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Joint User-Association and Resource-Allocation in Virtualized Wireless Networks

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ABSTRACT In this paper, we consider the down-link dynamic resource allocation in multi-cell virtualized wireless networks (VWNs) to support the users of different service providers (slices) within a specific region by a set of base stations (BSs) through orthogonal frequency division multiple access (OFDMA). In particular, we develop a joint BS assignment, sub-carrier, and power allocation algorithm to maximize the network sum rate, while satisfying the minimum required rate of each slice. Under the assumption that each user at each transmission instance can connect to no more than one BS, we introduce the user-association factor to represent the joint sub-carrier and BS assignment as the optimization variable vector in the problem formulation. Sub-carrier reuse is allowed in different cells, but not within one cell. As the proposed optimization problem is inherently non-convex and NP-hard, by applying the successive convex approximation (SCA) and complementary geometric programming (CGP), we develop an efficient two-step iterative approach with low computational complexity to solve the proposed problem. For a given problem, Step 1 derives the optimum user-association and subsequently, and for an obtained user-association, Step 2 finds the optimum power allocation. Simulation results demonstrate that the proposed iterative algorithm outperforms the traditional approach in which each user is assigned to the BS with the largest average value of signal strength, and then, joint sub-carrier and power allocation is obtained for the assigned users of each cell. Simulation results reveal a coverage improvement, offered by the proposed approach, of 57% and 71% for uniform and non-uniform users distribution, respectively, leading to higher spectrum efficiency for VWN.

INDEX TERMS Complementary geometric programming, successive convex approximation, joint user association and resource allocation, virtualized wireless networks.

I. INTRODUCTION

A. MOTIVATION

To increase the spectrum efficiency, the context of *virtualized wireless networks* (VWNs) is a promising approach in which the physical resources of one network provider, e.g., spectrum, power, and infrastructure, are shared among different service providers, also called slices [1]–[3]. Generally, each slice comprises of a set of users, and has its own quality-of-service (QoS) requirements. To harvest the potential advantages of VWN, effective resource allocation is a major concern, which has been drawing a lot of attention in recent years.

For instance, in [1], a resource management scheme is studied by introducing two types of slices, including rate-based and resource-based slices, where the minimum rate and minimum network resources are preserved for each slice, respectively. Furthermore, in [4], interactions among slices, network operator, and users are modeled as an auction game where the network operator is responsible for spectrum management on a higher level, and each slice focuses on QoS management for its own users. To preserve the QoS of slices from wireless channel fading, the admission control policy is proposed in [5], where the requirement of each slice is adjusted by the channel state information (CSI) of its own users. To extend the feasibility condition of VWN in order to support diverse QoS requirements, [6] considers the use of massive MIMO where the access point is equipped with a large number of antennas. In [7], the combination of time,

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space and elastic resource allocation for OFDMA systems is considered. Advantages of full-duplex transmission relay in VWN have been investigated in [8].

Generally, these works have focused on analyzing the resource allocation problem in a single-cell VWN scenario. However, in practice, the coverage of a specific region may require a set of BSs, in a multi-cell VWN scenario. The key question in such a multi-cell VWN scenario is how to allocate the resources to maintain the QoS of each slice, while improving the total performance of VWN over a specific region. In this paper, we consider a multi-cell OFDMA based VWN where the coverage of a specific region is provided by a set of BSs serving different groups of users belonging to different slices. The QoS of each slice is represented by its minimum reserved rate. Each user of each slice can be only served by one BS and this BS is not predetermined by the distance or by measuring the average received signal strength of BSs. Consequently, in this setup, the resource sets in the related optimization problem involve the *sets of BSs, sub-carriers* and *power* for each user belonging to each slice.

In this paper, the objective of proposed resource allocation problem is to maximize the total throughput of VWN in the specific region subject to power limitation of BSs, minimum required rate of each slice, and sub-carrier and BS assignment limitations. Based on the limitations of down-link OFDMA transmission, each sub-carrier can be assigned to one user within a cell and each user can be associated to only one BS. Since in this optimization problem, the sub-carrier assignment and BS association are inter-related, we introduce the user-association factor (UAF) that jointly determines the BS assignment and sub-carrier allocation as the optimization variable vector. Due to this user-association constraint and the inter-cell interference, the proposed optimization problem is non-convex and NP-hard, suffering from high computational complexity [9]. We apply the frameworks of complementary geometric programming (CGP) and successive convex approximation (SCA) [10]–[12], [36] to develop an efficient, iterative, two-step algorithm to solve the proposed problem. For a given power-allocation, Step 1 derives the optimum user-association solution, and subsequently, with this obtained user-association solution, Step 2 finds the optimal power allocation. This two-step optimization process is repeated until convergence. It can be shown that the simplified problem of each step still involves non-convex optimization problem. By applying various transformation and convexification techniques, we develop the analytical framework to transform the non-convex optimization problems encountered in each step into the equivalent lower-bound geometric programming (GP) problems, [12], which can be solved by available software packages, e.g., CVX [13].

Simulation results demonstrate that the proposed approach can significantly outperform the traditional scenario where the BS assignment is based on the largest average signal-to-interference-plus-noise ratio (SINR), and subsequent sub-carrier and power allocation is derived for the users of each cell. The simulation results reveal that considering UAF can increase the feasibility of resource allocation problem (i.e., the required rate of each slice will be satisfied with a higher probability as compared to the traditional approach). Specifically, the proposed algorithm can significantly increase the probability of achieving higher rates for the cell-edge users, resulting better coverage for the VWN.

B. RELATED WORKS

Our work in this paper lies along the intersection of two research contexts in resource allocation problems: 1) multi-cell OFDMA wireless networks, and 2) VWNs.

There exists a large body of research conducted in resource allocation for multi-cell OFDMA wireless networks. For example, in [14], the resource allocation in conventional OFDMA-based network is studied using BS-assignment based on the largest average received signal[1] strength from the BS at each user. An iterative algorithm for maximizing the weighted sum of minimum user rates in each BS is explored in [16]. Joint cell, channel and power allocation in multi-cell relay networks is explored in [17], where each user is assigned to the BS with the highest channel gain. In [18], a proportional fair resource allocation in a multi-cell OFDMA network is proposed aiming to maintain the quality of experience of users by considering a utility function based on the mean opinion score. In [19], joint scheduling of resource blocks, power allocation, and modulation and coding scheme in LTE-A system is considered by using the criteria of proportional fairness. A similar problem in OFDMA cognitive radio networks is studied in [20], where an iterative algorithm is proposed to solve the sub-carrier and power allocation. Similarly, in [21], a resource allocation problem for jointly optimizing the energy and spectral efficiency is proposed for a multi-cell OFDMA wireless network by considering an energy and spectral efficiency trade-off metric. In [22], the authors have considered an energy efficient resource allocation problem for a multi-cell OFDMA network in a conventional wireless network where the available values of channel state information (CSI) are imperfect. In [23], a resource allocation algorithm is proposed for a two-cell down-link OFDMA network with a fractional frequency reuse scheme among BSs.

In the aforementioned works (i.e., [14], [16]–[21], [23]),the BS assignment algorithm is separated from the sub-carrier allocation, while joint sub-carrier and power allocation is applied for multi-cell scenario. Compared to this approach, we consider UAF which jointly assigns the BS and sub-carrier for each user and then allocates power allocation using the derived UAF. Furthermore, we consider the implementation limitations of multi-cell OFDMA networks by proposing new constraints.

[1]This average is derived based on the measurement of users over one specific window in both idle and active phases, where the size of the measurement window of each user is adjusted based on the specification of wireless network standards [15].

As previously mentioned, the resource allocation in VWNs has received growing attention. In [2], different aspects of VWN including resource discovery and allocation as well as the research challenges have been discussed. Besides [1], [4]–[7], in [24], the challenge in allocating physical resource blocks (PRBs) to various slices in an LTE network has been addressed considering a single BS scenario. In [25], an opportunistic algorithm to allocate the resources to virtual operators is proposed by differentiating the resource requirements among operators as baseline and fluctuate requirements to ensures the minimum QoS requirements of each virtual operator. In [3], the concept of virtualization has been extended to a LTE network by considering virtual operators or slices each with various bandwidth requirements in terms of the physical resource blocks (PRBs) in LTE. To the best of our knowledge, the multi-cell scenario of VWNs has not been studied in the previous related works in this context.

C. STRUCTURE OF PAPER

The rest of this paper is organized as follows. In Section II, the system model and problem formulations are introduced. Section III presents the proposed two-step iterative algorithm for joint user-association and resource-allocation algorithm along with the mathematical background, the detailed explanation of iterative algorithms, and computational complexity analysis. Section IV demonstrates the simulation results and their detailed analysis, followed by concluding remarks in Section V.

II. SYSTEM MODEL

We consider the down-link transmission of a VWN where the coverage of a specific area is provided by a set of BSs, i.e., $\mathcal{M} = \{1, \ldots, M\}$. The total bandwidth of B Hz is divided into a set of sub-carriers, $\mathcal{K} = \{1, \ldots, K\}$ and shared by all BSs through orthogonal frequency-division multiple-access (OFDMA). The bandwidth of each sub-carrier, i.e., $B_c = \frac{B}{K}$, is assumed to be much smaller than the coherent bandwidth of the wireless channel so that the channel response in each sub-carrier is flat. This set of BSs serves a set of slices, $\mathcal{G} = \{1, \ldots, G\}$, where the slice g has a set of users $\mathcal{N}_g = \{1, \ldots, N_g\}$ and requests for a minimum reserved rate of R_g^{rsv} and $N = \sum_{g\in\mathcal{G}} N_g$ is the total number of users.

Let h_{m,k,n_g} and P_{m,k,n_g} be the channel power gain (also representing the channel state information (CSI)), and the allocated power, respectively, of the link from BS $m \in \mathcal{M}$ to user n_g of slice g on sub-carrier k. Due to the OFDMA limitation, each user is assigned to one BS, and to avoid intra-cell interference, orthogonal sub-carrier assignment is assumed among users in a cell. The binary-valued user-association factor (UAF) $\beta_{m,k,n_g} \in \{0, 1\}$ represents both sub-carrier allocation and BS assignment indicator for user n_g of slice g on sub-carrier k of BS m, i.e., $\beta_{m,k,n_g} = 1$ when BS m allocates sub-carrier k to user n_g, and $\beta_{m,k,n_g} = 0$, otherwise.

Consider $\mathbf{P} = \left[P_{m,k,n_g}\right]_{\forall m,g,n_g,k}$ and $\boldsymbol{\beta} = \left[\beta_{m,k,n_g}\right]_{\forall m,g,n_g,k}$ as the vectors of all transmit powers and UAFs of users, respectively. The rate of user n_g at sub-carrier k of BS m can be expressed as

$$R_{m,k,n_g}(\mathbf{P}) = \log_2\left[1 + \frac{P_{m,k,n_g} h_{m,k,n_g}}{\sigma^2 + I_{m,k,n_g}}\right], \tag{1}$$

where

$$I_{m,k,n_g} = \sum_{\forall m'\in\mathcal{M}, m'\neq m} \sum_{\forall g\in\mathcal{G}} \sum_{\forall n'_g\in\mathcal{N}_g, n'_g\neq n_g} P_{m',k,n'_g} h_{m,k,n'_g}$$

is the interference to user n_g in cell m and sub-carrier k, and σ^2 is the noise power. Without loss of generality, noise power is assumed to be equal for all users in all sub-carriers and BSs. From (1), the required minimum rate of slice $g \in \mathcal{G}$ can be represented as

$$\text{C1}: \sum_{m\in\mathcal{M}} \sum_{n_g\in\mathcal{N}_g} \sum_{k\in\mathcal{K}} \beta_{m,k,n_g} R_{m,k,n_g}(\mathbf{P}) \geq R_g^{\text{rsv}}, \quad \forall g \in \mathcal{G}.$$

We consider the maximum transmit power limitation of each BS as

$$\text{C2}: \sum_{g\in\mathcal{G}} \sum_{n_g\in\mathcal{N}_g} \sum_{k\in K} P_{m,k,n_g} \leq P_m^{\max}, \quad \forall m \in \mathcal{M},$$

where $P_m^{\max}$ is the maximum transmit power of BS m. Furthermore, the OFDMA exclusive sub-carrier allocation within each cell m can be expressed as

$$\text{C3}: \sum_{g\in\mathcal{G}} \sum_{n_g\in\mathcal{N}_g} \beta_{m,k,n_g} \leq 1, \quad \forall m \in \mathcal{M}, \quad \forall k \in \mathcal{K}.$$

In this setup, due to the limitation of multi-cell OFDMA, we restrict the access of each user by the following constraint

$$\text{C4}: \left[\sum_{k\in\mathcal{K}} \beta_{m,k,n_g}\right]\left[\sum_{\forall m'\neq m} \sum_{k\in\mathcal{K}} \beta_{m',k,n_g}\right] = 0, \quad \forall n_g \in \mathcal{N}_g, \forall g \in \mathcal{G}, \quad \forall m \in \mathcal{M}.$$

C4 implies that each user can be associated to only one BS. More specifically, C4 ensures when any sub-carrier k is assigned to user n_g by BS m, that user would not be assigned any sub-carriers by other BSs m'.

The joint power, sub-carrier and BS assignment can be formulated as

$$\max_{\boldsymbol{\beta},\mathbf{P}} \sum_{m\in\mathcal{M}} \sum_{g\in\mathcal{G}} \sum_{n_g\in\mathcal{N}_g} \sum_{k\in\mathcal{K}} \beta_{m,k,n_g} R_{m,k,n_g}(\mathbf{P}),$$
$$\text{subject to: C1 - C4.} \tag{2}$$

The optimization problem (2) has a non-convex objective function due to inter-cell interference and involves non-linear constraints with combination of continuous and binary variables, i.e., $\mathbf{P}$ and $\boldsymbol{\beta}$. In other words, (2) is a non-convex mixed-integer, NP-hard optimization problem [9]. Therefore, proposing an efficient algorithm with reasonable computational complexity is desirable.

Algorithm 1 Iterative Joint User-Association Factor (UAF) and Power Allocation Algorithm

Initialization: Set $t = 0$, and $\mathbf{P}(t = 0)$ such as power in each sub-carrier of BS m is $P_m^{\max}/K$.
Repeat: Set $t = t + 1$.
Step 1.A User Association:
Initialization for Step 1.A: Set $t_1 = 0$, $\boldsymbol{\beta}(t_1) = \boldsymbol{\beta}(t)$, $\mathbf{P}(t_1) = \mathbf{P}(t)$ and set arbitrary initial for $s_{m,n_g}(t_1)$.
  Repeat: Set $t_1 = t_1 + 1$.
    Step 1.A.1: Update $\lambda_{m,n_g}(t_1)$, $\alpha_{m,n_g}(t_1)$, $\nu_{m,k,n_g}(t_1)$, $\eta_{m,k,n_g}(t_1)$ and $\varphi_{m,k,n_g}(t_1)$ using (12)-(15) and (18)-(20),
    Step 1.A.2: Find optimal UAF in (16) using CVX [13],[2]
  Until $||\boldsymbol{\beta}^*(t_1) - \boldsymbol{\beta}^*(t_1 - 1)|| \leq \varepsilon_1$, set $\boldsymbol{\beta}(t) = \boldsymbol{\beta}^*(t_1)$.
Step 1.B Power Allocation:
Initialization for Step 1.B: Set $t_2 = 0$, $\boldsymbol{\beta}(t_2) = \boldsymbol{\beta}(t)$.
  Repeat: Set $t_2 = t_2 + 1$.
    Step 1.B.1: Update $\kappa_{m,k,n_g}(t_2)$, $\kappa_o(t_2)$ using (23) and (24),
    Step 1.B.2: Find optimal power allocation according to (25) using CVX [13], [34]
  Until $||\mathbf{P}^*(t_2) - \mathbf{P}^*(t_2 - 1)|| \ll \varepsilon_2$, set $\mathbf{P}(t) = \mathbf{P}^*(t_2)$.
Until $\|\boldsymbol{\beta}^*(t) - \boldsymbol{\beta}^*(t-1)\| \leq \varepsilon_1$, and $\|\mathbf{P}^*(t) - \mathbf{P}^*(t-1)\| \leq \varepsilon_2$.

III. TWO-STEP ITERATIVE ALGORITHM FOR JOINT USER-ASSOCIATION AND RESOURCE-ALLOCATION

To tackle the computational complexity of (2), we adopt an iterative approach to find the UAF and power allocation for each user in two steps as shown in Algorithm 1. In Step 1, for a given power allocation vector, the UAF is considered as the variable of the user-association problem and solved by Algorithm 1.A (to be discussed in detail in Section III.B).

This derived UAF is then used in Step 2 to find the corresponding allocated power as the solution of the power-allocation optimization problem by Algorithm 1.B (to be discussed in detail in Section III.C). Steps 1 and 2 are iteratively executed until both the current UAF and power allocation vector solutions are not much different from their values obtained in the previous iteration. In other words, the sequence of the UAF and power allocation vector solutions can be expressed as

$$\underbrace{\boldsymbol{\beta}(0) \rightarrow \mathbf{P}(0)}_{\text{Initialization}} \rightarrow \ldots \underbrace{\boldsymbol{\beta}^*(t) \rightarrow \mathbf{P}^*(t)}_{\text{Iteration } t} \rightarrow \underbrace{\boldsymbol{\beta}^* \rightarrow \mathbf{P}^*}_{\text{Optimal solution}}, \quad (3)$$

where $t > 0$ is the iteration number and $\boldsymbol{\beta}^*(t)$ and $\mathbf{P}^*(t)$ are the optimal values at the iteration t from convex transformation of related optimization problems in each step. The iterative procedure is stopped when

$$||\boldsymbol{\beta}^*(t) - \boldsymbol{\beta}^*(t-1)|| \leq \varepsilon_1 \text{ and } ||\mathbf{P}^*(t) - \mathbf{P}^*(t-1)|| \leq \varepsilon_2$$

where $0 < \varepsilon_1 \ll 1$ and $0 < \varepsilon_2 \ll 1$.

[2]CVX chooses its own initial value for vector $\boldsymbol{\beta}$ [13], which is applied for our algorithm to check the convergence condition.

Notably, both the user-association and power-allocation optimization problems are still non-convex and suffer from high computational complexity. To solve them efficiently, we apply complementary geometric programming (CGP) for each step [12] in which via different transformations and convexification approaches, the sequence of lower bound GP approximation of relative optimization problem is solved as described in detail in the following sections.

A. COMPLEMENTARY GEOMETRIC PROGRAMMING (CGP): A BRIEF REVIEW

Geometric programming (GP) is a class of non-linear optimization problems, which can be solved very efficiently via numerical methods [11]. Various resource allocation problems have been solved by converting them into GP problems to reach computationally tractable algorithms, e.g., [10], [11], [28]–[30]. The standard form of GP is defined as

$$\begin{aligned} &\min_{\mathbf{x}} f_0(\mathbf{x}), \\ &\text{subject to: } f_i(\mathbf{x}) \leq 1, \quad i = 0, 1, \cdots, I, \\ &\quad g_j(\mathbf{x}) = 1, \quad j = 0, 1, \cdots, J, \end{aligned} \quad (4)$$

where $\mathbf{x} = [x_1, x_2, \cdots, x_N]$ is a non-negative optimization variable vector, $g_j(\mathbf{x})$ for all j is a monomial function, i.e., $g_j(\mathbf{x}) = \prod_{n=1}^{N} c_{jn} x_n^{a_{jn}}$ where $c_{jn} > 0$, $a_{jn} \in \Re$, and $f_0(\mathbf{x})$ and $f_i(\mathbf{x})$ for all i are posynomial functions, i.e., $f_i(\mathbf{x}) = \sum_{k=1}^{K_i} \prod_{n=1}^{N} c_{ikn} x_n^{a_{ikn}}$. In (4), there are many restrictions on the equality and inequality constraints, which cannot be met for many practical problems related to the resource allocation of wireless networks such as the optimization problem considered in this paper. For example, in some cases, the equality constraints contain posynomial functions and/or inequality constraints contain the difference of two posynomial functions. Depending on the nature of the optimization problem, these types of problems belong to either one of classes of optimization problems such as generalized GP, signomial programming or complementary geometric programming (CGP). A CGP can be presented as

$$\begin{aligned} &\min_{\mathbf{x}} F_0(\mathbf{x}), \\ &\text{subject to: } F_i(\mathbf{x}) \leq 1, \quad i = 1, \cdots, I, \\ &\quad G_j(\mathbf{x}) = 1, \quad j = 1, \cdots, J, \end{aligned} \quad (5)$$

where $F_0(\mathbf{x}) = f_0^+(\mathbf{x}) - f_0^-(\mathbf{x})$, $F_i(\mathbf{x}) = \frac{f_i^+(\mathbf{x})}{f_i^-(\mathbf{x})}$, $i = 1, \cdots, I$ and $G_j(\mathbf{x}) = \frac{g_j(\mathbf{x})}{f_j(\mathbf{x})}$ (in which $f_0^+(\mathbf{x})$, and $f_0^-(\mathbf{x})$, $j = 0, 1, \cdots, J$, are posynomial functions), while $g_j(\mathbf{x})$ and $f_j(\mathbf{x})$ are monomial and posynomial functions [31], respectively.

One approach to solve (5) is to convert it into a sequence of standard GP problems [12] that can be solved to achieve a global solution. In other words, successive convex approximation (SCA) [32] can be applied, where

the non-convex optimization problem is approximated as a convex problem in each iteration. Specifically, arithmetic-geometric mean approximation (AGMA) can be applied to transform the non-posynomial functions to posynomial form, i.e., $F_i(\mathbf{x})$, and $G_j(\mathbf{x})$ to their posynomial and monomial approximations, respectively.

Using AGMA, at the iteration l, the approximated forms of $f_i^-(\mathbf{x}) = \sum_{k=1}^{K^{i^-}} g_k^{i^-}(\mathbf{x})$ and $f_j(\mathbf{x}) = \sum_{k=1}^{K_j} g_k^j(\mathbf{x})$ are

$$\widetilde{f}_i^-(\mathbf{x}(l)) = \prod_{k=1}^{K_{i^-}} \left(\frac{g_k^{i^-}(\mathbf{x}(l))}{\alpha_k^{i^-}(l)} \right)^{\alpha_k^{i^-}(l)}, \tag{6}$$

$$\text{and, } \widetilde{f}_j(\mathbf{x}(l)) = \prod_{k=1}^{K_j} \left(\frac{g_k^j(\mathbf{x}(l))}{\zeta_k^j(l)} \right)^{\zeta_k^j(l)}, \tag{7}$$

where $\alpha_k^{i^-}(l) = \frac{g_k^{i^-}(\mathbf{x}(l-1))}{f_i^-(\mathbf{x}(l-1))}$ and $\zeta_k^j(l) = \frac{g_k^j(\mathbf{x}(l-1))}{f_j(\mathbf{x}(l-1))}$. Subsequently, $\widetilde{F}_i(\mathbf{x}(l)) = \frac{f_i^+(\mathbf{x}(l))}{\widetilde{f}_i^-(\mathbf{x}(l))}$ and $\widetilde{G}_j(\mathbf{x}(l)) = \frac{g_j(\mathbf{x}(l))}{\widetilde{f}_j(\mathbf{x}(l))}$ are posynomial and monomial functions, respectively [12], and the optimization problem related to each iteration l of (5) becomes

$$\begin{aligned} &\min_{\mathbf{x}(l)} \quad \Xi + f_0^+(\mathbf{x}(l)) - f_0^-(\mathbf{x}(l)), \\ &\text{subject to: } \widetilde{F}_i(\mathbf{x}(l)) \le 1, \widetilde{G}_j(\mathbf{x}(l)) = 1, \\ &\qquad i = 1, \cdots, I, \quad j = 1, \cdots, J, \end{aligned} \tag{8}$$

where $\Xi \gg 1$ is a sufficiently large constant added to the objective function in (8) to keep it always positive [12]. However, the objective function of (8) still cannot satisfy the posynomial condition of (4). To reach the GP-based formulation for each iteration, we introduce the auxiliary variable $x_0 > 0$ for a linear objective function and use it to transform (8) into

$$\min_{\mathbf{x}_0(t)} x_0(l), \tag{9}$$

$$\begin{aligned} &\text{subject to: } \frac{\Xi + f_0^+(\mathbf{x}(l))}{f_0^-(\mathbf{x}(l)) + x_0} \le 1, \\ &\widetilde{F}_i(\mathbf{x}(l)) \le 1, \widetilde{G}_j(\mathbf{x}(l)) = 1, \\ &\qquad i = 0, 1, \cdots, I, j = 1, \cdots, J, \end{aligned} \tag{10}$$

where $\mathbf{x}_0(l) = [x_0(l), x_n(l), \cdots, x_0(l)]$. Similar to $F_i(\mathbf{x})$, term $\frac{\Xi + f_0^+(\mathbf{x}(l))}{f_0^-(\mathbf{x}(l)) + x_0}$ can be converted into posynomial function via AGMA, and finally, the resulting optimization problem has a GP-based structure and can be solved by efficient numerical algorithms, [12].

It has been shown that the solution obtained by the iterative algorithm based on the GP-based approximation of problem (5) can offer a performance very close to that of the optimal solution [12].

B. USER-ASSOCIATION PROBLEM

At the iteration t, with given $\mathbf{P}(t)$, we formulate the following user association optimization problem to maximize the sum rate,

$$\begin{aligned} &\max_{\boldsymbol{\beta}} \sum_{m\in\mathcal{M}} \sum_{g\in\mathcal{G}} \sum_{n_g\in\mathcal{N}_g} \sum_{k\in\mathcal{K}} \beta_{m,k,n_g} R_{m,k,n_g}(\mathbf{P}(t)), \\ &\text{subject to: } \quad \widetilde{\text{C}}1, \text{C3}, \text{C4}, \end{aligned} \tag{11}$$

where $R_{m,k,n_g}(\mathbf{P}(t))$ is computed by (1) with $\mathbf{P}(t)$ and

$$\widetilde{\text{C}}1: \sum_{m\in\mathcal{M}} \sum_{n_g\in\mathcal{N}_g} \sum_{k\in\mathcal{K}} \beta_{m,k,n_g} R_{m,k,n_g}(\mathbf{P}(t)) \ge R_g^{\text{TSV}}, \quad \forall g \in \mathcal{G}.$$

In (11), the only optimization variable is $\boldsymbol{\beta}$, and therefore, (11) has lower computational complexity than (2). However, it still suffers from the integer optimization variable $\boldsymbol{\beta}$. In addition, due to C4 and the objective function, (11) is still a non-convex optimization problem. To overcome the computational complexity of (11), we follow the following steps. We first relax the UAF variable to be continuous as $\beta_{m,k,n_g} \in [0, 1]$. Then, we apply the technique as described in Section III. A to convert (11) into the GP formulation by transforming C4 into its related linear constraints from **Proposition 1**, and the objective function to the monomial function from **Proposition 2**.

To have a standard GP formulation, the equality constraint in C4 should only involve monomial functions. In the following, we first relax C4 and then apply iterative AGMA algorithm (as in (6) and (7)) to get the monomial approximation for C4. Also, we show how we can convert the objective function of (6) into the standard form of GP.

Proposition 1: At iteration t_1 in solving (11), define $x_{m,n_g}(t_1) = \sum_{k\in\mathcal{K}} \beta_{m,k,n_g}(t_1)$ and $y_{n_g}(t_1) = \sum_{m\in\mathcal{M}} \sum_{k\in\mathcal{K}} \beta_{m,k,n_g}(t_1)$. C4 can be approximated by the following constraints.

$$\begin{aligned} \text{C4.1:} \quad & s_{m,n_g}^{-1}(t_1) + x_{m,n_g}(t_1) y_{n_g}(t_1) s_{m,n_g}^{-1}(t_1) \le 1, \\ & \forall n_g \in \mathcal{N}_g, \ \forall g \in \mathcal{G}, \quad \forall m \in \mathcal{M}, \end{aligned}$$

$$\begin{aligned} \text{C4.2:} \quad & \left[\frac{1}{\lambda_{m,n_g}(t_1)}\right]^{-\lambda_{m,n_g}(t_1)} s_{m,n_g}(t_1) \\ & \times \left[\frac{x_{m,n_g}^2(t_1)}{\alpha_{m,n_g}(t_1)}\right]^{-\alpha_{m,n_g}(t_1)} \le 1, \\ & \forall n_g \in \mathcal{N}_g, \ \forall g \in \mathcal{G}, \quad \forall m \in \mathcal{M}, \end{aligned}$$

$$\begin{aligned} \text{C4.3:} \quad & x_{m,n_g}(t_1) \prod_{k\in K} \left[\frac{\beta_{m,k,n_g}(t_1)}{\nu_{m,k,n_g}(t_1)}\right]^{-\nu_{m,k,n_g}(t_1)} = 1, \\ & \forall n_g \in \mathcal{N}_g, \ \forall g \in \mathcal{G}, \ \forall m \in \mathcal{M}, \end{aligned}$$

$$\begin{aligned} \text{C4.4:} \quad & y_{n_g}(t_1) \prod_{m\in\mathcal{M},k\in\mathcal{K}} \left[\frac{\beta_{m,k,n_g}(t_1)}{\eta_{m,k,n_g}(t_1)}\right]^{-\eta_{m,k,n_g}(t_1)} = 1, \\ & \forall n_g \in \mathcal{N}_g, \ \forall g \in \mathcal{G}, \ \forall m \in \mathcal{M}, \end{aligned}$$

where $s_{m,n_g}(t_1)$ is an auxiliary variable, and,

$$\lambda_{m,n_g}(t_1) = \frac{1}{x_{m,n_g}^2(t_1 - 1) + 1}, \tag{12}$$

$$\alpha_{m,n_g}(t_1) = \frac{x_{m,n_g}^2(t_1 - 1)}{x_{m,n_g}^2(t_1 - 1) + 1}, \tag{13}$$

$$\nu_{m,k,n_g}(t_1) = \frac{\beta_{m,k,n_g}(t_1-1)}{\sum_{k\in\mathcal{K}}\beta_{m,k,n_g}(t_1-1)}, \tag{14}$$

$$\eta_{m,k,n_g}(t_1) = \frac{\beta_{m,k,n_g}(t_1-1)}{\sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}}\beta_{m,k,n_g}(t_1-1)}, \tag{15}$$

for all $n_g \in \mathcal{N}_g$, $g \in \mathcal{G}$, and $m \in \mathcal{M}$.

Proof: See Appendix A. □

Based on C4.1-C4.4, C4 is transformed and represented by the approximated monomial equalities and posynomial inequalities. Next, we show how we can transform the objective function into the monomial function to reach the GP-based formulation for (11).

Proposition 2: Consider the auxiliary variable $x_0 > 0$ and $\Xi_1 \gg 1$. The user association problem (11) at each iteration t_1 can be transformed into the following standard GP problem

$$\min_{\boldsymbol{\beta}(t_1),\, x_0(t_1),\, s_{m,n_g}(t_1),\, x_{m,n_g}(t_1),\, y_{n_g}(t_1)} x_0(t_1), \tag{16}$$

subject to : C4.1-C4.4,

$$\Xi_1 \left[\frac{x_0(t_1)}{c_0(t_1)}\right]^{-c_0(t_1)} \prod_{m\in\mathcal{M},g\in\mathcal{G},n_g\in\mathcal{N}_g,k\in\mathcal{K}} \left[\frac{\beta_{m,k,n_g}(t_1)R_{m,k,n_g}(\mathbf{P}(t))}{c_{m,k,n_g}(t_1)}\right]^{-c_{m,k,n_g}(t_1)} \leq 1, \tag{17}$$

$\widetilde{\text{C}}1.1$: $R_g^{\text{rsv}} \times \prod_{m\in\mathcal{M},n_g\in\mathcal{N}_g,k\in\mathcal{K}} \left[\frac{\beta_{m,k,n_g}(t_1)R_{m,k,n_g}(\mathbf{P}(t))}{\varphi_{m,k,n_g}(t_1)}\right]^{-\varphi_{m,k,n_g}(t_1)} \leq 1,$

$\forall g \in \mathcal{G}$,

C3.1: $\sum_{g\in\mathcal{G}}\sum_{n_g\in\mathcal{N}_\mathcal{G}} \beta_{m,k,n_g}(t_1) \leq 1, \forall m \in \mathcal{M}, \quad \forall k \in \mathcal{K},$

where

$$\varphi_{m,k,n_g}(t_1) = \frac{\beta_{m,k,n_g}(t_1-1)R_{m,k,n_g}(\mathbf{P}(t))}{\sum_{m\in\mathcal{M}}\sum_{n_g\in\mathcal{N}_g}\sum_{k\in\mathcal{K}}\beta_{m,k,n_g}(t_1-1)R_{m,k,n_g}(\mathbf{P}(t))}, \quad \forall g \in \mathcal{G}, \tag{18}$$

and $c_{m,k,n_g}(t_1)$ and $c_0(t_1)$ are defined in (19) and (20) at the bottom of this page.

Proof: See Appendix B. □

Now, at each iteration, the optimization problem can be replaced by its GP approximation in (16). Iteratively, (16) will be solved until achieving the optimal value of UAF value as shown in Step 1.A of Algorithm 1.

Proposition 3: With AGMA, Step 1.A converges to a locally optimal solution that satisfies the KKT conditions of the original problem.

Proof: In [11], it is shown that the conditions for the convergence of the SCA are satisfied and guarantee that the solutions of the series of approximations by AGMA converges to a point that satisfies the KKT conditions of (11), i.e., a local maximum is attained [32]. □

C. POWER-ALLOCATION PROBLEM

For a given set of UAFs obtained from Step 1.A, the optimization problem can be formulated as

$$\max_{\mathbf{P}(t_2)} \sum_{m\in\mathcal{M}}\sum_{g\in\mathcal{G}}\sum_{n_g\in\mathcal{N}_g}\sum_{k\in\mathcal{K}} \beta_{m,k,n_g}(t)R_{m,k,n_g}(\mathbf{P}(t_2)) \tag{21}$$

subject to:

$$\widetilde{\text{C}}1.2 : \sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}}\sum_{n_g\in\mathcal{N}_g} \beta_{m,k,n_g}(t)R_{m,k,n_g}(\mathbf{P}(t_2)) \geq R_g^{\text{rsv}}, \quad \forall g \in \mathcal{G},$$

$$\widetilde{\text{C}}2.2 : \sum_{g\in\mathcal{G}}\sum_{n_g\in\mathcal{N}_g}\sum_{k\in\mathcal{K}} P_{m,k,n_g}(t_2) \leq P_m^{\max}, \quad \forall m \in \mathcal{M},$$

where t_2 is the iteration index. Due to interference in the objective function of $R_{m,k,n_g}(\mathbf{P}(t_2))$, (21) is a non-convex optimization problem. We again follow the approach of Section III.A to convert (21) into the GP optimization problem. First, we rewrite the objective of (21) as

$$\max_{\mathbf{P}(t_2)} \prod_{m\in\mathcal{M},g\in\mathcal{G},n_g\in\mathcal{N}_g,k\in\mathcal{K}} \gamma_{m,k,n_g}(\mathbf{P}(t_2)) \tag{22}$$

where

$$\gamma_{m,k,n_g}(\mathbf{P}(t_2)) = \frac{\sigma^2 + I_{m,k,n_g}(t_2) + P_{m,k,n_g}(t_2)h_{m,k,n_g}}{\sigma^2 + I_{m,k,n_g}(t_2)}$$

and

$$I_{m,k,n_g}(t_2) = \sum_{\forall m'\in\mathcal{M},m'\neq m}\sum_{\forall g\in\mathcal{G}}\sum_{\forall n'_g\in\mathcal{N}_g,n'_g\neq n_g} P_{m',k,n'_g}(t_2)h_{m,k,n'_g}.$$

$$c_{m,k,n_g}(t_1) = \frac{\beta_{m,k,n_g}(t_1-1)R_{m,k,n_g}(\mathbf{P}(t))}{x_0(t_1-1) + \sum_{m\in\mathcal{M}}\sum_{g\in\mathcal{G}}\sum_{n_g\in\mathcal{N}_g}\sum_{k\in\mathcal{K}}\beta_{m,k,n_g}(t_1-1)R_{m,k,n_g}(\mathbf{P}(t))} \tag{19}$$

$$c_0(t_1) = \frac{x_0(t_1-1)}{x_0(t_1-1) + \sum_{m\in\mathcal{M}}\sum_{g\in\mathcal{G}}\sum_{n_g\in\mathcal{N}_g}\sum_{k\in\mathcal{K}}\beta_{m,k,n_g}(t_1-1)R_{m,k,n_g}(\mathbf{P}(t))} \tag{20}$$

Now, from AGMA in Section III.B, $\gamma^{-1}_{m,k,n_g}(\mathbf{P}(t_2))$ can be approximated as

$$\widehat{\gamma}_{m,k,n_g}(\mathbf{P}(t_2)) = (\sigma^2 + I_{m,k,n_g}(t_2))\left(\frac{\sigma^2}{\kappa_o(t_2)}\right)^{-\kappa_o(t_2)} \times \prod_{m\in\mathcal{M},g\in\mathcal{G},n_g\in\mathcal{N}_g,k\in\mathcal{K}} \left(\frac{P_{m,k,n_g}(t_2)h_{m,k,n_g}}{\kappa_{m,k,n_g}(t_2)}\right)^{-\kappa_{m,k,n_g}(t_2)},$$

where

$$\kappa_{m,k,n_g}(t_2) = \frac{P_{m,k,n_g}(t_2-1)h_{m,k,n_g}}{\sigma^2 + \sum_{m\in\mathcal{M},n_g\in\mathcal{N}_g,g\in\mathcal{G}} P_{m,k,n_g}(t_2-1)h_{m,k,n_g}}, \quad (23)$$

and

$$\kappa_o(t_2) = \frac{\sigma^2}{\sigma^2 + \sum_{m\in\mathcal{M},n_g\in\mathcal{N}_g,g\in\mathcal{G}} P_{m,k,n_g}(t_2-1)h_{m,k,n_g}}. \quad (24)$$

Consequently, (21) is transformed into the following standard GP problem

$$\min_{\mathbf{P}(t_2)} \prod_{m\in\mathcal{M},g\in\mathcal{G},n_g\in\mathcal{N}_g,k\in\mathcal{K}} \widehat{\gamma}_{m,k,n_g}(\mathbf{P}(t_2)) \quad (25)$$

subject to:

$$\widetilde{\text{C}}1.2: \prod_{m\in\mathcal{M},g\in\mathcal{G},n_g\in\mathcal{N}_g,k\in\mathcal{K}} \widehat{\gamma}_{m,k,n_g}(\mathbf{P}(t_2)) \le 2^{-R^{\text{rsv}}_g}, \forall g\in\mathcal{G},$$

$$\widetilde{\text{C}}2.2: \sum_{g\in\mathcal{G}}\sum_{n_g\in\mathcal{N}_g}\sum_{k\in\mathcal{K}} P_{m,k,n_g}(t_2) \le P^{\max}_m, \quad \forall m\in\mathcal{M},$$

The overall optimization problem is iteratively solved as described in Step 1.B until the power vector converges, i.e., $||\mathbf{P}(t_2) - \mathbf{P}(t_2-1)|| \le \varepsilon_2$ where $0 < \varepsilon_2 \ll 1$. Note that Proposition III holds for Step 1.B.

IV. SIMULATION RESULTS

A. SIMULATION PARAMETERS

Let consider a multi-cell VWN scenario with $M = 4$ BSs and $K = 4$ sub-carriers serving $G = 2$ slices (service providers) in a 2×2 square area. The 4 BSs are located at coordinates: (0.5, 0.5), (0.5, 1.5), (1.5, 0.5) and (1.5, 1.5). The channel power gains are based on the path loss and Rayleigh fading model, i.e., $h_{m,k,n_g} = \chi_{m,k,n_g} d^{-b}_{m,n_g}$ where $b = 3$ is the path loss exponent, $d_{m,n_g} > 0$ is the normalized distance between the BS m and user n_g and χ_{m,k,n_g} is the exponential random variable with mean of 1 [15]. We use the noise power in a sub-carrier bandwidth as reference (i.e., normalized to 1 or 0 dB) and hence express transmit power or interference power in dB relative to noise power. For all of the simulations, we set $\Xi_1 = 10^7$ and $\varepsilon_1 = 10^{-5}$, $\varepsilon_2 = 10^{-6}$. In all of the following simulations, for each realization of network, when there exists no feasible solution for the system, i.e., C1-C4 cannot be satisfied simultaneously, the corresponding total rate is set to be zero. The simulation results are taken over the average of 100 different channel realizations. For all the following simulations, we set $R^{\text{rsv}} = R^{\text{rsv}}_g$ for all $g \in \mathcal{G}$ and $P^{\max} = P^{\max}_m$ for all $m \in \mathcal{M}$.

Algorithm 2

Initialization: Set $t_3 = 0$, BS assignment: user n_g is assigned to BS m based on the average received SINR.

Repeat: Set $t_3 = t_3 + 1$.

Step 2.A: Compute $\boldsymbol{\beta}'^*(t_3)$ by using Step 1.A except that the BS is assigned based on the signal strength.

Step 2.B: For a fixed $\boldsymbol{\beta}'^*(t_3)$, find the optimal power allocation $\mathbf{P}'(t_3)$ by using step **1.B**.

Until $||\boldsymbol{\beta}'^*(t_3) - \boldsymbol{\beta}'^*(t_3-1)|| \le \varepsilon_1$ and $||\mathbf{P}'^*(t_3) - \mathbf{P}'^*(t_3-1)|| \le \varepsilon_2$.

B. REFERENCE RESOURCE-ALLOCATION: ALGORITHM 2

For performance comparison, we take as reference, the traditional SINR-based joint sub-carrier and power allocation algorithm as summarized in Algorithm 2. Under the SINR criterion, the users are assigned to the BSs that yields the highest average received SINR. In this case, the resource allocation problem is formulated as

$$\max_{\boldsymbol{\beta}',\mathbf{P}} \sum_{m\in\mathcal{M}}\sum_{g\in\mathcal{G}}\sum_{n_g\in\mathcal{N}_g}\sum_{k\in\mathcal{K}} \beta'_{m,k,n_g} R_{m,k,n_g}(\mathbf{P}) \quad (26)$$

subject to: C1 - C3,

where $\boldsymbol{\beta}' = [\beta'_{m,k,n_g}]\forall_{m,k,n_g}$ and β'_{m,k,n_g} shows the sub-carrier allocation of user n_g on sub-carrier k when it is allocated to the BS m. Clearly, (26) is still highly non-convex. In order to show the importance and effects of defining UAF in this context, we apply the similar approach based on CGP to solve (26). In other words, Algorithm 2 is based on CGP and similar to Algorithm 1 introduced in Section III, except that, in Algorithm 2, (16) contains only C1-C3, i.e., C4 is removed, since BS-user association is based on the highest average received SINR. When the sub-carrier assignment is solved, the optimal power is derived from Step 1.B for (26). This iterative algorithm is terminated when the convergence conditions are met as summarized in Algorithm 2.

C. EVALUATION OF ALGORITHM 1 AND ALGORITHM 2

Primarily, we evaluate and compare the total rates achieved by Algorithm 1 and Algorithm 2 versus the number of sub-carriers and maximum transmit power in Figs. 1(a) and 1(b), respectively. We set $N_g = 4$ where the total $N = 8$ users in 2 slices are randomly located in the 2×2 square area according to a uniform distribution. The results in both Figs. 1(a) and 1(b) indicate that Algorithm 1 considerably outperforms Algorithm 2 for different values of R^{rsv}_g, K and $P^{\max}_m$.

From Fig. 1(a), it can be observed that the total rate is increased by increasing the total number of sub-carriers, K, due to the opportunistic nature of fading channels in wireless networks. As expected, with increasing $P^{\max}_m$, the total

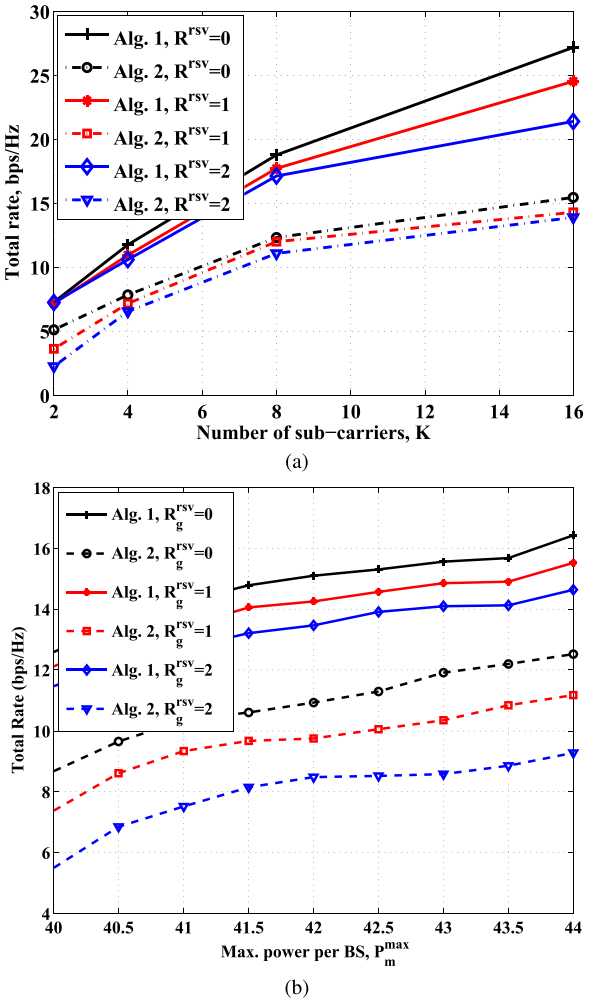


FIGURE 1. Total rate versus (a) number of sub-carriers, K, and (b) maximum transmit power per BS, $P^{\max}$ (in dB)

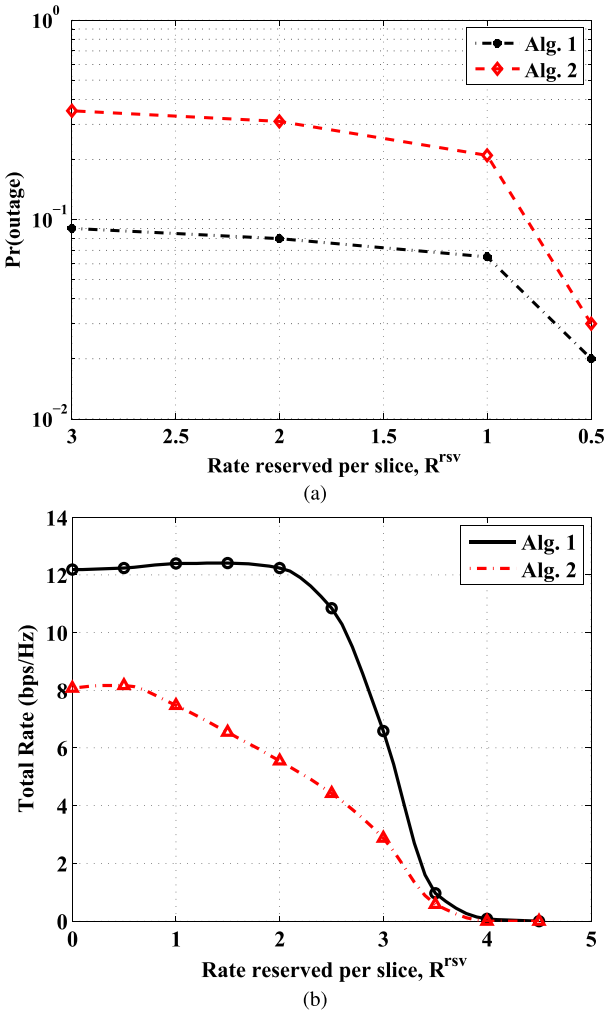


FIGURE 2. (a) Outage probability, and (b) total rate versus R^{rsv}.

achievable rate is also increased as shown in Fig. 1(b). Both figures indicate that by increasing the value of R_g^{rsv}, the total rate decreases because the feasibility region of resource allocation in (2) is reduced leading to less total average achieved rate. However, from Fig. 1(b), increasing R_g^{rsv} has considerable effect on the performance of Algorithm 2 as compared to Algorithm 1. It can be interpreted as Algorithm 1 can efficiently control interference between different cells compared to Algorithm 2. Therefore, the chance of feasible power allocation for larger values of R_g^{rsv} is increased by Algorithm 1. To study this point further, we consider the rate-outage probability of C1, expressed as

$$\Pr(\text{rate-outage}) = \Pr\{\sum_{m\in\mathcal{M}}\sum_{n_g\in\mathcal{N}_g}\sum_{k\in\mathcal{K}}\beta_{m,k,n_g}R_{m,k,n_g}(\mathbf{P},\boldsymbol{\beta}) \leq R_g^{\text{rsv}}\}, \quad \forall g\in\mathcal{G}.$$

Via Mont Carlo simulation, we compute Pr(rate-outage) of both Algorithm 1 and Algorithm 2 for the above-mentioned simulation setting, as depicted in Fig. 2(a) with $K = 8$ and $P_m^{\max} = 40$ dB for all $m \in \mathcal{M}$. The results demonstrate that as the rate reservation per slice R_g^{rsv} increases, the rate-outage probability of both Algorithm 1 and Algorithm 2 increases. However, Algorithm 2 has larger rate-outage probability compared to the outage probability of Algorithm 1, implying that the feasibility region of Algorithm 2 is smaller than that for Algorithm 1. On the other hand, Algorithm 1 can efficiently manage interference in the specific region between different cells as compared to Algorithm 2. It is mainly because Algorithm 1 has more degrees of freedom to choose the BS and allocate the sub-carriers among users of different slices while the BS assignment is predetermined in Algorithm 2. Therefore, the achieved rate of Algorithm 2 is less than that of Algorithm 1. With increasing R_g^{rsv}, the rate reduction of Algorithm 2 is greater than that of Algorithm 1, since Algorithm 2 cannot manage the interference between different BSs. Hence, Algorithm 2 cannot satisfy the minimum rate requirements of slices, leading to reduced VWN efficiency.

For the same setup, in Fig. 2(b), the total rate of Algorithms 1 and 2 are plotted for different values of R_g^{rsv}.

Fig. 2(b) clearly shows that Algorithm 1 yields higher rate than Algorithm 2. Note that in all the simulation results, when the problem is infeasible, i.e., there is no power and sub-carrier vectors that can meet the constraint C1 for all $g \in \mathcal{G}$, the achieved total rate is set to zero. These simulations highlight the importance of introducing UAF as the joint BS assignment and sub-carrier allocation in the multi-cell wireless networks to manage and control the interference between different cells.

D. COVERAGE ANALYSIS

In any cellular network, the coverage can be measured by SINR or achieved total rate of users at the cell boundaries. To study the performance of Algorithm 1 in terms of coverage, we consider the simulation setup similar to Fig. 3 where the majority of users are located in the cell-edge region, consequently, these users experience high interference from other BSs. Therefore, the achieved rate of each user is decreased, which can be considered as the worst-case scenario of coverage analysis.

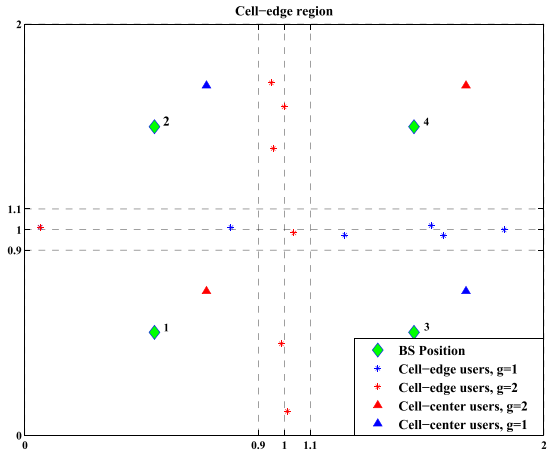
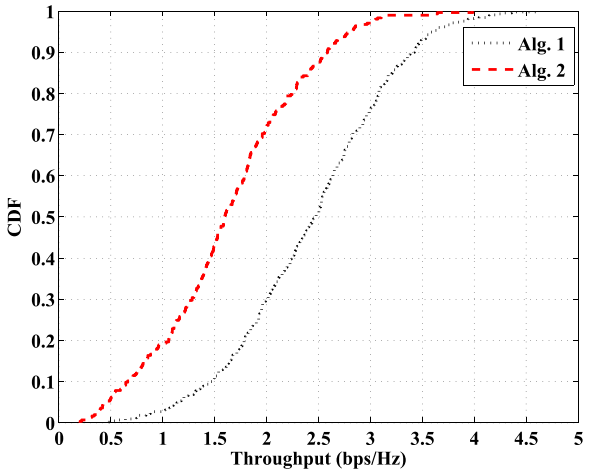


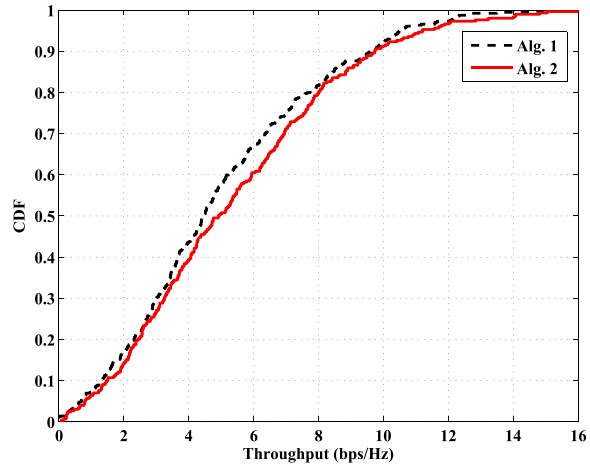
FIGURE 3. Illustration of network setup to investigate the coverage of multi-cell VWN.

The cumulative distribution function (CDF) of the total throughput of cell-edge users and cell-center users are depicted in Figs. 4(a) and 4(b), respectively, for both Algorithms 1 and 2. It can be seen that Algorithm 1 outperforms Algorithm 2 for the cell-edge users where 50% of users in the cell-edge achieve a rate of 2.5 bps/Hz by Algorithm 1, and around 1.5 bps/Hz via Algorithm 2. However, the performance of both algorithms are similar for the cell-center users. It is because via user-association in Algorithm 1, the interference among different cells can be controlled while Algorithm 2 cannot control the interference through the connectivity of users to different BS and it is pre-determined by the received SINR of reference signal. In other words, Algorithm 1 can provide better coverage for cell-edge users for multi-cellular VWN which is desirable from implementation perspective.

The performance is further investigated with respect to the number of users in the cell-edge in Figs. 5(a) and 5(b).



(a)



(b)

FIGURE 4. Rate distribution for (a) cell-edge, and (b) cell-center users.

Algorithm 1 can consistently improve the performance of cell-edge users and maintain desirable rate of each slice regardless of the user deployment density as compared to the Algorithm 2. For instance, with $N = 18$, for the uniform user distribution, the total rate is increased by 57% from 7 bps/Hz (by Algorithm 2) to 11 bps/Hz (by Algorithm 1) for cell-edge users and by 33% from 24 bps/Hz (by Algorithm 2) to 32 bps/Hz (by Algorithm 1) for cell-center users. For non-uniform user distribution, when $N = 32$, the rate is increased by 71% from 7 bps/Hz (by Algorithm 2) to 12 bps/Hz (by Algorithm 1) for cell-edge users and by 50% from 18 bps/Hz (by Algorithm 2) to 27 bps/Hz (by Algorithm 1) for cell-center users. These results show the efficiency of applying Algorithm 1 in increasing the coverage over the whole network.

E. OPTIMALITY GAP STUDY

We investigate the performance gap between the optimum solution (by exhaustive search) and the proposed Algorithm 1 for $K = 2$ sub-carriers and $N = 4$ users. Fig. 6 plots the

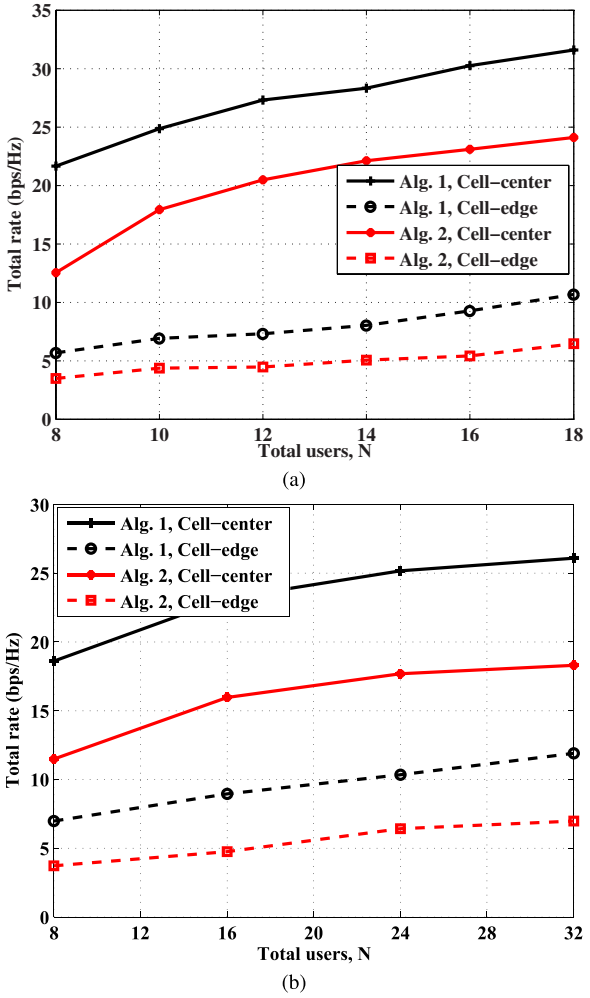


FIGURE 5. Total rate for (a) uniform user distribution, and (b) non-uniform user distribution.

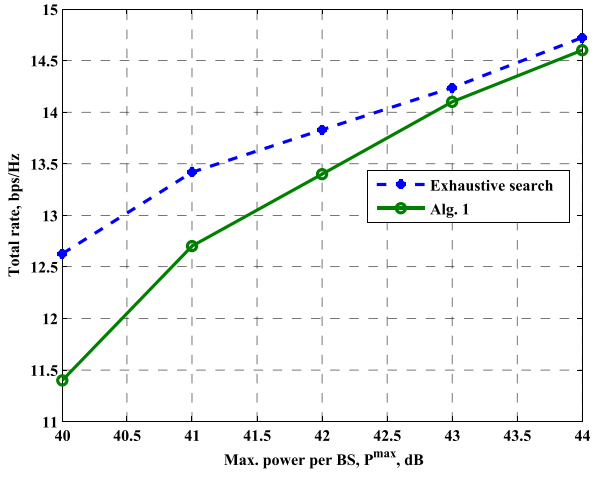


FIGURE 6. Total rate versus $P^{\max}$ for both exhaustive search and Algorithm 1.

total rate versus $P^{\max}$ for both Algorithm 1 and the exhaustive search. As seen in the figure, the performance of Algorithm 1 approaches the performance of exhaustive search as $P^{\max}$ increases because the AGMA approach to convexify the rate is the best fit approximation for the high SINR scenario.

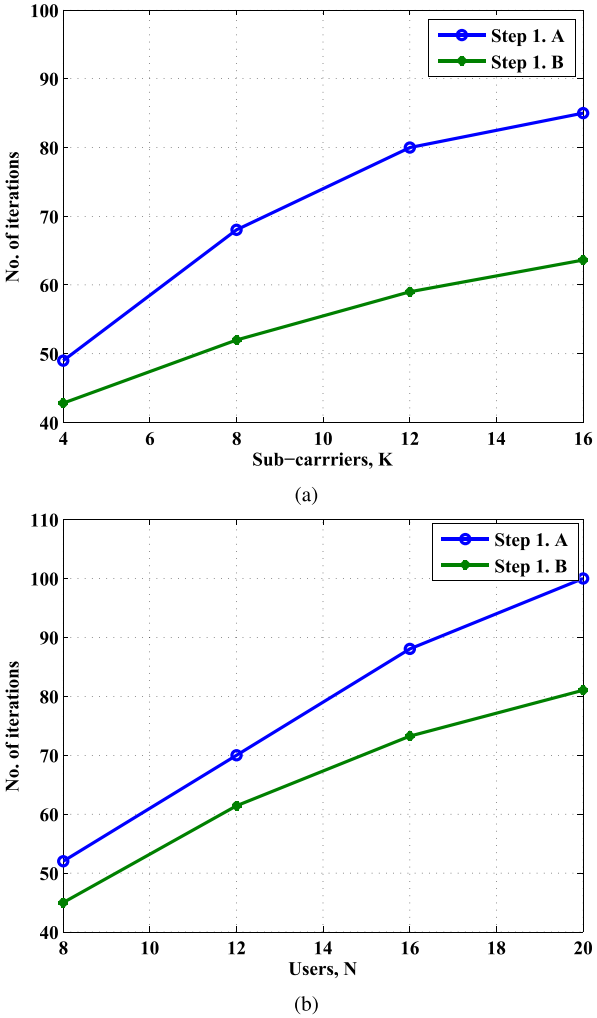


FIGURE 7. Number of required iterations for lower-level iterative algorithms versus (a) number of sub-carriers, K, and (b) total number of users, N.

F. COMPUTATIONAL COMPLEXITY AND CONVERGENCE ANALYSIS OF ALGORITHM 1

In this section, we investigate the computational complexity and the convergence of Algorithm 1. First, we focus on deriving the computational complexity of Algorithm 1 analytically. Since CVX is used to solve GP sub-problems with the interior point method in Steps 1.A and 1.B, the number of required iterations is $\frac{\log(c/(t^0\varrho))}{\log(\xi)}$ [34], where c is the total number of constraints in (16), t^0 is the initial point to approximate the accuracy of interior point method, $0<\varrho\ll 1$ is the stopping criterion for interior point method, and ξ is used for updating the accuracy of interior point method [34]. As previously discussed, the numbers of constraints in (16) are $c_1 = G + MK + 4MN + 1$ for Step 1.A and $c_2 = G + M$ for Step 1.B.

Moreover, in Steps 1.A and 1.B, for each iteration, the number of computations required to convert the non-convex

problems using AGMA into (16) and (22) is $i_1 = KM^2N + 6KMN + MKGN$ and $i_2 = GMKN + 2MKN$, respectively. Therefore, the order of computational complexity for each step is

- $i_1 \times \frac{\log(c_1/(t_1^0 \varrho_1))}{\log(\xi_1)}$ for Step 1. A,
- $i_2 \times \frac{\log(c_2/(t_2^0 \varrho_2))}{\log(\xi_2)}$ for Step 1. B.

Based on this analysis, the computational complexity of Step 1.A is significantly higher than that of Step 1.B. Moreover, Step 1.A is more sensitive to K and N than Step 1.B. Since Algorithm 1 is a type of block SCA algorithm [35], when (2) is feasible, the outer loop of Algorithm 1 is converged ([27, Proposition 6], [35, Th. 2]). For further investigation by simulation, in Fig. 7(a), the number of iterations required for convergence for Steps 1.A and 1.B versus the total number of sub-carriers, K, is plotted for $N = 8$ and $R^{\text{rsv}} = 2$ bps/Hz. Similarly, in Fig. 7(b), the number of iterations required for convergence versus the total number of users, i.e., N, for $K = 4$ sub-carriers is plotted in the case of Steps 1.A and 1.B. As N and K increase, the number of iterations required for convergence also increases. The computational complexity of Step 1.A is higher than that of Step 1.B because the total number of constraints for Step 1.A is higher than that for Step 1.B.

The major issue of Algorithm 1 is that (2) can be infeasible, e.g., due to deep fades and/or high interference and C1 cannot be met. Therefore, proposing the admission control policy to adjust R_g^{rsv} is of interest [5] which remains as a future work of this paper.

V. CONCLUSION

In this paper, we proposed the joint BS, sub-carrier and power allocation algorithm for multi-cell OFDMA based virtualized wireless networks (VWNs). In the proposed setup, we have considered a set of slices (service providers), each has a set of their own users and require a minimum reserved rate. We formulated the related optimization problem based on the new defined optimization variable, called user association factor (UAF), indicating the joint sub-carrier and BS assignment. To solve the proposed non-convex and NP-hard optimization problem, we followed an iterative approach where in each step, one set of optimization variables is derived. However, in each step, the optimization problem is non-convex and NP-hard. To derive the efficient algorithm to solve them, we apply the framework of iterative successive convex approximation via complementary geometric programming (CGP) to transform the non-convex optimization problem into the convex one. Then, to efficiently derive the solution, we applied CVX to solve the optimization problem of each step. Simulation results reveal that, via the proposed approach, the throughput and coverage of VWN, especially for the cell-edge users, are considerably improved as compared to the traditional scenario where the BS is assigned based on the maximum value of SINR.

APPENDIX A
PROOF OF PROPOSITION 1

From the definition of $x_{m,n_g}(t_1)$ and $y_{n_g}(t_1)$, C4 can be rewritten as for all $n_g \in \mathcal{N}_g$, $g \in \mathcal{G}$ and $m \in \mathcal{M}$

$$x_{m,n_g}(t_1)[y_{n_g}(t_1) - x_{m,n_g}(t_1)] = 0, \tag{27}$$

which is not a monomial function. (27) can be rewritten as $x_{m,n_g}(t_1)y_{n_g}(t_1) = x^2_{m,n_g}(t_1)$ and by adding 1 to both the left and right hand sides, we have $x_{m,n_g}(t_1)y_{n_g}(t_1) + 1 = 1 + x^2_{m,n_g}(t_1)$ for all $n_g \in \mathcal{N}_g$, $g \in \mathcal{G}$, and $m \in \mathcal{M}$. We define $s_{m,n_g}(t_1) \geq 0$ as an auxiliary variable to relax and convert (27) into the posynomial inequalities as follows [10]

$$x_{m,n_g}(t_1)y_{n_g}(t_1) + 1 \leq s_{m,n_g}(t_1) \leq 1 + x^2_{m,n_g}(t_1), \quad \forall n_g \in \mathcal{N}_g,\ \forall g \in \mathcal{G},\quad \forall m \in \mathcal{M}. \tag{28}$$

The above inequalities can be written as

$$\frac{x_{m,n_g}(t_1)y_{n_g}(t_1) + 1}{s_{m,n_g}(t_1)} \leq 1, \quad \frac{s_{m,n_g}(t_1)}{1 + x^2_{m,n_g}(t_1)} \leq 1.$$

Now, the above constraints can be approximated using AGMA approximation introduced in Section III. B. as

$$\text{C4.1: } s^{-1}_{m,n_g}(t_1) + x_{m,n_g}(t_1)y_{n_g}(t_1)s^{-1}_{m,n_g}(t_1) \leq 1, \tag{29}$$

$$\text{C4.2: } \Big[\frac{1}{\lambda_{m,n_g}(t_1)}\Big]^{-\lambda_{m,n_g}(t_1)} s_{m,n_g}(t_1) \times \Big[\frac{x^2_{m,n_g}(t_1)}{\alpha_{m,n_g}(t_1)}\Big]^{-\alpha_{m,n_g}(t_1)} \leq 1, \tag{30}$$

where $\lambda_{m,n_g}(t_1) = \frac{1}{x^2_{m,n_g}(t_1-1)+1}$ and $\alpha_{m,n_g}(t_1) = \frac{x^2_{m,n_g}(t_1-1)}{x^2_{m,n_g}(t_1-1)+1}$. Now, C4 can be replaced by the following constraints

$$\text{C4.1: } s^{-1}_{m,n_g}(t_1) + x_{m,n_g}(t_1)y_{n_g}(t_1)s^{-1}_{m,n_g}(t_1) \leq 1,$$

$$\text{C4.2: } \big[1/\lambda_{m,n_g}(t_1)\big]^{-\lambda_{m,n_g}(t_1)} s_{m,n_g}(t_1) \times \Big[\frac{x^2_{m,n_g}(t_1)}{\alpha_{m,n_g}(t_1)}\Big]^{-\alpha_{m,n_g}(t_1)} \leq 1,$$

$$\widehat{\text{C4.3}}: \quad x_{m,n_g}(t_1) = \sum_{k\in\mathcal{K}} \beta_{m,k,n_g}(t_1),$$

$$\widehat{\text{C4.4}}: \quad y_{n_g}(t_1) = \sum_{m\in\mathcal{M},k\in\mathcal{K}} \beta_{m,k,n_g}(t_1),$$

Note that via (28), the positive condition for the constraints of GP is met [31]. However, the equality constraints in $\widehat{\text{C4.3}}$ and $\widehat{\text{C4.4}}$ are not monomial since we have $x_{m,n_g}(t_1) - \sum_{k\in\mathcal{K}} \beta_{m,k,n_g}(t_1) = 0$ and $y_{n_g}(t_1) - \sum_{m\in\mathcal{M},k\in\mathcal{K}} \beta_{m,k,n_g}(t_1) = 0$, and, they have negative constraints. To convert $\widehat{\text{C4.3}}$ and $\widehat{\text{C4.4}}$ to the monomial functions, we again apply AGMA approximation presented in Section III.A as

$$\text{C4.3: } \quad x_{m,n_g}(t_1) \prod_{k\in K} \Big[\frac{\beta_{m,k,n_g}(t_1)}{\nu_{m,k,n_g}(t_1)}\Big]^{-\nu_{m,k,n_g}(t_1)} = 1,$$

$$\text{C4.4: } \quad y_{n_g}(t_1) \prod_{m\in\mathcal{M},k\in\mathcal{K}} \Big[\frac{\beta_{m,k,n_g}(t_1)}{\eta_{m,k,n_g}(t_1)}\Big]^{-\eta_{m,k,n_g}(t_1)} = 1$$

where $\nu_{m,k,n_g}(t_1)$ and $\eta_{m,k,n_g}(t_1)$ are defined in (14) and (15), respectively.

APPENDIX B
PROOF OF PROPOSITION 2

To reach the GP based formula for (11), we should have minimization over the objective function, i.e.,

$$\min_{\boldsymbol{\beta}(t_1)} \sum_{m\in\mathcal{M}} \sum_{g\in\mathcal{G}} \sum_{n_g\in\mathcal{N}_g} \sum_{k\in\mathcal{K}} -\beta_{m,k,n_g} R_{m,k,n_g}(\mathbf{P}(t)).$$

Clearly, we have negative terms on the objective function similar to our general formulation in (5). To meet the positive conditions of objective function in GP, we consider $\Xi_1 \gg 1$ and rewrite objective function as

$$\Xi_1 - \sum_{m\in\mathcal{M}} \sum_{g\in\mathcal{G}} \sum_{n_g\in\mathcal{N}_g} \sum_{k\in\mathcal{K}} \beta_{m,k,n_g}(t_1) R_{m,k,n_g}(\mathbf{P}(t))$$

which is always positive. Then, consider a positive auxiliary variable x_0, and rewrite the objective function with this new auxiliary variables

$$\frac{\Xi_1}{x_0 + \sum_{m\in\mathcal{M}} \sum_{g\in\mathcal{G}} \sum_{n_g\in\mathcal{N}_g} \sum_{k\in\mathcal{K}} \beta_{m,k,n_g}(t_1) R_{m,k,n_g}(\mathbf{P}(t))} \leq 1. \quad (31)$$

Now, (31) can be rewritten as the product of monomial functions based on the AGMA from Section III. B as

$$\Xi_1 \left[\frac{x_0}{c_0(t_1)}\right]^{c_0(t_1)} \prod_{m\in\mathcal{M},g\in\mathcal{G},n_g\in\mathcal{N}_g,k\in\mathcal{K}} \times \left[\frac{\beta_{m,k,n_g}(t_1) R_{m,k,n_g}(\mathbf{P}(t))}{c_{m,k,n_g(t_1)}}\right]^{c_{m,k,n_g}(t_1)} \leq 1, \quad (32)$$

where $c_{m,k,n_g}(t_1)$ and $c_0(t_1)$ are updated from (19) and (20), respectively. Therefore, the corresponding optimization problem can be transformed into (16).

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