ESTIMATION AND FILTERING FOR DIGITAL SIGNALS

BY

M. A. HUSAIN BSc

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Supervisor : Professor A. P. Clark
Department of Electronic and Electrical Engineering

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To my family whose endless love and encouragement inspired me to achieve greater success

Research is the process of going up alleys to see if they are blind

- Marston Bates

ABSTRACT

This thesis is concerned with the estimation of the sampled impulse response of a time-varying HF channel, over a synchronous serial data transmission system, using quaternary phase shift keyed signals operating at 2400 bauds, with an 1800 Hz carrier. It is also concerned with the adjustment of a linear filter ahead of a near maximum likelihood detector, to make the channel minimum phased, and hence remove phase distortion introduced over telephone channels for a 16 QAM data transmission system operating at 9600 bits/s.

HF modems employing near maximum likelihood detectors require an accurate knowledge of the sampled impulse response of the channel . A number of channel estimation techniques were studied for this application. The thesis provides a brief description of the ionospheric propagation medium, and the types of distortion encountered by data signals over such channels. It then presents an equivalent baseband model of the 2 sky-wave HF channel, with a 2 Hz frequency spread and a transmission delay of 1.1 ms between the two sky-waves. The results of computer simulation tests on the performance of the various estimators over a typical worst channel are presented.

An investigation was carried out into the adjustment of a linear filter at the receiver to make the channel minimum phased, and hence remove phase distortion introduced over typical channels in the British public switched telephone network. The filter is adjusted, by a novel technique employing the Gram Schmidt orthogonalization process.

The results confirm that the simple estimator could obtain an estimate of the sampled impulse response of the time-varying HF channel to an acceptable degree of accuracy and was therefore a cost effective estimator. The simple estimator is a modification of the conventional gradient estimator. The adaptive estimator has, however, proved to be more cost effective, as its performance is considerably better than the simple estimator, without an undue increase in

complexity. The most significant result obtained is that the modified simple estimator shows no improvement over the simple estimator. Another very significant result, is the numerical inaccuracy introduced into the filter tap gains by the orthogonalization process, for some telephone channels tested.

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GLOSSARY OF SYMBOLS

$a(t)$	Impulse response of a filter
$A(f)$	Frequency response of a filter
$\lvert A(f) \rvert$	Absolute value of $A(f)$
$a(t)*b(t)$	Convolution between $a(t)$ and $b(t)$
D_i	Tap gains of linear feedforward tranversal filter
e_i	Error in the estimated value of r_i
$E[.]$	Expectation operator
$g+1$	Number of samples in the sampled impulse response of the linear baseband channel
j	When not used as a subscript, $j = \sqrt{-1}$
$n(t)$	White Gaussian noise with zero mean and two sided power spectral density of $1/2\ N_0$
$n+1$	Number of taps of linear feedforward transversal filter
$1/2\ N_0$	Power spectral density of $n(t)$
$q_h(t)$	Statistically independent random processes
$\{q_{h,i}\}$	Sequence obtained by sampling $q_h(t)$
$\Re[.]$	Real part of a complex number
$r(t)$	Received signal
$\{r_i\}$	Sequence of received signal samples
r_i'	Estimated received signal sample
s_i	Data symbol
s_i'	Detected data symbol
Superscript *	Complex conjugate
Superscript T	Matrix (or Vector) transpose
T	Sampling interval
$w(t)$	Gaussian random process with zero mean
$y(t)$	Impulse response of linear baseband channel
Y_i	Sampled impulse response of linear baseband channel

Y'_i	Estimate of Y_i at time t=iT
$Y'_{i+1,i}$	One step prediction of Y_{i+1} at time t=iT
$Y''_{i+1,i}$	Estimate(prediction) of the rate of change of Y_{i+1} with respect to i
ξ_1	Mean Square error in the estimate (prediction) of Y_i
ξ_2	Mean Square normalized error in the estimate (prediction) of Y_i
ξ_i	Squared error in the estimate (prediction) of Y_i
ψ	Signal/Noise ratio
σ^2	Variance of w(t)
θ	Small positive quantity

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

One of the most striking of recent developments has been the rapid growth of digital data transmission systems [1,2]. The basic feature of a digital transmission system is that it transmits a series of separate data elements which carry the information to be transmitted [2]. Different media are used for digital data transmission, but the most important of these are the voice frequency channels over the telephone network and HF radio links [1-26].

Although telephone circuits with a passband ranging from 300 to 3000 Hz were originally designed for the transmission of speech [23-24], they play an important roll in data transmission, because of the existing worldwide telephone network. The radio frequency band in the region of 3 to 30 MHz is traditionally known as the High Frequency (HF) band [6]. At these frequencies, propagation of radio signals is achieved by ionospheric reflection from one or more layers of the ionosphere [1,4-5]. The problem of reliable transmission of digital information over the HF channel has always presented the communicator with a significant challenge. Despite the advent of satellite communication systems, HF still continues to be used owing to the fact that it is economical and flexible [6,27,52]. However, HF transmission is unpredictable due to the existence of multiple transmission paths caused by reflection from different layers of the ionosphere [1,3-22]. More recently, advances in efficient signal processing capabilities, have ensured a new wave of interest in data transmission over HF radio links [25-27].

The earliest attempts at HF radio transmission made use of serial asynchronous techniques such as manually transmitted and received Morse code (10 bits/s) and radio Teletype, RTTY (50 bits/s). The success of these low data rate transmissions, and their high information densities of 1-2 bits/s/Hz of available bandwidth, suggest that a simple extension of the channel bandwidth to 3 KHz voiceband channels and similar increase in data rates would allow HF data transmission at several Kilobits per second. Unfortunately, this gives poor results because of the nature of the HF medium.

Distortion of the high data rate signal is produced by dispersion in the HF channel, which is produced by the reception of several discrete paths with different transmission delays, each path fading independently. The spread can be several milliseconds. At higher data rates, the signal element duration is small, so that the multipath spread can be several sampling intervals long, resulting in a received element overlapping with one or more of the following elements and therefore interfering with them. This effect is known as intersymbol interference (ISI), and is the primary impediment to reliable high speed data transmission at high signal to noise ratios, over HF radio links and telephone networks [1,2,5].

Until recently, the preferred method of transmitting digital data at medium to high speeds (greater than 1200 bits/s) [25] has been to employ a number of low speed channels in parallel so as to avoid intersymbol interference. The data is split between a large number of subchannels equally spaced through the available bandwidth [25-26]. An alternative approach is to use serial transmission and employ some form of adaptive signal processing at the receiver. Comparison of the two transmission techniques at a speed of 2400 bits/s have suggested that the serial modem offers a better overall performance [26-27]. With increase in processing speeds of digital hardware and significant improvement in signal processing techniques, serial modems are now challenging the dominance of parallel modems for high speed applications.

Considerable advances have been achieved in the design of serial modems for HF radio links [1,2,25-33]. This has made it possible to achieve an increase in the highest practically obtainable transmission rate over a voiceband HF channel from 2400 to 9600 bits/s [1,2,25-33]. The increase has been possible by the development of more effective techniques for tracking the sampled impulse response of the time varying baseband channel [27,28,32,36-42], together with the development of more effective detection processes for handling the severe signal distortion introduced by the HF radio link [30,33,46,48,55]. Similarly, for serial modems over telephone channels, transmission of data at 9600 bits/s become possible due to more effective detection processes for dealing with the distortion introduced [29,43-46,48].

In the detection of serially transmitted digital data, the detector may adopt one of several strategies which are broadly classified into two groups [2]. The first group of detectors employ an equalizer to remove the intersymbol interference from the received signal, and then each of the data symbols can be detected independently [2,13,31,34-35,48,52]. This process results in using only part of the transmitted signal energy in the detection of a data symbol, with a consequent reduction in tolerance to additive noise. Equalization techniques used are the linear equalizer and the nonlinear equalizer (decision feedback equalizer) [2,34-35].

The second group of detection processes instead of removing the intersymbol interference, take full account of the distortion introduced by the channel thus using the entire transmitted energy in the detection process. This technique is known as the maximum likelihood detection process [2,44] and is the optimum detection process, in the sense that under the appropriate conditions it minimizes the probability of error in the detection of the whole message [44]. It is not feasible to implement the maximum likelihood detector in its true form because of the enormous memory requirements and equipment complexity. However, the Viterbi algorithm can achieve the same tolerance to noise as that of the maximum likelihood detector [2,44,45]. This selects as the detected message, the possible sequence of transmitted data symbols for which there is the minimum mean square difference between the samples of the corresponding received data signal, for the given signal distortion but in the absence of noise, and the samples of the signals actually received [44-45]. The Viterbi detector, requires a large amount of storage and causes an exponential increase in computational complexity, as the number of components in the sampled impulse response of the channel increases.

The Viterbi detector can be replaced by, with no significant loss in tolerance to additive white Gaussian noise, by a very much simpler detection process that is a development of the Viterbi algorithm [46,48]. These detectors are referred to as the near maximum likelihood detectors [29-30,33,46,48,55]. Without a linear feedforward transversal filter ahead of the detector, severe distortion introduced by HF and telephone circuits [55] prevent the satisfactory performance of the near maximum likelihood detectors unless they are further modified.

For the reliable operation of a near maximum likelihood detector at 9600 bits/s over a telephone channel, or 4800 bits/s over an HF radio link, a linear feedforward transversal filter must be used ahead of the detector [30,33,46,48]. The transversal filter is an allpass network with ideally an infinite number of taps, that adjusts the sampled impulse response of the channel and filter to be minimum phased, without changing the levels of the data signal and noise, which means that the absolute value of the frequency response of the channel and filter is the same as that of the channel itself [2,31,48]. The particular virtue of the operation just described is that it concentrates the energy of the sampled impulse response of the channel and filter towards the earliest samples, without, however, changing the signal/noise ratio at the output of the filter [2,34,48]. Hence, an NML detector with an adaptive filter ahead of the detector is therefore the most suitable detector for a time varying HF channel and for time invariant telephone channels.

Near maximum likelihood detectors require a knowledge of the sampled impulse response of the channel [46]. This may be achieved by using a channel estimator, which estimates the sampled impulse response of the channel from the received signal. When the detector and adaptive filter are provided with the correct channel estimate and when perfect operation of the filter can be assumed, the performance of the NML detector gives an upper bound to the performance obtained when the channel response must be estimated. Any error in the estimation of the channel affects the performance of the detector, and the adjustment of the filter. It is therefore imperative, that for the good performance of the detector (and hence HF modem), the channel estimator is able to make an accurate estimate of the sampled impulse response of the channel. The channel characteristics of a telephone circuit do not vary (or only vary very slowly) with time. These channels, therefore, come under the category of time invariant channels. The estimate of the impulse response of such a channel can be made quite accurately [37,43]. These estimators [37,43] are simple to implement, and provide accurate estimates provided that the channel does not vary rapidly with time. However, when there is rapid variation in the channel characteristics, as in the case of an HF radio link, a more sophisticated technique is required [27-28,32,36,38-42], the estimation process being performed adaptively such that the detector is held correctly adjusted at all times for the channel.

A channel estimator is basically a tapped delay line finite impulse response (FIR) filter with the filter tap coefficients forming the channel sampled impulse response. The input to the estimator is the current detected data and received sample and the output is an estimate/prediction of the channel sampled impulse response, ready for use by the detector at the next sampling instant. The tap coefficients of the filter are adjusted adaptively, according to a particular algorithm, in order to track a time varying channel. These can be broadly classified into least mean squares (LMS) and recursive least squares (RLS) algorithms. In the LMS algorithm, the tap coefficients of the filter are determined using the method of steepest descent [13,28,32,49-51,56]. The algorithm is simple and works adequately in a variety of applications but suffers from the disadvantage of having a slow convergence rate. The RLS algorithm [13,28,32,49-51] on the other hand, makes use of the input information to the channel estimator in such a way as to ensure optimality at every time instant.

Tests have shown that the conventional Kalman filter is not optimum for a typical HF channel [28,32]. New fast RLS algorithms have been developed [32] but these exhibit numerical instability. Similar adaptation algorithms can be used for adaptively adjusting the filter ahead of the near maximum likelihood detector. This suggests a concerted effort must be made to develop the simple LMS algorithm to satisfy the objectives of an efficient high speed modem.

1.2 OUTLINE OF THE INVESTIGATION

This thesis is concerned with the estimation of the sampled impulse response of a time varying HF channel, over a synchronous serial data transmission system, using quaternary phase shift keyed signals operating at 2400 bauds, with an 1800 Hz carrier. It is also concerned with the adjustment of a linear filter ahead of a near maximum likelihood detector, to make the channel minimum phased, and hence remove phase distortion introduced over telephone channels for a 16 QAM data transmission system operating at 9600 bits/s.

Chapter 2 contains a detailed description of HF radio channels. Firstly, the physical characteristics of the earth's atmosphere are considered and these lead to an understanding of the types of distortion occurring on such channels. Finally a model of the HF channel is presented suitable for computer simulation.

Chapter 3 describes a model of a synchronous serial QAM digital data transmission system, and presents an equivalent baseband model of a two skywave HF radio link.

In Chapter 4, a description of the simple gradient estimator as an HF channel estimator is provided.

Chapter 5 considers the adaptive estimators. These estimators are adaptive in the sense that the stepsize of the gradient algorithm are here adjusted to suit the channel.

In Chapter 6, a brief description is provided of the Modified estimator. These are a modification of the estimators discussed in chapter 4, in that a prediction is made of the error between the estimate of the channel and its prediction, which is then used to form an updated estimate of the channel. The modified estimator has therefore an extra predictor as compared to the simple estimator of chapter 4.

Chapter 7 presents a study of a novel technique that has been developed for adjusting the linear feedforward transversal filter ahead of the detector and at the same time estimating the sampled impulse response of the channel and filter. The filter ensures that the sampled impulse response of the channel and filter are made minimum phased, without affecting the signal to noise ratio at the output of the filter. The advantage of this scheme over other schemes, is that it does not require a root finding algorithm to obtain the location of the roots and replace them by the complex conjugate of their reciprocal. The filter has been tested over a time invariant channel.

CHAPTER 2

THE HF CHANNEL

2.1 INTRODUCTION

An HF channel as a transmission medium is still of great importance even after the introduction of several kinds of transmission media such as satellites, optical fibres, coaxial cables etc. For HF channels the transmission conditions are constantly changing, thereby resulting in the received signal depending on the transmission medium. Before an attempt is made to develop an HF channel model, it is essential that ionospheric propagation is well understood. In section 2.2 the composition of the ionosphere has been discussed in some detail. Section 2.3 provides an outline or the mechanism of radio propagation through the ionosphere. In section 2.4 a study is made of the various distortion introduced by the radio channel. Section 2.5 provides the characterization of the fading signal. Section 2.6 provides a detailed simulation of the HF channel. Finally in section 2.7 the results of the simulation are shown.

2.2 THE IONOSPHERE

The ionosphere extends between 50 and 700 Kms above the earth's surface, and is composed mainly of molecules and atoms of oxygen and nitrogen [3,4,6,7]. These molecules are progressively replaced by their respective atoms as the height increases [5]. Generally the electromagnetic radiation from the sun and, to a lesser extent, cosmic rays cause these molecules and atoms to be converted into ions and free electrons. A considerable contribution is made by meteorites travelling through this atmosphere [1,5]. The ionosphere is divided into three main regions, named D, E, F regions [4-7]. The degree of ionization and the density of ions present in these layers vary with height. The free electrons act as reflectors for HF radio waves, so variations in their density will result in variations in their ability to reflect radio signals.

The height of each region varies during the day, and with the seasons. The region of the ionosphere between 50 and 90 Kms is known as the D region. In this region the electron concentration is low (10^8 - 10^{10} electrons/m^3). The critical frequency defined as the highest frequency carrier frequency of a vertically incident ray which can be reflected by the layer, is given by [5,6],

$$f_o \approx 9\sqrt{N}$$

where N is the electron density. The critical frequency is low for the D region (100 - 700KHz). The molecules concentration is high (of the order of 10^{20} molecules/m^3) and the collision frequency between molecules and electrons is high ($5 \times 10^5 - 5 \times 10^6 per\, m^3/s$). Thus, for HF radio waves, the D region acts principally as an attenuator [5]. The D region appears after sunrise and during night time. At night, in the absence of solar radiation, very little ionization takes place and the D region virtually disappears, therefore not interfering with HF radio propagation [3], but background interference has increased [3].

The next region, called the E region is considered as existing from 90 Kms to approximately 160 Kms above the earth's surface [3-6]. Moreover ionization is found at an altitude of approximately 120 Kms and at this height the electron density is of the order of 10^{11} electron/m^3, which corresponds to a critical frequency around 4MHz [3-5]. Ionization begins at sunrise and maximum density occurs near noon with the seasonal maximum occurring in summer [4]. At night time there is still some ionization, due to imperfect ion recombination and to meteoric, but it is much weaker. The critical frequency drops about an order of magnitude from its daytime value [3,5]. The number of collisions between electrons and molecules is rather large [5] so there is still significant amount of absorption but not as much as in the D region [5]. The E region is useful for propagation support for distances upto 2000 Kms. Thin ionized layers with a maximum electron density greater than that of normal E are often found between 90 and 150 Kms above the earth, these layers are called sporadic E layers because of the variability in time and space of their occurrences [1,3-6]. The sporadic E layer is responsible for creating interference, and are capable of reflecting high frequencies [3,6].

The region above about 160 Kms is known as the F region. This region has been divided into two layers known as the F1 and F2 layers. The lower layer, F1, which exists between 160 and 200 Kms, displays different variations than the upper layer, F2 [3-6]. The F1 layer generally exists during daytime at about a height of 200 Kms. The F1 layer like the E region is strongly influenced by solar radiation. It reaches a maximum ionization level about an hour after noon, and its presence is generally only obvious during the summer [4]. At night the F1 and F2 layers merge and are termed simply, the F region [3,4,6]. Absorption in this layer is small [5]. The F1 layer is not considered as providing the basis for long distance communication [3].

The F2 layer is the highest ionospheric region, and is located between 200 - 450 Kms above the earth's surface [3-5]. The electron density in this region, is typically around 10^{12} electrons/m^3, maximum value generally occurs well after noon [4]. The critical frequency is between 5 and 10 MHz at middle latitude. During night time and sometimes during the day, particularly in winter, there is only a single F layer, as the two layers merge, and the critical frequency drops to 3 to 5 MHz [6]. The F2 layer is the most useful part of the ionosphere for HF radio communications both during day and night time. The use of single hop transmission, because of the much greater height of the F layer can provide support to a distance of 4000 Kms or more [3].

Figure 2.2.1 [13], shows the electron density profile for summer noon and midnight at middle latitudes. At any other location on the earth, the profile is expected to differ from that of Figure 2.2.1.

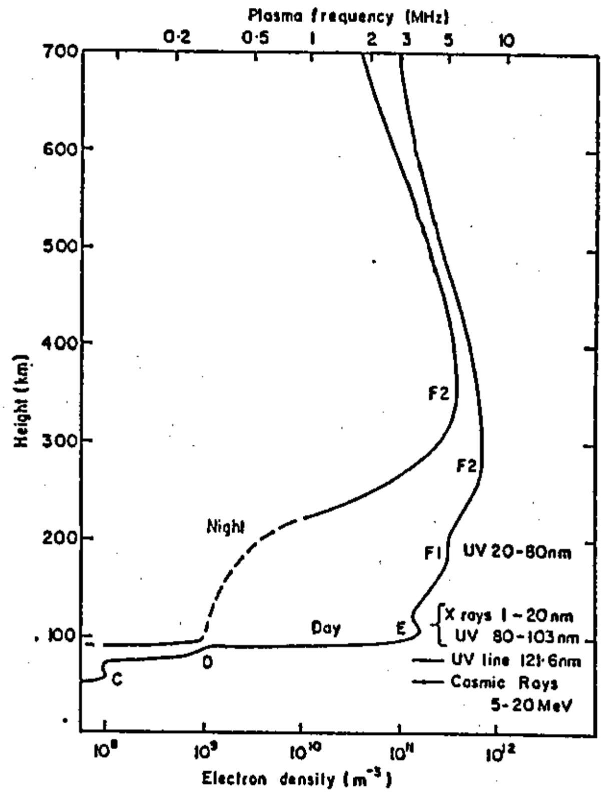


FIGURE 2.2.1 TYPICAL ELECTRON DENSITY DISTRIBUTION FOR SUMMER NOON AND MIDNIGHT CONDITIONS AT MID-LATITUDES

2.3 IONOSPHERIC RADIO PROPAGATION

The ionosphere affects the propagation of all waves, upto a frequency of about 50 MHz. Frequencies lower than approximately 30 MHz are propagated by reflection and frequencies between 30 and 50 MHz are propagated by scattering [5]. Refractive bending is the actual process by which HF radio waves are returned to earth. The refractive index, η, of the ionospheric layer changes continuously with its height. This is due to the dependence of η on the electron density of the ionised medium. The refractive index of the medium is given by [1,5],

$$\eta = \left(1 - \frac{81N}{f^2}\right)^{0.5} \tag{2.3.1}$$

where f is the frequency of the radio wave in Hz, and N is the number of free electrons per cubic metre(m^3).

A finely stratified region of the ionosphere with a constant refractive index of η_i in the i^{th} layer, is shown in Figure 2.3.1 to show more clearly the bending process. For a given angle of incidence of a radio wave meeting a reflecting layer, total internal reflection occurs when,

$$\mathrm{Sin}\,\theta_i = \left(1 - \frac{81N}{f^2}\right)^{0.5} \tag{2.3.2}$$

where θ_i is the angle of incidence of the wave measured from the normal.

The critical frequency, f_c is obtained when $\theta_i = 0$ (vertical incidence) and represents the highest reflectable frequency of the layer at this incidence, it is given by,

$$f_c = 9\sqrt{N_{max}} \tag{2.3.3}$$

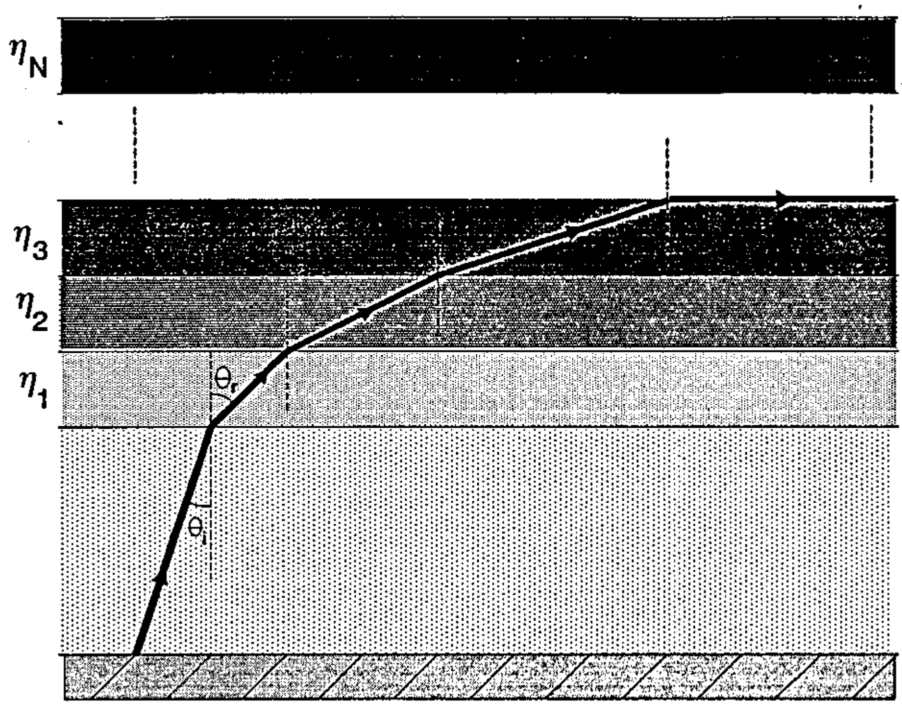


FIGURE 2.3.1 REFRACTIVE BENDING IN AN IONIZED MEDIUM

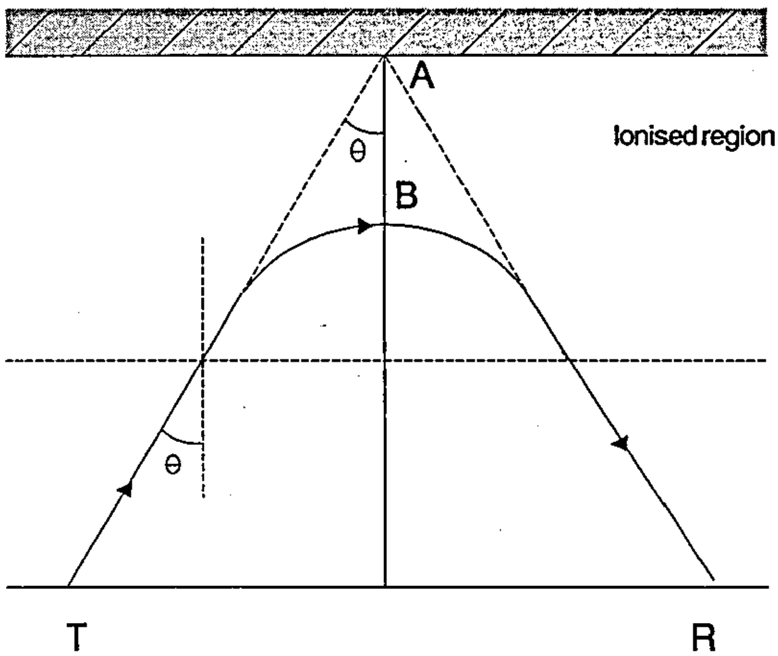


FIGURE 2.3.2 RELATION BETWEEN THE REAL AND VIRTUAL HEIGHT

where N_{max} is the maximum number of electrons per m^3 in the layer.

Higher frequencies can be reflected from this layer at other angles of incidence, but for any given angle of incidence, there is a maximum frequency at which reflection can take place and this is known as the maximum usable frequency (muf) and is related to the critical frequency, f_c, as,

$$muf = f_c \sec\theta_i \tag{2.3.4}$$

The term refraction and reflection have been used interchangeably to describe the process by which the radio wave is returned to the earth [5]. In figure 2.3.2 where a plane earth surface and a plane ionosphere is assumed, B is known as the real height, and A is called the virtual height, so that TBR is the actual ray path and TAR is the virtual ray part. Thus the actual ray path can be replaced by the virtual ray path in a medium of unit refractive index and reflected from a plane located at the virtual height. The refraction process at height B is equivalent to a mirror like reflection at height A [5].

For practical purposes, the model of the ionosphere can be reduced to the earth surface, and the E and F2 layers. According to their frequency, their angle of elevation and the critical frequencies of the layers, the waves can travel along different paths or by different ionospheric modes. Figure 2.3.3 illustrates some of the different modes of propagation.

The effect of the earth's magnetic field has been shown to split an incident wave on entering the ionized medium into two circularly polarized waves, the ordinary and extraordinary waves. The effect is known as magneto-ionic splitting. The two rays travel along different paths but they can sometimes recombine on leaving the ionized medium to give an elliptically polarized wave [1,5]. Magneto-ionic splitting is dependent on the operating frequency, the most noticeable splitting occurring at a frequency just below the muf of a layer.

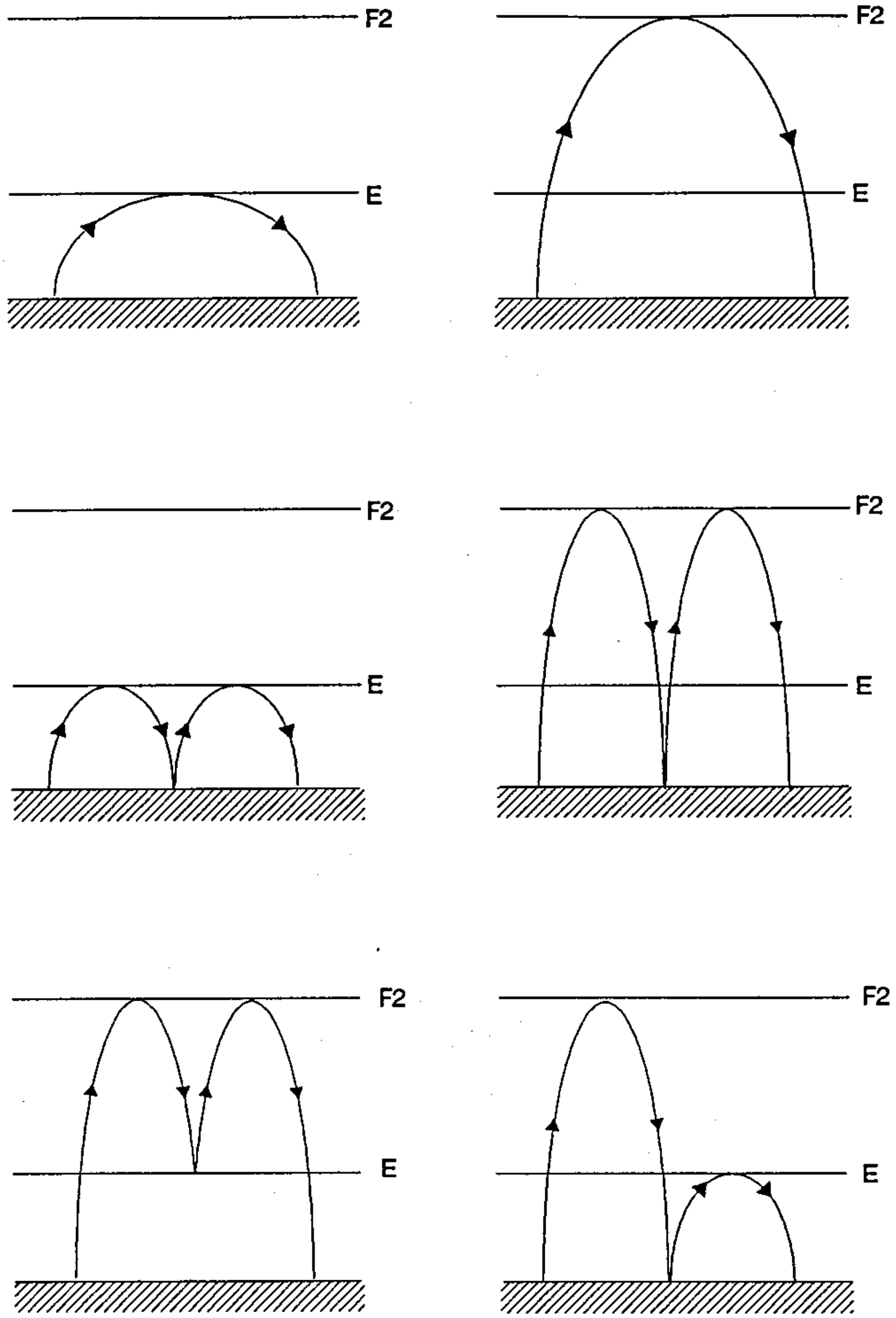


FIGURE 2.3.3 EXAMPLES OF DIFFERENT PATHS OF PROPAGATION

2.4 DISTORTION INTRODUCED BY HF CHANNELS

Over HF channels, the random variation of the ionosphere causes severe forms of distortion. In this section some of the distortions introduced are discussed.

2.4.1 MULTIPATH PROPAGATION AND TIME DISPERSION

A transmitted radio wave may be propagated to the receiver along one or more different paths, that is by multipath propagation. This is clearly illustrated in figure 2.3.3. As these paths have different lengths, the time taken by signals traversing these paths are different, hence when a short pulse of RF energy is transmitted, the received signal will have a profile such as that in figure 2.4.1.

In figure 2.4.1 the time between the reception of the first and last pulse is known as the time spread or time dispersion of the received signal. Different modes of propagation have different group delays and this difference in group delays causes time dispersion. When the reciprocal of the data transmission rate is comparable with the relative multipath delay, the signals received over the different modes overlap each other giving rise to intersymbol interference. The time dispersion is usually not greater than 3ms.

2.4.2 FREQUENCY DISPERSION

A shift in the frequency over a single propagation path can arise due to the movement of the ionospheric reflecting layer, and also the time variation of the electron density [2,10,11]. The Doppler shifts are relatively small at night. The Doppler shift is typically 0.01 to 1 Hz. However when the ionosphere is disturbed, large values of

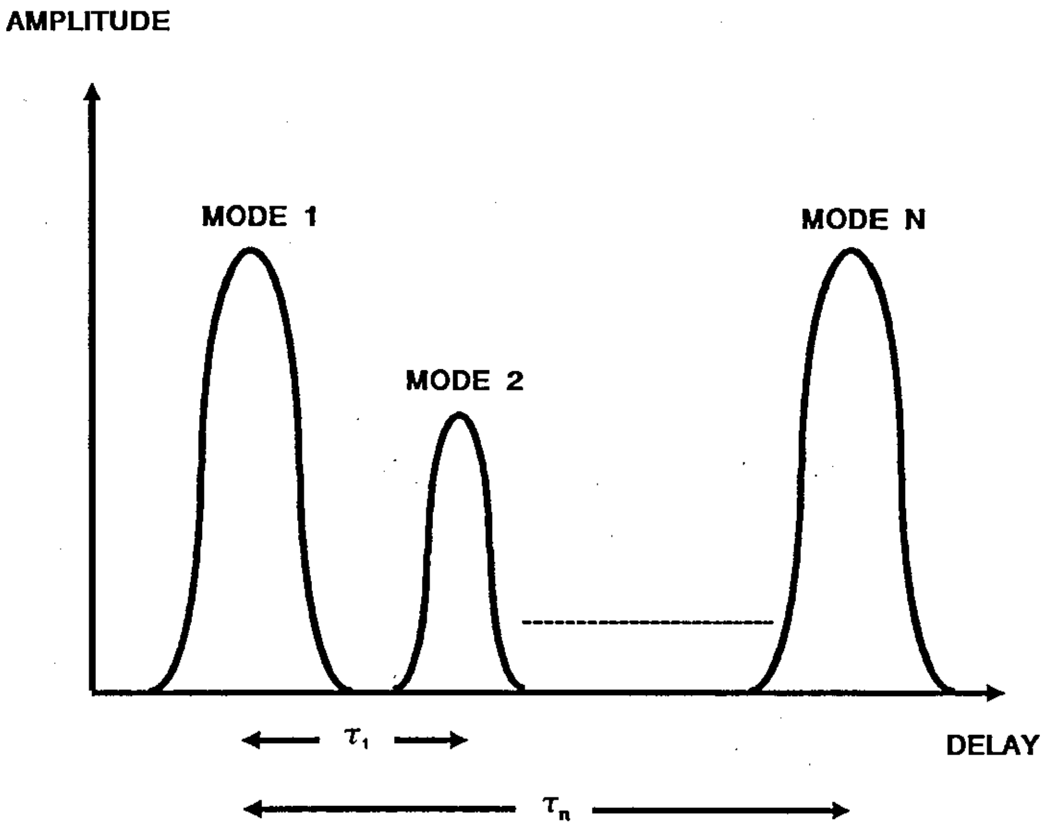


FIGURE 2.4.1 TYPICAL RESPONSE OF MULTIPATH CHANNELS

the order of 5 - 10 Hz are possible [2]. The shifts are greater for F mode propagation as compared to the E mode. The frequency shift on one propagation path is different from that on another path. This causes the received signal to be spread in frequency [2,10,11].

2.4.3 FADING

Sky-wave signals characteristically fluctuate in amplitude and phase. These random variations of the signal strength are referred to as fading and are a form of distortion experienced over HF channels. Fading can be classified as follows [1-2].

2.4.3.1 INTERFERENCE OR SELECTIVE FADING

An HF radio wave incident on the ionosphere, is received as several different rays, after having travelled over paths of different lengths, as a result of irregularities in the ionosphere (chapter 2.3). The received signal is the vector sum of the individual signals at the receiver. Movement of ionospheric regularities result in variations in the relative phases of the signals, thereby causing fluctuations in the resultant amplitude of the received signal. This is known as interference fading. When the individual signals are in phase, the amplitude of the received signal is a maximum.

As fading is frequency dependent, sidebands in a modulated wave fade differently due to the presence of a large number of frequency components. This distortion introduced in the modulated envelope is referred to as selective fading.

2.4.3.2 FLUTTER FADING

Flutter fading is caused as a result of severe ionospheric disturbances over the HF radio link. This is associated with the F region [1-2]. In this the variation in signal strength takes the form of a fast rhythmic beat. The fading period is very small (10 - 100ms) [1-2].

2.4.3.3 POLARIZATION FADING

The effect of the earth's magnetic field on the ionosphere, is to split radio waves into ordinary and extraordinary waves (section 2.3). The phase and amplitude of these waves change the polarization of the received signal to be elliptically polarized. The axes of the polarization ellipse rotate as a result of random variations in the propagation conditions. This results in polarization fading, which is only effective when both waves are present in approximately equal proportions [1-2].

2.4.3.4 ABSORPTION FADING

Absorption fading is caused by the variation in the absorption characteristics of the ionosphere with time. It is usually greatest at sunrise and sunset [2]. The fading period is of the order of one hour [2].

2.4.3.5 SKIP FADING

For a specific distance between two points, the highest frequency to be reflected is called the maximum usable frequency (muf). Skip fading is caused by the continuous variations of the muf [2], this could result in the operating frequency being greater than the muf at a particular instant and so penetrating the reflecting layer for a short period of time. During this time, communication is interrupted at the receiver. Skip fading can be avoided by working well below the muf [2].

2.5 CHARACTERIZATION OF FADING MULTIPATH CHANNELS

It is reasonable to characterize the time variant multipath channel statistically. The effects of the channel are examined, on a transmitted signal that is represented as

$$s(t) = \Re\left\{u(t)e^{j2\pi f_c t}\right\} \tag{2.5.1}$$

It is assumed that there are multiple propagation paths. Associated with each path is a propagation delay and an attenuation factor. Both the propagation delays and attenuation factors are time variant. Due to this multipath propagation, the received bandpass signal may be expressed in the form [12],

$$x(t) = \sum_n \alpha_n(t) s[t - \tau_n(t)] \tag{2.5.2}$$

where $\alpha_n(t)$ is the attenuation factor for the signal received on the nth path and $\tau_n(t)$ is the propagation delay for the nth path. Substituting s(t) from 2.5.1 into 2.5.2, results in,

$$x(t) = \Re\left(\left\{\sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} u[t - \tau_n(t)]\right\} e^{j2\pi f_c t}\right) \tag{2.5.3}$$

The equivalent lowpass received signal is

$$r(t) = \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} u[t - \tau_n(t)] \tag{2.5.4}$$

As r(t) is the response of an equivalent low pass channel to the equivalent lowpass signal u(t), the equivalent low pass channel is described by the time variant impulse response,

$$c(\tau;t) = \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} \delta[t - \tau_n(t)] \qquad (2.5.5)$$

For some channels it is more appropriate to consider the received signal as consisting of a continuum of multipath components. In such a case, the received signal x(t) is expressed in the integral form [12],

$$x(t) = \int_{-\infty}^{\infty} \alpha(\tau;t) s(t-\tau) d\tau \qquad (2.5.6)$$

where $\alpha(\tau, t)$ represents the attenuation of the signal component at delay τ and at time instant t

Equation 2.5.3 can be replaced by,

$$x(t) = \Re\left\{\left[\int_{-\infty}^{\infty} \alpha(\tau, t) e^{-j2\pi f_c \tau(t)} u(t-\tau) d\tau\right] e^{j2\pi f_c t}\right\} \qquad (2.5.7)$$

the equivalent lowpass received signal is,

$$r(t) = \int_{-\infty}^{\infty} \alpha(\tau, t) e^{-j2\pi f_c \tau(t)} u(t-\tau) d\tau \qquad (2.5.8)$$

In equation 2.5.8, the received signal r(t), is given by the convolution of u(t) with an equivalent lowpass time variant impulse response $c(\tau, t)$,

$$c(\tau, t) = \alpha(\tau, t) e^{-j2\pi f_c \tau} \qquad (2.5.9)$$

If an unmodulated carrier at frequency f_c is transmitted. Then u(t) = 1 for all t, and the received signal for the discrete case (equation 2.5.4), reduces to [12],

$$r(t) = \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)}$$

$$= \sum_n \alpha_n(t) e^{-j\theta_n(t)} \tag{2.5.10}$$

where $\theta_n(t) = 2\pi f_c \tau_n(t)$ for the continuous case the received signal reduces to,

$$r(t) = \int_{-\infty}^{\infty} \alpha(\tau, t) e^{-j2\pi f_c \tau(t)} d\tau \tag{2.5.11}$$

Thus the received signal consists of the sum of a number of time-variant vectors having amplitude $\alpha_n(t)$ and phase $\theta_n(t)$. Large dynamic changes in the medium are required for $\alpha_n(t)$ to change sufficiently to cause a significant change in the received signal. On the other hand $\theta_n(t)$, will change by 2π whenever τ_n changes by $1/f_c$. But $1/f_c$ is a small quantity and, hence, θ_n can change by 2π radians with relatively small motions of the medium [12]. It is expected that delays $\tau_n(t)$ and therefore $\theta_n(t)$, vary randomly. This implies that the received signal can be modelled as a random process.

When there are a large number of paths, each varying randomly and independently of each other the central limit theorem can be applied. That is, r(t) can be modelled as a complex Gaussian random process. This means that $c(\tau;t)$ is a complex valued Gaussian random process.

The multipath propagation model results in signal fading, the phenomenon is primarily a result of the time variations in the phases $\{\theta_n(t)\}$. That is, the randomly time invariant phases $\{\theta_n(t)\}$ associated with the vectors $\{\alpha_n e^{-j\theta_n}\}$ at times result in the vectors adding destructively. At other times the vectors $\{\alpha_n e^{-j\theta_n}\}$ add constructively, so that the received signal is large.

If the impulse response $c(\tau;t)$ is modelled as a zero mean complex valued Gaussian process, the envelope $c(\tau;t)$ at any instant t is Rayleigh distributed [12].

A single non-fading component may also be received, in this case the envelope has a rice distribution and the channel can no longer be modelled as having zero mean. In this case the envelope $c(\tau;t)$ has a Rice distribution and the channel is said to be a Ricean fading channel, Rayleigh distributed envelope fading has been observed often on HF channels and as a result is accepted as a model that best describes most HF channels [12].

The Rayleigh model, describes a continuous random variable, produced from two independent Gaussian random variables X and Y

$$m_x = m_y = 0 \qquad \sigma_x^2 = \sigma_y^2 = \sigma^2$$

where m and σ^2 are the mean and variance of X and Y.

Let

$$R = \sqrt{X^2 + Y^2}$$

Then R has a Rayleigh distribution and has a probability density function given by [13].

$$P_R(r) = \begin{cases} \dfrac{r}{\sigma^2} e^{\frac{-r^2}{2\sigma^2}} & 0 \le r \le \infty \\ 0 & r < 0 \end{cases} \tag{2.5.12}$$

Since R cannot be negative, by definition, it must have a nonzero mean value, even though X and Y have nonzero means. Figure 2.5.1 shows the pdf of a Rayleigh distribution.

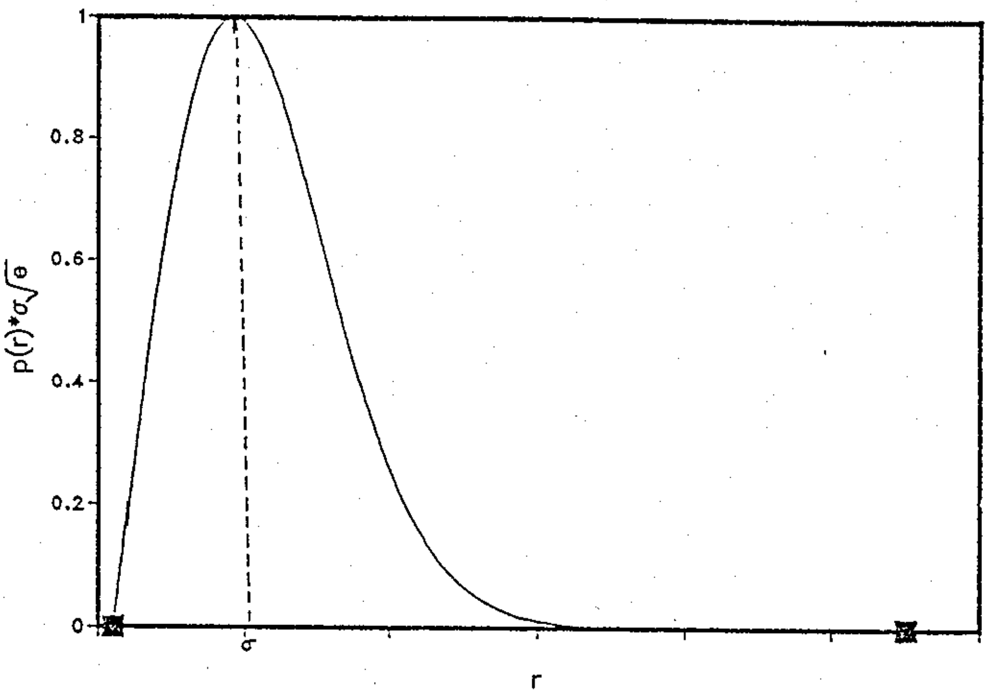


Fig. 2.5.1 – Rayleigh Probability Density Function

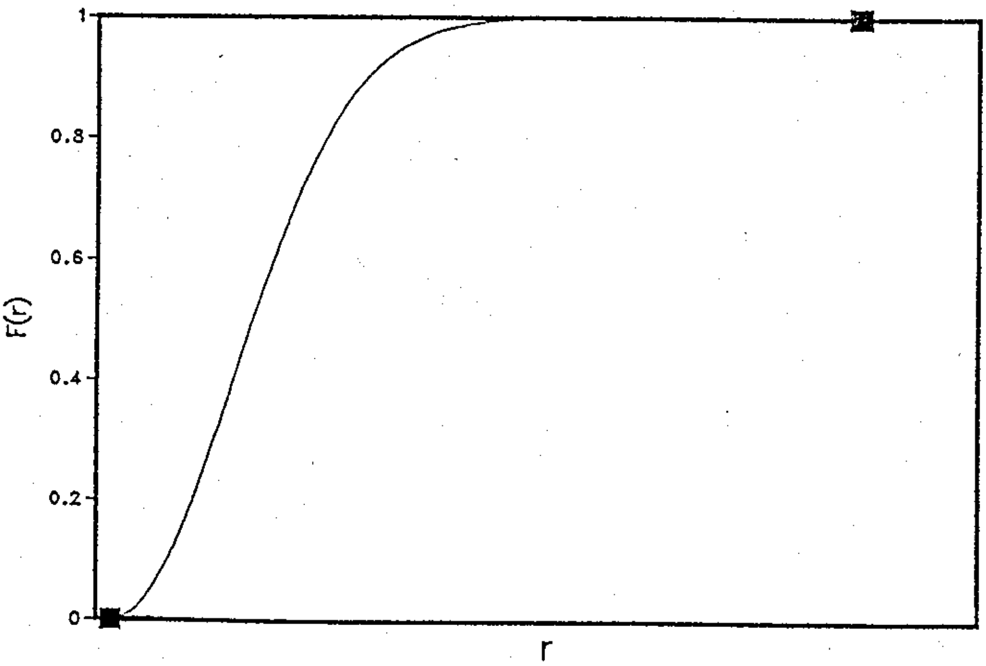


Fig. 2.5.2 – Rayleigh Cumulative Distribution Function

The cdf, of the Rayleigh distribution is given by,

$$f(r) = \int_0^r \frac{u}{\sigma^2} e^{\frac{-u^2}{2\sigma^2}} du$$

$$= 1 - e^{\frac{-r^2}{2\sigma^2}} \qquad (2.5.13)$$

Figure 2.5.2 shows the plot of the cumulative distribution function. The mean value of R is,

$$\bar{r} = \int_0^\infty r f(r) dr = \sqrt{\frac{\pi}{2}} \sigma \qquad (2.5.14)$$

The second moment of R is given by

$$E(R^2) = E(X^2 + Y^2) = E(x^2) + E(Y^2) \qquad (2.5.15)$$

where

$$E(X^2) = \sigma^2 + m_x^2$$

$$E(Y^2) = \sigma^2 + m_y^2 \qquad (2.5.16)$$

As m_x and m_y are zero, equation 2.5.15 becomes,

$$E(R^2) = \overline{r^2} = 2\sigma^2 \qquad (2.5.17)$$

The variance of R is,

$$\sigma_r^2 = E(R^2) - (E(R))^2$$
$$= \overline{r^2} - (\overline{r})^2 \tag{2.5.18}$$

from equations 2.5.14, 2.5.17 and 2.5.18,

$$\sigma_r^2 = \left(2 - \frac{\pi}{2}\right)\sigma^2 \tag{2.5.19}$$

2.6 SIMULATION OF AN HF CHANNEL

Their are two methods available for testing the performance of a transmission system for use on HF radio channels. Firstly, the system can be tested out over an actual channel. This method may be costly to implement and very time consuming, since any change required for the adjustments and/or improvements of the equipment could well involve alterations to the hardware. Also when several systems are to be compared, they have to be tested simultaneously because the same propagation and channel conditions are difficult to obtain at different times due to the random variations in time and frequency of the HF channel. To avoid such problems, the system is tested over a channel simulator. HF channel simulators, whether in hardware or software, are versatile in that a variety of channel conditions can simply be produced, and, if desired, these can be repeated any number of times with consistent results. Also the type and amount of distortion can be controlled so that any particular weakness of the system can be identified and studied in isolation. A channel simulator is accurate, and can simulate a large range of channel conditions. Also the performance of several systems under the same channel conditions can be compared, tests can also be repeated several times giving the same results.

Most of the simulator designs, are based on the tapped delay line to represent the HF channel, which has been proposed by Watterson et Al in reference 16. This model has been adopted unanimously by the International Radio Consultative committee (CCIR) of the International Telecommunications union (ITU). This investigation uses the tapped delay model as shown in figure 2.6.1.

The input signal is fed to an adjustable tapped delay line. Their are as many taps as their are modes of propagation. Rayleigh fading is then imposed on the delayed signals by multiplying each signal by a suitable tap gain function $Q_h(t)$. The resulting delayed and modulated signals from the received signal is the sum of the output of the delay line and an additive noise term $V_n(t)$ which represents the noise on HF channels.

In HF links, the most common additive noise is atmospheric noise, which is Gaussian. Hence a good tolerance to additive white Gaussian noise almost certainly means a good tolerance to atmospheric noise [17].

For one propagation path, the Rayleigh fading introduced by the skywave is modelled as shown in figure 2.6.2, where $q_1(t)$ and $q_2(t)$ are two random processes which must have the following properties [17]:

1) Each random process must be Gaussian with zero mean and the same variance

2) The random processes $q_1(t)$ and $q_2(t)$ must be statistically independent

3) the power spectrum of each random process must be Gaussian in shape, having the same rms frequency, f_{rms}.

The power spectra of $q_1(t)$ and $q_2(t)$ are given by,

$$Q_1(f)^2 = Q_2(t)^2 = \exp\left(-\frac{f^2}{2f_{rms}^2}\right) \tag{2.6.1}$$

and are shown in figure 2.6.3

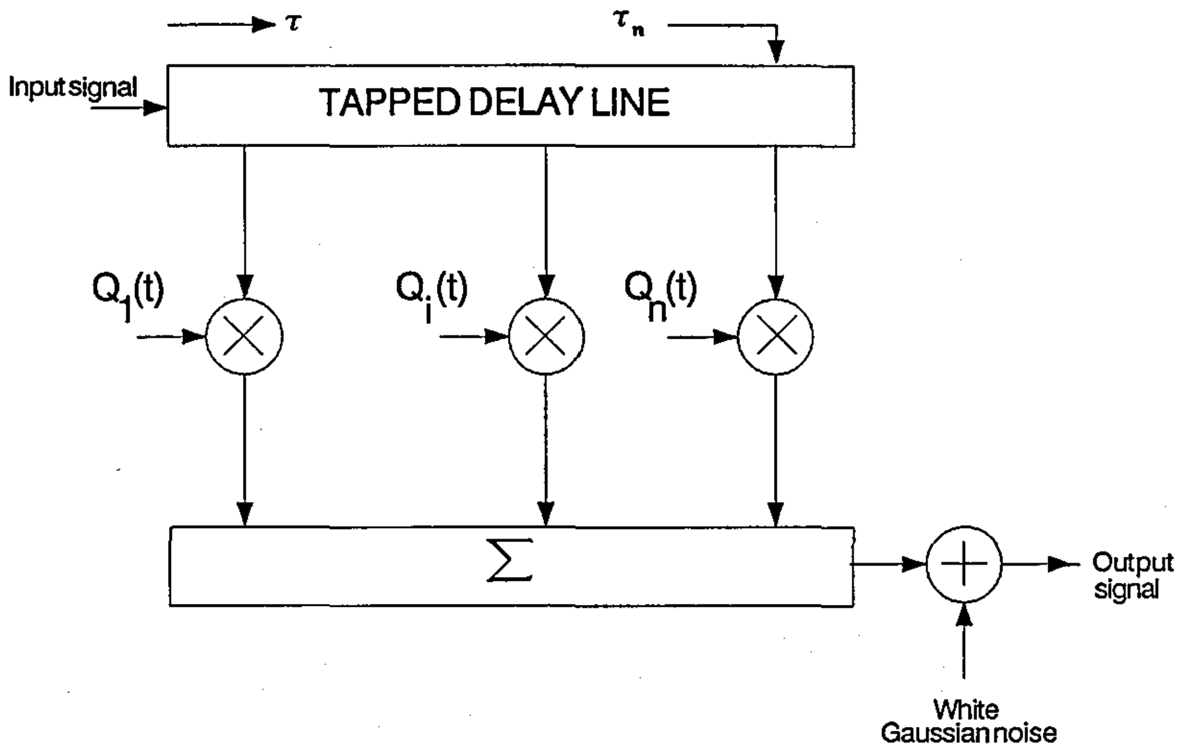


FIGURE 2.6.1 BLOCK DIAGRAM OF HF IONOSPHERIC CHANNEL MODEL

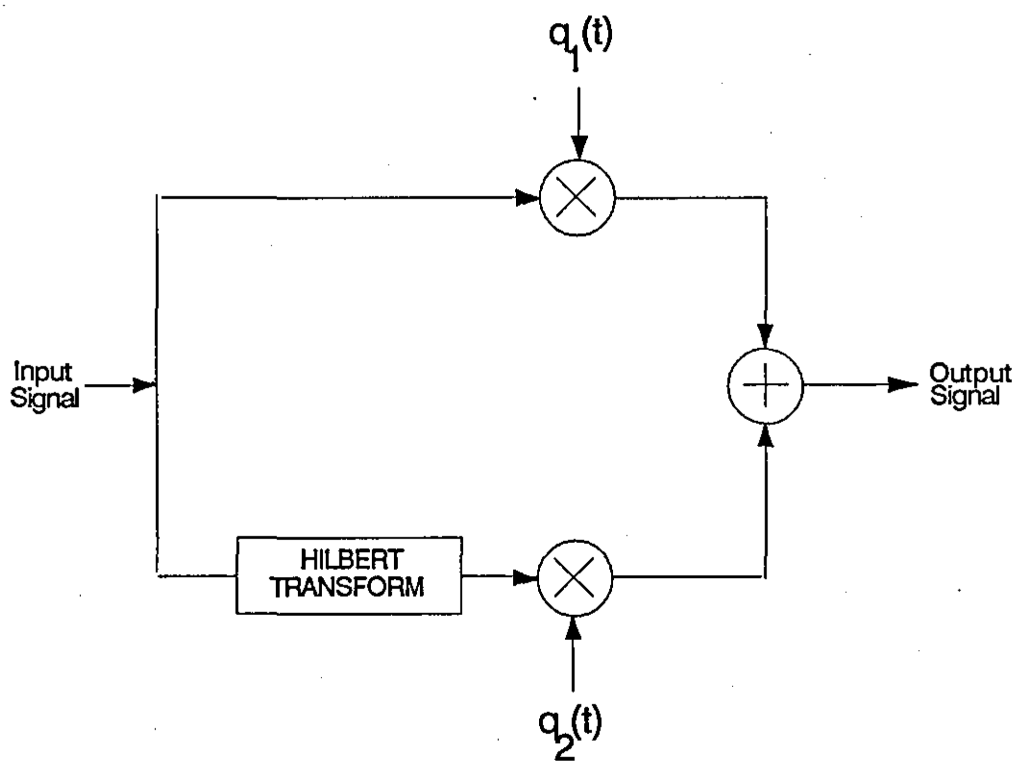


FIGURE 2.6.2 RAYLEIGH FADING INTRODUCED BY ONE SKYWAVE

The frequency spread f_{sp}, introduced by $q_1(t)$ and $q_2(t)$ into an unmodulated carrier is defined as the width of the power spectrum [17] and is given by

$$f_{sp} = 2f_{rms} \tag{2.6.2}$$

The rms frequency is related to the fading rate f_e, which is defined (for a single carrier) as the number of downward crossings per second of the envelope through the median value [8] according to the equation [8,17],

$$f_{rms} = \frac{f_e}{1.475} \tag{2.6.3}$$

from equations 2.6.2 and 2.6.3, f_{sp} is related to f_e by,

$$f_{sp} = 1.356 f_e \tag{2.6.4}$$

Reference 19 has recommended testing of HF modems on HF channels classified as good, moderate, poor and flutter conditions. Table 2.6.1 lists the parameters for these four channel conditions. The delay spread is usually upto 5ms. The HF channel model chosen was a 2 sky-wave channel with a frequency spread of 2Hz and a transmission delay of 1ms between the two sky-waves. These parameters represent a poor channel from table 2.6.1 [19].

The random process $q_1(t)$ is generated by filtering a zero mean white Gaussian noise waveform $v_1(t)$ as shown in figure 2.6.4. $q_2(t)$ is similarly generated but using a different Gaussian noise waveform $v_2(t)$, which is independent of 1. The power spectra of $q_1(t)$ and $q_2(t)$ is Gaussian, hence the filter should also have a Gaussian frequency response matching the power spectrum of $q_1(t)$ and $q_2(t)$. The power spectrum of each filter is given by equation 2.6.1, and is plotted in figure 2.6.3. The frequency response of the filter is given by

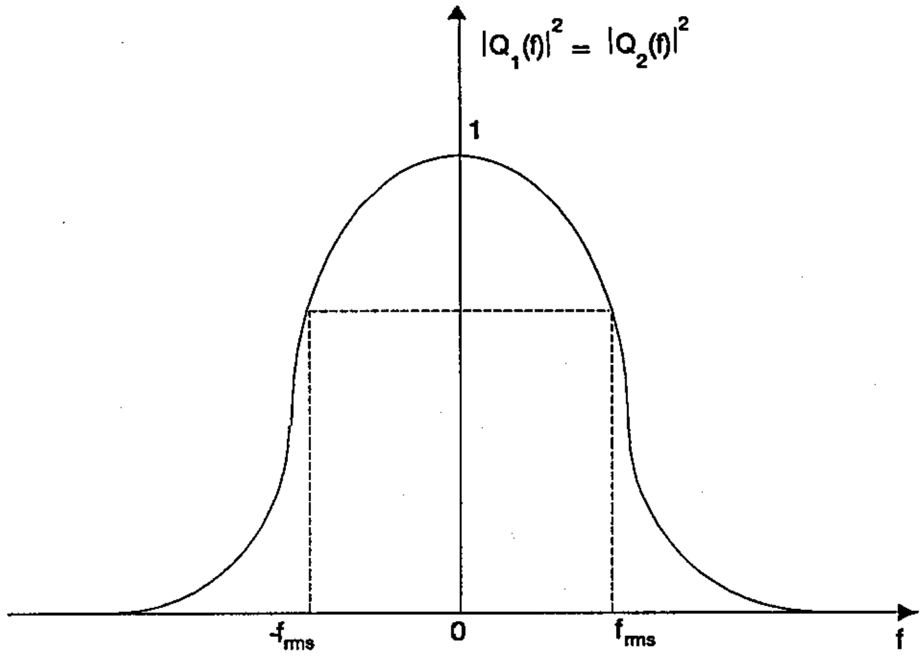


FIGURE 2.6.3 POWER SPECTRUM OF $q_1(t)$ AND $q_2(t)$

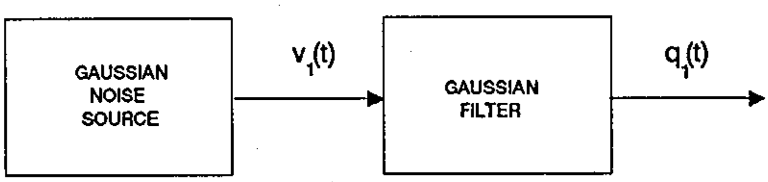


FIGURE 2.6.4 METHOD OF GENERATING $q_1(t)$

TABLE 2.6.1 HF CHANNEL PARAMETERS

Conditions	Differential time delay	Frequency spread
Good	0.5ms	0.1 Hz
Moderate	1ms	0.5 Hz
Poor	2ms	1 Hz
Flutter	0.5ms	10 Hz

TABLE 2.6.2 CHARACTERISTICS OF THE FIFTH ORDER BESSEL FILTER

Order of the filter, L	5
Frequency spread, f_{sp} (Hz)	2
Cut-off frequency, f_c (Hz)	1.1774
Filter poles in the s-plane p'_1 p'_2 , p'_3 p'_4 , p'_5	 $-11.1139 + j\,0$ $-10.2155 \pm j\,5.3110$ $-7.0847 \pm j\,10.8831$
Filter poles in the z-plane q_1 q_2, q_3 q_4, q_5	 $0.8948 + j\,0$ $0.9016 \pm j\,0.0479$ $-0.9261 \pm j\,0.1012$

$$F(f) = \exp\left(-\frac{f^2}{4f_{rms}^2}\right) \qquad (2.6.5)$$

A Bessel filter is used in the computer simulation tests to realize the required Gaussian spectral shaping of $q_h(t)$. The frequency and impulse response of the Bessel filter approaches Gaussian, when the order of the filter is sufficiently large [23]. In the simulation model there are 2 skywaves, and, therefore, it requires 4 random processes $q_h(t)$ for h=1,2...4. Each of the random processes are generated using a 5^{th} order Bessel filter [24]. Each of the random processes are generated such that they are statistically independent Gaussian random variables. The filter implemented is shown in figure 2.6.5 [26-30], and its tap values are given in table 2.6.2. The derivation of the tap values is given in Appendix A.

In a digital implementation of the HF model it is neither possible nor necessary to represent the fading signal $q_h(t)$ as continuous. Each of these signals must be represented as the corresponding sequence of discrete samples in time. From Nyquist's sampling theorem, the minimum sampling rate required in order to represent $q_h(t)$ is twice the highest frequency component of $q_h(t)$. As the fading signals have Gaussian spectra, they contain all frequencies. However, the model has an rms bandwidth of 1Hz, it can be adequately represented at a sampling frequency of 10 samples/second without any significant aliasing occurring. For testing a 2400 baud digital data modem, it is necessary that the channel samples are obtained at 4800 Hz. This means that $q_h(t)$ must be sampled at 4800Hz. Unfortunately, at this rate the filter poles are very close to the unit circle in the z plane, and to obtain the desired filter characteristics, these poles must be specified to a very high degree of accuracy, otherwise instability of the filter can occur [25]. This problem can be overcome by sampling the filters at a lower frequency and then interpolating between samples in order to obtain the required sampling rate. $q_h(t)$ has been sampled at 100Hz, high enough to satisfy the Nyquist sampling criterion for $q_h(t)$, yet limiting the degree of interpolation used and the need to have pole locations at an adequate distance from

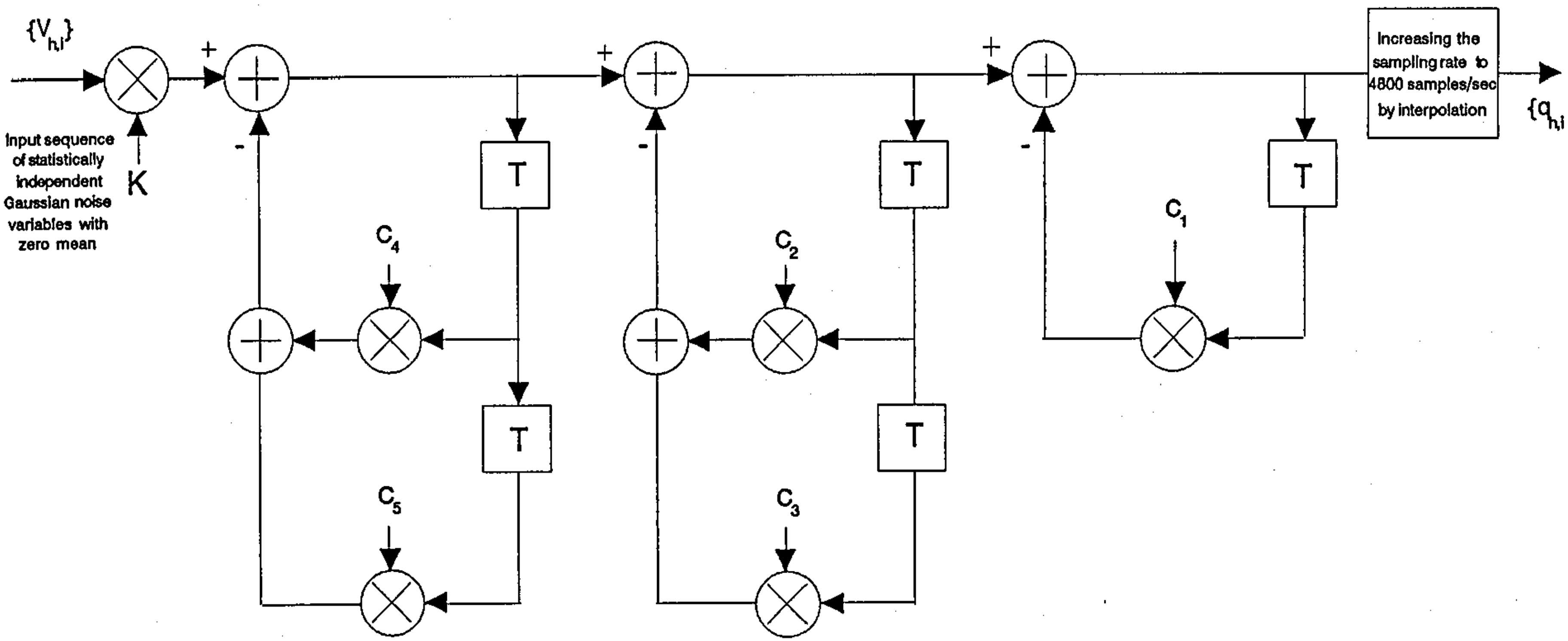


FIGURE 2.6.5 BLOCK DIAGRAM OF THE FIFTH ORDER BESSEL FILTER USED IN GENERATING THE FADING SEQUENCE

the unit circle. The samples were interpolated using linear interpolation (figure 2.6.6). Linear interpolation, is considered to be accurate at the sampling rate used [26]. The block diagram of the 2 skywaves HF channel model is given in figure 2.6.7.

2.7 COMPUTER SIMULATION

The results of the tests carried out on the simulated channel are summarized in figure 2.7.1 and table 2.7.1. Figure 2.7.1 compares the frequency response of the 5 pole Bessel filter used to generate the Gaussian random sequence with the theoretical Gaussian response given by equation 2.6.5. As can be seen, the simulated response agrees well with its theoretical response in the frequency band of interest.

Table 2.7.1, provides the mean value and variance of the sequences q_h for h=1, 2, ..., 4, for 15 different values of seed integer used by the random number generator. The fading sequences should have a characteristic, such that the mean value of the sequence is zero and that the variance is a 1/4. From table 2.7.1 it can be observed that this holds for the various sequences over the various seed integers tested.

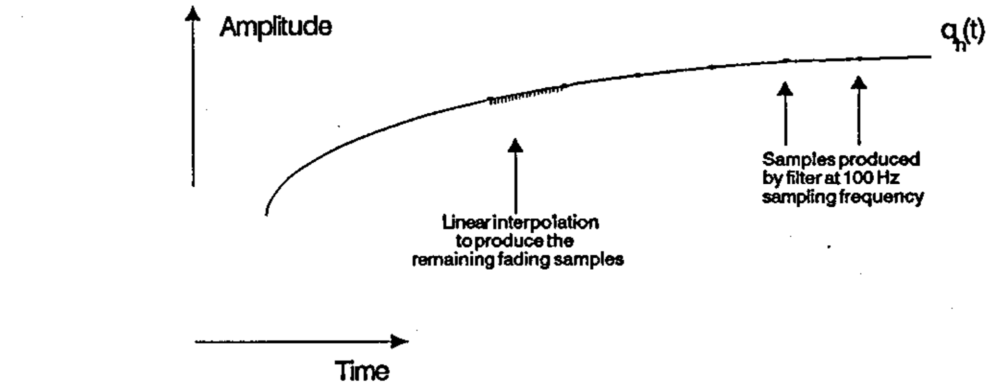


FIGURE 2.6.6 THE LINEAR INTERPOLATION PROCESS

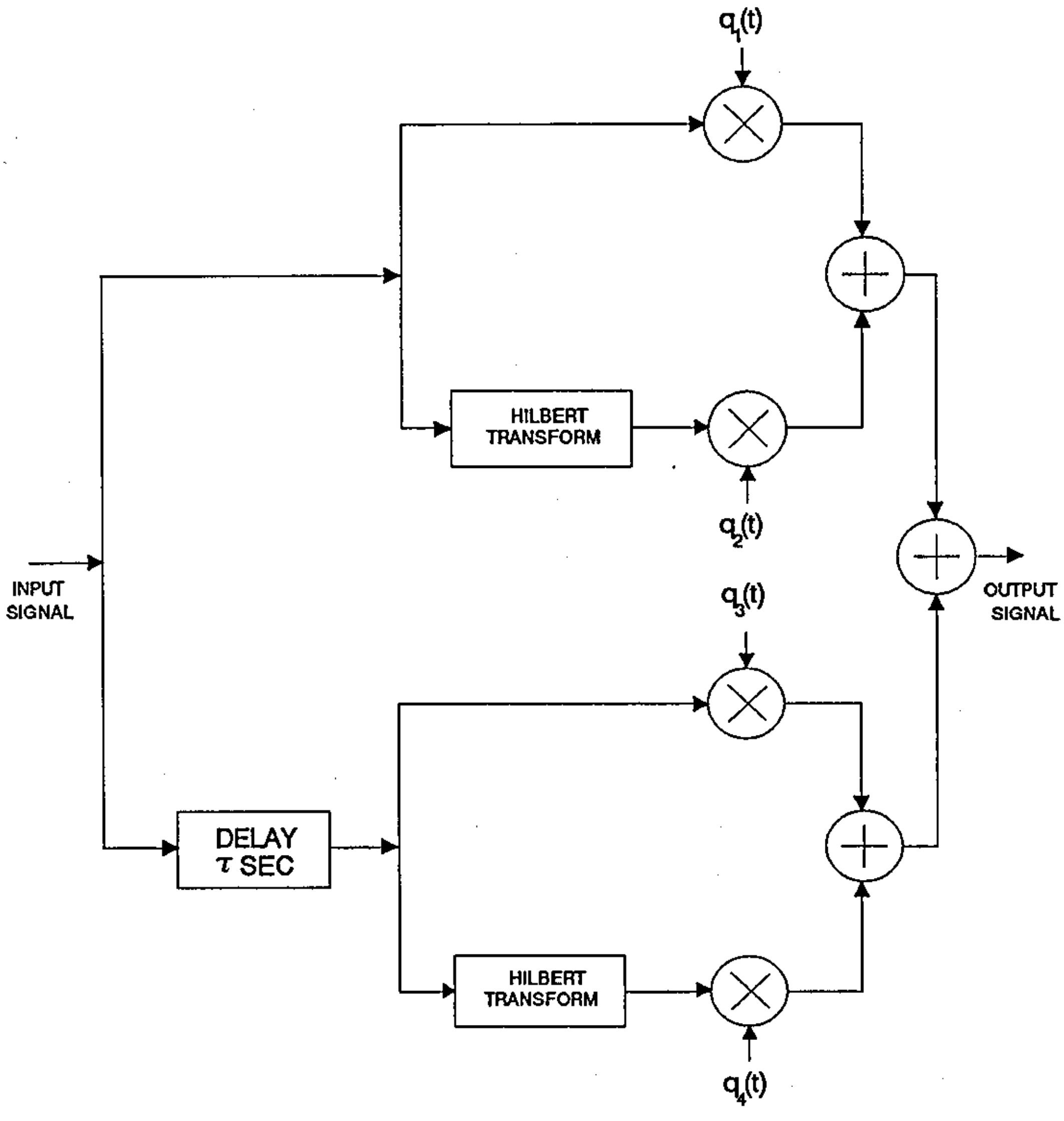


FIGURE 2.6.7 MODEL OF A TWO SKYWAVE HF RADIO LINK

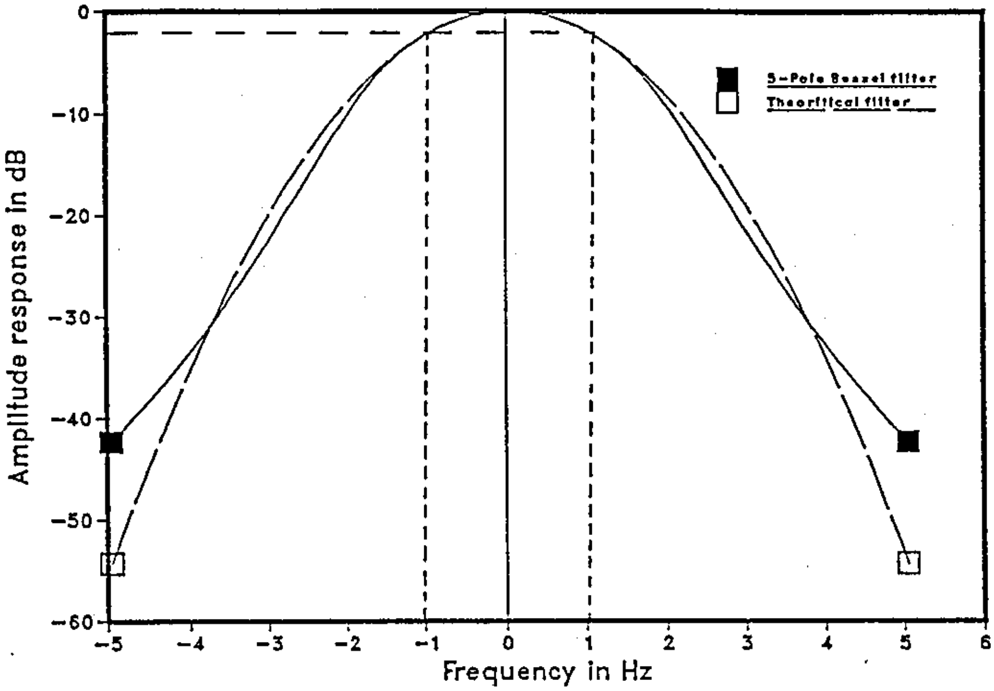


Fig. 2.7.1 – Frequency Response of Bessel Filter

TABLE 2.7.1 MEASURED CHARACTERISTIC OF THE FADING SEQUENCES USED TO MODEL A TWO SKYWAVES CHANNEL

Seed Integer	Mean value of q_h	Variance of q_h	Seed Integer	Mean value of q_h	Variance of q_h
10	-0.00365	0.21984	160	-0.10941	0.21976
	-0.04985	0.21712		0.03772	0.23891
	0.04806	0.19244		-0.00385	0.18136
	-0.03031	0.36125		0.08599	0.32926
15	-0.02403	0.18915	180	0.07719	0.20843
	-0.06007	0.24716		-0.04242	0.22257
	-0.00349	0.21467		-0.02769	0.24118
	-0.04328	0.32726		0.03324	0.22898
20	0.01910	0.26671	200	-0.01186	0.25722
	-0.03139	0.30948		-0.02712	0.26637
	-0.08034	0.23550		0.09060	0.31091
	-0.02148	0.27302		0.02133	0.21132
50	-0.02836	0.20762	220	-0.13397	0.32427
	-0.02034	0.20953		-0.04939	0.26870
	-0.04000	0.28467		0.01809	0.24870
	0.11585	0.22397		0.03406	0.26120
60	-0.04863	0.25649	300	0.04449	0.26363
	0.02514	0.26629		0.00642	0.29897
	0.07035	0.27238		-0.02668	0.24873
	-0.20028	0.30698		0.08960	0.27015
90	-0.02868	0.22320	400	0.04381	0.21265
	0.06335	0.20998		0.05576	0.20470
	0.01042	0.20734		-0.01458	0.24343
	0.07190	0.26982		0.02270	0.24933
120	0.10347	0.30076	500	-0.03408	0.29249
	0.13155	0.26585		0.09173	0.27020
	-0.00654	0.26051		0.00344	0.21935
	-0.09255	0.24363		0.00549	0.29339
140	0.00265	0.24242			
	-0.06944	0.21017			
	-0.12941	0.22474			
	0.03545	0.14814			

CHAPTER 3

MODEL OF THE DATA TRANSMISSION SYSTEM

3.1 INTRODUCTION

The information to be transmitted over a voice frequency channel is originally in the form of a baseband signal. The baseband signal cannot itself be transmitted satisfactorily over this type of channel because a significant fraction of the signal power will usually be lost in transmission. Also the received signal will normally be so severely distorted as to make satisfactory detection impossible [2]. The original baseband signal modulates a sinusoidal carrier. The frequency of the carrier is such that the spectrum of the transmitted data signal is placed in the available frequency band [1].

A digital communication system consists of a transmitter, transmission path and a receiver. The signal waveform s(t), at the input to the transmitter, carries the information to be transmitted. The transmission path is modified by the HF radio link. The receiver then reproduces the transmitted information from the distorted received signal at the input to the receiver. Data transmission systems can be classified as, serial and parallel.

In a serial data transmission system, the transmitted signal comprises a sequential stream of data elements whose frequency spectrum occupies the whole of the available frequency band. In a parallel data transmission system, two or more sequential streams of signal elements are sent simultaneously, and the spectrum of an individual data stream occupies only a part of the available bandwidth [1]. A serial transmission system is less complex than a parallel transmission system. In applications where a relatively high transmission rate is required over a given channel, a synchronous serial system is most commonly used system [2]. This model is assumed in the investigation.

3.2 DATA TRANSMISSION OVER A MODEL OF AN HF CHANNEL USING QAM

Figure 3.2.1 shows the model of the data transmission system which employs the HF radio channel as the transmission path. The signal at the input to the system is a sequence of signal elements $\sum_i s_i \delta(t - iT)$, where s_i is a complex valued data signal,

$$s_i = s_{0,i} + j s_{1,i} \tag{3.2.1}$$

where $j = \sqrt{-1}$ and $\{s_{0,i}\}$ and $\{s_{1,i}\}$ are statistically independent and equally likely to have any of their possible values $\pm 1 \pm j$.

Each of the two lowpass filter A' in the transmitter have a real valued impulse response $a'(t)$ and their function is to shape the transmitted signal spectrum so that it approximately fits the voice frequency band of the HF channel. The transmitted signal is then modulated using QAM with a carrier frequency of f_c. The QAM signal is then passed through an HF radio link. The radio transmitter uses single sideband modulation (SSB) to shift the voiceband spectrum back to the HF band, whereas the radio receiver linearly demodulates the received signal to return its spectrum back to the voiceband. It is assumed that the only form of noise present is white Gaussian noise n(t) which is added to the signal. The additive white Gaussian noise has zero mean and two sided power spectral density of 1/2 N_0. The bandpass filter C removes the noise frequencies outside the bandwidth of the data signal without unduly distorting it. The noisy, distorted and Rayleigh faded signal is then coherently

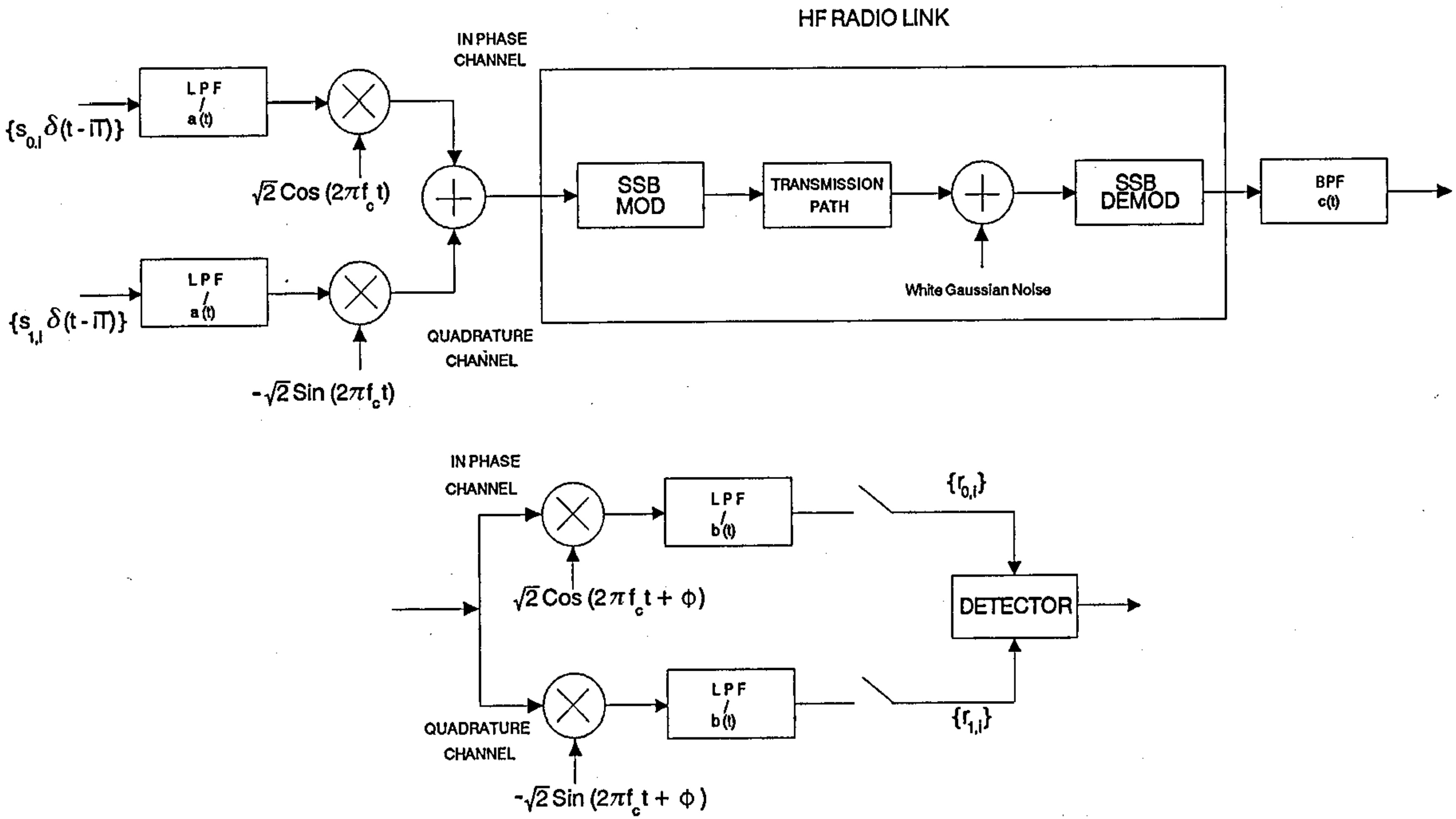


FIGURE 3.2.1 MODEL OF THE DATA TRANSMISSION SYSTEM OVER AN HF RADIO LINK

demodulated, the reference carrier being in synchronism with the average instantaneous carrier frequency of the received signal, thus eliminating any frequency offset in the received QAM signal. The output of the coherent demodulator is then low pass filtered by filter B, which removes high frequency components. The filtered output is then passed through a detector. The modulation and demodulation processes are linear, the only distortion introduced into the signal is due to the radio equipment filters and the HF channel.

In figure 3.2.1, the inphase and quadrature channels are real valued. When a QAM signal is transmitted over an equivalent linear baseband channel, an equivalent model of the data transmission system is shown in figure 3.2.2 [3-6].

The signals at the output of the two lowpass filters in the transmitter of fig 3.2.1, are

$$\sum_i s_{0,i} a'(t-iT) \& \sum_i s_{1,i} a'(t-iT) \tag{3.2.2}$$

and the signal $x_2(t)$ at the output of the adder is a real valued waveform and is given by

$$x_2(t) = \sqrt{2}\sum_i s_{0,i} a'(t-iT)\cos(2\pi f_c t) - \sqrt{2}\sum_i s_{1,i} a'(t-iT)\cos(2\pi f_c t) \tag{3.2.3}$$

The factor $\sqrt{2}$ in equation 3.2.3 ensures that the average power spectral level is unity for each of the two signals, $\sqrt{2}\cos(2\pi f_c t)$ and $-\sqrt{2}\sin(2\pi f_c t)$, when transmitted over an infinite period [3]. Therefore, the modulation process introduces no change in the signal level.

Equation 3.2.3 can be alternatively expressed as [5-7]

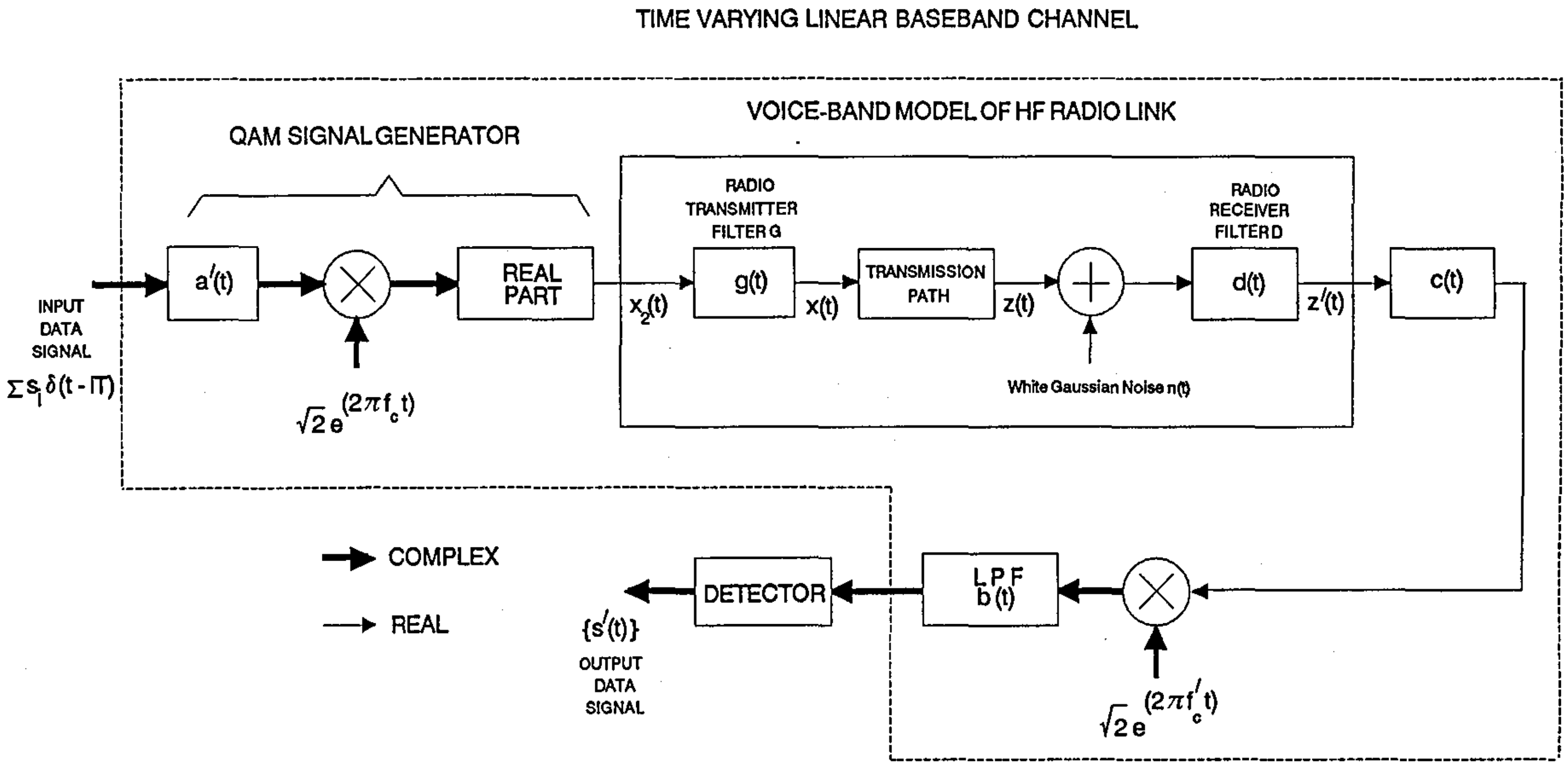


FIGURE 3.2.2 EQUIVALENT MODEL OF THE DATA TRANSMISSION SYSTEM

$$x_2(t) = \sqrt{2}\,\Re\left[\sum_i s_i a'(t - iT)e^{j2\pi f_c t}\right] \tag{3.2.4}$$

where

$$e^{j2\pi f_c t} = \cos(2\pi f_c t) + j\sin(2\pi f_c t)$$

$x_2(t)$ is fed to the radio equipment transmitter filter G in figure 3.2.2. Filter G has an impulse response of g(t) and a transfer function of G(f). The output of this filter, x(t) is real valued and is given by

$$x(t) = \Re[\sqrt{2}\sum_i s_i a'(t - iT)e^{j2\pi f_c t}] * g(t) \tag{3.2.5}$$

where * represents convolution.

Equation 3.2.5 can be written as

$$x(t) = \frac{1}{\sqrt{2}}\{\sum_i s_i a'(t - iT)e^{j2\pi f_c t} + \sum_i s_i^* a'(t - iT)e^{-j2\pi f_c t}\} * g(t)$$

(3.2.6)

Consider the convolution

$$\left(u_1(t)e^{-j2\pi f_c t}\right) * \left(u_2(t)e^{-j2\pi f_c t}\right) \tag{3.2.7}$$

By definition equation 3.2.7 can be written as

$$\int_{-\infty}^{\infty}\left(u_1(\tau)e^{-j2\pi f_c \tau}\right)\left(u_2(t-\tau)e^{-j2\pi f_c (t-\tau)}\right)d\tau = \int_{-\infty}^{\infty} u_1(\tau)u_2(t-\tau)e^{-j\pi f_c t}\,d\tau$$

(3.2.8)

Therefore, from equations 3.2.7 and 3.2.8,

$$(u_1(t)*u_2(t))e^{-j2\pi f_c t} = \left(u_1(t)e^{-j2\pi f_c t}\right)*\left(u_2(t)e^{-j2\pi f_c t}\right) \tag{3.2.9}$$

$$x(t) = \frac{1}{\sqrt{2}}\left\{\sum_i s_i a(t-iT)e^{j2\pi f_c t} + \sum_i s_i^* a^*(t-iT)e^{-j2\pi f_c t}\right\} \tag{3.2.10}$$

where

$$a(t-iT) = a'(t-iT)*\left[g(t)e^{-j2\pi f_c t}\right] \tag{3.2.11}$$

Equation 3.2.11 represents the overall filtering at the transmitter end, which includes the lowpass filter and the radio transmitter equipment filter which is a bandpass filter.

$a'(t-iT)$ and g(t) are real valued in equation 3.2.11 and, therefore, the complex conjugate of a(t-iT) is simply given by

$$a^*(t-iT) = a'(t-iT)*\left[g(t)e^{j2\pi f_c t}\right] \tag{3.2.12}$$

Figure 2.6.2 shows the Rayleigh fading introduced by a single skywave HF channel. Thus, when x(t) is fed into a single Rayleigh fading channel, the output from it would be

$$z'(t) = x(t)q_1(t) + \hat{x}(t)q_2(t) \tag{3.2.13}$$

where $q_1(t)$ and $q_2(t)$ are statistically independent Gaussian random processes that generate the fading and $\hat{x}(t)$ represents the Hilbert transform of x(t). The Hilbert transform of x(t) is given by the convolution of x(t) with the Hilbert transform filter i.e

$$\hat{x}(t) = x(t)*f(t) \qquad (3.2.14)$$

where f(t) is the impulse response of a Hilbert transform filter, whose Fourier transform is F(f) and is given by

$$\begin{aligned} F(f) &= j \qquad f<0 \\ &= 0 \qquad f=0 \\ &= -j \qquad f>0 \end{aligned} \qquad (3.2.15)$$

From equations 3.2.10 and 3.2.14

$$\hat{x}(t) = \frac{1}{\sqrt{2}}\{\sum_i s_i a(t-iT)e^{j2\pi f_c t} + \sum_i s_i^* a^*(t-iT)e^{-j2\pi f_c t}\}*f(t) \qquad (3.2.16)$$

and from equations 3.2.9 and 3.2.16

$$\hat{x}(t) = \frac{1}{\sqrt{2}}\sum_i s_i\left[a(t-iT)*f(t)e^{-j2\pi f_c t}\right]e^{j2\pi f_c t} + \frac{1}{\sqrt{2}}\sum_i s_i^*\left[a^*(t-iT)*f(t)e^{j2\pi f_c t}\right]e^{-j2\pi f_c t} \qquad (3.2.17)$$

The Fourier transform of $f(t)e^{-j2\pi f_c t}$ is F(f+f_c) and from equation 3.2.15 this has a value of -j over the frequency band -f_c to +f_c. On the other hand, the Fourier transform of $f(t)e^{j2\pi f_c t}$ is F(f-f_c) and this has a value of +j in the frequency band -f_c to +f_c. Moreover frequency response of a(t) is bandlimited to that of A'(f). Therefore, after taking the Fourier transform of equation 3.2.17, substituting the values of F(f+f_c) and f(f-f_c) from equation 3.2.15 and then taking the inverse Fourier transform of the resultant relation, equation 3.2.17 reduces to [4-7]

$$\hat{x}(t)=\frac{1}{\sqrt{2}}\{\sum_i -js_i a(t-iT)e^{j2\pi f_c t}+\sum_i js_i^* a^*(t-iT)e^{-j2\pi f_c t}\}$$

(3.2.18)

where s_i^* and $a^*(t)$ are the complex conjugates of s_i and a(t) respectively. For the sake of simplicity, the HF channel is now considered to be composed of two independent Rayleigh fading sky waves. The explanation can, however, be logically extended to any number of sky waves. For the two sky-wave channel, the relative delay between the two skywaves is taken to be τ seconds. x(t) is now fed to the HF channel and the output from the channel is given by,

$$z'(t)=[x(t)q_1(t)+\hat{x}(t)q_2(t)]+[x(t-\tau)q_3(t)+\hat{x}(t-\tau)q_4(t)]$$

(3.2.19)

From equations 3.2.10, 3.2.18 and 3.2.19

$$\begin{aligned} z(t)=\frac{1}{\sqrt{2}}\{&\sum_i s_i^* a^*(t-iT)\,[q_1(t)+jq_2(t)]\,e^{-j2\pi f_c t} \\ &+s_i a(t-iT)\,[q_1(t)-jq_2(t)]\,e^{j2\pi f_c t} \\ &+s_i^* a^*(t-\tau-iT)\,[q_3(t)+jq_4(t)]\,e^{-j2\pi f_c (t-\tau)} \\ &+s_i a(t-\tau-iT)\,[q_3(t)-jq_4(t)]\,e^{j2\pi f_c (t-\tau)}\} \end{aligned}$$

(3.2.20)

Let

$$\begin{aligned} h_i(t-iT)=&a(t-iT)\,[q_1(t)-jq_2(t)]+ \\ &a(t-\tau-iT)\,[q_3(t)-jq_4(t)]\,e^{-j2\pi f_c \tau} \end{aligned}$$

(3.2.21)

Thus, from equation 3.2.20 and 3.2.21,

$$z(t) = \frac{1}{\sqrt{2}}\left\{\sum_i s_i h_i(t - iT)e^{j2\pi f_c t} + \sum_i s_i^* h_i^*(t - iT)e^{-j2\pi f_c t}\right\}$$

(3.2.22)

If τ is assumed constant then $e^{-j2\pi f_c \tau}$ is a fixed complex scalar quantity with absolute value of 1 and, therefore, would not affect the statistical properties of $[q_3(t) - q_4(t)]\,e^{-j2\pi f_c \tau}$, bearing in mind that $q_h(t)$ are statistically independent with zero mean Gaussian random processes. Therefore, equation 3.2.21 can be simplified as

$$\begin{aligned} h_i(t - iT) &= a(t - iT)\,[q_1(t) - jq_2(t)] + \\ &\quad a(t - \tau - iT)\,[q_3(t) - jq_4(t)] \end{aligned} \qquad (3.2.23)$$

The output from the radio receiver filter (figure 3.2.2), whose sampled impulse response is d(t), is

$$z'(t) = z(t)*d(t) + n(t)*d(t) \qquad (3.2.24)$$

and the output of the linear demodulator in figure 3.2.2 is,

$$\begin{aligned} r(t) &= \sqrt{2}\left\{[z(t)*d(t)*c(t)]\,e^{-j2\pi f_c t}\right\}*b'(t) + \\ &\quad \sqrt{2}\left\{[n(t)*d(t)*c(t)]\,e^{-j2\pi f_c t}\right\}*b'(t) \end{aligned} \qquad (3.2.25)$$

Let

$$b(t) = \left\{[d(t)*c(t)]\,e^{-j2\pi f_c t}\right\}*b'(t) \qquad (3.2.26)$$

$$w(t) = \sqrt{2}\left\{[n(t)*d(t)*c(t)]\,e^{-j2\pi f_c t}\right\}*b'(t) \qquad (3.2.27)$$

where n(t) is a real value additive white Gaussian noise waveform comprising a two sided power spectral density of 0.5 N_0. w(t) in equation 3.2.27 represents a band limited, complex valued Gaussian noise waveform.

Combining equations 3.2.25, 3.2.26, 3.2.27,

$$r(t) = \sqrt{2}\left[z(t)e^{-j2\pi f_c' t}\right] * b(t) + w(t) \tag{3.2.28}$$

Equation 3.2.26 represents the overall filtering carried out on the signal at the receiver. Also it is assumed that the receiver is operating in synchronism with the transmitter and any constant phase difference between the reference carrier and the received signal is neglected (i.e $f_c = f_c'$). Then from equations 3.2.22 and 3.2.28,

$$r(t) = \sum_i \left[s_i h_i(t - iT) + s_i^* h_i^*(t - iT)e^{-j4\pi f_c t}\right] * b(t)$$
$$+ w(t) \tag{3.2.29}$$

The Gaussian shaped filter used to generate $q_h(t)$, has a frequency response that decreases sharply with f. $h_i(t\text{-}iT)$, which consists of the time invariant impulse response a(t) and the random components $q_h(t)$ can, therefore, be considered to be strictly bandlimited.

$$H(f) = 0 \qquad f > f_c \tag{3.2.30}$$

The second term in equation 3.2.29,

$$\left[s_i^* h_i^*(t - iT)e^{-j4\pi f_c t}\right]$$

is, therefore outside the passband of the low pass filter with an impulse response b(t). Hence

$$r(t) = \sum_i s_i h_i(t - iT) * b(t) + w(t) \tag{3.2.31}$$

Let

$$Y_i(t - iT) = h_i(t - iT) * b(t) \tag{3.2.32}$$

Then

$$r(t) = \sum_i s_i Y_i(t - iT) + w(t) \tag{3.2.33}$$

Combining equations 3.2.23 and 3.2.32, Y_i(t-iT) can be written as,

$$\begin{aligned} Y_i(t - iT) = \{ & a(t - iT)\,[q_1(t) - jq_2(t)] + \\ & a(t - \tau - iT)\,[q_3(t) - jq_4(t)]\} * b(t) \end{aligned} \tag{3.2.34}$$

Y_i(t-iT) is the impulse response of the equivalent time varying linear baseband channel. Figure 3.2.3 shows the baseband model of the QAM system over a two skywave HF radio link.

The average transmitted energy at the output of the transmitter filter in figure 3.2.3 is given by

$$E_t = E\left[\int_{-\infty}^{\infty} | s_i a(t - iT) |^2 \, dt\right] \tag{3.2.35}$$

Where E[.] represents the expected value of the quantity.

Let

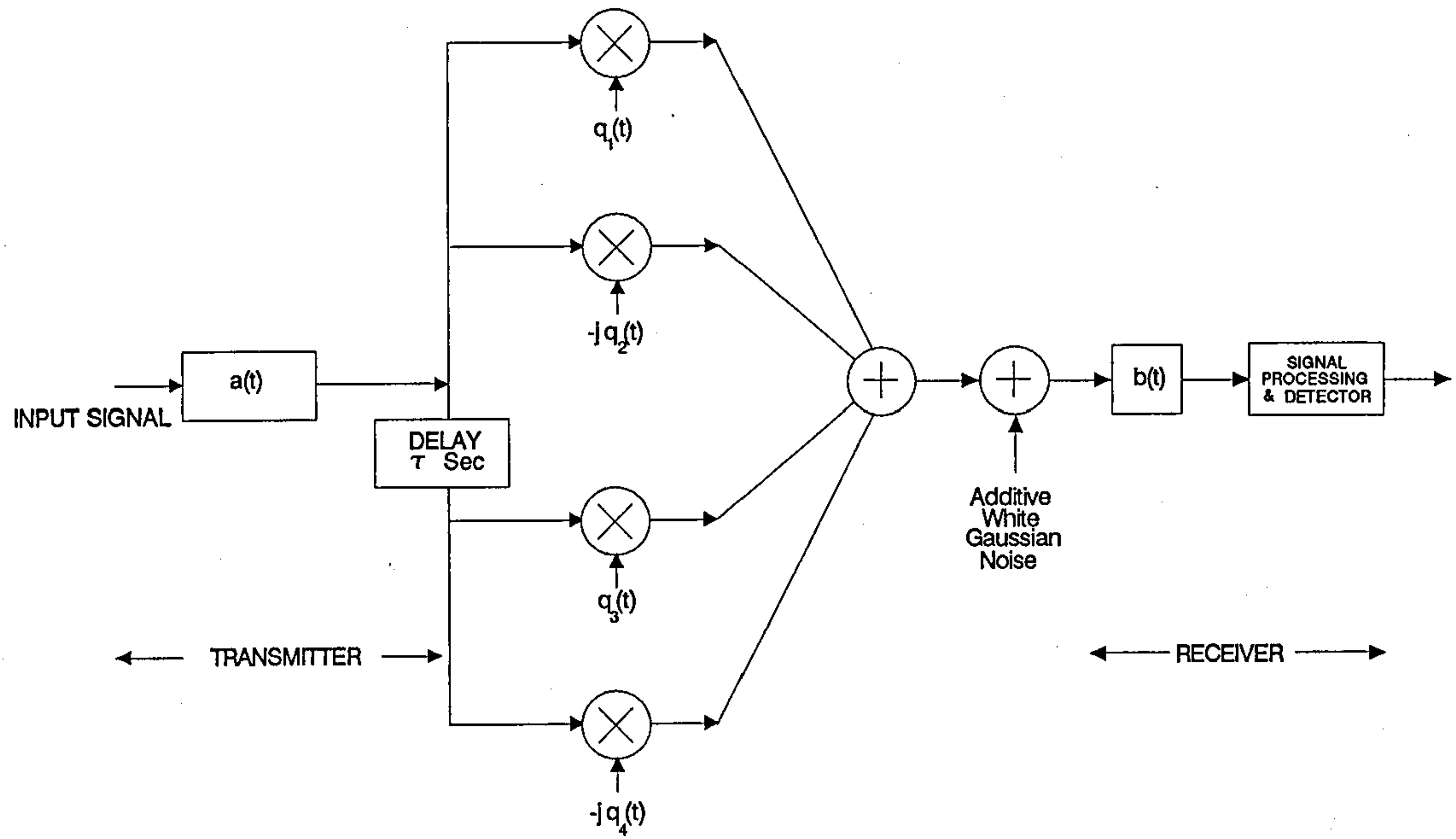


FIGURE 3.2.3 BASEBAND MODEL OF THE QAM SYSTEM OVER A TWO SKY-WAVE HF RADIO LINK

$$\overline{s_i^2} = E[|s_i|^2] \tag{3.2.36}$$

From Parseval's theorem, equation 3.2.35 can be written as

$$E_t = \overline{s_i^2} \int_{-\infty}^{\infty} |A(f)|^2 \, df \tag{3.2.37}$$

The average energy per signal element at the input of the receiver in figure 3.2.3 is given by

$$E_r = E \int_{-\infty}^{\infty} \{s_i a(t - iT) q_1(t) - j q_2(t) +$$

$$s_i a(t - \tau - iT) q_3(t) - j q_4(t)\}^2 dt$$

(3.2.38)

or

$$E_r = \overline{s_i^2} [\overline{q_1^2}(t) + \overline{q_2^2}(t) + \overline{q_3^2}(t) + \overline{q_4^2}(t)] \int_{-\infty}^{\infty} |A(f)|^2 \, df$$

(3.2.39)

If $\overline{q}_i^2(t) = \overline{q}_j^2(t)$, for i= 1, . . , 4, and for j = 1, . . , 4, and the sum of their variances is equal to unity than the average energy per signal element at the output of the transmitter filter and at the input to the receiver filter in figure 3.2.3 are equal and the HF channel, on average, does not introduce any attenuation or gain to the transmitted data signal and hence does not affect the signal/noise ratio of the system.

The signal/noise ratio is defined as

$$\psi = 10.0 \log_{10} \left(\frac{E_b}{\frac{1}{2} N_0} \right) \tag{3.2.40}$$

It has been shown in reference [8] that, for a QPSK signal,

$$\frac{E_b}{\frac{1}{2}N_0} = \frac{s_i^2}{2\sigma_i^2} \tag{3.2.41}$$

where σ_i^2 is the variance of the additive white Gaussian noise.

3.3 MODEL OF THE DATA TRANSMISSION SYSTEM SIMULATED

The baseband model of the data transmission system over a two sky-wave HF radio link is shown in figure 3.2.3. The impulse response of the linear baseband channel is time varying, and for a 2 skywave channel, it is given by

$$Y_i(t-iT) = \{a(t-iT)\,[q_1(t) - jq_2(t)] + a(t-\tau-iT)\,[q_3(t) - jq_4(t)]\} * b(t) \tag{3.3.1}$$

where

$$a(t) = a'(t) * \left[g(t)e^{-j2\pi f_c t}\right] \tag{3.3.2}$$

and

$$b(t) = \left\{[d(t) * c(t)]\, e^{-j2\pi f_c t}\right\} * b'(t) \tag{3.3.3}$$

a(t) is the impulse response of the overall transmitter filter A and b(t) is the impulse response of the overall receiver filter B in figure 3.2.3. Figure 3.3.1 - 3.3.3 show a combination of the equipment bandpass filters operating on the voiceband signal

[5-7]. Figure 3.3.1 shows the frequency characteristics of the radio filters G and D in cascade over the positive frequencies and table 3.3.1 shows the attenuation and group delay samples of the radio filters in cascade taken at 50 Hz frequency intervals. The radio filters used are the Clansman VRC 321 type, this being a typical radio filter used in a practical system. Figure 3.3.2 shows the frequency characteristics of the modem transmitter and receiver filters in cascade and in the passband of the QAM signal over positive frequencies. Table 3.3.2 shows the sampled values of the same characteristics taken at a frequency interval of 50 Hz. The frequency characteristic in figure 3.3.2 corresponds to the impulse response

$$\left\{a'(t)*\left[c(t)e^{-j2\pi f_c' t}\right]*b'(t)\right\}e^{j2\pi f_c t} \tag{3.3.4}$$

Figure 3.3.3 shows the frequency characteristics corresponding to the impulse response

$$\{a(t)*b(t)\}\,e^{j2\pi f_c' t} \tag{3.3.5}$$

Equation 3.3.5 is obtained from the convolution of equation 3.3.4 with $\{a(t)*b(t)\}$. The attenuation and group delay characteristics corresponding to each of a(t) and b(t), in equation 3.3.5, are obtained by shifting the frequency characteristics in figure 3.3.3, to the left by $f_c = 1800$ Hz, and dividing them by 2. A sequence $\{a'_k\}$ is obtained by taking the inverse DFT of the frequency characteristics at a sampling rate of 4800 samples/sec. This sequence is made minimum phase by replacing the roots outside the unit circle in the z-plane, by the complex conjugate of their reciprocals, giving the sequence $\{a''_k\}$ which, is at a sampling rate of 4800 samples/sec. The methods used for converting the impulse response to minimum phase are given elsewhere [9-10]. The sampled impulse response of the filters used are non-minimum phase which means that the responses rise slowly towards their respective peaks [4]. This has the effect of reducing the tolerance of the detection process to additive noise and also increasing the delay in detection of a data symbol [7]. Therefore, it is necessary

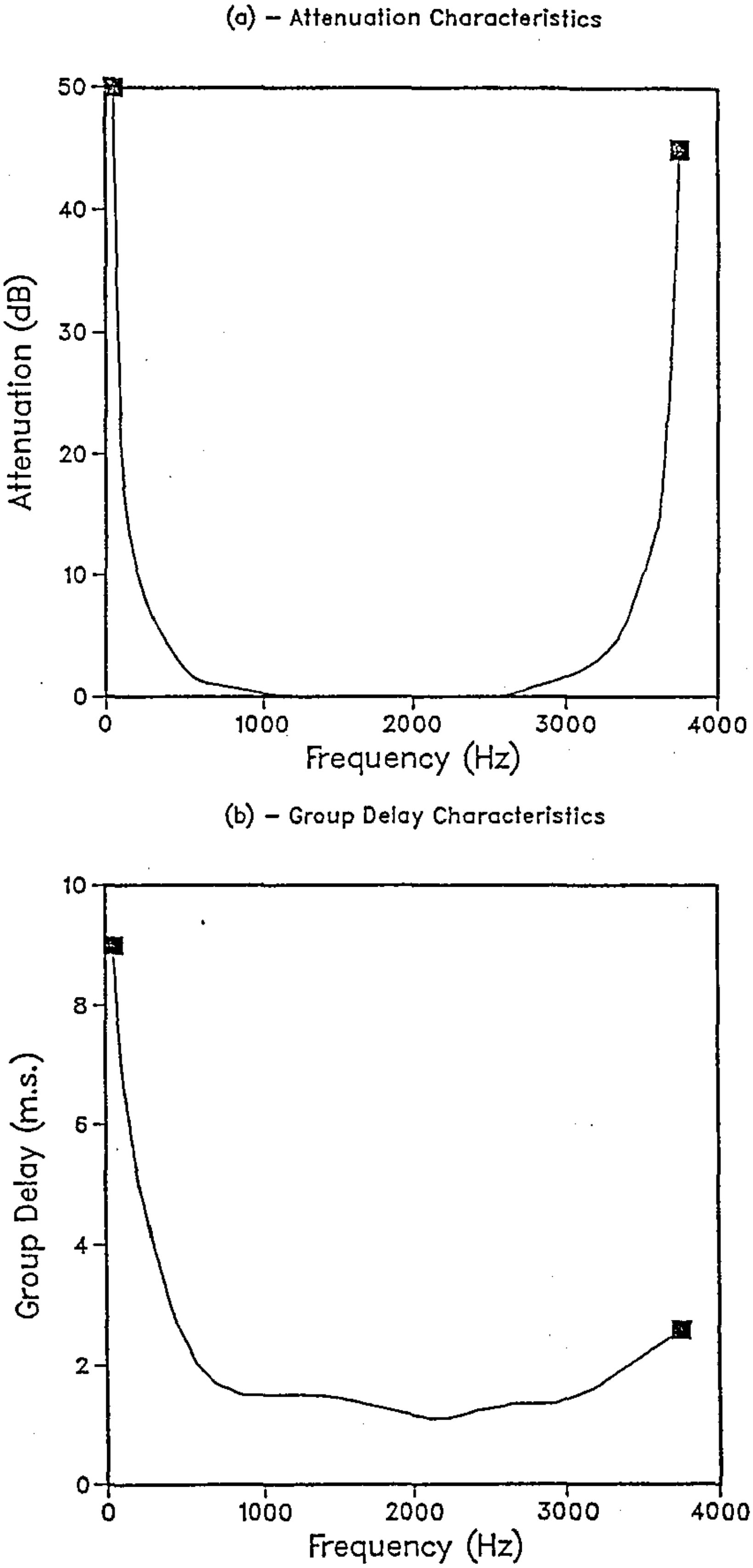


Fig. 3.3.1 – Frequency Characteristics of the Radio filters G and D in cascade over the positive Frequency

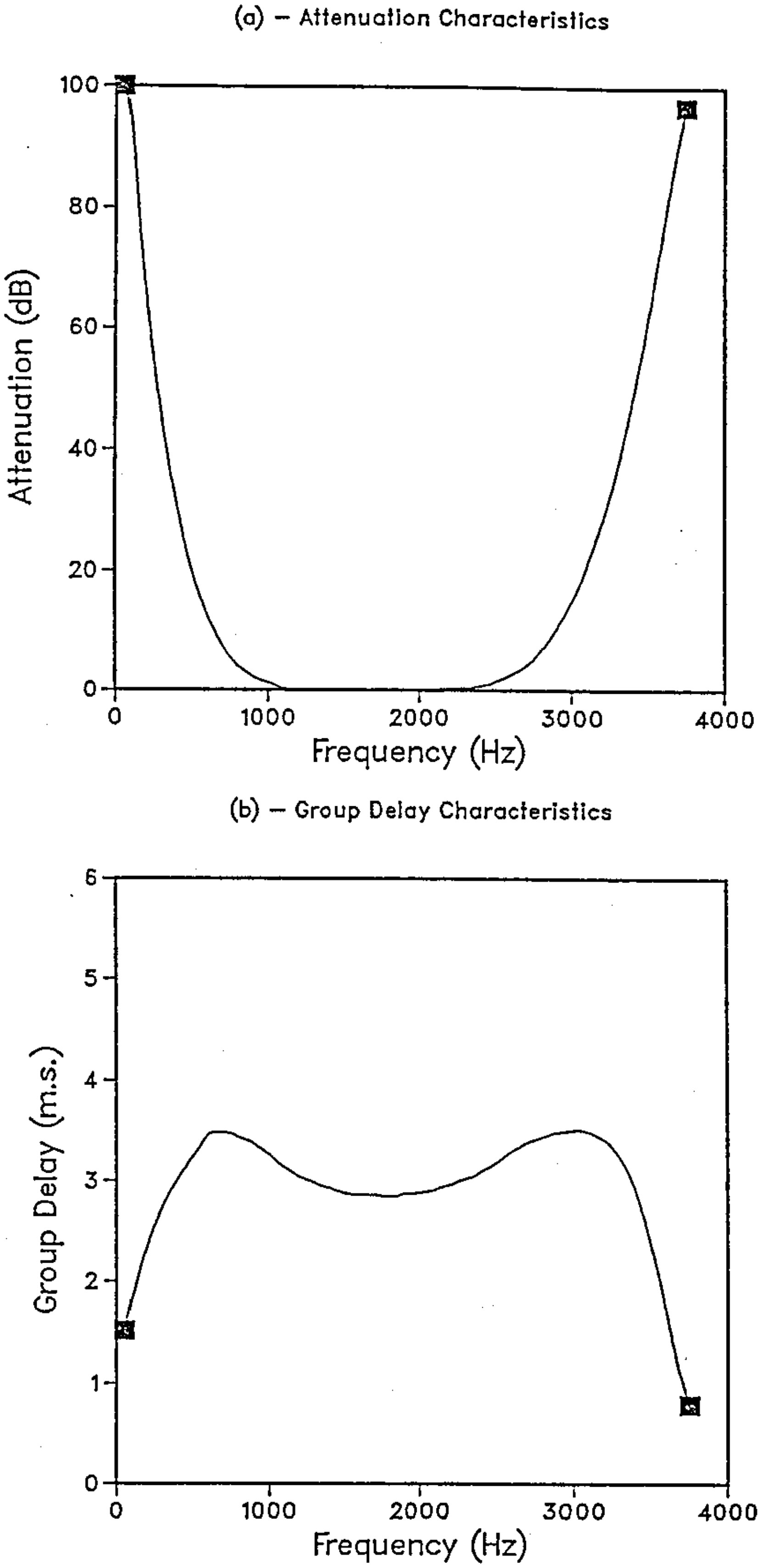


Fig. 3.3.2 – Filter Characteristics Corresponding to the impulse response
$\{a'(t)*[c(t)e^{-j2\pi f_o t}]*b'(t)e^{j2\pi f_o t}\}$

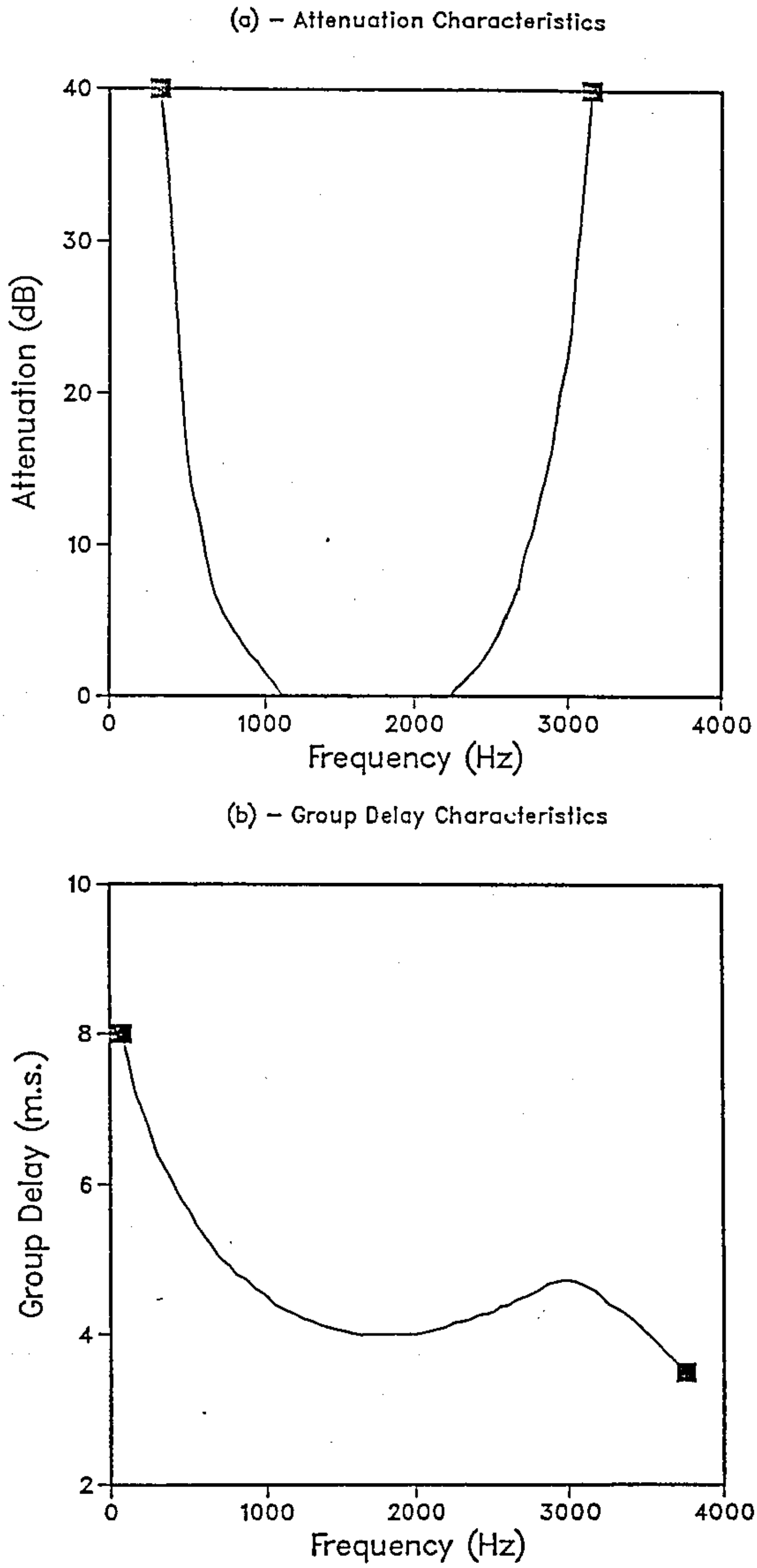


Fig. 3.3.3 – Filter Characteristics Corresponding To The Impulse Response $\{a(t)*b(t)\}e^{2\pi f_c t}$

TABLE 3.3.1 ATTENUATION AND GROUP DELAY CHARACTERISTICS OF RADIO FILTERS IN CASCADE

Frequency (Hz)	Attenuation dB	Group Delay msec	Frequency (Hz)	Attenuation dB	Group Delay msec
50	50.00	9.00	1950	0.00	1.18
100	21.00	7.00	2000	0.00	1.15
150	16.50	6.50	2050	0.00	1.13
200	13.00	5.30	2100	0.00	1.10
250	10.00	4.50	2150	0.00	1.10
300	7.60	3.90	2200	0.00	1.10
350	5.60	3.40	2250	0.00	1.12
400	4.10	2.90	2300	0.00	1.15
450	2.75	2.60	2350	0.00	1.18
500	2.00	2.35	2400	0.00	1.23
550	1.50	2.05	2450	0.00	1.25
600	1.25	1.90	2500	0.05	1.27
650	1.05	1.75	2550	0.10	1.29
700	0.95	1.65	2600	0.15	1.32
750	0.80	1.60	2650	0.30	1.35
800	0.70	1.55	2700	0.45	1.35
850	0.60	1.50	2750	0.65	1.35
900	0.50	1.50	2800	0.85	1.35
950	0.40	1.50	2850	1.02	1.35
1000	0.30	1.50	2900	1.20	1.35
1050	0.25	1.50	2950	1.42	1.35
1100	0.20	1.50	3000	1.65	1.38
1150	0.15	1.50	3050	1.90	1.40
1200	0.10	1.50	3100	2.20	1.50
1250	0.05	1.50	3150	2.60	1.58
1300	0.00	1.50	3200	3.00	1.66
1350	0.00	1.50	3250	3.50	1.75
1400	0.00	1.50	3300	4.00	1.83
1450	0.00	1.45	3350	5.25	1.92
1500	0.00	1.45	3400	6.50	2.00
1550	0.00	1.42	3450	8.25	2.08
1600	0.00	1.39	3500	10.00	2.16
1650	0.00	1.36	3550	12.00	2.25
1700	0.00	1.33	3600	14.00	2.33
1750	0.00	1.30	3650	20.00	2.41
1800	0.00	1.27	3700	30.00	2.50
1850	0.00	1.24	3750	45.00	2.58
1900	0.00	1.21			

TABLE 3.3.2 ATTENUATION AND GROUP DELAY CHARACTERISTICS OF EQUIPMENT FILTER

Frequency (Hz)	Attenuation dB	Group Delay msec	Frequency (Hz)	Attenuation dB	Group Delay msec
50	99.99	1.51	1950	0.00	2.88
100	93.79	1.83	2000	0.01	2.89
150	77.62	2.12	2050	0.05	2.90
200	64.73	2.37	2100	0.13	2.92
250	53.94	2.58	2150	0.23	2.95
300	44.70	2.76	2200	0.35	2.97
350	36.70	2.91	2250	0.40	3.00
400	30.40	3.03	2300	0.45	3.03
450	24.40	3.15	2350	0.57	3.05
500	19.50	3.26	2400	0.76	3.10
550	15.65	3.37	2450	0.93	3.15
600	12.30	3.47	2500	1.45	3.19
650	9.55	3.48	2550	1.97	3.25
700	7.30	3.48	2600	2.64	3.30
750	5.50	3.47	2650	3.25	3.35
800	4.10	3.43	2700	4.05	3.39
850	3.10	3.41	2750	5.20	3.42
900	2.20	3.37	2800	6.72	3.44
950	1.65	3.32	2850	8.20	3.47
1000	1.25	3.26	2900	10.25	3.49
1050	0.75	3.19	2950	12.45	3.50
1100	0.35	3.14	3000	14.95	3.51
1150	0.02	3.09	3050	17.70	3.51
1200	0.00	3.04	3100	21.10	3.49
1250	0.00	3.01	3150	24.80	3.45
1300	0.00	2.98	3200	28.60	3.41
1350	0.00	2.95	3250	32.83	3.33
1400	0.00	2.93	3300	37.63	3.22
1450	0.00	2.90	3350	43.10	3.08
1500	0.00	2.88	3400	49.15	2.89
1550	0.00	2.87	3450	55.55	2.65
1600	0.00	2.87	3500	62.30	2.36
1650	0.00	2.86	3550	69.55	2.05
1700	0.00	2.86	3600	76.75	1.69
1750	0.00	2.85	3650	84.05	1.33
1800	0.00	2.85	3700	90.85	1.03
1850	0.00	2.85	3750	96.80	0.79
1900	0.00	2.88			

to make the filters minimum phase. The DFT of $\{a''_k\}$ has been obtained with a sampling interval of 50 Hz and since the sampling rate is 4800 samples/second, there are 96 components in the DFT of $\{a''_k\}$. In order to explain different sampling phases, the $\{a''_k\}$ have been oversampled at 20 times the original sampling rate (i.e at a sampling rate of 96000 samples/seconds). This is done by inserting 1824 zero valued components, in the middle of the DFT of $\{a''_k\}$, this increasing the number of components from 96 to 1920, and the sampling rate from 4800 samples/second to 96000 samples/second. The inverse DFT of this expanded sequence, $\{a''_k\}$, gives the minimum phase impulse response of the minimum phase filters used in the channel model. Transmission filter A2, corresponds to a delay of 1.1 ms, with respect to A1. Similarly table 3.3.4 shows the minimum phase receiver filter. Appendix B gives the sampled impulse response of the transmitter and receiver filters at 96000 samples/second.

From equation 3.3.1, the sampled impulse response of the linear baseband channel is

$$Y_i(t-iT) = \{a''(t-iT)\,[q_1(t)-jq_2(t)] + a''(t-\tau-iT)\,[q_3(t)-jq_4(t)]\} * b''(t) \tag{3.3.6}$$

where $a''(t)$ and $b''(t)$ are minimum phase sampled impulse responses of the filters a(t) and b(t) respectively. The demodulated baseband signal r(t) at the output of the QAM system model comprises the stream of data elements $\{s_i y_i(t-iT)\}$ to which is added stationary zero mean complex valued baseband Gaussian noise waveform w(t). The waveform r(t) is sampled once per data symbol s_i, at the time instants iT. The sampling rate, is assumed to be correct. The delay in transmission is, for convenience, taken to be such that the first potentially non zero sample of a received signal element arrives with no delay. The complex valued sample of r(t) at time iT is now

TABLE 3.3.3 SAMPLED IMPULSE RESPONSE OF THE MINIMUM-PHASE TRANSMITTER FILTER SAMPLED AT 4800 SAMPLES/SECOND FOR A TWO SKYWAVE CHANNEL

TRANSMITTER FILTER A1		TRANSMITTER FILTER A2	
REAL PART	IMAGINARY PART	REAL PART	IMAGINARY PART
-0.1795896	2.3539405	0.0000000	0.0000000
-3.0773453	20.7590237	0.0000000	0.0000000
-9.9409021	45.5584592	0.0000000	0.0000000
-11.7869473	41.4909978	0.0000000	0.0000000
-3.4618271	8.7045826	0.0000000	0.0000000
4.4438154	-11.7869820	-1.6694374	13.2372707
3.0642536	-5.5819054	-7.8492148	39.6493461
-1.3596576	3.1582131	-12.3887079	46.9272219
-1.4973528	1.7365460	-6.6023157	19.2346609
0.2925598	-0.7776891	2.9408554	-8.8804125
0.5180829	-0.1292556	4.3005084	-9.0256163
-0.1842786	0.2880296	-0.3368383	1.6284281
-0.3167778	-0.2324818	-1.8014344	2.8139013
0.0021899	-0.2107548	-0.1433592	-0.4311352
-0.0443806	0.0392056	0.6242601	-0.4537174
0.0515533	0.0098505	0.0278577	0.3081762
		-0.0382007	-0.0772327
		-0.0416905	-0.3043271
		-0.0439705	0.0085057
		0.0749333	0.0093809
		-0.0594132	0.0094992

TABLE 3.3.4 SAMPLED IMPULSE RESPONSE OF THE MINIMUM-PHASE RECEIVER FILTER SAMPLED AT 4800 SAMPLES/SECOND FOR A TWO SKYWAVE CHANNEL

RECEIVER FILTER B	
REAL PART	IMAGINARY PART
-0.1795896	2.3539405
-3.0773453	20.7590237
-9.9409021	45.5584592
-11.7869473	41.4909978
-3.4618271	8.7045826
4.4438154	-11.7869820
3.0642536	-5.5819054
-1.3596576	3.1582131
-1.4973528	1.7365460
0.2925598	-0.7776891
0.5180829	-0.1292556
-0.1842786	0.2880296
-0.3167778	-0.2324818
0.0021899	-0.2107548
-0.0443806	0.0392056
0.0515533	0.0098505

$$r_i = \sum_{h=0}^{g} s_{i-h} y_{i,h} + w_i \tag{3.3.7}$$

and $y_{i,h} = 0$ for h<0 and h>g for practical purposes. The sequence of complex values given by the vector

$$Y_i = [y_{i,0} \quad y_{i,1} \quad \dots \quad y_{i,g}] \tag{3.3.8}$$

is taken to be the sampled impulse response of the linear baseband channel at time t=iT.

In the simulation tests, the vector Y_i is obtained by sampling the $\{y_i(t-iT)\}$ at a rate of 2400 samples/sec. Since the process is performed in the discrete time domain, and to avoid any aliasing likely to occur when any of the $q_1(t)$, $q_2(t)$, $q_3(t)$ or $q_4(t)$ is changing rapidly, the convolution in equation 3.3.6 is carried out, with a sampling rate of 4800 samples/sec which is well above the Nyquist rate for filters A and B. The functions $q_1(t)$, $q_2(t)$, $q_3(t)$ and $q_4(t)$ are generated at a sampling rate of 4800 samples/sec, as described in chapter 2. Let the three sequences A1, A2 and B represent the three impulse responses $a''(t), a''(t-\tau)$ and $b''(t)$ respectively sampled at 4800 samples/sec.

$$\begin{aligned} A1 &= [a''_{1,0} \quad a''_{1,1} \quad \dots \quad a''_{1,p}] \\ A2 &= [a''_{2,0} \quad a''_{2,1} \quad \dots \quad a''_{2,p}] \end{aligned} \tag{3.3.9}$$

and

$$B = [b''_0 \quad b''_1 \quad \dots \quad b''_p] \tag{3.3.10}$$

where

$$a''_{1,k} = a''\left(k\frac{T}{2}\right)$$

$$a''_{2,k} = a''\left(k\frac{T}{2} - \tau\right)$$

$$b''_k = b''\left(k\frac{T}{2}\right) \tag{3.3.11}$$

1/T is the data symbol rate of 2400 symbols/sec.

From figure 3.3.4, it can be assumed for practical purposes

$$a''(t) = b''(t) = 0 \qquad for \quad t < 0 \quad , \quad t > (p - p')T/2 \tag{3.3.12}$$

p' being an integer such that

$$\tau = p'\frac{T}{2} + \tau' \tag{3.3.13}$$

and $\tau' < \frac{T}{2}$. This implies that

$$a''_{1,k} = b''_k = 0 \quad for\, k < 0 \quad , \quad k > p - p' \tag{3.3.14}$$

$$a''_{2,k} = 0 \qquad for\, k \le p' \quad , \quad k > p \tag{3.3.15}$$

$p - p'$ is a fixed quantity.

The four components $q_1(t)$, $q_2(t)$, $q_3(t)$ and $q_4(t)$ are sampled, and the resultant samples up to time instant t=iT are represented by the four sequences:

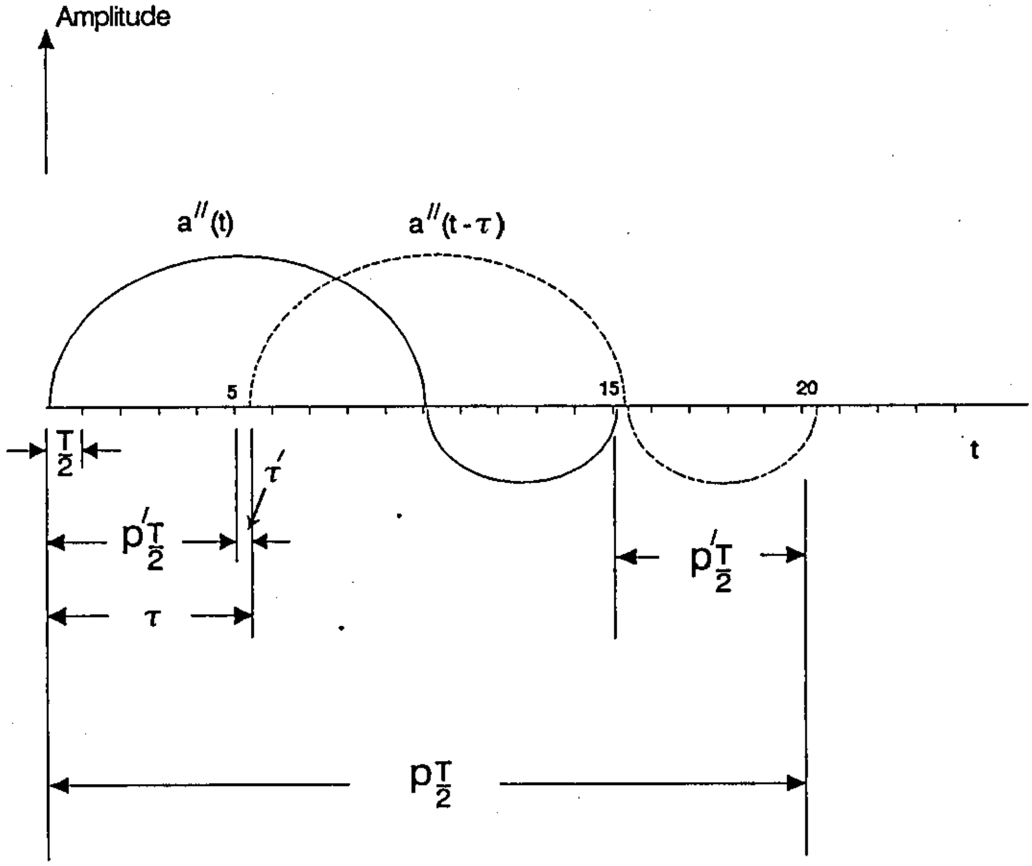


FIGURE 3.3.4 THE TIMING RELATIONSHIP BETWEEN THE TRANSMITTER FILTER IMPULSE RESPONSE (REAL OR IMAGINARY) AND ITS DELAYED VERSION

$$Q_{1,i} = [q_{1,1} \quad q_{2,2} \quad \cdots \quad q_{1,2i}]$$

$$Q_{2,1} = [q_{2,1} \quad q_{2,2} \quad \cdots \quad q_{2,2i}]$$

$$\cdot$$

$$Q_{4,i} = [q_{4,1} \quad q_{4,2} \quad \cdots \quad q_{4,2i}] \tag{3.3.16}$$

From equations 3.3.9-3.3.12 and 3.3.15, the components of the vector Y_i, in equation 3.3.8 can be written as

$$y_{i,h} = \left(\frac{T}{2}\right) \sum_{k=0}^{2} h[a''_{1,k}(q_{1,2(i-h)+k} - jq_{2,2(i-h)+k}) +$$

$$a''_{2,k}(q_{3,2(i-h)+k} - jq_{4,2(i-h)+k})]\, b''_{2h-k}$$

(3.3.17)

for h=0, 1, ..., g.

From equations 3.3.14 and 3.3.15 it can be shown that [5]

$$g = \frac{2p - p' + 1}{2} = \frac{2p_f + p' + 1}{2} \tag{3.3.18}$$

Hence g is a function of τ. For example for a delay τ of 1.1 msec and pf of 15 (Table 3.3.3 has 16 components), $p' = 5$, from equation 3.3.13, ($\tau' = 0.28$ T/2) hence g = 18, from equation 3.3.18.

3.4 MODEL OF THE SYSTEM CONFIGURATION USED FOR THE ESTIMATOR TESTS

Figure 3.4.1 and 3.4.2 show the model of the data transmission system used in the testing of the estimators. It shows in some detail the receiver configuration. In figure

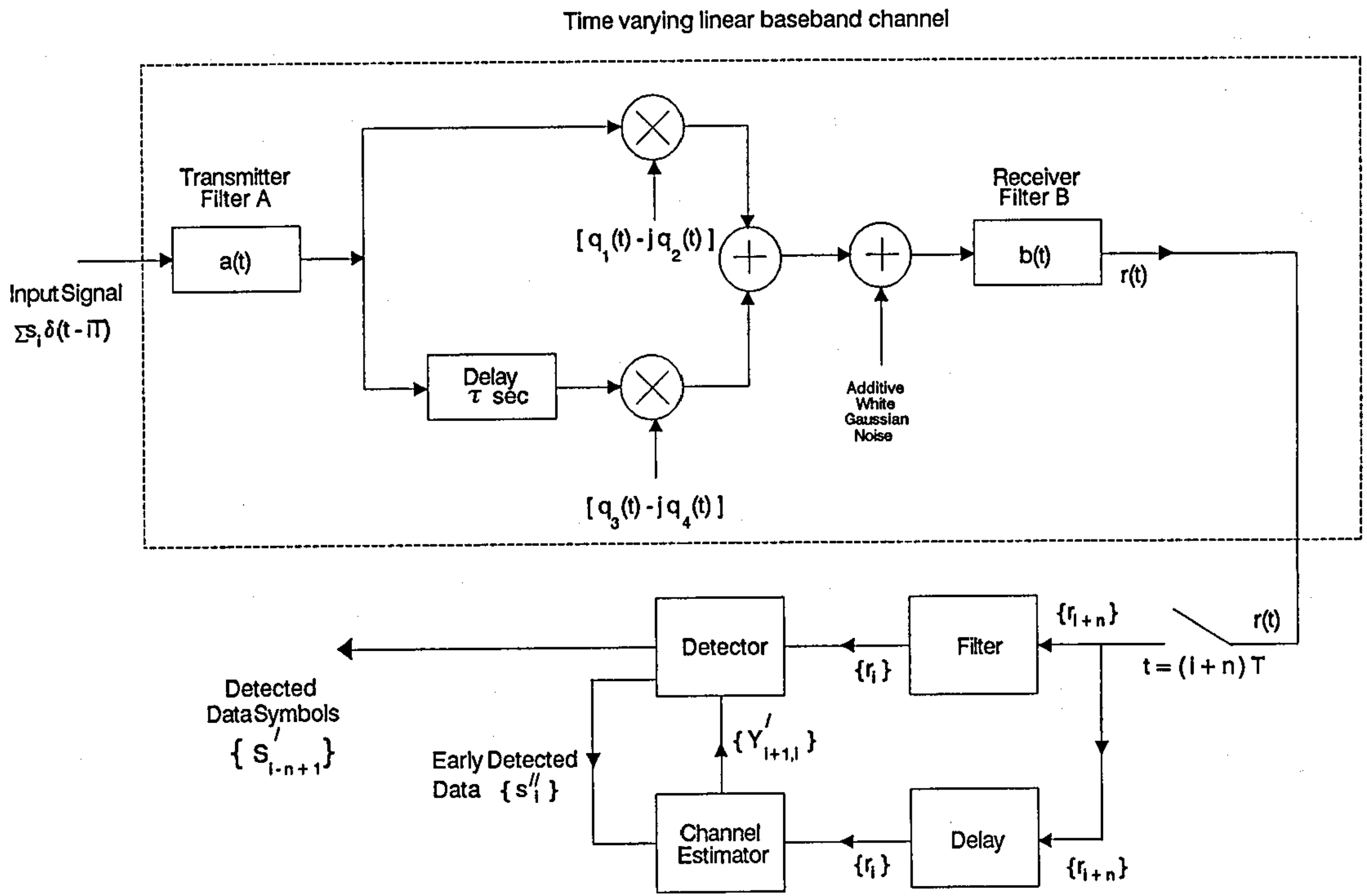


FIGURE 3.4.1 BASEBAND MODEL OF THE DATA TRANSMISSION SYSTEM OVER A TWO SKYWAVES HF RADIO LINK

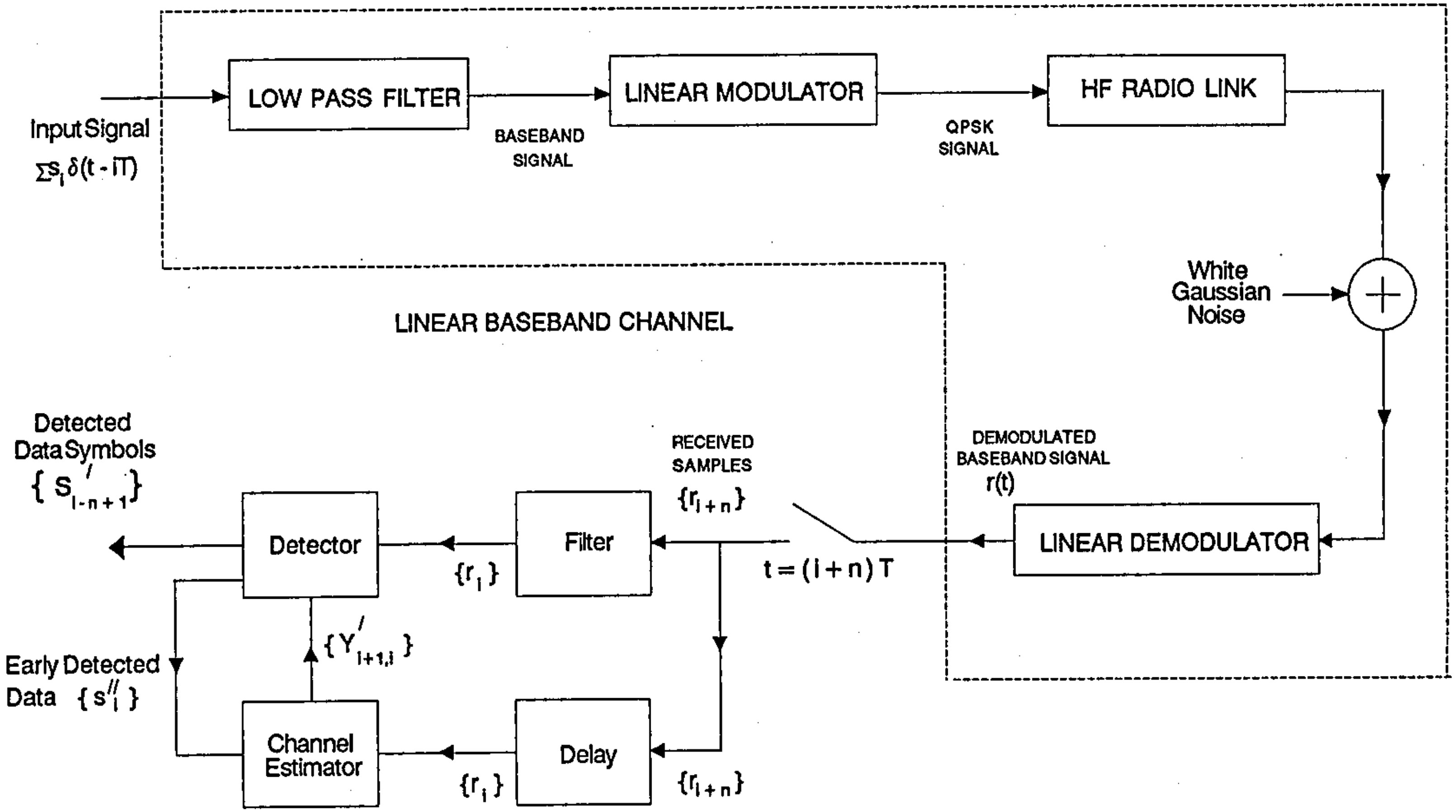


FIGURE 3.4.2 BASEBAND MODEL OF THE DATA TRANSMISSION SYSTEM

3.4.2 the output signal from the linear demodulator is a serial stream of real valued QPSK signal elements, with a carrier frequency of 1800 Hz and an element rate of 2400 bauds. Each signal element comprises the sum of two binary double sideband suppressed carrier amplitude modulated elements, with their carriers in phase quadrature, the binary values of the inphase and quadrature elements being determined respectively by the real and imaginary parts ($s_{0,i}$ and $s_{1,i}$) of the corresponding data symbol s_i. Thus the QPSK signal is handled as a Quadrature Amplitude Modulated (QAM) signal.

The channel considered in the estimator tests is an HF radio link, and is modelled as a two skywave Rayleigh fading channel with the delay between the two skywaves being 1.1 msecs. This model is based on the CCIR recommended model for poor channels [11], which considers the signal to be free from any frequency offsets (Doppler shift) as these are corrected for by special circuitry. The two skywaves consist of 4 Gaussian waveforms, which each have a variance of 0.25, to ensure that their is no gain or attenuation over the channel, and a root mean square bandwidth of 1 Hz. Thus the frequency spread of each skywave is 2 Hz. White Gaussian noise, with zero mean and a two sided power spectral density 1/2 N_0, is added to the data signal at the output of the HF radio link. The noise at the output of the linear baseband channel is bandlimited Gaussian noise.

The vector Y_i is the sampled impulse response of the linear baseband channel in figure 3.4.1 and 3.4.2. It has g+1 components, where g is 20 in the tests. The received samples $\{r_i\}$ are fed to an adaptive linear feedforward transversal filter. The latter is an allpass network that adjusts the sampled impulse response of the channel and filter to be minimum phase, without changing any amplitude distortion in the received signal [9-10]. The filter, in fact, maximises the ratio of the magnitude of the first few components of the resultant sampled impulse response to the output noise variance, when the noise components are statistically independent [10]. With the aid of an adaptive filter, a near optimum tolerance to noise can be achieved by means of a relatively simple detector, leading to a potentially cost effective system [5].

At time t=iT the received samples $\{r_i\}$ are fed to the detector. The detector introduces a delay of n-1 samples in the detected data symbol $\{s'_i\}$ obtained at its output. The detector also obtains at its output the early detected data symbols $\{s''_i\}$. This early detected data symbols have no delay in detection but have a higher error rate. The need for the early detected data symbols is due to the fact that the estimator requires an updated estimate of Y_i, which can be obtained using a one step prediction of Y_i, given by $Y'_{i,i-1}$.

The received samples $\{r_i\}$ are also fed to the channel estimator. The channel estimator uses the received samples

$$r_{i-g}, r_{i-g+1}, \ldots, r_i \tag{3.4.1}$$

together with the "early" detected data symbols

$$s''_{i-g}, s''_{i-g+1}, \ldots, s''_i \tag{3.4.2}$$

and the one step prediction of Y_i, given by

$$Y'_{i,i-1} = [y'_{i,i-1,0}, y'_{i,i-1,1}, \ldots, y'_{i,i-1,g}] \tag{3.4.3}$$

to form the updated estimate of Y_i, given by

$$Y'_i = [y'_{i,0}, y'_{i,1}, \ldots, y'_{i,g}] \tag{3.4.4}$$

which is then used by the predictor to form the one step prediction of Y_{i+1}, given by $Y'_{i+1,i}$. This is then used by the detector on receipt of r_{i+1} to obtain the early detected symbol s''_{i+1}. The process continues in a similar manner. Clearly any error in the estimation, will result in the detection process being in error, and so on.

The important advantage gained by the use of an adaptive filter ahead of the detector, is that it avoids the need for prediction over many sampling intervals, such as must be used in the absence of the filter [7]. The near maximum likelihood detector, without a filter, needs to make a prediction over many sampling intervals, if it is to obtain a satisfactory performance. Without the filter, the detector is not able to obtain the early detected data symbol s''_i, but some early detected data which lies between s''_i and s'_{i-n+1}. Prediction over many sampling intervals can considerably increase the error in the prediction [6-7].

In the tests carried out, it is assumed that the detection process is perfect, even at low signal/noise ratios, as the estimator not the detector is being tested, so that

$$s_i'' = s_i \tag{3.4.5}$$

for all $\{i\}$.

Most errors in the detection occur during the deeper fades and generally in long bursts. The channel estimate may then be degraded significantly, leading to more errors in the $\{s''_i\}$, which in turn further degrades the channel estimate, and so on, until the system collapses completely. It would therefore seem obvious that for a more reliable performance of the system, the channel estimate must obtain as accurately as possible the estimate/prediction of the channel.

Figure 3.4.3 shows the channel characteristics for the two skywave HF channel for various values of seed integer used in the random number generator. A typical worst channel, with a seed integer of 50, has been selected to test the performance of the various estimators studied. Table 3.4.1 shows the mean length of the channel for various seed integers tested.

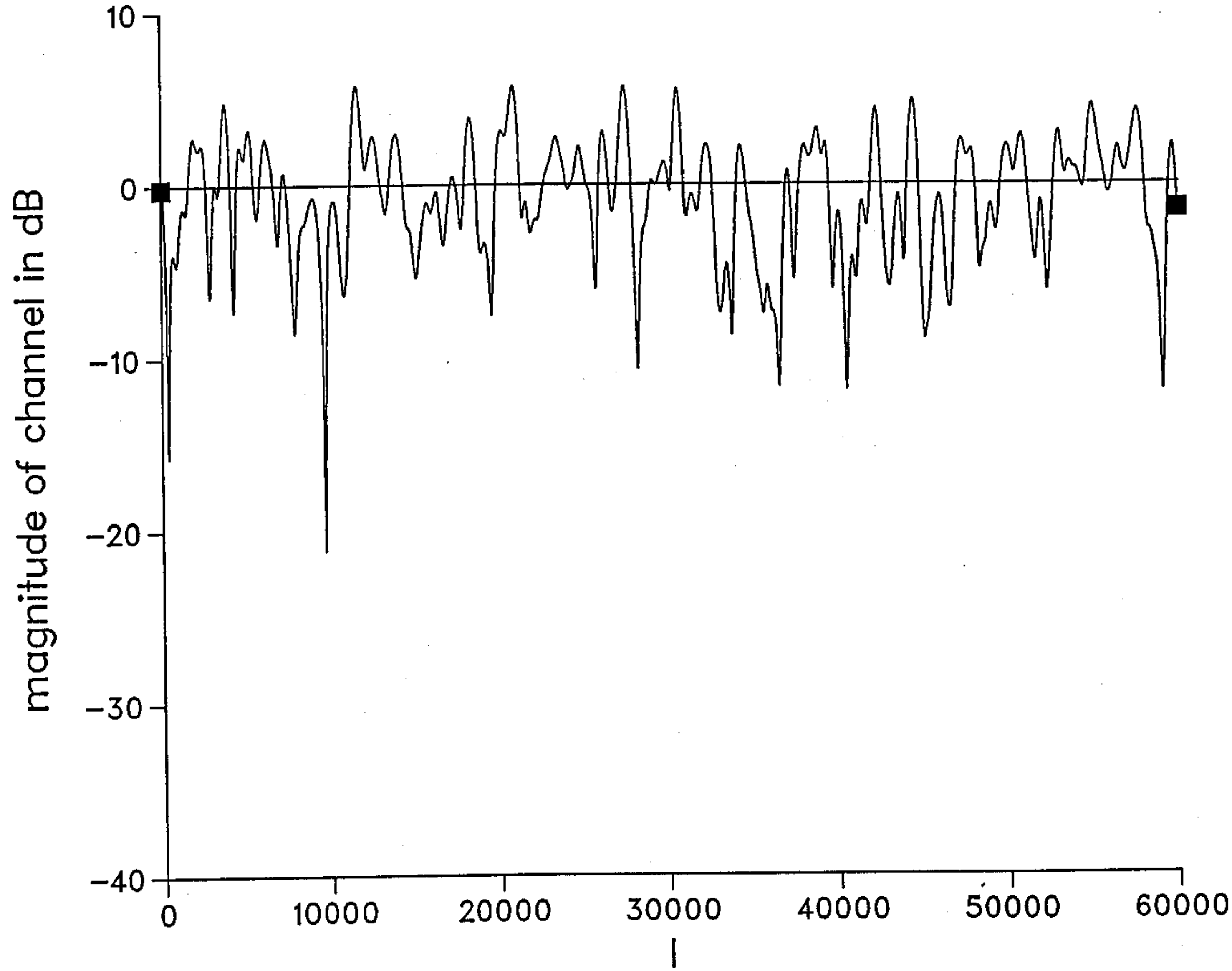


FIGURE 3.4.3 PLOT OF THE MAGNITUDE OF A TWO SKYWAVE HF CHANNEL VS TIME FOR SEED INTEGER = 10

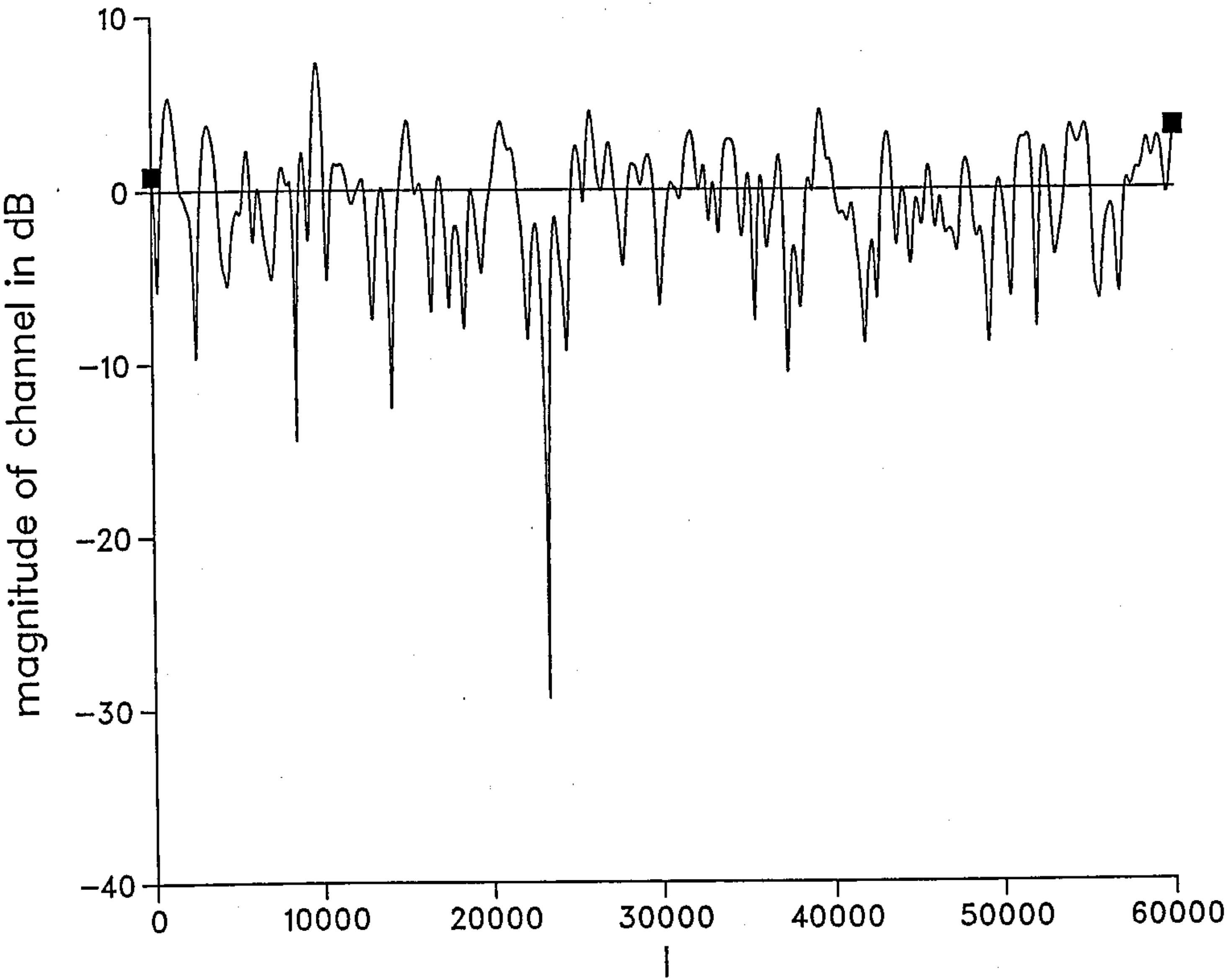


FIGURE 3.4.3 PLOT OF THE MAGNITUDE OF A TWO SKYWAVE HF CHANNEL VS TIME FOR SEED INTEGER = 15

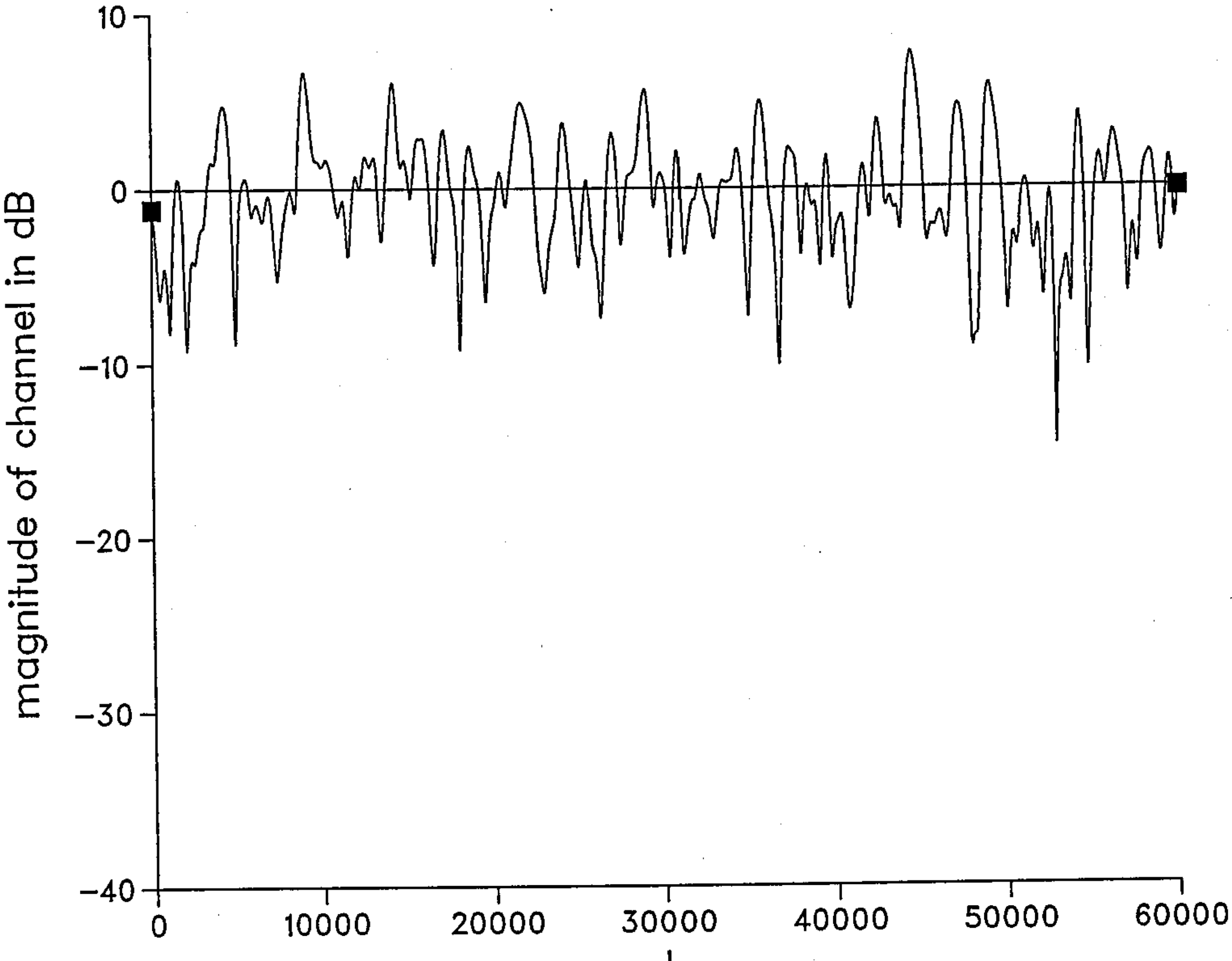


FIGURE 3.4.3 PLOT OF THE MAGNITUDE OF A TWO SKYWAVE HF CHANNEL VS TIME FOR SEED INTEGER = 20

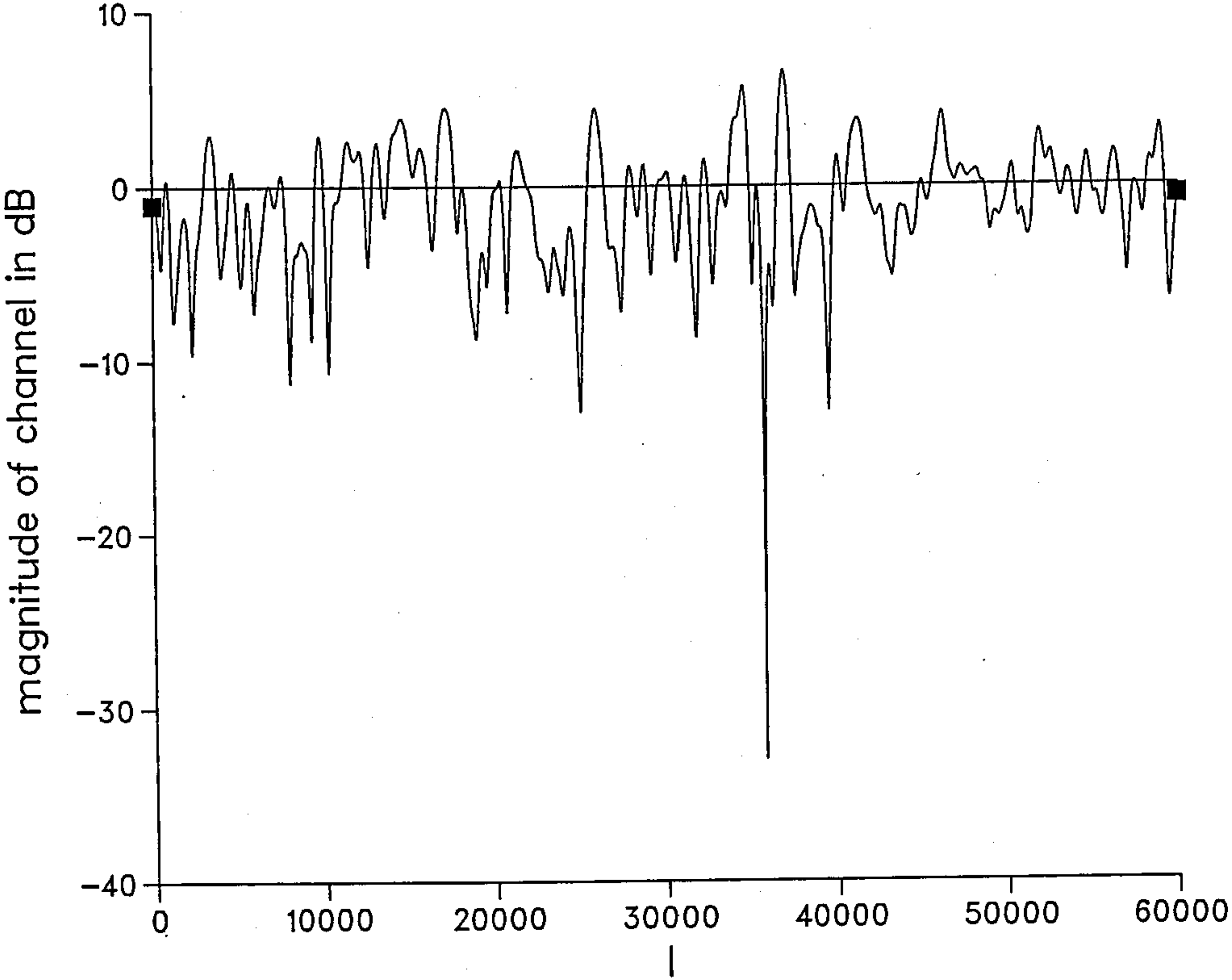


FIGURE 3.4.3 PLOT OF THE MAGNITUDE OF A TWO SKYWAVE HF CHANNEL VS TIME FOR SEED INTEGER = 50

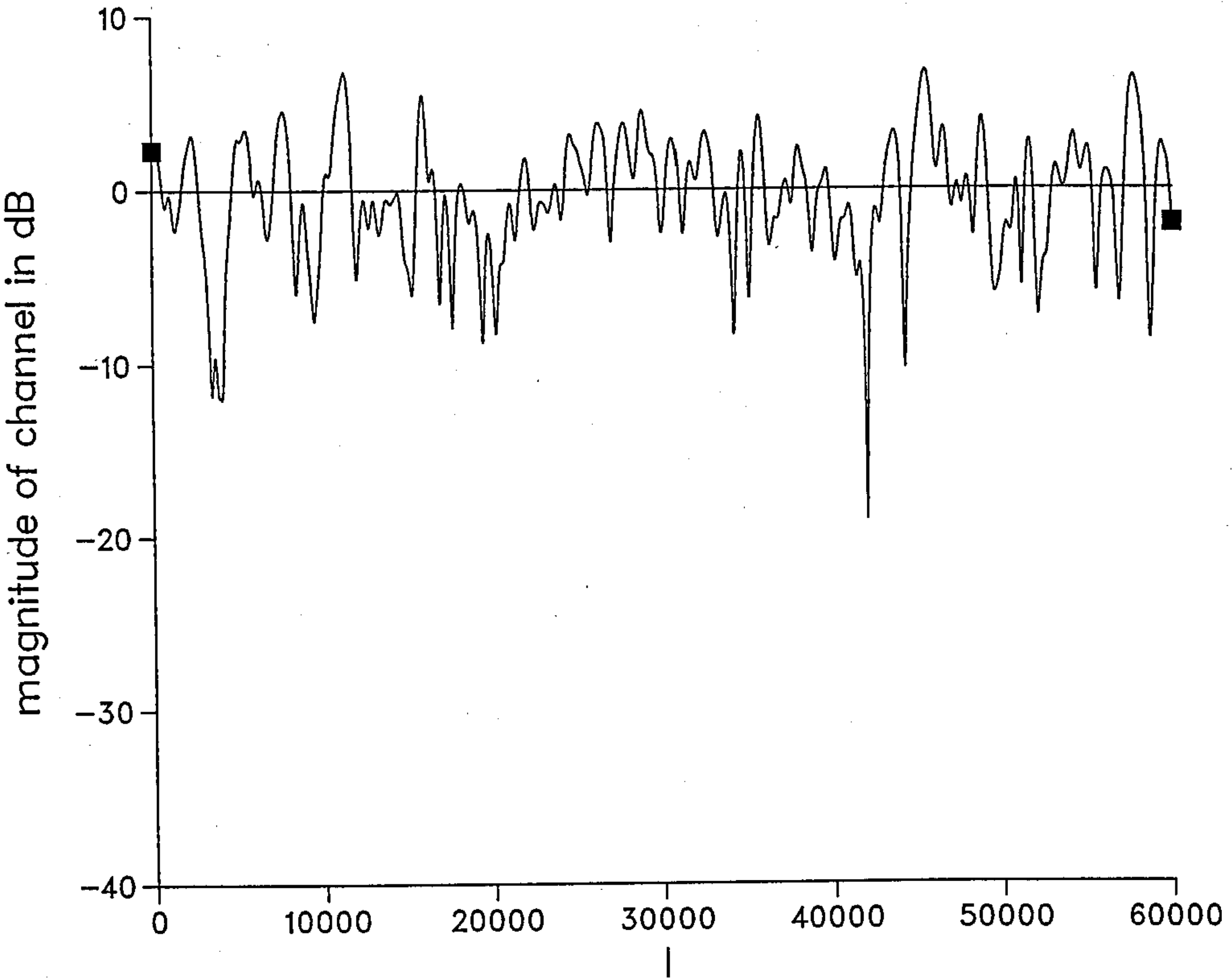


FIGURE 3.4.3 PLOT OF THE MAGNITUDE OF A TWO SKYWAVE HF CHANNEL VS TIME FOR SEED INTEGER = 60

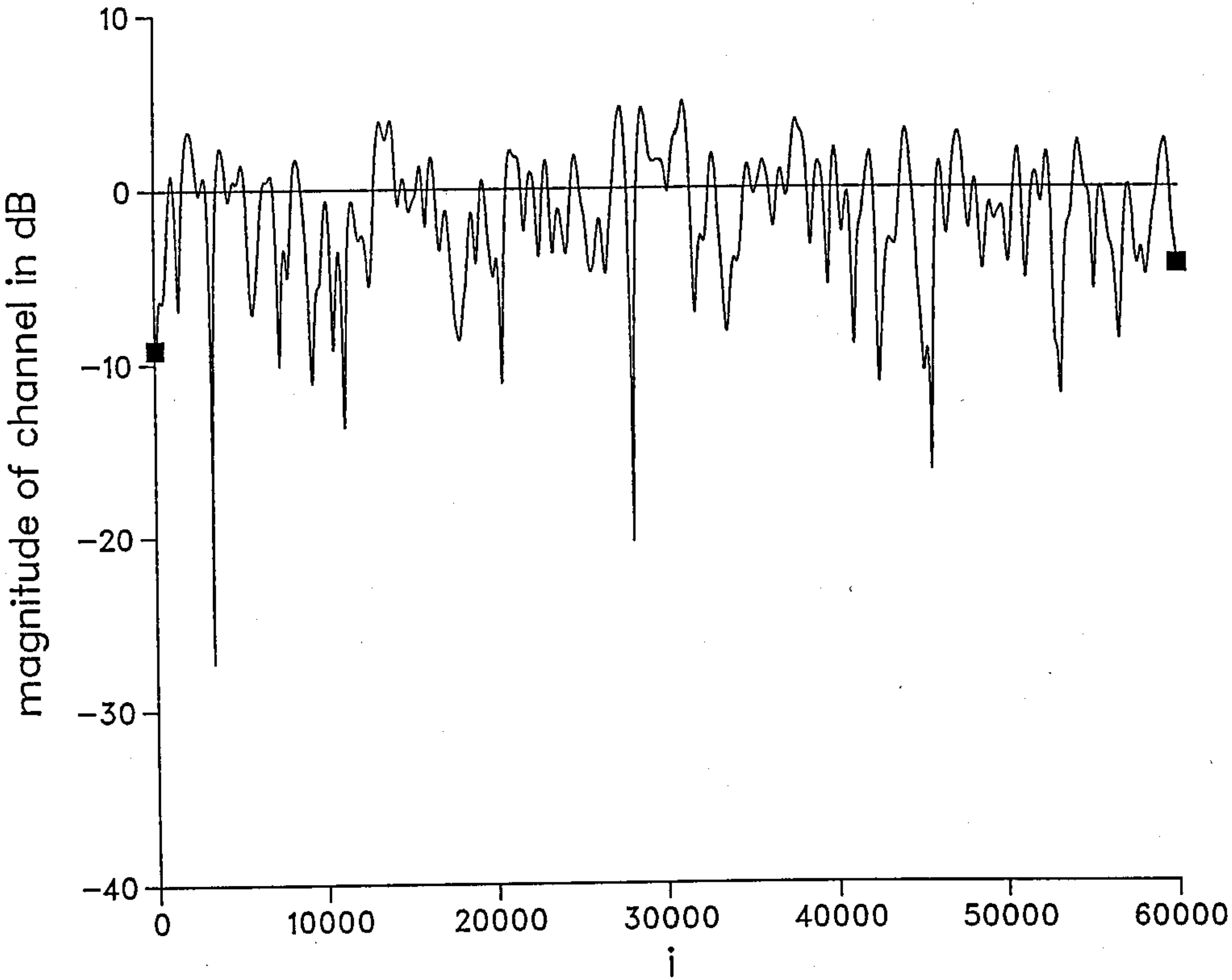


FIGURE 3.4.3 PLOT OF THE MAGITUDE OF A TWO SKYWAVE HF CHANNEL VS TIME FOR SEED INTEGER = 90

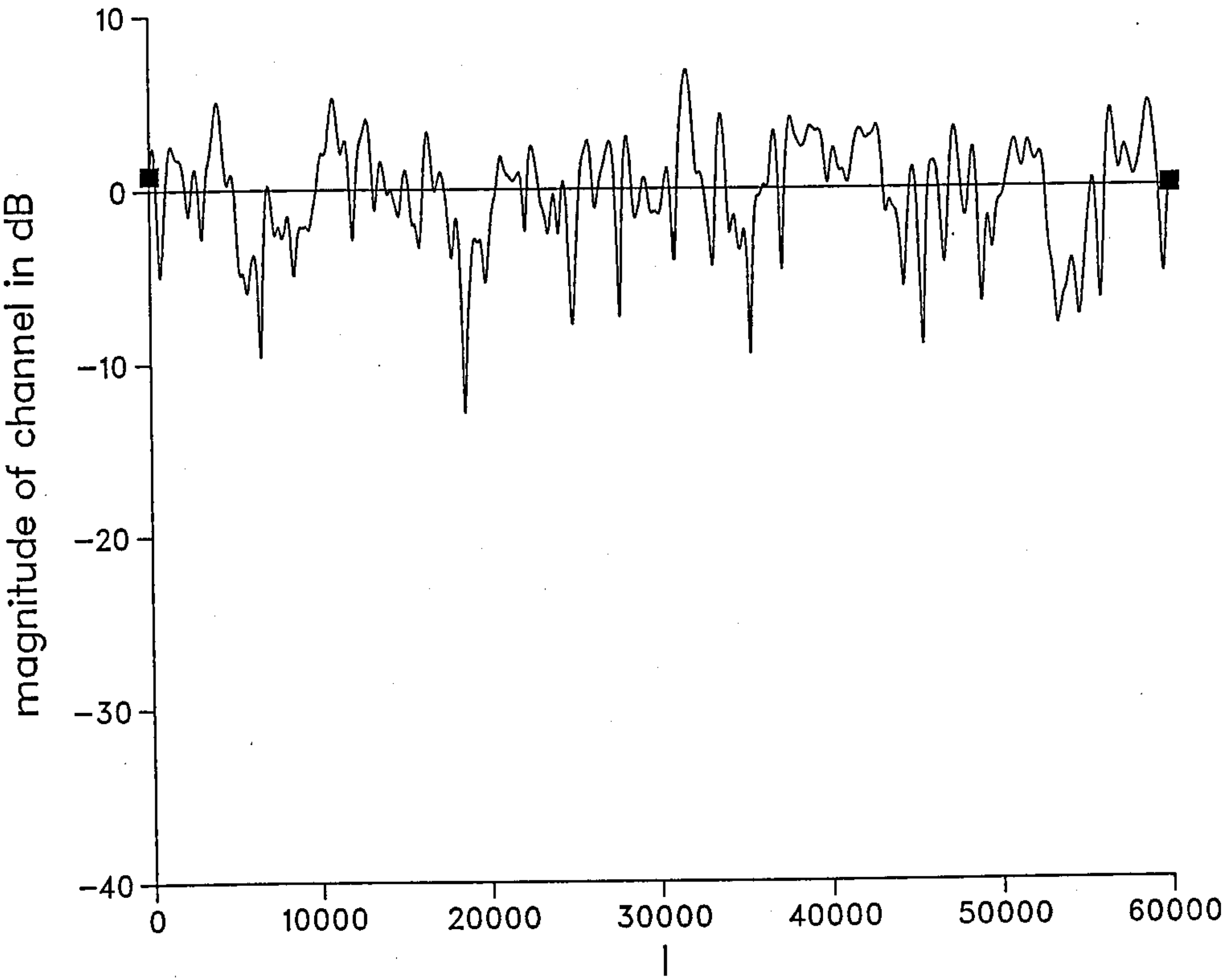


FIGURE 3.4.3 PLOT OF THE MAGNITUDE OF A TWO SKYWAVE HF CHANNEL VS TIME FOR SEED INTEGER = 120

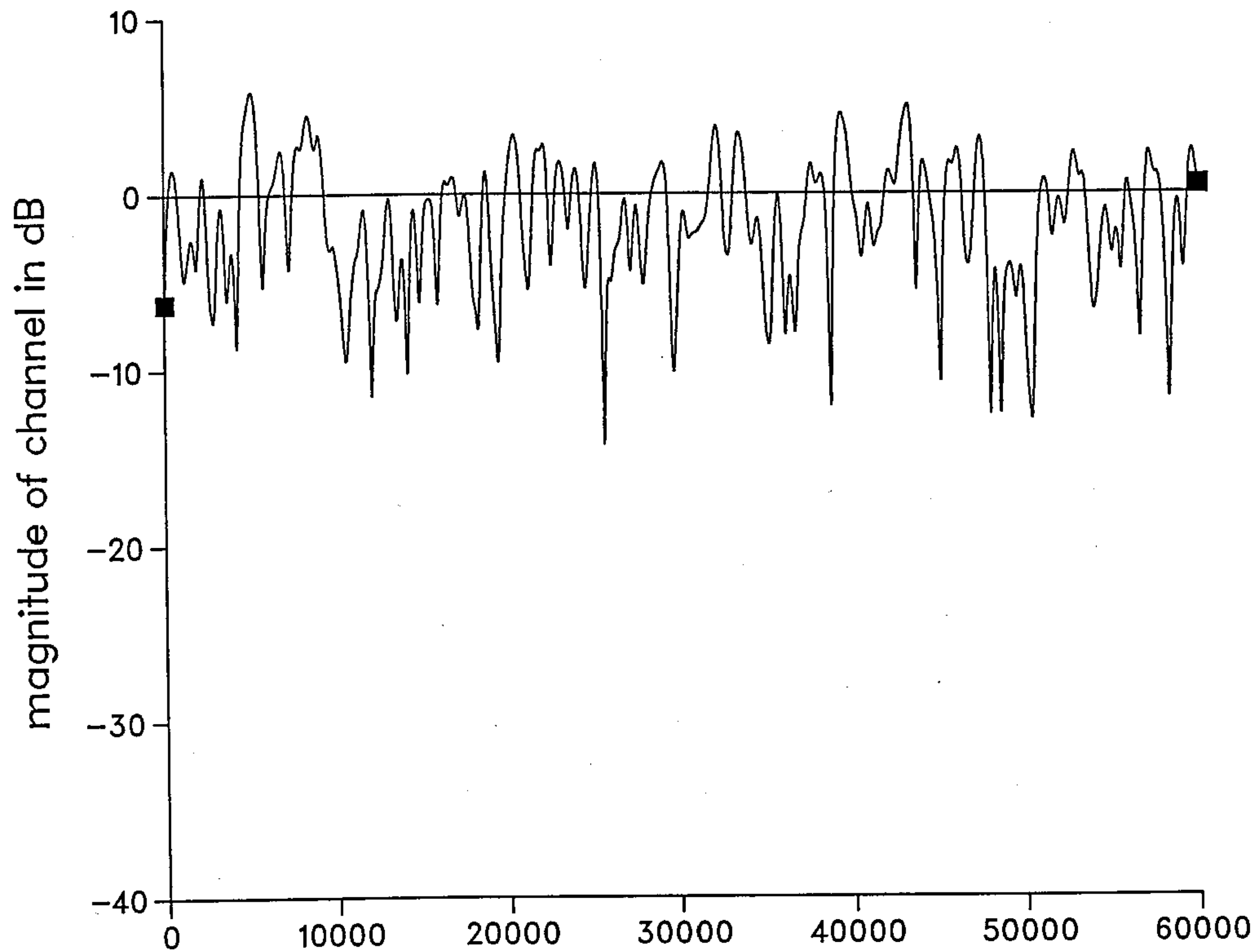


FIGURE 3.4.3 PLOT OF THE MAGNITUDE OF A TWO SKYWAVE HF CHANNEL VS TIME FOR SEED INTEGER = 140

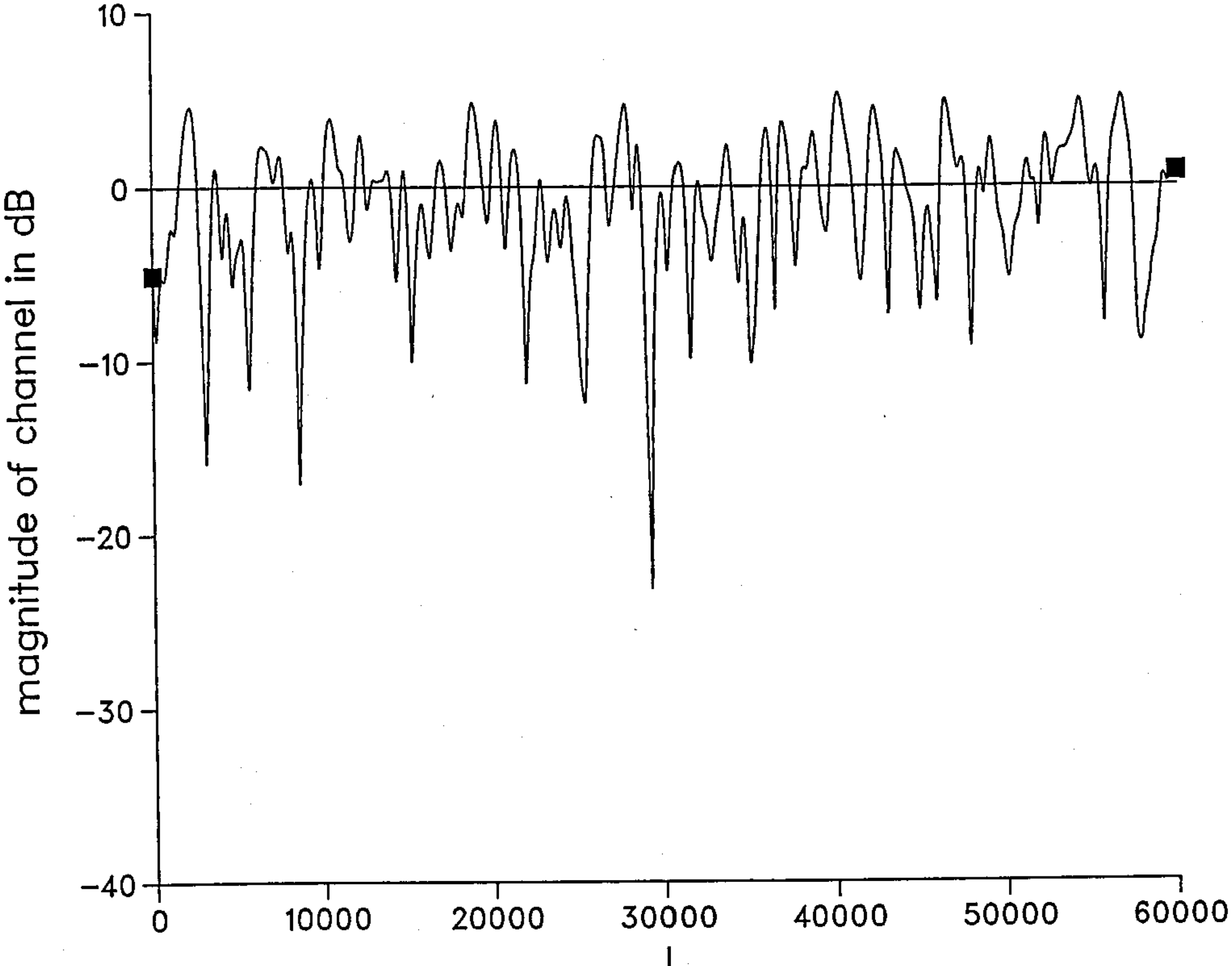


FIGURE 3.4.3 PLOT OF THE MAGNITUDE OF A TWO SKYWAVE HF CHANNEL VS TIME FOR SSEED INTEGER = 160

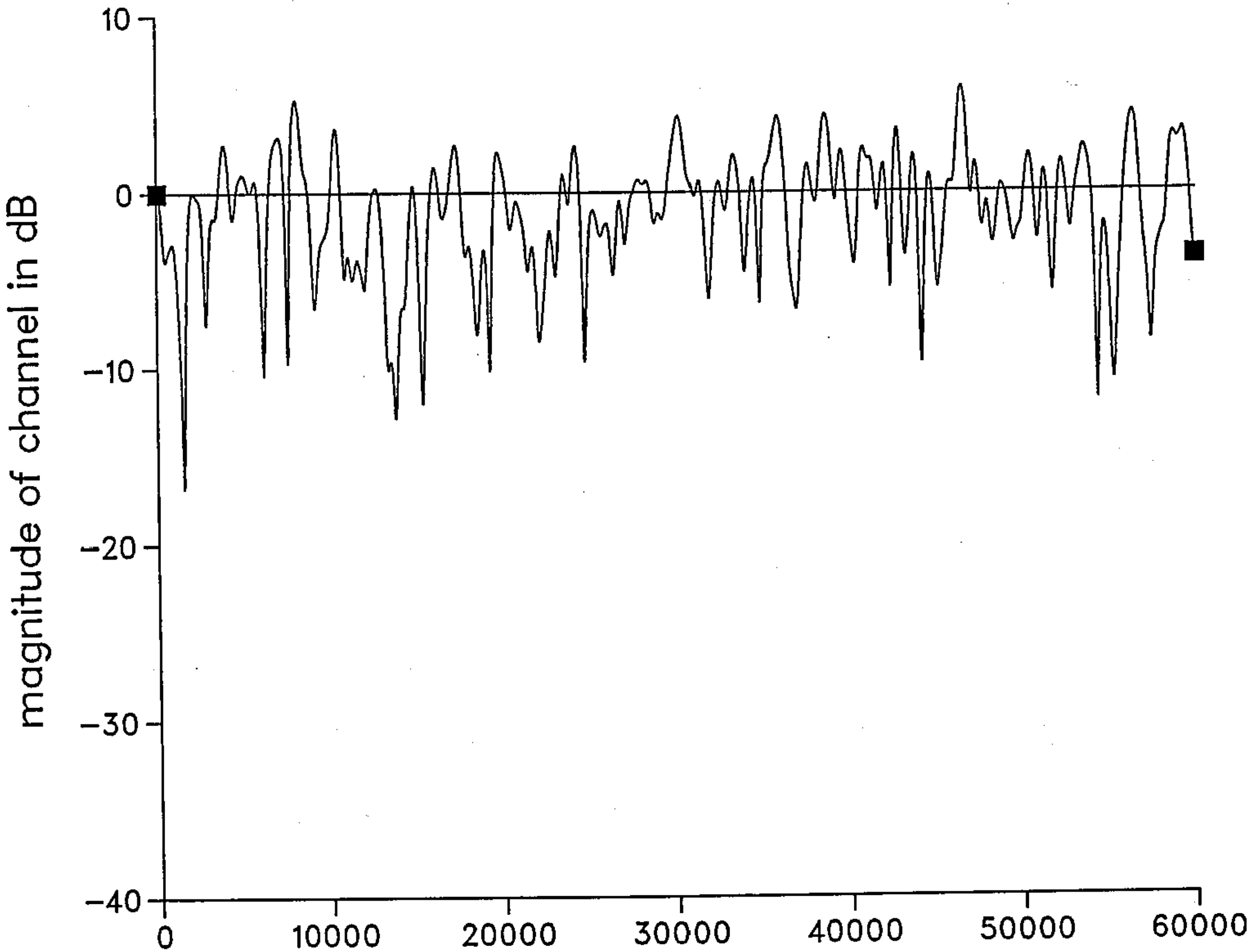


FIGURE 3.4.3 PLOT OF THE MAGNITUDE OF A TWO SKYWAVE HF CHANNEL VS TIME FOR SEED INTEGER = 180

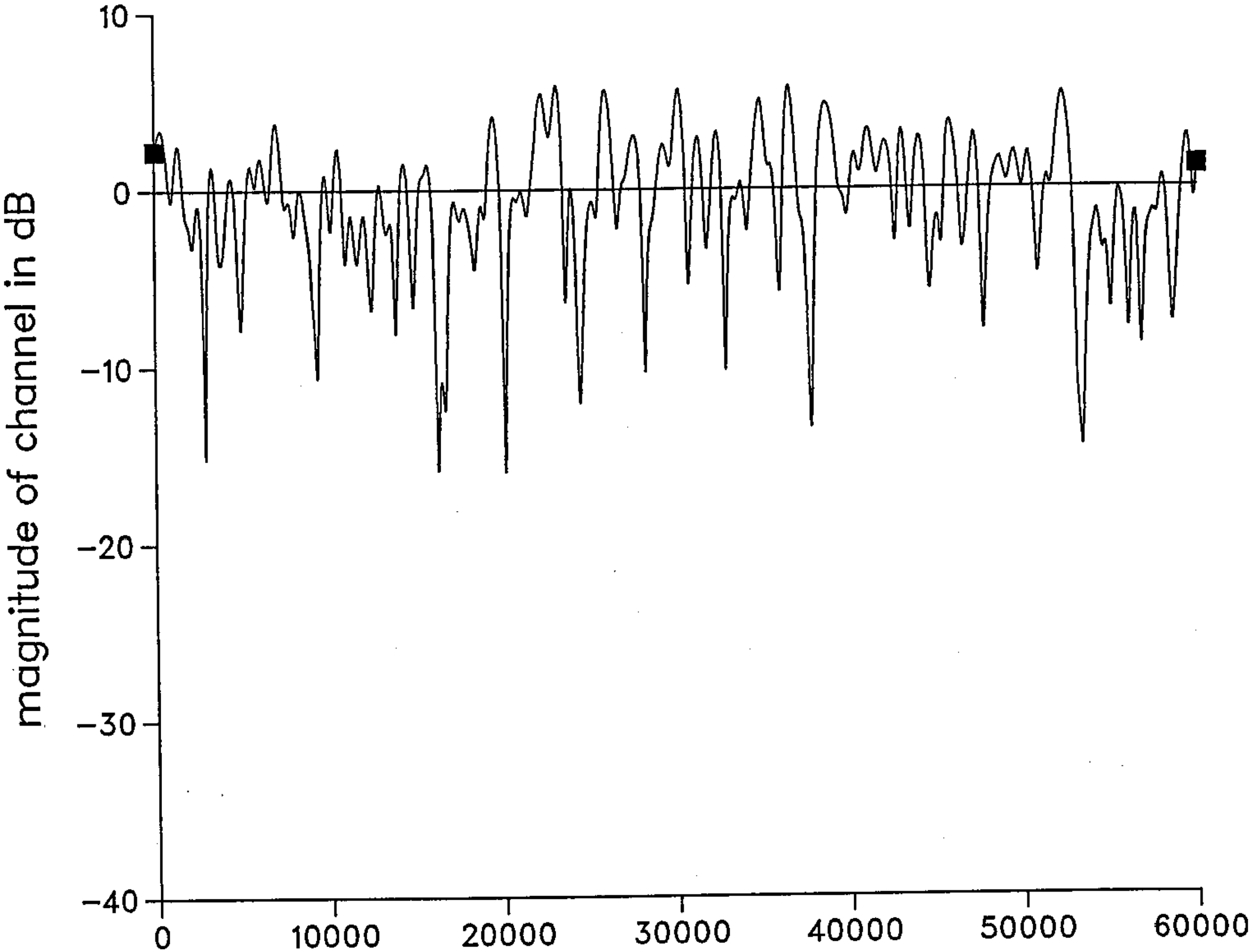


FIGURE 3.4.3 PLOT OF THE MAGNITUDE OF A TWO SKYWAVE HF CHANNEL VS TIME FOR SEED INTEGER = 200

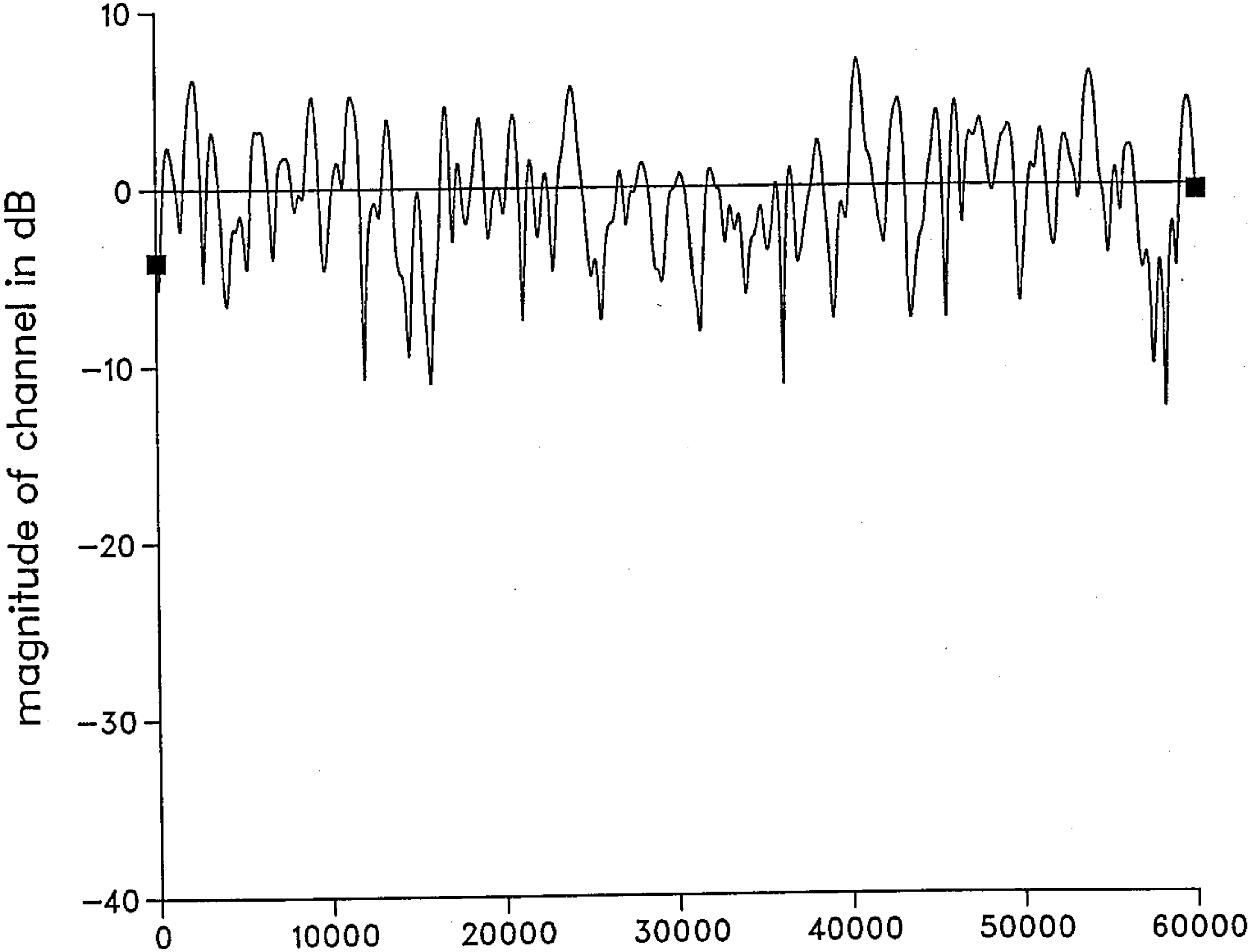


FIGURE 3.4.3 PLOT OF THE MAGNITUDE OF A TWO SKYWAVE HF CHANNEL VS TIME FOR SEED INTEGER = 220

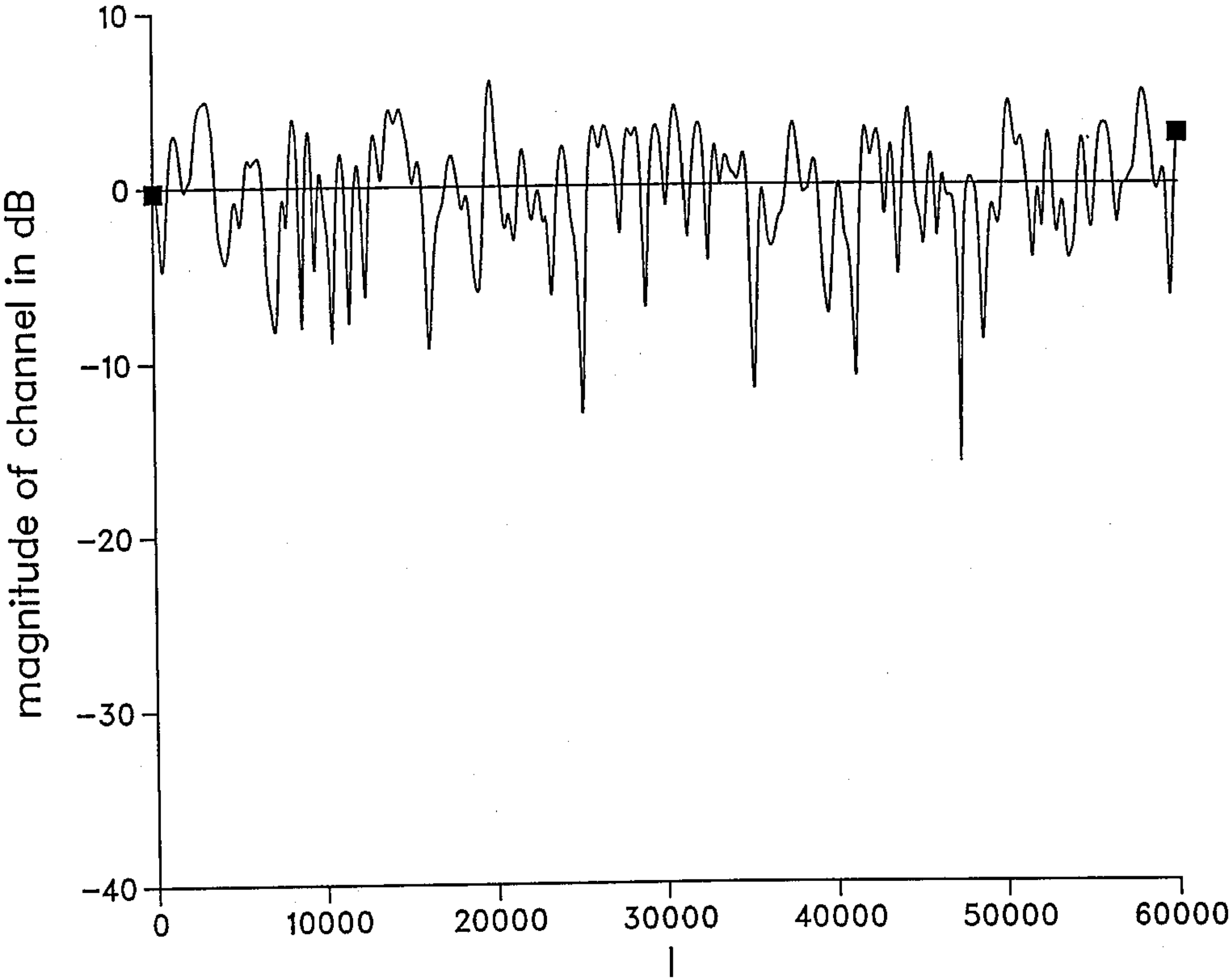


FIGURE 3.4.3 PLOT OF THE MAGNITUDE OF A TWO SKYWAVE HF CHANNEL VS TIME FOR SEED INTEGER = 300

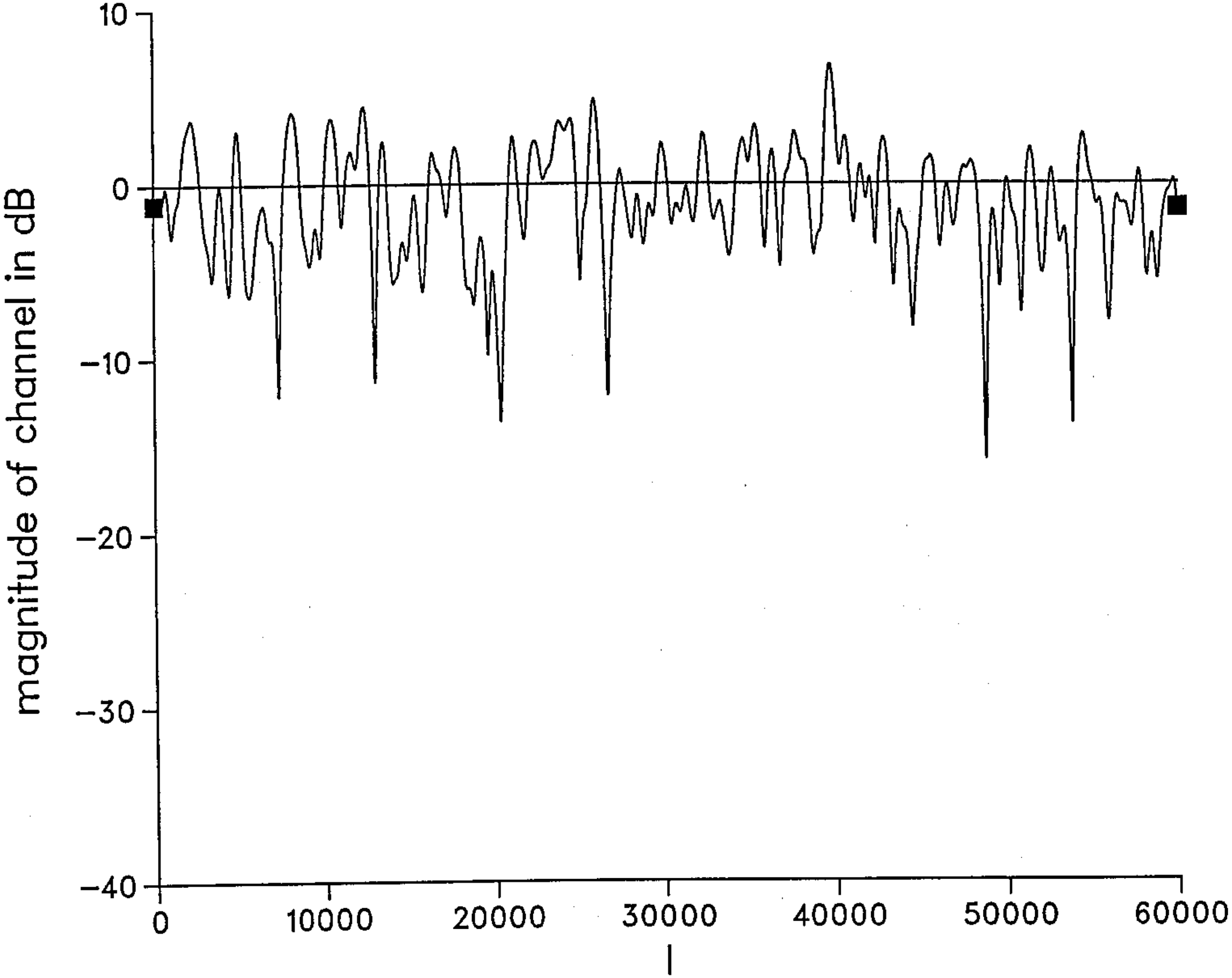


FIGURE 3.4.3 PLOT OF THE MAGNITUDE OF A TWO SKYWAVE HF CHANNEL VS TIME FOR SEED INTEGER = 400

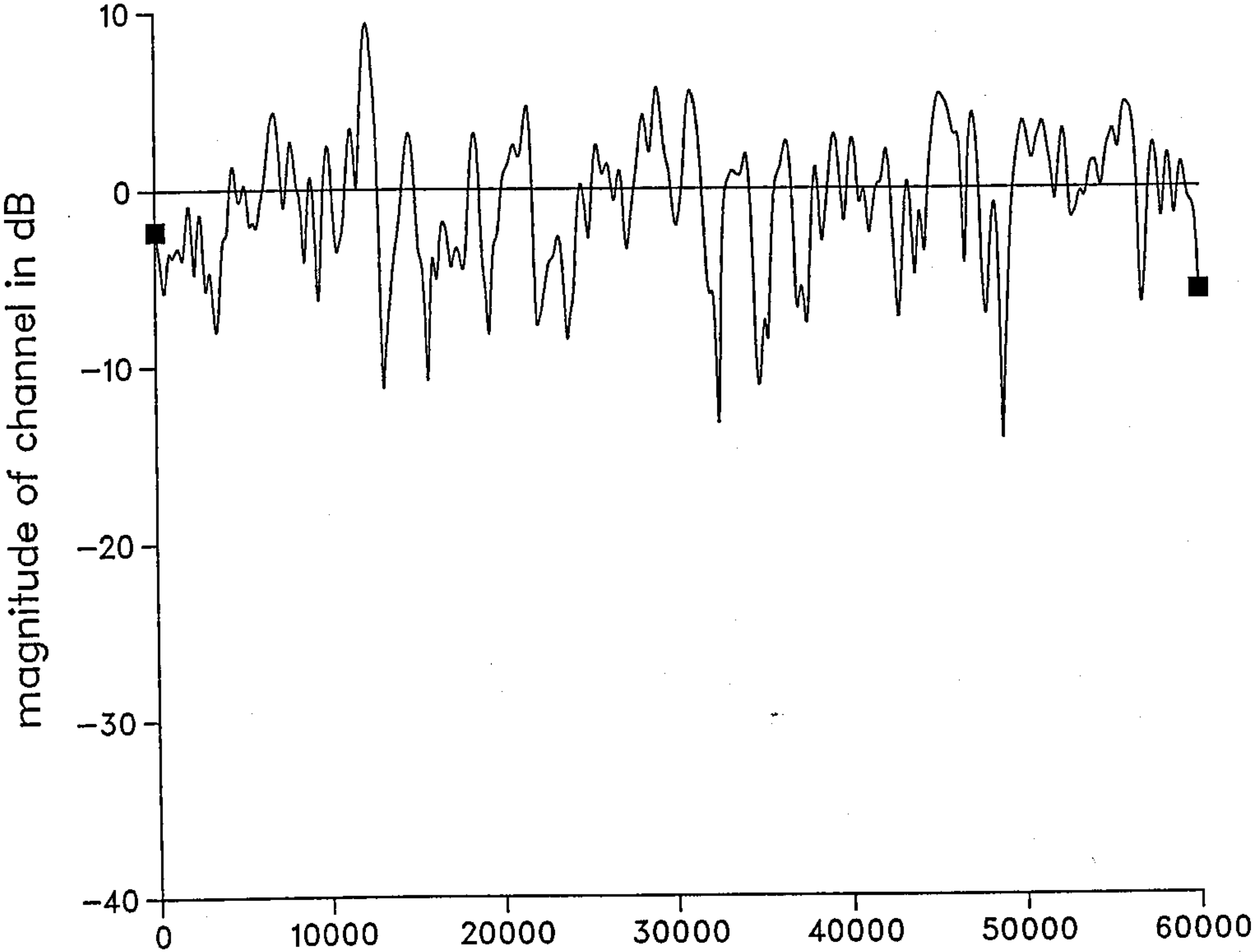


FIGURE 3.4.3 PLOT OF THE MAGNITUDE OF A TWO SKYWAVE HF CHANNEL VS TIME FOR SEED INTEGER = 500

TABLE 3.4.1 MEASURED CHARACTERISTIC OF THE FADING SEQUENCES USED TO MODEL A TWO SKYWAVES CHANNEL

Seed Integer	Mean lenght of channel	Deepest fade (dB)
10	1.0532	-21.2295
15	1.0225	-29.7387
20	1.1485	-14.7629
50	0.9682	-35.8800
60	1.1647	-19.0999
90	0.8943	-27.5285
120	1.1432	-12.9022
140	0.8803	-14.3120
160	1.0322	-23.1956
180	0.9567	-16.8310
200	1.0990	-15.9938
220	1.1522	-12.6362
300	1.1393	-15.9533
400	1.1782	-16.8382
500	1.1403	-14.2718

CHAPTER 4

SIMPLE ESTIMATOR

4.1 INTRODUCTION

In the transmission of digital data over a linear time varying baseband channel that introduces severe amplitude and phase distortion into the signal, a synchronous serial system is often used [8-9]. The received signal, after demodulation and filtering, is sampled once per data symbol and the resulting sample values are fed to the detector. it has been shown that, if additive white Gaussian noise is introduced at the channel output, a near maximum-likelihood detector that is held correctly adjusted for the channel can often achieve a much better tolerance to noise than the corresponding equalizer of optimum design [9,10]. The signal distortion introduced may be removed by means of a conventional near maximum likelihood detector, but the detector requires an accurate knowledge of the sampled impulse response of the channel if its full potential is to be achieved [1,2]. The advantage of this scheme is the relative ease with which an accurate estimate of the sampled impulse response of the linear baseband channel can be achieved [3-6]. Over HF radio link, since the characteristics of the channel are usually unknown prior to a transmission and also possibly vary with time, it is therefore necessary to estimate the sampled impulse response continuously so that the detector is held correctly adjusted for the channel [8-9].

In the following sections, a useful algorithm that can be used to estimate the sampled impulse response of a channel will be discussed. In section 4.2 an outline is given of the various assumptions made in the consideration of the data transmission system, this is necessary in order to simplify the description of the estimator. In section 4.3 an estimate of the channel is derived using the least mean square error criterion. In sections 4.4-4.5 a description of the estimator used in the tests is given. Section 4.6 provides the results of simulation tests carried out on the estimator to assess its performance.

4.2 MODEL OF SYSTEM

The model of the data transmission system assumed in the investigation is shown in Fig 3.4.2.

The signal at the input to the baseband channel is a sequence of regularly spaced impulses $\{s_i\delta(t-iT)\}$ where the s_i are assumed to be statistically independent and equally likely to have any one of its four values $\pm 1 \pm j(j=\sqrt{-1})$.

Transmission starts at time t=iT seconds. The linear baseband channel has an impulse response y(t) with an effective duration of less than (g+1)T seconds where g is a given positive integer. The only noise assumed to be introduced by the channel is stationary white Gaussian noise which is added to the data signal at the output of the transmission path such that the noise waveform w(t) at the output of the receiver filter is a bandlimited Gaussian noise. Thus the output signal form the baseband channel in Fig 3.4.2 is the waveform

$$r(t) = \sum_i s_i y(t-iT) + w(t) \tag{4.2.1}$$

where $s_i y(t-iT)$ is the i^{th} received signal element. The waveform r(t) is sampled once per received signal element, at the time instant {iT}, giving the received samples r_i, where,

$$r_i = \sum_{h=0}^{g} s_{i-h} y_{i,h} + w_i \tag{4.2.2}$$

$$= Y_i S_i^T \tag{4.2.3}$$

The delay in transmission is neglected so that $y_i = 0$ for i<0 and i>g. let Y represent the (g+1) component column vector of the sampled impulse response of the channel.

$$Y_i = [y_{i,0} y_{i,1} \cdots y_{i,g}] \tag{4.2.4}$$

The noise samples w_i are slightly correlated as they have been filtered at the receiver, they have zero mean and variance that is dependent on $0.5N_0$, and are statistically independent of the $\{s_i\}$.

The signal $\{r_i\}$ and s_i are fed to the channel estimator to give an estimate Y'_i of Y at time t=iT, where,

$$Y'_i = [y'_{i,0} y'_{i,1} \cdots y'_{i,g}] \tag{4.2.5}$$

The estimate Y'_i is used by the detector for the detection of the next data symbol on the receipt of r_{i+1}. Details of the channel model are given in chapter 3.

4.3 LEAST SQUARES ESTIMATOR

The sample value of the received signal taken at time t=kT is given by

$$r_k = \sum_{h=0}^{g} s_{k-h} y_h + w_k \tag{4.3.1}$$

where $\{y_h\}$, h=0,1,...,g are the g+1 unknown parameters. $\{y_h\}$ are the sampled impulse response of the channel and these are to be estimated. w_k is the noise component and is a statistically independent Gaussian random variable with zero mean and variance σ^2.

At time t=iT, i samples $\{r_k\}$ have been received and the corresponding i data symbols have been detected. Y'_i is the estimate of Y based on the i received samples $r_1, r_2, \ldots, r_i$. An estimate of the k^{th} received signal r_k is given by,

$$r'_k = \sum_{h=0}^{g} s_{k-h} y'_{i,h} \qquad k = 1, 2, \ldots i \qquad (4.3.2)$$

The error in the estimate of r_k is,

$$e_k = r_k - \sum_{h=0}^{g} s_{k-h} y'_{i,h} \qquad k = 1, 2, \ldots, i \qquad (4.3.3)$$

From equation 4.3.3 the sum of the square of the errors in the estimates of the $\{r_k\}$ upto time t=iT is given by,

$$M_i = \sum_{k=1}^{i} \left(r_k - \sum_{h=0}^{g} s_{k-h} y'_{i,h} \right)^2 \qquad (4.3.4)$$

The sequence of values $\{y'_{i,h}\}$ that minimizes M_i is the least square estimates of Y. Differentiating M_i with respect to each of the $\{y'_{i,h}\}$ gives,

$$\frac{\partial M_i}{\partial y'_{i,j}} = 2 \sum_{k=1}^{i} \left(r_k - \sum_{h=0}^{g} s_{k-h} y'_{i,h} \right) s_{k-j} \qquad j = 0, 1, .., g \qquad (4.3.5)$$

Defining the cross-correlation vector and the auto-correlation matrix respectively, as,

$$b_j = \sum_{k=1}^{i} r_k s_{k-j} \qquad j = 0, 1, \ldots, g \qquad (4.3.6)$$

and

$$q_{h,j}=\sum_{k=1}^{i} s_{k-h}s_{k-j} \quad h=0,1,\ldots,g \quad j=0,1,\ldots,g \tag{4.3.7}$$

The sequence of values $\{y'_{i,h}\}$ that minimizes M_i can be found by setting to zero the partial derivatives of M_i w.r.t $\{y'_{i,h}\}$,

$$2\sum_{k=1}^{i}\left(r_k-\sum_{h=0}^{g} s_{k-h}y'_{i,h}\right)s_{k-j}=0 \qquad j=0,1,\ldots,g$$

or

$$\sum_{h=0}^{g} y'_{i,h}\sum_{k=1}^{i} s_{k-h}s_{k-j}=\sum_{k=1}^{i} r_k s_{k-j} \qquad j=0,1,\ldots,g$$

or

$$\sum_{h=0}^{g} y'_{i,h}q_{h,j}=b_j \qquad j=0,1,\ldots,g \tag{4.3.8}$$

Let

$$B_i=[b_0 b_1 \ldots b_g]^T \tag{4.3.9}$$

and

$$C_i=\begin{pmatrix} s_1 & s_2 & \ldots & s_i \\ s_0 & s_1 & \ldots & s_{i-1} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ s_{1-g} & s_{2-g} & \ldots & s_{i-g} \end{pmatrix} \tag{4.3.10}$$

equation 4.3.8 can be written in matrix form as,

$$C_i C_i^T Y_i' = B_i \tag{4.3.11}$$

If $C_i C_i^T$ is non-singular, then it must have an inverse, so $\{Y'_i\}$

$$Y_i' = A_i B_i \tag{4.3.12}$$

where

$$A_i = (C_i C_i^T)^{-1} \tag{4.3.13}$$

If S_k is a (g+1) component column vector of the detected data symbols and is defined as,

$$\begin{aligned} S_k &= [s_k' s_{k-1}' \ldots s_{k-g}']^T \\ &= [s_k s_{k-1} \ldots s_{k-g}]^T \end{aligned} \tag{4.3.14}$$

it can be shown that,

$$C_i C_i^T = \sum_{k=1}^{i} s_k s_k^T \tag{4.3.15}$$

and

$$B_i = \sum_{k=1}^{i} r_k S_k \tag{4.3.16}$$

Therefore the least squares solution can be written as,

$$Y_i' = \left[\sum_{k=1}^{i} S_k S_k^T \right]^{-1} \sum_{k=1}^{i} r_k S_k \tag{4.3.17}$$

When i is large, equation 4.3.17 is replaced by,

$$E[S_k S_k^T] Y_i' = E[r_k S_k] \tag{4.3.18}$$

where E is the expectation vector. Equation 4.3.18 becomes,

$$\begin{pmatrix} E[s_k s_k] & E[s_k s_{k-1}] & \dots & E[s_k s_{k-g}] \\ E[s_{k-1} s_k] & E[s_{k-1} s_{k-1}] & \dots & E[s_{k-1} s_{k-g}] \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ E[s_{k-g} s_k] & E[s_{k-g} s_{k-1}] & \dots & E[s_{k-g} s_{k-g}] \end{pmatrix} \begin{pmatrix} y'_{i,0} \\ y'_{i,1} \\ \cdot \\ \cdot \\ \cdot \\ y'_{i,g} \end{pmatrix} = \begin{pmatrix} E[r_k s_k] \\ E[r_k s_{k-1}] \\ \cdot \\ \cdot \\ \cdot \\ E[r_k s_{k-g}] \end{pmatrix}$$

(4.3.19)

Since the $\{s_i\}$ are uncorrelated and also statistically independent of the $\{w_i\}$, equation 4.3.19 reduces to

$$\begin{pmatrix} E[s_k s_k] & 0 & \cdot\ \cdot\ \cdot & 0 \\ 0 & E[s_{k-1} s_{k-1}] & \cdot\ \cdot\ \cdot & 0 \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ 0 & \cdot & \cdot\ \cdot\ 0 & E[s_{k-g} s_{k-g}] \end{pmatrix} \begin{pmatrix} y'_{i,0} \\ y'_{i,1} \\ \cdot \\ \cdot \\ \cdot \\ y'_{i,g} \end{pmatrix} = \begin{pmatrix} y_0 & E[s_k s_k] \\ y_1 & E[s_k s_{k-1}] \\ & \cdot \\ & \cdot \\ & \cdot \\ y_g & E[s_k s_{k-g}] \end{pmatrix}$$

(4.3.20)

The inverse of the non-singular diagonal matrix on the left hand side of equation 4.3.20 is

$$\begin{pmatrix} (E[s_k s_k])^{-1} & 0 & \cdot\ \cdot\ \cdot & 0 \\ 0 & (E[s_{k-1} s_{k-1}])^{-1} & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ 0 & \cdot & \cdot\ \cdot\ 0 & (E[s_{k-g} s_{k-g}])^{-1} \end{pmatrix}$$

Premultiplying equation 4.3.20 by this inverse, gives,

$$Y_i' = Y \tag{4.3.21}$$

Hence, when i is large and the statistical properties of the $\{s_k\}$ and $\{w_k\}$ are met, the estimate Y_i' converges to the actual value of Y.

4.4 FEEDFORWARD TRANSVERSAL FILTER ESTIMATOR

Magee and Proakis [2] have developed a channel estimator, its structure identical to that of the linear feedback transversal equalizer [11]. The detected data symbols s_i' are fed to a linear feedforward transversal filter (fig 4.4.1), simulating the channel. The (g+1) tap gains of this filter are adjusted in such a way so as to minimise the mean square error between the actual received sample r_i and its estimate r_i' at the output of the estimator. The detected data symbols s_i' are assumed to be correct, so that $s_i' = s_i$ for each i. The g+1 tap gains form the estimate Y_i' of the sampled impulse response of the channel, after adjustment of the tap gains, on receipt of r_k.

The estimator operates as follows, each square marked T in Fig 4.4.1 is a store that holds the corresponding detected data symbols s_{i-h}'. Each time the stores are triggered, on the reception of a sample r_i, the stored values are shifted one place to the right. At time t=iT, the estimator is fed with the received sample r_i' and the detected data symbol s_i'. If Y_{i-1}' is the previous stored estimate of Y, then an estimate r_i' of r_i at the output of the estimator is given by,

$$r_i' = \sum_{h=0}^{g} s_{i-h}' y_{i-1,h}' \tag{4.4.1}$$

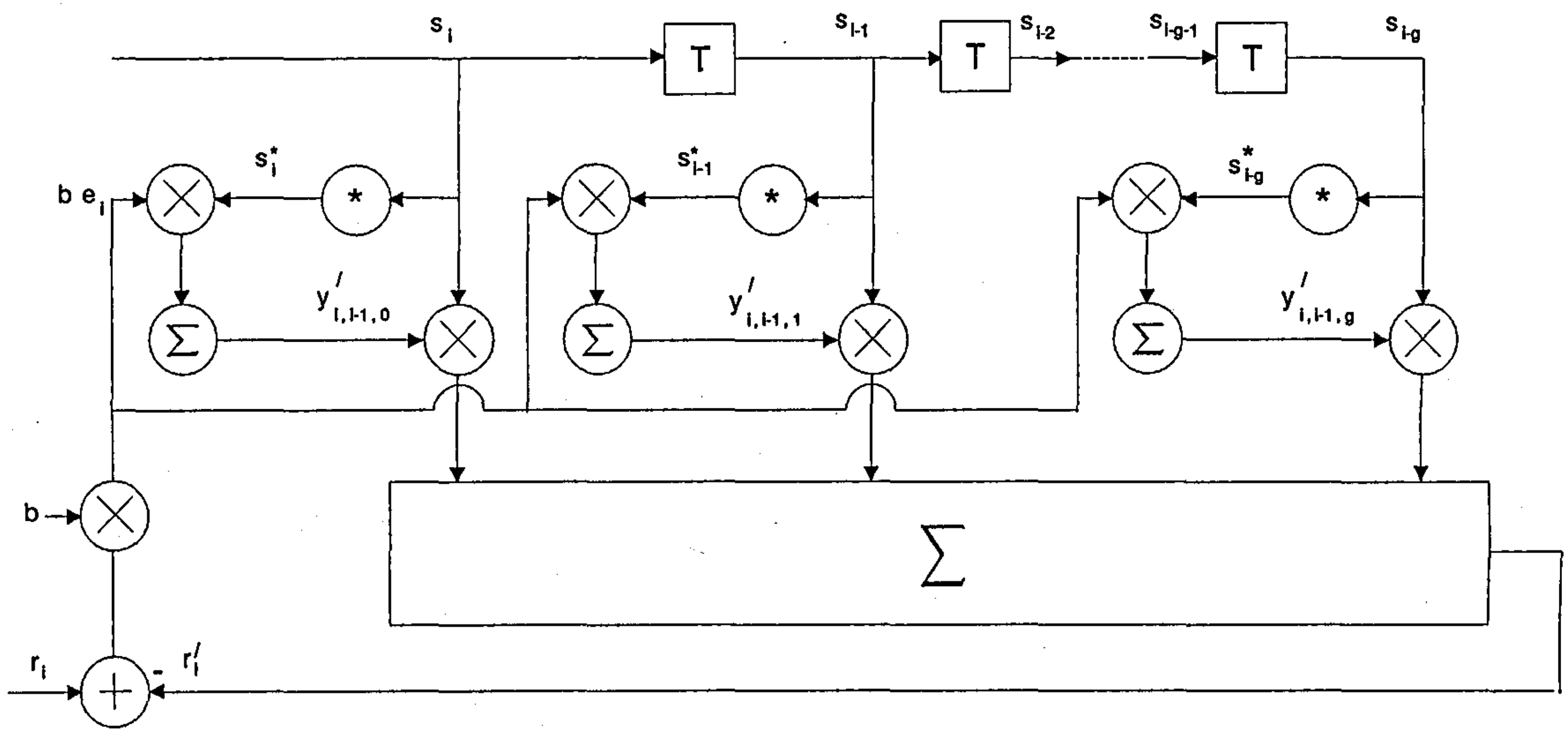


FIGURE 4.4.1 LINEAR FEEDFORWARD TRANSVERSAL FILTER

The error in the estimate is,

$$e_i = r_i - r_i'$$ (4.4.2)

which is then scaled by a small positive quantity b, and the resulting signal be_i is multiplied by s'_{i-h} for h=0,1,...,g, the products are added to the corresponding components of the previous estimate Y'_{i-1}, giving the new stored estimate Y'_i, where the $(i+1)^{th}$ component of Y'_i is given by,

$$y'_{i,h} = y'_{i-1,h} + be_i s'_{i-h}$$ (4.4.3)

Equation 4.4.3 is the steepest descent algorithm for adjusting the tap gains of the estimator [2]. When minimized in the absence of noise, the values of the tap gains are the values of the sampled impulse response of the channel (see section 4.3). The quantity b in eqn 4.4.3 is referred to as the step size of the estimator. The smaller the value of b, the smaller is the affect of additive noise on Y'_i but the slower is the response of Y'_i to changes in Y [1]. b need not necessarily be a constant. The feedforward transversal linear estimator can be easily implemented and can track with great ease slow variations in the channel response.

4.5 HF CHANNEL ESTIMATOR

In this section the linear feedforward transversal filter has been modified to handle the complex impulse response of an HF link, and enhanced by various prediction techniques [7]. In the estimator discussed in section 4.4 the estimator is derived from the immediate previous estimate (eqn 4.4.3). However in HF radio links, where the characteristics of the channel often vary rapidly, it is beneficial to make some sort of prediction of the sampled impulse response, and use this prediction in the estimation process [6].

4.5.1 SIMPLE ESTIMATOR

In section 4.4 the steepest descent method is used to obtain an estimate of Y_i, which is given by equation 4.4.3. The quantities in equation 4.4.3 are all real, whereas in an HF channel the input, output and the channel response are complex valued quantities, therefore equation 4.4.3 has to be modified to,

$$y'_{i,h} = y'_{i-1,h} + b e_i s'^{*}_{i-h} \tag{4.5.1.1}$$

This can be expressed in vector form as,

$$Y'_i = Y'_{i-1} + b e_i S'^{*}_i \tag{4.5.1.2}$$

where S'^{*}_i is a (g+1) component vector given by,

$$S'^{*}_i = \left[s'^{*}_i s'^{*}_{i-1} \ldots s'^{*}_{i-g} \right]^T \tag{4.5.1.3}$$

e_i is the error in the estimate of the i^{th} received signal r_i,

$$e_i = r_i - \sum_{h=0}^{g} s'_{i-h} y'_{i-1,h} \tag{4.5.1.4}$$

The estimator uses the prediction of Y_i when it is available, instead of Y'_{i-1} to give a better estimate Y'_i in equation 4.5.1.2. At time t=(i-1)T, a prediction is made of

Y_i, it is designated $Y'_{i,i-1}$. The estimator uses this prediction to form the estimate r'_i of r_i, which is given as,

$$r'_i = \sum_{h=0}^{g} s'_{i-h} y'_{i,i-1,h} \tag{4.5.1.5}$$

Equation 4.5.1.4 is modified, to give the error e_i,

$$e_i = r_i - \sum_{h=0}^{g} s'_{i-1} y'_{i,i-1,h} \tag{4.5.1.6}$$

The HF channel estimator is designed to be used with a near maximum likelihood detector (see Introduction). This detector introduces a delay of (n-1) sampling intervals in the detection of a symbol, so s_i is detected after the reception of r_{i+n-1}. Therefore, the received samples r_i, r_{i+1}, ..., r_{i+n-1} must be stored in a shift register, so that they are available at the appropriate time for the generation of the error signals e_i, e_{i+1}, ..., e_{i+n-1}. At time t=(i+n-1)T, the inputs to the channel estimator are s'_i and r_i which give the estimate Y'_i, which is then used by the detector on the receipt of r_{i+n} for the detection of s_{i+1}. Thus, the delay in estimation is n sampling intervals as Y'_i is only available on the receipt of r_{i+n}. Ideally Y'_{i+n} should be used in the detection of s_{i+1}, especially when n is large or the channel impulse response varies rapidly.

To summarize, there is a delay in detection of n-1 sampling intervals and a delay in estimation of n sampling intervals. On the receipt of r_{i+n}, with Y'_i known, two predictions $Y'_{i+1,i}$ of Y_{i+1} and $Y'_{i+n,i}$ of Y_{i+n} are required. $Y'_{i+1,i}$ is used by the estimator to obtain the next estimate Y'_{i+1} of Y_{i+1}. $Y'_{i+n,i}$ is used by the detector for the detection of s_{i+1}.

*However, with an adaptive filter ahead of the detector their is an important advantage gained, in that it avoids the need to predict over many sampling intervals [12], as the sampled impulse response of the channel anf filter is minimum phase [10]. The estimator needs to do only a one step prediction of the sampled impulse response of the channel [13-14].

4.5.2 LEAST SQUARES FADING MEMORY PREDICTION

The least squares fading memory prediction techniqne is used in the tests as it overcomes the limitations of the simple memory prediction and simple fading memory prediction [6]. In least squares fading memory prediction . The set of g+1 polynomials of given degree (0, 1 or 2) are determined, each of which gives the weighted least squares fit to the components in the corresponding location in the vectors Y'_i , $Y'_{i+1,i}$,..., these values are used at time t=(i+1)T or t=(i+n)T to determine the g+1 components of $Y'_{i+1,i}$ or $Y'_{i+n,i}$ respectively. The chosen polynomial is such that it gives the best fit to the sequence of past observations and the weighted sum of the squares of the error function is minimized [7]. In [7] the technique is applied to the prediction of the value of a variable parameter derived from past(noisy or inaccurate) observations of the parameter, the observations being unaffected by the prediction. The technique can also be applied to the prediction of a variable parameter from past updated estimates of the parameter, the prediction here influencing the subsequent updated estimates. The arrangement of degree-0, 1 and 2 least squares fading-memory prediction is given in Table 4.5.2.1.

where E_i is given by,

$$E_i = Y'_i - Y'_{i,i-1} \tag{4.5.2.1}$$

The quantities $Y''_{i+1,1}$ and $Y'''_{i+1,1}$ are functions of the first and second differentials of $Y'_{i+1,i}$, with respect to time. At start up the values of the various quantities in the arrangement in Table 4.5.2.1 are ,

TABLE 4.5.2.1 LEAST SQUARES FADING-MEMORY PREDICTION

DEGREE OF POLYNOMIAL	ONE STEP AND n STEP PREDICTION AT TIME t=iT
0	$Y'_{i+1,i} = Y'_{i,i-1} + (1-\theta)E_i$ $Y'_{i+n,i} = Y'_{i+1,i}$
1	$Y''_{i+1,i} = Y''_{i,i-1} + (1-\theta)^2 E_i$ $Y'_{i+1,i} = Y'_{i,i-1} + Y''_{i+1,i} + (1-\theta^2)E_i$ $Y'_{i+n,i} = Y'_{i+1,i} + (n-1)Y''_{i+1,i}$
2	$Y'''_{i+1,i} = Y'''_{i,i-1} + \frac{1}{2}(1-\theta)^3 E_i$ $Y''_{i+1,i} = Y''_{i,i-1} + 2Y'''_{i+1,i} + 1.5(1-\theta)^2(1+\theta)E_i$ $Y'_{i+1,i} = Y'_{i,i-1} + Y''_{i+1,i} - Y'''_{i+1,i} + (1-\theta^3)E_i$ $Y'_{i+n,i} = Y'_{i+1,i} + (n-1)Y''_{i+1,i} + (n-1)^2 Y'''_{i+1,i}$

$$Y'_{1,0} = Y'_1 \approx Y_1 \qquad (4.5.2.2)$$

$$Y''_{1,0} = Y'''_{1,0} = 0 \qquad (4.5.2.3)$$

The parameter θ is a real constant in the range 0 to 1. Increasing the value of θ towards 1, results in the weight factor that is used in fitting the polynomials to decrease more slowly with time, effectively involving more estimates in the prediction process.

4.6 RESULTS OF SIMULATION

Computer simulation tests have been carried out on the channel estimator described in section 4.5. The results of which are given in Tables 4.6.1-4.6.6 and Fig 4.6.1-4.6.16.

The signal/noise ratio (ψ), is defined as,

$$\psi = 10\log_{10}\frac{E_b}{\frac{1}{2}N_0} \qquad (4.6.1)$$

where E_b, is the average transmitted and received energy per bit at the input and output respectively of the HF radio link, and is unity. The two sided power spectral density of the white Gaussian noise at the output of the HF radio link is $0.5N_0$.

Every measurement has involved the transmission of 60000 data symbols. In error measurements the first 5000 received samples are ignored in order to eliminate any transient behaviour of the estimator which may be present at start up. The error measurements are made over the remaining 55000 received samples, signifying the region of steady state performance of the estimator. The error measurements are,

$$\xi_1 = 10\log_{10}\left(\frac{1}{55000}\sum_{i=5001}^{60000} |Y_i - Y'_{i,i-1}|^2\right) \tag{4.6.2}$$

and

$$\xi_2 = 10\log_{10}\left(\frac{1}{55000}\sum_{i=5001}^{60000} \frac{|Y_i - Y'_{i,i-1}|^2}{|Y_i|^2}\right) \tag{4.6.3}$$

where ξ_1 is called the mean square estimation error and is a measure of the actual error in $Y'_{i,i-1}$. ξ_2 is called the mean square normalized estimation error and is a measure of the normalized or relative error in $Y'_{i,i-1}$. ξ_2 is obtained to give a more realistic picture of the actual error in $Y'_{i,i-1}$ as the actual length of Y_i is taken into account in the error measurement.

The number of components (g+1) in the sampled impulse response of the channel is taken to be 21. At start up, it is assumed that $Y'_{1,0} = Y_1$, which is the actual channel sampled impulse response at the first sampling instant. In the tests b and θ have been optimized to a high degree of accuracy, so that the error in the estimation/prediction of the sampled impulse response of the channel, defined by equations 4.6.2-4.6.3, is minimized.

Tables 4.6.1-4.6.6 show the mean squared error for various prediction algorithms used in the tests. Figures 4.6.1-4.6.5 show the relative performance in the error measurements used for the various least squares fading memory prediction algorithms tested.

From the graphs in Figures 4.6.1-4.6.5, it can be seen that the degree 0 least squares fading memory prediction performs considerably worse than the degree 1 and degree 2 prediction algorithms, which is expected as the channel is time variant. At low signal/noise ratios (high noise levels), the degree 1 performs better than the degree 2 prediction algorithm, while at high signal/noise ratios (low noise level), the degree

2 performs better than the degree 1 prediction algorithm. The degree 2 predictor tries to obtain a quadratic least squares fit, while the degree 1 predictor obtains a linear least squares fit. The degree 2 predictor is able to track noise more effectively than a degree 1 predictor. At low signal/noise ratios (high noise levels), the degree 2 predictor can track the noisy channel sampled impulse response more accurately, thereby, resulting in a greater mean square error in estimation. The degree 1 is unable to track as accurately, thereby, resulting in an improvement in performance over the degree 2 predictor. At high signal/noise ratios (low noise levels), the degree 2 predictor, which obtains a quadratic fit, would be expected to perform better than a degree 1 predictor, which obtains a linear fit.

Figure 4.6.6 shows the channel characteristic of the two skywave HF channel for seed integer = 50. The first 5000 samples are neglected for the sake of convenience in comparison with figures 4.6.7-4.6.16.

Figures 4.6.7-4.6.10 show the steady state performance of the system described in section 4.5, using the degree 0, 1, 2 least squares fading memory prediction algorithms, respectively. The parameter in Figure 4.6.7-4.6.10, is the square of the error in $Y'_{i,i-1}$ measured in dB, and is,

$$\xi_i = 10\log_{10}|Y_i - Y'_{i,i-1}|^2 \tag{4.6.4}$$

From the graphs it can be observed that the linear feedforward estimator with degree 0, 1, 2 least squares fading memory prediction does not suffer from any instability.

Figures 4.6.11-4.6.16 show the steady state performance of the system described in section 4.5. The parameter in figure 4.6.11-4.6.16 is a measure of the normalized error, which takes into account the predicted value of the channel, $Y'_{i,i-1}$. From these graphs it can be observed that the error has a large value during a deep fade, as the actual magnitude of the channel becomes very small, and this error is obtained by normalizing the actual error by the magnitude of the channel.

The simple estimator with degree 1 prediction gives a good performance without an undue increase in complexity, confirming it to be a good estimation technique.

TABLE 4.6.1 MEAN SQUARE ERROR IN PREDICTION FOR THE SIMPLE ESTIMATOR WITH DEGREE 0 LEAST SQUARES PREDICTION

SNR	b	θ	$b(1-\theta)$	Mean square error in prediction
10	0.144	0.934	0.0095	-13.7874
20	0.204	0.913	0.0178	-19.3690
30	0.267	0.898	0.0272	-23.5012
40	0.276	0.886	0.0315	-25.1290
60	0.337	0.904	0.0324	-25.4310

TABLE 4.6.2 NORMALIZED MEAN SQUARE ERROR IN PREDICTION FOR THE SIMPLE ESTIMATOR WITH DEGREE 0 LEAST SQUARES FADING MEMORY PREDICTION

SNR	b	θ	$b(1-\theta)$	Normalized mean square error in prediction
10	0.162	0.934	0.0107	-9.0036
20	0.218	0.911	0.0194	-14.4878
30	0.252	0.89	0.0277	-18.0339
40	0.270	0.884	0.0313	-19.1483
60	0.335	0.904	0.0322	-19.3411

TABLE 4.6.3 MEAN SQUARE ERROR IN PREDICTION FOR THE SIMPLE ESTIMATOR WITH DEGREE 1 LEAST SQUARES PREDICTION

SNR	b	θ	Mean square error in prediction
10	0.212	0.984	-14.1945
20	0.152	0.970	-21.5964
30	0.142	0.955	-28.5332
40	0.123	0.934	-34.4636
60	0.109	0.912	-39.2525

TABLE 4.6.4 NORMALIZED MEAN SQUARE ERROR IN PREDICTION FOR THE SIMPLE ESTIMATOR WITH DEGREE 1 LEAST SQUARES FADING MEMORY PREDICTION

SNR	b	θ	Normalized mean square error in prediction
10	0.206	0.984	-9.9953
20	0.156	0.970	-17.7084
30	0.129	0.955	-24.9625
40	0.126	0.938	-30.6516
60	0.118	0.917	-35.6126

TABLE 4.6.5 MEAN SQUARE ERROR IN PREDICTION FOR THE SIMPLE ESTIMATOR WITH DEGREE 2 LEAST SQUARES PREDICTION

SNR	b	Θ	Mean square error in prediction
10	0.287	0.992	-13.3865
20	0.248	0.986	-21.0165
30	0.228	0.981	-28.4217
40	0.214	0.974	-35.1548
60	0.203	0.966	-42.6406

TABLE 4.6.6 NORMALIZED MEAN SQUARE ERROR IN PREDICTION FOR THR SIMPLE ESTIMATOR WITH DEGREE 2 LEAST SQUARES PREDICTION

SNR	b	Θ	Normalized mean square error in prediction
10	0.361	0.993	-8.9495
20	0.223	0.984	-16.2471
30	0.207	0.979	-23.8184
40	0.213	0.974	-30.2668
60	0.207	0.966	-37.3887

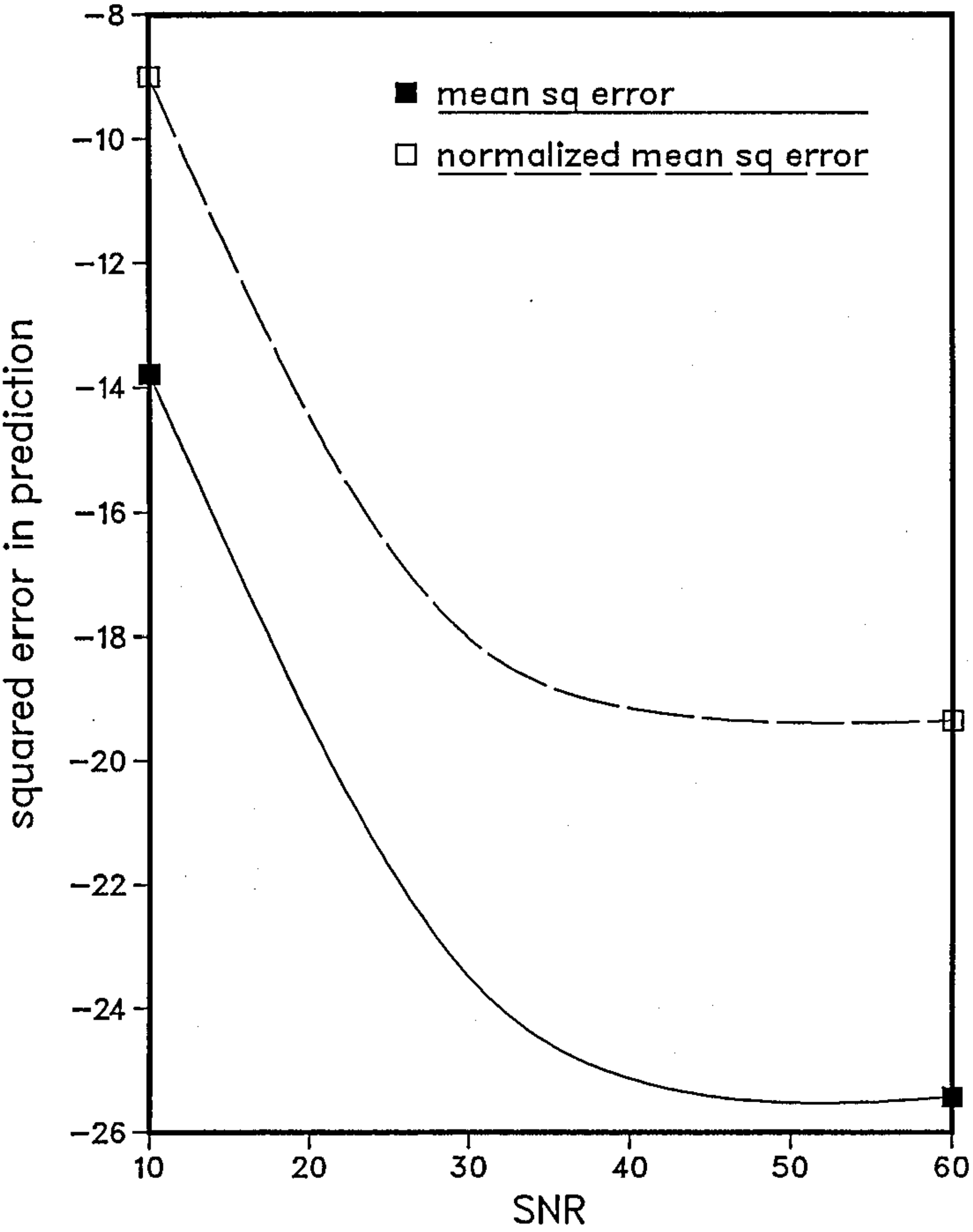


FIGURE 4.6.1 PLOT OF THE MEAN SQUARE AND NORMALIZED MEAN SQUARE ERROR IN PREDICTION FOR THE SIMPLE ESTIMATOR WITH DEGREE 0 PREDICTION

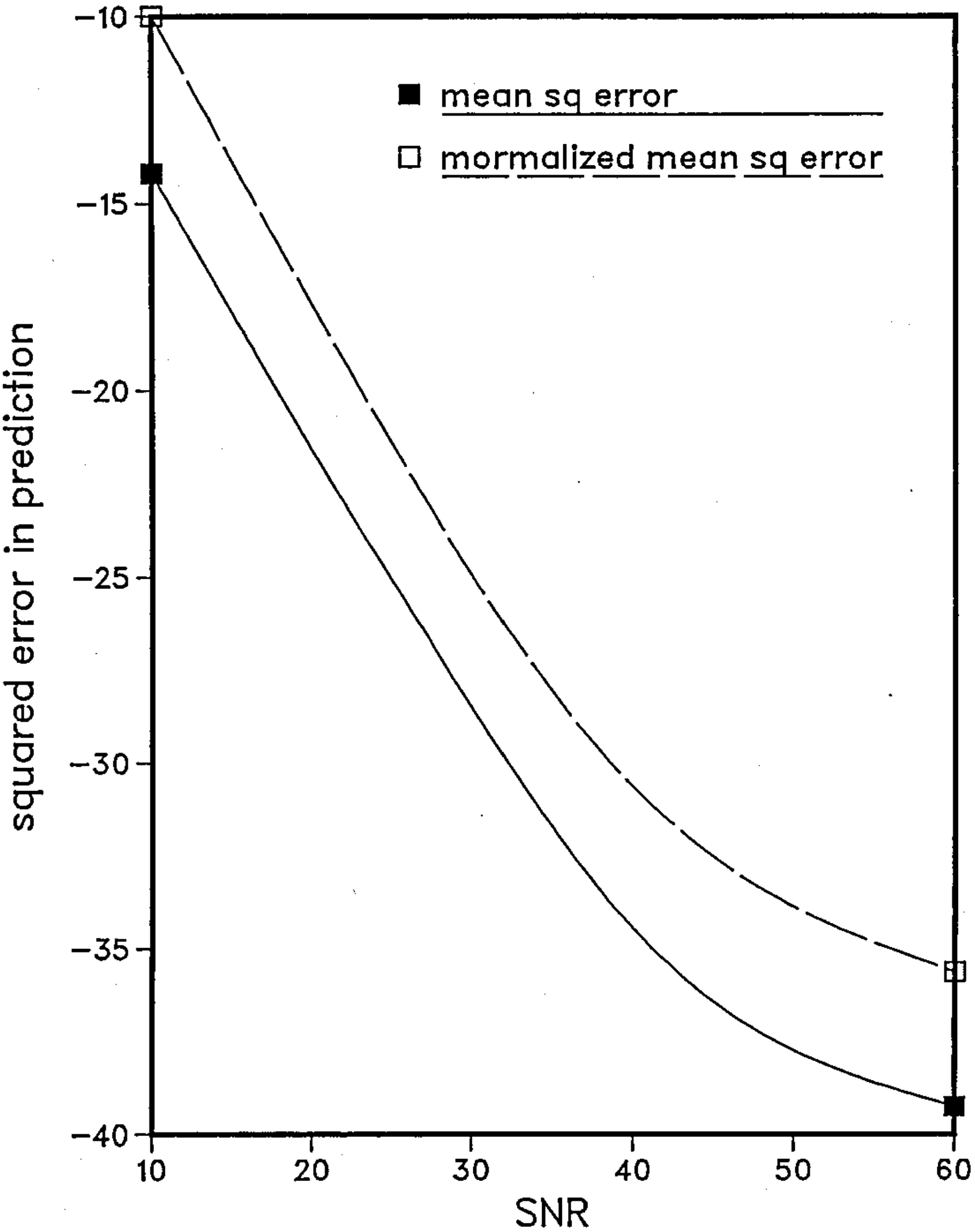


FIGURE 4.6.2 PLOT OF MEAN SQUARE AND NORMALIZED MEAN SQUARE ERROR IN PREDICTION FOR THE SIMPLE ESTIMATOR WITH DEGREE 1 PREDICTION

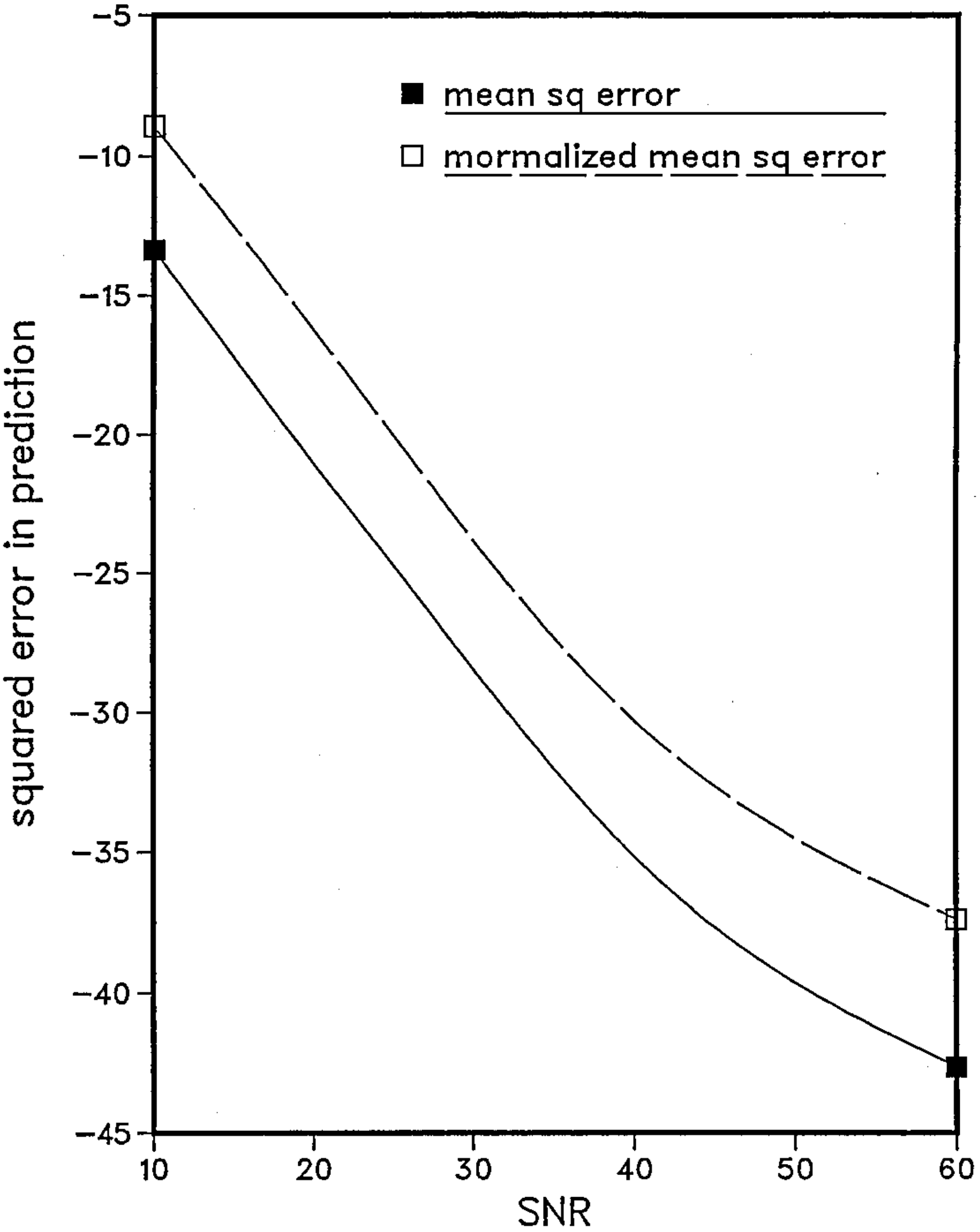


FIGURE 4.6.3 PLOT OF MEAN SQUARE AND NORMALIZED MEAN SQUARE ERROR IN PREDICTION FOR THE SIMPLE ESTIMATOR WITH DEGREE 2 PREDICTION

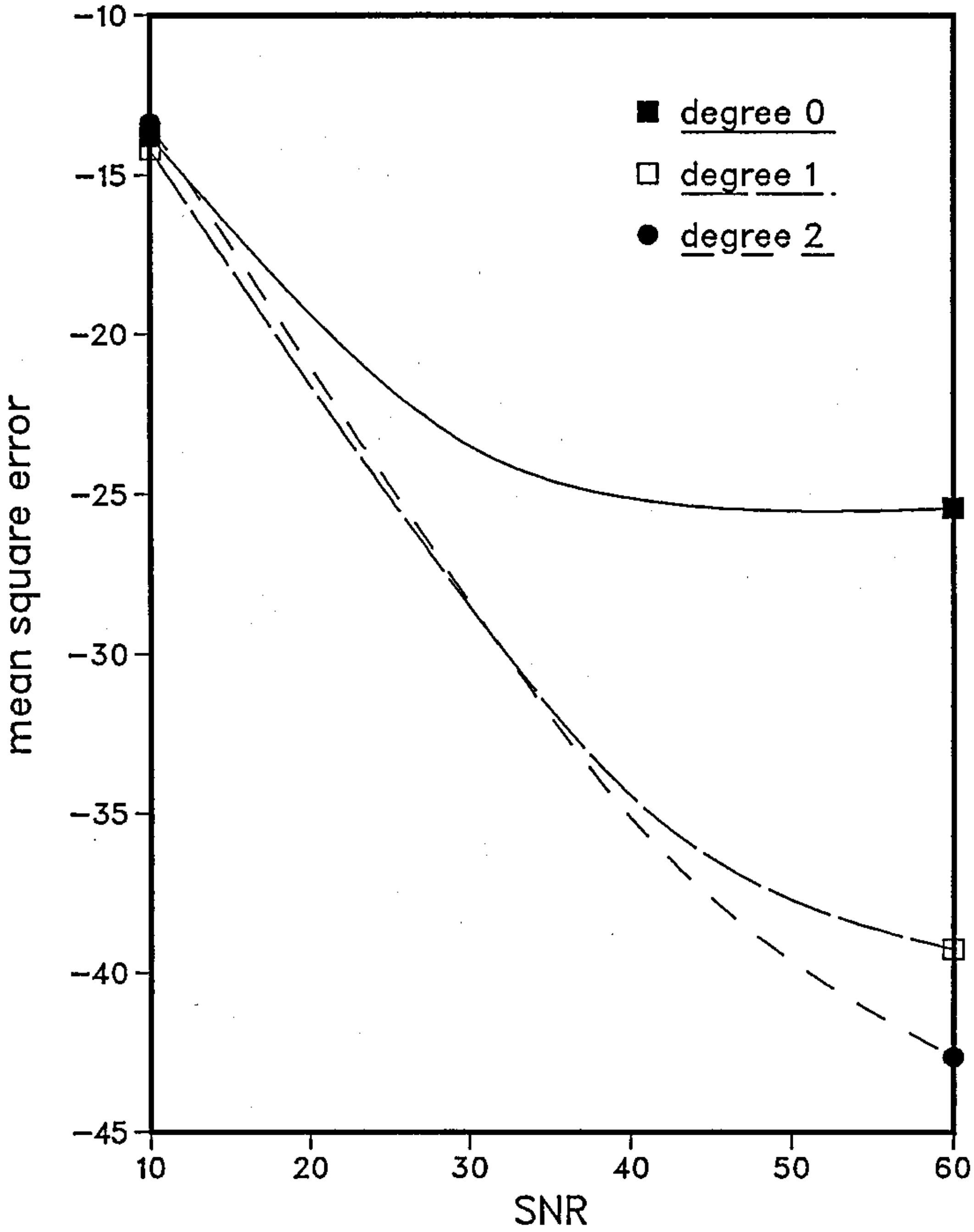


Fig 4.6.4 Plot of mean square error in prediction of channel using degree 0, 1, 2 prediction for a simple estimator

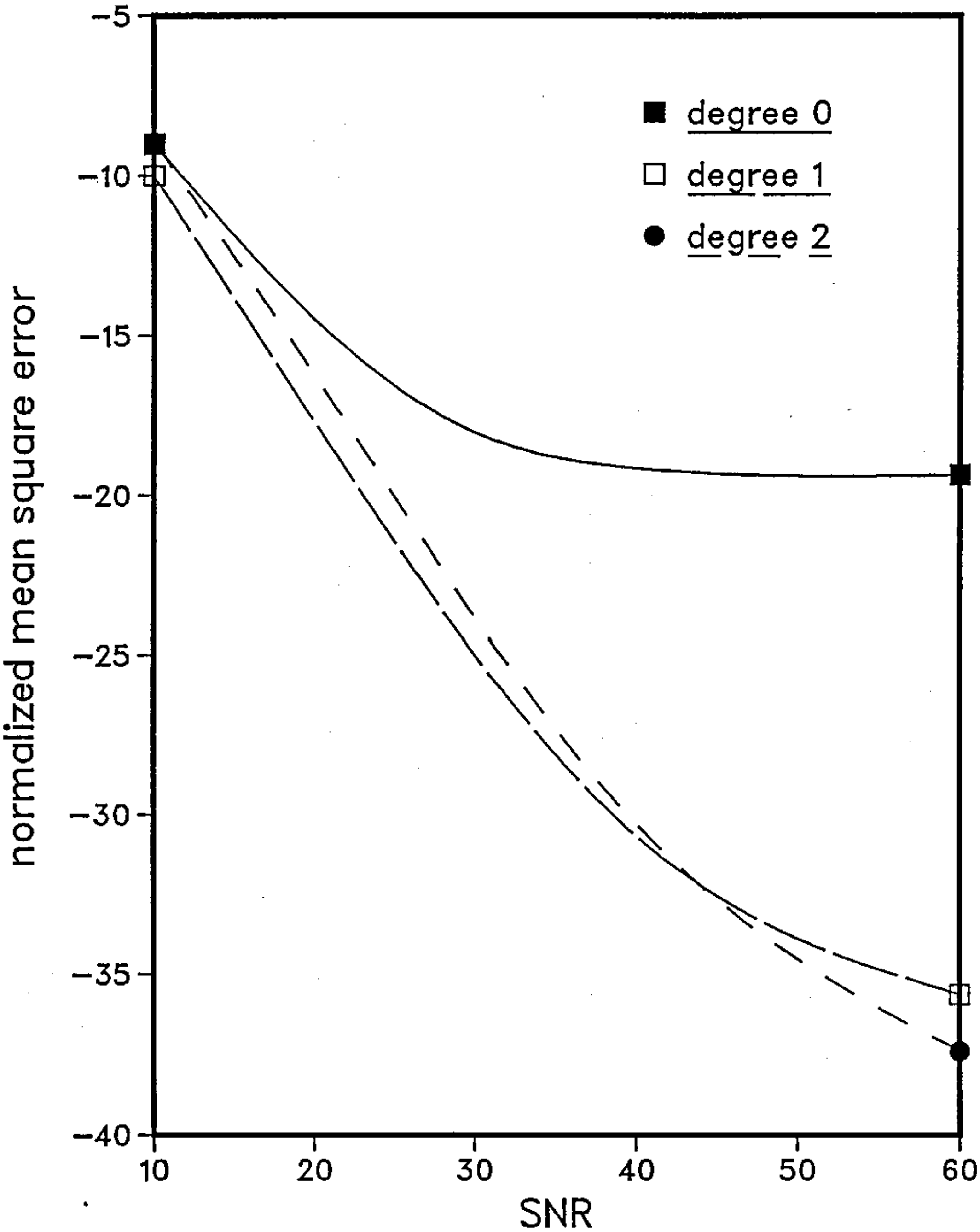


Fig 4.6.5 Plot of normalized mean square error in prediction of channel using degree 0, 1, 2 prediction for a simple estimator

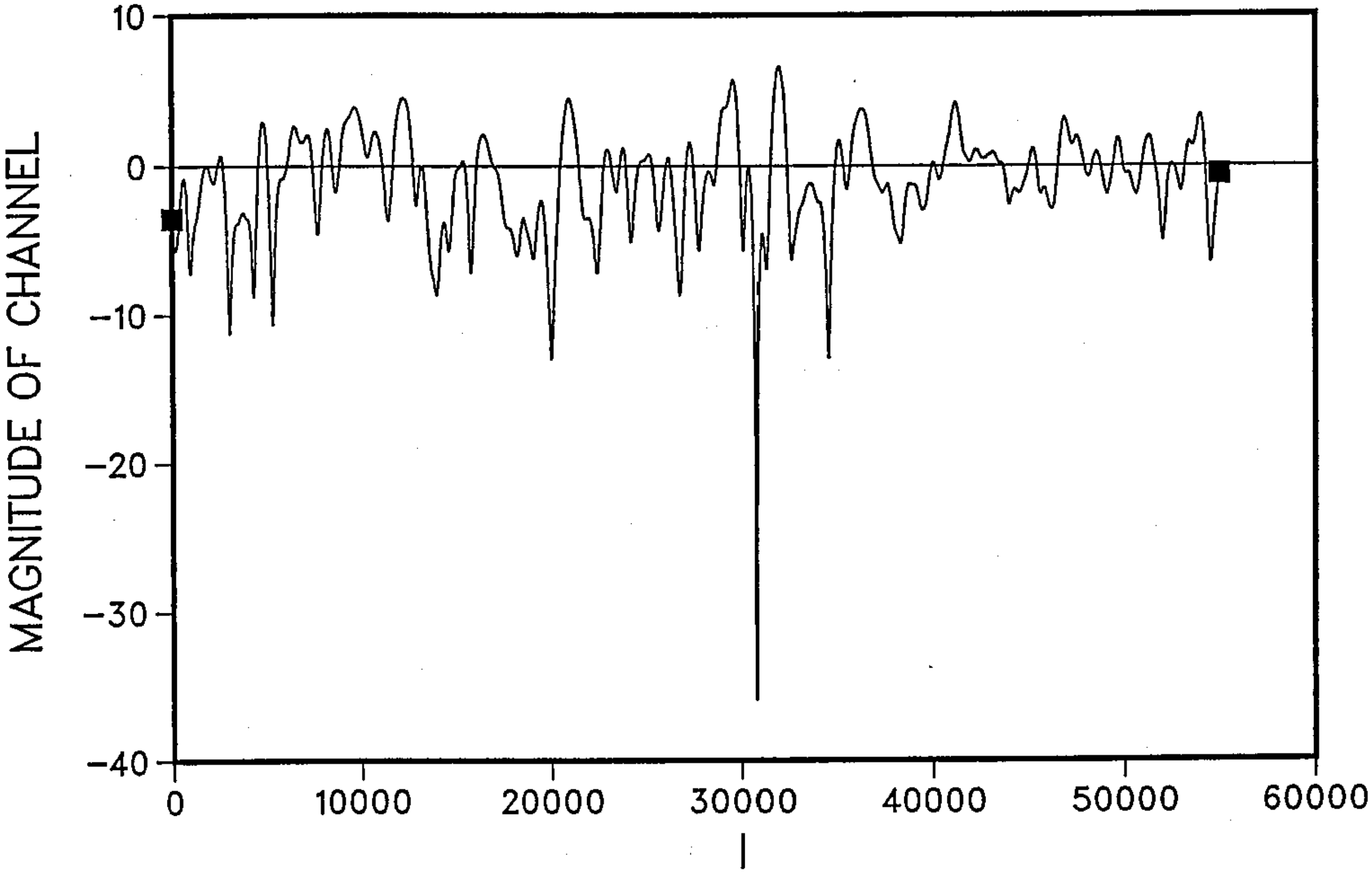


FIGURE 4.6.6 PLOT OF THE MAGNITUDE OF THE CHANNEL OVER WHICH ESTIMATOR WAS TESTED, SEED=50

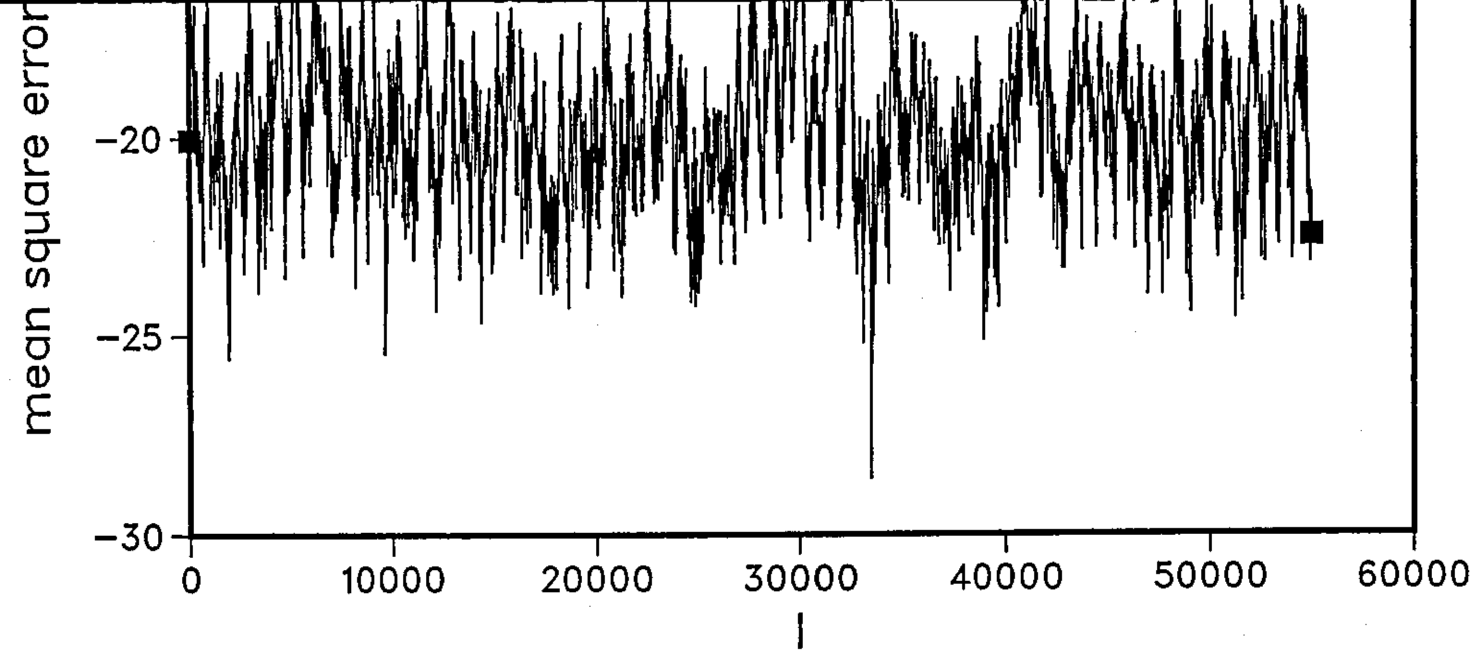
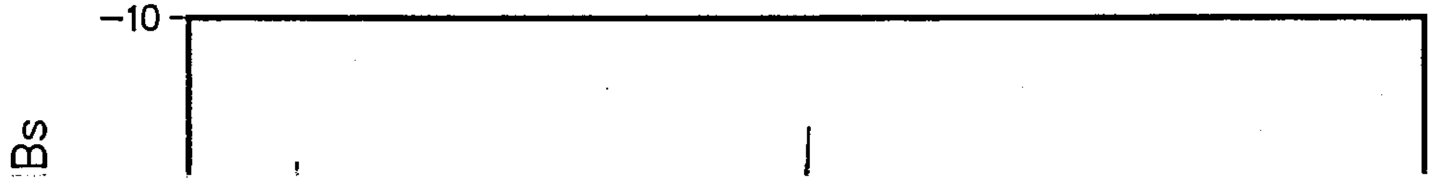


FIGURE 4.6.7 STEADY STATE PERFORMANCE OF THE SIMPLE ESTIMATOR WITH DEGREE 0 LEAST SQUARES FADING MEMORY PREDICTION, SNR = 20



Bs
−10

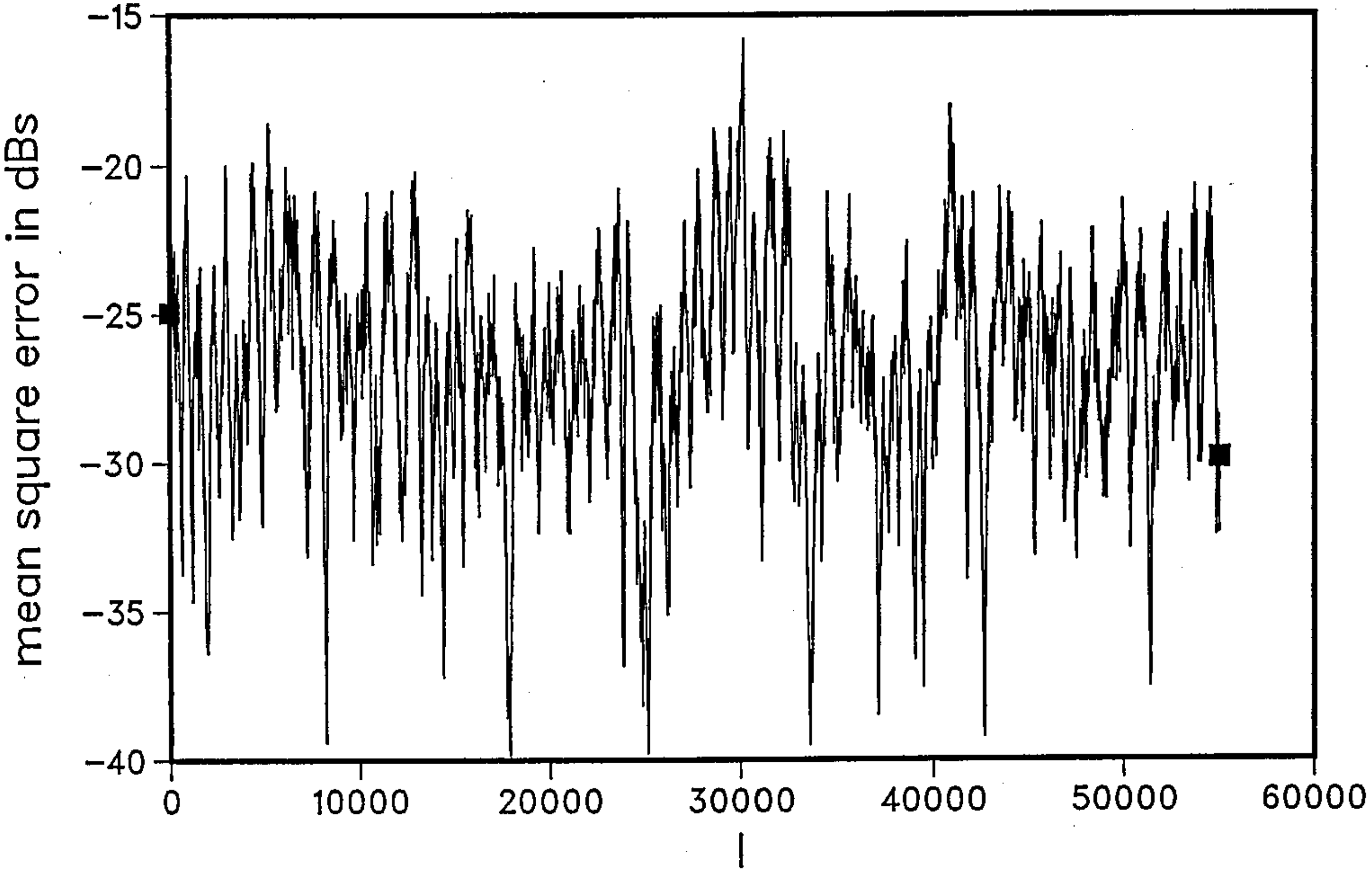


FIGURE 4.6.8 STEADY STATE PERFORMANCE OF THE SIMPLE ESTIMATOR WITH DEGREE 0 LEAST SQUARES FADING MEMORY PREDICTION, SNR = 60

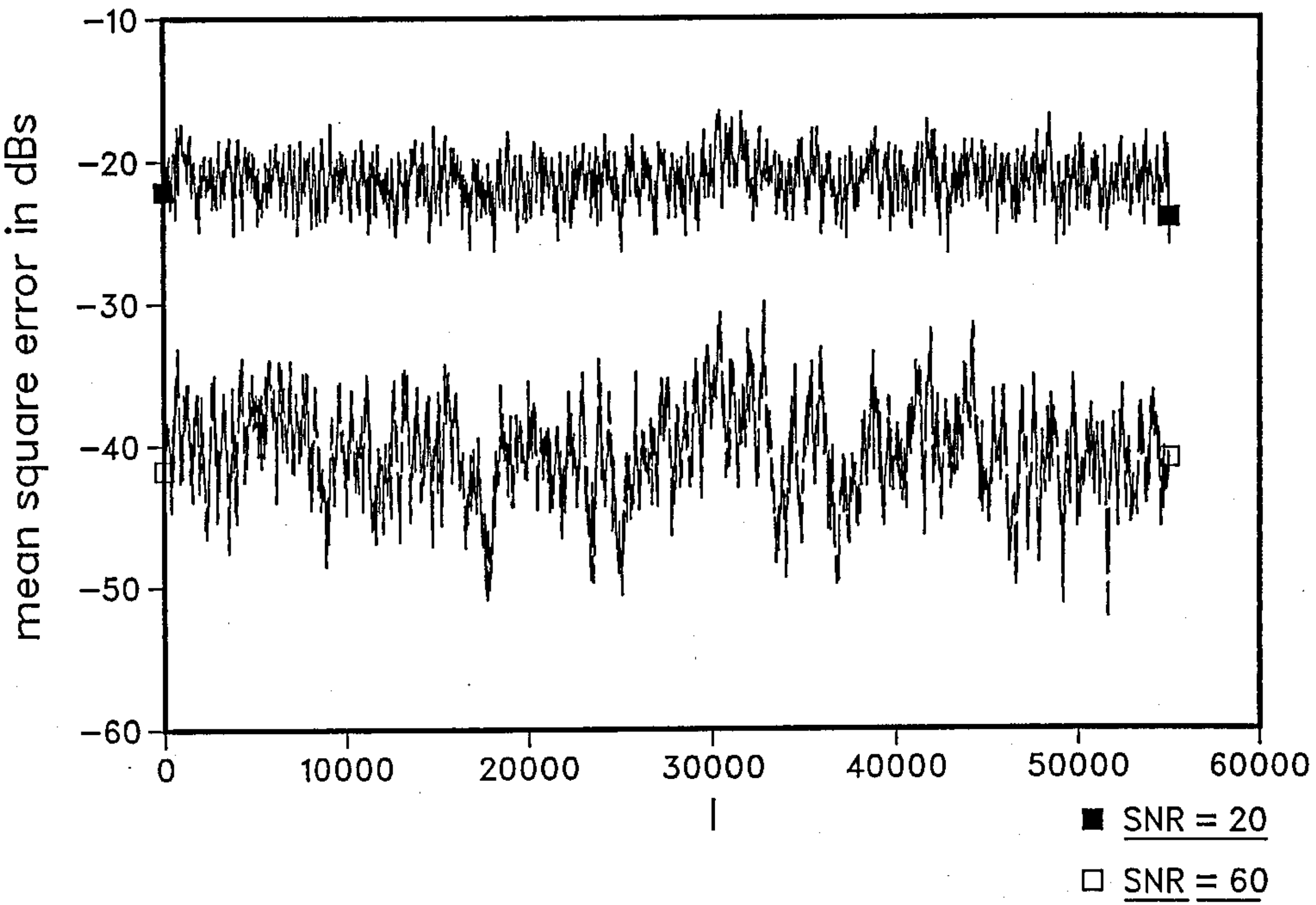


FIGURE 4.6.9 STEADY STATE PERFORMANCE OF THE SIMPLE ESTIMATOR WITH DEGREE 1 LEAST SQUARES FADING MEMORY PREDICTION, SNR = 20, 60

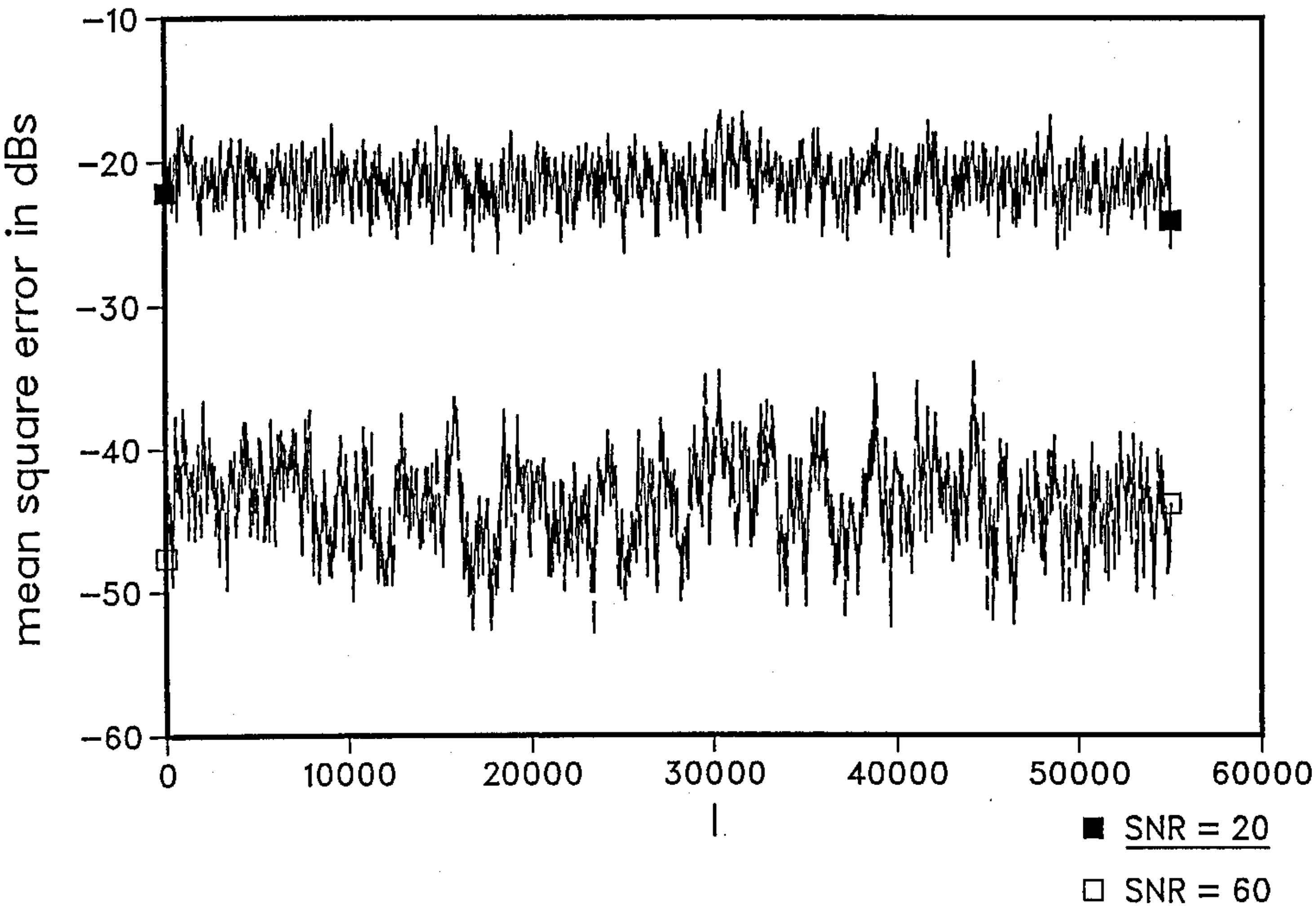


FIGURE 4.6.10 STEADY STATE PERFORMANCE OF THE SIMPLE ESTIMATOR WITH DEGREE 2 LEAST SQUARES FADING MEMORY PREDICTION, SNR =20,60

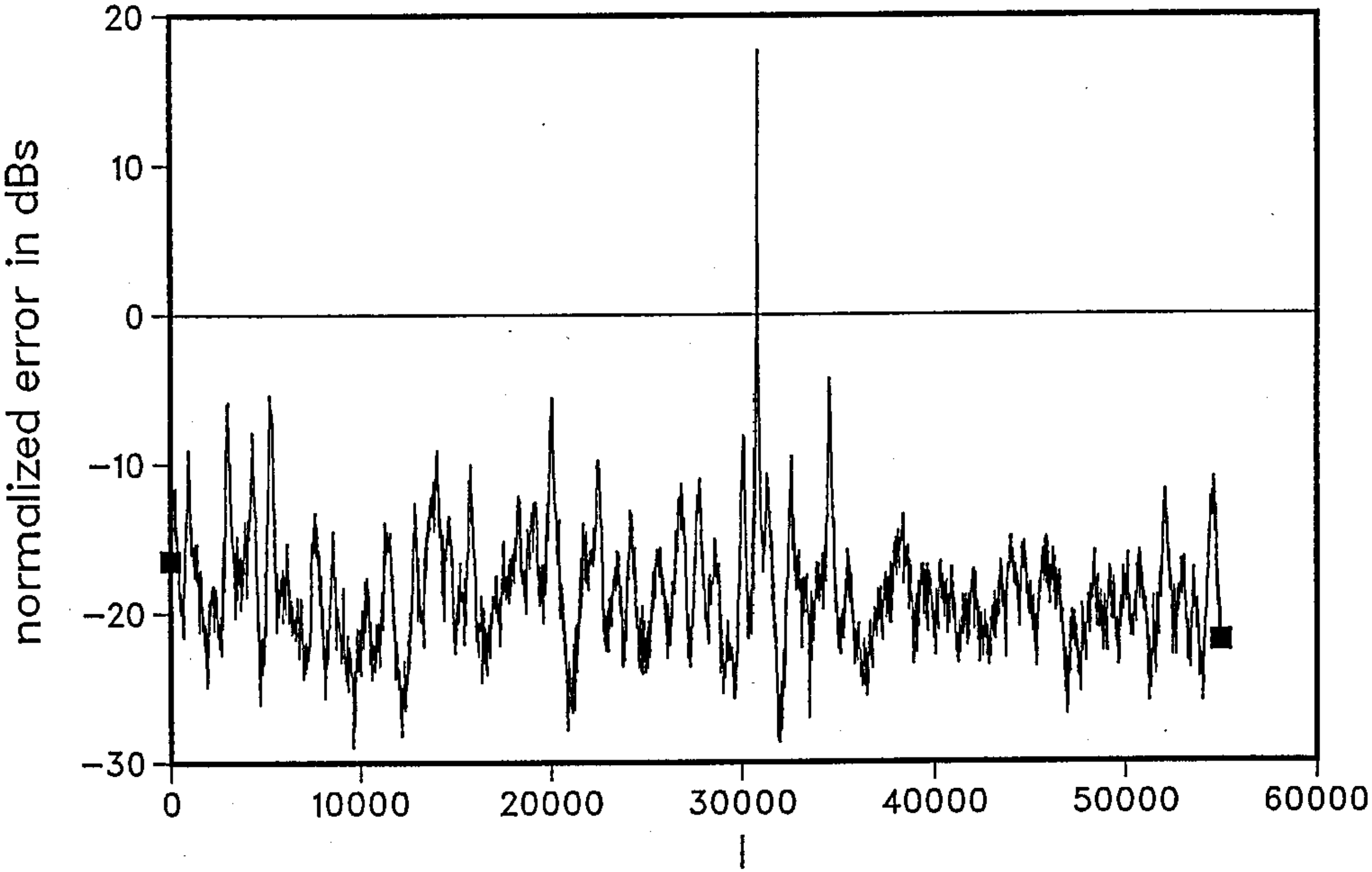


FIGURE 4.6.11 PLOT OF THE STEADY STATE NORMALIZED ERROR PERFORMANCE OF THE SIMPLE ESTIMATOR WITH DEGREE 0 LEAST SQUARES FADING MEMORY PREDICTION SNR = 20

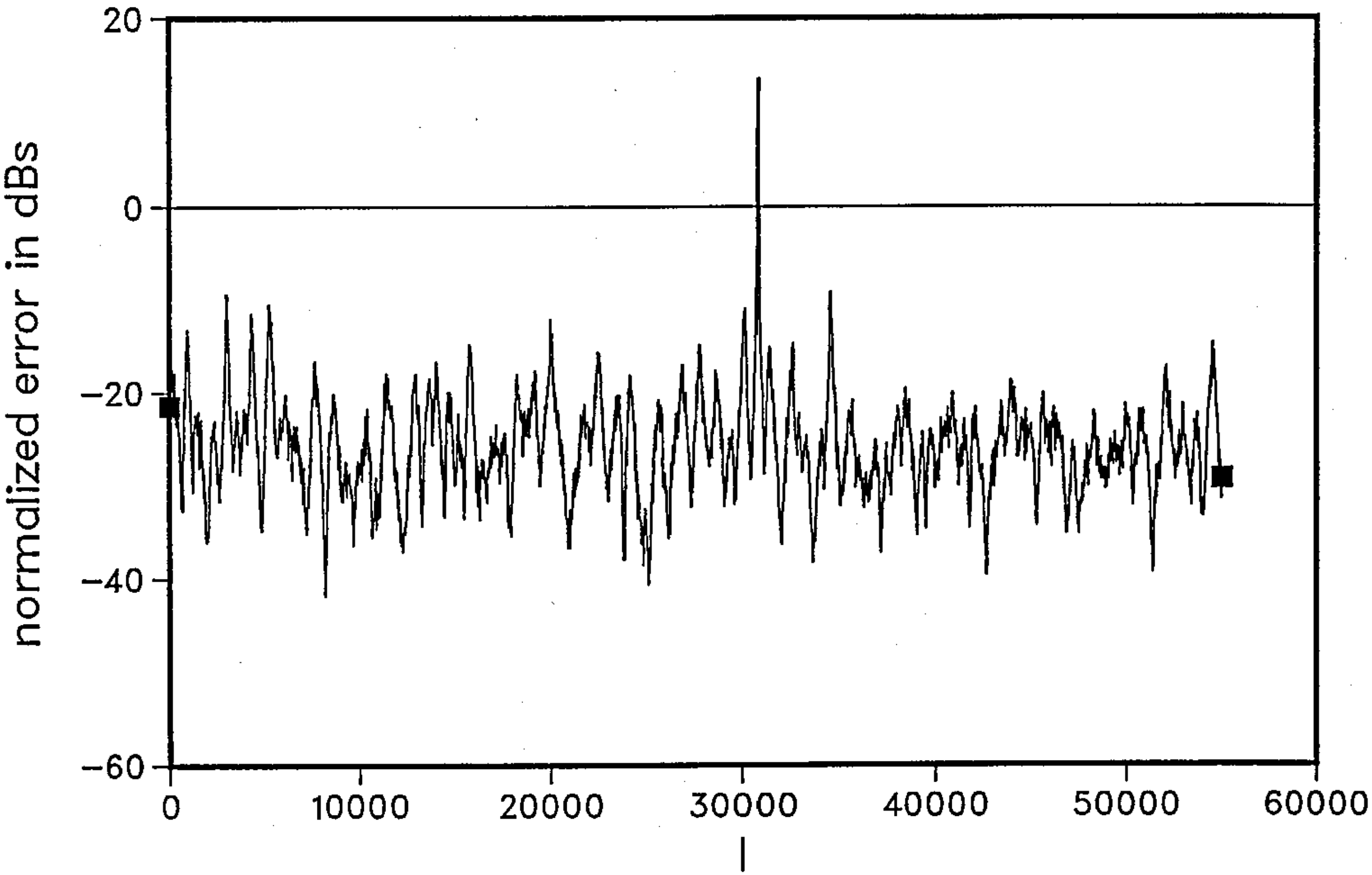


FIGURE 4.6.12 PLOT OF THE STEADY STATE NORMALIZED ERROR PERFORMANCE OF THE SIMPLE ESTIMATOR WITH DEGREE 0 LEAST SQUARES FADING MEMORY PREDICTION SNR = 60

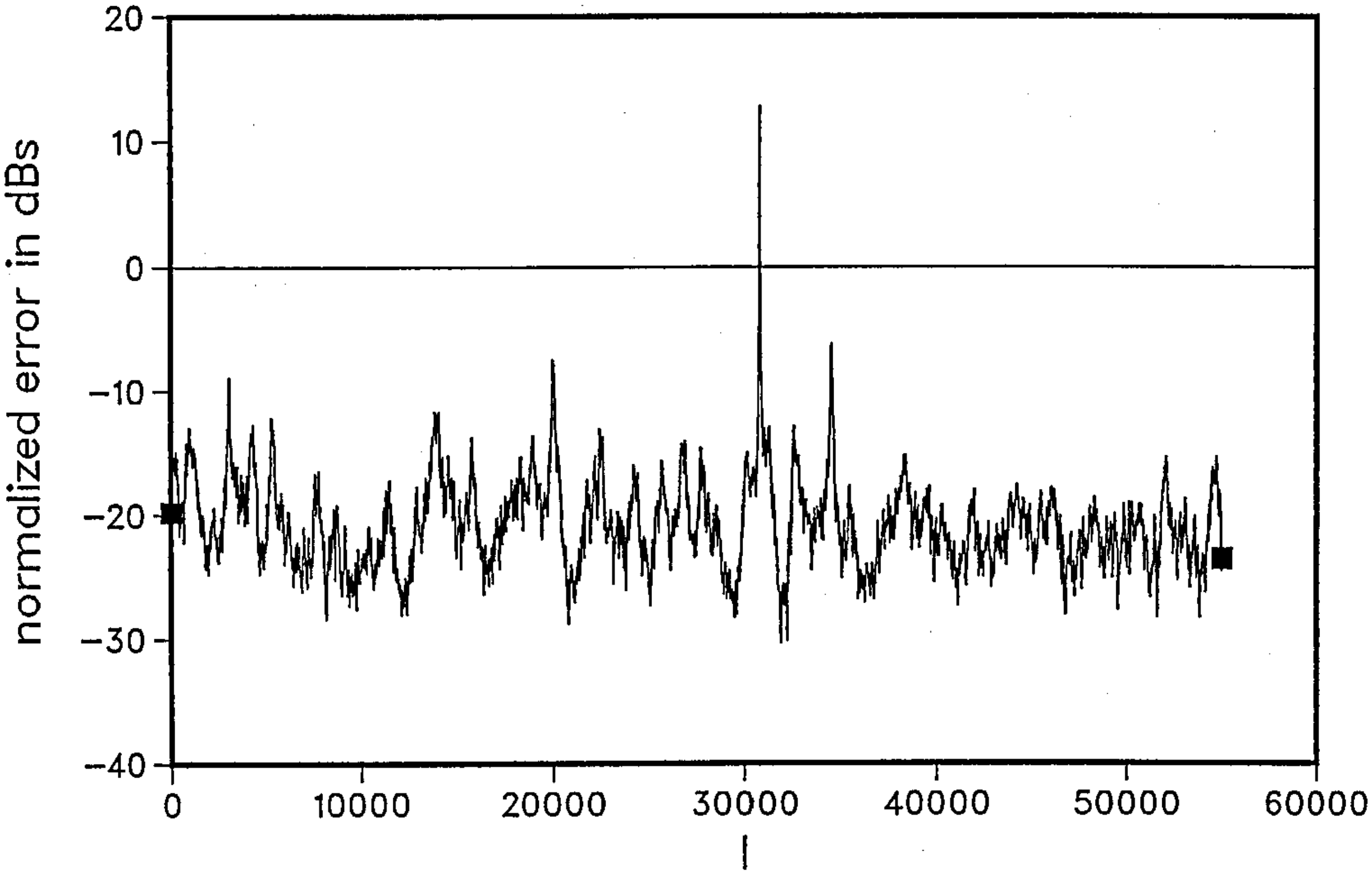


FIGURE 4.6.13 PLOT OF THE STEADY STATE NORMALIZED ERRROR PERFORMANCE OF THE SIMPLE ESTIMATOR WITH DEGREE 1 LEAST SQUARES FADING MEMORY PREDICTION, SNR = 20

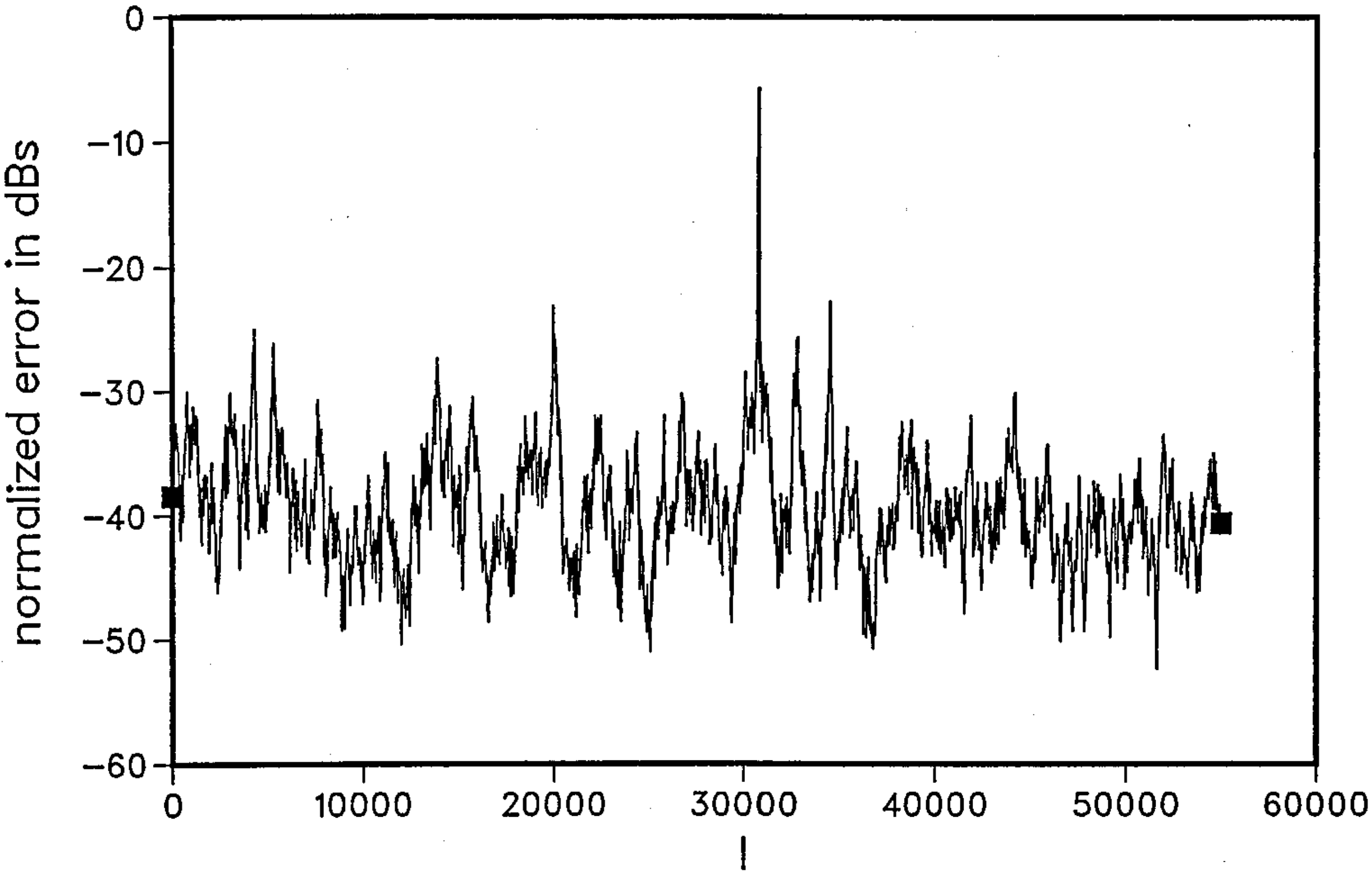


FIGURE 4.6.14 PLOT OF THE STEADY STATE NORMALIZED ERROR PERFORMANCE OF THE SIMPLE ESTIMATOR WITH DEGREE 1 LEAST SQUARES FADING MEMORY PREDICTION, SNR = 60

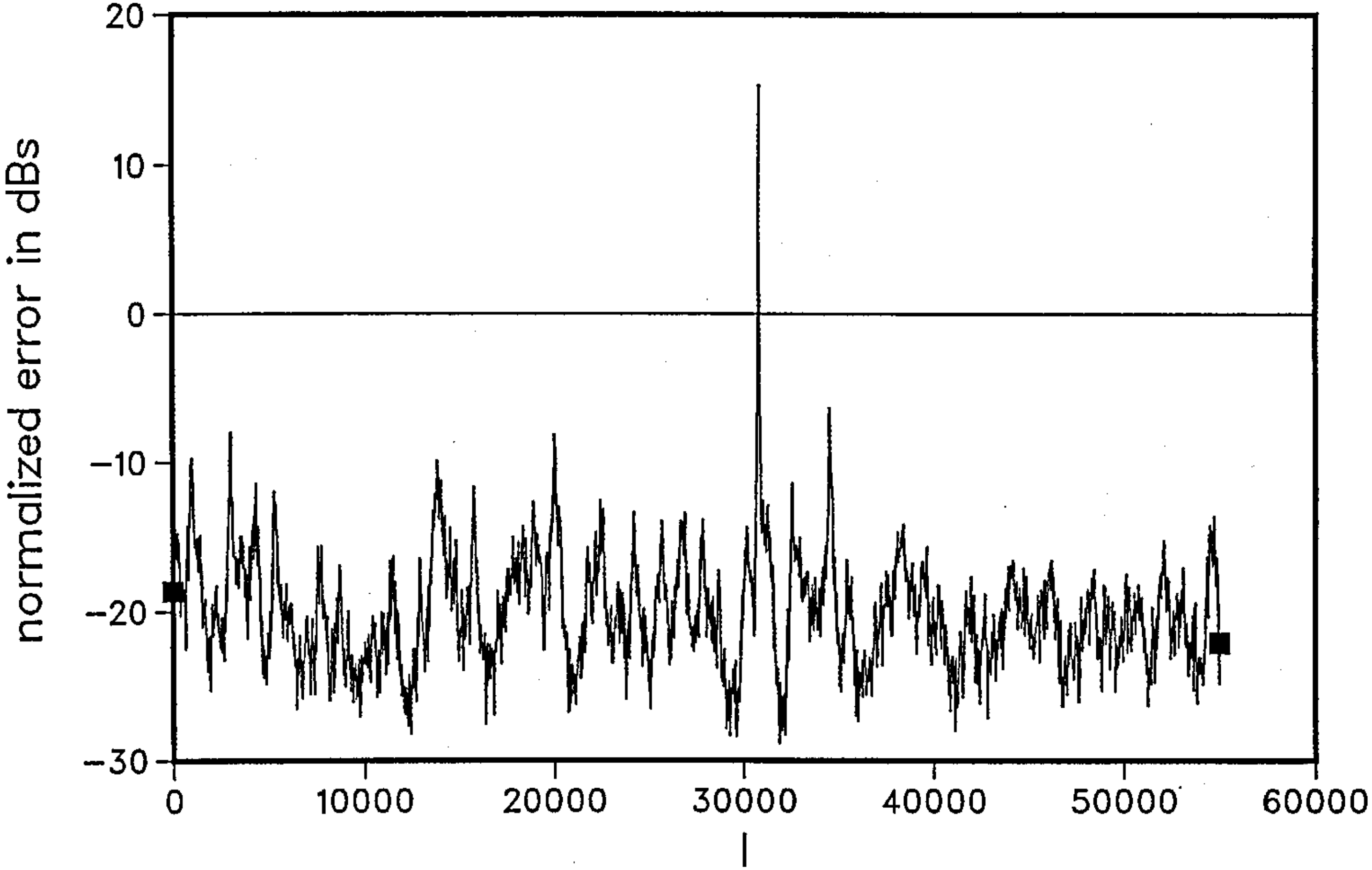


FIGURE 4.6.15 PLOT OF THE STEADY STATE NORMALIZED ERROR PERFORMANCE OF THE SIMPLE ESTIMATOR WITH DEGREE 2 LEAST SQUARES FADING MEMORY PREDICTION, SNR = 20

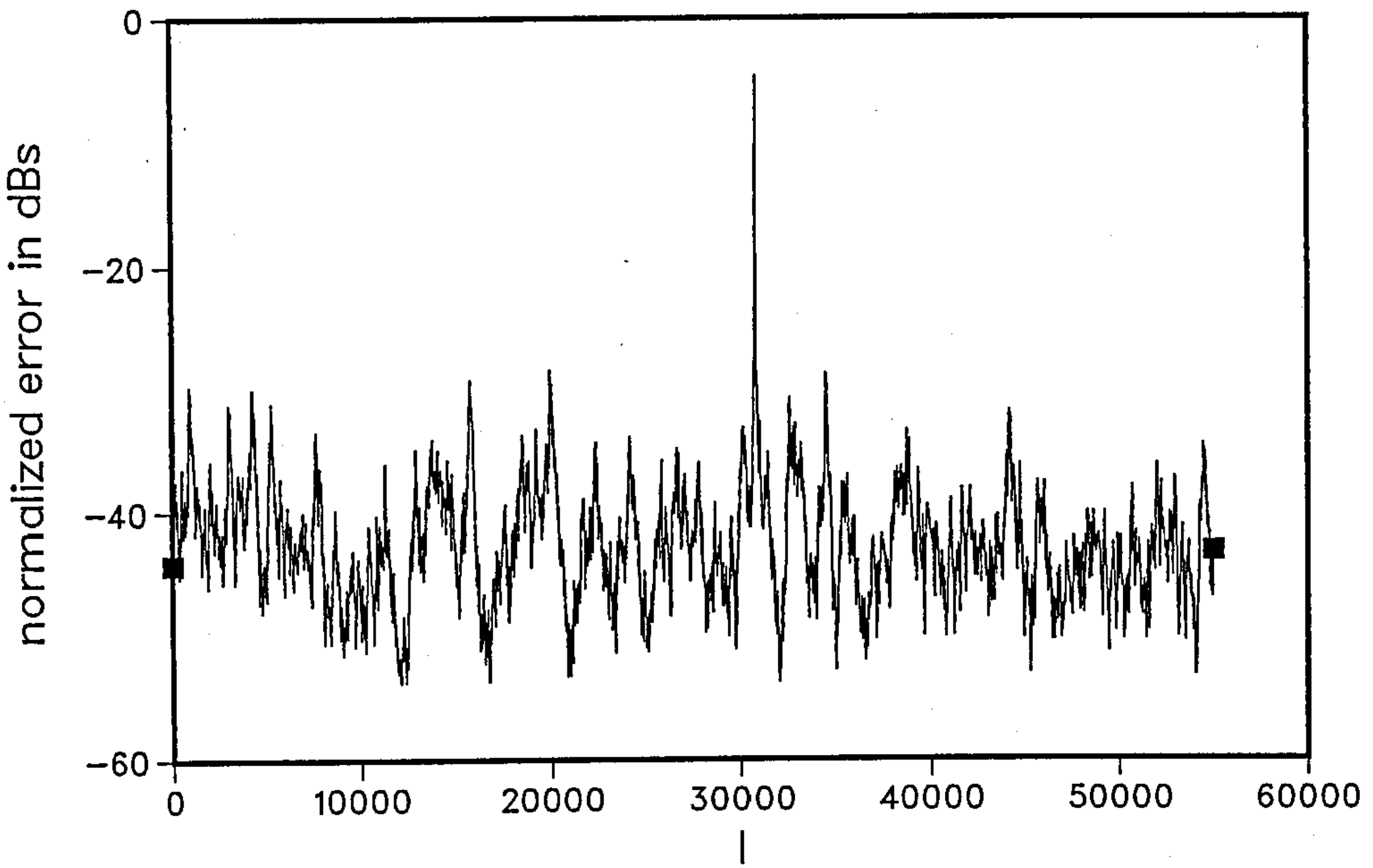


FIGURE 4.6.16 PLOT OF THE STEADY STATE NORMALIZED ERROR PERFORMANCE OF THE SIMPLE ESTIMATOR WITH DEGREE 2 LEAST SQUARES FADING MEMORY PREDICTION, SNR=60

CHAPTER 5

The man of virtue makes the difficulty to be overcome his first business, and success only a subsequent consideration

- Confucius

CHAPTER 5

ADAPTIVE CHANNEL ESTIMATOR

5.1 INTRODUCTION

In chapter 4 a simple estimator designed for a 4800 bits/s modem which employs a polynomial filter that gives a prediction of the channel sampled impulse response has been considered. This estimator is a development of the conventional gradient estimator [1-12]. Adaptive channel estimators are considered in this chapter, which are developments of the estimators in chapter 4. They are adaptive because they make no use of any prior knowledge of the channel and are able to track effectively an HF channel irrespective of the number of skywaves present in the fading channel [7-8]. In section 5.2 the model of the data transmission system used is described. Section 5.3 provides an insight into the need to use an adaptive estimator. In section 5.4-5.5 the adaptive channel estimator implemented is discussed. Section 5.6 gives the results of simulation tests carried out on the adaptive channel estimator.

5.2 MODEL OF SYSTEM

The model of the data transmission system assumed in the investigation is shown in figure 3.4.2.

The signal at the input to the baseband channel is a sequence of regularly spaced impulses $\{s_i\delta(t-iT)\}$ where the s_i are assumed to be statistically independent and equally likely to have any one of its four values $\pm1 \pm j(j=\sqrt{-1})$.

Transmission starts at time t=iT seconds. The linear baseband channel has an impulse response y(t) with an effective duration of less than (g+1)T seconds where g is a given positive integer. The only noise assumed to be introduced by the channel is stationary white Gaussian noise which is added to the data signal at the output of the transmission path such that the noise waveform w(t) at the output of the receiver filter is a bandlimited Gaussian noise. Thus the output signal form the baseband channel in Fig 3.2.1 is the waveform

$$r(t)=\sum_i s_i y(t-iT)+w(t) \tag{5.2.1}$$

where $s_i y(t-iT)$ is the i^{th} received signal element. The waveform r(t) is sampled once per received signal element, at the time instant {iT}, giving the received samples r_i , where,

$$r_i=\sum_{h=0}^{g} s_{i-h} y_{i,h}+w_i \tag{5.2.2}$$

$$=Y_i S_i^T+w_i \tag{5.2.3}$$

The delay in transmission is neglected so that $y_i = 0$ for i<0 and i>g. let Y_i represent the (g+1) component column vector of the sampled impulse response of the channel.

$$Y_i=[y_{i,0} y_{i,1} \cdots y_{i,g}] \tag{5.2.4}$$

The noise samples w_i are slightly correlated as they have been filtered at the receiver, they have zero mean and variance that is dependent on 0.5 N_0, and are statistically independent of the $\{s_i\}$.

The signal $\{r_i\}$ and s_i are fed to the channel estimator to give an estimate Y'_i of Y at time t=iT, where,

$$Y'_i = [y'_{i,0} y'_{i,1} \cdots y'_{i,g}] \tag{5.2.5}$$

The estimate Y'_i is used by the detector for the detection of the next data symbol on the receipt of r_{i+1}. Details regarding the channel model can be found in chapter 3.

5.3 ADAPTIVE ESTIMATOR SYSTEM 1

This system is a development of the simple estimator described in chapter 4. The estimator uses the same linear feedforward transversal filter as in chapter 4.5. Assuming correct detection so that the receiver knows the $\{s_i\}$.

$$S_i = [s_1 s_2 \ldots s_{i-g}] \tag{5.3.1}$$

The simple estimator (section 4.5) operates as follows. On the receipt of r_i, the estimator holds a one step prediction of Y_i, given by

$$Y'_{i,i-1} = [y'_{i,i-1,0} y'_{i,i-1,1} \cdots y'_{i,i-1,g}] \tag{5.3.2}$$

This prediction $Y'_{i,i-1}$ has been obtained at time t=(i-1)T, this is used by the estimator in place of Y'_{i-1} to obtain the estimate of r_i, given by

$$r'_i = \sum_{h=0}^{g} s'_{i-h} y'_{i,i-1,h} \tag{5.3.3}$$

e_i is the error in the estimate of the i^{th} received signal r_i,

$$e_i = r_i - \sum_{h=0}^{g} s'_{i-h} y'_{i,i-1,h} \tag{5.3.4}$$

The error signal is then used to form the updated estimate of Y_i, given by

$$Y_i' = Y_{i,i-1}' + b e_i S_i^* \tag{5.3.5}$$

where

$$Y_i' = [y_{i,0}' y_{i,1}' \cdots y_{i,g}'] \tag{5.3.6}$$

S_i^* is a (g+1) component vector given and is the complex conjugate of S_i, and b is an appropriate small positive real valued constant.

The predictor then forms the error vector,

$$\begin{aligned} E_i &= Y_i' - Y_{i,i-1}' \\ &= b e_i S_i^* \end{aligned} \tag{5.3.7}$$

the actual error in $Y'_{i,i-1}$ is,

$$Y_i - Y_{i,i-1}' \tag{5.3.8}$$

which is fed with $Y_{i,i-1}$ to the appropriate set of g+1 polynomial filters to form the one step prediction of Y_{i+1}, given by

$$Y_{i+1,i}' = [y_{i+1,i,0}' y_{i+1,i}' \cdots y_{i+1,i,g}'] \tag{5.3.9}$$

The polynomial filter may be degree 0, 1, 2 least squares fading memory prediction methods.

An estimate E_i' of the actual error in $Y'_{i,i-1}$ given by equation 5.3.8, can in principle, be derived from the degree 0 and degree 1 least squares fading memory prediction, which assume that the change in Y_i with i and the rate of change of Y_i with i are constant or only slowly varying with i, respectively. Thus for a degree 0 prediction, a significant source of error in the prediction $Y'_{i,i-1}$ is likely to be the velocity and acceleration in Y_i. While for a degree 1 prediction, a significant source of error in the prediction $Y'_{i,i-1}$ is likely to be the acceleration in Y_i [7,8].

If the only source of error in $Y'_{i,i-1}$ is due to the acceleration in Y_i, then

$$Y_i = Y'_{i,i-1} + c_i A_i \tag{5.3.10}$$

where c_i is a complex valued scalar and

$$\begin{aligned} A_i &= (Y_{i+1} - Y_i) - (Y_i - Y_{i-1}) \\ &= Y_{i+1} - 2Y_i + Y_{i-1} \end{aligned} \tag{5.3.11}$$

and from equations 5.3.8, 5.3.10

$$E_i = c_i A_i \tag{5.3.12}$$

An estimate of A_i is given by

$$A'_i = Y'_{i+1,i} - 2Y'_{i,i-1} + Y'_{i-1,i-2} \tag{5.3.13}$$

As $Y'_{i+1,i}$, $Y'_{i,i-1}$ and $Y'_{i-1,i-2}$ do not differ greatly, much of the difference between them is due to the noise, hence the relatively high noise level of A'_i. Thus, instead of using A'_i, the estimator uses

$$z_i = [z_{i,0} z_{i,1} \ldots z_{i,g}] \tag{5.3.14}$$

which is obtained from A'_i. Let $\alpha_{i,h}$ be the absolute value of the $(h+1)^{th}$ component of A'_i, for h=0, 1, ..., g. $z_{i,h}$ is a measure of the average value of $\alpha_{i,h}$ and is known as the fading memory average. It is given by

$$z_{i,h} = a \sum_{j=1}^{i} (1-a)^{i-j} \alpha_{j,h} \tag{5.3.15}$$

where a is a real valued constant such that 0<a<1 and j is an integer. Equation 5.3.15 can be implemented sequentially as follows

$$z_{i,h} = a \sum_{j=1}^{i-1} (1-a)^{i-j} \alpha_{j,h} + a\alpha_{i,h} \tag{5.3.16}$$

$$z_{i,h} = a(1-a) \sum_{j=1}^{i-1} (1-a)^{i-1-j} \alpha_{j,h} + a\alpha_{i,h} \tag{5.3.17}$$

from equations 5.3.15 and 5.3.17

$$z_{i,h} = (1-a) z_{i-1,h} + a\alpha_{i,h} \tag{5.3.18}$$

Since all components of z_i are real valued, whereas the components of A_i in equation 5.3.13 are, in general, complex valued, neither $Y'_{i,i-1} + Z_i$ nor $Y'_{i,i-1} + c_i Z_i$ could be used as a satisfactory updated estimate of Y_i in equation 5.3.10. Nevertheless, $z_{i,h}$ gives a measure of the magnitude $y_{i,h} - y'_{i,i-1,h}$ of the error in the component $y'_{i,i-1,h}$ of $Y'_{i,i-1}$.

For the most accurate tracking of a time varying channel, the step size employed in equation 5.3.5 should be permitted to vary from one component of $Y'_{i,i-1}$ to another, and should increase with the likely magnitude of the error in that component. Taking these considerations into account, equation 5.3.5 can be replaced by

$$y'_{i,h} = y'_{i,i-1,h} + b u_{i,h} e_i s^*_{i-h} \tag{5.3.19}$$

for h=0, 1, ...,g, where b is an appropriate small positive real valued constant, and

$$u_{i,h} = p(z_{i,h}) \tag{5.3.20}$$

$p(z_{i,h})$ is a monotonically non decreasing function. The parameter $u_{i,h}$ in equation 5.3.19 cannot be replaced by $z_{i,h}$ itself, for the following reasons. Firstly, no $u_{i,h}$ must be permitted to remain at zero for any significant period, since, if this occurs, the corresponding component in the prediction algorithm may become locked at zero, thus preventing any further change in the corresponding $y'_{i,i-1,h}$. Secondly, no $u_{i,h}$ should be permitted to become too large, in order to avoid possible instability of the algorithm given by equation 5.3.19. Thus the value of $u_{i,h}$ should be constrained such that

$$k_1 < u_{i,h} < k_2 \tag{5.3.21}$$

where k_1 and k_2 are appropriate positive real valued constants.

5.4 ADAPTIVE ESTIMATOR SYSTEM 2

This system does not attempt to measure the acceleration in Y_i directly, instead is uses the fact that the greater the maximum magnitude of any $y_{i,h}$, the greater is likely to be its maximum acceleration and hence the greater the probable value of the largest error in the corresponding prediction $y'_{i,i-1,h}$.

The estimator forms an estimate r'_i of the received signal r_i, given by equation 5.3.3. It then forms the error in the estimate of the received signal, given by equation 5.3.4. The estimator then forms the fading memory average $x_{i,h}^2$ of the mean square absolute value of $y'_{i,i-1,h}$ for h=0, 1, ..., g.

$$x^2_{i,h} = x^2_{i-1,h} + a\left(|\, y'_{i,i-1,h}\,|^2 - x^2_{i-1,h}\right) \tag{5.4.1}$$

where a is a positive real valued constant such that 0<a<1, and

$$x^2_{0,h} = |\, y'_{0,-1,h}\,|^2 = |\, y_{0,h}\,|^2 \tag{5.4.2}$$

for h=0, 1, ..., g,

The estimator next forms an update of $y'_{i,i-1,h}$ using equation 5.3.19, which is

$$y'_{i,h} = y'_{i,i-1,h} + b\,u_{i,h} e_i s^*_{i-h} \tag{5.4.3}$$

with

$$u_{i,h} = p(x^2_{i,h}) \tag{5.4.4}$$

where $u_{i,h}$ and $x_{i,h}^2$ are related according to figure 5.4.1 [7-8]. Over the curved portion of the relationship in figure 5.4.1

$$u_{i,h} = {x^2_{i,h}}^{\frac{1}{4}} = x^{\frac{1}{2}}_{i,h} \tag{5.4.5}$$

The value of $u_{i,h}$ should be constrained such that

$$k_1 < u_{i,h} < k_2 \tag{5.4.6}$$

where k_1 and k_2 are appropriate positive real valued constants.

Firstly, no $u_{i,h}$ must be permitted to remain at zero for any significant period, since, if this occurs, the corresponding component in the prediction algorithm may become locked at zero, thus preventing any further change in the corresponding $y'_{i,i-1,h}$. Secondly, no $u_{i,h}$ should be permitted to become too large, in order to avoid possible instability of the algorithm given by equation 5.3.19. It is not necessary to limit the maximum value of $u_{i,h}$ owing to the nonlinear variation of $u_{i,h}$ and $x_{i,h}^2$, which prevents $u_{i,h}$ from becoming too large as can be seen from figure 5.4.1.

The quantity k_0 is a small positive real valued constant, such that

$$d = k_0^4 \tag{5.4.7}$$

The adaptive estimator system 2 algorithm is tested for degree 0, degree 1 and degree 2 least squares fading memory prediction.

5.5 ADAPTIVE ESTIMATOR SYSTEM 3

System 3 is a simple modification of system 2. System 3 also makes use of the magnitude of the estimate of the $\{y_{i,h}\}$.

On receipt of r_i, the estimator holds in store the one-step prediction $Y'_{i,i-1}$ of Y_i. An estimate of the received signal is formed using equation 5.3.3. It then forms the error in the received signal, given by equation 5.3.4. It first forms the fading memory average $x_{i,h}^2$, of the mean square absolute value of $y'_{i,i-1,h}$ for h = 0, 1, ..., g.

$$x_{i,h}^2 = x_{i-1,h}^2 + a\left(|\, y'_{i,i-1,h}\,|^2 - x_{i-1,h}^2\right) \tag{5.5.1}$$

where a is a positive real valued constant such that 0<a<1, and

$$x_{0,h}^2 = |\, y'_{0,-1,h}\,|^2 = |\, y_{0,h}\,|^2 \tag{5.5.2}$$

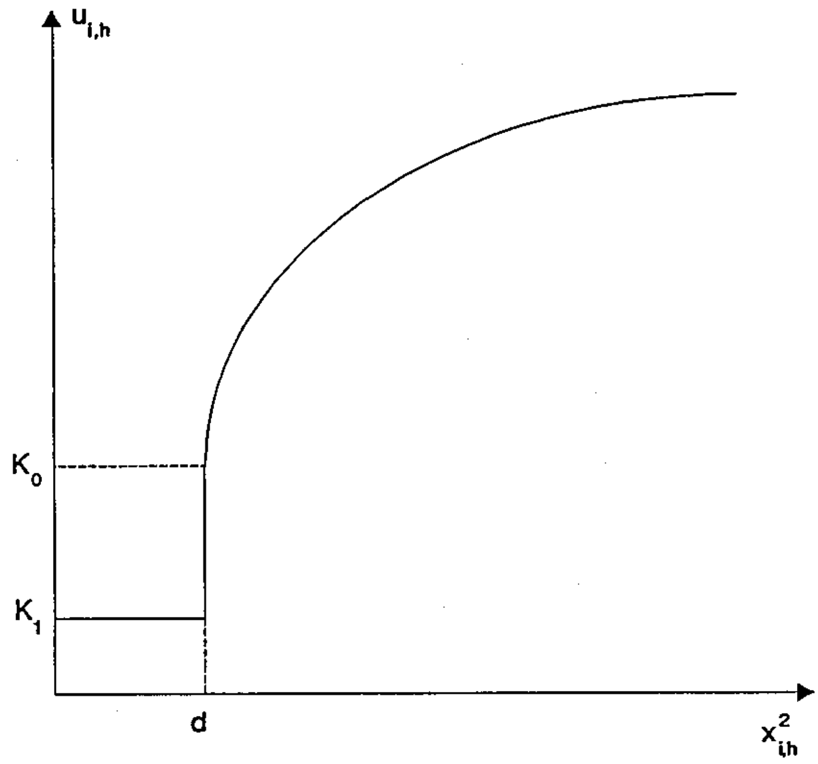


FIGURE 5.4.1 PLOT OF VARIATION OF $u_{i,h}$ WITH $x^2_{i,h}$

for h=0, 1, ..., g,

It then forms an update of $y'_{i,i-1,h}$ using equation 5.4.3. b is an appropriate scalar constant and $u_{i,h}$ and $x_{i,h}^2$ are related according to figure 5.5.1. Over the linear region $u_{i,h}$ and $x_{i,h}^2$ satisfy the relation

$$u_{i,h} = c x_{i,h} \tag{5.5.3}$$

c is an appropriate positive real valued constant. The value of $u_{i,h}$ is constrained such that

$$k_1 < u_{i,h} < k_2 \tag{5.5.4}$$

where k_1 and k_2 are appropriate positive real valued constants.

If a is set to unity in equation 5.5.1, no averaging is carried out, and equation 5.5.1 becomes

$$x_{i,h}^2 = | y'_{i,i-1,h} |^2 \tag{5.5.5}$$

System 3 is tested only for the degree 2 least squares fading memory prediction, to compare the performance of the degree 2 prediction for the different adaptive algorithms.

5.6 RESULTS OF SIMULATION TESTS

Computer simulation tests have been carried out on the channel estimator described in section 5.4-5.5. The results of which are given in Tables 5.6.1-5.6.8 and Fig 5.6.1-5.6.14.

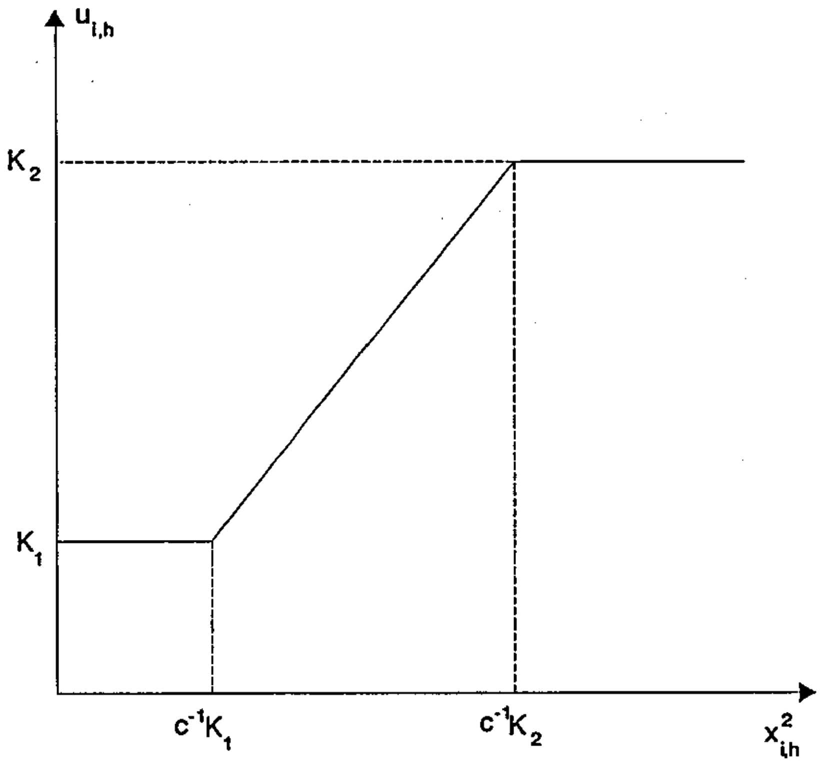


FIGURE 5.5.1 PLOT OF VARIATION OF $u_{i,h}$ WITH $x^2_{i,h}$

The signal/noise ratio (ψ), is defined as,

$$\psi = 10\log_{10}\frac{E_b}{\frac{1}{2}N_0} \tag{5.6.1}$$

where E_b, is the average transmitted and received energy per bit at the input and output respectively of the HF radio link, and is unity. The two sided power spectral density of the white Gaussian noise at the output of the HF radio link is $0.5N_0$.

Every measurement has involved the transmission of 60000 data symbols. In error measurements the first 5000 received samples are ignored in order to eliminate any transient behaviour of the estimator which may be present at start up. The error measurements are made over the remaining 55000 received samples, signifying the region of steady state performance of the estimator. The error measurements are,

$$\xi_1 = 10\log_{10}\left(\frac{1}{55000}\sum_{i=5001}^{60000} |Y_i - Y'_{i,i-1}|^2\right) \tag{5.6.2}$$

and

$$\xi_2 = 10\log_{10}\left(\frac{1}{55000}\sum_{i=5001}^{60000} \frac{|Y_i - Y'_{i,i-1}|^2}{|Y_i|^2}\right) \tag{5.6.3}$$

where ξ_1 is called the mean square estimation error and is a measure of the actual error in $Y'_{i,i-1}$. ξ_2 is called the mean square normalized estimation error and is a measure of the normalized or relative error in $Y'_{i,i-1}$. ξ_2 is obtained to give a more realistic picture of the actual error in $Y'_{i,i-1}$ as the actual length of Y_i is taken into account in the error measurement.

The number of components (g+1) in the sampled impulse response of the channel is taken to be 21. At start up, it is assumed that $Y'_{1,0} = Y_1$, which is the actual channel sampled impulse response at the first sampling instant. In the tests b, θ, k_0, c and k_2 have been optimized to a high degree of accuracy, so that the error in the estimation/prediction of the sampled impulse response of the channel, defined by equations 5.6.2-5.6.3, is minimized.

Tables 5.6.1-5.6.8 show the mean squared error for various prediction algorithms used in the tests. Figures 5.6.1-5.6.5 show the relative performance in the error measurements used for the various least squares fading memory prediction algorithms tested for system 2. Figure 5.6.6 shows the performance of system 3 tested with degree 2 least squares fading memory prediction.

The basic mechanism behind the improvement of system 2 with degree 1 prediction over the simple estimator in chapter 4.5, is at least in part, due to the fact that system 2 is better able to correct an error in $y'_{i,i-1}$ caused by an acceleration in Y_i. Similarly for system 2 with degree 0 prediction. From the plots it can be seen that the relative performance of the systems are not significantly affected by whether equation 5.6.2 or equation 5.6.3 is used for error measurements.

Figure 5.6.7 is a plot of the channel over which the estimator was tested, neglecting the first 5000 samples for the sake of convenience in comparison with figures 5.6.8-5.6.14.

Figures 5.6.8-5.6.10 show the steady state performance of the system 2 described in section 5.4, using the degree 0, 1, 2 least squares fading memory prediction algorithms, respectively. The parameter in Figure 5.6.8-5.6.10, is the square of the error in $Y'_{i,i-1}$ measured in dB, and is,

$$\xi_i = 10\log_{10}|Y_i - Y'_{i,i-1}|^2 \tag{5.6.4}$$

System 2 with degree 2 prediction did not perform adequately, the reason for this would seem obvious. The degree 2 least squares fading memory prediction process is more prone to unstability as compared to the corresponding degree 1 prediction process. From the characteristic of figure 5.4.1, it can be observed that for small values of $x_{i,h}^2$ in the region around d, $u_{i,h}$ shows a wide variation in its possible value. Hence for small values of $x_{i,h}^2$, the updated estimate and therefore the prediction could be degraded resulting in a poorer performance. Tests were carried out on system 2 with degree 2 prediction, and from the results it could be concluded that system 2 could not obtain an accurate prediction of Y_i at small values of $x_{i,h}^2$, thereby resulting in a poorer performance. This can be observed from figure 5.6.10.

Figure 5.6.11 shows the steady state performance of system 3 described in section 5.5, using the degree 2 least squares fading memory prediction algorithm. The parameter in Figure 5.6.11, is the square of the error in $Y'_{i,i-1}$ measured in dB, and is given by equation 5.6.4.

An attempt was made to correct this unstabilty, hence system 3. In system 3, the variation of $u_{i,h}$ with $x_{i,h}^2$ is given by the characteristic shown in figure 5.5.1, instead of figure 5.4.1. In figure 5.5.1, the less rapid variation in $u_{i,h}$ for small values of $x_{i,h}^2$, results in the system being more stable, hence an improved performance. From figure 5.6.11 it can be seen that the adaptive estimator system 3 with degree 2 least squares fading memory prediction does not suffer form any unstabilty problems.

TABLE 5.6.1 MEAN SQUARE ERROR IN PREDICTION FOR AN ADAPTIVE ESTIMATOR SYSTEM 2 WITH DEGREE 0 LEAST SQUARES PREDICTION

SNR	b	θ	K_0	$b(1-\theta)$	ξ_1	Improvement over Simple Estimator
10	1.0	0.975	0.086	0.025	-17.6085	3.8211
20	1.0	0.946	0.058	0.054	-23.7179	4.3489
30	1.0	0.903	0.037	0.097	-29.2381	5.7369
40	1.0	0.859	0.023	0.141	-33.2667	8.1377
60	1.0	0.839	0.018	0.161	-35.0521	9.6211

TABLE 5.6.2 NORMALIZED MEAN SQUARE ERROR IN PREDICTION FOR AN ADAPTIVE ESTIMATOR SYSTEM 2 WITH DEGREE 0 LEAST SQUARES FADING MEMORY PREDICTION

SNR	b	θ	K_0	$b(1-\theta)$	ξ_2	Improvement over Simple Estimator
10	1.0	0.967	0.100	0.033	-13.3426	4.3390
20	1.0	0.928	0.066	0.072	-19.4694	4.9816
30	1.0	0.877	0.041	0.123	-24.7926	6.7587
40	1.0	0.840	0.023	0.160	-28.2399	9.0916
60	1.0	0.831	0.016	0.169	-29.3020	9.9609

TABLE 5.6.3 MEAN SQUARE ERROR IN PREDICTION FOR AN ADAPTIVE ESTIMATOR SYSTEM 2 WITH DEGREE 1 LEAST SQUARES PREDICTION

SNR	b	Θ	K_0	ξ_1	Improvement over Simple Estimator
10	1.0	0.990	0.107	-18.1363	3.9418
20	1.0	0.982	0.070	-25.4006	3.8042
30	1.0	0.973	0.047	-32.6197	4.0865
40	1.0	0.959	0.032	-39.4409	4.9773
60	1.0	0.935	0.015	-48.8797	9.6272

TABLE 5.6.4 NORMALIZED MEAN SQUARE ERROR IN PREDICTION FOR AN ADAPTIVE ESTIMATOR SYSTEM 2 WITH DEGREE 1 LEAST SQUARES FADING MEMORY PREDICTION

SNR	b	Θ	K_0	ξ_2	Improvement over Simple Estimator
10	1.0	0.989	0.107	-14.4828	4.4875
20	1.0	0.981	0.069	-22.1792	4.4708
30	1.0	0.971	0.047	-29.2859	4.3234
40	1.0	0.959	0.033	-35.7905	5.1389
60	1.0	0.931	0.016	-45.3453	9.7327

TABLE 5.6.5 MEAN SQUARE ERROR IN PREDICTION FOR AN ADAPTIVE ESTIMATOR SYSTEM 2 WITH DEGREE 2 LEAST SQUARES PREDICTION

SNR	b	Θ	K_0	ξ_1	Improvement over Simple Estimator
10	1.0	0.993	0.134	-17.3647	3.9782
20	1.0	0.990	0.095	-23.9323	2.9158
30	1.0	0.986	0.061	-30.1142	1.6925
40	1.0	0.983	0.047	-34.5235	-0.6313
60	1.0	0.981	0.038	-35.9045	-6.7361

TABLE 5.6.6 NORMALIZED MEAN SQUARE ERROR IN PREDICTION FOR AN ADAPTIVE ESTIMATOR SYSTEM 2 WITH DEGREE 2 LEAST SQUARES FADING MEMORY PREDICTION

SNR	b	Θ	K_0	ξ_2	Improvement over Simple Estimator
10	1.0	0.993	0.134	-13.5811	4.6316
20	1.0	0.989	0.093	-19.6034	3.3563
30	1.0	0.986	0.055	-25.2070	1.3886
40	1.0	0.981	0.047	-29.3812	-0.8856
60	1.0	0.978	0.038	-31.6812	-5.7075

TABLE 5.6.7 MEAN SQUARE ERROR IN PREDICTION FOR AN ADAPTIVE ESTIMATOR SYSTEM 3 WITH DEGREE 2 LEAST SQUARES PREDICTION

SNR	b	θ	c	K_1	K_2	ξ_1	Improvement over simple estimator
10	1.0	0.994	9.2	10^{-5}	0.76	-16.6623	3.2758
20	1.0	0.990	17.0	10^{-5}	0.63	-23.7753	2.7588
30	1.0	0.984	46.4	10^{-5}	0.46	-31.0949	2.6732
40	1.0	0.973	53.9	10^{-4}	0.30	-38.0670	2.9122
60	1.0	0.967	92.1	10^{-4}	0.38	-46.4866	3.8460

TABLE 5.6.8 NORMALIZED MEAN SQUARE ERROR IN PREDICTION FOR AN ADAPTIVE ESTIMATOR SYSTEM 3 WITH DEGREE 2 LEAST SQUARES FADING MEMORY PREDICTION

SNR	b	θ	c	K_1	K_2	ξ_2	Improvement over simple estimator
10	1.0	0.994	9.8	10^{-5}	0.76	-13.2946	4.3451
20	1.0	0.990	17.0	10^{-5}	0.67	-19.6410	3.3939
30	1.0	0.983	46.4	10^{-5}	0.46	-27.2564	3.4380
40	1.0	0.973	53.6	10^{-4}	0.30	-34.1001	3.8333
60	1.0	0.967	92.9	10^{-4}	0.38	-42.8170	5.4283

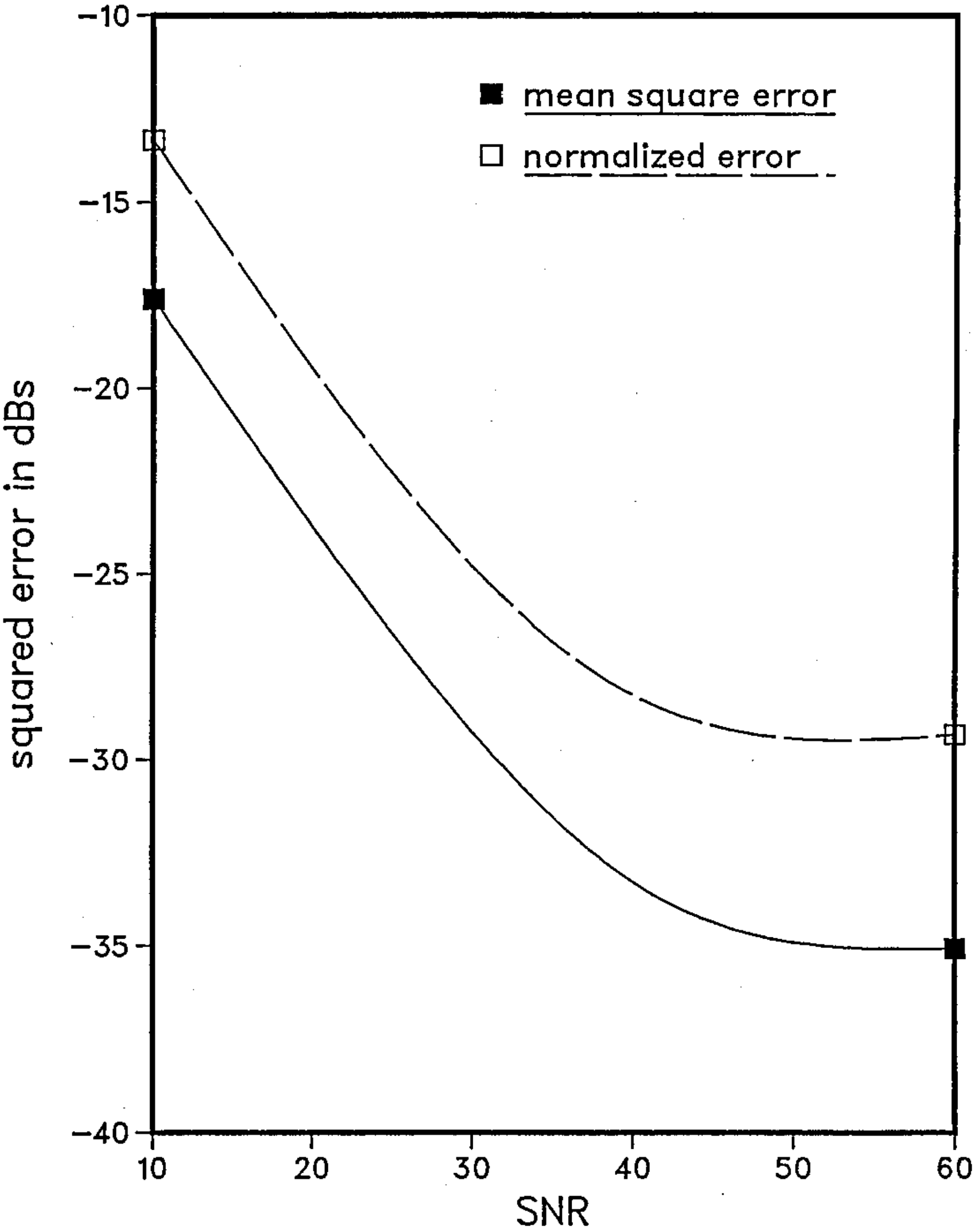


FIGURE 5.6.1 PLOT OF THE PERFORMANCE OF AN ADAPTIVE ESTIMATOR WITH DEGREE 0 LEAST SQUARES FADING MEMORY PREDICTION

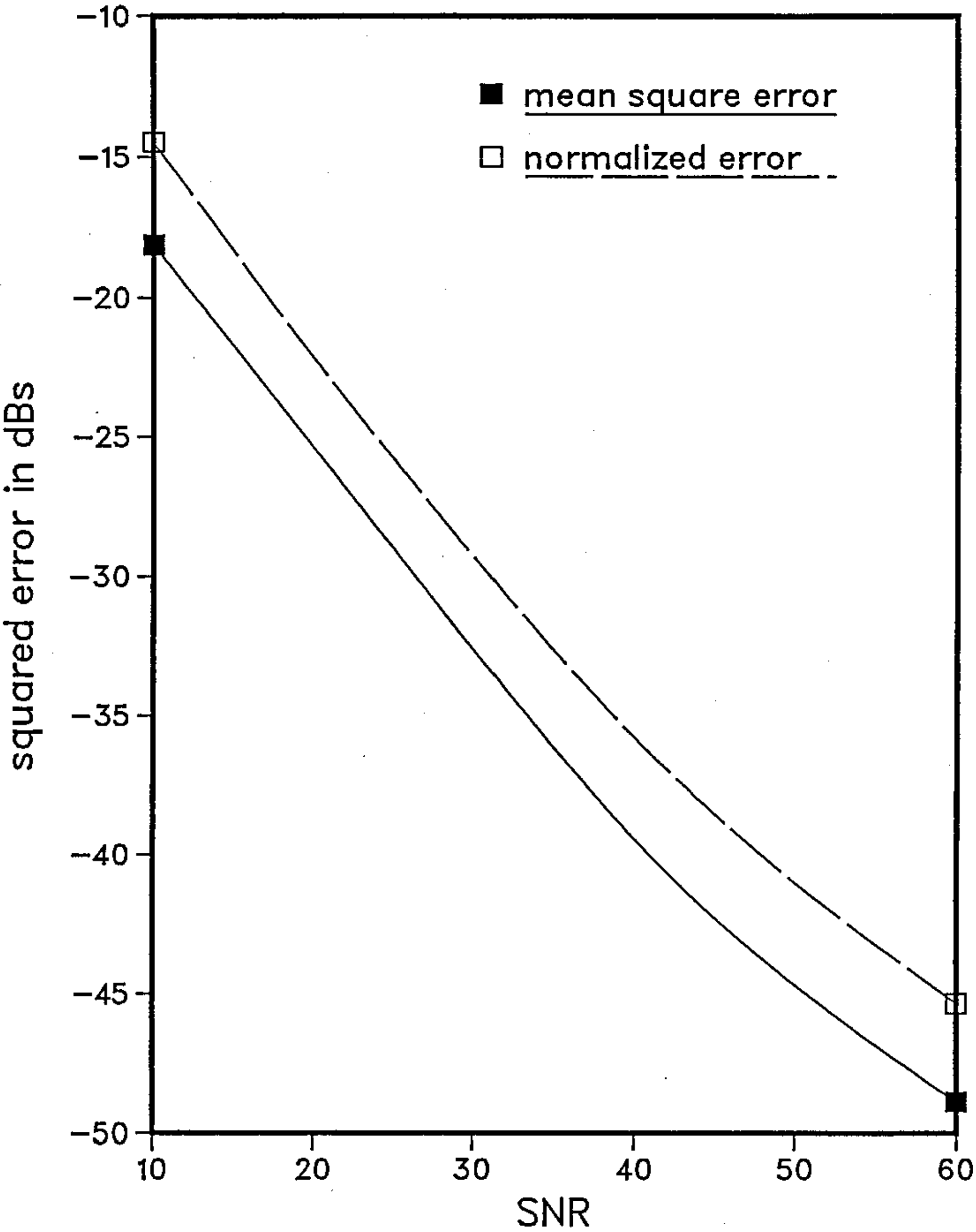


FIGURE 5.6.2 PLOT OF THE PERFORMANCE OF AN ADAPTIVE ESTIMATOR WITH DEGREE 1 LEAST SQUARES FADING MEMORY PREDICTION

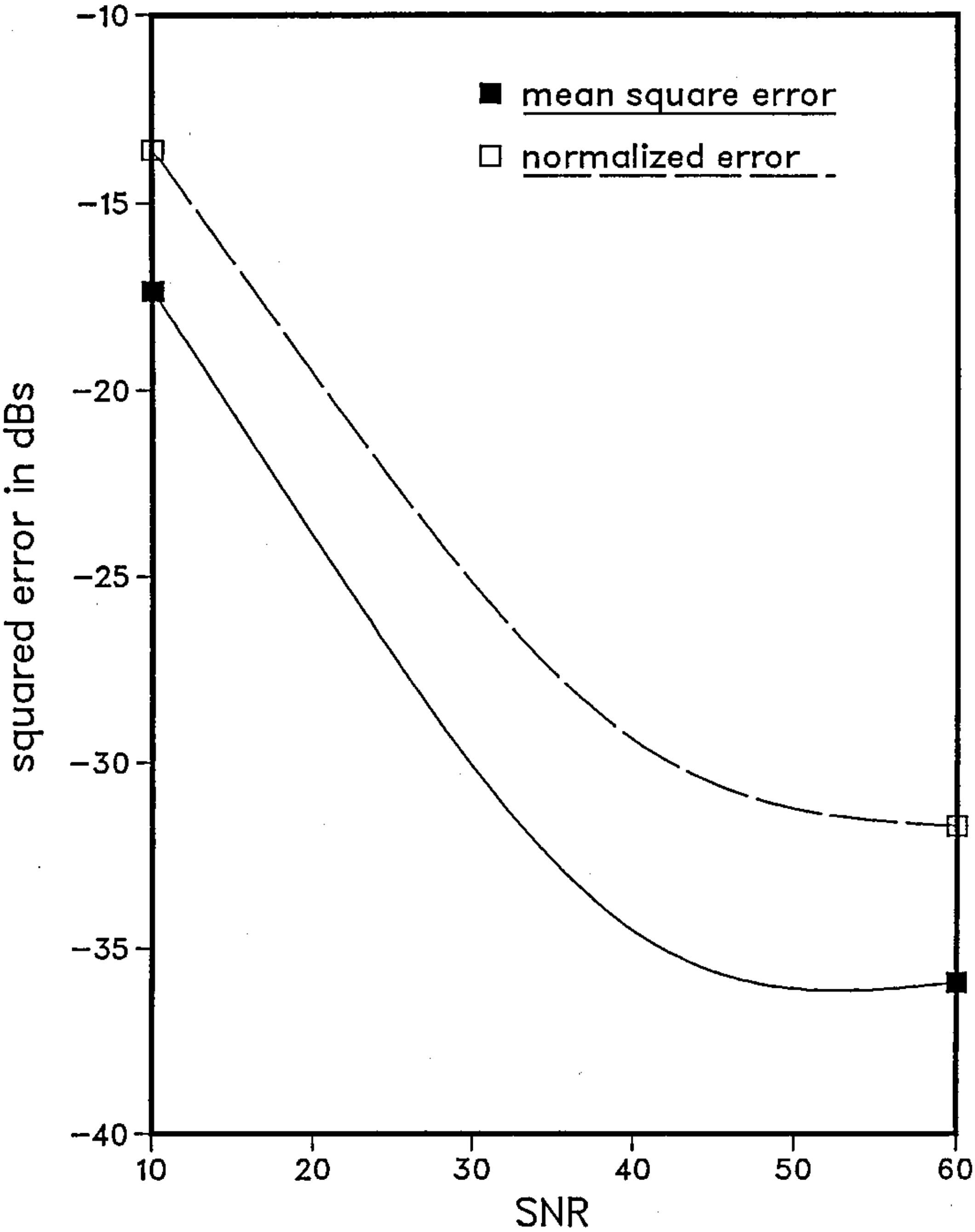


FIGURE 5.6.3 PLOT OF THE PERFORMANCE OF AN ADAPTIVE ESTIMATOR WITH DEGREE 2 LEAST SQUARES FADING MEMORY PREDICTION

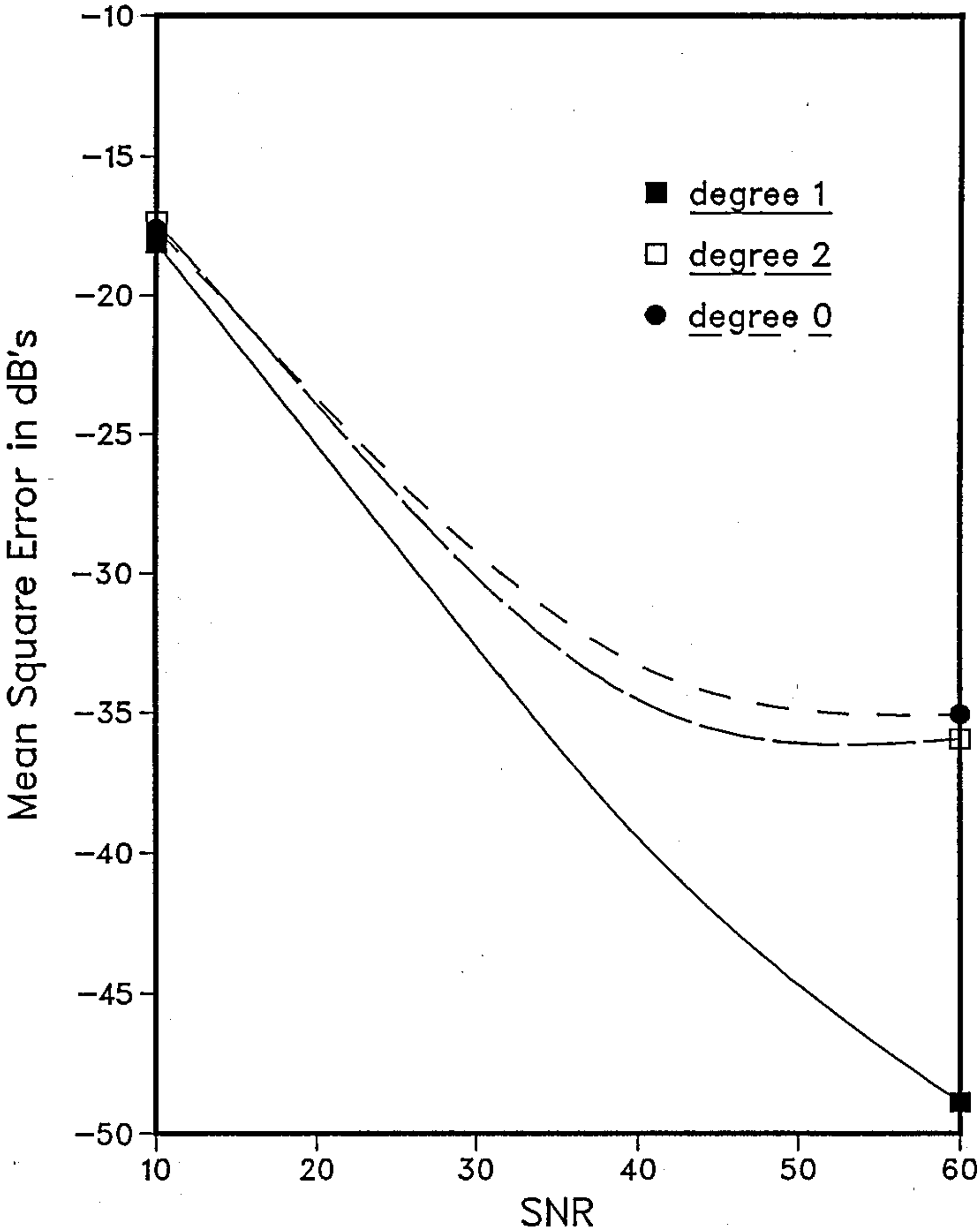


FIGURE 5.6.4 PLOT OF MEAN SQUARE ERROR IN PREDICTION FOR THE ADAPTIVE ESTIMATOR SYSTEM 2 WITH DEGREE 0,1,2 LEAST SQUARES FADING MEMORY PREDICTION

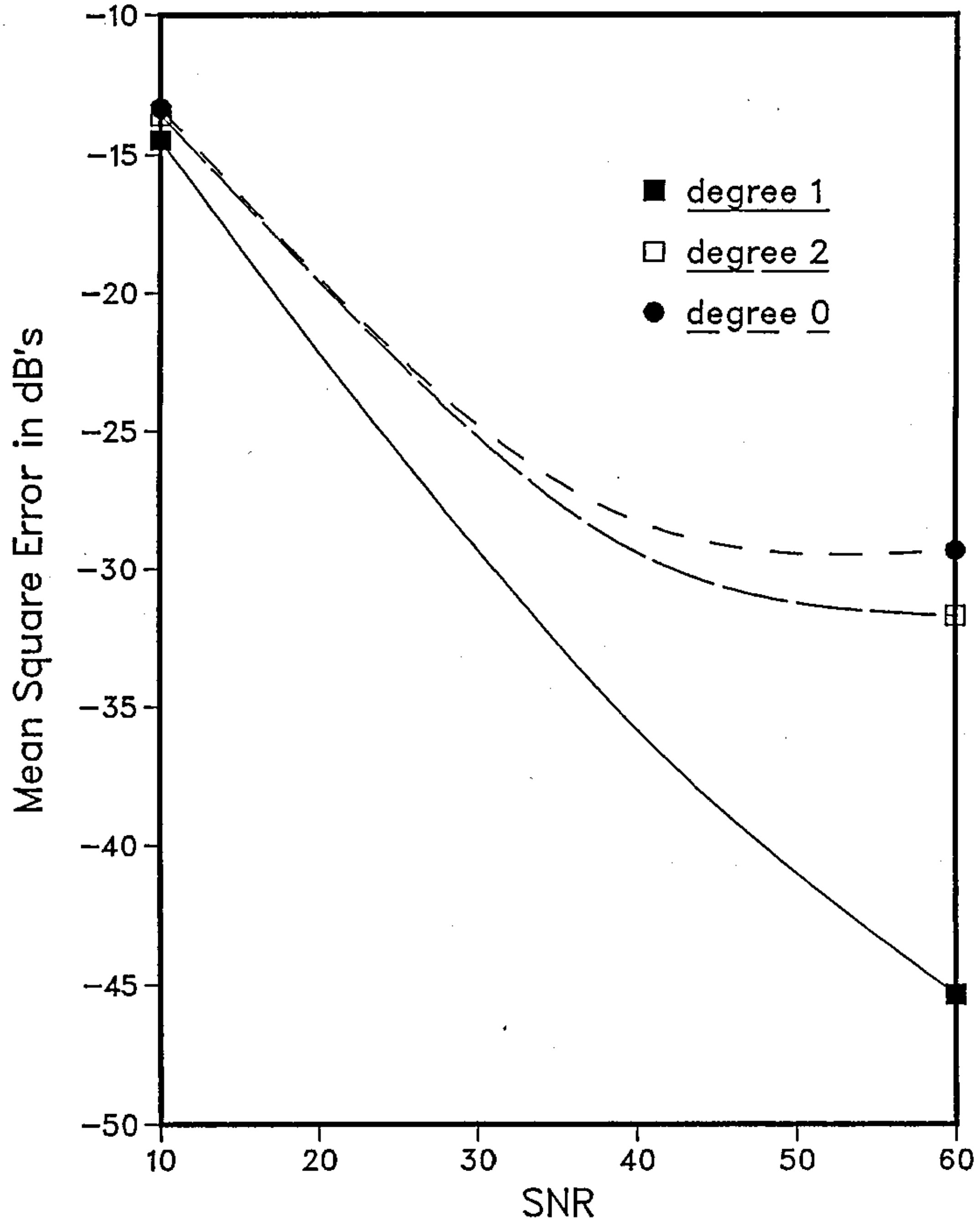


FIGURE 5.6.5 PLOT OF NORMALIZED MEAN SQUARE ERROR IN PREDICTION FOR THE ADAPTIVE ESTIMATOR SYSTEM 2 WITH DEGREE 0,1,2 LEAST SQUARES FADING MEMORY PREDICTION

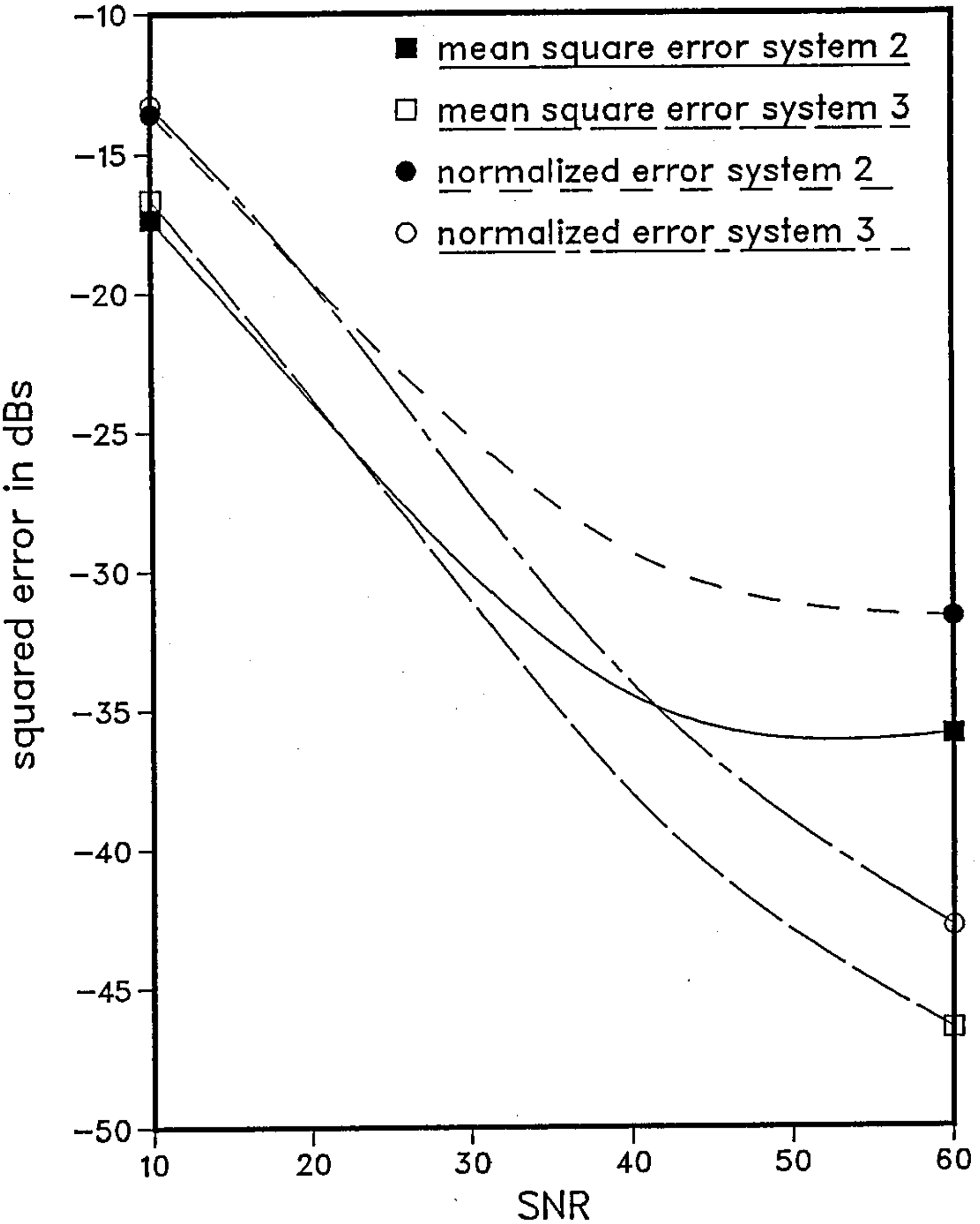


FIGURE 5.6.6 PLOT OF THE PERFORMANCE OF AN ADAPTIVE ESTIMATOR SYSTEM 2,3 WITH DEGREE 2 LEAST SQUARES FADING MEMORY PREDICTION

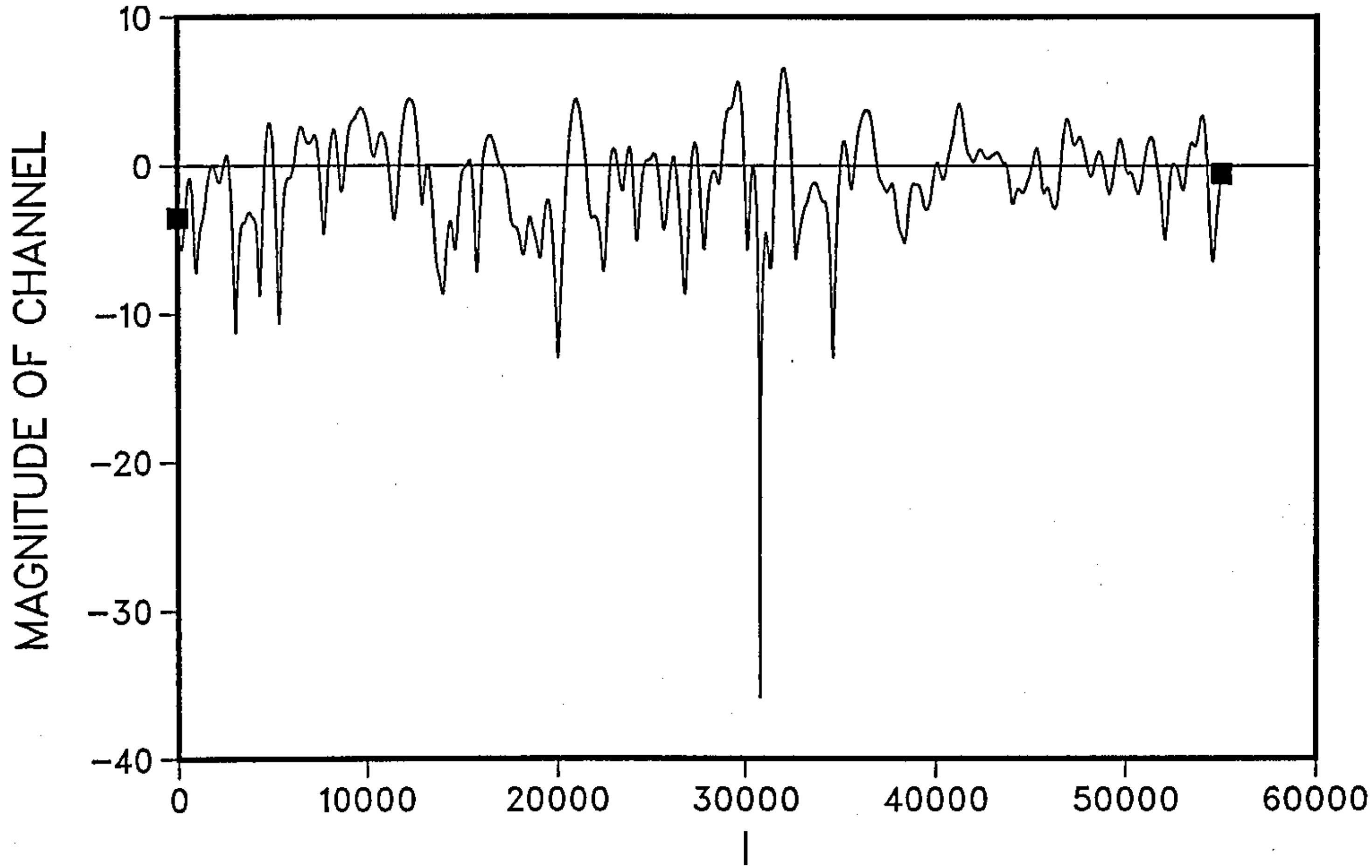


FIGURE 5.6.7 PLOT OF THE MAGNITUDE OF THE CHANNEL OVER WHICH ESTIMATOR WAS TESTED, SEED=50

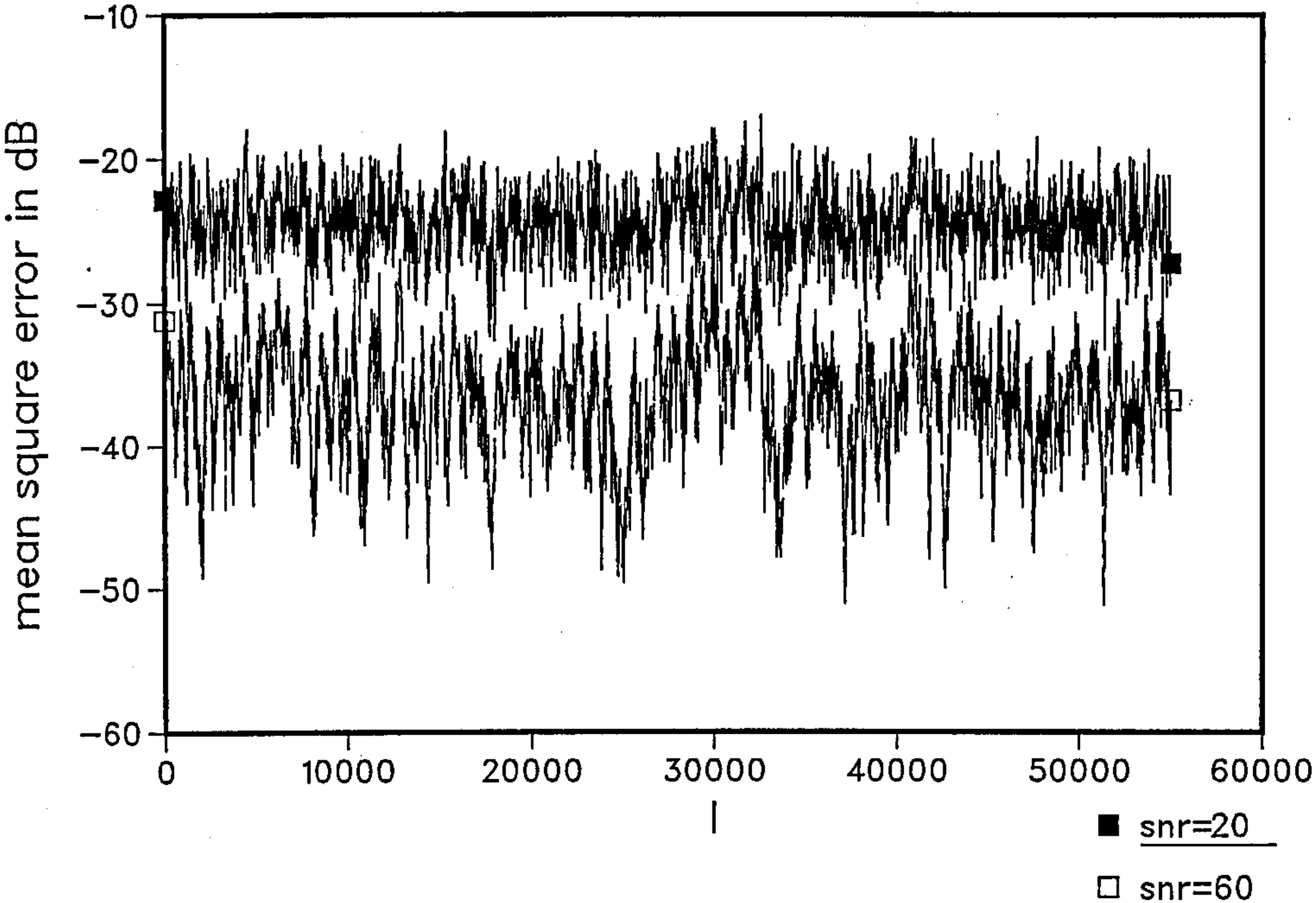


FIGURE 5.6.8 STEADY STATE PERFORMANCE OF THE ADAPTIVE ESTIMATOR
SYSTEM 2 WITH DEGREE 0 LEAST SQUARES FADING MEMORY PREDICTION SNR=20,60

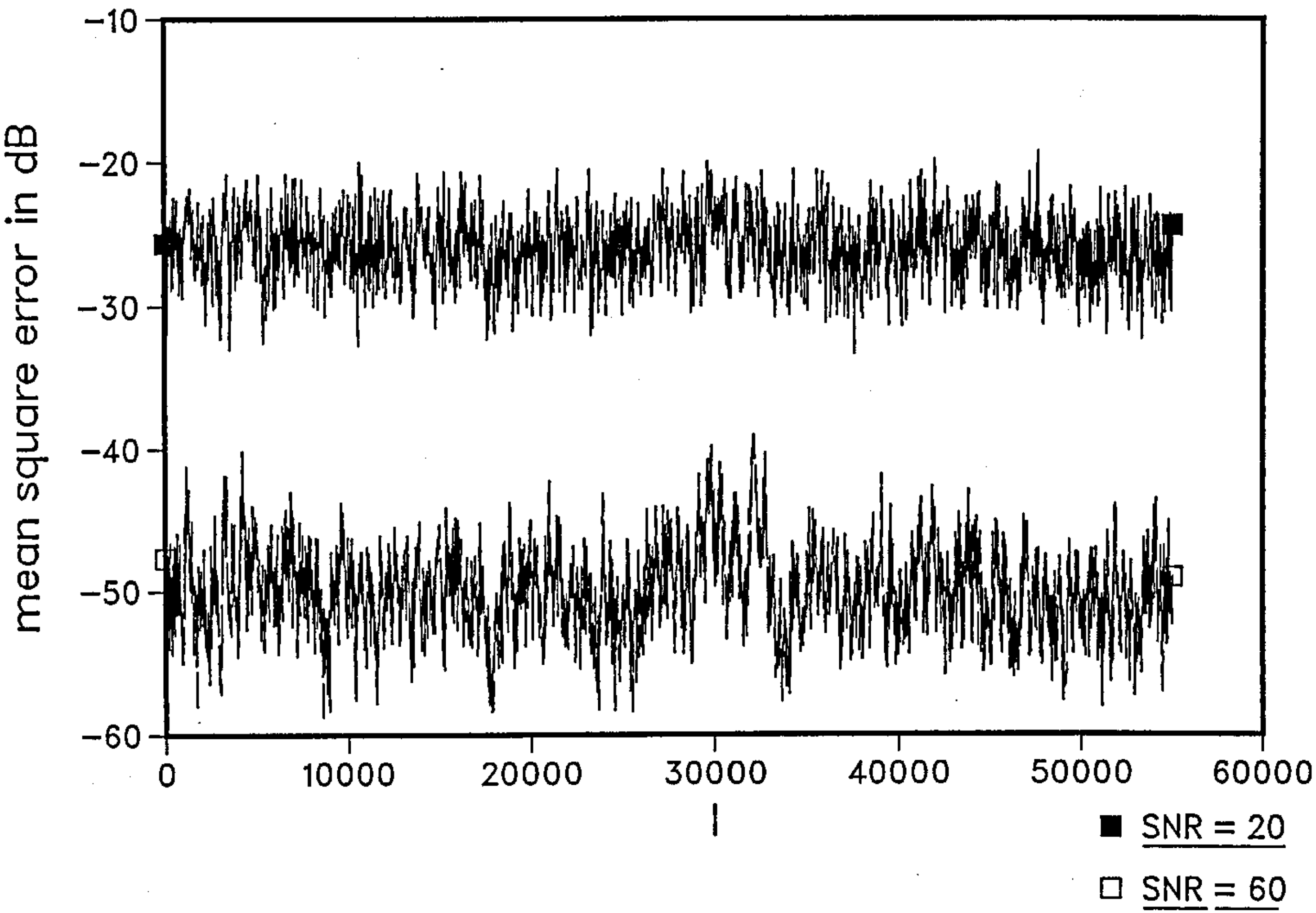


FIGURE 5.6.9 PLOT OF THE STEADY STATE PERFORMANCE OF THE ADAPTIVE ESTIMATOR SYSTEM 2 WITH DEGREE 1 LEAST SQUARES FADING MEMORY PREDICTION, SNR= 20,60

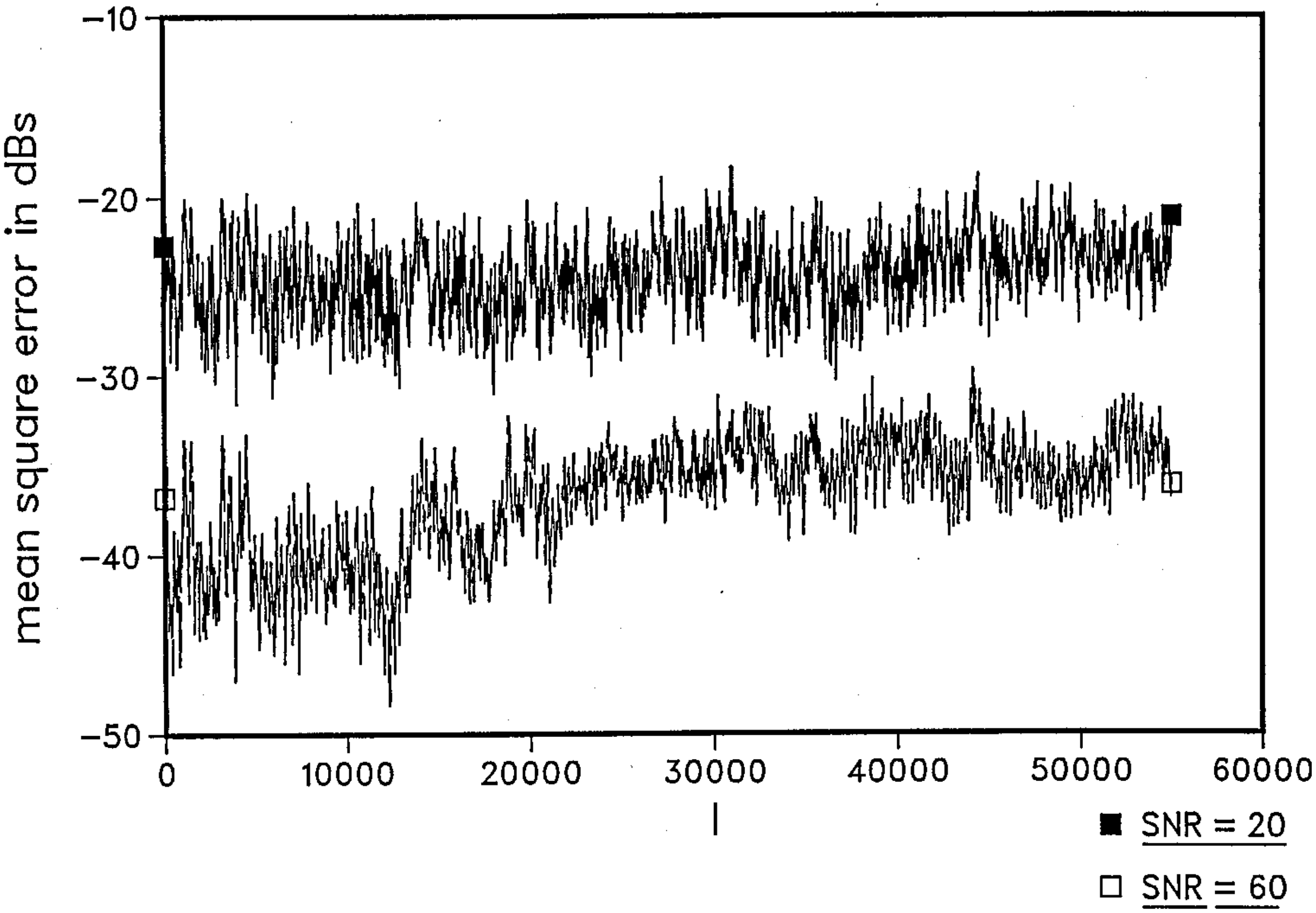


FIGURE 5.6.10 PLOT OF THE STEADY STATE PERFORMANCE OF THE ADAPTIVE ESTIMATOR SYSTEM 2 WITH DEGREE 2 LEAST SQUARES FADING MEMORY PREDICTION, SNR= 20,60

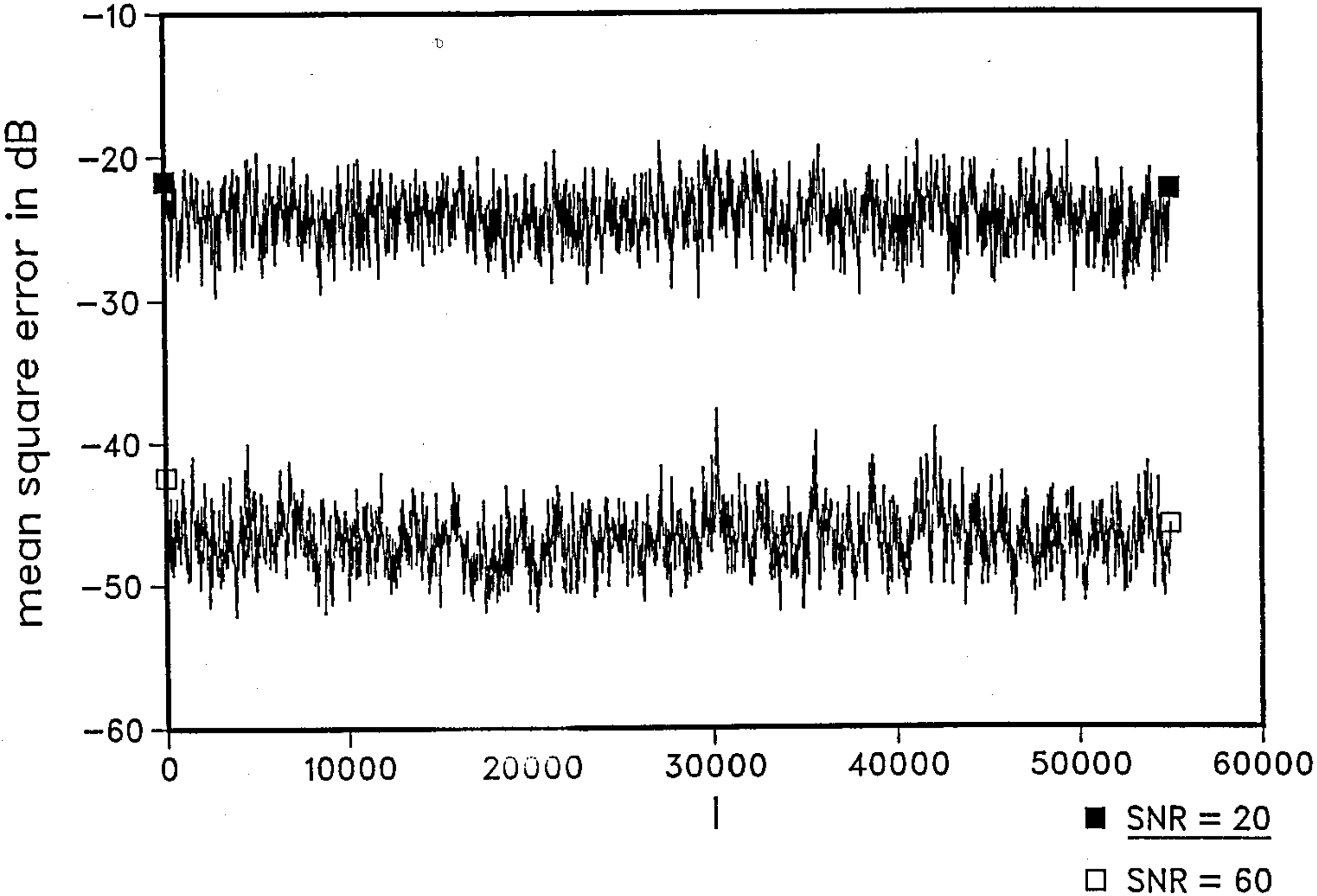


FIGURE 5.6.11 PLOT OF THE STEADY STATE PERFORMANCE OF THE ADAPTIVE ESTIMATOR SYSTEM 3 WITH DEGREE 2 LEAST SQUARES FADING MEMORY PREDICTION, SNR=20,60

CHAPTER 6

One sometime finds what one is not looking for

- Sir Alexander Fleming

CHAPTER 6

MODIFIED CHANNEL ESTIMATOR

6.1 INTRODUCTION

A simple estimator designed for a 4800 bits/s modem which employs a polynomial filter that gives a prediction of the channel sampled impulse response has been considered in chapter 4. This estimator is a development of the conventional gradient estimator [1-12]. The estimator studied in this chapter is a development of the simple estimator described in chapter 4, in that a prediction is made of the error between the estimate of the channel and its prediction, which is then used to from an updated estimate of the channel. The modified estimator therefore has one extra predictor as compared to the simple estimator of chapter 4.

In section 6.2 an outline is given of the various assumptions made in the consideration of the data transmission system. Section 6.3 provides a description of the various estimator configurations tested over the 2 skywave channel. Finally, section 6.4 provides the results of simulation tests carried out on the estimator to assess its performance.

6.2 MODEL OF SYSTEM

The model of the data transmission system assumed in the investigation is shown in figure 3.4.2.

The signal at the input to the baseband channel is a sequence of regularly spaced impulses $\{s_i\delta(t-iT)\}$ where the s_i are assumed to be statistically independent and equally likely to have any one of its four values $\pm 1 \pm j(j=\sqrt{-1})$.

Transmission starts at time t=iT seconds. The linear baseband channel has an impulse response y(t) with an effective duration of less than (g+1)T seconds where g is a given positive integer. The only noise assumed to be introduced by the channel is stationary white Gaussian noise which is added to the data signal at the output of the transmission path such that the noise waveform w(t) at the output of the receiver filter is a bandlimited Gaussian noise. Thus the output signal form the baseband channel in Fig 3.4.2 is the waveform

$$r(t) = \sum_i s_i y(t-iT) + w(t) \tag{6.2.1}$$

where $s_i y(t-iT)$ is the i^{th} received signal element. The waveform r(t) is sampled once per received signal element, at the time instant {iT}, giving the received samples r_i, where,

$$r_i = \sum_{h=0}^{g} s_{i-h} y_{i,h} + w_i \tag{6.2.2}$$

$$= Y_i S_i^T + w_i \tag{6.2.3}$$

The delay in transmission is neglected so that $y_i = 0$ for i<0 and i>g. let Y_i represent the (g+1) component column vector of the sampled impulse response of the channel.

$$Y_i = [y_{i,0} y_{i,1} \cdots y_{i,g}] \tag{6.2.4}$$

The noise samples w_i are slightly correlated as they have been filtered at the receiver, they have zero mean and variance that is dependent on $0.5N_0$, and are statistically independent of the $\{s_i\}$.

The signal $\{r_i\}$ and s_i are fed to the channel estimator to give an estimate Y'_i of Y at time t=iT, where,

$$Y'_i = [y'_{i,0} y'_{i,1} \cdots y'_{i,g}] \tag{6.2.5}$$

The estimate Y'_i is used by the detector for the detection of the next data symbol on the receipt of r_{i+1}. Details of the channel model are given in chapter 3.

6.3 MODIFIED ESTIMATOR

This system is a development of the simple estimator described in chapter 4. The estimator uses the same linear feedforward transversal filter as in chapter 4.5. Assuming correct detection so that the receiver knows the $\{s_i\}$.

$$S_i = [s_1 s_2 \ldots s_{i-g}] \tag{6.3.1}$$

The simple estimator (section 4.5) operates as follows. On the receipt of r_i, the estimator holds a one step prediction of Y_i, given by

$$Y'_{i,i-1} = [y'_{i,i-1,0} y'_{i,i-1,1} \cdots y'_{i,i-1,g}] \tag{6.3.2}$$

and it then forms an estimate of r_i, given by

$$r'_i = \sum_{h=0}^{g} s'_{i-h} y'_{i,i-1,h} \tag{6.3.3}$$

e_i is the error in the estimate of the i^{th} received signal r_i,

$$e_i = r_i - \sum_{h=0}^{g} s'_{i-h} y'_{i,i-1,h} \qquad (6.3.4)$$

The error signal is then used to form the updated estimate of Y_i, given by

$$Y'_i = Y'_{i,i-1} + b e_i S_i^* \qquad (6.3.5)$$

where

$$Y'_i = [y'_{i,0} y'_{i,1} ... y'_{i,g}] \qquad (6.3.6)$$

S_i^* is a (g+1) component vector given and is the complex conjugate of S_i, and b is an appropriate small positive real valued constant.

The predictor then forms the error vector,

$$\begin{aligned} E_i &= Y'_i - Y'_{i,i-1} \\ &= b e_i S_i^* \end{aligned} \qquad (6.3.7)$$

which is fed with $Y'_{i,i-1}$ to the appropriate set of g+1 polynomial filters to form the one step prediction of Y_{i+1}, given by

$$Y'_{i+1,i} = [y'_{i+1,i,0} y'_{i+1,i,1} ... y'_{i+1,i,g}] \qquad (6.3.8)$$

The polynomial filter may be degree 0, 1, 2 least squares fading memory prediction methods.

The possible weakness of this arrangement is that E_i is a scalar multiple of S_i^* and the latter has any one of a large number of different directions in the vector space, which means, of course, that most of the time, the correction $be_iS_i^*$ that is applied to $Y'_{i,i-1}$ to form Y'_i is not in the most ideal direction. If this weakness could be corrected, an improvement in achievement might be achieved.

The simplest and potentially most cost effective solution to this problem, is to filter the g+1 components of E_i through the appropriate degree 0, 1, or 2 polynomial filter [11], as follows.

Let $E'_{i,i-1}$ be a one step prediction of E_i, such that the measured error in $E'_{i,i-1}$ is

$$F_i = E_i - E'_{i,i-1} \tag{6.3.9)}$$

F_i is now fed, with $E'_{i,i-1}$ to the appropriate set of g+1 polynomial filters to form the one step prediction of E_{i+1}, given by $E'_{i+1,i}$. As before, the polynomial filters may be degree 0, 1, 2. E_i in equation 6.3.9 is, given by equation 6.3.7, so that Y'_i must be determined from equation 6.3.5, as before.

The polynomial filters determine, in addition to $E'_{i+1,i}$, the g+1 component vector cF_i, where $c = (1-\theta_2), (1-\theta_2^2)$ or $(1-\theta_2^3)$, depending upon whether the polynomial filters are degree 0, degree 1, or degree 2 least squares fading memory prediction processes, respectively. This prediction process, is referred to as the secondary prediction in the results presented in chapter 6.4. θ_2 is here a positive real-valued constant in the range 0 to 1.

An improved estimate of Y_i, which is better than Y'_i obtained from equation 6.3.5, should now be,

$$Z_i = Y'_{i,i-1} + E'_{i,i-1} + cF_i \tag{6.3.10}$$

and the measured error in $Y'_{i,i-1}$, is taken to be

$$\begin{aligned} X_i &= Z_i - Y'_{i,i-1} \\ &= E'_{i,i-1} + cF_i \end{aligned} \tag{6.3.11}$$

Finally, X_i and $Y'_{i,i-1}$ are fed to the appropriate g+1 polynomial filter, which may be degree 0, 1, 2 least squares fading memory prediction, to give the one step prediction of Y_{i+1}, which is $Y'_{i+1,i}$, this prediction process, being referred to as the main prediction in the results presented in chapter 6.4.

Z_i is the best estimate that can be made of Y_i, which is not $Y'_{i,i-1}+E'_{i+1,i}$, since $E'_{i+1,i}$ applies to time (i+1)T and not iT as it should, neither is $Y'_{i,i-1}+E'_{i,i-1}$ the best estimate of Y_i at time iT, as it makes no use of e_i (equation 6.3.4) which is now available from the feedforward transversal filter. Thus, Z_i must be given by equation 6.3.10, where $c=(1-\theta_2)$, $(1-\theta_2^2)$ or $(1-\theta_2^3)$, depending upon the degree of the polynomial filter. The least squares prediction algorithms are as in chapter 4, table 4.5.2.1.

To summarize the operation of the system, consider the particular case where all polynomial filters are degree 1. That is, the main and secondary prediction processes both implement degree 1 least squares fading memory prediction. The algorithm is as follows

$$Y'_i = Y'_{i,i-1} + b\,e_i S^*_i \tag{6.3.12}$$

$$\begin{aligned} E_i &= Y'_i - Y'_{i,i-1} \\ &= b\,e_i S^*_i \end{aligned} \tag{6.3.13}$$

$$F_i = E_i - E'_{i,i-1} \tag{6.3.14}$$

$$E''_{i+1,i} = E''_{i,i-1} + (1-\theta_2)^2 F_i \tag{6.3.15}$$

$$E'_{i+1,i} = E'_{i,i-1} + E''_{i+1,i} + (1-\theta_2^2) F_i \tag{6.3.16}$$

$$Z_i = Y'_{i,i-1} + E'_{i,i-1} + (1-\theta_2^2) F_i \tag{6.3.17}$$

$$X_i = Z_i - Y'_{i,i-1}$$
$$= E'_{i,i-1} + (1-\theta_2^2)F_i \tag{6.3.18}$$

$$Y''_{i+1,i} = Y''_{i,i-1} + (1-\theta)^2 X_i \tag{6.3.19}$$

$$Y'_{i+1,i} = Y'_{i,i-1} + Y''_{i+1,i} + (1-\theta^2)X_i \tag{6.3.20}$$

where, θ and θ_2 are between 0 and 1. The vectors $Y'_{i,i-1}$ and $E'_{i,i-1}$ are both available, having been evaluated during the previous iteration of the algorithm.

The corresponding all degree 0 algorithm, where both the main and secondary prediction are obtained using a degree 0 least squares fading memory prediction, operates as follows,

$$Y'_i = Y'_{i,i-1} + b\,e_i S_i^* \tag{6.3.21}$$

$$E_i = Y'_i - Y'_{i,i-1}$$
$$= b\,e_i S_i^* \tag{6.3.22}$$

$$F_i = E_i - E'_{i,i-1} \tag{6.3.23}$$

$$E'_{i+1,i} = E'_{i,i-1} + (1-\theta_2)F_i \tag{6.3.24}$$

$$Z_i = Y'_{i,i-1} + E'_{i,i-1} + (1-\theta_2)F_i \tag{6.3.25}$$
$$X_i = Z_i - Y'_{i,i-1}$$
$$= E'_{i,i-1} + (1-\theta_2)F_i \tag{6.3.26}$$

$$Y'_{i+1,i} = Y'_{i,i-1} + (1-\theta)X_i \tag{6.3.27}$$

In the case of a degree 2 least squares fading memory prediction being used to predict the error signal E_i, equation 6.3.25 is replaced by,

$$Z_i = Y'_{i,i-1} + E'_{i,i-1} + (1-\theta_2^3)F_i \qquad (6.3.28)$$

and equation 6.3.26 by,

$$\begin{aligned} X_i &= Z_i - Y'_{i,i-1} \\ &= E'_{i,i-1} + (1-\theta_2^3)F_i \end{aligned} \qquad (6.3.29)$$

with equation 6.3.24 being replaced with the corresponding degree 2 prediction algorithm.

The estimator was tested for different combination of prediction schemes. Their were 6 different schemes tested, where the prediction, $E'_{i,i-1}$, of the error signal E_i, was given by degree 0, 1, 2 least squares fading memory prediction and the prediction of Y_i which is $Y'_{i,i-1}$ was given by degree 0, 1 least squares fading memory prediction

6.4 RESULTS OF SIMULATION TESTS

Computer simulation tests have been carried out on the channel estimator described in section 6.3. The results of which are given in Tables 6.4.1-6.4.6 and Fig 6.4.1-6.4.13.

The signal/noise ratio (ψ), is defined as,

$$\psi = 10\log_{10}\frac{E_b}{\frac{1}{2}N_0} \qquad (6.4.1)$$

where E_b, is the average transmitted and received energy per bit at the input and output respectively of the HF radio link, and is unity. The two sided power spectral density of the white Gaussian noise at the output of the HF radio link is $0.5N_0$.

Every measurement has involved the transmission of 60000 data symbols. In error measurements the first 5000 received samples are ignored in order to eliminate any transient behaviour of the estimator which may be present at start up. The error measurements are made over the remaining 55000 received samples, signifying the region of steady state performance of the estimator. The error measurement is,

$$\xi_1 = 10\log_{10}\left(\frac{1}{55000}\sum_{i=5001}^{60000} | Y_i - Y'_{i,i-1} |^2\right) \qquad (6.4.2)$$

where ξ_1 is the mean square error in prediction.

The number of components (g+1) in the sampled impulse response of the channel is taken to be 21. At start up, it is assumed that $Y'_{1,0} = Y_0$, which is the actual channel sampled impulse response at the first sampling instant, also $E'_{1,0} = E_1$, which is the actual error between the estimate Y'_1 and the prediction $Y'_{1,0}$. In the tests b, θ, and θ_2 have been optimized to a high degree of accuracy, so that the error in the estimation/prediction of the sampled impulse response of the channel, defined by equation 6.4.2, is minimized.

Tables 6.4.1-6.4.6 show the mean squared error in prediction for various prediction algorithms used in the tests. Figures 6.4.1-6.4.6 show the relative performance in the error measurements used for the various least squares fading memory prediction algorithms tested.

Figure 6.4.7 is a plot of the channel over which the estimator was tested, neglecting the first 5000 samples for the sake of convenience in comparison with figures 6.4.8-6.4.13.

Figures 6.4.8-6.4.13 show the steady state performance of the system described in section 6.3, using the various configurations of the degree 0, 1, 2 least squares fading memory prediction algorithms. The parameter in Figure 6.4.7-6.4.13, is the square of the error in $Y'_{i,i-1}$ measured in dB, and is,

$$\xi_i = 10\log_{10} | Y_i - Y'_{i,i-1} |^2 \tag{6.4.3}$$

From the results, it could be observed that the modified estimator with degree 1 as the main prediction and degree 0 as the secondary prediction, when optimized, was the same as the simple estimator with degree 1 prediction (chapter 4.5). The modified estimator with degree 1 as the main prediction and degree 1 or 2 as the secondary prediction, gave the same error in prediction, when optimized, and also showed no improvement over the simple estimator with degree 1 prediction. This can be observed from tables 6.4.2-6.4.3, figures 6.4.2-6.4.3 and figures 6.4.9-6.4.10. The prediction errors, were equal to 4 decimal places accuracy.

The modified estimator with degree 0 as the main prediction and degree 1 or 2 as the secondary prediction, was observed to have a performance similar to that of the simple estimator with degree 0 prediction. This can be observed form tables 6.4.5-6.4.6, figures 6.4.5,6.4.6 and figures 6.4.12-6.4.13. The modified estimator with degree 0 as the secondary prediction showed a very slight improvement over the simple estimator with degree 0 prediction.

From the tests carried out, it could be observed that the modified estimator scheme did not show any improvement over the simple estimator. The reason for this performance, could be as a result of the fact that the error signal E_i, being filtered by the secondary prediction, consists mainly of noise, as it has already been filtered by the main prediction in the previous sampling instant. Therefore, an improvement in performance would not be expected as the secondary prediction is working on, basically noise, which is random in nature.

TABLE 6.4.1 MEAN SQUARE ERROR IN PREDICTION FOR THE MODIFIED ESTIMATOR WITH DEGREE 1 PREDICTION FOR THE MAIN PREDICTOR AND DEGREE 0 PREDICTION FOR THE SECONDARY PREDICTOR

SNR	b	θ	θ_2	ξ_1	Improvement over Simple Estimator
10	0.212	0.984	0	-14.1945	0
20	0.152	0.970	0	-21.5964	0
30	0.142	0.955	0	-28.5333	0
40	0.123	0.934	0	-34.4636	0
60	0.109	0.912	0	-39.2525	0

TABLE 6.4.2 MEAN SQUARE ERROR IN PREDICTION FOR THE MODIFIED ESTIMATOR WITH DEGREE 1 LEAST SQUARES FADING MEMORY PREDICTION FOR THE MAIN PREDICTOR AND DEGREE 1 LEAST SQUARES FADING MEMORY PREDICTION FOR THE SECONDARY PREDICTOR

SNR	b	θ	θ_2	ξ_1	Improvement over Simple Estimator
10	0.195	0.983	0.187	-14.1948	0.0003
20	0.144	0.969	0.025	-21.5969	0.0005
30	0.132	0.953	0.026	-28.5373	0.0040
40	0.120	0.933	0.137	-34.4650	0.0014
60	0.103	0.909	0.138	-39.2575	0.0050

TABLE 6.4.3 MEAN SQUARE ERROR IN PREDICTION FOR THE MODIFIED ESTIMATOR WITH DEGREE 1 LEAST SQUARES FADING MEMORY PREDICTION FOR THE MAIN PREDICTOR AND DEGREE 2 LEAST SQUARES FADING MEMORY PREDICTION FOR THE SECONDARY PREDICTOR

SNR	b	θ	θ_2	ξ_1	Improvement over Simple Estimator
10	0.195	0.983	0.356	-14.1949	0.0004
20	0.145	0.969	0.222	-21.5969	0.0005
30	0.132	0.953	0.220	-28.5374	0.0041
40	0.117	0.932	0.301	-34.4650	0.0014
60	0.106	0.910	0.269	-39.2575	0.0050

TABLE 6.4.4 MEAN SQUARE ERROR IN PREDICTION FOR THE MODIFIED ESTIMATOR WITH DEGREE 0 PREDICTION FOR THE MAIN PREDICTOR AND DEGREE 0 PREDICTION FOR THE SECONDARY PREDICTOR

SNR	b	θ	θ_2	ξ_1	Improvement over Simple Estimator
10	0.161	0.940	0.783	-13.7879	0.0005
20	0.215	0.914	0.841	-19.3850	0.0105
30	0.280	0.894	0.920	-23.6618	0.1606
40	0.331	0.890	0.930	-25.6038	0.4748
60	0.348	0.890	0.932	-26.0426	0.6119

TABLE 6.4.5 MEAN SQUARE ERROR IN PREDICTION FOR THE MODIFIED ESTIMATOR WITH DEGREE 0 LEAST SQUARES FADING MEMORY PREDICTION FOR THE MAIN PREDICTOR AND DEGREE 1 LEAST SQUARES FADING MEMORY PREDICTION FOR THE SECONDARY PREDICTOR

SNR	b	θ	θ_2	ξ_1	Improvement over Simple Estimator
10	0.190	0.95	0.177	-13.7872	-0.0002
20	0.197	0.91	0.212	-19.3684	-0.0006
30	0.224	0.88	0.223	-23.5007	-0.0005
40	0.242	0.87	0.171	-25.1296	-0.0006
60	0.249	0.87	0.132	-25.4315	-0.0005

TABLE 6.4.6 MEAN SQUARE ERROR IN PREDICTION FOR THE MODIFIED ESTIMATOR WITH DEGREE 0 LEAST SQUARES FADING MEMORY PREDICTION FOR THE MAIN PREDICTOR AND DEGREE 2 LEAST SQUARES FADING MEMORY PREDICTION FOR THE SECONDARY PREDICTOR

SNR	b	θ	θ_2	ξ_1	Improvement over Simple Estimator
10	0.111	0.914	0.317	-13.7870	-0.0004
20	0.183	0.903	0.332	-19.3684	-0.0006
30	0.269	0.900	0.333	-23.4998	-0.0014
40	0.314	0.900	0.336	-25.1262	-0.0028
60	0.323	0.900	0.336	-25.4273	-0.0037

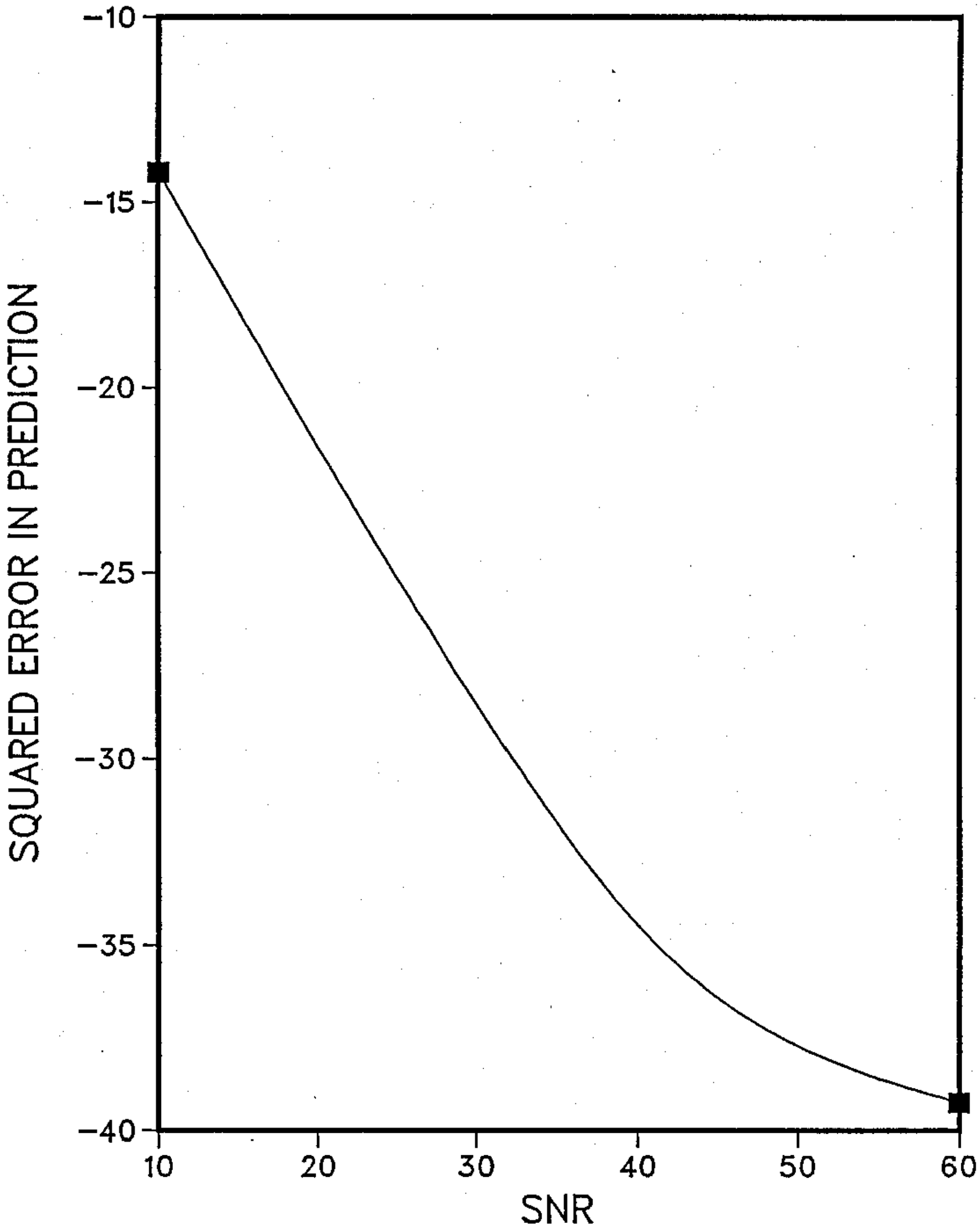


FIGURE 6.4.1 PLOT OF THE MEAN SQUARE ERROR IN PREDICTION FOR THE MODIFIED ESTIMATOR WITH DEGREE 0 AS THE SECONDARY PREDICTION AND DEGREE 1 AS THE MAIN PREDICTION

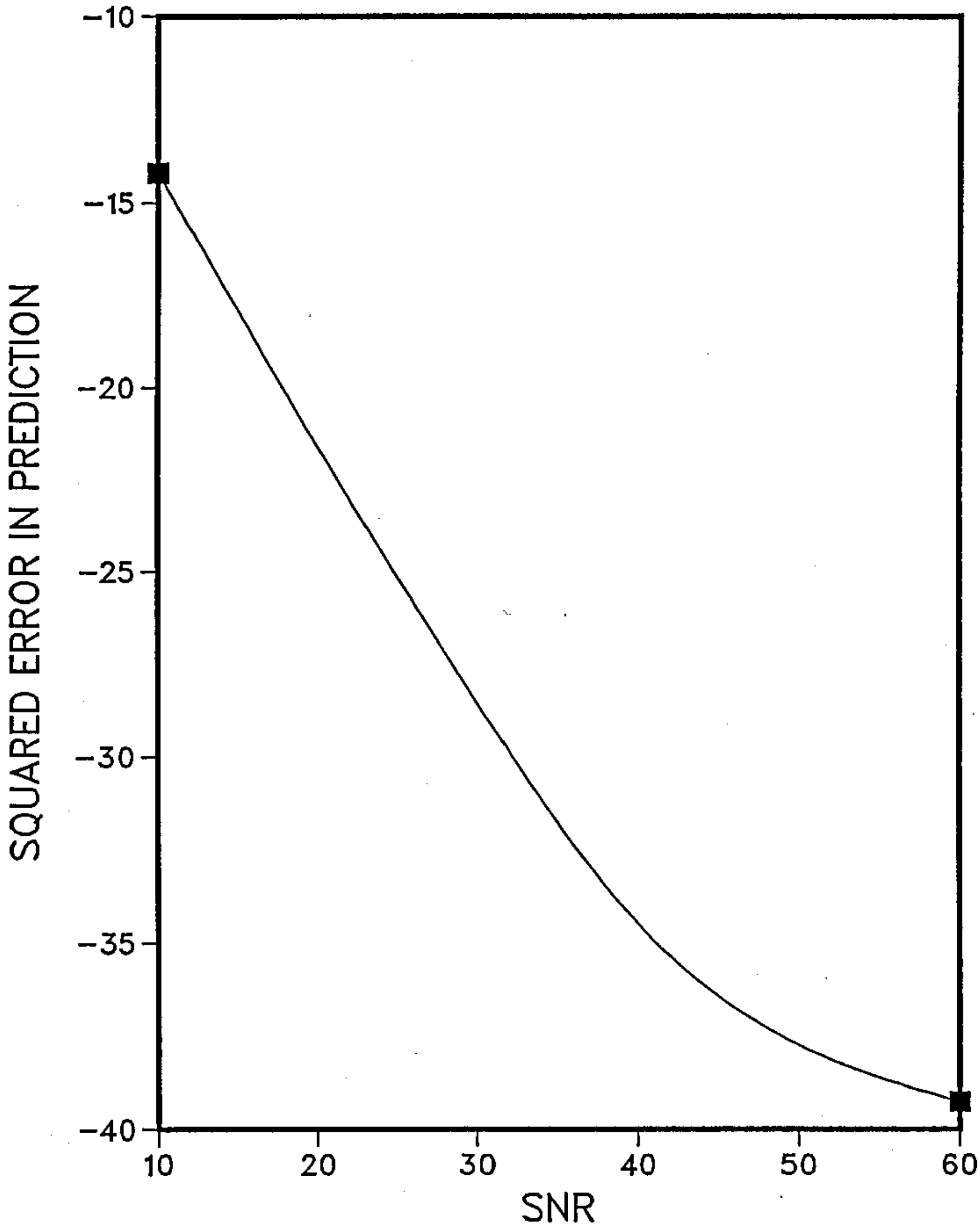


FIGURE 6.4.2 PLOT OF THE MEAN SQUARE ERROR IN PREDICTION FOR THE MODIFIED ESTIMATOR WITH DEGREE 1 AS THE SECONDARY PREDICTION AND DEGREE 1 AS THE MAIN PREDICTION

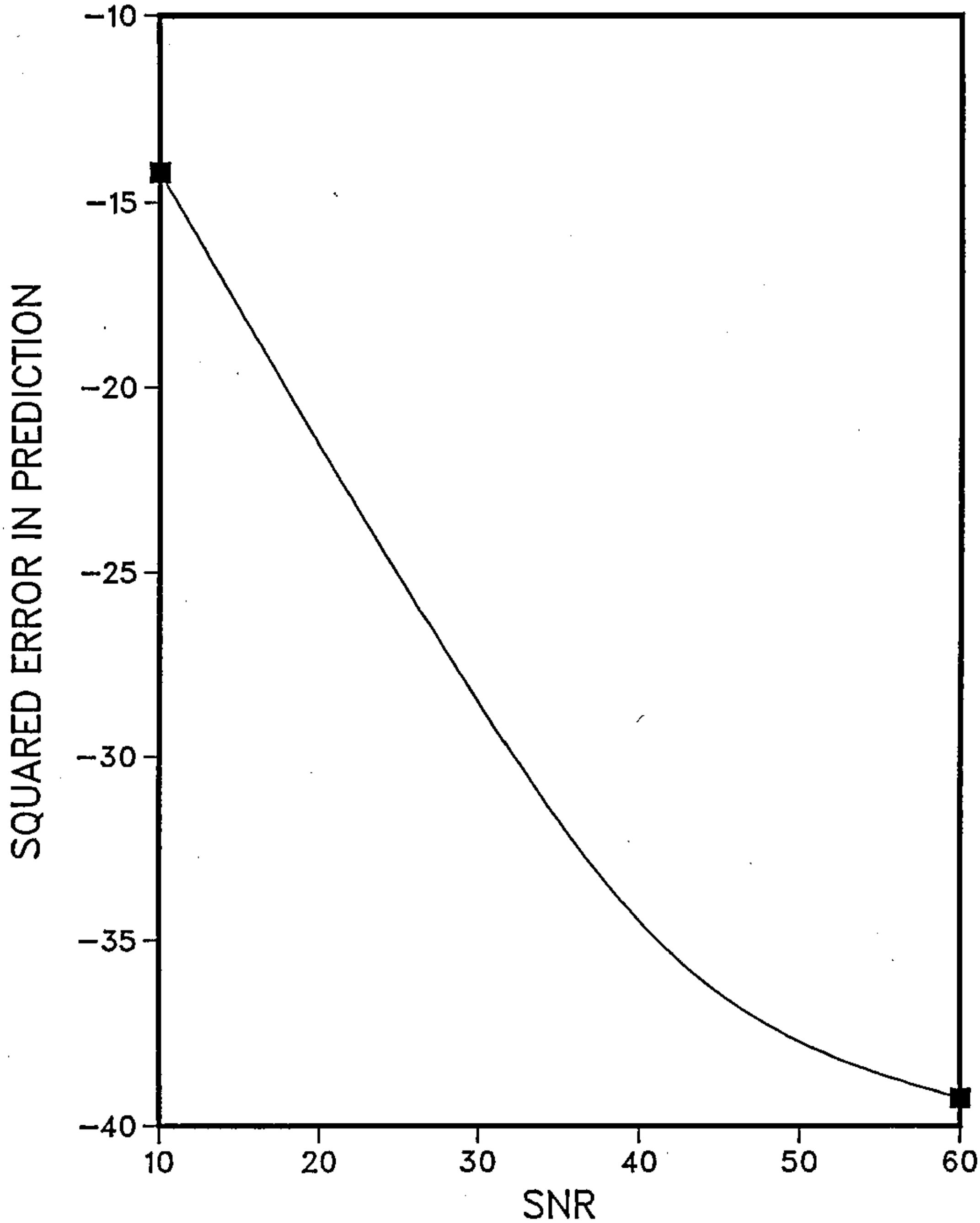


FIGURE 6.4.3 PLOT OF THE MEAN SQUARE ERROR IN PREDICTION FOR THE MODIFIED ESTIMATOR WITH DEGREE 2 AS THE SECONDARY PREDICTION AND DEGREE 1 AS THE MAIN PREDICTION

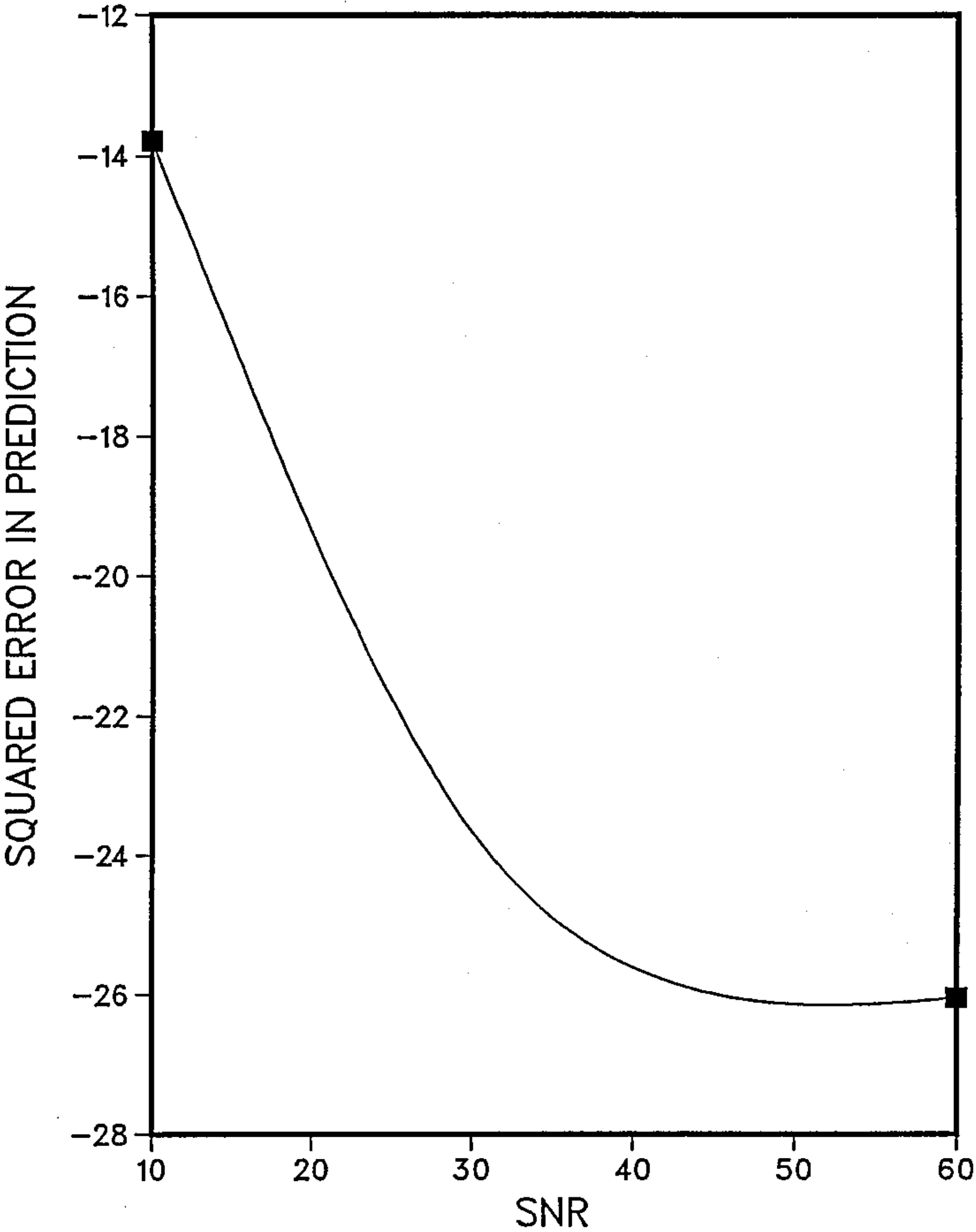


FIGURE 6.4.4 PLOT OF THE MEAN SQUARE ERROR IN PREDICTION FOR THE MODIFIED ESTIMATOR WITH DEGREE 0 AS THE SECONDARY PREDICTION AND DEGREE 0 AS THE MAIN PREDICTION

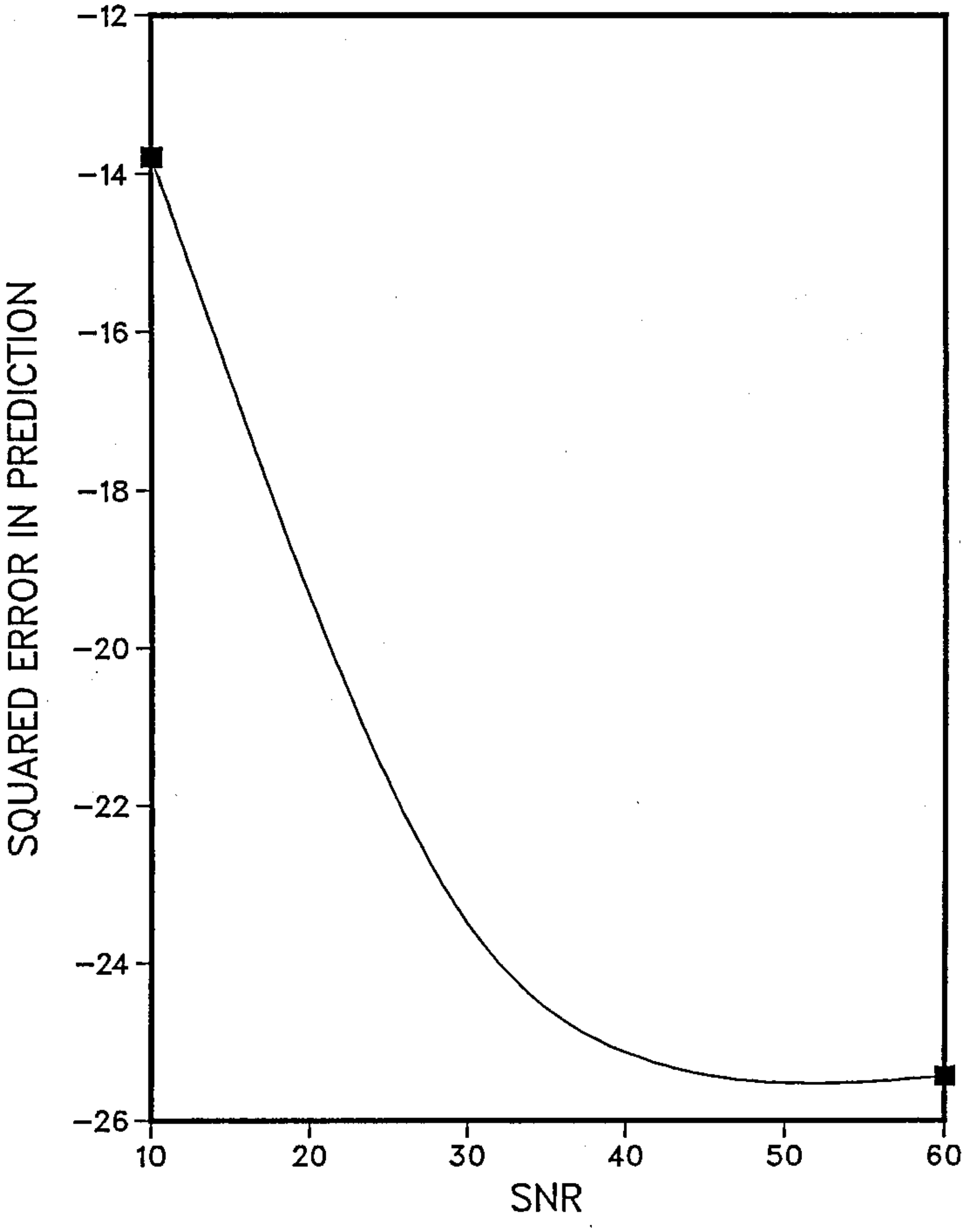


FIGURE 6.4.5 PLOT OF THE MEAN SQUARE ERROR IN PREDICTION FOR THE MODIFIED ESTIMATOR WITH DEGREE 1 AS THE SECONDARY PREDICTION AND DEGREE 0 AS THE MAIN PREDICTION

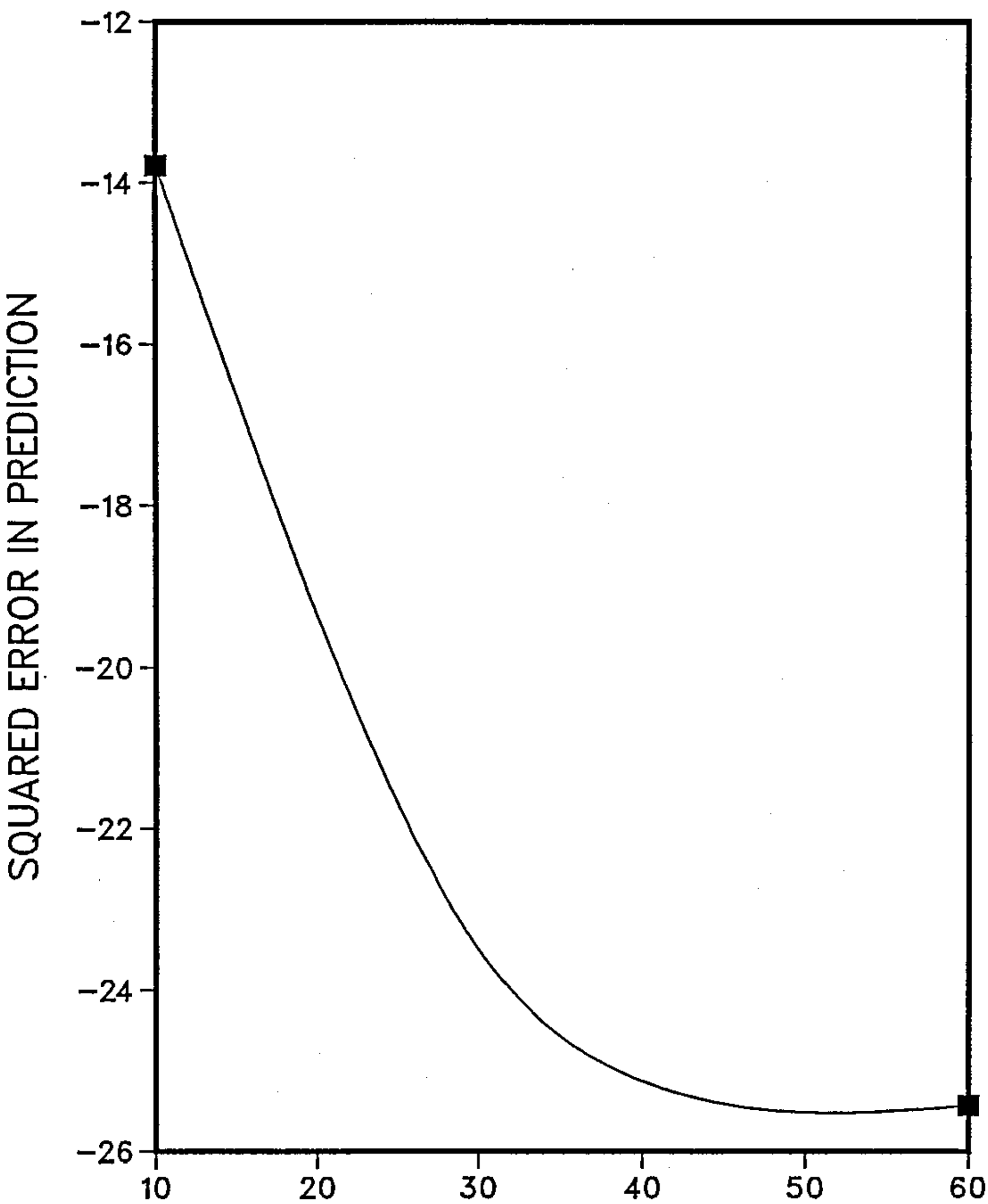


FIGURE 6.4.6 PLOT OF THE MEAN SQUARE ERROR IN PREDICTION FOR THE MODIFIED ESTIMATOR WITH DEGREE 2 AS THE SECONDARY PREDICTION AND DEGREE 0 AS THE MAIN PREDICTION

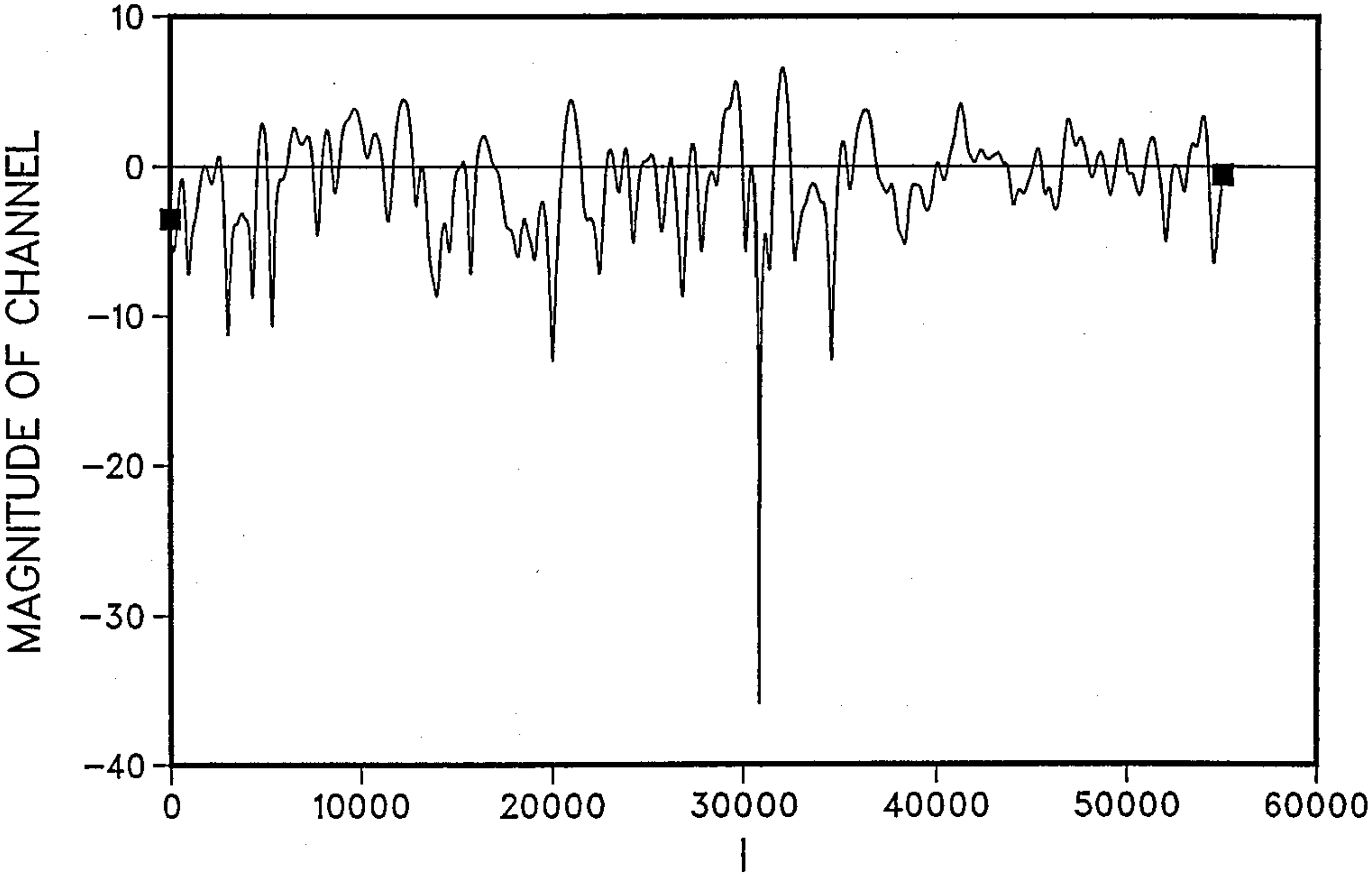


FIGURE 6.4.7 PLOT OF THE MAGNITUDE OF THE CHANNEL OVER WHICH ESTIMATOR WAS TESTED, SEED=50

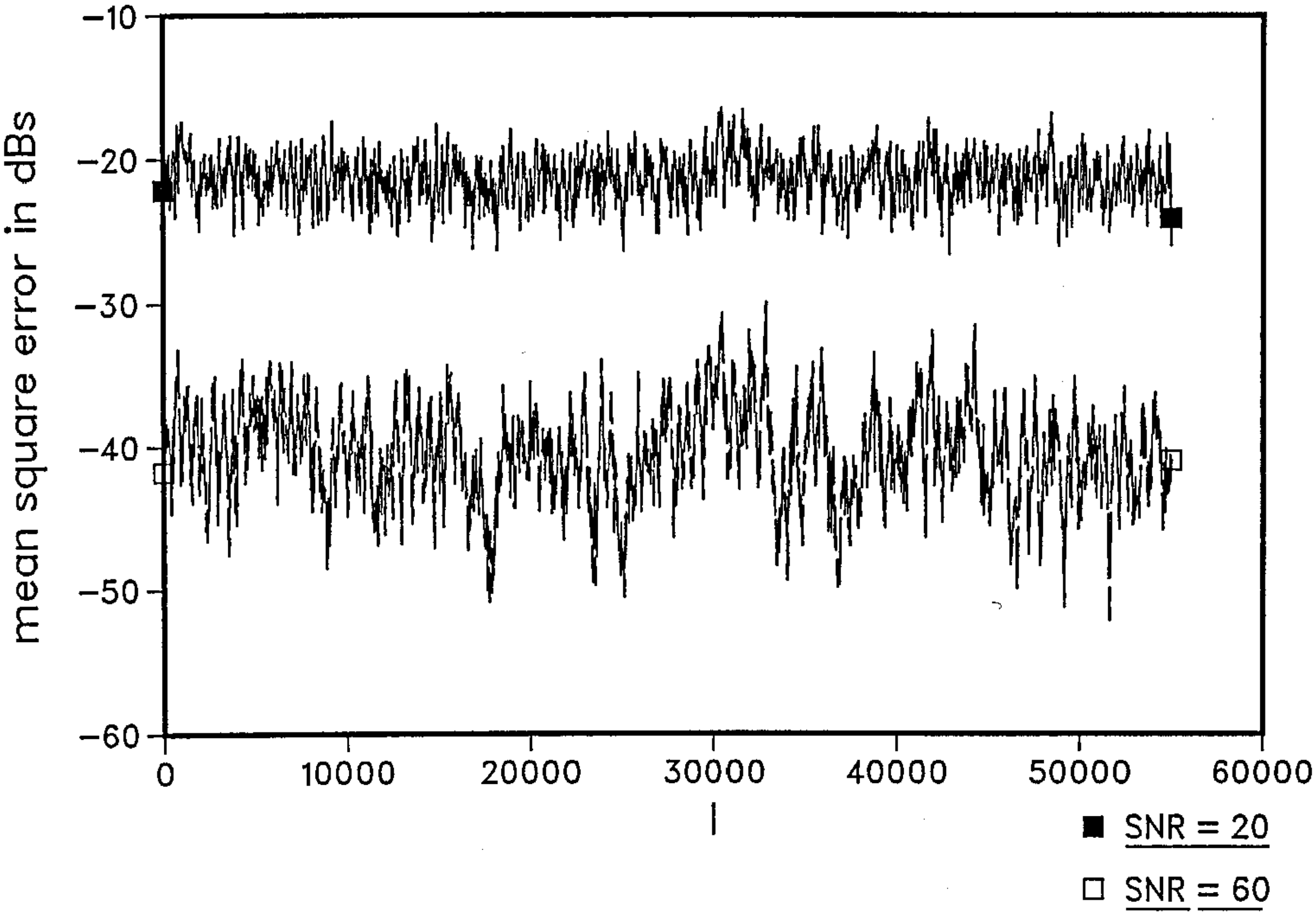


FIGURE 6.4.8 PLOT OF THE STEADY STATE PERFORMANCE OF THE MODIFIED ESTIMATOR WITH DEGREE 1 FOR THE MAIN PREDICTION AND DEGREE 0 FOR THE SECONDARY PREDICTION, SNR=20,60

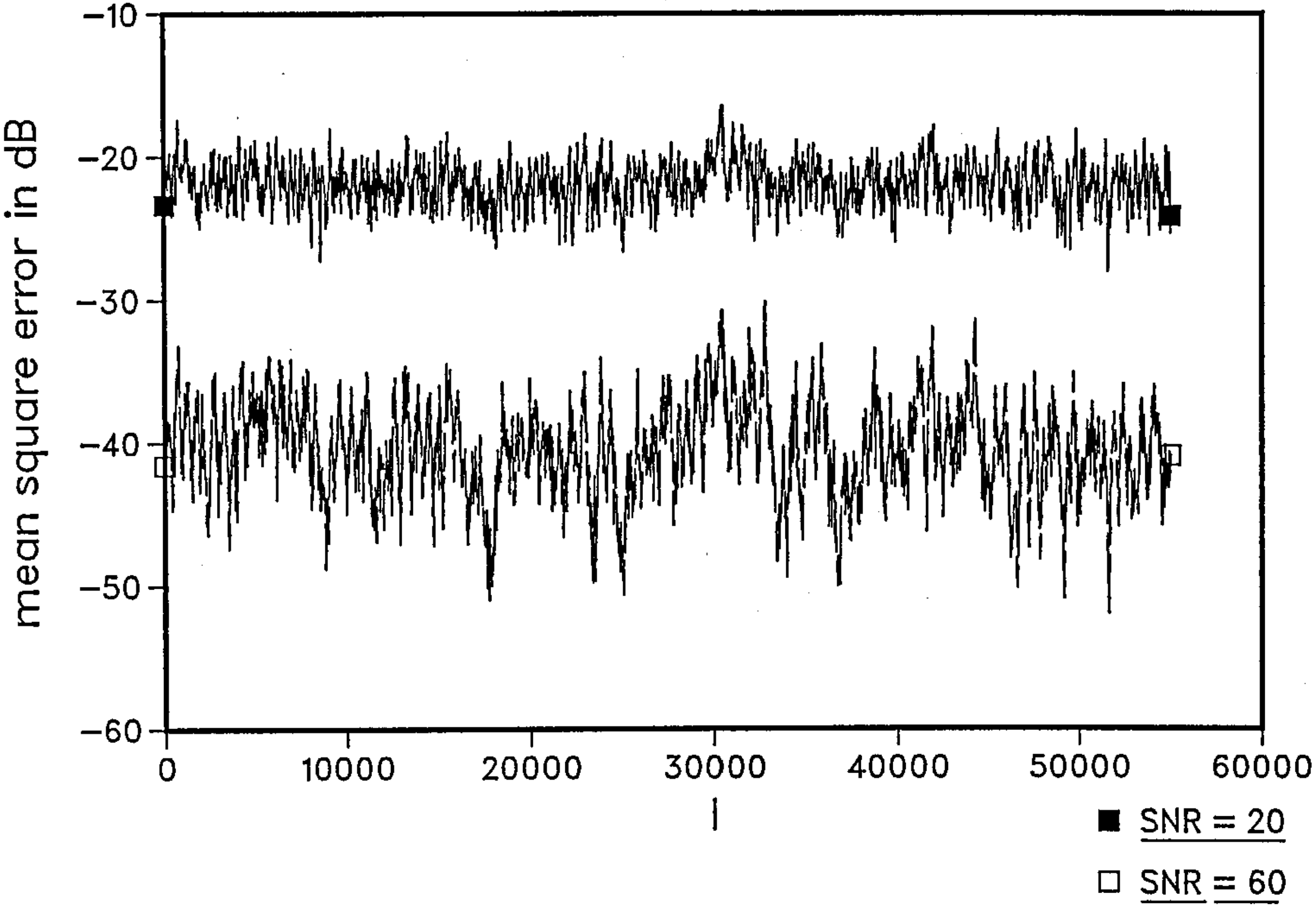


FIGURE 6.4.9 PLOT OF THE STEADY STATE PERFORMANCE OF THE MODIFIED ESTIMATOR WITH DEGREE 1 AS THE SECONDARY PREDICTION AND DEGREE 1 AS THE MAIN PREDICTION

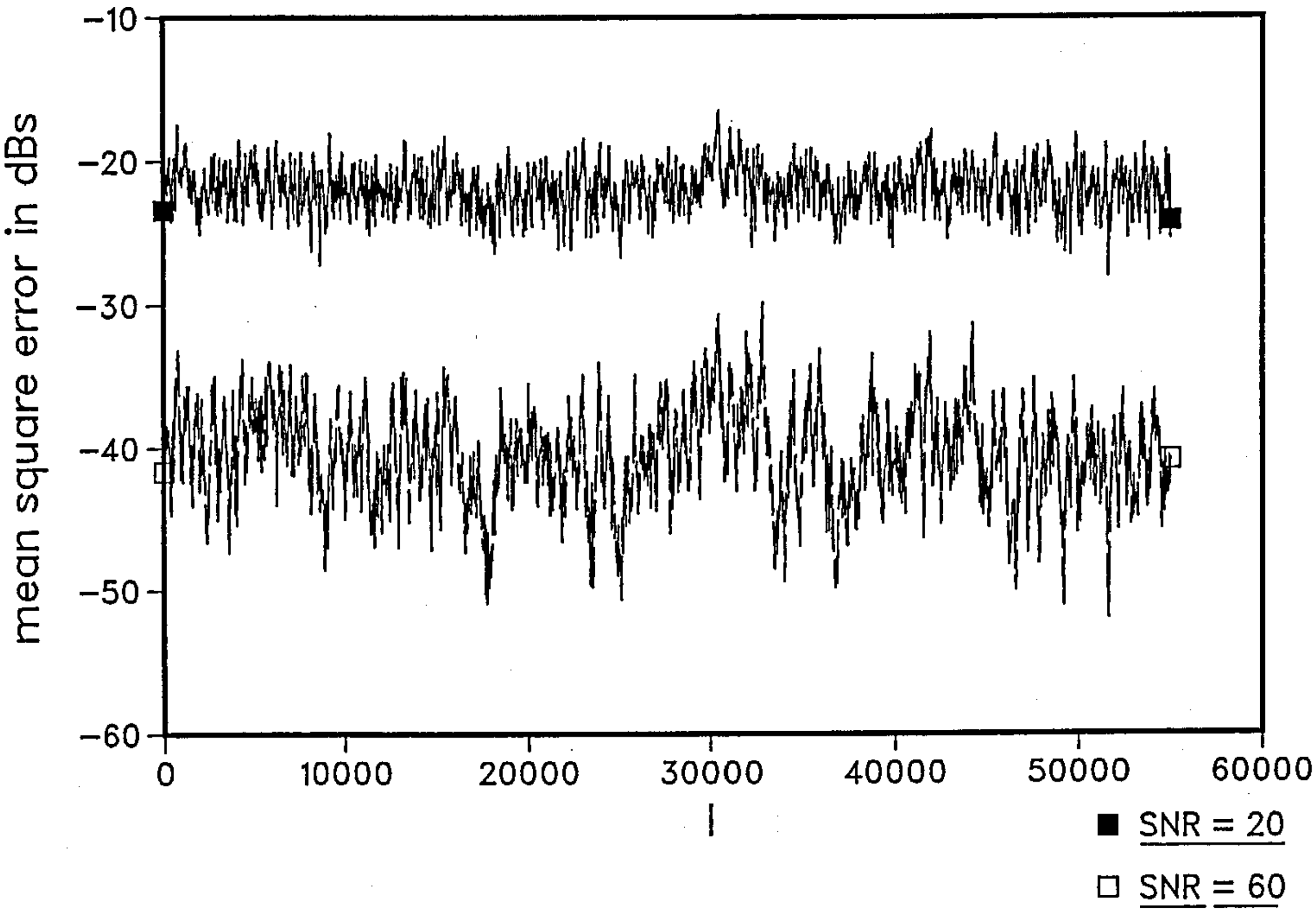


FIGURE 6.4.10 PLOT OF THE STEADY STATE PERFORMANCE OF THE MODIFIED ESTIMATOR WITH DEGREE 2 AS THE SECONDARY PREDICTION AND DEGREE 1 AS THE MAIN PREDICTION,SNR=20,60

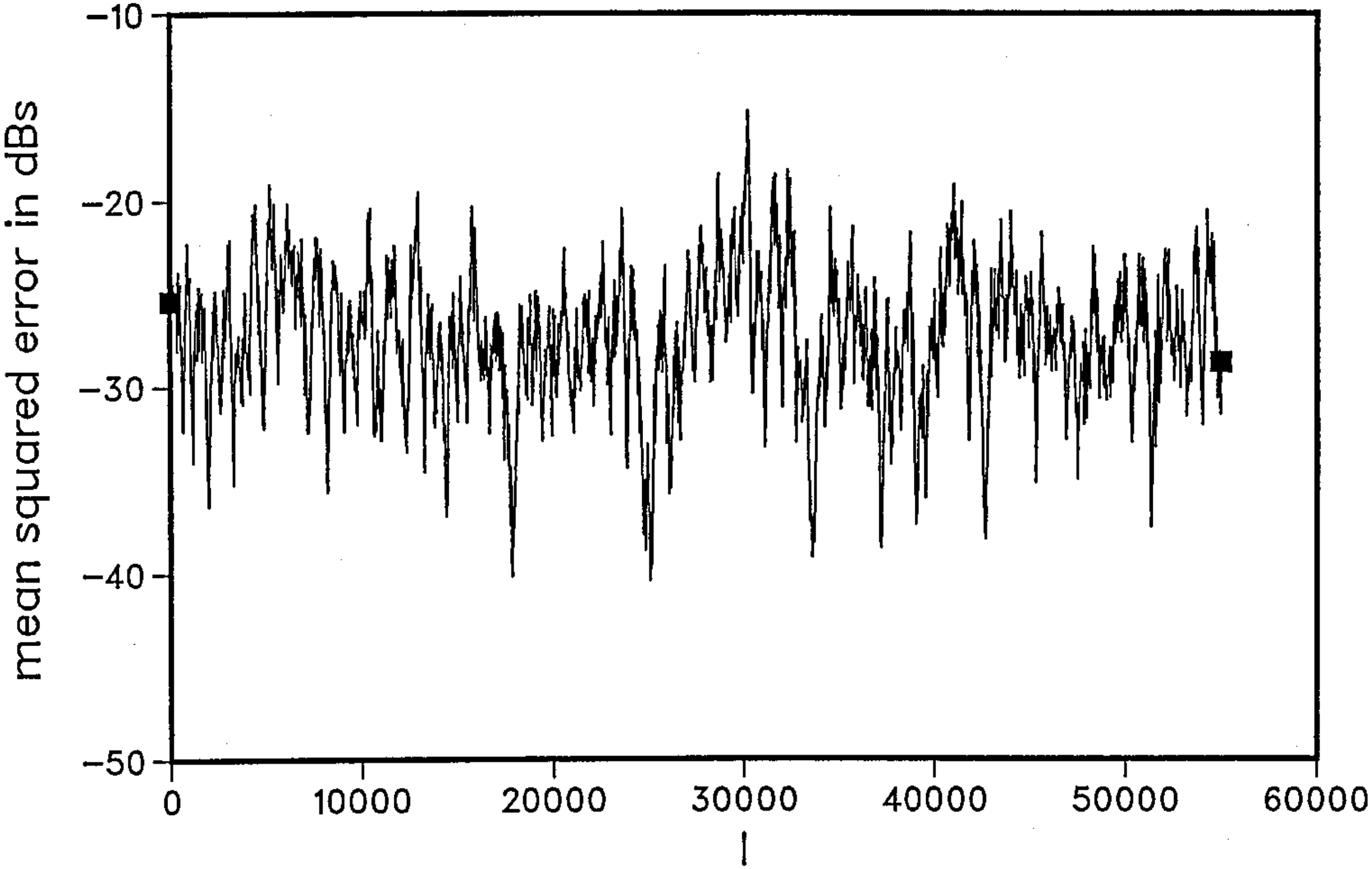


FIGURE 6.4.11 PLOT OF THE STEADY STATE PERFORMANCE OF THE MODIFIED ESTIMATOR WITH DEGREE 0 AS THE MAIN PREDICTION AND DEGREE 0 AS THE SECONDARY PREDICTION, SNR=60

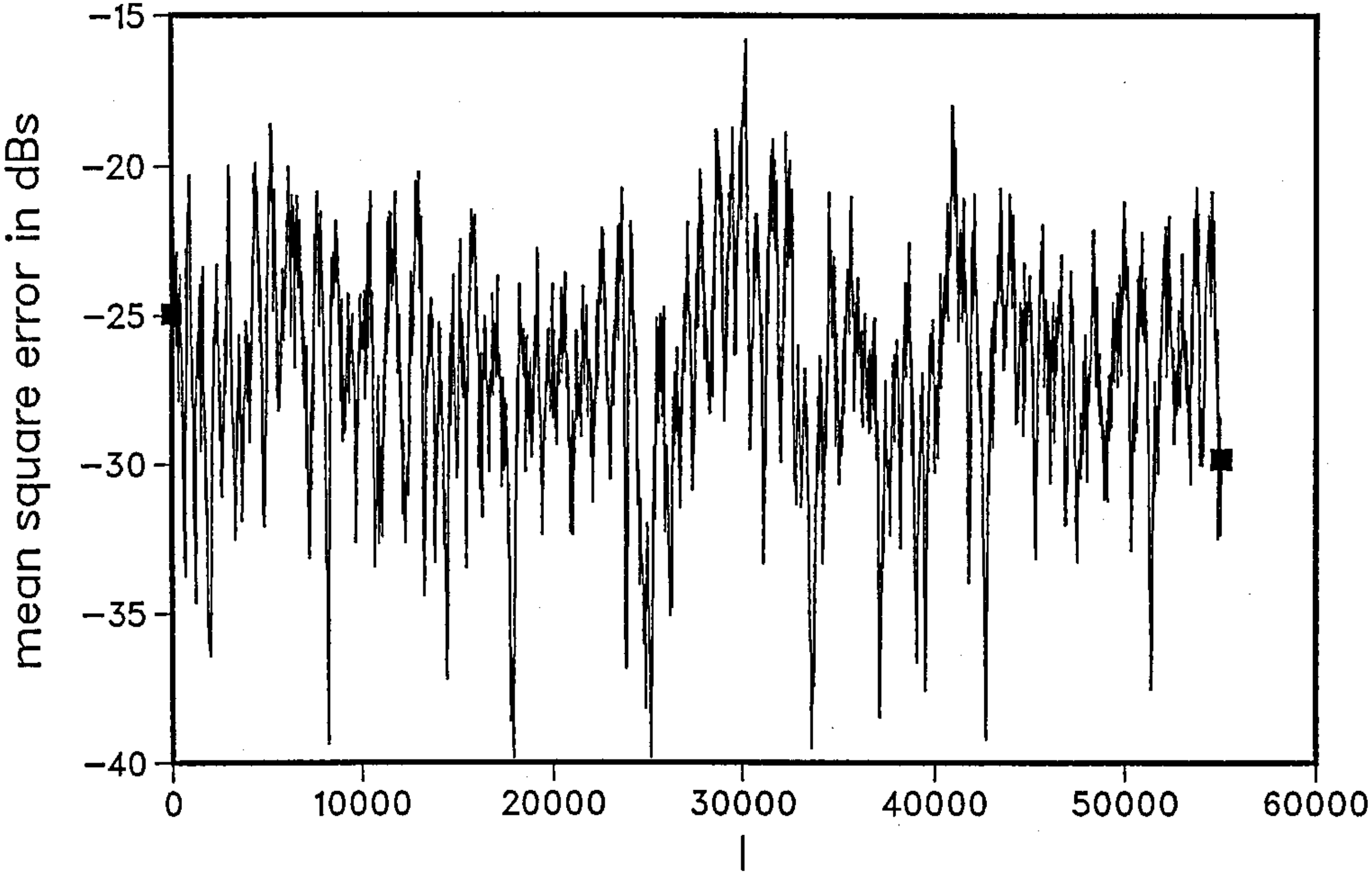


FIGURE 6.4.12 PLOT OF THE STEADY STATE PERFORMANCE OF THE MODIFIED ESTIMATOR WITH DEGREE 1 AS THE SECONDARY PREDICTION AND DEGREE 0 AS THE MAIN PREDICTION

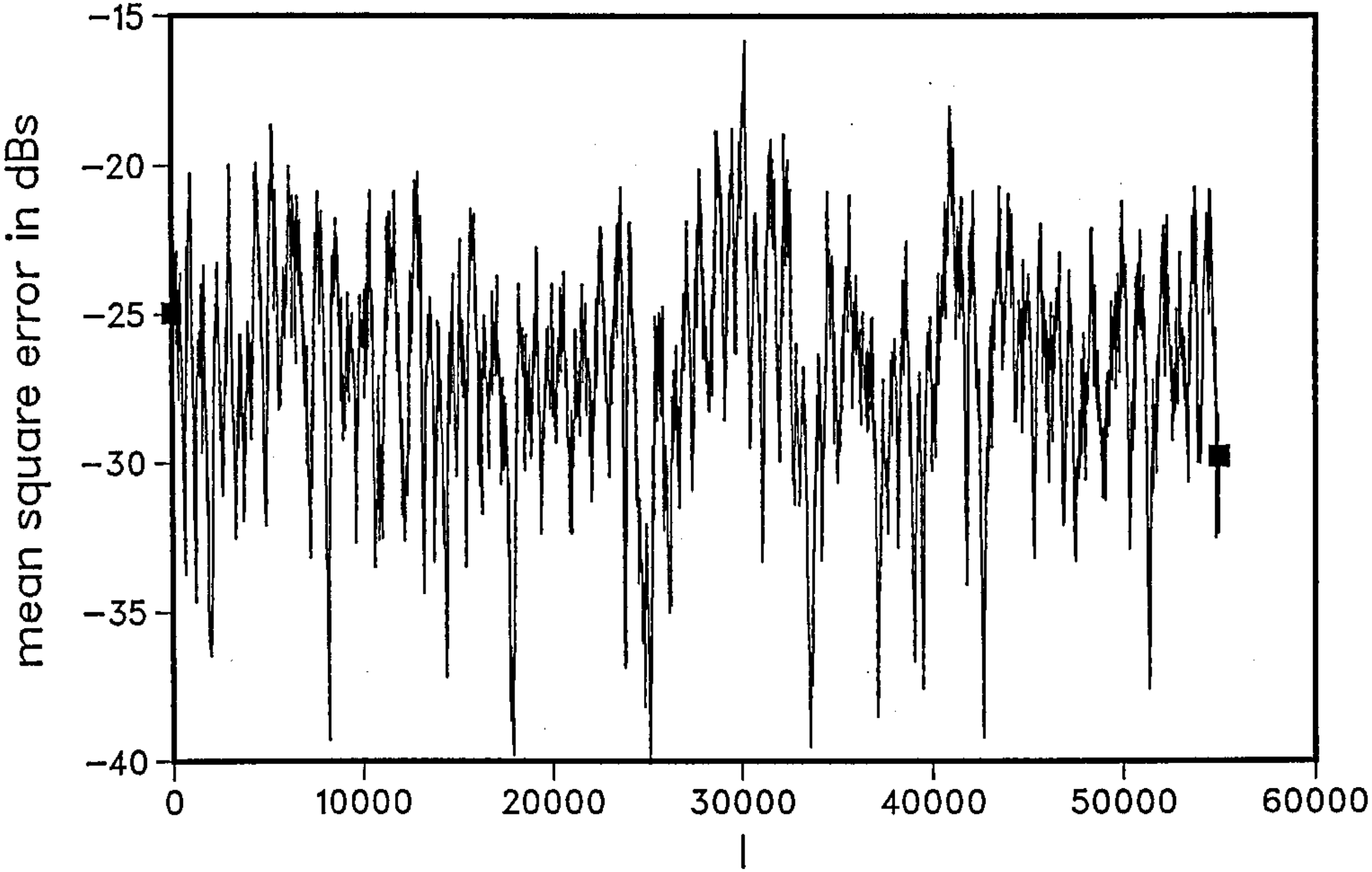


FIGURE 6.4.13 PLOT OF THE STEADY STATE PERFORMANCE OF THE MODIFIED ESTIMATOR WITH DEGREE 0 AS THE MAIN PREDICTOR AND DEGREE 2 AS THE SECONDARY PREDICTOR, SNR=60

CHAPTER 7

If we do not find anything pleasant, at least we shall find something new

- Francoise Marie Arouet

CHAPTER 7

ADAPTIVE FILTER

7.1 INTRODUCTION

The most common digital data communication link is the public switched telephone network. Telephone circuits introduce a wide range of linear distortions, but three distinct types can be identified : amplitude, group delay distortion and echoes [1]. Severe phase distortion introduced by the poorer telephone circuits prevents the satisfactory operation of the near maximum likelihood, unless the detector is further modified to a more complex system [5]. For reliable operation of any near maximum likelihood detector, in a 9600 bits/s data transmission system operating over a telephone channel, a linear feedforward transversal filter must be used ahead of the detector. The transversal filter is an allpass network, that adjusts the sampled impulse response of the channel and filter to be minimum phase, without changing any amplitude distortion introduced by the channel [2-4,6]. This concentrates the energy of the sample impulse response of the channel and filter towards the earliest samples.

This chapter describes a novel technique that has been developed for adjusting the linear feedforward transversal filter ahead of the detector, and at the same time, estimates the sampled impulse response of the channel and filter. The main advantage of this scheme, is that there is not a need to evaluate the roots of the channel, thereby eliminating the need for a separate root finding algorithm. The technique makes use of the Gram Schmidt orthogonalization process [7]. Section 7.2 provides a brief insight into telephone channels. Section 7.3 describes channel equalization. Section 7.4 outlines the Gram Schmidt orthogonalization process. Section 7.5 provides us with the actual implementation of the filter. The results of the implementation are given in section 7.6. Section 7.7 provides a description of the numerical problems encountered in the implementation of the filter as a result of the Gram Schmidt orthogonalization process. Finally, in section 7.8 and 7.9, further tests have been carried out to ascertain the reasons for the numerical errors encountered.

7.2 TELEPHONE CHANNELS

The bandlimited telephone circuit has a passband in the range 300 to 3000 Hz approximately which is available for the transmission of data at rates upto 19200 bits/s [1,2]. An important characteristic of telephone circuits is the wide range of different types of noise and distortion that may be experienced together with the widely varying severities of the different effects. This is as a result of a telephone circuit consisting of several links connected in cascade, each link having its own particular property. These corrupt the received signal, resulting in intersymbol interference, which results in a less reliable high speed data transmission [1,2,8].

Two telephone circuits, chosen to represent a wide range of telephone lines in the British Public Switched Telephone Network (BPSTN), have been used for computer simulation tests based on the QAM data transmission system operating at 9600 bits/s. These are designated as channel 3 and channel 4 [9].

The telephone circuits in channel 3 and 4 are close to the typical worst circuits (in the BPSTN) normally considered for the transmission of data at 9600 bits/s. The telephone circuit in channel 3 is close to the standard network N6 and introduces severe group delay distortions as well, as considerable attenuation distortions. The telephone circuit in channel 4 is close to the standard network N3 and introduces extremely severe group delay distortion [9]. The attenuation and group delay characteristic of the telephone circuits are shown in Figure 7.2.1 and Figure 7.2.2.

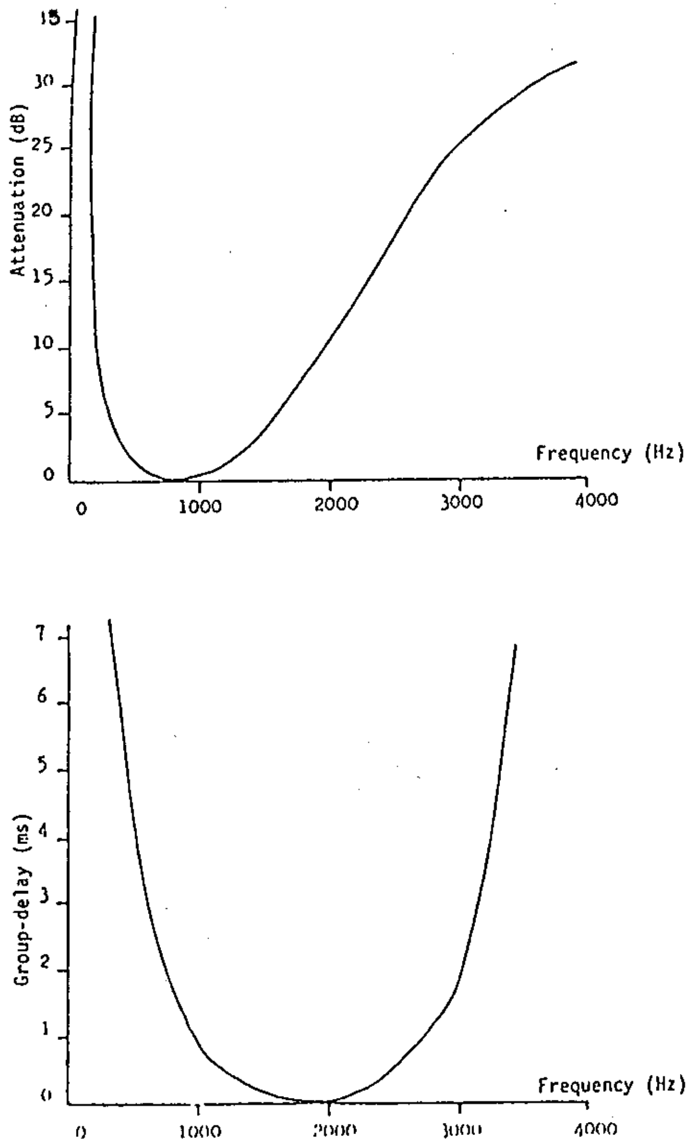


FIGURE 7.2.1 Attenuation and Group-delay characteristics of telephone circuit 3 (Table 7.2.1)

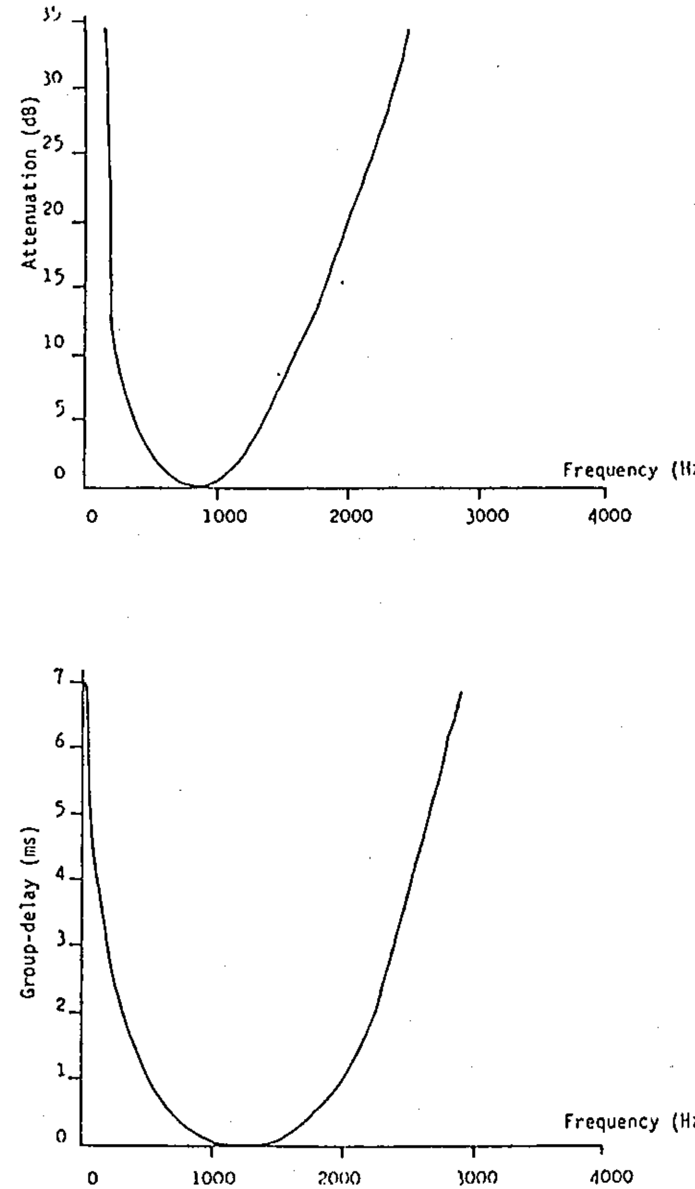


FIGURE 7.2.2 Attenuation and Group-delay characteristics of telephone circuit 4 (Table 7.2.2)

TABLE 7.2.1 IMPULSE RESPONSE OF TELEPHONE CHANNEL 3 AND ITS MINIMUM PHASED VERSION

Channel 3		Minimum phased channel 3	
Real	Imaginary	Real	Imaginary
0.0176	-0.0175	1.000000000000	0.000000000000
0.1381	-0.1252	0.460764577615	1.100433003616
0.4547	-0.1885	-0.582440040610	0.043597487245
0.5078	0.1622	0.157268400982	-0.172919181137
-0.1966	0.3505	-0.017502693381	0.087232147521
-0.2223	-0.2276	-0.002112883107	-0.019413851127
0.2797	-0.0158	-0.002100943217	0.008270752688
-0.1636	0.1352	-0.005118235443	-0.007506680616
0.0594	-0.1400	0.008046043854	0.005434517373
-0.0084	0.1111	-0.003896817352	-0.003543560324
-0.0105	-0.0817	-0.000114818018	0.001423445858
0.0152	0.0572	0.003849470664	-0.005560662102
-0.0131	-0.0406	-0.000922092690	0.002626529695
0.0060	0.0255	0.002256286865	-0.002664349578
0.0003	-0.0190	-0.000429691823	-0.000880528994
-0.0035	0.0116	0.001333981128	0.001823523637
0.0041	0.0078	-0.001644224660	-0.001317210282
-0.0031	0.0038	-0.000340511570	0.000005648129
0.0018	-0.0005	-0.000117638515	0.000119713819
-0.0018	-0.0005	0.000340720910	0.000178521579
0.0007	0.0007	-0.000709312095	0.001056849557
0.0004	0.0001	-0.000829483827	-0.000345507881
-0.0004	0.0001	0.000055730505	0.000202173398
-0.0001	0.0010	-0.000071641953	0.000714893619
0.0000	-0.0007	-0.000149351542	0.000371159729
0.0004	0.0008	-0.000019739511	0.000059785761

TABLE 7.2.2 IMPULSE RESPONSE OF TELEPHONE CHANNEL 4 AND ITS MINIMUM PHASED VERSION

Channel 4		Minimum phased channel 4	
Real	Imaginary	Real	Imaginary
-0.0038	-0.0049	1.000000000000	0.000000000000
0.0077	-0.0044	0.246056855104	1.980092667325
0.0094	0.0207	-1.719436570390	-0.202601581369
-0.0884	0.0355	0.674421718180	-0.792641959707
-0.1138	-0.2869	0.035740869047	0.506399849990
0.5546	-0.2255	-0.115537033599	-0.144963751908
0.1903	0.5813	0.029686525150	0.038535978654
-0.2861	-0.0892	-0.016852846469	-0.037043373228
0.2332	-0.0384	0.016071161647	0.018537904977
-0.0652	0.0428	-0.014501041022	-0.001223761962
0.0335	-0.0519	-0.000332014780	-0.002909058892
-0.0323	0.0170	0.002010935998	-0.003835867311
0.0044	-0.0023	-0.000147027127	0.000090206812
0.0054	0.0076	-0.002684284255	-0.007554547516
0.0008	-0.0051	0.006507570374	0.004660947979
-0.0056	0.0001	-0.003145677629	0.005167086324
0.0018	0.0032	-0.005981626998	-0.003084624838
-0.0009	-0.0015	0.005730632010	-0.001176100183
-0.0022	-0.0026	-0.001286562383	0.003781846255
0.0029	0.0019	-0.000865470558	0.000403423104
-0.0008	0.0009	-0.001071413311	0.000080235148
-0.0014	-0.0003	0.001221636935	-0.001649974249
0.0019	-0.0002	0.002325722568	0.000939184812
-0.0003	0.0005	-0.000446317835	0.001993089227
0.0007	0.0005	-0.001005067135	-0.000749564980
-0.0007	-0.0001	0.000595389442	-0.000293137583
0.0002	-0.0008	0.000098379873	0.000288331790
0.0006	0.0000	-0.000066358586	-0.000020473679
0.0002	0.0004	-0.000040887119	0.000003010859
-0.0001	-0.0004	0.000022018400	0.000009691860

TABLE 7.2.3 ROOTS OF TELEPHONE CHANNEL 3

Real	Imaginary	Magnitude
-4.064491140	1.520652710	4.339639719
-1.229764837	-1.361420158	1.834607969
-2.150508445	0.479560613	2.203330423
0.726549562	0.138326058	0.739600138
-1.343144107	-0.236738615	1.363847962
0.278672740	-0.699619312	0.753077472
0.025822867	0.719223972	0.719687392
-0.551831410	0.423187743	0.695417695
0.707556750	-0.284638570	0.762663536
-0.803334150	-0.489786700	0.940870218
0.530681454	0.462947812	0.704232549
-0.283533246	-0.702344387	0.757415830
-0.195261417	0.676930466	0.704529542
0.592612060	-0.455717692	0.747574523
-0.502466348	-0.627242038	0.803682154
0.238658833	0.674037181	0.715041370
0.072552416	-0.739593998	0.743144087
-0.655324104	0.189091603	0.682059613
0.636789079	0.321512572	0.713351712
-0.643262128	-0.155064051	0.661688012
0.746888223	-0.065332687	0.749740207
-0.379809864	0.591030490	0.702547203
0.455920193	-0.596026025	0.750406720
-0.107841739	-0.701090068	0.709335692
0.395458082	0.571966105	0.695364883
0.000000000	0.000000000	0.000000000

TABLE 7.2.4 ROOTS OF TELEPHONE CHANNEL 4

Real	Imaginary	Magnitude
-2.275477082	1.531197597	2.742692478
1.717696515	-0.798925879	1.894403304
2.213498116	1.862663948	2.892938107
-1.458929665	-0.612902350	1.582442750
0.659251848	-1.273485016	1.434007352
-0.745042338	-0.784404573	1.081840385
0.398993908	0.606874185	0.726286730
-0.063129693	-1.261717115	1.263295467
-0.566139575	0.444362051	0.719702474
0.750757384	-0.135908607	0.762959894
-0.225922335	0.684996651	0.721291421
0.104433722	-0.750769623	0.757998304
-0.463492686	-1.028153218	1.127796307
0.221732952	0.708914634	0.742782243
-0.731942646	-0.475617892	0.872898858
0.662063229	0.270225634	0.715087137
-0.689864250	0.087369603	0.695374813
0.665525443	-0.370063095	0.761492488
-0.395547412	0.609019175	0.726197020
0.516604970	-0.521214316	0.733856293
-0.752807688	-0.152274209	0.768053937
0.564445044	0.447378151	0.720239834
-0.209423715	-0.728637234	0.758136209
-0.010659208	0.749574250	0.749650035
0.347549721	-0.664464867	0.749869567
-0.632395842	0.245682420	0.678442593
-0.534213618	-0.504304541	0.734647711
0.397193606	0.320036812	0.510084622
0.735501372	0.078422588	0.739670447
0.000000000	0.000000000	0.000000000

The complex valued sampled impulse responses of the channels and the locations of all their roots are shown in Table 7.2.1 - 7.2.4. The resultant sampled impulse responses of the channel when they have been minimum phased are given in Tables 7.2.1-7.2.2. From Tables 7.2.3 - 7.2.4, it can be observed that channel 4 has 8 roots outside the unit circle, as compared to only 4 for channel 3. Channel 4, also seems to have its roots closer to the unit circle, SEE APPENDIX C.

In the tests to follow, channel 3 and 4 have been altered to slightly change the characteristic of the roots. The reason for this will become apparent, as the reader studies the proceeding sections in detail.

7.3 CHANNEL EQUALIZATION

In any serial data transmission system such as shown in Figure 7.3.1 where the channel introduces either one or both amplitude and phase distortion, the channel impulse response is time dispersive in that it has, for practical purposes, a duration of (g+1)T seconds where g is a positive integer and T seconds is the sampling interval. Thus if the sampled impulse response of the channel is a sequence of values $y_0, y_1, y_2, ..., y_g$ then the components of an individual received signal element for the data symbol s_i at the output of the sampler in Figure 2.3.1 are $s_iy_0, s_iy_1, s_iy_2, ..., s_iy_g$ which are received, in turn, at (g+1) consecutive sampling instants. Consequently, the received sample r_i at the output of the sampler not only contains s_i but also $s_{i-1}, s_{i-2}, s_{i-3}, ..., s_{i-g}$. This effect is caused by the overlapping of neighbouring signal-elements and is known as intersymbol interference. At high signal to noise ratios, such as in the voice channel, intersymbol interference becomes the primary impediment to the correct detection of the data symbols $\{s_i\}$.

One way of overcoming the effect of intersymbol interference in the received signal, thus enabling correct detection of the $\{s_i\}$ is by the use of equalizers [2,6,9,10]. The

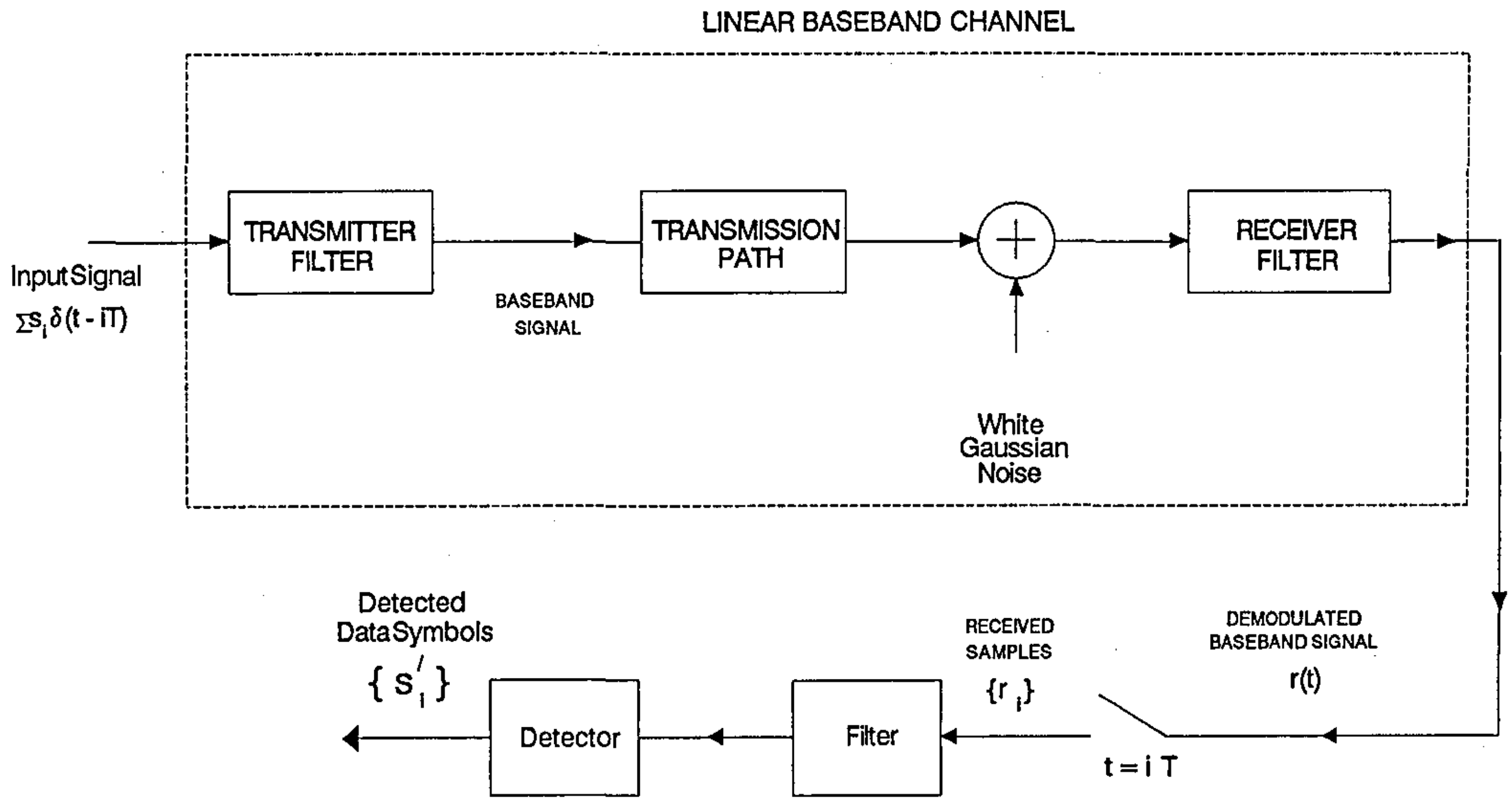


FIGURE 7.3.1 BASEBAND MODEL OF THE DATA TRANSMISSION SYSTEM

function of the equalizer is to correct the amplitude and phase distortions introduced by the channel and thereby restore the signal to being a copy of the original transmitted signal, hence acting as the inverse of the channel. The resultant received signal at the output of the equalizer, is then detected in the conventional manner as normally applied to a serial digital signal in the absence of intersymbol interference, by comparing the corresponding sample value with the appropriate threshold level (or levels). There are two main equalizer designs : linear and nonlinear equalizer [2,6,10].

Linear equalization is a process of linear filtering of the distorted signal by a transversal filter. The filter tap gains of the transversal filter could be chosen, such that the peak distortion is minimized. In this case the equalizer is called a zero forcing equalizer, and the equalizer simply acts as the inverse of the channel, so that the channel and filter introduce negligible distortion [2,6,9,10]. Another criteria is to choose the equalizer coefficients, so as to minimize the mean square error, due to both noise and intersymbol interference in the output signal. Therefore the equalizer maximizes the signal to distortion ratio at the equalizer output [2,6,9,10]. Figure 7.3.2 shows a block diagram of a linear transversal filter.

When a channel is equalized by a linear equalizer the tolerance to additive white Gaussian noise of the system is determined by the noise variance at the equalizer output [2], subject to the accurate equalization of the channel. The noise variance at the equalizer output is determined by the sum of the squares of the tap gains of the linear equalizer. When a channel contains both amplitude and phase distortions, an improvement in the tolerance to noise can be achieved by dividing the equalization between a linear (feedforward) an a pure nonlinear equalizer (feedback filter), resulting in the conventional nonlinear equalizer, which is as shown in Figure 7.3.3. This uses decision feedback to cancel the interference from symbols which were already detected. In a decision feedback equalizer, the ability of the feedback section to cancel the intersymbol interference, because of a number of past symbols, allows more freedom in the choice of the coefficients of the forward section. That is, the

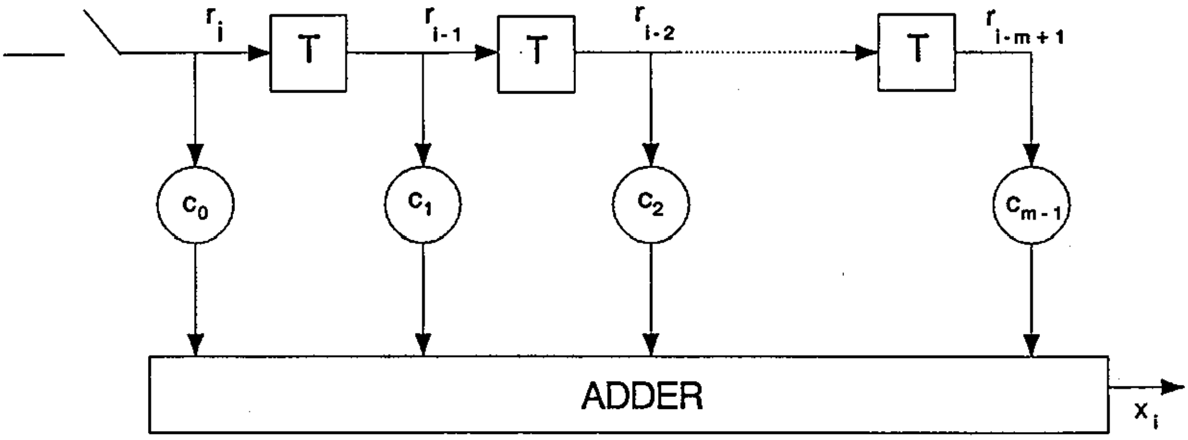


FIGURE 7.3.2 LINEAR FEEDFORWARD TRANSVERSAL EQUALIZER

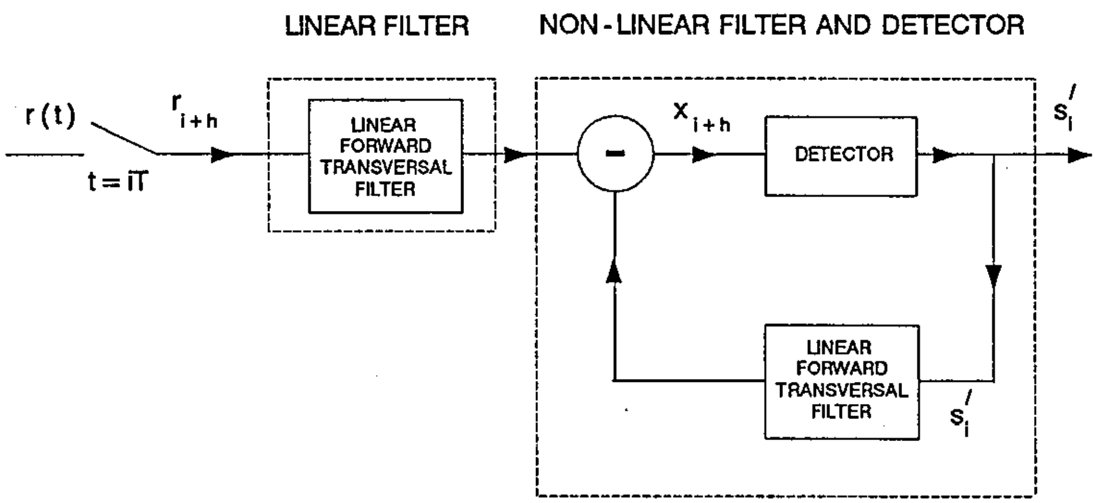


FIGURE 7.3.3 CONVENTIONAL NON-LINEAR EQUALIZER

forward section of a DFE need not approximate the inverse of the channel characteristic, and so avoid excessive noise enhancement, but a DFE is more prone to error propagation effects.

A brief description of the nonlinear equalizer was provided for completeness to give an insight into the various schemes used at the detector to correct or take into consideration the signal distortion introduced by the channel.

An alternative approach, is to modify the detection process to take account of the signal distortion that has been introduced by the channel, such as a maximum likelihood detector [2]. In maximum likelihood, instead of removing the intersymbol interference, the detector takes into full account of the intersymbol interference, thus using the entire transmitted energy in the detection process. A detection process employing the Viterbi algorithm can in practice achieve the same tolerance to noise as the maximum likelihood detection process [13,14]. These detectors are too complicated hence are not practical. One approach to solving this problem, would be to have a linear feedforward transversal filter at the detector input to reduce the number of components in the channel sampled impulse response, but this unfortunately, will result in the filter equalizing some of the amplitude distortion introduced by the channel, resulting in noise enhancement, hence an inferior performance [11,12].

A near maximum likelihood detector provides a practical alternative to the Viterbi detector [4]. It is a form of a reduced state Viterbi detector. Without an adaptive filter at the input to the detector, the severe phase distortion introduced over some telephone channels considerably degrade the performance of the system therefore requiring a more complex detector [12]. With this consideration, an adaptive linear filter was used ahead of the detector, to convert the sampled impulse response of the channel into a minimum phased sequence, without changing the amplitude distortion in the received signal. This concentrates the energy of the sampled impulse response in its first few components and thereby effectively reducing the number of components in the sampled impulse response. The linear filter, in essence, removes all roots (zeros) of the channel that lie outside the unit circle and replaces them by the complex conjugates of their reciprocals, while leaving all the remaining roots unchanged. Furthermore, the filter does not change the signal/noise ratio or any of the noise statistics. This process is a pure phase transformation and should not introduce any gain or attenuation in the signal energy. However, the first non zero component of the resultant sampled is real with value 1, and this requires all components of the sampled impulse response to be divided by the value of its first component. This is to ensure a less complex detector. This performs the same function as the whitened matched filter [13]. With which a Viterbi detector can achieve the best tolerant to noise.

Detection processes have been discussed briefly to understand the need for an adaptive filter ahead of a detector. More detailed analysis of detection schemes are available from a wide range of literature including refs [4,11-14], as these are beyond the scope of this thesis.

7.4 GRAM SCHMIDT ORTHOGONALIZATION PROCESS

If $Y_1, Y_2, \ldots, Y_m$ form a basis of an m dimensional vector space. Then,

$$Z_1 = Y_1$$

$$Z_2 = Y_2 - \frac{Z_1.Y_2}{Z_1.Z_1} Z_1$$

$$Z_3 = Y_3 - \frac{Z_2.Y_3}{Z_2.Z_2} Z_2 - \frac{Z_1.Y_3}{Z_1.Z_1} Z_1$$

....

$$Z_m = Y_m - \frac{Z_{m-1}.Y_m}{Z_{m-1}.Z_{m-1}} Z_{m-1} - \ldots - \frac{Z_1.Y_m}{Z_1.Z_1} Z_1$$

so by the gram Schmidt orthogonalization process there exists a set of orthogonal vectors $Z_1, Z_2, \ldots, Z_m$.

Let

$$G_i = \frac{Z_i}{|Z_i|} for\, i = 1, 2, \ldots, m$$

The vectors G_i are orthogonal and of equal length. They form an orthonormal basis of the m dimensional vector space.

1) Take $Z_1 = Y_1$

2) Take $Z_2 = Y_2 + a\, Z_1$

since Z_1, Z_2 are to be mutually orthogonal

$$\begin{aligned} Z_2.Z_1 &= (Y_2 + aZ_1).Z_1 \\ &= Y_2.Z_1 + aZ_1.Z_1 = 0 \end{aligned}$$

and

$$a = -\frac{Y_2.Z_1}{Z_1.Z_1}$$

Thus

$$Z_2 = Y_2 - \frac{Y_2.Z_1}{Z_1.Z_1} Z_1$$

3) Take $Z_3 = Y_3 + a\, Z_2 + b\, Z_1$

since Z_1, Z_2 and Z_3 are to be mutually orthogonal

$$Z_3.Z_1 = Y_3.Z_1 + aZ_2.Z_1 + bZ_1.Z_1$$
$$= Y_3.Z_1 + bZ_1.Z_1 = 0$$

$$Z_3.Z_2 = Y_3.Z_2 + aZ_2.Z_2 + bZ_1.Z_2$$
$$= Y_3.Z_2 + aZ_2.Z_2 = 0$$

then

$$a = -\frac{Y_3.Z_2}{Z_2.Z_2} \quad , \quad b = -\frac{Y_3.Z_1}{Z_1.Z_1}$$

and

$$Z_3 = Y_3 - \frac{Y_3.Z_2}{Z_2.Z_2}Z_2 - \frac{Y_3.Z_1}{Z_1.Z_1}Z_1$$

and so on..

If Y and Z are two vectors in a complex field such that

$$Y = [y_1 \quad y_2 \quad \dots \quad y_n]$$

and

$$Z = [z_1 \quad z_2 \quad \dots \quad z_n]$$

their dot product is defined as

$$Z.Y = Z\overline{Y}^T = z_1\overline{y}_1 + z_2\overline{y}_2 + \dots + z_n\overline{y}_n$$

The bar denotes a complex conjugate

7.5 IMPLEMENTATION OF FILTER

From the preceeding section, it has been observed that for the reliable operation of a near maximum likelihood detector in a 9600 bits/sec data transmission system over a telephone channel a linear feedforward transversal filter must be used ahead of the detector.

The filter used is shown in Figure 7.5.1. It has n+1 taps. The signals shown are those present at the time instant t=iT. Each square marked T is here a storage element that holds the corresponding sample value. The extreme left hand sample value at the input to the equalizer is that received at time t=iT, the next sample value is that received at time t=(i-1)T and so on, each storage element introducing a delay of T seconds.

A brief explanation of the filter implemented will now be presented. A matrix Y is formed form the channel sampled impulse response as explained below. Firstly, a row vector is formed by preceeding the channel sampled impulse response by a set of zeros, such that the total number of components of the row vector is n+1, where n+1 is the number of taps of the filter,

$$[\,0 \;\ldots\; 0 \;\; y_{i,g} \;\; y_{i,g-1} \;\ldots\; y_{i,1} \;\; y_{i,0}\,] \tag{7.5.1}$$

The matrix Y is then formed by shifting the row vector to the left until the first non-zero component in the row is the first component in the row, and then continue shifting to the left so that the non-zero components fall off the edge until there is only one non-zero component in the row. Similarly the row vector is shifted to the right until there is only one non-zero component left in the row. This matrix can be shown diagramatically in Figure 7.5.2. The matrix is an n+g+1 by n+1 component matrix, in other words it has n+g+1 rows and n+1 columns. The matrix can also be considered as consisting of n+g+1 row vectors.

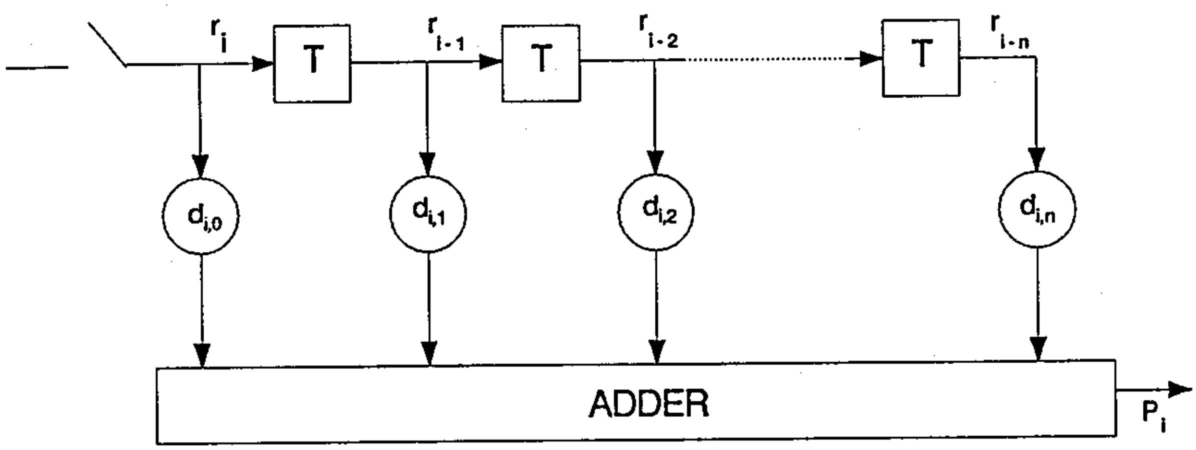


FIGURE 7.5.1 LINEAR FEEDFORWARD TRANSVERSAL EQUALIZER

$$
\begin{matrix}
y_{i,0} & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\
\cdot & & & & & & & & & & & \cdot \\
\cdot & & & & & & & & & & & \cdot \\
y_{i,g} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & y_{i,0} & 0 & \cdot & \cdot & \cdot & 0 \\
\cdot & & & & & & & & & & & \cdot \\
\cdot & & & & & & & & & & & \cdot \\
0 & 0 & y_{i,g} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & y_{i,0} & 0 & \cdot & 0 \\
0 & \cdot & 0 & y_{i,g} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & y_{i,0} & 0 & 0 \\
0 & \cdot & \cdot & 0 & y_{i,g} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & y_{i,0} & 0 \\
0 & \cdot & \cdot & \cdot & 0 & y_{i,g} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & y_{i,0} \\
0 & \cdot & \cdot & \cdot & \cdot & 0 & y_{i,g} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} \\
\cdot & & & & & & & & & & & \cdot \\
0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & y_{i,g}
\end{matrix}
$$

PRECURSORS

POSTCURSORS

$\xleftrightarrow{\quad n+1 \text{ TAPS} \quad}$

FIGURE 7.5.2 MATRIX Y

If the row vectors are considered,

$$[\, y_{i,g} \;\; y_{i,g-1} \;\cdots\; y_{i,1} \;\; y_{i,0} \;\; 0 \;\cdots\; 0 \,]$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$[\, 0 \;\cdots\; 0 \;\; y_{i,g} \;\; y_{i,g-1} \;\cdots\; y_{i,1} \;\; y_{i,0} \,] \tag{7.5.2}$$

Let the last row vector be Y_0, and the first as Y_{n-g}, then Y_0, Y_1 ... Y_{n-g} are a set of vectors which are not orthogonal to each other. The idea is to obtain a set of orthogonal vectors Z_0, Z_1 ... Z_{n-g}. These are obtained by using the Gram Schmidt orthogonalization process. The procedure is as follows,

$$Z_{n-g} = Y_{n-g}$$

$$Z_{n-g-1} = Y_{n-g-1} - \frac{Y_{n-g-1}.Z_{n-g}}{Z_{n-g}.Z_{n-g}} Z_{n-g}$$

$$Z_0 = Y_0 - \frac{Y_0.Z_1}{Z_1.Z_1} Z_1 \quad \ldots \quad - \frac{Y_0.Z_{n-g}}{Z_{n-g}.Z_{n-g}} Z_{n-g} \tag{7.5.3}$$

Z_0 being orthogonal to Z_1 ... Z_{n-g} hence Z_0 is orthogonal to the space spanned by Z_1, Z_2 ... , Z_{n-g} and so to every vector in the space, therefore Z_0 is orthogonal to Y_1, Y_2 ... Y_{n-g} and their dot products are zero.

The tap gains of the filter are given by the complex conjugate of Z_0. So if the signals Y_1, Y_2, ..., Y_{n-g} are lying on the filter taps, the output would be zero. If Y_0 was lying on the filter taps, the output would be a maximum. From the properties of the Gram

Schmidt orthogonalization process, the magnitude of the first non-zero component at the output is maximized, subject to the exact cancellation of the pre cursors, i.e previous symbols.

$$YD_i^T = Y\overline{Z}_0^T = Y.Z_0$$

$Y.Z_0$ gives the sampled impulse response of the channel and filter

If Y_i is the sampled impulse response of the channel and D_i is the sampled impulse response of the filter . The convolution of Y_i with D_i is given by $Y_i * D_i$, where

$$Y_i = [\, y_{i,0} \;\; y_{i,1} \;\; \cdots \;\; y_{i,g} \,] \qquad (7.5.4)$$

and

$$D_i = [\, d_{i,0} \;\; d_{i,1} \;\; \cdots \;\; d_{i,g} \,] \qquad (7.5.5)$$

For the sake of convenience in the description to follow, g is 2 since g+1 = 3 and n is 5 since n+1=6, and therefore the convolution of Y_i with D_i is,

$$\begin{array}{l}
d_{i,0}\, y_{i,0} \\
d_{i,1}\, y_{i,0} + d_{i,0}\, y_{i,1} \\
d_{i,2}\, y_{i,0} + d_{i,1}\, y_{i,1} + d_{i,0}\, y_{i,2} \\
d_{i,3}\, y_{i,0} + d_{i,2}\, y_{i,1} + d_{i,1}\, y_{i,2} \\
d_{i,4}\, y_{i,0} + d_{i,3}\, y_{i,1} + d_{i,2}\, y_{i,2} \\
d_{i,5}\, y_{i,0} + d_{i,4}\, y_{i,1} + d_{i,3}\, y_{i,2} \\
d_{i,5}\, y_{i,1} + d_{i,4}\, y_{i,2} \\
d_{i,5}\, y_{i,2}
\end{array} \qquad (7.5.6)$$

So the resultant channel and filter has n+g+1 i.e 2+5+1=8, components.

From Figure 7.5.1 the following analysis can be made. The received signal for no noise at time t=iT is ,

$$r_i = \sum_{h=0}^{g} s_{i-h} y_{i,h} \tag{7.5.7}$$

therefore

$$
\begin{aligned}
r_i &= s_i\, y_{i,0} + s_{i-1}y_{i,1} + s_{i-2}y_{i,2} \\
r_{i-1} &= s_{i-1}\, y_{i-1,0} + s_{i-2}y_{i-1,1} + s_{i-3}y_{i-1,2} \\
r_{i-2} &= s_{i-2}\, y_{i-2,0} + s_{i-3}y_{i-2,1} + s_{i-4}y_{i-2,2} \\
r_{i-3} &= s_{i-3}\, y_{i-3,0} + s_{i-4}y_{i-3,1} + s_{i-5}y_{i-3,2} \\
r_{i-4} &= s_{i-4}\, y_{i-4,0} + s_{i-5}y_{i-4,1} + s_{i-6}y_{i-4,2} \\
r_{i-5} &= s_{i-5}\, y_{i-5,0} + s_{i-6}y_{i-5,1} + s_{i-7}y_{i-5,2}
\end{aligned}
\tag{7.5.8}
$$

The signal, P_i, at the output of the filter at time t=iT, is given by

$$P_i = r_i\, d_{i,0} + r_{i-1}d_{i,1} + r_{i-2}d_{i,2} + r_{i-3}d_{i,3} + r_{i-4}d_{i,4} + r_{i-5}d_{i,5} \tag{7.5.9}$$

$$
\begin{aligned}
&= s_i\, [\, y_{i,0}d_{i,0} \,] \\
&+ s_{i-1}\, [\, y_{i,1}d_{i,0} + y_{i-1,0}d_{i,1} \,] \\
&+ s_{i-2}\, [\, y_{i,2}d_{i,0} + y_{i-1,1}d_{i,1} + y_{i-2,0}d_{i,2} \,] \\
&+ s_{i-3}\, [\, y_{i-1,2}d_{i,1} + y_{i-2,1}d_{i,2} + y_{i-3,0}d_{i,3} \,] \\
&+ s_{i-4}\, [\, y_{i-2,2}d_{i,2} + y_{i-3,1}d_{i,3} + y_{i-4,0}d_{i,4} \,] \\
&+ s_{i-5}\, [\, y_{i-3,2}d_{i,3} + y_{i-4,1}d_{i,4} + y_{i-5,0}d_{i,5} \,] \\
&+ s_{i-6}\, [\, y_{i-4,2}d_{i,4} + y_{i-5,1}d_{i,5} \,] \\
&+ s_{i-7}\, [\, y_{i-5,2}d_{i,5} \,]
\end{aligned}
\tag{7.5.10}
$$

P_i can also be written as Equation 7.5.11 shown below

$$P_i = s_i\, e_{i,0} + s_{i-1}e_{i,1} + s_{i-2}e_{i,2} + s_{i-3}e_{i,3} + s_{i-4}e_{i,4} + s_{i-5}e_{i,5} + s_{i-6}e_{i,5} + s_{i-7}e_{i,5} \tag{7.5.11}$$

where E_i is the sampled impulse response of the channel and filter

$$E_i = [\ e_{i,0}\ \ e_{i,1}\ \ e_{i,2}\ \ e_{i,3}\ \ e_{i,4}\ \ e_{i,5}\ \ e_{i,6}\ \ e_{i,7}\ ] \tag{7.5.12}$$

hence E_i can be obtained from Equations 7.5.10 and 7.5.11.

$$\begin{aligned}
&y_{i,0}d_{i,0} \\
&y_{i,1}d_{i,0} + y_{i-1,0}d_{i,1} \\
&y_{i,2}d_{i,0} + y_{i-1,1}d_{i,1} + y_{i-2,0}d_{i,2} \\
&y_{i-1,2}d_{i,1} + y_{i-2,1}d_{i,2} + y_{i-3,0}d_{i,3} \\
&y_{i-2,2}d_{i,2} + y_{i-3,1}d_{i,3} + y_{i-4,0}d_{i,4} \\
&y_{i-3,2}d_{i,3} + y_{i-4,1}d_{i,4} + y_{i-5,0}d_{i,5} \\
&y_{i-4,2}d_{i,4} + y_{i-5,1}d_{i,5} \\
&y_{i-5,2}d_{i,5}
\end{aligned} \tag{7.5.13}$$

For time invariant channels, the following conditions occur

$$\begin{aligned}
y_{i,0} &= y_{i-1,0} = y_{i-2,0} = \dots = y_{i-5,0} \\
y_{i,1} &= y_{i-1,1} = y_{i-2,1} = \dots = y_{i-5,1} \\
y_{i,2} &= y_{i-1,2} = y_{i-2,2} = \dots = y_{i-5,2}
\end{aligned} \tag{7.5.14}$$

hence the convolution of Y_i with D_i given by Equation 7.5.6 is the same as that obtained as the resultant sampled impulse response of the channel and filter given by Equation 7.5.13, provided y_{i-1} , y_{i-2} etc is replaced by y_i.

The sampled impulse response of the channel and filter for the filter implementation employing the Gram Schmidt orthogonalization process is given by $Y.Z_0$ and is

$$Y.Z_0 = Y\overline{Z}_0^T = \underline{YD_i^T} \tag{7.5.15}$$

$$
Y.Z_0 = \begin{pmatrix} y_{i,0} & 0 & 0 & 0 & 0 & 0 \\ y_{i,1} & y_{i,0} & 0 & 0 & 0 & 0 \\ y_{i,2} & y_{i,1} & y_{i,0} & 0 & 0 & 0 \\ 0 & y_{i,2} & y_{i,1} & y_{i,0} & 0 & 0 \\ 0 & 0 & y_{i,2} & y_{i,1} & y_{i,0} & 0 \\ 0 & 0 & 0 & y_{i,2} & y_{i,1} & y_{i,0} \\ 0 & 0 & 0 & 0 & y_{i,2} & y_{i,1} \\ 0 & 0 & 0 & 0 & 0 & y_{i,2} \end{pmatrix} \begin{pmatrix} d_{i,0} \\ d_{i,1} \\ d_{i,2} \\ d_{i,3} \\ d_{i,4} \\ d_{i,5} \end{pmatrix} \tag{7.5.16}
$$

which is

$$
\begin{array}{l}
d_{i,0}\, y_{i,0} \\
d_{i,1}\, y_{i,0} + d_{i,0}\, y_{i,1} \\
d_{i,2}\, y_{i,0} + d_{i,1}\, y_{i,1} + d_{i,0}\, y_{i,2} \\
d_{i,3}\, y_{i,0} + d_{i,2}\, y_{i,1} + d_{i,1}\, y_{i,2} \\
d_{i,4}\, y_{i,0} + d_{i,3}\, y_{i,1} + d_{i,2}\, y_{i,2} \\
d_{i,5}\, y_{i,0} + d_{i,4}\, y_{i,1} + d_{i,3}\, y_{i,2} \\
d_{i,5}\, y_{i,1} + d_{i,4}\, y_{i,2} \\
d_{i,5}\, y_{i,2}
\end{array} \tag{7.5.17}
$$

Equation 7.5.17, which is the sampled impulse response obtained by the filter design is the same as Equation 7.5.6 which is the convolution. Equation 7.5.17 is also the same as Equation 7.5.13 provided that the channel is time invariant and hence Equation 7.5.14 holds.

7.6 RESULTS OF SIMULATION TESTS

The adaptive filter employing the Gram Schmidt orthogonalization process was tested for two telephone channels: channel 3 and channel 4. Channel 3 had 26 components. The matrix Y was formed as described in the previous section, and then a set of vectors Z were obtained which were orthogonal to each other, where Z_0 is orthogonal to Y_1, Y_2, ... Y_m but responds to Y_0. The complex conjugate of Z_0 gave the tap gains of the filter. The dot product of Y and Z_0 gave the sampled impulse response of the channel and filter.

Results of the sampled impulse response of the channel and filter are given in tabulated form throughout this chapter. The first value being the first component of the sampled impulse response of the channel and filter combined, broken down into its real and imaginary parts. These are presented in Tables 7.6.1 and 7.6.2 for 50 and 80 taps respectively. For a filter with 50 taps, the sampled impulse response of the channel and filter has 75 (50+26-1) components. The first 49 components are known as the precursors and are the intersymbol interference of the symbols that have already been detected. The precursors can be broken down into 2 parts. The first is the dot product of Z_0 with the vectors Y, that are not orthogonal to each other, but by the Gram Schmidt process are made orthogonal to Z_0, hence $Y.Z_0$ for these vectors should become zero, so that the components of the sampled impulse response approximate to zero depending on the accuracy of the Gram Schmidt orthogonalization process. The second, is the inner product of Z_0 with the Y that are falling off the left edge of the matrix, as shown in Figure 7.6.1. The 50th component is that obtained at the output at time t=iT. The next 25 components are known as the postcursors, and are the inner product of the Z_0 with the Y that are falling off the right edge of the matrix as shown in Figure 7.6.1. For comparison between tables, the first value of the actual minimum phased channel obtained from NAG routines, is compared with $Y_0.Z_0$ (i.e YZ_0) obtained by the filter implementation, the second compared with the value proceeding $Y_0.Z_0$ (i.e YZ_{-1}) and so on.

The tests are carried out for a different number of taps in the filter and the results of these are presented in Table 7.6.5. The sampled impulse response of the channel and filter for the two channels tested came out to be minimum phased. The tests are repeated, but with channel 4 replacing channel 3 as the channel. The results of these are presented in Tables 7.6.3, 7.6.4 and 7.6.6.

$$
\begin{array}{cccccccccccc|l}
y_{i,0} & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \\
\cdot & & & & & & & & & & & \cdot & \text{PRE CURSORS 2} \\
\cdot & & & & & & & & & & & \cdot & \\
y_{i,25} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & y_{i,0} & 0 & \cdot & \cdot & \cdot & 0 & \\
\cdot & & & & & & & & & & & \cdot & \text{PRE CURSORS} \\
\cdot & & & & & & & & & & & \cdot & \\
0 & 0 & y_{i,25} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & y_{i,0} & 0 & \cdot & 0 & \text{PRE CURSORS 1} \\
0 & \cdot & 0 & y_{i,25} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & y_{i,0} & 0 & 0 & \\
0 & \cdot & \cdot & 0 & y_{i,25} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & y_{i,0} & 0 & \\
0 & \cdot & \cdot & \cdot & 0 & y_{i,25} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & y_{i,0} & \\
0 & \cdot & \cdot & \cdot & \cdot & 0 & y_{i,25} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & \\
\cdot & & & & & & & & & & & \cdot & \text{POST CURSORS} \\
0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & y_{i,25} &
\end{array}
$$

$\longleftarrow$ n + 1 TAPS $\longrightarrow$

FIGURE 7.6.1 MATRIX Y FOR CHANNEL 3

$$
\begin{array}{cccccccccccc}
y_{i,0} & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\
\cdot & & & & & & & & & & & \cdot \\
\cdot & & & & & & & & & & & \cdot \\
y_{i,29} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & y_{i,0} & 0 & \cdot & \cdot & \cdot & 0 \\
\cdot & & & & & & & & & & & \cdot \\
\cdot & & & & & & & & & & & \cdot \\
0 & 0 & y_{i,29} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & y_{i,0} & 0 & \cdot & 0 \\
0 & \cdot & 0 & y_{i,29} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & y_{i,0} & 0 & 0 \\
0 & \cdot & \cdot & 0 & y_{i,29} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & y_{i,0} & 0 \\
0 & \cdot & \cdot & \cdot & 0 & y_{i,29} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} & y_{i,0} \\
0 & \cdot & \cdot & \cdot & \cdot & 0 & y_{i,29} & \cdot & \cdot & \cdot & y_{i,2} & y_{i,1} \\
\cdot & & & & & & & & & & & \cdot \\
0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & y_{i,29}
\end{array}
$$

PRE CURSORS 2

PRE CURSORS 1

PRE CURSORS

POST CURSORS

$\longleftarrow$ n + 1 TAPS $\longrightarrow$

FIGURE 7.6.2 MATRIX Y FOR CHANNEL 4

TABLE 7.6.1 SAMPLED IMPULSE RESPONSE OF CHANNEL AND FILTER OBTAINED FROM FILTER WITH 50 TAPS EMPLOYING GRAM SCHMIDT ORTHOGONALIZATION PROCESS FOR TELEPHONE CHANNEL 3

1 ITERATION					
YZ_i	REAL	IMAGINARY	YZ_i	REAL	IMAGINARY
i 49	-0.000000745027	0.000001256978	11	0.000000000000	0.000000000000
48	-0.000002860127	0.000007575049	10	0.000000000000	0.000000000000
47	-0.000004729849	0.000007092683	9	0.000000000000	0.000000000000
46	0.000003455999	-0.000001165272	8	0.000000000000	0.000000000000
45	0.000014424642	0.000009136434	7	0.000000000000	0.000000000000
44	-0.000015572537	0.000013333343	6	0.000000000000	0.000000000000
43	0.000013589831	-0.000005139684	5	0.000000000000	0.000000000000
42	-0.000014954962	0.000002566463	4	0.000000000000	0.000000000000
41	0.000019995480	0.000014005451	3	0.000000000000	0.000000000000
40	0.000038011596	-0.000013559473	2	0.000000000000	0.000000000000
39	-0.000091135595	-0.000003494648	1	0.000000000000	0.000000000000
38	0.000104726670	0.000050159154	0	1.000000000000	0.000000000000
37	-0.000052506349	-0.000182517023	-1	0.457737696318	1.098589539359
36	-0.000085457178	0.000245677286	-2	-0.580276898553	0.042373379962
35	0.000252930164	-0.000328815936	-3	0.156778536071	-0.172053298832
34	-0.000428569033	0.000172158117	-4	-0.017481377209	0.087012299401
33	0.000600374214	0.000089405215	-5	-0.002023218873	-0.019480876058
32	-0.000712593631	-0.000476308616	-6	-0.002235433896	0.008343914319
31	0.000587705536	0.000781090735	-7	-0.004952443164	-0.007520801625
30	-0.000220264169	-0.000868096733	-8	0.007907757323	0.005396666964
29	0.000091826888	0.000641104860	-9	-0.003818692691	-0.003476595782
28	-0.000998694472	0.000989408227	-10	-0.000139250935	0.001349206875
27	0.002068698742	-0.005674041609	-11	0.003835585322	-0.005493317652
26	0.003072190406	0.014753568573	-12	-0.000899794300	0.002589088255
25	-0.020734186893	-0.006147760654	-13	0.002227716856	-0.002651602681
24	0.000000000000	0.000000000000	-14	-0.000412365020	-0.000872837778
23	0.000000000000	0.000000000000	-15	0.001323530554	0.001812301871
22	0.000000000000	0.000000000000	-16	-0.001642048538	-0.001311102371
21	0.000000000000	0.000000000000	-17	-0.000336742659	0.000006736984
20	0.000000000000	0.000000000000	-18	-0.000119101463	0.000117613652
19	0.000000000000	0.000000000000	-19	0.000341424329	0.000180776801
18	0.000000000000	0.000000000000	-20	-0.000709488093	0.001053537315
17	0.000000000000	0.000000000000	-21	-0.000826874085	-0.000344646227
16	0.000000000000	0.000000000000	-22	0.000055929420	0.000202794161
15	0.000000000000	0.000000000000	-23	-0.000071293601	0.000713591144
14	0.000000000000	0.000000000000	-24	-0.000148729153	0.000370004299
13	0.000000000000	0.000000000000	-25	-0.000019664916	0.000059559832
12	0.000000000000	0.000000000000			

TABLE 7.6.2 SAMPLED IMPULSE RESPONSE OF CHANNEL AND FILTER OBTAINED FROM FILTER WITH 80 TAPS EMPLOYING GRAM SCHMIDT ORTHOGONALIZATION PROCESS FOR TELEPHONE CHANNEL 3

1 ITERATION					
YZ_i	REAL	IMAGINARY	YZ_i	REAL	IMAGINARY
i 79	-0.000000041693	-0.000000216107	26	0.000000000000	0.000000000000
78	-0.000000433492	-0.000001134453	25	0.000000000000	0.000000000000
77	-0.000000162842	-0.000001241340	24	0.000000000000	0.000000000000
76	-0.000000255366	0.000000540500	23	0.000000000000	0.000000000000
75	-0.000002499667	0.000000477683	22	0.000000000000	0.000000000000
74	0.000000546122	-0.000003031901	21	0.000000000000	0.000000000000
73	-0.000000911781	0.000001982670	20	0.000000000000	0.000000000000
72	0.000001570043	-0.000001762292	19	0.000000000000	0.000000000000
71	-0.000003577408	0.000000558794	18	0.000000000000	0.000000000000
70	-0.000002733826	0.000005302848	17	0.000000000000	0.000000000000
69	0.000010854940	-0.000009025751	16	0.000000000000	0.000000000000
68	-0.000016726731	0.000005044798	15	0.000000000000	0.000000000000
67	0.000024599202	0.000014919675	14	0.000000000000	0.000000000000
66	-0.000016093652	-0.000036666007	13	0.000000000000	0.000000000000
65	0.000005179857	0.000063175449	12	0.000000000000	0.000000000000
64	0.000030482471	-0.000065186488	11	0.000000000000	0.000000000000
63	-0.000078824728	0.000052838417	10	0.000000000000	0.000000000000
62	0.000131089647	-0.000019588537	9	0.000000000000	0.000000000000
61	-0.000150917612	-0.000031008418	8	0.000000000000	0.000000000000
60	0.000116218340	0.000082034005	7	0.000000000000	0.000000000000
59	-0.000076090488	-0.000072389128	6	0.000000000000	0.000000000000
58	0.000007321882	-0.000202444535	5	0.000000000000	0.000000000000
57	0.000349293514	0.000849523189	4	0.000000000000	0.000000000000
56	-0.001846820181	-0.001375830685	3	0.000000000000	0.000000000000
55	0.003004477871	-0.001338832195	2	0.000000000000	0.000000000000
54	0.000000000000	0.000000000000	1	0.000000000000	0.000000000000
53	0.000000000000	0.000000000000	0	1.000000000000	0.000000000000
52	0.000000000000	0.000000000000	-1	0.460688889305	1.100386855260
51	0.000000000000	0.000000000000	-2	-0.582385932873	0.043567078147
50	0.000000000000	0.000000000000	-3	0.157256136028	-0.172897675478
49	0.000000000000	0.000000000000	-4	-0.017502190242	0.087226708774
48	0.000000000000	0.000000000000	-5	-0.002110601005	-0.019415523132
47	0.000000000000	0.000000000000	-6	-0.002104331312	0.008272545051
46	0.000000000000	0.000000000000	-7	-0.005114095129	-0.007506990165
45	0.000000000000	0.000000000000	-8	0.008042607109	0.005433542574
44	0.000000000000	0.000000000000	-9	-0.003894892092	-0.003541876599
43	0.000000000000	0.000000000000	-10	-0.000115408608	0.001421598300
42	0.000000000000	0.000000000000	-11	0.003849114573	-0.005558991026
41	0.000000000000	0.000000000000	-12	-0.000921536112	0.002625604879
40	0.000000000000	0.000000000000	-13	0.002255577756	-0.002664035485
39	0.000000000000	0.000000000000	-14	-0.000429262048	-0.000880336454
38	0.000000000000	0.000000000000	-15	0.001333722065	0.001823244930
37	0.000000000000	0.000000000000	-16	-0.001644169872	-0.001317059075
36	0.000000000000	0.000000000000	-17	-0.000340417858	0.000005675398
35	0.000000000000	0.000000000000	-18	-0.000117674408	0.000119661298
34	0.000000000000	0.000000000000	-19	0.000340738449	0.000178577719
33	0.000000000000	0.000000000000	-20	-0.000709316237	0.001056766632
32	0.000000000000	0.000000000000	-21	-0.000829418658	-0.000345486493
31	0.000000000000	0.000000000000	-22	0.000055735466	0.000202188727
30	0.000000000000	0.000000000000	-23	-0.000071633318	0.000714861001
29	0.000000000000	0.000000000000	-24	-0.000149336041	0.000371130900
28	0.000000000000	0.000000000000	-25	-0.000019737651	0.000059780128
27	0.000000000000	0.000000000000			

TABLE 7.6.3 SAMPLED IMPULSE RESPONSE OF CHANNEL AND FILTER OBTAINED FROM FILTER WITH 50 TAPS EMPLOYING GRAM SCHMIDT ORTHOGONALIZATION PROCESS FOR TELEPHONE CHANNEL 4

1 ITERATION					
YZ_i	REAL	IMAGINARY	YZ_i	REAL	IMAGINARY
49	0.000000549969	0.000000412148	9	0.000000000000	0.000000000000
48	-0.000000559676	-0.000001734132	8	0.000000000000	0.000000000001
47	-0.000002267755	0.000000242523	7	-0.000000000002	0.000000000003
46	0.000010137618	-0.000000413393	6	-0.000000000010	0.000000000001
45	-0.000006351793	0.000012076806	5	-0.000000000019	-0.000000000018
44	-0.000017904171	-0.000017672627	4	0.000000000005	-0.000000000062
43	0.000040149685	-0.000014202641	3	0.000000000123	-0.000000000074
42	0.000017651682	0.000022469636	2	0.000000000266	0.000000000145
41	-0.000011828985	0.000027081473	1	-0.000000000066	0.000000000534
40	0.000011794983	-0.000036057482	0	1.000000000000	0.000000000000
39	-0.000041908423	-0.000011686330	-1	0.243502962340	1.976857351364
38	0.000107647399	-0.000056643282	-2	-1.714546266257	-0.204823970050
37	-0.000067456523	0.000114575581	-3	0.673795628821	-0.788822058691
36	-0.000018897291	-0.000122256513	-4	0.034554279702	0.504544960977
35	-0.000013905444	-0.000058046036	-5	-0.114644904375	-0.144490695044
34	0.000179746066	0.000101201196	-6	0.029333123285	0.038382247988
33	-0.000270428318	-0.000140572052	-7	-0.016698875952	-0.036896278247
32	0.000096551131	0.000336549744	-8	0.016006701588	0.018483255352
31	0.000033927895	-0.000284431430	-9	-0.014488081137	-0.001236556305
30	-0.000320970110	0.000290813874	-10	-0.000307941945	-0.002863888570
29	0.000296601359	-0.000362289304	-11	0.002015802524	-0.003873248571
28	0.000096279196	0.000168255599	-12	-0.000193520563	0.000114203788
27	-0.001078575677	0.000019389991	-13	-0.002638060567	-0.007536161336
26	0.002193925969	0.000975466274	-14	0.006475787208	0.004645449420
25	-0.006230014365	-0.003148644837	-15	-0.003140913967	0.005166228060
24	0.015052120919	0.000905340119	-16	-0.005959550795	-0.003090772124
23	-0.020409886717	0.015248930209	-17	0.005718851288	-0.001161720917
22	-0.002464996409	-0.034132560431	-18	-0.001288341586	0.003767312284
21	0.025714647488	0.005714395830	-19	-0.000859039350	0.000407307075
20	0.000000000000	0.000000000000	-20	-0.001069091776	0.000074923377
19	0.000000000000	0.000000000000	-21	0.001219631309	-0.001645115378
18	0.000000000000	0.000000000000	-22	0.002322032351	0.000938091751
17	0.000000000000	0.000000000000	-23	-0.000448088528	0.001986419163
16	0.000000000000	0.000000000000	-24	-0.001001255932	-0.000748106341
15	0.000000000000	0.000000000000	-25	0.000594473527	-0.000291569014
14	0.000000000000	0.000000000000	-26	0.000097522095	0.000286921097
13	0.000000000000	0.000000000000	-27	-0.000066199708	-0.000020096691
12	0.000000000000	0.000000000000	-28	-0.000040600034	0.000002943623
11	0.000000000000	0.000000000000	-29	0.000021919640	0.000009648389
10	0.000000000000	0.000000000000			

TABLE 7.6.4 SAMPLED IMPULSE RESPONSE OF CHANNEL AND FILTER OBTAINED FROM FILTER WITH 80 TAPS EMPLOYING GRAM SCHMIDT ORTHOGONALIZATION PROCESS FOR TELEPHONE CHANNEL 4

1 ITERATION					
YZ_i	REAL	IMAGINARY	YZ_i	REAL	IMAGINARY
79	-0.000000098804	0.000000123056	24	-0.000000043699	0.000000008623
78	0.000000424057	-0.000000116842	23	-0.000000078439	-0.000000067863
77	-0.000000092555	-0.000000584580	22	0.000000009263	-0.000000241678
76	-0.000000116682	0.000002491308	21	0.000000455339	-0.000000342294
75	-0.000002197408	-0.000001683260	20	0.000001297161	0.000000305809
74	0.000003855605	-0.000004297442	19	0.000001482672	0.000002717601
73	0.000002134549	0.000010251063	18	-0.000002159257	0.000006915069
72	-0.000003806251	0.000003027171	17	-0.000014897790	0.000008133128
71	-0.000006632181	-0.000005010546	16	-0.000039504844	-0.000008869617
70	0.000007278963	0.000006413743	15	-0.000060188118	-0.000080323733
69	0.000005997111	-0.000011037036	14	0.000018939947	-0.000251396970
68	0.000005464769	0.000021451528	13	0.000447513614	-0.000444018372
67	-0.000023421317	-0.000010546397	12	0.001604402040	-0.000118055282
66	0.000034282710	-0.000008133429	11	0.003292730115	0.002555803508
65	0.000001796725	-0.000002156568	10	0.001625621811	0.010648067118
64	-0.000027541158	0.000046735536	9	-0.016306511043	0.021837246568
63	0.000052470006	-0.000063898961	8	-0.066323309238	0.008596953884
62	-0.000091175529	0.000004462647	7	-0.122927776458	-0.100441515117
61	0.000048840843	0.000055452316	6	-0.025170772737	-0.371763774773
60	0.000001040950	-0.000098922648	5	0.562882360622	-0.544196751818
59	0.000000759324	0.000023197199	4	1.662963010987	0.077740204572
58	-0.000021600460	0.000134081699	3	1.170206624134	2.100982022612
57	0.000111893998	-0.000342669016	2	0.758801858829	1.602856254617
56	-0.000416904212	0.000476309206	1	0.047482099472	2.537657199638
55	0.000933870428	-0.001211307030	0	1.000000000000	0.000000000000
54	-0.000384308750	0.003208033653	-1	0.899204151698	4.399318374184
53	-0.003353532193	-0.004690858181	-2	-4.121762130085	-0.485848242567
52	0.008001446250	-0.000042291534	-3	1.728167430345	-1.775021485090
51	-0.001495790015	0.005644199030	-4	-0.070647049959	1.189902596653
50	0.000000000000	0.000000000000	-5	-0.223107593382	-0.385481477780
49	0.000000000000	0.000000000000	-6	0.036105223259	0.118927932296
48	0.000000000000	0.000000000000	-7	-0.041782457807	-0.095098228935
47	0.000000000000	0.000000000000	-8	0.023104887166	0.042481791623
46	0.000000000000	0.000000000000	-9	-0.029841427331	-0.018800406746
45	0.000000000000	0.000000000000	-10	0.003046055537	-0.010798347013
44	0.000000000000	0.000000000000	-11	0.000969623570	-0.016432023365
43	0.000000000000	0.000000000000	-12	-0.002880148694	0.011108194997
42	0.000000000000	0.000000000000	-13	-0.003407832844	-0.012971822629
41	0.000000000000	0.000000000000	-14	0.008253873278	0.011356876942
40	0.000000000000	0.000000000000	-15	-0.002083391223	0.016972240266
39	0.000000000000	0.000000000000	-16	-0.016129949933	-0.005249194401
38	0.000000000000	0.000000000000	-17	0.013874261977	-0.000897658688
37	0.000000000000	-0.000000000001	-18	-0.001570747636	0.008521080716
36	0.000000000001	-0.000000000001	-19	0.002135576292	0.001149483870
35	0.000000000003	0.000000000001	-20	0.000410509515	0.001350012411
34	0.000000000004	0.000000000007	-21	0.003308350993	-0.001823291768
33	-0.000000000003	0.000000000019	-22	0.005120540772	0.001723632860
32	-0.000000000036	0.000000000027	-23	-0.000876484666	0.004465616584
31	-0.000000000106	-0.000000000014	-24	-0.001986453806	-0.001683197801
30	-0.000000000153	-0.000000000204	-25	0.001458625466	-0.000612658460
29	0.000000000103	-0.000000000601	-26	0.000069474143	0.000669451153
28	0.000000001221	-0.000000000779	-27	-0.000113415208	0.000026429068
27	0.000000003291	0.000000000965	-28	-0.000064019836	-0.000010176472
26	0.000000003245	0.000000007404	-29	0.000046616184	0.000020519089
25	-0.000000008669	0.000000016883			

TABLE 7.6.5 ERROR IN THE MINIMUM PHASE CHANNEL OBTAINED FROM THE ADAPTIVE FILTER EMPLOYING THE GRAM SCHMIDT ORTHOGONALIZATION PROCESS FOR CHANNEL 3

Taps	Pre Error 2	Pre Error 1	Pre Error	Post Error	Total Error
50	-31.325	-295.956	-31.325	-47.015	-31.209
60	-37.041	-295.157	-37.041	-57.817	-37.010
70	-42.356	-296.351	-42.356	-68.457	-42.345
80	-47.670	-297.623	-47.670	-79.058	-47.667
90	-52.967	-299.302	-52.967	-89.651	-52.967
100	-58.263	-299.250	-58.263	-100.241	-58.263

TABLE 7.6.6 ERROR IN THE MINIMUM PHASE CHANNEL OBTAINED FROM THE ADAPTIVE FILTER EMPLOYING THE GRAM SCHMIDT ORTHOGONALIZATION PROCESS FOR CHANNEL 4

Taps	Pre Error 2	Pre Error 1	Pre Error	Post Error	Total Error
50	-25.532	-183.912	-25.532	-41.745	-25.429
60	-33.409	-116.371	-33.409	-57.087	-33.390
70	-38.823	-48.727	-38.401	-42.760	-37.044
80	-38.394	12.770	12.770	11.693	15.275
90	-54.040	12.580	12.580	10.070	14.510
100	-70.780	13.110	13.110	10.790	15.110

TABLE 7.6.7 DOT PRODUCT OF ORTHOGONAL VECTORS OUTPUTED FROM FILTER FOR TELEPHONE CHANNEL 4

$Z_K \cdot Z_J$	J=35	J=41	J=20
	RL=-.568576677244856E-12	RL=0.178395673664037E-15	RL=0.356949746692324E-07
K=50	IM=-.362664340900387E-12	IM=-.675295534890833E-15	IM=0.791184624727702E-08
J=50	J=34	J=40	J=19
RL=0.994643570000000E+00	RL=-.178045109226481E-11	RL=-.763806666230645E-15	RL=0.714964720441160E-07
IM=0.000000000000000E+00	IM=0.309510727365618E-12	IM=-.123601085836377E-14	IM=-.489765804426241E-07
	J=33	J=39	J=18
	RL=-.277357972876521E-11	RL=-.349026786883007E-14	RL=0.367235862247403E-07
K=49	IM=0.393357572768517E-11	IM=-.755910214814757E-15	IM=-.203347482834019E-06
J=50	J=32	J=38	J=17
RL=-.349834215746021E-15	RL=0.250816684128938E-11	RL=-.675985506753843E-14	RL=-.265746642420186E-06
IM=0.111345154364318E-15	IM=0.121072056479576E-10	IM=0.433022724394185E-14	IM=-.413671748476088E-06
J=49	J=31	J=37	J=16
RL=0.349053535902088E+00	RL=0.266079542038782E-10	RL=-.416056553875519E-14	RL=-.116647315541547E-05
IM=0.000000000000000E+00	IM=0.157045797429292E-10	IM=0.182459833036170E-13	IM=-.341692521055883E-06
	J=30	J=36	J=15
	RL=0.710674340413933E-10	RL=0.220325494383855E-13	RL=-.275770876701246E-05
K=25	IM=-.241979548526982E-10	IM=0.369257785545768E-13	IM=0.129120191216753E-05
J=50	J=29	J=35	J=14
RL=-.111295894104479E-16	RL=0.586320217190655E-10	RL=0.962056048906912E-13	RL=-.304884938773478E-05
IM=-.260204815627377E-16	IM=-.164468408785715E-09	IM=0.271923539746046E-13	IM=0.699164171044452E-05
J=49	J=28	J=34	J=13
RL=-.455805369576484E-16	RL=-.220006165439636E-09	RL=0.206976152008866E-12	RL=0.560288999913502E-05
IM=0.122650370762423E-17	IM=-.310616702471997E-09	IM=-.107406869024761E-12	IM=0.183215209183693E-04
J=48	J=27	J=33	J=12
RL=-.293513856882560E-16	RL=-.714381601216384E-09	RL=0.179551806297920E-12	RL=0.420694746602534E-04
IM=-.150598222857078E-16	IM=0.748446507327265E-10	IM=-.518731896155468E-12	IM=0.258849392770574E-04
J=47	J=26	J=32	J=11
RL=0.825823242255053E-16	RL=-.226351821341016E-09	RL=-.548614347784730E-12	RL=0.123921619587382E-03
IM=0.297003632625248E-16	IM=0.948780339057667E-09	IM=-.117638652043460E-11	IM=-.246390624313081E-04
J=46	J=25	J=31	J=10
RL=-.103033087704013E-16	RL=0.106464454936743E+00	RL=-.289120227316692E-11	RL=0.184735334842315E-03
IM=0.650015201305318E-16	IM=0.000000000000000E+00	IM=-.105579885510500E-11	IM=-.265059556697624E-03
J=45		J=30	J=9
RL=-.373507648421256E-16		RL=-.656225882434449E-11	RL=-.148439839568888E-03
IM=-.226388189878551E-15	K=0	IM=0.318982525978084E-11	IM=-.780498709160374E-03
J=44	J=50	J=29	J=8
RL=-.105154058203938E-15	RL=0.302413261482447E-17	RL=-.502890624163689E-11	RL=-.159432069554722E-02
IM=-.109148665583189E-15	IM=0.642813303671289E-17	IM=0.164832373417785E-10	IM=-.108800057467495E-02
J=43	J=49	J=28	J=7
RL=-.830770761699965E-15	RL=0.270775257413518E-17	RL=0.210199107574727E-10	RL=-.436573664970101E-02
IM=0.102965113309996E-15	IM=0.681981960036899E-17	IM=0.346473584855224E-10	IM=0.952438834328936E-03
J=42	J=48	J=27	J=6
RL=-.676034571137313E-15	RL=0.243555059560113E-17	RL=0.936488930802467E-10	RL=-.545373703125484E-02
IM=0.982002565201590E-15	IM=0.231828947196562E-17	IM=0.155163846147480E-10	IM=0.886549479833967E-02
J=41	J=47	J=26	J=5
RL=0.432129951667779E-15	RL=-.116176656765236E-16	RL=0.167028962092897E-09	RL=0.582517384126584E-02
IM=0.328172412205133E-14	IM=-.113270274964969E-16	IM=-.145298939739858E-09	IM=0.189851952045688E-01
J=40	J=46	J=25	J=4
RL=0.482757280331758E-14	RL=0.669889241225864E-17	RL=-.197803745225714E-10	RL=0.392212777652992E-01
IM=0.415276739610074E-14	IM=-.793275471861214E-17	IM=-.515724195221514E-09	IM=0.221108181444603E-01
J=39	J=45	J=24	J=3
RL=0.155984535014822E-13	RL=-.101129381155059E-16	RL=-.978258843523035E-09	RL=0.341881125829654E-01
IM=0.128661107138552E-14	IM=0.403561874586748E-16	IM=-.701877976992345E-09	IM=-.168956502664386E-01
J=38	J=44	J=23	J=2
RL=0.320013223345105E-13	RL=0.722392049065830E-17	RL=-.267909360614807E-08	RL=0.143750156354736E+00
IM=-.195469329487336E-13	IM=0.229099515872933E-16	IM=0.773965961785116E-09	IM=0.101093030103064E-01
J=37	J=43	J=22	J=1
RL=0.348425581955287E-13	RL=0.151617772592887E-15	RL=-.254848082602491E-08	RL=-.194214646480755E-01
IM=-.905271313759239E-13	IM=0.470533154946868E-16	IM=0.607356473999600E-08	IM=-.992469078537458E-01
J=36	J=42	J=21	J=0
RL=-.752088183516366E-13	RL=0.204169403941328E-15	RL=0.737632146053361E-08	RL=0.295635228703710E+00
IM=-.239493970470656E-12	IM=-.109020128333443E-15	IM=0.137388610416560E-07	IM=0.000000000000000E+00

The performance of the filter is assessed by error measurements. Pre error 1 is the error due to precursors 1, as these for a true minimum phased channel are zero. Pre error 2 is the error due to precursors 2 (falling off the edge), as these for a true minimum phased channel are zero. The post error is the difference between the minimum phased channel obtained from the filter implementation and the actual minimum phased channel obtained from NAG routines. For channel 3 as the number of taps of the filter increased, the pre and post errors decrease as would be expected.

For channel 4, the performance of the system deteriorates as the number of taps increases until it finally collapses. This is as a result of pre error 1 increasing which is due to the Gram Schmidt process breaking down. The vector Z_0 is not orthogonal to the subspace spanned by the vectors Y_1, Y_2 etc, as Z_0 is not orthogonal to the subspace that spans the vectors Z_1, Z_2 etc. This can be observed form Table 7.6.7, as the dot product of two supposedly orthogonal Z vectors, is becoming increasingly larger, hence Z_0 is no more orthogonal to the subspace spanned by the remaining Z vectors. This loss of orthogonalization has posed a serious problem in the design of an efficient filter. The next section presents an overview of numerical problems encountered by the Gram Schmidt orthogonalization process and gives details of a solution to the problems encountered by the filter implemented for channel 4.

7.7 NUMERICAL PROBLEMS ENCOUNTERED BY THE FILTER DESIGN

In the previous section, it was observed that the Gram Schmidt orthogonalization process was breaking down, and therefore the required orthogonal vectors were losing orthogonality. This section presents a study of round-off errors that can be introduced into the orthogonalization process.

Numerical computations performed on computers can be either fixed or floating point computations [19]. Numerous texts are available discussing the various computational techniques. Due to the limited precision of the computer, round-off errors can occur in a computation.

The norm of a vector Y is taken to be

$$|Y| = (Y.Y)^{\frac{1}{2}} \tag{7.7.1}$$

A set of vectors Z_1, Z_2, ... is said to be orthogonal with respect to an inner product if and only if

$$Z_j.Z_k = \delta_{jk} = \begin{cases} 0 & if j \neq k \\ 1 & if j = k \end{cases} \tag{7.7.2}$$

A given vector y possesses an orthogonal expansion [18]

$$Y = \sum_{k=1}^{n} (Y.Z_k)Z_k \tag{7.7.3}$$

The Gram Schmidt process operating as described in section 7.4 results in,

$$Z_1 = Y_1$$

$$Z_2 = Y_2 - \frac{Z_1.Y_2}{Z_1.Z_1}Z_1$$

$$Z_3 = Y_3 - \frac{Z_2.Y_3}{Z_2.Z_2}Z_2 - \frac{Z_1.Y_3}{Z_1.Z_1}Z_1$$

....

$$Z_n = Y_n - \frac{Z_{n-1}.Y_n}{Z_{n-1}.Z_{n-1}}Z_{n-1} - \ldots - \frac{Z_1.Y_n}{Z_1.Z_1}Z_1$$

(7.7.4)

Round-off errors may be so severe in adverse conditions, as to produce a meaningless computation. Round-off may show its effects in several ways: the theoretically orthogonal vectors become less and less orthogonal as the computation proceeds. A good way of spotting round-off is as follows. In the computation of the orthogonal vectors Z from the Y's as described is section 7.4, for each Z, there will be computed the norm defined as $(Z.Z)^{0.5}$. Theoretically, the norm is monotonically decreasing, but it may be found that after decreasing for a while, it actually increases! This indicates the presence of serious round-off and the computation should be stopped at this point [18]. Table 7.7.1 shows the magnitude of Z, known as the norm. From that it can be observed that the norm is decreasing and then increases, thereby indicating the presence of round-off.

A method for alleviating some of the effects of round-off consists in a progressive "straightening out" of the orthogonal vectors [18]. Let us suppose that we have a system of n vectors Z_1, Z_2, ..., Z_{n-1}, Z'_n of which the first (n-1) are substantially orthogonal,

$$Z_i.Z_j = \delta_{ij} \qquad (i, j = 1, 2, ..., n-1) \tag{7.7.5}$$

whereas the n^{th} vector Z'_n is normal but slightly non-orthogonal to the first (n-1) vectors

$$Z'_n.Z_j = \varepsilon_j \qquad (j = 1, 2, ..., n-1) \tag{7.7.6}$$

$$Z'_n.Z'_n = 1 \tag{7.7.7}$$

and

$$Z_n = Z'_n + h \tag{7.7.8}$$

TABLE 7.7.1 MAGNITUDE OF EACH ORTHOGONAL VECTOR FOR THE FILTER IMPLEMENTED FOR CHANNEL 4

n	magnitude of Zn	n	magnitude of Zn
50	0.997318188945	24	0.326239850759
49	0.590807528644	23	0.326199637772
48	0.469219306001	22	0.326166374262
47	0.411332216056	21	0.326139099488
46	0.380836411095	20	0.326117111411
45	0.362328325475	19	0.326099664338
44	0.350734713387	18	0.326086099826
43	0.343247425318	17	0.326075526801
42	0.337787068860	16	0.326067071483
41	0.333984905762	15	0.326059995554
40	0.331483002959	14	0.326053673762
39	0.329960493200	13	0.326047657060
38	0.328939722537	12	0.326041763719
37	0.328210440414	11	0.326036029168
36	0.327702737179	10	0.326030716874
35	0.327421317133	9	0.326026745492
34	0.327261787043	8	0.326027869408
33	0.327127280401	7	0.326051436224
32	0.326998466847	6	0.326178150860
31	0.326866275257	5	0.326801722731
30	0.326731659456	4	0.329632416978
29	0.326618280657	3	0.343383416301
28	0.326514365307	2	0.387058007498
27	0.326423535260	1	0.529509548015
26	0.326349207190	0	0.543723485518
25	0.326288913291		

where Z_n is the true (improved) n^{th} orthogonal vector and h is a correction vector whose norm is assumed small. Expanding h in its orthogonal expansion from Equation 7.7.3 [18]

$$h = Z_n - Z_n' = \sum_{j=1}^{n-1} \frac{(h.Z_j)}{Z_j.Z_j} Z_j \quad + \quad \frac{(h.Z_n)}{Z_j.Z_j} Z_n \tag{7.7.9}$$

From Equation 7.7.6

$$(Z_n - h.Z_j) = \varepsilon_j \tag{7.7.10}$$

or

$$(Z_n.Z_j) - (h.Z_j) = \varepsilon_j \tag{7.7.11}$$

or

$$(h.Z_j) = -\varepsilon_j = -(Z_n'.Z_j) \qquad j = 1, 2, \ldots, n-1 \tag{7.7.12}$$

From Equation 7.7.7

$$(Z_n - h.Z_n - h) = 1 \tag{7.7.13}$$

or, neglecting (h.h),

$$(h.Z_n) = 0 \tag{7.7.14}$$

Thus

$$Z_n = Z_n' - \sum_{j=1}^{n-1} \frac{(Z_n'.Z_j)}{Z_j.Z_j} Z_j \tag{7.7.15}$$

The solution to this round-off problem is to re-orthogonalize the supposedly orthogonal vectors, resulting in vectors which are truly orthogonal [18,19]. This correction was performed in the implementation of the filter for channel 4 and the results of simulation tests are presented in Table 7.7.2, given on the next page

TABLE 7.7.2

ERROR IN THE MINIMUM PHASE CHANNEL OBTAINED FROM THE ADAPTIVE FILTER EMPLOYING THE GRAM SCHMIDT ORTHOGONALIZATION PROCESS FOR CHANNEL 4 FOR 2 ITERATIONS

Taps	Pre Error 2	Pre Error 1	Pre Error	Post Error	Total Error
50	-25.532	-310.723	-25.532	-41.745	-25.429
80	-47.68	-286.879	-47.68	-85.225	-47.679

As the uncorrected filter for channel 4 collapses completely for 80 taps (Table 7.6.8), the correction to the filter with 80 taps was made for channel 4, thereby resulting in the sampled impulse response of the channel and filter as shown in Table 7.7.3 with the correction being implemented. From Tables 7.7.2 and 7.7.3 it can be observed that the numerical problems experienced by the filter in section 7.6 have been taken care of by this rather minute change to the algorithm. The next section obtains modifications of the channels in section 7.6 for further testing of the filter.

7.8 RESULTS OF SIMULATION TESTS ON MODIFICATION TO CHANNELS

This section deals with the modification of the channels that were used in testing the filter in section 7.6, it also presents results of simulation tests carried out on these modified channels.

As the filter collapses for channel 4, most of the modifications made to the channels to test the filter were for channel 4. The roots of telephone channel 4 are presented in Table 7.2.4, from this it can be observed that channel 4 has 8 roots outside the unit circle. A possible reason why the filter for channel 4 collapses, might lie in the fact

TABLE 7.7.3 SAMPLED IMPULSE RESPONSE OF CHANNEL AND FILTER OBTAINED FROM FILTER WITH 80 TAPS EMPLOYING GRAM SCHMIDT ORTHOGONALIZATION PROCESS FOR TELEPHONE CHANNEL 4

2 ITERATION					
YZ_i	REAL	IMAGINARY	YZ_i	REAL	IMAGINARY
79	0.000000042406	0.000000033432	24	0.000000000000	0.000000000000
78	-0.000000042184	-0.000000145499	23	0.000000000000	0.000000000000
77	-0.000000199540	0.000000037701	22	0.000000000000	0.000000000000
76	0.000000861811	0.000000030864	21	0.000000000000	0.000000000000
75	-0.000000591987	0.000000745642	20	0.000000000000	0.000000000000
74	-0.000001478395	-0.000001293695	19	0.000000000000	0.000000000000
73	0.000003555799	-0.000000742894	18	0.000000000000	0.000000000000
72	0.000000961351	0.000001246663	17	0.000000000000	0.000000000000
71	-0.000001743476	0.000002368070	16	0.000000000000	0.000000000000
70	0.000002332975	-0.000002551294	15	0.000000000000	0.000000000000
69	-0.000003964370	-0.000002130326	14	0.000000000000	0.000000000000
68	0.000007349583	-0.000001625473	13	0.000000000000	0.000000000000
67	-0.000003323958	0.000007895983	12	0.000000000000	0.000000000000
66	-0.000003177399	-0.000012040415	11	0.000000000000	0.000000000000
65	-0.000000749961	0.000000129452	10	0.000000000000	0.000000000000
64	0.000016743781	0.000008914991	9	0.000000000000	0.000000000000
63	-0.000022982325	-0.000018409573	8	0.000000000000	0.000000000000
62	0.000001967640	0.000032646717	7	0.000000000000	0.000000000000
61	0.000020339222	-0.000018065662	6	0.000000000000	0.000000000000
60	-0.000036176092	-0.000001318068	5	0.000000000000	0.000000000000
59	0.000007933137	0.000003238638	4	0.000000000000	0.000000000000
58	0.000050479508	0.000004936441	3	0.000000000000	0.000000000000
57	-0.000124160990	-0.000041292109	2	0.000000000000	0.000000000000
56	0.000166413438	0.000151740719	1	0.000000000000	0.000000000000
55	-0.000410702835	-0.000327425925	0	1.000000000000	0.000000000000
54	0.001090523656	0.000127632859	-1	0.246038245463	1.980073385101
53	-0.001603632190	0.001167989668	-2	-1.719403873595	-0.202618164542
52	-0.000003425457	-0.002763311194	-3	0.674417562753	-0.792616301623
51	0.001909294890	0.000536251075	-4	0.035732938508	0.506387839181
50	0.000000000000	0.000000000000	-5	-0.115530894827	-0.144960795866
49	0.000000000000	0.000000000000	-6	0.029683929187	0.038534884023
48	0.000000000000	0.000000000000	-7	-0.016851733411	-0.037042230155
47	0.000000000000	0.000000000000	-8	0.016070810230	0.018537451773
46	0.000000000000	0.000000000000	-9	-0.014501048746	-0.001223896654
45	0.000000000000	0.000000000000	-10	-0.000331865491	-0.002908668526
44	0.000000000000	0.000000000000	-11	0.002011054799	-0.003836149061
43	0.000000000000	0.000000000000	-12	-0.000147400040	0.000090334694
42	0.000000000000	0.000000000000	-13	-0.002683958682	-0.007554377821
41	0.000000000000	0.000000000000	-14	0.006507380980	0.004660814180
40	0.000000000000	0.000000000000	-15	-0.003145676400	0.005167085809
39	0.000000000000	0.000000000000	-16	-0.005981470810	-0.003084660364
38	0.000000000000	0.000000000000	-17	0.005730559288	-0.001176008793
37	0.000000000000	0.000000000000	-18	-0.001286582795	0.003781747170
36	0.000000000000	0.000000000000	-19	-0.000865427714	0.000403454872
35	0.000000000000	0.000000000000	-20	-0.001071395553	0.000080195775
34	0.000000000000	0.000000000000	-21	0.001221620749	-0.001649941994
33	0.000000000000	0.000000000000	-22	0.002325699808	0.000939182053
32	0.000000000000	0.000000000000	-23	-0.000446329945	0.001993044139
31	0.000000000000	0.000000000000	-24	-0.001005042786	-0.000749556363
30	0.000000000000	0.000000000000	-25	0.000595383657	-0.000293126135
29	0.000000000000	0.000000000000	-26	0.000098374600	0.000288322318
28	0.000000000000	0.000000000000	-27	-0.000066357791	-0.000020471340
27	0.000000000000	0.000000000000	-28	-0.000040885244	0.000003010487
26	0.000000000000	0.000000000000	-29	0.000022017765	0.000009691580
25	0.000000000000	0.000000000000			

that some of the roots outside the unit circle are very close to the unit circle, as channel 4 introduces severe phase distortion. An approach which might lead to an improved performance is to put the root close to the unit circle inside it. Channel 4A was obtained from channel 4 by taking the root with magnitude 1.08, and putting it inside the unit circle thereby partially minimum phasing the channel. Channel 4B was similarly obtained but this time taking the roots at 1.08 and 1.127 and putting them into the unit circle. This process was repeated for each root, until channel 4G was obtained which put all roots outside the unit circle inside. A summary of the channels is as shown below

Channel 4A : root at 1.08 put inside circle

Channel 4B : roots at 1.08,1.127 put inside circle

Channel 4C : roots at 1.08,1.127,1.26 put inside circle

Channel 4D : roots at 1.08,1.127,1.26,1.43 put inside circle

Channel 4E : roots at 1.08,1.127,1.26,1.43,1.58 put inside circle

Channel 4F : roots at 1.08,1.127,1.26,1.43,1.58,1.89 put inside circle

Channel 4G : all roots put inside the circle

The sampled impulse response of the channels 4A-4G are given in Tables 7.8.1 and 7.8.2 and the roots of each of the channels are given in Table 7.8.3. These modified channels are fed to the filter to test the performance of the design. The results of which are presented in Table 7.8.4. From these results it can be observed that by replacing roots outside the unit circle by those inside, the filter performance was not enhanced, and the Gram Schmidt orthogonalization process was breaking down as in section 7.6. So by partial minimum phasing the channel performance was not improved, this suggests the performance of the filter is not affected by putting the roots outside the unit circle, inside it. This might seem strange at first but if the root just outside the unit circle, is put inside it will still be at the same distance from the unit circle. This possibly explains the poor performance of the modification to the channels. The next section deals with further modifications to the channel to test this possibility.

TABLE 7.8.1 REAL PART OF IMPULSE RESPONSE OF TELEPHONE CHANNELS 4A - 4G

CH4A	CH4B	CH4C	CH4D	CH4E	CH4F	CH4G
-0.004110993464	-0.004636363248	-0.005857096675	-0.008399119695	-0.013291126072	-0.025178753144	-0.199779295691
0.008170725097	0.008360211599	0.007141274268	0.000310901451	0.005829772938	-0.038865324996	0.460934892828
0.009660265869	0.011113619889	0.016178009529	0.022146950460	0.039828133895	0.006959345157	0.291315604376
-0.094654633773	-0.104170416471	-0.119612981318	-0.129451686932	-0.236389529770	-0.395899253551	-0.338928105588
-0.110952618886	-0.111256547123	-0.129127230611	-0.176411315044	-0.039815313551	-0.338375287577	0.123313452856
0.574925630655	0.598662390518	0.625505013352	0.601828247406	0.630084602628	0.479256092729	-0.014262225869
0.157031371644	0.122888716347	0.100279974424	0.053568661012	-0.159147820476	-0.124750834962	0.003996505941
-0.260144429973	-0.229379017128	-0.188473661090	-0.181487868551	-0.013287409118	-0.021333428297	-0.006175898930
0.223358186261	0.213865210045	0.176936330347	0.140336251410	0.046315614817	0.034336339736	0.001564867024
-0.064586002237	-0.069260282845	-0.075581538132	-0.065429975327	-0.023132765066	-0.014996185421	0.002581754266
0.031184852274	0.026144318580	0.028254640497	0.029638015429	0.009726030709	0.009515106645	-0.000683073403
-0.032696566647	-0.027870255413	-0.021146039081	-0.017537458935	-0.008664548457	-0.006758321855	-0.001389901709
0.007045699338	0.009118396805	0.007895972180	0.006542768539	0.004102554239	0.003085586742	0.000052611166
0.004959704089	0.002188736661	0.000066738829	-0.000382299405	0.000351411700	0.001104088542	-0.001409863659
-0.001000708884	-0.001034213253	-0.000215249357	0.000063132989	-0.000437922439	0.000727745837	-0.000099370344
-0.003964232247	-0.002553465033	-0.001554981388	-0.000790590893	-0.000585795002	0.000650775321	0.001959535115
0.001533083384	0.000758894562	-0.000065110193	-0.000036264618	-0.000048315285	0.000526579813	0.000400374579
-0.001590964840	-0.001781974071	-0.001962502085	-0.002035727911	-0.002048555391	-0.002187578533	-0.001447836965
-0.001386882812	-0.000842556888	-0.000111085907	0.000421093514	0.000825097837	-0.000285354522	0.001231270487
0.002482758170	0.002108026315	0.001767369361	0.001880108898	0.002201271830	0.001864367547	0.000276828983
-0.000752523161	-0.000552624754	-0.000484955790	-0.000251854806	-0.001501855584	-0.001579844247	0.000234715585
-0.001246259140	-0.001189126928	-0.000901160023	-0.000870519254	0.000428898707	-0.000240520867	-0.000669107871
0.001798156123	0.001682022406	0.001641922514	0.001291521172	0.000494246963	-0.000070128203	-0.000222687682
-0.000244720630	-0.000060054728	-0.000003363711	0.000114098744	0.000567024422	0.000431966899	0.000602605066
0.000686274561	0.000589503625	0.000389652218	0.000335061746	-0.000120601684	0.000008595268	0.000007696062
-0.000709975866	-0.000767244372	-0.000702810160	-0.000772132120	-0.000686575008	-0.000752599263	-0.000194461698
0.000169737729	0.000211456153	0.000242149556	-0.000007606546	0.000167731956	-0.000057416102	0.000054622932
0.000669943963	0.000712902181	0.000657682572	0.000506405476	0.000349636494	0.000295883559	0.000007982844
0.000132623925	0.000037411364	-0.000094179072	-0.000168744700	-0.000173791821	-0.000107830254	0.000008944028
-0.000092435078	-0.000081960791	-0.000064878560	-0.000045242837	-0.000028590505	-0.000015092090	-0.000001902099

TABLE 7.8.2 IMAGINARY PART OF IMPULSE RESPONSE OF TELEPHONE CHANNELS 4A - 4G

CH4A	CH4B	CH4C	CH4D	CH4E	CH4F	CH4G
-0.005301017888	-0.005978468399	-0.007552572029	-0.010830443818	-0.017138557304	-0.032467339581	-0.257610144443
-0.003715714042	-0.002579049104	-0.000320638383	0.001367031099	0.022075159036	0.016100816050	-0.458968260465
0.021306112397	0.022550517426	0.027718825856	0.046743577876	0.039570597892	0.116112063631	0.483419904491
0.036025117528	0.036313467657	0.035939320892	0.055773187431	0.057789366109	0.284278835371	-0.015384423791
-0.302620304155	-0.322508330314	-0.361000006108	-0.438799118784	-0.632519897791	-0.545740183596	-0.110375415807
-0.211553145563	-0.200238024077	-0.194481936533	-0.292221821935	0.011904156244	-0.087799765232	0.058724268171
0.582339163499	0.571239524581	0.532619591366	0.449608145937	0.271614766136	0.235776012672	-0.015346240706
-0.115233801864	-0.138591613876	-0.183949935323	-0.191404368677	-0.166348069958	-0.123868361039	0.011741963227
-0.020501516116	-0.007566566743	0.003836172605	0.027630264445	0.047799681080	0.040055155109	-0.007843583873
0.029013905882	0.011541109421	0.004313948731	0.001051964387	-0.016630780646	-0.013943338185	0.003980097575
-0.044901261357	-0.031734309447	-0.013596641853	-0.004584284596	0.006814803317	0.006264121304	0.000666700112
0.015930247196	0.015770775914	0.009657957869	0.004426383683	-0.001646177756	-0.002037905300	0.000248289357
-0.000421635949	-0.002391139393	-0.003997433708	-0.003699538247	-0.000853737642	-0.001879899219	0.000019854226
0.004615765610	0.003188726972	0.003305691629	0.002819168558	0.002315575956	0.001196654129	0.002200741037
-0.003697418940	-0.001479789164	0.000183046106	0.001147436633	0.000673165673	0.000497268813	-0.002607577048
0.000715659931	0.000124489942	-0.000689573749	-0.000588108661	-0.000562016953	-0.001587003586	-0.000221918398
0.002046758369	0.001556111531	0.001427254140	0.001601885256	0.002865457562	0.002444966373	0.002157171972
-0.000841345395	-0.000211579721	0.000560469597	0.001191976793	-0.000801844612	-0.000358553534	-0.001241308474
-0.002623566140	-0.002824809434	-0.002918149554	-0.002806849221	-0.001705804549	-0.002236541318	-0.000424103060
0.001712329651	0.001764154058	0.001570362467	0.001108135559	0.001311832426	0.000766075656	0.000142358411
0.001161592710	0.001292114394	0.001496816862	0.001330288904	0.000591905929	0.000686577080	0.000259977616
-0.000559944818	-0.000870860505	-0.001070135167	-0.001455650021	-0.001117631064	-0.000845805297	0.000014924626
-0.000043309218	0.000211126925	0.000240121932	0.000184975418	-0.000018812645	-0.000102057639	-0.000786759407
0.000402592869	0.000248296726	-0.000082657050	-0.000293582599	-0.000069294442	-0.000025052045	-0.000283201960
0.000559752180	0.000520303511	0.000619187478	0.000584083981	0.000451368786	0.000768894908	0.000408663054
-0.000211654300	-0.000230276582	-0.000206859520	-0.000276898251	-0.000354504795	0.000182158914	-0.000094815540
-0.000727286107	-0.000658886351	-0.000710354772	-0.000746939875	-0.000798331972	-0.000566309766	-0.000082946375
0.000002784515	-0.000053033111	-0.000149410310	-0.000155377499	0.000124445167	0.000103474652	0.000021184862
0.000340193450	0.000287174118	0.000251769934	0.000266456986	0.000078691169	0.000125036746	0.000009931429
-0.000369740311	-0.000327843165	-0.000259514241	-0.000180971346	-0.000114362018	-0.000060368359	-0.000007608396

TABLE 7.12 ROOTS OF TELEPHONE CHANNELS 4A - 4G

CH4A	CH4B	CH4C	CH4D	CH4E	CH4F	CH4G
2.742692478470	2.742692478470	2.742692478519	2.742692478497	2.742692478493	2.742692478493	0.872898858332
1.894403303907	1.894403303907	1.894403303910	1.894403303931	1.894403303926	0.527871000000	0.720239833935
2.892938106910	2.892938106917	2.892938106932	2.892938106938	2.892938106933	2.892938106933	0.791580454710
1.582442750481	1.582442750465	1.582442750498	1.582442750476	0.924350776436	0.924350776436	0.762959893661
1.434007352233	1.434007352236	1.434007352246	0.924350776389	0.739670447075	0.739670447075	0.719702473908
1.127796307295	1.263295467201	0.695374813401	0.726286730507	0.872898858302	0.872898858302	0.721291420481
0.742782243280	0.749650035219	0.720239834128	0.749869568063	0.749650035218	0.749650035218	0.749869567818
0.678442593053	0.761492488494	0.886684939071	0.721291420552	0.749869566052	0.749869566052	0.924350776410
0.762959893764	0.719702473871	0.749650035202	0.761492488523	0.726197019630	0.726197019630	0.742782243329
1.263295467194	0.924350776331	0.749869567321	0.768053936788	0.761492488561	0.761492488561	0.761492488498
0.721291420577	0.720239833992	0.726197019616	0.739670446934	0.695374813378	0.695374813378	0.695374813468
0.749869567387	0.768053936828	0.762959893749	0.719702473841	0.758136208958	0.758136208958	0.715087136613
0.768053936834	0.762959893741	0.924350776377	0.758136209173	0.726286730447	0.726286730447	0.768053936703
0.720239833990	0.721291420562	0.742782243275	0.742782243319	0.734647710942	0.734647710942	0.757998304673
0.924350776384	0.757998304231	0.791580454226	0.872898858368	0.720239834033	0.720239834033	0.749650035181
0.749650035234	0.872898858315	0.739670447073	0.715087136521	0.719702473826	0.719702473826	0.886684939000
0.761492488556	0.726286730411	0.768053936827	0.886684939023	0.762959893771	0.762959893771	0.739670447054
0.758136209357	0.886684939084	0.719702473839	0.726197019643	0.791580454225	0.791580454225	0.726197019611
0.719702473878	0.715087136644	0.761492488496	0.733856292695	0.742782243290	0.742782243290	0.734647711991
0.715087136661	0.678442593035	0.872898858370	0.678442592948	0.768053936877	0.768053936877	0.726286730577
0.872898858344	0.758136209297	0.726286730454	0.762959893688	0.733856292832	0.733856292832	0.678442593093
0.726286730427	0.742782243265	0.757998304079	0.757998304242	0.721291420534	0.721291420534	0.527870703564
0.757998304213	0.733856292600	0.721291420522	0.749650035235	0.757998304314	0.757998304314	0.697346494146
0.695374813440	0.695374813426	0.733856292564	0.720239833977	0.715087136709	0.715087136709	0.631934393346
0.739670447043	0.739670447029	0.678442592971	0.695374813371	0.678442592941	0.678442592941	0.758136209606
0.726197019663	0.726197019649	0.715087136762	0.791580454416	0.886684938982	0.886684938982	0.510084624581
0.733856292670	0.749869567392	0.734647711395	0.734647711038	0.510084622333	0.510084622333	0.733856293025
0.734647711381	0.734647711576	0.758136209218	0.510084622114	0.631934392852	0.631934392852	0.364605219827
0.510084622533	0.510084622293	0.510084621559	0.697346493179	0.697346496995	0.697346496995	0.345669334407
0.000000000000	0.000000000000	0.000000000000	0.000000000000	0.000000000000	0.000000000000	0.000000000000

TABLE 7.8.4 ERROR IN THE MINIMUM PHASE CHANNEL OBTAINED FROM THE ADAPTIVE FILTER EMPLOYING THE GRAM SCHMIDT ORTHOGONALIZATION PROCESS WITH 80 TAPS FOR CHANNELS 4A-G

Channel	Pre Error 2	Pre Error 1	Pre Error	Post Error	Total Error
ch4a	-41.702	9.291	9.291	5.724	10.874
ch4b	-42.750	8.226	8.226	3.366	9.454
ch4c	-43.757	-4.895	-4.899	1.215	2.167
ch4d	-35.815	17.454	17.455	20.704	22.387
ch4e	-50.444	1.398	1.398	-3.105	2.716
ch4f	-50.411	1.474	1.474	-3.185	2.751
ch4g	-47.798	3.717	3.717	-10.25	3.888

7.9 RESULTS OF SIMULATION TESTS ON FURTHER MODIFICATIONS TO CHANNELS

This section carries of from the last section, trying to find out the possible reason why the filter for channel 4 collapses. From the last section it was seen that putting the roots just outside the unit circle, inside, did not improve performance.

Channel 3, did not suffer from the problems experienced by channel 4, so channel 3 is modified to test whether the distance of the roots from the unit circle has any influence on the performance of the filter design. This can be tested by adding to channel 3, a root on the unit circle and then testing the performance of the filter. Channel 3 has 26 components, with 4 roots outside the unit circle. The modification to channel 3, known as channel 3H has 27 components. The sampled impulse response of the channel is given in Table 7.9.1 and the magnitude of the roots of the channel are given in Table 7.9.4. The minimum phase channel 3H is also given in Table 7.9.1. Table 7.9.5 gives the sampled impulse response of the channel and filter as obtained by the filter implemented. From Table 7.9.8, it can be observed that by adding a root on the unit circle to channel 3, the performance deteriorates as compared to that of channel 3 with an 80 tap filter, as shown in Table 7.6.5. So the performance of the filter is degraded by placing a root on the unit circle, or for that matter, very close to the unit circle, hence a perfectly working filter can collapse.

Similarly a filter that does not work for a particular channel, should be able to operate satisfactorily, by modifying the channel in such a way as to remove the roots that are close to the unit circle. This proposed modification is tested for channel 4. The root closest to the unit circle (i.e root with magnitude 1.0818) is removed from channel 4, to give channel 4J. This channel has 29 components instead of the 30 of channel 4 and only 7 roots outside the unit circle. The channel and its minimum phase version are given in Table 7.9.2. The magnitude of the roots of channel 4J are given in Table 7.9.4. Table 7.9.6 gives the sampled impulse response of the channel and filter as obtained by the filter implemented. From Tables 7.9.6 and 7.9.8, it can be observed that by removing the root closest to the unit circle, the performance of the filter is actually improved as compared to that of channel 4 with an 80 tap filter, as shown in Table 7.6.6. The orthogonalization process being performed accurately and the sampled impulse response of the channel and filter obtained by the filter design being close to the actual minimum phase channel obtained by implementing numerical techniques written by the Numerical Algorithms Group (NAG).

Channel 4 is further modified by removing the 3 closest roots to the unit circle, i.e the roots at 1.0818, 1.1278 and 0.8729, to give channel 4L. This channel has 27 components instead of the 30 of channel 4 and only 6 roots outside the unit circle. The channel and its minimum phase version are given in Table 7.9.3. The magnitude of the roots of channel 4L are given in Table 7.9.4. Table 7.9.7 gives the sampled impulse response of the channel and filter as obtained by the filter implemented. From Tables 7.9.7 and 7.9.8, it can be observed that by removing the root closest to the unit circle, the performance of the filter is actually improved as compared to that of channel 4 with an 80 tap filter, as shown in Table 7.6.6. The error performance of the filters implemented for each of these channels is given in Table 7.9.8.

TABLE 7.9.8

ERROR IN THE MINIMUM PHASE CHANNEL OBTAINED FROM THE ADAPTIVE FILTER EMPLOYING THE GRAM SCHMIDT ORTHOGONALIZATION PROCESS WITH 80 TAPS FOR CHANNELS 4J,4L,3H

Channel	Pre Error 2	Pre Error 1	Pre Error	Post Error	Total Error
ch4j	-73.220	-208.267	-73.220	-126.400	-73.223
ch4l	-119.608	-102.310	-102.200	-98.577	-97.022
ch3h	-30.913	-295.860	-30.973	-30.460	-27.699

TABLE 7.9.1 IMPULSE RESPONSE OF TELEPHONE CHANNEL 3H AND ITS MINIMUM PHASED VERSION

Channel 3H		Minimum phased Channel 3H	
Real	Imaginary	Real	Imaginary
0.017600000000	-0.017500000000	1.000000000000	0.000000000000
0.113280545279	-0.125270710697	-0.246342394486	0.393326031514
0.268518734246	-0.197621679940	-0.130126036752	-1.060336207273
0.052988795544	-0.026031875974	0.599943901726	0.208100145213
-0.440976169559	-0.123261671308	-0.250980634799	0.098298923287
0.164558224437	-0.336423763007	0.071945853117	-0.068720034310
0.275952333048	0.302327426749	-0.014334578328	0.023492456551
-0.372550110256	-0.051405529938	0.002215663044	-0.011869395909
0.270683563264	-0.119918161992	0.006357157620	0.014361683540
-0.149397130237	0.168092821951	-0.005743445935	-0.013075759155
0.073999283166	-0.154319886035	0.000134972489	0.006684588787
-0.035146016414	0.122395262828	0.004937187776	-0.006486001972
0.016598492828	-0.091794544780	-0.007576063178	0.003836525091
-0.013445441733	0.063471644402	0.004765542495	-0.003869568868
0.014088585956	-0.041273869621	-0.003909108160	-0.000591985005
-0.017147164562	0.024822900378	0.001015191022	0.002749989912
0.014777315279	-0.013527566474	-0.001298065740	-0.003549903916
-0.011514572968	0.006416295797	-0.000109277423	0.002099699425
0.006679038108	-0.000994974880	0.000127133422	0.000356498093
-0.003426346036	-0.001419239064	0.000508554400	0.000177054118
0.001619239064	0.002326346036	-0.000824004373	0.000689689572
0.000400000000	-0.000889949761	0.000419381390	-0.000591254044
-0.000612132092	-0.000253553486	0.000397953271	0.001033018226
0.000253553486	0.001212132092	0.000031908838	0.000532527971
0.000777817669	-0.001336396275	0.000406813246	-0.000083688009
-0.000094974880	0.001294974880	0.000348317638	-0.000097056355
0.000282842789	-0.000848528367	0.000056232874	-0.000028316982

TABLE 7.9.2 IMPULSE RESPONSE OF TELEPHONE CHANNEL 4J AND ITS MINIMUM PHASED VERSION

Channel 4J		Minimum phased Channel 4J	
Real	Imaginary	Real	Imaginary
-0.003800000000	-0.004900000000	1.000000000000	0.000000000000
0.006687578479	0.002231444833	-0.390525407195	1.309878408122
0.006167826424	0.013791711986	-0.592935747443	-0.774711166333
-0.082177009874	0.020386519406	0.532651531883	0.097919497908
-0.036583369381	-0.237628797778	-0.237708613670	0.087075319941
0.395459043508	-0.019760322672	0.094143174137	-0.041078614650
-0.119833817871	0.285822395012	-0.057774829616	0.001589778239
0.027381661446	-0.208151590784	0.020991090041	0.000666142892
0.049524443335	0.095203447441	0.003155075410	0.004045327489
-0.027419787559	-0.066977798886	-0.013798257318	-0.005913504279
0.001391210927	0.019509502625	0.004488409626	0.010103154434
-0.018033167974	0.001373322335	0.005924986620	-0.013275547267
0.018912713951	0.010822116133	-0.012816224055	0.004570190471
-0.000201855243	-0.015298154010	0.008537308410	-0.001874196981
-0.011049551256	0.006456108608	-0.000183242966	0.000132214663
0.007696584616	0.003957244277	-0.002940412847	0.005205716015
-0.000830200892	-0.005785550695	-0.000620845427	-0.004427784469
-0.004819677606	0.003461693593	0.003158251534	0.002058625902
0.004106232156	-0.001398531136	-0.001917303010	0.000354629591
-0.001256331025	-0.000278982372	0.000592711636	0.001462684987
-0.000082815244	0.002093325479	-0.000468406048	-0.001248148106
0.000303714940	-0.001794655453	0.000683298783	-0.000541474851
0.000265983567	0.000898858907	0.001527825625	0.000825933711
0.000206900018	-0.000378326668	-0.000865354909	0.000443334457
0.000249089559	0.000619576065	-0.000157063616	-0.000451802045
-0.000399583969	-0.000756997389	0.000392562296	0.000099739883
-0.000096085239	0.000077430597	-0.000084672265	-0.000038265186
0.000732324485	0.000017680628	-0.000038103179	0.000060634026
-0.000331743981	-0.000187611491	0.000024006735	-0.000010050336

TABLE 7.9.3 IMPULSE RESPONSE OF TELEPHONE CHANNEL 4L AND ITS MINIMUM PHASED VERSION

Channel 4L		Minimum phased Channel 4L	
Real	Imaginary	Real	Imaginary
-0.003800000000	-0.004900000000	1.000000000000	0.000000000000
0.003861754302	0.013803408183	-1.486870819230	0.025915822700
0.016971761376	-0.005552999152	1.188204403243	0.340984051256
-0.096807122519	-0.000187168902	-0.487461073353	-0.661051336198
0.076000445047	-0.109174333590	-0.151302022696	0.568894905853
0.125752461639	0.090628037093	0.427377893053	-0.175453645315
-0.228723292091	-0.101918234537	-0.334226776884	-0.172064415875
0.254556897932	0.148848354283	0.082596559183	0.270849818872
-0.164367047173	-0.258246986312	0.089383879265	-0.163533431178
-0.036323391275	0.263518934897	-0.104845369493	0.027315610084
0.165197433205	-0.119634579892	0.039932029392	0.027141987370
-0.144456258924	-0.029223868322	0.005546217765	-0.010883113370
0.055990658249	0.084334063225	-0.007405662466	-0.017970666022
0.009615914493	-0.064132555276	-0.014089536129	0.021812305119
-0.028543801609	0.026813389411	0.028650834569	-0.002141902378
0.021179484016	-0.004133680878	-0.022074691757	-0.015886173721
-0.010550393871	-0.000904614175	0.004918253413	0.019162566802
0.005582051391	-0.000818096567	0.007618500080	-0.010248813802
-0.006257169916	0.001315325860	-0.008190882397	0.000302805003
0.008241613331	0.002004159595	0.003018486862	0.004612786934
-0.006578590029	-0.006410844619	0.001412181242	-0.003879465888
0.001711834991	0.008042882007	-0.001961952680	0.000132605179
0.003091335427	-0.005849237128	0.001047649399	0.001662573087
-0.004202463622	0.001504276899	-0.000008833114	-0.001208084160
0.002306627268	0.001257030941	-0.000303340047	0.000278334841
-0.000432420148	-0.001414084115	0.000157305427	0.000048580638
-0.000137093542	0.000362052200	-0.000017552720	-0.000028680594

TABLE 7.9.4 ROOTS OF TELEPHONE CHANNELS 4J, 4L AND 3H

CH4J	CH4L	CH3H
2.742692626533	2.742692744822	4.339639718647
1.894402888617	1.894402706740	1.834607969330
2.892938238137	2.892937784885	2.203330423462
1.582442890597	1.582442800287	1.000000269994
1.434007185848	1.434007668436	1.363847962433
0.872898756408	0.695374852177	0.762663536208
0.726286863366	0.715088618273	0.803682154196
1.263295508127	1.263295504240	0.715041369461
0.719702042380	0.749650016691	0.753077471717
0.739670399890	0.757998126571	0.702547203044
1.127796371018	0.726197257530	0.749740207130
0.749649899426	0.739671292309	0.682059613216
0.757998029209	0.734647614315	0.757415830478
0.761492071310	0.733855499267	0.704232548761
0.721291160386	0.768053901760	0.940870218461
0.768053554711	0.742782256689	0.747574523457
0.762959625558	0.758136177394	0.704529542395
0.695374506807	0.726287883467	0.739600138430
0.720240310751	0.719702605318	0.661688011873
0.758135741573	0.761491606444	0.743144087257
0.726196594073	0.678442781017	0.719687392268
0.749869784102	0.762959854392	0.750406720007
0.742782182612	0.749869308829	0.695417694786
0.734646954896	0.721291552516	0.709335691932
0.715087427592	0.720241330512	0.695364882718
0.678442311691	0.510077829484	0.713351711740
0.733856547255	0.000000000000	0.000000000000
0.510081256999		
0.000000000000		

TABLE 7.9.5 SAMPLED IMPULSE RESPONSE OF CHANNEL AND FILTER OBTAINED FROM FILTER WITH 80 TAPS EMPLOYING GRAM SCHMIDT ORTHOGONALIZATION PROCESS FOR TELEPHONE CHANNEL 3H

1 ITERATION					
YZ_i	REAL	IMAGINARY	YZ_i	REAL	IMAGINARY
79	-0.000000964291	-0.000000853443	26	0.000000000000	0.000000000000
78	-0.000006663130	-0.000004624279	25	0.000000000000	0.000000000000
77	-0.000009346552	-0.000009539687	24	0.000000000000	0.000000000000
76	-0.000002384435	-0.000001955707	23	0.000000000000	0.000000000000
75	-0.000010020115	0.000014098054	22	0.000000000000	0.000000000000
74	-0.000018815988	-0.000009022675	21	0.000000000000	0.000000000000
73	-0.000000473826	-0.000002488421	20	0.000000000000	0.000000000000
72	0.000000622932	-0.000002942121	19	0.000000000000	0.000000000000
71	-0.000010789401	0.000004268481	18	0.000000000000	0.000000000000
70	-0.000009073680	0.000043774028	17	0.000000000000	0.000000000000
69	0.000015800635	-0.000051041745	16	0.000000000000	0.000000000000
68	-0.000027176426	0.000020893467	15	0.000000000000	0.000000000000
67	0.000100384551	0.000036654068	14	0.000000000000	0.000000000000
66	-0.000064113976	-0.000091517882	13	0.000000000000	0.000000000000
65	0.000105295879	0.000137452607	12	0.000000000000	0.000000000000
64	0.000053850407	-0.000131618501	11	0.000000000000	0.000000000000
63	-0.000162559651	0.000169103351	10	0.000000000000	0.000000000000
62	0.000278433516	-0.000165135436	9	0.000000000000	0.000000000000
61	-0.000247585950	0.000053148742	8	0.000000000000	0.000000000000
60	0.000080733656	0.000113451834	7	0.000000000000	0.000000000000
59	0.000078744122	0.000088451705	6	0.000000000000	0.000000000000
58	-0.001175778411	-0.001150636631	5	0.000000000000	0.000000000000
57	0.004295199875	0.001752318690	4	0.000000000000	0.000000000000
56	-0.010241988033	0.003428624125	3	0.000000000000	0.000000000000
55	-0.001943385797	-0.019683807156	2	0.000000000000	0.000000000000
54	0.011191591265	-0.011898916782	1	0.000000000000	0.000000000000
53	0.000000000000	0.000000000000	0	1.000000000000	0.000000000000
52	0.000000000000	0.000000000000	-1	-0.233765939355	0.405861704688
51	0.000000000000	0.000000000000	-2	-0.138119544587	-1.040579214079
50	0.000000000000	0.000000000000	-3	0.591983787125	0.201302607963
49	0.000000000000	0.000000000000	-4	-0.246795661917	0.098092741725
48	0.000000000000	0.000000000000	-5	0.070624561978	-0.067838184979
47	0.000000000000	0.000000000000	-6	-0.014122035609	0.023220280416
46	0.000000000000	0.000000000000	-7	0.002090957820	-0.011788826256
45	0.000000000000	0.000000000000	-8	0.006382785988	0.014197906618
44	0.000000000000	0.000000000000	-9	-0.005708605999	-0.012901126887
43	0.000000000000	0.000000000000	-10	0.000130898188	0.006587121369
42	0.000000000000	0.000000000000	-11	0.004915979116	-0.006467108553
41	0.000000000000	0.000000000000	-12	-0.007455412182	0.003813828254
40	0.000000000000	0.000000000000	-13	0.004719298618	-0.003848007747
39	0.000000000000	0.000000000000	-14	-0.003846283826	-0.000596549697
38	0.000000000000	0.000000000000	-15	0.001020696708	0.002732788306
37	0.000000000000	0.000000000000	-16	-0.001304286224	-0.003509656192
36	0.000000000000	0.000000000000	-17	-0.000113292347	0.002062314625
35	0.000000000000	0.000000000000	-18	0.000122732714	0.000352152093
34	0.000000000000	0.000000000000	-19	0.000505536668	0.000177179425
33	0.000000000000	0.000000000000	-20	-0.000821898464	0.000696132585
32	0.000000000000	0.000000000000	-21	0.000397106260	-0.000586774862
31	0.000000000000	0.000000000000	-22	0.000391822678	0.001018169397
30	0.000000000000	0.000000000000	-23	0.000030091236	0.000535748966
29	0.000000000000	0.000000000000	-24	0.000396899650	-0.000075577605
28	0.000000000000	0.000000000000	-25	0.000341734561	-0.000094255522
27	0.000000000000	0.000000000000	-26	0.000055226044	-0.000027809976

TABLE 7.9.6 SAMPLED IMPULSE RESPONSE OF THE CHANNEL AND FILTER OBTAINED FROM FILTER WITH 80 TAPS EMPLOYING GRAM SCHMIDT ORTHOGONALIZATION PROCESS FOR TELEPHONE CHANNEL 4J

1 ITERATION					
YZ_i	REAL	IMAGINARY	YZ_i	REAL	IMAGINARY
79	0.000000002912	-0.000000002072	25	0.000000000000	0.000000000000
78	-0.000000012783	-0.000000003694	24	0.000000000000	0.000000000000
77	-0.000000005020	0.000000031827	23	0.000000000000	0.000000000000
76	0.000000062959	-0.000000057807	22	0.000000000000	0.000000000000
75	-0.000000064438	-0.000000024604	21	0.000000000000	0.000000000000
74	-0.000000084735	0.000000201098	20	0.000000000000	0.000000000000
73	0.000000245674	-0.000000250102	19	0.000000000000	0.000000000000
72	-0.000000315018	-0.000000261412	18	0.000000000000	0.000000000000
71	0.000000023481	0.000000655847	17	0.000000000000	0.000000000000
70	0.000000632188	-0.000000565001	16	0.000000000000	0.000000000000
69	-0.000001289969	-0.000000222066	15	0.000000000000	0.000000000000
68	0.000000553045	0.000001223192	14	0.000000000000	0.000000000000
67	0.000001316887	-0.000001366470	13	0.000000000000	0.000000000000
66	-0.000002766511	-0.000000046819	12	0.000000000000	0.000000000000
65	0.000001243873	0.000002391284	11	0.000000000000	0.000000000000
64	0.000002366760	-0.000002791665	10	0.000000000000	0.000000000000
63	-0.000004740841	-0.000000796945	9	-0.000000000001	0.000000000000
62	0.000001961968	0.000006771056	8	0.000000000000	-0.000000000001
61	0.000008188156	-0.000007243745	7	0.000000000001	0.000000000000
60	-0.000015683514	-0.000004310484	6	0.000000000001	0.000000000001
59	0.000006229448	0.000022166014	5	-0.000000000001	0.000000000002
58	0.000020480390	-0.000026178346	4	-0.000000000004	-0.000000000001
57	-0.000044502640	0.000003553363	3	0.000000000000	-0.000000000009
56	0.000043366317	0.000036866090	2	0.000000000018	-0.000000000005
55	-0.000014175964	-0.000052377389	1	0.000000000009	0.000000000031
54	-0.000027596102	0.000001003781	0	1.000000000000	0.000000000000
53	0.000077825012	0.000091942666	-1	-0.390525581512	1.309878089754
52	-0.000119644014	-0.000087821593	-2	-0.592935496600	-0.774711055902
51	0.000000000000	0.000000000000	-3	0.532651439635	0.097919505367
50	0.000000000000	0.000000000000	-4	-0.237708608918	0.087075262752
49	0.000000000000	0.000000000000	-5	0.094143191173	-0.041078571440
48	0.000000000000	0.000000000000	-6	-0.057774826448	0.001589749867
47	0.000000000000	0.000000000000	-7	0.020991068828	0.000666139668
46	0.000000000000	0.000000000000	-8	0.003155065753	0.004045335971
45	0.000000000000	0.000000000000	-9	-0.013798252501	-0.005913520078
44	0.000000000000	0.000000000000	-10	0.004488383209	0.010103169381
43	0.000000000000	0.000000000000	-11	0.005924998469	-0.013275545338
42	0.000000000000	0.000000000000	-12	-0.012816238636	0.004570166563
41	0.000000000000	0.000000000000	-13	0.008537299984	-0.001874207131
40	0.000000000000	0.000000000000	-14	-0.000183246483	0.000132213430
39	0.000000000000	0.000000000000	-15	-0.002940414096	0.005205736166
38	0.000000000000	0.000000000000	-16	-0.000620850902	-0.004427797292
37	0.000000000000	0.000000000000	-17	0.003158280286	0.002058661360
36	0.000000000000	0.000000000000	-18	-0.001917323083	0.000354613350
35	0.000000000000	0.000000000000	-19	0.000592730691	0.001462699964
34	0.000000000000	0.000000000000	-20	-0.000468412624	-0.001248150555
33	0.000000000000	0.000000000000	-21	0.000683291679	-0.000541486536
32	0.000000000000	0.000000000000	-22	0.001527838857	0.000825933496
31	0.000000000000	0.000000000000	-23	-0.000865360639	0.000443334678
30	0.000000000000	0.000000000000	-24	-0.000157064338	-0.000451806762
29	0.000000000000	0.000000000000	-25	0.000392566369	0.000099741450
28	0.000000000000	0.000000000000	-26	-0.000084672834	-0.000038265545
27	0.000000000000	0.000000000000	-27	-0.000038103461	0.000060634462
26	0.000000000000	0.000000000000	-28	0.000024006973	-0.000010050485

TABLE 7.9.7 SAMPLED IMPULSE RESPONSE OF CHANNEL AND FILTER OBTAINED FROM FILTER WITH 80 TAPS EMPLOYING GRAM SCHMIDT ORTHOGONALIZATION PROCESS FOR TELEPHONE CHANNEL 4L

1 ITERATION					
YZ_i	REAL	IMAGINARY	YZ_i	REAL	IMAGINARY
i 79	0.000000000000	0.000000000015	26	0.000000000000	0.000000000000
78	0.000000000067	-0.000000000057	25	0.000000000000	0.000000000000
77	-0.000000000261	-0.000000000107	24	0.000000000000	-0.000000000001
76	0.000000000172	0.000000000789	23	0.000000000000	0.000000000001
75	0.000000000927	-0.000000001160	22	-0.000000000001	-0.000000000002
74	-0.000000002112	-0.000000000610	21	0.000000000002	0.000000000002
73	0.000000000558	0.000000003620	20	-0.000000000004	0.000000000001
72	0.000000004599	-0.000000002988	19	0.000000000004	-0.000000000003
71	-0.000000007057	-0.000000004261	18	-0.000000000003	-0.000000000004
70	-0.000000000067	0.000000011317	17	0.000000000022	0.000000000022
69	0.000000011653	-0.000000008640	16	-0.000000000089	0.000000000013
68	-0.000000015573	-0.000000002997	15	0.000000000082	-0.000000000174
67	0.000000006643	0.000000012123	14	0.000000000184	0.000000000232
66	0.000000006588	-0.000000010200	13	-0.000000000558	0.000000000140
65	-0.000000013004	-0.000000005195	12	0.000000000113	-0.000000000632
64	-0.000000005139	0.000000027270	11	0.000000000796	-0.000000000068
63	0.000000048627	-0.000000027203	10	-0.000000000488	0.000000001310
62	-0.000000092977	-0.000000031870	9	-0.000000000016	-0.000000002047
61	0.000000052340	0.000000147817	8	-0.000000000069	0.000000003884
60	0.000000112936	-0.000000175731	7	-0.000000008917	-0.000000008442
59	-0.000000258795	-0.000000005569	6	0.000000019725	-0.000000020450
58	0.000000126648	0.000000285986	5	0.000000075965	0.000000054002
57	0.000000272968	-0.000000279860	4	-0.000000196842	0.000000193057
56	-0.000000513946	-0.000000068224	3	-0.000000443433	-0.000000931440
55	0.000000397250	0.000000389737	2	0.000003099533	0.000001267991
54	-0.000000097781	-0.000000325135	1	-0.000006802710	-0.000000118575
53	0.000000000000	0.000000000000	0	1.000000000000	0.000000000000
52	0.000000000000	0.000000000000	-1	-1.486864987358	0.025914130514
51	0.000000000000	0.000000000000	-2	1.188197562964	0.340984233397
50	0.000000000000	0.000000000000	-3	-0.487456765589	-0.661048646415
49	0.000000000000	0.000000000000	-4	-0.151302641931	0.568891266781
48	0.000000000000	0.000000000000	-5	0.427376098109	-0.175451637849
47	0.000000000000	0.000000000000	-6	-0.334224704366	-0.172064255455
46	0.000000000000	0.000000000000	-7	0.082595666774	0.270848579908
45	0.000000000000	0.000000000000	-8	0.089383656039	-0.163532407660
44	0.000000000000	0.000000000000	-9	-0.104844788725	0.027315379092
43	0.000000000000	0.000000000000	-10	0.039931742653	0.027141767649
42	0.000000000000	0.000000000000	-11	0.005546150884	-0.010882969847
41	0.000000000000	0.000000000000	-12	-0.007405515572	-0.017970609885
40	0.000000000000	0.000000000000	-13	-0.014089574696	0.021812156669
39	0.000000000000	0.000000000000	-14	0.028650722172	-0.002141816537
38	0.000000000000	0.000000000000	-15	-0.022074574971	-0.015886149621
37	0.000000000000	0.000000000000	-16	0.004918205608	0.019162499335
36	0.000000000000	0.000000000000	-17	0.007618494813	-0.010248744407
35	0.000000000000	0.000000000000	-18	-0.008190848624	0.000302803644
34	0.000000000000	0.000000000000	-19	0.003018462747	0.004612771643
33	0.000000000000	0.000000000000	-20	0.001412178426	-0.003879447302
32	0.000000000000	0.000000000000	-21	-0.001961952846	0.000132603030
31	0.000000000000	0.000000000000	-22	0.001047644329	0.001662562357
30	0.000000000000	0.000000000000	-23	-0.000008829740	-0.001208081093
29	0.000000000000	0.000000000000	-24	-0.000303339999	0.000278334529
28	0.000000000000	0.000000000000	-25	0.000157305032	0.000048580881
27	0.000000000000	0.000000000000	-26	-0.000017552559	-0.000028680753

From the simulation tests carried out in this chapter it can not be conclusively stated but nonetheless we can draw a parallel between the error performance of the filter for a specific channel and the position of the roots of that channel. This suggests that the performance of the filter improves as the roots are further away form the unit circle.

CHAPTER 8

COMMENTS ON THE RESEARCH PROJECT

8.1 CONCLUSION

In this thesis several techniques for estimating the sampled impulse response of HF radio links over a serial data transmission system operating at 2400 Bauds/s were investigated. The channel estimators were based on the conventional gradient estimator design.

The estimation of the sampled impulse response of a time varying HF radio link could be performed to an acceptable degree of accuracy by employing the simple estimator. The simple estimator is a modification of the conventional gradient estimator, with prediction techniques to enhance its performance.

The adaptive estimator proved to be potentially the most cost effective of all the estimators considered in this thesis, as its performance is considerably better than the simple estimator, even at low signal-to-noise ratios, and this improvement is obtained without an undue increase in complexity. The adaptive estimator with degree-2 least squares fading memory prediction (system 2) was observed to be unstable and hence was modified. This resulted in system 3, which proved to be a more stable design.

The most significant result obtained is that the modified simple estimator (chapter 6) shows no improvement over the simple estimator. Thus confirming the simple estimator to be relatively cost effective.

The different measures of obtaining the performance of an estimator, namely, the mean square estimation error and the mean square normalized estimation error, did not affect the relative performance of the estimators. From the results of the simulation tests carried out on the estimators, it can be observed that the degree-1 least squares fading memory prediction is the most stable and, therefore, potentially the most suitable prediction mechanism for the proposed application.

This thesis is also concerned with the adjustment of a linear filter ahead of the detector, which makes the sampled impulse response of the channel a minimum phase. This linear filter has been tested over telephone channels for a 16-QAM data transmission system operating at 9600 bits/s. From the computer simulation results, it was observed, for some channels with severe amplitude and phase distortion, the implementation of the filter did suffer from numerical inaccuracies which had been introduced by the orthogonalization process. The nearness of the roots of a channel to the unit circle had a detrimental effect on the performance of the filter.

8.2 SUGGESTIONS FOR FURTHER INVESTIGATIONS

The adaptive estimator proved to be the most cost effective estimator studied. This stems largely from the fact that the step size of the gradient algorithm was adjusted in accordance with the magnitude of the channel component. There is a need however to further investigate the relationships by which the step sizes are adjusted. This should offer an improved performance as well as aid in the design of such a system in reality.

Attempts should be made to employ these estimation techniques for use with mobile radio communications.

This thesis was also concerned with the development of a novel technique for the adjustment of a linear filter ahead of a detector for a 9600 bits/s transmission system over telephone channels which are basically time-invariant.

Further possible work would be to modify the filter algorithm so as to be able to handle the roots closer to the unit circle, and then to try and apply this improved technique to time-varying channels such as HF radio links, where it is well known that channels can vary rapidly with time, and therefore, a more robust filter design is required.

APPENDIX A

DERIVATION OF RAYLEIGH FADING FILTER

A single Rayleigh fading propagation is modelled as in figure 2.6.2, where $q_1(t)$ and $q_2(t)$ are two Gaussian random processes with zero mean and the same variance. The shape of their power spectrum is Gaussian having the same rms frequency, f_{rms}. The power spectrum of $q_1(t)$ and $q_2(t)$ are given by, equation 2.6.1, as

$$Q_1(f)^2 = Q_2(f)^2 = \exp\left(-\frac{f^2}{2f_{rms}^2}\right) \qquad (A.1)$$

As is shown in figure 2.6.4, the random process $q_h(t)$ is generated by filtering a zero mean white Gaussian noise signal $v_h(t)$. The filter used in figure 2.6.4 has a Gaussian frequency response and is given by, equation 2.6.5, as

$$F(f) = \exp\left(-\frac{f^2}{4f_{rms}^2}\right) \qquad (A.2)$$

and the 3 dB cut-off frequency of the filter is

$$f_c = 1.17741 f_{rms} \qquad (A.3)$$

The rms frequency, f_{rms} and the frequency spread, f_{sp}, are related as follows

$$f_{sp} = 2f_{rms} \qquad (A.4)$$

Therefore, from equations A.3 and A.4,

$$f_c = 0.588705 f_{sp} \tag{A.5}$$

The impulse response and the magnitude response of a Bessel filter tends toward Gaussian as the order of the filter is increased [23]. A Bessel filter has, therefore, been used to obtain the Rayleigh fading filter in figure 2.6.4.

The Bessel filter has a transfer function of the form [23-25]

$$H(s) = \frac{d_0}{B_n(s)} \tag{A.6}$$

where $B_n(s)$ is the n^{th} order Bessel polynomial and d_0 is a normalizing constant of the form

$$d_0 = \frac{(2n)!}{2^2 n!} \tag{A.7}$$

$B_n(s)$ can be expressed in the form [32]

$$B_n(s) = \sum_{k=0}^{n} d_k s^k \qquad A(.8)$$

where

$$d_k = \frac{(2n-k)!}{2^{n-k} k!(n-k)!} \quad k = 0, 1, \ldots, n \tag{A.9}$$

A 5^{th} order Bessel filter has been chosen as a practical choice and thus n=5. Figure 2.7.1 compares the frequency response of this filter with that of the desired theoretical frequency response (Gaussian). It can be seen that a 5^{th} order Bessel filter has a frequency response that is Gaussian, at least in the range of interest.

Equation A.6 becomes

$$H(s)=\frac{945}{s^5+15s^4+105s^3+420s^2+945s+945} \qquad (A.10)$$

Factorizing the denominator in equation A.10 yield,

$$H(s)=\frac{945}{\prod_{i=1}^{5}(s-P_i)} \qquad (A.11)$$

where P_i are known as the poles of H(s) and are given by [24]

$$P_1=-3.64674$$

$$P_2,P_3=-3.35196\pm j1.74266$$

$$P_4,P_5=-2.32467\pm j3.57102 \qquad (A.12)$$

substituting $s=j\Omega$, in equation A.11, the frequency response of the Bessel filter is

$$H(j\omega)=\frac{945}{\prod_{i=1}^{5}(j\omega-P_i)} \qquad (A.13)$$

where, Ω is the angular frequency and $j=\sqrt{-1}$. When $\omega=\Omega_c$ rad/sec, the amplitude response of the 5^{th} order Bessel filter, drops by 3 dB from its peak value, Ω_c is called the 3 dB cut-off frequency and is given by [29]

$$\Omega_c=2.4274 rad/\sec \qquad (A.14)$$

One of the parameter of importance in the characterization of a channel is the frequency spread, f_{sp}. Therefore, it is desirable to express the cut-off frequency of the Bessel filter in terms of the frequency spread.

Let

$$\omega = C_0 \omega \qquad (A.15)$$

where

$$C_0 = \frac{\omega_c}{\Omega_c} = \frac{2\pi f_c}{\Omega_c} \qquad (A.16)$$

where f_c, from equation A.3, is the cut-off frequency of the desired filter.

Therefore, form equations A.14 and A.16,

$$C_0 = 2.58844 f_c \qquad (A.17)$$

Substituting the value of ω , from equation A.15, in equation A.13

$$H(j\omega) = \frac{945}{\prod_{i=1}^{5}\left(j\frac{\omega}{\Omega} - P_i\right)} \qquad (A.18)$$

Let

$$P_i^{\prime} = C_0 P_i \qquad (A.19)$$

Then, from equations A.18 and A.19,

$$H(j\omega) = \frac{945 C_0^5}{\prod_{i=1}^{5}(j\omega - P_i^{\prime})} \qquad (A.20)$$

and, from equations A.17 and A.20,

$$H(j\omega)=\frac{109805.0518f_c^5}{\prod_{i=1}^{5}(j\omega-P_i')} \qquad (A.21)$$

Equation A.21 can be expressed as,

$$H(j\omega)=\frac{d_0'}{\prod_{i=1}^{5}(s-P_i')} \qquad (A.22)$$

where, in equation A.22,

$$s=j\omega$$
$$d_0'=109805.0518f_c^5$$
$$P_i'=2.58844f_cP_i for\, i=1,2,..,5$$
$$(A.23)$$

Table A.1 summarizes all the parameters of the Bessel filter for a frequency spread of 2 Hz.

Equation A.22 is the transfer function of a 5[th] order Bessel filter. The analog filter is to be digitized for use in computer simulation. The method used for this is called the impulse invariant transformation method [23]. The important feature of this transformation is that, the impulse response of the resulting digital filter is a sampled version of the impulse response of the analog filter. in this technique the poles , in the s-plane, of equation A.22 are transformed to poles, in the z-plane [23], where T is the sampling interval.

Therefore, using the impulse invariant transformation method, Equation A.22 can be written as

$$H(z)=\frac{K}{\prod_{i=1}^{5}\left(1-e^{P_i T}z^{-1}\right)}$$

$$=\frac{K}{\prod_{i=1}^{5}(1-q_i z^{-1})} \qquad (A.24)$$

where in equation A.24, K is the D.C gain of the filter, q_i are the poles and are equal to

$$q_i = e^{P_i T} \qquad (A.25)$$

The z-plane poles obtained from equation A.24 for a frequency spread of 2 Hz are

$$q_1 = 0.8948$$

$$q_2, q_3 = 0.9016 \pm j0.0479$$

$$q_4, q_5 = 0.9261 \pm j0.1012 \qquad (A.26)$$

The digital filter is implemented as shown in figure 2.6.5 [26-30]. It comprises of a cascade of two 2-pole sections and a single pole section. Each of the 2-pole sections have complex conjugate poles and the single pole section has a real pole. The transfer function of the filter in figure 2.6.5 is, therefore,

$$H(z)=\frac{K}{(1-q_1z^{-1})\,\{(1-q_2z^{-1})(1-q_3z^{-1})\}\,\{(1-q_4z^{-1})(1-q_5z^{-1})\}}$$

(A.27)

$$=\frac{K}{(1-q_1z^{-1})\,\{(1-(q_2+q_3)z^{-1}+(q_2q_3)z^{-2})\}\,\{(1-(q_4+q_5)z^{-1}+(q_4q_5)z^{-2})\}}$$

(A.28)

where q_2, q_3 and q_4, q_5 are complex conjugate pairs. Therefore, from equation A.28 an figure 2.6.5, the filter coefficients $\{C_i\}$ are given by

$$C_1 = -q_1$$
$$C_2 = -(q_2 + q_3)$$
$$C_3 = q_2 q_3$$
$$C_4 = -(q_4 + q_5)$$
$$C_5 = q_4 q_5 \qquad (A.29)$$

The filter co-efficients obtained for a frequency spread of 2 Hz are listed in Table 2.6.2. The value of K, called the gain of the filter, in equation A.28, is chosen such that the $\{q_h(t)\}$ have a variance corresponding to $1/2n_s$, where n_s represents the number of skywaves. This ensures that the mean length of the channel sampled impulse response vector is equal to unity. The value of K used in the tests was 15822.

TABLE A.1 FIFTH ORDER ANALOG BESSEL FILTER FOR A FREQUENCY SPREAD OF 2 Hz

Frequency spread, f_{sp} (Hz)	2
Cut-off frequency, f_c (Hz)	1.1774
Constant $d_0^{/}$	248451.99
Filter poles in the s-plane $P_1^{/}$ $P_2^{/}, P_3^{/}$ $P_4^{/}, P_5^{/}$	 $-11.1139 + j\,0$ $-10.2155 \pm j\,5.3110$ $-7.0847 \pm j\,10.8831$

APPENDIX B

TRANSMITTER AND RECEIVER FILTER

Figure 3.3.3 shows the frequency characteristics of the combined equipment and radio filter. In order to obtain different sampling phases, the filter sampled impulse response has been obtained at a sampling rate that is 20 times higher than the original sampling rate (i.e at 96000 samples/sec). The oversampled transmitter and receiver filter impulse responses are given in table B.1 [6]. The two filters whose sampled impulse responses are $a_{1,k}$ and $a_{2,k}$, are required at 4800 samples/sec. This is obtained by taking every 20^{th} sample from the oversampled version. $a_{1,k}$ has (-0.1795896 + j 2.3539405) as its first sample and the other samples are obtained by taking every 20^{th} sample from the first sample.

$a_{2,k}$ is delayed 1.1 msecs with respect to $a_{1,k}$. This delay can be expressed as a fraction of the number of samples, p'

$$p' = \frac{1.1 \times 10^{-3}}{(1/4800)} = 5.28$$

The first sample of $a_{2,k}$ is delayed by 5.28 samples with respect to the first sample of $a_{1,k}$. It is however necessary to obtain the samples of the delayed filter at the sampling instants of the non delayed filter. This delay can be expressed as a whole number of samples and a fractional part (i.e 5 + 0.28). As the sampled response of the delayed filter would not be available at the 5^{th} sample of the non-delayed filter, the first component of $a_{2,k}$ is added to the (5+1) 6^{th} component of $a_{1,k}$. This leaves a discrepancy of (6-5.28) 0.72 sampling intervals. This discrepancy is taken into account by choosing from the oversampled version, the sample that is (0.72 x 96000/4800 = 14.4) 14 samples ahead of (-0.1995896 + j 2.3539405). Thus the first sample of $a_{2,k}$

is (-1.6694374 + j 13.2372707). The remaining samples of $a_{2,k}$ are obtained by choosing every 20^{th} sample from the oversampled version. The oversampled version of the receiver filter has been obtained at a different sampling phase to the transmitter filter. The first sample of the receiver filter impulse response is (-1.9417691 + j 1.3625952).

TABLE B . 1. - OVERSAMPLED TRANSMITTER & RECEIVER FILTER

(A) - IN PHASE RESPONSE OF TRANSMITTER FILTER SAMPLED AT 96000 SAMPLES/SEC.

-0.0002096	-0.0002887	-0.0003396	-0.0003572	-0.0003399
-0.0002893	-0.0002103	-0.0001106	0.0000000	0.0001107
0.0002107	0.0002901	0.0003411	0.0003588	0.0003413
0.0002904	0.0002111	0.0001110	0.0000000	-0.0001110
-0.0002113	-0.0002908	-0.0003419	-0.0003596	-0.0003420
-0.0002909	-0.0002114	-0.0001111	0.0000000	-0.0001111
0.0002113	0.0002908	0.0003418	0.0003593	0.0003416
0.0002905	0.0002110	0.0001109	0.0000000	-0.0001108
-0.0002106	-0.0002897	-0.0003404	-0.0003577	-0.0003399
-0.0002889	-0.0002098	-0.0001102	0.0000000	0.0001100
0.0002089	0.0002873	0.0003373	0.0003542	0.0003364
0.0002858	0.0002073	0.0001088	0.0000000	-0.0001085
-0.0002059	-0.0002829	-0.0003319	-0.0003483	-0.0003305
-0.0002805	-0.0002033	-0.0001066	0.0000000	0.0001060
0.0002011	0.0002760	0.0003234	0.0003389	0.0003212
0.0002722	0.0001970	0.0001032	0.0000000	-0.0001023
-0.0001937	-0.0002653	-0.0003103	-0.0003246	-0.0003070
-0.0002596	-0.0001875	-0.0000979	0.0000000	0.0000966
0.0001825	0.0002492	0.0002906	0.0003030	0.0002856
0.0002407	0.0001731	0.0000901	0.0000000	-0.0000881
-0.0001655	-0.0002249	-0.0002608	-0.0002703	-0.0002531
-0.0002118	-0.0001513	-0.0000781	0.0000000	0.0000750
0.0001394	0.0001873	0.0002146	0.0002194	0.0002024
0.0001666	0.0001168	0.0000591	0.0000000	-0.0000540
-0.0000975	-0.0001267	-0.0001395	-0.0001362	-0.0001190
-0.0000918	-0.0000594	-0.0000272	0.0000000	0.0000182
0.0000253	0.0000212	0.0000077	-0.0000113	-0.0000305
-0.0000440	-0.0000461	-0.0000322	0.0000000	0.0000502
0.0001148	0.0001868	0.0002566	0.0003124	0.0003414
0.0003319	0.0002744	0.0001636	0.0000000	-0.0002091
-0.0004487	-0.0006966	-0.0009238	-0.0010966	-0.0011788
-0.0011355	-0.0009363	-0.0005602	0.0000000	0.0007335
0.0016070	0.0025614	0.0035082	0.0043277	0.0048672
0.0049406	0.0043280	0.0027763	0.0000000	-0.0043177
-0.0105240	-0.0189960	-0.0301401	-0.0443918	-0.0622160
-0.0841076	-0.1105917	-0.1422238	-0.1795896	-0.2233037
-0.2740073	-0.3323647	-0.3990578	-0.4747796	-0.5602256
-0.6560838	-0.7630237	-0.8816842	-1.0126609	-1.1564934
-1.3136537	-1.4845339	-1.6694374	-1.8685694	-2.0820312
-2.3098150	-2.5518017	-2.8077597	-3.0773455	-3.3601064
-3.6554823	-3.9628099	-4.2813263	-4.6101723	-4.9483961
-5.2949561	-5.6487240	-6.0084869	-6.3729500	-6.7407393
-7.1104051	-7.4804261	-7.8492148	-8.2151244	-8.5764574
-8.9314755	-9.2784116	-9.6154822	-9.9409021	-10.2528991
-10.5497287	-10.8296895	-11.0911373	-11.3324982	-11.5522808
-11.7490864	-11.9216180	-12.0686870	-12.1892192	-12.2822589
-12.3469721	-12.3826494	-12.3887079	-12.3646935	-12.3102829
-12.2252865	-12.1096509	-11.9634618	-11.7869473	-11.5804806
-11.3445819	-11.0799204	-10.7873130	-10.4677274	-10.1222704
-9.7521911	-9.3588701	-8.9438114	-8.5086316	-8.0550490
-7.5848703	-7.0999776	-6.6023157	-6.0938784	-5.5766967
-5.0528272	-4.5243415	-3.9933167	-3.4618271	-2.9319357
-2.4056875	-1.8851016	-1.3721641	-0.8688202	-0.3769656
0.1015622	0.5649945	1.0116413	1.4399026	1.8482796
2.2353854	2.5999552	2.9408554	3.2570916	3.5478141
3.8123224	4.0500667	4.2606477	4.4438154	4.5994647
4.7276308	4.8284836	4.9023211	4.9495636	4.9707467

4.9665164	4.9376234	4.8849186	4.8093491	4.7119541
4.5938614	4.4562826	4.3005084	4.1279029	3.9398962
3.7379757	3.5236767	3.2985708	3.0642536	2.8223321
2.5744110	2.3220794	2.0668985	1.8103897	1.5540244
1.2992157	1.0473111	0.7995878	0.5572496	0.3214244
0.0931639	-0.1265567	-0.3368383	-0.5368582	-0.7258700
-0.9032046	-1.0682722	-1.2205644	-1.3596576	-1.4852158
-1.5969945	-1.6948431	-1.7787073	-1.8486294	-1.9047481
-1.9472954	-1.9765922	-1.9930423	-1.9971243	-1.9893827
-1.9704176	-1.9408745	-1.8014342	-1.8528030	-1.7957047
-1.7308727	-1.6590449	-1.5809597	-1.4973528	-1.4089564
-1.3164980	-1.2207005	-1.1222815	-1.0219528	-0.9204182
-0.8183711	-0.7164903	-0.6154345	-0.5158362	-0.4182949
-0.3233694	-0.2315715	-0.1433592	-0.0591323	0.0207712
0.0960762	0.1665695	0.2320973	0.2925598	0.3479056
0.3981238	0.4432363	0.4832902	0.5183509	0.5484959
0.5738109	0.5943870	0.6103202	0.6217122	0.6286729
0.6313238	0.6298011	0.6242601	0.6148771	0.6018517
0.5854067	0.5657863	0.5432531	0.5180829	0.4905592
0.4609658	0.4295803	0.3966679	0.3624759	0.3272300
0.2911328	0.2543641	0.2170838	0.1794357	0.1415545
0.1035718	0.0656237	0.0278577	-0.0095622	-0.0464514
-0.0826035	-0.1177907	-0.1517680	-0.1842786	-0.2150615
-0.2438612	-0.2704365	-0.2945709	-0.3160803	-0.3348201
-0.3506899	-0.3636354	-0.3736477	-0.3807599	-0.3850420
-0.3865939	-0.3855371	-0.3820071	-0.3761463	-0.3680978
-0.3580021	-0.3459950	-0.3322094	-0.3167778	-0.2998377
-0.2815380	-0.2620450	-0.2415492	-0.2202693	-0.1984549
0.1763861	-0.1543700	-0.1543700	-0.1118143	-0.0919475
-0.0734526	-0.0566187	-0.0416905	-0.0288560	-0.0182370
-0.0098828	-0.0037687	0.0002018	0.0021899	0.0024109
0.0011207	-0.0013998	-0.0048610	-0.0089801	-0.0134947
-0.0181736	-0.0228239	-0.0272931	-0.0314685	-0.0352709
-0.0386471	-0.0415585	-0.0439705	-0.0458416	-0.0471151
-0.0477136	-0.0475377	-0.0464696	-0.0443806	-0.0411434
-0.0366473	-0.0308152	-0.0236212	-0.0151063	-0.0053910
0.0053171	0.0167220	0.0284445	0.0400358	0.0509938
0.0608046	0.0689425	0.0749333	0.0783720	0.0789564
0.0765135	0.0710176	0.0626009	0.0515533	0.0383126
0.0234435	0.0076085	-0.0084686	-0.0240446	-0.0384004
-0.0508838	-0.0609485	-0.0681863	-0.0723509	-0.0733706
-0.0713496	-0.0665586	-0.0594132	-0.0504447	-0.0402683
-0.0295126	-0.0188364	-0.0088283	0.0000000	0.0072489
0.0126510	0.0160815	0.0175578	0.0172292	0.0153572
0.0122893	0.0084275	0.0041943	0.0000000	-0.0037884
-0.0068719	-0.0090381	-0.0101718	-0.0102567	-0.0093694
-0.0076666	-0.0053654	-0.0027205	0.0000000	0.0025394
0.0046743	0.0062320	0.0071031	0.0072476	0.0066944
0.0055350	0.0039116	0.0020017	0.0000000	-0.0019003
-0.0035251	-0.0047346	-0.0054343	-0.0055818	-0.0051885
-0.0043159	-0.0030676	-0.0015785	0.0000000	0.0015140
0.0028221	0.0038080	0.0043901	0.0045286	0.0042267
0.0035297	0.0025184	0.0013006	0.0000000	-0.0012561
-0.0023491	-0.0031798	-0.0036771	-0.0038043	-0.0035609
-0.0029819	-0.0021333	-0.0011046	0.0000000	0.0010721
0.0020098	0.0027267	0.0031602	0.0032766	0.0030735
0.0025790	0.0018487	0.0009591	0.0000000	-0.0009344

(B) - QUADRATURE RESPONSE OF TRANSMITTER FILTER
SAMPLED AT 96000 SAMPLES/SEC.

-0.0018960	-0.0026261	-0.0031067	-0.0032874	-0.0031465
-0.0026938	-0.0019699	-0.0010424	0.0000000	0.0010562
0.0020224	0.0028023	0.0033165	0.0035109	0.0033619
0.0028795	0.0021066	0.0011152	0.0000000	-0.0011310
-0.0021666	-0.0030035	-0.0035564	-0.0037666	-0.0036085
-0.0030923	-0.0022634	-0.0011988	0.0000000	0.0012171
0.0023328	0.0032356	0.0038332	0.0040621	0.0038938
0.0033386	0.0024451	0.0012959	0.0000000	-0.0013172
-0.0025262	-0.0035060	-0.0041563	-0.0044073	-0.0042275
-0.0036272	-0.0026583	-0.0014098	0.0000000	0.0014350
0.0027542	0.0038253	0.0045381	0.0048159	0.0046230
0.0039696	0.0029116	0.0015454	0.0000000	-0.0015757
-0.0030268	-0.0042076	-0.0049962	-0.0053068	-0.0050990
-0.0043825	-0.0032175	-0.0017095	0.0000000	0.0017465
0.0033584	0.0046736	0.0055556	0.0059076	0.0056827
0.0048899	0.0035943	0.0019120	0.0000000	-0.0019583
-0.0037705	-0.0052539	-0.0062539	-0.0066593	-0.0064148
-0.0055278	-0.0040692	-0.0021679	0.0000000	0.0022274
0.0042956	0.0059957	0.0071491	0.0076259	0.0073593
0.0063535	0.0046859	0.0025013	0.0000000	-0.0025804
-0.0049871	-0.0069761	-0.0083367	-0.0089133	-0.0086221
-0.0074620	-0.0055173	-0.0029528	0.0000000	0.0030629
0.0059365	0.0083288	0.0099837	0.0107080	0.0103921
0.0090242	0.0066959	0.0035966	0.0000000	-0.0037594
-0.0073159	-0.0103073	-0.0124094	-0.0133702	-0.0130372
-0.0113770	-0.0084850	-0.0045820	0.0000000	0.0048442
0.0094846	0.0134480	0.0162988	0.0176836	0.0173697
0.0152746	0.0114840	0.0062543	0.0000000	-0.0067345
-0.0133169	-0.0190802	-0.0233823	-0.0256677	-0.0255267
-0.0227450	-0.0173412	-0.0095856	0.0000000	0.0106649
0.0214730	0.0313657	0.0392410	0.0440432	0.0448589
0.0410111	0.0321468	0.0183105	0.0000000	-0.0218003
-0.0456111	-0.0694746	-0.0909950	-0.1074007	-0.1156220
-0.1123821	-0.0942916	-0.0579434	0.0000000	0.0827311
0.1932385	0.3342662	0.5082941	0.7175366	0.9639589
1.2493078	1.5751498	1.9429154	2.3539405	2.8095026
3.3108456	3.8591911	4.4557341	5.1016214	5.7979150
6.5455425	7.3452371	8.1974720	9.1023946	10.0597638
11.0688962	12.1286245	13.2372707	14.3926361	15.5920086
16.8321857	18.1095118	19.4199272	20.7590237	22.1221047
23.5042446	24.9003445	26.3051808	27.7134459	29.1197780
30.5187820	31.9050405	33.2731191	34.6175658	35.9329096
37.2136597	38.4543095	39.6493461	40.7932681	41.8806123
42.9059887	43.8641239	44.7499103	45.5584592	46.2851556
46.9257109	47.4762122	47.9331639	48.2935224	48.5547200
48.7146794	48.7718190	48.7250491	48.5737616	48.3178150
47.9575159	47.4936015	46.9272219	46.2599267	45.4936549
44.6307293	43.6738546	42.6261192	41.4909978	40.2723551
38.9744463	37.6019147	36.1597834	34.6534397	33.0886132
31.4713441	29.8079459	28.1049601	26.3691066	24.6072306
22.8262482	21.0330921	19.2346609	17.4377717	15.6491184
13.8752357	12.1224692	10.3969514	8.7045826	7.0510150
5.4416399	3.8815751	2.3756530	0.9284061	-0.4559482
-1.7735266	-3.0207994	-4.1946128	-5.2922114	-6.3112590
-7.2498590	-8.1065690	-8.8804125	-9.5708828	-10.1779415
-10.7020093	-11.1439499	-11.5050480	-11.7869820	-11.9917925
-12.1218489	-12.1798142	-12.1686116	-12.0913914	-11.9515022
-11.7524642	-11.4979467	-11.1917489	-10.8377827	-10.4400583
-10.0026703	-9.5297832	-9.0256163	-8.4944258	-7.9404850
-7.3680608	6.7813882	-6.1846416	-5.5819054	-4.9771435
-4.3741695	-3.7766193	-3.1879262	-2.6113002	-2.0497120
-1.5058820	-0.9822748	-0.4810978	-0.0043047	0.4463982

0.8695437	1.2638933	1.6284281	1.9623394	2.2650217
2.5360673	2.7752627	2.9825874	3.1582131	3.3025044
3.4160186	3.4995038	3.5538956	3.5803102	3.5800345
3.5545124	3.5053270	3.4341804	3.3428704	3.2332656
3.1072800	2.9668481	2.8139013	2.6503465	2.4780490
2.2988169	2.1143913	1.9264385	1.7365460	1.5462211
1.3568901	1.1698997	0.9865176	0.8079318	0.6352500
0.4694956	0.3116039	0.1624149	0.0226669	-0.1070118
-0.2261096	-0.3342379	-0.4311352	-0.5166680	-0.5908297
-0.6537350	-0.7056120	-0.7467906	-0.7776891	-0.7987986
-0.8106669	-0.8138825	-0.8090599	-0.7968257	-0.7778083
-0.7526319	-0.7219063	-0.6862301	-0.6461880	-0.6023532
-0.5552906	-0.5055596	-0.4537174	-0.4003201	-0.3459233
-0.2910796	-0.2363349	-0.1822225	-0.1292556	-0.0779191
-0.0286612	0.0181148	0.0620573	0.1028714	0.1403215
0.1742316	0.2044829	0.2310091	0.2537894	0.2728409
0.2882096	0.2999617	0.3081762	0.3129383	0.3143350
0.3124536	0.3073812	0.2992077	0.2880296	0.2739555
0.2571122	0.2376502	0.2157487	0.1916186	0.1655045
0.1376831	0.1084602	0.0781650	0.0471432	0.0157477
-0.0156703	-0.0467706	-0.0772327	-0.1067625	-0.1350979
-0.1620121	-0.1873135	-0.2108453	-0.2324818	-0.2521238
-0.2696933	-0.2851283	-0.2983775	-0.3093963	-0.3181453
-0.3245889	-0.3286970	-0.3304483	-0.3298346	-0.3268660
-0.3215770	-0.3140314	-0.3043271	-0.2925984	-0.2790169
-0.2637901	-0.2471575	-0.2293845	-0.2107548	-0.1915610
-0.1720951	-0.1526387	-0.1334542	-0.1147769	-0.0968097
-0.0797193	-0.0636352	-0.0486510	-0.0348275	-0.0221972
-0.0107706	-0.0005416	0.0085057	0.0163927	0.0231422
0.0287766	0.0333169	0.0367850	0.0392056	0.0406106
0.0410431	0.0405616	0.0392435	0.0371870	0.0345120
0.0313582	0.0278816	0.0242491	0.0206313	0.0171942
0.0140909	0.0114524	0.0093809	0.0079429	0.0071662
0.0070377	0.0075054	0.0084818	0.0098505	0.0114746
0.0132057	0.0148949	0.0164020	0.0176054	0.0184091
0.0187484	0.0185930	0.0194722	0.0168480	0.0153603
0.0135711	0.0115816	0.0094992	0.0074293	0.0054678
0.0036951	0.0021717	0.0009352	0.0000000	-0.0006413
-0.0010149	-0.0011618	-0.0011323	-0.0009808	-0.0007607
-0.0005197	-0.0002965	-0.0001182	0.0000000	0.0000550
0.0000538	0.0000111	-0.0000540	-0.0001207	-0.0001701
-0.0001877	-0.0001650	-0.0001006	0.0000000	0.0001250
0.0002584	0.0003820	0.0004777	0.0005303	0.0005291
0.0004698	0.0003546	0.0001930	0.0000000	-0.0002049
-0.0003999	-0.0005637	-0.0006772	-0.0007265	-0.0007039
-0.0006091	-0.0004496	-0.0002399	0.0000000	0.0002463
0.0004742	0.0006599	0.0007839	0.0008322	0.0007987
0.0006853	0.0005019	0.0002658	0.0000000	-0.0002695
-0.0005158	-0.0007142	-0.0008442	-0.0008922	-0.0008527
-0.0007287	-0.0005317	-0.0002807	0.0000000	0.0002828
0.0005397	0.0007452	0.0008787	0.0009265	0.0008835
0.0007534	0.0005486	0.0002891	0.0000000	-0.0002902

(C) - IN PHASE RESPONSE OF RECEIVER FILTER
SAMPLED AT 96000 SAMPLES/SEC.

0.0020834	0.0028840	0.0034098	0.0036060	0.0034493
0.0029515	0.0021571	0.0011408	0.0000000	-0.0011545
-0.0022093	-0.0030593	-0.0036185	-0.0038282	-0.0036635
-0.0031359	-0.0022927	-0.0012130	0.0000000	0.0012286
0.0023522	0.0032587	0.0038560	0.0040814	0.0039075
0.0033463	0.0024477	0.0012956	0.0000000	-0.0013136
-0.0025161	-0.0034875	-0.0041289	-0.0043724	-0.0041884
-0.0035888	-0.0026265	-0.0013910	0.0000000	0.0014119
0.0027060	0.0037529	0.0044457	0.0047108	0.0045154
0.0038713	0.0028351	0.0015025	0.0000000	-0.0015271
-0.0029286	-0.0040645	-0.0048183	-0.0051093	-0.0049009
-0.0042050	-0.0030818	-0.0016345	0.0000000	0.0016639
0.0031936	0.0044359	0.0052630	0.0055857	0.0053625
0.0046052	0.0033782	0.0017933	0.0000000	-0.0018291
-0.0035142	-0.0048862	-0.0058032	-0.0061655	-0.0059255
-0.0050943	-0.0037411	-0.0019883	0.0000000	0.0020327
0.0039102	0.0054437	0.0064736	0.0068867	0.0066276
0.0057057	0.0041961	0.0022333	0.0000000	-0.0022900
-0.0044121	-0.0061521	-0.0073280	-0.0078087	-0.0075278
-0.0064922	-0.0047831	-0.0025505	0.0000000	0.0026255
0.0050685	0.0070821	0.0084540	0.0090284	0.0087235
0.0075410	0.0055692	0.0029771	0.0000000	-0.0030806
-0.0059635	-0.0083562	-0.0100041	-0.0107162	-0.0103866
-0.0090078	-0.0066748	-0.0035805	0.0000000	0.0037323
0.0072530	0.0102042	0.0122676	0.0131981	0.0128502
0.0111969	0.0083378	0.0044955	0.0000000	-0.0047375
-0.0092603	-0.0131078	-0.0158593	-0.0171766	-0.0168415
-0.0147831	-0.0110937	-0.0060302	0.0000000	0.0064675
0.0127627	0.0182477	0.0223137	0.0244403	0.0242505
0.0215573	0.0163959	0.0090405	0.0000000	-0.0100061
-0.0200914	-0.0292647	-0.0365054	-0.0408489	-0.0414752
-0.0377947	-0.0295260	-0.0167591	0.0000000	0.0198065
0.0412785	0.0626214	0.0816758	0.0959832	0.1028664
0.0995193	0.0830984	0.0508120	0.0000000	-0.0718006
-0.1668071	-0.2869621	-0.4339259	-0.6090863	-0.8135869
-1.0483676	-1.3142129	-1.6118040	-1.9417691	-2.3047252
-2.7013103	-3.1322002	-3.5981102	-4.0997796	-4.6379411
-5.2132779	-5.8263700	-6.4776362	-7.1672740	-7.8952046
-8.6610237	-9.4639655	-10.3028792	-11.1762206	-12.0820595
-13.0181011	-13.9817192	-14.9699991	-15.9797864	-17.0077375
-18.0503691	-19.1041028	-20.1653032	-21.2303061	-22.2954378
-23.3570252	-24.4113966	-25.4548772	-26.4837800	-27.4943951
-28.4829812	-29.4457608	-30.3789209	-31.2786220	-32.1410152
-32.9622667	-33.7385899	-34.4662838	-35.1417733	-35.7616526
-36.3227256	-36.8220438	-37.2569380	-37.6250430	-37.9243142
-38.1530362	-38.3098230	-38.3936124	-38.4036547	-38.3394980
-38.2009723	-37.9881746	-37.7014554	-37.3414093	-36.9088698
-36.4049087	-35.8308386	-35.1882206	-34.4788717	-33.7048744
-32.8685841	-31.9726333	-31.0199326	-30.0136650	-28.9572744
-27.8544486	-26.7090951	-25.5253126	-24.3073586	-23.0596138
-21.7865466	-20.4926773	-19.1825454	-17.8606794	-16.5315711
-15.1996546	-13.8692888	-12.5447454	-11.2301982	-9.9297163
-8.6472568	-7.3866587	-6.1516341	-4.9457589	-3.7724599
-2.6349998	-1.5364595	-0.4797192	0.5325620	1.4979660
2.4143343	3.2797839	4.0927200	4.8518440	5.5561560
6.2049538	6.7978249	7.3346356	7.8155160	8.2408417
8.6112151	8.9274450	9.1905279	9.4016301	9.5620723
9.6733158	9.7369523	9.7546952	9.7283725	9.6599211
9.5513810	9.4048894	9.2226733	9.0070400	8.7603660
8.4850836	8.1836652	7.8586060	7.5124057	7.1475503
6.7664935	6.3716409	5.9653346	5.5498420	5.1273471
4.6999446	4.2696374	3.8383376	3.4078680	2.9799668

2.5562923	2.1384269	1.7278820	1.3260998	0.9344548
0.5542524	0.1867261	-0.1669666	-0.5057505	-0.8286388
-1.1347394	-1.4232612	-1.6935186	-1.9449349	-2.1770436
-2.3894877	-2.5820173	-2.7544853	-2.9068417	-3.0391275
-3.1514680	-3.2440658	-3.1371960	-3.3712008	-3.4064862
-3.4235203	-3.4228322	-3.4050122	-3.3707125	-3.3206485
-3.2555987	-3.1764049	-3.0839704	-2.9792569	-2.8632797
-2.7371004	-2.6018195	-2.4585658	-2.3084858	-2.1527328
-1.9924551	-1.8287851	-1.6628297	-1.4956608	-1.3283086
-1.1617554	-0.9969315	-0.8347123	-0.6759166	-0.5213057
-0.3715827	-0.2273931	-0.0893236	0.0420976	0.1664022
0.2831831	0.3920953	0.4928585	0.5852576	0.6691446
0.7444381	0.8111235	0.8692507	0.9189314	0.9603349
0.9936831	1.0192439	1.0373248	1.0482656	1.0524314
1.0502056	1.0419837	1.0281681	1.0091636	0.9853737
0.9571988	0.9250337	0.8892674	0.8502824	0.8084543
0.7641518	0.7177366	0.6695626	0.6199751	0.5693099
0.5178921	0.4660349	0.4140381	0.3621876	0.3107546
0.2599960	0.2101542	0.1614588	0.1141273	0.0683671
0.0243771	-0.0176514	-0.0575333	-0.0950895	-0.1301474
-0.1625418	-0.1921175	-0.2187313	-0.2422558	-0.2625825
-0.2796255	-0.2933242	-0.3036466	-0.3105902	-0.3141835
-0.3144855	-0.3115844	-0.3055961	-0.2966613	-0.2849425
-0.2706218	-0.2538974	-0.2349824	-0.2141025	-0.1914962
-0.1674140	-0.1421198	-0.1158908	-0.0890187	-0.0618098
-0.0345840	-0.0076731	0.0185830	0.0438410	0.0677611
0.0900148	0.1102940	0.1283201	0.1438535	0.1567017
0.1667264	0.1738479	0.1780475	0.1793672	0.1779058
0.1738134	0.1672828	0.1585399	0.1478332	0.1354230
0.1215723	0.1065385	0.0905683	0.0738947	0.0567369
0.0393033	0.0217962	0.0044183	-0.0126208	-0.0290985
-0.0447741	-0.0593875	-0.0726604	-0.0843018	-0.0940174
-0.1015230	-0.1065605	-0.1089161	-0.1084392	-0.1050604
-0.0988063	-0.0898108	-0.0783207	-0.0646936	-0.0493906
-0.0329595	-0.0160129	0.0007998	0.0168251	0.0314392
0.0440824	0.0542901	0.0617199	0.0661713	0.0675974
0.0661066	0.0619555	0.0555316	0.0473282	0.0379127
0.0278901	0.0178649	0.0084034	0.0000000	-0.0069515
-0.0121784	-0.0155414	-0.0170357	-0.0167846	-0.0150226
-0.0120718	-0.0083134	-0.0041553	0.0000000	0.0037858
0.0068974	0.0091119	0.0103004	0.0104327	0.0095730
0.0078684	0.0055313	0.0028173	0.0000000	-0.0026535
-0.0049063	-0.0065707	-0.0075227	-0.0077101	-0.0071534
-0.0059408	-0.0042170	-0.0021675	0.0000000	0.0020758
0.0038676	0.0052171	0.0060139	0.0062038	0.0057913
0.0048377	0.0034531	0.0017843	0.0000000	-0.0017257
-0.0032301	-0.0043763	-0.0050659	-0.0052469	-0.0049169
-0.0041225	-0.0029530	-0.0015311	0.0000000	0.0014904
0.0027980	0.0038019	0.0044133	0.0045833	0.0043063
0.0036196	0.0025991	0.0013507	0.0000000	-0.0013207
-0.0024846	-0.0033830	-0.0039349	-0.0040943	-0.0038540
-0.0032453	-0.0023344	-0.0012152	0.0000000	0.0011920

(D) - QUADRATURE RESPONSE OF RECEIVER FILTER SAMPLED AT 96000 SAMPLES/SEC.

-0.0012768	-0.0017684	-0.0020920	-0.0022137	-0.0021188
-0.0018140	-0.0013265	-0.0007019	0.0000000	0.0007112
0.0013618	0.0018869	0.0022331	0.0022331	0.0022636
0.0019388	0.0014184	0.0007509	0.0000000	-0.0007615
-0.0014588	-0.0020222	-0.0023944	-0.0023944	-0.0024294
-0.0020818	-0.0015237	-0.0008071	0.0000000	0.0008193
0.0015703	0.0021780	0.0025803	0.0025803	0.0026209
0.0022472	0.0016458	0.0008722	0.0000000	-0.0008865
-0.0017001	-0.0023595	-0.0027970	-0.0027970	-0.0028448
-0.0024407	-0.0017887	-0.0009486	0.0000000	0.0009655
0.0018529	0.0025734	0.0030529	0.0030529	0.0031097
0.0026701	0.0019583	0.0010394	0.0000000	-0.0010596
-0.0020353	-0.0028292	-0.0033593	-0.0033593	-0.0034280
-0.0029461	-0.0021628	-0.0011490	0.0000000	0.0011738
0.0022569	0.0031405	0.0037328	0.0037328	0.0038176
0.0032847	0.0024141	0.0012841	0.0000000	-0.0013149
-0.0025315	-0.0035271	-0.0041980	-0.0041980	-0.0043050
-0.0037093	-0.0027302	-0.0014543	0.0000000	0.0014938
0.0028805	0.0040200	0.0047926	0.0047926	0.0049319
0.0042571	0.0031392	0.0016754	0.0000000	-0.0017277
-0.0033384	-0.0046689	-0.0055784	-0.0055784	-0.0057667
-0.0049896	-0.0036884	-0.0019735	0.0000000	0.0020459
0.0039643	0.0055601	0.0066628	0.0066628	0.0069307
0.0060164	0.0044624	0.0023960	0.0000000	-0.0025024
-0.0048678	-0.0068550	-0.0082493	-0.0082493	-0.0086579
-0.0075514	-0.0056287	-0.0030378	0.0000000	0.0032077
0.0062762	0.0088926	0.0107699	0.0107699	0.0114595
0.0100688	0.0075633	0.0041152	0.0000000	-0.0044223
-0.0087353	-0.0125015	-0.0153018	-0.0153018	-0.0166617
-0.0148252	-0.0112861	-0.0062288	0.0000000	0.0069065
0.0138799	0.0202348	0.0252628	0.0252628	0.0287494
0.0262190	0.0204986	0.0116438	0.0000000	-0.0137805
-0.0287389	-0.0436259	-0.0569346	-0.0569346	-0.0717856
-0.0694849	-0.0580471	-0.0355096	0.0000000	0.0502174
0.1167072	0.2008434	0.3038040	0.3038040	0.5699953
0.7347397	0.9214035	1.1305184	1.3625952	1.6181574
1.8977660	2.2020347	2.5316323	2.5316323	3.2696998
3.6796437	4.1177943	4.5847515	5.0809810	5.6067709
6.1621927	6.7470708	7.3609602	7.3609602	8.6725928
9.3680549	10.0879953	10.8306611	11.5941040	12.3762123
13.1747412	13.9873413	14.8115810	14.8115810	16.4849309
17.3288771	18.1741338	19.0179670	19.8575654	20.6900291
21.5123589	22.3214508	23.1140945	23.1140945	24.6367063
25.3598096	26.0527816	26.7121061	27.3342937	27.9159203
28.4536656	28.9443490	29.3849631	29.3849631	30.1049827
30.3794641	30.5940557	30.7469241	30.8364944	30.8614495
30.8207268	30.7135164	30.5392595	30.5392595	29.9886380
29.6124372	29.1695346	28.6607020	28.0870086	27.4498335
26.7508758	25.9921623	25.1760495	25.1760495	23.3826834
22.4117411	21.3959867	20.3392687	19.2456627	18.1194377
16.9650207	15.7869608	14.5898947	14.5898947	12.1575352
10.9316736	9.7056198	8.4840207	7.2714615	6.0724514
4.8914092	3.7326488	2.6003648	2.6003648	0.4313078
-0.5978301	-1.5852635	-2.5276787	-3.4220063	-4.2654451
-5.0554846	-5.7899250	-6.4668938	-6.4668938	-7.6426318
-8.1393787	-8.5746090	-8.9481726	-9.2602472	-9.5113244
-9.7021921	-9.8339173	-9.9078276	-9.9078276	-9.8887123
-9.7994933	-9.6600448	-9.4727624	-9.2402193	-8.9651555
-8.6504685	-8.2992019	-7.9145327	-7.9145327	-7.0582687
-6.5935467	-6.1091243	-5.6085673	-5.0954462	-4.5733089
-4.0456518	-3.5158932	-2.9873479	-2.9873479	-1.9465057
-1.4401322	-0.9467899	-0.4690010	-0.0090979	0.4307804

0.8486902	1.2428835	1.6118081	1.6118081	2.2686229
2.5543929	2.8106573	3.0368596	3.2326498	3.3978877
3.5326443	3.6372023	3.7120534	3.7120534	3.7756159
3.7662970	3.7311846	3.6716792	3.5893140	3.4857337
3.3626719	3.2219288	3.0653490	3.0653490	2.7121606
2.5192911	2.3180335	2.1101940	1.8975352	1.6817692
1.4645527	1.2474814	1.0320863	1.0320863	0.6120931
0.4101854	0.2153215	0.0286218	-0.1488970	-0.3163297
-0.4728898	-0.6179151	-0.7508719	-0.7508719	-0.9790951
-1.0739389	-1.1558594	-1.2249389	-1.2813604	-1.3253953
-1.3573908	-1.3777569	-1.3869546	-1.3869546	-1.3738767
-1.3526856	-1.3224823	-1.2838523	-1.2373936	-1.1837168
-1.1234453	-1.0572163	-0.9856814	-0.9856814	-0.8293693
-0.7459592	-0.6599706	-0.5720984	-0.4830313	-0.3934445
-0.3039922	-0.2153002	-0.1279595	-0.1279595	0.0405073
0.1206639	0.1975324	0.2707403	0.3399563	0.4048860
0.4652678	0.5208688	0.5714807	0.5714807	0.6570132
0.6916247	0.7206306	0.7439369	0.7614804	0.7732345
0.7792146	0.7794833	0.7741548	0.7741548	0.7474360
0.7265489	0.7010670	0.6713682	0.6378702	0.6010234
0.5613013	0.5191920	0.4751887	0.4751887	0.3834522
0.3366648	0.2898653	0.2434774	0.1979014	0.1535145
0.1106709	0.0697030	0.0309219	0.0309219	-0.0389437
-0.0695178	-0.0968894	-0.1208743	-0.1413246	-0.1581338
-0.1712422	-0.1806410	-0.1863757	-0.1863757	-0.1873154
-0.1828896	-0.1755313	-0.1655443	-0.1532672	-0.1390640
-0.1233136	-0.1063995	-0.0886997	-0.0886997	-0.0523731
-0.0343991	-0.0169347	-0.0002249	0.0155204	0.0301249
0.0434430	0.0553573	0.0657745	0.0657745	0.0818470
0.0874120	0.0912970	0.0934988	0.0940330	0.0929364
0.0902695	0.0861192	0.0806009	0.0738595	0.0660688
0.0574296	0.0481657	0.0385182	0.0287380	0.0190781
0.0097832	0.0010815	-0.0068255	-0.0137700	-0.0196251
-0.0243087	-0.0277859	-0.0300688	-0.0312132	-0.0313141
-0.0304985	-0.0289175	-0.0267365	-0.0241266	-0.0212551
-0.0182778	-0.0153320	-0.0125319	-0.0099654	-0.0076930
-0.0057484	-0.0041414	-0.0028609	-0.0018799	-0.0011601
-0.0006569	-0.0003243	-0.0001182	0.0000000	0.0000622
0.0000920	0.0001053	0.0001109	0.0001122	0.0001085
0.0000975	0.0000765	0.0000439	0.0000000	-0.0000524
-0.0001082	-0.0001611	-0.0002036	-0.0002290	-0.0002318
-0.0002088	-0.0001599	-0.0000883	0.0000000	0.0000963
0.0001903	0.0002715	0.0003299	0.0003578	0.0003503
0.0003061	0.0002281	0.0001228	0.0000000	-0.0001281
-0.0002485	-0.0003484	-0.0004166	-0.0004453	-0.0004300
-0.0003711	-0.0002733	-0.0001456	0.0000000	0.0001491
0.0002867	0.0003987	0.0004734	0.0005024	0.0004821
0.0004136	0.0003029	0.0001605	0.0000000	-0.0001628
-0.0003116	-0.0004316	-0.0005104	-0.0005397	-0.0005161
-0.0004412	-0.0003222	-0.0001702	0.0000000	0.0001716
0.0003278	0.0004529	0.0005344	0.0005638	0.0005380
0.0004591	0.0003346	0.0001764	0.0000000	-0.0001773

APPENDIX C

SIGNAL DISTORTION

The signal distortion introduced by a channel sampled impulse response has been described in detail in reference 2, and only the more important results are quoted here.

The channel is said to introduce signal distortion whenever there are two or more non-zero components (y_h) in the sampled impulse response Y. Two types of distortion can be introduced by the sampled impulse response of a channel and these are the amplitude and phase distortion which are defined in terms of the discrete Fourier transform components of the corresponding sampled impulse response.

C.1 PHASE DISTORTION

The signal distortion introduced by a channel is defined to be pure phase distortion (that is no amplitude distortion or attenuation) when all the discrete Fourier components of the sampled impulse response of this channel have magnitude unity. To see the time domain equivalence of this definition, let V(z) be the z-transform of the channel sampled impulse response so that

$$V(z) = y_0 + y_1 z^{-1} \dots + y_g z^{-g}$$

Furthermore, let U(z) be the z transform of the sequence of values obtained by reversing the order of the complex conjugate of the sequence with z-transform V(z), the reversal being pivoted about the components at time t = 0. Thus,

$$U(z) = y_0^* + y_1^* z \dots + y_g^* z^g$$

It is shown in reference 2, that a channel with z-transform $V(z)$ introduces pure phase distortion when the sequence with z-transform $U(z)$ is the same as the sequence with z-transform $V^{-1}(z)$. That is, if the complex conjugate of the reversed sequence of a channel sampled impulse response is also the inverse of the sequence, then signal distortion introduced by this channel is a pure phase distortion.

Also it has been shown that the pure phase distortion represents an orthogonal transformation. Consequently, there is no inevitable loss in tolerance to noise when the received sequence is processed by the appropriate detection process. Hence phase distortion is not usually considered when assessing the distortion present in a sampled waveform, provided that a suitable detection process is used in the receiver which can reverse the orthogonal transformation.

Finally in the case of pure phase distortion each pole or zero of the z-transform of the sampled impulse response $V(z)$ is accompanied by a zero or pole, respectively, at the complex conjugate of the reciprocal value of z. Thus, when a filter introducing pure phase distortion is connected in cascade with a channel, and when the poles of the z-transform of the filter coincide with zeros of the z-transform of the channel, then the action of the filter is to replace these zeros by the corresponding set of zeros at the complex conjugate reciprocal values of z. Thus the replacement of one or more zeros in a z-transform, by zeros at the complex conjugate of the reciprocal values of z, is pure phase distortion and is therefore an orthogonal transformation.

C.2 AMPLITUDE DISTORTION

The signal distortion by a channel is defined to be pure amplitude distortion (that is no phase distortion or delay) when all the discrete Fourier transform components of the sampled impulse response of this channel are real-valued quantities. The time domain equivalence of this definition is that the components of the channel sampled impulse response y_0, y_1, y_g are Hermitian about its central component $y_{g/2}$ where g is an even value. Thus,

$$y_i = y_{g-i}^* \text{ for } i = 0, 1, \ldots ((g/2)-1)$$

It can be shown that a channel introducing pure amplitude distortion does not introduce an orthogonal transformation onto the transmitted signal and it normally reduces the best tolerance to additive white Gaussian noise regardless of the type of signal processor used at the receiver. Consequently, amplitude distortion is a much more important factor (than phase distortion) to be considered when assessing the severity of the signal distortion introduced by a channel.

Finally, all zeros of the z-transform of the sampled impulse response of the channel occur in complex conjugate reciprocal pairs when the channel introduces pure amplitude distortion.

APPENDIX D

APPENDIX D.1

THIS PROGRAM GENERATES A TWO SKYWAVE HF RAYLEIGH FADING CHANNEL WITH A FREQUENCY SPREAD OF 2 HZ AND A TIME DELAY OF 1.1 MSECS

```
      PROGRAM CHANEL1
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DOUBLE PRECISION CF(5),Q(4,3000)
      DOUBLE PRECISION TXR(16),TXI(16),TXDR(16),TXDI(16)
      DOUBLE PRECISION RXR(21),RXI(21),WSR(21),WSI(21)
      DOUBLE PRECISION QR(21,21),QI(21,21)
      DOUBLE PRECISION YR(21),YI(21),YRI(21),SQRTY(21)
      DOUBLE PRECISION CON(4),QQ(4),EQ(4),VQ(4),FMEAN(4),FVAR(4)

C     VALUES TO VARIABLES AND ARRAYS

      DATA CF /-1.80322972300000,0.81520668040000,-1.85218288200000,
     1     0.86788454580000,-0.89481307290000/
      DCG=15822
      DCG=1.0/DCG
      STDVN1=SQRT(1.0)
      NOSAM=2500
      INFD=50

      OPEN (25,FILE='CHNL1',FORM='UNFORMATTED')
C     OPEN (13,FILE='VCHSD50')


C     INITIALISING
      DL1=0.0
      DL2=0.0
      DL3=0.0
      DL4=0.0
      DL5=0.0

      JQ=50+NOSAM
      JQ1=JQ+1


      CALL G05CBF(INFD)

C     GENERATION OF Q1(T) AND Q2(T)

      DO 250 I=1,4
```

```
      JA=1

      TF3=0.0
      TVF3=0.0
      TF3G=0.0
      TVF3G=0.0

C     GENERATION OF THE BESSEL FILTER USED IN FILTERING THE
C     RANDOM NOISE TO OBTAIN A RAYLEIGH FADING MODEL

         DO 240 J=1,JQ1

         F0=G05DDF(0.0,STDVN1)
         F1=F0-(DL1*CF(1)+DL2*CF(2))
         F2=F1-(DL3*CF(3)+DL4*CF(4))
         F3=F2-(DL5*CF(5))

         F3DCG=F3*DCG

         DL5=F3
         DL4=DL3
         DL3=F2
         DL2=DL1
         DL1=F1

         IF(J.LE.50) GO TO 240

         Q(I,JA)=F3DCG
         JA=JA+1


         TF3=TF3+F3
         TVF3=TVF3+(F3**2)
         TF3G=TF3G+F3DCG
         TVF3G=TVF3G+(F3DCG**2)


240      CONTINUE


      EF3=TF3/(NOSAM+1)
      VARF3=TVF3/(NOSAM+1)
      EF3G=TF3G/(NOSAM+1)
      VARF3G=TVF3G/(NOSAM+1)
```

```
      WRITE(0,244)EF3,VARF3
      WRITE(0,246)EF3G,VARF3G
244   FORMAT('MEAN OF F3 =',1X,F10.5,2X,'VARIAN E OF F3 =',1X,F10.5)
246   FORMAT('MEAN OF F3DCG=',1X,F10.5,2X,'VAR OF F3DCG=',1X,F10.5)

250   CONTINUE

C     INITIALISING ARRAYS AND VARIABLES FOR MAIN PROGRAM

C     TRANSMITTER AND RECEIVER FILTER IMPULSE RESPONSE

      DATA TXR /-0.1795896, -3.0773455, -9.9409021,-11.7869473,
     1  -3.4618271, 4.4438154, 3.0642536, -1.3596576,
     1  -1.4973528, 0.2925598, 0.5180829, -0.1842786,
     1  -0.3167778, 0.0021899, -0.0443806, 0.0515533 /
      DATA TXI / 2.3539405, 20.7590237, 45.5584592, 41.4909978,
     1  8.7045826,-11.7869820, -5.5819054, 3.1582131,
     1  1.7365460, -0.7776891, -0.1292556, 0.2880296,
     1  -0.2324818, -0.2107548, 0.0392056, 0.0098505 /

      DATA TXDR / -1.6694374, -7.8492148,-12.3887079, -6.6023157,
     1  2.9408554, 4.3005084, -0.3368383, -1.9014342,
     1  -0.1433592, 0.6242601, 0.0278577, -0.3820071,
     1  -0.0416905, -0.0439705, 0.0749333, -0.0594132 /
      DATA TXDI / 13.2372707, 39.6493461, 46.9272219, 19.2346609,
     1  -8.8804125, -9.0256163, 1.6284281, 2.8139013,
     1  -0.4311352, -0.4537174, 0.3081762, -0.0772327,
     1  -0.3043271, 0.0085057, 0.0093809, 0.0094992 /


      DATA RXR / -1.9417691,-15.9797864,-35.1417733,-34.4788717,
     1  -11.2301982, 7.8155160, 7.5124057, -0.5057505,
     1  -3.3707125, -0.6759166, 1.0482656, 0.3621876,
     1  -0.3105902, 0.0438410, 0.0738947, -0.0646936,
     1  0.0000000, 0.0000000, 0.0000000, 0.0000000,
     1  0.0000000 /
      DATA RXI / 1.3625952, 11.5941040, 27.3342937, 28.0870086,
     1  7.2714615, -9.2602472, -5.0954462, 3.2326498,
     1  1.8975352, -1.2813604, -0.4830313, 0.7614804,
     1  0.1979014, -0.1532672, 0.0940330, -0.0312132,
     1  0.0000000, 0.0000000, 0.0000000, 0.0000000,
     1  0.0000000 /
```

```
      MLOOP=2500
      ISTEP=48
      STEP=1.0/ISTEP
      DEL1=1.1
      SAPRAT=2.4
      SFACT=1.0/(2.0*SAPRAT*1000)
      IDEL1=INT(SAPRAT*2*DEL1)
      KMPL=16
      KMP=IDEL1+KMPL
      KMP1=KMP-1
      ICOUNT=0
      DF=-1.0
      POS=-1.0

      DO 1010 I=1,KMP
      DO 1005 J=1,KMP
      QR(I,J)=0.0
      QI(I,J)=0.0
1005  CONTINUE
1010  CONTINUE

      DO 1020 I=1,4
      EQ(I)=0.0
      VQ(I)=0.0
      QQ(I)=0.0
1020  CONTINUE


C     ENTERING MAIN LOOP

      DO 9000 KMAIN=1,MLOOP
      IF (MOD(KMAIN,100).EQ.0) THEN
      PRINT*,'KMAIN=',KMAIN
      END IF

      DO 1510 I=1,4
      CON(I)=(Q(I,KMAIN+1)-Q(I,KMAIN))*STEP
1510  CONTINUE


C     ENTERING SECONDARY LOOP

      DO 8000 KSEC=1,ISTEP
```

```
      ICOUNT=ICOUNT+1
      COUNT=REAL(ICOUNT)

      DO 1520 I=1,4
      QQ(I)=Q(I,KMAIN)+((KSEC-1)*CON(I))
      EQ(I)=EQ(I)+QQ(I)
      VQ(I)=VQ(I)+(QQ(I)**2)
1520  CONTINUE

      QQ(2)=-QQ(2)
      QQ(4)=-QQ(4)


C     SHIFTING ARRAYS FOR CONVOLUTION

      DO 2010 I=1,KMP
      DO 2010 J=1,KMP1
      QR(I,KMP+1-J)=QR(I,KMP-J)
      QI(I,KMP+1-J)=QI(I,KMP-J)
2010  CONTINUE
      DO 2015 I=1,KMP
      QR(I,1)=0.0
      QI(I,1)=0.0
2015  CONTINUE


C     CONVOLUTION (TO OBTAIN IMPULSE RESPONSE OF CHANNEL), BEGINS

      DO 2020 I=1,KMPL
      QR(I,1)=TXR(I)*QQ(1)-TXI(I)*QQ(2)
      QI(I,1)=TXR(I)*QQ(2)+TXI(I)*QQ(1)
2020  CONTINUE
      DO 2030 I=1,KMPL
      QR(I+IDEL1,1)=QR(I+IDEL1,1)+TXDR(I)*QQ(3)-TXDI(I)*QQ(4)
      QI(I+IDEL1,1)=QI(I+IDEL1,1)+TXDR(I)*QQ(4)+TXDI(I)*QQ(3)
2030  CONTINUE
      POS=-POS
      IF(POS.LT.0.0) GO TO 8000

      IO=0
      JCOUNT=JCOUNT+1
      DCOUNT=REAL(JCOUNT)
```

```
      DO 2060 I=1,KMP,2
      IO=IO+1
      YR(IO)=0.0
      YI(IO)=0.0
       DO 2050 J=1,I
       YR(IO)=YR(IO)+QR(J,I+1-J)*RXR(I+1-J)-QI(J,I+1-J)*RXI(I+1-J)
       YI(IO)=YI(IO)+QI(J,I+1-J)*RXR(I+1-J)+QR(J,I+1-J)*RXI(I+1-J)
2050   CONTINUE
      YR(IO)=YR(IO)*SFACT
      YI(IO)=YI(IO)*SFACT
2060  CONTINUE

      IF(MOD(KMP,2).EQ.0)THEN
      GO TO 2070
      ELSE
      GO TO 2100
      END IF

2070  DO 2090 I=1,KMP1,2
      IO=IO+1
      YR(IO)=0.0
      YI(IO)=0.0
      MCONV=I+1
       DO 2080 J=MCONV,KMP
       YR(IO)=YR(IO)+QR(J,KMP+1+I-J)*RXR(KMP+1+I-J)
     1          -QI(J,KMP+1+I-J)*RXI(KMP+1+I-J)
       YI(IO)=YI(IO)+QI(J,KMP+1+I-J)*RXR(KMP+1+I-J)
     1          +QR(J,KMP+1+I-J)*RXI(KMP+1+I-J)
2080   CONTINUE
      YR(IO)=YR(IO)*SFACT
      YI(IO)=YI(IO)*SFACT
2090  CONTINUE

      GO TO 2150

2100  DO 2120 I=1,KMP1,2
      IO=IO+1
      YR(IO)=0.0
      YI(IO)=0.0
      MCONV=I+2
       DO 2110 J=MCONV,KMP
       YR(IO)=YR(IO)+QR(J,KMP+2+I-J)*RXR(KMP+2+I-J)
     1          -QI(J,KMP+2+I-J)*RXI(KMP+2+I-J)
       YI(IO)=YI(IO)+QI(J,KMP+2+I-J)*RXR(KMP+2+I-J)
     1          +QR(J,KMP+2+I-J)*RXI(KMP+2+I-J)
2110   CONTINUE
```

```
        YR(IO)=YR(IO)*SFACT
        YI(IO)=YI(IO)*SFACT
2120    CONTINUE

2150    CONTINUE


        DO 9100 I=1,4
        FMEAN(I)=EQ(I)/COUNT
        FVAR(I)=VQ(I)/COUNT
9100    CONTINUE

        VY=0.0
        DO 9110 I=1,21
        VY=VY+YR(I)*YR(I)+YI(I)*YI(I)
9110    CONTINUE
        TLGVY=10*LOG10(VY)
C       IF (MOD(ICOUNT,30).EQ.1) THEN
C       WRITE(13,9120) TLGVY
C9120   FORMAT(1H ,F12.8)
C       END IF
        IF (ICOUNT.GT.50) THEN
        IF (TLGVY.LT.DF) THEN
        DF=TLGVY
        END IF
        END IF



        TVY=TVY+VY


        WRITE(25)(YR(J),J=1,21)
        WRITE(25)(YI(J),J=1,21)

8000    CONTINUE

9000    CONTINUE

        STXR=0.0
        STXI=0.0
        DO 9190 I=1,16
        STXR=STXR+TXR(I)*TXR(I)
        STXI=STXI+TXI(I)*TXI(I)
```

```
9190    CONTINUE
        STXRI=STXR+STXI
        PRINT*,'STXRI=',STXRI

        SRXR=0.0
        SRXI=0.0
        DO 9195 I=1,21
        SRXR=SRXR+RXR(I)*RXR(I)
        SRXI=SRXI+RXI(I)*RXI(I)
9195    CONTINUE
        SRXRI=SRXR+SRXI
        PRINT*,'SRXRI=',SRXRI

        PRINT*,'INFD=',INFD
        AVRGVY=TVY*2.0/COUNT
        PRINT*,'AVRGVY=',AVRGVY

C       PRINTING RESULTS


        PRINT *,'COUNT=',COUNT
        PRINT*,'DF=',DF
        STOP
        END
```

APPENDIX D.2

PROGRAM THAT SIMULATES THE SIMPLE ESTIMATOR WITH LEAST SQUARES FADING MEMORY PREDICTION

```
      PROGRAM EST201
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DOUBLE PRECISION WFR(21),WFI(21),WDR(21),WDI(21)
      DOUBLE PRECISION YR(21),YI(21),Y1R(21),Y1I(21)
      DOUBLE PRECISION DGRDR(21),DGRDI(21)
      DOUBLE PRECISION ESTYR(21),ESTYI(21)
      DOUBLE PRECISION SR(21),SI(21),GRDR(21),GRDI(21),ERR(21),ERI(21)
C     VALUES TO VARIABLES AND ARRAYS

      INFD=70

      OPEN (16,FILE='D2SNR60N')

C     CALL G05CBF(INFD)

      DATA WFR / -0.0280463, -0.2308071, -0.5075768, -0.4980021,
     1    -0.1622055, 0.1128849, 0.1085069, -0.0073049,
     1    -0.0486855, -0.0097627, 0.0151408, 0.0052313,
     1    -0.0044861, 0.0006332, 0.0010673, -0.0009344,
     1     0.0000000, 0.0000000, 0.0000000, 0.0000000,
     1     0.0000000 /
      DATA WFI / 0.0196809, 0.1674616, 0.3948080, 0.4056800,
     1     0.1050267, -0.1337521, -0.0735970, 0.0466914,
     1     0.0274074, -0.0185076, -0.0069768, 0.0109986,
     1     0.0028584, -0.0022137, 0.0013582, -0.0004508,
     1     0.0000000, 0.0000000, 0.0000000, 0.0000000,
     1     0.0000000 /

      KMP=21
      KMP1=KMP-1
C     DO 1000 B=0.12,0.17,0.01
C     DO 1000 THETA=0.987,0.990,0.001
      CALL G05CBF(INFD)
      THETA=0.966
      B=0.207
      OPEN(25,FILE='CHNL1',FORM='UNFORMATTED')
      LCOUNT=0
      RRR=0.0
```

```
      WWW=0.0
      ERRTOT=0.0
      ERRNORM=0.0

      SNR=60.0
      STDVN=10.0**(-SNR/10.0)
      STDVN=SQRT(STDVN)

C     INITIALISING SR &SI

      DO 1100 J=1,KMP
      SR(J)=1.0
      SI(J)=1.0
1100  CONTINUE

C     INITIALISING FIRST DERIVATIVE IN PREDICTION ALGORITHM

      DO 1200 J=1,KMP
      GRDR(J)=0.0
      GRDI(J)=0.0
1200  CONTINUE

C     INITIALISING SECOND DERIVATIVE IN PRDICTION ALGORITHM

      DO 1250 J=1,KMP
      DGRDR(J)=0.0
      DGRDI(J)=0.0
1250  CONTINUE

C     INITIALISING NOISE DATA WDR &WDI

      DO 1300 J=1,21
      WDR(J)=0.0
      WDI(J)=0.0
1300  CONTINUE


      DO 1400 ICOUNT=1,60000
      READ(25)(YR(J),J=1,21)
      READ(25)(YI(J),J=1,21)


C     ESTIMATION OF IMPULSE RESPONSE

C     AT START UP OF TRANSMISSION
```

```
      IF (ICOUNT.EQ.1) THEN
      DO 3000 J=1,KMP
      Y1R(J)=YR(J)
      Y1I(J)=YI(J)
3000  CONTINUE
      END IF

C     ERROR IN ESTIMATION OF CHANNEL IMPULSE RESPONSE

      IF (ICOUNT.GT.5000) THEN
      LCOUNT=LCOUNT+1
      YERR=0.0
      YTOT=0.0
      DO 3100 J=1,KMP
      YERR=YERR+(YR(J)-Y1R(J))**2+(YI(J)-Y1I(J))**2
      YTOT=YTOT+YR(J)**2+YI(J)**2
3100  CONTINUE
      ERRTOT=ERRTOT+YERR
      YERRNOM=YERR/YTOT
      ERRNORM=ERRNORM+YERRNOM
      END IF

      IF (MOD(LCOUNT,15).EQ.1) THEN
      YERRNOM=10*LOG10(YERRNOM)
      WRITE(16,3150)LCOUNT,YERRNOM
3150  FORMAT(I5,',',F9.4)
      END IF

C     SHIFTING OF ARRAYS

      DO 3200 J=1,KMP1
      L=J+1
      SR(J)=SR(L)
      SI(J)=SI(L)
3200  CONTINUE

C     GENERATING QPSK DATA SYMBOLS

      XX=G05CAF(XX)
      IF (XX.LE.0.5) THEN
      SR(KMP)=-1.0
      ELSE
      SR(KMP)=1.0
      END IF
      XY=G05CAF(XY)
      IF (XY.LE.0.5) THEN
```

```
      SI(KMP)=-1.0
      ELSE
      SI(KMP)=1.0
      END IF

C     GENERATING NOISE AND CONVOLUTING WITH NOISE FILTER COEFFS

      DO 3300 J=1,KMP1
      L=J+1
      WDR(J)=WDR(L)
      WDI(J)=WDI(L)
3300  CONTINUE
      WDR(KMP)=G05DDF(0.0,STDVN)
      WDI(KMP)=G05DDF(0.0,STDVN)
      WR=0.0
      WI=0.0
      DO 3400 J=1,KMP
      K=KMP+1-J
      WR=WR+WDR(K)*WFR(J)-WDI(K)*WFI(J)
      WI=WI+WDR(K)*WFI(J)+WDI(K)*WFR(J)
3400  CONTINUE

C     CALCULATING RECEIVED SIGNAL

      RR=0.0
      RI=0.0
      DO 3500 J=1,KMP
      K=KMP+1-J
      RR=RR + SR(K)*YR(J)-SI(K)*YI(J)
      RI=RI + SR(K)*YI(J)+SI(K)*YR(J)
3500  CONTINUE
      RRR=RRR+RR*RR+RI*RI
      WWW=WWW+WR*WR+WI*WI

C     RECEIVED SIGNAL WITH NOISE

      RR=RR+WR
      RI=RI+WI

C     ESTIMATION OF RECEIVED SIGNAL

      ESTRR=0.0
      ESTRI=0.0
      DO 3600 J=1,KMP
      K=KMP+1-J
```

```
      ESTRR=ESTRR+SR(K)*Y1R(J)-SI(K)*Y1I(J)
      ESTRI=ESTRI+SR(K)*Y1I(J)+SI(K)*Y1R(J)
3600  CONTINUE

C     ERROR IN ESTIMATION OF RX'ED SIGNAL

      ERRRR=RR-ESTRR
      ERRRI=RI-ESTRI

C     UPDATING CHANNEL ESTIMATE USINF FEEDFORWARD EST

      DO 3700 J=1,KMP
      K=KMP+1-J
      ESTYR(J)=Y1R(J)+B*(ERRRR*SR(K)+ERRRI*SI(K))
      ESTYI(J)=Y1I(J)+B*(ERRRI*SR(K)-ERRRR*SI(K))
3700  CONTINUE

C     ERROR IN UPDATING

      DO 3800 J=1,KMP
      ERR(J)=ESTYR(J)-Y1R(J)
      ERI(J)=ESTYI(J)-Y1I(J)
3800  CONTINUE

C     PREDICTION USING DEGREE 1 LEAST SQUARES PREDICTION

      DO 3900 J=1,KMP
      DGRDR(J)=DGRDR(J)+0.5*((1-THETA)**3)*ERR(J)
      DGRDI(J)=DGRDI(J)+0.5*((1-THETA)**3)*ERI(J)
      GRDR(J)=GRDR(J)+2*DGRDR(J)+1.5*(((1-THETA)**2)
     1 *(1+THETA))*ERR(J)
      GRDI(J)=GRDI(J)+2*DGRDI(J)+1.5*(((1-THETA)**2)
     1 *(1+THETA))*ERI(J)
      Y1R(J)=Y1R(J)+GRDR(J)-DGRDR(J)+(1-THETA**3)*ERR(J)
      Y1I(J)=Y1I(J)+GRDI(J)-DGRDI(J)+(1-THETA**3)*ERI(J)
3900  CONTINUE


1400  CONTINUE


C     PRINTING RESULTS


      PRINT *,'ICOUNT=',ICOUNT
      ERRTOT=10.0*LOG10(ERRTOT/LCOUNT)
```

```
      ERRNORM=10.0*LOG10(ERRNORM/LCOUNT)
      PRINT*,'THETA=',THETA
      PRINT*,'B=',B
      PRINT*,'SNR=',SNR
      SNRE=10.0*LOG10(RRR/WWW)
      PRINT*,'SNRE=',SNRE
      PRINT*,'LCOUNT=',LCOUNT
      PRINT*,'MEAN SQ ERROR=',ERRTOT
      PRINT*,'NORM MEAN SQ ERROR=',ERRNORM
      CLOSE(25)
C1000 CONTINUE
      STOP
      END
```

APPENDIX D.3

PROGRAM THAT SIMULATES THE ADAPTIVE ESTIMATOR WITH DEGREE 2 LEAST SQUARES FADING MEMORY PREDICTION

```
      PROGRAM ADEST200
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DOUBLE PRECISION WFR(21),WFI(21),WDR(21),WDI(21)
      DOUBLE PRECISION YR(21),YI(21),Y1R(21),Y1I(21)
      DOUBLE PRECISION ESTYR(21),ESTYI(21)
      DOUBLE PRECISION SR(21),SI(21),GRDR(21),GRDI(21),ERR(21),ERI(21)
      DOUBLE PRECISION DGRDR(21),DGRDI(21)
      DOUBLE PRECISION B(21),BB(21)

C     VALUES TO VARIABLES AND ARRAYS

      INFD=70

C     CALL G05CBF(INFD)
      OPEN(11,FILE='AD2SR60')

      DATA WFR / -0.0280463, -0.2308071, -0.5075768, -0.4980021,
     1     -0.1622055,  0.1128849,  0.1085069, -0.0073049,
     1     -0.0486855, -0.0097627,  0.0151408,  0.0052313,
     1     -0.0044861,  0.0006332,  0.0010673, -0.0009344,
     1      0.0000000,  0.0000000,  0.0000000,  0.0000000,
     1      0.0000000 /
      DATA WFI /  0.0196809,  0.1674616,  0.3948080,  0.4056800,
     1      0.1050267, -0.1337521, -0.0735970,  0.0466914,
     1      0.0274074, -0.0185076, -0.0069768,  0.0109986,
     1      0.0028584, -0.0022137,  0.0013582, -0.0004508,
     1      0.0000000,  0.0000000,  0.0000000,  0.0000000,
     1      0.0000000 /

      KMP=21
      KMP1=KMP-1
C     DO 100 THETA=0.99,0.50,-0.05
C     DO 100 OK=0.15,0.05,-0.01
      THETA=0.981
      BEE=1.0
      ALFA=0.01
      OK=0.038
      CALL G05CBF(INFD)
      OPEN (25,FILE='CHNL1',FORM='UNFORMATTED')
      LCOUNT=0
```

```
      RRR=0.0
      WWW=0.0
      ERRTOT=0.0
      ERRNOM=0.0

      SNR=60.0
      STDVN=10.0**(-SNR/10.0)
      STDVN=SQRT(STDVN)

C     INITIALISING SR &SI

      DO 1100 J=1,KMP
      SR(J)=1.0
      SI(J)=1.0
      B(J)=0.0
      BB(J)=0.0
1100  CONTINUE

C     INITIALISING FIRST DERIVATIVE IN PREDICTION ALGORITHM

      DO 1200 J=1,KMP
      GRDR(J)=0.0
      GRDI(J)=0.0
1200  CONTINUE

C     INITIALISING SECOND DERIVATIVE IN PREDICTION ALGORITHM

      DO 1250 J=1,KMP
      DGRDR(J)=0.0
      DGRDI(J)=0.0
1250  CONTINUE

C     INITIALISING NOISE DATA WDR &WDI

      DO 1300 J=1,21
      WDR(J)=0.0
      WDI(J)=0.0
1300  CONTINUE


      DO 1405 ICOUNT=1,1000
      READ(25)(YR(J),J=1,21)
      READ(25)(YI(J),J=1,21)
1405  CONTINUE
```

```
      DO 1400 ICOUNT=1,59000
      READ(25)(YR(J),J=1,21)
      READ(25)(YI(J),J=1,21)

C     ESTIMATION OF IMPULSE RESPONSE

      IF (ICOUNT.EQ.1) THEN
      DO 3000 J=1,KMP
      Y1R(J)=YR(J)
      Y1I(J)=YI(J)
      BB(J)=Y1R(J)**2+Y1I(J)**2
3000  CONTINUE
      END IF

C     OBTAINING THE FADING MEMORY AVERAGE

      DO 3005 I=1,KMP
      YVAR=Y1R(I)**2+Y1I(I)**2
      BB(I)=(1.0-ALFA)*BB(I)+ALFA*YVAR
3005  CONTINUE

C     TRANSFORMING THE STEP SIZE USING THE RELATIONSHIP
C     GIVEN BY FIGURES 5.4.1 AND 5.5.1

      DO 3015 I=1,KMP
      B(I)=BB(I)**0.25
      IF(B(I).LT.OK) THEN
      B(I)=0.000001
      ENDIF
3015  CONTINUE

C     ERROR IN ESTIMATION OF CHANNEL IMPULSE RESPONSE

      IF (ICOUNT.GT.4000) THEN
      LCOUNT=LCOUNT+1
      YERR=0.0
      YTOT=0.0
      DO 3100 J=1,KMP
      YERR=YERR+(YR(J)-Y1R(J))**2+(YI(J)-Y1I(J))**2
      YTOT=YTOT+YR(J)**2+YI(J)**2
3100  CONTINUE
      ERRTOT=ERRTOT+YERR
      ERRNOM=ERRNOM+YERR/YTOT
      ENDIF
```

```
      IF(MOD(LCOUNT,15).EQ.1) THEN
      YERR=10.0*LOG10(YERR)
      WRITE(11,3150) LCOUNT,YERR
3150  FORMAT(I5,',',F9.4)
      END IF

C     SHIFTING OF ARRAYS

      DO 3200 J=1,KMP1
      L=J+1
      SR(J)=SR(L)
      SI(J)=SI(L)
3200  CONTINUE

C     GENERATING QPSK DATA SYMBOLS

      XX=G05CAF(XX)
      IF (XX.LE.0.5) THEN
      SR(KMP)=-1.0
      ELSE
      SR(KMP)=1.0
      END IF
      XY=G05CAF(XY)
      IF (XY.LE.0.5) THEN
      SI(KMP)=-1.0
      ELSE
      SI(KMP)=1.0
      END IF

C     GENERATING NOISE AND CONVOLUTING WITH NOISE FILTER COEFFS

      DO 3300 J=1,KMP1
      L=J+1
      WDR(J)=WDR(L)
      WDI(J)=WDI(L)
3300  CONTINUE
      WDR(KMP)=G05DDF(0.0,STDVN)
      WDI(KMP)=G05DDF(0.0,STDVN)
      WR=0.0
      WI=0.0
      DO 3400 J=1,KMP
      K=KMP+1-J
```

```
      WR=WR+WDR(K)*WFR(J)-WDI(K)*WFI(J)
      WI=WI+WDR(K)*WFI(J)+WDI(K)*WFR(J)
3400  CONTINUE

C     CALCULATING RECEIVED SIGNAL

      RR=0.0
      RI=0.0
      DO 3500 J=1,KMP
      K=KMP+1-J
      RR=RR + SR(K)*YR(J)-SI(K)*YI(J)
      RI=RI + SR(K)*YI(J)+SI(K)*YR(J)
3500  CONTINUE
      RRR=RRR+RR*RR+RI*RI
      WWW=WWW+WR*WR+WI*WI

C     RECEIVED SIGNAL WITH NOISE

      RR=RR+WR
      RI=RI+WI

C     ESTIMATION OF RECEIVED SIGNAL

      ESTRR=0.0
      ESTRI=0.0
      DO 3600 J=1,KMP
      K=KMP+1-J
      ESTRR=ESTRR+SR(K)*Y1R(J)-SI(K)*Y1I(J)
      ESTRI=ESTRI+SR(K)*Y1I(J)+SI(K)*Y1R(J)
3600  CONTINUE

C     ERROR IN ESTIMATION OF RX'ED SIGNAL

      ERRRR=RR-ESTRR
      ERRRI=RI-ESTRI

C     UPDATING CHANNEL ESTIMATE USINF FEEDFORWARD EST

      DO 3700 J=1,KMP
      K=KMP+1-J
      ESTYR(J)=Y1R(J)+B(J)*BEE*(ERRRR*SR(K)+ERRRI*SI(K))
      ESTYI(J)=Y1I(J)+B(J)*BEE*(ERRRI*SR(K)-ERRRR*SI(K))
3700  CONTINUE

C     ERROR IN UPDATING
```

```
      DO 3800 J=1,KMP
      ERR(J)=ESTYR(J)-Y1R(J)
      ERI(J)=ESTYI(J)-Y1I(J)
3800  CONTINUE

C     PREDICTION USING DEGREE 2 LEAST SQUARES PREDICTION

      DO 3900 J=1,KMP
      DGRDR(J)=DGRDR(J)+0.5*((1-THETA)**3)*ERR(J)
      DGRDI(J)=DGRDI(J)+0.5*((1-THETA)**3)*ERI(J)
      GRDR(J)=GRDR(J)+2*DGRDR(J)+1.5*(((1-THETA)**2)
     1 *(1+THETA))*ERR(J)
      GRDI(J)=GRDI(J)+2*DGRDI(J)+1.5*(((1-THETA)**2)
     1 *(1+THETA))*ERI(J)
      Y1R(J)=Y1R(J)+GRDR(J)-DGRDR(J)+(1-THETA**3)*ERR(J)
      Y1I(J)=Y1I(J)+GRDI(J)-DGRDI(J)+(1-THETA**3)*ERI(J)
3900  CONTINUE


1400  CONTINUE


C     PRINTING RESULTS


      ERRTOT=10.0*LOG10(ERRTOT/LCOUNT)
      ERRNOM=10.0*LOG10(ERRNOM/LCOUNT)
      PRINT*,'OK=',OK
      PRINT*,'THETA=',THETA
      PRINT*,'BEE=',BEE
      PRINT*,'SNR=',SNR
      SNRE=10.0*LOG10(RRR/WWW)
      PRINT*,'MEAN SQ ERROR=',ERRTOT
      PRINT *,'NORM.MEAN SQ ERROR=',ERRNOM
      PRINT*,'SNRE=',SNRE
      CLOSE(25)
C 100 CONTINUE
      STOP
      END
```

APPENDIX D.4

PROGRAM THAT SIMULATES THE MODIFIED ESTIMATOR WITH THE MAIN PREDICTION AS DEGREE 1 LEAST SQUARES FADING MEMORY PREDICTION AND THE SECONDARY PREDICTION AS DEGREE 1 LEAST SQUARES FADING MEMORY PREDICTION

```
      PROGRAM MD11EST0

      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DOUBLE PRECISION WFR(21),WFI(21),WDR(21),WDI(21)
      DOUBLE PRECISION YR(21),YI(21),Y1R(21),Y1I(21)
      DOUBLE PRECISION ESTYR(21),ESTYI(21),ESTMYR(21),ESTMYI(21)
      DOUBLE PRECISION SR(21),SI(21),GRDR(21),GRDI(21),ERR(21),ERI(21)
      DOUBLE PRECISION ERMR(21),ERMI(21)
      DOUBLE PRECISION ER1R(21),ER1I(21),FRDR(21),FRDI(21)
      DOUBLE PRECISION FERR(21),FERI(21)

C     VALUES TO VARIABLES AND ARRAYS

      INFD=70
      OPEN(11,FILE='MD11SR60')

C     CALL G05CBF(INFD)

      DATA WFR / -0.0280463, -0.2308071, -0.5075768, -0.4980021,
     1     -0.1622055, 0.1128849, 0.1085069, -0.0073049,
     1     -0.0486855, -0.0097627, 0.0151408, 0.0052313,
     1     -0.0044861, 0.0006332, 0.0010673, -0.0009344,
     1      0.0000000, 0.0000000, 0.0000000, 0.0000000,
     1      0.0000000 /
      DATA WFI / 0.0196809, 0.1674616, 0.3948080, 0.4056800,
     1      0.1050267, -0.1337521, -0.0735970, 0.0466914,
     1      0.0274074, -0.0185076, -0.0069768, 0.0109986,
     1      0.0028584, -0.0022137, 0.0013582, -0.0004508,
     1      0.0000000, 0.0000000, 0.0000000, 0.0000000,
     1      0.0000000 /
      KMP=21
      KMP1=KMP-1
      B=0.103
      THETA=0.909
      THETA2=0.138
      CALL G05CBF(INFD)
      OPEN (25,FILE='CHNL1',FORM='UNFORMATTED')
      LCOUNT=0
```

```
      RRR=0.0
      WWW=0.0
      ERRTOT=0.0


      SNR=60.0
      STDVN=10.0**(-SNR/10.0)
      STDVN=SQRT(STDVN)

C     INITIALISING SR &SI

      DO 1100 J=1,KMP
      SR(J)=1.0
      SI(J)=1.0
1100  CONTINUE

C     INITIALISING FIRST DERIVATIVE IN PREDICTION ALGORITHM

      DO 1200 J=1,KMP
      GRDR(J)=0.0
      GRDI(J)=0.0
1200  CONTINUE

C     INITIALISING DERIVATIVE IN THE SECOND PREDICTOR

      DO 1250 J=1,21
      FRDR(J)=0.0
      FRDI(J)=0.0
1250  CONTINUE

C     INITIALISING NOISE DATA WDR &WDI

      DO 1300 J=1,21
      WDR(J)=0.0
      WDI(J)=0.0
1300  CONTINUE


      DO 1400 ICOUNT=1,60000
      READ(25)(YR(J),J=1,21)
      READ(25)(YI(J),J=1,21)


C     ESTIMATION OF IMPULSE RESPONSE

C     AT START UP OF TRANSMISSION
```

```
      IF (ICOUNT.EQ.1) THEN
      DO 3000 J=1,KMP
      Y1R(J)=YR(J)
      Y1I(J)=YI(J)
3000  CONTINUE
      END IF

C     ERROR IN ESTIMATION OF CHANNEL IMPULSE RESPONSE

      IF (ICOUNT.GT.5000) THEN
      LCOUNT=LCOUNT+1
      YERR=0.0
      YTOT=0.0
      DO 3100 J=1,KMP
      YERR=YERR+(YR(J)-Y1R(J))**2+(YI(J)-Y1I(J))**2
      YTOT=YTOT+(YR(J)**2+YI(J)**2)
3100  CONTINUE
      ERRTOT=ERRTOT+YERR

      IF (MOD(LCOUNT,15).EQ.1) THEN
      YERR=10.0*LOG10(YERR)
      WRITE(11,3150) LCOUNT,YERR
3150  FORMAT(I5,',',F9.4)
      END IF

      END IF

C     SHIFTING OF ARRAYS

      DO 3200 J=1,KMP1
      L=J+1
      SR(J)=SR(L)
      SI(J)=SI(L)
3200  CONTINUE

C     GENERATING QPSK DATA SYMBOLS

      XX=G05CAF(XX)
      IF (XX.LE.0.5) THEN
      SR(KMP)=-1.0
      ELSE
      SR(KMP)=1.0
      END IF
      XY=G05CAF(XY)
      IF (XY.LE.0.5) THEN
      SI(KMP)=-1.0
```

```
      ELSE
      SI(KMP)=1.0
      END IF

C     GENERATING NOISE AND CONVOLUTING WITH NOISE FILTER COEFFS

      DO 3300 J=1,KMP1
      L=J+1
      WDR(J)=WDR(L)
      WDI(J)=WDI(L)
3300  CONTINUE
      WDR(KMP)=G05DDF(0.0,STDVN)
      WDI(KMP)=G05DDF(0.0,STDVN)
      WR=0.0
      WI=0.0
      DO 3400 J=1,KMP
      K=KMP+1-J
      WR=WR+WDR(K)*WFR(J)-WDI(K)*WFI(J)
      WI=WI+WDR(K)*WFI(J)+WDI(K)*WFR(J)
3400  CONTINUE

C     CALCULATING RECEIVED SIGNAL

      RR=0.0
      RI=0.0
      DO 3500 J=1,KMP
      K=KMP+1-J
      RR=RR + SR(K)*YR(J)-SI(K)*YI(J)
      RI=RI + SR(K)*YI(J)+SI(K)*YR(J)
3500  CONTINUE
      RRR=RRR+RR*RR+RI*RI
      WWW=WWW+WR*WR+WI*WI

C     RECEIVED SIGNAL WITH NOISE

      RR=RR+WR
      RI=RI+WI

C     ESTIMATION OF RECEIVED SIGNAL

      ESTRR=0.0
      ESTRI=0.0
      DO 3600 J=1,KMP
      K=KMP+1-J
```

```
      ESTRR=ESTRR+SR(K)*Y1R(J)-SI(K)*Y1I(J)
      ESTRI=ESTRI+SR(K)*Y1I(J)+SI(K)*Y1R(J)
3600  CONTINUE

C     ERROR IN ESTIMATION OF RX'ED SIGNAL

      ERRRR=RR-ESTRR
      ERRRI=RI-ESTRI

C     UPDATING CHANNEL ESTIMATE USINF FEEDFORWARD EST

      DO 3700 J=1,KMP
      K=KMP+1-J
      ESTYR(J)=Y1R(J)+B*(ERRRR*SR(K)+ERRRI*SI(K))
      ESTYI(J)=Y1I(J)+B*(ERRRI*SR(K)-ERRRR*SI(K))
3700  CONTINUE

C     ERROR IN UPDATING

      DO 3800 J=1,KMP
      ERR(J)=ESTYR(J)-Y1R(J)
      ERI(J)=ESTYI(J)-Y1I(J)
3800  CONTINUE

C     AT START UP FOR ICOUNT=1, ERR=ER1R  ETC

      IF (ICOUNT.EQ.1) THEN
      DO 3850 J=1,KMP
      ER1R(J)=ERR(J)
      ER1I(J)=ERI(J)
3850  CONTINUE
      END IF

C     ERROR IN PREDICTION OF E

      DO 3900 J=1,KMP
      FERR(J)=ERR(J)-ER1R(J)
      FERI(J)=ERI(J)-ER1I(J)
3900  CONTINUE

C     UPDATING CHANNEL ESTIMATE

      DO 4100 J=1,KMP
      ESTMYR(J)=Y1R(J)+ER1R(J)+(1-THETA2**2)*FERR(J)
      ESTMYI(J)=Y1I(J)+ER1I(J)+(1-THETA2**2)*FERI(J)
4100  CONTINUE
```

```
C       SECONDARY PREDICTION

C       ONE STEP PREDICTION OF E USING DEGREE 1

        DO 4000 J=1,KMP
        FRDR(J)=FRDR(J)+((1-THETA2)**2)*FERR(J)
        FRDI(J)=FRDI(J)+((1-THETA2)**2)*FERI(J)
        ER1R(J)=ER1R(J)+FRDR(J)+(1-THETA2**2)*FERR(J)
        ER1I(J)=ER1I(J)+FRDI(J)+(1-THETA2**2)*FERI(J)
4000    CONTINUE


C       ERROR IN UPDATING

        DO 4200 J=1,KMP
        ERMR(J)=ESTMYR(J)-Y1R(J)
        ERMI(J)=ESTMYI(J)-Y1I(J)
4200    CONTINUE


C       MAIN PREDICTION

C       PREDICTION USING DEGREE 1 LEAST SQUARES PREDICTION

        DO 5000 J=1,KMP
        GRDR(J)=GRDR(J)+((1-THETA)**2)*ERMR(J)
        GRDI(J)=GRDI(J)+((1-THETA)**2)*ERMI(J)
        Y1R(J)=Y1R(J)+GRDR(J)+(1-THETA**2)*ERMR(J)
        Y1I(J)=Y1I(J)+GRDI(J)+(1-THETA**2)*ERMI(J)
5000    CONTINUE


1400    CONTINUE

C       PRINTING RESULTS


        ERRTOT=10.0*LOG10(ERRTOT/LCOUNT)

C       ERRNORM=10.0*LOG10(ERRNORM/LCOUNT)

        PRINT*,'THETA2=',THETA2
        PRINT*,'THETA=',THETA
        PRINT*,'B=',B
```

```
      PRINT*,'SNR=',SNR
      SNRE=10.0*LOG10(RRR/WWW)
      PRINT*,'MEAN SQ ERROR=',ERRTOT

C     PRINT*,'NORMALIZED MEAN SQ EROR=',ERRNORM

      PRINT*,'SNRE=',SNRE
      CLOSE(25)

C100  CONTINUE

      STOP
      END
```

APPENDIX D.5

PROGRAM OBTAINS MINIMUM PHASE CHANNEL 4 FROM CHANNEL 4 USING NAG ROUTINES

```
      PROGRAM CHANEL4
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      PARAMETER (KMCONV=30)
      DOUBLE PRECISION YR(KMCONV),YI(KMCONV)
      DOUBLE PRECISION YNDIVR(KMCONV),YNDIVI(KMCONV)
      DOUBLE PRECISION YCOFR(KMCONV),YCOFI(KMCONV)
      DOUBLE PRECISION REZ(KMCONV),IMZ(KMCONV)
      DOUBLE PRECISION DUMR(KMCONV),DUMI(KMCONV)
      DOUBLE PRECISION YNAGR(KMCONV),YNAGI(KMCONV)

      OPEN(11,FILE='YREALCH4')
      OPEN(12,FILE='YIMGCH4')
      OPEN(8,FILE='YRMPCH4')
      OPEN(9,FILE='YIMPCH4')
      OPEN(13,FILE='ZREALCH4')
      OPEN(14,FILE='ZIMGCH4')
      OPEN(15,FILE='ROOTCH4')

C     VALUES TO VARIABLES AND ARRAYS

C     READ IN THE TELEPHONE CHANNEL 4

      READ(11,1000) (YR(J),J=1,KMCONV)
      READ(12,1000) (YI(J),J=1,KMCONV)
 1000 FORMAT(F8.4)

      ISEED=10
      UCIRC=1.05
      CALL G05CBF(ISEED)

      DO 1030 I=1,KMCONV
      REZ(I)=0.0
      IMZ(I)=0.0
 1030 CONTINUE


C     OBTAINING ROOTS OF Z-TRANSFORM OF CHANNEL

      DO 3000 I=1,KMCONV
      YNAGR(I)=YR(I)
      YNAGI(I)=YI(I)
 3000 CONTINUE
```

```
      NROOT=KMCONV
      TOL=X02AAF(1.0)
      IFAIL=0
      CALL C02ADF(YNAGR,YNAGI,NROOT,REZ,IMZ,TOL,IFAIL)

C     MANIPULATION OF ROOTS TO MAKE CHANNEL MINIMUM PHASE

      RPROD=1.0
      DO 3010 I=1,KMCONV
      RSQMOD=REZ(I)*REZ(I)+IMZ(I)*IMZ(I)
      RMOD=RSQMOD**0.5
      WRITE(13,3006) REZ(I)
      WRITE(14,3006) IMZ(I)
      WRITE(15,3006) RMOD
3006  FORMAT(F12.9)
      IF(RMOD.GT.UCIRC)THEN
      RPROD=RPROD*RMOD
      REZ(I)=REZ(I)/RSQMOD
      IMZ(I)=IMZ(I)/RSQMOD
      ELSE
      GO TO 3010
      END IF
3010  CONTINUE

C     RECONSTRUCTION OF CHANNEL

      DO 3020 I=1,KMCONV
      YCOFR(I)=0.0
      YCOFI(I)=0.0
3020  CONTINUE
      YCOFR(1)=1.0

      DO 3050 I=1,KMCONV-1
         DO 3030 J=1,KMCONV-1
         DUMR(J)=YCOFR(J)*REZ(I)-YCOFI(J)*IMZ(I)
         DUMI(J)=YCOFR(J)*IMZ(I)+YCOFI(J)*REZ(I)
3030     CONTINUE
         DO 3040 JJ=1,KMCONV-1
         JJ1=JJ+1
         YCOFR(JJ1)=YCOFR(JJ1)+DUMR(JJ)
         YCOFI(JJ1)=YCOFI(JJ1)+DUMI(JJ)
3040     CONTINUE
3050  CONTINUE
```

```
      DO 3060 I=2,KMCONV,2
      YCOFR(I)=-YCOFR(I)
      YCOFI(I)=-YCOFI(I)
3060  CONTINUE

      DO 3070 I=1,KMCONV
      YNDIVR(I)=(YCOFR(I)*YR(1)-YCOFI(I)*YI(1))*RPROD
      YNDIVI(I)=(YCOFR(I)*YI(1)+YCOFI(I)*YR(1))*RPROD
3070  CONTINUE


      YNDIVIR=YNDIVR(1)**2+YNDIVI(1)**2
      PRINT*,'YNDIVIR=',YNDIVIR

      DO 3075 I=KMCONV,1,-1
      YSUMR=0.0
      YSUMI=0.0
      YSUMR=(YNDIVR(I)*YNDIVR(1)+YNDIVI(I)*YNDIVI(1))/YNDIVIR
      YSUMI=(YNDIVI(I)*YNDIVR(1)-YNDIVR(I)*YNDIVI(1))/YNDIVIR
      YNDIVR(I)=YSUMR
      YNDIVI(I)=YSUMI
3075  CONTINUE


      WRITE(8,3080)(YNDIVR(I),I=1,KMCONV)
      WRITE(9,3080)(YNDIVI(I),I=1,KMCONV)
3080  FORMAT(F16.12)


C     CHECKING THE SUM OF THE SQUARES

      YSQ=0.0
      YYSQ=0.0
      DO 3090 I=1,KMCONV
      YSQ=YSQ+(YR(I)*YR(I)+YI(I)*YI(I))
      YYSQ=YYSQ+(YNDIVR(I)*YNDIVR(I)+YNDIVI(I)*YNDIVI(I))
3090  CONTINUE
      YDBQ=10.0*LOG10(YYSQ)


C     PRINTING RESULTS


      PRINT*,'YSQ=',YSQ
      PRINT*,'YYSQ=',YYSQ
      STOP
      END
```

APPENDIX D.6

PROGRAM OBTAINS CHANNEL4J FROM CHANNEL4 WITH ROOTS CLOSEST TO UNIT CIRCLE REMOVED

```
      PROGRAM CHANEL4J
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      PARAMETER (KMCONV=30)
      DOUBLE PRECISION YR(KMCONV),YI(KMCONV)
      DOUBLE PRECISION YNDIVR(KMCONV),YNDIVI(KMCONV)
      DOUBLE PRECISION YCOFR(KMCONV),YCOFI(KMCONV)
      DOUBLE PRECISION REZ(KMCONV),IMZ(KMCONV)
      DOUBLE PRECISION DUMR(KMCONV),DUMI(KMCONV)
      DOUBLE PRECISION YNAGR(KMCONV),YNAGI(KMCONV)
      DOUBLE PRECISION TREZ(KMCONV),TIMZ(KMCONV)

      OPEN(11,FILE='YREALCH4')
      OPEN(12,FILE='YIMGCH4')
      OPEN(13,FILE='ZRLCH4J')
      OPEN(14,FILE='ZIMGCH4J')
      OPEN(15,FILE='ROOTCH4J')
      OPEN(16,FILE='YRLCH4J')
      OPEN(17,FILE='YIMGCH4J')

C     VALUES TO VARIABLES AND ARRAYS

C     READ IN THE TELEPHONE CHANNEL 4

      READ(11,1000) (YR(J),J=1,KMCONV)
      READ(12,1000) (YI(J),J=1,KMCONV)
1000  FORMAT(F8.4)

      ISEED=10
      UCIRC=0.9
      UCIRC2=1.10
      CALL G05CBF(ISEED)

      DO 1030 I=1,KMCONV
      REZ(I)=0.0
      IMZ(I)=0.0
1030  CONTINUE


C     OBTAINING ROOTS OF Z-TRANSFORM OF CHANNEL
```

```
      DO 3000 I=1,KMCONV
      YNAGR(I)=YR(I)
      YNAGI(I)=YI(I)
3000  CONTINUE
      NROOT=KMCONV
      TOL=X02AAF(1.0)
      IFAIL=0
      CALL C02ADF(YNAGR,YNAGI,NROOT,REZ,IMZ,TOL,IFAIL)

C     MANIPULATION OF ROOTS TO MAKE CHANNEL MINIMUM PHASE

      K=1
      DO 3010 I=1,KMCONV
      RSQMOD=REZ(I)*REZ(I)+IMZ(I)*IMZ(I)
      RMOD=RSQMOD**0.5
      WRITE(13,3006) REZ(I)
      WRITE(14,3006) IMZ(I)
      WRITE(15,3006) RMOD
3006  FORMAT(F16.12)
      IF((RMOD.LE.UCIRC).OR.(RMOD.GE.UCIRC2)) THEN
      TREZ(K)=REZ(I)
      TIMZ(K)=IMZ(I)
      K = K + 1
      ELSE
      GO TO 3010
      END IF
3010  CONTINUE

      KMCONV2=KMCONV-1

C     RECONSTRUCTION OF CHANNEL

      DO 3020 I=1,KMCONV2
      YCOFR(I)=0.0
      YCOFI(I)=0.0
3020  CONTINUE
      YCOFR(1)=1.0

      DO 3050 I=1,KMCONV2-1
        DO 3030 J=1,KMCONV2-1
        DUMR(J)=YCOFR(J)*TREZ(I)-YCOFI(J)*TIMZ(I)
        DUMI(J)=YCOFR(J)*TIMZ(I)+YCOFI(J)*TREZ(I)
3030    CONTINUE
```

```
      DO 3040 JJ=1,KMCONV2-1
      JJ1=JJ+1
      YCOFR(JJ1)=YCOFR(JJ1)+DUMR(JJ)
      YCOFI(JJ1)=YCOFI(JJ1)+DUMI(JJ)
3040  CONTINUE
3050  CONTINUE


      DO 3060 I=2,KMCONV2,2
      YCOFR(I)=-YCOFR(I)
      YCOFI(I)=-YCOFI(I)
3060  CONTINUE

      DO 3070 I=1,KMCONV2
      YNDIVR(I)=(YCOFR(I)*YR(1)-YCOFI(I)*YI(1))
      YNDIVI(I)=(YCOFR(I)*YI(1)+YCOFI(I)*YR(1))
3070  CONTINUE


C     YNDIVIR=YNDIVR(1)**2+YNDIVI(1)**2
C     PRINT*,'YNDIVIR=',YNDIVIR

C     DO 3075 I=KMCONV,1,-1
C     YSUMR=0.0
C     YSUMI=0.0
C     YSUMR=(YNDIVR(I)*YNDIVR(1)+YNDIVI(I)*YNDIVI(1))/YNDIVIR
C     YSUMI=(YNDIVI(I)*YNDIVR(1)-YNDIVR(I)*YNDIVI(1))/YNDIVIR
C     YNDIVR(I)=YSUMR
C     YNDIVI(I)=YSUMI
C3075 CONTINUE


      WRITE(16,3080)(YNDIVR(I),I=1,KMCONV2)
      WRITE(17,3080)(YNDIVI(I),I=1,KMCONV2)
3080  FORMAT(F16.12)


C     CHECKING THE SUM OF THE SQUARES

      YSQ=0.0
      YYSQ=0.0
      DO 3090 I=1,KMCONV
      YSQ=YSQ+(YR(I)*YR(I)+YI(I)*YI(I))
3090  CONTINUE
```

```
      DO 3100 I=1,KMCONV2
      YYSQ=YYSQ+(YNDIVR(I)*YNDIVR(I)+YNDIVI(I)*YNDIVI(I))
3100  CONTINUE

C     PRINTING RESULTS

      PRINT*,'YSQ=',YSQ
      PRINT*,'YYSQ=',YYSQ
      STOP
      END
```

APPENDIX D.7

PROGRAM GENERATES CHANNEL 3H FROM CHANNEL 3 WITH AN EXTRA ROOT LYING ON THE UNIT CIRCLE

```
      PROGRAM CHANEL3H
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      PARAMETER (KMCONV=27)
      DOUBLE PRECISION YR(KMCONV),YI(KMCONV)
      DOUBLE PRECISION YNDIVR(KMCONV),YNDIVI(KMCONV)
      DOUBLE PRECISION YCOFR(KMCONV),YCOFI(KMCONV)
      DOUBLE PRECISION REZ(KMCONV),IMZ(KMCONV)
      DOUBLE PRECISION DUMR(KMCONV),DUMI(KMCONV)
      DOUBLE PRECISION YNAGR(KMCONV),YNAGI(KMCONV)
      DOUBLE PRECISION TYR(KMCONV),TYI(KMCONV)

      OPEN(11,FILE='YREALCH3')
      OPEN(12,FILE='YIMGCH3')
      OPEN(8,FILE='YRMPCH3H')
      OPEN(9,FILE='YIMPCH3H')
      OPEN(13,FILE='ZRLCH3H')
      OPEN(14,FILE='ZIMGCH3H')
      OPEN(15,FILE='ROOTCH3H')
      OPEN(16,FILE='YRLCH3H')
      OPEN(17,FILE='YIMGCH3H')

C     VALUES TO VARIABLES AND ARRAYS

C     READ IN THE TELEPHONE CHANNEL 3

      READ(11,1000) (YR(J),J=1,26)
      READ(12,1000) (YI(J),J=1,26)
1000  FORMAT(F8.4)

      ISEED=10
      UCIRC=1.05
      UCIRC2=10.0
      CALL G05CBF(ISEED)

      DO 1030 I=1,KMCONV
      REZ(I)=0.0
      IMZ(I)=0.0
1030  CONTINUE
```

```
      SQR2=SQRT(2.0)
      ROOTUNIT=1/SQR2
      B30R=-ROOTUNIT
      B30I=-ROOTUNIT

      DO 1050 I=1,KMCONV-1
      TYR(I)=YR(I)
      TYI(I)=YI(I)
1050  CONTINUE

      TYR(KMCONV)=0.0
      TYI(KMCONV)=0.0

      DO 1100 I=2,KMCONV
      TYR(I)=TYR(I) + B30R*YR(I-1) - B30I*YI(I-1)
      TYI(I)=TYI(I) + B30R*YI(I-1) + B30I*YR(I-1)
1100  CONTINUE

      DO 1125 I=1,KMCONV
      WRITE(16,1150) TYR(I)
      WRITE(17,1150) TYI(I)
1150  FORMAT(F16.12)
1125  CONTINUE


C     OBTAINING ROOTS OF Z-TRANSFORM OF CHANNEL

      DO 3000 I=1,KMCONV
      YNAGR(I)=TYR(I)
      YNAGI(I)=TYI(I)
3000  CONTINUE
      NROOT=KMCONV
      TOL=X02AAF(1.0)
      IFAIL=0
      CALL C02ADF(YNAGR,YNAGI,NROOT,REZ,IMZ,TOL,IFAIL)


C     MANIPULATION OF ROOTS TO MAKE CHANNEL MINIMUM PHASE


      RPROD=1.0
      DO 3010 I=1,KMCONV
      RSQMOD=REZ(I)*REZ(I)+IMZ(I)*IMZ(I)
      RMOD=RSQMOD**0.5
      WRITE(13,3006) REZ(I)
      WRITE(14,3006) IMZ(I)
```

```
      WRITE(15,3006) RMOD
3006  FORMAT(F16.12)
      IF (RMOD.GT.UCIRC) THEN
      RPROD=RPROD*RMOD
      REZ(I)=REZ(I)/RSQMOD
      IMZ(I)=IMZ(I)/RSQMOD
      ELSE
      GO TO 3010
      END IF
3010  CONTINUE


C     RECONSTRUCTION OF CHANNEL

      DO 3020 I=1,KMCONV
      YCOFR(I)=0.0
      YCOFI(I)=0.0
3020  CONTINUE
      YCOFR(1)=1.0

      DO 3050 I=1,KMCONV-1
        DO 3030 J=1,KMCONV-1
        DUMR(J)=YCOFR(J)*REZ(I)-YCOFI(J)*IMZ(I)
        DUMI(J)=YCOFR(J)*IMZ(I)+YCOFI(J)*REZ(I)
3030    CONTINUE
        DO 3040 JJ=1,KMCONV-1
        JJ1=JJ+1
        YCOFR(JJ1)=YCOFR(JJ1)+DUMR(JJ)
        YCOFI(JJ1)=YCOFI(JJ1)+DUMI(JJ)
3040    CONTINUE
3050  CONTINUE


      DO 3060 I=2,KMCONV,2
      YCOFR(I)=-YCOFR(I)
      YCOFI(I)=-YCOFI(I)
3060  CONTINUE

      DO 3070 I=1,KMCONV
      YNDIVR(I)=(YCOFR(I)*TYR(1)-YCOFI(I)*TYI(1))*RPROD
      YNDIVI(I)=(YCOFR(I)*TYI(1)+YCOFI(I)*TYR(1))*RPROD
3070  CONTINUE


      YNDIVIR=YNDIVR(1)**2+YNDIVI(1)**2
      PRINT*,'YNDIVIR=',YNDIVIR
```

```
      DO 3075 I=KMCONV,1,-1
      YSUMR=0.0
      YSUMI=0.0
      YSUMR=(YNDIVR(I)*YNDIVR(1)+YNDIVI(I)*YNDIVI(1))/YNDIVIR
      YSUMI=(YNDIVI(I)*YNDIVR(1)-YNDIVR(I)*YNDIVI(1))/YNDIVIR
      YNDIVR(I)=YSUMR
      YNDIVI(I)=YSUMI
3075  CONTINUE

      WRITE(8,3080)(YNDIVR(I),I=1,KMCONV)
      WRITE(9,3080)(YNDIVI(I),I=1,KMCONV)
3080  FORMAT(F16.12)

C     CHECKING THE SUM OF THE SQUARES

      YSQ=0.0
      YYSQ=0.0
      DO 3090 I=1,KMCONV
      YSQ=YSQ+(TYR(I)*TYR(I)+TYI(I)*TYI(I))
      YYSQ=YYSQ+(YNDIVR(I)*YNDIVR(I)+YNDIVI(I)*YNDIVI(I))
3090  CONTINUE
      YDBQ=10.0*LOG10(YYSQ)

C     PRINTING RESULTS

      PRINT*,'YSQ=',YSQ
      PRINT*,'YYSQ=',YYSQ
      STOP
      END
```

APPENDIX D.8

PROGRAM SIMULATES THE FILTER WITH CHANNEL 4 AS THE CHANNEL

```
      PROGRAM ADFCH4
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      PARAMETER (NT=80,NY=79)
      DOUBLE PRECISION ZR(-30:NY,NT),ZI(-30:NY,NT)
      DOUBLE PRECISION YR(-30:NY,NT),YI(-30:NY,NT)
      DOUBLE PRECISION ZYR(-30:NY),ZYI(-30:NY)
      DOUBLE PRECISION ZYPRODR(0:NY,0:NY),ZYPRODI(0:NY,0:NY)
      DOUBLE PRECISION ZZPRODR(0:NY),ZZPRODI(0:NY)
      DOUBLE PRECISION ACTMPYR(-30:0),ACTMPYI(-30:0)
C     CALL XUFLOW(0)

      OPEN(18,FILE='MAGZCH4S')
      OPEN(11,FILE='YREALCH4')
      OPEN(12,FILE='YIMGCH4')
      OPEN(13,FILE='GRMPCH4S')
      OPEN(14,FILE='GIMPCH4S')
      OPEN(15,FILE='YRMPCH4')
      OPEN(16,FILE='YIMPCH4')
      OPEN(17,FILE='DPRCH4S')
      OPEN(10,FILE='Z0CH4S')

C     NO OF TAPS=NT
C     NOS OF CHANNEL IMPULSE REPONSE(I.E Y)=NY
C     NT=50
C     NY=49
C     INITIALISING YR AND YI TO ZERO

      DO 100 J=0,NY
      DO 200 I=1,NT
      YR(J,I)=0.0
      YI(J,I)=0.0
200   CONTINUE
100   CONTINUE

C     READING IN THE ACTUAL MINIMUM PHASED CHANNEL FOR CHANNEL4

      READ(15,150) (ACTMPYR(I),I=0,-29,-1)
      READ(16,150) (ACTMPYI(I),I=0,-29,-1)
150   FORMAT(F16.12)

C     READING IN THE TELEPHONE CHANNEL 4 WHICH HAS 30 COMPONENTS
```

```
      READ(11,250) (YR(0,I),I=NT,NT-29,-1)
      READ(12,250) (YI(0,I),I=NT,NT-29,-1)
250   FORMAT(F8.4)

      SUM=0.0
      DO 260 I=1,NT
      SUM=SUM+YR(0,I)**2+YI(0,I)**2
260   CONTINUE
      PRINT*,'SUM=',SUM

C     ****************************************************************
C     OBTAINING THE Y MATRIX WHICH IS AN NT+29 X NT MATRIX
C     ****************************************************************

C     Y30 0 ............................ 0
C     .                 .                .
C     .                 .                .
C     Y1 Y2 Y3..........Y30  0...........0
C     0   Y1 Y2 Y3.......Y29 Y30  0......0
C     .                 .                .
C     .                 .                .
C     0...............0  Y1  Y2..........Y30
C Y0 = 0..................0   Y1  Y2 ...... Y29
C Y-1 = .                   .               .
C     .                   .               .
C     0................................0  Y1

      DO 300 I=1,NY
      DO 400 J=2,NT
      YR(I,J-1)=YR(I-1,J)
      YI(I,J-1)=YI(I-1,J)
      YR(I,NT)=0.0
      YI(I,NT)=0.0
400   CONTINUE
300   CONTINUE

      DO 350 I=-1,-29,-1
      DO 450 J=NT-1,1,-1
      YR(I,J+1)=YR(I+1,J)
      YI(I,J+1)=YI(I+1,J)
450   CONTINUE
350   CONTINUE
```

```
C       ****************************************************
C       GRAM SCHMIDT ORTHOGONALIZATION PROCESS
C       ****************************************************

C       Y(0), Y(1),.., Y(NY-29) ARE NOT ORTOGONAL SO THE GRAM SCHMIDT
C       PROCESS IS USED TO OBTAIN Z(0), Z(1),.., Z(NY-29) WHICH ARE
C       ORTHOGONAL Z(0), Z(1),.., Z(NY-29) ARE THE ORTHOGONAL VECTORS

C       NEXT DO LOOP SETS ALL ZYPRODS AND ZZPRODS TO ZERO
C       ZYPRODR(J,K) IS THE REAL PART OF Z(J).Y(K)
C       ZZPRODR(J,K) IS REAL PART OF Z(J).Z(K)
C       ZZPRODI(J,K) IS THE IMAGINARY PART OF Z(J).Z(K)

        DO 500 J=0,NY
        DO 600 I=0,NT-1
        ZYPRODR(J,I)=0.0
        ZYPRODI(J,I)=0.0
600     CONTINUE
        ZZPRODR(J)=0.0
        ZZPRODI(J)=0.0
500     CONTINUE

        DO 700 J=NY-29,0,-1
        IF (J.EQ.(NY-29)) THEN

        DO 800 I=1,NT
        ZR(J,I)=YR(J,I)
        ZI(J,I)=YI(J,I)
800     CONTINUE

        ELSE

        DO 900 I=1,NT
        SUM1R=0.0
        SUM1I=0.0

        DO 1000 K=J+1,NY-29
        A=ZYPRODR(K,J)
        B=ZYPRODI(K,J)
        C=ZZPRODR(K)
        D=ZZPRODI(K)

        GSREAL=((A*C+B*D)/(C**2+D**2))
        GSIMG=((B*C-A*D)/(C**2+D**2))
C       GSREAL=A/C
C       GSIMG=B/C
```

```
C       PRINT*,'A=',A
C       PRINT*,'B=',B
C       PRINT*,'C=',C
C       PRINT*,'GSREAL=',GSREAL
C       PRINT*,'GSIMG=',GSIMG

        SUM1R=SUM1R+(GSREAL*ZR(K,I)-GSIMG*ZI(K,I))
        SUM1I=SUM1I+(GSREAL*ZI(K,I)+GSIMG*ZR(K,I))
 1000   CONTINUE

C       PRINT*,'J=',J
C       PRINT*,'I=',I
C       PRINT*,'SUM1R=',SUM1R
C       PRINT*,'SUM1I=',SUM1I
C       PRINT*,'YR(J,I)=',YR(J,I)
C       PRINT*,'YI(J,I)=',YI(J,I)
C       PRINT*,'ZR(J,I)=',ZR(J,I)
C       PRINT*,'ZI(J,I)=',ZI(J,I)

        ZR(J,I)=YR(J,I)-SUM1R
        ZI(J,I)=YI(J,I)-SUM1I

 900    CONTINUE

        END IF

        IF (J.NE.0) THEN

        DO 1100 I=1,NT
        ZZPRODR(J)=ZZPRODR(J)+ZR(J,I)*ZR(J,I)+ZI(J,I)*ZI(J,I)
C       ZZPRODI(J)=ZZPRODI(J)+ZR(J,I)*ZI(J,I)-ZR(J,I)*ZI(J,I)
 1100   CONTINUE

        DO 1200 K=J,NY-29

        DO 1250 I=1,NT
        ZYPRODR(K,J-1)=ZYPRODR(K,J-1)+ZR(K,I)*YR(J-1,I)+ZI(K,I)*YI(J-1,I)
        ZYPRODI(K,J-1)=ZYPRODI(K,J-1)+ZR(K,I)*YI(J-1,I)-ZI(K,I)*YR(J-1,I)
 1250   CONTINUE

 1200   CONTINUE

        END IF

 700    CONTINUE
```

```
C       ***************************************
C       END OF GRAM SCHMIDT PROCESS
C       ***************************************


C       DOT PRODUCT CHECK OF SUPPOSEDLY ORTHOGONAL VECTORS Y.Z(0)

        DO 1380 K=NY-29,0,-1

        WRITE(17,1355) K
1355    FORMAT('K=',I2)

        DO 1370 J=NY-29,K,-1

        CHECK=0.0
        CHECK1=0.0
        DO 1300 I=1,NT
        CHECK=CHECK+ZR(J,I)*ZR(K,I)+ZI(J,I)*ZI(K,I)
        CHECK1=CHECK1+ZR(J,I)*ZI(K,I)-ZI(J,I)*ZR(K,I)
1300    CONTINUE

        WRITE(17,1360) J
1360    FORMAT('J=',I2)

        WRITE(17,1365) CHECK
        WRITE(17,1366) CHECK1
1365    FORMAT('CHECK=',E21.15)
1366    FORMAT('CHECK1=',E21.15)

1370    CONTINUE
1380    CONTINUE

C       ************************************************
C       OBTAINING THE SAMPLED IMPULSE RESPONSE
C       OF THE CHANNEL AND FILTER
C       ************************************************

        DO 1400 J=-29,NY

        SUM2R=0.0
        SUM2I=0.0

        DO 1500 I=1,NT
        SUM2R=SUM2R+ZR(0,I)*YR(J,I)+ZI(0,I)*YI(J,I)
        SUM2I=SUM2I-ZR(0,I)*YI(J,I)+ZI(0,I)*YR(J,I)
1500    CONTINUE
```

```
      ZYR(J)=SUM2R
      ZYI(J)=SUM2I

1400  CONTINUE

      DO 1600 J=-29,-1
      ZYI(J)=ZYI(J)/ZYR(0)
      ZYR(J)=ZYR(J)/ZYR(0)
1600  CONTINUE

      DO 1650 J=1,NY
      ZYR(J)=ZYR(J)/ZYR(0)
      ZYI(J)=ZYI(J)/ZYR(0)
C     PRINT*,'ZYR=',ZYR(J)
1650  CONTINUE

      ZYI(0)=ZYI(0)/ZYR(0)
      ZYR(0)=ZYR(0)/ZYR(0)

      WRITE(13,1700)(ZYR(J),J=NY,-29,-1)
      WRITE(14,1750)(ZYI(J),J=NY,-29,-1)
1700  FORMAT(1X,'ZYR(J)=',1X,F16.12)
1750  FORMAT(1X,'ZYI(J)=',1X,F16.12)

C     *********************************************
C     OBTAINING THE TAP GAINS OF THE FILTER
C     *********************************************

      DO 1800 J=1,NT
      ZI(0,J)=-ZI(0,J)
1800  CONTINUE

      WRITE(10,1850) (ZR(0,I),I=1,NT)
      WRITE(10,1850)  (ZI(0,I),I=1,NT)
1850  FORMAT(F16.12)

C     **********************************
C     CHECKS ON FILTER
C     **********************************

C     MAGNITUDE OF Z VECTORS CHECK

      DO 1870 J=NY-29,0,-1
      CHKZ=0.0

      WRITE(18,1855) J
```

```
1855    FORMAT('J=',I2)

        DO 1875 I=1,NT
C       CHKZ=0.0
        CHKZ=CHKZ+ZR(J,I)**2+ZI(J,I)**2
        SRTCHKZ=CHKZ**0.5
1875    CONTINUE

        WRITE(18,1860) SRTCHKZ
1860    FORMAT('MAG Z=',F16.12)
1870    CONTINUE

C       ANGLE BETWEEN ANY TWO Y AND ANY TWO Z VECTORS CHECK

        DO 2100 J=NY-29,0,-1
C       PRINT*,'J=',J

        DO 2200 K=NY-29,J,-1
C       PRINT*,'K=',K

        ANGLEYR=0.0
        ANGLEYI=0.0
        ANGLEZR=0.0
        ANGLEZI=0.0
        AMAGYJ=0.0
        AMAGYK=0.0
        AMAGZJ=0.0
        AMAGZK=0.0

        DO 2300 I=1,NT
        ANGLEYR=ANGLEYR+YR(J,I)*YR(K,I)+YI(J,I)*YI(K,I)
        ANGLEYI=ANGLEYI-YR(J,I)*YI(K,I)+YI(J,I)*YR(K,I)
        AMAGYJ=AMAGYJ+YR(J,I)**2+YI(J,I)**2
        AMAGYK=AMAGYK+YR(K,I)**2+YI(K,I)**2

        ANGLEZR=ANGLEZR+ZR(J,I)*ZR(K,I)+ZI(J,I)*ZI(K,I)
        ANGLEZI=ANGLEZI-ZR(J,I)*ZI(K,I)+ZI(J,I)*ZR(K,I)
        AMAGZJ=AMAGZJ+ZR(J,I)**2+ZI(J,I)**2
        AMAGZK=AMAGZK+ZR(K,I)**2+ZI(K,I)**2
2300    CONTINUE

        ANGLEY=(ANGLEYR**2+ANGLEYI**2)**0.5
        ANGLEZ=(ANGLEZR**2+ANGLEZI**2)**0.5
        AMAGYJK=(AMAGYJ*AMAGYK)**0.5
        AMAGZJK=(AMAGZJ*AMAGZK)**0.5
```

```
      ANGLEY=ANGLEY/AMAGYJK
      ANGLEZ=ANGLEZ/AMAGZJK
      ANGLEY=ACOS(ANGLEY)
      ANGLEZ=ACOS(ANGLEZ)
C     PRINT*,'ANGLE B/W Y',ANGLEY
C     PRINT*,'ANGLE B/W Z',ANGLEZ

 2200 CONTINUE
 2100 CONTINUE

C     *********************************
C     ERROR MEASUREMENTS
C     *********************************

      PREERR=0.0
      DO 1900 J=1,NY
      PREERR=PREERR+ZYR(J)**2+ZYI(J)**2
C     PRINT*,'PREERROR=',PREERR
 1900 CONTINUE

      PREERR1=0.0
      DO 1925 J=1,NY-29
      PREERR1=PREERR1+ZYR(J)**2+ZYI(J)**2
C     PRINT*,'PREERR1=',PREERR1
 1925 CONTINUE

      PREERR2=0.0
      DO 1950 J=NY-28,NY
      PREERR2=PREERR2+ZYR(J)**2+ZYI(J)**2
C     PRINT*,'PREERR2=',PREERR2
 1950 CONTINUE

      POSTERR=0.0
      DO 2000 J=-29,0
      POSTERR=POSTERR+(ZYR(J)-ACTMPYR(J))**2+(ZYI(J)-ACTMPYI(J))**2
C     PRINT*,'POSTERR=',POSTERR
 2000 CONTINUE

      TOTERR=PREERR+POSTERR

      PREERR=10*LOG10(PREERR)
      PREERR1=10*LOG10(PREERR1)
      PREERR2=10*LOG10(PREERR2)
      POSTERR=10*LOG10(POSTERR)
      TOTERR=10*LOG10(TOTERR)
```

```
PRINT*,'PRE ERROR=',PREERR
PRINT*,'PRE ERROR1=',PREERR1
PRINT*,'PRE ERROR2=',PREERR2
PRINT*,'POST ERROR=',POSTERR
PRINT*,'TOT ERROR=',TOTERR


STOP
END
```

APPENDIX D.9

PROGRAM THAT SIMULATES FILTER FOR CHANNEL 3

```
      PROGRAM ADFCH3
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      PARAMETER (NT=50,NY=49)
      DOUBLE PRECISION ZR(-30:NY,NT),ZI(-30:NY,NT)
      DOUBLE PRECISION YR(-30:NY,NT),YI(-30:NY,NT)
      DOUBLE PRECISION ZYR(-30:NY),ZYI(-30:NY)
      DOUBLE PRECISION ZYPRODR(0:NY,0:NY),ZYPRODI(0:NY,0:NY)
      DOUBLE PRECISION ZZPRODR(0:NY),ZZPRODI(0:NY)
      DOUBLE PRECISION ACTMPYR(-29:0),ACTMPYI(-29:0)
      CALL XUFLOW(0)

      OPEN(10,FILE='Z0CH3')
      OPEN(11,FILE='YREALCH3')
      OPEN(12,FILE='YIMGCH3')
      OPEN(13,FILE='GRMPCH3')
      OPEN(14,FILE='GIMPCH3')
      OPEN(15,FILE='YRMPCH3')
      OPEN(16,FILE='YIMPCH3')
      OPEN(17,FILE='DPRCH3')
      OPEN(18,FILE='MAGZCH3')

C     NO OF TAPS=NT
C     NOS OF CHANNEL IMPULSE REPONSE(I.E Y)=NY
C     NT=50
C     NY=49

      DO 100 J=0,NY
      DO 200 I=1,NT
      YR(J,I)=0.0
      YI(J,I)=0.0
200   CONTINUE
100   CONTINUE

      READ(15,150) (ACTMPYR(I),I=0,-25,-1)
      READ(16,150) (ACTMPYI(I),I=0,-25,-1)
150   FORMAT(F16.12)


      READ(11,250) (YR(0,I),I=NT,NT-25,-1)
      READ(12,250) (YI(0,I),I=NT,NT-25,-1)
250   FORMAT(F8.4)
```

```
        DO 300 I=1,NY
        DO 400 J=2,NT
        YR(I,J-1)=YR(I-1,J)
        YI(I,J-1)=YI(I-1,J)
        YR(I,NT)=0.0
        YI(I,NT)=0.0
400     CONTINUE
300     CONTINUE

        DO 350 I=-1,-25,-1
        DO 450 J=NT-1,1,-1
        YR(I,J+1)=YR(I+1,J)
        YI(I,J+1)=YI(I+1,J)
450     CONTINUE
350     CONTINUE


C       ************************************************
C       GRAM SCHMIDT ORTHOGONALIZATION PROCESS
C       ************************************************

C       Z(0),Z(1),Z(2)..Z(NY-25) ARE THE ORTHOGONAL VECTORS

C       NEXT DO LOOP SETS ALL ZYPRODS TO ZERO
C       ZYPROD(0,0)=Z0.Y0

        DO 500 J=0,NY
        DO 600 I=0,NT-1
        ZYPRODR(J,I)=0.0
        ZYPRODI(J,I)=0.0
600     CONTINUE
        ZZPRODR(J)=0.0
        ZZPRODI(J)=0.0
500     CONTINUE

        DO 700 J=NY-25,0,-1
        IF (J.EQ.(NY-25)) THEN

        DO 800 I=1,NT
        ZR(J,I)=YR(J,I)
        ZI(J,I)=YI(J,I)
800     CONTINUE

        ELSE

        DO 900 I=1,NT
```

```
      SUM1R=0.0
      SUM1I=0.0

      DO 1000 K=J+1,NY-25
      A=ZYPRODR(K,J)
      B=ZYPRODI(K,J)
      C=ZZPRODR(K)
      D=ZZPRODI(K)

      GSREAL=((A*C+B*D)/(C**2+D**2))
      GSIMG=((B*C-A*D)/(C**2+D**2))
C     PRINT*,'GSREAL=',GSREAL
C     PRINT*,'GSIMG=',GSIMG

      SUM1R=SUM1R+(GSREAL*ZR(K,I)-GSIMG*ZI(K,I))
      SUM1I=SUM1I+(GSREAL*ZI(K,I)+GSIMG*ZR(K,I))
1000  CONTINUE

      ZR(J,I)=YR(J,I)-SUM1R
      ZI(J,I)=YI(J,I)-SUM1I

900   CONTINUE

      END IF

      IF (J.NE.0) THEN

      DO 1100 I=1,NT
      ZZPRODR(J)=ZZPRODR(J)+ZR(J,I)*ZR(J,I)+ZI(J,I)*ZI(J,I)
      ZZPRODI(J)=ZZPRODI(J)+ZR(J,I)*ZI(J,I)-ZR(J,I)*ZI(J,I)
1100  CONTINUE

      DO 1200 K=J,NY-25

      DO 1250 I=1,NT
      ZYPRODR(K,J-1)=ZYPRODR(K,J-1)+ZR(K,I)*YR(J-1,I)+ZI(K,I)*YI(J-1,I)
      ZYPRODI(K,J-1)=ZYPRODI(K,J-1)+ZR(K,I)*YI(J-1,I)-ZI(K,I)*YR(J-1,I)
1250  CONTINUE

1200  CONTINUE

      END IF

700   CONTINUE

      DO 1325 K=NY-25,0,-1
```

```
      WRITE(17,1355) K
1355  FORMAT('K=',I2)

      DO 1350 J=NY-25,K,-1

      CHECK=0.0
      CHECK1=0.0
      DO 1300 I=1,NT
      CHECK=CHECK+ZR(J,I)*ZR(K,I)+ZI(J,I)*ZI(K,I)
      CHECK1=CHECK1-ZR(J,I)*ZI(K,I)+ZI(J,I)*ZR(K,I)
1300  CONTINUE

      WRITE(17,1360) J
1360  FORMAT('J=',I2)

      WRITE(17,1365) CHECK
      WRITE(17,1370) CHECK1
1365  FORMAT('CHECK=',E21.15)
1370  FORMAT('CHECK1=',E21.15)

1350  CONTINUE
1325  CONTINUE

      DO 1400 J=-25,NY

      SUM2R=0.0
      SUM2I=0.0

      DO 1500 I=1,NT
      SUM2R=SUM2R+ZR(0,I)*YR(J,I)+ZI(0,I)*YI(J,I)
      SUM2I=SUM2I+ZR(0,I)*YI(J,I)-ZI(0,I)*YR(J,I)
1500  CONTINUE

      ZYR(J)=SUM2R
      ZYI(J)=SUM2I

1400  CONTINUE

      DO 1600 J=-25,-1
      ZYI(J)=ZYI(J)/ZYR(0)
      ZYR(J)=ZYR(J)/ZYR(0)
1600  CONTINUE
```

```
      DO 1650 J=1,NY
      ZYR(J)=ZYR(J)/ZYR(0)
      ZYI(J)=ZYI(J)/ZYR(0)
1650  CONTINUE

      ZYI(0)=ZYI(0)/ZYR(0)
      ZYR(0)=ZYR(0)/ZYR(0)

      WRITE(13,1700)(ZYR(J),J=NY,-25,-1)
      WRITE(14,1750)(ZYI(J),J=NY,-25,-1)
1700  FORMAT(1X,'ZYR(J)=',1X,F16.12)
1750  FORMAT(1X,'ZYI(J)=',1X,F16.12)

      DO 1800 J=1,NT
      ZI(0,J)=-ZI(0,J)
1800  CONTINUE

      WRITE(10,1850) (ZR(0,I),I=1,NT)
      WRITE(10,1850) (ZI(0,I),I=1,NT)
1850  FORMAT(F16.12)

      DO 1870 J=NY-25,0,-1
      CHKZ=0.0

      WRITE(18,1855) J
1855  FORMAT('J=',I2)

      DO 1875 I=1,NT
C     CHKZ=0.0
      CHKZ=CHKZ+ZR(J,I)**2+ZI(J,I)**2
      SRTCHKZ=CHKZ**0.5
1875  CONTINUE

      WRITE(18,1860) SRTCHKZ
1860  FORMAT('MAG Z=',F16.12)

1870  CONTINUE

      DO 2100 J=NY-25,0,-1
C     PRINT*,'J=',J

      DO 2200 K=NY-25,J,-1
C     PRINT*,'K=',K
```

```
      ANGLER=0.0
      ANGLEI=0.0
      AMAGYJ=0.0
      AMAGYK=0.0

      DO 2300 I=1,NT
      ANGLER=ANGLER+YR(J,I)*YR(K,I)+YI(J,I)*YI(K,I)
      ANGLEI=ANGLEI-YR(J,I)*YI(K,I)+YI(J,I)*YR(K,I)
      AMAGYJ=AMAGYJ+YR(J,I)**2+YI(J,I)**2
      AMAGYK=AMAGYK+YR(K,I)**2+YI(K,I)**2
2300  CONTINUE

      AMAGYJK=(AMAGYJ*AMAGYK)**0.5
      ANGLE=(ANGLER**2+ANGLEI**2)**0.5
      ANGLE=ANGLE/AMAGYJK
      ANGLE=ACOS(ANGLE)
      ANGLE=90*ANGLE/3.14159
C     PRINT*,'ANGLE=',ANGLE

2200  CONTINUE
2100  CONTINUE

      PREERR=0.0
      DO 1900 J=1,NY
      PREERR=PREERR+ZYR(J)**2+ZYI(J)**2
C     PRINT*,'PREERROR=',PREERR
1900  CONTINUE

      PREERR1=0.0
      DO 1925 J=1,NY-25
      PREERR1=PREERR1+ZYR(J)**2+ZYI(J)**2
C     PRINT*,'PREERR1=',PREERR1
1925  CONTINUE

      PREERR2=0.0
      DO 1950 J=NY-24,NY
      PREERR2=PREERR2+ZYR(J)**2+ZYI(J)**2
C     PRINT*,'PREERR2=',PREERR2
1950  CONTINUE

      POSTERR=0.0
      DO 2000 J=-25,0
```

```
      POSTERR=POSTERR+(ZYR(J)-ACTMPYR(J))**2+(ZYI(J)-ACTMPYI(J))**2
C     PRINT*,'POSTERR=',POSTERR
 2000 CONTINUE

      TOTERR=PREERR+POSTERR

      PREERR=10*LOG10(PREERR)
      PREERR1=10*LOG10(PREERR1)
      PREERR2=10*LOG10(PREERR2)
      POSTERR=10*LOG10(POSTERR)
      TOTERR=10*LOG10(TOTERR)

      PRINT*,'PRE ERROR=',PREERR
      PRINT*,'PRE ERROR1=',PREERR1
      PRINT*,'PRE ERROR2=',PREERR2
      PRINT*,'POST ERROR=',POSTERR
      PRINT*,'TOT ERROR=',TOTERR


      STOP
      END
```

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