{~d} 0 / \mathrm{fs}$
g 25
$p i=3.141592654 d 0$
c ..SAMPLE THE IMPULSE RESPONSE
do 1 i=0, $\mathrm{G}, 1$
$h(i)=d s q r t(f e) *(d s i n(p i *(2.0 d 0 * i * t s * f e+0.5 d 0))$

* /(pi*(2.0do*i*ts*ie + 0.5d0))
* $+\quad d \sin (p i *(2.0 d 0 * i * t s * f e-0.5 d 0))$
continue
do $2 i=-g,-1,1$
$h(i)=h(-i$
continue
**scale all tap gains so that mean-square power ah(0)=1 $\operatorname{ah(0)}=0.0 \mathrm{~d} 0$
do $991 \mathrm{i}=-\mathrm{e},+\mathrm{E}, \mathrm{l}$
$a h(0)=a h(0)+h(i) * h(1)$
do $993 \mathrm{i}=-\mathrm{g},+\mathrm{g}, \mathrm{I}$
$h(i)=h(i) / d s q r t(a h(0))$ contlnue
c
** output sampled ir to file
if (fs.eq.(2.OdO*fe)) then
pen(unitic?, file='rc_s2', form='formatted')
elseif(fs.eq. (4.0d0:fes) then
open(unit=7, file='rc_st', form='formatted') elgeif(fs.eq.(B.Odo*fe)) then
pentunit=7, file='rc_s $8^{\prime}$, form='formatted')
elself(fs.eq.(16.0do*fe)) then
opentunit: $=7$, file='rc_si6', forme'formatted')
endif
write(7,*) 2*et1
urite(7,*) (h(i), $i=-\mathbf{g},+\mathrm{E}, \mathrm{i})$
close(7)

```
* INTERPOLATE HC FOORTRAN
* INTERPOLATE_RC.FORTRAN
c.....7........... 20........10........40........50. . . . . . . 60. . . . . . . . 70.
    program interpolate_rc
        double precision xt-500:5000), y(-500:5000)
        double precision g(-960:960), a(320.6)
        double precision pi, alpha
        integer isampl, inter, num, i, k, n
        *********************************************************
        ***Program to generate the tap-gains of a cosine roll-off
        *** intergolatinefilter, with roll-off factor alpha (see
        **The interpolating filter impulse response is sampled
        ** to give the coerricients of the "inter " num" matrix
        *** "a". Matrix multiplication gives the interpolated
        *** samples.
        pi=3.14159265440
        alpha=0.60d0
        I sampl =2
        inter=2
    num=6
    do 991 i=0,isampl. }
        x(i) = 0.0d0
        y(i)}=0.0\textrm{do
    continue
    ***sample the interpolating filter impulse response
    *** from times "-(num/2)Ts to tornum/2)Ts"
    *** i.e. from samples "-(num/2)*inter to +(num/2)*inter"
    *"*Note that this impulse response is:-
    *** symmetrical about time=0
    *** equal to zero at times=-3Ts, -2Ts, -Ts, Ts, 2Ts, etc.
    *** equal to 1 at t1me=0
```



```
    do 1:1 i=1,(num/2)*inter,l
    (i) (dsin(pi*dble(i)/dble(inter)) (pindble(i)/dble(inter)))*
    * (dcos(alpha*pi*dble(i)/dble(inter))/
    (1.0d0 - 4.0dO*(alpha*dble(i)/dble(inter))**2))
    11 continue
    e(0)=1.0do
    (112 i=inter,(num/2)*inter.inter
    g ( i ) = 0 . 0 d 0
    12 continue
    do 1122 i=1,(num/2)*inter,l
    e(-i)=g(i)
    1122 continue
    ***form the matrix a
    do 121 k=1,inter.
    do }122\textrm{n}=1,\textrm{num},
        a(k,n)=g((k-1) - ((num/2)+1-n)*inter)
    122
    continue
    12
c
****output to file
    **output to rile
    if (inter.eq.10) then
    nnenfunitt=10
```

elseif(inter.eq. 20 ) then
open(unit=10, file='intera20', form='formatted') elself(inter.eq. 40 ) then
open(unit=10, file='interatio', form='formatied') elseif (inter.eq. 80 ) then
open(unit=10, flle='intera80', form='formatted') elseif(inter.eq. 160 ) then
opencunit =10, file='Interal60', form='formatted')
Plise
PRINT*,'ERROR IN inter.
stop
endif
write (10.*) num
write (10,*) ( $(a(k, n), n=1$, num, 1), $k=1$, inter, 1$)$ close (10)
stop

## **************

* SIM2.FORTRAN
*************
$\qquad$
program sim2
nteger n,k,i.e
double precision tt, fm, s, pl, prodio:10000), e, r, h(0:200) louble precision t(0:400), at (0:400)
$p f=3.141592651$ do
$\mathrm{tt}=1 / 600.0 \mathrm{do}$
$\mathrm{fm}=80.0 \mathrm{~d}$
 Print*,' with sampling period of ', tt,' seconds.

COMPUTE AREA (INTEGRAL) FOR $n$ STRIPS
do $1 n=10000,10000,5000$
$s=3.141592654 d 0 / 2.0 \mathrm{~d} 0 / \mathrm{dble}(n)$
in*loop to calc each $h(k)$ where $h(k)=h(k T$
do $11 \mathrm{k}=0, \mathrm{e}, 1$
***calc. fn. value at edges of each strip do $111 i=0, n-1,1$
prodif) $=d \cos (2.0 d 0 * p i * f m * d b l e(k) * t t * d s i n(d b l e(i) * s))$

* contínue
$\operatorname{prod}(n)=0.040$
***rind area
e=0.0do
do $112 \quad i=1, n-1,2$
e $=\mathrm{e}+\operatorname{prod}(\mathrm{i}$
$r=0.0 d 0$
$r=0.000$
do $113 \mathrm{i}=2, \mathrm{n}-2,2$
$r=r+p r o d i l i$
113 continue
s/3.0d0*(prod $(0)+\operatorname{prod}(n)+40 d 0 * e+20 d 0 * r$
1 continue
c
c ....time delay impulse response to make it physically realisable do $58 \quad i=0, g-1,{ }^{\prime}$
(i) $=h(\mathrm{e}-\mathrm{i})$
58 continue
$(\mathrm{e})=\mathrm{n}(0)$
o $581 \quad i=e+1, e+e$.
$t(i)=h(i-t$
581 continue
... OUTPUT FILTER COEFFS TO FILE
opencunit =7, file='omvermon8o', formiformatted')
urite (7,*) fm, $2 * \mathrm{~g}+1$
wite (7,*) t
close(7)


## ********************* <br> MESTIMPRED.FORTRAN * <br> \section*{*********************}


integer isettl,isampl
parameter (isettl $=1200,1 \mathrm{sampl}=6000+1 \operatorname{sett})$
double precision $s x(0: 60000,2)$, sy $(0: 60000,2)$
ouble precision $x(0,6000,2,2), y(0: 60000,2,2$
integer iobbr
nieger iobbc
nteger ispsym, ipacl, imetrt,irtsym, inter
nteger isinum,isig, ianum,iant
ouble precision snrdb, svar
haracter modsc
ouble precision sxdet(0:60000,2), sydet(0:60000,2)
nteger imetes
ouble precision xest(0:60000,2.2), yest (0:60000.2.2)
ouble precision resest, ryest
ouble precigion ex, ey, b,btemp
neger iordp
e precigion xpred(0:60000,2,2), ypred(0:60000,2,2)
double precision theta, th1, th2, th3, th4, th5, the, thy
ouble precision the, the
ouble precision
ouble precision ezx, ezy
ouble precision $20 x(2,2), z 0 y(2,2)$
double precision $2 \times(2,2)$. 21y(2,2)
double precision $z 3 \times(2,2), 23 \times(2,2)$
double precision c(0:5)
nteger imete, imetp
double precision sqerr(isampl), msqer (0:2
double precision seb(isampl), msebl0:2)
double precision templ, temp2
integer ib ibmax ithet, ithmax isnr, isnmax
integer ib,ibmax. Het,ithmax. isnr,ismax
ouble precision plotp(21,20,7), plote(21,20.7
neeger imetb
Modsh=.16-pt
odschar QAM.
average transmitted energy per bit,
if (modsch eq 'QPSK') then
var = 1.0 do
elgeif(modsch.eq.'16-pt QAM') then
svar=2.5do
else
Print*.' modulation scheme not avallable
stop
iginum=1
ianum=1
|pacl $=1$
**if iobbc=0 then read in $s, r, y$ arrays from stored files
**else run subroutine sbbchan
\#\#else
lobbc=1
**retralning method imetrt
imetrt=0
**Assume $10 \%$ retraining, unless no retraining (i (1metrt.eq 0) then

## irtsym=0

lse
irtsym $=$ \{pac $\mid / 10$
endif
inter $=20$
ispsym=1
***if imetes=1 then do NOT use predn. fed back in the estimator
***if imetes=2 then use predn. fed back in the estimator
****calculate estimation error if imete:
imete $=0$
***calculate prediction error if imetp=
imetp=1
***output riles if iofile=1
or 11e=0
**PRINT system constants for this run
Print. ianum, ANTENNAE.
Print*.isinum,', modsch, signal.(S).
Print*." Packet Length $=$ ', ipacl
Printmetrt.eq.0) then
Priseif No retrainine
Printa, IDEAL retrainine
Print*, 10\% retraining therefore , irtsym, retraining symbolg.
else
Print., Retrainine method $\cdot$, imetrt
Print*', 10\% retraining therefore , irtsym, retraining symbols' endif
Print*,'Run for ', isampl/ispsym,' symbols'
Print*' 'including , isettl ' samples for transient to settle doun Print*,' and ', ispsym,' samples per symbol'
if timete.eq. '0)
Print*, DO NOT calculate egtimation error'
else
Print.'. Calculate estimation error' endit
if (imetp.eq. i) then
Print*,'Calculate prediction error
else
Print*.'DO NOT calculate prediction error
endif
.DO FOR DIFFERENT SNRS
isnr=0
do $5 \operatorname{snrdb}=20.0 \mathrm{~d} 0,26.0 \mathrm{~d} 0,10.0 \mathrm{dO}$
if ((snrdb.gt. 75.0d0).and.(snrdb.1t. 1999.0d0)) eoto 5
ignr=isnrti
Print*,' $S N R=1, s n r d b, d^{\prime}$
**read in files of signal, channel, received signal
if (iobbc .eq. 0) then
opencunit=? filez'sxio', form='formatted'
open(unit=8, file='syio.', form='rormatted')
open(unit=9, file='xio', form='rormatted')
open(unit=10, file='yio' form='formatted'
read (7,*) ( $(s x(i, i s i g), i=0, i s a m p l, 1), j s i g=1$,isinum, 1$)$
read ( $8, *$ ) ( (sy(i,isig), i=0,isampl,i), isig=i,isinum, 1 )
read (9,*) ((ix(i,isig,iant), $i=0$, isampl,i), isig=1,isinum,i).
iant=1, ianum, 1 )
read(10,*) ((y(i,isig,iant), i=0,isampl,i), isie=1,isinum,i)
close (7)
close(8)
close(9)
close(10)
openfunit=19, filez'rxio', form='formatted'
apen(unit=12 file='ryio' form='rormatted
read (11,*) ( $r$ rx(i,iant), $i=0, i$ ampl,i), iant=1,ianum, 1
read (12,*) ( $(r y(i, i a n t), i=0, i s a m p l, 1), i a n t=1$, ianum. $)$
close(11)
close(12
else
Print.' SuBroutine sbbchan'
call sbbchantisampl, ipacl,irtsym,inter,snrdb.
modsch,svar,isinum, ianum, $\mathrm{rx}, \mathrm{ry}, \mathrm{sx}, \mathrm{sy}, \mathrm{x}, \mathrm{y}$ )
Print*,'RETURNED sbbchan'
endit
do 9991 iant=1, ianum, 1
if (iant.eq.1) Print*, ANTENNA a'
ff(iant.eq.2) Print*.'ANTENNA b

if (i.eq. 3; i=isampl-
do 9995 isig=1,isinum, 1
printe, signal ,isig
Print 900t,i,sx(i,isig),sy(i,isig), x(i,isig,iant),y(i,isig,iant
rmat(16, 12x, 2f9.1, 2f12.4)
continue
Print 9002, rx(i,iant), ry(i,iant)
ormat (10x, 2f9.4)
continue
**assume perfect detection i.e. detected s = actual s
do 99 lsig=1,isinum,
do $99 \mathrm{i}=1$, isampl,
sxdet(i,isig) $=s x(f, i s i g)$
sydet(i,isig) $=s y(i, i s i g)$
..REPEAT RUN FOR: - 0 "UNBIASED ESTIMATE", 1 "NO FEEDGACK" AND . 2 "FEEUBACK" (FROM PRED.TO EST.)
do 4 imetes=2,2,1
if (imeteg.eq. 0). and. (isinum.ne. 1)) then
Print*,'CANNOT DO UNBIASED ESTIMATES FOR ',isinum,' SIGNALS
stop
endif
.. REPEAT RUN FOR EACH ORDER PREDICTOR IN TURN
***if iordp $=-1$ NO prediction
***elseif iordp=0,1,2,3 least-squares fad-mem polynomial pred
**elseif iordp=12,13,..,16 Taylor predictors, p=2,3...,6 respec.
do 1 iordp=1.1,1
. . REPEAT RUN for each different value of d $1 b=0$
do 2 btemp $=0.025 d 0,0.0451 \mathrm{~d} 0,0.005 \mathrm{~d} 0$ $1 b=1 b+1$
** 1.e. for i6-pt QAM, constellation is not constant envelope
***else ifnetb=0 : constant b=btemp
1 metb=0
$\mathrm{b}=\mathrm{btemp}$
. REPEAT RUN FOR EACH DIfferent value of theta
do 3 theta $=0.25 d 0,0.901 \mathrm{~d} 0,0.05 d 0$ ithet =ithet +1
if (imetes.ea. 0 ) then
Print*, Unbiased estimates.
elseif (imetes.eq.1) then
Print*' NO feedback from predictor to Gradient estimator'
elseif (imetes.eq. 2 ) then
Print*, 'USE feedback from predictor to Cradient egtimator' endif
if (imetes.ne. 0) then
if iimetb.eq. i) then
czzzzPrint*,
Printa, CAREFUL... VARIABLE $b=$, btemp, *[si**2]
Print*, Pote : multiply NOT divide'
else
Print*.' ... CONSTANT $b=$, b
endif

## endif

if liordp.eq. -1) then
Print*.'NO PKEDICTION'
( (iordp.ge.0).and. (iordp.le.3)) then
Print*,'LSFM Predictor .... order= 'iordp,' theta= , theta
Print*. 'Taylor predictor, order $p=1$, $\operatorname{lordp-10}$
endir
**Channel estimate at $t=0$ is assumed to be unknown
** Initialise predictor similarly
do 199 iant=1, ianum, 1
do 199 isig=1, isinum, 1
xest(0,isig,iant) $=0.0 \mathrm{~d} 0$
yest $(0,1$ gig.iant $)=0.0 \mathrm{~d} 0$
xpred (1,isig,iant) $=0.0 d 0$
ypred ( 1, isig, lant) $=0.0 \mathrm{~d} 0$
z0x(isic.iant) $=0.0 d 0$
zoy(1sig, iant) $=0.0 \mathrm{~d} 0$
z1x(isig,iant) $=0.0 \mathrm{~d} 0$
zly(isig, iant) $=0.0 d 0$
$22 x(\operatorname{sig}$, ant $)=0.0 \mathrm{~d} 0$
$23 \times(i s i e$ iant) $=0.0 \mathrm{~d} 0$
23(isig iant) $=0.040$
continue
.. Channel estimator
tho $=1.0 d 0-$ theta
th1 $=(1.0 d 0-$ theta) $(1.0 d 0-$ theta $)$
th2 $=1$. Odo - thetantheta
th3 $=0.5 d 0 *(1.0 d 0-$ theta)**3

ths $=1.0 d 0-$ theta** 3
th6 $=($ ( $1.0 \mathrm{do}-$ theta) $* * 4) / 6.0 \mathrm{~d} 0$
th7 $=((1.0 d 0-$ theta $) * * 3) *(1.0 d 0+$ theta
th8 = (1.Od0 - theta)*(1.Od0 - theta)*(11.0d0 + 14.0d0*theta

th9 $=1.0 d 0-$ theta** 4
**Set-up c(0,1,..5) for Taylor predictor
if (iordp.le.3) then
elseif(iordp.eq.12) then
$\mathrm{c}(0)=2.0 \mathrm{~d} 0$
$c(0)=2.0 \mathrm{do}$
$c(1)=-1.0+0$
$\mathrm{c}(2)=0.0 \mathrm{~d} 0$
$c(2)=0.0 \mathrm{~d} 0$
$c(3)=0.0 \mathrm{~d} 0$
$c(4)=0.0 \mathrm{~d} 0$
$\begin{array}{ll}c(4) & =0.0 \mathrm{~d} 0 \\ c(5) & =0.0 \mathrm{~d} 0\end{array}$
elgeif(iordp.eq.13) then
$\mathrm{c}(0)=3.0 \mathrm{~d} 0$
$c(0)=3.0 \mathrm{~d} 0$
$c(2)=1.0 \mathrm{~d} 0$
$c(2)=1.0 \mathrm{~d} 0$
$c(3)=0.0 n 0$
$c(5)=0.0 \mathrm{~d} 0$
$c(5)=0.0 \mathrm{~d} 0$
elseif(iordp.eq.14) then
$c(0)=4.0 \mathrm{dO}$
$c(0)=4.0 \mathrm{~d} 0$
$c(1)=-6.0 \mathrm{~d} 0$
$c(2)=4.0 \mathrm{~d} 0$
$c(2)=4.0 \mathrm{~d} 0$
$c(3)=-1.0 \mathrm{~d} 0$
$c(3)=-1.0 \mathrm{~d} 0$
$c(4)=0.0 \mathrm{~d} 0$
$c(4)=0.0 \mathrm{~d} 0$
$c(5)=0.0 d 0$
elseif(iordp.eq.15) then
$c(0)=5.0 \mathrm{do}$
$c(0)=5.0 \mathrm{do}$
$c(1)=-10.0 \mathrm{~d} 0$
$c(1)=-10.0 \mathrm{do}$
$c(2)=10.0 \mathrm{~d} 0$
$c(2)=10.0 \mathrm{dO}$
$c(3)=-5.0 \mathrm{~d} 0$
$c(4)=1.0 \mathrm{~d} 0$
$c(5)=0.0 \mathrm{~d} 0$
elseif(iordp.eq. 16 ) then
$c(0)=6.0 \mathrm{do}$
$c(0)=6.0 \mathrm{~d} 0 \mathrm{do}$
$c(2)=20.0 \mathrm{~d} 0$
$c(3)=-15.000$
$c(4)=6.0 \mathrm{do}$
$c(4)=6.0 \mathrm{do}$
$c(5)=-1.0 \mathrm{~d} 0$
end if
if (iordp.ee. 12) Print*, c
do 111 i=5,isampl. 1
. Channel. Estimator
do 320 iant $=1$,ianum, 1
if (imetes.eq. O) then
"**unbiased estimator for 1 signal
=1.Odo/(sxdet(i,1)*sxdet(i,1) +sydet(i,i)*sydet(i,1)
$e 1 s e$
$b=0.5 \mathrm{~d} 0$
endif
xest(i,i,iant) = b*(sxdet(i,l)*rx(i,iant) + sydet(i, l)*ry(i,iant))
yest(i,1,iant) = b*(sxdet(i,i)*ry(i,iant) - sydet(i,i)*rx(i,iant))
else
**"gradient estimator
rxest $=0.0 \mathrm{~d} 0$
ryest $=0.0 \mathrm{do}$
do 3211 isig=1,isinum,
if (imetes.eq.1) then
rxest = rxest
*
sxdet(i,isig)*xest(i-1,isig,iant) -
sydet(i,isig)*yest(i-1,isig,iant)
ryest $=$ ryest
*
sxdet(ifisif)*yest(i-1,isig,iont) +
sydet(i,islg)mxest(i-1,isig,iant)
elseif(imetes.eq.2) then

```
        rxest = rxest (',
    ryest = ryest +
    * sxdet(i,isig)*ypred(i,isig,iant)
        sydet(i,isig)*xpred(i,isig,iant)
    *
    21: continue
        ex = rx(i,iant) - rxest
        ey = ry(i,iant) - ryest
        do 3212 isie=1,isinum,
        if (imetb.eq. 1) then
czzzzz b = btemp/(sxdet(i,isig)**2 + sydet(i,isig)**2
        b = btemp*(sxdet(i,isig)**2 + sydet(i,isig)**2
        endif
    xest(f,isig.iant) =xesti(i),isig.iant) +
    **(exnsxdet(i,isig) + ey*sydet(i,isig))
    yest(f,igig,iant) = yest(i-1,isig,iant) +
    * b*(-ex*gydet(i,isig) + ey*sxdet(i,isig))
                    elseif(lmetes.eq.2) then
        xest(i,isie,iant)=xpred(i,isig,iant) +
    b*(ex*sxdet(i,isig) + ey*sydet(i,isig))
    yest(i,isig,iantl (ypred(i,isig,iont)
    * bu(-ex*sydet(i,isig) + ey*sxdet(f,isig))
        continue
        endil
*"{imetes endif
c...PREDICTOR
        do 3213 isig=1,iginum,?
        ezx = xest(i.isig,iant) - xpred(i,isig.iant)
        ezy = yest(i,isig,iant) - ypredii,isig,iant 
            if (iordp.eq. -1) then
mo predictio
    xpred(j+1,isje,iant) = xest(i,isig,iant)
        ypred(i+l,isig,iant) = yest(i,isig,iant)
                elseif(iordp.eq.0), then
...degree
        xpred(i+1,isig,iant) = xpred(i,isig,iant) + tho*ezx
        ypred(i+1,isig,iant) = ypredif,isig,iant) + tho*ezy
                elseif(iordp.eq.1) then
    ...degree
        ziv(la(c,iant) = 21x(isig,iant) + thi*ezx
        (ant) = ziy(isig(iant) + thl"ezy
        xpred(i+1,isig,jant) = xpred(i,isig,iant) + zix(isig,iant)
    th2*ezx
    ypred(i+1,isig,iant) = ypred(i,isig,iant) + ziy(isig,iant) +
                                    th2mezy
                    elseif(iordp.eq.2) then
#..degree 2
    z2x(isig,iant) = z2x(isig,iant) + th3*ezx
    z2y(isig,iant) = z2y(isig,iant) + ths*ezy
    2*(isie,iant. = 2rx(tsie,i
        zly(isig,iant) = z1y(isig,iant) + 2.0q0*z2y(isig,iant) +
    *
                                    i,isig iant) + 21x(isig iant)
    xpred(i+1,isig,iant)= xpredin,isie,iant)+ th5*ezx
    vored(i+1.isig.iant) = ypred(i.isic,iant) + zly(isig,iant) -
                        z2x(isic,iant) + th5*ezx
```

* 

22y(isie, iant) + th5*ezy
c...degree 3
elseif tiordp.eq. 3 ) then

z3y(isig,iant) $=23 y(i g i g, i a n t)+t h 6 * e z y$
$z 2 x(i s i g, i a n t)=22 x(i s i g, i a n t)+3.0 d 0 * z 3 x(i s i g, i a n t)+t h 7 * e z x$
z2y(isig,iant) $=22 y(i s i g, i a n t)+3.0 d 0 * z j y(i s i e . i a n t)$, th7*ezy
zix(isig,iant) $=21 x(i s i g, i a n t)+2.0 d 0 * z 2 x(i s i g, i a n t)-$
*ziy(isig,iant) $=3.0 d 0 * 23 x(i s i g, \operatorname{lant})+$ th8*ezx
zly(isig,iant) $=21 y(i s i g, i a n t)+2.0 d 0 * z 2 y(t \operatorname{sig}, i a n t)-$
xpred(i+1,isig,iant) $=x p r e d(i, i g i e, i a n t)+2 l x(i s i g, i a n t)-$
ypred(itl, z2xisig,iant) + z3x(isig,ianti + thynezx
ypred $(i+1, i s i g, i a n t)=y p r e d(i, i s i g, i a n t)+z l y(i s i g, i a n t)$.
z2y(isig,iant) + z3y(isie,iant) t thgerezy elseif (iordp.ge.12) then
. . Taylor
elseif (iordp.ge.12) then

* c(1)*xest(i-i,isig,iant) +c(2)ixest(i-2,isig,iant) *
* c(3)*xest(i-3,isig,iant) + c(4)*xest(i-4,isie.iant) +
* c(5)*xest(l-5,isig,iant)
ypred (i+1,isig,iant) =c(0)*yest(i.isig,iant) +
c $\quad c(1)$ \#yest(i-1,isig,iant) + c(2)*yest(it-2,isig, iant) +
* c(3)*yest(i-3,isig,iant) + c(4)*yest(i-4.isig.iant) +
* c(5)nyest(i-5,isig,iant)

213 continue
320
continue
if (imete.eq. 1) then
. OBTAIN SQUARE-ERROR CURVE \& MEAN-SQUARE-ERROR VALUE ..ESTIMATION ERROR
Print", ESTIMATION ERROR.
Print*, MSE(e) over . (isampl-isettl)/ispsym, samples is.
print*,' (and MSE as a fraction of the actual value).
***initialise
$m s q e r(0)=0.00$
mseb10)=0.0a0
do ${ }^{2}$.
po 211 i=isettlol
do 211 , isampl, ispsym
sqerr(1)=0.00
do 212 iant =
sqerr(i) = samum, 1
seb(i) $=\operatorname{seb}(i)+$
(x(i,isig,iant) - xest(i,isig,iant))**2

212 * (x(i,isie, iant)*x(i,isig,iant) $+y(i, i s i q, i a n t) * y(i, i s i e, i a n t))$
msqer(isig) $=0.0 \mathrm{do}$
mseb(isig) $=0.0 \mathrm{do}$
do 222 i=isettl+i, isampl, ispaym
msqer(iaig) $=$ msqer(isig) + sqerr(i)
mseb(igig) $=$ mseb(isig) + seb(i)
continue
msqer(isig) $=$ msqer(isig)/dble(isampi-isetti)/dble(ispsym)
msqer(isig) $=10.0 d 0 * a l o g i o(m s q e r(i s i g))$
mseb(isig) = mseb(isig)/dble(isampl-isettl)/dble(ispsym)
mseb(isig) = 10.0dondlogio(mseb(isig))
msqer( 0 ) = msqer $(0)+\operatorname{msqer}(i s i e)$
mseb(0) $=$ mseb(0) + mseblisic
pRINT 9031, isig. msqertisig)
pRINT 9032, mseb(isig)
9031 format ( SIGNAL $=$, ,i3, MSE $=, f 15.10, \mathrm{~dB}^{\prime}$ )
9032 format (4日x,'(MSEB= , f15.10,' dB)')
210 continue
msqer(0) $=m s q e r(0) / d b l e(i s i n u m)$
mseb(0) = mseb(o)/dble(isinum)
Print 9033, isinum, msqer(0)
print 9034, mseb(0)
9033 format(' Average MSE over ', i3.' signalsx , fis.10,' dB')
9034 format $\left(52 x\right.$. ' $(=, \text { f15.10, } \mathrm{dB})^{\prime}$
*Anstore MSEe in array
plote(ithet,ib,isnr) $=m s q e r(0)$
endif
c
c
if imetp.eq. 1 ) then
..PREDICTION ERROR
Print*,' PREDICTION ERROR'
Print*,'mean square error over ', isampl-isettl,' samples is'
print".' (and MSE as a fraction or the actual value).
**initlalise
$\operatorname{msqer}(0)=0.0 \mathrm{dO}$
$m s e b(0)=0.0 \mathrm{~d} 0$
do 410 isiciei,isinum,
Print*,' sienal '.isig
do $411 t=1$ sett $1+1$, isampl, 1
sqerr(i) $=0.0 \mathrm{dO}$
seb(i) $=0.0 \mathrm{~d} 0$
do 412 lant $=1$, lanum, 1
: sqerr(i) = sqerr(i)

* (x(i,isie.iant) - xpred(i,isig,iant))**2+
seb(i) $=\operatorname{seb}(i)+$
c $c$ ( $\quad$ (x(i,isig,iant) - xpred(i,isig,fant) $) *$ * +
*c $c \quad c \quad l y(i, i s i g, l a n t)-y p r e d(i, i s i g, l a n t)) * * 2)$,
(x(i,isig.lant)*x(i,isig,iant) +y(i,jsig,iant)*y(i,isig,iant))
412 continue
msqer(isig) $=0.0 \mathrm{~d} 0$
mseb(isig) $=0.0 d 0$
do 422 isisettl+1,isampl, 1
msqer(isig) $=$ msqer(isig) + sqerr(i)
mseb(isig) $=$ mseb(isig) + seb(i)
msqer(isig) $=$ msqer(isig)/dble(isampl-isettl)
if (msqer(isig).eq. 0.0 O 0$) \mathrm{msqer(1sig)=1.0d-10}$
msqer(isig) = 10.0do*dioglo(msqer(isig))
mseb(isig) $=$ mseb(isig)/dble(isampl-isetti)
msqer ( 0 ) = msqer(0)/dblelisinum
mseb(0) $=$ mseb $(0) / d b l e(1 s t n u m)$
Print 9033. isinum, msqer(0)
print 9034, mgeb(0)
***store MSEp in array
plotplithet,ib.isnr) $=m s q e r(0)$
endif
continue
ithmax = ithet
continue
ibmax=ib
continue
continue
contlnue
isnmax = 1 snr
... OUTPUT MSE RESULTS TO Files
Print*, oufrut MSEe PESULTS TO FILE ... ploteo
open(unit=7, file='ploteo')
urite(7.) ithmax, proxe isnmax
urite( $7, *$ ) ((iplote (ithet,ib,isnr), ithet=1,ithmax, $)$
ib=1,ibmax,1\},isnr=1,isnmax,i)
close ${ }^{7}$
endif
if (imetp.eq. 1) then

Print*,' ouTPUT MSEp RESULTS TO FILE ... plotpo'
opentunit=7, file='plotpo')
write(7.*) ithmax, ibmax, isnmax
urite(7.*) (( $(\mathrm{plot} p(i t h e t, i b, i s n r), ~ i t h e t=1, i t h m a x, 1)$,
close(7)
ib=1,ibmax,i), isnr=1,isnmax,is
if (iofile.eq. 1$)$ then
. OUTPUT RESULTS TO FILES
opencunit=7, file='xesto', form='formatted')
opentunit= ${ }^{\prime}$, flle='yesto', form='rormatted')
urite(7, $)$ (i(xestifisig,iant), i=0,isampl,i), isig=i,isinum,1),
urite( 8,4 (iant=1,ianum,1)
closel7
close(8)
open(unit =7, file='xpred3o', form='formatted')
pencunit = $B$, file='ypred3o, form='formathed'
urite(7,*) (f(xpred(i,isig,iant), $i=0,1$ sampl,i), isig=i,isinum, 1$)$

* iant=1,ianum,i)

closel7
close(
endif
stop


## KAI.MEST.FORTRAN

******************
program kalmes
30...... . 40 . . . . . . 50 . . . . . . . 60 . . . . . . . 70.

## initialise

integer isettl, isampl
parameter(isettl=1200, 1 sampl $=1$ isettl+6000)
double precision $r x(0: 60000,2)$, ry(0:60000,2)
double precision sx(0:60000,2), sy(0:60000,2)
double precision x(0:isampl,2,2), y(0:isampl,2,2)
double precision sxdet(0:isampl,2), sydet(0:isampl,2)
double precision xest 0 : isampl,2,2), yestio:isampl,2,2
double precision xpred (0:1sampl,2,2), ypred(0:isampl,2,2)
double precision rxest,ryest, ex,ey
double precision $p x, p y, k x, k y$, omega, spsx, spsy
double precision sqerr(lisampl), msqer(0:2)
double precision seb(isampl), mseb(0:2)
double precision plotp(1,40,7), plote(1,40,7)
integer i,ispsym, inter, isig,isinum, iant,ianum
characterk9 modsch
integer ipaci, imetrt,irtsym
integer imete,imetp
integer iobbc, iofile
double precision svar, snrdb
integer isnr,isnmax, iome, lommax
modsch='QPSK'
***svar is the average transmitted energy per bit,
*** needed in calcn. of noise variance
if (modsch.eq.'QPSK') then
svar=1.0d0
elseif(modsch.eq.'16-pt QAM') then
svar=2.5do
else
Print*,' modulation scheme not avallable.
stop
endif
Isinum=1
janum=1
if ((isinum.ne. 1$)$.or. (ianum.ne.1)) then
PRINT*, 'NOT set up for 2 signals or two antennas.
stop
endif
ipacl $=1$
ipacl=1
***if iobbc=0 then read in $s, r, y$ arrays from stored files
***else
lobbe $=1$
***ifor no retraining set imetrt=0
***for ideal retraining set imetrt=1
***for slope retraining:- by best straight line set imetrt=3
***for slope retraining:- subtracting one from other set imetrt=2
imetrt = 0
***Assume $10 \%$ retraining, unless no retraining
if (imetrt.eq.0) then
irtsym=0
else
irtsym $=$ ipacl/10
***assume unknown initial channel conditions, and allow
*** isettl symbols for the estimators to settle down
***assume fading is generated at 600 samples per sec.
*** this must be interpolated to glve inter*600 samples per sec.
inter $=20$
ispsym=1
***calculate estimation error if imete=1
imete =1
***calculate prediction error if imetp=1
imetp=0
***output files if iofile=1
iofile=0
***PRINT system constants for this run
Print*, lanum,' ANTENNAE,
Print*, isinum,' ',modsch, SIGNAL(S)'
Print",' Packet Length $=$ ', ipacl
if (imetrt.eq.0) then
Print*, NO retraining'
elseif(imetrt.eq.1) then
Print*.' IDEAL retrainine,
Print*,'. $10 \%$ retraining therefore , irtsym,' retraining symbols. else
Print*,' Retraining method , imetrt
Print*,' $10 \%$ retraining therefore , irtsym,' retraining symbols' endif
Print
Print*, 'Run for ', isampl*ispsym,' symbols'
Print*, ispsym,' samples per symbol'
if (imete.eq. 0) then
Print*, DO NOT calculate estimation error'
else
Print..' Calculate estimation error'
endif
if (imetp.eq. 1) then
Print*, Calculate prediction error'
else
Print*,' DO NOT calculate prediction error'
endif
. .DO FOR DIFFERENT SNRS

## isnr=0

do $5 \sin d b=5.0 \mathrm{~d} 0,36.0 \mathrm{~d} 0,10.0 \mathrm{~d} 0$
(f ((snrdb.et. 75.0d0).and.(snrdb.1t. 1999.0d0)) goto 5 ignr=isnrti
Print*
Print*:
Print,, SNR $=$, snrdb, ${ }^{\prime} B^{\prime}$,
Print*,' SNR $=$, snrdb, dB'
***read in flles of signal, channel, received signal if (iobbc .eq. 0 ) then
Print* ' PEAD iN FILES.
open(unit=7, file='sxio', form='formatted')
open (unit $=$, file=, syio.' form=, Pormatted',
open(unit $=6$, rile='syio, form='rormatted
opencunit=9, file='yio: form='rormatted')
read ( $7, *$ ) ( $(s x(i, i s i g), i=0, i s a m p l, 1), i s i g=1, i s i n u m, 1)$



```
* iant=1, ianum, 1
```

read (10,*) ((y(i,isig,iant), $1=0$, isampl, 1 ), isig=1,isinum, 1$)$,
close(7)
close(8)
close (9)
close(10)
open(unit=11, file='rxio', form='formatted')
opentunit=12, file='ryio', form='rormatted')
read (11,*) ( (rx(i,fant), $i=0, i s a m p l, 1)$, iant $=1$, fanciä, 1
read(12,") ((ry(i,iant), i=0,isampl,i), iant=1,ianum,1)
close (11)
close(12)
Print*, subroutine sbbchan'
call sbbchanlisampl,ipacl,irtsym, inter, snrdb,

* modsch, svar, isinum, ianum, $r x, r y, s x, s y, x, y$ )

Print*, RETURNED sbbchan'
endif
do 9991. Lant:1. lanum, 1
Print
(iant.eq. 1 ) Print*.'ANTENNA a

do $9993 i=1$, isampl,
if (i.eq. 3) $i=i s a m p l-1$
do 9995 isie: , isinum,
Print*,'SIGNAL •isic
Print 9001, i,sx(i,isig),sy(i,isig), x(f,isig.iant),y(i,isig,iant)
continue $2 x, 2$.1, $2 f 12.4$
continue
Print 9002, rx(i,iant), ry(i,iant)
9002 format.10x, 2 f9.4
9993 continue
9991 continue
***assume perfect detection i.e. detected $s=a c t u a l s$
do 99 isig=1.isinum,
do 99 i=1,isampl:1
sxdet(i,isic) $=s x(i, t s i e$
sydet(i.isif) $=s y(i, i s i g$
continue
REPEAT RUN fOR DIFFERENT VALUES OF CONSTANT w iome =0
do 1 omega=0.05d0,0.951d0,0.05d0
iome = iome +1
x = 1.0do/omee
py = 0.0do
.. REPEAT RUN FOR EACH ORDER PREDICTOR IN TURN
$\cdots$. REPEAT RUN FOR EACH ORDPE-1 NO prediction
do 2 iordp $=-1,-1$,
Print*,' ---.....-.
Print".'Kalman Estimator,
omega= ', omega
Print". 'Kalman Estimato
do 111 i=1
do 111 l=1, isampl, 1
rxest = sxdet(i,isig)*xest(i-1,isig.iant)
sydet(i,isig)*yest(if-1,isig,iant)
ryest $=$ sxdet (i, isig)*yest (i-1,isig,iant $)$
sudetifisig)*xest(i-l.isin.iant)
ex $=$ rx(i,iant) - rxest
ey $=$ ry(i,iant) - ryes
merlisie) $=0.0 \mathrm{~d} 0$
mseblisle) $=0.0 \mathrm{~d} 0$
do 222 l=isettl +1 , isampl,
msqer(isig) $=\operatorname{msqer}(i s i g)+s q e r r(i)$
mseb(islg) $=$ mseb(ligig) +seb(i)
msqer(isig) $=$ msqer(isig)/dbletisampl-isetil)
msqer(isig) $=10$. Odondioglo(msqer(isig)
mseb(isig) $=$ mseb(isigi/dblelisampl-isettl
mseb(isig) $=10.040 * d l o g 10(m s e b(i s i g i)$
msqer(0) $=\operatorname{msqer}(0)+m s q e r(i s i c)$
mgeb $(0)=m \operatorname{meb}(0)+m s e b(i g i g)$
pRINT 9031, isig msqer(isig)
pRINT 9032, mseb(istg)

```
9031 format(' SIGNAL = ',i3.' MSE= ',r15.10,' dB')
9032 format(48x,'(MSEB= ',f15.10.' dB)')
210 continue
msqer(0) = msqer(0)/dblelisinum
mseb(0) = mseb(0)/dble(lsinum)
Print 9033, isinum, msqer(0)
print 9034, msebi0
    9033 format(' Average MSE over ',i3,' gignals= ',f15.10,' dB')
    9034 format(52x,' (=,.f15.10,' dB)')
c M**store MSEE in array
    MSE (n) =moqerio
    endif
    2 \mp@code { c o n t i n u e }
    continue
    commax=lome
    continue
    isnmax = ismr
c
f (imete.eq TO FILES
    Print*, OUTPUT MSFe RESULTS TO FILE ... ploteo
    open(unit=7, file='ploteo')
    write(7,*) 'I',iommax,isnmax
    urite(7.*) ((plote(1, lome,isnr).
    close(7)
        iome=1,iommax,1), isnr=1,isnmax,1)
        endif
c
    stop
    end
* SINEWAVE.FORTRAN
* SINEWAVE.FORTRAN
.....7..........20........ . 30.... . . . .40. . . . . . . 50. . . . . . . 60. . . . . . . . }7
program sinewave
```


## initlalise

double prectsion $\times 1(0: 7001)$, xest $1(0: 7001)$ double precision $x(0: 7001)$, dc(0:7001) double precision xpredi(0:7001 double precision double precision double precision double precision double precision double precision double precision
double precision double precision double precision integer no, n90,
double precision double precision
double precision double precision ds90 dention thetdc double precision $c, k c, k c 2, k c 3$, thetad, dsipre, thlid, thi2d double precision errds, ds2pre, th21d. th22d, th23d
integer iordds double precision double precision
double precision double precision double precision double precision double precision double precision double precision Integer iorde double precision double precision
double precision double prec
integer $j$
double precision
integer intion
integer imetho
intart $=15$
isampl $=2200+1$ start
pl $=3.141592654 \mathrm{dO}$
thetaa=0.765d0
thetaa=0.1.00do
thetad $=0.995 \mathrm{~d} 0$
thetae= 0.845 d 0
forde=2
iordas=2
iordds $=2$
ime tho = 2
***subtract kx*sine wave
$k x=0.740 d 0$
***subtract koc*dc eajn
$\mathrm{kdc}=2.0 \mathrm{~d} 0$
***scale the value of $b$
$\mathrm{kb}=1.000 \mathrm{~d} 0$
***scale the value of $c$
$k c=1.000 \mathrm{~d} 0$
***scale the value of $c$ samples
$k c 3=1.0 \mathrm{~d} 0$
**read in this fading sequence and its estimate
Print*, 'READ IN FADING SEQUENCE AND ITS ESTIMATE.
open(unit=7, file='sino', form='formatited'
read(7,*) (xi(i), $1=1$, isampl,1)
close (7)
opencunit=', file='sinesto', form='formatied'
read (7,*) (xesti(i), $i=1, i$ isampl,i)
close (7)
***do for different values of theta? AND k?
do 2 thetad $=1.025 d 0,1.202 \mathrm{~d} 0.0 .500 \mathrm{do}$
do $3 \mathrm{kx}=1.000 \mathrm{~d} 0,1.003 \mathrm{~d} 0,0.050 \mathrm{~d} 0$
**initialise predictor constats
as2pre $=0.0 \mathrm{~d} 0$
aspred $=0.0 \mathrm{0d0}$
aspred $=0.0 \mathrm{~d}$
thita $=(1.0 d 0-$ thetaa) $11.0 d 0$ - thetaa
thl2a $=1.0 \mathrm{do}-$ thetaa*thetaa
th2 0 . 5 d 0 (1. 01.
)*(1.0dO-thetaa)*(1.0dO+thetaa)

- thetaa**3
dc2pre=0.0d0
dcpred=0.0d0
th21dc $=0.50$
(1.0d0 - thetdc)**3
h2dc $=1.5 d 0 *(1.0 d 0-$ thetdc $) *(1.0 d 0-$ thetdc $) *(1.0 d 0+$ thetdc $)$
th23dc $=1.0 \mathrm{~d} 00^{\circ}$ thetdc**
e2pred=0.0d0
elpred=0.0do
epred (istart) $=0.0 \mathrm{~d} 0$
thile $=(1.0 d 0$ - thetae) $*(1.0 d 0-$ thetae $)$
thize $=1.0 d 0$ - thetae*thetae
th2le 0.50 wil.0d0
(ae)*(1.Od0-thetae)*(1.0d0+thetae)
- thetaen" 3
dsipre=0.0d0
silpre=0.0d0
ds $90 p=0.040$
$\mathrm{ds} 270 \mathrm{p}=0.0 \mathrm{~d} 0$

thi2d $=1.0 d 0$ - thetad\#thetad
th12d $=1.0 d 0-(t h e t a d * t h e t a d$
$\operatorname{th} 21 d=0.5 d 0 *(1.0 d 0-$ thetad $) * 3$
th22d $=1.5 d 0 *(1.0 d 0$-thetad)*(1.Odo-thetad)*(1.OdO+thetad
th23d $=1.0 \mathrm{~d} 0-$ thetad**3
th23d = 1.00
***start up
dee270 $=-30.00$
deg $270=-30.0$
deg9p=55.0d0
deg9p=55.0d0
a $90=1$. 0 d 0
asipre $=0.0 \mathrm{~d} 0$
aspred $=0.9 \mathrm{~d} 0$
aspred $=0.9 \mathrm{~d} 0$
dspred $=80.0 \mathrm{do}$ dspred=80.0d0 $d s 90 p=80.000$
tot $b=20.72 d 0$
totb $2=-21.84 \mathrm{dO}$
$+202=-21.840$
totc $=-7.93 \mathrm{~d} 0$
totc2 $2=-18.43 \mathrm{do}$
$\operatorname{totc} 3=16.25 \mathrm{~d} 0$
totch $=+80.0 \mathrm{~d} 0 / \mathrm{pi}$
totq=20.0dO
sum0ㅍ․ 37.46
sum90 97.46 do
sum180 $=-13.46 \mathrm{~d} 0$
sum270 $=-13.46 d 0$
n $0=40$
n) $80=40$
n270 $=40$
$\mathrm{dc} 0 \mathrm{p}=0.3 \mathrm{~d} 0$
$d c 90 p=0.3 d 0$
$\mathrm{dc} 180 \mathrm{p}=0.3 \mathrm{~d} 0$
$d c$
$d \quad 270 p=0.3 d 0$
..FADING PREDICTOR INC. SINE WAVE PREDICTOR
Print., 'fading predictor Inc. SINE WAVE PREDICTOR.
do : i=istart, isampl 1
..TAKE APPROPRIATE PATH IN PROGRAM DEPENDINC ON THE ANGIE
.. of the sine wave component
***if i is .ge. the predicted odegree point
if (iquad.eq. 270 ), and. (i.ge.degop)) then
***ESTIMATE the peak at the PREVIOUS 270degrees
a270 = kb*5.27898do *(totb +(2.0d0/pi)*totb2)/totp
$\mathrm{a} 270 \mathrm{z}=\mathrm{kb*5.27898d}$
$\mathrm{~b}=\mathrm{a} 270$ - aspred
***PREDICT the peak at the NEXT 90degrees
if (iordas.eq.1) then
...degree ।
asipre = asipre + thl1a*b
+ thila*b
aspred = aspred + asipre + thi2anb
else
gree 2
as2pre $=$ as2pre + th21a*b
asipre = as1pre + 2.0d0*as2pre + th22a*b
aspred $=$ aspred + asipre - as2pre + th23a*b
endif
***reset totals, sum, sum count and quadrant
totb=0.0do
totb2 $=0.0 \mathrm{~d} 0$
$\operatorname{totp}=0.0 \mathrm{dO}$
sumo
no $0=0$
iquad $=0$
do $1001 \mathrm{j}=\mathrm{deg} 180, \operatorname{deg} 270,1$
plot1(j) = a270* dsin(pi/ds270*(j-dep180+ds270))
continue
***if i is .ge, the predicted 90degree point
elseif((lquad.eq.0) and. (1.ge.deg9p)) then
***ESTIMATE ds between PREVIOUS 27Odegrees and HERE 90degrees
*** Therefore estimate deg90 (and dego)
***irstly find $c$ in rad. Then SUBTRACT $c$ in samples from dspred
$c=$ datan(kc*5.27898do *(totc -(2.0d0/pi)"totc2)/totc3)
c = -c*dspred/pi
deg90 = deg9p + c
ds90 = deg90 - deg270
\#**PREDICT ds between HI:RE 90derrees and NEXT 2loderfees
c *** Therefore predict decz7p (and deel8p) errds $=$ ds $90-\mathrm{ds} 90 \mathrm{p}$
if (iordds.eq.1) then
c ...decree
dslpre $=$ dsipre + thlldnerrds
ds270p $=$ ds90p + dsipre * th12deerrds
elseif(iordds.eq.2) then
c $\quad$..degree 2
ds2pre = ds2pre + th21d*errds
dsipre = ds1pre + 2.0d0\#ds2pre + th22dnerrds
ds270p = ds90p + ds1pre - ds2pre + th23d*errds endif
dspred=ds270p+kc3*c
dec27p $=$ deg9p + dspred
deg18p $=$ deg $9 p+0.5 d 0 * d s p r e d$
c ***reset totals, sum, sum count and quadrant
totc $=0.0 \mathrm{dO}$
$\operatorname{totc} 2=0.0 \mathrm{~d} 0$
totc $3=0.0 \mathrm{~d} 0$
totect =0.0d0
totq=0.0d0
sum90 $=0.0 \mathrm{~d} 0$
n90 $=0$
iquad = 90
do 1002 jrdeg 270. dego. 1
plot1(j) = a270* dsin(pi/ds90 *(j-deg270+1.5d0*ds90))
continue
***it i is . Ge. the predicted lifodegree point
elself(tiquad.eq.90) and. (i.ge.degitp)) then
**ESTIMATE the peak at the PREVIOUS 90degrees
a90 $=k b \pm 5.27898 d 0 *($ totb $-(2.0 d 0 / p i) \# t o t b 2) / t o t p$
$b=a 90-$ aspred
c . **PREDICT the peak at the NEXT 2\%0degrees
c If (iordas.eq.l) then
c ...degree
asipre = asipre + thlyanb
aspred $=$ aspred + asipre + thl2a*b
else
c ...degree 2
as2pre $=$ as2pre,$~ t h 21 a * b$
as1pre = asipre + 2.OdO*as2pre + th22a*b
aspred = aspred + as1pre - as2pre + th23a*b
endif
c ***reset totals, sum, sum count and quadrant
totb $=0.0 \mathrm{do}$
$\operatorname{totb2}=0.0 \mathrm{~d} 0$
totp $=0.0 \mathrm{do}$
$\operatorname{sum} 180=0.0 \mathrm{~d} 0$
n1 $80=0$
1 quad $=180$
do $1003 \mathrm{j}=\mathrm{dego,deg} 90,1$
ploti(j) = a90* dsin(pi/ds90 *(j-dego)) continue
**ilf i is .ee. the predicted 270degree point
elseif((iquad.eq. 180 ). and. (i.ge.deg27p)) then
***ESTMATE ds between Previous yodegrees and HERE 270degrees
** Therefore estimate dee 270 (and degi80)
**firstiy find cin rad. Then SUBTRACT c in samples from dspred $c=$ datan (kce5.27898d0 *(totc +(2.0d0/pi)ntotc2)/totc3) $c=-c$ algmpert/nj
deg270 $=\operatorname{deg} 27 \mathrm{p}+\mathrm{c}$
ds270 = deg270 - deg90
deg180 $=\operatorname{deg} 90+0.5 \mathrm{~d} 0 * d s 270$
c ***PREDICT ds betheen HERE 270degrees and NEXT 90degrees
c *** Threrfore predict deg9p (and degop)
errds $=\mathrm{ds} 270-\mathrm{ds270p}$
if (liordds.eq.1) then
c ...degree
dsipre z dslpre + thild*errds
ds90p = ds270p+ds1pre + th12d*errds
elseif (iordds.eq.2) then
c ...degree 2
ds2pre $=$ ds2pre + th21d*errds
ds1pre $=$ ds1pre +2.0 donds2pre + th22d*errds
ds90p $=d s 270 p+d s 1 p r e-d s 2 p r e+t h 23 d x e r r d s$
endif
dspred $=d s 90 p+\mathrm{Hc} 3 * \mathrm{c}$
deg9p $=$ deg27p + dspred
degop $=$ deg27p $+0.5 d 0 * d s p r e d$
c ***reset totals, sum, sum count and quadrant
totc $=0.0 \mathrm{~d} 0$
totc $2=0.0 \mathrm{dO}$
totc3 $=0.0 \mathrm{~d} 0$
totch $=0.0 \mathrm{~d} 0$
totq=0.0do
sum270 $=0.0 \mathrm{~d} 0$
$n 270=0$
quad $=270$

continue
else
c endiculate thetas (i) and thetas (i+1)
if (iqquad.eq. 0 ) then
thets $0=\mathrm{pi} / \mathrm{dspred} *(i-d e g 0 p)$
thets $1=$ pi/dspred $(1+1-$ degOp $)$
elseif(iquad.eq. 90 ). or. (iquad.eq. 180 )) then
thetso = pi/dspred (i - deg9p + 0.5do*dspred)
thets $1=$ pi/dspred *(i+i - deg9p + 0.SdO*dspred)
else
thetso $=$ pi/dspred *(i - deg27p + 1.5d0*dspred)
thetsi=pi/dspred *(i+i-deg27p + 1.5do*dspred endif
c ...SINE WAVE PREDICTION
$x(i)=($ aspred $) * d s i n(t h e t s o)$
$x(i)=$ aspred)*asin(thetso)
$x(i+1)=($ aspred)
RESIDUAL FADING PREDICTOR
if (imetho.eq. 1 ) then
e = xesti(d) - kx*xif) -kdc*dc(i)
elseif (imetho.eq.2) then
= xesti(ij)-kx*x(i)
else
*** predictor
errpe $=$ e - epred(i)
if (iorde.eq.1) then
c ...degree ;
elpred = elpred + thlle*errpe
epred $(i+1)=$ epred(i) + elpred + thl2e*errpe


## .degree 2

e2pred $=$ e2pred + th2lenerrpe
elpred $=$ e1pred $+2.0 d 0 * e 2 p r e d+$ th2ze*erpe
epred $(i+1)=$ epred(i) + elpred - e?pred + th23e*errpe
endif
. FROM TIME IT
if (imetho.eq.1) then
xpredi(i+1)=kx*x(i+1)+epred(i+1)+kdc\#de(i+i)
elseif (imetho.eq.2) then
xpredi $(i+1)=k x * x(i+1)+\operatorname{epred}(i+1)$
else
end if
**for amplitude update
cotb $=$ totb + xesti(i)ndsin(thetso)
totb2-totb2 + xestill
(thets0)*dsin(thets0)
**for phase update
otc=tolc + xesti(i)ndcostthetso
rotc2=とotc2 + xestl(i)
totc $3=$ totc 3 + xesti(i)*dsin(thetso)
otct $=$ totch + dcos(thetsO)
totq=totq + dcos(thetso)*dcos(thet.s0)
continue
.CALCULATE PREDICTION SQUARED ERRORS
Print*, PREDICTION SQUARED ERRORS.
do $32 i 1=i s t a r t$, isampl,i

* (x1(i) - xpredi(i))**2
continue
error
msepre=0.0do
do 331 I=1start,isampl;
msepre $=$ msepre + sepred(i)
msepre $=$ msepre/dble(isampl-istart
Print..'kx= ',kx.' kdc = ',kdc
Printa, kx , kc=
thetaa = ', thetaa, thetdc $=$, thetdc.' thetad $=$ ', thetad
Print*,'the
Print*,' Mean-squared-error over ', isampl-istart,' samples is'
pre $=\cdot$, msepre
continue
c ...OUTPUT RESULTS
open(unit =7, file='xo', form='formatted')
open(unit=7, file='xo', forma'rormatled $\begin{aligned} & \text { open(unjt=8, file='xpredlo', form='formated') }\end{aligned}$
urite (7,*) (x(i), i=1,isampi,i)

cloge(7)
close(8) file='plotio, form='formatted'
urite(7,*) (ploti(i), $i=1$, isampl,i)
crite(7;


## stop

end

## ******************** <br> EQUALIZERZ.FORTRAN

c..... 7
7.............. . 20. zer 2
c
double precision double precision double precision double precision double precision double precision double precision double prectision isampl $=1000$
isampl $=$
**read
**read in files of signal, channel, recelved sipnal opencunit=7, file='sxo', form='formatted') pentunit=8, file='syo', form='formatted
penfunitio file='yo: form=, formatted ,
pen(unit=11, file='rxo' form='formatted'

(
reade*) (sy(i) $i=0$ isampl 1
read (9,*) (x(i), $i=0$, isampl, i)
read (10, *) (y (i), $i=0$ isampl 1$)$
read (11, ) (rxcii, $i=0$ isampi $)$
read (12, *) (ry(i), $i=0, i=a m p, 1)$
read(17;") (ry(1), $1=0,13 a m p 1,1)$
close(7)
close(8)
close(9)
close(9)
close(10)
close(10)
close(111)
close(12)
***form inverse-channel coefficients cx/cy
do $98 \mathrm{i}=0,1 \mathrm{sampl}, 1$
$c x(i)=x(i) /(f(i) * x(i))+(y(i) x y(i)))$
$c y(i)=-y(i) /(1 x(i) * x(i))+(y(i) * y(i)))$
continue
**inverse-channel estimate at $t=0$ is assumed to be exact cxest $(0)=c \times(0)$
cxest $(0)=c x(0)$
cyest 0$)=c y(0)$
***assume perfect detection i.e. estimates of $s=a c t u a l s$
***assume perfect
do $99 \quad i=1$, isampl, 1
sxest(i) $=3 x(i)$
sxest(i) $=\operatorname{sx(i)}$
syest(i)
continue
...CHANNEL EQUALIZER
do 111 i=1, isampl, 1
$x x(i)=r x(f) * c x e s t(i-1)-r y(i) * c y e s t(i-1)$
$x y(i)=r x(i)+c y e s t(i-1)+r y(i)+c x e s t(i-1)$
ex(i) $=x x(i)-s x e s t(i)$
ey(i) $=x y(i)-$ syest(i)
cxest(i) $=$ cxest(i-1) - a*( ex(i)*rx(i) + ey(i)*ry(i) cyest(i) = cyesti(i-i)-a:(-ex(i)nry(i) +ey(i)arx(i))

```
...OBTAIN SQUARE-ERROR CURVE & MEAN-SQUARE-ERROR VALUE
        do 211 i=1, isampl.l
        sqerr(i) = (ex(i)))**2 +(ey(i))**2
    211 sqerr(i)
continue . 0.0do
msqerr = 0.0d0
do 212 i=1,isampl.1
msqerr = msqerr + sqerr(i)
212 continue
msqerr = msqerr/dble(isampl)
!print*,'mean square error over ',isampl.' samples is ',msqerr
c
...OUTPUT RESULITS TO FILES
    open(unit=7, file='cxo', form='formatted')
    open(unit=8, flle='cyo', form='formatted')
    open(unit=9, file*'cxesto', form='formatted')
    open(undt=10, flle='cyesto', form='formatted'
    write(7,*) (cx(i), i=1,isampl,1)
    urite(8,*) (cy(i),i=1,isampl,i)
    urite(9,*) (cxestr(i), (=1,isampl,1)
    write(10,*) (cyest(i), i=1,isampl.1)
    close(7)
    close(9)
    close(9)
    cloge(10)
c
end
```



