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DIFFERENCES IN FURTHER AND HIGHER EDUCATION ARISING FROM TRADITIONAL/MODERN MATHEMATICAL BACKGROUNDS

by

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ABSTRACT

Most mathematics classes in further and higher education nowadays contain students from widely differing mathematical backgrounds. Some students have followed a modern mathematics course at school whilst others have studied traditional mathematics prior to their entry to college or university. These differences in mathematical backgrounds can account for differing mathematical performances and attitudes. This dissertation is an attemptto see if different mathematical backgrounds affect students' performances and attitudes.

Some evaluation studies have been attempted with regard to the effect of modern and traditional mathematics on the performances of school children. These are reviewed in Chapter 2 which follows the introduction to the dissertation. It is shown that the studies produced contradictory opinions as to the relative merits of traditional and modern mathematics.

The particular problems confronting students and mathematics teachers in further and higher education are discussed in Chapter 3.

It was decided to attempt evaluation studies on the effect of differing mathematical backgrounds on students' performances in further and higher education. The examination results of samples of students from Derby College of Further Education and Loughborough University of Technology were analysed. The results obtained showed significant differences in performance between those students from a modern mathematics background and those from a traditional mathematics background. The evaluation studies are discussed in detail in Chapters 4 and 5.

A questionnaire was designed to show students' attitudes to mathematics. This was presented to various classes of students from both modern and traditional mathematics backgrounds at Derby College of Further Education. It was found that there was no noticeable difference in attitude between the two groups of students. Chapter 6 discusses in detail the questionnaire and the responses received.

The author's personal views on the problems highlighted are contained in the dissertation's final chapter.

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CHAPTER 1

INTRODUCTION

Great changes are taking place in the schools, colleges and universities of Britain today. New courses are constantly being introduced and existing courses regularly modified. Mathematics courses are certainly no exception - indeed at the present time courses in mathematics are the ones which are subject to some of the greatest reappraisals.

These changes have arisen in part from the tendency of mathematics to grow and reorganise itself and from educators to question seriously why we teach mathematics. They have led to the introduction of many new mathematics schemes and projects, some of which are now firmly established in many schools. These courses, even though many are now over ten years old are still referred to as 'modern' maths courses. They are to some extent new in content in their use of mathematical language and symbolism. The implied revolution in teaching methods and educational objectives however is their most important attribute. Thus modern mathematics is very different in outlook from the type of mathematics taught exclusively in schools before the late 1950's - 'traditional' mathematics.

Any change in mathematics teaching no matter howwell intentioned is certain to cause argument both for and against change. Many teachers, lecturers, industrialists and parents have presented heated arguments on the relative merits of 'traditional' and 'modern' mathematics. Attempts have been made to present the point of view of both sides (1) and the 'great debate' is still continuing.

Many schools have been operating modern mathematics courses for many years and mathematics teachers are not nowadays finding too many problems in providing their pupils with the mathematical education thought most desirable by the school. Further and higher educational establishments however are not so fortunate. It is only very recently that lecturers in further and higher education have encountered students who have followed a modern maths course; these students often following the same course and in the same classes as students from schools adopting a 'traditional' approach to their mathematics teaching. This is causing difficulties and it is the purpose of this dissertation to look in detail at the problems of modern mathematics in further and higher education.

(1) Modern Mathematics - The Great Debate
Institute of Mathematics and its Applications Bulletin
Vol. 9 Number 8 August 1973 P. 238 - 252

CHAPTER 2

A REVIEW OF MODERN MATHEMATICS EVALUATION STUDIES

In attempting to review the problems of modern mathematics in further education it seems appropriate to first research what earlier work has been done by previous investigators on the evaluation of modern as against traditional mathematics courses. Though much time and money have been, and are being, spent on developing new approaches to the teaching of mathematics, very little of this effort has been apportioned to the problem of evaluation. In particular there has been little or no systematic attempt to compare the degree of genuine mathematical understanding gained by students following traditional with that achieved by those following modern syllabuses. What few studies have been undertaken have been almost exclusively concerned with mathematics in schools as distinct from mathematics in further or higher education. These studies are relevant however and they provide a foundation for original work to be later discussed on the specific problems of modern and traditional mathematics in further and higher education.

One of the earliest evaluation studies on the effects of traditional and modern courses was attempted by G.S. Gopal Rao (1). He decided to assess students' general educational growth rather than ascertain if and how far they had mastered specific parts of the subject.

A claim for the modern maths courses is that pupils are provided with greater opportunities to develop their critical thinking ability.

Rao postulated a hypothesis that so far as critical thinking ability was concerned there was no difference between comparable groups of boys and girls studying modern mathematics and traditional mathematics. To test this hypothesis five tests of critical thinking which have been employed by other investigators were employed. These were:-

1. Analysis of data - a verbal test of inference involving both inductive and deductive reasoning.
2. Abstraction - a test claimed to be an illustration of the eduction of relations and correlates.
3. Classification and Inference embodying fairly advanced ideas of the process of classification.
4. Sign changes.
5. Symbol manipulation.

The tests were given to two samples of boys and girls in non-selective secondary schools, one sample following a modern mathematics course, the other a traditional mathematics course.

The results showed that in four of the five tests the experimental (modern maths) group showed themselves to be significantly better than the control (traditional) group. Even in the other test the experimental group fared better than the control group, though the difference in mean scores was not found to be statistically significant.

Rao also looked at another intellectual objective of education, that of creative ability or creative thinking ability. Advocates of modern maths teaching believe that modern maths courses enable pupils to discover, inquire and look for answers which are not definite, specific or pre-determined. Traditional mathematics, many believe, lays too much stress on what Guilford (2) has called 'convergent thinking'. This is considered far less beneficial than the development of 'divergent thinking' in pupils. As it appeared that a child studying modern mathematics is being given greater opportunities to engage in this divergent thinking, Rao decided to try and find out if this freedom had resulted in the greater development of creative thinking ability. It was again postulated that there would be no significant difference between comparable groups of pupils studying modern and traditional maths. This hypothesis was investigated by means of five tests. These again had been used by previous investigators and were tests involving first and last letters, uses of words, making up problems, hidden words and concept formation.

It was found that in three out of five tests the experimental group (modern maths) performed significantly better than the control group (traditional maths). In the other two tests the experimental group still did better than the control group although the results were not statistically significant.

It would appear from this survey therefore that so far as the intellectual outcomes of mathematics teaching, in the fields of critical and creative thinking are concerned, pupils who are taught modern maths are significantly better than those who are taught traditional maths.

In the same work Rao looked at the attitudes shown towards mathematics by the two groups of students. Forty statements of the type "I think a knowledge of mathematics is very desirable because it helps us to understand so many other subjects" and "There are times when I hate the sight and sound of maths" were given to the pupils who had to indicate whether they agreed or disagreed with the statements.

Rao found that there was no significant difference between the two groups. He concluded therefore that the expectations or claims of the advocates of change that the new mathematics induces better attitudes towards the subject on the parts of the pupil are not borne out. Rao stressed however that this may be a rash conclusion to come to since changes in attitudes are not brought about either easily or quickly.

Another early evaluation study was undertaken by the International Project for the Evaluation of Educational Achievement (IEA) in 1965. Some 12,000 English pupils attending over 400 secondary schools of all types were tested in mathematics and additional information was collected about them and their teachers and schools. The results and findings were edited by D.A. Pidgeon (3). Teachers were asked to state, not only whether they had attended any recent course or lecture dealing with 'new mathematics', but also whether they were at present basing any of their teaching on it. According to their replies, the teachers and the pupils they taught were divided into two groups. The pupils were given comprehensive mathematics tests involving modern and traditional items suited to their ages. It was discovered that the pupils of teachers who said they were teaching the 'new mathematics' produced an overall superior performance in the mathematics test compared with the pupils of teachers who said they were not teaching it. This superiority, however, was no more marked in the sub-tests involving set theory or specific 'new mathematics' items than it was on any group of items. Further analyses revealed that the teachers who taught the 'new mathematics' tended to be specialist teachers teaching only mathematics, and the results showed clearly that the pupils of the specialist teacher produced a highly superior performance in the mathematics test. It is questionable therefore if any significance can be attached to this early survey.

The sixties appear to have produced very very few evaluation studies, schools and teachers undoubtedly being concerned with the introduction and teaching of modern mathematics rather than its evaluation. By the 1970's however as the various mathematics projects became approved and adopted further researchers began to question their effect in the schools.

Richards and Bolton (4) attempted to clarify some of the relationships involved in the issue of discovery methods in mathematics and convergent and divergent cognitive performance. They investigated 265 children (ages 10 and 11) in the final year of three junior schools. The schools were comparable for intelligence and social class. The time allocated in each school to mathematics was exactly the same, one hour per day. Thus, the schools differed principally in the manner in which they taught mathematics. In school A a conscious attempt was made to 'keep a balance' between traditional and modern methods, in School B the mathematics teaching was largely traditional and orientated to the whole class whereas School C, as a school in one of the Nuffield pilot areas, was enthusiastically committed to a modern, discovery approach.

Tests were selected to measure a wide range of abilities. Thus, as well as the measures of convergent and divergent thinking, Richards and Bolton aimed to select mathematics tests which ranged from convergent to divergent poles. A test of attitudes to school subjects, including mathematics was also given to assess the relationship of this variable to teaching method and cognitive style.

It is not necessary to present the Varimax analysis made by the authors of the results but their conclusions are interesting. They found that the mathematical attainments of children taught by discovery methods in the sample were significantly lower than those of children taught either by traditional methods or by a combination of discovery and traditional methods. In mechanical and problem arithmetic both control schools were clearly superior to School C, though this was not entirely surprising in view of the greater attention they paid to mastery of routine skills and the fact that teaching in School C deliberately avoided problems involving computation.

The NFER Intermediate Mathematics Test was thought initially to be the test most likely to do justice to School C as it is especially designed in a non-traditional form to stress understanding and avoid routine calculation. School C, however, had the lowest performance of the three schools.

The performance of School C on the Easy Problems Test however was equivalent to that of the other two schools. This test was designed with reference to M. Wertheimer's work (Productive Thinking (1961) London: Tavistock) which involves the subject understanding the structure of the problem to find an 'easy' solution, rather than the application of role learning. It is possible that the inclusion of other, similar measures might do more justice to the effects of the discovery approach.

There was no significant differences between the schools in liking for mathematics.

Richards and Bolton believe that there is a need for more attention to be given to the acquisition of mathematical skills in children taught by newer methods. They finally concluded that a combination of traditional and discovery methods is the most effective. This lends weight to much earlier work done by Torrance (5) who also suggested that we must determine which kinds of information are learned most economically by authority and which by creative means.

R. Skemp and S. Mellin-Olsen have made a preliminary evaluation study performed simultaneously in England and Norway, with support from the Nuffield Foundation in this country and the NAVF, Norway (6). They looked at the qualities of instrumental and relational understanding. The first of these is a limited kind of understanding when pupils (and also teachers) think they understand something if they can get the right answers to a given category of questions, without necessarily knowing why the method works. Relational understanding on the other hand means not only knowing what to do but why - that is, it includes knowledge of the underlying mathematical relationships and properties. It is the only kind of mathematical understanding a mathematician would accept.

Skemp and Mellin-Olsen were also interested in pupils' ability to penetrate beyond the superficial appearance of a problem to the deeper mathematical content. They therefore devised a test consisting of 10 items in all, each in two parts. The first part of each item was intended to sample pupils' choice of relational or instrumental explanations, and the second part their 'superficial' or 'deep' (i.e. more penetrating) perception of mathematical problems.

The test was given, in Norway, to 177 pupils in eight classes, four of whom were following modern and four traditional syllabuses. In England it was given to 316 pupils, in 14 classes in five different schools, some of whom were following modern syllabuses, some traditional and some a middle-of-the-road syllabus in which the emphasis was on improving the method of teaching rather than innovations of content.

The clearest results were obtained from the Norwegian schools, since these were very much alike in all ways except for the syllabuses followed. The results showed that the choice of a 'modern' or a 'traditional' syllabus did not by itself make any significant difference to the quality of mathematical thinking in either of the two aspects sampled.

The English results pointed to the same conclusion though more tentatively. Inter-class and inter-school differences were high and consequently the results were not as conclusive as the Norwegian ones.

Skemp and Mellin-Olsen finally concluded that the innovations in syllabus which are currently being made do not by themselves make any difference to the quality of pupils' mathematical thinking, in either of the two ways which were sampled: and that the old saying "what matters most is not what you teach but how you teach it" is supported by their study.

M. Preston (7) investigated the attitudes shown towards mathematics by pupils following modern and traditional mathematics courses. He used an inventory of 40 items which divided into three categories. The definition of these three factors was stated as:-

Factor A: tending to see mathematics as an algorithmic, mechanical and somewhat stereotyped subject.

Factor B: tending to use mathematics in an open-ended, intuitive and heuristic setting.

Factor C: representing commitment, interest and application to mathematics.

The 40 items were printed in the form of a questionnaire which was given to 2,690 C.S.E. pupils in 35 schools. Of these schools 23 were following a completely traditional maths course, the rest adopting one of the modern maths courses (8 schools - SMP, 2 - SMG, 1 - MME, 1 - Westminster).

The results obtained provide information about the effect of the SMP course. Pupils following this course had significantly different attitudes from the norm. The level of Factor B mean score indicated that these pupils see mathematics in a wider context of application, that they have a more strongly developed sense of intuition and their approach to problems allows greater flexibility. However the level of commitment and interest was found to be significantly lower. The numbers of schools and children taking other nationally developed syllabuses was not large enough to provide satisfactory means for comparison.

This difference in attitude between students undertaking the SMP course and those students following traditional courses is contrary to the results found in the previously mentioned work of Richards and Bolton who found no difference in attitude. However Preston was investigating a sample of secondary school pupils - Richards and Bolton junior school pupils.

Preston also found that girls see mathematics in a rather restricted and predictable environment. Their level of interest and commitment was significantly lower than for males. He believes that boys show a greater appreciation of the variety of approaches and situations to be found in mathematics and that they are likely to be more intuitive.

In a further attitudes study, G.S. Gopal Rao (8) used a sample of 137 junior school children between 10 and 11 years old and 300 secondary school children between 11 and 15 years old. The children were given a paired comparison questionnaire, a Thurstone type questionnaire and open-ended questions. Some of the children were also given a structured interview. A Likert scale questionnaire covered their attitudes to school, and a Guttman questionnaire was given to a number of their parents.[1]

1 A Thurstone scale is made up of about twenty independent statements of opinion about a particular issue. Each statement has a numerical scale value determined by its average judged position on the continuum. A person's attitude on the issue is measured by asking him to check those statements with which he agrees.

One of the practical drawbacks of the Thurstone scale is that its construction is extremely laborious and time consuming. The Likert scale copes with this problem. A person's attitude is measured by asking him to indicate the extent of his agreement or disagreement with each item. This is done by having the person rate each item on a five-point scale of response (strongly agree, agree, undecided, disagree, strongly disagree).

A third scaling technique is based on the assunption that a single, unidimensional trait can be measured by a set of statements which are ordered along a continuum of "difficulty of acceptance". That is, the statements range from those which are easy for most people to accept to those which few persons would endorse. Guttman presents sample subjects with an initial set of items and records the extent to which they respond to the items with specified answer patterns. The subject may either accept none of the items in the set, accept item A only, accept items A and B only and so on.

Rao found that junior school children had developed their attitudes to mathematics by the time they had reached 10 or 11 years old. He found no sex difference in attitudes to mathematics but boys showed a marked decline in attitude after age 13. Attitude was found not to correlate significantly with I.Q. but did reflect the children's general attitude to school. Parental attitude and peer group attitudes did affect a child's attitude to mathematics.

A final attitude study in school children worthy of mention is that made by J. Selkirk (9). He found that attitudes of students (aged 16 to 18) in one county towards mathematics were generally worse than corresponding attitudes in any other subject. The results were very similar for students studying modern and traditional mathematics courses. Selkirk suggested that this might be due to the inherent difficulty of the subject, of the dissimilarity between present and earlier courses and a tendency to select mathematics only as a back-up for more favoured subjects.

One would hope that the evaluation studies mentioned would enable the drawing of some positive conclusions. Unfortunately they do not. In both the cognitive and affective domains no broad areas of agreement exist. Indeed the studies appear contradictory - some researchers concluding modern maths a better school mathematics course than traditional maths, others the opposite and others advocating a 'compromise' or 'middle of the road' approach to mathematics teaching. The reasons for the discrepancies are not simply due to sample sizes, the schools chosen or the tests used but more fundamental issues of what is being evaluated. Few projects have issued formal statements of basic aims and fewer still have progressed to detailed objectives. It is indeed difficult to divorce the content of a mathematics syllabus from the way the syllabus is presented by the teacher. The personality, enthusiasm and attitude of the mathematics teachers in the schools obviously has a great effect on the 'quality' of their mathematics teaching.

There are therefore so many factors affecting the quality of mathematics taught that one must be cautious in any interpretation of evaluation studies. Indeed since the basis of any evaluation is chosen by the evaluator alone, it may not be acceptable to anyone who wishes to make use of the evaluation.

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(7) M. Preston - The Measurement of Affective Behaviour in C.S.E. Mathematics (University of Leicester, Ph.D. Thesis, 1972).

(8) G.S. Gopal Rao - A Study of the Attitudes towards Mathematics of Junior and Secondary School Pupils in a New Town, and of Factors Relating to Them. (University of London, Ph.D. Thesis 1973).

(9) J. Selkirk - 'An Enquiry into Attitude and Performance among Pupils taking Advanced Level Mathematics'. Unpublished report. (University of Newcastle-upon-Tyne School of Education 1972).

CHAPTER 3

MATHEMATICS IN FURTHER AND HIGHER EDUCATION

Further education nowadays has emerged as a distinct sector of the British educational system, subsuming what used to be known as technical education. Strictly speaking, further education is all education which takes place beyond compulsory school-leaving age; in practice the term is used for colleges which cater for this stage of education other than autonomous degree-awarding universities and colleges of education. The term 'higher education' is used for these universities and colleges of education. There is however a certain amount of overlap between the various sectors of our educational system so that there are higher education facilities within the further education sector.

The term 'technical education' implied an obvious vocationalism and is really too narrow to cover the broad spectrum of work we now find in further education. Until comparatively recently the dominant theme of further education could be summed up as vocationalism and this implied training for a job rather than education for vocation. Few students were engaged in full-time further edudation, most of the students had already started their full-time paid employment and were intent on obtaining vocational qualifications.

In 1958, S.F. Cotgrave in 'Technical Education and Social Change' (1) stated:

> "The achievement of a paper qualification is the immediate objective of the majority of students attending vocational courses at the technical colleges. They do so prompted by the hope that the possession of a certificate or diploma will lead to a better job or increased job security".

Over the past two decades the number of full-time teachers and students in further education has increased dramatically, and there has been more serious consideration of the educational value as well as the vocational relevance of courses.

At the same time there has been considerable discussion about the aims of education in the sphere of higher education. The universities currently are anxious not to be regarded as "ivory towers"; in fact, recent university foundations have deliberately promoted the concept that they are anxious to make their work highly relevant to industry and the professions. This is certainly true with mathematics courses, and in looking at the effect of modern mathematics in higher and further education establishments it is necessary to bear in mind the educational aims of these institutions.

The Aims of University Education

One of the best known statements of the aims of university education was made by J.H. Newman (2). He put forward the Platonic concept that all knowledge is one, that education is striving for perfection , for identification with the supreme 'idea' of knowledge. He was aware of the need for specialised study but held that concentration upon one subject should be complemented by a recognition of the claims of other subjects. The emphasis in Newman is upon the liberalising function of university study: the supreme aim is the cultivation of habits of intellectual inquiry, not the acquisition of specific professional techniques.

Whilst it is easy enough to accept in principle much of what Newman wrote nearly a century ago, nowadays it is much more difficult to ensure a practical expression of his themes. Not all the subjects of university study now fit so easily into the liberal tradition and the claims of vocation become more and more insistent.

Sir Sydney Caine (3) sees four basic elements in the pattern of university activity. These are:-

a) Scholarship - the acquisition of existing knowledge for its own sake.

b) Research - additions to knowledge.

c) Mind-building - intellectual curiosity and moral leadership.

d) Training - study as a preparation for particular professions or vocations.

University education has become subject-centred, and departmentalism is a characteristic of British universities. The Robbins Report (4) criticised the exaggerated dominance of the special honours school which emphasises concentration upon one subject. It is fair to add, however, that more recent university foundations are trying to break away from this excessive specialisation at undergraduate level. The intellectual excellence emphasised by Newman has certainly been accepted as of central importance in the twentieth-century development of British universities, but in the recent years of rapid expansion it is doubtful if the broader liberal aims expressed by Newman have received as much attention.

Beard, Healey and Holloway in their 'Objectives in Higher Education' (5) point out that objectives in higher education are necessarily largely determined by the society catered for. They see the present purpose of higher education in this country as to produce individuals, experts in various specialities, to maintain and to advance knowledge in an increasing number of fields. As the number of specialised fields increases so does the importance of communication and this may lead to problems if higher education is not sufficiently broadly based.

These discussed aims reveal therefore that there are well-established fundamental principles inherent in the British approach to university education. The universities have strong intellectual traditions, and, whilst training students for a vocation may be desirable, the claims of utility are not alone sufficient to justify the study of a subject.

This is true with mathematics. The true intuitive understanding of mathematics comes from a knowledge of many mathematical structures, not from physical models which may be too limited in scope. Undergraduates reading mathematics as a single subject are involved with this emphasis on abstraction and the perception of broad mathematical patterns. For these students, the following of a modern maths course (prior to their university education) would present no problems for their undergraduate maths studies. Indeed, one could argue that such students are at an advantage as opposed to fellow students who have followed a less abstract, traditional, maths course at school.

The problems may occur, however, for those undergraduates not studying for a mathematics degree. Students studying science, art, and in particular engineering subjects are more concerned with the applications of mathematics. These students see their mathematics as having a more utilitarian attraction. Criticism has been heard that students who have taken the SMP and other modern curricula have less technique than others who have followed a more traditional course. This criticism is investigated in a later chapter.

The Aims of Further Education

Because of the tremendously wide range of student abilities catered for in further education it is even more difficult to list the aims of further education than those of higher education. One general point arises, however, when the aims of the two types of institutions are compared: i.e. further education is more utilitarian and geared to the vocational needs of the student. D.F. Bratchell (6), in looking at universities and further education colleges stated:

> "The strong intellectual traditions of the universities cannot apply throughout further education, which is more comprehensive in character; this does not imply that students in further education should be taught to do things in parrot fashion. The emphasis in colleges of further education is utilitarian and there are none of the inhibitions about the inclusion of subjects for study which exist in the universities; relevance to industrial and commercial needs is the criterion".

Bristow in his book 'Inside the Colleges of Further Education' (7) suggested four broad objectives of these colleges:

1. To assist to the full the personal development of students, mentally, physically, and morally.
2. To enable students to pass their examinations.
3. To educate students so that they can take their place in a rapidly evolving and increasingly technological society.
4. To produce future citizens who are sensible, confident, courteous and happy.

Further education is not merely an extension of school work; its role is to complement the work of schools and universities. Furthermore, further education now has the task of ensuring that students are not only trained for their immediate future, but are also given a background education which enables them to adapt to the changing environment and retrain for new and possibly very different jobs several times in their working careers. The emphasis placed on examinations severely hampers those teaching in further education from looking to the future sufficiently. Even Bristow places the passing of examinations as second in the order of objectives; lecturers in further education often place it first.

A working party of the Mathematics Advisory Unit of the School of Education at Nottingham University have prepared a discussion report on mathematics in further education (8). This saw the general aims of those teaching mathematics in further education courses as being:

1. To reinforce the relevant mathematical techniques which the student has acquired at school before entering the further education course.
2. To provide additional manipulative skills required in the technological or commercial course of study and possibly in association with the industrial or commercial training being concurrently undertaken.
3. To develop the concepts and principles of more advanced mathematics (beyond the CSE or GCE 'O' level) associated with progressive vocational courses.

The working party listed two further objectives for mathematics in craft courses, namely:

1. To provide drill and practice in the application of concepts and principles to the particular applied technology.

2. To fit the student for the mathematical content of the examination in the applied subject.

The introduction of modern mathematics in schools has led to the expectation that there will be difficulties when students who have studied modern mathematics arrive in further education colleges. It is of interest therefore to compare college examination results for students who have followed a traditional, and others who have followed a modern, mathematics course in school. This is attempted in the next chapter.

Problems Arising in Further Education due to the Introduction of Modern Mathematics Courses in Schools

The working party of the Nottingham University Mathematics Advisory Unit (8) anticipate the following problems with regard to mathematics teaching in further education:

1. Difficulties in communication where students use different mathematical language and symbols.

2. Lack of manipulative skill in algebra, arithmetic and trigonometry from 'modern' students.

3. Mixed classes of 'traditional' and 'modern' students with inadequate common background.

4. Discontent among 'modern' students with 'traditional' methods.

Many lecturers in further education are quite honest about their ignorance of modern mathematics, its objectives and its language, but are a little bemused about how they should remedy it. Since college managements and external examining boards appear unprepared to suggest solutions to the problems involved, individual lecturers feel no great pressure to exert themselves to remedy their ignorance. A problem confronting those who desire to bridge the gap is the absence of suitable texts relating modern mathematics to further education courses. Specialist teachers of mathematics who might be able to propose solutions to such problems are rather thin on the ground in local colleges and any but the largest area colleges.

The further education lecturer is steeped in the philosophy of relating his mathematics teaching to the real life of the student at work. In most colleges, mathematics is taught to students of science, engineering, construction, catering, etc. by specialists in these fields, who tend to view mathematics only as a convenient language for expressing truths of their discipline. They regard four figure tables, Pythagoras, trigonometry, etc. as essential equipment for the craftsman or technician at his place of work. They are not particularly sympathetic to a structural, axiomatic approach to mathematics which they regard as 'a variety of useful techniques'.

Thus, confronted with a modern secondary course such as SMP, they tend to see it as a series of irrelevancies offering the student nothing much in the way of techniques for doing his job.

L.M. Cantor and I.F. Roberts in 'Further Education in England and Wales' (9) point out that the teaching problems in connection with students undertaking non-advanced courses are considerable because it may be impossible to cover the syllabus in the limited time available. Because a course (particularly a part-time day release course) has a syllabus on which examination questions have to be answered, a certain amount of basic information has to be learned. This is often assembled by the teacher in a logical sequence and either written out on the board or dictated. Such teaching methods are seen by many teachers, and their students, as the necessary and inevitable consequence of 'having to get through the stuff in time'. This certainly applies to mathematics courses in further education.

A further fundamental difficulty, and source of confusion for the further education lecturer on this issue, is the change in character of the student population entering further education, which has coincided with the introduction of modern mathematics at secondary school. The School Mathematics Project pamphlet 'Manipulative Skills in School Mathematics' (10) points out that the greatly increased proportion of eighteen-year olds going into tertiary education has led not only to a decrease in the overall ability of the average undergraduate mathematical class but also to a chain-reaction throughout the rest of tertiary and further education, industrial training, craft apprenticeship and so on. With the emergence of the polytechnics and the expansion of university education, this means most of the more academic students now enter directly into these institutions and never come near the local colleges. Despite this, the further education net widens and more and more students who have not attempted CSE come into the colleges. Perhaps these students have difficulty not with modern maths particularly but with maths per se. Certainly, the further education lecturer finds himself teaching technique, as opposed to understanding, not from any desire so to do, but as a last resort if the student is to survive the rigours of the terminal examination.

With regard to further education syllabuses and examinations, there is no doubt that their style generally implies a traditional approach to traditional topics. Some new topics, e.g. statistics, probability, and linear programming have recently been introduced into some ONC and OND syllabuses and seem likely to be introduced into the proposed new TEC syllabuses. However, as long as the time is so heavily restricted and the examination paramount, it is hard to see how mathematics in many further education courses can be anything but a collection of vocationally orientated computational and manipulative techniques. It can rarely be a medium for assisting in the achievement of the other broad aims of further education discussed earlier.

Even if more time were available, examining bodies and professional institutions would probably still need to be convinced of the relevance of modern mathematics courses to the needs of the craft and technician student.

(1) S.F. Cotgrave - 'Technical Education and Social Change' Allen and Unwin 1958.

(2) J.H. Newman - 'The Scope and Nature of University Education'. Everyman edition, Dent 1919.

(3) Sir Sydney Caine - 'British Universities, Purposes and Prospects' Bodley Head 1969.

(4) Lord Robbins - Report on Higher Education Ministry of Education, H.M.S.O. 1963.

(5) Beard, Healey and Holloway - 'Objectives in Higher Education' Society for Research into Higher Education Ltd. 2nd Edition, January 1974.

(6) D. F. Bratchell - 'The Aims and Organisation of Further Education' Pergamon Press 1968. P. 47.

(7) A. Bristow - 'Inside the Colleges of Further Education' H.M.S.O., 1970.

(8) Modern Mathematics in School and Further Education - A Discussion Report Prepared by a Working Party of the Mathematics Advisory Unit of the School of Education, Nottingham University. June 1973.

(9) L.M. Cantor and I.F. Roberts - 'Further Education in England and Wales' Routledge and Kegan Paul, 2nd Edition, 1972 P. 191.

(10) 'Manipulative Skills in School Mathematics' The School Mathematics Project 1974.

CHAPTER 4

A COMPARISON OF STUDENTS' MATHEMATICAL PERFORMANCES IN FURTHER EDUCATION

It has been mentioned in Chapter 1 that very little research has been attempted on the performances of students from different mathematical backgrounds in further education. Although the school evaluation studies discussed showed no consistent results it was still thought advisable to attempt an evaluation study for students in further education.

Over the past few years, industry has been concerned at the arithmetical shortcomings of its entrants from schools and colleges. The Engineering Industry Training Board is particularly concerned about lack of numeracy among intending craftsmen. The Board has voiced its anxiety, justifiably or otherwise, over the possible effects of the teaching of modern mathematics syllabuses on the students' ability in computation and algebraic manipulation (1).

Ruth M. Rees (2) has studied the difficulties experienced in mathematics by craft and technician students in further education. It was found that the concern over mathematics in further education is valid. The research also showed that it is possible to diagnose difficulties in mathematics and pin-point their nature. No comparison was made in this study, however, between students from traditional and modern mathematics backgrounds.

Other publications worthy of mention connected with the mathematical problems of students in further education are the proceedings of two conferences on 'Mathematical Shortcomings at the School/Employment Interface' (3). These were initiated by the Shell Centre for Mathematical Education and convened by the Institute of Mathematics and its Applications. These conferences enabled industry, colleges and schools to discuss at length the innumeracy of craft apprentices joining the engineering industry and whether modern or traditional mathematics syllabuses affect this lack of numeracy. It is fair to state that no unanimously agreed conclusion was achieved as to the most effective mathematics syllabus for aspiring engineering students. Nevertheless the proceedings indicate that there is a need for discussion and research on and into students' mathematical performances in further education.

It was decided to analyse statistically the mathematics examination results achieved by a sample of students at Derby College of Further Education. Although many secondary schools in Derbyshire have been using a modern mathematics syllabus (notably SMP) for many years, it is only since 1974 that a significant number of students from a modern mathematics background have filtered through to the College of Further Education. The full time 'A' level course was chosen as the course on which to make an evaluation study, mainly because this course contains students from both traditional and modern mathematics backgrounds but also because the author is actively concerned with teaching that course.

In September 1974, all students enrolling for a course in 'A' level mathematics were questioned on their mathematics background. Out of the 52 students who were accepted onto the course, 9 had followed a modern mathematics course at secondary school, all of them obtaining a pass at 'O' level in modern mathematics. The remaining 43 students had followed a completely traditional mathematics course, the majority of them obtaining 'O' level mathematics - the remainder obtaining the Certificate of Secondary Education (CSE) Grade 1. For the purposes of the evaluation study the students were split into three groups: group 1 comprising students who had followed a modern mathematics course; group 2 - students who had obtained a 'good' traditional 'O' level pass (grades A, B or C); and group 3 comprising students who had a 'bad' traditional mathematics result ('O' level grade D or E or CSE grade 1)[1]. The students were not grouped in this way for their mathematics teaching at college - the 'A' level maths classes contained a mixture of students from all three groups.

1 There is no suggestion in this grouping that a CSE grade 1 should be considered inferior to an 'O' level pass. The students who had obtained CSE had not covered such a comprehensive mathematics syllabus as the 'O' level students (e.g. none had studied calculus). It was therefore considered sensible to include these CSE students in group 3.

The 'A' level maths course at Derby College of Further Education is designed to meet the requirements of the syllabus laid down by the Associated Examining Board. This is essentially a traditional 'A' level syllabus. Students can pass 'A' level pure maths by taking the Board's papers I and II; and 'A' level applied maths by taking the Board's papers III and IV. At the end of the student's first year of the two year course an internal examination on the first year's work is set. It was on the results of this examination that the evaluation study was made.

The mean mark obtained by each of the three groups is given below:-

Total number of students = 52

Group 1 (modern) 9 students	Group 2 (good traditional) 31 students	Group 3 (bad traditional) 12 students
52.9 %	65.5 %	54.4 %

An F test was carried out first in order to test the hypothesis that there is no significant difference between the mean marks of each group. The full details of the results obtained and the statistics involved are given in Appendix A. It was found that there was a significant difference between the groups at the 5% level.

Because of this significance a t test was then applied to test for significance between each of the groups, (again the details are given in Appendix A). The test showed that even at the 2.5% level there was a significant difference between groups 1 and 2 and between groups 2 and 3. There was no significant difference between groups 1 and 3.

The conclusion drawn from this exercise is that students with a 'good' traditional 'O' level performed significantly better at this stage of this course than students with a 'bad' traditional 'O' level or with a modern 'O' level. There is no significant difference between the performance of the latter two groups of students.

Caution has to be taken in drawing these conclusions. Marks were only recorded for one set of examinations. It would be interesting and useful to compare the 'A' level results of the same students after another year's study and see if there was still a significant difference.

Further, no attempt was made to compare the degree of genuine mathematical understanding gained by the students. In the final analysis, however, the majority of 'A' level students in further education would firmly support the view that it is their 'A' level mark which is all important to them. The evaluation study as carried out seems therefore to be relevant. What is open to question, however, is the size of the sample. A group of 52 students of which only 9 had followed a modern mathematics course is small. This unfortunately was the largest group of students from differing mathematical backgrounds at the College.

(1) 'Skill' - A publication by the Engineering Industry Training Board. Editor, P.S.D. Hodgkinson No. 15 (1975).

(2) Ruth M. Rees - Mathematics in Further Education. Difficulties Experienced by Craft and Technician Students. Further Education Group, Brunel University. Hutchinson Educational (1973).

(3) Proceedings of the Conferences on 'Mathematical Shortcomings at the School/Employment Interface'. Interface 1 - July 1974 and Interface 2 - July 1975 Institute of Mathematics and its Applications.

CHAPTER 5

A COMPARISON OF STUDENTS' MATHEMATICAL PERFORMANCES IN HIGHER EDUCATION

Because the sample chosen for the evaluation study discussed in Chapter 4 was small, it was thought advisable to collect information from a different, larger sample and make a second evaluation study. Furthermore the results obtained from the first study relate to students in further education. It was thought useful therefore to see whether there is any difference in mathematical performance by students from differing mathematical backgrounds in the field of higher education.

At the beginning of the academic year 1974/75, 1st Year engineering students at Loughborough University of Technology were asked to give details of their mathematical background prior to admission. These details were determined by means of the form reproduced in Appendix B which each student was asked to complete. From the replies the students were divided into three groups. These were:-

Group 1: Students who had obtained a pass at GCE 'A' level on a modern mathematics syllabus.

Group 2: Students who had obtained a pass at GCE 'A' level on a traditional mathematics syllabus.

Group 3: Students who had been accepted onto their course by virtue of obtaining an Ordinary or Higher National Certificate or Diploma (or equivalent).

This division was only made for the purposes of the later statistical analysis. For the tuition purposes of the University the students were not so grouped.

At the end of their first year the results obtained by the students in their 1st Year mathematics examinations were recorded. Students reading Electrical and Electronic Engineering, Chemical Engineering, Civil Engineering, Management Studies, and Engineering Science and Technology took a different mathematics examination from those students reading Mechanical Engineering and Aeronautical and Automotive Engineering. It was necessary therefore to analyse the two groups separately. The results of 264 students were analysed. In fact more students filled in the original form relating to mathematics background, and more sat the examinations. Unfortunately about 10% of the forms returned did not contain the student's name, and there were results which could not be correlated with a particular background. The results from the first mathematics examination are summarised below:-

Studies

<u>Electrical, Chemical and Civil Engineering, Management Services, Engineering Science and Technology</u>

Total number of students = 144

Mean Score (%)		
Group 1 (modern 'A' level) 23 students	Group 2 (Traditional 'A' level) 92 students	Group 3 (ONC, HNC or equivalent) 29 students
51.9	63.6	54.9

The complete results and statistical analysis are shown in Appendix C.

An F test on these results showed there was a significant difference (at the 2.5% level) between the mean marks.

A t test showed a significant difference between groups 1 and 2 at the 2.5% level. A comparison of groups 1 and 3 showed no significant difference at this level. Finally there is a significant difference between groups 2 and 3 at this same level.

It is possible to conclude therefore that the students who had followed a traditional 'A' level course prior to their studies at the University performed significantly better in mathematics than those students who had followed a modern 'A' level course or those who had followed an ONC, HNC or equivalent course.

The second mathematics examination gave the following results:-

<u>Mechanical, Aeronautical and Automotive Engineering</u>

Total number of students = 120

Mean Score (%)		
Group 1 (modern 'A' level) 17 students	Group 2 (traditional 'A' level) 72 students	Group 3 (ONC, HNC or equivalent) 31 students
48.9	56.6	39.5

The complete data is given in Appendix C.

The F test again showed a significant difference between the marks (at the 2.5% level).

In this case, however, the t test showed no significant difference between groups 1 and 2 and between groups 1 and 3. There is a significant difference at the same 2.5% level between groups 2 and 3.

It appears from this study therefore that once again a traditional 'A' level mathematics course is a better preparation for these University examinations than an ONC or HNC course or equivalent. Modern mathematics students did not perform as well as the traditional mathematics students but in this case there is no statistically significant difference.

The results support the conclusions derived in Chapter 4 relating to students in further education. They also appear to lend weight to an evaluation study similar to this one attempted by J. Hunter (1) who is involved as an author in the Scottish Mathematics Group modern syllabus. Assessing the work of two groups of university students, one trained on a traditional syllabus and the other on the Scottish modern syllabus, he found little difference between the two groups after one term's work, the traditional background students performing slightly better. After three terms this gap had widened considerably, the new syllabus appearing to have done little to improve mathematical understanding. Hunter pointed out that the modern background students' difficulties were due mainly to poorer manipulative technique and lack of knowledge in trigonometry.

Revision of the new syllabus will hopefully remedy this.

It would be of interest to analyse the Loughborough University students' examination marks at the end of their second year in 1976 to see if any significant differences still exist. One may speculate that the modern background students need longer than one year to perform as effectively (or better) than their traditional background companions, on this type of course.

(1) J. Hunter - Some Aspects of Syllabus Development, Evaluation and Revision, illustrated by the work of the Scottish Mathematics Group. (Department of Mathematics, University of Glasgow) (1971).

CHAPTER 6

THE ATTITUDES SHOWN BY STUDENTS IN FURTHER EDUCATION TO MATHEMATICS

Several of the evaluation studies discussed in Chapter 1 looked at school-children's attitudes towards mathematics. Although the results obtained by the different researchers pointed to no common conclusion, it was still thought useful to attempt a survey comparing attitudes to mathematics shown by students from modern and traditional mathematics backgrounds. Furthermore none of the discussed studies had looked at the attitudes of students in further education.

Now attitudes are enduring predispositions that are learned rather than innate. Thus, even though attitudes are not momentarily transient, they are susceptible to change. An attitude survey to mathematics in further education would therefore be an original piece of work which might provide useful information for those concerned with the teaching of mathematics in these establishments.

A draft questionnaire was devised to try and indicate students' attitudes to mathematics. This was given to one class of students in further education studying 'A' level mathematics. After completion, discussion took place with the students regarding any difficulties they experienced in answering the questions.

This showed up certain ambiguities in the questions and the vocabulary used. As a result the questionnaire was slightly modified. The final modified form of the questionnaire is shown in Appendix D.

During November 1975, copies of this questionnaire were given to all the students studying mathematics on the full-time 'A' level course at Derby College of Further Education. To give a bigger sample the questionnaire was also given to students from two evening classes studying 'A' level mathematics at the College.

174 students answered the questionnaire. Of these 41 students had followed a modern mathematics course at school, the other 133 following a traditional mathematics course prior to their studies at College.

The main part of the questionnaire consisted of opinion statements to which the students had to give the extent of their agreement or disagreement. Because of its relative ease, a Likert scale was chosen as the method of assessing the students' attitudes. The responses can vary from 1, indicating strong agreement, to 5, indicating strong disagreement. The Likert scale does not assume equal intervals between scale values. For example it is quite possible that the difference between 'agree' and 'strongly agree' is much larger than the difference between 'agree' and 'undecided'.

The scale then provides information on the ordering of students' attitudes but it is unable to indicate how close or far apart different attitudes might be. No statistical analysis was therefore performed on the results - it being considered fairer to simply present the results obtained.

The opinion statements given and the replies received expressed on a percentage basis are set out below:-

a) There are too many formulae to learn in mathematics.

	Strongly agree	Agree	Undecided	Disagree	Strongly disagree
Response	1	2	3	4	5
Traditional Students %	10	33	22	30	5
Modern Students %	7	39	15	39	0
Overall %	9	34	20	33	4

b) There is too much time spent on lecturing and not enough time allowed for us to find out the results ourselves.

	Strongly agree	Agree	Undecided	Disagree	Strongly disagree
Response	1	2	3	4	5
Traditional Students %	1	11	18	59	11
Modern Students %	3	10	28	52	7
Overall %	2	11	20	57	10

c) I have difficulty understanding mathematical notation.

	Strongly agree	Agree	Undecided	Disagree	Strongly disagree
Response	1	2	3	4	5
Traditional Students %	1	11	11	54	23
Modern Students %	0	14	14	62	10
Overall %	0	12	12	55	21

d) In general the maths problems we have to solve are boring.

	Strongly agree	Agree	Undecided	Disagree	Strongly disagree
Response	1	2	3	4	5
Traditional Students %	2	6	16	57	19
Modern Students%	3	24	17	35	21
Overall %	2	11	16	51	20

e) The way maths is taught here is not as interesting as the way maths was taught at school.

	Strongly agree	Agree	Undecided	Disagree	Strongly disagree
Response	1	2	3	4	5
Traditional Students %	2	1	11	57	29
Modern Students %	7	3	14	52	24
Overall %	3	2	12	56	27

f) Mathematics is not as interesting as science.

	Strongly agree	Agree	Undecided	Disagree	Strongly disagree
Response	1	2	3	4	5
Traditional Students %	5	13	12	43	27
Modern Students %	4	25	11	29	31
Overall %	5	16	12	39	28

g) I wish we were told more about the practical applications of maths.

	Strongly agree	Agree	Undecided	Disagree	Strongly disagree
Response	1	2	3	4	5
Traditional Students %	20	45	18	15	2
Modern Students %	40	27	23	7	3
Overall %	25	40	20	13	2

h) Apart from being on the syllabus, I cannot see the point of some of the maths we are taught.

	Strongly agree	Agree	Undecided	Disagree	Strongly disagree
Response	1	2	3	4	5
Traditional Students %	16	39	17	21	7
Modern Students %	21	48	10	14	7
Overall %	17	42	15	19	7

i) An 'A' level maths course nowadays ought to include some work on computers.

	Strongly agree	Agree	Undecided	Disagree	Strongly disagree
Response	1	2	3	4	5
Traditional Students %	11	44	23	17	5
Modern Students %	21	38	21	17	3
Overall %	14	42	22	17	5

j) We seem to be taught 'technique' rather than 'understanding'.

	Strongly agree	Agree	Undecided	Disagree	Strongly disagree
Response	1	2	3	4	5
Traditional Students %	7	31	25	30	7
Modern Students %	18	21	18	36	7
Overall %	10	29	23	31	7

k) Mathematics is an academic exercise of little practical value.

	Strongly agree	Agree	Undecided	Disagree	Strongly disagree
Response	1	2	3	4	5
Traditional Students %	1	5	10	40	44
Modern Students %	0	3	7	45	45
Overall %	0	5	9	42	44

The results show quite convincingly that there is very little difference in attitude between students from modern and traditional mathematics backgrounds. There appears no noticeable difference in the responses to this questionnaire except possibly to statements d), f) and h).

With regard to statement d), 27% of modern students agreed or strongly agreed that the mathematics problems they had to solve were boring whilst only 8% of the traditional students agreed.

Replies to statement f) showed that 29% of modern students agreed with the statement that mathematics is not as interesting as science. Only 18% of traditional students agreed with this statement.

A greater percentage of modern students do not see the point of some of the mathematics they are taught. 69% of modern students agreed with statement h) as opposed to 55% of traditional students.

When the questionnaire was formulated, however, several of the statements were considered 'key' statements which might show quite quickly if there was any noticeable difference in attitude between the two types of students. These 'key' statements were thought to be statements b), e) and j). If the percentage results for 'agreement' and 'disagreement' are combined, the following summary results:-

Statement		Agree	Disagree
b) There is too much time spent on lecturing and not enough time allowed for us to find out the results ourselves.	Modern %	13	59
	Traditional %	12	70
e) The way maths is taught here is not as interesting as the way maths was taught at school	Modern %	10	76
	Traditional %	3	86
j) We seem to be taught 'technique' rather than 'understanding'	Modern %	39	43
	Traditional %	38	37

There appears to be no noticeable difference in the above responses.

The two remaining items on the questionnaire also supported the proposition that there is very little difference in attitude to mathematics between the modern and traditional students following this particular 'A' level course.

Question 5 asked students to give the most-liked and least-liked topic from Algebra, Calculus, Co-ordinate Geometry and Trigonometry. The responses summarised on a percentage basis are given below:-

Most-Liked Topic

	Algebra	Calculus	Co-ordinate Geometry	Trigonometry
Modern students %	32	21	32	15
Traditional students %	35	38	7	20

Least-Liked Topic

	Algebra	Calculus	Co-ordinate Geometry	Trigonometry
Modern students %	32	4	28	36
Traditional students %	20	21	25	34

The students appeared to have widely differing opinions with regard to their like and dislike of certain topics. No definite conclusions therefore can really be drawn from these figures.

The final item on the questionnaire asked students if they thought 'A' level mathematics examinations ought to have a time limit. 61% of traditional background students said 'Yes'; 39% said 'No'. The modern students were divided in almost exactly the same proportion 62% saying 'Yes', 38% saying 'No'. There is certainly no indication of a difference in attitude between the two groups with regard to this question. The question was included however mainly for interest rather than using it to determine attitudes. It is indeed interesting that such a large percentage of further education students are in favour of examinations with a definite time limit.

CHAPTER 7

CONCLUSION - A PERSONAL VIEW

The introduction of modern mathematics in schools has caused many teachers to seriously question the type of mathematics they are currently teaching. It has been mentioned that although much time and money have been spent on the implementation of modern mathematics in schools, very little work has been done on assessing the effects of such schemes on pupils' mathematical performances and attitudes. Whilst research of this nature is difficult and time-consuming, it is surely worthwhile.

The researches discussed in the first chapter showed that the introduction of modern mathematics in schools has not proved to be the unqualified success many of the various projects' instigators had hoped. No obvious conclusions were reached as to which school mathematics course is the most suitable for the majority of pupils. The reasons for this are complicated and there appears room for further, more extensive, research on the effects of various types of mathematics teaching.

The fact that British schools have a tremendous amount of freedom in the choice of their mathematics syllabuses has given rise to a variety of different schemes and projects. Some of these must of necessity be more applicable to certain types of pupil than others.

Thus different and contradictory mathematical performances from pupils might be expected. If we had the more uniform educational system adopted in some countries, for example Scotland or France, the evaluation studies might point to more definite conclusions and recommendations. As long as the admirable freedom of educational choice in this country continues, however, all that has been definitely concluded is that the 'quality' of mathematical teaching is all important. In the hands of an enthusiastic and energetic teacher, mathematics, whether it be modern or traditional can come to life and thrive even in the most unpromising soil.

This dissertation was primarily concerned, however, with the effect of differing mathematical backgrounds on students in further and higher education. In this field surprisingly little research has been attempted. The discussed aims of mathematics teaching in further and higher education (Chapter 2) show that problems can occur when students enter these establishments from different mathematical backgrounds.

In higher education it has been shown that students from a modern 'A' level mathematics background studying engineering at Loughborough University did not perform as well in their first year examinations as students from a traditional 'A' level mathematics background. This suggests that one might question the mathematics grounding given at school to aspiring engineering students. I have stated that it is inadvisable to make specific conclusions from the results of only one evaluation study, but nevertheless the results were significant and the implications ought to be considered. It is true that many university students have the natural ability and talent to achieve success no matter what their mathematical background. Surely, however, we ought to be giving students the mathematical background most suited to their ambitions and aspirations.

In further education the differences are more disturbing. It has been shown that students on the 'A' level course at Derby College of Further Education from a modern 'O' level background did not achieve such high examination results as students from a good 'O' level traditional background. This may not be surprising since the mathematics teaching at the College is essentially traditional in nature. Nevertheless for the optimum development of all the students' understanding of mathematics, some modification of teaching methods in the light of the students' previous backgrounds might be considered desirable.

There was no noticeable difference in attitude between students from modern and traditional mathematics backgrounds.

There are, I am afraid, too many prejudged views from teachers, lecturers, parents and industrialists with regard to modern and traditional mathematics. There is a lack of flexibility by some 'traditional' lecturers in further and higher education to adapt to change, and a contemptuous attitude by some 'modern' advocates in not admitting that modern mathematics courses may not always be ideal for students moving onto studies in further and higher education.

A compromise course between traditional and modern mathematics is generally considered impracticable and unnecessary - the modern mathematics courses being allegedly a blend of the best of the old and the new. I feel, however, that some compromises in mathematics syllabuses ought to be affected. The less able students especially might respond to traditional methods (even down to such allegedly abhorrent practices as 'rote' learning) if they can be convinced of their use in passing the terminal examinations that colleges or industry might ask them to sit.

I am very much disturbed about the inadequacy of communication between mathematics teachers in schools, lecturers in further and higher education and the employers of those who use mathematics in their jobs. Meetings are clearly a step in the right direction but, ultimately, real progress can only be made through a sincere desire for mutual understanding and co-operation at a local level, with the interests of the students primarily in mind.

The most sensible policy is surely to embrace the new thinking in mathematics wholeheartedly; exploit its advantages to the full but react quickly and effectively to compensate for its disadvantages as and when they become apparent. This dissertation has, I hope, pointed out some of the disadvantages and problems with regard to mathematics teaching in the fields of further and higher education.

If this work causes any teacher, lecturer, examiner or employer to think even more carefully about the mathematics teaching taking place in our schools and colleges, it has been worthwhile.

APPENDIX A

1st Year Examination Results of Students Following an 'A' Level Course in Further Education and their Statistical Analysis

% Marks					
Group 1 Modern	Group 2 Good Traditional			Group 3 Bad Traditional	
41	93	71	72	67	67
33	78	83	20	67	
59	54	52	54	61	
53	60	46	69	58	
46	48	82	80	43	
75	78	72	28	28	
60	75	79	84	80	
58	31	79	59	57	
51	75	69	68	42	
	80	84		35	
	51	57		48	
Mean = 52.9	Mean = 65.5			Mean = 54.4	

Suppose there are k mathematics background groups, each group being replicated n_j times. Let x_{ij} denote the mark obtained by the ith student in the jth group.

Put $T._j = \sum_{1}^{nj} x_{ij}$ = total mark in the $_j$th group

$\bar{x}._j = \frac{T._j}{n_j}$ = mean mark in the $_j$th group (sample)

$T.. = \sum_{1}^{k} T._j$ = total mark for sample

$\bar{x}.. = \frac{T..}{N}$ (where $N = \sum_{1}^{k} n_j$)

Let μ_j be the population mean for the $_j$th group

The hypothesis

$H_o: \mu_1 = \mu_2 = \mu_3$ is tested against the hypothesis

$H_1: \mu_1 \neq \mu_j$ for at least one $i \neq j$

$(x_{ij} - \bar{x}..) = (\bar{x}_{ij} - \bar{x}..) + (x_{ij} - \bar{x}._j)$

giving $\sum_{j=1}^{k} \sum_{i=1}^{nj} (x_{ij} - \bar{x}..)^2 = \sum_{j=1}^{k} \sum_{i=1}^{nj} (\bar{x}_{ij} - \bar{x}..)^2$

$$+ \sum_{j=1}^{k} \sum_{i=1}^{nj} (x_{ij} - \bar{x}._j)^2$$

i.e. $SS_T = SS_B + SS_W$

where SS_T is a measure of the overall variation of N observations

SS_B is a measure of the variation of the k sample means

SS_W is a measure of the variation within the individual groups

Assuming the samples are drawn from a population $N(\mu, \sigma)$ then $\frac{SSW}{N-k}$ is an unbiassed estimate of the population variance σ^2.

Assuming the hypothesis to be true $\frac{SS_B}{k-1}$ is an unbiassed estimate

of σ^2. So $\frac{SS_B}{k-1} \Big/ \frac{SS_W}{N-k}$ obey the F- distribution with k - 1,

N - k degrees of freedom.

For the results given:-

j	1	2	3	
$T_{\cdot j}$	476	2031	653	3160 = $T_{\cdot\cdot}$
n_j	9	31	12	52 = N
$\bar{x}_{\cdot j}$	52.9	65.5	54.4	60.8 = $\bar{x}_{\cdot\cdot}$

$$\sum_{j=1}^{k} \sum_{i=1}^{n_j} x_{ij}^2 = 207,154$$

$$T_{\cdot\cdot}^2/_N = 192030.8$$

$$\sum_{j=1}^{k} \frac{T_{\cdot j}^2}{n_j} = 193,772$$

giving $$SS_T = \sum_{j=1}^{k} \sum_{i=1}^{n_j} x_{ij}^2 - \frac{T_{\cdot\cdot}^2}{N} = 15,124$$

$$SS_B = \sum_{j=1}^{k} \frac{T_{ij}^2}{n_j} - \frac{T_{\cdot\cdot}^2}{N} = 1,741$$

$$SS_W = SS_T - SS_B = 13,383$$

Analysis of Variance Table

	Sum Squares	Degrees of Freedom	Mean square	Mean square ratio
Between Groups	1741	2	870.5	3.19
Residuals (within)	13383	49	273.1	
	15124	51		

Using the percentage points on the F distribution, the hypothesis can be accepted at the 2.5% and 1% level but can be rejected at the 5% level.

i.e. There is a significant difference between the groups at the 5% level.

Given two groups P and Q the hypothesis that group P does not differ from Q is the null hypothesis that both are $N(\mu, \sigma^2)$. The statistic

$$t = \frac{|\bar{x}_p - \bar{x}_q|}{S\sqrt{\frac{1}{n_p} + \frac{1}{n_q}}}$$

obeys a t distribution with degrees of freedom that of S.

$\bar{x}_p(\bar{x}_q)$ = sample mean of group P(Q)

$n_p(n_q)$ = number of replicates in same P (Q)

$E(S^2) = \sigma^2$

Now S^2 can be estimated from the two treatments P, Q by $$S^2 = \frac{\sum_{i=1}^{n_p} (x_{ip} - \bar{x}..)^2 + \sum_{i=1}^{n_q} (x_{iq} - \bar{x}..)^2}{n_p + n_q - 2}$$

with $(n_p + n_q - 2)$ d.f. but a better estimate of S^2 (in the present problem) is given by

$$S^2 = \frac{SS_W}{N - k} = 273.1 \text{ with } 49 \text{ d.f.}$$

Thus t has to be compared against $t_{49} \simeq 2.01$ at 2.5% on one tail.

a) <u>Groups 1 vs. 2</u>

$\bar{x}_p = 52.9$ $n_p = 9$

$\bar{x}_q = 65.5$ $n_q = 31$

gives $t = 5.32$

which <u>is</u> significant at 2.5%

b) <u>Groups 1 vs. 3</u>

$\bar{x}_p = 52.9$ $n_p = 9$

$\bar{x}_q = 54.4$ $n_q = 12$

gives $t = 0.467$

which <u>is not</u> significant at 2.5%

c) <u>Groups 2 vs. 3</u>

$\bar{x}_p = 65.5$ $n_p = 31$

$\bar{x}_q = 54.4$ $n_q = 12$

gives $t = 4.688$

which <u>is</u> significant at 2.5%

APPENDIX B

Evaluation Study in Higher Education

LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY

Name ....

Department/Course ....

....

Date of Course Entry .... Course Year Entered 1st/2nd

MATHEMATICS QUALIFICATIONS

	Date Exam was taken	Grade	Syllabus: Give exam ref & state whether sullabus was mod or trad
C.S.E.			
'O' Level			
'A' Level			
ONC/D or HNC/D			
Any other			

School or College
prior to entry

APPENDIX C

1st Year Examination Results of Students studying Engineering in Higher Education and their Statistical Analysis

Examination 1: Electrical, Chemical and Civil Engineering, Management Services, Engineering Science and Technology.

Total number of students = 144

% Marks								
Group 1 (modern 'A' level)		Group 2 (traditional 'A' level)					Group 3 (ONC, HNC or equivalent)	
79	22	83	62	61	82	77	80	38
31	59	90	64	69	61	50	70	25
75	58	70	69	89	54	55	97	34
87	33	96	63	48	46	54	64	36
63		52	90	78	61	40	60	54
61		99	76	20	63	73	73	50
62		66	75	66	47	45	56	79
54		69	85	56	63	68	50	41
67		77	30	53	41	88	51	51
78		85	90	61	48	62	44	75
48		72	84	76	81	38	35	
51		73	80	76	63	57	58	
60		73	51	45	49	59	61	
19		69	50	53	50	70	54	
63		20	88	43	64	38	31	
42		78	99	48	58	72	58	
38		47	63	35	52		69	
42		84	73	75	24		43	
32		72	52	74	51		56	
Mean = 51.9		Mean = 63.6					Mean = 54.9	

For the results given:-

j	1	2	3	
$T_{\cdot j}$	1194	5849	1593	8636 = $T_{\cdot\cdot}$
n_j	23	92	29	144 = N
$\bar{x}_{\cdot j}$	51.91	63.58	54.93	59.97 = $\bar{x}_{\cdot\cdot}$

$$\sum_{j=1}^{k} \sum_{i=1}^{n_j} x_{ij}^2 = 563,550$$

$$\frac{T_{\cdot\cdot}^2}{N} = 517,920$$

$$\sum_{j=1}^{k} \frac{T_{\cdot j}^2}{n_j} = 521,345.8$$

Thus $SS_T = \sum_{j=1}^{k} \sum_{i=1}^{n_j} x_{ij}^2 - \frac{T_{\cdot\cdot}^2}{N}$

$= 45,630$

$SS_B = \sum_{j=1}^{k} \frac{T\ ij}{n_j} - \frac{T_{\cdot\cdot}^2}{N}$

$= 3,425.8$

$SS_W = SS_T - SS_B = 42,204.2$

Analysis of Variance Table

	Sum squares	Degrees of freedom	Mean square	Mean square ratio
Between Groups	3425.8	2	1712.9	5.84
Residuals (within)	42,204.2	144	293.08	
	45630.0	146		

Using the percentage points on the F distribution, the hypothesis that there is no significant difference between the groups can be rejected at the 5% level.

i.e. There is a significant difference between the mean marks of the groups at the 5% level.

<u>t test</u> Comparing against $t_{143} \simeq 1.98$ at 2.5% on one tail

a) <u>Groups 1 vs. 2</u>

$\bar{x}_p = 51.91 \qquad n_p = 23$

$\bar{x}_q = 63.58 \qquad n_q = 92$

gives $t = 2.925$

which <u>is</u> significant at 2.5%

b) <u>Groups 1 vs. 3</u>

$\bar{x}_p = 51.91 \qquad n_p = 23$

$\bar{x}_q = 54.93 \qquad n_q = 29$

gives $t = 0.632$

which <u>is not</u> significant at 2.5%

c) <u>Groups 2 vs. 3</u>

$\bar{x}_p = 63.58 \qquad n_p = 92$

$\bar{x}_q = 54.93 \qquad n_q - 29$

gives $t = 2.373$

which <u>is</u> significant at 2.5%

Examination 2 : Mechanical, Aeronautical and Automotive Engineering.

Total number of students = 120

% Marks						
Group 1 (modern 'A' level)	Group 2 (traditional 'A' level)				Group 3 (ONC, HNC or equivalent)	
51	69	47	90	58	34	12
61	63	81	53	43	54	20
45	56	75	67	44	42	17
45	22	63	62		39	35
58	14	67	32		38	7
49	68	60	22		46	51
89	63	80	40		39	10
20	61	46	66		59	51
42	81	37	51		63	
45	56	53	71		42	
47	85	37	66		86	
65	77	37	65		44	
42	81	94	71		41	
42	40	73	12		77	
43	57	31	60		42	
50	67	60	78		51	
38	58	88	44		9	
	52	49	20		63	
	82	80	47		52	
	58	69	29		51	
	25	49	87		19	
	52	68	50		4	
	48	37	31		27	
Mean = 48.9	Mean = 56.6				Mean = 39.5	

For the results given:-

j	1	2	3	
$T_{.j}$	832	4075	1225	6132 = $T_{..}$
n_j	17	72	31	120 = N
$\bar{x}_{ij}$	48.9	56.6	39.5	51.1 = $\bar{x}_{..}$

$$\sum_{j=1}^{k} \sum_{i=1}^{n_j} x_{ij}^2 = 365,414$$

$$T_{..}^2/N = 313,345.2$$

$$\sum_{j=1}^{k} \frac{T_{ij}^2}{n_j} = 319,760.1$$

$$\text{Thus } SS_T = \sum_{j=1}^{k} \sum_{i=1}^{n_j} x_{ij}^2 - \frac{T_{..}^2}{N}$$

$$= 52,069$$

$$SS_B = \sum_{j=1}^{k} \frac{T_{ij}^2}{n_j} - \frac{T_{..}^2}{N}$$

$$= 6,414.9$$

$$SS_W = SS_T - SS_B = 45,654$$

Analysis of Variance Table

	Sum squares	Degrees of freedom	Mean square	Mean square ratio
Between Groups	6414.9	2	3207.45	8.43
Residuals (within)	45,654	120	380.45	
	52068.9	122		

Using the percentage points on the F distribution, the hypothesis that there is no significant difference between the groups can be rejected at the 5% level.

i.e. There is a significant difference between the mean marks of the groups at the 5% level.

<u>t test</u> Comparing against $t_{117} \simeq 1.98$ at 2.5% on one tail.

a) <u>Groups 1 vs. 2</u>

$\bar{x}_p = 48.9 \qquad n_p = 17$

$\bar{x}_q = 56.6 \qquad n_q = 72$

gives $t = 1.47$

which <u>is not</u> significant at 2.5%

b) <u>Groups 1 vs. 3</u>

$\bar{x}_p = 48.9 \qquad n_p = 17$

$\bar{x}_q = 39.5 \qquad n_q = 31$

gives $t = 1.59$

which <u>is not</u> significant at 2.5%

c) <u>Groups 2 vs. 3</u>

$\bar{x}_p = 56.6 \qquad n_p = 72$

$\bar{x}_q = 39.5 \qquad n_q = 31$

gives $t = 4.13$

which <u>is</u> significant at 2.5%

APPENDIX D

Attitudes to Mathematics in Further Education

STUDENT QUESTIONNAIRE

This questionnaire is part of a piece of research on the teaching of mathematics in Colleges. Please answer the questions by expressing your own opinions.

There are no right or wrong answers. You do not have to fill in your name and the replies you give will be completely confidential.

If you do not understand anything please ask.

Tick the appropriate box.

1. Male ☐ Female ☐

2. Mathematics examinations passed before entering this college.

   G.C. 'O' level ☐ C.S.E. ☐

   Overseas Certificate ☐ Any other ☐

3. Did you follow a MODERN or a TRADITIONAL maths course at school?

   Modern ☐ Traditional ☐

   (N.B. Please ask if you are not sure about this question).

4. Below you will find statements relating to the 'A' level maths course you are now taking. Put a number 1, 2, 3, 4, or 5 against each statement in the box provided.

   Put 1 if you strongly agree with the statement
   Put 2 if you agree with the statement
   Put 3 if you are undecided (i.e. you neither agree nor disagree with the statement)
   Put 4 if you disagree with the statement
   Put 5 if you strongly disagree with the statement

a) There are too many formulae to learn in mathematics ☐

b) There is too much time spent on lecturing and not enough time allowed for us to find out the results ourselves. ☐

c) I have difficulty understanding mathematical notation. ☐

d) In general the maths problems we have to solve are boring. ☐

e) The way maths is taught here is not as interesting as the way maths was taught at school. ☐

f) Mathematics is not as interesting as science.

g) I wish we were told more about the practical applications of maths.

h) Apart from being on the syllabus, I cannot see the point of some of the maths we are taught.

i) An A level maths course nowadays ought to include some work on computers.

j) We seem to be taught 'technique' rather than 'understanding'.

k) Mathematics is an academic exercise of little practical value.

5. Leaving out influences due to different lecturers, which part of the 'A' level maths syllabus do you find the most interesting?

Is it Algebra/ Calculus/ Co-ordinate Geometry/ or Trigonometry? ------------

Which do you find the least interesting? ------------

6. Do you think 'A' level maths examinations should have a time limit?

Yes ☐ No ☐

Thank you for your co-operation

M.J. BARRETT

15.11.75