Ordinary level mathematics syllabuses

PUBLISHER

Loughborough University of Technology.

ORDINARY LEVEL MATHEMATICS SYLLABUSES

by

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A Master's Dissertation submitted in
partial fulfilment of the requirements
for the award of the degree of M.A.
in Curriculum Studies of the
Loughborough University of Technology,
January 1982.

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ABSTRACT

The secondary school examination system has become, rightly or wrongly, one of the most important aspects of education today. Chapter one describes how examinations began in the nineteenth century and traces their development up to the present day.

Over the last century there has been a tremendous shift in emphasis regarding the style and content of mathematics teaching in schools. In Chapter two these curriculum changes are discussed in detail.

The twenty four Ordinary level Mathematics syllabuses (listed in the appendices) at present available in England and Wales form the subject of the next five chapters. For the purposes of this investigation I have categorised the syllabuses under the headings "traditional", "modern" and "traditional/modern". This is explained in Chapter three which also describes the major characteristics of all the syllabuses. Chapters four, five and six compare the actual content of the syllabuses under the given headings.

As well as major differences in content between the syllabuses, there are also various methods of examination. These are detailed in the next chapter which also looks at the interpretation of the syllabus by the examiners

of a topic which appears in all of the syllabuses, namely graphical work.

A contentious issue at the present time involves the place of calculators in the teaching of mathematics. Chapter eight examines the arguments for and against the use of calculators both in mathematics lessons and in mathematics examinations.

The ninth chapter looks at the progress of the sixteen plus examination proposals to date, and consideration is given as to whether or not the sixteen plus examination will suitably replace the G.C.E. O level (and C.S.E.) examinations in mathematics.

ACKNOWLEDGEMENTS.

I would like to thank my supervisor, Mrs. M. E. O. Hughes for her valuable help and guidance. My thanks also to my wife, Marie for her support throughout the course, and for typing the manuscript.

CONTENTS

CHAPTER 1

A HISTORY OF THE MAJOR DEVELOPMENTS IN THE SCHOOL EXAMINATION SYSTEM.

It is assumed by many people that written examinations have had a long history, but in reality they are comparatively recent in origin. Written examinations were first used in the American universities but as Roach (1971)[34] says, "Public examinations were one of the great discoveries of nineteenth century Englishmen". At the beginning of the nineteenth century there were virtually no public written examinations in this country. Although Cambridge University had had some form of written examination in some subjects since 1722, all degrees were awarded on oral examination and public dissertation, until the Oxford Public Examinations Statute of 1800 and the Cambridge Classical Tripos of 1824.

This was followed in 1838 by "London Matriculation", one examination which covered entry at that time, to all the courses at London University. This differed from Oxford and Cambridge Universities where most of the colleges set their own entrance test. The importance of the London University Matriculation Examination was that it came to be regarded as a public leaving examination for secondary school pupils whether or not they intended to go to university. (In 1858 the University of London became a worldwide examining body independent from its colleges).

This was the beginning of the examination revolution. They rapidly became a major tool of social policy being used to recruit men for government service, to select the ablest students in the universities, to control the work of the secondary schools, and to regulate grants to elementary schools under the Revised Code of 1862. Everyone got on the examinations bandwagon. There were few areas of public life upon which examinations had not touched, as can be seen from the many satirical comments at the time, for example in Gilbert and Sullivan's "Iolanthe" of 1882,

> "Peers shall teem in Christendom,
> And a Duke's exalted station
> Be attainable by Competitive Examination "

Roach (1971)[34] outlines three reasons he sees for this remarkable growth in public examinations. The first was to provide a means of maintaining academic standards and an incentive for raising them, especially in universities. Morris (1961)[29] quotes the report of the Oxford University Commision of 1850 which claimed it was the introduction into the university of properly conducted examinations which "first raised the studies of the University from their abject state", resulting in larger numbers of students and examinees.

The second reason for the growth was their usefulness in deciding the fitness of candidates for public office or for a profession. There was now a model of professional efficiency upon which to base one's judgement rather than an ideal of general culture and intellectual attainment.

The third reason, which is probably more true of the early twentieth century than the nineteenth, was a matter of social conscience. Open competitive tests could serve to bring forward larger numbers of able children from low social classes and give them opportunities to advance themselves. This remedy for social inequality was the basic philosophy behind the free place system for transfer between elementary and grammar school of the early twentieth century. However, as the Taunton Commission notes,[29] open competition tends to favour those who have had the best (and therefore usually the most expensive) education. In this sense open competition can have the reverse effects to those intended, which indicates that equality is not an administrative commodity.

The growth of the middle class in Britain has been cited as the main cause for the growth in the numbers of written public examinations by Montgomery (1978).[27] During the nineteenth century the replacement of nepotism and patronage by competitive examinations in the Civil Service and the Forces, led to a demand for better education and a rapid expansion in the "private" school system. These schools required some indication of standards achieved and examinations met this need. The first examination system was that of the College of Preceptors (a group of private school teachers) who began operating a full scale examination system in 1854. Originally it was intended that teachers would set their own examinations for their pupils and carry them out themselves, with moderation from the College's examiners. However this was discontinued in 1853 when it was thought that parents would not place much trust in such a scheme.

The College of Preceptors had started out with the ambitious objective of providing a professional standard of qualifications for teachers themselves. However few teachers were willing to present themselves for examination(only 53 in 1870), so their main effort was sustained in the college diplomas for pupils (1571 in 1870).[27] The College of Preceptors was important in that it paved the way for other school examinations notably the University Local examinations.

Following the development of an examination system in the private area came the development of a system in the elementary schools. It was the Newcastle Commission (1856-61) that suggested there should be the introduction of "a searching examination conducted by a competent body, of every child in each of those schools to which grants are paid."[29] Originally only the teaching of reading, writing and arithmetic qualified the school for a grant but later on other subjects were included. However all of them had to be ones which could be tested by an examination. Morris (1961)[29] quotes the H.M.I. report of the time which concluded that "all studies of the classroom must be those wherein progress can be definitely measured by examination." This is the first evidence of the constricting effect examinations were to have on the curriculum.

In the secondary schools, the beginnings of the modern system can be traced back to 1858 when Oxford and then Cambridge formed their Local Examination Boards. Initially the examinations were held in a public hall hired for the occasion quite separate from the schools and only boys were allowed to enter. Girls were admitted to Cambridge Locals in 1865 after a trial two year period and to those of Oxford in 1870. There was an almost missionary zeal about the new extension work of the Universities, the examining of school pupils. It was believed that examining would both raise the standards of the schools, and bring forward a more numerous and a better trained group of entrants for the universities themselves.

However opinions were mixed. As early as 1868 the dangers of examinations for schools were being recognised in the Report of the Taunton Commission.[34] It criticised the Locals for being too difficult for most pupils (out of 750 candidates for the first Oxford examination of 1858, only 280 were successful due to the fact that candidates had to pass every subject in the group.) Also as only small numbers of pupils were entered, private coaching was encouraged at the expense of the majority of pupils. The Taunton Commission suggested that whole classes should be entered for examinations rather than individuals, thus preparing the way for school examinations.

A controversial point suggested was that the state should be the only authority able to set up Examination Boards which would then be administered locally perhaps by the universities. The headmasters of the public schools at the time were horrified at the thought of this state interference in academic matters and formed the Headmasters' Conference which successfully opposed the notion.

The Headmasters' Conference was also significant in that it turned to the two universities Oxford and Cambridge to assist in the setting up of more meaningful examinations for their schools. This resulted in the formation of the Oxford and Cambridge Schools Examination Board in 1873. It held its first examination, the Higher School Certificate, for boys of 18 and over in 1874, (girls were admitted in 1879). A Lower Certificate for 16 year olds followed in 1884, which became the School Certificate in 1905. The Joint Board is unlike the other boards in that it is not associated with any particular area of the country and it was set up in a different manner. It still maintains traditions of links with the private sector of education in that over 90% of schools taking its examinations are independent.

The London University Examination Board continued its development in 1902 when it decided to only allow its examinations to be held in schools that were inspected and approved by it. The other Boards soon followed this example.

To meet the demand for examinations, further boards were established. Petch (1953)[31] in his book "Fifty Years of Examining" traces the history of the Northern Universities Joint Matriculation Board and shows how it developed from the Preliminary Examination of the Victoria University and how it overcame the initial alienation from the schools, "the schools...... thought (with some reason, although with some exaggeration) that we were poaching on their work and taking away their pupils."

By the beginning of the twentieth century the school examination system left much to be desired. The Boards were essentially part time organisations using examining methods more appropriate to undergraduates than school pupils. Schools entered their more able pupils for one examination after another, as one examination board would frequently fail to recognise the examination of another for qualifying purposes.

The Taunton Commission's concern for the danger of examinations to schools was echoed in 1911 by the Report of the Consultative Committee on Examinations in Secondary Schools.[29] In a comprehensive document they outline the advantages and disadvantages of examinations and their effects on pupils and teachers. It concludes that external examinations are not only necessary but desirable in secondary schools but with the proviso that "we are equally convinced that if the admitted advantages of external examinations are to be secured and the dangers of them minimised, such examinations should be subject to most stringent regulations as to their number, the age at which they are taken and their general character."

The Report also suggested that teachers themselves should be able to submit their own syllabuses to the board for them to set their examinations on it (similar to the Mode 2 C.S.E. system today)," where such a claim is admitted, we think that special examination papers should be prepared without extra cost to the school." Unfortunately this did not happen for many years.

As a result of this report a new examination system was introduced for secondary schools: the School Certificate for 16 year old pupils, and the Higher School Certificate for 18 year olds. In 1917 the Board of Education officially recognised the examining boards as the examiners but maintained the right to control the curriculum by laying down the subjects to be included. It controlled standards by setting up the Secondary Schools Examination Council (S.S.E.C.)to investigate and compare different examining boards and report back to the Board. (The S.S.E.C. later became the Schools Council in 1964). Surprisingly the universities reserved the right to draw up independently their own syllabuses. Central authority, unlike other countries, did not take the chance to have a more direct influence on what was taught in a particular subject, which has led to the multiplicity of different O level syllabuses in Mathematics for example (see later). The S.S.E.C. did have the desired effect and the lack of uniformity of standards amongst the different examining boards diminished during the 1920's.

To obtain a School Certificate and for Matriculation purposes, passes in five groups of subjects were required and these had to be at least one from each particular group of subjects. To obtain a Higher School Certificate a candidate needed passes in at least three principal or two principal and two or three subsiduary subjects.

This system was continued until just after the war. The 1938 Spens Report[25] had argued against it but the Second World War prevented any action being taken. The Report did not like the grouping of subjects and preferred individual performances in each subject to be recorded, (over 30% of pupils failed the School Certificate because of one subject). Also it did not like the narrowing of the curriculum the examination caused. The Norwood Report[24] of 1943 reiterated the Spens Report in calling for a change in the existing examination system. It also considered the school curriculum as a whole, and many of the recent changes in the school curriculum have their origins in the "ahead of its time" Norwood Report. For example, it supported internal assessment by teachers, coordinated by the H.M.I's as an alternative to external examinations. However over twenty years were to pass before this became reality with the Schools Council and the C.S.E. examination.

The Secondary Schools Examination Council modified and extended the Norwood Report findings in 1947. It recommended the establishment of an examination at three levels, Ordinary, Advanced and Scholarship to be available to pupils who were at least sixteen on the first of September of that year. The examinations were also to be available to those who were not pupils of a secondary school, and this gave a great boost to adult education in later years.

The Ordinary level papers were to provide a "reasonable test" for pupils who had studied a subject as part of a general course upto the age of sixteen, or for those who had studied the subject in a non-specialist way in the sixth form or after leaving school. Advanced level papers were for candidates who had taken the subject as a specialist study for two years in the sixth form. Scholarship papers were to provide an opportunity for gifted pupils to show distinctive merit. These were adapted, in 1962, into the Special Papers of today.

The group system was abolished and all subjects at all three levels were to be optional. Candidates passing in at least one subject were to be awarded a General Certificate of Education. The pass standard at O level was to be set at the old credit level in the School Certificate. A pass at A level was to be roughly equivalent to a pass in a Principal subject in the Higher School Certificate. There is some doubt however as to whether these standards were achieved as no absolute standard can be applied, partly due to the change in examination and the simultaneous changes in the school population.

The first examinations for this new scheme were held in 1951. Initially they were just pass or fail, but later grades were introduced. For example, in 1960 grades for A level were set as follows: A (distinction) for the top 10%, B (good) for the next 15%, C (better than average), D (average) and E (pass) for a further 45%. Those candidates who just missed a pass grade were awarded an O level pass.

At the same time as the General Certificate of Education was introduced, a new examining body was established. This was the Associated Examining Board, the only English Board not affiliated to a university or a group of universities. It was originally set up to provide Ordinary Level syllabuses for those subjects particularly suited to Secondary Modern and Technical College students in the City and Guilds tradition. It still provides a wide range of practical and general subjects such as Photography, Building Practice and European Studies, although now it also provides a full range of "academic" subjects at all three levels in addition to these.

However there was one major problem remaining with the new system. The G.C.E. O level was not appropriate to pupils in the secondary modern schools that had been set up with the 1944 Act. Only one child in five across the whole ability range was capable of obtaining at least four passes at O level. Those children who were unable to cope with the G.C.E. found themselves without any national qualification for which to aim. This led to a growing demand for a new kind of examination more suitable for the majority of pupils.

In 1955, the Minister of Education issued Circular 289[21] in which he refused to set up a new general examination of national standing for secondary schools, or lower the Ordinary Level standard. He opposed the widespread use of privately organised external examinations but encouraged the organisation of individual school internal examinations.

He reinforced this viewpoint in Circular 326[22] in 1957 but agreed to the setting up of regionally organised external examinations for sixteen year olds as an aid to selection for further education at a Technical College. However unofficial examinations started to fill the gap. The long established College of Preceptors (which was originally set up to help private schools) formulated a new Certificate Examination for 15 year olds in 1953. This was almost monopolised by state school candidates, the number of entrants increasing to 21,500 in 1963. Other examinations were developed by other bodies such as the

Royal Society of Arts, until there was a multiplicity of different examinations.

Belatedly the Minister of Education set up a committee to report on the situation, the Beloe Commission. They found that entries for external examinations below O level were growing at an "alarming" rate in spite of the Minister's objections in Circulars 289 and 326, and recommended a new examination.

This led to the introduction of the Certificate of Secondary Education in 1965. It differs from the G.C.E. in that it is not a pass/fail examination, but has five grades overall with two reference grades. Grade 1 is defined as being of such a standard that the candidate "might reasonably have secured a pass in the O level of the G.C.E. exam." had he followed a course of study leading to that examination. Grade 4 represented the standard which a sixteen year old pupil of average ability might be expected to achieve after a five year course of secondary education. However there were difficulties associated with this grade and definition so it was decided that the grade 4 standard should be restated as the concensus standard established and continually applied by the C.S.E. boards. The numbers of candidates to receive grades 2 and 3 were to be approximately equal in size, and grade 5 was to be awarded to candidates whose performance was below grade 4, but sufficiently of merit to enable assessment to be made in terms of the examination. Performances which could not were ungraded and these do not appear on the certificate. The C.S.E. is organised on a regional basis and a pupil must take the examination of the Regional Board whose area he is in.

The double examination system, O level and C.S.E., since 1965 has inevitably led to difficulties for teachers over which examination to enter pupils for, especially since the C.S.E. Grade 1 is still regarded with suspicion in some quarters, notably industry. This has led to the present trend towards joint sixteen-plus examinations, several of which are in operation at the moment. I will discuss these more fully in a later chapter, (Chapter 9).

The situation in the sixth form is particularly complicated with the wide range of ability pupils now staying on at school. This is likely to become even more pronounced in the future, with the current economic climate. The G.C.E. examining boards responded to this change by introducing a wider range of Alternative Ordinary level subjects. These are designed for candidates of greater maturity than is normally expected at Ordinary level".

A proposed change was the introduction of the C.E.E. (Certificate of Extended Education) for sixth formers and many C.S.E. boards are still running feasibility studies, but the idea has not been accepted officially. This examination also "overlaps" the G.C.E. O level examination. It is likely that the C.E.E. will be dropped in favour of the sixteen-plus examination.

This then is the present state of the examination system in 1981. In the future I can envisage a common examination at the sixteen-plus level replacing all the G.C.E. O level, C.S.E., C.E.E. and other examinations. However great care will have to be taken to ensure that the new examination is realistic for the lower abilities and yet will give a meaningful test for those pupils who previously took an O level course in order to proceed to A level courses.

CHAPTER 2

RECENT DEVELOPMENTS IN MATHEMATICS TEACHING AND EXAMINING.

The introduction of Mathematics as an integral part of the school curriculum coincided roughly with the establishment of external examinations. This perhaps tended to fix a pattern of mathematics which does not seem now to be very well thought out. Only Arithmetic, Algebra and Geometry were taught, often in separate classes and possibly with three different teachers. The arithmetic was very commercial, the algebra was very formal and the geometry consisted entirely of Euclid.

The choice of Euclid's Elements for school study was probably partly due to the headmasters of most schools having a classical background, (the books form a classic masterpiece) and mainly because of their reputation for developing the powers of logical reasoning. However Euclid's work was not written for schoolboys and it became apparent that "much time was being wasted, and that boys were not learning any geometry and were not receiving a training in logic."[23] The Schools Inquiry Commission of 1868 also reports a dissatisfaction with the use of Euclid to teach geometry.

In the same year J.M. Wilson, the senior mathematical master of Rugby School, produced his textbook on geometry, specially designed to be more suitable than Euclid for the grammar schools. The book sold well and aroused great interest (and some criticism).

As a result of the dissatisfaction with Euclid, several teachers got together and formed the Association for the Improvement of Geometrical Teaching. It very soon ceased to confine its attention to geometry and in 1897 changed its name to "The Mathematical Association." This body has had a great influence on the teaching of mathematics right upto the present day through "The Mathematical Gazette" and other publications. One effect of their influence was that by 1888 Oxford and Cambridge allowed proofs other than Euclid's, provided that Euclid's order was not violated, in all geometry questions.

The next development was in 1901 when the Teaching Committee of the Mathematical Association was formed. It proposed a reform of the whole school curriculum in elementary mathematics, and not simply geometry. Eventually in 1903 the major examining bodies agreed to accept any proof of propositions instead of the standard Euclid proof, so long as these formed part of a systematic treatment of the subject. This led to many textbooks being launched, written from the new point of view. Some authors and teachers misunderstood the position and there was a small amount of confusion in the examinations for a time. This caused a certain amount of (fortunately unsuccessful) reaction against the reforms.[23]

An important event was the publication in 1909 by the

Board of Education of Circular 711 on the Teaching of Geometry and Graphic Algebra, which had great influence on the teaching of these topics. It was followed by two comprehensive volumes in 1912 entitled "Special Reports on the Teaching of Mathematics in the United Kingdom." These thirty nine papers were on every aspect of teaching and covered the whole age range from the infant school to open scholarship work. They did not become widely known probably due to a combination of the time needed to read them thoroughly and also because of the outbreak of the First World War.

Another notable event was the formation of the Science Masters Association at the turn of the century. Its growth in size and influence helped widen the scope and efficiency of science teaching and indirectly encouraged the study of mathematics as the "queen of sciences."

The Mathematical Association was also growing at this time and it continued to increase its influence through the wide circulation of the Mathematical Gazette and various reports. One of these reports in 1919 was significant in that it considered the place of mathematics in a liberal education,"a boy's educational course at school should fit him for citizenship in the broadest sense of the word.......in so far as mathematics is concerned, his education should enable him not only to apply his mathematics to practical affairs but also to have some appreciation of those greater problems of the world, the solution of which depends on mathematics and science."[23]

The interval between the two World Wars seemed to be one of consolidation. The Teaching Committee of the Mathematical Association produced several reports on Geometry (1923,1938), Mechanics (1930), Arithmetic (1932), Algebra (1934) and several other general documents on the aims and objectives of teaching mathematics. The number of pupils taking the School Certificate rose from 23,000 in 1918 to 77,000 in 1938 and 90% of these offered Elementary Mathematics as an option.

The Spens Report[25] of 1938 however criticized the way mathematics was then taught. It said there was a tendency to stress secondary rather than primary aims, and to emphasise extraneous rather than intrinsic values, "instead of giving broad views (mathematics teaching) has concentrated too much upon the kind of methods and problems that have been sometimes stigmatised as "low cunning"."

The Norwood Report[24] of 1941 was kinder. It acknowledged the great progress made in the teaching of mathematics during the present century, but hoped that further progress would be made, especially in the first three years of the secondary school, when a pupil's attitude was crystallised. The Committee suggested that all pupils should take mathematics for at least three years in "sets" to allow for different abilities. It recommended that many pupils would be better employed on a course in mathematics less exacting than the usual "Elementary Mathematics", where stress could be laid on practical illustrations and applications. Arithmetic would include some elementary trigonometry, the algebra would include the formation of arithmetical problems in algebraic language, and the geometry would include no formal proofs but some investigation of geometrical properties. For all pupils it recommended more ""use of the globes", involving a little spherical geometry with some astronomy and navigation, which are of increasing interest in an age of flying."[24]

Both reports seem satisfied with the way mathematics had become more unified over the years. The Spens Report[25] maintains "the various branches of the subject have coalesced, dead matter has been pruned away; the course has gained in unity and embraces content which some years ago was reserved only for advanced students." This is difficult to understand as examination boards still insisted on separate papers for Arithmetic, Algebra and Geometry, and hence school timetables still had separate lessons for these three with probably different teachers.

Dissatisfaction with the existing syllabuses and examination papers, particularly for weaker pupils, was growing however. Geometry was still felt to be too extensive and formal. In 1943 a conference was called by the Cambridge Local Examinations Syndicate to consider an alternative syllabus in geometry that they had prepared. All the examining boards and the Mathematical Association were represented. The conference decided that the Cambridge suggestions were in the right direction, but the principles behind the proposals for geometry could be equally applied to the whole subject and that the traditional division of mathematics into Arithmetic, Algebra and Geometry was too restrictive at school level. A small committee under the chairmanship of Professor G.B. Jeffery was set up to prepare a new syllabus.

Six months later the committee presented its suggestions to a second meeting of the Conference which decided to issue the new syllabus for the consideration of the examination boards as an alternative to the existing syllabuses. All of the examining boards accepted the new "alternative" or " Jeffery" syllabus with slight modifications and set up examination papers for it. This coincided with the 1944 Education Act, which made it compulsory for pupils to study some mathematics. The new syllabus was not of a lower standard, although it was easier because heavy manipulation in arithmetic and algebra was cut out and formal geometry reduced.

However it was more exacting due to the fusion of geometry with trigonometry and the inclusion of simple ideas of calculus. The demands for the removal of barriers between branches of mathematics were met and emphasised by including mixed questions on each paper. This very significant option became more and more popular, until today, when only one board, London, offers mathematics examinations under the three old headings, and this syllabus is for use by overseas candidates only.

The next major development came during the nineteen-sixties, the so called "Modern Mathematics Revolution", and it coincided with the development of comprehensive schools. For a long time mathematicians had been critical of school mathematics as a foundation for further study. Professor H.R. Pitt wrote in the Mathematical Gazette[32] that "in a quite literal sense, students often do not know what the subject is about." He said that after leaving school most pupils were not fully aware of the axiomatic and deductive nature of mathematical reasoning, the axiomatic foundations of its main branches or the structural relationships between them. Associated with this was a general weakness of logical skill and uncertainty about the nature of applied mathematics and the relationship between mathematics and the physical world. Professor Pitt makes several recommendations including teaching "the basic mathematical concepts (sets and set operations, correspondence, functions and mappings, order, algebraic structure, etc.) as soon as they can be understood."[32] Several new mathematical projects were devised along these lines as a result of comments such as these.

The title "Modern Mathematics" is sometimes a confusing one as there are two main interpretationsof this term. There is the modern approach to mathematics and the modern content of a mathematics syllabus. Most of the experimental schemes of work that have been developed in this and other countries have both modern approaches and modern contents, which are of course both related.

Throughout history there have probably always been teachers who have devised new and interesting methods of teaching. However most teachers until recently used the traditional method of rote learning and formal methods. This theory was ideal in the conditions prevalent at the end of the nineteenth and beginning of the twentieth centuries. With extremely large classes, untrained teachers and severe discipline, schools were seen as "knowledge shops" and teachers as "information-mongers". Children learned everything by rote, lists, dates, etc. Mathematics consisted largely of learning tables of number and the tables of length, volume weights and some of Euclid's elements. At the primary school, the 11+ examination when it was introduced consisted entirely in tests in English and Arithmetic. There was a natural tendency for the teaching in these subjects to be limited to the preparation for the examination which had a constricting effect, as children were drilled in techniques, often without meaning or understanding.

Those pupils who succeeded at the 11+ passed to the secondary schools where a similar pattern dominated the programmes there, designed to lead the children through the Schools Certificate, Matriculation or later , the G.C.E.

Hence there were two factors which have adversely affected the development of mathematics in schools. It is perhaps the most suitable subject in the curriculum for examination work and also most of the secondary and external examinations have been dominated by Universities and their traditional methods. The "Modern Mathematics" pioneers however saw mathematics not as a set of rules to be learnt, a set of techniques to be mastered and a set of facts to be regurgitated at will, but that the aim of the study of mathematics was the <u>understanding</u> and recognition of certain numerical and spatial relationships and the development of a feeling of satisfaction from this recognition. Several influential mathematicians designed experimental courses to achieve these objectives.

The most famous experiment was the S.M.P. (School Mathematics Project).[15] It was instigated and dircted by Professor Thwaites and resulted from a conference he called in 1961.

The Oxford and Cambridge Joint Examination Board agreed to set an O level examination paper for the S.M.P. and the first papers were set in 1964. Out of seven schools originally taking part it was significant that six were Public Schools. Various reasons have been suggested to account for this; these schools may have been able to attract more able mathematics teachers, or they may be more responsive to change than other secondary schools, or, more likely, their staff had more free time and money available with which to experiment compared with their secondary counterparts.

The S.M.P. syllabus itself was initially concerned with O level and A level examinations leading on to more advanced mathematics. The inclusion of a particular topic depended largely upon its relevance and requirement by modern day application. Many of the ideas and topics have been extended since through the S.M.P. letter series textbooks for less academic pupils, but primarily the approach was designed specifically for able pupils.

A quite different approach to modern mathematics was taken by Professor Skemp of Leicester University, in his Psychology and Mathematics Project.[15] It was not concerned primarily with introducing "modern" topics as was S.M.P., but with whether the concepts taught were fundamental to the understanding of mathematics. However, since most types of modern mathematics are largely concerned with mathematical structure, in both schemes there are common topics such as sets, mappings, functions, number systems etc. This Leicestershire project prepared pupils for the old syllabus 2 of the Joint Matriculation Board which is now syllabus C.

Cyril Hope of Worcester Training College initiated another syllabus of a modern approach to mathematics. This was the Midland Mathematical Experiment.[15] It was concerned in the construction of a new syllabus which:

(i) takes notice of contemporary mathematics

(ii) includes contemporary uses of mathematics in industry, science, etc.

(iii) puts mathematics into a setting which the pupils recognise as within their experience of the twentieth century.

(iv) is taught in the light of educational developments of the past thirty years paying due attention to providing background experience, aiming at insight into structure and encouraging pupils to recognise the patterns into which mathematical ideas fall.

A number of new topics are included, but the main innovation here was the giving of opportunities to pupils to investigate various mathematical situations themselves. This project tries to avoid giving the pupils the impression that the whole content of mathematics is already prescribed and that their problems are not important or they have nothing to find out. The pupils studying this course meet occasions when they are able to say or write what they think with a good chance of achieving some originality based on sound thinking accompanied by reason.

The above three projects typify the approaches taken by modern mathematics pioneers. Other projects combine elements from these, such as S.M.G. (Scottish Mathematics Group), M.E.I (Mathematics in Education and Industry), and others.

There have been many criticisms of aspects of modern mathematics. D.A. Quadling[33] writing in the Mathematical Gazette in 1975 suggested that as a result of the so-called "reforms" there was "an increasing danger that the trivial, the irrelevant and the plain wrong will become permanent features in our mathematics syllabuses- at least until the reformers of the next millenium try once again to restore sanity and balance." He finds disturbing the ease with which codification of a course into a list of topics for examination can distort the teaching of the subject.

He agrees that the basic concepts of "set" and "function" help to give a more balanced, unified view of mathematics, but cannot see the point in asking, in what is for many candidates their last ever examination in mathematics, questions such as:

"If $\mathcal{E} = \{1,2,3,4,5,6\}$, $R = \{1,2,3\}$, $S = \{4,5,6\}$, What is $R' \cap S$?"

He suggests that teachers use the idea of a set in teaching mathematics, but it should not have the status of an examination topic.

There is also the pedantry which some people have mistaken for the essence of modern mathematics. It was helpful to find out that pupils'errors and misunderstandings sometimes stem from lax use of technical terms, e.g. "area of a circle" instead of "area inside a circle". However it is ridiculous to replace "the function $f(x) = x^2 - 3x + 1$" with "the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2 - 3x + 1 \; \forall x \in \mathbb{R}$". Creative mathematics would soon grind to a halt.

There are other criticisms of aspects of modern mathematics, and it seems that at the present time we are at a turning point. In their recent book "Curriculum Development and Mathematics" Howson, Keitel and Kilpatrick[16] write: "In official circles, there is now little enthusiasm for curriculum development: its stock is low. As a result many large scale initiatives have ended and there has been no support forthcoming for further work."

This is not to suggest the reforms of the nineteen-sixties have failed. In fact the current problems seem to have stemmed from their great success. Many teachers who initially eyed the new programmes of S.M.P., M.E.I. and M.M.E. with suspicion are now finding a place for a more open approach in their own teaching. Almost every pupil studies at least one modern topic during his or her education in a variety of courses.

This proliferation of courses in Britain is very different from most European countries where generally one course is universally followed, although many courses have been developed in the U.S.A. where the separate states are autonomous. However the British approach, while it will take much longer to settle to a generally acceptable common core should eventually result in a more acceptable solution. The proposals for a common examination system at 16+ gives us another opportunity to reappraise the aims of mathematics teaching and the needs of our pupils in the last two decades of the twentieth century. Hopefully the new syllabus will set the content of school mathematics in a framework which finds no place for a dichotomy between "modern" and "traditional" mathematics. Instead we need to combine the best of what both stand for. It will be interesting to see if we get it right this time.

CHAPTER 3

THE VARIOUS ORDINARY LEVEL MATHEMATICS SYLLABUSES AT PRESENT AVAILABLE IN ENGLAND AND WALES.

At present there are twenty four different "straight" Ordinary Level Mathematics syllabuses. This excludes the Ordinary level syllabuses in Statistics, Mathematics and Statistics, Commercial Mathematics, Additional Mathematics (Pure, Pure and Applied, and Pure and Statistics) and Mathematical Studies which are also offered by many boards. This would take the total to over fifty.

These twenty four syllabuses are provided by the eight English and Welsh G.C.E. Boards and by the Mathematics in Education and Industry Project (M.E.I.), the School Mathematics Project (S.M.P.) and the Midland Mathematical Experiment (M.M.E.). The examinations on the three projects are open to all schools although the first two are conducted by the Oxford and Cambridge Schools Examination Board (O&C) and the third by the Joint Matriculation Board (J.M.B.), in accordance with the normal inter board project examination procedure. Apart from these three cases, teachers are restricted to entering their pupils for examinations of the G.C.E. boards(s) with which the school is registered as a centre. In practice therefore the choice of examination is limited.

The Associated Examining Board (A.E.B.) offer six linked mathematics syllabuses. All candidates study a common core syllabus for 50% of the final examination mark. This consists of mainly traditional mathematics but also including some modern topics, taken together with one of the six optional syllabuses: "General A" has an integrated approach to traditional mathematics with the various branches, arithmetic, algebra, mensuration, trigonometry, geometry and calculus being tested separately or together in a question. "General B" is a traditional syllabus without calculus divided into two sections, section A- arithmetic and algebra, and section B - mensuration, geometry and trigonometry being tested separately. (This paper is the only one in England and Wales to have survived the integrating influence). "General C" is a blend of modern and traditional mathematics following the same content as the common core syllabus. "Modern" is an integrated syllabus of modern topics with sets, structure, vectors, matrices, graphs and calculus featuring here. The "Commercial" option presents arithmetic, algebra and statistics in a commercial setting and the "Technical" option relates traditional mathematics to the technical or technical building areas. Calculators may be used only in the optional papers.

The University of Cambridge Local Examinations Syndicate

(Cambridge) offer three mathematics syllabuses. Syllabus B is very traditional with arithmetic, mensuration, algebra, graphical work, formal geometry (including proofs) and trigonometry. There is no calculus. Syllabus C has an integrated approach. The emphasis here is on the "understanding of basic mathematical concepts and their application rather than on skill in performing lengthy manipulations." It covers modern topics and again there is no calculus. Calculators may be used in any of the papers for syllabuses B and C but not paper 1 of syllabus D.

The Joint Matriculation Board (J.M.B.) offers two syllabuses. Syllabus B has a traditional approach and includes calculus. Proofs of geometry theorems are not required. However, "a sound appreciation of their properties is expected". Syllabus C is a comprehensive modern syllabus which was initially based on Profeesor Skemp's Leicester based "Understanding Mathematics" project mentioned in Chapter 2. It includes a large section on probability and a comprehensive calculus part, as well as the usual modern topics.

J.M.B. also set the examinations for the Midland Mathematical Experiment (M.M.E.). This has a very modern approach throughout and includes geometry by vectors, sets, calculus, statistics and probability in addition to arithmetic and algebra. Calculators are allowed in all of the papers set by J.M.B.

The Oxford and Cambridge Schools Examination Board (O.&C.) offer only one syllabus themselves which combines the traditional topics such as formal geometry (with proofs) together with modern topics such as sets and vectors. However there is no calculus, probability or statistics. Calculators are allowed in Paper two only. O.&C. also set examinations for the three projects Mathematics in Education and Industry (M.E.I.), School Mathematics Project (S.M.P.) and the Scottish Mathematics Group (S.M.G.).

The M.E.I. syllabus is in two parts. There is a core syllabus including modern topics such as transformation geometry, matrices, probability and statistics, but also a section on formal geometry based on five basic theorems. Candidates are asked for full statements of these but are not expected to learn the proofs. There is also an optional supplementary syllabus covering elementary ideas of modern algebraic structure, logic, electronic computers and an extension of the statistics part of the core syllabus to include standard deviation and the normal distribution, to make the course more flexible for teachers and candidates.

The emphasis of the S.M.P. examination is on the "understanding of simple basic mathematical concepts and their application". They offer two syllabuses, N and C. Both of these have a common Paper one in which calculating aids are not allowed. There is then a choice for the second paper. Paper 2C allows the use of an electronic calculator (in fact they are essential for it) while Paper 2N allows only the use of tables and slide rule. The two syllabuses are similar except that syllabus C specifically mentions teaching the use of the calculator compared to teaching the use of the slide rule in syllabus N. Also spherical geometry related to longitude and latitude in syllabus N is replaced in syllabus C by flowcharts and iterative processes. Both syllabuses include all of the modern topics apart from calculus and there is no formal geometry or formal algebra.

O.&C. also set examinations for schools which follow the S.M.G. pattern of work. This is based on the syllabus for the Scottish Certificate of Education Examination in Mathematics. The aims of this syllabus are to enable pupils to "express a response to a mathematical situation clearly and logically, formulate a mathematical model and respond to a routine mathematical problem". This comprehensive syllabus includes most modern and traditional topics with the exception of formal proofs in geometry and probability and statistics. A calculator is necessary in Paper 2 for the iteration questions, but is not allowed in Paper 1.

The Oxford Delegacy of Local Examinations (Oxford) offer two syllabuses. In neither may calculators be used. The compulsory part of the first syllabus (4851) is traditional including arithmetic, mensuration, algebra, formal geometry (with proofs), trigonometry and practical applications of these. However there are six optional topics designed to "give some flexibilty of approach to teachers and candidates". These are calculus, probability and statistics, set algebra, inequalities and linear programming, matrices and transformation geometry. No guidance is given as to how many of the optional topics it is advisable to study.

The second Oxford syllabus (4582) is recommended as being "particularly suitable for candidates who have followed any one of a variety of schemes of modern mathematics, eg. S.M.P., M.M.E.,M.E.I., etc." Again the emphasis here is on "the understanding of basic mathematical concepts and their applications rather than on skill in performing lengthy manipulations." It follows the same pattern as the first syllabus in that it has a compulsory part. This includes modern and traditional topics, excluding vectors, non-right-angle trigonometry, formal geometry and calculus. These form the four optional topics.

The University of London Schools Examinations Department (London) provide three syllabuses. Syllabus B has been recently revised and was introduced to "bring together the modern and traditional approaches to Ordinary level Mathematics". It will replace the other two syllabuses (C and D) in 1983. It includes most modern and traditional topics apart from proofs of theorems in formal geometry. Calculators are allowed only in the second examination paper.

Syllabus C is mainly modern. When it was first introduced it was decided to make the syllabus very full and comprehensive so as to give the teachers a relatively free hand in choosing what they would teach. The papers give a wide choice of questions and candidates were not expected to have covered the whole syllabus. The syllabus was then revised to permit a fuller teaching of a smaller number of topics, those topics which had proved unpopular being deleted together with others which were not "altogether in keeping with the spirit of modern mathematics." It is now a blend of mainly modern topics with extra algebra. Both syllabuses B and C allow calculators in the second examination paper only.

Syllabus D is traditional in nature. It includes arithmetic, algebra, graphical work, formal geometry and trigonometry and practical applications. Calculators may be used in either paper.

The Southern Universities Joint Board for School Examinations (S.U.J.B.) offer three syllabuses. Syllabus A is very traditional and includes no modern topics, not even calculus. On the other hand Syllabus B is very modern. In the geometry and trigonometry section it states, "formal proofs are not required, any appropriate method, Euclidean, vector, transformation or matrix may be used." There are three optional topics in addition to the modern core. These are statistics, matrices and vectors, and further geometry and trigonometry for which formal proofs are required. Candidates should have studied two of these optional topics.

The Welsh Joint Education Committee (W.J.E.C.) provide one Ordinary Level syllabus,although they provide two sets of examination papers per session. Candidates take two papers which include questions on arithmetic, statistics, algebra (including modern topics of sets, vectors and matrices) geometry and trigonometry. In addition candidates may also offer an optional paper of two and a half hours either in Further Trigonometry or in Coordinate Geometry and Calculus. I wondered whether sitting an optional paper would increase a candidate's chances of passing the examination (it does not give any details in the syllabus). However the Head of the Examinations Department, Mr. H. Cook assures me; "There is no obvious advantage to be gained by sitting the optional papers other than to provide the opportunity of greater self-satisfaction for the candidates and possibly another chance to demonstrate one's mathematical ability." This unusual idea is not found elsewhere in other boards' examinations.

A summary of the content of the various O level syllabuses provided by all the boards is given in Appendix B.

In the next few chapters I shall look in more detail at the differences in content between the various O level syllabuses, (Chapters 4,5,6). In Chapter 7, I shall consider the methods of examination and interpretation of some parts of the syllabuses as indicated by either the 1980 or 1981 papers (where available).

For the purposes of this investigation I have categorised the syllabuses myself under the titles "traditional", "modern", "traditional/modern", according to the topics listed, and also the approaches advocated. This is a personal division and is, as such arbitrary. However I feel my method is justified as the difference in syllabus content is quite marked, and the majority of syllabuses fit neatly into one of these three categories.

To compare the actual content of these syllabuses I shall look first at the "traditional" then at the "modern" and finally at the "traditional/modern" syllabuses. I will take certain topics and see if they are included and, if so, to what extent. In a later chapter I shall consider the actual questions set on some of these topics to see the way in which the examiners have interpreted the syllabuses.

CHAPTER 4

THE CONTENT OF THE TRADITIONAL MATHEMATICS SYLLABUSES IN 1981.

There are five syllabuses to be considered here. I am including the Oxford syllabus 4851, which has a traditional core with modern options, as it is not necessary to teach any of the options for the final examination (Paper 1 is based on the core syllabus, Paper 2 has five questions from the core and three questions from the optional part; candidates are expected to answer any three questions).

The other four syllabuses are Cambridge syllabus B, J.M.B. syllabus B, London syllabus D, and S.U.J.B. syllabus A.

Algebra.

There is good agreement here between the various boards. All of the syllabuses include constructing, interpreting and manipulation of a formula; the use of fractional and negative indices; common factors, factors of the difference of two squares and trinomial factors; manipulation of fractions; simple equations, quadratic equations (including those with irrational roots) and linear simultaneous equations in two variables.

London also include solution of simultaneous equations with one linear and one quadratic, as well as the use of arithmetic and geometric progressions.

The remainder theorem occurs in Oxford's syllabus only.

Graphical Work.

All of the syllabuses include sketch graphs of relations such as "V varies as x^3", "y is inversely proportional to x", as well as graphical treatment of the function $y = Ax^3+Bx^2+Cx+D+E/x+F/x^2$ where not less than three of the constants are zero. With the exception of Cambridge, all of the syllabuses ask for the determination of the gradients by calculation as well as by drawing. J.M.B. include "the area "under" a graph by drawing", which probably means counting squares or using the trapezium rule.

Calculus.

Calculus is not included in the Cambridge syllabus and it is an optional topic in the Oxford syllabus. Oxford limits the differentiation and integration of algebraic forms to the function $y = A+Bx+Cx^2+Dx^3$. The others include differentiation and integration of any integer powers of x (excluding integration for the power of -1).

All those syllabuses which include calculus mention its application to gradients, rates of change, maxima and minima, area under a curve and applications to kinematics. London and J.M.B. also feature volumes of revolution about both main axes.

Trigonometry.

All of the boards ask for the sine, cosine and tangent of angles between 0° and 180° only. The Sine Rule and Cosine Rule are also included. Three dimentional trigonometry is a general application with the angle between a line and a plane and the angle between two planes in all of the syllabuses, as is spherical geometry, longitude and latitude.

Circle Geometry.

There is good agreement here between the various boards. All mention the perpendicularity of tangent and radius, symmetry properties, the alternate segment theorem, the intersecting chord theorems and the angle properties of a circle including cyclic quadrilaterals.

Oxford also specifically mention "the distance between the centres of circles in contact".

Geometrical Proofs

J.M.B. alone does not require proofs of theorems although "a sound appreciation of their properties is expected". The other boards have selected certain ones from their geometry content which they must consider important, ranging from four in the Oxford syllabus to eleven in the Cambridge syllabus. There is no general agreement over which proofs are chosen, except that all ask for the alternate segment theorem proof.

For example, London ask for these proofs: exterior angle property and angle sum property of a triangle; parallelograms on the same base and between the same parallels have the same area; the angle at the centre of a circle is twice any angle at the circumference standing on the same arc; the alternate segment theorem, the intersecting chord theorem for an internal point and an external point, the relationship between areas of similar triangles; the bisector of any angle divides the opposite side in the ratio of the sides containing the angle.

Some of the Cambridge proofs are quite involved and they are the only board to include proofs for Pythagoras, the Sine Rule and Cosine Rule for any triangle.

Constructions.

S.U.J.B. and J.M.B. do not require formal ruler - and - compasses constructions, although questions may require "accurate drawing". The others include bisection of angles and straight lines, construction of perpendiculars, inscribed and circumscribed circles, 60°, 45°, 30°.

London and Oxford include the constructions of a segment containing a given angle and tangents from an external point to a circle, while Cambridge includes division of a straight line into proportional parts. London also ask for the unusual constructions of a triangle equal in area to a quadrilateral and a square equal in area to a rectangle.

CHAPTER 5

THE CONTENT OF THE MODERN MATHEMATICS SYLLABUSES IN 1981.

There are nine syllabuses to be considered here. I am including the Oxford syllabus 4852, which has a modern core and modern and traditional options, as it is not necessary to teach any of the options for the final examination (Paper 1 is based on the core syllabus, Paper 2 has ten questions from the core and four questions from the optional part; candidates are expected to answer any six questions).

I am also including the S.U.J.B. syllabus B. There are three optional topics; statistics, matrices and vectors and formal geometry and trigonometry. Candidates are expected to have studied two of these, therefore it is possible to avoid the traditional topic which includes formal proofs and spherical geometry.

The other seven syllabuses are Cambridge syllabus C, J.M.B. syllabus C, London syllabus C, M.M.E., M.E.I., S.M.G. and S.M.P. (both syllabuses).

Vectors.

All of the syllabuses include some work on vectors, their multiplication by scalar quantities, addition and subtraction, although it is an optional topic in the syllabuses of S.U.J.B. and Oxford. With the exception of S.M.G. also mentioned is their combination with 2x2 matrices to represent transformations of the Euclidean plane.

Many syllabuses specifically mention the use of the results: (i) $\underline{a}=\underline{b} \Rightarrow \underline{a}=\underline{b}$ and $\underline{a}/\!/\underline{b}$

(ii) $h\underline{a}=k\underline{b} \Rightarrow \underline{a}/\!/\underline{b}$ or $h=k=0$

in proving properties of equivalence, parallelism and incidence in rectilinear figures. However M.M.E. go further. They expect candidates to know about the scalar (dot) product of two vectors and its use in the mensuration of rectilinear figures as follows: $\underline{a}^2-\underline{b}^2 = (\underline{a}+\underline{b}).(\underline{a}-\underline{b})$ and $(\underline{a}+\underline{b})^2 = \underline{a}^2+2\underline{a}\underline{b}+\underline{b}^2$.

M.E.I. and Cambridge include simple applications of vectors to forces and combinations of velocities or displacements, with solution by drawing and by calculation.

Transformation Geometry.

There is no general agreement here between the syllabuses. J.M.B. and S.U.J.B. include the use of 2x2 matrices in geometrical transformations, but not in the context of reflection, rotation, etc. The others include reflection, rotation, translation, reflective and rotational symmetry and, with the exception of M.E.I., enlargement.

M.M.E. also includes shearing, whereas Cambridge, Oxford and S.M.P. include this and also the transformation of stretching. Combinations of transformations are important and pupils are expected to be able to recognise transformations given either directly or in the form of coordinates. The fundamental properties of rectilinear figures can usually be assumed but precise descriptions of the transformations used have to be given, for example, "a rotation through a quarter turn clockwise about the point (3,5)" rather than "a clockwise rotation".

There is agreement overall with those boards involved for the notation, "if T(a)=b, and M(b)=c, then MT(a)=c".

Matrices.

Apart from S.M.G., all of the syllabuses mention the use of matrices as stores of information and include addition, multiplication, the use of matrices in representing transformations in the Euclidean plane, and the calculation of the determinant and inverse matrix of non-singular 2 x 2 matrices. Also with the exception of Cambridge and S.U.J.B., the application of matrices to the solution of simultaneous linear equations in two unknowns.

M.M.E. include, in addition, the multiplication of an nxn matrix by a vector with simple applications to costing problems.

Probability Theory.

With the exception of S.U.J.B., all of the boards include some aspects of probability theory in their syllabuses, ranging from the tabulation and diagrammatical representation of sets of all possibilities, and probabilities of combined events, (using two dimentional possibility tables and tree diagrams), to formal statements of the addition law for mutually exclusive events and the product law for independent events.

J.M.B. extend this latter section to include the generalisation: $P(A\cup B) = P(A)+P(B)-P(A\cap B)$.

M.M.E. alone include the topic of probability distributions from graphical data with related problems.

Sets.

Every modern syllabus includes work on the ideas and symbolism of sets and set algebra. These symbols are standard with all of the boards:

$\cup, \cap, A', \subset, \supset, \mathscr{E}, \{\} = \emptyset, n(A), \notin, \in, \Rightarrow, \Leftarrow, \Leftrightarrow,$

$\{x$: x satisfies a condition$\}$,

representing union, intersection, complement, subset, empty and universal sets, etc.

The use of Venn diagrams in simple, logical problems is also standard for all syllabuses.

In addition, M.M.E. mention the application of sets to simple switching problems and commercial situations, while J.M.B. and London include operation tables for sets with associated ideas of closure, identity and inverse elements.

Networks and Topology.

Only two syllabuses, S.M.P. and London mention this modern topic, and their content is the same. Included are the calculation of route and incidence matrices and their combination, simple applications of the matrices involved, and the idea of topological equivalence of networks involving nodes, arcs and regions.

Statistics.

All of the boards include some work on statistics in their syllabuses, although it is an optional topic with S.U.J.B. Included in all syllabuses are the collection, description, interpretation and criticism of data. M.M.E. specifically mention the interpretation of data collected from simple sample experiments; for example, "as commonly encounted in elementary biology field courses."

Also included in all syllabuses are the graphical representation of data by bar charts, pie charts, histograms and cumulative frequency diagrams. Most syllabuses include the calculation of mean and median and with the exception of M.E.I. and S.M.G., the quartiles and interquartile range.

J.M.B. includes in addition, the mean absolute deviation and determination of percentiles by computational as well as graphical methods. S.U.J.B. include the use of weighted means and S.M.G. the use of index numbers.

M.E.I., in an extension of their basic syllabus, include the meaning of standard deviation and its calculation in simple cases. Also, the use of a simple form of table for the area under the Normal curve. For example, given the parameters of a distribution and the limits of acceptance of a member of it, to find the proportions likely to be rejected as "too large" or "too small".

Graphical Work.

All of the boards include the use of rectangular cartesian coordinates in two dimensions. Only S.M.P. extends this to three dimensions and they also include polar coordinates in two dimensions.

There are several graphical topics common to all of the syllabuses. These are the solution of linear and quadratic equations, simultaneous linear equations, simultaneous linear inequalities, the applications of inequalities to linear programming, and with the exception of M.E.I., finding approximate gradients.

Determining the area under a curve by counting squares or by the trapezium rule is also well supported, with only M.E.I. and S.M.G. leaving this out. London, S.M.G., J.M.B. and M.M.E. include the determination by inspection of zeroes, maximum and minimum turning points, and greatest and least values of a function given its graph.

A few syllabuses specifically mention finding the values of the constants in the equation of straight lines in the form $y = mx+c$, and $x/a + y/b = 1$. Oxford and M.M.E. include the graphs of cos x and sin x, and S.M.G. includes the construction of a "best fitting" straight line graph from experimental data. I was surprised not to find this final topic in more modern syllabuses, as it demonstrates a very useful technique, and it is all too often left to the science department to teach.

Trigonometry.

There is some variation here. All of the syllabuses include the sine, cosine and tangent of acute angles and their application to the solution of right angled triangles. Also most boards include problems in three dimensions to be solved by calculation and drawing, including the angle between a line and a plane. Fewer boards specify the angle between two planes, those which do are Cambridge and Oxford.

All of the syllabuses apart from M.E.I., J.M.B. and S.U.J.B. extend the concept of the ratios from 90° to 360° including the graphs of these functions with related problems on them. Oxford, London and M.M.E. also include the use of radians from 0 to 2π.

The Sine and Cosine Rules are only in the syllabuses of Cambridge and S.M.G., and they are optional in Oxford and S.U.J.B. However a few syllabuses state that candidates may use the rules if they are familiar with them.

CHAPTER 6

THE CONTENT OF A TRADITIONAL/MODERN MATHEMATICS SYLLABUS IN 1981.

There are many syllabuses which have been developed in recent years to try and combine the best of traditional and modern mathematics. These are the syllabuses that I have mentioned previously, those which do not fit readily into either of the traditional or modern groupings.

As it would of course be impractical to combine into one syllabus the whole of a traditional syllabus with the whole of a modern syllabus, compromises have to be made. Some topics must be left out, others treated in a less thorough way.

Even with this limitation the present trend seems to be towards integration. For example, London have recently devised a new syllabus, syllabus B, which it says, "has been introduced to bring together the modern and traditional approaches to Ordinary level Mathematics." This new syllabus will replace both London's existing syllabuses C and D in June 1983. Other boards, for example Cambridge, have been modifying their syllabuses as well in order to present a more consistent content.

The traditional/modern syllabuses are indicated in Appendix B. It is likely that these syllabuses will become increasingly important over the next few years. The sixteen plus examination proposals for mathematics are based on similar ideas. Therefore instead of itemising the differences between these syllabuses as in the previous two chapters, I am going to look at a typical example of the integrated approach, the O&C syllabus 4600. I will give a more detailed account of its content, and where appropriate, its ommissions.

Arithmetic

This section includes the following topics: The ordinary processes of arithmetic including the use of bases other than 10. SI units. Fractions, decimals, ratio, proportion and percentages. The use of logarithms and graphs in arithmetical problems. The meaning, but not the manipulation of, fractional and negative indices. Standard form.

Mensuration.

Included here are: The rectangle, triangle, circle and figures derived from them. The cube, rectangular block, wedge, pyramid, cylinder, cone and sphere. The meaning of density.

Spherical Geometry

Included here are: Length of arc and area of sector of a circle as fractions of circumference and area of a circle. Easy questions on longitude and latitude. The nautical mile and knot.

Algebra

This section comprises: Construction of a formula. Easy questions on the simplification of algebraic expressions, such as might arise in dealing with practical problems, substitution, change of subject of a formula. Solution of linear and quadratic equations and inequalities and of two simultaneous equations. Factorisation, simple algebraic fractions.

Vectors

Included here are: The meaning of a vector, scalar multiples of vectors and two dimensional problems only. There is no work on matrices and related problems on transformations in the Euclidean plane.

Sets

This section includes the notation and idea of a set, union, intersection, complement, subset, empty and universal set. Venn diagrams and their use in simple logical problems.

Graphical Work

Included here are : Cartesian coordinates in two dimensions only. The distance between two points, gradient of a straight line, equation of a straight line in the form $y=mx+c$ or $y-y_1=m(x-x_1)$. Graphs of simple functions including sine, cosine and tangent and inequalities. Linear programming. Area under a curve and sketch graphs are not included.

Constructions

No formal constructions are involved, such as construction of angles without a protractor. However the use of scale drawing to solve practical problems in two dimensions are included.

Trigonometry

This section comprises: Sine, cosine and tangent of angles between 0° and 180° only. Applications to easy problems on heights and distances, and elementary mensuration of plane and solid figures. Angle between a line and a plane (but not the angle between two planes). Area of a triangle = $\frac{1}{2}bc \sin A$. The sine and cosine rules for any triangle.

Formal Geometry.

Six formal proofs are required. These are: The exterior angle property and angle sum of a triangle. The angle which an arc of a circle subtends at the centre is double that which it subtends at a circumference. Angles in the same segment of a circle are equal. The opposite angles of a cyclic quadrilateral are supplementary and an exterior angle equals the interior opposite angle.

The alternate segment theorem. The intersecting chord theorem for an external point ($OP.OQ = OR.OS = OT^2$) and the analogous property for an internal point.

The remainder of the geometry section is similar to that of a traditional syllabus including the angle bisector of a triangle theorem, Pythagoras, the midpoint theorem for a triangle, tests for congruence and similarity, symmetry, area and volume of similar shapes, the angle in a semicircle is 90° and other common circle theorems.

Summary of Main Ommissions.

Topics that have been ommitted which are usually found in either a traditional or a modern syllabus are as follows:

1. Calculus
2. Formal ruler and compass constructions
3. Trigonometry of angles between 180° and 360°, the angle between two planes.
4. The significance of area "under" a graph.
5. Transformation geometry, reflection, rotation, translation, etc.
6. Using vectors to prove properties of equivalence, parallelism and incidence in rectilinear figure
7. Matrix work
8. Networks and topology
9. Statistics
10. Probability

CHAPTER 7

METHOD OF EXAMINATION AND INTERPRETATION OF THE SYLLABUSES BY THE EXAMINERS.

From the previous four chapters it can be seen that there are, in some cases, fundamental differences in content between the various syllabuses offered at the present time. It will be no surprise that there are also major differences in the method of examination.

Only the J.M.B. and S.M.G. give any indication of the specific objectives of the examinations. The other boards just give the aims of the syllabus, and some do not even include these.

J.M.B. gives eight items of knowledge and ability which the examinations are designed to test. These range from knowledge of mathematical terminology, and the ability to recognise appropriate mathematical methods in a given situation, to the ability to make logical deductions and evaluate and interpret mathematical results. These eight items form a hierarchical structure, in which the earlier items are tested most frequently. No weighting is given to each of these abilities as "they are inter-dependent and interrelated."

J.M.B. states that the marks allocated in the examination fall broadly into two categories as follows, "marks for the appreciation and application of an appropriate method, and marks consequential on the application of an appropriate method given for the accuracy of manipulative and numerical work."

The S.M.G. syllabus states that each examination paper will contain questions of varying difficulty to test four ability levels, knowledge (1), comprehension (2), application (3) and analysis/evaluation (4). It gives a reference grid to show the approximate weighting of these ability levels with the four sections of the syllabus as follows:

	Syllabus Section 1	2	3	4	Total
Ability					
1	8	5	3	1	17
2	7	5	1	1	16
3	2	1	1	1	5
4	1	1	0	0	2
Total	18	17	7	3	40

One factor which affects the questions set in examinations is the provision or otherwise of a list of definitions or formulae. Boards which do this explicitly are Oxford, London and M.M.E. Other boards sometimes give the required formulae at the end of the question that needs it. Obviously this affects the type of question that can be set. Candidates cannot be asked to state a standard result contained in the formula sheet but they can be asked a question which requires the use of it.

This particular issue of whether or not to provide standard formulae is another facet of the debate between those who follow the traditional view that formulae should be learnt, and those who take the modern view. This is that providing a list of formulae is a good idea as, if a person needs a formula in real life he may look it up, and it is better to test the understanding of concepts in a mathematics examination, rather than just memory.

Another important difference between the examination papers is the approval by some boards of the use of electronic calculators. This topic is discussed in full in chapter 8. At this stage it can be said there is no consistency here. Some boards such as J.M.B. allow the use of a calculator ("Questions are set which give no advantage to the calculator user"). A calculator is essential in the S.M.P. Paper 2C. Other boards such as Oxford and S.U.J.B. have a total ban on their use. The remainder of the boards fall somewhere in between these extremes. A summary of the regulations of the boards relating to calculator usage is given in Appendix A.

The actual time candidates spend in examination sessions varies in addition from board to board. Most opt for two two-and-a-half hour sessions. However O&C, M.E.I. and S.M.G. have two two-hour sessions. London in their syllabuses B and D have a one-and-a-quarter hour paper 1 and a two-and-a-half hour paper 2.

This means that the total examination time varies from three-and-three-quarter hours to five hours. Putting this another way, it means that a particular pupil may have to have a $33\frac{1}{3}\%$ longer examination time than a pupil at a nearby school who takes his O level mathematics examinations with a different board. A summary of the various times for the examination papers by the boards is given in Appendix C.

Many of the boards have started putting the total marks allocated to a particular question at the side of it, or at the beginning of a section. Some boards such as J.M.B. go further and specify the mark for every part of the question. This is useful in a number of ways. It gives some candidates more confidence when they know exactly how many marks they could gain. Also if a particular part of a question is worth only one mark then there must be an obvious way to tackle the problem, giving the hint to the candidates not to spend a long time on this particular part.

There are many variations on the way the examination papers are structured. Many boards set papers with a first section usually of short questions for about half of the available marks. In the second section a choice is given. Candidates answer three out of five or four out of six questions. If there is an optional part of the syllabus the choice is extended in the second section (three out of seven questions in the M.E.I. papers). Syllabuses which follow this pattern are Cambridge B, M.M.E., J.M.B. B and C, O.&C., M.E.I., Oxford 4851, S.U.J.B. A and B, and W.J.E.C.

A.E.B. and London give a wholly multiple choice paper one, with many short questions. There are twenty five for the A.E.B. syllabus in one hour, fifty in one and a quarter hours for London syllabuses B and D, and sixty in one and a half hours for London syllabus C. A.E.B. continue this with a one and a half hour paper with twenty four short-answer questions, and then a two section paper as outlined above. London follow with a similar two section paper. Some of the other boards already mentioned include a short multiple choice part in the compulsory first sections of their papers.

S.M.G., S.M.P. and Oxford syllabus 4852 set a paper one including up to thirty short-answer questions with the condition that candidates must answer as many as possible in the time allowed. Paper two contains longer questions. Oxford candidates must answer six out of fourteen questions (this includes five questions on the optional topics). S.M.G. candidates must answer as many questions as possible out of the ten given while S.M.P. candidates have a two section paper with a choice in section B.

The Cambridge Syllabus C and D are unusual. Syllabus C paper two contains two sections in the usual pattern. In the second section of the paper one there are twelve questions. Pupils may answer all twelve questions but only their best eight will be counted. This method removes pressure from the pupils of deciding in the examination room which questions are their best ones. However there may be a tendency for a pupil to keep going on to the next question rather than persevering with the ones initially chosen.

Cambridge Syllabus D paper two also contains two sections in the usual pattern. Paper one contains twenty-nine short answer questions with the instruction "all may be attempted". However it does not state how many could be done to gain full marks.

No "reading time" is provided for any G.C.E. O level mathematics examination, unlike many C.S.E. examination papers (ten minutes is the usual time).

Having described the format of the various examination papers, I shall now consider some of the actual questions set. The papers to be used are from the June 1981 examinations, excepting the M.M.E., J.M.B., London, W.J.E.C. and S.U.J.B. papers for which only the 1980 ones were available. Also, only specimen papers for A.E.B.'s new syllabuses were available.

There are twenty four different sets of examination papers involved. As a discussion of each individual question would be more relevant to a much longer study, I shall confine myself to a look at the following aspect of graphical work: The construction of a curve or a straight line from a given equation and the interpretation of information from it. This topic is in all of the syllabuses, modern, traditional and modern/traditional. This does not include linear programming questions which appear in the modern syllabuses only. All of the questions are in the choice sections unless indicated.

The papers for the A.E.B. syllabuses General A and Commercial include one question on graphs. They give all the points corresponding to a relationship of the form $y = ax+b$, ask for the graph to be plotted and then estimation of the values of the constants a and b. There are then further interpolation parts in the question.

The A.E.B. Technical syllabus paper includes a similar question using the form $y^2 = ax+b$, and also another question. This involves the construction of a table of values and the drawing of the graph $y = 1/10(8x-x^2)$.

The area "under the curve" is then asked for, using both the trapezium rule and the mid-ordinate rule.

The A.E.B. General C syllabus paper involves finding the points of intersection of the curve $y = 1/2x^2-2x-1$ with the line $y = 2/3x-2$, and the gradient of the curve by drawing a tangent at $x = 3$.

The A.E.B. Modern syllabus paper includes one question on the plotting of a distance/time graph for a stone thrown vertically from experimental data. The General B syllabus paper does not include any question on this topic.

On the Cambridge syllabus B paper is one question for finding the value of the intersection of the curve $y = 20/x^2$ with the line $2y = x+8$. The Cambridge syllabus C question on this topic consists of drawing the relationship between profit (y) and number of articles produced (x) given by $y = 4x-x^2-1$. The graph is used to find maximum profit and minimum number of articles produced for a given profit. Then a profit line given by y/x is drawn to find the smallest number of articles to be produced to make £1 profit per article.

A compulsory question in the paper one for the Cambridge syllabus D gives the graph of the quadratic equation $y = x^2-5x+5$ and asks for an estimate of the solution of the equation $x^2-5x+5 = 0$, and the gradient of the given curve by the drawing of a tangent at (1,1).

The M.M.E. paper one question involves finding the intersection points of the two curves $y = 20/(x+1)$ and $y = 2+1/4x^2$ and estimating the range of values for which $3 < 20/(x+1) < 6$.

The paper two also has a graph question, this time to plot the function $y = 13+5\sin x^\circ$, and estimate its gradient at a given point.

The J.M.B. syllabus B graph question is on paper one and follows the same pattern as the first M.M.E. question with the functions $y = 5x-x^2$ and $6y+5x = 30$.

There is a compulsory question on paper one of the J.M.B. syllabus C involving drawing the graph of $y = -3\cos x^\circ$ for x between 0° and 360°. The paper two graph question includes drawing the graphs, and finding the intersection points of the curves $y = x\log_{10}x$ and $y = 1/x$.

The O.&C. graph question asks candidates to draw three graphs using the same axes. These are $y = 6/(x+1)$, $y = 5-x$ and $y = 2x$. The range of values for which $5-x > 6/(x+1)$ and the positive root of the equation $x^2+x-3 = 0$ are then to be estimated from these graphs.

M.E.I. set one graph question on paper one. Again three graphs are drawn: $y = 6/(x-3)$, $y = x$, $y = 3x-12$. Only the intersection points are required from the graphs.

On the compulsory part of the S.M.G. paper one, candidates are asked to recognise a quadratic graph from four possible choices. There are no further graph questions.

The S.M.P. compulsory first paper includes reading an intersection point from a given graph of two straight lines. Paper 2N's compulsory section has a sketch of a sine curve. Candidates have to estimate the values of "a" and "b" from the given equation $y = a+b\sin x^{\circ}$. Paper 2C, for which a calculator is essential, asks interpolation questions when the graphs of $y = 7.5D/(7.5-D)$ and $y = 7.5D/(7.5+D)$ have been drawn.

The Oxford syllabus 4852 has a compulsory paper one question which involves recognising a cosine curve of the form $y = p\cos qx+r$ from amongst five alternatives given the graph of the function. The paper two question involves drawing the curve $y = x^3-6x^2+20$ and then using the trapezium rule (or other method) to calculate an area enclosed by the graph.

The Oxford syllabus 4851 graph question asks for the drawing and the intersection points of the curve $y = 3x^2-x^3$ with the line $y = x+1$.

There is a similar question on the London syllabus B paper two but with simpler functions, given by $f(x) = 1/x$ and $g(x) = 4-x$. The graph question in the London syllabus C paper gives a velocity/time graph and asks for estimates including maximum velocity, acceleration at a point, and the distance travelled in a certain time by using integration or an approximate method.

The syllabus D paper one question from London asks candidates in the compulsory section to recognise the graph of the function $y = 2x-x^2$ from five choices. The paper two question involves finding ranges of values of x which satisfy given conditions for the curve $y = x+12/x-4$, and points of intersection with the line specified as having gradient -0.8 and passing through the point (10,0).

On the S.U.J.B. syllabus A paper one is a velocity/time graph question similar to the one for London syllabus C. On the paper two, the graph question asks for the curves $y = x^2-5x+10$ and $y = 6-3/x$ to be drawn. The intersection points are required as well as the minimum point of the first curve by inspection.

Paper two of the S.U.J.B. syllabus B has a graph question with the function $y = 1/5x^3$. The equations $1/5x^3 = 2$ and $x^3 = -20$ then have to be solved. The final part of the question asks for the gradient of a chord of the curve to be estimated.

The W.J.E.C. papers include a question which involves drawing the graphs of the functions $y = -2x^2+5x+4$ and $2y = 3x-1$, estimating the maximum value of the first equation and finding approximate solutions of the equations $-2x^2+5x+2 = 0$ and $-2x^2+5x+4 = 1/2(3x-1)$.

This concludes the examination of the given papers for inclusion of the graphical work specified. On the whole this topic was adequately examined in the optional sections by most boards. However, I would have preferred to have seen more short questions in the compulsory parts of the papers. The drawing of a graph and the interpretation of information from it is a very useful mathematical skill which is relevant to many other school subjects. It is also a topic which can be taught at many levels of understanding, which makes it an obvious choice for inclusion in any proposed sixteen plus syllabus.

CHAPTER 8

THE ARGUMENTS FOR AND AGAINST THE USE OF CALCULATORS IN (i) MATHEMATICS LESSONS (ii) MATHEMATICS EXAMINATIONS.

(i) For several years, electronic pocket calculators have been commonplace in everyday life, at home, business and in higher education. However many schools do not allow their pupils to use them in any lessons. Others allow pupils to use calculators in science but not in mathematics. As there is no doubt that the calculator is here to stay, and very soon most pupils will have access to at least one, it is very important that a concensus of opinion is reached amongst teachers regarding the place the calculator should have in mathematical education at school level. Many teachers are suspicious of them and raise questions about their possible effects on arithmetic skills. Others have accepted them wholeheartedly and become very enthusiastic about new approaches to traditional mathematical topics.

The calculator was accepted in the United States earlier than in Britain and there have been more investigations there on the effects of the calculators in schools. M. Suydam is a director of the Calculator Information Centre at Ohio State University and he carried out an extensive survey on the opinions of educators and teachers in America regarding the use of calculators in schools.[44] No similar survey has been carried out in Britain but the arguments listed for and against are indicative of the attitudes here as outlined in many British articles.

Suydam summarises the most frequently given reasons in favour of using calculators as follows:

(1) They aid in computation,

(2) They facilitate understanding and concept formation,

(3) They lessen the need for memorisation,

(4) They help in problem solving,

(5) They motivate students in mathematics,

(6) They aid in exploring, understanding and learning, algorithmic processes,

(7) They encourage discovery and exploration,

(8) They exist.

The supporters of calculators see them, not only as quick convenient aids to computation, but also as important tools for getting across key mathematical concepts. Kaner (1980)[17] in his article "The Calculator: An Imperative in Secondary Schools" stresses that the calculator has rendered pencil and paper computation almost redundant and claims, "the barrier of poor arithmetic skills, that has kept so many children from effective study in the sciences, geography, home economics, etc.

has now been swept away."[17] He maintains that all that is required is a small supply of calculators in each classroom. Many more examples can be studied in the same time with a calculator this giving the concept a better chance of being learnt.

Blakeley (1980)[5] shows in his article "The Calculator Mathematics Curriculum of the Future" how, although pencil and paper algorithms may be removed from the curriculum, it does not mean that pupils will be introduced to fewer algorithms. Using a calculator, "there are many more powerful and more general, waiting to be explored." He suggests that using the calculator we can teach the Polya method of problem solving, moving from "guessing" i.e. unstructured trial and error, to guided trial and error, search methods, and more general iterative procedures. He states that pupils of the future should be able to use a calculator to help them:
(a) follow an algorithm
(b) modify an algorithm to produce an alternative result
(c) design algorithms, involving them in the analysis of problems.
He gives examples of how this can be carried out.

Motivation can be greater when the child is using a calculator. A good mathematical example of this is given by Halberstam (1978)[14] in his article "In Praise of Arithmetic." He encourages children to look for patterns in arithmetic using a calculator similar to the one below:

$$1 \times 9 + 2 = 11$$
$$12 \times 9 + 2 = 111$$
$$123 \times 9 + 2 = 1111$$
$$1234 \times 9 + 2 = 11111$$

$$1234567 \times 9 + 2 = 11111111$$

Without a calculator the arithmetic involved would be too time consuming. Examples such as these, Halberstam maintains, as well as motivating, "add much to the enjoyment of discovery and the appreciation of number.

Langham (1977)[18] reports this to be particularly true with low achievers in mathematics, pupils of low ability or those who fail to realise their full potential for various reasons. However such claims as these can only be based on impression, as yet there is no clear indication that "calculator" pupils do better (or worse) than "non-calculator" pupils. What is important, however, is that in none of the studies have any adverse effects of calculator use been observed. Some of the investigations claim other relevant benefits as well as improvement of motivation, and improved learning of problem solving techniques, such as helping low ability pupils to compete more successfully with those of higher ability.

From 1976-78, Bell and others from the Shell Centre for Mathematical Education at Nottingham University did research into the usage of calculators in a primary school.[3] They tried out different approaches using calculators, to teach the children early number concepts, symbols and notation, number facts, grouping, place value and notation, fractions, negative numbers, computational skills and applications of number operations. The Report identified seven areas where the calculator could be used profitably in teaching mathematics: forecasting and checking, generation of examples and generalisation, game playing based on the calculator, the allowing of numbers of realistic size to be handled, provoking the study of new concepts, exposing misunderstanding of existing ideas and exploring the calculator itself.

The study concluded by stating the need for long term studies on the effects of calculators, but in their opinion it found: "the presence of calculating machines in the primary school does not appear to prevent children from learning to calculate: on the contrary the calculator appears positively to encourage and aid the process." [3]

At the secondary level the School Mathematics Project

(S.M.P.) Computing in Mathematics Group[11] carried out a similar investigation in seven secondary schools with forty calculators in each. They came to conclusions which echoed the findings at primary level. Moreover the report was the starting point for an important development; the S.M.P. Mathematics O level was made available in "calculator" form in 1979. (see later for a complete discussion of this.).

Many educators feel that calculators will have a profound effect on the design of school mathematics courses in the future, just as the introduction of slates, paper, pencils and later printing did in earlier times. They believe pencil and paper algorithms and topics such as vulgar fractions will decrease in importance while others such as estimation and approximation will increase in importance. New topics like iteration will be included and other topics such as decimal multiplication and division will be taught earlier.

This question of the new content of Mathematics was considered by the working group into the usage of calculators chaired by D.R. Green of Loughborough University, the report of which is in his article "The Content of Mathematics- What should be Retained, What should Go and What should be Added."[12] They decided calculator usage should be taught all the way through school, rather than just from time to time. They point out that calculation efficiency should be stressed, there is no need to use the calculator to work out such questions as 102-97 =5, the "short-cuts" should be taught; "efficiency with such.......... simple calculations is what the citizen of the future requires - and the harder calculations can be left to the machine." [12]

Also, as the calculator saves classroom time there should be no need to exclude topics from the present syllabus for this reason. However more topics could be included due to the extra time available such as statistics, errors and accuracy, finance and perhaps iteration. Trigonometry becomes more meaningful when using a calculator. "Getting the right answer, and doing so before forgetting what the question was, are powerful allies!"

The report concludes by stating that if the calculator is accepted, there will be a change of emphasis and attitude in the mathematical topics rather than a change in syllabus content. It claims in addition, that the introduction of the calculator will help children towards the view that mathematics models the real world, so assisting them to a better understanding and appreciation of the subject.

The other side of the argument is that calculators are not desirable in schools. Suydam (1978)[44] lists the most common reasons given against the use of calculators in schools as follows:

(1) They are not available to all,

(2) They could be used as substitutes for developing computational skills,

(3) They may give a false impression of mathematics - that it only involves computation and is largely mechanical,

(4) There is insufficient research on their effects,

(5) They lead to maintenance and security problems.

The concern is that younger and less able pupils will not gain basic arithmetical skills, and will become overdependent on the calculator, leading to a generation of innumerate adults. Employers, especially in the engineering industry, have complained in recent years about the "decline" in mathematical ability amongst schoolchildren. The "basic" tests they set have been done very poorly. In the opinion of these people calculators will only make matters worse and the only solution is to ban their use in schools completely.

Another school of thought would like to see, not a total ban on calculators, but the postponement of their use until the child has developed some of the basic pencil and paper skills. Blakeley (1980)[5] seems to be of this viewpoint. He suggets that the following "mental" skills should remain:

(1) an ability with single digit arithmetic,

(2) facility with powers of 10,

(3) understanding of place value,

(4) number "sense" which may further be broken down into an awareness of arithmetic operations and awareness of number size.

Further to the last point he considers it sensible to "expect our pupils to say how many pages it is reasonable to find in a paperback novel, a telephone directory, an encyclopaedia, and to have some idea of the floor area of a living room, or a complete flat or house." [5]

One problem which still remains is that of who should pay for calculators in schools, the Education Authority or parents. Especially in the present economic climate it seems unlikely that schools will provide each pupil with one. The school will probably have a few classroom sets but rely on pupils bringing their own, i.e. parents buying them for their children. However, the supporters of this argument will say, this will discriminate between poorer and richer families, giving an unfair advantage to the well-off families who will be able to afford better calculators, and their children will have them at home.

The debate on whether calculators should be allowed in mathematics lessons and under what conditions, if any, will no doubt go on for some time. To some extent the Examining Boards have taken the initiative allowing the use of calculators in several G.C.E. O level and C.S.E. examinations. The next step is up to the schools.

(ii) The question of whether or not to allow the use of calculators in mathematics examinations is a complicated one, as there are many factors involved. The issue has been investigated by a working party led by N.G. Warwick[45] of the University of London School Examinations Department and their findings raise some interesting points.

They suggest it is partly pressure from industry as well as for educational reasons that some questions and papers at C.S.E. and G.C.E. O level do not allow calculating aids. This leads to a consideration of what objectives a mathematics examination is set to evaluate. In a paper which is designed to test arithmetic skill and computational ability should the calculator be allowed? Of course if the calculator was fully integrated into the teaching of mathematics, the objectives of the teacher and examiner would have to change and such a paper would allow calculating aids and test efficiency of calculation, rather than pencil and paper algorithms. The previously mentioned 102-97=5 is a simple example of such efficiency, quicker to work out mentally than using a calculator.

Fairness is also a factor. Should calculators be allowed in examinations if not every candidate has one? Does an expensive machine give an advantage over a basic model?

There are two approaches in operation at the moment to avoid this problem. Some boards such as J.M.B. try to devise questions which do not give an advantage to those candidates with calculators. Their regulations state "no significant advantage" will be gained from the use of calculators. However this is difficult and must inevitably lead to a reduction in numerical content. The Warwick Report[45] mentioned earlier also points out that some questions may now state numerical results rather than ask for them to be calculated, causing familiar topics to be examined from a different starting point and perhaps, increasing the difficulty of questions set. This would mean that a candidate would need to understand thoroughly the concepts involved, and be able to "switch-on" to a problem immediately without doing any preliminary work.

An interesting angle on the question of fairness is taken by the Associated Examining Board. In an interview with "Where"[19] magazine in 1977, they point out that better aids to examination success have always been available to those who can afford them; "good books, good resource materials, better facilities, etc., are all available to some children and not to others." However the Board itself does not allow the use of calculators in two out of the three papers it sets at O level.

The second approach to the problem of fairness is to have two separate examination systems set at the same time, which is the approach advocated by the School Mathematics Project. At O level they set a common paper 1 (no calculators allowed), then a paper 2C (calculators <u>must</u> be used) or a paper 2N (calculators must <u>not</u> be used). This seems to be good short term measure, allowing the choice to be made by the schoolteacher, but eventually a decision will have to be made one way or the other.

The current position of the various G.C.E. Boards on the use of calculators in their O level examinations can be seen in Appendix A. They fall into four groups:

(1) Oxford Local, S.U.J.B. and W.J.E.C. do not allow the use of calculators at all.

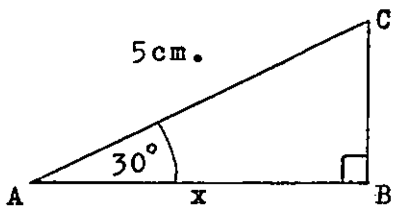
(2) Oxford and Cambridge Joint Board do not allow calculators to be used in paper 1, but do allow them in paper 2. This is also true for the project examinations that they supervise; M.E.1 and S.M.G. (based on the Scottish syllabus A). A.E.B. do not allow them in papers 1 and 2, but do allow them in paper 3.

(3) London and J.M.B. allow calculators to be used in all papers. This is also true for the M.M.E. project that J.M.B. supervises.

(4) S.M.P. have the dual scheme involving papers 2N and 2C as mentioned above.

(Cambridge Local offer three syllabuses, two of which fall into category 3 and one in category 2).

In all "calculator" papers apart from S.M.P. Paper 2C questions are set in which they maintain calculators should not be used, although of course there can be no verification that candidates do not "check" their answers using a calculator. For example in the following question which states that "calculators or tables should not be used";



Find AB.

x can be worked out as $\frac{5\sqrt{3}}{2}$, and then x can be worked out normally using cos $30^{\circ} = 0.86603$ verifying that $\frac{5\sqrt{3}}{2} \simeq 4.33$.

The boards which allow calculators usually have conditions relating to their use. For example, A.E.B. states that no prepared programs in any form, such as magnetic cards etc. must be taken into the examination room. Instruction manuals or other document explaining the function of the calculator or giving formulae or tables of figures are banned. Candidates are responsible for the maintenance of the calculator during the examination and cannot be given advice about repair. Lastly they must be silent.

In comparison with the O level regulations the C.S.E. regulations mainly have a total ban on the use of calculators or else they severely restrict their use in mathematics examinations. Their concern seems to be about standards of numeracy and fairness. Those boards which do allow calculators in some examination papers impose conditions similar to those above.

It would be to everyones' advantage if the Examination Boards could reach a concensus of opinion on the use of calculators. Until they do the teacher should not disregard calculators as a valuable aid to teaching because they do not feature in the examination his pupils are taking. In the words of Blakeley,[4] writing in the Times Educational Supplement, "Mathematics teachers cannot ignore the calculator if they wish to retain credibility in the eyes of their pupils".

CHAPTER 9

THE "SIXTEEN PLUS" EXAMINATION PROPOSALS.

One of the factors which will dramatically affect O level mathematics syllabuses in the future is the proposal for a common system of examining at 16, the so called "16+ examinations."

The development of this lobby can be traced back to 1966, after the C.S.E. had been in use only a short time, although it is true to say that many people anticipated the difficulties which the introduction of C.S.E. would involve. The Joint 16+ G.C.E./C.S.E. Committee set up by the Schools Council at this time (see Schools Council, 1966) reported that the C.S.E. had directly led to "two separate systems of examining the educational attainment of pupils aged about 16."[36] It expressed concern that "the groups (of pupils) do not meet at a clearly divided line... which makes it impossible to be confident about the allocation of border zone pupils to one group or the other." They did not suggest altering the dual examination set-up but envisaged a common system of grades recorded on the same certificate.

This was not accepted for two reasons. It was felt that the C.S.E. grading system had only been in operation for a short while and although there were difficulties convincing people that a C.S.E. Grade 1 was equivalent to a G.C.E. O level pass, it would be premature to alter it. Also many people did not want to replace the pass/fail concept of the G.C.E. O level with grades which described different levels of attainment.

However by 1968 the movement had gained strength. Even common certification or a common grading system was not enough. The Steering Committee of the Schools Council urged them to investigate the possibility of having a <u>single</u> examination at 16+, to cater for at least the ability range then provided for by the G.C.E. O level and C.S.E. examinations. In 1970 the governing council accepted this recommendation and a working party was set up. Their findings were reported in the Schools Council Examination Bulletin 23, "A Common System of Examining at 16+."[37] It investigated many of the issues surrounding the 16+ controversy and optimistically set a time scale for the feasibility studies to be completed and anticipated the first examinations totally under the new system by 1977. Successive Departments of Education have delayed the progress as well as the fact that there have been unforeseen problems to overcome, so the earliest date possible now is 1986.

The Report makes many recommendations and suggestions. It looks at the aims and objectives of a common system and says it "should be based on the view that the curriculum comes first and that the purpose of the examination system is to assess the work and attainments of pupils in appropriate subjects and subject areas." It should combine what is best in both the C.S.E. and the G.C.E. examinations, particularly providing for the continuing development of a variety of modes of examining (eg. C.S.E. Mode 3). Moreover the examinations should be "largely controlled by teachers."[37]

The range of abilities to be catered for by the new examination is recommended to be from the 40th percentile to the top of the range of ability with no upper or lower age limit imposed. The pass/fail concept should not be included in the new system, but there should be an unclassified category. Both the candidate and the user of the certificates should be provided with as much valid information as the examination is capable of yielding, allowing the user to exercise his own discretion in fixing any qualifying levels that may be appropriate to his own purposes. Some safeguard, however, is needed against a "worthless or unassessable performance." In addition there should be two examination sessions per year as with most G.C.E. O level and many C.S.E. examinations at the moment.

The Report discourages any simultaneous introduction of a profile method of reporting results as this could be "damaging to the establishment of confidence in the new system."[37]

Finally further studies were recommended to investigate the technical problems of examining over a wide range of abilities, the optimum number of grades to be awarded (probably between five and nine), and the specific problems relating to Mode 3 assessment.

As a result of this bulletin, G.C.E. and C.S.E. boards linked together in groups of two or three to form "consortia". These set up working parties to carry out the feasibility and developmental studies into fourteen of the sixteen subjects or subject areas suggested in Bulletin 23. A Central Examinations Research and Development Unit[39] (C.E.R.D.U.) was established in 1971 to "be responsible for the planning and coordination of feasibility and development studies covering the whole field of secondary school examinations" (Schools Council, 1975). As well as looking after the overall strategy this body was to undertake some of the necessary research itself.

The Schools Council received many comments on Examination Bulletin 23. These came from G.C.E. and C.S.E. boards, teachers' associations and other interested parties. An "objective summary" of these comments was published as Schools Council Pamphlet Number 12 "A review of comments on Examination Bulletin 23."[39] According to this publication there was no widespread opposition to the proposal for a common examination, in fact there was a great deal of support, although some people doubted the need for change.

Others had reservations about particular features of the proposals, whilst not being opposed in principle to the whole concept.

Taking into account these criticisms and comments the Schools Council published "Arguments for a Common System of Examining at 16+" in 1973. The advantages of the common system are outlined in detail.

Historically there was perhaps a justification for the dual system when there were two types of secondary school, but even then the distinction between "academic" and "non academic" pupils was suspect. (Compare the varying percentages of pupils chosen for the grammar schools by local authorities). A more likely explanation is that there is a normal distribution of ability, and the division of pupils into groups which are given different labels is bound to influence pupils' development and could have a "self-fulfilling prophecy" effect.

Now that comprehensive schools are almost universally accepted it seems illogical to continue with two types of examination in which the overlap is ill-defined. It is this overlap which causes many of the problems, especially the Grade 1 C.S.E. being equivalent to a pass at O level. (A Grade 1 C.S.E. is defined as being of such a standard that the candidate might reasonably have secured a pass in the O level of the G.C.E. examination had he been following a course of study leading to that examination). This has never been accepted by many industrialists, despite the monitoring studies on comparability carried out by the N.F.E.R. who did confirm that C.S.E. Grade 1 was equivalent to an O level pass. Some teachers as well do not accept these findings. In many schools I know pupils following an O level mathematics course are "double-entered" for both examinations if there is any possibility that they will "fail" the O level. The 1981 double entry pupils in one school achieved these results:

11 pupils got Grade D O level, Grade 1 C.S.E.
9 pupils got Grade E O level, Grade 1 C.S.E.
7 pupils got Grade U O level, Grade 1 C.S.E.
3 pupils got Grade C O level, Grade 2 C.S.E.

In other words 30 pupils following an O level course in Mathematics were double-entered for C.S.E. and O level. Twenty seven of these got a Grade 1 C.S.E. This grade means that they would have probably attained a Grade C or better at O level had they been following an O level course. However these pupils had been following an O level course and all of them got less than a Grade C. Only three out of the thirty pupils got an O level pass together with a Grade 2 C.S.E.

Admittedly this is small sample, and further research would be necessary before making any comment but the results are disturbing. A pupil going to an employer with a Grade 1 C.S.E. and a U Grade O level when the examinations were taken at the same time by a pupil on an O level course does nothing for the case that a Grade 1 C.S.E. is equivalent to at least a Grade C at O level.

Another difficulty is that teachers often have to make early decisions on which examination a pupil is going to be entered for, as the syllabuses can be very different. Changes between sets during preparation for examinations can involve a lot of work for teacher and pupil. This is particularly true in the Leicestershire comprehensive scheme where pupils transfer at 14 from as many as three High Schools to one Upper School. The Upper School is dependent on information of varying quality about a particular child's ability. Once a child has been allocated to a set it becomes harder for them to move to a different set due to the pressure of numbers as well as the different work covered.

With two examination systems there are double the administrative duties involved with regard to pupil entry, different candidate requirements and different forms. The school's organisation is disrupted for two sessions during May and June, instead of just one. Finally it is uneconomical to have so many examination boards when just a few would suffice.

The disadvantages outlined above could be avoided if the new proposals were accepted. In particular the advantages to be gained by adopting the new system are listed by the 1973 Report as follows:

(1) the elimination of the need for early decisions on courses leading to different examinations with related problems of class size, distortion of the curriculum and unnecessary divisions between teaching groups;

(2) the simplification of the examination system for users of the certificate;

(3) the elimination of dual entry;

(4) the facilitation of developments in the curriculum;

(5) the easing of problems of administration and organisation within the school;

(6) the reduction of time spent in examinations and of schools' resources devoted to examining;

(7) the more economical and flexible use of resources by examining boards;

(8) at least a partial solution of comparability of standards.

These are all strong reasons in favour of a combined system, and would seem to present an overwhelmingly good case for the 16+ examination to be adopted immediately. However there are many problems yet to be resolved and there are disadvantages of a common system.

One problem is, who is to run the new examination? Both teachers and the universities could be loth to give up the control they have had in respectively the C.S.E. and G.C.E. O level examinations. This is coupled with the fact that the two examinations have a different emphasis. Only C.S.E. courses have the option of continual assessment and monitoring of pupils progress over the whole course, compared with the G.C.E. O levels' almost entirely examination bias.

Critics of the 16+ idea list many disadvantages. They say it will not be possible to maintain the standard of work at present obtained from pupils, especially more able pupils who will not be "stretched" sufficiently with an examination designed to cater for at least the top sixty per cent of pupils (this being the percentage of pupils the C.S.E. and G.C.E. O level examinations were originally designed for). However at several Leicestershire Upper Schools there is a policy of entering at least ninety per cent of pupils for one of these examinations in mathematics alone. The concern is that standards of numeracy and literacy will suffer.

The argument that the solution to this problem is to set, say, four papers for each subject with able pupils taking Papers 1 and 2, "average" pupils taking Papers 2 and 3 and the weak pupils taking Papers 3 and 4, introduces the problem of moderation of the difficulty of questions used in the various papers. Also there is the problem of deciding which papers a pupil should take (back to one of the problems the common examination was trying to eliminate). The moderation between boards of different regions is not so much of a problem, as the various boards in operation at the moment seem to be fairly compatible. This was demonstrated by the Cross Moderation Exercise in Mathematics 1979 conducted by the C.S.E. Boards Research Working Group.[7] They compared G.C.E. O level and C.S.E. mathematics examinations, and their findings showed, on the whole, an acceptable standard of consistency from Board to Board and at the O level/ Grade 1 C.S.E. interface.

Also to be resolved before the 16+ idea is accepted is the question of where A levels would stand in relation to the new examination. Would these remain the province of universities or would some new system be advocated here?

All these points for and against the 16+ system apply to mathematics; even more so in some cases because Mathematics along with English is compulsory for most pupils. Today a mathematics qualification of some sort is required for many jobs, so there is always pressure on teachers to enter pupils for some examination. The concern is, can a reliable and valid examination be set for the majority of the school ability scale?

After trial examinations taken by 68,500 candidates in

1974 the Schools Council analysed the results and in 1976 recommended the common system of examinations at 16+ to the Education Secretary. However a few months later the Education Secretary, Shirley Williams, said that an independent study was needed, and in 1977 the Waddell Committee was set up to investigate fully the feasibility and the implications of a common system. This was seen as a time delaying tactic especially when the Committee just reiterated the findings of the Schools Council and said the system was feasible. However it did recommend central coordination with regional authorities and a target date of 1985. This was later dropped by the new Tory Government in 1980 in favour of loose groups of boards with strong national criteria, including G.C.E. board veto on top grades awarded in any common examination. This then is the situation at the moment, with the Regional Groups producing documents for consultation.

One of these groups, the Midland Examining Group, has just published the Interim Report of the 16+ Subject Working Party in Mathematics[20] which is quite interesting. It is now circulating schools for comments.

In line with other schemes the working party has decided on an integrated approach to mathematics rather than preserve the modern-traditional dichotomy. However they have not yet considered the possibility of providing options such as Commercial Mathematics or Statistics or a separate examination in Arithmetic. Presumably these will follow later.

The group identify four main learning aims, educational, practical, professional and appreciative. These aims should enable pupils to:

(1) increase intellectual curiosity, develop mathematical language as a means of communication, orally and in writing, and explore mathematical ways of thinking;

(2) acquire skills and knowledge necessary in relation to number, measure and space in mathematical situations that the students may meet in life;

(3) acquire skills and knowledge pertinent to other disciplines so that the students can respond to the demands in society and the needs of the world of work;

(4) appreciate the power, pattern and structure of mathematics through the satisfaction and confidence derived from the understanding of concepts and the mastery of skills.

From these broad aims eleven assessment objectives are developed which are defined in terms of behaviour of pupils, such as "the schemes of assessment will test the ability of candidates to recognise the appropriate calculation for a given situation."

The Working Party offer two patterns of assessment for consideration, both of which are based on the award of a grade between one and seven inclusive. The syllabus for the "chain" pattern of assessment is in three parts, lower level, intermediate level and higher level. An example of the difference in levels is in the section on number:

Lower Level - approximations and estimates; significant figures and decimal places,

Intermediate Level - limits of accuracy,

Higher Level - percentage error.

In addition the intermediate level contains the whole of the lower level and the higher level contains the whole of the intermediate and lower level syllabuses. Four papers are set and a candidate takes two consecutive papers as follows:

PAPERS	GRADES AVAILABLE
1 and 2	5,6,7 (4 exceptionally)
2 and 3	3,4,5 (2,6,7 exceptionally)
3 and 4	1,2 (lower grades can be used)

Paper 1 would contain only short questions, but papers 2,3 and 4 would contain longer and shorter questions. Papers 1 and 2 would be based on the Lower Level syllabus, while papers 3 and 4 would be based on the intermediate and higher level syllabuses respectively. An interesting point is that the group suggest there should be no choice of questions within any paper.

The second pattern for assessment termed "petal" involves candidates being entered for a Paper 1 and one of papers 2,3 and 4 or Paper 1 together with school based assessment as follows:

PAPERS	GRADES AVAILABLE
1 and 2	5,6,7 (4 exceptionally)
1 and 3	3,4,5 (2,6,7 exceptionally)
1 and 4	1,2 (lower grades can be used)
1 and school based assessment	All grades, provided the appropriate criteria are met.

The syllabus for the "petal" system of assessment is in two parts, main and extended. An example of the difference in levels is in the matrices section:

Main syllabus - algebra of 2 x 2 matrices including identity and zero matrices.

Extended syllabus - the determinant and its use in testing for singularity. The inverse of a non-singular matrix. Solution of simple matrix equations of the form AX = Y.

Papers 1,2 and 3 would be based on the Main syllabus and Paper 4 on the Main and Extended syllabus. Again there would be no choice in any paper.

Both of these systems have the same problem of comparability of questions and papers. However I feel the "petal" scheme will probably be the one adopted if the proposals are carried out, for two reasons. There is a common paper one for all abilities which will help in the assessment of the papers which follow. Also there is the option of school based assessment which continues the Mode 3 C.S.E. tradition. On the other hand the problems of setting a common paper one across the ability range will be tremendous. Will it be fair for the intelligent pupil who works slowly, and does not manage to finish in time, although he could have done all of the questions eventually? Lower ability pupils could become demoralised in addition by a paper in which they can only attempt fifty per cent of the questions. When sample examination papers are available for both schemes it will be easier to judge their success.

The move towards reducing question choice is probably a sound one. Traditionally this has been upheld for two reasons. Teachers are given freedom to concentrate on the portions of the syllabus in which they are particularly interested, and candidates are allowed to concentrate on particular topics in which they are able to show themselves to the best advantage.

However, the assumption is made, although implicitly, that a candidate will be able to be compared with others taking the same examination, whatever the combination of questions he attempts on an examination paper. This implies a form of comparability between individual questions and hence combinations of questions. Can this really be done in an examination such as Oxford Local O level Mathematics 4852 Paper 2 where candidates answer six out of fourteen questions, which gives 3003 possible combinations of choices? Can the solving of a problem involving areas and volumes be on a par with the plotting of a graph and the solving of quadratic equations with it? Such questions may be attempted by different groups of pupils who perform differently on the paper as a whole. If all the "better" candidates attempt a particular question, the marks will be higher overall on this question and an "adjusting marks exercise" becomes very difficult.

On the other hand, removing question choice, although it improves reliability, raises other problems. If the number of questions is small, say six, each question plays an important part in a pupil's overall mark. Differences between candidates who have revised carefully the questions to come up will be magnified. "Question spotting" would become more important as a result. Also questions which would stretch the brighter pupils would not be included, as the average candidate would not answer such a question very well.

The Midland Examining Group seem to have made up their minds on this issue with regard to mathematics (no choice being available at all). However, final parts of questions will probably be discriminating towards brighter pupils which should alleviate some of the problems mentioned.

This then is the state of things at the moment. The whole subject of a common examination system is still being investigated fully. Decisions have been taken, and a sixteen plus examination seems the only logical way forward. However there is much consultation still to take place, and several factors (such as a change in Government) could still affect such a move. The dual system of G.C.E. O level and C.S.E. will be with us for at least the next five years.

Footnote.

As this work is being compiled, the Times Educational Supplement of 16/10/81 reports that the Government is expected to announce soon a new target date for the merger which will probably be 1987.

CHAPTER 10

CONCLUSION

To draw conclusions on such a diverse topic as mathematics syllabuses and examinations is difficult as thereare so many factors involved. However I feel that my research has brought to light some important points that require amplification.

The chapters on the various Ordinary level syllabuses at present available illustrate the sometimes quite major differences in content and assessment procedures between syllabuses of different boards, and, between syllabuses from the same board. It is difficult to justify such diversity.

A pupil whose parents have to change jobs and move to another area can be at a considerable disadvantage if he has to change from a modern to a traditional syllabus in mathematics, or vice versa. This is particularly true if the change takes place within two or three years of a G.C.E. or C.S.E. examination.

In addition, pupils leaving school at sixteen and going on to further education, apprenticeships, or jobs can find that they have "missed out" topics which are relevant to their present needs. Their new employers or lecturers may be unaware of this. Alternatively, other institutions may find it necessary to give "remedial" courses. This problem becomes even more acute at Advanced level.

In an attempt to resolve this dichotomy between "modern" and "traditional" mathematics, several examination boards have developed integrated modern/traditional Ordinary level syllabuses, as mentioned in Chapter six. However the London G.C.E. board has taken the lead and gone one step further. It has developed a system of combined traditional/modern syllabuses at <u>all</u> levels of G.C.E. From June 1984, there will be only one main route through the examinatin syllabuses offered. The Ordinary level mathematics syllabus B, first examined in June 1980 and the well established Alternative Ordinary combined syllabus are to be followed from June 1982 by a new set of combined Advanced level mathematics syllabuses. These will be based closely on the proposals recently published by the Standing Conference on University Entrance (S.C.U.E.).

Other G.C.E. boards are developing syllabuses and examination structures similar to the one described above, but of course there will be slight variations, if only to justify these boards' independence. Hopefully these new syllabuses will alleviate the problems of the pupil who has to change schools at a critical period in his education.

A partial assessment by multiple choice examination techniques have been included in London's new examination scheme. These enable a comprehensive coverage of the syllabus with a variety of question types. This partially overcomes the problems of question choice in examinations as mentioned in Chapter nine, although London have retained the choice element in at least one paper. This seems to be a good compromise.

There is no doubt that the pocket electronic calculator is here to stay. Present and future generations of schoolchildren will almost certainly make use of calculators at some time in their lives. Most of the G.C.E. examination boards now allow the use of calculators in at least one of their examination papers (the exceptions being Oxford, S.U.J.B. and W.J.E.C.). A calculator is essential now in the S.M.P. syllabus C and also in the rapidly growing C.E.E. subject "Mathematics with Applications" which has been developed and offered by five of the G.C.E. boards on an inter-board basis. These two syllabuses point the way to the future where the sensible use of a reasonably priced calculator is an essential aid to calculation, as a servant rather than a master.

It would be to everyone's advantage if a more consistent approach to calculators could be reached. However I feel that further research is necessary regarding the acquisition of basic skills and also how the calculator can be used more efficiently to improve the teaching of mathematics at all levels of ability. In-service training courses could then be arranged to pass on this information.

The trend towards combined traditional/modern syllabuses and examinations has influenced the proposals for the new sixteen plus examination system. The advantages and disadvantages of such a scheme have been outlined in Chapter nine. To me it seems the only logical way forward from the present G.C.E./C.S.E. difficulties, and the differences in syllabus content mentioned earlier. However there are tremendous problems.

At the time of writing the four groups of examination boards that were set up in England are still finding it difficult to come to agreement on how to organise and divide up the work between them. There are problems over who is to devise, mark and run the new examinations, and how the entry fees should be shared out.[4]

Unfortunately this has diverted energy away from the work of creating draft criteria for the new syllabuses, many of which are still awaiting completion. Concern has been expressed that there will not be enough time with the present time scale to discuss fully the content and aims of these syllabuses.[4]

In the meantime the feasibility studies go on. Over a million joint G.C.E./C.S.E. examinations have already been taken and the numbers entered each year are increasing (by a factor of fifteen per cent in the case of the Northern C.S.E. and J.M.B. joint schemes).

Hopefully more and better consultation with teachers, lecturers and other interested parties will take place soon before the system is finalised. It is a unique opportunity which should not be missed.

We are presently in a wave of recession with related demands for more qualifications to chase the fewer jobs on offer. There seems to be at the present time a "back to basics" lobby and for "questions related to the world of work". I hope that the advances made in mathematics teaching and examining over the last century will not be damaged by attitudes such as these, and that future examination syllabuses can continue to stimulate and encourage good mathematics teaching.

Appendix A

Summary of the Regulations of the G.C.E. Boards on the Use of Calculators in O level Mathematics. (based on 1981 regulations)

Board	Syllabuses	Comments
1.A.E.B.	100,1,2,3,4,5	Not allowed in Papers 1 and 2. Allowed in Paper 3, (candidates take one of six options, 50% of final mark)
2.Cambridge	B	Allowed.
	C	Allowed.
	D	Not allowed in Paper 1. Allowed in Paper 2.
3.J.M.B.	B	Allowed.
	C	Allowed.
4.O & C	4600	Not allowed in Paper 1. Allowed in Paper 2.
5.Oxford	4851	Not allowed.
	4852	Not allowed.
6.London	B	Not allowed in Paper 1. Allowed in Paper 2.

Board	Syllabuses	Comments
London	C	Not allowed in Paper 1. Allowed in Paper 2.
	D	Allowed.
7.S.U.J.B.	A	Not allowed.
	B	Not allowed.
8.W.J.E.C.	0131	Not allowed.
9.(J.M.B.)	M.M.E.	Allowed.
10.(O & C)	S.M.P.	Not allowed in Paper 1. Not allowed in Paper 2N. Allowed in Paper 2C, (candidates take one of 2N or 2C).
	S.M.G.	Not allowed in Paper 1. Allowed in Paper 2.
	M.E.1	Not allowed in Paper 1. Allowed in Paper 2.

Appendix B

Summary of the Content of the Various Ordinary Level Mathematics Syllabuses (based on 1981 syllabuses).

Board	Syllabus	Content
1. A.E.B.	100"General A"	Paper 2 Traditional.
	101"General B"	Paper 2 Traditional.
	102"General C"	Paper 2 Trad./Mod.
	103"Commercial"	Paper 2 Traditional in a commercial setting.
	104"Technical"	Paper 2 Traditional in a technical setting.
		(All of the above have a Trad./Mod. Paper 1.)
2. Cambridge	B	Traditional.
	C	Modern.
	D	Trad./Mod.
3. J.M.B.	B	Traditional.
	C	Modern.
4. O.&C.	4600	Trad./Mod.
5. Oxford	4851	Traditional, modern options.
	4852	Modern, Traditional options.

Board	Syllabuses	Content
6. London	B	Trad./Mod.
	C	Modern.
	D	Traditional.
7. S.U.J.B.	A	Traditional.
	B	Modern, Trad./Mod. options.
8. W.J.E.C.	0131	Trad./Mod., optional extra paper in Trigonometry or Coordinate Geometry and Calculus.
9. (J.M.B.)	M.M.E.	Modern.
10. (O.&C.)	S.M.P.	Modern.
	M.E.I.	Modern.
	S.M.G.	Modern.

Appendix C

Summary of the Total Examination Time Allocated by the Various Boards.

Board	Syllabuses	Total Examination Time
1. A.E.B.	100,1,2,3,4,5	5
2. Cambridge	B,C,D	5
3. J.M.B.	B,C	5
4. O.&C.	4600	4
5. Oxford	4851,4852	5
6. London	B,D	$3\frac{3}{4}$
	C	4
7. S.U.J.B.	A,B	5
8. W.J.E.C.	0131	5
9. (J.M.B.)	M.M.E.	5
10. (O.&C.)	M.E.I.	4
	S.M.P.	5
	S.M.G.	4

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