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## A theoretical study of wind flow over hills

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## A THEORETICAL STUDY OF WIND FLOW OVER HILLS

## by

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## SUMMARY

In order to determine the forces on a structure placed upon an obstacle it is necessary to know as accurately as possible the velocity of the wind at every point of the structure. The earth's boundary layer may be considered as consisting of a mean velocity together with a random variation about the mean, and the aim of the present work is to determine the change in the mean velocity profile of the wind as it passes over an arbitrarily shaped obstacle.

The first method considered was to modify the classical approach, which calculates the flow of a uniform stream about an obstacle, for a boundary layer type profile. This proved unsuitable and attention was turned to the solution of the governing equations of motion. These were solved, using a numerical method, to enable the flow to be calculated over an arbitrary shape. The flow as computed for two hill shapes is presented and the results agree with a qualitative consideration of the problem.

To study the effects of separation, the flow was calculated for a family of shapes and a range of Reynolds numbers. Graphs of the streamlines and velocity profiles are presented for some typical cases.


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## CHAPTER 1: INTRODUCTION

A better knowledge of the power of the natural wind has become more and more important as the height of man made structures has increased. Moreover their susceptibility to wind damage has been further increased by the necessity of designing them within an economic limit by keeping the safety factors involved to a minimum. Various natural disasters from the 01d Brighton Pier in 1836, through the Tay Bridge and Tacoma Narrows failures to the recent collapse of the cooling towers at Ferrybridge have emphasised that the power of the natural wind must not be underestimated. It is thus necessary to know, as accurately as possible, the velocity of the wind at each part of the structure.

As the wind blows over level ground the velocity profije develops and then maintains a constant shape of the boundary layer type. However, if there are obstacles such as a hill, or on a smaller scale a road or railway embankment, the profile undergoes a change of shape. Since tall masts for television transmissions tend to be sited on the tops of hills, and large vehicles, which may be affected by the wind, use the embankments, a problem of some importance is that of the change in wind speed profile over such an obstacle. .

The forces due to the natural wind may be thought of as consisting of two types:
(1) Those due to the action of an average velocity
(2) Those due to the variations of the velocity about this average。

The latter has obviously to be treated as a statistical problem, since the variations are random, while the former may be considered along deterministic lines.

Work has been done ${ }^{l}$ to deternine the effect of the random variations on the structures but there seems to be no method of predicting the change in flow as the wind passes over an obstacle of arbitrary shape. It is the purpose of the present work to determine such a method.

A further problem which can be considered once a solution has been found is that of separation of the flow. This may occur on the lee side of the hill and if it occurs it alters the flow over the brow of the hill. The conventional method used to predict the onset of separation is that the skin friction becomes zero. As will be shown (Chapter 2) the problem is not amenable to an analytic solution and no general criterion can be applicd to find the onset of separation. However the method of solution enables the point of separation to be determined for particular hill shapes, while also determining the flow within the separation bubble.

### 1.1 Previous work

The classical method of predicting the flow of an inviscid fluid over a body is that due to Rankine ${ }^{2}$ 。 By using a combination of sources and sinks together with a uniform velocity profile it is possible to calculate the changes of a uniform airflow over a hill shape. This method is however not accurate enough as in this case the velocity of the air actually increases towards the boundary of the body, rather than decreasing to zero as in practice.

The first attempts at the problem were to modify this method to take into consideration a boundary layer type profile. These are presented in Chapter 2, and although the results were not satisfactory they did enable a 'feel' for the problem to be obtained.

With the failure of this method to predict the flow near a boundary attention was turned to the solution of the governing Navier-Stokes equations. These are difficult to solve in all but a few special problems, the main difficulty being that they are non-linear second order simultaneous differential equations. In many of the exact solutions the quadratic terms are identically zero (as in flow along a pipe) or the non-linear terms can be neglected in comparison with the viscous terms (as in Stokes flow past a sphere). However in the present case there will be a region near the solid surface where the inertia terms and viscous terms are of the same order of magnitude so that neither can be neglected.

In 1904 Prandtl initiated the concept of a boundary layer. It is based upon the assumption that the action of viscosity on the flow past a solid body is confined to a thin layer of fluid close to the solid boundary, the motion outside the bowdary layer being of the inviscid flow type. Inside this thin layer the viscous and inertia terms are considered to be of the same order of magnitude and the velocity gradient perpendicular to the surface is very large compared with the downstream velocity gradient. These assumptions enable the equations of motion to be simplified, : although their solution in the general case is still very difficult. However the progress of aeronautics led to great interest in the flow of a fluid near a surface and many techniques were developed for their solution, or approximate solution ${ }^{3}$. Prandtl's simplifying assumption is that the thickness of the boundary layer is small compared with the radius of curvature of the surface, and this is assumed in most of the work.

The height of the boundary layer for the natural wind extends to at least 300 metres or more depending upon the type of underlying terrain, while the size of the obstacles may be considerably less than this. In this layer neither the viscous or inertia terms may be neglected, but the vertical velocity gradient may be of the same order of magnitude as the downstream velocity gradient, and thus the conventional boundary layer theory, as introduced by Prandtl, is not applicable to the present work. The velocity profile of the wind is of the boundary layer type (zero at the surface, increasing to the free stream velocity) and the problem may be referred to as being of a boundary layer type.

Attention was turned to the problem of wind flow over hills when glider pilots frequently reported what appeared to be systems of standing waves in the lee of the hills. In a series of papers Long (Ref. 5 i , ii, iii) investigated, both theoretically and experimentally, the conditions under which lee waves may be set up. He considered steady two-dimensional incompressible flow, and assumed that upstream from the obstacle the density varied linearly with height and that $U^{2} \rho=$ constant, where $U$ is the velocity, $\rho$ the density, and both are functions of $y$ 。 Using these approximations the partial differential equations reduced to a linear partial differential equation, which was solved by analytical techniques. The
disadvantage of the method, however, was that the shape of the obstacle was determined from the solution, by replacing suitable streamlines by solid boundaries. Thus the direct calculation of flow over an arbitrary shape was excluded. For the shapes that did result his theoretical solutions agreed with the practical results obtained.

His work generated interest in the flow of a stratified fluid over obstacles, but it was not until 1967 when Drazin and Moore ${ }^{6}$ published their work that the flow over a prescribed obstacle was possible. They used the same assumptions as Long, but recognised the resulting equations as similar to one derived in diffraction theory, and using methods developed in this field they calculated flows over a thin vertical strip. This was further developed by Davis ${ }^{7}$ who used a numerical method for the direct integration of the equations using Green's functions.

Work was done by Onishi ${ }^{8}$ who combined a stratified flow with the boundary layer equations and obtained an approximate solution for the flow over a ridge, which was described by an analytic function. Although no lee waves were present, the flow separated on the downstream side of the ridge, which would be expected for the steep ridge considered.

The large amount of theory that has been developed for the flow of a stratified flujd over an obstacle is not applicable to the problem at hand. The winds which generate the important forces (from a wind loading viewpoint) on a structure are those having a high velocity. At these high speeds there is so much mechanical stirring of the atmosphere that any density stratification that existed is destroyed, and the air may be thought of as having uniform density。

The only available work found on non-stratified flow is that published by Imai et al ${ }^{9}$. He assumes that the flow may be thought of as consisting of a boundary layer adjoining the surface, and a free atmosphere located over it. The flow is solved for the inviscid layer over the surface, and the results used as boundary conditions for the boundary layer equations. By assuming that the velocities just outside the boundary layer are the same as those just inside the boundary layer the equations are reduced to those of a heat conduction type. In his solution he arbitrarily limits the
boundary layer height to a constant value over any type of terrain. This approximation is difficult to justify. He shows that for obstacles with a gradual slope there is a drop in wind velocity behind the hill, which would be expected, while for obstacles with steep slopes a velocity greater than that obtained in the upper air was produced near the surface in front of the hill, and separation occurred behind it. Although these results are not actually impossible phenomena, they cannot be expected from an equation of the heat conduction type (since for this type of equation no such changes in the value of the function can be expected downstream of the initial conditions, for the values of the function at any position depend upon those immediately before it) and indicate that the method is not reliable for obstacles with steep slopes.

From the work studied it appears that all the approximations necessary in order to solve the Navier-Stokes equations limit tho flow to areas which are not applicable to the problem at hand, and attention was focussed on the two dimensional Navier-Stokes equations for incompressible flow. Wi.th the advent of high speed digital computers less emphasis has been placed upon obtaining exact analytical solutions. Numerical schemes, using finite difference techmiques, have been developed to obtain approximate. solutions to the full equations, and it is upon these methods that the present work has been based.

The earliest analytical approach to the problem of a two-dimensional incompressible flow over an obstacle is that due to Rankine. The method he adopted was to superimpose a velocity field consisting of a constant velocity parallel to the horizontal axis on that of a source, and for steady flow the resulting streamlines may be considered as the trajectories of the fluid particles, as is shown below.

Milne-Thompson ${ }^{10}$. uses complex variable theory to calculate the flow of a uniform strean over a circular mound or ditch, and this work has been further extended by Lahiri ${ }^{11}$ who considers the case of uniform shearing motion of a fluid past a circular projection. In both cases, however, the flow is of the inviscid type, and not that found under natural conditions. Schubert ${ }^{12}$ does consider the flow of a viscous fluid over circular obstacles. He too uses a complex transformation but limits the problem to one where the Reynolds number becomes small and the velocity profije may be considered linear.

None of these methods gives an accurate description of the flow as it occurs naturally, and attempts were made (Chapter 2.2) to modify Rankine's original method to take into account a boundary layer type profile.

### 2.1 Source and a uniform stream

This is the method proposed by Rankine and has been used by Glauert ${ }^{2}$ to examine the flow of an airstream, which is assumed to be inviscid, over a level plane and then over an obstacle.

The equation of continuity of an incompressible fluid is

$$
\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0
$$

and this enables the stream function, $\Psi$, to be defined as

$$
u=\frac{\partial \Psi}{\partial x} ; \quad v=-\frac{\partial \Psi}{\partial y}
$$

where $x$ and $y$ are mutually perpendicular axes, $u$ is the velocity parallel to the $x$ axis, and $\nabla$ is the velocity parallel to the $y$ axis.

Ihus the streanfunction for a uniform stream of velocity $U$ parallel to the $x$ axis, going from left to right is given by

$$
\begin{equation*}
\psi_{U}=-U_{y} \tag{2.1}
\end{equation*}
$$

A source is a point where fluid is created at a uniform rate, and flovs uniformly in all directions along radial lines, the velocity being proportional to the distance from the centre. If $m$ is the rate at which fluid is being produced i.e. the strength, then the streamfunction is given by

$$
\begin{equation*}
\Psi_{S}=\frac{m}{2 \pi} \arctan \left(\frac{y}{x}\right) \tag{2.2}
\end{equation*}
$$

The streamfunction for the combined systems is given by the sum of Equation (2.1) and Equation (2.2). Thus

$$
\begin{align*}
\Psi & =\Psi_{V}+\Psi_{S} \\
& =-U y+\frac{m}{2 \pi} \arctan \left(\frac{y}{x}\right) \tag{2.3}
\end{align*}
$$

The stramlines for this flow are given in Fig. 1.

The velocity at any point can be calculated from Equation (2.3)

$$
\begin{gather*}
u=-U+\frac{m}{2 \pi} \frac{x}{x^{2}+y^{2}}  \tag{2.4}\\
v=\frac{m}{2 \pi} \frac{y}{x^{2}+y^{2}} \tag{2.5}
\end{gather*}
$$

The stagnation point is the point at which the velocity is zero, and it can be seen from Equation (2.5) that this occurs on $y=0$, and there is no flow across the x axis. From Equation (2.4) the stagnation point is given by

$$
\mathrm{x}=\frac{\mathrm{m}}{\mathrm{U} 2 \pi}
$$

This is the point at which the stream velocity and the velocity due to the source neutralize each other and is the point A in Fig.l. The streamline passing through this point is called the dividing streamline, since all the flow contained within it is due entirely to the source and the flow outside is due to the uniform stream. Thus we could replace this streamline wi.th a solid shape and Equation (2.3) would give the flow around this obstacle.

From Equations (2.4) and (2.5) the velocity profiles may be found and are drawn in Pig.2. This shows that the velocity profile only resembles a boundary layer type profile near the foot of the hill. On the boundary there are two velocity components, $U$, a horizontal velocity and
$\frac{m}{2 \pi}\left(x^{2}+y^{2}\right)^{\frac{1}{2}}$ a veloci.ty due to the source. The total velocity at this point is then a tangent to the hill and can actually exceed $U$. With a real boundary layer type velocity profile, the total velocity on the obstacle must be zero.

### 2.2 Source and a boundary layer type profile

As a uniform velocity profile dees not represent the natural wind conditions very accurately it was thought that a velocity profile more like that expected in nature should be tried. Consideration of Fig. 3 shows that a profile given by

$$
\frac{u}{v}=\tanh (y)
$$

is of the boundary layer type, lying between a laminar and turbulent profile. Here $u$ is the velocity at height $y$, and $U$ is the mainstream velocity which is supposed, for the sake of convenience, to be unity. If the edge of the boundary layer is considered to be 0.99 U then its height ( $\delta$ ) is 2.65 .

Thus the streamfunction for this velocity profile becomes

$$
\begin{equation*}
\Psi_{S}=-\log \cosh (y)+f(x) \tag{2.6}
\end{equation*}
$$

where $f(x)$ is an arbitrary function of integration, and at our disposal.
Suppose initally we put

$$
f(x)=0
$$

If a source is now introduced at the origin, the streamfunction for the combination, following the previous case, is given by

$$
\begin{equation*}
\Psi=-\log \cosh y+\frac{m}{2 \pi} \arctan \frac{y}{x} \tag{2.7}
\end{equation*}
$$

The streamlines using this equation are given in Fig. 4.

The components of velocity become

$$
\begin{gather*}
u=-\tanh y+\frac{m}{2 \pi} \frac{x}{\left(x^{2}+y^{2}\right)}  \tag{2.8}\\
-12-
\end{gather*}
$$

$$
\begin{equation*}
v=\frac{m}{2^{\pi}} \frac{y}{x^{2}+y^{2}} \tag{2.9}
\end{equation*}
$$

Equation (2.9) shows that again there is no flow across the x axis, and from Equation (2.8) the stagnation point is seen to be at infinity. Thus the hill, as determined by the dividing streamline has a much gentler slope than that given by a uniform velocity profile, the height diminishing to zero at infinity.

Using Equations (2.8) and (2.9) the velocity profiles may be drawn and these are shown in Fig.5. These profiles are closer to reality than those previously obtained, but still do not satisfy the condition that the velocity component must be zcro on the grcund i.e. the divi.ding streamline.

In obtaining the streamfunction for the velocity profile used the arbitrary function of integration $f(x)$ was assumed to be zero. If this is not the case and it is used to obtain a value such that the velocity components satisfy the boundary conditions, the flov pattern may go further to modelling the real situation.

From Equations (2.2) and (2.6) the streamfunction for the combined motion becomes

$$
\begin{equation*}
y=-\log \cosh y+\frac{m}{2 r} \arctan \frac{y}{x}+f(x) \tag{2.10}
\end{equation*}
$$

and the velocity components are
and

$$
\begin{align*}
& u=\frac{\partial \Psi}{\partial y}=-\tanh y+\frac{m}{2 \pi} \frac{x}{x^{2}+y^{2}}  \tag{2.11}\\
& v=\frac{-\partial \Psi}{\partial x}=\frac{m}{2 \pi} \frac{y}{x^{2}+y^{2}}-f^{\prime}(x) \tag{2.12}
\end{align*}
$$

From Equation (2.11), using the boundary condition $u=0$

$$
\begin{equation*}
x=\frac{m \pm{\sqrt{\dot{m}^{2}-(4 \pi y \tanh y)}}_{4 \pi \tanh y}^{2}}{4} \tag{2.13}
\end{equation*}
$$

and from Equation (2.12)

$$
\begin{equation*}
f^{\prime}(x)=\frac{m}{2 \pi} \frac{y}{x^{2}+y^{2}} \tag{2.14}
\end{equation*}
$$

Equation (2.13) permits the values of $x$ and $y$ to be found such that $u=0$ is satisfied.

These are plotted, for various values of $m=1,4,10,20$ in Fig. 6. Although a singularity has been introduced at the origin, a fair representation may still be expected upstream from the maximum of the curves.

It appears from Equation (2.14) that $f^{\prime}(x)$, and hence $f(x)$, is a function of both $x$ and $y$, whercas it is actually only a function of $x$. In theory it is possible to obtain, from Equation (2.13) y as a function of $x$, and if this is substituted into Equation (2.14) it becones a function of $x$ alone. However Equation (2.13) is not easily solved for $y$ in terms of $x$, and the values of $x$ have to be found at each value of $y$, and both values substituted into Equation 2.14) to obtain $f^{\prime}(x)$ at each value of $x$. This has been done and the results, for the same valucs of $m$ as before, are shown in Fig. 7 .

The dividing streamline is obtained by letting $\psi=0$. Upon putting this condition into Equation (2.10) one obtains

$$
f(x)=+\log \cosh y-\frac{m}{2 \pi} \quad \arctan \left(\frac{y}{x}\right)
$$

where $y$ is given by Equation (2.13). The function $f(x)$ is shown in Fig. 8.

Thus knowledge has been obtained about the arbitrary function $f(x)$ and the flow may be drawn such that the conditions $u=0, v=0$ are satisfjed upon the boundary, given by $\psi=0$. Figure 9 shows the streamlines drawn for such a motion, for a value of $m=20$. The streamlines group closer together as the maximum of the streamline $\psi=0$ is approached, indicating an increase in velocity.

From Equations (2.11), (2.12) and (2.14) the velocity profiles may be drawn, and these are shown in Fig.10. Although the conditions at the boundary are satisfied the velocity profile becomes shallower as the brow is reached, instead of becoming steeper, as would be expected from qualitative considerations of flow around an obstacle. Further if the edge of the boundary layer is regarded as the point given by $u=0.99 \mathrm{U}$, where U is the mainstream velocity, the depth of the boundary layer undergoes a remarkable change of height, as indicated in Fig.11, one that would not at all be expected of the real boundary layer in the earth's surface.

By definition a source is a point of infinite velocity, while in order to satisfy the boundary conditions it is necessary to impose zero velocity upon
the origin where the source is situated. This anomaly is further displayed in that all the strcamlines produced pass through the origin, this in itself would not be scrious if the effects were negligible further away from the origin. However it is also shown in the height of the boundary layer, becoming unduly large, and the velocity profiles not becoming as steep as would be expected. Thus it appears that it is incompatible to have the conditions $u=0 ; \quad v=0$ on $\Psi=0$ in coexistence with a source and boundary layer type profile of the form $u=\tanh y$.

Thus it is not possible to model a boundary layer passing over an obstacle in this way. Although the work done on trying to extend Rankine's method to incorporate a boundary layer type profile produced no positive results it did enable a fuller understanding of the problem to be obtained.

### 3.1 Structure of the atmosphere

The failure of the previous analytical methods to describe the change in velocity profile which would be similar to that expected for the flow of the natural wind over an obstacle led to a closer examination of the nature of the atmospheric boundary layer.

In discussing the details of a general air flow, it is convenient to consider the atmosphere to be divided into a number of horizontal layers, each with different characteristics as shown in Fig.12. In the upper atmosphere the effects of friction may be j.gnored and the estimation of wind force is based on the assumption that the speed is adjusted to maintain a balance involving only the pressure gradient and the forces arising from the rotation of the earth. This velocity is knom as the geostrophic velocity and occurs at heights between 500 and 1000 metres. The layer immediately below this is known at the planetary boundary layer, where the transition from the flow near the surface to frictionless flow takes place. In this layer the wind is influenced by a combination of the surface friction, any variation of density gradient which is present, and the effects of the earth's rotation i.e., the Coriolis force. Below this is a further layer which may extend up to 100 metres. Within this the wind structure is influenced by the nature of the surface and the vertical gradient of temperature. This layer is referred to as the surface boundary layer. In a thin layer adjoining the surface the flow is determined by the roughness parameters of the underlying surface, and it is called the dynamic sublayer, which is several metres deep.

These factors combine to produce a turbulent boundary layer which consists of a steady mean motion upon which is superimposed a complicated secondary, or cddy, motion of a random nature and is quite unlike most turbulent boundary layers normally dealt with in engineering. It is impossible to obtain exact solutions to the equations of the motion which satisfy all of these conditions. However when considering the flow of the wind with reference to structural loading problems not all of these conditions exist, and the governing equations are much simpler in form.

The most severe forces are imposed upon structures under high wind conditions, and it is this requisite which enables some simplifications of the equations to be made. With this condition of high speeds the friction caused by the earth's surface is very large, and so much mechanical stixring of the atmosphere is present that any thermal or density gradients which exist are minimal and need not be considered in the derivation of the equations. The forces produced by high wind conditions near to the surface are much greater than those due to the earth's station and thus the Corioli's forces may be neglected when deriving the equations of motion. Also the effects of the compressibility of the air need not be considered since the velocities concerned are not large enough for any compression to take place, and the atmosphere may be regarded as incompressiblea Thus the very complex structure of the earth's boundary layer may be reduced to one where the dominating influences are those of velocity and viscosityo In addition experimental work (see Davenport ${ }^{23}$ ) has been carried out to determine the height of the boundary layer under such conditions, for varying underlying terrains, the shape of the velocity profile, and height, of the boundary layer being determined by the fetch (see section 4.1 )。

As stated earlier the turbulent motion may be thought of as consisting of a mean motion with random fluctuations about this mean. Much work has been done on the forces produced by the fluctuations, but little is available on the nature of the mean motion of the wind over obstacles. This mean motion may itself be susceptible to changes with time, but for the purposes of the present work it will be considered to be invariant with time. The maximum velocities obtained do vary but over a relatively long period of time compared with the response time of the structures concemed. Winds may take many hours to reach their highest velocity, and this velocity may be maintained for a period of several hours with the forces gradually imposing themselves upon the structure until a maximum is reached. Thus there are no sudden changes of the mean velocity which affect the structure and the mean motion may be thought of as being steady.

In the physical situation the motion is three dimensional in nature, and equations may be derived to model this. However the full three dimensional equations are extremely difficult to solve and in order to obtain a solution to the problem the motion is considered to occur in two dimensions
only. In many cases, such as railway embankments, this may be qui.te an accurate approximation; but in others it may not be so acceptable. By considering the energy of the system it can be seen that it has one dimension less through which to act in the two dimensional case, and this will provide a limiting solution of the three dimensional one。

Thus from the very complex description of the atmosphere first presented it is possible, by various approximations, to obtain a much simplex idealization of the airflov, and it becomes more like the boundary layers considered in aeronautics. As shown previously the approximations used in this field axe not applicable to the case of wind flow over hills, and the two dimensional Navier-Stokes equations for an incompressible fluid are needed for a description of the flow.

### 3.2 Derivation of the equations

If $u$ and $v$ are the components of velocities in the $x$ and $y$ directions respectively then the Navier-Stokes equations of motion for a steady viscous incompressible fluid in the absence of external forces have the form

$$
\begin{equation*}
\rho\left(u \frac{\partial u}{\partial x}+r \frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}\right) \tag{3.1}
\end{equation*}
$$

and

$$
\begin{equation*}
\rho\left(u \frac{\partial y}{\partial x}+v \frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^{2} v}{\partial x^{2}}+\frac{\partial^{2} v}{\partial y^{2}}\right) \tag{3.2}
\end{equation*}
$$

where $\frac{\partial p}{\partial x}$ and $\frac{\partial p}{\partial y}$ are the pressure gradients along the axes, $\rho$ the density of the fluid and $\mu$ the viscosity.

Since the motion is turbulent the velocities $u$ and $v$ may be divided into a mean value and a small fluctuation about the mean.

Thus

$$
\begin{align*}
& u=\bar{u}+u^{\prime}  \tag{3.3}\\
& v=\bar{v} \cdot+v^{\prime} \tag{3.4}
\end{align*}
$$

where a bar denotes the mean value and a dash denotes the fluctuation, and
where the mean value $\bar{u}$ is defincd over an interval of time ' (see Fig.13) as

$$
\vec{u}=\frac{1}{T} \int_{t-\frac{1}{2} T}^{t+\frac{1}{2} T} u d t
$$

and

$$
\overline{\mathrm{V}}=\frac{1}{T} \int_{t-\frac{1}{2} T}^{t+\frac{1}{2} T} \nabla d t
$$

In steady mean flow we have

$$
\begin{gathered}
\overline{\bar{u}}=\bar{u} \text { and } \overline{\vec{v}}=\bar{v} \\
\text { and } \quad \bar{u}^{\prime}=\int_{t-\frac{1}{2} T}^{t+\frac{1}{2} T}(u-\bar{u}) d t=\bar{u}-\bar{u}=0
\end{gathered}
$$

and similarly

$$
\bar{\nabla}^{\prime}=0
$$

Thus in Figo 13 the parts of the curve that lie above $u=\overrightarrow{\mathrm{u}}$ are equal to the parts that lie below it.

The terms on the right hand side of equations (3.1) and (3.2) are the stresses in the fluid and may be expressed in the. form

$$
-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} y_{j}}{\partial y^{2}}\right)=\frac{\partial p_{x x}}{\partial x}+\frac{\partial p_{x y}}{\partial y}
$$

and

$$
-\frac{\partial x}{\partial y}+\mu\left(\frac{\partial^{2} v}{\partial x^{2}}+\frac{\partial^{2} y}{\partial y^{2}}\right)=\frac{\partial p_{x y}}{\partial x} \quad \frac{\partial p_{y y}}{\partial y}
$$

where $p_{x x}$ are the viscous stresses in the $x$ direction, $p_{x y}$ in the direction etc, across a plane normal to the x axis.

Upon substituting into Equations (3.1) and (3.2) and transforming one obtains
and

$$
\begin{align*}
& 0=\frac{\partial}{\partial x}\left(p_{x x}-\rho u^{2}\right)+\frac{\partial}{\partial y}\left(p_{x y}-\rho u v^{\prime}\right)  \tag{3.5}\\
& 0=\frac{\partial}{\partial x}\left(p_{x y}-\rho u v\right)+\frac{\partial}{\partial y}\left(p_{y y}-\rho v^{2}\right) \tag{3.6}
\end{align*}
$$

Now these equations are assumed valid at any time within the turbulent flow, and substituting Equation (3.3) and Equation (3.4) we get, after some simplification the equations of mean motion

$$
\begin{align*}
& 0=\frac{\partial}{\partial x}\left(\bar{p}_{x x}-\rho \bar{u}^{2}-\rho \bar{u}^{\prime} 2\right)+\frac{\partial}{\partial y}\left(\bar{p}_{x y}-\rho \bar{u} \bar{v}-\rho \overline{u^{\prime} v^{\prime}}\right)  \tag{3.7}\\
& 0=\frac{\partial}{\partial x}\left(\bar{p}_{x y}-\rho \bar{u} \bar{v}-\rho \overline{u^{\prime} v^{\prime}}\right)+\frac{\partial}{\partial y}\left(\bar{p}_{y y}-\rho \bar{v}^{2}-\rho \bar{v}^{2}\right) \tag{3.8}
\end{align*}
$$

Equations (3.7) and (3.8) have the same form as (3.5) and (3.6) if $\bar{u}$ replaces $u$, etc., and

$$
\left.\begin{array}{l}
\text { the viscous stress } \dot{p}_{x x} \text { is replaced by } \overline{p_{x x}}-\rho \overline{u^{2}}  \tag{3.9}\\
\text { the viscous stress } p_{x y} \text { is replaced by } \overline{\bar{p}_{x y}}-\rho_{u}^{\prime} u
\end{array}\right\}
$$

and the other terms similarly.

The additional stresses shown in Equation (3.9) are called Reynolds or eddy stresses and in turbulent motion generally outweigh in importance the purely viscous stresses.

In order to proceed further with the problem it is necessary to assume some expression for these eddy stresses. .From the relationships in ordinary viscous flow that the viscous stresses are proportional to the velocity gradient, a similar assumption is used for the eddy stresses. Further it is physically reasonable since the greater the velocity gradient the greater the degree of turbulence.

Thus we have

$$
\begin{aligned}
& -\bar{u}^{2}=(v+K) \frac{\partial \bar{u}}{\partial x} \\
& -\overline{v^{\prime}} 2=(v+K) \frac{\partial \bar{v}}{\partial y} \\
& -\overline{u^{\prime} v^{\prime}}=(v+K) \frac{\partial \bar{u}}{\partial y} \\
& -\overline{\bar{y}^{\prime} u^{\prime}}=(v+K) \frac{\partial \bar{v}}{\partial x}
\end{aligned}
$$

Here $K$ is the eddy viscosity and $v=\frac{\mu}{\rho}$ is the Kinematic viscosity. In the atmosphere however molecular friction is negligible compared with turbulent friction and the expressions for the eddy stresses become

$$
\begin{array}{ll}
-\bar{u} \prime^{2}=K \frac{\partial \bar{u}}{\partial x} & -\bar{v}^{\prime 2}=K \frac{\partial \bar{v}}{\partial y} \\
-\overline{u^{\prime} v^{\prime}}=K \frac{\partial u}{\partial y} & -\overline{v_{i}^{\prime} u^{\prime}}=K \frac{\partial v}{\partial x}
\end{array}
$$

If we also neglect mean viscous stresses like $\overline{\mathrm{p}}_{\mathrm{x}}$ compared with terms like $\overline{u^{\prime} v^{\prime}}$ etc., and putting

$$
\frac{\partial \bar{p}_{x x}}{\partial x}=\frac{\overline{\partial p}}{\partial x} \quad \text { and } \quad \frac{\partial \bar{p}_{y y y}}{\partial y}=\frac{\partial \bar{p}}{\partial y}
$$

the following is obtained

$$
\begin{align*}
& \overline{\dot{u}} \frac{\partial \bar{u}}{\partial x}+\bar{v} \frac{\partial \bar{u}}{\partial y}=-\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x}+\frac{\partial}{\partial x}\left(K \frac{\partial \bar{u}}{\partial x}\right)+\frac{\partial}{\partial y}\left(K \frac{\partial \bar{u}}{\partial y}\right)  \tag{3.10}\\
& \bar{u} \frac{\partial \bar{y}}{\partial x}+v \frac{\partial \bar{v}}{\partial y}=-\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y}+\frac{\partial}{\partial x}\left(K \frac{\partial \bar{v}}{\partial x}\right)+\frac{\partial}{\partial y}\left(K \frac{\partial \bar{y}}{\partial y}\right) \tag{3.11}
\end{align*}
$$

This set of equations for the mean motion in a turbulent flow is identical with those for laminar flow except that molecular viscosity has been replaced by the eddy viscosity.

The continuity equation for the full turbulent motion is

$$
\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0
$$

and for the mean motion this becomes

$$
\begin{equation*}
\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}=0 \tag{3.12}
\end{equation*}
$$

Since we are only concerned with the mean quantities the 'bar' indicating the average values will be dropped in further expressions.

Before proceeding further and trying to solve the Equations (3.10) and (3.11) for the unknowns $u$ and $v$ either $K$ and $P$ must be eliminated from them or their values must be known throughout the region of interest.

No exact knowledge of the value of the eddy shearing stress is available, and, as we are regarding the atmosphere as a fully developed turbulent boundary layer, we may assume its value is constant, as is its molecular counterpart in laminar flow. Thus a knowledge of K has been assumed. However the pressure still remains to be found and no supposition can be made as to how it varies. Now that $K$ is a known function we have two equations and three unknowns. The pressure can be eliminated by combining these equations into a single one and introducing two new variables defined in terms of the velocities.

The first of these is called the streamfunction, denoted by $\Psi$, and may be derived from the continuity Equation (3.12). Thus we define $\Psi$ by

$$
\begin{equation*}
u=\frac{\partial \Psi}{\partial y} \quad ; \quad v=-\frac{\partial \Psi}{\partial x} \tag{3.13}
\end{equation*}
$$

and the continuity equation is satisfied.

The second is known as the vorticity, denoted by $w$, and is a function of the derivatives of $u$ and $v$. It is defined by

$$
\begin{equation*}
w=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y} \tag{3.14}
\end{equation*}
$$

It can be seen that the streamfunction and vorticity can be expressed in the one relation

$$
\begin{align*}
\nabla^{2} \Psi & =-w \\
\text { where } \nabla^{2} & \equiv \frac{\partial^{2}}{\partial x} 2+\frac{\partial^{2}}{\partial y^{2}} \tag{3.15}
\end{align*}
$$

If we differentiate Equation (3.10) with respect to $y$ and Equation (3.11) with respect to $x$ and subtract, the following equation is obtained

$$
\begin{equation*}
\nabla^{2} w=\frac{1}{K}\left(\frac{\partial \Psi}{\partial y} \frac{\partial w}{\partial x}-\frac{\partial \dot{w}}{\partial x} \frac{\partial w}{\partial y}\right) \tag{3.16}
\end{equation*}
$$

Thus the problem of solving Equations (3.1) and (3.2) is reduced to solving Equations (3.15) and (3.16).

The velocities and lengths may be put in non dimensional form as follows:

$$
\begin{array}{ll}
u=\frac{\mathrm{u}}{\mathrm{U}} & v=\frac{v}{U_{0}} \\
y=\frac{y}{l} & x=\frac{x}{l}
\end{array}
$$

If this is done the Equations (3.15) and (3.16) may be expressed in the following form

$$
\begin{gather*}
\nabla^{2}=-w  \tag{3.17}\\
\nabla^{2} w=R \frac{\partial \Psi}{\partial y} \frac{\partial w}{\partial x}-\frac{\partial \Psi}{\partial x} \frac{\partial w}{\partial y} \tag{3.18}
\end{gather*}
$$

where $R$ is the eddy Reynolds number and is given by

$$
\begin{equation*}
\mathrm{R}=\frac{\mathrm{W}_{0} \ell}{\mathrm{~K}} \tag{3.18a}
\end{equation*}
$$

where Uo is a reference velocity
$\ell$ is a reference length
and $\quad K$ the eddy viscosity.

### 3.3 Boundary conditions

Equations (3.17) and (3.18) are known as simultaneous non-linear elliptic partial differential equations, and for their solution it is necessary to know the boundary conditions on all four sides of a closed boundary. This means that for the equations to be solved the values of $\Psi$ and $W$, or their first derivatives, have to be given at all points on the boundary.

For the flow of air over a hill there is only one natural boundary available, this is the ground. We can, however, impose three other 'boundaries' on the motion. Although these are called boundaries they are not physical barriers but rather positions at which the values of the required variables are known.

Consider Fig.14. This shows an idealized drawing of an isolated obstacle of arbitrary shape, given by the line $A B C$. For such a configuration it seems reasonable to assume that the velocity profile $A E$, upwind of the obstacle, is known and also that this profile is resumed after a certain distance downwind of the obstacle (profile at line CD). As for the other two boundary conditions the lower one is that the velocity is known to be zero on the ground ABC. For the remaining condition it may be expected that at some height (line ED) above the ground the hill has minimal effect on the flow. Although Fig. 12 shows this height the same as the boundary layer height this need not be the case, it may be greater or less.

This knowledge of the velocities, via Equations (3.17) and (3.18) enable the streamfunction and vorticity to be computed at the boundaries. At the lower boundary $A B C$ the vorticity must be found as part of the solution, as at this boundary it may be produced or destroyed by the action of viscosity.

### 3.4 Solution of the equations

Equations (3.17) and (3.18) are a slightly simplified version of the Navier-Stokes equations given in Equation (3.1) and Equation (3.2), and contain all the difficulties of solution that are inherent in their original form. As indicated in the introduction only in a few special cases are analytic solutions possible, and the problem at hand does not fall into any of these categories and an effort must be made to solve them by other than analytic means.

With the development of high speed digital computers it is now widely recognised that finite difference methods are of practical value for solving fluid flow problems which cannot be handled by classical methods ${ }^{13,14}$. It supposes that the field under consideration may be divided into a finite number of points and that the equations which describe the flow are valid at each of these points. The equations are expressed in terms of these discrete points and then solved to obtain a solution at each point. Thus rather than a continuous solution being obtained valid for any position in the field, a solution is obtained which satisfies the equations at each of a finite number of points.

Probably the first attempt to integrate the Navier-Stokes equations numerically was that of $\mathrm{Thom}^{15}$, as far back as 1933. He managed to obtain a solution for the wake formed behind a cylinder placed in a uniform stream, by using a conformal transformation to transform the cylinder into a straight line. In this transformed plane central differences combined with an iterative relaxation technique were employed to solve two second order simultaneous equations, for streamfunction and vorticity, for a Reynolds number of 10. The computation, which was of course perfonned by hand, proved to be very cumbersome, and further work was not pursued until the development of digital computers in the 1950's. Then new papers appeared on the subject, notably those associated with the names Kawaguti ${ }^{16}$ and Allen and Southwell ${ }^{17}$. Thom's practice of solving two simultaneous equations has been followed in the majority of cases ever since together with the central difference representation of first order derivatives introduced by him. All the cases that were considered dealt with low Reynolds numbers flow up to 100, since divergence occurred at higher values.

However, by the use of severe under relaxation Burggraf ${ }^{18}$ applied the same formulation to obtain solutions for flows in a square cavity at Reynolds numbers up to 400. Large computation times of the order of 30 minutes were necessary, and it was further noted that the number of iterations increased with Reynolds number.

No further inroads were made on the subject until work published, almost simultaneously, by Runchal and Wolfstein ${ }^{19}$ and Greenspan ${ }^{20}$ showed that by using a particular combination of forward and backward difference schemes,
combined with central differences it was possible to obtain solutions up to Reynolds numbers of 10,000 for the flow in a square cavity, and in the case of the former authors, for impinging jet flow. Further it was reported that the computations did not take an excessive amount of computer time, being in the case of Runchal 1 minute of IBM 7090 for 40,000 point iterations, and for Greenspan up to 10 minutes of CDC 3600 for a solution to converge.

With the success of these independent results it was decided to use their method of approximating the equations for the case of boundary layer flow over an arbitrarily shaped obstacle.

### 3.5 Finite difference representations

In order to obtain discrete points the field is divided into a mesh. This is achieved as follows. Suppose we have, as in Fig. 15 a field $A B C D$. Then if lines are drawn parallel to $A B$ and to $A D$ the area may be thought of as consisting of the finite number of points of nodes given by the intersections of these lines. In order to achieve ease of computation and greater accuracy in the approximations made the distance between each grid line parallel to $A B$ is the same, and similarly for the lines parallel to AD .

Suppose there is a function $f$ which has to satisfy some given equation throughout the area of interest, then it is assumed to satisfy the equation at each node of the field. Suppose as in Fig. 15 one such node is designated by 0 , and is surrounded by the four other nodes $1,2,3$ and 4. Further. suppose that the function $f$ may be represented at these points by the discrete values $f_{0}, f_{1}, f_{2}, f_{3}$ and $f_{4}$ respectively. In addition to the actual values it may also be required to know the derivatives of $f$, and a choice is available of how to obtain these derivatives.

Suppose the distance between each grid line in the X direction is given by $\Delta x$ and in the $Y$ direction by $\Delta y$, then we may derive, by means of a Taylor Series expansion the following approximations for the partial derivative of $f$ with respect to $x$ :
(a) using forward differences

$$
\left.\frac{\partial f}{\partial x}\right|_{0}=\frac{f 1-f_{0}}{\Delta x}
$$

(b) using backward differences

$$
\left.\frac{\partial f}{\partial x}\right|_{0}=\frac{f_{0}-f_{3}}{\Delta x}
$$

(c) using central differences

$$
\left.\frac{\partial f}{\partial x}\right|_{0}=\frac{f_{1}-f_{3}}{2(\Delta x)}
$$

where $\left.\frac{\partial f}{\partial x}\right|_{0}$ indicates the partial derivative of $f$ with respect to $x$ evaluated at the point o.

In the case of the one sided differences (a) and (b) the error terms are of order $\Delta x$ while in the central differences (c) they are of order ( $\Delta x)^{2}$, as $\Delta \mathrm{x} \rightarrow$ o (see Ref.14). Thus it can be seen that the central finite differences are far more accurate, and where possible should always be used.

The second partial differential of $f$ with respect to $X$ at o may also be derived, again using a Taylor series expansion.

$$
\left.\frac{\partial^{2} f}{\partial x^{2}}\right|_{0}=\frac{f j-2 f_{0}+f_{3}}{(\Delta x)^{2}}
$$

which has an error of order $(\Delta x)^{2}$.
Similar expressions to those above may be derived for

$$
\left.\frac{\partial f}{\partial y}\right|_{0} \text { and }\left.\frac{\partial^{2} f}{\partial y^{2}}\right|_{0}
$$

### 3.6 Application to Naviex-Stokes equations

The method used to transform Equations (3.17) and (3.18) into finite difference form is the following. Equation (3.17) is approximated using central differences throughout, and the left hand side of Equation (3.18)
is handled using the same approach. The right hand side of Equation (3.18) is formulated somewhat differently and the choice made is to obtain the gradients of the vorticity by a uni-directional difference scheme in such a way that the finite differences always go backwards with reference to the direction of flow. The term of the finite difference equations may be put into matrix form, and for convergence it is necessary that the diagonal terms of this matrix dominate the others and Runchal and Wolfstein (Ref. 12 iii) show that the particular combination of finite differences used above produces the required diagonal dominance of the matrix.

For example consider a node where, at any particular instant, the flow is in the negative $x$-direction and in the positive $y$-direction, then the vorticity gradients are represented by forward differences in the $x$-direction and backward differences in the $y$-direction, thus:

$$
\begin{aligned}
& \frac{\partial_{W}}{\partial x}=\frac{w_{1}-w_{0}}{\Delta x} \\
& \frac{\partial_{W}}{\partial y}=\frac{w_{0}-w_{4}}{\Delta y}
\end{aligned}
$$

Since we have $u=\frac{\partial \Psi}{\partial y}$ and $v=-\frac{\partial \Psi}{\partial x}$ Equation (3.18) may be written in the form

$$
\nabla^{2} w=R\left(u \frac{\partial w}{\partial y}+v \frac{\partial_{w}}{\partial x}\right)
$$

Using the selective finite difference approximations as indicated Equations (3.17) and (3.18) may be expressed, for every nodal point, as follows:

$$
\begin{gathered}
\frac{\Psi_{1}-2 \Psi_{0}+\Psi_{3}}{\left(\Delta_{\mathrm{x}}\right)^{2}}+\frac{\Psi_{2}-2 \Psi_{0}+w_{4}=-\mathrm{w}_{0}}{(\Delta \mathrm{y})^{2}} \\
\frac{\mathrm{w}_{1}-2 \mathrm{w}_{0}+\mathrm{w}_{3}}{(\Delta \mathrm{x})^{2}}+\frac{\mathrm{w}_{2}-2 \mathrm{w}_{0}+\mathrm{w}_{4}}{(\Delta \mathrm{y})^{2}}=\mathrm{R}\left(\frac{|\mathrm{u}|\left(\mathrm{w}_{0}-\mathrm{w}_{A}\right)}{\Delta_{\mathrm{x}}}+\frac{|\mathrm{v}|\left(\mathrm{w}_{0}-\mathrm{wB}\right)}{\Delta y}\right)(3.20)
\end{gathered}
$$

where if

$$
\begin{array}{cc}
u>0 & A=3 \\
u<0 & A=1 \\
v>0 & B=4 \\
v<0 & B=2 \\
-28 & -
\end{array}
$$

and $u$ and $v$ are formed using central finite difference form, and are given by

$$
\begin{aligned}
& u=\frac{\Phi_{2}-\Psi_{4}}{2 \Delta y} \\
& v=\frac{\Phi_{3}-\Phi}{2 \Delta x}
\end{aligned}
$$

Equation (3.19) may be used to find the value of the streamfunction at the position $o$ and Equation (3.20) to find the vorticity.

Thus

$$
\begin{gather*}
w_{0}=\frac{\frac{\psi_{1}+\psi_{3}}{(\Delta x)^{2}}+\frac{\phi_{2}+\Psi_{4}}{(\Delta y)^{2}}+w_{0}}{\left(\frac{1}{(\Delta x)^{2}}+\frac{1}{(\Delta y)^{2}}\right)}  \tag{3.21}\\
, w_{0}=\frac{\frac{1}{R}\left(\frac{w_{1}+w_{3}}{(\Delta x)^{2}}+\frac{w_{2}+w_{4}}{(\Delta y)^{2}}\right)+\frac{|u| w_{A}}{\Delta x}+\frac{\mid v w_{B}}{\Delta y}}{\frac{2}{R}\left(\frac{1}{(\Delta x)^{2}}+\frac{1}{(\Delta y)^{2}}\right)+\frac{|u|}{\Delta x}+\frac{|\nabla|}{\Delta y}} \tag{3.22}
\end{gather*}
$$

and
where the values of subscripts $A$ and $B$ are given above.
It is thus possible to find the streamfunction and vorticity at every node of the mesh.

### 3.7 Description of the irregular boundary

The approximations which have been derived previously require knowledge of the values of the function at five points of the grid ioe. the central point and its four neighbours. From the description of the boundary conditions given in section (3.3) it is possible to a rrange for the boundary conditions upstream and downstream of the obstacle to be placed along a grid line, as is the case of the upper boundary also. For the lower boundary, which may be completely arbitrary, the grid points will rarely coincide with the boundary itself, and methods are needed both to define the lower boundary and evaluate the vorticity on it and hence to calculate the vorticity and streamfunction at points adjacent to it.

In order to obtain these requirements the following scheme was adopted. In Fig. 17(A) suppose CD is the boundary and the vertical grid lines are numbered, left to right $1 \ldots \ldots \mathrm{~N}_{1}$. Any mesh line i must intersect the
boundary at one point. The height of this point from a horizontal datum may be regarded as forming a component of a vector $\underline{B}$, which may be referred to as a vertical distance vector. Further each horizontal grid line, numbered $1 . \ldots . M_{1}$, intersects the boundary at two points. From Fig.17(b) the distances from a vertical datum line may be expressed by Aij, $i=1,2 ; j=1 \ldots \ldots M_{1}$ and this may be referred to as a horizontal distance matrix A. 。

Thus it is possible to define the boundary in terms of $\underset{A}{A}$ and $\underline{B}$. We may however proceed one stage further and use as our datum lines the initial grid lines which form the edge of the mesh, and state the lengths Aij, $\mathbf{i}=1,2 ; j=1 \ldots \ldots M_{1}$ and $B i, i=1 \ldots \ldots N_{1}$ in terms of the number grid lines, or fractions thereof, in the horizontal or vertical direction. It is now possible to define whether a point is on the boundary, adjacent to it, or otherwise.

Suppose the node under consideration is the one denoted by ( $i, j$ ) where $\mathrm{l} \leq \mathrm{i} \leq \mathrm{N}_{1}$ and $\mathrm{l} \leq \mathrm{j} \leq \mathrm{M}_{1}$ then the terms 'boundary point', 'weak internal point' and 'strongly internal point' may be defined as follows:
(1) The point is called a boundary point if $\mathrm{j}=\mathrm{Bi}$ (which implies

$$
\left.i=A_{1} j \text { or } i=A_{2} j\right)
$$

(2) The point is called weakly internal if $j-B i<1$ or $A_{1} j-i<1$ or $i-A_{2} j<1$.
(3) The point is called strongly internal if $j-B i \geq 1$ and either $A_{1} j-i \geq 1$ or $i-A_{2} j \geq 1$.

In (2) and (3) the condition which must be considered to hold of either $A_{1} j-i$ or $i-A_{2} j$ depends upon which side of the obstacle is being considered. If, for example, the left hand side is being considered, then $A_{1} j-i$ is of the order of $l$ while $i-A_{2} j$ will be much larger than unity. A similar comparison holds for $A_{2} j$ and the right hand side。

The finite difference formulae which have been derived so far are obviously only applicable to points which are classed as strongly internal, and additional approximations have to be made for points which fall into the other two categories.

### 3.8 Derivations of approximations for points close to the boundary

For points classed as weakly internal the following method was adopted. Suppose a node has been reached, as in Fig.18, which lies to the left hand side of the hill, and may be classed as a weakly internal point, which has two arms of the mesh shorter than the normal distance $\Delta x$ or $\Delta y$. For complete generality suppose that the other arms are also shorter, and their lengths are given by $\theta_{1} \Delta x, \theta_{2} \Delta y, \theta_{3} \Delta x$ and $\theta_{4} \Delta y$ where $\Delta x$ and $\Delta y$ are the mesh increments and $0<\theta_{1}, \theta_{2}, \theta_{3}, \theta_{4}<1$ are the distances from the node under consideration to the boundary. Of course such a situation should never arise since the mesh would then be far too coarse for any meaningful results to be obtained. At most only two of $\theta_{1}, \theta_{2}, \theta_{3}$, and $\theta_{4}$ should be less than unity, the others being equạl to it。

Suppose we may write Laplace's equation as,

$$
\begin{equation*}
\nabla^{2} l_{0}=\alpha_{0} f_{0}+\alpha_{1} f_{1}+\alpha_{2} f_{2}+\alpha_{3} f_{3}+\alpha_{4} f_{4} \tag{3.23}
\end{equation*}
$$

where $f$ is any function and $\nabla^{2} f$ o means $\frac{\partial^{2} f}{\partial x^{2}}+\frac{\partial^{2} f}{\partial y^{2}}$ evaluated at the point o. Using a Taylor series $f_{1}, f_{2}, f_{3}$ and $f_{4}$ may be expanded about the node to obtain:

$$
\left.\begin{array}{l}
f_{1}=f_{0}+\left.\theta_{1} \Delta x \frac{\partial f}{\partial x}\right|_{0}+\left.\frac{\left(\theta_{1} \Delta x\right)^{2}}{2} \frac{\partial^{2} f}{\partial x^{2}}\right|_{0} \\
f_{2}=f_{0}+\left.\theta_{2} \Delta y \frac{\partial f}{\partial y}\right|_{0}+\frac{\left(\theta_{2} \Delta y\right)^{2}}{2} \\
\left.\frac{\partial^{2} f}{\partial y^{2}}\right|_{0}  \tag{3.24}\\
f_{3}=f_{0}+\left.\theta_{3} \Delta x \frac{\partial f}{\partial x}\right|_{0}+\left.\frac{\left(\theta_{3} \Delta x\right)^{2}}{2} \frac{\partial^{2} f}{\partial x^{2}}\right|_{0} \\
f_{4}=f_{0}+\left.\theta_{4} \Delta y \frac{\partial f}{\partial y}\right|_{0}+\left.\frac{\left(\theta_{4} \Delta y\right)^{2}}{2} \frac{\partial^{2} f}{\partial y^{2}}\right|_{0}
\end{array}\right\}
$$

Further terms in the expansions have been neglected as these are of third order and higher.

Substituting Equations (3.23) into (3.24) and equating coefficients of like terms one obtains

$$
\left.\begin{array}{l}
\alpha_{1}=\frac{2}{(\Delta x)^{2} \theta_{1}\left(\theta_{1}+\theta_{3}\right)} \\
\alpha_{2}=\frac{2}{(\Delta y)^{2} \theta_{2}\left(\theta_{2}+\theta_{4}\right)} \\
\alpha_{3}=\frac{2}{(\Delta x)^{2} \theta_{3}\left(\theta_{3}+\theta_{1}\right)}  \tag{3.25}\\
\alpha_{4}=\frac{2}{(\Delta y)^{2} \theta_{4}\left(\theta_{2}+\theta_{4}\right)} \\
\alpha_{0}=\sum_{1}^{4} \alpha_{1}
\end{array}\right\}
$$

Thus an expression for Laplace's equation has been obtained where the grid lines do not necessarily coincide with the boundary.

Equation (3.19) may now be used to obtain the streamfunction at weakly internal points of the mesh. The finite difference form now becomes

$$
\psi_{0}=\frac{\alpha_{1} \psi_{1}+\alpha_{2} \psi_{2}+\alpha_{3} \psi_{3}+\alpha_{4} \Psi_{4}-w_{0}}{-\alpha_{0}}
$$

Equation (3.20) may be expressed similarly in finite difference form as

$$
\alpha_{0 w_{0}}+\alpha_{1 w_{1}}+\alpha_{2 w_{2}}+\alpha_{3 w_{3}}+\alpha_{4 w_{4}}=R\left(u \frac{w_{0}-w_{A}}{\theta_{A} \Delta x}+\frac{y w_{0}-w_{B}}{\theta B \Delta y}\right)
$$

where for

| $u>0$ |  |  |
| :--- | :--- | :--- |
| $u<0$ |  |  |
| $v$ | $>0$ |  |
| $v<0$ |  | $A=3$ |
|  | $B=1$ |  |
|  | $B=2$ |  |

with $u=\frac{\partial \Psi}{\partial y}$ which in finite difference form becomes $u \dot{\doteqdot} \frac{\Psi_{2}-\Psi_{4}}{\left(\theta_{2}+\theta_{4}\right)} \Delta y$
and $\quad v=\frac{-\partial \Psi}{\partial x}$ becoming $\left.v \dot{\doteqdot} \frac{\Psi_{3}-\Psi_{1}}{\left(\theta_{1}+\theta_{3}\right.}\right) \Delta x$
Thus for weakly internal points the expressions for the vorticity may be approximated as follows:

$$
w_{0}=\frac{\frac{1}{R}\left(\alpha_{1} w_{1}+\alpha_{2} w_{2}+\alpha_{3} w_{3}+\alpha_{4} w_{4}\right)+\frac{|u| w_{A}}{\theta_{A} \Delta x}+\frac{|v| w_{B}}{\theta B_{A Y}}}{-\frac{\alpha 0}{R}+\frac{|u|}{\partial_{A} \Delta x}+\frac{|v|}{\theta_{B A}}}
$$

where subscripts $A$ and $B$ are given above, and $\alpha_{i}, i=0 \ldots \ldots 4$ are defined in Equations (3.25).

The values of $\theta_{1} \ldots \ldots \theta_{4}$ may be found as follows

$$
\begin{aligned}
& \theta_{1}=A_{1 j}-i \quad \theta_{2}=J \\
& \theta_{3}=i-A_{2 j} \quad \theta_{4}=B_{i}-j
\end{aligned}
$$

If one of $\theta_{1}, \theta_{3}$ or $\theta_{4}$ is greater than unity then it is given the value one. Further if $\theta_{1}=\theta_{2}=\theta_{3}=\theta_{4}=1$ then the approximations derived above reduce to those given in Equations (3.21) and (3.22) for strongly internal points.

For the boundary points the value of the streamfunction remains constant upon the boundary and only the vorticity needs to be calculated upon the lower boundary as indicated in Chapter 3.3.

For points upstream and downstream of the hill, where the boundary may coincide with the initial grid line a simple approximation may be derived, using Equation (3.19).

Suppose that Laplaces equation may be expressed as follows

$$
\nabla^{2} \Psi=\alpha_{0} \Psi_{0}+\alpha_{1} \Psi_{1}+\alpha_{2} \Psi_{2}+\alpha_{3} \Psi_{3}+\alpha_{4} \frac{\partial \Psi}{\partial y}
$$

where the points 0,1, 2 and 3 are as shown in Fig.16.
if $\Psi_{1}, \Psi_{2}$ and $\Psi_{3}$ are expanded in a Taylor series, substituted into the equations above, the coefficients of either side equated and the resulting simultaneous equations solved, as for the weakly internal points, one obtains

$$
\begin{aligned}
& \alpha_{0}=-2\left(\frac{1}{(\Delta x)^{2}}+\frac{1}{(\Delta y)^{2}}\right) \\
& \alpha_{1}=\frac{1}{(\Delta x)^{2}} \\
& \alpha_{:_{2}}=\frac{2}{(\Delta y)^{2}} \\
& \alpha_{3}=\frac{1}{(\Delta x)^{2}} \\
& \alpha_{4}=\frac{2}{(\Delta y)}
\end{aligned}
$$

Thus an expression for vorticity at the boundary point 0 is obtained as follows:

$$
\begin{equation*}
W_{0}=-2 \Psi_{0}\left(\frac{1}{(\Delta x)^{2}}+\frac{1}{(\Delta y)^{2}}\right)+\frac{\Psi 1}{(\Delta x)^{2}}+\frac{2 \Psi_{2}}{(\Delta y)^{2}}+\frac{\Psi 3}{(\Delta x)^{2}}-\frac{2}{\Delta y} \frac{\partial \Psi}{\partial y} \tag{3.26}
\end{equation*}
$$

where the streamfunction values and its derivatives are known on the boundary.

While Equation (3.26) is adequate for boundaries coinciding with a mesh line, the vorticity remains to be evaluated on the irregular boundary of the hill. This may be divided into three classes
(i) for boundary points like $A_{1} j \quad j=1 \ldots . M_{1}$
(ii) for boundary points like $A_{2} j \quad j=1 \ldots \ldots M_{2}$
(iii) for boundary points like $\mathrm{Bi} \quad \mathrm{i}=1 \ldots \ldots \mathrm{~N}_{1}$

In all three cases Equation (3.19) is expressed in finite difference form, and thus the vorticity is found on the boundary.
(i) for boundary points $A_{1}{ }^{j}$

Consider Fig.19(a). Suppose the point given by ( $i, j$ ) is known to be weakly internal then $\theta_{2}=A_{1} j-i$.

Suppose Laplaces equation may be expressed as

$$
\left.\nabla^{2} \Psi\right|_{0}=\alpha_{0} \Psi_{0}+\alpha_{1} \Psi_{1}+\alpha_{2} \Psi_{2}+\left.\alpha_{3} \frac{\partial \Psi}{\partial x}\right|_{0}+\left.\alpha_{4} \frac{\partial \psi}{\partial y}\right|_{0}
$$

$\Psi_{1}$ and $\Psi_{2}$ are expanded in a Taylor series about the point 0 to obtain

$$
\Psi_{1}=\Psi_{0}+\left.\Delta_{y} \frac{\partial \psi}{\partial y}\right|_{0}+\left.(\Delta y)^{2} \frac{\partial^{2} \psi}{\partial y^{2}}\right|_{0}
$$

and

$$
\Psi_{2}=\Psi_{0}-\left.\theta_{2} \Delta_{\mathrm{x}} \frac{\partial \Psi}{\partial \mathrm{x}}\right|_{0}+\left.\left(\theta_{2} \Delta_{\mathrm{x}}\right)^{2} \frac{\partial^{2} \Psi}{\partial \mathrm{x}^{2}}\right|_{0}
$$

where terms of the third order and higher have been neglected.
Substituting into the above and equating coefficients the following is obtained

$$
\begin{aligned}
\alpha_{1} & =\frac{2}{(\Delta y)^{2}} & \alpha_{2}=\frac{2}{\left(\theta_{2} \Delta_{x}\right)^{2}} \\
\alpha_{3} & =\frac{2}{\theta_{2} \Delta_{x}} & \alpha_{4}=-\frac{2}{\Delta y} \\
& =-\frac{4}{1} \alpha_{i} &
\end{aligned}
$$

It is known on the boundary that $\Psi=0$ and $\frac{\partial \Psi}{\partial x}=\frac{\partial \Psi}{\partial y}=0$, so that the following equation is valid for vorticity

$$
\begin{equation*}
w_{0}=-\frac{2 \psi_{1}}{(\Delta y)^{2}}-\frac{2 \Psi_{2}}{\left(\theta_{2} \Delta_{x}\right)^{2}} \tag{3.27}
\end{equation*}
$$

For this case point 2 coincides with a node, and hence the streamfunction is known at this point, but position 1 is not a node, and the streamfunction remains to be evaluated at this point. $\Psi_{1}$ is obtained by using linear interpolation between the values of the streamfunction at the nodes marked 3 and 4. If the boundary lies between these the value given to the $\theta^{\prime}$ s in Fig.19(a) is altered to the correct value. As the nodes 'inside' the hill are of no interest the value of the streamfunction at them may remain at zero throughout the calculation, and as this is the value of the streamfunction at the boundary, the value may be used instead of the boundary value.

Thus

$$
\Psi_{1}=\frac{\theta_{2} \Psi_{4}+\left(\theta_{2}^{\prime}-\theta_{2}\right) \Psi_{3}}{\theta_{2}^{1}}
$$

where $\theta_{2}{ }^{1}$ is given by

$$
\theta_{2}^{f}=\min \left\{A_{1 j+1}-i ; 1\right\}
$$

Hence

$$
\begin{equation*}
w_{0}=-\frac{2}{(\Delta y)^{2}}\left(\frac{\theta_{2} \Psi_{4}+\left(\theta_{2}^{\prime}-\theta_{2}\right) \Psi_{3}}{\theta_{2}^{\prime}}\right)-\frac{2}{\left(\theta_{2} \Delta_{x}\right)^{2}} \Psi_{2} \tag{3.28}
\end{equation*}
$$

(ii) for boundary points $A_{2}{ }^{j}$

A similar approach may be adopted for these points and a result similar to Equation (3.27) may be obtained. In this case it is of the form

$$
w_{0}=-\frac{2}{\left(\theta_{3} \Delta_{x}\right)^{2}} \Psi_{1} \frac{2}{\left(\Delta_{y}\right)^{2}} \Psi_{2}
$$

where $\theta_{3}$ is given by $\theta_{3}=i-A_{2}$ (see Fig. 19(b))
In this case point 1 coincides with a node but not point 2. Again linear interpolation is used to find the value of the streamfunction at this point.

Thus

$$
\Psi_{2}=\frac{\psi_{3}\left(\theta 3^{\prime}-\theta_{3}\right)+\theta_{3} \psi_{4}}{\theta_{3}}
$$

where

$$
\theta_{3}=\min \left(i-A_{21} j+1 ; 1\right)
$$

Hence the value of the vorticity is given by

$$
\begin{equation*}
w_{0}=-\frac{2}{(\Delta y)^{2}}\left(\frac{\Psi_{3}\left(\theta_{3}^{\prime}-\theta_{3}\right)+\theta_{3} \Psi_{4}}{\theta_{3}^{\prime}}\right)-\frac{2 \Psi_{1}}{\left(\theta_{3} \Delta x\right)^{2}} \tag{3.29}
\end{equation*}
$$

(iii) for boundary points Bi

Again the same method is used to arrive at the equation for the vorticity in the form

$$
w_{o}=-\frac{2}{\left(\theta_{1} \Delta y\right)^{2}} \quad \Psi_{1}-\frac{2}{(\Delta x)^{2}} \Psi_{2}
$$

where $\theta_{1}=\mathbf{j}-\mathrm{Bi} \quad$ (see. Fig。19(c))
Here point 1 coincides with a node and point 2 does not. By linear interpolation the streamfunction at position 2 is given by

$$
\Psi_{2}=\frac{\theta_{1} \Psi_{1}+\left(\theta_{1}^{!}-\theta_{1}\right) \Psi_{3}}{\theta_{1}^{\prime}}
$$

where

$$
\theta_{1}^{1}=\min \left\{j-B_{i-1} ; 1\right\}
$$

Hence

$$
W_{0}=-\frac{2 \Psi_{1}}{\left(\theta_{1} \Delta_{y}\right)^{2}}-\frac{2 \Psi_{2}}{(\Delta x)^{2}}\left\{\frac{\theta_{1} \Psi_{4}+\theta_{1}-{ }_{1}{ }_{1} \Psi_{3}}{\theta_{1}^{!}}\right\}
$$

The remarks noted in case (i) for the value of the streamfunction on the boundary if needed apply equally as well to the situations found in (ii) and (iii).

### 3.9 Iteration technique

With the approximations for the equations derived for points away from and close to the boundary it remains for the equations to be solved and the solutions obtained.

The finite difference equations obtained rely for their solution on a knowledge of the function at its neighbouring points. This is of course, except on the boundary, not known and some form of iteration will be needed in order to obtain a solution. Two methods were used, one for the values away from the boundary, and another though similar, for points on the boundary.

The basic method which was chosen is known as a successive relaxation iterative technique, and is used as follows. Initially a guessed solution (usually zero) is given to all the values of the function throughout the mesh, and a new value, using Equations (3.21) and (3.22) is calculated at each node. This value is then assumed to be incorrect and is adjusted according to the following relation

$$
f n=\left(1-R_{f}\right) f_{n-1}+R_{f} f_{n c}
$$

where $f_{n-1}$ is the value of the function at the node before any new value is computed, fnc is the value calculated according to either Equation (3.21) or Equation (3.22) and fn is then given as the latest value of the node. $\mathrm{R}_{\mathrm{f}}$ is known as the relaxation parameter and has a value lying between 0 and
2. For a value of $R_{f}$ equal to unity, no relaxation is performed, for $R_{f}$ $>1$ the process is known as over-relaxation, and for $R_{f}<l$ the process is called under-relaxation. For linear differential equations it is possible to find analytically, what the optimum value of $R_{f}$ is in order to achieve the fastest convergence possible. For non-linear equations no such method is available and the best value has to be decided upon by trial and error.

A range of values of $R_{\Psi}$ and $R_{W}$ were tried and it was found that convergence was achieved in the least number of iterations if the following values were assigned to the relaxation factors

$$
R_{\psi}=1.6 \quad R_{W}=1.0
$$

The process for the main part of the mesh is thus known as an overrelaxation method.

For the boundary values of $w$ the best value of $R_{w B}$ to achieve convergence was found to be 0.1 , and in this case the process is known as an underrelaxation technique.

The measure of the error used between two values of fn and fn-l is given by

$$
\epsilon=\frac{f_{n}-f_{n}-1}{f_{n}}
$$

If this error is within the permissible maximum then the process is said to have converged. If however it is larger than necessary the new values of the variables are computed throughout the mesh until it becomes within the accepted range. For all the results in the following chapters a value of 0.005 was given to $\epsilon$.

### 4.1 Choice of velocity profiles

Before proceeding to use the approximation obtained to calculate flows over obstacles it is necessary to establish what form the boundary conditions stated in section (3.3) will take when referred to physical situations.

The first to be determined is the shape of the velocity profile taken by the earth's boundary layer, as this is needed to establish the boundary conditions both upstream and downstream of the obstacle.

It is possible ${ }^{21}$ by the use of basic dimensional considerations combined with a mixing length hypothesis to obtain a logarithmic law for the increase of velocity with height. The relationship obtained is

$$
\frac{u\left(y_{1}\right)}{u\left(y_{2}\right)}=\text { constant } x \log \left(\frac{y_{1}}{y_{2}}\right)
$$

The constant term contains factors relating to the roughness parameters of the underlying terrain. Further assumptions may be made which lead to more complicated expressions of a similar form.

In practice, however, it has been found ${ }^{22,23}$ that it is possible to obtain a good fit with experimental values of average velocity by means of a simple power law of the form

$$
\begin{equation*}
\frac{u\left(y_{1}\right)}{u\left(y_{2}\right)}=\left(\frac{y_{1}}{y_{2}}\right)^{a} \tag{4.1}
\end{equation*}
$$

where a is an exponent depending upon the type of terrain. Equation (4.1) relates the velocity at height $y$, to the velocity. at a height $y_{2}$. Usually the expression is standardised by allowing $y_{2}$ to become a reference height, either the geostrophic height, or a standard height of ten metres. In practice the latter height is usually taken, although in the situation being considered in the present work it will be more convenient to use the geostrophic height, since this forms the upper limit of the boundary layer. Thus the power law becomes

$$
\begin{equation*}
\frac{u(y)}{u g}=\left(\frac{y}{y_{g}}\right)^{a} \tag{4.2}
\end{equation*}
$$

where $U_{g}$ is the geostrophic velocity at height $\ddot{y}_{g}$ 。

Davenport ${ }^{23}$ collected mean wind profiles measurements made by various workers in a wide range of countries and terrains, and these are summarised in Table 1.

Table 1

| Type of terrain | Power law <br> exponent | Geostrophic <br> height <br> yg (metres) |
| :---: | :---: | :---: |
| (i) Open terrain with few obstacles e.g. <br> open grass or farmland with few trees. <br> (ii) Terrain uniformly covered with obstacles <br> 10 to 15 m high e.g. woodland and shrub. <br> (iii) Terrain with large and irregular objects <br> e.g. centres of cities. | 0.16 | 300 |

For terrains (i) and (ii) the results have been substantiated by a wide range of reliable measurements, while for (iii) it has been difficult to obtain measurements above the urban areas, where the obstruction height may be up to 60 metres high. However for the purposes at hand this situation is highly unlikely to occur and need not be considered further.

Besides knowledge of the velocity profile upstream and downstrean of the obstacle it is also required to know at what distances these profiles are unaffected by the hill, and at what height the hill has no effect upon the flow at the upper edges of the boundary layer. No definite information was found to be available upon these topics but it seems reasonable to assume, in the first instance, that at a distance of five times its height upstream the hill will not influence the shape of the velocity profile, and that it will take up to ten times its height for the profile to return to its original shape. In addition it has been assumed that the hill will have a minimal effect upon the flow at above ten times its height.

With the boundary conditions determined along the four sides of the control volume, two parameters remain to be chosen before the computer program can be used. One is the shape of the lower boundary, and the other the value of the Reynolds number.

### 4.2 Reynolds number

From Equation (3.18a) the eddy Reynolds number is given by

$$
\mathrm{R}=\frac{\mathrm{U}_{0} \ell}{\mathrm{~K}}
$$

where
Uo is a reference velocity
$\ell$ a reference length
and $K$ the eddy viscosity

An advantage of using a power law for the velocity profile is that the mean wind speed at the top of the boundary layer may be used as a reference velocity in the calculation of Reynolas number, as well as in the velocity profile itself. Also the distances used for reference lengths may be the same, and in order to complete the specification of the Reynolds number a value is needed for the eddy viscosity.

There is no fixed standard value for the eddy viscosity, and suggestions as to its size range from $8.6 \times 10^{3} \mathrm{~cm}^{2} \mathrm{sec}^{-1}$ (Ref.21) through $1.25 \times 10^{4} \mathrm{~cm}^{2}$ $\mathrm{sec}^{-1}$ (Ref.8) and $107 \mathrm{~cm}^{2} \mathrm{sec}^{-1}$ (Ref.24) to $10^{9} \mathrm{~cm}^{2} \mathrm{sec}^{-1}$ (Ref.25). Obviously each of these will produce grossly different Reynolds numbers, and the corresponding flow pattern from each of them would be different.

For the initial calculations a value falling in the middle of the range was used, and this value, together with the representative velocity and length used are

$$
\begin{aligned}
& \mathrm{U} 0=3 \times 10^{3} \mathrm{~cm} \mathrm{sec}^{-1} \\
& \ell=3 \times 10^{4} \mathrm{~cm} \\
& \mathrm{~K}=10^{6} \mathrm{~cm}^{2} \mathrm{sec}^{-1}
\end{aligned}
$$

This gives a Reynolds number of 100 , which is of the same order as the value used by Imai et al ${ }^{9}$ in their calculations.

### 4.3 Choice of lower boundary

A feature of the formulation of the approximations to the equation is that any arbitrary shape may be used for the lower boundary. The shape used
in the first instance is that given in Fig.20(a). This shows a smoothly changing surface, with no discontinuities and presents a suitable test case for the computer program.

### 4.4 Theoretical results

For the first calculations the vertical distance was divided into a mesh consisting of thirty grid points, which implies for a boundary layer height of 300 metres the distance between grid points corresponds to 10 metres. In the horizontal direction thirty grid points were also employed, which gives a separation of grid points of 20 metres.

For an initial power law exponent of 0.16 the streamlines were calculated and are given in Fig.20. The obvious feature of the flow pattern is that the influence of the hill on the flow does not extend to the full height considered, but rather its effect on the streamline pattern is only apparent up to approximately four times its height, which is of the same order as that found by Onishi ${ }^{8}$.

The distance between the grid points of ten metres was imposed in order to keep computation time to within a reasonable limit while still allowing the flow pattern to be determined to within a reasonable degree of accuracy. With the above result however it is possible to obtain a smaller distance between the grid points without an increase in the number of grid points used. The program was thus rerun with the separation of the grid point only 2.5 metres apart in the vertical direction, while the number in the horizontal direction was doubled allowing a distance between nodes of ten metres. In addition to computing the streamlines the velocity profiles were also calculated, and these are shown in Figs.20(b), (c). The stations $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ and E shown are the velocity profiles at the corresponding positions in Fig.20(a).

These show that as the hill is approached the velocity profile becomes gentler until the crest of the hill is reached when there is a very rapid j.ncrease in mean wind speed from ground level. This is followed by a region where there is a slight decrease in the velocity which is finally followed by the region where the influence of the hill is insignificant and the normal increase of wind speed with height in the boundary layer is resumed. These results agree with the qualitative considerations of the problem given by Harris ${ }^{22}$.

Further calculations were performed for a power law exponent of 0.28 , and the same general flow pattern is exhibited as that for an exponent of 0.16 . However, as can be expected, because of the gentler slope of the initial velocity profile, the increase in velocity over the brow of the hill is not so large, as can be seen from Fig. 20(c).

Similar calculations were performed for the streamlines and velocity profiles over a shape similar to a motorway or railway embankment, and the results are given in Fig.21. These show that the profile at the upwind edge of the flat top is of the same pattern as that which occurs at the brow of the hill. At the downwind edge, however, the profile has become less steep, and the point of maximum wind speed has moved higher. This is to be expected as the velocity profile will tend to change its shape to the form of the initial profile as it moves across the top of the embankment.

Figures 22 and 23 show in much greater detail the comparison between velocity profiles for the hill shapes shown in Figs. 20 and 21 respectively. These indicate that the rapid increase in wind speed occurs in the lower 30 metres of the boundary layer, and in the case of an exponent of 0.16 the maximum velocity reached is just greater than the reference value outside the boundary layer, while for an exponent of 0.28 the maximum value in this region is only three-quarters of reference value.

Position C in Figs. 22 and 23 show that the maximum velocity reached in this region is approximately double the value of the velocity at a corresponding height far from the hill. This implies that when structures are being sited on the brow of a hill it is not sufficient to suppose that the forces may be calculated from the velocities upstream of the hill.

The results have a bearing upon meteorological measurements made on site. Normally only two or three measurements are taken, and a power law fitted to these. In hilly areas it is only possible for measurements to be taken in the lower part of the boundary. The above results indicate that it would not be a true representation of the whole of the velocity profile to fit a pover law, as the measurements are taken in the area where a large increase in wind velocity occurs.

## CHAPTER 5: FLOW SEPARATION ON HILLS

Flow separation occurs when the air passing over a hill crest is decelerated by an adverse pressure gradient such as occurs on the lee face of a hill. In the examples given in the preceding section the hills considered are very smoothly shaped and the pressure gradient present is not large enough to cause the flow to separate from the surface. In practice there will exist shapes where this requirement is satisfied and interest lies in the prediction of when separation will occur together with a description of the flow within the separated region.

The relationship used in conventional boundary layer theory to determine the onset of separation is (Ref.3)

$$
\left(\frac{\partial u}{\partial y}\right)_{y=0}=0
$$

Up to this point the equations are adequate to describe the flow, but are not valid within the separation bubble, and no indication can be given of the flow in this region by using them.

It has been shown earlier (section (1.1)) that the approximations used to obtain the conventional boundary layer equations are not valid for the type of flow being considered here. While the above relationship is satisfied in all types of two dimensional flow ${ }^{26}$ and is normally used to define the point of separation, in the present work the use of a finite grid mean that it is not possible to calculate accurately the point at which it occurs. The criterion that has been employed here is to determine when the value of the streamline given to the boundary of the hill divides into two, the area enclosed by the two parts of this streamline being the separation bubble.

The use of a numerical method cannot give the general solution to the problem but only a solution in any one particular case. However by studying several cases tentative suggestions may be advanced as to what shape of hill may produce separation, and what the velocities are likely to be within the separated region.

### 5.1 Choice of profile

The shapes of hills which are conducive to separation are many and varied and it would not be possible to perform calculations upon all of them. It was decided it would be more appropriate to select an analytical function with several parameters which control its shape, and vary these parameters in order to produce a family of curves. It will thus be possible to noti.ce how the flow changes with a gradually changing hill profile.

The general form of the equation for the shape that was chosen is given by

$$
y=\frac{\cdots h b^{2}}{b^{2}+(x-a)^{2}}
$$

where:
a merely determines the position of the hill relative to the origin.
$h$ determines the maximum height, occurring at the point $x=a$.
b determines the 'peakedness' of the hill.

The two parameters of interest are obviously $b$ and $h$, or more particularly the ratio $b / h$. From the general expression for the hill it can be seen that as $b$ increases so the sharpness of the hill decreases.

A value of 0.3 was assigned to $h$, while $b$ was allowed to assume the values $0.2,0.3,0.4$ and 0.6 . The corresponding hill shapes are given in Figs.24-27.

### 5.2 Boundary conditions

The initial velocity profile was taken to be of the power law type given by Equation (4.2), an exponent of 0.16 being used for all the calculations.

The distance in which the downstream velocity profile resumed its original shape was taken as ten hill heights from the peak of the hill, and the upstrean velocity profile was located at five hill heights from the peak, although in the cases where the hill is less peaked these distances had to be increased in order that the boundary conditions were far enough removed from the lower slopes to allow any influence of the hill to become insignificant.

From the results obtained in section (4.4) the influence of the hill was taken not to extend above five times its height.

### 5.3 Reynolds number

With such widely reported values for the eddy viscosity (see section (4.2)) a single value of Reynolds number could not be justified and its value was allowed to vary from 10 to 1000 for each of the values of $b$ considered.

### 5.4. Discussion of results

The results for a Reynolds number of 10 are shown in Figs.24-27 for the values of $b=0.2,0.3,0.4$ and 0.6 respectively. In all of the graphs of the streamfunction the stations A-B-C-D-E correspond to the same stations in the graphs of the velocity profiles.

Point A gives, for comparison, the initial velocity profile corresponding to a value of the power exponent of 0.16 .

Position B shows that for all the values of $b$, as the hill is approached so the velocity profiles become gentler, until the top of the hill is reached (position C) when there is a very rapid increase in mean wind speed from ground level. This is followed by a region where there is a slight decrease in the velocity which is finally followed by the region where the influence of the hill is insignificant and the normal increase of wind speed with height in the boundary layer is resumed.

For points $D$ and $E$ downstream of the brow of the hill the difference between the profiles is more marked. Only in the case $b=0.2$ does separation occur and here the maximum velocity in the reverse direction is 0.075 of the reference velocity. In other cases for an increase of the value of $b$ the velocity profile is found to be much steeper than the velocity profile far upstream. In both the separated and non-separated cases the velocity does not exceed the reference velocity.

The Reynolds number was increased by steps of 10 , and for a value of 100 the results are given in Figs.28-31. From the plots of the streamlines it. can be seen that separation also occurs for the cases $b=0.3$ and $b=0.4$, while for $\mathrm{b}=0.2$ the scparation bubble has grown in both length and height. As the hill becomes shallower the general effect is for the separated region behind the hill to become smaller, which is as would be expected from qualitative considerations. At the stations before the brow of the hill the velocity profiles in all cases are somewhat similar to the case for a

Reynolds number of 10. However, due to the onset of separation, the profiles in the lee of the hill are noticeably different. For $b=0.2$ the maximum velocity attained in the reverse direction is 0.15 of the reference value, while for $b=0.3$ and 0.4 it is 0.09 and 0.07 respectively. Another feature of these profiles is that the maximum velocity reached in the forward direction is greater than the reference velocity, and this occurs at a height greater than the height of the brow of the hill. After this maximum is reached the velocity falls away to a value less than the reference value, and then starts to increase again, until the reference value is attained. In the case of $b=0.6$ where no separation occurs the velocity in the lee of the hill never exceeds that of the reference velocity and it can be seen irom Fig. 31 that the velocities near the ground are very much reduced as compared with their value at a corresponding height far away from the hill.

The Reynolds number was then increased further, in steps of 100 , up to a maximum value of 1000. Where separation occurred at a value of 100 , the length of the separation bubble increased with Reynolds number, at the same time growing in the vertical direction, as illustrated in Fig. 32 (a), (b), (c) for $b=0.2$ and values of Reynolds number of 200, 300 and 400 . For the cases of $b=0.6$ where no separation occurred at the lower Reynolds number, a region of reverse flow now appeared in the lee of the hill, first occurring at values of $R=200$ for $b=0.6$. In this case also as the Reynolds number was increased so did the separated region, and the form of the velocity profiles was similar to those obtained for the cases of lower values of $b$ at lower values of Reynolds number.

In order for the equation to be solved it was necessary (see Chapter (3.3)) to know the boundary conditions on all four sides of a control volume. This was satisfied by assuming that the velocity profile downstrean of the peak returned, after a suitable distance, to its upstream distribution. At the higher values of Reynolds number this may occur at a very large distance downstream, much larger than could be accommodated into the present finite difference scheme, in order that reasonable computation times may be achieved. Thus it was necessary to place the boundary at an artifically short distance downstream. For each value of Reynolds number the distance of this boundary from the peak was gradually increased and the following effects were noticed.

For values of $b=0.2,0.3$ and 0.4 at low values of Reynolds number when separation occurred, the length of the separation bubble was not changed, implying that the boundary was placed far enough downstream not to affect the flow. However at the higher values of Reynolds number (commencing at $R=500$ for $b=0.2$ and increasing for increasing values of $b$ ) the length of the wake becomes dependent upon the distance downstream of the boundary, for as this increased so did the length of the region of reversed flow, or until the situation, shown for $b=0.2$ in Fig. $32(\mathrm{~d})$ is reached, where the separation region appears to continue indefinitely. The velocity profiles behind the hill are similar to those found in the separated region when it is of finite size. These profiles are propagated throughout the whole of the separated region, as shown in Fig.32(d).

From these results it can be seen that the peakedness of the hill and Reynolds number are the critical parameters in forming separation of the flow. In the present instance the parameter $b$ performs the function of determining the peakedness, effectively describing the 'width' of the hill. In order to form a more meaningful parameter this must be related to the height $h$. Thus the resulting describing the peakedness can be given as $b / h$. From the calculations already performed it is possible to plot b/h against Reynolds number, and this is given in Fig.33. Thus once the ratio of width to height has been determined for a hill it is possible to find at what Reynolds number separation will first occur.

### 5.5. Choice of Reynolds number

In the foregoing work a range of Reynolds numbers was chosen, since knowledge of the exact values of the eddy viscosity is so uncertain. For the practising engineer, who may only need the velocities for a single hill it would not be practicable to conduct a series of tests covering the whole range of Reynolds numbers, and he would much prefer to use first a single value. No definite conclusions may be drawn from the present study but it is suggested that a value of 100 would give reasonable results. The flow produced from using this value falls between the two extreme values of 10 , producing negligible wake, and 1000 producing a very large wake. Further it will be noticed that the velocities at the brow of, and just in the lee of the hill, for a value of Reynolds number of 100 , differ very little from those at the higher value. Thus for a structure erected near the top of the hill a value of 100 would give results of the correct order.

## CHAPTER 6: CONCLUSTONS

The classical method of determining the wind flow over a hill is to assume that the velocity profile of the wind is uniform. In practice this is not so, for the velocity on the ground is zero and the profile of the wind is of a boundary layer shape. Attenpts were made to modify this method by using a profile given by the Equation

$$
\frac{\mathrm{u}}{\mathrm{U}}=\tanh (\mathrm{y})
$$

In order to satisfy the boundary conditions of zero velocity upon the ground a large increase in the height of the boundary layer resulted, one that would not be expected in a practical situation. A further defect of the approach is that the shape of the hill cannot be determined 'a priori', it being given as a feature of the solution. In view of this latter defect it was not thought worthwhile spending more time trying to modify the solution in order to obtain a more appropriate height of upper boundary.

Attention was then turned to the equations of motion, which are valid throughout a region, independent of the shape of the boundary. These equations cannot be solved analytically and a numerical approach was adopted. The problem of how to specify the lower boundary was overcome by expressing it in terms of the finite difference grid used to solve the equations, and approximations were obtained for grid points on, or adjacent to, the irregular boundary.

A sample hill was then chosen to test the program, and the results obtained agree with a qualitative consideration of the problem. Although the obstacle has been termed a hill throughout it is possible for another shape to be used, and the flow for a shape similar to a railway embankment or road embankment was computed. In both cases the most striking of the results is that the velocity at the top of the obstacle exceeds that of the reference velocity, and is several times greater than the velocity at a corresponding height far away from the hill.

The flow was then calculated for a family of hills and a range of Reynolds numbers in order to study the effects of separation. It was noted that when separation occurred the velocities just outside the separation bubble
were larger than the reference velocity. For the more peaked hills at a higher Reynolds number the wake was found to continue for a very large distance downstream.

It was decided that a value of Reynolds number of approximately 100 seemed to be most appropriate for the calculations, and the results given for this value around the brow of, and just in the lee of, the hill do not vary significantly from those for a much higher value.

If it is desired to erect a structure on the top, or in the lee of, a hill, the present study shows that the maximum velocity the structure will have to withstand is several times larger than the corresponding velocity at the same height from the ground of the velocity profile far from the hill.

### 6.1 Further work

Although a study of the flow in two dimensions is in many cases sufficient to enable the wind speeds to be calculated in a given situation there are clearly other cases when three dimensional effects are marked. The basic equations of motion are available for three dimensional boundary layex flow but their general solution is at present impossible, even on the largest computers, and some simplifications have to be made to the equations to obtain solutions in a reasonable time。 Imai ${ }^{9}$, for example, imposed some fairly drastic simplifications to the equations including the assumption that the boundary layer height was constant regardless of the terrain.

Since the full three dimensional equations are, at the moment, intractable it is proposed that an attempt be made to evaluate the effects of a hill upon three dimensional flow, using the work for two dimensional flow. For the inviscid case the flow may be expressed by means of Laplace's equation, and this can be solved in both two and three dimensions. For the flow around arbitrary shapes no general analytic flow is possible, but numerical techniques can be employed to solve the equation.

The method proposed to evaluate three dimensional effects is to calculate velocity profiles in inviscid flow for both two and three dimensional shapes, with the same centre line profile, e.g., flow around an ellipsoid
with a fixed cross-section in a vertical plane in the wind direction and variable length across wind. This will enable the reduction in wind speed due to three dimensional effects to be calculated.

Having obtained the change for inviscid ilow it is possible to calculate, using the techniques established in the present work, the velocity profiles for a boundary layer flow over the two dimensional obstacle and find how the inviscid solution is modified when the flow is of a boundary layer type. This will involve finding correction factors for the inviscid velocity profiles in order that it may be modified to a boundary layer flow.

Using these modifications it should be possible to correct the three dimensional inviscid solution in order that it becomes a boundary layer type profile. If a suitable method can be evaluated it should be generally applicable and can be extended to the case of any shape of hill by obtaining a numerical solution for the case of inviscid flow.

The author is indebted to the Electrical Research Association and the Construction Industry Research and Information Association for their financial support for the work.

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## APFENDIX: TIDE COMPUTER, PROGRAM

As stated in Chapter 3 the calculations that are needed to solve the equations cannot be attempted by hand, and a computer program was written to perform this task. A complete list of the statements, as written for the Loughborough University's I.C.L. 1904A computer, is shown in Fig. 36. In addition to this a flow chart showing, in brief, the sequence of operations, the call statements and the main functions of the various sections is given in Fig. 34.

From an examination of the flowchart it can be seen that the program may be subdivided into the following five broad sections:
(a) Input of data
(b) Calculation of streamfunction
(c) Calculation of vorticity
(d) Calculation of velocities
(e) Output of results

Each of these sections will now be discussed in more detail.

## (a) Input of data

The following nethod was found acceptable for the organization of the necessary data for input to the computer.

The following have to be decided
(i) The velocity at the upper edge (signified by VB). The velocity used is a non-dimensional reference velocity, and for flow from right to left it should take the form -1.0 , and for flow from left to right +1.0 . The actual value of the velocity is used to obtain the value of Reynolds number, as below.
(ii) The Reynolds number, using the appropriate values of velocity, length and eddy viscosity。
(iii) The distance of the upstream and downstream boundaries from the hill crest, and the height of the undisturbed layer.
(iv) A unit length in the horizontal and vertical directions, and thus the length of the sides of the control area formed in (iii) in terms of unit lengths. These are symbolised by $H L$ and $V L$ in the horizontal and vertical directions respectively. For example, suppose the height of the boundary layer is 450 metres and a unit length is taken as 150 metres then the height of the control area will be 3 units. If the horizontal distance between boundaries is 2700 metres and the unit is taken as 450 metres then the length of the control area will be 6 units.
(v) The actual length the unit represents, denoted by HUL and VUL in the horizontal and vertical directions. For this purpose the largest length involved must be reduced to the form n.m $\times 10^{\ell}$ where $\mathrm{n}, \mathrm{m}$, and $\ell$ are integers ( $\geqslant 0$ ) and $H U$ or $V L L$ is given by the value n.m. In the above example the longest length is 2700 metres and this becomes $2.7 \times 10^{3}$ metres, and the other lengths become $0.45 \times 10^{3}$ and $0.15 \times 10^{3}$ metres. Thus the horizontal unit length HUL is 0.45 and VUL is 0.15 .
(vi) A suitable division of the unit length for the mesh size, denoted by HDUL and VDUL. In the example if 10 divisions per unit length is decided upon in the horizontal direction then the actual distance the mesh lines are apart is given by $\frac{1}{10} 0.45 \times 10^{3}=45$ metres. If 10 was also chosen for VDUL then the separation of the vertical grid lines is $\frac{1}{10} 0.15 \times 10^{3}=$ $=15$ metres.
(vii) The grid is drawn and the lines so obtained are numbered, commencing at the bottom left hand corner with the value 1. Referring to Fig. 35 the points are numbered l......Nl and l......Ml in the horizontal and vertical directions respectively. Thus in the example the values would be l......61 and 1......31. The values of the grid lines which enclose the hill are noted. From Fig. 35 these are denoted by N2, N3 and M2.
(viii) The values of the hill are read off in terms of the vertical grid lines from N2 to N3. These are the values which comprise the vertical distance vector $\mathrm{B}_{\text {。 }}$ Also the values of the left and right hand side of the hill are noted by using the horjzontal grid lines. These are the values for the horizontal distance vectors $\mathrm{Al}^{2}$ and $\mathrm{A}_{2}$. The value 0.0 is
given to $A l_{1}$ and $A 2_{1}$ and all other values of $A 1$ and $A 2$ which do not intersect the hill, for example $\mathrm{Al} \mathrm{m}_{\mathrm{m}}$ and $\mathrm{A} 2_{\mathrm{m} 2}{ }^{*}$
(ix) A value of the power exponent (A) in the expression for the initial velocity profile.
(x) The proportion of the boundary layer under consideration (denoted by PRO).

The order and format of the data cards are
(i) VB in format FO. 0
(ii) Reynolds number in format E 9.3
(iii) N2, N3, M2 in format 3 IO
(iv) Array $B$ in format 10 FO 0
(v) Array Al in format 10 FO 0
(vi) Array A2 in format 10 FO 0
(vii) HL, HUL and HDUL in format 3 FO. 0
(viii) VL, VUL and VDUL in format 3 FO. 0
(ix) A in format FO 0
(x) PRO in format FO .0

## (b) Calculation of streamfunction

For the purposes of computing the streamfunction the control area is subdivided into four compartments, as described in (a) and shown in Fig. 35. In Sections 1, 3 and 4 the calculation is straightforward, as the boundaries coincide with the nodal points and Equation (3.21) may be used at each node. This is performed by means of the subroutine INTERIORPSI. In section 2 the process is not so simple as the boundary of the hill does not coincide with the grid lines and a test has to be made to decide whether the point is weakly or stiongly internal (section(3.7)) For strongly internal points the subroutine is used, whereas for weakly internal points the approximations given in section (3.8) are employed. The values of the streanfunction are stored in the two dimensional array $P_{0}$ The maximum error for the whole grid is stored in the second element of the area RSDU。

## (c) Calculation of vorticity

The vorticity has to be evaluated on the boundary, and this is performed first, using the approximations derived in section ( 3.8 ). The values of the vorticity everywhere else are computed in a similar manner to the streamfunction vajues, except that Equation (3.22) is used by means of the subroutine INTERIOROMEGA. The values of the vorticity are stored in the two dimensional array 0. The maximum error for the whole net is stored in the first element of the array RSDU。

## (d) Calculation of velocities

Before the velocities are calculated a check is made on the convergence of the procedure. If the maximum exror is outside the allowable margin then control is transferred to the beginning of the evaluation of the streamfunction stage and the process repeated until convergence has been achieved.

Once this has been accomplished the velocities are calculated. This is done by means of the streamfunction, using the approximations given by the expressions following Equation (3.20). For any horizontal station this calculation is performed for all the vertical stations above it, the horizontal velocities being stored in the one dimensional array UC, and the vertical velocities are stored also in a one dimensional array VC. Also computed is the total velocity at each point, and the angle this velocity makes with the horizontal. The results are printed out before the next horizontal station is reached.
(e) Output of results

The values of all initial data supplied are outputted. The streamfunction is printed at every point of the mesh, moving in a horizontal direction, giving all the values in the vertical direction for each horizontal station. In a similar manner the values of vorticity are printed, and finally values of velocity are outputted in the manner already described.

## Al SOME FORTRAN VARTABLES

It would not be possible to explain all the FORTRAN symbols used in the program; there are too many of them. However, all the important symbols are included here. Symbols which have been omitted are usually durmy ones, or are such that their meaning is clear after inspection of the statements near to their place of origin.

Fortran Symbol
$P(I, J)$
$0(I, J)$
$\mathrm{B}(\mathrm{I}), \mathrm{Al}(\mathrm{J}), \mathrm{A} 2(\mathrm{~J})$

OMEGAAI (J), OMEGA2 (J) OMEGAB(I)
$\mathrm{UC}(\mathrm{J}), \mathrm{VC}(\mathrm{J}), \operatorname{TOT}(\mathrm{J})$

DX

DY

EPSILON

RSDU

## Significance

An array containing the streamfunction at each node; I denoting the location in the horizontal direction, $J$ the location in the vertical direction。

An array containing the vorticity at each node. Arrays containing the co-ordinates of the hill, in terms of the grid lines.

Arrays containing the values of vorticity on the hill itself.

Arrays containing the horizontal, vertical and total velocity, and the angle of this total. with the horizontal axis, at each vertical node above a horizontal station.

Increment of the mesh in the horizontal direction.

Increment of the mesh in the vertical direction. The magnitude of the maximum allowable error.

An array containing the maximum error for either variable through the whole of the mesh.

ITER

RPSI

ROMEGA

GAMMA

UB

R

N2, N3, M2

HL

VL

HUL

VUL

HDUL

VDUL

PRO

A

## A2 COMPUTER TIME TAKFN

For a mesh consisting of 1200 pts., and for 150 iterations (approximately the number required for convergence - this of course varies from problem to problem) the time taken is approximately 12 minutes on an ICL 1904A computer.

## REITERENCES

1 C.I.R。I.A: Seminar on the design of Wind Sensitive Structures C.I.R.I.A, London, 1970.

2 Glauert H: 'Elements of Aerofoil and Airscrew Theory'.
3 Rosenhead L: 'Laminar Boundary Layers', 1963.
4 Goldstein S: 'Modern Developments in Fluid Dynamics', Vol.I, 1965.
5 Long R R:
(i) Some Aspects of Flow of Stratified Fluids: I Tellus 51953
(ii) Some Aspects of Flow of Stratified Fluids: II Tellus 61954
(iii) Some Aspects of Flow of Stratified Fluids: III Tellus $\underline{7} 1955$

6 Draxin P G and Moore D W: 'Steady Two Dimensional Flow of Fluid of Variable Density over an Obstacle'. Journal of Fluid Mechanics, Vol.28, 1967.

7 Davis R E: 'The Two Dimensional Flow of a Stratified Fluid over an Obstacle', Journal of Fluid Mechanics, Vol.36, 1969.

8 Onishi G: 'A Study of Approximate Solution for the Airflow over a Ridge in a Viscous Atmosphere'. Sci. Rep. Tohoku Univ. (Series 5, Geophysics) 15, 1, 1963.

9 Imai I, Munakata T, Karbe T, Kaneko T: 'Variations in Wind Velocity distribution due to Topography', Fluid Mechanics News (Iyuriki News) 9, I2 (In Japanese).

10 Milne-Thompson L M: 'Theoretical Hydrodynamics'.
11 Lahiri S: 'On the Uniforn Shearing Motion of a Fluid past a Projection'. Indian Journal of Mech. Math. 2, 1, 1964.

12 Schubert G: 'Viscous Incompressible Flow over Wall Projections and Depressions', AIAA Journal 5, 2, 1967.

13 Barakat H Z and Clark J A: 'Analytical and Experimental Study of the Transient Laminar Natural Convection Flows in Partially Filled Containers', Proc. Third Int. Heat Transfer Conference, Chicago, 1966, (Vol.2, Paper 57).

14 Forsythe G E and Wasow W R: ${ }^{\text {T Finite Difference Methods for Partial }}$ Differential Equations', Wiley, New York, 1960 。

15 Thom A: 'The Flow Past Circular Cylinders at Low Speeds', Proc. Royal Society, London A144, 1933.

16 KawagutiM: 'Numerical Solution of Navier-Stokes Equations for the Flow in a Two Dimensional Cavity', Journal Physical Society of Japan, Vol. 16 II, 1961.

17 de G Allen D N and Southwell R V: 'Relaxation Methods Applied to Determine the Motion, in Two Dimensions, of a Viscous Fluid Past a Fixed Cylinder.' Quart. Journal Mech. and Applied Math. Vol.8, 2, 1955.

18 Burgraaf $0 \mathrm{R}:$ 'Analytical and Numerical Studies of the Structure of Steady Separated Flows'. Journal of Fluid Mechanics, Vol.24, 1, 1966.

19 Runchal A K and Wolfstein M:
(i) 'Numerical Integration Procedure for the Steady State NavierStokes Equations', Journal Mech. Eng。Sci. Vol.11, 5, 1969.
(ii) (With D B Spalding) 'The Numerical Solution of the Elliptic Equations for Transport of Vorticity, Heat and Matter in Two Dimensional Flows', Dept. of Mech. Eng. Imperial College, London, Ref.SF/TN/14, 1969.

20 Greenspan D: 'Lectures on the Numerical Solution of Linear, Singular and Non-Linear Differential Equations', Prentice-Hall, 1968.

21 Sutton 0: 'Micrometeorology', McGraw-Hill, 1953.
22 Harris R I: 'The Nature of Wind', Paper No.3, Seminar on the Design of Wind Sensitive Structures C.I.R.I.A, 1970.

23 Davenport A G: 'A Rationale for the Determination of Basic Wind Design Velocities'. Journal Structural Division, Proc ASCE, May, 1960.

24 Chopra K P and Hubert L F: 'Mesoscale Eddies in Wake of Islands', Journal of the Atmospheric Sciences, Vol.22, 1965.

25 Heffter G L: 'The Variation of Horizontal Diffusion Parameters and Travel Periods of One Hour or Longer'. Journal Applied Meteorology, Vol.4, 1965.

26 Maskell E C: 'Flow Separation in Three Dimensions', Royal Aircraft Establishment Report No. Aero 2565, 1955.


FIG. 1 STREAMLINES FOR THE COMBINATION OF SOURCE AND UNIFORM STREAM


FIG. 2 VELOCITY PROFILES FOR COMBINATION OF FIG. 1


FIG. 3 COMPARISON OF VELOCITY PROFILES


FIG4 STREAMLINES FOR THE COMBINATION
OF SOURCE AND TANH $(y)$ PROFILE




FIG. 7 GRAPH OF $F^{\prime}(x) V . X$ FOR VARIOUS VALUES OF $M$


FIG. 8 GRAPH OF $F(X)$ V. $X$ FOR VARIOUS VALUES OF M


FIG. 9 STREAMLINES FOR SOURCE \& TANH (Y) PROFILE SUCH THAT U=0 \&V=0 $O N \Psi=0 . M=20$


FIG.IO VELOCITY PROFILES FOR COMBINATION OF FIG. 9


FIG. 11 VARIATION IN DEPTH OF BOUNDARY LAYER


FIG. 12 REGIONS OF AIR MOVEMENTS

$t$
FIG 13 DEFINITION OF MEAN VELOCITY


FIG 14 ILLUSTRATION OF BOUNDARY CONDITIONS


FIG. 15 ILLUSTRATION OF GRID AND NUMBERING OF 5 POINTS FOR FINITE DIFFERENCE APPROX.


FIG. 16 NUMBERING OF SECTION OF GRID FOR
EVALUATING VORTICITY ON THE LOWER
BOUNDARY

(A) VERTICAL DISTANCE VECTOR B

(B) HORIZONTAL DISTANCE MATRIX A

FIG. 17 DEFINITION OF BOUNDARY

FIG. 18 NOTATION FOR APPROXIMATIONS FOR WEAKLY INTERNAL POINTS

(A) POINTS LIKE Aij

(i) L.H.S

(B) POINTS LIKE Azj

(ii) R.H.S
(C) POINTS LIKE Bi

FIG. 19 NOTATION FOR APPROXIMATIONS TO VORTICITY ON BOUNDARY

(A) STREAMLINES FOR EXPONENT 0.16

(B) VELOCITY PROFILES FOR EXPONENT 0.16

(C) VELOCITY PROFILES FOR EXPONENT 0.28

FIG. 20 RESULTS FOR A SAMPLE HILL SHAPE

(A) STREAMLINES FOR EXPONENT 0.16

(B) VELOCITY PROFILES FOR EXPONENT 0.16

(c) VELOCITY PROFILES FOR EXPONENT 0.28

FIG. 21 RESULTS FOR AN EMBANKMENT SHAPE

SYMBOL POWER EXPONENT

| $x$ | 0.16 |
| :--- | :--- |
| 0 | 0.28 |



FIG. 22 COMPARISON BETWEEN VELOCITY
PROFILES FOR HILL OF FIG. 20


FIG. 23 COMPARISON BETWEEN VELOCITY
PROFILES FOR HILL OF FIG. 21
$\square$


STREAMLINES


VELOCITY PROFILES

FIG. 24 RESULTS OF CASE $b=0.2, R=10$
POWER EXPONENT 0.16


FIG. 25 RESULTS OF CASE $b=0.3, R=10$
POWER EXPONENT 0.16



## VELOCITY PROFILES

FIG.2G RESULTS OF CASE $b=0.4 \quad R=10$
POWER EXPONENT 0.16


FIG. 27 RESULTS OF CASE $b=0.6 \quad R=10$
POWER EXPONENT 0.16


FIG. 28 RESULTS FOR CASE $b=0.2, R=100$
POWER EXPONENT 0.16
$\qquad$

$\qquad$

$\qquad$ STREAMLINES


VELOCITY PROFILES

FIG. 29 RESULTS OF CASE $b=0.3, R=100$
POWER EXPONENT 0.16
$\qquad$


STREAMLINES


VELOCITY PROFILES

FIG. 30 RESULTS OF CASE $b=0.4 \quad R=100$ POWER EXPONENT 0.16


FIG. 31 RESULTS OF CASE $b=0.6 \quad \mathrm{R}=100$ POWER EXPONENT 0.16

$A \quad R=200$

$\qquad$


B $\quad R=300$
$\qquad$


$$
\text { C } \quad R=400
$$

FIG. 32 STREAMLINES FOR CASE $b=0.2$, VARIOUS VALUES OF $R$.


STREAMLINES


VELOCITY PROFILES
FIG. $32 d$ RESULTS OF CASE $b=0.2, R=1000$
POWER EXPONENT 0.16


FIG 33 GRAPH OF REYNOLDS NO. $V$ b/h FOR ONSET OF SEPARATION


FIG. 34 FLOW DIAGRAM OF COMPUTER PROGRAM


FIG. 34 CONTD.


FIG. 34 CONTD.


FIG. 35 DIVISION OF CONTROL AREA
FOR COMPUTATIONS

MASTER HILIL
ПIIENSTOU B(51), A1(51), A? (51), OMFGAB(51). OMEGAA1(59).OMEGAAZ(5१)
9
$\operatorname{HC}(51) \cdot V C(51), \operatorname{VTOT}(51)$ ANG(51)

COH1HORS SHBT/RPSI.AR
COHION/SUBC/ROHEGA,R:R
RE: AYATION PARAMETERS
PPSi=í 6
ARHA, RDCI
QOMFGA=1 0
PT: =1, mOMMEGA
READ(i. . a)UR
rean reynolds No.
QEAD(1.そ)
pean limits of hill
REND(T.1)N?.N3.N2
N iG HORIZ.indicatorim is Vegt indicator
pean boidmatry values of Hill
QEAD(i, S)(B(T),I=N?,N3)
OEAD(1.5)(AT(J), $1=1, M 2)$
DEAD(4.5)(A2(J), $1=1,42)$
HL. -HORTZ. IfNGTH
HDHImHATZ.NO.DIV. IUNIT LENGYH
HMUI.HORTZ.NO. UNTT LENGTHS
SIMIIARLY FOR VL....fTC V VERT
REAO(1, 4)HI.HUL. YDUL

GAICILLATE NO MESH POINTS
HWIL $=$ HI_ HUL

y!limeravi.
AT:VDUI*VNIIL+1.

$2 x=19 x+411$
nX2: $0 \mathrm{DX}+\mathrm{ny}$
$D Y=9$. /VAll


nXY? ? ? $10 \times 2+7 . / D Y 2$
INIT:ALISE ARRAYS TO ZERO

20 100. 1mi.il9
$p\left(1, \int\right)=n$
$n(1, j)=n$
100 COHTIMAE
$\mathrm{N}=\mathrm{Hi}-\mathrm{i}$
$\mathrm{M}=11 \mathrm{q}=1$
MAXIT=3nn
c
READ PONER L.AN EXPONFNT, PROPORTION OF BOUNDARY LAYER
TEAB(T.) TA
RERD(9.) PDPO
WRTPE(?.10)
URIME(?, 11)MI,VL,HUL,VUL, HDUL, VDIII,N1,M1, DX,DY
WRTTE(?.2)

URiTF(?.44)(I, B(I),I=1, M1)
URETE(9.45)R.A.PRO
calcillate initial condition

$\mathrm{R} M=11$
no 101.121.41

QJ $=(J-9) / R 4$
IfC（只）＝UR＊（4J＊＊A）
$o(1 ; A)=\ln n *(R, I * *(1 .+A)) /(1 .+A)$

101 COHTTNUF
$n(1,1)=-3 . * p(1,2) / D Y$ ？
$201021=2.4$
O（1．a）$=(\|C(J-1)-\| C(J+1)) /(2 . \operatorname{DY})$
102 COHTTNUE
O（1，MI）＝（UC（M）－UC（H1））／DY
ก0 103 ． $1: 1.14$
n（N4，J） $\operatorname{mon}(1, . J)$
$D(H \overline{1}, \mathrm{~J})=\mathrm{D}(1, \mathrm{~J})$
$103-0$ शTIMUF
$20104 \quad r=2$ ，
O（I，Mi）$=0(1, M 1)$
$D(1, M 1):=0\left(1, M_{1}^{\prime}\right)$
104 CONTINUF
SET VALUF OF MAXIMUM ERROR
EPSYION：O． 105
ITFQ
111 CONTINリょ
TTER＝IYEQ＊1
CAICHLATEPSt
REGINN i
GALL INTFRIORPSI（2，N？m1，2．M2）
RFGInN 2
no $105 \mathrm{~A}=2, \mathrm{~N}$ ？
ก0 $105 \quad 1=N ?, N 3$
SEE YF POINT IS OF INTEREST
TF（J．L．E R（I））GOTO 105
B1：$A B S(J=B(I))$
R2：ARS（A4（A）$-I)$
A3：ABS（T－A？（1））

IF（日1．！T 1．O．UR．B2．LT．1．O．OR．B3．1．T．1．0）GO TO 106
DOINT IS STRONGLY INTERNAL
CALL INTFRTURPSI（I，I，J，J）
GO TO 105
POIVT IS WFAKLY INTERNAL
106 THFTAR：10
THETAL＝RT
IF（BígT 1．0）THETA4＝1．0
THETA1＝R？
TF（AR GTT 1 ）THETA1＝1．0
THETAZ $=$ RT
PF（BS．AT 1．0）THFTAB＝1．0


AL 彐ョ？．（（ DX 2＊THETAB＊（THETA1＋THETAZ））

$A L_{\text {时 }}\left(A A_{1}+A L 2+A I .3+A L 4\right)$
$T 1:=(A I, 1 * P(I+1, J)+A I 2 * P(I, I+1)+A I 3 * P(I-1, J)+A L 4 * P(I, J-1)+0(I, J))$
7：ロ（1，J）
$D(I ; J)=A R * P(I, J)+R P S T * T 1 / A L .0$
PFO（I．N．EQ：U（O）GOTO 107
PS：1．－7／D（F，J）
107 COUTJNHF
IF（ARS（DS）－ABS（RSDU（2））） 105.105 .668
GO3 RSDH（？）$=0 \mathrm{~S}$
？SDU（6）$=9$
－SDil？ 7
905 COHTIM！f

CALL INTMRTORPSI（N3＋1，N，？，M2）
CALI．INTERYORPSI（2，N，H2＋1，M）
CAISHLATE BOINNARY VAIUES OF OMEGA

GAlliti＝n
） 0 1 $\cap$ ？ $1: 2$ ，id
FF（is（j）$=Q .0 .0)$ GOTO 150
$1=\Pi(I)$
$J=1+1$
$\mathrm{BI}=\mathrm{A}=\mathrm{B}$（Y）
A4：$=1$ ．
TF（R（I）GT B（Im1））GOTO 151
TF（ $B 4 . G T(J-B(I+1)) \quad B L=I-B(I+1)$
$B S!2=(Q 1+P(I+1, J+1)+(B 4-B 1) * P(I+i, J)) / B 4$
H0 TO 15？
151 TF（AK，Gr（dmB（Im1）））B4＝d＝B（I＝1）
DST？$=(B q+P(I-1, J=1)+(B 4 \sim R 1) * P(I-1, J)) / B 4$
15？ALP1＝？／（B1＊R1＊DY2）
ALP？＝ス $0 / 0 \times 2$
$7=$ OHEGAR：I）
OMEGAE（Y）ニー（ALPY＊P（I．J）＊ALP2＊PSI？）
ПHEGAS＇Y）＝7＋GAMMA＊（OMEGAR（I）－Z）
150 COUYINHE
$D O=m P(!, Y) * D X Y ?$
Dizp $(1+1,1) / D X$ ？

D3：？（I－5．1）／DX？
$7=0$（T， 11
$0(I: 1)=m+P 1+P 2+P 3+P 0)$
П（I．A）$=7$ AGAMMA＊（O（I．1）＝Z）
103 COHYYHUF
$00 \quad 109 \quad 1=2$ ． 1
TF（A：（J）EQ．O．O）GOTO 110
「：Aク（J）
TF？$E \cap A\{(J)) I=1=1$
ก2：ABS（A？（J）－ 1 ）
n4：1．
？F（BG．GT $\operatorname{ARS}(A 1(1+1)-1)) \quad B 4=A B S(A 1(d+1)-1)$
$\mathrm{DS}[1=(n) * P(i+1, j+1) *(B 4=B 2) * P(I, 1+1)) / B 4$
$A L P M=? / \cap Y ?$
ALD2 $=2$（ $\quad$（B？＊$B 2 * D \times 2$ ）
$7=$ OHEGAA＂（J）

OMEGAAM（．1）＝Z＋GAMMA＋（OHEGAA1（J）－Z）
110 TF（A）（I）EO．O．O）GO TO 109
$=\therefore$－ 2 （
$T=9+1$
73：1－A？（．1）
34：19．
PFiBK．GT（I－A2（J＊1）））$B 4=I=A 2(J+1)$
$D S I=(B 3 * P(I-1, J+1)+(B 4-R 3) * P(I, I+1)) / B 4$
（ $L P Y=2 . /(B 3 * R 3 * D \times 2)$
ALD？：2 1 YY？
$7=O H E G A A$（I）
OMEGAAD（I）$=-(A L P 1 * P(1, J)+A 1 . P 2 * P S \div ?)$
OHEGAA）（！）＝7＋GAMMA＊（OMEGAA2（J）－7）
109 OOTTYNUF
20 11？． $1-2 . \mathrm{A}$
$00=\mathrm{mP}(1.1) * D X Y$ ？

22： $2(1,1+1) / D Y ?$
$0 \measuredangle: p(1,1-1) / D Y$ ？
$7=0$（1，J）
n（1； 1$)=1+P 0+D 1+P ?+P 4)$
$0(A ; J\rangle=7+G A M M A *(0(i, J)-Z)$



○i：$: p(2,+19) / i) \times 2$
$03=p(1,+1) / n \times 2$

$9=0(1 . M 1)$


$50 \quad 115 \quad 1=2$ ， 11
ni：M1）：$n\left(1, M_{1}\right)$
113 लOHTY MU：
C，AIGILATF OMFGA REGBON 1
AALL INT：RTOROHEGA（2，N2－A．2，M2） REGINN 2
10 120 $1-1,17$
no－T？YON？，NS
TG DOINT UF INTEPEST
；$F\left(\begin{array}{ll}\text {（I）} & \mathrm{FO} . \mathrm{d}) \\ (1, J)=O M E G A B(I)\end{array}\right.$
FEJ L． F （i））GOTO 120
Bी：ABS（AB（I））
T2：ARS（A：（J）-1$)$
33：ARS（7mA）（J））
IS DOINY HEAKEY OR STRONGLY TNTERNAL

POFNT IS STRONGLV INTERNAL
r．Ali THTERIUROHEGA（I，I，J，J）
GOTO 90n
DOIMT IS WEAKLV INTERNAL


n4－0（1，1．．1）
60 70 172

OदTHPEGAD（I）

n2＝0（5． $1+1$ ）
？F！B2．1F 1．u）（OO TO 12.4

THETA1＝1 0
$01=1)(I+1 . J)$
－ 1 FTAS＝1 0
ก3：n（9－i．J）

124 THETA：＝ 5
ก1：OMEGAA1（J）
THF：TAB＝1 0
03ッ0（I～1 J）

125 THFTA1＝10
ก1：$=0(I+1, J)$
THETAS＝0\％
ПЗ二〇MEGAA2（J）
126 OOHTINH：
ALA\＆？．（ $\cap$ X？＊THETA1＊（THETA1＊THETAZ））

1L．Be？．（ $\cap X$ ）＊THETA3＊（THETAY＋THETAZ））

$A L \cap_{\text {ta }}=(A 19+A L \lambda+A 17+A L 6)$
$11=(P(1,1+1)-D(I, 1-1)) /\left(\left(T H E T A 2+Y^{\mu} F T A 4\right) * D Y\right)$
$V=\sim(D(T+1, I)=P(I=1+J)) /((T H E T A I+r H E T A S) * n X)$
FF（1）977．929．129
$12 ? \cap A=01$
PA：＝THETA 1
GO rn $9 \rightarrow 0$
$128 \cap A=05$
PA：THETAT
129 ：$\%$（V） 130131,131
130 ก日：（1）
TB：THEYA？
90 TO 1₹2
$13198=04$
TB：＇fHETA
132 $71=(\mathrm{AL} 1+\cap 1+A 12 * 0)+A \mathrm{~L} 3 * 03+\mathrm{AL} 4 * 04) / \mathrm{R}$
r2：ARS（IN＊$\because A /(T A * D X)+A B S(V) * O_{B} /(T G * D Y)$
$T 3=m A D / R+A B S(U) /(T A * D X)+A B S(V) /(T R * D Y)$
$\because \because O(1 . J)$
O（I，J）$n R 1 * 0(T, J) \mu R O M F G A *(T Y+T 2) / T 3$
TFSO（I，N．EG 0．0）GO TO 133
RS＝1，$-7 / \cap(1,1)$
133 Bolly NiJr
TF（ABS（7a）－ABS（RGOU（1）））120，120，679
679 QSDIG（1）：2S
$S$ SD： 3 （3）$=7$
OSDU（O）＝．1
120 CONTJNIF
$\therefore A \operatorname{it}$［MTERIORUIIEGA（N3＋1，N，2，M2） RFGION 4
CAIIL INTERYOROMEGA（2，N，M2＋1，M） CHACK FUP GONVERGANCF
RES：O． 0
$70134 K=1.2$
TF（ABS（RFS）．IT，ARS（RSDU（K）））RES＝RSDU（K）
तSDURK）＝ก．$n$
134 COUTIN1IF
TF（ABS（AES）．GT．EPSILON）60 TO 111
PRIHT OUT RESULTS
URITF（？．16）
no $135 \quad 1=1$ ，N1
URTTE（2．17）I
MR：TE（？．4 8）（D（I，J）．J＝1，M1）
135 SOHTPNUF
IJRITF（？．9）
no $136 \quad 1:=1.11$
HRTTE（ .4 ．7）I

136 COHTYMUF
IRTYE（つ．，1）

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                        calgilate veiocitifes
```

    VTOT(H1)=ARS (UB)
    リC（M1）＝11R
$20937 \quad i=1.111$
$\| C(I 19)=(D(Y, M 1)=D(I, M)) / D Y$
$20 \quad 1.38 \quad .1=2.4$
TF？
$B 1=A P S(1-B(I))$
ก2：$=A B S(A 1(1)=I)$
n3＝ABS（T－A）（I））
IS DOINT UEAKLY OR STRONGLY ENTERNAL
TF！BI．IT．1．0．OR Q2．LT．1．0．OR．B3．1T．1．0）GO TO 140 POTM IS STRONGLV INTERNAL
：F（I．ER 1．OR．I．EO．N1）GO TO 141
GO YO 147
149 Y（ $1 . j=0 n$
00 ro 1／23

14？ $1 / C(, j):=(D(I, j+1)-D(I, J-1)) /(2$, 由DY）
90 YO $4 \%$

$!1 C(J)=(D(1, J+1)-D(I, d-1)) /(2, * D Y:$
rO TO 1 14
145 COHyINUE
UC（J）$=-($（B1－1．$) * P(I, d) / B 1-B 1 * P(I . j+1) /(1 .+B 1)) / D Y$
146 ：F（AP．IF 1 ）GO TO 147
IF（B3．1：1．）GO TO 148
UC（，I）$=(0(I-1, J)-p(I+\eta, J)) /(2, * D X)$
GO TO 1.6
147 VC（J）$=4((B 2-1) * P.(I, 1) / B 2-B 2 * P(I \cdots 1, J) /(1 .+B 2)) / D X$
GO TO 1 LK
$148 \cup C(1)=m(R 3 * p(1+1 . J) /(1,+83)+(1, * R 3) * p(I, J) / B 3) / D X$
$144 \operatorname{lC}(\mathrm{i})=1 \mathrm{C}(\mathrm{d}) / \mathrm{UC}(\mathrm{M} 9)$
$V T O_{T}^{*}(1)=S Q R T(V C(J) * * 2+U C(J) * 由 2)$
AU：ABS（IIC（J））
$A V=A R S(\cup C(1))$
TF（All．FO O ）GO TO $1 \angle 9$
ANG（J）＝ATAN（AV／AII）＊180．13．14159
HO TO $\mathrm{F} \boldsymbol{\mathrm { F }}$
139 VC（1）：＝0
$\| C(j)=0$
VTロテ（J）$=0$ 。
14.9 ANS（．1）＝0

933 COHTYH1！
IC（H1）＝HE（MI）／IC（MI）
URTYF（？．72）I
WRTTE（ 2,33$)$
$H R \div T E(2,34)(U C(J), J=1, H 1)$
URTTF（ 2.25 ）
URTTE（？．24）（VC（J），N＝1，（H1）
URETE（？，つ6）
URTYE（？．，24）（VTOY（J），J＝1，M1）
！IRITF（？， 27$)$
JRITE（2．24）（ANG（．1），$d=1, M 1)$
137 CO：IVTNHF
a ORHAT（ZYO）
2．GORMAT（EN．O）
3 EORMAT（FO． 3 ）
4．CORAAT（za）．O）
$\therefore$ OOR：－AT（AnFO．n）
in cOR！IAT（141， $4 \times 32 H$ INFORMATION CONCERNING THE MESH， $1,3 X, 33 H$

 1 18H HORTZ．UNIT IENGTH，1nX，F4．1．OX，17H VERT．INIT LFNGTH，1OX，F4． 1
 3rNGYH，गX，F4．1．t．
4 ？ 4 H HORTZ．NO．OF MFSH POTNTS， $4 X, T\rangle, 19 X, 23 H$ VERT．NO OF MESH POINT 5 4×＂：12．1．



13 EOPNATIAX，2H ．I，OX．2OH L．H．S．VALIIE OF HIIL．9X．2UH R．H．S．VALUF OF


 1？7X，Fム 2／5イH PROPORTTON OF B．L．THICKNESS UNDFR CONSIOERAYION IS．

```
    2.F染2.111)
1G =ORMAT(aY1, 25H VALUES OF STREAMFIINGTION,I,25H
    1-m.....1)
17 FOR:IAT(1.14H HORIZ.POSN,2X,I2)
13 cOR:1AT(\operatorname{lng(2X,E\O 3))}
```



```
21 FOR:HAT(1H1)
2.? FORMAT:1.2OH HORTZOHTAL POSITION.?X,12I)
23 :OD:IAT(APH HORTZ VFIOCITIES)
2% =0RHAT(1n(2x.FB.&))
25 cor:IAT(agh VERT.VELOCITIES)
26 =ORMAT(aTH TOTAL VELOCITIES)
27 FORIAT (GY ANGLE)
    STOD
    END
```

egment, length 2663 , name hill

SURQOUTTME INTERIORPSI(I1.12.J1..I?)

COHION/SHBT/RPSI,AR
no 1 J.n.1..12
no i $\mathrm{f}=\mathrm{ri}, \mathrm{r} 2$
TP1: $(P(1+1, j)+P(I-1,1)) / D X 2$
$T P ?=(P(1,7+3)+P(1, J-1)) / D Y 2$
-P.jatpaurpzan(T, I)
$\Rightarrow \mathrm{PD}(\mathrm{T}, \mathrm{J})$
$D(T, 1)=A R * D(T, J) \not R R S I * T P 3 / D X Y Z$

DS:1-~7/D(!, 1)
2. COHTMAIF
iF(ABS (DG)mABS(RSDU(2))) 1.1 .668
663 osDij(2):os
RSD! (6):T

- Stile 7 ) $=1$

1 rontimisf
DETIRN
ENT

EGMENT, LENG?H i8O, NAME INTERIORPSI

SUBROUTTAE INTERIOROHEGA(I1.12.J9.J2)
COMMON/GON/P(51.51), O(51.51), DX, OX2, DY, DY2.DXYZ, EPSILON,RSDU(9)
COLIION/ SURO/ROMEGA,RT,R
$: 01$ J=.19.12
DO 1 I=71, T2
$\|=(D(1,1+1)-p(I, d-1)) /(2, * D Y)$
$V=(D(I-1 d)=D(X+1, d)) /(2 * D X)$
1F(U)?,マ.3
$20 A=0(I * 1 . J)$
ro ro 4
3 0A: O (1~9.j)
4 $7 F(V) 5.6 .6$
5. OB:OO(I. . $1 \div 1$ )

00 \% ?
$6 \cap B=0(1,1-1)$
$7>1=(0(1 \div n, 1) \div 0(1 \sim 1, j)) / D \times 2$
$T 2=(0(1.1+1)+0(1, d-1)) / D Y 2$.
$T 3:(T H+T ?) / R$
$T 4=A B S(11) * 0$ A/ $D X$
T5\#ABS?V!*OG/DV
$\operatorname{CB}=A R S(H) / D X+A B S(V) / D Y$
7ヵ0 (1.J)
O(I, J) =R1* $\cap(I, J)+R O M F G A *(T 3+T 4+T 5) /(D X Y 2 / R+T 6)$
TF(O(1.IN.EQ.D.) GOTO B
RS: $9,-7 / 0(1,1)$
3 ro:TinNur
FF(ABS(DC) mABS (RSDU(1))) 1.1.679
$670 \cdot P S D!(1)=0 S$
RSDH(3) $=$ ?
RST1)(O) $=1$
a COHTINIF
RETURN
END

EGMENT, LENGTH 312, NAME INTERIOROMEGA

