Multi-Variable Control of a High Redundancy Actuator

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Abstract: The High Redundancy Actuator project deals with the construction of an actuator from many redundant actuation elements. Whilst this promises a high degree of fault tolerance, the high number of components also poses a unique challenge from a control perspective.

This paper shows how the state space model of a stack of actuation elements in series can be separated into a high dimensional internal and a low dimensional external subspace. Once the internal states are decoupled and damped, the behaviour is dominated by the few states of the external subspace. This means that the high redundancy actuator with many redundant elements behaves just like a conventional single actuator.
Keywords: electromagnetic actuation, fault tolerance, multi-variable control

1 High Redundancy Actuation

High Redundancy Actuation (HRA) is a novel concept of designing a fault tolerant actuator that comprises a relatively large number of actuation elements (see Figure 1). As a result, faults in the individual elements can be inherently accommodated without resulting in a failure of the complete actuator system.[1]

The concept of the High Redundancy Actuation (HRA) is inspired by the human musculature. A muscle is composed of many individual muscle cells, each of which provides only a minute contribution to the force and the travel of the muscle. These properties allow the muscle as a whole to be highly resilient to damage of individual cells. The aim of this project is not to replicate muscles, but to use the same principle of co-operation with existing technology to provide intrinsic fault tolerance.

An important feature of the High Redundancy Actuator is that the elements are connected both in parallel and in series. While the parallel arrangement is commonly used, the serial configuration is rarely employed, because it is perceived to be less efficient. However, the use of elements in series is the only configuration that can deal with the lock-up of an element. In a parallel configuration, this would immediately render all elements useless, but in the series configuration it only leads to a slight reduction of available travel (see Steffen et al. 2007b, 2008 for details).

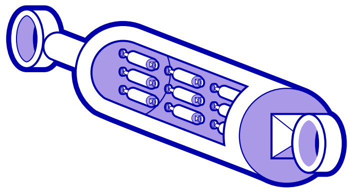

Figure 1: High Redundancy Actuator

2 Motivation

Because the parallel configuration is already well studied, this paper focuses on the use of elements in series. This is more challenging from a control point of view, because each element is a moving mass, and the model needs to describe the position and speed of each mass separately. For example, the element at the bottom of the assembly experiences a higher load, because it needs to move all the other elements in addition to the load.

For the envisioned number of elements (10x10 or more), this may lead to a model with hundreds of states, which would be too complex even for advanced multi-variable control approaches. Thus the goal of this paper is to reduce the model complexity to a level comparable to a conventional actuator.

The basic idea is to split the travel equally between all actuation elements. If this is achieved, the states of the elements are no longer individual variables, and they can all be reduced into a single simple model. In other words: because the whole system behaves like a single conventional actuator, a simple conventional actuator model is sufficient to describe it.

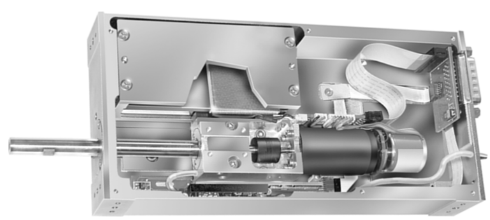

Figure 2: Electromechanical actuator

[1]This project is a cooperation of the Control Systems group at Loughborough University, the Systems Engineering and Innovation Centre (SEIC), and the actuator supplier SMAC Europe limited. The project is funded by the Engineering and Physical Sciences Research Council (EPSRC) of the UK under reference EP/D078350/1.

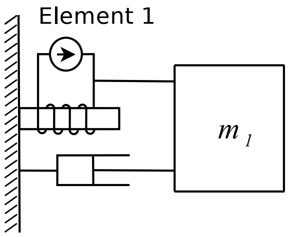

Figure 3: Dynamic components of a single element

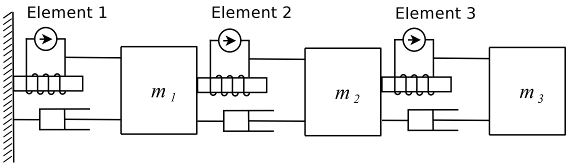

Figure 4: 3 Elements in Series

3 System Model

The basic components of an electromechanical actuation element are shown in Figure 3. From a modelling perspective, it is a typical single mass system, which can be described by NEWTONian mechanics. Three forces act upon the mass: the electromagnetic force $F_{el} = ki$, the damping force $F_d = dv$, and the spring force $F_s = rx$ (see Davies et al. 2008 for more details). Together, they lead to the second order differential equation

$$m\ddot{x} = ki - d\dot{x} - rx \quad .$$

Choosing x and $\dot{x}$ as states leads to the model:

$$\frac{d}{dt}\begin{pmatrix} \dot{x} \\ x \end{pmatrix} = \begin{pmatrix} -\frac{d}{m} & -\frac{r}{m} \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \dot{x} \\ x \end{pmatrix} + \begin{pmatrix} \frac{k}{m} \\ 0 \end{pmatrix} i \quad .$$

In the case of several actuation elements in series, each element creates forces between neighbouring masses, so each mass is subject to forces from both sides. The resulting model for three actuation elements (as shown in Figure 4) is:

$$\frac{d}{dt}\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{i} \tag{1}$$

with $\alpha = \frac{1}{m_1} + \frac{1}{m_2}$, $\beta = \frac{1}{m_2} + \frac{1}{m_3}$,

$$\mathbf{A} = \begin{pmatrix} -\frac{d_1}{m_1} & -\frac{r_1}{m_1} & \frac{d_2}{m_1} & \frac{r_2}{m_1} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{d_1}{m_1} & \frac{r_1}{m_1} & -\alpha d_2 & -\alpha r_2 & \frac{d_3}{m_2} & \frac{r_3}{m_2} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{d_2}{m_2} & \frac{r_2}{m_2} & -\beta d_3 & -\beta r_3 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} \frac{k_1}{m_1} & -\frac{k_2}{m_1} & 0 \\ 0 & 0 & 0 \\ -\frac{k_1}{m_1} & \alpha k_2 & -\frac{k_3}{m_2} \\ 0 & 0 & 0 \\ 0 & -\frac{k_2}{m_2} & \beta k_3 \\ 0 & 0 & 0 \end{pmatrix} \quad ,$$

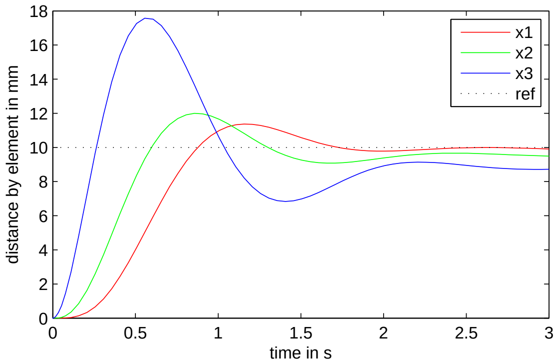

Figure 5: Delay between elements

state $\mathbf{x} = (\dot{x}_1\ x_1\ \dot{x}_2\ x_2\ \dot{x}_3\ x_3)^T$, and input $\mathbf{i} = (i_1\ i_2\ i_3)^T$. The overall extension is $y = x_1 + x_2 + x_3$.

The main problem with this system is that the elements are not used in an equal way. If the same input is used for all three elements, the top element (x_3) moves first, because it has the lightest load. Then the middle element (x_2) begins to move, and finally the element on the base (x_1) will respond. So the step moves through the system like a longitudinal wave.

This is shown in Figure 5 for a nominal system with $d_i = 2$, $r_i = \frac{1}{2}$, $k = 1$, $m_1 = m_2 = 0.5$ and $m_3 = 1$. A simple single input/single output (SISO) proportional controller with a phase lead compensator is used

$$K(s) = 2\frac{0.4s+1}{4s+1} \quad , \tag{2}$$

and a reference step of 30 mm (10 mm per element) is simulated. Since this kind of wave propagation complicates the control of the actuator, the next two sections present ways to eliminate the time delay and to align the movement of all elements.

4 Parameter Tuning

One of the goals of the HRA is to spread the travel equally between the elements

$$x_1 = x_2 = x_3 \quad . \tag{3}$$

This cannot be achieved directly, but the model can be tuned for

$$\ddot{x}_1 = \ddot{x}_2 = \ddot{x}_3 \tag{4}$$

if x_i and $\dot{x}_i$ are equal. Since the system is linear, it is sufficient to satisfy Equation (4) for the two basis vectors $\mathbf{x}_v = (1\,0\,1\,0\,1\,0)^T$ and $\mathbf{x}_p = (0\,1\,0\,1\,0\,1)^T$ (assuming $\mathbf{i} = \mathbf{0}$ for now).[2]

If the mechanical parameter are used for the tuning, this leads to the two equations

$$-\frac{d_1}{m_1} + \frac{d_2}{m_1} = \frac{d_1}{m_1} - \frac{d_2}{m_1} - \frac{d_2}{m_2} + \frac{d_3}{m_2} = \frac{d_2}{m_2} - \frac{d_3}{m_2} - \frac{d_3}{m_3}$$

[2]For more details on the geometric approach and invariants see Wonham, 1985, Basile and Marro, 1992.

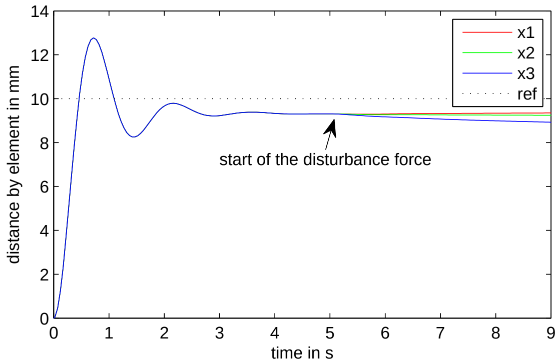

Figure 6: Step response after parameter tuning

$$-\frac{r_1}{m_1}+\frac{r_2}{m_1}=\frac{r_1}{m_1}-\frac{r_2}{m_1}-\frac{r_2}{m_2}+\frac{r_3}{m_2}=\frac{r_2}{m_2}-\frac{r_3}{m_2}-\frac{r_3}{m_3}$$

which solve to

$$d_2 = d_3+\frac{2m_2}{3m_3}d_3 \quad (5)$$

$$d_1 = d_2+\frac{1m_1}{3m_3}d_3 \quad (6)$$

$$r_2 = r_3+\frac{2m_2}{3m_3}r_3 \quad (7)$$

$$r_1 = r_2+\frac{1m_1}{3m_3}r_3 \quad . \quad (8)$$

Under these conditions, the system has two modes that satisfy the condition of equal spread of travel. It is necessary to align the input signal with these modes. Using the same approach, the result is

$$k_2 = k_3+\frac{2m_2}{3m_3}k_3 \quad (9)$$

$$k_1 = k_2+\frac{1m_1}{3m_3}k_3 \quad (10)$$

assuming all inputs are equal ($i_1 = i_2 = i_3$).

The result of this tuning is shown in Figure 6. The parameters of the third element are equal to the step response in Figure 5, and the other elements are tuned accordingly. Clearly the delay between the elements has been eliminated, and they all respond at the same time. The disturbance response (at $t = 5$) still deviates slightly between the elements, but the difference is small and not significant for most practical purposes.

5 Tuning Using Feedback

Since the tuning of mechanical parameters is not always possible, the second approach for travel equalisation uses feedback. The inputs i_1 and i_2 will receive proportional feedback based on the acceleration

$$a = \ddot{x}_1+\ddot{x}_2+\ddot{x}_3 = -\frac{d_3}{m_3}\dot{x}_3-\frac{r_3}{m_3}x_3+\frac{k_3}{m_3}i_3 \quad (11)$$

of the load m_3, and feed-forward from i_3:

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} a_3 + \begin{pmatrix} i_3 \\ i_3 \end{pmatrix} \quad (12)$$

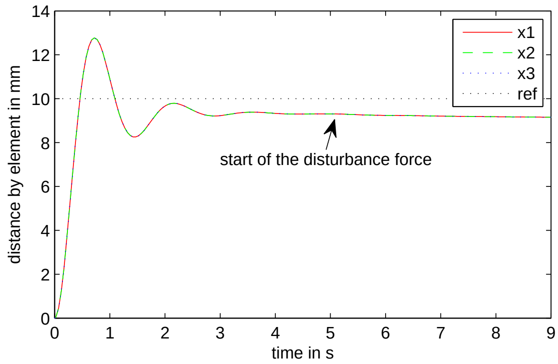

Figure 7: Step response after acceleration feedback

where $f_1, f_2 \in \mathbb{R}$ are coefficients to be determined. This leads to an augmented model

$$\frac{d}{dt}\mathbf{x} = \mathbf{A}_F\mathbf{x}+\mathbf{B}_F i_3 \quad (13)$$

(the matrices are omitted for space reasons), and again the requirement is Equation (4). This is similar to a disturbance decoupling problem [Commault et al., 1997], but the two subspaces are already defined, which simplifies the solution. Assuming that all elements are equal ($d_1 = d_2 = d_3$, $k_1 = k_2 = k_3$ and $m_1 = m_2$), and using $\mathbf{x} = \mathbf{x}_v$, $i_3 = 0$ this leads to the equation

$$f_1 - f_2 = 2f_2 - f_1 = \frac{m_1}{k_1} - f_2 \quad (14)$$

with the solution

$$f_1 = \frac{m_1}{k_2} \quad , \quad f_2 = \frac{2}{3}f_1 \quad . \quad (15)$$

The alignment is also satisfied for the second basis vector $\mathbf{x}_p$ and the input, as long as the springs ($r_1 = r_2 = r_3$) and force constants ($k_1 = k_2 = k_3$) are equal.

Again the subspaces representing unequal extensions of the elements have been decoupled from the input. With this solution, the decoupling also includes disturbance forces on the load m_3, because they are measured via a_3 and distributed equally over the elements. The simulation result is shown in Figure 7: all elements show the same response both to the set-point change and to the disturbance force.

6 State Reduction

To separate the two different subspaces, the following state transformation can be used:

$$\mathbf{T} = \begin{pmatrix} 1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 \\ 1 & 0 & -3 & 0 & -\frac{3m_3}{2m_2} & 0 \\ 0 & 1 & 0 & -3 & 0 & \frac{3m_3}{2m_2} \\ 1 & 0 & 1 & 0 & 1+\frac{3m_3}{2m_2} & 0 \\ 0 & 1 & 0 & 1 & 0 & 1+\frac{3m_3}{2m_2} \end{pmatrix}^{-1} \quad (16)$$

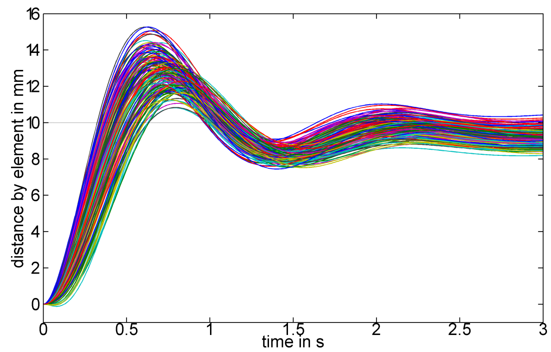

Figure 8: Responses with 5% parameter tolerances

The first two columns obviously span the part of the state space with equal extent and velocity, while the remaining columns span the remaining modes of the system. Applying this transformation to the tuned system from Section 4 (and the system from Section 5 gives similar results) leads to the model

$$\frac{dt}{d}\mathbf{x}' = \mathbf{A}'\mathbf{x}' + \mathbf{B}'i \tag{17}$$

with the new state $\mathbf{x}' = \mathbf{T}\mathbf{x}$ and matrices

$$\begin{aligned}\mathbf{A}' &= \begin{pmatrix} -\frac{d}{3m_3} & -\frac{r}{3m_3} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_1 & a_2 & a_3 & a_4 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & a_5 & a_6 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \\ \mathbf{B}' &= (k\,0\,0\,0\,0\,0)^T \quad .\end{aligned}$$

As this model clearly shows, the dynamics of the system are determined by first two states. The other states are not excited by the input (as can be seen by the zeros in $\mathbf{A}'$ and $\mathbf{B}'$), so they remain close to zero.

7 Robustness

If there are parameter tolerances or faults in the system, the model of the system changes, and the decoupling is no longer perfect. So a slight change in behaviour is expected. However, since the secondary modes are fast and well damped, they will not have a significant influence on the overall behaviour. The influence of random 5% parameter tolerances is shown in Figure 8, and the behaviour after a lock-up of the right element $x_3 = 0$ is shown in Figure 9. As predicted, the alignment between the elements is still very close. For a more detailed analysis see Steffen et al. [2007a].

8 Conclusion

Two methods have been presented that can equalise the motion of the elements in the HRA. The resulting behaviour is identical to a single classical actuator, and the state space model can be reduced to two states. Although the method has been demonstrated for three elements, it scales well and it can easily be applied to arbitrary complex configuration. The robustness to parameter variations and faults has been demonstrated using an example, but a more detailed analysis is a matter of continuing research.

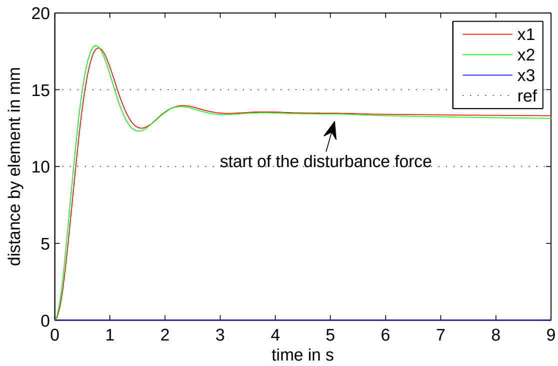

Figure 9: Response with single element looked up

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