# The biomechanics of twisting somersaults. Part II: contact twist

# The biomechanics of twisting somersaults. Part II: Contact twist

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### Abstract

A simulation model and a rigid body model are used to investigate twisting initiated during the takeoff or contact phase. It is shown that it is possible to produce a full twist solely by building up angular momentum in the arms during the contact phase. This method is only half as effective as building up momentum in the whole body during contact. The introduction of twist into a somersault changes the somersault rate by less than 1%. By timing arm adduction appropriately, it is possible to take advantage of nutation and boost the initial value of the tilt angle and so obtain a greater twist rate. Twist may be stopped by the action of piking, since the motion changes from the twisting mode to the wobbling mode of rigid body motion. Transition to and from these two modes can be used to increase or decrease the tilt angle and twist rate.

Keywords: Twist, somersault, model, simulation, biomechanics.

## Introduction

Twist may be initiated either prior to takeoff or during the aerial phase of a somersault. Twist produced during the takeoff phase, in which the feet are still in contact with the diving board, trampoline bed, tumbling surface or ski ramp, will be referred to as *contact twist* in this paper. Other terms for contact twist include 'transfer of momentum twist' (Rackham, 1960), 'torque twist' (Frohlich, 1979), 'angular momentum twist' (Frohlich, 1980), 'twist from the board' (Eaves, 1969) and 'inertial twist' (Kosa and Kamimura, 1972).

It has been proposed that the initiation of twist during takeoff is achieved by twisting the head, arms and shoulders (Aaron, 1977), the upper body (Batterman, 1974) or the legs (Smith, 1980, 1981). Regarding the contribution of arm movements, Valliere (1976) commented: 'Contrary to what certain people still believe, it is not the transfer of momentum from the arm to the rest of the body that initiates the movement.' Thus the question arises as to what contribution the arms are capable of making to the twist.

Bunn (1972) held the view that the introduction of twist results in a faster somersault. If this is the case, then it is of interest to know why the somersault rate changes and by how much. Rackham (1960) stated that a diver can slow but cannot stop the twist by stretching his arms out sideways and piking his body. If this is so, then the performance of movements such as a full-in back-out, in which there is twist in the first somersault but not in the second, requires explanation. Eaves (1960) stated that taking twist from the board leads to tilt at the end of a dive. How divers can cope with this problem and enter the water with a straight body position without further twisting needs to be explained.

These questions, raised in the coaching literature, do not appear to have been addressed by research studies. In this paper, contact twist is considered from a theoretical viewpoint and such questions are answered using both the rigid body model developed in Part I and the 11-segment simulation model of Yeadon et al. (1990).

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# Transfer of momentum

Contact twist is initiated while the feet are in contact with the takeoff surface. During this contact phase, body segments are set in motion so that, at the instant of takeoff, there is angular momentum about the longitudinal axis of the body. Subsequently, the relative segmental motions cease and the whole body assumes an angular velocity that is determined by the conservation of angular momentum. This process is known as transfer of momentum, since initially only certain segments possess angular momentum, whereas once the relative segmental motions cease, all body segments possess angular momentum.

Expressions will be obtained for the angular momenta associated with relative segmental movements. Three cases will be considered:

(a) motions of the arms $A$ and $B$ relative to the chest $C$;

(b) torsion of the chest $C$ relative to the thorax $T$;

(c) twisting of the whole body $F$ relative to the feet.

Case (c) will be used as an approximation to the situation in which torsion occurs at the ankles, knees and hips.

## Angular momentum due to motion of the arms

An idealized situation is envisaged in which the arms move in a horizontal plane relative to the remainder of the body which is vertical and not rotating. Figure 1 shows a plan view of the left arm $A$, right arm $B$, left shoulder centre $S$ and right shoulder centre $R$. The midpoint of $R$ and $S$ is represented by $N$.
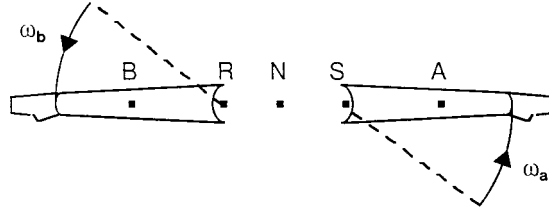


Figure 1: Plan view of arm rotation showing arms $A$ and $B$, shoulder centres $S$ and $R$ with midpoint $N$.

The angular momentum of the left arm about a vertical axis through $N$ is given by:

$$h_a = I_{aa}\omega_a + m_a v_a d(AN)$$

where $I_{aa}$ is the moment of inertia of the arm about a transverse axis through its mass centre, $\omega_a$ is the angular velocity of the arm, $m_a$ is the mass of the arm, $v_a$ is the velocity of the mass centre of the arm and $d(AN)$ is the distance from the mass centre of $A$ to the midpoint of $N$ of the shoulder centres. Since the shoulder centre $S$ remains at rest during the motion,

$$v_a = \omega_a d(AS)$$

so that:

$$h_a = [I_{aa} + m_a d(AS)d(AN)]\omega_a$$

The right arm will possess an equal amount of angular momentum if it also has angular velocity $\omega_a$, so that the total momentum may be written as:

$$h_{ab} = I_{ab}\omega_a$$

where $I_{ab} = 2[I_{aa} + m_a d(AS)d(AN)]$

## Angular momentum due to motion of the chest

In this movement, the chest and arms rotate about a vertical axis relative to the remainder of the body which does not rotate during the contact phase. Figure 2 shows a plan view of the left arm $A$, the right arm $B$ and the chest $C$. During the motion the supra segment $D$, which comprises $A$, $B$ and $C$, moves as a rigid unit so that the total angular momentum is given by:

$$h_d = I_{dd}\omega_c$$

where $I_{dd}$ is the moment of inertia of $D$ about a vertical axis through $N$ and $\omega_c$ is the angular velocity of the chest
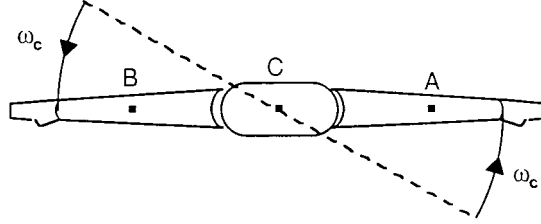


Figure 2: Plan view of chest torsion showing arms $A$, $B$ and chest $C$.

$I_{dd}$ may be calculated as $I_{dd} = I_{an} + I_{bn} + I_{cc}$ where $I_{an}$ is the moment of inertia of the left arm $A$ about a vertical axis through $N$ and may be evaluated using the theorem of parallel axes. $I_{bn}$ and $I_{cc}$ are the moments of inertia of the right arm $B$ and chest $C$ about the same axis.

## Angular momentum of the whole body

If the whole body rotates about the twist axis $\mathbf{f}_3$, the angular momentum will be:

$$h_f = I_{ff}\omega_f$$

where $I_{ff}$ is the moment of inertia of the body about $\mathbf{f}_3$, and $\omega_f$ is the angular velocity.

## Comparison of angular momenta

The angular momenta of the three motions are $h_{ab}$, $h_d$ and $h_f$ where:

$$h_{ab} = I_{ab}\omega_a$$
$$h_d = I_{dd}\omega_c$$
$$h_f = I_{ff}\omega_f$$

For the segmental inertia parameters of an elite trampolinist, $I_{ab}$, $I_{dd}$, and $I_{ff}$ take the values:

$$I_{ab} = 1.11 \quad I_{dd} = 1.84 \quad I_{ff} = 2.17 \quad (\text{kg} \cdot \text{m}^2)$$

where the arms are abducted at right angles to the body in each case. Thus for equal angular velocities $\omega_a$, $\omega_c$, and $\omega_f$, the angular momenta $h_{ab}$, $h_d$ and $h_f$ will be proportional to 1.11, 1.84 and 2.17, and so using the arms will be about half as effective as using the whole body to initiate the twist.

Once the arms are adducted, the moment of inertia of the body about the long axis will be $I = 0.70$ (kg· m$^2$) and, assuming that there is no somersault, the twist rate may be calculated as $h/I$ so that for the three cases the final twist rates will be:

$$I_{ab}\omega_a/I = 1.59\omega_a$$
$$I_{dd}\omega_c/I = 2.63\omega_c$$
$$I_{ff}\omega_f/I = 3.10\omega_f$$

In order to calculate the number of twists that can be produced by using the arms, an estimate of the maximum value of $\omega_a$ is needed. During a filmed movement in which an elite trampolinist adducted his arms through 180°, the maximum angular velocity of the arms was 2.4 rev s$^{-1}$. Fenn et al. (1931) obtained a maximum angular velocity of 1.9 rev s$^{-1}$ for arm swings parallel to the sagittal plane. In the proposed movement of the arms in a horizontal plane, the arm position shown in (Fig. 1) will maximize $I_{ab}$, but will reduce the maximum $\omega_a$, value since the arms are near the limit of their range of movement.

If $\omega_a$, is taken to be 1 rev s$^{-1}$, the resulting twist rate will be initially $I_{ab}\omega_a/I_{ff} = 0.51$ rev s$^{-1}$, increasing to $I_{ab}\omega_a/I = 1.59$ rev s$^{-1}$ once the arms are adducted. Even if the flight time is only 1 s and if half of this time is allowed for arm adduction and abduction, the total twist will be greater than one revolution.

These calculations indicate that it should be possible to produce a full twist in a vertical jump solely by building up angular momentum in the arms during the takeoff. In practice, it is to be expected that the chest or even the whole body will be used for the build up of angular momentum, since the moment of inertia of the moving, unit will be larger.

## Combining twist with somersault

In the preceding analysis, the twist rates were calculated on the assumption that there was no somersault. If twist and somersault occur together, it is of interest to see whether the twist rate is affected by the presence of somersault and whether the somersault rate is affected by the presence of twist. As a first approximation, the body will be modelled as a rigid rod with principal moments of inertia $A = B > C$.

### The motion of a rod

The description of the twisting mode in Part I showed that the body spins at a constant rate $p$ about the body axis $\mathbf{f}_3$, which makes a constant angle $\alpha$ with the plane normal to the angular momentum vector $\mathbf{h}$ and precesses about $\mathbf{h}$ at the constant rate $\Omega$ (Fig. 3). $\Omega$, $p$, $\alpha$ and $h$ are related by equations (24) and (25) derived in Part I as:

$$\Omega = h/A \tag{1}$$

$$p = \Omega(A/C - 1)\sin\alpha \tag{2}$$

so that equation 2 may be rewritten in the form:

$$p = h\sin\alpha(1/C - 1/A) \tag{3}$$

Suppose that the initial values of the somersault, tilt and twist angles are zero, so that the body axes $\mathbf{f}_1$, $\mathbf{f}_2$ and $\mathbf{f}_3$ are coincident with the inertial axes $\mathbf{i}_1$, $\mathbf{i}_2$ and $\mathbf{i}_3$ and let the angular momentum vector $\mathbf{h}$ lie in the plane $\mathbf{i}_1\mathbf{i}_3$ (Fig. 3) so that the components of $\mathbf{h}$ in the inertial frame $i$ will be:

$$h_1 = h\cos\alpha \quad h_2 = 0 \quad h_3 = h\sin\alpha$$

The somersault angle $\phi$ is the angle between the planes $\mathbf{i}_1\mathbf{f}_3$, and $\mathbf{i}_1\mathbf{i}_3$, and so the somersault rate $\phi$ is the rate of precession of the body axis $\mathbf{f}_3$, about both $\mathbf{i}_1$ and $\mathbf{h}$ (Fig. 4). As a consequence, the average somersault rate will be equal to the precession rate $\Omega$. The apparent tilt angle $\theta$ is the angle between $\mathbf{f}_3$, and the vertical plane $\mathbf{i}_2\mathbf{i}_3$ which is normal to $\mathbf{i}_1$. Since $\mathbf{f}_3$, makes a constant angle $\alpha$ with the plane normal to $\mathbf{h}$, the apparent tilt angle after a half a somersault will be $2\alpha$ (Fig. 4). Thus half a somersault about $\mathbf{i}_1$ followed by tilt through an angle $2\alpha$ about $\mathbf{f}_2$ is equivalent to half a revolution of precession about $\mathbf{h}$. In order for the final orientations to be equivalent, the twist angle $\psi$ must equal the angle of spin about $\mathbf{f}_3$, and so the average twist rate will be equal to the spin rate $p$.

If $\alpha = 45°$, then after half a revolution of precession about $\mathbf{h}$, the body will be horizontal as shown in Fig. 5. This position is equivalent to half a somersault and 90° of apparent tilt, although a complete cycle of precession is barely recognizable as a twisting somersault.

If $\alpha > 45°$, the body will never reach a horizontal position and the somersault angle will vary between approximately -$\alpha_1$ and $\alpha_1$, while the apparent tilt angle will oscillate between 0 and $2\alpha_1$ where
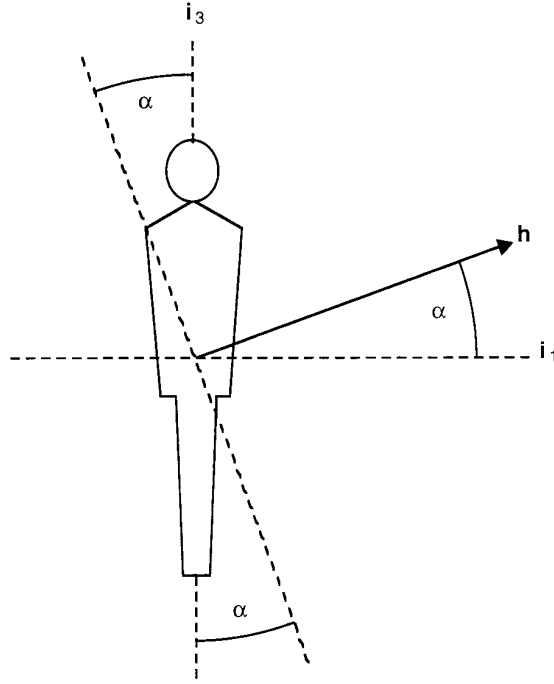
Figure 3: Precession of a rod about the angular momentum vector **h**.



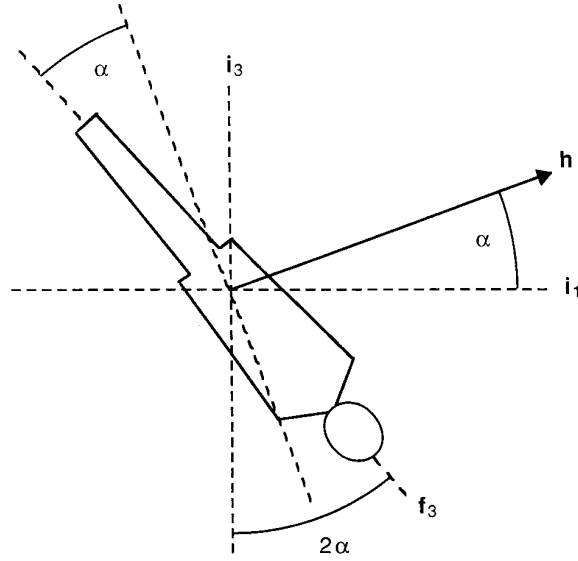Figure 4: Precession of a rod when angle $\alpha < 45°$.

$\alpha_1 = 90° - \alpha$ (Fig. 6). One revolution of precession about **h** will be perceived as a full twist and so the average twist rate will be $n$, where:

$$n = p + \Omega = h \sin \alpha (1/C - 1/A) + h/A$$

and when $\alpha = 90°$ the motion will be a pure twist with twist rate:
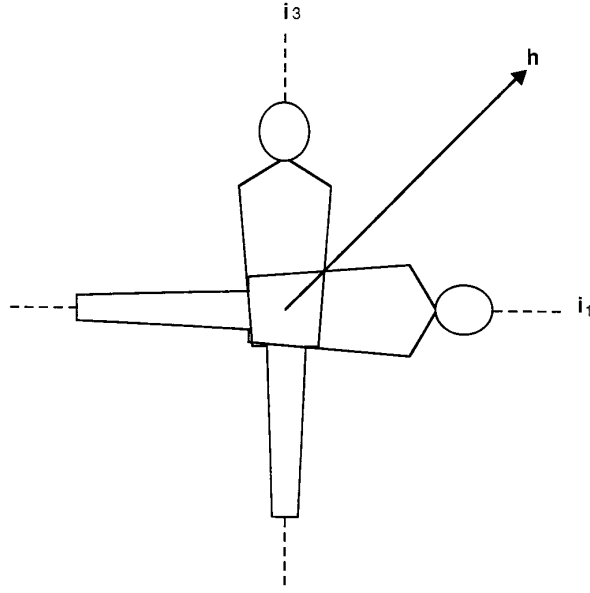
$$n = h/C$$

Three different motions may now be compared:

Figure 5: Precession of a rod when angle $\alpha = 45°$.



Figure 6: Precession of a rod when angle $\alpha = 45°$.

(a) $\alpha = 0$ for which the motion comprises somersault without twist with the constant somersault rate $h_1/A$.

(b) $0 < \alpha < 45°$ for which the motion is a twisting somersault with average somersault rate $\Omega$ and average twist rate $p$ where:

(1): $\Omega = h/A$

(3): $p = h_3(1/C - 1/A)$

(c) $\alpha = 90°$ for which the motion comprises twist without somersault with the constant twist rate $h_3/C$.

The introduction of the angular momentum component $h_3$ changes the plain somersault (a) into the twisting somersault (b) and the somersault rate increases from $h_1/A$ to $h/A$, where $h^2 = h_1{}^2 + h_3{}^2$. Thus the introduction of twist increases the somersault rate. The introduction of the angular momentum component $h_1$, changes the plain twist (c) into the twisting somersault (b) and the twist rate decreases from $h_3/C$ to $h_3(1/C - 1/A)$. Thus the introduction of somersault decreases the twist rate.

To see whether the same results occur when the whole body principal moments of inertia $A$ and $B$ are only approximately equal, consider the simulation CT1 shown in Fig. 7. The body remains in a fixed configuration and behaves like a rigid body. Initially, the apparent tilt angle between the longitudinal axis and the vertical somersault plane is zero and the angular momentum vector $\mathbf{h}$ has a normalized value of 1 which would produce one somersault if it were directed parallel to the lateral body axis $\mathbf{f}_1$.
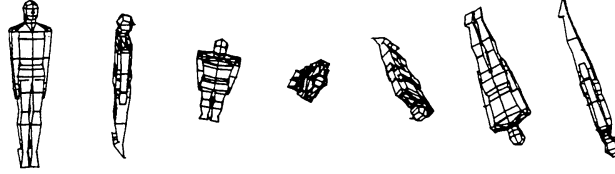


Figure 7: The rigid body simulation CT1.

The average somersault rate is 0.98 rev per unit time and the average twist rate is 2.51 rev per unit time. These values will be compared with the somersault rate of a plain somersault and the twist rate of a plain twist.

The principal moments of inertia have values $A = 11.01$, $B = 10.56$ and $C = 0.70$ (kg· m$^2$) and the angular momentum $\mathbf{h}$ has a normalized value of 1, which means that $h/B$ is equivalent to 1 rev per unit time.

The somersault rate of a plain somersault is $h_1/B = h\cos\beta/B$, which is equivalent to 0.985 rev per unit time. The twist rate of a plain twist is $h_3/C = h\sin\beta/C$, which is equivalent to 2.62 rev per unit time. Thus the introduction of somersault into a plain twist decreases the twist rate from 2.62 to 2.51 rev per unit time and the introduction of twist into a plain somersault decreases the somersault rate from 0.985 to 0.98 rev per unit time.

The reason for the decrease in somersault rate is that in simulation CT1 the somersault rate is approximately $h/I$, where $I = (A + B)/2$, so that although the angular momentum increases by 1.5% from $h_1 = 0.985h$ to $1.0h$, this is more than offset by an increase of 2.1% in the inertia term from $B = 10.56$ to $I = 10.785$ (kg· m$^2$).

## The nutation effect

In the twisting mode, the angle between the twist axis $\mathbf{f}_3$, and the plane normal to the angular momentum vector increases from $\beta$ to $\alpha$ as the twist angle increases by a quarter twist. This variation is known as nutation and although the increase in angle is only about half a degree in simulation CT1, the effect will be more pronounced when a wide arm position is employed. Let the movement CT2 be defined as follows:

Initially, the arms are extended laterally and the angle $\beta$ is 10°. In this position the principal moments of inertia become:

$$A = 13.45 \quad B = 11.52 \quad C = 2.17 \quad (\text{kg} \cdot \text{m}^2)$$

The angle between $\mathbf{f}_3$ and the plane normal to the angular momentum vector will increase from $\beta$ to $\alpha$ where equation (16) in Part I gives the relation:

$$(1/C - 1/A)\cos^2\alpha = (1/C - 1/B)\cos^2\beta \tag{4}$$

from which $\alpha = 14.4°$.

If the arms are now rapidly adducted at the quarter twist position, the principal moments of inertia will become:

$$A_1 = 11.01 \quad B_1 = 10.56 \quad C_1 = 0.70$$

and the angle between $\mathbf{f}_3$, and the plane normal to the angular momentum vector will vary between $\alpha$ and $\beta_1$ where:

$$(4): \quad (1/C_1 - 1/A_1)\cos^2\alpha = (1/C_1 - 1/B_1)\cos^2\beta_1$$

from which $\beta_1 = 14.0°$.

If the angular momentum components for the movement CT2 are the same as for simulation CT1 and the initial apparent tilt and twist angles are zero, the following results are obtained. In simulation CT2, the apparent tilt angle increases from zero to a maximum value of 24.2°, which lies between the theoretical limits of $(\beta + \beta_1) = 24.0°$ and $(\beta + \alpha) = 24.4°$. It should be noted that this maximum is reached after approximately half a somersault and the precise value is dependent upon the twist position at that time. The twist rate is initially slow because of the arm abduction, but once the arms are adducted at the quarter twist position the body twists at an average rate of 3.45 rev per unit time, which is 1.38 times the twist rate of simulation CT1. The somersault rate has an average value of 0.84 rev per unit time during the first quarter twist and becomes 0.99 rev per unit time once the arms are adducted, so that the final somersault rate is within 1% of the somersault rate in simulation CT1.

In simulation CT1, the arms may be considered to be initially extended laterally and to be instantaneously adducted at the start of the motion (Fig. 8a), whereas in simulation CT2, the instantaneous arm adduction occurs at the quarter twist position (Fig. 8b). This delaying of the arm adduction produces an increase of about 4° in the angle between axis $\mathbf{f}_3$, and the plane normal to the angular momentum vector and so the final twist rate is correspondingly greater for simulation CT2. For a rod, equation (2) shows that the twist rate is proportional to $\sin\alpha$, so that the ratio of the twist rates in simulations CT2 and CT1 will be approximately $(\sin 14.4°/\sin 10.4°) = 1.38$, which is in agreement with the value obtained from simulation.

In practice, the arms cannot be adducted instantaneously and so the final twist rate will be less than the theoretical value of 3.45 rev per unit time obtained in simulation CT2. Suppose that the arms are adducted between times $t = 0.1T$ and $t = 0.4T$, where $T$ is the flight time. If $T$ is near the 1.6 s flight time of a typical trampoline movement, this allows an ample 0.5 s for the arm adduction. The resulting motion is given by the simulation CT3 (Fig. 8c). After the arm adduction, the twist rate has an average value of 3.35 rev per unit time, which is 1.33 times the twist rate of CT1 and 0.97 of the theoretical maximum attained in CT2. The simulations CT1, CT2 and CT3 are compared in Fig. 8.
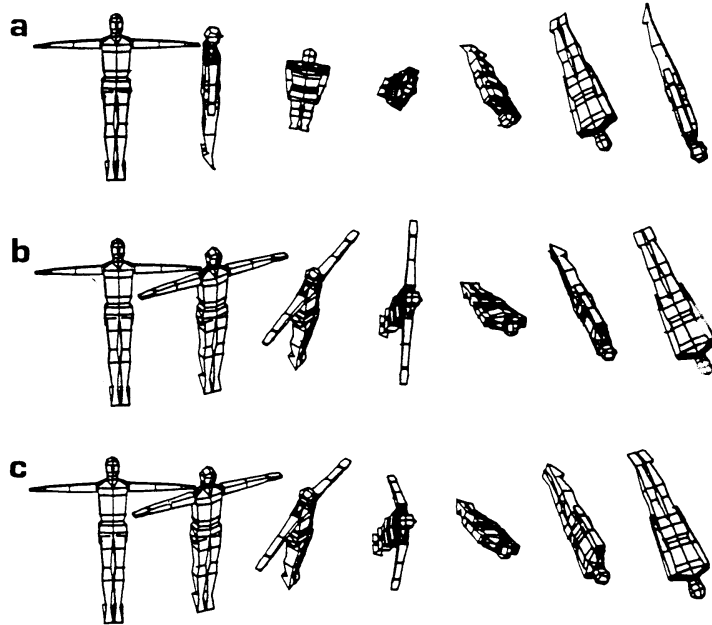


Figure 8: Comparison of contact twist simulations (a) CT1, (b) CT2 and (c) CT3, which have different timings of arm adduction.

This technique of delaying the arm adduction in order to increase the nutation and subsequent twist

rate will be referred to as the nutation technique. The nutation effect will be greatest when the arms are extended laterally, as in simulation CT2, and the initial tilt angle $\beta$ is small. Equation (4) with $A = 13.45$, $B = 11.52$ and $C = 2.17$ (kg· m$^2$) and $\cos\beta = 1$ gives $\alpha = 10.4°$.

In practice, the increase in tilt angle will be less than $10.4°$, since very small values of $\beta$ will result in an excessive time for the first quarter twist. When $\beta = 3°$, the above equation gives $\alpha = 10.6°$, which corresponds to a nutation effect of $7.6°$. Thus the maximum nutation effect will be about $7°$.

## Stopping the twist

Equations (1) and (2) show that for motion in the twisting mode, the twist rate is approximately $p = h\sin\alpha(1/C - 1/A)$. When the body pikes, $C$ will increase and $A$ will decrease so that the twist rate will decrease. This is shown in simulation CT4, which is similar to CT3 but somersaults in the opposite direction. Initially angle $\beta = 10°$, but this increases to $12.6°$ as the arms are adducted between $t = 0.1$ and $t = 0.4$ and the nutation effect is exploited. From $t = 0.7$ to $t = 1.0$, the arms are abducted and the body pikes so that the twist rate is reduced from about 3 rev per unit time to 0.75 rev per unit time. At $t = 1.0$ the residual tilt angle is less than $2°$ and the twist angle is within $6°$ of two twists so that the movement is perceived as a double twisting backward somersault with no apparent tilt upon landing (Fig. 9). The twisting mode of motion is maintained throughout this movement and so the twist rate is always strictly positive.
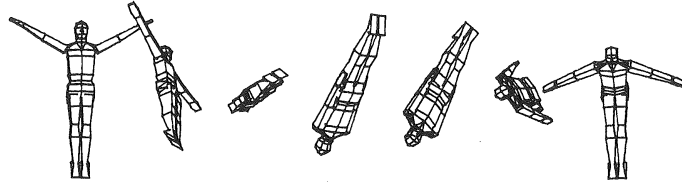


Figure 9: The simulation CT4 of a somersault with double twist.

In order to stop the twist, it is necessary to change from the twisting mode to the wobbling mode. The critical value of $\alpha$ corresponding to the singular solution is given as $\alpha_0$, defined in Part I by equation (13) as:

$$\cos^2\alpha_0 = A(B - C)/B(A - C) \tag{5}$$

If $\alpha > \alpha_0$ the motion will be in the twisting mode and if $\alpha < \alpha_0$ the motion will be in the wobbling mode. The values of $\alpha_0$ may be calculated for different pike angles by using the above equation (Table 1). In these piked positions, the arms are close to the legs and axis $\mathbf{f}_1$ corresponds to the maximum principal moment of inertia providing the pike angle is less than $137°$. If the arms were to be extended laterally, the axis $\mathbf{f}_1$, would correspond to the maximum moment of inertia only when the pike angle is between about $40°$ and $80°$, so that this position is not particularly suitable for changing the motion from a twisting somersault into a wobbling somersault.

Simulation CT5 shows how the wobbling mode may be used to obtain a tilt angle close to zero at the $1\frac{1}{2}$ somersault position (Fig. 10). The nutation effect is used to boost the angle $\beta$ from an initial value of $10°$ to $12.6°$, which corresponds to $\alpha = 13°$. As the body pikes, the value of $\alpha$ undergoes little change since the action of piking produces little change in the directions of the principal axes. Once the pike angle reaches $137°$, the angles $\alpha$ and $\beta$ become equal and, although they diverge as the pike deepens, angle $\alpha$ is largely unaffected since the piking occurs near to the $1\frac{1}{2}$ twist position. As the pike angle decreases, the critical value $\alpha_0$ increases (Table 1), so that when the pike angle is about $90°$ the condition $\alpha < \alpha_0$ applies and the motion changes from the twisting mode to the wobbling mode. In the wobbling mode, the angle between the twist axis $\mathbf{f}_3$ and the plane normal to the angular momentum vector oscillates between $+\alpha$ and $-\alpha$. In simulation CT5, the disc mode is maintained until this angle reaches about $-10°$, so that with the body extended at the $1\frac{1}{2}$ somersault position the apparent tilt is close to zero (Fig. 10).

Table 1: Values of the angle $\alpha_0$ for different pike angles

| Pike angle | $\alpha_0$ |
|---|---|
| 120° | 6° |
| 110° | 9° |
| 100° | 13° |
| 90° | 19° |
| 80° | 26° |
| 70° | 35° |
| 60° | 50° |

Note: For $\alpha > \alpha_0$ motion is in the twisting mode; for $\alpha < \alpha_0$ motion is in the wobbling mode. Arm positions are close to the body.
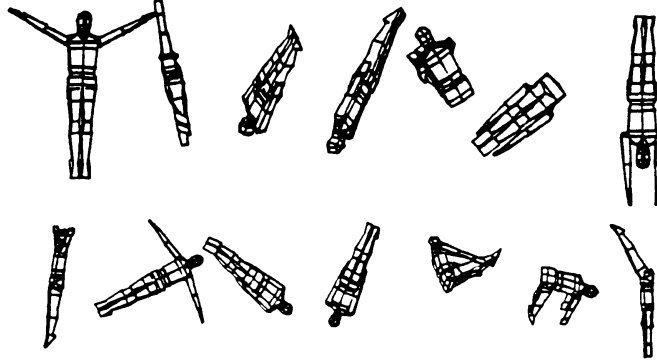


Figure 10: The simulation CT5 of a $1\frac{1}{2}$ somersault dive with $1\frac{1}{2}$ twists.

At the start of the simulation, angle $\beta = 10°$ and the body twists to the left while, at the end, angle $\beta = -10°$ and the body is twisting to the right. It is possible to arrange for the final twist rate to be zero by extending from the pike earlier so that the final value of $\beta$ is zero, but this will result in an apparent tilt angle of $-10°$. Thus there is no ideal solution for $1\frac{1}{2}$ somersault dives with contact twist and a compromise has to be made between a zero angle of tilt with a non-zero twist rate and a zero twist rate with a non-zero apparent angle of tilt.

## Oscillation in the wobbling mode

Let the angle between the twist axis $\mathbf{f}_3$, and the plane normal to the angular momentum vector (known as the invariable plane) be referred to as $\theta_1$. Figure 11a shows the effect that piking has on angle $\theta_1$. Initially, motion is in the twisting mode with $\theta_1$ oscillating between $\beta = 10°$ and $\alpha = 10.5°$ as the body twists in the straight position. At time $t = 0.1$, when the half twist position is reached, the body pikes rapidly to an angle of $90°$ and the motion changes to the wobbling mode. The angle $\theta_1$ then oscillates between $\alpha_1$ and $-\alpha_1$ where $\alpha_1 = 10°$, while the twist angle $\psi$ oscillates about the half twist position with amplitude $33°$.

If the body extends from the pike after one-quarter of an oscillation, the angle $\theta_1$ will be zero and the twist angle will be $213°$. The subsequent motion will approximate to the singular solution with the initial twist rate being zero. If the body extends from the pike after half an oscillation, the angle $\theta_1$ will be $-10°$ and the subsequent twisting somersault will be similar to the original motion but with the twist in the opposite direction. This was the technique used in simulation CT5.

Figure 11b shows the effect of piking earlier at $t = 0.08$, when the twist has reached only 0.4 rev. In the subsequent motion in the wobbling mode, $\theta_1$ initially increases and reaches a maximum value

## piking at 0.5 twists



## piking at 0.4 twists
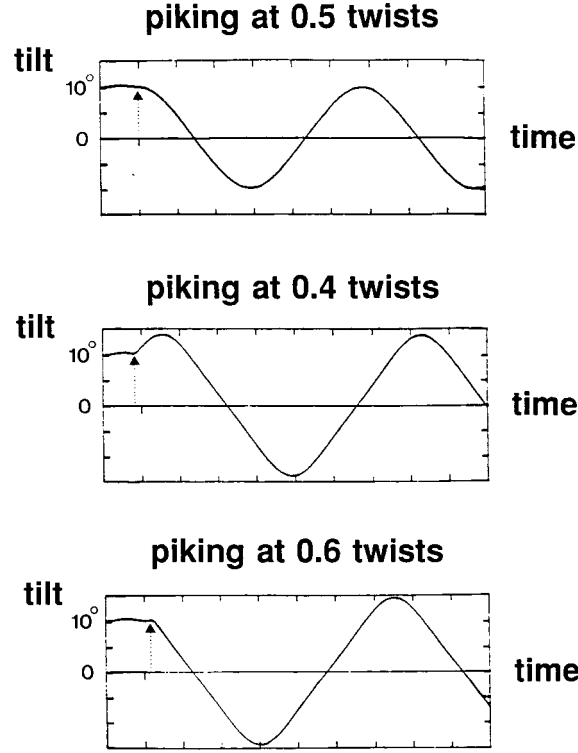


## piking at 0.6 twists



Figure 11: Wobbling mode oscillations of the tilt angle $\theta$, corresponding to various times of piking around the half twist position.

of $\alpha_1 \approx 14°$. Figure 11c shows the effect of piking later at $t = 0.12$, when the twist is 0.6 rev. In the subsequent motion, $\theta_1$ oscillates with amplitude $\alpha_1 \approx 14°$ and the twist angle $\psi$ oscillates with amplitude $49°$.

It should be noted that although the piking movements were made rapidly, the same effects can be produced using longer times for piking, since it is a matter of being early or late relative to the half twist position that produces (b) or (c). This variation in the response of $\theta_1$ to the timing of the pike provides a means of control. By piking early as in (b), extension may be made when $\theta_1$ is near the maximum so that the wobbling mode is used to boost $\theta_1$ and the twist rate. By piking late as in (c), $\theta_1$ may be reduced to zero more rapidly than in (a).

Another effect of piking early or late is to increase the oscillation period. In (a) one oscillation of $\theta_1$ occurs in 1.8 somersaults, while in (b) and (c) this increases to 2.2 somersaults. In theory, the oscillation period may be extended indefinitely by rapidly piking at the quarter twist position in such a way that $\alpha_1$ is equal to the critical value $\alpha_0$ which corresponds to the singular solution of rigid body motion. The oscillation period is a function of the pike angle as shown in Table 2 for $\alpha_1 = 10°$. For a pike angle of about 108°, the critical value $\alpha_0$, is near 10° so that the motion approaches the singular solution and the oscillation period becomes large.

Thus the timing and depth of the pike will affect the phase angle, amplitude and time period of $\theta_1$ and enable diverse movements to be produced. Any technique capable of producing a wide range of responses also introduces problems of control. If small differences in technique produce large differences in the subsequent motion, then it will become necessary to modify technique during flight in order to produce a given movement.

The oscillation effect can increase the angle $\theta_1$ to values approaching $\alpha_0$, which will be large when the pike angle is small (Table 1). For a pike angle of 90°, the critical value $\alpha_0 = 19°$ and so it is possible to boost the angle $\theta_1$ to more than 20° providing a pike angle of less than 90° is adopted.

Table 2: Oscillation periods for different pike angles

| Pike angle | Oscillation period (somersaults) |
|:---:|:---:|
| 60° | 1.56 |
| 70° | 1.57 |
| 80° | 1.63 |
| 90° | 1.81 |
| 100° | 2.13 |

Note: The amplitude of the tilt oscillation is 10°.

## Discussion

The segmental inertia parameter values of one elite male trampolinist have been used in the calculations and simulations pertaining to contact twists. While the values of the inertia parameters do not affect the occurrence of momentum transfer, nutation in the twisting mode and oscillation in the wobbling mode, the magnitudes of these effects are dependent upon the individual's inertia values. Some idea of individual variation may be obtained by comparison with the corresponding results for two other trampolinists, one male and one female.

The largest variation occurs in the value of $I_{ab}/I$, which gives the ratio of the twist rate after momentum transfer to the initial angular velocity of the arms. The value for the female is some 20% lower than that used in the analysis and this difference is due in part to a smaller relative arm mass (2%) but arises primarily from her shorter arm length (90%), the value of $I_{ab}$ being approximately proportional to the square of arm length.

The ratios of whole body principal moments of inertia of the additional two subjects, for various pike and arm positions, all lie within about 10% of the corresponding ratios used. As a consequence, the inertia characteristics are very similar to those of the elite trampolinist:

(a) When piking, the directions of the whole body principal axes change by less than 4° (for pike angles greater than 80°).

(b) Nutation effects lie within 1° of those obtained.

(c) The angles of pike required to change a given motion from the twisting mode to the wobbling mode lie within 10° of those obtained.

(d) The angles of pike corresponding to a given period of wobbling mode oscillation lie within 10° of those obtained.

The nutation effect in the twisting mode and the effects which can be produced using the wobbling mode all require a delay before changing the arm or pike angles. Although the nutation effect is greatest when the arms are adducted rapidly at the quarter twist position, simulation CT3 has shown that a relatively slow arm movement can produce a twist rate within 3% of the theoretical maximum. Similarly, the duration of the piking movement has little effect on the wobbling mode oscillations. Thus it is not the duration but the timing of the internal movement that is of importance. As a consequence, the ability of an individual to make rapid configurational changes is of marginal advantage, whereas the ability to make internal movements at the appropriate time is crucial.

In the Introduction, four questions were raised. These have been answered in the following way.

1. It is possible to produce a full twist solely by building up angular momentum in the arms during the takeoff phase.

2. The introduction of twist into a somersault changes the somersault rate by less than 1%.

3. It is possible to stop the twist in a somersault by piking so that the motion changes from the twisting mode to the wobbling mode.

4. In a dive where contact twist is used, the apparent tilt angle at entry can be reduced to zero by piking to take advantage of wobbling mode oscillation.

   Thus from a theoretical perspective, the use of contact twist appears to be a viable technique. This is supported by the experimental study of Moore (1951), in which full twisting somersaults were performed on trampoline with the relative movements of body segments restricted using splints and plaster casts. It was found that as more body parts were immobilized, the trampolinist twisted further during the contact phase. This result also suggests that the trampolinist used other techniques when his relative movement was not restricted. Since the initiation of twist during the contact phase has the disadvantage that twist will still be present if the body is correctly aligned for landing, it may be speculated that aerial techniques will be used in preference to contact techniques. This speculation presupposes that aerial techniques are capable of producing substantial twist during a somersault, a matter that is considered in Part III. The extent to which contact and aerial techniques are normally used by trampolinists, gymnasts and divers has yet to be established. This question will be addressed in Part IV, where a method is presented for the partitioning of twisting performances into contributions from contact and aerial techniques.

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