## Appendix:

The explicit form of the Bezier curves is as follows:

|  |  |  |
| --- | --- | --- |
|  | $$B\_{n}\left(t\right)=\sum\_{i=0}^{n}\left(\begin{matrix}n\\i\end{matrix}\right)\left(1-t\right)^{n-i}t^{i}P\_{i}, 0\leq t\leq 1.$$ | (A1) |

The order of a Bezier curve can be calculated according to the number of control points that are used to generate the curve. The order of a Bezier curve is related to the control points with the following equation:

|  |  |  |
| --- | --- | --- |
|  | $$n=N\_{P}-1.$$ | (A2) |

If the order of the curve $n$ is taken as 2, the Bezier curve for the *kth* fibre becomes quadratic:

|  |  |  |
| --- | --- | --- |
|  | $B\_{n}^{k}\left(t\right)=\sum\_{i=0}^{2}\left(\begin{matrix}2\\i\end{matrix}\right)\left(1-t\right)^{2-i}t^{i}P\_{i}^{k}$, $k=1,…,N.$ | (A3) |

In Eq. (A1), $\left(\begin{matrix}n\\i\end{matrix}\right)$ is the binomial expansion which can be calculated as,

|  |  |  |
| --- | --- | --- |
|  | $$\left(\begin{matrix}n\\i\end{matrix}\right)=\frac{n!}{i!\left(n-i\right)!}.$$ | (A4) |

The control points of the *kth* fibre ($P\_{i}^{k}$), used to generate the fibrous networks in the two-dimensional case, are provided as $P\_{0}^{k}=(x\_{0}^{k},y\_{0}^{k})$, $P\_{1}=(x\_{1}^{k},y\_{1}^{k})$ and $P\_{2}=(x\_{2}^{k},y\_{2}^{k})$.

If control points ($P\_{i}^{k}$) are substituted into Eq. (A3), the following equation is obtained explicitly:

|  |  |  |
| --- | --- | --- |
|  | $$B\_{2}^{k}\left(t\right)=\left(x\_{0}^{k}-2x\_{1}^{k}+x\_{2}^{k}\right)t^{2}+\left(-2x\_{0}^{k}+2x\_{1}^{k}\right)t+x\_{0}^{k}$$$+\left(y\_{0}^{k}-2y\_{1}^{k}+y\_{2}^{k}\right)t^{2}+\left(-2y\_{0}^{k}+2y\_{1}^{k}\right)t+y\_{0}^{k}$, $k=1,…,N.$ | (A5) |

The mathematical equation of each fibre can be written according to the parametric form of the curve in 2D space as

|  |  |  |
| --- | --- | --- |
|  | $\vec{B}\_{2}^{k}\left(t\right)=\vec{r}^{k}(t)$, $k=1,…,N,$ | (A6) |

where $\vec{r}^{k}(t)=x^{k}\left(t\right)\vec{i}+y^{k}\left(t\right)\vec{j}$. Here, $\vec{r}^{k}(t)$ is the path of the fibre in vector form, $x^{k}$(t) and $y^{k}$(t) indicate $x^{k}$ and $y^{k}$ components of this fibre path, respectively. After some mathematical manipulations, the fibre path for the *kth* fibre is written based on the quadratic Bezier curve:

|  |  |  |
| --- | --- | --- |
|  | $\vec{B}\_{2}^{k}\left(t\right)=\left\{\left(x\_{0}^{k}-2x\_{1}^{k}+x\_{2}^{k}\right)t^{2}+\left(-2x\_{0}^{k}+2x\_{1}^{k}\right)t+x\_{0}^{k}\right\}\rightharpoonaccent{i}+\left\{\left(y\_{0}^{k}-2y\_{1}^{k}+y\_{2}^{k}\right)t^{2}+\left(-2y\_{0}^{k}+2y\_{1}^{k}\right)t+y\_{0}^{k}\right\}\rightharpoonaccent{j}$, $k=1,…,N.$ | (A7) |