This item was submitted to Loughborough's Research Repository by the author.
Items in Figshare are protected by copyright, with all rights reserved, unless otherwise indicated.

An isotropic hyperelasto-plastic model for soil with non-linear compressibility at low stress

PLEASE CITE THE PUBLISHED VERSION

LICENCE

CC BY-NC 4.0

REPOSITORY RECORD

Jimenez-Diaz, Luis C., William M. Coombs, and Charles Augarde. 2021. “An Isotropic Hyperelasto-plastic Model for Soil with Non-linear Compressibility at Low Stress”. Loughborough University. https://doi.org/10.17028/rd.lboro.14588430.v1.

An isotropic hyperelasto-plastic model for soil with non-linear compressibility at low stress

Luis C. Jimenez-Diaz[1*], William M. Coombs[1], Charles Augarde[1]

[1] Department of Engineering, Durham University, Durham, UK. luis.c.jimenez-diaz@durham.ac.uk, w.m.coombs@durham.ac.uk, charles.augarde@durham.ac.uk

Abstract

The compressibility of soils at very low stresses is of significance in geotechnical engineering problems at the seabed such as cable trenching. At low stresses, compressibility of soil is highly non-linear and in a Critical State modelling framework, best expressed by a non-linear Normal Compression Line (NCL). To date very few constitutive models have incorporated this behaviour. The aim of this study is to capture the non-linear compressibility of clays via incorporating a compression index dependent on the current stress within the hyperplastic formulation of Coombs and Crouch [1]. The proposed formulation is consistent with the observed non-linear consolidation data of soils during one-dimensional consolidation at low stresses. The validity of this approach is examined using experimental data to demonstrate that the model with this added feature provides satisfactory predictions of uniaxial monotonic elasto-plastic behaviour. The advantage of the proposed model is that the non-linear compressibility of soil may be calibrated from a standard oedometer test and then used to predict mechanical properties of soils obtained from tests such as, uniaxial loading-unloading tests.

Key words: *compressibility; hyperplasticity; Critical State Soil Mechanics; Constitutive Modelling*

1 Introduction

Reproducing the non-linear compressibility of soil under low stress levels is needed to better model offshore soil behaviour near the seabed. The model proposed by Yao et al. [2] does include non-linear compressibility at low stress, but for granular soils where the Normal Compression Line (NCL) is non-unique. Another model for low stress has been proposed, but for structured soils via incorporating an initial degree of structure parameter[3]. These models have been focused on the response of a particular soil type without addressing the fact that reconstituted (destructured) soils exhibit a non-linear NCL at low stresses. In this paper, non-linear compressibility of soils is included by having the compression index dependent on the current stress state. The limitations of modelling soils at low stress was studied by merging an existing hyperplastic model [1] with features such as, non-linear elasticity and the consideration of non-linear consolidation of soil.

2 Non-linear compressibility

The vast majority of Critical State constitutive models are not designed for low stresses, that is less than 100 kPa of vertical stress [5]. Following Skempton's assumption, the Normal Compression Line (NCL) is linear in a bi-logarithmic specific volume-vertical stress, $\ln \upsilon - \ln \sigma_v$, compression plane. However it should not follow a linear consolidation line at low stresses, since it would predict an infinite specific volume [5]. The initial consolidation of soils at low stresses was modelled, in this paper, by using Butterfield's [6] reloading curve and calibrated with respect to consolidation data from [7, 8] in $\upsilon - \sigma_v$ compression plane, using MATLAB curve fitting tool. The compression index, $\lambda(\sigma_v)$, or the gradient of the non-linear compression curve in bi-logarithmic space can be expressed as

$$\lambda(\sigma_v) = C_r \left(\frac{\sigma_v}{\sigma_v^b}\right)^{(\alpha+1)} \tag{1}$$

Where, σ_v, is the vertical stress, C_r initial slope of the curve, σ_v^b stress reference of initial slope and α controls the slope of the curve.
The main limitation of using Butterfield's reloading curve to represent the initial compressibility of soils is it doesn't have a limiting compression index and tends exponentially towards infinity. The initial compressibility index should be bounded between the rage of the swelling index and the compression index obtained from the linear NCL [3]. In this research, we follow a different approach and propose a hyperbolic tangent function to represent a non-linear NCL, with the form

$$\lambda(p) = \lambda_{min} + (\lambda_{max} - \lambda_{min})\frac{1}{2}\tanh\left(\frac{\sigma_v - \sigma_v^T}{\sigma_v^L} + 1\right), \tag{2}$$

where, λ_{min} is the initial compressibility, λ_{max} final elasto-plastic compressibility, σ_v^T transition stress and σ_v^L is the transition length. The main advantage of the proposed hyperbolic tangent function over Butterfield's compression curve [6] is that it is bounded between a minimum compression index equal to the swelling index ($\lambda_{min} = \kappa$) and a maximum compression index equal to the compression index from a straight NCL ($\lambda_{max} = \lambda$).

3 Experimental comparison

A selection of structured soils (Guang-shen clay and Fjord sediment)[7] and reconstituted clays (Kaolin, Lianyungang clay and Pleistocene clay) [8] were used to demonstrate that compression behaviour follows a non-linear consolidation line (NCL) at low stresses. The oedometer obtained experimental results are shown in Figure 1. There is a clear transition from an initial low compressibility to a gradual increase in compressibility while yielding and past a point of inflection of the NCL. The reduction in gradient of the NCL (λ) looks as if the soil has been previously overconsolidated, although that is not the case, thus exhibiting an apparent overconsolidation at low stresses [9]. The NCL's based on the proposed formulation and Butterfield's approach are well calibrated to the consolidation data, also shown in Figure 1. It is expected that soils with high water content (w%) to have the point of inflection at low stresses and the curvature of the NCL to reduce [8]. For the consolidation of Lianyungang clay with high water content ($w = 146\%$), a non-linear NCL was better captured from the proposed formulation than based on Butterfield's which captured a linear NCL, shown in Figure 1 a.
The range values of the compressibility index, λ, obtained in a bi-logarithmic, $\ln v - \ln \sigma_v$, compression plane from these tests are shown in Figure 2. Both formulations result with an increase in compressibility from low stresses to higher stress levels. However, major differences result from Butterfield's formulation, obtaining λ values tending exponentially as vertical stresses increase. Compared with the proposed formulation, definite λ values are obtained at higher stresses. This is significant, since soil does not progressively become less stiff with increasing stress and should instead follow a constant gradient equal to a linear NCL at high stresses. At the other extreme at low stresses, for the consolidation of Pleistocene stiff clay in Figure 2 a, the compressibility index results obtained from Butterfield's formulation produce a null value, than compared with the proposed formulation. A null limiting value is not representative for soils, since soils are not incompressible and will cause numerical errors in the constitutive model at a very low stress.

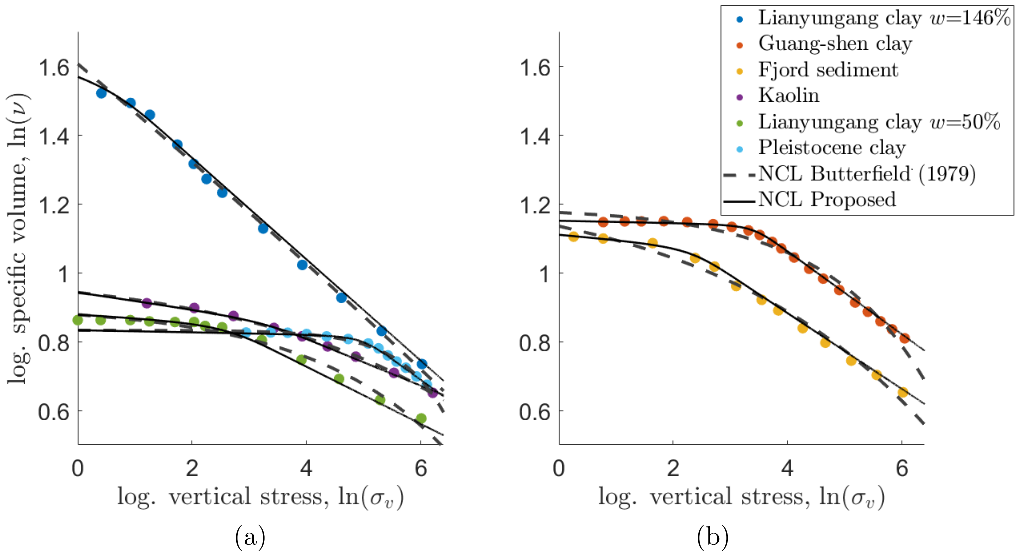

Figure 1: NCLs in bi-log compression plane: (a) reconstituted soils (b) structured soils

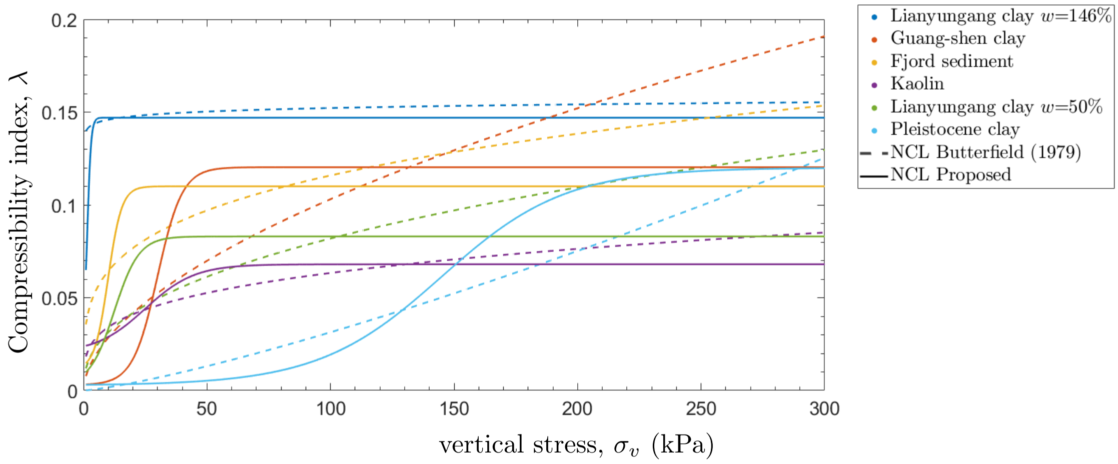

Figure 2: Variation of compressibility index, λ, with current stress, σ_v

4 Conclusions

The consolidation of reconstituted and structured soils were used to represent their non-linear compression behaviour. The non-linear compressibility of soils at low stresses is not due to soil structure alone, since is present in reconstituted soils as well and is an intrinsic property of soil. Physically, the voids in soils, expressed via the specific volume, ν, cannot grow infinitely large at low stresses, suggested by a linear NCL. A non-linear NCL was proposed and shown to reproduce soils compression behaviour satisfactory than compared with Butterfield's formulation. Compressibility values at extreme ranges from Butterfield's formulations do not represent soil behaviour well, since soils are not entirely incompressible ($\lambda = 0$) at minimum stress levels and do not increase in compressibility at high stresses. The advantages of the proposed formulation is that the compressibility index can be calibrated based on theoretical limits and obtain a reliable model for soils at low stresses without compromising its values at higher stresses. The resulting compressibility index variation with current stress from the proposed formulation can then be incorporated into a hyperelasto-plastic model to predict more reliably the mechanical properties of soils at low stresses.

Acknowledgments

Research supported by the National Council of Science and Technology (CONACyT).

References

[1] W. M. Coombs, R. S. Crouch (2011): Algorithmic issues for three-invariant hyperplastic critical state models. *Computer Methods in Applied Mechanics and Engineering* 200 (25), 2297- 2318.

[2] Y. Yao, N. Wang, D. Chen (2020): UH model for granular soils considering low confining pressure. *Acta Geotechnica* https://doi.org/10.1007/s11440-020-01084-7

[3] C. Yang, J. P. Carter, D. Sheng (2014): Description of compression behaviour of structured soils and its application. *Canadian Geotechnical Journal* 51(8), 921-933.

[4] R. Lagioia and A. Panteghini.(2019): The difficult challenge of modelling the non-linear elastic behaviour of soils within a theoretically sound framework. *International Journal for Numerical and Analytical Methods in Geomechanics* 43(11), 1978-1994.

[5] M. D. Liu, D.W. Airey, B. Indraratna Z. Zhuang and S. Horpibulsuk (2019). An extended modified cam clay model for improved accuracy at low and high-end stress levels. *Marine Georesources & Geotechnology* 38 (4), 1–14.

[6] R. Butterfield (1979): A natural compression law for soils (an advance on e-log p'). *Géotechnique* 29 (4), 469-480.

[7] M. D. Liu, J. P. Carter (1999): Virgin compression of structured soils. *Géotechnique* 49 (1), 43-57.

[8] M. D. Liu, Z. Zhuang, S. Horpibulsuk (2013): Estimation of the compression behaviour of reconstituted clays. *Engineering Geology* 167 (1) 84-94.

[9] A. Anandarajah, D. Lavoie (2002): Numerical simulation of the microstructure and compression behavior of eckernförde bay sediments. *Marine Geology* 182 (1) 3-27.