Mathematics at the 16+ school–work interface

TUESDAY - This - Put In about Questionnaire - Views on O'level CSE - Requirements

WEDNESDAY

THURSDAY

What Wrong with Examination system - O'level CSE - NO plan - Not any - What Employers Want

MATHEMATICS AT THE GCSE 16+ SCHOOL/WORK INTERFACE

by

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Abstract

This piece of work is a study of the mathematical shortcomings of the 16+ school-leaver. Most pupils who obtain high O-level grades go on to further education while those who do not take an external examination usually obtain jobs in which there is no requirement for mathematical expertise. The emphasis here has, therefore, been on the pupils taking CSE courses or obtaining a grade B or lower at O-level. These are the pupils who will leave school at 16+ and will apply for jobs which require the use of certain mathematical techniques.

As much of the criticism of school-leavers' mathematical abilities has centred on 'modern' mathematics, the first chapter gives a brief resumé of the reasons for the changes made in both mathematics syllabuses and teaching methods in schools since the early 1960s.

During 1973/4 the 'backlash' against the teaching of mathematics reached its height and Chapter 2 is an attempt to give an indication of the sort of dialogue conducted at that time. In order to provide a forum in which the problems of the 16+ 'interface' could be more rationally discussed, the Institute of Mathematics and its Applications supported a number of conferences during 1974/5 where speakers representing both employers and educational institutions advanced their points of view. Chapter 3 outlines the employers' case, the published evidence that supports it and the teachers' reply.

Chapter 4 contains an attempt to construct a list of topics in which employers require their trainees to be proficient and which should, therefore, appear in both school and examination board syllabuses.

The final chapter suggests some initiatives which could be taken by various bodies - the schools, the teachers, the examination boards and the employers - to improve the transfer from mathematics in school to mathematics at work.

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Chapter 1

Why 'Modern' Mathematics?

It may appear paradoxical to open an investigation into a problem which is exercising the minds of both educationalists and industrialists at the present time with a statement of the philosophy underlying the type of mathematics teaching that has developed during the last twenty years, but as much of the criticism of the mathematical abilities of school-leavers has laid the blame at the door of modern teaching methods I feel that it is important that we bear in mind the background which led to their development.

During the early 1960s it was recognised that education in England faced a decade of ferment under the influence of three major factors; the growing number of children in the school age range, the widening desire for higher education and the shift in emphasis from the arts to the sciences in higher education. Mathematics was seen as being in the vanguard of this ferment for a number of reasons: mathematics is the common factor in all scientific study and as science becomes more important, so does mathematics; the distinction between mathematics as an intellectual activity and mathematics as a problem-solving technique was likely to disappear; the shortage of mathematics teachers in schools and universities looked as if it would become steadily more crippling unless remedies were undertaken.

The tripartite conferences between industrialists, school teachers and university lecturers at Oxford in 1957, Liverpool in 1959 and, particularly, at Southampton in 1961 led to a new initiative being taken in the direction of modernising the school mathematics syllabus. It is important to remember the significance industry and commerce attached to these initiatives. In addition to the number of people concerned with the use of mathematics in business who took part in the conferences, over twenty large industrial and commercial concerns provided the finance to enable the Southampton Conference to take place.

A number of decisions were taken at Southampton which were to have lasting effects and their results will be referred to again and again in later chapters. Despite the realisation that in order to satisfactorily staff our secondary schools with adequately qualified and trained mathematics graduates we would have to take the entire output of mathematics graduates from British universities for two years, the conference assumed the existence of teachers of sufficient quality and quantity, and then considered the content of mathematical education and the spirit in which it should be taught.

Careful consideration was given to the nature of mathematics as a two-faced subject having a dual role of academic discipline and technologically significant study - as a study in its own right and as a tool to be used. It was considered as harmful to divide mathematics into such sections as 'pure' and 'applied' as these divisions were at most temporary with topics switching across the pure/applied barrier constantly with mathematicians trained in either pure or applied mathematics being unable to adapt to the changing needs of society. What was aimed for was a structure of mathematical education which, while being a basis on which courses for the future could be built, would also be responsive to short-term changes in the mathematical needs of the country as a whole.

Bryan Thwaites (1961) stated his aims without reservation, "The teaching profession is highly conservative - and on the whole rightly so, for the maintenance of many high and certain absolute standards is its greatest responsibility. Yet change there must be." It was realised that by considering school mathematics in its relation to producing graduates, the innovators were restricting their consideration to those in the sixth-form who would go on to university, but as the first priority was seen to be an adequate supply of suitably qualified graduate teachers this was thought to be reasonable.

The basis of the mathematics course to O-level was redefined. It was seen as a unified course using set language from the earliest years emphasising clarity of thought, precise use of language and placing emphasis on algebraic structure rather than technique. "A greater understanding of the structure will inevitably enable the acquisition of techniques to be made" (Thwaites (1961)). This, in one sentence, summarizes the philosophy which was to form the basis for the far-ranging innovations that were to appear in school mathematics over the next ten years.

The publication of the Crowther Report in 1963 added weight to the movement for change in the structure of mathematical education. It was here that 'numeracy' was first defined and advocated as a universally desirable attribute. "Just as by 'literacy' . . . we mean much more than . . . the ability to read and write, so by 'numeracy' we mean more than the mere ability to manipulate the rule of three." Numeracy was to be considered as a part of the basic equipment of every educated person, to enable them to think quantitively, to realise how far our problems are ones of degree even when they appear to be ones of kind and to avoid the statistical fallacies which are as widespread and as dangerous as the logical fallacies which come under the heading of illiteracy. The Newsome Report of the same year reinforced this view. "An exclusive diet of the three Rs just does not work." Numeracy involves more than a single skill in computation and the manipulation of symbols. It includes "the basic mathematical equipment for successful everyday living."

The influences noted above led to two major changes taking place simultaneously in many of our schools. New mathematical concepts and new ways of looking at mathematics were introduced to pupils at an age which would not previously have been considered possible. At the same time there was a move towards the principle of encouraging children to find out for themselves what was previously told to them. The aim was that whereas previously children were 'told' first and then 'did' afterwards, now they 'did' first and were then led to formulate their findings. J.T. Combridge (1968) in an explanation of this approach quotes a 'diehard' who wrote, "I well remember the excellent teaching I received in primary schools . . . all the basic arithmetic processes . . . were effectively drilled into me well before the age of eleven. Admittedly I see now that I did not understand what I was doing, but that did not matter in the least. The point is that I could do it." Combridge points out that this process was designed to produce human beings programmed to act like comptometers or cash-registers, an aim which it achieved admirably. But in the 1960s stores and offices were able to employ women and girls who were failures in the drill technique; they only had to tap out the correct figures on a machine and it would 'do' the sum and print the answer. The drilling process also made many people allergic to mathematics from an early age, a wastage which could be afforded when the only function of mathematicians was to teach more mathematicians but not when industry was starved of mathematical expertise.

From the mid-1960s courses advocating this new philosophy were published in some profusion until by the mid-1970s a majority of secondary schools in England could be said to be following a 'modern' mathematics course. By 1975, in fact, the School Mathematics Project could claim that half of the schools in England and Wales were using their texts while a quarter of all O-level mathematics candidates sat their examination papers.

The last fifteen years, then, have seen school mathematics courses reacting swiftly to the criticisms levelled at them by both the educational establishment and society at large during the early 1960s. Most schools now follow courses which were developed to satisfy an apparent need. Why, then, is there a constant expression of concern from employers about the mathematical standards of our school-leavers? As Professor Armitage pointed out in 1974, "Expressions of concern at the decline in standards of numeracy are as old as the teaching of arithmetic, but in recent years the clamour has grown." This dissertation is an attempt to answer three questions concerning this clamour: What are the criticisms? Are they valid? What can be done to answer them?

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Chapter 2

General Criticisms

On 29th April, 1977 the Times Educational Supplement carried a report by the Confederation of British Industry in Wales on the results of tests given to school-leavers who had applied for craft apprenticeships. Only 37% could divide 966 by 7 and nearly 90% were baffled by the multiplication of fractions. Only one in a hundred could answer the question, "A man is paid £1.75 as a basic rate and is given a bonus of 14p. What percentage of the basic rate is the bonus?" The report concludes, "It is evident that a sizeable proportion of school-leavers are ill-equipped not only to find work, but also to deal with quite simple matters arising outside work. In short, they are at a great social disadvantage." This was only the latest in a series of similar reports from various authorities who had all found that the mathematical abilities of the school-leavers applying to them for employment were either not up to the standard required or at least not as high as they had been able to demand in previous years.

Most of the reports are couched in more general terms than those of the Confederation of British Industry in Wales but all seem to offer a similar comment: "The school-leavers of today are not as able mathematically as they used to be. The schools are falling down on their job."

The most vociferous exchanges were probably those that appeared in the Times Educational Supplement during the months on either side of Christmas 1973. The spark which ignited the tinder-box was provided by a letter from Donald Sherriff on 23rd November.

> "When in my accounts class a girl in the 5th year taking accelerated O-level this January asks me how to take 20% trade discount off an invoice; or a 'radio nut' in the fourth has not got the maths to calculate impedance though he can read a transistor circuit like a book; or when an A-level economist though he or she has got through O-level maths, is quite ignorant of a simple geometrical progression, I inwardly curse the New Maths."

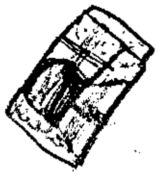
An article entitled 'Bring Back the Maths of Yesteryear' on 1st February brought together the comments of a number of critics. Paul Hodgkinson of the Engineering Industry Training Board stated that "Industry resents having to pay people to teach school-leavers to count" and Mr. G. Taylor, a training officer from Yorkshire, reported that schools and firms had first met in his area to discuss standards of mathematics in 1968 but that teachers failed to see the industrialists' point of view. In the same article Brian Longbottom of the Shipbuilding I.T.B. said, "We have a constant complaint from our craft training managers that we need the old maths for drawing plans and for development work," while Mr. C. Van der Meuten of the Air Transport I.T.B. noted that written examinations used to select apprentices had shown up a drop in standards during recent years and teachers in tertiary education were reported as saying that the problem was "not so much that students have never met the required mathematics in school but rather that they cannot in fact make satisfactory use of it."

Mathematical educators were not slow to answer these criticisms in the same way that they had been presented. In an article headed 'Maths critics barking up the wrong tree' on 8th February Claude Birtwistle, the Mathematics Inspector for Lancashire, said that employers were "trotting out the same old hardy annuals that had been heard for the last 25 years." How could modern mathematics effect the standard of so many school-leavers when only 25% of schools were using modern syllabuses? "I thought industry had gone metric anyway. If they want it measured in decimals, then they should measure it in decimals." Similarly David Fielker of the Abbey Wood Mathematics Centre claimed that "most apprentices get tested on a lot of arithmetic and when they prove deficient they are given a six-week remedial course. They then get on to the factory floor and find that they don't need any of it."

In the letters column of 15th February the teachers had their say. Robert Sutton asked "On the conversion of $\frac{7}{16}$ to a decimal, have engineers heard of metrication, conversion-tables, slide rules or computers?" D.T. Taylor claimed that "As a maths teacher . . . it is my job to provide a broadly based mathematical experience to all my pupils and not a narrowly defined arithmetic course as the E.I.T.B. and others seem to expect" and G.N. Bailey said that there was a great need for the critics of modern mathematics to get to know more about it. "Problems arise because the pupil or student has one set of tools which could be used on a problem while his mentor only knows another set."

In the calmer atmosphere of July 1974 Mr. D. Davies of the British Aircraft Corporation gives a typical outline of the problem as seen from the position of the industrialist. He finds youngsters of fifteen who want to be craft apprentices taking a mathematics test paper and getting most of the questions wrong. Not only do they get them wrong, however, they have not the slightest idea how to attempt them. The boys are not very bright, but they are brighter than their performance in mathematics indicates. Something has been done to them during their school years so that they are not able to perform to the standard one would expect. Davies lays the blame at a number of doors: the more free-and-easy atmosphere in modern schools; the removal of the motivation for youngsters to qualify for a job by the provision of the safety-net of the Welfare State; the shortage of good quality teachers. But he does not blame modern syllabuses as such, which makes him unusual among the critics.

J.C. Carroll (1974) of the Engineering Industry Training Board finds similar problems. Most engineering



trainees start their courses with basic training in manufacturing methods. While graduates and those entrants with A- or O-level mathematics find no insurmountable difficulties with the arithmetic used in basic training, those entrants below the O-level threshold present particular problems. Increasingly training officers are finding that they are arriving ill-equipped to do the simple arithmetic that is required during basic training. Trainees follow a technical education course parallel to their training and their mathematical skills are developed ahead for the needs arising in the later, more specialized training. But there is no time to organise an arithmetic course before the start of basic training as from the first day in the training workshop the trainees need to be able to do calculations.

Both Davies and Carroll say that their organisations, although not equipped to do so, undertake remedial measures in arithmetic for their trainees by either organising special classes for them at local colleges of further education or by taking skilled craftsman trainers away from their principal work to provide help with arithmetic. These measures do have good results, but employers consider that they constitute a waste of manpower when the schools could do the job so much more efficiently.

Many of the critics find evidence to substantiate their claims in the results of selection tests which they give to applicants for apprenticeships.

D.G. Dean (1975) of Westland Helicopters finds that applicants to his company have recurring weaknesses in the conversion of decimals to fractions, the conversion of fractions to decimals, the addition of fractions, metric conversions, general appreciation of metric quantities, formulas of simple geometric shapes, the manipulation of simple formulas and the use of logarithms.

Dean quotes an article from the Engineering Industry Publication, 'Skill,' in which a company director said that his son, who had gained a high grade in 'modern' O-level mathematics, was unable to do simple calculations. He was seated with a young engineer involved in simple tasks of addition, multiplication and percentages. The boy had an understanding of all of these tasks but could not do them accurately, or at speed, or repetitively. It was considered that before the boy was able to cope with a normal job at the age of sixteen, he would need a course of remedial treatment. The fray was then joined by the Group Training Officer of the Nottingham District:

> "Every year this problem arises within our member companies, and in spite of discussions with Youth Advisory Officers and Careers Masters over the years, I am afraid it's a case of no progress. . . . Consequently those responsible for training . . . have an unnecessary delay in the initial training period."

closely followed by his Training Manager:

> "Ask the applicant to express the formula graphically for the classification into sets of left-handed people with warts over 32 in the Home Counties in 1936, and they'll do a perfect job; ask them to express 7/16 as a decimal and they don't know where to start."

and then by H.J. Smith of Myford Limited of Beeston:

> "Our experience over the past years has been the lack of knowledge of the three R's of boys coming into engineering as apprentices (particularly mathematics, with written English coming a close second). Our own training instructor has the job of coping with remedial instruction. He spends valuable time instead of being able to concentrate entirely on the job he is paid for - teaching the basic principles of engineering."

Dean ends with the statement that the answer to the questions, "Is the new mathematics relevant?" and "Does modern mathematics prepare the children to support themselves and their families?" is a definite "No."

The problem is not confined to those who leave school at sixteen, as is confirmed by a medical school lecturer:

> "These young medics worry me. They will do the sums and cheerfully submit an answer which they ought to know must be wrong. Of course, so long as they are out by an order of a thousand times or so, we are all safe - when

> they reach for the syringe they just will not find one big enough, and they will smell a rat. But if they are only two or three times out - well, do not go into hospital in the next few years, that is my professional advice."

This comment, quoted by Lindsay (1975), appears to be more serious than the preceding ones. He maintains that not only can't school-leavers 'do' the calculations, but also that they have no feeling for orders of magnitude.

The above are just a few of the more coherent voices selected from among the many which have been raised in criticism of school mathematics during the last five years or so. During one period in late 1973 and early 1974 sections of the press seemed to be conducting a campaign of vitriolic abuse against the teaching profession as a whole and mathematics teachers in particular. For some years now it has been common for mathematics teachers in normal social gatherings to be wary of revealing their occupation lest their ears be belaboured with the views of people working in industry and commerce concerning the appalling mathematical ability of school-leavers and the irrelevance of modern syllabuses and teaching methods to the outside world. Fortunately, that situation has now been alleviated somewhat, mainly as a result of the events to be covered by the next chapter.

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Chapter 3

Both Sides Meet

The Employers Make their Case - The Schools Reply

Background

The type of criticism presented in the previous chapter is widespread but not very helpful to either the teacher or the employer. What the teacher needs to know is the areas in which school-leavers are being seen to be deficient, the specific problems that trainees encounter in industry and if the skills that employers now find lacking in trainees are, in fact, essential to their progress. Teachers during the early 1970s were also rather concerned that employers did not seem to appreciate the changes that had taken place in the teaching of mathematics during the previous ten years. Many experienced mathematics teachers, accepting the need for change outlined in Chapter 1, had studied the new syllabuses, attended courses on the new ways of teaching the subject and, with the encouragement of H.M.I.s and local authority advisers, modified the mathematics courses in their schools to take account of the needs of modern society as they saw it. Now here were seemingly reactionary employers, apparently without taking the trouble to acquaint themselves with the philosophy behind or the methods used in modern mathematics courses, using the press as a vehicle in which to criticise the schools for not supplying their particular needs. On what evidence were these criticisms based? Teachers had the suspicion that employers were basing their findings on the results of pupils' performances on well established selection tests which bore no relation either to modern teaching methods or to the needs of the employer. It was also suspected that the critics were not aware of the increasing number of school-leavers going on to higher education, with the subsequent reduction in the number of able school-leavers at sixteen.

The need was clearly for a forum in which both employers and educationalists could meet, put their individual points of view and discuss their differences with the object of providing each side with a better understanding of the other's problems. Such a forum was provided by three conferences convened during 1974 and 1975. Two of these, 'Mathematical Needs of School Leavers Entering Employment: 1 and II,' were held at Nottingham University in July 1974 and July 1975, while the third, 'Mathematics at the School/Industry Interface,' took place at Yeovil in March 1975.

Although the reports of these conferences in the educational press caused many people in both industry and the schools to consider for the first time their position regarding mathematics for the sixteen-plus school-leaver, it would not be true to consider that these were the first initiatives in the field. Since the early 1960s the Schools and Industry Committee of the Mathematical Association has encouraged and sponsored liason between teachers and industrialists through local initiatives. Groups working in London, Bath, Bristol, Nottingham and Sheffield have made useful advances in the relating of school mathematics to mathematics at work. The results of these initiatives, however, were usually only published locally and it was not until the reports of the 1974/5 conferences were made generally available in published form by the Institute of Mathematics and its Applications that both teachers and industrialists could study the problems in some depth and evaluate the evidence produced.

Mathematical Deficiencies

Ruth Rees of Brunel University spoke at all three of the conferences about her research into the mathematical difficulties experienced in further education by craft and technician apprentices. She provided evidence of the lack of skills generally referred to by employers and went on to attempt an analysis of these difficulties. She studied 17000 first year City and Guilds craft engineering papers and found that the overall performance on the questions involving calculating was worse than the performance on other questions to a considerable degree and that where possible those questions involving calculations were avoided. In fact, many of the candidates passed the examinations while failing in calculations and even without attempting any calculation items (Rees (1973)).

In order to attempt a definition of the regions of difficulty found in this research, it was decided to construct a diagnostic multiple-choice test with responses selected to pinpoint the nature of the difficulties. This test was given to a range of first year craft and technician students and a first year O.N.C. group as a comparison. The test discriminated between the different groups with craft students performing less well than technician students who, in turn, performed less well than O.N.C. students. Similarities did exist, however, between the groups and these were revealed by using an item analysis of the facility with which each of the groups answered specific questions correctly. There was a 'common core' of twelve test items for which craft and technician students both gave their worst performance. Even for the O.N.C. student group where the facility values were generally higher, ten of these twelve items showed a weakness in performance. For all of these 'common core' items the craft groups selected a specific incorrect response with greater frequency than the correct response. The technician groups showed this tendency in seven of the twelve items. The items which constitute this 'common core' with the most common incorrect responses are shown in Table 1 which is abstracted from Rees (1973) Tables 4 and 8.

As a result of her investigations Mrs. Rees concluded that the areas in which craft and technician apprentices found most difficulty in further education were:

(i) the concept of area proportional to $(\text{dimension})^2$ for similar figures, particularly circles;

(ii) operations with numbers less than unity;

(iii) orders of magnitude;

(iv) proportion;

Table 1

Calculations Test: Analysis of most common wrong responses for 'common core' items

Item	Facility	Most common wrong response	Frequency of wrong response	Comment
$\frac{7}{16}$ correct to 2 d.p.	30%	0.43	40%	significance missed of 'correct to 2 d.p.'
Square root of 0.9	13%	0.3	50%	lack of appreciation of numbers less than 1
Diameter of circles in ratio 1:2, ratio of areas?	27%	1:2	42%	assumption that area $\propto$ dimension
In $1m^2$ there are ...?	26%	$1000mm^2$	31%	as above plus possible confusion with powers
In $1m^2$ there are ...?	25%	10^2mm^2	25%	as above
5° 36' written in decimals is ...	18%	5.36°	35%	assumption that $1^\circ = 100'$
Diameter of base of cone is 30mm. Area of base?	20%	$15\pi^2 mm^2$	32%	lack of knowledge of area formulas
Brass is 7 parts copper and 3 parts zinc. Weight of brass containing 39g of zinc?	33%	91g	37%	found weight of one component only
Length of metal to make pipe inside diameter 200mm, wall thickness 10mm?	18%	none of these	33%	lack of item understanding and circum. of circle
$\frac{1}{R} = \frac{1}{2} + \frac{1}{6}$ R = ?	16%	$\frac{2}{3}$	52%	added correctly did not invert
1mm expands .00001mm/$^\circ$C. Expansion of 20mm for 5°C?	34%	.00055	32%	confusion because of craft context
Drill 600 rpm, feed rate 0.15 mm/rev. Feed rate per sec?	30%	0.015	32%	as above

(v) reciprocals;

(vi) conversion of angles in degrees and minutes into decimal form;

(vii) calculation in a technical context. (Rees (1974))

Here, probably for the first time, was evidence from carefully conducted research of the existence of specific deficiencies in the mathematical abilities of school-leavers and the nature of these difficulties.

At the Nottingham Conference in 1975 further evidence was provided by Lieutenant Allan of the Royal Navy who is involved with the training of electrical artificers and mechanicians. The training is both thorough and expensive and so there exists a need to identify training risks at an early stage. As part of a programme to identify these training risks the records of all trainees have been kept on a computer file for three years. A study of these records has indicated that the academic abilities of the recruits have decreased at a faster rate than their technical abilities. On entry to the basic training establishment it was found that although all of the recruits had achieved high enough aggregate scores on tests of basic intelligence, mechanical aptitude, mathematical ability and spacial reasoning to be accepted into the Navy as artificer apprentices, an increasing number exhibited below acceptable standards of mathematical ability. In 1971 this accounted for 8% of the intake, in 1972 for 30% and in 1973 for 52%. Over the three years a mean of nearly

40% of the intake were considered to have lower than acceptable levels of mathematical ability. This is quite alarming when one considers that entry to the Royal Navy as artificer apprentices is limited to boys with at least C.S.E. grade 2 in mathematics. Yet, after remedial training and extra tuition in mathematics, only 3% of the intake over the three years have failed the course as a result of academic ability. It appears that the mathematical shortcomings are not due to any lack of ability on the part of the recruits but are the result of some lack in their education (Allan (1975)).

Lieutenant Allan also indicated that some of the areas of difficulty were common to many recruits. The major difficulties occured with:

(i) transposition of formulas;

(ii) algebra as generalised arithmetic;

(iii) the ability to divide fractions and compound numbers;

(iv) the concept of size of numbers and the placing of a decimal point;

(v) trigonometry - difficulties in concepts and ratios;

(vi) geometry - generally poor but especially with those who did 'modern' mathematics at school;

(vii) manipulation - the majority of those who did 'modern' mathematics at school were unhappy because of their lack of practice in manipulation.

Elizabeth Mann teaches in a college of further education where lecturers were finding that they could not teach their course material adequately because of the lack of mathematical ability in their students. Catering students find costing calculations difficult or impossible; students on a child-care course where, as part of the course, they go into local primary schools and find that they cannot answer the childrens' questions about the work they are doing on fractions and decimals; mature students on a pre-teacher training course doing A-levels in arts subjects who realise that, as primary teachers, they will have to teach mathematics but are not confident with number work; A- and O-level mathematics students who cannot do the arithmetical part of their questions - the pass-rate for O-level mathematics in the college was 10%; the science staff complaining that students cannot tackle chemistry and physics calculations.

A test was devised for general studies students taking O- and A-levels as a preliminary to embarking on careers such as nursing, child-care, medical ancillary work or laboratory technician work. The test included questions on the four rules with decimals, the conversion of fractions to decimals, the idea of percentage without calculations, ratio and metric measures (Mann (1975)). Mrs. Mann's results are given in Table 2. Without seeing the test itself it is difficult to judge what is considered to be a 'deficiency' but Mrs. Mann claims that her results correlate highly with those of Ruth Rees on the same subject matter. Study of Figure 9, Table 7 and Appendix 1 of Rees (1973) would indicate that this is only true to a certain extent. I have summarised what I would judge Mrs. Rees' findings to be in Table 3.

Table 2

Deficiencies exhibited by general studies further education students on mathematics achievement test

Decifiency	Percentage of students
four rules of number (mainly division but some subtraction)	14
four rules of fractions	68
four rules of decimals	57
fraction/decimal conversion	88
percentage	65
ratio	58
metric measures	74

Table 3

Deficiencies exhibited by the whole further education sample in the calculations test

Deficiency	Percentage of students	Comments
four rules of number	18	Fairly close to Mann's finding
four rules of fractions	24	Rees only tested addition
four rules of decimals	62	Fairly close to Mann's finding
fraction/decimal conversion	70	But 40% of Rees' students converted correctly making the mistake with the decimal places
percentage	32	Only half of Mann's finding
ratio	67	Fairly close
metric measures	-	Not specifically tested by Rees

Despite the discrepancies between Tables 2 and 3, Mann had clearly identified a problem of some magnitude. A 'Basic Mathematics' service was initiated in her college where students could go to obtain remedial help. The test was used to identify problem areas which were then worked on, usually in a one-to-one teaching situation. Of the 160 students who have been through the Basic Mathematics Department, all except two have made what Mrs. Mann describes as "astonishing progress." She has found that in the course of one year for a few periods each week students can be helped to become confident in all of the topics covered by her test.

The human element in the numeracy problem is considered my Mrs. Mann to be very important. She gives examples of the many young people of 17 or 18 who come to her greatly distressed, feeling inadequate in handling numbers and believing that they cannot train for the job they want to do. Others say to her, "I have never learnt to divide," and are also distressed. They learned at the primary school to divide by repeated subtraction but never became proficient with a more efficient method. Now in further education when they are faced with a calculation like 9948 ÷ 12, the science staff laugh at them for trying to do it by repeated subtraction in the margin. The student knows that this is an inefficient method, but does not know a better way.

Not only school mathematics courses are found to be deficient by employers, G.C.E. boards are criticised as well. At the Nottingham Conference of 1975 Flight Lieutenant I.E. Cooke of RAF Cosford produced evidence that the possession of an O-level grade in mathematics was no longer an accurate prediction of suitability for further education courses. (Lindsay (1975)). Recruits at RAF Cosford follow an O.N.C. course in which the mathematics is biased towards electronics. In the past the assumption has been made that entrants with O-level mathematics will have the necessary manipulative skills to follow this course. In the entries for the previous two years this assumption was found not to be valid. The trend over the previous three years appeared to be clear:

Year 1	Year 2	Year 3
Very little remedial work.	Remediation required for a small number, particularly those who studied modern mathematics.	A lot more remedial work, again dominated by the ex-modern maths students.
Two members only studied modern mathematics.	Failures by SMP O-level grade 3 and by CSE grade 1 entrants.	Three Scottish Board O-level grade C failed.
Marks in ONC correlate positively with O-level grades.	Marks on ONC still correlate positively with O-level grades.	Not as clear a correlation. Ex-SMP students marks scattered randomly, but this could be due to remedial work.
	SMP candidates scored bottom marks in their grade category.	

From the observations in the table it is clear that the recruits likely to fail their O.N.C. are either those having obtained a low grade at O-level on a traditional syllabus or those having studied a modern syllabus, irrespective of the

O-level grade obtained. This finding, which is not untypical of other comments from further education, must be of concern to the schools.

Here were, then, facts to back up the impressionistic opinions of the type found in Chapter 2. Ruth Rees and Elizabeth Mann had produced clear evidence of deficiencies in the mathematical abilities of further education students and the two representatives from the Armed Forces had convincing reasons for believing that the situation was deteriorating rapidly over a fairly short period of time. Having identified a problem, the task was now to identify the causes, name the culprits and institute the remedies.

The Teachers

The obvious candidate for blame in this situation is the teacher. As Bryan Thwaites said in 1974,

> "Report after report, over the last 15 years, has warned of the inevitable decline in the mathematical competence of the generations of schoolchildren in the 1970s and 1980s, which would follow the steady decline in mathematical capabilities of those teaching mathematics in schools. There are many schools now whose staffing problems are such that it is hardly relevant to talk of standards of skill of any kind."

This view was supported by a primary school teacher, J.W.G. Boucher (1975), who said that,

> "You can have numerous fine schemes, and I do not think there is anything wrong with Nuffield Mathematics, I do not think there is anything wrong with SMP, but I do think there is something wrong with us as teachers."

While bearing in mind that both of the spokesmen above have close connections with two modern projects, Professor Thwaites as founding director of SMP and Mr. Boucher as a teacher deeply involved in the development of Nuffield Mathematics, and as such may have a vested interest in diverting some of the criticism levelled at their projects in the direction of the teachers, it is clear that many teachers are doing one of two things. They are either teaching as they were taught or they are using the publications of one of the modern projects as a 'bible,' following slavishly the instructions of the teachers' guide and giving all of their pupils the same work to do regardless of their ability. The teachers who do this, unfortunately, often have no choice. Their lack of knowledge of, confidence in or feeling for mathematics is such that no other course is open to them.

Evidence for this lack of mathematical ability amongst school teachers is contained in a follow-up study carried out by Ruth Rees (1974) with approximately 600 teacher trainees. On being given the same calculations test as the further education students (see page 22) the trainee teachers experienced particular difficulty with the same 'common core' items and also tended to select the same wrong responses as the F.E. students. The teachers as a group only managed to score a mean of 36 correct items out of the fifty in the test with the primary group scoring a mean of 32.

As part of the same study the teachers were asked about their qualifications in and attitude to mathematics. Table 4 shows the results for the whole sample

Table 4

Attitude of 504 student teachers to mathematics

I like	I tolerate	I dislike	No reply
34%	45%	18%	3%

with comparatively large proportions either actively disliking or only tolerating mathematics. Of course, many of these trainees will not teach mathematics in schools and so Mrs. Rees studied the responses separately for primary specialists. The responses, with the mean score of each group out of fifty in the test are shown in Table 5.

Table 5

Attitude of 107 student primary teachers to mathematics

Attitude	Percentage having attitude	Mean score of those having attitude
I like	41	36.5
I tolerate	38	30.3
I dislike	21	24.5

A more serious situation is evident here. These students will all spend some time each day teaching mathematics to young children. Yet 1 in 5 of them dislike mathematics and

3 in 5 tolerate it at most. The correlation between their attitude to the subject and their performance in it can be clearly seen from Table 5; is it unreasonable to suppose that their teaching skill will be similarly related to their attitude?

As an apologia for the teaching profession, a number of 'mitigating circumstances' were cited at the first Nottingham Conference (Lindsay (1974)). Teachers have not only had to cope with innovations in syllabuses and methods during the last decade, but have been involved in many cases in the reorganisation of secondary education which has provided a further distraction. New teaching methods take time to assimilate as innovation makes it more difficult for the teacher to pace himself and get the right emphasis when he first adopts new ways of teaching. When it was common for mathematics to be taught in the same way for many years, the teacher was usually following a familiar routine in which he knew where emphasis needed to be laid and where he could expect difficulties to arise.

The attitude of society to mathematics and teachers must have some effect on how pupils in schools react to the subject. During the last decade society has progressively become more inclined to express views opposed to the imposition of authority. For the school-pupil the obvious target for him to rebel against is the schoolteacher. How can one expect children to accept that in order to perform well in mathematics it is necessary to exercise discipline over both ones time and ones thoughts when the common ethos of society as a whole is that the good things of life are obtained by 'doing your own thing' or 'taking the easy way out?' The reaction of many youngsters must be that if the learning of mathematics requires so much effort, "I'll not bother, thank you." This attitude is strengthened by the attitude to the subject that they encounter from adults. From the parent who, seeing his child struggling over his mathematics homework, remarks "Never mind, son, I was never any good at maths in school and I've done all right for myself haven't I?" to the public figure who openly admits that any mathematical argument is beyond him, the young person is constantly assured that although it would be very nice if he was able to do mathematics, it does not really matter if he cannot. The teacher often appears to the pupil to be the lone voice in opposition to these views.

The teacher can only hope to counteract the forces acting in society against the attributes necessary for mathematical competence if he is able to appear to children as a person of some authority with something interesting to say. While having increasing competition from the mass media for the attention of his pupils, the teacher is at the same time having his authority questioned by responsible people on all sides. Many of the critics are sympathetic and constructive but often the criticism is of the form, "I earn more than a teacher and am therefore more important to society. My views on the education of children thus carry more weight than those of the teacher," or "Teachers go from school to college and then back to school. What do they know about life in the real world?" Thus sniped at from all sides, the teacher finds himself in a beleagured position, observed by his pupils to be under criticism and often unable to earn their respect for his views and interests.

The environment of supermarket tills, bank computer terminals and advertisements for hand-held calculators suggests that computation is a dying art. Yet in reality the use of calculating aids depends upon an apprenticeship in written calculation. In industry computers, calculators, slide-rules, nomographs, graphs and tables are not available for every calculation and often simple ad hoc calculations by hand and brain are what is required. It would make the teachers' task in motivating pupils much easier if industrialists could emphasize this when making public pronouncements on the inadequacy of school-leavers instead of concentrating their remarks on the shortcomings of the educational system as a whole and teachers in particular.

These, then, were the types of argument advanced concerning the quality and the problems of teachers at Nottingham in 1974. However, although Ruth Rees was able to show that mathematical deficiencies did exist in prospective teachers and others were quite willing to accept that the poor quality of teachers, to some extent at least, accounted for the poor performance of school-leavers, there was at that time no evidence available to indicate whether this was true. Since that time one attempt has been made to provide such evidence.

A four-year research project at the Chelsea College of Science and Technology by David Mathews investigating the relevance of school learning experience to the performance of craft and technician trainees during their first year of training in the engineering industry is in the process of being completed at the present time. An interim report has, however, been published by the E.I.T.B.(1977) in which some of the main findings of the research are available. The study was conducted by administering a battery of tests and questionnaires to a large number of craft and technician trainees during their first year of training. A measure of the type of mathematics teaching experienced by the trainees was obtained from the answers to the questionnaires and this compared with performance on tests and in training. The numbers who were classified as having experienced mathematics 'probably badly taught' were reported to be not very high and there was a low reported incidence of features considered to be typical of 'bad' teaching. The example is given of "only" 92 out of 498 trainees responding "no" to the question "Did your maths teacher usually mark your work thoroughly?" In another question about the frequency of marking work, "Responses for 494 trainees show that for the great majority work was reported to be marked either every lesson or once or twice a week, and only 87 replied that it was marked no more often than once or twice a term." As both of these responses were made by 18% of the sample, the conclusion that the incidence of 'badly taught' mathematics was low would seem to be at variance with the evidence. "How low is low?" would seem to be a pertinent question. Perhaps the full report, when published, will provide a more detailed account.

In addition to reservations concerning the definition of 'badly taught' mathematics, the correlation between performance in the initial arithmetic test given to the trainees and those trainees 'probably badly taught mathematics' was only -.19. The report concludes that "the incidence of features included in the 'probably badly taught' variable is detectably damaging to performance in arithmetic" but this statement is not supported by the evidence presented.

It is intuitively clear that the quality of teaching will effect the performance of school-leavers in mathematics. What is less clear is the magnitude of the problem. While Ruth Rees finds that large numbers of teacher-trainees are not confident in their mathematical ability, David Mathews apparently finds that poor teaching does not account for poor performance in a significant number of cases. While it would be unfair to be over critical of Mathews' results before the full report is published, it does seem that his method of identifying poor teaching by noting the responses of ex-pupils to questions concerning the setting and marking of work is open to some degree of inaccuracy. What seems to be lacking when one attempts to quantify the extent to which the performance of school-leavers in mathematics is deficient due to the poor quality of the teaching received, is research into the actual problem itself. There is evidence of the qualifications and attitudes of teachers but none of their effectiveness. Does a mathematics teacher with good academic qualifications necessarily make a better teacher than one without? The assumption in written papers seems to be that he does, but it must be the personal experience of most teachers that the highly qualified graduate is often less able to communicate his knowledge to pupils, especially during the early years of schooling, than the enthusiastic teacher with only minimal mathematical qualifications but with the ability to communicate his enthusiasm to children. Until someone is able to draw up a profile of the 'effective teacher' it will be impossible, with any degree of certainty, to determine the extent to which the problems that we are considering here are the result of what is commonly referred to as the poor quality of the teaching profession.

'Modern' Mathematics

Probably the most common cause cited for the decline in the ability of school-leavers to perform as well in recent years as their forbears did is the advent of 'modern' mathematics. Ever since the events outlined in Chapter 1 led to the establishment in many schools of new courses, there have been vociferous critics of the innovations. As early as 1968 Dr. J.M. Hammersley, the convenor of the Oxford Conference, which could be said to have provided the first stimulus for reform, produced a scathing attack on what he described as "the enfeeblement of mathematical skills by 'modern mathematics' and by similar soft intellectual trash in schools and universities" (Hammersley (1968)). He considered the new courses to be too concerned with 'pure' mathematics and with those pupils who would go on to study mathematics at university. To him the case for modern mathematics in the classroom must rest on the needs which the school sees for society as a whole. But, as W.W. Sawyer (1965) pointed out a few years earlier, it is difficult to define what the mathematical needs of society are. The problem is complicated by teaching being essentially for the future. With teachers having a working life of forty years and the children they are teaching having one of fifty years, what the trainee teachers of today are learning may have effects on what is happening ninety years from now.

Both Lieutenant Allan (page 25) and Flight Lieutenant Cook (page 30) in their evidence to the 1975 Nottingham Conference indicated that they found the mathematical deficiencies of their entrants having followed modern syllabuses to be greater than those who had followed a more traditional course. Other evidence, however, is to the contrary. Ruth Rees (1974) finds no difference between weaknesses exhibited by students studying traditional and modern mathematics.

A. Fitzgerald (1976) similarly found in a pilot study in the Birmingham car industry that the results of selection tests used by the industry did not favour either traditional or modern candidates at O-level. An investigation conducted in Southampton by the John Lewis Partnership was likewise unable to find any significant correlation between the difficulties experienced at the start of training and the type of syllabus followed by the trainee (Southampton Area Education Office (1974)).

The most recent work in this field is again found in David Mathews' study (E.I.T.B.(1977)). From the trainees' answers in the questionnaire a 'maths innovative' variable was constructed. This was an aggregate of the responses which indicated that pupils had frequent experience of teaching methods popularly associated with 'modern' mathematics.- use of work cards, project work, work in groups and the use of certain equipment. While most of the trainees had experienced some of these approaches, 'traditional' methods were far more common during the last year of school at least. The 'maths innovative' variable showed an insignificant correlation with the arithmetic test scores at the beginning of training. The correlations between the 'maths innovative' variable and other measures indicated that the more intelligent trainees tended to experience more innovative methods but, as Mathews says, "There is certainly evidence enough to counsel restraint in the condemnation of innovative methods as the culprits for poor mathematical attainment."

A separate part of the questionnaire asked about the study at school of topics usually associated with 'modern' mathematics - sets, vectors, matrices, motion geometry, computers and statistics. Again it was found that most trainees had experienced a mixed course consisting of both 'traditional' and 'modern' topics but when the number of 'modern' topics studied was correlated against the arithmetic test scores a low positive correlation was obtained. This was above what would be expected when the scores on the test of general intelligence were taken into account. While this would seem to indicate that those trainees who had studied more 'modern' topics had higher arithmetical ability on leaving school, another finding of the study was that those who had studied a wider range of topics, whether 'modern' or 'traditional', had also tended to score higher in arithmetic. The important variable thus seems to be the range of topics studied and not their nature. What Mathews' work does support, however, is that the cause of deficiencies in arithmetical ability is not the study of 'modern' mathematics at school.

While bearing in mind the findings of the previous three paragraphs, it is generally acknowledged that there has been a swing away from computation in the 'modern' mathematics courses. All the old arithmetic and algebra is still there in the modern syllabus; what may be lacking is the old drill and practice exercises which have, in some cases, been sacrificed in order to enable a wider syllabus to be studied. Both Professor Thwaites and Dr. Alan Rogerson of S.M.P. spoke at the Nottingham Conferences in defence of the modern approach to teaching mathematics and indicated the ways in which they thought the curriculum development projects could influence the relationship between school mathematics and mathematics at work.

Bryan Thwaites (1974) assured the conference that he considered the question of manipulative skills in mathematics to be an important one as the subject could not be practiced with any confidence, satisfaction or success unless the basic technical skills were readily at hand. He thought, however, that the complexity of the question was often overlooked by people from industry who feel dissatisfied with the apparent skills of the school-leaver. The level of skill required for any particular manipulation depends fundamentally on the purpose to which the individual will put the manipulation and this tends to mitigate against the construction of norms of skill desirable for different types of manipulation. The very specific skills which often form the basis for discussions of manipulation in mathematics appear often to be disconcertingly straightforward; $123 + 234 = 357$ only requires the knowledge that $1 + 2 = 3$, $2 + 3 = 5$ and $3 + 4 = 7$, but this is a very limited view of the addition of three-digit numbers.

The fact remains, however, that people outside the educational sphere are not satisfied with the quality of the products of the educational system and Thwaites recognises this. He notes that the balance in the mathematics classroom since the mid-1960s has been strongly affected by materials which have laid greater stress on mathematical ideas, understanding and applications than did earlier syllabuses. Now that teachers have become used to using these materials, they may reconsider their syllabuses, perhaps adjusting the balance. The School Mathematics Project has realised that the early books in their series were short on drill and revision exercises but the balance began to be restored as the writing evolved and extended. Dr. Rogerson pointed out that supplementary booklets containing reinforcement in basic arithmetic and algebraic skills were being produced by S.M.P. in order to alleviate some of the difficulties encountered in this area. It remains a fact of life, however, that we are not likely to return to the type of partitioned, didactic courses that were common in the late 1950s. What is required is a reappraisal of the present courses, adjusting the balance where necessary to bring them into line with modern requirements. As a guide to teachers in making this adjustment, S.M.P. have produced a table of manipulative skills and the ages by which they believe children should be able to show a 'high degree of proficiency' in them. This table is included as an Appendix.

The Demise of the 11+

One factor considered by Thwaites (1974) as an influence on the achievement in mathematics of pupils was the widespread disappearance of the eleven-plus examination. With no norm of attainment at a child's half-way point in education a valuable monitoring device was destroyed and the time scales for detecting and correcting deficiencies was lengthened. The new freedom in primary schools may have encouraged them to concentrate more on spacial work at the expense of manipulation. Neville Bennett (1976) tends to support this view. He found that in primary schools there was noticeably higher progress in mathematics made by pupils from areas having an eleven-plus examination than from those who did not. His results, however, have to be treated with some circumspection as very few of his informal schools were in selective areas and analysis of the progress made in selective and non-selective areas by children being taught by formal, mixed or informal methods shows very little variation in mathematical progress. Bennett's results are shown in Table 6.

Table 6

Gain over expected mathematical progress in selective and non-selective areas of pupils being taught by different teaching styles

(The number of schools in each category is shown in brackets)

	11-plus	No 11-plus
Formal schools	+1.8 (5)	+1.0 (7)
Mixed schools	-2.0 (5)	-2.3 (7)
Informal schools	-0.4 (3)	-1.0 (10)
All schools	+0.1 (13)	-0.8 (24)

Whether the existence of an eleven-plus examination does or does not effect the mathematical abilities of pupils entering secondary education, it clearly has other effects which make the secondary school teacher's task more difficult. When there was a universal standard test at the age of eleven for all children of an area, the secondary schools could be sure that the type of item examined in that test would be covered by the primary schools to the extent that their children would have had plenty of practice in it. Now, without such a yardstick and with larger secondary schools taking pupils from up to a dozen primary schools, the first year of secondary education often tends to take the form of a revision course in basic number work merely to ensure that those pupils who have not had the opportunity of becoming proficient in such work at the primary school can now remedy that deficiency.

As the controversy concerning the mathematical inadequacies of school-leavers has gained momentum during the last five years many teachers have entertained the suspicion that the lack of ability noted by employers is the result of one of, or a combination of, two factors. Either the tests on which the employer is basing his findings are not relevant to modern teaching in schools or the applicants for employment in various grades are now of a different general ability level than previously.

Are we considering the same people as 20 years ago?

The second factor was considered at the Nottingham Conference of 1974 by both Bryan Thwaites and Mr. D. Davies. Thwaites calls this the "redistribution of ability" where the greatly increased proportion of sixteen year-olds staying at school and qualifying for university entrance has led not only to a decrease in the overall ability of the average undergraduate

mathematics class but also to a chain reaction throughout the rest of tertiary and further education, industrial training, craft apprenticeship and so on. In other words, with the expansion of educational opportunity the bright pupil who used to leave school at fifteen and become a craft apprentice is now able to obtain higher education and so will no longer be available for employment in the less socially exalted spheres of the skilled craftsman. These positions are now filled from school-leavers, the overall ability of whom will be inevitably lower than it was twenty years ago. Davies uses a piece of elastic to illustrate the same point. The elastic, representing the whole school population at sixteen, is held vertically against a wall and fixed at the bottom. If you draw an arrow on the wall pointing to the pupils who were applying for craft apprenticeships twenty years ago and then start pouring people into universities the top of the elastic will be stretched upwards while the bottom stays fixed. The arrow on the wall is now opposite a lower level on the elastic than it was before.
The availability of higher education was also cited by J.C. Carroll (1974) at the same conference. "In some areas, where there is little tradition of sixth-form work, there are still considerable numbers of able 16 year olds entering industry. But in other places, one sees an increasing tendency for the school-leaving age of less able youngsters to rise and it becomes increasingly difficult to recruit young people with the right kind of arithmetical ability."

This reasoning was accepted at the first Nottingham Conference but a year later doubts were expressed as to its validity. Mr. Carroll (Lindsay (1975)), despite his remarks a year earlier, said that employers were seeing a fall in the level of results produced by applicants on arithmetic selection tests. In many firms the applicants were still recruited either because there was no one else or on the basis of other indicators such as spacial and mechanical aptitude tests, school reports and performance at interview. After remedial teaching at the training centre, these poor candidates often reached the required mathematical standard.

> "One concludes that it is not the fundamental arithmetic ability of the average candidate which had changed but rather that there has been a significant change in the process through which that ability is made manifest and developed."

Robert Lindsay of the Shell Centre, Nottingham also reported in 1975 that the frequent response to enquiries is that after a battery of tests have been used and the individual and collective profiles drawn, while the other tests show very much the same results as before, the results of the arithmetic tests have progressively declined. This type of comment must bring into focus the whole question of the validity of the arithmetic selection tests used by employers.

Employers' selection tests

As Carroll (1974) points out, these tests are used to give a general indication of the overall ability of an applicant, perhaps as part of a test battery, and also to estimate the extent to which the applicant possesses the specific skills needed to cope with basic training. He also points out that as the tests are usually based upon 'traditional' arithmetic they can give wrong indications of general ability when completed by pupils doing 'modern' mathematics. The tests are produced by organisations such as the National Institute of Industrial Psychologists (until NIIP was absorbed into NFER) and the National Foundation for Educational Research who validate the tests, provide literature concerning the uses to which the tests can be put and train industrial staff in their administration. NFER will in fact not supply tests to organisations who do not have the requisite number of staff trained in the administration of their tests. As the training is quite expensive, this does lead to some organisations who are not able to buy up to date tests from NFER reproducing old NIIP tests, making their own modifications to take account of decimalisation and metrication. One large organisation of my acquaintance does this and still produces a 'mathematical age' score for their applicants based on the NIIP formula devised for use with the original test. Their concern is that many pupils from schools while producing 'average' scores on the other tests in their battery turn out to have 'mathematical ages' of only twelve or thirteen!

It appears that in many companies identical test batteries are used year after year and so have tended to acquire an air of authority which it is difficult to justify. Changes in syllabuses and teaching methods in schools must have a profound effect on the value of test batteries used in industrial selection. Test devisers in validating their tests are aware of this and take it into account when publishing their literature, but if up to date material is not used the validity of the test results obtained by the use of out of date tests and methods must be open to some doubt.

The major concern of teachers regarding the mathematics tests given by employers is their mystique. The validity of the tests is maintained by the tests being shown to no one except the candidate at the time of testing. This means that neither the teacher nor the pupil has any knowledge of the test that the pupil is about to take. As most of the tests are administered during the early Spring, there is nothing much the teacher can do about this situation except to continue following his syllabus towards the C.S.E. or G.C.E. examinations of the Summer. Only by good fortune will the pupil be in practice for the items included in the test. This invalidates the test for any other use than to give a measure of the transitory state of the pupil in respect of a small part of his school work. Holders of this view would maintain that if these mathematics tests are relevant to the job the pupil has applied for, we could expect that when he is employed he will be in practice in the items included in the test. The employer will, therefore, obtain a more realistic picture from the test of how the pupil will perform as an employee if he is in practice for the test. Why not publish a specimen test paper for all to see? This would prove advantageous to the employer as teachers would then ensure that the items on the test were well covered.

On the other hand, the items included in most of these tests are of a fairly elementary nature and should be part of the 'inbuilt equipment' of the average school pupil of sixteen. The tests are very susceptible to the effects of teaching, however, and as such their predictive qualities or ability to measure mathematical aptitude could be invalidated by making their contents available for general perusal. J.J. Benson of the British Aircraft Corporation (Preston) relates a significant experience:

> "After testing we decided to inform the candidates who had failed in mathematics. One headmaster, who had had six such letters brought to his notice, listened incredulously to an account of their mistakes. On learning that they would have been accepted but for their mathematics, he begged for them a second chance, with the result that, after additional work, several weeks later, their scores were doubled and they were accepted."
>
> (Lindsay (1975)).

In his report of the second Nottingham Conference, Robert Lindsay says that the time is ripe for employers to say to applicants, "Here is a specimen mathematics test paper . . . We are giving you fair notice well in advance of the date of the selection test, but if you cannot perform convincingly on the day, then we shall both know that an apprenticeship would be a doubtful venture for you as well as for us." Lindsay claims that in this way the air of mystery and suspicion would be cleared and that schools would then be able to keep in step with changes in the requirements of employers as reflected in their tests. Whether this recommendation would be acceptable to many employers, is I would consider, open to some doubt. In the present climate of suspicion between employers and the schools, would not the former suspect the latter of taking the easy way out if such a solution to the problem were adopted? Surely the employers' complaint is that the items on their tests should be so well known to their trainees that practice in their performance should not be necessary.

Here, then, during 1974 and 1975 both the employers and the educationalists came face to face and presented their views to each other. Although the official reports of the conferences indicate that the exchanges were both constructive and understanding, other observers gained the impression that little progress had been made towards getting away from the practice of 'sniping at the other side from the safety of ones own ivory tower' (see, for example, Tammage (1975) and Green (1976)). Whatever the immediate impressions from the conferences were, however, they have provided a catalyst which, during the last three years, has led to much more thought being given, on both sides of the fence, to the way in which employers select their trainees and the way in which mathematics is taught in the later years of secondary education. If nothing else, some progress must have been made if it is true, as Robert Lindsay (1975) says, that there was a concord from the first Nottingham Conference that "Ambitious craftsmen, employers, teachers and society are all agreed that skill without understanding is in the long run utterly unacceptable as preparation for future innovation or preferment."

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Chapter 4

The Mathematical Needs of the 16+ School-leaver

One factor which became apparent during the exchanges of 1974 and 1975 was the increasing need for teachers, particularly in secondary schools, to become more aware of the mathematical requirements of industry and commerce. Not only was this felt at a national level but locally many forward-thinking industrial concerns were actively initiating measures whereby teachers attended courses at factories where they could see mathematics at work. The leaders in this field have generally been the engineering concerns who have said, "come and see your ex-pupils using the type of mathematics we require; talk to our instructors; talk to our apprentices; and then let us discuss ways in which the present situation can be improved."

As John Crank (1975) pointed out at the Yeovil Conference, there are many occupations in which the need for mathematics is small. He maintained that bus or train drivers, policemen, musicians, doctors, dentists, nurses and TV stars need little more than basic arithmetic and that even craft apprentices need only a very limited range of skills. Information received personally from employers concerned with fields traditionally considered to need a high degree of mathematical skill from their employees have confirmed this viewpoint. The Personnel Manager for the Nottingham Area of the National Westminster Bank comments, "From our own experience, the applicants we recruit generally meet our requirements with regard to Mathematical ability but our standard is not set high and . . . a flair for figures is not now a prerequisite for Bank staff," and Mr. Phillpott of the Nottingham Office of the Prudential Assurance Company says that, "We have not consciously identified any problems with the mathematical abilities of school-leavers who have been recruited to our staff." There are some sectors of commerce, however, who have indicated that their recruits are not showing the types of abilities which they consider essential. Attempts have been made in the last five years to identify these areas of mathematical expertise in different occupations.

During the early 1970s Margaret Hayman carried out a survey of the job opportunities available to school-leavers and the mathematical requirements of these jobs. Her findings were published in the International Journal of Mathematical Education for Science and Technology (1972) and referred to in her presidential address to the Mathematical Association in April 1975. The replies she received from employers indicated that their requirements would be met if school-leavers could show ability in the following areas:

1. Accurate addition and subtraction of numbers, money and time.
2. Facility in mental arithmetic, particularly money.
3. Multiplication and division of whole numbers and money.
4. The understanding of fractions, decimals and percentages.
5. Degrees of accuracy and approximate answers.
6. The use of calculators and mathematical tables.

7. Formulae for and measurements of length, area, volume and weight.
8. Accurate use of geometrical instruments for length, angles and circles.
9. Scale drawing and proportion of areas and volumes.
10. Elementary geometry of shapes, solids and loci.
11. Use of mathematical notation ($=$, $<$, $>$, $\approx$, x^n) and formulae.
12. Binary numbers and data sorting.
13. Collection, evaluation and representation of statistical data.
14. Use and interpretation of graphs.
15. Elementary probability.
16. Analysis of problems and the presentation of ideas or instructions in concise form, either in words or mathematical symbols.

The colleges of further education indicated that the above list would also provide a suitable starting point for entry to City and Guilds courses in engineering. Close scrutiny of Margaret Hayman's findings, however, indicate that her results may not be totally reliable. The table from which she abstracts the above list of requirements contains a number of controversial entries. The first two entries, for instance, are:

Job	Mathematics necessary
1. Hairdresser	Assistants - none; Cashier - money sums.
2. Punch card operator	'Modern maths', particularly binary numbers and simple problems on sets and data sorting.

But a visit to any hairdressing salon will reveal that at least a good knowledge of ratio and proportion is required for all employees using dyes, shampoos or rinses. The instructions for the use of these preparations are often given as a proportion or ratio by volume, and any mistakes made in their dilution or mixing could lead to the most embarrassing results.

The punch card operator, on the other hand, while exercising a specialist skill which, as in all clerical jobs, is carried out more efficiently with some knowledge of numbers, has no particular need for a knowledge of 'modern maths' or binary arithmetic. Her skill is in being able to accurately and quickly associate what she reads with what she types.

Confusion between the concepts involved in a task, and the skills needed to carry it out, is a common pit fall in studies of the mathematical requirements of employees. It is mentioned by Fensham and Davison (1972) in a report of work done by student teachers in Australia. Their students spend a few days on a 'mathematical hunt' in an industrial company trying to find as much mathematics as possible. Although many interesting mathematical ideas were found in their investigations, it was clear that once the operation had been set up by someone with mathematical ability, it could often then be carried out by semi-skilled or unskilled workers. For example, in a paint factory the basic ingredients of the paint are in proportion to each other in a predetermined way but the operatives who mix the paint work from tables and charts that give them the required proportions, and in a food processing factory a quite unskilled operative is able to work a quality control system. At five-minute intervals the next twenty-four '2 oz.' cans of paste are taken from the production line and weighed in grammes. The weights are recorded as in Table 7 indicating whether the cans are over or under weight.

Table 7: Weight control sheet for 2 oz. cans

Time	9.10			9.15			9.20		
Weight	F	+	-	F	+	-	F	+	-
7									
6									
5									
+ 4									
3	1	3		11	6				
2	111	6		1111	8		11111	10	
1	11111	5		11111111	8		1111111	7	
0	1111111111	-	-	1111	-	-	11111	-	-
1	11111		5	111		3	1111111		7
2				111		6			
3									
- 4									
5									
6									
7									
Sub-total	+14-5			+22-9			+17-7		
Total	+9			+13			+10		
Mean, Range	0.37 , 4			0.54 , 5			0.42 , 3		

The operative then computes the mean and range of the weights and plots them on charts for the quality control staff to observe. It is clear from this example that although the operative needs to be able to weigh accurately, multiply by small numbers, add directed numbers and divide by 24, her mathematical ability does not have to encompass the realms of the sampling techniques being used.

Similar errors in assessing the mathematical attributes needed for a particular occupation are often made by looking theoretically at the job instead of asking the man on the 'shop floor' what he actually does. In the foundry, for example, one would imagine that the moulder would need to calculate the volume of a casting in order to determine the amount of molten metal required and the pressures exerted by the molten metal in order to decide whether to pour 'from the top' or 'from the bottom.' In practice, however, the foundry-worker very rarely does these calculations; he learns by experience during his apprenticeship to estimate the amount of metal required and to recognise the types of casting which will require to be poured in different ways.

In addition to Mrs. Hayman's work, many other attempts have been made to construct a basic syllabus that would satisfy the needs of industry and commerce. Most people's lists seem to contain a common core on which everyone agrees, but this is augmented in a way that reflects the vested interests of the individual compiling the list. For example, D.G. Dean (1975) of Westland Helicopters constructed a list in which the items were similar to but not identical to those in Margaret Hayman's list on page 55. He suggests that the basic requirements from industry are that the school-leaver must:

(i) have the ability to do simple sums;

(ii) know and be able to use the principles of arithmetic;

(iii) have the ability to work in metric and imperial notation, including fractions and decimals.

He goes on to tabulate the basic foundations that are needed for industry to build upon:

1. Knowledge of tables.
2. Addition, subtraction, multiplication and division of four-figure numbers.
3. Conversion - fractions into decimals, etc.
4. Conversion - metric into imperial, etc.
5. Calculation of percentages.
6. Area and volume of simple shapes and solids such as squares, rectangles, right-angled triangles and circles.
7. Transposition of formulae.
8. Calculation of powers and roots.
9. Use of logarithms for the above.
10. Solving simple trigonometry.

At the 1974 Nottingham Conference a "necessary core" syllabus was devised which, with minor additions, I have found to be acceptable to all employers of 16+ leavers to whom I have shown it. This 'Common Core' syllabus, which was considered to be a desirable common component of every CSE/GCE O-level syllabus, is as follows:

The 'Common Core' Syllabus in Mathematics

1. Basic arithmetic. 4 rules. Squares. Tables.
2. Powers of ten. Place value.
3. Units of measurement. Dimension.
4. Ratio and proportion.
5. Concentration, mixtures, percentages.
6. Graphs and functionality; i.e. how one thing may depend on another.
7. Orders of magnitude. Estimation. Rough checks.
8. Flow diagrams for a scheme of work.
9. Organisation of calculations on paper. Neatness, care, checking.
10. Use of formulae: substitution and transposition.
11. Fractions and decimals.
12. Experience of a variety of calculating aids.
13. Properties of simple plane and solid shapes.
14. Trigonometry of the right-angled triangle.
15. Modelling: i.e. how one piece of mathematics describes several different situations.
16. Playing around with a problem. Strategies and tactics of problem solving.

(Lindsay (1974)).

In the remainder of this chapter I will consider the items in this 'Common Core' together with their applications to different occupations and then consider any additions that may be necessary to the syllabus to make it universally acceptable.

1. Basic Arithmetic

The major requirement of all employers is that their recruits should exhibit the ability to perform accurately and quickly the four basic arithmetic operations. Even the banks and insurance companies, the most undemanding of employers, in regard to their mathematical requirements, need their employees to be able to add and subtract money. A basic 'feeling' for number is what many employers find lacking in the young people they interview for jobs. The interviewee may be able to perform a particular type of calculation but the method employed is so stereotyped and unapplicable to its use in the execution of the job that it is useless (see, for example, the following change calculation and the percentage calculation on page 71).

Although many of these criticisms are probably justified, we must beware of the employer who assumes that a method is unapplicable simply because it is one that is not familiar to him. The personnel officer of a large departmental store told me that she was appalled at the method employed by a school-leaver to calculate the change required from £1 for a 37½p purchase. On being provided with a sheet of paper the girl produced:

$$\begin{array}{rcccc} & \cancel{1} & {}^{1}\cancel{0}^{9} & {}^{1}\cancel{0}^{9} & 1 \\ - & & 3 & 7 & \frac{1}{2} \\ \hline & & 6 & 2 & \frac{1}{2} \\ \hline \end{array}$$

I pointed out that the girl had used the decomposition method of subtracting which is the method usually taught in schools today. The personnel officer considered this to be a most complicated way of doing the calculation. I asked her how she would have performed the same sum. She produced:

$$\begin{array}{r} 1\ {}^{1}0\ {}^{1}0\ \ 1 \\ -\ 1\ 1^{3}\ 1^{7}\ \tfrac{1}{2} \\ \hline 6\ \ 2\ \tfrac{1}{2} \\ \hline \end{array}$$

the equally cumbersome 'equal addition' method. She maintained that this method was far superior to the decomposition method as it could be easily understood. The most significant aspect of the conversation to me was the total irrelevance of both methods to the question asked. How many shop assistants ever subtract when giving change? Every one that I have encountered always 'adds on' to obtain the amount of change, often counting the money out of the till as she does so. From the same personnel officer, however, comes confirmation of the truth of a story which I had always considered to be apocryphal when I heard it from other sources. I am assured that it was at this Nottingham store that a young girl, calculating the total value of 172 items at £1 each, pressed the £1 button on the adding machine 172 times.

Problems in basic arithmetic can lead to unfortunate misunderstandings between trainees and older people in places of work. It was at Rolls-Royce in Derby that an apprentice, making a component for a piece of machinery, asked the foreman how many of the components he was to make. The foreman pointed out that it could be calculated from the information on the drawing where it said that each machine needed eight of the components and that six machines were to be made. "How many is that?" he asked the apprentice. ". . . 40?" came the tentative reply. The foreman patiently explained that 6 x 8 = 48. The apprentice still did not really understand but produced the forty-eight components. When he had finished he went to the foreman and asked, "What do you want me to do with the eight spares?" The older man, unable to comprehend such ignorance, assumed that the apprentice was being sarcastic and it took the intervention of a training instructor to heal relationships between the two men.

Wherever one enquires, the first requirement of trainees is inevitably stated as the facility to deal with the four basic operations with whole numbers. I have received practically identical comments from the fields of engineering, retailing, laboratory work, nursing and agriculture with the most serious difficulties being encountered in the first of these. The types of calculation encountered here are, 'if a component is 50mm long including any cutting allowance, how much bar is required to make 500 components?' and 'if a round bar is to be reduced from 50mm to 46mm on a lathe, by how much should the cutting tool be advanced?' (Carroll (1974)). Figure 1 shows that the latter calculation involves subtraction and division by 2, and it is performing this arithmetic, not the formulation of the problem, that many apprentices apparently find difficult. As Carroll remarks, "A trainee could use any method for the solution of the problem and provided he produced an accurate job in the time allowed we would not question his arithmetic."

D = original diameter

d = finished diameter

Tool advance = $\frac{D - d}{2}$

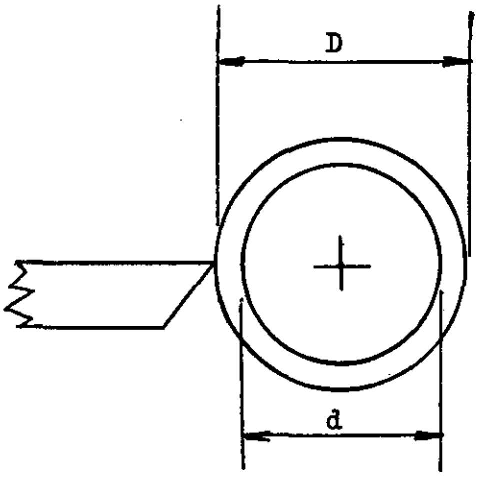


Figure 1

From conversations with employers, it appears that the basic requirement is the ability to be confident with the four rules in numbers and to be able to apply different methods of doing the calculations to different situations, using quick methods where these are appropriate.

2. Powers of Ten and Place Value

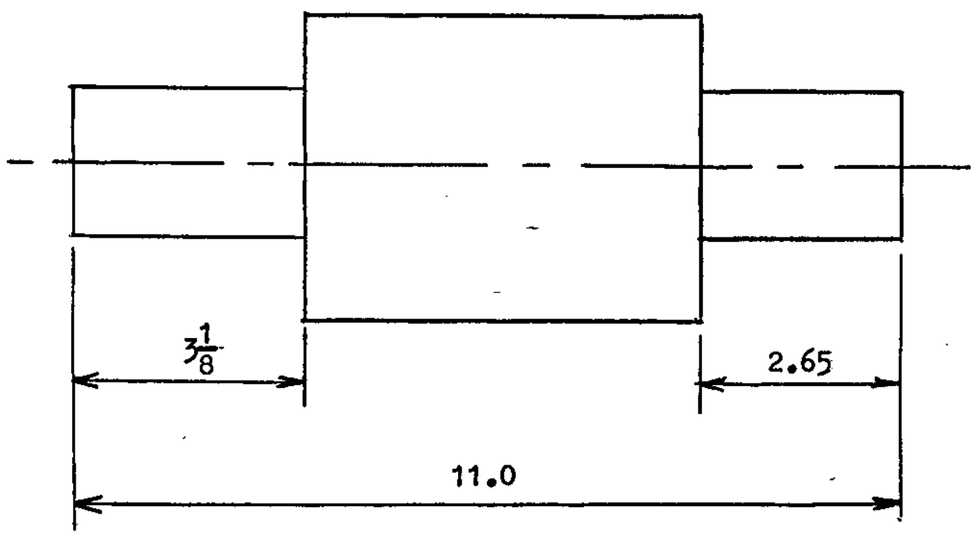
A thorough understanding of the powers of ten and the system of place value is not only important to progress in learning mathematics, but is useful in many commercial or industrial situations. When multiplying or dividing by large or small numbers an appreciation of the place value system allied with the use of powers of ten can make calculations much quicker. In occupations like nursing and laboratory work where SI units are the only ones in use it is essential that the relationship between the different units, which are always expressed in powers of ten, is second nature. For example, a laboratory assistant is often entrusted with conducting an experiment which may last over a week. The weights of the vessels in which the chemicals are measured at the beginning of the experiment are often of a different order of magnitude to the weights of the chemicals they contain. If, because of confusion between, say, 520.3g and 52.03g, the weight of the vessel is subtracted wrongly, the whole week's work is wasted.

3. Units of Measurement and Dimensions

The matter of units of measurement is one of utter confusion at the moment. It seems to be the opinion of many teachers that all of industry now only requires metric measurements and so these are the only ones consistently used in schools. In fact very few industries have totally converted to the metric system. The engineering industry has two main reasons for not doing so. The capital expenditure involved in replacing machinery is so large that many machines in use today are over twenty-five years old and still have a long working life in front of them. They are callibrated in imperial measures and so trainees must be able to work in these dimensions in order to operate them. Also, structures and engineering products that were made many years ago are still in use and need servicing with spare parts. For example, Stanton and Staveley of Ilkeston are still making pipe sections to replace ones first laid over fifty years ago. These, clearly, have to be constructed to imperial dimensions. Whether the conversion from imperial to metric is carried out in the drawing office or on the workshop floor seems to vary from company to company, but the manufacture of spare parts for ageing machinery can lead to some intricate anomalies. For example, it is often clear from the dimensions of a drawing that the part to be made is to fit an imperial machine. Dimensions like 50.8mm, which is obviously a conversion from 2 inches, frequently crop up. If the part is to be made on an imperially callibrated machine, the 50.8mm has to be reconverted to 2 inches.

A particularly confused situation with regard to metric/imperial measures exists for the police. The length of skid marks, position of vehicles and width of road at the scene of a motor accident is always measured in yards, feet and inches. This is because in giving evidence in court the policeman could not guarantee that a magistrate or jury could visualize the distances involved if metric measurements were used. On the other hand, vehicle use regulations are specified in metric units and lorry drivers are often charged with driving a vehicle in excess of 33 metres in length. A further complication is found in pathologists' or forensic scientists' reports which are always presented in metric units but which have to be converted into imperial if they are to be used as evidence in court.

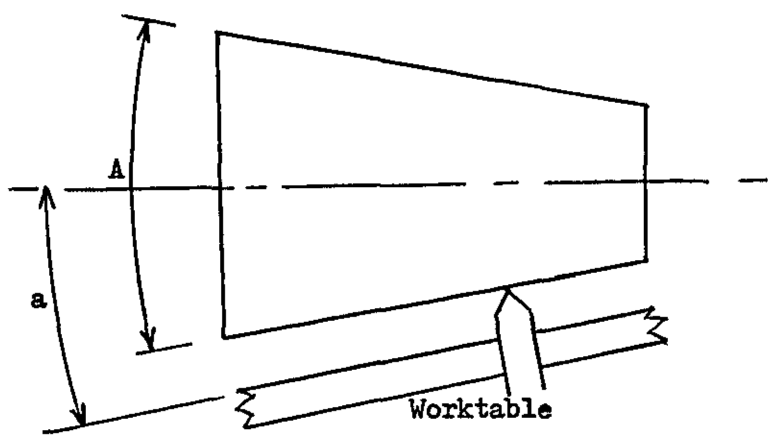
It is difficult to gain a concensus on the use of vulgar fractions as dimensions. Carroll (1974) gives an example of both vulgar and decimal fractions being used on the same drawing to enable different tolerances to be conveniently expressed (Figure 2). In the engineering concerns that I have visited, however, all dimensions are given as decimals with each one being given a specific tolerance. A common factor in all of industry is the units in which dimensions are expressed. All concerns follow the instructions of B.S.308 which states that "all dimensions shall be expressed in millimetres." This simple instruction may seem strange to teachers used to dealing with millimetres, centimetres and metres but engineers find that specifying all dimensions in the same units leads to fewer errors being made. Even large pipe sections $3\frac{1}{2}$ metres in diameter and $7\frac{3}{4}$ metres long would have their dimensions specified as 3500mm and 7750mm.



All dimensions in inches.
Tolerances: fractional $\pm$ 1/16; decimal $\pm$ 0.005.

Figure 2

Dimensions concerning angles are always expressed in industry as degrees, minutes and seconds. As Ruth Rees found (page 25), this is an area where trainees are deficient. An example was shown to me by Mr. J. Smith at Rolls-Royce of the way in which a taper is constructed. In precision work the taper would be specified by the angle made by the taper ($11^{\circ}\ 1'\ 4''$). To turn the taper on a lathe the operative would have to divide this angle by 2 to determine the angle to be made by his work-table (Figure 3). Many trainees do not know that $60'' = 1'$ and $60' = 1^{\circ}$ and so cannot carry out the calculation.



A - angle of taper ($11^{\circ}\ 1'\ 4''$)
a - angle of worktable - half of angle of taper ($5^{\circ}\ 30'\ 32''$)

Figure 3

4. and 5. Ratio, Proportion and Percentage

The applications for these three topics is legion. In laboratory work where small-scale experiments are carried out to test new products, the accurate application of proportion and ratio concepts is essential. The engineering craftsman also finds ratio a very important idea and uses it often to ease his work. When drilling holes of various sizes the craftsman does not need to calculate the drill speed for each hole. He works out the most suitable speed for a particular sized hole and then will relate the drill speed to drill size by using inverse ratios - double the drill size, halve the drill speed. Perhaps the most notorious use of ratios in engineering is concerned with the operation of a dividing head which is part of the equipment for a milling machine. It is used for milling slots, grooves, splines and teeth which are equally spaced round the circumference of a cylindrical workpiece. The gearing is so arranged that 40 turns of the dividing head will cause the work to be rotated through one revolution. If eight flutes are to be milled into the handle of a screwdriver, the dividing head needs to be rotated through $\frac{40}{8} = 5$ turns between each milling operation in order to position the flutes correctly (Shell Centre/EITB (1977)). To enable the circumference of the work to be divided into a number of parts which does not divide 40, the dividing head has holes equally spaced around a series of concentric circles with a different number of holes in each circle. By choosing an appropriate circle the work can be moved through any fraction of a single revolution. To mill three equispaced slots in a workpiece, 40 is again divided by 3 to give $13\frac{1}{3}$ revolutions of the dividing head. To move the head through one-third of a revolution the 27-hole circle is selected and the handle adjusted to rotate through nine of these holes. To produce the three equispaced slots, then, the dividing head is turned through thirteen full turns and nine holes on the 27-hole circle between each milling operation (Carroll (1974)).

Even in the retailing trade a knowledge of proportion can be useful. "Batteries are packed in boxes of 4 which cost 36p; how much for 7 batteries?" Often young people apparently do not know how to go about solving this sort of problem. Again, when considering the application of the concept of percentage, the need is seen for a thorough knowledge of the subject to enable a flexible approach to be made to any question. Staff discount at Pearson Bros. of Nottingham is 17½%. If this is attempted for a purchase of £8.40 by the usual method taught in schools, a quite cumbersome calculation results:

$$\frac{17\frac{1}{2}}{100} \times \frac{840}{1} = \frac{\overset{7}{\cancel{35}}}{\underset{\cancel{4}1}{\cancel{200}}} \times \frac{\overset{21}{\cancel{840}}}{1} = 147\text{p} = \underline{£1.47}$$

The efficient way to perform this calculation would be:

10% of £8.40	=	84p
5% of £8.40	=	42p
2½% of £8.40	=	21p
17½% of £8.40	=	£1.47

but this can only be used by people with a thorough understanding of percentage. Of course, when the same employee has to calculate his superannuation contribution at 4% this method does not work and so another strategy has to be used; 'multiply by four and shift the decimal point two places to the left?'

6. Graphs

Graphs showing the relationship between two variables are often used in engineering. The electrical engineer constructs graphs for motors showing their torque for different speeds and the lathe operator draws graphs of cutting speeds for different diameter pieces of work in different materials to cut down on his need for calculation. The basic necessity of these graphs is that they are accurately plotted and that the scales are sensibly selected.

7. Orders of Magnitude, Estimates and Rough Checks

The ability to estimate the order of the answer to a calculation seems to be lacking in many of the trainees in industry. Nonsensical answers are often produced when a 'feeling' for the calculation would have indicated that something had gone wrong. Mr. Lowe of Diversey Laboratories tells of a laboratory assistant who produced a mean figure for five trials of a new preparation that was larger than any of the figures for the individual trials. When he pointed out, on looking at the figures, that the mean figure could not be correct, the laboratory assistant was unable to see how he could tell that it was wrong without carrying out the calculation.

8. Flow Diagrams

The types of flow diagrams required by industry are generally of a simpler type than those usually included in school courses. They are often used either to show the flow of work through a production process or to describe the operation of a new machine.

9. Organisation of Calculations on Paper

A common criticism of trainees, particularly from the machine skills instructors, is their reluctance to take care with the organisation of a calculation on paper. Often a jumble of apparently unconnected calculations is produced with a wrong answer at the bottom. The identification of the area of the mistake or mistakes is clearly difficult in such a situation. The neatness and care taken in setting out a calculation in full is highly regarded by instructors, not only for their own convenience, but for the benefit of the trainee in building up a store of calculations to which reference may be made later. The care taken over calculations should extend to the careful checking of the correctness of any answers obtained. This should involve the substitution back into formulae of results found, re-performing calculations by a different method and by seeing if the results of the calculations fit the practical situation to which it applies.

10. Formulae

Although formulae are used widely in engineering, their major use seems to be in the substitution of numbers into a formula to determine a particular measurement. Very little transposition is required. The types of formulae used are those to determine the cutting angle for different sizes of square thread and the determination of cutting speeds for different types of material. For example, the formula for determining the speed of rotation of a drill or lathe is found from the following formula:

$$N = \frac{1000C}{\pi D} \qquad \begin{array}{l} N - \text{cutting speed (revs/min)} \\ C - \text{cutting speed (metres/min)} \\ D - \text{diameter (mm)} \end{array}$$

The cutting speed in metres per minute would be read from tables and substituted into the formula with the diameter of the work or the drill to obtain the optimum cutting speed in revolutions per minute, which could then be set on the lathe or drilling machine. Of course, most machines have only a limited number of different speeds available and in practice the formula is simplified to

$$N = \frac{300C}{D}$$

which is found to be accurate enough for most purposes. Although many craft instructors have bemoaned the poor performance of trainees in the transposition of formulae, what is generally required is not that the formula be transposed, but that an equation be solved. An example is given in the Shell Centre/EITB booklet where the height of a cylindrical tank of volume 16 cubic feet and radius 1 foot is required. The formula is

$$\text{Volume} = \pi r^2 h$$

and when the known values are substituted,

$$16 = 3.142 \times 1 \times h.$$

This is now a simple equation in h. There is no need to go through the process of transposing the formula to

$$h = \frac{V}{\pi r^2}$$

and substituting the values in. Although the transposition of formulae appears on the syllabuses of the college of further education courses followed by trainees, it is very rare that it is needed in the place of work.

11. <u>Fractions and Decimals</u>

The use of vulgar fractions has already been touched upon while talking about ratios and percentages, but it is clear from both published material and personal experience in talking to industrial instructors that fractions are far from dead. Many of the material and reamer sizes quoted in manuals are in fractions of an inch and often work has to be made to fit machinery originally made in imperial dimensions using fractions. These fractions are almost exclusively given as multiples of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ and $\frac{1}{64}$ and, in practice, their decimal equivalents are soon committed to memory by constant use. The major requirement of both industry and commerce is that trainees are able to inter-transpose vulgar fractions, decimal fractions and percentages with accuracy and speed for the more common values. The application of the four rules to both decimal and vulgar fractions should also be known.

12. <u>Calculating Aids</u>

The use of calculating aids in places of work varies from industry to industry and from factory to factory. During personal research I have found no common consensus on the needs for various aids even within the same industry. The most controversial piece of equipment is clearly the electronic pocket calculator. Some jobs, like laboratory work, require the constant use of a calculator and every employee has a personal one available. Mr. M.P. Corby of Diversey Laboratories feels that school-leavers should at least have some appreciation of the different types of machine available and their suitability for different types of application. A similar comment, that trainees should "have been introduced to modern pocket calculators," was also, rather surprisingly, received from the principal of a college of agriculture. In the retailing trade, however, these machines are rather frowned upon on the shop floor. Apparently a customer is far more impressed if the hire-purchase deposit and repayments are worked out mentally or on paper than if a calculator is resorted to. Even in specialist areas like furnishing, ready-reckoners and pencil-and-paper calculations are found to be more acceptable.

In the engineering industry a variety of calculating aids is used. Technicians in design and drawing offices almost invariably use electronic calculators but the position in the workshop is less clear. At Stanton and Staveley logarithm and trigonometrical tables are used exclusively. It is maintained by their instructors that the workshop is a far too messy place for delicate pieces of electronic machinery to be left lying about. With filings, lubricating oils and cooling fluids in abundance it is felt that electronic calculators would not be very reliable. There is also the problem of security with employees not usually being stationed in one place of work for a long period of time. With the manufacture of an article possibly requiring the operative to move from bench to milling machine to drill to lathe and back to bench, either the calculator has to be carried in overall pockets or left unattended where it is liable to theft. A book of tables, on the other hand, is not effected by the ravages of filings or liquids (a wipe over with a rag restores its readability) and can be carried quite easily in overall pockets. At Rolls-Royce in Derby, however, most of the craft apprentices have their own calculators and use them extensively in the workshops.

In any discussion of calculating aids only one fact is certain; the slide-rule is obsolete. Nowhere could I find a place of employment where it was used.

13. Geometry

Geometrical ability in industry seems to be limited to a knowledge of the properties of plane and solid figures. The rectangle, triangle, circle, cylinder and prisms seem to be the most important figures to study. The laying-out of work in both the sheet-metal and engineering workshop requires a thorough knowledge of both plane and solid shapes (see for example Carroll (1974) p. 98 and Shell Centre/EITB (1977) p. 23) while the patternmaker needs to be able to construct quite complex geometrical structures. The theorem of Pythagoras, both for the determination of dimensions and for the construction of right-angles is a useful tool for the craftsman.

14. Trigonometry

The 'Common Core' lists the trigonometry of the right-angled triangle as a necessary study for school-leavers. All engineering employees use trigonometry to a greater or lesser extent. The craftsman in the workshop probably only needs a knowledge of the relationships for the right-angled triangle but others require a little more. The technician in the drawing office needs to be able to apply the trigonometry of the right-angled triangle to quite complicated two and three dimensional figures and the patternmaker has specific needs for specialist knowledge. For example, when a pattern for a circular casting is to be made, it is not constructed from one piece of wood but from several segmental parts. In order to determine the length of each segment a table of chords is used, the calculations being performed using either a calculator or logarithms.

15. and 16. Modelling and Problem Solving

Many times while talking to apprentice instructors I heard the comment, "They know the mathematics, but don't know how to apply it." In the drawing office at Rolls-Royce for example, it was noted that when an apprentice was in difficulties with a problem he just sat and stared at it instead of getting a piece of paper and 'playing around' with it. Employers seem to value very highly the ability of trainees to solve problems and it is thought that the schools could give far more experience to their pupils in the positive steps that can be taken when a problem appears to be unsolvable by standard methods.

Additions to the 'Common Core'

While showing this list of topics to employers my attention ~~has~~ only been drawn to one major and three minor omissions. The major omission is the lack of any mention of the solution of simple equations. As I indicated when dealing with the transposition of formulae, this seems to be a very useful technique which is used quite extensively in industry. Electrical engineers in addition have a need for the ability to solve simultaneous equations when dealing with Kirchhoff's laws while laboratory technicians require a working knowledge of statistical methods. This can be based upon an understanding of the mean, median and mode with an appreciation of the need for a measure of spread. Both the agricultural worker and the nurse require to be able to appreciate measures of mensuration, which is not specifically tabulated in the 'Common Core.'

Accuracy of Calculations

One important general difference between the mathematics taught in schools and the mathematics required by industry is in the matter of accuracy. This is probably best summed up in the comment made to me by Mr. J. Keatley, the workshop instructor at Stanton and Staveley,

> "In school if a pupil gets all of a question correct except the last part of the calculation you give him eight out of ten, and quite rightly too. In industry, however, a mistake in the third decimal place means scrap."

A similar comment could be made of wages clerks, shop assistants or nurses. There is often not a 'nearly right' answer in these occupations. It is either correct to the required degree of accuracy or it is wrong.

In view of the observations concerning the 'Common Core' made above, I consider that a suitable syllabus which should form part of every O-level and C.S.E. course would be:

1. <u>Operations on Whole Numbers</u>

   Confident, accurate and reasonably fast calculation of addition, subtraction, multiplication and division sums.

   Squaring and finding square roots.

   Short methods of calculation.

   Different methods of performing the same sum.

   The selection of the most suitable method from the repertoire for a particular application.

2. <u>Units of Measurement</u>

   SI units of length, mass and capacity (mm and m emphasised in length).

   Imperial units of length, mass and capacity.

   Calculations using SI units.

   Conversion of Imperial to SI and vice versa.

   Angles in degrees and minutes.

3. <u>Fractions and Decimals</u>

   Addition, subtraction, multiplication and division of simple vulgar fractions (multiples of halves, quarters, eighths, sixteenths, thirty-seconds, sixty-fourths, tenths, fifths, thirds and twelfths).

   Appreciation of place value in base ten and powers of ten.

   Addition, subtraction, multiplication and division of decimal fractions.

   Conversion of vulgar fractions to decimal fractions and vice versa.

   Fractional and decimal amounts of quantities.

4. Order of Magnitude

Estimation of calculation's results.

Pre-estimation - the answer to this calculation should come out to be about . . .

Post-estimation - the answer appears to be about right.

Degrees of accuracy.

5. Ratio and Proportion

Direct and indirect proportion.

Mixtures and concentrations.

Increasing and decreasing mixtures in a fixed ratio.

6. Percentage

Relationship of percentage to vulgar and decimal fractions.

Expression of one quantity as a percentage of another.

Increasing and decreasing quantities by a percentage.

Short methods of finding percentages.

7. Algebra

Solution of simple equations.

Substitution in formulae.

8. Graphs

Functionality - how one thing may depend on another.

Sensible selection of scales.

Construction of graphs to show relationships between two quantities.

Use of graphs as ready-reckoner.

9. Geometry

Properties of square, rectangle, triangle, circle, cube, cuboid, recular prism, cylinder, cone and sphere.

Pythagoras' theorem.

Accurate drawing to solve problems graphically.

10. Trigonometry

Sine, cosine and tangent relationship in right-angled triangle.

Application of these ratios to two and three dimensional problems.

11. Mensuration

Area of rectangle, triangle, parallelogram, circle and shapes made from these figures.

Volume of cuboid, prisms and cylinder.

Circumference of circle and surface area of prisms.

12. Statistics

Representation and interpretation of data.

The mean, median and mode - their uses.

A measure of spread (range or inter-quartile range).

13. Flow Diagrams

Flow diagrams for work processing and machine operation.

14. Calculating Aids

Practical applications of ready-reckoners, graphs, nomograms, books of tables and logarithms.

Practice in the use of above.

An appreciation of the uses and limitations of electronic calculators.

An appreciation of the computer, its uses and limitations.

15. Organisation of Calculations on Paper

Neatness, logical presentation, care and checking.

16. Problem Solving

Strategies and tactics of problem solving.

'Playing around' with a problem.

17. Modelling

The way in which a piece of mathematics can be applied to several different applications.

In addition to the above, O-level courses, which will be followed by pupils intending to become technicians or more skilled craftsmen like electricians, should include the solution of simultaneous equations in two unknowns, the solution of quadratic equations and the application of these techniques to problem solving.

If the topics in the above list were well understood by the majority of school-leavers within the O-level/C.S.E. range I think that all employers would be highly satisfied with their trainees' mathematical performance. While not all employers require the same set of skills from their employees, from personal interviews with training personnel in engineering, retailing, banking, insurance, agriculture, the police, nursing and the chemical industry, I would consider that the syllabus should be comprehensive enough to serve the needs of all sixteen-plus school-leavers. This, then, is what the employers require. How their requirements can be met is the subject of the next chapter.

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Chapter 5

The Remedies

The previous chapters clearly indicate that there is something lacking in mathematics at the 16+ school/work interface. The schools are certainly not providing pupils with the types of skills that employers require and the employers are not, in many cases, aware of the type of curriculum being used in present-day schools. In order to remedy this situation it seems that there has to be a concerted effort during the next few years, not by one body alone, whether educational, governmental or commercial, but by all parties concerned with mathematics, both at school and at work, to attempt to find a way out of the present empasse. I am sure that the will is there; what is lacking in many cases is the knowledge of what is required. During my visits to places of employment in connection with this piece of work it was most encouraging to find that all of the people concerned with training in both industry and commerce to whom I spoke were keen to talk to me about their problems and prepared to go to great lengths and much trouble to ensure that I obtained the right sort of information from them. Even more significant was their willingness to listen to the teacher's point of view. In many cases they were as interested to hear what was being taught in schools as I was to hear what was being done in industry.

for GCSE

The will is there, then, what is to be done? I have attempted to draw up recommendations for various fields within this area but would emphasise that the most important of these is the last one of cooperation, for without that the reforms made in one area may well be nullified by actions in another.

The School Curriculum

As per bw As M.G. Kendall pointed out in 1968, we should aim at three levels of competence in our education in mathematics. Firstly, every citizen should have an arithmetic facility sufficient to conduct his ordinary affairs and a statistical facility which will enable him to avoid being misled by the use of numerical evidence. Secondly there is the level of competence needed by people who will use mathematical techniques as part of their employment. This will include the craftsman, the nurse, the shop assistant, the hairdresser, the office-worker and the draughtsman who use particular mathematical skills in their jobs to enable them to exercise their primary expertise. The third level of competence is that required by the person whose job will involve extensive use of mathematical methods. People such as systems analysists, statisticians, production managers and designers need the mathematical techniques and knowledge without which their jobs would be impossible.

The reforms detailed in Chapter 1 were aimed at the third type of competence and have had considerable success in achieving their objectives. There are now apparently enough mathematicians graduating from our universities to satisfy the needs of industry and commerce (Lindsay (1974)) but this comparative success with the motivation of able pupils towards higher education must not be allowed to divert our attention away from the question of what success the modern syllabus has had with the 16+ school-leaver. For it is here, at Kendall's second level of competence, that the deficiencies seem to be evident.

One factor is generally agreed; there is no question of going back to the old syllabuses of the 1950s. On the basis of the evidence at the moment (see pages 40 to 42) there does not seem any reason to equate the lack of mathematical skills directly with the advent of modern syllabuses in secondary schools but it was pointed out at the Yeovil Conference (Crank (1975)) that there may be a need to modify the emphasis placed on different aspects of mathematics within the broad framework of the school syllabus. The urge during the last twenty years has been to get away from rote learning and excessive drilling techniques and to concentrate on the ideas and structure of mathematics itself. It seems to me that, especially in the case of the 'average' pupil, the pendulum has swung too far. To answer the criticism from industry and higher education that youngsters seem to know about mathematics but cannot 'do' it, there needs to be a shift of emphasis. It is a fine aim to ensure that all children really understand what they are doing before they attempt it, but is this ideal too abstract for many children to achieve? Margaret Hayman (1975) quotes evidence to show that it is beyond the capabilities of many pupils to cope with the abstract lines of reasoning required in modern syllabuses and the personal experience of many teachers must show the same result. We all must, at some time, have had the experience of leading children carefully through a line of discovery during which only some of the class have managed to successfully reach the conclusion of the work. The tactic resorted to is then often to tell the rest of the class of the results of the work and show them how to use it. The relief felt by the children when they realise that the concept was not as difficult as they thought, and the sense of achievement experienced by them when they manage to correctly apply the principle to a series of examples, often purely by rote, is clear to see. They are not really concerned about whether Pythagoras' Theorem can satisfy the criteria of rigorous proof. What they need is the confirmation that the result is true for a number of triangles drawn by them, reassurance from the teacher that their results could be applied to all right-angled triangles, some success in applying the theorem to examples and an appreciation of the uses of it in solving real problems.

One of the problems in teaching mathematics to secondary school pupils is the dichotomy between the two types of pupil being taught. On the one hand are the able pupils who will go on to study mathematics at a higher level and use it as an integral part of their employment, and on the other are the pupils who find mathematics not easy but will require some mathematical skills in the performance of their chosen career. For the former category the modern syllabuses with their rigour and imagination provide an excellent course, but for the latter a more flexible approach is necessary. What they need, basically, is a thorough grounding in the topics recommended in Chapter 4 plus an understanding of other areas considered to be necessary for an appreciation of the nature of mathematics itself. While the able pupil will become proficient in the 'common core' topics with very little drill and practice, this is not true of the majority of pupils. What they require is constant revision and practice in these basic techniques of manipulation so that they do not get forgotten. SMP has recognised this and has produced supplementary books of practice examples to be used with their texts.

The schools, then, could enhance the manipulative ability of their pupils by adopting the following strategy. Firstly draw up a list of topics and techniques which it is considered desirable that children of different abilities should know by the time they leave school or go into the sixth-form at 16+. Then decide which of these items should be known for various ability ranges by certain ages (see Appendix for an example). A programme of regular practice could then be introduced so that the techniques learned so far could be reinforced and not so easily forgotten. I see nothing wrong with a weekly session of 'Revision Exercises' containing one or two examples of all of the techniques learned to date. By this method not only could pupils gain confidence with particular manipulations which they know, but the teacher would have a regular source from which to monitor the pupils having difficulties with particular topics. It is my experience that children, far from finding this sort of exercise a chore, look forward to the challenge that such a regime offers. They are keen to improve their personal score from the previous week and are insistent in seeking help with items that they are finding difficult.

One significant factor of the teaching of mathematics during the last fifteen years has been the increasing tendency to teach more and more at an earlier age. It is not now uncommon to find primary children coping with the complexities of sets and geometry, and twelve-year-olds struggling with the laws of commutivity and associativity. Perhaps the time is now ripe to reverse this trend and defer teaching some topics until a later stage. J.W.G. Boucher (1975) told the Nottingham Conference that as far as primary mathematical education was concerned, "One thing I have learnt over the last fourteen years is that perhaps we are now trying to do too much, far too much." Ruth Rees (1975) is also of the opinion that the reason why some topics are found difficult by pupils in school is that they are taught too early.

The whole primary and secondary curriculum in mathematics needs to be reconsidered in the light of the remarks of the previous few paragraphs. We must not throw out all that is new lest the 'baby be disposed of with the bathwater' but, for the majority of our pupils we must restructure the contents of our syllabuses and our methods of instruction. Boucher maintains that in the primary schools two topics should be predominant, counting and place value, "If you get these two things right, you have a chance with the rest. But we try to go far too quickly at the beginning, so we store up for ourselves, and more importantly for our children more problems." The same could be said for secondary education. We try to make our pupils run before they are able to walk. Let us take the work much more slowly with our 'average' pupils, allowing ample time for both reinforcement and revision of basic manipulative skills. And finally, let us take note of Ruth Rees' observation that we should recognise that there is an interplay between understanding and drill in the learning process.

The Teachers

From the observations made in previous chapters it would appear that teachers and employers see mathematics teaching in schools in quite different lights. An attempt was made some years ago to see if this was, in fact, true. In 1970 A.J. Bishop and D.I. McIntyre published a study into the attitudes of both teachers and employers to the emphases which should be placed on secondary school mathematics. 131 schools and 71 employers replied to their questionnaire placing six possible emphases in order of importance. The results for both the schools and the employers are shown in Table 8. While one may argue with the ranking given to the different emphases by either of the groups, it must be accepted that there appears to be little divergence between the schools and employers in the general type of mathematics which they consider desirable in the secondary school. As a second part of their research Bishop and McIntyre asked the same respondents to rate on a one to five scale the importance of 50 topics representative of different types and branches of mathematics. Again a considerable measure of overall agreement was noted between the two groups with a rank-order correlation of 0.74 being obtained between the teachers' and employers' ratings of importance. The teachers and employers, then, seem to agree on the general type of course to be followed by secondary school pupils. What appears to be controversial is the emphasis to be placed during such a course. Is it to be the understanding of mathematical concepts, the learning of computational techniques by rote and drill, or a mixture of the two?

Table 8: Ranking of emphases to be placed on secondary school mathematics
(Bishop and McIntyre 1970)

Emphasis	Ranking by schools	Ranking by employers
Applicability to everyday life	2nd	2nd
Foundation for more advanced mathematics	5th	5th
An enjoyable and satisfying activity	1st	4th
A tool for a person's expected occupation	4th	3rd
Foundation for scientific study	6th	6th
Training children to think logically	3rd	1st

Ruth Rees (1974) maintains that the acceptance by teachers that a problem exists is half the battle won and that enough work is available to help teachers in providing their own solutions. She cites the need for positive teaching of number, the awareness that number work sometimes involves more mathematical reasoning than hitherto appreciated and the realisation that the reinforcement of concepts quickly and frequently throughout the school life of pupils is essential. Teachers must be aware of the efficiency of their methods of instruction and constantly seek confirmation that what they are teaching their pupils is, in fact, being learnt. This is particularly true of the primary school teachers who, as both Rees (1974) and Haylock (1977) have found, are in many cases not very confident with mathematics themselves. There must come a point during most courses of learning when the teacher must say, "Some of these children are never going to be able to work their way through all of the preliminary stages leading to the crucial part of this topic. How can I devise a simplified lesson which will show them the main points and teach them how to use the technique?" The learning of multiplication tables is a good example of the way in which a topic has been neglected. It is most desirable that primary school children should learn the basis of multiplying by adding the same integer a number of times, but when a fifteen-year-old still uses this same technique, something has gone wrong. There must come a time, preferably before the age of eleven, when a concerted effort is made to ensure that all children are able to accurately and reasonably quickly recall the basic 36 facts of the multiplication table.

This is hard work in some cases, but I think that it is more desirable than the present situation where children in secondary schools find mathematics so difficult, not because they cannot understand the concepts but because they cannot handle the numerical manipulations needed.

During the earlier years of secondary schooling it is very tempting to shelve problems with basic number work while the more interesting and challenging areas of topology, sets and motion geometry are investigated. In the upper part of the school there is always the C.S.E. or O-level syllabus to be covered. But what is the use of 'covering' the syllabus for any year if the children cannot use the techniques they are learning because of their inability to calculate accurately and confidently? What is the use of knowing how to find the inverse of a matrix if you always get the wrong answer because of your inability to multiply fractions?

The question of what makes an effective teacher must be left open. It seems from published information that no concerted effort has yet been made to determine what attributes or techniques make some teachers more effective in teaching their pupils mathematics than others. At the moment the colleges of education in both their selection procedures and training programmes must be working in the dark. Do they have any clear idea of the type of person they should be looking for from their applicants and do they know what techniques and attitudes they should be fostering in their students? There is no published evidence to indicate that they do. Should not H.M.I.s and local authority advisers be looking at effective teachers, not only by the criteria of producing good external examination results but in providing the type of school-leaver that employers would like to employ, and attempting to observe the ways in which their methods differ from those of the ineffective teacher?

Comes straight after what employers want

The Examination Boards

There has been much public discussion of recent years concerning public examinations. The structure of the examination system as a whole embracing G.C.E., C.S.E., a common examination at 16+, C.E.E., N and F has been discussed both in public and in private at great length. What has been lacking is a similar concern for the internal structure of the public examinations - their syllabuses and their examination papers. There must be something wrong with an examination where a 'pass' can be obtained by a candidate who got more wrong than right, and where an employer does not know from the certificate in which areas the candidate is considered to be proficient.

What is, then, the role of the C.S.E. and O-level examination in mathematics? For the pupil going on to study mathematics or science at A-level it is used as an indication of the ability to cope with sixth-form work. For the pupil who leaves school at 16+, however, the O-level or C.S.E. grade is used by many employers as an indication that the pupil has achieved a certain standard of competence in mathematics. Increasingly these employers are finding that school-leavers, having gained a good grade in C.S.E. or G.C.E., are unable to perform the quite routine arithmetic or algebraic manipulations which are needed during their training. There are a number of reasons for this.

The interpretation of examination results has become much more difficult of recent years with the increase in the number of options offered especially in C.S.E. Most G.C.E. boards offer at least two mathematics syllabuses, some of which usually concentrate more on the type of manipulations that employers consider desirable while others tend to emphasise the understanding of mathematical concepts and lay less stress on the ability to calculate and carry out algebraic manipulation. With the C.S.E. boards the position is even more confused. Nine of the 14 boards offer more than one mathematics syllabus with one having as many as six options (Graham (1977)). How is an employer to know whether an applicant with a grade 1 C.S.E. certificate in mathematics is competent in statistics, money management, navigation or mechanics? The situation is not quite as complicated as it may seem, however, for each of the syllabuses contains a compulsory 'common core' of topics which are examined, usually in Paper 1, for all candidates. J.D. Graham has analysed these 'common cores' for all of the 14 examination boards and finds at least 13 of them stipulate that the following should have been studied:

> number systems; use of logarithms; percentages; mensuration in 2-dimensions; Pythagoras' Theorem; symmetry; linear equations; symultaneous linear equations; graphical representation of data; four rules of number; number bases; approximations and accuracy; averages; ratio; 3-dimensional mensuration; 2-dimensional shapes; special triangles (equilateral, isosceles, right-angled, etc.); similarity; literal representation of algebraic processes; trigonometrical ratios; compass directions and bearings.

Two factors seem to be significant to me on perusal of this list. The first is the absence of any reference to vulgar or decimal fractions. This, I feel, must be an oversight on Graham's part as they do not even appear in his Table 2 where he presents a full analysis of the topics covered by each of the C.S.E. boards. The second factor which I find surprising is the high degree of agreement of this list with the 'common core' found to be satisfactory by employers (see pages 80 to 83). The only items of any significance which appear on the 'common core' of Chapter 4 and which are omitted from the 'common core' syllabuses of nearly all of the C.S.E. boards are graphs, flow diagrams and a wider appreciation of calculating aids.

The topics considered to be important by employers are, therefore, nearly all included in the compulsory part of practically every C.S.E. syllabus. Why, then, are pupils with good grades being shown to be deficient in these topics? The answer appears to be in the surfeit of choice allowed by most examination boards. A poor performance on the compulsory part of the syllabus may be redressed by a good performance on other topics. The overall grade, therefore, gives no assurance that the compulsory topics have been mastered. Lindsay (1975) suggests two solutions to this problem. Examiners might adopt the strategy of devising a separate section of the examination paper which would test the candidates knowledge and ability in the compulsory 'common core' items. The questions would be simple but the pass mark high (Lindsay suggests 80%). A pass in this part of the examination would then be a necessary condition for passing the whole of the mathematics examination. This suggestion has, however, been overtaken by events. The results of O-level and C.S.E. examinations are no longer given on a pass/fail basis, but the candidates are graded A or 1 to U. Lindsay's second suggestion would be perfectly feasible. He again recommends the setting of a separate 'common core' paper of the same type. The fact that a candidate has passed this part of the examination would be indicated on the certificate by an endorsement, rather in the way that a speed endorsement is given for typewriting examinations. The candidate would then be graded on the aggregate score of the whole examination to give his mathematics grade, but this would be endorsed or not, depending on his performance on the 'common core' paper. What this endorsement would be called is open to some doubt (Numeracy?).

The situation is complicated at 16+ by both C.S.E. and G.C.E. boards offering Mode 3 syllabuses. Although these are moderated by the examination boards, many of them employ novel methods of assessment and may not contain as much of the compulsory part of the syllabus as Mode 1. In order for the scheme outlined in the previous paragraph to work it would be necessary for all Mode 3 candidates to sit the special 'common core' paper.

The Employers

During the second Nottingham Conference it was suggested that, as the schools seemed to be inadequate in providing the particular types of mathematical expertise needed by industry, it would be sensible to channel more resources into remedial mathematics teaching during industrial training. It had been claimed by both Carroll (1974) and Allan (1975) that remedial work in mathematics for trainees who, at entry, did not have the required abilities was highly successful. The obvious course would seem to be to encourage this work as the highly motivated trainee, seeing the need for the mathematics he is learning, would be far more likely to do well than he would at school where the need for particular mathematical skills is not always evident.

The employers, however, saw some disadvantages in this scheme. Money would not be the only expense incurred. Time would also be a crucial factor. Apprenticeships are now usually only of four years' duration, having been shortened at the one end by ROSLA and at the other by the Trade Unions and any time lost to remedial work would be a further loss of irreplaceable training time. This would inevitably reduce the standard of craftsmanship to the disadvantage of both industry and the individual apprentice.

This does not mean that employers are averse to taking responsibility for teaching any mathematics at all. They fully accept their responsibility for teaching the particular applications of mathematics to their own sphere of operations and would, in fact, object to the schools attempting to do this. Their objection is to having to teach basic techniques which are applicable to many occupations and which they consider should be taught in schools. Mr. J. Keatley of Stanton and Staveley again sums up the situation succinctly when he says, "I enjoy teaching industrial mathematics but object to teaching basics. While industrial mathematics cannot be taught by anyone who has not worked in industry, we have not the expertise available to teach apprentices basic arithmetic."

Many employers now invite teachers into their factories so that they can see the types of mathematics required by industry. This is all for the good. What seems to be lacking is any concerted movement in the opposite direction. As well as saying "This is the type of mathematics we need" they should be asking "What are you doing in schools nowadays that we may be able to use?" The secondary mathematics curriculum has changed so much during the last fifteen years that very few industrial instructors can have an accurate idea of the type of mathematical education their future apprentices are receiving. Perhaps if they came into schools with an open, enquiring attitude they may find parts of our syllabuses on which they could build during vocational training.

Perhaps such visits would also help to rid employers of the common misconception that teachers could use 'real' examples from industry as motivation in schools. Table 8 (page 91) draws attention to the difference in attitude between teachers and employers on this point. Teachers, who have to interest children in mathematics, naturally rated mathematics as an 'enjoyable and satisfying activity' very highly while employers tended to rate it lower. Apprentice instructors who see their trainees working out a practical mathematical problem and then trying out the solution on a piece of metal so that the correct result gets instant reinforcement, probably need to be convinced that the practical utility of a piece of mathematics will not necessarily motivate a pupil in school. This is not at all obvious to people who have not had the experience of teaching in a school. With apprentices relevance is all important, but with school-pupils the use of 'real' examples is often regarded more as an annoying complication.

And finally, those infamous selection tests. Here, surely, is an area where the employers must take some action. Local initiatives have shown what can be done and two of them are quoted by Lindsay (1975). In Hillingdon the mathematics adviser has a team of employers and teachers who have created a question-bank which some of the local employers have now decided to use in their own tests. A similar exercise some years ago in Sheffield was able to up-date employers' tests so that there was an improvement in both the candidates' results and the tests' efficiency as selectors.

Cooperation

If a lasting solution is to be found to the present unsatisfactory position it will need the cooperation of a number of bodies. The Department of Education and Science, the Local Authorities, the H.M.I.s and L.E.A. advisers, the teachers, the Industrial Training Boards, the Colleges of Education and the Colleges of Further Education all have a contribution to make and a point of view to be considered. At a national level, a colloquium of representatives from each of these bodies meeting in the glare of publicity such an event would attract, could well lead us back to the days of 1973 with each participant making his points with a view, not to a solution of the problem, but to ensuring that his opinion received a good 'press.'

I see nothing wrong with conferences like those organised by the I.M.A. at Nottingham and Yeovil and by C.A.M.E.T. at Loughborough in 1976 which are reported in the educational press and which produce reasoned publications aimed at providing a fair view of the whole spectrum. These should, however, only be the catalysts for local initiatives which I see as being the most promising areas for reform.

Many such local schemes are already in operation. In 1975, for instance, the Bournmouth Working Party on 'From School to Industry' published its recommendations in the Journal of the Southern Scientific and Technological Forum. Recognising that the real problems existed in the area of the C.S.E. examinations, they agreed that remedies could only be found by continuous cooperation at three levels: at the teacher level where more information should be available about the type of mathematics required in employment; at C.S.E. moderation level where representatives of further education and employers should take part; and at C.S.E. organisation level where F.E. and employers' representatives should sit on subject panels where decisions are taken on syllabuses and examinations.

In Peterborough a consortium consisting of the Group Training Manager of Baker Perkins, the Cambridgeshire Senior Mathematics Adviser, teachers from schools and further education colleges, the careers service and training officers met to consider the difficulties encountered by the 16+ school-leaver and as a result are instigating a joint Mode 3 syllabus in which the further education college introduces fifth-form pupils to the realities of engineering craftsmanship, thus motivating them to learn the relevant mathematics at school.

These local cooperative ventures in Bournmouth and Peterborough, alongside those previously mentioned in Hillingdon and Sheffield are where the solutions will be found to the present problems. What is needed is someone in each area with the drive and initiative to get such groups working on the particular difficulties of their own 16+ interface. The task is not easy as, for such an initiative to produce satisfactory results or acceptable recommendations, it must represent the requirements and views of the schools, the colleges of further education, the employers and the careers service. The results of such deliberations must then be made available for the perusal of a wider audience so that any national initiatives which would facilitate the introduction of recommended reforms could be undertaken.

Above all we must realise that the present disenchantment on the part of employers with the mathematical abilities of the 16+ school-leaver cannot be ascribed to any one cause. Thus, there is no one solution. A situation of mutual respect between the school-teacher and the industrial instructor who both are concerned for the future careers of their respective pupils and apprentices must be the basis on which any attempts at cooperation are founded.

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Appendix

Manipulative Skills up to the age of 16

(Extract from Thwaites (1974) pp 74 - 78)

The following table shows the ages by which we believe children, of the three indicated ability levels, should acquire a high degree of proficiency in various manipulative skills.

	By age of about 13	By age of about 14	By age of about 16
The most able: those likely to get a good grade at O-level	1 - 13	14 - 18	19 - 21
The able: those likely to get a pass at O-level or grade 1 CSE	1 - 7	8 - 13	14 - 19
The average: those likely to be graded at CSE	1 - 3	5 - 8	9 - 12

The numerals in the table refer to mathematical processes as follows:

1. Recall and use of the addition and multiplication of integers up to 10.
2. Manipulation of simple* arithmetic fractions.
3. Simple problems involving percentages.
4. Simple mental arithmetic with integers, involving 'carrying' one number in the head, using +, −, x, ÷.

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* It is difficult to avoid using the word "simple" in such a list. Broadly by "simple" we mean "very simple."

5. Addition, subtraction, long multiplication and division, normally of numbers with up to three significant figures (including decimals).
6. The four arithmetic operations of directed numbers.
7. Simple use and applications of units of length, area, volume, mass and decimal money.
8. Simple ratio calculations.
9. Estimation of size of answers to numerical calculations (including decimals) using 'rounding.'
10. Solution of simple linear equations, e.g., $ax + b = c$; $px + q = rx + s$, where the coefficients are numbers.
11. Manipulation of simple formulae involving +, −, x, ÷ and use of brackets.
12. Use of slide rule or calculator for multiplication and division.
13. Use of standard form, with positive indices, in simple multiplication and division.
14. Use of standard form, with negative indices, in simple multiplication and division.
15. Squares and square roots using slide rule and tables.
16. Simple applications of sine, cosine, tangent and the use of the corresponding tables.
17. Solution of simultaneous linear equations in two unknowns by a non-graphical method.
18. Interpretation of the graph of $y = ax + b$.
19. Simple examples of the applications of proportion.
20. Use of logarithms for simple calculations.
21. Interpretation of graph for $y = ax^n + b$, and use of linear graphs for non-linear relations.

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