Loughborough University

This item was submitted to Loughborough's Research Repository by the author.
Items in Figshare are protected by copyright, with all rights reserved, unless otherwise indicated.

Time crystallinity in dissipative Floquet systems

PLEASE CITE THE PUBLISHED VERSION

https://doi.org/10.1103/physrevresearch.2.022002

PUBLISHER

American Physical Society (APS)

VERSION

VoR (Version of Record)

PUBLISHER STATEMENT

This is an Open Access Article. It is published by American Physical Society under the Creative Commons Attribution 4.0 Unported Licence (CC BY). Full details of this licence are available at: http://creativecommons.org/licenses/by/4.0/

LICENCE

CC BY 4.0

REPOSITORY RECORD

Lazarides, Achilleas, Sthitadhi Roy, Francesco Piazza, and Roderich Moessner. 2020. “Time Crystallinity in Dissipative Floquet Systems”. Loughborough University. https://hdl.handle.net/2134/14270849.v1.

Rapid Communications

Time crystallinity in dissipative Floquet systems

Achilleas Lazarides,[1,2] Sthitadhi Roy,[3,4] Francesco Piazza,[1] and Roderich Moessner[1]

[1]*Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Straße 38, 01187 Dresden, Germany*

[2]*Interdisciplinary Centre for Mathematical Modelling and Department of Mathematical Sciences, Loughborough University, Loughborough, Leicestershire LE11 3TU, United Kingdom*

[3]*Physical and Theoretical Chemistry, Oxford University, South Parks Road, Oxford OX1 3QZ, United Kingdom*

[4]*Rudolf Peierls Centre for Theoretical Physics, Clarendon Laboratory, Oxford University, Parks Road, Oxford OX1 3PU, United Kingdom*

(Received 29 April 2019; accepted 5 March 2020; published 3 April 2020)

We investigate the conditions under which periodically driven quantum systems subject to dissipation exhibit a stable subharmonic response. Noting that coupling to a bath introduces not only cooling but also noise, we point out that a system subject to the latter for the entire cycle tends to lose coherence of the subharmonic oscillations, and thereby the long-time temporal symmetry breaking. We provide an example of a short-ranged two-dimensional system which does not suffer from this and therefore displays persistent subharmonic oscillations stabilized by the dissipation. We also show that this is fundamentally different from the disordered discrete time crystal previously found in closed systems, both conceptually and in its phenomenology. The framework we develop here clarifies how fully connected models constitute a special case where subharmonic oscillations are stable in the thermodynamic limit.

DOI: 10.1103/PhysRevResearch.2.022002

Introduction. Understanding how statistical mechanics emerges in closed quantum many-body systems undergoing coherent dynamics with time-independent Hamiltonians has been one of the major themes of physics research over the last few decades. More recently, attention has been focused on closed systems with time-periodic ("Floquet") Hamiltonians, where fundamentally novel out-of-equilibrium phases describable in macroscopic terms have been discovered; none more prominent than the π-spin glass (SG) also termed the discrete time crystal (DTC) [1–10].

Generically, a major obstacle to working with Floquet systems is that they suffer heat death due to an unbounded increase of entropy, approaching an infinite-temperature state [11–13]. This can be avoided via disorder-induced localization [14,15] or coupling the system to an external environment draining energy from the system. The former, used in Ref. [1], additionally endows the Floquet eigenstates with a discrete-symmetry broken spatial glassy order. Crucially, the eigenstates connected by the symmetry are separated in quasienergy by π/T with T the driving period, leading to subharmonic oscillation of an appropriate local observable. Reference [16] shows that an external Markovian environment, unless explicitly fine-tuned, destroys the delicate coherence required for the subharmonic oscillations, driving towards a mixture with no spatial order (time dependent or not).

Here we analyze general dissipative Floquet systems in a different setting where the ordered phases, time-crystalline or otherwise, are *stabilized* by the dissipation, and which would be entirely absent without it. The mechanism, manifestly different from that of the π-SG, involves a periodic rotation between two "sectors" of a Hilbert space followed by dissipative "cooling" to states distinguishable by a measure such as magnetization.

This intuitively appealing picture ignores the possibility that the dissipation also generates noise, leading to loss of phase coherence in the oscillations. In terms of the density matrix of the system, the noise leads to the probability distribution of the observable broadening, which we argue destroys the DTC. We demonstrate this failure mode in a one-dimensional (1D) chain and show how the broadening may be avoided in 2D leading to a DTC. This broadening mechanism can be absent altogether in mean-field dynamics such as for fully connected models [17–20].

How a dissipative DTC can exist. First, let us describe the general arguments for the stability or instability of DTCs to dissipation before giving specific examples. For a visual demonstration, see Fig. 1.

(1) The Hilbert space is divided into different (for concreteness, two) sectors, $\mathcal{H}_{\pm}$, which could be symmetry sectors or simply based on an empirical criterion based on the expectation value of an observable, and

(2) each of these sectors has a manifold of states $\{|G_{\pm}\rangle\}$ which possess *quantum order* characterized by an observable, $\hat{M}$. Crucially,

(3) the two ordered manifolds (groups of states), $\{|G_{\pm}\rangle\}$, cannot be connected to each other via local operators, and

(4) the expectation value m of the order parameter, $\hat{M}$, is sufficiently narrowly distributed over the states within each of the manifolds that the two distributions for the

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Open access publication funded by the Max Planck Society.

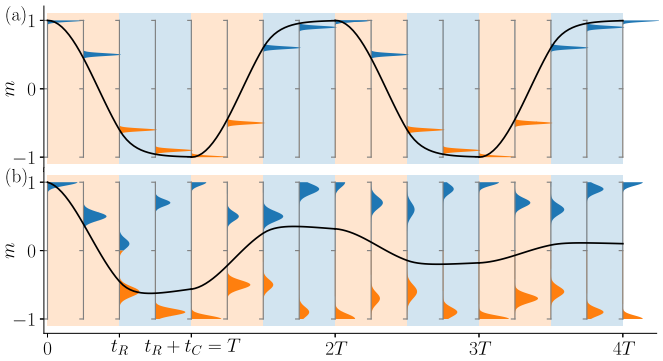


FIG. 1. Schematic of how a dissipative DTC can (a) survive and (b) die: Evolution of the distribution and the mean (black line) magnetization of a quantum spin system with time. Within a period (of T) a unitary process effecting global spin rotation and nonunitary cooling processes act together for t_R, whereas for t_C, the cooling processes act alone. (a) and (b) show an example where the magnetization distribution does not broaden in time. Consequently in (a) the weight is always on the *correct* magnetization sector leading to the persistent subharmonic oscillations, whereas in (b), the cooling processes split the distribution as such that there is weight left behind in the *wrong* sector which eventually kills the DTC.

two manifolds do not overlap (up to exponentially small corrections).

In Fig. 1, the two sectors are the positive- and negative-m halves of the vertical axis. Consider now a two-step dissipative Floquet protocol such that

(1) In the first step ("rotation phase"), the system evolves under the simultaneous action of a Hermitian rotation operator H_R and the dissipation. In the absence of dissipation the evolution over the step is unitary and given by $U_R(\theta)$, mapping neighbourhoods of the ground state manifold of one sector to states of the other sector. For a particular θ_*, $U_R(\theta_*)$ maps $\{|G_\pm\rangle\}$ exactly onto $\{|G_\mp\rangle\}$.

(2) In the second step ("cooling phase"), the system is governed by a Hamiltonian for which $\{|G_\pm\rangle\}$ are ground state manifolds as well as by the same dissipation processes.

The dissipative processes cool the system so that under their sole influence all states in the $\mathcal{H}_\pm$ sector would be driven to $\{|G_\pm\rangle\}$.

Such a quantum system, initialized in either of the ground state manifolds, shows a time-crystalline response trivially if $\theta = \theta_*$, as the expectation value of $\hat{M}$ oscillates stroboscopically between that in $\{|G_+\rangle\}$ and $\{|G_-\rangle\}$ with a period twice that of the Floquet drive, *provided the dissipation is inactive in the rotation cycle*. The dissipation is expected to make the temporal order robust to deviations of θ from θ^*: if the unitary rotation does not take states from the ground state manifold of one sector (say $\{|G_+\rangle\}$) entirely to $\{|G_-\rangle\}$ but admixes nearby excited states in the other sector, the cooling step of the drive will push the weight back towards $\{|G_-\rangle\}$. Thus cooling kills off the excitations left behind by the imperfect rotation, stabilizing a DTC.

How a dissipative DTC can die. In a broad sense, the dissipative processes have three effects:

(1) *Hindering rotation during rotation phase*. Recall that $\{|G_\pm\rangle\}$ are not connected via local operators. U then naturally has the form of a global rotation of the degrees of freedom. During the rotation, the state must become excited; this is opposed by the dissipation, which cools it back down making the rotation process less effective. Therefore rotation with dissipation is less effective than without, trapping (part of) the weight in the *wrong* sector. This is unfavorable to the presence of a stable DTC.

(2) *Correcting error caused by imperfect rotation during cooling phase*. Imperfect rotation potentially leaves the state in the correct sector, but not in the $|G_\pm\rangle$ manifolds; dissipation corrects this, favoring the DTC.

(3) *Broadening the distribution during both phases*. The rotation and the cooling acting in conjunction can, for short times, increase the width of the distribution of the state's overlaps with the excited states such that the resulting state is spread over both the sectors. In the following cooling cycle of the Floquet drive, the weights in each sector get pushed to their respective ground state manifolds, resulting in a finite weight in the *wrong* sector (see later discussion and Fig. 1). This is generally fatal to the DTC.

Of the three, the third (broadening) invariably causes the DTC signal to decay eventually. In its absence, a stable DTC phase is possible with the first two mechanisms determining the parameter regime of the stability. Let us note that while dissipation is favorable for the temporal order in the cooling cycle, it is detrimental in the rotation cycle, and it is *a priori* not obvious whether increasing the strength of the dissipation favors or disfavors the temporal order.

In what follows, we introduce a microscopic model and give three examples of dissipative processes. First we show that dissipation cleanly separating the two sectors but broadening the magnetization distribution leads to decay of the oscillation. We then introduce spatially local dissipation processes and show that (1) in 1D they fail to separate the two sectors, cause broadening, and lead to a decaying oscillation. In 2D, they cleanly separate the sectors and do not result in broadening, so that in this case a stable DTC appears.

Quantum spin systems. To analyze the above ideas in a concrete setting, we consider a system of spins 1/2, first in 1D. Using as basis states product states of σ^z (henceforth denoted as $\{|\alpha\rangle\}$), the $\mathcal{H}_\pm$ can be taken as the set of product states $\{|\alpha_\pm\rangle\}$ satisfying $\langle\alpha_\pm|\hat{M}|\alpha_\pm\rangle \gtrless 0$, respectively, with $\hat{M} = \sum_\ell \sigma^z_\ell$ [21]. This is a natural choice for a system described by an Ising Hamiltonian

$$\hat{H}_{\text{TFIM}} = -\sum_\ell \sigma^z_\ell \sigma^z_{\ell+1} + g\sum_\ell \sigma^x_\ell, \tag{1}$$

as in the limit of $g \to 0$, $\{|\alpha\rangle\}$ is a possible set of eigenstates while for $g \neq 0$, $|G_\pm\rangle$ are adiabatically connected to the $|\Uparrow\rangle$ (all-up) and $|\Downarrow\rangle$ (all-down) states as long as the Hamiltonian is in the ferromagnetic phase, $|g| < 1$. Note that so defining the two sectors allows us to label the basis states with the magnetization density $m = \langle\hat{M}\rangle/N$ (N being the system size).

The unitary operator U_R, which in the thermodynamic limit maps states $\{|G_+\rangle\} \leftrightarrow \{|\alpha_-\rangle\}$, is given by $U_R(\theta) = \exp[-i\theta\sum_\ell \sigma^x_\ell]$ with $\theta \in (\pi/4, \pi/2]$ and is produced by the action of the Hamiltonian $H_R = \frac{\theta}{t_R}\sum_j \sigma^x_j$ over time t_R. It follows that for $\theta_* = \pi/2$, $U_R(\theta_*)$ precisely maps the all-up state to the all-down state. In fact, since the ground state

of the system breaks the $\mathbb{Z}_2$ symmetry of the Hamiltonian spontaneously, $U_R(\theta_*)$ connects the two ground states exactly throughout the ferromagnetic phase. As anticipated, the rotation $U_R(\theta)$ is manifestly a nonlocal operation. In the thermodynamic limit, a product state with definite magnetization m is mapped to a (in the σ^z basis, nonproduct) state with definite magnetization $m\cos(2\theta)$, so that the rotation operation does not result in broadening.

Depending on the dissipative processes involved, this model can show decaying (bottom of Fig. 1) or persistent (top) subharmonic oscillations depending on whether broadening occurs or not. In the following, we use three explicit Markovian dissipative processes to show how the absence (presence) of broadening due to them favors (disfavors) the persistence of the temporal order.

Lindblad dynamics. Focusing on Markovian dissipative processes, the equation of motion for the density matrix of the system is governed by the Lindblad equation

$$\partial_t \rho = -i[\hat{H}(t), \rho] + \sum_i \left(\hat{L}_i \rho \hat{L}_i^\dagger - \frac{1}{2}\{\hat{L}_i^\dagger \hat{L}_i, \rho\} \right), \qquad (2)$$

where $H(t)$ is the time-dependent (in our case, time-periodic) Hamiltonian, $\{L_i\}$ is the set of time-independent quantum jump operators which arise due to the coupling to the dissipative environment, and i runs over all operators acting on the system. Our binary Floquet protocol with period $T = t_C + t_R$ is

$$\hat{H}(t) = \begin{cases} \frac{\theta}{t_R} \sum_\ell \sigma_\ell^x; \, 0 \leqslant t < t_R \\ \hat{H}_{\mathrm{TFIM}}; \, t_R \leqslant t < t_R + t_C. \end{cases} \qquad (3)$$

We focus mostly on the $g \to 0$ limit of $\hat{H}_{\mathrm{TFIM}}$.

Direct jump operators. To demonstrate the deleterious effects of broadening we begin by considering a set of jump operators $\{L_\alpha\}$:

$$L_\alpha = \sqrt{\gamma}[\Theta(m_\alpha)\,|\Uparrow\rangle\,\langle\alpha| + \Theta(-m_\alpha)\,|\Downarrow\rangle\,\langle\alpha|], \qquad (4)$$

where m_α denotes the magnetization of the product state $|\alpha\rangle$ and γ is a scalar parameter fixing the rate. These jump operators take the weight from any diagonal product state and transfer it *directly* to the ground state of the corresponding sector, providing very efficient cooling. However, they broaden the distribution leading to decay of the oscillatory signal. To show this explicitly, we study the magnetization of the system starting from the $|\Uparrow\rangle$ state, using translational invariance to access large system sizes [22]. Increasing γt_R leads to shorter lifetime, while increasing γt_C to longer, so that dissipation has opposite effects during the two parts of the driving (Fig. 2).

The magnetization vanishes with time due to the state stroboscopically being in a mixture of both ground state manifolds, with opposite magnetizations. One then expects that an observable finite and equal in both the ground states will remain finite in a statistical mixture of the two. Such an observable is the correlator

$$C(t) = \frac{1}{N^2} \sum_{\ell \neq r} \mathrm{Tr}\big[\rho(t)\sigma_\ell^z \sigma_r^z\big], \qquad (5)$$

persistent oscillations of which are shown in the rightmost column of Fig. 2. This is a fundamental difference between

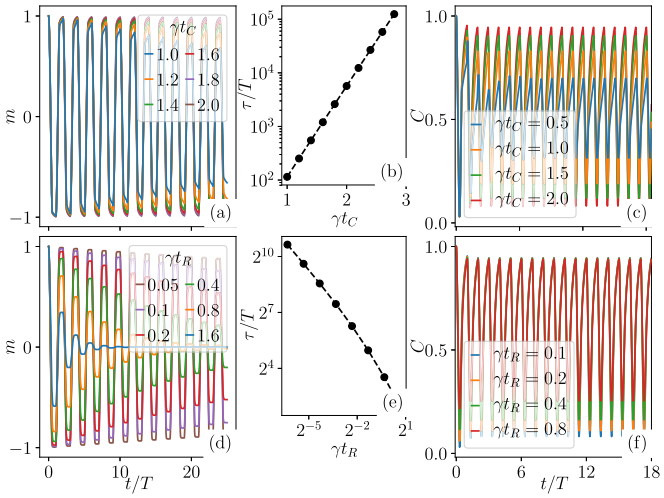


FIG. 2. Direct jump operators: The first column shows the *instability* of the subharmonic oscillations for the jump operators (4) for (a) $\gamma t_R = 0.1$ and different values of γt_C and (d) $\gamma t_C = 10$ and different values of γt_R. The lifetimes τ are shown in the second column, revealing an exponentially increasing and at least polynomially decreasing lifetime with γt_C and γt_R, respectively. The data is for $N = 51$, and $\theta = 0.45\pi$ and $\theta = \pi/2$ for the top and bottom rows, respectively. *Stability* of the persistent oscillations of the correlation function C, Eq. (5), is shown in the third column. For fixed γt_R [$=0.1$ in (c)], the oscillation amplitude increases with γt_C as dissipation corrects the error induced by imperfect rotations. By contrast, for a fixed γt_C [$=2$ in (f)], the amplitude decreases with γt_R as dissipation hinders the rotation. Here $N = 51$ and $\theta = 0.4\pi$.

this dissipative Floquet phase and the π-spin glass, where persistent oscillations of C imply those of an initially finite m. This order is also *induced* by dissipation, as evidenced by the increase of the amplitude with γt_C.

The direct jump operators demonstrate that the broadening of the distribution in magnetization is fatal to the time-crystalline order, *even when the dissipation cleanly separates the two sectors*.

Domain-wall annihilating jump operators. We now introduce a set of jump operators which avoid broadening in a natural way. These operators cause dynamics that only move domain walls (DWs): a freestanding DW can move but not disappear. However, two DWs can move into each other and annihilate. Such dynamics are fundamentally different in the 1D and 2D cases.

Denoting the neighbors of a site ℓ by $\{r_\ell\}$, to each product state $|\alpha\rangle$ and site ℓ there corresponds a jump operator

$$L_{\alpha,\ell} = \sqrt{\gamma}\left(\prod_{i \neq \ell} \mathcal{P}_{i,s_i^\alpha} \right) \times \begin{cases} \sigma_\ell^x, & M_{\{r_\ell\},\alpha} = 0 \\ \sigma_\ell^\pm, & \mathrm{sgn}\big(M_{\{r_\ell\},\alpha}\big) = \pm 1, \end{cases} \qquad (6)$$

where $M_{\{r_\ell\},\alpha}$ is the net magnetization of the spins in $\{r_\ell\}$ and $\mathcal{P}_{i,s_i^\alpha}$ is a projector onto the spin at site i in spin state $s_i^\alpha = \uparrow / \downarrow$. The corresponding Lindblad dynamics along the diagonal

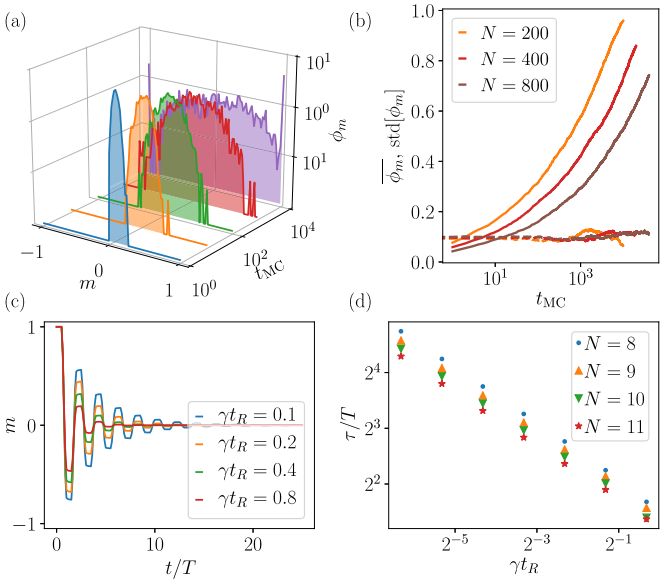


FIG. 3. Instability of the DTC to domain-wall annihilating operators in 1D: (a) Results obtained from classical Monte Carlo in one dimension show that the distribution of the state in magnetization for the jump operators in Eq. (6) broadens with time. (b) The mean magnetization (dashed lines) stays constant over Monte Carlo times, whereas the standard deviation (solid lines) grows, indicating broadening of the distribution. (c),(d) Numerically solving the Lindblad equation with the time-periodic Hamiltonian (3) and the jump operators (6) shows an exponential decay of the time-crystalline order with polynomially decreasing lifetime with γt_R. For the numerical solutions, $\theta = \pi/2$ and $\gamma t_C = 100$.

is governed by a Pauli master equation

$$\partial_t \rho_{\alpha\alpha} = \sum_\beta \gamma_{\alpha\beta}\rho_{\beta\beta} - \left(\sum_\beta \gamma_{\beta\alpha}\right)\rho_{\alpha\alpha} \qquad (7)$$

with the γ determined according to the rules above while the off-diagonals decay exponentially.

In 1D, the dynamics along the diagonal amounts to the DWs executing a random walk, i.e., diffusing. The probability distribution of magnetization starting from a sharp value m broadens at short times (Fig. 3, top left), and at long times becomes bimodal with two peaks at ± 1 of height such that $-p(-1) + p(1) = m$ (Fig. 3, top right), as found by solving Eq. (7) using a classical kinetic Monte Carlo approach. The resulting destruction of the DTC is shown in the bottom two panels.

For $d \geqslant 2$, this type of domain-wall dynamics eventually eliminates the minority phase by effectively causing a line (or surface) tension, tending to minimize the area of the interface between two nonconserved phases and suppressing interfacial fluctuations in the thermodynamic limit, far from the bulk critical point [23] (see Fig. 4 for two examples of allowed transitions and the Supplemental Material [22] for a demonstration of how the dynamics minimizes the interface length) so that the dissipation cleanly separates the two sectors. In the Supplemental Material we also show results for finite g.

In general, local dissipative processes lowering the energy of (ferromagnetic) Ising-type Hamiltonians in $d \geqslant 2$ will

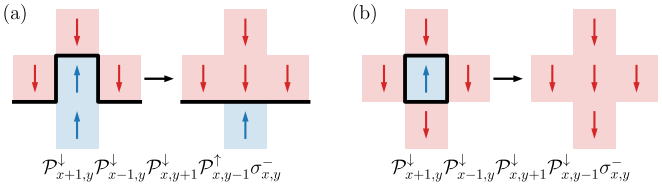


FIG. 4. Examples of the local jump operators leading to a persistent DTC in two dimensions: In general and for individual product states, these operators tend to either decrease the length of domain walls, such as the transition shown on the left, or cause minority regions to vanish, as on the right.

behave in a qualitatively similar way. Less obviously, the dissipative dynamics does not broaden the magnetization distribution starting from a state sharp in magnetization, Fig. 5. It then follows that a DTC phase may be stable; this is supported by the lower panels of Fig. 5 where a rapid rotation is shown to result in persistent subharmonic oscillations, while a slow rotation in a ferromagnetic phase in which the magnetization never changes sign.

In order to show this explicitly on finite-sized systems a numerical solution of the full Lindblad equation is desirable. Details of the numerical approach are given in the Supplemental Material. Figure 4 shows examples of allowed transitions in our dynamics, while the resulting system dynamics is displayed in Fig. 5, clearly showing a regime of persistent oscillations.

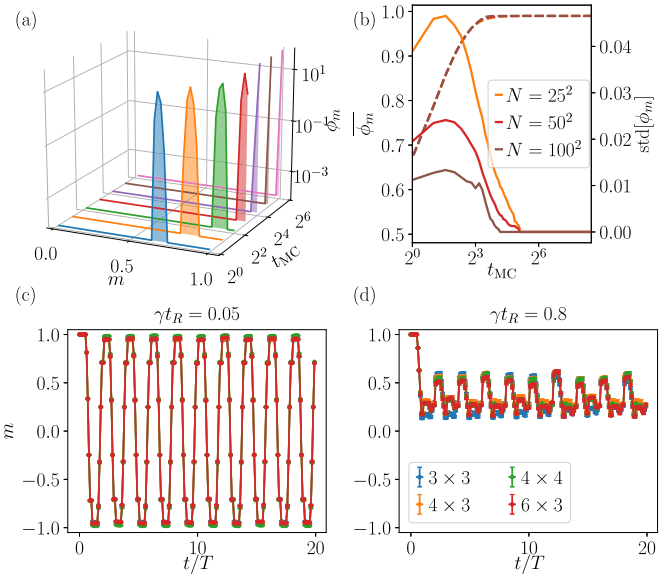


FIG. 5. Stability of the DTC to domain-wall annihilating operators in 2D: (a) Results obtained from classical Monte Carlo in two dimensions for the jump operators in Eq. (6). The magnetization distribution remains sharp. (b) The mean magnetization (dashed lines) saturates to unity, whereas the standard deviation (solid lines) decreases with system size. (c),(d) Numerically solving the Lindblad equation for square lattices (of size $N_x \times N_y$) shows a persistent time-crystalline response of the magnetization for low γt_R, and an oscillating ferromagnet at high γt_R. For the numerical solutions, $\theta = \pi/2$, $\gamma t_C = 100$, and $\gamma' = \gamma/10$.

Conclusions and outlook. We have discussed general mechanisms leading to dissipative (de)stabilization of DTCs in *disorder-free* dissipative Floquet systems, and have shown that any driving protocol such that there is no broadening during either rotation or cooling may display a regime of persistent subharmonic oscillations.

Our example of a dissipation-stabilized DTC is completely distinct, relying on a fundamentally different mechanism, from the π-spin glass introduced in [1,16]. Phenomenologically, a crucial difference between the two is that in the latter the oscillations in the magnetization can decay even though its correlation function, $C(t)$, oscillates persistently; in the former, one implies the other.

While the mechanism of obtaining period doubling from periodic switching between distinct sectors of Hilbert space is intuitively transparent, our analysis of its failure modes we believe also sheds light on recent work finding stable subharmonic oscillations [17–19]. In these systems the Hamiltonian is fully connected, typically leading to a stochastic description with noise vanishing with diverging system size, thus a density matrix with no broadening over the time evolution [24,25].

We believe that, generally, treatments for short-range models based on approximate mean-field and other approaches involving only a few effective degrees of freedom may erroneously find stable time-crystalline behavior by neglecting this mechanism. Our proposal is that the role provided by long-range interactions can, however, be replaced by the effectively macroscopic rigidity of the ordered component of a symmetry-broken system.

Finally, we have only considered Markovian dissipation. An open and interesting problem is to understand whether the physics unveiled here is changed qualitatively in the non-Markovian case, and whether there are non-fine-tuned non-Markovian environments that lead to interesting new examples of oscillatory dynamics in quantum systems.

Acknowledgments. This research was partly developed with funding from the Defense Advanced Research Projects Agency (DARPA) via the DRINQS program. The views, opinions, and/or findings contained herein are those of the authors and should not be interpreted as necessarily representing the official views or policies, either expressed or implied, of the Department of Defense or the U.S. Government. This work was in part supported by EPSRC Grant No. EP/N01930X/1.

[1] V. Khemani, A. Lazarides, R. Moessner, and S. L. Sondhi, Phase Structure of Driven Quantum Systems, Phys. Rev. Lett. **116**, 250401 (2016).

[2] D. V. Else, B. Bauer, and C. Nayak, Floquet Time Crystals, Phys. Rev. Lett. **117**, 090402 (2016).

[3] R. Moessner and S. L. Sondhi, Equilibration and order in quantum Floquet matter, Nat. Phys. **13**, 424 (2017).

[4] K. Sacha, Modeling spontaneous breaking of time-translation symmetry, Phys. Rev. A **91**, 033617 (2015).

[5] N. Y. Yao, A. C. Potter, I.-D. Potirniche, and A. Vishwanath, Discrete Time Crystals: Rigidity, Criticality, and Realizations, Phys. Rev. Lett. **118**, 030401 (2017).

[6] J. Zhang, P. W. Hess, A. Kyprianidis, P. Becker, A. Lee, J. Smith, G. Pagano, I.-D. Potirniche, A. C. Potter, A. Vishwanath *et al.*, Observation of a discrete time crystal, Nature (London) **543**, 217 (2017).

[7] S. Choi, J. Choi, R. Landig, G. Kucsko, H. Zhou, J. Isoya, F. Jelezko, S. Onoda, H. Sumiya, V. Khemani *et al.*, Observation of discrete time-crystalline order in a disordered dipolar many-body system, Nature (London) **543**, 221 (2017).

[8] S. Pal, N. Nishad, T. S. Mahesh, and G. J. Sreejith, Temporal Order in Periodically Driven Spins in Star-Shaped Clusters, Phys. Rev. Lett. **120**, 180602 (2018).

[9] J. Rovny, R. L. Blum, and S. E. Barrett, Observation of Discrete-Time-Crystal Signatures in an Ordered Dipolar Many-Body System, Phys. Rev. Lett. **120**, 180603 (2018).

[10] J. O'Sullivan, O. Lunt, C. W. Zollitsch, M. L. W. Thewalt, J. J. L. Morton, and A. Pal, Dissipative discrete time crystals, arXiv:1807.09884.

[11] L. D'Alessio and M. Rigol, Long-Time Behavior of Isolated Periodically Driven Interacting Lattice Systems, Phys. Rev. X **4**, 041048 (2014).

[12] A. Lazarides, A. Das, and R. Moessner, Equilibrium states of generic quantum systems subject to periodic driving, Phys. Rev. E **90**, 012110 (2014).

[13] P. Ponte, A. Chandran, Z. Papić, and D. A. Abanin, Periodically driven ergodic and many-body localized quantum systems, Ann. Phys. **353**, 196 (2015).

[14] A. Lazarides, A. Das, and R. Moessner, Fate of Many-Body Localization Under Periodic Driving, Phys. Rev. Lett. **115**, 030402 (2015).

[15] P. Ponte, Z. Papić, F. Huveneers, and D. A. Abanin, Many-Body Localization in Periodically Driven Systems, Phys. Rev. Lett. **114**, 140401 (2015).

[16] A. Lazarides and R. Moessner, Fate of a discrete time crystal in an open system, Phys. Rev. B **95**, 195135 (2017).

[17] A. Russomanno, F. Iemini, M. Dalmonte, and R. Fazio, Floquet time crystal in the Lipkin-Meshkov-Glick model, Phys. Rev. B **95**, 214307 (2017).

[18] Z. Gong, R. Hamazaki, and M. Ueda, Discrete Time-Crystalline Order in Cavity and Circuit QED Systems, Phys. Rev. Lett. **120**, 040404 (2018).

[19] F. M. Gambetta, F. Carollo, M. Marcuzzi, J. P. Garrahan, and I. Lesanovsky, Discrete Time Crystals in the Absence of Manifest Symmetries or Disorder in Open Quantum Systems, Phys. Rev. Lett. **122**, 015701 (2019).

[20] N. Y. Yao, C. Nayak, L. Balents, and M. P. Zaletel, Classical discrete time crystals, Nat. Phys. (2020), doi: 10.1038/s41567-019-0782-3.

[21] Modulo the ambiguity for the product states with zero magnetization; we choose to put half of them in the first sector and half of them in the other. The choice has no bearing on the subsequent dynamics.

[22] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevResearch.2.022002 for material supporting main article.

[23] A. O. Parry, J. M. Romero-Enrique, and A. Lazarides, Nonlocality and Short-Range Wetting Phenomena, Phys. Rev. Lett. **93**, 086104 (2004).

[24] J. Gelhausen and M. Buchhold, Dissipative Dicke model with collective atomic decay: Bistability, noise-driven activation, and the nonthermal first-order superradiance transition, Phys. Rev. A **97**, 023807 (2018).

[25] F. Benatti, F. Carollo, R. Floreanini, and H. Narnhofer, Quantum spin chain dissipative mean-field dynamics, J. Phys. A: Math. Theor. **51**, 325001 (2018).