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## A history of the development of mathematics teaching in England from before the sixteenth century to the beginning of the twentieth century

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A HISTORY OF THE DEVELOPIAENT OF MATHEMATICS TEICHING IN ENGIAND FROLI BEFORE THE SIXTEENTH CENTURY TO THE BEGINNING UF THE T.iENTIETH CENTURY.

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A Master's Dissertation submitted in partial fulfilment of the requirements for the award of the Degree of M.Sc. in Ifathematical Education of the Loughborough Unıversity of Hechnology, December, 1975.

Supervisor: D.R. Green. M.Sc., M.Ed., P.G.C.E.
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## Abstrdct

The dim of the alssertation is to trace the nistory of the development of the teaching of mathematics, within the stated period, with reference to the secondary sector of education.

The development of mathematics teaching is looked at in relationship to the whole educational development auring the period, both being influenced by social, cultural, educational, technological and economic torces; these have been related where possible in the study. Beginning with the era prior to the blxteenth century the concept of a liberal eaucation $10 r$ the upper classes in relationship to mathematical education $1 s$ discussed and the influence that the unlversities of Oxiord and Cambridge exerted on this development. The mathematics teaching that took place in the endowea schools is diso considered in relatıonship to the $1 \mathrm{~m}-$ portance attached to the teaching of the classics. Through the l7th, 18th and l9th centuries two important aspects of mathematics, those of arıthmetic, for the supply of clerks, and geometry as a training in mental reasoning, are discussed with the internal and external factors that promoted elements of reform in mathematical education.

The later period of the 19 th century is discussed under three main points; the mathematics that developed through the provision of elementary education, and the subsequent development of secondary education; the reform in geometry teaching and the attempt to remove the influence of Euclid's Elements; the effect of the Industrial Revolution with its effect on post-prımary and technıcal education, resulting in a reform for a more practical mathematics.

The development of mathematıcal text books during the whole period is also considered. Although certain books are introduced and discussed throughout where relevant to the study, a section on mathematıcal text books with photocoples of certain pages is included, lllustrating the actual mathematics that may have reached the classroom, under the influence of the varıous phılosophies. The final section is a summary of the period from the start of the 20th century up to the 1970's. It discusses the educational changes and the period of consolidation in mathematics teaching up to the Second world iar. The post-war perıod is seen as a perıod of great change, partıcularly with the rise of 'modern' mathematics and new projects in the 1960's being followed by both the consolıdation and the cratıcism of the $1970^{\circ}$ s.

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MIr. R. 1 . Kirk, Librarian, University of
Leicester School of Education, for the loan of source material and text books.
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## Introduction

In presenting a history of the development of mathematıcs teaching it is thought necessary to first define the bounds of this particular dissertation. The content will be primarily based on the following three areas of study:
(a) the history of mathematics teaching in England (and Wales where Acts of Parliament are concerned) and the educational, cultural and social forces that may have helped to shape the direction of that teaching.
(b) The age range which is now recognised as being called "secondary", le. 11 years to 18 years of age. However, since the use of the word 'elementary' with reference to school education ${ }^{l}$ could technocally include pupils below this age range there will be an area of overlap of ages, particularly in the education that took place in the nineteenth and previous centuries, hence what would now be termed 'primary' education - 7 to 11 years, might necessarily be included.
(c) In terms of a starting point in time the era prior to the establishment of many of the endowed grammar schools during the Sixteenth and Seventeenth centaurles was chosen as it seems to set the tradition of an educational pattern that was to persist until the Twentieth century. References to other forces though
$l_{\text {Suggested }}$ nomenclature for primary and secondary education given in 'Education of the Adolescent' Hadow Report 1926. Sec. 98.
wall relate to points outside these three bounds; for instance the influence of European and American educational changes and mathematical teachıng; the influence of the Unıversities on secondary education; the appeal to the mathematics of Greek antiquity and Platonic ideals which satisfied the classical foundations of the grammar schools and well known public schools.

The end point of the study will be the start of the 20th century and pass through the periods during whlch varıous changes have taken place. These changes whic $\ddagger$ have taken place in mathematical teaching throughout the period mentioned above have been continuous, but happenıng at dıfferent rates during certain time periods. It was decided to dıvide the hıstory into periods, but recognising the fact that educational and cultural forces are always overlapping and intermingling within these periods. The dates of the periods were chosen as being approprlately close enough to slgnlficant events such as the passing of important education Acts in Parlıament, the beginnings of reform movements in mathematics or conference and committee reports. Sometimes a measure taken in the field of education as a whole coincided with a movement for mathematıcal reform in pedagogical terms; thıs will be amplıfıed in the more detailed descrıption of each perıod.

These perıods are as follows:

1) Pre-Sixteenth century up to 1800
2) 1800 to 1870
3) 1870 to 1902

The next section deals with many of the mathematics books that are mentioned in the preceding text. There are others included and a brief commentary on elther the work itself, or the chosen page which has been photocopled is given. Fihally, a resumé of the developments since ly02 are included to complete the overall stuay.

Prior to the Sixteenth century up to 1800
It is hoped in this section to introduce the general view of education at this tıme and see the part that mathematics teaching played in this, looking at some of the content and the method of teaching.

Hathematics as a subject in schools today occupies an $1 \mathrm{~m}-$ portant place in the curriculum; $1 t 1 s$ often stressed, perhaps not always justıfiably by some employers, parents and teachers, as a. 'necessary' subject using as a criteria its utılitarıan value only and requirement in later vocational trainıng or employment. This present day importance may give the mpression that mathematics, or at least certain branches of it, always played a major part elther in schools, unıversıties, or as private tultion where the social position of the pupil was able to accommodate this. It is acknowledged that the core of the curriculum before and during the sixteenth century tor middle and upper class society at least, was mainly a study of Latın and Greei culture which were an 'essential toundation for a lıberal,gentlemanly education... to be found in the wrıtıngs of Caesar and Ovid,Horace and Vırgıl

Aeschylus and Sophocles' (ref: 1. P.249). According to A.N. Whitehead, 'In the past, classics reagned supreme throughout the whole sphere of higher education. There were no rivals; and accordingly all students were steeped In the classics throughout their school life and its domination at the Universities was only challenged by the narrow disciplane of mathematıcs' (ret: 2). It is necessary here to describe the state of education prior to and during the slxteenth century and the forces that shaped the 'classic' tradition in the curriculum; for this we need to look first at the unlversities of Oxford and Cambridge, founded in the twelfth and thirteenth centuries respectively, and the early 'grammar' schools ${ }^{1}$ often assoclated with the establıshed Church through foundations and statutes drawn up by distinguished ecclesiastics or lay noblemen during the 14 th and 15 th centuries.

The influence of Renaissance thinking and the rise of humanism, the recapturing of the 'Golden Age of Antiquity' meant that as a part of a medieval 'hıgher' education (here begınnıng though from the age of 11 to 13 years of age) the 'trivium' of grammar, rhetoric and dialectic were taught in the grammar schools. At the unlversities the student would then continue with the 'quadrıvium', arithmetic (numbers absolute), music (numbers applied), geometry (magnitudes at rest) and astronomy (magnitudes in motion); this took a period of 7 years. The conception of the Seven Iiberal Arts was developed
$I_{\text {First }}$ use in english,'gramer schole' in 1387 by John of Trevisa in his translation of Ralph Higden's 'rolychronicon' (Source S.J.Curtis, ref. 6).
from Plato and then the late Latin wraters Augustane (5th century) and Boethius (6th century)who also advocated this course of study. In the Unıversıties, medıcıne was added later. The mathematıcs of the quadrıvium until the midale of the 13 th century amounted to very $\perp$ ittle, but the mater$1 a l$ was greatly extended by the introduction into western Europe of 'algorism' (i.e. arithmetic based on the HinduArabic system of numeration, ) probably helped by the work of Leonardo of Pisa through the 'Liber abaci', 1202 (Book of the Abacus, revisea again in l228), and of Arabic translatıons of Euclid, Archımedes,Apollonıus and Ptolemy. Foster Vatson (ref: 3) states that anongst the mediaeval writers of books on arithmetic used in England in teaching the subject were Bede about 730, Alculn about 760, John Peccam 1260, Slmon Bredon 1370 , Richard Wallangłord 1326 , Thomas de Bradwarain(e)1349 and John Kıllingworth l360. The source of this old arıthmetic was the Roman scholar, theologian and philosopher Severınus Boethıus (480-524); according to De Morgan (ref: 4.p.4)"the arıthmetic of Boethius was the classical work of the middle ages. It consists of staternents of the commonest properties of numbers, under a great many classifications to each of ihlch a name is given. The second work, of arithmetical rules, shows the very $10 w$ state of the art. It takes pages upon pages to explain the simple rules, though no examples are ventured on which have more than three figures".De Morgan (ref:4, p.xx)also points out that in Boethius' Arıthmetic the highest number was about lifty wath no indication given as to whether reckoning was performed by abacus, finners or on pen and paper. Some care must be taken when interpreting De liorgan as he was compiling this work when scientıfic blblıography was in its
intancy. At Oxtord Unıversity in the Statutes about 1408 , for proficiency in arithmetic the student was to have studied Boethius for one term and again in the 1431 Statutes he is still the one prescribed arithmetic. De Morgan (ref: 4, p. xix) points to the fact that the Arabic method was common in England in the 13 th Century and that Roger Bacon recommends it as a study (a practical study) for the clergy; he emphasizes the point that with the Arabic system they were reading an al gorism as distinguished from arithmetic (i.e. the number symbols) and that the organised rules of computation always went with the arablc system and not the Roman system; this means the introduction of place value, the use of the zero symbol and the performing of an operation using symbols. The spread of the new system to the schools and universities, particularly in England was probably a slow process; one should remember, though, that as yet printed books were not avaılable ${ }^{l}$, and that 'pen and paper' arıthmetrc was not always preferred to the abacus for commercial calculatıons. Varlous sources indıcate a different rate of infiltration. According to Smeltzer (ref: 5) the Hindu-Arabic numerals were rarely used in England, France and Germany untıl the middle of the fifteenth century, and even in the first half of the slxteenth century most English Merchants continued to use Roman numerals in keeping their accounts. In referring to W.W. Rouse Ball's "Cambrıdge Mathematıcs" (p.5) Foster

[^0]Watson (ref: 3, p.291) states that "the system of arıthmetic called algorisms..........was used from the middle of the thirteenth century for nearly all mathematical tables, for trade purposes partıcularly by Italıans, for calculations of almanacs, which books generally included an explanation of the system". This view partially supports De Morgan; however, it would indrcate that they probably mean its use in the commercial world rather than in teaching. S.J. Curtis (ret: 6 p.90) informs us that the arabic notation, although gainıng popularity on the Continent in the 15 th century, did not come into use in England until the following century, (but see the section on text books) and that the first authentic use of the Arablc notation is sald to be at St. Andrew's Scotland at the close of the 15 th century. (source not indicated). The new notation had met some opposition. Even in Italy in 1299 for Instance the merchants in Florence were forbidden to use it and were ordered to use Roman numerals; this may have been due to conservatism, perhaps Church influenced at the time, or due to misunderstanding and fraudulent alteration of the new numerals. Some impetus for change in the arithmetical notation and system of computation in England may have come from the merchant classes, the landowners and the gentleman farmers who wished for a more practical mathematics capable of dealing with problems of accounts, land measure and surveying. Later during the 16 th century (Henry VIII and Ellzabeth I's reign) the need for mathematics (taken to mean more than just arith-
metac and geometry) would be required in navigation, surveyıng, military supplies, fortifications and ballıstics as England began to look abroad for trade and conquests; the influence of this would have affected onemght suppose the curriculum in the schools.

However, forces which opposed the change to a more balanced education which would have included more mathematics could be due to (1) the desire of the middle class to become accepted by the nobility; this would mean having an educatıon similar to the upper classes; one which was blassed towards the classlcs and gentlemanly pursults, and (1i) the influence of two Humanist Renaissance treati es on education. The first was by Sir Thomas Elyot in 1531 called "The Boke named the Governor". It was dedicated to the King and its main theme was the education of "a gentleman which is to have authority in the public weal". From an early age, 7 years, the pupil, under a tutor, vould be instructed in Latin and Greek and atiention given to the reading of classical writers in both prose and poetry; at fourteen he would study writers on rhetoric; the Greek and Roman hıstorians would gıve hım experiences of milıtary, polltical and moral wlsdom. Elyot also recommends the quadrivium subjects of geometry and astronomy to be studied. At seventeen he should study philosophy (Arıstotle, Cıcero and the 0ld Testament) and on his coming of age a study of law to suit his life for public service to his country. Along with these academic studies he would receive a trainıng in wrestling, rumning, swimming, hunting,
and above all, archery.
A later work was "The Scolemaster" by published after his death in 1568. He was tutor to the young Princess Ellzabeth and emphasizes the teaching of Latin through the Roman writers and also the study of Greek. Thus in this educational envaronment the rising merchant and land owning middle classes :ere placed in a dilemma; they washed for an education for their sons similar to that of the nobilıty, but often this would not be suitable or useful for many of their occupations which would contain practical mathematics after school and unıversıty. Obviously there were arguments put forward by some 'educators' for the nobllıty to receive more practical applications of mathematics in their training, but these were not powerful enough to seriously change the pattern (see later). The dichotomy of 'Pure' mathematics and 'Applied' mathematics also seems to have existed in the teaching of the subject even at this early stage of development.

Teaching went on in other schools besides the grammar schools; there were merchant and gild schools who gave instruction for artısans and skılled craftsman, but which also taught Latın grammar; Charity schools founded by wealthy indıviduals but not necessarily connected with a church also gave a 'grammar' school education. llost of these allowed pupils of different classes irom a localısed area (given in the Statutes) and, like the grammar schools, had places for the 'poor', or relatively poor (the sons of tradesmen, country gentry) but not the destitute.
S.J. Curtis (ret: 6 p.48) in quoting from A.F. Leach 'English schools at the Reformation' (Constable 1896) states: "that occasionally bright boys were snatched out of the ranks of the real poor and turned into clerics, to become lawyers, cıvil servants, bishops is not to be doubted. But it was the maddle classes, whether country or town, the younger sons of the nobilaty, or farmers, the lesser land-holders, the prosperous tradesmen, who created a demand for education and furnished the occupants of Grammar Schools".

Since these schools collectıvely supplied the universıtıes with students then the curriculum of the schools was effectavely determined by the type of education that the unlversitıes were glving. Since this was essentıally a classical one, as we have seen, the sltuation was reinforced. One might compare this with the present day and conclude that the same situation still applıes.

Unfortunately, with the dissolution of the monastries by Henry VIII in the Act of 1545 and the Charities Act by Edward VI in 1547 which removed the Guild Schools and the Charity schools, followed by the Injunctions of Queen Elızabeth in 1559 many of the schools disappeared, but many more were eventually set up as well through further endowments. ${ }^{1}$

[^1]The Act of 1559 virtually ensured that the State, through the established Church (the Church of England) and its religious teaching influenced and controlled the grammar schools now formed.

It has been necessary to emphasize that it was against this background of a classical educational inertia and of political and rellgıous changes that the progress of the teaching of mathematics struggled to develop. In the instance that has been cited it had taken over three hundred years for the Arabic-Hındu notation to be recognised as superior and, even allowing for the lack of printed books and poor communication, lt surely represents an unwallıgness to change the status quo. It was left to individuals to state the case and produce suitable written books 'on 'mathematics', and methods of instruction, or groups of indıviduals and isolated establishments to evolve some advancement in mathematics teaching. Some of these will be looked at with reference to the teaching of arıthmetic and geometry mainly, but obviously a collectıve approach to 'mathematics' as a whole will be attempted.

It was not until Edward VI's Statutes of 1549 that Boethius disappeared at 0xford, and the works of Tunstall (or Tonstall) and Cardan were substıtuted using the new system. Gırolamo Cardano (Cardan), an Italıan, wrote 'Practica

Arıthmetic et Mensurandi Singularis' (Iflan 1539) and Cuthbert Tunstall (1474-1559), Bishop of London (1522) wrote 'de Arte Supputandı むıbrı Quatuor', which was published in 1522.
De Morgarn (ref: 4) has words of high praise for botr these works, the latter being 'the most classical which ever was wratten on the subject in Latin, both in purıty of style and goodness of matter'; but for all this the work was very little known by succeeding English writers. It was suggested by iv. F. Bushell in his presidential address to the Mathematical Associatıon in April 1947: "A century of School Mathematics'(ref: 7) that some of the problems in school arithmetic can be traced back to Tunstall, as it was he who introduced the problem of the cistern with three pipes emptying $1 t$ ın various times $1 n d ı v i d u a l l y$ and it is required to find the time when they are all open together. He does not comment, however, on whether he views this type of problem wath the same entnusiasm as De Morgan did one hundred years earlier.

At Cambridge in 1549 there was an attempt at academic reorganisation; the new entrant (now about 16 years of age) was first to be taught mathematics as giving the best general training followed by 2 years of dialectics and phılosophy for the Bachelor degree; to complete the Master's degree further study uncluded astronomy, perspective, Greek and more
philosophy. Thus, here was some suggestion of including the teaching of mathematics to the students; unfortunately when the Elizabethan code of 1570 further strengthened the influence of the Crown and the establlshed Church the curriculum was recast to exclude mathematics from the Bachelor's course in favour of logic and rhetoric. Logic in this sense meant the syllogistic logic based on the works of Arıstotle later to be superseded by a difterent system based on the logic of Ramus. The liaster's course remained virtually unaltered. The professorial system had been establıshed during Henry VIII's relgn, but in practical terms most teaching was performed by college tutors (i.e.students who had recently ganned a Master's degree)and the professors contracted out of actual teaching.

The lectures consisted of elther dictation of textbooks, or were dialectical or purely formal and traditıonal (commentary on a well known work of antiquity). The student had to dispute publically some thesis he asserted and attack those asserted by others (called 'Keeping his acts'); he was examıned ultımately by detending a thesis through disputations which were, of course, conducted in Latın. the disputatıons were not finally abolıshed untıl 1839, some 300 years later, but after about 1770 they were used only to classify the candidates into elght classes. This background of mathematıcs at the unıversitıes being lımıted as $1 t$ was, would have adversely influenced the teaching of the subject in the grammar schools which drew their senıor, and head
masters from the universities, although much of the favourable influence can be attributed also to graduates who became physicians, 'gentlemen of means', ecclesiastics or politicians. Some of the well known Englush mathematicians of this period in the l6th and l7th centuries, who contributed to the teaching of the subject by producing text books on mathematics were as follows: Robert Recorde (1510-1558) taught at Cambridge and was also a physıcian after 1545 to Edward VI and Queen Mary. Recorde will be looked at in more detail later. Henry Brıggs, who was the first Savılıan Professor of Geometry (1619-1630) at Oxford, made John Napier's discoveries known and first introduced the decimal notation in 1617 (Rouse-Ball's opinıon). Thomas Harriot (1560-1621) helped develop algebra and his book 'Artis Analytıcal Praxıs' was published in 1631, after his death. Villıam Oughtred (1574-1660) who was an Episcopal Minister systematised elementary arithmetic, algebra and trigonometry. He wrote ' Clavis lilathematicae' (1631) which gave an account of arıthmetıc and algebra and 'Trigonometria' (1657) which treated plane and spherical trigonometry. John Nallis (1616-1703) was the most influential English mathematician before Isaac Newton and became Savilian professor in geometry at Oxford in 1649; he had works published from the "Arıthmetıca Infinitorıum' (1655) to his 'Treatise on Algebra' (1685) amongst others and contributed to the orlgins of calculus. According to S.H. Hollingdale (ref:8) it was Wallis who took nis B.A at Emmanuel College in 1637,
and recorded that "I had none to direct me what books to read, or what to seek, or in what method to proceed. For mathematıcs (at the tıme, with us) was scarce looked upon as academical studıes, but rather mechanıcal; as the busıness of trades, merchants, seaman, carpenters, surveyors of lands or the lake" (no source glven). One sees here again the low regard for mathematics and the poor teaching methods, if any, that went with the subject. As will be seen later geometry was a more acceptable subject in the universitıes, but even this was taught in a rather haphazard manner, although Oxford preceded Cambridge in this until after 1660, the Restoration period, when Cambridge became predomınant for its mathematics teaching through Barrow and Newton. Outside the unlversities there was a slmılar attitude throughout the later part of the 16 th century and the early 17 th century; mathematics still could make no definıte headway. Some contemporary views of the times showed a mediocre acceptance and for differıng reasons as it will de seen. Sir Humphrey Gılbert's scheme for Queen Elizabeth's Academy (c.1572) saw the man function of mathematics as a study with a view to practical ends in the service of the State, partıcularly for noblemen, $1 . e . \quad$ usefulness in war, judıcıal or the diplomatic services.

Richard Mulcaster in has work 'Posıtions' (1581) proposed a college for the teaching of mathematics; he rates the teachıng of mathematıcs very highly having studied hımself at Cambridge (in arithmetic and geometry including Euclıd). He points out that mathematıcs was studied in ancient Greece that by its nature $1 t$ is "sensıble even to simple people", It is able to demonstrate truth, $1 l l u s t r a t e ~ u n d e r s t a n d i n g$
and logic and ls useful to the professions and trades; he dıd not, however, even introduce arithmetıc into Merchant

Taylors School (founded 1561) where he was the high master. Lord Herbert of Cherbury (1583-1648) in his Mutobıography' which is a scheme of education for a gentleman, suggests arithmetic as being useful for keeping accounts, but, whilst geometry is not as important $1 t$ ls useful for understanding fortificatıons and milıtary subjects; on the whole though the place assigned to mathematics is not an important one. Lord Bacon's vlews in his 'Advancement of Learning' (1605) were that pure mathematics ( here meaning geometry and pure number) was suitable as an exercise for training in sharp thinking and subtle mental dexterity. Henry Peacham (1576-1643), a Cambridge graduate wrote a book suitable for gentlemen of the Court called the 'Complete Gentleman' (1622): In It he says nothing about arithmetic, but extols geometry as a subject "worthy the contemplation and the practice of the greatest prances". He malnly puts forward the case for its use in war and dealing with problems arising on a gentleman's estate. Robert Burton in his 'Anatomy of Melancholy' (1621) puts forward an impassioned plea for mathematics. He is quoted as saying (F. Watson Ref: 3, p.279): "Such is the excellency of mathematics that all those ornaments and childish bubbles of wealth are not worthy to be compared with them". He has an all-embracing view of what 'mathematics' means though; as well as geometry and arithmetic he includes perspective,optıcs, astronomy, mechanics, music and applicatıons to navigation, architecture
and a whole host of subjects. In spite of all these views If we look at the timetable of a typical Elizabethan grammar school for 1598 (see page 23.) gıven by Curtıs (ref: 6) and taken from the Schools Inquiry Commission Report (1868) Volume VII, we see that the only mathematics teaching that appeared on the official timetable $1 s$ lısted as arithmetıc at the end of the saturday afternoon period in classes $I$ and II. The higher classes (older pupils) appeared not to have any arithmetıc at all and the tımetable is completely dominated by the classics. Thus the foundations of what might now be termed 'school mathematics teaching', in partıcular in arithmetic and geometry, were laid half way through the l6th century; al though the teaching was limited in scope a consideration of these two subjects will now be given. As an influence on the teaching of mathematics at school level it is to Robert Recorde that one must probably look first. He was the tirst to write mathematıcal and astronomical works in English and his mathematics texts were used in England for more than a century. According to S.J.curtis (ref: 6, p.102) evidence indicates that the number of people who were able to read and write English in the l6ih century was far greater than $1 t$ was prevlously thought, $1 m p l y i n_{s}$ that elementary schools and non-grammar schools were more widespread than was formerly believed and a basic education of reading, writing and arithmetic was given to many people. Recorde's first b,ok was a popular arithmetac 'The Ground(e) of Artes' (c.1542); he also wrote an astronomy text 'The Castle of Knowledge' (1551). Hıs 'Pathewale to Knowledge' (15ヶl?) was an abridged

# TIME-TABLE OF A TYPICAL ELIZABETHAN GRAMMAR-SCHOOL, 1598 <br> (Schools Inquiry Commission, Vol. VII, pp. 262-3) 

Classes III, IV, and V were taught by the master; Classes I and II by the usher In winter the school closed at 4 p m .


| Class III 7-11 am $1-5 \mathrm{pm}$ | Lecture on the letters of Ascham, or Sturm's Cicero's Letters, ${ }^{3}$ or Terence <br> Paraphrase of a sentence. <br> Latin Syntax or Greek Grammar or Figures of Sysenbrote ${ }^{1}$ Home lessons and exercises given out and prepared | Lecture on Ascham, etc., as on Monday Vulgaita in Prose <br> Latin Syntax, etc , as on Monday. | Lectures on Palengenius, or the Psalms of Hess Paraphrase of a sentence <br> Latin Syntax, etc, as on Monday | Lecture on Palengenius or the Psalms of Hess <br> Half-hol:day | Vulgana in Prose, and repetition of the week's lectures <br> Repetition contunued. <br> Lecture on Erasmus' Apophthegms | Examination in lecture of previous afternoon. <br> Catechism and New Testament. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Class II <br> 7-11 a m | Lecture on Colloquies of Erasmus or on Dialogues of Corderius | Lecture, etc , same as on Monday | Lecture on the Cato sentor, or Cato junior | Lecture, etc, same as on Wednesday | Repetition of the week's lectures | Examifation in lecture of previous afternoon. |
| © $1-5 \mathrm{pm}$. | Translations from English into Latun. Home lessons and exercises given out and prepared. | Translations as on Mondays | Translations as on Mondays. | Half-holiday | Repetition controued. Lecture on Esop's Fables. | Writmg out the Catechism 10 English. Arithmettc. |
| $\begin{gathered} \text { Class I } \\ 7-11 \mathrm{a} \text { a } \end{gathered}$ | The Royal Grammar. | The Royal Grammar. | The Royal Grammar | The Royal Grammar. | Repetition of tre work of the week | Examination in lecture of previous afternoon. |
| 1.5 pm. | The English Testament, or the Psalms of David, in English | As on Monday. | As on Monday | Half-holiday | Repetition continued. Lecture on Esop's Fables. | Wrating out the Catechism 10 English. <br> Arithmetic. |

[^2]version of Euclıd's 'Elements' and in hıs notable work 'The Whetstone of intte' (1557) he furst proposed the use of the symbol for the equals sign, 'a parre of parallells'. Recorde's use of the vemacular in place of Latin probably contributed to the popularaty of has books; he also wrote In dialogue form as this seemed to him the easlest method of instruction because the Master had to meet the difficulties of the pupil. Accordıng to De Morgan (Ret: 4) the last edrtion that he had seen of 'Ground of Artes' was dated 1699 published by E. Hatton of London, indicating some 150 years of use. Arıthmetic was most of ten taught though outside the academic institution framework by writing masters, special tutors or as extra currıcula tuition, partıcularly in the grammar and public schools. Nine 'Public' schools were designated up to this time, which, by their Statutes, could admıt pupils from any location in England or abroad. These were Eton, Winchester, Westmanster, Charterhouse, St. Paul's, Merchant Taylors, Harrow, Rugby and Shrewsbury. In these the arithmetic teachers were of a lower status than the grammar master and were paid a lower salary. About Recorde's book Rouse-Ball (Cambridge Mathematics) states that "The work 1s the best treatise on arithmetic produced in that century". De Morgan wishes that it had been his own first book of arithmetic according to F. Vatson (Ref:3)(p.300) but in the introduction to his 'Works on Arithmetac' (Ref:4, p.xxi) he puts Recorde as the inftator of the commercial school of arıthmetic and criticises $1 t$ by statıng: "To the
commercial school of arithmeticians we owe the destruction of demonstrative arithmetic in this country........ It never was much the habit of arithmeticlans to prove their rules'. There was too much emphasis on money problems and sets of rules which were simply mechanically obeyed; the trend tended to continue amongst the popular arithmetics through the work of Edward Cocker and other writers (see later). Recorde describes arithmetic by counters and also arithmetic worked with the pen. His method of using counters was based on horizontal lınes belng given a certain place indicated by:


* = thousands lme


A counter above a line meant 5 of the particular unit on the Inne below it. Using this counter arithmetic he deals with the four rules in number and for money sums (see text book section later). He points out the 'merchants use' of the rules and methods of audıtors accounts. Thus it was possible that this counters method could be easlly taught by any teacher who had mastered the mechanles of $1 t$, or by a master to his apprentice or by a writing master even at an 'elementary' school. The use of a pen and numbers Recorde describes as 'cyphering', whereas the other technıque became known as 'casting accounts' or simply 'accounts' or 'accompts'.

The rules of arithmetical manıpulation in Recorde's book are adroitly pointed out by Alan Robson in his presıdential address to the Mathematical Association in May 1949 (ref:9). In this he criticizes the use of mechanical rules to obtain answers without understanding and quotes an example of Recorde's description of applying the golden rule of three direct, pointing out that there was also 'a backer rule of three, a double rule of three direct and backer, and a golden rule of three for fractions'. So rote arıthmetic was the order of the day. The system of casting accounts was an added subject in some schools in the 16 th and 17 th centuries. F. iatson (ref: 3, p.304-305) lists examples of this in extracts of varlous Statutes? Conslder the following two examples:

1597 Alderham (Herts).......For the free instruction of 60 scholars in purity of life, manners and rellgion and in Latın, English, writing, cyphering and accounts. 1624 Kırkby-Ireleth (Lancs) Founder: Giles Brownrıgge. Grammar, writing, cyphering and accounts. The term 'accounts' may indicate that this was the arithmetic of counters to distinguish it from arithmetac using the pen. (cyphering). An interesting entry given in Appendix IV (P.40) of the Schools Inquiry Commission (1868) includes the following concerning the purpose of the original foundation of Rolleston Grammar School (c.l520,Staffs).

[^3]'Any scholar dull and utterly unable to learn grammar (obviously Latin is meant) to be taught reading, writing and accounts'.

This pronouncement precedes Recorde's book by 20 years and It is a further indication of the low regard for basic subJects including arıthmetic. Recorde hımself has to make on appeal in the preface of the 'Ground of Artes' (the King Edward VI edıtion) in order to Justrfy why arıthmetac should even be taught. He gives the antiquity of its use in Anclent Brıtain (parallelling arguments about Anclent Greece) and also its practical value in all departments of experience particularly with reference to the coinage of the realm. Recorde's book 'The Whetstone of illtte' included the extraction of roots and, as he puts $1 t$, "The Cossike practise, wath the rule of Equation'. The cossic art was the old name of algebra and this book was the first English work on the subject which also became a popular text. There was apparently no particular shortage of arithmetic texts after the time of Recorde, relatrve to the amount of output to be expected at that time. De Horgan (ref: 4 p. 19 to 48) lists over one hundred texts that he had inspected from 1544 to 1677, some by Continental authors (for example Tartaglia, Ramus, Stevinus, Vieta) and others by English authors (for example Buckley, Digses, Blundervile, Oughtred, Iioore, Wingate, , Vallis). Prıvate teachers of mathematics would have resorted to the use of such text books. In 1650 for anstance the second edition of 'Arıthmetique made easie' by Edmund Wingate (lst edition 1629) was edıted by John Kersey who inserted his own prospectus as a private teacher
(including his 'school' address). It included arıthmetic, algebra, geometry, mensuration, navigation, construction of sun dials and mathematical instruments and chırography (i.e. handwriting) reflecting the broad scope of 'mathematics teaching' at that time (see F.latson (ref:3, p. 324-326, tor detalls).

In the teaching of geometry during this and later periods the main text book of instruction was Euclid's 'Elements'. However, Robert Recorde is credited wath the first English work on geometry, 'ihe Pathway to Knowledge', published by R.alolfe, London 1551. This was essentially a work in two parts containing practical geometrical problems and the way of working them in the first part and descristions, but not demonstrations, of the theorems of Euchid (books l-ıv) in the second part, without keeping to Euclid's order. The first English translation of Euclid's Elements is credited to Fenry Billingsley (later Lord Mayor of Iondon) and appeared in 1570. The preface was written by John Dee (152'7-1608) an alchemist, astrologer and mathematician in the courts of both Mary Iudor and Elizabeth I; he encouraged any interest in mathematics and gave instruction to pilots and navigators and, it was once thought, did most of the Euclid transtation himself having lectured on Euclid in Paris in 1550. Other evidence, however, tends to contradict this view. For instance, in the book 'Early editions of Euclid's Elements' by Charles Thomas-stanturd printed for the Bibllographical Society, London 1926, it is suggested that an old Oxtord Unıversity mathemetician named Whytehead
who lıved at Billingsley's house may have assisted Billingsley in the translation. Thomas-Stamford could find no trace of John Dee's 'turgid style' of writing asserting atself in the editorial notes. Alan Robson (ref: 9) in his presidential address suspects that not many copies of Billingsley's Euclid found their way into the schools of England and that Newton as late as 1661 at Cambridge knew little geometry and afterwards used a copy of Barrow's Latin translation. De Ilorgan attached much lmportance to the introduction of Euclad because it represented the demonstrative art of mathematics with rigid investigations, whereas arithmetic and algebra had become merely sets of mechanical rules. In the British Almanac and Companion 1837 he in fact states that "the science of arithmetic was sought in the seventh and following books of Euclid" (quoted F.ilatson (ref:3. p.335) and not, he implies, in the commercial applications of arıthmetic.

Leonard Digges was another of the leading mathematicians of the l6th century. He wrote "A geometrical practise, named Pantometria....." contaınıng plane and solıd geometry with an addition by his son John Digges on the five regular Platonic solıds and published in 1571. It is possible that through the translation of the geometry of P.Ramus ${ }^{1}$ (first written in Latin and printed in 1569) by Thomas Hood in 1590, that geometry was taught in the Unıversities, albeit to a limited extent. The name of Ramus carried welght,

[^4]partıcularly at Cambridge and the dissemınatıon of Buclıdian geometry may have helped the introduction of the algebralc geometry of Descartes which came later.

As was mentioned previously the first Professor of Geometry, Henry Brıggs, was establıshed in 1619 by Sır Henry Savile (1)49-1622) at Oxford. Briggs had previously been a Fellow of St. John's College Cambrıdge and in 1596 was appointed Professor of Geometry at the newly founded Gresham College in London. This movement from one establishment to another by Briggs and by other mathematicians as well, could only have increased the spread of knowledge and exchange of ideas. The regulations drawn up by Savile in 1619 called for the Professor of geometry to "expound publacly the thirteen books of Euclid's elements"amongst other things and this persistence of Euclid was to last well into the l9th century. Reinforced by the educationalists' views of Peacham and Bacon (also Miton in 1644) geometry was acceptable to the upper classes and, eventually, held its place in a weaker context beside the classical curriculum given in the public schools. William Kempe (head master of Plymouth grammar school) also suggested the teaching of geometry as a school subject in his book 'Educatıon of Chıldren' (1588). The preparatory schools (dealing with pupils of what now would be called primary schools) offered no geometry, but mainly dealt with the basıc teaching of reading, writing and arıthmetic and were often run as pravate schools, or, as a separate school attached to a grammar school. The teaching of geometry then would have been mainly by rote learning and repetition of the order and method given in Euclıd's Elements; this, $1 t$ may be thought, encouraged men-
tal discipline, purity and order in mathematical demonstration, whilst practical geometry would offer some useful applications in certain professions.

The revival of mathematics in the Unıversities in the l7th century may be attributed mainly to the influence of the works of Descartes (for instance, 'Discourse on lifethod, 1637) and Newton (Prıncipla, 1687) with increased actıvity in the field of algebra, analytical geometry, mechanics and calculus (theory of fluxions). However, the influence on school mathematics as a whole was not felt for some time if we are to Judge by the School's Inquiry Commission Reports later in the l9th century.

In 1673 an attempt at producing a mathematics curriculum, in the modern sense of the word, was begun by Sir Jonas Moore for use at the first separate mathemetical school which had then been founded at Christ's Hospital, London, and was endowed by Richard Aldworth with $£ 7,000$. The original purpose of the school was to provide young people to be apprenticed to the captains in the Navy and hence tended to be biassed towards arithmetic and navigation in the wldest sense; Charles II granted an additional Charter and the promise of financial help under the persuasion of powerful school governors which Included Samuel Pepys and Christopher Wren.

De Morgan (ref: 4 p.43) cıtes two works by Moore prior to 1073; the first is 'Moore's Arıthmetack (1650) and the second 'Moore's Arithmetick in two books' (1660), the second part being algebra. De Morgan thanks high ly of both of these. Phe book written for Christ's Hospital ls listed as 'A New Systeme of the Mathematrcks' (1681) in two volumes,
the first beang an abrıdged version of the eaplier arıthmetac. The whole system was in nine sections, of which Moore contributed the first four before he died. The system itself is more fully described in the article by A.Robson (ref: 9) but the followng is a last of the nine sections:

1. Arithmetic - this included all of Recorde's work $1 n-$ cluding proportion and decimals and logarithms.
2. Practical geometry - various definıtıons, less formal than Euclid's wath geometrical problems and use of ruler and compasses.
3. Trigonometry - plane and spherical with chiefly the solution of triangles.
4. Cusmography - divided into astronomy and geography. 5. Navigation - this section was written by Peter Perkans a master at the school.
5. Doctrine of the Sphere - written by John Flamsteed the Astronomer Royal, described their concept of the Unıverse.
6. Astronomical Tables - these were also provided by Flamsteed.
7. The Arithmetic of Specres or Algebra and
8. The Chief Fropositions of Euclıd.

According to Robson these last two sections were makeshift publicatıons and Section 9 was really an English version of Barrow's Euclid limited to Books I $七$ VI, XI and XII. It was some time before the mathematical school was merged with the other schools and in 1715 the senlor boys of the grammar
school went to the mathematical school for two hours a day so that they might be better prepared tor Unıversity. In spite of having all the advantages (including the support of Halley and Newton, the mathematical school failed to become a great success according to Watson (ret: s p.313). He glves the pretentiousness of the curriculum and inefficient teachers and poor management along with a lack of personal impulse as the reasons. A point though, raised by Robson, concerned the craticism by some of the governors that Hoore's system was not written in Latin and should be translated as soon as possible. This lack of enthuslasm simply because it was not in Latın may have contrıbuted to its fallure; an algebra book in Latin that was used at the Mathematical school is shown later (photostat pp 135-136). It was published in 1709 and written by John Alexander with an appendix by Humphrey Ditton. The photostat pages show the methods of solution of simple equations and indicate that quadratic and cubic equations are discussed later in the book. This seems to andicate the hagh level to which the school aspired, but whether it was actually reached is debatable: Compared though wath other mathematics teaching in the other established grammar schools this was probably in advance of anything at that time. Dartmouth Grammar School appointed a master to teach inglish, Mathematıcs and Navıgation in 1679 whllst other mathematics schools were founded at a later date. Sir Joseph 'Villıamson's mathematical school vas established at Rochester (1701) where they had a lower master who taught arıthmetic as far as decımal fractions and the extraction of cube roots, and when the pupils were ready
they were taught algebra, geometry, trigonometry and the use of globes by an upper master. The first upper master was John Colson who afterwards became Lucasian Professor at Cambrıdge which must reflect the possibllity of a high standard being taught. Later stıll, Neale's Mathematical School, London (1705) was established and the mathematics content of Kıng Edward's Grammar School Bırmıngham (1743) was wadened to give a more modern looking curriculum. Another source of mathematical progress in the l7th century may have arisen through the teaching in the dissenting charıty schools and the dissenting academics which arose after the varlous Acts of Parliament had prevented nonconformists from holding public office or positions in the established Church ${ }^{1}$. Entrants to Oxford and Cambridge would have had to be communcant members of the Church of England; likewise many of the grammar schools were established through Church foundations and thus teachers who became non-conformists would be forced to leave their posts. Vatson (ref:3, p.314) recounts of such a person in Adam Hartindale; having been ejected from his living as a mınister and his teaching post (after 1662) he later went to Ifanchester where the readmaster of Manchester Granmar School sent to hım his best boys for instruction in mathematics.
${ }^{1}$ Often called the "Clarendon Code" (1661-1665).

The dissenters in general broadened their scope of education to provide trainino for the secular professions and for business as well as for thelr own minlstry. Being less dependent on the need to teach the classics they provided more mathematics and science teaching in their curricula. One example of a mathematical curriculum at such a private school in Pontefract in 1743 is given by $H$.B. Griffiths and A.G. Howson in 'Hathematics: Society and Curricula' (ref:l0 p.ll). It indicates that arithmetic, accompts, algebra, geometry, trigonometry, fluxions, globes and mathematical instruments were all included in the course of instruction. Further examples of these curricula and discussion of the role of the dissenters in education in England can be found summarised in 'Scientific Progress and Relıgıous Dissent' (an Open Unıversıty Publıcation) (ref: ll). Theır schools must have given some mpetus to mathematıcs teaching in a broader sense and helped the development of the science and technology that would lead up to the time of the Industrial Revolution. This does not mean that the other universities neglected entirely the teaching of mathematics; in the l9th century as wall be pointed out the standard was extremely variable in order to obtain the various classes of a Cambridge degree.

The late 17 th and l8th centuries produced quite a number of mathematics text books which would have been used by teachers. The most famous, or infamous, according to De IIorgan, was by Edward Cocker, fırst publıshed in 1677.

It was called 'Cocker's Arıthmetick' and was published by John Hawkins, a writing master in Southwark; the preface also states that the book was "......commended to the World by many eminent fathematicians and writıng-masters in and near London". This book is called 'the parent of l9th century books'by Bushell (ref: 7) and he records that the 53rd edition was published in 1750. De Morgan (ref: 4 p.56-62) writes a lengthy article about this book and condemns it as a 'patchwork collection' and states: "I am of the opinion that a very great deterioration in elementary works on arıthmetic is to be traced from the time at which the book called Cocker began to prevail". The book itself has nothing original in $1 t$ and is an amalgam of the works by Recorde, Inngate, Bridges and Moore, including similar wording of examples and questions. The explanations are lengthy, over-elaborate and sometimes complicated compared to the previous authors, but the popularity of the book ensured its long life. Bushell remarks on the fact that an old Victorian schoolmaster jokingly asked him if he used Cocker, and Bushell lett school in 1903! De Morgan's attack was again based on the view that an understanding of the process is necessary rather than merely providing a set of rules; this theme nas been echoed in many of the schemes in the 20th century. Bushell attributes the first use of the decimal point to Cocker, whereas De liorgan in his introduction (ref: 4), pages xxill to $x x v i$, writes about the origin of the decimal point notation and does not credit it to Cocker. (see later section on mathematics text books).

From 1700 to 1800 many more books were published reflecting the teaching that must have been taking place in the schools. Many of the books were by teachers who produced their own system, based on previous works and used in their own school which probably would have been private or attached to a grammar school through visiting pupils. Some of the most well known texts wall be mentioned, but further details of other texts can be found for instance in De Morgan (ref: 4) and in "The Teaching of Arathmetic through 400 years 15351935" by Florence Yeldham (1936) amongst others. The content, and hence the actual mathematics that was taught, did not develop significantly through these books; it consisted of the four rules, numeration, reduction, proportion, mixtures (allıgation), practice, progressions, vulgarfractions and roots. Logarıthms and decimals were also included in many texts as were sections on square and cube roots; others included geometric progressions (tor problems on compound interest) and sections on mensuration. Special topics such as purchasing, treehold estates, stocks and duodecimals are dealt with as well. In some texts algebra is used to demonstrate the general principle, but in other cases algebraic methods are used without derivation of formula and the user was expected to apply them mechanically. An early work was "The Young Mathematicians Guide' by John Ward (lst edition 1707); this comenced with arıthmetic and included algebra later. Two very popular books were "The Schoolmaster's Assistant' by Thomas Dilworth (1743) and
"The Tutor's Asslstant" by Francis :/alkingame (1751). Dilworth used the catechetical method similar to Recorde which tended to emphasize a mechanical approach to learning and the introduction is shown in the next section. An interesting point in the introduction $1 s$ the distinction shown between 'theoretical arıthmetıc' which 1 s considered a science and 'practical arithmetic' which is useful in business and is regarded as an art. Dilworth gave many worked examples and gave the answers to his exercises vhereas Wingate, Cocker and Ward did not. The popularity of a book could possibly be judged by the number of editions published; De Morgan quotes the 22 nd edition as being published in 1784. The 32nd Yearbook of the National Councıl of Teachers of Mathematics (Vashington 1970) entitled "A History of Mathematics Education in the United States and Canada' gives Dilworth's book as the most popular 18 th century arithmetic in hmerica and states that there were fiftyeight American editions up to 1832 (p.14). According to
 England is known after about 1800. The wo rk by Walkingame is the subject of an article by P.J. allis (ref:l2) who aptly describes the book as 'an early best seller'. The first edition was an 1751 whilst the latest known of any edition was published in 1882 (publisher Spottiswood). In the preface of the book the author states that the book grew out of the rules and questions which had been drawn up "for the use of my own school" and he claimed that the book was used in "almost every school of emmence". The length of its publication time would indicate that there was a great demand for the work. Even De Morgan (ref:4, p.80) (who admits to not knowing who Walkingame was)states
that "thls book ls by far the most used of all the school books and deserves to stand high among them". The book itself followed the same general content described above, except that it was not in dialogue form, rather it gave worked examples to illustrate the rules. Walkingame also used algebraic symbols without explanation and the student was left to apply the formula. In a sense the development of mathematical teaching was $2 n f l u e n c e d ~ a d v e r s e l y ~ b y ~ e c o n o m i c ~$ factors. Having found a successful text book formula it is probable that the publishers would continue to publish the book in its successful form with only mınor alterations. Thus, other books which copied a successful work were encouraged and perhaps prevented the development of texts which would have ralsed the level of mathematics understanding rather than continue to reanforce the development of mechanıcal skılls. A similar book, but without answers which were included in a Key, was called "The Mutor's Guide, being a complete system of Arıthmetıc" by Charles Vyse (1770). Some of the problems are written in the form of poems and his book was noted for its 'muse of arithmetic'. Another well known work was "The Scholar's Guide to Arıthmetac" by John Bonnycastle (1780). This did attempt some understanding of the processes by including algebralc demonstrations along with the worked examples. Finally in this collection, which according to De Morgan would have adorned the shelves of a country schoolmaster about 1800, there are the books by Thomas Kelth, "The Complete practıcal Arithmetician" (1788) and his abrıdged version "The New Schoolmaster's Assistant'(1791). These required a Key in order to obtain the answers to the set problems. Apart from the
ossified contents and the mechanıcal rigidity in the methods of working another point seems apparent in the titles themselves. The books seem to have been wratten for instructing the teacher as much as for instructing the pupil; perhaps this is why the titles of ten included the idea of them being a 'tutor's guıde'. Certainly the mathematical abılıty of some of the teachers would have been limited having not recelved an adequate education in mathematics themselves. This was confirmed later in reports on education during the century that followed. A comparison to this situation might be drawn in the 20 th century when 'modern mathematics' schemes were being taught wath many teachers learning along with the pupils.
The geometry of the period centred around the use of Euclid's Elements and practical geometry in relationship to mensuration and surveying. However, analytical geometry, based on the algebralc geometry of Descartes and geometry related to the calculus were also developed in the unlversities. Isaac Barrow had produced a translation of Euclid's Elements 'The Whole Fifteen books' (1660) and this was re-issued six times in the early part of the l8th century. He also wrote 'Lectiones Geometricae' (1670: "Geometrical Lectures") which contained elements similar to the calculus later developed by Leibniz. Other editions of Euclid ancluded the complete 0xford edition of 1703, in Greek and Latin, by David Gregory and the most popular source by Robert Simson in Latin and

Englısh in 1756 which contalned only Books I-vi, $x i, x i l$ and the 'Data'. An edition published in 1781 (Volume One) and 1788 (Volume Two) by James Willamson included all the books of Euclid and was a really accurate translation of the Greek. A continental edition of Euclid was published at the Hague written in French by Koenıg and Blasslere in 1762. This was to provide the Ioundation for the famous English edition by Modhunter in the l9th century (see later). An interesting contribution $1 s$ polnted out by Griffiths and Howson (rei:l0 p.91) In the work 'Elements de Geometrie' by Alexis Claıraut as early as 1741. This book apparently broke away from the usual textbook presentation trying to introduce an element of discovery in the teaching of geometry. The accepted teaching pattern would have been the strict presentation of a theorem as per Euclid usually followed by an example and exerclse on the theorem. Someılmes the 'follow-up' would be a utılitarıan application of the theorem. The schools of this perlod, particularly those over-loaded with a classical curriculum would have offered no trigonometry, unless as a sllght extension of practical geometry and the 'theory of fluxions' (calculus) would have been left untouched. Two further influences on mathematics during the later part of the l8th century were the Ecole Polytechnıque of Paris in 1794 (as a part of the French Revolution) and the rise of private instruction with an increase in mathematical content at the higher degree level in the Unlversities. The former had established an outstanding array of mathematicians (Laplace and Lagrange for instance) which helped to increase the overall knowledge level, whilst the latter made mathematıcs a more acceptable subject along with classics as a
part of a degree. In spite of a good class degree including in the final set of questions such things as the eleventh and twelfth books of Iuclid, spherical trigonometry, the hlgher parts of algebra and Newton's 'Prınclpia' followed by questions on philosophy (Senate House examination l/'2) the lowest degree (poll degree) merely required two books of Euclid, simple and quadratic equations and the early part of Paley's 'Moral Phalosophy' (1799). Reprints of examınation questions for 1185 , 1786 and 1802 are included in the appendix of the artıcle by Hollingdale (ret: 8) which indicate a considerable standard for the better cand dates (shown on the photostat, page 43).

The repercussions of these influences wuld not have been felt in the schools for many decades to come and in spite of the mathematical studies at Cambridge (until 1857 even the classlcists had to satisfy the examiners in mathematics for an honours degree), classics studies and teachers of classics always held predominant positions in the grammar and public schools. This last viewpoint was reinforced by a Court decision of Lord Eldon's in 1805 which gave Judgment to the effect that only Latin and Greek could be legally taught in certain grammar schools in order to respect the founder's :ishes. (See Curtis (ref6) pages 124-125 for detarls).

He had attended Newcastle grammar school in 1765 and only in 1820 was instruction begun in other subjects which included mathematics.

A brief mention must also be made about other schools of the period. These were (i) Schools of Industry, a type of char $\downarrow$ ty school for the poorer classes which had a severely

## Appendix II

The Structure of the Cambridge Problem Papers set in 1802, with selected Questions
Monday morning Problems (15 questons)
First and second classes (i e., the expectant wranglers)
5. Determine the evolute to the logarithmic spiral.
10. Find the principal focus of a globule of water placed in arr.
14. Two places, $A$ and $B$, are so situated that when the sun is in the northern tropic it rises an hour sooner at $A$ than at $B$, and when the sun is in the southern tropic it rises an hour later at $A$ than at $B$. Required the latitudes of the places
15 From what point in the periphery of an ellipse may an elastic body be so projected as to return to the same point, after three successive reflections at the curve, having in its course described a parallelogram?
Monday afternoon Problems ( 20 questions)
Third and fourth classes (1.e, the expectant senior optimes)
3. Given a dechnation of the sun and the latitude of the place, to find the duration of twilght.
10 Find the velocity with which arr rushes into an exhausted receiver
17. If half of the earth were taken off by the impulse of a comet, what change would be produced in the moon's orbit?
20 Find the proportion between the centrupetal and centrifugal forces in a curve; and apply the expression to the reciprocal spiral.
Monday afiernoon Pioblems ( 15 questions)
Fifth and sixth classes (i e., the expectant jumor optmes)
2 Every section of a sphere is a circle-Required a proof.
9 In a given circle to inscribe an equilateral triangle.
12. Given the latutude of the place and the sun's meridian altutude, to find the declination.
Monday ce eminn Problems ( 26 questions)
First, second, thrid and fourth classes
4. If $Q$ represent the length of a quadrant, whose radius
is $R$, and the force vary $1 / D^{2}$, the time of descent half way to the centre of force: the time through the remainıng half:: $Q+R Q-R$. Required a proof.
9. Would Venus ever appear retrograde according to the Tychonic system?
13. Transform the equation $x^{n}-p x^{n-1}+q x^{n-2}-$ \&c. $=0$ into one, whose roots are the reciprocals of the sum of every $n-1$ roots of the original equation.
15. Construct the equation $a^{2} y-x^{2} y-a^{3}=0$.

Tuesday mornng Problems ( 14 questions)

## FIrst and second classes

1. Inscribe the greatest cone in a given spheroid.
2. Draw an asymptote to the elliptic spiral.

10 Find the fluent $v x \dot{x}$, where
$v=$ hyp $\log .\left(x+\sqrt{\left.x^{2}+a^{2}\right)}\right.$.
13. Required the equation to a curve, whose subtangent is equal to $n$ times its abscissa.
Tuesday afternoon Pioblems (19 questions)
Third and fourth classes
5 Divide a given line into two parts, such that their product multypled by their difference may be a maximum.
8 Find the area of a curve whose equation is $y=a^{3} /\left(a^{2}-x^{2}\right)$.
12. Determine the limits within which an eclipse of the sun or moon may be expected, and show what is the greatest number of both which can happen in one year.
17. Deduce Newton's general expression in Scct. 9, for the force in the moveable orbit.
Tuesday afternoon Problems (19 questions)
rifth and sixth classes
7 Gren the sine of an angle, to find the sine of twice that angle
12. Find the ratio of $P W$ when every string in a system of pullies is fastened to the weight.
16. Find the fluxion of $\sqrt{a^{3}+x^{3}-\sqrt{a^{2}}-x^{2}}$

Tuesday et ching Problems ( 22 questions)
First, second, third and foun th classes
3. Investigate the value of the circumference of a curcle whose radus is unty.
17 Sum the series

$$
1-\frac{1}{2^{3}}+\frac{1}{2^{5}}-\frac{1}{2^{7}}+\& c . \text { ad } i n f .
$$

and also to $n$ terms.
restricted utılitarıan curriculum of practıcal actıvities with occasional reading, writing, arithmetic. These had orlganated in the late 17 th century $a_{n}$ d further developed in the last decades of the l8th century along with the Poor Law system, one supposes as a useful supply of labour for the partıcular industry. The education recelved was very meagre and the arithmetrc, even $1 f$ it was taught, was of a very elementary standard. (土i) Schools of the Charıty School Movement and the Sunday School Movement. These were again founded in the early l8th century by relıgious organisations in order to promote the teaching of the catechism and rellgious instruction, mainly to the poorer classes. However, they also did include reading, writing and arıthmetıc in a lımıted sense. Robert Raıkes is credited with the first Sunday School proper at Gloucester in 1780. (111) Private Schools and Commercial Academies. These schools, which offered an alternative education to the classical grammar school contınued to thrive with varying numbers of pupils. They ranged from small village schools to larger commercial establishments, which may have taken in boarders, and charged fees varying from 5 shıllıngs a quarter to 60 guıneas. Although only glving the details of a local area the curriculum of some of these schools is given in 'Educatıon in Leıcestershıre, 1540-1940') edıted by Brian Simon (Leicester University Press 1968 p. 103-129) and would seem to be fairly representative of such schools. With regard to mathematics teaching the content advertised at the Commercial Academy in Bond Street, Leicester in 1788 Is stated as offering algebra, geometry, mensuration, gauging, trigonometry, surveying, use of globes, along with the normal
arithmetic suitable for a commercial education. Thus, as we move into the 19 th century the following state of affalrs (which was to resist change) seems to have generally crystallısed. The Universitıes, particularly Cambrıdge, were advancing the level of hagher mathematics but the knowledge did not seem to be transmitted outside the higher academic circles. The public and grammar schools did not include mathematics as a part of the official curriculum and only geometry was regarded as a suitable mental training, partıcularly if Euclid was taught. Arıthmetic was taught, but outside the establishment, often by writing masters. Ifany pravate schools (later also to be called public schools), apprentice schools, and dissenting schools were teaching a broader course of 'mathematıcs', but often it was applied to commerce or had utilitarian applications. The content of the mathematics that was taught to certan scholars was also a function of the following; the social class that the pupil occupled in soclety, the avallabillty of a good capable mathematics teacher and the adequacy of the text books that were used for instruction. Inherent in this system was also engendered the apparent dichotomy of what we now often call 'pure mathematics' and 'applied mathematics' and the harmful effects that may have arisen from this philosophy.

The period 1800 to 1870

During this period major points of influence in education seem to emerge overall. These could be described as: (I) The efforts of voluntary and religious bodies both within and without the established church to have more control in the education of the poor.
(11 )An increase in Government participation in the development of education through various Acts of Parliament, monetary grants and commissions of inquiry culminating in the Forster Elementary Education Act of 1870 .
(ill )The effect of the Industrial Revolution; giving rise to a more powerful $1 n d u s t r ı a l ı s t ~ m i d d l e-c l a s s, ~$ the need for more skilled artisans in industry, more clerks and a literate lower class and the expansion of urban dwelling as people migrated from the country. A short but concise history of education during this period can be found in the book 'Nineteenth Century Education' by Eric Midwinter (ref:l3) which includes a large bibllography for further study.

Another source of the history of education which ls a simple guide to the important Parliamentary Acts is the booklet by J. May and A. Greer 'A Students' Guide to the Development of Education in England and Wales (ref:l4), whilst the Hadow Report (1926) "The Education of the Adolescent" (ref: l5a) also contains a useful summary of the $1 m p o r t a n t$ developments from 1800 to 1918.

The historical development of the traditional curriculum mainly with reference to the grammar schools is dealt with farrly compactly in the Spens Report (1938), 'Secondary Education with special reference to Grammar Schools and Technical Hıgh Schools' (ref:l5b).

From the varlous reports and documents it is necessary to glean through and see to what extent the above factors may have affected the teaching of mathematics.

The Monitorial System of Joseph Lancaster (1788-1838) and Andrew Bell (1753-1832) at the beginning of the l9th century was surtable for the mass teaching of the poorer classes in reading, writing and arithmetic. The basıc idea was that the monitors (older pupils mannly) who were instructed by one master should go and instruct the rest of the pupils. The average age of the monitors was about 12 years; they instructed the pupils mechanıcally in what to do on a primitıve stimulus-response system. Joseph Lancaster is reported to have sald the following (from I.Corston's book "A Brief Sketch of the Life of Joseph Lancaster', quoted in Madwanter (ref:I3) (p.75): "Any boy of eight years old, who can barely read writing, and numerate well, $1 s$, by means of the gulde containing the sums, and the key thereto, qualified to teach the first four rules of arithmetrc, simple and compound, if the key is correct, whth as much accuracy as mathematicians who may have kept school for twenty years".

The system thus provided a cheap method of educating the lower classes since it was deemed easy to apply. It also satısfied the bellef that this type of education and the content of the 3 R's was the only educational level to which the lower classes
should be exposed. Midwinter (ref:13, p.76) refers us to D.Salmon's book 'Lancaster's Experiments and Bell's Improvements' (193'2, where Bell is purported to state: "There is a risk of elevating them (the lower classes he means) from the drudgery of daily labour above thear condition and thereby render them discontented and unhappy in their lot'. Arıthmetic teaching would have been directed towards the vocational alm of producing semı-skilled artisans or perhaps later on, clerks.

One must also bear in mand that the education being described applies mainly to the pre-primary stage, using present day terminology. However, this method of elementary educatıon eventually became the pattern for pupils up to 13 and 14 years of age as pupils stayed on later in the century, where they could afford the cost.

The foundation of two educational organlsations, the British and Forelgn school Society (1808) which supported Lancaster and the National Society for Promoting Education of the Poor In the principles of the Establıshed Church (1811) which folLowed the system of Bell, saw an increase in the provision of education for the poor. Statistics 1 or schools built by the National Soclety show that 125 schools contained ll7,000 pupils in 1817 (ref:14, p,6). By 1851 this had reached 17,015 schools containıng 956,000 pupils. The teaching of the liturgy of the Church of England and the catechism was a part of the instruction in these schools; perhaps there $1 s$ a weak link between this and the catechetical method of teaching arithmetic which vias so popular at that time. Relıgious instruction, however,

poor, than arithmetıc teaching which would have enabled them to compete for clerlcal and other skilled jobs. Influenced by the example of Scotland, Prussia, France and other continental states, a small group of thinkers led by Joseph Bentham and Francis Place tried to introduce a wider school curriculum for the class mmediately above the very poor. This was the "Chrestomathic scheme' for the higher education of children from 7 to 14 years of age and was propounded in Bentham's work 'Chrestomathia' (part I) published In 1816. It was not gıven much support at the time, but some schools of the British Society and the National Society $1 n-$ troduced geometry, trlgonometry and 'technlcal' drawing into the mathematıcs teaching during the l820's, for the older pupils. Up to 1833 the new schools which had been established were wholly supported by voluntary contributions and school fees; there were unsuccessful attemots in Parliament to overcome relıgious antagonism to State education and interference (1807, 1820 and 1833). However, in 1833 the Government, for the i'irst time, made a grant of $£ 20,000$ avallable to be applied to the erection of school houses. Much earlier than this the State had intervened in the education of the very poor through Peel's Factory Act,1802, whereby an employer was required to provide anstruction in the three R's for the first four years of a seven year apprenticeshıp. It falled because of the lack of an inspectorate to enforce it. With State intervention there also came many further acts and inquiry commissions resulting in inspectorates and reports suggesting reforms. A summary of the Acts can be found in

Iildwinter (ref:13) Chapter 4: "The State Intervenes', pp. 31-38.

Important though was the provision made in 1839 and 1846 by a select Committee to adminıster grants to finance a pupil-teacher scheme (Dr. James Kay-Shuttleworth was the orlginator) and to help in the running costs of the schools. The Political and religious disorder of the midule century period prevented a settled educational policy and with it any chance of reform in the curricula of the schools. The effect of the monitorial system is recorded in 'Report of the Committee of Council on Education for 1845, Vol. I, pp.163-4, by F.C. Cook, H. M.I. Report on Schools in the Eastern District' (ref:16, p.7). The pupils had a 'more or less facility in working the ordinary rules of arithmetic to proportion or practıce, but with lıttle or no insight into its principles'. The theme of mechanical skill versus understanding was prevalent in the l9th century as it had been previously and was to continue to the present day. The generally poor conditions in the lower class schools and the ineffective teachers are reported as well.

In the 1850-51 Report to the same committee the Rev. in. Mitchell writes of one school 'The master and mistress are totally ancompetent......Only two of the boys could work any sums. Nine of the others attempted addition, but failed entarely' (ref: 16, p23).

One school which was an exception (although there were others) was the Natıonal School at Kıng's Somborne in Hampshire. In the 1847 commattee report it is stated: "The boys in the

First Class took algebra and the first book of Euclid and mensuration was taught as an application of the principles of geometry' (ref: 15a,p6). "The school also offered a wide variety of subjects including natural sclence and retained a large proportion of pupils over the age of ll years. Eventually more pupils would sta; on to the age of 13 and 14 years which made it necessary to remove the word 'elementary' education and replace it by 'prımary'and 'secondary' education as two separate, but continuous, systems. Matthew Arnold in this perlod as an H.M.I. was introducing the term 'secondary education' Into English discourse as a result of his comparatıve studies of European Educational systems. In order to see if this level of education was influenced by other sectors of public education $1 t$ ls first necessary to look at the field of hlgher education as it applied to the alfferent classes in soclety, and then to look at the development of the elementary, public and grammar schools in relationship to higher education during this period from 1800.

The development of higher education durang the early part of the l9th century was also provided through the establishment of instıtutions other than the Oxford and Cambridge unıversities. Mechanics' Institutes were formed which represented the aspirations of perhaps the skilled working class who were without formal higher education, but required to know more about science and mathematics in particular because of the growing industrial state.

The London Mechanıcs' Institute (now Bırkbeck College) was established in December 1823. The movement was supported by the dissenting clergy and the followers of Bentham. The instıtutes began to fall the needs of the skilled working class after a decade or so because of the illıteracy of the class for which they were intended and the 'invasion' by the maddle class (cleras and shopmen) into their lectures. The institutes were distributed all over England and Nales with large concentrations in the North and were more concerned with sclence teaching than offering a full curriculum. For a distribution map of institutes taken from the 1851 census see D.S.I. Cardwell (Ref: 17. p.75). By 1850 there were 610 of these Institutes in England and 12 in Tales with a total membership of over 600,000. For the middle class the foundations laid by Gresham College (opened in 1596) in London enhanced the establishment of Unlversity College in 1826, again by non-Anglıcan founders. This lead to the foundation by the Anglicans of King's College, London, 1 n 1828, as a rival which would also offer a curriculum of classics, mathematics, languages and the sciences.

The first appointed Professor of liathematics at University College was, in fact, Augustus de irorgan. The courses at Kıng's College were Just as 'lıberal' as those at University College, but also included religıous instruction. The two rıval organisations existed unhappily untıl, as a compromise, the Government created the Unıversity of London in 1836, which had statutory powers to affillate any colleges which had reached sufficiently hlgh standards of scholarshıp. By 1844, as well as Unıversity and Kıng's, twenty other colleges were affillated. This relationship disappeared when, in 1858, the University of Lonaon became an examination board and offered $1 t s$ degree examinations on an 'open' basis, thus providing a measure of respectability for the later provincial unlversity colleges in the Northern towns (Owen's College, l85l, Manchester, for example). The North in fact had uts first unıversity founded at Durham in 1832; It was modelled though on Oxford and Cambridge to provide an education for the wealthier classes. Oxford and Cambridge did manage some measures of reform; the Cambridge mathematıcs tripos had been in existence since 1747, whilst 0xford introduced degree examinations in mathematics in 1800, uut even so the teaching staff were predominantly clerical and mews differed as to the importance of a mathematics education at the unıversitıes. It has been remarked previously that Cambridge evolved a great tradition for mathematical studres. The great mathematicians at. Cambridge - Woodhouse, Herschel, Whewell, Maule, Babbage and Peacock, tried to advance the study of analytıcal methods similar to the continental approach and founded the Analytical Society (1812) for this purpose. The opinıons of Peacock and Whewell were an mpportant influence in justıfying the educa-
tional value of mathematics; even so, they still perpetrated the established view of a non-utılıtarıan 'Liberal' approach, particularly for the upper and middle classes; just the classes, in fact, who were exposed to the higher level of mathematıcs, but who had no incentive to apply it to any practical situations.

Accoraing to Cardwell (Ref: l'/. p.54) Dr. Peacock (who was Dean of Ely) writing in 184l, observed that of the two classes of students at Cambridge the very wealthy would tollow no profession and the others would enter the professions of law, medicine and divanıty.

Dr. Whewell in his Essay 'Of a Liberal Education' (Iondon 1845)
argued the case for a liberal education of the upper clases based on mathematics (the geometrical form used in Newton's 'Principıa' was advocated).

He considered the stuay of mathematics as a unique mental disclpline (c.f.Bacon 1622) and drew attention to the number of eminent judges who had been highly placed in the Mathematıcs Tripos.

Cardwell (Ref.17. p. 55 ) also notes that between 1800 and 1850 no fewer than 43 men who subsequently became bishops were also successful in the Tripos. The specialised course, as it was then considered, of the lathematıcs Trıpos was criticısed as being too narrow a basis for a proper education. One critic writing under the pseudonym of 'B.A.' wrote in 'University Education' (London 1842) that the degree was suitable for 'engıneers, archıtects and artillerymen'.

Augustus de Morgan writing in the 'Quarterly Journal of Education' (1832) on the 'State of the Mathematical and Physical Scıences in the Unıversity of Oxfora' stated the view that a degree should not be obtained on the strength of a person's ability in one branch of knowledge alone. One can infer that professional mathematicians and mathematics teachers were not in great demand even though the universities at this time had produced some of the great names of l9th century mathematics and men of high quality were on the Unıversity mathematics teaching staff.

Apart from those previously mentioned one can list the followang, who were iranglers during the mid l9th century: George Green (1832), G.G. Stokes (1841), Arthur Cayley (184火), William Thomson (1845), J.C. Haxwell and E. Fouth (1854). The retorms in the universities during the 1850's and later as a result of the Royal Commission reports, whilst liberalising the curricula and giving broader courses with more choice in the Honours schools, actually increased the duration and difficulty of the Mathematics Tripos.

The route to higher education, whether it was to the Institutes, the Colleges or the Universitıes was via the analogous path in secondary education - that is, through the elementary schools, the grammar and private schools or the public schools. What type of mathematics education was taking place at this level to complement the hagher level ?

The be nefits of the education offered by the Mechanics' Instıtutes was not fulfilled as far as the majority of the workang class were concerned. The elementary education offered had been extremely variable in content and quality and with regard to mathematics it seems likely that only arithmetic
of a commercial type had been the staple diet. It has been pointed out previously, however, that a number of schools were improving the general standard of education and retaining older pupils. Gosden quotes from the Report of the Committee of Councll on Education for 1847-8, Vol. I, where the Rev. H. Moseley reported on Kıngs Sombourne School, Hampshıre, (ref:16, p.174); he found that algebra was taught to 21 boys of the first class including the two pupil teachers, and geometry to 11 of these pupils'. The Rev. Moseley states: "Nine of them have solved correctly a proposition in the first book of Euclid, beyond which book none have yet advanced". However, this particular school was the exception rather than the general rule.

In 1858 a Royal Commission under the Duke of Newcastle inquired into the state of popular education $1 n$ England and Wales and published $1 t s$ report in 1861 (The Newcastle Commission) (ref:18). The reports of some of the assistant commissloners in varlous districts indicates on the whole an unsatisfactory state of education for the working classes, except where the schools were inspected and tranned teachers were used. Apart from the poorly trained teachers and the inadequate buildings the school education was limited in its curriculum. In Volume II of the Commissioner's Reports arıthmetıc ls usually included as one of the '3 R's'.

Concerning the maning distracts of the West Madands Mr. Coode states (ref: 18, p.305): "The education they (the parents) at all care about is reading wath spelling, writing and arlthmetic. A clergyman in the nelghbourhood, a great
linguist and good mathematiclan, thinking they maght wish to advance themselves.....especially of mathematics to help them in their engineering, put out circulars for evening classes: he had not one application".

Mr. Foster (ref: 18, p.371) reporting on company schools in the mining districts of Durham, and writing about a group of "seven lads 13 to $17 \frac{1}{2}$ years of age" states that they "are well aware that there is no chance of getting promotion without being good scholars. Four of them had been all through the arithmetic; three only as far as compound interest". This indlcates that compound interest was a minlmum level in this case; these pupils were attending the evening school of the company. Evenıng schools were popular in the Nest Midands as well. Mr. Coode reports (P. 299) that "Arıthmetic and geometry are frequently pursued. Book-keeping, as opening the way to a lucratıve and respectable advancement, is a very common object of study".

The Rev. James Fraser reporting about a private school in the South iौest (ref: 18, p.36) notes that the pupils are taught to repeat the multipllcation table and "that there is very little wakening of intelligence". In another district the parents he states (Ret:18, p.59) are 1 mpressed by "the production of a clpherıng book......and the sums neatly set out". At another school (ref: 18, p.102) he writes of "the set mechanical manner in which arıthmetical processes are elaborated, the pupil's mind never seeming able to make a short cut for itself". Mr. J. Jenkins in his report on schools in wales (ref: 18, pp.557-558) comments on the absurdity of some of the numbers used in 'addition of money', e.g. the addition of ten lines involving amounts such as $\begin{aligned} & \text { f } 540,976 \text {. He also reported on }\end{aligned}$
pupils who stated that they were working at proportion, fractıons and decimals but who could not even obtain correct answers to problems involvang simple addıtion. Later he states that the cause of the defect is due to lack of repetitıon of exercises in the tundamental processes. This does seem a contrast to present day thought where understanding is deemed more important and also differs from the views of some of the other commissioners in the inquiry, who thought that too much mechanical repetition was unnecessary. The Newcastle Commission found then a need for a greater concentration on teaching the elementary skills of the 'three R's' (which in terms of mathematics teaching simply meant arithmetic) and a slmplificatıon of the grant system to aid schools.

In 1862 a system of grants based on the attendance of puplis and the performance in annual examinations was put forward by Robert Lowe, the Vice President of the Committee of Council on Education. This 'Revised Code' on grants to maintain schools continued, with various modifications until nearly the end of the l9th century. It became known as the 'payment by results' system; the pupils had to reach certaın levels of attaınment in reading, writing and arithmetic for the grants to be claimed. There were six standards inltially given in the report (ref:19) the arithmetic being stated as follows:

Standard I - Form on blackboard or slate, from dictation figures up to 20; add and subtract figures up to 10 orally from examples on blackboard.

Standard II - A sum in simple addition or subtraction and the multıplication table.

Standard III- A sum in any simple rule as far as short dıvisıon (inclusive).

Standard IV - A sum in compound rules (money).
Standard V - A sum in compound rules (common weights and measures).

Standard VI - A sum in practice or bills of parcels. The majority of these skills can be seen demonstrated in the photostat coples of pupil's exercise books of about this time in the later section on text books. It must be remembered that the lower standards were tor 'primary' school pupils, although the top standards would have been reached by pupils of early 'secondary school' age (up to about 14 years of age). The criticisms that followed in the wake of the Revised Code were based on the following factors: (i) that it encouraged the excessive use of rote learning and development of mechanical skills, and (il) that it prevented the development of a wader curriculum and, in particular, actually stopped the mathematical development that was beginning to take place in some of the more 'progressive' schools. Gosden (ref: 16, p.29) reports from a book 'Some Habits and Customs of the Working Classes' by A.Journeyman Engineer (1867). Here it is stated that the son of a skillea artisan in a British Soclety School when he is eight years old would have night lessons (i.e. homework) involving 'the working of half a dozen sums and the learning by heart of a table of welghts and measures'. By the time he is twelve years old "His night sums wall now be in the higher rules of arlthmetic, which he finds exceedingly
difficult from his having been forced through the earlier rules without beang taught the principles of their applıcation". Hore powerful criticism came from Matthew Arnold, the govern: ment inspector, in his General Report for the Year 1869 (given in Gosden (Ref: 16, p.37). Arnold denlgrates the 'payments by results' system in stating that "The object being to ensure that on a given day a chıld shall be able to turn out, worked right, two out of three sums of a certan sort, he is taught the mechanical rule by which sums of this sort are worked out and sedulously practised all the year round in working them; arithmetical princlples he is not taught, or introduced into the sclence of arithmetic".

Schools which had encouraged some pupils to stay to 14 years of age and had studied perhaps algebra and geometry were more or less forced to concentrate on realising the Revised Code standards for the majoraty of the pupils in order to obtain the Government grant. Supporters of the system malntained that it allowed a basic minımum standard of education to be reached by the majority of the population rather than allowing just a few to be glven a hıgher standard. The above arguments concerning both pedagogy and phılosophy are stıll being fought out at the present time.

Two other Royal Commissions were anstigated durıng the 1860's and these looked into the state of affairs as they existed in the Public schools and the Grammar schools.

Although mathematical education will be looked at in greater detall, 1 ts relationshıp to the rest of the curriculum cannot be 1 gnored. For the upper and, to some extent, the middle classes the basis of education rested upon the idea of a
'lıberal' education evolved from the 'seven lıberal arts' concept discussed previously. However, by the l9th century thls idea had become an education based prımarily on the study of the classics, Roman and Greek writers and the history of antiquity. An excellent summary of this is given by Dr. R. F. Young in the Spens Report (Ref: 15b, Appendix II, pp. 403414) entrtled 'Note by the Secretary on the development of the conception of General Lıberal Educatıon'. The Publıc Schools and Grammar Schools, It has been stated, provided the entrants to the ancient Universities; in many cases the day pupils at the Grammar Schools wo uld leave at the age of 13 years approxımately and obtain a more vocatıonal training at a commercial academy, particularly in the North of England; the boarders would most likely enter university. $W_{1}$ th Oxford and Cambrıdge both demanding Mathematics as well as Classics for the B.A. degree after 1802 and with an honours class list in mathematics at 0xford in 1807 It would be thought that mathematics would have held a more prominent place in the curriculum of these schools. However, the predominance of the influence of the Classics in the unıversities, plus the Statutes $1 m p l i e d$ in the foundations of the schools prevented any serious changes even later into the l9th century. In a paper read to the Internatıonal Congress of Mathematics in Rome in April 1908, Mr. C. Godfrey (Ref: 20, p.250) in describing the teaching of mathematics in English Public Schoolis comments on the lack of freedom in mathematical teaching due to the demands of the examinatıons set by the Unıversities and the Civil Service Commissioners (for Army Officers) in the previous decade. In fact the dominance of the examination largely determined the curriculum after 1850 when the College
of Preceptors (1853), the Indlan Clvil Service and the Home Givil Service (1855) and the London Matriculation, Oxford Local and Cambridge Local (1858) examinatıons were introduced. It is worth noting that mathematical text books of this later period often included in the preface the examınations for which the book was sultable, whereas earlier books had tended to emphasize the need to learn mathematics as a subject worthy of studying for its own sake and practical value (see text book section). The Clarendon Report (published 1864) which was the Royal Commission Inquiry into the nine public schools has in Its reports some correspondence from notable people on the subject of the type of curriculum which should have been included in the public schools. For instance, in Vol. II (appendix) (ref: 21, p.42) the Right Hon. T. Gladstone, H.P. states "I hope you (the commissioners) wall hold by affirmation........the propositıon that classıcal trainıng is the proper basis of a liberal education". (Aug. 29 186x). Other letters of correspondence, notably those by the Rev. W. Whewell (pp.43-47) insisted on a wider curriculum, particularly with reference to mathematics and science. Whewell reiterated his paper 'of a Liberal Education' (lo45, but republished version in 1850).

Lefevre (llth Feb. l863) included the mathematical requirements for matriculation at London Unıversity. On the whole the level required in arıthmetıc and algebra was fairly low by modern standards, e.g. the hlghest level includes geometric progressions. In geometry the first four books of Euclid and
the propertaes of carcles were requared. It ls worth noting that the subjects of examination for entry to the Royal Milıtary Academy, woolwich, (1862) given in the report (ref: 21, p. 42) included in the mathematics section such topics as spherical trigonometry, co-ordinate geometry and integral and differential calculus. A knowleage of statics, dynamics and hydrostatics was also required. In the latter examination it was also slgnificant that the marns alloted to mathematıcs $(3,500)$ exceeded the marks given to the Classics (3,000). This was different, as whll be seen by the marks appropriated to these subjects in both the public and grammar schools. This hlgher standard of mathematics for mılitary training may have been influenced by the early developments in France which had taken place under Napoleon (179b-1802) and the possible growth of Prussian mılıtary education. Earlıer criticlsms of the endowed schools had come from James Pillans (1/78-1864) Professor of Humanaty at Edınburgh Unıversity (who had been a private tutor at Eton) in his 'Rationale of Discipline' (written 1823) and also by Thomas Wyse (1791-1862) in his 'Education Reform' (1837) where he writes of "the dy husks of ancient learning" given unnecessarıly to the middle class pupils in the Grammar Schools: Some of the early experamental private schools may have been influenced by the work of Pestalozzl (1746-1827). The Mill Hıll School founded by the Congregationalists in 1807 had a wide curriculum and included in the mathematics course were algebra, Euclid and trlgonometry; others were at Bruce Castle, Tottenham and Hazelwood School, Birmingham (1825):

Some elements of reform took place at Shrewsbury Public School under Dr. Samuel Butler, Head Master from 1798-1836 and at Rugby under Dr. Thomas Arnold, Head lhaster from 18281842. Classics still formed the core of the curriculum, but other subjects were added; mathematics, though, was often taught by visiting masters or classical form masters. The predominance of the classics still held for a long time after the passing of the Grammar School Act (1840) entatled 'An Act for Improving the Condition and Extending the Benefits of Grammar Schools'. Arnold's work at Rugby had restored the prestige of the large boarding schools among the middle class who welcomed the social and moral training which they offered. A conslderable number of new boarding schools were thus established from 1840; Cheltenham College (1841) and Clufton College (1862) being Just two of them. These schools were descrabed in the Public Schools Commission (1864) as proprietary schools and were less expensive to the middle class than the more famous schools. Reports on some of these schools were included in Vol. II, Appendix (ref: 21, pp.509-581) and dealt with Marlborough (1843), Wellington (1853) and Cheltenha: Colleges. A statement from the head master of the City of London School (1837) was also included. Cheltenham College had a 'Modern' department which was designed to provide a more vocational training for government, commercial and milıtary posts, partıcularly for Woolwich and Sandhurst which, as has been stated, demanded a high mathematical standard for entry. The main study was mathematics, and natural sciences were included, but Greek was omitted. The lower mıdde classes tended to use the private boarding or day schools
which were mainly commercial schools and were noted by the Schools Inquiry Commission (1868) to be lacking in standards of teaching. The mathematics taught in these was generally arithmetic, but assoclated subjects such as book-keeping, land surveying and mensuration may have been taught where the pupils were sons of tradesmen, farmers or artisans in different localities.

The Government in 1861 appointed a Royal Commission with Lord Clarendon as Chaırman to inquire into the administration of nine great Public Schools - those mentioned previously. The Report, published in 1864, found that the course of study provided in the nine Public Schools was sound and valuable in its main elements; Latin and Greek, but was lacking in breadth and flexibulıty. The Commissloners strongly supported the classlcal tradition and adhered to the concept that it should occupy the most time in the curriculum. Other subjects were subsidiary but they recommended that mathematics should include arıthmetic, geometry, algebra and plane trigonometry. The Commissioners in fact were taking as their model a modified form of the Prussian 'Gymnasium' of the perıod with its neohumanıstic ldeal derıved from the reforms introduced by V. Von Humboldt in 1809. The actual report was quite lengthy and only a sample of the evidence wall be considered with regard to the teaching of mathematics. In Vol. IV; Evidence, Part 2 (ref: 22) various witnesses from the following schools: Charterhouse, St. Paul's, Merchant Taylors', Harrow, Rugby and Shrewsbury gave evidence relating to mathematics that had been taught and was being taught at the time of the inquiry.

Each entry is accompanıed by a Summary of the evidence at the end of the Report on each school. The Rev. S.F.Walliams a mathematical master at Charterhouse reported on his mode of teaching (ref: 22, p.42) (Section ll51) ${ }^{\text {l"The subjects...... }}$ under consideration are Euclid, Arithmetıc, Algebra and Trigonometry with the higher class......Euclıd I require them to say by word of mouth.......n arıthmetic and algebra I give questions pointing out to the class the difficulties......." Later he reported that the higher level boys studied differential calculus, of which he disapproved because it used up time which could have been spent on other topics (1172). Mechanics were learnt (statics) without the ald of calculus (1173). Conlc sections were learnt analytıcally rather than geometrically as they had been in the past (1175). The pupils had a slight practical knowledge of trigonometry (ll77). Charterhouse in fact seems to present a reasonable level of mathematics teaching overall. In the Vol. II Appendix (ref: 21) there are cross references to the dıvasions (i.e. classes of subjects) listed under Table B (pp.333-386) and also the actual methods of teaching, books used and methods of evaluation in Table C (pp.387-455) for all the schools in the Report. Table D in the Report (pp.456-486) gave references to the employment of time in lessons and the work done, whilst Table E (pp. 487-500) listed some of the private work undertaken with special tutors outsideschool hours. In Table B for Charterhouse entries are quite explicit about the level of mathematics
$I_{\text {These }}$ numbers reter to the sections in the Report for the particular witness.
taught in the various divisions; for instance, in the Upper Mathematical School, 2nd division, algebra, arıthmetic and four books of Euclid are taught. One boy can be seen as aged 14 years 2 months in the upper school, lst division - the hlghest level of mathematics whilst another boy of 18 years is in the third division where the primary rules of arithmetic were taught and Euclıd's definıtıons (ref: 2l, p.3ヶ0). This implies that the 'streaming' that took place was based on the abılity of the pupil only.

In Table C (pp.408-409) some of the text books used at Charterhouse are Listed. These were 'Euclıd' by R.potts, 'Arithmetıc' by J. Colenso, 'Algebra' by Todhunter and Colenso, 'Trigonometry' by CoLenso and Todhunter, 'Conic Sections and Difterential Calculus' by Todhunter, 'Statics' by Goodwin and Whlliams and Wrigley's Examples. Some of these text books are included in the later section. The arithmetic by Colenso was the one which ousted the work by Cocker according to N. Bushell (ref: 7). Une outstanding feature noted from Table $\mathcal{C}$ for all the schools though was the overwhelming dominance of the Classics in the form of the Latin and Greek authors, the learning by heart of verses and prose for recitation around the class; any mathematics that was taught was completely submerged. Of the two really well known public schools, Eton and Harrow, the Iormer presents an abysmal state of mathematics teaching in both content and method. The replies to a set of printed questions on various topics are given in vol. II Appendix (ref: 21, pp.93-1/'2) for Eton and they make fascinating reading concerning the teaching of mathematics.

The number of mathematic masters was glven by the Rev. H. Snow (ret: 21, p.p. 14フ-144).

| Year | Classical hasters |  |
| :---: | :---: | :---: |
| 1812 | 10 | -- |
| 1841 | 14 | 1 |
| 1849 | 17 | 5 |
| 1857 | 19 | 7 |
| 1861 | 23 | 8 |

The report given by the Rev. S.T. Hawtrey, the Mathematical Assistant liaster ( $\mathrm{pp} .154-163$ ) relates the history of mathematics from the early writing and arithinetic master Mr. W. Hexter, up to the year when he was appointed in 1837 by his cousin Dr. Hawtrey who was the headmaster at that time. (Havtrey had graduated in 1832 as llth wrangler). He does, in fact, describe Eton as essentially a 'classical school'. According to Bushell (ref: 7) Hawtrey placed Euclid on a pedestal; "he divided the books of the world into three classes: Class I - the Blble; Class II - Euclid; ClassIII - all the rest"

Bushell (ref: 7) also descrıbes a book of Euclıd's propositions wrıtten by Hawtrey designed to ancrease tneır logical deductıve abilıtıes. In Hawtrey's description of his boys studying Euclid he quotes from his 'Narrative Essay on a Iıberal Education' as follows: "Look into those boy's faces ! They are merry hearted......The interest and pleasure felt in the exerclse of their mental faculties in following out intellectual truth.....tell me if you do not thank that Euclad is dongg them a great good". Perhaps, a rather sincere but mistaken
enthusiasm. Hawtrey gave the mınımum mathematıcal standard in an appendix to his report and these were basically the four rules, weights and measures, compound rules and reduction, vulgar fractions and the rule of three. This was required by
a fourth form boy. The lower division of the fifth form were required to know decimal fractions, proportion and interest plus the above. This was really a basic arithmetic syllabus and he left the work on Euclid and algebra to the middle and upper divislons. For instance in the upper division he asked for the lst book of Euclid, a knowledge of algebraic fractions, equations of the first degree and simultaneous equations. This was not a partıcularly high standard compared with the other schools and very narrow compared with a present day syllabus. The Rev. F.J. Ottley, another assistant in the mathematical school (but not the mathematical assistant master) listed the books in use; they were J. Colenso's arithmetac, algebra and trigonometry and R. Pott's Euclid. Little mention was made of the trigonometry although a part of his examination papers were in that subject. Many other mathematical masters report on their teaching, but also they all comment on theur low status. Mathematics was not a compulsory subject until 1851 according to $F$. Ottley (ref: 21, p.165) when each boy had to recelve three hours a week instructaon. Ottley also puts the blame on non-effectıve work in mathematics due to the fact that "mathematical masters were from the first placed in a position of inferiority as regards authority in the school".
liany of the boys received private tuition from the mathematical masters; this achieved two objectives: (i) it helped the income of the lower paid masters (for unstance the Hathematical assistant Hawtrey received $£ 1,386$ per annum whalst the first assistant received on 1 £ 270 per annum); and (11) it helped prepare some of the boys for the competitive examinations for the miltary colleges (c.f. the entrance standard for $W$ Woolw ch College, ref: $21, ~ p .42)$. Mathematical masters were not even entered on the lists published annually at election as entered in the table siven by the head master, the Rev. C.O. Goodford (ref: 2l, p.ll2).

The Classical Assistant Hasters were aware of this discrimınation and from their comments imp̃led a wish to change the position to one involving equalıty of authority. A similar situation occurred at the other schools. The senior mathematical masters at Harrow who gave evidence at the time of the report (1862) were J.F. Marıllier and the Rev. R. Maddemist, although there were two others. The history of mathematics at Harrow was brlefly described by Marillier who had been there for 44 years (since 1819) having taken over from a private writing master at that time. When he entered in 1819 the sixth form alone were reading Euclid once a week with the head master and the other boys had voluntary private lessons. In 1837, John Colenso (later Bishop of Natal) was appointed as well and the whole school recerved compulsory mathematics educatıon. The 'whole school' in this context vould be quite small as between 1837 and 1837 the number of boys in the school varied from b0 to 460. At the time of the report Maddlemist had been at Harrow for 17 years and according to Bushell (ref: 7) was of sımılar
disposition as Hawtrey and also required a perfect rendering of Euclıd. Hidalemıst (ref: 22, p. 212 sec .1280 ) gave the topics reached by the sixth form as arithmetic, algebra and Euclid, mainly, with a smaller number doing trigonometry, conic section and mechanıcs; he also gave private tuation in differentıal calculus. The time alotted to mathematics was 6 hours a week, although only 3 hours of this was in class teaching. Concerning the classucs-mathematics conflıct at Harrow, Marıllier describes the classical masters as "antagonistic" and pointed out that in the marks alloted to the classes only one fifth of the total was given to mathematics whilst classics were given the other four fifths. Another master, the Rev. H.N. Natson, described the overall standard reached by the tame the boys left (ref: 22, p.216); he reported that they were all famılıar with arıthmetic, one thırd leit wath very little knowledge of algebra and about one quarter nad studied trigonometry. However, most of the sixth form had stualed 6 books of Euclid, which does indicate a surfert of geometry. On the whole though, Harrow, as far as mathematıcs teachıng was concerned had developed a better organisation and standard than Eton in this subject.

The mathematrcal teaching at Rugby was in a simılar state. Accoraing to the evidence gaven $1 n$ the report (ref: $22, \mathrm{p} .280$ ) mathematics was taught to all the school probably on the appolntment of the master, the Rev. R.B. Mayor, in 184h. The other mathematıcs master was J.M. ,ilson, later to become Canon J. M. Wilson, who was appointed in 18っ9. He was one of the founder members of the assoclation now known as the Nathematical Assocıation in 1871 which will be discussed later. The Head Master, Dr. Temple, also taught some of the mathematics to the
sixth form. Apparently durang 1828 or 1829 Dr . Arnold contributed £う0 a year towards providing a mathematical master (ret: 2., p.244), although he expected the classical masters to teach mathematics and modern languages and also had a writing master to teach arithmetıc. One interesting comment was gaven by Dr. Temple (ref: 22, p.250) concerning the numbers in a mathematıcs class; "A man can easily teach 50 (boys) considering how much he 1 s supposed to do for them".

One concession was that a mathematical master had equal authority compared to a classical master. At the tame of the report the boys in the lower school (below the age of 14 years), were taught arıthmetic for about one hour a week by a writing master. By the time a boy left at the age of 18 years he would have gone through the whole arithmetic, algebra to the end of progressions and the first four books of Euclid. Some boys in the sixth would have reached differential calculus and the conic sections. However, as J. Wilson pointed out, 10 of has pupils out of 26 washed to go to Woolwach, but they would not be able to go direct from Rugby and he wished that a second school on modern lines could have been established at Rugby. The Rev. Mayor made a final comment which indicates the low overall mathematıcal standard reached (apart from Euclid) when he said: "The standard now required for Woolwich is altogether beyond boys of $18^{\prime \prime}$.

At Shrewsbury the books used by some of the sixth form were Bernard Smith's Arithmetic, 'rodhunter's Algebra and Trigonometry, Drew's Conıc Sections, Parkınson's Mechanıcs, Todhunter's Differential and Integral Calculus, Colenso's Algebra and Pott's Euclid. In the method section it was stated: "Every boy's proficiency in Euclid $1 s$ tested both vava voce (with
the "blackboard") and by written examination in the lessons". Bushell (ref: 7) describes the method of learning Euclid by having the boys 'reciting' round the class one line at a time some proposition wath complete accuracy during"that frigid hour before breakfast". The boys he stated, finlsh school knowing nothing but the sterile repetition of propositıons. The same pattern seemed dominant in the other schools and a brief look at Table $C$ (ref: 21) concerning the text books and instructional methods, plus the evidence given by varlous witnesses (ref: 22) vould suffice to glve an indication of the state of mathematics teaching in the public schools at this tıme. In fact, as Bushell (ref: 7) and Canon J.h. Wilson (ref: 23) both pointed out, this was continued up until the first decade of the 20 th century when Oxford and Cambrıdge no longer required Euclid for their entrance examinations.

A brief summary of the Public Schools report with regard to mathematics teaching was given by A.J. Siddons (who had been to Harrow as a school boy and started teaching at Harrow in 1899) in his Presidential address to the mathematical Association $1 n$ January 1936 (ref: 24). The title of his address was called 'Progress' and related to the early teaching of mathematics and the developments that took place in the final quarter of the l9th century. His conclusions confirm the above mentioned facts, namely, that "the few best mathematicians in varıous (public) schools did trigonometry mechanics, conic sections and in some schools differential calculus; but the majority of boys seem to have done nothing beyond about four books of Euclid, some algebra and arithmetic and the work was very mechanical" (ref: 24, p.12)

The next report published in 1868, known as "The Schools Inquiry Commission" (ref: 25) looked at the type of education that was beang given in schools other than the nine public schools and those schools considered in the Newcastle Commission of 1861. The Report looked at what were termed 'endowed schools'; these were divided into what the 1842 digest called 'Grammar Schools' (where Latin and Greek were taught) and 'IVon-classical Schools'. The inquiry also looked at proprietary schools and private schools; these filled the void where endowed schools were sparsely distrıbuted and, to many parents, offered a more useful, commercial education than the classıcal based endowed schools. The difference between the two was glven in the report (ref: 25, p.310); basically proprietary schools were similar in concept to the private schools, but were not actually owned by the master or mistress who taught in them, as were the private schools (see ref: $25, \mathrm{p} .283$ ). The report also classifiled schools into three grades which were directly related to the social class of the pupil and his parents. the classification system described in the report began by basing the school grading, list 2nd and $3 r d$, on the length of time that a pupil was expected to recelve education. Thus, the lst grade schools were for pupils intending to receive education up to and beyond 18 years of age; the 2nd grade for those up to 16 years and the 3rd grade for those up to about 14 years. This correlated heavily with the social rank that the school served. This was an important factor in determining the type of mathematical education that a pupil would receive. The three grades implied a different course of study; the following table is taken from the report (ref: 25) Appendix $I I(b)$ page 12.

| School |  | Type of Instruction | School Life |
| :---: | :---: | :---: | :---: |
| Furst Grade | $A_{1}$ | Training tor Universities and learned professions. | to 18 and upwards. |
|  | $\mathrm{A}_{2}$ | Tralning for Mercantile and Fianufacturing business. | 17 or 18 |
| Second Grade | $\mathrm{B}_{1}$ | Training for Trade and Agriculture. | 15 or 16 |
| Third Grade | $\mathrm{B}_{2}$ | Training of small or decayed Trades people's famılıes. | 13 or 14 |
|  | $\mathrm{c}_{1}$ | Upper Artızans |  |
| Below the range of the commission. | $\mathrm{C}_{2}$ | Labourers | Usually 12 or 13. |

A full description of these grades was given (ref: 25, pp.1622) and in essence the type of school considered by the first grade was contained in the Public Schools Commission (ref:22) including the newer foundations such as Cheltenham College. The startling conclusions reached by practically all the witnesses was that Latin should be retanned as the major language, even including it as a large part of the curriculum in the 3rd grade schools. As some witnesses stated "They would teach Latin if only for two years, and even to peasants, if peasants could be induced to learn it". (ref: 25, p.23). There were other witnesses (Canon inoseley being one) who advocated that artizans shou d have a thorough knowledge of thesr own handrcrafts and own language, even if it meant less 'cultıvation' (ref: 25, p. 23 to 26), but these were in a minority. The report also commented on the number of people amongst the mercantile classes who patronised the pravate schools because they offered a 'modern education' and less classics, but the Commissioners were reluctant to see the time allowed for classics to be dimınıshed. Thus, the 2nd and 3rd grade
schools retalned Latin, pernaps against the washes of many of the parents; as one witness stated they wished for "a clerk's education; namely, a thorough knowledge of arıthmetic and ability to write a good letter". The summary of the mathematics teaching in all the schools in Vol.I of the Report concluaes that mathematics should have recelved more attention and that it was not taught properly, being generally unsatistactory in the schools considered. Arithmetic was considerea of prime importance; the report stated "wo one can doubt the value of geometry as an exercise in severe reasoning; ana digebra, though inferıor to geometry...gives completeness to arıthmetic" (ret: 2y, p.30.). Euclid, accoraing to the report, was almost the only text book used in teaching geometry. It was considered ill-advised for beginners to pursue suchia and :lrurifilth, secretary to the British Association, stated that too much time was given wo Eucमld and that many boys had read slx books of it who knew notning of geometry. Professor Key wished to get rid of E'uclid altogether and callea it an lllogical book. With regard to Euclia little notice was taken of liatthew Arnold's inquiry into the european education system for the upper and middle classes which he had conauctea trom Aprif 1865 and published especially ior the Commission in 1866. The French and German schools had ceased to use Eucमld in lts 'pure' form since the work by Legenure: 'Elements de Geometrie' in 1794, had simplifiea and rearranged the propositions to create a more effective textbook. A Scottish Commission of inquiry
by iir. Fearon (1866) also found that geometry in Scottish schools was far more practıcally blassed and included mensuration. This pre-occupation with geometry seemed worth considering because it brought about the first mild reform movement in mathematics teaching a few years later in 1871. The teaching of arithmetic in some endowed schools was described as 'clumsy and unscientıfic' (ref: 25, p.134) and the algebra standard rarely reached quadratic equations (ref: 25 p.137). In one district only 12 boys out of 109 had studied plane trigonometry and only a small mınority had been over the six books of Euclid (ref: 25, p.136). In the private schools the amount of time given to arithmetic fared better than the grammar schools and as one witness pointed out (ref: 25, p.286): "rhe great stress was lald on the arithmetic and one third of the time was glven to it. In all private schools arithmetic appears to be, if not really, yet professedly, the leading study". The Commissioners remarked on the fact that mathematıcs was neglected, implying that geometry and algebra were hardly taught at all. The mechanical teaching of arithmetic rules was described at a private school at Gateshead by Mir. Hammond (ref: 25, p.288): "Ihere was no exercise of thought or reflection in the process; all was effected by mere strength of memory......The application of rules and processes was instantaneous......the instruction in arıthmetic was oral........probably not a single princıple was understood". The master apparently did not approve the system, but explained that the merchants on the quayside required it and preferred boys in their offices from schools where 'ciphering books were in vogue'. Although the grammar school boys may have shown more command of mental abılıties the courses of commerclal arithmetic gave the private school boy an advantage in
obtainıng employment and this was what many parents required. There is to some extent a similar situation existing today With employers and parents requiring evidence of a higher level of numerıcal abılıty in children when they leave school. The Gateshead School went so far as to exclude history, geography, mathematics and languages in its narrow curriculum. An important point was that in most private schools Latin, If lt was laught at all, was subordinate to the commercial modern course subjects. In general, the private schools catered for the second and third grade type of education and were regarded by the Commissioners as the poorest schools in relatıonship to the qualıty of education. On the other hand, the first grade proprietary schools gave a classlcal education of the highest standard but allowed more time for the study of mathematics and modern languages. Such schools the report stated: "commenced with the establishment of Universitj College and King's College' London (ref: 25, p.310). Later colleges such as Cheltenham and ilarlborough became a part of this system and these have been dealt with previously. However, some of the second and third grade proprietary schools were established for a less wealthy section of the community. Second grade for farmers, for example, and third grade for clerks, small shopkeepers and upper artızans. They often arose out of a Mechanics Instututes or denominational schools set up by dissenting clergy. The report came down in 1 avour overall on the attempt by these schools to give a wide curriculum and stated: "The books, too, and methods used in these schools are better than in the endowed schools of the third grade and so is the teachıng". Mathematics as a complete suoject was taught,
and not just arithmetic, and many of the upper boys gained places at Oxford or Cambrıdge and the University of London, as shown by the examination lists in the report, Appendix VII, Table IX (Oxford) (P. 172 ) and Table XIII (Cambridge) (p.1/6).(Ref. 25).

Perhaps an unfortunate development that emerged was the establishment of the three grades of schools which served as a basis tor later systoms of education.

Before looking at the crisis which was reached about the 1870's it wall be necessary to Look brıefly at another development which was evolving within the educational system - that Is the education of girls. In the elementary schools described in the Newcastle Commission (ref: 18) the education avallable was for boys and girls mainly up to 12 years of age in the labouring classes. A土ter that many left to become employed in factories or become apprentices unless they continued to a higher level of education. In Encrland, as in many other countries of Western Europe, girls of the upper and middle classes were mannly educated privately up to about 1845 and the traditional education consisted of forelgn languages and 'accomplishments', such as music, needlework and conversation. An abılity to provide dıvertisement and amusement by solving mathematical problems of, at first, an elementary kind may have been regarded as an 'accomplishment' for girls. R.C. Archibald (ref: 26) in his article 'Notes on some Minor Englısh Mathematical Serıals' noted that the 'Lady's Dıary' started in London in 1704 was described as being 'designed for the sole use of the female sex'. It contained mathematical problems as well as poems and other articles and appeared under various names and forms for the next 168 years. This could be considered as a form of in-
filtration of mathematics into the luves of women in the upper and middle classes even if it was not accepted explicatly.

The limited number of endowed schools for girls (20 only are given in ref: 25 p .565 ) meant that the middle classes had little cholce for obtaining a classical grammar school education for their girls even if they desired it. As the Iaunton Commission pointed out (ref: 25, p.546) the middle class parents were indifferent to a wider education for girls and required "what is showy and superficially attractive". Thus, arithmetic and mathematics were taught, but in a very unsatisfactory manner in most cases and the results for glrls, therefore, were often inferior to those of the boys. One watness, in comparing boys and girls, found the latter "not so quick and accurate in arıthmetic, algebra and Euclıd" (ref: 25, p.549). The report also commented that "Arithmetic is spoken of as the "weak point" in women teachers". However, this may hive been the method of teaching or the attitude engendered by the teachers tremselves; for instance, in the garl's department of the Liverpool Ins ii tute arithmetic was one of the branches in which 'hıgh excellence' was obtained and at the Berwick Corporation Academy where urls and boys were taught by masters the ;urls proved themselves to be superior to the boys in the examinations (ref: 25, p. 550 footnote 1). Gosden (ref: 16, p.150) gives an extract from an arıthmetic book especially written for girls described as "The Young Ladıes' New Guide to Arithmetic" by John Grieg, London 1835, which would probably have been used at many of the girls private schools. The book was nothing special and based on commercial arıthmetic.

The movement for the higher education of women began with an attempt to provide training for women who antended to teach. Governesses were granted certificates in 1846 by the Governesses' Benevolent Association. This lead to the foundation of Queen's College London (1848), the London Collegiate School (1850), Bedford College, London (1849) The Cheltenham College of Young Ladies (1853) and later Girton College (1873). Only by 1880 though did London Unıversity admit women to their degree courses. The curriculum in the girls' schools used the traditional model of the boys' schools as a basis except that there was less emphasis on the Classics, but music, needlework and dancing were included. A further influence on the curriculum was the permanent acceptance of girls to take the Cambridge local examinations in l868, whıch gave a guıde to possible study. For many parents and supporters of women's education, examinations provided both motivatıon and justification $10 r$ girls to study and for them to extend thelr education both in quality and quantity. The Schools Inquiry Commission (1868) recommended for girls' schools slmılar subjects as in boys schools and also mathematics as an effective means of trainang the mental faculties, (the lıberal education concept) although it was realised that if women ever wished to enter the professions, examinations would have to be passed and an element of vocational training was necessary. An lmmediate effect of the Commission report was the Endowed Schools Act of 1869 which enabled funds of educational trusts to be used for girls' education. Compared with the small number of girls schools in 1868, by 1897 there were 86 endowed schools for girls and 31 endowed schools for boys and girls. The
continuing education of girls was one of the important factors in the development of the educational system during the period from 1870 to the beginning of the 20 th century. With reference to the section on the mathematics text books of this period it ls possible to evaluate the type of mathematıcs that was actually taught. Whereas the social and cultural forces mentioned previously would have exerted an overall influence on the educational system, the text books represented the overt influence on what was used by the teacher in the classroom. To a great extent the text books simply continued the tradition established in the previous century by emphasizing the commercial aspect of arithmetic. The Rev. Joyce's 'Practical Arıthmetic' (see later section) published in 1816 was typical of this era and Francis Walkıngame's 'The Tutor's Assistant', it has been stated, continued publication until 1861.
'The Elements of Arıthmetic' by Augustus de Morgan was pubIlshed in 1830 and was an attempt to give explanations of the rules of arıthmetical operations rather than just the tradıtional approach of detınition, rule, lllustration and example.

It is worth noting that John Bonnycastle, who was Professor of Mathematics at the Royal Milıtary Academy, woolwich in the early part of the 19 th century, produced a series of text books on arithmetac, algebra, geometry, astronomy and trigonometry. Bearing in mand the high standard of entry required for Woolwach his books exhibited this standard, in particular by giving proofs and demonstrations of any formula that had been derlved (see examples in the later section). J. Colenso's books on algebra and arıthmetic published about

1843 became the recognised text books for the public schools as did the 'Euclid's Elements' by R. Potts, first published in 1845 and $I$. Todhunter's in 1862.

A geometry book which attempted to give a practical approach to geometry based on Euclid was wrıtten by George Darley and published in 1844. The vogue tor ciphering books during this period in the National schools and Private schools is illustrated by many of the examples shown in the following section which date from 1806 to 1845. The text books, as can be seen, during the 1870 to 1900 period did not change drastically. Rather, the algebralc text books tended to increase the manipulative content and the arithmetac books increased the number of examples to exagoerated amounts (see Cusack's 'Arıthmetıc' for instance).

One type of text book that did not seem to be written was a suitable 'practical mathematıcs' book in the sense that the pure mathematics was appiled to probiems in technology, engineering and science. This is one aspect that will be looked at in the 101 lowins perıod.

The commencement of this period had already seen the introduction of the Endowed Schools Act in 1869 and its effect on establishing the education of girls in the Grammar schools towards the end of the 19 th century. However, other factors of intluence had evolved and the time was ready for changes to take place in education as a whole and in certain branches of mathematics teaching. Before developing these other factors they may be briefly described as:
(1) The Elementary Education Act of 1870
(2) The interest and awareness in the growth of technical education, and
(3) A reappraisal of the role of Euclid in geometry teaching and the foundation of the Association for the Improvement of Geonetrical Teaching (A.I.G.T.) later to become the Mathematical Association. To recapitulate the state of affarrs: the Schools' Inquiry Commission had defined three grades of schools by leaving age' although these correlated roughly to the type of educa_ tion received by the varıous classes of society which supported the respective schools. The working classes and the lower middle classes would have recelved a mathematical education probably based on commercial arithmetic with perhaps some practical geometry and mensuration for the skilled artızans. Book-keeping and accounts sultable for clerks and the use of cipherıns books would have been popular as vocational demands by the parents were beang met. A large proportion of the middle class, whilst supporting the classics, still wished for a practical mathematıcs, mercantile arithmetic, to be the 'bread and butter' of mathematical
education, although some geometry, algebra and trigonometry was attempted, although geometry often meant pure Euclid. The upper classes, at the public schools, would have recerved arithmetic which enabled them to cope with commercial applications if required, some algebra and the books of Euclid which occupied an important place in developing mental reasoning. Those entering the military schools would havc recelved by today's standards the highest mathematical education possible including the calculus, but these were the exception rather than the rule.

In this development there seemed to be a gap left by the lack of mathematically trained engıneers, both at the higher abılıty level and at the lower level. The apprenticeshıp system of training (often lasting for seven years), the 'rule of thumb' methods used by some englneers and skilled workers plus the lack of a sound elementary education beyond 12 or 13 years of age meant that few skilled workers would have had any basic mathematical trainıng in their particular field. Some text books had earlıer establıshed the arithmetic suitable for tradesmen (see later Willam Hawney's 'The Complete Measurer'; De Morgan (ref: 4) gives 1717 as the first edition) but it was through milıtary training that mathematıcs was first 'applıed' and later required in the training of the 'physicist' (this word not being defined as it ls now defined). Neither of the latter vocations were possible for the majority of the working class and the relevant mathematics was not therefore required. The majorıty of public schools and the upper classes had concentrated on meeting the entry requirements of the universitles and the grammar schools had done likewise, whılst the private schools
had mannly developed along the commercial arıthmetic route for the middle class who were that much poorer. Thus, the three points previously mentioned tended to associate with the various schools and classes; the geometry reforms were levelled at the grammar, public schools and the universities although the need for hagher 'technical' educatıon was recognised as well due to external factors hlghlıghting the lssue. The elementary education Act and the need for an extension of technical education applied mainly to the lower classes. First, the problem of the need for geometry reform had been recognised by some of the leading mathematics teachers at the public and grammar schools. The Association for the Improvement of Geometry Teaching (A.I.G.'I.) was founded by teachers dissatısłled with the way in whlch geometry was being taught and was dictated by the use of Euclid's Elements. The founder members included, amongst others, the Rev. J.M. Wilson, a science master at Rugby (1859) and senior mathematical master in 1868. (see Ref:23), R.B. Haywood, mathematics master at Harrow in the 1870's and Rawdon Levett, Chief Mathematical Master at Kıng Edward's School, Blrmıngham (1869-1903). An interesting letter by Rawdon Levett to 'Nature' (26th May 1870) is reproduced in 'Mathematıcs: Society and Curricula' by Griffiths and Howson (ref: 10 p.128). In that letter he appealed for the organisation of an 'Anti-Euclid Association' and asked for the 'rank and file' to co-operate and form such an association. The early history of the Association was given by Canon J.i. Wilson in the lifathematical Gazette (ref:23); other writers referred to previously, i.e. Robson (ref: 9 ), Bushell (ref:7), Godfrey (ref: 20) and Siddons (ref:24) all mention
the foundation of the A.I.G.T. in themr articles in other editions of the Mathematical Gazette. A special centenary edition of this publication Volume LV, No.392, March 1971, also gives details of the history as does the article by A.W. Siddons 'The A.I.G.T. and the H.A.' published in No. 300, July 1948.

The Public Schools Commission report (ref: 21 ) had made it plain that boys might have worked for years at Euclid, knew Euclid perfectly, but knew next to nothing of the method or the results of geometry. A better method was required for teaching geometry. Augustus de IIorgan had criticised Euclid as a text book of geometry in the 1849 ' Companion to the British Almanack', although later he came to the defence of Euclid as a system of geometry when ililson brought out his own book on geometry.

Dr. Temple who had been a leading member of the Covmission and was head master of Rugby asked Wilson to prepare a text book on geometry more surtable than Euclid, which he did in 1868. Wilson had explained to Dr. Pemple that in Euclid no hypothetical constructions were allowed; which meant that It could not be assumed that a line had a mıdde point unless by using a stralght edge and compass it could be proved as such. WIlson glves detalls of the features of this nev book (ref:23) which was the first challenge to Euclid as a system of 'pure' geometry and these teatures were rezterated at the foundation meeting of the A.I.G.T. held at a Conference on the 17 th January 1871. Resolution 6 stated the $\pm 011$ wing: "........in any new text book -
(a) the following princlples, only partially or not at all recognised by Euclid, should be adopted:-
(i) Hypothetical constructions
(11) The arıthmetical definition of proportion (11i) Superposition
(Iv) The conception of a moving point and of a revolving line
(b) the following lamıtations should be removed:
(1) The restriction of the number of axioms to those which admat oir no proof
(11) The restriction which excludes all angles not less than two right angles.
(c) modern terms, such as locus, projection etc., should be introduced".

These points were taken up later by the Committee of the A.I.G.T. and expanded further (see Siddons, Ref: 24, pp.16-17). In conjunction wath other members, ancluding W.Done Bushell, F.S. Marshall and J.F. Moulton, a serles of syllabuses were drawn up and one was published in 1875 after about four years work. From present day hindsight this speed of reform and the content of reform does not seem great. It should be remembered, though, that educational change during that period met with considerable resistance. Sıddons reported (ref: 24) "the question in Geometry proposed at the Matriculation Examination of the Unıversity of London in June 1876, deviated from the old Euclidean type so far as to provoke a spirited controversy in the columns of the 'Times'".

It has to be appreciated that geometry examınation papers consisted of questions mainly asking for exact reproductions of

Euclıd's propositıons and very few riders.
In 1874 in the pass part of the Oxford and Cambridge Joint Board's Certificate there was not a sıngle geometrical exer-
case. The A.I.G.T. published a text book in 1884 and had previously set up in 1881 a committee to report on Solid Geometry, Higher Plane Geometry and Geometrical Conics. The syllabus produced in 1875 had produced strong opposition from no less than De Morgan, Lewis Caroll (C.L.Dodgson) and I. Podhunter, the latier having published his own 'Euclid's Elements' (see later section) in 1862 which was a popular text book in the public schools. In 1887 the Association was stıll trying to persuade the universities to remove the restrictions of Euclid in the entrance and pass examinations. Siddons (ref:24) refers to a meeting of the Mathematical Board at Cambridge in 1887 when Professor Cayley stated: "the proper way to learn geometry is to start with ' $n$ ' dimensions and then come down to 2 and 3 dimensions". Cayley was also an ardent admirer of Euclid and at another meeting in 1889 he even suggested striking out Simson's additions to Euclid and keeping strictly to the original treatise. Sldaons (ref:24) also pointed out that the syllabus and the text book produced by the A.I.G.T. were more rigorous and sound than Euclid but it was not a prelimınary study of geometry and perhaps it was also because of this that it did not produce the required change quickly enough in teaching the subject. Siddons started teaching at Harrow in 1899; he states (ref:24, p.19): "I had no special dırections about what algebra and arıthmetic I should do but I was told that at half term the boys would have a paper on Euclid, Book III, the paper consisting of entirely propositions". The criticlsms of Euclid's Elements as a course an pure geometry by the A.I.G.T. were most likely almed at two points; these were (1) It was unsultable for
the majority of pupils as a singular approach to geometry, and (11) it tended to force out a whole host of other important branches of mathematics (trigonometry was only taught to a small minorıty). The text book by Todhunter previously mentıoned for example was in its l8th edıtion in 1887. It was an extremely competent book with an historical account of Euclid's Elements and varıous translations, notes on Euclid glven on which the book was based (pp.250-25l). An overview of the 'Elements' for each book 1 to 6 and 11 and 12 was also given along with explanatory notes and alternatıve proofs, corrections and references to sources of research papers on pure geometry (p.252-291). The photocopy in the next section also shows some exercises taken from school and unıversity papers included in the book; they are, in general, problems based on the stated books and propositions, which tends to dispute the claim that all the work was solely the repetition of the propositions. The preface of the book though was totally commıtted in its support for Euclid; it stated: "the habits of mind which the study of pure geometry tends to form, furnish an advantageous corrective for some of the evils resultang from an exclusive devotion to Analysis". The A.I.G.T. had wldened its basls to include all branches of elementary mathematıcs by 1881 and in April 1894 publlshed the first copy of "The Mathematical Gazette. A terminal journal 1 or students and teachers". The Gazette presented articles from a wlde range of material, but did not necessarıly concentrate on applications of teachıng.

In 1897 the A.I.G.I. changed its name to the 'Mathematical Association' (which it has remained ever since) and continued to play an important role in developing the course of mathematıcal education and teaching.

At the turn of the century, in September 1901, the British Assoclation met at Glasgow and Professor John Perry read a paper condemning most of the teacıing in schools; this will be considered in more detall later. ' In January 1902 the Mathematıcal Associalion appointed Its fırst Teaching Committee which included recent members of the A.I.G.T and in May 1902 It came to the conclusion that It was thought hopeless to try and abolısh Euclid as an examınation text book. This was to some extent a disappointment but it was proposed that changes could be made without violating Euclid's order and it was hoped that this would be accepted by the Unıversitıes. However, at a meeting of a Syndicate at Cambridge in May 1903 to consider mathematıcs in the pass examination, Professor Forsyth expressed the opinion that here was an opportunity to remove the bondage of Euclid completely, (ref: $24, \mathrm{p} .20$ ) and a schedule was drawn up which included the following statement: "Any proof of a Proposition shall be accepted which appears to the Examiners to form part of a systematac treatment of the subject". The Senate accepted the report of the Syndicate and the first examinations were held at Cambridge under the new regulations in March 1904; As Siddon's put it: 'Euclıd as a text book ceased to dominate English education' (ref:24).

In 1903 the Uxiora local examinations, the Civil Service Commission and Oxford Unıversity had glven the same Ireedom to candıates and once the unlversıtıes had nade this decision schools were free to adopt any system of geometry. At tirst there was little departure from Euclia and a number of reports were published Later by the Board of Education (H. M.S.O. 1y09 and 1914) and by the Nathematical Assoclatıon (1923), giving
further recommendations about the teaching of geometry. As Iar as geometry was concerned some measure of retiorm had been achleved and more important the Mathematical Association had been firmly establıshed in the last 30 years of the I9th century. It should be remembered, though, that the other branches of mathematics had been developing in the unlversities quate fully during the l9th century in both the old and new establishments. (For detalls of examinations 'Mathematical Examinations at Oxford' by A.I. Dixon in Vol. 27, part II of Board of Educatron Reports - "The Ieaching of Mathematics in the Unıted Kingdom', H.H.S.O. London 1912, where a lıst of examination subjects 15 given for 1852, 1877, 1884, 1904 and 1913).

Two new universities (called 'red brick'), Manchester and Birmingham, gained their Charters in 1880 and 1900 respectively and whilst not offering specialised courses as in the present day definition they were alıgned to varıous fields of study. Hollingdale (ref: 8 ) noted that in 1900 a mathematical specialist at Birmingham spent less than half his time on mathematics. The emphasis that had been placed on geometry had been established by tradition in the public schools and grammar schools through to the universities as a part of a 'code of behaviour' belonging to the liberal education concept, and a training for moral, physical and mental discipline, (note the way that rodhunter referred to the 'evils' resulting from a study of Analysis). Even earlier in 1837 Walliam Whewell in his work 'On the Prınciples of English Unıversity Education' (ref: 27, pp. 128-129) whilst recognusing the importance of mathematics as an intellectual discipline stated "we cannot give to our mathematical
studies their true dignity, without showing the place they hold in the progress of science"; he then qualified his views later by statıng: "practıcal knowledge such as cıvil engineering, practical arts and trades, and the like....... can never stand in the place of a really liberal education". Hence, pure mathematics in general was still regarded as belonging to this concept and applications to engineering and scientific studies during this important partof Engirsh history, the Industrial Revolution, would be.looked upon with disrespect.

Bearing this in mand, the other two points raised, that of the 1870 Elementary Education Act and the need for an increase in technical education can be looked at, and the ultimate effect they had on mathematics teaching. Mathematical education during, and prior to, this period did not develop solely in isolation, but was often linked to a scientific and technical education. It is hoped to link this development with the overall educational development during this period.

To set the scene, the Great Exhibition at Crystal Palace in 1851 had demonstrated a certain superiority in British industry over the Continent. However, there were people who thought that such a state of affairs would not last, particularly as theoretical knowledge in engineering was not being developed in England. For instance, Willam Fairbairn In a lecture to the members of the Manchester Mechanics ${ }^{1}$ Institution in Narch 1852 (ref:27, pp.262-266) stated:
"How very few of our best practitıoners in Architecture, Clvil and Mechanıcal Engıneerıng are acquainted with the rudiments, or even with the simplest theoretical rules of their professions". He also advocated a better educatıon for the skilled worker in industry; he stated"we must rear for future service a more intellisent and better educated class of foreman, managers and workmen........offer them, at a cheap rate, such rudimentary and theoretical knowledge as wall qualıfy them for the due performance of themr duties". Unfortunately, the elementary education recelved by the working class of society in the Natıonal schools was not high enough for advantage to be taken of this and many of the middle class were recelving, as has been seen, an educatıon steeped in classics at the grammar schools or heavily blassed towards commercial arıthmetic. The recognition of the need for a sound mathematical training for engineers was realısed by 'T.J.M. Rankıne in his work "A Manual of Applled Mechanics" in 1858 (ref: 27, pp.266-271). He wrote "In the origınal discovery of a proposition of practical utality, by deduction from general princlples and from experımental data, a complex algebraical investigation $1 s$ often not merely useful, but indispensable,...........the more thoroughly a scientıfic man has studled the higher mathematics..........the better qualıfied does he become to free the exposition and application of scientifac principles from mathematical intricacy". Although Rankine was thinking of unıversity level mathematıcs the study was necessary raght down the line to the elementary level.

Further pressure for change came from ihe effect of the Paris Exhibition in 1867; the Continental industries had lmproved greatly compared to England and this was partially attributed to new skills and advances in technology and a superior education in this field. In a letter to the Committee of Council for Education in 1867 by B. Samuelson, M.P., concerning the technical education in countries abroad the work of tho Polytechnic School of Stuttgart was mentioned (ref: 27,pp. 137-140). He noted that the school consisted of a (first) mathematıcal division, with two divisıons including a mercantile class and a (second) technical dıvision. The age of admissi on to the mathematical division was 16 years minimum and the entrance examination required a knowledge of quadratics, the use of logarıthms,wath plane trigonometry. He later stated: "It wlll not be supposed that mathematics ends ith the mathematical division......the study of pure mathematics occuples a prominent position in every sub-division". Although it could be argued that this type of education was perhaps too vocational the scholars were also taught French, essay writing and history as well as the practical side of thelr technıcal subject, l.e. architecture, clvil or mechanical engineering and chemical technology. Comparın'f this with the Endowed Grammar School achievements at 16 years of age which had been seen by the Taunton Commission it must have made people realise why England had not advanced in technology. Samuelson was then on a select committee in 1868 appointed to inquire into scientific instruction and which later led
to the appointment of the Royal Commission on Scientific Instruction (the 'Devonshire Commission') from 1870-75 for the same purpose. The development of education during the period as a whole is described in more detall in Curtis (ref:6, Chapters VIII and IX pp. 272-321). Gosden gives several references to primary sources (ref: 16, pp.33-142), whilst both the Hadow Report (ref: 15a, pp.14-27) and the Spens Report (rof: l5b, pp.47-56) gave useful summaries appertaining to elementary and secondary education, the latter giving detalls of technical educ ${ }^{\text {tion. The }}$. Tevelopment of technıcal education is also descrıbed by P. I.Musgrave In two artıcles: "The Definıtion of Technical Education 18601910" and "Constant Factors in the Demand for Technical Education, 1860-1960". (See ref: 31 p. 65 and p.143). A very detalled discussion of one area is glven in 'Education in Leicestershıre 1540-1940' (ref: 32 pp.156-194). The 1870 Elementary Education Act (Forster's Education Act) created local School Boards to fill the 'gaps' in the voluntary system and establish compulsory attendance. This was not really enforced and exceptions were included and not until 1876 (Sandon's Act) and 1880 (fiundella's Act) was the mınımum leaving age established at 10 years old. Fees were stıll payable but the Boards paid for the really poor. The effect of the Acts was to cause 'Tops' to the elementary schools and as far as mathematical teaching was concerned the c'illdren who voluntarily stayed on beyond the terminal age recelved instruction to a higher grade.

The Hadow Report (ref: l5a, p.15) indicated Lancaster Natıonal School and Oswestry National School as two such schools; the former offered a great deal of mathematics whilst the latter ofıered algebra, practıcal mensuration, geometrical drawing and mechanıcs. However, it seems significant that the boys who left at 15 or 16 years of age usually went ınto clerkshıps and merchants' offıces where only commercial arithmetic would have been required. Many elementary schools in the 1870 to 1880 period had * pupilsvabove 13 years of age; the Education Department then created a Standard VII of the Revised Code in 1882 to cope with these pupils. Eventually these 'higher grade schools' were teaching not only arithmetic, but algebra, plane geometry, projections and mechanics. The Department of Science and Art had been founded in 1853 at South Kensington by the Government and set examinations and paid grants to institutions that prepared candidates for their examınatıons.Courses offered included mathematics and practacal geometry and some of the endowed schools were offering these courses, and recelving grants up to the 1880 's along with the hosher srade elementary schools. In 1872 the Royal Society of Arts was founded which offered a series of vocational examınations and later introduced the first strıctly technological examinatıons. The Royal Commission on Sclentafic Instruction from 1870-75 found that it was theoretical instruction that was required, not practical instruction in technıcal subjects; at this tıme
scientific instruction and technological education were synonomous, the latter phrase not being used untıl later. Other institutions such as the City and Guilds of London Institute founded in 1880 offered simılar courses designed to bridge the gap between formal school work and the practical/theoretical needs of industry.

John Perry, who was later Professor of Mechanics and IIathematics at the Royal College of Science London was giving lectures on 'Practical Mathematıcs' at City Guilds Technical College Finsbury, London, in 1881; these lectures and others were incorporated into a Board of Education Report in 1899 entitled 'Summary of Lectures on Practical Matheratics' and wese a model for other text books on practical mathematıcs, includıng Perry's own book 'Practical Hathematics' published in 1899.

In 1882 the Regent Street Polytecharc was founded and admitted students from 16 years of age; Gosden (ref: 16, p.137) glves a lıst of courses that were studred which included Bracklayıng, Electrical Engineering, Plumbing and Carrıage Buılding, all of which included some form of mathematics, e.s. practical, plane and solıd geometry, algebra an mechanlcs.

A Royal Commission on Technlcal Instruction was appointed and the results publashed in 1884; the Report recognised the difference between an elementary education and a postelementary education, encouragang the enrichment of the curriculum as it vas taking place in the higher grade
elementary schools. It also commented upon the use of instruction in practical geometry to chilaren whose occupations were to be artisans or workmen. 1 his followed on from the reforms suggested in geometry teachIng; unfortunately it tended to produce a stereotyped form of constructional geometry with pupıls simply learnlng how to use ruler and compasses. The use of concrete operations to introduce the concepts of formal geometry seemed to have been forgotuen and many unsatisfactory text books were produced with numerous rules of constructions. An example shown in the neat section is Rawle's 'Practical Geometry' published in 1888 which in its own way was an excellent book of constructions. The frontisplece of that book included a reference to its being suitable for the Science and Art Departments examination in practical geometry.

Arıthmetıc was still based on rules and mechanical repetıtıons; the examples were often very difficult and numerous; typical were such questions as the following:

1. Multıply 4 tons $12 \mathrm{cwts}$.l qr. 10 lbs . by 573.
2. Express in Its lowest terms $\frac{114977}{220501 .}$
3. Express 2.7683 tons in tons, cwts, qrs and lbs. 4. Find the value of $365 \frac{1}{4}$ articles at $\& 4$. 7s. $9 \frac{1}{4} \mathrm{~d}$. 5. If 160 men in $12 \frac{1}{2}$ days of 11 hrs. each can dig a trench 230 yds long, $5 \frac{1}{2}$ ydis. wlde and $1 \frac{1}{2}$ yds.deep, In how many days of 8 hrs each can 96 men dig a trench 207 yds long by $3 \frac{1}{2}$ yds. by 1 yd ?

Sometimes as many as 500 questions in different sets were given on a partıcular toplc, vulgar fractions tor instance; some examples are shown in the next section. Algebra was just as manıpulatıve; one book, Hall and Knlght's 'Elementary Algebra (1885) is shown later as a
 lengthy questions that occupled time rather than teachins now concepts. Many of the examples may have been satisfactory, but it vas the excesslve number and complications that would have been somewhat a waste of trme. Since many of the teachers of mathematics were probably not mathematıcs 'specialists' a poor text book could have been used in conjunction with a poor teacher and the effect reinforced.
$3 y$ the 1880's elementary education had been established and many chıldren were receiving a sound basic educatıon in reading, wrıting and arıthmetıc. In 1888 the final report of the Commissioners appointed to inquire into the workings of the Elementary Eduçtion Act was published (the Cross Commission). They observed the need for a wider curriculum than Just the $S$ R's and suggested that it should be lıberalısed by the addıtıon of other subjects including science and technical instruction. They also suggested that new County Councils should be responsible for the planning and building of new schools. The higher grade elementary schools had stabilised themselves and later developea into 'Senior Standard Schools' in the early 1890's, but the Cross Commission wished for Endowed Schools to be encouraged rather than the former.

The line between what was 'elementary education' and 'post elementary' had become rather ill-defined now, particularly as many middle class and prosperous working class parents were taking advantage of the hlgher grade schools and the education that they offered (see Hadow Report p.22). I'he schools were still fee paying at this tame (until the 1891 Elementary Education Act introduced free education) and certainly the endowed schools would have been more expensıve and, to many people, stall not offering a 'useful' education for obtaining employment afterwards. The aftermath of the Royal Conmission in 1884 was the Technical Instruction Act l889; this empowered local bodies, the new Counclls, to aid the supply of technical and manual instructıon in schools and institutes. Endowed Schools (grammar schools) recelved this ald as well; many had been reorganised after the Endowed Schools Act of 1869 and science and modern languages were included in the curraculum; the Act could not be fully amplemented though because of lack of funds. The wide range of schools and the variation of standards called for a new look at the organisation of what was now called secondary education. In 1894 a Royal Commission on Secondary Education was appointed and its report was published in 1895 (usually called the Bryce Commission) (ref: 28). The report stated "the boundary line whlch divides secondary from elementary education is not easy to draw in the abstract and in the concrete can hardly be said to exist" (ref:28 p.2). The Bryce Commission was more concerned with organisation and administration, rather than with the actual
teaching content and methods. They recommended that one central authority should supervise all of secondary educatıon whilst local authorıties should be responsible for secondary and technical education within their respectrve areas. The existing authorıties, the Education Department, the Sclence and Art Department and the educational functions of the Charlty Commissions, would all be merged in the new central authority.

Following on from the three grade classıficatıon system defined by the Schools Inquiry Commission 1868 (ref: 25) the Bryce Commission also noted that "The rapid growth and success of higher grade board schools, especially in great
 secondary education at a cheap rate" (ref: 28, p.79). They consolıdated the three grade system and 'First Grade' schools were essentially those which fostered 'a learned or literary, professional or cultured class' (ref: 28, p.138). The Report acknowledged that mathematıcs, whilst more allied to scientific subjects, ought to enter into a literary course. The 'Second Grade' schools were more concerned wath the mercantile side, busıness, arıthmetic, modern languages or concerned with industrial sciences. The 'Thırd Grade' were for hıgher handıcrafts and the commerce of the shop or town. Technical education in the secondary schools would serve as a broad preparation for the upper levels of the labour force in order to face the competition in andustry from the Continent. This
reorganisation was reflected in the mathematics text books and in the curriculum for a partıcular school which was determaned by its grade. The Bryce Commission reported on the curricula of various schools in the different grades. These indicate the proportion of time allotted to mathematics and other subjects at that tame and some are recorded below; (from Ref: 29, Vol. IX, Appendix, Statıstıcal tables pp. 404-423, Bryce Commission). Listed across the page are Hours of Instruction per Subject.

Order of Subjects Latin / Greek /Arıthmetıc/Mathematics
(A) Ist grade endowed.
(1) Rugby

| Classical | I | 6 | 8 | - | 4 |
| :--- | :--- | :--- | :--- | :--- | ---: |
| Specralısts |  | 2 | 2 | - | 14 |

(2) Manchester Grammar

| Classical Ia | 8 | 8 | - | 1 |
| :--- | :--- | :--- | :--- | ---: |
| Specralıst | 3 | 2 | - | 13 |

(3) Kıng Edward Grammar Blrmingham. Classical I $5 \quad 5 \quad$ - $\quad 6$
(B) Ist grade proprietary

Liverpool College (Upper)

| Classical | II | 8 | 6 | - | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Modern | II | 4 | 4 | - | 7 |

(C) 2nd grade enaowed

Bedford Modern

| Form II | 4 | - | 2 | 4 |
| :--- | :--- | :--- | :--- | :--- |
| Commercial II | - | - | 4 | 4 |

(D) 2nd grade proprletary

Leeds Liodern
Form I 2 - - 9
(E) 3rd grade endowed

Wellingborough Grammar
Form I $4 \frac{1}{2}$ - $5 \quad l_{2}^{1}$
(F) 3rd Grade

Leeds, Central Higher grade Board School (Boys' Dept.)
4 th year pupils 5 - 7
Some girls' schools were also Listed.
(G) Ist grade endowed.

Bedford High School tor Girls
Form I 4 2 - $4 \frac{1}{2}$
(H) Ist grade proprietary

Cheltenham Ladies College
Form I 3 l - 6 $\frac{1}{2}$
(I) 2nd grade endowed

Camden School for Girls N.W.
Form I - - $3 \frac{1}{2}$ -
(J) 3rd grade

Ieeds Central Higher grade Board School (Girls Dept.)
4 th year 5 - - 6
(K) Prıvate school.

Miss Metcalie, Hıghfield, London
Form 1 - - $1 \frac{1}{2}$ -

It can be seen that mathematics was placed on an equal footing with the classics in all but the special forms and that arıthmetıc was taught in the lover grade schools. Leeds Central School was an exceptıonally good higher grade school but stıll modelled itself on the endowed grammar schools and included 5 hours Latin a week.

The text books prior to $18 y 5$ (approximately) have reterences In the pretace to 'for the use of schools', whereas most of those aiter 1895 refer to 'secondary schools'. One series 'Gill's Algebra' produced two versions: an 'Imperial A」gebra' (1888) for 'mıadle class schools' and 'Algebra' for 'Secondary and Elementary Schools' (clrca 1895). Whe book 'Introduction to Algebra' by G. Chrystal, first published in 1898 has in its frontisplece 'for the use of secondary schools and technical colleges'. It stated in the preface that it was intended as an introduction to his larger text book on algebra used in the highest classes in the endowed schools and he identıfıed himself "with the reforming party of mathematical teachers, whether academic or (I suppose I must say) technical". Notıce now that a division had been implied between the two types of teacher as well as the type of material used in the text books. McDougall's 'Complete Arıthmetıc' (circa 1897) was another commercial arithmetic book that was described as sultable for secondary schools. Extracts from these books are shown in the later section.

Certanly the three grade classıfıcation would mean that the commercial arithmetic aspect of mathematics teaching would continue and unfortunately 'technical' mathematics
would have been denigrated to the second and third grade secondary technical schools whlch were of lower status than the secondary grammar schools. Musgrave (ref: 31) recounts the dılemma that technıcal education was presenting to some employers; many of them preferred to have their skllled workers advance vaa the shop floor training system rather than have 'bookish, college trained lads' in their employment. The older unlversities were wary of developing 'applıed' courses without the liberal education concept. The opportunity of integrating a 'technical education' into the classical system may have been lost at thls point In time by the three grade classificatıon which tended to perpetrate a divisive educational system. The 1897 code abolıshed the 'payments by results' system of $g$ rants and the 1899 Education Act now set up a national Board of Education as recommended by the Bryce Commission in 1895 , which would supervise all elementary and secondery education.

Although beyond the scope of thas study the Board of Education publıshed $1 n 1912$ a serıes of 'Special Reports on Education Subjects'; Vol. 26 and 27 of the reports were parts $I$ and II respectively of "The Teaching of IVathematics in the United Kingdom". Of relevance here is the photocopy shown on the following page taken from Vol. 26 Which shows the arithmetic standards from I to VII for the years 1894 to 1905 which would have been used in the elementary schools. It ls, as can be seen, extremely blassed

26 Teaching of Mathematzes in London Elementary Schools.


APPENDIT I.

# Board of Edccation. <br> Standards of Examination in Arithmetic. 

## Scheme D.*

Siandard $I$.
The four simple rules Divisors anl multiphers not exceeding 6 No number higher than 99 to be exployed in the questions or requured in the answers.

## Standard II.

Compound rules (money) Dirisors and multipliers not exceeding 12. Sums of money in the questrons and answers not to exceed $10 l$

## Standard III.

Simple rules and compound rules (money). Dirisors and multipliers not esceedng 99 No number lugher than 99,929 to be employed in the question on required in the answer. Sums of money in the questions and answers not to exceed $99 l$.

## Standard IV

Compound rules applied to the followng weigints and measures (leugth, weight, capacity, time) In length, sards, feet, and inches, in weight, tons, cwts, qrs, Ibs , ozs, in caracty. צaluns , quarts, pints; in time, days, hours, munutes, seconds-are the only terms that will be required in this, and in the fifth standard Divisors and multipleers not to exceed 99 .

## Standard $V$.

Vulgar fractions (simple fractions only). Practice Bills of parcels Common weights and measures.

Standard VI.
Decımal fractions (excluding recurring decimala) Simple proportaon or single rule of three by the method cif mity Calculation of simple interest upon a given principal Common werghts and measures. Mensuration of rectangles and rectangular solds, the extraction of square and cube roots is not required (Boys only)

Standard VII
Vulgar and decimal fiactions Arerages anl percentages Investments of savings. Consols

[^5]towards the commercial aspect of arıthmetic.
Although the Mathematical Association had begun a certain measure of reform an important event was the address given by Professor John Perry of the Royal College of Science, London, at the British Asso ciation meeting in Glasgow in September 1901. The subsequent discussion at the meeting included some of the leading mathematicians and teachers at that time. (rof: 30). Included amongst the speakers were Professor Forsyth, Pafessor Silvanus Thompson, Professor Alfred Lodge and Professor $G$. Minchin, whilst wrıtten remarks came from Oliver Heaviside, Lord Kelvan, Professor Horace Lamb and Professor David Eugene Smith, to name but a few. With such a weight of mathematical authority behind it the report of the discussion was able to influence the thinking in mathematical education after this period, although the effect did not happen immediately. On reading parts of the address one notices the aggressive style of delivery, a rather informal approach, which must have been somewhat unusual for that period. Perry began by reminding them that he was about to repeat all he had said in 1880 in a paper read at the Royal Society of Arts. He putforward the case that mathematics had a great utility value and was not just a study of something useless. He listed 8 points why he thought the study of mathematics was useful; of these 8 points only 2 actually refer to the ald given by mathematics to the study of science and engineering. The others, in fact, restate some old ideas; for example, he uses the expressions 'glving mental pleasure',
'producıng logical ways of thınking' and 'convincing man that he is one of hichest beings'. (Thls list, in full, $1 s$ also given by Griffiths and Howson, Ref: 10, p.17). He later pleaded for a more practical character in mathematical teaching; he suggested that the majority of Euclid could be assumed; for a stuay of natural phenomena the results of the greatest mathematicians were needed. He crıtıclsed engıneers who condemned 'all computation more complex than that of the housekeeper'; 'the study of hıgher mathematics' he stated 'has got to be a very useful thing'. Later in the report he gave a course of study in elementary mathematics and advanced mathematics,which was essentially a similar one to a pure mathematics course,but recommendations were included that they should be applied to mensuration problems, mechanics and physics. In the discussion that followed many of the speakers repeated previous arguments; Professor Hudson stated 'the fault in teaching arithmetic is that of not attending to general princıples and teaching instead of partıcular rules'; Professor Everett 'the teaching of geometry has been too pedantic': Professor rhompson did not agree that Euclid should be abolished from the schools; Professor Minchin stated 'There can be no satistactory progress in the teaching of mathematics in this country until Euclid is got rid of'. There were in fact many references to Euclid, both by speakers and in the written replies showing that it was still a major debating point. Another point that was contınually referred to was the examınation system
and how often this inhıbıted the type of mathematics teaching in the schools. The report ended with a reply by Professor Perry based on the criticisms; it Iasted ten points of agreement though and pointed to the desire for an immedate large reform in the teaching of mathematics. Grifiiths and Howson (ref: 10, p.18) suggest that Perry and others were mainly concerned with the mathematics that should have been taught to an intellectual and social $\dot{e} l i t e$ and left in abeyance the question of what mathematics should have been taught to the working classes. It was pointed out by a speaker at the discussion, Mrs. Nathanlel Cohen, that Sır John Gorst in his presidentıal address to the Educational Section of the British Association had ascribed to the unıversities and public schools a large measure of influence in shaping public opinıon on educatıon. She stated "on the unaversities, therefore, lies the onus of reform". (ref: 30, p.25).

In a written statement Professor G.B. Mathews stated "The best hope I can see Ior a real and immeaiate improvement depends upon the action of influential schools". (ref: 30 p.96). Perry though, in his reply, stated "I assert that we want reform of a system......of which examinations are only a part. Is the retorm to come from above or below? In Germany reforms always come from above and from the middle". Grif土Iths and Howson seem to be looking at that earlıer perıod with the hindsight of a class-conscious awareness of a later era; reform from the top would have probably been consldered as the natural order of things. The reforms that Perry envisaged would have been passed down the system and distorted so that the use of 'Practical'
mathematics at this time and in the succeeding two deades became the vogue.

There were numerous text books produced; probably the most famous serıes was by Frank Castle; 'Practıcal Mathematics for Beginners' (first edition 1901), 'Elementary Practical Mathematics' and 'ilorkshop Mathematics' (1900) being three books of this perlod. One example is shown in the next section, stıll 1 n $u s e 40$ years after publication.

As a development in general education an important Act was passed in 1902. The 1902 Education Act (The Balfour Act) gave the Board of Eaucation more power over secondary education and empowered the newly created Part II Local Education Authorıties to ald higher education and provide new secondary schools. The 'higher grade' schools had been virtually pronounced $2 l l e g a l$ by the 'Cockerton ruling' in 1901 and the 1902 Act enabled many of these schools (and Pupil Teacher centres) to be converted into Councll Secondary Schools. This was an important stage in secondary education; these new ifunicipal Secondary Schools, influenced by the tradıtion of the higher srade schools were able to create a more modern curriculum with scientific studies, forelgn languages, mathematics and practical instruction evenly balanced over a four year course up to 14 or 15 years of age.

To summarise; The period 1870 to 1902 was an era of consıderable government intervention $n$ education, particularly with reference to the establishment of the secondary sector. In terms of mathematics teaching there was an attempt to reconcıle two phılosophies; at the higher level one was
the ap proach to mathematics as an abstract reasoning and as a part of a liberal education, whilst the other recognised Its utılıty value and ultamate importance in science and engineering. At the lower level in the schools an awareness that rote learning, either in the form of the rules of commercial arlthmetic or the repetation of Euclid's theorems, was not good mathematics. Rather, mathematics was to be thought of as offering a sense of pleasure in solving problems and at the same time as a useful practical subject in solving practical problems. It is probably true to comment that the integration of these two phılosophies did not take place and 'pure' and 'applıed' mathematics continued as separate entıtıes. Further to this there may have been confusion or misinterpretation of the definition of what 'practical' mathematics meant at secondary school level and at other levels. Did the Perry view really intend at to mean that mathematics should be applied to real problem solving and modelling practical sıtuations ? To many text book writers and probably teachers as well, it seems to have been construed as meaning a physical action supplied to an already established system of pure mathematics. For instance, using a ruler, compasses and protractor to allow concrete proofs of Euclid's theorems, or using squared paper in plotting functions as graphs and physically measuring and calculating areas and volumes. This type of mathematics would then have found outlet via the secondary schools through to the institutes and technical schools whilst a more 'pure' form of reformed mathematics would continue through the endowed schools and through to the unlversıtıes, albelt now in a modified form.

The mathematics text book has been an important factor In the development of the actual content of the teaching at what would be the classroom level. Many would regard this as the all important feature of mathematics teaching: however, the text book is often the last vital lank in the chain of the whole philosophy and concept of mathematics teaching from which it would have evolved. The following list is arranged prımarıly in chronological order, but there are periods of overlap. It $1 s$ hoped that the text books can now be related to the periods previously described to see if they reflect any of the ldeas prevalent at that tame.

It is not intended that the list should be in the form of a strict bıbliography.

The page references refer to photocoples within the main text.

Pages 115-122.

Photocopies taken from "Rara Arıthmetıca" by D.E.Smıth, Ginn and Co. London and Boston 1908.

These show: (1) some of the works of Boethius; in many cases the copres have Arabic numerals inscribed, but the 1300 manuscript shows the original Roman numerils still bein。used. Other pages show his figurate numbers and his treatment of proportion.
(11) The title page of Cuthbert Tonstall's 'De Arte Supputandi Libri Quattuor' 1522.
(iil) The title page and addition by counter reckoning from Robert Recorde's 'The Ground of Artes' 1558.
(1v) Extracts from two early manuscripts of Euclid's Elements.

Page 123.
Frontispiece from Henry Billingsley's 'Euclid' first published in 1570. Photocopy from 'Early Editions of Euclid!s $\operatorname{Elements}$ ' by Charles Thomas-Stanford. Illustrated lionographs ijo. XX, The Blbliographical Socrety, London 1926.



## RARA ARITHMETICA

GINN AND COMPANY PVBLISHERS BOSTON AND LONDON MDCCCCVIII



Patit I. Fkovi i whescripi of puithics, c. 1300

## ANICIUS MANLIUS SEVERINUS BOETHIUS <br> Ed pr rif8 <br> Augsburg, 1488.

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Noyigtixi, and from the later algorismus (p 5). Boethus gave an claborate theory of ratios and devoted much attention to figurate numbers, such as the trangular, square, pentagonal, and cuble (Sce Fig. 13 ) 'The work was the standard $m$ the Church schools thioughout the Middle Ages.


Fig 14. Lisi two likis of hir 1488 boethus

## BOETHIUS

Ed. pi of the Arithmenc, 1488
Vence, 1491-92

Title See $\mathrm{F}_{1} \mathrm{~g}$ I 5
Colophon 'Impreffis venetys per Jounne de Forli-//uio et Gregorium fratics Anno falutis $\mathrm{M}^{\prime} / / \operatorname{cccc}$ lxxxxj de $\mathrm{x} \mid \mathrm{vj}$, menfis Mart1]' (F 352, r)

Onf 256 (220 as numbered in the book), at the end of the Geometry is the following: 'Venctijs Impreffum Boetıj opus $p$ Joäneş 2 Gre//gonū de gregonijs fratres fecticı exitu ad finē vfclz pductü// aceuratıffimeqs̃ emédatū Anno humane reftaurations. // 1492 die 18 Augufti. Auguftino Barbadico Screniff//mo Venctiarum pincipe Rem pu lenc̈te'

Discriptoon F ©, $21.7 \times 32 \mathrm{~cm}$, printed in double columns, each beng $66 \times 24 \mathrm{I} \mathrm{cm} .3 \mathrm{ff}$ unnumb. +345 numb. +1 blank $=349 \mathrm{ff}, 66-70 \mathrm{ll}$ Venice, $1491-92$ (see the colophons).

Edithons This is the caitto pronceps of the works of Bocthius For other editions sec p 27





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Fig. 15. Titie pag.f, $1491-92^{-}$borihils
BOETHIUS. Ed pr of the Arithmetic, 1488 Paris, 1503 See p. 25
Tatle. 'In hoc libro contenta.// Epitome/compendofacis introductio in hbros// Arithmeticos diun Seuerm lBoetij. adiecto fa-//mulari commentario dilucidata // l'axis numerandi certis qubufdam regulis //confticta //Introductio in Geometnam

## RARA ARITHMETICA

(c. iq96), an anonymous trealise entuted 'De arte numerädı cōxd putile icipit feliciter. (Q)uomam rogatus a pluribz compedium or, numerandı ac breuë tractalutu . . .'

Works of 1497. Boethus, p. 27, 1488 (the colophon of the 14ヶ:


Fig 38. Rithmimachia, 1496 boethius
edition has the meorrect date Mcccclisxvin), Suseth, 1497 and 149 S, $\mathrm{P}^{-10,} \mathrm{c}_{-}^{-1}+480$.

Works of 1498 Amanus, p 32, i4 88 , Bradwardin, p 61, 1495 Chiarini, p. 1i, 148x; Anonymous, 'Enchiridion swe tractatus de numeris integris, fractis,' etc, $4^{\circ}$ (doubtless the work published at Deventer in 1499, p. 67).

has been extencled through the seventeenth century London， c 1542 ； b ， $1543,8^{\circ}$ ； $1 \mathrm{~b}, 1549,8^{\circ}$ ； $\mathrm{lb}, \mathrm{I} 55 \mathrm{I}, 8^{\circ}$ ； $\mathrm{ib}, 1556$ ； 1552，London， $1558,8^{\circ}$（here desenibed）， $1 \mathrm{~b}, \mathrm{I} 56 \mathrm{I}, 8^{\circ}$（the earhest seen by De Morcan）； 1570 ；London， $1571, S^{\circ}$ ；ib， 1573 ；ib， 1577；1b 1579， $8^{\circ}$（p 217）；1b，1582， $8^{\circ}$（the Mclis edition）；

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Fig．iu5．Thitr pagi oh hae $155^{8}$ ricorde．
ib， $1586,8^{\circ}, 1 \mathrm{~b}, \mathrm{I} 590,8^{\circ}$（the Dee and Mellis edition）； 1 b ， 1594， $\mathrm{S}^{\circ}$（p．207）；1b， $1596,8^{\circ}(\mathrm{p}) 219$ ）；1b，1618， $8^{\circ} ; 1 \mathrm{~b}, 1623$ ， $8^{\circ}$ ； 1636 ；London， $1646,8^{\circ}$（p 210）；sb．，c． 1646 （p 220）； 1652 ； 1654；I．ondon，1662， $8^{\circ}$（p．220）；1b．，1668， $8^{\circ}$（p．221）；1673； I699 Presumably all of these wete published at London．

## ADDITION．

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Bernardmo Bald testifies to the estecm on which the author of this worh was held，in the following words＇Di patraa Fiorentino far Pawolo il quale，per l＇eccellenza ch＇egll hebbe ne le Matematiche，lacenato il proprio cognome，ful chamato di tuttial Geometra．Come apunto fra gla anticht auenne ad Apollono P＇rgeo＇

This mani x rept is promaniy a treatise on arithmetic．The writur， howeler，left a number of blank pages at thd end，and these have been filled in from ume to time by wartous owners．



Page 125 and reverse.
'Mr. 'Ingate's Arlthmetrck', 'Knowledge and Practice of Common Arıthmetick' by Edmund lingate. Ist edition 1629, London.

The edition, photocoples as shown,ls the 13 th edition, and although no publishıng date is given it was probably publishea before 1720. Jingate's book was virtually rewritten by three writers, J. Kersey, George Shelley and James Dodson (See De Iorgan, Ref: 4, p.(3). This edition of 640 pages was enlarged by John Kersey and the table of contents andicates those chapters altered by Kersey and those composed by Kersey (indicated by an asterisk ${ }^{*}$ )

The Supplement was written by George Shelley a writing master at Christ's Hospital. The photocopy shows the use of the dotfor a decimal point which was, in fact, introduced by Kersey, and not by 'ingate in 1629. The extraction of the cube root by the method indicated shows the laborious mechanical procedure to be carried out as far as the twentreth stage. The book contained the usual four rules, fractions, compound quantities and commercial applications of arithmetic and was highly regarded by teachers. 'the particular edition photocopled had the name and date 'Thomas Giles 1800' entered In the front cover showing 2ts use for over 80 years at least.


M, Book I. 276 VII. Let the whole Rcfivend, except the firit place thercef towards the tight hand (to wit, the place of watts) be efteemed as a Seviden'; then demanding how often the filt figure (towards the let hard) of the Di-

125 vifor is containcel in the cortefpondent pait of the Dividend, and obferving in that behe If the Rules befoct taught in Divifion, write the Aufiver in the Quotient: So if I ask how often 7 ' (the firit figuie of the Divifor towatds the left hand) is contaned in 32 (the corrcfpondent part of the Dividend placed above) the Anfwer will be 4 ; wherefore I write 4 in the Quotient, as you fee in the Example.
XVIII. Having drawn another line under the Work, multiply the triple fquare
$\therefore$ :
$157464 .(54$
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765 Divifor.
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 before fubicribed (as is directed in the fourtenth Rule.) by the figure laft placed in the quotient, and fubfaibe this Produck under the faid triple fquare; (to wit, unts urder units, tens under tens, óc.) So 75 being multiplied by 4, the Product is 300 ; which 1 fubfribe under 75 (the $t_{1}$ iple Square) and the work will ftand as you fee in the Margin.

XIX: Multiply

Chap. XXXIII, The Cube Root. 277
XIX. Multiply the figure laft placed in the quotient frit by it felf, and then the Product by the triple number before fubscribed(as is disected in the I sth Rule of this, Chapter; ) thrs done, fubrcribe the laftProduct under the faid triple number (to wit, units under units, tens under tens, \&c.) fo 4 bcing fquared or muluplied by it
molf, the Piodnct is 16 , which beng multipled by the tiple number 15 , the Product is $24^{\circ}$ this theretorelfubicribe under thed the work will flaud as and fee.
XX. Subrcribe the Cube of the figure lat placed in the quotient under theRefolvend, 10 fuch ifanner that the fir ft place of this Cube (to wit, the place of untes, may fatand under the place of wnits in the Refolvend: So 64 beng the, Cube or 4 , I write it under the Rerolvend $324^{6} 4$, in fueh which is that the figure ' 4 , whe whe in the place of under the figure 64 ,may stand und in the place 4, which is feated inthe plat Ob fervetheWorkin the Margin,


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157+64 (54.
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$32+64$ Kcfolverai.
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figure lat placed

Pages 128 and 129
'Cocker's Arıthmetica' by Edward Cocker 'Perused and published by John Hawkins, Vri ting Inaster, London. The first edıtion was published in 1677 and this photocopy is from the 37 th edition dated 1720. Its popularity may have stemmed from the extremely detalled explanation of every step, although at times the work becomes verbose. Some reminiscent traces of Recorde's dialogue form still persist, whilst the quantıties dealt with at times seem radiculous. The question shown on reduction for instance asks tor 'How many years in 438657540 minutes ?'

The section on subtraction of fractions shows the type of 'rule' arıthmetic that was beang taught.

Pages 130 and 131 and reverse.
'Decımal Arıthmetıck' by Edward Cocker lst edıtion about 1684. The 6th edition 1729 , London 448 pages.

This edition was 'perused, corrected and published' by John Hawkins, Vriting master at Southwark. There were addıtional sectıons on 'Artıfıcial Arıthmetıck' explaining the use of logarithms and 'Algebraical Arıthmetic..' The latter was called the 'mysterious art' and was basea on the algebra of Kersey and others. The photocoples show:
(1) The supposed_trrst_use-of-the-decimal-point compared to the systems of other authors. (p.3).
(2) The extraction of the cube root. Only the rules were given and this shows the $1 /$ th and 18th stages.
(3) The recognition of Oughtred, Jonas Moore and Kersey as the source of Cocker's chapters on algebra.
(4) Examples using the set rules glven in the previous chapter. IJote the very short stages of working that are given. De Morgan ald not rate this book very highly and regarded it as a copy of all the other authors without any improvements. (see ref: 4).



98


Chap. Io.
The Cube Root.
XVII. Multiply the triple Square of the Roor; fubcribed as is before directed in the twelfth Rule of this Chapter, by the Figure laft placed in the Root, and place the Product under the Number laft fubfribed, which is the Product of the fquare of the Figure laft placed in the Root multiplied by the fand triple Root, in fuch manner that the Place of Units of this may fland under the Place of Tens in that; as in this Example, the triple Square of the Root is 108, which multiplied by 4, the Figure laft placed in the Root, the Product is 432 , which I place under 288, the Number lat fubfcribed, in fuch order that the Figure 2, in the place of Units of the faid laft Product, may fand under 8 , which is in the place of Tens of the faid Number laft fubfcribed, and then the Work will ftand as followeth.

262144

262144 (64
216
46144 Refolvend.
18 Triple Root.
108 Triple Square of the Root.
1098 Divifor.
$6_{4}$ Cube of 4.
288 Square of 4 in the tripic Root.
432 . Triple Square of the Root in 4 .
XVIII. Draw another Line under the Work, and add the three Numbers together, that were laft placed under the Divifor, in the fame order as they there ftand, and let their Sum be called the Subtrahend, which let be fubtratted out of the Refolvend, nothing the Remainder; fo in this Example I add 64, 288,432 , together in the fame order as they ftand, and their Sum is 46144, the Subtrahend, which I fubtrait out of 46144 , the Refolvend, and there is nothing remaineth, fo the whole Work is finimed, and I find the Cube Root of 262144 to be $\sigma_{4}$, without any Remainder See the whole Worls as followeth.

342: The Rule of Tbree, \&c. Chap. $力$ :
Term by $\frac{d}{x}$ the fecond Tçrm, the Product will be $\frac{d f}{g}$, which being divided by $\frac{\text { di }+c^{-}}{d}$, the firt Term, the Quotient will be $\frac{d d^{2} f}{\mathrm{~L}_{5}+\mathrm{a}^{c}}$, which is the fourth Proportional fought: For ${ }_{2}$,

$$
\frac{d b+c}{d}: \frac{d}{1}:: \frac{f}{g}: \frac{d f f}{d g_{g}+g^{c}}
$$

I fhall not need here to give any Examples in the inverfe Rule of Proportion in the Algebrack Quantities, the manner of the Operation bcing the fame with the former; only the Proportion !ows backward, as in the Rule of Three Inverfe in Vulgar Arithmetck.

## C H A P. X.

A Collection of fome eafy Queftions
roberein the Rules bitberto deliwherein the Rercifed, taken out of Mr . Oughtred's Clavis Mathematica, Cbap. 11. Sir Jonas More's Arithmetick in Species, Cbap. 10. and Mr. Kerfey's Elements of Algebra, Chap. 10. of the Firft Book.
I. THERE arctwo Quantities or Numbers, leffer ise $(=2)$ What is their Sum? What their Difference? What the Product of their Multiplication ? What the Quotient of the greater dirided by the lefler? What the Quotient of the lef: fer divided by the greater? What the Sum of their Squares? What the Differchce of their Squares? What is the Sum of thicir Sum, and ind ference? What is the Difle Product of their Sunt Difference? What is the Square of thicir Smm? and Differcnce ? What the Square of them What

344 2ueftions to Excrcife Chap. 1o. What the Square of their Difference? What the Square of their Product ?

1. The Sum of the Quantities? propofed is
$\} a+e$
2. Their Difference is $a-\infty$
3. Their Product by Multi-? plication
4. The Quote of the Greater $\left\{\begin{array}{l}\text { a } \\ \text { divided by the Leffer }\end{array}\right.$
5. The Quore of the Leffer $\} \frac{\bullet}{\text { by the Greater }}$
6. The Sam of their Squares aatee
7. The Difference of their $\}$ Squares
\}an-ee
8. The Sum of their Sum and $\}$ Difference $\quad$,
9. The Difference of their $\}_{\text {, }}$ 2e
10. The Product of their Sum? and Differencé $\quad\}^{a n-e e}$
iy. The Square of their Sum $a n+2 a e+s e$ 12. The Square of the Dificr
11. The Square of thear Prod.

$$
\begin{aligned}
& a \pi+2 a e+f e \\
& a a-2 . z e+c e
\end{aligned}
$$

ance
II. There are two Quantities whofe Sum is $b(=12)$ and the greater of them is $a(=S)$ i demand what is the Lefier ? What their Dificrence? What is the Product of their Multeplication? What is the Sum of their Squares? What the Difference, of their'Squares?

1. The Leffer is
2. Their Difference is
3. The Product is

Chap. $\overline{1} 0$. Algebraical Aritbmetick. 345
4. The Sum of their Squares is $\quad 2 a a-2 b a+b b$ 5. The Difference of their
$\}$ $2 b a-b b$ Squares is
III. There are two Quantities or Numbers whofe Difference is $d(\rightleftharpoons 4)$ and the greater of whom is $a(=8)$ I demand what is the Leffer? What is their Sum? What their Rectangle or Product? What the Sum of their Squares? What the Difterence of their Squares?

1. The Difference or Excels? $\left.\begin{array}{l}\text { being fubtracted from the } \\ \text { greater, gises the leffer }\end{array}\right\}$
greater, gires
Their Sum is
2. Their Product or Rectan-? gle is
3. The Sum of their Squares is
4. The Difference of their?

Squares is
IV. There are two Numbers, Magnitudes, or Quantities, whereof the Ratio of the greater to the leffer is as $r$ to $s$, (or as 3 to 2) and the gre ter of them is-a ( $=12$.) I demand what their DiffeLeffer? What is their Sum? Wetangle, or Product? rence ${ }^{2}$ What their Requagle, and what the Difference of, ther Squares?

1. The Leffer is by the Rule of Three
2. Their Sum is
3. Their Difierence is

4. Thcir

Page 133 and reverse.
"The Young INathematician's Guide: Introduction to Mathematics' by John Vard.

5 th edition $1 /(\mathcal{C B}$, Lst eartion about L'/06. London, 456 pages.

A complete work which had sections on arithnetic, algebra, geometry (including trigonometry), conic sections, infinlte series and practical gauging. Ihe arithmetic section gave aefinıtions followed by a rule and many examples but no problems to be solved. ward used the comma in writing decamals although he did mention that a point was used by some authors. He did not demonstrate his rules of arıthmetic by using algebra (see Bonnycastle), but relled on mechanıcal methods. He gave as one example Innaing the cube root of $916,379,602, y 89,073,900,279,630$, 298,890 (page 133) which compared to Cocker was short and only took two sides of working. the algebra section dealt with solutions of equations up to biquaaratic; bimple and compouna interest was also included in this section, but there was no reterence to the use of logarithms. I'he geometry section was interesting because many of the proofs usea partial algebralc methods; in proving Pythagoras' Theorem Ior instance ivard acknowledged that there were several ways of proof but stated "none more eary to be unaerituod by a learner than that which I fhall here propofe". In this book Euclid had been abandoned some 200 years too early. The photocopy shows his treatment of the conlc sections which was geometric rather than algebraic and also a part of the algebra section where rules for dealing with fractions were gaven.

## Chap. 3. Of of ractional ondantitieg.

When whole Quantities are to be fet down Frachon-wwife, Suljcribe an Unit for the Denomanator.
'Thus $a b$, is $\frac{a b}{x}$. And $a a-b b$, is $\frac{a a-b b}{1}$ \&c.

Sect. 4. To gbbatuntr, or 9Reduce Fractional guantities into their Lowelt Denomination.

## 踏le.

Divide botb : We Nunzerater and Denominator by their greatef . 1 minon $\mathcal{D}_{1}$ vifor, viz by fuch Quantaties as are found zn botb;

Thus $\frac{a c c}{d c}$ is $\frac{a d}{d}$. "And $\frac{a b b b}{a b c}$ is $\frac{b b}{c}$. - Alfo $a+\frac{b d c}{b c}=a+\bar{d}$.
In fuch fingle Fractions as thefe ; the common Dive fors (if there fe any) arceafily difcover'd by Inffeetion only ; but in Componand Irestions it often proves very tronblefone, and mult be done uther by Divodnag the NTuncrator by the Denomanzutor, until nothr g Reirams, when that can be done: Or elfe finding their rommon Neafure; by Dividug the Dencranator by the Numeritor, and the Numeratior by the Remander, and fo on as in

Examples.

Suppofe $\frac{\text { anc-aad }}{c d-A d}$ were to be re, 1 ced Iower.


$$
\frac{a c^{r} c-a n d}{0}
$$

In this İvamfle at fo haffones that the Nuncrato is Divided If ? If by the Denormator; but in the Newt it's otherwife, and



$$
\begin{array}{ll}
\text { I irlt } a a+2 a b+b b) & a a-a b b \\
& a a n+2 a b \rightarrow a b
\end{array}
$$

Thea - $2 a, b-2 a b b) \quad a a+2 a b+b b$

$$
-2 a a b-2 a b b \text { the Rcmaindor. }
$$

$$
\left(-\frac{1}{2 b}-\frac{1}{2 a}\right.
$$

$\quad \pi+6$ $\qquad$
aly $+1 b$
$\frac{a b+b b}{0}$

$$
0
$$



## Setion $x$.

 to it ate acgulated, by the help of one general Tbeormin


- As the Riridarele of any trin Abdiffis : is to the

Silure of ary ()rdimate whith diuides thrm : : So
is the Reckangle of any otber tato Abrilla's: To
the Squatu of balf that Urdinate robich diwinis t)


Let the annex'd Figure reprefent a Right Cone, cut thro'beeh Sides by the Right-lne $T S$; then will the Plan of that Scetion be an Ellipfis, (by Sect. 3. chap. I.) $T S$ will be the Tranfverfe Diameter NCN and $b$ a $b$ will be Ordinntes rigbtly apply'd; as beforc.

- Again, if the Lincs $D d$ and $K k$ be parallel to the Cone's Bafe, they Srct. 2, chap. 1.) Then will $\triangle T C K$ and $T a D$ be alike. Alfo $\triangle S a d$ and $\triangle S C k$ will be alike.


Ergo 1 \{Sa:ad::Sc:ck
And $2 T C: C K:: T a: a D\}$ per Theorem 13.
$\because 3 S a \times C k=a d \times S C$
$\therefore \quad \therefore \quad T a \times C K=T C \times a D$

But And $\left.^{2} C K \times C_{k} \div \square N C\right\}$
$7 a D \times a d=\square b a\} p e r$ Lemma Sect. 2.
Then for $C K \times C k$, and $a D \times a d$, take $\square N C$ and $\square 1 . a^{\prime}$ $5,6,7\} 8 S n \times T n \times \square N C=T C \times S C \times \square b a$ Yer $A x i o \% s$. Hence $10^{\prime} S a \times T a: \square b a: T C \times S C: \square N C$. Secpageioq. Q.E.D.
( 1 ap. 2. contermine the cilliplig.







Arin, From the Point (inthe E/hpfor Peripbery) drow
 1 lims $B C$.
"मansll $\Delta B$ C a and $\Delta C f a$ be alike.


And fo for any other $A b \int_{C l} \int f a$ s, and their Semi-ordinate.
Thefe Proportions being found to be the true and common Pro-- ties of every Elhofis, all that is father requard in (or about) - "Sethon myy be catily dediced from them.
 of any Ellipis.
There are feveral Ways of findung the Ifrus Rectrinn, but I thin unc fo enfie, and thews it fo planly to be the Fhrd pronWhane an the $E / / i p$ fis, as the f.ll wing

SAs the Trminerfe Dimeter: asin Proportion to Eholenti. $\left\{\begin{array}{l}\text { As the Conjugte }: \text { : fo is the Conjugate : to the } \\ \text { the } \\ \text { Latus Rectum. }\end{array}\right.$
in (in the following $F_{g}$ ) TS $\cdot N_{n t}: N_{r} \cdot$ LR the Latus Recfum.

From the laft Proportions take cither of the Antecedents, and s Confequent, viz, cithar $T C \times S C: \square N C$, or $T a \times S a: \square b a$, Bbb

## Pages 135 and 136.

"Synopsis Algebraica" by John Alexander (Second edıtıon amended) published in 1709 , London and used at the Christ's Hospital Mathematical School. The appendix was by Humphrey Ditton. Note that the work was in Latin and this would have met with the approval of many of the governors of the school. The pages shown ( pp .46 -47) indicate methods of solution of simple equatıons, but quadratıc, cubıc and bıquadratıc equations are dealt with later. The book had 120 pages.

Page 137.
"The Complete Measurer or the "hole Art of Measuring" by WIllıam Hawney. lst edition l717. 10th editıon 1761. The frontisplece stated "very useful for all tradesmen, especlally carpenters, bricklayers, plasterers, pranters, jolners, glazıers, masons etc."

A large work of 346 pages and all the answers were given including detailed worked examples. The method of explanation was by the rule, followed by an example and problems. Compare this to Bonnycastle's work on geometry later where proofs of formula were given. Hawney was a schoolmaster who kept a school and was a private teacher of mathematics at Lydd in Kent.
46. De Equationtm Algebrataraios
Si requalibus æqualia addantur vel fubducantur, aggregata ac refidua erunt æqualia.
Si æqualia per æqualia multiplicentur vel dividantur, quæ oriuntur erunt æqualia.
Fqualium radices homogenex, erunt rquales.
Porro æquationes rion funt unius 'generis, fed tot earum exiftunt clafles, quot gradus poteftatum feu dimenfionum iu fsala Quantitatum progteflionali. Unde otiuntur equationes,

1. Simplices, $\}$ \{3. Cubica,
2. Quadratice, $\}$ \{ 4. Quindrato-quadr. \&c.
Quarum fingulia fingulas "quafi Algebrápartes conftitudnt-
Reductio Equationum (potiffimum fimplicium feu, primarum) ad fimplicifimam formam perficitur, $\because, \cdots, i$,


[Nota Additionis itaque \& Subtractionis hujus compendium erit Tranfofitio Quantitatum ex una parte æquationis in alteram facienda femper fub figno contrario.]

$$
\begin{aligned}
\text { Sit } a-b-x & \ddots \\
\text { erit } a-2 b & =2 x^{\circ}
\end{aligned}
$$

3. Multuplicatione.

$$
\begin{array}{ll}
\text { Sit }\binom{\frac{x}{a}=b}{a \text { mult. } a} & \frac{\mathrm{Q}}{3 b}=a-c ; \\
\text { erit } \frac{\pi x}{x}=a b & \text { mult. per } 3 b \\
\text { oc eft } x=a b & \text { erit } \mathrm{Q}=3 a b-3 b c
\end{array}
$$

$$
4 \text { Divifione. }
$$

$$
\begin{array}{ll}
\text { Sit } x^{2}=6 x & 3 x^{2}=9 x \\
\text { erit } x=6 & \frac{3 x \text { div. }}{x=3}
\end{array}
$$

180: Menfuration of Solids. PartII: To meafure a Cylindroid; that is, a Fruftum of a Cone, baving its Bafes parallel to each other, but unilke.

## The RULE.

ToO the longct Diameter of the greater Bafe, add half the longert Diameter of the leffer Bafe, and multuply the Sum by the fhorteft Diameter of the greater Bafe, and referve the Produck.
Then, to the longeft Diameter of the leffer Eafe, add half the longeft Diameter of the greater Bafe, and muluply the Sum by the fhorteft Diameter of the leffer Bafe, and add the Product of the former referved Sum, and that Sum will be the triple Square of 2 mean Diameter; which multupled by 7854 , and that Product multiplicd by a third Part of the Height, the Product is the folid Content.
Exam. Let ABCD be a Cylindroid, vhole Bottom-bufe is an Oval, the tranf. verfe Diameter be. ing 44 Inches; and the conjugate Dia meter if Inches; and the fupper Bafe is a Curcle, whore Dameter is 26 Inches; and the Height of the Frufum is 9 Feet; the Soladity is recuured. To 44 (the greater Diameter of the low. er Bafe) add 13 (half 7

Chap. 2. Menfuration of Solids. 181
the Diameter of the greater Bafe, the Sum is 57 , which multiphed by 14 (the conjugate Diameter of the gicater Eafe), the ProduQ is 798 ; which referve. Then to 26 (the Diameter of the leffer Bafe) add 22 (half the traverfe Dameter of the greater Bafe), and the Sum is 48 , which mult'pled by 26 (the Diameter of the leffer Bafe), the Product is 1248 , to which add the former referved Product, the Sum is 2046 , which miltuplied by. 7854 , the Product is 1606928 4, which multppied by 3 (a third Part of the Height), the Product is 4820.7852 , which divided by 144 , the Quois 3347 Feet, the folld Content. See the Work.



Page 139.
"Arithmetıck in theory and Practice" by John Hıll 2nd edition L716, Iondon. Recommended by H. Ditton. lst edition 1712.

The preface by Ditton (at Christ's Hospital) stated that Hill "has done more and nuch better than Wingate, Cocker or Leybourn" in arıthmetic. The book presented arithmetic through the usual format of rules and examples of which there were many. There was also a short section which included problems involving algebra. The photocopy shows the arithmetacal treatment of progressions without recourse to an algebraic treatment. The book also included an excellent table of 8 figure logarithms for the integers from 1 to l0,000. The book was quite a large work having 378 pages excludıng the logarıthms.

## Aritbmetical Progreffion．

Product of 5 ，the preceding number of Terms by 3 ，the common Excels．
Hence may arife this Corollary．
That of the Common Excefs be multiflied by the Nur． ber of Terms Minis Unity，＇and to the Product the leze Term be added，the Sum is equal to the greatet．

## THEOREM II．

If Three Numbers be in Arotbmetical Progrefion；the double of the Mean is equal to the Sum of the Extreams．
So $2,4,6$ ，are Three Numbers in Aribunctical Pregreft on，and the double of the Mean 4，is equal to the Sumd the two Extreams 2 and 6 ．
THEOREM III.

If Four Numbers are in Axitbmetical Piogieflon，the Sum of the two Means is equal to the Sum of the two Ex treams．
So 7，11，15，19，are Four Numbersin Arthmetical Por greffon，and the Sum of the Two Means， 11 and 15,4 equal to the Sum of the two Extreanis 7 ，and 19.

THEOREM IV．
In any Arttbmetical Progrefion，any Term doubled is equal to the Sum of any orther two Terms equally Diftas．

$$
E X A M P L E
$$

$$
3,8,12,18,(23) 28,33,38,43
$$

In the annexed Aisthmetical Proyieffion，the double d 23 is equal to the Sum of 3 ，and $43,0_{1}$ of 8 and 38 ，or a 13 ，and 33 ，or of $: 3$ ，and 28 ，all Numbers which are eque！ ly Diftant．

## THEOREM V．

In any Aritbmetical Progreffion，the Sum of any tin－ Terms，is equal to the Sum of any other two Terms d like Diftance from them．

$$
E \times A M P L E .
$$

8，11，14，17，20，23，26， 29.
In the amm xed Piogreflion，the Sum of 14 and 23,5 equal to the Sum of 8 and 29 ，or of 11 and 26 ，or of 14 aud 20 ；all beng ailike Difunt．

## －THEOREM VI．

1：In any Arithmetical Progrefion whatfoever，if the Sum of the greateft and leaft Terms，be mulcupled by＇the Number of Terms，and the Product divided by 2，the Quonent is equal to the Sum of all the Terms．
Quonent is the Sum of the greateft and leatt，be mul－ tinhed by $\frac{x}{3}$ the Number of Terms，the Product is equal． to the Sum of all the Terms．
3．Or if the half Sum of the greateft and lcaft Terms， be multiplied by the Number of Terms，the Product is equal to the Sum of all the Terms．
4．Or the middle Number（when the Progrefiion is dd）mulapled by the Number of Terms，gives the Sum co all the Terms．

$$
E X A M P L E
$$

$3,6,9,12,15,18,21$.


T．HEOREM VII．
In a Progrefion of natural Numbers，as $1,2,3,4$, ， $\mathcal{G}_{c}$ ，if tic laf Term be muluplicd by the nexx greater，one balf of We Product is equal to the Sum of the whole Progrefion．

$$
1,2,3,4,5,6,7
$$

So the Product of 7 by the next greater 8 ，gives 56 ， rich half of which is 28 ，which is the Sum of the whole Prugreflion，

## Pages 141, 142, 143 .

"The Schoolmaster's Assistant, being a Compendum of Arıthmetic both practical and theoretical" by Thomas Dilworth. lst edıtion 1743, this edition (13th) 1765, London.

The photocopies shown include the introduction, the single rule of three (proportion) and the final stages in an example of extracting a cube root. The dialogue form $2 s$ well illustrated; the cube root extraction method, whilst being essentially a mechanical routine, does have the algebraically equivalent steps included (note that eee means e cubed).

Note that the number of digits in some of the examples would tax the accuracy given by an electronic calculator. Dilworth included the answers to all his examples which was a good feature.

A useful table that he included for power calculations was all the powers up to the fifteen power for the integers from l to 9. Thıs edition extended to 192 pages.

THE
Schoolmafters Affifant


Of Ailthmetic in Whole Numbers.

## The INTRODUCTION. Of Aritbmetic in generati.

2. TTHAT : Arthmetic?
A. In, Ahre'co is the Art or Science of copputing by Numbers, etther ubole or in Fiactoons.

A Nustrer is one me more Quantites, anfivering to the Quetion, fors …: ?
Q laytar dirnmercon Whole Numbers?
 Nurbers to be equre C anatites, and not divaded into Parts.

Q lifat artar, ntion Fiactons)
A. Arthmetic is $F_{1 \text { tifitns, fuppofes its Numbers to be the }}$ Parts of foune ent re cimintuty.

$A$ Both in Thery ind Practug.
Q ${ }^{\prime \prime \prime} /$ it $_{\text {t }}$ Theorencal Arithmetic?
A Tiscritcal Antiontto confiders the Nature and ravier of Numburs, ad de rontrates the Reafon of Praftcailopry rations. Aud in this Senfe Arithmetic is a Scence. Q i/hiar is Prấcal Ari hmetre?
A. Priftecal Arithutco is that which feews the Methed of working by Nu nbers, io as may be, mut afefal und expeditious for Bufizets And in th's Senfe withraetic is cat trt. Q What es tie Nature of all Arithnétical Operationz?
A. The Nature of all Aretherethal oprwaits is,-by fore Quatuits thit are riven, to find our ouberish are reciured.

Q "thech are tre furdment it Rates on inthmer 4 ?
 ston and $D x$ fion.

## B

DILWORTH'S SCHCOL:'ASTERS ASSISTANT. English arithmetics in dialogue and catechetical form date back to Robert Recorde, c. 1540.




Tbe Schoolmasters Alfitant.
(7) The next Figure in the Root, viz. 3, found as before is alfo called e ; then again 3 ane +3 zea + ece as the other Subtrahend, or Nurnber to be fubducted; thus,

$$
\frac{189}{90}
$$

$$
\frac{189}{2052}=3 \operatorname{cita}
$$

$4441 \stackrel{\circ}{94} 947^{\circ}(76.3$ Anfwer

1491 | $\frac{343}{101194}$ |
| :--- |
| 05976 |
| Refolvend |
| Subtrabond |

$$
\begin{aligned}
& 101194 \text { Kefowcha } \\
& 95976 \text { Subtrabend }
\end{aligned}
$$

173508)5218 947 Refolvend

5218947 Subtrabent
$\qquad$

1. What is the Cube of 64 ? Anfw. 262.144
2. What is the Cube of .13 ? Anfu. coz197
3. What is the Cube of 41.1? Aifw. 69426531
4. What is the Cube of .c9? Anfw. .000729
5. What is the Cube of 007? Allfi. .000000343
6. What is the Cube-Root $\}$ of go:2.3121G1? - - - 7. What is the Cube-Root $\}$ 7. What is the Cube-Root $\}$ 12181.7012?
7. What is the Cube-Root of 6121800121?

-     - 

9. Wiat is the Cube-Root $\}$ of 7121.1021698? $-\quad-1$
10. What is the Cube-Root of 12000.812161 ? - II What is the Cube-Root 2 II, What isi85128!?-- of 1218128 What is the Cube-Root $\}$ of .00697612:8 $\qquad$ Anfov. 19.67 Anfw. 1967 t

An/w. $39.41+$
Anfw. 19.238+
Anfo. 22.89 +
Anfw. $495{ }^{\circ}$
Anfw..19107+
13. If a cubical liece of rimber be 41 Inches long, 42 Inches bioad, and 41 Inches deep; how many cubical linches doth it contans? Anfw. G8921 cubucal Inches.
14. Suppofe

$$
\begin{aligned}
& \begin{aligned}
17328 & =3 \\
3 & =e
\end{aligned} \\
& 51984=3 \text { ane } \\
& \text { cee viz. } 3=27 \\
& 2052=3 \mathrm{cea} \\
& 27=e e e \\
& \text { Sub. } \frac{27}{5218947}=3 \text { ane }+3 \text { eea }+ \text { ece } \\
& \begin{array}{l}
3=e \\
3=e \\
9=e 6
\end{array} \\
& \frac{3}{27=3}= \\
& 76=2
\end{aligned}
$$

Page 145 and reverse.
"Euclid's Elements" by Robert Samson, Glasgow ll56 (last edition).

This was the popular version by Samson, in Latin, used in the Universities and from which further translations were made (R. Rot's version for instance l845). The photocopy shows the frontispiece and a part of book I, Pythagoras' Theorem.

Page 146 and reverse.
"The Scholar's Guide to Arithmetic", 'A complete execcase book for the use of schools' by John Bonnycastle. (Professor of mathematics, Royal lifilitary Academy, :Toolwich).

This edition 1808. 212 pages. List edition 1780 . Although he gave the rules of working problems Bonnycastle also gave algebraic demonstrations; in the example shown, that of finding a cube root, he took only 7 stages compared with Cocker's lo steps. He also gave an approxmation method (with an acknowledgment to Newton and Simpson - see RuLe 2 of the photocopy), even showing the algebraic proof and demonstrating the fast convergence of the method. He also showed the extraction of any root using the binomial expansion and gave examples. A later section of the book dealt with the properties of numbers (based on Euclid). Bonnycastle did not neglect the commercial arithmetic either, but struck a good balance between this and demonstrative arithmetic. All the answers were given to every problem and another feature was a detailed subject index on the Last page. Altogether a superior book for Its time.

## EEEMENTORUNT

## LIBRI PRIORES SEX,

ITEM

## UNDECIMUS ET DUODECIMUS,

## EXVERSIONELATINA

FEDERICICOMMANDINI;

Sublatis iis quibus olim Libri hi a Theone, alifve, Vitiati funt, Et quibufdam Euclidis Demonftrationibus Reftitutis,

AROBERTOSIMSON, M.D.
In Academia Glafguenfi Mathefeos Profeffore.

GLASGUAE,
INAEDIBUSAGADEMIGIS
EXCUDEBANT ROBERTUS ET ANDREAS FOULIS ACADEMIAE TYPOGRAPHI
M.DCC.LVI.

# LIBER PRIMUS. 

PROP. XLVII. THÉOR.
N reciangulis triangulis, quod a latere rectum angulum fubrendente defcribitur, quadratum aequale eft quadratis quae a lateribus rectum angulum continentibus defcribuntur.

Sit tringulum rectangulum $A B C$, reftum habens $B A C$ angulum; dico quadratum a recta $B C$ aequale effe quadratis $a b$ ipfis $B A, A C$ defrriptis.

Defribstra enim a BC quidem quadratum ${ }^{2} \mathrm{BDEC}$, ab ipfis vero a. 46. r. $\mathrm{BA}, \mathrm{AC}$ quadrata $\mathrm{GB}, \mathrm{HC}$; perque A alterutri ipfarum $\mathrm{BD}, \mathrm{CE}$ parallela ducatur ${ }^{\mathrm{b}} \mathrm{AL}$, et $\mathrm{AD}, \mathrm{FC}$ jungantur. Quoniam igitur uterque b. 3r. r. angulorum BAC, BAG rectus eft , ad aliquam rectam lineam $B A$, et ad punctum in ea $A$ duae rectac lineac $A C, A G$ non ad eafdem partes pofitac, angulos qui deinceps funt duobus rectis aequales efficiunt; in directum igitur eft CA ipfi AG ${ }^{\text {d }}$. eadem ratione, et $A B$ ipfi $A F$ eft in directum. et quoniam angulus DBC eft aequalis angulo FBA, rectus enim uterque

c. Def. 30 .
d. I4. I. cit, communis apponatur ABC , totus igitur $D B A$ angulus toti $F B C$ eft aequalis ${ }^{c}$. et quoniam duae $A B$, e. Ax. 2 . BD duabus $\mathrm{FB}, \mathrm{BC}$ aequales funt, altera alteri, et angulus DBA aequalis angulo FBC ; erit et bafis AD bafi $F C$ aequalis, et ABD triangulum triangulo $F B C$ aequale $f$. eftque trianguli quidem $A B D$ duplumf.4. 1.

BL

To Extraet the Square Root.

## Example.

'Required the fquare root of 14876.2357 .

$$
\begin{aligned}
& \text { 2429123523 } \\
& \text { 9121861 } \\
& \begin{array}{r}
24386 / 166257 \\
6 / 146316
\end{array}
\end{aligned}
$$

$$
\begin{aligned}
& \begin{array}{r}
-428 \\
-185
\end{array} \\
& 15 \\
& \text { Anf. } 121.06517 \text { s ine root required, }
\end{aligned}
$$

3. What is the figuare root of 1000 g 29 ?
4. What is the fuuare root of 152399025 ?
5. What is the fquare root of stojio60gizt

Then $\overline{a+b+c^{2}}=a^{2}+2 a b+b^{2}+2 a c+2 b c+c^{2}$, and the mane ner of ninding $a, b$, and $c$, will be as before, thus :
2fl, divifot a) $a^{2}$
2f. divifor $2 a+b) 2 a b+l 2$

$$
2 a b+b^{2}
$$

3d, divifor $2 a+2 b+c) 2 a c+2 b c+c^{2}$

$$
2 a c+2 b c+c^{2}
$$

Now, the operation, in cach of theis cafes, exactly agrect with the of purtods whatever. of purtods whatever.
6. Whet

The Ixtraction or the CubeRoot. 167
6. What is the fquare root of 368863 ?
7. What is the fquire root of 31721812 ?
8. What is the fquare root of .00032754 .607.34092, Evc,
Anf. 1.78106, E®c.
of. $00032754!$
of $\frac{s}{1}$ ? Anf .01809 10. What is the fquare root of $6_{3}^{2}$ ? 11. What is the fquare root of Io? Anf 3.162277, عٌ

## The Extraction of the Cube Root.

## RUEE.

1. Separate the given number into periods of three figures each, by putting a point over every thard figure from the place of units.
2. Find the greateft cube in the firt period, and fet its root on the riglit hand of the given rumber, after the manner of a quotient figure in divifion
3. Subtract the cube thus found from the faid period, and to the remainder annex the following period; and call this the refluent.

4 Under the refolvend, put-the triple root, and its triple fquare, the latter beng removed one place to the left, and call their fun the divifor
5. Seek how ofien the divifor may be had in the refolvend, exclufive of the place of units, and fet the refult an the quotient
C. Under the divifor, put the cube of the han quotiert figure, the fyuate root of at maluphed by the triple 100 t , and the thple of it by the fauare of the 100 t , each removed one place to the lfft , and call therr fum the fubtrahend.
7. Suberact the fubtrahend from the refolvend, and to the remander bring down the next period for a new refolvend, with whelr proceed as bofore, and fo on whe whole is finithed.

* The reafon of pointing the given number, as directed in the rule, is obvious tom Coroll 2, to the lemma made ufe of in demonftrating the finre root; and the rett of the operation will be buft underitood from the tollowing analyt cal procelo.
Suppofe $N$, tie given number, to confite of two periods, and let the . figures in the root be denoted by $a$ and 6 .


## igz TheEstraction of the Cuge Root.

Note. The hame rule mu't be obferved for continuing the oreration, and pointing for decimals as in the fquare 1000.

## ExAMPLES.

1. Required the cube root of 4822854 \%

$$
\begin{aligned}
& 4 \dot{822 \dot{8} 54 \dot{4}(364} \\
& 27 \\
& 21228 \text { refolvend. } \\
& 9 \text { tuple of } 3 . \\
& 27 \\
& \text { retp }{ }^{2} \text { e jquare of } 3 \text {. } \\
& 279 \text { di:ifor, } \\
& 216 \text { cule of } 6 . \\
& 324 \text { firuare of } 6 x \text { bs the trinle of } 3 . \\
& 162 \text { triphic of } 6 \times \frac{8}{6} \text { the foxere of } 3 \text {. } \\
& \text { 39556 fuberaherd. } \\
& 1572544 \text { ficond rifol uind. } \\
& 108 \text { truple of } 36 . \\
& 398 \mathrm{~s} \text { triple Jquare of } 36 . \\
& 38983 \text { farond duvifor. } \\
& 6_{4} \text { nobe of } 4 . \\
& 1728 \text { fquare of } 4 \times \text { by the inple of } 36 \\
& 15552 \text { trogle of } 4 \times \text { by the fiuare of } 36 \text {. } \\
& 1572544 \text { facond fuitralint. } \\
& \text { Ar. } 3^{6} 4=\text { root requined. }
\end{aligned}
$$

Whit is the cule root of 380017 ?

Then $\bar{a}+b^{3}=a^{3}+a^{r} b+3 a b^{2}+b^{2}=\mathrm{N}=$ prom nuaber, ond to find the cube root of N is the func as to find the cube root of $a^{3}+3^{2} b+3 a^{2}+6$, the methed of domg which is as fallows:

The Extraction of tue Cubergot.
163
4. What is the cube root of 27054036008 ?
5. Required the cube roor of 12251532723 , Anf. 3002
6. What is the cube root of 127085327232 ? . Anf. $49^{6} 0^{\circ}$ 6. What is the cube root of $1+6708483$ ? AN. 52.74
7. What is the cube root of 171.15775 to6? AIIf. 5555 , E.c.
 9. What is the cube root of $13^{4}$; 10. What is the cube root of $4 \frac{10}{2} \lambda$
41. What is the cube root of $\frac{3}{5}$ ?
A. $8.87,8_{6}$

## R U L E 2 *。

1. Find, by triais, the nearef rutional cube to the given numbet, and call it the affumed cube.

2 Then, as twice the afomed culse added to the guen number, is to twice the given number added to the andued cube, fo is the root of the affimed cube to the root requined nearly.
3. And by taking the cube of the 100 thus found, for th: aniumed cube, and repeating the operation, the root will be had to a full greater degree of cxactnefs.


And in the fame manner moy the root of a guantity confifing of any number of pericods whatever be found.

* The metheds ufually gicn for extracting the cule root are fo ex. cedinely tedious and diffecte to be remmbered, that anthmetici ins 'ave long wifhed for a fiot enfy tule that would be morc reaty and consement in pactice. Sur Ifatc Neruten, Mr. Simpion, Mr, Enorfon, ind

No photocopy glven.
"The Tutor's Guide, being a complete system of Arithmetic" by Charles vyse. Lst edıtion 1771, London. To make arıthmetic more interestıng vyse wrote many of the examples in verse; for instance: When first the marriage knot was tied

Between my wife and me,
IIy age to hers we found agreed
As three tames three to three;
But when ten years and half ten years
We man and wife had been,
Her age came up as near to mine
As eight is to slxteen.
Now tell me, pray,
What were our ages on our weadins aay ?
Of this book De Morgan (ref:4) wrote in 1847: "If a new edition were published some of the examples must be omitted as rather opposed to modern ldeas of decency". It was a very popular text book apparently.

Page 148.
"The Key to the 'Iutor's Guide" by Charles Vyse. 5th edition, 1791, London, 374 pages.

I'his 'Key' containea not only the answers but solutions and extra hints and ru^es. 'the 'Tutor's Guide' was crowded with examples which explains the large number of pages in the key. The preface contanned letters of recommendation from schoolmasters throughout England. Note in the decimal adaition problems that Vyse used a comma for the decimal 'point' and did not always give the correct answers.


Page 151
"An introduction to Algebra, (for the use of schools and other places of Public mducation").

John Bonnycastle. 12th edition 182 'L, London (Appendix was added in 1815). 260 pages.

A comprehensıve book on higher algebra agdin by the Professor of Mathematics at the koyal Milıtary academy, Woolwach.

The photocopy shows a page of questions on series which would not disgrace a present uay text book. 'l'se whole book haa demonstrations and proofs of theorems and ample questions with the answers provided.

Bonnycastle acknowleaged throushout references to other works by mathematiclans and citea works by Newton, MacLaurin, bupson, Emerson, clalraut, Euler, Lagrange and Lacrolx, indlcating probably his own wide sphere of influence. I'he pretace gave no mention of any examinations $10 r$ which the book would be suitable, another teature of this period.

## rage 152

" $\operatorname{lntroduction~to~Mensuration~and~Practical~Geometry,~with~}$ notes contanning the reason of every rule.

John Bonnycastle 1lth edrtion 1816, London. 300 pages. Bonnycastle in the pretace acknowleaged the work of Hawney; however, apart from the usual method of presentation - rule, examples, problems, he also gave algebralc proofs of the tormula used. The example shown in the photocopy illustrates the use of the calcuus terms, 土luxion and iluent
and 'dot' notation (trom Newton) in the general p-oof ot the volume of a pyramid. All the answers to problems were given although many were unrealistic for that time being given to $1 / \perp 0 \cup, 000$ part of an inch.

v
$\stackrel{N}{N}$
敫
To find the foldity of a cone or 1 yramed.

## RULE.

Multiply the area of the bafe by the petpenidenlar heiglit of the cone, or pyiamet, and $\frac{4}{3}$ of the product will be the folldity. ( $m$ )
eyamples.

1. Regruired the fulidity of the cone $\mathrm{c} s \mathrm{p}$, where dameter $\triangle \mathrm{a}$ is 20 , and its perpendicular hembit cs 21 .
(m) Demon. Let cs $\quad=a, c s=x$, und $\Delta=\operatorname{arca}$ of the ba* of the conle
3hen $a^{2}\left(o s^{2}\right) \cdot x^{2}\left(c s^{2}\right) \cdots \Delta B^{2}: E D^{2}(b y$ fini. $\Delta s): A$ : $A Z^{2},(-$ area of the circle $B D)$ becaucoall carcles are as tho fquares of their clameters.

But $\frac{\lambda^{2}}{a^{2}} \times \dot{x}=$ flurwon of the cone $a \varepsilon n$, and its fluent $=\frac{A 2^{3}}{3 a^{2}}$; whech, when $x=a$, becomes $\frac{a}{3}=A \times \frac{a}{3}$ for the foludity of the whole cone, bemg the fame as in the rule.

 $\frac{A r^{2}}{a^{-}}$(arca of the polygoriab) beciufe all fimlar figures are az the fyuares of their libe fides.
But $\frac{A A^{2}}{a^{2}} \times \dot{x}=$ fuxion of the PS ramide $c$ a $b$, and ts correct fluent $=\wedge \times \frac{a}{a}$, as un the cone, which rule is generar, let the figure of the bafe be wint it moy.
or SOLIDS.
163


Here. 785 s
$400=$ fyuare of A s.
$314.1600=$ arsa of the base.
24

## 12566100

6283200
3)7539 8100
$251328=$ folidity required.
2. Required the folidity of the hexagonal pyramid c a $\mathrm{B}_{\mathrm{c}} \mathrm{c}$, each of the equal fides of its bafe boe jui 40 , and the perpendicular height es 60 .


Here, by the table, before given for polygons,

## Page 154

"A System of Practıcal Arıthmetic" by Rev. J.Joyce, lst eartion 1816 Lonaon.

A commercial arithmetic book containing many examples on the four rules but also including decimals, fractions, proportion, logarithms (土or compouna interest calculations) and stocks and bills of exchange. The pretace does not suggest the book was suitable for examination work which had not taken hold at this time. The photocopy shows the very long examples on addıtion of money which supposedly would have been a useful training for clerks. No answers were given as these were publıshed in a separate book called a 'Key' which is descrıbed below.

## Page 155

'Key to a system of Practıcal Arıthmetic' by Rev. J.Joyce. Publishing date would have been after 1816 .

This is the answer book to the above text and would have been invaluable to any schoolmaster judging by the length of some of the addition problems. The Idea of a 'key' book was sometimes popular and even in the 20 th century Durell's famous 'Geometry' had a key and even Later the S.M.P. series provided a separate 'Answers and Hints' set of books.

为



COMHOUND ADDITION.



| 5. 65716101 | $\begin{array}{r}10.10116 \\ 27215 \\ \hline 15\end{array}$ | $\begin{array}{ll}723 \\ 966 & 10 \\ 81 \\ 81\end{array}$ | $\begin{array}{lll}477 & 16 & 4 \\ 305 & 15 & 21\end{array}$ |
| :---: | :---: | :---: | :---: |
| 879 <br> 874 <br> 14 | - BB9 17 10 10 | $89015{ }^{24}$ | $736511^{24}$ |
| 91012104 | 6171924 | 21510101 | 6021192 |
| 1311911 | 308107 | 5ge lf 9 | 5051351 |
| 235.7 6! | 64316104 | 4701974 | 937170 |
| 43618 34 | 770 0-bi | : 74t1世 0 \% | 4111017 |
| 53795 | 065 177 | ctg 157 ? | 7001901 |
| 6731110 | 49013 01 | 50315115 | 6721111 |
| 82010 \% | $15010 \quad 0$ | $15010{ }^{\circ}$ | $40 \quad 010$ |

40. 421271 12.92 76550264 $900089 \underset{\sim}{2} \quad 2$ 408103179 $375151 \quad 3 \quad 10$ 269101861 50 50ikS 11 ot $91064910 \quad 6$
41. 00114216102 $\begin{array}{lll}24611 & 5 & 95 \\ 21\end{array}$ ala3sa ib 7
 315512 17 0! GIRSs0 12 $91 \therefore 271910$ 42071310
89129117
$\qquad$
L. s. $d$ 51. 45671611 $493415 \quad 9$ 2765 16 1Ct 957610112 2.49718 1234 $10 \quad 8 \frac{2}{5}$ $56781610^{\circ}$ 437689 27941541 $\begin{array}{ll}7521 & 1210\}\end{array}$ IIG4 13 180517 19612 $\begin{array}{lll}1961 \\ 9450 & 15 & 11\end{array}$ 2416 ic 104 17061410 1,325108 bti7s 10113 4932166 20050 51
 $d$. 53. $356712 \quad 9$ 79601710 1234 157 6.78128 01231410 4567 13 11\} 601217 ! 11569 $89110 \quad 19$ $2845 \quad 6 \quad 3$ $678012 \quad 5$ 2:15 13 11 67891608 ci 321510 $34,6 \quad 19 \quad 5!$ 751107 $23151+111$ 6752120 $4315127 \frac{1}{4}$ 210580

- From these tarce examples the preceptor may malse an alnest


## 30

COMPOUND ADDITION
Questions, 太c p 55
What is meant by Compound Addation 7
What is the rule for Compourd Addition? Iuplam the mode of operation by the evample Illustrate this rule by an example Note

$$
\text { Ausucrs to Evamples, p } 56,--59
$$

| Ex | - $L$ | s. d | Lr | $L$ |  |  | Lx |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 298 | 175 | 2. | 314 | 13 | 10 | L | 420 |  |  |
| 4. | . 319 | 211 | 5 | 316 | 13 | 9 | 6. | 205 | 18 |  |
| 7 | 405 | $1410 \frac{1}{2}$ | 8. | 321 | 2 | 12 | 9 | 35 | 11 |  |
| 10 | 253 | $2{ }^{2} 8$ | 11. | 351 | 12 | 9. | 12 | 133 | 17 |  |
| 13 | 350 | $12{ }^{4 \frac{1}{2}}$ | 11. | 339 |  | $10 \frac{3}{4}$ | 15. | 392 | 6 |  |
| 10 | 417 | 2114 | 17 | 602 | 2 | ${ }^{4}$ | 18 | 443 | 0 |  |
| 13. | . 2358 | $18^{-}{ }_{\sim}^{1}$ | 20 | 391 | 19 | 10 | 21. | 16 | ${ }^{3}$ |  |
| 22 | 364 | 3 9 | 23. | 362 | 3 | 7 | 21 | 490 | 13 |  |
| 35 | 4066 | $210 \frac{3}{4}$ | 26 | 4097 | 12 | $7 \frac{1}{2}$ | 27. | 1130 | 6 |  |
| 23. | 3886 | $10 \quad 1 \frac{1}{2}$ | 29 | 4230 |  | $1 \frac{1}{2}$ | 30 | 4762 | 1 |  |
| 31 | 5771 | $88^{2}$ | 92 | 4799 | 3 | $9{ }^{1}$ | 33. | 4089 | 1 |  |
| 34. | 318 | $9 \quad 2{ }^{2}$ | 35 | 338 | 9 | $1 \stackrel{1}{2}$ | 36. | 471 | 4 |  |
| 37. | 395 | 29 | 88 | 376 |  | $11^{2}$ | 39 | csi | , |  |
| 40. | 341 | $210{ }_{4}^{*}$ | 41. | 418 |  | $5{ }_{5}^{1}$ | 42 | 356 | 14 |  |
| 43 | 4635 | 1823 | 4 | 5981 | 7 | $0{ }_{4}^{7}$ | 5 | 6138 | 7 |  |
|  | 5559 | 178 | 47 | 6005 | 14 | ${ }^{2}$ |  | 5721 |  |  |
|  | Ex. 49 | 4205895 | 19 | T ${ }_{4}$ |  |  | 901365 | 514 |  |  |
|  | Ex 51. | 78130 | 6 | $5 \frac{1}{2}$ |  |  | 101415 | 514 | + |  |
|  | Ex 53. | 101035 |  | $6 \frac{1}{2}$ |  |  | 1014 |  |  |  |

## Ansuens to Examples in Thoy Warot, p 60

 Cx. 1. 3975 ll 8023 dwt $23595 \mathrm{lb} 11 \mathrm{oz} 1 \mathrm{dwt} 29 g r$ Ex. 33509 lb 3020 dut $4310 \mathrm{lb} 10 z 0$ dut 10 d Ex 5230 l lb 10 oz 7 dut 620010713 dut 18 gr Ex 7. 519 lh 904 ledwt 8. 401020 dut 21 gr[^6] fmples, as mas be fon d necessar

## COMPOLND ADDITRON.

## 31

Ansucrs to Examples in Aloirdupozs Wright, p. 61
19 Ix.
$3014 \mathrm{lb} 8020 \mathrm{dr} \quad 2.2751$ tons, 9 cwt 3 fr Glb. 2007 lb $6023 \mathrm{cr} \quad 4.3152$ tons. 10 cx .1 q 1.11 h . 3247 tons, 14 cwt 2 qr .6 .2402 cwt 2 qr .24 lb

8485 lb 13020 drs
Anulters to Erar'ples in Apothccares Werght, p 69.
人 Ex
$12590 \mathrm{lb}, 7 \mathrm{cz} 4 \mathrm{dr} \quad 2.299 \mathrm{oz} 2 \mathrm{dr} .2 \mathrm{scr} .14 \mathrm{gr}$.
3 301glb 1 oz 5 dr 1 se 4 gr 4457 lb 1 oz .6 dr

- 1030 Jar liscrup

62313 dr 2 sc .9 gr.
409tb 8070 dr
Ansuens to Examples an Cloth Meusare, p $6 \mathbf{a}$
1 x 1.4106 yds 3 qr .1 nl Ex. 2.3570 E e. 0 qr 3 nl .
 if $; 3103 \mathrm{Ee} 1 \mathrm{qr} 0 \mathrm{nl}$ Ex. 6. $3735 \mathrm{E} . \mathrm{c} .4 \mathrm{qr} 2 \mathrm{nl}$.


Ausuts to Eiampics in Long Mctasue, p. 63.
1:
La.
14155 m 6 fur 3403 vus. 24080 yll 1 ft .1 m 0 bc . $3170 \dot{y}$ lea 2 m iftre3p 4191 lea 0 miles, 7 fur. : 395 fm up $2 y \mathrm{ds} \quad 04703 \mathrm{p} 4 \frac{1}{2} y \mathrm{ds} 1 \mathrm{ft}$

67 ft .2 in 2 bc
Ansucrs to Examples on Lund Mcasure, p 63-4
! 1 31G6ac 1 r up. Ex. $21177 \mathrm{ac}, 2 \mathrm{r} 28 \mathrm{p}$.


Ea. 6 113ac. 2 r 11 p .
Is Ansuas to Exam,iles on II ane Mersine, p Gi.


 314 57l 241 !
-

## Page 157

"The Advanced stage of Arithmetrc" by James Perry, London 1829.

This book, according to the frontıspiece was based on the 'Perrian System' of education and was a part of a whole method of teachıng. It $1 s$ unbellevably bad; the examples shown such as 'If 4.173982065 currants are eaten in London in a week, how many are eaten there in ll weeks ?' point out the unreal nature of the questions. If a closer look is taken at the numbers used in the photocopy shown they appear to be made up by re-arranging the digits 0 to 9 in a random sequence. Note also that the 37 th lesson and the 194 question was reached on page 21 the whole book containing 96 pages including the rules of arıthmetıc.

## Page 158

'Private Key to Principle of Equations part of the Advanced Stage of Arıthmetic' by James Perry, London 1829. Thıs was the ney to the book previously described and seems to indicate that a comercial venture could be realised by selling two books instead of one, if the text book provided such awkward examples.



Pages 161 and 162
'The Young Man's Best Companion' by John Dougall, Kınnersley 1815.

This text book was probably a 'teach yourself' book intended for perhaps the upper working class and could have been used by scholars of institutes. It deals with everything that might be useful and, according to the preface presented to the reader, "a general introduction to knowledge applicable to various businesses and occupations in ordınary life".

The sections shown include trigonometry and a section on logarıthms. Other sections include commercial arıthmetic, navigation, surveyıng, geometry, mensuration, book-keepıng and algebra.

Reverse of Page 158
'Intellectual Arıthmetrc, with a Key' by a teacher of Youth.' 7th edition 1840. Published Hodson, London, for the Amerıcan Literature Co. ( 184 pages).

This book was written by iVarren Colburn (1793-1833) an eminent American mathematical educator of the 19 th century. In this Brıtish edition has name was not given, but the work was ldentical to an Amerıcan copy entıtled 'First lessons in Arlthmetic on the Plan of Pestalozzl, with some improvements' (1821) quoted in 'Readings in the History of Mathematics Education' by J.K. Bidwelम $a_{n} d$ R. G. Clason (IN.C.T.M.1970).

The answers were in the 'Key' section with hints where necessary; since the book contaned many short written questions of the type snown it was suitable, Colburn suggested, for pupils who could not understand the baslc arıthmetical processes.

Even in 1913 It was selling several thousand copies a year in America.
tis them phace mother set of numbers increasmig by arthmetteal
 1han the number numedrately Leloue it, as $1,2,3,4,5,6,7,3$, o, Sc. thas conous eficet and property will be foumd that, by smply alkine together these last numbers, and obselving that numbice of the finst vet opposite to which thes sum stands, we at onte discover the product nhich wond hase been obtaned by the multephation of the two quantities rit the set of geometried
 pord This will be mone minellighbe from a connderation of the t.able.

I'able 1st.

| 3 | 2 | 4 | 3 | 16 | 32 | 61 | 128 | 256 | 512 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 3 | 3 | 3 | 4 | 3 | 6 | 7 | 8 | 9 |

'The upper row of mambes in this fable comotins a set mereasm,
 namber is double its predecesson: thus 2 are double 1,3 dombt,
 cremmer arntmetically br the addition of ome ; and these lomer monbers an monese of the momber of sedupleations, by whath
 rppat fov bank the ontimal stock of the whole, before the dunblary
 leen doubled. I m the lowed maks that: m the upper has bern spe doulded, 2 an he lover low points out that 4 an the uppet las bern furice donbled; a shows that the conserpoment
 resull of 1 nume tmes donbled.

$$
\text { 'Tublo }{ }^{\text {nind. }}
$$

| 1 | 10 | 100 | 1000 | 10,000 | 100,1000 | $1,000,000$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 2 | 3 | 1 | 3 | 6 |

Arbut in this table the nmbers in the upper row proced in $d$ tentold geomutijal ptopostion, each steccednes number bemg fas
 moredsmg an an authmetical proportm by one, pont ont bow orien the uppel aumbers oppasite to each hive undergone thas tenteld waltupladica. 'Dis' seao or noumht as plased opposice to the onemin mumber $Y$ : but 2 malicates the mamber dbove it to be puod?ced by two multipheatoms by $10:$ for 10 tames 1 :ate 10 , mul 10 thes 1 th ais 100. The malex $:$ shows that 10,000 is pro dued b) 1 multuplicestous by 10 ; and 6 marks.the six multiphad dionst,y 10 , by whel 1,1 inctesed to une millom, In both the



 the njpat row, $A$ an evanghe of the mature add use of there
logalithmic numbers, let us suppose it ware requard to thed the product of 4 multiplied by 8 in table 1 st. Uldel 4 in the upped row or the natural numbers we have 2 for ats $\log$ anthu, and under the matual number 8 we have 3 for its logmatha Then addang these two lownithms 2 and 3 togethen, we have 0 for the loganthan of the prodnct; and over 5 we ind 32 the matual product foms maltinglyng 4 by 8. Again, to multiply 1 by 10, and then product agan by $t$, we add togethen the three loganimms 2,4 , and 3 , standmig respectively under 4,16 , and 8 ; and the sum 0 is the lonanthm an the lower row, above whech is 512 , the product ot the auccessive multupheations of 4 into 16 , and therr product by 3 . In the vime Way an table 2md, when the matual numbers tugnent an a tentold proportion, if we wanted to know the pioduct of 10 by 1000 , we nould smply ard togrethea the mdeses under 10 and 1000 , whach are 1 and 3 , and above 4 then sum, we wonld find 10,000 , the prodet requmed. If we wanted to discoser the value of 10 multupited by 100 , and their poolut by 1000 , we hase only to add togenter the loganthms is the lower 10 w consespondme to those natun 1 numbers, viz. 1,9 , and 3 , amounting to 0 , over wheh lognithm wa find $1,000,000$, the ultmate product lecpured.
On the other hand, to divide any given monber by another, all we hase to do is thom the loguthm of the greater mumber to sulh-
 the faotiont thus lit it le requated to divide 512 by 8 an thble lst. Fiont 9 the logathm standmer mader the dmalend 512 , wh-


 divalen el, we tahe asay the logarathan at the diveon, when the

To suluare any mumber, as 8 , we have only wo donble di" logardim m table 1 st, whach is 3 making $G$, above wheh we haver of wheh is the squate of 8 : and the cube of 8 will be found by t.king thene tumes its logatilim 3 or 9 wheh stands unter 512 the whe ef 3 Ag:'n, to citact the squate root of any given sum

 the cube root of any tuantity as 512 wall be tutund by mesty daking the that pat of ats logarathin 9 , whech as 3 , the logantim of 8 the cube 200 of 312 thit was equared.
From the spechuens bere exmbited the reader whll be canaled to torm a notion of she mature and bes on logarshms; of which the alculation, to compose tables applienthe to a macat extent of rumbers up fom umty, is an operation of podegtotis fabour: but
 for thear not over ertatedil posterify: selt of ample and at caste







the point $S$ in the circumference, and dividing the quadrant if $S$ I anto two equal parts, each must of course contain the half of no, or 45 degrecs: let also $\mathrm{C} S$ be produced untul it meet the taneme 33 Dis the point $\Gamma$ : from $S$ draw $S B_{j}$ joming the extremuties of the are S B, and also S L perpendicular to the racius C D. We have now these lines belonging to the are $S B$, that is to the angle $S C B$, nanely, $\mathrm{C} S$ or C 13 the radtus, S B the choid, S L the sine, S II the co-sine, B T the tangent, and C 'I the secant of an arch, or an angle of $45^{\circ}$. Now, let the thace lines C S, S 13 , and 13 C form a trangle $C S B$; if we make the other augular ponts 13 and $S$ the centits of circles os ares, we may obthin other sets of smalar hase, mater or smaller in preporton to the gagle to wheh they sevenaly pelong: and hence it win follow that the sides of trangles mint bean a certam popontion to the opposite anyles.

If in a aght-angled trimgle, as $\triangle 13 \mathrm{C}$, fig. 31, fiom the angle ut $A$ as a ecntre, with the wase $A \mathrm{E}$ for madrus, an ore Cebe descibed, the perpendicular C $B$ will be the lungent, and the bypothense $\Lambda^{\prime} 13$, wit be the scemin of the argle In $A \mathrm{E}$. Similat effects will be pro duced by making the per. pemhenlar 1 CC Clie radhas. On the other land, at he hypothenuse A B be made radius, the sades $A C$, and C IB, will become the sater of the angles to what these sides are respectiody oppostic.
Agrin, in all oblique-angled thangle's, such as $A C B$, ins. 32, the sides are to cach otior in the propontion of th. 4 sincs of the angles respectively opposite to esich: that is to say, the side $\Delta C$ is to the sade C B, as thr sme of the angle at $\mathrm{B}, \mathrm{Oj}$. posite to AC , is to the sime of the augle at $A$, op. posite to $\mathrm{C} B$; and the side $A B$ is to the side $A C$, as the sine of the angle at $C$, opponte to $1 C$, is to the sine of the angle at $B$, opposite to $\Lambda C$. Irm there propertics it follows thit, out of the three sides and thes angles of wheh every trangle concists; it ang three be given, (one of them hewever ,hway to bie a stac) the other three parts may be dacovered by calculatoon. [Sece Alache on Logarimams.]
Having thus generally tated the matue of bigonomedy, we mey now proced to give a eloot account of ats applecation, an men
sery inportant banches of practical knowledge, namely, in the nausurcment of the herghts and distances of parteculiar vbjects, usually, but too restrictedly called Practical Geomety, Mensuation of Suifnces and Solds, Measurement of Artheers' Work, Gauging, Land surveying, and Navigation.

## Or Practiche Geometry.

1st. Let $A B$ in fig. 33, represent the wall of a tower, of which itis required to dascover the herght, without actunl measurement.
 suppose an observer to be pineed at C , with his feet on precisely the same level with IS the botom of the tower, and his cye at $e, 5$ fett from the ground the horizontal line $e m$, will then be the level of his eye. The distance of the observen fiom the tower, that is $C B_{\text {。 }}$ equal to $e m$; is measured on the groand 130 deet: and with a common quadrantor other proper instument, the angle of the clevation of the tows, that is, the angle fumed by the horizoutal lime c $m$, and the lane of stent from the cye at $e$, to the top of the tover at $A$, is found to be 20 deg . 50 mmutes. The tower A $D$ standag pependiculaty on the base $C B$ or $e m, A$ em is a reght-angled thangle, of whinh we know the bise and the angle of clevtion at $c$. If then from $c$ as a centre, with hat base $e$ in for radin, an are be described, $m$, $I$ will be the thgent of the ange at $c$. We have now oltaned three lenme of a propontion, comsequently the fouth may soon be found. Thas poporion is as the logaitheric adats of any chelef always equal to the sine of a
 (romsidered to be 10 , is to the tangent of the angle at $e=23^{\circ} 50^{\circ}$, so ts the base e $m=C \mathrm{~B}=130$ fect, cornesponding to the whans. to the perpendiculan $n \Lambda$, conesponding to that harent. Or stated in this way:

Radius
Eine $90^{\circ} 00^{\circ}$ 10.00000

$$
\begin{aligned}
& \overline{11.87509} \\
& -10.00000 \\
& m A=75=1.07509
\end{aligned}
$$

But the point $m$ being clevated $s$ fect albove the botton of the tover, equal to the height of the obsemver's eyc at $c$, above the level line C 13, that quantity must be alded to the of feet : whose found; giving 80 feet lor the whole height of the tower from the round; which was the thang requared to be kuown.

Page 164
'The Complete Measurer or Practical Geometry and Mensuration' by Thomas Kerth. 320 pages.

The photocopy is from a 'new' edition enlarged by Samuel Maynard published in 1839, London.

The preface of the 1824 edition by Kelth referred to the lst edition of 1798 which was the work by Hawney but extended. The work was quite comprehensive and ancluded algebraic demonstrations of the rules, but not proofs. Some of the numbers used were impractical; one answer gave the radius of a sphere inscribing a tetrahedron as 2.0412413 Inches.
3. What is the solidity of the segment of a sphere whose diameter $\mathrm{s} \mathrm{D}=20$ fect, and the height of the segment $\mathrm{o} D=5$ feet? Ans. $654 \cdot 5$ cubic feet.
4. The dameter of a sphere $\mathrm{FD}=21$, what is the soldity of a segment thereof, whose herght o $D=45^{\prime}$ Ans. $572 \cdot 5566$.
5. The diameter of the base of a segment of a sphere $\wedge B=28$, and the height of the segment $O D=$ 6.5 ; required the solidty? Ans. 2141 '93285.

## PROBLEM XIV

## To find the Solddty of the Frustum or Zone of a Sphere.

Ruie I*. To the sum of the squares of the radii

- Denonstration C ll the height of the greater segment m, and the height of the less $h$; also $R$ the radius of the greater base, and $r$ that of the less, then it is evident the differcuce between the soludities of these two segments will be the soldity of the zone. Hence $\left[\left(3 n^{2}+1^{2}\right) \times\right.$ ir $\left.\times 5236\right]-\left(3,2+h^{2}\right)$
 Put $a=1 t-h$ the brcalth of the zone, and $D$ the dhameter of
the splure. then, by thic proputy of the urele, ( $\mathrm{p}-\mathrm{n}) \times \mathrm{n}=\mathrm{h}^{3}$, the splure then, by the proputy of the urcle, $(b-n) \times n=h^{2}$, and $(\mathrm{p}-h) \times h=r^{2}$ from the former of these $\mathrm{n}=\mathrm{K}^{2}+{11^{2}}^{2}$, and from the latter, $D=\frac{r^{2}+h^{2}}{h}$, thertonc, $\frac{R^{2}+11^{2}}{\mu}=\frac{\prime^{2}+h^{2}}{h}$, and from above, $a=1 t-h$ revermmate the values of in wind $h$, and the above theorem $\left[\left(3 n^{3} 11+11^{3}\right)-\left(3 r^{2} h+h^{3}\right)\right] \times 5236$ will become $\left(\mathrm{k}^{2}+r^{2}+\frac{a^{2}}{3}\right) \times a \times 15703$
If one of the radu pass through the centre, as in the gone EFDC, then $n^{2}=\frac{D^{3}}{4}=\cos +00^{2}=r^{2}+a^{2}$, hence the last theorem becomes $\left(r^{2}+\frac{2}{3} a^{2}\right) \times a \times 31410=\left(\frac{1}{D^{2}}-\frac{1}{3} a^{2}\right)$
$\times a \times 3 \mathrm{~J} 116$
Hence $\left(r^{2}+\frac{2}{3} a^{2}\right) \times a \times 6.2832=\left(\frac{1}{2} D^{2}-\frac{1}{3} a^{2}\right) \times a \times 62832$
of the two ends, add $\frac{1}{3}$ of the square of their dis tance, or the height of the zone; this sum multuplied by the heght of the zone, and the product agan by $1 \cdot 5708$, will give the solidity.


## Rule II. For the middle zone of a sphere.

To the square of the drameter of the end, add two-thards of the square of the height; multiply this sum by the hegght, and then by 7854 , for the solndity.

## Or thus,

'From the square of the dameter of the sphere, subtract one-third of the square of the herght of the middle zone; muluply the remander by the height, and then by 7854 , for the solidity.
Example 1. Required the soludity of the zone of a sphere efdc, whose greater dameter $\mathrm{x} P=20$ mehes, less dameter $\mathrm{c} d=15$ mehes, and distance between the ends, or helght $\mathrm{G} 0=10$ inches


By Rulc I. $\left[\left(\mathrm{rc}^{2}+\mathrm{co}^{2}\right)+\frac{1}{3} \mathrm{GO}^{2}\right] \times \mathrm{cox} 1 \cdot 5708$ $=\left[\left(10^{2}+7 \cdot 5^{2}\right)+\left(10^{2}-3\right)\right] \times 10 \times 1 \cdot 5708=[(100+$ $502.5)+(100-3)] \times 15 \cdot 708=(15(\cdot 2.5+33334)$ $\times 15.708=18058_{5}^{2} \times 15.708=2977.975$ cubsc anches, the soludity required.
2. Required the solndity of the midde zone $A B D C$ of a sphere, whose diameter $\mathrm{r}=22$ mehes, the top and bottom dameters C ? and $\wedge \mathrm{B}$ of the zone being each 16971 inches, and the height $n 0=14$ mehes?

Ans. 4003 4912 cubic inches.
woll express the solduty of the mudde zone CDBA , beng double of the former, where $a$ is half the altutude, and $r=$ half the diameter of eqch end. Put a for the whole altitude, and $d=2 r$ the dinmeter of ench end, and the theoremis become ( $a^{2}+\frac{3}{3} \Lambda^{2}$ ) $\times \Lambda \times 7834=\left(\mathrm{D}^{2}-\frac{1}{3} \mathrm{~A}^{2}\right) \times \Delta \times 785$

## Page 167

"A System of Popular Geometry, containing in a few lessons so much of the elements of Euclid as is necessary and sufficient for a right understanding of every art and science" by George Darley. Thıs edıtıon 1844, Ist edition 1826, London. 128 pages.

This was an attempt to simplify and re-arrange Euclid and considering the date of publication seemed quite in front of its time. Darley put some thought into the geometry even challenging the 12 th axıom, the parallel axiom. The pretace suggested the use of the book for three classes of student. (1) Those in public or private schools, (II) those whose education had been neglected, (ill) for artists and mechanics. Darley also stated that it could be used for a fourth class 'If it was the custom', but stated that ladies would probabiy never have needed this book.

He also wrote a popular algebra and a popular trigonometry book at this time.

Reverse of page 167
"The Elements of Arithmetrc" by Augustus de Iforgan, 4 th edition 1850 , lst edition 1830 , London. 166 pages. This was De Morgan's answer to all the books on arithmetic that he had critıcızed. He gives quite lengthy explanations and was caretul to give each stage in detall with proofs and demonstrations, not just rules.

Later in the book he gave algebralc aemonstrations on the general properties of numbers and a chapter on permutations and combinations which was falrly uncommon. However, he did include the usual commercial arithmetıc as well. The photocopy shows his approach to proportion rather than the rigid rules glven by many authors, as in the 'rule of three' varlatıons. Another important point to note is that in contrast to many of the later text books of that period he does not give many set questions. He only glves 7 problems on long division for instance (c.f. Cusack's Arıthmetic in the 1880's which gave 26 dıffıcult examples).

Also the angle alr will be cqual to the angle crs, and the angle $\mathrm{bi} I$ to the angle me, as the whole angles themselves are gren equal. Hence by Pait II. tho
 arches on which the angrles AEI, cris staud, willhe equal, and also the arelies on which the angles BLt, Dr K stand; thercfore the whole arch arb will be equal to the whole arch chd. These were the assertions, \&e.

Ant. 85 In equal curcles, the angles whoch stand upon equal anches are equal, urhether they be at the centres of the circumfer ences.

Let acb, crid, bo two equal cincles, with the angles a and in at the centres, as in fig 1, -or the angles E and $\mathbf{F}$ at the circumferences, as in figs. 2, 3, 4,-standing upon cqual arches aib, cKd. Then also these angles at g and $\mathrm{H}, \mathrm{x}$ and r , are equal
Dev PartI. In fig. 1 , as the sides $A B$ and $C D$ aro equal, by Arr. 83 ; and as AG, GB are respectively equal to cir, un, by Ant. $\$ 1$,-the angles at a and in are equal, by Art. 6.
Part II. In fig. 2, (the equal arches Aib, ckD being semacirclos, ) then the segments alb, crd, are also senncurcles and consequently the angles at E and r , by Ars. 72 , are both right ones. IIence, by Art 8, they are equal.
PartiII. Infigs 3and 4, (the arches beng either less or geaterthan semucircles,) the angles G and it at tho respective contresarecepual, as in Part I., Henco m fig 3 , the angles at s and F

are equal, becauso they aro halves of those at $c$ and $u$, loy Arr 71. Also for tho same reason, thie angles at $I$ and $K$, in fig 4 , are equal. But the angles at I and r are together equal to two
 1 ight angles, by Art. 69, and therefore equal to the angles. at $k$ and $F$ together. Hence, taking away from both sudes the equal angles at 1 and r , the angles at a and r reman equal
These were the nssentions, \&c. [Seo Note IFP.]
Aur. 86. These latter four aticles, it is evident, aro true for the same circle as well as equal ones.
Art. s7. In a circle parallel chords intorcept equal arches

Let $A B, C D$, be two parallel chords in the cucho Andc. Then the arches $A C$, $B D$, are equal.
Dim. Draw ad, and the angles cda, 1 ib are equal, by Ant. 12. Hence, by Art 86 and 84 , the arches ac and ud are
 equal. Thus, \&c.

Pros. XIV To divide a given arch of a carcle anto tuo equal parts
Let adc be a given arch It is 1equired to divide it minto two equal parts.
Cons Draw the right line ac, jommg the extremitics of the given anch. Divide. Ac equally at the point D , by Prob $V$., and fiom the point d rase ds perpendicular to ac, by Pros VII. Then a will be the middle point of the arch $A B C$
Dem. Draw the nght limes $A \mathrm{D}$ and cb. In the thangles $A \mathrm{db}$, CDB, since the side DB is common, and since AD is equal to DC, and the angle ADB equal to the angle CDn, by construction,-therfore by Anr. I, AB is equal to de IIence, by Art. SG and 82 , the arch $A B$ is equal to the arch nc . 'Ther, Cc .
which $b$ is of $c$. It is plain that $a$ is greater than, equal to, or less than $b$, according as $c$ is greater than, equal to, or less than $c l$.
179. Four numbets, $a, b, c$, and $d$, being pioportional in the order written, $a$ and $d$ are called the eatrenes, and $b$ and $c$ the means of the proportion. Cor convenience, we will call the two estremes, or the two means, smilar terms, and an extreme and a mean, disszmilar terms, Thus, $a$ and $d$ are similar, and so are $b$ and $c$; whle $a$ and $b, a$ and $c$, $d$ and $b, d$ and $c$, are dissimilar. It is customary to express the proportion by placing dots between the numbers, thus,

$$
a \cdot b \cdot c \cdot d
$$

180. Dqual mmbers wall still reman equal when they have been mereased, dimimslied, multiplied, or divided, by equal quantities. This amounts to saying, that if $a=b$ and $p=q, a+p=b+q, a-p=b$ $-q, a p=b q$, and $\frac{a}{p}=\frac{b}{p}$. It 28 also evident, that $a+p-p, a-p+p$, $\frac{a p}{p}$, and $\frac{a}{p} \times p$, are all equal to $a$.
181. The product of the extremes is equal to the product of the means. Let $\frac{a}{b}=\frac{c}{d}$, and multiply these equal numbers by the product $b d$. Then, $\frac{a}{b} \times b d=\frac{a b d}{b}(116)=a d$, and $\frac{c}{d} \times b d=\frac{c b d}{d}=c b$. hence (100), $a d=b c$. This, $6,8,21$, and $\neg S$, no opopotional, sme $8-\frac{3}{4}=\frac{3 \times 7}{4 \times 7}=\frac{21}{25}(180)$, tund it apocus thit $6 \times 28=8 \times 21$, wime both products ate $16 S$.

182 If the product of two numbers be equal to the product of two others, these numbers are proportional in any onder whatever, provided the numbers in the same product ane so placed as to lee smblar terms that is, if $a b \in p$ g, we have the followng pormortions $\sim$

| $\dot{a} \cdot p \cdot q: b$ | $p: a \cdot b a$ |
| :--- | :--- |
| $a: q \cdots p: b$ | $p \cdot b \cdot a \cdot q$ |
| $b: p \cdot q \cdot a$ | $q: a \cdot b: p$ |
| $b \cdot q: p \cdot a$ | $q: b \cdot a \cdot p$ |

To prove any one of these, divide both $a b$ and $p a$ by the product of its second and fourth teims; for evample, to shew the truth of $a^{\prime} q \cdot p . b$,
diude both $a b$ and $p q$ by $b q$. Then, $\frac{a b}{b q}=\frac{a}{q}$, and $\frac{p q}{b q}=\frac{p}{b}$; bence (180), $\frac{a}{q}=\frac{p}{b}$, or $a \cdot q:: p$. . The pupl should not fail to prove every one of the eight cases, and to venfy them by some simple ex. amples, such as $1 \times 6=2 \times 3$, which gives $1: 2: 3: 6,3: 1:: 6$. $2, \& c$.
183. Hence, if fou numbers are proportional, they are also proportronal in any other order, provided it be such that similar terms still remain similar. For since, when $\frac{a}{b}=\frac{c}{d}$, it follows (181) that $a d=b c$; all the proportions which follow from $a d=b c$, by the last artuche, follow also from $\frac{a}{b}=\frac{c}{d}$
184. From (114) it follows that $1+\frac{a}{b}=\frac{b+a}{b}$, and if $\frac{a}{b}$ be less than I , $1-\frac{a}{b}=\frac{b-a}{b}$, while if $\frac{a}{b}$ be greater than one, $\frac{a}{b}-\mathrm{x}=\frac{a-b}{b}$. Also (122), if $\frac{a+b}{b}$ be divided by $\frac{a-b}{b}$ the result is $\frac{a+b}{a-b}$. Hence, $a, b, c$, and $d$, beng proportionals, we may obtan other proportions, thus.

$$
\text { Let } \begin{aligned}
& \frac{a}{b}=\frac{c}{d} \\
& \text { Then (114) } 1+\frac{a}{b}=1+\frac{c}{d} \\
& \text { or } \quad \frac{a+b}{b}=\frac{c+d}{d} \\
& \text { or } a+l: l \because a+d: a
\end{aligned}
$$

That is, the sum of the first and second is to the sccond, ns the sum of the thrd and fourth is to the fourth. For brevity, we shall not state in words any more of theso proportions, sunce the pupil will easily supply what is wanting.

Resuming the proportion $a: b:, c: d$

$$
\begin{aligned}
\text { or } \frac{a}{b} & =\frac{c}{d} \\
3-\frac{a}{b} & =1-\frac{c}{d}, \text { if } \frac{a}{b} \text { is less than } \mathrm{I}, \\
\text { or } \frac{b-a}{b} & =\frac{d-c}{d}
\end{aligned}
$$

that $18, b-a: b: d-c \quad d$

$$
\text { or, } a-b: b \cdot c-d: d, \text { if } \frac{a}{b} \text { is greater thian } 1 .
$$

Page 169
'Mathematics I' by Augustus de liorgan 1831, Iondon. This book was not a text book but rather a descriptive mathematics text without problems, containing detalled explanations of varıous processes in arıthmetic and algebra. It contanned a section on the study and difficulties of mathematics for both the teacher and the pupil and could almost be the first serious work on mathematical education, De llorgan himself using the latter description. As can be seen by the photocopy from the arithmetic and algebra section, although a rule was given $\ddagger$ or solving quadratıc equations, each stage was explained rather than just mechanically set down.

Page 170
'Arithmetic for the use of schools' by Rev. J. $\begin{gathered}\text {. Colenso }\end{gathered}$ published Longmans London 1843.

The pretace mentions the use of the book in public scnools. It is commercial in outlook and rules are mainly given, although there was some explanation to the rules. Colenso stated in the preface "Most books on arithmetic specially fall in conveying any clear ldea to the pupil of the reason of the steps taken". The photocopy shown of the rule of ihree however states "This explanation..... aoes not at all profess to give the true reason for so stating" which was rather the opposite.
tion whose roots shall be any given
numbers, 2 and 3 for mstance. Ve or $\quad x^{2}-6 x+9=9-8$, numbers, 2 and 3 for mstance. We must make $-p=2+3$, or $p=-5$, and $q=2 \times 3$, or 6 . The equation

$$
x^{2}-5 x+6=0
$$

209. The rule for solving guadratic equations is as follows. clear the equation of fiactions of there be any 110]; remove all the terms contanning he unknoun quantity into one membet, and all those that do not contann at anto anothe7, collect into one the coefficients of the square of the unknown quantaty $f$ more than one, and also the
cooflictents of the smple power of the copflicients of the stmple power of the unknown quanthty; divide every torm $1 n$ both members by the coefficient of the square of the unknown quantily; to each member of the cquation as it $n 070$ stands add the square of half the coeflectent of the simple power of the unknown quantity; eatract the square root of each member, and write the results as equal to each other, prefixing. the double sign to the one whach does not contain the unknown quantity; the quadiatic equation is then reduced to a simple one, and may be solved accondangly.
210. The following are examples of questions producing quadratic equations.

The sum of two numbers is 6 , and the sum of their reciprocals is $\frac{3}{4}$, what whence

$$
(x-3)^{2}=1,
$$

$$
x-S= \pm 1 ;
$$

and this gives us

$$
x=3 \pm 1, \text { or } x=4, \text { or } 2 .
$$

If 4 be taken for the one number, the other is $6-4$, or 2 , and 2 and 4 ait the numbers that satisfy the question since $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$.

It is observable, that in this case the two roots of the cquation 2 and 4 ar the two numbers sought. This is neeessarily the case; for whichever of the numbers is called $x$, the other is $6-7$ and there is no way of distingushmg that $x$ is to stand for one of the numbers rather than the other Everyicason, then, that exusts to make one of the numbers satisfy the equation apphes equally to the other. It is theietore impossible that the equation can be satisfied by one of the numbers and not by the other.
211. To find two numbers such thet their sum is 10 , and the sum of ther squales 58. Call the one number $x$ then the other is $10-x$. By the ques-
tion

$$
x^{3}+(10-x)^{2}=58
$$

$2 x-20 x+100=58$.
are the numbers $x$; then, as in the one of is $6-x$, and taking the sum of ther recmocils [96] we have by the ques. thon

$$
\frac{1}{x}-\frac{1}{6-x}=-\frac{3}{4}
$$

Multeplymg both members of this equation by $x(6-x)$ it becomes

$$
\begin{gathered}
6-x+x=\frac{3}{4}\left(6 x-x^{2}\right) \\
\frac{3}{4}\left(6 x-x^{2}=6\right.
\end{gathered}
$$

or
and multplying both members by $-\frac{4}{3}$,

$$
z^{2}-6 x=-3 .
$$

To each nember add $\left(\frac{6}{2}\right)^{2}$, or 9 , and we have

Carrying 100 to the other member of the equation and dividng by 2 this becomes

$$
x^{2}-10 x=-21
$$

Add 25 to each member and we have

$$
x^{2}-10 x+25=25-21
$$

or

$$
(x-5)^{2}=4
$$

$$
x-5= \pm 2
$$

which gives $x=3$, or 7. We find which gives $x=3$, or 7 . We find
$3^{2}+7^{2}=9+49=58$. For the same $x^{2}+7=9+49=58$. For the same eason as in the last example, the two Supe the two numbers sought
Suppose that it had been requaed to find two numbers such that while ther sum is 10 , the sum of their squars should be 20. As before, we should have
and $2 a^{2}-20 x+100=20$,

Alding 25 to each member, this be. each of these pays 2 s . more than his comes

$$
x^{9}-10 x+25=-15
$$

whinch gives us

$$
x=5+\sqrt{-15}
$$

or

$$
x=5-\sqrt{-15}
$$

Here the values of $x$ are impossible [at. 201], which shows that there are no numbers that can satisfy the conditions given, but that the question contains something absurd and contraductory. This is plainly the case, for we cannot divide 10 into two parts such that the sum of their squares shall be less than 50. The magmary vilues of howe sa and, as before, the two loots of the efuation are the twe expressions souyht.
leturning to [206] the roots of a quadratic equation will be impossible whenever $\frac{p^{2}}{4}-q$ is a negative quantity. Now $\frac{p^{2}}{4}$ is in its nature positive, since
every square number is always positive Theiefore the roots are imposslble only when $q$ is positıve, and greater than $\frac{p^{2}}{4}$. When $q$ is a negative quan. hity, such as $-m$, the expression under the radical sign becomes $\frac{p^{*}}{4}-(-m)$, or $\frac{p^{2}}{4}+m$, a quantity essentially positive, and thencfore when $q$ is negative the loots are always possible. If one root be impossible the other must be imposshble also, for the two roots always consist of the sa
different signs.
212. A. company of persons spend 3l. 10s. at a tavern Four of them go away without paying, in consequence of wheh each of the others has to pay $2 s$. more than lus share. How many persons were there in the company, and what was the proper share of each? Call the number of persons $x$. They spend 70 shillings, so that the proper share of each is $\frac{70}{x}$ shlumgs. But there are only $x-4$ who pay, so that every one of these pays $\frac{90}{x-4}$ shillings. Now
proper share, therefore

$$
\frac{70}{x-4}=\frac{70}{x}+2
$$

Multiply each member by $x \cdot(x-4)$ and this becomes
$70 x=70 x-280+2 x^{2}-8 x$, or, when reduced to the general form

$$
x^{2}-4 x=140 \text {. }
$$

Adding 4 to each member we have $x^{2}-4 x+4=144$
which gives
whence

$$
x=14, \text { or }-10
$$

The positive value of $x, 14$, satisfies the question; for when the number in company is 14 the pioper share of each is $\frac{70}{14}$ or 5 shillings, while what ench of those who remain actually pays is $\frac{70}{10}$, 017 shilhngs, which is 2 shillings more. With respect to the negative value, -10 , it also satishes the equation, since

$$
\frac{70}{-10-4}=\frac{70}{-10}+2
$$

As we have stated the question, the sums $\frac{70}{x-4}$ and $\frac{70}{x}$ are to be pand by the company, when they are posilive numbers. Theretore when they becone negrative numbers, that is, when w becomes negatise, they tue stims to be recesved by the company. But when we introduced a into the equation, we intioduced it quite gemer aly, as any quanity enther positive or negrative that would satisfy the equation, and therefore the equation, as we stated at, necessarily applied as much to the case in which the company were to receave, as to that in which they were to pay the moncy. When they are to secure the money the question must be a.tered to this- A company of persons are entitled to have 70 shillings distimbuted among hem, their number is incieased by tou, in consequence of which the shate of cach is diminshed by two shillugs. how many persons were there at first? This question is, algebracally spreithing, the same as the foimur, with tie wigis of the difterent numbers changed, ith answer to at is 10, the negame inswer answer to it is the changed. Not only, then, do the two


Page 172 and reverse.
"The Tutor's nssistant or Comic Figures of Arıthmetac" by Alired Crowquall (A.H. Forrester) London 1843.

Thıs was Walkıngames arıthmetıc distorted and with
drawings. The usual 'definitıons, rule, example and problems' format was adopted and the comic $\pm 1 g u r e s ~ i n-~$ troduced to make the presentation Less Iormal (see also S.I.P. book later). The book was a wholly commercial arithmetic and the photocopy shows examples on Tare and Trett (from busıness examples) De liorgan (ref: 4)stated about this book "the joke will remain on hand too long for the learner".

-
 3nth fion waste, dut, \&e., made by the merchant to the buye.
Clow is an nllowance of 2 lb . for cevery 3 ewt., or adsth part for waste on a fow articles, but is now vely seldom made.
Sutrese is when only patt of the allowances have been deducted from the gross.
Nors - Draf is an allowance in consequence of weighing goods on the quays in very large quantities, so that the welght may not prove defictent when sold agan in snallor quantities, for retal sale ; it is deducted in the first instance, as the goods, are weighed, and It is deducted in the finst mastance, as
the rem under eatered as gross werght.
The deduction sanctioned by Government for Draft, is 1 lb . on goods not exceeding $1 \mathrm{cwt} ; 2 \mathrm{lb}$. from 1 to $2 \mathrm{cwt}, 3 \mathrm{lb}$. from 2 to 3 cwt . , 4 lb from 3 to 10 cut .; 7 lb . from 10 to 18 cwt ., and 9 lb . from 18 to 30 cwt . and upwards, but among private individuals, on goods not subject to such control, the patties concerned mutually determine the allowance
Tret and Cloff are in a manner synonymous, ench being for wáste or dast, und as some articles are subject to a greater portion of it than others, Tret has not been deemed suffuent, and the add. tomal allowance of Cloff prevals on goods of particular descuptrons; it is not uussual to enter it waste, dust, or rubbibh, \&e Whenever two or more of the deductions are similar, that 15 , so mach per cut or per hidd \&e., they should be culculated compontly, as contributing to cepedition
Contormable to arecent regulation at the Custom House an London, all the deductions ure discontinued but Tare, and on very particular occeassons dust or rubbinh, but they reman in practice among merchants to a certan extent.
It may here be observed, that in computing the allowances for Tace, Tret, \&c., we may reject any fraction less than $\frac{1}{\text { of }}$ a lb . and yet obtan an answer suticiently exact for practical purposes. In busmess, the unform practice is to reject any fraction less than $\frac{1}{2} \mathrm{lb}$.
business, the uniform practice is to reject any fraction less than
and to allow 1 lb . when the fraction 1 s cqual to $\frac{\mathrm{a}}{\mathrm{l}} \mathrm{lb}$. or upwards.
(घ) EN $(5)$


Ruse : $-H^{r}$ ruen the Thare is at so much prer bug, barrol, \&c.: multiply the number of baçe, bartels, \&e. by the tare, nud subhact tho product fiom the gross; the remainden 14 tha net.

In 7 fials of rasing, each weighing 5 ewt. 2 qis. 5 lb. gross, the at 23 lb per fial, how much net weight?

Ans. 37 cwt I qr. 14 lb.


What is the net weight of 25 hogsheads of tobacco, weighing gross 163 cwt .2 qrs. 15 lb ., tare 100 lb . per hogshead? ${ }^{2}$ Ans. 141 cwt. 1 qr .7 lb .

In 16 bags of pepper, each 85 lb .4 oz . gross, tare per bag 3 lb 5 oz. ; how many pounds net? Ans. 1311 .

Rule 2 - When the Tare is at so much in the whole gross weeght: subtract the given taie from the gross, the lemander is the net.

What is the net weight of 5 hogsheads of tobacco, weighing gross 75 cwt 1 qr .14 lb ., tare in the whole $752^{\circ}$ ? Ans. 68 cwt .2 qrs. 18 lb In 75 barrels of figs, cach 2 qrs. 27 lb gross, total tare 507 lb ; how much net weight? Ans. 50 cwt. 1 qr .

Ruls 3-7\%hen the Tare es at so much per crot. divide the werght by the aliquot parts of a cwt., which subtract fiom the gross; the remander is the net.
In 25 barrels of figs, each 2 cwt. 1 qr. gross, tare 16 1b. per cwt, how much net werght? Ans. 48 cwt. 24 lb



Pages 174, 175 and reverse.
"Euclid's Elements of Geometry" by R.Potts, lst edition 1845 Lonaon, based on the 'eucमia' of Vr. Simson. This photocopy $1 s$ from the 1861 edition. 'rhe pretace gives an interesting history of the translations of Euclia and makes an appeal for the study of mathematics (geometry at least) as a means of 'developing and cultivating the reason'. A part of the preface and the contents are shown as well as the Pythagoras theorem prool and some questions on the $4 i$ th proposition. 'lhis book was a popular one, used in the universities and the better Public schools. The preface justafies the concept of the part that mathematics has to play in a liberal education.
thus acequired, will be necessaryly confined to the consideration of lines, angles, surfices and solids. The process of deduction pursucd in Ccometry from cettain admitted principles and possible consturctions to their consequences, and the rigidly exact compaison of those conscquences with known and established truths, can scarcely fail of producing such habits of mind as will influence most bencficially our reasonings on all subjects that may come before us.

In support of the views here maintained, that Gcometrical studies form one of the most suitable and proper introductory clements of a scientific cducation, we may add tho judgment of a distinguished living witer, the author of "Tho IIistory and Philosophy of the Inductive Sciences," who has shewn, in his "Yhoughits on tho Study of Mathematics," that mathematical studic, juliciously pusucd, form ono of the most effectivo monns of devcloping and cultevating tho eason: and that "the object of a liberal edlucation is to levelope the whole mental sytem. of man;-to make his speculative inferenecs coincide with his practical convictions;-to conable him to 1 ender a renson for the belicf that is in. him, and not to leave him in the condition of Solomou's sluggand, who is wisor in his own conceit than soven mon that cen render a reason." To this wo may subjoin that of Mr. John Stuant Mill, which he has recorded in his invaluable System of Logic, (Vol. 11. p. 180) in the following tems. "The value of Mathematical instruction as a preparation for those more difficult investigations (physiology, socicty, rovernment, \&e.) consists in the applicability not of its doctrines, but of its method. Mathematies will ever remain the most perfect typo of the Deductive Mretholin general; and the applications of Mathematics to the simpler brancles of physics, furnish the only school in which philosophers can effectually learn the most difficalt and important portion of their art, the cmployment of the lavs of simpler phenomema for cxplaining and piedicting those of the more complex. These grounds are quite sufficient for decming mathematical training an indispensable basis of real scientific cducation, and icgading, with Plato, one who is à $\gamma c \omega \mu$ ćrp $\eta$ ros, as wanting in one of the most essential qualifications for the successful cultivatum of the higher toanches of phulosophy."
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## CONTENTS.

anglo, and shall bo equal to a given rectilnoal figure; viz by applying to the given straght line a parallelogram equal to tho first trianglo $A B D$, (I. 44.) and havmg an angle equal to tho given anglo.

## PROPOSITION XLVI. PROBLETI.

## To describe a squaro upon a given straglt lino.

Lut $A B$ be the green straight line


It is required to deschbo a squaro upon $A B$
From tho point $A$ draw $A C$ at right ancles to $A B$; (i. 11.)

$$
\text { mako } A D \text { equel to } A B \text {, (I. 3.) }
$$

through tho point $D$ danw $D E$ parallol to $A D$; ( 131. )
and through $B$, draw $B E$ pasallel to $A D$, mooing $D E$ in $E$;
the coforo $A B E D$ is $n$ parallologiara,
Whonco $A B$ is equal to $D E$, nid $A D$ to $D E$; (1. 31.)

$$
\text { lut } A D \text { os cqual to } A D \text {, }
$$

thorofore the four lines $1 B, B L, E D, D A$ no oqun to one anothor, and tho parallelogiam $A B E D$ is oqualatoral.
It has hikewiso all its anglos right angries;
suce $A D$ meets tho parallols $A B, J D$,
thereforo the angles $B A D, A D E$ aro equal to two xi ght angles; (r. 29.) but $B A D$ is a mght anglo; (constr.) therefuro also ADE' is a viglit anglo.
But the opposito angles of parallelogians aro czual; (r. 34.)
therofore oach of tho opponto angles $A B E, B E D$ as a right anglo; when cfore tho firme $A B E D$ is noctangular,
and it has been provol to bo equataterd;
thereforo the figure $A B E D$ is a squave, (dof. 30.)
and it is describod upon the given straght line $A B$. G.v $F$.
Cor. Henco, crery parallologram that has ono of its angles a right anglo, has all its angles right angles.

## PROPOSITION XLVII. TIEOIREM.

In any right-anylel triangle, the sfamo which is decembed upon the sido subthandigy tho vipht angle, as equal to the squarcs desented upon the sudes woluch contann the aght anglo.

Iot $A B C$ bo a dirht-angled tonnglo, having the ipht anglo BAC.
Thon tho squano doscribed upen Hio sislo $A C$, shat! bo oqual to tho squares described upon $B .1, \ldots$


On $B C$ describo the squan $B D E C$ ( ( 46 ) and on $B .1, A C$ tho squares $G B, I T C$
through $A$ draw $A L$ pasallel to $1 B D$ or $C E$; (r. 31.)

$$
\text { and joun } A D, F^{\prime} C \text {. }
$$

Thon bocause the anglo $B A C$ is a ight anglo, (hyp)
and that tho anglo $B A G$ is a might anrle, (def 30 )
the two staright hnes $A C, A G$ upon tho opposite sides of $A B$, mako wath it at tho point $A$, the adjacont angles equal to two right angles,
thereforo $C A$ is in the samo stiaught line mith $A G$. (I 14)
For the same reason, $B A$ and $A I I$ aro in the same stratght hino
And because the anglo $D B C$ is equal to the angle $F 1 B A$,
each of them boing a right angle,
add to oach of these equals tho anglo $A B C$,
therofore the whole angle $A B D$ is equal to the whole arigle $F B C$ (ax. 2)
And becauso the two sides $A B, B D$ aro equal to the two sades $F P$, $B C$, each to each, and tho nucluded anglo $A B D$ is equal to the inchudud anglo $F B C$,
therefore tho base $A D$ is equil to tho base $F C$, (1 4.)
and tho trimnglo $A B D$ to tho tuangle $F B C$
Now the parallelogram $/ B L$ is double of tho thamerlo $A B D$, ( 14 ) becauso they aro upon tho samo baso 73]), and botweon tho suluno parallols $B D, A L$;
aleo tho square $G D$ is doublo of the triangle $F B C$,
bocauso these also rae upon the samo baso $F B$, and betwoen the samo parallicls $F B, G C$.
But the doubles of equals are equal to one another; (ax 6)
therefore the parallelogrom $B L$ is equal to tho squano $G B$.
Sumilarly, by joinng $A E, B K$, st ean be proved,
that the parallologiam $C L$ 1s oqual to tho syuaro IIC.

- Thorefore the whole squave $B D E C$ is equal to the two squacs $G B$,

MC; (ax.2)
and the square $B D E C$ is described upon the strangt line $B C$, and tho squares $C B, \Pi C$, upon $A B, A C$ :
therofore tho square upon tho side $B C$, is equal to tho squares upon the sides $A B, A C$.


## PROPOSITION XLMIIL, TIEOREM.


 these too sules is a right anglo.

Tot the equano demenewl upein $B C$, ono of than skins of tho himgho


90 How is th shewn that equal tringgles upon the same base on inon equal bikes, have equal altutudes, whether they are situated on the same side or upon oppouste hides of the same straght lino
91. In Lac. r. 37, 38, if the trangies are not towards tho eame parts, shicw that the straght line joining the vertiecs of the triangles as bisected by the hno contrinug the bascs.
92. If the complements (fig. Luc. r. 43) be squanes, determino their relation to the whole parallclogrem.
93 What 18 meant by a paralielogram beng applied to a straight line?

1. Is the construction of Euc. 1. 15, peffectly general ?
2. Define a squaic without includung superfluous corditions, and explan the mode of constructing a square upon a given straight line in conformity with such a defimition.
( 00. The sum of the angle
convesse true? If not, why?
3. Concciving a square to be a figure bounded by four equal straight lines not nececsarily in the same plane, what condition respecting the angles is necessary to complete the defintions
4. In Eucld 1. 17, why 15 it necessary to prove that one side of each square descubed upon each of the sudes contaning the night angle, should be in the same str ught lane with the other side of the triangle?
5. On what assumption is an analogy shewn to cerst between the product of two equal numbers and the surfacc of a qquare ?
6. Is the triangle whose sudes are $3,4,5$ right-angled, or not ?
7. Can the side and dagonal of a square be represented sumultaneousiy by niny finte numbers.
102 By means of Euc. I. 47, the squanc roots of the natural numbers, $1,2,3$ 4, Se may be represented by straight lines.
8. Prove the 17 th Prop. of Book r . by describing the squares on the sides towards the hypotenuse, and shewing that they are divided by the sides of the square on the hypotenuse anto segments which may be so placed as to corer exactly that square.
9. If Euclad rr. 2, be assumed, cnunciate the form in which Euc 1. 47 may be cipressed
105 Classify all the properties of triangles and parallelograms, proved in the First Book of Euclid
10. Nention any propositions in Dook r. which are included in more gencial oncs thech follow.
11. Degnnung with the forty-seventh proposition of the First Book of Luch'd's Dlements, trace backwards how many of the propositions of the book are necessary to the proof.
12. How are converse propositions gencrally proveds Do you know of any except on to this general rule ?
109 Trhang soludity as a fundamental idea of Geometry, how would you define a supeffices, a lane, a poont?
13. What gencral classification may be made of the Propositions contaned in the First Book of Euchd?

## DEFINITIONS.

## I.

Drery ight-angled parallelogram is called a rectangle, and is said to bo contaned by any two of tho staight linos wluch contain ono of the right anglos

## II.

In every parnliclogram, any of tho narallolograms about a diametor together with tho two complemonts, is callod a giomon.

"Thus the parallelogram $I I G$ together with the conיplements $A T, T C$, 13 the gunmon, which is more briefly espmesed by the letters $A G K$, or $A H C$, whinh are nt the opposite angles of the paaliclogiams whech make the gromon."

## PROPOSITION I TIIEOREM.

If thero bo tioo straight lines, one of whoh is durded anto any number. of parts; the rectangle contancel by the two straught hurs, as equal to the rectangles contanid by the unduvded line, and the sevcral paits of tho rectangles cont
divided lunc.

Let $A$ and $B C$ bo two straight lines;
and lot $B C$ be drided into any parts $B D, D L, E C$, in the points $D, P$
Then the rectangle contaned by the straight lincs if and $B C$, shall be oqual to tho rectangle contamed by 1 and $D D$, together with that contaned by $A$ and $D E$, and that contamed by $A$ and EC.


From the point $D$, draw $B T$ at right angles to $B C$, (土. 11) and malre $B G$ equal to $A$; (土. 3 )

## Page 177

'Elements of Algebra (part l) for the use of Schools' $\mathrm{b}_{f}$ Rev. J. Colenso , Longmans, London. This edition 1 s the 15 th and dated 1860, although the Ist edition may have appeared in 1849. This was another standard text in many of the better schools. The 'simple' equatıons shown $\quad$ ndicate the manıpulative type of algebra that was popular in text books and often rather extreme. Another example from the book on long division (p.2l) is:
'Question 8

$$
\text { Divade } x^{4}+y^{4}-z^{4}+2 x^{2} y^{2}-2 z^{2}-1 \text { by } x^{2}+y^{2}-z^{2}-1
$$

## Page 178

"First book of Arithmetic for the use of Schools" (revised edition) 1857. Dublin.

This was produced by the Commissioners of National Educa_ tion in Ireland and provided a cheap, useful arithmetic text book. It was used in many of the elementary schools inspected by the Newcastle $C_{0}$ mmission $\ln 1859$ and was referred to as the 'Irısh' series in that report. The answers were given and the book was quite physically small with 143 pages of text.

## Ex 13.

1. $\frac{x+\frac{6}{7}}{7}-\frac{x}{2}=\frac{4}{35 x}-\frac{5 x}{14}$.
2. $\frac{2 x}{3}-\frac{1-\frac{1}{4 x}}{4 x}=\frac{x-1}{2}+\frac{x}{6}$.
3. $\frac{8 x+5}{11}+\frac{7 x-3}{6 x+2}=\frac{4 x+6}{7}$.
4. $\frac{2(4 x+3)}{x+3}+\frac{3}{x+1}=8$.
5. $\frac{a x}{b(x+c)}+\frac{l x}{a(x+c)}=1$.
c. $\frac{x-3}{x+2}=\frac{1}{2}+\frac{x-3}{2 x-1}$.
6. $\frac{6 x+a}{4 x+b}=\frac{3 x-b}{2 x-a}$.
S. $\left.\frac{x-3}{3} \frac{(x-1)}{36}+\frac{31}{3}=\frac{3-1}{2}-2\right)$.
7. $\frac{3-1 x}{3(3-x)}+\frac{1}{2(1-x)}=11$.
8. $\frac{1}{a b-a x}+\frac{1}{b c-b x}=\frac{1}{a c-a x}$.
9. $\frac{(2 x+3) x}{2 x+1}+\frac{1}{3 x}=x+1$.
10. $\frac{2 x+a}{3(x-a)}+\frac{3 x-a}{2(x+a)}=2$.
$13 \geq\left\{3 x-\frac{n}{3}(1+x)\right\}+\frac{1-\frac{1}{6} x}{5!}=\frac{22_{5}^{2}+\frac{1}{2} 5(x-1)}{2 s_{5}^{2}}$.
11. $\frac{x}{a+x}=\frac{a+x}{x}-\frac{2 a-z}{2 x}$.
12. $\frac{x+4}{3 x+5}+1 \frac{1}{8}=\frac{3 x+8}{2 x+3}$.
13. $\int_{5}^{1}(11 x-13)+\frac{3}{7}(10 x+3)-3\left(5 x-23_{3}^{5}\right)=28 \frac{1}{4}-\frac{1}{2}(17 x+4)$.
14. $\frac{10 x+17}{18}-\frac{12 x+2}{13 x-16}=\frac{5 x-4}{9}$. 18. $\frac{x+1 \frac{1}{2}}{3}-\frac{10-x}{33}=\frac{1-\frac{1}{3} x}{11}-\frac{1}{11}$,

15. $\frac{6 x+13}{15}-\frac{3 x+5}{5 x-25}=\frac{2 t}{3}, \quad 21 \frac{x-7}{x+7}=\frac{2 x-15}{2 x-6}-\frac{1}{2(x+7)}$.
16. $\frac{139 x+1}{3 r+1}+\frac{8 r+5}{x-1}=52$
$23 \frac{7 x+1}{x-1}=\frac{35}{9} \cdot \frac{x+1}{x+2}+3 x$.
17. $\frac{17}{6 x+17}-\frac{10}{3 x-10}=\frac{1}{1-2 x}, \quad$ 25. $\frac{11}{12 x+11}+\frac{5}{6 x+5}=\frac{7}{4 x} \frac{7}{4}$.
$20 \frac{3}{3} x-\frac{(2 x-3)-\frac{1}{2}(3 x-1)}{3(x-1)}=\frac{3}{2} \cdot \frac{r^{4}-\frac{1}{3} x+2}{3 x-2}$.
18. $1_{1}^{1}\left(7 x+0^{2}\right)+1_{2}^{1}\left\{\left(11 x-\frac{1}{-}\left(x-1 \frac{1}{2}\right)\right\}=\frac{1}{6}(3 x+1)+2_{2}^{2}\{(43 x-2(2-8 x)\}\right.$
19. $\frac{6 x-7 \frac{1}{2}}{13-2 x}+2 x+\frac{1+10 x}{21}=42_{2}^{5}-\frac{125-8 x}{3}$.
20. $4 x-3(x-2)-\left[2 x-\left(\frac{1}{1} x-\frac{1}{2}\left\{10-\frac{3}{2}(x+4)\right\}\right)\right]=\frac{3}{2}(x+2)$,
21. $\frac{\hat{0}-5 x}{15}-\frac{7-2 x^{2}}{11(x-1)}=\frac{1+3 x}{21}-\frac{2 x-21}{6}+\frac{1}{105}$.
22. The following ate addational Problems in Simple Equations, presenting somewhat mole of difficulty thum those given under (41).

IN. 1. A fish was caught whose tail weighed 0 lles; his head wheghed as much as his tand and helf his body, and his body welylied as much as his fead and tani. What dud the fish wergh,
It is sometimes convenient to tahe $x$ to appesent, rot the quantity actually demarded in the question, but somo other undiown quartity on whel thes one deperels. It is only experaence, howercy, and pactice whuch can sugrest these cares, but thes example is one of tlem.
Let $x=$ werght of body;
$\therefore 9+!r=$ werght of tan $+\underline{l}$ body $=$ weight of head; but the body wambes as much as lead and tul,

$$
\therefore x=\left(0+\frac{1}{2} x\right)+9 \text {, whence } x=36 \text {, wcight of botly; }
$$

$\cdot 9+\frac{1}{2} r=27$, verght of had
and the whole fish werghed $27+36+0=72$ lus.
Ex 2. A gamester at one sitiog lost ? of his money, and then won 10s; at a sccond he lost 3 of the 1 emander, and then won 3 , rnd now he has 3 gumeas leit. INow much money had he at fust?

$$
\text { Inet } x=\text { number of shinhers le had at fist; }
$$


he then won 10 s , and had, thenetons, $\frac{1}{2} \tau+10 \mathrm{~m}$ hand,
 aud he then wan 38 , and so has $j(1, x+10)+3$ shather,
whel, by the question, is equel to 3 guncon, on cos,

En. 3. Find a number such thet if E $_{4}$ of it be subtiacted fiom 20), ind $x_{2}$ of the $2 e m a i n d e t$ from ${ }^{2}$ of the ontgimen number, 12 theses the second remainder shall be half the onarime number.

Let $x=$ the number;
$\therefore 20-\frac{3}{b} x=1$ st 2 cmainder , and $\frac{1}{2} x-\frac{5}{2}\left(20-\frac{3}{3} x\right)=$ Ind remainder; $\therefore 12\left\}-S_{1}\left(20-\frac{1}{8} x\right)\right\}=\frac{1}{2} x$, by the question, whence $x=2 k$.
Ex. 4. A cestain number consists of two digits whose difference is 3; ond, af the digits be inverted, the number so fomed will be: of the former: find the cliginal number.



Page 180 and reverse.
'Elenents of Euclid' - 'for the use of schools and colleges' by I. Todhunter lst edition 1862.

This edition 1887. 400 pages.
This work was the one frequently mentioned in the Public Schools Commission Report of 1864. It dealt with the first 6 books and portions of books 11 and 12. There were also sections on the history of Euclad Elements and notes grvins alternatrve methods of proof. A section of exercises contained questions from college and university examinations and some of these are shown on the photocopy.

## EUCLID'S ELLGMENTS.

## PROPOSIIION 47. THEOREM.

In any $n g h t$-angled to iangle, the square whach is described on the side subtending the mght angle is equal to the squares described on the sides which contain the right angle.

Inet $A B C$ be a right-angled triangle, having the right nngle $B A C$ the square described on the side $B C$ shall bo equal to the scuares described on the sides $B A, A C$.

On $D C$ describo tho syauro $B D E C$, and on $B A, A C$ desenibo tho squares $G B, I H C$ [I 40 though $A$ diaw $A L$ CD: $[131$ CD;
[I 31
and join $A D, F C$
Then, because tho anglo $B A C 13$ a right anclo, [IIypothess and that the angle $B A G$ is also a right anglo, [Dfinintion 30 .
tho two sts ught limes $A C, A G$, on tho opposito sides of $A B$, mink with it at tho point $A$ the adjacent angles equin to two melit angles,
thereforo $C$ is in the samo siraight how whith $A O$. [I I4. For the sume cason, $A B$ and $A I I$ aro in the samo straygt his.

Now the anglo $D E C$ is equal to the anglo $F B A$, for cach of them is a inght angle.
[Axom 11 . Add to each tho anglo $A B O$.
Therefor the whole anglo $D B A$ is equal to the whole angle $F B C$
[Axiom 2.
And because the two sides $A B, B D$ are counl to the two bides $F B, B C$, each to each; [Defunzon 50 and the angle $D B A$ is equal to the anglo $F D C$;
thonefore the tranglo $A B D$ is equal to tho tuanglo
[I. 4.

Now the parallelogram $B L$ is double of the trianglo $A B D$, because they are on the same base $B D$, and between the sime parallels $B D, A L$.
[I 41
And the square $G B$ is double of the triangle $F B C$, because they are on the same base $F B$, and between the same prallels $F B, G C$. II. 41 13ut tho doubles of equals are equal to one another. [ $1 x$ B. Therefore the parallelogram $B L$ is equal to the square $G D$

In the same manner, by joinng $A E, B K$, it can bo shemn, that the parallologram $C L$ is equal to the square $C H$ The efore the whole squaso $B D L C$ is equal to tho two squares $G B, I I C$
[42iom 2. And the squane $B D E C_{1 s}$ described on $B C$, and tho squares $A$ nd the squate $B D E$
$G B, I I C$ on $B A, A C$
Therefore the square described on the sido $B C$ is equal to the squares desenbed on the sides $B B A, A C$.

TYacreforo, in any right-angled triangle \&c. Q E D

## PROPOSITION 43 THEOREM

If the squane described on one of the sades of a tri angle be equal to the squares described on the other tueo angle be equal the squares described on the other tato
sides of $2 t$, the angle contained by theso two sides is a light angle.

Let tho square described on $B C$, one of tho sides of tro tringglo $A B O$, be equal to tho squanes deseribed on the other sutes $B A, A C$. tho anglo $13 A C$ shall bo a nirgt angle.
Erom tho point $A$ drav $A D$ at right angles to $A C$; [III. and mako $A D$ equal to $B A$; [I, 3. and jom $D C$.

Then because $D A$ is equal to $B A$, the squaro on $D A$ is equal to tho fquare on $B A$.
To each of theso add, the square on $A C$.
Therefore the squares on $D A, A C$ are equal to the squares on $B A, A O$.
[Axiom 2.
4-2
$\therefore \therefore$ 为


## I 46 to 48

125 On the sides $A C, B C$ of a trianglo $A B C$, squares $A C D L, B C F I I$ are descubed: shew that the strayght lincs $A F$ and $B D$ aro cequal
126. The squase on the side subtending an acuto angle of a tranglo is less than the squares on the sades contaming the acutc anglo
127. Tho square on the sado subtendeng an obtuse angle of a tinuglo is gicater than tho squares on the sides contaning the obtuse mingle.

128 If the square on one side of a triangle bo less than tho symarcs on the other two sides, the angle conta red by theso sides is an acute angle, if greater, an obtiso angle
129 A shaght lino is drawn intersecting the two sides of a right-angled triangle, and cach of the acute angles is joned with the points where thus straight lino inter sects the sides respectirely opposite to them. shew that the squarcs on the joming stiaight lines ano together equal to the square on the hypotenuse and the square on the straght no drawn miallel to it
130 If any pount $P$ be jomed to $A, B, C, D$ the angular points of a rectangle, the sfluares on $P^{\prime} A$ and $P C$ aro together equal to the squares on $P B$ and $P D$
131. In a right-angled trangle if the squire on one of tho sides contanurs the richt angle bo thirco tames tho squate on the other, and from the rimht angle two straight squas bo diawn, ono to bisect tho onposito sido straight other perpenderviar to that side, these straight lines dunde the right angle into the ee equal parts

132 If $A B C$ be a tıinglo whoso angle $A$ is a rimht angle, and $B E, C F$ bo drawn bisecting the opposito sides iespectively, shew that four times the stum of the squares on $B E$ and $C F$ is equal to five times the squane on $D C$
133. On the hypotenuse $B C$, and tho sides $C A, A B$ of a rephtangled trianglo $A B C$, squares $B D E C, A F$ and $A G$ are described. shew that the squales on $D G$ and

## II. 1 to 11 .

134. A straight lino is divided into two parts, shew that if twice the rectangle of tho paits is equal to tho sum of the squares described on the parts, the stragint hine is bisected

135 Divide a given straight lime into two puts such that the rectanglo contaned by them shall bo tho greatest possible
136. Construct a rectanglo equal to tho difference of two given squaces
137. Dhyude a given staryht line into two purts such that the sum of tho squarcs cas the two parts may bo tho least possible
138. Show that the squane on the sum of two stranght lanes together with the squaro on ther dufference is doublo the squares on the two shayght mes
139. Divide a given stragith lino into two pats such that the sum of ther squates slaall be equal to a given square
140. Dipide a given sta aight line into two pats such that the square on one of them may be double the square on the other In figuro of II. 11 if CII be produced to meet $B F$ at $L$, show that $C L$ is at riglit angles to $B F^{\prime}$
142. In the figule of II 11 if $B E$ and $C I I$ mect at $O$, shew that $A O$ is at rught angles to C/I.
143 Show that in a strenght lue dinded as in II. In the iectanglo contamed by the sum and rufference of tho parts is equal to the rectangle contuned by the paits.

## If 12 to 14.

144. The square on the base of an isosceles trianglo is equal to time the rectanglo contained by etther sido and by the stranglat hne intricepted betwecn the perpendicular lot fall on tho side from tho opposite anglo and the extremity of the basc
145 In any traangle the sum of the squares on the sides is equal to twice tho square on half the bise together waih twice the squane on the straught line drawn fiom the vertex to the maddle point of the basc.

## Pages 183 to 191

A selection of pages from exercise books and ciphering books dated from 1798 to 1860.

Pages 183 and 184
1798 exercise book. Two pages from a book which showed that the pupil used a text book called the 'Youth's Infallıble Instructor'.

Problems in the book indicate that apart from arithmetic problems questions on algebra (solutions of simple equations), mensuration, slmple geometry and trigonometry were also tackled. The photocopy shows a form of code used throughout the book where each Letter represented a certain number. A problem in long division was worked out using this complicated procedure ! The second photo copy shows a mensuration problem solved.

## Page 185

1806 exercise book, 12 year old boy.
This copy shows the commercial arithmetic aspect, being concerned with tare and trett.

Page 186
1809 exercise book.
All commercial arıthmetıc problems by a l2 year old gırl.
Page 187 and reverse.
1824 and 1858 clphering books showing multiplication problems and addition of money problems. In these books the problems were already written down and the pupil merely completed the working according to the rules given in the example.

## Page 188

1845 exercise book used in a private boarding schoo1. The photocopy shown is an arithmetic problem on proportion.

Page 189
1849 exercise book showing calculations on Wine, Ale and Beer llieasures'.
Page 190 and reverse.
An 1860 exercise book; the photocopy shows the extraction of a cube root. The reverse copy is from an $184 \perp$ exercise book showing a calculation using proportion.
Page 191 and reverse.
An 1860 exercise book; the pupll gave his age as 14 years and the photocopy shows proportion and a bill of sale.

In general the commercial arithmetic aspect of mathematics teaching represented by the above examples was the major content of these exercise books.
$\stackrel{\infty}{\infty}$
c Hroremere＇Prefictilei）

（1）．Fouthis Enfalletites
e mestructor：


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＂いい！＂に．．．＂








 chaquods louglet iened es viller
$\therefore$ 'tt on much fur lox, luy, larert Sil

- (t se mesthper cont wi.



WII Sarath Thornfoon
Mountit of Eluzabeth ithirkly Aurilisi" nso,
 15 ithun firich imourte9-a a







 y Crieants - al ... - Cll



CONTRACTIONS.
RULE $I$.
When there are Ciphers at the right-hand of one or both Factors, multiply the other Figures only, and place as many Ciphers to the night of the product as are in both Furtors.

EXAMPLES.


RULE II.
When there are Ciphers in any other part of the Multipher besides at the right-hand, place dot under meath and proceed as before, observing that you heep $n 2$ mind to place the first figure of ce I'roduct eacuctly under the Figure you are multiplying by.

EXAMPLES.
No. 19.
No. 20.
Mlutitijly 3450042
OS

| $\frac{2300400}{18800168}$ |
| :--- |
| 10350126 |
| 6900084 |
| 7936476616800 |

No. 21.
No. 22.
Woutioly
84391
$3114 \%$


 $\qquad$

$$
\frac{5525}{3 c 5}
$$

$$
\frac{625}{\frac{549}{329}} 5
$$

$$
\begin{aligned}
& \frac{7595}{5136} \\
& \therefore \frac{1.500}{9005} \\
& 483 \\
& \text { 劣年年告 } \\
& \text { Cuns 今, } 2 \text {, } 11.14 \ldots 2011+
\end{aligned}
$$

$$
\begin{aligned}
& \frac{26}{119.3429} \\
& 2.90619 \\
& 3
\end{aligned}
$$

$$
\begin{aligned}
& \frac{0.15 \% 0}{123170} \\
& \text { ノおゲインず }
\end{aligned}
$$



Mivee, - -te and Buer Hecusuras









$$
\begin{aligned}
& \begin{array}{l}
\frac{106}{350} \\
\frac{35-5}{60-5} \sin
\end{array} \\
& \text { 4 } \frac{0}{060}
\end{aligned}
$$

$$
\begin{aligned}
& 33 \times 35^{2}=2,245 \\
& \text { に, }
\end{aligned}
$$

$$
\begin{aligned}
& 1^{2}<.3=3.353^{3} 4 \\
& 1 \times 3 \times 夕^{2}=2 / 23 \\
& \text { 3世゙多 } \\
& 11^{2}<5^{3}-10,3
\end{aligned}
$$

$$
\begin{aligned}
& 1115^{2} \times .3-w 1117+1 / 1+3
\end{aligned}
$$

$$
\begin{aligned}
& \frac{20}{151} \\
& 204 \\
& 6 \mathrm{CH} \\
& 3.62 \\
& 3624 \\
& \cdots=-
\end{aligned}
$$

Boule


Third.

When the "free is more than one shelling an g las than two take the fount on harts south 1.0 much of the given frocee as as onnere: thai ia shilling which ago to the given quantity and guide lo 20. it will. goa the answer es

Qu. Onturtura
2106
$\frac{40}{18954}$

$$
\begin{aligned}
& \frac{8424}{183194} \\
& 12) \frac{25798.6}{2612149.108} \\
& 2109.9 .108
\end{aligned}
$$

Sheest- Mlongers.

On. Gharles Goss

$$
\text { Brught of el amsiel Groont Duly } 6 \div 1 / 85 \%
$$

d: $\sigma_{0} f_{i} d: \partial_{i}$
8 Ataf Gambuage Bintermanat 0.6 perth $0.4 \ldots 0$



2 Warmichshire oomit 15 ffmmat 0.3 men 0.3 .9
12 It of ereass sheese manat o. 6 men 0.6
E $\because 3.1 \neq .7$

## Page 193

'Algebra - for the use of colleges and Schools' by I. I'odhunter, Mackillan, London.

Photocopy edıtıon lø79. Lst edition c.l853.
A large work consisting of 608 pages.
The pretace stated "complicated problems have been excluded because they consume time".

However, the photocopy shows a lypical example of manıpulatıve algebra. Another problem ended with the pupil having to solve $738 x^{2}-8431 x+20216=0$ which did not agree with the intention in the preface. The book contalnea sections on probability and the theory of numbers which may have been only used by the mathematıcs specialısts in the public schools where the book was popular. In his introduction Todhunter stated that he had studied עe Norgan's Algebra Ior ideas in writing his own text book.

Reverse of Pase 193
"The Imperial Algebra - Mext book Ior madale class
schools and candidates for Public Examınations'. George Gill and Sons, London 1888, 335 pages.
l'hls book was malnly a guide for examınatıon work with questions from various papers as shown on the photocopy The title, descriptron, length and price indicated that ıt was surtable for a better class (or grade) of school; the book containea maınly rules and manıpulatıve technıques.

$$
\text { 6. } 31\left\{\frac{24-5 x}{x+1}+\frac{5-6 x}{x+4}\right\}+370=20\left\{\frac{17-7 x}{x+2}+\frac{8 x+55}{x+3}\right\}
$$

$$
\text { Hero } 31\left\{\frac{24-5 x}{x+1}+\frac{5-6 x}{x+4}+11\right\}=29\left\{\frac{17-7 x}{x+2}+\frac{8 x+55}{x+3}-1\right\}
$$

$$
\text { or } 31\left\{\frac{24-5 x}{x+1}+5+\frac{5-6 x}{x+4}+6\right\}=20\left\{\frac{17-7 x}{x+2}+7+\frac{8 x+55}{x+3}-8\right\}
$$

$$
\text { therefore } 31\left\{\frac{20}{x+1}+\frac{20}{x+4}\right\}=20\left\{\frac{31}{x+2}+\frac{31}{x+3}\right\} \text {; }
$$

$$
\text { therefor } 0 \quad \frac{1}{x+1}+\frac{1}{x+4}=\frac{1}{x+2}+\frac{1}{x+3}
$$

therefore

$$
\frac{1}{x+1}-\frac{1}{x+2}=\frac{1}{x+3}-\frac{1}{x+4}
$$

therefore $\quad(x+1)(x+2)=(x+3)(x+4)$;

$$
\text { thel fole } \quad 3 x+2=7 x+12
$$

thercfore

$$
4 x=-10
$$

therefore

$$
x=-2 \frac{1}{2}
$$

$$
\text { 7. } \frac{1}{5} \frac{(x+1)(x-3)}{(x+2)(x-4)}+\frac{1}{9} \frac{(x+3)(x-5)}{(x+4)(x-6)}-\frac{2}{13} \frac{(x+5)(x-7)}{(x+6)(x-8)}=\frac{92}{58 j}
$$

It is clear that the numerator and denominator of each fiachon involves the cxpucssion $x^{2}-2 x$, put thenefore $(x-1)^{2}=y$; then the equation becomes

$$
\begin{gathered}
\frac{1}{5} \frac{y-4}{y-9}+\frac{1 y-16}{9}-\frac{2}{13} \frac{y-36}{y-10}=\frac{92}{555} \\
\frac{1}{5}+\frac{1}{9}-\frac{y}{13}=\frac{92}{555}
\end{gathered}
$$

subtracting con esponding terms, we have

$$
\frac{1}{5} \frac{5}{y-9}+\frac{1}{9} \frac{0}{y-25}-\frac{2}{13} \frac{13}{y-49}=0
$$

that is

$$
\frac{1}{y-3}+\frac{1}{y-25}-\frac{2}{y-49}=0
$$

therefore
that is

$$
\frac{1}{y-9}-\frac{1}{y-40}=\frac{1}{y-49}-\frac{1}{y-25}
$$

the efore
that is

$$
\frac{-40}{y-9}=\frac{24}{y-25}
$$

$$
3(y-9)+5(y-25)=0
$$

$$
\mathrm{S} y=152,
$$

therefore

$$
y=10 \text { and } x=1 \text { н } \sqrt{ }(10)
$$

8. 

$$
x \cdot \frac{x+3 a}{c+32}=\sqrt{ } /(n c) \frac{a+3 x}{2+3 c}
$$

$$
\text { Hero } \frac{2^{\frac{1}{2}} x+3 c}{a^{\frac{1}{3}} a+3 x}=\frac{c^{\frac{3}{3}}}{a^{\frac{3}{2}} x+3 c} \text {, that is } \frac{x^{\frac{3}{3}}+3 c a x^{\frac{3}{3}}}{a^{\frac{3}{3}}+3 a^{\frac{1}{2} x}}=\frac{c^{\frac{3}{3}}+3 c^{\frac{1}{4}} x}{2^{\frac{4}{4}}+3 a^{\frac{1}{2}}} \text {; }
$$

adding and subtracting the numenator and denominator of cach filuction, we have $\frac{\left(x^{\frac{1}{2}}+a^{\frac{3}{3}}\right)^{3}}{\left(x^{\frac{1}{2}}-a^{\frac{1}{2}}\right)^{3}}=\frac{\left(c^{\frac{1}{6}}+a^{\frac{1}{2}}\right)^{3}}{\left(c^{\frac{1}{2}}-x^{\frac{1}{2}}\right)^{3}}$,
therefore $\quad \frac{x^{\frac{1}{3}}+a^{\frac{1}{2}}}{x^{\frac{3}{2}}-a^{\frac{1}{2}}}=\frac{c^{\frac{1}{2}}+x^{\frac{1}{2}}}{c^{\frac{1}{2}}-x^{\frac{1}{2}}}$; therefore $\frac{x}{a}=\frac{c}{x}$,
therefore $\quad x= \pm \sqrt{ }(a c)$

$$
\text { 0. } \quad(x+a)\left(1+\frac{1}{a^{2}+a^{2}}\right)+\sqrt{ }(2 a x)\left(1-\frac{1}{x^{2}+a^{2}}\right)=2 .
$$

Heno

$$
\{x+\sqrt{ }(2 \alpha x)+a\}+\frac{x-}{\sqrt{ }(2 a x)+a} \lambda^{2}+a^{2}-2
$$

therefore $\quad x+\sqrt{ }(2 a x)+a+\frac{1}{x+\sqrt{(2 a x)}+a}=2$;
therefone $\{x+\sqrt{ }(2 a x)+a\}^{2}-2\{x+\sqrt{ }(2 a x)+a\}+1=0$;
therefore $\quad x+a+\sqrt{ }(2 a x)=1$,
therefore $\quad(x+a)^{2}-2(x+a)+1=2 a x$;
therefore $\quad x^{2}-2 x+1=2 a-a^{2}$;
thenefors $\quad x=1 \pm \sqrt{ }\left(2 a-a^{2}\right)$.

Excrcise LXXI.
Solve tho following equations -

1. $\frac{b^{2}}{x}+c=\frac{a^{2}}{c}$.
2. $\frac{x}{a b}+\frac{x}{b c}+\frac{x}{c a}=a+b+c$
3. $\frac{9 x}{-} \frac{8}{7}+\frac{x+2}{4}+\frac{2}{9}=0$.

$$
\begin{aligned}
& \text { Pupl Teachers } \\
& \begin{array}{l|l}
x-2 a \\
2 \bar{b}-\frac{x}{x}+\frac{x-a}{b-x}=-2 & \text { 6. } \frac{x+a x-b x}{a-b}=\frac{c x-1 l}{c} . \\
5 \frac{x}{a}-\frac{d x}{c}+3 a b=1 . & 7 \cdot \frac{7 \cdot 1-3 x}{5}-8 x=944
\end{array} .
\end{aligned}
$$

8. $\frac{a^{3}-x^{3}}{a-x}-a^{2}-x^{2}=4 a(a-x)$.
9. $\frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}=b$.
10. $\frac{(x+a)(x+b)}{x+a+b}=\frac{(x+c)(x+d)}{x+c+d}$

$$
\sqrt{50-4 x} .
$$

12. $\frac{a-b}{x-c}=\frac{a+b}{x+2 c}$

$$
\text { 13. } \sqrt{ }(2 x)+\sqrt{ }(2 x-3)=3
$$

11. $\frac{4}{\frac{\overline{3}}{3}(x-1)}-\frac{3}{\frac{1}{4}(x-2)}+\frac{51}{12}(x-1)(x-27 x-2)=0$.
12. $\sqrt{a-x}-\sqrt{b-x}=\sqrt{a+b-4 x}$.
13. $\frac{3 x-2 a}{a-b}-\frac{2 x-3 a}{a+b}=1 . \mid$ 17. $\frac{a x-b^{2}}{a-b}-\frac{b x-a^{2}}{a+b}=\frac{a b}{b-a}$.
oxford Local
14. $\left.\frac{a\left(d^{2}+x^{2}\right)}{d x}=a c+\frac{a x}{d} . \quad \right\rvert\, 10 . \sqrt{x+25}-\sqrt{ } x=1$.
15. $\frac{2 x-a}{b}-\frac{3 x-b}{a}-\frac{3 a^{2}-8 b^{2}}{a b}=0$
$2115+\sqrt{x+7}=19$.
Cambruje Local

| 22. $\frac{a}{l a}+\frac{b}{a x}=\frac{1}{a}+\frac{1}{b}$. | 21. $a \frac{x-b}{a-b}+\frac{x-a}{b-a}=1$. |
| :--- | :--- |
| 23. $\frac{a x-b}{c}+\frac{b x-c}{a}+\frac{c x-a}{b}=0$. | 25. $\frac{x+1}{a+1}+\frac{x+2}{a+2}=2$. |

26. $\frac{x-\frac{1}{a}}{c}+\frac{x-\frac{1}{b}}{a}+\frac{x-\frac{1}{c}}{b}=0$.
$27 a+x+\sqrt{2 a x+x^{2}}=1$.
$28(x-a)(2-2 b)(x-3 c)=2^{3}-4 a b c-x^{2}(a+2 b+3 c)+3 c^{2}(a+20)$
27. $\frac{a-5}{a-6}-\frac{a-6}{a-7}=\frac{a-1}{a-2}-\frac{a-2}{a-3}$. Find the value of $a$.

30 If $x: x+3: 3 \cdot 4$. Find $x$.
31. If $x+1 \cdot x-3: \cdot x-4 \cdot x-6$. Find $x$.
32. $\frac{a+x}{a^{2}+a x+x^{2}}+\frac{a-x}{a^{2}-a x+x^{2}}=\frac{3 a}{x\left(a^{4}+a^{2} x^{2}+x^{4}\right)}$.
33. $\dot{6} x+75 x-16 x=x-58 \dot{3} x+5$.

3edical Preliminary
31. $\sqrt{ }(x+2)+\sqrt{ }(x-14)=8$
$35 x-3+\sqrt{4 x^{2}-3 x-4}=3 x-4$.
36. If $n x-a: n x-b \cdot: m x-b: n x-a$, find $x$
37. $x(x+2)^{3}=(x+3)(x+1)^{3}$.

South Kchungtom
$38(x+3)^{2}-3 x(1 r-1)=5 \iota^{2}-(4 x-5)^{2}$
39. $(x+a)(x+3 a)(x+6 a)=(x+2 a)(c+4 a)^{2}$.
40. $\frac{1}{3}(075-x)+\frac{1}{5}(047+2 x)=\left(3-\frac{1}{15}\right) x_{0}$
41. $\frac{x}{x+a}-\frac{x-a}{x}=\frac{a^{2}}{(x-a)^{2}}$.
42. $\sqrt{x+1}+\sqrt{x-1}=\frac{2}{\sqrt{x+1}}$.
43. $\sqrt{9 x+10}-\sqrt{4 x+7}=\sqrt{x+11}$.

Various Examinations
44. $\frac{6 x-7 \frac{1}{3}+2 x+\frac{1}{13-2 x}+26 x}{25}=4 \frac{5}{15}-\frac{128-8 x}{3}$.
$45 \sqrt{ }(3+2)+\sqrt{ } x=\frac{6}{\sqrt{(3+x)}}$.

Page 196 and reverse
'Practical Plane and Solid Geometry' by John Rawle revised and enlarged edition, 15 th edition 1888. Simpkin, Marshall ci Co. Iondon. 130 pages. This book was aesignea for the use of candidates tor examinations of the science and Art Department, Rawle being an examiner in this subject. The rirst edition date was not 11 stea but the 15 th edition indicated that 140,000 copies had been sold previously. The whole book consisted of aetinitions, constructions according to set rules and problems; in fact, very much akin to the way in which arithmetic had developed. The preface implied the view that technical instruction had become equated with mere education in practical skills. Page 197 and reverse.

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'Elementary Algebra for schools' by H. Hall and b.Knlght
1st eartion 188\zeta. This edition 1942 Lhacmıम1an & Co.
London (55b pages).
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The length of time of publication of this work indicated by the above dates shows the 'popularit.' of the work. Looking at the photocopy of the manıpulative algebra which was featured throughout, il would not have been too popular with schoolboys ! The methods were explained throughout and there were numerous workea examples illustrating each stage but the complexity and number of problems of sımılar type were taken to an excess (there
were 234 questions on surds). The preface pointed to a new feature introduced in 1907 - a section on graphs and plotting graphs, no doubt influenced by the various reports recommending more 'practical' mathematics issued at the start of the 20 th. century.

## CLASS-SHEET 13،

DTFFINIXIONS, etc.:-201.-An Ellipse is a plane figure, contaned by one contanuous curved lune (Fig 266 ) Its greatest length, A B, is called the Major or Transverse Axis, or the Longest Diameter, Its grentest Diameter. The major and minor aves bisect ench other at right angles in a point L, culled the Centre. Any fine drewn through $k$, and terminted by the curie at each extremity, is $i$ diameter, as $H M$, or N $O$, but A B is the longest, and CD, the shortest dhameter that can be diawn in an ellpse $A l l$ diameters are bisected at $E$, and eveh dirmeter divides the cllipse into two equal and smmar puts.
202.- On the mujor axis, A B, cquadstint from E, are siturted two frxed points $F$ and $G$ wach of these points is called a Focus (plural focz). The positions of the foct are regulated by the length of ench axis They are so placed that, if from any point, II, in the curve we draw lines to eacll focus, the two lanes are together
cgual to the length of the major atis. Thus, $\mathrm{HF}, \mathrm{HG}$, equal to the lengf of
together equl $A B$

203.- $\boldsymbol{\Lambda}$ Tangent to an ellipse is a straght ine that touclies the curve in one pomt, is the line 1 J, at II, the point of wintact


205.-The mijor and minor nyes of in clltpse divide the figure fato four equal and mmilar parts, as $\Lambda \mathrm{L}, \mathrm{C},\left(\mathrm{CE} \mathrm{B}_{1} \mathrm{BL} \mathrm{L}\right)$ and D D A .
200.-If from anv potat It in the cirve, we thaty a dimeter If M, and then
 are c that Conjugate Diamotors.
207.-Lllipses can be thann in infinte variety, ns to length and wdith the
 mati be equal. but the neare tha ate to cach other in length, the closed toes the

 and thengi whatly turnagy it round, bath hathy any of the sinfite the form of in

208.-- An thipe is ofien inaceurntely cillulan mat, but the latter is a disturet
 other, like thie shete of an usis.
209.- - Pormplery is the bound try line of a curvilumeal fivure. Thus, the perphery of 2 cirule is to urcumfurcme, and the periphery of an ellipse is its curved boundary.

PROBLEM 141.-To find the Foci of an Rllipse; and then to draw the Pliptio Curve by moans of intersecting arce, the major axis $P Q$, and mino axis $T V$, being given.
1.-With T-one end of the minor ans-as centre, axis-as rechens, strihe are Y, cutting the major anis at $\mathrm{F}^{2}, \mathrm{~F}^{2}$. These founts ale the requtuel tot.
2,-1Between $\mathrm{F}^{1}$ and $\mathrm{X}_{\text {, mar }}$ any number of points 1 , 2. 3. 4.
3.-With centics $F^{1}, F^{2}$, and ralius P 1, strke arcs a $a, a, a$. With the sime cut arcs $a, a, a$ rans $Q 1$, cut arcs $a, a, a, a$, at $b$,
 - Wha, b.

Wini each focus as centre, and rachus $P$ 2, strike vics $c, c, c, c$. With -In the came way use points 3 and cut thesc arcs $d$, $d$, 1
In the sme way use points 3 and 4 to get $g, g, h, h$. Ihrough pomen b, $d, g, h$, draw the culye of the ellirsc.
Note (a). - The ponts 1, 2, 3,, , my be at any dint nuce apret, but it is more convenuent auses confuesons decriase in tengeh towards 1. Do not muhe the arcs too long, is this $X^{\text {(b) }}$ The major axis and foct being given, to find the minor ayss - Bisect $P Q$ at X. With X P or XQ as raduuq, and the

PROBLTM 112 -Draw the curvo of an mllipse, by means of intorsocting lines. The longths of tho major and minor asos, $\Lambda B, C D$, are given.
5,-Mrect CDat E Brect ABnt X, byapeipen-
duulir. Mabe X $G$. X H, ench equal to EC or ED
2.-Wath centrot G, II, nud 1athus X A, vithe aus at K, K, K, K. Witl contics $B, B$, and mulias $X G$, cut thise ancs at L, M, N, 0 .
3. $-\mathrm{Mm} \mathrm{M}, \mathrm{M} O, O \mathrm{~N}$, A N, BM, BO, A X, BX cachintothe same mumpr of cyual parts
 - say four.

 med 2 G at $x$, Jhrough 3 , daw a he to mect 3 Gat $x$.
255.-In Solid Geomotry, or Orthographio Projection, the visual mys are parallifs, and nn object is represented exactly the sime size, by SCAlr, no and sections, machucry can be constructed, and a house or a ship can be buif.

258 - Suppose we take a plece of drawing-paper, and tack a portion of 1 t $A B X Y$, to 2 ectecal wall ( $\Gamma_{1 g}$ 362), and tach the remaming portion, $C D X Y$

 intersecting line, or ground ine.
(This line is sometimes lettered, I L, for intirsectivg Line) G HE FIJ is a rectangular block placed upon the shelf, with two of its fices parallel to the wall If we magine the eve to be vectically over cucry part of lhe block, at the same tume, rect angle, GK L H, whych 15 the plan of the block Now, suppose the position of the cye to be changed, and tet us look forward in a direction perpendicular to A B X Y, the eyc being opposite to cvery part of the block, at the same time. Then, pt E is opposite to, or is projected nt, $\mathrm{pt} c^{\prime}$, on the vertical plane A B X Y, pt F is profected nt $f^{\prime}$, pt G, at $S^{\prime}$, and H , at $h^{\prime}$. The rectangle, $c^{\prime} f^{\prime \prime} b^{\prime} h^{\prime}$, is the elevation
257.-It is evident that it would ise impossible to make drawngs upon a piece of paper folded in the manner shewn in I ig 362 . If ue talae the paper from the
 appear is indicated in rig 362 . The operntions cxplaned in Tig 362, can be performed upon this firt shect of paper. We draw the intersecting line, X Y ( 5 jg 363), at any convensent position, horizont illy, zcross the paper. All above $X X$, represents the vertical plane of projection, and all belozv $X Y$, represents the horizontal plane of projection. We draw fievations (hortiontal froccitens) upon the HORILON IAL PLANE OF PROJECTION.

Note - An Ortfagrafher frajection neens the correct representution of an object upon aplane. The proicturs-the lues by which we obtann the profection-are drawn perpen
 Fige 382,903 , and for many of the following dingrims, the student should foid a piece of
cardboard to represent the yert and hor planes of projection, and place the model in lts proper position. This will help the student io sce how the croblemi is to te werked
(B) PROJECTIONS OE LINES, PLANES, AND SOLIDS.

The diagrans in this Chatter are drawn to a redured scale. They showld be oprent rulu -ine in the duagrams.
TIG. 364.- -1 B C D repre sents the draving paper. $\lambda$ Y ${ }_{1}$ s the inhersecting line $A B X Y$ is the aerlical plame, for clrva. touss XYCD is the hortsontal plane, for plans.
Projections of a Lino, 11" long, prin. to one or both planes of projection.- I.--Line co 19 the clev. of a line, parl. standing verticallt upontic it P (hortzontal plane). Get its plinn


Draw $e^{\prime} c$, perp. to XY . Ihen $e$ s the requred plan of the line $\epsilon^{\prime} f$.
II.- $g^{h} h$ is the plan of a hor. line, perp. to the V. P. Get ats elev. Draw $g^{\prime}{ }^{\prime}$ Io $X$. Then $f$ is the End Elev, of line $g$ h.
III. $-z^{\prime} j^{\prime}$ is the elev of a lane purl to the II $P$ and V P. Get ats plan Drw IV. (projectors) from $z^{\prime}, J^{\prime}$, perp to X X. Draw the PIAN $2 \%$ pul to X X
IV.-Projections of a Square of $11^{\prime \prime}$ sides -The square, $k^{\prime} l^{\prime \prime} m^{\prime} n \prime$ stands on the $H \mathrm{P}$, and is parl to the
$k^{\prime}, l^{\prime}$. Draw the rcan $\& l$, parl to XY .
V. $\llcorner 0$ o is the plin of a square, sinnding vertically on one side, and perp. to the V. P. Get its elev. Draw projec. from 0 . Make the klev. $o^{\prime} q^{\prime}=0 p$.
VI. $-r^{\prime} s^{\prime}$ is the elcy of a square, parl to the $\mathrm{H} P$, and with 2 sides perp. to the V. P. Get its plan. Draw projecs. from $r^{\prime}, s^{\prime}$. Draw $r s$, parl to $X \mathrm{Y}$. Mahe $r t, s u=r s$. Join $t u$. Then $r s t u$ is the riAN of the square $r s^{\prime}$.
Note (a)-The plas and ele, of the same point are ahvoys in a line drwwn at Richt of $f^{\prime} f^{\prime}$ or $\xi^{\prime} h$, the pian of $\epsilon^{\prime} f^{\prime}$ is a pont $c^{\prime}$, and the elev of $g$ is a point $\xi^{\prime}$, - If $c$ is the




 clined to the $H$ P. or V P $-I$ - $a^{\prime} b^{\prime}{ }^{25}$,
the elev of a line, inclined at $60^{\circ}$ to the II $1{ }^{2}$., the elce of a line, incimed at is, pts $a^{\prime}, b^{\prime}$, nire
but parl. to the V. P. that equidstant from the V. P. Get the pian Draw projecs from $a^{\prime}$
II. $-c d$ is the pl in of a her line, inclincil I $15^{\circ}$ to the V. P Get the eler Draw propes from $c, d$. Draw the elev. $c^{\prime} d^{\prime}$, purl to $X Y$

III.-A Square, of $1 l^{\prime \prime}$ sides, inclined
 Get its plan Draw projecs from $c^{\prime}, f^{\prime}$. Draw $c f$, pirl to X X. Mike $c$ sit $f=$ $\prime^{\prime} f^{\prime}$. Join $g h$, then $e f g^{h}$ is the $\mathrm{PI} A N$ of the square $c^{\prime} f^{\prime}$
IV. $\boldsymbol{m}^{2} \mathcal{I} f$ the plan of a square, standing on the H P , and inclined at $30^{\circ}$ to
 ${ }^{\prime} j^{\prime}$ ' then $z^{\prime} y^{\prime} k^{\prime} b^{\prime}$ is the requred EfFY,
2

Hz $\qquad$ $\sin x$
An

N

250
ALGEBRA
Example. Find the value of

$$
\frac{a}{(a-b)(a-c)(x-a)}+\frac{b}{(b-c)(b-a)(x-b)}+\frac{c}{(c-a)(c-b)(x-c)} .
$$

Changing the slgn of one factor in each denominator, so as to preserve cyche order, we get for the lowest common denominator,

$$
(a-b)(b-c)(c-a)(x-a)(x-b)(x-c)
$$

The whole expression has for its numerator
$-[a(b-c)(x-b)(x-c)+. .+\quad]$
or

$$
\begin{aligned}
& -\left[a(b-c)\left\{x^{2}-(b+c) x+b c\right\}+\right.\text {. } \\
& \text { according to powers of } x \text {; thus }
\end{aligned}
$$

Arrange it according to powers of $x$; thus
coefficient of $x^{2}=-\{a(b-c)+b(c-a)+c(a-b)\}$

$$
=0 ;
$$

coefficient of $x=\left\{a\left(b^{2}-c^{2}\right)+b\left(c^{2}-a^{2}\right)+c\left(a^{2}-b^{2}\right)\right\}$

$$
\begin{equation*}
=(b-c)(c-a)(a-b) ; \tag{Art2.3}
\end{equation*}
$$

terms which do not contan $x$

$$
=-\{a b c(b-c)+a b c(c-a)+a b c(a-b)\}
$$

$$
=-a b c\{b-c+c-a+a-b\}
$$

$=0$.
Hence the expression $=\frac{(b-c)(c-a)(a-b) x}{(b-c)(c-a)(a-b)(x-a)(x-b)}$

$$
=\frac{x}{(x-a)(x-b)(x-c)} .
$$

Note. In examples of this hind the work will be much fachitated of tho student accustoms himsolf to readily writing down the follow mg equatents.

$$
\begin{aligned}
(b-c)+(c-a)+(a-b) & =0 . \\
a(b-c)+b(c-a)+c(a-b) & =0 .
\end{aligned}
$$

$$
a^{2}(b-c)+b^{2}(c-a)+c^{2}(a-b)=-(a-b)(b-c)(c-a) \text {. }
$$

$$
b c(b-c)+c a(c-a)+a b(a-b)=-(a-b)(b-c)(c-a)
$$

$$
a\left(b^{3}-c^{2}\right)+b\left(c^{2}-a^{2}\right)+c\left(a^{2}-b^{2}\right)=(a-b)(b-c)(c-a)
$$

Some of the identities in Examples xxix $c$ may also bo re membered with advantage.
*EXAMPLES XXIX. d

1. $\frac{a}{(a-b)(a-c)}+\frac{b}{(b-c)(b-a)}+\frac{c}{(c-a)(c-b)}$.
2. $\frac{b c}{(a-b)(a-c)}+\frac{c a}{(b-c)(b-a)}+\frac{a b}{(c-a)(c-b)}$.

## 5II] <br> CYCLIC ORDER

3. $\frac{a^{2}}{(a-b)(a-c)}+\frac{b^{2}}{(b-c)(b-a)}+\frac{c^{2}}{(c-a)(c-b)}$.
4. $\frac{a^{3}}{(a-b)(a-c)}+\frac{b^{3}}{(b-c)(b-a)}+\frac{c^{3}}{(c-a)(c-b)}$
5. $\frac{a(b+c)}{(a-b)(c-a)}+\frac{b(a+c)}{(a-b)(b-c)}+\frac{c(a+b)}{(c-a)(b-c)}$.
6. $\frac{1}{a(a-b)(a-c)}+\frac{1}{b(b-c)(b-a)}+\frac{1}{c(c-a)(c-b)}$.
7. $\frac{b c}{a\left(a^{2}-b^{2}\right)\left(a^{2}-c^{2}\right)}+\frac{c a}{b\left(b^{2}-c^{2}\right)\left(b^{2}-a^{2}\right)}+\frac{a b}{c\left(c^{2}-a^{2}\right)\left(c^{2}-b^{2}\right)}$.
8. $\frac{(x-b)(x-c)}{(a-b)(a-c)}+\frac{(x-c)(x-a)}{(b-c)(b-a)}+\frac{(x-a)(x-b)}{(c-a)(c-b)}$
9. $\frac{b c(a+d)}{(a-b)(a-c)}+\frac{c a(b+d)}{(b-c)(b-a)}+\frac{a b(c+d)}{(c-a)(c-b)}$
10. $\frac{1}{(a-b)(a-c)(x-a)}+\frac{1}{(b-c)(b-a)(x-b)}+\frac{1}{(c-a)(c-b)(x-c)}$.
11. $\frac{a^{2}}{(a-b)(a-c)(x+a)}+\frac{b^{2}}{(b-c)(b-a)(c+b)}+\frac{c^{2}}{(c-a)(c-b)(x+c)}$.
12. $a^{2} \frac{(a+b)(a+c)}{(a-b)(a-c)}+b^{2} \frac{(b+c)(b+a)}{(b-c)(b-a)}+c^{2} \frac{(c+a)(c+b)}{(c-a)(c-b)}$.
13. $\frac{a^{3}(b-c)+b^{3}(c-a)+c^{3}(a-b)}{(b-c)^{3}+(c-a)^{3}+(a-b)^{3}}$.
14. $a^{2}(b-c)+b^{2}(c-a)+c^{2}(a-b)+2(a-b)(b-c)(c-a)$.
15. $\frac{a^{2}(b-c)+b(c-a)+c}{(b-c)^{3}+(c-a)^{3}+(a-b)^{3}}$
16. $\frac{a^{3}(b-c)+b^{3}(c-a)+c^{3}(a-b)}{a^{2}(b-c)+b^{2}(c-a)+c^{2}(a-b)}$
17. $a^{2}(b-c)^{5}+b^{3}(c-a)^{3}+c^{2}(a-b)^{3}$.
18. $\frac{\frac{1}{a}(b-c)+\frac{1}{b}(c-a)+\frac{1}{c}(a-b)}{\frac{1}{a}\left(\frac{1}{b^{2}}-\frac{1}{c^{3}}\right)+\frac{1}{b}\left(\frac{1}{c^{2}}-\frac{1}{a^{2}}\right)+\frac{1}{c}\left(\frac{1}{a^{2}}-\frac{1}{b^{2}}\right)}$.
19. $\frac{a^{2}\left(\frac{1}{c^{2}}-\frac{1}{b^{2}}\right)+b^{2}\left(\frac{1}{a^{2}}-\frac{1}{c^{2}}\right)+c^{2}\left(\frac{1}{b^{2}}-\frac{1}{a^{2}}\right)}{\frac{1}{b c}\left(\frac{1}{c}-\frac{1}{b}\right)+\frac{1}{c a}\left(\frac{1}{a}-\frac{1}{c}\right)+\frac{1}{a b}\left(\frac{1}{b}-\frac{1}{a}\right)}$.


Pages 200 and 201
"Arıthmetic" by $\mathcal{C}$. Pendelbury.
1st eaition 1886. I'his eartion 1908. G. Bell London. l'hls very popular text book presented "the sclence of Arlthmetic as is neeuful for school use and for the Civil service and other examinations" accoraing to the pretace. It servea the needs of what was callea a 'clerk's euucation' and, as shown by the photocopies, gave what could now be regaraed as ridiculous problems. Note the square root example No. 56 asking for the square root of 1.57 to 22 decimal places. l'he questions on compound proportion involve the artificial situation of cows chewing at a unlıorm rate some grass which is frowing uniformly. Yenaelbury also produced a book "The Shılling Arıthmetıc" which 1 s reputed to have sold $1,000,000$ copies. Finally, he was one time secretary of the Mathematical Association.

No photocopy given.
"The Oxford and Cambridge Algebra for use of Secondary and Elementary Schools". 'Suitable for candidates for publıc examınations' published by George Gill London. (crrca 1895). 200 pages.

This was a version simılar to Gıll's Imperial Algebra but with easler questions and less content made for the lower grade schools. Extensive sets of rules were given; for instance when dealing with the factorisation of trionomial expressions 9 cases are dealt with separately in a mechanıcal method. Questions were gıven from many examınation
papers; a question from the Lundon School Board Exam for scholarshıps 1886 was:

Solve $\frac{x-2}{6}+\frac{5 x-1}{16}=\frac{3 x+1}{8}$

| (2v) | $10010 \pm 416388(23)$ | -11851219. |  | 012x19 |
| :---: | :---: | :---: | :---: | :---: |
|  | . $0013010721 . \quad$ (26) | $\cdot 00000961$ |  | 000051 |
| (28) | .00010201. (29) | 00017961. | (J) | -0003972181. |
| (31) | -0003418801. (32) | 0000001840 |  | 0000005181 |
| (34) | .0000021904. (35) | 0000063001. |  | 2 todylucs |
| (37) | 5 to 1 pluecs. (38) | 15 io 1 pluces. |  | I3 Ln a plue |
|  | 1.6 to 4 places. (11) | 1 to 1 phaces. |  | $\text { to } 11$ |
| (43) | 77 to 4 pluces. (41) | 002 to 1 place |  | 45 to 1 phices |
| (46) | 3250 to 4 placos. | (17) 3 |  |  |
| (48) | - 00521 to 4 places | (19) 5013 | 0 | dues |
| (50) | -029si to 4 places. | (51) 3 to | lac |  |
| (52) | 6 4.31 to 8 pluces. | (53) 101 | prac |  |
| (54) | . 001728 to 10 pluces | (5J) 12 to |  |  |
|  | 157 to 22 placcs. |  |  |  |

1 to 22 places
XL (d). THE SQUARE ROOT OF A VULGAR FRACTION
322. We know that " $\times \stackrel{0}{0}-\frac{1}{4}$, and genchally that the squat of any vulgat fraction is also if faction, whose numentor is the square of the riven numeator, and denominator the stiane of the meon denominator

Conversely the sfunte toot of $\frac{4}{4}$ as $\frac{4}{3}$, and genem.illy, wheneren the numeraton and denommator ane both semues, the toot of the faction is a fataction whose numerator is the root of the ornimal numeritor, and denommator the noot of the wiswni denommator
 to ats lowest terms

Eivemple: Find tho squenc iout of an.

$$
\sqrt{\frac{40}{206}}=\frac{\sqrt{49}}{\sqrt{256}}={ }^{7}
$$

323. To determine the squuc soot of anived number we expess the number in the form of an impoper fration, aud if the denominator is a square we procced as in Ait 322

Ea ample 1 . Find the squane root of 201 .

$$
\sqrt{20\}}=\sqrt{\frac{8!}{4}}=\frac{9}{2}=4 \frac{4}{2}
$$

Sumple is. Lind the squterc oot of $188_{y}$.

$$
\sqrt{168}=\sqrt{167}=\frac{\sqrt{167}}{3}=\frac{1}{3} \sqrt{1} 67 .
$$

Wo now find $\sqrt{ } 167$ to 5 placess, suppose, and divida it by 3.
(11) Fund the damater of a sphene whose volumo is $179^{\circ} \mathrm{cu}$. incheq. ( $\pi=\frac{-1}{7}$ )
(15) A lull of iron, 4 in in chameter, werghs 2 lb . ; find the werght of a ball whose diameter is 12 m .
(10) The ater of the base of a cylindrical vessel is 59.319 sq inches and ats herght is 8 inches; what would be the heaght of a perfect cube of equal volume?
(17) The sides of the base of a triangular pusm are 51,40 and 13 unches, and the height 15 is inclies; find the edge of a cube which is of the sume volume as the prism.
(18) The diameter of a sphere is 6 feet; what will be the edge of $i$ cube of the sume volme?

## XLIX. PASTURE WITH GROWING GRASS.

386. Examples of the followng kind should be noticed, in whach a pastuio is used for feeding cattle, and the concunent growth of the grass 15 taken into account.

Lisamples The giass on a certain pasture, growneng un form 1 y, arould be consumed by 70 cows in 24 days, and by 30 coves in bo dxys Ifow many cous wowld consume it $2 n 40$ days?

Ongmak icrop +21 dny $x^{\prime}$ growth $=70 \times 24 \times$ daly con cumption per cow, and " " +60 " $\quad=30 \times 00 \times$

and $36 ", \quad==(96 \times r e c q 410-1800) \times, " \quad ;\}$ $\therefore \quad 96 \times 1 \mathrm{eqd} 110,-1800=1800-1080$;
$\therefore \quad 96 \times 1$ eqd no $\quad=3600-1680=1020$;
seqd. no, of co.vs $==1 \mathfrak{S}^{2}{ }^{2}=20$.
Evample in Hath the same inta as in Example i, find for hosv long one day's growth of yrass would fced a cow, and for how long the oriqinal crop reulld do so. Find also how long it would tale 20 cons to cechraus the prasture.
(1) Ouginal capin +60 days' gowth $=30 \times 60 \times$ covis daly consumption, and " " $421, \quad "=70 \times 21 \times$
$\begin{array}{ccccc}\therefore 36 " & " & =(1800-1680) \times " & " & " \\ \therefore 30 " & " & =120 \times & " & ? \\ \therefore 30 & ;\end{array}$ $\begin{array}{clllll}\therefore 30 & " & =120 x & " & \text { " } \\ \therefore 1 " & " & =3 j x & " & " & ;\end{array}$ $\therefore$ one day's growth would fed a cow for $3 \frac{1}{3}$ daps.
380.1 Pasture witil arowing cirasq.
(ii) Agan, 30 ding' giowth $=120 \times$ con's duly convinition,
$\begin{array}{rlrl}\therefore 12 " & =40 x & " & =400 x\end{array}$
$\begin{aligned} \text { C0 } " \quad " & =200 \times \\ \text { ongmal clop } & =(1800-200) \times "\end{aligned}$

$$
=1600 \times
$$

",
$\therefore$ the original crop alone would feed a cow for 1600 dily $y$
(iil) Orig crop $+(1$ cql no $) \times$ d.tys' growth $=20 \times 1$ cqi. no $\times$ cuw's danly $\}$ comsumption,

$$
\begin{aligned}
& \text { - } 1000+\text { tecid no } \times 33=20 \times 1 \text { rqd no ; } \\
& \therefore(20-3 j) \times r e q d ~ n o=1600 \text {, } \\
& \therefore \quad 50 \times \text { read no }=4800 \\
& \therefore 20 \text { cows would axhmest the pastme in } 90 \text { days. }
\end{aligned}
$$

## EXAMPLES CCVIIT.

(1) In a pasture ground wheron grass grows momormly, and wheh contains severai day's gowth of gides, 20 cows cat all thos ghass in 7 days. If 25 cows be kept on the sume pastune ground, the fiats would last for 9 days. Irow many cows may he allowed to giaye so that all the grass may be eaten up in 6 days?
(2) If the grass on a centain pisture erows umfontioy and would be consumed by 20 sheep in 90 days and by 30 therp at 60 days, by how many shecep would il be combmed an 21 days?
 grass in 7 weeks, and 95 shecp would do 6 m 9 weeks, how many shicep would exhaust it in 6 weeks?
(1) The umfomly growing gras on at centrin meadow womk be consumed by 150 oxen in 15 diys and by 120 oxen $m 20$ diys How many oxen are pat in, if 30 ane temoved after 8 diys and the whole of the grass is consumed m \& days more?
(5) If 17 horses would owhaust a field of unformly gowing ghass in 30 diys, and 19 hotses would do so in $2 f$ diays, how many horses must be put in to cat up the whole in 8 clays, if 4 of them are remaved after 0 days?
(6) If 19 sheep consumo the unfomly growing gins on $n$ field m 13 days, and 16 of them would consumes the ghath on the same field in 16 days; in how many days woukl 107 sheep exhaust the fichl?


Page 203 and reverse.
'Complete Arathmetic'.
McDougall's Educatıonal Company Ltd. London (1897). 160 pages.

A commercial arithmetic book mainly consisting of problems and examination papers. The photocopy shows typical questions from various examinations and suggests that the whole teaching of mathematics was a preparation for these papers. Despite the plea for simpler numerical examples many of the questions involve lengthy working.

## Page 204

'Introduction to Algebra' For the use of Secondary Schools and Technical Colleges by $G$. Chrystal. 4th edition 1927, Black, London. 412 pages.

First published in May 1898 as a 'practical' mathematics text book. There is very little of a technical nature, but a lot of pure algebra. The preface states that graphical illustratıons are gıven to demonstrate mathematical notions, such as a contınuously varying function. The chapter on graphs (Chap.25) is informative and contains numerous sketches of rational functions but is hardly practical in its applications to any real problems. The photocopy shows problems leading to the solution of linear equations. Question 36 is the infamous 'pipes fıllıng a cistern' problem.

## IV．Leaving Certuficate－Lower Grade．

1．Simplify－
（1）$\frac{1444 \times 0133}{.0057 \times 1.805}$ ；
（2） 32 j of $£ \mathrm{l}, 3 \mathrm{~s} 4 \mathrm{~d}-0013$ of $2 \mathrm{~s} .6 \mathrm{~d}-031$ of 2 s .7 d ．
2．Express 41 yds． 9 ms as（1）a vulgar，（2）a decmal fraction of a furlong

3．One bookseller allows his customers 2d．in the shalling discount，and，in addition， 5 per cent on the remainng puce， another simply allows 20 per cent．on the published price of the books．Find which terms are the best for the customer by consideing the case when the published price of the books is $£ 6$ ．

4．Find the sum which，along with simple interest at 4 per cent．per annum，will amount of $£ 92,16$ s in four years．
5 A path $4 \frac{1}{2} f t$ ．wide is made round a lawn $12 y d s$ long and 5 yds wide How many squa＇e yards are thete in the surface covered by the path？

## V．Oxford Jumor Local Examination．

A 1．Sumphfy－
（1） $139 \mathrm{I}_{\mathrm{I}}^{5}-127{ }_{\mathrm{I} 5}^{4}$ ，（2）$\frac{7}{8} 3098$ ；（3） $342 \times 36$ ．
2．Multuply $£ 18,13 \mathrm{~s} 9 \mathrm{~d}$ ．by $5 \frac{2}{3}$ ，and find the value of -421875 of $£ 5$.
3．Express SI584 drs．in cwts．qrs，etc．
4．Find the rent of 52 ac． 3 ro $10 \mathrm{p}^{\mathrm{ls}}$ ，at $£ 3,9 \mathrm{~s}$ ． 4 d ． an acre．
5 If 5 oz． 11 dwts ．of siver is worth $\mathcal{L 1}, 7 \mathrm{~s} 9 \mathrm{~d}$ ，find the value of 1 lb .17 dwts .12 grs．

6．Find the simple interest on $£ 5400$ for 3 months at $2 \frac{1}{4}$ per cent．per annum
B 7．An Indan official starts with an income of 350 rupees a month，the rupee beng worth ls． 9 d ．，and recerves annually au increase of pay of 20 rupees a month．Find by how
mach per cent．his income is nominally the rupee has in the meantime fallen to 1s．3d
8 Find what amount must be invested in the $2 \frac{1}{2}$ per cents at 953 to produce a clear income of $£ 100$ a year，brokerage being $\frac{1}{8}$ per cent，and income tax being $6 d$ in the pound．
9．Find，to four places of decimals，the square root of $\frac{22}{7}$ ．
10．A bankrupt＇s debts are $£ 5317,5 s$ ，and his assets are £4076，11s 2d．How much can ho pay in the pound after defraying the expenses connected with the bankruptcy，which amount to 10 per cent．of the debts？
11．A boat＇s crew can row a four－mile course in a tidal river in 20 min ．in still water，and in 16 min with the tide．How long would it take to row the course against the tide ？
12．A cistern is fed by a pipe running continuously for the whole of the 24 hours at the rate of 2 gals．per hour．What， at lenst，must be the capacity of the cistern so that every day 24 gals may be drawn off unformly between 6 and $10 \mathrm{Am}, 12$ gals．between 2 and 3 p．m，and 12 gals between 6 and 10 pm ，and so that the cistern shall never overflow， and never contann less than 4 gals．？

## VI．Oxford Junior Local Examination．

A 1．Simplify－
（1） $2 \frac{1}{2}-3 \frac{1}{3}-4 \frac{1}{4}+5 \frac{1}{6}$ ；（2） $3 \frac{3}{17} \times 4 \frac{7}{11}-3 \frac{15}{2}$ ．
2．（1）Reduce 01875 and 0227 to vulgar fractions in therr lowest terms ；（2）find the value of 0625 of $£ 3,4 \mathrm{~s}$ ．

3．An estate of 1416 ac .2 ro 16 pls．was divided into allotments，each 4 ac． 3 ro 27 pls．in area．How many allotments were made？

4 Find the cost of 2 tons 4 cwts． 3 qrs． 21 lbs ，at £14，16s．8d．per cwt ．
5．If 9 lbs． 9 oz 10 dwts of silver cost $£ 32,6 \mathrm{~s}$ ． 3 d ，what is the value of 3 lbs .4 oz .3
6．Find the simple interest on $£ 236,6$ s． 8 d ．for $2 \frac{1}{2}$ years， at 3 per cent．per annum．



Page 206
'Arıthmetic Theoretical and Practical - For Certificate students, Scholarshıp candıdates, Pupıl teachers, Civıl Service Commission' by James Cusack. lst editıon 1896, 2nd edition 1909. Clty of Eondon Book Depot, London.

A large work of 727 pages which gave lengthy explanations and an excessive number of long questions. For example, there were 26 questions on long division. For instance 'divide 235062991380 by 928765'. I'here were 56 questions on the addition of fractions alone and there were 500 mlscellaneous problems in the final chapter. The photocopy shows the treatment of compound proportion. Note how the book emphasized the preparction for vafious examinatıons.

Proceed as follows -
$\Delta$ can do the whole work in 4 days $\therefore \frac{3}{3}$ of the work in 1 day. $B$ can do the whole work in 6 days $\therefore \frac{1}{8}$ of the nork in 1 day. $C$ can do the whole work in 8 days $\therefore \frac{1}{8}$ of the work in 1 day. Hence, A, B, and C together can do $\frac{7}{4}+\frac{1}{6}+\frac{1}{8}=\frac{13}{2} \frac{3}{4}$ of work in I day ; and hence this question an proportion -

$$
\begin{aligned}
& \text { Ans 111 } 14 \text { days } \\
& \therefore x=1 \frac{1}{4}
\end{aligned}
$$

397. A and $B$ together can do a piece of work in 10 duys; $B$ and $C$ together can do the same work in 12 days; $A$ and $C$ together can do the same work in is days, in what tume can $\Delta$ alone do the work ?
$A$ and $B$ do work in 10 days $\therefore \frac{1}{10}$ of work in 1 day.
$B$ and $C$ do work in 12 days $\therefore \frac{1}{12}$ of work in 1 day.
$A$ and $C$ do work in 15 days $\therefore \frac{1}{15}$ of work in 1 day.
Hence $A+B+B+C+A+C$ do $\frac{1}{10}+\frac{1}{12}+\frac{1}{15}$ in 1 day, That is, trice $A+$ twice $B+$ twace $C$ can do $\frac{18}{6}=\frac{1}{6}$ in 1 day. Hence $A+B+C$ can do $A \div 2=\frac{d}{} \ln 1$ day.

$$
\text { But } \quad \mathrm{B}+\mathrm{C} \text { can do } \quad \mathrm{T}^{2} \ln 1 \text { day. }
$$

$\therefore$ By subtraction, $A$ can do $\frac{1}{8}-\frac{1}{12}=\frac{2}{27}$ nn 1 day.
And hence, by an easy proportion, A can do the whole work in 21 days Ans.
898. A cistern has 3 tupu, $\Lambda, B$, and $C$. A alone could fill it in 8 nims. - Balone could fillit in 12 mms. . C alono could empty it in 15 mins if the castem to quite empty and the the co tape bo openced how long will it be till the cistern is quate full?

$$
\text { A can fill the cistcram } 8 \text { mins. } \therefore \frac{1}{8} \text { of it in } 1 \text { min. }
$$

$B$ can fill the cistern in 12 mins. $\therefore \frac{1}{1 r}$ of at in 1 min ,
$\therefore$ A and $B$ together fill $\frac{1}{8}+\frac{1}{15}=\frac{5}{2} \frac{5}{2}$ m 1 min.
But what C enptres

$$
=\frac{2}{15} \text { in } 1 \mathrm{~mm} .
$$

$=\frac{i_{5}^{2}}{2} 1 \mathrm{~mm}$.
Therefore the net quantity filled in $1 \mathrm{~min}=\frac{5}{24}-\mathrm{a}^{2} 5$

$$
=\frac{28}{1+0}-\frac{1}{280}=\frac{19}{120} .
$$

## Hence this casy question in proportion :-

$$
\begin{aligned}
& \text { If } \frac{17}{10} \text { be filled in } 1 \text { min } \\
& 120,7 \quad 2, \quad \text { As } \frac{17}{120}: \frac{120}{120}: 1: 1 \\
& \text { Ans. } 7 \frac{1}{7} \mathrm{mins} \text {, } \\
& \text { * } x=7 \frac{1}{1},
\end{aligned}
$$

Exercise 250.

## Work.

1. $\triangle$ and $B$ can do a piece of work in 12 days, $B$ and $C$ can do it in 16 days, and A and $C$ can do it in 18 days. How long would 1 t occupy $A, B$, and $C$, working together?
2. $A$ and $B$ can do a plece of work in 8 days, $B$ and $C$ can do it in 12 days, and $C$ alone can do it in 30 days. How long would it take $\Delta$ to complete the work alone?
3. $A$ can do a piece of work in 6 days and $B$ can do it in 8 days. If $A, B$, and $C$ work together the nork is done in 2 days. How long would it tako C alone?
4. A vessel can be empticd by a pipe in 15 mins. and filled by a tap in 20 minutes. If both are opened together when the vessel is full how long will it bo before the vessel is empty?
5. A pipe $A$ can empty a tank in 15 minutes, a tap $B$ can fill it in 18 manutes, and another tap $C$ can fill it in 24 minutes. If all are opened together when the tank 18 half full, will it get empty or full, and how long will be required?
C. A can do a plece of work in 3 hours, B takes 4 hours; C takes as long as $A$ and $B$ together. How long will it take $B$ and $O$ to complete the work together?
6. $A$ and $B$ together can do a piece of work in $5 \frac{2}{5}$ dayg. $A$ doen twice as much work as 13 in a given timo, find how long A alono would take to do the work.
7. If B oundo $\hat{3}$, and $\mathrm{C}\{$ of the work $\Lambda, \mathrm{B}$, and C , can do working together, how much can A do in a divy, supposing that together they can finish the work $2 n 8$ days?
8. A's rate of working is to B's ass $\frac{1}{2}: \frac{1}{3} ; \mathrm{B}^{\prime} \mathrm{s}$ is to $\mathrm{C}^{\prime} \mathrm{s}$ as $\frac{1}{3}: \frac{1}{5}$; C's 18 to $D$ 's as $\frac{1}{6}: \frac{1}{T}$. If $A$ could do a prece of work alone in 420 hours, how long would it take $A, B, C$, and $D$, working tegether?
9. 4 men and 6 boys do a work in 24 days, 4 boys and 6 men do the same work in 21 days. In what time could 8 men and 8 boys do at?
10. A can do a plece of work in 6 hours, $B$ can do the same in 8 hours, and C can do it in 10 hours. If the three work together how long will they take to finsh the work ?

Page 208
'Practical Mathematics for Beginners'.
Frank Castle. MacMillan Jondon 1921. lst edition 1901. The book in fact seems a mixture of some 'pure' mathematics and pseudo-practical problems (even of the path around a lawn type). However, there are some practical details enclosed, such as the use of a planimeter for finding irregular areas. Graphical work is a feature of the book and problems of the type shown, finding the law relatıng two quantities by plotting a graph, were classed as practical mathematics although out of the 10 examples shown only 3 actually refer to physical conditıons. The extensive use of the book is indicated by the fact that the owner wrote the date '1943' in the front cover whilst on a course at a technical college. Page 209
'Elementary Geometry' by C. Godtrey and A. Sidaons. Cambrıdge Press lst editıon 1903.

This edition 1936 (reprinted 31 times since 1903). This book is included as an example at the beginning of the 20th century to lllustrate the 'new' approach to geometry based on experimental procedure and theoretical discovery. The pupils were allowed hypothetical constructions but as the authors point out in the 1903 preface the sequence of theorems was Euclidean in form but simplıfied by the omission of non-essentials. A poor feature was the very small print used.

180 PRACTICAL MATIYEMA'CICS FOR BEGINNERS
3 A series of observed values of $n$ and $V$ uic given. Find the relation in each case between $n$ and $\log N$.
(1)

| $r$ | 25 | 5 | 75 | 1 | 125 | 16 | 175 | 2 | 220 | 25 | 275 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $N$ | 154 | 180 | 265 | 875 | 485 | 685 | 835 | 1185 | 1535 | 1835 | 2435 |

(a)

| $n$ | 25 | 5 | 73 | 1 | 195 | 15 | 175 | 20 | 25 | 30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\lambda^{4}$ | 145 | 186 | 235 | 290 | 355 | 495 | $5 J$ | 653 | 1115 | 1515 |

(111)

| $n$ | 25 | 5 | 75 | 1 | 125 | 15 | 175 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $N$ | 115 | 145 | 185 | 235 | 300 | 355 | 400 | 605 |

4 An electric light station when making its maximum ontput of 600 kulowatts uses 1020 lbs of coal per hour When ats load factor is 30 per cent (that is, when its ouiput is $600 \times 30-100$ ) it uses 1026 lhs of coul per hom What will be the proboble consumption of coal per hour when the load factor 1515 per cent?

5 Plot on sfumed puper tha following obsetved values of $A$ and
 the percontage etror in the obsenved valato of $B$ when $A$ is 150 .

$$
\begin{array}{|c|c|c|c|c|c|c|c|c|c|}
\hline 1 & 0 & 50 & 100 & 150 & 200 & 250 & 300 & 350 & 100 \\
\hline 1 & 0-2 & 74 & 83 & 05 & 103 & 116 & 124 & 130 & 145 \\
\hline
\end{array}
$$

6 The following oloserved values of $M$ and $N$ are supposed to be related by a lineni law $M=a+b N$, but there are emors of observation lind by plotting the values of $M$ and $N$ the most probable values of $a$ and $b$.

| $Y$ | 25 | 35 | 44 | 58 | 7.5 | 96 | 120 | 151 | 183 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $M$ | 136 | 176 | 202 | 28 | 0 | 35 | 5 | 474 | 561 | 746 |

EXPRCISES.
7. (1) The following values, which we may call $x$ and $y$, werc measured Thus when a was found to be $1, y$ was found to be 123 .

| $x$ | 1 | $1 \cdot 8$ | 28 | 39 | 51 | 60 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 223 | 327 | $\cdot 525$ | 750 | 910 | 1095 |

It is known that there is a law hike-

$$
y=a+b x
$$

connecting these quantities, but the obseived values are slightly wrong. Plot the values of $x$ and $y$ on squared paper, find the most likely values of $a$ and $b$, and write down the law of the lame
(in)

| $a$ | 05 | 17 | 30 | 47 | 57 | 71 | 87 | 90 | 106 | 118 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 148 | 180 | 265 | 320 | 388 | 430 | 529 | 502 | 611 | 652 |

State the probuble erior in the measured value of $y$ when $\imath=87$.
8 In the amexed table, values of $L$, the length of a ligmad colum, and $I$, its time of vibratiou, are given The relation between $L$ and $T^{2}$ is gaven by $L=a T^{2}+b$, fine $a$ and $b$

| $L$ | 24 | 28 | 30 | 32 | 34 | 36 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $T$ | 106 | 123 | 129 | 134 | 138 | 132 |

9 It is known that tha followng velues of a nati $y$ ane comuer terl
 in the givera atuos Dotermane tho mont poblable vaitu of $a$ and $b$.

| 1 | 18 | 98 | 54 | 133 | -455 | -111 | -65 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 5 | 0 | 7 | 8 | 0 | 10 | 11 |

10 The following measurements were made at an leketue lesht Station under stady conditions of output

IV 18 the waght in pounds of feed water per hom, and $P$ the electric power, in kilowatts, given out by the station When $P$ was $50, W$ was found to be 3500 , and when $P$ was $100, W$ was found to be 5100 .

If it is known that the following law is nearly true-

$$
W=a+b P
$$

find $a$ and $b$, also find $W$ when $P$ is 70 kilowatts State the value of $W-P$ meach of the three cases.
\&Ex. 1231. From a point ou tho dingonal of a squaro, lines PR, QS aro drawn parallel to the sides, $P, Q, R, S$ bemg on the bides. Prove that these forr points are euncyche

EEx. 1432. O is the centro of a curcle, $C D A$ dinmoter, and $A B$ a ohord perpendeular to $C D$. If $B$ is joncd to any yoint $E$ in $C D$, and $B E$ pro. duced to meet the circle ngan in $F$, then $A, O, E, F$ aio concyche.

Ex 1433. Show how to constract a right angled trangle, given the adius of tho sn-cribed cacle, and an acute angle of tho triangle.
EEx. 1434. Two circles tonch at $A$. Throngh $A$ aro drawn straight lines PAQ, RAS, cuttung the carcles in $P, Q$ and $R, S$ Prove that $P R$ is puallel to $Q S$. (Drow tangent at $A$. Comparo Dx 1425.)
+Ex. 1435. Iwo circles cut at $P, Q$. A, a yount on the onocirole, is joined to $P, Q$; and these lines are produced to rect tho other cuclo in $B, C$. Prove that $8 C$ is parallel to the tangent at $A$. (Compare Ex. 1125.)
fEx. 1436. A chold $A B$ of a circlo biscets tho angle between the diametor though $A$, and the porpendicular fion $A$ upon the tangent at $B$.
tre 437 . $A B C D$ is a cyclo quadriaternl, whose dangonals intersect at $E$ : a circic is drawn though $A, B$ and $E$. Provo thet tho tangent to this circle at $E$ is parnilel to $C D$.
tEx. 2430. $A B, A C$ no two chords of a crrelo; $B D 18$ dawn pmalled to ih) trusent at $\wedge$, to mect $A C$ in $D$, prove that $\angle A B D$ is equal or suppiomentiry to $\angle B C D$. Henco show that tho carclo though $B, C$ and $D$ toulies $A B$ at $B$.

Siction $X$ Ares and Anarm at mhe Cmoumbminole
4fEx, 1430. Dinw a curclo of radus 25 in ; draw a dannoter $O P_{5}$ nud a tangent $A O B$ as in fir, 273, Dwido $\angle A O P$ mio five equall parts, swo $\angle B O P_{5}$. Neasure the chorls $O P_{1}$, $P_{1} P_{1}$, . eto. What ancle does $P_{3} P_{y}$ sabtend at tho centro of tho carcle? I'sore that $O P_{1} P_{3}$, eto. are the vertices of $a$ reguln decsegon.

TEx. 1440 . In the fig. of Dx. 1439 draw riy straight The cutting acress the Eet of lines $O P_{3}, O P_{2}, O P_{3}$, eto. Is tlus lino diy-ded into cqual parts?

fig. 278.

TEx, 1412. Taho $\Omega$ point $O, 1$ in. from tho centio of a cucle of radus -ax, diav throwgh 0 a dinmeter and $a$ set of chords making nngles of $15^{\circ}$ with ono nnother. Find by measuroment whether those chords divide the circumiferenco into oqual arcs.

TVx. 1442. Would tho croumicrenco bo divided unto equal pres it the Ex. 122. Wow many arcs would theso bo?
TEx. 1443. Provo that equal ares or chords of a circio subtend cqual (or guppiementary) ancles at a point on tho
Diaw a figura to lllustrate tho enso of supplementary ancles
Prove the conversc.
NoTe. In the following each ciscs (lix. $1415-1462$ ) the studsnt sadvised to make use of "the angle subtended at the cincumfercnce"
十Ex. 1444. Drav a regular pentigon $A B C D E$ in a circlo. Provo that tho anglo $A$ is trisected by $A C, A D$.
†Ex. 1.445. $A B C D E$ is o remular pantagon.
(i) Provo that $A B$ is prrallel to $E C$ (Join $A C$.)
(i1) At what angla do $B D, C E$, intcrscet? (ui) Prove that $\triangle A$
move that $A^{\prime}$ CXD, CDE are equangular
(iv) If $B D, C E$ mect at $X$, provo the prclo at $A$ is parallel to $B E$.
(v) Prove that the tangent to the circlo at $A$ is parallel to $B$.
[Uso III, 14.]
Fix. 1416. $A B, C D$ aro paralich chouds of in arcle lioyo that แо $A C=$ mo $B D$.

$A D$ is parallet or oqual to CB.
FEx. 1440. $A O B, C O D$ aro two chords of a caclo, intersecting at right
angles. Shov that arc $A C+$ are $B D=\operatorname{arc} C B+\operatorname{arc} D A$.
+Ex. 1449. Through a given point draw a chord of arlo so that the minor segment cut off may bo a Eiven circo possible
tEx. 1450. Provo that in fig. 278

$$
\mathfrak{a r c} O P_{1}=\operatorname{arc} P_{1} P_{2} .
$$


fig. 279.

Ex. 1451. In fig. 279 what fractions of Ex. 1451. arch $A B, B C, C D, D A, B C D$ ?

Page 211 and reverse
'School Mathematics Project' Book T4. C.U.P. 1965. This shows an example from the newer type of text book from a 'modern' mathematıcs scheme of the 1960's. The actual topic content, that of matrices, was new for secondary level and the informal approach leading to more formal definltıons illustrates the method. This serles $2 s$ distinct from many traditional books was written by a group of authors, rather than one ındividual.

|  | Type of biscut |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| $\quad$ Box | Chocolate | Thin wine | Shortbread | Lincoln cream |
| Holly Berres | 8 | 10 | 6 | 4 |
| Slesh Bells | 12 | 0 | 15 | 9 |
| Reindece Special | 16 | 8 | 12 | 10 |

Such an array of numbers is a coded set of instructions for the packers, who will soon remember what each row and each column refers to; all they need is the array:

$$
\left(\begin{array}{rrrr}
8 & 10 & 6 & 4 \\
12 & 0 & 15 & 9 \\
16 & 8 & 12 & 10
\end{array}\right)
$$

An array like thes is called a matrix; it can have any number of rows and any number of columns, matrices, like biscuit tins, come in all shapes and sizes.

It is convenient to have a definite convention by which the shape of a matrix is described. The above matrix has 3 rows (corresponding to the three types of box involved) and 4 columns (corresponding to the four types of biscuit); it is described as a $3 \times 4$ (read as 'three by four') matrix. A $4 \times 3$ ('four by three') matrix, on the other hand would have 4 rows and 3 columns, which is different. We could, of course, have interchanged rows and columns in the above matrix, writing the types of biscuit in rows and the different kinds of gift-box in columns. This would yield the array

$$
\left(\begin{array}{rrr}
8 & 12 & 16 \\
10 & 0 & 8 \\
6 & 15 & 12 \\
4 & 9 & 10
\end{array}\right)
$$

which gives exactly the same information; nevertheless, it is a different matrix.
The following are all matrices:

$$
\begin{aligned}
& \left(\begin{array}{lll}
1 & 2 & 3 \\
4 & 5 & 6
\end{array}\right), \quad\left(\begin{array}{r}
3 \\
-1 \\
2
\end{array}\right), \quad\left(\begin{array}{ll}
0 & 1
\end{array}\right), \quad\left(\begin{array}{lr}
2 & -3 \\
3 & -1
\end{array}\right), \\
& (6), \quad\left(\begin{array}{ll}
a & b \\
c & d
\end{array}\right), \quad\left(\begin{array}{lll}
x & y & z
\end{array}\right), \quad\left(\begin{array}{rr}
4 & 2 \\
-1 & -2 \\
3 & 6 \\
5 & 10
\end{array}\right),
\end{aligned}
$$

A matrix with $m$ rows and $n$ columns is said to have order $m \times n$.

## 3

MATRICES


And bade his messengers ride forth, East and west and south and north, To summon his array
thomas babington macaulay, Lays of Ancient Rome

## 1. MATRICES

There are a great many occasions when a large quantity of numerical data has to be stored for use. Very often this data is most conveniently arranged in a table with rows and columns
For example, in the packing department of Messrs Freak Bean's Biscuit Factory there might be a list posted to help the workers assembling Christmas Gift Boxes, as follows.

From 1902 to the $1970^{\prime}$ s.
The following is a summary of the many developments that took place during this period.

The earlier part of this perıod was one which continued the reforms suggested by Perry, and the Mathematical Association took the inltiative by producing a series of reports on the teaching of elementary mathematics from 1902 to 1908. These were on Geometry, Arithmetic and Algebra, Elementary Mechanıcs, Advanced School Mathematics, the course required for Entrance Scholarships at the Universities and Mathematics in Preparatory schools.

Many text books produced about this time reflected the suggestions in the reports; Godfrey and Siddons book 'Elementary Geometry Practical and Theoretical' and 'Elementary Algebra' by W. Baker and A.Bourne (lst editıons 1904), both followed the recommendations of the Mathematical Association. The Board of Education in its Handbook of Suggestions (1905) lald stress on the fact that the numbers supplied to pupils for calculations at the 'elementary' level were still too large; by 1910 some progress may have taken place because the Revised Code was no Longer applicable. The Board of Education published a special report called 'The teaching of Geometry and Graphıc Algebra' (Circular No. 711) in 1909; this eventually impressed itself on the Oxford and Cambrıdge Local Examinations by 1914. Another Board of Education paper, Circular 851, was issued, again on 'Geometry' in 1914. This stressed the need for both inturtive and practical work to go alongside formal proofs in the teaching of geometry. The arguments
concerning Euclid were restated again $n$ this document which was essentially for preparatory schools and better secondary schools. The Mathematical Association contınued to produce reports from various committees on all aspects of teachıng mathematics, maınly at secondary level, and in different types of school from the beginning of the I'wentieth century. Lists of their publications can be found in many of the older issues of the 'Gazette' since many are now out of print and have obviously been superseded by more upaated reports. Some of these reports were 'The teaching of Mathematıcs $1 n$ Publıc and Secondary Schools' (1919), 'Geometry' (1923), 'Lechanics' (1930) 'Arıthmetıc' (1932), 'Algebra' (1934) and again 'Geometry' (1938). The Board of Education reports in 1912 'The teaching of Mathematics in the U.K!' Vol. 26 and Vol. 27 as previously mentioned, had contributıons written by some notable authors of various text books such as Godfrey, Siddons, Barnard, and C.V. Durell. The Latter's name became almost a legend in many grammar schools; his text books included arithmetıc, geometry, algebra, trigonometry and general mathematics. During the 1930's and 1940's another serıes was publishea wath the tıtles 'New Algebra for bchools (1930) and 'New Geometry for Schools' (1939) and so on which trled to reflect changes in sy1labuses or courses drawn up after various recommendations in reports. In fact, apart from changes in geometry, the inclusion of calculus and mechanlcs in the courses, much of the general nature of the mathematics and the text
books remained unaltered before the Second world Var; the period had been one mainly of consolidation, particularly with reference to the grammar schools and public schools, where much of the retorm had originally been inltated. The background against which the mathematics was developing was one where the government showed an interest in the elementary and secondary sectors of education. IPhrough the Hadow Report 'the education of the Adolescent' (1926) and the Spens Report 'Secondary Education with special reference to grammar schools and technical high schools' (1999) there was a call for a broader, more flexıble curriculum. In mathematics this suggested that more actuvity and discovery methods could be used (Maria Montessori being an innovator in this field) and that a more integrated approach to the separate subjects of arıthmetic, trigonometry, algebra and geometry could be used. The Spens report, like many previous reports, pointed out the defects in mathematics where manipulation and 'tricks' replaced genuine understanding and the over emphasis on rigorous proof and abstraction replaced some intuition and experiment. This theme was one which had been continually repeated throughout mathematical education and no doubt will continue in the future. Another point ralsed by the Spens report, again a debatable issue today, was the influence that external examinations had on determining the content and teaching methods in schools. Although the Second World War intervened and removed some of the class barriers that existed, the Norvard Report 'Report of the Secondary School Examınatıons Council on

Curriculum and Examinations in Secondary Schools' (1943) tended to reinforce the Spens recommendations of three types of schools, or going further back the type of schools described by the Schools Inquiry Commission (1868) and then by the Bryce Commission (1895). The apparently logical conclusion was then to 'construct' three types of pupil to fit each school as shown by the following descriptions of each school and its pupil (The Norwood Report quoted by Graffiths and Howson, Ref: 10, pp.55-56) the modern school........'deals more easlly with concrete things than with ideas'; the technıcal school......... 'abilıties lie markedly in the field of applıed science or applied art'; the grammar school (a copy of the public school generally).......'ınterested in learning for its own sake......can follow a piece of connected reasoninfr'. Rather thaı the introduction of a tripartite system it seems to be the culmination of such a system where the underlying philosophy had been laid in previous centurles. The mathematical education had tended to reinforce the system as well with its 'pure' and 'liberal' concepts and 'applied' and 'practical' aspects being attached to different schools and further and higher educational institutes. Published text books were suitable I'or alfferent types of schools and pupils; it mlght even be hypothesised that the three stages of learning geometry $A, B$ and $C$ as described in the Mathematical Association's report on Geometry in 1923 would fit each type of pupil

In the three schools. The 'A' stage being a practical type of geometry suitable for the secondary modern pupil, the 'B' stage being more enriched, applicable but not fully formal for the technical boy, whilst stage 'C', being formal and axiomatic, would be suitable for the grammar school boy who could supposedly reason logically. The challenge to formulate a suitable curriculum for mathematıcs was undertaken by a joint commıttee under $G$. B. Jeffery which met a number of times in late 1943, and in 1944. It contained representatives from the Mathematical Associatıon, some teachers' Unıons, Unıversitıes and some examination boards. The resulting report, the Jeffery Report (1944) on Schooम Certificate Mathematics recommended a less formal approach to geometry (in fact a fusion with trigonometry) and more integration of algebra and arithmetic. Examination papers were recommended to be 'mıxed' and not n separate subject sections. (It maght be worth commenting that as late as 1970 the London University Board were still offering the 'O' level G.C.E. Mathematıcs syllabus $A$ with three separate sections). The report also proposed the introduction of calculus in the optional parts of the examination paper.

The recurrence of various themes in mathematical reports has already been noted; the Jeffery report restated another one in the tollowing manner: "The whole syllabus is inspired by the desure to brang mathematics more closely into relation with the life and experıence of the pupıl". The Hadow Report (Ref: 15a, p.175) (1926) had put it this way, though in more general terms: "the courses of instruction........should be used to connect the school work
with the interests arising from the social and industrial environment of the pupil". Both these comments indicate a new awareness in education - that of a pupil-centred approach in the teaching method. To some extent mathematics, unless it was to be thought of for its vocational uses, was essentially subject-centred in its outlook. liany of the modern projects in mathematics after 1960 have tried to emphasize this idea of making mathematics relevant to the pupil.

The 1944 Eaucation Act was the next main event which confirmed the recommendations of the Norwood report almost by default, since the Act removed the word 'elementary' education from the educational vocabulary but did not really define how the secondary level was to be organised. In mathematics teaching many of the pre-war text books had been republished and were being used by inexperlenced teachers. To some extent there was little change in the type of mathematics since before the war and in present day terminology this would be called the 'traditıonal' syllabus.

When the General Certificate of Education was introduced in 1951 the School Certificate syllabus which had been operating was naturally the basis for 1 t. The adherence to the older type of mathematics and the rigid methods of teaching were, in fact, often in confluct with the theorles of learning that emerged from the 1940 's to the 60's from the work of some notable educational pschologists. Some of the work was particularly applied
to mathematics such as that by Piàget, Drenes, Gattegno and Stern, whilst the work of Bruner and Bloom could be applied to mathematics as well as other subjects. In summary the theorles revolvei around the different conceptual levels at which learning took place, the use of discovery methods in learning and the use of a 'structure' underlying the baslc concepts. These theorles helped to reshape the approach and content of mathematical education during the 1960 's and $70^{\prime} \mathrm{s}$.

Technological and economic pressures similar to those during the industrial revolution in the l9th century were other factors which helped to reshape the mathematics curriculum. It is almost a part of mathematical 'folklore' that the launching of the Kussian Sputnix in 1957 was the catalyst that caused the Americans to reform their whole mathematical programme on the lines of the 'new' mathematics aready beang followed in the Unıted States in many experimental projects (The Illinois project for instance was begun in 1952). They tackled the problem in a more national way with the formation of the wchool Mathematics Study Group (S.M.S.G.) at Stanford Unıversity in Californıa in 1958.

The momentum for change did not take place in England untıl the early l960's when a fresh look at the content of the mathematics that was being taught took place. Prior to this the Crowther Report, '15 to 18' in 1959 and the Newsom Report 'Half our future' in 1965 had both stressed a plea for numeracy and the latter also pointed
out a greater need for a practical mathematical and scientıfıc curriculum, partıcularly for the less able pupal. In hagher education both the Robbins Committee (1963) and the Dainton Report (1968) both stressed the need for more well trained scientists, mathematicians and technologists; the whole range of mathematics teaching was under revlev during this period.

Another impetus for change had come from a meeting held by the member nations of the Organisation for European Economic Co-operation (O.E.E.C.) in 1959. Ihis was the 'Royaumont Conference' which had speakers and discussions organised under three headings which were: (1) New Thinking in Mathematics, (2) New Thinking in Mathematics saucation and (3) The Implementation of Retorm. The Conference was Jolned by representatives from Canada and the United States who gave the experience of therr own already established reforms. One comment worth relating was the famous retort by Professor Dieudonné that 'euclid must go' in trying to crystallise a symptom of the stagnation that mathematical education had reached in general. However, it has been seen that he was not the first person to utter such a challenging edict. This Conference, and others similar to it (such as the Oxford Conference in 1957 and the Liverpool Conference in 1959) all looking at the need for reform in mathematics teaching, lead the way for a series of mathematics projects manly during
the years 1960 to 1967. These were admırably summarised in a book issued by the Mathematical Association called 'Mathematics Projects in British Secondary Schools' (Bell 1968). The projects were often insplred or guided by the views of an indıvidual or a group of individuals; once again this reflected on the relative freedom of the English educational system from centralısed Governmental. control. To many laymen this may have presented itself as too much diversity in education, and to many teachers it must have presented a drlemma as to which was the best 'modern' mathematics course (as it was now unfortunately labelled) to adopt. In one aspect there was much that was sımılar; topics such as set theory, groups, matrıces, statistics, number theory, mathematical logic, transformations and the use of computers in mathematics had been introduced into the content of all the projects, along with the retention of some of the 'traditional' topics. The underlying ideas though in the structure of the course itself and methods of presentation may have been different. To glve some indication of the cholce of project during these years the followin ${ }_{0}$ is a brief list with the name of the important individual behind the project:

1960: The Contemporary School Mathematics or St. Dunstan's Project (C.S.M.), Professor G. Matthews.

1961: The Midands Mathematical Experiment (M.M. T.)
Cyrıl Hope and R.H. Collins.
1961: The School Mathematics Project (S.M.P.)
Professor Bryan Thialtes.

1961: The Swansea Scheme (S.S.) The Department of Pure Mathematıcs at Swansea University College.

1962: The Manchester Mathematics Group (M.M.G.)
F. Gorner, Hanchester School of Education.

1962: The Psychology and Nathematics Project (Psif_)
Dr. Skemp (and later Dr. Dienes).
1963- The Ifathematics in Education and Industry 1965:

Project (M.E.I.) Mathematıcal Association.
1963: The Scottish lathematıcs Group (S.I.G.)
Scottish Education Department.
1964: The Shropshire Mathematics Experiment Project (S.M.E.) Seachers and Local Authority in Shropshire.

1967: Mathenatics for the Majority Project (M.I.P.) Schools Council, Exeter Unıversity Educatıon Dept.

1971: Mathematics for the Majority Extension Project Exeter University - director P. Kamer.

This ls by no means a complete list of all the mathematics projects that were or are, being concelved. A new teacher may have been bewlldered by these, but for the fact that the projects were usually centred around a particular area of the country so in fact no decision had to be made. Another point is that many of the projects began as a course sultable for the G.C.E stream and the better mathematicians in the grammar and public schools. Subsequently some of the original materıal was modified to suit the
pupıls takıng the C.S.E. examınation ${ }^{1}$, after the material had been approved by the Boards, and was in many cases perhaps unsuitable.

An example of the content from a text book from the S.M.P. course has been given in the last section for comparison to some of the older text books; one point to note Immediately is the informallty of approach in introducing the topic of matrices to arouse the pupil's immediate attention perhaps.

Two other important organisations during this period were The Schools Council (1964) and the Assoclation of Teachers of Mathematıcs (A.T.M.) (1962). The former has played an extremely important part in defining the structure and content of examinations at the secondary level and ensurinc a parıty of standards between the various boards. Themr reports and working papers on mathematics indicate how some rapport is maintained between the actual teaching side of education and the administration through the H.II.I.'s.

The A.T.M. Like the Mathematical Association, publishes a journal 'Mathematıcs Teaching' amongst its other book publlcatıons. It presents many articles concerned with the actual practical level of classroom teaching. The research in the 1970's in mathematical education continues to be malnly in the field of syllabus and content

[^7]development and curriculum studres. A useful summary and bibllography of this is glven by A.J. Bishop in an article 'Trends in Research in Mathematical Education' in the A.T.I. publication 'Ilathematics Teaching' (No. 58, Spring 1972); whilst a short appraisal of secondary school mathematıcs is given by T.J. Fletcher (H.M.I) in an article 'Secondary Mathematics Today' in 'Trenas in sducation' No. 35 October 1974. The reforms of the $60^{\prime} \mathrm{s}$ and $10^{\prime}$ s have not been without their crıtıcs; Dr. Hammersley in the Bulletın of the I.I.A. in 1968 and S.H. Froome in the Iamous Black Paper Two both thought that mathematics had been weakenea by the introduction of modern mathematics into schools. Corresponaence in the rimes Education 1 Supplement (24.1.15)
'All at sea over new mathematıcs' was quite crıtıcal about the effects of the ne: mathematıcs. External critıcs were certainly many employers who complained about apprentices or clerks who could not ao smple arithmetic; many parents complained that their children 'couldn't do sums'. Whe very topic that had been over emphasized auring the 19 th century, commercial and practical drithmetıc, was now, one nundred years later, being critıcısed as being inadequitely taught. The S.II.P. project brought out in $19 / 4$ a reappraisal of the role of numeracy in the mathematical content of the project. I'he original enthuslasm and drive of the teachers in the early deys of the projects may have added to the Idea that maybe 'modern' mathematics would be a panacea for the llls of traartional
mathematics teachıng. It cannot go unnoticea either that the rajority of the projects iegan as courses for the mathematical élite, the better schools or the top streams; in terms of applications in industry the cdll for modern mathematics at the Liverpool Conference (1959) 'Mathematics in Eaucotion and Inuustry' indicated that it would be more usetul at the top level rather than on the workshop Iloor. Perhaps this was the turning of a Iull curcle; not mathematics for the lelsured élite, but mathematıcs for the technological élıte ? Another point of dispute, although more recent, is whether the teaching about computers which was glven ds a simple introduction in the earlier projects, should be the responsibilıty of the mathematics teacher. Although there is a place ior the use of a computer, or simllar aevice, in nathematicel eaucation, both as a tool and as an 'instructor', computer equcation as a subject in lts own right and with a place in the curriculum now requires more specially trainea teachers. whe role of the computer, or programmable calculator, in mathematical education is a toplc for serious study in its own right, and no doubt the arguments on 1 ts use will continue for some time (see articles by Bryan Thwaites in the Bualetin of the I.M.A. (ı) Dec. 1967 entitled '1984: Mathematıcs
$\Leftrightarrow \quad$ Computers' and(1i) Sept./Oct. 1974 'Ten Years to go to 1984 ${ }^{1}$ ).

In the history of the development of mathematics teaching there has always been, to some degree, criticism of both
the method and content from within, and external to, the actual teaching situation. Sometrmes the pressure for reform has been inltiated by the teachers themselves; at other tames economic, educatıonal or social factors have caused the disturbance for reform. Whatever the case, it is unlikely that there wall be anything matching the reform initiated by the introduction of the 'modern' mathematics projects of the 1960's. Rather, $1 t$ is lıkely that modıfıcatıons will emerge from existing projects as experience and feedback is gained from the whole field of mathematics teaching and more emphasis will be placed on the abılıty and angenuity of the teacher to generate excitement in mathematics teachıng.

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[^0]:    $I_{\text {Wooden }}$ block printing about 1500. Printang press at Gutenberg 1539.

[^1]:    ${ }^{l_{\text {See }}}$ List in Appendix IV of the Schools Inquiry Commıssion. Vol.I. Reports (1868)

[^2]:    1 J. Susenbrotus, a German, who died in 1543. Wrote an eptome of Rhetoric in 1540
    2 Justmian's Institutes.
    ${ }^{2}$ Sustinian's Institutes.

[^3]:    1
    He takes these from Howard Staunton's 'Great Schools of England'.

[^4]:    $I_{\text {P. Ramus }}$ -
    Latınızed name of Pierre de la Ramée (1515-1572) a French phılosopher and logician.

[^5]:    *This scheme, which appeared annually mathe Code from 1894 to 1005 , is no longer unpd in the examinations for Certaficates of Proficiency, but has been replaced by the Sjllabus of Examination in Anthryetic, nhich is to bo tound in
    Schedule VI. of the present Code

[^6]:    - Note, In thas and all the toliowing sections, the miph ustration of onc or moleca

[^7]:    1•The Certificate of Secondary Education first introduced in 1965.

