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Exploitation of Quasi-Orthogonal Space Time Block Codes in Virtual Antenna Arrays: Part I - Theoretical Capacity and Throughput Gains

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Abstract—A full-rate and full-diversity closed-loop quasi-orthogonal space time block coding scheme pioneered by Toker, Lambotaran and Chambers is proposed for application in virtual antenna arrays. The theoretical capacity and throughput gains are evaluated as a function of signal-to-noise ratio. It is shown that the scheme has particular benefits in both ergodic and non-ergodic channel environments, and outperforms virtual antenna arrays based solely upon conventional orthogonal space time block codes.

Index Terms— Cooperative diversity, quasi-orthogonal space-time block codes (QOSTBCs), closed-loop phase angle feedback, multi-input multi-output (MIMO), virtual antenna arrays (VAAs).

I. INTRODUCTION

In the next generation of wireless communication systems there will be an increased demand for wireless systems that can meet the quality-of-service (QoS) and data capacity specifications demanded by future applications, for example video-on-demand. Over a single transmission link the wireless radio channel may experience fading resulting in significant fluctuations in signal power at the receiver. Diversity can be used to stabilize the received signal power by introducing multiple copies of the information bearing signal over ideally independent realizations of a fading channel. Spatial diversity, through the use of multiple antennas at the transmit and/or receiver side, offers a spectrally efficient alternative to temporal or frequency diversity assuming antenna elements are adequately spaced [1].

In practical wireless communication systems, incorporating cellular handsets or wireless sensor nodes, it may not be practical or even feasible to employ multiple antenna elements on the host due to the physical size of a device and power constraints. To overcome these constraints researchers have been investigating *user cooperation* (also referred to as *cooperative diversity*), inspired by the work in [2], as a novel technique for extracting spatial diversity in situations where *simple* network nodes may only employ a single transceiver antenna element. Through collaboration with other nodes virtual antenna arrays (VAAs) can be formed through joint encoding and decoding in a distributed manner, whereby improvement in capacity and throughput can be achieved.

In this paper we focus on a simple parallel relay channel illustrated in Fig.1, several cooperative schemes have been pioneered that are applicable to the network model [3], however this paper focuses exclusively on space-time block codes (STBCs) because of their simple extension to distributed coding, simple maximum-likelihood decoding and spectral efficiency over simple repetition schemes. Our contribution is to exploit a novel full-rate and full-diversity closed-loop quasi-orthogonal space time block code (CL-QO-STBC) scheme [5] within this parallel relay channel.

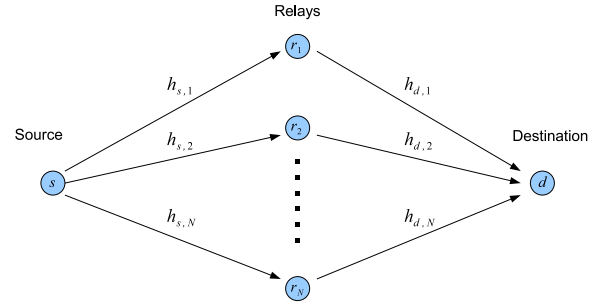


Fig. 1. Simple two-stage relay architecture

In part I of this contribution the paper is organized as follows. Section II introduces the system model description of the two-stage relaying protocol. Sections III and IV give a brief analytical overview of the proposed scheme based on normalized capacity and throughput arguments in ergodic and non-ergodic channels. Section V demonstrates the improved performance of the proposed scheme over conventional space-time block coding techniques, followed by a brief discussion of the throughput and capacity benefits of the proposed scheme at a typical operating signal-to-noise ratio of 10dB.

In part II further implementation-specific analysis of the proposed CL-QO-STBC scheme is included where the results in part I are supported by Monte Carlo simulations.

II. SYSTEM MODEL

In our model we consider a simple two-stage relay network depicted in Fig.1, with a single source and destination communicating via cooperating relay nodes. All participating nodes communicate using a single antenna configuration over narrow-band flat-fading channels $h_{i,j}$, where $i \in \{s, d\}$ represents the channel from the source or destination to a particular relay node $j \in \{1, 2, \dots, N\}$ up to the maximum number of participating relays N . In the context of this work all random channel parameters $h_{i,j}$ are assumed to be zero mean circular symmetric complex Gaussian (ZMCSG) random

variables with unity variance (i.e. Rayleigh fading).

In this scheme we use a simple two-step protocol over two time-frames, i.e. time division multiple access (TDMA). In the first time-frame the source node broadcasts $\sqrt{P_s}s_k$ to the relay nodes over a number of symbol intervals depending on the size of the STBC coding matrix; where P_s is the source transmit power and $s \in Z$ is the transmitted symbol. Assuming perfect cooperation among relay nodes, i.e. perfect synchronization and error free inter-relay communication using no channel resources (bandwidth/time), the distributed channel vector \mathbf{h}_{sr} can take the form,

$$\mathbf{h}_{sr} = [h_{s,1} h_{s,2} \dots h_{s,N}]^T \quad (1)$$

which enables the received signal in the presence of Gaussian noise denoted by \mathbf{n} (i.e. ZMCSCG random variables with variance N_0) to be compactly represented in vector form over all the relay terminals,

$$\mathbf{y}_{sr} = \sqrt{P_s} \mathbf{h}_{sr} s + \mathbf{n} \quad (2)$$

Adopting a decode-and-forward (DF) strategy at the relay terminals and assuming full channel-state-information (CSI) is available at all receiving nodes, then simple maximal-ratio-combining (MRC) followed by maximum-likelihood decoding is the optimum approach for the best distributed estimation of the transmitted symbol,

$$z = \mathbf{h}^H \mathbf{y}_{sr} = \|\mathbf{h}\|^2 s + \mathbf{h}^H \mathbf{n} \quad (3)$$

$$\hat{s} = \arg \min_{\hat{s} \in S} \|\hat{s} - z\|^2 \quad (4)$$

where $(\cdot)^H$ and $\|\cdot\|^2$ denote Hermitian and Euclidean distance operators, \hat{s} represents the symbol estimate restricted to a finite set of signal constellations S (i.e. BPSK/QPSK).

In the next time-frame each relay node synchronously transmits the estimated symbols in the first time-frame according to a pre-allocated column of a STBC matrix. In this implementation the Jafarkhani QO-STBC scheme [6] with four participating relays is used,

$$\begin{bmatrix} y_1 \\ y_2^* \\ y_3^* \\ y_4 \end{bmatrix} = \begin{bmatrix} h_{d,1} & h_{d,2} & h_{d,3} & h_{d,4} \\ h_{d,2}^* & -h_{d,1}^* & h_{d,4}^* & -h_{d,3}^* \\ h_{d,3}^* & h_{d,4}^* & -h_{d,1}^* & -h_{d,2}^* \\ h_{d,4} & -h_{d,3} & -h_{d,2} & h_{d,1} \end{bmatrix} \begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \hat{s}_3 \\ \hat{s}_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \\ n_3^* \\ n_4 \end{bmatrix} \quad (5)$$

$$\mathbf{y}_{rd} = \mathbf{H}_{rd} \hat{\mathbf{s}} + \mathbf{n}$$

where \mathbf{y}_{rd} , \mathbf{H}_{rd} and $\hat{\mathbf{s}}$ denote the vector representation of the received symbols over four symbol intervals, the channel matrix representation between the relays and destination nodes after space-time coding at the relay terminals and the estimated symbols after the first-stage; note $(\cdot)^*$ denotes complex conjugate operation. However, in [6] Jafarkhani notes that full order diversity cannot be accomplished due to inter-symbol-interference (ISI) between estimated symbols, illustrated by the α terms below, which after combining at the destination receiver results in a matrix of the following form,

$$\mathbf{H}_{rd}^H \mathbf{H}_{rd} = \begin{bmatrix} \gamma & 0 & 0 & \alpha \\ 0 & \gamma & -\alpha & 0 \\ 0 & -\alpha & \gamma & 0 \\ \alpha & 0 & 0 & \gamma \end{bmatrix} \quad (6)$$

For a more detailed analysis the interested reader is referred to [6]. However, Toker et al. [5] discovered that a simple phase rotation θ and ϕ on the complex symbols transmitted from nodes 3 and 4

cancels the ISI and provides 4th order diversity, at the expense of requiring a feedback channel.

The remainder of this paper assumes perfect ISI cancellation through phase rotation at the relaying nodes and analyzes the maximum normalized capacity and throughput advantage of the proposed scheme for both ergodic and non-ergodic channels. The ensuing theoretical analysis is outlined on the assumption that perfect CSI is available at the receiver and feedback eliminates ISI enabling a simple analytical approach to be applied to the analysis of the CL-QO-STBC scheme that would be used for traditional orthogonal space-time block codes (OSTBCs)

III. ERGODIC CHANNELS

Ergodic channels experience the same moments of the random fading process from codeword to codeword, when infinite length codewords assumed by Shannon [4] are used. For flat-fading multiple input multiple output (MIMO) channels, orthogonalized through the use of OSTBCs, the normalized capacity can be derived by evaluating the integral,

$$C = R \int_0^\infty \log_2 \left(1 + \frac{1}{R} \frac{\lambda}{t} \frac{S}{N} \right) pdf_\lambda(\lambda) d\lambda \quad (7)$$

where S , N , R , t and λ denote respectively the signal power, noise power, STBC rate, number of transmit antennas and the random channel gain parameter. The pdf of λ can be shown to follow a chi-square distribution when each sub-channel (channels between nodes) assumes a complex Gaussian distribution with zero mean and unity variance,

$$pdf_\lambda(\lambda) = \frac{1}{\Gamma(u)} \lambda^{u-1} e^{-\lambda} \quad (8)$$

where $\Gamma(\cdot)$ and u denote the *ordinary gamma function* and the number of sub-channels respectively. Solutions to the capacity integral (7) have been derived by Dohler [7](Eq.2.45) in an iterative form which we use extensively in this paper to evaluate the normalized capacity of various STBC configurations over a single relaying link; see Section V.

However, the above analysis is only comprehensive when evaluating the capacity of a single wireless hop. In multi-stage wireless networks the average end-to-end capacity is determined by the hop with the minimum capacity. Even in the simple model illustrated in Fig.1, the capacity of each stage is unequal. One method to equalize the capacity of the relaying stages is to fractionally allocate all available bandwidth α_j and power β_j resources, resulting in a new capacity expression for each relaying stage v incorporating the fractional resource parameters,

$$C_v = \alpha_v R_v E_{\lambda_v} \{ \log_2(1 + \lambda_v \frac{1}{t_v} \frac{\beta_v}{\alpha_v} \frac{S}{N}) \} \quad (9)$$

In [7] Dohler shows that a simple approximation $[\log(1+x) = \sqrt{x}]$ performed on the normalized capacity expression for orthogonal-MIMO (O-MIMO) channels (7) yields [7](Eq.2.105),

$$C \approx \frac{\sqrt{S}}{\sqrt{N}} \frac{\sqrt{R}}{\sqrt{t}} \frac{\Gamma(u + \frac{1}{2})}{\Gamma(u)} = \frac{\sqrt{S}}{\sqrt{N}} \Lambda(t, u, R) \quad (10)$$

this can then be extended to include the fractional resource allocation parameters α_v and β_v ; i.e. approximation of (9),

$$C_v \approx \alpha_v \sqrt{\frac{\beta_v}{\alpha_v}} \frac{\sqrt{S}}{\sqrt{N}} \Lambda(t, u, R) \quad (11)$$

This result (11) then enables [approximate] explicit fractional resource allocation avoiding lengthy numerical optimizations. Firstly, to acquire α_v, β_v the ratio of resource allocation parameters needs to be resolved for each relay stage $v, w \in \{1, 2\}$ [7](Eq.3.22),

$$\frac{\beta_v}{\alpha_v} = 2 \cdot \frac{\Lambda^{\frac{2}{3}}(t_{w \neq v}, u_{w \neq v}, R_{w \neq v})}{\sum_{k=1}^2 \Lambda^{\frac{2}{3}}(t_k, u_k, R_k)} \quad (12)$$

using this intermediate result the bandwidth α_v parameters for each stage may be calculated using [7](Eq.3.13),

$$\alpha_v = \frac{R_{w \neq v} E_{\lambda_{w \neq v}} \{\log_2(1 + \lambda_v \frac{\gamma_{w \neq v}}{t_{w \neq v}} \frac{\beta_{w \neq v}}{\alpha_{w \neq v}} \frac{S}{N})\}}{\sum_{k=1}^2 R_k E_{\lambda_k} \{\log_2(1 + \lambda_k \frac{\gamma_k}{t_k} \frac{\beta_k}{\alpha_k} \frac{S}{N})\}} \quad (13)$$

The power allocations β_v for the various stages can then be calculated by substituting the values for α_v into (12).

In Section V results will demonstrate the advantages of fractional resource allocation for various space-time coding schemes implemented at the relay terminals.

IV. NON-ERGODIC CHANNELS

In contrast to ergodic fading channels, the complex channel coefficients λ in non-ergodic channels are fixed at the start of a transmission. Hence, there is a non-zero probability, referred to as the *outage probability*, that the channel cannot support a given rate ϕ . For O-MIMO channels utilizing OSTBCs at the transmitter the probability that the channel is in outage is expressed as,

$$P_{out}(\phi) = \frac{\gamma(u, \hat{\lambda})}{\Gamma(u)} \quad (14)$$

where $\gamma(\cdot, \cdot)$ denotes the *lower incomplete gamma function* [7]. Expression (14) can therefore be evaluated by substituting the following expression for the upper integral limit $\hat{\lambda}$,

$$\hat{\lambda} = \frac{2^{\frac{\phi}{R}} - 1}{\frac{1}{R} \frac{1}{t} \frac{S}{N}} \quad (15)$$

For non-ergodic channels the term capacity is essentially meaningless because the channel is fixed at the start of transmission and has a non-zero probability $P_{out}(\phi)$ that the channel cannot support a given rate; therefore, normalized throughput Φ is a more appropriate metric,

$$\Phi = (1 - P_{out}(\phi)) \cdot \phi \quad (16)$$

$$= (1 - P_{out}(\phi)) \cdot R \log_2 \left(1 + \frac{1}{R} \frac{\hat{\lambda}}{t} \frac{S}{N} \right) \quad (17)$$

Clearly, there exists an optimum transmit rate ϕ which maximizes throughput, however this proves intractable to determine with the outage expression (8). Interestingly, in [7] the analysis is simplified by introducing an approximation for (14) in the form of,

$$\gamma(u, \hat{\lambda})/\Gamma(u) \approx a \hat{\lambda}^b \quad (18)$$

where a and b are calculated to provide optimal curve fitting to the outage probability for various OSTBC schemes and values of $\hat{\lambda}$. It was shown in [7] that this simple approximation yields an optimal operating outage probability that maximizes the throughput,

$$P_{out} \approx \frac{1}{1 + 2b} \quad (19)$$

which is invariant to changes in signal-to-noise ratio (SNR) at the receiver.

As with ergodic channels, throughput in multi-stage non-ergodic channels is also determined by the weakest hop, i.e. the hop with minimum throughput. Therefore to maximize end-to-end throughput, the throughput at each individual stage needs to be equalized and maximized through fractional resource allocation. After simple manipulations of the expressions (17), (18) and (19) along with insertion of fractional resource allocation parameters α and β a throughput expression for stage v maybe expressed as [7](Eq.3.48),

$$\Phi_v = \alpha_v \cdot R_v \cdot \left(\frac{2b}{1 + 2b} \right) \cdot \log_2 \left(1 + \left(\frac{1}{a(1 + 2b)} \right)^{\frac{1}{b}} \frac{\beta_v}{\alpha_v} \frac{1}{R_v} \frac{1}{t_v} \frac{S}{N} \right) \quad (20)$$

then applying the approximation $\log(1 + x) = \sqrt{x}$,

$$\Phi_v = \alpha_v \cdot \sqrt{R_v} \sqrt{\frac{\beta_v}{\alpha_v}} \sqrt{\frac{1}{t_v}} \sqrt{\frac{S}{N}} \left(\frac{2b}{1 + 2b} \right) \cdot \left(\frac{1}{a(1 + 2b)} \right)^{\frac{1}{b}} \quad (21)$$

results in a similar expression to (11) [adjusting for constants], therefore enabling the utilization of fractional resource allocation strategies that were adopted for ergodic channels [7].

The benefits of resource allocation to maximize throughput in non-ergodic channels will be demonstrated in Section V.

V. SIMULATION RESULTS AND DISCUSSION

In our simulations, we consider a simple two-stage relay network as depicted in Fig.1. Sub-channel coefficients and additive noise at receiving nodes are assumed to be ZMCSCG random variables and only the narrow-band case is examined. To facilitate an accurate comparison of distributed-STBC schemes at the second stage, a system with four receiving nodes and the simple combining scheme (3) is assumed for the first stage.

In our first simulation, Fig.2, the normalized capacity (ergodic channels) of OSTBCs over a single stage is illustrated with varying values of signal-to-noise ratio (SNR). Clearly, the CL-QO-STBC with perfect phase rotation at the transmitters provides the optimal capacity performance. Note that an 'equivalent' Gaussian channel has been included as an upper bound to performance in this flat-fading environment.

In a two-stage implementation, as illustrated in Fig.1 with four relay nodes, the proposed CL-QO-STBC scheme as shown in Fig.3 provides increased channel capacity over all other OSTBC schemes in ergodic channels independent of SNR at the receivers. Assuming an operating SNR of 10dB at all receiving nodes and implementing fractional resource allocation, our scheme delivers a 20% capacity improvement over the non-optimized Alamouti case, i.e. nearest competitor scheme.

Our third simulation, Fig.4, demonstrates that the proposed CL-QO-STBC scheme, operating in a single-stage non-ergodic channel, provides the highest achievable rate ϕ in applications that require reliable transmission, i.e. $P_{out}(\phi) \leq 0.5$. Also, the dominance of our scheme in this range eradicates the need for a STBC scheme selection algorithm to ensure optimal transmission rates.

Extending the previous simulation to the two-stage network architecture, the CL-QO-STBC scheme, as illustrated in Fig.5, provides the maximum throughput Φ over all OSTBC schemes over the entire SNR range. Again assuming an operating SNR of 10dB the CL-QO-STBC scheme delivers a 15% improvement in throughput, and when fractional resource optimization is used the improvement increases

to 40% over the nearest non-optimized OSTBC competitor scheme, clearly strengthening the case for using CL-QO-STBC in distributed VAAs.

This paper shows the possible improvements in both capacity and throughput achievable by implementing the CL-QO-STBC scheme in relay networks over conventional OSTBC schemes. Part II of this contribution extends this work through Monte-Carlo simulation studies.

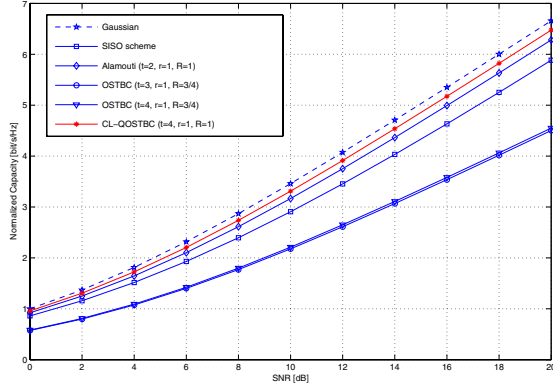


Fig. 2. Normalized capacity vs. SNR - single stage ergodic channels

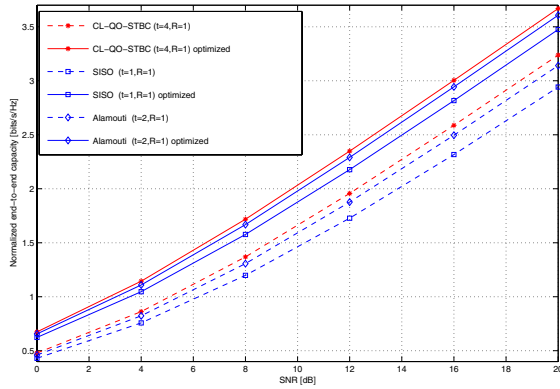


Fig. 3. Normalized capacity vs. SNR two-stage ergodic channel configuration (fractional resource optimization considered)

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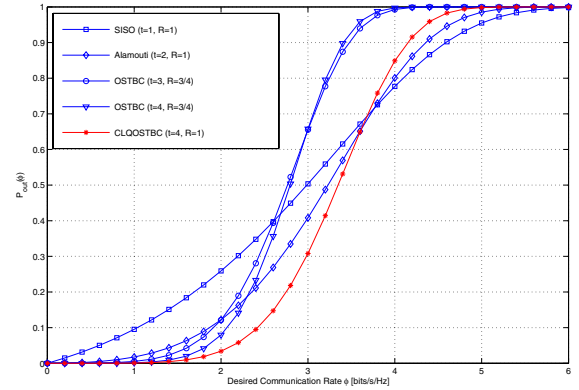


Fig. 4. Outage probability $P_{out}(\phi)$ vs. normalized transmission rate ϕ - single stage non-ergodic channels (SNR=10dB)

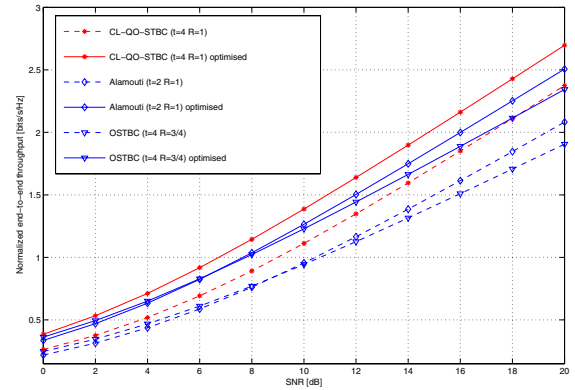


Fig. 5. Normalized throughput Φ vs. SNR - two-stage non-ergodic channel configuration (fractional resource optimization considered)

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