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A two segment simulation model of long horse vaulting

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ABSTRACT

Optimum preflight characteristics of the Hecht and handspring somersault vaults were determined using a two segment simulation model. The model comprised an arm segment and a body segment connected by a frictionless pin joint, simulating the vault from Reuther board takeoff through to landing. During horse contact shoulder torque was set to zero in the model. Five independent preflight variables were varied over realistic ranges and an objective function was maximised to find the optimum preflight for each vault. The Hecht vault required a low trajectory of the mass centre during preflight with a low vertical velocity of the mass centre and a low angular velocity of the body at horse contact. In contrast the optimum handspring somersault required a high preflight trajectory with a high angular velocity of the body and a high vertical velocity at horse contact. Despite the simplicity of the model, the optimum preflights were similar to those used in competitive performances.

INTRODUCTION

Long horse vaults can be divided into two distinct groups (Takei, 1988): (a) continuous rotation vaults in which the somersault rotation continues in the same direction throughout the vault (e.g. handspring vault), and (b) counter-rotation vaults in which the direction of rotation is reversed during contact with the horse (e.g. Hecht vault). Most of the research into vaulting has used statistical analysis of film data to understand the mechanics and techniques used in vaulting (Brüggeman 1987; Dainis, 1979; Dillman *et al.*, 1985, Kerwin *et al.*, 1993; Kwon *et al.*, 1990; Takei, 1988, 1989, 1991; Takei and Kim, 1990). These studies have described the actual techniques used by gymnasts performing continuous rotation vaults and have identified the characteristics of successful performance. Takei (1988), for example, showed that a high horizontal velocity during preflight (the flight onto the horse) and a large gain in vertical velocity while in contact with the horse were important determinants for successful performance of the handspring somersault vault.

Dainis (1981), Gervais (1994) and Sprigings and Yeadon (1997) used theoretical models in their analysis of vaulting. Gervais recorded three trials of the handspring somersault vault which were used as a basis for varying technique. Optimum technique was determined by maximising an objective function based upon height and distance achieved during postflight (the flight from the horse). Dainis used a three segment model to show the relationships between the preflight variables and the outcome of the vault for handspring vaults. In this model assumptions were made concerning the contact phase, and the model cannot be used for counter-rotation vaults (Dainis, 1981). Sprigings and Yeadon (1997) showed that it is theoretically possible to perform the Hecht vault without using shoulder torque during the contact phase. The model required high horizontal preflight velocities, assumed that the arms remained in a fixed orientation relative to the body during postflight with the optimum solution achieving a vertical landing position. In practice gymnasts use

arm circling in postflight to aid rotation and land at a suitable angle behind the vertical so that they do not need to step forwards or backwards.

The ability to perform a vault is dependent on a number of factors including the preflight parameters at horse contact, the elastic properties of the horse and the gymnast, and the joint torques exerted while in contact with the horse. This paper will highlight the importance of the preflight phase by investigating the difference between optimum preflight conditions for the Hecht and handspring somersault vaults using an inelastic torque-free two segment simulation model.

METHOD

Anthropometric measurements available on 11 elite male gymnasts from previous studies and the mathematical model of Yeadon (1990a) were used to calculate segmental masses, mass centre locations, link lengths and moments of inertia. The inertia data were then normalised to a total body mass of 62.88 kg and a standing height of 1.67 m corresponding to the average male gymnast from the 1988 Olympic Games (Takei and Kim, 1990). Normalisation was carried out using the procedure of Dapena (1978) in which segment mass is assumed to be proportional to body mass, segment length proportional to standing height and transverse segmental moment of inertia proportional to mass times length squared. In addition the normalised inertia data were averaged to give a single inertia data set from which inertia parameters were calculated for a simulation model comprising an arm segment and a body segment (Table 1).

Table 1. Means and standard deviations of average normalised segmental inertia parameters

	arm	body
[kg]	$M_a = 7.14 \pm 0.59$	$M_b = 55.74 \pm 0.59$
length [m]	$c = 0.524 \pm 0.03$	$d = 1.508 \pm 0.04$
CM location [m]	$a = 0.276 \pm 0.03$	$b = 0.467 \pm 0.02$
moment of inertia [kg.m ²]	$I_a = 0.214 \pm 0.05$	$I_b = 9.115 \pm 0.42$

Note: See Fig 1. for nomenclature

Simulation model

A planar two segment computer simulation model was used to simulate a vault from Reuther board takeoff, through horse contact, until landing. Input to the model comprised the values of five variables at the time of horse contact while output variables described the postflight phase (Table 2). The conditions at horse contact were used as model input rather than initial conditions at Reuther board takeoff in order to ensure that all simulations contacted the horse. The conditions at Reuther board takeoff were back-calculated from the horse contact conditions and were used as initial conditions for the simulations. To describe the orientation and configuration of the model the shoulder angle α was defined as the angle between the two segments and the body angle ϕ as the angle of the body segment (shoulders S

to feet F) relative to the horizontal (Fig. 1). The mechanical analysis of the vault was divided into preflight, impact, postflight and landing phases.

Table 2. Input and output variables to the simulation model

Input variables at horse contact	Symbol	Output variables from the model	symbol
horizontal velocity of the mass centre	u	maximum height of the mass centre	h
vertical velocity of the mass centre	v	landing distance of the mass centre	d
body angle	ϕ	landing angle	γ
shoulder angle	α		
angular velocity of the body	ω		

Preflight

The preflight phase is from Reuther board takeoff to the initial contact of the hands with the horse. The motion of the model during this phase was determined by the orientation and configuration of the segments, the velocity of the mass centre and the angular velocity of the body at takeoff from the Reuther board. Projectile equations of motion (1, 2) and conservation of angular momentum (equation (3)) were used to calculate the orientation and location of the segments at a given time.

$$\text{horizontally} \quad x = x_o + ut \quad (1)$$

$$\text{vertically} \quad z = z_o + vt - \frac{1}{2}gt^2 \quad (2)$$

where (x_o, z_o) and (x, z) are the locations of the mass centre at takeoff from the Reuther board and at time t , (u, v) is the velocity of the mass centre at takeoff from the Reuther board, and g = acceleration due to gravity.

$$L = I\omega \quad (3)$$

where L = (constant) angular momentum about the mass centre, I = moment of inertia about a transverse axis through the mass centre, and ω = angular velocity about a transverse axis. Since a rigid configuration is maintained by the model during the preflight phase, I is constant and hence ω is also constant.

An iterative integration process was used to calculate the motion of the model. The step size (0.005 s) was selected so that further reductions in step size led to changes in the output variables h , d and γ that were smaller than 0.01m, 0.01m and 1° respectively to ensure that integration errors were small. A rigid configuration was maintained throughout the preflight phase and the orientations of the segments were determined at the time the hands (i.e. the end of the arm segment) reached horse level. Due to the stepwise procedure used, the hands were above the horse at the end of one iteration and below the horse surface at the next iteration. To calculate the time of horse contact linear interpolation between iterations was used.

Impact and contact

The model made contact with the horse when the hands reached the level of the horse. At the initial impact of the model with the horse, the velocity of the hands was reduced to zero instantaneously and the angular velocities of the two segments immediately after impact were determined using conservation of angular momentum (equations (4) and (5)). Since the duration of impact is assumed to be zero the torques due to the segment weights will not change the angular momentum.

The angular momentum L_o of the whole system about the hand contact point O is conserved since the external impulsive reaction forces act through O and do not produce an impulsive torque about O.

$$\text{Before impact:} \quad L_o = I_g \omega_g + M_g v_g d_{og} \quad (4a)$$

$$\text{After impact:} \quad L_o = I_a \omega'_a + I_b \omega'_b + M_a \omega'_a d_{oa}^2 + M_b v'_b d_{ob} \quad (4b)$$

The angular momentum L_s of the body segment about the shoulder point S is conserved since the external impulsive reaction forces act through S and do not produce an impulsive torque about S.

$$\text{Before impact:} \quad L_s = I_b \omega_b + M_b u_b d_{sb} \quad (5a)$$

$$\text{After impact:} \quad L_s = I_b \omega'_b + M_b u'_b d_{sb} \quad (5b)$$

where I = moment of inertia about a transverse axis, ω and ω' = the angular velocity before and after impact respectively, M = mass, d_{ij} = distance from point i to j , v and v' = the velocity component perpendicular to the line joining the mass centre to O before and after impact respectively, and u and u' = the velocity component perpendicular to the line joining the mass centre to S before and after impact respectively. The subscripts g , a and b represent the whole body, the arm segment and the body segment respectively.

During the remainder of the contact phase it was assumed that there was no torque at the shoulders and only the torque due to gravity affected the rotation of the model (Fig. 1).

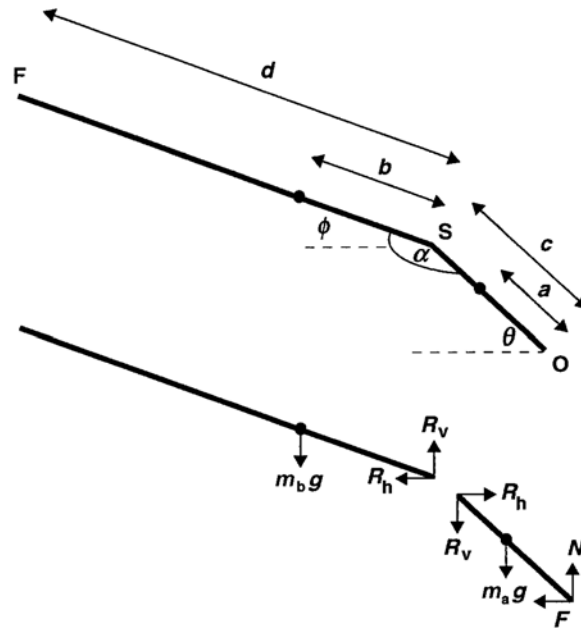


Figure 1. Two segment model of the contact phase.

Newton's Second Law was used to calculate the angular acceleration of each segment and the vertical reaction force at the hands (equations (6) - (11)). To solve the six equations in six unknowns, a linear least squares technique was used (Stewart, 1973). A second order Runge-Kutta method was used to advance the solution of the second order system of differential equations one step. Takeoff from the horse occurred when the vertical reaction force N at the hands reached zero.

Resolving perpendicular to the arm segment:

$$R_h \sin \theta - R_v \cos \theta + N \cos \theta - F \sin \theta - M_a g \cos \theta = M_a a \ddot{\theta} \quad (6)$$

Resolving parallel to the arm segment:

$$R_h \cos \theta + R_v \sin \theta - N \sin \theta - F \cos \theta + M_a g \sin \theta = M_a a \dot{\theta}^2 \quad (7)$$

Taking moments about the mass centre of the arm segment:

$$R_h(c - a) \sin \theta - R_v(c - a) \cos \theta - Na \cos \theta + Fa \sin \theta = I_a \ddot{\theta} \quad (8)$$

Resolving perpendicular to the body segment:

$$R_v \cos \phi - R_h \sin \phi - M_b g \cos \phi = M_b [b \ddot{\phi} + c \ddot{\theta} \cos(\theta - \phi) - c \dot{\theta}^2 \sin(\theta - \phi)] \quad (9)$$

Resolving parallel to the body segment:

$$-R_v \sin \phi - R_h \cos \phi + M_b g \sin \phi = M_b [b \ddot{\phi} + c \dot{\theta}^2 \cos(\theta - \phi) + c \ddot{\theta} \sin(\theta - \phi)] \quad (10)$$

Taking moments about the mass centre of the body segment:

$$R_h b \sin \phi - R_v b \cos \phi = I_b \ddot{\phi} \quad (11)$$

where: a = distance from hands to mass centre of arm segment, b = distance from shoulders to mass centre of body segment, c = length of arms, θ = angle of arm segment above the horizontal, ϕ = angle of the body segment above the horizontal, M = mass, I = moment of inertia about a transverse axis, N = normal reaction force at hands, F = frictional force at hands, and R_v , R_h = vertical and horizontal reaction forces at the shoulder joint. The subscripts a and b represent the arm and body segments respectively.

Postflight

The motion in the postflight phase was determined by the orientation and configuration of the segments, the velocity of the mass centre and the angular velocity of the two segments at the moment the vertical reaction force became zero. For the Hecht vault the arms were allowed to continue circling forwards relative to the body at a constant angular velocity determined at takeoff from the horse. For the straight handspring somersault vault the arms were moved to the sides of the body at takeoff from the horse and remained there for the whole of the postflight phase. Projectile equations (1) and (2) were used to calculate mass centre kinematics, and conservation of angular momentum was used to determine the angular velocity of the body using equation (12). The postflight phase ended when the feet reached the level of the mat. Linear interpolation between iterations was used to calculate the time of landing.

The conservation of angular momentum for the postflight may be written as:

$$L = I_\phi \dot{\phi} + I_\alpha \dot{\alpha} \quad (12)$$

where α = the angle between the arm and body segments, and I_ϕ and I_α are functions of α (Yeadon, 1990b). I_ϕ and I_α were calculated by considering the cases (a) $\dot{\alpha} = 0$ and (b) $\dot{\phi} = 0$, giving:

$$I_\phi = I_g$$

$$I_\alpha = I_a + \frac{M_a M_b}{M_a + M_b} \left[(c - a)^2 + (c - a)b \cos \alpha \right]$$

where a = the distance from the point of contact with the horse to the mass centre of the arm segment, b = the distance from the shoulder joint to the mass centre of the body segment and c = the length of the arm segment. The subscripts a and b represent the arm and body segments respectively.

Landing

The landing of the gymnast was modelled as an instantaneous impact followed by a contact phase similar to the treatment of horse contact. During contact the orientation of the body relative to the vertical is described by an angle γ with the convention that γ is negative when the gymnast needs to rotate forwards in order to reach a vertical position. The initial value of γ at impact with the landing mat will be referred to as the ‘landing angle’. After impact the two segment model was treated as a single rigid body in the landing configuration which rotated about the contact point with the mat, with gravitational torque used to slow down the rotation of the model. Newton's Second Law was used to calculate the angular acceleration of the single segment using equations (13) - (15). To solve the three equations in three unknowns a linear least squares technique was used (Stewart, 1973). A second order Runge-Kutta method was used for the numerical integration to calculate the body angle γ until a vertical position was reached or the body stopped rotating forwards.

Resolving perpendicular to the line of the body:

$$-N \sin \gamma - F \cos \gamma + M_g g \sin \gamma = M_g d \ddot{\gamma} \quad (13)$$

Resolving parallel to the line of the body:

$$-N \cos \gamma + F \sin \gamma + M_g g \cos \gamma = M_g d \dot{\gamma}^2 \quad (14)$$

Taking moments about the contact point O:

$$M_g g d \sin \gamma = (I_g + M_g d^2) \ddot{\gamma} \quad (15)$$

where d = fixed distance between the mass centre and O.

Evaluation of the two segment impact model

During the modelling of the horse contact phase it was assumed that the horse and gymnast are inelastic and that no torque is exerted at the shoulder joint. These assumptions were made since it was thought that the preflight characteristics are the major determinants of postflight performance. In order to test whether these assumptions are reasonable a simulation was based on the preflight characteristics of the highest scoring Hecht vault from a national competition (Yeadon *et al.*, 1998). The postflight characteristics of the simulation after impact were compared with the video analysis of the actual performance in order to assess whether there was reasonable agreement between them.

Simulations

Simulations were carried out to determine the optimum preflight conditions for the Hecht and the straight handspring somersault vault. Realistic input values to the model at takeoff from the Reuther board were required in order to optimise the Hecht and the handspring somersault vault. In addition for the Hecht vault a body angle of at least 20° above the horizontal at horse contact was required by the International Gymnastics Federation in order for no deductions to be made to the score.

Realistic input values to simulation model

Reported data from the 1988 Olympic Games (Takei and Kim, 1990) and the 1993 Canadian National Championships (Yeadon *et al.*, 1998) were used to set realistic limits on the preflight variables for both vaults (Table 3). For both vaults upper limits were required for the horizontal preflight velocity since, when optimising the vaults, it was found that faster preflights produced greater height, distance and rotation. For the handspring somersault the maximum horizontal preflight velocity of 6.05 m.s^{-1} for Olympic gymnasts was used (Takei and Kim, 1990).

For the Hecht vault the only available data was a maximum horizontal preflight velocity of 6.13 m.s^{-1} reported by Yeadon *et al.*, (1998) for Canadian gymnasts. Since this value was likely to be less than the maximum horizontal preflight velocity achievable by Olympic gymnasts it was increased in proportion to the approach speeds of Olympic and Canadian gymnasts. The mean approach speed of Olympic gymnasts was 7.93 m.s^{-1} compared with 7.14 m.s^{-1} for the Canadians. The upper limit for the horizontal preflight velocity of Olympic gymnasts was estimated using this procedure to be 6.81 m.s^{-1} .

The body angles at takeoff from the Reuther board were taken from film data of handspring somersault vaults (Takei and Kim, 1990) and video data of Hecht vaults (Yeadon *et al.*, 1998). For the Hecht vault the largest observed body lean at Reuther board takeoff of 31.1° from the vertical was chosen since the greater the body lean the less the required rotation in preflight and the easier it is to produce the backwards rotation. For the handspring somersault vault, the average body lean from Takei and Kim (1990) was used and the angular velocity of the body was limited to a realistic value since faster preflight rotations resulted in greater height, distance and rotation. Takei and Kim (1990) reported a mean preflight angular velocity value. To estimate a maximum value the mean plus two standard deviations was used resulting in a maximum angular velocity of 8.13 rad.s^{-1} .

Table 3. Realistic limits for input variables at takeoff from the Reuther board

Input variables	Hecht vault	handspring somersault vault
horizontal velocity of the mass centre	$< 6.81 \text{ m.s}^{-1}$	$< 6.05 \text{ m.s}^{-1}$
vertical velocity of the mass centre	no limit	$< 4.52 \text{ m.s}^{-1}$
body angle	-59° to -79°	-68° to -83°
shoulder angle	no limit	no limit
angular velocity of the body	no limit	$< 8.13 \text{ rad.s}^{-1}$

Criteria for a successful Hecht simulation

For a successful Hecht simulation the body angle should be at least 20° above the horizontal at horse contact, the foot clearance needs to be positive otherwise the gymnast would have to pike or straddle to miss the horse. The distance travelled by the mass centre should be at least 2.5 m past the end of the horse (as per International Gymnastics Federation regulations) and the landing angle needs to be such that the angular velocity after impact is close to zero. Simulation was used to estimate the required landing angle γ_0 behind the vertical. An objective function f defined in equation (16) was maximised where there were a number of possible solutions that satisfied all the criteria for a successful simulation. The objective function was based on the body angle ϕ at horse contact, maximum height h of the mass centre, landing angle γ and the landing distance d (Table 4). The coefficients for each variable in the equation were based upon the variation obtained from video analysis of competitive Hecht vaults (Yeadon *et al.*, 1998). This choice of coefficients implies that the standard deviations of ϕ , h , d and γ obtained from competitive performances will produce equal contributions to the variation in the objective function f . To ensure that the feet missed the horse a penalty function defined in equation (17) was used. A quadratic penalty function was used rather than the inequality constraint $\delta > 0$ in order to ensure that there were solutions when starting the optimisation procedure. The coefficient of δ needs to be sufficiently large to ensure that $\delta > 0$ in the optimal solution. A value of 1000 was chosen and corresponds to a distance $\delta = -0.03$ m in order to make a contribution equal to that made by each of ϕ , h , d and γ when varying over their respective standard deviations.

$$\text{for } \delta \geq 0.00 \text{ m} \quad f = \frac{\phi}{\sigma_\phi} + \frac{h}{\sigma_h} + \frac{d}{\sigma_d} - \frac{|\gamma - \gamma_0|}{\sigma_\gamma} \quad (16)$$

$$\text{for } \delta < 0.00 \text{ m} \quad f = \frac{\phi}{\sigma_\phi} + \frac{h}{\sigma_h} + \frac{d}{\sigma_d} - \frac{|\gamma - \gamma_0|}{\sigma_\gamma} - 1000\delta^2 \quad (17)$$

where ϕ = body angle at horse contact, h = maximum height of the mass centre during postflight, d = landing distance of the mass centre past the end of the horse, γ = landing angle, δ = foot clearance, and σ = standard deviation in each variable.

Criteria for a successful handspring somersault simulation

For a successful handspring somersault simulation the path of the mass centre should be high with the distance travelled by the mass centre at least 2.5 m past the end of the horse as required by the International Gymnastics Federation in order for there to be no deductions from the score. In addition the landing angle needs to be such that the angular velocity after impact is close to zero. Simulation was used to estimate the required landing angle γ_0 behind the vertical. An objective function f defined by equation (18) was maximised where there was a range of possible solutions that satisfied all the criteria for a successful simulation. The objective function was based upon the maximum height h of the mass centre, the landing angle γ and the landing distance d . The coefficients for each variable were based upon the variation in each variable (Table 4) obtained from analysis of competitive handspring somersault vaults (Takei and Kim, 1990) in order to give appropriate weighting to each variable.

$$f = \frac{h}{\sigma_h} + \frac{d}{\sigma_d} - \frac{|\gamma - \gamma_0|}{\sigma_\gamma} \quad (18)$$

where h = the maximum height of the mass centre during postflight, d = the landing distance of the mass centre past the end of the horse, γ = landing angle, and σ = standard deviation in each variable.

Optimisation Procedure

For the Hecht vault the horizontal preflight velocity was fixed at 6.81 m.s^{-1} and the body lean at Reuther board takeoff was fixed at 31.1° from the vertical while the body angle ϕ , the shoulder angle α and the vertical velocity v of the mass centre at horse contact were allowed to vary. For given values of ϕ , α and v at horse contact the corresponding values at Reuther board takeoff were obtained using the equations of projectile motion. These values at Reuther board takeoff were then used as the initial values of a simulation. For a fixed body angle ϕ , optimum values of α and v were obtained by first finding the value of α that maximised the objective function for a fixed v and then optimising v for this value of α . This procedure was iterated until the solution converged.

For the handspring somersault vault the horizontal preflight velocity was fixed at 6.05 m.s^{-1} , the preflight angular velocity was fixed at 8.13 rad.s^{-1} and the body lean at Reuther board takeoff was fixed at 14.4° from the vertical. The shoulder angle α and the vertical velocity v of the mass centre at horse contact were allowed to vary. For a given value of v at horse contact, projectile equations were used to calculate v at Reuther board takeoff. The values of ϕ , α and v at Reuther board takeoff were used as the initial values of a simulation. Optimum values of α and v were calculated iteratively as described above.

Table 4. Standard deviations for variables used in the objective functions

variable	Hecht vault (Yeadon <i>et al.</i> , 1998)	handspring somersault vault (Takei and Kim, 1990)
body angle at horse contact	5.6°	not used
maximum height of the mass centre	0.11 m	0.08 m
landing distance	0.29 m	0.34 m
landing angle	13.3°	4.2°

RESULTS

The preflight characteristics of the highest scoring video recorded Hecht vault, which was used to evaluate the modelling of the contact phase, are shown in Table 5. These values were used as input to the simulation of the contact phase. The values of these variables at the end of the simulated contact phase are similar to the corresponding values from video analysis with the exception of the shoulder angle α which remains unchanged by the instantaneous simulated impact (Table 5). The postflight performance of the model, however, will depend

primarily on u , v and ω which are in reasonable agreement with actual performance values. This suggests that the assumptions of inelastic impact and zero shoulder torque do not produce major errors.

Table 5. Comparison of simulation and video analysis for the Hecht vault

variable	end of preflight	start of postflight simulation	video
horizontal velocity of the mass centre [m.s^{-1}]	6.01	3.51	4.18
vertical velocity of the mass centre [m.s^{-1}]	0.96	3.07	2.99
body angle [$^{\circ}$]	2.9	2.9	4.5
shoulder angle [$^{\circ}$]	141.7	141.7	111.5
angular velocity of the body [rad. s^{-1}]	2.92	-2.44	-2.18

In all simulations the instantaneous impact with the horse resulted in angular velocities for which the vertical reaction force at the hands was negative so that takeoff was immediate in all cases. Simulations of the landing phase indicated that in order to produce a stationary vertical landing the required landing angle γ_0 was close to -25° for the Hecht and -35° for the handspring somersault. The precise value of γ_0 is, of course, a function of the linear and angular velocities at landing.

Within the realistic ranges of values used for the input variables (Table 3) there was no combination of input variables that satisfied all the requirements for a successful Hecht simulation. To achieve a landing angle of $\gamma = -25^{\circ}$ and a positive foot clearance required a body angle of 14° below the horizontal at horse contact. For body angles higher than this the amount of backwards rotation progressively became less. Table 6 shows the best vaults for a range of body angles at horse contact which minimised equation (16) over realistic ranges of the input variables (Table 3). For a zero body angle at horse contact, the best landing angle was $\gamma = -6^{\circ}$ (Fig. 2) and for a 20° body angle at horse contact the best landing angle was $\gamma = 8^{\circ}$. To achieve a landing angle $\gamma = -25^{\circ}$, a positive foot clearance and a landing at least 2.5 m past the end of the horse while having a body angle of 20° above the horizontal at horse contact required an increase in the upper limit for the horizontal preflight velocity from 6.81 m.s^{-1} to 7.3 m.s^{-1} .

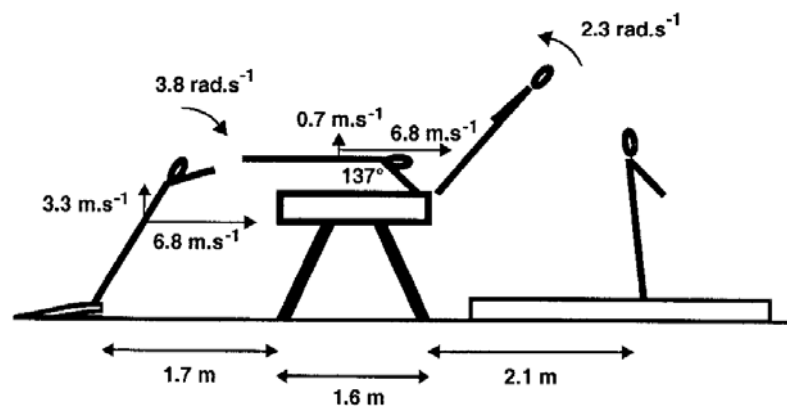


Figure 2. Optimum Hecht vault for a horizontal body position at horse contact.

Table 6. Optimum Hecht vaults for different body angles at horse contact

input variables at horse contact					output variables			
ϕ	α	v	u	ω	δ	γ	d	h
[°]	[°]	[m.s ⁻¹]	[m.s ⁻¹]	[rad.s ⁻¹]	[m]	[°]	[m]	[m]
30.0	148.6	-2.28	6.81	2.20	0.00	10.5	1.88	2.12
20.0	143.6	-1.18	6.81	2.60	0.00	8.0	1.98	2.11
10.0	139.9	-0.21	6.81	3.11	0.00	2.4	2.05	2.14
0.0	136.6	0.66	6.81	3.77	0.00	-6.4	2.09	2.20
-5.0	133.8	1.02	6.81	4.07	0.00	-12.4	2.17	2.22
-10.0	130.9	1.36	6.81	4.40	0.00	-19.6	2.26	2.25
-14.0	129.0	1.64	6.81	4.71	0.00	-25.3	2.32	2.28

The contribution of circling the arms to the production of rotation during postflight was found for the optimised Hecht solution by using a modified simulation in which the arms moved to the sides of the body at takeoff from the horse and remained there throughout postflight. This resulted in 17% less rotation of the body with the landing angle changing from -6° to $+10^\circ$.

A range of input variables satisfied the criteria for a successful handspring somersault simulation. The optimum handspring somersault was determined by maximising the objective function defined by equation (18). Figure 3 shows the optimum simulation with a landing distance of 3.5 m and a landing angle of $\gamma = -35^\circ$ for a horizontal preflight velocity of 6.05 m.s^{-1} and an angular velocity of the body of 8.13 rad.s^{-1} . Reducing the limiting values for horizontal preflight velocity and angular velocity of the body by one standard deviation resulted in an optimum solution which was 3° short on rotation at landing with a landing distance of 2.43 m.

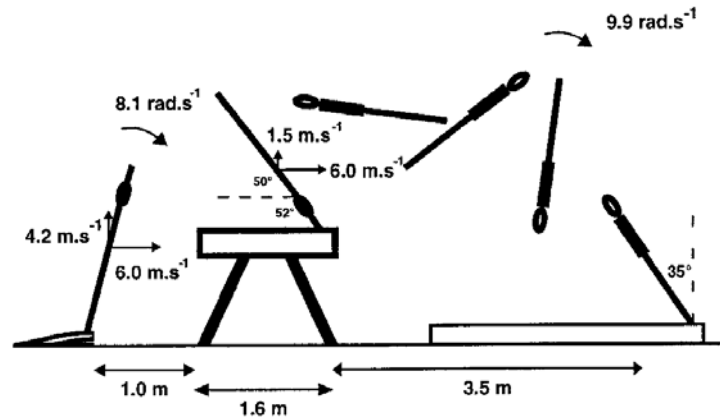


Figure 3. Optimum straight handspring somersault simulation.

Table 7 shows the variation in performance arising from using the 11 normalised inertia data sets in simulations with the optimum preflight parameters for the Hecht vault. The handspring somersault simulation was optimised for each of the 11 normalised inertia data sets. Table 8 shows the variation in the optimum preflight variables for the 11 normalised inertia sets.

Table 7. Variation in performance of the Hecht vault for 11 sets of inertia parameters

	foot clearance [m]	maximum height [m]	landing angle [°]	landing distance [m]
average set	0.00	2.28	-25.3	2.32
minimum	-0.04	2.22	9.4	2.13
maximum	0.09	2.37	-50.4	2.63
mean	0.01	2.28	-26.1	2.36
σ	0.05	0.05	17.6	0.16

Table 8. Variation in optimum preflight variables for the handspring somersault vault for 11 sets of inertia parameters

	optimised preflight variables at horse contact					postflight variables		
	ϕ [°]	α [°]	v [m.s ⁻¹]	u [m.s ⁻¹]	ω [rad.s ⁻¹]	γ [°]	h [m]	d [m]
average set	50.0	178.0	1.53	6.05	8.13	-35.0	2.74	3.50
minimum	48.2	174.0	1.36	6.05	8.13	-35.0	2.60	3.07
maximum	52.8	184.4	1.79	6.05	8.13	-35.1	2.81	3.81
mean	50.2	178.1	1.53	6.05	8.13	-35.0	2.73	3.52
σ	1.5	3.1	0.18	0.00	0.00	0.0	0.06	0.22

Figures 4, 5, 6 and 7 show the effect of varying one preflight variable while keeping the others constant. In each of these figures one preflight variable is varied over a range of values while the remaining preflight variables are set to their optimum values for a Hecht vault with a body angle at horse contact of 10° above the horizontal.

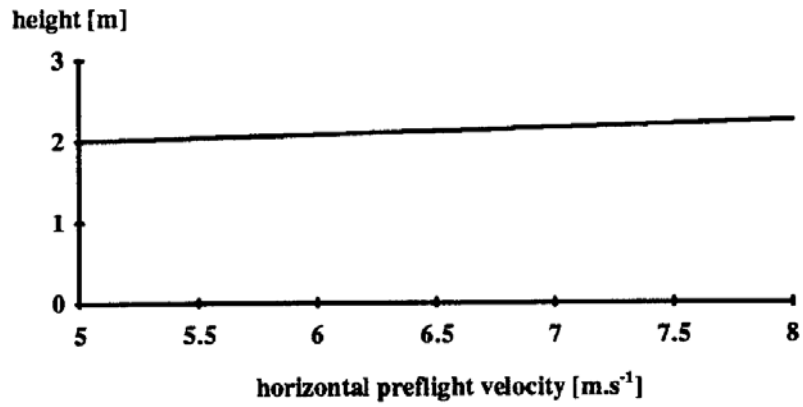


Figure 4. The effect of the horizontal preflight velocity on the maximum height reached by the mass centre during the postflight of the Hecht vault.

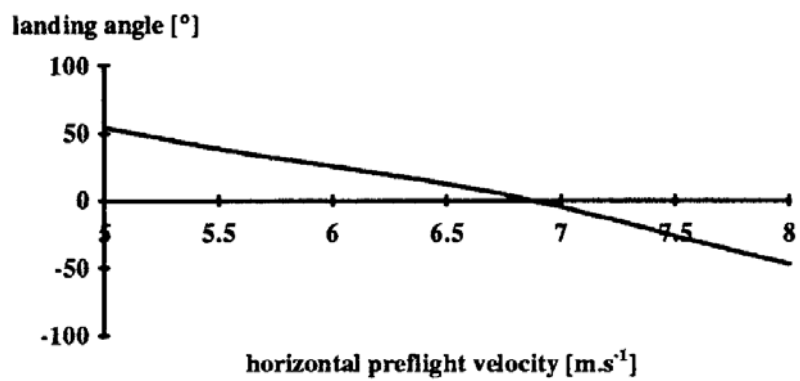


Figure 5. The effect of the horizontal preflight velocity on the landing angle for the Hecht vault.

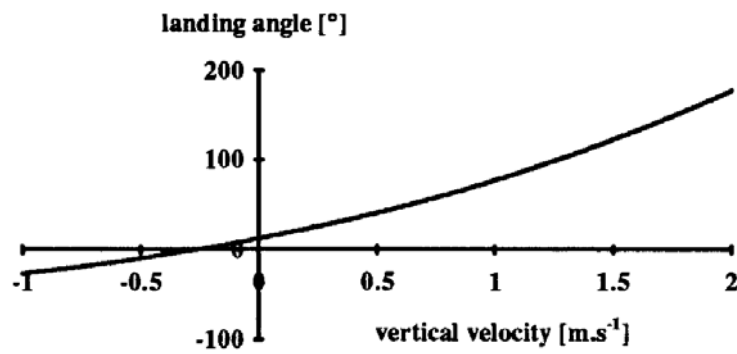


Figure 6. The effect of the vertical velocity at horse contact on the landing angle for the Hecht vault.

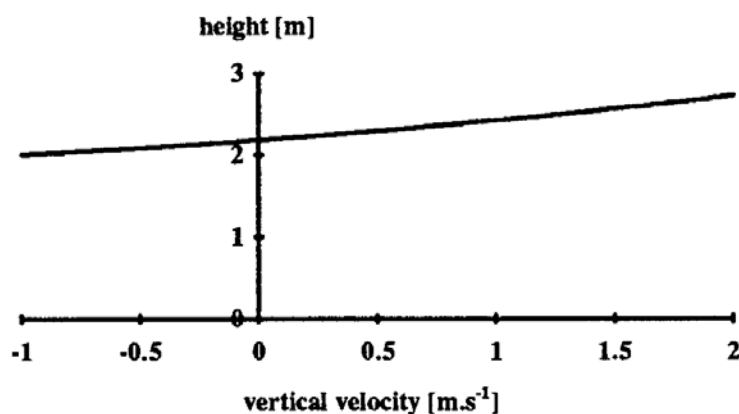


Figure 7. The effect of altering the vertical velocity at horse contact on the maximum height reached by the mass centre during the postflight of the Hecht vault.

DISCUSSION

To optimise the Hecht required finding the correct combination of preflight variables at horse contact which would produce the necessary height, distance and backwards rotation to perform the vault. With the limits set on the preflight variables it was not possible to satisfy all three requirements as the preflight variables had different effects on the postflight variables (Fig. 4 - 7). For example increasing the vertical velocity at horse contact increases the height reached by the mass centre but decreases the backwards rotation (Fig. 6, 7) so that the required landing angle of -25° is not achieved for vertical velocities greater than 0.5 m.s^{-1} .

For a body angle of 0° at horse contact it was possible to produce 83% of the required backwards rotation by optimising the conditions at horse contact within realistic limits using the simulation model. This suggests that most of the backwards rotation required to perform the Hecht vault comes from the correct preflight and not from other factors such as shoulder torque which must make relatively smaller contributions.

The optimised Hecht simulation for a zero body angle at horse contact (Fig. 2) had similar preflight characteristics and a similar performance to those found by Yeadon *et al.* (1998) for the Canadian championships. At takeoff from the board, the optimised Hecht simulation had a similar vertical velocity (3.3 m.s^{-1} compared with the mean 3.5 m.s^{-1}). The Canadian gymnasts generated less rotation at takeoff (1.8 rad.s^{-1} compared with 2.3 rad.s^{-1}) which had the effect of keeping the feet higher for longer to miss the horse. The vertical velocity of the mass centre, the angular velocity of the body and the shoulder angle at horse contact were found to be similar for the optimised simulation and the mean values of the competition performances (0.7 m.s^{-1} and 1.00 m.s^{-1} ; 3.8 rad.s^{-1} and 3.4 rad.s^{-1} ; 137° and 140° respectively). For the optimised Hecht simulation the arms rotated forwards through approximately 1.5 revolutions which was similar to the maximum of the competitive performances.

A lower vertical velocity and angular velocity at horse contact resulted in more backwards rotation during postflight (Fig. 6) but also resulted in a lower height reached by the mass centre in postflight (Fig. 7). Thus a fine balance was required between producing backwards rotation or height by selection of the vertical velocity at horse contact. Horizontal velocity had an advantageous effect on height and backwards rotation as an increase in

horizontal velocity increases height, distance and backwards rotation for the Hecht vault (Fig. 4, 5).

The preflight of the optimised handspring somersault simulation was similar to the handspring somersault vaults from the 1988 Olympic games (Takei and Kim, 1990). At contact with the horse the body angle for the optimised handspring somersault simulation was 50° above the horizontal compared with a mean body angle of 30° for the Olympic handspring somersault vaults (Takei and Kim, 1990). For the optimised simulation, the body angle at takeoff was also 50° as the contact was instantaneous, but for the Olympic performances the gymnasts body angle at takeoff was 92° due to the rotation of the body during the contact phase. The larger body angle at horse contact for the optimised simulation affected the vertical velocity at touchdown on the horse. A higher body angle at horse contact requires a relatively longer preflight time for the optimised simulation, which results in a lower vertical velocity at horse contact (1.5 m.s^{-1} compared with 2.4 m.s^{-1}). The horizontal and vertical velocities of the mass centre at takeoff from the horse were 4.5 m.s^{-1} and 3.7 m.s^{-1} respectively for the optimised simulation which are similar to the maximum values of 4.1 m.s^{-1} and 3.8 m.s^{-1} for the Olympic gymnasts (Takei and Kim, 1990).

From simulations of handspring somersault vaults a set of relationships were found equivalent to those shown in Figures 4 - 7 for the Hecht vault. A greater horizontal preflight velocity produced greater height, distance and rotation. The generation of more rotation by increasing the horizontal preflight velocity has been demonstrated for both the Hecht and handspring somersault vaults, despite the fact that the rotation is reversed for the Hecht vault. The reason for this is related to the body position and shoulder angle. For the Hecht the shoulder angle tends to close during contact due to the much smaller shoulder angle at contact ($\alpha = 137^\circ$, Fig. 2) whereas for the handspring somersault the shoulder angle tends to open due to the almost straight shoulder angle at contact ($\alpha = 178^\circ$, Fig. 3). Thus increasing the horizontal preflight velocity for the Hecht produces a faster closing of the shoulder angle and for the handspring somersault produces a faster opening of the shoulder angle. The effect of the vertical velocity at horse contact on the handspring vault is very similar to the Hecht vault (Fig. 6, 7), with the higher the vertical velocity at horse touchdown the greater the vertical velocity and angular velocity at horse takeoff. These relationships identified for the handspring somersault vault will also hold for other continuous rotation vaults such as the Yurchenko, Tsukahara and Kasamatsu vaults.

The optimum Hecht and handspring somersault simulations had quite different preflights. At takeoff from the board the optimum Hecht simulation had a higher horizontal velocity, a lower vertical velocity and much less angular velocity (Fig. 2, 3). The reason for this is that a low angular momentum value at horse contact facilitates the reversal of the direction of rotation while in contact with the horse. In contrast for the handspring somersault vault, a high angular velocity and high vertical velocity are required at horse contact to give a high angular momentum value. At contact with the horse the Hecht requires a low body angle so as to maximise the production of backwards rotation while for the handspring vault a high body angle is required to reduce the amount of angular momentum lost. A shoulder angle of 137° was used for the Hecht (Fig. 2) compared with a shoulder angle close to 180° for the handspring somersault vault (Fig. 3) since a smaller shoulder angle helps the reversal in the direction of rotation.

The variation in performance of the simulation model for the Hecht vault from using the 11 different inertia parameter data sets with the optimum preflight variables obtained for the average inertia data set (Table 7) demonstrates that the postflight performance is sensitive to

the inertia characteristics of the gymnast for a given set of preflight variables. This result is also true for the handspring somersault simulations. On the other hand optimising the preflight variables for each of the 11 inertia data sets for the handspring somersault vault (Table 8) shows that the optimum postflight performance is not sensitive to the inertia parameters used. This result is also true for the Hecht simulations. Thus a gymnast's inertia parameters do not affect the level of performance that is attainable but do affect the technique used to achieve this level.

Although the model presented is a simple representation of gymnastic vaulting, it has been shown to reproduce the main features of the vault and forms a basis for a mechanical understanding of the mechanisms underlying vaulting. The major simplifications of the model occur for the contact phase, where it is assumed that there is no shoulder torque used during horse contact, and that the impact with the horse is inelastic.

Assuming that the model does not allow shoulder torque during the contact phase may appear to be an over-simplification. For the Hecht vault a high angular velocity of the arms is generated during the impact with the horse which results in a fast change in shoulder angle (Yeadon *et al.*, 1998 report a shoulder angular velocity of over 7.85 rad.s^{-1} at takeoff from the horse). Since the maximum torque that can be exerted at a joint decreases for high angular velocities, the torque that the gymnast can exert around the shoulder joint will be limited for much of the contact phase. As a consequence it may be expected that shoulder torque makes a minor contribution to performance. In order to estimate the magnitude of the contribution to the reversal in angular momentum that might be produced by the use of shoulder torque, a hypothetical simulation was carried out using a two segment model with shoulder torque. To isolate the effect of shoulder torque, the effects of the impact with the horse, segment weight and gravitational torque were removed by carrying out a simulation without gravity from a stationary position in contact with the horse. Mean values for the contact time (0.11 s), the angular velocities of the arms relative to the body at contact (1.99 rad.s^{-1}) and at takeoff (8.11 rad.s^{-1}) were calculated from data on 27 Hecht vaults (Yeadon *et al.*, 1998). It was assumed that the angular velocity increased linearly with time in order to calculate the torque at any given time during the hypothetical simulation. The relationship between maximum torque and shoulder angle and angular velocity was determined using measurements on a gymnast with a Kim-Com isokinetic dynamometer (King *et al.*, 1996). In the hypothetical simulation the shoulder torque was set equal to the maximum possible torque at the corresponding time of the Hecht contact phase. The simulation showed that about 10% of the angular momentum change during a Hecht takeoff could be produced by shoulder torque. This supports the idea that the majority of reversal in the angular momentum is produced by the correct preflight and body position at contact with the horse.

Assuming an inelastic impact with the horse results in an instantaneous reduction of the hand velocity to zero at impact and a subsequent immediate takeoff from the horse. In actual performances (Yeadon *et al.*, 1998) the contact time is around 0.1s. For a Hecht vault model the introduction of elasticity at the hand-horse interface would result in a finite contact time and an upwards component of hand velocity at takeoff from the horse together with an increase in the counter-rotation angular velocity of the body.

During preflight it was assumed that the model maintains a fixed configuration. This is realistic in that it is a fair representation of the technique used by many gymnasts although techniques do vary. Some gymnasts maintain a relatively fixed body configuration during preflight while others start the downwards motion of the arms relative to the trunk prior to horse contact.

The assumption of a fixed optimum landing angle γ_0 for each type of vault was made to simplify the optimisation procedure. The resulting optimum simulations were not affected greatly by this assumption. In the case of the Hecht vault it was not possible to satisfy the requirement for a successful simulation and so the use of a fixed landing angle requirement had no effect. In the case of the handspring somersault the optimum simulation had a landing angle which was not only close to -35° but was also within 1° of an ideal landing. The fixed landing angles of -25° and -35° for the Hecht and handspring somersault vaults compare well with the mean values of $-28^\circ \pm 16^\circ$ and $-34^\circ \pm 4^\circ$ from Yeadon *et al.* (1998) and Takei and Kim (1990) for Hecht and handspring somersault vaults respectively. In actual landings small inaccuracies in the landing angle may be corrected by rotating the arms forwards or backwards. This may give some flexibility in the range of landing angles that can be accommodated but the mean landing angles will be close to optimum as indicated by the above results.

Using a model comprising two rigid segments linked by a pin joint suggests that the majority of the backwards rotation during the postflight of a Hecht vault is a result of a low trajectory of the mass centre during preflight with a low vertical velocity of the mass centre and a low angular velocity of the body at horse contact. Similarly the majority of the postflight rotation of a handspring somersault is a result of a high preflight trajectory with a high angular velocity of the body and a high vertical velocity at horse contact. Despite the assumption of inelasticity and zero shoulder torque during horse contact, the two segment model was able to determine optimum preflight characteristics which were in close agreement with observed values for two competitive vaults. This demonstrates the importance of the preflight characteristics for vaulting performance and indicates that the model, although simplified, incorporates the main elements of vaulting. In the future the model will be modified in order to quantify the contributions of elasticity and shoulder torque to the translational and rotational performance characteristics of these vaults.

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