Developing multivariate time series models to examine the interrelations between police enforcement, traffic violations, and traffic crashes

Developing Multivariate Time Series Models to Examine the Interrelations between Police Enforcement, Traffic Violations, and Traffic Crashes

Mingjie Feng[a,b,c], Xuesong Wang[a,b*], Mohammed Quddus[c]

[a] *School of Transportation Engineering, Tongji University, Shanghai, 201804, China*
[b] *The Key Laboratory of Road and Traffic Engineering, Ministry of Education, Shanghai, 201804, China*
[c] *School of Architecture, Building and Civil Engineering, Loughborough University, Loughborough LE11 3TU, United Kingdom*
** Corresponding author. Tel: +86 21 69583946. E-mail address: wangxs@tongji.edu.cn*

Abstract

Safer roads and police enforcement are closely associated since the latter directly encourages road users to improve their behavior by complying with basic traffic rules and laws. Understanding the relationships between police enforcement, driving behavior, and traffic safety is a prerequisite for optimizing enforcement strategies. However, there is a dearth of research on the contemporaneous relationships between these three parameters. Using multivariate time series techniques, this study provides an in-depth analysis of contemporaneous relationships and dynamic interactions among police enforcement, traffic violations, and traffic crashes. The amount of police patrol time per day was used as a surrogate measure for police enforcement intensity. A vector autoregressive (VAR) model was first used to examine the influences of exogenous factors including weather conditions and holidays. Based on the findings of the VAR model, a structural vector autoregressive (SVAR) model was developed to determine contemporaneous effects; the Granger causality test was employed to detect any *dynamic* interactions between the three parameters. The results indicated that traffic crashes and violations had weekly variation and were significantly impacted by holiday and weather conditions, while police patrol time was not impacted. A contemporaneous negative impact of police patrol time was found in traffic crashes: each 1% increase in police patrol time was associated with a 0.15% decrease in contemporaneous crash frequency. The findings from the Granger causality test demonstrated that police patrol time did not Granger-cause traffic crashes, but crashes Granger-caused police patrol time. The significant bidirectional interactions in conditional variances of police enforcement, traffic violations, and traffic crashes further confirm the necessity to analyze the three simultaneously. The findings of this study are expected to assist the relevant traffic authorities in devising policies and strategies such as optimal police patrol scheduling.

***Keywords*:** Police Enforcement; Traffic Safety; Time Series Model; Granger Causality; Structural Vector Autoregressive Model

1. Introduction

Road traffic injury has become the 8th global cause of death in 2016, a fact that emphasizes the urgency of improving traffic safety (World Health Organization, 2018). Traditionally, countermeasures promoting traffic safety involve three aspects: engineering, education, and enforcement (Rothengatter, 1982). A great deal of engineering research has focused on the impact on traffic safety of roadway geometric features, traffic control strategies, and traffic operational characteristics by constructing safety performance functions (Lord and Mannering, 2010; Savolainen et al., 2011), resulting in the publication of well-known guidelines for improving traffic safety from the perspective of engineering (Administration, 2009; Bonneson, 2010; Harwood et al., 2010). However, research on traffic enforcement and its corresponding applications are scattered and require further investigation.

Traffic enforcement is implemented by relevant government departments and applied to road users, with the purpose of maintaining desirable traffic behavior through a process of surveillance, prosecution, and penalization (Rothengatter, 1982). The traffic violation, that is, an undesirable driver behavior that is illegal, can be understood as a link between police enforcement and crashes, as violations are influenced by police enforcement and also have potential to cause crashes. During the traffic safety analysis work conducted by Shanghai's police department, the relationships between police enforcement, traffic violations, and traffic crashes were found to be complex, as well as endogenous to each other, as demonstrated in Figure 1 (the causalities presented by arrows need to be investigated).

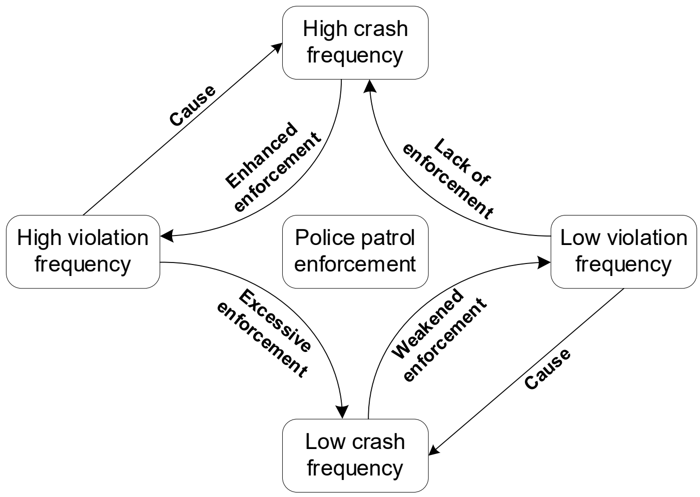


Figure 1. Possible relationships between police enforcement, traffic violations, and crashes

As Figure 1 illustrates, there arise multiple combinations of high and low crash frequency, and high and low traffic violation frequency, with different potential causalities represented by arrows. For example, a road section with a high crash level and few violations might indicate a lack of police enforcement. On the other hand, a road section might have a high frequency of crashes as well as a high frequency of violations. This combination could imply that more frequent violations cause more frequent crashes, or it could imply that the high crash level motivates the traffic police to enhance their enforcement activities and subsequently apprehend more violators. These underlying interrelationships are contemporary and are referred to as contemporaneous relationships in this study.

Apart from the contemporaneous relationships, police enforcement, traffic violations, and traffic crashes are endogenous to each other, resulting in dynamic interactions between them. For instance, if freeway segments with high historical traffic crashes have attracted tougher police enforcement, the enhanced enforcement will result in further interaction between the police enforcement and traffic crash variables. While stricter enforcement may reduce contemporaneous crashes, it may also be a response to the higher historical crash rates (Makowsky and Stratmann, 2011). The interactions between the current value and the lags of police enforcement, traffic violations, and crashes are referred to in this study as *dynamic* interactions.

Considering the contemporaneous relationships and dynamic interactions among police enforcement, traffic violations, and traffic crashes, these variables are best studied simultaneously. However, existing research has mainly investigated the impact of police enforcement either on crash data or on traffic violations, while it has rarely studied the three interconnected parameters simultaneously, nor has it considered endogeneity. Not controlling for the endogeneity may lead to a bias, and thus an inaccurate estimate of the effect of police enforcement.

Time series data is a common data structure obtained when measuring the level of police enforcement with surrogate variables such as the amount of police patrol time (Tay, 2005; Makowsky and Stratmann, 2011; Terrill et al., 2016). Yet most previous studies have used cross-sectional models to analyze the effects of police enforcement on traffic safety within a single period. When applying cross-sectional models to time series data, the results can be misinterpreted because time series data usually suffer from serial correlation. To handle this problem, time series modeling is recommended to reveal the complex interrelationships that evolve over time (Lavrenz et al., 2018). Compared to traditional cross-sectional models, causal-inference modeling or time series modeling based on time series data has shown better causality/inference capacity (Mannering et al., 2020).

This study aims to examine the contemporaneous relationships and potential dynamic interactions between police enforcement, traffic violations, and traffic crashes using multivariate time series modeling techniques with *daily* as the temporal unit. The amount of police patrol time is used as a surrogate measurement for police enforcement intensity. A vector autoregressive (VAR) model is used to examine the influences of exogenous factors including weather conditions and holidays. A structural vector autoregressive (SVAR) model is then developed based on the VAR findings to determine contemporaneous relationships; and the Granger causality test is employed to detect any dynamic interactions between the three variables.

In addition to weather conditions and holidays, however, a plethora of other exogenous factors may influence the endogenous variables, and the impossibility of accessing all such factors can lead to unobserved heterogeneity (Mannering et al., 2016). Unobserved heterogeneity can, in turn, result in heteroscedasticity, a common phenomenon in time series modeling in which the variance of error terms is time-dependent. In such circumstances, a multivariate generalized autoregressive conditional heteroscedasticity (GARCH) model is also utilized to analyze the relationships between the variances of errors of the endogenous variables.

2. Literature review

2.1 Relations between police enforcement, driving behavior, and traffic safety

The majority of previous studies have focused on the impact of police enforcement either on traffic safety or on driving behaviors, resulting in a dearth of research on the contemporaneous relationships between these three parameters as well as their inherent endogeneity. Some studies have used the number of traffic citations as a measure of traffic enforcement intensity to investigate the impact of enforcement on traffic safety. For instance, Terrill et al. (2016) used ordinal least squares to examine the relationship between citations and crashes; finding that the number of citations correlated with the number of crashes, they concluded that citing traffic violators is a preventive measure for crashes. Makowsky and Stratmann (2011) used municipal budgetary shortfalls as an instrumental variable to control for endogeneity while identifying the effects of traffic citations on traffic safety. They found that motor vehicle crashes and crash-related injuries decrease as the number of tickets written increases. Rezapour Mashhadi et al. (2017) classified traffic citations into ten categories and developed a negative binomial model to identify the causal impact of various violations on crash frequency. Their results showed that an increase in speeding and seat belt citations significantly reduced the number of crashes.

In addition to traffic citations, other indices have also been selected as surrogates to analyze the effect of police enforcement. By developing a Poisson regression model, Tay (2005) found that the percentage of drivers apprehended and the number of breath tests administered for drunk driving had a negative impact on the number of serious injury crashes. Rezapour et al. (2018) used allocated enforcement budget, number of sworn officers, and time spent on patrolling as representatives of enforcement. The researchers employed simple linear regression models to analyze the influence of the three enforcement indicators on the highway fatality rate. Although a limitation of their study was that data was only obtained from eight states, i.e., their regression models were comprised of only eight observations, they found that time spent on patrolling was the best indicator of crashes. Abaza (2018) analyzed the relationship between highway police patrol time and traffic crashes using binary logistic models and Poisson models, and found that the number of crashes decreased with police presence.

Connections between traffic enforcement and both traffic violations and aggressive driving behavior have been shown in a number of studies (Elliott and Broughton, 2005; Bendak, 2005; Walter et al., 2011; Montella et al., 2015). Stanojević et al. (2013) compared the attitude and behavior of drivers in two regions, one with traffic enforcement and the other without, and found that the lack of enforcement resulted in an increase in speeding, drunk driving, the committing of aggressive and ordinary violations, and failing to wear seat belts. Using a survey on speed choice, Ryeng (2012) found that substantially increasing enforcement has significant reducing effects on individual speed choice. By combining roadside surveys and web surveys, Johnson (2016) concluded that a high-visibility enforcement campaign can reduce driving after drinking. Work by Pantangi et al. (2019) provides some evidence that a high-visibility enforcement program also has the ability to decrease the likelihood of speeding behavior. Findings reported by Stanojević et al. (2018) indicate that an absence of police enforcement was a significant predictor of aggressive and risky driving, leading the authors to suggest spending limited resources on police enforcement rather than on publicity campaigns.

Most of the aforementioned research has found that enhancing police enforcement can reduce traffic crashes or undesirable driving behavior, while neglecting to consider endogeneity can lead to inaccurate estimation of the effect of police enforcement.

2.2 Time series modeling

The widely used cross-sectional models in the above research consider the effect of enforcement on traffic safety within a single period. However, time series data is a common data structure obtained when assessing the effects of enforcement (Tay, 2005; Makowsky and Stratmann, 2011; Terrill et al., 2016; Abaza, 2018). In the previously mentioned study by Makowsky and Stratmann (2011), both crash and enforcement data were aggregated to a monthly level and collected for 22 months. Tay (2005) used the percentage of drivers apprehended and the number of breath tests to represent enforcement intensity, and included in the study's Poisson regression model a series of dichotomous variables to manage the monthly seasonal effects on crashes; the results were found to be statistically significant. Since serial correlation is often present in time series data, however, using cross-sectional models may cause inefficient estimates of the parameters (Quddus, 2008). Therefore, single-equation autoregressive methods such as autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) models have been adopted in traffic safety research, especially for determining the effects of an intervention (Vernon et al., 2004; Masten, 2007; Neyens et al., 2008; Carnis and Blais, 2013; Lavrenz et al., 2018).

The vector autoregressive (VAR) model was derived (Sims, 1986) from univariate time series models to account for the interactions among multiple endogenous time series variables, but its application in traffic safety research is rare. Beenstock and Gafni (2000) proposed a theoretical framework based on the VAR model to investigate the effect of national policy on the downward trend in Israel's crash rate. Serhiyenko et al. (2014) applied a VAR model to four pedestrian crash injury levels and found significant effects of time trends and seasonal variations on pedestrian crashes. However, the VAR model has a limitation, which is the inability to capture the contemporaneous relations between endogenous variables. To make up for this shortcoming, the structural vector autoregressive (SVAR) model was developed and has been widely used (Pfaff, 2008; Hu et al., 2018). Because they account for endogeneity, the VAR and SVAR models are helpful for examining the dynamic interactions and contemporaneous relations between police enforcement, traffic violations, and crashes.

Theoretically, the error terms of endogenous variables in the above time series models are assumed to be identically and normally distributed with zero mean and time-invariant variance. In reality, however, most time series data have heteroscedasticity; that is, the variance of error terms is time-dependent (Tsay, 2013). In financial analysis, forecasting the future volatility (i.e., variance) of a series is usually vital to assess the risk associated with certain assets (Lütkepohl, 2005). Ignoring the heteroscedasticity can cause inferior predictive performance. To this end, the generalized autoregressive conditional heteroscedasticity (GARCH) model (Bollerslev, 1986), derived from the autoregressive conditional heteroscedasticity (ARCH) model (Engle, 1982), has been widely applied in economics in order to allow past residuals and past conditional variances to be predictors of the current conditional variances. This method of risk assessment and prediction has also found application in traffic safety management. Ye et al. (2012) and Quddus (2018), for example, used an extension of the univariate GARCH model to improve traffic safety prediction.

In the case of multiple time series, especially those series that have a close relationship, it can be intuitively conjectured that the volatility of one series has an impact on the volatility of another series (Lütkepohl, 2005). In such cases, multivariate extensions of ARCH and GARCH models are of more interest. Commonly used multivariate models for conditional heteroscedasticity include the exponentially weighted moving average (EWMA) model (Tsay, 2013), the BEKK model (Engle and Kroner, 1995), and the dynamic conditional correlation (DCC) model (Tse and Tsui, 2002). Readers are referred to Asai et al. (2006) and Bauwens et al. (2012) for more detail.

3. Data preparation

3.1 Endogenous variables

This study focused on the entire freeway system in Shanghai, China. Police patrolling time, traffic violation, and crash data for the year 2018 were collected from the Shanghai Traffic Police Department. Patrol time was used to represent police enforcement intensity: all traffic police have a portable Global Positioning System (GPS) electronic equipment that updates their positions' information every five seconds as they patrol freeways. Patrol time was thus extracted from officers' positioning data, and calculated by multiplying the number of updated GPS coordinates.

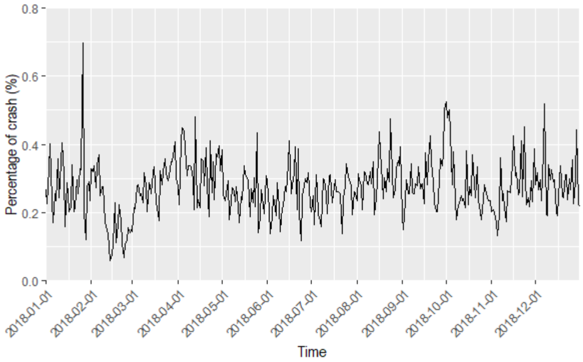
Traffic violations were classified into two categories according to their sources. The first is the technology-detected violation, which is identified automatically by the electronic policing system. The technology is installed along certain segments of the freeway system, and operates 24 hours a day. The second is the police-detected violation, which is identified by traffic police officers during their daily patrolling activities. Both technology-detected and police-detected violations are considered to be only samples of violations, as it is not possible to capture all the violations on the entire freeway system. The difference is that technology-detected violations are captured at fixed locations and monitored the whole day, while police-detected violations are captured on random sites, and their detection is influenced by the time of traffic police officers spent on patrol. Therefore, technology-detected violations are more representative of all driver violations, while police-detected violations are more reflective of traffic police enforcement intensity.

Traffic crashes, technology-detected violations, police-detected violations, and police patrol time were set as endogenous variables since they are dependent on each other. These four time series variables were obtained from 1 January to 31 December 2018, and were aggregated at the daily level, that is, the number of crashes, number of detections of the two types of violation, and number of hours police spent patrolling Shanghai's freeways were counted for each day. Table 1 gives the descriptions or units of the four variables. Of the 365 observation days, the mean crash frequency was 106.8, the mean frequencies of technology-detected violations and police-detected violations were 2,965 and 3,523 respectively, and the average police patrol time was 340.24 hours.

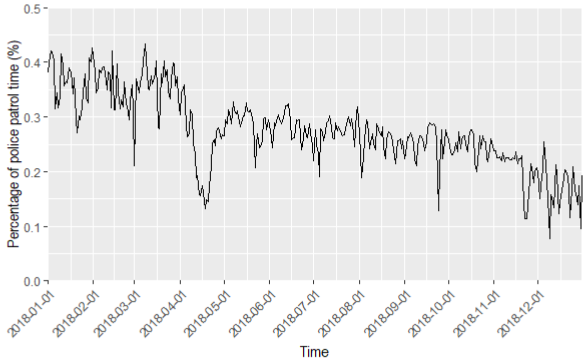
Table 1. Descriptions or units of four endogenous variables

Endogenous variable	**Description or unit**
Traffic crashes	Daily number of traffic crashes
Technology-detected violations	Daily number of technology-detected violations
Police-detected violations	Daily number of police-detected violations
Police patrol time	Daily number of hours spent on patrolling freeways

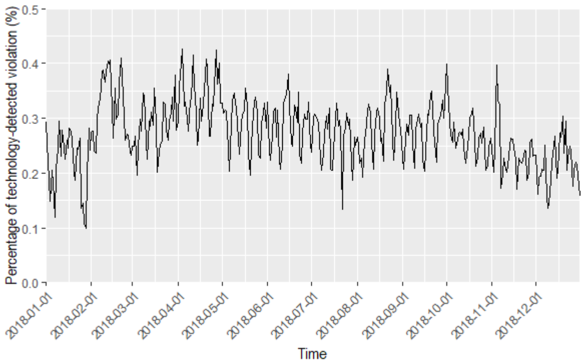
Figure 2 shows the daily distribution of the four endogenous variables, presented in percentages because of data confidentiality. Officers' patrol time shows an obvious downtrend over the year, which was the result of staff shortage. For the same reason, police-detected violations also shows a downward trend. Traffic crashes and technology-detected violations were relatively stable.



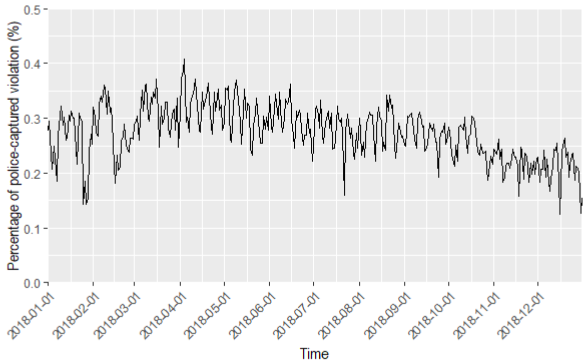
(a) Traffic crashes



(b) Police patrol time



(c) Technology-detected violations



(d) Police-detected violations

Figure 2. Daily distributions of traffic crashes, police patrol time, technology-detected violations, and police-detected violations

3.2 Exogenous variables

In addition to the four endogenous variables, holiday and weather conditions were also obtained for the study period. These two conditions were used as exogenous variables because they may affect the endogenous variables, but are not directly affected by them.

Holiday was selected as a variable because additional vacation travel results in increased traffic volume, making the presence of a holiday an important factor in traffic safety and enforcement. In this study, seven statutory public holidays, periods that encompass 27 days in total, were taken into consideration. They are New Year's Day, Spring Festival, Qingming Festival, Labour Day, Dragon Boat Festival, Mid-Autumn Festival, and National Day.

Weather is also a significant factor contributing to traffic crash frequency (Xing et.al., 2019). The Shanghai daily weather in the study period from January 1 to December 31, 2018, was obtained from an online weather database (http://www.tianqihoubao.com/lishi/shanghai.html), and was initially classified as sunny, cloudy, rainy, and snowy. Pavement conditions, however, are closely related to weather conditions and are important in traffic safety. Ahmed et al. (2011) observed that the increased crash risk in the snowy season was related to snowy, icy, and slushy pavement conditions; in 2012, Ahmed et al. (2012) found that the likelihood of a crash could, indeed, be doubled in these conditions. Based on numerous crash samples, Sun et al. (2019) also found that icy pavement was one of the primary causes of Chinese freeway tunnel crashes.

The winter temperature in Shanghai is rarely below freezing. Consequently, the freeway pavement conditions on snowy days (mostly light snow) with temperatures above 0℃ are more similar to those on rainy days, while the pavement on snowy days with temperatures below 0℃ is more consistent with the snowy and icy conditions in previous research. In Shanghai freeways' 2018 data, the average traffic crash frequency on icy days was 1.78 times the frequency on sunny days, indicating the significant influence of the icy pavement. Therefore, days with snow were classed as snowy if the temperature was over 0℃; if the temperature was below 0℃, both snowy and rainy days were classed as icy. In 2018, 2 of Shanghai's 5 snowy days, but none of its 105 rainy days, were below 0 . Consequently, 105 days were classified as rainy, 3 as snowy, and 2 as icy. Since icy and snowy pavement is a significant hazard for traffic safety, these conditions were included in the model despite their small sample size. Descriptive statistics on holiday and weather conditions are shown in Table 2.

Table 2. Descriptive statistics of holiday and weather conditions

Variables	Description	Summary Statistics
Holiday	0: not holiday	Observations: 338
	1: holiday	Observations: 27
Weather	1: sunny	Observations: 50
	2: cloudy	Observations: 205
	3: rainy	Observations: 105
	4: snowy	Observations: 3
	5: icy	Observations: 2

Note: As observations were made once per day, the total number of holiday observations is 365, as is the total number of weather observations.

4. Methodology

4.1 Vector autoregressive model

The vector autoregressive (VAR) model was first introduced by Sims (1986) to examine the dynamic interactions among interrelated time series data. VAR models include an equation for each variable, which explain each variable's evolution with its own lags and the lags of other variables, so that all the variables are symmetrically treated as endogenous (Alsaedi and Tularam, 2019). The VAR model can be expressed as:

$$\boldsymbol{y}_t = \boldsymbol{A}_1\boldsymbol{y}_{t-1} + \cdots + \boldsymbol{A}_p\boldsymbol{y}_{t-p} + \boldsymbol{B}\boldsymbol{x}_t + \boldsymbol{\varepsilon}_t \quad t = 1, 2, \cdots, T \tag{1}$$

where y_t is a vector of k endogenous variables, $\boldsymbol{A}_p$ is a matrix of k autoregressive coefficients at lag p, x_t is a vector of d exogenous variables, and $\boldsymbol{B}$ is a matrix of d coefficients on x_t. ε_t is a $(k \times 1)$ vector of reduced form shocks or error terms, which are assumed to contain no serial correlation and which are identically and normally distributed. T is the sample size. For clarity, Eq. (1) can also be expressed as:

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{k,t} \end{pmatrix} = \boldsymbol{A}_1 \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \\ \vdots \\ y_{k,t-1} \end{pmatrix} + \boldsymbol{A}_2 \begin{pmatrix} y_{1,t-2} \\ y_{2,t-2} \\ \vdots \\ y_{k,t-2} \end{pmatrix} + \cdots + \boldsymbol{A}_p \begin{pmatrix} y_{1,t-p} \\ y_{2,t-p} \\ \vdots \\ y_{k,t-p} \end{pmatrix} + \boldsymbol{B} \begin{pmatrix} x_{1,t} \\ x_{2,t} \\ \vdots \\ x_{d,t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{k,t} \end{pmatrix} \quad t = 1, 2, \cdots, T \tag{2}$$

Selecting the optimal lag length p before constructing the VAR model is important because a trade-off is always involved in the selection of the number of lags. Specifically, too few lags may lead to poor model specification, while too many may lead to the loss of degrees of freedom (Alsaedi and Tularam, 2019). The Akaike information criterion (AIC) was used to determine the optimal lag length, as suggested by Lütkepohl (2005).

Establishing a stationary time series is a precondition of developing a VAR model, as the estimation of the econometric model based on non-stationary time series may cause misleading results (Koitsiwe and Adachi, 2015). Before developing the VAR model, the augmented Dickey-Fuller (ADF) test (Dickey and Fuller, 1981) was used to check the stationarity properties of the endogenous variables. The time series is believed to be stationary when the ADF t-statistic is lower than the Mackinnon critical value at the 5% significance level.

4.2 Granger Causality Test

An advantage of the VAR model is its ability to perform the Granger causality test to examine the direction of causality among the endogenous variables (Granger, 1969). Granger causality assesses whether one variable precedes another in a time series. If the lags of a variable, for example Y_2, help predict another variable Y_1, then Y_2 is said to Granger-cause Y_1 (Alsaedi and Tularam, 2019). This method requires estimation of the multivariate VAR model (Damos, 2016). Taking two variables as an example:

$$Y_{1,t} = \alpha_{1,1}Y_{1,t-1} + \cdots + \alpha_{1,p}Y_{1,t-p} + \varepsilon_{r,t} \quad t = 1, 2, \cdots, T \tag{3}$$

$$Y_{1,t} = \alpha_{1,1}Y_{1,t-1} + \cdots + \alpha_{1,p}Y_{1,t-p} + \alpha_{2,1}Y_{2,t-1} + \cdots + \alpha_{2,p}Y_{2,t-p} + \varepsilon_{u,t} \quad t = 1, 2, \cdots, T \tag{4}$$

where $\varepsilon_{r,t}$ and $\varepsilon_{u,t}$ are uncorrelated disturbances-residuals and p is the maximum number of lagged observations included in the VAR model. $\alpha_{1,n}$ and $\alpha_{2,n}$ $\left(n=1,\ldots,p\right)$ are coefficients of the model. If $\alpha_{2,1}=\alpha_{2,2}=\cdots=\alpha_{2,p}=0$, variable Y_2 does not Granger-cause variable Y_1; otherwise, variable Y_2 Granger-causes variable Y_1. To identify the significance of Granger causality, the Wald-test has been widely used:

Null hypothesis: $\alpha_{2,1}=\alpha_{2,2}=\cdots=\alpha_{2,p}=0$

Alternative hypothesis: $\alpha_{2,j}\neq 0$, for at least one value of j, $1\leq j\leq p$

The χ^2 statistic is:

$$S=\frac{T\left(RSS_0-RSS_1\right)}{RSS_1}\sim\chi^2\left(p\right) \tag{5}$$

where $RSS_0=\sum_{t=1}^{T}\hat{\varepsilon}_{r,t}^{\,2}$ and $RSS_1=\sum_{t=1}^{T}\hat{\varepsilon}_{u,t}^{\,2}$. If S is greater than the χ_p^2 critical value, the null hypothesis is rejected, that is, variable Y_2 Granger-causes variable Y_1; otherwise, the null hypothesis cannot be rejected. The Granger causality test was used in this study to examine the dynamic interactions that may exist among police enforcement, traffic violations, and crashes.

4.3 Structural vector autoregressive model

The Granger causality test can assess the influence of the lags of many variables, but has limitations in measuring contemporaneous relationships among endogenous variables. These relationships are hidden in the error term $\boldsymbol{\varepsilon}_t$ of the VAR model and cannot be interpreted directly. To identify the contemporaneous relationships, structural restrictions are required (Lütkepohl, 2005). The structural vector autoregressive (SVAR) model can be expressed as follows:

$$\boldsymbol{A}_0\boldsymbol{y}_t=\boldsymbol{\Gamma}_1\boldsymbol{y}_{t-1}+\cdots+\boldsymbol{\Gamma}_p\boldsymbol{y}_{t-p}+\boldsymbol{u}_t\quad t=1,2,\cdots,T \tag{6}$$

To estimate the SVAR model, the reduced form is determined by multiplying Eq. (6) by an inverse matrix $\boldsymbol{A}_0^{-1}$:

$$\begin{aligned}\boldsymbol{y}_t&=\boldsymbol{A}_0^{-1}\boldsymbol{\Gamma}_1\boldsymbol{y}_{t-1}+\cdots+\boldsymbol{A}_0^{-1}\boldsymbol{\Gamma}_p\boldsymbol{y}_{t-p}+\boldsymbol{A}_0^{-1}\boldsymbol{u}_t\\&=\boldsymbol{A}_1^{*}\boldsymbol{y}_{t-1}+\cdots+\boldsymbol{A}_p^{*}\boldsymbol{y}_{t-p}+\boldsymbol{\varepsilon}_t\qquad t=1,2,\cdots,T\end{aligned} \tag{7}$$

where $\boldsymbol{A}_j^{*}=\boldsymbol{A}_0^{-1}\boldsymbol{\Gamma}_j$ $\left(j=1,...,p\right)$ and $\boldsymbol{\varepsilon}_t=\boldsymbol{A}_0^{-1}\boldsymbol{u}_t$, so the reduced form shocks $\boldsymbol{\varepsilon}_t$ are linear combinations of the structural shocks $\boldsymbol{u}_t$. Matrix $\boldsymbol{A}_0$ captures the contemporaneous effects of one variable on another, that is,

$$A_0 = \begin{pmatrix} 1 & a_{12} & \cdots & a_{1k} \\ a_{21} & 1 & \cdots & a_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ a_{k1} & a_{k2} & \cdots & 1 \end{pmatrix} \tag{8}$$

To make the SVAR model identifiable, in other words, to ensure precise inference is possible, it is necessary to impose restrictions such as assuming that specific endogenous variables have no contemporaneous effects on the other endogenous variables, or that some contemporaneous effects can be determined based on experience. If there are k endogenous variables, $k(k-1)/2$ restrictions are required to make the SVAR model identifiable. Cholesky decomposition is the most commonly used restriction method, in which A_0 is identified as a lower triangular matrix. When adopting Cholesky decomposition, the ordering of endogenous variables requires special attention.

In this study, there are four endogenous variables, so $k=4$. Because the main purpose of this study is to investigate the influence of police enforcement on traffic safety, the endogenous variables are ordered as follows: police patrol time, technology-detected violations, police-detected violations, and traffic crashes. In this way, the impact of patrol time and violations on crashes can be explored. Following Cholesky decomposition, elements above the diagonal in A_0 are set to be zero, that is, $a_{12}=a_{13}=a_{14}=a_{23}=a_{24}=a_{34}=0$ (Lütkepohl, 2005). Matrix A_0 is thus expressed as:

$$A_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 \\ a_{41} & a_{42} & a_{43} & 1 \end{pmatrix} \tag{9}$$

The off-diagonal elements in this matrix capture the contemporaneous effects of one variable on another. For instance, a_{21} measures the contemporaneous impact of police patrol time on technology-detected violations, whereas a_{12} measures the contemporaneous impact of technology-detected violations on police patrol time. All other elements in A_0 are defined analogously.

4.4 Test for heteroscedasticity

In VAR models, the errors $\varepsilon_t (t=1,2,\ldots,T)$ are assumed to be identically and normally distributed with a zero mean and a time-invariant covariance matrix. However, autocorrelation may exist in the squared errors; this conditional heteroscedasticity makes the errors still serially dependent. To test for conditional heteroscedasticity, the ARCH Lagrange Multiplier (ARCH-LM) test and the Ljung-Box Q test are frequently employed.

4.4.1 ARCH-LM test

The Lagrange multiplier test procedure was introduced by Engle (1982). Define the return residual series as $e_t = y_t - \hat{y}_t \ (t=1,2,\ldots,T)$, whereupon the regression is:

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \cdots + \alpha_m e_{t-m}^2 + z_t \tag{10}$$

where z_t is a white noise process. The null hypothesis of the ARCH-LM test is that there is no conditional heteroscedasticity in the return residuals; that is, $\alpha_0 = \alpha_1 = \cdots = \alpha_m = 0$. Referring to Tsay (2005), the test statistic is:

$$F = \frac{(SSR_0 - SSR_1)/m}{SSR_1/(T-2m-1)} \tag{11}$$

where $SSR_0 = \sum_{t=m+1}^{T}(e_t^2 - \frac{1}{T}\sum_{t=1}^{T} e_t^2)^2$ and $SSR_1 = \sum_{t=m+1}^{T} \hat{z}_t^2$. The F statistic asymptotically follows a χ^2 distribution with m degrees of freedom. If F is greater than the χ_m^2 critical value, the null hypothesis is rejected, and the return residual series is confirmed to have conditional heteroscedasticity.

4.4.2 Ljung-Box Q test

The Ljung-Box Q test, named after Ljung and Box (1978), tests for autocorrelation at multiple lags. For a Ljung-Box Q test based on the Q-statistic at lag m, its null hypothesis is that the time series being tested has no autocorrelation up to order m. The Q-statistic at lag m of the return residual series e_t is calculated as:

$$Q(m) = T(T+2)\sum_{t=1}^{m} \frac{\hat{\rho}_t^2}{T-t} \tag{12}$$

$$\hat{\rho}_t = \frac{\sum_{t=m+1}^{T}(e_t - \overline{e})(e_{t-m} - \overline{e})}{\sum_{t=1}^{T}(e_t - \overline{e})^2} \tag{13}$$

where T is the number of observations and $\hat{\rho}_t$ is the autocorrelation at lag t. Under the null hypothesis, the Q-statistic asymptotically follows a χ^2 distribution with m degrees of freedom. Therefore, the null hypothesis will be rejected if the $Q(m)$ exceeds the χ_m^2 critical value. By replacing e_t with e_t^2 in Eq. (13), the autocorrelations in the squared residuals can be tested.

4.5 BEKK-GARCH model

The ARCH-LM and Ljung-Box Q test results (shown in Section 5) indicate that return residuals of this study's VAR model have conditional heteroscedasticity. To handle the conditional heteroscedasticity and examine the relationships between volatilities of the four endogenous time series variables, the BEKK-GARCH model was adopted.

The BEKK-GARCH model is a multivariate extension of the univariate GARCH model. For the univariate GARCH (*p*, *q*) model, let ε_t denote the return residuals of a time series. The ε_t is split into a time-dependent standard deviation σ_t and a stochastic piece z_t, that is, $\varepsilon_t = \sigma_t z_t$. z_t is a strong white noise process. The current variance σ_t^2 is modeled as a function of the ARCH terms ε^2 (past squared return residuals) and the GARCH terms σ^2 (past conditional variances):

$$\sigma_t^2 = c + \alpha_1\varepsilon_{t-1}^2 + \cdots + \alpha_q\varepsilon_{t-q}^2 + \beta_1\sigma_{t-1}^2 + \cdots + \beta_{t-p}\sigma_{t-p}^2 \tag{14}$$

where *p* is the order of GARCH terms and *q* is the order of ARCH terms. Previous researchers have proved that the GARCH(1,1) model often provides a parsimonious description of the data

(Bollerslev, 1986; McCurdy and Morgan, 1988). In other words, the univariate GARCH(1,1) model is sufficient for most conditions.

To investigate the four endogenous variables in this study, the multivariate BEKK-GARCH(1,1) model was more appropriate. In the BEKK-GARCH(1,1) model, the error term $\boldsymbol{\varepsilon}_t$ of the mean model (i.e., the VAR model) is written as:

$$\boldsymbol{\varepsilon}_t = \boldsymbol{H}_t^{1/2} \boldsymbol{z}_t \tag{15}$$

where $\boldsymbol{H}_t$ is the conditional covariance matrix of $\boldsymbol{\varepsilon}_t$, $\boldsymbol{H}_t^{1/2}$ is the positive-definite square-root matrix of $\boldsymbol{H}_t$, and $\boldsymbol{z}_t$ is a sequence of independent and identically distributed random vectors that $\boldsymbol{z}_t \sim N(0, \boldsymbol{I}_k)$. The variance equation of the BEKK-GARCH(1,1) model with four time series is:

$$\boldsymbol{H}_t = \boldsymbol{C}'\boldsymbol{C} + \boldsymbol{D}'\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1}\boldsymbol{D} + \boldsymbol{E}'\boldsymbol{H}_{t-1}\boldsymbol{E} \tag{16}$$

$$\boldsymbol{C} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ 0 & c_{22} & c_{23} & c_{24} \\ 0 & 0 & c_{33} & c_{34} \\ 0 & 0 & 0 & c_{44} \end{pmatrix} \quad \boldsymbol{D} = \begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{pmatrix} \quad \boldsymbol{E} = \begin{pmatrix} e_{11} & e_{12} & e_{13} & e_{14} \\ e_{21} & e_{22} & e_{23} & e_{24} \\ e_{31} & e_{32} & e_{33} & e_{34} \\ e_{41} & e_{42} & e_{43} & e_{44} \end{pmatrix}$$

where $\boldsymbol{C}$ is an upper triangular matrix, and $\boldsymbol{D}$ and $\boldsymbol{E}$ are 4×4 matrices. The diagonal elements of $\boldsymbol{D}$ examine the ARCH effects, and the diagonal elements of $\boldsymbol{E}$ evaluate the GARCH effects. In the off-diagonal elements of $\boldsymbol{D}$ and $\boldsymbol{E}$, d_{ij} and e_{ij} $(i \neq j)$ estimate the ARCH and GARCH effects of time series i on time series j.

5. Modeling results

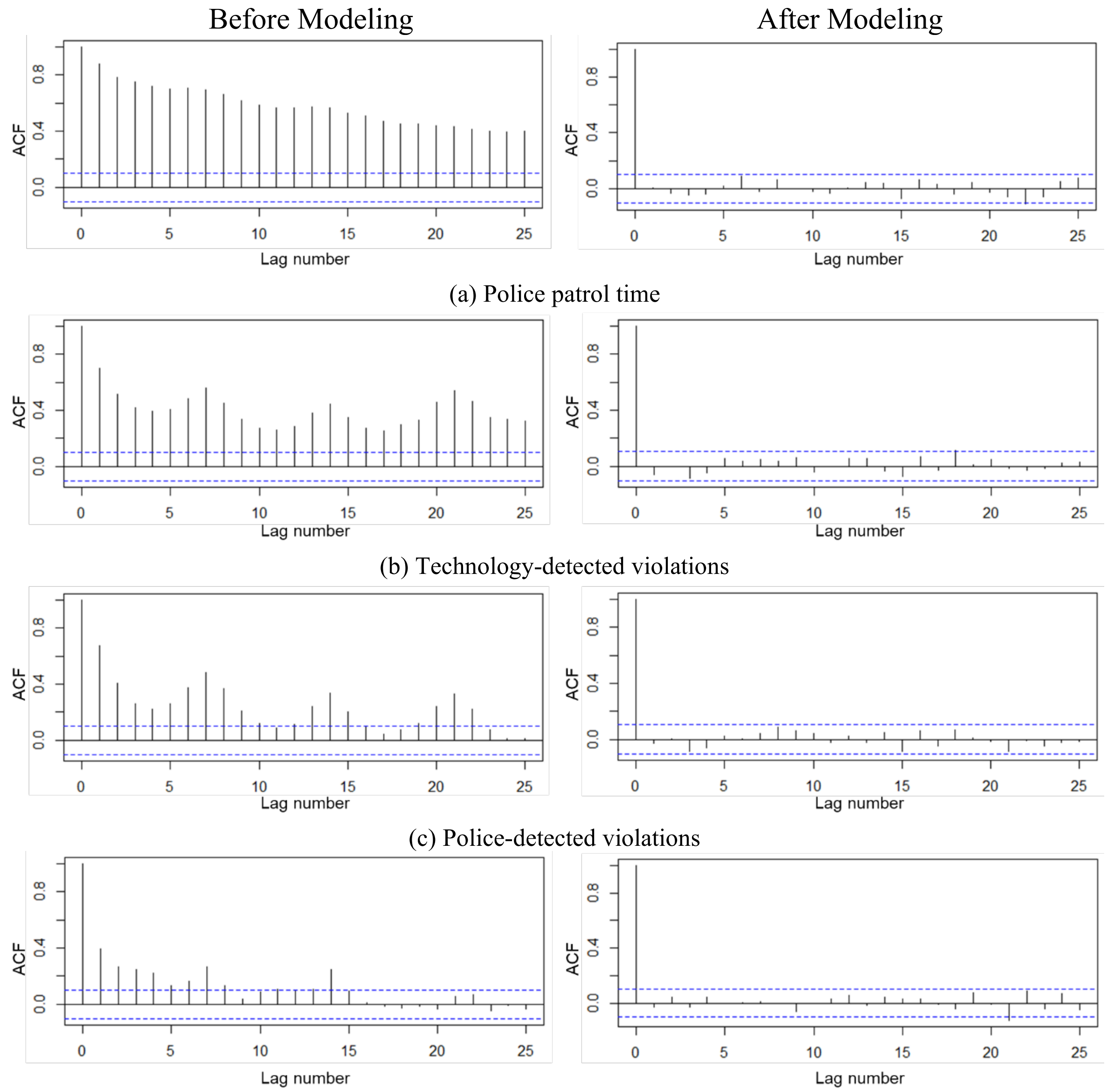
The *vars* package in R® was used to develop the VAR and SVAR models, and the *aod* package was adopted to conduct the Granger causality test. The BEKK-GARCH model was estimated using WinRATS®. Results of the VAR model are discussed first to reveal the impact of the exogenous variables on the endogenous variables. The VAR results subsection is followed by the results of the SVAR model, which demonstrate the contemporaneous relations between endogenous variables. This subsection is followed by the results of the Granger causality test, which capture the dynamic interactions between endogenous variables. Finally, the relationships between conditional variances of the endogenous variables are examined through results of the BEKK-GARCH model.

5.1 VAR modeling results

Before modeling, the plots of the autocorrelation function (ACF) (Tsay, 2005) were used to test any serial correlation in the endogenous variables. As is obvious in the left-side panel of Figure 3, the four endogenous time series are autocorrelated, emphasizing the necessity of achieving stationarity in time series modeling. To detect the components, the four endogenous time series were decomposed, and were found to have constants, time trends, and evidence of weekly variation. After removing the trend component and weekly variation, the remaining random parts of the four variables became stationary, as confirmed by the ACF plots and the

ADF test. Therefore, when determining the lag order for the VAR model, the time trends and weekly variations were considered. The lag length of 3, that is, the VAR model with lag 1, lag 2 and lag 3, was found to have the lowest AIC value and was thus selected as optimal.

The four-variable VAR model was initially developed by including the constants, time trends, weekly variations, the exogenous variables (holiday and weather conditions), and 3 lags of each endogenous variable (police patrol time, technology-detected violations, police-detected violations, and traffic crashes) simultaneously. However, several variables were found to be statistically nonsignificant. To make the model more accurate, the *restrict* function in the *vars* package was used to set a restriction matrix to constrain the coefficients of nonsignificant variables to zero. Finally, all the variables included in the VAR model showed statistical significance. The VAR modeling results are displayed in Table 3. Based on these results, the ACF plots were redrawn for the return residuals, as illustrated in the right-side panel of Figure 3. Now, the time series show no autocorrelation. In addition, the results of the ADF test also show that the return residuals have no unit roots. Consequently, the VAR(3) model produces white noise here, which indicates its rationality.



(a) Police patrol time

(b) Technology-detected violations

(c) Police-detected violations

(d) Traffic crashes

Figure 3. ACF plots of the four endogenous variables before and after modeling

Table 3. Parameter estimates of the VAR(3) model

Variable	Police patrol time	Technology-detected violations	Police-detected violations	Traffic crashes
Constant	85.374***	603.829***	1524.445***	-
Trend	-0.122***	-	-1.382***	0.058***
Weekly variation (reference: Sunday)				
Monday	32.754***	673.939***	647.006***	26.069***
Tuesday	-	723.422***	654.161***	22.459***
Wednesday	20.528***	551.382***	476.662***	24.255***
Thursday	-	389.464***	328.093***	14.389***
Friday	9.459*	558.752***	463.765***	37.397***
Saturday	-	-	-	6.213*
Holiday (reference: not holiday)				
Holiday	-	133.767*	-	4.192***
Weather condition (reference: sunny)				
Cloudy	-	-	-	-6.487**
Rainy	-	-156.897***	-183.348***	-
Snowy	-	-	-454.833**	-20.449*
Icy	-	-1109.791***	-1415.637***	92.207***
Lags of endogenous variables				
Police patrol time.lag1	0.680***	-	-	-
Police patrol time.lag2	-	-0.124**	-0.618*	-
Police patrol time.lag3	0.119**	-	-	-
Technology-detected violations.lag1	-	0.606***	-	-
Technology-detected violations.lag2	-	-	-	0.001**
Technology-detected violations.lag3	-	-	-	-
Police-detected violations.lag1	-	-	0.487***	0.011***
Police-detected violations.lag2	-	0.159***	0.229***	-
Police-detected violations.lag3	-	-	-	-
Traffic crashes.lag1	-	-	-	0.318***
Traffic crashes.lag2	-	-	-	0.166***
Traffic crashes.lag3	0.040**	-	-	0.112**

Note: *** indicates significance at the 1% level.

** indicates significance at the 5% level.

* indicates significance at the 10% level.

The VAR model results show a significant constant for police patrol time, evidence that Shanghai traffic police officers patrolled the freeway system every day. The significant downward trend in patrol time over the year, reflected by the negative trend parameter, was due to inadequate resources or staff shortage. The dummy variables for weekly variation show that compared to Sunday, police officers spent more time patrolling freeways on Monday, Wednesday, and Friday, but the impact of holidays and adverse weather conditions on patrol time were not statistically significant. Among the endogenous variables, only the lags of traffic crashes (and of course, patrol time) were useful for predicting patrol time; both technology-detected violations and police-detected violations turned out to be unhelpful.

The technology-detected violation variable also has a positive constant term, indicating that its mean is significantly different from zero. Its nonsignificant trend term means that technology-detected violations have no deterministic time trend. The positive coefficients of Monday to Friday variables imply that technology-detected violations are more likely to occur on weekdays than on Sunday, whereas the nonsignificant coefficient of Saturday signals that drivers break fewer traffic laws on Saturday and Sunday, which might be due to the absence of work or time pressure on the weekend. More violations tend to occur on holidays, possibly owing to increased traffic volume because of vacation travel. The negative coefficients of rainy and icy days may be due to drivers concerned about traffic safety in adverse weather curtailing their car travel; a related reason may be that drivers who must travel in adverse weather are more willing to obey traffic rules. The endogenous variables found to help predict technology-detected violations were police patrol time and police-detected violations.

Similar to technology-detected violations, the police-detected violation variable also has a significant constant term. The negative coefficient of the trend term demonstrates that police-detected violations had a significant downward trend, which is logically consistent with the downward trend in police patrol time because police-detected violations are identified by police officers during their daily patrols. The weekly variation in police-detected violations is similar to that of technology-detected violations: Monday to Friday were likely to have more police-detected violations than Saturday and Sunday. In addition to the weekday work and time pressure that likely affected drivers' tendency to aggressive driving behavior, another potential reason for the weekly variation in police-detected violations is the smaller number of traffic police officers on duty on weekends. Rainy, snowy, and icy days also had significantly negative influences on police-detected violations. The lags of police-detected violations, traffic crashes, and police patrol time were somewhat helpful for predicting police-detected violations.

Traffic crashes showed a positive trend over the year, with a significant coefficient of 0.05. The six dummy variables for weekly variation were all significant, meaning that compared to Sunday, more traffic crashes were likely to occur from Monday to Saturday. Holidays had a significantly positive relationship with crashes, that is, the Shanghai freeway system tended to have more crash occurrences during holidays. When compared with sunny days, cloudy days had fewer traffic crashes, which is possibly due to more moderate sunlight reducing associated glare. Contrary to popular belief, rainy days did not show any significant impact on crashes. Potential reasons are that drivers are more cautious on rainy days, or that commuters choose subways rather than driving because of concerns about traffic jams. Snowy days show similar results, but the small sample of snowy days might be the reason for the negative relationship. Icy days were found to be significantly and positively related to crashes, as expected, as icy pavement reduces friction and makes it difficult to control a vehicle. Of the endogenous variables, the coefficients of the lags of technology-detected violations and police-detected violations were significant, meaning that past violations of both types are helpful traffic crash predictors.

5.2 SVAR modeling results

Matrix A_0, introduced earlier, was added on the left side of the VAR model. Thus, the coefficients in matrix A_0 change signs when they are moved to the right side of Eq. (6). That is, a negative coefficient in matrix A_0 implies a positive contemporaneous effect, and vice versa. Table 4 displays the inverse value of parameter estimates of matrix A_0 in the SVAR model: a negative coefficient in Table 4 means a negative contemporaneous effect.

Table 4. Contemporaneous interactions

Variable	Police patrol time	Technology-detected violations	Police-detected violations	Traffic crashes
Police patrol time	1	0	0	0
Technology-detected violations	0.102	1	0	0
Police-detected violations	0.887	0.671	1	0
Traffic crashes	-0.150	0.012	-0.017	1

The contemporaneous relationship between police patrol time and technology-detected violations is interesting, in that as patrol time increases, technology-detected violations also increase. This positive relationship might be the result of traffic officers' experience, specifically, that they increase their patrols on those days that have incurred more violations in the past. Police-detected violations are more strongly and positively affected by police patrol time because they are directly identified by traffic police officers, so they can to some extent reflect traffic police enforcement intensity. The two types of traffic violations have a positive contemporaneous relationship.

Table 4 shows that traffic crashes are negatively impacted by contemporaneous police patrol time and police-detected violations. Each 1% increase in police patrol time tends to generate a 0.15% decrease in contemporaneous crash frequency. One possible reason is that more traffic violations are intercepted before they cause a crash, and another is that drivers break fewer traffic rules when they are aware of patrolling traffic police. In contrast to police-detected violations, technology-detected violations were found to have a positive influence on crashes. As stated in the data preparation section, technology-detected violations are less influenced by police visibility. Even though the detector positions of the electronic policing system are possibly known to the public, technology-detected violations are more representative of drivers' ordinary behaviors than are police-detected violations. Consequently, the contemporaneous positive relationship between technology-detected violations and traffic crashes is reasonable because traffic violations are usually the causes of crashes.

5.3 Granger causality test

The Granger causality test was used to identify the dynamic interactions between police patrol time, technology-detected violations, police-detected violations, and traffic crashes. Table 5 presents the test results. When the χ^2 statistic is greater than its critical value at the significance value of 10%, the corresponding null hypothesis can be rejected, that is, the Granger causality is significant. Figure 4 illustrates the dynamic interactions: solid arrows represent significant Granger causality, while dotted arrows mean that the results are not statistically significant.

Table 5. Granger causality test results

Causal Variable	Null Hypothesis	χ^2 statistic	p-value	Conclusion
Police patrol time	Police patrol time does not Granger-cause technology-detected violations	7.5	0.059	Reject
	Police patrol time does not Granger-cause police-detected violations	6.8	0.078	Reject
	Police patrol time does not Granger-cause traffic crashes	1.1	0.79	Accept
Technology-detected violations	Technology-detected violations do not Granger-cause police patrol time	3.7	0.3	Accept
	Technology-detected violations do not Granger-cause police-detected violations	6.5	0.089	Reject
	Technology-detected violations do not Granger-cause traffic crashes	6.4	0.093	Reject
Police-detected violations	Police-detected violations do not Granger-cause police patrol time	2.4	0.49	Accept
	Police-detected violations do not Granger-cause technology-detected violations	7.1	0.069	Reject
	Police-detected violations do not Granger-cause traffic crashes	15	0.0018	Reject
Traffic crashes	Traffic crashes do not Granger-cause police patrol time	7.9	0.049	Reject
	Traffic crashes do not Granger-cause technology-detected violations	2.3	0.51	Accept
	Traffic crashes do not Granger-cause police-detected violations	3.7	0.3	Accept

Note: In the Wald-test, the degree of freedom is 3.

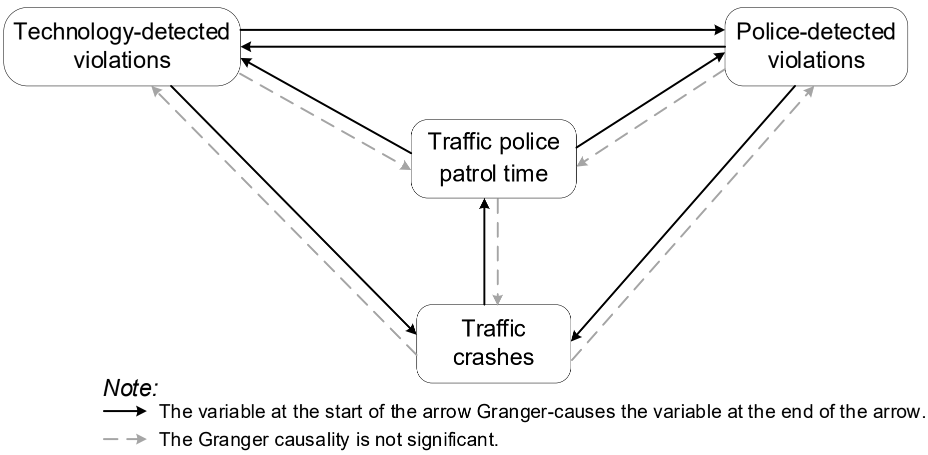


Figure 4. Granger causalities

Granger causality assesses whether changes in one variable are potentially the cause of changes in another; that is, Granger causality reflects the dynamic interactions between endogenous variables. As can be seen in Figure 4, there is a two-way Granger causality between technology-detected violations and police-detected violations. Additionally, both violation types Granger-cause traffic crashes, meaning that a change in the number of violations can affect crash frequency. As expected, traffic crashes do not Granger-cause technology-detected violations and police-detected violations, because traffic crashes are usually the results of violations, not the causes.

Interestingly, police patrol time does not Granger-cause traffic crashes, but Granger causality exists in the opposite direction: crashes Granger-cause traffic police patrolling. The significant coefficient of the lag of crashes in the model for police patrol time (as presented in Table 3) is positive, meaning that an increase in crashes will cause an increase in subsequent traffic law enforcement. As previously conjectured, the probable reason is that police officers adjust their daily patrols according to historical traffic crash information.

The two types of violations do not Granger-cause police patrol time, but police patrol time Granger-causes changes in both. The significant coefficients of the lags of police patrol time in the models for technology-detected violations and police-detected violations are negative, indicating that increased police patrol intensity can reduce subsequent traffic violations.

5.4 BEKK-GARCH model

Table 6 displays results of the ARCH-LM test and Ljung-Box Q test, which were applied to detect conditional heteroscedasticity. The ARCH-LM test statistics of police patrol time, police-detected violations, and traffic crashes show rejected null hypotheses; that is, return residuals of these three time series exhibit conditional heteroscedasticity. The Ljung-Box Q statistics demonstrate that, in all four endogenous time series, there is no autocorrelation in return residuals. These results are the same as those of the post-modeling ACF plots in Figure 3, but the squared return residuals of police patrol time and technology-detected violations are autocorrelated. These test results confirmed the necessity of establishing the GARCH model.

Table 6. Conditional heteroscedasticity test results for return residuals of the VAR model

Statistics	Police patrol time	Technology-detected violations	Police-detected violations	Traffic crashes
ARCH-LM(1)	7.832 (0.005)***	1.066 (0.302)	5.391 (0.020)**	10.37 (0.001)***
Q(10)	6.546 (0.767)	10.184 (0.424)	9.933 (0.446)	3.499 (0.967)
Q(20)	13.749 (0.843)	18.925 (0.527)	22.777 (0.299)	9.509 (0.976)
$Q^2(10)$	18.524 (0.047)**	22.705 (0.008)***	15.854 (0.104)	15.427 (0.117)
$Q^2(20)$	32.881 (0.035)**	28.044 (0.076)*	24.842 (0.208)	20.964 (0.399)

Note: Q is the Ljung-Box Q statistics of return residuals.
Q^2 is the Ljung-Box Q statistics of squared return residuals.
Values in parentheses are the corresponding p values.
*** Rejected the null hypothesis at the 1% level.
** Rejected the null hypothesis at the 5% level.
* Rejected the null hypothesis at the 10% level.

The results of the BEKK-GARCH(1,1) model are shown in Table 7. In matrix $\boldsymbol{D}$, the significant diagonal elements d_{11} (-0.510) and d_{22} (0.148) reveal that past squared return residuals of police patrol time and technology-detected violations have significant impact on their own current conditional variance, while the ARCH effects were not found in police-detected violations and traffic crashes. Similarly, in matrix $\boldsymbol{E}$, diagonal elements e_{11}, e_{22}, e_{33}, and e_{44} are all significant at the 1% level, demonstrating the existence of significant GARCH effects in the four time series. The past conditional variances of police patrol time, technology-detected violations, police-detected violations, and traffic crashes are shown to have significant impact on their current conditional variances.

The significant off-diagonal elements of matrices $\boldsymbol{D}$ and $\boldsymbol{E}$ reflect the ARCH and GARCH effects between the four endogenous time series. For instance, the off-diagonal element e_{12} (-1.982) of matrix $\boldsymbol{E}$ means that the change in past conditional variance of technology-detected violations has a negative influence on the current conditional variance of police patrol time. To detect the interactions between conditional variances of the four time series, the joint significance test was conducted for off-diagonal elements of matrices $\boldsymbol{D}$ and $\boldsymbol{E}$. Results in Table 8 show that there are significant bidirectional interactions in the conditional variances of police patrol time, technology-detected violations, police-detected violations, and traffic crashes.

The ARCH-LM test and Ljung-Box Q test were conducted again on the residuals of the BEKK-GARCH model to assess the adequacy of the BEKK model. As can be seen in Table 9, the results show no conditional heteroscedasticity nor autocorrelation in the residuals, which demonstrates the rationality of the BEKK-GARCH(1,1) model.

Table 7. Parameter estimates of the BEKK-GARCH model

Matrix	**Parameter estimates**			
$\boldsymbol{C}$	3.261 [0.838]	-92.367 [-2.601]***	-78.555 [-2.314]***	-8.016 [-2.384]***
		0.031 [0.0002]	0.095 [7.199]	0.013 [0.001]
			-0.022 [-0.0002]	-0.002 [-0.0002]
				0.0007 [0.000]
$\boldsymbol{D}$	-0.510 [-5.343]***	-0.187 [-0.265]	-1.199 [-1.873]*	-0.147 [-2.743]***
	-0.007 [-1.262]	0.148 [2.144]**	-0.036 [-0.588]	-0.014 [-2.624]***
	0.015 [2.136]**	-0.243 [-2.882]***	-0.088 [-1.314]	-0.001 [-0.075]
	-0.084 [-0.918]	-0.989 [-0.963]	-1.151 [-1.309]	-0.002 [-0.030]
$\boldsymbol{E}$	0.826 [20.072]***	-1.982 [-9.865]***	-1.374 [-4.366]***	-0.256 [-7.838]***
	0.028 [5.609]***	1.140 [25.379]***	0.267 [6.085]***	0.010 [2.520]**
	-0.009 [-1.551]	-0.309 [-5.407]***	0.754 [16.126]***	-0.007 [-1.521]
	0.241 [4.807]***	-3.096 [-5.087]***	-2.224 [-4.527]***	0.844 [18.978]***

Note: Values in square brackets are the corresponding T statistics.

*** indicates significance at the 1% level.

** indicates significance at the 5% level.

* indicates significance at the 10% level.

Table 8. Test results for bidirectional interactions in conditional variances

Bidirectional interactions between conditional variances	Test result
Police patrol time and technology-detected violations	H$_0$: $d_{12}=e_{12}=d_{21}=e_{21}=0$; $\chi^2 = 109.067^{***}$
Police patrol time and police-detected violations	H$_0$: $d_{13}=e_{13}=d_{31}=e_{31}=0$; $\chi^2 = 47.752^{***}$
Police patrol time and traffic crashes	H$_0$: $d_{14}=e_{14}=d_{41}=e_{41}=0$; $\chi^2 = 71.178^{***}$
Technology-detected violations and police-detected violations	H$_0$: $d_{23}=e_{23}=d_{32}=e_{32}=0$; $\chi^2 = 55.699^{***}$
Technology-detected violations and traffic crashes	H$_0$: $d_{24}=e_{24}=d_{42}=e_{42}=0$; $\chi^2 = 30.021^{***}$
Police-detected violations and traffic crashes	H$_0$: $d_{34}=e_{34}=d_{43}=e_{43}=0$; $\chi^2 = 25.521^{***}$

Note: *** Rejected the null hypothesis at the 1% level.

Table 9. Conditional heteroscedasticity test results for residuals of the BEKK-GARCH model

Statistics	Police patrol time	Technology-detected violations	Police-detected violations	Traffic crashes
ARCH-LM(1)	0.207 (0.649)	1.136 (0.287)	0.018 (0.893)	0.101 (0.750)
Q(10)	6.019 (0.814)	8.924 (0.539)	11.037 (0.355)	2.212 (0.994)
Q(20)	9.419 (0.978)	17.328 (0.632)	22.056 (0.338)	10.420 (0.960)
$Q^2(10)$	1.706 (0.998)	6.144 (0.803)	8.512 (0.579)	5.727 (0.838)
$Q^2(20)$	4.103 (0.999)	15.736 (0.985)	19.148 (0.512)	13.427 (0.858)

Note: Values in parentheses are the corresponding p values.

6. Discussion

6.1 Inconsistent responses to exogenous factors

The VAR model results revealed that the four endogenous variables had inconsistent responses to the exogenous factors including time trends, weekly variation, holiday, and weather conditions. First, crash frequency had a slightly upward trend over the year, while police patrol time had a downward trend. Second, the occurrences of traffic crashes and violations significantly increased on weekdays, while patrol time only showed a significant increase on Monday, Wednesday, and Friday. Third, significant changes in crash and violation frequency were found as holiday and weather conditions varied, while police patrol time showed no change. These results indicate that traffic crashes and violations are more volatile in response to external factors, while police enforcement intensity is relatively stable. As enhanced police patrol intensity was found to help reduce contemporaneous traffic crashes, the discordance between police patrol time and traffic safety emphasizes the necessity for traffic police to strengthen their enforcement and adjust their daily patrol strategy according to the temporal distributions of crashes and violations.

6.2 Significance of Granger causality

It is important to emphasize that Granger causality assesses whether the lags of one variable, X, help predict another variable, Y. The presence of Granger causality does not guarantee that X causes Y, but suggests that X may be causing, or is at least significantly correlated with Y (Lavrenz et al., 2018). For instance, police-detected violations and technology-detected violations are certainly not the cause of each other; thus the two-way

Granger causality between them is believed to be the result of their high correlation, reflected by the correlation coefficient of a significant 0.77. Traffic crashes were found to be the Granger-cause of police patrol time, but this finding is not due to the correlation between them, which is not significant at only -0.05. It is more reasonable to assume that the change in patrol time results from police departments taking notice of recent changes in crashes, and consequently making adjustments to their next patrols. In other words, past crashes have a significant influence on current patrol time. Using the VAR model to test Granger causality is thus a useful way to examine the various dynamic interactions among traffic enforcement, violations, and crashes.

6.3 Technology-detected violations vs. police-detected violations

The close connection between traffic police patrols and police-detected violations motivated the division, in this study, of violations according to their sources. As stated in the data preparation section, technology-detected violations are more representative of drivers' ordinary behaviors, while police-detected violations can somewhat reflect police enforcement intensity. The contemporaneous relationships between traffic crashes and both technology-detected and police-detected violations differ with the presence of police officers: more technology-detected violations are contemporaneous with more crashes, while more police-detected violations are contemporaneous with fewer crashes. The degree to which increases in technology-detected violations and crashes occur contemporaneously reveals the causal relationship from traffic violations to crashes, while the increase in police-detected violations coinciding with a decrease in crashes points to the important role of police patrolling, and explains why more intense police patrolling can reduce contemporaneous traffic crashes.

6.4 Unobserved heterogeneity, temporal instability, and heteroscedasticity

Just as Bhat and Zhao (2002) noted that spatial heteroscedasticity is heterogeneity in the variance of the unobserved component across spatial units, the temporal heteroscedasticity in return residuals in this study is due in part to those unobserved factors in each daily unit. In other words, not accounting for unobserved heterogeneity has, in some part, led to this study's heteroscedasticity in return residuals. For example, unannounced police department inspections might not be included in the model because of data unavailability, but may nevertheless result in a larger than normal variation of traffic police patrol time on those days. Temporal instability is another possible reason for conditional heteroscedasticity. Drivers may experience, for instance, cognitive shifts in their attitudes towards traffic safety and enforcement over time, as they gather more information about their driving environment (Mannering, 2018). Their propensity to aggressive driving such as speeding or drunk driving can change with these shifts, which can result in variation in the frequency of traffic crashes. Consequently, the relationships between traffic enforcement intensity and crashes can vary over time.

The BEKK-GARCH modeling showed significant bidirectional interactions between conditional variances of police patrol time, technology-detected violations, police-detected violations, and traffic crashes. These bidirectional interactions point to the existence of important factors, such as traffic volume, that simultaneously influenced the four endogenous variables but were not included in the VAR model. Knowing this was a limitation of the current study, the dummy variable representing holidays was used in an effort to supplement the level of traffic volume. Although the present work has utilized the statistical method to compensate for the lack of an exposure variable, there is still the possibility of the persistence of unobserved heterogeneity and temporal instability. Therefore, it should be noted that conclusions with regard to the model's results are obtained under these conditions, so ought to be carefully interpreted. If, in future studies, more data on currently unobserved factors become available, it is recommended to include them in the models. Improvement of the model structure is an alternative approach to relieve heteroscedasticity. For example, random effects or random parameters can be introduced to manage unobserved heterogeneity (Mannering et al., 2016). To address temporal instability, Mannering (2018) also proposes a method that allows a parameter to be a function of factors that impact its time-variant characteristics. However, these methods significantly increase the complexity of the modeling process, especially in time series modeling.

7. Conclusions

To our knowledge, this study, *for the first time*, aims to reveal the dynamic interactions and contemporaneous relationships between police enforcement, traffic violations, and traffic crashes using vector autoregressive models. To summarize, traffic police patrol time was used to represent police enforcement intensity, and traffic violations were classified as technology-detected and police-detected violations. Seven Granger causalities have been identified between police patrol time, technology-detected violations, police-detected violations, and traffic crashes. Differing from the traditional concept of causality, these Granger causalities are usually reflections of the dynamic interactions hidden within the system of the four endogenous variables, in which traffic police officers and drivers play an important role. This deeper understanding of dynamic interactions is conducive to further optimization of police enforcement.

Conditional heteroscedasticity, which can to some extent be attributed to unobserved heterogeneity and temporal instability, was found in return residuals of the VAR model. The significant bidirectional interactions between the conditional variances of police patrol time, technology-detected violations, police-detected violations, and traffic crashes further confirm the necessity to analyze the three simultaneously. At the same time, these interactions imply the absence of factors that simultaneously impact the four time series. Investigating such influencing factors can be accomplished with more exposure data availability and improvement in model structure.

The exogenous variables including time trend, weekly variation, holidays, and weather conditions were also investigated, and were found to significantly impact traffic crashes and violations. Since both crashes and violations increase on holidays, the authors of this study make the recommendation to control holiday traffic volume or adjust traffic distribution on freeways through management measures such as ramp metering or variable message signs. Although driver concern about traffic congestion on rainy and snowy days seems to lead to less travel, icy weather conditions pose another great threat to freeway traffic safety. To alleviate the influence of icy pavement, icy road warnings should be issued to the public in advance, and freeway maintenance authorities are advised to deal with these conditions promptly.

As police patrol has been found to significantly reduce contemporaneous crashes, freeway traffic safety can be further improved by enhancing the role of traffic enforcement. From the temporal aspect, deployment of more police to patrol freeways on Tuesdays and Thursdays is encouraged, as is adjusting the vacations of police officers to ensure sufficient patrol intensity during holidays. Making full use of the electronic police system is another measure for consideration. The Shanghai Traffic Police Department has recently gained the ability to quickly query driver information from its internal database. Once a traffic violation such as an illegally parked vehicle is detected by surveillance cameras, the monitoring police officer can immediately contact and warn the driver by phone call. With the full use of the electronic police system, traffic police officers are recommended, from the spatial aspect, to patrol more on those freeway segments that lack surveillance cameras.

This study enhances the understanding of the interactions between enforcement, traffic violations, and crashes, an important step toward guiding police strategies given limited resources. The findings from this study are expected to assist any traffic police department in developing strategies and guidelines for optimizing resources and enforcement in traffic safety management.

Acknowledgment

This study was sponsored by the International Science & Technology Cooperation Program of China (2017YFE0134500), the Science and Technology Commission of Shanghai Municipality (18DZ1200200), and the 111 Project (B17032).

Author Contributions

The authors confirm contribution to the paper as follows: study conception and design, data collection, analysis and interpretation of results, and draft manuscript preparation were provided by Mingjie Feng, Xuesong Wang, and Mohammed Quddus. The authors reviewed the results and approved the final version of the manuscript.

References

Abaza, O., 2018. Highway patrol investment levels vs. crash outcomes. Alaska Department of Transportation and Public Facilities.

Ahmed, M., Abdel-Aty, M., Yu, R., 2012. Assessment of interaction of crash occurrence, mountainous freeway geometry, real-time weather, and traffic data. Transportation research record 2280(1): 51-59.

Ahmed, M., Huang, H., Abdel-Aty, M., Guevara, B., 2011. Exploring a Bayesian hierarchical approach for developing safety performance functions for a mountainous freeway. Accident Analysis & Prevention 43(4): 1581-1589.

Alsaedi, Y.H., Tularam, G.A., 2019. The relationship between electricity consumption, peak load and gdp in saudi arabia: A var analysis. Mathematics and Computers in Simulation.

Asai, M., McAleer, M., Yu, J., 2006. Multivariate stochastic volatility: a review. Econometric Reviews 25: 145–175.

Bauwens L., Hafner C.M., Laurent S., 2012. Handbook of volatility models and their applications. John Wiley & Sons.

Beenstock, M., Gafni, D., 2000. Globalization in road safety: Explaining the downward trend in road accident rates in a single country (israel). Accident Analysis & Prevention 32 (1), 71-84.

Bendak, S., 2005. Seat belt utilization in saudi arabia and its impact on road accident injuries. Accident Analysis & Prevention 37 (2), 367-371.

Bhat C., Zhao H., 2002. The spatial analysis of activity stop generation. Transportation Research Part B: Methodological 36(6): 557-575.

Bollerslev, T., 1986. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics 31, 307-328

Bonneson, J.A., 2010. Highway safety manual. Washington, DC: American Association of State Highway and Transportation Officials.

Carnis, L., Blais, E., 2013. An assessment of the safety effects of the french speed camera program. Accident Analysis & Prevention 51, 301-309.
Damos, P., 2016. Using multivariate cross correlations, granger causality and graphical models to quantify spatiotemporal synchronization and causality between pest populations. BMC ecology 16 (1), 33.
Dickey, D.A., Fuller, W.A., 1981. Likelihood ratio statistics for autoregressive time series with a unit root. Econometrica: Journal of the Econometric Society, 1057-1072.
Elliott, M., Broughton, J., 2005. How methods and levels of policing affect road casualty rates TRL Limited London.
Engle, R.F., 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. Econometrica 50, 987-1007.
Engle, R.F., Kroner, K.F., 1995. Multivariate simultaneous generalized ARCH. Econometric Theory 11: 122–150.
Federal Highway Administration, 2009. Manual on uniform traffic control devices. U.S. Department of Transportation. Washington, DC.
Granger, C.W., 1969. Investigating causal relations by econometric models and cross-spectral methods. Econometrica: Journal of the Econometric Society, 424-438.
Harwood, D.W., Torbic, D.J., Richard, K.R., Meyer, M.M., 2010. *SafetyAnalyst^{TM}*: Software tools for safety management of specific highway sites. Turner-Fairbank Highway Research Center.
Hu, C., Liu, X., Pan, B., Chen, B., Xia, X., 2018. Asymmetric impact of oil price shock on stock market in China: A combination analysis based on SVAR model and NARDL model. Emerging Markets Finance and Trade 54(8): 1693-1705.
Johnson, M.B., 2016. A successful high-visibility enforcement intervention targeting underage drinking drivers. Addiction 111 (7), 1196–1202.
Koitsiwe, K., Adachi, T., 2015. Australia mining boom and dutch disease: Analysis using var method. Procedia Economics and Finance 30, 401-408.
Lavrenz, S.M., Vlahogianni, E.I., Gkritza, K., Ke, Y., 2018. Time series modeling in traffic safety research. Accident Analysis & Prevention 117, 368-380.
Ljung, G.M., Box, G.E.P., 1978. On a measure of lack of fit in time series models. Biometrika 65(2): 297-303.
Lord, D., Mannering, F., 2010. The statistical analysis of crash-frequency data: A review and assessment of methodological alternatives. Transportation Research Part A: Policy and Practice 44 (5), 291-305.
Lütkepohl, H., 2005. New introduction to multiple time series analysis. Springer Science & Business Media.
Makowsky, M.D., Stratmann, T., 2011. More tickets, fewer accidents: How cash-strapped towns make for safer roads. The Journal of Law and Economics 54 (4), 863-888.
Mannering, F., 2018. Temporal instability and the analysis of highway accident data. Analytic Methods in Accident Research 17: 1-13.
Mannering, F., Bhat, C.R., Shankar, V., Abdel-Aty, M., 2020. Big data, traditional data and the tradeoffs between prediction and causality in highway-safety analysis. Analytic

Methods in Accident Research 25, 100113.

Mannering, F., Shankar, V., Bhat, C.R., 2016. Unobserved heterogeneity and the statistical analysis of highway accident data. Analytic Methods in Accident Research 11: 1-16.

Masten, S.V., 2007. Do states upgrading to primary enforcement of safety belt laws experience increased daytime and nighttime belt use? Accident Analysis & Prevention 39 (6), 1131-1139.

McCurdy, T.H., Morgan, I.G., 1988. Testing the martingale hypothesis in Deutschmark futures with models specifying the form of heteroskedasticity. Journal of Applied Econometrics 3, 187-202.

Montella, A., Punzo, V., Chiaradonna, S., Mauriello, F., Montanino, M., 2015. Point-to-point speed enforcement systems: Speed limits design criteria and analysis of drivers' compliance. Transportation Research Part C: Emerging Technologies 53: 1-18.

Neyens, D.M., Donmez, B., Boyle, L.N., 2008. The iowa graduated driver licensing program: Effectiveness in reducing crashes of teenage drivers. Journal of Safety Research 39 (4), 383-390.

Pantangi, S.S., Fountas, G., Sarwar, M.T., Anastasopoulos, P.C., Blatt, A., Majka, K., Pierowicz, J., Mohan, S.B., 2019. A preliminary investigation of the effectiveness of high visibility enforcement programs using naturalistic driving study data: a grouped random parameters approach. Analytic Methods in Accident Research 21: 1-12.

Pfaff, B., 2008. VAR, SVAR and SVEC models: Implementation within R package vars. Journal of Statistical Software 27(4), 1-32.

Quddus, M.A., 2008. Time series count data models: An empirical application to traffic accidents. Accident Analysis & Prevention 40 (5), 1732-1741.

Quddus M.A., 2018. Time-series regression models for analysing transport safety data. Safe Mobility: Challenges, Methodology and Solutions 279.

Rezapour, M., Wulff, S.S., Ksaibati, K., 2018. Effectiveness of enforcement resources in the highway patrol in reducing fatality rates. IATSS Research 42 (4), 259-264.

Rezapour Mashhadi, M.M., Saha, P., Ksaibati, K., 2017. Impact of traffic enforcement on traffic safety. International Journal of Police Science & Management 19 (4), 238-246.

Rothengatter, T., 1982. The effects of police surveillance and law enforcement on driver behaviour. Current Psychological Reviews 2 (3), 349-358.

Ryeng, E.O., 2012. The effect of sanctions and police enforcement on drivers' choice of speed. Accident Analysis & Prevention 45, 446-454.

Savolainen, P.T., Mannering, F.L., Lord, D., Quddus, M.A., 2011. The statistical analysis of highway crash-injury severities: A review and assessment of methodological alternatives. Accident Analysis & Prevention 43 (5), 1666-1676.

Serhiyenko, V., Ivan, J.N., Ravishanker, N., Islam, M.S., 2014. Dynamic compositional modeling of pedestrian crash counts on urban roads in connecticut. Accident Analysis & Prevention 64, 78-85.

Sims, C.A., 1986. Are forecasting models usable for policy analysis? Quarterly Review (Win), 2-16.

Stanojević, P., Jovanović, D., Lajunen, T., 2013. Influence of traffic enforcement on the attitudes and behavior of drivers. Accident Analysis & Prevention 52, 29-38.

Stanojević, P., Sullman, M.J., Jovanović, D., Stanojević, D., 2018. The impact of police presence on angry and aggressive driving. Accident Analysis & Prevention 110, 93-100.

Sun, H., Wang, Q., Zhang, P., Zhong, Y., Yue, X., 2019. Spatialtemporal characteristics of tunnel traffic accidents in China from 2001 to present. Advances in Civil Engineering, 2019.

Tay, R., 2005. General and specific deterrent effects of traffic enforcement: Do we have to catch offenders to reduce crashes? Journal of Transport Economics and Policy 39 (2), 209-224.

Terrill, T.T., Mashhadi, M.M.R., Ksaibati, K., 2016. Developing a tool to help highway patrol in allocating resources to crashes. International Journal of Police Science & Management 18 (4), 231-241.

Tsay R.S., 2005. Analysis of financial time series. John wiley & sons.

Tsay R.S., 2013. Multivariate time series analysis: with R and financial applications. John Wiley & Sons.

Tse, Y.K., Tsui, A.K.C., 2002. A multivariate GARCH model with time-varying correlations. Journal of Business and Economic Statistics 20: 351–362.

Vernon, D.D., Cook, L.J., Peterson, K.J., Dean, J.M., 2004. Effect of repeal of the national maximum speed limit law on occurrence of crashes, injury crashes, and fatal crashes on utah highways. Accident Analysis & Prevention 36 (2), 223-229.

Walter, L., Broughton, J., Knowles, J., 2011. The effects of increased police enforcement along a route in london. Accident Analysis & Prevention 43 (3), 1219-1227.

World Health Organization, 2018. Global status report on road safety 2018. Geneva, Switzerland.

Xing, F., Huang, H., Zhan, Z., Zhai, X., Ou, C., Sze, N.N., Hon, K.K., 2019. Hourly associations between weather factors and traffic crashes: Non-linear and lag effects. Analytic Methods in Accident Research 24: 100109.

Ye, F., Garcia, T. P., Pourahmadi, M., Lord, D., 2012. Extension of a negative binomial GARCH model: Analyzing the effects of gasoline price on fatal crashes in Texas. Transportation Research Record, 2279, 31–39.