Mathematics in the sixth form: non-A-Level courses

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MATHEMATICS IN THE SIXTH FORM:

NON A-LEVEL COURSES

by

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ABSTRACT

The English school sixth form, during the last 100 years, has undergone significant change. Section I of this report traces the development of the sixth form and examines why mathematics should be taught to all students after the end of compulsory education.

Section II examines the mathematics courses available to the "traditional" arts and social science sixth form student. The main problem with this group of students, who already have O-level mathematics, is not just one of providing a compulsory course but of producing a course that the student sees as interesting and worthy of his study time. Various Schools Council projects are discussed as well as an evaluation of the mathematical content of A-level general studies.

The situation with the "new sixth" form is different. Most have limited or perhaps no mathematical qualifications. The problem is to provide a course that is acceptable to employers and further education establishments and is within the ability range of the student. Section III discusses both the single subject examinations like the G.C.E., C.S.E., C.E.E. and the proposed G.C.S.E., and the pre-vocational courses like the new C.P.V.E. courses that are usually considered appropriate to the "new sixth form" student.

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CONTENTS

SECTION I

In this section, I shall trace the development of the English School Sixth Form from the academic education of the sons and later daughters of the middle class in the nineteenth century (Chapter 1) to the mixed ability sixth form of today (Chapter 2). Chapter 3 will discuss why mathematics occupies such a large part of the secondary school curriculum, while the final chapter will consider the reasons for increasing the amount of mathematics that should be taught in the sixth form.

CHAPTER 1

The Growth of the English School Sixth Form 1850-1960

Secondary education during the nineteenth century was essentially the education of the sons, and later the daughters of the middle class. This was expressed quite clearly by the Schools Inquiry Commission reporting in 1868. They distinguished three grades of secondary education, corresponding roughly to separate grades of society, and described them carefully not only in terms of the occupations of the parents, but also of the leaving age of the pupils. "Education can at present be classified as that which is to stop at about 14, that which is to stop at about 16, and that which is to continue till 18 or 19; and for convenience we shall call these the Third, the Second and the First Grade respectively. It is obvious that these distinctions correspond roughly but by no means exactly to the gradations of society. Those who can afford to pay more for their childrens education will also as a general rule continue that education for a longer time." (Schools Inquiry Commission 1868; p.16)[77] It is from these First Grade schools that we inherit our view of the nineteenth century sixth form as being identified with academic learning and close connections with the ancient universities and being concerned with character training and social responsibility, ideals encapsulated in the phrase "godliness and good learning".

This division of schools into grades was repeated by the Bryce Commission in 1895. "First Grade schools are those whose special function is the formation of a learned and a professional cultured class. This is the class whose school continues to 18 or 19 and ends in the Universities." (Sec. Educ. Rep. Pt III. 1895; p.138)[5] The academic record of these schools was often formidable.

The Manchester Grammar School was sending 50 pupils a year to university in the 1890's and St. Paul's School was winning more open awards to Oxford and Cambridge than any other two schools put together. Only 8 of Yorkshire's 36 grammar schools, for example, were found worthy of the First Grade. Far more numerous were Second Grade schools, while they might send a few pupils to the local university, their main function was the education of men with a view to some form of commercial or industrial life.

The First and Second Grade schools were intended to meet the demands of all the wealthier parts of the community. Third Grade schools, while not intended for the average working class, belonged to a class distinctly lower, the small tenant farmer or the small tradesman, and whose function was the training of pupils for the higher handicrafts or the commerce of the shop and the town.

Some grammar schools, however, were placed with the Higher Grade Elementary Schools in the Third Grade and to further complicate the matter a few Higher Grade schools stepped beyond their limits, continuing the education of their best pupils for some time after the age of 15, preparing them for scholarship competition or for matriculation at the local university college.

However very few Boards could afford prolonged schooling beyond 14, but what was more disturbing was the number of grammar school pupils who stayed on no longer. Some had no choice, being in schools little more than Higher Elementary and even Second Grade schools, probably through lack of funds, found difficulty in providing more than the barest amount of advanced work.

The Education Act of 1902 gave Local Education Authorities powers to provide grant aided secondary education, since so many grammar schools had been crippled by lack of funds. Aid for these schools became available from the rates but in order to obtain this aid after the passing of the free place regulations of 1907 grammar schools had to take 25% of their intake from the elementary schools thus making them accessible to all classes. Although there was vigorous development in secondary education up to World War I, there was much unwillingness by parents to allow their sons to remain at school after the minimum school leaving age of fourteen. Such a struggle to maintain even a 4 year course, due to many students being legally allowed to leave early, as normal in grammar schools meant that there was still no solid base from which sixth form work could grow.

Whereas previously secondary schools had received no grants for pupils over 16, the new regulations in 1907 allowed £5 a year for each pupil aged 12-18, which meant a sharp increase in school resources, but no provision was made for the special expense of advanced work. Sixth form sizes however remained small; "One of the chief duties of secondary schools is to pass on to the Universities a supply of pupils well prepared to do degree work. Yet the number of such pupils was often so small as to make class teaching impossible" (Great Britain. Board of Education 1913)[25] It was clear that while the sixth form had associations of dignity and weight in the public and large urban grammar schools, most grammar schools were hampered by lack of resources and numbers.

In 1917 the Board of Education announced a special grant for sixth form courses which were deemed of sufficient standard as entry for a university honours course. It was not until 1935, that additional grants were given for all pupils under the age of 19 who were following courses beyond School Certificate level. Hundreds of schools benefitted, for the first time, from the new system which provided a strong financial support for expanding their sixth form.

Between 1920 and 1926 the percentage of pupils leaving the secondary schools without sitting the School Certificate had dropped from 50% to 30% (it was still the same in 1938).

Even this was a great improvement on the situation before 1914, but when so many pupils left school early, there was no obvious passage into the sixth form and the advance work in schools was still restricted by the narrow base from which it grew.

Unlike today, employers did little to persuade pupils to stay on at school for a sixth form education. They would usually give no seniority for the Higher School Certificate. A sixth form course was certainly not the sure passport to success that it later seemed to be. There was still strong pressure to regard 16 as the normal leaving age for those not clearly bound for the universities. Also Higher Education was also assumed to be for the few and this assumption, together with the financial burdens involved, did much to check sixth form growth.

Although by 1937, 44% of pupils in grant aided secondary schools had free places it was not always the most able who were chosen. Many free places were rejected by parents unable to support a long schooling. Some able children from poorer homes were never entered at all. The odds were heavily against children from working class homes. The difference widened still more within the school. Even in the late 30's working class children could hardly drift into the sixth form. Entry was unusual enough to need clear evidence of academic ability and uncommon ambition in parents. It was "professional and middle class parents who might see 18 as the age when their own children almost irrespective of ability should leave school". (Crowther 1959; p.62)[12] Sixth formers then were not necessarily the intellectual elite of their age group, but were very much a social elite.

The Education Act of 1944 made it, for the first time, the statutory duty of every local education authority to make available, throughout its area, efficient facilities for secondary education. It was these post-war years which saw the brief flourishing of the classic grammar school sixth form. This increase is shown in table 1.

	Number (1000's)	% increase over 1947
1947	32.0	-
1955	45.0	41
1956	49.0	53
1957	51.3	60
1958	53.2	66

Table 1
Numbers of Pupils aged
17 in all Types of Schools

(Crowther 1959. p.226)[12]

The full employment of these years placed a premium on better qualifications. Parents had less financial worries and were able and willing to keep their children on at school. Employers began to give extra seniority to A-level qualifications. But still the major prize was the passport to Higher Education with grants becoming easier and finally mandatory in 1962 to those with 2 A-level passes. Schools Council Working Paper No. 5 (1965)[75] described the sixth form as a "subgroup which often constitutes a society in its own right". The sixth form was a world of privilege, private study, prefectship and specialisation with entrance depending upon, usually, five O-level passes and the right attitude and ability to benefit from sixth form study.

Although academic ability was to determine the education provided, the measured ability at 11, for selection, was so closely connected to the social background that the grammar schools remained predominantly middle class. The process of social selection continued and intensified during the years 11 to 16, making the sixth form less socially mixed than the main school. In 1958, 95% of all pupils aged 17 were in state aided or independant grammar schools. "The great expansion that has taken place has been on a relatively small and fixed base of those already in secondary schools. ... Among the sixth formers there are a few, but only a few, who completed the first stage of their secondary education to the Ordinary Level standard in Modern Schools and then transferred to a grammar school sixth form. But, although this group is statistically insignificant, it derives importance from the fact that it foreshadows what can be expected when a much larger proportion of boys and girls are in schools where there is the possibility of their taking the General Certificate of Education." (Crowther 1959; p.229)[12] Comprehensive Education certainly widened this base. Not only did it allow more able pupils to take General Certificate of Education examinations as inferred by Crowther, but it has produced a large increase in the sixth formers Crowther called "Sixth formers with a Difference", the "non academic" or "New Sixth". The next chapter will discuss the effect that this increase of pupils has had on the sixth form of today.

CHAPTER 2

The Sixth Form Today

The post war system of 16-19 education was relatively simple with schools concerned with general education leading to School Certificate and Higher School Certificate (later O and A Levels) and Further Education being essentially vocational in character. Today, twenty years on from Crowther the situation is much more complex. G.C.E. work in colleges has developed substantially and full time study for pre-vocational qualifications has become more common. Schools too have changed, with courses of a less academic nature meeting the needs of pupils not taking A-Levels. Fig. 1 shows that the majority of 16-19 year olds who remain in full time education do so in schools.

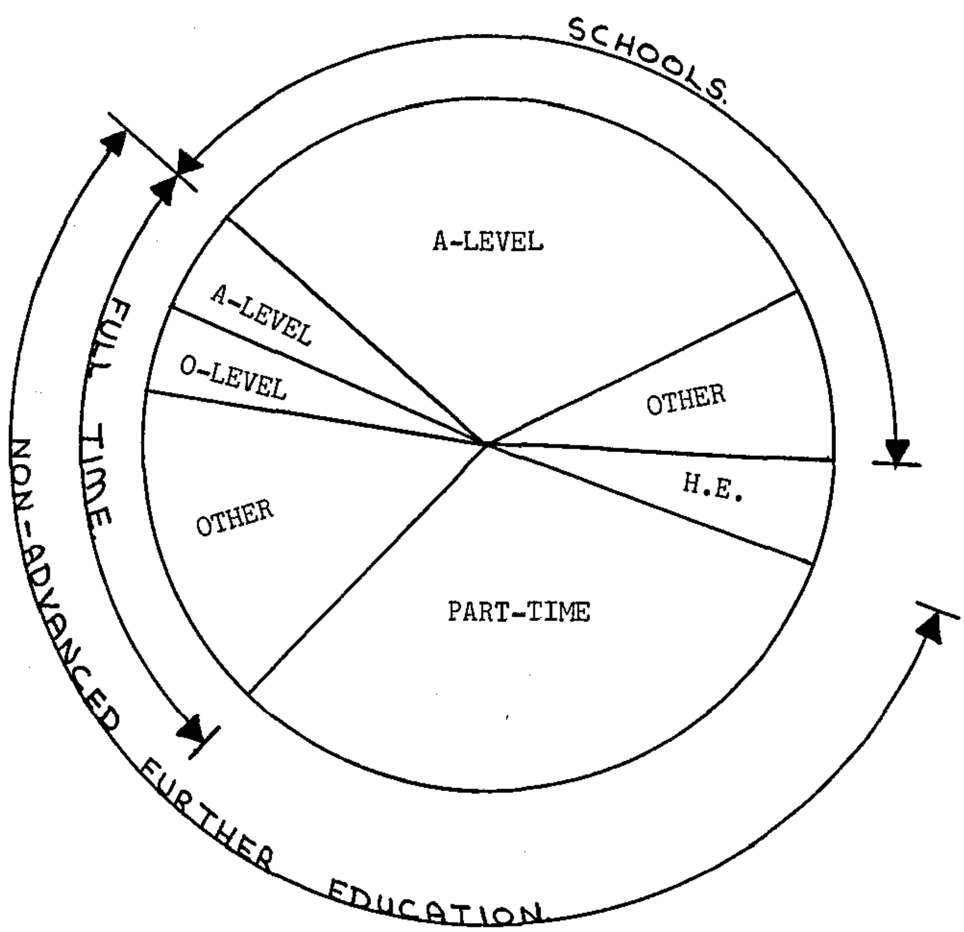


Fig. 1

16-19 year olds in schools, non advanced further education and Higher Education in 1979-80. (Macfarlane Report 1980; p.9)[47]

The number of students taking G.C.E. courses in further education rose by a third from 1973-74 to reach nearly 70,000 in 1978-79. In schools numbers have shown a proportionally smaller rise but increased from 310,000 to 380,000 in the same time. "New Sixth Formers" taking non-A-level courses accounted for one in five in schools in 1979-80. However, although their numbers had grown faster than those of A-level pupils, the total number of students taking non-GCE courses in colleges of further education had grown faster still and at nearly 150,000 was twice as numerous as "new sixth formers" in schools.

The picture that emerges is of former distinctions beginning to break down. Those who stayed at school beyond the statutory leaving age tended traditionally to be academically orientated, reflecting the fact that until secondary school organisation most sixth form pupils were in grammar schools. Now that sixth forms have become more widely available they cater significantly for young people without academic aspirations. Many of these students may need courses with a strong pre-vocational character, such as those leading to City and Guilds of London Institute, Royal Society of Arts, Business Education Council and Technical Education Council qualifications.

Macfarlane (1980)[47] classifies those who continue their full time education into four groups:

"D. those staying on with a view to proceeding to Higher Education in due course;

E. those seeking an essential vocational qualification to fit them to enter employment at some stage up to 18 (but perhaps with the further prospect of proceeding to higher education later if they so decide;

F. those who do not wish to be committed to a specific vocational objective, but who wish to continue their general education, personal development and pre-employment preparation;

G. those who require remedial education to enhance their employment and life prospects."

(Macfarlane 1980; p.13)[47]

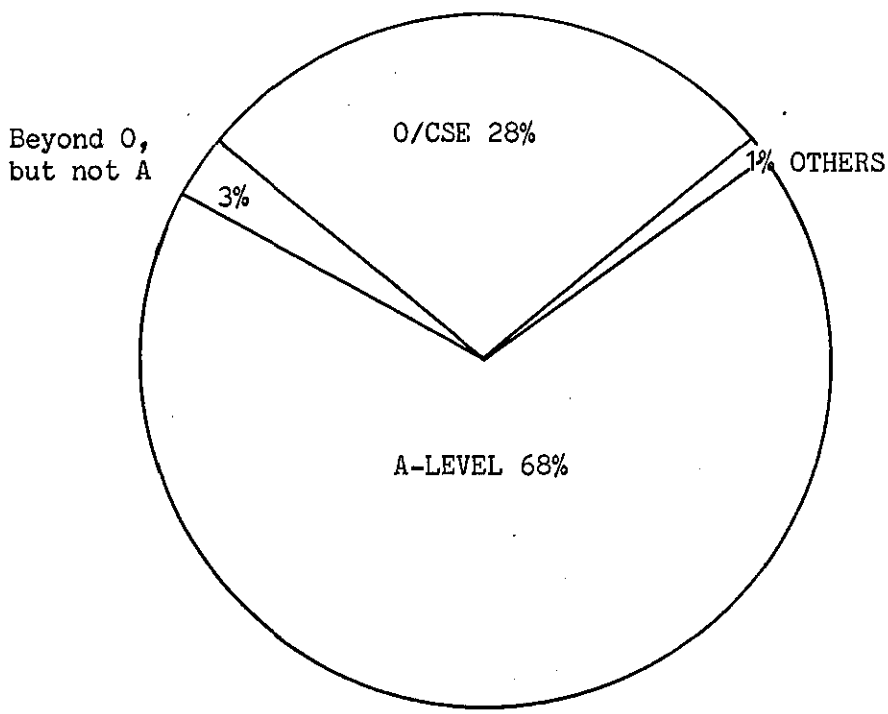


Fig. 2

Courses of Study followed by 16 year old in Maintained Schools

(GB.DES 1981)[30] (Appendix 1)

Group D was Crowther's sixth form and is still a large component of all sixth forms. Fig. 2 shows that 68% of sixth form students are studying for one or more A-level but nearly all will be taking at least one or more O-levels and/or CSE's. The traditional 3-A-level sixth form group will certainly not be the 68% of Fig. 2, but will be mainly in the shaded 34% of Fig. 3 with General Studies or the odd retake being the O-level, plus a few from the A,O and CSE group.

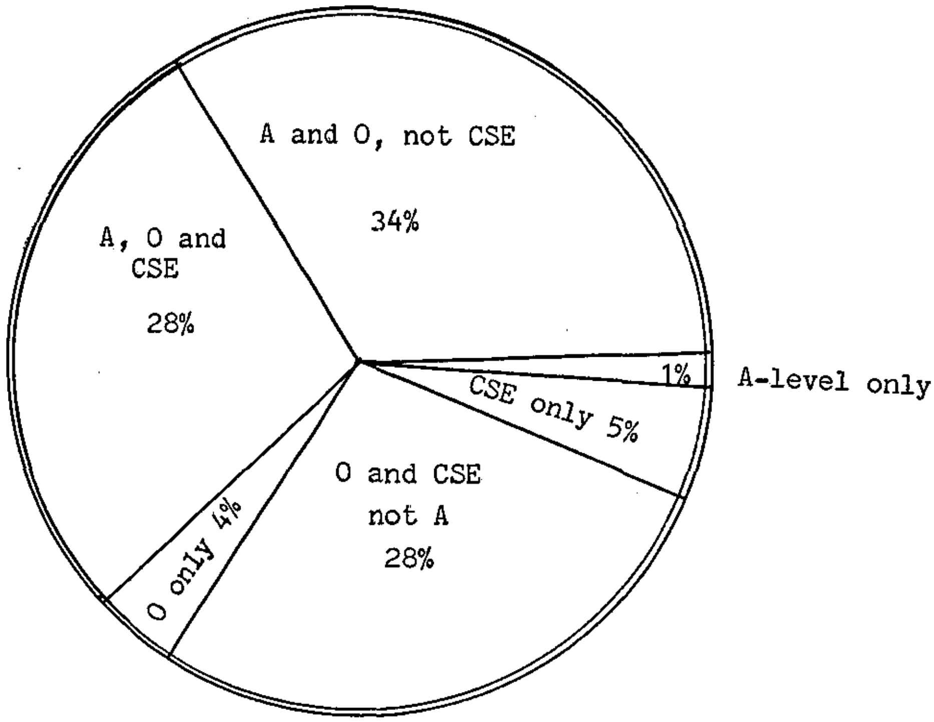


Fig. 3

GCE's and CSE's attempted by school leavers 16 years of age and over

(GB.DES 1981)[30] (Appendix 2)

On the borderline between Macfarlane's groups D and F, come A-level candidates who have no clear view of the future. Some are able and motivated generally, some are able but unmotivated, while some will struggle throughout the course and be among the 35% who obtain not more than one pass grade at A-level. They are also the candidates most likely to drop out. According to Watkins (1982)[105] about 12% of those who embark on a 2 year A-level programme never reach the second year of the course.

Group E is found predominantly in technical colleges although a few sixth forms offer vocational courses such as secretarial and City and Guilds Foundation Courses, but these are ones usually situated at a distance from a technical college.

Group F has been described for some time as the "New Sixth". For some, the sixth form is used to repeat examinations failed a year earlier and for some, whose development has been slower, to attempt qualifications that abler pupils have achieved in five years. Many are well motivated and not only succeed in their one year course, but then embark on a 2 or 3 A-level course. Some, however, have already reached a plateau, and will neither retrieve past failure nor achieve in a sixth year what others have achieved in five.

Group G includes young people who have had difficulties in reaching reasonable standards of numeracy, communication and life skills and need time to catch up. The Warnock Report (1978)[104] emphasised the need for providing post-16 education for this group with special educational needs. They need special attention and the provision made for other groups may not always be adequate for their needs.

Obviously, any categorisation of students involves over-simplification. There is a continuum of ability and motivation and an infinite variety of individual needs. According to Pring (1980, p.25)[59] the sixth form is "a continuum from those who are capable of academic and disinterested study to those who may be barely literate and who are there because there is nowhere else to go."

Unfortunately there is still no parity of esteem between courses in the 16-19 sector. Whatever the reason, whether it be due to ignorance about other courses or just a high respect for academic qualifications, the majority of pupils, parents and employers see passing the GCE examination as showing greater all-round ability than success in any other school or technical college examination. Many young people pursue academic courses beyond the age of 16 when their interests and prospects would be better served if they opted for vocational or pre-vocational courses. Such choice requires positive decisions and is only likely if students and parents are fully and fairly informed about the range of opportunities available and the implications of their choice. Schools have a moral duty to discuss and advise students as to what might be in their long-term interest and either provide independently or with further education colleges the range of courses required to prepare all students adequately for future life.

CHAPTER 3

Secondary Mathematics

There seems general agreement that every pupil should study mathematics up to the end of compulsory schooling. Table 2 shows that at least 96% of boys and 88% of girls were studying mathematics up to the age of sixteen in 1966. The raising of the school leaving age in 1972 will almost certainly have increased the slightly lower fifth year figures so that today about 98% of pupils will be studying mathematics up to the end of compulsory education.

(i) Boys and Girls Schools

Year of Study	1	2	3	4	5
Boys	98.6	98.4	98.6	98.7	97.3
Girls	98.4	98.7	97.9	97.8	92.5

(ii) Co-Educational Schools

Year of Study	1	2	3	4	5
Boys	96.3	97.2	97.5	97.8	96.4
Girls	97.1	97.6	97.7	95.0	88.2

Table 2

Percentage of Secondary Pupils studying Mathematics

(G.B.DES 1966)[31]

Together with English, mathematics is regarded by most people as being essential for normal life. As such, mathematics and English are nearly always given the largest amount of teaching time. It is now intended to consider why so much time is given to mathematics in the pre-sixth period in the hope that this will shed some light on the paradoxical situation that occurs after the age of sixteen i.e. that only a few studens continue their mathematical education after this age.

The teaching of mathematics has often been justified on the basis that mathematics is a tool of everyday life. Plato[57] stated that mathematics is the "one thing that all occupations, practical, intellectual, or scientific make use of - one of the first things we must all learn". Boehm (1958; p.8)[4] sees mathematics as an essential element in the cultural heritage of the Western world. The Newsom Report (1968; p.51)[53] states that "mathematics and science, closely related, are the basis of the most revolutionary of recent developments in society and in the everyday lives of all young people". Cockcroft (1980, p.1)[8] found that many perceived the usefulness of mathematics in terms of the arithmetic skills which are needed for use at home or in the office or workshop" and "as the basis of scientific development and modern technology".

Bell (1937; p.2)[2] argues that this is only one of the many functions of mathematics. "It must not be imagined that the sole function of mathematics is to serve science. Mathematics has a light and wisdom of its own and it will richly reward any intelligent human being trying to catch a glimpse of what mathematics means to itself."

A further justification for teaching mathematics has evolved from the conception of mathematics as a game with its own rules and without any responsibility to external criteria. From this viewpoint, mathematics is mainly a matter of puzzles, paradoxes and problem solving a sort of healthy mental exercise. Wheeler (1963; p.140)[107] sees the fault in this argument as "it would tend to predetermine an approach that would treat mathematics devoid of its applications". Cockcroft (1982; p.171)[8] also sees applications as an essential part of mathematics teaching. " We believe that all A-level mathematics courses should contain some substantial element of 'applied mathematics' so that all who are studying the subject, whether for its own sake or because of its usefulness as a 'service subject' are able to gain a balanced view of mathematics." However, this and the previous justification both carry with them the strong implication that mathematics is, can be, or ought to be enjoyable. "Games and puzzles" according to Cockcroft (1982; p.2)[8] "provide enjoyment and also, in many cases, lead to increased mathematical understanding."

The close association between mathematics and logic has given rise to the justification of mathematics teaching as a form of training in logical thought. Cockcroft (1982)[8] however, does not see mathematics as the only subject providing logical thought and "the need to develop these powers does not in itself constitute a sufficient reason for studying mathematics rather than other things".

A further justification often given is that mathematics is the only true international language. Taylor (1969; p.76)[95] states that "a language is more than just a means of communication, it is a vehicle of thought. No real thought is possible without a language, and with a rich and varied language such as mathematics both clear thinking and accurate communication of ideas are easier." According to the Mathematical Association (1955)[51] "a child lives in a community which uses the language of numbers and measurements and shapes in everyday talk". While Cockcroft (1982; p.3)[8] states "that foremost among them (the reasons for teaching mathematics) is the fact that mathematics can be used as a powerful means of communication - to represent, to explain and to predict". There are other reasons for teaching mathematics but the above reasons, although some cannot stand alone, provide a powerful argument for providing a mathematical education.

No doubt strong arguments could be given for the teaching of other subjects, but why does mathematics command a larger part of the total curriculum than most other subjects? Firstly, mathematics has historically occupied a larger amount of curriculum time because it was one of the few original subjects on the curriculum.

Plato (p.38)[57] suggested the study of five mathematical disciplines (arithmetic, plane and solid geometry, astronomy and harmonics), while in medieval times the Quadrivium consisted of arithmetic, geometry, music and astronomy. Also the elementary schools, especially after the Revised Code of 1862 with "payment by results" concentrated on the "3 R's". Other subjects have had to compete for what curriculum time was left: however, they have not been able to catch up with mathematics, presumably because the reasons for teaching it are too important for it to be displaced.

Secondly, the time allocated for the teaching of mathematics has seldom been seen to be enough to make the majority of students proficient. This is undoubtedly due to the very real difficulty most people experience in the learning of mathematics. Cockcroft (1982; p.67)[8] sees mathematics as "a difficult subject both to teach and learn". Newsom (1968; p.48)[53] agrees, but states that "most people have a greater capacity for mathematical understanding than they are aware of, and a large reservoir of undeveloped mathematical competence exists among youngsters of ordinary ability which good teaching and an enlightened approach could reveal". Newsom was no doubt referring to the "Rote" method of learning which gave little understanding of mathematical principles. Cockcroft (1982; p.70)[8] states: "There are certainly some things in mathematics which need to be learned by heart but we do not believe that it should ever be necessary in the teaching of mathematics to commit things to memory without at the same time seeking to develop a proper understanding of the mathematics to which they relate." Both Gogne (1965; pp.142-156)[22] and Skemp (1971)[80] have developed the theory that knowledge is organised in hierarchies of principles. Hart (1981)[32] has shown this empirically and established hierarchies of understanding for eleven major topics which appear on the school mathematics curricula. Consequently, if a student is to understand mathematics, he must be presented with the subject matter in a logically sequential way.

The final part of this chapter will look at the implications for the mathematics teacher and subsequent chapters will discuss how present sixth form mathematics courses cater for these needs. According to Cockcroft (1982; p.4)[8] "the most important task of the mathematics teaching is to make each pupil aware that mathematics provides him with a powerful means of communication". As such the relevance of simple figures, letters, charts, graphs and diagrams to the outside world needs understanding. This relevant understanding should enable the pupil to see the beauty of mathematics and according to Servais (1971; p.16)[78] "the fear and anxiety often raised in them should be removed". Quadling (1978; p.171)[60] also saw the emphasis for many teachers and users of mathematics "as moving from 'mathematics as a discipline' and 'mathematics as a culture' to 'mathematics as a language' and specifically the language of patterns".

Secondly, mathematical applications now appear in many subjects on the school curriculum. In some cases this use is new to the subject and according to Hart (1981)[33] "the traditional complaint of science teachers - that the pupil cannot do the mathematics - is now being echoed by teachers in many other disciplines such as geography and economics. An assumption by other subject teachers that mathematics is composed of comparatively easily mastered skills leads to a gross mismatch between the demands of the mathematics and other subject lesson. Hastily teaching the required mathematics may solve the immediate problem for the subject teacher, but it is the same mathematics that will break down and cause the child to fail later". According to Cockcroft (1982; p.4)[8] "it is the task of the mathematics teacher to provide each pupil with such mathematics as may be needed for his study of other subjects".

Thirdly, mathematics should be made enjoyable and not expressed as a difficult subject, but unfortunately this is not usually the case. Every subject has its hidden curriculum and according to Lawton (1978; p.117)[44] from mathematics lessons pupils may learn, in addition to, or sometimes instead of, the intended skills, that

"(i) Mathematics is largely done in silence and often in solitude.

(ii) Mathematics is done with pencil and paper in classrooms in schools.

(iii) Mathematics is not part of everyday life or popular culture; what is learnt in school classrooms is rarely reflected in the press, or on radio or television programmes.

(iv) Mathematics is inescapably boring and unremittingly difficult.

Not all of these, if any, would be accepted as desirable and intentional outcomes of learning mathematics."

Finally, mathematics should be taught with regard to the requirements of adult life, employment and further and higher education. A full discussion will not be given as these were the terms of reference of the Cockcroft Report. However, the major points of this Report will be briefly summarised.

<u>For Adult Life</u>

"We would include among the mathematical needs of adult life the ability to read numbers and to count, to tell the time, to pay for purchases and to give change, to weigh and to measure, to understand straightforward timetables and simple graphs and charts and to carry out any necessary calculations associated with these." (para. 32)

"to have a feeling for numbers which permits sensible estimation and approximation" (para. 33)

"Most important of all is the need to have sufficient confidence to make effective use of whatever mathematical skill and understanding is possessed whether this be little or much." (para. 34)

For Employment

"Almost all the mathematics which young people need to use, whatever their job, is included within all the existing O-level and CSE Mode 1 syllabuses." (para. 68)

"it is possible to summarise a very large part of the mathematical needs of employment as a feeling for measurement" (para. 85)

For Further Study

The standard of entry is diverse from no formal mathematical qualification to good A-level grades.

It has been shown in this chapter why mathematics occupies such a substantial part of the school curriculum up to the age of 16 together with the implications for the mathematics teacher. Subsequent sections will consider how sixth form courses cater for these needs.

CHAPTER 4

Mathematics in the Sixth Form

In most developed countries students continue their mathematical education to a much later age than that at which compulsory education finishes in England. In Germany students follow courses leading to the Abitur (qualification for entry to Further Education) and must study some subjects from each of the three subject areas - arts, mathematics, science and social science. In France there is a common core involving mathematics for all students in the year equivalent to the English first year sixth form. A similar situation occurs in the United States of America where 70% of the pre-university age group are involved in mathematics compared with 12% in England[10]. It is often argued that the need for other countries to continue to teach mathematics is due to the lower level of mathematical attainment of pupils of age 16 in these countries. In the case of the U.S.A. the students may be of a lower academic standard than those in this country. Although this was shown by the C.E.E.B.[9] tests, the same is not true throughout the developed world. Wilson (1983)[109] found that in the U.S.S.R. where pupils follow a common core until they are 17, "roughly 95% of children are taught mathematics including calculus to a standard comparable to O-level". In Germany Wood (1983)[110] found that "A comparison of standards between British and German pupils shows that British pupils appear to trail a full two years behind their German counterparts," ... "although mathematics standards between the top 25% are broadly similar."

If as Chapter 3 suggests, the study of mathematics is so important, a point taken more seriously in many other countries, why then is so little mathematical education provided for the non-specialist sixth former in this country? The answer is aptly summed up by Walker (1980)[103] who sees the sixth form student as only interested in examination success. A similar view was given by the DES (1982; p.1)[28] who stated that "During the last decade many changes in mathematical provisions in the sixth form have been called for; ... some of them arising from the ... increasing demands for a mathematical qualification. It is difficult to see how with examination results determining entry to a career, whether direct or via Further Education, the situation is likely to change, unless of course the whole system of 18+ examinations is changed." At present the mathematical requirements are either O-level/CSE as a general qualification or A-level for a particular career or as a University/College entrance requirement. Little, if any, credit is given for intermediate mathematical qualifications. The remainder of this chapter will consider why mathematics should be taught to the non-specialist sixth former.

The work of Piaget (see page 73), implies that for a pupil to grasp a particular concept he must have reached that level of understanding. Although this increase in understanding may be only small (note page 75), it may nevertheless allow previously unknown work to be mastered as well as allowing fresh work to be undertaken. According to Wilson (1983)[109] this idea is taken to an extreme in the U.S.S.R. "with little attempt to adopt a spiral development. The attitude seems to be that it is better to wait until you can do a topic "properly" and then do it. Similarly the Mathematical Association (1965; p.2)[50] have stated that "mathematics is an attitude, a way of thought and demands a certain level of maturity. To understand the relevance of mathematics today and its relation to other fields of knowledge, mathematics must be studied seriously. This need not involve a high degree of technical specialisation; it does mean, however, that the study of mathematics should not be cut off suddenly after O-level".

Secondly, it is often wrongly assumed that the level of mathematics required by the non-specialist is aptly catered for by O-level. Even Cockcroft (1982; p.19)[8] implies this when he states that "almost all the mathematics which young people need to use, whatever their job, is included within all the existing O-level and CSE Mode 1 syllabuses", although he also says that "Some art graduates who had gained O-level passes in mathematics were nevertheless so aware of a lack of confident understanding of the subject that their career choices were seriously reduced as a result of their determination to avoid mathematics". Howson and Eraunt (1969; p. 6)[38] also say that "many non-specialists will later assume managerial and administrative roles and it is essential that they should not do this entirely without knowledge of the applications of mathematics to commerce and industry and what is even more important, lacking the confidence to attempt to follow an argument expressed in mathematical terms. In addition, university students reading such subjects as geography, biology and economics are faced with an increasing need for mathematics." The latter point is further emphasised by the ever increasing number of post O-level text books with titles such as "Mathematics for Geographers", Mathematics for Business Studies", etc.

Finally, is the need to produce primary teachers who are able to teach mathematics confidently and with enthusiasm. It is during these early years that "a child's attitude to mathematics is often becoming fixed and will determine the way in which he will approach mathematics at the secondary stage. He may thoroughly enjoy his work in mathematics, or he may be counting the days until he can stop attending mathematics lessons. ... He may be well on the way to mastering some of the mathematician's skills, or he may already see mathematics as an area of work which he cannot understand and in which he always experiences failure".[8] The Royal Society Report (1976)[62] says that

"During his professional life a teacher of mathematics may influence for good or ill the attitudes to mathematics of several thousand young people, and decisively affect many of their career choices. It is therefore necessary that mathematics should not only be taught to all pupils, but well taught. All pupils should have the opportunity of studying mathematics in the company of enthusiastic and well qualified mathematics teachers".

Mathematics in primary schools is almost always taught by the class teacher and only a minority of primary teachers study mathematics as a main subject during their initial training. "In spite of the great efforts which have been made over recent years, it is still the case that too many teachers have to teach mathematics without knowing enough about the subject, or about current ideas of teaching it." (DES. 1979)[27] "It was disturbing to find that in nearly a quarter of the primary school lessons, teachers showed signs of insecurity in the subject being taught... This insecurity in some cases led to the choice of unsuitable materials, unrealistic tasks for pupils in which the teacher could offer little help and failure to recognise opportunities to extend or deepen children's understanding and skills." (1983)[96]

"Most courses of training for primary teachers aim at equipping them to sustain a broad curriculum as class teacher. But the expertise of the primary teacher is heavily weighted towards the humanities and aesthetic subjects. Although mathematics is a central element in courses of training for primary teachers, it is often the most difficult for students." (1983)[96] In 1978 Gardiner (1978)[23] stated that at primary level "it is well known that about 40% of new entrants to the profession each year have not even O-level mathematics". Although all new entrants must now possess at least and O-level or equivalent in mathematics, many will have found O-level difficult and most will have not studied mathematics during their two years in the sixth form. As such if all students had had some type of mathematical education beyond O-level it would be expected that they would enter their B.Ed. more numerate and correspondingly graduate into the teaching profession with greater mathematical knowledge.

It has been shown in this chapter why mathematics should be taught to the non-specialist sixth former. Subsequent chapters will consider the types of mathematics courses available.

SECTION II

In this section I shall be considering the mathematical education of the traditional 3 A-level non-mathematics specialist. It is assumed in this section that the academic students being considered have a pass in O-level mathematics. Although some of this group will not have obtained this qualification and will be either retaking or converting from CSE these courses will be considered in section III. As stated on page 12, "the sixth form is a continuum" and to divide it into exact sub-groups is not possible.

Chapter 5 looks at the traditional A-level curriculum with particular reference to "specialisation" and "minority time". Chapter 6 looks at the various general studies, A-level courses of the different examination boards and examines their mathematical components. Chapter 7 examines in detail the mathematical component of the Joint Matriculation Board advanced level general studies syllabus while chapter 8 considers other post O-level courses. The final chapter in this section makes recommendations as to the type of mathematics course best suited to this target group.

CHAPTER 5

The Traditional A-Level Curriculum

"When a pupil enters the Sixth Form he becomes a specialist - that is to say, the subjects of his serious intellectual study are confined to 2 or 3. They are usually interlocking and are chosen, at least for potential university candidates, with an eye on faculty requirements." (Crowther, 1959; para. 377)[12] With the amount of knowledge available to man specialisation is unavoidable if one is to become an expert in a given field, but the main argument is when it should begin. In England it begins at 15 or 16 with the selection of 3 advanced level subjects. Neither in Western Europe nor in North America is the situation similar. On the continent of Europe, there is no question of dropping altogether the study of languages or history or mathematics or science, while in the United States of America the seventeen year old in High School takes a wide range of subjects.

Many educationalists argue against sixth form specialisation. It is said to be too early because a decision has to be made before a pupil is old enough to know his own mind. Early preferences are often reflections of the qualities of the teaching, or of the teacher rather than of the subject or the pupil's needs and it is surely better to leave a decision, which is of lifelong importance, until it can be made on more substantial ground. Specialisation is also said to begin too early because it takes place before a pupil has had time to reach in his other studies and in his own psychological development the stage at which his education can safely be left to him to develop out of his own interests.

The other major complaint against specialisation is that it is on too narrow a front. Hirst (1965)[35] argues that education is concerned with the development of knowledge and this knowledge consists of distinct types. "The development of mind has been marked by the progressive differentiation in human consciousness of some seven or eight distinguishable cognitive structures, each of which involves the making of a distinctive form of reasoned judgement and is therefore a unique expression of man's rationality. This is to say that all knowledge and understanding is logically locatable within, I suggest, mathematics, the physical sciences, knowledge of person, literature and the fine arts, morals, religion and philosophy." (Hirst, 1974)[36] Hence, the educated man should be familiar with all the different kinds of knowledge. Without such general education, the specific in depth studies will doubtless produce a clever person, but according to Hirst not an educated one.

This philosophical view is shared by the Schools Council. When it was established in 1964, one of the first decisions made was that the changing character of the sixth form and the changing society in which pupils would live demanded a complete reappraisal of the sixth form curriculum and examinations. According to Working Paper 45 (1972)[74] there was overwhelming feeling among the professional bodies, industrialists, Colleges of Education and Colleges of Further Education that sixth form students were getting too narrow an education and that far more attention ought to be given to literacy, numeracy and the ability to speak well. The best preparation for the world of work, it was felt, was one which was broad enough and general enough to enable people to be adaptable later, since it is widely accepted that the worker of the future will need to be retrained more than once during his working life. Therefore, there is no point in sending him out from school or college so specialised as to be unable to meet the challenges and changes of the future.

Working Paper 45 (1972)[74] (also S.C.P.21)[71] spells out the elements necessary for a balanced education as:

Communication Skills	1.	Literacy (and related oracy).
	2.	Numeracy.
Knowledge and Understanding (Cognitive)	3.	A knowledge and understanding of the natural and physical environment.
	4.	A knowledge and understanding of human beings and their social environment.
Affective Qualities	5.	A developing moral sensibility.
	6.	A developing aesthetic sensibility.
Expressive Qualities	7.	Fashioning the environment (the creative arts and the creative aspects of technology).
	8.	Physical education in its widest sense.

The argument for specialisation is that pupils are ready and eager by the time they are sixteen to get down to serious study. Crowther (1959; para.333)[12] calls this "subject-mindedness". The student has been looking forward to being a science specialist or an historian and whatever hinders specialisation to him is a waste of time. He wants to get down to the serious study of some one aspect of human knowledge which with the one-sided enthusiasm of the young will obscure all other fields of endeavour.

A second argument is that concentration on a limited field leads naturally to study in depth. "As he goes deeper and deeper, he acquires self-confidence in his growing mastery of the subject. ... No longer does he accumulate largely isolated pieces of information and separate unrelated skills. ... His subject is no longer something that he must learn, he begins to feel himself the master of it." (Crowther, 1959; para. 388)[12] Since the process of intellectual growth demands a great deal of concentrated time, Crowther (1959; para. 391)[12] argues that this virtually enforces specialisation because the time left for other studies is bound to be small.

Although, as seen, Crowther supports specialisation he rejects the argument that specialisation at school is necessary if pupils are to be brought up to the necessary standards to get university places. "We are, however, very much concerned to make it clear that an arbitrary fixing by the universities of the stage that a boy or girl should have reached by 18 (standards which are determined by counting back from the degree standard) should not be allowed to determine the nature and content of education given in pre-university years since less than half of boys and girls in the sixth form will be going to university." (Crowther, 1959; para. 392)[12] However, Watkins (1982; p.32)[105] sees this as the main reason why the sixth form curriculum has remained so specialised. "If A-levels were to be replaced by a less specialised examination, the universities have argued it would be necessary either to extend the first degree course to four years, or to acquiesce in a significant reduction in academic standards, neither of which could be contemplated." Also according to the Schools Council, the main argument causing the rejection of their N and F proposals was as Watkins described, although additional pressure came from many teachers who did not want to lose the more advanced part of their teaching.

Although, as mentioned, Crowther was in favour of specialisation, he did see the need for some compensatory balance to be introduced into the curriculum, by what he called "minority time". "One of the main functions of minority time should be to provide a complementary element in the curriculum, and if it is properly used, we think it will provide a remedy for some of the most serious of the justified complaints about the dangers of over specialisation." (Crowther, 1959; para. 397)[12] The common elements were summarised as religious education and all that goes to the formation of moral standards, art, music and physical education, alongside literacy for scientists and numeracy for arts specialists. The provision of general studies, as it is usually called, is rarely accorded high priority when scarce manpower resources are being allocated. In 1976 the General Studies Association reported that a planned course of general studies is not usually the norm. Schools Council Examinations Bulletin No. 38 (1978)[65] concluded that by far the most common arrangement appears to be a programme which depends upon the interests of and enthusiasm of teachers who are available to become involved once the other demands of the curriculum have been met.

However, if the balanced two facet model of Crowther or the eight facet model of Hirst is to be met, both specialist and general courses have to make their contribution. It is in the next two chapters that we shall discuss the role of the mathematics department in helping to provide this balanced curriculum.

CHAPTER 6

General Studies

The pioneers of General Studies, like Geraldine Lack of Roseberry Grammar School, saw it as the intellectual core of an intellectual education appropriate to the highly motivated and able sixth former who flocked to schools like hers in the 1960's. Although GCE General Studies examinations were available, they were not seen as essential for student motivation or course credibility. The last decade has seen the number and variety of GCE General Studies examinations together with the number of pupils entering them on the increase. Table 3 shows a 106% increase in the pass rate from 1971 to 1981.

Year	No. of A-level passes in General Studies
1971	11,842
1974	14,753
1975	16,512
1976	22,311
1977	19,643
1978	20,244
1979	22,625
1980	23,140
1981	24,349

Table 3
(G.B.DES 1981)[30]

Until recently only two Boards, J.M.B. and Cambridge, offered A-level General Studies. Now there are five: J.M.B., Cambridge, Oxford, A.E.B. and London. Walker (1980)[103] sees this trend towards examinations as an attempt to legalise the non-examination time spent in the classroom to teachers, the community and the sixth former.

He sees the students' reason for entering the sixth form as more utilitarian. "They do not seek education, they want qualifications. They do not look for intellectual stimulation, they want good jobs."

It is in such an atmosphere that General Studies has to be taught. Not all schools follow GCE courses; some like St. Austell Sixth Form College, have thriving non-examination courses, but the trend is towards an A-level qualification, although there is still much argument for and against examining this subject. The decision by most Universities and Polytechnics to accept General Studies as a third A-level if only two stated subjects are required, should make more courses examinable[24].

The Schools Council Sixth Form Survey (1971; p.177)[73] found that 67% of sixth formers were taking some kind of General Studies or discussion lessons. However, Table 4 shows that only 13% of the Arts sixth and 16% of the Social Science sixth were studying mathematics. The figures for the Science sixth cannot be used to determine the percentage of students studying mathematics, as most will be studying this subject at A-level, the 11% being mainly the Physics, Chemistry and Biology students. The Sixth Form survey (1971; p.38)[73] also found that "The A-level general studies examination was primarily being taken as an additional subject by those who were preparing three or, in a few cases, four other subjects for A-level. Of those taking a total of four or more A-level subjects, 58% were taking general studies, while only 2% of those taking three A-levels were doing so and only 1% of those taking one or two A-levels". These figures seem to infer that Advanced level general studies is taken almost exclusively by the traditional A-level sixth former and that more general studdies courses are becoming examinable. As such the remainder of this chapter will investigate the mathematical content of the various general studies syllabuses.

Subjects taken in general studies and non-examination courses	Science	Arts	Social Science with or without Arts	Mixed
	%	%	%	%
Art	23	24	26	23
Music	31	38	33	28
Drama	9	9	11	7
Spoken English	6	7	8	6
English	50	31	31	42
Languages	18	11	9	15
History	16	13	19	13
Geography	5	6	4	6
Religious instruction	72	73	74	72
Mathematics	11	13	16	10
Science	17	27	34	22
Sociology	5	5	10	4
Philosophy	11	10	16	10
Politics	13	13	17	16
Economics	8	6	8	8
Sports	85	84	84	87
Handicrafts	10	8	6	8
Current affairs and social problems	42	47	48	43
Local community service	17	16	17	18
Weighted base: Pupils taking three or more A levels	(1095)	(951.5)	(311)	(433.5)

Note: Subjects taken by less than 5% of pupils are omitted from this table.

Table 4

Subjects taken in general studies non-examination courses by pupils taking 3 or more A levels. (School Council, 1971)[73]

1. London

According to the syllabus the subject "concentrates upon the interdisciplinary characteristics of General Studies ... this consists of topics that require a range of disciplines for their understanding and full appreciation". (Univ. of London, 1984; p.271)[99] The topics examined are Science and Society, France - an area study, the Modern Movement and the potentialities and limitations for mankind on our earth. The only mention of mathematics is:

a. In objective 1 "It is expected that a student who completes the course of General Studies will be able to use a variety of communication skills: numerical, verbal and literate." (University of London, 1984; p.271)[99]

b. The syllabus mentions "The treatment of the topics selected ... often requires an introduction to the methods of quantitative analysis." (University of London, 1984; p.272)[99]

c. Section 1 (time allocation 30 mins) will consist of 3 short-answer questions based on attached abstracts. One of the questions will normally involve numeracy.

Examination of specimen papers[98] shows that the numerical component consists of the interpretation of statistical data given by pie charts, bar charts, graphs and tables, together with simple numerical manipulation including percentages. Although it cannot be denied that the interpretation of data is an important aspect of mathematical education, it is nevertheless only a small part of the mathematics curriculum.

2. Associated Examining Board

This course is based on the broad perspective "Man in his Environment". As with London, little mention is made of mathematics, except for:

a. "The course of study will enable students to extend the depth and range of their knowledge and skills in relation to literacy, numeracy and oracy." (AEB, 1985; p.91)[1]

b. Objective (g) states "interpret the use of mathematical information". (AEB, 1985; p.91)[1]

c. An essential component of a course of General Studies is the ability not only to comprehend but to communicate. One of the types of communication given is "Mathematical - by the use of statistics, graphs and histograms, etc." (AEB, 1985; p.91)[1]

d. Section D of paper I will consist of 3 questions incorporating simple mathematical/numerical aspects taken from any section of the Syllabus.

3. <u>Cambridge</u>

This consists of five compulsory papers with paper 4 lasting 2 hours and consisting of comprehension, numeracy and logical reasoning. Examination by the writer of past papers showed one question out of five as requiring mathematical reasoning. It was again considered that the mathematics in this paper was minimal although the introduction of logical reasoning was considered worthwhile and interesting.

4. <u>Oxford</u>

This consists of two papers each of 4 sections. The syllabus states that "Numeracy is not specified as a subject for study, but section D of the second paper consists of 4 tests of numeracy of which candidates must answer one." (University of Oxford, 1984; p.280)[100] The syllabus then specifies the range of mathematical ability the candidate is expected to demonstrate. That is:

a. ability to extract data required for calculation from descriptive paragraphs or tables of data or simple graphs of one variable plotted against another, to cope with redundant data if applicable and to understand the arithmetic processes which must be applied to the data to obtain the required answer.

b. an appreciation of the degree of accuracy which can be justified in the calculation and ability to handle very large numbers (tens or hundreds of millions) without error.

c. the necessary judgement to make sensible decisions based on the answers obtained.

d. the ability to deal with the following mathematical processes:

(i) addition and subtraction;
(ii) repeated multiplication and division to an accuracy of 3s.F;
(iii) percentage in any form;
(iv) slopes or small rotations in the form "1 to x" including the reading of rates of change from a graph;
(v) plotting and interpretation of simple graphs of one variable.
(vi) preparation of simple plans or maps.
(vii) interpretation of codes or simple cyphers, given clear descriptions;
(viii) ratio, proportion and variation;
(ix) an understanding of the commoner systems of weights, measures and money and their standard abbreviations.

Although the mathematical component of this syllabus is exactly defined, examination reveals it to be rather narrow and of a low level of difficulty.

5. Joint Matriculation Board

According to the syllabus the course is designed to permit the widest possible interpretation of general studies so as to allow for the varying needs of pupils, interests of teachers and the resources of different centres. "The examination is intended for pupils who are completing two or more years study in the sixth form after having pursued courses at the ordinary level. In particular it will be assumed that candidates have followed or are following courses at roughly the ordinary level in a foreign language, mathematics and science." (JMB, 1984; p.21)[42] The examination consists of 2 papers each of 5 questions. One of the questions is on mathematical reasoning. Examination of past papers shows the mathematical reasoning question to consist of approximately 20 multi-choice questions where the candidate selects one correct answer from five.

Although the advantages of general studies as an inter-disciplinary course linked to the outside world is recognised, it is not the intention in this report to examine the aims of A-level general studies examinations and to see if the mathematics so examined meets these aims. The reasons why mathematics should be taught after the end of compulsory education have been discussed in previous chapters. It is the aim of this section of the report to see if general studies provides sufficient mathematics to meet these needs. I have in this chapter described the mathematical component of the various Advanced level syllabuses. Although it is accepted that a detailed examination of each syllabus would be interesting and a detailed comparison worthwhile, such a survey is not within the scope of this report.

The examination of the general studies syllabuses and past specimen papers briefly described in this chapter, leads this writer to believe that the mathematical component of general studies papers varies from little more than being able to read and interpret statistical data as in the case of London to being able to answer mathematical questions of about O-level standard as with the Joint Matriculation Board. It was therefore thought that the general studies course did not offer a suitable mathematical course for the traditional sixth form student who already possessed O-level mathematics. As such, it was decided to carry out a detailed examination of the syllabus with apparently the largest mathematical component. This survey of the Joint Matriculation Board syllabus and questions is given in the following chapter.

CHAPTER 7

J.M.B. Advanced Level General Studies

In this chapter I shall investigate the mathematical component of the J.M.B. A-level General Studies paper to see if this syllabus provides a satisfactory level of mathematics for the 3 A-level non-mathematics specialist sixth form student.

It was first decided to consider the type of questions that had been set in recent examinations. Six past papers were investigated. Each question was examined to see if it fell within the range of the modern or traditional O-level syllabus. Table 5 gives the results of this survey. To facilitate recording and to show the breadth of the general studies mathematics syllabus it was decided to allocate each question a topic name. Certain questions required the use of more than one mathematical technique and therefore needed proficiency in more than one topic. Ninety percent of the topics were within the S.M.P. syllabus and 73% were within the traditional mathematics O-level syllabus. Obviously these figures are meaningless if the questions within each examination were of different degrees of difficult. It was therefore decided to test a sample of pupils, to compare the level of difficulty between general studies mathematics and O-level mathematics.

The sample of pupils came from the Henry Fanshawe School, Dronfield, a 14-18 Upper School having a fourth year of 450, a fifth year of 450 and a sixth form of 300. Mathematics is taught in two equal half year bands, each containing 9 sets. S.M.P. is used throughout the 4th and 5th year. Sets I take the O-level in November of the 5th year and the AEB AO examination in the following June. Sets 2 and 3 take O-level in June, while sets 4, 5 and 6 and sets 7, 8 and 9 take CSE mode I and limited grade mode III respectively. The school obtained results above the national average with 37.5% passing (A-C) O-level mathematics in June 1983.

Topic	General Studies						O-level Maths.	
	1977	1978	1980	1981	1982	1983	SMP	Traditional
Quadratic equations	/	/	/				/	/
Solution of equations		/		/		/	/	/
Substituting in equations	/			/		/	/	/
Simultaneous equations	/	/	/			/	/	/
Other simple algebra			/		/	/	/	/
Logs(manipulation)				/			/	/
Logs(Theory of)	/				/			
Ratio				/	/		/	/
Percentage	/			/	/	/	/	/
Metric system				/			/	/
Arithmetic	/	/	/	/	/	/	/	/
Pythagorus Th.			/	/		/	/	/
Volume				/			/	/
Area		/	/		/	/	/	/
Averages					/		/	/
Interest					/		/	/
Geometry of Triangle		/			/		/	/
Degrees of Accuracy						/	/	/
Trig. in surd form	/					/		/
Co-ord.Geometry				/	/			
Graphs	/	/	/	/	/	/	/	/
Operators			/				/	
Indices			/		/	/	/	/
3D figures		/	/			/	/	/
Inequalities	/	/			/	/	/	
Binary	/				/		/	
Probability	/						/	
Vector Add.	/						/	
Rotations		/					/	

Table 5

Analysis of the Mathematics found in the J.M.B. General Studies Examination

The 49 students in the two set ones, were given the mathematics section (Appendix 3) of the 1982 J.M.B. general studies paper. There was no special significance in selecting this paper. The examination papers were found to vary little from year to year. It was decided to select the most mathematically able fifth formers for the following reasons:

(i) The test was given in October 1983, just one week after this particular group of pupils had taken their mock O-level mathematics examination. It was, therefore, possible to compare the results from these two examinations.

(ii) The selected group would be sitting their O-level examination within 4 weeks of taking this test. It could, therefore, be expected that the sutdents' knowledge of the O-level mathematics syllabus would be near to its maximum.

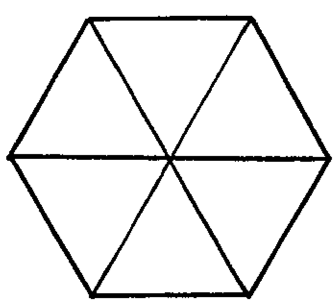
(iii) This group would consist of those pupils most likely to

a) fully understand the O-level syllabus;
b) be entering the academic sixth form.

The results of both tests are given in Appendix 4). These show that 84% of pupils obtained a mark of over 45% on the general studies paper. This seems to indicate that the majority of pupils with S.M.P. O-level mathematics should pass the mathematics component of general studies, if it were taken at the same time as their O-level mathematics examination. This seems to agree with the syllabus requirements "that candidates have followed or are following courses at roughly the ordinary level in ... mathematics" (J.M.B. 1984)[42]. However, of these mathematically able pupils (average mark in O-level mock 65%) only 55% got a mark of over 50%, 35% over 55% and only 25% over 60%. This seems to indicate that the general studies questions were found to be more difficult than the O-level questions. This conclusion is based on the assumption that the general studies test was taken seriously by the students. The test was taken under the identical conditions as the mock O-level examination and all students were briefed as to the importance of the results. The test was supervised by this writer who considers that the students were working to the best of their ability. In order to check this hypothesis the results were plotted (Fig. 4) and the regression line of general studies results on O-level results (y on x) was obtained. This indicates that the average student would need an O-level mark of 56% to obtain 45% on general studies. As such the O-level grade C student would probably not achieve a pass on the general studies mathematics paper.

It was, therefore, decided to investigate whether this difference in marks could be due to poor results on certain types of questions that were very different from typical O-level questions. Fig. 5 shows the number of correct responses to individual questions in the paper. Questions with a mark less than 40% were therefore investigated.

Question 8



The diagram represents a regular hexagon of side one metre. What is the area of the hexagon in square metres?

A $2\sqrt{3}$

B 3

C $\frac{3\sqrt{3}}{2}$

D $3\sqrt{3}$

E 6

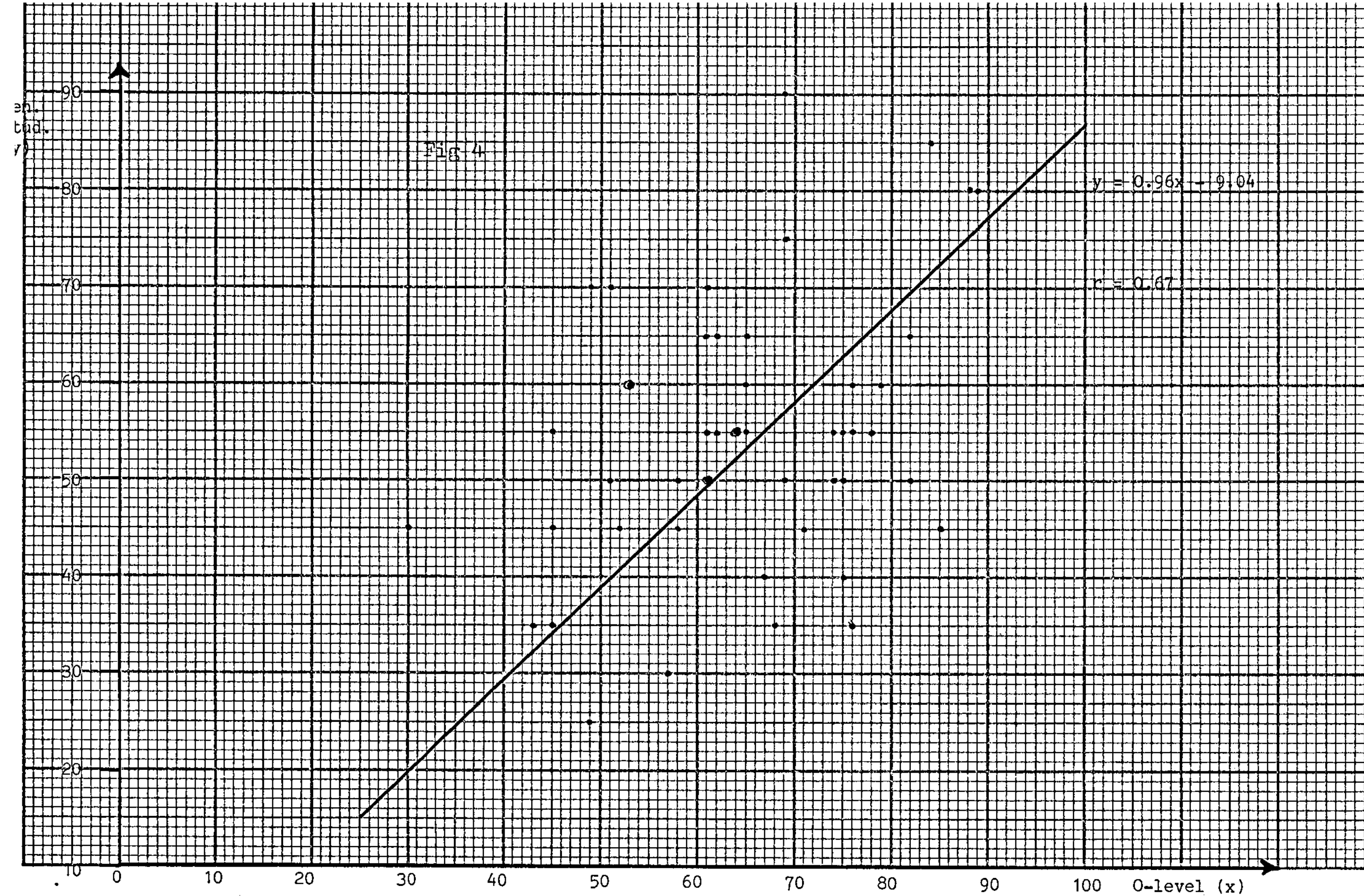


Fig 4
y = 0.96x - 9.04
r = 0.67
O-level (x)
0
10
20
30
40
50
60
70
80
90
100
10
20
30
40
50
60
70
80
90

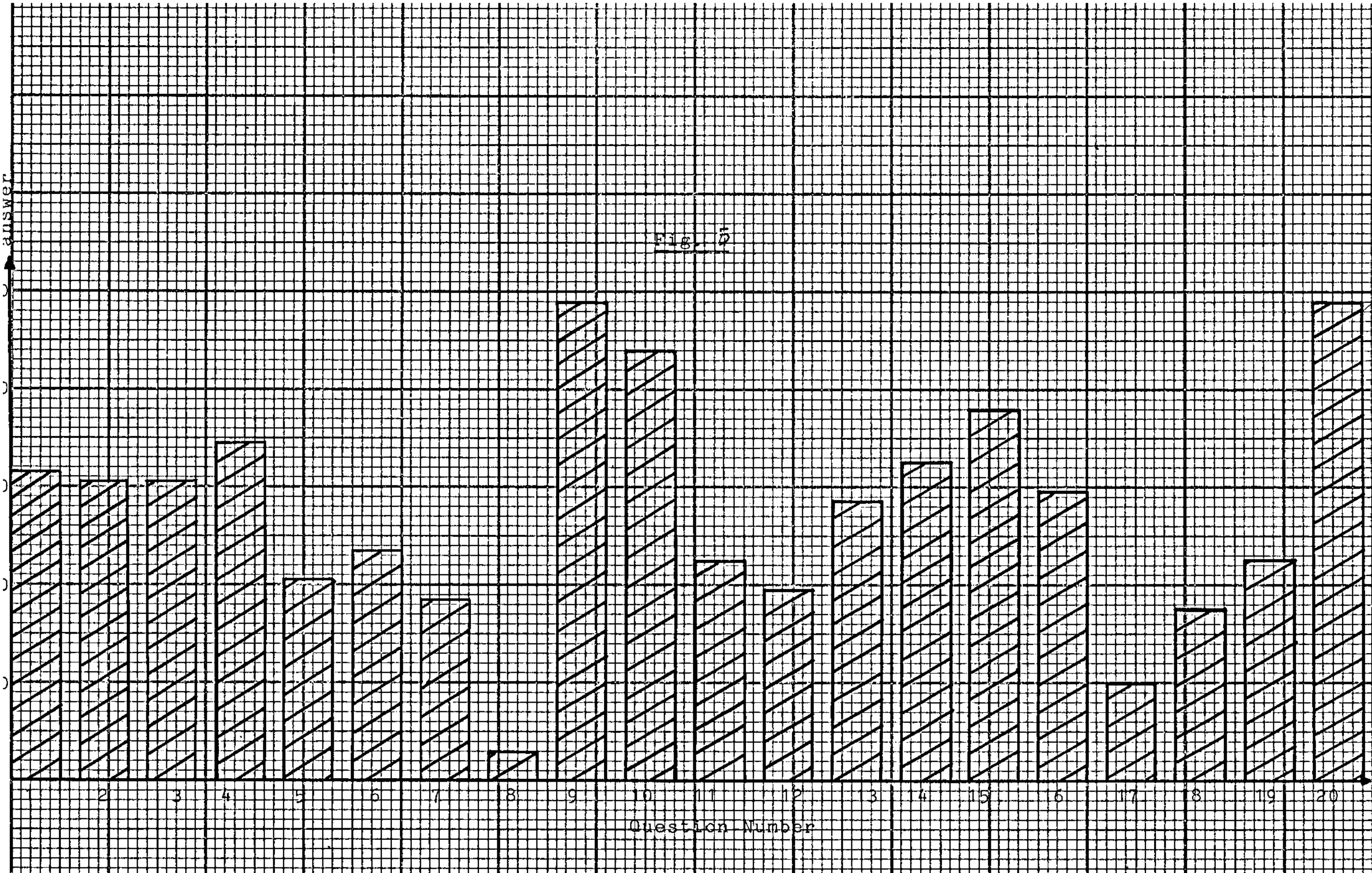


Fig. 5
answer
1
2
3
4
5
6
7
8
9
10
11
12
3
4
15
6
17
18
19
20
Question Number

Answered	A	-	6%	
	B	-	57%	
	C	-	6%	*
	D	-	6%	
	E	-	14%	
No answer		-	11%	

It was thought surprising that only 6% of students got this question correct. On verbally examining the pupils it was found that the difficulty was not with expressing sin 60 in surd form, but incorrect use of the formula for the area of a triangle. Most of the 57% who had given answer B had used the slant height rather than the percendicular height. Since finding the area of a triangle is a relatively simple part of the O-level syllabus, it is difficult to understand why such a poor response to this question was obtained. A possible explanation is that as all the pupils came from the same school this section of the syllabus had either not been recently taught or badly taught in the first place.

<u>Question 17</u>

A logarithm is the exponent to what a given base must be raised to obtain the number whose logarithm is required. An antilogarithm is

A the logarithm raised to the power of 10.

B the reciprocal of the logarithm.

C the number such that the base will equal the logarithm.

D the number obtained when the logarithm is raised to the power of the base.

E the number obtained when the base is raised to the power of the logarithm.

Answered	A	-	6%	
	B	-	26%	
	C	-	4%	
	D	-	20%	
	E	-	20%	*
No answer		-	24%	

Although the use of logarithms is taught to pupils at ordinary level, the principle of logarithms as stated as an "exponent to which a given base must be raised" would be foreign. As such a wide spread of answers would be expected. It could be argued that one should be able to obtain the correct answer from just reading the question but as the results show, such abstract reasoning is beyond most pupils. It is this writer's opinion that this question is "playing with words" and is totally unsuitable at this level.

Question 18

Given that $a > 0$ which one of the following statements could be false?

A $a^0 = 1$

B $a^p a^q = a^{p+q}$

C $a^{\frac{1}{p}} = \frac{1}{a^p}$

D $\frac{a^p}{a^q} = a^{p-q}$

E $(a^p)^q = a^{pq}$

Answered	A	-	13%	
	B	-	8%	
	C	-	37%	*
	D	-	10%	
	E	-	17%	
No answer		-	15%	

In the writer's opinion this is a typical general studies question. That is, it consisted of mathematics that was taught at O-level but presented in an unfamiliar way. Although the laws of indices are used extensively at O-level, very rarely are letters used as indices. The same students were later orally examined on this question and when p and q were replaced by numbers the majority obtained the correct answer.

Question 7

The numbers a, b, c, p and q are such that a:b = p:1 and b:c = 2q:1. What is the value of the ratio a:c?

A $2pq:1$

B $\frac{p}{2q}:1$

C $\frac{2p}{q}:1$

D $p + 2q:1$

E $2(p + q):1$

Answered	A	-	37%	*
	B	-	18%	
	C	-	17%	
	D	-	8%	
	E	-	16%	
No answer		-	14%	

Question 12

In x games of football an average of p goals was scored and in a different set of y games an average of q goals was scored. What was the average number of goals scored in all these games?

A $\frac{px + qy}{p + q}$

B $\frac{pq}{xy}$

C $\frac{px + qy}{x + y}$

D $\frac{p + q}{2}$

E $\frac{px + qy}{2}$

Answered	A	-	24%	
	B	-	4%	
	C	-	39%	*
	D	-	10%	
	E	-	11%	
No answer		-	12%	

Questions 7 and 12

Both these questions were again within the O-level syllabus but in the writer's opinion the extra difficulty was probably in unfamiliar presentation. Being able to use both ratio and averages is well within the ability of the pupils examined. It is also recognised that both these questions together with number 18 used letters whereas O-level questions would probably have used numbers. This added difficulty is also recognised. It was therefore concluded that, with the exception of question 17, this 10% mark difference could not be due to poor results on non O-level type questions.

Another possible reason for this difference could be due to the fact that this sample had spent the last 8 weeks practising past O-level papers. Therefore, even if the questions were of the same difficulty, the students' unfamiliarity with general studies questions could account for this difference. In order to test this hypothesis, a test (Appendix 5) consisting of a mixture of O-level and general studies questions from just one topic, sets, was given to 78 pupils in sets 2 and 3. These students were just about to finish the O-level syllabus but had not yet seen any past O-level papers. The results (Appendix 6) show a correct response of 46% on general studies and 45% on SMP O-level. This suggests that both general studies and O-level questions are of about equal difficulty. It is recognised by the writer that the results obtained were from a very limited sample and that a fuller analysis of all the topics would have produced more accurate results. However, the writer feels that it is correct to conclude that the mathematics required for JMB A-level general studies, with the exception of a very few topics, is very similar to that needed for SMP O-level mathematics. Furthermore, the questions are set at approximately the same level of difficulty although the presentation may be different.

Even accepting the above argument, it does not necessarily follow that there is no benefit to be gained from teaching the mathematical component of general studies. It has been shown in previous chapters that there is a need for sixth form non-mathematics specialists to be confident and proficient in mathematics. In the writer's opinion, if mathematics is not taught in the sixth form, the mathematical ability of the vast majority of these students will considerably regress. As such, even if the general studies syllabus consists of no new topics, continued teaching should improve proficiency and confidence. In order to test this hypothesis, 28 second year sixth form students were given the test in Appendix 3 and the results compared with their O-level grades. None of these pupils had received any mathematical teaching since the end of their 5th year. Appendix 7 shows that the average mark of the 8 students with an O-level grade B was only 42% and of the 19 students with an O-level grade C only 36%. Both these marks are about 20% lower than the O-level grade suggests. This 20%, or 10% if compared with the initial test results, certainly shows a considerable deterioration in the mathematical ability of this group.

It has been shown in this chapter that

a. the mathematical knowledge required for the mathematical section of the JMB A-level general studies paper is very similar to that required for SMP O-level mathematics.

b. the mathematical ability of non-specialist sixth formers is lower at the end of their sixth form course than it was at the end of the fifth form.

The next chapter will consider other mathematical courses that are available for the non-specialist sixth former while in the final chapter the writer will review and recommend how mathematics can be taught to this group of pupils.

CHAPTER 8

Post O-Level Mathematics Courses

In this chapter I shall be reviewing the major mathematics courses that are available for the sixth form non-specialist.

8.1 Schools Council Sixth Form Project - Mathematics Applicable

The project (1969-78) was initially set up to review the content of sixth form mathematics and to develop new materials for sixth formers, taking into account the needs of those who do not wish to specialise in mathematics but needed some additional mathematical knowledge for their other courses. The approach which the project adopted was that of projective modelling - using mathematical models to explore the implications of proposals and hypotheses about the world. Using this idea a series of student books was produced as a post O-level course in mathematics. The materials are designed to encourage students to recognise patterns by using mathematics to explore different possibilities. When a new concept is introduced, it is illustrated by means of modelling real or hypothetical situations which could occur in everyday life. Ideas are then consolidated through further examples and problems. Although specifically talking about I-level courses Cockcroft (1980; p.180)[8] states that courses "should illustrate the many ways in which mathematics can be applied ... we believe that use could be made of some of the ideas which are contained in Mathematics Applicable".

Up to 1982, the project designed its own AO examination which had some unusual features that were an attempt to test students' ability to extract the mathematics from a situation. Each paper follows through a particular problem and contains ten concealed hints, so if a candidate is stuck on a question the concealing paper can be torn off to use the hint - though by using (unsealing) any hints they incur a "penalty". This writer has reservations about this wondering if students would use the hints unnecessarily to "check" their answers through feelings of insecurity. Past problems have included a fire escape system, a luminescent paper for notice boards, a butter-oil mixture and a proposed extendible car.

The course consists of eight books which are written to make the student think about a wide range of mathematical patterns and learn how to explore the mathematical possibilities of the situation. Mathematics topics include:

1. Indices, indicial models, indicial equations.
2. Binomial expansion for positive integral index.
3. Rectangular cartesian co-ordinates: the idea of point-sets representing lines, circles and other simple curves defined by open sentences. Gradient, mid-points, distances and intersections. Easy problems on loci.
4. Generating formulae for simple sentences: summation of arithmetic and geometric series.
5. The idea of a limit: "tending to zero", "tending to infinity".
6. Differentiation and integration of powers of x. Applications of differentiation to rates of change, maxima, minima and points of inflexion. Application of integration to areas under curves and to volumes. Chain rule and differentiation of products.
7. The idea of linear and quadratic models: solution of linear and quadratic equations. Conditions for the existence of real roots of a quadratic equation.
8. The use of e as the limit of $(1 + {}^1/_n)^n$ as $n \to \infty$. The function $x \to e^x$ and the series of $e^{\kappa x}$. Definition and use of natural logarithms. The differential equation $\frac{dy}{dx} = \kappa y$.

9. Probability laws: probability tree diagrams. The ideas of probability and frequency distributions and their graphical representation. Mean and standard deviation of a distribution. Binomial and normal probability distributions.
10. Force and torque. Newton's laws of motion in linear and rotational form. Simple kinematics and the motion of projectiles. Concept of kinetic energy: simple ideas of work and energy.
11. Vector addition and subtraction. Multiplication of a vector by a scalar. Relative velocity. Component and resultant forces.

The teachers' guide recommends that if the course is taken over one year, then four timetabled lessons of 35 to 40 minutes per week are needed. If taken over two years, three timetabled lessons are used. Although "the one-year and two-year course have been run in various schools and colleges on less than this, but the result is a certain degree of strain of getting through the course" (Schools Council)[69]

Although the syllabus content is not unlike other post O-level courses, it is the method of presentation that is unique. It is intended that this modelling approach will improve motivation. "Arts" students aged 16 to 19 have received many hundreds of hours of instruction in mathematics. They have been told again and again that mathematics is important. But ten years' experience and a growing capacity for critical judgement leads many of them to a view of mathematics as being distinctly less exciting, less meanginful and less purposeful than the official wisdom allows. This is the stubborn reality which many teachers of non-specialists aged 16-19 know they have to face. Mathematics Applicable tries to improve motivation by using horizontal relevamce rather than vertical relevance. Vertical relevance is relevance to things which will be met, or fully understood later on. So the student can only be told about this.

Horizontal relevance is to things in the student's current mental world, so it is possible to show relevance of this kind.

According to the teachers' notes[69] "stop telling them mathematics is exciting and purposive and start showing them. Start using mathematics as a modelling kit to explore the implications of intrinsically interesting ideas." By suitable discussion it should be possible to ensure that quite a lot of mathematics gets taught. Of course, one of the troubles with modelling is that it leads all over the place, it does not lead where the teacher wants to go. However, mathematics applicable was written as a highly structured course. The material was not written as a free-wheeling exercise in modelling. It was chosen to do a particular job. The modelling was developed to be built into the structure of the course and not vice versa.

The use of mathematical modelling, however structured, causes changes in the pattern and manner of organisation of mathematical teaching. First, once one has embarked on a modelling style of mathematics in the classroom, one cannot easily fill gaps in the curriculum with merely formal exercises. Once one has broached the idea with a given set of students that mathematics is a modelling kit, the alternative (that it is not) ceases for many to be a viable alternative. Secondly, one needs to build up a style of work over a period of time in which students develop their perceptions of the real world and of the mathematical patterns embedded in it. These requirements point to the introduction of a less teacher-centred style of work than has often been adopted in mathematics with sixth form students. The third point is that the teacher cannot expect of maintain the same tempo when the student is working in an applicable style as one could on a formal course. As such more time is needed to cover the same material although the mathematics will probably be better understood, understanding being a commodity which is in short supply in many English sixth forms. Finally, one would not expect to achieve the same amount of rigour as one could on a formal course. However, it is the writer's view that it is mathematical understanding, appreciation and application, not rigour, that are the requirements of a mathematics course for non-mathematics specialist sixth form students.

Currently the work of the project has been taken over by a voluntary group, the Mathematics Applicable Group (Appendix 8), which was formed in September 1978 to extend the use of modelling approaches. Unfortunately since 1982 there has been no AO examination, but the Mathematics Applicable Group have been offering their own non-GCE examination in Applicable Mathematics. Although this will probably prevent many students from attempting the complete course, this writer does not see this change as a major catastrophe.

The majority of sixth form non-mathematics specialists will not want or be able to devote 3 or 4 timetabled lessons a week to mathematics. Even if such time were available, this writer feels that a broader range of teaching methods should be used. The main advantage of this course is in providing the teacher with material that is presented in an unusual way: modelling material that can be selected and used as an integral part of a sixth form mathematics course for this target group.

8.2 The Continuing Mathematics Project

This project was designed for students in the age range 16-19 who do not wish to study mathematics as a main subject beyond O-level, but who require some mathematics to support further studies in subjects such as biology, geography, economics, sociology and psychology. The material is largely self-instructional, needing the minimum of teacher intervention. It is modular in form, each unit consisting of about 30 pages of text, together with a summary of the ground covered and a post test. Students are expected to attempt questions as they are put, without reference to the answers which appear below the question. It is this writer's experience that although the more diligent student will use the text correctly, the majority spend little time puzzling over a question before using the answer. The list of units is given in Table 6, while a precis of each unit is given in Appendix 9.

The Schools Council (1980)[64] state that "the material has been effectively used in general studies courses and in A-level courses in biology, geography and other subjects which use mathematics". An obvious problem for the mathematics teacher is in selecting units that are relevant to a particular subject and as such, close liaison between subjects is essential. Table 7 gives the units which the writer considers, and has used, as suitable for five of the most commonly taught subjects.

Unlike the Statistical Education Project 16-19, each unit is not aimed at a particular target subject. Direct relevance is therefore not always possible. Even when it is, it is quite often the student rather than the mathematics teacher who first sees the connection with the student's main subjects of study. Motivation can therefore oscillate with the most difficult part of the mathematics teacher's job being educating himself as to relating the work to the students' main subjects. Nevertheless, this course offers a very versatile method of teaching, allowing students in the same class to proceed along different paths and when used effectively by schools it can promote in students a sense of responsibility for their own learning.

8.3 Statistical Education Project 16-19

This project is still in its evaluation stage and has another 12 months still to run. It is located at the University of Sheffield with Peter Holmes as director. The aim of the project is to produce units to be used by sixth form non-mathematics specialists, every unit being written by a different subject specialist. Present units include titles such as:

1. Growing Up.
2. Mendelian Genetics I.
3. Development and Organisation.
4. Population.
5. Bracken.
6. Statistics in Business.
7. Consumer Behaviour: Expenditure Patterns.
8. Meeting the Budget.
9. Sales Forecasting.
10. Looking at Data.
11. Rivers and Streams.
12. Statistics through Experiments in Psychology.
13. Agriculture and Climate.
14. Limb Dominance.

Whereas most statistical texts concentrate on method and then give illustrations, this project introduces statistics as a tool to solve practical problems. The units are supposed to be self-instructional with any added teaching being related to its application. As such, the project Director recommends that the teaching be done by other subject specialists rather than mathematics teachers. The first unit in the science series states that "the purpose of these science units is to introduce and use the statistical concepts and techniques required by A-level science

TABLE 6

The List of CMP Units

Category 1 (single, independent revision units)	
1	Indices
2	Positive and Negative Numbers
3	$y = mx + c$
4	Inequalities
5	Working with Ratios
6	Transformation of Formulae
7	Trigonometry of the Right Angled Triangle
8	Trigonometry of the General Triangle
9	An Introduction to Trigonometrical Graphs
10	An Introduction to Sets
11	${}_nC_r$ - Calculation and Use
12	Computation with Logarithms 1
13	Computation with Logarithms 2
14	Theory of Logarithms 1
15	Theory of Logarithms 2
16	Using Logarithms to Determine Relationships
Category 2 (introductory units)	
17	Flowcharts and Algorithms
18	Critical Path Analysis
19	Further Critical Path Analysis
20	Systems
21	Linear Programming
22	Information and Coding 1
Category 3 (short courses of work)	
23, 24, 25	Information and Coding 2, 3, 4
26, 27, 28, 29, 30	Elementary Calculus 1, 2, 3, 4, 5
31, 32, 33	Mathematics in Geography 1, 2, 3
34, 35, 36, 37	Descriptive Statistics 1, 2, 3, 4
38, 39, 40	Probability 1, 2, 3
41, 42, 43	Hypothesis Testing - non-parametric methods, 1, 2, 3
44, 45, 46, 47, 48, 49	Parametric Statistics 1, 2, 3, 4, 5, 6
50	The χ^2 Test
51	Correlation
52, 53	Maths for Biologists 1, 2

TABLE 7

Continuing Mathematics Project - Selection of Units for use with Biology, Physics, Chemistry, Geography and Economics Students

	Biology	
34	Descriptive statistics	1
35	" "	2
36	" "	3
41	Hypothesis testing	1
42	" "	2
43	" "	3
44	Parametric statistics	1
45	" "	2
46	" "	3
47	" "	4
48	" "	5
49	" "	6
50	The χ^2 Test	
51	Correlation	
26	Elementary Calculus	1
27	" "	2
28	" "	3
29	" "	4
30	" "	5
30A	" "	6
52	Mathematics for Biologists	1
53	" " "	2

Physics		Chemistry
1	Indices	1
5	Working with ratio	5
6	Transformation of formulae	6

Continued

Table 7 (Continued)

Physics			Chemistry
26	Elementary Calculus	1	26
27	" "	2	27
28	" "	3	28
29	" "	4	29
30	" "	5	30
14	Theory of logs	1	14
15	" " "	2	15
34	Descriptive stats.	1	34
35	" "	2	35
36	" "	3	36
41	Hypothesis Testing	1	
42	" "	2	
43	" "	3	
50	The χ^2 test		
	Probability	1	38
	"	2	39
	Parametric stats.	5	48
	" "	6	49

	Geography	
31	Mathematics in Geography	1
32	" " "	2
33	" " "	3
33A	" " "	4
34	Descriptive Statistics	1
35	" "	2
36	" "	3
37	" "	4
38	Probability	1
39	"	2
40	"	3

Continued

Table 7 (Continued)

	Geography	
41	Hypothesis Testing	1
42	" "	2
43	" "	3
45	Parametric Statistics	2
46	" "	3
47	" "	4
48	" "	5
49	" "	6
50	The χ^2 test	
51	Correlation	
	Economics	
34	Descriptive Statistics	1
35	" "	2
36	" "	3
37	" "	4
38	Probability	1
39	"	2
40	"	3
41	Hypothesis testing	1
42	Correlation	

courses in the context of practical work in science. The starting point for each unit is a scientific rather than a statistical problem, the statistical work is only introduced insofar as it is relevant to the scientific work being considered for the emphasis will be on understanding principles rather than arithmetic calculation."[85]

Although the teaching of statistics by the user subject can be highly beneficial and as there should be "co-operation between all those in schools who make use of statistics in their teaching" the writer feels that with the limitations on A-level teaching time most subjects will not spend significant time on these units. Even in the trial schools, many of the evaluation forms were returned by mathematics staff. Nevertheless this project is providing for the 16-19 age range useful material that emphasises the practical approach to the teaching of statistics, material that can provide the statistical element of any non-specialist sixth form course and present it in such a way as to be relevant to a student's main area of study.

The last three chapters have discussed various mathematics courses available to the non-specialist sixth form student. The final chapter in this section will make recommendations as to how mathematics should be taught to this group of students.

CHAPTER 9

Mathematics for Non-Specialists: Resumé and Recommendations

Section I has shown the need for a mathematical education while previous chapters in this section have discussed the mathematical courses available to the non-mathematics specialist. This chapter will discuss how mathematics can be taught to this group of students so as to meet this need for greater numeracy. In the wake of Cockcroft (1982)[8] the word numeracy seems "to imply the possession of two attributes. The first of these is an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his everyday life. The second is an ability to have some appreciation and understanding of information which is presented in everyday terms: in other words to cope confidently with the mathematical needs of adult life." Although this is an adequate definition for the majority of school leavers, the traditional sixth form are an academic elite and as such they perhaps need mathematical ability above that of basic numeracy.

It is the concept of numeracy first introduced in the Crowther Report (1959)[12] that this writer considers to be relevant to the academic sixth form. Crowther sees numeracy as "an understanding of the scientific approach to the study of phenomena-observation, hypothesis, experiment and verification ... the need to think quantitatively, to realise how far our problems are problems of degree even when they appear as problems of kind" and he sees statistical ignorance and statistical fallacies as just as dangerous as the logical fallacies which come under the heading of illiteracy. "However able a boy may be, if his numeracy has stopped short at the usual fifth form level he is in danger of relapsing into innumeracy." [12] It is the prevention of this relapse as much as anything else that should be provided for by sixth form courses.

The largest single attempt at sixth form curriculum change in the last decade has been the proposed N and F levels. Although these proposals have been rejected, certain aspects are considered relevant to this report. In the N and F proposals it was recommended that sixth form courses should consist of five subjects which would be examined at two levels, the Normal, N, and Further, F, level. N-level courses would take about half the study time of an existing A-level course and F-level about three-quarters of this study time for an A-level course. It was intended that the teaching relationship between N and F levels would be such that the choice of F-levels could be delayed to the end of the first year of study in the sixth form with the possible implication of this delayed choice being a more general curriculum. As such the Working Party sought indications of the subject choices likely to be made by students under a two-level five-subject system. Table 8 shows the number of students taking each subject at A-level and the number of students who said they would have chosen this subject at N or F level in 19 trial schools. Therefore, under the new proposed system, the number of students studying mathematics in the sixth form would have increased from about 33% to 50% of the total traditional sixth form population. Although there are, as expected, considerably more students studying mathematics "there has been a tendancy for the A-level "league leaders" to be caught up by other subjects. English and mathematics are still the leaders but the gap between these subjects and geography, history and economics, all of which have increased their recruitment considerably, has narrowed[66]." However, even with a sixth form curriculum based on five rather than three subjects, only about 50% of students would be studying mathematics. The question to be answered is how can more arts and social science sixth form students be persuaded to study mathematics?

One factor found by the N and F level Working Party is the nature of the syllabus. "School 15 ... offered a 'Use

School	English A	English N+F	Maths (1st year) A	Maths (1st year) N+F	History A	History N+F	Geography A	Geography N+F	French A	French N+F	Biology A	Biology N+F	Physics A	Physics N+F	Chemistry A	Chemistry N+F	Art A	Art N+F	Economics A	Economics N+F	Total in 10 most popular subjects A	Total in 10 most popular subjects N+F
1	6	21	1	9	7	17	4	10	3	7	1	5	1	4	2	3	4	5	0	6	29	87
2	10	20	10	14	4	10	14	25	6	8	5	7	6	8	5	7	1	3	13	16	74	118
3	23	35	14	25	14	27	8	19	2'	6	7	19	7	12	6	8	8	13	0	0	89	164
4	8	14	10	16	7	9	3	11	5	12	4	8	9	12	10	13	6	7	5	14	67	16
5	35	52	10	35	15	30	13	43	14	21	13	41	19	22	20	22	11	17	0	0	150	283
6	9	24	14	27	5	10	3	4	0	2	2	8	13	19	6	11	5	10	0	7	57	122
7	13	43	34	59	13	33	34	49	3	6	11	17	33	38	21	29	8	20	18	20	188	314
8	14	29	18	28	18	27	23	39	10	24	11	25	18	27	17	23	1	7	11	30	141	259
9	29	48	19	34	16	33	23	49	21	27	9	40	12	16	5	12	11	22	24	33	169	314
10	58	76	34	47	22	35	6	22	14	30	14	24	23	31	16	25	9	30	41	61	237	381
11	67	101	29	49	29	69	13	45	64	83	23	49	19	30	16	21	7	13	35	58	302	518
12	33	55	29	44	16	23	25	35	13	17	20	41	23	25	22	25	16	39	18	30	215	334
13	48	75	22	37	20	58	11	52	10	21	22	34	19	26	23	21	15	19	6	22	196	365
14	59	80	34	52	21	51	35	73	14	22	23	39	15	22	25	36	11	19	41	71	278	465
15	56	91	42	79	59	97	20	75	29	54	46	63	24	29	41	49	22	48	53	77	392	662
16	33	49	63	87	27	51	39	54	31	45	38	70	40	46	40	52	13	27	44	83	368	564
17	105	151	109	155	63	97	85	136	39	73	54	108	68	101	47	66	33	61	55	121	658	1069
18	88	150	74	89	81	103	52	69	46	59	40	124	58	71	43	61	22	17	49	67	553	810
19	168	248	208	267	117	225	116	235	76	123	94	191	116	147	85	118	54	116	126	267	1160	1937

School	Most popular 2nd language A	Most popular 2nd language N+F	Sociology A	Sociology N+F	Home economics A	Home economics N+F	Technical drawing A	Technical drawing N+F	Maths (2nd year) A	Maths (2nd year) N+F	RE A	RE N+F	Music A	Music N+F	Politics A	Politics N+F	Other A	Other N+F	TOTAL A	TOTAL N+F
1	1	3	0	7	0	1	0	0	0	0	8	14	0	0	0	0	2	2	40	114
2	4	9	0	13	1	3	3	4	0	0	0	0	0	0	10	19	10	26	102	192
3	4	9	6	27	2	5	0	0	2	0	0	0	1	1	0	0	5	12	109	220
4	2	3	0	0	1	3	3	4	0	9	0	0	3	3	0	0	0	2	76	140
5	12	30	0	0	2	12	0	2	0	0	14	10	4	5	0	0	3	11	185	353
6	0	2	0	7	0	0	0	1	5	13	0	0	0	0	2	5	0	13	64	163
7	4	5	18	7	0	0	2	2	3	0	0	2	0	1	0	0	17	22	232	353
8	3	5	1	0	0	0	0	0	3	0	0	0	0	8	0	0	61	17	209	289
9	6	12	1	0	4	8	1	6	6	8	7	7	3	7	12	33	15	34	224	429
10	7	9	13	21	2	4	1	7	13	16	0	0	1	0	14	15	1	65	289	518
11	16	26	0	0	9	25	0	0	5	2	5	13	4	9	0	0	147	63	488	656
12	4	8	1	31	11	23	3	0	3	7	9	24	0	6	0	0	2	85	248	518
13	30	58	0	0	5	17	2	5	0	0	13	18	5	28	2	0	2	17	255	508
14	4	12	1	1	2	10	2	15	5	2	2	9	4	9	6	22	8	27	312	572
15	22	17	0	0	10	35	0	0	6	6	8	19	3	13	23	26	9	72	473	850
16	10	23	0	32	0	16	0	0	0	9	3	5	2	6	1	0	27	105	411	760
17	11	23	0	1	6	23	0	0	4	15	1	20	7	18	0	0	113	116	800	1285
18	21	46	0	35	7	14	2	18	28	26	8	20	7	9	0	0	15	46	641	1024
19	32	51	111	294	17	37	29	58	31	77	29	78	24	40	15	23	60	249	1516	2834

* The figures are based on information supplied by students. Since a number of students in each school failed to fill in forms, some of the figures are slight underestimates.

Table 8

Number of students taking each subject at A-level and the number who said they would have chosen this subject at N or F level in 19 trial schools. (Schools Council 1980)[67] of Mathematics' course with N-level students particularly in mind which recruited well." Similarly, courses with particular students in mind like the Continuing Mathematics Project and the trial Statistical Education Project 16-19, should provide the motivation for mathematics that many non-specialists lack, by showing direct relevance to a student's main subjects of interest. These, or similar, courses are taught in many schools, but they are nearly always taught to scientists not studying mathematics and to a lesser extent to geography and economics students. Such courses certainly increase the mathematical ability of the students, but as they cater for such a limited population, do little for the overall mathematical ability of the arts and social science sixth form.

In trying to design a mathematical course that will be accepted by this target group, it is worth noting a point made by the N and F level Working Party who, although largely concerned with the curriculum, stated that "it was clear to them that curricular change could only be brought about by redesigning the system of examinations at 18+".[66] This seems to concur with a view held by this writer, that the sixth form system is based solely on examination success. Since redesigning the system of examinations is not within the scope of any school or college, the only feasible alternative is to find a way to teach mathematics within the present examination system. The academic sixth form student is almost solely concerned with A-level examinations, success in these usually being essential for University or Polytechnic entry or for entry to a chosen career. It is this writer's opinion that any lesser qualification will not be accepted by these students as worthy of their time, effort and commitment. It is therefore concluded that if mathematics is to be taught successfully it must be part of a course that provides A-level certification.

Obviously A-level mathematics would fit this criteria, but is totally unsuitable from both the time allocation required and the mathematical ability and interests of the students. The only suitable alternative seems to be to teach mathematics as an integral part of an Advanced Level general studies course. Many schools offer general studies but some select syllabuses that contain little or no mathematics while others who select syllabuses that contain mathematics, do not teach the subject. It is therefore recommended, after consideration of the various syllabuses in Chapter 6, that schools should select the J.M.B. syllabus. It was further shown in Chapter 7, that the mathematical component of this syllabus is very similar to O-level mathematics. It is therefore not recommended that the O-level syllabus be retaught as lack of motivation and interest will certainly follow. Student motivation should not only be due to the possibility of A-level certification but also because of the way the subject is taught. The widely used method of just teaching to past examination papers should be rejected. Revision of any past topics should be by way of new methods of teaching such as investigations, models, or through the introduction of new and interesting topics which can be shown as relevant to the student's future needs. Use can be made of the various schemes mentioned in Chapter 8, plus mathematical games, computer and video assisted learning including simple Open University extracts such as titles from the History of Mathematics Course.

For the well organised and enthusiastic mathematics department, this area of the curriculum offers the ideal situation for innovation. Unfortunately, a well-planned and presented course, by an experienced mathematics specialist, is not currently the norm, but odd lessons by various members of the mathematics department who happen to have the odd lesson free are. It is this area of the curriculum that is usually ignored by most schools and it is an area that contains a wealth of opportunity, How many of a school's best O-level mathematics candidates do not study the subject after the fifth form? The number as all mathematics teachers will agree, is certainly not minimal. Not only should we be trying to prevent the mathematics regression mentioned in Chapter 7, but we should be improving the mathematical ability and understanding, so that in the future students should not leave the sixth form innumerate.

SECTION III

The final section of this report will look at Mathematics Courses for what has been described for some time as the "New Sixth Form". Unlike the traditional sixth form, the vast majority of these students have few, if any, O-level qualifications. Advanced level courses are totally unsuitable, while the majority of these students do not wish to be committed to a specific vocational objective. They wish to obtain from the sixth form "an improvement in their general education, personal development, and pre-employment preparation" (Macfarelane, N. 1980)[47]

With this growing number of young people, of a wide range of ability, continuing their full time education beyond the age of 16 the school has to match, as best it can, the abilities and aspirations of these students to the courses available. Although this writer accepts that some schools produce their own courses, these courses always lead to some form of external validation, whether it is City and Guilds, BEC, CEE, O-level, etc. or some combination of the examinations available for to be fully effective, the period of full-time education after the age of 16 must give students an opportunity to acquire a qualification. Such courses give the young people concerned a clear goal and increased commitment. Syllabuses and programmes leading to an examination provide a framework on which teachers can construct an appropriate course, and the systematic assessment of performance helps employers in recruiting and aids those who have to decide on applications to more advanced courses.

At present post-16 full-time courses in England and Wales for this target group are of two types:

(i) single subject examinations such as the General Certificate of Education, the Certificate of Secondary Education and the experimental and now rejected Certificate of Extended Education.

(ii) the various pre-vocational certificates offered by further education examining bodies.

Chapters 10 and 11 will discuss the single subject examinations and the pre-vocational certificates respectively.

CHAPTER 10

Single Subject Examinations

After A-level, O-level represents the next most common examination taken in the Sixth Form and is used for a variety of purposes. It may be used to support two A-levels, either because the student is unable to sustain three A-levels, or because he prefers to take only two A-levels and an ancillary O-level is required by the school to produce an acceptable timetable. A second category may be a student using O-level as a means of consolidating basic knowledge on the way to A-levels in subjects not previously studied.

The final two categories, of which English and mathematics are by far the most common subjects taught, is for those students trying to overcome earlier failures in a subject essential for a qualification and those converting from C.S.E. to O-level. It is often in these sixth form O-level courses that the limitations of time and staffing are most apparent. Both Cockcroft and the DES (1982)[1] found that quite often the time allowance was meagre since students could not attend all the periods arranged. On occasions classes catered for students taking O-level as well as C.S.E. Furthermore some of the students were taking an examination in November and some in the following June. Additional complications arose where students have previously followed different syllabuses, either GCE or CSE, of the same or different examination bodies. In such circumstances the task of the teacher is difficult and can lead to a temptation merely to practise past examination papers in the hope that improved performance will result.

In the writer's view the majority of those converting or retaking were unable or considered unable to obtain an O-level pass at 16, otherwise they would have either passed the examination or have been entered for it by their school, although increased motivation caused by a non-compulsory

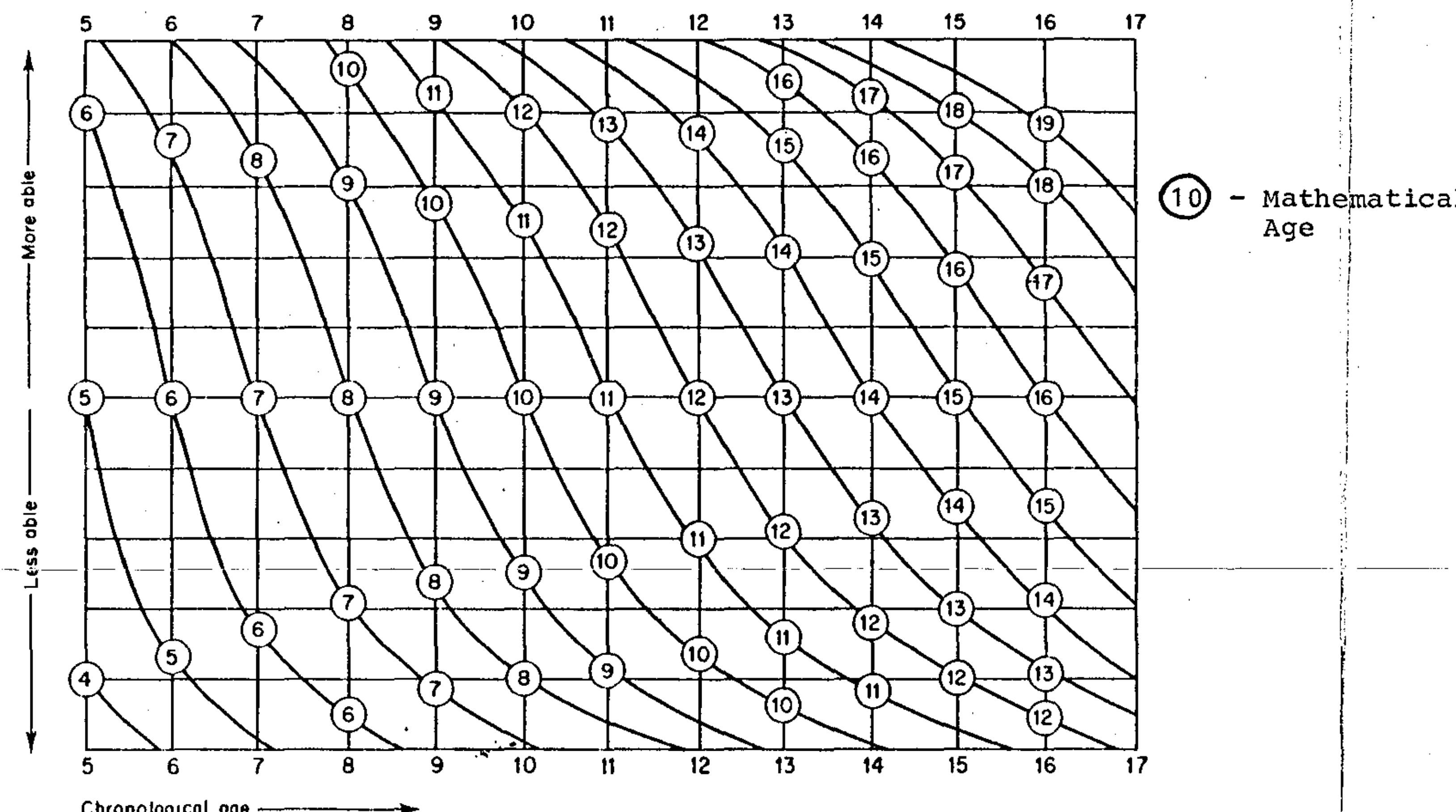


Fig. 6

Model Relating Chronological Age and Mathematical Age. (Cable, 1972)[6] course together with an increasing maturity may improve performance. It will now be argued why it is unreasonable to expect a student's level of attainment to increase sufficiently in one year to make a pass probable. Hart (1981: p.209)[32] states that understanding improves only slightly with age. Similar results to those obtained by the CSMS Project were shown on a model by Cable (1972)[6] (Fig. 5). It shows how pupils of the same chronological age can have sidely different mathematical ages. For example, some pupils of chronological age 16 can have mathematical ability the same as pupils aged under 12, while others that of pupils aged over 19. This model compares chronological age with mathematical age (ability). Interesting features of this model show that assuming O-level is taken at 16+ by the top 25%, it is an examination for children of mathematical age 17½+. More important is the ability gap between this top 25% and the children of average and below average ability who will be following O-level courses in the sixth form. The majority of these pupils will never reach the level of mathematical ability of the top 25% of 16 year olds.

Of further significance is the standard of the O-level questions in relation to the development of the student. Piaget identifies various stages of development through which each child must pass. Average ages are sometimes given for the transition from one stage to the next, with the transition from concrete to formal operational thinking at a mental or mathematical age of about 14 to 15 years. The development stage of the child limits the type of problem he can tackle and to some extent the type of topic worth studying.

According to Piaget, concrete operations constitute mental activities which correspond to physical actions which we can do, or imagine ourselves doing, upon objects or systems of objects in the real world. One of the key differences between concrete and formal operations lies in the nature of the data which the thinker has to handle and the kind of model available to support this thinking. Whereas an important characteristic of the data which can be represented by a thinker using concrete operations is that it can be represented by a concrete model, this kind of representation is no longer necessary for a thinker able to use formal operations. A further important characteristic which distinguishes formal from concrete thinking is concerned with the direction in which thought tends. The concrete operational thinker can start with the real world and make (concrete) models of non-real but logically possible systems, but in general he does not work in the opposite direction. This, however, is what the formal operational thinker does. He considers whole sets of possible cases and singles out suitable actual ones for consideration or some sort of test. That is, thought goes from the possible to the real.

Malpas and Brown (1973)[48] considered 720 mathematics O-level items from two different mathematics syllabuses and assessed them for formal and concrete demands. Their results (Fig. 7) show that the majority of O-level questions are at the stage of formal operational thinking. The formal operations were further divided into two categories, FO1 and FO2, so that straightforward items requiring formal operations were placed in FO1 and items requiring more complex operations of a formal type were classified as FO2.

Category	CO	FO1	FO2	All
Syllabus C	74	167	119	360
Syllabus D	48	133	179	358

Fig. 7

Number of Items in Each Category of Agreed Classifications for University of London O-level Syllabuses C & D

Since many below average students will only have just reached this stage of formal operational thinking it is unsatisfactory to expect them to succeed in an examination requiring formal reasoning. Also of interest is the fact that the modern syllabus C has significantly fewer questions in the formal operations category than the more traditional syllabus D, an important fact in syllabus selection.

Table 9 relates a pupil's chance of success at O-level against his previous qualification in the same subject.

O-level Retakes		Grade on 1st attempt			
		D	E	U	all retakes
Number of Entries		88	93	63	253
Pass rate		60%	52%	37%	50%
CSE to O-level conversion:		CSE Grade already obtained			
		1-2	3	4,5,U	all conversions
Number of Entries		182	136	83	401
Pass rate		46%	32%	23%	36%
New Subject					
Number of Entries	456				
Pass rate	36%				

Table 9

Results of O-level Examinations by Students in Sample

(Dean and Steeds, 1982)[13]

The pass rate in the retake category was 50% compared with 36% for both the new and conversion categories. However, as Table 9 shows, the pass rate in the retake subjects varied considerably according to the grade obtained on the first attempt. A similar pattern was detected in the CSE to O-level conversion where pass rates were slightly higher in subjects in which students had previously obtained a grade 1 or 2 CSE. From these figures one can produce a rough scale of a student's likelihood of passing a subject at O-level given their previous performance in the same subject. This is, in descending order:

O-level D
O-level E
CSE Grades 1 and 2
O-levelU/New subject/CSE Grade 3
CSE Grades 4, 5 or U

With the possible exception of those with previous O-level grade D or E, the O-level seems an unsatisfactory course for the remainder. Table 10 shows that the pass rate in mathematics along with geography is significantly lower than the other major subjects, emphasising that in mathematics, retakes and conversions are unsatisfactory.

Subject	Pass Rate (A-C)
English	40%
Mathematics	31%
Geography	28%
Biology	46%
History	40%
English literature	51%
Others	40%
All	39%

Table 10

Results of O-levels taken by Students in Sample
(Dean and Steeds, p.93)[13]

Rather than retake or convert from CSE to O-level, many students have been taking the CEE pilot examination. However, the decision not to implement the proposals for the introduction of the CEE, together with the fact that the new pre-vocational course is not a single subject examination but a package, will probably, especially in mathematics where a qualification is often deemed essential, increase the number of pupils either retaking or converting. With most of the CEE students being in the group having previous qualifications of CSE grade 3 or below, a more unsatisfactory situation could well be expected.

Although the experimental CEE examination is not to be implemented, it is of value to consider some of the arguments for its recognition. At the time of its proposals, the Schools Council[63] stated that the CEE was intended "primarily for pupils who have obtained CSE with grades 2-4", and identified three groups of potential CEE candidates:

"(i) sixth-formers not taking A-levels, for whom the CEE should be primarily designed.

(ii) traditional sixth-formers taking A-levels. A considerable number of these were expected to complement their course by one or more CEE subjects; and

(iii) some students in further education who might opt for CEE in place of O-level courses."

The examination was a single subject examination, separately certificated and primarily for candidates spending one year in the sixth form. The main purpose behind CEE was to provide an examination appropriate to the vocationally uncommitted 17 year old, for whom A-levels were too demanding and the retaking of O-levels and CSE a recipe for continued failure and boredom. The emphasis was thus on the increased maturity of the 17 year old and on a general preparation for adult life rather than on specific vocational preparation for a trade or industry.

Working Paper 46[74] states that "It cannot be assumed without question that the pupil continuing a subject in the sixth form needs to go on to material that demands a greater conceptual understanding. Another possible way is to widen the area of study. Even in linear subjects such as mathematics and science it is possible when one reaches the age of 16 to study alternative branches of the subject which are not more conceptually difficult but may be more appropriate for the older pupil". This is one of the main benefits of this examination. Watkins (1982; p.41)[105] also states that many colleges use CEE in mathematics and English rather than O-level, because the syllabuses are more varied and the style of examining more appropriate at this age, while the likelihood of obtaining an O-level equivalent is strong. This writer interprets Watkins as inferring that the increased motivation caused by a different style of learning will increase student proficiency. Without doubt the major obstacle to the development of this examination has been the lack of national validation.

In 1979, besides recommending the approval of the CEE examinations on a pilot basis, the Keohane Report (1979)[43], on the assumption that over two-thirds of those taking CEE pilot examinations expect to seek employment on completion of their studies at 17 plus, recommended that the examination should actively prepare students for employment. Had this report been accepted, the CEE would have been different from its present form. It would have been a free standing examination with its own certification and it would have been unrelated to either CSE or O-level. There were to have been three gradings: pass with merit, pass, and fail and in addition there would have been compulsory proficiency tests in English and Mathematics.

The writer doubts the wisdom of trying to separate the CEE totally from the GCE O-level. Unless the new examination were to obtain immediate acceptance, many employers would either, especially in subjects like English and Mathematics, demand O-level certification or a certain pass on the CEE that they deemed as equivalent. The idea behind the proficiency tests was certainly to make the CEE more comparable to courses in further education but in the writer's opinion its big advantage was individual subject certification that made a direct comparison with O-level possible.

Table 11 shows that slightly more students in the sample by Dean (1982; p.97)[13] entered CSE than CEE. "However, the total number of subjects was lower and students took on average only 1.9 subjects at CSE. Mathematics was the most popular subject and there were more than twice as many entries in this subject as English, the next most popular. Indeed, CSE appears to be used in some sixth forms purely or mainly as a means of enabling students to obtain some form of qualification in mathematics. ... The mean grade in mathematics was 3.8 ... and other subjects 3.1..." This seems to confirm the demand for a mathematical qualification. The low percentage of students (9%) attempting CSE infers that it is not well-recognised by either schools or students.

	Schools	6th Form Colleges	All Institutions
Mainly O-level Courses	64	76	66
Mainly CEE	7	6	6
Mainly CSE	10	2	9
Secretarial Courses	17	10	16
City and Guilds Foundation Courses	2	2	2
Other Courses	1	4	2

Table 11

Distribution of non-A-level students on different courses (percentage). (Dean and Steeds; p.45)[13]

The whole system of 16+ examinations is currently in a state of insecurity. The dual system of GCE and CSE creates a number of difficulties which are just as great if not greater in the sixth form as in the fifth form. "The dual grading scheme does not match any natural division between the aspirations or abilities of the young people concerned; CSE certificates have not been accepted in the way in which holders have been led to expect or which reflects the attainment of many holders by comparison with the attainment of those who are awarded equivalent O-level grades, and this may reduce the motivation of pupils. The dual grading scheme confuses schools, pupils, parents and employers; it contributes to a harmful tendency to enter pupils for unsuitable courses; and it is inefficient because it involves duplication of effort by the examination boards and because some pupils enter both GCE and CSE examinations in the same subject, sometimes on syllabuses which are widely divergent." (G.B. DES.)[26]

The new proposed examination, probably called the General Certificate of Secondary Education (GCSE) would be awarded on a single scale of 7 grades. Fig. 8 shows it in relationship to GCE and CSE.

GCE O-level	GCSE	CSE
A	1	1
B	2	
C	3	
D	4	2
E	5	3
	6	4
	7	5

Fig. 8

Comparison of proposed GCSE grades with GCE O-level and CSE

The main advantage of this system is that it allows for candidates to be placed on a single grading scale.

The major problem, however, is to construct an examination that assesses the attainment of candidates spanning a wide range of ability. The Waddell Report (1978, p.29)[102] considering various studies on how to examine mathematics to this wide ability range, concludes "that in mathematics a common examination comprising only written papers taken by all the candidates cannot be satisfactory for both extremes of the ability range". The mathematics subject panel for the Midlands Examining Group have selected a method of assessment available at three levels, namely Lower, Intermediate and Higher as shown in Fig. 9 . At each level, the papers will be designed for the target group indicated by the grades in the table. In exceptional circumstances, the next two grades below those indicated may be awarded for candidates entered for the Intermediate and Higher Levels.

Level	Intended Target Group	Papers to be Set	
Lower (L)	GCE Grade E CSE Grades 3,4,5	Paper I(A) (50%) (1½ hours)	Paper II (50%) (2 hours)
Intermediate (I)	GCE Grades C,D,E CSE Grades 1,2,3	Paper I(A/B) (50%) (2 hours)	Paper III (50%) (2½ hours)
Higher (H)	GCE Grades A,B,C CSE Grade 1	Paper I(B/C) (50%) (2½ hours)	Paper IV (50%) (2½ hours)

Fig. 9

Proposed GCSE grading scheme of the Mathematics Panel of the Midlands Examining Group. (Midland Examination Group 1983)[52]

The use of alternative papers or alternative combinations of papers would mean that candidates would have to decide with the help of their teachers which to tackle. Parential pressure coupled with pupils' aspirations and employers' demands may cause many pupils to attempt the combination of papers which are too difficult for them. Whereas the present system has the O-level as the generally required goal, so probably will be the equivalent grade 3 in the single system. Although double entry is often unsatisfactory, it does provide a safety net for many borderline pupils. Although sixth form students should already have a certain grade, the problem is no way easier. Many enter the sixth form hoping to improve previous grades significantly and as such will be demanding entry for the higher combination of papers. But as mentioned on page 73 many will never reach the level of understanding required to attempt these more difficult questions satisfactorily.

It has been shown in this chapter that the main priority for most students of mathematics in this target group is to obtain an O-level or its equivalent. Although the GCSE may alter the systems of examining this group, the writer believes that with our society being strongly examination orientated there will still be strong pressure to obtain the equivalent of an O-level pass.

CHAPTER 11

Pre-Vocational Courses

The introduction to this final section showed post-16 full-time courses as leading to two types of qualification. These qualifications, taken as a whole, make a valuable contribution to preparing young people for further and higher education and for employment and adult life. "They do not, however, meet fully the needs of young people of widely varying ability, but usually with modest examination achievements at 16+, who have set their sights on employment rather than higher education, but have not yet formed a clear idea of the kind of job they might tackle successfully, or are not yet ready to embark on a specific course of vocational education or training." (G.B.DES: p.1)[29] For this group of students, single subject examinations do not provide the coherent programme that is required nor do they have the required practical slant. As such, the various existing pre-vocational courses are more appropriate.

Since 1970, many new pre-vocational examinations have been introduced by organisations such as the Further Education Regional Examining Bodies (Certificate of Further Education), the City and Guilds of London Institute (Foundation courses), the Business Education Council and the Royal Society of Arts. To date, however, these courses according to the survey by Dean (Table 11) have only been used by between 2 and 4 percent of non-A-level students in schools and sixth form colleges. The probable reason for this is adequately seen from a statement by the study group of the Further Education Curriculum Review and Development Unit (1982; p.6)[18]. They concluded that there was "a provision of courses so tangled as to confound the investigator, let alone the undecided 16 year old" and argued for a rationalisation of provision in order to provide a chance of countering the dominant influence of GCE O-level as currency in the minds of young people and employers. As such a single national qualification was needed.

It was against this background that the Government decided to establish a new national qualification. The target group "excludes young people who have the potential to take two or more A-level examinations, and those who have a clear vocational objective which would best be pursued by immediately following a vocational course in further education. The Secretaries of State consider that the target group should be further limited by excluding those who are properly advised to devote their main effort to obtaining or improving GCE O-level or CSE qualifications with a view to gaining access to particular courses or to certain categories of employment"(1982; p.3)[29]. As such the Government has not been able to define the target group positively and it has to be deduced from the above statement about those for whom it is not intended. In terms of ability the range is enormous but in terms of numbers it is likely to be small. The Government's estimate of 80,000 initially may prove to be optimistic in view of the introduction of the Manpower Services Commission's New Training Initiative which guarantees all unemployed school-leavers a period of training and financial support - a somewhat ironic twist in view of the Government's declared intention of rationalising education and training provision for the seventeen year olds.

According to the D.E.S. the aims of this course should offer a broad programme of general education with emphasis on its various types of employment, develop personal attributes such as self-motivation, adaptability, self-reliance, a sense of responsibility and an ability to work with others, and help each student to discover what kind of job he or she might expect to tackle with success. The course will consist of a common core that will occupy some 60% of student time. The core would include written and spoken English, mathematics, aspects of science and technology, and their applications in the modern world; and studies designed to give a broad understanding of citizenship and its responsibilities, the way the country earns its living and the nature of our institutions.

It could easily be argued that there does not appear to be much that is new in the above and that most of the young people will have had some experience in all of these areas. In earlier sections it has been argued that repeat courses usually fail to motivate pupils and that examination results are usually poor. It is difficult to see how grouping all the elements into a common core will improve motivation and performance. This problem is particularly acute in mathematics where sutdents have been studying this subject, many with little success and enjoymnet, for the last twelve years. Schools will need to plan their course carefully to make sure it is not just a rehash of what has been taught in the last two years. Section I of this report argued that mathematics should be taught to all students after the end of compulsory education and as such this writer welcomes mathematics being part of a compulsory common core. However, one has serious reservations about schools developing interesting and worthwhile mathematics courses, given the relatively small numbers and the widely differing abilities of students. Many students could well be demoralised with either their inability to cope or the repetition of work easily mastered years earlier.

The main objective of this course is to prepare young people more effectively for employment and adult life. As such the new certificate would be awarded to all those who complete the course which records performance across the whole range of work. "It is not proposed to establish any general equivalence between the assessment of performance on a CPVE course and the grades awarded in GCE or CSE examinations. The case for the new qualification rests on the unsuitability of GCE and CSE syllabuses as the basis for the pre-vocational courses which are needed by people in the target group." (G.B.DES: p.6)[29] The long term success of the CPVE will depend largely on the extent to which those with the qualifications are successful in obtaining employment as a result of having obtained this certificate. One cannot help thinking that the CPVE student could well be at a disadvantage compared with his GCE/CSE or N.T.I. colleagues, the former because these qualifications, especially the GCE, are well-established and recognised and the latter because of the greater amount of work experience.

Any discussion on Pre-Vocational Courses cannot be complete without a mention of the F.E.U. document "A Basis of Choice" (ABC). The inspiration of the CPVE seems to owe much to this document but the execution seems to have departed some way from it. For example, although ABC states that the target group would have "a wide spread of ability" (p. 6)[18] it does limit the top end as compared with the CPVE. Whereas the CPVE excludes some students of O-level calibre,"it excludes those who are properly advised to devote their main effort to obtaining or improving GCE or CSE qualifications" (p.13)[29]. ABC excludes all such students: "We think that the course should be aimed at those young people who are thought not to be equipped to follow a one year O-level course successfully" (p.12)[18]. This more restricted target group enabled ABC to specify the curriculum in terms of objectives. The mathematical objectives are given in Appendix 10. To attempt this with a wider ability band as envisaged by the CPVE would, in the writer's view, lead to the problems mentioned earlier of motivation and relevance. It is also considered doubtful whether many students capable of O-level certification would attempt this new course. The introduction of the GCSE may also reduce the range of this target group. As mentioned earlier, many students in the sixth form have little motivation for restudying CSE courses but a single examination, such as the GCSE will probably be more acceptable. Therefore the spread of ability within the CPVE may not be as great as envisaged.

There is no doubt that there is an urgent need for proper courses and assessment procedures for this group of young people. The Government's declared intention to do something about this problem is to be admired. However, only time will tell whether such a course will be accepted by students and more importantly, employers as recognition by the latter is essential if it is to be worthwhile for the former.

CHAPTER 12

Conclusion

Although the majority of mathematics courses taught to non-specialist sixth form students are unsatisfactory, the problems of providing suitable courses for the traditional academic sixth form student are completely different from those of the "New Sixth Form" student. The traditional sixth form student is attempting to follow a path that has hardly changed since the introduction of the General Certificate of Education in 1951. Many new ideas, like the N and F levels, have been considered but all have been rejected and the problems associated with specialisation and "minority time" are the same today as they were at the time of Crowther in 1959.

It was recommended in Chapter 9 that in order to provide student motivation, mathematics should be taught to the traditional A-level student as part of a well-planned general studies course. There are, in this writer's view, two main problems for schools providing suitable general studies courses. The first problem is one that is within the capability of individual schools to remedy. General studies should be given the same status as other A-level subjects and organised and timetabled accordingly. Although many schools give little time and commitment to this subject, the increasing acceptance of A-level general studies as an entry qualification to universities and polytechnics should see many more schools providing well-organised courses.

The second problem is one of syllabus content. Chapter 6 has shown that the majority of A-level syllabuses require little if any mathematical knowledge. Even the J.M.B. syllabus, the one considered to have the largest mathematical component, contained questions of only about O-level standard. It is therefore difficult to see how without a complete reorganisation of the traditional A-level curriculum or a dramatic change in the content of A-level general studies, these students are going to achieve a worthwhile post-O-level mathematical education.

Although due to more schools providing well-organised general studies courses the future may see a few more pupils leaving the sixth form just as numerate as when they entered, the majority of these pupils will leave less numerate than they were at sixteen, a totally unsatisfactory situation for a group that will provide many of the country's leading figures not only in the Arts but in all walks of life.

Although unsatisfactory, the mathematical curriculum for the traditional sixth form student is stable compared to that of the "new sixth form" student. Also whereas mathematical certification for the traditional sixth form student, since O-level has been obtained, is not usually a consideration, a recognised mathematical qualification is the main and often only consideration for the "new sixth form" student. The problem with "new sixth form" students is in providing a mathematical course or part of a course, that is recognised by society yet within the ability of the student. Chapter 10 has argued that the O-level is society's symbol of proficiency while the work of Dean and Steeds (1982) has shown that for pupils entering the sixth form with C.S.E. grade 3 and below, the probability of O-level success is low.

It is these pupils that are the greatest cause for concern. This situation is made worse by the rejection of the C.E.E. causing more students to attempt O-level mathematics. Although the new C.P.V.E. is intended for this ability range, Chapter 11 has shown this writer's reservations about its success. The whole system of English secondary school education is based on academic excellence measured by individual subject examinations. It is this writer's view that if pre-vocational courses are to be accepted by society and especially by employers, then to introduce these courses after the end of compulsory education for students who are deemed not capable of success in individual examinations, is, with today's high unemployment, giving them a difficult position from which to succeed.

The future seems uncertain. Besides the combining of the G.C.E. and C.S.E. into a common examination, the rejection of the C.E.E. and the introduction of the C.P.V.E. and Y.T.S., there is the new proposed change in the secondary curriculum that was announced in January 1984and is aimed to take effect from 1989 (T.E.S. 1984)[96A]. Only time will tell how effective these changes will be, but change is needed as the present system offers little for those sixth form students with average and below average academic ability.

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APPENDIX 1

TABLE A17/81 [A17/80]
COURSES OF STUDY FOLLOWED BY OLDER PUPILS

ALL SCHOOLS[1]: 1. BOYS AND GIRLS

January 1981

	Age of pupils at 31 August 1980	Pupils following these courses[2]: Beyond GCE A level	GCE A level: Year of course: 1st year	GCE A level: Year of course: 2nd or later year[3]	GCE A level: Total	Beyond O level/CSE but not at or beyond A level	GCE O Level/CSE	Other pupils
MAINTAINED SCHOOLS								
Modern	15	-	5	7	12	35	45,141	3,270
	16	-	1,183	62	1,245	81	2,404	79
	17	-	52	813	865	7	142	7
	18	-	2	49	51	-	5	-
	19 and over	-	-	5	5	-	1	-
	Total	-	1,242	936	2,178	123	47,693	3,356
Grammar	15	-	500	5	505	1	21,014	64
	16	-	13,992	480	14,472	70	1,094	15
	17	24	539	12,390	12,929	15	55	5
	18	97	10	852	862	1	2	1
	19 and over	3	1	28	29	-	-	-
	Total	124	15,042	13,755	28,797	87	22,165	85
Comprehensive	15	-	1,581	45	1,626	379	597,487	34,835
	16	16	104,869	1,919	106,788	5,569	48,310	1,945
	17	51	5,441	86,853	92,294	644	4,561	276
	18	225	325	8,506	8,831	44	343	21
	19 and over	8	45	453	498	14	56	2
	Total	300	112,261	97,776	210,037	6,650	650,757	37,079
Technical and other secondary	15	-	12	2	14	-	8,054	244
	16	-	1,823	19	1,842	109	877	10
	17	-	97	1,450	1,547	24	78	4
	18	3	8	94	102	-	10	1
	19 and over	-	2	7	9	-	-	-
	Total	3	1,942	1,572	3,514	133	9,019	259
All maintained secondary schools	15	-	2,098	59	2,157	415	671,696	38,413
	16	16	121,867	2,480	124,347	5,829	52,685	2,049
	17	75	6,129	101,506	107,635	690	4,836	292
	18	325	345	9,501	9,846	45	360	23
	19 and over	11	48	493	541	14	57	2
	Total	427	130,487	114,039	244,526	6,993	729,634	40,779
Independent schools[4]	15	-	4,793	279	5,072	349	41,866	2,449
	16	20	24,748	5,103	29,851	374	5,266	913
	17	181	2,098	22,952	25,050	101	775	559
	18	85	365	2,152	2,517	20	181	140
	19 and over	7	148	473	621	5	110	74
	Total	293	32,152	30,959	63,111	849	48,198	4,135
All schools[1]	15	-	6,891	338	7,229	764	713,562	40,862
	16	36	146,615	7,583	154,198	6,203	57,951	2,962
	17	256	8,227	124,458	132,685	791	5,611	851
	18	410	710	11,653	12,363	65	541	163
	19 and over	18	196	966	1,162	19	167	76
	Total	720	162,639	144,998	307,637	7,842	777,832	44,914

1. Excluding special schools
2. Each pupil is included once only according to the highest level of course being followed
3. Mainly 2nd year A level pupils
4. Partly estimated

APPENDIX 2

SCHOOL LEAVERS DURING ACADEMIC YEA

Details of leavers by age

(i) Age at 31 August 1978

Table 1 (1)

	Boys					Girls			
	15	16	17	18 and over	Total	15	16	17	18 and o
Leavers who had attempted CSE or GCE examinations									
Numbers who had attempted									
A level only	—	0·03	0·39	0·12	0·64	—	0·06	0·68	0·1
A and O levels, but not CSE	0·14	2·61	32·88	5·53	41·16	0·14	1·78	28·30	2·3
A level and CSE but not O level	0·04	0·06	0·09	0·02	0·20	0·03	0·02	0·11	–
A and O levels and CSE	0·14	0·69	21·91	3·79	26·52	0·15	0·85	22·99	1·9
O level only	9·89	4·41	0·97	0·13	15·41	10·17	3·77	0·89	0·1
O level and CSE only	100·53	20·15	2·91	0·23	123·83	107·53	26·89	4·27	0·4
CSE only	117·26	3·72	0·41	0·06	121·46	102·64	5·52	0·50	0·0
Total attempting CSE or GCE	228·02	31·65	59·57	9·88	329·12	220·66	39·00	57·73	5·0
Leavers who had attempted no CSE or GCE examination	46·73	1·02	0·13	0·01	47·90	36·54	1·16	0·14	0·0
Leavers with A level passes									
3 or more higher grade combinations[1]	0·10	1·52	17·01	3·12	21·75	0·02	0·91	13·20	1·1
3 or more other passes	0·01	0·60	12·46	1·74	14·80	—	0·46	10·03	0·6
2 A level passes	0·05	0·45	12·06	1·89	14·44	0·03	0·46	13·21	1·0
1 A level pass	0·07	0·44	8·33	1·49	10·34	0·14	0·50	9·34	0·9
1 or more A level passes	0·23	3·00	49·86	8·25	61·34	0·19	2·33	45·78	3·8
No A level passes									
5 or more higher grade O level or CSE results[2]	17·94	7·05	5·64	0·89	31·52	22·97	8·56	6·14	0·5
1–4 higher grades	76·51	14·91	3·23	0·62	95·27	82·07	18·73	4·77	0·6
No higher grades									
1 or more other grades	128·64	6·53	0·77	0·10	136·04	111·85	9·22	0·96	0·1
No GCE or CSE qualifications	51·44	1·18	0·20	0·05	52·86	40·12	1·32	0·23	0·0
Leavers entering									
Degree courses	0·05	1·35	26·23	5·08	32·72	0·02	0·92	18·66	1·97
Teacher training courses	—	0·01	0·66	0·11	0·78	0·06	0·08	2·66	0·26
HND/HNC courses	—	0·03	1·00	0·39	1·41	—	0·02	0·92	0·06
OND/ONC courses	1·18	0·46	0·13	—	1·78	0·99	0·20	0·07	0·01
Catering courses	1·39	0·11	0·15	0·03	1·67	3·52	0·44	0·28	—
Nursing courses	0·03	0·02	—	—	0·05	4·18	0·66	0·50	0·02
Secretarial courses	0·13	—	—	0·01	0·14	13·70	1·82	1·71	0·12
GCE A level courses	4·28	1·08	2·00	0·26	7·61	5·70	1·08	1·60	0·15
GCE O level courses	4·44	0·43	0·14	0·01	5·02	5·69	0·46	0·15	0·01
Other further education courses									
Science and engineering[3]	2·88	0·55	0·72	0·12	4·27	0·77	0·21	0·64	0·02
Education social studies and professional courses[3]	2·07	0·41	1·31	0·20	3·98	8·72	1·27	1·60	0·07
Arts[3]	1·26	0·30	1·15	0·08	2·80	2·45	0·70	2·19	0·16
Miscellaneous and other courses	1·73	0·40	0·32	0·06	2·51	4·07	0·63	0·68	0·09
All full-time further education	19·46	5·15	33·80	6·34	64·76	49·88	8·39	31·59	2·94
Temporary employment pending entry to									
Degree courses	0·05	0·52	2·08	0·11	2·14	—	0·26	1·78	0·05
Teacher training courses	—	—	0·01	0·01	0·02	—	0·02	0·06	—
Other courses	0·20	0·18	0·38	0·02	0·78	0·07	0·09	0·32	—
Total	0·25	0·70	2·45	0·14	3·54	0·07	0·37	2·16	0·05
Employment	234·69	23·53	19·06	2·30	279·58	185·92	27·75	20·49	1·69
Destination not known	20·35	3·30	4·39	1·11	29·15	21·32	3·65	3·63	0·64
Leavers leaving in									
Autumn term, 1978	0·20	3·75	2·25	2·62	8·82	0·29	4·27	1·38	1·02
Spring term, 1979	25·30	2·98	0·60	0·44	29·33	19·34	3·45	0·69	0·39
Summer term, 1979 in									
April	1·14	0·19	0·07	0·02	1·42	1·00	0·26	0·02	—
May	137·77	3·45	3·15	0·28	144·65	126·05	4·05	4·18	0·23
June	53·64	8·43	20·79	2·45	85·31	56·89	11·81	23·50	1·39
July	56·32	13·76	32·72	4·07	106·87	53·33	16·17	28·07	2·09
August	0·40	0·11	0·12	0·01	0·65	0·33	0·15	0·04	—
Total Summer Term 1979	249·25	25·94	56·85	6·83	338·87	237·58	32·44	55·80	3·71
All leavers	274·75	32·68	59·70	9·89	377·02	257·20	40·16	57·87	5·11

(ii) Age at 31 December 1978

	Boys					Girls			
All leavers	188·70	108·82	50·08	29·43	377·02	176·42	108·57	52·72	22·64

(iii) Age on leaving school

	School leaving age	Others aged 16	17	18 and over	Total	School leaving age	Others aged 16	17	18 and over
All leavers	183·51	100·64	33·15	59·72	377·02	169·08	99·33	38·98	52·95

[1] For a definition of higher grade A level passes see paragraph 23 of the explanatory notes.
[2] For a definition of higher grade O level (including O level passes on A level papers) or CSE results see paragraph 50 of the explanatory notes.
[3] Including an element of City and Guilds courses

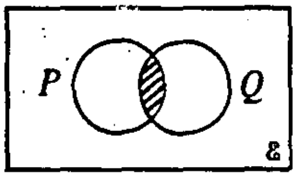
Source: S

QUESTION 4

For each of Questions **4.1** to **4.20** choose the answer you consider the best of the alternatives offered in **A, B, C, D** and **E**. You are reminded that graph paper is available on request from the Supervisor.

Questions 4.1 to 4.3

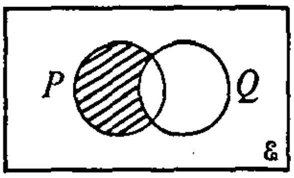
In each of the following Venn diagrams a region is shaded if and only if it is known to be empty.

Thus  indicates that $P \cap Q$ is empty. This can be written $P \cap Q = \emptyset$

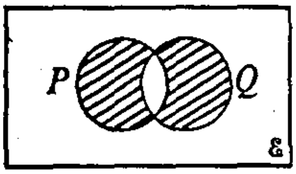
In each of Questions **4.1** to **4.3** you are given a Venn diagram. Choose from the list **A** to **E** the relationship which it represents.

A $Q' = \emptyset$
B $P = Q$
C $Q \subset P$
D $P \cup Q = \&$
E $P \cap Q' = \emptyset$

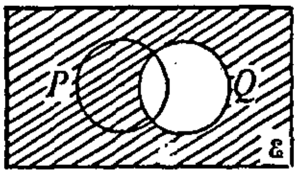
4.1



4.2



4.3



QUESTION 4 *(continued)*

Questions 4.4 to 4.6

In each of Questions 4.4 to 4.6 an expression has been rearranged by four stages to give the result on the final line. The rearrangement may contain an error.

Answer

A if the error appears in line (i).
B if the error appears in line (ii).
C if the error appears in line (iii).
D if the error appears in line (iv).
E if no error appears.

4.4 $x^2-y^2 = 2xy$

(i) $x^2-2xy-y^2 = 0$

(ii) $(x-y)^2 = 0$

(iii) $x-y = 0$

(iv) $x = y$

4.5 $x-y = \frac{1}{x+y}$

(i) $(x-y)(x+y) = 1$

(ii) $x^2-y^2 = 1$

(iii) $x^2 = y^2 + 1$

(iv) $x = y+1$

4.6 p and q are positive numbers where $\frac{2}{p} - \frac{3}{q} > 0$

(i) $\frac{2}{p} > \frac{3}{q}$

(ii) $\frac{p}{2} > \frac{q}{3}$

(iii) $\frac{p}{q} > \frac{2}{3}$

(iv) $\frac{q}{p} < \frac{3}{2}$

QUESTION 4 (*continued*)

4.7 The numbers a, b, c, p and q are such that $a{:}b = p{:}1$ and $b{:}c = 2q{:}1$. What is the value of the ratio $a{:}c$?

A $2pq{:}1$

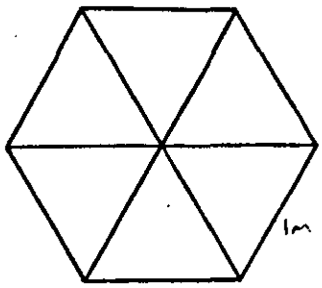
B $\frac{p}{2q}{:}1$

C $\frac{2p}{q}{:}1$

D $p+2q{:}1$

E $2(p+q){:}1$

4.8



The diagram represents a regular hexagon of side one metre. What is the area of the hexagon in square metres?

A $2\sqrt{3}$

B 3

C $\frac{3\sqrt{3}}{2}$

D $3\sqrt{3}$

E 6

Questions 4.9 to 4.11

Five points have coordinates as follows.

A (3,2)

B (3,0)

C (−2,−2)

D (−3,0)

E (−1,4)

4.9 Which of these points is farthest from the origin?

4.10 Which of these points is farthest from the line whose equation is $x+y = 0$?

4.11 Four of these points lie on a single circle of centre (0,1). Which point does not lie on this circle?

QUESTION 4 (*continued*)

4.12 In x games of football an average of p goals was scored and in a different set of y games an average of q goals was scored. What was the average number of goals scored in all these games?

A $\frac{px+qy}{p+q}$

B $\frac{pq}{xy}$

C $\frac{px+qy}{x+y}$

D $\frac{p+q}{2}$

E $\frac{px+qy}{2}$

4.13 In a game a coin is tossed three times by each player. Each "head" counts +2 points and each "tail" -1 po
The player's total score is the sum of the points gained in the three tosses.
What is the number of different scores between -3 and $+6$ which cannot be achieved?

A 3
B 4
C 5
D 6
E 7

4.14 The shaded area in the diagram is described by the set

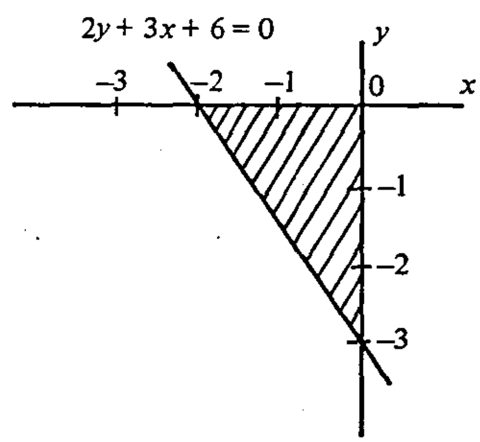
A $\left\{x < 0, y = 0, y \geqslant \left(\frac{3x}{2} + 3\right)\right\}$.

B $\left\{x < 0, y < 0, y < \left(\frac{3x}{2} + 3\right)\right\}$.

C $\left\{x \leqslant 0, y < 0, y = \left(\frac{3x}{2} + 3\right)\right\}$.

D $\left\{x = 0, y = 0, y = -\left(\frac{3x}{2} + 3\right)\right\}$.

E $\left\{x \leqslant 0, y \leqslant 0, y \geqslant -\left(\frac{3x}{2} + 3\right)\right\}$.



4.15 Working entirely in binary numbers, $(1\,0\,1)^{11}$ is the same as

A 101+101.
B 101×11.
C 101×101.
D 101×101×101.
E 1111111.

4.16 A sum of money is invested for 1 year at 15% interest. The interest is subject to tax at a rate of 30%. The *effective* rate of interest (i.e. taking tax into account) on the investment is

A 12%.
B 10½%.
C 15%.
D 4½%.
E 3%.

QUESTION 4 *(continued)*

4.17 A logarithm is the exponent to which a given base must be raised to obtain the number whose logarithm is required.

An antilogarithm is

A the logarithm raised to the power of 10.
B the reciprocal of the logarithm.
C the number such that the base will equal the logarithm.
D the number obtained when the logarithm is raised to the power of the base.
E the number obtained when the base is raised to the power of the logarithm.

4.18 Given that $a > 0$ which one of the following statements could be false?

A $a^0 = 1$
B $a^p a^q = a^{p+q}$
C $a^{\frac{1}{p}} = \frac{1}{a^p}$
D $\frac{a^p}{a^q} = a^{p-q}$
E $(a^p)^q = a^{pq}$

Questions 4.19 and 4.20

In each of Questions **4.19** and **4.20** you are given two statements, *P* and *Q*.

Answer

A if it is *certain* that *P* and *Q* are both true.
B if it is *certain* that *P* is true and *Q* is false.
C if it is *certain* that *P* is false and *Q* is true.
D if it is *certain* that *P* and *Q* are both false.
E if none of the answers **A, B, C** and **D** is correct.

4.19 *P*: A triangle must contain exactly one obtuse angle.
Q: A triangle must contain exactly two acute angles.

4.20 n is an odd integer. *P*: $3n+1$ is bound to be even.
Q: $2n-1$ is bound to be odd.

APPENDIX 4

Results of the Mathematical section of the 1983 general studies examination and the November 1982 S.M.P. Mathematics O-level examination given to 49 mathematically able 5th form pupils of the Henry Fanshawe School, Dronfield

Pupil No.	Pupil Sex	Gen.Stud. (y) Maths. %	SMP (x) Maths. %	Pupil No.	Pupil Sex	Gen.Stud. (y) Maths. %	SMP (x) Maths. %
1	M	25	49	26	F	55	45
2	M	30	57	27	F	55	65
3	M	35	45	28	F	55	61
4	M	35	68	29	M	55	74
5	M	35	76	30	F	55	76
6	F	35	43	31	F	55	75
7	F	40	67	32	F	55	64
8	M	40	75	33	F	60	65
9	M	45	71	34	M	60	79
10	M	45	58	35	F	60	53
11	M	45	85	36	F	60	76
12	F	45	30	37	F	60	53
13	M	45	52	38	F	65	61
14	M	45	45	39	F	65	65
15	M	50	82	40	F	65	62
16	M	50	61	41	M	65	82
17	F	50	51	42	M	70	61
18	M	50	61	43	F	70	51
19	F	50	58	44	F	70	49
20	M	50	74	45	F	75	67
21	F	50	75	46	M	80	88
22	F	50	69	47	M	80	89
23	M	55	62	48	M	85	84
24	M	55	78	49	M	90	69
25	F	55	64				

Contd.

Appendix 4 (Continued)

Σy	=	2600	Σx	=	3170	Σxy	=	176130
Σx^2	=	213326	Σy^2	=	155375			
$\bar{x}$	=	64.694	$(\bar{x})^2$	=	4185.3			
$\bar{y}$	=	53.061	$(\bar{y})^2$	=	2815.49			

Calculation of the least squares regression line of y on x

$$\Sigma y = a_1 \Sigma x + n b_1$$

$$\Sigma xy = a_1 \Sigma x + b_1 \Sigma x$$

Solving simultaneously for a_1 and b_1

$$a = 0.96 \qquad b = -9.04$$

Therefore $y = 0.96x - 9.04$

Calculation of product moment correlation coefficient

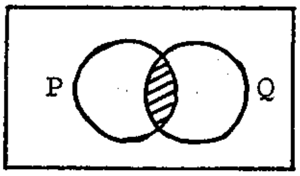
$$r = \frac{\frac{1}{n}\Sigma xy - \bar{x}\bar{y}}{\sqrt{\frac{1}{n}\Sigma x^2 - \bar{x}^2}\ \sqrt{\frac{1}{n}\Sigma y^2 - \bar{y}^2}}$$

$$= \pm 0.67$$

APPENDIX 5

In each of the following Venn diagrams a region is shaded if and only if it is known to be empty.

Thus

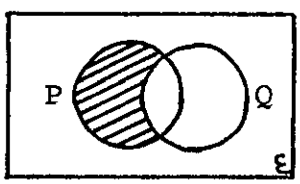


indicates that $P \cap Q$ is empty.
This can be written $P \cap Q = \emptyset$

In each of Questions 4.1 to 4.3 you are given a Venn diagram. Choose from the list A to E the relationship which it represents.

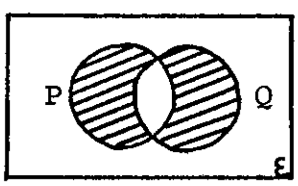
A $\quad Q' = \emptyset$
B $\quad P = Q$
C $\quad Q \subset P$
D $\quad P \cup Q = \mathcal{E}$
E $\quad P \cap Q' = \emptyset$

1



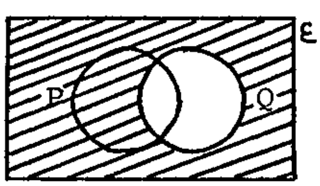
Gen St

2



Gen St

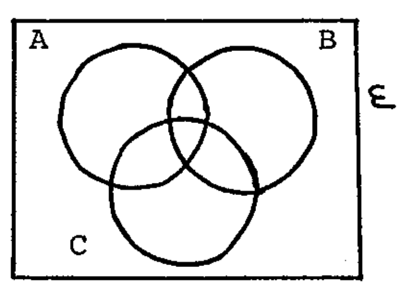
3



Gen St

4 In the Venn diagram A, B and C are subsets of $\mathcal{E}$. Draw the diagram and shade the region which represents the set $A' \cap (B \cup C)$.

SMP

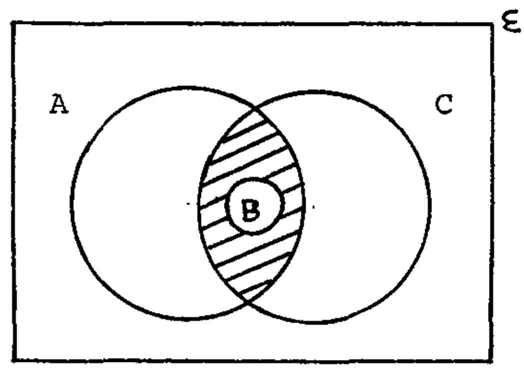


APPENDIX 5 (Contd.)

SMP 5 The sets A, B and C are shown in the Venn diagram.

(i) Write down an expression for the shaded region in the form

.... $\cap$ .... $\cap$ ....



SMP (ii) If $\mathcal{E}$ = {positive integers less than 40}

A = {even numbers}

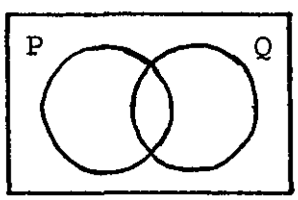
B = {6, 18, 30}

C = {multiples of 3}

list the elements represented by the shaded region.

6 The operation * is defined on any two sets X and Y by X * Y = X' $\cap$ Y.

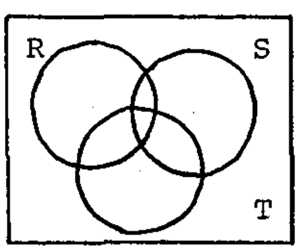
SMP (i)



(a) Copy this diagram and shade the region representing P * Q.

SMP (b) Make another copy of the diagram and shade the region representing Q * P.

(ii) V = R * (S * T)



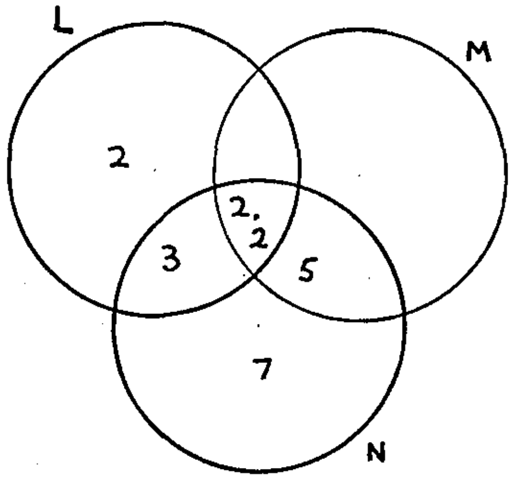
(a) Copy this diagram. Show and label clearly the region representing V.

Gen St 7 In the diagram the prime factors of the numbers 24, 20, 420 have been arranged in overlapping sets L, M, N. The factors 3 and 5 are common to the overlaps between L and N and between M and N respectively and not to L and M. Consider the numbers 36(P), 96(Q), 210(R) in the same way. Which factor(s) is(are) common to the overlap between P and Q but not to P and R or Q and R?

Contd.

APPENDIX 5 (Contd.)

A	2
B	2,2,2
C	2,3
D	3



APPENDIX 6

Results of Test on Sets

Pupil Number	General Studies	SMP	Pupil Number	General Studies	SMP
1	1	4	41	0	3
2	3	1	42	2	6
3	2	0	43	1	0
4	2	4	44	2	5
5	2	0	45	2	4
6	2	0	46	5	2
7	1	1	47	3	5
8	1	1	48	3	6
9	2	2	49	3	6
10	2	2	50	3	6
11	2	2	51	2	5
12	3	6	52	2	3
13	2	6	53	1	3
14	2	3	54	2	0
15	2	0	55	2	2
16	1	2	56	3	6
17	2	1	57	3	6
18	2	0	58	3	3
19	2	0	59	1	4
20	3	3	60	0	2
21	3	1	61	1	0
22	2	5	62	1	0
23	3	5	63	1	0
24	3	5	64	1	0
25	0	6	65	2	0
26	3	5	66	2	0
27		4	67	3	4
28	1	5	68	3	3
29	1	6	69	1	0
30	3	1	70	1	5
31	1	0	71	1	0
32	2	1	72	1	4
33	2	5	73	2	5
34	1	0	74	1	0
35	2	2	75	1	0
36	1	3			
37	1	3	Total	139	201
38	1	2			
39	1	3			
40	1	3			

Number of Gen. Stud. Questions $= 4 \times 75 = 300$

" " SMP " $= 6 \times 75 = 450$

% correct Gen. Stud. $= \frac{139}{300} \times 100 = 46\%$

% " SMP $= \frac{201}{450} \times 100 = 45\%$

APPENDIX 7

Results of the Mathematical Section of the 1983 general studies examination given to 28 second year 6th form non-mathematical specialist students

Pupil No.	O-level Grade	Gen. Stud.	Pupil No.	O-level Grade	Gen. Stud.
1	A	55%	19	C	35%
2	B	55%	20	C	35%
3	B	55%	21	C	30%
4	B	45%	22	C	30%
5	B	40%	23	C	30%
6	B	40%	24	C	30%
7	B	40%	25	C	25%
8	B	35%	26	C	25%
9	B	30%	27	C	25%
10	C	55%	28	C	25%
11	C	50%			
12	C	50%			
13	C	45%			
14	C	45%			
15	C	40%			
16	C	40%			
17	C	35%			
18	C	35%			

		Gen.Stud.	O-level approx.	
Average Mark.	Grade A =	55%		(1 pupil)
	B =	42.5%	60	(8 pupils)
	C =	36%	50	(19 pupils)

APPENDIX 8

Mathematics Applicable Group

In addition to its local dissemination activities, the group holds a workshop each summer at Reading University. Reports from two of these workshops, "Modelling with Trigonometry" and "Modelling with Probability" have been published.

Further details of the group's work and publications are available from the secretary:

F. Mays,
29, Elgin Mansions,
London,
W9 1JG

APPENDIX 9

THE CONTENT OF THE UNITS

1 Indices and Standard Form

A text and a diagnostic test. Introduces index notation. Deals with the manipulation of numerical and algebraic statements expressed in index form.

Provides the background necessary to an appreciation of 'Computation by Logarithms' and the 'Theory of Logarithms'.

2 Positive and Negative Numbers

A text and pre-test. Explains how signed numbers behave in mixed expressions involving all four rules.

Provides background to a large number of other CMP units.

3 'y = mx + c'

A text, and a tape/filmstrip introduction to the significance of the values of m and c. Students are then taught to sketch a line given its equation, and to determine m and c when given two or more values of x and y which are related linearly.

Pre-requisites include elementary ideas of co-ordinates and an ability to manipulate directed numbers. The unit forms a useful background unit for 'Linear Programming' and for 'Using Logarithms'.

4 Inequalities

A text and a diagnostic test. Deals with the solution of linear inequalities in one variable, the graphical representation of linear inequalities in one or two variables, and with the determination of regions satisfying two or more orderings.

Pre-requisites include a knowledge of directed numbers, an ability to represent linear equations graphically, and an ability to solve simple linear equations.

The unit prepares the ground for 'Linear Programming'.

5 Working with Ratios

A text, a tape/filmstrip introduction to the 'game-board' technique and a pre-test.

Concentrates on the manipulation of expressions of the form $\frac{A}{B} = \frac{C}{D}$.

Pre-requisites include some previous familiarity with simple equations and ratios.

6 Transformation of Formulae

A text and pre-test. Builds up to problems like : Find h, if $\frac{Ah^n}{B} + \frac{C}{D} = \frac{E}{F}$.

Pre-requisites - a mastery of unit 5

7 The Trigonometry of the Right Angled Triangle

A text and a diagnostic test. All six ratios dealt with in calculating sides and/or angles of right angled triangles.

Pre-requisites familarity with 'similar' triangles and an ability to use log tables (see units 12 and 13)

8 The Trigonometry of the General Triangle

Text only. Covers the application of sine and cosine rules to the solution inclu ing the ambiguous case of triangles given any three of its six elements. Proofs of rules and explanation of the sine, cosine and tangent of obtuse angles are given in Appendices. Unit 6 is a pre-requisite.

9 An Introduction to Trigonometrical Graphs

Text only. Covers graphs of sine, cosine, and tangent of angles in degrees and radians to the level of a f(bx) where f is sin, or cos or tan. Solution of y = af(bx) where y is known. Graphs of A sin ax + B sin bx, and similarly for cosine. Pre-requisite is unit 8.

10 An Introduction to Sets

A text only. Provides an introduction to the language of sets and to operations with sets. Appropriate for revision, or for a student unfamiliar with set notation.

The unit may be needed as a pre-requisite to the Project's Probability Course and the sequence on Information and Coding, which makes limited use of 'set' language.

11 ${}_nC_r$ - Calculation and Use

A text and a tape/filmstrip introduction to the ideas of listing possible combinations by means of Pascal's triangle. The text gives practice with the notation and shows applications.

The unit provides background necessary for the Project's units on the binomial distribution, especially units 42, 45 and 46.

12 Computation with Logarithms Part 1
13 Computation with Logarithms Part 2

Designed as a single unit, but divided for convenience into two parts. Text only

May be used by students having no knowledge of logarithms, but is so designed that students with partial knowledge are directed to sections relevant to their needs. Covers the multiplication, division, powers and roots of number >1 and <1. A sheet of tables is provided.

Pre-requisites include a knowledge of simple powers and roots, and of indices in fractional and decimal form.

14 The Theory of Logarithms Part 1

15 The Theory of Logarithms Part 2

Designed as a single unit, but divided for convenience into two parts. Text only More demanding than most revision units. Appropriate for students preparing for 'additional' and 'A' level G.C.E. in mathematics.

The concept of a logarithm is approached via numbers which are positive integer powers of two. This leads to the principle on which a slide rule works and so t logs to base two, to base 10, and finally to natural logs. The equivalence of an index expression and its corresponding log form are shown, and the formula for changing from one base to another is derived.

Pre-requisites include an ability to multiply, divide, raise to a power and find roots using log. tables. Familiarity with index notation and with 'rules' for man pulation is also called for.

The pre-requisites may be met by using the units 'Indices and Standard Form', a 'Computation by Logs'. Applications are discussed in 'Using Logarithms to Determine Relationships'.

16 Using Logarithms to Determine Relationships

A text only. More demanding than most revision units.

Examples of exponential growth and decay are introduced. The unit then discusse how experimental results can be sorted into those conforming to an exponential law of growth and those related in other ways. The advantages of using logarithmic graph paper are shown.

Pre-requisites include an ability to interpret $y = mx + c$ in terms of gradient and intercept, and the elementary theory of logarithms. Units 3, 12, 13, 14 and 15 relevant.

17 Flowcharts and Algorithms

A text only.

Introduces elementary flowchart conventions and shows how to read and interpret a flowchart. Flowcharts are used to express simple algorithms e.g. an iterative process for determining a square root. The second part of the unit uses a flowchart to play a game. Various ideas arising from the game are then developed in the text.

18 Critical Path Analysis

A text and a game 'Circumbendibus', for which the network sheets, cards and indicator boards are provided.

Aims to familiarise the student with the diagrammatic conventions and the terminology of Critical Path Analysis. In the first part, simple examples are discussed and the student is shown how to identify the critical path. The game is designed to consolidate the student's understanding and to provide motivation for further study.

The unit provides essential background to the follow-up unit, 'Further Critical Path Analysis'.

19 Further Critical Path Analysis

A text only, which provides further analysis of networks with emphasis on two types of float - free and total float. Gantt charts are used to discuss float before a discussion of 'early' and 'late event' times.

Pre-requisites include a knowledge of elementary Critical Path Analysis terminology, and an ability to identify the critical path in a given network.

The unit 'Critical Path Analysis' provides the necessary background.

20 Systems

Text only. An elementary introduction to the concepts and terminology associated with systems. In addition to the introduction of important basic ideas the unit provides a wide range of examples.

Pre-requisites include some knowledge of coordinates.

21 Linear Programming

A text only. Shows how simple restrictions on the values of one or two quantities may be expressed in the form of algebraic inequalities and by regions on a graph. Regions satisfying several conditions are identified, and methods given for determining the maximum or minimum value of a linear expression.

Pre-requisites include an ability to draw the line corresponding to an equation of the form $y = mx + c$, and an ability to express simple inequalities graphically.

The units 3 and 4 provide the necessary background.

THE INFORMATION AND CODING SEQUENCE (4 units)

The sequence aims to show how the symbols (generally letters, spaces between words, punctuation marks, etc) which make up a message, can be quantified according to their frequency of occurrence and how they can be encoded and transmitted according to measurable levels of efficiency.

The sequence is of interest, in that it shows how language is susceptible to mathematical analysis. It is also of value as an introduction to information and communication theory.

Each unit is in the form of a programmed text. While the first may stand on its own, forming, in the context of Category 2, an introduction to the subject, it is suggested that students going beyond the first unit should complete all four units in order to obtain a coherent picture. Further details of the title and content of each unit are provided below.

22 1 : Binary Codes and 'bits' of Information

Shows how to 'quantify' information in the simplest situation, where the symbols used to send messages are used with equal frequency. Calculates for this simplified model the 'uncertainty' associated with a set of message units and the 'information content' of a message.

Although explanation is provided in the text, students will find the work easier to follow, if they are familiar with the function $x \longrightarrow 2^x$ and with the idea of logarithms to the base 2.

23 2 : More 'bits' of Information

Moves into the reality of language usage, and develops a more sophisticated measure of the 'average information content' of a message, recognising that in practice the symbols used in the English language occur with unequal probability.

Some acquaintance with the Σ notation an advantage; familiarity with elementary probability, including the addition rule for mutually exclusive events, also useful (see Unit 39).

24 3 : Redundancy - Friend and foe

Introduces the concept of 'redundancy', its causes, the way it is measured, and some of the advantages to be obtained from it. Calculates the amount of 'redundancy' attributable to variation in 'letter frequency' and to 'diagram structure'.

Some calculating aid, preferably an electronic calculator should, if at all possible be available. However, the student working with only a slide-rule or with log tables can manage, although he is at a disadvantage.

25 4 : Getting the message across

Concentrates on the themes of (i) coding for efficiency (the methods of Fano and Huffman are introduced) and (ii) coding for the detection and correction of errors. The Morse code is subjected to rigorous scrutiny.

Exercises in this text also involve a good deal of computation, so some form of computing aid is again recommended.

THE ELEMENTARY CALCULUS SEQUENCE (5 units)

By considering the relationship between rate and cumulative graphs, the link between integration and differentiation is established at an early stage. The work in the units will enable the student to differentiate and integrate polynomials, to calculate the area under a curve, to find stationary values of a function and to apply these techniques to simple problems in the fields of economics and biology.

There is a programmed text for each unit. The final unit has a tape/book section which provides applications of calculus methods in economics and biology.

The five units are :

26 'Introducing Calculus'

27 'An Introduction to Notation' (Integration)

28 'Polynomials, Negative Areas and Constants of Integration'

29 'Further Notation' (Differentiation)

30 'Mathematical Models'.

The pre-requisites for this sequence are usually provided in 'traditional' or 'modern' CSE or 'O' level courses. Interpreting graphs of elementary functions is particularly important. Facility in handling directed numbers is crucial.

MATHEMATICS IN GEOGRAPHY (3 units)

The 'new' geography is employing some mathematical techniques in a very enterprising manner. The units under this heading exemplify this usage of mathematics in geography and cover topics required in some of the new 'A' level syllabuses. The units are not sequential.

31 Nearest Neighbour Analysis

This particular technique is concerned with obtaining a fairly precise estimate of the degree to which a pattern of settlements is clustered (e.g. towns in the industrial north) or regularly distributed (e.g. the market towns of East Anglia) or merely distributed in a random fashion.

By reducing the 'settlement pattern' to a single 'nearest neighbour statistic' the geographer is able to make an objective and precise assessment of the pattern of towns in a particular area. The unit makes much use of map work which ensures that the mathematics does not dominate the geography. All the maps ar supplied except for a suggested 'field work' question where the student is encouraged to analyse the settlement pattern of his own particular area.

32 Rank Size Rule

This empirical rule about the pattern of urban populations is followed to a greate or lesser extent by many countries of the world. Although logarithmic graph pap is used to illustrate the rule, every attempt is made to help the student with thi: In fact, this text could well serve as a gentle introduction to logarithms graph paper even if the Rank Size Rule itself is not an essential part of the syllabus. (See Unit 16)

The unit also explains how some insight may be gained into the pattern of urban population for a country which does not follow the rule. Very often, the reason why a model does not fit a situation are at least as interesting as why it does.

No previous knowledge is required.

33 Network Analysis

Relates the concept of 'connectivity' to topological equivalence. Defines connectivity in numerical terms, and calculates the α and β indices. Discusses accessibility and the mean associated number. Applies theory to maps of Anglesey and of a border county West of Berwick upon Tweed.

STATISTICS

An increasingly wide range of subjects require students to use statistical techniques. Yet with changes taking place in the statistical content of school courses at 'O' and C.S.E. level, students enter sixth forms or higher education with varying degrees of exposure to statistics, some being unfamiliar with a histogram, while others may be used to applying the χ^2 test. In this rather complex situation, it would appear that independent learning materials could be especially useful. The Project has consequently concentrated its effort on this particular branch of mathematics and has developed a total of 18 units in a variety of sequences.

THE DESCRIPTIVE STATISTICS SEQUENCE (4 units)

34 Descriptive Statistics 1 : Presenting Statistics
In three parts: Parts 1 and 2 consist of a tape/filmstrip, followed by an assignment. Part 3 is a programmed text.

Introduces the histogram, emphasising that frequency is represented by area. Contrasts with the bar chart, where frequency is shown by length. Discusses the distinction between 'discrete' and 'continuous' data. Gives methods for tabulating and illustrating data. The illustrations in the filmstrip encourage the student to be sensitive to the shapes of distributions.

35 Descriptive Statistics 2 : Modes and Medians
Also in three parts: each part is in the form of a tape/filmstrip, followed by an assignment.

Introduces two kinds of 'average' - the mode and the median. Discusses methods of determining these values for data in both tabular and graphical form. Points out the need for a measure of spread. Introduces the interquartile range. Finally discusses the determination and use of percentiles.

36 Descriptive Statistics 3 : Mean and Standard Deviation
Text only. Introduces the mean and standard deviation as alternative measures of 'position' and spread. Discusses methods for computing them, including cases where the data has been tabulated with class-intervals. (Coding methods are discussed in an appendix.) Ends with an informal look at how the value of the mean and standard deviation can provide an understanding of the nature of the distribution.

37 Descriptive Statistics 4 : Some Experiments
The fourth and last unit of the Sequence is designed to provide students with an opportunity to apply the techniques they will have learnt in the earlier units of the Sequence to data they are invited to gather for themselves. Some simple equipment is needed, namely dice, a pack of playing cards, some darts and a large sheet of graph paper.

THE PROBABILITY SEQUENCE (3 units)

This introduction to the ideas and basic rules of probability uses the language and notation of sets. It is anticipated that an increasing proportion of students will have the necessary background knowledge, but for students requiring an introduction to sets, the CMP unit 'An Introduction to Sets' should prove helpful. Apart from this, the pre-requisites are few. Each unit involves a programmed text.

38 Probability 1 : Introducing Probability
Introduces a probability scale, and, through a range of examples, shows how the probability of given events may be determined either from experimental long run frequency or from intuitive considerations such as symmetry.

39 Probability 2 : Some Basic Rules of Probability
Amplifies the notion of complementary events, and discusses the result $p(E) + p(E') = 1$. Derives and uses $p(A \cup B) = p(A) + p(B)$, the probability pattern for mutually exclusive events.

40 Probability 3 : More Rules of Probability

Develops the idea of a path diagram as a means of representing a sequence of events. The endpoints after more than one stage in a path diagram are shown t(correspond with what are referred to as AND events. Students are shown how the probabilities of AND events can be found by multiplying together the probab- ilities of the 'basic' events along a path in the diagram. The meaning of conditional probability is discussed, and the usual notation is used to express the value of p ($A \cap B$) where A, B, are dependent.

THE HYPOTHESIS TESTING SEQUENCE - non-parametric methods (3 units)

The sequence aims to introduce students to the basic ideas of hypothesis testing by a route which makes more modest demands on their time and mathematical ability than does the more traditional road of the Binomial, Normal and other standard distributions. This approach might be particularly appropriate for students of say, Biology and Geography.

41 Hypothesis Testing 1 : The Wilcoxon Rank Sum Test

Introduces the basic concepts of 'null hypothesis', 'significance' and 'interpretatio of a test result' in the context of the Wilcoxon Rank Sum Test, the non-parametri counterpart to the well-known parametric 't' test. Uses the test to assess the significance of the differences between two independent samples.

The main pre-requisite is a grasp of elementary ideas in probability, such as those contained in the CMP unit 'Introducing Probability'.

42 Hypothesis Testing 2 : The Binomial Test and the Sign Test

Deals with the Binomial test as a test of significance rather than as a means of evaluating binomial probabilities. Places emphasis on answering a question of th sort "is 8 heads out of 10 throws of a coin a significantly high proportion ?" rather than a question of the form "what is the probability of obtaining 8 heads from 10 throws of a coin ?"

Uses the test in the context of examples based on opinion polls, simple genetic situations and birth rates. In the form of the Sign test, some sociological examples are considered. An appendix deals with binomial situations other than those in which $p = \frac{1}{2}$, using examples taken mainly from genetics.

43 Hypothesis Testing 3 : The Wilcoxon Matched Pairs Test and the Runs Test

The final unit of the sequence introduces two further tests, namely, the Runs tes and the Wilcoxon Matched Pairs test. The first of these tests uses the same so of data as the Binomial test, but concerns itself with the order in which the two possible outcomes occur. The Wilcoxon Matched pairs test is parallel to the Ra Sum test encountered in Unit 1, but the new test enables the student to study two samples in which, for each measurement in sample 1, there is a corresponding measurement in sample 2. Enabling students to distinguish between matched and independent samples is an important objective of the unit.

An additional purpose of the unit is to ensure that the student consolidates what h has learned in the sequence as a whole; consequently there are more exercises than usual in this unit.

THE PARAMETRIC STATISTICS SEQUENCE (6 units)

Familiarity is assumed with the content of the CMP units Descriptive Statistics 1 to 3 and with Probability 1 to 3.

Although the full sequence comprises six units, it may be that courses consisting of fewer units will meet the needs of particular students. The 'map' of CMP statistics units may be found useful in planning a course. A synopsis of each unit is provided below. Each unit consists of a programmed test; the second unit also involves a game.

44 1 : Populations and Samples
Establishes a framework on which to build, in subsequent units, the practical details of significance testing. Distinguishes between the concepts of a 'population' and of a 'sample', and also between 'parameter' and 'statistic'. Shows how random numbers are used to select a sample. Discusses the importance of sample size, and introduces informally the two basic results for the distribution of the means of samples.

45 2 : The Binomial Probability Distribution
Uses the game 'Crown and Anchor' as an introduction to binomial probability and the binomial distribution. Discusses the nature of the game, and gives criteria for identifying binomial situations. Shows how to determine, by the use of path diagrams in simple cases, the number of successes, and hence the probability of that number of successes. Uses binomial probability tables to determine the probabilities of 'r or more successes' etc. Ends with the exploration of a simple quality control problem.

46 3 : Hypotheses and Ideas of Significance
Introduces the key ideas of significance testing in the context of examples based on the binomial probability distribution. Explains the null hypothesis and the alternative hypothesis. Explores the ideas of significance levels for one-tailed and two-tailed tests. Supplementary examples (and detailed solutions) are provided at the end of the unit.

47 4 : From the Binomial to the Normal Distribution
Introduces the normal distribution as an approximation to the binomial distribution $(q + p)^n$, for large values of n. Shows how it can help when its binomial counterpart fails either because of the limitations of binomial tables or because the form of the binomial result gives rise to difficult problems of manipulation.

On completing the unit, students will be expected to be able to use a table of areas under the standard normal curve to estimate the probability of 'r or more' ('or fewer') binomial successes and also to use the standard normal distribution in a hypothesis testing situation which is essentially binomial.

48 5 : The Normal Distribution and the Means of Samples
Takes the ideas developed in Unit 44 a step further. Reviews and extends the $\frac{\sigma}{\sqrt{n}}$ rule to the normal distribution, so that significance testing can be extended to a new range of problems.

The second half of the unit is concerned with the distribution of the difference between the means of pairs of samples taken from two independent populations.

49 6 : Significance Testing and the Problem of Small Samples.
Concentrates on the problem of small samples and the use of Students' t-distributions. Describes the way the standard deviation of a population can be estimated from a sample, emphasising that the estimate may not be a good one. The doubt about this estimate leads naturally to a description of t-distributions, and eventually to their use in significance testing situations.

The last part of the unit deals with confidence limits for a population mean estimated from a sample.

INDEPENDENT STATISTICS UNITS

50 The χ^2 test

The χ^2 test is one of the most frequently used statistical techniques in biology, geography and the social sciences. The unit concentrates upon the practicalities of using the test - stating a suitable hypothesis, calculating the χ^2 value, using significance levels and drawing a correct conclusion from the χ^2 value in terms of the experiment or the testing situation.

The emphasis is upon using the χ^2 test as a tool. Little attempt is made to explain the formula for χ^2 or even the notion of degrees of freedom. The unit, which is in the form of a programmed text, ends with a comprehensive exercise, which contains some questions of a general nature and interest. No previous statistical knowledge beyond ideas of elementary probability are required.

51 Correlation
The Spearman rank correlation coefficient is probably the most well-known of the non-parametric techniques. This unit takes the student step-by-step through the calculation of the coefficient and provides sufficient practice for mastery of the correlation formula.

In addition, great emphasis is placed on an understanding of the various kinds of correlation (positive, negative, zero) as well as the importance of distinguishing cause from effect between two variables. The text contains many exercises and examples covering geography, economics, as well as general interest. No previous statistical knowledge is required.

Mathematics for Biology (2 units)

52 Growth in Biological Populations
Text only. Formulates the exponential law of growth from cell division, and expresses the equation of growth in both exponential and logarithmic forms. From these equations rates of growth are calculated from data in various forms. The logarithmic graph is used to test if growth obeys exponential law.

The pre-requisites required for this unit are a knowledge of the rules of logarithms and their use for calculation; familiarity with the straight line graph, ($y = mx + c$), and the meaning of the parameters m and c; and (not essential) a knowledge of flowcharts. The latter is used as an alternative explanation in certain parts of the text.

53 Population Dynamics
Text only. Builds on unit 52 and aims to give facility in translating biological problems into mathematical models and visa versa. Uses the concept of rate of change to form differential equations, and discusses the solution of the equation for exponential growth. Provides a basis for further reading into the subject.

APPENDIX 10

Aim 6

To bring about a level of achievement in literacy numeracy and graphicacy appropriate to ability, and adequate to meet the basic demands of contemporary society.

The students should be able to

6.1 **Numeracy**

6.1.1 Add, subtract, multiply and divide whole numbers, basic fractions and basic decimals.

6.1.2 Given an awareness of their own abilities and future needs, decide when to use calculators, tables, pencil and paper or mental arithmetic, and be prepared to learn any required techniques.
What does their intended job require (a) of necessity, (b) for convenience?
What are their personal needs?

6.1.3 Interpret place value.

6.1.4 Convert fractions to decimals and vice-versa.

6.1.5 Use standard units of measurement, read graduated scales, and make approximate conversions between imperial and metric units.
eg. through the interpretation of plans and preparation of materials in the course of a project.

6.1.6 Read the 24-hour clock, and train/bus timetables. Make estimations of time.
eg. planning journeys (ref. 8.2).

6.1.7 Interpret and use tables of figures.
eg. bank statement, temperature conversion, football results.

APPENDIX 10 (Contd.)

6.1.8 Make approximations and estimations, and assess the accuracy of results obtained by a calculator.

6.1.9 Apply and interpret ratio and proportion. *eg. preparing mixtures, scale on maps, wage increases.*

6.1.10 Calculate averages. *eg. average wage rates.*

6.1.11 Calculate percentages. *eg. VAT, discounts, tax rates.*

6.1.12 Make elementary algebraic substitutions. *eg. using a formula to determine one unknown.*

6.1.13 Describe the properties of common shapes, and measure angles.

6.2 **Graphicacy**

6.2.1 Make calculations and estimations of

perimeter, area and volume of right-angled figures, circles and cylinders. *eg. estimating quantities of materials.*

6.2.2 Interpret and present graphs, charts and maps, choosing an appropriate form for their purpose. *eg. understanding of gradient of axes; pi-charts, bar-charts, etc. in the context of newspapers, textbooks, student projects (ref. 10.1).*

6.2.3 Appreciate perspective in space, photography, etc.