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# Correction of laser Doppler vibrometry measurements affected by steering mirror vibration

Ben Halkon<sup>a</sup> and Steve Rothberg<sup>b</sup>

## Abstract

*The laser Doppler vibrometer (LDV) is now well-established as an effective non-contact alternative to traditional contacting vibration transducers. LDVs are technically well suited to general application but they offer special benefits in a variety of challenging measurement scenarios. A limitation in this respect is sensitivity to vibration of the instrument itself or of any steering optics used to orient the probe laser beam. Making use of a general vectorial framework for modelling the measured velocity, this paper will present a mathematical treatment of the velocity measured in the scenario where the laser beam direction is manipulated by a vibrating mirror. It will be shown that, by knowing the steering mirror vibration, it is possible to completely correct for the perturbation of the measured signal.*

*A complementary experimental investigation is described. The LDV, the target and the mirror were relatively carefully aligned with respect to one another enabling three alternative angles of 90°, 60° and 30° between the instrument and the target vibration direction. The vibrating target and the steering mirror assemblies were each instrumented with an accelerometer; the target measurement being the reference or “true” measurement while the mirror measurement is used to perform the required correction to the LDV measurement. Simultaneous measurements were taken with either the target or the mirror vibrating at “high” and “low” broadband levels; the LDV is shown to over-estimate in the mirror vibration only cases by over 22000 and 11000% respectively. Post-processing steps are presented which enabled the measurement to be corrected by circa 35dB.*

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## Introduction

The laser Doppler vibrometer (LDV) is now well-established as an effective non-contact alternative to traditional contacting vibration transducers such as piezoelectric accelerometers.

Commercially available LDVs are technically well suited to general application but they offer special benefits in a variety of challenging scenarios including measurements on hot, light, rotating or remote surfaces. Care should be taken when interpreting the measured signal which can be subject to measurement uncertainty from two sources in particular, namely the sensitivity of the measurement to motions other than that intended when target rotation is involved and to laser speckle noise [1-4].

A further potential limitation of the technique is sensitivity to vibration of the instrument itself or of any steering optics (usually mirrors) which may have been used to orient the probe laser beam in order to address the measurement region(s) of interest. The typical approach taken in the presence of significant such motions is to attempt to isolate the instrument or steering optics from the motions to minimise the impact but this will clearly have limited success and will be dependent on the inevitably compromised ability of the isolation [5]. While many LDV experts have anecdotally commented for some time on the possibility of measuring and then compensating for these motions, none have reported having done it for real until now.

### Modelling the measured velocity

A LDV measures surface velocity at the incidence point in the direction of the incident laser beam [1-2]. The completely general laser beam orientation can be written as any initial alignment, in the case shown in Figure 1 the negative  $x$ -direction, with two rotation matrices,  $[y, \beta_1]$  and  $[z, \gamma_1]$ , i.e.  $\beta_1$  around the  $y$ -axis and  $\gamma_1$  around the  $z$ -axis [2], applied in order to give the final arbitrary orientation:

$$\hat{b}_1 = [\hat{x} \ \hat{y} \ \hat{z}][z, \gamma_1][y, \beta_1][-1 \ 0 \ 0]^T \quad (1.1a)$$

For the case under consideration here, both rotations are zero and so:

$$\hat{b}_1 = -\hat{x} \quad (1.1b)$$

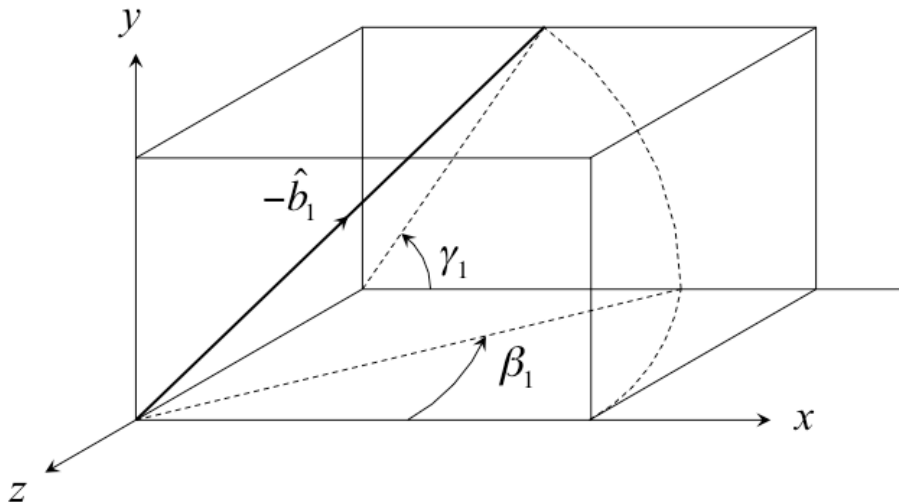
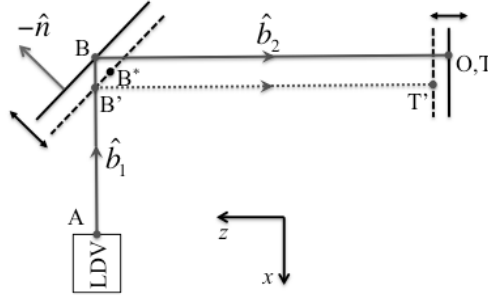


Figure 1: Arbitrary incident laser beam orientation through rotations  $\beta_1$  and  $\gamma_1$



**Figure 2:** Schematic showing translational vibration measurement using an angled steering mirror.

Consider the scenario, as shown schematically in Figure 2, where a steering mirror, which may itself be undergoing vibratory motion, is used to deflect the laser beam in the direction of the target. Point A, an arbitrarily chosen point on the line of laser beam, can be written in vector form as:

$$\overrightarrow{OA} = [\hat{x} \ \hat{y} \ \hat{z}][x_A \ 0 \ z_A]^T \quad (1.2)$$

while point B is the point on the mirror where the laser beam is incident in the absence of any vibration; we use this point as the reference position and:

$$\overrightarrow{OB} = [\hat{x} \ \hat{y} \ \hat{z}][0 \ 0 \ z_B]^T \quad (1.3)$$

It is useful to define coordinate axes fixed in the mirror, i.e.  $[\hat{x}_m \ \hat{y}_m \ \hat{z}_m]$ , and these can be used to define the point  $B^*$ , to which reference point B moves following mirror motion:

$$\overrightarrow{OB^*} = \overrightarrow{OB} + [\hat{x}_m \ \hat{y}_m \ \hat{z}_m][BB^*_{mx} \ BB^*_{my} \ BB^*_{mz}]^T \quad (1.4)$$

The mirror's translational vibration velocity,  $\overrightarrow{V_{B^*}}$ , can also be written in terms of its components,  $V_{B^*x_m}$ ,  $V_{B^*y_m}$  and  $V_{B^*z_m}$ , in the mirror coordinate system:

$$\overrightarrow{V_{B^*}} = [\hat{x}_m \ \hat{y}_m \ \hat{z}_m][V_{B^*x_m} \ V_{B^*y_m} \ V_{B^*z_m}]^T \quad (1.5)$$

The surface normal for the mirror,  $\hat{n}$ , can be written very simply in the mirror coordinate system:

$$\hat{n} = \hat{z}_m \quad (1.6a)$$

or it can be written in the global coordinate system with an initial orientation in the  $z$ -direction modified by a rotation around the  $y$ -axis using a rotation matrix, i.e.:

$$\hat{n} = [\hat{x} \ \hat{y} \ \hat{z}][y, \beta][0 \ 0 \ 1]^T \quad (1.6b)$$

In Figure 2,  $\beta = \frac{3\pi}{4}$  but the analysis will be kept general. The vibration of the mirror affects the location at which the laser beam is incident on the mirror, point  $B'$ , which can be obtained from three equations. The first is a vector triangle involving the laser beam path to the mirror:

$$\overrightarrow{OB'} = \overrightarrow{OA} + |\overrightarrow{AB'}| \hat{b}_1 \quad (1.7a)$$

The second is a vector triangle involving the reference position on the mirror during vibration and the point of incidence on the mirror:

$$\overrightarrow{OB'} = \overrightarrow{OB^*} + \overrightarrow{B^*B'} \quad (1.7b)$$

The third notes that the vector  $\overrightarrow{B^*B'}$  lies in the plane of the mirror:

$$\overrightarrow{B^*B'} \cdot \hat{n} = 0 \quad (1.7c)$$

These equations can be combined to:

$$\overrightarrow{OB'} = \overrightarrow{OA} + \left[ \frac{(\overrightarrow{OB^*} - \overrightarrow{OA}) \cdot \hat{n}}{\hat{b}_1 \cdot \hat{n}} \right] \hat{b}_1 \quad (1.7d)$$

The laser beam direction after reflection at the mirror is found from [2]:

$$\hat{b}_2 = \hat{b}_1 - 2(\hat{b}_1 \cdot \hat{n})\hat{n} \quad (1.8)$$

The point  $T'$  in the target plane is then found in a similar, but slightly simpler, manner, to  $\overrightarrow{OB'}$  using just two vector equations. The first is the vector triangle involving the laser beam path while the second notes that the position  $\overrightarrow{OT'}$  lies in the  $xy$  plane:

$$\overrightarrow{OT'} = \overrightarrow{OB'} + \left| \overrightarrow{B'T'} \right| \hat{b}_2 \quad (1.9a)$$

$$\overrightarrow{OT'} \cdot \hat{z} = 0 \quad (1.9b)$$

leading to:

$$\overrightarrow{OT'} = \overrightarrow{OB'} - \left[ \frac{\overrightarrow{OB'} \cdot \hat{z}}{\hat{b}_2 \cdot \hat{z}} \right] \hat{b}_2 \quad (1.9c)$$

The total measured velocity is the sum of the measured velocities from points  $B'$  and  $T'$  and can be written as [2]:

$$U_m = (\hat{b}_2 - \hat{b}_1) \cdot \overrightarrow{V_{B'}} - \hat{b}_2 \cdot \overrightarrow{V_{T'}} \quad (1.10)$$

The surface velocity for the mirror can be re-written [2] and the component of measured velocity associated with the mirror,  $U_{mB'}$ , can then be written:

$$U_{mB'} = (\hat{b}_2 - \hat{b}_1) \cdot (\overrightarrow{V_{B'}} + \overrightarrow{\omega_m} \times \overrightarrow{B^*B'}) \quad (1.11)$$

In this analysis, we are considering only translational mirror vibrations so, the rotation of the mirror,  $\overrightarrow{\omega_m} = 0$ . Using equations (1.6a) and (1.8), this simplifies to:

$$U_{mB'} = -2(\hat{b}_1 \cdot \hat{n})(\hat{z}_m \cdot \overrightarrow{V_{B^*}}) \quad (1.12a)$$

Equation (1.12a) shows how the contribution of the mirror vibration to a measurement can be corrected solely by making a measurement of vibration in the direction of the mirror normal. This indicates that a practical and simple correction can be performed, for example with a single axis accelerometer attached to the back face of the mirror at the location of the laser beam incidence but aligned with the mirror normal.

Equation (1.12a) simplifies further, using equations (1.1b), (1.5) and (1.6b), to show the effect of geometry explicitly:

$$U_{mB'} = 2 \sin \beta V_{B^*z_m} \quad (1.12b)$$

A corrected measurement of velocity,  $U_{corr}$ , is therefore relatively straightforwardly given by:

$$U_{corr} = U_m - 2 \sin \beta V_{B^*z_m} \quad (1.13)$$

## Experimental investigation

### *Instrumentation configuration*

As shown in Figure 2, the experimental arrangement employed involved the use of a pair of electrodynamic shakers to generate the target and steering mirror motion. In both cases, the shaker was connected via a stiff bar to a structure mounted onto a linear bearing such that translational vibration in a single direction only was isolated. For the case of the target, an accelerometer was mounted onto the vibrating structure directly at the point of laser beam incidence. For the mirror, the accelerometer was mounted on the reverse side of the rigid plate onto which the mirror was mounted, directly behind the point of laser beam incidence. In both cases care was taken to ensure that the positive sensor direction was such that the measured signals were in direct agreement with that of the LDV. Each shaker was each driven by an LDS

10W power amplifier with a signal generated within the multi-channel data acquisition system that was used to capture the measured signals.



**Figure 3:** *Experimental arrangement for 90° LDV to target surface/vibration direction scenario.*

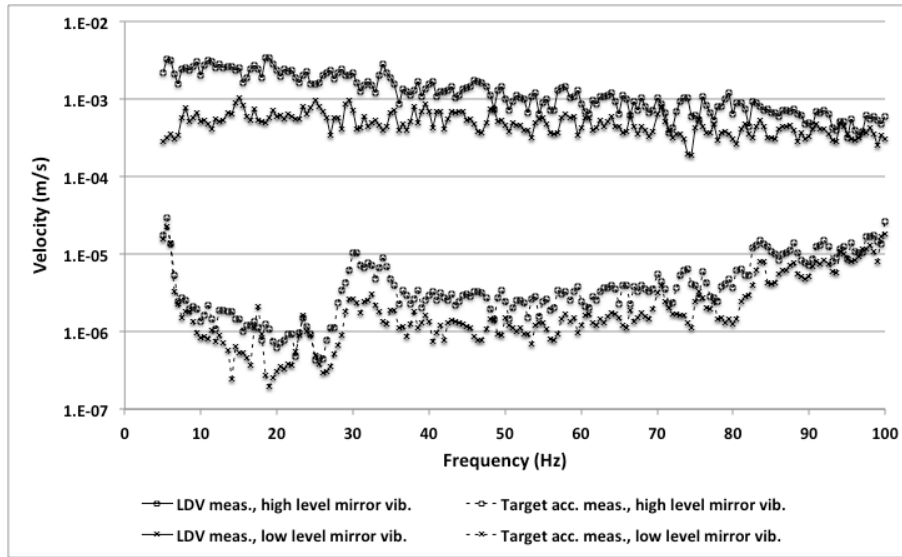
The LDV was mounted on a readily commercially available tripod which included various bubble spirit level indicators to enable initial positioning of the device such that the laser beam direction was nominally in a horizontal plane. Combination of height adjustment in the tripod and screw adjusters within the tripod head enabled fine adjustment of the instrument in five degrees of freedom:  $y, z, \alpha, \beta, \gamma$ . In the arrangement shown in Figure 2 the nominal angle between the instrument and the target surface/vibration direction is nominally 90° while the steering mirror surface/vibration direction bisecting that. Care was taken, in accordance with that which could be attributed to the experienced LDV practitioner, to ensure the certainty of the angles to within of the order of  $\pm 2^\circ$ . In practical terms, this involved firstly aligning the instrument and the target relative to one another using the orthogonal channels on the bench followed by adjusting the steering mirror position and orientation in order that the incident (from the LDV) and reflected (to the target) laser beams were aligned along the vertices of a stiff card template. The LDV – target stand-off distance was specified in accordance with the manufacturer's recommendations, in this case 400 mm.

Both accelerometer channel sensitivities were adjusted relative to the LDV output, in the case of the mirror accelerometer by temporarily mounting it on the target in place of the target accelerometer. In order to achieve this relative adjustment, broadband (white) random excitation over a 512 Hz frequency range with a 0.5 Hz resolution was applied to the target assembly shaker. The accelerometer and LDV signals were obtained with Hann windowing and were captured with a single spectrum of each extracted from the acquisition system in real and imaginary parts. The accelerometer signal was integrated in the frequency domain in software and the revised accelerometer sensitivity was calculated by multiplying the original value by the

mean of the ratios of the magnitudes between 5 and 100 Hz. Following adjustment of the specified accelerometer sensitivity, a resulting revised accelerometer-LDV magnitude ratio well within 1% of unity was achieved in both the target and steering mirror cases. Small variations from unity are to be expected, of course, due to experimental noise on the signals.

### *Accelerometer signal time delay corrections*

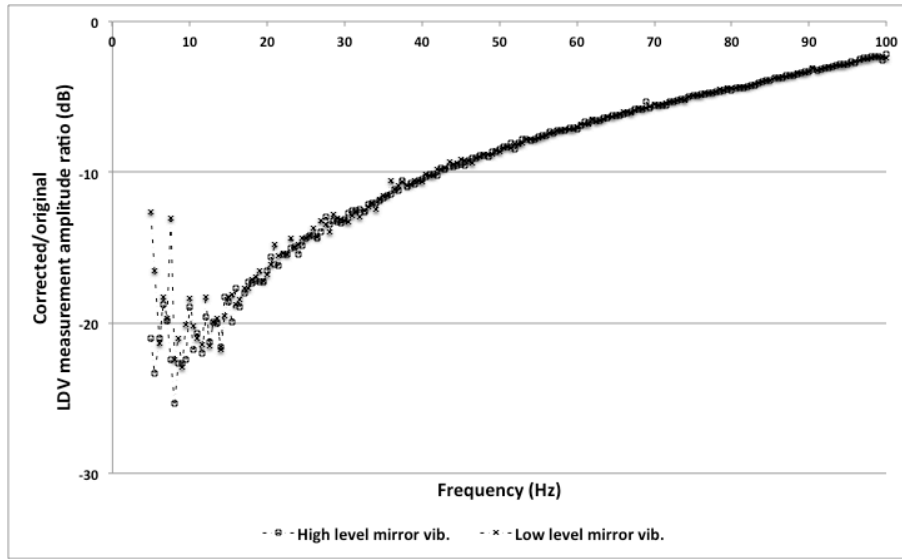
With the specified accelerometer sensitivities duly adjusted, five sets of consecutive frequency spectra were captured for each of four scenarios: target vibration “high” ( $60 \times 10^{-3}$  g RMS) and “low” ( $30 \times 10^{-3}$  g RMS) level with nominally zero steering mirror vibration and, the inverse, steering mirror vibration high ( $50 \times 10^{-3}$  g RMS) and low ( $25 \times 10^{-3}$  g RMS) level with nominally zero target vibration. For the nominally zero steering mirror vibration cases, the LDV measurement is in very close agreement with the (integrated) reference accelerometer measurement as should be expected; differences of 1.3% and 1.2% are observed for the high and low *target* vibration cases respectively. For the nominally zero target vibration cases, however, the difference between the measurements is, in accordance with the mathematical description presented in the previous section, significant. Figure 3 shows the significance of the LDV measurement error with respect to the reference accelerometer target motion measurement; here differences of over 22000% and 11000% for the high and low steering mirror vibration cases respectively are clearly evident.



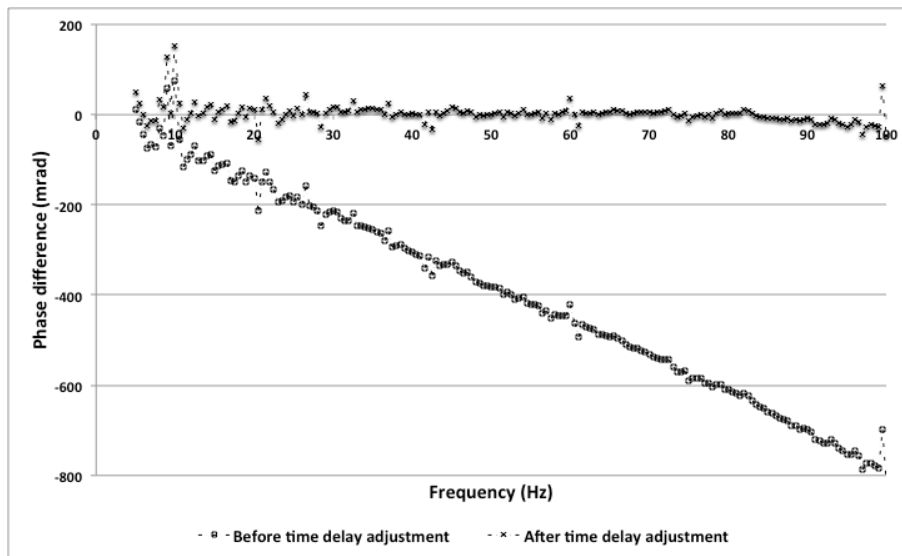
**Figure 4:** LDV and target accelerometer measurements for the zero target vibration cases.

With reference to Equation (1.13), the LDV measurement can be corrected by subtraction of the product of a constant based on the experimental arrangement ( $2 \sin \beta$ ;  $\beta = 90^\circ$  in this set-up) and the (measured) component of the steering mirror vibration normal to the mirror ( $V_B \cdot z_m$ ). Figure 5 shows the mean (of the 5 runs) resulting ratio of the corrected to the original LDV measurement amplitude (presented in dB) for both the high and low level steering mirror vibration scenarios. Here the mean (in the 5-100 Hz range) reduction is c10dB in both cases. The observed reduction in improvement of the corrected signal with increasing frequency was not initially expected.

Upon closer inspection, however, a phase difference, an example of which is shown in Figure 6 for the high steering mirror vibration case, increasing proportionally with frequency, between the corrected signals and the uncorrected equivalents can be observed and it is this that is attributable to the frequency-dependent reduction in the improvement of the corrected signal. Since any hardware integration phase delays were avoided by performing the accelerometer signal integration in software as already described, it was hypothesised that finite time delays between the accelerometer and LDV signals, caused by differences in the specific nature of the signal conditioning and amplification electronics, are the cause of the increasing phase differences.



**Figure 5:** LDV measurement amplitude correction ratio for the zero target vibration cases.



**Figure 6:** Example integrated accelerometer to LDV signal phase difference before and after time delay adjustment.

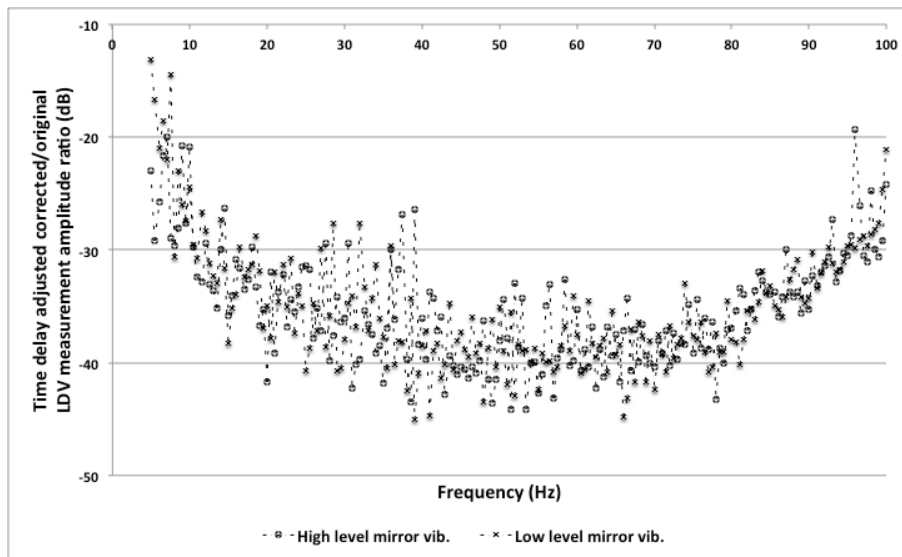


Further correction of the two accelerometer signals in terms of adjustment for these time delays was performed on a frequency-by-frequency basis since, a frequency-independent time delay will, of course, be manifested as a frequency-dependent phase shift. The mean (of the 5 runs) (unwrapped) phase difference per frequency was first determined followed by the mean phase difference (in the range 5 to 100 Hz). As is shown in Table 1, this mean value was approximately 400 mrad for all four scenarios. Dividing this by the mean angular frequency in that same 5 to 100 Hz range clearly leads to an equivalent mean inter-channel time delay; approximately 1.2 ms for both accelerometer channels (similar electronics). The measured accelerometer signals were subsequently phase adjusted, again, on a per-frequency basis, by subtracting the appropriate phase delay. The revised mean phase differences between the *time delay corrected* accelerometer and LDV signals were, as expected, zero in all four scenarios as also shown in Table 1.

**Table 1:** Inter-channel time delay/phase correction statistics

Target vibration		Steering mirror vibration		Phase diff. wrt LDV (mrad)		Equivalent time delay (ms)		Corr. phase diff wrt. LDV (mrad)	
H/L/off	Nominal RMS (x10 <sup>-3</sup> g)	H/L/off	Nominal RMS (x10 <sup>-3</sup> g)	Mean	(SD)	Mean	(SD)	Mean	(SD)
H	60	off	0	-402.4	217.2	-1.220	0.176	0.0	12.5
L	30	off	0	-396.8	216.5	-1.203	0.107	0.0	15.6
off	0	H	50	-402.5	218.9	-1.220	0.205	0.0	14.1
off	0	L	25	-392.5	215.0	-1.190	0.173	0.0	14.1

Using the *time delay adjusted* mirror accelerometer signal to perform the previously described LDV measurement correction in accordance with Equation (1.13) leads to the significantly improved mean ratio of the corrected / original LDV measurement amplitude for both the high and low level steering mirror vibration scenarios that is presented in Figure 6. Here the mean (in the 5-100 Hz range) reduction is c35dB in both cases.



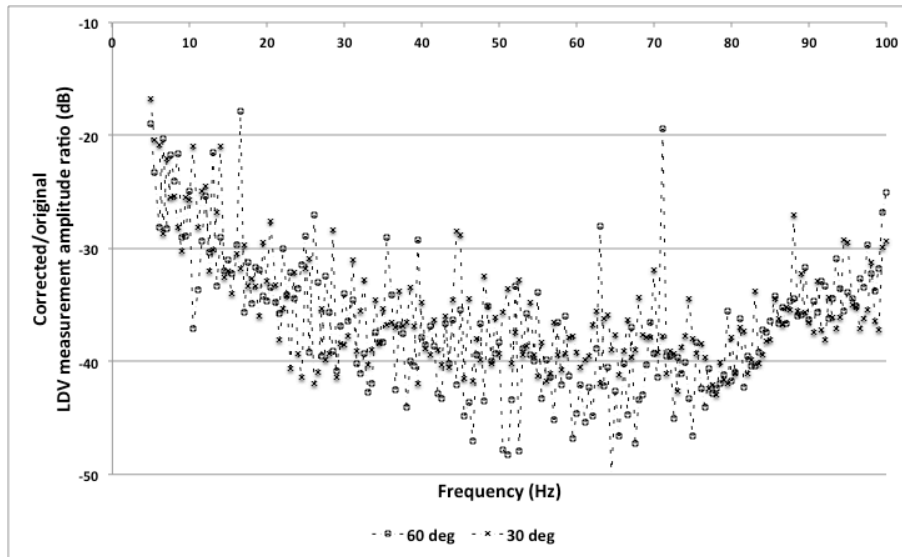
**Figure 7:** LDV measurement amplitude correction ratio for the zero target vibration cases after time delay adjustment.

## Alternative angle measurements

In addition to the arrangement shown in Figure 2, in which the angle between the laser beam and the target surface normal is nominally  $90^\circ$ , repositioning of the instrument and appropriate reorientation of the steering mirror mounting enabled angles of  $60^\circ$  and  $30^\circ$  to be arranged for. Data collection was repeated for these alternative angles for the high steering mirror vibration case only and, as can be seen in Table 2, similar values for the phase differences between the accelerometer with respect to the LDV signal were observed as should be expected. Converting into equivalent time delay, correcting the accelerometer signal and determining the subsequent mean phase difference also shows the same outcome as was found for the  $90^\circ$  scenario. Figure 7 shows the LDV signal reduction for the high steering mirror vibration case; again, the mean reduction in the 5-100 Hz range is 35dB in both cases.

**Table 2:** Inter-channel time delay/phase correction statistics; alternative angles

Angle	Target vibration		Steering mirror vibration		Phase diff. wrt LDV (mrad)		Equivalent time delay (ms)		Corr. phase diff wrt. LDV (mrad)	
	H/L/off	Nominal RMS (x10 <sup>-3</sup> g)	H/L/off	Nominal RMS (x10 <sup>-3</sup> g)	Mean	(SD)	Mean	(SD)	Mean	(SD)
60	H	60	off	0	-399.8	221.6	-1.212	0.105	0.0	20.6
60	off	0	H	50	-398.4	216.2	-1.206	0.230	0.0	16.2
30	H	60	off	0	-399.7	222.0	-1.212	0.242	0.0	22.7
30	off	0	H	50	-399.6	215.6	-1.211	0.204	0.0	12.0



**Figure 8:** LDV measurement amplitude correction ratio for the high steering mirror / zero target vibration case after time delay adjustment; alternative angles.

## Conclusions

This paper has presented, for the first time, a general mathematical treatment of the total velocity measured when a vibrating steering mirror is used to direct a LDV towards the vibrating measurement surface of interest. As has been shown, by making a measurement of the normal

vibration of the steering mirror, it is possible to completely correct for the potentially significant inaccuracy in the measured LDV signal due to the mirror motion. An experimental investigation in which the target and steering mirror vibration were independently controllable was presented. The instrument orientation, i.e. the laser beam direction, with respect to the target was varied between 90°, 60° and 30° with the steering mirror orientation being revised accordingly.

The accelerometer-instrumented target and mirror assemblies were subjected to broadband vibration at two different levels. Adjustment of the (integrated) accelerometer signals in terms of their amplitude relative to the LDV signal was performed by adjusting the specified sensitivities within the acquisition system with the resulting outputs in agreement to within 1.5%. Phase differences, due to the differing signal conditioning electronics in the accelerometer channels with respect to the LDV, were found to be of the order of 400 mrad (mean across the 5-100 Hz range). These differences were completely eliminated by adjusting the accelerometer signals using an equivalent time delay of circa 1.2ms. The LDV signals, over-estimating the nominally zero target vibration by at least 10000% for all of the steering mirror vibration only cases, were able to be corrected by applying the mathematical treatment presented. The over-estimation was reduced consistently by of the order of 35dB. This important, but in many respects intermediate, result will enable future investigations to focus on the, arguably more useful and industrially relevant, scenarios in which there is simultaneous steering mirror and target vibration.

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