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The emergence of children's natural number concepts: Current theoretical challenges

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Abstract
Learning the meaning of number words is a lengthy and error-prone process. In this review, we highlight outstanding issues related to current accounts of children's acquisition of symbolic number knowledge. We maintain that, despite the ability to identify and label small numerical quantities, children do not understand initially that number words refer only to sets of discrete countable items, not to other non-numerical dimensions. We question the presence of a sudden change in children's understanding of cardinality, and we report the limits of the give-a-number task. We also highlight that children are still learning the directional property of the counting list, even after acquiring the cardinality principle. Finally, we discuss the role that the Approximate Number System may have in supporting the acquisition of symbolic numbers. We call for improvements in methodological tools and refinement in theoretical understanding of how children learn natural numbers.

KEYWORDS
Approximate Number System, counting, learning, mathematical cognition

Knowing how to count, represent, and think about natural numbers is often conceptualized as an eminently human achievement of higher cognition and has been investigated as a major developmental milestone throughout the history of developmental science. Children's counting knowledge moves through a series of lengthy and error-prone stages, which are commonly tracked using the give-a-number task (GaN; see Figure 1a; Wynn, 1992), whereby children are asked to give a number of objects corresponding to a specific number (e.g., "Can I have two tomatoes, please?"). At around 2 years, children are often considered *pre-number-knowers* because they cannot give any number accurately, even if they can recite the counting list.

Older children succeed in trials asking for "one," but not larger numbers (1-knowers). Then, after a few months, they can give two when asked for "two" (2-knowers). After another few months, children become "3-knowers" and then "4-knowers." Together, these children are often labelled *subset knowers* because their knowledge of number words remains limited to a subset of the counting list. Sometime around their fourth birthday, children can reliably give any requested number, and are considered cardinal-principle knowers (CP-knowers) because they know the cardinal meanings of number words in their counting list (e.g., Carey, 2009; Sarnecka, 2015). Children at this stage are assumed to fully understand the conceptual basis of number, including the fact that adding one item to a set leads to the next number word in the counting list (i.e., the successor function; Carey, 2009).

An increasing number of researchers have questioned whether CP-knowers truly understand the conceptual

Abbreviations: ANS, Approximate Number System; CP, cardinal-principle; FC, fast cards; GaN, give-a-number.

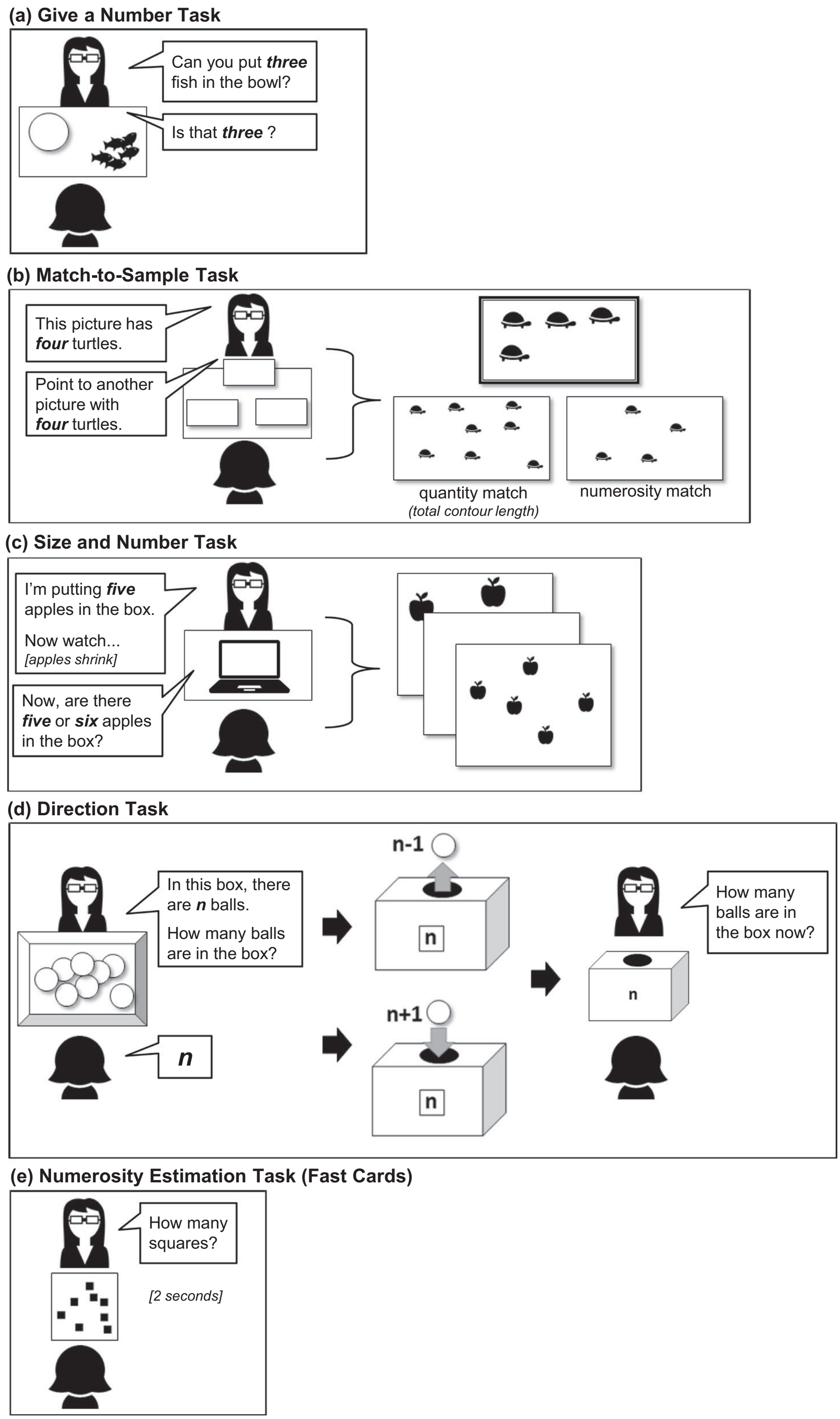

FIGURE 1 Numerical tasks. In the give-a-number task (a), the experimenter asks the child to give several target numerosities. After each trial, the child is usually asked to confirm their response (e.g., "Is this three?"). In the match-to-sample task (b), the experimenter presents and labels a sample picture, then asks the child to identify which of two additional pictures matches the sample. Trials vary to ask about properties of individual items in the set (e.g., "green" turtles) and numerical properties of the set (e.g., "four" turtles). In the size and number task (c), children view a screen with a set of items that, once labelled by the experimenter (e.g., "five apples"), transforms in size or number. Afterwards, children are asked whether the number word label has changed. In the direction task (d), the child is presented with a box containing n items. Then, the box is closed and the experimenter adds or removes one item using a hole on the top. The child tells the number of items that are now in the box. In the numerosity estimation task (e) (frequently known as fast cards), children estimate the numerosity of briefly presented visual sets

bases of symbolic number (Davidson et al., 2012; Le Corre & Carey, 2007; Sella & Lucangeli, 2020), whether different components of their knowledge emerge concurrently (Slusser et al., 2013; Slusser & Sarnecka, 2011), and whether the GaN task is used optimally to monitor their number knowledge (Krajcsi, 2021). In this article, we review some persistent and some recent uncertainties about this classic description of the development of symbolic number knowledge.

First, we review recent evidence revealing a parallel learning trajectory for children's understanding of more general properties of number words (i.e., that number words denote the number of items in a set rather than referring to other nonnumerical dimensions). Second, we discuss methodological challenges with the GaN task, which also call into question the idea that development culminates in a sudden change sometime after children become 3- or 4-knowers. Third, we review evidence that children still struggle with mastering the successor function before *and after* acquiring the cardinality principle. Finally, we examine the relation between children's intuitive number sense—the Approximate Number System (ANS; see Odic & Starr, 2018)—and emerging natural number knowledge.

WHEN DO CHILDREN UNDERSTAND THAT NUMBER WORDS ARE USED TO LABEL NUMEROSITIES?

Research investigating children's developing understanding of individual number words, including studies likely to use the GaN task, often isolates numerosity as the quantitative dimension of interest. But with clear evidence that infants and young children attend to both numerical and nonnumerical dimensions of quantity (see Feigenson et al., 2004), these studies may fail to detect early tendencies to map number words onto representations of continuous, rather than discrete and countable, quantities. In fact, children who know just one or two number words, as determined by the GaN task, do not initially understand that higher number words (such as *five* or *six*) are similarly constrained to sets of discrete, countable objects (as opposed to continuous substances, such as water or sand; Slusser et al., 2013).

Match-to-sample tasks can be used to understand more fully children's early interpretations of number word by, for example, pitting numerosity against nonnumerical dimensions of quantity (e.g., summed spatial extent measured as total surface area or contour length). For example, in one study, children were introduced to a sample picture ("This picture has five turtles") and then two additional pictures ("Find another picture with five turtles"; Slusser & Sarnecka, 2011). One picture displayed the same number of items as the sample picture, but with half the overall spatial extent (five small turtles). The other displayed twice the number of items with the same overall spatial extent (10 small turtles; see Figure 1b). Children learned to connect number words to the numerosity of a set, rather than summed continuous spatial extent or properties of individuals in the set, only as they progressed through the GaN knower levels.

It can be difficult if not impossible to account for all relevant dimensions of quantity in any one task (especially when using sets of identically sized items in which dimensions of continuous quantity increase proportionally with each item). However, computer animation can be used to manipulate the relation between continuous and discrete quantities.

For example, in another study (Slusser & Cravalho, 2020), a computer-animated version of the transform sets task (Sarnecka & Gelman, 2004) was created to determine whether there is a period in development during which children think a change in total surface area should change the number word label (e.g., Slusser & Cravalho, 2020). In this version of the task (size and number task; see Figure 1c), children viewed a display of objects ("Look, there are five pumpkins on the screen"). Then the set transformed in some way, and the child was asked a test question ("Now, are there five or six pumpkins?"). Half the trials introduced a transformation in summed spatial extent—the items doubled in total surface area or shrank to half their original size. The other trials introduced a transformation in both number and summed spatial extent as one object was either added or removed from the set. In line with findings discussed earlier, emerging evidence suggests that children do not initially understand that the number label changes if, and only if, the number of the items in a set change.

Together, these findings highlight the need for nuanced assessments that appropriately account for (rather than control for) other dimensions of quantity. Such assessments could identify when children ascribe exclusive meanings to number words—individually and as a class of words with unique semantic constraints.

DOES THE GaN TASK ASSESS CHILDREN'S CARDINAL KNOWLEDGE PROPERLY?

In the original version of the GaN task (Wynn, 1990), the experimenter asks the child to give several target numerosities (e.g., "Place five toy fishes in a large bowl"; see Figure 1a). After each response, the experimenter prompts the child to recount the given set to ensure that no performance error has been made. In case of mismatch, the experimenter asks for the same numerosity again. The task starts by asking for a small number (e.g., one or two items) and after a successful trial, the experimenter requests the successive number. After an unsuccessful trial, the experimenter asks for a preceding number until the largest number is found that has been given incorrectly two of three times. This titration method defines the limit of the children's cardinal knowledge. There are alternative versions of GaN task, but it is assumed that different versions yield equivalent results (Krajcsi, 2021).

Recent studies have questioned whether the GaN task is a reliable or valid measure of children's cardinal knowledge. First, some CP-knowers can accurately give only up to five, six, seven, and so forth objects, numbers that are smaller than the limit of their counting list. Accordingly, these children can be categorized as 5-knowers, 6-knowers, and so on (Gunderson et al., 2015; Marchand & Barner, 2021; Mussolin et al., 2012). Therefore, the transition to the status of CP-knowers appears prolonged compared to former assumptions.

Second, some reports have suggested that children may have partial knowledge of numbers that are beyond their knower level (Barner & Bachrach, 2010; Gunderson et al., 2015; O'Rear et al., 2020; Wagner et al., 2019). In this light, the titration method may fail to capture the limit of children's cardinal knowledge precisely because numbers with partial knowledge may be categorized either as known or unknown depending on random noise. Third, while it is assumed that various versions of the GaN task yield similar results, this is not necessarily the case. For example, omitting recounting instruction may increase performance errors (Krajcsi, 2021), which leads to a biased assessment of cardinal knowledge.

Fourth, evaluation methods influence the determination of children's knower level. For example, studies that consider a deviation of 1 from the target number as a correct response (Le Corre et al., 2006) would inevitably lead to a different assessment of the knower level than studies that require exact responses. Finally, while the reliability of the GaN task measured in the whole scale is high (i.e., it is unlikely that a child will be radically miscategorized—for example, a 1-knower will not be categorized as CP-knower), the reliability of the specific knower levels is low (i.e., children may get the label of the neighboring category—for example, a 4-knower may be recognized as CP-knower; Marchand & Barner, 2021). The low reliability of specific number knowledge is problematic in many studies that contrast 4-knowers and CP-knowers.

In Figure 2, we present a more extensive list of factors that may influence the assessment of cardinal knowledge in the GaN task. All these factors should be considered when using the GaN task and explored to optimize the task to increase its reliability and validity.

DO CP-KNOWERS MASTER THE SUCCESSOR AND PREDECESSOR FUNCTIONS?

In principle, CP-knowers should understand the directional property of the counting list. That is, adding one item to a set leads to the next number word in the counting list (i.e., successor function; $n + 1$), whereas removing one item leads to the preceding number word (i.e., predecessor function; $n - 1$). Despite the cardinality principle being a prerequisite to learning the successor function (Spaepen et al., 2018), not all CP-knowers understand the successor function (Davidson et al., 2012; Sarnecka & Carey, 2008), and it takes approximately 2 years after becoming CP-knowers to generalize the

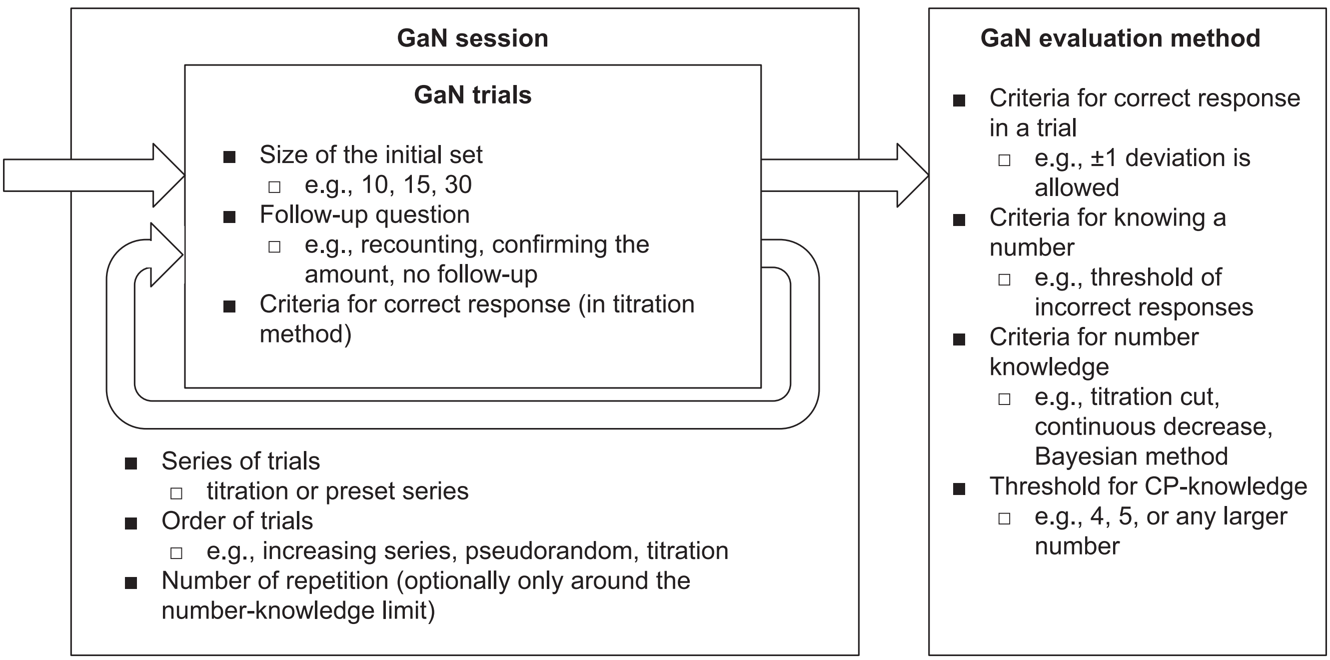

FIGURE 2 Various factors and their variations that may influence the measured number knowledge in the give-a-number (GaN) task

successor function to all the numbers (i.e., infinity; Cheung et al., 2017).

Recently, researchers devised a direction task (see Figure 1d) in which the experimenter showed children an opaque box containing few (e.g., two) or many (e.g., eight) objects (Sella & Lucangeli, 2020; Sella et al., 2020). Then, the experimenter added ($n + 1$) or removed ($n - 1$) one object from the box, and asked children to indicate the number of objects inside the box after the manipulation. When only a few objects were inside the box, as in the 2 + 1 and 2 − 1 trials, children could keep track of individual objects and responded correctly. But when many objects were in the box, children could not track individual objects, but instead had to know that adding or removing one object led to the preceding or successive number word, respectively. Most CP-knowers correctly responded $n + 1$ when one item was added to a box containing few or many objects. When one object was removed from the box, CP-knowers correctly responded $n - 1$ if the box contained few objects, but failed when the box initially contained many objects. In case of error, CP-knowers frequently responded $n + 1$ even though an object was removed from, not added to, the box.

Two reasons could explain these findings. On the one hand, when objects in the box could not be tracked, some CP-knowers applied a blind counting up procedure that led to the wrong response in $n - 1$ trials, but to the correct response in the $n + 1$ trials, thereby overestimating children's mastering of the successor function and, more generally, their understanding of the directional property of the counting list. On the other hand, CP-knowers knew that the correct response was $n - 1$ when an object was removed from a set, but they could not inhibit the strong tendency to count up or lacked sufficient experience moving up and down on the counting list. Accordingly, some CP-knowers attempted to get to $n - 1$ by counting from one but stopped too early at $n - 2$. Others responded with $n + 2$ because they knew that $n + 1$ could not be the right response. The analysis of responses revealed within-subject variability, with some children failing in some trials but surprisingly offering correct response on others. Taken together, these results suggest inexperience in accessing the counting list rather than a lack of conceptual understanding.

Overall, it appears that CP-knowers, despite a perfect performance in the GaN task, have a limited comprehension of the directional property of the counting list (conceptual knowledge) or show a fragile ability to move up and down the list (procedural knowledge). These abilities should not be overlooked because they are relevant for establishing exact symbolic numerical knowledge. Accordingly, accuracy on $n - 1$ trials of the direction task emerged, among other numerical skills, as one of the best correlates of number word magnitude comparison, after controlling for domain-general processes (i.e., memory). Moreover, a short training on indicating the preceding and successive number in the counting list significantly improved numerical skills in preschool children (Xu & Lefevre, 2016).

WHAT IS THE ROLE OF THE ANS IN NATURAL NUMBER ACQUISITION?

Before children achieve mastery over natural number concepts, they also have access to at least two other types of perceptual number representations: a parallel individuation system that allows them to represent three to four objects precisely, and an ANS, which is broadly shared ontologically with other animals and which represents any number quickly but imprecisely (Feigenson et al., 2004; Piazza, 2010). An ongoing matter of debate is whether and if the ANS contributes to children's natural number concepts (see Carey, 2009; Krajcsi et al., 2020; Levine & Baillargeon, 2016; Piantadosi et al., 2012; van-Marle, 2015).

Le Corre and Carey (2007) famously argued that the ANS plays no role in the initial acquisition of natural numbers, while the parallel individuation system does (see also Carey & Barner, 2019). They found that, even among children who were classified as CP-knowers, some could not attach number words to quickly presented sets of four or more dots (fast cards [FC] task)—terming them "nonmappers"—suggesting that the link between number words and the ANS emerges only after natural numbers are fully acquired. The ANS has also been argued to not be a necessary contributor to natural number concepts because, unlike natural numbers, the real-numbered format of the ANS lacks a successor function (Carey, 2009; Carey & Barner, 2019). Also, the mapping between the ANS and number words even later in life is malleable and susceptible to feedback (e.g., even adults label a set of 20 dots as sometimes "fifteen," sometimes "twenty," sometimes "twenty-five"), making the ANS potentially too imprecise to inspire exact number concepts (Izard & Dehaene, 2008; Sullivan & Barner, 2013).

Both ideas have recently been challenged. In one study (Shusterman et al., 2016), the FC task (see Figure 1e) produced unreliable results: Children who were tracked longitudinally sometimes appropriately gave higher number words for more numerous arrays of dots and sometimes failed to do so, contrary to the idea that the FC task measures a reliable nonmapper status. In another study (Odic et al., 2015), children who failed to give higher number words for more numerous displays of dots nevertheless *produced* higher approximate number estimates if the experimenter gave them a number word and asked them to quickly tap a particular number of times without counting. Hence, it is unclear whether the nonmappers truly exist or if they are an artifact of an unreliable FC task.

Even outside the FC task, in another study (Shusterman et al., 2016), children's ANS acuity improved in tandem with the acquisition of CP-Knower status, while in yet another study (Gunderson et al., 2015), children used number words approximately even outside of their knower level. Finally, individual differences in the ANS longitudinally predicted number word knowledge, both before *and* after cardinal principle acquisition, in another study (vanMarle et al., 2016). Together, these findings suggest that the link between the approximate and exact number representations might emerge earlier than originally proposed (Le Corre & Carey, 2007), and that the lack of mapping among some CP-knowers may be a by-product of the task rather than of the children's ability.

The real-numbered nature of the ANS might also be a red herring: While this format would preclude the inference of the successor function and the cardinality principle, other key features of the natural numbers—that they refer to *numbers*, the arithmetic operations that can be carried over them—might all be supported by the ANS. For example, in studies, the ANS might be vital in children attending to number over other magnitudes (Fuhs et al., 2018; Silver et al., 2020; vanMarle, 2015), and adding and subtracting within the ANS shows the same ordinal asymmetry as children do when reasoning about successors and predecessors (de Hevia et al., 2014; Macchi Cassia et al., 2012). Especially when considered alongside the idea that CP-knowers might not have actually mastered the full range of natural number meanings (Davidson et al., 2012), the ANS might play a more long-term and nuanced role in helping guide children toward several key features of number words. This is especially important since other systems, such as parallel individuation, only implicitly represent numbers (Feigenson et al., 2004). Together, these findings point to a potentially key role of the ANS in guiding children toward some of the necessary features natural numbers must possess, even though, as a result of its analog format, the ANS may not be *sufficient* for the acquisition of full-blown natural number concepts.

DISCUSSION

Recent findings shine new light on the acquisition of numerical knowledge in young children, revealing a more complex developmental trajectory than previously thought. In Table 1, we summarize the reviewed evidence, adding questions that need to be further investigated.

Children apparently show an understanding of small number words. But the content of these number representations is surprisingly nonnumeric: They still conflate number words with other adjectives and quantifiers, such as the color or size of individual items in a set. Only at later stages, do children understand that number words refer exclusively to the number of elements in the set. Titration, different evaluation methods, recounting instruction, and low reliability of the GaN task compromise the assessment of children's numerical knowledge, making it difficult to establish the presence of a sudden change in performance when children become CP-knowers. Despite a ceiling performance in the GaN task, children still commit errors when performing the direction task, thereby displaying prolonged learning before mastering the successor and predecessor functions. To the extent that the GaN task measures children's knowledge of cardinality, this principle is not the final one children learn when acquiring number words. Finally, while natural number concepts are unlikely to be derived solely from the ANS, emerging work has shown that subset knowers already display a link between the two systems, much earlier than previously suggested.

We maintain that designing valid, reliable, and strategy-revealing behavioral tasks represents a first step in answering these and other outstanding research questions. We need tasks that measure specific numerical knowledge while minimizing the influence of other components that are closely related to the target skill. For instance, before adding or removing one element from the box in the unit task, we must ensure that children recall the number of objects currently within the box (i.e., memory check). This simple question minimizes the influence of memory and inattention on numerical performance. We must ensure that the chosen numerical tasks have high test-retest reliability and internal consistency. Newly designed tasks should be administered at least twice on consecutive days to assess their reliability. Finally, open-answer tasks, rather than forced-choice tasks, can uncover children's reasoning and strategy. For instance, the pattern of error in the GaN task reveals how subset knowers may lack the exact numerical meaning of "eighth," but they never give "two" because they know that "eight" is not "two."

Researchers can use the newly designed behavioral tasks with other methods within longitudinal and experimental research designs. Eye-tracking technology can provide novel insights on cognitive processing by measuring the focus of attention when completing numerical tasks. The repeated assessment in short time intervals, as in microgenetic studies, would unveil how numerical skills develop in relation to each other, and determine the order in which knowledge is constructed and whether clusters of children follow different developmental trajectories (Cahoon et al., 2021). Additionally, these learning pathways may vary within cultures and cross-culturally, depending on many factors, such as the linguist structure (e.g., single-plural distinction; see Almoammer et al., 2013) and the presence of other numerical representations to which number words can be mapped (e.g., Arabic numerals; see Jiménez Lira et al., 2017; Marinova et al., 2021). Combining diverse research methods within appropriate research designs would improve our theoretical understanding of how children learn natural numbers.

TABLE 1 Summary of the main knowledge components and related outstanding questions

Knowledge component	Task(s) and findings	Outstanding questions
Number words refer to numerosity	Match-to-sample task (Figure 1b) Size and number task (Figure 1c) Subset knowers often do not understand that number words refer exclusively to numerosity (not characteristics of individual items or other dimensions of quantity, such as total surface area)	Do children initially conflate number words with other adjectives or quantifiers? What is the role of other representations, such as the ANS, of quantity in guiding children's attention towards numerosity?
The cardinality principle	Give-a-number task (GaN; Figure 1a) Children understand the first four numbers relatively slowly (subset knowers), then they seemingly understand how any larger numbers should be handled (cardinality principle-knowers). Still, several assumptions behind the task do not hold, and some results that rely on the GaN task may not be entirely reliable or valid	When assessing children's numerical knowledge, what systematic biases may be introduced with various versions of the GaN task? What is the largest number that subset knowers know if it is not 4? Are new numbers learned gradually? If so, what directs this gradual learning? What is the ideal version of the GaN task that can capture the latest key findings of numerical knowledge development?
Directional property of the count list	Direction task (Figure 1d) Children ultimately learn that adding one item to the set corresponds to the next number word in the counting list (i.e., $n + 1$; successor function), whereas removing one item leads to the preceding number word (i.e., $n - 1$; predecessor function) CP-knowers perform well on $n + 1$ trials for small and large numerical quantities. On the $n - 1$ trials, performance is accurate for small numerical quantities, but for large numerical quantities, children frequently respond by saying $n + 1$	Do successor and predecessor functions follow different developmental trajectories? Does responding with $n + 1$ in the $n - 1$ trials of the Direction task reflect a blind counting forward procedure (i.e., lack of conceptual understanding)? Alternatively, do children know that they should move backwards in the counting list to answer the $n - 1$ trials but struggle to complete the task (i.e., lack of experience in accessing and moving backwards on the counting list)? Following, what is the role of domain-general factors such as inhibition and (verbal) working memory when completing the Direction task? Why are there symmetries between ordinality effects in counting and in the ANS?
Mapping number words to other number representations	Numerosity estimation task (Figure 1e) CP-Knowers frequently fail to map number words to quickly presented sets of 4 or more dots, but succeed when asked to "produce" a number of taps when given a specific number	Do "nonmappers" truly exist, or are they an artifact of the fast cards task? Why are there correlations between growth in ANS and acquisition of cardinality principle?

Abbreviation: ANS, Approximate Number System.

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

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