Blind adaptive channel shortening with a generalized lag-hopping algorithm which employs squared auto-correlation minimization [GLHSAM].

BLIND A DAPTIVE CHANNEL SHORTENING WITH A GENERALIZED LAG-HOPPING ALGORITHM WHICH EMPLOYS SQUARED AUTO-CORRELATION MINIMIZATION [GLHSAM]

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ABSTRACT

A generalized blind lag-hopping adaptive channel shortening (GLHSAM) algorithm based upon squared auto-correlation minimization is proposed. This algorithm provides the ability to select a level of complexity at each iteration between the sum-squared autocorrelation minimization (SAM) algorithm due to Martin and Johnson and the single lag autocorrelation minimization (SLAM) algorithm proposed by Nawaz and Chambers whilst guaranteeing convergence to high signal to interference ratio (SIR). At each iteration a number of unique lags are chosen randomly from the available range so that on the average GLHSAM has the same cost as the SAM algorithm. The performance of the proposed GLHSAM algorithm is confirmed through simulation studies.

1. INTRODUCTION

Channel shortening is a generalization of equalization, since equalization amounts to shortening the channel to length one. Channel shortening to a length greater than one is frequently used to facilitate equalization in systems employing multicarrier modulation (MCM) [7]. In multicarrier modulation (MCM) systems, such as asymmetrical digital subscriber line (ADSL) transceivers, each symbol consists of samples to be transmitted to the receiver plus an acyclic prefix (CP) of length v [2]. The CP is the last v samples of the original N samples to be transmitted. The CP is inserted between blocks to combat inter-symbol interference (ISI) and inter-channel interference (ICI). The length of the CP should at least be equal to the order of the channel impulse response. At the receiver the CP is removed and the remaining N samples are then processed by the receiver. Since the efficiency of the transceiver is reduced by the introduction of the CP it is therefore desirable either to make v as small as possible or to choose a large N. Selecting large N will increase the computational complexity, system delay, and memory requirements of the transceiver. To overcome these problems a short time-domain equalizer (TEQ), usually an FIR filter, can be placed in the front end of the receiver, to shorten the impulse response of the effective channel [9]. This approach is much more simplistic than frequency domain approach in [10]and therefore easier for realization and hence the focus of this work The length of the shortened impulse response filter and CP are usually fixed a priori and not changed from channel to channel. A low complexity blind adaptive algorithm to design a TEQ, called sum-squared auto-correlation minimization (SAM) was proposed in [3] which achieves channel shortening by minimizing the sum-squared autocorrelation terms of the effective channel impulse response outside a window of a desired length. The drawback with SAM is that it has a significant computational complexity. SLAM [4], on the other hand, achieves channel shortening by minimizing the squared value of only a single autocorrelation at a lag greater than the guard interval. The drawback with the SLAM cost is that a low value does not necessarily guarantee convergence to high SIR [5]. Our contribution is therefore to propose a new channel shortening algorithm with random lag selection which has complexity at each iteration between that of SLAM and SAM. Channel shortening filter design is a widely investigated topic in the literature. The minimum mean squared error (MMSE) methods, initially proposed in the context of the maximum likelihood sequence estimation problem [7] in the 1970s, were later adapted to MCM [2] following the advances in OFDM in the 1990s. Many other shortening techniques have been developed including the maximum shortening signal-to-noise ratio (MSSNR) method [8]. An intensive survey of these techniques has been composed by Martin, et al., in [10]. The organization of this paper is as follows. In Section 2 we demonstrate the system model. In Section 3 we discuss the idea of minimizing the autocorrelation at a random lag. In Section 4 we develop the gradient descent implementing of the GLHSAM algorithm. In Section 5 we discuss the SIR performance for the SAM, SLAM, and GLHSAM algorithms. In Section 6 we provide the comparative simulations between SAM and GLHSAM and in section 7 we draw our conclusions.

2. SYSTEM MODEL

The system model is composed of an input source signal $x(n)$ typically drawn from a finite alphabet transmitted through a linear finite-impulse-response (FIR) channel $\mathbf{h}$ of length (L_h+1) taps, $r(n)$ is the received signal, which will be filtered through a TEQ with an L_w+1 impulse response vector $\mathbf{w}$ to obtain the output sequence $y(n)$. For convenience in this work we assume real signals but generalization to the complex case is straight-forward. We denote $\mathbf{c}=\mathbf{h}*\mathbf{w}$ as the shortened or effective channel, assuming $\mathbf{w}$ is in steady-state, where * denotes discrete time convolution. We also assume that $2L_c < N$ holds. The signal $v(n)$ is a zero-mean, i.i.d. noise sequence, uncorrelated with the source sequence with variance σ_v^2. The received sequence $r(n)$ is

$$r(n)=\sum_{k=0}^{L_h} h(k)x(n-k)+v(n) \tag{1}$$

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and the output of the TEQ $y(n)$ is given by

$$y(n) = \sum_{k=0}^{L_w} w(k)r(n-k) = \mathbf{w}^T\mathbf{r}_n \tag{2}$$

where $\mathbf{r}_n = [r(n)r(n-1)...r(n-L_\mathbf{w})]^T$ and $\mathbf{w}$ represents the impulse response vector of the TEQ $\mathbf{w} = [w_0w_1w_2...w_{L_w}]^T$.

3. SAM AND SLAM COST FUNCTIONS

The notion of SAM is founded on the fact that for the effective channel to have zero taps outside a window of size $(v+1)$ its autocorrelation values must be zero outside a window of size $(2v+1)$. In SAM the auto-correlation sequence of the combined channel-equalizer impulse response becomes

$$R_{cc}(l) = \sum_{k=0}^{Lc} c(k)c(k-l) \tag{3}$$

and for the shortened channel, the following must hold$R_{cc}(l) = 0, \forall |l| > v$ The cost function J_{v+1} in SAM is defined on the basis of minimizing the sum-squared autocorrelation terms, i.e.,

$$J_{v+1} = \sum_{l=v+1}^{L_c} R_{cc}(l)^2 \tag{4}$$

Conversely,SLAM is based on the fact that a single autocorrelation at a lag greater than the guard interval provides a measure of the presence of the channel outside the desired guard interval, hence minimizing only this single autocorrelation is particularly applicable to subscriber line channels which are essentially minimum phase. In SLAM the autocorrelation sequence of the combined channel-equalizer impulse response is also given by[3] and for a shortened channel, it must satisfy$R_{cc}(l) = 0, l = v+1$ .In this case the cost function J_{v+1} in SLAM is defined based upon minimizing the squared-auto-correlation of the effective channel only at lag $l = v+1$, i.e.,

$$J_{v+1} = R_{cc}(l)^2, \quad l = v+1 \tag{5}$$

In[5], however, it has been pointed out that minimizing (8) does not guarantee high SIR for certain combined channel and shortener responses. To overcome this problem our contribution is to generalize a lag hopping version of SLAM, where the lag parameter in (8) is chosen at random to lie within the range $v+1, ..., L_c$, with equal probability of selecting anyone lag, to the case of selecting randomly, but uniquely, any number of lags between 1 and $L_c - v$, so that on average the cost is identical to (5) when implemented in an adaptive learning algorithm. The computational complexity at each iteration of the algorithm could therefore be chosen between that of SLAM and SAM.

4. GLHSAM ADAPTIVE ALGORITHM

The steepest gradient-descent algorithm to minimize the SAM cost J_{V+1} becomes

$$\mathbf{w}^{new} = \mathbf{w}^{old} - \mu\nabla_\mathbf{w} \sum_{l=v+1}^{L_c} (E[y(n)y(n-l)])^2 \tag{6}$$

where l is the lag index, μ denotes the step size, and ∇_w represents the gradient with respect to $\mathbf{w}$. We define the instantaneous cost function, where expectation operation is replaced by a moving average over a user-specified window of length N, as

$$J_{v+1}^{inst}(k) = \sum_{l=v+1}^{L_c} \left\{ \sum_{n=kN}^{(k+1)N-1} \frac{y(n)y(n-l)}{N} \right\}^2 \tag{7}$$

where N is a design parameter and it should be large enough to yield a reliable estimate of the expectation, but no larger, as the algorithm complexity is proportional to N. The gradient descent algorithm becomes

$$\begin{aligned} \mathbf{w}(k+1) &= \mathbf{w}(k) - 2\mu \sum_{l=v+1}^{L_c} \left\{ \sum_{n=kn}^{(k+1)N-1} \frac{y(n)y(n-l)}{N} \right\} \\ &\times \left\{ \nabla_w \left(\sum_{n=kN}^{(k+1)N-1} \frac{y(n)y(n-l)}{N} \right) \right\} \end{aligned} \tag{8}$$

and for GLHSAM this is given by:

$$\begin{aligned} \mathbf{w}(k+1) &= \mathbf{w}(k) - 2\mu \sum_{L=l_1}^{L_{N_{LAGS}}} \left\{ \sum_{n=kn}^{(k+1)N-1} \frac{y(n)y(n-l)}{N} \right\} \\ &\times \left\{ \left(\sum_{n=kN}^{(k+1)N-1} \frac{y(n)\mathbf{r}_{n-l} + y(n-l)\mathbf{r}(n)}{N} \right) \right\} \end{aligned} \tag{9}$$

where $l_1...l_{N_{LAGS}}$ are chosen to be unique and to be drawn with uniform probability from the available lags, initially $v+1, ..., L_c$. The number of lags, $L_{N_{LAGS}}$, can be chosen over the range $1, ..., L_c - v$, and when $N_{LAGS} = 1$, the algorithm takes the form of a lag-hopping version of SLAM and where $N_{LAGS} = L_c$, the algorithm is identical to SAM. The key advantage of the random lag hopping in the proposed GLHSAM algorithm is that as $k \to \infty$ the average cost which is minimized is identical to that of SAM, and thereby should retain the same convergence properties.

5. SIR PERFORMANCE

In [5], the authors provide an expression for the signal to interference ratio (SIR) achieved in the output $y(n)$ when the TDE is based on the blind channel shortening metrics of SAM, sum absolute autocorrelation minimization (SAM), SLAM and the signal to interference power ratio in $x(n)$. They defined it to be

$$SIR = \frac{\sum_{l=-v}^{v} |R_{cc}(l)|^2}{\sum_{l=-N}^{-v+1} |R_{cc}(l)|^2 + \sum_{l=v+1}^{N} |R_{cc}(l)|^2} \tag{10}$$

It should be noted that the denominator in this expression is the SAM cost considering those combined z-domain responses only which satisfy the unit energy constraint, the following relation can be derived as

$$\begin{aligned} SIR(dB) &= 10log_{10}\left(\sum_{l=-v}^{v} |R_{cc}(l)|^2 \right) - 10log_{10}(J_s) \\ &= 10log_{10}\left(1 + 2\sum_{l=1}^{v} |R_{cc}(l)|^2 \right) - 10log_{10}(J_s) \\ &\geq -J_s(dB) \end{aligned} \tag{11}$$

where $J_s(.)$ dentes the SAM cost function, and a low SAM cost can be guaranteed to a high SIR at the output of the matched filter. SLAM design provide no such under-bound on the SIR performance, [5]. Our algorithm (GLH-SAM)eliminates the problem of SLAM by selecting the lags randomly, so that a low average GLHSAM cost, achieved through recursive learning, will be identical to a low SAM cost which guarantees to give a high SIR at the output of the matched filter, as on the average algorithm employs all the lags as in SAM.

6. SIMULATION

The Matlab code at [6] was developed to simulate all algorithms. The cyclic prefix had length 32. The FFT size $N_{fft} = 512$, the TEQ had 16 taps and the channel used was the test ADSL channel CSA loop 1 provided at [6]. The noise was chosen such that $\delta_x^2\|c\|^2/\delta_v^2 = 40dB$ where $\|.\|$ denotes the Euclidean norm; and 75 OFDM symbols were employed. The step size for SAM and SLAM were 5 and 600 to match the energy in the update equation; whereas for GLHSLAM was 600, when one lag was chosen and 582 for 15 lags. All algorithms ware compared with the maximum shortening SNR solution and the matched filter bound (MFB) on capacity, which assumes no ICI. In Figures 2, 3 and 4 the shortened channels of the GLHSAM, SAM and SLAM algorithms are compared with the original channel. All algorithms successfully shorten the channel. In Figures 5, 6 and 7, GLHSAM with one lag and 15 lags, together with SAM, are compared in terms of achievable bit rate as a function of averaging block number. Importantly, the speed with which the algorithm reaches the best performances increases as the number of lags increases in GLHSAM, for example in Figure 5 this is approximately 1080 blocks whereas with 15 lags it is approximately 100 blocks. This performance is even better than that of the SAM algorithm shown in Finger 7 which takes approximately 230 blocks, and our proposed GLHSAM algorithm has substantially reduced complexity as 15 lags are used rather than $L_c - v$ in SAM.

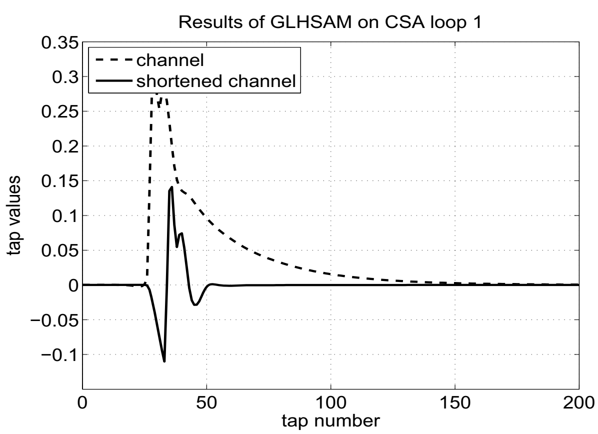

Figure 1: Channel (dashed) and shortened channel (solid) impulse response of GLHSAM with one lag algorithm.

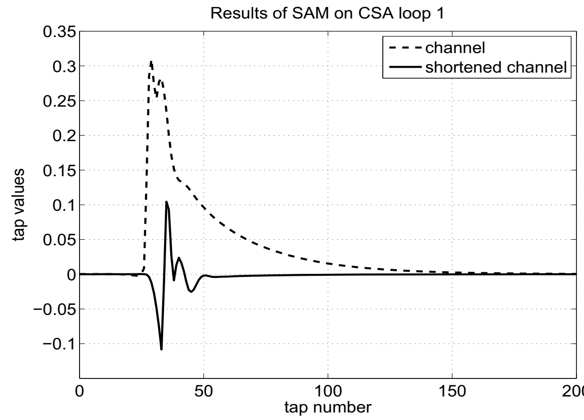

Figure 2: Channel (dashed) and shortened channel (solid) impulse response of SAM algorithm.

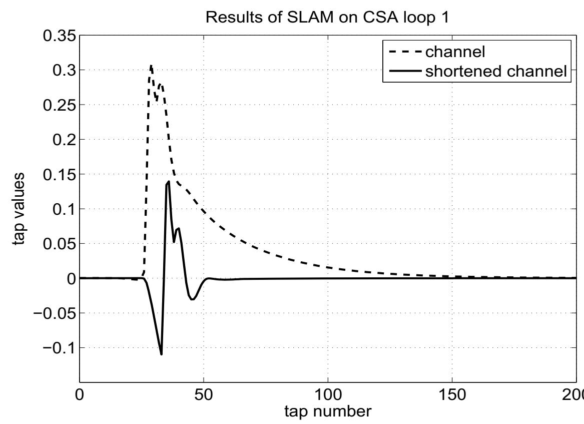

Figure 3: Channel (dashed) and shortened channel (solid) impulse response of SLAM algorithm.

7. CONCLUSION

A new generalized lag hopping blind channel shortening algorithm has been proposed. This algorithm allows the user to select performance between the SAM and SLAM algorithms whilst trading off computationally complexity. Importantly, the disadvantage of SLAM in terms of the SIR performance has been overcome by the proposed algorithm. Future work is considering other methods to enhance the convergence performance of GLHSAM.

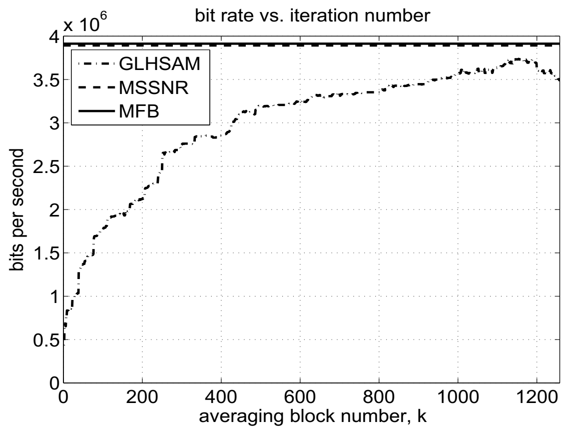

Figure 4: Achievable bit rate versus iteration number at 40 dB SNR of GLHSAM with one lag algorithm.

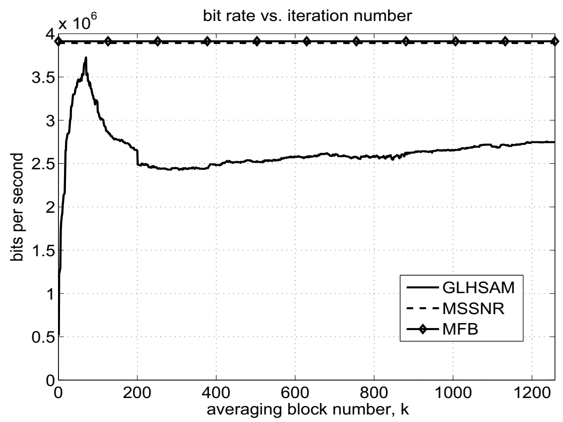

Figure 5: Achievable bit rate versus iteration number at 40 dB SNR of GHLSLAM with 15 lags algorithm.

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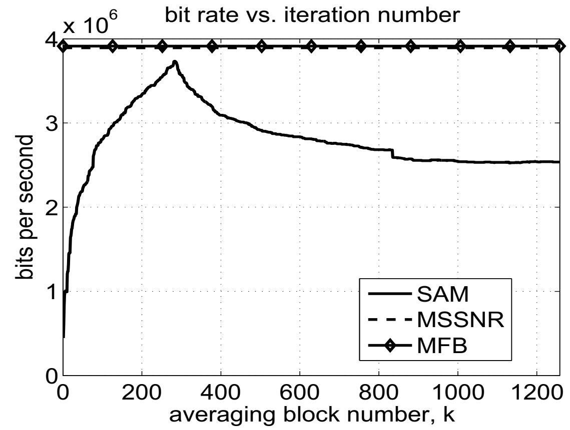

Figure 6: Achievable bit rate versus iteration number at 40 dB SNR of SAM algorithm.

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