Readability and mathematics

PUBLISHER

Loughborough University of Technology

REPOSITORY RECORD

J. Perry, Andrew. 2021. “Readability and Mathematics”. Loughborough University.
https://doi.org/10.26174/thesis.lboro.14229446.v1.

Readability and Mathematics

by

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A Master's dissertation submitted in partial fulfilment of the requirements for the award of the degree of M.Sc in Mathematical education of the Loughborough University of Technology, January 1986

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A review of the issues involved in the readability of the texts in general and mathematics in particular.

The dissertation considers the three aspects of readability, motivation, legibility and language factors, and considers particularly the measurement of the latter bu use of readability formulae, and the cloze procedure.

The first chapters look at these ideas as they apply to prose in general, since this sort of prose exists in most textbooks, including Mathematics.

The main section reviews the particular problems involved in reading Mathematics texts caused by the inclusion of symbolic language and illustrations and by the non-standard use of English, syntactically and lexically. The penultimate chapter again looks at readability measuring procedures with particular reference to their application to mathematics texts.

The dissertation is completed by a review of three mathematics courses using the measures of readability discussed with accompanying comments upon the legibility and the style of presentation.

Acknowledgements

My thanks to Mr Costello for his advice and assistance.

Declaration

I declare that this is entirely my own work.

CONTENTS

Chapter 1

There has been over recent years an increasing use of individualised learning schemes across the curriculum partly as a response to mixed ability teaching. The majority of these schemes make some use of workcards as an instructional tool and this means that they must be read by the pupil and understood sufficiently for learning to take place. In this situation a much greater burden is imposed on the writer of such texts than was the case for the writer of the more traditional textbook which is usually intended more to provide back up material and exercises to support a teacher's exposition. He must attempt to present the topic in such a way that the reader understands it and gains from it independently. The teacher must also exercise more care with the selection of appropriate materials for the ability of the class. Therefore to both these groups of people the study of readability is important, as the Bullock report states:

"...The effect of modern approaches in many subjects is to put a higher premium than ever on the ablity to read. There is increasing use of assignment cards and worksheets. All too often these and the tasks they prescribe make no allowance for individual differences in reading ability, and the advice given to subject departments should include a concern for readability levels in the materials being used"(1)

Though it is not difficult to understand the concept of the readability of a text it is worthy of more consideration than is generally the case. There is a general agreement amongst the authorities in this field that there are three aspects to the readabiltiy of a text, the level of difficulty of the language, the legibility of the printing and the interest of the reader in the content, as Dale and Chall state:

"In the broadest sense, readability is the sum total (including interactions) of all those elements within a given piece of printed material that affects the success which a group of readers have with it. The success in the extent to which they understand it, read it at optimum speed and find it interesting"(2)

Different people may lay differing emphasis on each of the three factors and research generally concentrates on one of the factors, though acknowledging their interaction. For example Harrison cites the 'pterodactyl phenomenon' experienced by Junior school teachers, where seven and eight year old pupils cope successfully with reference books of a quite difficult level because they are interested in dinosaurs. Similarly many textbooks and reference works are printed in a small, close type that is difficult to read, so that a casual reader would probably not bother to try to read it, yet a student may well be motivated to struggle through, either through interest in the subject matter or extrinsic pressure such as examinations.

If, as I think we must, we accept the readability of text is an important consideration then some time must be given to the examination of the factors involved and possible tests of readability that can be undertaken before a book or card is issued to a pupil. The 5% test, where a book is considered too difficult if a pupil makes 5% or more errors in reading it aloud ,and the five finger test where a book is considered too difficult if there are five or more words on a page that a pupil cannot understand, which Harrison mentions, though they may have their uses in teaching reading, seem unsatisfactory as measures of readability since they rely on the failure of the pupil to cope with a text. I shall therefore spend some time looking individually at the three aspects of readability mentioned previously.

i) <u>Legibilty</u>

Under this heading we must consider the various aspects of text format that affect the ease of reading. Unfortunately there are a large number of variables that can be studied and there is difficulty in drawing any firm conclusions from much of the research.

Obviously one may have a subjective response to the format of a book, saying that a text is aesthetically satisfying and easy to follow or otherwise but much of the hard evidence available seems to be at times rather contradictory. There are however a number of conclusions tthat can safely be drawn regarding some of the factors involved.

a) Print size and style

The size of type is measured in points, one point being 1/72 of an inch, and being measured from the top of a b to the bottom of a p. As one would expect large type is recommended for young or beginning readers and smaller type for older readers. Experimental results have shown that for beginning readers 14 to 18 point is the most effective whilst for the middle school to adult age range 10 to 12 point is recommended, and 11 point considered the optimal size. 8 point print is considered to be the smallest size that should be used even for adult readers.

The additional space put between lines, so that the bottom of a p does not meet the top of a b on the line below is called leading, and it is recommended that there should be $1\frac{1}{2}$ or 2 points of leading to significantly assist reading in the recommended print size.

The style of print used does not appear to have a great deal of effect though mathematics and science textbooks have particular problems in ensuring that there is no confusion introduced by the type face. For example there should be no uncertaintity over the letters I, l, and the figure 1,or with x and *. The notation used for fractions is also a frequent problem, the common fractions such as a half, or a quarter etc. may well be written in the usual way $\frac{1}{2}$, $\frac{1}{4}$ etc but less common fractions may be written using the solidus, /, e.g. four fifths written 4/5. It may not be clear, particularly for a weak reader, what is meant by this and the inconsistencies within the text exacerbates the problem.

At one time serifs, the small strokes at the end of certain letters, were considered to give a more adult appearence but increasingly sans serif type, literally without serifs, is being used, shedding the image gained by its use to reflect the sort of characters children are expected to write. There is no evidence to suggest that either type is preferable in terms of legibility.

Theories regarding the way people, particulaly fluent readers, read have shown that they use overall word shape as a cue , so that continuous upper case type is read more slowly. Possibly for the same reason italic type has been shown to be more difficult to read than roman (the type used here), but it is quite acceptable if used occasionally for emphasis, though Tinker, one of the chief researchers in this area, recommends bold type face for emphasis in preference to italic.

Of course an improvement in the choice of printing style does not necessarily make the text itself easier and one cannot reject a text purely on the grounds of an unsatisfactory type face, yet it is a factor that should be borne in mind, particularly by the printers of books. As Harrison says:

"... it would be unrealistic to expect any great gains in reading speed or comprehension as a result of using more legible texts. Less legible texts do lower the readers motivation, but more legible texts will not turn a poor reader into a good one overnight"(3)

b) Colour and Illustration

Any deviation from black print on a white background has, not surprisingly, been shown to reduce the legibility of print. This is because of the high contrast between the black and the white. However the use of coloured inks and paper, while it may decrease the legibility it could equally improve the motivation. This increase in motivation will only occur if the colour change is used for emphasis and its novelty value and then care must be taken over the choice of colour. To again quote Harrison:

"The remoteness of legibility research from the classroom may be judged from the fact that no published research has been widely reported on that colour which has played a dominant role in education over the past decade_ spirit duplicator purple."

Shuard and Rothery (1984) divide illustrations into three classes, decorative, related but non-essential, and essential. Decorative illustrations serve to break up the text and may increase motivational aspects but do not play any instuctional or helpful role. Related illustrations have an important role in not only breaking up the page but assist in the reading of the words by giving clues and embodiments. Essential illustrations occur often in many forms of textbooks, and are an integral part of the subject matter to be learnt. Unfortunately students do not often distinguish between these three and therefore may not give sufficient weight to the essential illustrations, treating them in the same manner as decorative illustrations. In other cases the illustrations may actually be counter productive in that they distract the reader from the essential parts of the text and all that is recalled are the illustrations.

ii) Interest and Motivation.

The interest one has in a particular book is usually in one of two forms, though a book may fulfil both roles at different times, it is either interesting from an an intellectual standpoint in that it imparts information or stimulates the intellect, or from an emotional standpoint in that it gives enjoyment or arouses pleasurable feelings.

Of course the two aspects may be combined, one may get emotional satisfaction from reading for information, and equally one may read for pleasure and gain information. For example one reads a novel essentially for pleasure yet may well gain a great deal of information, and one may read a textbook in an area of particular interest and find emotional satisfaction in it.

The interest in a book of whatever nature may come from a variety of sources. The most powerful of these are the self generated, intrinsic ones. These may lead to reading of complex and variable material, or maybe to a narrow band of similar works. As has already been mentioned these may lead to such a high level of motivation that one reads text much more difficult than that to which we are normally accustomed. Eventually the pleasure in achievement in reading may become a factor in itself. External factors, though less powerful, are perhaps more easily controlled, or even contrived. In children, parental or teacher approval can provide stimulus for interest in reading. Similarly the requirements of examinations or a particular job can provide incentives to reading, though unfortunately often little pleasure may be derived from the reading. One may argue that in this case the reader is behaving out of compulsion rather than interest. Klare describes this attitude towards the task of learning as a set to learn. The effect of a weak set to learn is to reduce the level of comprehension on more readable passages, measured by the speed at which the text was read.

The ability of the reader is also an interactive factor in this area. Harrison reports that:

"there were no significant differences between ability groups in comprehension of stories rated as 'high interest'...(and) poorer readers did comparatively worse on passages they rated as uninteresting"(4)

In other words the effect of interest is more pronounced in readers of lesser ability than in readers of higher ability.

Because these factors are personal to the individual reader it is impossible in practice to predict their effect in any particular case and though a very high degree of motivation may override other factors it does not occur very frequently. It is clear however that motivation and interest can make a difference to the ability of the reader to understand text though Colin Harrison states that:

" results support the widely held theoretical view that two years is approximately the jump in reading level which a highly motivated reader can make"(3)

Even more difficult is the assesment of what a reader will find interesting though some work of a general nature over how attitudes to topics vary with age and sex have been undertaken. These do not help the writers of subject textbooks however since their content is predetermined, they must seek to improve the motivation through the style of presentation and the linguistic style.

Language Factors

Probably the factors that first spring to mind when one thinks of 'readability' are those to do with language. The way that a text is written, the words used and the structure would seem to be the most obvious constraint on its readability. It is in this area in particular that there has been a large amount of research done to predict the readability of a text using some sort of formula. The two factors most often considered in the construction of these formulae is the vocabulary and the syntax involved in the text.

The two two variables considered for vocabulary are either the word length or the number of words not contained in a particular list. The first of these, word length, coincides with the the familiar statement 'I don't understand this, there are too many long words.' What is meant by the term 'long words' is in fact difficult words, which are clearly not the same. However a number of formulae with an acceptable level of reliability do use this as a measurable variable in the formulae, arguing that although for specific cases short words are not necessarily easy and long words difficult in the long run this would be the case and that more difficult texts would be inclined to make use of longer words. Also longer words tend to have a more abstract meaning than the shorter words and are used less frequently in speech. The word length in these cases is usually measured by simply counting letters or counting syllables in words selected from the text according to some rules that are part of the conditions of use.

The idea of checking off words that are not on a list of common words has a great deal of intuitive appeal and should certainly give some idea of the difficulty of the text. There are however a number of problems in their use;

(i)the majority of work that has been done in this area has been in America, so the word lists have an American bias and as yet there does not seem to be a British equivalent of Dale's 3,000 and 769 word lists.

(ii)the word lists are not always easy to use because there are certain rules to be considered if a word is to be considered as not on the list.e.g. all regular plurals of nouns on the list are considered as familiar, such as streets or companies, being plurals of street and company that are on the list, are considered familiar, but irregular plurals, unless on the list in their own right, are to be considered as unfamiliar, so thieves is considered unfamiliar although thief is on the list.

(iii)the effort of checking the words in a word list, though it may be simplified by use of a computer program, is fairly time consuming, and most of us do not have access to such a program. (iv) Some of the words that are on the list have many meanings and though some will be familiar there is a possibilty that a text could be using a familiar word with an unfamiliar meaning.

A problem that occurs with both methods of assessing the difficulty of words is that at times concepts that are difficult are described in words that are themselves quite straightforward. Harrison (1980) quotes the example of a 'black hole in space' which is readable at a quite elementary level yet describes a concept that is difficult to understand for the majority of the population. Some studies taking this into account such as Morris and Halverson (1938) Ideas Analysis Technique or Chall (1958) idea density have taken place but have failed to be reliable.

The measurement of the difficulty created by syntax is even more fraught with problems. Harrison (1980) tabulates five of the main types of difficulty as related to syntax:

(i) The use of a passive, rather than active, verb makes a sentence more difficult to read and recall, so that

'the book was liked by thirty people '

is more difficult than

'thirty people liked the book'

(ii) Forming an abstract noun from an active verb increases the difficulty. For example

'The translation of the book was done by experts'

is harder than

'Experts translated the book'

(iii) The use of conditional verbs, such as might or could, leads to poor understanding. In an experiment where lecturers gave talks on similar subjects to parallel groups those who avoided the use of qualifiers and what were called 'probability words' were found to be more successful in putting across their ideas.

(iv) In general the more clauses there are in a sentence the more difficult it is to understand. I am sure we have all experienced the style of writing, or speaking, in which the main theme of the sentence is lost in the depth of clauses that are produced. This style of writing puts too much strain on the short term memory and information processing capacity, yet it is a favoured form of writing in many texts, particularly in the writing of questions e.g.

"Describe, in detail, a computer application with which you are familiar. Include details of data collection and output devices used, details of the central processor relevant to the application chosen, storage devices and any important human elements of the system." (A.E.B. 1978 specimen question.)

(v) Though sentence length is the most frequently used measure of syntactic difficulty it is sometimes the case that the compression of a sentence actually makes the comprehension more difficult since the use of references in a sentence to other parts of the text makes extra demands on the short term memory.

Though all of these are useful guides to the complexity of the syntax in a text there are considerable problems in the use of them as reliable guides to syntax difficulty. They are extremely time consuming to use since the text must be read with considerable care if a valid conclusion is to be reached, and judgements are bound to be subjective in nature relying as they do on the opinion of the reader. They are however useful criteria to be born in mind in the writing of any text.

However various studies have found that there exists a strong correlation between complexity of syntax and sentence length, and since the latter is far easier to measure, and is objective, it is the variable most often used in readability formule as a measure of syntax difficulty. Klare states:

"Though sentences can be evaluated in several ways, a simple count of length is generally sufficient either by hand or machine. Sentence complexity is probably the real causal factor in difficulty, but length correlates very highly with complexity and is much easier to count"(5)

For both of the variables discussed above it is quite easy to produce counter examples to indicate a lack of validity for their use as measurements, long sentences that are easier to understand than short, sentences made up of long words that are easier to understand than those made up of short words. This does not however mean that they are not useful measures of readability in the long run. They are not however recommended as models around which to base the writing of text. Though it is certainly worth bearing these constraints in mind when writing for children or adults, to write with these as the foremost constraints will produce an invalid conclusion as to the readability of the text, and probably a stilted and boring piece of writing.

In this area of readabilty we are concerned with the match between text and reader, and the ability of the text to convey a meaning to the reader.It is therefore important to look briefly at what is meant when we say that a reader would understand the text. The measurement of comprehension is in itself an area fraught with difficulties but a number of criteria for understanding have been developed and used quite extensively. If one assumes that the comprehension test used is valid and reliable the scores most often quoted are:-

(i) 90% correct answers is taken to be the independent level, that is a student could read and understand the text without external support.

(ii) 75%-90% correct responses is taken to be the instructional level. In this case the reader would need guidance and support if he is to benefit from reading the text.

(iii) Lower than 75% correct responses is called the frustration level. Here the reader would be expected to gain very little from the text, even with support and guidance.

The method used for derivation of the majority of these formulae for predicting the readability of text use a 50% criterion i.e. a reader with the given reading age (or more commonly grade level) would be expected to get half of the answers correct on a comprehension test on the passage being measured. The formulae were created by analysing results obtained on sets of standard reading tests in America. Regression analysis was then used to find the co_efficients of the variables required in the formula. When these formulae were then measured against pooled expert judgements there was found to be a quite high correlation, which suggests that they are a valid method of ranking texts in order of difficulty. In the Effective Use of Reading project a validation study found correlations of around 0.7 when most of the formulae were compared with teacher judgements. This would imply that most of the formulae have good predictive value.

There are a large number of these formulae (in 1963 Klare listed thirty one established formulae, and admitted that this was by no means exhaustive, and since then more have been developed) so it would be beyond the scope of this work to survey even a small fraction of them. I shall, in the next chapter, therefore outline a small sample which illustrate some of the important points in this type of procedure.

Chapter 2

In this chapter I shall look at a few of the methods used to assess readability of books, most of which are formulae but I shall outline a few other techniques that may be used in the assessment of readability.

The Dale-Chall formula

This was quite an early attempt at the derivation of a formula of this type, being developed in 1948. The procedure requires the selection of 100 word samples at intervals throughout the book, every tenth page is recommended. The formula is then as follows:

$$X=0.1579x + 0.0496y + 3.6365$$

where x is the number of words to be counted as unfamiliar, y is the average number of words per sentence and X is the resulting U.S. grade score. To convert from grade scores to age levels one simply adds five. Later research suggested to Dale and Chall that the formula slightly underestimated the difficulty of more advanced texts and so provided a correction factor. This results in the following table quoted by Harrison (1980)

Dale-Chall formula score	Corrected Age Levels
4.9 and below	9 and below
5.0 - 5.9	10 - 11
6.0 - 6.9	12 - 13
7.0 - 7.9	14 - 15
8.0 - 8.9	16 - 17
9.0 - 9.9	College
10.0 and above	College graduate

The method of counting the unfamiliar words is to use a list of familiar words (the Dale list of 3,000 words) and a set of rules.

The formula has been quite popular though it sufffers from some problems as to the interpretation of the rules and is quite time consuming to calculate. It has however proved to be the most consistently valid formula producing high correlations with both experts assessments and reading tests. Unfortunately the difficulties of application outweigh these advantages unless the user has plenty of time or access to a suitable computer and software.

The Flesch formula

This is also an early attempt to produce a formula, though this time the original intention was to produce an index of adult reading ease/difficulty. the formula then produces a score out of 100, with a difficult work producing a low score, and a simple one producing a score close to 100. The factors used are similar to those in the Dale-Chall formula, using systematically selected 100 word samples, but the difficulty of the words is measured by the number of syllables per 100 words. The formula is:

$$\text{Reading Ease Score} = 206.836 - 0.846x - 1.015y$$

(x is described above and y is as described for the Dale-Chall formula). Flesch provided a transformation table to convert these results to U.S. grade scores, and as before adding 5 will give an age level.

Reading Ease Score (r.e.s)	Grade Level
Over 70	(150 - r.e.s.)/10
60 - 70	(110 - r.e.s.)/5
50 - 60	(93 - r.e.s.)/3.33
Under 50	(140 - r.e.s.)/6.66

Though it is simpler to decide on the number of syllables than the words outside the Dale list there can be differences of interpretation over the number of syllables per word, though a reasonable rule of thumb has been found to be the number of vowel sounds per word. The correlations between this and expert judgements have ranged from .61 to .84 quite within the range of other readability measures but not quite as good as Dale-Chall, but it is slightly easier to calculate.

The McLaughlin SMOG Grading

This is the simplest to calculate of the formulae since it uses a single variable, p, the number of poly-syllabic (three or more syllables) words in 30 sentences, 10 taken from the beginning 10 from the middle, and 10 from the end of the text. The final calculation is

$$\text{U.K. reading level} = 8 + \sqrt{p}$$

and $\sqrt{p}$ is taken to the nearest whole number. The acronym SMOG (Simple Measure Of Gobbledegook) owes something to Gunnings FOG index (Frequency of Gobbledegook) which contained a poly-syllabic word measure as well as words per sentence. With the use of a simple calculator this is by far the easiest formula to calculate. Though the only explicit variable considered is word length, measured as poly-syllabic words, implicit in the use of thirty sentences instead of groups of 100 words is a sentence length variable. Unlike the majority of the other formulations this is intended to correlate with 100% comprehension of text. This much higher criterion means that the formula produces scores suggesting much greater difficulty for a particular text than the others mentioned. Since this formula is a more recent development there have been less studies regarding its validity and the Effective Use of Reading survey found it to be less valid and a less accurate age level predictor, than the two mentioned so far, but this is to some extent compensated for by the ease of use, being the easiest of the nine measures they considered.

The FORCAST formula

This formula is even more recent than the SMOG formula being developed in 1973 by Tom Sticht. This formula is the only one that involves no use of a sentence variable and so has no measure of syntactic difficulty. This, theoretically at least, must rob it of some predictive value but does make it easy to calculate and in certain cases it may be the only formula applicable. Its early development was aimed towards functional literacy and in particular the measurement of technical manuals and forms, and for this type of material, which often contains very little discernible sentence structure, this is the only method of assessment mentioned so far that is suitable. The formula is:

U.K. reading level = 25 - x/10

where x is the number of single syllable words in a passage of 150 words.

Clearly this can be of no use for assessing material for younger readers since even if every word is single syllable the reading level predicted is 10 but within its particular area of application it is virtually the only formula applicable, and this type of text is not usually aimed at younger readers but at adults.

The effective use of reading survey considered it to be the least valid, and one of the least accurate predictors over an 8 - 16 range of the nine considered.

The Fry Graph

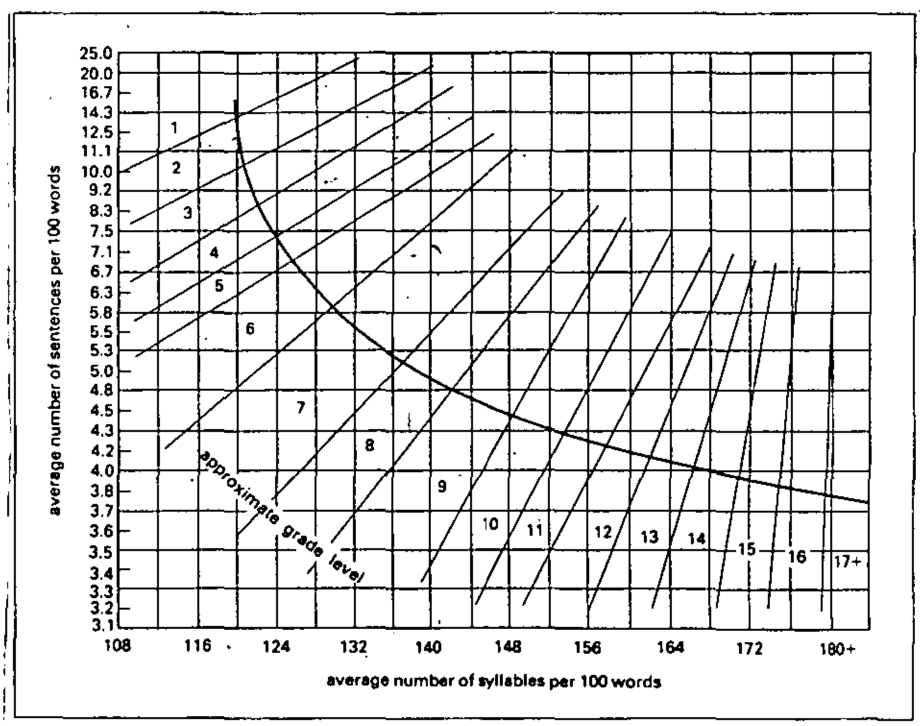


Figure 7.2 The Fry Readability Graph (after Harrison, *Readability in the Classroom*, 1980).

One method of avoiding any problems of calculation is to present the information in the form of a graph. This is the approach taken by Fry.The variables used are similar to those in the previous formulae but are used to produce a point on Cartesian axes with x, the number of syllables per 100 words and y, the average number of sentences per 100 words. Once again to obtain age levels in the U.K. one adds 5 to the grade levels. The use of the graph eliminates the need for any mathematical calculation, though with the use of calculators this should not be a tremendous advantage, and has produced high correlations when compared with Dale-Chall and other readability measures. The structure of the graph does give additional information since points in the top right hand corner imply the text consists of short sentences with difficult vocabulary and those in the bottom left suggest easy vocabulary but long sentences.

All of these methods of calculating readability measures concentrate exclusively on the text taking no account of legibility or motivation, both of which affect considerably the overall readability of text. There is one technique that attempts to take reader factors as well as text factors into account and that is Cloze procedure. This term was first used by Taylor to describe a method of testing comprehension and takes its name from ideas in Gestalt psychology. These suggest that we tend to mentally complete patterns, and so see, for example, a broken circle as a complete one. This concept is called closure. Taylor took the term and altered it slightly to describe his procedure of requiring readers to complete gaps in sentences by using information contained in the remainder of the paragraph .(Later research has suggested that it is a rather inaccurate use of the theory of closure since the completion of meaning relies more on reason and context than on completion of pattern.) When a fluent reader reads a passage he does not read letters, or even words, individually but samples only part of the information and predicts the rest. Studies have shown that they in fact only focus on about 80% of the text, and when asked to read passages with deliberate mistakes in them, such as forever spelt foyever, will unconciously read the correct form. This is due to the grammatical and vocabulary constraints on language that the fluent reader has absorbed through experience. Obviously the better the reader, or the easier the text, the more simple is the completion of passages by the use of these constraints. This is the basis on which cloze procedure is used as a measure of a readers ability, and as a measure of readability. It is of course the latter that is of interest here.

The method adopted is quite straightforward. Three passages of at least 250 words are selected, from the middle, beginning and end ofthe book. Each passage must start at the beginning of a paragraph and the first sentence left intact. From then on words are deleted at regular intervals, usually every fifth or seventh word, with uniform sized gaps and the readers asked to complete the passages taking as much time as they need. The test is marked by comparing the original with the guesses of the readers and a percentage score calculated. The resulting percentage is then intended to suggest the level at which those readers can use the book. The research so far undertaken suggests that, broadly speaking, a range of 60% and above corresponds to an independent level, 40% - 60% an instructional level, and below 40% a frustration level.

To the constructor of a cloze test two issues may be of interest, the rate of deletion and the method of scoring. The rate of every fifth word is popular in research work since the higher the deletion rate the shorter the passage length required to obtain a satisfactory degree of reliability (around 50 deletions), but higher rates than 1 in 5 make the test too difficult because there is insufficient information remaining for anything but guess work. Evidence suggests that for rates of more than 1 in 7 there is little improvement in the readers scores, suggesting that they do not gain from the extra information available. Between rates of 1 in 5 and 1 in 7 there is little to choose since results have varied between tests and between subject areas. The method of scoring is to only accept verbatim responses, and misspellings. This is faster than synonym scoring and generally more reliable though there are sometimes difficulties whether a word is misspelt or incorrect grammar, e.g. is where for were a misspelling? Early studies comparing the two methods of scoring showed a high correlation between scores obtained using the two methods, and the extra effort involved in synonym scoring hardly seems worthwhile.

While the use of readability measures and cloze procedure methods are certainly not mutually exclusive the latter does seem to have some advantages over the former.

i) The interaction between reader and text is more involved in this method of measurement and so can more accurately match a reader to a book. Readability formulae exclude any reader factors such as motivation which have been shown to considerably affect nominal reading ages. Another reader factor is the backgound knowledge, or subject specific knowledge, that the reader brings to the text. In many cases works in subject areas contain technical terms that the reader is expected to know from previous work. These words, being outside the mainstream language, will raise the readability level given by a formula thus not giving a true measure of difficulty.

ii) If it is possible to copy exactly the print style and layout of a text whilst leaving gaps for missing words any problems with legibility of print are also involved in the measurement using cloze procedure.

iii) The score given by cloze procedure is a much finer tool, showing at which level a particular group of readers should be using the text, whereas the formulae simply give a reading age, or grade score. As Rye (1982) states:

"Cloze Procedure is a much more subtle readability measure (than formulae) and reflect a persons understanding of the text. Cloze scores can be interpreted in the light of reference points based on different levels of comprehension. These scores enable a teacher to evaluate the readability of a book for the whole class, for the individual, and in comparison to other books"

However Cloze procedure can measure readability but cannot predict it, which is an advantage of the various readability formulae, and is rather more time consuming than the computation of a formula score. It is important therefore to carefully construct, administer and mark the test if valid conclusions are to be reached.

Chapter 3

The prose that occurs within a textbook fulfils one of a number of different roles, each requiring a different response from the reader. Shuard and Rothery identify five different categories:

1) Exposition, which is the explanation of concepts and methods. It is in this area that any new vocabulary is likely to be introduced, and notation and rules are explained. This exposition must be read carefully, and remembered since the information contained in it often is used later, possibly without reinforcement. In the individual learning situation this has to take the place of a teacher's explanation and, like the teacher, needs to make the situation clear to the weakest and the brightest in the class. This is a terribly difficult task since the author has little influence over the use to which his material is put and is always aiming at a much wider audience than the classroom teacher.

2) Instruction, which tells the reader to do something, write, draw, copy and complete, evaluate etc. These have also to be read carefully but must also be acted upon. Very often unless an instruction is part of an exercise the reader is inclined to ignore it and the concept the task is intended to clarify remains cloudy. The term copy and complete has been shown to be one that means very little to readers, though it is much used, in general the reader is not sure what he is intended to do in order to complete the table or diagram. Evaluate is also a source of confusion, I have found that children even of fifth year 'O' level standard need reminding that evaluate is a means of saying 'work out the answer to' in most situations in which it is used. Students have to be familiar with this usage since it is much used in examinations, but I see no reason why some other more familiar terminology should not be used, since evaluate is hardly an important mathematical term, nor would ignorance of it

greatly impoverish their language, particularly since its usage in OE is slightly different.

3) Exercises and examples, which are familiar to all who have ever been in a Mathematics classroom. Many of the older mathematical textbooks, and some of the newer, consist of nothing but exercises and examples for the reader to do. Often they are merely routine repetitions of some learned technique but in more modern textbooks examples and exercises are used to stretch and extend the knowledge, in the obvious form of an investigation type question, or less obviously by leading the reader to a discovery of some underlying structure or pattern. This more oblique method can lead to problems because frequently the reader does not realise the importance of his discovery. Being familiar with the standard type of exercise often all that concerns him is whether his answer is correct and little thought is given to any possible significance there is in his answer. Of course if the answer he gets is wrong he has no possibility of discovering the particular pattern or concept, or he may even discover one that is not correct. This may cause problems later when the discovered information is relied upon for some development. In general I would approve of discovery methods, as long as the reader is informed of the importance of his discovery, and not left in doubt over its correctness.

4) Peripheral writing, which does not literally mean writing around the edge of the page , though occasionally this is the form it takes. Rather the type of writing meant is the introductory remarks, historical asides etc. that are intended to motivate the reader in some way and keep him going, rather in the manner of a compere at a variety show. The content is rarely of much importance and could be skipped over without losing any essential information. It can therefore be read in a fairly cursory way.

5) Signals, which consist of the chapter headings, orderings, and such like that may not be read in the

conventional sense but provide clues to the content or guides to the way the reader is intended to proceed through the page. That they are not read is evidenced by the number of pupils who work down an exercise where they are intended to work across, or vice versa. I find that in any class there is always a small group who realise after three or four problems that thay are working in the wrong direction. This is particularly strange if, as is usually the case, the correct direction of working is the conventional left to right orientation. Often chapter headings are used to introduce new vocabulary, as well as telling the reader the subject of the chapter or section. Also frequently the signal is in the type of typeface used, in S.M.P. for example, italic typeface is used to indicate that a new word or phrase is being introduced and the surrounding text is meant to define it. Though the typeface does draw attention to the word or phrase the uninformed reader may not realise that he is being told the meaning in this way, and may well feel that he does not understand the text because he does not understand the new words. In fact what he does not understand is the signal.

It is fairly easy for the advanced reader to work through a page of text and identify the type of text; exposition, instruction, exercise, peripheral writing, or signal, and treat each in the most appropriate manner. This is not however quite so easy for the less experienced reader and there is a great danger of them glossing over an important piece of exposition whilst giving much more concentration to a piece of peripheral writing, and maybe ignoring a signal altogether. This is obviously going to lead to a lack of adequate understanding and a failure to satisfactorily complete any work resulting from that text.

For whatever purpose the prose in a Mathematics text is used there are a number of problems regarding its readability. Along with the problems and conditions considered in the previous chapters Mathematics has some characteristics peculiar to itself, complicated by the use of graphs, formulae, diagrams and a different use of English. Kane states:

" All mathematics textbooks are written in more than one language. Each contains portions in a natural language, such as English, together with portions in one or more additional language, such as Hindu-Arabic numeration, various algebraic notational systems, the language of sentational calculus and the like"(6)

In an earlier article he differentates between English in common use, which he called Ordinary English, and that in mathematics, Mathematical English:

" Mathematical English (ME) is a hybrid language. It is composed of Ordinary English (OE) co_mingled with various brands of highly stylized formal symbol systems.... ME and OE exhibit different characteristics and consequently may require different skills on the part of readers to achieve levels of reading comprehension" (7)

The text, quite apart from diagrams, differs in the areas both of vocabulary and syntax.

The difference in vocabulary consists of two types; the words that are technical terms, exclusive to mathematics and those that are in common use but either have a more strict meaning in mathematics, or one not related to the Ordinary English usage.

Words of the first type, such as trapezium, congruent, coefficient etc. are met only in a mathematical context and must be learnt from either the teacher or the text book. It is highly unlikely that any of the terms will be used in the pupils usual speech, which is usually a fairly restricted part of the English language anyway, and so the rarity itself will cause a problem. Also, unfortunately, many of the words betray the origins of much of mathematics in Greek culture, e.g polygon, or Latin, eg binary, or even Arabic, e.g Algorithm, none of which are familiar to the majority of our students. This use of Latin, Greek, and Arabic words and roots of words to give names to concepts may have been useful when the educated man wrote in Latin and was well versed in Greek, but the present day student has no recourse to any reinforcement from the structure of the word, he must simply learn the word, its meaning, and far too often even how to pronounce it. That pupils do not understand these words is clear from the work of Otterburn and Nicholson who found for example, that of 300 pupils tested who were following a C.S.E. course only 22% could indicate that they understood the word symmetry either by a description or by drawing a diagram and only 11% could show that they understood trapezium.

Often these words are a crucial part of the text and, though they may have been defined earlier, the reader will be unable to get any meaning from text containing such words if their meanings are not readily available to him. Shuard and Rothery state:

" Even when the text give definitions it may not reinforce the new vocabulary..... The most difficult words are often the ones which it is most important for the pupil to read with meaning" (8)

An answer to this problem of a sort is to eliminate where possible all technical terms. This may make the text easier to read but has only short term advantages. As has been stated above these technical terms have often a crucial role and the pupil cannot proceed without knowing them. So, though it is desirable to avoid the use of unnecessary technical terms, (for example I have managed quite well up to now without knowing what a minuend and subtrahend are!) the important technical terms have to be constantly reinforced.

Words of the second type, such as similar, product, intersection, that are used in ordinary English with a different meaning or emphasis are very common in mathematical English. The main source of confusion here is the difference in meaning. For example Kath Hart quotes an interview with a pupil of secondary school age:

" Interviewer: Do you know what volume means.

Child: Yes.

Interviewer: Could you explain to me what it means?

Child: Yes, its what is on the knob on the television set."(9)

In this case there is very little obvious connection between the mathematical usage and the English usage yet confusion has occurred between the two. There is quite a large group of these words in which the two usages are dissimilar for example integration in OE has little relationship to its use in calculus, though fortunately this is not in the vocabulary of the majority of school students, but words such as product, difference etc are sufficiently common in every day language to cause similar problems to that illustrated above. In their survey of 300 pupils following a C.S.E. course Otterburn and Nicholson found that the word product had the fifth highest percentage of confused responses (20%), 'beaten' by multiple (34% confused), a large number of these confusing it with multiply or factor, kite (32% confused), parallelogram and rhombus (both 22% confused). This perhaps indicates that though students felt that the word was familiar they were unclear as to its mathematical meaning.

" the usual errors were to confuse the word product with sum (addition) and there were twenty cases of this, or difference (subtraction) for which there were nine cases. Quite a number of pupils described its everyday use and gave something produced or an equivalent expression"(11)

Sometimes the difference is more of emphasis or strictness as in the case of the word similar which in common usage is a synonym for resembles but in mathematics the meaning is much more strict. In an OE sense all triangles could said to be similar, which is certainly not the case in ME. Again the word average which in Mathematical English means representative value and can take one of many forms is either used in OE to imply mean, or as is often the case, is used without any specification as to the meaning.

A student unfamiliar with these differences will be at a considerable disadvantage in tackling a standard ME text, and since he is familiar with the words will either remain unaware of the problem unless it is specifically highlighted or will believe, quite wrongly that he has understood the text because he has read the words successfully. The situation may even occur where the pupil has read the text, thinks that he understands each individual word, but finds that the text makes no sense to him, this is bound to be a very disturbing experience and is highly unlikely to improve motivation. The typical response when this situation is discovered is to accuse the mathematics text of a peculiar perverseness and ask why the text cannot use the 'proper' meanings of words.

A further problem comes with the connotations that words bring with them from the OE usage. When a word is used in normal prose it is intended to convey a number of ideas and subtle nuances of meaning and suggest others of a connecting nature, in Mathematics however words take on one meaning alone, generally universally accepted, but occasionally peculiar to the author. The latter is a source of annoyance both to the experienced reader and to the novice and is to be avoided wherever possible.

In OE the context in which the word is found generally will point the reader in the direction of the correct meaning, mathematics however tends to be weak in contextural clues, the precise meaning intended by the author must be understood without aid from the rest of the sentence.

Certain words used in mathematics have derogatory connotations when used in OE such as vulgar, as in vulgar fractions (though this usage as tended to be superseded by improper) or irrational, some have connotations in terms of levels of difficulty, such as

Simple Interest (should Compound Interest be termed Hard Interest instead) or Complex numbers (which must be difficult since they are also imaginary.)

Superficially the sentence structure in mathematics is the same as in OE, after all they are both intended to be read by the same target group. There are however a number of differences, partly due to the nature of the subject and partly due to the writers.

There is a tendency for ME to be written in a particularly dense form, so that there are relatively large numbers of ideas to understand in each paragraph, with few adjectives and each word being important to the overall meaning. This makes the scanning of a page for the relevant information in the way that one would in a novel or history textbook for example particularly difficult. As was previously stated research has shown that quick readers tend not to read all the words but scan the shapes of words and infer meanings from that, this is why lower case Roman is usually easier to read than upper case, and read a small percentage of the text inferring the rest from the portion actually read. This is dangerous when reading mathematics because of the low level of redundancy.

Writers of mathematics text books are generally inclined to write in a terse style, with short sentences and many connectives. The sentences may each contain only one idea and will certainly follow a logical order with implications form one sentence to the next. As Harrison states

"By training, mathematicians are encouraged to prefer elegance and conciseness, but this in turn may lead them to write textbook prose which contains less redundancy than is present in other subject areas"(3)

Though this may not cause particular difficulties regarding the understanding it does cast doubt on readability formulas that make use of sentence as a crucial variable.

Many texts mix symbolism in with prose, often unavoidably, and this makes the reading of the text quite difficult. If the meaning of the symbol is not completely understood the sentence containing it will not be understood either. Even if the meaning is understood it is often difficult to extract the meaning from the sentence because of the presence of the symbol.

Yet another difference in style that mathematics text have from those in the Arts and Humanities, but is shared with other science and tecnical subjects is the way in which ideas are presented. Very often new terminology or ideas are presented and having previously been explained, or seem to occur incidently. e.g. "In fact, we can join up the points by drawing a line with a ruler (see Figure 5). This straight line is called the graph of the relation.(Relations whose graphs are straight lines are sometimes called linear relations.)" (S.M.P. Book C)

In this sort of example the reader is expected to refer back for an explanation where the normal practice is to refer forward for explanations. If a reader is prepared to accept this technique and re-read to get the explanation there is no problem, however in many cases a new word or expression is encountered and the reader halts, feeling unable to continue and not realising that the new term has already been explained. At this stage he either reports elsewhere for explanation or stops altogether , even if the explanation of the term is simple. Some texts, such as the S.M.P. lettered books, often introduce a mathematical term then put an explanation, or more common equivalent, in brackets afterwards such as, vertices (corners). In my experience few readers get as far as reading the word or explanation in brackets, prefering to ask for help. This casual method of introducing new terminology does not appear to fix the new words in the readers mind so that subsequent occurence are no better understood than the initial one.

The style in which questions are asked is often a source of problems to the reader. The mathematics question writer seems extremely fond of conditional questions e.g. If 7 bananas cost 56p, how much would 20 cost? This has been shown to be much more difficult to understand than a question written in the present tense and it usually is no more difficult to write in that form e.g. 7 bananas cost 56p. How much do 20 cost? Pupils also often find difficulty with the multi-part type question such as

"Use your graph to find how much petrol can be bought for:

(a) 80p; (b) 55p (c) 185p (d) 112½p " (S.M.P. Book D)

In this case they either look for a pattern in their answers or, more usually are not sure what to do with the last three parts to the question.

There is also, in more modern textbooks, a fondness for the rhetorical question. This is perhaps an attempt to improve motivation or reflect a more conversational style, however it generally results in confusion. Pupils responses tend to be along the lines of "Do I have to answer this?" or unsatisfactory responses such as "No!" as a response to "Can you see a relationship ?" Though an attempt to improve the motivation contained within a text is quite laudable this method does not really work. Rhetorical questons in a teachers discussion of a topic can cause problems but in this situation at least the teacher is in a position to remedy the confusion immediately, and at least he knows what he intends to elicit with the question. This is certainly not always clear even to the member of staff using the book and often the only response the teacher can give the child is to ignore it, which does not set a particularly good example. Recognising this problem in the Mathematics Applicable project Ormell used a special symbol to suggest that a question was rhetorical and explained it in the chapter entitled 'How to Use this Book', stating :

"◎ indicates a rhetorical question. These questions are not intended to be answered immediately. They are posed in order to clarify mathematical challenges or to introduce discussions of the purpose of the current work."(12)

Since this work is a part of a sixth form course certain amount of maturity and understanding is to be expected from the students and they should be capable of dealing with language at this level of sophistication. It may not be all that successful to place this in a preface chapter since there is a tendency for those not to be read. Rhetorical questions are not however restricted to these texts and generally texts do not make the same early explanation of the nature of a rhetorical question, and it is doubtful if the average student would understand it if they did.

These problems of style are quite a common feature in textbooks, and often examinations. Either by training or nature mathematicians seek the most elegant solution to any particular problem, and this can often be interpreted as the shortest solution. This attitude is, of course, carried through into any written work that they attempt, including textbooks. As the mathematics becomes more advanced, at sixth form level for example so the text becomes terser and much more difficult to understand. As proof becomes an important part of the work at this level it becomes more a case of how much to leave out and how complete to make any explanation. The result is that for all but the most able the textbook is useless as a primary learning tool. It can only be used as back up for the teacher and as a source of adequate examples. The user needs to have an understanding of the subject before he can understand the text, which seems somehow to be the wrong way round.

Chapter 4

Yet another problem with the reading of mathematics is the interpretation of the conventional coding system used with the symbolism. In the reading of symbol free text though a word may be unfamiliar in its written form it is usually possible to take an informed guess as to its pronunciation, and maybe to its meaning through the context in which it appears. The verbalising of the written word then gives an aid to its meaning. The use of symbolism adds an extra dimension to the problem, since first one must identify a symbol with the words it replaces, if it can be adequately expressed in words, and then with the ideas these words are intended to convey, and generally though there may be clues in the context in which the symbol appears there is unlikely to be much clue in the actual symbol. Skemp (1971) distinguishes ten different functions of symbols; Communication, Recording Knowledge, formation of new concepts, making multiple classifications straightforward, explanation, making possible reflective activity, helping to show structure, making routine manipulations automatic, recovering information and understanding creative mental activities. Most of these are inter-related, particularly with the first and it is primarily this role that is of interest here.

Often symbols can represent more than one word, for example + can represent plus ,add ,more than and many others and in context the correct word is often clear, but classroom uses and textbook tend to be context free and though the child may identify the operation with the symbol he may not possibly identify a meaning. Matthews (1981) found that though 75% of 11 year old pupils could use - as take away, only 20% could use it as difference. All four of the symbols for the 'four rules' operations have a number of interpretations and a child needs to have fully grasped the concepts underlying the four rules if he is to read the correct meaning into their use.

To make the matter of comprehension even worse rarely do symbols occur individually and the spatial arrangement of symbols is also intended to convey a meaning dependent on the context in which they appear. Generally this meaning has developed from a convention rather than any particularly logical rationale. For example at an early stage children learn that the positioning of the numerals has significance, the place value concept, and that 25 and 52 have different meanings. These meanings may not fit the conventions of speech in a particular language, in most western cultures the ordering of written text is left to right so the order of 2 and 5 in twenty five fit the conventions of the English, French and Italian languages but not German whilst the ordering of the digits in the numbers from 11 to 19 inclusive does not fit the language conventions in any of the languages. (At least German is consistent in its mis-matching.) The child has to come to terms with this and when this is mastered, or possibly before, he meets the conventions of algebra where juxtaposition implies multiplication (5a meaning 5 times a) or fractions where a fraction and whole number next to each other implies addition ($6\frac{1}{2}$ meaning 6+ $\frac{1}{2}$). Yet further along this road dx does not mean d times x but a small increase in x, and though dy/dx looks like by/bx in the first example there is no way in which the d's can be cancelled.

The coding system used in algebra is a considerable source of difficulties for children. Both the APU and the CSMS work found that the majority of children had only a very shallow knowledge of the conventions, and were using the letters in a different form to that intended. Kuchemann in his work on algebra for the CSMS found that chidren operated on the letters in algebra in six different ways of ascending order of complexity:-

i) Letter evaluated, the letter is considered to have a numerical value from the beginning of the problem, which enables children to deal with such things as $a + 5 = 8$, find a, but not find a if $12a + 5 = 9a + 8$ where one has to operate with the letter a as an unknown.

ii) Letter ignored, in certain situations it is possible to solve a problem simply ignoring the letters, e.g. if a + b = 43 find a + b + 2. In this situation it is possibly the most appropriate technique towards a solution. In an answer that requires a letter as part of it, it is clearly not applicable. If a pupil is relatively successful with this technique in solving problems he is likely to apply it to written material, i.e. ignoring the letters in any algebraic work.

iii) Letter as object, the letter is used as shorthand for an object, or standing for an object itself. This is encouraged in a number of text books concerned with the early stages of algebra where one has exercises to collect like and unlike termsthat say for example;

5 oranges + 3 apples + 2 oranges + 6 apples = 7 oranges + 9 apples

so

$$5o + 3a + 2o + 6a = 7o + 9a$$

No doubt this is done with the best intentions and in the above situation may be an acceptable technique but in later work when it is essential to distinguish between the object and the number of that object it is inappropriate. This becomes particularly obvious in dealing with linear programming questions at both 'O' level and C.S.E. where 'twenty four glasses equals one bottle' is written algabraically as;

$$b = 24g$$

instead of;

$$g = 24b$$

Perhaps if the idea of number of bottles and number of sheep was emphasised so that the last statement becomes

' the number of glasses is twenty four times the number of bottles'

there would be more success, and though the use of the initial letter may be convenient if the algebra is understood it does encourage this particular misuse.

Obviously if the pupil is writing word problems in this incorrect fashion he is also reading algebraic problems in the same incorrect way. In an item testing this concept only 10% of 14 year old pupils could answer correctly.

Of the pupils tested only 9% of the 15 year old pupils and 6% of the 15 year old pupils could be said to have reached a greater level of understanding than this and could

iv) use a letter as a specific unknown and operate on it directly.

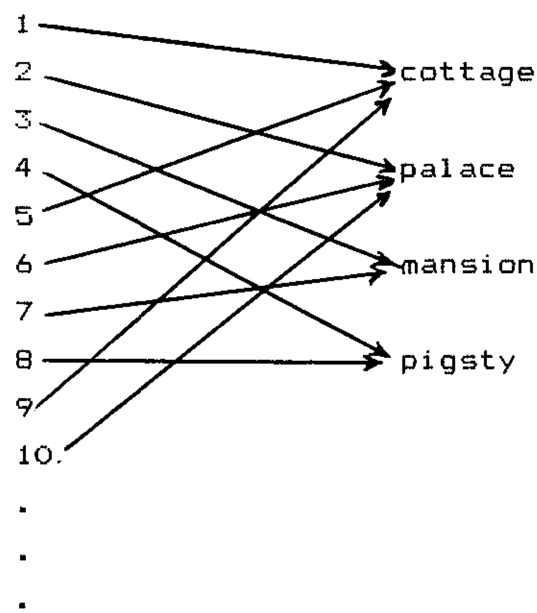
v) use a letter as a generalised number and recognise that a letter could take several different values rather than just one.

vi) use a letter as a variable and recognise that it represents a range of unspecified values, and a relationship is seen to exist between and two sets of values.

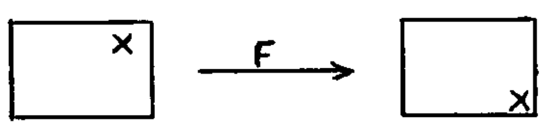
Since a large proportion of the skills required in secondary school algebra require pupils to operate with letters in the form of (iv), (v), and (vi), yet the majority of our pupils are operating with algebra in the forms (i), (ii) and (iii), there seems a distinct mis-match between our requirements and pupils capabilities and there would appear to be fairly sound grounds for assuming that most of the pupils do not understand a great deal of what they read if it contains algebraic notation, or perhaps more dangerously think that they do understand when in fact they do not.

A symbol that falls into both the field of symbolic language and the field of graphic language is the arrow. This is used with a wide variety of interpretations. Within one particular text (S.M.P. Book D) an arrow is used to indicate: (see overleaf)

i) a mapping as

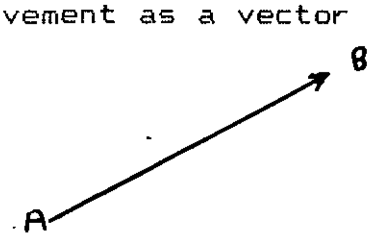


ii) a movement, labelled with a letter



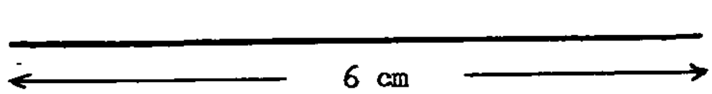
iii) a functional mapping, as $x \longrightarrow 2x$

iv) a movement as a vector

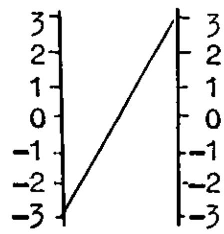


v) a linking line in a flow diagram indicating an order in which operations are to be performed.

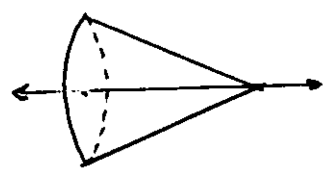
vi) the length of a line using the conventions of technical drawing.



vii) arrow diagrams to represent relationships and mappings



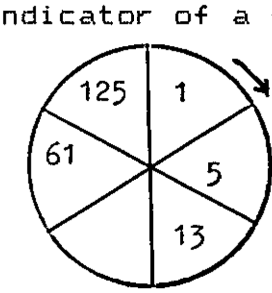
viii) as an axis of symmetry



ix) as a pointer labelling a particular position.

x) as movements, through a maze or up steps.

xi) as an indicator of a direction of movement.



For the majority of these situations there is very little to differentiatebetween one meaning of a line and another and the reader must judge from the context of the situation exactly what the arrow is intended to represent, to the mature reader this is usually obvious, I wonder how obvious it is to the less mature reader for whom it is intended. Some of the uses are conventional, common to all text books as in the notation $f:x \longrightarrow 4x - 5$, other notations are peculiar to a particular author or series of texts. Even the conventional ones as in the example given may be verbalised in a number of ways, each having equivalent meaning, but possibly not obviously so to the inexperienced. If a teacher reads such a combination of symbols as 'f maps x onto 4x − 5 on one occasion, then later reads the combination as the function that takes x to $4x - 5$ clearly the same operation is implied but the listener may think that the two diagrams that look superficially the same are in fact in some way different, adding further to the mysteries of the subject.

Chapter 5

Shuard and Rothery state that:

"the main structure of any piece of mathematical writing is provided by the prose text; illustrative material such as pictures, graphs and diagrams, is normally structured in relation to the prose reading matter."(8)

Though the first part of this statement is generally correct there are cases of texts where the main structure is provided by diagrams, particularly in the area of Euclidean geometry, and in more advanced forms of mathematical writing the structure may be provided by the symbolism, in either case prose text may be entirely absent, or at least minimal. Overall though, particularly for text books in general use in lower than 'A' level courses the above statement is correct, as is also the case for the positioning of illustrative material, though this may in part be dictated by the dimensions of the page and the costs both in terms of space and money.

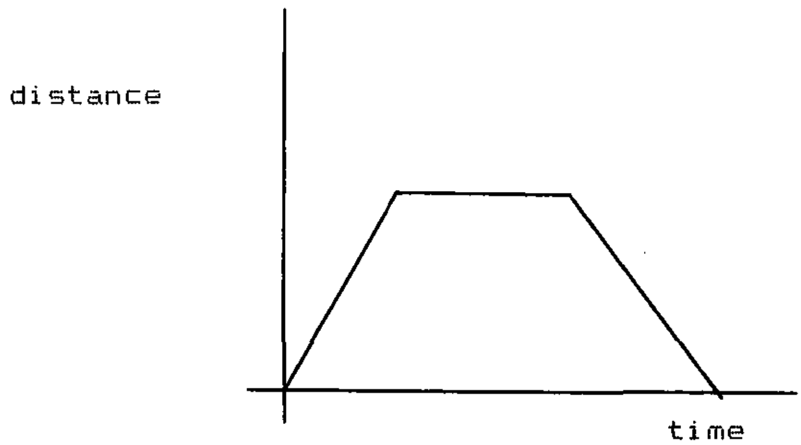
The positioning of diagrammatic material is important whatever the nature of the material. Whalley and Fleming found that in two versions of illustrated technical items on electronics students reading a text in which a diagram immediately followed the line of text which referred to it reported that the text was clearer and more easily understood than one in which the diagrams were arranged to give a balanced layout of text and diagrams. During the reading of the text the students eye movements were monitored and it was found that students spent 35% of their reading time on the diagrams in the first format as opposed to 15% for those using the second, original, format. Clearly the original was laid out to some aesthetic criteria, to provide the best looking page, and, though this may have provided a more attractive, and motivating, text, this advantage was far outweighed by the advantage of easy reference to a relevant diagram. In primary school and in the early years of secondary school children appear confused by the use of instructions such as '(see figure 5)' perhaps they do not understand the use of 'figure' referring to a diagram but since this terminology is not confined to mathematics but is quite widespread, this seems doubtful. It seems more likely that they are reflecting the results previously mentioned and are finding the reference to a diagram positioned elsewhere on the page a distraction, and probably paying insufficient attention to it when they eventually find it.

One method of classifying illustrative material is to relate it to its importance to the surrounding text. This produces three classes:

i) Decorative illustrations which break up the text and help to present a less forbiding appearence to the page, but serve little or no purpose apart from this. At times they may be used to put some sort of context upon a topic or question but do not actually impart information or contribute to the solution of a problem. The role they play is quite important but may be misleading since in some cases the reader is not sure what degree of consideration should be given to the illustration. Texts vary as to the amount of use made of this sort of material, for example the S.M.P. letter series have few purely decorative illustrations apart from the chapter headings, whilst the Oxford Comprehensive Mathematics series, which was published at about the same time, makes frequent use of this sort of thing with for example a picture of a bust of Pythagoras in the section on the theorem of Pythagoras, and frequent slightly jokey cartoons. Though these illustrations do lighten the scene a little they can also be rather a distraction and at time present a rather cluttered appearence to the page.

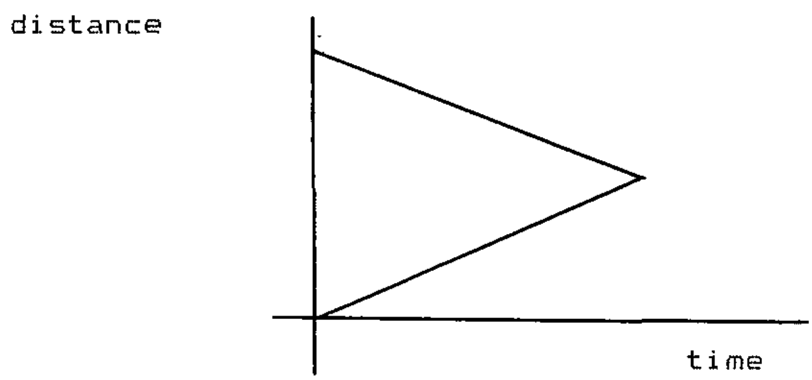
ii) Related but non essential illustrations, as the title suggests, are not crucial to the understanding of the text but are related to it and may provide extensions or embodiments of the ideas contained in the prose. In many cases a picture can help the reader to visualise the situation the prose is attempting to describe giving a visual embodiment of an abstact concept or a picture that helps to put a meaning to some difficult words, for example a selection of drawings of isosceles triangles to illustate the essential properties and give the reader an idea of what one looks like. This sort of illustation does not need a detailed examination but a global processing is sufficient.

iii) Essential illustrations generally consist of graphs, diagrams or tables. In the case of this type of illustration a prose description may appear in the text but often this is very difficult to do since to describe the diagram successfully is either impossible or so confusing as to make the effort not worthwhile. Graphs are an important part of this section and unfortunately are not well understood. We are all familiar with the pupil insists on plotting points in the wrong order, demonstrating a failure to learn the convention, or appreciate the importance of it, but more important than this is the failure of graphs to communicate the message they are intended to to the majority of the population of schools and probably the adult population as well. Daphne Kerslake in the C.S.M.S. report found a wealth of misunderstanding over graphs, over the importance of correct labelling of the axes, with both a suitable axis and a suitable scale, and, more importantly for the reading of a mathematical text, over the interpretation of a line or point on a line, for example a distance/time graph, or velocity/time graph. The interpretation that many children place on these shows that they do not consider the horizontal axis to indicate the passage of time, but interpret the graph as a picture of the journey so that;



(figure i)

is interpreted as climbing a hill, walking along the top, then down again, followed by climbing again, and;



(figure ii)

is interpreted as a journey north-east followed by a movement north-west. These children are wrongly interpreting a conventional coding system as a pictorial illustration, and in the context of a textbook would not give it the necessary attention, and if they did they would wrongly interpret the graph. Problems arise later in pupils who do not understand the convention that the controlled variable should always be plotted horizontally so that graphs as in figure (ii) do not make any sense, in that they suggest that two or more different results may occur from the same reading at the same instant. In essentially linear relationships there is no problem if the axes are incorrectly designated but for non-linear relationships considerable confusion can occur in the interpretation.

Meredith states:

"One of the factors which make graphic organisation so powerful is that it can draw simultaneously on a number of different codes and so achieve great economy of expression.... Some codes are more obvious than others. Some are so little obvious as to be invisible. The most important example of this tacit symbolism is the relative position of the elements in a diagram. The centre, the top, the bottom, the sides, the proximity or distance of the elements, all of these can and should signify important relations."(13)

These relations may be obvious to an experienced mathematician but are probably not so obvious to anyone else. This ignorance of the message a graph or diagram is meant to convey is amply demonstrated by the amount of poor diagrammatic representation prevalent in many areas of a pupil's experience, both in media presentations that are intended to be informative, but far too often end up being misleading, or confusing and sadly often in textbooks. I am not always certain whether or not the former are deliberately misleading or are honestly unaware of the implications of thir misleading graphs and diagrams. The biggest problems come in the drawing of graphs where axes are scaled in an inconsistent manner to emphasise an increase or decrease, or broken to give the same effect, lines are drawn between points assuming knowledge of the intervening period where none exists or even continuity where none exists. In diagrams the biggest problem is in the use of scale, in using textbooks children often assume that a diagram is drawn to scale and this is not always the case, in Mathematics textbooks particularly diagrams are often deliberately not drawn to scale. When they are often a mistake is made in increasing or decreasing in a proportion, commonly the dimensions of a diagram are doubled to represent a doubling in size where in fact if the diagram is two dimensional this represents a quadrupling in size, and if it represents three dimensions an multiplying by eight. With this sort of misuse, deliberate or otherwise, when a pupil is confronted with one in a mathematics textbook it is hardly surprising that he does not appreciate the subtle, or not so subtle, nuances of meaning the diagram is intended to convey.

As Shuard and Rothery state;

"If a child does not understand all that is implied by an illustration in one of his reading books his grasp of the story may not be substantially impaired. However, if he cannot read the graphic language of his mathematics book as well as the words and symbols, he will not be able to get the full message from the page"(8)

The tendency carried over from other subjects is to spend very little time in processing diagrammatic material and this is a vital part of any mathematics text which if given insufficient attention will lead to incomplete understanding.

Chapter 6

For the reasons indicated earlier the application of readability formulae to mathematics texts can produce rather misleading results. However they have been used in a number of pieces of research including the Effective Use of Reading survey (LUNZER) and Jones'," The Usability of Mathematics Textbooks as Found in Third Year Junior Classes." I suggest however that results obtained by these methods be treated with caution, and a degree of judgement made in their application. The work by Jones seems to suggest that the Fry readability graph, which is generally considered to give scores rather higher thsn they should be, in the case of mathematics texts give a realistic age score.

In answer to the problem of applying readability tests to mathematics texts Kane, Byrne and Hater in America have looked at the production of readability formulae specifically designed to be applied to mathematics texts. As with the formulae for prose they suffer from the disadvantage of being American biased, so that the language considered unfamiliar in America may be considered familiar here and vice versa, they also are rather difficult to get hold of in detail. One quoted by Shuard and Rothery is as follows;

Predicted Readability = -0.15A + 0.10B - 0.42C - 0.17D + 35.52

where

A is the number of words not on the Dale list of 3000 common words, and not on the list of Mathematics words known to 80% of children.

B is the number of changes from a mathematics token to a word token and vice versa.

C is the number of mathematical terms not on the list of mathematics words known to 80% of children, plus the number of symbols not on the list of symbols known to 90% of children.

D is the number of question marks.

It is suggested that ten samples of text be taken, each consisting of 400 tokens, mathematical and otherwise. The formula does not give a reading age but a score that enables comparison of texts, the higher the score the easier the text. The large sample required 400 tokens compared to 100 words for most formulae, would make this an extremely time consuming exercises to perform. It is also quite fortunate that the only people likely to use this formula are mathematics teachers since I am sure many other subject specialists would be overawed by a formula containing four variables, and each multiplied by a decimal.

In the course of the research underlying the production of these formulae Kane, Byrne and Hater studied the symbols familiar to 12 and 13 year old children and found that the only ones known to 90% of children were +, -, x, \$, ÷, %, C, and the numerals. If one replaces \$ and C by £ and p then the only ommision I find surprising is =, though pupils tend to use it with cavalier abandon even up to sixth form level, they are mostly familiar with it. I have yet to discover what they consider it to mean, since they say 'equals', or are baffled by the sheer stupidity of a teacher asking "What does that mean?" and pointing to the sign =, yet they are quite happy to use it as some form of connection, writing at sixth form level such things as

$$y = 2x \cos x$$

$$= 2 \cos x - 2x \sin x$$

when differentiating

or lower down the school

$$3y + 7 = 25$$
$$= 25 - 7$$
$$= 18$$
$$= 18 \div 3$$
$$y = 6$$

Showing perhaps an awareness of the necessary operatios but rather less awareness of the use of the equals sign.

Shuard and Rothery noted that the mentioned readabilty formula is not only not culture free, a problem that should be relatively easily rectified, but is also not curriculum free. This is because different curricula will place differing emphasis in the wide field of mathematics and each area of the subject has specialised terminology that should be relatively familiar to students of that area, and unfamiliar to others, also the oreder in which topics are studied will affect when new terminology is introduced, so that the variable that measures word difficulty, if it uses a word list technique, would need a different list for every course, and since different texts may also take topics in differing orders, alist for each text, totally defeating the object of readability formulae.

Though the application of readablity formulae is fraught with problems, whether or not they are specifically oriented towards mathematics, there are occasions when some measure of the pupils understanding of the text is important. An alternative that was described in the previous chapter is to use the cloze procedure, or at least an adaptation of it for mathematics. Research, particularly by Kane and Hater (14), has found that it is an acceptable measurement for

Mathematical English if a few adaptations are made. In their study a deletion rate of five was used and deleted words or symbols were replaced by underlines, so that;

Example: Use the quadratic formula to solve :

$$a^2 + \frac{1}{2} = 16$$

becomes

Example: Use the _______ formula to solve:

$$a\text{---} + \frac{1}{2} \ \_\_\_ \ 16$$

Because the ordering of mathematical language is not necessarily left to right there can be some confusion over the ordering of symbols and words in the text. This was in part overcome by using the convention that the ordering would be as the text is read, not necessarily as written. If the order was not clear from the text then an arbitrary decision was made and then that ordering was used consistently.

Five different tests, each beginning with a different word or symbol deleted, were used, for five different texts giving in all 25 experimental groups, and then a number of cross validation studies undertaken. They found that:

" There is a high correlation between cloze and comprehension tests. The results supported the hypothesisthat cloze tests adapted for Mathematical English passages are valid predictors of reading comprehension .... since scores on cloze tests of different forms do not differ significantly for a passage it appears that the representativeness of a passage can be determined by using any fifth word deletion pattern" (14)

In her earlier work Hater (1969) made a distinction between mathematical tokens and word tokens the latter being words as used in Ordinary English prose and the former a piece of mathematical symbolism. These include letters as used in algebra, numbers and signs so that 7 − 3 contains three tokens, 7, −, and 3, and so does 1/2, 1, /, and 2. The cloze procedure then adopted was to delete every fifth token, mathematical or word tokens, and replace word tokens with long lines, and replace word tokens with short lines. This is the procedure adopted in the previous example and allows a subject to tell whether a word token or mathematical token is required to fill a particular gap.

Chapter 7

If, as I think we must, we accept that the ease with which our pupils read the mathematics textbooks that they are expected to use is important then we must consider what is the most suitable method available at present to measure that readability. In the absence of a formula particularly designed for mathematics texts used in schools we are left with using one of the methods designed for use with ordinary English. The methods I decided to use are

i) the Flesch formulae, because it has been found to be one of the best,in terms of validity and reliability, of the formulae available for ordinary English. Though the Dale-Chall formula is generally considered to be more reliable than the Flesch in the particular application to mathematics, and in other technical areas, the use of a word list as a measure of the word difficulty is likely to make it much less valid since so much of the language is specific to the subject it is unlikely to be on any list of common words but previous usage should have made it familiar. For this reason the Flesch formula which uses number of syllables as the variable for measuring the difficulty of words, was used.

ii) the SMOG grading, because of its relative simplicity in use and because the final result is intended to correlate with a 100% comprehension of text. Though a 100% comprehension may be difficult to imagine I feel a comprehension level of greater than 50% is needed if these texts are intended to be used without too much teacher intervention. The accepted level for work with support and guidance is 75% so 50% seems to be a less than acceptable standard.

iii) the FORCAST formula, because this was specifically designed for use with technical manuals and is intended to be a measure of functional literacy so would seem appropriate for mathematics texts. It also has no measure of syntactic difficulty and could therefore be useful in the type of text that consists largely of multi-part questions. It must however be borne in mind that the text under consideration are in the lowest range of the formula's applicability.

iv) the Fry graph, because again it is quite easy to use and other workers in this area have considered that the higher gradings it tends to give are more appropriate for mithematics texts.

v) the Cloze procedure, because there is at least an adaptation of this that is considered appropriate to mathematics texts and as has been stated previously it is the only method that also measures the interaction of the reader with the text, and apart from personal and reported observation, it is the only measure that uses the text with children.

The texts I chose to consider are concerned mainly with the lower years of the secondary school and are intended to be used across the whole ability range. The reason for choosing lower school texts was that generally these cover the same sort of ground and are the most likely to be issued as class texts. From third year onwards there is much more setting in mathematics classrooms and a much more eclectic use of text books. For this reason they are used far more as a support of the teacher so it is less crucial that they should be easily read, though still desirable. The text considered are the first four S.M.P. letter books, A, B, C, D, which are intended to be covered in the first two years of a secondary school, the Oxford Comprehensive Mathematics Books 1 and 2, which are aimed at a similar age and ability range and are contempory to the S.M.P. letter books, and the levels 1 to 4 of the S.M.P. 11-16 which are aimed at the same population but are a much more modern development.

The original S.M.P. course broke new ground in a number of ways, not least of which was the approach to the material. The mathematics textbooks that preceded S.M.P. consisted mostly of examples with little text that the pupil was expected to read, it was more a case of repeating a technique on a large number of similar examples. The S.M.P. books however were intended to be used in a much different way and had a different content and layout. The original course was written for 'O' level students and as a demand grew the letter books were written for all abilities in the early years of secondary school with brighter children expected to cover the material at a faster rate and so be able to eventually cover the work in books X, Y, and Z and take 'O' level. The original

texts were in a large format and bound in hardback. Though attractive in appearance they suffered from more wear and tear than would be expected because of their size, so the letter series was printed in a smaller format with two books for each year with soft covers to keep down the cost. Unfortunately there are now a large number of pupils who do not use bags at all and the soft covers are an encouragement for the books to be rolled up and pushed into pockets and such like. This has meant that the books still suffer from more wear and tear than they ought to. It is however probably the textbook most commonly found in schools, even if it is not the most used. One reason why it is not used in the way originally intended is that experience has shown that pupils find difficulty in reading and understanding it. The print style is generally a Roman sans serif type face with 10 point print and 3 point leading for the majority of the text, so although the print is slightly small for the intended reader there is a very spacious appearence to the page. This is further emphasised by the frequent use of diagrams. The text is exclusively black on white, with red being used on diagrams, italic print is used for words the writers intend to emphasise, often in the defining of a word or phrase. Chapter headings and section headings are given in bold type, and the latter are in upper case only. The overall appearance of the text is clear and uncluttered. Though there is considerable use made of diagrams there are few that are purely decorative apart from the chapter headings, and by far the greatest proportion would be considered essential visual material.

The general style of the text is to introduce the topic with a section of exposition that may or may not include instructions to the reader. This often causes difficulties to pupils who do not feel that they need to do anything if there is not a specific exercise. There is quite a lot of use made of rhetorical questions and open questions which in themselves cause poblems since the weaker student is often unhappy about questions in mathematics that do not have a right or wrong answer. The exposition is followed by exercise that include a high proportion of non-routine problems and word problems. This has been a source of complaint in the past since there is very little routine to establish a procedure in the minds of the weaker student before they are asked to extend it. This in part was answered by the introduction of the supplementary books.

The Oxford Comprehensive Mathematics scheme has a single book for each year, Books 1 and 2 for the first two years are for all abilities and have yellow covers, Books 3,4, and 5 have blue covers for the 'O' level course and green covers for the C.S.E. course. The type face is slightly larger than S.M.P., 13 points, with a smaller leading of 1½ points. The print is once again fairly exclusively black on white but occasionally a rule is printed in black on grey with the change of colour intended to act as a signal. Unfortunately this is often not the case. Italic print is used for emphasis and heavy type is used for definitions e.g. The statements $x>1$, $x<8$ are **inequalities.** There is much more use of colour in diagrams with blue, yellow, and green being used. There is in all much more use made of illustrations with few pages being without at least one diagram. The diagrams fall into all three categories, essential, non-essential but related, and decorative, the latter consisting of cartoons, such as a question relating to the average of a batsmens scores being accompanied by a cartoon of a ferocious bowler approaching a batsman working out his average score, and more serious pictures, such as on the same page as the previous example a picture of a man trapping and weighing a squirrel accompanying a question on mean weights. There is nothing apart from the intelligence of the reader to distinguish between the essential and purely decorative illustration which could be quite a serious fault. The approach to the teaching material itself is also quite different. There is rarely any introductory material but the reader must discover the necessary information by proceeding through a series of exercises that include the expository material as well as instructions. There is quite a lot of use made of open and rhetorical questions. In spite of the use made of a large amount of illustrative material the actual text is quite dry. Again there is not a great deal of repetitive type exercises but the Books 1 and 2 and the C.S.E. Books have accompanying workbooks.

The 11-16 course takes an entirely different approach being work booklet based with a considerable amount of ancillary material, worksheets, review books, investigations and stretchers. The course is organised into four levels and with the last three levels there are extension booklets to supplement the main course. The course is intended for all abilities to work at their own pace, as with the earlier letter books, with the brighter child reaching level 4 after two years and the less able not proceeding beyond level 2. The scheme then leads into three levels of books catering for different levels of ability. The type face is not sans serif and is 12 points with 4 points leading again producing a very spacious look to the pages. There is however a considerable use made of a different type styles with some print done in a cursive script and some in an irregular written style that mimics handwriting. The printing is not exclusively black on white, there is some black on pale blue, some black on red, some red on white, but the majority in each booklet is black on white. The change of colour is not used for emphasis, merely as a change to maintain interest. As can be deduced from the earlier sentence there is considerable use of colour across the scheme but each booklet contains only one colour, though sometimes in different shades. This would, I imagine, keep down printing costs somewhat since each booklet requires only one colour apart from black. There is a great deal of use of illustration, some of it purely decorative but the majority in the other two classes. Much use is made of strip cartoons, to show the reader how something should be done, for example in level 1 the reader is shown how to build a framework of a cube in a series of pictures showing the job being done, or to give information and set questions. There is little formal exposition most of the exposition being done in the form of cartoons and instructions. When a difficult word is introduced a phonetic breakdown follows it to help the reader pronounce the word. e.g. This model is called a **regular octahedron.** (Say oct-a-heed-ron). As you will see from this example heavy type is used to introduce new terminology.

This form of presentation takes some getting used to for both teacher and pupil since so much of the required information is contained in a form that the reader is used to processing globally rather than in detail, but once familiar with it there is quite a good motivational aspect to the scheme and the student learns to process all illustrations carefully for the information they contain. The mathematics contained is either of a practical nature directly or put into some context for the reader. As is mentioned later the sentences were in general very short, even when not constrained to be short by the nature of the presentation, and questions of a conditional nature are not used. e.g. Alan and his family are staying at the Europa Hotel. Alan sees that it has 8 floors. There are 30 windows on each floor. How many windows are there altogether in the Europa Hotel?

In more traditional texts this would be asked something like ' If a Hotel has 8 floors, and 30 windows on each floor, how many windows does it have altogether?'

The difficulties in applying the techniques for measuring readability become even clearer when one attempts to use them on these textbooks. The Flesch formula proved to be the most difficult to apply, as was expected. The initial requirement of passages of 100 words taken every ten pages was far from easy to satisfy since many pages in all of the texts under consideration contained little or no text and those that contained more text also contained a large number of symbols. I finally decided to count the symbols as one would the words they replaced wherever possible, so that for example cm. was read as centimetre, = as equals etc. This is not entirely satisfactory since it presumes knowledge of the meaning of the symbols and in the case where one symbol can take a number of meanings, + for example, only one can be considered, but seemed to be the only satisfactory way of proceeding since symbolic language is such a vital part of the subject. Fortunately at this level the prose generally dominates over the other types of language apart from in exercises, for this reason it proved easier to apply the technique to the more junior books, to S.M.P. Book A rather than Book D, and level 1 rather than level 4 in the 11-16 course for example.

Calculation of the S.M.O.G grading was much easier as was indicated earlier. There was much less difficulty in counting the polysyllabic words than counting the number of syllables and less samples were required, so the gathering of the initial data was less time consuming. The calculation itself was obviously easier and in general lead to a readability grading higher than the other techniques. This is noticably not true for two of the S.M.P. 11-16 levels, probably due to the short sentences. Where in all the other text books 10 sentences contained well over 100 words in these on only one occasion did more than 100 words occur, and that was only 118, and the mean was 96.5.

The FORCAST grading did not prove to be any easier than using the SMOG grading though it as possible to take into account labelling of diagrams in a more convenient manner. The lack of a sentence length variable, explicit or implicit, meant that the short sentences in the 11-16 course did not affect the grading and this was the only measure where this series did not achieve a significantly lower reading grade.

The Fry graph was easy to use since the collection of data had already been done for the Flesch formula, the words per sentence being the inverse of the number of sentences per 100 words (though the averages of these two are not inverses) and since it is easier to count the former than the later this is the way the data was originally collected for the calculation of the Flesch formula. The added information contained in the graph as to the reason why a particular text had the given grading proved interesting.

In the application of cloze procedure it is desirable if the text is an exact copy of the original with lines where words have been deleted. When I considered the texts in question this proved to be impossible. Though it was possible to copy the texts with spaces where words had been deleted the length of the gap was too small for the reader to insert a word of their choice, and was proportional to the length of word deleted. This made it difficult to distinguish between a gap left for a mathematical token and one left for a word token. Consequently I made typed copies of the necessary passages with diagrams inserted in the correct positions, a deleted word token was shown by _______ and a mathematics token by ___. Each text had three passages taken from it, one from the beginning, middle and end, each of 250 tokens with a deletion rate of 1 in 5. The pupils used were two mixed ability second year tutor sets with a total of 50 pupils aged from 12 years 3 months to 13 years 3 months. Before completing the cloze procedure questions they were given a reading test to ascertain their mean reading age. Their mean age was 12.70 years (standard deviation 0.29) and their mean reading age 12.36 (standard deviation 1.96) so all of the texts under consideration are aimed at that population. The results obtained are shown below.

TEXT BOOK	FLESCH	SMOG	FORCAST	FRY	CLOZE	S.D.
S.M.P. Book A	12.59	14	13.6	12	49.2	13.5
S.M.P. Book B	12.73	14	13.5	12	42.5	13.9
S.M.P. Book C	12.02	13	13.5	12	45.3	13.4
S.M.P. Book D	12.74	15	13.7	12	44.2	12.8
S.M.P. 11-16						
level 1	11.75	13	13.4	8	58.4	16.0
level 2	12.3	13	13.5	8	54.6	14.5
level 3	10.42	10	13.1	7	56.8	12.8
level 4	11.68	11	13.8	9	49.3	13.0
Oxford Comp.						
Book 1	11.97	13	13.7	10	40.6	12.5
Book 2	12.14	13	14.13	11	42.4	13.4

When one considers the readability in mathematics one must first be constrained by the content, however easy to read a text is there is no point in purchasing it if it does not fulfil your requirements. Nor is it possible to look at individual texts in isolation since one logically must buy the whole course or at least a unit to cover a couple of years so one must consider all the texts that make up your requirements. Experience of using the above texts have shown that the S.M.P. 11-16 is the easiest to read for understanding by the target population and the Oxford Comprehensive Mathematics the most difficult. All the readability measures gave the 11-16 as the easiest to read, though not always with any great significance, but the other two showed in all but the cloze procedure little difference. This seemed to be the best discriminator of the five tried and reflects experience quite well. The only text that approaches the score required to use the material at an independent level, 55% and above, is the 11-16 course. The S.M.P.

letter books would appear to be usable at the instructional level, 40% to 55%, where additional support is required from some external source and the Oxford comprehensive scheme is perilously close to being at the frustration level. Against its good level of discrimination is the time taken to administer and mark the tests and the necessity of using the texts with a class.

When using the Fry graph all of the texts showed that there was a tendency for shorter sentences with long words, in the 11-16 course very short sentences. This may give a lower than expected grading when a text consisting of a large number of exercises, which tend to have relatively short sentences, is investigated. The use of a graph rather than a formula is not a great advantage, if a user is capable of working out an average then he should also be capable of working out a formula. However the use of a formula tends to give a rather spurious accuracy to the results and the unwary may put too much faith in the accuracy of the result. Though the results for the Flesch formula are quoted to 2 decimal places the actual formula gives results to a much larger number, at least six decimal places, and when one considers that the standard deviation on the reading ease score was between 8 and 12, on scores offrom 70 to 90, there can be little faith in even the first decimal place. There is no danger of this occurring if the Fry graph is used, and the additional information is quite useful. The contention that the Fry graph gives unreasonably high gradings does not appear to be born out by results shown here.

Experience would suggest that the figure given by the FORCAST formula to be quite reasonable for the S.M.P. letter series and the Oxford Comprehensive Mathematics series but it does not show up the relative ease of the 11-16 booklets. The lack of a syntax variable in the form of a sentence length measurement reduces its validity rather too much, particularly with material for younger readers. However this does give the best correlation with the cloze procedure scores, of 0.61.

Though the use of a formula to give a judgement must, on this evidence, be considered unreliable the use of one such as the Flesch formula, or the Fry graph, does concentrate the mind quite well on the readability of the text. By taking the frequent measurements throughout a book one becomes more aware of the problems that may arise in using the text with a class and can make a better judgement as to its readability than if one simply read through the text. The relative simplicity of the 11-16 text shows that at least some of the writers of modern material for use in classrooms are considering the ease the students have in reading the material. It is a matter that has been ignored far too long by writers of texts and by the teachers who issue them to students. For far too long the only criteria has been the content, it is clear that the presentation both in looks and style are important and should be given more thought.

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