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# Variable Rate and Variable Power MQAM System Based on Bayesian Bit Error Rate and Channel Estimation Techniques

Lay Teen Ong, Mohammad Shikh-Bahaei, and Jonathon A. Chambers

Abstract—The impact of inaccurate channel state information at the transmitter for a variable rate variable power multilevel quadrature amplitude modulation (VRVP-MQAM) system over a Rayleigh flat-fading channel is investigated. A system model is proposed with rate and power adaptation based on the estimates of instantaneous signal-to-noise ratio (SNR) and bit error rate (BER). A pilot symbol assisted modulation scheme is used for SNR estimation. The BER estimator is derived using a maximum a posteriori approach and a simplified closed-form solution is obtained as a function of only the second order statistical characterization of the channel state imperfection. Based on the proposed system model, rate and power adaptation is derived for the optimization of spectral efficiency subject to an average power constraint and an instantaneous BER requirement. The performance of the VRVP-MQAM system under imperfect channel state information (CSI) is evaluated. We show that the proposed VRVP-MQAM system that employs optimal solutions based on the statistical characterization of CSI imperfection achieves a higher spectral efficiency as compared to an ideal CSI assumption based method.

Index Terms—Adaptive modulation, adaptive power, BER, channel estimation, flat-fading channel, MQAM.

## I. INTRODUCTION

INK adaptation is one of the promising approaches to increase spectral efficiency of wireless channels [1]−[9]. In [2] ergodic capacity of a single-user flat-fading channel is obtained when optimal and suboptimal rate and power adaptation are used. The work in [4] and [5] examined a variable rate and variable power MQAM (VRVP-MQAM) system with optimal solutions derived assuming ideal channel state information (CSI) is available at the transmitter. Adaptive transmission under imperfect CSI has been studied in the literature, for instance in [4], [7], [8], [10]–[14]. In [4], the effect of channel estimation error and delay on the bit error rate (BER) performance of the VRVP-MQAM system is also analyzed. The effect of imperfect channel estimation on a VRVP-MQAM system is studied in [7] wherein an instantaneous signal-to-noise ratio (SNR) estimate based on the minimum mean-square error criterion was considered.

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Transmitter adaptation using discrete power and/or rate levels has also been considered in a number of works, and more recently, such schemes have been analysed for systems with coding [15]–[17]. Most of these works have used SNR as a system performance indicator. However SNR measurements may not be a direct measure of quality for a mobile system with time-varying channel characteristics due to fading. Therefore power adaptation based on a BER parameter, which is considered to be a more direct representation of the measure of quality has been proposed in a number of published works, e.g. in [18] and [19]. Outer-loop power control based upon BER or FER (frame erasure rate) in W-CDMA systems is a practical example of this kind of power adaptation [20]. Joint optimization of BER-based adaptive modulation and outer loop SNR target was considered in [21].

In this work, we provide a VRVP-MQAM scheme using a BER estimate based on the maximum a posteriori (MAP) criterion, which does not assume exact knowledge of SNR. Instead, we assume the correlation between the true SNR and its estimate is known. Based on the proposed MAP-based BER estimate, we derive analytical expressions for optimal rate and power adaptation to maximize spectral efficiency while satisfying an average power constraint and an instantaneous BER target. We consider a pilot symbol assisted modulation (PSAM) method for channel estimation [22] at the receiver and consider the second order statistical characteristics of the channel imperfection (i.e. correlation coefficient) for transmitter adaptation.

The remainder of this paper is organized as follows. In Section II we describe the proposed system model. We derive the BER estimator in Section III. Then we obtain the analytical expressions for optimum rate and power adaptation in Section IV. In Section V, we evaluate the performance of the proposed VRVP-MQAM system under two scenarios: an ideal CSI scenario and a partial CSI scenario. Finally, conclusions are drawn in Section VI.

## II. SYSTEM MODEL

We adopt the system model shown in Fig. 1, and consider a single user flat-fading channel. The channel is modelled in discrete-time, denoted by index i, with statistically stationary and ergodic time-varying gain  $\alpha\left(i\right)$ , and zero mean additive white Gaussian noise  $n\left(i\right)$ . We consider a PSAM [22] method for the estimation of the channel gain  $\alpha\left(i\right)$ . At time i, based on the channel gain and its respective SNR, the channel estimator has knowledge of the instantaneous received SNR estimate, the average SNR estimate and the correlation coefficient between the true channel SNR and its estimate. We also assume that this information is fed back to the transmitter error-free and without delay. By incorporating the autocorrelation

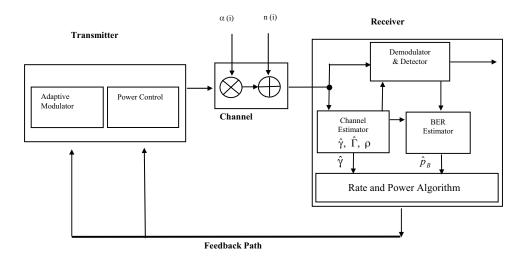


Fig. 1. Block diagram of the proposed variable rate variable power MQAM system based on BER and SNR estimates. CSI parameters include: instantaneous SNR estimate  $\hat{\gamma}$ , average SNR estimate  $\hat{\Gamma}$ , correlation coefficient  $\rho$  and BER estimate  $\hat{p}_B$ .

function of the fading process, the extension of this work to the case with non-zero feedback delay between receiver and transmitter is straightforward. For a constant average transmit power  $\bar{S}$ , the instantaneous received SNR  $\gamma(i)$  is defined as  $\gamma(i) = \bar{S}|\alpha(i)|^2/(N_0B)$  and its estimate is defined as  $\hat{\gamma}(i) = \bar{S}|\hat{\alpha}(i)|^2/(N_0B)$ , where  $\hat{\alpha}(i)$  is the estimate of  $\alpha(i)$ , B denotes the received signal bandwidth and  $N_0$  denotes the noise power spectral density. The average received SNR will be denoted by  $\Gamma = E[\gamma]$  and its estimate by  $\hat{\Gamma} = E[\hat{\gamma}]$ , where E[.] denotes the statistical expectation operator. At time i, the transmitter adapts to channel variation by adjusting its transmitted power  $S(\hat{\gamma}(i))$  based on information related to  $\hat{\gamma}(i)$  fed back from the receiver. With power adaptation, the estimate of SNR at the receiver can be written as  $\sigma(\hat{\gamma}(i)) =$  $\hat{\gamma}(i) S(\hat{\gamma}(i))/\bar{S}^{1}$  We denote the instantaneous BER by  $p_{B}(i)$ and obtain an estimate thereof,  $\hat{p}_B(i)$ , using the Bayesian estimation approach [23]. For simplicity and due to stationarity of the channel, we shall omit the discrete time reference i in the remaining sections.

The transmitter adapts its rate by adjusting the constellation size M of the MQAM scheme based on  $\hat{\gamma}$  and a required instantaneous BER target. With data sent at  $k\left(\hat{\gamma}\right) = \log_2\left(M\left(\hat{\gamma}\right)\right)$  bits/symbol, the instantaneous data rate is  $k\left(\hat{\gamma}\right)/T_s$  bits/sec (bps), where  $T_s$  denotes the symbol period duration. The spectral efficiency of an MQAM scheme can be expressed as its average data rate per unit bandwidth R/B. Assuming Nyquist data pulses of duration  $T_s = 1/B$ , the spectral efficiency (in bps/Hz) can be written as

$$\frac{R}{B} = \int_{\hat{\gamma}} k(\hat{\gamma}) f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma},\tag{1}$$

where  $f_{\hat{\gamma}}(\hat{\gamma})$  denotes the probability density function (PDF) of  $\hat{\gamma}$ . We aim to obtain the optimal rate and power adaptation in order to maximize spectral efficiency (1) subject to an average

<sup>1</sup>Note that  $\sigma(\hat{\gamma}(i))$  is a function of both variation in transmit power  $S(\hat{\gamma}(i))$  and channel gain,  $\hat{\alpha}(i)$ , whereas  $\hat{\gamma}(i)$  is only a function of variation in channel gain,  $\hat{\alpha}(i)$ , for a constant transmit power.

power constraint

$$E[S(\hat{\gamma})] = \int_{\hat{\gamma}} S(\hat{\gamma}) f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} \le \bar{S}$$
 (2)

and an instantaneous BER requirement. However our proposed algorithm will be based on the estimates of SNR and BER values.

# III. BER ESTIMATE

Our objective is to obtain an expression for the BER estimate  $\hat{p}_B$  in terms of the observation of  $\hat{\gamma}$ . The instantaneous BER  $(p_B)$  will be estimated by adopting the MAP-optimal approach. Hence, the  $p_B$  that maximizes the conditional PDF of  $p_B$  given  $\hat{\gamma},~f_{p_B|\hat{\gamma}}\left(p_B|\hat{\gamma}\right),$  will be derived using Bayes' theorem [24]. Therefore we maximize the function  $l_{MAP}\left(p_B\right)=f_{p_B,\hat{\gamma}}\left(p_B,\hat{\gamma}\right),$  where  $f_{p_B,\hat{\gamma}}\left(p_B,\hat{\gamma}\right)$  is the joint PDF of  $p_B$  and  $\hat{\gamma}.$  The estimate of  $p_B$  can thus be obtained from the solution to  $\frac{\partial l_{MAP}}{\partial p_B}=0.$  We will next derive the closed form expressions for  $f_{p_B,\hat{\gamma}}\left(p_B,\hat{\gamma}\right)$  and  $\hat{p}_B.$ 

Assume the value of the SNR estimate  $\hat{\gamma}$  is known through a particular channel gain estimator (where  $\hat{\gamma} \neq \gamma$ ). The transmitter will adapt its power  $S(\hat{\gamma})$  and rate  $k(\hat{\gamma})$  relative to a BER target (BERT) based on the SNR estimate  $\hat{\gamma}$  instead of the actual SNR,  $\gamma$ . Using the generic approximation of the BER expression for an MQAM scheme [4, eqn (42)], the instantaneous BER at the receiver can be expressed as

$$p_B(\gamma, \hat{\gamma}) = c_1 \exp\left(-\frac{c_2 \gamma}{M(\hat{\gamma}) - 1} \frac{S(\hat{\gamma})}{\bar{S}}\right), \quad (3)$$

where  $c_1$  and  $c_2$  are positive real numbers [4], [5]. Let  $\sigma$  be a particular value of  $\sigma(\hat{\gamma})$  at the receiver, and M be the corresponding value of  $M(\hat{\gamma})$  determined at the transmitter. The corresponding instantaneous BER at the receiver will be

$$p_B = c_1 \exp\left(-\frac{c_2 \sigma}{M - 1} \frac{\gamma}{\hat{\gamma}}\right). \tag{4}$$

Here we consider a Rayleigh flat-fading channel and incorporate the PSAM method for channel estimation where  $\hat{\alpha}$  is derived as the magnitude of a weighted sum of zero mean complex random variables. Hence  $\alpha$  and  $\hat{\alpha}$  have a bivariate Rayleigh distribution [25]. Since the SNR ( $\gamma$  and  $\hat{\gamma}$ ) is expressed as a function of the channel gain ( $\alpha$  and  $\hat{\alpha}$  respectively), the joint PDF of  $\gamma$  and  $\hat{\gamma}$  can be derived (using the transformation of random variables [24, chpt. 6]) as

$$f_{\gamma,\hat{\gamma}}(\gamma,\hat{\gamma}) = \frac{1}{(1-\rho)\,\Gamma\hat{\Gamma}} I_0\left(\frac{2\sqrt{\rho}}{1-\rho}\sqrt{\frac{\gamma\hat{\gamma}}{\Gamma\hat{\Gamma}}}\right)$$

$$\cdot \exp\left(-\frac{1}{(1-\rho)}\left(\frac{\gamma}{\Gamma} + \frac{\hat{\gamma}}{\hat{\Gamma}}\right)\right) u(\gamma) u(\hat{\gamma}), \tag{5}$$

which is a bivariate gamma distribution [11].  $I_0\left(.\right)$  is the zero order modified Bessel function and  $u\left(.\right)$  is the unit step function. The parameter  $\rho$  denotes the correlation coefficient between  $\gamma$  and  $\hat{\gamma}$  which we assume to be available at the transmitter. We note that under the above conditions,  $\gamma$  and  $\hat{\gamma}$  are exponentially-distributed with average values  $\Gamma$  and  $\hat{\Gamma}$  respectively. That is, we have  $f_{\gamma}(\gamma) = \frac{1}{\Gamma} \exp\left(-\frac{\gamma}{\Gamma}\right)$  and  $f_{\hat{\gamma}}(\hat{\gamma}) = \frac{1}{\hat{\Gamma}} \exp\left(-\frac{\hat{\gamma}}{\Gamma}\right)$ . In practical situations, the information about  $\rho$  at the transmitter is available through its estimate in the receiver. For a system employing PSAM with MAP-optimal prediction of SNR at pilot symbol instants under Rayleigh fading having Jakes spectrum,  $\rho$  has been analytically derived in [11] as a function of normalized Doppler spread, namely the Doppler frequency  $f_d$  times symbol duration  $T_s$ .

By exploiting (4) and (5), and through a transformation of random variables, a closed form expression can be obtained for  $f_{p_B,\hat{\gamma}}(p_B,\hat{\gamma})$ 

$$f_{p_B,\hat{\gamma}}(p_B,\hat{\gamma}) = c(p_B) I_0(b(p_B)\hat{\gamma}) \exp(-a(p_B)\hat{\gamma}),$$
 (6)

where

$$a(p_B) = \frac{1}{1 - \rho} \left( \frac{-(M - 1)}{\sigma c_2} \ln \left( \frac{p_B}{c_1} \right) \frac{1}{\Gamma} + \frac{1}{\hat{\Gamma}} \right), \quad (7)$$

$$b(p_B) = \frac{2\sqrt{\rho}}{1-\rho} \sqrt{\frac{-(M-1)}{\sigma c_2} \ln\left(\frac{p_B}{c_1}\right) \frac{1}{\Gamma\hat{\Gamma}}},$$
 (8)

$$c(p_B) = \frac{(M-1)\hat{\gamma}}{p_B c_2 \sigma (1-\rho) \Gamma \hat{\Gamma}}.$$
 (9)

Based on (6), and omitting terms that are not functions of  $p_B$ , the final form of the MAP function can be expressed as

$$I_{MAP} = \frac{1}{p_B} I_0 (b(p_B) \hat{\gamma}) \exp(-a(p_B) \hat{\gamma}).$$
 (10)

To perform the operation  $\frac{\partial I_{MAP}}{\partial p_B}$ , (10) will be approximated [26] as

$$I_{MAP_{eq}} = \frac{1}{p_B} \frac{\exp\left(b\left(p_B\right)\hat{\gamma}\right)}{\sqrt{2\pi b\left(p_B\right)\hat{\gamma}}} \exp\left(-a\left(p_B\right)\hat{\gamma}\right), \tag{11}$$

where  $I_{MAP_{eq}}$  denotes the approximate form of the MAP function for large values of SNR estimate  $\hat{\gamma}$ . Finally, an approximate closed form expression for  $\hat{p}_B$  is obtained as

$$\hat{p}_B \approx c_1 \exp\left(\frac{\rho \frac{-\sigma c_2}{M-1} \frac{\Gamma}{\hat{\Gamma}}}{\left((1-\rho) \frac{\sigma c_2}{M-1} \frac{\Gamma}{\hat{\gamma}} - 1\right)^2}\right), \qquad \hat{\gamma} > \hat{\gamma}_{th},$$

where  $\hat{\gamma}_{th}$  can be obtained from  $\hat{\gamma}_{th} \triangleq (1 - \rho) \left(-\ln\left(\frac{BERT}{c_1}\right)\right) \Gamma$ . Exploiting numerical analysis to

obtain the maximum value of  $I_{MAP}$  and the corresponding BER estimate using the exact expression (10) leads to the result that realistic  $\hat{p}_B$  values greater than zero exist only for the range of  $\hat{\gamma} > \hat{\gamma}_{th}$ . Otherwise the optimum value for  $\hat{p}_B$  will be zero. Therefore transmission will be stopped for  $\hat{\gamma} \leq \hat{\gamma}_{th}$ , as there is no practical and reliable BER estimate (and hence no reliable channel) available in that case.

#### IV. OPTIMAL RATE AND POWER ADAPTATION

For a required BERT, and with only the knowledge of channel estimate available,  $M(\hat{\gamma})$  is derived according to [4]

$$M(\hat{\gamma}) = 1 + \frac{c_2 \hat{\gamma}}{-\ln(BERT/c_1)} \frac{S(\hat{\gamma})}{\bar{S}}.$$
 (13)

We note from (13) that for a given BERT and  $\hat{\gamma}$ ,  $\frac{-\sigma c_2}{M-1} = \ln{(BERT/c_1)}$ . Using (12) and (13), we can obtain an estimate of BER:

$$\hat{p}_{B}\left(\hat{\gamma}\right) = c_{1} \exp\left(\frac{\ln\left(BERT/c_{1}\right)\frac{\Gamma}{\hat{\Gamma}}\rho}{\left(-\ln\left(BERT/c_{1}\right)\frac{\Gamma}{\hat{\gamma}}\left(1-\rho\right)-1\right)^{2}}\right).$$
(14)

Considering that the estimates  $\hat{\gamma}$  and  $\hat{p}_B(\hat{\gamma})$  are available at the receiver and transmitter, the corresponding MQAM constellation size  $M_{\hat{p}_B}(\hat{\gamma})$  will be

$$M_{\hat{p}_{B}}(\hat{\gamma}) = 1 + \frac{c_{2}\hat{\gamma}}{-\ln(\hat{p}_{B}(\hat{\gamma})/c_{1})} \frac{S_{\hat{p}_{B}}(\hat{\gamma})}{\bar{S}}, \quad (15)$$

where  $S_{\hat{p}_B(\hat{\gamma})}$  is the power variation at the transmitter. For  $\hat{\gamma} > \hat{\gamma}_{th}$ , we substitute (14) into (15) and obtain the adjusted M as

$$M_{\hat{p}_B}(\hat{\gamma}) = 1 + \frac{K\hat{\Gamma}}{\rho\Gamma} \left\{ \frac{c_2\Gamma}{K\hat{\gamma}} (1 - \rho) - 1 \right\}^2 \frac{S_{\hat{p}_B}(\hat{\gamma})}{\bar{S}} \hat{\gamma}, \quad (16)$$

where  $K = \frac{c_2}{-\ln\left(\frac{BERT}{c_1}\right)}$ . The corresponding transmission rate is  $k_{\hat{p}_B}(\hat{\gamma}) = \log_2\left(M_{\hat{p}_B}(\hat{\gamma})\right)$ .

For maximizing spectral efficiency subject to the average power constraint  $\bar{S}$ , the corresponding power control  $S_{\hat{p}_B}\left(\hat{\gamma}\right)$  obtained using a Lagrangian method is

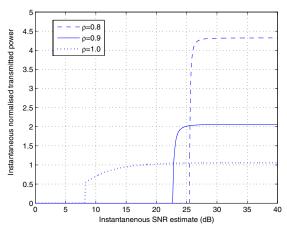
$$\frac{S_{\hat{p}_B}(\hat{\gamma})}{\bar{S}} = \begin{cases}
U - \frac{\frac{\Gamma}{\hat{\Gamma}}\rho}{K\hat{\gamma}\left\{\frac{c_2}{K}\frac{\Gamma}{\hat{\gamma}}(1-\rho)-1\right\}^2}, & S_{\hat{p}_B}(\hat{\gamma}) \ge 0, k_{\hat{p}_B}(\hat{\gamma}) \ge 1, \\
0, & \text{otherwise,} 
\end{cases} (17)$$

where U is a constant value found through numerical search such that the average power constraint (2) is satisfied. Subsequently, the optimal rate adaptation is

$$k_{\hat{p}_{B}}(\hat{\gamma}) = \begin{cases} \log_{2} \left[ \frac{K\hat{\Gamma}}{\rho \Gamma} \left\{ \frac{c_{2}}{K} \frac{\Gamma}{\hat{\gamma}} (1 - \rho) - 1 \right\}^{2} \hat{\gamma} U \right], & k_{\hat{p}_{B}}(\hat{\gamma}) \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$
(18)

Note that  $k_{\hat{p}_B}(\hat{\gamma}) \geq 1$  corresponds to a realistic MQAM constellation size  $M \geq 2$ . For  $S_{\hat{p}_B}(\hat{\gamma}) \geq 0$  and  $k_{\hat{p}_B}(\hat{\gamma}) \geq 1$ , the SNR cutoff threshold can be shown to be

$$\hat{\gamma}_0 = \hat{\gamma}_{th} + \chi,\tag{19}$$



(a) Instantaneous normalised transmitted power

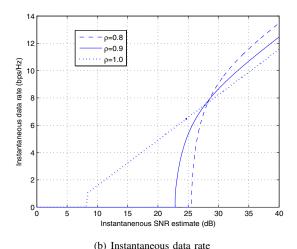


Fig. 2. VRVP-MQAM-CSI system: instantaneous power  $\frac{S_{\hat{p}_B}(\hat{\gamma})}{S}$  and rate  $k_{\hat{p}_B}(\hat{\gamma})$  adaptations as a function of instantaneous SNR estimate  $\hat{\gamma}$ .

where  $\chi \triangleq \frac{\Gamma}{2KU\hat{\Gamma}} \left[ 2\rho + 2\sqrt{\rho \left( 2Uc_2\hat{\Gamma} - 2Uc_2\hat{\Gamma}\rho + \rho \right)} \right]$ which is  $\geq 0$ . Hence,  $S_{\hat{p}_B}(\hat{\gamma}) \geq 0$  and  $k_{\hat{p}_B}(\hat{\gamma}) \geq 1$  imply  $\hat{\gamma} \geq \hat{\gamma}_0$  and transmission is allowed. These conditions also verify the appropriateness of using the derived BER estimator (12) since  $\hat{\gamma}_0$  is always  $> \hat{\gamma}_{th}$ .

#### V. NUMERICAL RESULTS

We evaluate the performance of the system over channel variations modelled by Rayleigh fading and we assume that the PSAM technique [25] is used for deriving the channel estimate. For a system employing PSAM under fairly common channel conditions and SNR estimation criterion,  $\rho$  has been shown [11] to be obtained from  $\rho = \dot{\Gamma}/\Gamma$ . Therefore it is reasonable to assume that  $\hat{\Gamma} = \rho \Gamma$  for our numerical results. We assume  $M \geq 2$  can be a non-integer value and consider a BER target of  $10^{-3}$ , and set  $c_1 = 0.2$  and  $c_2 = 1.5$ .

Based on the analytical expressions derived in Section IV, we evaluate the performance of our proposed VRVP-MQAM system that employs adaptations based on CSI imperfection and a MAP-optimal BER estimate. We refer to this system as 'VRVP-MQAM-CSI'. We further compare the performance of this system with that of two other MQAM systems: 1) a VRVP-MQAM system that employs adaptations based on an ideal CSI assumption which we refer to as 'VRVP-MQAM', and 2) a nonadaptive transmission system that employs a constant-rate constant-power MQAM (CRCP-MQAM) scheme [4].

#### A. Instantaneous Rate and Power in VRVP-MQAM-CSI

We represent the perfect-CSI scenario by  $\rho = 1$  whereas the scenario with CSI imperfection will be represented by  $\rho < 1$ . For  $\Gamma=25$  dB, the sets of solutions  $\frac{S_{\hat{p}_B}(\hat{\gamma})}{S}$  and  $k_{\hat{p}_B}(\hat{\gamma})$  for  $\rho = 0.8, 0.9, 1.0$  are obtained using (17) and (18) respectively. The corresponding values for U are obtained numerically. The computed results are illustrated in Fig. 2(a) and Fig. 2(b) respectively. At  $\rho = 1$ , the transmit power variation  $\frac{S_{\hat{p}_B}(\hat{\gamma})}{\bar{c}}$ follows a smooth water-filling profile for  $\hat{\gamma}$  beyond a cutoff

value  $\hat{\gamma}_0$ , and the rate  $k_{\hat{p}_B}(\hat{\gamma})$  increases linearly as  $\hat{\gamma}$  increases beyond  $\hat{\gamma}_0$ . No data transmission is allowed for  $\hat{\gamma}$  below  $\hat{\gamma}_0$ . Indeed, when  $\rho = 1$ , the adaptations turn into the rate and power adaptations observed in [4] with adaptation based on assumption of ideal CSI. There is, however, no transmission in our plot when k(.) < 1 but the corresponding result in [4] uses the transmission rate limit of k(.) = 0. For  $\rho < 1$ , higher cutoff SNR values are derived in the rate and power adaptation plots.

As depicted in Fig. 2(a) and 2(b), the power and rate adaptation schemes adapt to the CSI-imperfection by transmitting at a higher power level and a higher transmission rates as  $\rho < 1$  for larger SNR values. It is noted that the total average transmitted power is still maintained at S.

# B. Instantaneous Rate and Power in VRVP-MQAM

In this section, we show the effect of channel imperfections on the performance of a VRVP-MQAM system based on an ideal CSI assumption. An ideal CSI is assumed by considering  $\rho=1,\,\gamma=\hat{\gamma},$  and  $\hat{\Gamma}=\Gamma.$  We note that by substituting  $\rho=1,$  $\gamma = \hat{\gamma}$ , and  $\hat{\Gamma} = \Gamma$  into (17) and (18), the respective transmit power and rate adaptations are

$$\frac{S(\hat{\gamma})}{\bar{S}} = U - \frac{1}{K\hat{\gamma}}, \tag{20}$$

$$k(\hat{\gamma}) = \log_2[K\hat{\gamma}U], \tag{21}$$

$$k(\hat{\gamma}) = \log_2 \left[ K \hat{\gamma} U \right], \tag{21}$$

where  $S(\hat{\gamma}) \ge 0$  and  $k(\hat{\gamma}) \ge 1$ . In fact, optimal solutions (20) - (21) are those of [4], corresponding to a VRVP-MQAM scheme based on actual SNR  $(\gamma, \Gamma)$  knowledge.

To investigate the impact of CSI imperfection (i.e.  $\rho$  <  $1, \ddot{\Gamma} = \rho \Gamma, \dot{\gamma} \neq \gamma$ ) on the 'ideal-assumed' system, we first perform numerical search for U at  $\rho = 1$  by using (20) in the power constraint formula (2), and determine U numerically. Next, for  $\rho$  < 1, we use (17) and (18) respectively for power and rate adaptations for  $\rho < 1$ , but always adopt the value of U that was obtained at  $\rho = 1$ . We compute numerical results for a set of  $\rho$  values, and at  $\hat{\Gamma} = 25$  dB. The numerical results are illustrated in Fig. 3(a) and Fig. 3(b). For  $\rho = 1$ ,

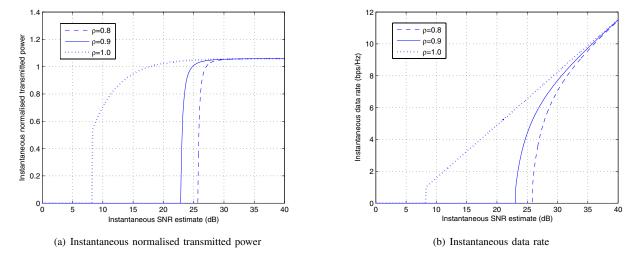


Fig. 3. VRVP-MQAM system: instantaneous power  $\frac{S_{\hat{p}_B}(\hat{\gamma})}{S}$  and rate  $k_{\hat{p}_B}(\hat{\gamma})$  adaptations as a function of instantaneous SNR estimate  $\hat{\gamma}$ .

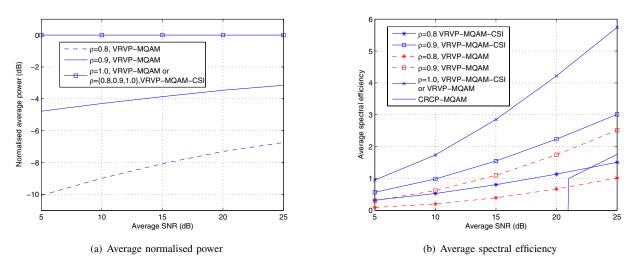


Fig. 4. Average normalised power and spectral efficiency vesus average SNR  $\Gamma$  for different values of correlation coefficient  $\rho$ . Comparison of VRVP-MQAM-CSI, VRVP-MQAM and CRCP-MQAM systems.

rate and power have a water-filling nature. For  $\rho < 1$ , higher  $\hat{\gamma}_0$  values are derived in the rate and power adaptation plots. Finally, we note that unlike in the VRVP-MQAM-CSI scheme, the power and rate adaptation curves converge to the same value at higher SNRs since rate and power adaptations are not adapting to CSI imperfection. It is therefore clear that the resulting average transmitted power will vary across channel imperfections, which will be verified in section V-D.

# C. Instantaneous Rate and Power in CRCP-MQAM

In the CRCP-MQAM system, constant rate and power are transmitted at all times. For comparison with the other two systems, the transmit power is set to  $\bar{S}$ . The number of constellation points M, restricted to be  $\geq 2$ , is obtained for a given  $\Gamma$  value such that the average BER is equal to BERT. Subsequently, the corresponding spectral efficiency is  $\log_2(M)$ .

#### D. Average Power and Spectral Efficiency

The normalized average power, which is expressed as  $\sum_{\hat{\rho}_B}^{\infty} \left[ S_{\hat{\rho}_B}(\hat{\gamma}) / \bar{S} \right] f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma}$ , is shown in Fig. 4(a) for the VRVP-MQAM-CSI and the VRVP-MQAM systems. In the perfect-CSI assumed VRVP-MQAM system, constant average power is maintained at  $\rho = 1$ , but variation in the average power occurs for  $\rho < 1$ . On the other hand, the VRVP-MQAM-CSI system maintains constant average power for all  $\rho$  values since the system is adapting to the  $\rho$  variations. Though not shown in the figure, the average power for the CRCP-MQAM scheme is clearly constant from  $\Gamma > \sim 21$ dB, which corresponds to  $M \geq 2$ . These results confirm that the VRVP-MQAM-CSI system has appropriately exploited the power resource variation in an imperfect CSI scenario, and results in a higher spectral efficiency performance over a wide range of SNRs and for practical ranges of  $\rho$ , as we will show in Fig. 4(b).

Fig. 4(b) shows the spectral efficiency performance for VRVP-MQAM-CSI, VRVP-MQAM and CRCP-MQAM sys-

tems. The figure confirms that the spectral efficiency for the VRVP-MQAM-CSI and the VRVP-MQAM systems declines as  $\rho$  decrease, and both schemes converge to the same performance in the perfect CSI scenario ( $\rho=1.0$ ). However, it is noted that the VRVP-MQAM-CSI system outperforms the VRVP-MQAM system for all channel imperfection scenarios ( $\rho<1$ ). In comparison to the CRCP-MQAM system, the results illustrate that the VRVP-MQAM-CSI system provides a better performance for larger values of  $\rho$ , and for small  $\rho$  the CRCP-MQAM system would be a better alternative.

# VI. CONCLUSION

We investigated the impact of imperfect channel estimation for a variable rate variable power MQAM system over a Rayleigh flat-fading channel. We have proposed a variable rate and power system model based on the estimates of the instantaneous SNR and BER using, respectively, a PSAM technique and a MAP-optimal BER estimator. Based on our proposed model, we derived optimal solutions for the rate and power algorithms, and the solutions were compared to an ideal CSI assumption based VRVP-MQAM system and a nonadaptive MQAM system. Our proposed system achieves a higher spectral efficiency as compared to the ideal CSI assumption based VRVP-MQAM system in all cases, and performs better than the nonadaptive MQAM system under reasonable channel imperfection scenarios.

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