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Nonlinear dynamics of structures with propagating cracks

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<u>*Summary*</u>. The aim of this paper is to study the evolution of nonlinear dynamics of structures with a propagating crack. A method of simulation used for analysis of dynamics of the cracked structure is based on a combination of an analytical technique and Matlab-Simulink-based simulations. As an example, a model of a cracked bar subjected to longitudinal excitation is used to analyse its nonlinear response as a way to monitor the structural health as crack propagates.

Introduction

Aerospace and power-plant industries face large losses due to failure of mechanical components. The cause of structural failure is often linked to crack propagation taking place at locations of stress concentrators such as notches, abrupt changes in a cross-section etc. To understand dynamics of the cracked structure, different researchers tried to address this issue by considering their various characteristics such as natural frequency, damping and mode shape [1, 2]. Most of these researches were based on a linear vibration theory and few considered so called *breathing cracks,* which induce nonlinearity in the structures under consideration. In this paper, the focus is on nonlinear dynamics of a structure with a propagating breathing crack. To model this problem, two factors are considered: one is the well-established fact that a propagating crack reduces stiffness and another is the generation of a contact force due to interaction of crack faces. A method of simulation for studying dynamics of structures with propagating cracks is described based on a combination of an analytic technique and Matlab-Simulink-based computations developed in [3]. A new substructure named *a virtual linear system* is introduced, which helps to distinguish clearly the influence of geometrical and dynamical factors on the structural behaviour. This helps in the application of methods of modal analysis of the substructures and nonlinear integral equations to structural modelling. As an example, a model of a cracked bar subjected to longitudinal excitation is used to analyse its nonlinear behaviour as a way to monitor structural health.

Modelling of fixed-fixed bar with propagating crack

Consider a uniform fixed-fixed bar of steel of length $x = l$ under longitudinal excitation as shown in Figure 1. It is assumed that a crack is initiated at the right end of the bar, splitting the bar into two subsystems: one is the cantilever bar fixed at the left end, which is a *virtual linear system*, and another is an impact pair of interacting faces of the crack. Figure 2 shows the schematic of the *virtual linear system*, under longitudinal excitation capable to move without any obstruction, preserving its linear behavior. In real, splitted fixed –fixed bar under longitudinal excitation this free end of the bar interacts with the fixation boundary on the right with a very small gap of Δ between this fixation and the cross-section of the cantilever bar. To model the structure with a propagating crack, two factors are considered: one is the well-established fact that the propagating crack reduces stiffness of the fixation, which is modelled as the linear spring force (F_k) at the right end of the bar, and another is generation of a contact force due to interaction of crack's faces modelled as limiters. Figure 3 shows the schematic model of the fixed – fixed bar with a developing crack on the right-hand end of fixation.
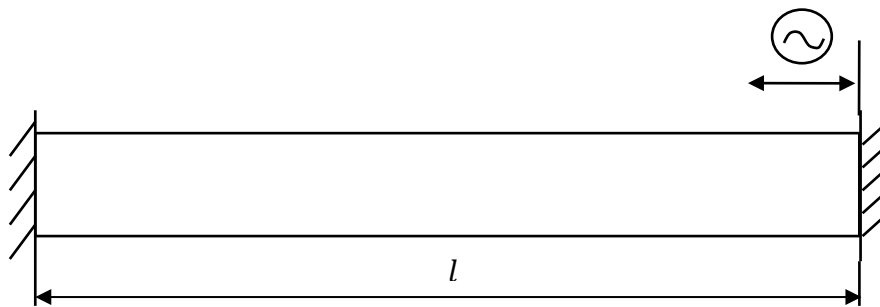

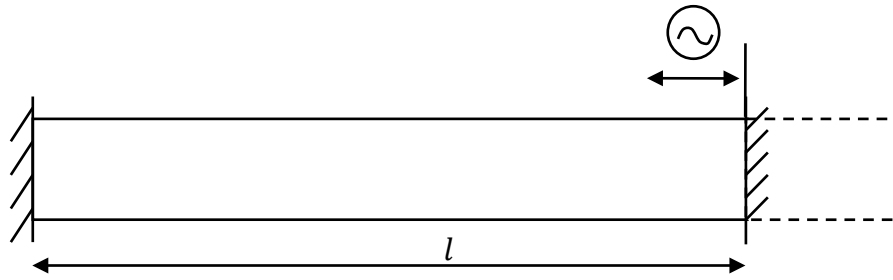Figure 1 Schematic of fixed-fixed bar
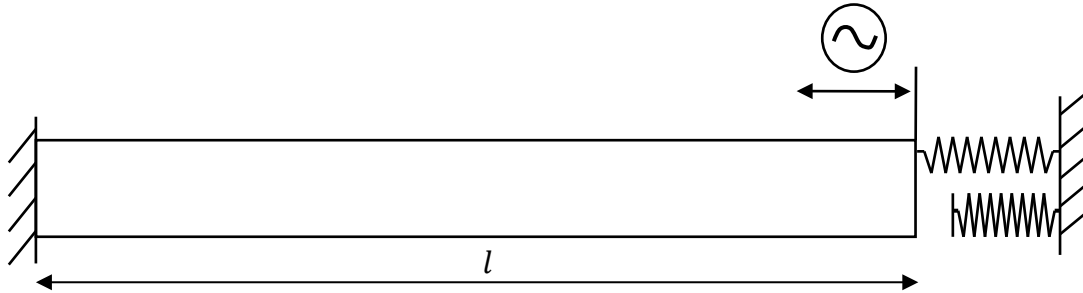
Figure 2 Schematic of virtual linear system

Figure 3 Schematic model of cracked bar with propagating crack

To model the reduction in stiffness of the structure with crack, a spring connection is introduced between the bar and the fixed boundary on the right-hand end having linear variable stiffness ('stiffness of fixation'). Generation of the contact force due to crack-face interaction is modelled with a gap Δ between the cross section of the bar's right-hand end and another linear spring fixed at the right-hand boundary working as a limiter at the respective end of the bar. This interaction between the crack faces transforms the continuous process into successive impulses, modulated by the velocity of the input process at the instant, when the process reaches a threshold value (closing of the crack). Such nonlinear components of the structural scheme are called *impact elements* [4]. These elements have positional impulse effects, which specifically combine the effects of the relayed and impulse elements. Figure 4 shows an example of temporal transformations of deformation $u(x,t)$ into the force $F[u(x,t)]$ by an impact element with force characteristic $F(u)$. This force characteristic of contact
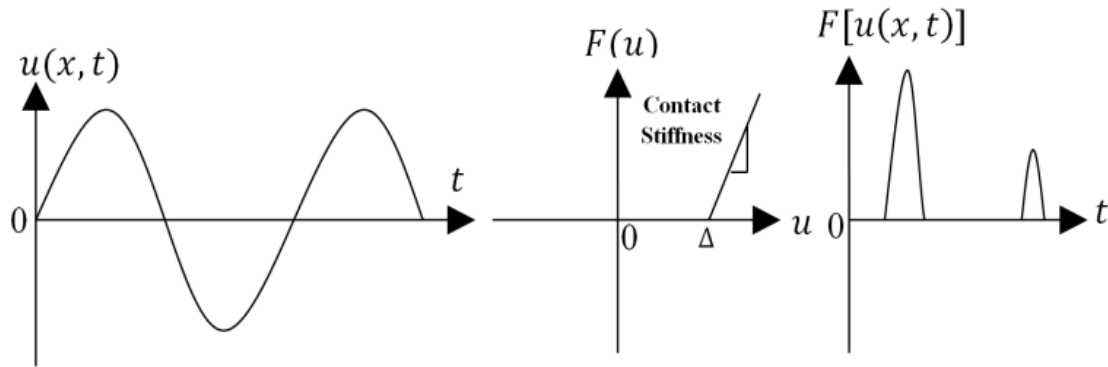

Figure 4 Contact-force characteristics

interaction is given by an expression called *static force characteristics of the impact pair* [4]:

$$F(u) = \psi(u - \Delta)\eta(u - \Delta) , \tag{1}$$

where ψ is the linearized coefficient of contact stiffness,

$$\eta(u) = \begin{cases} 0, & u < 0 \\ 1, & u \geq 0 \end{cases} . \tag{2}$$

As a result, one spring, which is a limiter, is responsible for generation of a slow changing contact force $(F_\epsilon[u(t), \dot{u}(t)])$ due to interaction of crack faces. Another spring – between the cantilever bar's right-hand end's cross section and the fixation boundary, has variable spring stiffness responsible for a slow change in stiffness of the

structure, when crack propagates leading to generation of a linear spring force (F_k). When the cantilever bar interacts with the limiter and as the crack propagates, vibration displacement of the cantilever bar at any arbitrary section x from its fixed end is defined as a function of $u(x, t)$. By introducing the contact force characteristic $F_\in(u, \dot{u})$, which is a slowly changing function, changes in stiffness due to the propagating crack introduced as a linear spring force (F_k) and a receptance operator $L_{ll}(s)$, coupling displacement $u(x, t)$ to the force acting at $x = l$, the operator equation for the unknown function $u(l, t)$ can be written in the following form:

$$u(l, t) = L_{ll}(s)P_l(t) - L_{ll}(s)[F_\in(u, \dot{u}) + F_k] . \tag{3}$$

The receptance in Eq. (3) consists of the infinite number of vibration modes and can be written as given in [4]:

$$L(s) = L_{ll}(s) = \sum_{v=1}^{\infty} \frac{A_v(l)A_v(l)}{s^2 + 2r_v\Omega_v s + \Omega_v^2} \tag{4}$$

where $A_v(l)$ are the *modal shape functions* of the virtual linear system, Ω_v are its natural frequencies, s is the complex variable, ω is the frequency of excitation and $r_v = \chi\Omega_v/4\pi\omega$ is energy dissipation in the form of internal friction in the material, in which χ is the absorption coefficient responsible for internal damping of the material.

Simulation results

Simulations of the system were performed using Matlab-Simulink, considering Eqs. 3 and 4. For simulation purposes only, the first ten modes of vibration were taken into consideration in the receptance operator, giving accurate numerical results. To simulate the linear response of this system, contact stiffness for the limiter was considered as 1% of stiffness of the bar and stiffness of fixation was considered as 600% of that of the bar. Figure 5 shows the first three resonant frequencies of the linear the system.

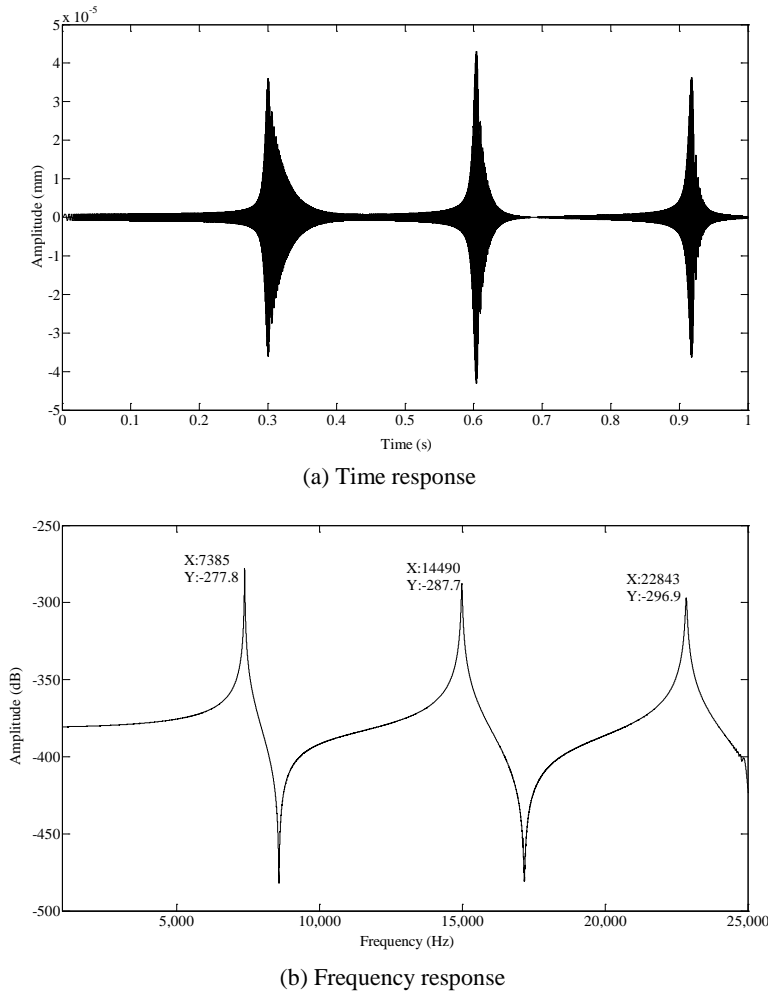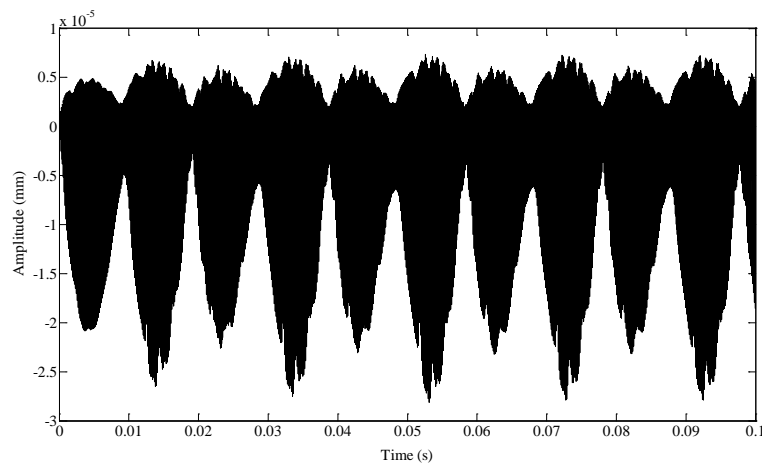(a) Time response

(b) Frequency response

Figure 5 Linear response of the system

The first linear resonant frequency is used for exciting the system as the crack propagates. Table 1 show the parameters used in simulations. As the crack propagates, stiffness of fixation is decreased and the contact stiffness is kept constant.
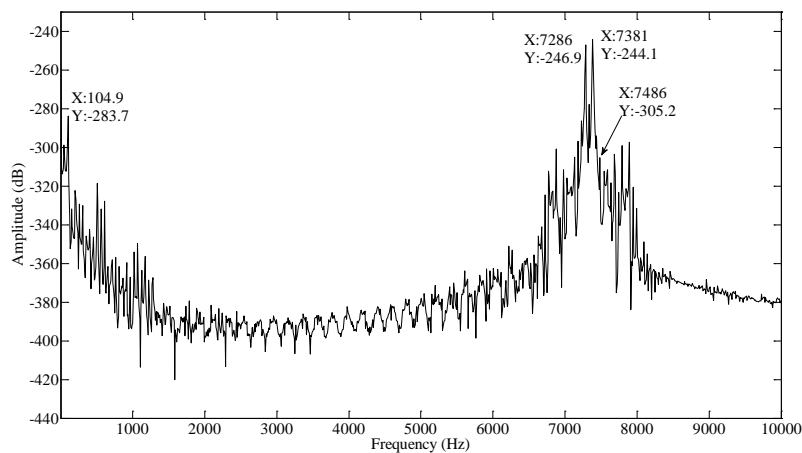
Stiffness of fixation	Contact stiffness
420%	600%
240%	600%

Table 1 Simulation parameters (as percentage of the bar stiffness)

As the crack propagates, stiffness of fixation (spring stiffness) decreases; in this case it is reduced from the initial magnitude (600% of stiffness of the bar) to 420%. Figure 6a shows the modulation in the time response. The frequency response (Fig. 6b) shows the presence of low frequency, side-band frequency along with the frequency of excitation due to the crack-face interaction.
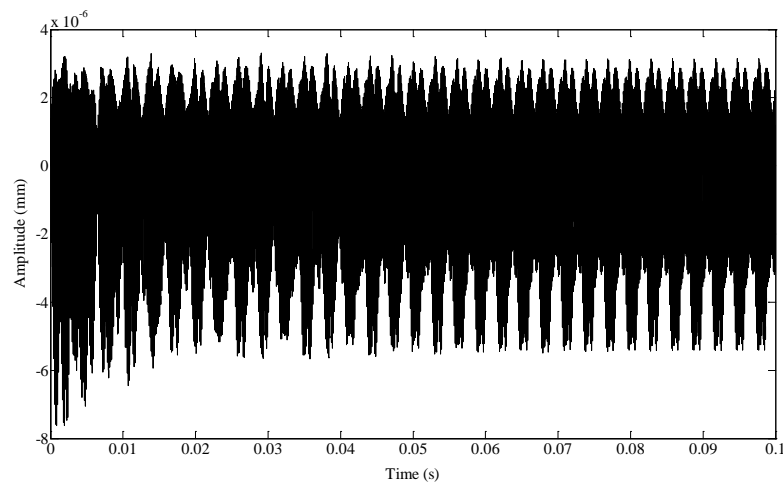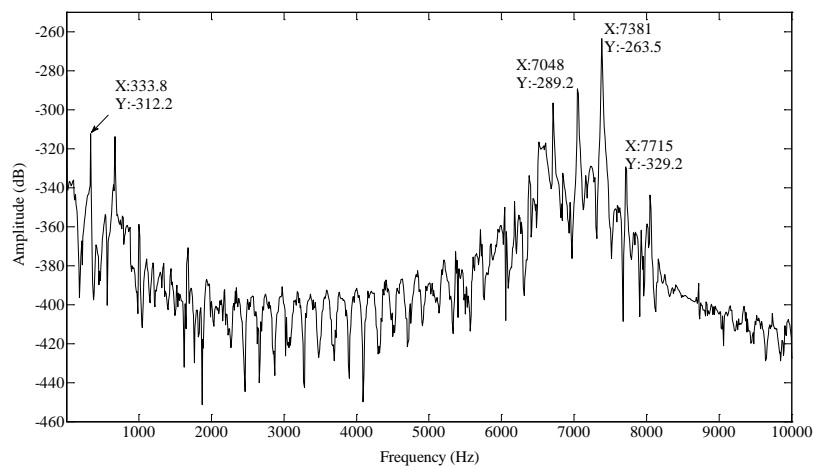

(a) Time response

(b) Frequency response

Figure 6 Nonlinear response of system at stiffness of fixation of 420%

Similarly, as the crack propagates further, the response is obtained by decreasing stiffness of fixation to 240 % of that of the bar, keeping the contact stiffness constant. It can be observed from Figure 7a that there is intense modulation in the time response as crack propagates. Also, the frequency response (Figure 7b) shows the presence of low frequency, side-band frequency along with the frequency of excitation.

(a) Time response

(b) Frequency response

Figure 7 Nonlinear response of system at stiffness of fixation of 240%

Conclusions

The concept of receptance operator along with the theory of modal analysis to generate the Matlab-Simulink model of the structures with developing discontinuities is developed. From the simulation results, it can be concluded that as the crack propagates in the bar, intense modulation is observed in the time response. This indicates that nonlinear transformation of the modulated signal by the crack generates a low-frequency component along with side-band frequencies; this can be a good indicator for crack detection.

Acknowledgement

References
[1] Dimarogonas A. (1996) Vibration of cracked structures: A state of the art review, *Engineering Fracture Mechanics*,**55**,831-857
[2] Sinou J.J. (2009) A Review of Damage Detection and Health Monitoring of Mechanical Systems from Changes in the Measurement of Linear and Nonlinear Vibrations. *Mechanical Vibrations: Measurements, Effects and Control* , 643-702
[3] Hiwarkar V.R., Babitsky V.I, Silberschmidt V.V. (2011) On the Modelling of Dynamic Structures with Discontinuities. *Nonlinear Dynamics* **67** 2651–2659.
[4] Babitsky, V. (1998) Theory of Vibro-impact Systems and Applications. Springer, Berlin (Revised translation from Russian, Nauka, Moscow, 1978)